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REINFORCED CONCRETE RAILWAY STRUCTURES

BY

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EDITOR'S PREFACE

ALTHOUGH Reinforced Concrete has been hitherto more extensively utilised for railway purposes on the Continent and in America, it has of recent years been more largely adopted in this country for certain classes of structures. It is not claimed in this book that this material is universally applicable or suitable for all kinds of construction, and in many cases the nature of the site or the conditions of erection might render it inadvisable to use it; but, on the other hand, it has many desirable features, more especially durability and economy of maintenance. These advantages and deficiencies have been clearly set forth by the Author in the opening chapter.

The formulæ given are comparatively few in number, are reduced to simple forms for facility of practical application, and by the aid of the curve diagrams calculations can be made with ease and expedition. A considerable number of worked-out examples have been included, covering the great majority of the problems which would occur in practice, and it is hoped that this will simplify the application of the formulæ. For convenience of reference the notation adopted and the formulæ have been collected together in the last chapter, and an explanation is there given of the method of using the diagrams.

G. M.

AUTHOR'S PREFACE

It has been the author's aim to describe in this book the generally accepted principles and processes upon which the design and construction of reinforced concrete structures depend, and more especially those structures which come within the practice of the railway engineer.

The use of complicated formulæ and calculations with which the literature of this subject has so frequently been associated has, as far as possible, been avoided, and attention has been concentrated upon arriving at results as simply as possible and presenting them in a convenient manner for use in the design of the various structures. The reader's attention is drawn to Figs. 10, 11, 14, 15 and 17, and the explanatory notes at the end of Chapter X., which the author has found useful and speedy for proportioning the parts of beams and other members subjected to bending. They are quite sufficiently accurate for all practical purposes, for it must not be forgotten that in applying the theory of bending to reinforced concrete beams in the generally accepted manner, very broad assumptions are made.

On the other hand, the careful determination of reactions and bending moments, in the case of many reinforced concrete structures, is rendered very necessary, and at the same time more difficult, owing to their monolithic character. In this connection, and in determining the stresses in arches, for instance some of the problems presented have necessarily required mathematical treatment, but, where possible, graphic methods of procedure have been adopted.

The method of dealing with flat arches, and the determination of the stresses in sleepers, have been previously described by the author in *The Engineer* and *The Engineering Review* and his thanks are due to the Editors of these periodicals for permission to reproduce some of the illustrations and descriptive matter here.

Much of the matter relating to materials and labour has been given in the form of extracts from specifications of some of the works described, with a view to making this part of the book of the greatest practical utility.

For the illustrations and descriptions of some of the works described the author is indebted to the engineers or architects of the railways concerned, to the partners of the late Mr. L. G. Mouchel in the case of works carried out in Hennebique ferro concrete and to the Indented Bar and Concrete Engineering Company and the Trussed Concrete Steel Company in the case of structures reinforced with their respective specialities and carried out to their designs.

J. D. W. B.

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REINFORCED CONCRETE RAILWAY STRUCTURES

CHAPTER I

PRELIMINARY CONSIDERATIONS

DEVELOPMENT.

THE component parts of reinforced concrete are well known to railway engineers, and have been largely used by them for railway structures for many years. Lime concrete and iron, the forerunners of cement concrete and steel, entered into the construction of works in connection with the earliest railways, and many of these are still standing as monuments to the skilful design and excellent materials employed in the building of them. The deservedly high reputation won by the earliest railway engineers, and the manner in which the structures built by them have stood the test of time and in many cases proved equal to carrying far heavier loads than those for which they were designed, have probably had a good deal to do with the conservatism of railway engineering practice. For instance, the types of bridges adopted in present-day designs have been gradually evolved from the early structures and there have been few radical changes in bridge construction. -But there is little doubt that the most important reason why railway engineering tends to lag behind the enterprise of commercial engineering lies in the fact that the rigorous conditions of loading to which railway structures are subject, and the public nature of the service they are called upon to perform, must cause any departure from stereotyped practice to be viewed with extreme caution.

Considerations of economy must give place to the need for absolute security.

Without calling the material so formed reinforced concrete and, indeed, before this material was in anything like general use, old rails were embedded in concrete structures in an arbitrary manner to increase their strength. In the use of steel for railway bridges and buildings, it has become a very common practice to encase the steelwork in brickwork or concrete or both, with the object of protecting the metal from corrosion and eliminating the cost of painting. In both these cases certain of the advantages, resulting from the use of steel and concrete in combination, have been recognised. On many of the railways in the British Isles the use of reinforced concrete as a material of construction has gone beyond the experimental stage and most of our railways have given it a trial. Construction in this material certainly costs no more than steel construction and very often results in considerable economy.

The principal advantage lies in the greatly-reduced cost of the maintenance of reinforced concrete structures as compared with those of timber, steel, or brickwork for instance. In many cases it is stated that the cost of maintenance over a number of years has been *nil*. But even if in course of time the surface of the concrete should be damaged in places, the cost of making good such places should be far less than that of pointing a brick structure throughout, and incomparably less than scraping and painting steelwork at frequent intervals.

The building in of steel joists and girders with brickwork or concrete is in many cases only an expedient and, in the case of heavy girders of large span, a clumsy expedient for saving painting charges. Until quite recently this object was only partially obtained, because the under side of girder flanges and the edges of the plates still remained exposed, and the practice now adopted of covering these with a thin layer of reinforced concrete, as shown in Fig. 1, is costly, and requires to be carefully carried out to produce a satisfactory result.

For nearly ten years in this country, and perhaps fifteen years abroad, reinforced concrete has been competing with this type of construction with increasing success, and it is believed that the maintenance question will be the determining factor resulting in considerable use being made of this material for railway purposes.

Difficulties.

Much of the work done by railway engineers at the present time consists in renewing existing structures and in widening or extending existing lines or works in connection with them, and this fact presents some difficulties to the free use of reinforced concrete, and, on the other hand, other difficulties

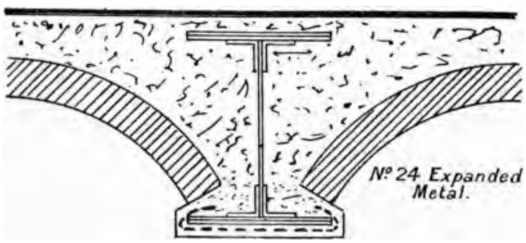


Fig. 1.—Encased Girderwork.

which construction in this material is particularly suited to overcome.

A very frequent difficulty met with in work of this class is the small amount of space available for construction, particularly as regards the depths of floors. It is only possible to design a reinforced concrete floor of shallow construction to carry a heavy load by the use of a large percentage of compression reinforcement, and this means a loss of economy. But apart from the depth required for the permanent construction, there is the temporary timber work to be considered, and the space required for this is not always available for the period which must be allowed, in the first place for the construction of the work, and subsequently for the concrete to properly set. This difficulty can sometimes be got over by constructing the work at a convenient site on one side and erecting it in position after the concrete has set.

Apart from considerations of space, work done between trains in the neighbourhood of running lines must needs be done hurriedly. It is well known that the strength of a reinforced concrete structure depends very much on the care with which the concrete is worked into complete contact with the reinforcement, and it is particularly desirable that this portion of the work should not be scamped in any way. When practicable, therefore, it is far better for the constructional part of the work to be carried out away from the lines, and in the case of small road bridges or footbridges over the line for instance, it is quite possible to do this without any great increase in the cost of construction. This method of procedure also possesses the great advantage that if at any time alterations have to be made the work can be taken down and re-erected elsewhere. From a railway point of view the monolithic character of reinforced concrete constructed *in situ* is in this sense a disadvantage. It is very difficult and often impossible to see clearly even ten years ahead, and work carried out at the commencement of such a period, may, at the end of it, have to be demolished to make room for more extensive premises or rival interests which have attained greater importance. In the case of a monolithic structure demolition generally means destruction, and in the case of a reinforced concrete structure which has stood for some years, this destruction would be a matter of great difficulty.

These are some of the reasons which are advanced against the use of reinforced concrete for railway purposes, and encourage a preference for steel-girders or beams, or brickwork in the form of arches; but it seems probable that the difficulties mentioned can be, and indeed are being overcome.

Advantages.

Turning to the advantages of the material, it is probably the experience of most railway engineers that simple structures of square form are the exception rather than the rule. Presumably because they were the last in the field, railway companies at the best of times have to be very accommodating; and especially when undertaking widening works in crowded districts, it is more often than not the case that the structures which have to be erected are tied by existing works of irregular form. If steelwork is employed in such circumstances as these, accurate dimensions must be taken in the first place, a vast amount of detailing is required in making working drawings and similarly in setting out the work. The construction of a brick arch as a covering to an irregular enclosure presents difficulties when there are two sides or abutments parallel and the other two not parallel, but it is quite impracticable to adopt this form of construction when the irregularity extends to all four sides.

For such cases as these reinforced concrete possesses undoubted advantages, provided that the circumstances permit of a monolithic structure. The fact of the centering being formed on the site, ensures the work fitting properly without the necessity of accurate measurement beforehand. The rods are easily cut to the required lengths, and the concrete can readily be moulded into any desired form. Skew angles can be formed almost as readily as if they were square, and there is little difficulty introduced because the beams are not parallel or the structure bounded by other than straight lines. By the use of reinforced concrete beams and slabs moulded beforehand, bridge floors, platforms, and buildings have been very rapidly constructed, either completely with these units, or in conjunction with steel girders or framework. For instance, on the South Eastern and Chatham Railway, bridge floors have been constructed of longitudinal beams, moulded beforehand, set side by side, and in some of the roofing over the District Railway, hollow reinforced concrete blocks of arched form have been set on the lower flanges of rolled steel joists and backed with concrete to form the foundation of the road above.

Encased Steelwork.

Mention has been made of the practice of encasing steelwork in brickwork or concrete as a means of protection from corrosion. The results of measurements of the deflection of bridge girders treated in this manner, when compared with the calculated deflection of the girder alone, make it quite evident that the filling adds considerably to the strength of the steel structure. In designing floors of this type it is not usual to take any account of this additional strength, but it is nevertheless interesting to investigate the probable nature of the stresses in the composite structure.

The construction illustrated in Fig. 2 presents a simple case. The floor which is shown in cross section in this illustration consists of 7 in. by 4 in. by 16 lbs. British standard beams, spaced 2 ft. centres and encased in a slab of concrete 10 in. deep. It is evident that under load the concrete suffers the same deflection as the joists, and provided this strain is not sufficient to cause disintegration of the concrete, the latter must carry a portion of the load. Also, if the concrete continues to adhere to the steelwork under load, the two must

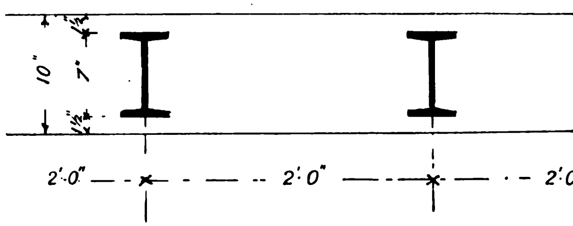


FIG. 2.—Steel Joists and Concrete.

act together as a composite beam. The working moment of resistance of a 2 ft. width of this floor, calculated on the basis adopted in practice of the steel joist carrying all the load, and a maximum stress of 6½ tons per square inch, is 72·8 inch tons. Upon the hypothesis of the section of the floor under consideration acting as a composite or reinforced concrete beam, and adopting the rules generally assumed for calculating beams of this material (see p. 21), the stresses in the various members of the structure corresponding to a bending moment of 72·8 inch tons are approximately as follows :—

Maximum compression in concrete,	440 lbs. per square inch.			
Maximum compression in steel	3,000 lbs. ,,	,,	,,	
	1·3 tons ,,	,,	,,	
Maximum tension in steel	13,000 lbs. ,,	,,	,,	
	5·8 tons ,,	,,	,,	

If the adhesion between the concrete and the steel should break down under the live load, or dead load added after the concrete has set—as it is very likely to do because the surface offered by the steel is small compared with its cross-sectional area—the construction practically becomes a series of steel beams alternating with concrete beams keyed into one another in such a manner that adjacent beams would of necessity be subject to practically the same deflection under a distributed load. The proportion of the total load carried by each member would therefore depend on the product of the moment of inertia of its cross section and the modulus of elasticity of the material. If the stresses corresponding to a bending moment of 72·8 inch tons per 2 ft. width of beam are calculated on this basis the results are roughly as follows:—

Maximum compression and tension in steel,
3,600 lbs. per square inch.
1·6 tons " " "

Maximum compression and tension in concrete,
330 lbs. per square inch.

No concrete would stand a tensile stress of this magnitude, and if the conditions assumed occurred in practice, the concrete would crack and become useless as a beam, and all the load would be thrown upon the steel beams.

These figures, which are calculated on values of the moduli of elasticity of concrete and steel bearing to one another the ratio of 1 to 15, are given to show the weakness of the type of construction shown in Fig. 2. It is not an economical form of construction, but is convenient because the shuttering for the concrete can be suspended from the joists and no props are required. When constructed in this way a certain amount of stress is set up in the steel joists, due to their own weight and that of the shuttering and the wet concrete, without any corresponding stress in the concrete, which can therefore only be stressed by additional dead load and live load. This fact probably explains why the construction illustrated in Fig. 2 behaves quite satisfactorily in practice; but even when constructed in this way it is doubtful whether a structure of uniform strength is obtained, and, if tested to destruction, the concrete would probably crack long before the breaking load for the steel joists was reached. This form of construction has been perfected by the introduction of steel girders, specially designed to act as reinforcement for concrete floors, and known as Fawcett's Mon'lithcrete girders. Large diamond-shaped apertures are stamped out from the web, leaving the two flanges rigidly connected together by inclined ties sloping up towards each end as shown in Fig. 3. A much more efficient bond between the steel and the concrete is thus ensured, and greater economy can be obtained by making the tension flange of larger area than the top or compression flange. The concrete is further reinforced by twisted hoop-

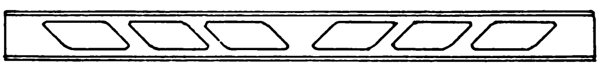


FIG. 3.—Fawcett's Mon'lithcrete Girder.

iron strands passing through the apertures in the webs and crossing the main reinforcement at right angles.

REINFORCED CONCRETE.

Without at this stage going into the methods of calculation adopted, which will be treated in a subsequent chapter, it is of interest to compare the steel beam and concrete floor illustrated in Fig. 2 with a reinforced concrete tee beam construction designed to have the same calculated safe moment of resistance, namely 72·8 inch tons per 2 ft. width of floor. The dimensions of the concrete and the size of the reinforcing rods shown in Fig. 4 fulfil this condition, and are based upon values of 500 lbs. per square inch for the working compressive stress in the concrete, and 14,500 lbs. (approximately 6·5 tons) per square inch for the working tensile stress in the steel. This construction occupies a little more room than the steel-beam floor, the overall depth as drawn being 1 ft. 1½ in. in the former case as against 10 in. in the latter case. A shallower reinforced concrete floor can be designed, but the heavier reinforcement required entails a loss of economy. The estimated weight of the steel beam and concrete construction in this case is 10·4 cwt. per square yard, and of the reinforced concrete construction as shown in Fig. 4, 5·7 cwt.

per square yard, exclusive in each case of paving or wearing surface. The saving in weight by adopting tee beam construction is thus something like 45 per cent., and on account of this decreased dead load the depth of the floor can either be reduced somewhat, or the safe live load can be correspondingly increased.

The estimated cost of the steel beams and concrete filling is about seventeen shillings per square yard, and of the tee beam construction in concrete reinforced with round rods, from ten to twelve shillings per square yard. These figures do not include anything for special wearing surface, or for paving, but the estimated saving on the cost of the floor itself is from

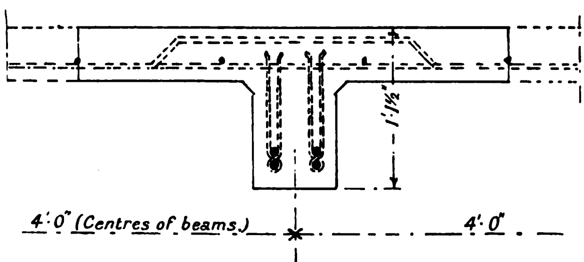


FIG. 4.—Tee Beam.

20 to 30 per cent., and amounts to a considerable sum when large areas have to be covered. Referring to the estimated cost of the steel beam and concrete construction, rather more than nine shillings represents the approximate value of the steel beams per square yard, and the remaining eight shillings the value of the concrete filling per square yard. If the joists are not encased in concrete some addition is required to complete the floor, and the estimated cost of that part of the structure which is designed to carry the load is much the same in each case. The additional cost of the construction first considered is due to the concrete filling which is provided in many cases solely to protect the steelwork from corrosion and eliminate the cost of painting.

The author has extended the comparison to further cases of beams, and other structures and materials, such as timber and brickwork,* and finds it to be very generally true that the cost of reinforced concrete beams and columns is much the same as that of steel or timber units designed in accordance with established practice to carry the same load; *but whereas the steel and timber beams generally require the addition of some kind of flooring to complete the structure, apart from the question of protecting the steelwork, the reinforced concrete construction forms a complete floor in itself with all the steelwork protected from corrosion* without any additional cost.

It is these features of reinforced concrete construction which make it a more economical material to use than steel or timber, because there is in many cases a saving in first cost, apart from the question of maintenance, and secondly a substantial saving either of the money which would have to be expended in first cost to encase the steelwork and so eliminate the cost of painting, or of the subsequent cost of painting the structure from time to time, or renewing the timber or steelwork.

There is little doubt, too, that the steelwork is far more efficiently protected in a properly designed reinforced concrete structure than in an encased steelwork structure. In the first case it is only at one or two points that the minimum cover of, for example, 1 in. of concrete occurs, but in the second case large surfaces have to be protected with a similar thickness and to apply this in an efficient manner to the underside of a girder, for instance, is an expensive process.

Working Stresses.

The working stresses commonly adopted in designing reinforced concrete structures vary from 500 to 600 lbs. per square inch, as regards the compressive stress in the concrete, and 16,000 lbs. per square inch is a very usual figure for the tensile stress in the steel, but higher values such as 750 lbs. per square inch for the concrete, and 20,000 lbs. per square inch are used in certain cases. Railway Engineers are not

* "Reinforced Concrete for Railway Engineering Works," *The Engineer*, September 29th, October 27th, November 3rd, 1911.

likely to adopt such high values as these. Although not bound to adopt as the limiting stress for steelwork, the value of 6·5 tons per square inch laid down by the Board of Trade, except for structures carrying passenger lines, it is a very common practice to do so, and it is not likely that this value would often be exceeded for structures subject to a vibratory load. This value of 6·5 tons per square inch, or the practical equivalent 14,500 lbs. per square inch, gives a factor of safety of a little over four when divided into 28 tons per square inch. The latter figure is generally the lower limit of the breaking stress specified for mild steel test prices.

The breaking strength of concrete of course depends upon the age of the specimen tested, as well as upon the constituents and upon the proportions in which they are mixed. A very usual value adopted for the breaking compressive stress of concrete mixed in the proportions of 1 part cement, 2 parts sand, and 4 parts of suitable aggregate, is 2,250 lbs. per square inch, and good concrete of this kind should possess this strength after four weeks. A working stress of 500 lbs. per square inch gives, as in the case of the steel tensile stress, a factor of safety of four with a little to the good, and these values would appear to compare favourably with the practice of railway engineering construction in other materials. *When the value of 500 lbs. per square inch is converted into its equivalent of 32 tons per square foot, and compared with the usual working stresses of 5-12 tons per square foot adopted for concrete piers and foundations, not reinforced, it is at once apparent how great an advance has been made in the amount of reliance placed on this material.* It also becomes evident that the very best materials must be used and mixed in the most careful manner, to produce a concrete capable of working at such a stress as this.

A number of experiments on test blocks, to determine the crushing strength of cement concrete mixed in various proportions, have been carried out at Watertown Arsenal, U.S.A. Specimens were tested seven days, one month, two months, and six months after manufacture, and the results of these tests in the case of 1 : 2 : 4 and 1 : 3 : 6 concrete, are plotted in

Fig. 5. The values used for plotting the curves are indicated by the small circles, and in each case represent the mean of four tests.

Striking Centres.

It is clear from these curves that under ordinary circumstances concrete attains a very fair strength at the age of four

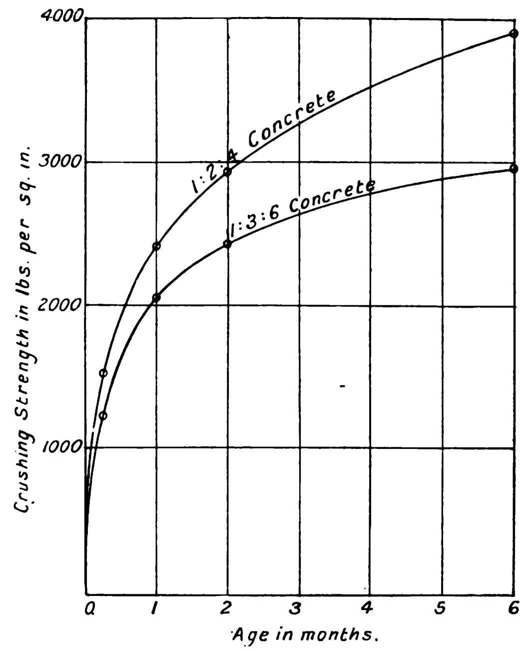


Fig. 5.—Strength of Concrete.

weeks, and at the termination of this period the supports to beams can generally be removed, provided the setting has not been hindered by frost or bad weather. Vertical sheeting and the moulds for the sides of beams are generally removed one week after mixing, or even less, as this facilitates the setting of the concrete, and enables a fine finished face to be obtained by working in cement grout with a piece of wood or steel float. If the boxings of columns are removed as early as this, care must be taken that these members are not subjected to excessive load.

The concrete blocks for laboratory tests are naturally carefully made, and being small expose a considerable area of surface relatively to their bulk. The concrete under these circumstances will in all probability set much quicker than that in the work. When making blocks for testing purposes it is obviously desirable that the moulds should be filled from the mixing platform for the ordinary work, and as far as possible be allowed to set under the same conditions, in order that the results may provide a trustworthy indication of the strength of the work and of the time which should be allowed to elapse before the supports are struck. The curves in Fig. 5 illustrate very clearly that the strength of a concrete structure steadily increases with age, a very valuable advantage, particularly in the case of railway works, which so frequently as time goes on have to perform heavier duty than that for which they were originally designed. In the case of the 1 : 2 : 4 concrete the curve indicates an increase of strength of about 5 per cent. per month at six months and the strength is then nearly 4,000 lbs. per square inch.

Specification of Materials.

The British Standard Specification for Portland Cement is so generally adopted for defining the quality of the cement to be used that little need be added on this subject, except that a slow-setting cement is generally used to minimise the risk of joints being formed between the several layers or stages in which the work is carried out. Sand must be clean and sharp, and washed, if necessary, to remove earthy and other impurities. In order that the percentage of voids may be as small as possible the grains should be of various sizes, but as the cement consists of very fine particles it is more useful to the end in view for the sand to be coarse rather than fine. The aggregate used will, of course, depend largely on the locality, and the choice of materials is by no means confined. Thames or pit ballast, clean gravel, broken granite or hard close-grained stone are generally preferred. Broken brick, furnace clinker or cinders are sometimes used, the latter more particularly for floors on account of the light weight, fire-resisting and sound-proof properties of concrete mixed with this material.

It is most important that the material should be clean and properly graded, and it is advantageous for the stones to be cubical in form rather than needle-shaped or flaky. If the percentage of voids in broken stone with the particles mostly of one size, is determined, it is found to be about 50 per cent. By properly grading the material this may be reduced to from 45 per cent. to 30 per cent. In order to obtain a strong concrete it is absolutely essential that all the voids should be filled, and in the case of reinforced concrete work and all work requiring a finished surface, that there should be some mortar to the good for coating purposes and for working to the surface. Provided this is done the strength of the concrete, other things being equal, depends on the strength of the mortar.

In some specifications it is usual to define the proportions of the cement and aggregate exactly, and determine the proportion of sand as sufficient to fill the voids in the aggregate. In such a specification the strength of the mortar is not absolutely defined and depends upon the grading of the material, being poorer in strength if this is badly carried out and richer in strength if the sizes of the stones are thoroughly well assorted.

For reinforced concrete work in which the strength of the concrete is of the greatest importance, a more rigid specification would appear to be far preferable, and is generally adopted, the constituents, cement, sand, and aggregate being mixed in such proportions by volume as $1:1\frac{1}{2}:3$, $1:2:4$, or $1:3:6$ as the particular kind of work may require. A very common specification for the size of the particles of the aggregate is to pass through a sieve with $\frac{3}{4}$-in. mesh and be retained on a sieve with $\frac{3}{16}$-in. or $\frac{1}{8}$-in. mesh. A 1-in. mesh is sometimes used for the larger sieve for heavier work. The specification is sometimes worded in such a way as to ensure the particles being roughly cubical in form and capable of being passed "in any direction" through a ring of, say, 1 in. internal diameter. Such a specification would be exceedingly difficult to enforce, and is therefore of doubtful utility. Extracts from specifications for reinforced concrete are given in the subsequent chapters in which actual works are described.

The reinforcement used assumes a variety of forms, including ordinary round or square bars bent to the required shapes and bound together with fine annealed wire; various special bars, such as the Kahn trussed bar, the expanded steel bar having shear members rigidly attached to them; various bars designed to obtain a mechanical bond, such as the patent indented bar, Kahn ribbed bar, Thatcher bar, and twisted bars; various lattice reinforcements designed for use in slabs, such as expanded steel, Clinton electrically cross-welded steel wire reinforcement, Johnson's steel wire lattice, &c. These are so well known, and so completely described in the handbooks issued by the firms manufacturing them, that there is no need to further describe them. Ordinary round or square bars may be used in various special ways, as in the Hennebique, Coignet, Considère, Paragon, and other systems. These lists are by no means exhaustive, but serve to show the growth of reinforced concrete construction in the comparatively short space of less than ten years.

Mild steel is generally specified for the reinforcement, having an ultimate tensile strength of from 28 to 32 tons per square inch, with an elongation of at least 20 per cent. on an 8-in. gauge length. Sometimes a rather greater elongation is specified, and in some cases limiting values are set down for the elastic limit. In the process of manufacturing expanded metal from mild steel the ultimate tensile strength is raised from 28 to 32 tons per square inch to 32 to 36 tons per square inch, and the elastic limit is raised from about 16 tons per square inch to about 26 tons per square inch. Indented bars generally have an ultimate tensile strength of about 40 tons per square inch, and an elastic limit of over 22 tons per square inch, but they can be supplied to conform to the usual specification for mild steel.

In all cases the reinforcement should be perfectly clean, not painted. In rare cases the rods are coated with cement before the concrete is placed. The specification of the reinforcement is considered in more detail in subsequent chapters, wherein particular works are described.

CHAPTER II

BENDING STRESSES

THEORY.

MANY of the earlier examples of reinforced concrete railway structures in this country were designed, in so far as the details and the arrangement and amount of the reinforcement are concerned, by the late Mr. L. G. Mouchel and his partners on the Hennebique system. The formulæ employed in these cases and in all work designed on the Hennebique system are to a large extent empirical and based on the extensive experimental investigations and practical experience of M. Hennebique.

The formulæ in more general use are deduced theoretically from certain assumptions as to the behaviour of the two materials as indicated by the results of experiments on trial beams. Recommendations have been made by various authorities as to the method of calculating the stresses in beams of reinforced concrete, and these authorities very generally agree that the tensile resistance of the concrete may be omitted without appreciable error, and that the strains produced in the various parts of the effective members of the structure may be taken as proportional to the distance of such part from the neutral axis of the beam.

Very careful series of experiments were made at McGill University, Toronto, with the object of ascertaining to what extent these assumed conditions are fulfilled in practice. The tests were made on two sets of beams, the dimensions of one set being 6 in. × 8 in. × 6 ft. $4\frac{1}{2}$ in. long and of the other set 8 in. × $11\frac{3}{4}$ in. × 10 ft. 6 in. long. The beams were loaded at the third points, the span between centres of bearings being 6 ft. in the case of the smaller beams and 10 ft. in the case of the larger beams. The concrete was mixed in the proportions of one part of cement, two parts of sand, and four parts of trap rock crushed to pass through a $\frac{3}{4}$-in. ring. The reinforcement consisted of patent bars by Kahn, Johnson, and Ransome. One of the larger beams selected as a typical case is shown in section in Fig. 6. The reinforcement consisted of five $\frac{1}{2}$-in. Johnson rods with a total cross-sectional area of ·90 sq. in. which is 1·02 per cent. of the effective area of the beam. This beam was tested after 56 days and failed at a load of 20,500 lbs. As this beam was tested with the tension side uppermost and the load applied from below, the weight of the beam, which is about 1,000 lbs., should be subtracted from this.

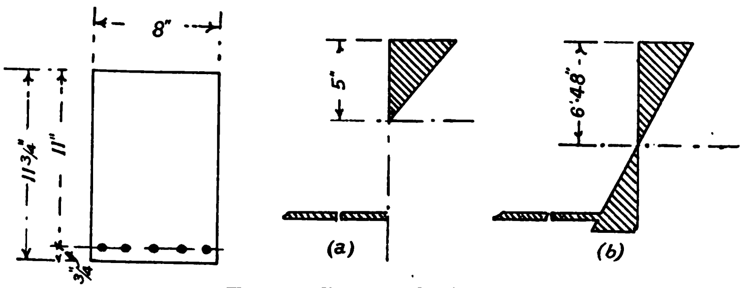


FIG. 6.—Rectangular Beam.

By means of delicate reflecting extensometers, the strain was measured at five horizontal layers of the beam as the loading proceeded, and by plotting the results of these measurements it was possible to locate the positions of the neutral axis corresponding to the increasing values of the load, and obtain a graphic record of the variation of the strain at the different layers with the distance of each layer from the neutral axis. For small loads the neutral axis was found to be at 52 per cent. of the depth of the beam, measured from the compression surface. Between loads varying from about one-seventh to about one-third of the ultimate load, the position of the neutral axis changed until its distance from the compression surface was 41 per cent. of the total depth of the beam, and then remained in practically this same position throughout the remainder of the test. The strains measured were found to be so nearly proportional to the distance from the neutral axis, as to justify the assumption of linear proportionality in making calculations.

In the early stages of the loading of these test beams, the tensile resistance of the concrete was evidently an important

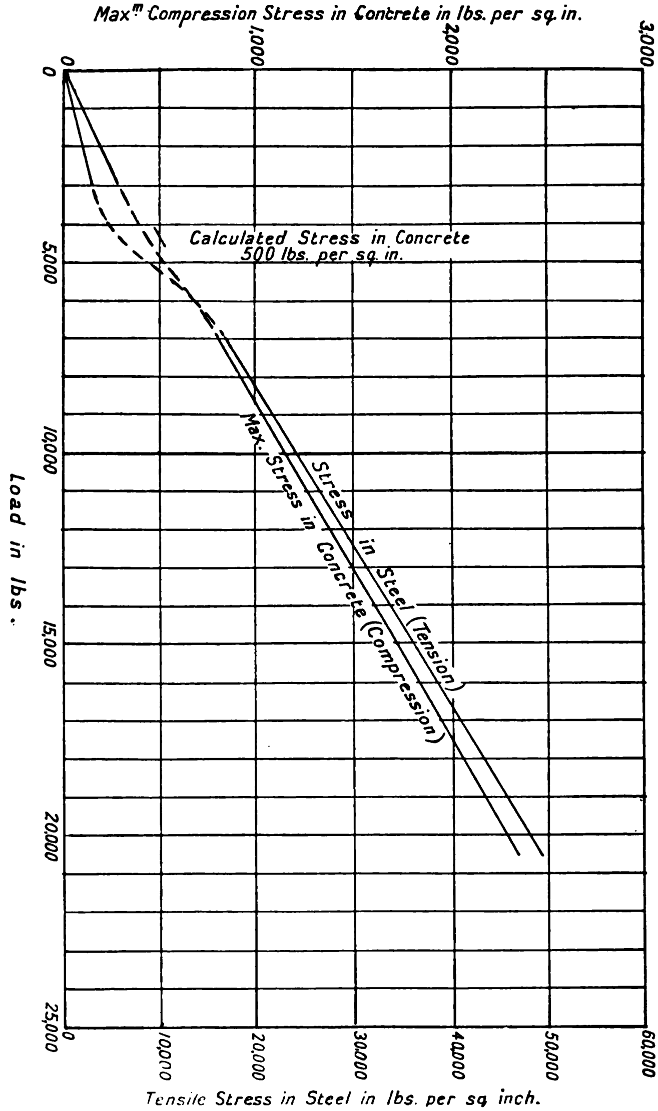


FIG. 7.—Variation of Stresses.

factor in determining the moment of resistance, and the total stress acting on a cross section of the beam may be represented by the shaded area in Fig. 6B. Assuming a straight line law of proportionality for the variation of stress throughout the cross section, the position of the neutral axis under these circumstances is calculated to be 55 per cent. of the total depth measured from the compression surface.

If the conventional method of calculation is adopted and the tensile resistance of the concrete neglected, the distribution of stress is as shown in Fig. 6A.

The neutral axis under these conditions is nearer the compression surface and in the case of a beam with 1 per cent. of reinforcement the distance is calculated to be 38 per cent. of the total depth. The natural inference is that for small loads the tensile resistance of the concrete is developed almost to the full extent while for larger loads it is almost valueless.

The values of the maximum stresses in the concrete and the tensile stress in the steel in each case can readily be calculated as functions of the load, and these values are plotted in Fig. 7. For values of the load between zero and one-seventh of the ultimate load, the tensile resistance of the concrete has been taken into account, and for values of the load from one-third of the breaking load upwards, the tensile resistance of the concrete has been neglected. Between one-seventh and one-third of the ultimate load is a transition period during which the tensile resistance of the concrete apparently breaks down and the values of the stresses may be interpolated approximately by connecting the parts of the diagram already determined by the dotted lines as shown in Fig. 7. Under ordinary circumstances the working load occurs in this transition period and the conventional method of calculation—*i.e.*, neglecting tension—therefore errs slightly on the side of safety. A beam designed to have a factor of safety of about $4\frac{1}{2}$ would appear to have a reserve of strength of from 20 per cent. to 50 per cent., the higher percentage applying to the steel.

Concrete in Tension.

It may be noticed that the calculated maximum tensile stress in the concrete on the basis of the stress distribution sketched in Fig. 6B, corresponding to one-seventh of the breaking load is

230 lbs. per square inch, which is about the ultimate strength of this class of concrete in tension. It is at this point that the neutral axis commences to rise, presumably on account of the tensile resistance of the concrete breaking down. The experiments of Considère indicate very strongly, however, that the concrete does not actually fail in the sense of sustaining fracture.

By means of a weighted lever attached to the top end of a column consisting of one part cement to three-and-a-half parts of sharp sand, and reinforced with wires ·04 in. in diameter on one side only, Considère produced a tensile stress, constant throughout the length of the reinforcement, of about 20 tons per square inch of the section of the wires, while the non-reinforced side was in compression. Subsequently the lever was repeatedly loaded more than 100,000 times with loads sufficient to stress the steel from 6·3 to 13·3 tons per square inch. A careful examination of the concrete surrounding the reinforcement revealed no signs of fracture; and a specimen of this concrete containing no reinforcement was sawn out and tested in tension and sustained a load of 317 lbs. per square inch before fracture took place. Concrete when reinforced was thus proved to be capable of sustaining a strain or elongation nearly ten times as great as that corresponding to the breaking point of the same material not reinforced. The explanation put forward is that the presence of the steel prevents the local yield which in many materials may be observed to precede fracture. When the yield point of the concrete is reached it cannot sustain additional load without a largely increased strain, and this the steel prevents and in doing so receives the load which the concrete is incapable of carrying. Other engineers declare that hair cracks do occur in the concrete if the tensile stress in the steel is allowed to reach too high a value, and so long as there is any doubt on this point there would certainly appear to be strong reason to adopt for the working stress a figure no higher than the 14,500 lbs. or 6·5 tons per square inch referred to in the previous chapter.

The importance of this point lies in the fact that once cracks develop in a reinforced concrete structure, the length of its life is very doubtful, especially if the structure is subject to a rolling load or vibration. The possible effect of moisture reaching the steelwork must also give rise to suspicion of the safety of a structure in which cracks have appeared, even if these do not become sufficiently bad to cause the concrete to break away.

Neutral Axis.

The formulæ which may be deduced from the conditions assumed to maintain in a reinforced concrete beam are some-

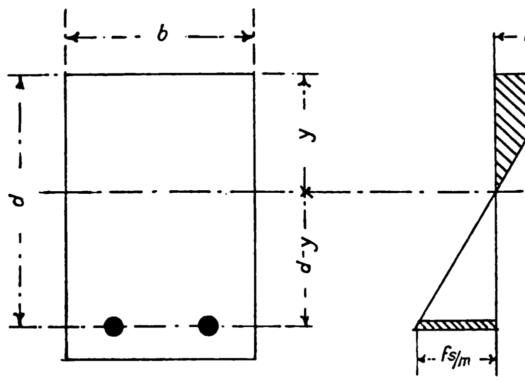


Fig. 8.—Beam with Single Reinforcement.

what complicated in appearance, but capable of simple interpretation. A beam containing only tensile reinforcement presents the simplest case, and the reinforcement is generally expressed as a percentage, or else as a proportion, p, of the effective area of the cross section of the beam. The effective area of the beam is the product of the breadth b, and the depth d, of the centre of the reinforcement below the compressive surface of the beam.

The important quantities to consider in designing a reinforced concrete beam are the moment of resistance and the maximum stresses. The calculation of the latter involves the determination of the depth of the neutral axis below the compression surface of the beam.

If this is denoted by y, Fig. 8, the maximum compression stress in the concrete

$$f_c \propto y$$

while the tensile stress in the steel

$$f_s \propto m\,(d - y)$$

m representing the ratio of the modulus of elasticity of steel to that of concrete, and generally taken as 15.

Therefore $$\frac{f_s}{f_c} = \frac{m\,(d - y)}{y} = \frac{m\left(1 - \frac{y}{d}\right)}{\frac{y}{d}}$$

The total compressive stress in the concrete must equal the total tensile stress in the steel, therefore, $\frac{1}{2}\, b\, y\, f_c = p\, b\, d\, f_s$

or, $$\frac{f_s}{f_c} = \frac{y}{2\,p\,d} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

By equating these two expressions for this ratio the value of $\frac{y}{d}$ may be obtained in terms of p and the constant m

Thus $$\frac{y}{d} = \sqrt{p^2\,m^2 + 2\,p\,m} - p\,m \quad . \quad . \quad . \quad (2)$$

Example 1.—Calculate the position of the neutral axis, and the ratio of the tensile stress in the steel to the maximum compressive stress in the concrete, in the test beam of which the cross section is sketched in Fig. 9, assuming the modulus of elasticity of steel to be 15 times that of concrete.

Breadth, 4 in.

Effective depth (from compression surface to centre of reinforcement) 11 in.

Effective area, 44 sq. in.

Area of reinforcement (1) $\frac{3}{4}$ inch diameter bar, ·44 sq. in.

Proportion of reinforcement $\frac{\cdot 44}{44} = \cdot 01$.

From (2) $\frac{y}{d} = \sqrt{\cdot 01^2 \times 15^2 + 2 \times \cdot 01 \times 15} - \cdot 01 \times 15$

$= \cdot 42$.

$y = \cdot 42 \times 11 = 4{\cdot}62$ in.

$=$ distance of neutral axis from compression surface.

From (1) $\frac{f_s}{f_c} = \frac{\cdot 42}{2 \times \cdot 01} = 21$.

It is thus evident that in a beam containing only tensile reinforcement, the position of the neutral axis, and therefore the ratio of the tensile stress in the steel to the maximum compressive stress in the concrete depend only on the proportion of the reinforcement. *Consequently there is only one proportion* *of reinforcement, placed all in tension, which will permit definite working stresses in the steel and in the concrete to be reached simultaneously.* If the value of the working tensile stress in the steel f_s is fixed at 14,500 lbs. per square inch, and the working compressive stress f_c in the concrete at 500 lbs. per square inch, the ratio $f_s \div f_c = 29$ and the proportion of the reinforcement must be about ·6 per cent. of the effective area $b\ d$, that is, p must equal ·006.

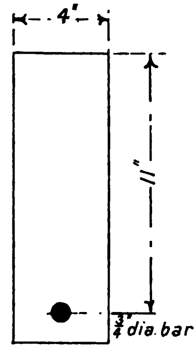


FIG. 9.—Example 1.

MOMENT OF RESISTANCE.

If the proportion of the reinforcement is less than ·6 per cent. the moment of resistance of the beam is determined by the working tensile stress in the steel and is equal to

$$M.R. = p\left(1 - \frac{1}{3} \cdot \frac{y}{d}\right) b\ d^2 f_s \quad . \quad . \quad (3)$$

This can be written $K\ b\ d^2 f_s$, K depending on the percentage of reinforcement and having the values plotted in Fig. 10, p. 24.

If the proportion of the reinforcement is more than ·6 per cent. the moment of resistance of the beam is determined by the working compressive stress in the concrete and is equal to

$$M.R. = \tfrac{1}{2}\frac{y}{d}\left(1 - \frac{1}{3} \cdot \frac{y}{d}\right) b\ d^2 f_c \quad . \quad . \quad (4)$$

EXAMPLE 2.—Calculate the working moment of resistance of the beam described in the previous example corresponding to working stresses as under :—

Concrete in compression, 500 lbs. per sq. in.

Steel in tension, 14,500 lbs. per sq. in.

As there is more than ·6 per cent. of reinforcement, the critical proportion corresponding to these working stresses, the maximum limit for the concrete will be reached before that for the steel and the moment of resistance is determined from (4).

$$M.R. = \tfrac{1}{2} \times \cdot 42 \left(1 - \frac{\cdot 42}{3}\right) 4 \times 11^2 \times 500$$
$$= 43{,}700 \text{ in. lbs.}$$

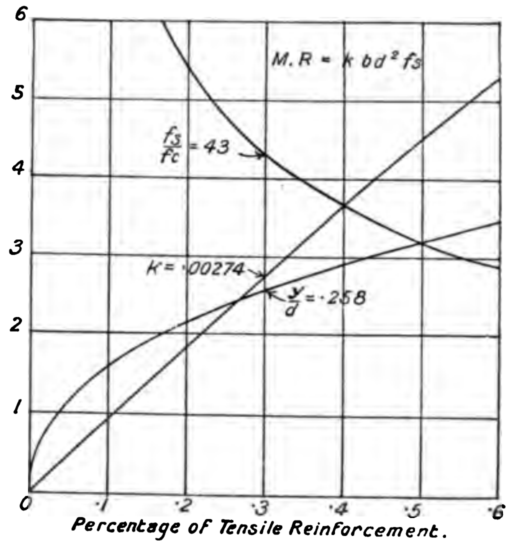


FIG. 10.—Curve of Constants, Single Reinforcement.

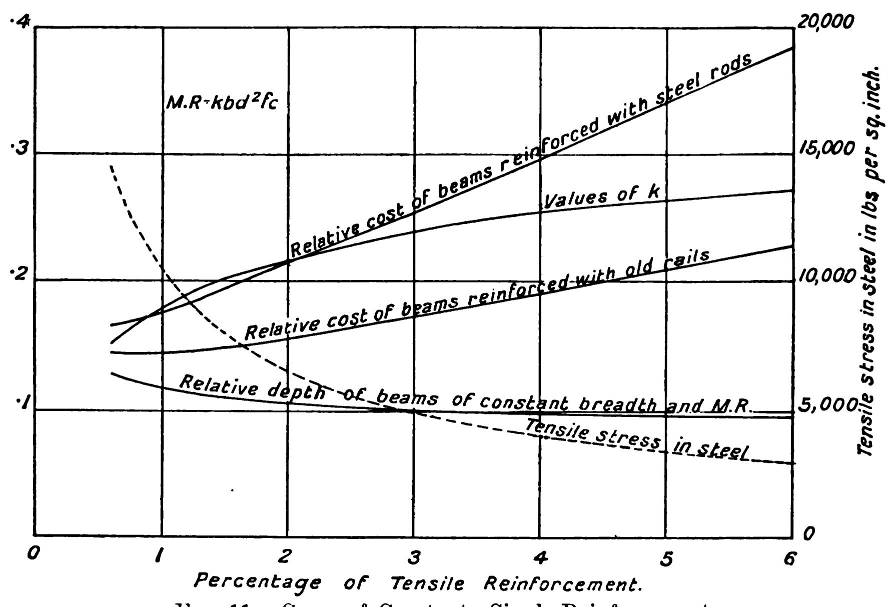


FIG. 11.—Curve of Constants, Single Reinforcement.

Equation (4) can be written as $M.R. = k\ b\ d^2\ f_c$ where k is a constant depending only on the percentage of reinforcement. The values of k are plotted in diagram form in Fig. 11, and on the same diagram the values of the stress in the steel corresponding to a maximum compressive stress of 500 lbs. per square inch in the concrete are also indicated. It will be noticed that the value of k increases but slowly with the percentage of reinforcement, suggesting that it is not profitable to employ a large percentage of reinforcement entirely in the tension member of the beam.

Example 3.—Using the diagram Fig. 11, calculate the depth of a lintel to span an opening of 8 ft. and carry a distributed load of 5 tons. The breadth is 14 in. and the depth must be a whole number of brick courses of 3 in. each, including $1\frac{1}{2}$ in. cover to the centre of the reinforcement. Calculate also the area of reinforcement required and the maximum stress in the steel. Working stresses 500 lbs. per square inch compression in concrete and 14,500 lbs. per square inch tension in steel.

Maximum bending moment, $\dfrac{5 \times 2{,}240 \times 96}{8} = 134{,}400$ in. lbs.

To find the approximate effective depth d.

The working stresses are reached simultaneously when there is ·6 per cent. of reinforcement and the factor k determining the moment of resistance is ·15 (Fig. 11.)

$$M.R. = \cdot 15 \times 14 \times d^2 \times 500 = 134{,}400 = \text{B.M.}$$
$$d^2 = 128 \text{ in. squared.}$$
$$d = 11{\cdot}3 \text{ in.}$$

The total depth must therefore be 12 in. and the effective depth, allowing $1\frac{1}{2}$ in. cover, 10·5 in.

The factor k then becomes $134{,}400 \div (14 \times 10{\cdot}5^2 \times 500)$
$= \cdot 174.$

The required percentage of reinforcement is ·9 per cent. and the tension in the steel 11,200 lbs. per square inch (Fig. 11.)

·9 per cent. of $14 \times 10{\cdot}5 = 1{\cdot}32$ sq. in., say (four) $\frac{3}{4}$ in. diameter bars.

Example 4.—Using the diagram Fig. 10, determine the amount of reinforcement required to make a 4-in. concrete platform coping, sufficiently strong to span a 3-ft. opening (effective span 3 ft. 6 in.) and carry a super load of $1\frac{1}{4}$ cwts. per square foot. The working tensile stress for steel is to be taken at 14,500 lbs. per square inch and the centre of the reinforcement placed $1\frac{1}{4}$ in. from the bottom surface. Calculate also the maximum compression in the concrete.

Consider a width of 12 in.

Dead load, $3 \cdot 5 \times 1 \times \cdot 33 \times 156$ lbs. $= 182$
Super load, $3 \cdot 5 \times 1 \times 140$ lbs. $= 490$ } 672 lbs.

Maximum bending moment $= \frac{672 \times 42}{8} = 3,530$ in. lbs.

Moment of resistance $= K \times 12 \times 2\frac{3}{4}^2 \times 14,500$ in. lbs.

K = ·0027, corresponding to ·3 per cent. of reinforcement, and $\frac{f_s}{f_c} = 43$ (Fig. 10).

·3 per cent. of $12 \times 2\frac{3}{4} = \cdot 1$ sq. in. per ft. width, say $\frac{1}{4}$ in. diameter rods, 6 in. centres.

Maximum compression in concrete $= \frac{14,500}{43} = 337$ lbs. per sq. in.

With the exception of lintels and sills, simple beams similar to that illustrated in Fig. 8, are not of common occurrence in reinforced concrete structures, but the formulæ relating to this case are nevertheless very frequently used in designing slab floors, a convenient width of which may be treated as a beam.

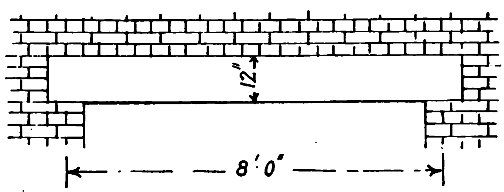


FIG. 12.—Lintel.

In this case the breadth of the beam is fixed by the width of floor for which the load has been calculated, and the only possible variables are the depth and the percentage of reinforcement. For the moment of resistance to be constant $\frac{y}{d}\left(1 - \frac{1}{3} \cdot \frac{y}{d}\right) d^2$ occurring in equation (4) must be constant or d must vary inversely as $\sqrt{\frac{y}{d}\left(1 - \frac{1}{3} \cdot \frac{y}{d}\right)}$, a function of the percentage of reinforcement which may be evaluated by means of equation 2. These values of d, which represent the relative depths of beams of constant breadth and constant moment of resistance, when reinforced with varying percentages of tensile reinforcement, are plotted in the curve so marked in Fig. 11, and as the breadth is constant the curve also represents the relative areas of cross section, which are a measure of the volumes of concrete required.

Relative Cost.

An average price for concrete as used for this class of work, including labour in mixing and ramming and profit, but excluding shuttering which would be much the same in all cases, is about twenty-eight shillings per cube yard, and for the steel reinforcement laid in the work about £12 per ton. The price of the steel works out about fifty times as much volume for volume as the price of the concrete, and by increasing the ordinates of the curve in Fig. 11 by fifty times the percentage of reinforcement corresponding to each ordinate considered, a curve is obtained which gives the relative cost of beams of constant breadth and moment of resistance for varying percentages of tensile reinforcement. This curve shows that with concrete and steel at the relative prices named, the most economical percentage of reinforcement is ·6 per cent. or that which permits the working stresses in the concrete and steel to be reached simultaneously.

Railway engineers have generally on hand a stock of second-hand rails, and provided the design permits of steel in this form being used, the price of the reinforcement becomes very much less. It is estimated that old steel rails, as taken out of sidings, could be laid in the work at an outside cost of £5 10*s.* per ton, which, on a volume basis, is twenty-three times the cost of concrete at 28*s.* per cube yard. To suit these conditions the ordinates of the relative depth curve must be increased by twenty-three times the corresponding percentage of reinforcement, and the result indicates the most economical proportion of reinforcement in this case to be about ·9 per cent., while considerably larger percentages may be used without any great loss of economy. As, in many cases, a high percentage of reinforcement has to be resorted to, in order to develop the necessary moment of resistance within the depth available for construction, it is possible that considerable use might be made of old rails for reinforcing concrete, and some designs of structures reinforced in this way will be considered in subsequent chapters.

Double Reinforcement.

Generally speaking, a high percentage of tensile reinforcement only is not an economical arrangement, and when a comparatively high moment of resistance is required in a floor, of which the depth is limited, it is in nearly all cases more economical to place some of the steel in the compression part of the beam as shown in Fig. 13. This has

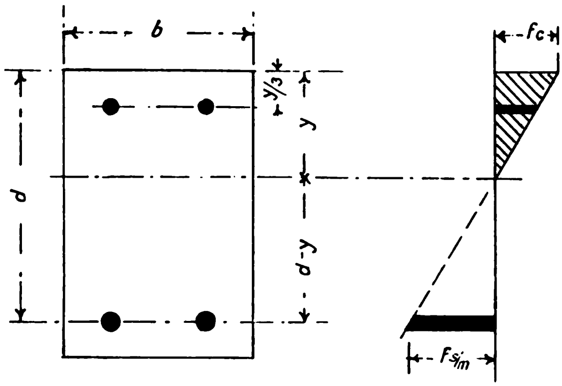


FIG. 13.—Beam with Double Reinforcement.

the effect of raising the neutral axis and so increasing the tensile stress in the steel relatively to the maximum compressive stress in the concrete. The calculations are simplified if the compression reinforcement is placed symmetrically about the line of action of the resultant of the compressive stresses in the concrete, that is, at one-third of the distance of the neutral axis from the compression surface of the beam. The compressive stress in this steel is then equal to $\frac{2}{3}\ m\ f_c$, and if p' is the proportion of the compressive reinforcement the equation of the compressive and tensile stresses becomes

$$\tfrac{1}{2}\ b\ y\ f_c + \tfrac{2}{3}\ p'\ b\ d\ m\ f_c = p\ b\ d\ f_s$$

or,
$$\frac{f_s}{f_c} = \frac{y}{2\ p\ d} + \frac{2\ m\ p'}{3p} \quad . \quad . \quad . \quad (5)$$

As in the case of tensile reinforcement

$$\frac{f_s}{f_c} \text{ also equals } \frac{m\left(1 - \frac{y}{d}\right)}{\frac{y}{d}} \text{ and by}$$

equating these two values of this ratio the position of the neutral axis is fixed thus—

$$\frac{y}{d} = \sqrt{m^2\,(p + \tfrac{2}{3}\,p')^2 + 2\,m\,p} - m\,(p + \tfrac{2}{3}\,p') \quad . \quad (6)$$

EXAMPLE 5.—Find the position of the neutral axis and the ratio of the tensile stress due to bending to the maximum compression stress due to bending in a square column 14 in. × 14 in. when subjected to bending action parallel to two of the sides. The column is reinforced with $\frac{7}{8}$ths inch diameter rods, the centre of each of which is $1\frac{3}{4}$ in. from each of the two nearest sides.

Breadth, 14 in.

Effective depth, $12\frac{1}{4}$ in.

Proportion of tension reinforcement, $\dfrac{2 \times \cdot 6}{14 \times 12\frac{1}{4}} = \cdot 007.$

Proportion of compression reinforcement $= \cdot 007.$

$$y/d = \sqrt{15^2 (\cdot 007 + \tfrac{2}{3} \times \cdot 007)^2 + 2 \times 15 \times \cdot 007} - 15 (\cdot 007 + \tfrac{2}{3} \times \cdot 007) = \cdot 31.$$

$$y = \cdot 31 \times 12\tfrac{1}{4} = 3\cdot 8 \text{ in.}$$

$$\frac{f_s}{f_c} = \frac{\cdot 31}{\cdot 014} + \frac{2 \times \cdot 007 \times 15}{3 \times \cdot 007} = 32\cdot 5.$$

NOTE.—*According to the formula the compression reinforcement is assumed to be about $1\frac{1}{4}$ in. from the compression surface. As it is actually $1\frac{3}{4}$ in. from the surface a certain amount of error is introduced.*

The moment of resistance of the beam in terms of the compressive stresses is

$$M.R. = \left(\tfrac{1}{2}\, b\, d^2 \frac{y}{d} f_c + \tfrac{2}{3}\, p'\, m\, b\, d^2 f_c\right)\left(1 - \frac{y}{3\, d}\right) \quad . \; . \; . \; (7)$$

Practically speaking, the maximum moment of resistance is obtained with any given total percentage of reinforcement when this is disposed between the tension and compression parts of the beam in the proportion which secures that the maximum compressive stress in the concrete and the tensile stress in the steel shall in each case be equal to the working stress. It is these maximum values of the moment of resistance with which one is chiefly concerned, and the relative proportions of tensile and compressive reinforcement which must be adopted in order to obtain them, and also their values are indicated in Fig. 14. The values of the maximum moments are given as coefficients in the formula

$$M.R. = k\, b\, d^2 f_c,$$

and for purposes of comparison the values of k for the same total percentage of reinforcement placed all in the tension part of the beam are plotted on the same diagram by a dotted line.

The total percentage of reinforcement is read on the lower horizontal scale, and the percentage of reinforcement which must be placed in the compression part of the beam to secure

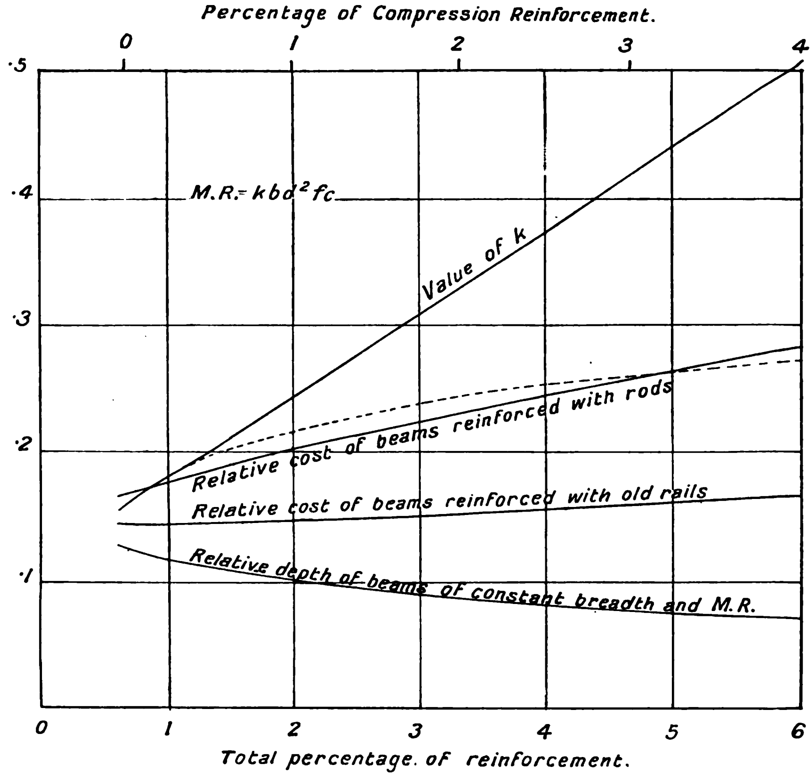


FIG. 14.—Curve of Constants, Double Reinforcement.

the maximum moment of resistance is indicated by the scale at the top of the diagram. It will be noticed, that for the higher percentages of reinforcement the maximum values of k greatly exceed the values obtained with tensile reinforcement only; also that the higher curve is for all practical purposes a straight line. The value of the maximum moment of resistance obtainable with a given total percentage of reinforcement

P may *therefore* be represented by the empirical relation

$$M.R. = (\cdot 11 + \cdot 066\,P)\,b\,d^2 f_c \quad . \quad . \quad . \quad . \quad . \quad (8)$$

since the straight line referred to, when produced, cuts the vertical axis of co-ordinates at a value of $k = \cdot 11$, and the tangent of the angle the straight line makes with the horizontal axis of co-ordinates $= \cdot 066$.

EXAMPLE 6.—A beam carrying an external wall and a floor has a span of 11 ft. and the total load is 16,000 lbs. The breadth is fixed at 9 in. and the effective depth at 15 in. Using the diagram, Fig. 14, determine the areas of tension and compression reinforcement required in order that the maximum stresses shall be 500 lbs. per square inch compression for concrete and 14,500 lbs. per square inch tension for steel.

$$\text{Maximum bending moment} = \frac{16{,}000 \times 132}{8} = 264{,}000 \text{ in. lbs.}$$

$$\text{Moment of resistance} = k \times 9 \times 15^2 \times 500.$$

$$k = \cdot 26.$$

Then from Fig. 14.

Tension reinforcement, 1 per cent. of $9 \times 15 = 1 \cdot 35$ sq. in.
Compression reinforcement, $1\frac{1}{4}$ per cent. of $9 \times 15 = 1 \cdot 69$ sq. in.

The compressive stress in the steel placed near the top of an ordinary beam will never exceed under the conditions assumed $\frac{2}{3} \times 15 \times 500 = 5{,}000$ lbs. per square inch. It is, therefore, rather remarkable that with 6 per cent., for example, of total reinforcement, 4 per cent. should have to be placed as compressive reinforcement working at this comparatively low stress in order to obtain the maximum possible moment of resistance. But by doing this the stress in the remaining 2 per cent. of reinforcement, which is left in the tension member of the beam, is raised from 2,950 lbs. per square inch to 14,500 lbs. per square inch, and by the raising of the neutral axis the arm of the resisting couple has been increased from $\cdot 76\,d$ to $\cdot 89\,d$. *The figures applying to such a high percentage of reinforcement as this must be accepted with some reserve because the theory has been built up on the results of tests carried out upon beams reinforced with a much smaller proportion of steel.*

In certain cases such as columns, arches, &c., in which either surface of the member may become in tension when subjected to bending action, the proportion of tensile reinforcement is equal to the proportion of compression reinforcement. To meet this condition, values determining the position of the neutral axis, the ratios of the stresses, and the moment of

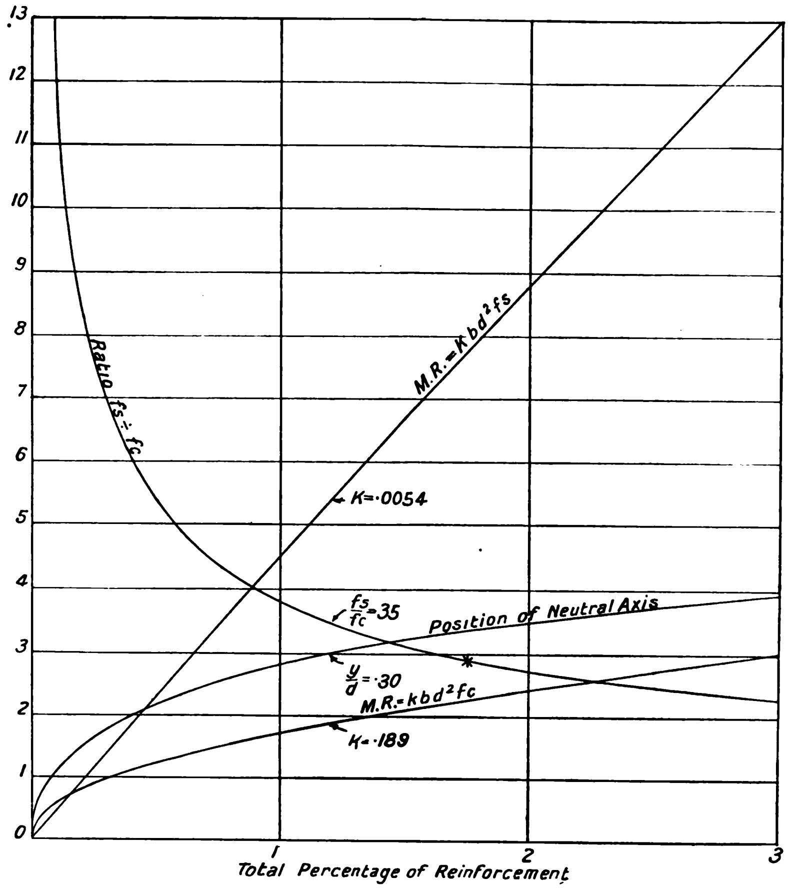


FIG. 15.—Equal Areas of Tension and Compression Reinforcement.

resistance in terms of the working stress for the steel as well as for the concrete, when there are equal percentages of tension and compression reinforcement, are plotted in Fig. 15.

The relative depth of beams with double reinforcement and constant breadth and moment of resistance is shown by the curve so marked in Fig. 14. From this the curves of relative cost of beams, reinforced with various total percentages of steel placed in the most economical way, have been obtained in the same way as in Fig. 11, both for ordinary steel-rods and also for old rails. These curves indicate that shallow floors reinforced with a high percentage of old rails can be used without any great loss of economy, and the most economical percentage would appear to be about $\frac{7}{8}$ths per cent. Also that a shallow beam reinforced with 6 per cent. of old rails costs no more than a deeper beam reinforced with ·6 per cent. of ordinary steel-rods.

Tee Beams.

In practice the beams most commonly occurring are of T section, as shown in Fig. 4, part of the slab floor supported by the beams being included in the compression member of the beam. Except in the case of a very deep beam supporting a thin slab, a tee beam may be considered as an ordinary beam in which a portion of the unnecessary concrete in the tension part has been omitted. In the particular case mentioned the neutral axis falls considerably below the underside of the slab, and this case requires special treatment. It is quite evident that in the case of tee beams it is still more economical to use a deeper floor sparingly reinforced, because as the depth of the beam increases, the area of concrete saved between the beams and under the slab floor becomes a greater percentage of the whole area. Also in the deeper beam with a small percentage of reinforcement the concrete is relatively the more expensive part of the composite structure and a reduction of this part naturally results in increased economy.

The author has selected at random half a dozen floors constructed with tee beams and calculated the area of the concrete saved below the slabs and between the beams, as a percentage of the whole area. The average saving in these six cases works out at 60 per cent., the lowest value being 54 per cent. and the highest value 64 per cent. In Fig. 11 the relative cost of a rectangular beam reinforced with ·6 per cent. of steel in the form of rods is shown to be ·166 of which ·128 represents the cost of the concrete and ·038 the cost of the steel. When the percentage of tensile reinforcement is increased to 3 per cent., for instance, the relative cost is shown to be increased to ·255, that is by 54 per cent., and of this ·255, ·102 represents the cost of the concrete and ·153 the cost of the steel. In the case of a tee beam with ·6 per cent. of reinforcement the relative cost is represented approximately by

$$\frac{40}{100} \times \cdot 128 + \cdot 038 = \cdot 089$$

i.e., cost of concrete, less 60 per cent. saving + cost of steel; and when the percentage of reinforcement is increased to 3 per cent. the relative cost is increased to

$$\frac{40}{100} \times \cdot 102 + \cdot 153 = \cdot 194$$

that is by 118 per cent. This percentage would in practice be somewhat greater, because in arriving at the cost of the sparingly reinforced beam the proportion of concrete saved should have been taken at rather more than 60 per cent., while in the case of the larger percentage of reinforcement the proportion of concrete saved by tee beam construction should have been estimated at rather less than 60 per cent.

Similarly in the case of a rectangular beam reinforced with ·6 per cent. of old rails, the relative cost is shown in Fig. 11 to be ·146 as compared with ·152 which is the relative cost of a beam of the same breadth and moment of resistance, when reinforced in tension and compression with 3 per cent. of old rails. The increase in the cost in this case is only 4 per cent. but in the case of tee beams similarly reinforced the approximate relative cost of the deeper beam, sparingly reinforced, is $\left(\frac{40}{100} \times \cdot 128 + \cdot 018\right)$ ·069 and of the shallower beam with 3 per cent. of total reinforcement $\left(\frac{40}{100} \times \cdot 090 + \cdot 062\right)$ ·098 showing an increase in cost amounting to 42 per cent.

It is quite evident then that with tee beams a small percentage of reinforcement provides by far the most economical construction, and larger percentages should only be used when the required moment of resistance cannot otherwise be obtained within the depth available for the construction of the floor.

CHAPTER III

SHEAR STRESS

FORMULÆ.

THE first cracks to appear in reinforced concrete test beams are nearly always due to diagonal tension, resulting from shear stress, because local slipping of the reinforcement causes the tendency to shear to concentrate at certain points. The vibration to which so many railway structures are subjected increases this tendency for local slipping to occur, and it is, therefore, very essential that such structures should be adequately reinforced to resist shear stress. Numerous tests have been carried out, some with the object of investigating the nature of these shear stresses, others with the object of demonstrating the virtue of particular forms of shear reinforcement.

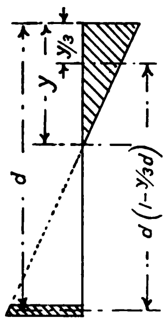


FIG. 16.—Diagram of Couple Resisting Bending.

In the previous chapter expressions were obtained for the total compressive stress, $\frac{1}{2}byf_c$, and the tensile stress, $pbdf_s$, constituting the equal forces providing the moment of the couple resisting bending, in a beam reinforced in tension only. The length of the arm of the couple was also determined and found to be $(d - y/3)$ or $d\left(1 - \frac{y}{3d}\right)$ see Fig. 16. The moment of resistance at any section can be written

$$M = p\,b\,d\,f_s\,d\left(1 - \frac{y}{3\,d}\right) \quad . \quad . \quad . \quad (3)$$

and at a section a unit distance away

$$M' = p\,b\,d\,f_s'\,d\left(1 - \frac{y}{3\,d}\right).$$

Inches and pounds are the units conventionally used in reinforced concrete calculations, and as one inch will in general

D 2

be a very small part of the whole span, the shear throughout this length may be taken as constant, and equal to the difference between the bending moments at the sections at either end of it, since such difference represents the rate of change of the bending moment at the section considered, which by definition is equal to the total shear. The total shear F, therefore $= M - M_1 = p\,b\,d\,(f_s - f_s')\,d\left(1 - \frac{y}{3\,d}\right)$ and $p\,b\,d$ $(f_s - f_s')$ is equal to the bond stress, or the resistance offered by the rods to slipping through the concrete, per unit of length. If the intensity of this stress per square inch is represented by u, and there are n rods each having a circumference πt,

$$p\,b\,d\,(f_s - f_s') = n\,\pi\,t\,u$$

and $$F = d\,(1 - y/3\,d)\,n\,\pi\,t\,u,$$

or $$u = \frac{F}{d\,(1 - y/3\,d)\,n\,\pi\,t} \quad . \quad . \quad . \quad (9)$$

This equation provides a means of proportioning the size and the number of the rods so as to keep u sufficiently small to prevent the possibility of the rods slipping through the concrete.

EXAMPLE 7.—The maximum shear on a reinforced concrete platform flag 3 ft. wide is 2,530 lbs. The reinforcement amounts to ·6 per cent. of the effective area and is $4\frac{3}{8}$ in. below the top surface of the flag. The tendency of the rods to slip through the concrete must not exceed 50 lbs. per square inch. Find the maximum diameter of the rods and their distance apart.

From (9) $$50 = \frac{2{,}530}{4\frac{3}{8}\left(1 - \frac{\cdot 33}{3}\right)n\,\pi\,t}$$

$\therefore$ $$n\,\pi\,t = 13.$$

Also $$\frac{n\,\pi\,t^2}{4} = \cdot 006 \times 36 \times 4\tfrac{3}{8},$$

$\therefore$ $$n\,\pi\,t^2 = 3{\cdot}78$$

$$t = \frac{3{\cdot}78}{13} = \cdot 29 \text{ in., say No. 1 gauge, area}$$

·07 sq. in., 13 of these would be required to make up the area, and the spacing is therefore $\frac{36}{13}$ or $2\frac{3}{4}$ in.

The shear stress in the concrete immediately above the rods must also be equal to $pbd\ (f_s - f_s')$ and if the intensity of this stress, which is spread over an area $b \times 1$, is represented by s

$$F = b\ s\ d\ \left(1 - \frac{y}{3\ d}\right).$$

If the tensile resistance of the concrete is neglected, as it has been in arriving at the preceding formulæ, the shear stress in the concrete between the steel reinforcement and the neutral axis must be constant and will then diminish in value from the neutral axis upwards and become zero at the compression edge of the beam. The symbol s, therefore, represents the value of the shear stress throughout a considerable part of a section of the beam, and the design of the web reinforcement is generally based upon this value of s as determined by the equation

$$s = \frac{F}{b\ d\ \left(1 - \frac{y}{3\ d}\right)} \quad . \quad . \quad . \quad (10)$$

Example 8.—Find the maximum intensity of shear stress in the lintel described in Example 3.

$$s = \frac{5{,}600}{14 \times 10{\cdot}5\ \left(1 - \frac{{\cdot}40}{3}\right)}$$

$$= 44 \text{ lbs. per square inch.}$$

$\cdot 40$ is the value of y/d corresponding to $\cdot 9$ per cent. in Fig. 17.

$d\ \left(1 - \frac{y}{3\ d}\right)$ is the distance between the centre of the reinforcement and the resultant of the compression stresses and this distance depends only upon the percentage of reinforcement. The values of y/d determining the position of the neutral axis corresponding to various percentages of tensile reinforcement are plotted in diagram form in Fig. 17 and the values of $\left(1 - \frac{y}{3\ d}\right)$ are plotted on the same diagram. It is these values which enter into the calculations of the intensity of shear stress.

Ultimate Shear Stress.

The question of web reinforcement has been very completely investigated by Prof. Talbot, who carried out an extensive

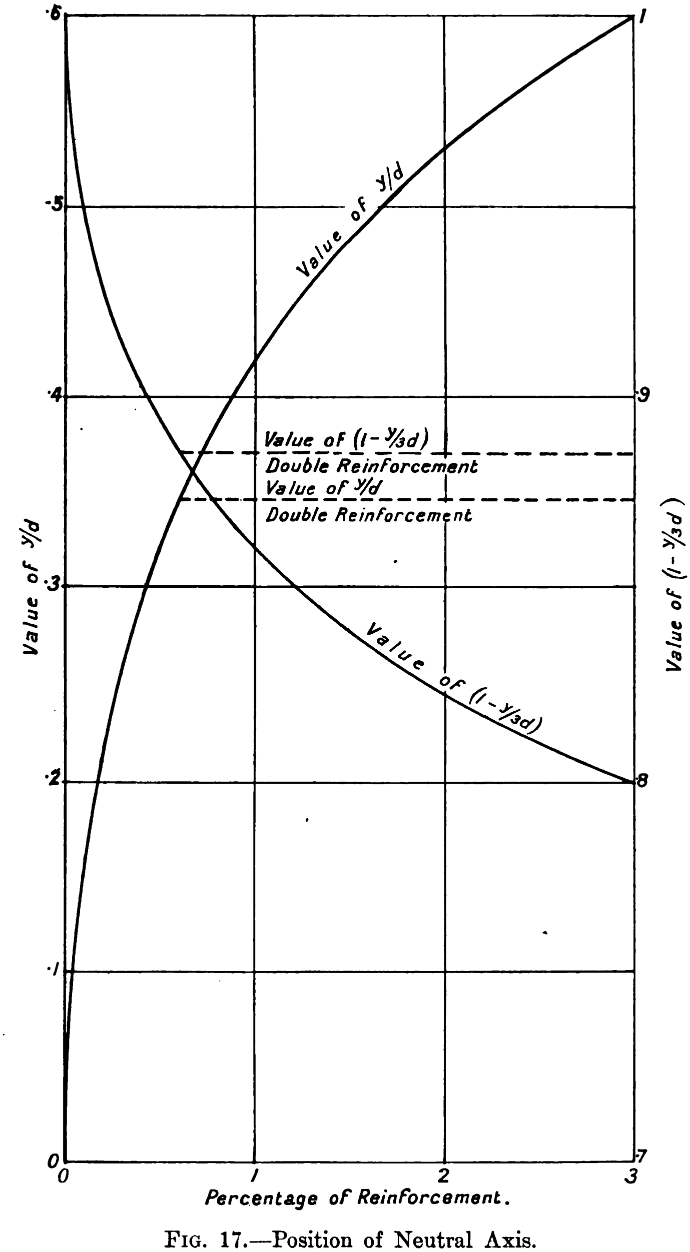


FIG. 17.—Position of Neutral Axis.

series of tests on beams, without web reinforcement and with various forms of web reinforcement, at the Engineering Experiment Station in connection with the University of Illinois, U.S.A., during the years 1907 and 1908. The experiments covered a very wide range and were planned to give information of a varied character. Possibly concrete mixed in the proportions of one part cement, two parts sand and four parts of aggregate represents the quality in most common use and a number of beams made with concrete of this strength and reinforced with from 1 per cent. to 1·5 per cent. of straight rods were tested, generally after 60 days. In about 40 tests the value of *s* calculated for the breaking load varied from 100 lbs. per square inch to 200 lbs. per square inch. The beams were loaded at the one third points and the span was in some cases 6 ft. and in other cases 12 ft. The average value works out at about 150 lbs. per square inch and this is equivalent to about 125 lbs. per square inch at 28 days. *It would appear, therefore, that the working value of the shear stress for* 1 : 2 : 4 *concrete should not be taken at a higher value than* 30 *lbs. per square inch,* although the recommendations of various authorities favour a value as high as 60 lbs. per square inch. The ultimate shear stress developed in the 12 ft. spans was generally speaking a good deal less than that developed in the 6 ft. spans, all the beams being of the same section, namely, 8 in. wide and 10 in. deep from the compression surface of the concrete to the centre of the reinforcement. The beams containing the higher percentage of reinforcement gave better results than those with the lower percentage, and as in practice the proportion of steel would often be even less than that provided in these test beams, *a value of* 30 *lbs. per square inch would certainly appear to be the maximum shearing stress that should be permitted in beams containing no web reinforcement.*

It may be that the method of concentrating the load at two points is not conducive to good results in tests designed to determine shear resistance and better results would possibly be obtained with a distributed load. As most laboratory tests however are carried out with this third point loading it is difficult to ascertain whether higher values of shear resistance are likely to be developed under the condition of uniformly distributed loading generally occurring in practice.

In some tests carried out by Messrs. David Kirkaldy and Son for the purpose of comparing beams reinforced with expanded steel-bars with others reinforced with straight plain bars only, the latter failed by diagonal shear at loads giving an average value of about 100 lbs. per square inch for the maximum shear stress. Similar beams reinforced with expanded steel-bars failed by direct tension at the middle of the beam. Prof. Talbot points out that failure by diagonal tension in beams without web reinforcement generally occurs suddenly and without warning, and in consequence of this, emphasises the importance of using effective web reinforcement in all cases in which the shear stress cannot be kept at a low value.

Depth of Beams.

In ordinary beams carrying a uniformly distributed load and designed for definite working stresses in the concrete and in the steel, the intensity of the shear stress is simply a function of the ratio of the depth of the beam to the span. For instance, in the case of a beam supported at the ends and carrying a uniformly distributed load of w lbs. per lineal inch and reinforced with ·6 per cent. of tensile reinforcement in the shape of straight rods, the intensity of shear stress

$$s = \frac{\frac{1}{2} w l}{\cdot 88\, b\, d} = \frac{\cdot 57\, w\, l}{b\, d},$$

·88 being the value of $\left(1 - \frac{y}{3d}\right)$ corresponding to ·6 per cent. in Fig. 17.

The maximum bending moment at the *centre* of the span $= \frac{w\, l^2}{8}$ and the moment of resistance $= \cdot 15\, b\, d^2 f_c$, the figure ·15 being obtained from Fig. 11.

Equating the bending moment and the moment of resistance

$$\frac{w\, l^2}{8} = \cdot 15\, b\, d^2 f_c,$$

and ∴ $$\frac{w\,l}{b\,d} = \frac{1{\cdot}2\,d\,f_c}{l},$$

or $$\frac{{\cdot}57\,w\,l}{b\,d} = \frac{{\cdot}68\,d\,f_c}{l},$$

and this is equal to s. If s is taken at 30 lbs. per square inch and f_c at 500 lbs. per square inch

$$30 = \frac{{\cdot}68\,d \times 500}{l}$$

$$\text{and} \quad \frac{d}{l} = \frac{1}{11} \text{ about.}$$

Therefore, in ordinary beams uniformly loaded, the shear stress will not exceed 30 lbs. per square inch *if the depth from the top of the beam or slab to the centre of the reinforcement is less than one eleventh of the span*, and ordinary beams or slabs which fulfil this condition need not be provided with stirrups or shear members.

Reinforced concrete beams are very generally continuous over the support and the usual practice in designing them is to estimate the maximum bending moment as equal to $\frac{w\,l^2}{10}$ or even $\frac{w\,l^2}{12}$. In the former case the shear stress will not exceed 30 lbs. per square inch unless *the beam is deeper than one-fourteenth of the span*. At first sight it would appear that the deeper beam would be stronger in shear, but the necessity for a beam twice as deep, for instance, for a particular span, implies a load two squared or four times as great, and the intensity of the shear stress will therefore be twice as great.

Beams deeper than the limits referred to, therefore, will always need to be reinforced with stirrups or bent up rods or other form of web reinforcement, in order that the beam may be as strong to resist shearing action as it is to resist bending.

Shear Reinforcement.

Turning up some of the rods towards the ends of the beam, as shown in Fig. 18, provides a simple and effective means of reinforcing the web, and at the same time brings some of the reinforcement into the top plane of the beam at the ends, where, by reason of continuity or complete or partial fixture, tension stresses may be developed. For slabs this method is very generally adopted more perhaps on account of preserving continuity over the supports than of increasing the resistance to shear, because as already stated this last is in the case of shallow slabs generally amply provided by the concrete itself.

It is rather remarkable that in the tests carried out at the Illinois University Experiment Station the beams in which some of the rods *were* turned up failed on the average when the calculated maximum shearing stress was only about 220 lbs. per square inch and these beams moreover, were tested after 60 days and in general had a larger percentage of reinforcement than is common in practice. In some of these tests the ends of the rods turned up were provided with screw threads, washer plates and nuts to anchor the rods firmly into the concrete, and most of the beams treated in this manner failed by tension at the middle and not by diagonal shear. The practice of splitting the ends of rods, or bending the ends to a right angle, which is very commonly adopted, is likely therefore to increase the value of shear reinforcement treated in this way. The beams in which rods were turned up at more than one point gave better results than those in which all the shear reinforcement was concentrated in one diagonal plane, and this seems to point to the value of a good distribution of the web reinforcement.

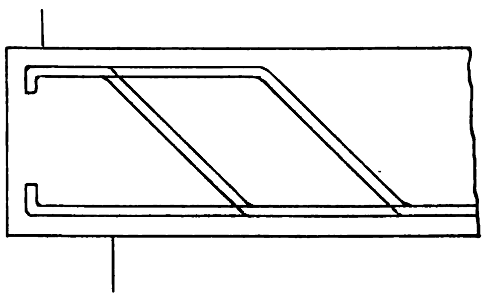


FIG. 18.—Shear Reinforcement.
Bent up Bars.

The values of the shear resistance obtained with beams reinforced with straight rods and stirrups averaged about the same as those with turned up rods, namely, 220 lbs. per square inch for 1 : 2 : 4 concrete after 60 days. After 28 days this would only be equal to about 180 lbs. per square inch and *these figures indicate* *very forcibly the importance of keeping the estimated shear stresses as low as possible*, when designing reinforced concrete beams, and providing where necessary adequate web reinforcement.

Prof. Talbot concludes from his experiments that very little stress is developed in vertical stirrups until a diagonal crack has formed. These members therefore, when used in practice, are in the nature of a "lay by" or safeguard, and would only appear to come into operation if the concrete is not able to resist the shear stress unaided. It is very desirable that the concrete should not crack, and, as in test beams with rods bent up the loads producing the first discernible crack were considerably higher than in those with vertical stirrups, this first method of reinforcing the web has a considerable advantage. This fact has been recognised in the construction of many patent stirrups which are made in such a way that when fitted on to the main reinforcing rods the loose ends lie in diagonal planes of the beam.

Design.

As it is so uncertain what proportion of the stress is taken by bent up rods or stirrups, any method of calculating the size and spacing of these members can only be regarded as an arbitrary means of comparing one case with another. One method is to proportion the section of the steel to take the whole of the shear stress; another method is to provide sufficient steel to resist the surplus shear over and above that which the concrete is safely capable of resisting.

The recommendations contained in the second report of the Joint Committee on Reinforced Concrete, appointed by the Royal Institution of British Architects, are in this sense, and a value of 60 lbs. per square inch is assumed for the working resistance of the concrete to shear; but a note is added to the effect that in important cases when extra strength is required the resistance of the concrete to shear should be disregarded.

The maximum intensity of the shear stress on a vertical plane has been determined as

$$s = \frac{F}{b\,d\left(1 - \dfrac{y}{3\,d}\right)} \quad . \quad . \quad . \quad (10)$$

and this value, therefore, also represents the maximum intensity of shear stress on any horizontal plane in the lower part of the beam at the section where the total shear is equal to F. The shear stress per lineal inch therefore equals

$$b\,s = \frac{F}{d\left(1 - \frac{y}{3\,d}\right)} = \frac{F}{j\,d}$$

the value of j being given in Fig. 17. If vertical stirrups (Fig. 19),

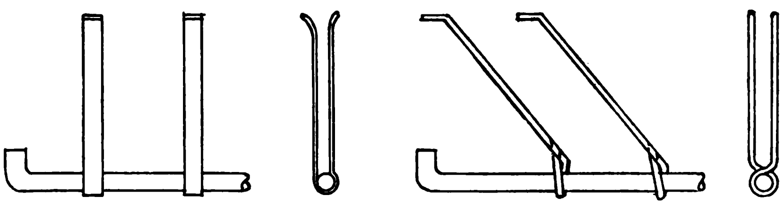


FIG. 19.—Shear Reinforcement. Stirrups.

each having a total cross sectional area a, and spaced x in. apart at this point, are to be proportioned to resist the whole of this shear stress; the unit shear stress in the steel will equal

$$s' = \frac{F\,x}{a\,j\,d}$$

or,
$$x = \frac{s'\,a\,j\,d}{F} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The value of s' commonly used is 75 per cent. of the working tensile stress adopted for the main steel reinforcement.

EXAMPLE 9.—Fig. 20 shows the cross section of the floor of a coal gantry. The span of the cross girders is 28 ft., they are 8 ft. apart,

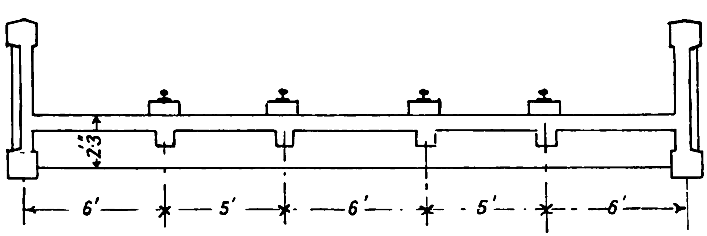


FIG. 20.—Floor of Coal Gantry.

12 in. wide, 2 ft. 3 in. deep overall, and there is $2\frac{1}{2}$ in. of concrete below the centre of the reinforcement. The rail bearers divide the

length of the cross girders into five bays. In the end bays some of the shear is resisted by the main bars turned up, but in the bays under each "four foot" the only shear reinforcement consists of stirrups of $\frac{5}{16}$ in. diameter. Assuming the reinforcement to resist the whole of shear stress determine the spacing of the stirrups. Working shear stress for steel 11,000 lbs. per square inch. Live load 1 ton per lineal foot on each track. Dead load 1 cwt. per square foot. Proportion of longitudinal reinforcement ·6 per cent. Shear at section, 5 ft. 6 ins. from centre.

$$\begin{aligned} \text{Live load, } 8 \times \quad 1{,}120 &= 8{,}960 \text{ lbs.} \\ \text{Dead load, } 8 \times 5{\cdot}5 \times 112 &= 4{,}928 \text{ lbs.} \\ &\overline{13{,}888 \text{ lbs.}} \end{aligned}$$

$$\text{Area of } \tfrac{5}{16} \text{ inch diameter rod} = {\cdot}077 \text{ sq. in.}$$

From equation (11)

$$x = \frac{11{,}000 \times 2 \times {\cdot}077 \times {\cdot}89 \times 24{\cdot}5}{13{,}888},$$

say $2\frac{3}{4}$ in.

The stirrups can be arranged in pairs or groups of three on the various longitudinal rods, as convenient, so as to leave about 6 in. or about 9 in. spaces for pouring in the concrete.

It is recommended in the report previously referred to, that when shear members are inclined at an angle of about 45 degs. to the horizontal, the cross sectional area may be decreased in the proportion of $\frac{1}{\sqrt{2}}$. There is in common use a simple graphic method of determining the theoretical spacing of shear members of constant cross section throughout the length of a beam when once the spacing of these members at the ends of the beam, where the shear stress reaches a maximum, has been fixed. It is based on the rule defining shear as the rate of change of bending moment, and worked out from the

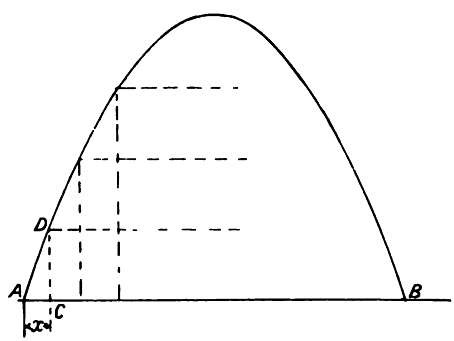


FIG. 21.—Spacing of Stirrups.

bending moment diagram, see Fig. 21. The spacing x for the end stirrups is marked off at the ends of the span in the bending moment diagram, and a vertical line CD erected. CD represents the change in bending moment throughout the length x, and the total working shear stress for one stirrup or web member. Other horizontal lines are drawn at equal distances CD from one another and verticals are drawn through the intersections of these lines with the bending moment curve. These verticals determine the theoretically correct spacing for stirrups or shear members of constant section. Strictly speaking these should be placed at the centre of the spaces thus determined. This method is of course only applicable to a beam carrying a fixed load. In the case of beams subject to a moving load the condition of loading producing the maximum shear at any section will need to be considered.

PRINCIPAL STRESSES.

The experiments referred to in Chapter II. indicated that at the centre of a beam the tensile resistance of the concrete, corresponding to working loads, is on the point of breaking down and it is therefore neglected. At the ends of a beam, however, the stresses due to bending are generally small and under these conditions the concrete is undoubtedly in tension.

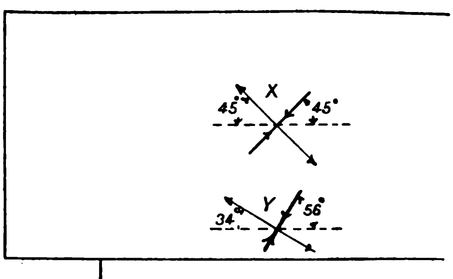


FIG. 22.—Principal Stresses.

In parts of a beam subject only to a vertical and horizontal shear stress, say of 60 lbs. per square inch, the resultant principal stresses, tension and compression, are each of intensity equal to 60 lbs. per square inch and act in directions making angles of 45 per cent. with the axis of the beam as shown at x in Fig. 22. If in addition to this shear stress there is also tension acting in a horizontal direction of intensity equal to 80 lbs. per square inch the resultant principal stresses are given by the roots of the equation

$$q\,(q - 80) = 60^2.$$

That is, the resultant tension is 112 lbs. per square inch, and the resultant compression 32 lbs. per square inch, and the directions which are shown at *Y* in Fig. 22 are determined by the relation:—

$$\tan 2\,\tau = \frac{2 \times 60}{80}.$$

The two values of τ obtained from this equation determine the directions of the resultant stresses relatively to the axis of the beam. These stresses are always at right angles to one another, and the smaller angle applies in this case to the resultant tensile stress.

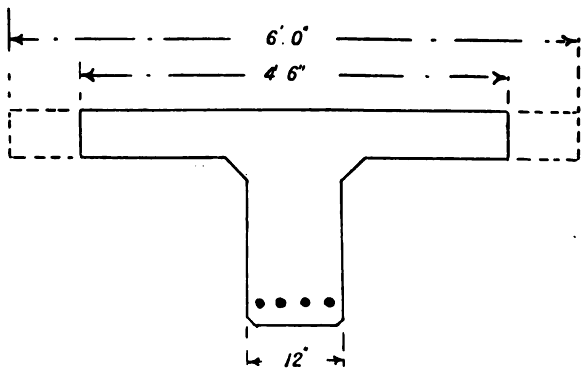


FIG. 23.—Tee Beam.

By working out the values and directions of these principal stresses at several points in a beam or similar structure, a good idea is obtained of where high tension stresses are likely to occur, and reinforcement should always be provided in a suitable direction to augment the resistance of the concrete at such points. It is claimed for the Hennebique system of reinforced concrete construction that out of more than a thousand completed buildings and other structures in the United Kingdom there has not been one failure, and it is probable that this success is largely due to the numerous stirrups and secondary members employed, apart from the main reinforcement.

TEE BEAMS.

Tee beams will in most cases require shear reinforcement because the area of the concrete resisting shear is considerably restricted. A typical tee beam which was designed for a 27 ft. span, is shown in Fig. 23. The recommendations contained in the R.I.B.A. reports with regard to the calculation of tee beams are that the width of the slab floor, considered as forming the compression member of the beam, should not exceed one-third of the span, three-quarters of the distance between the centres of the beams, or five times the width of the lower part of the beam. The second criterion will determine the theoretical width of the beam shown in Fig. 23 as 4 ft. 6 in., and the resistance to compression is assumed to be uniformly distributed over this width. The neutral axis is estimated to be well below the under side of the slab and as the compression in the small area below the slab and above the neutral axis is usually neglected, the intensity of shear stress in the lower part of the beam may be regarded as constant. Immediately above the under side of the slab the intensity of shear stress diminishes in the ratio of 1 : 4·5 and between this point and the top of the beam further diminishes to zero according to the ordinates of a parabola as shown in Fig. 24. The sudden change in stress at the junction of the slab and the beam is avoided to a certain extent if the sharp angles are rounded or splayed off as shown in Fig. 23. The stirrups or other form of web reinforcement provided to meet the high shear stress in the lower part of the beam must obviously be carried well up into the slab in order to effect any useful purpose, because the maximum shear stress is maintained right up to the junction of the beam with the slab.

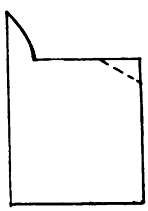


FIG. 24. Distribution of Shear Stress.

SPECIAL SYSTEMS.

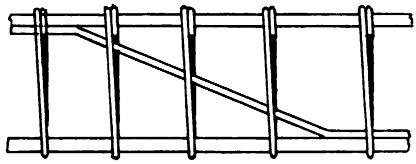


FIG. 25. Bent Bars and Stirrups.

In some systems the shear members form part of the mainbars, as in the Kahn bar and the expanded steel-bar Fawcett's Mon'lithcrete girders, illustrated in Fig. 3 really consist of tension, compression and shear reinforcement all in one. In the Coignet system vertical stirrups are wrapped round the tension and compression main rods, and in addition smaller rods are bent up as in Fig. 25, and the reinforcement thus forms a complete frame. One of the greatest difficulties in designing the shear reinforcement of beams, and the same difficulty applies to the hooping of columns, is to avoid the possibility of infringing several patents. Numbers of these have been granted in this country, and it would appear to be difficult to bend a piece of wire without infringing one or other of them. Possibly the best way out of the difficulty is with good grace to purchase these members from one or other of the firms supplying them, taking care that the price paid covers the necessary licence to use the stirrups or hoops without liability of infringing any master patent.

Bond Stress.

Closely allied to shear stress is the bond stress between the reinforcement and the surrounding concrete. An expression was obtained for the intensity of this stress on p. 36. It may be noticed that in Considère's experiment described on p. 20, care was taken to eliminate shear and obtain a condition of pure bending. That is, the tension produced in the steel was constant throughout the length of the test-piece, and under these conditions it was possible to stress the steel up to 20 tons per square inch without causing cracks in the surrounding concrete. In the ordinary test beam and in practice, however, the stress in the steel varies, and under these conditions it is found that cracks occur in the concrete when the stress in the embedded steel is very much less than 20 tons per square inch. The variation in the total stress in the steel between any two sections is balanced by the bond stress spread over the circumference of the rods between these sections, and as the stress due to bending generally varies most rapidly at the ends of a beam, it is at these points that the bond stress attains a maximum value.

Reference has been made to some of the beams tested at the Illinois University Experiment Station in which the ends of the inclined rods were screwed and provided with washer plates and nuts to prevent the rods pulling out of the concrete. The beams so treated failed by direct tension at the centre and not by giving way at the ends, and the inference is that the cracks, which occurred at the ends of beams in which the rods were not provided with anchorages, were in great measure due to the steel slipping in the concrete near the ends where the bond stress is greatest. In the case of the beams in which the rods were anchored, slip may, and probably does, still take place locally, resulting in what is virtually a trussed beam, and the stress in the tie rods is more or less constant throughout the length of the beam. The practice of turning rods up towards the ends and fish-tailing them or bending them at the extremities has therefore much to commend it, and is likely to form an adequate safeguard against absolute failure by diagonal tension.

The ultimate adhesion between concrete and steel appears to vary very considerably and to depend very largely on the manner in which it is determined. The value of the working adhesion is sometimes taken as equal to the working shear stress in the concrete, on the assumption that some of the latter sticks to the steel in the event of failure, and therefore such failure is really due to the shearing of the concrete. In the London County Council regulations for reinforced concrete construction the bond stress is allowed to reach a maximum value of 100 lbs. per square inch, but the ends of ordinary smooth bars are required to be bent over or fish-tailed. The natural adhesion between the reinforcement and the surrounding concrete may be augmented in various ways, as, for instance, by employing special bars securing a mechanical bond such as the indented bars, Kahn rib bars, Thatcher bars, or twisted bars, or by using stirrups, which are or can be rigidly connected to the main reinforcement, as in the case of Kahn trussed bars, expanded steel bars and various patent stirrups used in conjunction with ordinary round or square rods. The bond stresses may also be kept low by using a greater number of rods of small section instead of a few large ones, because in the former case the exposed surface of the rods is greater relatively to the cross sectional area.

CHAPTER IV

FLOORS AND BUILDINGS

Scope.

The construction of floors and buildings constitutes part of the railway engineer's work in providing platforms, loading decks, wharves, high-level stations, goods warehouses, offices, &c. Reinforced concrete has been extensively used in the construction of floors for loading and storage purposes, and for buildings generally. Probably the earliest experience in this country in the use of this material on anything like a large scale was obtained in the construction of buildings for transport purposes, as, for instance, the transit sheds built for the Manchester Ship Canal Company.

In this connection the material possesses very great advantages: It is clean, does not harbour vermin, and the risk of damage by fire is very small. For many years it has been a common practice to substitute steel and jack arch, or similar flooring, for wooden-joists and floor-boards in the pursuit of these advantages, coupled with the object of eliminating maintenance. Reinforced concrete floors are lighter and can be constructed at less cost than steel and jack arch, or steel and concrete floors, and the surface can easily be laid to falls and drains provided to facilitate washing down. These properties render the material particularly suitable for the construction of warehouses, loading decks and similar structures, and considerable use has already been made of it in these connections.

Flags of York stone, artificial stone or cement concrete, not reinforced, have long been in common use for paving laid on a solid foundation or to bridge over small openings. When cement concrete or artificial stone flags are reinforced, their strength is very considerably increased, and they can be used to span much longer distances without any increase in the thickness. The construction of floors on this principle possesses very great advantages. In floors carried on jack arching, for instance, the wearing surface has still to be provided after the arches have been backed up with concrete to form a level foundation, and this construction, therefore, occupies

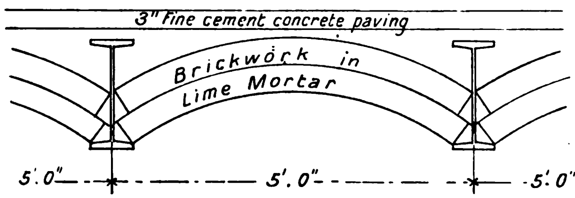


FIG. 26.—Jack Arch Floor.

a fair amount of space besides being costly. An example of this type of flooring in common use for bridge and warehouse floors is shown in Fig. 26. This floor consists of two half-brick rings of brickwork in lime mortar, and 3 in. of cement concrete paving, and, when the soffit is faced with blue brick, costs about 11*s.* a super yard exclusive of the joists. A single half-brick ring built in cement mortar, with the same thickness of paving, would cost nearly 10*s.* a super yard. A reinforced concrete flag $5\frac{1}{2}$ in. thick is sufficiently strong for ordinary loads to span about 7 ft., which is the maximum spacing over which the jack arch construction would be used, and the cost of this is about 7*s.* per super yard.

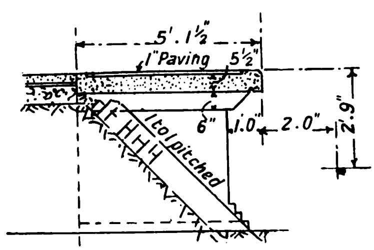


FIG. 27.—Reinforced Concrete Platform. Section.

PLATFORMS.

At this price a platform can be built more economically by constructing piers at frequent intervals and bridging over the gaps with flags, than by building the usual platform wall to retain earth filling and laying the paving on the solid foundation thus formed. Figs. 27 and 28 show in section and in elevation a platform constructed in this way, the piers being 8 ft. centres and the flags $5\frac{1}{2}$ in. thick. The standard platform construction is shown for purposes of comparison in Fig. 29. The back of the platform is constructed in the same way in either case, and the differences are confined to a width

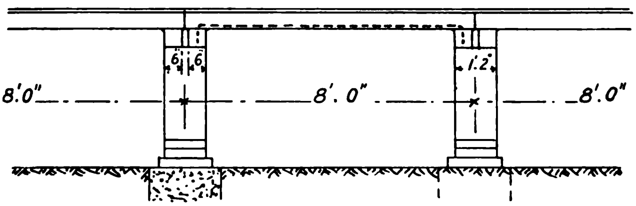


FIG. 28.—Reinforced Concrete Platform. Elevation.

of about 5 ft. measured from the platform edge. The cost of this portion complete in the case of the reinforced concrete construction was 1*l*. 10*s*. 3*d*. per lineal yard. The cost of the same width of platform constructed as shown in Fig. 29 at the same basis prices is estimated to be 2*l*. 4*s*. 2*d*. per lineal yard. The somewhat high prices are largely due to the special hard non-slip flags in the latter case, and the provision of a comparable wearing surface in the former case.

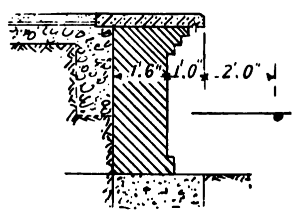


FIG. 29.—Standard Platform.

The ribs beneath the flags at the bearings are provided in order that the face of the brick piers may be set back 1 ft. from the nose of the platform as required by the Board of Trade regulations, and also serve to distribute the end reactions uniformly over the whole width of flag. The flags are liable to bear at one or two points, or even at diagonally opposite corners, and in a plain slab this would result in a considerable loss of strength.

As regards the longitudinal reinforcement a convenient width of the slab, say 1 ft., is designed as a simple beam as follows:—

Span (between centres of bearings), 7 ft. 4 in. = 88 in.
Width considered, 12 in.
Dead load, 7·3 ft. × 1 ft. × ·54 ft. × 145 lbs. = 572
Live load, 7·3 ft. × 1 ft. × 140 lbs. = 1,022

1,594 lbs.

Maximum bending moment $\dfrac{1{,}594 \times 88}{8}$

$= 17{,}500$ in. lbs.

Moment of resistance with ·6 per cent. of reinforcement (Fig. 11),

$$= \cdot 15 \times 12 \times d^2 \times 500$$
$$d^2 = 17{,}500 \div \cdot 15 \times \cdot 12 \times 500$$
$$= 19 \cdot 4$$
$$d = 4 \cdot 4 \text{ in.}$$

·6 per cent. of 12 × 4·4 = ·31 sq. in.

These slabs are reinforced with Johnson's steel-wire lattice, the cross sectional area of the longitudinal wires being ·29 sq. in. per foot of width. The specification requires that all steel shall have at least 1 in. of concrete covering, and the flags were made 5½ in. deep to achieve this. The area of the reinforcement being a trifle bare means a somewhat higher unit stress in the steel than 14,500 lbs. per square inch, but with fine wires such as those employed—the longitudinal wires are No. 10 gauge—there is no harm in this, as the bond stresses are quite small.

It is rather necessary to examine the tendency of the reinforcement to slip in the case of slabs, because as the span is generally small the maximum stress is developed in a comparatively short length.

In the platform flags illustrated in Figs. 27 and 28 the reinforcement is turned down at the ends into the ribs, and this provides further security against slipping. Even in flags supported on two sides only it is always desirable to provide transverse reinforcement. The reactions from a concentrated load at the centre of a square slab, for instance, are distributed more or less uniformly over the whole width of the bearings, and this distribution can only be secured at the expense of transverse stresses in the slab. In a slab of definite dimensions and liable to be subjected to a known concentrated load the requisite amount of transverse reinforcement can be calculated. Generally speaking, however, the transverse reinforcement is determined in quite an arbitrary manner, and in the case of the Johnson's wire lattice used to reinforce the platform flags the cross wires are No. 11 gauge and spaced 4 in. apart.

CONTINUOUS SPANS.

Slab floors are far more commonly formed *in situ* and supported by rolled steel joists or reinforced concrete beams.

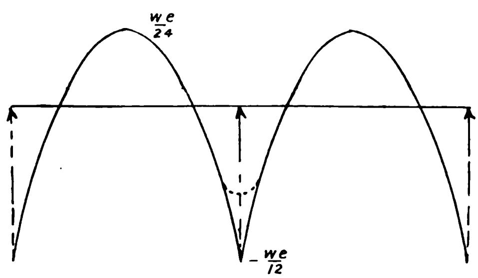


FIG. 30.—Bending Moment Diagram for Equal Continuous Spans, with Fixed Ends. Uniformly Distributed Load.

In either case the slabs are generally continuous over the supports and require to be designed accordingly.

If a number of equal continuous spans are loaded uniformly with a total load W on each span of length l, and the ends are fixed at the extreme supports, there is a positive bending moment at the centre of the spans equal to $\frac{Wl}{24}$, and a negative bending moment over the supports equal to $-\frac{Wl}{12}$, as shown in Fig. 30. If some spans are covered with live load and others unloaded, these values may be varied somewhat, and in standard practice it is usual to provide for a maximum positive bending moment of $\frac{Wl}{12}$ at the centres of the spans, or even $\frac{Wl}{10}$.

The bending moment diagram for the case of a uniformly distributed load on five equal continuous spans, with the

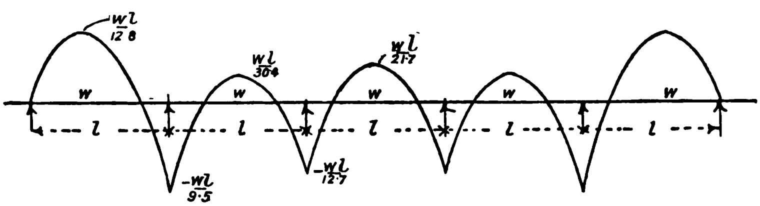


FIG. 31.—Five Spans Fully Loaded.

extreme ends supported only and not fixed, is drawn in Fig. 31. In this case the positive bending moment in the middle of a span reaches a maximum value of $\frac{Wl}{12{\cdot}8}$ and the negative bending moment over a support a maximum value of $-\frac{Wl}{9{\cdot}5}$. If the live load is twice the dead load, and only the first, third and

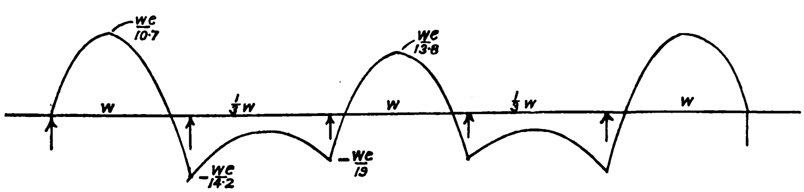


FIG. 32.—Alternate Spans, Dead Load only.

fifth spans are covered with live load, the bending moment diagram is as shown in Fig. 32. This condition is quite likely to occur in an office, for instance, with bookcases or presses against the wall and heavy desks in the middle of the room, and the result is increased positive bending moments in the loaded spans. These diagrams (Figs. 31 and 32) are quite sufficient to show the necessity of adequately providing for the negative bending moments over supports, and also providing for a possible positive bending moment at the middle of the spans at least equal to $\frac{Wl}{12}$ or in end bays $\frac{Wl}{10}$.

In determining the negative bending moments, the supports are assumed to be without breadth, and the values obtained would therefore never be realised in practice. The effect of the definite breadth of the support is to round off the sharp points of the bending moment diagram as shown by the dotted line in Fig. 30, and considerably diminish the maximum value. The practice of only turning up alternate rods to lie near the top surface of slabs over supports and leaving the intermediate rods straight, is, in most cases, therefore, quite sufficient to ensure continuity without subjecting the steel to any higher tensile stress than that developed in the middle of the span. The manufacturers of expanded steel recommend the provision of separate sheets of the metal over the supports, of length equal to ·4 times the span, and the lower sheet is kept in one plane and extends throughout the slab and is joined where necessary over the supports or beams. With some forms of wire-lattice reinforcement it is practicable to constrain the fabric to lie in the lower plane of the slab in the middle of the spans and near the top surface over the supports.

Slabs Supported on Four Sides.

Reference has been made to the necessity for providing transverse reinforcement in all slabs, although the precise amount required cannot in all cases be readily ascertained. It very frequently happens that slabs are supported on four sides, and if the length of rectangle enclosed by the four supports is not greater than twice the width a considerable increase in strength results from the additional support. *It seems fairly well established by experiment that a square slab supported on four sides will carry twice the load that a similar slab will carry when supported on two sides only.*

In the case of a square reinforced concrete slab supported on four sides the reinforcement will need to be the same in both directions, and the amount can be calculated from half the bending moment corresponding to the load to be carried, the length of the span, and the condition of the ends. Various rules have been framed for determining the strength of rectangular slabs supported on four sides, but these differ considerably, and it is quite evident that a sound theory as to the distribution of stress in rectangular slabs has not yet been propounded.

The rules for designing slabs as used in practice assume, for all intents and purposes, that a larger proportion of the total weight carried by the slab is transmitted by ordinary beam action to the long supports and a smaller proportion in a similar way to the short supports, the proportion depending upon the ratio of the length to the breadth. Fig. 33 shows a slab with sides of length a and b respectively. If, of the total uniformly distributed load on the slab W, W_1 is carried by the slab acting as a beam of span a and breadth b, and W_2 is carried by the slab acting as a beam of span b and breadth a, the common deflection δ at the centre of the slab may be written

$$\delta = \frac{5\ W_1\ a^3}{32\ b\ h^3\ E},$$

or,

$$\delta = \frac{5\ W_2\ b^3}{32\ a\ h^3\ E}.$$

$$\therefore \quad \frac{W_1\ a^3}{b} = \frac{W_2\ b^3}{a},$$

or,

$$\frac{W_1}{W_2} = \frac{a^4}{b^4},$$

and

$$\frac{W_1}{W} = \frac{a^4}{a^4 + b^4} \quad . \quad . \quad . \quad . \quad (12)$$

while

$$\frac{W_2}{W} = \frac{b^4}{a^4 + b^4} \quad . \quad . \quad . \quad . \quad (13)$$

Professor Grasshof and Professor Rankine formulated this theory of the action of rectangular slabs supported on four sides, and the values of the ratios (12) and (13) are plotted in Fig. 34 for values of $\frac{a}{b}$ from 1 to 2. This theory might reasonably be supposed to apply to the strips marked x in Fig. 33, but the strip marked y, for instance, is obviously in a very different condition under load and would scarcely deflect at all. The values of the factors adopted by the French Government in the instructions issued to the Ingénieurs des Ponts et Chaussées are lower than those of Grasshof and Rankine, and in this sense may be said to take into account the decreased strain of

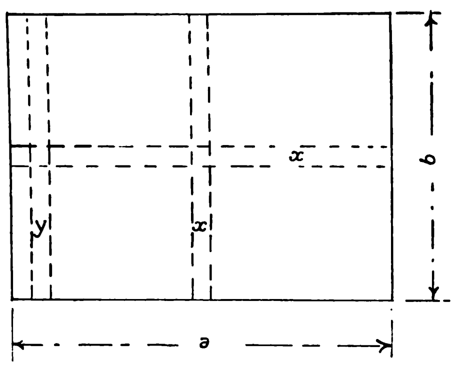


FIG. 33.—Rectangular Slab.

parts of the slab near the supports in similar positions to the strip marked y. These values are also plotted in Fig. 34, or they may be calculated from the expressions $\frac{a^4}{a^4 + 2\,b^4}$ and $\frac{b^4}{b^4 + 2\,a^4}$.

It will be seen that for a square slab the value of the ratio is ·33, and this is hardly in keeping with the experimental evidence that square slabs supported on four sides carry twice the ultimate load as compared with similar slabs supported on two sides. Square slabs loaded with a uniformly distributed load, when tested to destruction, generally fail by direct tension along the diagonals or by diagonal shear along the dotted lines in Fig. 35, and this fact would seem to imply that the greatest stresses are produced on diagonal planes and not upon planes parallel to the supports. Bach's theory is based on this hypothesis, and the values of the maximum diagonal stresses determined by this method are in magnitude equal to the mean values of the maximum stresses determined by the rules recommended by the French Government. It is not convenient to place the reinforcement diagonally, and steel bars are scarcely ever so arranged in practice.

Probably Grasshof and Rankine's factors are the most

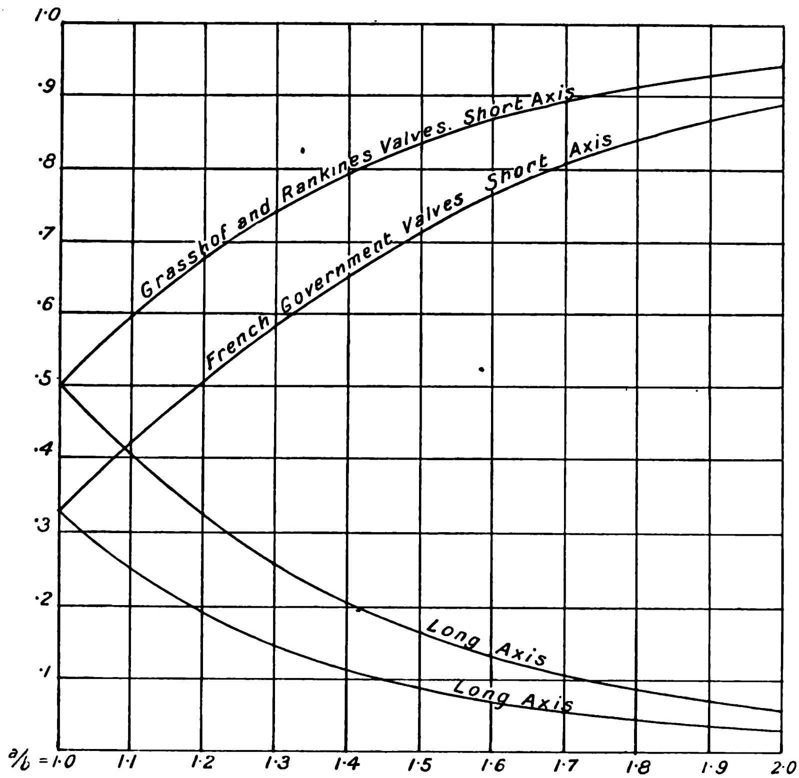


FIG. 34.—Factors for Rectangular Slabs.

reliable to use, at any rate for designing the reinforcement of the middle portions of the slab, and as there is so much uncertainty as to the real distribution of stress, the plan generally adopted in practice of maintaining the size and spacing of the rods, so determined, throughout the slab, would appear to be fully justified.

Grasshof and Rankine's theory is based upon the assumption that the slab is equally strong in either direction, and it is rather important to investigate the probable effect of providing less reinforcement parallel to the long axis of the slab, and as this is placed above the shorter bars in most systems, the effective depth is also less in this direction. With a

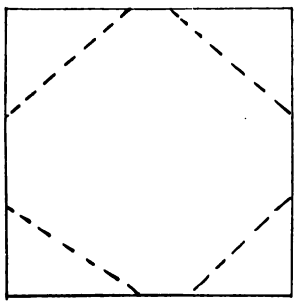


FIG. 35.—Fracture of Square Slab, supported on four sides by Diagonal Shear.

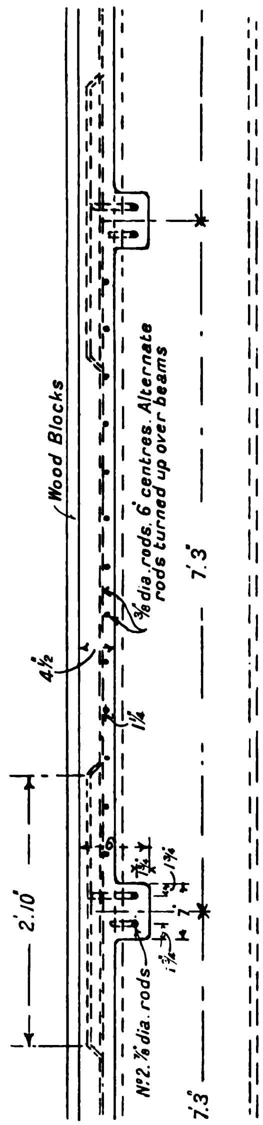


FIG. 36.—Office Floor.

weaker section a given deflection is produced by a smaller load, and it therefore seems probable that more load will be transmitted in the direction of the short axis in a concrete slab with heavier reinforcement in this direction than in a similar slab composed of a material equally strong in either direction.

FLOORS OF BUILDINGS.

In a large building, of which the shell is constructed of brick or other block work and the floors of reinforced concrete, it is very convenient to have steel main girders, because these serve to tie the walls together during building and enable the shell to be finished without waiting for each floor to be completely constructed as it is reached. The main girders being of steel also simplifies the temporary shuttering for the floor, as this can be supported on the lower flanges of the girders, thus saving continuous propping from floor to floor from the ground upwards.

An example of a reinforced concrete floor constructed on this principle is shown in Fig. 36. The main girders consist of 18 in. × 7 in. British standard beams of 28 ft. span and spaced 7 ft. 3 in. apart. The concrete beams at right angles to these are also 7 ft. 3 in. apart. The section (Fig. 36) is taken across these beams and shows the main steel beams in elevation. The slabs are $4\frac{1}{2}$ in. thick over all, reinforced with $\frac{3}{8}$ in. diameter round rods 6 in. apart in each direction.

The calculation of the maximum stresses in the concrete and in the steel according to Grasshof's and Rankine's theory follows :—

Strength of floor for a live load of 140 lbs. per super foot.

Size of slabs, 7 ft. 3 in. × 7 ft. 3 in.

Depth from compression surface to centre of reinforcement, $2\frac{7}{8}$ in.

Percentage of reinforcement $\dfrac{\cdot 11 \times 100}{6 \times 2\frac{7}{8}} = \cdot 64$ per cent.

Weight of slab	$= 7{\cdot}25 \times 7{\cdot}25 \times {\cdot}38 \times 156$ lbs. =	3,070
Wood floor	$= 7{\cdot}25 \times 7{\cdot}25 \times {\cdot}12 \times 56$ lbs. =	370
Live load	$= 7{\cdot}25 \times 7{\cdot}25 \times {\cdot}140$ lbs. =	7,350
		10,790 lbs.

Maximum bending moment, $\cdot 5 \times \dfrac{10{,}790 \times 87}{10} = 46{,}900$ in. lbs.

Moment of resistance, $\cdot 15 \times 87 \times (2\frac{7}{8})^2 \times f_c$.

Maximum compression in concrete, $f_c = \dfrac{46{,}900}{\cdot 15 \times 87 \times 2\frac{7}{8}^2} =$ 430 lbs. per square inch.

Maximum tension in steel, $27 \times 430 = 11{,}600$ lbs. per square inch.

Maximum bond stress $\frac{2 f_s a}{\frac{1}{2} l \pi t} = \frac{2 \times 11{,}600 \times \cdot 11}{43 \cdot 5 \times 1 \cdot 18} =$ 50 lbs. per square inch.

Note:—This method of determining the bond stress, as an alternative to the formula given on p. 36, is convenient in the case of a distributed load.

If the super load to be carried by a floor of this sort were actually uniformly distributed it would be quite reasonable to assume the maximum load carried by one beam to be that covering the hatched area in Fig. 37 and equal to half the area of one square bay. If the distribution is also assumed to

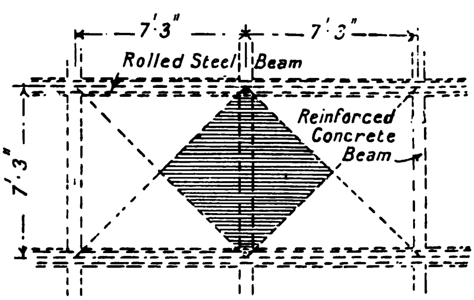


FIG. 37.—Plan of Office Floor.

follow the hatched area, the maximum bending moment for supported, not fixed, ends in terms of W, the load on one square bay, would be $\frac{Wl}{12}$. Probably the actual distribution is much more uniform than this, in which case the maximum bending moment would be less. It must not be forgotten, however, that in practice the actual loading, although equivalent to 140 lbs. per square foot when recorded over a reasonable area, might be concentrated in the vicinity of a beam.

In calculating the strength of the beams shown in Figs. 36 and 37, the whole load covering one square bay was assumed to be carried by one beam, and the maximum bending moment calculated from the formula $\frac{Wl}{10}$, in order to include the two end bays as well as the three interior bays. It is certainly advisable that these beams should be made of ample strength and stiffness in order that the slabs may receive in practice, as in theory, support on four sides.

Strength of beams :—

Span, 7 ft. 3 in. = 87 in.

Effective width the minimum of these values :—

Span divided by 3 = $\frac{87}{\cdot 3}$ = 29 in.

Three-fourths spacing of beams = $\frac{3}{4} \times 87$ = 65 in.

Five times width of beam = 5×7 = 35 in.

That is 29 in.

Effective depth, $7\frac{1}{4}$ in.

Percentage of reinforcement $\frac{100}{29 \times 7\frac{1}{4}} \times 1{\cdot}2 = {\cdot}57$ per cent.

Load :—

Weight of beam, $7{\cdot}25 \times {\cdot}6 \times {\cdot}4 \times 156$ =	280 lbs.
Weight of slab and live load (previous calculation)	10,790 lbs.
	11,070 lbs.

Maximum bending moment, say

$$\frac{11{,}070 \times 87}{10} = 96{,}300 \text{ in. lbs.}$$

Moment of resistance = $\cdot 15 \times 29 \times (7\frac{1}{4})^2 \times f_c$ (Fig. 11).

Maximum compression in concrete = $\frac{96{,}300}{\cdot 15 \times \cdot 29 \times 7\frac{1}{4}^2}$ = 420 lbs. per square inch.

Maximum tension in steel 30 (Fig. 10) $\times$ 420 = 12,600 lbs. per square inch.

The concrete was specified to be composed of one measure of slow setting Portland cement, two measures of clean sharp sand, and four measures of hard broken stone, granite chippings or Thames ballast to pass through a screen with $\frac{3}{4}$-in., mesh and to be retained on a screen with $\frac{3}{16}$-in. mesh.

Old Rail Reinforcement.

The above floor forms part of the first floor of a large office in connection with a railway goods depôt. A cartway occupies a portion of the ground floor, and the site of this had previously been filled in with some 10 ft. of rubbish. As an alternative to removing all this and filling in with reliable material the concrete foundation to the wood block paving was reinforced

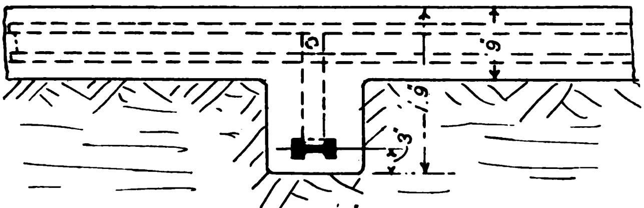


FIG. 38.—Foundation to Cartway. Part Longitudinal Section.

with old rails running longitudinally and strengthened transversely by means of concrete beams, formed in trenches and also reinforced with old rails. Portions of the longitudinal section and the cross section are shown in Figs. 38 and 39, respectively. The transverse beams bear on the walls of the

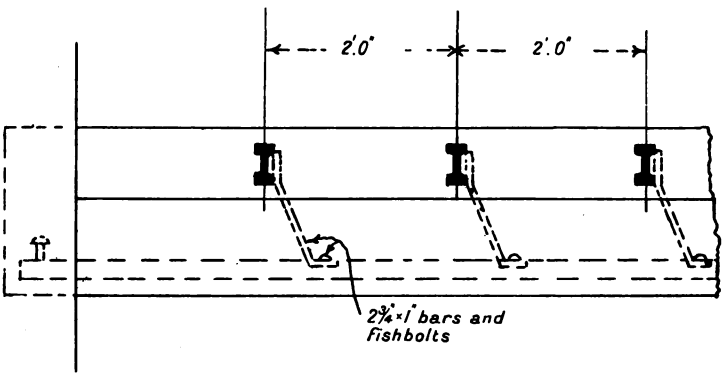


FIG. 39.—Foundation to Cartway. Part Cross Section.

building, which are carried down to a firm foundation, and although there is filling up to the underside of the concrete, no reliance was placed on this in calculating the strength of the floor, except that a weaker concrete was used. These calculations are given in outline as an example of the use of old rail reinforcement.

The average weight of the old rails is 68 lbs. per yard, and the area of the cross section 6·67 sq. in., of which 2·6 sq. in. is in each flange.

Strength of foundation to cartway under 20-ton road roller.

Slabs :—

Span, 7 ft. 6 in. = 90 in.

Width considered, 2 ft. 6 in. = 30 in.

Effective depth, 7 in.

Percentage of tensile reinforcement, say $\frac{2{\cdot}6 \times 100}{24 \times 7} =$ 1·5 per cent.

Percentage of compression reinforcement = 1·5 per cent.

Dead load :—

Concrete	$7{\cdot}5 \times 2{\cdot}5 \times {\cdot}75 \times 156$ lbs. =	2,200
Wood blocks	$7{\cdot}5 \times 2{\cdot}5 \times {\cdot}4 \times 56$ lbs. =	400
		2,600

Maximum bending moment from dead load $\frac{2600 \times 90}{12} =$ 19,500 in. lbs.

Live load :—

$6\frac{3}{4}$ tons distributed over a square 2 ft. 6 in. × 2 ft. 6 in.

Maximum bending moment :—

$$\frac{8}{12} \times 3\tfrac{3}{8} \times 2240\,(45 - 7{\cdot}5) = 189{,}000 \text{ in. lbs.}$$

The factor $\frac{8}{12}$ is introduced to allow for the continuity of the floor slabs over the beams.

Total bending moment, 208,500 in. lbs.

Moment of resistance, $\cdot 3 \times 30 \times 7^2 \times f_c$ (Fig. 15).

Maximum compression in concrete $= \frac{208{,}500}{\cdot 3 \times 30 \times 7^2} =$ 470 lbs. per square inch.

Maximum tension in steel, 23 × 470 = 10,800 lbs. per square inch.

The factors ·3 in. the expression for the moment of resistance and 23 in the ratio of tensile stress in steel to maximum compressive stress in concrete are obtained from Fig. 15.

Strength of beams :—

Span, 17 ft. = 204 in.

Effective width, 5 × 12 = 60 in.

Effective depth, 18 in.

Percentage of reinforcement $= \dfrac{6{\cdot}67 \times 100}{60 \times 18} = {\cdot}6$ per cent.

Dead load :—

Concrete	17 ×	1 ×	1 ×	156 =	2,650	
	17 ×	7·5 ×	·75 ×	156 =	14,900	
Wood blocks	17 ×	7·5 ×	·4 ×	56 =	2,850	
					20,400	lbs.

Live load :—

13·5 tons axle, wheels 6 ft. 6 in. centres.

13·5 tons = 30,240 lbs., 15,120 lbs. on each wheel.

To obtain the maximum bending moment half the dead load is added to one of these wheel loads,

$$15{,}120 + \frac{20{,}400}{2} = 25{,}320 \text{ lbs.}$$

Maximum bending moment =

$\left\{(25{,}320 + 15{,}120)\ 204 - 15{,}120 \times 78\right\}^2 \div 4 \times 40{,}440 \times 204 = 1{,}510{,}000$ in. lbs.

Moment of resistance $= {\cdot}15 \times 60 \times 18^2 \times f_c$.

Maximum compression in concrete $= \dfrac{1{,}510{,}000}{{\cdot}15 \times 60 \times 18^2} =$ 520 lbs. per square inch.

Maximum tension in steel = 29 × 520 = 15,100 lbs. per square inch.

A weaker concrete was specified for this work, namely, one measure of Portland cement to six measures of hard stone of such a size as to pass through a screen of 1-in. mesh and be retained on a screen with ¼-in. mesh, and three parts of sand.

Lintels.

Lintels provide one of the few instances of rectangular beams simply supported at the ends, occurring in reinforced concrete construction. One of these is sketched in Figs. 40

and 41, and the calculation of the stresses in this lintel serve to further illustrate the use of beam formulæ in reinforced concrete design. In this case the lintel was designed to match an existing stone band running round some station buildings and, in consequence, the depth of the beam was fixed and the breadth was also restricted by the existing conditions to 7½ in.

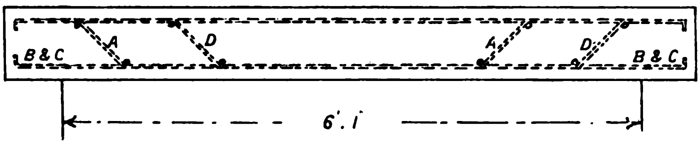


FIG. 40.—Elevation of Lintel.

The calculations which follow determine the percentage of reinforcement required to develop sufficient strength in a beam of these dimensions to carry the superimposed load of a few courses of brickwork, and an area of slated roofing and ceiling,

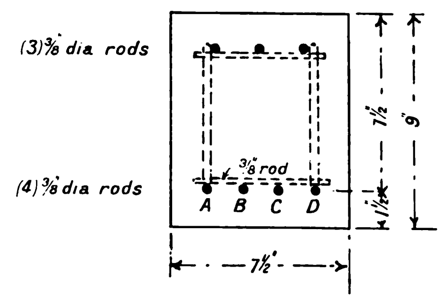


FIG. 41.—Section of Lintel.

reckoned as equivalent to a uniformly distributed load of 40 lbs. per super foot.

Span between centres of bearings, 6·7 ft. = 80 in.

Breadth of lintel, 7½ in.

Effective depth, 7½ in.

Load :—

Own weight, 6·7 × ·75 × ·6 × 156 lbs.	=	470
Brick wall, 6·7 × 2·25 × ·75 × 140 lbs.	=	1,580
Allowance for roof and ceilings, 6·7 × 8 × 40 lbs.	=	2,140
		4,190

Maximum bending moment, $\frac{4{,}190 \times 80}{8} = 41{,}900$ in. lbs.

Moment of resistance $= k \times 7{\cdot}5 \times 7{\cdot}5^2 \times 500$

$$k = \frac{41{,}900}{7{\cdot}5 \times 7{\cdot}5^2 \times 500} = {\cdot}20.$$

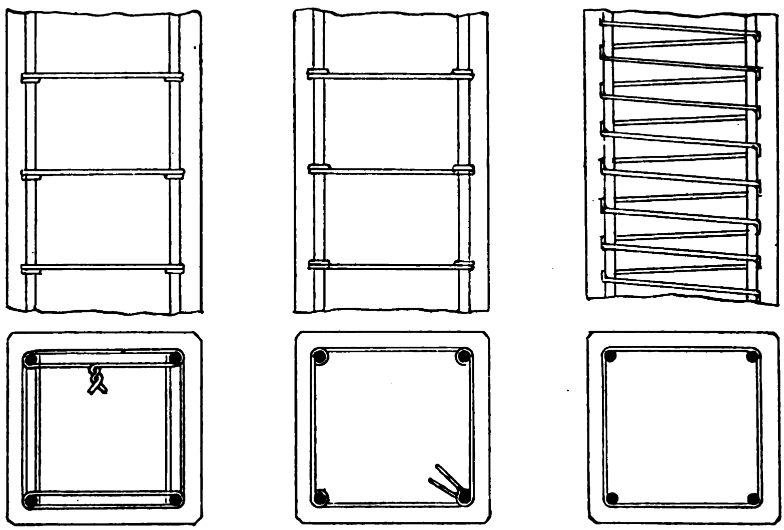
From Fig. 14 it will be seen that this factor is obtained with a total reinforcement of 1·3 per cent., of which ·8 per cent. is placed as tension and ·5 per cent. as compression reinforcement.

·8 per cent. $\times 7\frac{1}{2} \times 7\frac{1}{2} =$ ·45 sq. in., say (4) $\frac{3}{8}$ in. diameter rods.

·5 per cent. $\times 7\frac{1}{2} \times 7\frac{1}{2} =$ ·28 sq. in., say (3) $\frac{3}{8}$ in. diameter rods.

Columns.

The condition of stress in a loaded column is readily provided for in reinforced concrete construction. In short columns, not subject to bending action, experiment shows the correct form of reinforcement to be in the nature of a cage, surrounding a concrete core and preventing the latter from bursting under the influence of compression. This steel-cage is formed of longitudinal bars in conjunction with various forms of lateral bracing as shown in Fig. 42, most of which are protected by patent. The stress produced in the longitudinal reinforcement, as compared with the stress in the surrounding concrete, is known when once the ratio of the modulus of elasticity of the two materials has been determined, and is therefore a function of the total load, the area of the cross section of the column, and the percentage of reinforcement. The stress produced in the lateral reinforcement presents a far more difficult problem, but M. Considère has ascertained by experiment that spiral winding is 2·4 times as effective as the same weight of steel in the form of longitudinal rods if the breaking load is taken in each case as the criterion of strength. The results of Prof. Talbot's experiments agree very closely with this figure. A very ingenious argument has been propounded to show that as regards working strength longitudinal reinforcement is far more effective relatively to lateral reinforcement than these figures appear to indicate. It is pointed out that as the breaking load is reached Poisson's ratio, *i.e.*, the ratio of lateral strain to axial strain is relatively high, but for working loads is quite small, and therefore, it is only in resisting loads approaching the breaking load that lateral reinforcement becomes so effective, and that lateral bracing, therefore, is comparable in its action to vertical



Links. *Hoops.* *Spiral Winding.*

FIG. 42.—Columns.

stirrups in beams and is to a certain extent in the nature of a lay-by.

In columns of shorter length than, say, 18 diameters, not subjected to bending and with laterals more than half a diameter apart, the area of cross section A increased by 14 times the area of longitudinal reinforcement PA and multiplied by the working compressive stress for the concrete f_c is usually taken as the safe working load W,

That is $W = A\,(1 + 14\,P)\,f_c$ . . . . . . (14)

P is generally made not less than ·008 of A or ·8 per cent. The diameter of the lateral bracing is generally from $\frac{3}{16}$ in. to $\frac{3}{8}$ in.

EXAMPLE 10.—Find the size of a short square column and the area of reinforcement required to carry a central load of 60,000 lbs. Proportion of longitudinal reinforcement to be ·8 per cent. of the cross sectional area and the compression stress in the concrete not to exceed 500 lbs. per square inch.

From (14). $60{,}000 = A\,(1 + \overline{14 \times \cdot 008})\,500.$

$A = 108$ sq. in.

Side of column, say $10\frac{1}{2}$ in.

Area of reinforcement $= \cdot 008 \times 108 = \cdot 87$ sq. in., say four $\frac{9}{16}$ in. diameter bars.

In the second report of the R.I.B.A. Joint Committee and also in the London County Council regulations, the additional strength obtained by lateral hooping of small pitch is very carefully considered, and under favourable circumstances the working compressive strength of the concrete may be increased by 66 per cent. by using about 2 per cent. of special winding of pitch not greater than one-fifth of the diameter of the hooped core.

Columns have sometimes to resist bending as well as pure compression. In certain cases the bending moment can readily be calculated and depends only on the arrangement of the applied loads. The gate post shown in Fig. 118, and the columns carrying the tracks for the overhead cranes in Fig. 43 are examples of this kind.

Columns of greater length than 18 diameters will be very rare in reinforced concrete construction, but in long slender columns some bending should certainly be assumed, apart from any known eccentricity of the loading, to allow for inaccuracies of workmanship and the deflection produced in long thin struts under direct compression.

In other cases the maximum bending moment in the column depends not only on the applied loads but also on the relative stiffness of the column and the adjoining members, such as beams, of the structure of which the column forms a part. The calculation then becomes more difficult and similar in form to the problem presented on p. 137.

EXAMPLE 11.—Design a column to carry a load of 50,000 lbs., and to resist a bending moment of 80,000 in. lbs., the proportion of reinforcement being 1 per cent. of the cross sectional area and the working compressive resistance of the concrete 500 lbs. per square inch.

Assuming, in order to make a trial shot, that the maximum compressive stress due to bending is equal to the direct stress, the column will have to be about 14 in. sq.

From (14). Direct compression in concrete $= 50{,}000 \div 196\ (1 + \overline{14 \times \cdot 01}) = 224$ lbs. per square inch.

The centre of the steel will need to have 2 in. of concrete covering and 1 per cent. of $14 \times 14 = \frac{14}{12}$, or $1{\cdot}17$ per cent. of 14×12.

From Fig. 15, resistance to bending $= \cdot 187 \times 14 \times 12^2 \times f_c = 80{,}000$ inch lbs.

Therefore maximum compressive stress due to bending $= 80{,}000 \div (\cdot 187 \times 14 \times 12^2) = 212$ lbs. per square inch, and the total compressive stress is 436 lbs. per square inch.

If the column was made 13 in. $\times$ 13 in., the direct stress would be increased by about 20 per cent. and the stress due to bending still more, so that the total stress would be well over 500 lbs. per square inch. Without going to half-inches then, a 14-in. column is the smallest which will fulfil the conditions.

Example 12.—Referring to the previous example determine the amount of reinforcement required to bring a 13-in. square column up to the required strength.

The direct compression is approximately $\frac{14^2}{13^2} \times 224 = 260$ lbs. per square inch. Therefore the maximum compressive stress due to bending must not exceed 240 lbs. per square inch.

$$k \times 13 \times 11^2 \times 240 = 80{,}000$$
$$\therefore k = \cdot 21$$

corresponding to $1\frac{1}{2}$ per cent. of reinforcement (Fig. 15).

$1\frac{1}{2}$ per cent. of $13 \times 11 = 2{\cdot}14$ sq. in., say (4) $\frac{7}{8}$ in. diameter bars.

The amount of bending assumed in this example is equivalent to an eccentricity of the whole load of 1·6 in., and the stress due to bending is nearly half the working stress. This shows the serious effect of quite a small amount of initial curvature in a column or eccentricity of loading. On the other hand Example 11 illustrates the comparatively large increase of strength obtained by increasing a 13-in. sq. column to 14 in. × 14 in., and a small increase in the section is, therefore, all that is necessary to allow for inaccuracies of workmanship or a small deviation of the load from the centre line.

The ground floor of warehouses and goods sheds constructed for railway purposes has generally to be laid out with lines of rails, decks and roadways, and the accommodation of these and the necessary turntables and cranes restricts within narrow limits the possible positions of the columns carrying the upper floors, and entails the use of beams of considerable span. This fact, added to the heavy live loads which have to be provided for in designing floors to be used for storage purposes, often makes it very difficult to plan a building which can be satisfactorily constructed in reinforced concrete.

The difficulty has been overcome either by using steel for the main girders and stanchions, and reinforced concrete for the secondary beams and floor slabs, or by very heavily reinforcing the concrete, for instance, with a riveted steel skeleton properly designed to resist all the tensile and a proportion of the compressive stresses. The Trafalgar Goods Warehouse, Newcastle-on-Tyne, illustrated in Fig. 43, is an example of the successful construction of long span, heavily loaded beams in reinforced concrete. One of the difficulties met with in long buildings constructed entirely of reinforced concrete has been the tendency of the thin walls to crack, and the recommendation has been made that the supports and floors should be built as a monolith, and the walls filled in with brickwork, independent concrete panels, or other suitable material.

There is a fine example of a reinforced concrete goods station and warehouse at New Bridge Street, Newcastle, on the North Eastern Railway Company's system, constructed of Hennebique ferro-concrete. This is shown in outline in the section, Fig. 43. The floor at rail level is carried on five longitudinal rows of columns of reinforced concrete and two longitudinal abutments of ordinary concrete, dividing it into six bays, of which the greatest is 35 ft. wide. The span of the longitudinal beams is about 32 ft. 8 in., spaced from 5 to 8 ft.

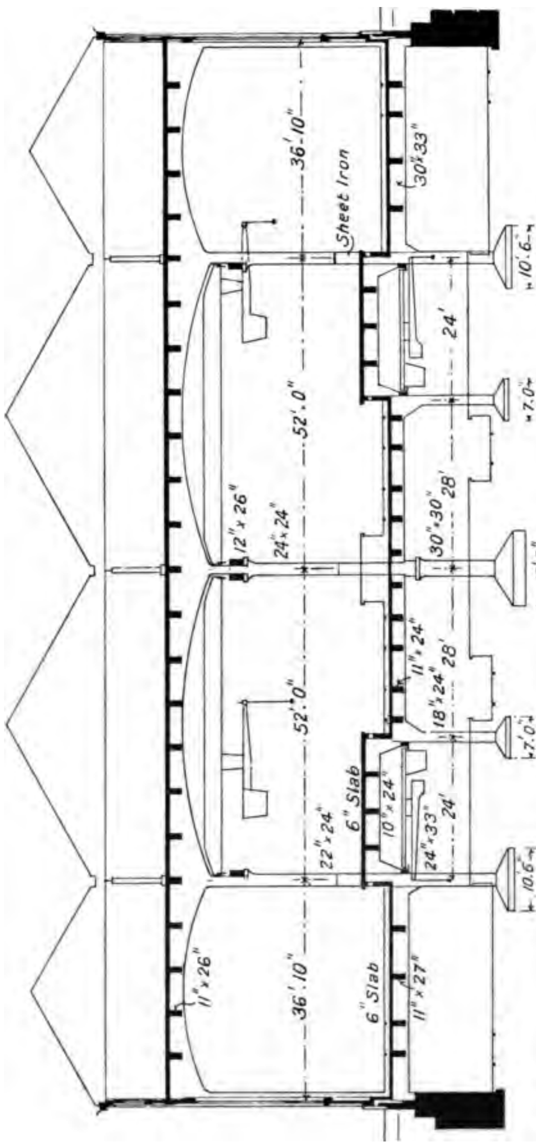


FIG. 43.—New Bridge Street Goods Station and Warehouse.

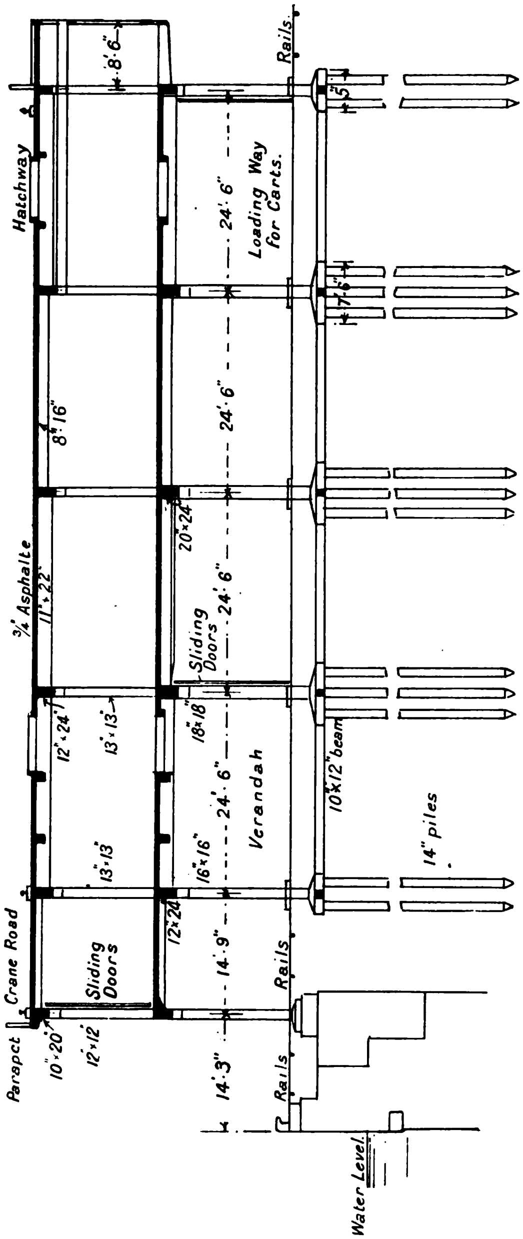


FIG. 44.—Goods Shed at Canons Marsh.

apart. This floor is designed to carry a dead load of 3 cwt. per square foot and a super load of the same amount, except where rails occur, when the moving load consists of 76-ton locomotives.

Some of the columns carrying this floor also support runways for travelling cranes serving the platforms and lines in the basement beneath. The largest of these columns, designed to carry a load of 1,105 tons, is 30 in. square, and the base is 12 ft. 6 in. square. The sizes of the principal beams are shown on the section.

The floor above is carried on the side walls and three longitudinal rows of columns, dividing the building into four bays, the two inside bays being 52 ft. wide, and the two outside 36 ft. 10 in. wide. The transverse beams carrying this floor have arched soffits. They are about 3 ft. deep at the centre, and are designed to carry a load of 400 tons each. Travelling cranes work in the two middle bays supported on longitudinal reinforced concrete beams. The span of the longitudinal floor beams is about 32 ft. 8 in.: they are spaced about 7 ft. 6 in. apart centre to centre, and are 11 in. wide by 31 in. total depth. The floor slab is 5 in. thick, the floor being designed to carry a super load of 3 cwt. per square foot. The roof is of timber.

Fig. 44 is a cross section of Goods Shed No. 1 at the Great Western Railway Bristol Dock, Canons Marsh. This is also constructed of Hennebique ferro-concrete, from the original plans of Mr. W. W. Squire. The foundation is on ferro-concrete piles, carried down about 40 ft. below the normal water level in the harbour. The first floor and the flat roof are designed to carry a super load of 3 cwt. per square foot. A travelling crane runs along the side of the roof nearest to the quay wall. The spans and sizes of the beams and the sizes of the columns are shown on the drawing.

Jetties and Gantries.

The King's Dock, Swansea, presents numerous examples of reinforced concrete construction for railway purposes. On the north side the Great Western Railway Company have the use of a reinforced concrete quay 750 ft. long, with tip jetties and high level viaducts.

The reinforced concrete quay on the south side, which is 1,100 ft. in length, is shared by the Midland Railway, the Swansea Harbour Trust, and the Rhondda and Swansea Bay Railway. The late Mr. P. W. Meik, M.I.C.E., and Mr. A. O.

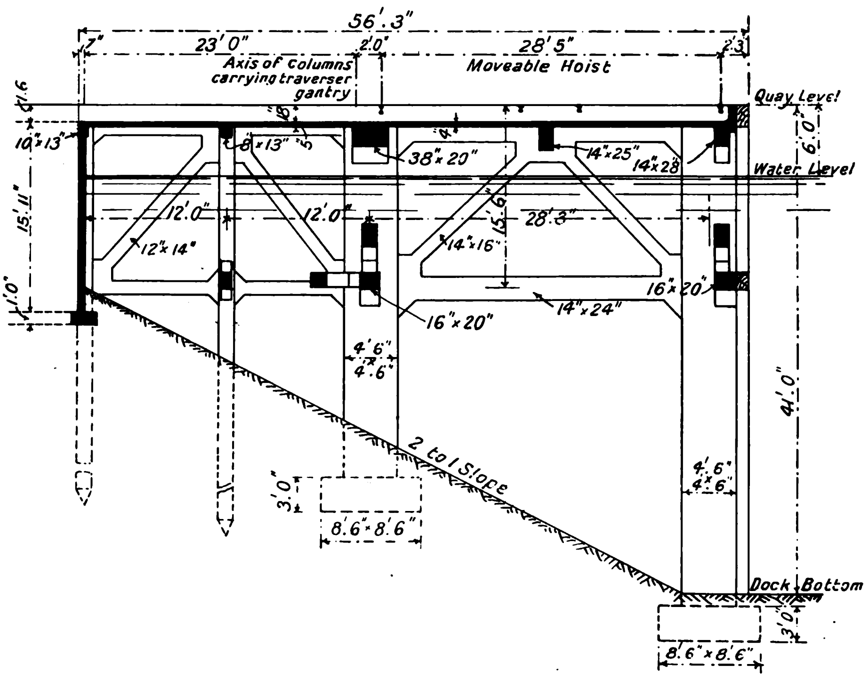


FIG. 45.—King's Dock, Swansea.

Schenk, M.I.C.E., acting on behalf of the Swansea Harbour Trust, were the engineers for this work, which is carried out in Hennebique ferro-concrete. A typical section showing the construction of the quays is reproduced in Fig. 45. The height from the dock bottom to the quay level is 41 ft. The width of the quays generally is about 56 ft., and the jetties project into the dock by about the same amount. Timber fenders are fixed along the front of the quay and round the jetties.

The gantries for carrying away at the high level the empty trucks from the coal hoists fixed along the south quay of the

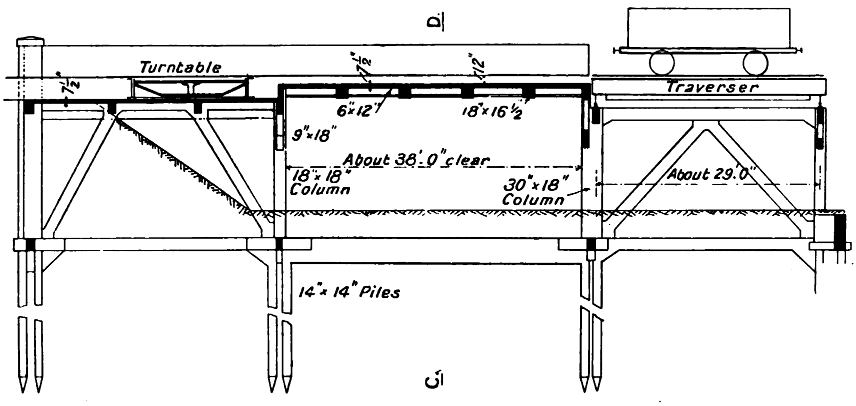


FIG. 46.—Coal Gantry, Immingham Dook.

Great Central Railway Company's Immingham Dock are constructed in Hennebique ferro-concrete. One of these gantries is shown in Figs. 46—47. This is No. 7 gantry (there are eight in all), and is worked with a movable hoist. The gantry carries two tracks, one by which the full waggons arrive from the gravity sidings and the other by which the empties are returned by gravitation under the high level serving roads to the empty storage sidings. The width between the parapets is increased at the back to accommodate the turntable on the full waggon road shown in section A B. The longest main girders are about 40 ft. span, and the longest cross girders the same length. The foundation consists of 14-in. square piles

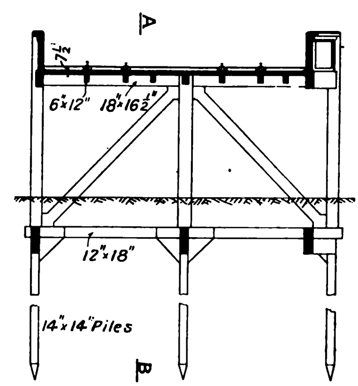


FIG. 47.—Coal Gantry, Immingham Dock.

arranged in pairs under each column. The floor is designed to carry 30-ton (total weight) four-wheeled waggons, 19 ft. in length over buffers, with a 9-ft. wheel base, but the turn-table and traverser will accommodate longer waggons than these.

CHAPTER V

FOUNDATIONS AND RAFTS

PILES.

THE suitability of concrete for works below ground level has long been established. In some cases the function of such works is to transmit the load in the most economical manner down to a firm foundation, and in other cases to sufficiently distribute the load over an indifferent foundation. For the carrying out of both of these purposes the substitution of reinforced concrete for plain mass concrete frequently results in a stronger, lighter, and more economical structure.

Reinforced concrete piles are sometimes used in place of wooden piles when the advantage of greater permanence justifies the increased first cost, or the circumstances are such that timber would not be a suitable material to use. The foundations of railway structures built on a recently-tipped bank can very conveniently be carried down to the natural ground by means of reinforced concrete construction. If the bank has already been tipped, piles provide the most ready means of reaching firm ground, and as a situation of this sort is very likely to be alternately wet and dry, timber is not a desirable material to use.

The station buildings at the new Stonebridge Park Station, which is built on a newly-tipped bank, are founded upon piles in this way, the heads of the piles being connected together transversely by tie rods surrounded by concrete passing under the permanent way.

It will be noticed that many of the buildings and bridges illustrated in the previous and subsequent chapters are founded on piles of reinforced concrete, and the necessity of providing rigid foundations for structures of a monolithic character, particularly those in which advantage is taken of

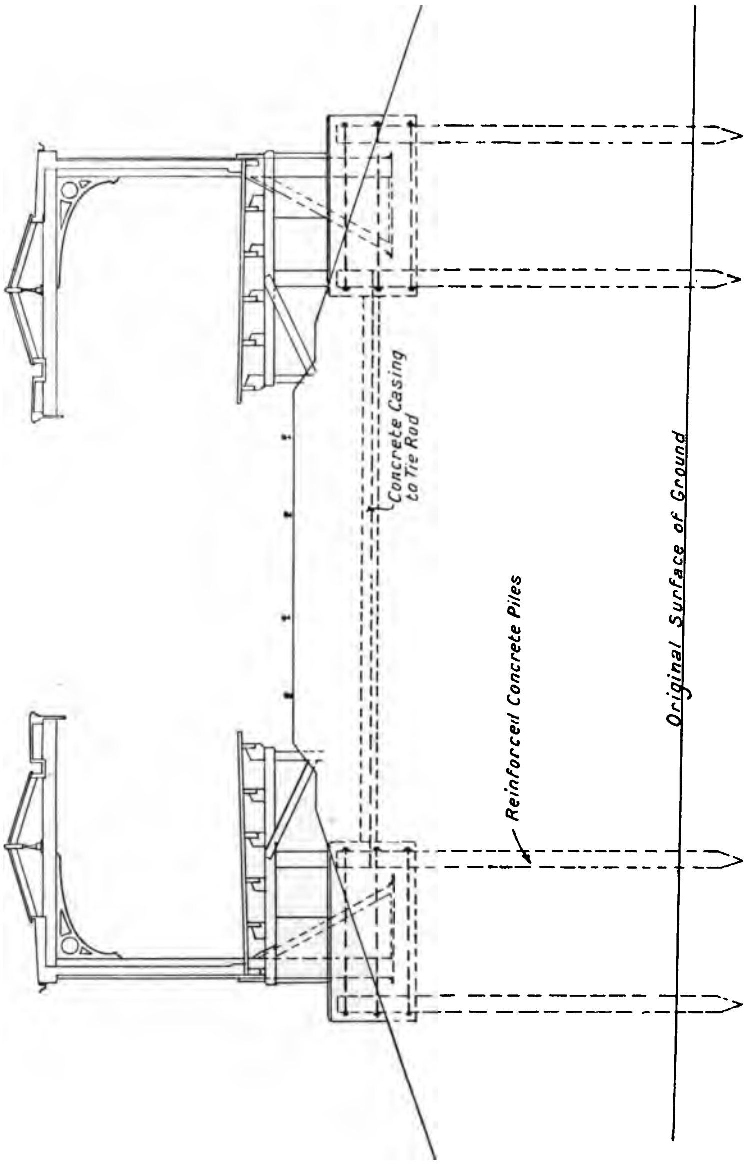


FIG. 48.—Section of Railway Station on Bank (Stonebridge Park).

the increased strength resulting from the continuity of adjoining members, is obvious.

The piles most commonly used are of square section, varying from 10 in. × 10 in. to 14 in. × 14 in., reinforced with four or sometimes eight longitudinal rods and transverse links or wrappings. These are spaced closer together at the ends, where they may be, for instance, 3 or 4 in. apart, than at the middle, where the spacing may be from 6 to 10 in. The diameter of the wire used for bracing is generally about $\frac{3}{16}$ in. or $\frac{1}{4}$ in. In one system, practised by Messrs. D. G. Somerville & Co., two wires of about $\frac{1}{8}$ in. diameter are twisted together with tourniquets, thus causing the lateral bracing to

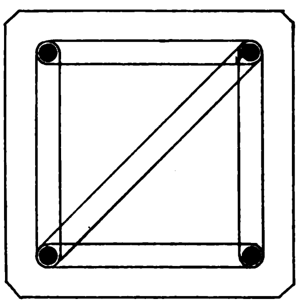


FIG. 49.

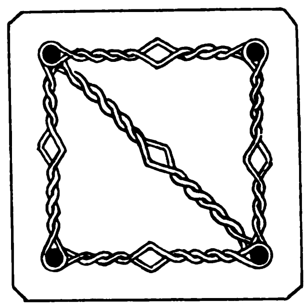


FIG. 50.

Sections of Pile.

tightly grip the main rods. By the addition of diagonal links at intervals the complete reinforcement is given such rigidity that it can readily be moved about and lowered into the moulds without displacing any of the members. Fig. 49 shows the wire as first wound somewhat slackly, and Fig. 50 shows the lateral bracing as it appears when the wires have been twisted together.

The proportion of the longitudinal reinforcement depends largely on the length, and the stresses produced in the pile by its own weight when acting as a beam during lifting will need to be considered. These stresses are minimised by slinging the piles at points about one-third of the length from the top end. As a rule, the proportion of the longitudinal reinforcement will not be less than 1 per cent. of the cross sectional area. In some special piles the reinforcement takes the form of a rolled steel joist or four angles braced together. Others are of circular form with rods arranged round the circumference. Sheet piles are constructed of rectangular form with perhaps six rods, one near each corner and one near the middle of each of the longer sides.

The concrete for piles is often a richer mixture than would be used for beams or floors, as there is a large amount of surface compared with the volume, and the material is subjected, during driving, to considerable shock. One part of cement, one and a half parts of sand, and three parts of the aggregate for the middle part of the pile, with a somewhat smaller proportion of aggregate near the ends represents not uncommon practice in this respect. Pile shoes commonly employed are very similar to those used for wooden piles, with the exception that the straps are turned into the concrete, and a recess is provided in the base for housing the ends of the rods. A common form is shown in Fig. 51.

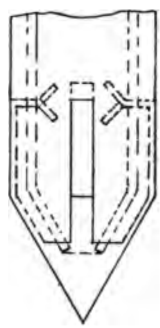


FIG. 51.—Pile Shoe.

One of the difficulties in connection with reinforced concrete piles arises from the time required for maturing before they can be safely handled or driven. This period must be at least from 3 to 4 weeks, while the piles themselves under ordinary circumstances require some little time for construction. As these members are generally required right at the commencement of the work of which they form a part, the delay which would be caused by the use of reinforced concrete piles not infrequently constitutes a serious objection to their use. At present reinforced concrete piles are not in such general demand as to warrant manufacturers holding stocks of them, apart from the difficulty occasioned by the variation in the lengths required.

Except under very favourable circumstances the cost of reinforced concrete piles averages from 8*s*. to 9*s*. per cube foot, including driving, rather more for small piles, rather less for larger ones. An average price for wooden piles, including driving, is from 4*s.* to 5*s.* per cube foot, so that the increased cost of the more permanent construction is considerable. In certain cases where the piles are driven down to rock or other hard foundation, somewhat greater loads might be put upon them if the material is reinforced concrete instead of timber, but in cases where the load carrying capacity of the pile depends largely upon friction, reinforced concrete has not this advantage. It is interesting to note here that a pile of circular section offers considerably less surface than one of square section possessing the same cross sectional area. A circular pile should therefore be easier to drive, but under some circumstances might not be capable of carrying so great a load as a square pile.

The cost of an ordinary cement concrete foundation, not reinforced, including excavation, of fair depth, may be taken at 19*s.* per cube yard, that is about one-twelfth of the cost, volume for volume, of reinforced concrete piles, including driving. Therefore for the same money that two 12 in. × 12 in. piles would cost, a concrete pier 24 sq. ft. in section, or, say, 8 ft. × 3 ft., could be provided, which would carry as great a load as the two piles, if a very good foundation were obtainable at a reasonable depth below the surface.

PIERS.

Fig. 52 shows an example of a foundation to a goods yard warehouse built upon a tipped bank from 10 to 20 ft. in height. The piers were constructed before the bank was tipped, and are practically of plain mass concrete, as there are only four light rods in each pier. The walls of the warehouse, which is 200 ft. long, are built upon reinforced concrete beams supported by these piers. The under side of the beams coincides with the formation level of the bank, and the only shuttering required for their construction consisted of sideboards. The intentions with regard to the building having been altered after the piers were put in, it became necessary to design the beams for somewhat heavy loads, amounting approximately to 28 tons distributed over a 10-ft. span. The beams are 2 ft. in depth and 1 ft. 10½ in. in width. The longitudinal reinforcement consists principally of ⅞ in. diameter rods, arranged as shown in Fig. 52. The transverse rods are ⅜ in. diameter, and there are two rods of this size running longitudinally at the neutral axis of the beam, provided with a view to further prevent the opening up of cracks likely to occur in beams of this length. The ⅞-in. diameter rods near the top of the beams are joined and overlapped where necessary at the middle of the spans. Those near the bottom of

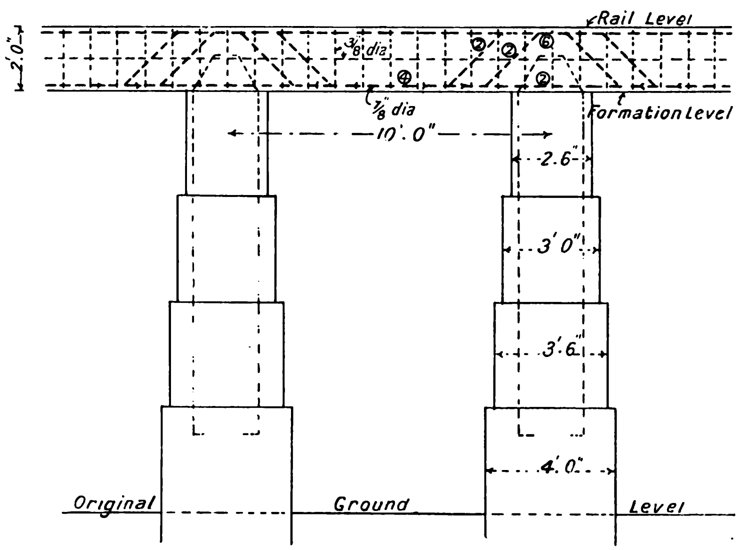


FIG. 52.—Foundation for Warehouse on Bank.

the beams are joined and overlapped where necessary over the piers.

Another example of a reinforced concrete foundation is shown in Fig. 53. The portion included in the illustration supports part of the extension of an engine shed at Immingham, Great Central Railway. Here not only the walls but the whole floor is carried on a stiffened slab of concrete and buttressed piers of the same material. The reinforcement consists of Kahn Trussed and Rib Bars disposed as shown in the illustration.

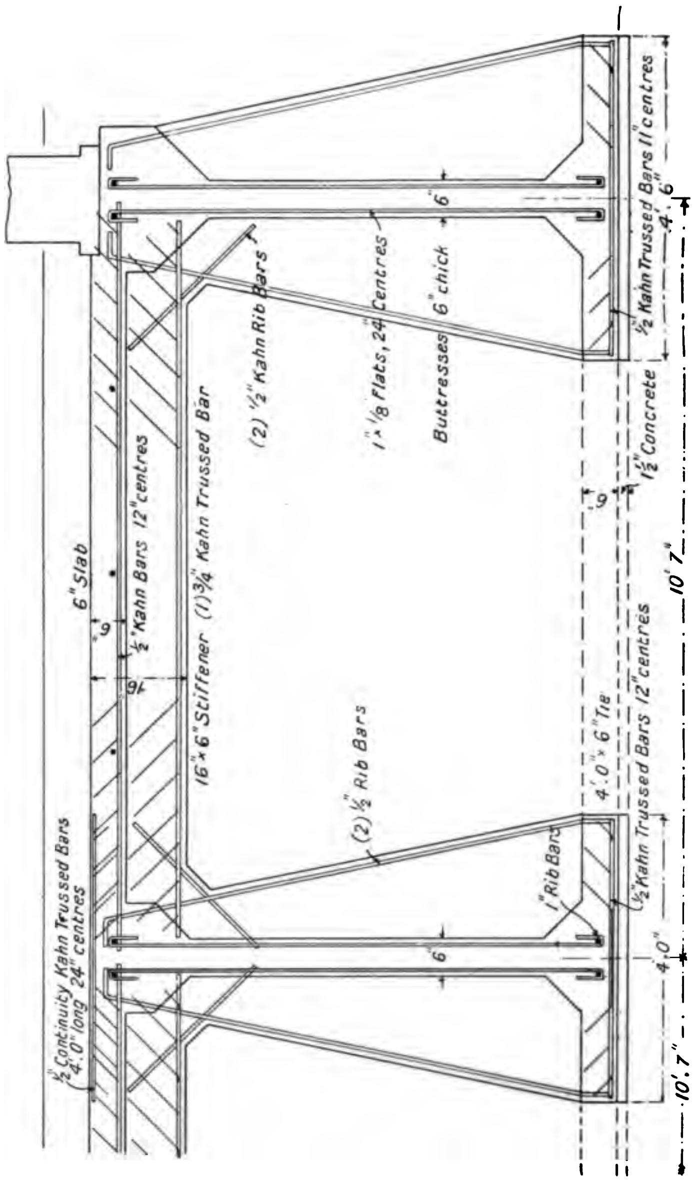


FIG. 53.—Engine Shed Foundation at Immingham.

SPREAD FOUNDATIONS TO PIERS.

The design of a spread foundation or raft is attended with rather more difficulty than the previous cases, because of the unknown variation in the intensity of the earth pressure which causes the principal stresses in such a foundation. A railway sleeper, strictly speaking, presents a case of this sort, which is considered somewhat fully in Chapter IX. The results there obtained are sufficient to show that very little error is introduced in the case of a symmetrical structure by assuming the earth pressure on the underside to be uniformly distributed over the whole area, and as regards the strength of the structure itself this error is on the side of additional safety.

In determining the required area, however, of a large spread foundation, from the total load and the known safe pressure for the particular soil built upon, it should be borne in mind that the maximum pressure must exceed the average pressure by a varying amount, which, in the case of the wooden sleeper considered in Chapter IX., is found to be 10 per cent.

The spread foundation to a wall or a pier of some length presents perhaps the simplest case, because consideration of the principal stresses may be confined to a plane section such as that shown in Fig. 54. If b is the width of the wall in inches, B the width of the foundation in inches, and W the load in pounds per lineal foot on the foundation, the earth pressure on the under side may be assumed to be $\frac{12\,W}{B}$ lbs. per square foot. Considering a 1-ft. length of the wall the maximum shear which occurs at sections AA′ and CC′ is equal to

$$\frac{B-b}{2} \times \frac{W}{B} \text{ lbs.},$$

and the maximum bending moment which occurs at the centre of the foundation is equal to

$$\frac{W}{2}\left(\frac{B}{4} - \frac{b}{4}\right) = \frac{W}{8}(B - b).$$

The variation in the shear and bending moment is readily obtained by graphic integration of the load diagram, Fig. 54. The shear and bending moment diagrams are shown below. The thickness of concrete required and the amount of reinforcement can then be obtained in the same way as for slab floors.

In the case of a wall 1 ft. 6 in. thick carrying at ground level 6 tons per lineal foot, a foundation 6 ft. wide will distribute the load so that the average pressure does not exceed 1 ton per square foot. The maximum shear is $\frac{(72 - 18)}{2} \times \frac{6 \times 2{,}240}{72} = 5{,}040$ lbs. per lineal foot. The maximum bending moment is $\frac{6 \times 2{,}240}{8}(72 - 18) = 90{,}700$ in. lbs. per lineal foot. In order that the working stresses may be 500 lbs. per square inch compression for concrete and 14,500 lbs. per square inch tension for steel, the percentage of reinforcement must be ·6 per cent., and the expression determining the moment of resistance is then

$$\cdot 15 \times 12 \times d^2 \times 500 \text{ in. lbs.} \quad \text{See p. 23.}$$

This is equal to 90,700 in. lbs., the maximum bending moment, when $d = 10$ in. The overall depth will need to be 12 in., and the area of reinforcement required is $\cdot 006 \times 10 \times 12 = \cdot 72$ sq. in. per lineal foot of wall; ⅝th in. diameter rods spaced 5 in. apart between centres will secure this amount. The maximum intensity of shear stress is

$$\frac{5{,}040}{\cdot 9 \times 10 \times 12} = 47 \text{ lbs. per square inch.} \quad \text{See p. 43,}$$

a sufficiently high value to make it advisable to provide some shear reinforcement.

As the maximum shear occurs within such short distances from the maximum bending moment it is not practicable to turn up any of the rods provided to resist bending stresses. If sufficient steel is to be provided to resist the whole of the tension component of the shear stress the area required will be

$$\frac{5{,}040}{1 \cdot 41 \times 12{,}000} = \cdot 3 \text{ sq. in. per lineal foot,}$$

where 12,000 is the working shear resistance of the steel in lbs. per square inch and 1·41 or $\sqrt{2}$ the factor which allows for the increased value of the metal when inclined at 45 deg. as shown in Fig. 54. If the bent rods are attached to every

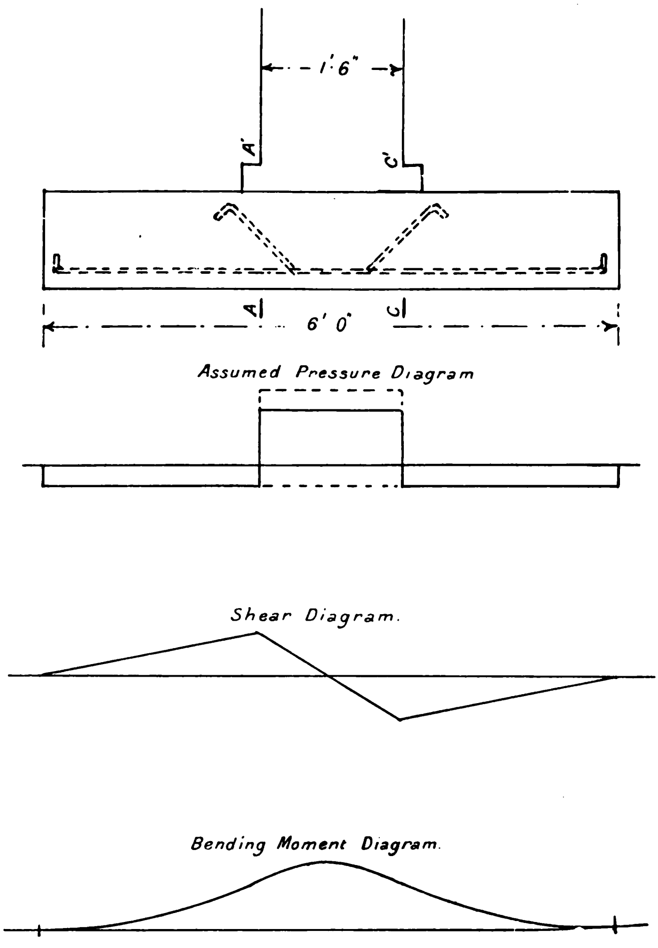


FIG. 54.—Wall or Pier Foundation.

other one of the straight rods that is spaced 10 in. apart, the area of each should be ·25 sq. in.; although $\frac{5}{8}$th in. diameter rods have a rather greater area than this (·30 sq. in.), $\frac{1}{2}$ in. diameter rods would be too small, and there is an advantage in keeping all the bars of one size. Some longitudinal reinforcement will be required to link up the transverse units and resist stresses due to inequalities in the upward pressure. Four of these, $\frac{3}{8}$th in. in diameter should be ample for this purpose.

Column Foundations.

The case of a column foundation is complicated because the stresses are not confined to one direction but radiate in all directions to and from the centre. In some designs the principle embodied in the familiar steel-joist grillage is followed, and the load from the column is first distributed in one direction by means of a beam and subsequently to the slab upon which this beam rests. More frequently the foundation consists of a plain slab only, as shown in plan and elevation in Fig. 56. If such a foundation failed under excessive upward bending action the manner of failure would almost inevitably be the breaking off of one or more corners because on the sections thus exposed the bending moment would be a maximum and the moment of resistance a minimum. There would appear to be an advantage then in laying the reinforcing rods diagonally, or better still radially, if this was not impracticable, because of the number of rods that would have to cross one another at the centre, but, as in the similar case of a square slab floor supported on four sides, this is seldom done in practice on account of the inconvenience of providing the rods of many different lengths.

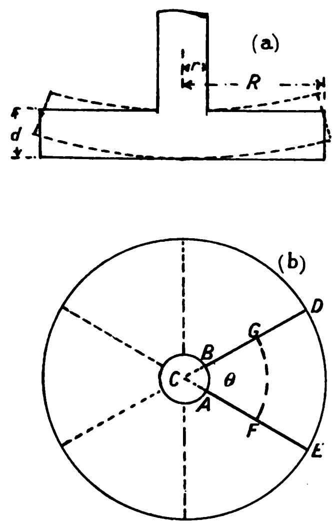


FIG. 55.—Circular Column Foundation.

The simplest example of a column foundation from a theoretical point of view is a circular column resting on a circular base as in Fig. 55. The base could then be divided up into any number of similar cantilevers which in plan increase in width towards the free ends as shown by the radial lines in Fig. 55.

Looking at the sectional elevation, Fig. 55, these cantilevers when stressed assume the form shown to an exaggerated extent by the dotted lines and the lower edge of the foundation, and similarly the circumference of any concentric circle traced on the under side of the slab increases in length. The length of the top edge is similarly compressed. It is evident, therefore, that the strength of the slab is increased, firstly by the resistance offered to these changes in circumferential length, secondly by the frictional resistance between the concrete and the earth upon which it rests, and thirdly because the upward pressure, instead of being uniformly distributed as assumed, diminishes towards the outer edges where its effect is greatest. These factors make it very difficult to accurately determine the stresses in a plain slab foundation, but some approximation can be made with regard to the effect of the circumferential stresses upon the strength of the foundation and the magnitude of these stresses.

If w is the load in lbs. per square inch spread over the foundation sketched in Fig. 55, and CDE is a segment of the circular base including an angle θ radians, the total upward pressure on the portion $ABDE = \frac{1}{2}(R^2 - r^2)\,\theta\, w$, and the bending moment on a vertical section represented in plan by the line AB is approximately

$$\tfrac{1}{2}(R^2 - r^2)\,\theta w \times \frac{R - r}{2} = \tfrac{1}{4}\,\theta w\,(R^2 - r^2)(R - r).$$

Let f be the maximum radial extreme fibre stress at AB, and Cf the average value of the circumferential extreme fibre stress along BD or AE, then the moment of the resistance offered to the upward bending moment on AB is

$$\tfrac{1}{6}\,r\theta d^2 f + \tfrac{1}{6}(R - r)\,d^2 Cf \times 2 \sin\frac{\theta}{2};$$

when θ is small this becomes

$$\tfrac{1}{6}\,r\theta d^2 f + \tfrac{1}{6}(R - r)\,\theta d^2 Cf.$$

This is assuming that the foundation slab is of uniform depth, d, throughout. The value of the constant C depends

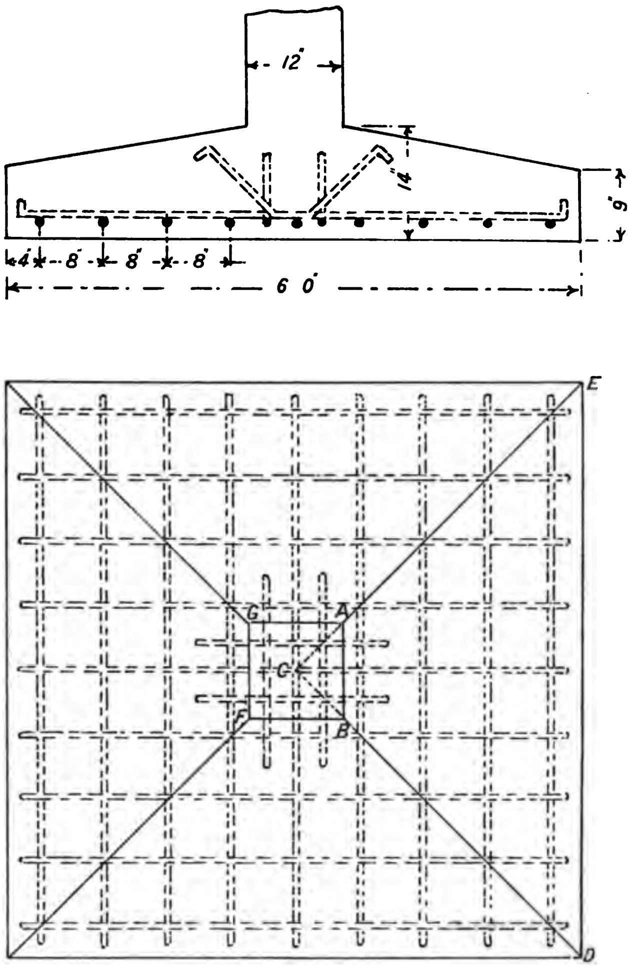


FIG. 56.—Square Column Foundation.

upon the variation in the circumferential stress. If it was possible for the maximum or skin stress along any radius to have a uniform value f from the centre to the circumference, and the radius x of any concentric circle was increased or decreased by an amount δ in consequence of this stress, f could be written $E\frac{\delta}{x}$, which is the same as $E\frac{2\pi\delta}{2\pi x}$. $\frac{2\pi\delta}{2\pi x}$ is the circumferential strain, and, therefore, under the supposed conditions, the circumferential stress is everywhere the same as the radial stress. The bending moment on sections such as AB, FG, &c., can readily be calculated, and the variation from the centre to the circumference is such, that, taking into consideration the usual practice of decreasing the

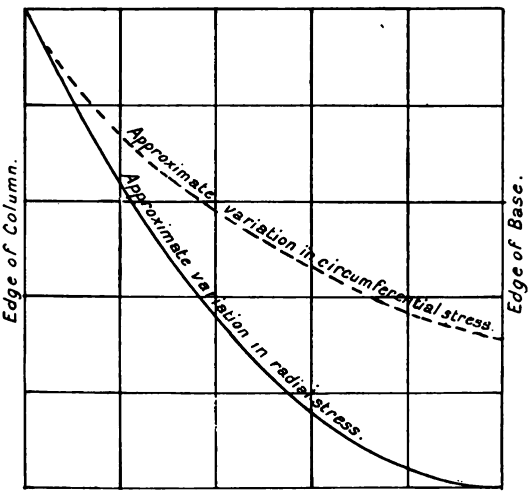


FIG. 57.—Radial and Circumferential Stress.

depth of the slab towards the edges, the value of the maximum radial stress approximately follows the ordinates of a parabola with its axis vertical and its apex at the extreme edge of the slab, as shown in Fig. 57. Under these conditions the circumferential stress varies according to the ordinates of the dotted curve in the same figure, and taking into consideration the decreased depth at the edges the value of the constant C in the formula may without appreciable error be written as ·5. This formula then becomes :—

Moment of resistance $= \frac{1}{6}\left(\frac{R+r}{2}\right)\theta d^2 f$, d being the depth of the foundation at the section considered and $\left(\frac{R+r}{2}\right)\theta$ the average width of the portion of the cantilever beyond this section.

This result, although only approximate, appears reasonable, and, in the case when the forces on half the base are considered, is equivalent to assuming the maximum bending stresses on the cross section through the centre of the foundation as constant throughout the portion of the foundation immediately under the column, and decreasing uniformly in either direction to zero at the edge of the foundation.

In the case of a square column terminating in a square slab the form of the base lends itself to division into four cantilevers as shown in Fig. 56, and consideration of an actual example presents the readiest means of explaining the method of proceeding with the design.

The load on the column is 36 tons, and the safe bearing pressure of the earth 1 ton per square foot. The section of the column is 12 in. square, and the base will need to be 6 ft. sq. Working stresses: 500 lbs. per square inch compression for concrete, 14,500 lbs. per square inch tension for steel. In determining the upward bending moment on a section of the base represented in plan by the side of the column *AB*, the total pressure acting on the under side of the trapezium *ABDE* can, without practical error, be assessed at 9 tons, or 20,160 lbs. acting at the centre of gravity of the triangle *CDE*.

Bending Moment on $AB = 20{,}160\ (24 - 6)$

say 363,000 in. lbs.

The moment of resistance $\cdot 15 \times \frac{12 + 72}{2} \times d^2 \times 500$ reaches this value when $d^2 = 115$ or $d = 10\frac{3}{4}$ in.

The area of reinforcement required per foot

$= \cdot 006 \times 12 \times 10 \cdot 75.$

$= \cdot 77$ sq. in.

$\frac{13}{16}$th in. diameter rods 8 in. apart will obtain this result. To allow for these rods crossing one another at right angles, about 3 in. should be added to the effective depth, making a total of, say, 14 in. Theoretically, the foundation would be sufficiently strong if this was decreased to just the 3 in. at the edges, but in practice this would certainly be increased to 6 in., and generally rather more.

The maximum shear stress will occur under the sides of the column *ABFG* in Fig. 56. The total shear is 35 tons or 78,400 lbs., and the area resisting this $4 \times 12 \times 14 = 672$ sq. in. This intensity of shear stress amounting to 117 lbs. per square inch is too high for the concrete to resist unaided, and steel should be provided to take about three quarters of the total or 58,800 lbs. If the working shear stress for steel is taken at 11,000 lbs. per square inch (about three quarters of the figure for tension), and the shear reinforcement is inclined at 45 deg. so as to increase its efficiency in the ratio of $\sqrt{2} : 1$ the total area of steel required is

$$\frac{58{,}800}{11{,}000 \times \sqrt{2}} = 3 \cdot 8 \text{ sq. in.,}$$

or ·95 sq. in. to each side of the square. Two $\frac{3}{4}$ in. diameter rods would only provide ·88 sq. in., and it will be more convenient to use rods of the same size as the main reinforcement, namely $\frac{13}{16}$th in. in diameter, the area of two of which is 1·04 sq. in.

If the horizontal shear were constant throughout, these turned up rods would have to be repeated at intervals equal to the depth of the slab. Taking into account the diminishing depth of the slab towards the edges, the first of these additional rods would occur at distances of 12 in. from the side of the columns, and in plan their middle points would lie on the sides of a square 3 ft. each way. The total shear on the section indicated by the lines of this square is $(6^2 - 3^2)$ tons, or 56,000 lbs. The total area of the section is $4 \times 36 \times 12 = 1{,}728$ sq. in., and the intensity of the shear stress is therefore 32·4 lbs. per square inch. No further shear reinforcement is required therefore, as the concrete is quite capable of resisting the stress unaided. The complete reinforcement is shown in plan and in sectional elevation in Fig. 56. In the trapezium *ABDE*, for instance, the bars at right angles to AB chiefly resist the radial tensile stresses, and the bars parallel to AB chiefly resist the circumferential tensile stresses. In the neighbourhood of the diagonals, however, both radial and circumferential tensile stresses are resisted by the framework formed by the intersection of the two sets of bars.

Rafts.

In another type of spread foundation the complete structure may be built on one raft. An example of this construction

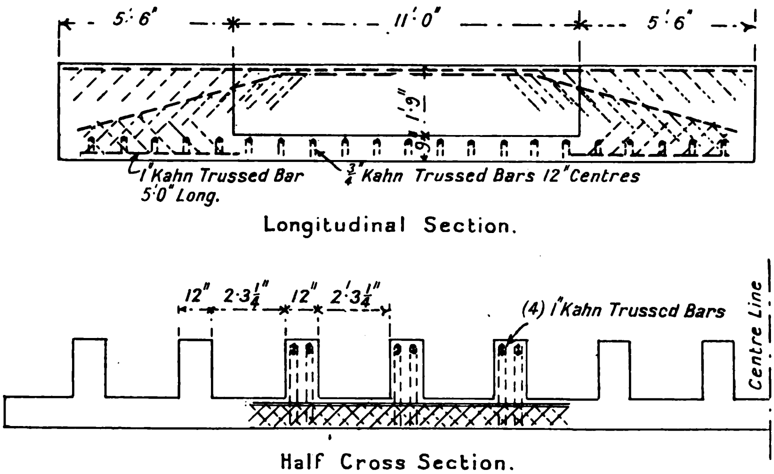


Fig. 58.—Raft.

is seen in Fig. 58, which shows the details of the raft upon which one of the bridges of the Swansea District Lines, Great Western Railway, is built. This is a bridge over a road of about 12-ft. span, and the raft passes under the road from one to the other abutment. The abutments themselves are built on a solid bed of concrete 2 ft. 6 in. deep, but under the road the raft assumes the form of a series of inverted tee beams 2 ft. 6 in. deep, by 12 in. wide, and spaced about 3 ft. 3 in. apart between centres. The thickness of the continuous bottom slab is 9 in. Each main beam is reinforced with four 1-in. Kahn Bars. The slab is reinforced in a transverse direction with ¾-in. Kahn Bars spaced 12 in.

apart between centres. If such a raft is assumed to distribute the load uniformly over the whole foundation, the stresses are readily determined in the same way as for a bridge carrying the same uniform load. The uniform upward pressure takes

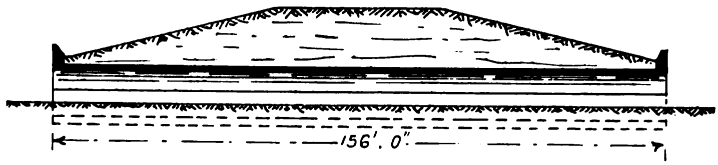


FIG. 59.—Longitudinal Section of Culvert.

the place of the uniform load, and the loads transferred to the raft from the abutments supply the reactions.

UNDERGROUND SUBWAYS.

In any structure of considerable length subject to other than uniform loading, dangerous stresses are likely to be set up owing to differences between the distribution of the load and the upward pressure, unless the foundation is particularly rigid as, for instance, in the case of a rock foundation. Long culverts or subways constructed of mass concrete through high embankments, which are much more heavily loaded at the middle where the bank reaches its full height, than at the ends situated near the feet of the slopes, have not infrequently been found to develop cracks, which are in all probability due to this cause. Fig. 59 illustrates a case in point of a concrete culvert 156 ft. in length, and carrying about twice as much load at the middle as at the ends; and although one abutment is 12 ft. in depth and the other abutment 15 ft. in depth, extensive cracks appeared at the bottom of these near the

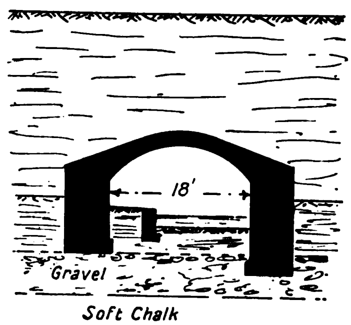


FIG. 60.—Cross Section.

middle of the length before the bank had quite been tipped to its full height of 15 ft. above the top of the arch. The gravel bed upon which the foundations rest overlays a considerable depth of soft chalk, as shown in Fig. 60, and the more heavily loaded portion of the culvert at the centre in all probability settled more than the end portions because the bearing area is practically of constant width throughout the whole length.

The resistance offered by the structure to the bending produced by this unequal settlement distributes the load more uniformly over the whole foundation to such an extent that the moment of the resistance equals the nett moment of the load and the upward pressure. Unless the actual amount of settlement can be ascertained at various points it is not possible to determine the stresses, but in the present case, if

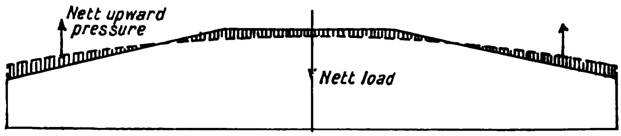


FIG. 61.—Pressure Diagram.

the settlement at the centre was only a quarter of an inch more than at the ends, the distribution of the upward pressure would be approximately as shown by the curved line in Fig. 61, and the maximum tensile stress at the bottom and at the middle of the abutments would be about 13·3 tons per square foot or 200 lbs. per square inch, and quite sufficient to account for the cracks which occurred here. The structure described is practically in the same condition as a ship subjected to a "sagging" moment. In a case of this sort three or four old rails laid longitudinally at the bottom of each abutment, and properly fished or lapped at the joints, would probably prevent cracking, but it is certainly preferable when building a monolithic structure on a yielding foundation to endeavour to proportion the bearing area so that the pressure on the foundation is of uniform intensity. Only light reinforcement would then be required to prevent the opening up of small cracks due to local inequalities of settlement.

CHAPTER VI

RETAINING WALLS

Advantages of Reinforced Concrete Construction.

One of the difficulties experienced in the construction of retaining walls of concrete, not reinforced, is the common occurrence of cracks and the difficulty of preventing these, especially in walls of any length. Such expedients as dividing the length of a wall into sections, separated at the face by a yielding material such as pitch, or embedding old rails in the concrete, have been tried with fair success. The use of reinforcement is able to do much more than merely prevent the formation of cracks, however, and a concrete wall if properly reinforced and provided with an adequate base may be very considerably reduced in thickness and still retain sufficient stability and strength to resist the pressure of the earth retained. Advantages claimed for retaining walls of reinforced concrete are, therefore, the absence of cracks and the lightness of the construction; and a third claim is added that the use of this material results in considerable economy; but it is somewhat doubtful whether this third claim is realised to any great extent in practice. It is true the volume of concrete required is greatly reduced, but the cost of shuttering is increased and also the cost per cube yard of the concrete. The cost of the reinforcement runs away with a large part of the saving effected by the reduction in bulk of the concrete.

Earth Pressure.

It is generally admitted that Rankine's values for the lateral earth pressure on retaining walls are too high. In fact, most of the walls designed by the rules of thumb in common use in practice, if examined on the basis of Rankine's values, would be found to be unstable. Rankine's analysis takes no account of the friction between the earth and the back of the wall as a factor tending to prevent overturning, and although the value of this factor cannot be exactly determined it is quite safe to allow for some additional stability due to this friction. Another method which is coming into use is to determine the maximum reaction which would have to be exerted by the wall to prevent the earth slipping on any plane steeper than the angle of repose.

In Fig. 62 *EAB* represents the angle of repose ϕ, and *AC* any steeper plane making an angle θ with the

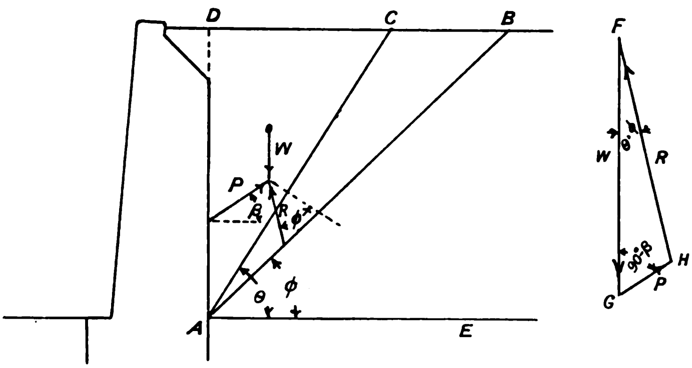


FIG. 62.—Earth Pressure on Retaining Wall.

horizontal along which it is assumed slipping would occur but for the resistance offered by the wall. The forces keeping the wedge of earth *ACD* in equilibrium are its weight *W*, shown as concentrated at the centre of gravity, the resultant reaction, *R*, of the plane *AC* inclined at an angle ϕ to the normal to the plane, and the resultant reaction, *P*, of the wall, inclined at an unknown angle β to the normal to the wall. On the analogy of fluid pressure this force *P* is assumed to act at a point one third of the height of the wall from the base. In the case of a very rough wall it is not unreasonable to assume β equal to ϕ the angle of repose, and this assumption is frequently made. The minimum value of the coefficient of friction for earth on a masonry or concrete surface is probably about ·33 or tan 18 deg. and if β is taken as 18 deg. any error introduced will be on the safe side. By the side of Fig. 62 is shown the triangle of forces FGH in which FG represents the weight, GH the required pressure on the wall and HF the reaction of the plane along which slipping is prevented. The angle FGH is 90 deg.$-\beta$, taken here as 72 deg. The angle GFH is $\theta - \phi$. The value of P, or GH, depends then on the weight of earth and this angle $\theta - \phi$. While $\theta - \phi$ increases of course as θ increases, the weight of earth decreases, and the simplest way of determining for which value of θ the pressure P will have a maximum value is to draw the triangle of forces corresponding to gradually increasing angles and plot the results on squared paper. From this plot, the angle of θ corresponding to the maximum pressure, and the value of this pressure, can be determined with sufficient accuracy for all practical purposes.

With wet earth or wet sand, for which the angle of repose may be taken as about 22 deg., the maximum pressure is obtained when θ is 50 deg., and its value per foot run of wall in terms of w the weight per cubic foot of the earth and h the height of the wall is $\cdot 20\ wh^2$

With dry earth or dry clay for which the angle of repose is about 30 deg., the maximum pressure is obtained when $\theta =$ 55 deg. and its value is

$$\cdot 15\ wh^2.$$

With gravel or stiff clay the angle of repose is about 45 deg., and the maximum pressure is obtained when $\theta = 66$ deg. and its value is

$$\cdot 09\ wh^2.$$

These figures have been calculated on the assumption that the surface of the ground retained is level with the top of the wall.

When the wall to be designed is surcharged the value of the maximum pressure can be determined in the same way to suit the particular condition of surcharge.

It is a very common practice with walls of brickwork or mass concrete to make the thickness at the bottom equal to one third of the height, unless the earth to be retained is very poor and likely to exert excessive pressure, in which case the thickness may be somewhat increased. Such a wall is shown in Fig. 63. It is 20 ft. high above the concrete foundation, and its thickness at the top of the footings is 6 ft., which is just one third of the height above this point. In the most

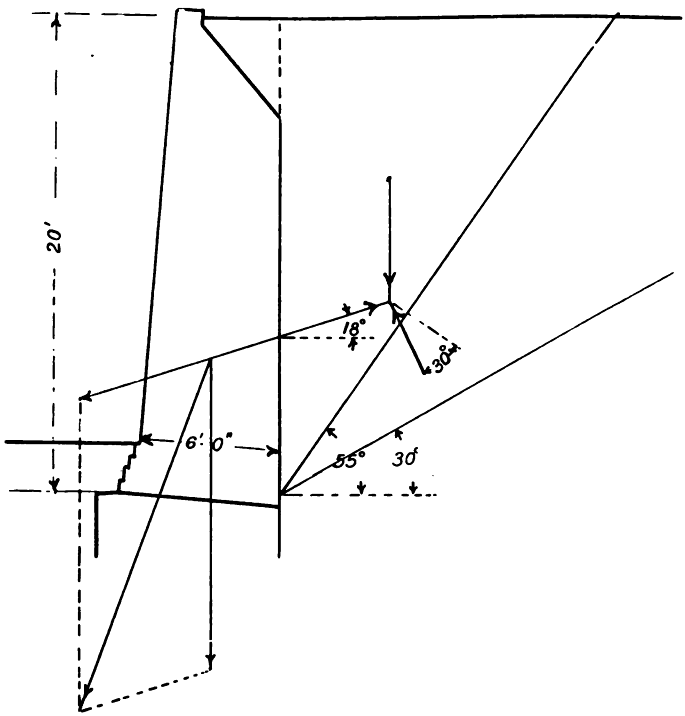


FIG. 63.—Stability of Retaining Wall.

general case the angle of repose will be about 30 degs. and the total earth pressure per lineal foot of wall

$$= \cdot 15 \times 120 \times 20^2$$
$$= 7{,}200 \text{ lbs.}$$

The weight of the wall and its foundation per lineal foot is estimated at 16,200 lbs. The resultant of these two forces is shown on the diagram, and it will be noticed that at the bottom of the wall it lies just outside the middle third of the thickness.

Such a wall as this is known to be quite stable under general conditions, and it would therefore appear reasonable to proportion the parts of reinforced concrete walls in accordance with these estimated values of the lateral earth pressure.

Walls with Counterforts.

Types of reinforced concrete retaining walls are shown in Figs. 64 and 70. These are 20 ft. in height above the base in each case. The counterforts in Fig. 64 are 12 ft. apart and designed as cantilevers, and the walls between them are designed as beams with fixed ends. The surface of the ground is level with the top of the wall, and if the angle of repose of the earth is taken as 30 degs., and its weight 120 lbs. per cube foot, the total pressure on the wall per foot run is

$$\cdot 15 \times 120 \times 20^2 = 7{,}200 \text{ lbs.}$$

inclined at an angle of 18 degs. with the normal to the wall. If this is resolved into two components, normal and parallel to the wall respectively, the normal component is 7,200 cos. 18 degs. = 6,850 lbs.

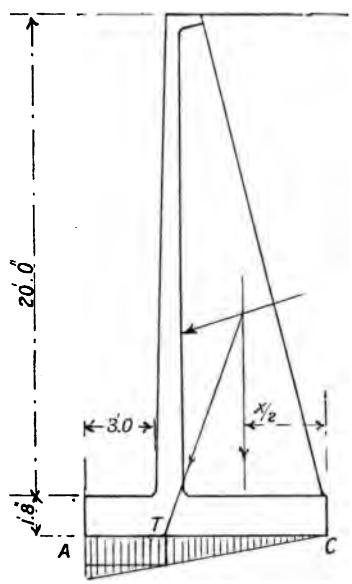


Fig. 64.—Wall with Counterfort.

The average normal pressure per square foot is therefore 6850 ÷ 20 = 343 lbs., and as the maximum pressure at the bottom of the wall is twice the average it is 685 lbs. per square foot. *It is interesting to note that the maximum pressure under the same conditions, as determined by Rankine's formula, is*

$$120 \times 20 \times \frac{1 - \sin 30 \text{ degs.}}{1 + \sin 30 \text{ degs.}}$$
$$= 800 \text{ lbs. per square foot.}$$

This pressure of 685 lbs. per square foot at the bottom of the wall is reduced by 34·3 lbs. per square foot for each foot of height and becomes zero at the top.

Calculations for the wall as a beam with fixed ends, follow :—

Working stresses :—Compression in concrete 500 lbs. per square inch.
Tension in steel 14,500 lbs. per square inch.

At the bottom :—

Span = 12 ft. = 144 in.
Width considered, 12 in.
Total pressure $685 \times 12' = 8{,}220$ lbs.
Maximum bending moment $\frac{8220 \times 144}{12} = 98{,}640$ in. lbs.
Moment of resistance $= \cdot 15 \times 12 \times d^2 \times 500$.
This is equal to 98,640 in. lbs. when $d = 10 \cdot 5$ in.
Reinforcement required, ·6 per cent $= \cdot 006 \times 12 \times 10 \cdot 5 = \cdot 75$ square inches per foot.
For instance, $\frac{9}{16}$ in. diameter rods, 4 in. pitch.

NOTE.—*Near the bottom the wall receives considerable lateral support from the base, and in some designs the rods are spaced further apart right at the bottom than they are a little higher up.*

15 feet from the top :—

Total pressure, $514 \times 12 = 6{,}168$ lbs.

$$\text{Maximum bending moment} = \frac{6168 \times 144}{12}.$$
$$= 74{,}000 \text{ in. lbs.}$$
$$= \cdot 15 \times 12 \times d^2 \times 500.$$
$$d = 9 \cdot 1 \text{ ins.}$$

Reinforcement, $\cdot 006 \times 12 \times 9 \cdot 1 = \cdot 66$ sq. in. per foot.
For instance, $\frac{9}{16}$ in. diameter rods, $4\frac{1}{2}$ in. pitch.

10 feet from the top :—

Total pressure, $343 \times 12 = 4110$ lbs.

$$\text{Maximum bending moment} = \frac{4110 \times 144}{12}.$$
$$= 49{,}320 \text{ in. lbs.}$$
$$= \cdot 15 \times 12 \times d^2 \times 500.$$
$$d = 7 \cdot 4 \text{ in.}$$

Reinforcement, $\cdot 006 \times 12 \times 7 \cdot 4 = \cdot 53$ sq. in. per foot.
$\frac{9}{16}$ in. diameter rods, $5\frac{1}{2}$ in. pitch.

5 ft. from the top:—

Total pressure, $171 \times 12 = 2{,}050$ lbs.

$$\text{Maximum bending moment} = \frac{2050 \times 144}{12}.$$
$$= 246{,}000 \text{ in. lbs.}$$
$$= \cdot 15 \times 12 \times d^2 \times 500.$$
$$= 5\cdot 2 \text{ in.}$$

Reinforcement, $\cdot 006 \times 12 \times 5\cdot 2 = \cdot 37$ sq. in.

$\frac{9}{16}$ in. diameter rods, 8 in. pitch.

To provide the necessary cover for the reinforcement these thicknesses will have to be increased by about two inches.

It will be noticed that the required effective thickness of the wall varies with the square root of the bending moment; and therefore with the square root of the distance from the top of the wall. These thicknesses are shown in diagram form in Fig. 65 and follow the ordinates of a parabola having its apex at the top of the wall. It is obviously impossible to vary the thickness of the wall in this way in practice and the difficulty may be got over by making the lower half of the wall of uniform thickness and tapering the top half, or by allowing a somewhat greater thickness than is actually required at the base and decreasing the thickness of the wall uniformly throughout. This last method has been adopted in the present instance, the thickness of the wall near the top being $6\frac{1}{2}$ in. overall and at the bottom 13 in. overall as shown in Fig. 65. Except at the top, the amount of concrete, over and above that which is theoretically necessary, is very small.

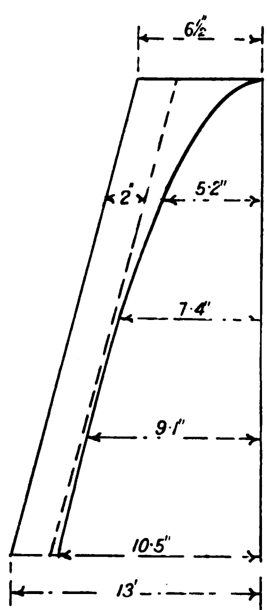


FIG. 65. — Theoretical Thickness of Wall.

The horizontal reinforcing rods at the front face may be jointed at the counterforts, and other rods provided at the back to resist the negative bending at these points due to continuity of beam. The length of these should be about ·4 times the spacing between the counterforts or in this case 5 ft. This is generally found to be more satisfactory than bending the one set of rods from the front to the back of the wall where the counterforts occur. The stress in these short rods is developed very quickly, so that it is important to make their diameter sufficiently small to provide sufficient surface to keep the bond stress within working limits. With $\frac{9}{16}$ in diameter rods the area is ·25 sq. in. and the maximum tensile stress.

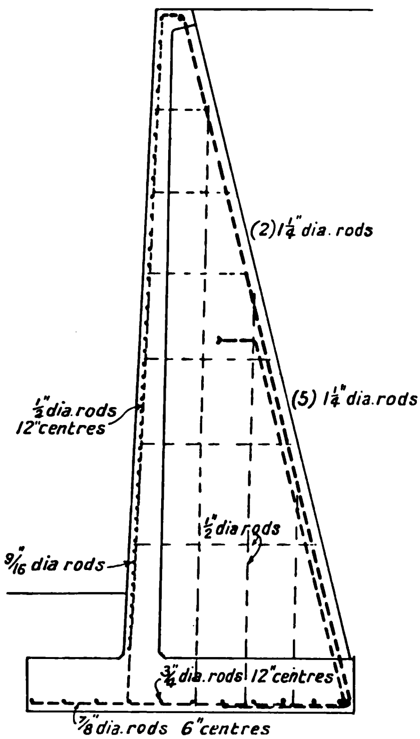


FIG. 66.—Reinforcement of Wall with Counterforts.

$$·25 \times 14{,}500 = 3{,}620 \text{ lbs.}$$

The circumference of a $\frac{9}{16}$ in. diameter rod is ·177 in. and the length in which the maximum stress is developed is 30 in. The adhesion required to be developed between the concrete and steel is, therefore,

$$\frac{3620}{30 \times 1·77} = 68 \text{ lbs. per square inch.}$$

The vertical rods shown in Fig. 66 serve a similar purpose to the transverse reinforcement in slabs, and help to distribute excessive local pressures. They also resist stresses due to uneven settlement, and prevent the formation of cracks which might otherwise arise from this cause.

Design of Base.

Certain of the dimensions of the wall shown in Fig. 66 have to be determined in an arbitrary manner. For instance, the width of the projecting toe, the depth of the bottom slab, and the total width of the bottom slab, are interdependent quantities, and either the first or the last of these three dimensions must be assumed before the others can be determined. In the present case the projection of the toe was made 3 ft., and the thickness of the slab, which cannot be exactly determined at this stage, assumed to be 20 in. The total amount of earth pressure on a 1 ft. length of the wall is 7,200 lbs. inclined at an angle of 18 degs. with the horizontal, and acting at a point ($20 \div 3$) 6 ft. 8 in. from the top of the slab, or 8 ft. 4 in. above the bottom of the slab. If the total width of the base slab is $(3 + x)$ feet, the weight of the wall and earth resting on it, taking concrete and earth all at 120 lbs. per cube foot, is approximately $21{\cdot}7 \times x \times 120$ lbs. $= 2600\,x$ acting at a distance $\frac{x}{2}$ ft. from the back of the slab. The earth pressure may be resolved into a horizontal component 7200 cos. 18 degs. = 6850 lbs., and a vertical component 7200 sin. 18 degs. = 2220 lbs. These forces are shown in diagram form in Fig. 64, and in order that the wall may be in equilibrium the moment of the weight about the one-third point of the base marked T in Fig. 64 must be at least equal to the overturning moment of the earth pressure about the same point. That is

$$2600x\left(\frac{6+2x}{3} - \frac{x}{2}\right) = 6850 \times 8{\cdot}33 + 2220\left(\frac{6+2x}{3} - x + 1\right)$$

or $x = 7{\cdot}1$ ft., say 7 ft. 1 in. The weight of the concrete toe is neglected, and is something to the good in maintaining the stability of the structure. The stresses in the base can now be determined.

As the wall is designed to be just stable there will be no upward pressure on the base at the back edge C, and the maximum pressure at the front edge A will be just twice the average.

The weight of earth resting on
a 1 ft. length of the wall is . $20 \times 6 \times 120 = 14{,}400$ lbs.
The weight of the wall itself is $\left.\begin{array}{r} 20 \times 8 \times 156 \\ 10{\cdot}1 \times 1{\cdot}67 \times 156 \end{array}\right\} = 5{,}100$ lbs.

19,500 lbs.

The average pressure on the foundation is, therefore, $19{,}500 \div 10{\cdot}1 = 1930$ lbs. per square foot and the maximum pressure at A, 3860 lbs. per square foot. At the front of the wall the pressure indicated by the shaded diagram in Fig. 64 is resisted by the projecting toe acting as a cantilever, the effective length of which may be taken as 3·5 ft. At a point 3·5 ft. back from the end the pressure $= \frac{6{\cdot}6}{10{\cdot}1} \times 3860 =$ 2520 lbs. per square foot. The maximum bending moment produced at the assumed support of the cantilever by this pressure, of varying intensity, is readily obtained by dividing the pressure diagram into a rectangular portion whose ordinate is 2520 lbs., and a triangular portion with a maximum ordinate of $3860 - 2520 = 1340$ lbs. The value of this bending moment is—

$$\tfrac{1}{2} \times 2520 \times 3{\cdot}5 \times 3.5 = 15400 \text{ ft. lbs.} = 184{,}800 \text{ in. lbs.}$$
$$\tfrac{1}{2} \times 1340 \times 3{\cdot}5 \times \tfrac{2}{3} \times 3{\cdot}5 = 5500 \text{ ft. lbs.} = 66{,}000 \text{ in. lbs.}$$

250,800 in. lbs.

Adopting the same working stresses as before, namely, 500 lbs. per square inch for the maximum compressive stress on the concrete, and 14,500 lbs. per square inch for the tensile stress in the steel, the moment of resistance can be written

$$\cdot 15 \times 12 \times d^2 \times 500 \text{ in. lb.}$$

and this is equal to 250,800 in. lbs. when $d = 16{\cdot}7$ in. The dimension of 1 ft. 8 in. assumed for the thickness of the bottom slab, therefore allows plenty of cover for the transverse reinforcement, and the longitudinal rods placed above. The latter prevent the opening up of cracks, and resist stresses, due to the upward pull exerted by the tension members of the counterforts, and uneven settlement. The rods provided for this purpose may be $\frac{3}{4}$ in. in diameter and 12 in. apart.

With the working stresses adopted, the transverse reinforcement will need to be ·6 per cent. The area of steel required per foot run, therefore, is $·006 \times 12 \times 16·7 = 1·2$ sq. in. This area is provided by two $\frac{7}{8}$ in. diameter bars spaced 6 in. apart, and these are carried right through to the back of the slab.

Design of Counterfort.

The counterforts which are 12 ft. apart are treated as cantilevers of T section. On account of their great depth the

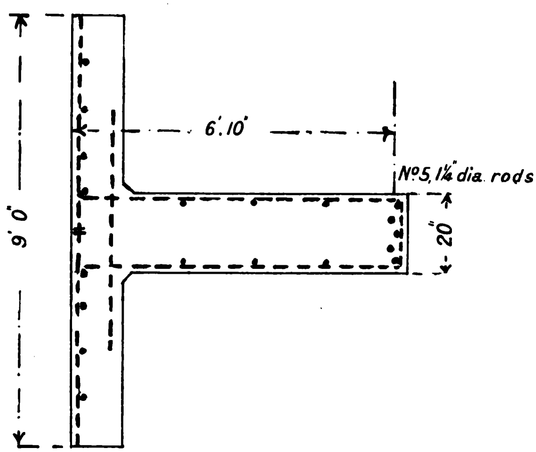


Fig. 67.—Section of Cantilever.

percentage of reinforcement required will be but small, and the neutral axis will therefore in all probability lie within the slab. The effective width of the slab may be taken as three quarters of the distance between the counterforts, that is 9 ft. The effective depth, allowing 3 in. from the back of the counterfort to the centre of the reinforcement is 6 ft. 10 in. at the plane of maximum bending moment where the counterfort joins the base. The effective section of the counterfort at this plane is shown in Fig. 67. As the percentage of reinforcement required will be but small, the stress in the steel will be high compared with the compression in the concrete. Under these circumstances the moment of resistance of the cantilever at the section considered will be determined by the working stress in the steel, and equals $K \times 108 \times 82^2 \times 14{,}500$. The maximum bending moment is

$$12 \times 6850 \times 80 = 6{,}576{,}000 \text{ in. lbs.}$$

Therefore $K = \cdot 00062$, and from Fig. 10, the percentage of reinforcement will need to be ·068 per cent., and the value of y/d is ·13. The neutral axis is therefore $\cdot 13 \times 82 = 10{\cdot}7$ in. from the front of the wall, that is, within the thickness of the wall as assumed. ·068 per cent. of $108 \times 82 = 6{\cdot}0$ sq. in., number 5 rods 1¼ in. in diameter is 6·1 sq. in., and the stress in the steel will then be $\frac{6\cdot 0}{6\cdot 1} \times 14{,}500 = 14{,}300$ lbs. per square inch. The arrangement of the five 1¼ in. diameter rods is shown in Fig. 67, and to accommodate these the width of the counterfort is made 20 in.

At a plane halfway up the height of the counterfort the effective depth is reduced to $\frac{82 + 15}{2} = 48{\cdot}5$ in., and the effective width can only be reasonably assumed to be one half of that taken at the bottom, that is 4 ft. 6 in. = 54 in. The earth pressure on a 12 ft. length of the wall above this section is $12 \times \frac{6850}{4} = 20550$ lbs., and the bending moment at the section is $20550 \times 40 = 822{,}000$ in. lbs. The calculation of the strength of the counterfort at this point proceeds as follows :—

$$\begin{aligned} K &= 822{,}000 \div (54 \times 48{\cdot}5^2 \times 14{,}500) \\ &= \cdot 00045 \end{aligned}$$

corresponding to ·05 per cent. of reinforcement and a value of $y/d = \cdot 115$ (Fig. 10).

The neutral axis is therefore ($\cdot 115 \times 48{\cdot}5$) 5·6 in. from the front of the wall and still within its thickness which is about 10 in. halfway up the wall,

$$\cdot 05 \text{ per cent. of } 54 \times 48{\cdot}5 = 1{\cdot}31 \text{ sq. in.}$$

It is not necessary then for all the rods to be taken to the top of the counterfort. In Fig. 66 the two inner rods are stopped about halfway and the ends turned inwards.

At the section of the counterfort adjoining the base the effective area as regards shear resistance is

$$20\,(82 - 3{\cdot}6) = 1570 \text{ sq. in.}$$

The total shear at this section is $6850 \times 12 = 82{,}200$ lbs.

$$82{,}200 \div 1570$$
$$= 52 \text{ lbs. per square inch.}$$

Additional resistance to shear is provided by the horizontal and vertical $\frac{1}{2}$ in. diameter rods shown in Figs. 66 and 67. These rods embrace the main reinforcement of the counterforts, and their ends are carried right into the wall and base respectively, and bent to obtain anchorage.

The stress in the *main* reinforcing rods reaches its maximum value at the bottom where these terminate, so that it is very essential that these rods should be well anchored to the base. A convenient means of doing this is to bend the ends to pass under the longitudinal rods in the base and splice them with soft iron wire to the transverse reinforcement. This upward pull, exerted by the reinforcement of each cantilever upon the base, balanced as it is by the weight of earth resting upon the base, sets up longitudinal bending action on the base, which is resisted by the longitudinal rods of $\frac{3}{4}$ in. diameter and 15 in. apart.

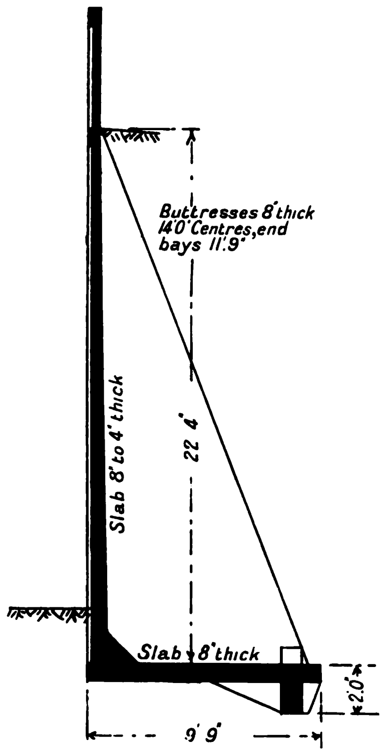


Fig. 68.—Wall with Counterforts.

Walls of the counterfort type are sometimes designed without any projecting toe, and there is then no need to make the base very thick, as the pull of the counterforts upon the back of the slab can be met by the provision of a longitudinal beam.

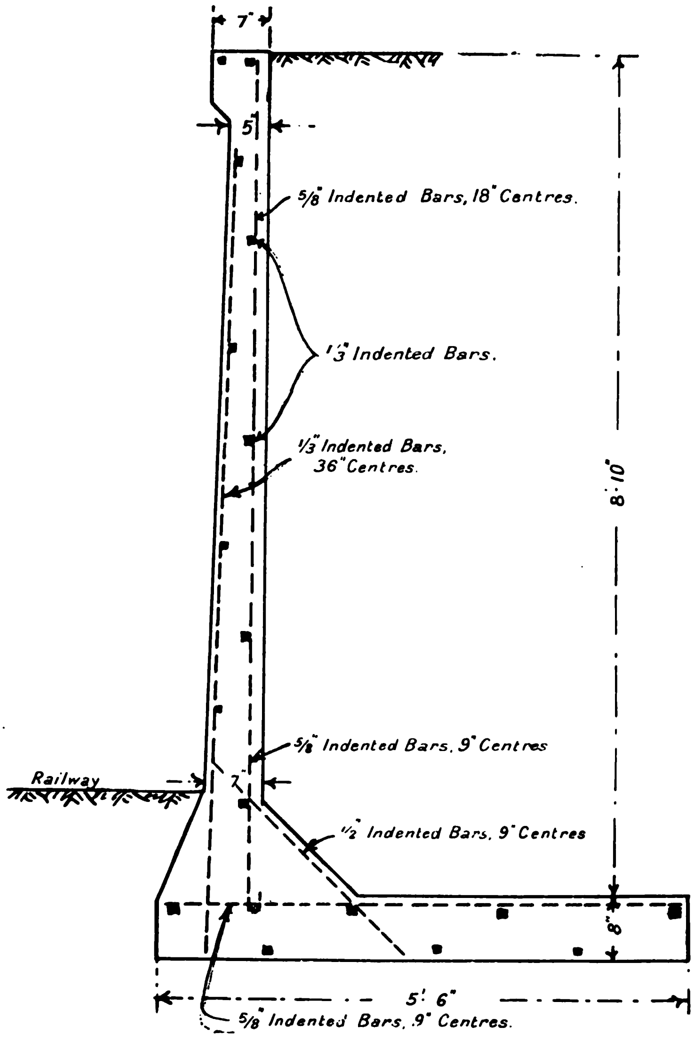


FIG. 69.—Retaining Wall at Fenton.

Such a wall is shown in section in Fig. 68. This wall was designed by the Indented Bar and Concrete Engineering Co.

and reinforced with Indented Bars. It is 20 ft. high above ground level, exclusive of the parapet, and the counterforts are 14 ft. apart, centre to centre, with the exception of the end bays, which are 11 ft. 9 in. The width of the base is 10 ft. The front wall is only 8 in. thick at the bottom and 4 in. at the top, there being compression reinforcement in addition to the tension reinforcement. The thickness of the base slab and of the counterforts is 8 in. The beam at the back of the base is 2 ft. deep overall and 10 in. wide.

Walls without Counterforts.

Another wall designed by the same Company is shown in section and part elevation in Fig. 69. It was constructed at Fenton, on the North Stafford Railway, to retain a road running alongside the rails. This wall is 900 ft. in length and 9 ft. 6 in. deep overall. It is of the plain cantilever type, without counterforts, and is 7 in. thick at the bottom and 5 in. near the top below the coping. The bottom slab is 5 ft. 6 in. wide and 8 in. thick. This wall is reinforced with Indented Bars as shown in the illustrations, and the parapet, which is not shown, consists of iron railings fixed to the top of the wall.

These types of wall are most suitable to retain a tipped bank, because, if used in a cutting, there would be so much excavation which would afterwards have to be replaced. Moreover, if built entirely on the Railway Company's property, there would be a considerable waste of space. The wall shown in Fig. 70 is free from these disadvantages, the excavation all being useful and the base entirely on the railway side of the wall and below the formation level. The pressure on the foundation is also considerably less, but this loss of weight necessitates the provision of a wider base to ensure stability. The calculations are comparatively simple, a convenient length of the wall, say 1 ft., being designed as a cantilever.

Maximum bending moment at the base $= 6{,}850 \times 80 =$ 548,000 in. lbs.

Moment of resistance $\cdot 15 \times 12 \times d^2 \times 500$

$d = 24\cdot 7$ in.

·6 per cent. of reinforcement = ·006 × 24·7 × 12 = 1·78 sq. in. per foot.

No. 3, $\frac{7}{8}$ in. diameter bars 4 in. centres will suffice.

The thickness of the wall at the bottom is made 2 ft. 3 in. to allow the bars to be imbedded 2 in. in the concrete.

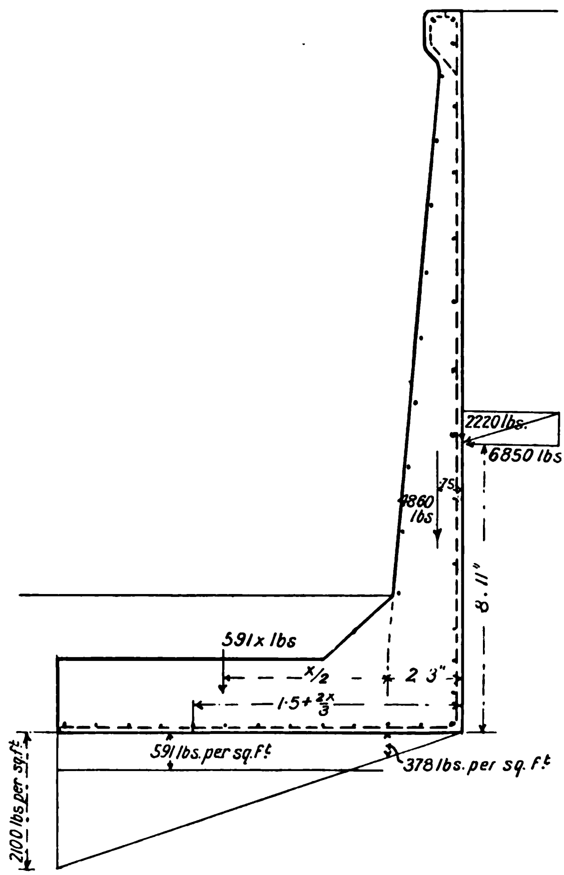


Fig. 70.—Wall without Counterforts.

The bending moment at any horizontal section through the wall is proportional to the cube of the distance from the top of the wall. The required thickness of the wall is proportional to the square root of the bending moment, and therefore to the square root of the cube of the distance from the top of the wall, and the theoretically correct section is shown in Fig. 71. Obviously this cannot be adopted in practice, and the thickness of the wall drawn in Fig. 70 is 6 in. at the top, uniformly increasing to 2 ft. 3 in. at the bottom. The back of the wall is vertical and the front face battered, and the latter is corbelled out at the top to form a coping, and splayed at the bottom to form a strong gusset between the wall and its base. It is not necessary to carry all the rods up to the top of the wall, and the levels at which alternate rods in the first place, and three out of four rods in the second place, may be discontinued, can only be determined by trial.

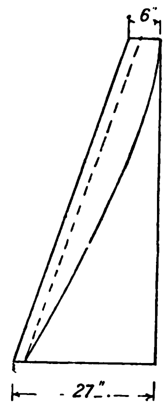


FIG. 71.—Theoretical Thickness of Wall.

For instance, halfway down the wall the total thickness is 1 ft. $4\frac{1}{2}$ in., and the effective thickness 14·5 in. The bending moment is $548{,}000 \div 2^3 = 68{,}500$ in. lbs.

If the rods were spaced 16 in. centres here the percentage of reinforcement would be

$$\frac{100}{16 \times 14{\cdot}5} \times {\cdot}6 = {\cdot}26 \text{ per cent.}$$

As this is less than the ·6 per cent. corresponding to the working stresses of 500 lbs. per square inch in the concrete, and 14,500 lbs. per square inch in the steel, adopted in designing the wall, the reinforcement will be the weaker member.

The value of K corresponding to ·26 per cent. of reinforcement is ·0024 (Fig. 10). Therefore

$$\cdot 0024 \times 12 \times 14{\cdot}5^2 \times f_s = 68{,}500$$

$$\text{and } f_s = 11{,}300 \text{ lbs. per square inch.}$$

Fifteen feet from the top of the wall the bending moment is $548{,}000 \times (\frac{3}{4})^3 = 231{,}000$ in. lbs. per foot run. The effective thickness of the wall is 19·8 in., and the percentage of reinforcement, if the rods are spaced 8 in., $\frac{100}{8 \times 19{\cdot}8} \times {\cdot}6 = {\cdot}38$ per cent.

$$K = {\cdot}0035 \text{ (Fig. 10).}$$

$$\cdot 0035 \times 12 \times 19{\cdot}8^2 \times f_s = 231{,}000$$

$$f_s = 14{,}000 \text{ lbs. per square inch.}$$

Accordingly the 4 in. spacing may extend for the bottom 5 ft. of the wall, the 8 in. spacing from 5 to 10 ft. up, and the 16 in. spacing for the top half.

Design of Base.

In a wall of this kind stability is obtained by the weight of the wall itself and the earth resting on the base and by the friction between the earth and the back of the wall. These forces are indicated in diagram form in Fig. 70.

The thickness of base cannot be exactly determined until the width has been ascertained, but it must be at least as thick as the wall at the bottom. If it is assumed to be of equal thickness to the wall at this point, namely 2 ft. 3 in., the weight per square foot will be approximately as follows:—

Concrete	2·25	× 156	=	351 lbs.
Earth	2·0	× 120	=	240 lbs.
		Total		591 lbs.

If the required width of the base to prevent overturning is x feet clear of the thickness of the wall, its weight per lineal foot of wall is

$$591x \text{ lbs.}$$

The weight of the wall per lineal foot is

$$22{\cdot}25 \times 1{\cdot}4 \times 156 \text{ lbs.} = 4{,}860 \text{ lbs.}$$

The horizontal component of the earth pressure per lineal foot of wall has already been determined, and is

6,850 lbs.

while the vertical component is also the same as in the case of the wall with counterforts, namely

2,220 lbs.

The resultant of this earth pressure is assumed to act at a point 6 ft. 8 in. from the top of the base, or 8 ft. 11 in. from the underside. Taking moments as before, about the third point of the base nearest to the toe, which is

$$\left(1{\cdot}5 + \frac{2x}{3}\right) \text{ ft.}$$

from the back of the wall

$$4{,}860\left(1{\cdot}5+\frac{2x}{3}-{\cdot}75\right)+591x\left(1{\cdot}5+\frac{2x}{3}-2{\cdot}25-\frac{x}{2}\right)+$$
$$2{,}220\left(1{\cdot}5+\frac{2x}{3}\right)=6{,}850\times 8{\cdot}92,$$

which equation is satisfied when

$$x=10{\cdot}2, \text{ say } 10 \text{ ft. } 3 \text{ in.},$$

making the total width of the base of the wall 12 ft. 6 in. The sum total of all the vertical forces is

	lbs.
	4,860
$591\times 10{\cdot}25=$	6,060
	2,220
	13,140

and the average pressure on the foundation

$$13{,}140\div 12{\cdot}5=1{,}050 \text{ lbs. per square foot.}$$

The distribution of this pressure is shown in Fig. 70, and varies from zero at the heel to a maximum intensity of 2,100 lbs. per square foot at the toe. The maximum stress produced in the base by this pressure, the resultant of which acts at the third point nearest the toe, less the weight of the base and the earth upon it, will occur just clear of the wall. The intensity of the upward pressure at this point is $\frac{2{\cdot}2}{12{\cdot}5}\times 2{,}100=378$ lbs. per square foot.

The calculation of the bending moment is simplified by deducting this 378 lbs. per square foot from the upward pressure throughout and deducting a like amount from the weight of the concrete and earth, which is 591 lbs. per square foot, to balance matters. This leaves on the 10 ft. 3 in. width of the base in front of the section considered an upward pressure of uniformly increasing intensity varying from 0 to $(2{,}100-378)=1{,}722$ lbs. per square foot and a uniform load of $(591-378)=213$ lbs. per square foot. The bending moment per lineal foot due to these forces is

$\frac{1}{2} \times 1{,}722 \times 10{\cdot}25 \times \frac{2}{3} \times 10{\cdot}25 =$

60,300 ft. lbs. = 723,600 in. lbs.

less $213 \times 10{\cdot}25 \times \frac{1}{2} \times 10{\cdot}25 =$

11,200 ft. lbs. = 134,400 „ „

589,200 „ „

As would be expected this is not much in excess of the maximum bending moment produced in a corresponding length of the wall, which was found to be

548,000 in. lbs.

and to require a thickness of 2 ft. 3 in. of concrete reinforced with $\frac{7}{8}$ in. diameter rods, spaced 4 in. centres, and as additional strength is given by the fillet connecting wall and base, the latter will be amply strong if made 2 ft. 3 in. in thickness and reinforced by bending at right angles the vertical rods in the wall.

As before auxiliary rods will be required at right angles to the main reinforcement and these may be of $\frac{1}{2}$ in. diameter and 12 in. apart. Of these, some at any rate should be placed near the front face of the wall to prevent the opening up of surface cracks due to shrinkage, temperature changes, or variation in the lateral pressure. Often vertical rods are placed here too, as in the case of the wall illustrated in Fig. 69. The wall is then calculated as a cantilever with double reinforcement. In a wall of the type shown in Fig. 69, that is without counterforts and the base behind the wall, the weight of earth resting on the base is greater than the upward pressure at the back of the base and the transverse reinforcement of the base slab is required near the top surface.

Relative Cost.

The relative cost of these different types of retaining wall naturally depends very largely on the individual case and its locality, but comparative estimates based on average prices give some indication of the relative merits of the various types as regards economy.

The estimated cost per lineal yard of the brick wall, 20 ft.

high, illustrated in Fig. 63, suitable for retaining a tipped bank, is arrived at as follows :—

			£	s.	d.
2½ cube yards,	Excavation in foundations	@ 2/6		6	3
1⅓ " "	Concrete in foundations .	@ 12/-		16	0
10¾ " "	Brickwork in lime mortar .	@ 17/-	9	2	9
6 square yards	Extra for facing with blue brick	. @ 2/-		12	0
3 lineal feet,	Blue brick coping .	. @ 1/6		4	6
⅐ cube yard,	Cement concrete splay .	. @ 20/-		2	10
	Estimated cost per lineal yard		£11	4	4

In comparing this with the estimated cost of the reinforced concrete wall with counterforts 12 ft. apart, of which Fig. 66 is a section, it is convenient to consider a 12 ft. length of the wall.

			£	s.	d.
16½ cube yards,	Excavation in foundations	@ 2/6	2	1	3
19 " "	Cement concrete . .	. @ 28/-	26	12	0
1 ton	Steel rods . .	. @ £13	13	0	0
70 square yards,	Shuttering . .	. @ 1/9	6	2	6
			£47	15	9
	Estimated cost per lineal yard		£11	19	0

This works out at 1*s.* 10½*d.* per cube foot of concrete all complete, and must be regarded as a low if not a minimum price. Under favourable circumstances a mass concrete wall, similar in section to Fig. 63, can be built for less than £9 per lineal yard, and consequently there would not appear to be any great economy to be obtained by adopting reinforced concrete for the construction of retaining walls, or similarly the abutments of bridges.

In the case of a retaining wall built in a cutting, where a large amount of the excavation must be strongly timbered, the cost per lineal yard naturally increases. In cutting, the brick wall (Fig. 63) is estimated to cost £15 7*s.* 0*d.* per lineal yard, while the cost of the same length of the L shaped

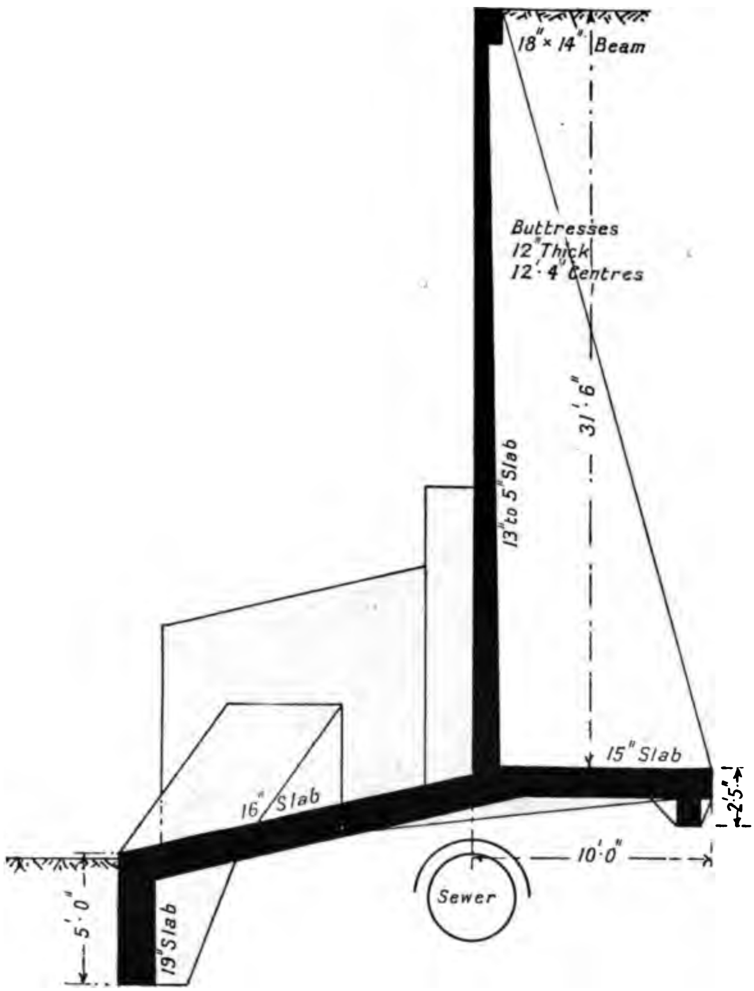


FIG. 72.—Combined Wall and Coal Bunkers.

reinforced concrete wall, drawn in Fig. 70, is estimated as follows:—

			£	s.	d.
31 cube yards,	Excavation in timbered trenches . . .	@ 3/6	5	8	6
6·6 „ „	Cement concrete . . .	@ 28/-	9	4	9
7 square yards,	Shuttering . . .	@ 1/9		12	3
5·1 cwt.	Steel rods . . .	@ 13/-	3	6	3
	Estimated cost per lineal yard		£18	11	9

This works out at 2*s.* 1*d.* per cube foot of concrete all complete. The estimated cost of this wall, exclusive of the additional excavation, is £14 17*s.* 0*d.* per lineal yard, and is therefore not as economical as the type of wall previously considered, in which the weight of earth resting on the base assists in preserving stability. The only advantage in using an L shaped wall is economy of space or land which, under certain circumstances, might justify the increased cost of construction.

Combined Wall and Coal Bunker.

An interesting example of a reinforced concrete retaining wall for railway purposes is illustrated in Fig. 72. This wall, which is reinforced with Kahn Bars, retains coal sidings adjoining a boiler house, and the projecting toe is designed as a series of coal bunkers. The construction was made more difficult by the existence of a sewer practically on the line of the wall which was protected on top by ordinary concrete. The wall is 31 ft. 6 in. high, and the counterforts are 12 ft. 4 in. apart. The thickness of the vertical slab varies from 13 in. at the bottom to 5 in. at the top. The projecting toe is 16 in. thick, but this is strengthened transversely every 12 ft. 4 in. by the partition above, and longitudinally throughout by the beam at the toe. The base slab at the back of the wall, which is 15 in. thick, is also strengthened by a longitudinal beam as shown. The total width of the base is nearly 25 ft., 15 ft. of this being in front of the wall.

CHAPTER VII

BRIDGES

Types.

There are very few examples of reinforced concrete bridges carrying lines of railway in this country, but a fair number of road bridges over various railways. Many of these have a very pleasing appearance and serve to show that it is by no means unavoidable that a reinforced concrete bridge should be an eyesore.

In this chapter some examples of bridges consisting of beams supported by abutments or columns will be considered, and arched bridges will be treated in the following chapter. The simplest type of bridge consists of beams running longitudinally with the ends resting upon abutments of mass concrete or brickwork. A bridge of this description costs about the same as a brick arch bridge of the same span, width and height and probably 25 per cent. less than a bridge consisting of longitudinal steel girders and jack arch flooring. A reinforced concrete bridge thus possesses the advantages that it can be constructed in a less depth from rail level to road level than a brick arch, and at the same first cost, and with the same freedom from subsequent maintenance charges. In larger span bridges where the depth available for construction is small the parapets may be designed as beams with double reinforcement, and the road carried on transverse beams or slab floor.

Footbridge.

Footbridges can be economically designed on this principle, because in this case the span of the floor from parapet to parapet is quite short and requires but little depth, thus effecting a saving in the number of steps required. The elevation and cross

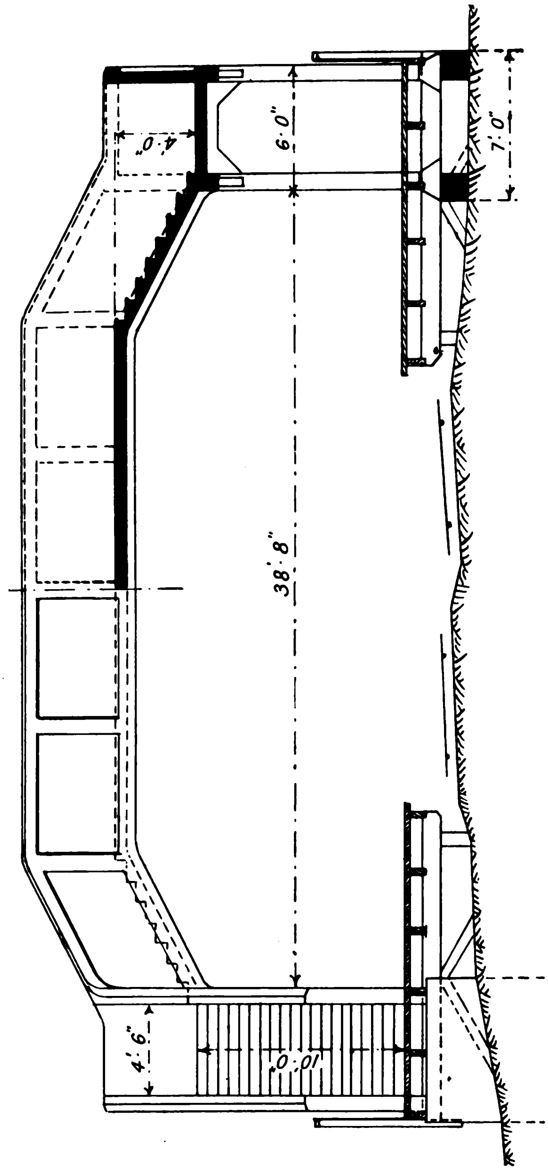


FIG. 73.—Elevation of Footbridge.

section of a station footbridge of this kind are shown in Figs. 73 and 74. A further economy is effected by ramping the ends of the girders and placing some of the steps at these points. In this way the remaining steps at each end can be

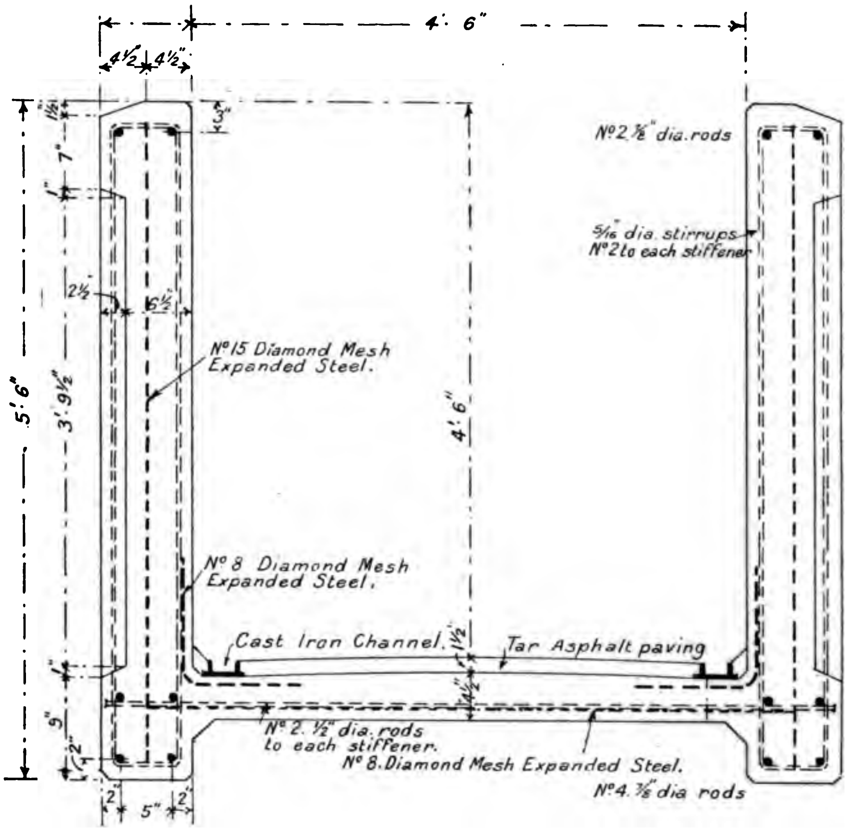


FIG. 74.—Cross Section of Superstructure.

arranged in one flight, which will not exceed in height the maximum of 10 ft. allowed by the Board of Trade Regulations.

Of the four $\frac{7}{8}$ in. diameter rods in the lower member, two are turned up at each end, one in each of the two end panels, which they cross diagonally. There are also vertical stirrups of $\frac{5}{16}$ in. diameter, to each stiffener and expanded steel of light section throughout the web, for the shear stress in a beam of this span is somewhat high.

The floor is reinforced with No. 8. Diamond Mesh Expanded Steel, the cross sectional area of which is equal to ·6 per cent. of the effective area of the concrete. The calculation of the main beams presented some difficulty on account of their arched form coupled with the absence of abutments rigid enough to resist the thrust arising from arch action.

In the absence of this rigidity there is no doubt that the bending moment at the centre is very nearly as great as it would be in the case of a straight beam, and the bending moment at this point was calculated with the aid of the formula,

$$\frac{W\,l}{10}$$

the factor 10 being adopted in view of the partial fixture obtained at the ends.

It is with regard to the stresses at the ends that some doubt exists, because in a girder with fixed ends the bending would be negative here, that is, tension at the top, while in an arch of similar form to the main beams of this footbridge, with fixed ends, the bending would be positive at these ends, that is, tension at the bottom. Furthermore the absence of rigidity does not affect the stresses at the ends so much as at the centre, so that it appears likely that the stresses at the ends are not excessive.

To test this the author made some scale models of the beams in plaster of Paris and loaded these in the manner indicated in Fig. 75 until they failed. In the first model (Fig. 75) the section was uniform throughout. In the second (Fig. 76) the depth was decreased at the ends, in the ratio of $1 : \sqrt{2}$ so that the strength at the ends was just half the strength at the centre. The beams were supported at the ends on glass plates which in turn rested upon several thicknesses of cloth, so that there was perfect freedom for the ends to move laterally. In loading the beams at the third points, as shown in Fig. 75, as much load was placed in equal portions upon the bearings, as at the centre, in order to obtain a certain degree of fixture, as regards bending, at the ends. The method of failure is shown in the sketches, and the fact that in each case fracture occurred simultaneously under or near each point of support indicates that plaster of Paris is quite a reliable material for the purpose, and capable of giving uniform results. In the second model the crack at the bearing occurred after the failure at the points of application of the load, and was caused by the weights falling on to the left-hand end of the beam after the middle portion had given way. These results indicate that the bending moment at the end in this particular case is less than half that at the centre, and rather confirms the theory that

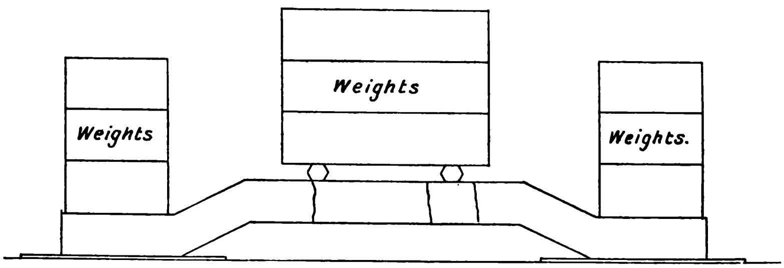


Fig. 75.—Model Beam of Uniform Section.

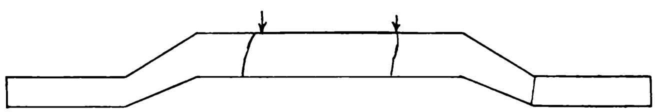


Fig. 76.—Model Beam with Reduced Section at Ends.

arch action tends to diminish the bending moment at the ends of a beam of this form.

In the footbridge illustrated in Figs. 73 and 74, there are two $\frac{7}{8}$ in. diameter rods at the top and two at the bottom, near the ends, and in the strings to the lower flights of steps, which are of similar section, there are two $\frac{7}{8}$ in. diameter rods at the bottom and one at the top. The percentage of reinforcement is somewhat small, because the section of the concrete is practically the minimum that could be adopted to provide a parapet of the required height and possess the necessary lateral stiffness. The compressive stresses in the concrete, therefore, are not high and are calculated to vary between 300 and 400 lbs. per square inch.

Specification.

The specification for the concrete provides for one measure of Portland cement to four measures of granite or whinstone from an approved quarry properly graded and broken to such a size as to be left on a screen with a $\frac{3}{16}$ in. mesh and capable of being passed in any direction through a ring $\frac{3}{4}$ in. in internal diameter, including only a sufficient quantity of clean, sharp sand to fill all cavities.

The steps were specified to be finished while green with a 1 in. layer of cement mortar mixed with $\frac{3}{8}$ in. screened Hard York non-slip stone chippings.

Notes on the drawing and clauses in the specification provide for "all reinforcement to be rigidly fixed in position so that it cannot be disturbed while the concrete is being placed. No steel to have at any point less than 1 in. of concrete covering. All bars where they intersect to be bound together with annealed wire about $\frac{1}{16}$ in. in diameter. The boxings of columns and side-boards of beams must not be removed until the concrete has had at least one week to set. The centering must not be struck until at least six weeks after the completion of the work." This period is rather longer than usual, but was considered advisable in view of the position of the structure relatively to running lines of railway. To allow for the temporary centering the lowest part of the permanent structure over the rails was kept 8 in. above the minimum structure gauge, that is, 14 ft. 11 in. above rail level. This, together with the recess under the floor, which is 6 in. deep, permitted the use of fairly stout timber beams for the temporary centering.

Quoting again from the specification :—" The concrete must be placed in position as rapidly as possible after mixing and thoroughly tamped into position up to the shuttering and round the steel reinforcement. Care must be taken at the end of every stage, in which the concrete may be allowed to set for more than one hour, to thoroughly cleanse it from all dirt and loose matter and well saturate it with water and wash with a thin grout of neat cement before proceeding with the adjoining work. The whole of the steel is to be thoroughly scraped to remove all dirt, rust, and scale immediately before it is used in the works.

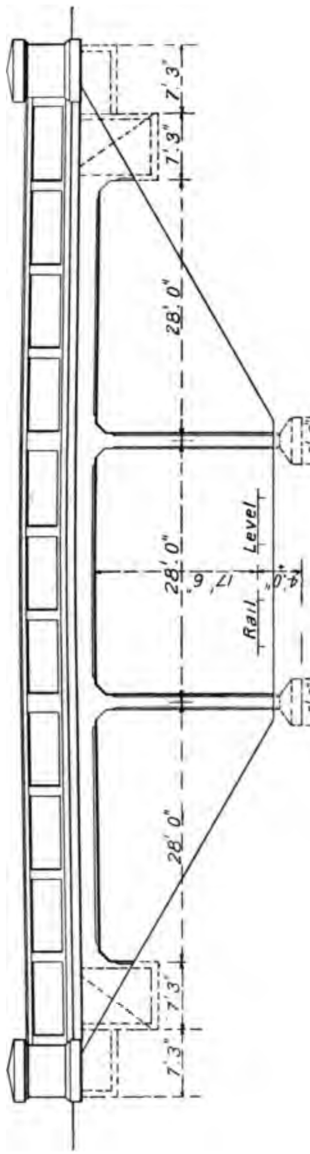


FIG. 77.—Elevation of Overbridge.

"The boards for shuttering must not in any case be less than $1\frac{1}{2}$ in. in thickness, and must be planed and both edges shot so as to fit tightly at the joints, and be thoroughly cleansed and the vertical surfaces properly greased and the horizontal surfaces covered with oil-paper each time they are used. Care must be taken that the boards are set to ensure the faces of the concrete being regular." No rendering was allowed except where necessary to fill in cavities unavoidably formed, the exposed surfaces being rubbed smooth with pieces of wood while the concrete was still green.

BRIDGE OVER CUTTING.

Another example of a small bridge over a railway is shown in Figs. 77 and 78, the views presenting the elevation and cross section respectively, of a bridge carrying a roadway 12 ft. wide over the Audley line of the North Staffordshire Railway, which, at the point crossed, is in cutting. There is plenty of room for construction, permitting the beams to be placed below the floor. In this way the columns are decreased in height and a very pleasing appearance is obtained. The parapets are also of reinforced concrete with deeply recessed panels.

The reinforcement consists of patent Indented Bars. The bracing of the columns is shown in the cross section and the method of constructing the abutments at the tops of the

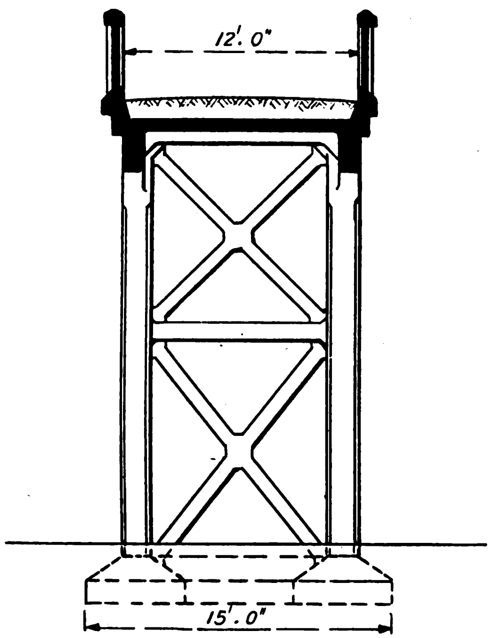


FIG. 78.—Cross Section of Overbridge.

slopes with vertical and horizontal slabs is indicated in the elevation.

BRIDGE UNDER RAILWAY.

Figs. 79 and 80 show in longitudinal section and part cross section one of the few reinforced concrete railway underbridges which have been constructed in the British Isles. These bridges, for there are three of them side by side, carrying eight tracks in all, serve to connect the main lines of the Great Western Railway with the locomotive and carriage depot at St. Philip's Marsh, Bristol. They are constructed of

Hennebique ferro-concrete throughout, and are of the girder type supported on walls built up from pile foundations. The clear span is 36 ft. and the bridges were designed to carry 80-ton locomotives. When tested under 75-ton locomotives the maximum deflection was very little more than $\frac{1}{8}$ of an inch.

The total depth of the longitudinal beams including the 7 in. slab floor is 33 in., or about $\frac{1}{13}$ of the clear square span. As all the girders are built slightly on the skew the

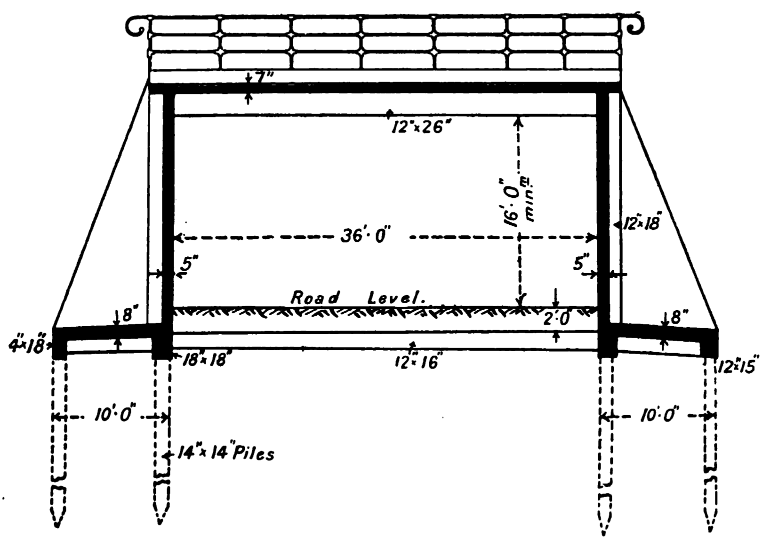


FIG. 79.—Victoria Street Bridge, Bristol. Longitudinal Section.

actual span is rather more than 36 ft. The spacing of the main beams varies somewhat to suit the lines of rails. Generally speaking, there is a beam under each rail so that the beams are from 5 to 6 ft. apart between centres. The counterforts to the abutments occur under every other beam, the alternate beams being carried on columns 12 in. × 18 in. in section. The thickness of the abutment slabs is 5 in. The piles are 14 in. square and 41 ft. in length. Connecting the heads of these together transversely and passing under the road are 12 in. × 16 in. beams spaced about 10 ft. apart.

Highway Bridges.

Another Hennebique bridge of similar construction is shown in Figs. 81 and 82. This is a highway bridge over the Avonmouth and Filton Railway. The road is 25 ft. wide between the brick parapets and the clear span of the beams is 27 ft. The beams, which are 4 ft. 6 in. apart, are 2 ft. deep × 9 in.

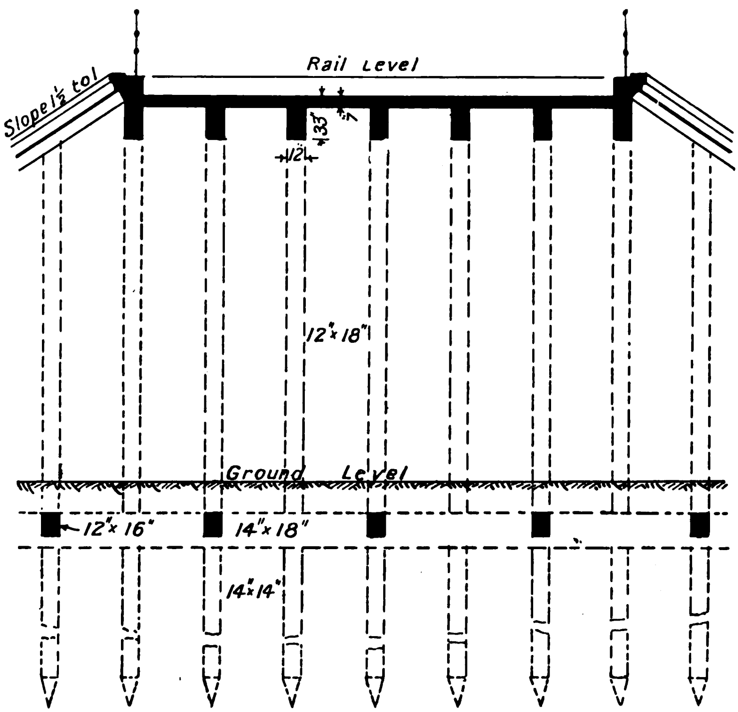


Fig. 80.—Victoria Street Bridge, Bristol. Cross Section.

wide and the thickness of the slab floor is 6 in. The abutments are of considerable height as the formation level of the railway is about 6 ft. above the original surface of the ground. The abutments are founded on piles 14 in. square and 21 ft. in length.

There are several examples of Hennebique ferro-concrete construction at the Great Central Railway Company's Immingham

Dock. These include four bridges carrying the high level serving roads over the empty waggon sidings, designed to carry the heaviest locomotives on the G. C. R. and gantries at the coal hoists for carrying empty trucks away over the quay. A road bridge consisting of two spans of about 35 ft. each is illustrated in Figs. 83 and 84. This bridge was designed to carry two 15-ton traction engines, two 40-ton boiler trolleys

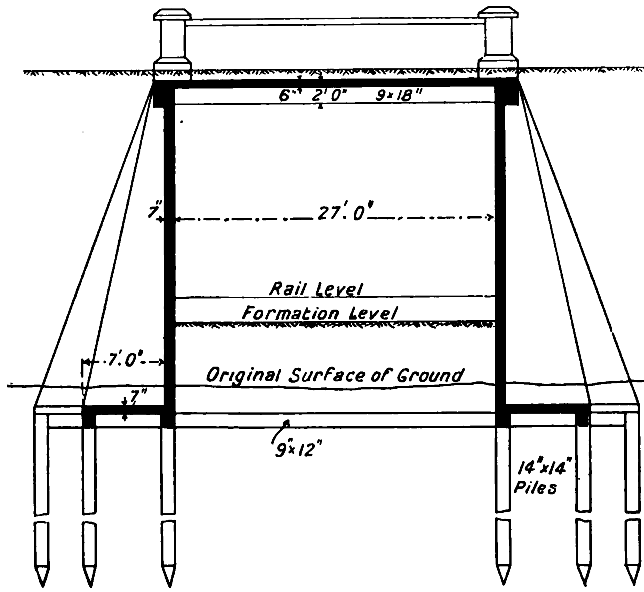


FIG. 81.—Bridge over Avonmouth and Filton Railway. Longitudinal Section.

and in addition 1 cwt. per super foot on the remaining uncovered portions of the bridge, a very heavy load.

The road is 40 ft. wide between the parapets and is carried on five main beams rather over 10 ft. apart between centres. These are 30 in. deep, including the 6 in. slab floor, and 18 in. wide. There are cross beams 18 in. deep by 6 in. wide spaced about 3 ft. 9 in. apart between centres. The centre support consists of five columns 20 in. square built up from a double row of 14 in. piles. The counterforts to the abutments are about 10 ft. apart, and the 5 in. slab walls are stiffened by intermediate

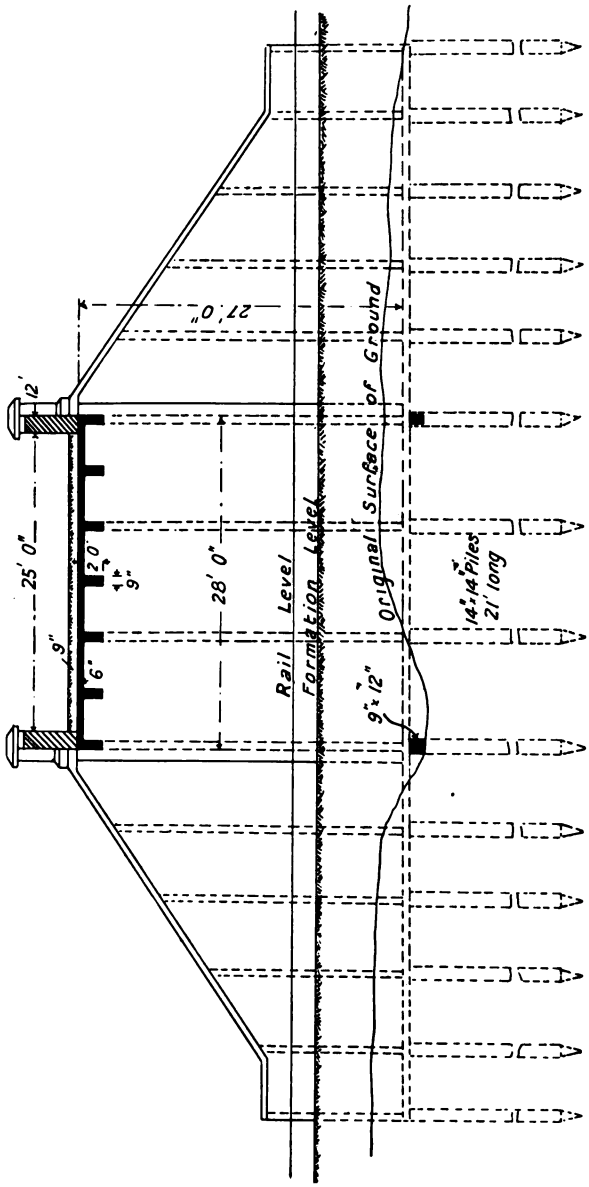


FIG. 82.—Bridge over Avonmouth and Filton Railway. Cross Section.

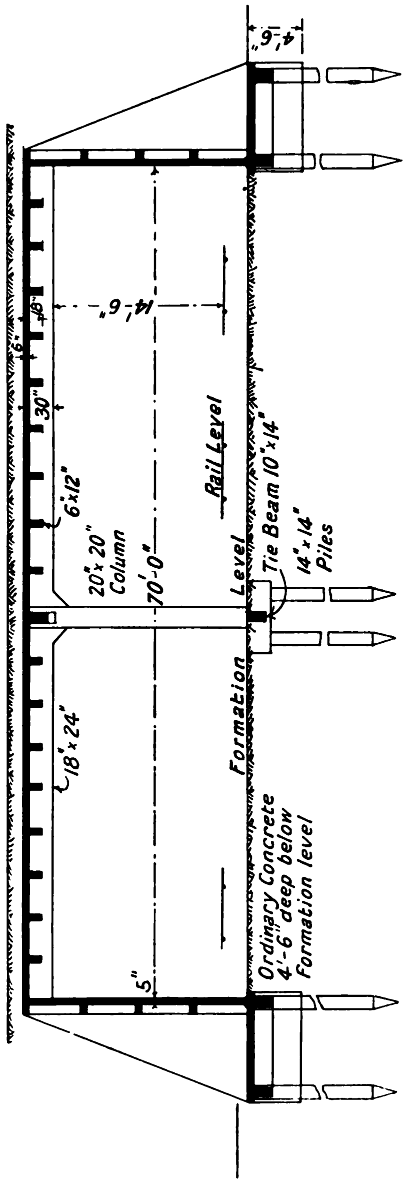


FIG. 83.—Highway Bridge at Immingham Dock. Longitudinal Section.

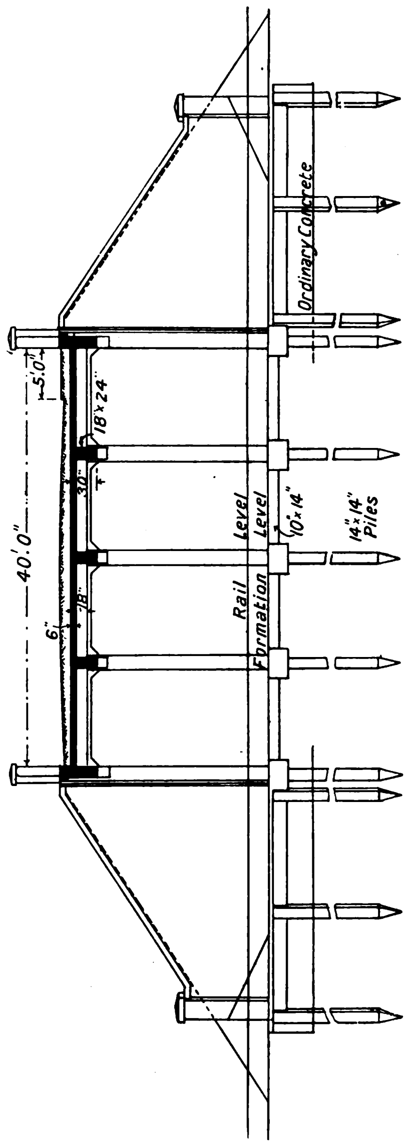


FIG. 84.—Highway Bridge at Immingham Dock. Cross Section.

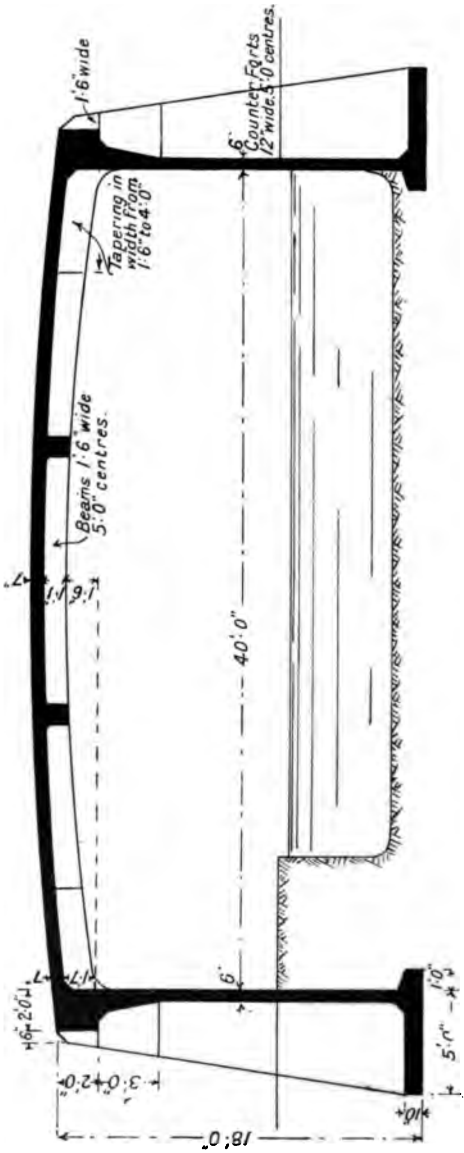


FIG. 85.—Canal Bridge at Burnley.

uprights and horizontal beams. The abutments are also founded on Hennebique ferro-concrete piles 14 in. square.

One of the advantages of reinforced concrete construction, and at the same time one of the difficulties as regards design, is illustrated in the bridge of which Fig. 85 is a longitudinal section. This is one of two bridges spanning the canal at Burnley and carrying the tramway, constructed for the Burnley Corporation from designs prepared by the Patent Indented Bar and Concrete Engineering Co. The span of this particular bridge is 40 ft. and the overall depth of the beams at the centre is only 1 ft. 8 in. or $\frac{1}{24}$ of the span. This shallow construction was made possible by moulding the

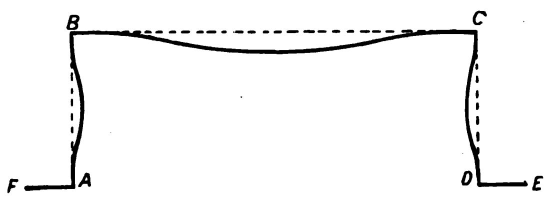
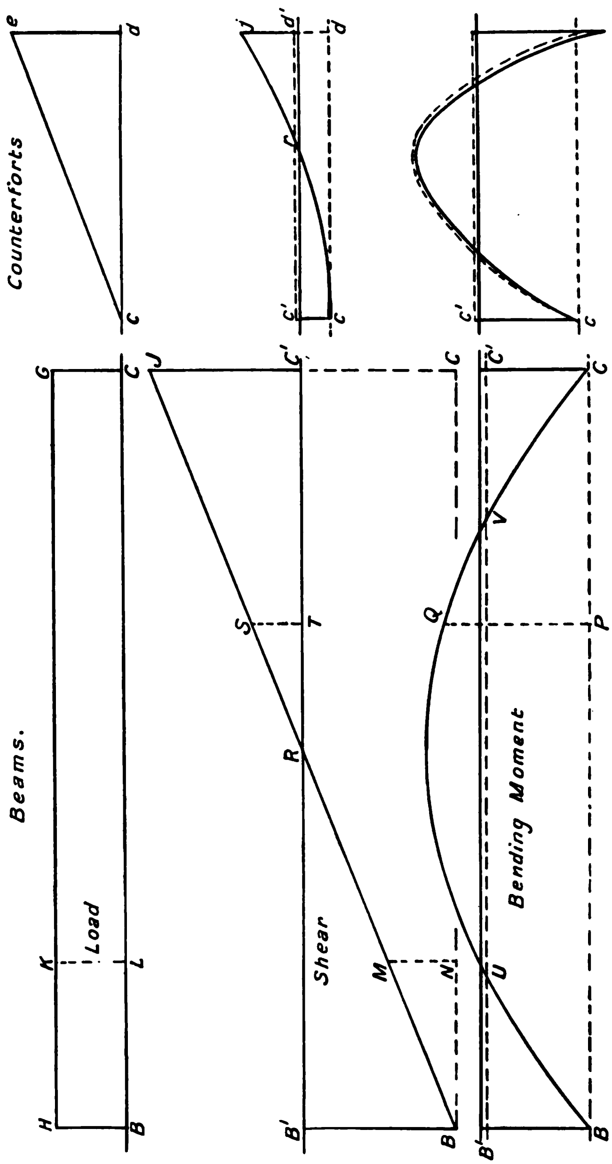


FIG. 86.—Diagram Exaggerating Deformation.

beams in one with the abutments and designing the reinforcement, which consists of indented bars, and is of necessity very heavy, so as to obtain complete continuity.

GRAPHIC DETERMINATION OF STRESSES.

The determination of the stresses in a structure of this sort under the combined influences of dead and live load and earth pressure, presents a distinctly interesting problem which, although capable of a more elegant solution, can be solved in a comparatively simple manner graphically. The deformation produced in a structure of this sort is shown exaggerated in Fig. 86. The span remains constant for all practical purposes and so does the length of the supports. The ends A and D will be fixed, provided that the width of base AF and DE and the weight of earth above is sufficient to prevent overturning under the influence of the bending moment at these points.



Counterforts
Beams.
Load
Shear
Bending Moment

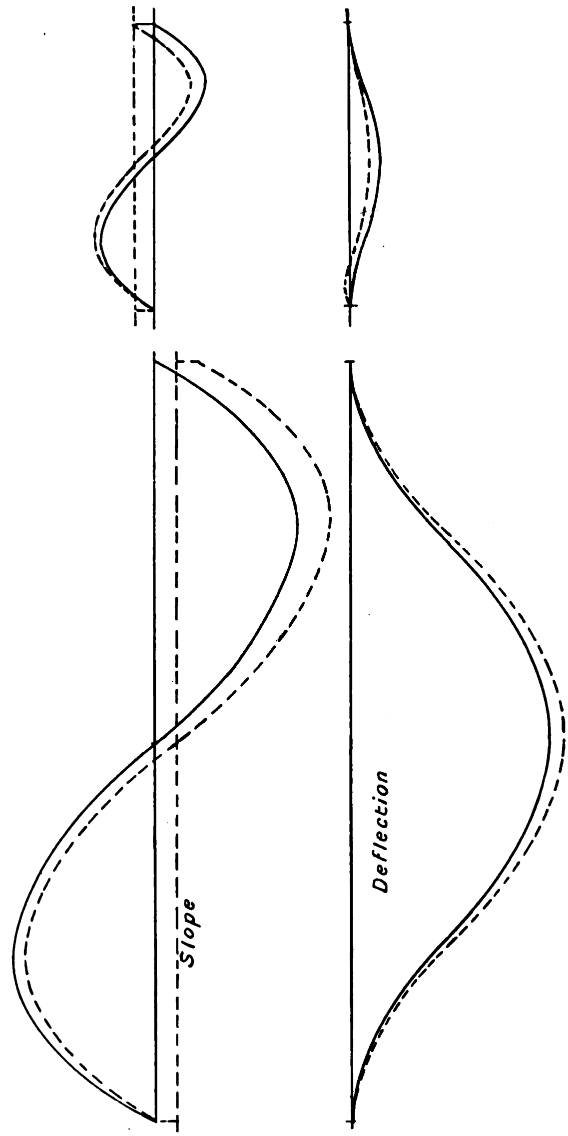


FIG. 87.—Graphic Determination of Stresses.

As regards the beam BC, if the live load is assumed to be uniformly distributed, the total load may be represented to a convenient scale by the rectangle $BCGH$ in Fig. 87. Below is shown the shear diagram and the bending moment diagram. These may be obtained successively by the process of graphic integration, that is MN, for instance, in the shear diagram, is equal to the area $BLKH$ of the load diagram. To obtain the actual shear diagram the datum $B'C'$ has to be substituted for the original datum BC equivalent to adding a constant in quantitative integration, and the value of the constant BB' is obtained in this case from the knowledge that the reactions BB' and JC' at the abutments are each equal to half the total load CJ.

Similarly in the bending moment diagram, PQ is equal to the area $B'RB$ minus the area RTS, areas above and below the datum being of opposite sign. The correct alteration of the datum to complete the bending moment diagram is dependent upon knowing the degree of fixity of the ends. If the ends are perfectly fixed the slope of the beam at each of these points is zero, and the graphic integration of the bending moment diagram can only yield this result if the areas $BB'U$ and $CC'V$ below the datum are together equal to the area UQV above the datum. From the value of the area of a parabola it follows that BB' or CC' must be equal to $\frac{2}{3}$ of the maximum ordinate. Similarly the deflection curve is obtained by graphic integration of the slope diagram. To obtain actual values of the slope or deflection, and, if the section of the beam varies, to obtain correct relative values of these quantities, it is necessary to divide the ordinates of the bending moment diagram by the moment of inertia of the section and the modulus of elasticity of the material.

The pressure on the counterforts instead of being a uniformly distributed load is a uniformly increasing load varying from zero at the top to a maximum value at the bottom, which may be determined in the manner described in the chapter on Retaining Walls. This pressure may therefore be represented by a triangle, and the shear diagram becomes a parabola, but the correct datum for this shear diagram can only be determined by trial.

If, however, the ends are perfectly fixed, cc', the reaction at the top end will be $\frac{3}{10}$ of the total load dj. On this basis the subsequent diagrams relating to bending moment, slope and deflection have been prepared, the ordinate cc', determining the corrected datum for the bending moment diagram, being equal to $\frac{wl^3}{30}$ where l is the height of the counterfort and wl the maximum pressure at the base. The quantity $\frac{wl^3}{30}$ represents the bending moment at the top of the counterfort, provided both top and bottom are fixed in direction as well as position. This can only occur if $\frac{wl^3}{30}$ referred to the counterfort is equal to $\frac{w_1 l_1^2}{12}$ referred to the beam and giving the value of the bending moment at its ends if these are perfectly fixed. Obviously these bending moments must be equal, and if they are not so, for the mutual condition of fixed ends the datums must be amended so that (1) the bending moments CC_1 and cc_1 are equal; (2) the slope at the end of the beam is the same as the slope at the top of the counterfort, having made due correction in the bending moment diagrams for variation in section; (3) the deflection at each end of the beam and at top and bottom of the counterfort must be zero. In the diagrams, Fig. 87, the bending moment at the end of the beam, CC', is a little greater than the bending moment at the top of the counterfort, cc'. The datum of the bending moment diagram for the beam $B'C'$, must therefore be lowered slightly. As a result, the slope at each end of the beam as registered in the following diagram will, instead of being zero, have a small value, indicated by the dotted curve and datum, and the maximum deflection will be somewhat increased. In the diagrams applying to the counterfort the datum of the shear diagram $c'd'$, must be raised a trifle so as to give the required bending moment and slope at the top end and yield a deflection curve registering zero deflection at the top and at the bottom, and also at a third point near the top.

Widening an Existing Bridge.

A cross section and elevation of a road bridge over a railway, widened by means of a reinforced concrete construction, are shown in Figs. 88 and 89. The road crosses the Chester and

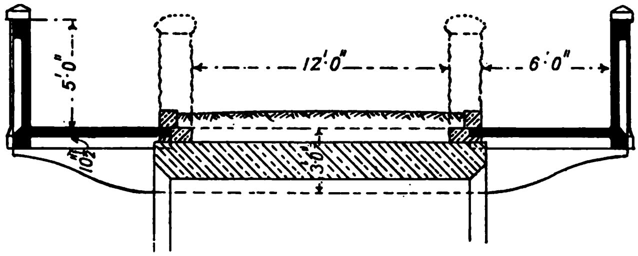


Fig. 88.—Widening an Existing Bridge. Cross Section.

Holyhead Railway at Conway, and was originally only 12 ft. wide between the parapets with no footpath. The bridge consisted of three stone arches of considerable height above the railway and heavy stone parapets, which are shown dotted

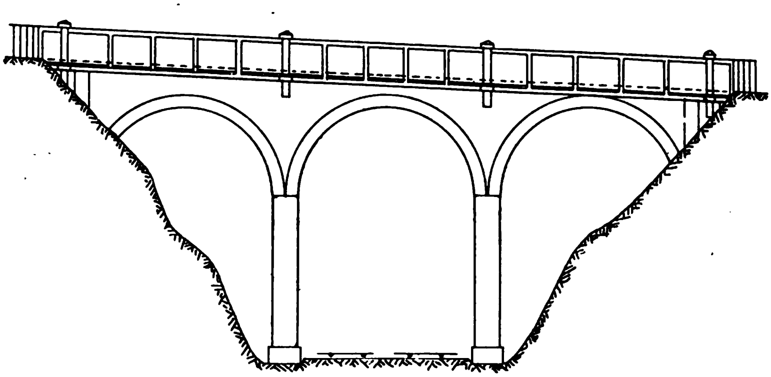


Fig. 89.—Widening an Existing Bridge. Elevation.

in Fig. 88. These were removed and cantilevers constructed at each pier and the two abutments, carrying parapets, which serve also as main girders supporting the floor slabs. By this means a footpath 6 ft. wide was provided at each side of the road. The overall length of the cantilevers is 28 ft. 6 in.

and these take bearings on the spandril walls of the bridge 6 ft. 9 in. from each end. The span of the main girders is about 30 ft. The widening was undertaken by the Conway Corporation and carried out from designs prepared by Messrs. Mouchel and Partners on the Hennebique system. There was no objection on behalf of the Railway Company, because the weight of the additional structure, together with a reasonable allowance for live load on the footpaths, was no more than that of the old stone parapets removed. Both the cantilevers and the main girders are reinforced with compression as well as tension reinforcement.

Bridge at King's Cross.

A reinforced concrete bridge has recently been constructed over the Metropolitan Railway at King's Cross, carrying a

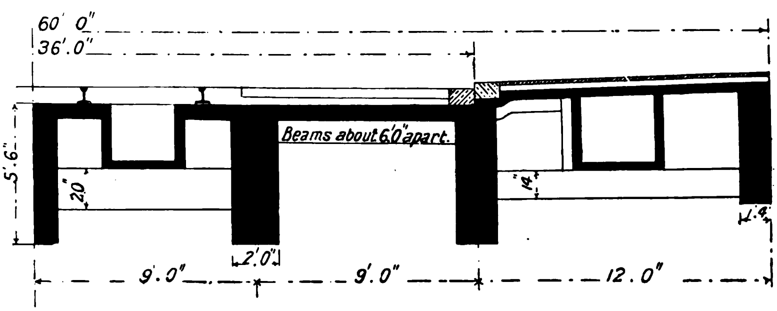


Fig. 90.—Bridge at King's Cross.

roadway and providing a connection between the L. C. C. tramways along Gray's Inn Road and Pentonville Road. This consists of two main spans, one of 53 ft. 6 in. over the electric lines and platforms, and one of 30 ft. over the pair of lines alongside. A half cross section through the floor of the larger span is shown in Fig. 90. The construction is on the Coignet system throughout. The largest beams are about 5 ft. 6 in. deep and 2 ft. wide and very heavily reinforced. For instance, the compression reinforcement consists of 8 round rods each $1\frac{7}{16}$ in. diameter. The tensile reinforcement for the centre beam of all comprises, at the middle of the span, four groups of

7 round rods $1\frac{1}{8}$ in. in diameter, and in addition 4 round rods $1\frac{1}{16}$ in. in diameter. Some of these are turned up at the ends and bent over the compression rods, and vertical stirrups, passing under the lower rods and wrapped round the upper rods, are provided throughout the entire length in accordance with the Coignet principle of reinforcing beams. There is a pipe duct under each footway carried by rectangular beams about 6 ft. apart. Provision for the tramway conduit is made in a similar manner. There are no parapets, as the station buildings are approached from the footpath on the one side and flats are in course of construction on the other side. This is a skew bridge and in consequence the reinforced concrete columns between the two larger spans are diamond-shaped in section. The other ends of the beams are carried on brick piers forming part of the old work, and the pockets in the original retaining walls are covered over with reinforced concrete construction similar in type to the section illustrated in Fig. 90. The proportions used for the mixture of the concrete were as follows:—

1 bag of cement (224 lbs.).
$4\frac{3}{4}$ cub. ft. of sand.
$9\frac{1}{2}$ cub. ft. of aggregate.

This is approximately a $1 : 1\frac{1}{2} : 3$ mixture. Ham River ballast was used for the aggregate, which varied in size from $\frac{1}{4}$ in. to $\frac{3}{4}$ in.

CHAPTER VIII

ARCHED BRIDGES

General.

For small spans there is probably no cheaper form of bridge of a permanent character than a brick, stone, or concrete arch, according to locality, and when there is headway enough and to spare, this construction has in the past represented the standard type both for bridges under and over the line. Not only is the first cost less than for a steel bridge with jack arch floor, but under favourable circumstances the cost of maintenance has been practically *nil.* It is found, however, that under the heavy express traffic of to-day, this happy state of affairs is not always maintained, and arches show an increasing tendency to develop cracks. For small spans the introduction of light reinforcement, without reducing the proportions of the arch to any great extent, might be tried with advantage as a means of prevention distinctly preferable to the unsightly cure of tie bolts and large washer plates introduced after the damage is done, while for larger spans the thickness of a concrete arch containing a normal percentage of reinforcement has proved quite sufficient to withstand the rough usage of railway traffic. For instance, on the Continent, several bridges of considerable span and of the bowstring type have been constructed under the running lines.

Reinforced concrete is a peculiarly suitable material for the construction of arches, and does away with the necessity of making the arch sufficiently thick to eliminate tension, while retaining a comparatively cheap material to resist the direct thrust. Reference has already been made to the properties of double reinforcement. In an arch, as in a column, double reinforcement is a necessity, because under variations of load either surface may be in tension. It will be seen in

Fig. 15 that with small percentages of double reinforcement the stress in the steel due to bending is high compared with the maximum compression in the concrete due to bending, but in the arch and column this inequality is compensated by the direct thrust, which increases the compression in the concrete, and reduces the tension in the steel, thus enabling normal working stresses to be adopted for each material while retaining a small percentage of reinforcement. As in the case of beams, so with arches it will be found that a thicker arch sparingly reinforced is, under ordinary circumstances, more economical than a thin arch containing a high percentage of reinforcement.

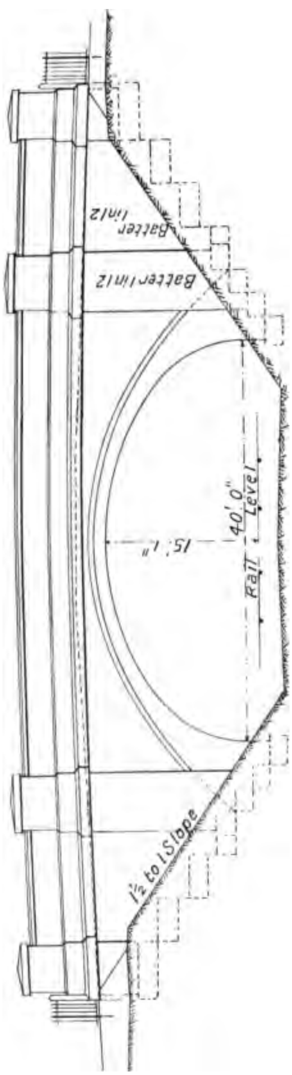


Fig. 91.—Elevation of Arched Bridge.

Solid Arch Bridge.

An example of the simplest type of arch bridge is illustrated in Figs. 91 and 92. Several of these bridges have been constructed to carry roads 40 ft. in width over a new line of railway. The intrados is not a perfectly true ellipse, but is struck with three centres, the somewhat flat curve at the crown being determined by the limited depth available. The span is 40 ft., and the semi minor axis 14 ft., the maximum height from rail level to soffit being 15 ft. 1 in. The thickness of the arches at the crown is 1 ft. 3 in., and at the change of curvature 1 ft. 9 in. The thickness of the abutments as they appear on plan is 7 ft., and the foundation is on sandstone. Mass concrete was used for the wing walls and concrete blocks for the parapets.

The main reinforcement of the arches consists of Kahn Bars with some ordinary round binding rods parallel to the axes of the arches. The main reinforcement is placed symmetrically about the centre line throughout, and except at the springings is 2 in. from each surface. At the crown, and extending each way to the change of curvature, the bars are $\frac{3}{4}$ in. square, spaced 1 ft. apart centre to centre, with wings 12 in. long. At the springings the bars are 1 in. square, spaced 1 ft. 6 in. apart, centre to centre, with wings 18 in. long. The section of the bars was changed in order to obtain longer wings at the springings, 12 in. and 18 in. being the respective standard lengths of wings for $\frac{3}{4}$ in. and 1 in. bars.

Specification.

The cement concrete in the foundation of the arches below the formation surface of slopes was specified to be composed of one measure of Portland cement, three measures of clean, sharp sand, and six measures of the aggregate. The cement concrete in arches above the formation surface of slopes to be composed of one measure of Portland cement, two measures of clean sharp sand and four measures of the aggregate. The aggregate in each case to consist of properly graded granite or hard close grained stone approved by the engineer, crushed to pass in any direction through a ring 1 in. in internal diameter and retained on a screen with a $\frac{3}{16}$-in. mesh and washed if necessary. The proportions to be accurately measured in a gauging box made for the purpose. The concrete must be thoroughly mixed to a rather wet consistency upon a close wooden platform or by an approved mixing machine. The concrete must in all cases be well rammed round the steel reinforcement. The finish to the exposed surfaces was specified to be obtained by working the "fat" to the front by means of hoop iron swords.

As regards the reinforcement, it was specified that the steel reinforcing bars must be free from oil, grease, dirt, and paint,

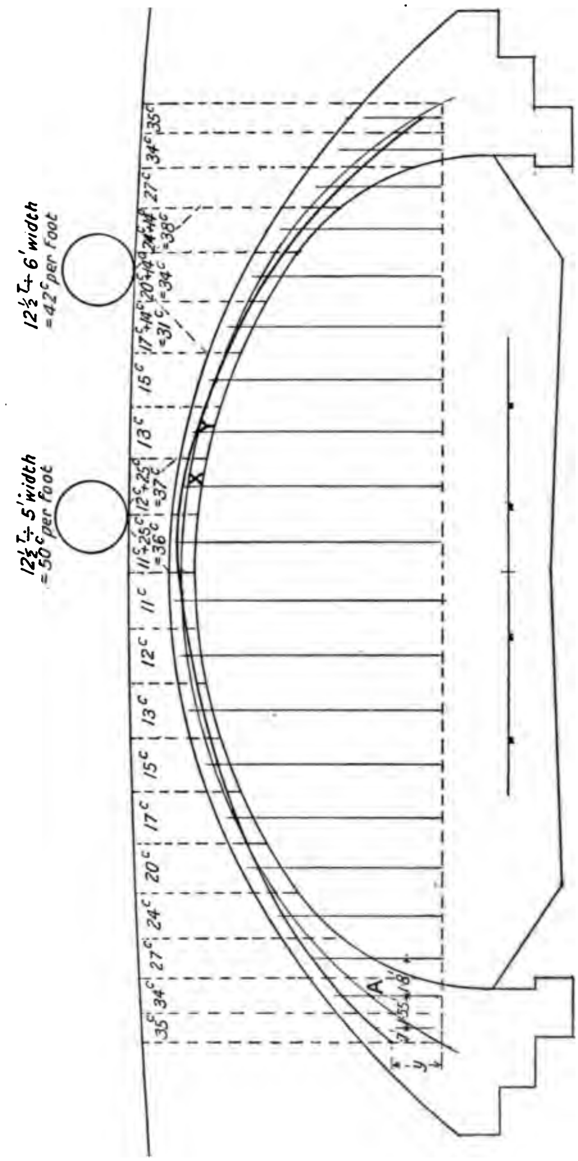


FIG. 92.—Longitudinal Section of Arched Bridge.

and all rust must be removed by thoroughly cleaning with wire brushes immediately before the concreting is carried out. All steel rods and bars must be firmly fixed to the centering by means of temporary templates to ensure each part of the reinforcement remaining firmly in the position indicated for it on the drawings. No steel must have at any point less than 1 in. of concrete covering. The bars must in all cases be carefully bent to uniform curves, the wings being set to a true angle of 45 degs. with the main bar.

These bridges cost just about the same as a brick arch bridge of the same width and height, but only a flat brick arch with a rise of from $\frac{1}{4}$ to $\frac{1}{5}$ of the usual span for a double line bridge, namely, 26 ft., could be built with the head room of 18 ft. available for the construction of these concrete arches. The first cost of a steel-girder bridge with jack arch floor and brick abutments and wings would probably be about 20 per cent. higher, and in addition there is the cost of maintenance.

Bending Moments, &c.

The calculation of the stresses in an arch of this kind presents some difficulty, and in view of the fact that comparatively few books deal with this subject, is worth reproducing somewhat fully. The bridges were required to be sufficiently strong to carry a 50-ton boiler trolley with a wheel base of 12 ft. by 6 ft., and the load equally divided between the four wheels.

Width of arch considered 1 ft.

Effective span 45 ft. between extreme points on centre line.

Length of centre line 54·5 ft. divided into twenty equal parts.

Estimated dead loads vertically over each part as shown by dotted lines in Fig. 92, averaging the weight of concrete, filling, and metalling at $1\frac{1}{4}$ cwt. per cube foot.

20·5	× 1·4	× $1\frac{1}{4}$ cwt.,	say,	35 cwt.
16	× 1·7	× $1\frac{1}{4}$,,	,,	34 ,,
11·4	× 1·9	× $1\frac{1}{4}$,,	,,	27 ,,
8·7	× 2·2	× $1\frac{1}{4}$,,	,,	24 ,,
7·0	× 2·3	× $1\frac{1}{4}$,,	,,	20 ,,

5·7	× 2·4	× $1\frac{1}{4}$ cwt.,	say,	17 cwt.
4·7	× 2·6	× $1\frac{1}{4}$,,	,,	15 ,,
3·95	× 2·6	× $1\frac{1}{4}$,,	,,	13 ,,
3·45	× 2·7	× $1\frac{1}{4}$,,	,,	12 ,,
3·2	× 2·7	× $1\frac{1}{4}$,,	,,	11 ,,

The boiler trolley was placed to the right of the centre line as shown in Fig. 92, and the reason for the varying amount of distribution assumed is indicated by the dotted lines. Having regard to the amount of the filling, the $12\frac{1}{2}$-ton wheel load

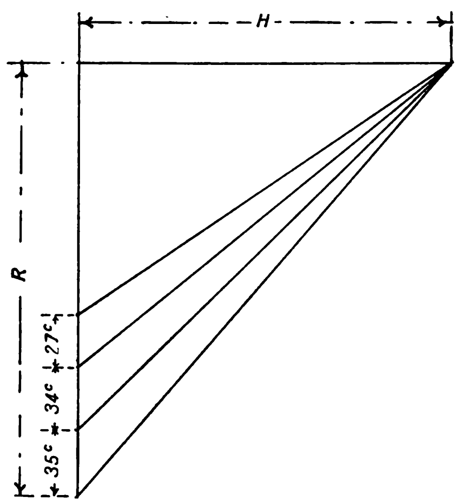


FIG. 93.—Force Polygon.

near the centre line was assumed to be distributed over a width of 5 ft. across the bridge and over two divisions longitudinally, that is a length of 5·4 ft. The wheel load furthest from the centre line, under which there is a greater depth of filling, was assumed to be distributed over a width of 6 ft. across the bridge, which is the spacing between the pair of wheels, and over three divisions longitudinally.

If the horizontal thrust H, and the vertical reaction R at the left abutment were known, the force polygon sketched in Fig. 93 could be drawn, and if one point on the resultant thrust line were known, this line could be drawn in its correct position relatively to the centre line of the arch by means of the well-known funicular polygon construction.

Taking the line joining the ends of the centre line of the arch as a horizontal datum, let y be the ordinate of the line of resultant thrust at the left abutment. The slope of this line at this point is $\frac{R}{H}$ so that the ordinate at the centre line of the first division of the arch is $y + \cdot 7\, \frac{R}{H}$.

Proceeding to the right the slope of the thrust line according to the graphic construction is $\frac{R - 35}{H}$, and the ordinate at the centre line of the second division is $y + \cdot 7\, \frac{R}{H} + 1\cdot 55\, \frac{R - 35}{H}$. Similarly the ordinate at the centre line of the third division is $y + \cdot 7\, \frac{R}{H} + 1\cdot 55\, \frac{R - 35}{H} + 1\cdot 8\, \frac{R - 35 - 34}{H}$. The remaining ordinates are readily worked out in tabular form on p. 152. The values of y, R, and H can now be ascertained from properties of the elastic arch with fixed ends, which need not be gone into here.

The ordinates of the resultant thrust line with reference to the centre line obtained in the last column of Table I., when multiplied by the horizontal thrust H, are a measure of the varying bending moments throughout the arch. With regard to any one of the twenty equal divisions into which the centre line of the arch is divided, and within the limits of which the bending moment is assumed to be constant, the amount of bending of the arch is directly proportional to the length of the division and the magnitude of the bending moment, and inversely proportional to the moment of inertia of the average section. The length of each division is constant, and can be eliminated, but the measures of the bending moment, namely, the values in column 8 of Table I., must be divided by the moment of inertia of the section at the centre of each division. For instance, at the crown, the moment of inertia, taking into account the greater modulus of elasticity of the steel is :— $\frac{1}{12} \times 12 \times 15^3 + 2 \times \cdot 75 \times 5\cdot 5^2 \times 15 = 4{,}060$ in. units. Instead

Table I.

(1) Distance between Centre Lines.	(2) Distance from Left Abutment.	(3) Loads.	(4) Sum of Loads.	(5) (1)×(4).	(6) Sum of (5).	(7) Ordinates of Centre Line.	(8) Ordinates of Resultant Thrust Line with Reference to Centre Line.
·7							
	·7	35				1·2	$y + {\cdot}7\frac{R}{H} - 1{\cdot}2$
1·55			35	54			
	2·25	34			54	3·4	$y + 2{\cdot}25\frac{R}{H} - \frac{54}{H} - 3{\cdot}4$
1·8			69	124			
	4·05	27			178	5·4	$y + 4{\cdot}05\frac{R}{H} - \frac{178}{H} - 5{\cdot}4$
2·05			96	197			
	6·1	24			375	7·15	$y + 6{\cdot}1\frac{R}{H} - \frac{375}{H} - 7{\cdot}15$
2·25			120	270			
	8·35	20			645	8·65	$y + 8{\cdot}35\frac{R}{H} - \frac{645}{H} - 8{\cdot}65$
2·35			140	329			
	10·7	17			974	9·9	$y + 10{\cdot}7\frac{R}{H} - \frac{974}{H} - 9{\cdot}9$
2·5			157	392			
	13·2	15			1366	10·9	$y + 13{\cdot}2\frac{R}{H} - \frac{1366}{H} - 10{\cdot}9$
2·6			172	447			
	15·8	13			1813	11·7	$y + 15{\cdot}8\frac{R}{H} - \frac{1813}{H} - 11{\cdot}7$
2·65			185	491			
	18·45	12			2304	12·2	$y + 18{\cdot}45\frac{R}{H} - \frac{2304}{H} - 12{\cdot}2$
2·7			197	532			
	21·15	11			2836	12·45	$y + 21{\cdot}15\frac{R}{H} - \frac{2836}{H} - 12{\cdot}45$
2·7			208	562			
	23·85	36			3398	12·45	$y + 23{\cdot}85\frac{R}{H} - \frac{3398}{H} - 12{\cdot}45$
2·7			244	659			
	26·55	37			4057	12·2	$y + 26{\cdot}55\frac{R}{H} - \frac{4057}{H} - 12{\cdot}2$
2·65			281	745			
	29·2	13			4802	11·7	$y + 29{\cdot}2\frac{R}{H} - \frac{4802}{H} - 11{\cdot}7$
2·6			294	765			
	31·8	15			5567	10·9	$y + 31{\cdot}8\frac{R}{H} - \frac{5567}{H} - 10{\cdot}9$
2·5			309	772			
	34·3	31			6339	9·9	$y + 34{\cdot}3\frac{R}{H} - \frac{6339}{H} - 9{\cdot}9$
2·35			340	799			
	36·65	34			7138	8·65	$y + 36{\cdot}65\frac{R}{H} - \frac{7138}{H} - 8{\cdot}65$
2·25			374	842			
	38·9	38			7980	7·15	$y + 38{\cdot}9\frac{R}{H} - \frac{7980}{H} - 7{\cdot}15$
2·05			412	845			
	40·95	27			8825	5·4	$y + 40{\cdot}95\frac{R}{H} - \frac{8825}{H} - 5{\cdot}4$
1·8			439	790			
	42·75	34			9615	3·4	$y + 42{\cdot}75\frac{R}{H} - \frac{9615}{H} - 3{\cdot}4$
1·55			473	733			
	44·3	35			10348	1·2	$y + 44{\cdot}3\frac{R}{H} - \frac{10348}{H} - 1{\cdot}2$

of dividing by these large numbers it is more convenient to multiply by equivalent factors, calling the factor corresponding to the largest divisor, 1. These are given in the first column of Table II. Obviously the total amount of bending throughout an arch with fixed ends is, in the algebraical sense, zero. Therefore the summation of the quantities in column 2 of Table II., which represent bending moments corrected for varying moment of inertia of section, may be equated to zero. Further, it has been proved that the sum of the products of these quantities and the distances of the respective sections from either abutment is zero, and also that the sum of the products of these quantities and the ordinates of the centre line to the arch, measured from any convenient horizontal datum, is zero.

In this way three equations are obtained in y, $\frac{R}{H}$, and $\frac{1}{H}$, thus:—

$$544{\cdot}6\, y + 12256\, \frac{R}{H} - \frac{1884000}{H} - 5895 = 0$$

$$5895\, y + 132700\, \frac{R}{H} - \frac{19983100}{H} - 65957 = 0$$

$$12256\, y + 322823\, \frac{R}{H} - \frac{53127000}{H} - 132633 = 0$$

which when simplified become

$$y + 22{\cdot}5\, \frac{R}{H} - \frac{3460}{H} = 10{\cdot}82$$

$$y + 22{\cdot}5\, \frac{R}{H} - \frac{3390}{H} = 11{\cdot}18$$

$$y + 26{\cdot}3\, \frac{R}{H} - \frac{4330}{H} = 10{\cdot}82$$

and determine the values

$$y = 2{\cdot}36 \text{ ft.}$$

$$\frac{R}{H} = 1{\cdot}16$$

$$H = 196{\cdot}1 \text{ cwt.}$$

By substituting these values in the general expressions for

TABLE II.		TABLE III.		TABLE IV.	
1. Factor to correct for varying section.	(2.) Ordinates of Thrust Line relatively to Centre Line corrected for varying section.	(1.) Distances from left abutment.	(2.) Corrected Ordinates of Thrust Line relatively to Centre Line multiplied by Distances from Left Abutment.	(1.) Ordinates of Centre Line.	(2.) Corrected Ordinates of Thrust Line relatively to Centre Line multiplied by Ordinates of Centre Line.
1	$1\,y + \cdot7\frac{R}{H} - 1\cdot2$	·7	$\cdot7y + \cdot5\frac{R}{H} - \cdot8$	1·2	$1\cdot2y + \cdot8\frac{R}{H} - 1\cdot4$
2·5	$2\cdot5y + 5\cdot6\frac{R}{H} - \frac{135}{H} - 8\cdot5$	2·25	$5\cdot6y + 12\cdot6\frac{R}{H} - \frac{304}{H} - 19\cdot1$	3·4	$8\cdot5y + 19\cdot1\frac{R}{H} - \frac{459}{H} - 28\cdot9$
6·8	$6\cdot8y + 27\cdot6\frac{R}{H} - \frac{1210}{H} - 36\cdot7$	4·05	$27\cdot5y + 111\cdot7\frac{R}{H} - \frac{4900}{H} - 149$	5·4	$36\cdot7y + 149\frac{R}{H} - \frac{6535}{H} - 198\cdot2$
16	$16\,y + 97\cdot5\frac{R}{H} - \frac{6000}{H} - 114\cdot5$	6·1	$97\cdot6y + 595\frac{R}{H} - \frac{36600}{H} - 697$	7·15	$114\cdot5y + 697\frac{R}{H} - \frac{42900}{H} - 819$
24	$24\,y + 200\cdot5\frac{R}{H} - \frac{15480}{H} - 207\cdot5$	8·35	$200\cdot4y + 1675\frac{R}{H} - \frac{129200}{H} - 1734$	8·65	$207\cdot5y + 1734\frac{R}{H} - \frac{134000}{H} - 1795$
30	$30\,y + 321\frac{R}{H} - \frac{29220}{H} - 297$	10·7	$321\,y + 3435\frac{R}{H} - \frac{312800}{H} - 3175$	9·9	$297\,y + 3175\frac{R}{H} - \frac{280000}{H} - 2938$
39	$39\,y + 515\frac{R}{H} - \frac{53300}{H} - 425$	18·2	$515\,y + 6800\frac{R}{H} - \frac{704000}{H} - 5615$	10·9	$425\,y + 5615\frac{R}{H} - \frac{581000}{H} - 4633$
46	$46\,y + 727\frac{R}{H} - \frac{83500}{H} - 538$	15·8	$727\,y + 11480\frac{R}{H} - \frac{1320000}{H} - 8510$	11·7	$538\,y + 8510\frac{R}{H} - \frac{978000}{H} - 6295$
51	$51\,y + 942\frac{R}{H} - \frac{117500}{H} - 622$	18·45	$942\,y + 17400\frac{R}{H} - \frac{2160000}{H} - 11490$	12·2	$622\,y + 11490\frac{R}{H} - \frac{1434000}{H} - 7590$

56	$56y + 1185\frac{R}{H} - \frac{158800}{H} - 697$	21·15	$1185y + 25070\frac{R}{H} - \frac{3360000}{H} - 14750$	12·45	$697y + 14750\frac{R}{H} - \frac{1977000}{H} - 8680$
56	$56y + 1335\frac{R}{H} - \frac{190200}{H} - 697$	23·85	$1335y + 31850\frac{R}{H} - \frac{4540000}{H} - 16620$	12·45	$697y + 16620\frac{R}{H} - \frac{2370000}{H} - 8680$
51	$51y + 1355\frac{R}{H} - \frac{206700}{H} - 622$	26·55	$1355y + 36000\frac{R}{H} - \frac{5485000}{H} - 16530$	12·2	$622y + 16530\frac{R}{H} - \frac{2520000}{H} - 7590$
46	$46y + 1343\frac{R}{H} - \frac{221000}{H} - 538$	29·2	$1343y + 39250\frac{R}{H} - \frac{6460000}{H} - 15730$	11·7	$538y + 15730\frac{R}{H} - \frac{2585000}{H} - 6295$
39	$39y + 1240\frac{R}{H} - \frac{217200}{H} - 425$	31·8	$1240y + 39450\frac{R}{H} - \frac{6900000}{H} - 13520$	10·9	$425y + 13520\frac{R}{H} - \frac{2370000}{H} - 4633$
30	$30y + 1029\frac{R}{H} - \frac{190200}{H} - 297$	34·3	$1029y + 35300\frac{R}{H} - \frac{6520000}{H} - 10175$	9·9	$297y + 10175\frac{R}{H} - \frac{1880000}{H} - 2938$
24	$24y + 880\frac{R}{H} - \frac{171500}{H} - 207{\cdot}5$	36·65	$880y + 32220\frac{R}{H} - \frac{6280000}{H} - 7610$	8·65	$207{\cdot}5y + 7610\frac{R}{H} - \frac{1483000}{H} - 1795$
16	$16y + 623\frac{R}{H} - \frac{127700}{H} - 114{\cdot}5$	38·9	$623y + 24230\frac{R}{H} - \frac{4970000}{H} - 4455$	7·15	$114{\cdot}5y + 4455\frac{R}{H} - \frac{914000}{H} - 819$
6·8	$6{\cdot}8y + 278{\cdot}5\frac{R}{H} - \frac{60000}{H} - 36{\cdot}7$	40·95	$278{\cdot}5y + 11410\frac{R}{H} - \frac{2450000}{H} - 1503$	5·4	$36{\cdot}7y + 1503\frac{R}{H} - \frac{324000}{H} - 1982$
2·5	$2{\cdot}5y + 106{\cdot}9\frac{R}{H} - \frac{24030}{H} - 8{\cdot}5$	42·75	$106{\cdot}9y + 4570\frac{R}{H} - \frac{1027000}{H} - 363{\cdot}5$	3·4	$8{\cdot}5y + 363{\cdot}5\frac{R}{H} - \frac{81800}{H} - 28{\cdot}9$
1	$1y + 44{\cdot}3\frac{R}{H} - \frac{10348}{H} - 1{\cdot}2$	44·36	$44{\cdot}3y + 1963\frac{R}{H} - \frac{458200}{H} - 53{\cdot}2$	1·2	$1{\cdot}2y + 53{\cdot}2\frac{R}{H} - \frac{1241}{H} - 1{\cdot}4$
Sum	$544{\cdot}6y + 12256\frac{R}{H} - \frac{1884000}{H} - 5895$	Sum	$12256y + 322823\frac{R}{H} - \frac{53127000}{H} - 132700$	Sum	$5895y + 132700\frac{R}{H} - \frac{19983100}{H} - 65957$

the resultant thrust line given in Table I., the location of this line can be determined with more accuracy than is possible with the graphic method. The resultant thrust line plotted in Fig. 92 was determined in this manner.

The figures in Tables II., III., IV. have been given in full in order to show that the calculations are not as involved as they appear at first sight. For instance, the coefficients of y in Table IV. are the same as the last column of numericals in Table II., and the coefficients of y in Table III. are the same as the coefficients of $\frac{R}{H}$ in Table II. Also the last column of numericals in Table III. are the same as the coefficients of $\frac{R}{H}$ in Table IV. In the equations obtained by simplifying the three summations it will be noticed that the numbers 22·5 and 10·82 each occur twice. This is not a coincidence, and will always occur in a arch having springings at the same level and a centre line symmetrical about the crown. 22·5 is the half span, and that it should be such can be verified from a consideration of the operations by which this figure is obtained.

In an arch of uniform section the value 10·82 would be the average ordinate, but in the present case this is qualified by the variation in the thickness of the arch throughout its length. If the line of resultant thrust plotted on Fig. 92 is examined, it will be found that although it is deflected above the centre line by the wheel load near the crown, it is below the centre line of the arch underneath the wheel load near the right abutment, and this fact, together with the elliptical shape of the arch, suggested that a worse result might be obtained by placing the wheels symmetrically about the crown.

The calculation under these conditions is very much simplified, because everything is symmetrical about the crown, and only half the arch need be considered. R is known to be half the total load, and the second equation obtained from the sum of the products of ordinates relatively to the centre line, and their distance from the left abutment, would yield the same result as the first equation, and need not be considered. This leaves two equations to determine the values of H and y. It was found, however, that this loading did not give such high bending moments as the first arrangement.

Calculation of Stresses.

The maximum stresses produced in the concrete and in the steel at any section are determined by considering the combined effect of the bending moment and the direct thrust at that section. To obtain the magnitude of the resultant thrust,

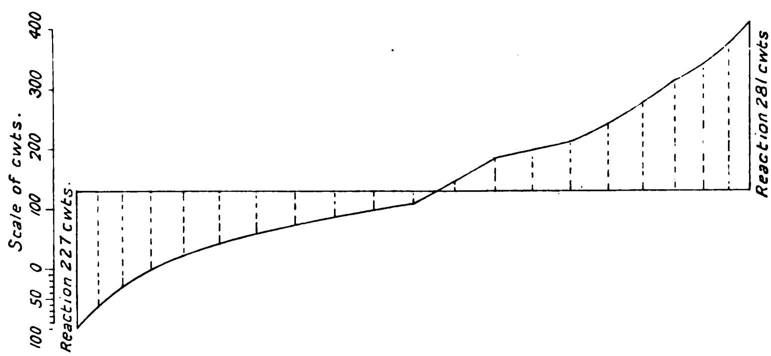


Fig. 94.—Shear Diagram.

the horizontal component, which is constant throughout and equal to 196 cwt. with the loading indicated in Fig. 92, must be combined with the vertical component or shear which varies throughout the arch. This variation is shown in diagram form in Fig. 94, which is plotted from the ascertained value of the vertical reaction at the left abutment,

$$R = 1{\cdot}16 \times 196 = 227 \text{ cwt.},$$

and the loads supported by each division of the arch which are given in Fig. 92.

The maximum bending moment near the crown of the arch occurs at X in Fig. 92, and is equal to

$$196 \times {\cdot}32 = 62{\cdot}7 \text{ ft. cwt.} = 84{,}300 \text{ in. lbs.}$$

The shear at this section is practically *nil*, so that the amount of the direct compression is 196 cwt. or 22,000 lbs. The section resisting these stresses is drawn in Fig. 95.

Width of section = 12 in.

Total depth of section = 15 in.

Effective depth of section = 13 in.

Area of reinforcement at top and also at bottom = ·52 sq. in. = ·33 per cent.

Moment of resistance to bending,

$$\cdot 0030 \times 12 \times 13^2 \times fs. \quad \text{(Fig. 15.)}$$

Tension in steel due to bending,

$$84{,}300 \div \cdot 0030 \times 12 \times 13^2 = 13{,}900 \text{ lbs. per square inch.}$$

Maximum compression in concrete due to bending,

$$13{,}900 \div 46 \cdot 8 = 297 \text{ lbs. per square inch.} \quad \text{(Fig. 15.)}$$

Direct compression in concrete,

$$22{,}000 \div (12 \times 15 + 14 \times 2 \times \cdot 52) = 113 \text{ lbs. per square inch.}$$

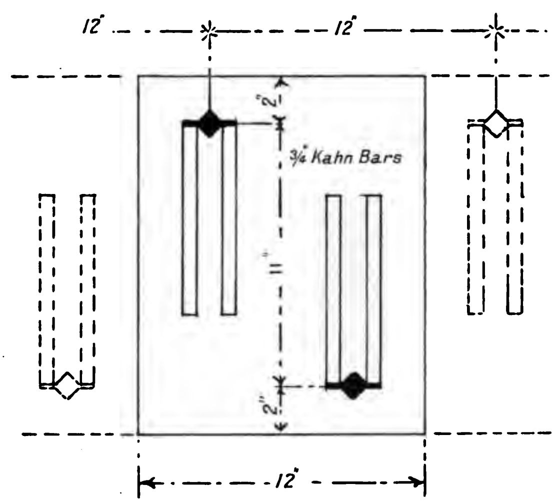


FIG. 95.—Section of Arch near Crown.

Direct compression in steel = 113 × 15 = 1,700 lbs. per square inch.

Maximum compression in concrete (total) = 410 lbs. per square inch.

Nett tension in steel (13,900—1,700) = 12,200 lbs. per square inch.

The bending moment at the weakest point near the springing, marked *A* in Fig. 92, is

$$196 \times 1{\cdot}3 = 255 \text{ ft. cwt.} = 343{,}000 \text{ in. lbs.}$$

The shear at this section is 175 cwt., and the resultant thrust $\sqrt{196^2 + 175^2} = 263$ cwt. $= 29{,}500$ lbs. Fig. 96 shows the section resisting this combined bending moment and thrust.

Width of section, 12 in.

Total depth, 44 in.

Effective depth, 40 in.

Area of reinforcement at top and also at bottom,

$$\cdot 66 \text{ sq. in} = \cdot 14 \text{ per cent.}$$

Moment of resistance to bending,

$$\cdot 0013 \times 12 \times 40^2 \times fs. \quad \text{(Fig. 15.)}$$

Tension in steel due to bending, *fs*.

$$343{,}000 \div \cdot 0013 \times 12 \times 40^2 = 13{,}700 \text{ lbs. per square inch.}$$

Maximum compression in concrete due to bending,

$$13{,}700 \div 50{\cdot}8 \text{ (Fig. 15)} = 270 \text{ lbs. per square inch.}$$

Direct compression in concrete,

$$29{,}500 \div (12 \times 44 + 14 \times 2 \times \cdot 66) = 54 \text{ lbs. per square inch.}$$

Direct compression in steel,

$$54 \times 15 = 800 \text{ lbs. per square inch.}$$

Maximum compression in concrete (total),

$$270 + 54 = 324 \text{ lbs. per square inch.}$$

Nett tension in steel,

$$13{,}700 - 800 = 12{,}900 \text{ lbs. per square inch.}$$

The length of overlap required where bars join, in order that the resistance to slipping may be comparable with the working tensile resistance of the bar, is 5 ft. 0 in. Thus, working pull on 1 in. square bar

$$= 14{,}500 \times 1 = 14{,}500 \text{ lbs.}$$

Circumference of bar, 4 in.

Length of overlap required $= 14{,}500 \div (4 \times 60) = 60$ in.

The rigidly-attached wings provide in addition a mechanical bond which fully justifies the adoption of as high a value as 60 lbs. per square inch for the working bond stress.

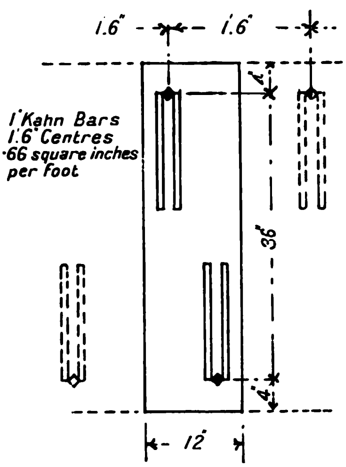


FIG. 96.—Section of Arch near Springing.

The only shear stress on laminal and radial sections, is due to the variation of bending stresses throughout the arch, and this shear will be a maximum where the rate of variation of bending stress is greatest. For instance, the ordinate of the thrust line relatively to the centre line at X, in Fig. 92, is ·32 ft., while at the adjoining section Y it is ·05 ft. The tensile stress in the steel due to bending, corresponding to the ordinate ·32 ft., was found to be 13,900 lbs. per square inch.

The tensile stress in the steel due to bending corresponding to the ordinate ·05 ft. is therefore $\frac{\cdot 05}{\cdot 32} \times 13{,}900 = 2{,}200$ lbs. per square inch. The total stress due to bending in one bar

$$\text{at } X \text{ is } \cdot 52 \times 13{,}900 = 7{,}200 \text{ lbs.}$$

and

$$\text{at } Y \text{ is } \cdot 52 \times \ \ 2{,}200 = 1{,}100 \text{ lbs.}$$

The distance between these sections is 2·65 ft., and in this length there are approximately 5 wings, each having a cross-sectional area of about ·12 sq. in. As these are inclined at 45 degs. in the right direction the area may be increased in the proportion of $\sqrt{2} : 1$, making a total area of

$$5 \times \cdot 12 \times 1 \cdot 41 = \cdot 84 \text{ sq. in.}$$

The stress in the steel $= (7{,}200 - 1{,}100) \div \cdot 84 = 7{,}300$ lbs. per square inch, assuming all the shear stress to be resisted by these diagonal members.

Arched Rib Footbridge.

An interesting example of an arched bridge is shown in Figs. 97 and 98. These footbridges, for there are two of them, were constructed for convenience as three-pin arches. The arched ribs, and the floor and steps supported by them, were constructed at a convenient site near by, and, after the elapse of at least four weeks, erected in position. The reason of this was twofold. Firstly, to do away with the necessity of keeping centering over the line for a considerable period, and to avoid raising the bridge in order to provide room for this centering. Secondly, to obtain a structure which it would be possible to remove without completely destroying it in the process.

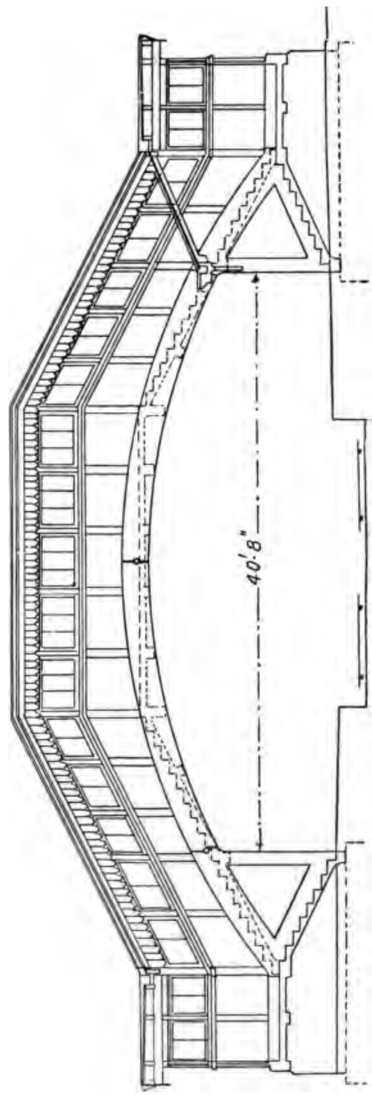


Fig. 97.—Elevation of Footbridge.

The sides and roofing of the footbridges were constructed *in situ*. In the arched ribs vertical gas tube ferrules were cast to form sockets for the reinforcing rods in the lower portion of the sides, which consisted of fine cement concrete rendering on Expanded Metal Lathing. Stiffening columns were formed at intervals by grouping four rods together, and the top was finished with a concrete cill reinforced with two $\frac{3}{8}$ in. diameter rods. This portion of the sides extends about 4 ft. up from the floor, and the upper part is glazed. The roof is of corrugated iron supported on timber beams and posts.

The span of the arches between the centres of the pins at

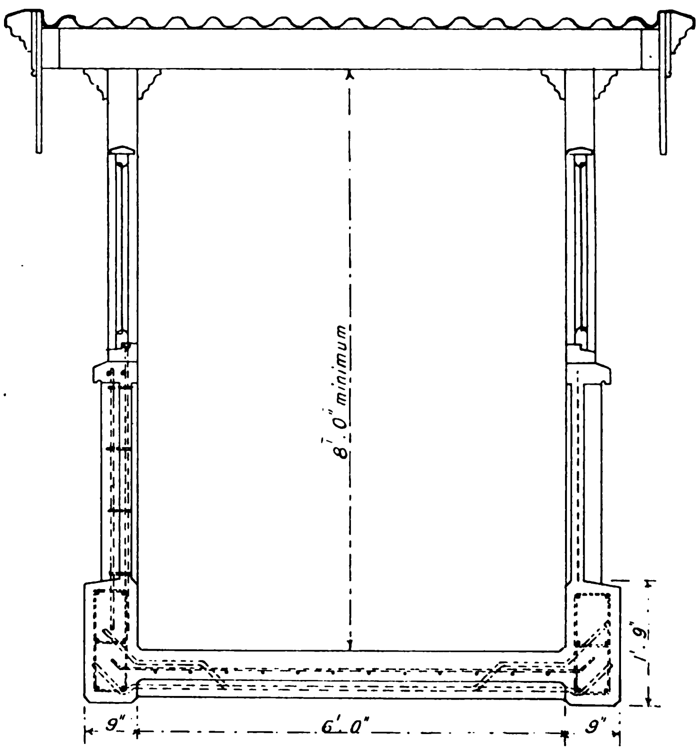


FIG. 98.—Cross Section of Footbridge.

the springings is 40 ft. 8 in., and the rise of the centre line at the crown is 4 ft. 10 in. There are no pins in the exact sense of the word, the bearings being in the form of a knuckle joint. The lower member at each abutment is made in two sections with chipping fillets to admit of adjustment.

The contractor had the option of (*a*) erecting the ribs separately and constructing the floor between them on the site, or (b) constructing the ribs and the floor between them, for each half of the bridge on one side. The weight of each unit to be lifted in the latter case, comprising the arched ribs, floor and steps, for the half span was estimated at $8\frac{1}{2}$ tons. The first alternative was adopted in carrying out the work. The total dead load for the whole span is about 27·5 tons. The live load allowed for is 140 lbs. per super foot of floor and treads of steps. The abutments are partly of reinforced concrete resting upon mass concrete piers.

Specification.

"The cement concrete in piers and foundations to same" was specified "to be composed of one measure of Portland cement, three measures of clean sharp sand, and six measures of the aggregate. The cement concrete in arched ribs, columns, floors, landings, and steps to be composed of one measure of Portland cement, two measures of clean sharp sand, and four measures of the aggregate. The aggregate in each case to consist of properly graded granite or hard close-grained stone, approved by the engineer, broken to pass through a screen with a 1 in. mesh in the case of the piers and $\frac{3}{4}$ in. mesh in the case of the reinforced concrete work, and retained in each case on a screen with a $\frac{3}{16}$ in. mesh, and to be washed if required by the engineer.

"The proportions are to be accurately measured in a gauging box made for the purpose, and the concrete must be thoroughly mixed upon a close wooden platform to a quaking consistency, and must in all cases be well rammed round the steel reinforcement. Great care must be taken that the steel reinforcement is not displaced."

The shuttering for the concrete piers must not be less than 2 ft. deep and $2\frac{1}{2}$ in. in thickness, and they must be sufficiently strutted and supported so as to entirely prevent any bulging of the boards. The shuttering for the concrete beams, floors, landings, and steps must be of sufficient strength to carry the weight of the concrete without deflection, and in no case less than $1\frac{1}{2}$ in. in thickness. The shuttering for the sides of the concrete arched ribs and the boxings for columns must be properly strutted and of sufficient strength to prevent bulging, and provided with fillets to leave a chamfer on the edges of the work, as shown on the drawings. One side of each boxing is to be fixed in short lengths as the concreting proceeds. The timber used in the construction of all boxings and shuttering must be planed on the face in contact with the concrete, with edges shot and set with care to ensure a regular and even surface wherever the face of the concrete will be exposed to view. All boxings and shutterings are to be thoroughly cleaned, wetted, and properly limewashed, greased, covered with oil-paper, or treated with other approved method before use each time they are used. The contractor must supply 6 in. cube test blocks of the concrete as mixed for the work, and these will be tested from time to time by the engineer. The time at which the centering of the arched ribs and beams may be removed will be decided by the engineer from the results obtained. The vertical boxings and shuttering are not to be removed within 48 hours from the time the whole of the concrete is laid in, and the centering for the concrete arched ribs and beams must not in any case be removed within 28 days from the time the concrete is laid in. The contractor may remove the shuttering to the floors, landings, and steps after 14 days from the time the concrete is laid in, provided that in so doing he does not interfere with the centering of the arched ribs and beams and supports to same.

If any part of the reinforced concrete face work shows a rough or uneven surface when the shuttering, boxing, or centering is removed, such uneven portions must be immediately well washed over with thin grout composed of one measure of Portland cement to one measure of clean sharp sand carefully applied with a brush so as to entirely remove such rough or uneven surface, but if any part of the face of the piers shows a rough or uneven surface when the sheeting is removed, such uneven portions must be cut out wedge-shape in squares to a depth of 4 in. and refilled with concrete composed of two parts of granite chippings broken so as to pass in any direction through a ring $\frac{3}{4}$ in. in internal diameter to one of Portland cement, including only a sufficient quantity of clean sharp sand to fill all cavities. The concrete piers must be carried out in regular courses parallel to the top sloping surface, not exceeding 2 ft. in height. Care must be taken that at the end of every stage in which the concrete may be allowed to set for more than two hours the front edges for a thickness of 4 in. shall be left to a perfectly straight line, the inner portion of the layer being left with a rough uneven surface sloped downward from the faces, which must be thoroughly cleansed from all dirt and loose matter and well saturated with water before proceeding with the layer above.

The reinforced concrete portion of each footbridge and its abutments consists of four parts or members connected together by cast-iron bearing pieces, and the whole of the concreting for any one member must, if possible, be completed during one continuous period in regular radial courses, and in the case of the columns regular level courses not exceeding 2 ft. in height. If at the end of any such stage it should be necessary to allow the concrete to set for more than two hours, before proceeding with the next stage the surface must be thoroughly cleansed from all dirt and loose matter, well saturated with water, and washed over with thin neat cement grout before proceeding with the layer above, and the top of the concrete piers must be similarly treated before the concreting for the reinforced portion of the work is commenced.

If the concrete for the landings or floors cannot be completed at one time, the junction is to be made over the central concrete pier in the case of the landings, and over the centre of a beam in the case of the floors. The concrete where exposed to view and from 1 ft. below the surface of the ground must in all cases be finished with a fine face, great care being taken to see that the concrete "fat" is well and continuously worked to the front by means of hoop iron swords. No broken stone must be allowed to appear on the face. The top surface of the arched ribs and other portions not in contact with shuttering, boxing, or centering must in all cases be finished with smooth and regular surfaces true to form. The concrete walls, arched ribs, floors, steps and landings must be carefully protected from injury while setting.

The fine cement concrete rendering on expanded metal lathing in partitions to the sides of footbridges is to be carried out *in situ*, and is to be composed of four measures of finely crushed granite passed through a sieve of $\frac{3}{8}$ in. mesh and retained upon a sieve of $\frac{3}{16}$ in. mesh, and thoroughly washed, including a sufficient quantity of sand to fill all cavities, and one measure of Portland cement gauged and mixed in a similar manner to the cement concrete, but with a sufficient quantity of clean cowhair.

The rendering is to be put on in two layers, and finished with smooth floated surfaces.

The steel reinforcing rods must be kept under cover until required for use in the work, and must be free from oil, grease, dirt, and, except where otherwise specified, paint and all rust must be removed by thoroughly cleaning with wire brushes immediately before the concreting is carried out. All steel-rods must be firmly fixed to the centering by means of temporary templates, and to the cast-iron bearing pieces by means of tapered spikes, to ensure each part of the reinforcement remaining firmly in the position indicated for it on the drawings. No steel, except expanded metal lathing, is to have at any point less than 1 in. of concrete covering. The bars must in all cases be carefully bent to uniform curves, and where turned up must be set to a true angle of 45 degs. The prices in the schedule of quantities must include all royalties, necessary templates, sweeping to curves, fixing, and tightly binding with wire at intersections and overlaps, and fixing with tapered spikes to cast-iron bearing pieces, and all cleaning.

The expanded metal lathing for partitions to sides of footbridges to be tightly bound with annealed wire to the round rod reinforcement. Temporary supports to be placed horizontally on one side of the partition, while the reverse side is being rendered and allowed to set. The steel reinforcement for partition work is to be thoroughly cleaned with wire brushes and afterwards immediately painted one coat of red lead paint.

Bending Moments and Reactions.

The calculation of the maximum stresses in arches of this kind is comparatively simple, because the section is constant throughout and the curvature sufficiently flat to justify the assumption that the centre line follows a parabola instead of a circular arc. After erection the clearances between the various members were grouted up, so that the structure can scarcely be assumed to act as a three-pin arch, in so far as live load stresses are concerned. The actual condition of the ends is partial fixture, and the only reliable method of procedure was to make the ribs of sufficient strength to resist at all points

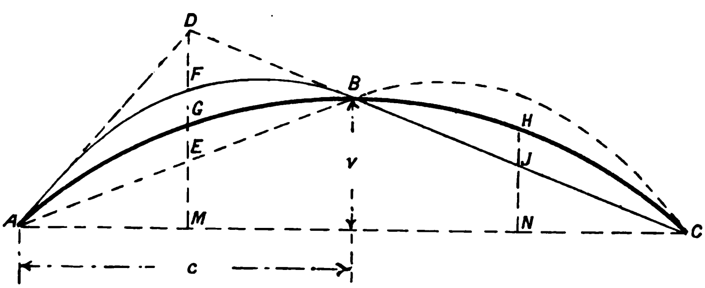


Fig. 99.—Three-pin Arch.

the maximum stresses as calculated on the basis of (*a*) a three-pin structure, and (*b*) an arch with fixed ends. When a parabolic arch is completely covered with a uniformly distributed live load in addition to the dead load which is practically uniformly distributed, the line of pressure for all practical purposes coincides with the centre line of the arch, no matter whether there are two or three pins or fixed ends. The rib is then in simple compression. The bending stresses caused by unsymmetrical loading will in general be a maximum when one half of the arch is covered with live load and the other half unloaded. In the simplest case of the three-hinged arch the line of pressure corresponding to this condition of live load will be obtained as follows:—In Fig. 99, *AB* being the loaded half of the arch, join *CB*, and produce to cut a vertical line through the centre of the load at *D*. This will be at one quarter of the span. Join AD. The line of pressure is completed by constructing a parabola with AD and DB as tangents at A and B, and is shown by the full line in Fig. 99.

For a two-pin arch the line of resultant pressure will be similar in form, but it is not at first sight certain that it will pass through the centre line of the arch at the crown. If, however, the right-hand half of the arch be loaded in place of the left, the line of pressure will be shown by the dotted line. These two combined together must give as a resultant the line of pressure due to the completely loaded arch which coincides with the centre line; and, therefore, for the two-pin arch the line of thrust for the arch half loaded must intersect the centre line at the crown of the arch.

Also the maximum sagging moment which occurs at the centre of the loaded half of the arch, and is represented by FG, must equal the maximum hogging moment occurring at the centre of the unloaded half, represented by HJ. This also follows from the property of the parabola; for FE and EM being each equal to $\frac{1}{2}v$ and GM equal to $\frac{3}{4}v$, FG and HJ will each be equal to $\frac{1}{4}v$. The vertical components of the reactions R_1 and R_2 are respectively $\frac{3}{4}W$ and $\frac{1}{4}W$, and the horizontal component throughout the arch, H, is equal to $\frac{1}{4}W \times c \div v$, W being the total load on half the arch, c the half span, and v the rise of the centre line of the arch. The maximum sagging and hogging moments referred to above will therefore equal

$$\left(\frac{1}{4}W \times c \div v\right) \times \frac{1}{4}v = \frac{1}{16}Wc.$$

In the case of the fixed end arch, by the same reasoning, the line of thrust for the same condition of loading must intersect the centre line of the crown, and referring to Fig. 100, ordinate AP at the left abutment must be equal to the ordinate CQ at the right abutment. The value of AP is, however, unknown and must be worked out from the properties of an elastic arch with fixed ends to which reference has already been made.

This can be done in general terms applying to any parabola, the value of AP or CQ being $\frac{v}{4}$ or one quarter of the rise of the centre line of the arch, and the line of resultant thrust will therefore cut the centre line of the arch at points marking the quarter of each half span. The value of the bending moments

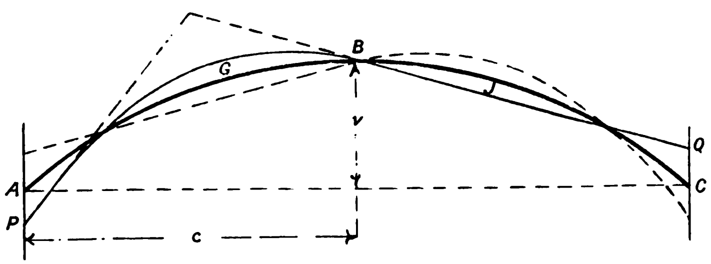


FIG. 100.—Fixed End Arch.

at J and G are readily ascertained from the properties of the parabola and equal $\frac{9}{64}\,vH$.

Similarly the values of the reactions R_1 and R_2 and the horizontal thrust may be determined

Thus,
$$R_1 = \frac{13}{16}\,W$$
$$R_2 = \frac{3}{16}\,W$$
$$H = \frac{Wc}{4v}.$$

The maximum bending moment at the springing is therefore
$$\frac{Wc}{4v} \times \frac{v}{4} = \frac{1}{16}\,Wc,$$
and at points J and G (Fig. 100),
$$\frac{Wc}{4v} \times \frac{9v}{64} = \frac{9}{256}\,Wc.$$

It is thus seen that in the fixed end arch the maximum bending moment for this condition of loading is the same as in the two-hinged arch, the difference being that in the former case the maximum occurs at the springings, and in the latter at the quarter spans. In the footbridges illustrated in Figs. 97 and 98 the section of the arched ribs was made constant throughout and sufficiently strong to resist this maximum bending moment of $\frac{1}{16}$ Wc in addition to the maximum direct compression. This will appear from the calculations which follow. The section of one rib showing the reinforcement which consisted of Patent Indented Bars appears in Fig. 101.

Calculation of Stresses.

Span between centres of pins, 40 ft. 8 in.
Rise between centres of pins, 4 ft. 10 in.
Width of one rib, 9 in.
Effective depth, 18·5 in.
Overall depth, 21 in.
Area of reinforcement near each surface $= 2 \times {\cdot}25 = {\cdot}5$ sq. in.

$$\text{Percentage} = \frac{{\cdot}5 \times 100}{9 \times 18{\cdot}5} = {\cdot}3 \text{ per cent.}$$

Total area of reinforcement $= 2 \times {\cdot}5 + 2 \times {\cdot}06 = 1{\cdot}12$ sq. in.

Effective area resisting direct compression
$= (9 \times 21 + 14 \times 1{\cdot}12) = 205$ sq. in.

Dead load on one rib

Rib $42{\cdot}5 \times {\cdot}75 \times 1{\cdot}75 \times 156$ lbs. =	8,700
Floor $22 \times 3 \times {\cdot}4 \times 156$ lbs. =	4,120
Steps $18 \times 3 \times {\cdot}5 \times 156$ lbs. =	4,210
Beams $3 \times {\cdot}75 \times 5{\cdot}6 \times 156$ lbs. =	1,960
Total amount for erection purposes	18,990
Parapet $40{\cdot}7 \times 4{\cdot}2 \times 35$ lbs. =	5,990
Glazed sides $40{\cdot}7 \times 80$ lbs. =	3,260
Roofing and snow $40{\cdot}7 \times 4{\cdot}5 \times 13$ lbs. =	2,380
	30,620 lbs.

Live load on half span $20{\cdot}4 \times 3 \times 140$ lbs. $= 8{,}570$ lbs.

Horizontal thrust due to dead load

$$\frac{30{,}620 \times 40{\cdot}7}{8 \times 4{\cdot}8} = 32{,}500 \text{ lbs.}$$

Horizontal thrust due to live load

$$\tfrac{1}{4} \times 8{,}570 \times 20{\cdot}4 \div 4{\cdot}8 = 9{,}080 \text{ lbs.}$$

Total horizontal thrust 41,580 lbs.

Maximum bending moment $= \frac{1}{16} \times 8{,}570 \times 20{\cdot}4 = 10{,}900$ ft. lbs. $= 131{,}000$ in. lbs.

Total shear at quarter span

$$\frac{30{,}620}{4} + \frac{2}{3} \times \frac{3}{4} \times 8{,}570 = 11{,}940 \text{ lbs.}$$

Magnitude of resultant thrust at this point

$$\sqrt{41{,}580^2 + 11{,}940^2} = 43{,}300 \text{ lbs.}$$

Direct compression stress in concrete $= \dfrac{43{,}300}{205} = 210$ lbs. per square inch.

Direct compression stress in steel $= 15 \times 210 = 3{,}200$ lbs. per square inch.

Moment of resistance to bending

$$= {\cdot}135 \times 9 \times 18{\cdot}5^2 \times fc = 131{,}000 \text{ in. lbs. (Fig. 15.)}$$

Maximum compression stress in concrete due to bending

$$131{,}000 \div {\cdot}135 \times 9 \times 18{\cdot}5^2 = 315 \text{ lbs. per square inch.}$$

Maximum tensile stress in steel due to bending

$$46{\cdot}6 \times 315 = 14{,}700 \text{ lbs. per square inch (Fig. 15).}$$

Total compression stress in concrete	315	
	210	
	525	lbs. per square inch.
Nett tensile stress in steel	14,700	
	3,200	
	11,500	lbs. per square inch.

The only shear stress is that due to the variation in the bending stresses. Assuming the ends of the rib to be fixed, and therefore the maximum stress due to bending to occur at the ends, 14,700 lbs. per square inch tensile stress is developed in a length of 20·4 ÷ 4 = 5·1 ft. = 61 in.

Total stress in steel = ·5 × 14,700 = 7,350 lbs.

Bond stress 7,350 ÷ (61 × 2 × 2) = 30 lbs. per square inch.

In a length of 5 ft. there are six of the bracing members shown in Fig. 101, presenting a total cross sectional area in a plane parallel to the neutral axis of 6 × 2 × ·06 = ·7 sq. in. Assuming the steel to resist all the shear stress the intensity of shear stress is

7,350 ÷ ·7 = 10,500 lbs. per square inch.

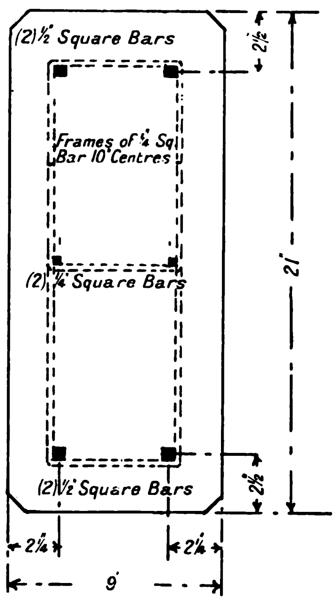


Fig. 101.—Section of Arched Rib.

The spacing of these lateral members is also dependent upon obtaining an efficient cage to resist the bursting tendency of the concrete under compression.

The maximum tensile stress in the steel is rather low, but during erection the ribs and the floor between them were liable to be picked up by means of a pair of lifting eyes one to each rib, and sufficient reinforcement had to be provided to meet this condition of a double cantilever. The tensile reinforcement in the neighbourhood of the lifting eyes, where the maximum stresses occur during erection, is augmented by the ¾ in. diameter rod in which the eye is formed and which is continued in each direction for a sufficient distance near the top surface and then turned down into the rib. It will be seen from the elevation (Fig. 97) that the floor is near the top of the rib at this point, and the longitudinal reinforcement of the floor also helps to rest the tensile stresses during erection.

Maximum Erection Stresses.

Area of tensile reinforcement at lifting eye.

(2) $\frac{1}{2}''$ Indented bars	=	·5
(1) $\frac{3}{4}''$ round rod	=	·44
say (1) $\frac{3}{8}''$ „ „ in floor	=	·11
		1·05

Percentage of reinforcement $\frac{1\cdot05 \times 100}{18\cdot5 \times 9} = \cdot6$ per cent.

„ „ compression reinforcement = ·3 per cent.

Maximum bending moment (one rib)

$$\frac{18{,}990}{4} \times \frac{40\cdot7}{8} = 24{,}100 \text{ ft. lbs.}$$

$$= 289{,}000 \text{ in. lbs.}$$

Moment of resistance $= \cdot17 \times 9 \times 18\cdot5^2 \times f_c$.

Maximum compression stress in concrete $= \frac{289{,}000}{\cdot17 \times 9 \times 18\cdot5^2}$
= 550 lbs. per square inch.

Maximum tensile stress in steel $= 31\cdot7 \times 550 = 17{,}400$ lbs. per square inch.

Concentrated Loads.

The stress due to concentrated loads is very easily determined in the case of a flat arch with fixed ends. In Fig. 102 POQ is the line of resultant thrust due to the load at O, and is fixed by the values

$$AP = \frac{2}{15} \times \frac{c - 5b}{c - b} \times v.$$

$$MO = \frac{6}{5} v.$$

$$CQ = \frac{2}{15} \times \frac{c + 5b}{c + b} \times v.$$

When AP is negative, it is measured below the springing line, and it will be seen that MO is independent of b, the distance of the load from the centre of the span, and therefore the point O will always lie on a horizontal line $\frac{1}{5}v$ above the centre line at the crown. Any given load produces a maximum bending moment in the arched rib when $b = \cdot 45c$ and to obtain the worst result from a system of concentrated loads these should be placed on half the span with this position as a centre. The horizontal component of the thrust due to

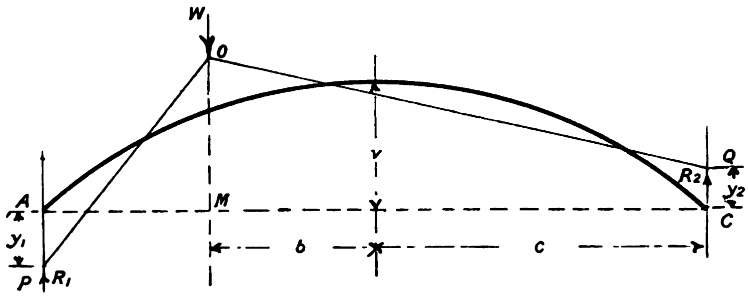


FIG. 102.—Fixed End Arch with Concentrated Load.

a concentrated load W, is $15W(c^2 - b^2)^2 \div 32vc^3$, and the reactions will be obtained from the relations:—

$$\frac{R_1}{H} = \frac{MO - AP}{c - b}.$$

$$\frac{R_2}{H} = \frac{MO - CQ}{c + b}.$$

To simplify the determination of the different values, the curves in Fig. 103 and Fig. 104 have been plotted. The ordinates in Fig. 103, when multiplied by v, give the value of AP (y_1) and CQ (y_2) for varying ratios of $b \div c$. In Fig. 104 the values of y_2 are plotted to 10 times the scale and in addition are two curves for finding the value of H_1, the horizontal thrust, and R_2, the smaller reaction. To obtain the horizontal thrust for any required value of $b \div c$ the ordinate of the curve marked H must be multiplied by $\frac{Wc}{v}$, and to obtain the reaction R_2 the ordinate of the curve so marked has to be multiplied by W. If there are several concentrated loads the location of the thrust line and the magnitude of the horizontal component may be determined for each singly, and the resultant thrust line plotted, and the magnitude of the

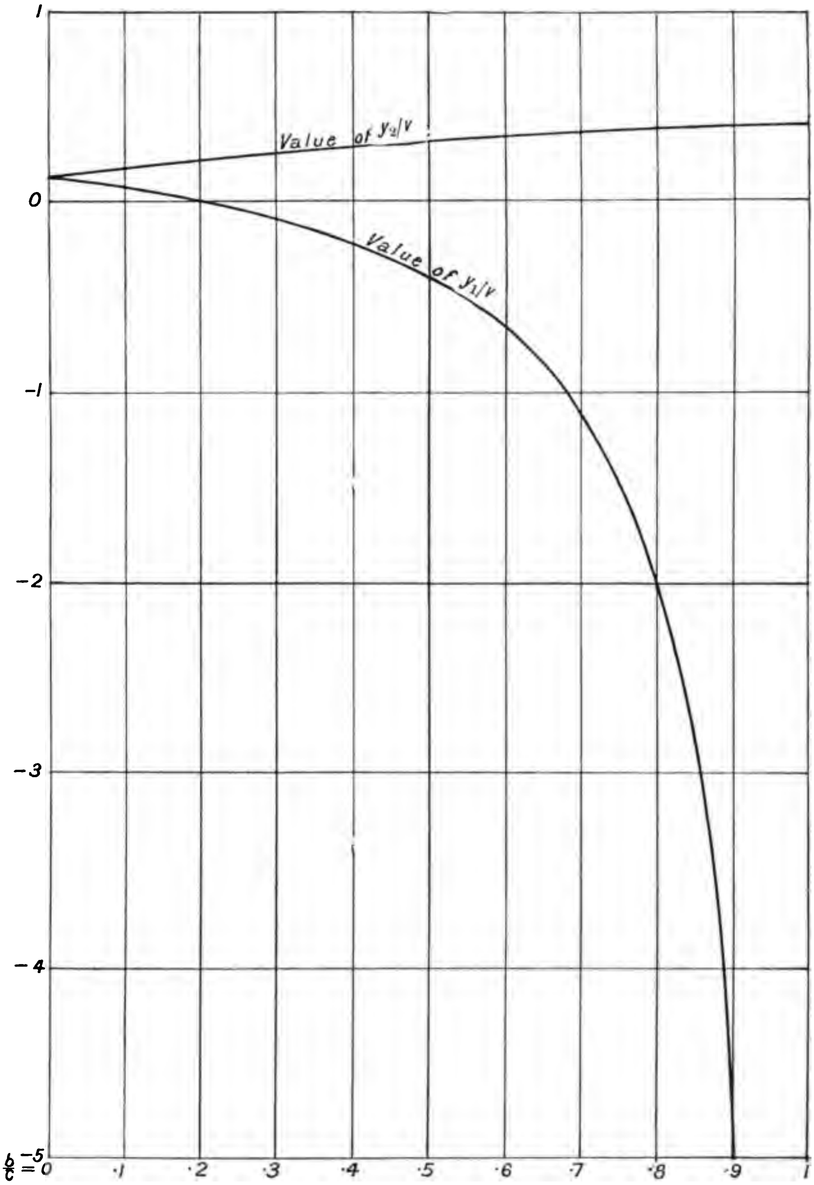


FIG. 103.—Curve Determining the Line of Thrust for Concentrated Load.

combined horizontal component ascertained, by the ordinary rules for finding the resultant of two or more coplanar

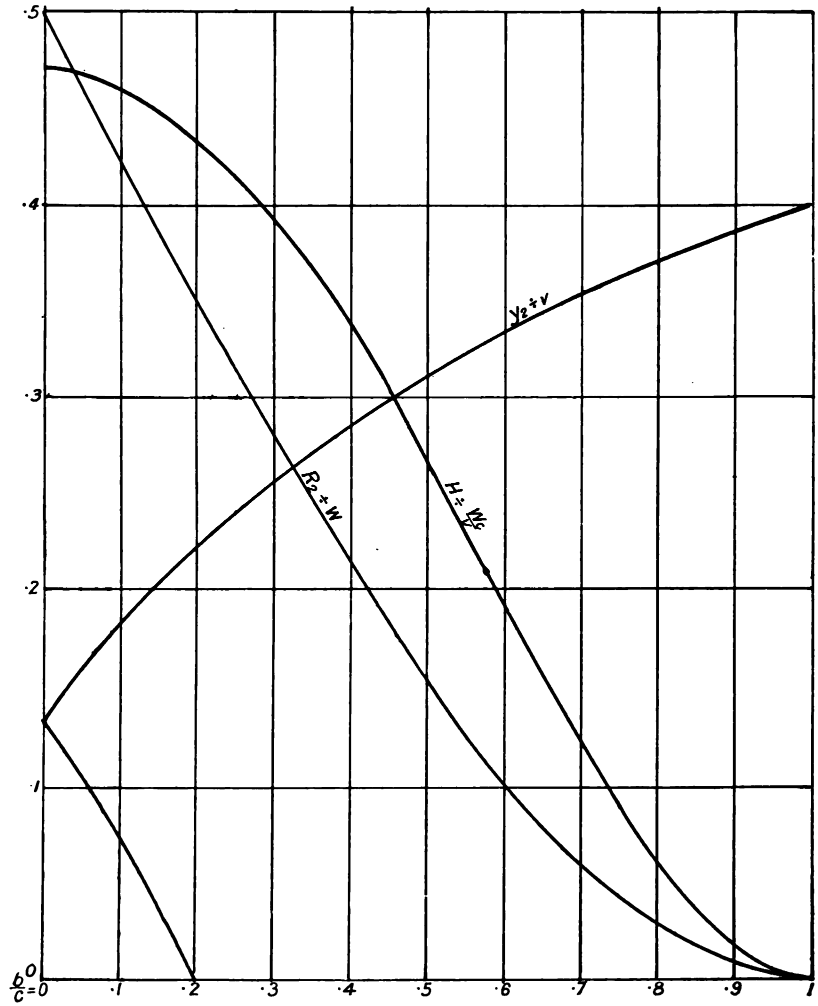


FIG. 104.—Horizontal Thrust and Vertical Reaction for Concentrated Load.

forces, the positions and horizontal components of which are known.*

* The Author's article, "Steel Arched Ribs for Railway Bridges," in *The Engineer*, December 9th, 1910.

Examples of Arched Bridges.

The footbridge over the London and South Western Railway at Bollo Lane, illustrated in Figs. 105 and 106, presents

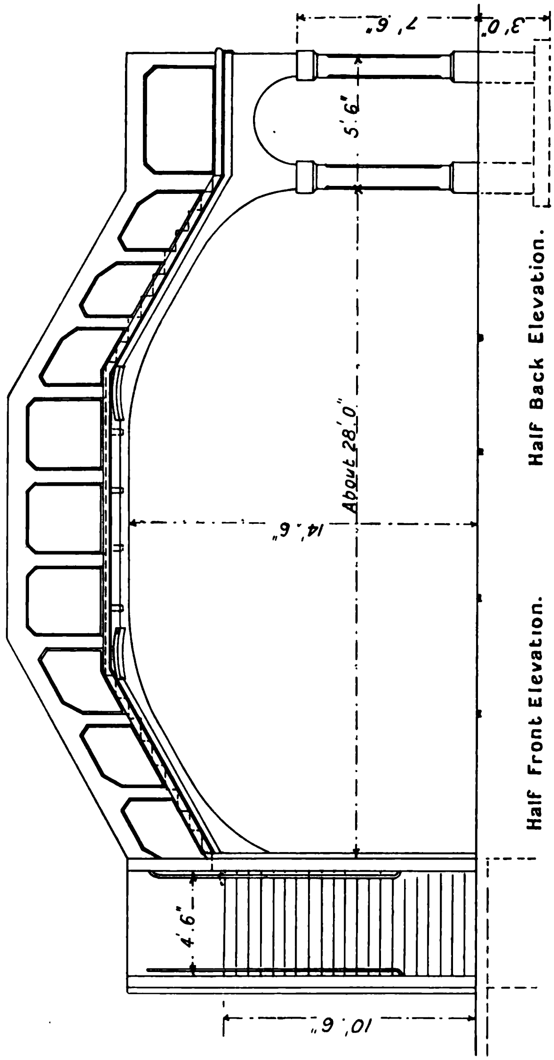


Fig. 105.—Footbridge at Bollo Lane, London and South Western Railway.

another example of arched construction. This bridge is of Hennebique ferro concrete and presents a pleasing appearance.

The main dimensions appear on the elevation and enlarged cross section.

The bridge illustrated in Figs. 107 and 108, also of Hennebique ferro concrete, enjoys the distinction of being the first reinforced concrete bridge to be passed by the Board of Trade for passenger service. This bridge is under the Avonmouth and Filton Railway, Great Western Railway, and is built for a double line of rails. The span is 24 ft. and the thickness of the arch 8 in. at the crown and 14 in. at the springings. The counterforts to the abutments are about 7 ft. apart and

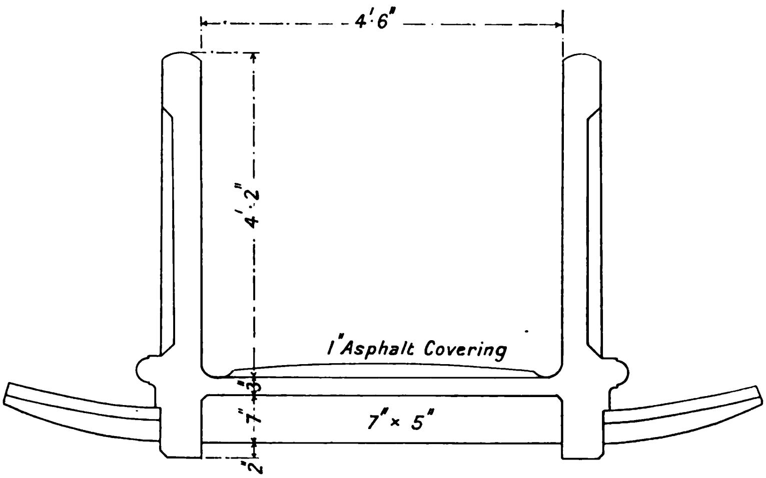


FIG. 106.—Footbridge at Bollo Lane. Cross Section of Superstructure.

the thickness of the walls is 7 in. The abutments are founded upon 14-in. piles and are connected together below ground level by three 9-in. × 12-in. tie beams. The bridge was designed for 80-ton locomotives and carries in addition a considerable amount of filling.

In another type of arched bridge which has been largely used the floor is constructed as a deck, consisting of slabs and beams, supported by columns resting on two or more arched ribs. Several large spans have been constructed in this way, resulting in structures distinctly pleasing in appearance. There is a bridge of this type carrying an occupation road over the Cheshire Lines Committee's main line from Liverpool to Manchester at Farnworth. The clear span of the arches is 57 ft. and the rise 8 ft. 3 in. The road is 12 ft. wide between parapets and there are three ribs. The depth of these is 2 ft. at the crown and 2 ft. 4 in. at the springings, and the width is 12 in. in the case of the outside ribs, and 14 in. in the case of the centre rib. The columns are

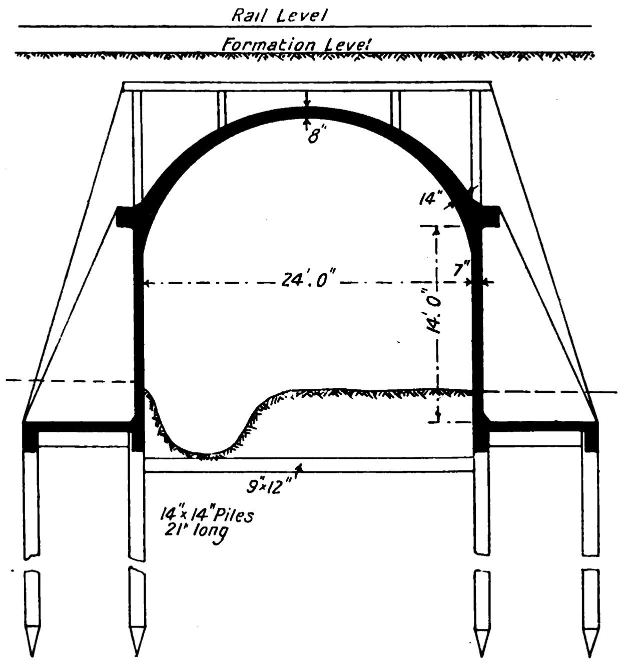


FIG. 107.—Underbridge, Avonmouth and Filton Railway. Longitudinal Section.

12 in. × 9 in. generally and the end posts 12 in. square. The deck beams are 15 in. deep overall × 12 in. wide and the slab floor 10 in. thick overall. The reinforcement consists of Kahn Trussed Bars and Kahn Rib Bars, and the proportions of the concrete constituents in the order Portland cement, sand, granite chippings are 1 : $1\frac{1}{2}$: $3\frac{1}{2}$. The parapets are of reinforced concrete, and the abutments of mass concrete faced with masonry. The bridge is shown in outline in Fig. 109.

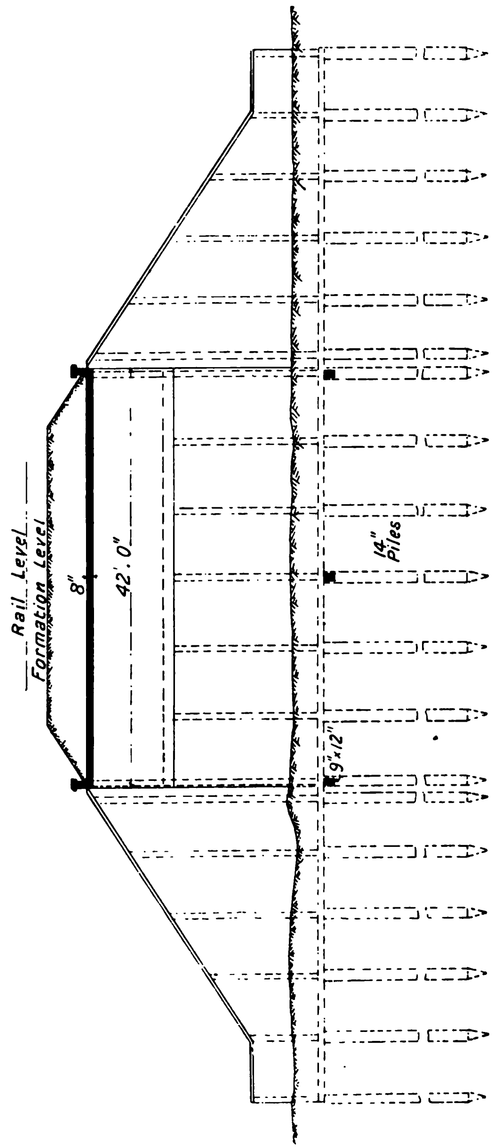


FIG. 108.—Underbridge, Avonmouth and Filton Railway. Cross Section.

CULVERTS.

Concrete tubes are now used for a variety of purposes, not only as drainage pipes and culverts but also in the construction of manholes, cesspits, &c., in connection with these

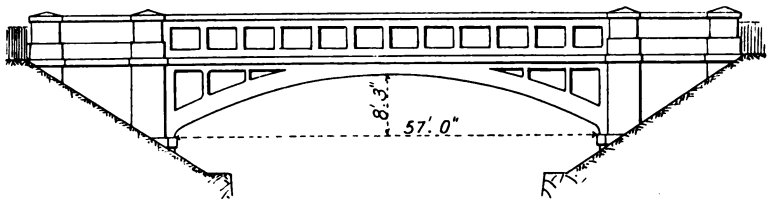


FIG. 109.—Road Bridge at Farnworth. Cheshire Lines Committee.

and drainage systems generally. For the small tubes reinforcement is not found to be necessary, and in larger culverts constructed *in situ* it is generally more economical to

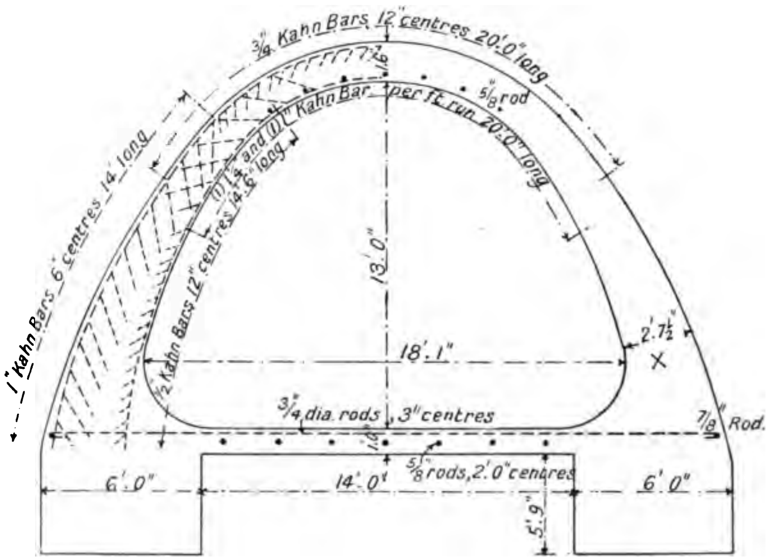


FIG. 110.—Kilton Culvert, North Eastern Railway.

increase the thickness of the concrete rather than obtain additional strength by reinforcement. Larger tubes moulded elsewhere are usually reinforced, and attention has already been called to the need for longitudinal reinforcement in long culverts constructed *in situ*. Some large culverts have been constructed, through railway banks, in reinforced concrete. The section of one of these, at Kilton, on the North Eastern Railway, is shown in Fig. 110. This culvert is 435 ft. in length, of which 275 ft. in the middle is of the section shown. Towards the end the thickness at the crown diminishes 2 in. at a time to 12 in. at the inlet and outlet, and the thickness at X similarly diminishes to 1 ft. 9 in. at the ends. The reinforcement consists principally of Kahn Trussed Bars, and the sizes and the spacing of these are indicated on the section. At the end the spacing of the bars is increased considerably.

CHAPTER IX

SLEEPERS, FENCE POSTS, &c.

Sleepers.

At present the use of reinforced concrete sleepers in this country can only be regarded as experimental. There is considerable difficulty in the first place in designing a sleeper of sufficient strength if the depth is not to exceed that of the standard wooden sleeper, namely, 5 in., and the amount of metal required creates further difficulty in providing the holes for spikes and screws. In America and on the Continent reinforced concrete sleepers have been tried with more success, possibly on account of the greater depth employed, 8 in. in some cases, and also because the use of flat-bottomed rails simplifies the fastenings. The sleepers which have been tried in this country are rather more costly in the first place than wooden sleepers and about three times as heavy. Their weight, about $3\frac{1}{2}$ cwt. each, or nearly $4\frac{1}{2}$ cwt. when two chairs are added, makes them extremely difficult to handle. Unless they remain perfectly sound under traffic their one advantage is lost, and the difficulty of turning out sleepers in large numbers at the lowest possible cost, and at the same time maintaining uniform perfection of workmanship, would appear to be very great. Unless this last point is observed it is certain that under the punishment of express traffic sooner or later cracks will appear, and the concrete will begin to break away and the reinforcement will be exposed to corrosion. In attempting to design a sleeper of a depth not greater than 5 in. it is necessary to bring the steel as near to the surface as possible, and this introduces the danger that in the process of manufacture the reinforcement may either come into actual contact with the mould or else be so near the surface that a crack is formed immediately the sleeper is subjected to load and vibration. Sufficient experience has been obtained to show that very great care will have to be exercised in the design of a successful reinforced concrete sleeper, and in order to reduce the cost and the weight to the absolute safe minimum the materials must be disposed in the best possible way.

LOADS.

This opens up the interesting problem of determining the nature and the amount of the stresses produced in a sleeper under traffic. The first point to determine is the maximum amount of load transmitted to the ballast by one sleeper when the heaviest engines are passing over the track. This depends upon the stiffness of the rail and the amount of the depression of the sleeper into the ballast. The author has carefully observed this depression, which is quite visible, both for wooden sleepers and for reinforced concrete sleepers. The observations were made by gumming strips of paper, ruled with divisions one-tenth of an inch apart, to the web of the rail at points as near to the centre of the sleepers as the chairs would allow, and reading the maximum movement under traffic, relatively to the cross hair in a telescope, set up some 20 or 30 ft. away and focussed upon the paper strip. In the case of a line laid with rails of British standard section and weighing 95 lb. per lineal yard, and reinforced concrete sleepers, as shown in Fig. 115, and 2 ft. 6 in. apart, the maximum depression was in most cases ·08 in., the lowest reading being ·06 in. and the highest ·09 in. There was no appreciable difference between sleepers adjacent to a joint in the rails, and the ordinary sleepers, and none between the observations made on a low bank as against those made in a shallow cutting. With wooden sleepers the results were not so uniform, but the average maximum reading was ·16 in. As the result of a careful analysis* the author concludes that under four-coupled engines, with 20 tons on each coupled axle, and six-coupled engines, with 15 tons on each of the coupled axles, the maximum

* "Stresses in Rail Sleepers," *The Engineering Review*, December 15th, 1911.

load carried by one sleeper will not exceed 10 tons. Although in actual practice the loads are applied to and withdrawn from a sleeper perhaps ten times in one second, the influence of a wheel load upon any particular sleeper is felt before the load reaches that sleeper, so that the effect produced is that of a load increasing from zero to its full value.

In observing the depressions, it was not possible for the eye to pick out the rapidly repeated effect of each wheel of the engine, but the maximum movement could readily be gauged. Most of the trains were made up of long bogie coaches; the result being sets of four axles alternating with long spaces. As these spaces came opposite the sleeper under observation the rail could be seen to recover its position, and as each cluster of wheels passed over, a maximum depression was observed in some cases equal to that produced by the engine. In other cases the subsequent depressions were not so great, depending probably upon the extent to which the passage of the loads synchronised with the vibrations set up in the rail. These, however, are evidently speedily damped, because almost immediately after the passage of the train the rail came to rest in its original position. In observing the deflections of bridge girders, also it is noticeable that the deflection due to an engine passing over at speed is not much, and not always any more than that due to the engine at rest, so that in determining the effect of moving loads upon permanent way it is probably more correct to consider the loads as gradually attaining their full value rather than as suddenly applied.

The importance of the point lies in the fact that a load applied at its full value before any resistance is set up would depress the sleeper into the ballast, if the latter were subject to elastic compression directly proportional to the resistance set up, just twice as much as a load gradually applied; *and the maximum pressure on the under side of the sleeper would be just twice as great as the applied load.*

The observations described, however, would not appear to indicate that this effect is produced, and from the cases considered it seems reasonable to suppose that the maximum load on one sleeper does not exceed 10 tons, that is 5 tons from each rail. *It must not be forgotten, however, that this is "live" load directly applied to the member under consideration.*

Stresses in Memel Sleeper.

The stresses which these loads will produce in a sleeper must of necessity be largely dependent on the manner in which the sleeper is bedded on the ballast. The usual practice with wooden sleepers is to pack the finer material under the chairs with the "beater," or flat end of the pick, for a space of about 20 in. on either side of each rail, until the sleeper is firmly embedded, and to tightly pack the remainder, that is, about 6 in. at each end, and about 18 in. in the middle, with the shovel.

This is done with the object of preventing the sleeper taking a decided bearing at the ends or at the middle.

The maximum depression of a wooden sleeper has been observed to be about ·16 in., whereas the net deflection of the sleeper under the rail with reference to the middle and the ends, to secure distribution of a load of 10 tons over the whole length of the sleeper would be about ·03 in. It is evident, therefore, that the whole of the sleeper must be depressed into the ballast, and therefore the whole of the under side of the sleeper must be subjected to pressure. The problem is to determine the distribution of this pressure, and from this the stresses produced in the sleeper. If the assumption that the resistance to depression offered by the ballast is directly proportional to the amount of depression is correct; and further, if the distribution of resisting pressure, which will cause at all points a depression directly proportional to the pressure at the point, can be found, then the problem is solved. It is known that the depression at the ends and at the middle of a standard wooden sleeper will be about $\left(\frac{\cdot 16 - \cdot 03}{\cdot 16}\right) = \frac{13}{16}$ths of that under the rails, and therefore the resistance at the ends of the sleeper and at the middle will be about $\frac{13}{16}$ths of that immediately under the rails. The sinuous curve representing the distribution of pressure in Fig. 111, has been drawn to secure this relation and at the same time give a total pressure on the under side of the sleeper of 10 tons. The average pressure per foot on a sleeper 9 ft. long being 1·11 tons, that at the end and middle has been taken at $\left(\frac{13}{14\cdot5} \times 1\cdot11\right)$ about 1 ton per foot, while under the rail the pressure per foot has been assumed to be $\left(\frac{16}{14\cdot5} \times 1\cdot11\right)$ about 1·22 tons. The applied load of 5 tons at each rail is shown uniformly distributed over the

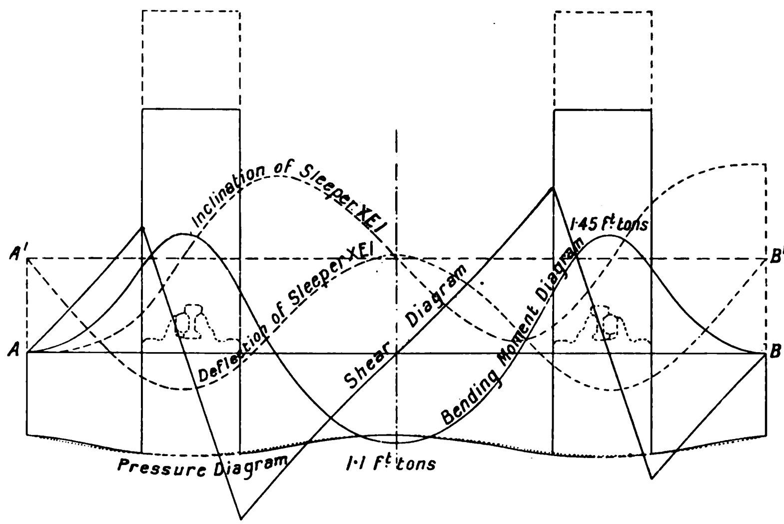


FIG. 111.—Bending Moment Diagram, &c., for Memel Sleeper.

length of a chair 14 in. From this load diagram a shear diagram may be obtained by graphic integration, and the bending moment diagram may be obtained from the shear diagram by a similar process. For instance, working from the end of the sleeper where the shear and the bending moment are each known to be zero, the ordinate of any point on the shear diagram is the area of the load diagram to the left of the point, areas above the datum being counted positive, and areas below the datum negative. When the middle of the sleeper is reached, the shear is found to be zero, because the positive and negative loads balance. Similarly, any point on the bending moment diagram is obtained by taking the area of the shear diagram to the left of the point considered, and at the centre of the sleeper there is found to be a negative or upward bending moment. The scales originally adopted were 1 in. equals 1 ft. for length, and 1 sq. in. equals 1 ton for load, and the bending moment diagram was plotted to a vertical scale of 1 in. equals 1 ft. ton. The diagrams are reproduced in Fig. 111 about half this size. By continuing the process of integration the slope of the bent sleeper and the net deflection at any point is obtained, but to secure actual results the ordinates of these diagrams must be divided by the modulus of elasticity of the material, and the moment of inertia of the section reduced to feet and ton units. In proceeding from the slope diagram to the deflection diagram it is necessary to change the datum from AB to A^1B^1 equivalent to adding a constant in the process of integration. This can readily be done, because the sleeper is known to be horizontal at the middle, and the curve will therefore intersect the true datum A^1B^1 at this point. The maximum ordinate of the deflection curve is 1·6 ft. ton units. The modulus of elasticity of fir is about 130,000 tons per square foot, and the moment of inertia of the section of a 10-in. × 5-in. sleeper is

$$\frac{1}{12} \times \frac{10}{12} \times \left(\frac{5}{12}\right)^3 = \cdot 0050 \text{ ft. units.}$$

The maximum net deflection of the sleeper under the chair with reference to the end is therefore

$$\begin{aligned} 1\cdot 6 \div (130{,}000 \times \cdot 0050) &= \cdot 0025 \text{ ft.} \\ &= \cdot 03 \text{ in.} \end{aligned}$$

The value assumed for the maximum net deflection of the sleeper is thus justified by this result, and, as a means of comparing the depression of the sleeper at other points with the resistance assumed, the two are plotted side by side to suitable scales. It will be seen that the dotted line, which shows the position of the depressed sleeper relatively to the datum AB, scarcely departs from the pressure diagram, showing that with the distribution of pressure represented by this diagram, the condition that the depression shall be proportional to the resistance offered is fulfilled.

The maximum bending moment which occurs under the chair is 1·45 ft. tons, or 17·4 in. tons. The modulus of the section of a 10-in. × 5-in. sleeper deducting two ⅞ths in. holes is

$$\frac{1}{6} \times 8{\cdot}25 \times 5^2 = 34{\cdot}4 \text{ in. units,}$$

resulting in a maximum fibre stress of

$$17{\cdot}4 \div 34{\cdot}4 = {\cdot}5 \text{ ton per square inch.}$$

This gives a factor of safety of about 4. The maximum shear is 2 tons, and as there are no holes where this occurs the average shear stress per square inch of section is $2 \div 50 = {\cdot}04$ tons per square inch, equivalent to a maximum shear stress at the neutral axis of ·06 tons per square inch. This gives a factor of safety along the grain of about 4·7. The maximum pressure on the ballast is 1·22 tons per lineal foot, or for a sleeper 10 in. wide, 1·46 tons per square foot. The diagrams shown in Fig. 111 were not obtained at the first trial. In the first instance, a distribution of pressure varying from about ·9 tons per lineal foot at the ends and middle to 1·3 tons per lineal foot under the chairs was adopted. The bending moment diagram obtained varied scarcely at all from that given in Fig. 111, but the resulting deflection curve did not agree with the assumed distribution of pressure, which was corrected accordingly.

Mention is made of this to show that the result would not be greatly affected if in practice the relation between the depression and the pressure was somewhat different from that assumed.

The upward bending moment at the middle of the sleeper of 1·1 ft. tons is not very much less than the downward bending moments under the chairs, and it is here that the direct stresses are most likely to be increased by secondary stresses due to vibration. It is therefore very essential to avoid all possibility of the sleeper taking any bearing at the centre in the nature of a fulcrum, especially with reinforced concrete sleepers which are generally reduced in section throughout the middle portion of their length. For this reason it is common practice with such sleepers to pack the ballast only under the chair-seats or widened portions leaving the ballast quite slack at the centre, or even raking it out, until there is a clear space of an inch or so between the under side of the sleeper and the ballast beneath it.

STRESSES IN REINFORCED CONCRETE SLEEPERS.

A reinforced concrete sleeper intended to be packed in this way is shown in Fig. 112, the chair-seats being 3 ft. long and packed throughout this length, and the space between them not packed, 2 ft. long. The dotted line shows

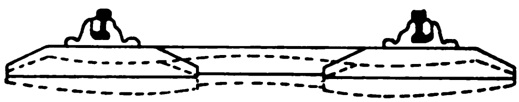


FIG. 112.—Reinforced Concrete Sleeper.

to an exaggerated scale the form taken by the sleeper when loaded. The resistance to bending offered by the middle portion causes the sleeper to bear more heavily on the ballast inside the rails than outside, as indicated by the pressure diagram in Fig. 113. The distribution shown in this figure was obtained by the process of making a trial shot to determine the form of the depressed sleeper and amending the pressure diagram to agree with it.

From this amended pressure line the diagrams in Fig. 113 were obtained by successive integration as before, and it will be seen that the form of the depressed sleeper shown by the dotted line agrees sufficiently closely with the pressure diagram to secure practical fulfilment of the straight line relation between depression and pressure. In passing from the bending moment diagram to that giving the slope of the bent sleeper it was necessary to make a correction for the reduced moment of inertia of the section of the sleeper at the middle. The maximum downward bending moment under the chairs with this method of packing is ·97 ft. tons, as compared with the maximum of 1·45 ft. tons obtained with a sleeper 9 ft. long packed throughout its length.

The upward bending moment at the centre is only ·2 ft. tons and considerably less than the maximum of 1·1 ft. tons in the previous case. The maximum shear is also less, being 1·55 tons as compared with 2·0 tons. The pressure per

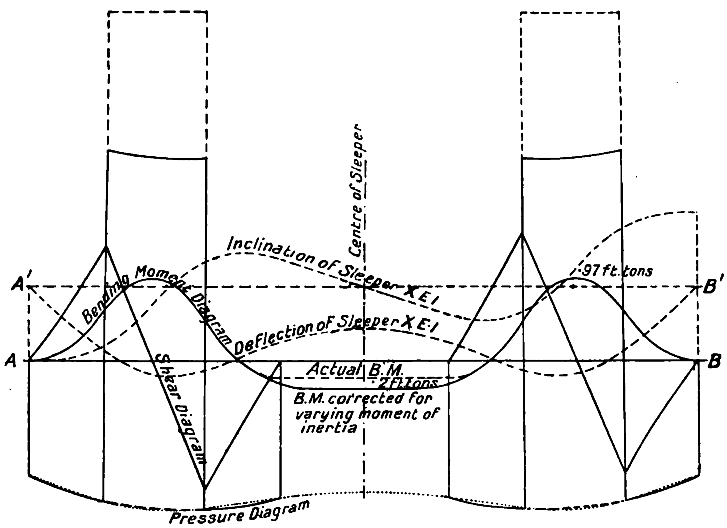


FIG. 113.—Bending Moment Diagram, &c., for Reinforced Concrete Sleeper.

lineal foot on the ballast is more, the maximum being 1·75 tons as compared with 1·22 tons in the previous case. It is not probable, or even possible, that with nothing to guide him the platelayer will accurately pack an equal distance on either side of the chair, and it is interesting to see what is the effect if the packing is carried further on the insides of the rails than on the outsides, and *vice versa*.

In Fig. 114 the full line shows the bending moment diagram obtained in Fig. 113. The thick lines indicate the extent of packing, namely, 3 ft. under each chair.

The dotted line above the full line shows how the bending moments are influenced if the packing is carried about 2½ in. further on the outsides of the rails and restricted about

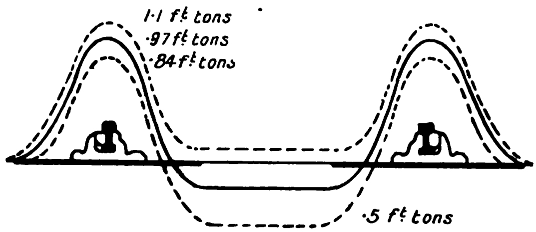


FIG. 114.—Result of Variation in Packing.

2½ in. on the insides. Under this condition there is no upward bending moment at the middle of the sleeper, and the

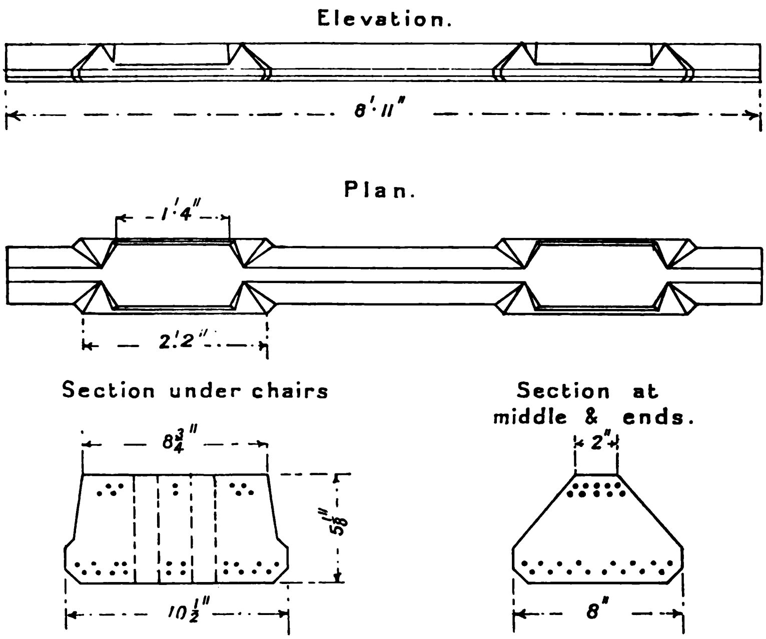


FIG. 115.—Improved Construction Company's Sleeper.

maximum downward bending moment under the chairs is increased from ·97 ft. tons to 1·1 ft. tons. The lower dotted line shows the bending moment diagram obtained if the packing is carried about $2\frac{1}{2}$ in. further on the insides and restricted by equal amounts on the outsides. In this case the upward bending moment at the centre is increased to ·5 ft. tons. In order to define the area which it is intended should be packed the chair-seats are sometimes made to project below the middle portion of the sleeper as shown in Fig. 112, this being one of the many improvements in concrete sleepers protected by patent in this country.

Mention has been made of the secondary stresses which may be produced, particularly in the middle part of the sleeper by vibration. But even when a clear space is maintained beneath the sleepers at the middle it is usual to fill up with ballast between the sleepers, and this ballast bearing on the sides must tend to quickly damp vibrations and confine them within a limited amplitude.

As the ordinary fir sleepers appear to have a factor of safety of about 4 it would seem to be reasonable to design reinforced concrete sleepers on similar lines, and the theoretical strength of the examples shown in Figs. 115 and 116 is approximately in accordance with this standard.

Examples of Reinforced Concrete Sleepers.

Fig. 115 shows the details of a sleeper manufactured by the Improved Construction Co. Ltd. by the Jagger process on oscillating and vibrating tables. It is claimed that this process ensures absolute homogeneity of the concrete, and contact of the latter with the steel and the fine appearance of sleepers manufactured in this way proves how perfectly the concrete fills the mould. It is also possible to do with much less cement and sand than in the case of hand or ordinary machine mixed concrete, because the vibratory motion of the table brings the particles of the aggregate into such close contact that the percentage of voids is very much decreased, and any excess of mortar, if used, would be brought to the surface by the action of the machine. The average composition of the concrete as mixed on these tables is 1 part of cement, 2 parts of close grained sand, and from 4 to 5 parts are broken stone. The weight of one of these sleepers for 4 ft. $8\frac{1}{2}$ in. gauge is $3\frac{1}{2}$ cwt. including 35 lbs. of steel. The longitudinal round rods shown in the section are $\frac{3}{16}$ in. diameter and the longitudinal lattice bracing rods are of the same size. Wooden ferrules are driven into holes moulded in the sleeper to receive the spikes and screws for fastening the chairs. These sleepers have been considerably used on the Continent

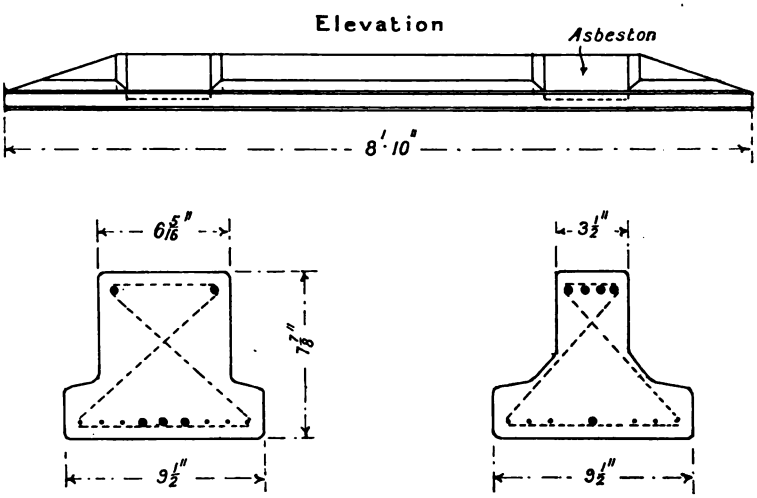


FIG. 116.—Wolle's Asbeston Sleeper.

and have been tried experimentally on some of the main lines in this country.

Fig. 116 shows the design of a sleeper manufactured by Herr Rudolf Wolle of Zurich, which has been used in Saxony. One of the characteristics is the incorporation of two blocks of a material called "Asbeston," one under each rail seat, to receive the spikes or screws. This is found to secure a very satisfactory fastening. The asbeston practically becomes a part of the concrete sleeper, and greater force is required to withdraw spikes from this material than from timber. The gauge is 4 ft. $8\frac{1}{2}$ in. and the length of the sleeper 8 ft. 10 in. The width of the bearing surface is $9\frac{1}{2}$ in. throughout, but the width of the rail seat is only $6\frac{5}{16}$ in., flat bottomed rails being used. The depth is $7\frac{7}{8}$ in. and it is this increased depth

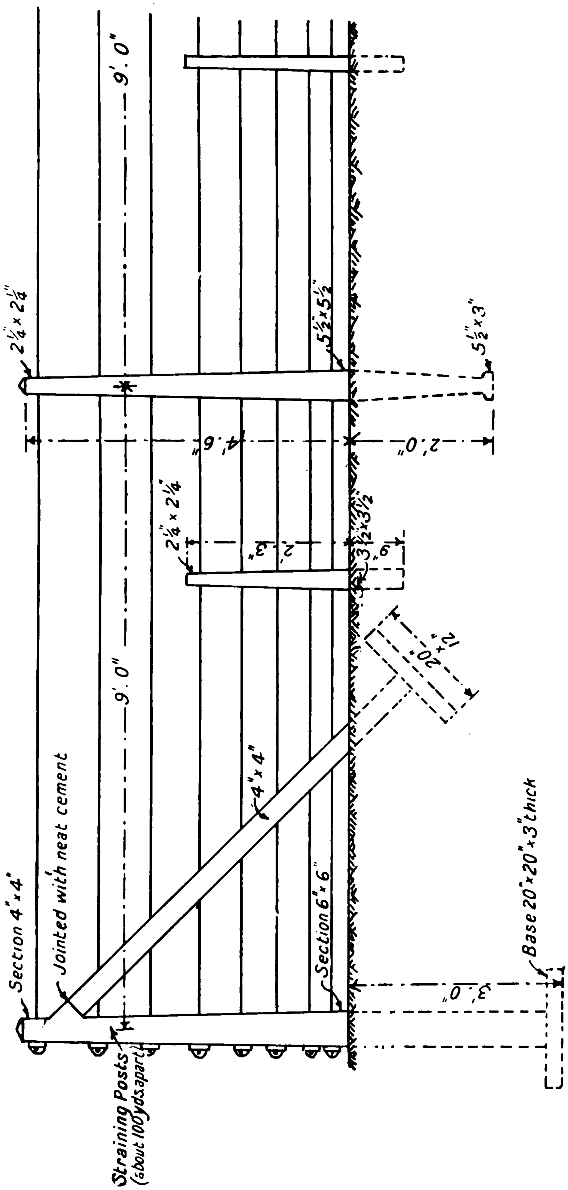


FIG. 117.—Reinforced Concrete Fence Posts.

which enables the sleeper to be designed with a comparatively small amount of reinforcement, leaving room for the "asbeston" blocks, the extent of which is shown in the elevation. The weight of one of these sleepers is 3·8 cwt. including 28 lbs. of reinforcement. This consists mainly of five larger rods rather more than $\frac{3}{8}$ of an inch in diameter and six smaller rods rather more

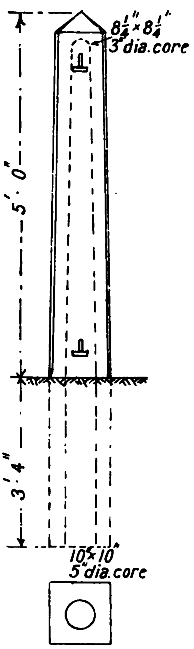


FIG. 118.—Gate Post.

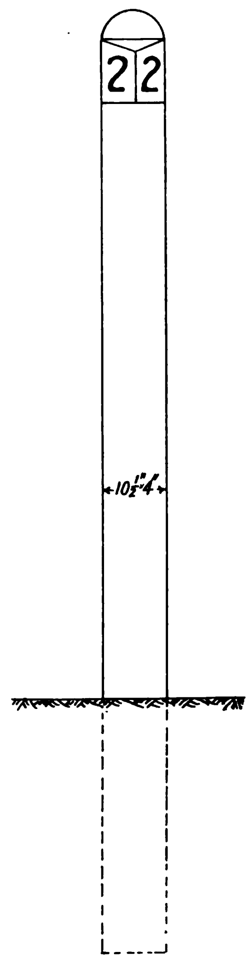


FIG. 119.—Mile Post.

than $\frac{3}{16}$ of an inch in diameter. Under the chair-seats two of the larger rods dip down from the top to the bottom of the sleeper. At these points the proportion of the bottom or tension reinforcement is $1\frac{1}{4}$ per cent. of the effective cross-sectional area, and the proportion of the top or compression reinforcement is ·5 per cent. of the effective area. In the shallower sleeper illustrated in Fig. 115, under the chair-seats, these proportions are $1\frac{1}{2}$ per cent. and $\frac{3}{4}$ per cent. respectively.

Posts for Fences, &c.

Fences consisting of reinforced concrete posts and seven or eight strands of strained wire are being used more and more

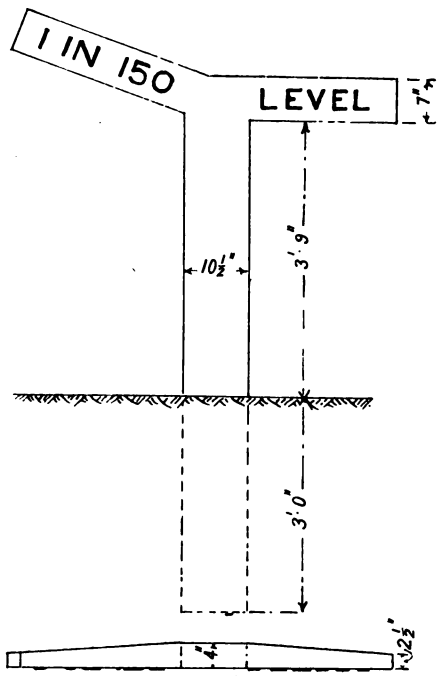
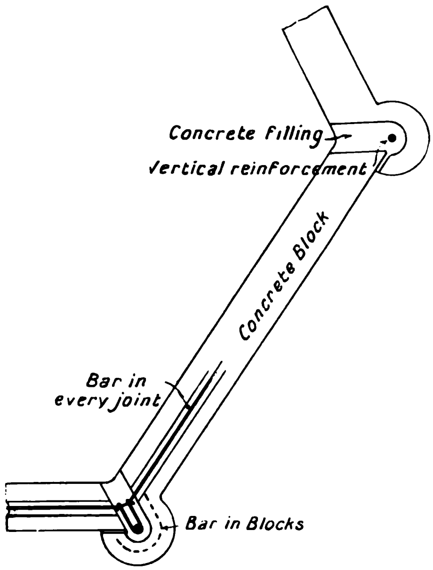


Fig. 120.—Gradient Post.

for railway purposes. The first cost is very little above that of the usual post and rail wooden fencing, and the result is more lasting, at any rate in so far as the posts are concerned. A type of fence in common use is shown in Fig. 117, this being a registered design of Mr. E. F. W. Grimshaw. The straining post shown is suitable for the commencement or end of a line of fencing. In the case of intermediate straining posts, the struts are frequently omitted and two posts linked together at the top by steel straps with the winders between them. Wires of from $\frac{5}{16}$ in. to $\frac{3}{16}$ in. diameter are used to reinforce these posts, four of the larger size being used for the straining and main posts and the smaller size for the "prick" posts, bound together at intervals with wires of about $\frac{1}{8}$ in. diameter. The



121.—Units for Chimney Construction.

use of reinforced concrete posts is not confined to boundary fencing.

Cattle-pens have been constructed with posts of this material used in conjunction with gas tube railings. Fig. 118 shows a gate post which is of square section with a hollow core. These are frequently used for field gates, and accommodation level-crossing gates. On some of the more recently constructed lines reinforced concrete mile and gradient posts have been employed and examples of these are illustrated in Figs. 119 and 120. They are practically everlasting and require less painting, and possess a great advantage over wooden posts, which are bound to rot in time below the ground line.

Chimneys.

The advent of electric traction on our railways brings the building of power stations and the necessary chimneys within the practice of the railway engineer. Reinforced concrete chimneys constructed *in situ* have not proved successful on account of the liability to crack under the great heat. An ingenious method of building chimneys of specially shaped concrete units, which permit of the structure as a whole being suitably reinforced with steel, has however been devised.

The blocks are shaped and put together as shown in Fig. 121. The spaces or clearances at the angles which are filled in with concrete serve a two-fold purpose. They provide chases for the vertical reinforcement near the outside of the chimney where this is most required, and the means for obtaining the battered faces without the necessity of specially shaping each block. The blocks are reinforced in themselves and the bed joints are grooved to accommodate continuous horizontal bars. These are connected by loops to the vertical reinforcement, and the whole structure is thus completely reinforced. The vertical ribs formed by the rounded covers to the angles give the chimney a pleasing appearance, and suitable cornices and copings can readily be formed. The lower part of these chimneys is lined with firebrick, but at the top the concrete is left unprotected.

Water Towers.

Several water towers have been constructed, for railway purposes, of reinforced concrete, and the material is happily chosen as regards the appearance, utility and freedom from maintenance charges of the resulting structures. Fig. 122 is a sectional elevation of a tank to hold 40,000 gallons, constructed at Heaton, on the North Eastern Railway, of Hennebique ferro concrete. The height overall is about 60 ft., the dimensions of the circular tank being 21 ft. in diameter and 19 ft. in depth. The thickness of the bottom is 6½ in. and of the sides 5½ in. There are eight uprights of square section

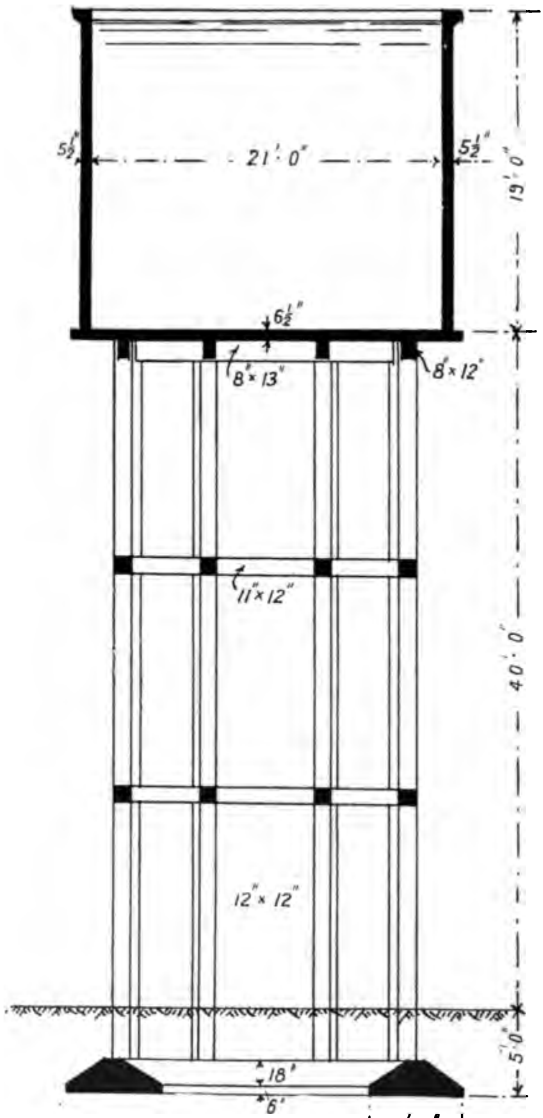


FIG. 122.—Water Tank at Heaton.

12 in. × 12 in. connected together at the head by a circumferential beam and four transverse beams, and similarly braced at two intermediate points. The cill is circular in plan and the section is shown in Fig. 122. Fig. 123 shows a section of

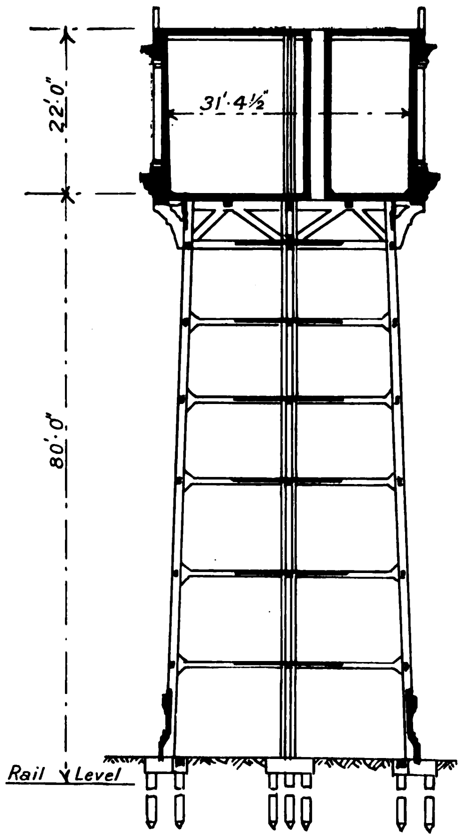


FIG. 123.—Water Tower at York.

the larger of two water towers erected for the North Eastern Railway Company at York. These tanks which are reinforced principally with Kahn Bars hold 100,000 gallons and 20,000 gallons respectively. The larger tank is octagonal in shape and 31 ft. 4 in. internal diameter. It is 21 ft. in depth, and the bottom of it is about 80 ft. above ground level. The weight, about 450 tons of water, is carried on eight outside columns which measure 36 in. × 15 in. at the base, and one central column, octagonal in shape and 21 in. in diameter. These columns are braced horizontally at six intermediate points, as shown, and the bracing also supports platforms which serve to give access by means of ladders to the tank. The foundation consists of three piles under each column. It will be noticed that the tank is covered and the top is reached by means of a circular shaft through the tank.

CHAPTER X

SUMMARY OF NOTATION EMPLOYED

A Area of column section.
a Area of stirrup section.
a Longer axis of rectangular slab.
B Width of foundation.
b Shorter axis of rectangular slab.
b Breadth of rectangular beam or effective breadth of tee beam.
b Thickness of wall or pier.
b Distance of concentrated load from centre of span.
c Half-span.
d Effective depth of beam or other member subjected to bending.
F Total shear.
f_c Maximum compression stress in concrete.
f_s Tensile stress in steel.
H Horizontal component of thrust in arch.
h Height of retaining wall.
jd $\left(1 - \frac{y}{3d}\right) d$, arm of couple of stresses resisting bending.
K A constant determining moment of resistance to bending in terms of tensile stress in steel.
k A constant determining moment of resistance to bending in terms of compression stress in concrete.
l Span of beam.
MR Moment of resistance.
m Ratio of modulus of elasticity of steel to that of concrete, taken as 15.
n Number of reinforcing rods.
p Proportion of tension reinforcement.

p' Proportion of compression reinforcement.
P Proportion of total reinforcement.
P Earth pressure on retaining wall.
q Principal stresses.
R Vertical components of reactions at abutments of arches.
R Reaction of slope upon wedge of earth slipping on same.
R Radius of circular foundation.
r Radius of circular column.
s Maximum shear stress in concrete.
s' Shear stress in steel.
t Diameter of reinforcing rod.
u Bond stress between steel and concrete.
v Rise of centre line of arch.
W Total load or concentrated load.
W_1 Portion of load on slab transmitted in the direction of short axis.
W_2 Portion of load on slab transmitted in the direction of long axis.
w Distributed load per unit of length.
x Spacing of stirrups.
y Depth of neutral axis of beam below compression surface.
y Ordinate of a point on the line of thrust for a loaded arch referenced to a convenient horizontal datum.
β Inclination of resultant earth pressure with normal to back of retaining wall.
τ Inclination of principal stresses with neutral axis.
θ Segment of circular foundation.
θ Angle of earth slope upon which slipping is assumed to occur.
ϕ Angle of repose.

Summary of Principal Formulæ.

Resistance to bending single reinforcement:—

Ratio of stresses, $\frac{f_s}{f_c} = \frac{y}{2pd}$ . . . . . . (1)

Neutral axis, $\frac{y}{d} = \sqrt{p^2m^2 + 2\,pm} - pm$ . . . . (2)

Moment of resistance,

$$MR = p\left(1 - \frac{y}{3d}\right)bd^2f_s \quad . \quad . \quad . \quad (3)$$

$$= Kbd^2f_s;$$

$$MR = \tfrac{1}{2}\frac{y}{d}\left(1 - \frac{y}{3d}\right)bd^2f_c \quad . \quad . \quad . \quad (4)$$

$$= kbd^2f_c.$$

Resistance to bending, double reinforcement :—

Ratio of stresses, $\frac{f_s}{f_c} = \frac{y}{2\,pd} + \frac{2}{3}\frac{mp'}{p}$ . . . . (5)

Neutral axis,

$$\frac{y}{d} = \sqrt{m^2\left(p + \tfrac{2}{3}\,p'\right)^2 + 2\,mp} - m\left(p + \tfrac{2}{3}\,p'\right) \quad . \quad (6)$$

Moment of resistance,

$$MR = \left(\tfrac{1}{2}\frac{y}{d} + \tfrac{2}{3}\,p'm\right)\left(1 - \frac{y}{3d}\right)bd^2f_c \quad . \quad . \quad (7)$$

When p and p' are so disposed that $\frac{f_s}{f_c} = 29$

$$MR = (\cdot 11 + \cdot 066\,P)\,bd^2f_c \quad . \quad . \quad . \quad (8)$$

Resistance to shear :—

Bond stress, $u = \dfrac{F}{d\left(1 - \frac{y}{3d}\right)n\pi t}$ . . . . (9)

Shear stress, $s = \dfrac{F}{bd\left(1 - \frac{y}{3d}\right)}$ . . . . (10)

Spacing of stirrups, $x = \dfrac{s'a\left(1 - \frac{y}{3d}\right)d}{F}$ . . . . (11)

Slabs :—

Proportion of load transmitted in the direction of short axis,

$$\frac{W_1}{W} = \frac{a^4}{a^4 + b^4} \quad . \quad . \quad . \quad . \quad (12)$$

Proportion of load transmitted in the direction of long axis,

$$\frac{W_2}{W} = \frac{b^4}{a^4 + b^4} \quad . \quad . \quad . \quad . \quad (13)$$

Short columns:—

$$W = A\,(1 + 14\,P)\,f_c \quad . \quad . \quad . \quad (14)$$

Use of the Squared Paper Diagrams.

The results of many of the preceding formulæ have been worked out and plotted in the form of diagrams, see Figs. 10, 11, 14, 15, 17 and 34, pp. 24, 30, 32, 38 and 60, also plate at end of Chap. X.

It will be noticed that on the same diagrams, quantities of very different numerical value are represented. Consequently, the vertical scale on the diagram must be used for each curve in conjunction with the written value of one particular ordinate on that curve, which determines the position of the decimal point or the number of O's required.

Beams with Tension Reinforcement Only.

When the working tensile stress in the steel and the working compressive stress in the concrete have been decided upon, and the ratio of the former divided by the latter determined, the critical percentage of reinforcement, that is, the percentage which permits these working stresses to be reached simultaneously, can be read off from the curve marked $\frac{f_s}{f_c}$ in Fig. 10.

For instance, when the percentage of tensile reinforcement is ·5 the ratio $\frac{f_s}{f_c} = 32$.

The factor K determining the moment of resistance which $= K \times$ breadth in inches $\times$ (effective depth in inches)^2 $\times$ working tensile stress in steel in lbs. per square inch, *i.e.*,

$$MR = Kbd^2f_s$$

can also be read off from the curve marked K on the same diagram.

Thus with ·5 per cent. of tensile reinforcement $K = {\cdot}0045$.

With a percentage of reinforcement lower than the critical percentage, the working stress in the steel is reached before the working compression stress in the concrete, and the ratio of these stresses, and the factor determining the moment of resistance, for any percentage of tensile reinforcement between 0 and ·6 can be read off from the curves so marked in Fig. 10. With a larger percentage of reinforcement than the critical percentage, the working moment of resistance depends upon the working compression stress in the concrete, f_c, which is reached before the working tensile stress in the steel. The factor k determining this moment of resistance

$$MR = kbd^2 f_c$$

can be read off from the curve so marked in Fig. 11. The position of the neutral axis, corresponding to various percentages of tension reinforcement, is recorded in Figs. 10 and 17. Thus, with 1 per cent. of reinforcement

$$\frac{y}{d} = \frac{\text{depth of neutral axis below compression surface}}{\text{effective depth}} =$$

·418 (Fig. 17).

Beams with Double Reinforcement.

The full line, rising and practically straight, in Fig. 14, determines the values of the moment of resistance when compression reinforcement is provided in addition to tension reinforcement, and the proportions are such as to result in the working compression stress in the concrete and the working tensile stress in the steel being reached simultaneously. The total percentage of reinforcement is read on the lower horizontal scale, and the percentage of compression reinforcement on the upper horizontal scale. The difference between these two gives the percentage of tension reinforcement.

The results recorded in Fig. 14 have been worked out for the following values of f_s and f_c :—

f_s = working tensile stress in steel = 14,500 lbs. per square inch,
f_c = working compression stress in concrete 500 lbs. per square inch, and they apply equally well for working stresses which bear to one another the ratio 29.

The value of k determines the moment of resistance in terms of the maximum compression stress in the concrete, thus :—

$$MR = kbd^2f_c.$$

In Fig. 10 it will be seen that when the ratio of the tensile stress in the steel to the maximum compression stress in the concrete equals 29, the neutral axis is ·34d below the compression surface, and so long as the neutral axis remains in this position the ratio of the stresses will be 29. If it is desired to use compression reinforcement, with working stresses bearing to one another a different ratio, the altered position of the neutral axis can be obtained from Fig. 10, and the relative proportions of compression and tension reinforcement can be worked out by substituting this altered value of $\frac{y}{d}$ in equation 6, p. 28.

Equal Percentages of Tension and Compression Reinforcement.

In columns and arches which may be subjected to bending in addition to direct compression, the proportions of tension and compression reinforcement will, more often than not, be equal. Fig. 15 will enable the reader to determine the position of the neutral axis, the ratio of the maximum stresses, and the factor determining the moment of resistance in terms of f_s or f_c corresponding to the percentages of total reinforcement marked on the lower horizontal scale. The net or working tensile stress in the steel will be the tensile stress due to bending, f_s, minus the direct compression in the steel, usually taken as fifteen times the direct compression in the concrete. The net or working compression stress in the concrete will be the maximum compression stress due to bending, f_c, plus the compression in the concrete.

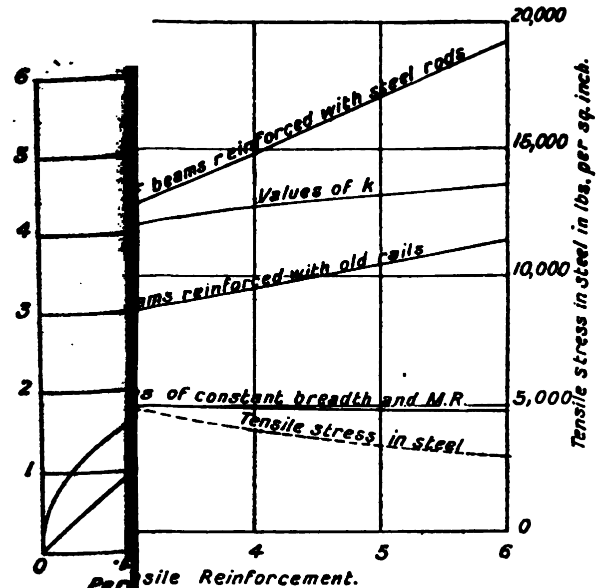
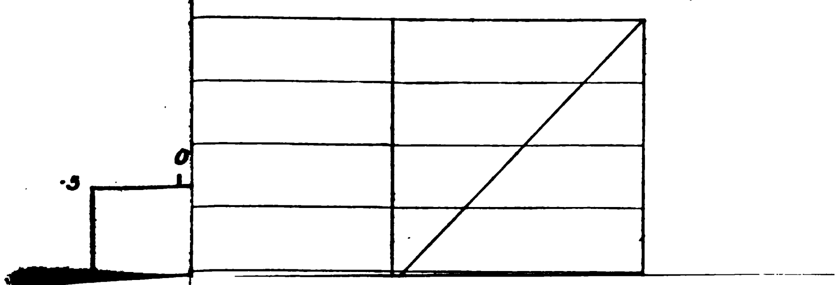


FIG. 10.—Constants, Single Reinforcement.



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