REINFORCED CONCRETE

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WITH 512 ILLUSTRATIONS AND DIAGRAMS

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M. HENNEBIQUE'S COUNTRY HOUSE AT BOURG-LA-REINE.

This house is built entirely of reinforced concrete, and is an excellent example of the boldness of design which is rendered possible by the use of this material. The main tower, weighing about 200 tons, projects 13·12 feet, and is carried mainly by the two intersecting cantilevers. On the right is a balcony which will be used for a winter garden. It is supported on cantilevers 8·20 feet in length. All the roofs are flat, and are laid out as hanging gardens. At the extreme right of the building, not seen in the illustration, there is another projecting tower containing a water tank of 4,855 gallons capacity, with a circular stairway passing through the centre.

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INTRODUCTION

In the following pages the author has endeavoured to place before engineers, architects and others a complete treatment of the subject of reinforced concrete in so far as is possible at the present day. It is hoped that the book, which is believed to be the first of the kind to appear in the English language, will be found useful by the many engineers and architects who already take an interest in this form of construction, and who wish to investigate it, and that it may also be the means of bringing to the notice of others a material whose usefulness and economy in suitable cases are beyond dispute.

All the subject matter has been so arranged as to facilitate reference as much as possible. The several systems used up to the present have been placed in alphabetical order, so that any particular one may be readily found when desired.

It is believed that the part relating to the calculations covers all forms of construction in as concise and clear a manner as possible. The formulæ for slabs and beams, although giving somewhat smaller dimensions than those recommended by M. Christophe in *Le Beton Armé* (a standard French work on the subject), are still well on the side of safety, and it is hoped that the tables and diagrams may be of use in saving the labour necessary in making the requisite calculations. The subject of arches has been dealt with in as condensed a form as possible compatible with a clear demonstration of the methods adopted for locating the pressure curve. The graphical method for finding the stresses to be resisted in domed coverings is believed to be entirely new, and greatly simplifies the treatment of these structures.

It has been considered advisable to illustrate the book very fully, in order that all the subject matter, where possible, may be rendered clearer, and that a true idea may be formed of the remarkable adaptability of reinforced concrete for constructional purposes.

The author is indebted to two French works, *Le Beton Armé*, by M. P. Christophe, and *Ciment Armé*, by MM. C. Berger et V. Guillerme, but mostly to *Le Beton Armé*, in which M. Christophe has given a most excellent and complete treatment of the subject, and which is deservedly considered a standard work on reinforced concrete. Many foreign, American and English sources of information have been drawn upon, and the work has been rendered easier by the universal courtesy and kindness of the several patentees of the different systems and their representatives.

Acknowledgments of the use of articles from the various periodicals have been made in the body of the book, and it is hoped that such acknowledgment has not in any case been omitted through an oversight.

The author is also greatly indebted to Mr. W. Dunn, F.R.I.B.A., and to Mr. G. W. Herdman, Assoc.M.Inst.C.E. for their kindness in looking through his MS. and proofs, and for many valuable suggestions.

CHAS. F. MARSH.

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to a greater degree, but that by combining the two materials we protect the iron and give to small sections of metal a usefulness which they cannot possess by themselves, and further, that as each material acts in its own way in resisting the stresses, it is not really reinforced concrete but iron and concrete combined in such a manner as to counteract the imposed stresses.

This is true, but undoubtedly the main object of the combination is to strengthen the concrete slab, beam, column, arch, etc., to a greater degree, and certainly the first idea of using this combination arose from the desire to strengthen concrete.

ERRATA.

p. 202, 3rd line. "boats" should read "bolts."

p. 219, 3rd line. "3·67 × 106" should read "$3·67 \times 10^6$."

p. 236, line below, Fig. 282. "34" should read "340."

p. 258, table XLVI. first line. "0·13" should read "1·3."

In both lists of symbols, "ω_B" should read "ω_u" and "ω_u" should read "ω_s."

p. 293, fourth paragraph 4th line. "0·8" should read "0·08."

p. 293, equation [19a]. "$P - \Delta c +$" &c. should read "$P = \Delta c +$" &c.

p. 299, Fig. 317. The upper reinforcement should be omitted.

p. 303, table LVII. 5th column. "j^2" should read "γ^2."

p. 336, first and second lines. "$\frac{1}{6}$" should read "$\frac{5}{6}$" and "$\frac{1}{10}$" should read "$\frac{9}{10}$."

p. 338, 4th paragraph from bottom. After "equations [19] to [27]" add "with the diagrams and tables."

p. 340, 3rd paragraph from bottom. After "equations [70] to [97]" add "with the diagrams and tables."

p. 346, 4th paragraph, 2nd line. "stress" should read "resistance."

p. 347, line before equation [2]. "u'" should read "u."

p. 351, paragraph after equation [28]. "rib" should read "slab."

p. 359, equation [12]. "σ" should read "ω_s" and "k_m" should read "k_s."

p. 362, first word. "line" should read "lie."

p. 365, after equation [2]. "bending moment" should read "moment of resistance."

p. 366, 3rd paragraph. After "ϵ_f" insert "can be tabulated."

p. 368, third paragraph. "15·41" should read "15·46."

p. 374, 2nd line. After "direct tension on" read "a unit length of."

p. 383, 7th line. "$a,$" should read "$a_{,}$."

p. 385, equation [11] commences "$\Sigma\,(y\,\theta) =$"; equation [12] commences "$\Sigma\,(x\,\theta =$"); equation [15] "$-$" should read "$=$".

Part I

GENERAL REVIEW OF THE SUBJECT

Name

THERE is much difference of opinion as to the proper name to describe the combination of iron and concrete as a material for building. It is called armoured concrete, ferro-concrete, concrete steel, and reinforced concrete, and undoubtedly something can be said in favour of each of these.

On looking up the word armour in the dictionary it will be found to mean " to fit out with implements," " to equip." The French call this material *béton armé*, which translated literally means armoured or armed concrete, but in English we have become accustomed to distinguish between the meanings of " armed " and " armoured." When we wish to convey the idea of being " fitted out with implements," or " equipped," we use the word " armed," and we use " armoured " as meaning rather " protected " or " defended " by some kind of covering.

It appears better then not to use the term " armoured concrete," as although " armoured " really means " equipped," we have given it a rather different signification of late years.

Ferro-concrete, or iron concrete, is probably a better term, but a concrete formed with iron chippings or filings embedded could equally well be called ferro-concrete, but would hardly serve the same purposes as the material under consideration. Ferro-concrete, then, is not entirely suited to describe this material.

In the United States the name of concrete-steel is generally applied to this combination, but it appears even less suitable than that of ferro-concrete, since it might refer to a sample of steel; and further, it does not include the combination of concrete and wrought iron, whereas the term iron includes both wrought iron and steel.

" To reinforce " means " to add new strength, force, power, or weight to "; to " strengthen to a greater degree." This certainly describes the material to perfection. " Concrete strengthened to a greater degree " seems to be the very description we require.

Some say, however, that this combination is not merely concrete strengthened to a greater degree, but that by combining the two materials we protect the iron and give to small sections of metal a usefulness which they cannot possess by themselves, and further, that as each material acts in its own way in resisting the stresses, it is not really reinforced concrete but iron and concrete combined in such a manner as to counteract the imposed stresses.

This is true, but undoubtedly the main object of the combination is to strengthen the concrete slab, beam, column, arch, etc., to a greater degree, and certainly the first idea of using this combination arose from the desire to strengthen concrete.

On these accounts the name Reinforced Concrete has been taken as the title of this book, and the material is referred to throughout under this name.

All Embedded Ironwork not necessarily a True Reinforcement

Reinforced concrete properly so called consists of introducing iron or steel sections into the substance of the concrete in such a manner and in such positions as to take up those stresses which cannot be resisted by the concrete unaided.

A joist surrounded by concrete is not reinforced concrete, but rather protected ironwork, as the joist bears all the stresses.

A reinforcement employed in an unscientific manner greatly reduces the economy of this form of construction, as in such a case there will always be a waste of material and also a reduction, or rather no increase to the elasticity of the concrete in tension, a property which will be referred to later.

Doubts as to Proper Theoretical Treatment

To arrive at the true principles on which to base theories for obtaining the areas of the concrete and reinforcing sections, many tests have been made and many theoretic formulae have been established, but they have been carried into practice in very few instances, and consequently have seldom been subjected to the test of practical working.

Although much has been done of late years by experiments, and mathematical reasoning based on the elastic properties of the materials, there are still some doubts to be cleared away and phenomena to be explained.

Theory based on the elasticity of the materials, which is of necessity the true one, has not been used up to the present time by a great majority of the patentees or constructors of the several systems of reinforced concrete, nearly all of these using more or less empirical formulae, and some no formula at all.

The properties of the materials when combined are still under dispute, some engineers maintaining that the concrete by having a reinforcement of iron or steel obtains different properties than those it has when alone. The behaviour of reinforced concrete in works and in tests made to elucidate this point certainly seems to point in this direction, but there still remains a great deal of diversity of opinion upon this and other matters relating to the theoretical treatment of reinforced concrete.

Systems

At the present time there are about fifty different systems of reinforced concrete, many of which appear to have been adopted to avoid the infringement of earlier patents.

Some of these systems employ peculiar sections of reinforcing metal, some use ordinary sections in peculiar dispositions, while others have both peculiar sections and methods of employing them.

All, however, attempt to effect the same object, that of placing the metal so as to counteract those stresses which the concrete is least able to resist.

Short History of the Development of the use of Reinforced Concrete

The first employment of reinforced concrete has been attributed to a French gardener, M. Joseph Monier, who in 1867 or thereabouts made big pots for shrubs, etc., of concrete, with a metal reinforcement, with the idea of reducing the thickness. Reinforced concrete was undoubtedly known before that date; an exhibit of this material being shown at the Paris Exhibition of 1855, and several methods of its application having been proposed by a French engineer, M. François Coignet, as early as 1861.

M. Monier, was however, certainly the first to employ reinforced concrete in a large way, and having satisfied himself as to its usefulness, he patented his system, which he employed for tanks, ponds, floors, etc., and later for small bridges. For some years it was only employed in France, and even there, as the construction was only based on experience and practical rule of thumb, it was used in a very limited manner. In 1879 Monier exhibited his system at the Antwerp Exhibition, where it was noticed by Herr G. A. Wayss, who bought the German patents and formed the company of G. A. Wayss & Co., of Berlin and Frankfort, to work the system. Experiments were made, and a thorough study of the subject was undertaken, which proved very clearly the advantages to be gained by this form of construction, and principles were arrived at on which its application should be based. This naturally opened up a much larger field of usefulness to the invention, and it became more and more employed, branches for the construction being established not only in Germany but also in other European countries, notably in Vienna, in which town Herr G. A. Wayss himself established a business. The system was now used not only for small arches, floors, tanks, etc., but also for reservoirs, pipes and similar structures. It was not until much later, however, that reinforced concrete beams and bridges of large span were constructed; in fact, not until the subject had received much more extended study.

When the Monier system of construction became generally known on the Continent, other systems of reinforced concrete were brought out differing from the Monier and from one another in the methods employed and the form of reinforcement. During this period several systems were introduced in the United States, amongst which those of Messrs. Hyatt and Ransome may be mentioned. The formulae employed by the first users of this form of construction were all purely empirical, not taking into account the true action of the concrete and metal in the resistance, and up to the present day these or similar formulae are employed for most of the systems.

M. François Hennebique and M. Paul Cottancin were the first to use beams or ribs of reinforced concrete, and to initiate the principles on which they are designed.

Now that reinforced concrete has become one of the ordinary forms of construction on the Continent, and in America, it is receiving more attention from engineers; experiments are being made, and valuable information obtained on the behaviour of the materials, and we may hope, at no very distant date, to know sufficient on the subject to be able to establish formulae based on the true properties of the materials, which will enable us to calculate the thickness of the piece and the area of the reinforcements with much greater accuracy than at the present time, although even now formulae are used which are based on the elasticity of the two materials.

The Employment of Reinforced Concrete in England

It is unfortunate that engineers and architects in England are so conservative, one might almost say prejudiced in their ideas, that many of them will not use this form of construction, even though their Continental and American *confrères* have proved to them so clearly its usefulness and economy, and above all, its safety ; having shown that it may be employed with perfect confidence, and that by its use cheaper, lighter and more durable structures may be erected than those built employing the old methods.

Year by year the demand for cheap construction is increasing, and if we can replace iron, steel, masonry, etc., by reinforced concrete, reduce the thickness of retaining walls by the use of this material, and form of it light coverings to our service reservoirs, we shall lower the cost of construction and maintenance, and lose none of the stability thereby.

Our bridges would be lighter, more graceful and cheaper, if instead of using brick or stone we employed reinforced concrete in suitable cases. What has been done by Continental and American engineers can be done by English, and we surely should take our place in the progress of the world, and not stand aside while others are, not only experimenting with and obtaining valuable information about reinforced concrete, but are also employing it for many important structures.

Before reinforced concrete can come into general use, it will be necessary to amend our building laws and Local Government Board regulations, in order that structures of this material can be constructed with economy. Under the present Metropolitan Building Act it is impossible to construct with reinforced concrete so as to obtain the economy which this class of material allows. The provincial building bye-laws militate in a like manner against the use of this form of construction, and until they are altered the employment of reinforced concrete must be limited to those structures which fall outside the jurisdiction of the several authorities.

In Germany, Austria and Switzerland, special clauses are inserted in the regulations for buildings to cover construction in reinforced concrete, and in France such buildings are allowed, subject to the investigation of proper authorities.

In Switzerland, all such structures must be of sufficient strength to pass the test of Professor Ritter's formulae [vide page 359].

In Berlin, structures must be calculated to bear an extra load of ten times their own weight, and the strength of the reinforcement must be calculated irrespective of the resistance of the concrete to tensile stress.

In Düsseldorf the concrete must be mixed in such proportion as to afford a resistance to compression of at least 2,130 pounds per square inch after 28 days. The calculations must be made, however, only allowing a resistance of 436 pounds per square inch.

If the strength is increased the safe stress may be increased in proportion. The stress on the reinforcement must not exceed 12,443 pounds per square inch in compression and tension for pieces subjected to bending, nor 9,955 pounds per square inch under direct compression. Where steel rods are employed this stress may be raised to 14,220 and 11,376 pounds per square inch respectively. The stress in the reinforcement must be calculated while bearing in mind the difference in the coefficients of elasticity of the concrete and metal. Columns on the Hennebique system are to be calculated, not only by their compressive strength, but also with regard to flexibility under changes of temperature. For this purpose the following equations may be employed :—$I = 60PL^2$ in case of a central load and $I = 100PL^2$ where the load is slightly eccentric ; I meaning the minimum moment of inertia of the full section of the column in centimetres, P the load in tons, and the L the free length in metres.

If the eccentricity of the load is relatively great, calculation must be made by formulae for composite flexure.

In Dresden the safe resistance of the concrete must be taken at 355 pounds per square inch; while the metal must be allowed a safe stress of 12,443 pounds per square inch in tension and compression in pieces subjected to bending, and a safe compressive stress of 9,955 pounds per square inch under direct compression. The rule for columns is the same as above.

In Frankfort, roofs and floors must be able to support ten times their own weight without any appreciable deformation. The construction of columns in reinforced concrete is prohibited. The tensile and compressive resistance to bending must be supplied entirely by the reinforcements.

In Hamburg the same rules apply for bending and direct compression as those of Düsseldorf. In the case of structures exposed to vibrations, an extra allowance of 20 per cent. must be allowed in the calculated sizes. The reinforcement must be fitted to cope with the crushing strain. The rules for columns are the same as those of Düsseldorf.

In Vienna, the plans must indicate the mode of execution, and emanate from certified engineers or architects taking full responsibility for the construction. For the reinforcements only the best laminated metal must be used, the maximum safe stress allowed being 14,220 pounds per square inch in tension and 10,665 pounds per square inch in compression.

For the concrete, Portland cement of perfectly constant volume, and of the best quality only, is to be used; absolutely clean river sand and pure water must likewise be employed. The safe compression on the concrete is not to exceed 355 pounds per square inch, and a test of quality may be demanded at any time. The concrete is to be mixed in the proportions of 836 pounds of cement to one cubic yard of sand, or about one to three. The making of concrete when the temperature is below zero[1] is not allowed. Four weeks must elapse after completion before the structures can be taken into use.

The Bureau of Building for the Borough of Manhattan, New York, has recently issued regulations, as to the use of reinforced concrete for floor and column construction.[2] The main particulars of these regulations are given below :—

(1) Reinforced concrete is not to be applied for fireproof buildings, unless satisfactory fire and water tests have been made under the supervision of the Bureau.

(2) Complete drawings and specifications, showing all details of the construction, shall be filed with the superintendent of the Bureau.

(3) The execution of the work shall be confided to workmen who shall be under the control of a competent foreman.

(4) The concrete shall be mixed in the proportions of one of cement, two of sand and four of stone or shingle; or the proportions may be such that the crushing resistance of the concrete shall not be less than 2,000 pounds per square inch after hardening for twenty-eight days.

The concrete must be what is usually known as a wet mixture.

Only high grade Portland cements shall be permitted which must satisfy prescribed conditions (generally, in accordance with the standard tests decided upon by the Committee of the American Society of Civil Engineers).

[1] Centigrade scale.
[2] These regulations were published in *The Engineering Record*, October 10, 1903.

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The sand must be clean, sharp grit sand, free from loam or dirt and shall not be finer than the standard sample of the Bureau of Buildings.

The stone shall be a clean, broken trap rock or shingle of a size that will pass through a ¾ inch ring. In case it is wished to employ any other material or other kind of stone, samples must be submitted and approved by the Bureau.

(5) Reinforced concrete shall be designed that the stresses in the concrete and steel shall not exceed the following limits :—

Kind of Stress.	Allowed stress in pounds per square inch.
Extreme fibre stress on concrete in compression	500
Shearing stress on concrete	50
Concrete in direct compression	350
Tensile stress in steel	16,000
Shearing stress in steel	10,000
Adhesion of concrete to steel	Not greater than shearing strength of the concrete

The ratio of the moduli of elasticity of concrete and steel shall be taken as 1 to 12.

(6) The following assumptions shall guide in the determination of the bending moments due to the external forces.

(*a*) Beams shall be considered as simply supported at the ends, no allowance being made for continuous construction over the supports.

(*b*) Floor slabs when constructed continuous and provided with an upper reinforcement at the top over the supports, may be treated as continuous beams. The bending moment for uniformly distributed loads being taken at not less than $\frac{W\,L}{10}$, except in the case of slabs square in plan, when $\frac{W\,L}{20}$ may be allowed, if these are reinforced in both directions.

The floor slab to the extent of not more than ten times the width of any beam, may be taken as part of that beam.

(7) The moment of resistance under transverse loads shall be determined by formulæ based on the following assumptions :—

(*a*) The bond between the concrete and steel is sufficient to make the two materials act together as a homogeneous solid.

(*b*) The strain on any fibre is directly proportional to the distance of that fibre from the neutral axis.

(*c*) The modulus of elasticity of the concrete remains constant within the limits of the working stresses.

(*d*) The tensile strength of the concrete shall not be considered.

(8) When the shearing stress developed in any part of a construction exceeds the safe working strength of the concrete, a sufficient amount of steel shall be introduced in such a position that the deficiency in the resistance to shear is provided for.

(9) When the safe limit of adhesion between the concrete and steel is exceeded, some provision must be made for transmitting the strength of the steel to the concrete.

(10) Reinforced concrete may be used for columns in which the ratio of length to least side or diameter does not exceed twelve. The reinforcing rods must be

tied together at intervals of not more than the least side or diameter of the column.

(11) The constructor must be prepared to make load tests on any portion of the construction within a reasonable time after erection and as often as may be required. The tests must show that the structure will sustain, without any sign of failure, a load of three times that for which it was designed.

These bye-laws may not be all that can be desired, but they are a step in the right direction, although they generally err on the side of safety, and none of them appear to include walls.

Buildings of reinforced concrete have been erected in England, but, generally after a great deal of trouble with the authorities ; and it seems a pity that suitable rules should not be formulated, and be brought into general use, which would cover this class of construction.

Advantages and Disadvantages in the use of Reinforced Concrete

The use of this form of construction has many advantages and few disadvantages. This may seem at first sight a somewhat wild statement, but, when we come to look into the subject thoroughly, it will be seen that there is little doubt as to its truth.

Fire-resisting Qualities. — Reinforced concrete is undoubtedly the best material for fireproof construction since the concrete protects the embedded skeleton by reason of its low conductivity of heat.

The employment of unprotected ironwork has time after time caused the almost complete destruction of buildings, in consequence of the joists or girders being badly distorted by the heat. The danger to life is great, not only for the inmates of such a building, but also for the fire and salvage men.

Timber of large scantling is considered far safer for use in the construction of buildings than unprotected ironwork, as the charring of the wood acts as a preventative to further combustion, by forming a non-conductive coating.

Where timber is employed there are always, however, joists, rafters, etc., of small scantling, and these take fire quickly and are not sufficiently large to protect themselves by charring. The sparks from these timbers are scattered around, and cause the fire to spread to the surrounding buildings as well as through the building in which it originated.

The metal skeleton employed for reinforced concrete floors, being nearly always used in the form of a network of more or less fine mesh, has a marked effect in preventing the concrete from cracking either under the influence of the heat of the fire itself or under the rapid cooling action of the water thrown upon it.

Ordinary concrete may stand the action of fire without absolutely giving way, but, when a stream of water is applied to the hot surface the cohesion of the concrete is rapidly destroyed.

The finer the mesh the less will be the disintegration, but with any ordinary floor reinforcement, the cracking of the concrete is so little that the floor will be as strong after the fire as before.

The effect of the cracking of the concrete subjected to tensile stress is referred to later on [page 21], and it is clearly shown that the structure will still be perfectly safe even when this portion of the concrete is badly cracked.

That buildings of reinforced concrete are habitable directly after a severe conflagration, has been proved again and again, and this, it must be admitted, is an immense advantage. The fittings may have to be replaced, but the buildings themselves need little repair, if any.

Yet another advantage accruing to the use of reinforced concrete for fireproof structures is, that its very small powers of heat conduction will enable the firemen, etc., to enter, without inconvenience, and with perfect safety, the rooms just above and around those in which a fire is raging.

The coefficient of expansion of iron and concrete under the action of heat being practically identical, there are no internal stresses set up between the iron and the concrete by differences in expansion and contraction. The opinion of Continental and American fire-brigade officers is strongly in favour of the employment of reinforced concrete for buildings, and many English officials have the same opinion.

Below will be found some accounts of experiments on the resistance of reinforced concrete buildings against fire, and also accounts of fires which have occurred in structures of this material, and of tests of such buildings after the fire. Many such experiments and reports could be cited, since exhaustive tests have been carried out on most of the systems. All these experiments show in the same manner the remarkable fire-resisting qualities of reinforced concrete. One need only add that there appears most ample proof that for fireproof construction reinforced concrete and brickwork are far ahead of any other building materials.

OFFICIAL REPORT OF COMMANDER WELSCH, OF THE GHENT FIRE BRIGADE

"The fresh experiments with which we proceeded on September 28 to ascertain the resistance to fire of the patent concrete building erected on the Hennebique system, and of window glass reinforced with a metallic web (Siemens system), manufactured at Neusattl, Bohemia, have confirmed in every respect the results obtained by the first trial on the 9th of this month (September).

"To begin with, it is astonishing that a building which had already been subjected to severe tests, concerning both superloads and resistance to fire, should have been able to withstand a second time with such success a similar ordeal of even greater severity.

"Not only has the first floor supported a superload of 47 cwt. per square yard, and the roof a superload of 22 cwt. per square yard, without showing any appreciable deflection (hardly ·02 in.), but it had to undergo also the effects of an extremely violent fire, fed and maintained during two consecutive hours on the ground and first floors.

"All the competent authorities—engineers, architects, builders, army officials, among whom was Major Vanden Borren, specially appointed by the War Minister—who witnessed those tests, marvelled at the manner in which the materials stood the trial. From place to place some crevices from ·03 in. to ·07 in. were produced in the walls, but without any deformation; and this morning (September 29) these cracks had closed up.

"The radiation of heat through the floors was never sufficiently strong to prevent access to the balcony on the first floor nor to the roof. Moreover, during the whole of the trials it was never sufficient to prevent the laying of the hand on the outside surfaces.

"We must lay stress on the admirable manner in which the glass, reinforced with metallic web, stood the experiments. In the account of the first trials of the 9th inst., I had already explained how the glass behaved during the fire. Yesterday it again stood the test, notwithstanding greater heat and a more extended trial. Some cracks have been added to those produced in the previous experiments, but no pieces of the glass have become detached, nor have they allowed flames or smoke to pass through them.

"To give an approximate idea of the heat, a wooden staircase which had been constructed outside the building at about eight inches from one of these panes of glass took fire twice during the experiments, by reason of the heat radiating through the glass. At the end of two hours we extinguished the fires, taking particular care to direct the jets of water on the pillars, the walls and the ceilings, all of which stood the test admirably. The plastering alone has somewhat suffered, but it will cause very little expense to restore the building to its original appearance.

"Externally, if it were not for the cracks in the armoured glass, nobody could tell that the building had sustained the trial of two severe conflagrations.

"We cannot insist too much on the fact that we are here in the presence of materials the judicious use of which will diminish considerably the catastrophes occasioned by fire.

"The insurance offices will be the first to benefit by this, and we should recommend them to encourage that mode of construction by granting special conditions to industries, public works, and to private individuals who will make use of them.

"Ghent, *September* 29, 1899. (*Signed*) "COMMANDER WELSCH."

With reference to the tests referred to in the above report, as applied to the floors, the following details are interesting :—

Previous to the second trial by fire on September 28, it was determined to test the floor of the building to ascertain if the previous conflagration had had any injurious effect on the material or the mode of construction.

A load of pig-iron was spread over the first floor equal to 423 lbs. to the square foot, and the apparatus registered a deflection of ·02 in. The load was then increased to 585 lbs. to the square foot, and a deflection of ·065 in. was recorded. This load was left in position for some time without any increase in the deflection, thus conclusively proving that the floor had in no way become weakened.

Later, on the same day, at the request of Major Vanden Borren, of the Engineers, who was specially delegated by the Minister of War, further tests were made.

The flat reinforced-concrete roof was subjected to a superload equal to 274 lbs. per square foot, and on one-half of the area of the first floor the load of 585 lbs. to the square foot was retained. Combustible materials, such as timber, coal soaked with petroleum, petroleum, etc., were liberally distributed over the other half of the first floor as well as on the ground floor. The materials were lighted simultaneously and burned fiercely for over two hours, when they were extinguished in the manner described by Commander Welsch.

On a close examination being made on the 29th, when the building had sufficiently cooled, it was found that no disintegration of the concrete had taken place either inside or outside.

The apparatus placed on the roof had recorded a deflection during the fire to the extent of ·79 in., but it had on the 29th regained its former level, notwithstanding the fact that the superload was still in position, showing how little effect the fire had on the steel bars encased in the concrete.

In addition, pieces of zinc, copper and phosphor-bronze, which had been secured to the ceilings, had disappeared, while the glass of the bottles containing petroleum was found molten, proving that a very high temperature had been attained.

To give a further example of the resistance of reinforced concrete to the action of fire.

The part of a mill at Court St. Etienne, Belgium, where a fire took place covered an area of about 1,200 square yards, and was constructed entirely in reinforced-concrete on the Hennebique system in 1898.

In July, 1901, a fire fed by the inflammable nature of the contents of the building took place and raged for about two hours.

After the conflagration, on examination it was found that the plastering to the walls and ceilings alone had suffered—the building itself was practically uninjured. A test of the floors was, however, made.

The floor chosen for the purpose of the test was originally constructed to carry a safe distributed load of 90 lbs. per square foot, and prior to handing over the building after the completion of the work, it had been tested with a load of 135 lbs., or 50 per cent. greater than that for which it was calculated.

It was decided on this occasion to adopt a still more stringent test, and a load of 200 lbs. per square foot was applied, with the followed results :—

After the Construction, Dec. 17, 1898. Superload 135 lbs. per sq. foot.				After the fire, July 20, 1901. Superload 200 lbs. per sq. foot.		
Span		Deflection.		Span.		Deflection.
ft.	in.	in.		ft.	in.	in.
14	7	·02	..	14	7	·02
22	8	·07	..	22	8	·031

After removing the loads there was no permanent set.

The result of the above tests fully demonstrates the fact that, notwithstanding the expansion and contraction caused by fire and water, the floors had lost none of their original elasticity and strength ; in fact, that, owing to the time which had elapsed since the first test, they had become better set and stronger.

Protection of the Metal from Rust.—It is undoubted that in reinforced concrete the skeleton is perfectly protected against rusting. It must be remembered, however, that for this form of construction the best materials must be used, and the concrete properly and thoroughly mixed, and well worked and rammed around the reinforcement, so as to be free from cracks and voids.

Sometimes where the larger diameter rods, etc., are used, the iron is brushed over with a cream of neat cement before being embedded, to ensure the thoroughness of the protecting coat, but where small sections are employed the concrete is mixed fairly dry, and is rammed thoroughly around the skeleton with iron rammers, so that it is of a very close and impermeable nature.

That reinforced concrete requires special care is a fact admitted by all, but the same applies more or less to all forms of construction, and this special care is well compensated for by the durability obtained. There appears to be a chemical action between the cement and the iron, forming a coating of silicate of iron on the reinforcement, which not only protects it from oxidization, but also removes any little rust that may be on it when placed in the concrete, and gives a greater adhesion between the two materials. The coating protects the reinforcement against oxidization, even when there is a slight passage of water through the concrete.

It is not to be denied that steel and iron embedded in concrete have in some few cases been known to have become rusted, but in such cases it will always be found that the concrete is of a porous nature, and that it has not been well rammed around the iron, and consequently the protective coating has in places not been formed. Even with porous concrete of furnace ashes, if this layer is obtained, the metal will remain perfectly protected even when the concrete is exposed to continuous moisture,

When steel and iron are employed alone, however well they may be maintained, there are always places where moisture lodges, causing oxidization, and the extra care required in the maintenance of a steel or iron structure very greatly exceeds that for the proper initial protection of the metal in a structure of reinforced concrete.

Many instances might be cited proving the thorough protection of metal embedded in concrete. Perhaps the most remarkable is the case mentioned by Herr von Empergner, of the discovery of rods embedded in concrete under water for four hundred years coming out free from rust. An interesting experiment was conducted by Mr. E. Ransome of New York, to test the preservation of metal when embedded in concrete. He partly embedded some hoop-iron in concrete blocks, which were left exposed to sea air for many years. When the exposed iron had rusted completely away, the blocks were cut open and the embedded metal was found to be entirely free from rust.

The experiments carried out by M. Breuillie at La Chaînette and described in the *Annales des Ponts et Chaussées* are extremely interesting, proving conclusively the protection of metal when embedded in concrete. A description of these tests was published in the *Engineering Record*, September 20, 1902.

Reference can also be made to the results of an official inquiry into reinforced concrete pipes [page 12].

It is sometimes recommended that the concrete should be mixed wet where it is applied to the metal, but in practice it is very difficult to make the concrete wet around the reinforcement and moderately dry elsewhere, and it has been found that for all ordinary cases a fairly dry concrete, well rammed, will form a protective coating on the steel or iron, and moderately dry concrete, well rammed, will acquire its maximum strength much quicker than a wet mixture. If mixed too wet the concrete probably never attains its full strength. In special cases the reinforcement can be brushed over with a grout before being placed in the work. It is also sometimes stated that the metal must be thoroughly clean before being embedded in the concrete, but this does not appear to be borne out by facts. In some tests made to elucidate this point, the curious fact presented itself that not only does rusty iron become clean when embedded in concrete, but that it becomes more effectively protected against oxidization than clean iron which has been similarly treated. A rusty and a clean nail were both embedded in the same concrete block and left for over three years; on being taken out the rusted nail had become free from rust. Both nails, together with a new nail, were then placed in water; the new nail rapidly became rusted. The nail which was rusty when first embedded in the concrete block showed no signs of rust a month after being placed in water, except at one place, where it had been scraped with a penknife before being immersed; the other nail, after resisting the action of water for a few days, showed signs of rusting, which increased with time. This nail rusted more on the thin edge than on the broad edge, which only showed slight signs of rust after being a month in water; the new nail also rusted more on the thin edge than on the flat edge, but the oxidization was more marked. These tests seem to show that the protective coating formed on clean iron will dissolve in water, but that if the iron is slightly oxidized before it is embedded, some chemical combination in the coating resists solution in water. This is a curious result, but it was very clearly demonstrated by the behaviour of the iron nails. A further series of experiments of the same nature, and with similar results, have also been made. Three nails (polished, untreated, and rusty) were placed in a block of concrete, which was submerged in water for three months. The nails were then taken out and exposed to the weather; the one which was originally rusty stood the test best, and the polished nail showed signs of rusting first.

Impermeability.—The resistance of reinforced concrete to the penetration of water is one of its most advantageous properties, but, where such resistance is required, the concrete should be of sand and cement only, and in proportions not more than three to one. No shingle or broken stone should be used in the mixture except in comparatively thick walls, where concrete of shingle or broken stone may be employed for the backing.

It appears that free carbonic acid has a bad effect on the concrete, but, when the water is still, a little carbonic acid in solution will do no harm. There is hardly any hurtful action except in the case of mineral waters; but even in this case, as a rich mortar is necessitated by the requirements of impermeability, the action is very slow, and the re-coating of the face may be considered as ordinary maintenance. This property of reinforced concrete gives it a distinct advantage over bricks and masonry for aqueducts and service reservoirs, tanks, etc., as there is no need, when it is used, to employ puddle or asphalte to prevent leakage. With proper care reinforced concrete will retain water with a fifteen-foot head without any sweating.

For certain industries, where vats and big tubs are used, reinforced concrete has proved itself immensely superior to timber for their construction, and it has been found to withstand the action of alkalies and acids far better than iron or wood. If, however, these liquids are in a very concentrated state, it is better to use a special lining, such as glass slabs, as concentrated acid and alkaline solutions will act upon the concrete.

Running water has no effect upon reinforced concrete mixed as recommended, and in pipes of this material the nodules so prevalent in iron pipes are not formed. It is well, however, to keep concentrated solutions of nitric or hydrochloric acid or of alkalies from the pipes, unless they are frequently flushed with fairly pure water. Reinforced concrete pipes should not be employed for hot water.

With reference to the use of this material for pipes and reservoirs and to its impermeability to water, the diagram (Fig. 1) is interesting, as showing how the leakage decreased in a conduit.[1]

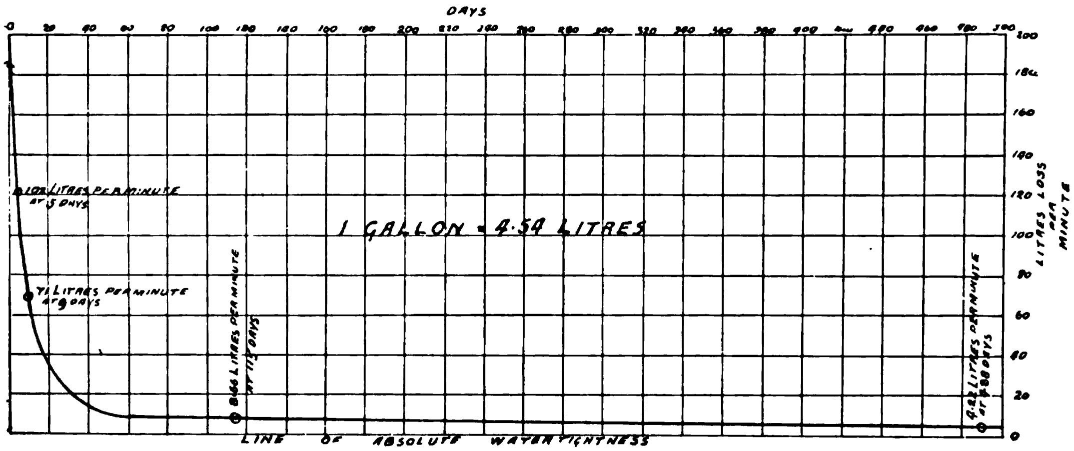


DIAGRAM SHOWING LOSS FROM A CONDUIT 710 YARDS LONG, 31½ INCHES DIAMETER & 1·46 INCHES THICK UNDER A HEAD OF 23 FEET

FIG. 1

The results of an official inquiry into the behaviour of reinforced concrete water pipes at Grenoble is also of interest.

CONCLUSIONS ARRIVED AT BY AN OFFICIAL INQUIRY INTO THE USE OF REINFORCED CONCRETE PIPES BY THE CITY OF GRENOBLE.

The city authorities had laid in 1886 a line of reinforced concrete water pipes 330 ft. in length.

The pipes have all the time resisted, and still resist, the normal pressure of 80 ft. head of water. The length of each section of pipe is 6 ft. 3 in., its thickness 1⅝ in., and its internal diameter 12 in.

The metal skeleton of these pipes is formed by thirty longitudinal rods ¼ in. diameter and by an internal $\frac{5}{32}$ in. spiral wire, also an external ¼ in. spiral wire.

The sections of the pipes weigh 88 lb. each. They are connected together with reinforced concrete rings.

On February 2, 1901, a length of 16 ft. of these pipes was raised. Two of the joint rings were broken so as to set free two lengths of pipe which had been lying under 3 ft. of ballast.

A close examination of these pieces established the following facts :—

1st. The irreproachable state of preservation of the pipes, in whch there was found a slight calcareous deposit about $\frac{1}{16}$ in. thick. They did not show the least fissure, either internally or externally.

2nd. There existed no trace of oxidation from the metal. The binding in wire which connected the longitudinal rods was absolutely free from oxidation.

3rd. The adherence between the metal and the cement concrete constituting the body of the pipe was such that, despite the thinness of the concrete (1⅝ in.), they could only be separated by heavy blows from a sledge-hammer.

[1] The decrease in the leakage with age is probably due to the water liberating small particles of cement while passing through the mortar. These it tends to bring to the surface, but in the passage carbonates of lime are formed which block the pores.

It has been suggested that a reasonable proportion of ordinary *slaked* lime should be added to the cement, which would supply sufficient lime to ensure the closing up of the pores. It has been shown by certain experiments carried out by Professor De Smedt, that this lime would have no injurious effect on the mortar, only causing a slight retardation of the setting. Vide also Appendix II.

4th. When struck with the hammer these pipes evinced remarkable sonority, such as might be obtained from a sound cast-iron pipe.

5th. The detached fragments of the cement concrete showed very sharp angles.

6th. The Water Committee of the City Council declared that this line of pipes had required no repairs since it was set in place in 1886.

The adaptability of this material for the construction of pipes and culverts is well shown in Fig. 2, which is a section of a Monier culvert constructed in Venezuela, to conform to the surface of the ground.

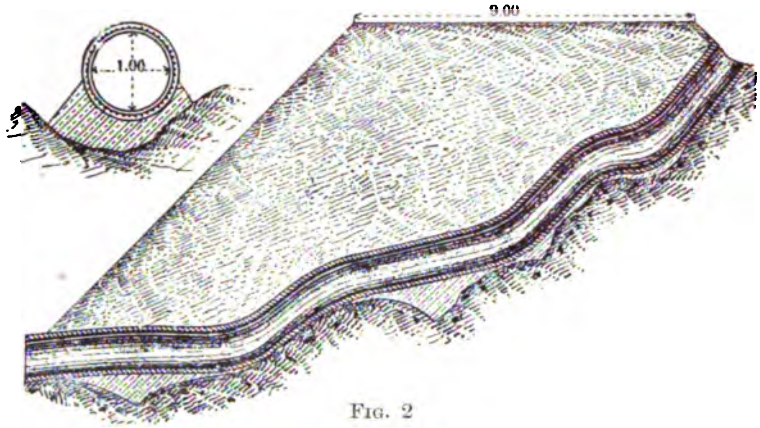


FIG. 2

For roofs to buildings, the advantage gained by the use of reinforced concrete is very evident.

A sugar store at Calais, with a large area of flat roof, has been covered with about 12 in. of soil, with open drains under, so as not to allow the soil to become overcharged with moisture. A good hay crop is obtained from this roof every year.

Articles have appeared recently in several professional papers quoting statements made to the effect that reinforced concrete pipes are not suitable for water supply.

The special accusations are as follows :—

(1) *That the reinforced concrete water mains laid in the United States between* 1870 *and* 1876 *have been replaced by cast-iron pipes.*

These mains were not of reinforced concrete, but were made of sheet iron, with a covering of cement.

(2) *That the concrete is attacked by bacteria.*

This may be true for very poor concretes, which are easily decomposed, but has not been found to be the case with reinforced concrete as constructed at the present day. It is extremely doubtful whether any bacteria to be found in pured water will decompose concrete. Filter beds and reservoirs have been constructed of this material for some time past, and there does not appear to have been any noticeable deterioration from this cause.

(3) *That reinforced concrete pipes are permeable, with the reasoning that when mortar is poor, and little compressed, it becomes very porous, and water, even under slight pressure, runs through it.*

Reinforced concrete pipes are made of specially rich mortar, which is well consolidated. They are not porous, and can be made to withstand a considerable head without perceptible leakage. The special pipes, with caoutchouc or metal linings embedded, are made to withstand high pressures, and do not leak at all.

(4) *That reinforced concrete pipes are constructed in yards with more or less clean materials, while cast-iron pipes are moulded at very high temperatures, which averts all contamination.*

If the reinforced concrete pipes are made with the materials that are considered necessary for this class of construction, there is very little fear of contamination from unclean materials.

(5) *That the joints of reinforced concrete pipes allow the water to be contaminated by liquid manure from dung and cesspools.*

The joints of reinforced concrete pipes are carefully made, and will not allow infiltration.

Cast-iron pipes may be better for water supply than reinforced concrete pipes, but not for any of the reasons stated above.

It is also stated that six French towns have replaced concrete mains by cast-iron pipes.

These towns have been communicated with. One of the letters was returned, the town being apparently unknown. Two of the letters have not been answered. Of the three others—

One states that the pipes are of concrete not reinforced, and were laid in June, 1869, the length of piping being 4,153 yards. Of this 193 yards was relaid with iron pipes in 1893. The small importance (i.e. length) of the branch was the sole reason for the preference for iron piping. The change appears to have been made because an increase of diameter was required.

Another answer to the inquiry states that the conduits were not of reinforced concrete ; that some were of a U section, covered with a flat slab, and some circular.

The original length laid in 1887 was four miles. The whole length was relaid in 1900 with metal pipes, the reason for the change being that the cement conduits were not water-tight, either by reason of expansion cracks during the setting, or shrinkage cracks at the joints. Roots of trees grew into the conduits, completely blocking them.

This would be impossible with a properly constructed pipe line of reinforced concrete.

The remaining answer was to the effect that the town in question had never used cement pipes for water mains.

With regard to the use of pipes of this material it may be interesting to mention that during the year ending August, 1903, about 2½ miles of reinforced concrete pipes were laid for the Brussels water supply, varying from 23½ to 31½ inches diameter for a maximum head of 138 feet,[1] and the laying of a further 2 miles commenced in April, 1904.

Durability.—The durability of structures of reinforced concrete is well established. One may almost say that the cost of maintenance is nil, it being well known that the resistance of concrete increases with time.

In this respect reinforced concrete compares very favourably with steel and iron structures, whose durability certainly decreases with time, both on account of fatigue, the action of which is even now not fully understood, and has to be allowed for largely in the factors of safety used in designing these structures; and also on account of oxidation, which will always occur in places, however carefully the structure may be kept painted. Steel and iron structures require almost continual supervision and painting.

It is said that some portion of the Forth Bridge is always under the painter's

[1] A considerable length of these pipes have been in use for over a year and have given complete satisfaction. A description of these pipe lines is given page 455.

brush. This is the same in a less degree with structures of less magnitude, since if ironwork is not constantly repainted it will rapidly deteriorate. Brick or stone structures, of course, require little care in maintenance, but reinforced concrete is, in the majority of cases, applied as a substitute for some form of metal structure, and where used instead of brickwork or masonry it has the advantage of greater lightness and economy.

No boring animals work their way into reinforced concrete, and structures of this material are consequently free from rats and similar vermin, and no insects can find refuge in it, as in the case of timber. When old Baker Street Station was being pulled down the timber was found to be simply alive with fleas; this would be impossible in a structure of reinforced concrete.

Reinforced concrete will not harbour microbes, as it is perfectly free from pores, and thus in the matter of hygiene it is an excellent material for buildings such as fever hospitals, mortuaries, abattoirs, etc., as well as for ordinary factories, warehouses and dwelling houses. For jetties, wharves, and similar structures, the employment of reinforced concrete is immensely superior to timber or iron, as it cannot decay or oxidize.

It has far more resistance to abrasion from shipping or other floating objects than timber, and the resistance it offers against the attack of marine worms and insects is an advantage which will be at once evident. It may be mentioned that the Memel timber at Bell Rock was discovered by Stephenson to be destroyed by the *Lemnoria Terebans* at the rate of 1 in. per annum, and at Lowestoft piles were eaten at the rate of 3 in. per annum. The *Teredo Navalis* grows to as large a size as 2 ft. long and ¾ in. diameter, and is very destructive to marine timber work.

Weight.—Reinforced concrete is heavier as a building material than steel or ironwork for supporting the same load. This causes its own weight to be a greater percentage of the total load than in the case of a steel or iron structure. The high percentage of dead to live load will prevent the use of reinforced concrete for bridges of very large span, although it is an advantage in other respects.

The excess of weight is, in a great measure, compensated for in the use of reinforced concrete, by reason of its employment enabling us to do away with most of the auxiliary parts necessary in a steel or iron structure.

The flooring and beams of reinforced concrete act together in resisting the stresses, and there are no tie rods, cross girders, jack arches, floor plates, flooring, etc., to be supported, which in a structure of steel or iron always add considerably to the dead load.

In road bridges of reinforced concrete, the decking slab needs only to be covered with a coating of asphalte, or other paving, to make the road surface, whereas in a steel bridge there must be a decking of some sort, such as trough flooring, and this must be filled or covered with concrete before the road surface can be formed.

When compared with brick or stone piers, abutments, arches, or walls, reinforced concrete has about three to five times less volume, and therefore has considerably less weight.

In abutments and retaining walls, it is evident that this material will need a much less thickness, and therefore volume, than that required for similar works in brick, stone or plain concrete. The comparative lightness of arches of reinforced concrete enables structures to be erected which are impossible with ordinary masonry or brickwork. This is shown very clearly in Fig. 3, which is a view of a bridge of

98·4 feet span, and shows well the extreme lightness which can be obtained in bridges of this material.

The lightness of columns and walls is a great advantage to a building, and the employment of reinforced concrete slabs or rafts in place of the usual foundations will greatly reduce the necessary excavation, besides in some instances enabling a building to be erected at a very small cost for foundations, where, if ordinary materials had been employed, the work would be very costly, if not impossible.

FIG. 3.

Resistance to Stresses. — The resistance of concrete to imposed stresses is often said to be so uncertain that the employment of reinforced concrete must be attended by great risks. This might be refuted by simply referring to the many tests of structures already erected and in use, for if there is such risk it certainly does not appear to be shown in existing works.

More can, however, be said on this subject. When briquettes of small dimensions and of neat cement are tested, in which the amount of water used has been carefully measured, great diversity of results are obtained when different persons test the same cement. There is no doubt on this point. It will be found, however, that the personal difference in results is much less marked in sand tests than in those of neat cement.

In fact, with the same sand, cement and water, and using the same measured proportions of ingredients, the difference in the results of tests made by different persons is small. Some experiments made to test this are given in Table 1.

For reinforced concrete, sand and shingle or broken stone are used, and it is extremely doubtful whether there is much variation in the strength of such concrete, if it is made of a finely ground and cool cement with clean aggregate and water. It is certain that any variation in the strength does not amount to so much as to cause an appreciable difference to the safety of the structure with such co-efficients of resistance as are usually allowed in the calculations for reinforced concrete. The resistance of the concrete to tension is generally ignored, and where it is allowed for a very low value is given to it.

TABLE I

Tests showing the variation of results obtained in testing neat cement and 3 to 1 mortar, the briquettes being made by persons of varying experience in cement testing. Briquettes broke after 28 days.

Description of composition of briquettes	Made by a boy totally unacquainted with cement testing		Made by a man accustomed to cement testing, but who had not done any testing for some time		Made by a man accustomed to cement testing and in practice	
	Tensile strength in pounds per square inch	Average in pounds per square inch	Tensile strength in pounds per square inch	Average in pounds per square inch	Tensile strength in pounds per square inch	Average in pounds per square inch
Neat Cement with 20% of water	450 440 470	453	600 600 580	592	640 620 610	623
3 of Sand to 1 of cement, with 10% of water to sand and cement	160 165 160	162	145 145 145	145	155 160 150	155

Although its resistance may be ignored in tension, still the concrete has to take up increments of resistance from the iron sections produced by the longitudinal shearing forces, and the ability of the concrete to resist these stresses must be assured. Many constructors neglect this also, and assume that the concrete is able to resist them, but this is not always the case, and these resistances should be carefully looked into and provided for where necessary. At the same time it must be stated that one of the curious properties of reinforced concrete seems to indicate that the concrete in tension can bear safely stresses up to its ultimate breaking strength. It has, in fact, been found that the concrete may be strained up to its ultimate tensile resistance without cracking, and even under further stress it appears that the stresses in the concrete remain constant, all further stress being taken by the rods. The concrete, however, still stretches. In other words, after the ultimate tensile strength of the concrete is reached its modulus of elasticity becomes nil.

These remarkable properties have not been as yet fully explained, but that they exist has been shown by careful tests made by M. Considère, which will be referred to later.

In nearly all cases the concrete is only considered to act in compression, and the reinforcement is calculated as taking all the direct tensile stresses and in many cases the tensile shearing stresses also.

It is an admitted fact that the variation in the resistance of concrete, and neat cement to compression, is less than that in the tensile resistance. The concrete when reinforced appears also to gain in some way further resistance to compression, and in almost all cases of tests to failure of reinforced concrete under bending, the test pieces have failed first by the cracking of the concrete in tension or by shearing. There appears, therefore, to be no cause to fear failure by reason of any uncertainty of the resistance.

Hardness and Impenetrability.—The hardness of reinforced concrete, which has always a large percentage of cement, is a disadvantage in some particulars. For instance, nails cannot be inserted easily, and when anything has to be secured to it in this manner, provision must be provided when moulding, although holes can be made after the concrete has set if required. This disadvantage is really more apparent than real. With a little care fixtures or wood blocks, where the strength will not be affected, can be moulded in, and in special cases cinder concrete may be employed if care is taken in mixing and depositing that the metal is well protected ; but in this case the strength is greatly reduced, since the crushing resistance of the aggregate rules the crushing strength of the concrete.

Moulding.—The advantage gained by being able to mould reinforced concrete into any required shape is very great. Many parts of a structure can be moulded on the ground before being placed in position, thus greatly reducing the necessary false work,as these pieces merely require to be propped up while a junction is moulded between them and the contiguous parts. Skew bridges require no more care in execution than those on the square ; the cutting of checks at the springing is of course unnecessary, there are no courses to require special care in setting out and no stones to be dressed to template.

Concrete piles are a feature of special interest, the ease with which they can be moulded, lengthened *in situ*, or connected to columns, or deckings forming one continuous whole, has given them a position for structural purposes which no other kind of similar support can attain to.

Ornamental parts of a structure can be moulded as one with the rest, and, in fact, are often made to take their part in the resistance to stresses.

Appearance. — The appearance of a reinforced concrete structure can be made almost anything we require by ornamental mouldings, and if the surface of the concrete after it has been rendered is splashed over with grout from a whitewash brush, or similar contrivance, a slightly roughened surface is formed which has a stone-like appearance, and hides the hair cracks which will form in the rendering. This is often done, and is very effective. There are also many other ways of treating the surface.

The outsides of reinforced buildings can be formed of columns and lintels, or crossbeams, and have window and door openings formed in reinforced concrete, the rest of the filling in of the bays being done in ornamental brick or stone work. This is a favourite form of construction in France, and if properly treated can be made to look very well.

In arched bridges the arch can be of reinforced concrete, and the spandril walls may either be of an open arched form, or may be formed in brick, stone, or concrete. Large arches should be hinged at the springings and centre, which, besides being an advantage in other ways, rather improves the appearance. Such parts of a structure as are of reinforced concrete are sometimes cut to represent stone, but such treatment is seldom advisable. The fact that it is concrete will be still apparent, and there is no artistic reason for disguising it. A brick or stone face may be put on a reinforced concrete bridge, but as reinforced arches are usually made with a small proportion of rise to span, it is much better to leave the arch at least to show as concrete, since if it is faced with stone or brick it will give the appearance of instability. The rise of arched bridges in reinforced concrete may be slight as compared with the span, and the thickness of the arch can be made very much less than in the case of masonry or brickwork.

These facts combine to make arched bridges of this material of singularly graceful proportions, as can be seen by Fig. 3 and other views of bridges erected of this material at the end of the book. The handsome buildings that have been erected of reinforced concrete and brickwork can also be seen from the illustrations.

The ease of construction, and the lightness of domes of these materials gives a great scope to artistic treatment of buildings.

The Monolithic Nature of Reinforced Concrete Structures.—A structure of reinforced concrete is not an assemblage of parts connected together in a more or less thorough manner, but is a united whole, each part being one with the neighbouring pieces. This intimate connexion gives a strength to a structure unknown prior to its introduction, and also affords an almost perfect resistance to vibrations. This resistance is perhaps most noticeable in buildings containing machinery, where the absence of vibration is not only beneficial to the building itself, but also to the machinery contained therein. This runs more smoothly, and not being subject to external vibration, has a longer life. Fast running machinery, such as that used for working dynamos or centrifugal pumps, is specially benefited by being placed in buildings of this material.

For structures on bad or swampy ground, reinforced concrete and brickwork have a peculiar advantage. A reinforced platform under the building, supported if necessary, on reinforced piles, being an ideal foundation in such cases, and their monolithic nature enables them to resist the stresses caused by unequal settlement. Such a building is also lighter than one of ordinary brickwork or masonry, which is an advantage. The description of the destruction of the San Marino Pavilion, erected for the Paris Exhibition of 1900 by M. Cottançin is given below, and clearly shows the monolithic nature of such structures.

Account of the Destruction of the Pavilion of the Republic of San Marino, Built for the Paris Exhibition of 1900, on the Cottançin System.[1]

This pavilion was erected at the foot of the Eiffel Tower on foundations of the cellular box type, largely employed in the structures erected by M. Cottançin.

The framework of the building consisted of four groups of vertical and arched members, tied together with horizontal ties, the whole being formed of reinforced cored brickwork and concrete. At one corner a space was formed for a staircase, and at another for a turret (Fig. 4). [The supports at P, O, N, and R are omitted in this Figure]. At the back of the building the uprights I and K were placed to give greater rigidity.

At each corner, i.e. A, B, C, and D; E, H, and I; M, L, and K, and P, O, N, and R, there was an area of brickwork of 186 square inches.

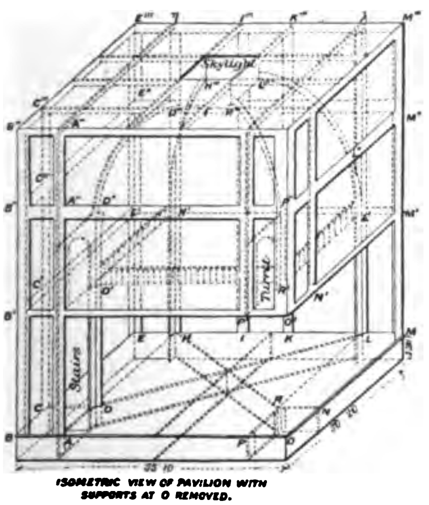


Fig. 4.

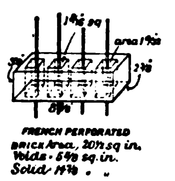


Fig. 5.

The weight of the building was 118 tons, and therefore each group carried about 29·5 tons, equal to 355 pounds per square inch, or about 33 per cent. of the ultimate crushing strength of the bricks—omitting the mortar cores.

The groups were braced together by horizontal courses of reinforced brickwork, forming rectangular frames from 8 to 16 inches thick (Fig. 4).

At the height of 15·09 feet above the bottom floor 4 cored brick arches, 3½ inches thick and 8⅝ inches deep, sprung from he uprights D′, H′, L′ and R′ (Fig. 4).

[1] *Engineering*, March 21, 1900.

The upper ends of these arches abutted against a square frame for the central skylight about 39 feet above the bottom floor. This frame also received the ends of the roof ribs, which abutted at the top, while the arches abutted at the bottom (Fig. 4). Each group of supports had 48 reinforcing wires, and the total sectional area of steel in each group was 8·31 square inches or a weight of 3·6 pounds per foot of height.

The weight of steel for the whole building was only 13·8 pounds per foot of height.

The panels of the walls were filled in with thin slabs of plaster of paris. When the building had to be removed, a severe fire test was first carried out, without any signs of failure, even when played upon by the firemen's hoses; no cracks showed between the cored brickwork and the plaster panels, although the glass in the skylight had melted and the portions of the cored concrete were covered with "vassy" cement, though the rendering was the only part affected.

After the fire test, the demolition was commenced by knocking away the support to the turret at O (Fig. 4).

After five blows with a ram weighing 1,540 pounds, and a stroke of 3¼ feet, 12 to 16 inches in height was broken away, leaving half the weight, or 59 tons, supported on 119½ square inches of cored brickwork.

This area of the reinforced brickwork was consequently subjected to a stress of about 1,100 pounds per square inch.

The support at R was next battered in, leaving only 47 square inches of support for the 59 tons weight, or about 2,844 pounds per square inch of support.

The support P was partly broken away, leaving only 4·7 square inches of support intact, for which the apparent stress amounted to 12·7 tons per square inch. Failure only occurred when this 4.7 square inches was knocked in, leaving no support under the turret. It is well to mention that in all probability part of the weight was carried cantilever-wise by the groups at A, B, C, D and M, L, K, the cantilever end being anchored by the group E, H, I, but in any case the upper part of the corner at N, O, P and R not failing until its whole support was removed, shows the immense resistance of reinforced concrete and brickwork.

When the failure occurred, the upper portions P′ P″ and N′ N″ gave way first, the flying arches being forced out at the corners at M″ B″ and E″ by snapping the wire cores (12 at each point).

The uprights at M″, B″, and E″ were forced outside the building but remained perfectly intact from the level of the springing of the arches, to the point of failure.

The whole storey above B′ O′ M′ and E′ came down bodily, and the flat roof dropped vertically on to the ground floor, carrying the first floor gallery with it.

The displacement in the direction of the under cut corner at O was only from 1·64 to 2·28 feet, and the bottom floor retained its horizontal position, suffering little or no deformation, and presenting only a few slight cracks.

These tests also prove the great strength and stability of this form of construction. They also demonstrate how each part of a reinforced concrete or brickwork structure will support others, and prove that great resistance still remains, even when only a part is left standing.

The effect of similar tests on a structure formed of the usual building materials need not be enlarged upon. A structure of steel or iron, when partly demolished, loses its resisting qualities almost entirely on account of the want of rigidity at the joints, and ironwork built into brickwork or masonry would not show very great resistance if subjected to the treatment applied to this building of reinforced concrete.

Resistance to Shocks.—The small vibration produced in a structure of reinforced concrete clearly points to the further advantage of resistance to shocks.

The driving of concrete piles 30 or 40 feet long without any shattering clearly proves that the resistance of reinforced concrete to shocks is very great.

The very short period of vibration of floors of this material, when subjected to the action of a falling load, is clearly brought out by the experiments detailed below.

In a test carried out recently in Paris by the engineers of the Paris and Orleans Railway Company, at their electric works at Austerlitz Station, two floors had been constructed, one in reinforced concrete, the other with rolled joists and brick jack arches, each with the same bearing, and calculated for a similar free load. These floors were tested in order to ascertain the result of shocks.

The dead weight of the floors were—

Iron and brick floor . . .	100 lbs. per sq. ft.
Reinforced concrete floor . .	62 ,, ,,

A weight of 112 lbs. dropped from a height of 6 ft. 6 ins. on the iron and brick floor produced vibrations of $\frac{5}{16}$ in. amplitude, lasting two seconds, while a weight of 220 lbs. falling 13 ft. on to the reinforced concrete floor only caused vibrations of $\frac{1}{16}$ in., lasting $\frac{5}{7}$ths of a second.

The short period and smallness of the vibrations in a reinforced concrete structure, protect it in a great measure from the gradual fatigue, which is so noticeable in structural metal work.

It cannot be definitely stated that constant vibrations, such as those sustained by railway bridges, would not ultimately have a detrimental effect on reinforced concrete. Time only can demonstrate this, but, it is certain that the vibrations will be comparatively small to those occurring in steel bridges, and it is extremely probable that the combination of concrete and steel may be found to give excellent results. It would, however, be foolish to assert definitely that such would be the case, until sufficient time has been allowed to elapse and a correct judgment obtained.

The great resistance offered by reinforced concrete to shocks and vibrations, renders it peculiarly appropriate for the construction of military works, such as forts and shelters.

Notice of the Approach of Failure.—A structure of reinforced concrete will seldom collapse suddenly, but will give plenty of warning before giving way. This has been proved again and again in experiments and tests.

The behaviour of a test T-beam at Calais may be mentioned, as bearing on this valuable property of reinforced concrete. This test beam had been calculated for a load of four tons. In November, 1898, it was loaded with 34 tons of rails, with the result that it cracked at the centre of the span; four very noticeable fissures extending well into the upper portion of the beam. Since that date (nearly six years ago) it has been left with the 34 tons upon it, equal to 8½ times the calculated load, without any increase in its deflection or in the size of the cracks.

M. Christophe, in his book, *Le Béton Armé*, quotes the results of experiments carried out by the Commission on Arches of the Society of Austrian Engineers and Architects, in a series of tests to rupture on arches of 23 metres, or about 75½ feet span, and constructed of different materials.

It is interesting to note that for the several materials tested the excess of the loads at the time of breaking to those when the first crack appeared were as follows—

For Ashlar	30 per cent.
For Brickwork	59 per cent.
For Ordinary Concrete . .	31 per cent.
For Reinforced Concrete . .	86 per cent.

These tests, which were carried out by a responsible authority, show very clearly that the failure of a reinforced concrete structure is very gradual, and that the resistance is still very great after the first crack has appeared.

The Effect of Atmospheric Changes.—The effect of changes of temperature on concrete has been referred to when dealing with the fire-resisting qualities; but smaller changes of temperature than these have, as is well known, a marked effect on concrete, and the richer in cement we make the mixture the worse would be this effect.

Also the humidity of the atmosphere affects the concrete considerably, dampness causing an elongation, and dryness a contraction.

The variation in volume of concrete under changes of humidity appears to be even more affected by the richness of the mixture than under that change of temperature. This change of volume is clearly a disadvantage in reinforced concrete construction, as it is a *sine qua non* that rich mixtures should be used. Fortunately, however, the reinforcing rods and network appear to act as preventatives against the cracking of the surface of the concrete, and it is seldom found that cracks are formed where the reinforcement is placed near the face exposed to changes in temperature and humidity.

In buildings, except in special instances, the concrete is not exposed to sudden changes of this character, under normal conditions, and in most structures the surface can be covered with some non-conducting medium, such as soil, or paving.

In retaining walls and similar structures, vertical timber expansion strips can be used, as they do not affect the stability, except in the case of walls curved in plan, in which they should not be used.

In bridges, and cases where beams and floorings are not fixed but free on their support, there is, of course, a certain freedom to expansion, and contraction.

In large arches of reinforced concrete it is well for this reason to place hinges at the abutments and centre.

It is advisable in special cases to place a light mesh of ironwork near an exposed surface, to prevent cracking from temperature and humidity changes.

As in the casting of iron, quick changes of thickness should be avoided, and all angles should be rounded. Panels in bridge parapets, and faces of buildings, etc., should have splay mouldings where possible, to reduce the thickness gradually.

When placing the concrete in position, it is well to keep it damp during the setting, since the slower the setting action the better the concrete will be able to resist the action of changes of temperature and humidity. In times of great heat or dryness it is specially advisable to keep the surface moist and protected from the sun's rays, so that it may not set too quickly. The whole body of the concrete should, as far as possible, set together to prevent internal stresses being set up.

It is better to avoid concreting, if possible, during severe frosts, since although the effects appear to be only temporary, tests have been published showing a decrease of strength when briquettes have been frozen.

The frost seems only to retard the setting and does not appear to have any effect on the final strength of the concrete. Some tests made to discover the effect of frost on concrete are given below.

Twelve briquettes were made of sand and cement mixed three to one with 10 per cent. of water on December 20, 1901. Six were placed in an exposed place in the open, and taken out of their moulds on the 23rd, being still left in the open. The remainder were left indoors, and placed in water on the 23rd. All were broken on January 19, 1902, with the following results. The average tensile strength of those left in the open was 249 lbs. per square inch, and for those left indoors 249 lbs. also.

The Temperatures registered were as follows—

Date of reading, taken at 9·0 a.m.	Min.	Max.
Dec. 21	22	34
„ 23	24	34
„ 24	28	42
„ 30	28	50
Jan. 1, 1902	28	52

Many bridges have been constructed during severe frosts in the United States without any detrimental effect on the concrete.

Evenness of Temperature and Deadening of Sound in Buildings of Reinforced Concrete.—The poor conducting qualities of concrete keep the rooms in buildings of reinforced concrete at a very equable temperature, which is a special advantage where they are partially in the roof, and in ordinary buildings are unbearably hot in the summer and excessively cold in the winter. The advantages from this quality in the case of a fire have been referred to (p. 8). The effect on sound is the reverse by consequence of the walls being thin, and where single walls are employed sound penetrates in a marked degree.

It is very usual, however, to employ double walls in this form of construction, as these may be made very thin, and can be united together by bonding-in cross-ties, so that the two walls act as one, in resisting the stresses. Floors are also made double, the ceiling slab being separate from the flooring slab, and the same method is employed for roofs.

Ease and Rapidity of Erection.—Ease and rapidity in the erection of a reinforced concrete structure is an immense advantage: walls can be moulded of concrete far quicker than they can be built of stone or bricks.

Floors and roofs are moulded at the same time as their supporting beams.

Bridges can be erected, and roofing to reservoirs constructed, in a far shorter time when made of reinforced concrete than of brick, stone, or iron.

The materials are easily procured, and require no such treatment as that of dressing stone or the making of girders to dimensions.

The iron or steel work is almost entirely in the form of round rods, hoop iron, or wire, being only in a few instances of special section. It is in most cases simply laid in place, and perhaps tied with wire at crossing-places, and requires only ordinary simple smith-work which can be done on a portable forge at the site of the works, or the pieces may be bent to the required shape and cut to necessary dimensions before delivery.

In some few instances, a small quantity of bolts are required, but this only applies to a few methods of reinforcement.

Special Precautions Necessary.—Great care and judgment are required in the selection and preparation of material and preparing of temporary work for the erection of reinforced concrete structures. The cement must be of the very best, finely ground and cool. The aggregate must be of proper size for the different kinds of work, and be perfectly clean. The concrete when mixed should be moderately dry. The water should be quite clean, and the concrete thoroughly and carefully mixed and deposited.

The falsework requires special care and forethought, so that it may be as economical as possible, since it forms a large item in the total cost of a reinforced concrete structure. The concrete must be thoroughly well rammed, especially round the reinforcement, as it is very essential that there shall be no pores, and that the concrete shall be thoroughly homogeneous.

Great care is necessary in the placing and keeping of the reinforcement in position, as the strength of the structure mainly depends on the skeleton being in its calculated position. Any welding or bending must be done with great care, so that no appreciable strength is lost thereby.

Both foremen and labourers must be carefully selected, and the foreman especially trained to apply the care and thought required, in order that he may see that the structure is exactly as designed, and that all fixtures, etc., are properly moulded in the places assigned to them. A careless labourer should be dismissed at once, as there must be no risk of bad workmanship.

That great care is required must not be lost sight of, and it would be foolish to deny that such care must be taken. But the benefits derived from this form of construction amply repay the necessity of employing selected men at a slight increase on the usual rate of wages. A foreman, who should thoroughly understand smithwork and timbering, a good carpenter, and blacksmith, with the labourers specially selected, are all that is required, and no other tradesmen are necessary except in the case of reinforced brickwork, and in this instance the employment of bricklayers is compensated for by there being no carpentry for falsework.

Economy.—The economy of the use of reinforced concrete in suitable cases is undoubted. The thinness of the walls, floors, etc., more than compensate for the extra cost of the metallic skeleton, and the care required in mixing and depositing the concrete, and in the placing of the reinforcement. The comparative lightness of structures of this material causes much saving in the foundations in very many instances.

The large amount of falsework needed for the proper moulding forms a considerable item in the cost of a structure of reinforced concrete, but this falsework is usually of an extremely simple form, and requires no special treatment, and very few timbers of large scantling. Many of the parts of a building can be moulded on the ground before erection, which greatly reduces the amount of falsework required. The comparative lightness of arches of reinforced concrete, compared to similar masonry and brickwork structures, allows the centring to be much lighter than that generally employed.

If too much concrete has been mixed for immediate use, it appears that it may be used even after it has been mixed for some hours, providing the cement is sufficiently slow-setting, and the atmospheric conditions do not incline to excessive heat or dryness. This is clearly shown by a set of experiments carried out to ascertain the loss of strength caused by mixing mortar several hours before using it.

The description of these experiments was published in a paper on the subject of construction in concrete and reinforced concrete, which appeared in vol. cxlix. of the "Minutes of Proceedings of the Institution of Civil Engineers," and the results are given in Table II.

The first tests were of mortar, as mixed by the bricklayers; the rest were of special mixtures of 3 of sand to 1 of cement, and 10 per cent. of water to the weight of the cement and sand. The cement was slow-setting, the final set taking place in from 5½ to 7 hours. The tests were made 28 days after moulding.

Some further tests to elucidate the same point were made in 1902 by Mr. C. G. Streels, of Sioux City, U.S.A., and are given in Table III.

TABLE II.

COMPARATIVE TENSILE STRENGTHS IN LBS. PER SQUARE INCH.

Freshly mixed	Left standing		Knocked up every 10 Minutes	
	After 1 hour	After 2 hours	After 1 hour	After 2 hours
185	186	180	186	170
226	233	231	223	236
	After 2½ hours	After 3 hours	After 2½ hours	After 3 hours
203	203	203	196	196
	After 3½ hours	After 4 hours	After 3½ hours	After 4 hours
158	146	161	163	168
	After 4½ hours	After 5 hours	After 4½ hours	After 5 hours
200	153	118	158	120

TABLE III.

RESULTS OF TESTS MADE IN 1902 ON EFFECT OF CONTINUOUS MIXING OF PORTLAND CEMENT MORTARS BY C. G. STREELS, ASSISTANT CITY ENGINEER, SIOUX CITY, U.S.A.

Sand through sieve of 20 meshes per lin. inch, and retained on sieve with 30 meshes per lin. inch. 99 oz. of cement, 220 oz. of sand, and 24 oz. of water, or 7·77 per cent. of the cement and sand.

No. of briquettes.	Continuously mixed for hrs.	mins.	Average tensile strength lbs. per sq. inch.
4	0	15	294
2	0	30	278
2	0	45	282
2	1	00	243*
2	1	25	275
2	1	55	283
2	2	25	287*
2	2	55	314
2	3	25	326*
2	3	55	372
2	4	25	334†
2	4	55	384
2	5	25	264
2	5	55	249
2	6	25	308*
2	7	25	217
2	8	25	255
4	8	55	236
4	8	55	220*
2	8	55	215*

* 2 oz. water added. † 3 oz. water added.

These results show that no waste of concrete need occur.

The fact that no tradesmen need be employed on structures of reinforced concrete, except ordinary carpenters and smiths, causes a distinct saving in cost.

For reinforced brickwork, bricklayers must of course be employed, but in this case, as has been said before, no carpenters are required. There is a saving of about 20 to 30 per cent. in the cost of a concrete column reinforced with longitudinal rods over that for a steel column to support the same load, and the saving will be greater when a hooping of spirally-wound wire with a series of light longitudinal rods is used as a reinforcement. For beams the saving is very nearly as great. There is also a considerable saving in maintenance. And it must be also borne in mind that iron and steel columns and beams are frequently surrounded with concrete or other material for protective purposes. When used in the place of masonry or brickwork there is also a saving in cost, on account of the comparative thinness of the walls and arches.

Compared with timberwork, as when used for wharves or jetties, reinforced concrete is of course the more expensive material. Against this must be placed the fact that a structure of reinforced concrete is practically everlasting, whereas had it been made of timber its life would be only of short duration.

The place of reinforced concrete as a building material may be well represented by drawing two circles to overlap one another, one representing structural iron and steel work and the other masonry. The area formed by the overlapping of the two circles, which represents the common province of these two forms of construction, and where they may be used in combination, is the special field of reinforced concrete. Within this area it will be cheaper than steel or iron by reason of the simplicity of construction and cheapness of materials, and more economical than masonry by reason of its comparative smallness of weight and quantity; while outside it is unable to compete with iron and steelwork by reason of its weight, or with ordinary masonry by reason of the cost of construction.

Doubts as to Proper Method of Calculation.—The doubts that still remain as to the principles on which the true formulae for obtaining the sections of reinforcement and concrete should be based, will, it is to be hoped, soon be cleared away. Infinite care and labour have been, and are being, spent to remove this difficulty. Elaborate experiments and researches have been carried out, and to-day we certainly know much more about this subject than was known a few years ago.

Until we thoroughly understand the properties of the materials employed, when used in combination, we must do our best with the knowledge we already possess, and use the best formulae that we can obtain. This need of true formulae, however, need not prevent us from employing reinforced concrete and masonry. The formulae already in use give us structures that in every way resist the stresses they have to bear, and although a saving might possibly be effected if we knew more, the structures we can design with our present knowledge will be perfectly safe and undoubtedly economical. The knowledge of the properties of *iron and steel* has been greatly extended in the past few years, but we are not yet unanimous in our opinions respecting all their properties, and in the design of iron and steel structures there is doubtless still a great deal to be learned, and to provide against all contingencies we are in the habit of allowing large factors of safety. In the present formulae used in designing reinforced concrete structures, it only amounts to very much the same thing, and as we learn more, these formulae will doubtless be improved.

The Construction of Reinforced Concrete mainly in the Hands of a Few Firms.—Reinforced concrete construction is at present mainly in the hands of the patentees of the different systems, and of firms who have patented some detail of construction and make a speciality of this kind of work. It appears at first sight difficult to design and carry out a structure of this nature, without infringing some of the many patents. It will be found, however, that the really valid patents are, generally speaking, those for some small detail, and that the main principles are not patented, and in fact are unpatentable. There are many ways of embedding iron or steel in concrete to obtain the results required, and if one form is patented, and the patent is valid, there are other forms which could be used just as well.

It will be found that the principal firms who carry out this form of construction do not rely so much on their patent systems for bringing them business, as upon the fact that they thoroughly understand the work, and have in their employ men who have become thoroughly accustomed to use the care and forethought which is undoubtedly required. They also know how to use their materials to the best advantage, and having given special attention to the study of this form of construction, they are able to do the work better, and cheaper, than others who are not so intimate with it.

At first sight it may seem a disadvantage that the construction in reinforced concrete and masonry should be in a great measure in the hands of a few firms. We must remember, however, that the bringing of reinforced concrete and masonry into general use is mainly, if not entirely, due to the men who have studied the subject more or less thoroughly, and who in consequence brought out the several systems.

It is advisable in many ways that the firms constructing in these materials should, at present, be employed to do any work of this kind, as they thoroughly understand the methods and have suitable men in their employ. If work of this character is undertaken by those not well accustomed to its special requirements, they will probably find that it will cost more than if done by a good firm who make a speciality of the work, and they will also find that men must be trained to use the special care which is necessary in the erection of reinforced concrete or masonry structures, and that although ordinary labourers with good supervision are certainly all that are required, it will still take some time to bring an ordinary navvy to understand that the concreting he is employed upon must be treated in a very different manner to that he has been accustomed to. Even when reinforced concrete has come into general use, it will probably be found advisable to employ some recognized firm who is accustomed to the work, when a structure of this material is to be erected.

Accidents.—It cannot be denied that accidents have happened with structures of reinforced concrete; a notable instance being the fall of the footbridge over the Avenue de Suffren, for the Paris Exhibition, causing the death of eight persons and the injury of eight others.

This bridge was completed and the centring removed, but before it was opened for traffic it fell with no load but its own weight. The failure was attributed at the official inquiry mainly to two causes—

1. The weakness of the columns, most of which were only 11·8 inches square, and several of which were frequently struck by carriages passing along the roadway.

2. The bridge, though originally designed to be straight throughout, had subsequently to have the side spans placed on the skew to avoid certain trees; the oblique stresses caused by this alteration were not provided against, the structure being erected as at first designed.

The columns were stated to have been originally designed 16 inches square, whereas all but four were constructed 11·8 inches square. It also appeared that a letter was written the day before the accident by the designer, pointing out that the columns were too weak, as the reinforcement was not exactly as designed, and had not been inserted in a satisfactory manner, and that unless they were strengthened he could not be responsible for their safety. It seems probable, therefore, that this accident was mainly caused by carelessness in the workmanship, due to the urgency for speed in completion.

A large percentage of the few accidents which have occurred with this form of construction have been due to the premature striking of the falsework. The cause of some has been traced to the use of improper materials, and others to insufficient rigidity of the falsework, or to the concrete having been subjected to vibrations while setting. It has seldom, if ever, happened that a failure has been due to the inherent weakness of the structure.

It cannot be too strongly insisted on that proper care must be exercised in the workmanship and selection of materials, but there is no necessity to condemn the use of reinforced concrete on this account, and the multitude of satisfactory structures that have been erected sufficiently proves its utility, and economy.

Part II

SYSTEMS EMPLOYED

General Remarks

Before commencing the description of the various systems, it may be advisable to briefly point out the various types of reinforcement that are used, and the reasons for their adoption. This treatment of the subject will be very general, and deal with columns, beams, arches and pipes only, since most of the details of construction are related to one or the other.

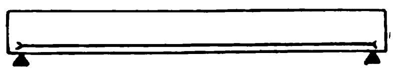


FIG. 6.

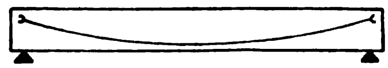


FIG. 7.

Columns.—These are usually reinforced with longitudinal bars tied together at intervals by wire or flat iron cross-pieces. Recently, however, the advantage of adding a spirally wound hooping to prevent the swelling of the concrete has been demonstrated by M. Considère and others, and such a reinforcement has been tried in some few cases with good results.

Beams.[1]—As a general rule it is found that the concrete is sufficient to resist the compressive stresses, and we have therefore to supply the reinforcements to resist the tensile stresses only. In the case of a freely supported beam, the reinforcements are consequently placed as shown in Fig. 6. Or, since the curve of bending moments is parabolic under a uniformly distributed load, they may be placed as in Fig. 7, a disposition which increases the efficiency. It will be noticed, however, that if the reinforcement is placed as shown in Fig. 7, the bottom of the beam may take a similar curve to that of the beam in Fig. 8. If the compressive

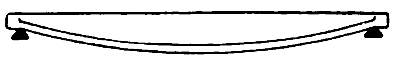


FIG. 8.

stresses are considered too great for the concrete, straight reinforcements are added along the top of the beam.

When we come to consider the case of a built-in beam we find it more complicated. There is a bending moment at the supports in the opposite direction to that at the centre of the span. A reinforcement must therefore be placed near the upper surface over the supports extending some distance towards the centre of the span, as well as the reinforcement at the bottom of the beam.

The straight bottom reinforcement of Fig. 6 is consequently retained, and two

[1] M. Christophe employs a similar method of general treatment for beams and arches in his book, *Le Béton Armé.*

short upper reinforcements added, as shown in Fig. 9. The upper reinforcement is sometimes continued across the span, serving, at the centre, to resist any excess

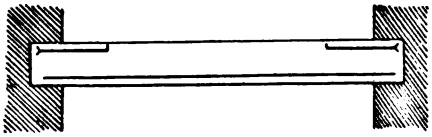


FIG. 9.

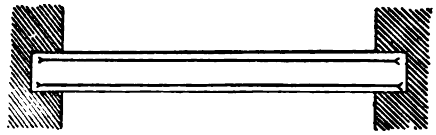


FIG. 10.

of compression which cannot be safely taken up by the concrete; in the same manner the bottom rod will resist this excess near the supports. This method is shown in Fig. 10.

A further method of resisting the stresses in a built-in beam is to employ one series of reinforcements, bent as shown in Fig. 11, so as to follow the path of the tensile stresses; but in this case there is severe compressive stress on the bottom

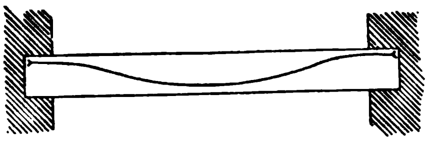


FIG. 11.

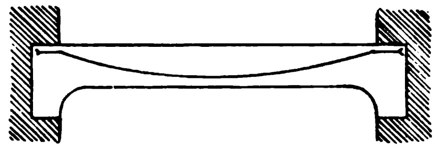


FIG. 12.

of the beam near the supports due to building-in, and it is advisable to form the ends of the beam in the manner shown in Fig. 12, so as to gain extra resistance by increasing the area of the concrete in compression.

A series of bottom straight rods are frequently added, extending through the whole length of the beam, as shown in Fig. 13. These will increase the tensile resistance at the centre of the span, and the compressive resistance near the supports. If further tensile resistance is required at the top near and over the supports, some constructors add the short reinforcements shown in Fig. 9 to the arrangement of Fig. 13, or, when there are several spans, the ends of the bent-up rods are made to overlap, which produces the same result.

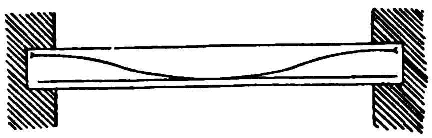


FIG. 13.

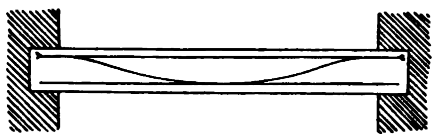


FIG. 14.

If the concrete at the centre of the span has not sufficient area in compression to resist the stresses, a series of straight rods are added, extending completely across the span as shown in Fig. 14.

Arches.—These may have either a curved or flat extrados, but both types may be considered together. The most simple method employed to reinforce an arch is to place a reinforcement near the intrados throughout the whole span. The stresses in an arch are mostly compressive, and the greatest tendency to tension is at the intrados at the crown or at the haunches under a uniformly distributed or moving load. Such a reinforcement is shown in Fig. 15.

The failure of an arch is first indicated by cracks in the intrados at the crown, and in the extrados near the springing; the single reinforcement of Fig. 15 is therefore frequently considered insufficient, and reinforcements are added at the springings as shown in Fig. 16, or, for greater security, since it is difficult to

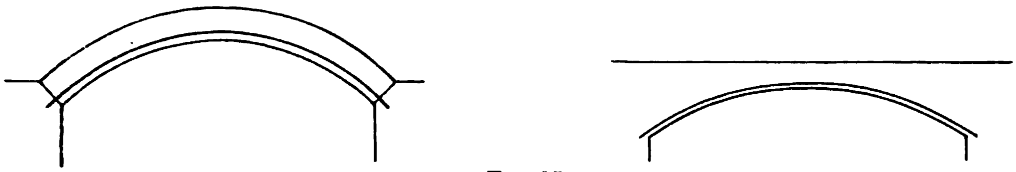


FIG. 15.

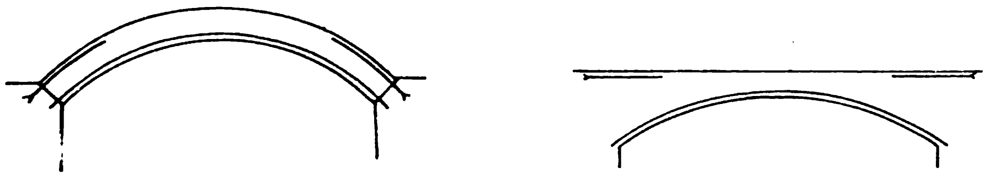


FIG. 16.

decide on the length of these extrados reinforcements, these upper bars are continued throughout the whole span, as in Fig. 17. This form of reinforcement is rendered the more necessary on account of the tensile stress produced at the extrados under changes of temperature.

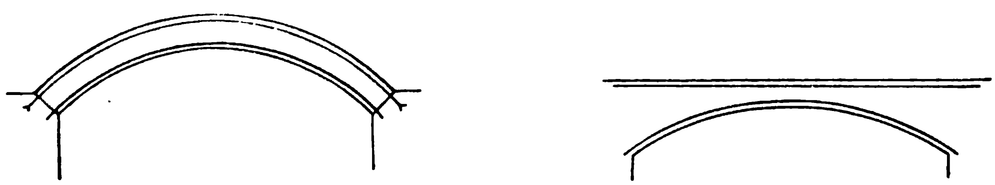


FIG. 17.

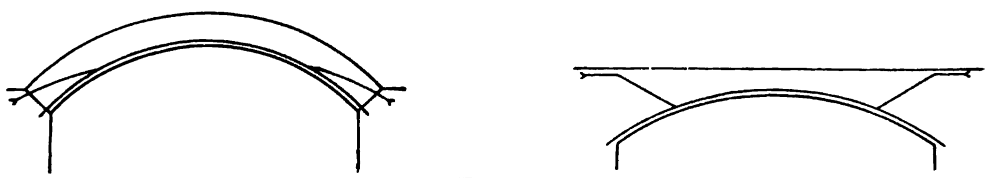


FIG. 18.

A further method adopted is to retain the bottom reinforcements throughout, and incline the upper reinforcements shown in Fig. 16, so as to be near the extrados at the springing, as shown in Fig. 18.

When this form is combined with a series of reinforcements extending throughout the whole length of the extrados, we have a complete reinforcement which will resist any stresses which may be induced. The combination is shown in Fig. 19.

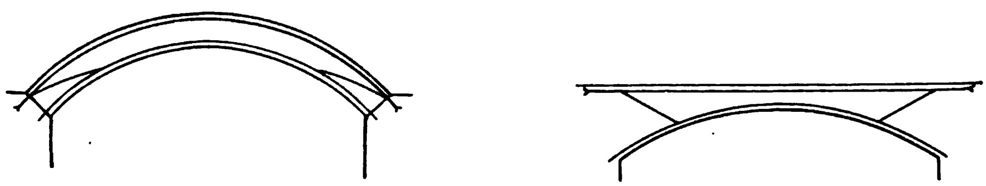


FIG. 19.

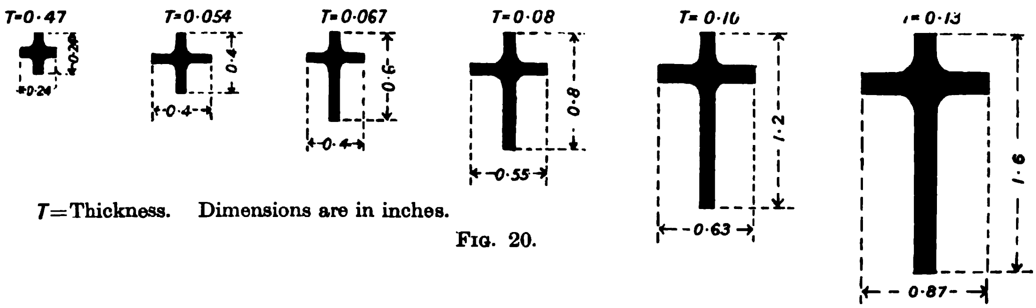
The types of beam and arch reinforcements (Figs. 6 to 19) which are reproduced from M. Christophe's *Beton Armé,* show the general methods adopted by the various systems, and will be recognized when we come to describe the various forms of construction in detail. In beams there are also shearing stresses to be resisted, and consequently various methods of transverse reinforcement are employed, either vertically or inclined, or both combined. The bent reinforcements also assist in taking up the shearing stresses, and in the case of a uniformly distributed load those shown in Figs. 7 and 8 resist them entirely. The shearing stresses are very slight in arches which do not require special reinforcements for the purposes of their resistance. Many constructors, however, place transverse reinforcements in the vertical plane near the springings as a precautionary measure. In any case it is advisable to tie the main reinforcements of arches securely together, for the better resistance to the swelling of the concrete under compression.

Pipes and Circular Reservoirs.—These are reinforced with hoops or spiral windings of metal, together with longitudinal distribution bars, placed internally or externally, according to the direction of the pressure.

It will be impossible to describe all the different types of construction adopted by every system ; but it is hoped that by giving a short account of the leading features of each, a fair idea may be obtained of the general scope.

Bonna System

This system was introduced by M. Bonna of 78, Rue d'Anjou, Paris, in 1893, and is perhaps best known on account of the manufacture of pipes, of which a speciality is made. One of the distinctive features of M. Bonna's system is the use of special steel sections in the form of a Latin cross. These are employed in all the forms of construction, and have an increased resistance due to the smallness of the sections and the rolling. Fig. 20 shows the sizes generally used. M. Bonna



T=Thickness. Dimensions are in inches.

FIG. 20.

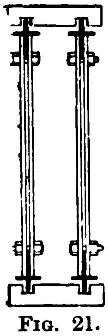


FIG. 21.

also uses ordinary angles and tees and has lately adopted a double-cross section like a small rolled joist, with its web produced through the flanges in both directions. All the reinforcements for a building are built up before the concrete is deposited, being secured by a few bolts. Fig. 21 shows the usual method of beam reinforcement. The main bars of the cross type are connected by vertical braces of double flat bars. They are also tied together, as shown, by horizontal transverse flat bars notched out to receive the short ends of the cross sections. When the double-cross sections are used, the vertical ties are of course omitted.

All the reinforcements of the various elements (primary and secondary, beams, columns and supports) are secured together. The columns are reinforced by profile bars tied together by horizontal flats secured to the main bars by bolts or rivets. The reinforcements, being thus all tied together, serve to support the falsework, men and materials during construction.

The floors are usually reinforced in the same manner as the beams with cross-shaped bars at the top and bottom secured together by verticals of flat iron, and held transversely by upright notched flat bars extending across the whole width of the slab.

The Pipes and Reservoirs constructed by M. Bonna are reinforced with spiral or circular hooping, the spiral form being always employed when the cross-shaped reinforcements are sufficiently small. Against the hooping are placed longitudinal distribution rods, which are notched out to receive the hoop bars. Either one or two series of reinforcements are employed, according to the size of the pipes and the pressure upon them. When pressures of more than 50 feet head are to be resisted, a sheet steel tube is embedded in the concrete to ensure impermeability, or sometimes this tube is placed in the inside of the pipe. Fig. 22 shows a simple spirally wound pipe reinforcement.

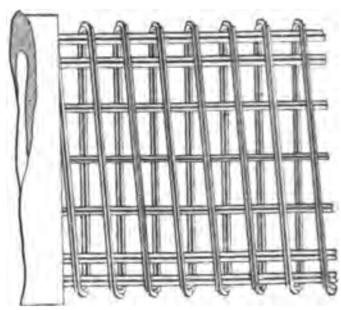


FIG. 22

The methods of construction of the pipes and reservoirs are more completely dealt with when treating of "Practical Construction" [pp. 179 and 199 to 202.]

A mixture of quick and slow setting cement is used for the concrete of the pipes and reservoirs.

Straight and arched bridges are also constructed in this system.

Among the works carried out by M. Bonna the following may be mentioned.

The Pipes carrying the sewage of Paris through the Argenteuil Gallery and the distribution system of the Achères Sewage Farm.—The pipes through the gallery were 5·91 feet in diameter, and were constructed with a steel tube lining. The length of this pipe line was 1,640 yards. The distribution pipes for the Achères Sewage Farm were of diameters varying from 15·75 inches to 43·3 inches. They were formed with a tube of steel sheet embedded in the centre of the thickness of the shell. The head on these pipes was 131·2 feet, and their total length was nearly 25 miles.

The distribution systems of Méry, Pierrelaye and Triel (districts of Paris).—The pipes are of diameters varying between 11·81 and 43·3 inches, and some of 6·56 feet diameter. The total length of these branches is 74·6 miles. M. Bonna completed this work in the extraordinary short time of eight months, the mean progress being 547 yards a day.

The works of the Cie Centrale des Emeris et Produits a Polir, situated in the Boulevard Sérurier, Paris.—A building 492 feet long and 66 feet wide, with ground floor, one floor above, and a pitched roof, the whole being of reinforced concrete.

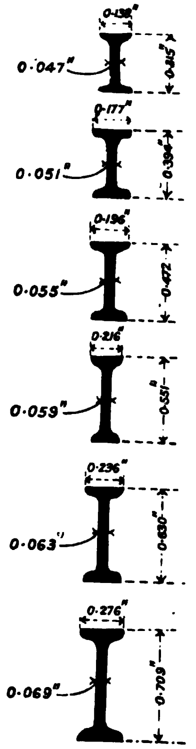
Workshop for MM. Sautter-Harlé et Cie, 35, *Rue de la Fédération.*—Two galleries run round this building, being supported on columns. A travelling overhead crane works at the level of the first gallery, the rails being carried on cantilevers projecting from the beams supporting the front of the gallery. The whole building was constructed of reinforced concrete.

Arched Bridge over the Canal du Midi, at Toulouse.—This bridge is formed of six arched ribs 48·2 feet span, and 4·33 feet rise, the width of the ribs being 9·84 inches, and the thickness at the crown, including the decking, being 2 feet. The total width of the bridge is 26¼ feet, but the two footways, 3·28 feet wide, are supported on corbels projecting from the outer arched ribs. The ribs are formed with open spandrels towards the abutments. They are reinforced with profile bars and vertical bracing similar to those described for beams. The piers between the openings in the spandrels are also reinforced as columns. The decking and footways have round rod reinforcements placed longitudinally and transversely, in the manner of a Monier network.

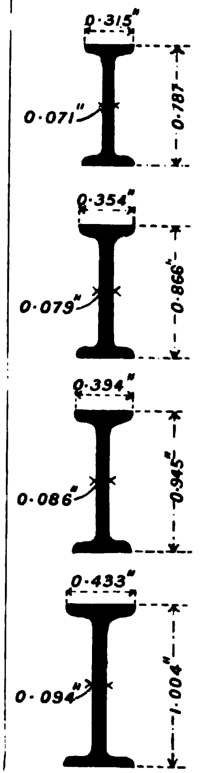
Bordenave System

In 1887 M. Bordenave, of 28, Rue de Lyon, Paris, brought out his system which is confined to the construction of pipes, sewers and reservoirs.

Special small I-sections of steel are employed for the reinforcements, together with round rods for secondary reinforcements, and for the floors and covers to reservoirs.



AREA SQ.R INCHES	WEIGHT POUNDS.	SAFE STRESS POUNDS. PER SQ.R INCH.
0·023	0·080	
0·033	0 113	21·330
0·042	0·142	19·196
0·051	0·174	17·419
0·061	0·209	15·997
0·075	0·256	14·931



AREA SQ.R INCHES	WEIGHT POUNDS	SAFE STRESS POUNDS PER SQ.R INCH
0·090	0·307	14·220
0·111	0·380	13·153
0·135	0·466	12·442
0·161	0·547	12·087

FIG. 23

Fig. 23 gives the sizes, weights[1] and safe resistance of the special I sections. The small size of these sections causes them to gain a considerable increase of resistance from the rolling. The hooping of pipes is wound spirally, the distribution bars resting against the spirals, and being tied to them with wire ties. M. Bordenave

[1] The weights are per lineal foot.

uses quick setting cement for all his works. A full description of the methods of construction of pipes and reservoirs is given when treating of "Practical Construction" [pp. 176, 177, 199 and 200].

Amongst the pipe lines constructed by M. Bordenave, *the Conduit of Bone* (*Algeria*), may be mentioned. The pipes for this conduit are 23·6 inches diameter, and are under a head of from 56 to 79 feet. The total length is about 18½ miles.

Boussiron et Garric System

The system patented by MM. Boussiron et Garric, of 16, Rue Milton, Paris, is used for entire buildings, also for reservoirs, aqueducts, arches, etc.

For floors of small span, a simple slab is employed with no supporting beams. If this is merely supported on the edges it is reinforced with a series of rods running from wall to wall about an inch from the lower surface. If the slab is built in, every other rod is bent up near the supports. When the slab has to bear concentrated loads, a further series of rods is added running perpendicularly to the former. This arrangement is shown in Fig. 24. When the spans are great the floors are constructed with main and secondary beams, either with or without a ceiling slab. The construction is of the monolithic type, in which the beams and floor slabs are formed together.

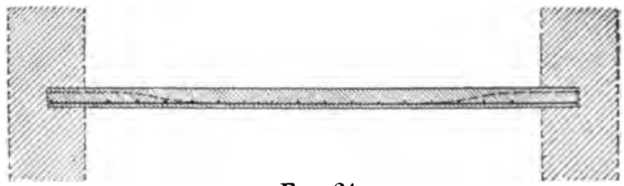


FIG. 24

The floor slabs are reinforced in the same manner as described above for floors which may have to bear concentrated loads, and as shown in Fig. 24.

The beams, if freely supported, are reinforced with two or more longitudinal rods near the bottom, and hoop-iron V-shaped stirrups arranged as shown in Fig. 25, passing under the longitudinals and turning inwards near the upper surface of the concrete. The stirrups are placed further and further apart from the supports towards the centre of the span. If the beams are built in, rods are also placed near the top surface extending for about one-fifth of the span from the supports, being anchored back to the walls, and extending into the adjoining slab at intermediate supports. Hoop-iron stirrups are also inserted in the same manner as for freely supported beams. This arrangement is shown in Fig. 26. The upper reinforcements are bent at the walls into the shape of a U in plan, the two branches forming the

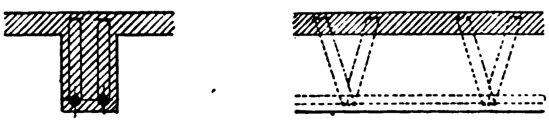


FIG. 25

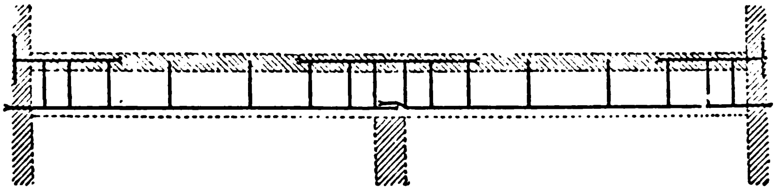


FIG. 26

projecting rods, and the curved portion receiving the anchor rod. When the floors are formed with hidden beams and a ceiling, the longitudinal rods of the lower slab are bent round the lower rods of the beams, as shown in Fig. 27. The ceiling slab has also a further reinforcement of rods perpendicular to those shown in the figure. When rectangular beams are used without a floor slab, they are reinforced in the same manner as the floor beams.

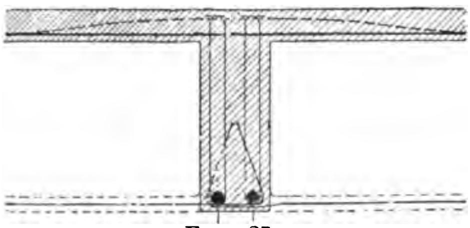


FIG. 27

Columns are reinforced with four vertical rods tied together by wire loops, as shown in Fig. 28.

Elevated Circular Reservoirs are formed with a series of hooped circular reinforcements, formed in two halves overlapping at the joints. These are placed at the centre of the thickness of the shell. Inside the hoops, and bearing against them, are a series of vertical distribution rods. The bottoms of the reservoirs are usually formed flat, in a similar manner to that already described for floors. At other times they are spherical, with a rise at the centre, in which case hoops of flat bars are placed at the bottom of the walls to take the thrust.

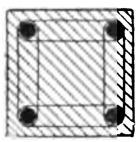


FIG. 28

In this system the calculations are made allowing for the differences of the coefficients of elasticity of the two materials, and for pieces subjected to bending the stress strain curve of the concrete in compression is considered to be a straight line. The tensile resistance of the concrete is neglected.

Bramigk System

The firm of Bramigk of Dessau (Anhalt) construct floors up to 13 feet span, in the manner shown in Fig. 29. The concrete is moulded round pipes of concrete or earthenware, and is reinforced by flat bars 0·30 inches thick, running in the direction of the pipes. Sometimes round rods are used, being placed near the lower surface, and between the pipes. The concrete is 7·87 inches thick, and is either covered with linoleum, or, as shown on the left side of the figure, in which case wood strips are embedded in the concrete and floor boards nailed to them.

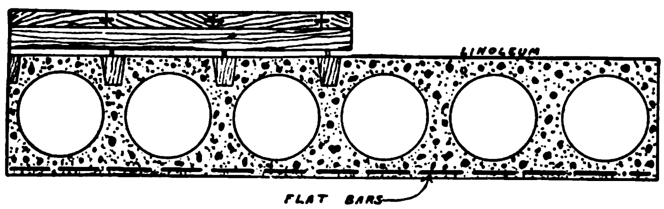


FIG. 29

Chaudy System

This system is employed by the Société des Travaux en Ciment de la Plaine-Saint-Denis, 15, Rue du Louvre, Paris, who also use the Monier system for their constructions.

In his calculations M. Chaudy considers his reinforced concrete *beams* as consisting of two metallic members, and neglects the concrete, except for tying the members together, and resisting the compressive stresses due to shearing. His reinforcements are consequently always symmetrical. M. Chaudy also connects the two rods of his elementary beam by a stirrup of round iron, which he bends over the top rod so as to tie the upper and lower reinforcements together; he does not use a stirrup open at the top, since he is of the opinion that it is essential to form a rigid connexion between the two members. Fig. 30 shows an elementary beam with the stirrup, which is bent, as shown on the left, so as to prevent any sliding through the concrete.

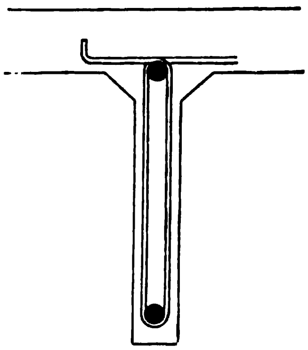


FIG. 30

In most cases it is necessary to have several upper and lower rods. In this case the rods are placed side by side at the top and bottom, and the whole series are embraced by a hooping of flat iron, as shown (Fig. 31). These hoops are made in advance, being joined by a cover plate and bolts, as shown. The rods are tied to the hoops at their proper spacing by wire ties, so as to hold all the reinforcements in position during the execution of the work. The distance apart of the hoops is calculated, so that if the reinforcements were stripped of concrete, the top rods could not bend under the direct compressive stress upon them. M. Chaudy also uses a beam reinforcement consisting of angle irons back to back for the top and bottom members, these being connected by vertical flat bars secured by rivets, as shown (Fig. 32) but he prefers the reinforcements of round iron, as there is no loss of metal due to rivet holes.

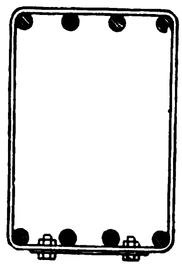


FIG. 31

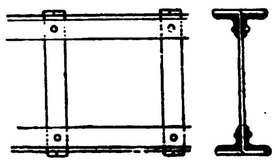


FIG. 32

Floors are considered as a series of beams connected to one another. They are reinforced either with two or one series of rods, held together transversely by rods bent in the form of a rack, which, besides their office as reinforcements, form an excellent gauge for the proper spacing of the rods. Fig. 33 shows a floor with a double series of rods, and Fig. 34 is a view of a slab with one set of rods near the lower surface. The racks are made before the rods are sent to the work. When a floor has several bays, the reinforcements are overlapped across the beams; and since M. Chaudy is of the opinion that a considerable length of overlap is necessary to ensure proper continuity, he bends the rods up or down, as shown (Fig. 35), thus giving them sufficient hold in the concrete without a great length of overlap.

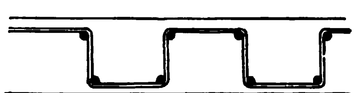


FIG. 33

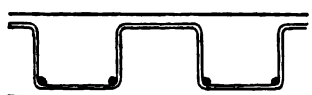


FIG. 34

Columns are reinforced with vertical rods tied together with a hooping similar to that shown in Fig. 31.

Walls are formed in the same manner as slabs, having one or two series of vertical rods, according as to whether the pressure may act from one or both sides. These are held in position by longitudinal rods in the form of racks, similar to those shown (Figs. 33 and 34).

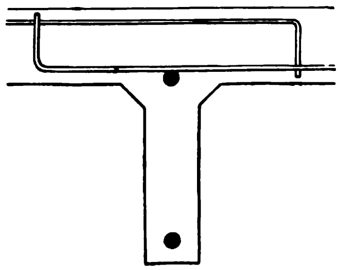


FIG. 35

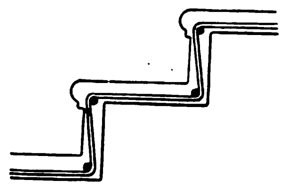


FIG. 36

Sometimes the racks, instead of being bent at right angles, are formed with a slightly acute angle like that of the *stair* reinforcement shown (Fig. 36). This illustration requires no further description, the arrangement being clearly indicated in the figure. Sometimes *stairways* are formed with a slab underneath, carrying the steps. In this case the slab is reinforced with bent rods following the steps, and a cross rod in each bend ; the steps being formed of plain concrete.

The Société des Travaux en Ciment de la Plaine-Saint-Denis have carried out a considerable quantity of work in reinforced concrete, including floors, stairways, aqueducts, reservoirs, and large wine tanks, etc. For large reservoirs they use I, T, and L iron reinforcements.

The falsework is partially supported by the reinforcements, and a saving in the cost of construction is thereby obtained. The proper spacing of the reinforcements is always ensured, and they are all securely held in position during the deposition and ramming of the concrete. This is a distinct advantage, as it guards against any want of care in the execution of the work.

16. The System of La Société des Chaux et Ciments de Crèches (Saône-et-Loire)

The form of reinforcement adopted by this firm consists of one straight round rod along the bottom of the beam, and a rod of small section along the top. The upper rod rests on the floor rods, which are of small sectional area running across the beams. The upper and lower rods of the beams are tied together by transverse reinforcements of iron wire. The upper rod is not used with an idea of offering resistance to compression, but merely to form a support for the transverse reinforcements. A section of this beam is shown (Fig. 37).

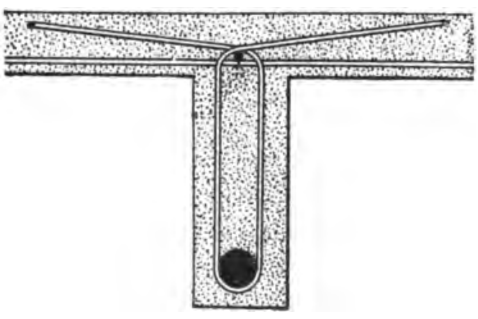


FIG. 37

Reservoirs and aqueducts are also constructed by this firm.

The "Chicago" Fireproof Floor System

This system of fireproof floor construction was brought out by Mr. Theo. Kandeler, of Chicago.

The main beams employed for this type of floor are the ordinary rolled joist; to these, upright channel irons are attached, forming secondary beams.

The reinforcement for the floor consists of a series of flat steel bars about 5 feet long and 3½ inches wide, set vertically, and spaced 2 feet apart. On the bars are laid 2 × 3 inch bevelled wooden nailing strips, and between the bars sheet iron arches are sprung, having 2½ inches rise. This metal work is fitted together in the shops in sections 5 feet wide and 10 feet long.

In the finished flooring the sections rest on the rolled joists and the channel iron cross-beams as shown (Fig. 38), and are covered with concrete.

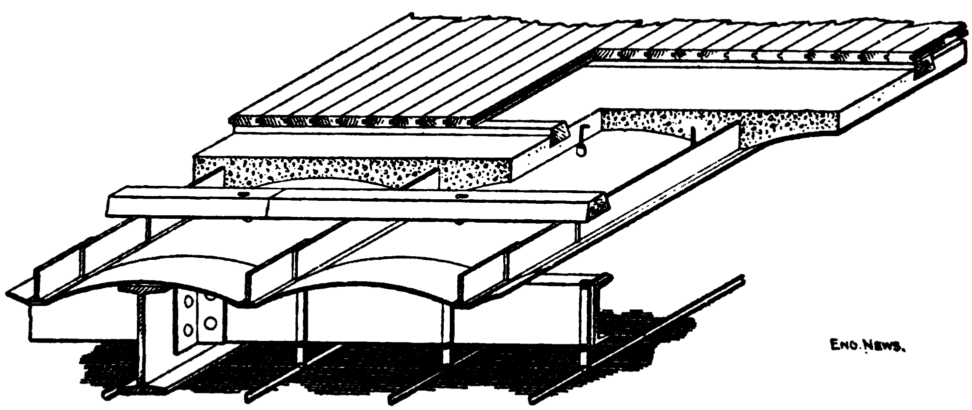


FIG. 38

The ceiling is formed like that of the Roebling system [p. 101] on a wire mesh suspended from the channel irons. The cost of this floor is about 3*s*. per square foot.

Coignet System

M. François Coignet was the first to point out the advantages that would result from the combination of metal and concrete, but he did not put his ideas into practice.

His son, Edmund Coignet, placed before the Société des Ingénieurs Civils, in 1888, his ideas as to the disposition of the two materials, the metal to resist the tension, and the concrete the compression. With M. N de Tedesco he deduced rational methods of calculation based on his experimental studies. These calculations took into consideration the differences in the coefficients of elasticity of the two materials. His communication to the above-mentioned society in March, 1894, "Au calcul des ouvrages en ciment armé avec ossature métallique," was considerably read and formed the point of departure of new studies and theories. Since that date M. Coignet's ideas have been in a great measure confirmed by the studies and experiments of such authorities as MM. Stellet, Lefort, Harel de la Noë, Considère, Résal, Von Empergner, and others.

The *floors* of the Coignet system (20 Rue de Londres, Paris), vary from 2 to 7·87 inches in thickness, and span, either the distance between two walls, or if this is too great, the distance between intermediate beams.

The reinforcement is either double or single.

The single reinforcement consists of longitudinal and transverse rods crossing one another at right angles near the lower surface of the concrete. The double reinforcement consists of two series of crossing-rods, one near the lower and the other near the upper surface of the slab. The upper longitudinals are always of less diameter than the lower ones, and the two are connected by stirrups of round iron.

The transverse rods are of small diameter, and are placed above the longitudinals, both for the upper and lower reinforcements. These are generally spaced 5·63 inches apart, and are about 0·24 inches in diameter. The longitudinals are spaced about 3·94 inches apart. The crossing rods are tied together at every intersection with annealed wire.

The *beams* of this system are considered as of T section, the portion of the floor slab, acting with the beam proper, being taken, on either side, as half the distance between the beams.

M. Coignet always uses a double reinforcement, as he does not consider that there is sufficient guarantee of the proper distribution of the stresses on the width of the table of a T beam or of the perfect connexion between the leg and the table.

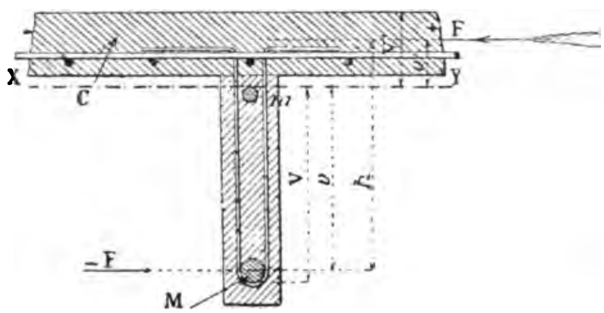


FIG. 39

He uses a section such as shown (Fig. 39) with a lower reinforcement of greater sectional area than the upper, and connected to the floor slab with stirrups of round rods. He does not always use a single rod at the top and bottom, as shown, but frequently has two or more in the width of the beam.

The branches of the stirrups are frequently twisted together over the tops of the upper rods, so as to tie the upper and lower rods together. A transverse reinforcement, as shown (Fig. 40), is also sometimes adopted.

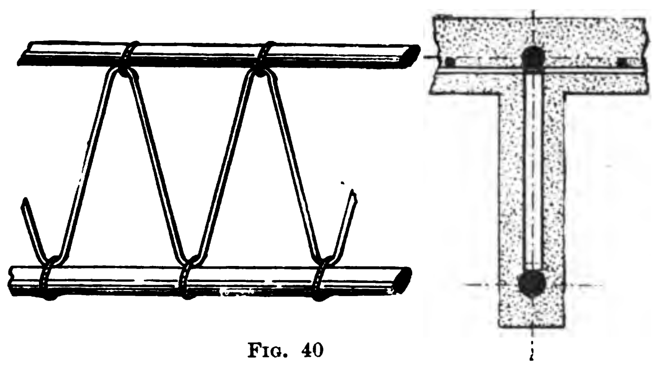


FIG. 40

Hollow floors are formed in several ways. The ceiling slabs are sometimes moulded in advance with vertical wires at certain distances apart, which are moulded into the beams and hold up the ceiling slab; sometimes the reinforcement of the ceiling slab is formed of a network of galvanized iron wire or expanded metal, supported by iron rods, and tied to them by annealed wire. The plaster is put on this slab in place, one man holding up a board near the under surface, while another applies the plaster from above.

A further method of constructing the hollow floors is to use timber for the floor, and mould the ceiling slabs and beams together *in situ*, the rods of the ceiling slabs, running perpendicularly to the beams, being passed over the lower reinforcements of the latter.

Square columns are reinforced with four rods, one in each corner. These are surrounded by wire hoops embracing all the rods, their overlapping ends being wrapped with annealed wire. The hoops are spaced about 7¾ inches apart.

Foundation blocks are reinforced with crossing rods near the lower surface, and have two ribs of reinforced concrete, crossing one another at right angles, projecting from the upper surface.

Walls.—M. Coignet generally constructs his walls of uprights and lintels, forming the bays with reinforced slabs or other filling.

The stairways of this system are formed with stringers reinforced as ordinary beams, the steps being formed on a reinforced slab. The reinforcing rods of the slab running across the stairway are passed under the lower rods, and then over the upper rods of the stringers. They are then bent down almost to the bottom surface, and thus form the stirrups for the stringers and at the same time tie the slab to them.

Retaining walls are formed of a face slab with ribs at the back in the same manner as those shown in the Hennebique system (Fig. 92).

The back edge of the foundation slab is formed like a beam, and the inclined outer rods of the ribs are carried down through this heel. These rods are tied to the vertical rods in the wall slab by stirrups of small round iron.

For the walls of underground reservoirs M. Coignet has patented a very excellent arrangement.

The walls are hollow and are formed of a double series of reinforced slabs arched in plan; one set of the arches being convex towards the ground, and the other set towards the water.

The arches abut against small reinforced counterforts, which have an arched form in cross-section to economize material. The corners of the reservoirs are formed with walls of much greater thickness than the arched slabs, and act as abutments.

The pipes constructed on the Coignet system are reinforced as shown (Fig. 41).

Elevated circular reservoirs are reinforced in much the same way as pipes, the hoops being spaced farther apart as the pressure diminishes. If they become too

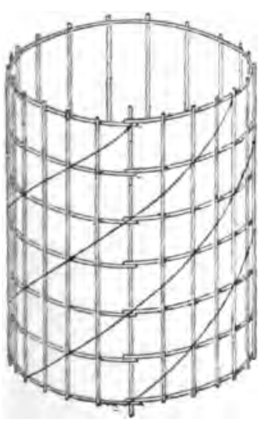


Fig. 41

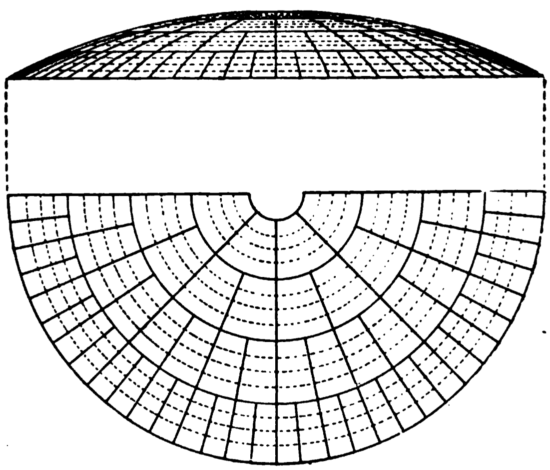


Fig. 42

near together towards the bottom, the sectional area is increased and also the spacing. The vertical reinforcements are formed of rods 0·24 inches diameter, with a rigid upright, generally formed of a small channel iron, every 6·56 feet. The top and bottom hoops are replaced by flat bars, and these are connected to the channel irons to form a rigid skeleton, on which the rest of the reinforcement is formed. The positions of the hoops are marked on the uprights, and those of the rod uprights are marked on the flat iron hoops at the top and bottom. Fig. 42 shows a spherical cover to a reservoir. The bottoms are frequently also spherical, being convex towards the water.

M. Coignet has also constructed piles reinforced with vertical rods, held by double square hoops, one inside and the other outside; the latter is shrunk on to hold the inner hoop in position.

M. Coignet has constructed many buildings, reservoirs, aqueducts, conduits, arched and straight bridges, etc. Among these may be mentioned—

La Palais d'Électricité at the Paris Exhibition, 1900. Illustrated and described [p. 421].

A Building for the Societé "Aux Classes Laborieuses," 83, Rue du Faubourg-Saint-Martin, Paris, a four-storied building, which has a frontage of 73·8 feet, and a depth of 164 feet.

The Aqueduc d'Achères, consisting of 614 yards of circular section 9·84 feet internal diameter, and 5,687 yards of gallery with an elliptical arch of 17·25 feet span. This gallery is illustrated and described [p. 457].

Grain Silos for M. Abel Leblanc, to contain 6,628 cubic feet, or 5,675 bushels.

M. Coignet also designed a reinforced concrete arched bridge for spanning the Seine, where the Pont Alexandre III has been erected, but this design was passed over in favour of the present steel bridge, as the authorities at that time (1894) had not sufficient confidence in reinforced concrete.

Columbian Fire-Proofing System

The double-cross section recently adopted by M. Bonna is used by the Columbian Fire-proofing Company, and by its employment the vertical flat iron ties connecting the two bars of the ordinary cross form are eliminated. This firm uses rolled joists embedded in concrete for beams, the double-cross floor reinforcements being held in place by flat iron inverted stirrups, placed over the top flanges of the joists. The vertical legs of the stirrups are slotted out in the shape of the double cross-sections, and the floor bars housed in the slotted holes.

The bottom flanges of the rolled joists are specially protected against fire by troughs of concrete, in the sides of which pieces of hoop iron are embedded, with their ends projecting. These are clipped round the flange of the joist, and hold the troughs against the bottom surface. The hollows thus formed leave an air space between the concrete and the under side of the flanges.

Cottançin System

This system is one of the oldest, M. Cottançin [47, Boulevard Diderot, Paris] having taken out his patents in 1889. Since that time it has been employed very largely both in France and other countries. M. Cottançin considers that the adherence between the metal and concrete is an entirely unreliable quality which should be totally neglected. He is also of the opinion that the tensile reinforcements must, in consequence of the continual vibrations, tend to become loosened by a molecular disintegration of the concrete.

M. Cottançin was the first to appreciate the advantage that would accrue from the reinforcement of concrete by hooping. As early as 1889 he placed before himself the problem of so enclosing the small particles of concrete, that this material should always be subjected to compression, while the metal acted constantly in tension. His idea was to produce a perfect and continuous metallic mesh with invariable connexions. He further attempted to annul all tensile stresses in the concrete while neglecting always its adherence to the metal.

His reinforcements consist of a woven network, which enchains small particles of concrete so that they can only act in compression while the network acts in tension. Both materials are compelled in this manner to act at their best advantage, and he considers that he can apply the ordinary formula for metal girders, since he assumes that with this form of reinforcement the varying elastic properties of the two materials may be neglected.

M. Cottançin came to the conclusion that to obtain the best results the thickness of his slabs and beams must be small, since, if a particle of concrete is held firmly, in one plane only, by a network near the centre of its thickness, it can be readily seen that the greater the thickness the more tendency there will be for the concrete between the enclosing wires to disintegrate at its ends by wedge-shaped pieces breaking away under the lateral compression. He, therefore, never exceeds 2·36 inches for the thickness given to his slabs, or ribs, and uses concrete with a high proportion of cement.

If we consider a small prism of concrete imprisoned between the meshes of metal, we see at once that when it tries to expand the tendency is for the mesh to elongate, and consequently to become of less width if we neglect the small elastic deformations.

M. Cottançin considers that with the Monier form of network, which is only tied at the intersection, the meshes will give way to this tendency, and that this is even more probable with the networks for which no ties are used, if the adherence is neglected. To obtain a firm junction at the crossing of the wires, he therefore employs a woven mesh, as shown (Fig. 43).

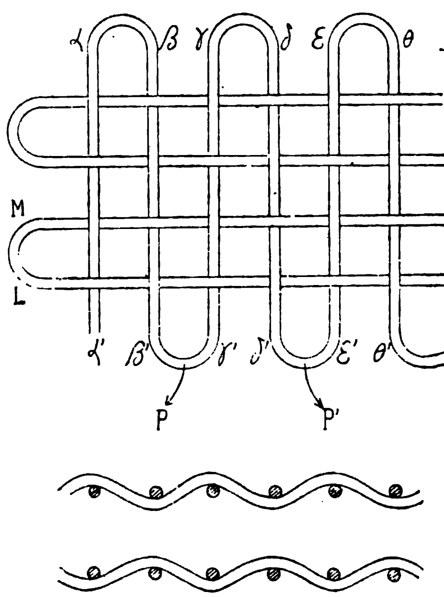


FIG. 43

It would seem at first sight that the weaving would not prevent the sliding of one wire over another, but M. Cottançin points out that with the woven network there is no need for an absolute fastening together of the interlacing wires.

If we consider two neighbouring wires running in the same direction, they are on opposite sides of the crossing wire. Now the forces on the inner sides of each of these tend to force them against the crossing wires, and in opposite directions, but movements in these directions are prevented by the crossing wire itself and the concrete in the neighbouring meshes.

M. Cottançin does not employ slotted sheet metal or "expanded metal" since, though he admits that it would effect the same object as his woven mesh, he believes that the slots have a tendency to become elongated under repeated stresses.

It would appear from the results of experiments that the meshes of "expanded metal" do close up under tensile stress, but this closing up seems to produce some such effect as that claimed by M. Cottançin.

The mesh of the Cottançin network is varied according to the intensity of the stresses on the various portions, and the resistance of the slabs is in this manner made proportional to the imposed stress at every place. Such a disposition would be difficult of attainment with any slotted metal plate or "expanded metal." M. Cottançin claims that in his system it is the metal that imprisons the concrete, whereas in all others it is the concrete that imprisons the metal.

For floor slabs the form of reinforcement generally adopted by M. Cottançin is that shown in Fig. 43, although he has patented several other kinds of network more or less complicated. The floor slabs are always thin, and are generally strengthened at frequent intervals by ribs, called by M. Cottançin "spinal-stiffeners" (*epine-contreforts*). These stiffening ribs enable spans of great width to be constructed, and, though they produce an extremely light effect, have proved themselves to possess ample powers of resistance. The ribs are placed either above or below the slab which they stiffen and support. Their employment enables M. Cottançin to place at each point the necessary quantity of metal for the resistance of the stresses due to external loading, and to add to the moment of inertia at what would otherwise be weak places. The spinal-stiffeners are placed crossing one another, and wonderfully graceful effects are thus obtained, while always so disposing them that they increase the strength at the places which require the most resistance.

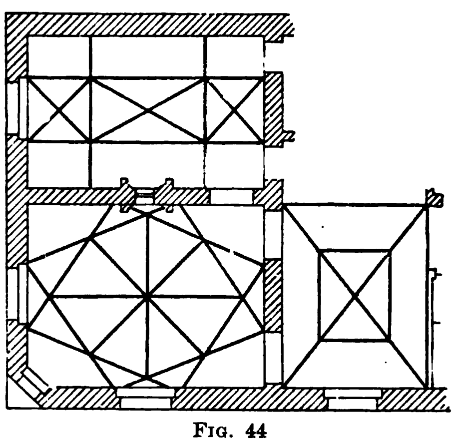


FIG. 44

Fig. 44 shows some methods of placing the ribs for Cottançin floors. A framing rib also follows the perimeter of the walls of each room. The reinforcement of the stiffening ribs is similar to that for the slabs, the continuous woven network being placed vertically, and interwoven with that of the supported slab, by the loops projecting from the rib being threaded through the network of the slab and then through one another before the concrete of the slab is deposited. Where lights are required in the flooring, glass slabs with grooves along the edges are employed, the wires of the woven network passing along the grooves, which are filled in with mortar. Fig. 45 shows a floor during construction and before the slab concrete was deposited.

The ribs on the Cottançin system are considered as N-girders, of which the joints are absolutely fixed, the mesh forming the tension bracing and the concrete the compression bars. In addition to the metal basket work, flat bars are placed longitudinally at the top and bottom, the network being woven round them.

Fig. 46 shows a Cottançin floor slab and rib. The moment of inertia is frequently increased by forming nosed projections on the bottom of the rib, as shown. According to M. Cottançin, it is not necessary to find the position of the neutral axis, since

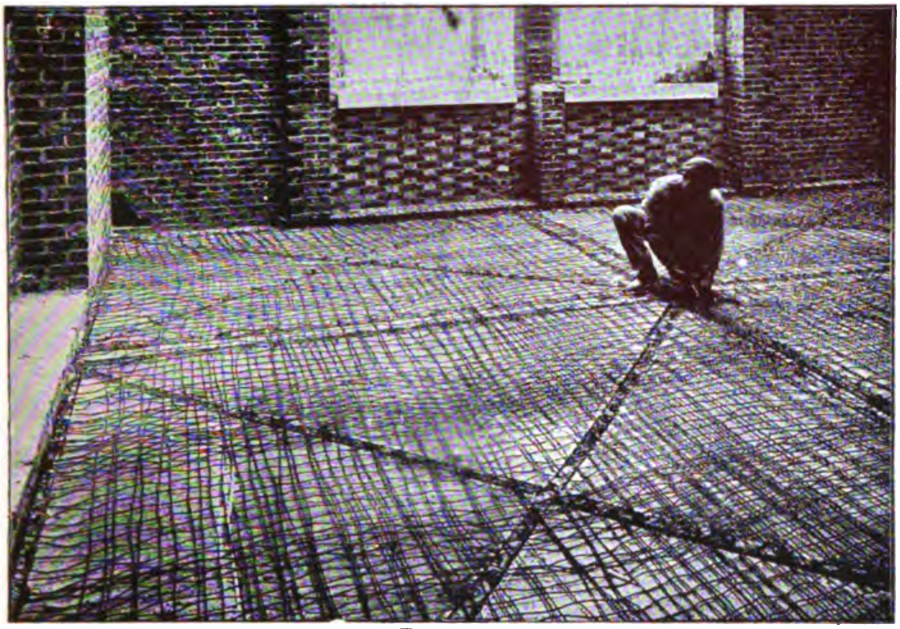


FIG. 45

the concrete acts in compression throughout the whole depth. In reality he considers his ribs as similar to a series of N-girders super-imposed on one another, of which the flanges are represented by the wires parallel to the bottom edge, and claims that its formation may be expected to greatly reduce the elastic deformations. The remarkable strength of the Cottançin ribs is demonstrated by the fact that whereas a rib reinforced with 26·4 pounds of metal will bear a load of 200 pounds per lineal foot with only a powdering of the concrete, the same amount of metal in the form of a rolled joist would fail under a load of 33 pounds per lineal foot, the span in both cases being 14·76 feet.

Many interesting comparative tests have been carried out by various authorities, which show the wonderful resistance for Cottançin floors.

The columns and walls on this system are frequently constructed of cored bricks or tiles, the vertical wires being passed through the holes, and the longitudinal or horizontal wires lying in the bed joints, being interwoven with the vertical wires. The holes are grouted in with cement mortar.

Thin partition walls are built of tiles placed on end having one vertical hole. The upright wires are threaded through the holes in one course and the joints of the next, the cross wires always passing along the joints. For thicker *partitions* or *outer walls*, a cored brick ($8\frac{5}{8} \times 2\frac{3}{8} \times 3\frac{1}{8}$ inches), Fig. 47, is used having four holes $1\frac{3}{16}$ inches square.

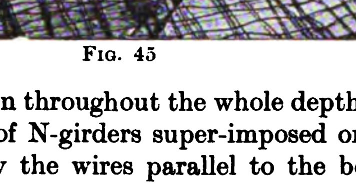


FIG. 46

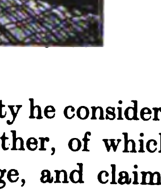


FIG. 47

These are built with the vertical wires passing through the holes, and the horizontal wires along the bed joints.

If the walls have to support considerable loads, they are strengthened with spinal-stiffeners of cored brickwork or concrete. Cellular walls are frequently used externally. These have vertical cored cross walls tying the two main walls together. All these are reinforced with upright and longitudinal wires, those of the cross walls being tied to the main reinforcements. A flat bar of iron is embedded in the bottom and top bed-joints of all walls and also at the levels of the floor slabs, to which the ends of the vertical wires are attached. Similar bars are also placed vertically at the ends of partitions, if the walls against which they abut are not reinforced. The *columns* are usually square, and are built of cored bricks with wire reinforcements.

Roofs are very often formed with intersecting ribs like the floors, but these generally have a slight rise at the centre of the span. The effects that are produced by the intersecting ribs are wonderfully artistic, as can be seen from the illustrations (Figs. 396 and 397.)

M. Cottançin also constructs roofs of the Mansard type, with a double roof slab for the portion with the steep pitch. The upper part has inclined slabs forming the roof, and a horizontal slab which constitutes the ceiling for the room below, these slabs being connected together with lattice ribs of reinforced concrete.

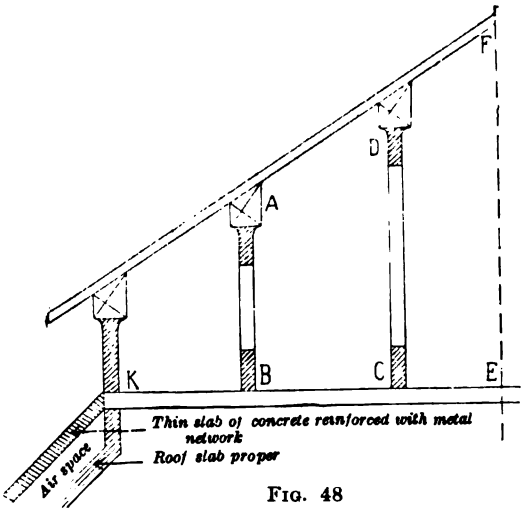


FIG. 48

This method of construction produces a very stiff roof, and also has the great advantage of keeping the attic story at an even temperature by reason of the double roofs. Fig. 48 shows a roof of this type.

Domes of large span are also built by M. Cottançin with slabs and intersecting spinal-stiffeners. Two slabs are usually employed with the spinal-stiffeners between them, thus leaving a hollow space between the two shells, The triangulation of the stiffening ribs offers a great resistance to deformation, and also carries a great portion of the weight directly on to the supports when these are supplied by columns instead of a continuous wall.

The ribs do not cross at the centre of the bays, but at about the level of the ring of greatest horizontal thrust, thus considerably lessening the stresses at this point. Fig. 395 shows a view of a dome.

When a gallery or a similar structure has to be constructed of such forms as shown in Figs. 49 and 50, M. Cottançin frames the opening with a rib following the curvature, and places the spinal-stiffeners tangentially to the curves.

A favourite method of constructing *foundations* of this system, is the formation of box-like cells by intersecting walls of cored brickwork or concrete. These have proved very efficient in soft ground, and also for the foundations for machinery. This disposition of crossing walls with a covering slab forms a series of inverted boxes, in which the earth is compressed, since it cannot spread out laterally, being held by the reinforced walls. The whole foundation consequently acts together as a raft, and is thus able to support heavy loads, although the ground may be very

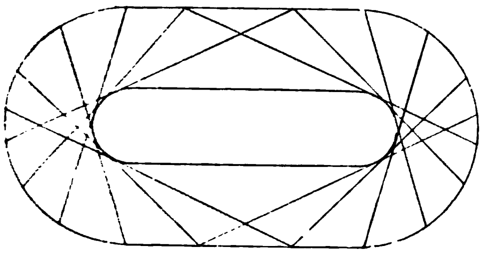


FIG. 49

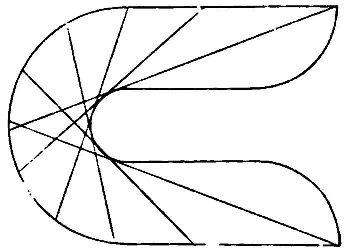


FIG. 50

soft. A further advantage is the equalization of any settlement which may occur, since one portion of the foundation cannot settle without the rest following. This type of foundation is shown in Fig. 51, which is a photograph of the foundations for a 500 H.-P. Cockrell gas-engine at the Paris Exhibition of 1900. The intersecting cored brick walls are arranged in various manners according to the nature of the foundations.

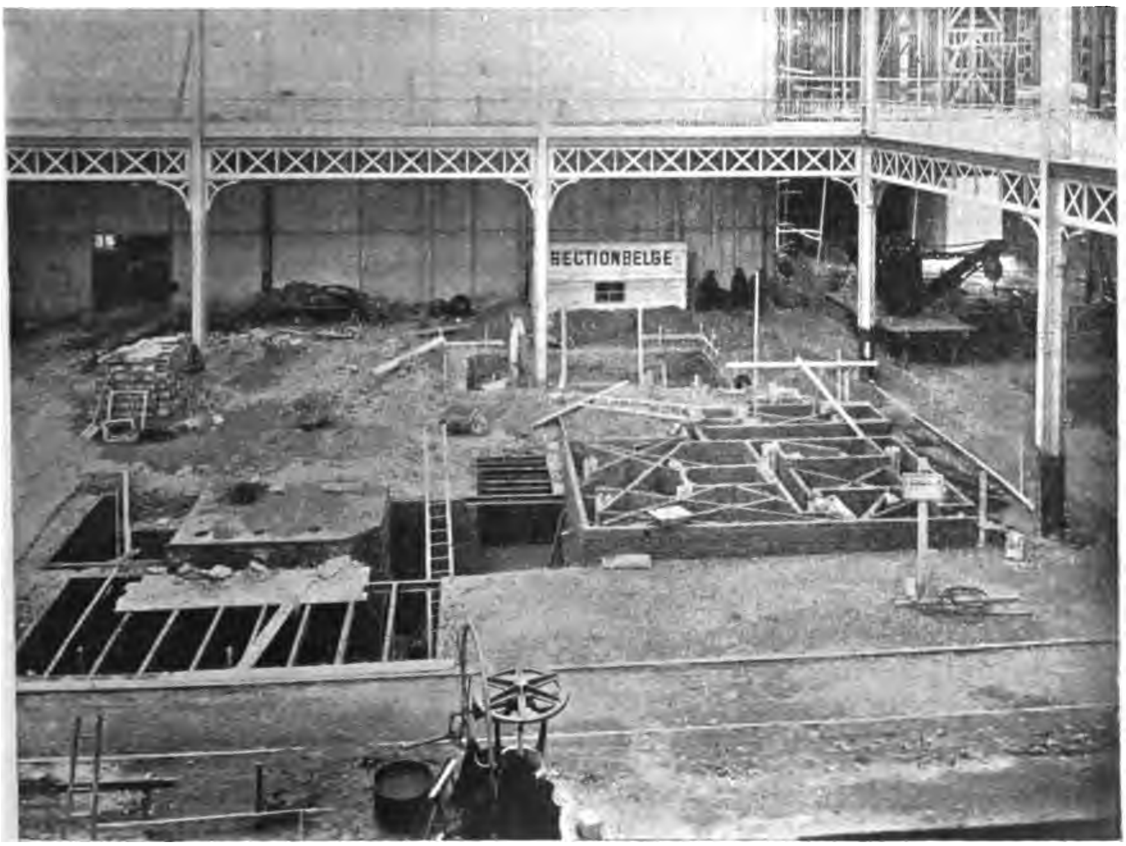


FIG. 51

Pipes constructed by M. Cottançin are reinforced with a woven network. The circular wires are wound spirally, and the longitudinals are woven through, these being bent at the ends and woven through again in the opposite direction, thus forming a continuous basket work tube of the length of a pipe. This reinforcement is placed near the outer surface of the shell, as M. Cottançin considers that being in this position it will cause the concrete to be compressed, since any expansion is resisted by the metal.

M. Cottançin produces perfect impermeability by the use of a very thin sheet of metal, which is not attacked by acids. This is in narrow strips, and is wound spirally on itself so as to form a tube. An insertion of vegetable fibre or wire gauze embedded in an agglutinant is placed between the folds, where they overlap. This tube which prevents any percolation is sheathed in a thin sheet of gutta-percha or a similar material, and protected by canvas. It is then embedded between an inner and outer layer of concrete, both layers being reinforced with a Cottançin network, as shown in Fig. 52.

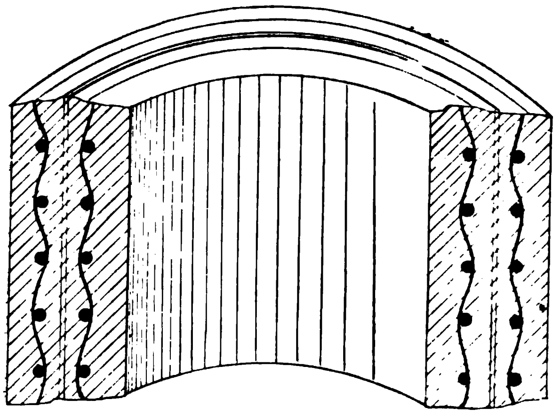


FIG. 52

Rectangular reservoirs are constructed of walls of concrete, reinforced with a woven mesh, and strengthened by vertical and horizontal stiffening-ribs. When the depth is small only one set of horizontal stiffening-ribs, passing round the perimeter, are employed; these are placed near the bottom and tie in the vertical ribs; further ribs are carried across the floor slab from the feet of the vertical ribs. The coverings are often formed of a groined slab with intersecting arched ribs of cored brickwork or concrete supported on columns. A view of such a covering is shown Fig. 411.

Elevated circular reservoirs have usually slightly dished bottoms. The supports are placed at equal distances round the circumference, and the stiffening-ribs of the bottom are placed in three series, crossing one another at the centre, being parallel to the sides of an equilateral triangle inscribed in the circle. That is—three ribs cross one another at the centre, making angles of 60°; their ends are joined by a series of ribs forming a regular hexagon inscribed in the circle, the spinal-stiffeners thus forming six triangular bays.

The English representative for this system is Mr. A. Vye-Parminter, architect, 27, Avenue des Acacias, Paris, who is also part owner of the English patents and is frequently in this country In America M. Cottançin has formed the American Cottançin Construction Company [332, East 35th Street, New York] to exploit his patents.

Coularou System

The method of *beam* reinforcement adopted in this system is shown in Fig. 53. The bottom rods extend throughout the whole length, while those along the top remain parallel to the upper surface till approaching the centre of the span. They are then bent down at an angle of 45°, and are hooked round the lower reinforcing rods. Inclined transverse rods are also used, as shown in Fig. 53. They are hooked round the main rods in the manner indicated in Fig. 54. The floor rods pass under the upper reinforcements of the beams.

Fig. 55 shows the form of *hollow floor*. The beams are fairly close together;

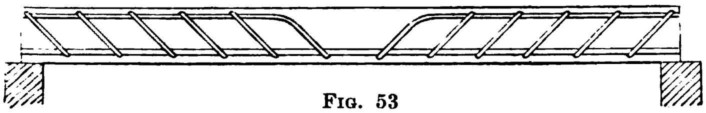


FIG. 53

and all the parts are moulded *in situ*. The space between the floor and ceiling slabs is filled in with cinder concrete, or the floor is moulded on hollow concrete cores, which are left in.

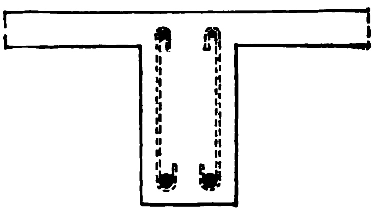


FIG. 54

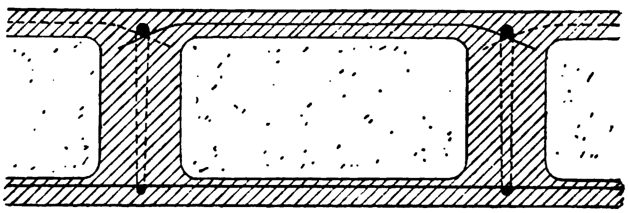


FIG. 55

Floor slabs are reinforced with rods crossing one another at right angles, and the concrete is mixed in the proportions of 673 pounds of cement per cubic yard of gravel passing a 0·20 screen, the fine sand being excluded.

Stairs on this system are formed with stringers reinforced in the same manner as the beams. The slabs carrying the steps are reinforced as floors, with rods crossing at right angles. The steps are not reinforced.

Square columns are reinforced with four vertical rods held together by horizontal rods hooked like the transverse beam reinforcements. These are spaced at equal distances apart.

Walls and roofs are reinforced in the same manner as the floor slabs.

Dégon System

M. Dégon had been employed for some years carrying out works in reinforced concrete, prior to bringing out his own system, and had in consequence all the advantages of practical experience.

His experience led him to believe that insufficient resistance was given by hoop-iron stirrups acting entirely by themselves. He considers that there should be a complete tying together of the various elements of the skeleton, and that with the ordinary stirrup it is very difficult to be sure of proper contact with the enveloped bar, and in consequence the proper tie to the compressive portion of the beam is unassured. He further believes that if a void is formed between the bar and the stirrup, it is practically impossible to fill it with concrete.

M. Dégon also considers that it is wise to provide against premature striking of the falsework, which, if the rods are not thoroughly tied together, may cause abnormal settlements. The reinforcement employed in the *beams* of this system is consequently of the double type, composed of two or more sets of round rods, the upper ones being generally $\frac{7}{10}$ the sectional area of those at the bottom. The bottom rods are bent up at the ends, so that those at the top may be hooked round them, as shown in Fig. 56. The transverse reinforcements are bent in several different forms, the most simple being a rectangle with a wavy bottom member, the lower reinforcements resting in the depressions.

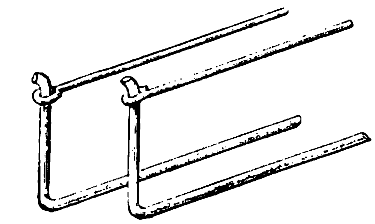


FIG. 56

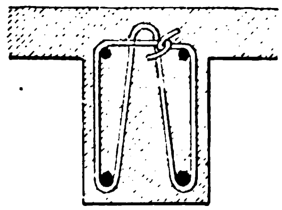


FIG. 57

Another form is shown in Fig. 57, embracing two bottom and two top rods ; a third top rod is sometimes added, passing through the top loop.

If the rods used for the transverse reinforcement are short, their ends are hooked together in the manner shown in Fig. 57 ; but if they are long, one end is hooked round a longitudinal rod, and the other extremity is prolonged into the floor, being connected by a hook to the slab reinforcement of the adjoining bay, as

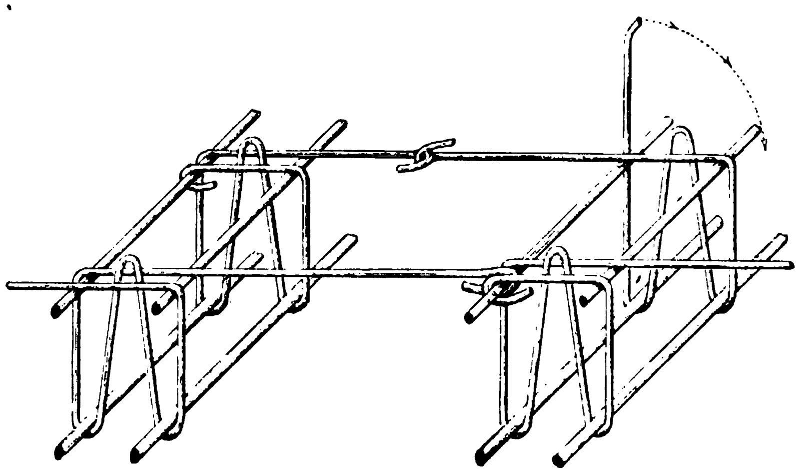


FIG. 58

shown (Fig. 58). When the rectangular transverse reinforcement is employed, a further rod is placed longitudinally between the main reinforcements and bent in snake-like curves, the bottom folds passing under the bottom of the transverse rectangular reinforcements, and the upper folds extending into the upper surface of the concrete.

The *floors* of this system are reinforced with rods running across the beams, with snake-like transverse rods, as shown (Figs. 59 and 60). Fig. 59 also shows a further form of transverse beam reinforcement.

The *columns* are reinforced with four longitudinal rods held together by star-shaped wire ties, the ends of which are turned down to obtain a firm hold in the concrete, as shown (Fig. 61).

Walls are reinforced in different ways, according to the direction of the stresses. When the wall acts entirely under direct compression, it is reinforced by central vertical rods and two series of snake-like longitudinal reinforcements, the alternate ones being placed half of the distance between the loops to the side of that below, the loops clasping the upright rods alternately on either side.

When the pressure acts only on one side, the vertical rods are placed at this

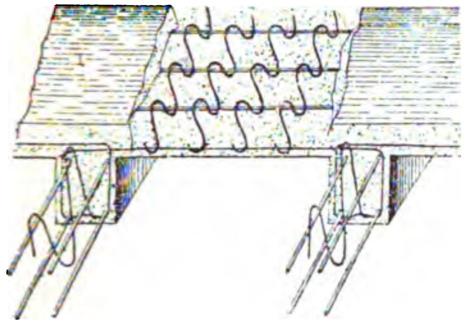


FIG. 59

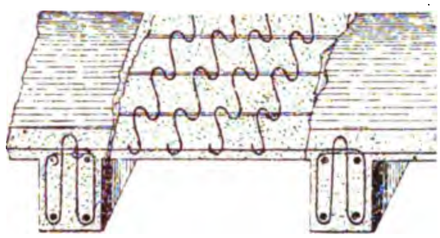


FIG. 60

side, and only one series of snake-like longitudinal reinforcement is employed the vertical rods bearing against the folds of the bent rods.

When the pressure may be on either side, two series of vertical rods are used, one on either side of the wall. The longitudinal snake-like reinforcement is also in two series, placed alternately in the height of the wall, each series bearing against one set of vertical rods. This type is shown in Fig. 62.

Another type of wall reinforcement consists of three series of longitudinal snake-like reinforcements only, placed alternately in the height of the wall, and each being set half a loop to the side of the one below.

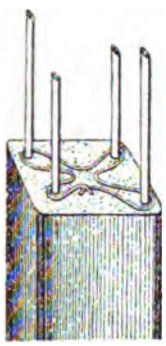


FIG. 61

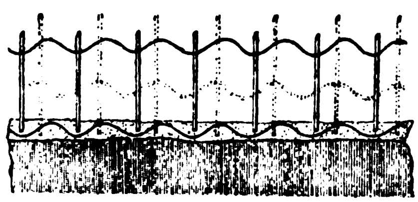


FIG. 62

The *pipes* of this system are reinforced in the same manner as walls with longitudinal rods, either placed near the outer surface or at the centre of the shell, the circular reinforcement consisting of hoops with snake-like folds, in one series or in two series placed alternately in the length of the pipe. When the longitudinal rods are near the outer surface, and either one or two series of circular reinforcements are employed, the longitudinal rods bear against the outer folds of the hoops; but, with central longitudinals, the hoops are so placed that the loops of one series bear against the outer side of the longitudinals, and those of the other series against the inner side.

There is a great advantage in this system, in that all the several reinforcements are thoroughly tied together, so that they will offer a great resistance in themselves without the aid of the concrete.

Among the works carried out by M. Dégon on his system may be cited—
Floors and Silo for M. Pagnier at Buironfosse.
Floor of a Bakehouse at Amiens.
Covered Silo at Momignies (Nord).
A Starch Manufactory at Marcoing (Nord).

Demay System

The system devised by MM. Demay frères, of Rheims, is peculiar in the employment of flat bars for the purposes of main reinforcements, and in the very thorough manner in which all the reinforcements are secured together. A double reinforcement is used for *beams*, the upper and lower bars being connected by 0·04 × 0·40 inches flat transverse reinforcements, which are passed round the main bars and secured to them at their terminations, as shown (Fig. 63). The upper bars are

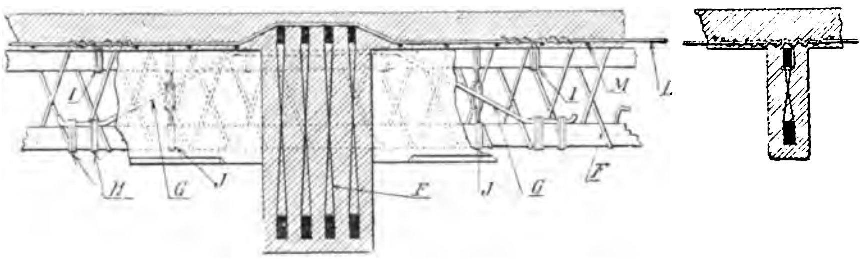


Fig. 63

generally half the sectional area of the lower bars, but a symmetrical reinforcement is sometimes employed. Fig. 63 shows their ordinary *floor* construction with main and secondary beams. The rods G (Fig. 63) are inserted to resist the shearing near the supports, and are tied to the main reinforcements with a wire wrapping, as shown.

The main reinforcements of the consecutive secondary beams are tied together in the width of the main beams, and their upper bars tied to the similar bars of the main beams. The upper reinforcements of the main and secondary beams are wrapped with pieces of annealed wire I (Fig. 63) at intervals of about 12 inches. The ends of these wires project beyond the top of the beam, and are tied to the floor rods before the concrete for the slab is deposited.

To keep the main bars of the beams in position while the concrete is being deposited, the upper and lower reinforcements are tied together at intervals of about 2·62 to 3·28 feet by rods of 0·24 inches diameter, J (Fig. 63).

The floor reinforcement consists of a network of round rods, the mesh being usually 3·94 × 7·88 inches, the rods M (Fig. 63) being 3·94 inches apart; these are tied to the upper reinforcements of the beams, as described above, and are calculated to resist the maximum bending moment. The rods L (Fig. 63) are 7·88 inches apart, and from 0·197 to 0·236 inches diameter.

The two series of rods are fastened together with wire ties at every other intersection in both directions.

It will be seen that all the reinforcements are very thoroughly tied together.

MM. Demay Frères have also constructed floors in the manner shown (Fig. 64) for the cellars of a brewery at La Capelle, thereby forming isolated areas.

Columns.—The columns constructed on this system are reinforced with round rods, which are carried up into the beams, and one of them is carried through the floor and attached to a rod of the column above. The reinforcements are carried

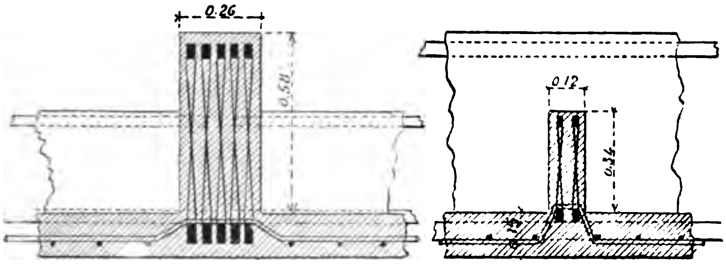


FIG. 64

down into the foundation blocks, and bent out at right angles. In the height of the column the vertical rods are tied together by a hooping of thin flat or hoop iron.

Walls.—The reinforcement for walls is also formed of round rods, after the manner of a Monier reinforcement. Where counterforts or strengthening ribs are employed for retaining walls similar to those shown (Fig. 92), flat reinforcements are used. One bar is placed vertically in the wall, and another inclined near the outer edge of the strengthening rib; these two reinforcing bars are tied together by a diagonal bracing of thin rods.

MM. Demay Frère have constructed buildings, reservoirs and silos, amongst which may be mentioned—

- Grain Warehouses at Odessa (Russia).
- A Brewery at La Capelle.
- A Reservoir containing 221,000 gallons for MM. Vve. Pommery et Cie, at Rheims.
- A Circular Covered Elevated Reservoir at Arcis-en-Brie, containing 22,100 gallons, having an internal diameter of 16·24 feet.

Donath System

The *beams* for the floors constructed by Jul Donath and Co., of 115 Linienstrasse, Berlin, are formed of rolled joists. There are several different methods of forming

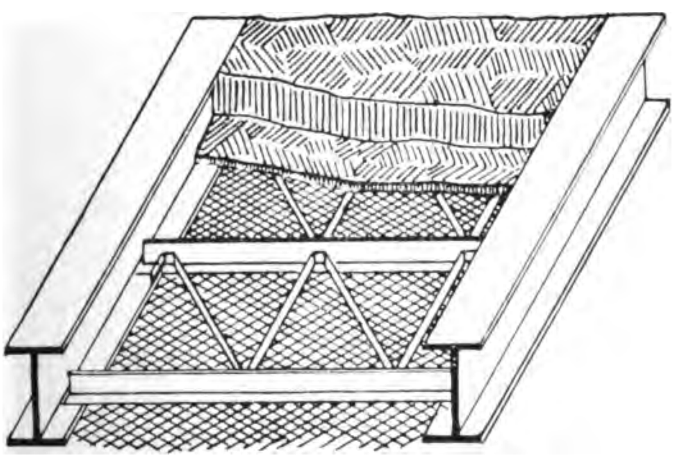


FIG. 65

floor slabs. The method, shown in Fig. 65, consists of laying transversely between the joists, T or I-irons 0·79 to 1·18 inches deep, spaced up to 8 inches apart, and tied together by hoop iron as shown. To the bottom of this skeleton a small mesh network of wire is attached, which receives the concrete, no timber centring being required.

Messrs. Donath sometimes construct floors of concrete, in which flat bars or pieces of sheet iron bent to the form of an S are embedded, running across from joist to joist, as shown in Fig. 66. Another type of flooring used by this firm is

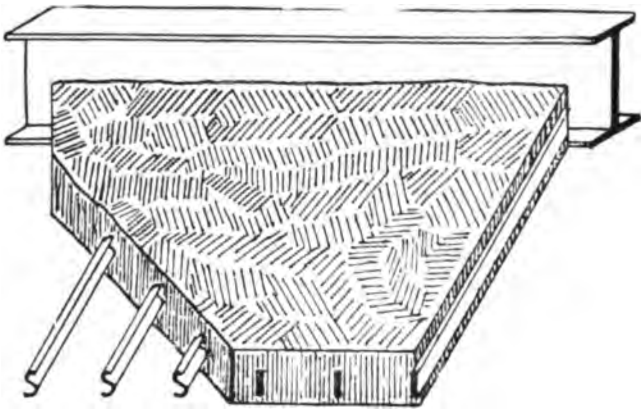


FIG. 66

formed of special blocks, about 4 inches thick, moulded in advance. These fit into one another in such a manner that the floor is self-supporting. The blocks are not reinforced, but iron rods or the sheet iron S-sections are placed in the joints between neighbouring blocks (Fig. 67), either all or alternate joints being thus reinforced.

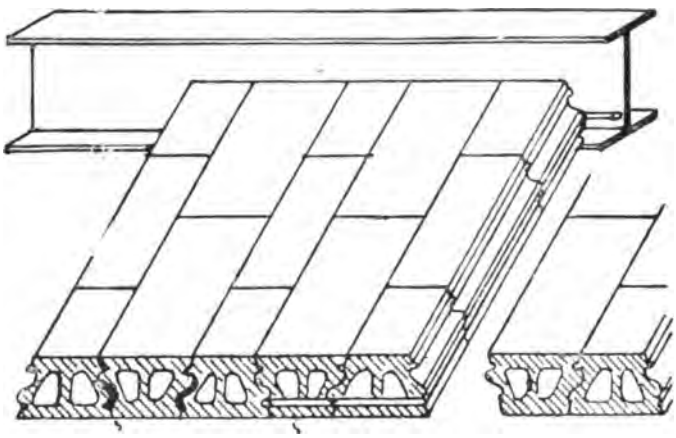


FIG. 67

The S-sections are specially recommended, as they have a tendency to close up when the floor is in use, and obtain a firmer hold on the blocks. Rods may also be placed in a transverse direction, if necessary.

With this form of floor, no centring is required, and great resistance is obtained. The mortar of the joints forms a protection to the embedded reinforcements.

Self-supporting *partition walls*, from 16 to 20 feet long, are also constructed by Messrs. Donath of their hollow moulded blocks, 4 inches thick, reinforced along the three bottom joints with iron rods, similar reinforcements being used over door or other openings. Walls between 20 or 33 feet long and up to 15 feet in height are constructed in the same manner, but have inclined ties of round or flat iron embedded in them, in addition to the longitudinal reinforcements.

"Expanded Metal" System

The "Expanded Metal" (Fig. 68), invented by Mr. J. F. Golding, and exploited by the New Expanded Metal Company, York Mansion, York Street, S.W. is a most excellent reinforcement for many forms of construction. It is not, however, adapted for the construction of beams, and has not up to the present time been employed as a column reinforcement, although if the joint could be contrived it would lend itself to the reinforcement of hooped columns. For reinforcing floor slabs, and similar pieces, it forms an excellent substitute for the network of rods usually adopted, and the extra initial cost is generally counterbalanced by the saving in labour due to its easy manipulation.

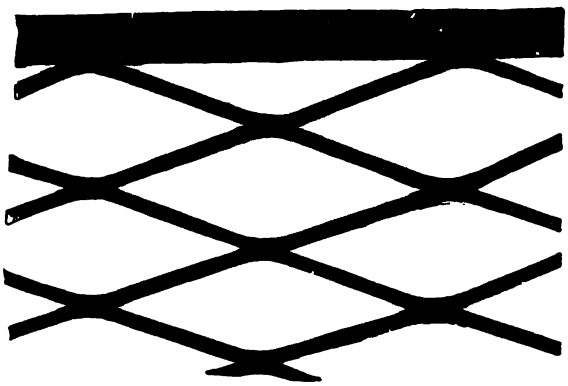


FIG. 68

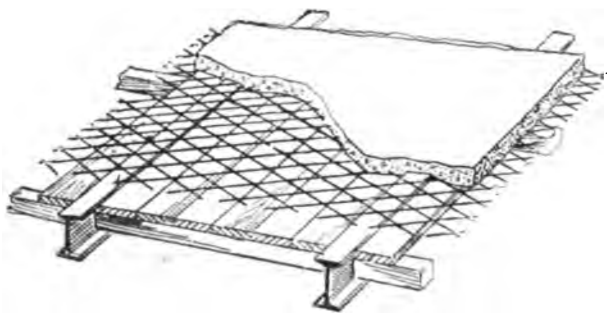
A beneficial action appears to be set up in the slab, due to the tendency of the meshes to close up under bending. It can be easily seen that if the meshes close up by a lengthening of the longer and a shortening of the shorter diagonal, the area within the mesh must become reduced, and consequently the enclosed concrete will be compressed. This is a very valuable property, since, if the sheets are so placed that the greatest tensile stresses act in the direction of the longest diagonal, it causes the concrete in the lower portion to be under compression, instead of tensile stresses, as is the case with most methods of reinforcement. The sheets are made in many different strengths, so that almost any size of piece can be economically reinforced. Where one sheet is not of sufficient length or width, a slight overlapping of the sheets in the concrete gives the necessary continuity, since its form renders any slipping through the mass of concrete practically impossible.

The extreme usefulness of "expanded metal" for reinforcing portions of works constructed of plain concrete, where any special strength is required locally, is generally well known, and no attempt will be made to detail such uses, but its special application to structures formed entirely of reinforced concrete will alone be dealt with.

The Expanded Metal Company will undertake the erection of structures reinforced with this material, and are always ready to give all the necessary information to clients wishing to do the work themselves. They have also devised several typical methods for the formation of floors, walls, stairs, covered reservoirs, etc.

When "expanded metal" is employed as a reinforcement for floors the main beams are usually formed of rolled joists.

The Company recommend that the area of metal compared with that of the concrete forming the slab should be ½ per cent., which would seem about the proper theoretical percentage for an economical use of material [p.305] They further recommend that the concrete should be mixed in the proportions of 1 of Portland cement to 4 or 5 parts of clean aggregate containing 33 per cent. of sand, and all of which will pass a ¾-inch screen.



Expanded Metal Tension Band Coring laid on centering and Concrete Floor partly laid.
FIG. 69

Flat floors.—There are two general types of flat floor construction. Fig. 69 shows the ordinary floor in which the slab rests on the upper flanges of the rolled joists, leaving the joists exposed. They are usually, however, either filled in and surrounded with concrete, or protected by a layer of plaster on a lathing of "expanded metal."

In Fig. 70 the floor slab again rests on the top flanges of the rolled joists, and, in addition, a lathing, on which a plaster ceiling is formed, is suspended from the bottom flanges by means of a series of upright flat bars, to which the lathing is attached by special clips. The bars are suspended from the joists by the arrangement shown in Fig. 71. This figure also shows the clips for attaching the lathing to the bars. The flat floors are adapted for spans up to 12 feet between the beams.

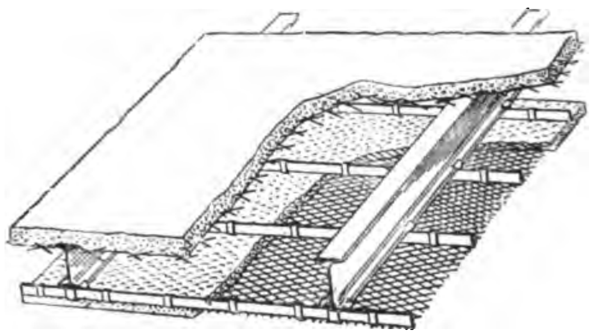


FIG. 70

Floor supported by arched secondary ribs.—This form of floor is a very favourite one, as it produces a very light and economical floor for spans of from 12 to 25 feet between the main beams, and 4 to 8 feet between the arched ribs.

The main beams are formed of rolled joists, the secondary beams being arched ribs of curved steel channel bars, filled in with concrete, which is continued upwards to support the floor slab. The channels are usually 6 inches wide and weigh 12 pounds per lineal foot, and are formed with a rise of $\frac{1}{12}$ the span. It is stated that the total weight of this form of floor need not exceed from 25 to 30 pounds per square foot.

Fig. 72 shows this form of floor at the bottom and the hollow flat floor at the top. If a hollow floor is desired, upright flat bars are attached crossing the arched channel irons and along the bottom flanges of the rolled joists. The "expanded metal" lathing is suspended from these bars, and a plaster ceiling slab is formed following the curve of the channel ribs and passing under the bearing joists.

Fig. 72 also shows the method adopted for the formation of *solid partition walls*. For these steel rods about ¼ inch diameter are placed vertically and attached to the upper and lower flanges of the bottom and top floor joists by special clips

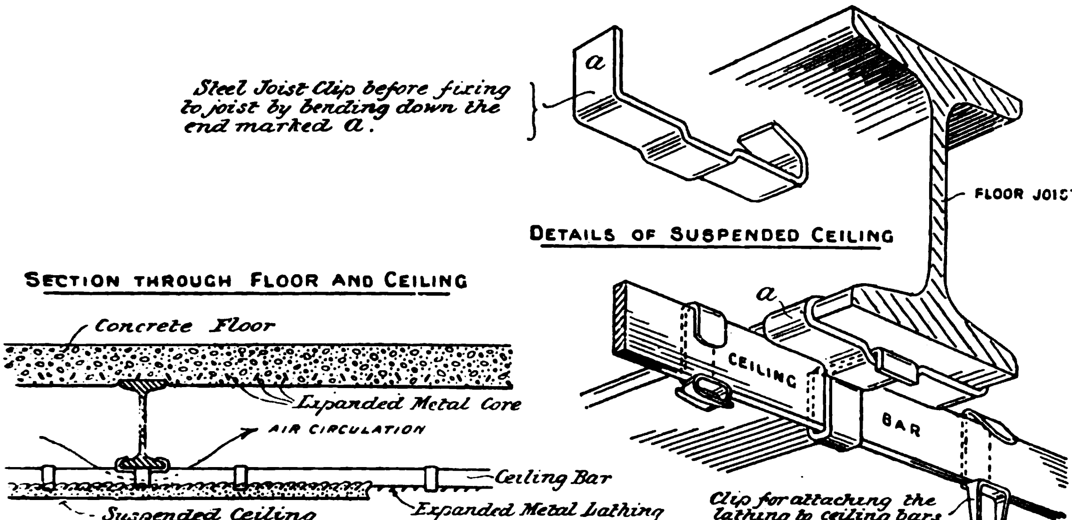


FIG. 71

and hooks and tightening nuts, as shown in Fig. 72. These are spaced about 12 inches apart, and the "expanded metal" lathing is attached to them by wire ties, or sometimes the rods are threaded through the meshes. A plaster slab about 2 inches in thickness is then formed on the lathing in two layers, one on each side, and the partition is complete. When door openings are required, a vertical rod is placed on each side and attached to the door frame with staples before the lathing is placed in position. It is recommended that the lathing should be strutted up temporarily until the plaster on one side is set.

This form of partition only weighs about 18 pounds per square foot, and is said to save about 66 per cent. of space and 50 per cent. of weight compared with a half-brick wall.

The framing employed for a *hollow partition wall* consists of pairs of light angle irons placed vertically and held at the

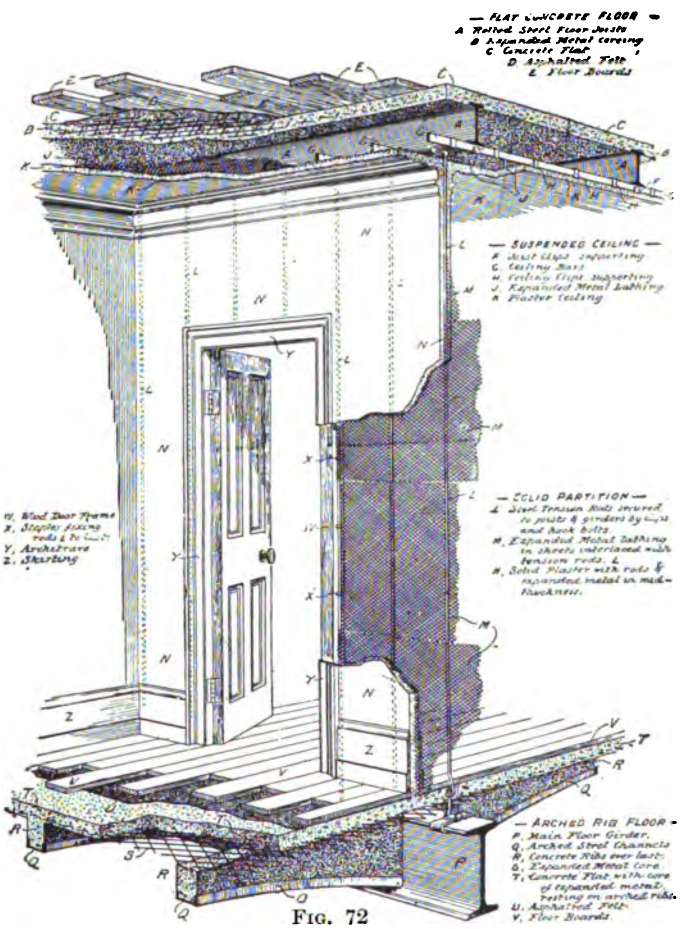


FIG. 72

proper distance apart by a series of distance pieces, those at the top and bottom being formed of angle iron and the intermediates of flat iron. When the partition is placed between two of the upper floor joists, the upper horizontal piece of angle iron bears against the concrete of the top floor, whereas if it is placed under a joist, the top table of this angle is longer than the vertical portion and is clipped round the bottom flange of the joist.

The top bearing angle is always made with slotted bolt holes, so that the length of the upright may be adjusted to bear against both floors. At the foot the angle iron distance pieces rest on the concrete of the bottom floor. Sheets of "expanded metal" lathing are attached to both sides of the framed uprights by special clips, and are covered with plaster. The door frames are secured to the angle irons by coach screws.

The method of forming *external double walls* is really a form of protected steel framing, the steel skeleton being built up of rolled joists in the ordinary way. To this framing, rods of about ¼-inch diameter are attached, one end being simply hooked over the flanges, and the other fastened by special clips, so that they may be tightened up by means of nuts. The rods are attached on both sides, and spaced about 12 inches apart. The "expanded metal" lathing is secured to the rods by wire ties, and plaster applied, forming a thin layer on each side of the framing.

Columns are formed of hollow circular cast-iron sections, or any form of rolled section may also be used, joists and the cross section being those most frequently adopted. These are covered with a lathing of "expanded metal," on which a protective covering of plaster is applied, the space between the

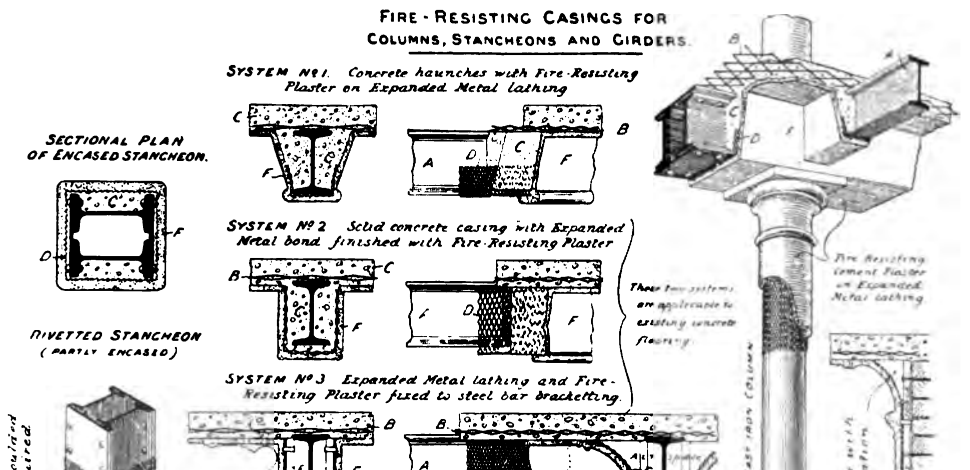


FIG. 73

lathing and the metal being generally filled in with concrete. The beams are protected in the same manner, with "expanded metal" lathing attached to flat bars on edge, which are fastened on to the joists. Sometimes the flat bars are dispensed with, and the joists are completely surrounded with concrete, over which a layer of plaster is formed on a lathing of "expanded metal."

Several types of column and girder protective coverings are shown (Fig. 73).

Stairways and the tiers for theatre seats are formed as shown (Fig. 74). Rolled joists are used as stringers, on these stepped bearers of bent flat iron are riveted.

The risers, which are formed of light built-up Z-girders, with T-iron stiffeners, rest on the bearers. The treads are constructed of flat bars on edge placed 12 inches apart, with a quarter turn at each end to form a bearing on the angle irons of the risers.

To both these skeletons "expanded metal" sheets are attached, on which the concrete and plaster slabs are formed.

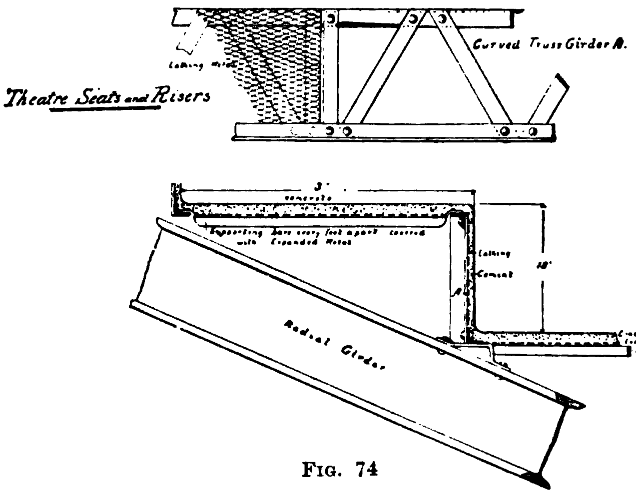


FIG. 74

Fig. 74 shows views of the circle seats and risers of the Royal York Theatre of Varieties, Southampton. In this case the slabs for the seats were made $2\frac{1}{2}$ inches thick, while the thickness of the plaster covering on the lathing for the risers was $1\frac{1}{2}$ inches.

"Expanded metal" is a very convenient reinforcement for domes and similar coverings. It is also frequently used as a lathing for ornamental plaster work.

Both *circular and rectangular tanks and reservoirs* have been built of concrete reinforced with "expanded metal."

When rectangular or of large diameter, vertical uprights of section iron are employed for the main reinforcement of the walls, "expanded metal" sheets being secured to them.

The roofs are frequently supported on arched ribs of channel iron in a similar way to the arched floors already described.

The channels spring from the flanges of rolled joists embedded in the walls at the springings. Concrete ribs are carried up from the channel irons, having a flat extrados, on which the roof slab rests, this slab being reinforced in the usual way with "expanded metal" sheets. To lighten the concrete ribs, a series of circular holes are frequently formed in them. The channels are tied together transversely by angle irons attached to flat projecting bars riveted to their webs.

Spans of 100 feet or more can be constructed on this method. Small spans may be executed in the same manner as already described for flat or arched floors.

Habrich or Thomas and Steinhoff System

This system was brought out by Herr Habrich, and is constructed by the firms of Thomas and Steinhoff, of Mülheim-s.-Ruhr, and Aug. Potthoff, of Münster (Westphalia).

The reinforcements consist of twisted flat bars and rolled joists. Several types of *flat floors* are employed for spans up to 6·56 feet.

Fig. 75 shows the method adopted when the supporting joists are visible. This type is constructed for spans between 3·28 and 6·56 feet.

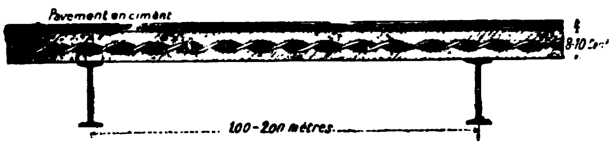


FIG. 75

Fig. 76 is a method used when the joists are partially embedded in the floor slab, and are filled in with concrete above the bottom flange.

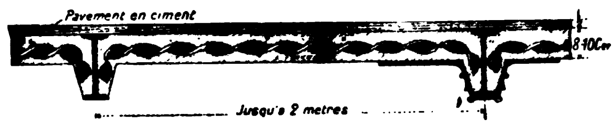


FIG. 76

Another similar floor has the under surface of the concrete curved, where it fills in the bottom of the bearing joists. A plaster ceiling is formed following the underside of the floor slabs and covering the bottom flanges of the joists.

Fig. 77 shows a floor where the slab rests on the lower flanges of the joists, a

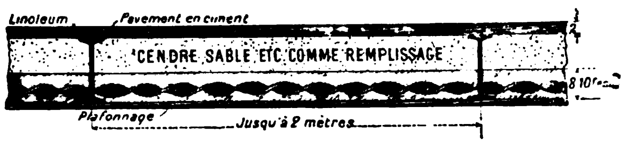


FIG. 77

filling of cinder concrete being formed between the top of the slab and the floor paving. If boards are used for the floor, nailing strips are embedded in the cinder concrete filling.

A form of floor more seldom used is that shown in Fig. 78. Here the floor is reinforced with rolled joists, the twisted bars being used to tie the concrete together.

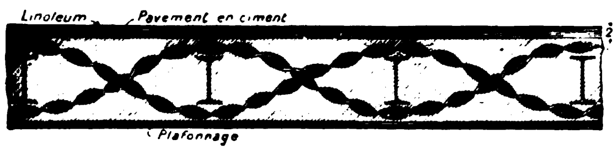


FIG. 78

Flat floors of the types represented in Figs. 75 to 77 are not used for spans greater than 6·56 feet or to support greater loads than 100 pounds per square foot. When greater spans up to 16·4 feet are required an *arched floor* slab is used, as shown (Fig. 79), with a rise of $\frac{1}{12}$ the span. For a 16·4 feet span the thicknesses of the arch at the crown and springings are 3·94 and 5·91 to 6·30 inches respectively.

The flat bars used in this system are 1·38 × 0·063 inches, or 1·57 × 0·059 inches; for flat floors these are spaced from 1·10 to 0·47 feet apart, and for arched floors the spacing varies from 0·82 to 0·55 feet.

The concrete is usually mixed in the proportions of 1 of cement to 8 or 9 of sand and shingle or cinders; sometimes 1 of cement to 3 of sand and 4 of broken stone is used.

Arched roofs are also constructed in this system, the arch springing from

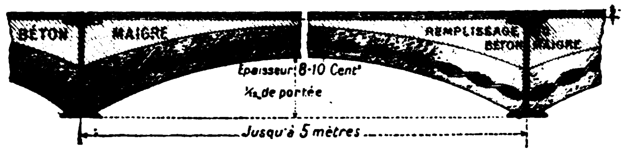


FIG. 79

channel irons laid on the top of the walls, and tied together from wall to wall with iron tie-bars of double angles. Vertical tie-rods are also placed at various intervals across the span, being embedded in the concrete at their upper ends. *Domes* have also been constructed, of which a ring at the bottom and a portion at the top are solid, the remaining portion being constructed of open bays with ribs of reinforced concrete between.

Hennebique System

M. Hennebique constructed floors of reinforced concrete in 1879, and has been connected with this form of construction since that date.

His system, however, was not brought out until 1892, when he became almost exclusively a constructor in this material. M. Hennebique was one of the first to introduce the reinforced concrete beam, and has been frequently represented as its original inventor, although Coignet and Cottançin in France, Möller in Germany, and Ransome in America introduced this method of construction at about the same time.

Floors.—The floors on this system are formed in several different ways, that most commonly employed being the flat single floor with exposed beams. A general view of this type of floor with its supporting beams and columns is very clearly shown (Fig. 80). The floor slab is shown here with only one series of reinforcing rods, but both longitudinal and transverse rods are usually employed.

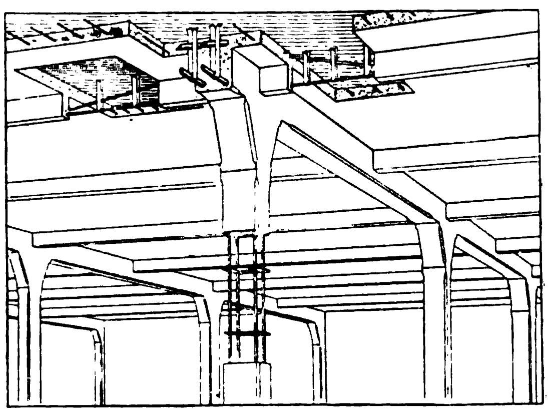


FIG. 80

The floor rods are in two series, one bent up to pass over the support near the upper surface of the slab, and the other set straight throughout and being embedded near the lower surface. The rods are placed alternately, one straight and the next bent up over the supports. The hoop-iron stirrups, which are a feature of this system, are placed near the supports to resist the shearing stresses. These stirrups are formed as shown (Fig. 81), and pass under the rods, their extremities being slightly bent out and terminating near the upper surface of the concrete.

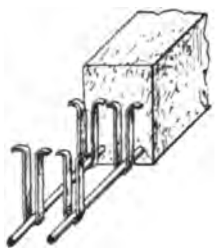


FIG. 81

Fig. 82 shows the general method employed for reinforcing floor slabs.

When the slabs are freely supported, the bent-up rods are omitted.

The beams are reinforced in the same manner as slabs with straight and bent-up rods, but in this case the straight rods are placed near the bottom surface, and the bent rods above them. The ends of the rods are carried over the supports and some distance into the adjoining beam or wall. The main stirrups of the beams are spaced further and further apart from the support towards the centre of the span as the shearing stresses diminish in a

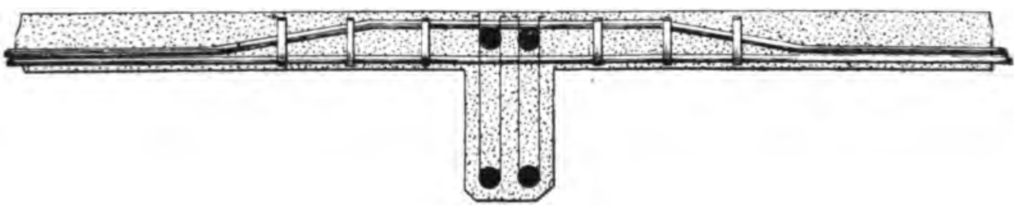


FIG. 82

like manner; a further series of short stirrups are placed over the ends of the bent-up rods to secure them firmly to the concrete.

Fig. 83 shows the general arrangement of the reinforcements of beams.

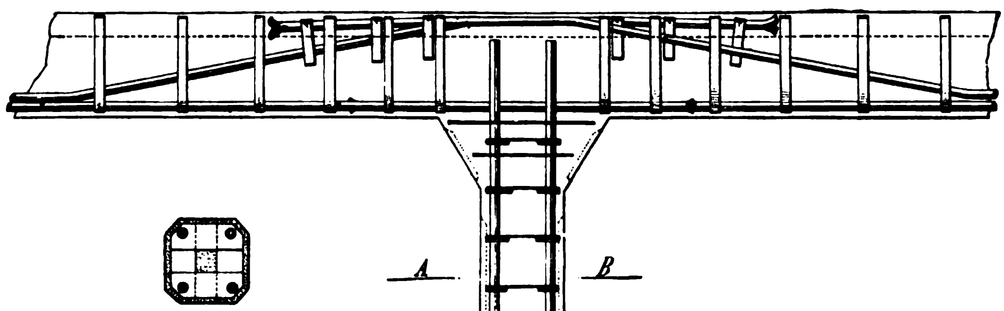


FIG. 83

When the depth of the beam is too small to obtain the necessary compressive resistance from the concrete alone, a further series of straight rods is employed near the upper surface.

The beams and floor slab are monolithic, and in consequence the beams are designed as being of T-section. When these are freely supported the bent-up rods are omitted.

The *column* reinforcement consists of four or more vertical rods tied together by flat plates or wire ties at frequent intervals. The flat plates are very seldom employed at the present time, preference being given to the wire ties.

Fig. 84 shows the general arrangement of the column reinforcement. The rods of the columns are carried up well into the beams, as shown in Fig. 83.

A splay is formed on the tops of the columns to help the beam in resisting the compressive stresses which are induced at the bottom over the supports. Special rods are placed in this portion, either horizontally, as shown in Fig. 83, or inclined parallel to the face of the splay.

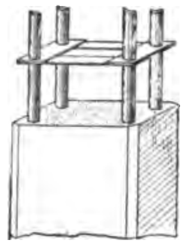


FIG. 84

The bottom of the column rods abut against a plate embedded in the foundation blocks, as shown (Fig. 85). The columns are formed of various cross-sections, and the number of rods is also varied.

The *foundation blocks* are formed so that each side acts as a cantilever, and are generally reinforced with two sets of rods crossing one another, with supporting stirrups as shown (Fig. 85). If the load is great, several series of crossing rods or flat bars are employed.

The *walls* on the Hennebique system, when the pressure may occur on both

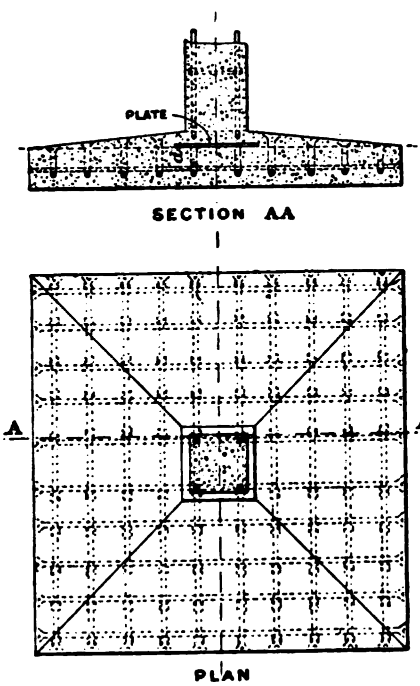


FIG. 85

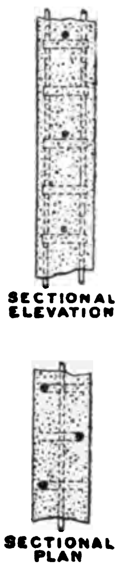


FIG. 86

sides, are reinforced with two series of vertical rods, one near each face, each set being tied to the opposite face with stirrups, longitudinal rods being placed in the centre of the wall. The general method of wall reinforcement is shown (Fig. 86). If the pressure will only be exerted from one side, the vertical rods are only placed near this face.

Flat roofs are constructed in the same manner as floors, but other forms of roof are frequently employed and are reinforced in various ways according to the nature of the stresses. A favourite form of construction for factories, warehouses, and similar buildings is shown (Fig. 87).

The faces of the buildings are formed in bays, which are filled in either with reinforced concrete or with brickwork. The main columns of the face, together with

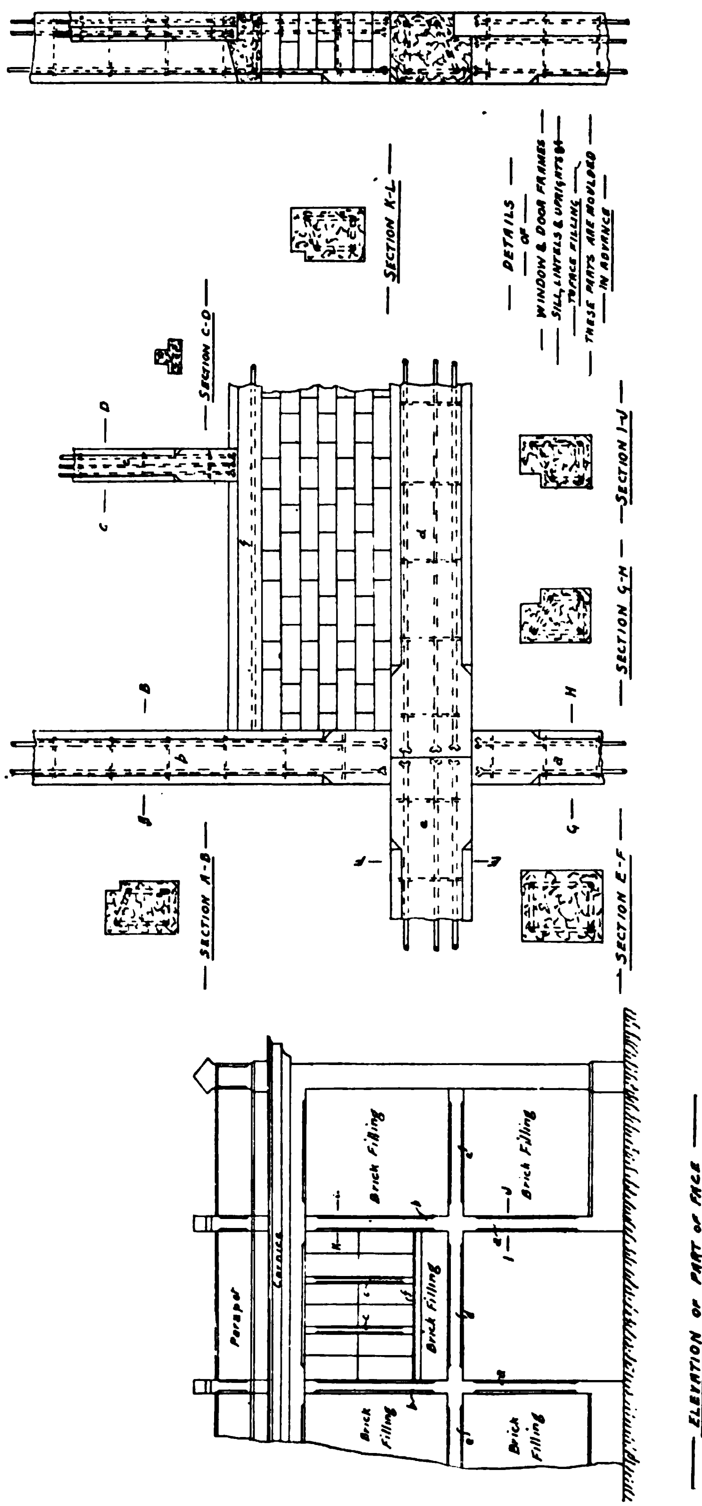
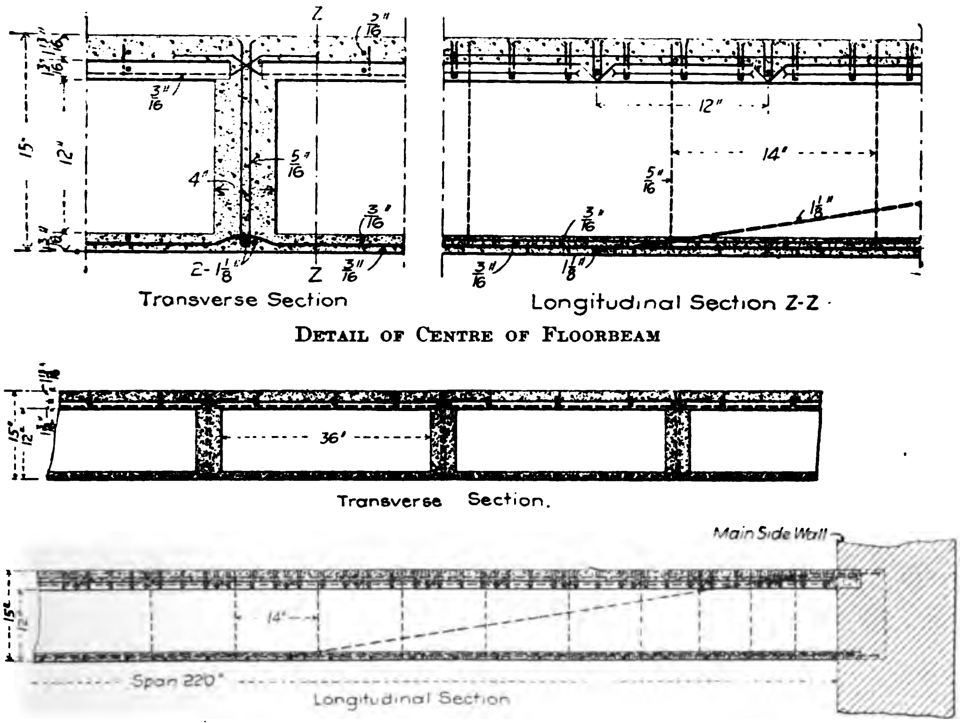

SECTION A-B
SECTION C-D
SECTION K-L
SECTION E-F
SECTION G-H
SECTION I-J
DETAILS
OF
WINDOW & DOOR FRAMES
SILL, LINTELS & UPRIGHTS &
TERRACE FILLING
THESE PARTS ARE MOULDED
IN ADVANCE
Parapet
Cornice
Brick Filling
Brick Filling
Brick Filling
Brick Filling
Brick Filling
ELEVATION OF PART OF FACE

FIG. 87

the floors, roof, and internal columns, are formed in place, but the window and door frames, sills, lintels, and lesser uprights of the façade are moulded in advance and lifted into their proper positions as the work proceeds.

Double floors with hidden beams are also frequently used, the ceiling slab and beams being moulded in place, while the floor slab is generally formed on the ground and lifted into place as the work proceeds. Fig. 88 shows a floor of this type.



Upper Floors.

FIG. 88

Double walls are also used with vertical cross walls at certain intervals, tying the two portions together. The floor slabs in this case are carried through the cavity and are connected to the outer wall.

Stairways are constructed in many ways; a common method is that shown (Fig. 89). The foot and top of the stairs are finished against beams of the landing slabs; the reinforcement consists of bent and straight rods arranged in a similar manner to that described for floor slabs. The stirrups are placed at each step, and tie the reinforcement to the concrete of the step.

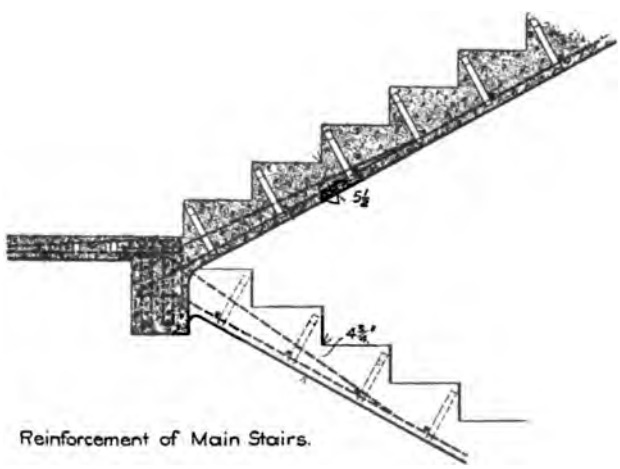


FIG. 89

Overhanging stairs, supported cantilever-wise from the walls, are formed in different ways. In one method the slab carrying the steps is reinforced as a cantilever in the manner shown (Fig. 90), the main rod being tied well back into the wall at its two ends.

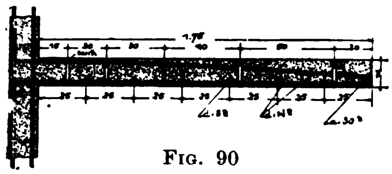
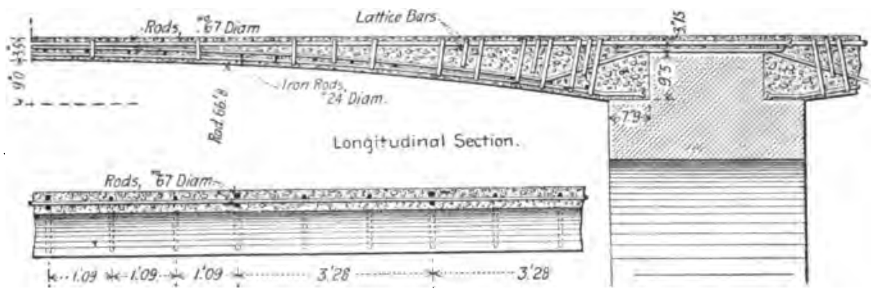


FIG. 90

A longitudinal rod of large diameter is placed near the bottom at the outer edge of the slab, being clasped by the main rods. This acts as a kind of stringer, and is carried some feet into the concrete of the landings. The slab is also reinforced with small longitudinal rods resting on the lower arm of the main reinforcements. Stirrups are placed at increasing intervals apart from the support towards the end.

Another method employed for overhanging stairs is to carry the upper portion of the main cantilever rod into the steps themselves, which makes a much stiffer stairway. In other respects the reinforcement is very much the same as described above.

Arched floors of the type shown (Fig. 91) are sometimes adopted. In this case



Cross Section at Crown.

FIG. 91

there are three series of longitudinal rods, one set being straight and placed near the top surface, another being curved to follow the intrados, while the third follows the intrados through the central portion of the span, but is bent up near the supports, and passes over these in the neighbourhood of the upper surface. All the longitudinal reinforcements are well tied together, and to the opposite face by cross-ties. A series of transverse rods is also employed, being placed just above the lower reinforcement.

The concrete of the arches is sometimes carried over the supports, the bottom rods overlapping in the same manner as those at the top.

Different types of *retaining walls* are shown in Fig. 92. These walls are formed of a thin slab with ribs at the back, both the slabs and ribs being connected to a foundation slab. In the walls shown, there is also a rib in the front at the toes, but this is sometimes omitted.

For very high walls, as that shown (Profile No. 1), a second foundation slab is placed at a higher level than the bottom of the wall, to increase the resistance. The tendency of the wall being to overturn at its toe, the ribs at the back are in tension, the wall being virtually held up by the weight of the earth on the foundation slabs.

The wall slab itself has only to support the thrust of the ground between the ribs, and acts in very much the same manner as a floor slab. The profiles shown in Fig. 92

are those used for the construction of the retaining wall for the Quay Debilly, Paris.

M. Hennebique was the first to introduce reinforced concrete *piles*, commencing to use them in 1896, since which date he has had many imitators.

The solid square pile is formed as shown (Fig. 93), excepting that the tube for a water jet is very seldom employed. The reinforcement is very similar to that of the columns, but the wire cross-ties are placed nearer together. The tops of the rods are generally about 2 inches below the concrete at the head. The toe is formed with an ordinary pile shoe, the straps of which are turned in at the top to form a hold in the concrete. The rods are bent in at the foot, and bear against the bottom of the shoe.

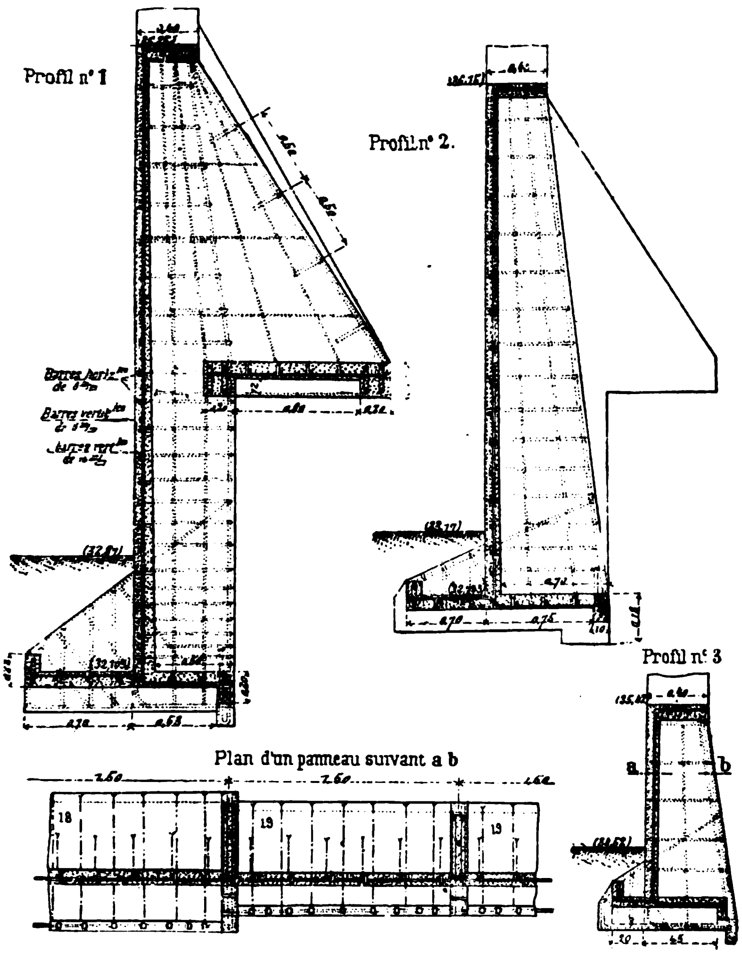


Fig. 92

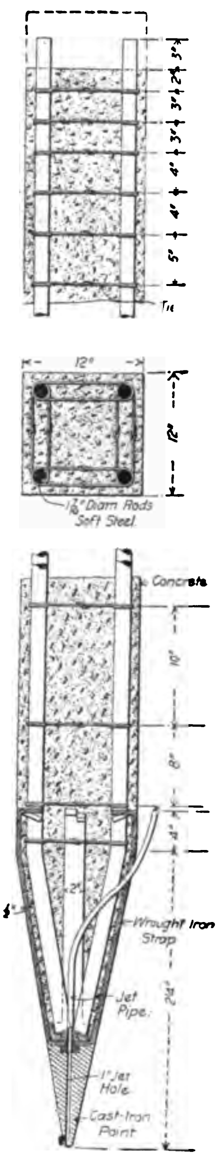


Fig. 93

Sheet piles are constructed in much the same manner, being generally reinforced with six vertical rods tied together, in both directions, in the horizontal planes. A semi-circular chase is formed down the edges of each pile, and the hole formed by these chases when the piles are driven side by side is filled in with grout. The piles can be made of any length, and are also occasionally lengthened by the concrete being broken away for some distance, and fresh rods inserted overlapping the old ones, after which a further length of concrete is added; but this is far from good practice and is very seldom resorted to.

14 × 14-inch piles have been driven with a 2-ton monkey having a drop of six feet, and the driving of 12 × 12-inch piles with a 30-cwt. monkey and a drop of four feet is quite usual. The piles are connected to the capping by having the concrete broken away for some distance down, so that the pile rods may penetrate into the concrete of the capping. A general view showing the manner of constructing retaining walls with pile foundations is shown (Fig. 94).

A hollow pile has lately been introduced, of which a sketch is shown

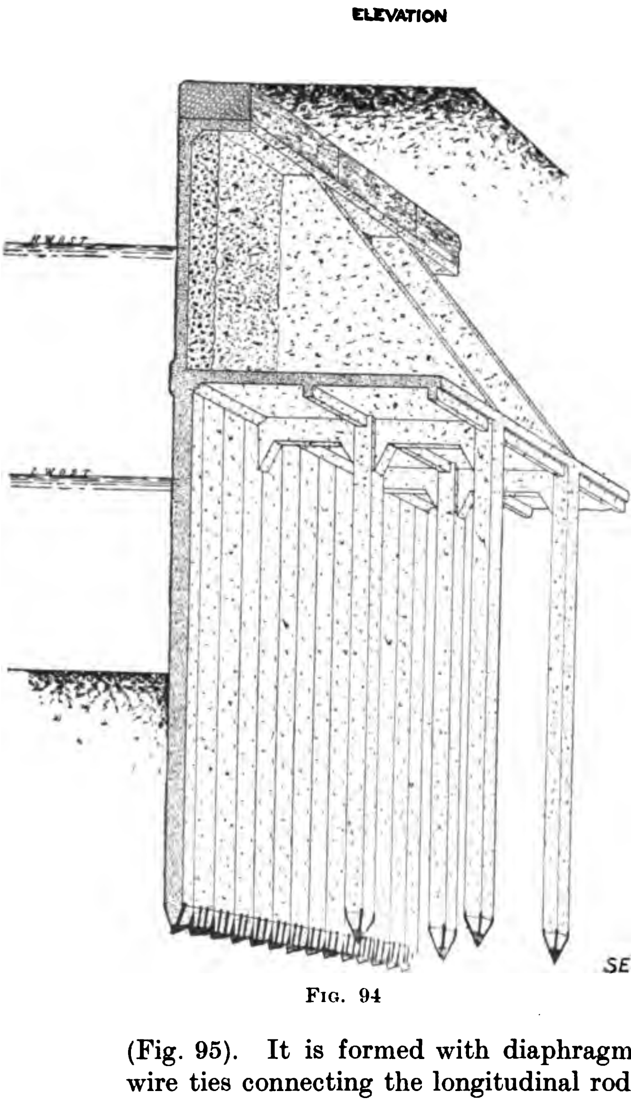


FIG. 94

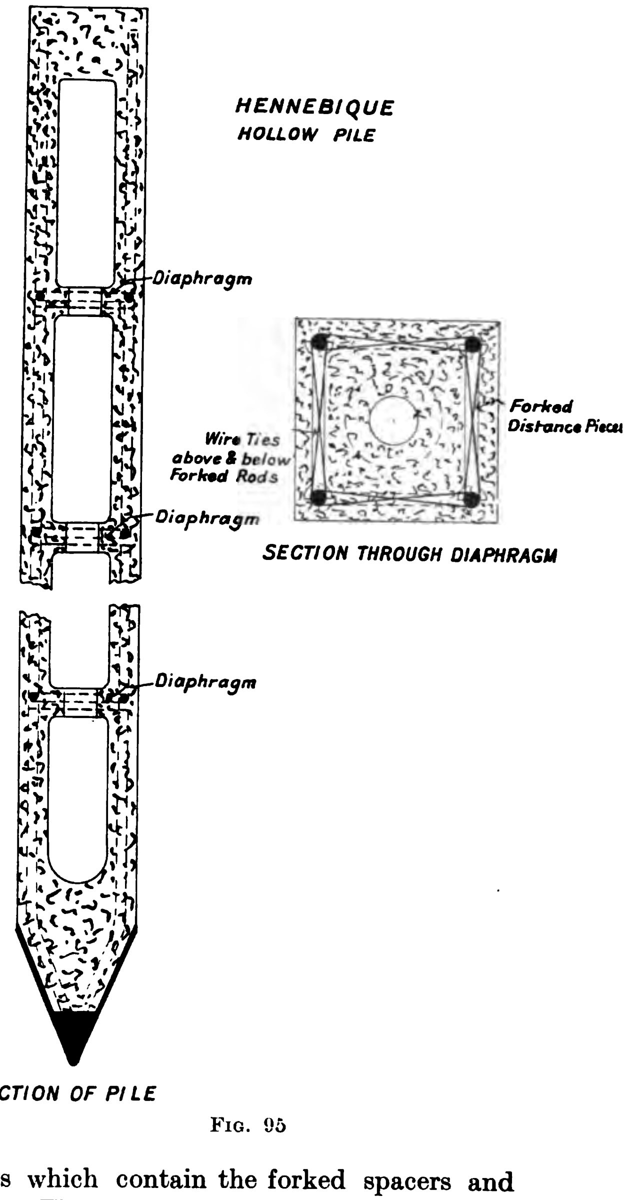
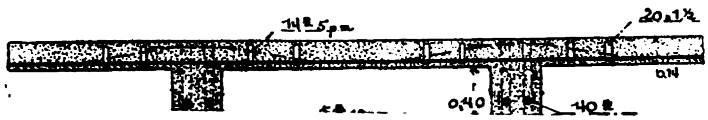


FIG. 95

(Fig. 95). It is formed with diaphragms which contain the forked spacers and wire ties connecting the longitudinal rods. The main advantage gained by its use is the lightness for transport purposes.

Several types of *arched bridges* are employed by M. Hennebique. A favourite form is constructed with arched ribs, reinforced in the same manner as shown in Fig. 91, supporting the decking either directly or with the intervention of cross beams. In some cases the ribs have a curved extrados, and the spandrils are open, being formed of columns resting on the ribs and connected at the top by transverse and longitudinal beams or arches. The transverse reinforcement in the vertical plane for large arches is arranged as shown in Fig. 91, thus tying the upper and lower rods securely together. In another type the ribs are hidden by having slabs at the intrados and extrados, in much the same way as in a flat floor with hidden beams. The arch formed on this type has the appearance of a solid arch. Fig. 96 shows a longitudinal section at the springing and a cross-



posite face by stirrups.

Page 68.—Since publication I have been informed by Mr. L. G. Mouchel that the patent hollow pile here figured is not a Hennebique pile, but that the inventor and owner is Mr. Mouchel.

C.F.M.

Columns.[1]

Internal	9*s*. 1*d*. " " "
External	5*s*. 9*d*. " " "
Floors (including beams)	2*s*. 1*d*. per square foot.
Flat Roof " "	1*s*. 4*d*. " " "

[1] These prices were given in a paper published in the Proceedings of the Institution of Civil Engineers, vol. cxlix. Part III.

concrete being broken away for some distance, and fresh rods inserted overlapping the old ones, after which a further length of concrete is added; but this is far from good practice and is very seldom resorted to.

14 × 14-inch piles have been driven with a 2-ton monkey having a drop of six feet, and the driving of 12 × 12-inch piles with a 30-cwt. monkey and a drop of four feet is quite usual. The piles are connected to the capping by having the concrete broken away for some distance down, so that the pile rods may penetrate into the concrete of the capping. A general view showing the manner of constructing retaining walls with pile foundations is shown (Fig. 94).

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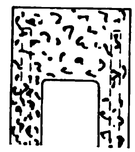


HENNEBIQUE HOLLOW PILE

Page 69.—It should have been stated that the prices given include driving.

Mr. L. G. Mouchel (Mr. Hennebique's Agent in the United Kingdom) states that these prices are not correct. Current prices are considerably lower.

C.F. M

FIG. 95

(Fig. 95). It is formed with diaphragms which contain the forked spacers and wire ties connecting the longitudinal rods. The main advantage gained by its use is the lightness for transport purposes.

Several types of *arched bridges* are employed by M. Hennebique. A favourite form is constructed with arched ribs, reinforced in the same manner as shown in Fig. 91, supporting the decking either directly or with the intervention of cross beams. In some cases the ribs have a curved extrados, and the spandrils are open, being formed of columns resting on the ribs and connected at the top by transverse and longitudinal beams or arches. The transverse reinforcement in the vertical plane for large arches is arranged as shown in Fig. 91, thus tying the upper and lower rods securely together. In another type the ribs are hidden by having slabs at the intrados and extrados, in much the same way as in a flat floor with hidden beams. The arch formed on this type has the appearance of a solid arch. Fig. 96 shows a longitudinal section at the springing and a cross-section at the crown of an arch of this type.

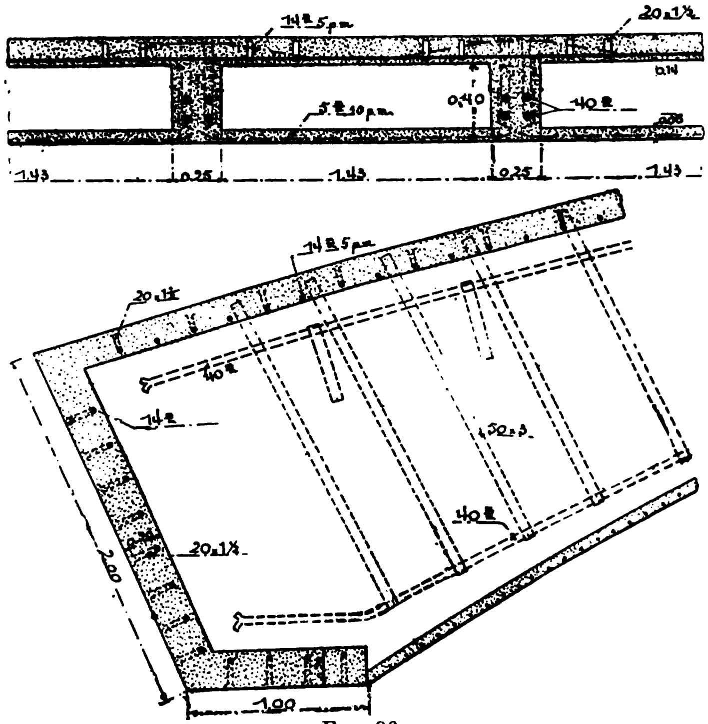


FIG. 96

Corbels or cantilevers are also employed by M. Hennebique. These are generally straight along the top, while the under side is formed to a curve. They have a reinforcement of upper and lower rods, the upper rods being anchored back, and both sets of rods being tied securely to the opposite face by stirrups.

The reinforcements for *pipes and circular reservoirs* consists, as in the Monier system, of spirals or hoops of round rods, with a series of distribution rods bearing against the circular reinforcements.

The price of the work naturally depends on many conditions, and it is impossible to give prices, which can be applied generally. As a rough guide, however, it may be stated that a roof for a single-stooled warehouse at Calais,[1] including the beams and supporting columns, cost about 1*s.* 11*d.* per square foot. For a structure erected in England the following prices may be given, but they must be considered as only an approximate rough guide, as they only apply to a single case—

Piles.[1]	
10 × 10 ins.	5*s.* 6*d.* per lineal foot.
16 × 16 ins.	9*s.* 3*d.* ,, ,, ,,
Columns.[1]	
Internal	9*s.* 1*d.* ,, ,, ,,
External	5*s.* 9*d.* ,, ,, ,,
Floors (including beams)	2*s.* 1*d.* per square foot.
Flat Roof ,, ,,	1*s.* 4*d.* ,, ,, ,,

[1] These prices were given in a paper published in the Proceedings of the Institution of Civil Engineers, vol. cxlix. Part III.

This system is represented in England by M. L. G. Mouchel, 38 Victoria Street, Westminster. All the work is done by licensed constructors, but the designs and specifications are prepared by M. Mouchel, and the works are generally supervised by him during construction.

Amongst the licensed constructors of this system, Messrs. John Aird & Sons (contractors) and Messrs. W. Cubitt & Co. (builders) may be mentioned. Both these and the other firms who have taken up this form of construction thoroughly appreciate the care necessary in the execution of the work and selection of the materials.

The New York agent for M. Hennebique is Mr. R. Baffrey, 1,123 Broadway, New York City, U.S.A.

M. Hennebique's chief offices are at 1, Rue Danton, Paris, occupying part of a house built entirely on this system.

Hyatt System

This was one of the first American systems employed for floors. It is not employed to any great extent at the present day, and is chiefly interesting from an historical point of view.

Only floor slabs were at first used, these being reinforced with longitudinals of flat bars on edge, which were pierced with holes, through which round rods were threaded, as shown (Fig. 97). At a later date M. Jackson applied the same principle to beams as shown (Fig. 98).

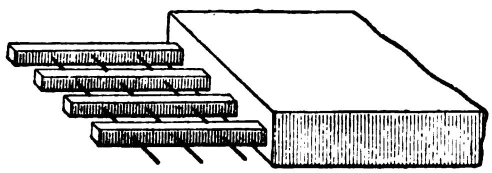


Fig. 97

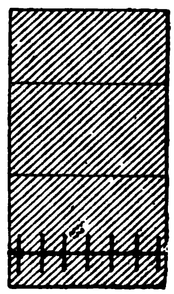


Fig. 98

Kahn System.

This system has only been introduced quite recently by Mr. Julius Kahn, of Detroit, Michigan, U.S.A.

The chief novelty is the form given to the reinforcement. The main bottom longitudinal bars are rolled of a diamond section with projecting wings on either side. These wings are slotted off along the edge of the diamond for certain distances and are bent up to an angle of about 45° to form the reinforcements resisting the shearing stresses ; these are consequently rigidly connected to the main bottom bars, which is a very excellent and desirable feature.

The three principal advantages claimed for the employment of this form of reinforcement are :—

(1) The reinforcements in the vertical plane are rigidly connected to those in the horizontal plane, resisting the tendency of the concrete to shear along the longitudinals.

(2) The projections formed by the wings where they remain a part of the main bar, help in resisting any sliding action of the steel through the concrete.

(3) The shearing bars being inclined are better able to resist the shear than similar reinforcements placed vertically.

The bars ordinarily used are of the following sizes :—

Side of square bar, forming diamond. Inches.	Thickness of wings. Inches.	Total width across wings. Inches.	Weight per lineal foot. Pounds.
$\frac{1}{2} \times \frac{1}{2}$	$\frac{1}{8}$	$1\frac{1}{2}$	1·4
$\frac{3}{4} \times \frac{3}{4}$	$\frac{3}{16}$	$2\frac{3}{16}$	2·7
1×1	$\frac{1}{4}$	3	4·8
$1\frac{1}{4} \times 1\frac{1}{4}$	$\frac{1}{4}$	$3\frac{3}{4}$	6·9

A form of reinforcement used for lintels consists of a flat plate with a series of flat bars, bent to an angle of 135°, rivetted to it.

The beams are formed of a smaller width at the bottom than at the top, which causes a small saving in the amount of concrete used.

Kindle System

This system was suggested in 1891 by Mr. Kindle, of Pittsburg, U.S.A., and is chiefly interesting from an historical point of view.

The main beams consisted of rolled joists and the secondary beams of tiles reinforced with suspension straps of flat iron, on which rested a series of rods as shown (Fig. 99).

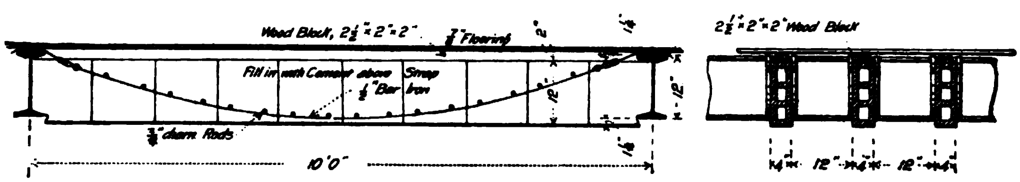


Fig. 99

The floors and ceilings were formed of cement slabs carried on the reinforced tiles.

Fig. 100 shows cross-section through the floor, and the method of attaching

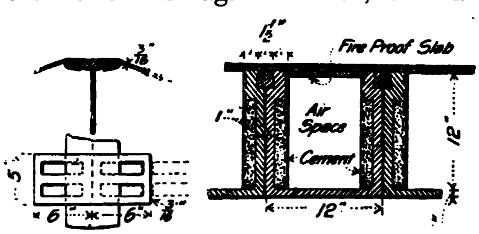


Fig. 100

the straps to the flanges of the rolled joists, portions of the strap being slotted out on three sides, and bent over in the form of hooks.

Klett System

This system of floor or roof construction is patented by the Vereinigte Maschinenfabrik, Augsburg-und, Maschinenbaugesellschaft, Nürnberg, of 100, Katzwanger Strasse, Nürnberg, Germany, and represented in England by F. H. Rudolph, 68, Victoria Street, Westminster.

Rolled joists, or light girders of T-irons and diagonal bracing, are employed for beams, and these have sufficient strength in themselves to support the centreing and the weight of the men and materials during the formation of the slabs.

The bottom of the concrete of the *floor* is formed with curves near the supporting beams or walls to increase the compressive resistance of the underside at these places.

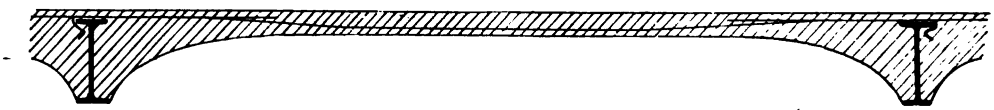


FIG. 103

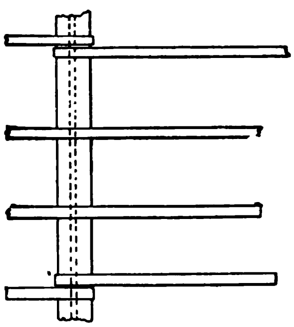


FIG. 104

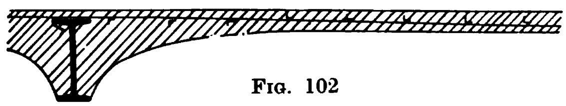


FIG. 102

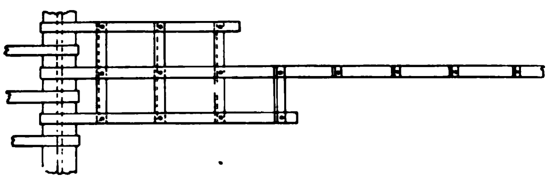


FIG. 101

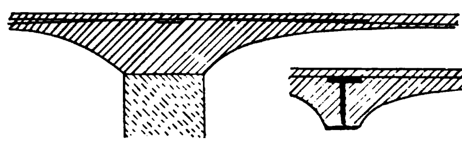


FIG. 105

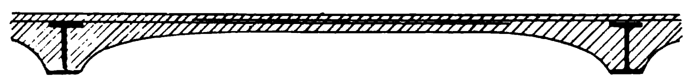


FIG. 106

The reinforcement of the slab consists of flat iron rods, some extending from beam to beam, being either attached to the top flanges by being bent round, or extend over into the adjoining slab. These bars are depressed towards the centre of the span in the manner of suspension bars.

Further flat bars are placed between these, and extend for various distances into the slab as shown (Fig. 101). The bars are usually connected in the transverse direction by angle irons, as shown (Figs. 101 and 102), and further short pieces of angle iron are attached to the through bars. In Figs. 101 and 102 the short bars follow the same curvature as the through bars and all the reinforcements, are hooked over the top flange of the supporting beams. Fig. 103 shows a form of floor reinforcement without the transverse angle irons, in which the short bars are horizontal. Fig. 104 shows a plan of the reinforcements of a floor of this type, in which the through bars are hooked over the beams, while the short reinforcements extend into the adjoining slab. When the slabs pass over a wall the bars are inserted, as shown (Fig. 105).

Sometimes the intermediate bars are continued so as to extend beyond the centre of the slabs, to assist in resisting the compressive stress, as shown (Fig. 106), or they are carried right through from beam to beam or through several bays. The main suspension bars are not shown in this figure.

Koenen System

The object aimed at in the Koenen System of reinforced concrete floors (represented in London by Mr. R. B. Roxby, 18, Featherstone Buildings, Holborn, W.C.), is that of obtaining practically an absolute building in by securely anchoring the reinforcements to the walls and girders. It was introduced in 1898, and has been extensively employed on the Continent.

Rolled joists are employed as beams in this system, and are embedded in concrete.

The reinforcement of the floor slab consists of a series of round suspension rods securely fixed to the top flanges of the joists by being hooked over, or firmly anchored back to the walls. These suspension rods follow the curve of the bending moments for a built-in slab, and consequently take up the shearing stresses.

The concrete of the *floor* is arched near the supports to offer more resistance to the compressive stresses, which are greatest at this point, in the case of a perfectly built-in piece.

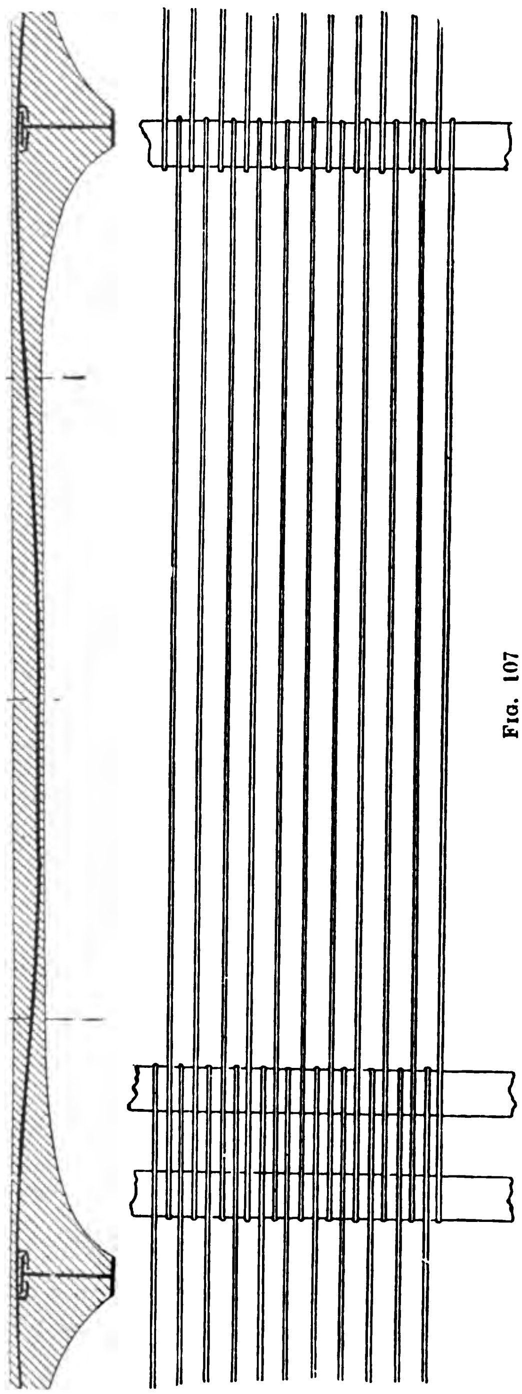


Fig. 107

Fig. 107 shows a longitudinal section and plan of the centre bay of a floor on this system.

There are several methods adopted in forming the floors. Fig. 108 shows the reinforcement for a floor-slab where the reinforcing rods are not anchored back to the wall; in this case the rods follow the lower surface to the centre of the span, and from this point are bent up and hooked over the top flange of supporting joists.

Fig. 109 shows the arrangement of the reinforcement when there are no joists, but the floor continues on the further side of a wall; the rods have the same curved form as when joists are employed, and pass through the wall near the upper surface of the floor slab, being continued into the next floor.

The anchors to the walls are formed by hooked rods attached to vertical pieces

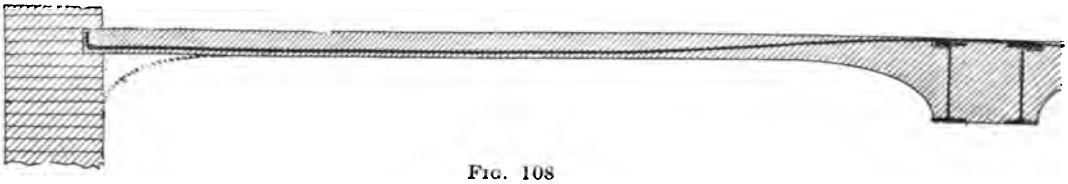


FIG. 108

of iron built into the wall. At the outer end the anchor rods are hooked round a flat iron bar, which runs along the face of the wall, and receives the hooked ends of the suspension rods. Sometimes, instead of passing the suspension rods through

FIG. 109

a division wall into the next floor, a series of anchor rods are built through the wall, supporting longitudinal flat bars on both sides in the same manner as described above.

These floors have been subjected to severe load and fire tests with good results. They form an economical flooring for spans of from 6 to 20 feet.

Locher System

The *beams* of this system are entirely different from any other, in that the reinforcements are placed so as to follow the direction of the lines taken by the combined tensile stresses in a beam freely supported at the ends. The reinforcements consist of flat bars laid on their widest side. They are placed in layers, each bar being horizontal through the centre of the span, and are bent up at a different distance from the supports. Fig. 110 shows the longitudinal and Fig. 111 the cross-section of a beam.

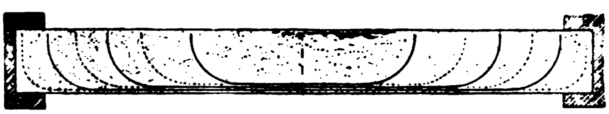


FIG. 110

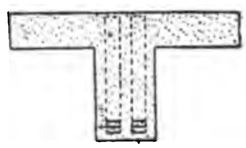


FIG. 111

It will be seen that the beam is shown built in at the ends. This form of reinforcement, though it would wholly resist all the direct tensile stresses and also the shearing stresses in a freely supported beam, should be somewhat altered for a built-in beam; since the bent-up bars do not provide for the tensile stress induced in the upper portion by the building in.

Maciachini System

Signor Maciachini, engineer and constructor of Milan, had, until a recent date, occupied himself with constructions in reinforced concrete on the Walser Gerard System. The interesting experiments of M. Considère encouraged Signor Maciachini to introduce a system of reinforcement in the construction of beams based on the hooping of concrete. Signor Maciachini's object has been to discover the best method to affect such a hooping to benefit the concrete both on the compressive and tensile side of a beam.

The efficient hooping is not easily effected for a beam, since the moulding must be done horizontally if formed *in situ*, and the fabrication in advance of such a piece causes the loss of many advantages. To obtain a hooping practicable and at the same time effectual, for a beam moulded *in situ*, Signor Maciachini forms his reinforcement as shown (Fig. 112).

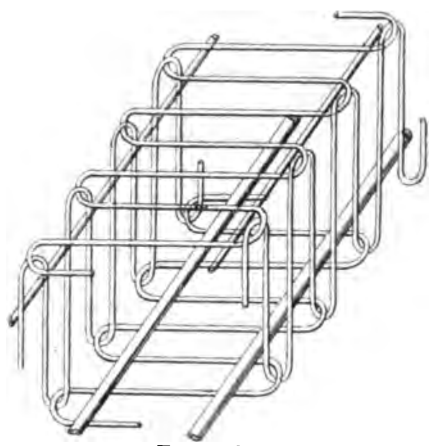


FIG. 112

The hooping wires of a suitable diameter are as long as possible, and are bent up and down before being placed in position, the height being that of the width or depth of the beam, less about 1·6 inches to allow for a covering of 0·8 inches of concrete on all sides.

The bottom and side hoopings are placed together, in the manner shown (Fig. 113), so that when they are all connected these reinforcements appear, as shown (Fig. 114).

After 0·8 inches of concrete has been deposited, the hoopings are put in position with the bottom rods at the angles. The filling is then brought up and well rammed,

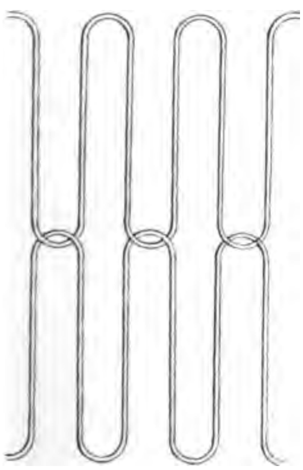


FIG. 113

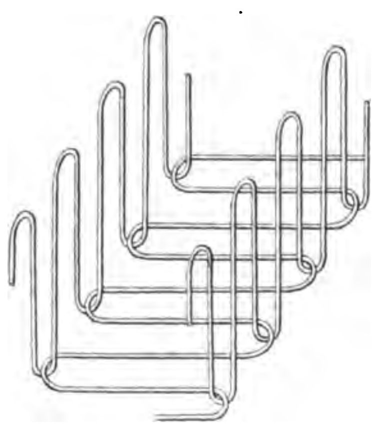


FIG. 114

until it reaches the level of the top rods. These are then put in place and the top portion of the hooping is threaded through the top loops of the sides and bent backwards and forwards, as shown (Fig. 112).

After this operation is completed the remainder of the concrete is added.

Fig. 115 is a cross-section of a beam of this form.

Signor Maciachini occasionally adds a further vertical reinforcement, as shown (Fig. 117.)

Fig. 117 is a longitudinal section of a freely supported beam reinforced on the Maciachini system. To discover what advantage is gained by this method of

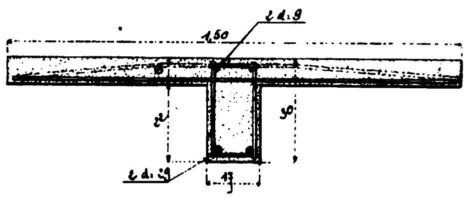


FIG. 115

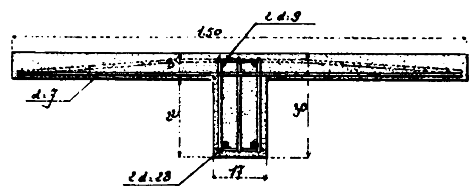


FIG. 116

reinforcement, beams as shown in cross-section in Figs. 115 and 116 and of the dimensions given on the figures were tested against a beam of the ordinary Hennebique type of dimensions shown in Fig. 118.

The weights of reinforcement and proportions for the concrete were as follows—

TABLE IV.

	Weight of Reinforcement Pounds	Proportions of Concrete		
		Cement Pounds	Sand Cubic yards	Stone Cubic yards
Fig. 115	170·28	710	0·33	0·66
Fig. 116	170·72	,,	,,	,,
Fig. 118	173·14	505	0·33	0·66

The total depth of the beams (Figs. 115 and 116) was 12 inches, and that of the beam (Fig. 118) was 14 inches.

The tests were made twenty days after moulding, which perhaps gave a slight advantage to the beams of the richer concrete. The table below gives the deflections measured for the three beams under increasing loads, and clearly shows the advantage gained by the hooping, and also that the addition of the central vertical reinforcement adds considerably to the stiffness.

TABLE V.

Uniformly Distributed Load Pounds per square foot	Deflection at Centre in Millimetres.		
	Beam Fig. 115	Beam Fig. 116	Beam Fig. 118
41·75	0·000	0·000	0·000
62·19	0·000	0·000	0·220
82·63	0·100	0·000	0·890
103·07	0·170	0·110	0·610
123·50	0·250	0·190	1·140
133·83	0·300	0·260	1·196
144·05	0·570	0·310	2·780
154·27	0·690	0·390	The tests were not continued for these loads.
164·50	0·850	0·590	
184·93	0·910	0·690	
205·37	0·990	0·860	

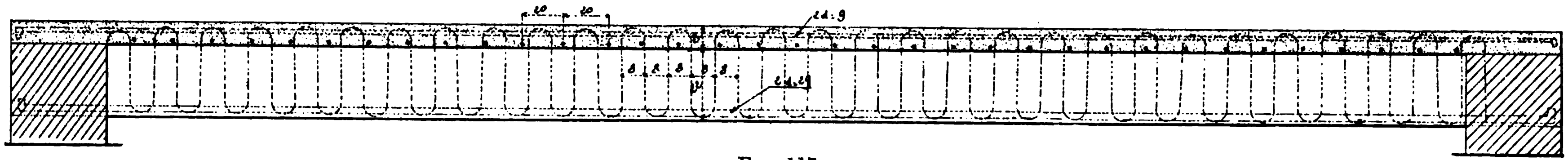


FIG. 117

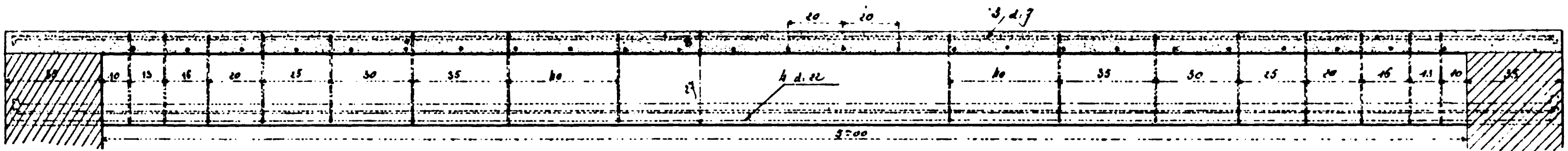


FIG. 118

It will be noticed that under a load of 144·05 pounds per square foot the deflection of beam (Fig. 118) was nearly five times as great as that of beam (Fig. 115), and over nine times as great as that of beam (Fig. 116), and that the deflection of beam (Fig. 115) was nearly double that of beam (Fig. 116).

The concrete for the two beams of the Maciachini system being of richer proportions would increase their stiffness, but this would only account for a small portion of the difference in the deflections.

Signor Maciachini has also patented a *column* reinforcement formed in the same manner as that shown (Fig. 113) with looped wires. Fig. 119 shows this form of reinforcement.

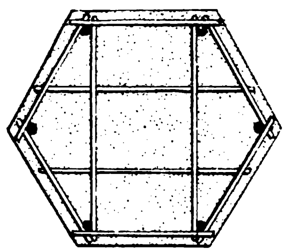


FIG. 119

It seems that the effect of the hooping of the Maciachini beams would be improved if further longitudinal rods of small section were placed along the top and the sides to act as distribution rods.

M. Considère found it beneficial to use longitudinal rods for his hooped columns.

It appears also that the comparative width of the loops might allow them to close up somewhat, allowing the hooping to stretch with the swelling, but it is probable that the " adhesion "of the concrete would prevent such a movement.

This system is represented in England by Mr. C. H. Reynolds, Clevelands, Teddington, S.W.

Matrai System

Herr Matrai, an engineer of Hungary, and formerly a Professor at the Polytechnic of Budapest, was for some time employed in the exploitation of the Monier patents, and had the supervision of many important works. The failure of a bridge, which to all appearances had been constructed under the best conditions, led him to direct his attention to the invention of a system which was not dependent on the execution or the materials of which the concrete was made. In this system the metallic skeleton of beams and floors is designed to resist all the stresses, and the resistance of the concrete is entirely neglected.

In the Matrai system steel wires are employed which are given the curves which they would naturally take under the load. These wires, or the cables into which they are sometimes twisted, resist entirely by tension. The employment of steel wires of the best quality and of small diameter allows the adoption of a safe stress of 21,350 to 28,450 pounds per square inch.

As the concrete is only used as a filling and for distributing the load over the reinforcement, an economical mixture may be used such as a lime concrete or a poor cement concrete.

M. Matrai likens his system to a spider's web firmly attached to the points of support.

This system has been employed with success in Hungary, Austria, Russia, Italy, France and America.

Beams.—M. Matrai employs rolled joists or light built-up N-Girders as beams. These he further strengthens by a steel wire suspension cable on either side. These cables are suspended from loops either anchored back into the walls, as shown

(Fig. 120), or simply passed over the end of the beam as shown (Fig. 121). The suspension cables act entirely in tension, and relieve the girders considerably.

The drop of the cables is usually the depth of the beam, and as they require three or four times less metal than an ordinary beam having the depth of their drop

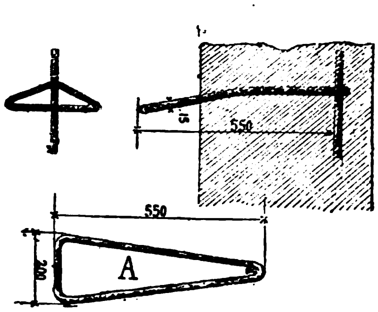


Fig. 120

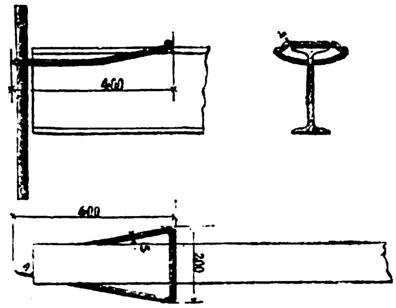


Fig. 121

to support the same load, they form a very economical reinforcement. The cables and girders are generally so designed that each carries half the total load.

Floors.—The reinforcement of Matrai floors is formed of suspension wires or small cables, in various manners, according to the requirements. There are always some running across the span, and also at least two diagonally, these last being generally cables.

The usual methods are shown in Figs. 122 and 123. The object of the diagonal

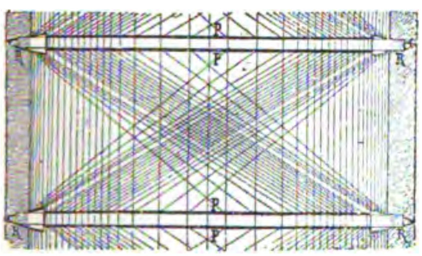


Fig. 122

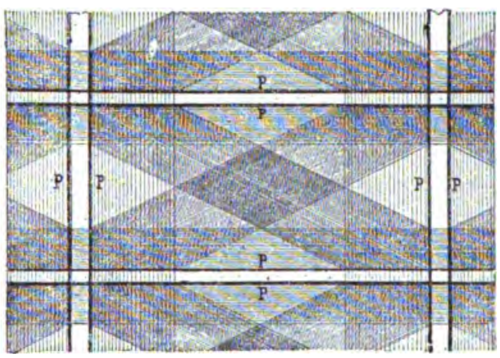


Fig. 123

crossing wires is very ingenious. It will be noticed that their ends are attached at different points along the beams; the effect of this disposition is to equalize the bending moments on the beams, as shown in Figs. 124 and 125.

Fig. 124 shows the effect on the diagram of bending moments caused by attaching these diagonal wires at increasing distances apart from the supports towards the centre, as in Fig. 122, and Fig. 125 that of bringing all the attachments to the two ends, as shown in Fig. 123.

Considering Figs. 122 and 124. The beam supports two loads of Q in such a manner that the forces are represented by the ordinates of the curve $G\ H\ I\ L\ K$, the area $G\ H\ I\ L\ K\ D\ C$ being *double* the rectangle $E\ F\ D\ C$, which represents a load P equal to Q, but uniformly distributed. The centre of gravity of the loads Q are at a distance of $\frac{l}{8}$ from the extremities of the beam. In the bottom part of the figure, the curves of the bending moments correspond to the loads $2\ Q$ and P.

It can be seen that the ordinates of the curves $a\,b_1\,c\,d_1\,e$ for the load $2\,Q$, are very nearly constant and equal to $c\,f$ for some distance each side of the centre, while

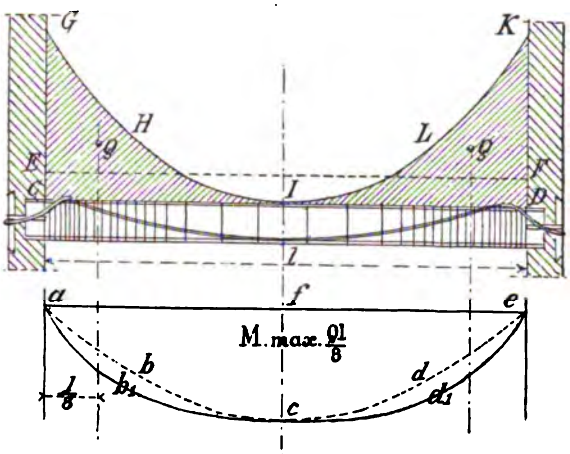


FIG. 124

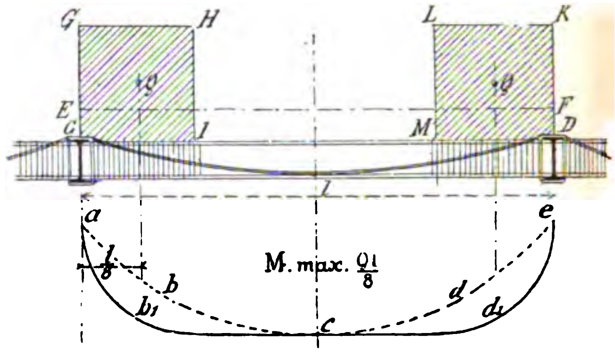


FIG. 125

those of the curve $a\,b\,c\,d\,e$, corresponding to a beam loaded uniformly with a load $P = \frac{2\,Q}{2}$, decrease rapidly towards the supports.

It is evident, then, that if the load $2\,Q$ is distributed over the extremities of the span in the above manner, it is possible to employ a beam of half the resistance necessary for the same load uniformly distributed.

If the loads are distributed as shown in Figs. 125 and 123 following the ordinates of the rectangles $G\,H\,I\,C$ and $L\,K\,D\,M$, of which the total area equals $2\,Q$, if the centres of gravity of these two rectangles fall at a distance of $\frac{l}{8}$ from the supports, we obtain the curve of bending moments $a\,b_1\,c\,d_1\,e$ which further equalizes

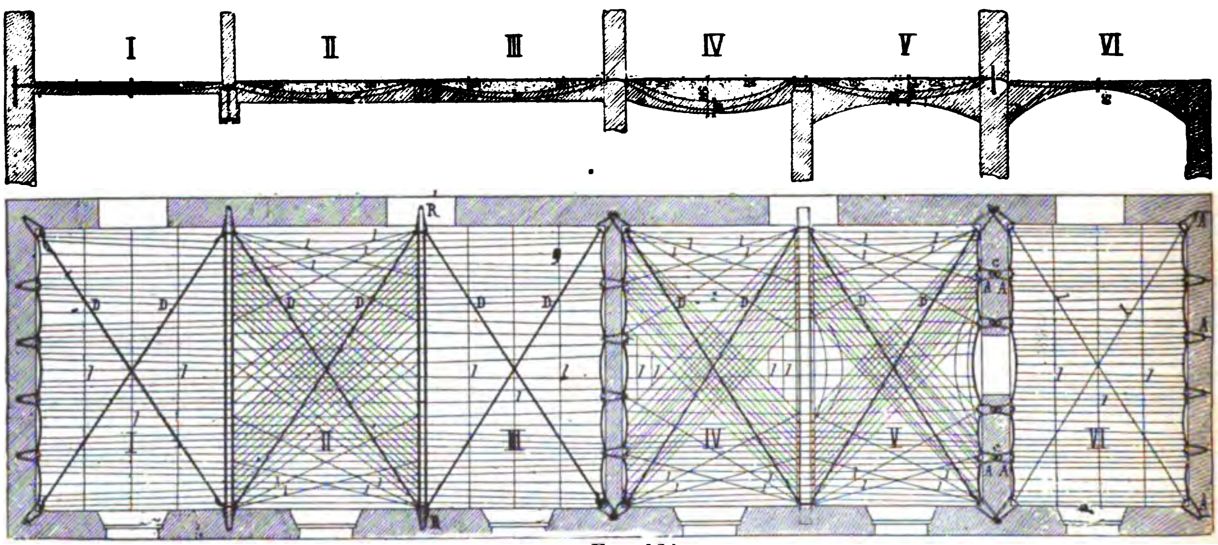


FIG. 126

the stresses on the beam. In practice this effect is obtained by attaching the wires at equal spaces apart on the two extreme quarters of the beams.

Fig. 126 shows various methods adopted for the disposition of the cables and wires.

The bays marked *I* and *III* have the wires placed so as to distribute the load equally over the beams, which only support half the load, being assisted by the diagonal cables, which are calculated to take the other half.

In bays *II*, *IV* and *V* the diagonal cables are not sufficient to transmit half the load to the extremities of the beams, and they are suplemented by diagonal wires.

The transverse wires in bay *VI* are sufficient to distribute the load equally over the walls, and the diagonals and longitudinals are only employed to intertie the transverse wires.

The upper surface of the concrete for the floor slabs is usually hollow, following the curve of the wires. This hollow is filled in with cinder or coke breeze concrete, into which nails can be driven if necessary to fix floor boards, or pieces of timber may be embedded in this filling, to which the floor boards may be nailed.

The lower surface of the concrete around the reinforcements is generally flat, and supplementary short loops of wire are fastened to the main wires near the beams, to support the concrete where it thickens out.

When it is formed to an arch, as in bays *V* and *VI* (Fig. 126), there is no need to have any auxiliary supporting wires.

It may also be curved, following the curve of the suspension wires, as in bay *IV* (Fig. 126).

The tension in the cables and wires is calculated by the usual formula—$T = \frac{P\,L}{8\,v}$, where *T* is the tensile stress, *L* the span, and v the drop.

The floor suspension wires are fixed to the beams by simply enrolling them round, but when they have to be attached to walls they are fastened to cables supported by loops anchored into the wall by vertical rods, or passing through it, as shown in bays *I*, *IV*, *V*, and *VI* (Fig. 126). The main diagonal cables are secured direct to loops either anchored in to the walls or passing round the ends of the beams.

Columns.—Herr Matrai, as a general rule, considers the concrete of his columns as resisting the vertical stress and the steel for resisting the lateral stresses only. But if necessary he reinforces the concrete by vertical I-irons. He does not attempt to reduce the sectional area of his columns, for the reason that he considers the appearance is better if the columns are of fairly large size, and by employing columns of large sectional area he can make them of a poorer concrete, and thus gain in economy.

Fig. 127 shows the reinforcement of an ordinary column without any vertical bars; the reinforcements are in the form of parabolic cables, with a drop nearly equal to the width of the column, secured to a ring at the top and to

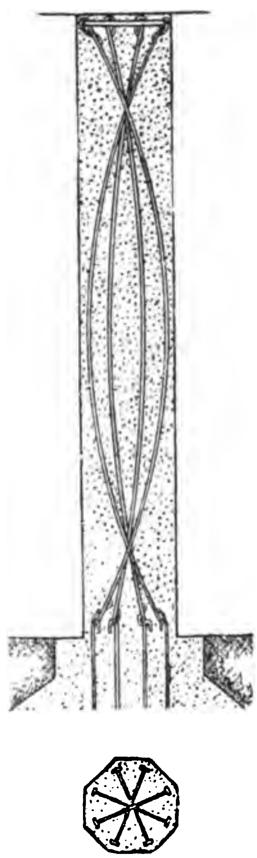


Fig. 127

vertical hooked bars, extending into the foundation, at the bottom. The parabolic cables are considered necessary to prevent any lateral flexure.

If the column of concrete only is not sufficient to resist the vertical stresses, small rolled joists are placed vertically in the concrete, as shown (Fig. 128), the extremities of the parabolic cables being attached to them. Herr Matrai also employs a hollow column of the form shown (Fig. 129). The parabolic cables in this case are placed as shown within the thickness of the shell, and further cables are placed on each side of each joist, their drop being equal to the depth of the web.

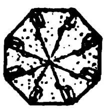


FIG. 128

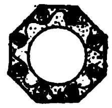


FIG. 129

Walls.—The reinforcement for walls is arranged as shown in Fig. 130. The vertical stresses are resisted by the concrete, and the small I-bars and the lateral stresses by parabolic cables attached at their extremities to the joists. If the thrust is only exerted on one face it is only necessary to employ one set of cables, but if the thrust may be in both directions two sets are inserted.

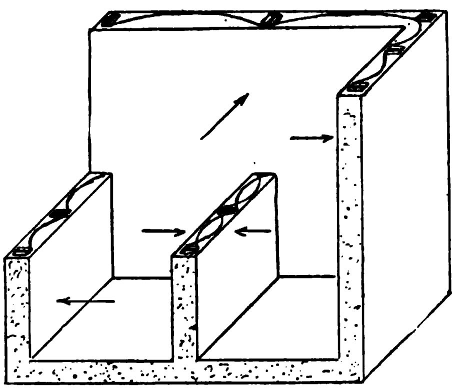


FIG. 130

The concrete is also held together by a diagonal network when only compression is anticipated, but if bending may occur the wires of the network are placed vertically and horizontally.

If the wall has an opening in it a suspension cable with a deep drop is supported from the ends of the upper beam, and embedded in the concrete of the wall.

Stairs.—The stairways of this system are formed very much in the same way as floors, the loads supported by the steps being carried by parabolic wires to supports, to which they are attached.

If the stairway is supported on stringers, these are reinforced with angles, channels, tees or joists, to which the parabolic wires are attached, whereas if the stairway is between two walls the supporting bars are placed transversely, being built into the side walls at their ends and spaced about one to each two steps, the parabolic wires running longitudinally and being attached to them.

The steps themselves, if supported by stringers, are formed of plain concrete being supported on a slab below, which contains the reinforcements ; but when the main bars run transversely they are embedded in the concrete forming the steps. Sometimes when the stairway is between walls, the parabolic wires, running transversely, are anchored direct to the walls.

Some of the structures carried out by Herr Matrai are detailed below—

Le Globe Céleste at the Paris Exhibition of 1900.

Le Palais des Manufactures National Exhibition of 1900.

The Covering over the Moulineaux Railway cuttings opposite the Eiffel Tower.

Coverings over the Metropolitan Stations in Paris.

Le Maison d'Education de la Légion d'Honneur at Saint Denis.

Cement Silos at Chantemelle.

A Spherical Reservoir Bottom of 20 feet diameter.

And many others.

SYSTEMS EMPLOYED

Melan System

This system was devised by Professor Melan, of the Polytechnic School of Brünn, Austria, in 1892, and has been largely employed for floors and bridges in Austria, and also in America, where many important bridges have been constructed by Herr F. Von Emperger, who became the agent for this system in the United States.

The *floors* are of the arched type and are reinforced with rolled joists, as shown (Fig. 131). The tie rod shown here is, however, generally omitted. The supporting beams are always formed of rolled joists. The arch proper is formed with a curved extrados, and of concrete, in the proportions of about 1 to 2 to 4; this concrete is carried round the supporting joists. The filling above the arches is mixed with larger proportions of sand and stone. The rise of the floor arches varies from $\frac{1}{10}$ to $\frac{1}{15}$ the span.

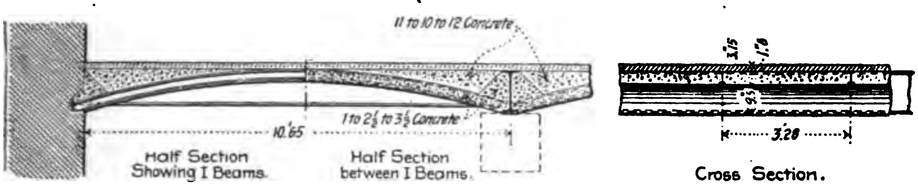


Fig. 131

The reinforcing joists are spaced about 3¼ feet apart, and are wedged tight against the webs of the beams. For smaller spans than 10 feet the reinforcing sections are formed of T-irons.

The table given below shows the details for various spans :—

TABLE VI.

Span Feet	Depth of Joist Inches	Weight Pounds per Foot	Distance apart Inches	Weight of Floor per Sq. Foot Pounds	Depth of Concrete Arch in Inches
10 to 12	3	6	40	1·8	4
12 to 16	4	6	40	1·8	4½
16 to 20	4	7½	40	2·25	4½
20 to 24	5	10	50	2·4	5½

The arched bridges on this system are constructed with reinforcements of rolled joists or light built-up girders. The built-up girders vary in depth, being deeper at the springings than at the crown, and sometimes being thickened out at the haunches. This form of reinforcement lends itself to hinged arches, a form of construction very frequently adopted The joists or girders are spaced from 2·46 to 3·28 feet apart.

This system is constructed by the Concrete-Steel Engineering Co., Park Row Buildings, New York.

Metropolitan Fireproof Construction

This system has been employed in America, and consists of beams of I-section, over which small wire suspension cables are stretched, each formed of two galvanized wires twisted together. The distance between the cables varies with the load to be carried.

Round rods are laid across the cables parallel with the beams at the centre of each span, to obtain a uniform deflection. The cables are continued over several supports, being secured at their ends by loops of heavy wire hooked over the flanges of the beams or anchored back to a wall. When the reinforcement is in position the centreing is erected and a composition, the principal ingredients of which are plaster of paris and wood chips in the proportions of 75 per cent. to 25 per cent. by weight, is poured in, and solidifies in a few minutes. The concrete which embeds the joists is supported on wire netting passed round the flanges.

If a *ceiling* is required, iron bars are laid across from beam to beam, resting on the bottom flange. A wire netting is placed on these and a thin layer of the above composition is poured over.

Möller System

This system was brought out by Professor Möller, of the Polytechnic School of Brunswick, in 1894, and has been employed considerably in Germany for floors from 16·42 to 32·8 feet span, and also for bridges and coverings. It is constructed by the firm of Drenckhahn und Südhop, of Brunswick.

The *floor or decking* slab is reinforced with rolled joists, and is supported by fish-bellied beams at certain intervals apart. Generally the spacing of the beams is in the neighbourhood of 4 feet.

The *beam* reinforcements consist of flat bars placed as shown (Fig. 132). These are firmly anchored back to the walls or abutments, as shown (Fig. 133), and

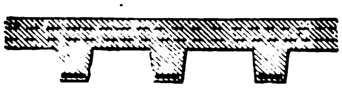


FIG. 132

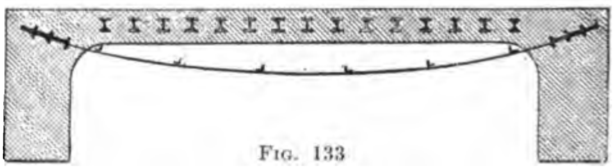


FIG. 133

have short pieces of angle-iron secured to them at equal distances apart, to resist the longitudinal shears of the flat bars through the concrete. These angles are shown in Fig. 133.

Bridges and coverings are constructed in the same manner. The *river and canal bank protections* on this system are reinforced by round rods placed horizontally and passed over pieces of stone between the anchorings, which are formed of posts driven perpendicularly into the bank from 1·64 to 2·46 feet apart. These slope coverings cost about 1*s*. 11*d*. to 3*s*. 1*d*. per square yard.

Monier System

This system, for which the patents have lapsed, is used at the present day by many firms.

These employ the Monier type of reinforcement for slabs, walls, pipes, reservoirs, etc., but construct beams in various ways. Some of them also adopt peculiar arrangements for the reinforcements to resist shearing, and for other parts of the work.

The original firm to buy up the Monier patents was G. A. Wayss & Co., of Berlin, becoming at a later date the "Actien Gesellschaft für Beton und Monierbau," of Berlin, and the firm of Wayss und Fratag whose chief office is at Neustadt an der Haardt. Herr G. A. Wayss also formed a branch establishment at 13, Wallfischgasse, Vienna, and other towns.

Under the heading of the *Monier System*, the methods of construction employed by Herr Wayss & Co. must be dealt with. The special features of the other firms using this type of reinforcement will be dealt with separately.

The reinforcement of slabs consists of rods crossing one another at right angles, and tied together at their intersections with annealed wire, as shown (Fig. 134).

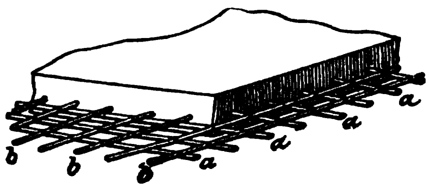


FIG. 134

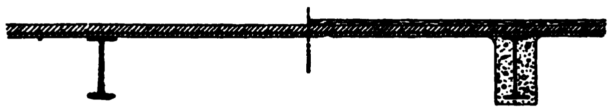


FIG. 135

The original *flat floors* were supported on rolled joists in various ways some of which are shown (Figs. 135 to 137). Fig. 135 shows a floor slab supported on rolled joists, encased with concrete or left unprotected and showing below the floors.

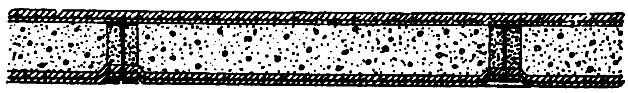


FIG. 136

Figs. 136 and 137 are types of hollow floors with ceilings. The ceiling slab is bent up so as to bear on the bottom flanges of the joists, and a wire network is placed across below the flanges to receive a layer of plaster, which fills in the hollows along the

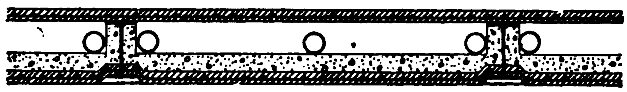


FIG. 137

bottoms of the joists. The space between the floor and ceiling is filled with a poor concrete in Fig. 136, and left hollow to carry pipes, etc., in Fig. 137.

Sometimes timber beams are used instead of joists in which case the ceiling slabs are carried through level, and are fixed to the bottoms of the beams with screws.

The floor slabs are occasionally replaced by boarded floors, either nailed to timber bearers placed across the joists or to a filling of cinder concrete.

Several types of *arched floors* supported on rolled joists are also employed.

Fig. 138 represents a floor with a reinforced flat slab and arched ceiling. The

FIG. 138

central portion of the arch merges into the flat floor slab. A similar floor is formed with the arch continued through, without the straight portion at the centre of the span.

This type of floor is also constructed of an arched slab only; the level floor being formed of boards or plain concrete. When boards are used they are nailed either to longitudinal timbers resting on the concrete filling over the arch or to the filling itself, which is made of cinder concrete. Fig. 139 represents a double floor with two arched slabs, one resting on the lower flanges of the joists and the

FIG. 139

other bearing against the webs of the joists just below the top flanges, and supported on a filling of rich concrete extending for a short distance on each side of the webs.

Fig. 140 shows an arched floor with a flat ceiling slab suspended below by

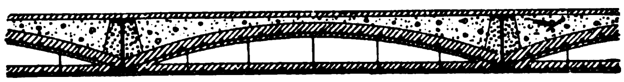


FIG. 140

rods tied into the arch supporting the floor.

All these slabs, flat or arched, are reinforced with the Monier network near their lower surfaces.

Herren Wayss und Fratag have of late years adopted the reinforced concrete *beam*. They use round rods, one set being straight along the bottom and another set bent up near the supports. Further short straight rods are added near the top

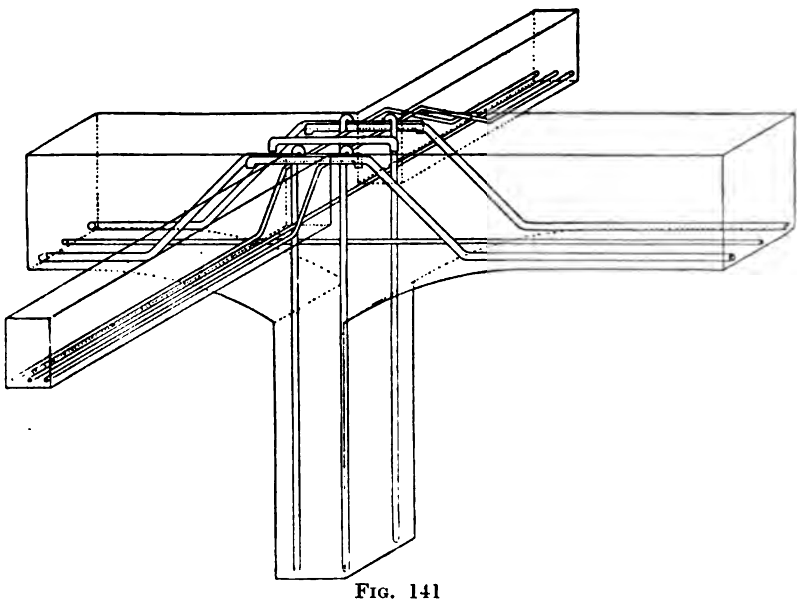


FIG. 141

surface over the supports, extending for some distance on both sides. No form of stirrup or similar reinforcement is used except in special cases. Occasionally a set of rods is placed near the upper surface throughout the whole span. Fig. 141 shows the general arrangement of the reinforcements for beams.

The ordinary *stairs* on this system are supported on rolled joists, as stringers, as shown (Fig. 142). The treads, risers and ends of the steps are all reinforced with crossing rods.

Fig. 143 shows a winding stairway in which the treads and risers are reinforced and are well tied into the wall.

Herren Wayss und Fratag employ Ritter's methods of calculation at the present time, but up to a recent date they used empirical formulæ.

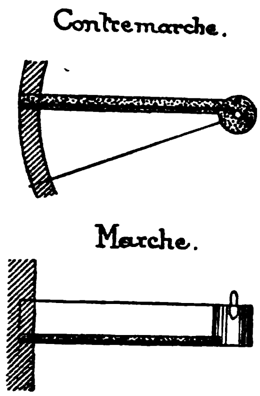


Fig. 143

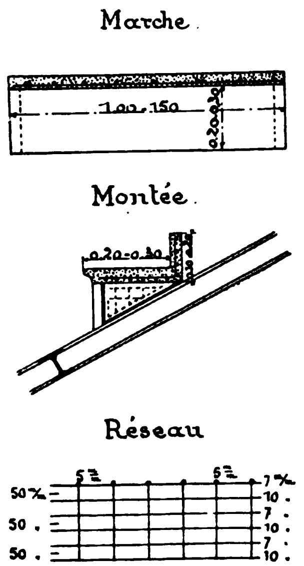


Fig. 142

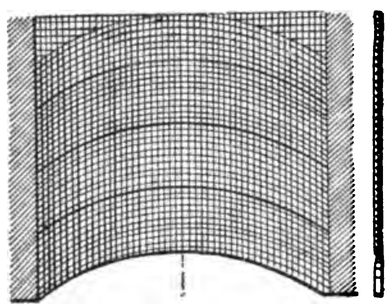


Fig. 144

Partition walls are reinforced with vertical and longitudinal rods, forming a network similar to that for slabs. The longitudinal rods are sometimes bent up, as shown (Fig. 144).

Columns are also employed reinforced with vertical rods tied together with wire hoops.

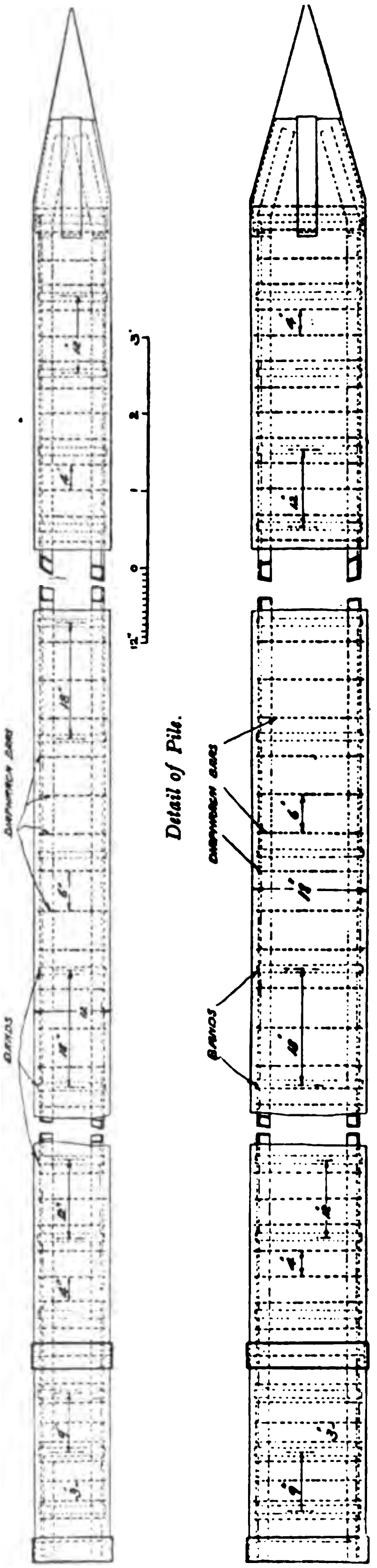
The *arched* bridges on this system have a curved extrados. If of small span, the "Monier" network is only placed near the intrados. For larger spans additional networks are placed for a short distance from the abutments near the extrados, or inclined from the intrados to the extrados near the springings. Both the intrados and extrados are also frequently reinforced throughout the whole span.

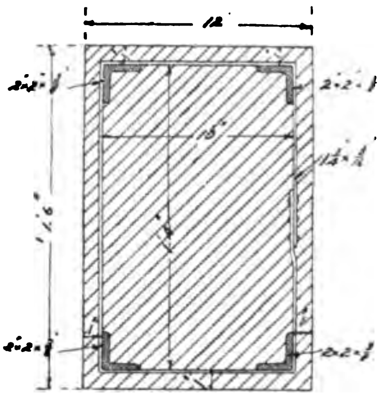
Retaining walls are constructed of thin vertical walls with counterfort ribs and foundation slabs, all reinforced with networks of rods. Sometimes the counterforts are connected by arches.

The pipes and sewers are reinforced with either one or two networks of rods, according to the pressure. The small pipes are sometimes formed with a socket but the joints are usually made with a collar.

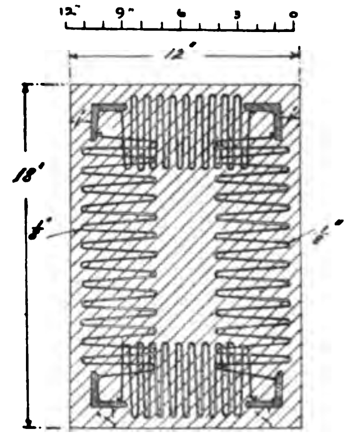
The reservoirs, silos and similar structures are reinforced with networks; the sizes of the hoopings are varied according to the pressure, but the spacing is generally kept the same. Several standard sizes of rods are used, and these are interspersed in various ways, so as to obtain the requisite area of metal in the height under consideration.

The firm of Herren Wayss und Fratag is represented in London by Herr H. C. Werner, 4, Westminster Palace Gardens, Artillery Row, S.W. The name adopted by the English firm is *The Armoured Concrete Company*, of which Mr. A. Johnston, M.I.C.E., M.I.M.E., is the Managing Director.





Section showing Angle-iron and Band.



Section showing Angle and Diaphragm Bars.

FIG. 145

Mr. Johnston has designed a reinforced concrete pile, views of which are given (Fig. 145). The object of the diaphragm bars is to resist the shear. They appear a somewhat uneconomical form of reinforcement. The wire would be more usefully employed if wound round outside the angle irons to prevent the swelling of the concrete. The length of wire in one diaphragm would encircle the angle irons nearly eight times.

Mueller, Marx & Co.'s System

Mueller, Marx and Company, of 212 and 213, Greifswalder Strasse, Berlin, brought out their system of reinforced concrete floor construction in 1895, since which date it has been largely employed.

Rolled joists are always used for beams, being embedded in concrete.

Several types of *floors* are constructed on this system. Fig. 146 shows the usual

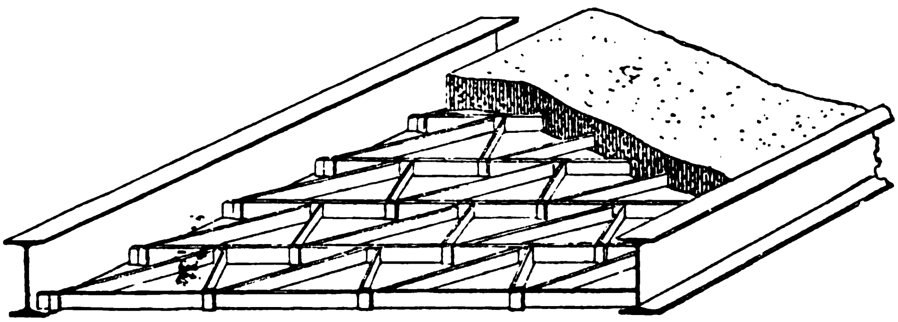


FIG. 146

form for flat floors of small span; these are reinforced with upright flat bars extending from joist to joist, and resting on the lower flanges. These are tied together by a zig-zag reinforcement of similar bars secured by thin iron clips.

This form of zig-zag reinforcements is used in all the types of flooring constructed by Messrs. Mueller, Marx and Company. Fig. 147 shows another form of flat floor

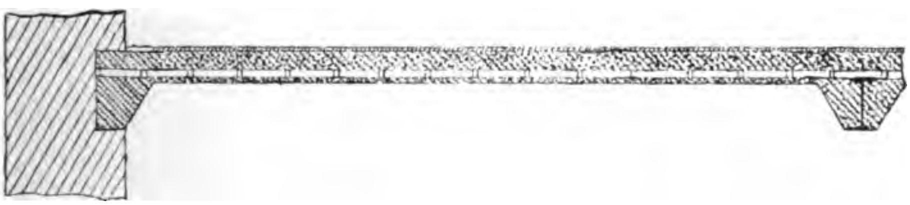


FIG. 147

in which the main bars of upright flat iron pass over the top of the rolled joists, and are embedded in concrete at their junction with the walls, forming a partial building in. The floor slab is bevelled at the wall to give greater resistance to the compressive stresses developed by the building in.

Fig. 148 shows a form of arched floor, the upright flat bars in this case following the curve of the intrados and abutting on the web of the supporting joists close to the bottom flange.

Floors reinforced with suspension bars of upright flat iron are shown in Figs. 149 and 150. Here the reinforcements are well anchored back into the walls, and formed with a quarter twist at the supporting joists, being securely hooked over the

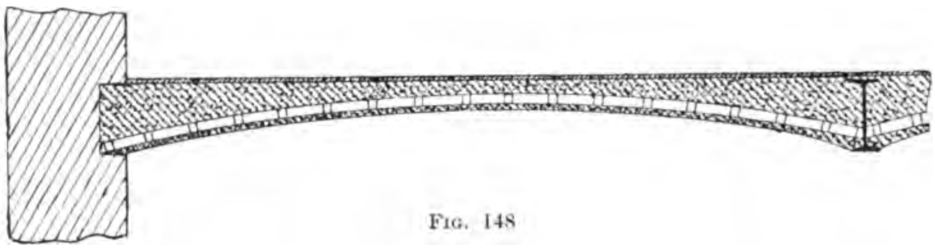


FIG. 148

top flange. The fastening to the walls is formed by a series of horizontal anchor bars, which are hooked round vertical rods built into the wall. At their outer ends the

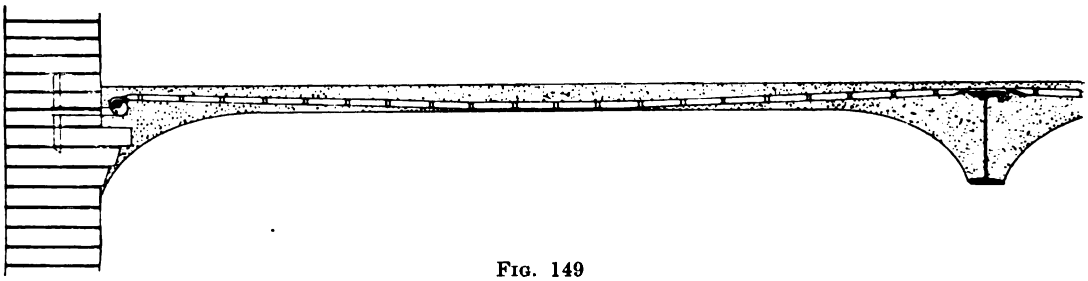


FIG. 149

anchor rods hold a longitudinal rod to which the suspension bars are secured. Fig. 149 shows a floor of several spans, and Fig. 150 a single span floor.

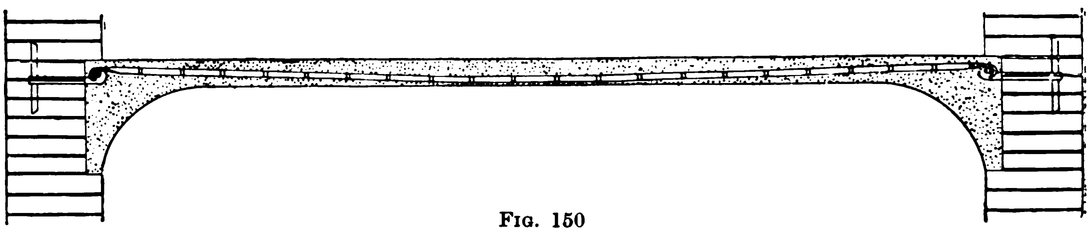


FIG. 150

The concrete is moulded with a curve at the under side at the walls and supporting joists to resist the excess of compressive stress at these points, caused by the building in of the ends due to the anchoring of the reinforcements. In Fig. 149 the wall is corbelled out to receive the floor slab. This method is one frequently adopted, and is sometimes combined with a building in at the top. In this case a small rolled joist resting on the corbelling, and anchored back to the wall, receives the hooked end of the reinforcements on its upper flange.

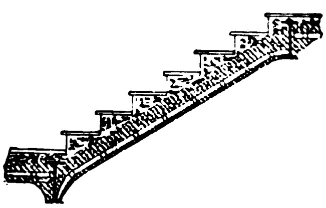


FIG. 151

These floors have been subject to severe fire trials with excellent results.

The stairs on the Mueller, Marx system are supported entirely by the slabs on which the steps are formed. These slabs are reinforced with upright flat bars as shown (Fig. 151), the bars rest on the bottom flanges of rolled joists at the floors and landings, and are tied together by the zig-zag ties in the same manner as floors (Fig. 146).

A further notable invention of this firm is the construction of *abutments to bridges*, in the manner shown in Fig. 152. This is an excellent arrangement, and is

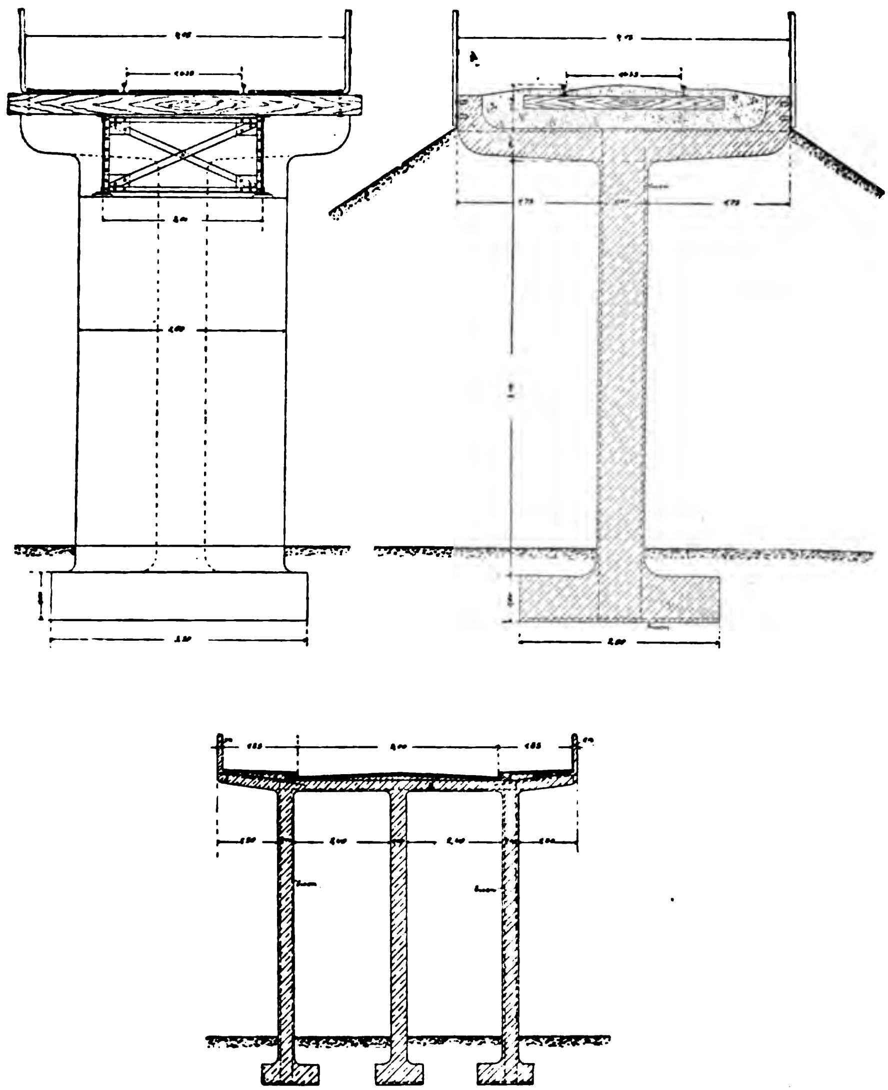


Fig. 152

rendered feasible by the use of reinforced concrete. The saving in cost compared with the usual heavy masonry abutments is said to be 36 to 39 per cent. These abutments are formed with a screen wall at the face, behind which the road or railway is carried in a trough of reinforced concrete, supported on one or more longitudinal walls of the same material, as shown in Fig. 152. These walls are connected to reinforced foundation slabs extending for their entire length. Fig. 153 shows a longitudinal section of this form of abutment and a sectional plan of the supporting walls.

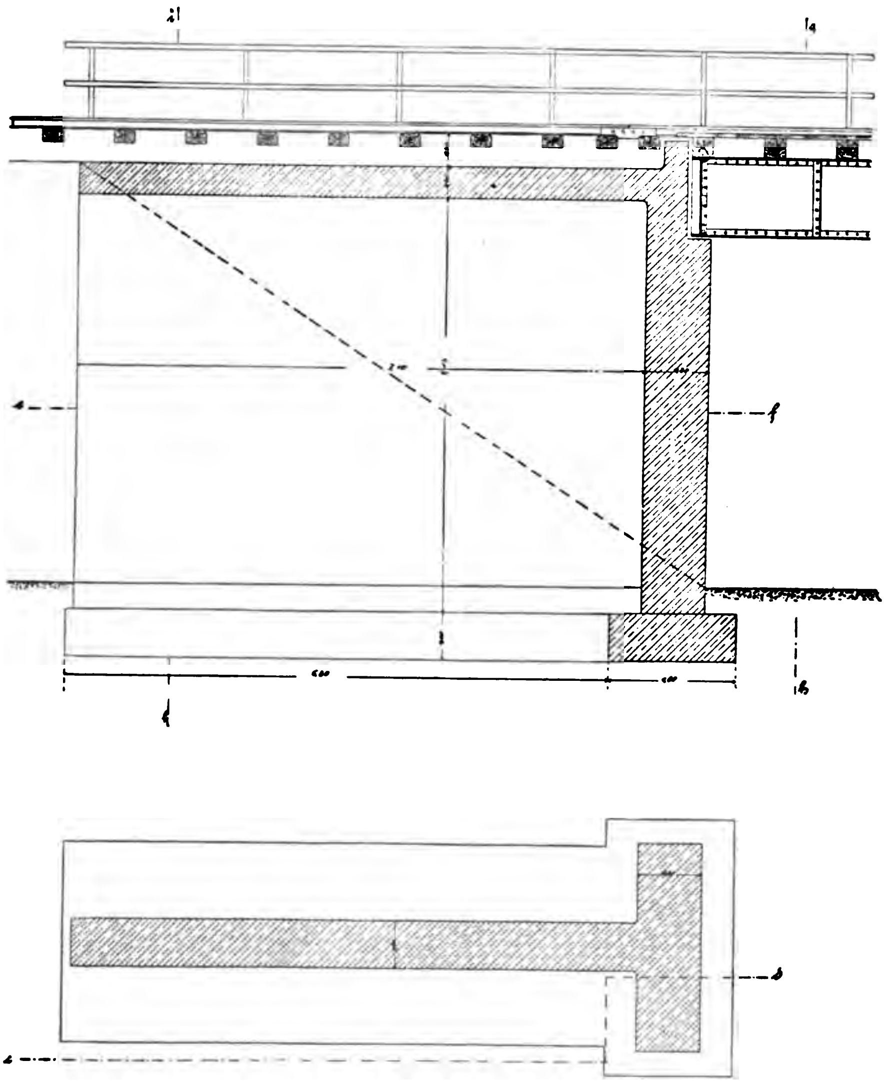


FIG. 153

Pavin de Lafarge System

La Société J. et A. Pavin de Lafarge et du Teil (Ardèche) are well known French manufacturers of limes and cements, who have developed a system of reinforced concrete. They do not now construct in it themselves, but will supply their clients with all information necessary for carrying out the work.

Beams.—La Société Pavin de Lafarge consider that the building in of beams is always more or less perfect, and that the adherence of the mortar causes even a freely supported beam to be partially built in. They use a double reinforcement tied together by transverse reinforcements, as shown in Fig. 154.

Floors.—The floors of this system are usually double, i.e. with floor and ceiling slabs, the beams being hidden (Fig. 155), the ceiling slabs and beams being con-

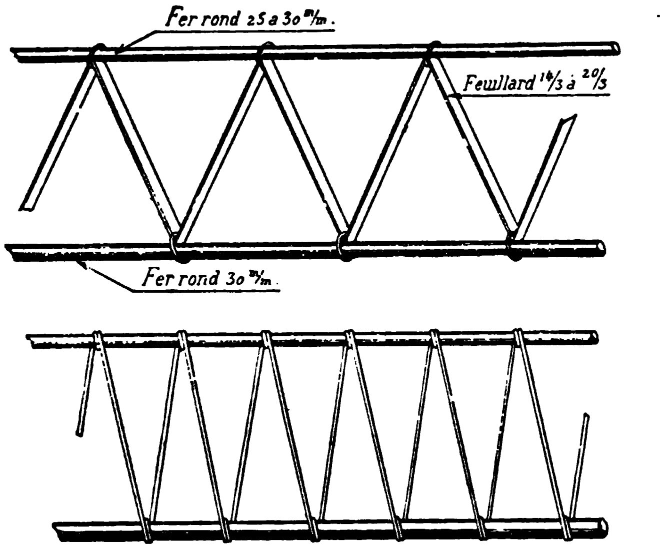


FIG. 154

structed *in situ*, and the floor slab made on the ground and placed in position when the beams have been brought up to the level of its underside. The reinforcement of the slabs is composed of longitudinal and transverse rods.

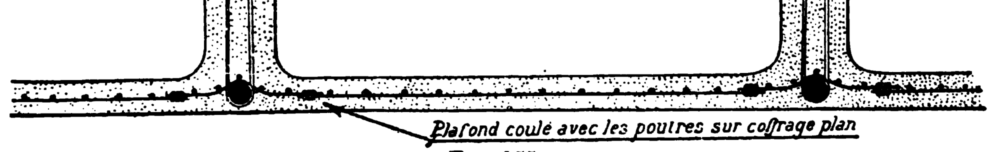
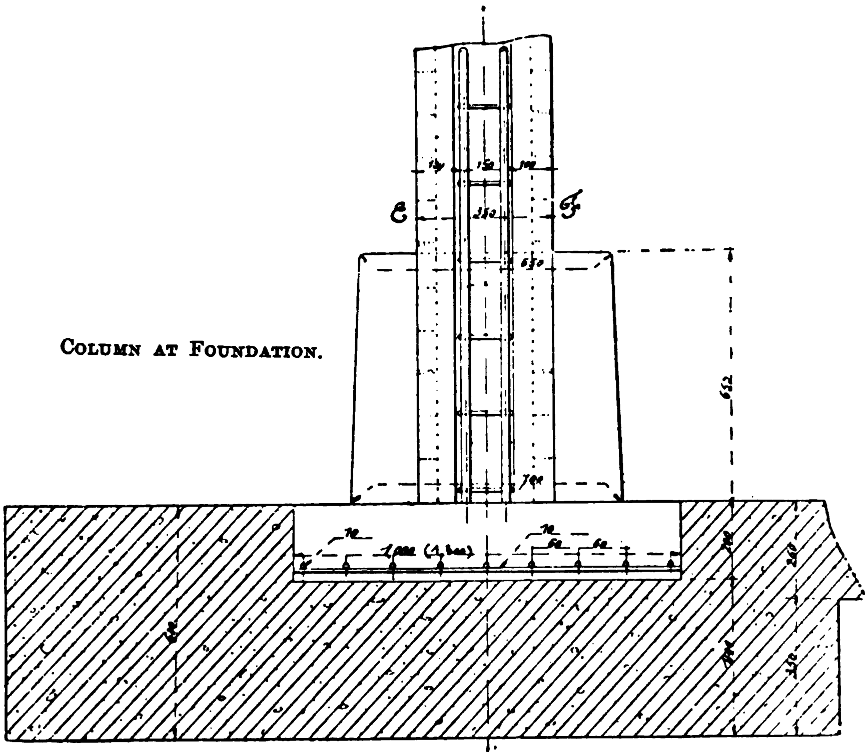


FIG. 155

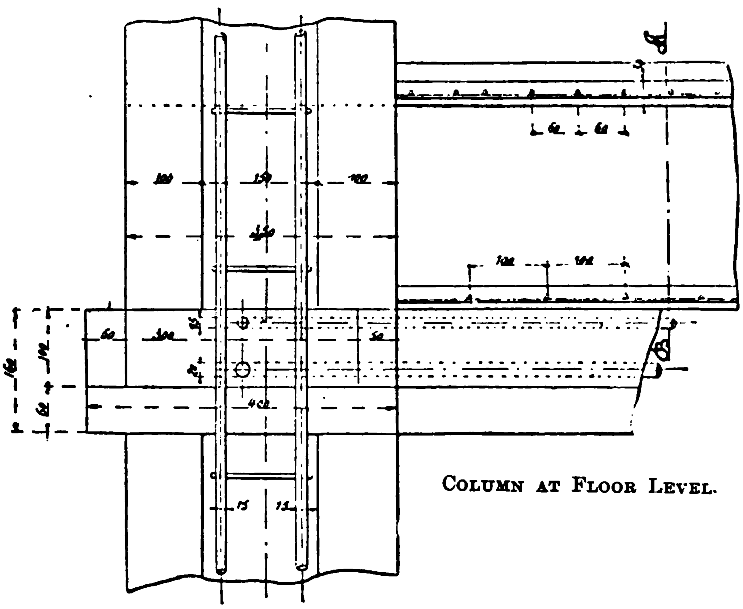
Columns.—The columns are generally made of square section, and are reinforced with four vertical rods with wire cross-ties at frequent intervals. These are supported on reinforced foundation blocks, to which they are united.

Figs. 156 and 157 show the details of a lock house constructed on this system at Lyons by the Canal Company.

Fig. 156 shows sections of one of the columns at the foundation and at the level of the floor, the bottom view being on a larger scale than the top.



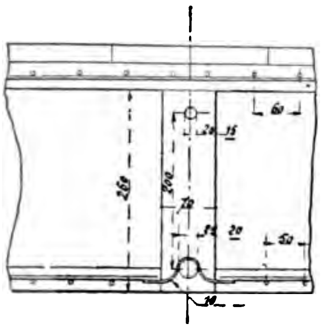
Column at Foundation.



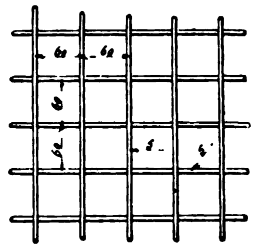
Column at Floor Level.

Fig. 156

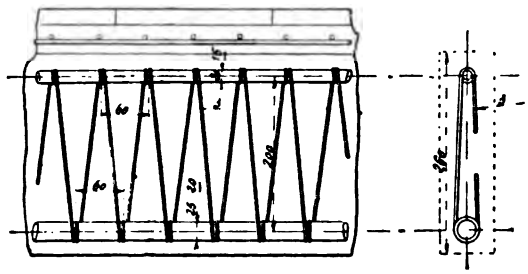
Fig. 157 gives a section of one of the beams with its floor and ceiling slabs and the network of the ceiling slab. The floor network is very similar, excepting that the longitudinal rods are nearer together and the transverse rods farther apart, the



SECTION A B OF THE BEAMS.



NETWORK OF THE CEILING SLAB.



REINFORCEMENT OF THE BEAMS.

FIG. 157

diameter of the transverse rods being double that of the longitudinals. The bottom view shows a longitudinal section of the floor and beam and a cross-section of the reinforcement.

Pipes, Sewers, Conduits and Reservoirs.—The pipes and sewers of this system are reinforced with longitudinal rods, round which a spiral circular reinforcement is wound if the section of the iron permits. When the circular reinforcement cannot be wound spirally, it is formed by a series of hoops placed at equal distances apart.

The lengths of piping thus constructed vary from 3·28 to 6·56 feet, according to the diameter of the pipe. The lengths are connected in the trench by collars of reinforced concrete and special expansion joints, as explained (p. 200).

Fig. 158 shows a longitudinal and transverse section of a pipe, and Fig. 159 an elevation and longitudinal and transverse sections of a horseshoe shaped conduit.

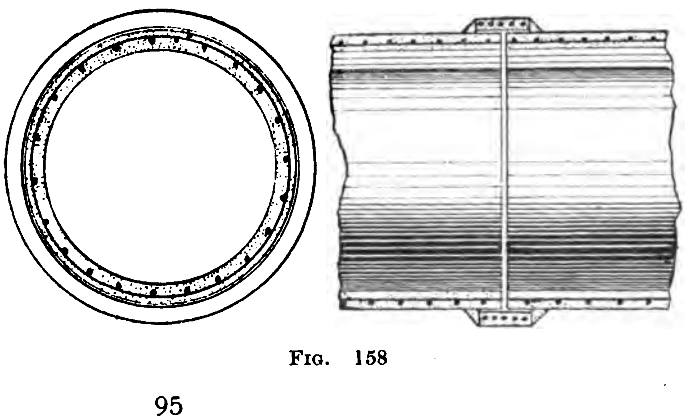


FIG. 158

MM. Pavin de Lafarge have also constructed bridged conduits supported on piers, of the type shown in Fig. 160. The aqueduct shown in the figure had a centre span of 26·24 feet, and side spans of 13·12 feet, the depth and width inside being 2·62 and

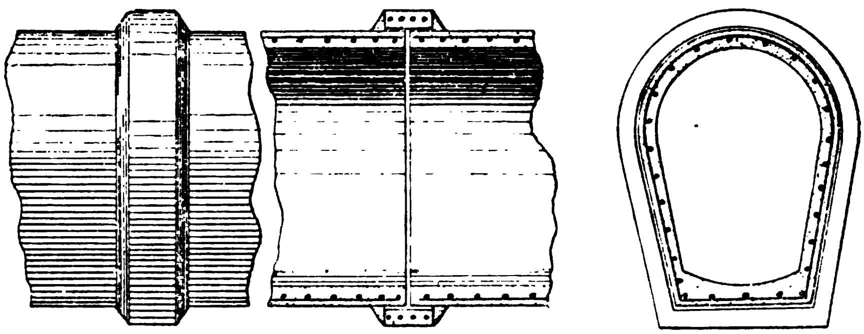


FIG. 159

2·30 feet respectively. The tops of the side walls were connected in places by cross-ties of reinforced concrete, as shown.

The *circular reservoirs* constructed on this system are formed, as shown (Fig. 161), if of large diameter, and are reinforced with flat or channel bars, placed vertically and in the form of hoops, spaced further and further apart from bottom to top. This skeleton is secured together by a few bolts. Between these bars a trellis work of vertical and circular round rods is placed.

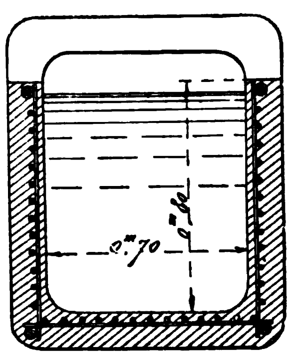


FIG. 160

If the reservoir is covered, the vertical bars and rods are bent over to form the radial reinforcement of the roof. These terminate by being attached to an iron plate at the centre.

Circular wine tanks holding from 2,200 to 3,300 gallons are constructed in the same way, excepting that the hoops are placed at equal distances apart, and the roof has only a single span.

The main hoops are calculated to resist half the pressure, and the secondary hoops of round iron to resist the other half.

The area of the reinforcements in the domed roofs is calculated by M. Godard's formulae, which are as follows:—If $2W$ is the total load supported by the walls, 2ϕ the central angle, v the rise, R the half span, Q the total horizontal thrust in a radial direction, and P the total tangential thrust on a vertical section—

$$Q = \frac{2\,W R}{3\,v}$$

$$P = W \operatorname{Sin} \phi + Q \operatorname{Cos} \phi.$$

And if a hoop is placed at the springing to support the total thrust of the dome, the tension in this ring is $T = \frac{Q}{2\pi}$

Arches.—For small arches not exceeding 30 to 45 feet span, MM. Pavin de

Lafarge employ a single network and use M. Godard's formulae. Employing the same symbols as those for domes, excepting that Q in this case is the horizontal

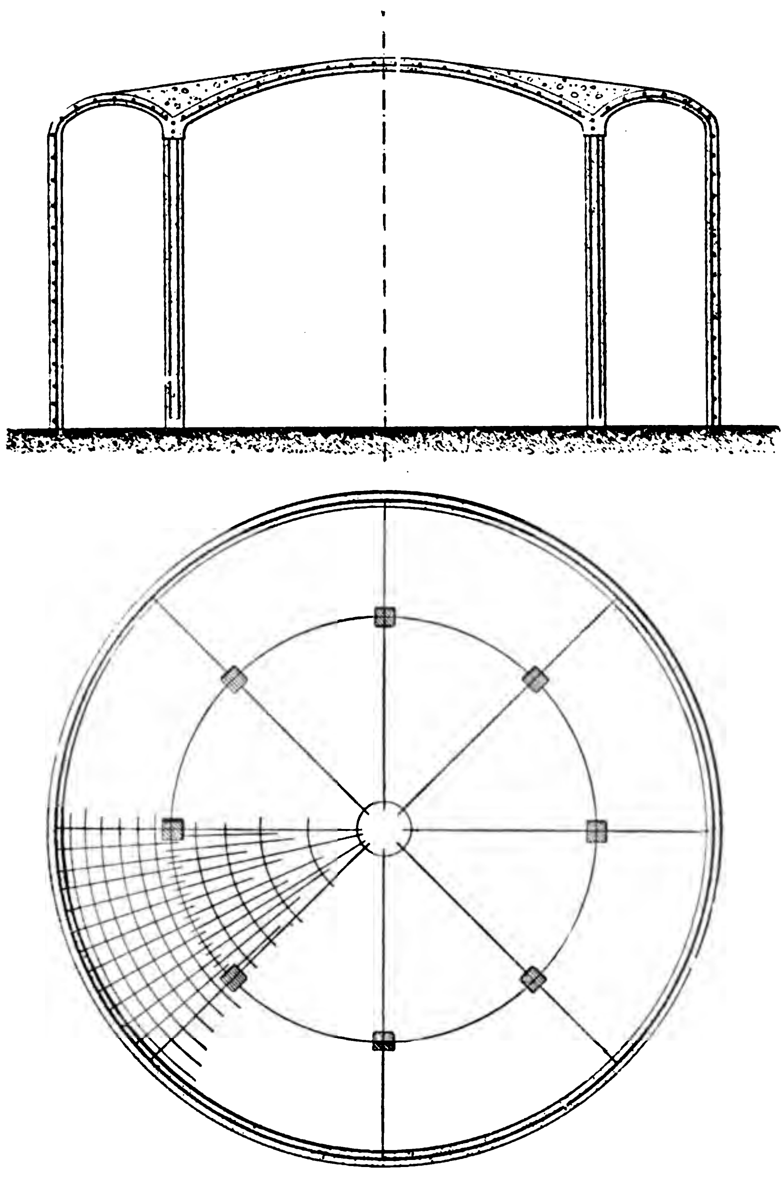


FIG. 161

thrust and P the tangential thrust both per lineal metre—

$$Q = \frac{W\,R}{2\,v}, \text{ and}$$

$$P = W \operatorname{Sin} \phi + Q \operatorname{Cos} \phi$$

the value found for P gives the sectional area of metal required per metre width of the arch.

M. Piketty

M. Piketty is so far original in that he has never instituted a system. He considers that the varying circumstances all require special treatment, and adapts his reinforcements to the exigencies of each case.

M. Piketty considers that a double reinforcement is absolutely indispensable in every case, mainly for the reason that it is impossible to tell whether the building in is perfect or only partial, also for an attachment for the transverse reinforcements, for increasing the compressive resistance and for the resistance of secondary tensile stresses which may be induced by permanent deformations of the concrete.

The beam of reinforced concrete, according to M. Piketty, must be considered as two layers, one represented by the tensile reinforcement, and the other by the portion of the concrete in compression. He believes that it is not sufficient to rely upon the concrete only to unite these two layers, as its resistance to shearing is comparatively small. It is therefore necessary to insert transverse reinforcements in the vertical plane, and for a perfect connexion these must be securely attached to an upper longitudinal reinforcement, on account of the elevated position of the neutral axis which causes the depth of the beam under compression to be small. He is also of the opinion that for rectangular beams the concrete is insufficient to resist the compressive stresses, and consequently that a symmetrical reinforcement should be employed. When, however, the beam is of the T-form and the floor slab aids in resisting the compressive stresses, the upper longitudinal reinforcement may be greatly reduced.

M. Piketty prefers round rods to flat bars or hoop iron for reinforcements, as the flats separate the concrete for a greater width. The transverse reinforcements should, according to M. Piketty, be able to resist the tensile stress caused by shearing. He therefore places them at an angle of 30° near the supports where the shearing force is greatest, while towards the centre, where it is greatly reduced and under rolling loads may induce tensile stress in either direction. he places them vertically. This disposition is shown in Fig. 162, the transverse reinforcements being inclined

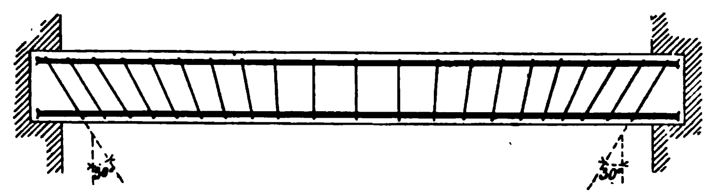


Fig. 162

at an angle of 30° to the vertical near the supports, and approaching the verticals gradually as the centre of the span is approached.

When M. Piketty foresees the existence of shearing stress of considerable magnitude he adopts a different method, and places vertical transverse reinforcements throughout the whole span, and also over the supports, to which he adds in the neighbourhood of the supports inclined rods of larger section than that of the vertical reinforcements, placing them at an angle of 30° to the horizontal. These he secures firmly to the longitudinal rods, as shown (Fig. 163).

The inclined transverse reinforcements are firmly secured to the bottom longitudinals, either by being looped behind a ring shrunk or clamped on to the longitudinal, or by being held in a small notch cut in the underside.

M. Piketty, in his calculations, neglects the tensile resistance of the concrete, and considers the stress strain curve of the concrete in compression as a straight line. He further takes into consideration the different values of the coefficient of

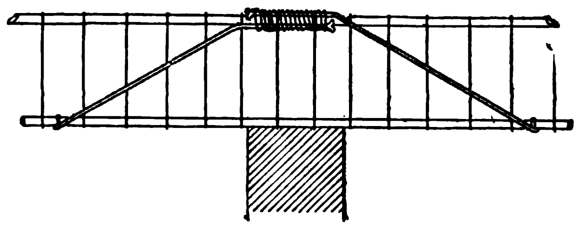


FIG. 163

elasticity of the two materials. M. Piketty has constructed reservoirs and arched and straight bridges, besides floors, with their supporting columns. He has also constructed with reinforced brickwork and masonry.

Rabitz System.

Herr Rabitz, of Berlin, uses ordinary galvanized wire networks for the purpose of reinforcement. These have either diamond or hexagonal shaped meshes, and are provided in rolls about 3¼ feet wide.

Floors are of the double type, and are either formed of reinforced floor and ceiling slabs, or more generally the ceiling slab only is formed of plaster on the wire mesh, the floor being of the ordinary construction.

The air space between the ceiling and floor forms a protection against fire, and as an extra precaution the ceiling slab is generally covered with a layer of cinders.

The wire mesh is stretched tightly across the span and attached to beams of timber or rolled joists, the ceiling slab being frequently further supported by intermediate suspension rods.

The thickness of the ceiling slabs varies between 0·79 and 1·18 inches.

Partition walls are formed of plaster on the galvanized network, which is stiffened along its edges with rods 0·39 inches diameter, and stretched tightly, being attached between pairs of angle irons back to back. These angle irons are secured by hooks or screws to uprights of iron or timber forming the framework.

For long partitions intermediate stiffeners of rods, or angle irons back to back, are employed and also diagonal rods.

The thickness of the partitions is usually about 2 inches.

Double partitions are also constructed of two single thicknesses of 1·18 inches, and a space of 2 inches, the bays being framed with timbers having grooves in which rods of 0·31 inches diameter are placed, the networks being attached to these. Sometimes the bays are framed with iron to which the networks are fixed by hooks. When the partitions abut against masonry walls the galvanized mesh is attached to pieces of timber dovetailed into the masonry.

Outer walls are constructed in a similar manner to that described for partition walls.

A speciality of this system is the *protection of slopes* to reservoirs and water channels by reinforced slabs, a form of construction introduced by Herr Rabitz in 1898.

Ransome System

This system, invented by Mr. E. G. Ransome, was one of the first to be introduced in America, and is largely used in the United States at the present day.

The reinforcing bars are of square section, and are twisted cold, with a varying number of twists per lineal yard. This treatment greatly increases the ultimate strength and elastic limit, and prevents any tendency of the reinforcements to sliding. The system is employed for complete buildings, and also for straight and arched bridges.

The *beams and floor slabs* are constructed together and are frequently very similar to those of the Hennebique system, except that square twisted bars are used for the reinforcements in the vertical plane. A special arrangement for a floor with exposed beams is shown in Fig. 164. The bays are generally square,

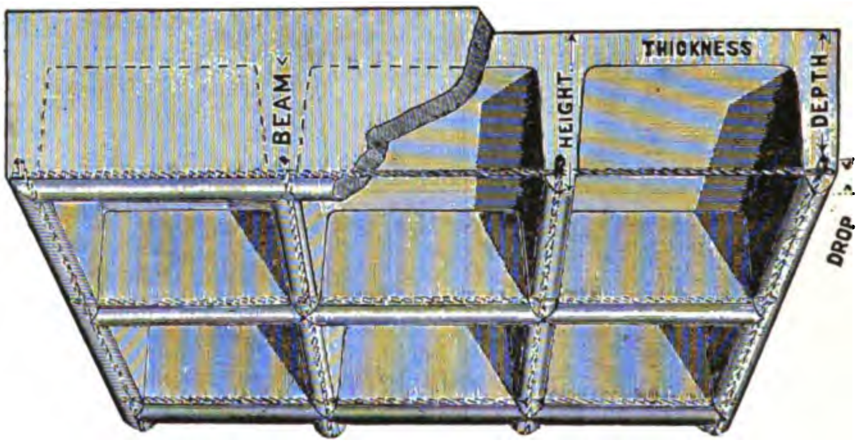


Fig. 164

and the beams, both longitudinal and transverse, have the same depth, and are usually reinforced with one twisted steel bar along the bottom. The floor slabs are reinforced with a series of bars near the under surface, with a few cross-bars spaced some distance apart.

Fig. 165 shows a floor with a ceiling supported from the bottoms of the beams, and Fig. 166 the method for attaching the timbers to which the ceiling laths are nailed. Floors with longitudinal and cross beams of the ordinary form, the bays being rectangular, are also employed.

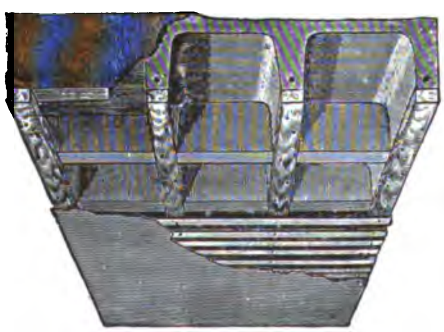


Fig. 165

Fig. 166

The *walls and columns* on the Ransome system are reinforced with vertical twisted steel bars.

Foundation blocks are reinforced by bars crossing at right angles and diagonally.

Arched bridges are reinforced with one series of bars near the intrados, and these are united where lengthening is necessary with sleeves which screw on to the twists of the bars. The Aberthaw *pavement lights*, reinforced with Ransome bars, are being largely employed. A description of these will be found (p. 495). Many very fine buildings, important bridges and tall chimney stacks have been constructed in the United States on this system by various licensed constructors, amongst which may be mentioned the Aberthaw Construction Company, of 8, Beacon Street, Boston, U.S.A.; Messrs. A. Mönsted and Company, Milwaukee; the Donnelly Contracting Company, Buffalo, N.Y.; the Ransome Construction Company, Philadelphia, Pa.; and The Ransome Concrete Company, of 26, Broadway, New York, which is the head office of the firm.

Roebling System

The Roebling Construction Company, of 121, Liberty Street, New York, make a special feature of fireproof floors, partitions, and columns. They have carried out exhaustive tests on their method of construction, with very satisfactory results.

They employ rolled joists as beams, and also for columns, but sometimes form these latter of a built-up section composed of plates and *Z*-bars. Both the beams and columns are protected by concrete and a wire mesh covered with plaster. Several methods are used in the construction of floors.

Fig. 167 is a general view of their *arch* construction with flat ceilings.

This consists of a wire mesh arch, stiffened by steel rods woven in, which is sprung between the secondary floor beams, and abuts into the seat formed by the web and lower flange of the I-beams. On this wire centreing the Portland cement concrete is deposited.

The ceiling consists of a system of supporting rods or flat bars set on edge, attached to the lower flanges of the secondary floor beams by a patent clip, which sets them below the bottom flanges. Under these rods or bars, and securely laced to them, is the Roebling standard wire lathing, with quarter-inch steel rods woven in every 7½ inches, as stiffening ribs crossing the supporting rods at right angles. This lower mesh receives the ceiling plaster. The figure shows the con-

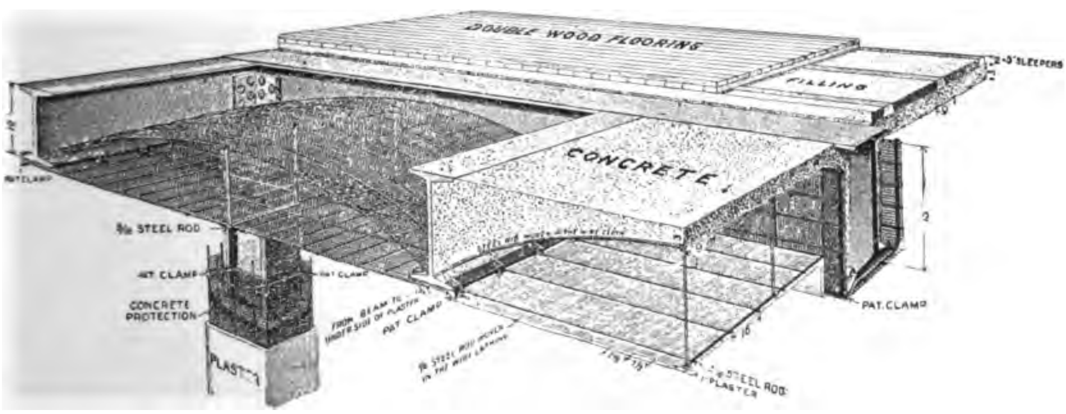


Fig. 167

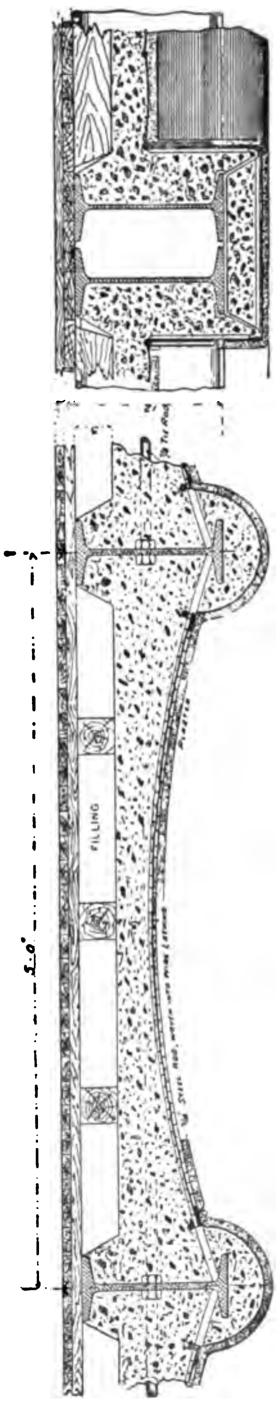


FIG. 169

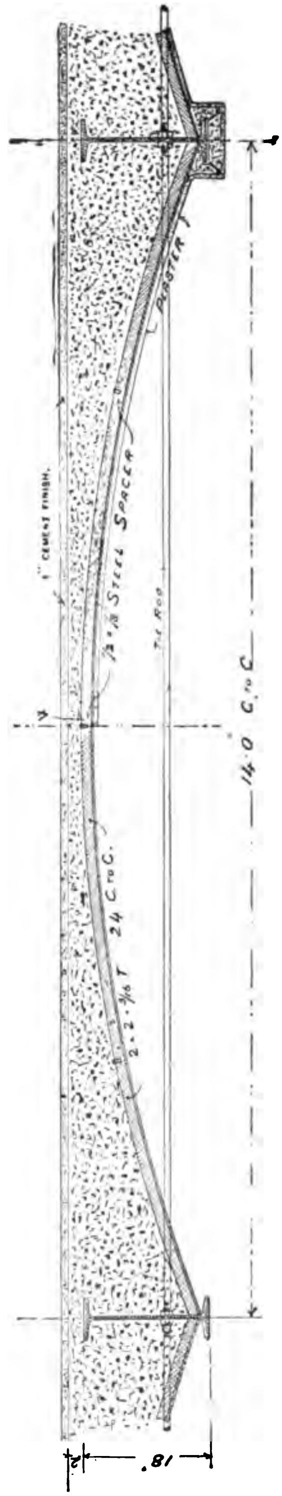
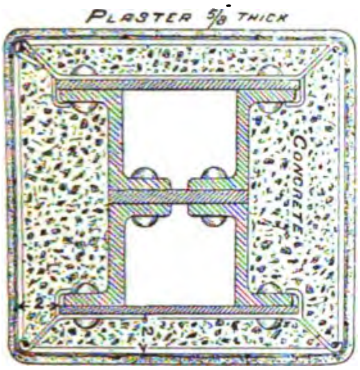


FIG. 170

struction very clearly. Fig. 168 shows a section of a supporting column and of main beam.

Sometimes the ceiling is arched, and the cost thereby reduced. When the beams



Typical Column Section.

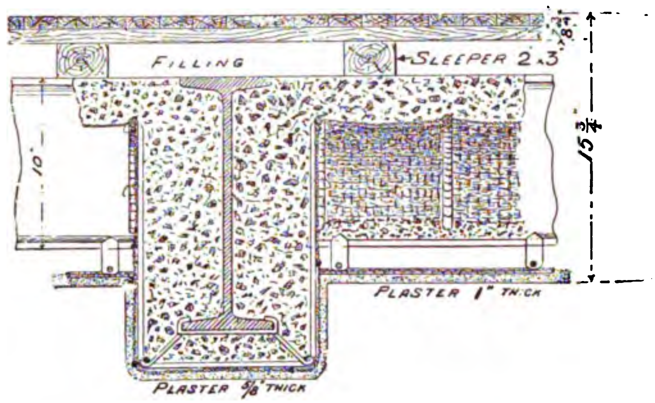


FIG. 168

are not too far apart and no piping or wiring is required below the floor, the bearers for the flooring are depressed, as shown (Fig. 169). The section here shows double joists for the main beams, but, frequently, only one is used. If the beams are farther apart, or a hollow beneath the floor and across the secondary joists is necessary, the concrete filling is carried up to the top of the I-beams and the bearing timbers for the floor boards run over the top flanges.

Another form of *arched floor* is shown (Fig. 170). Here curved T-section ribs are used instead of the steel rods in the arch netting. The Tees are spaced two feet apart, and held rigidly in position by means of steel spacers. Wire lathing, with a woven-in stiffening rib is laid between the Tees and laced to them, the concrete being laid on this netting in the usual manner, the webs of the T-ribs being embedded. This form of construction is used where the beams are more than 10 feet apart. In spans greater than 12 feet, and where loads greater than 500 pounds per square foot have to be supported, the tie rods are specially designed to resist the thrust.

Special specifications for sizes of rods, etc., are issued for each type of floor.

Flat floors are constructed in much the same manner. Fig. 171 shows a general

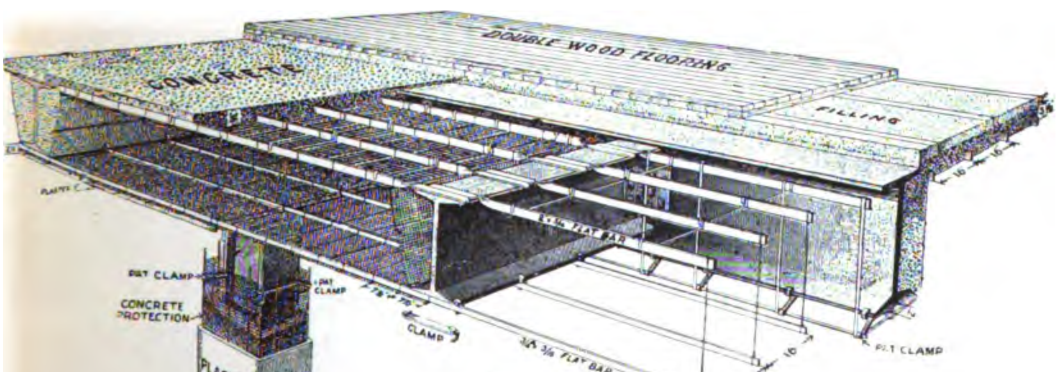


FIG. 171

view of such a floor. A light iron framework spans the interval between the joists, consisting of flat iron or steel bars set on edge and spaced 16 inches centre to centre, with a quarter turn at both ends where the bars rest on the beams. Spacers of half oval iron are placed at suitable intervals to separate and brace the bars.

The Roebling standard wire lathing, with the quarter-inch steel stiffening rib woven in every 7½ inches, is attached to the underside of the bars, the stiffening ribs running cross-wise under the main bars and laced to them at every intersection. Cinder concrete is deposited on the wire lathing, which thoroughly embeds the light framework.

The ceiling construction is the same as that described for the arched floors

The beams and columns are treated in the same manner as those for the arch construction.

As in the case of the arched floors, there are several types of flat construction.

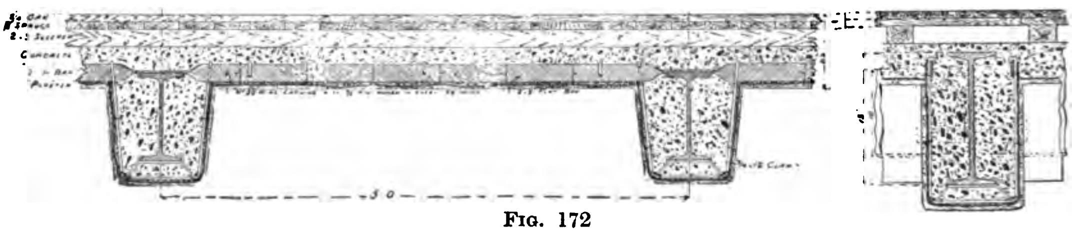


FIG. 172

The cost is sometimes reduced by allowing the beams to show and forming the ceiling just below the floor slab, as shown (Fig. 172).

Another form adopted for light floors of large spans is shown (Fig. 173).

The flat bars are here bent downwards two inches or more at the centre of the span.

This type of floor is said to be more economical than any other when the joists

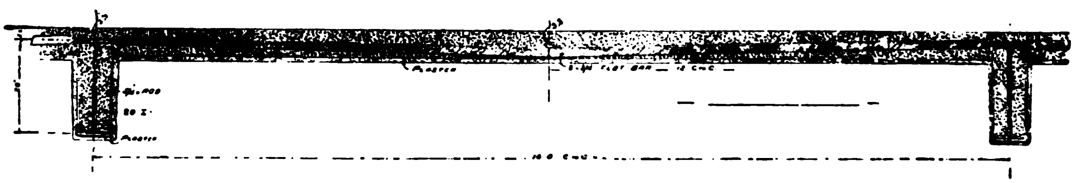


FIG. 173

are more than 9 to 10 feet apart centre to centre. It has been used for spans up to 22 feet, giving every satisfaction.

If cinder concrete is used for filling over the network no bearers are required, the floor boards being nailed to the concrete direct.

The bearers may be depressed as described for the arch construction, and shown (Fig. 169). In this case the underside of the concrete slab is finished just below the bottom of the beams.

Partition walls.—Messrs. Roebling form their solid partition walls of concrete with vertical channel irons and their patent wire lathing with steel stiffening ribs, as shown (Fig. 174). The concrete is of cinders, and will therefore receive nails; no furring is required, and convenient vertical spaces can be left for piping, wires, speaking tubes, etc. The manner usually adopted for forming the door frames is clearly shown in the section.

Hollow partitions with no concrete filling are also used, the vertical channel irons being replaced by flat bars. The door framing in this case is formed in the same manner as for the solid partitions.

Another form of partition is constructed with only one layer of wire lathing with vertical channel irons of light section, the whole thickness being filled with plaster.

Outer walls. — These are formed of brick or stone, and have an outer layer of plaster on wire lathing, supported by vertical V-irons, leaving an air space between the plaster and the main wall.

The wire lathing also lends itself to the formation of ornamental coverings to beams and similar internal decoration.

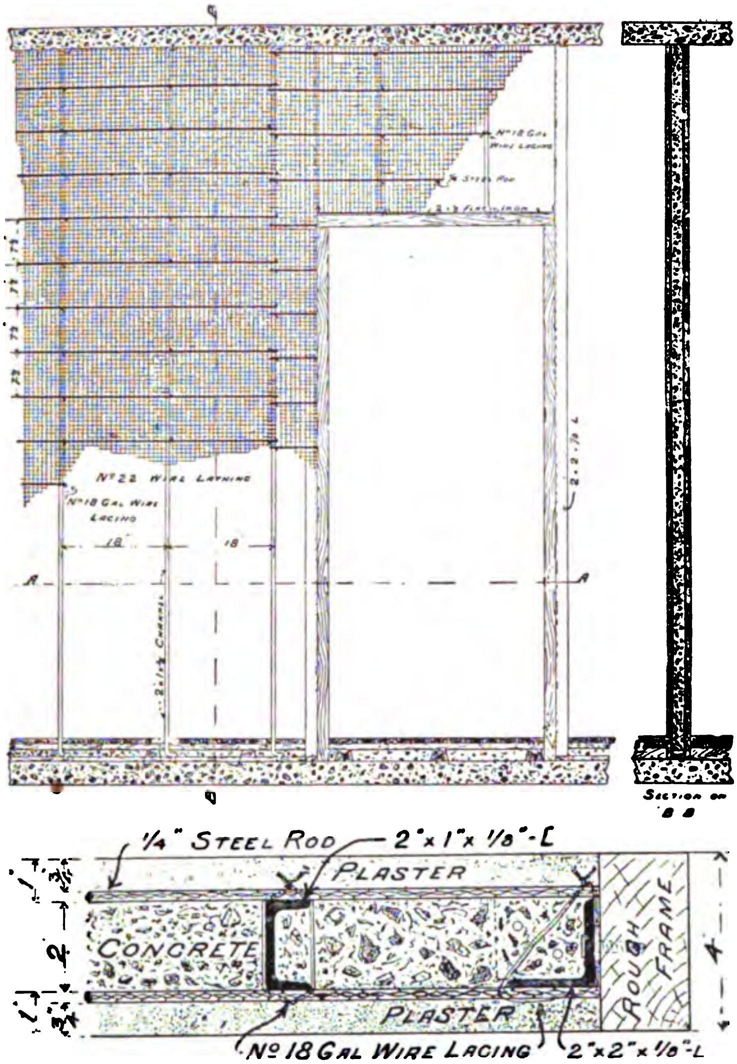


Fig. 174

Arches, domes and alcoves up to 12 feet span are also constructed of the stiffened wire lathing.

Sanders System

This system is constructed by the Amsterdamsche Fabriek von cement-ijzerwerken, of 108 Wittenburgerstratt, Amsterdam. The Monier system is adopted generally for all works ; but the beams, slabs, columns, etc., are designed according to the calculations established by Herr L. A. Sanders. Herr Sanders considers that the stress-strain curve of the concrete in compression and tension is parabolic, and allows a certain resistance for the concrete in tension. He takes into account the respective coefficients of elasticity of the materials. To simplify his calculations he uses straight line stress strain curves, these lines cutting through the parabolic curves so as to equalize the triangular and parabolic areas. Round bars are used for all the *beam* and *slab* reinforcements.

The transverse shearing reinforcements usually embrace two bottom main bars, and are hooked over those at the top.

Sometimes in floors two sets of transverse rods are used, passing along near the bottom and top surfaces for distances covering two rods. They are then bent up or down, as the case may be, continuing near the opposite surfaces for a similar distance, when they are bent again, and extend in this sinuous form across the whole width of the slab.

Many important works have been carried out by this firm, including floors, stairs, subways, reservoirs, bridges, etc.

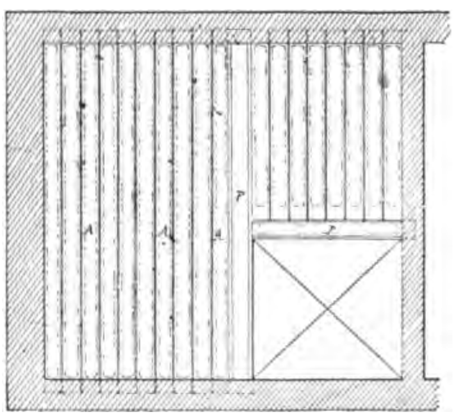
Siegwart System

This system is employed in France, Switzerland and Italy, mostly for floors of small span. The Italian and German representatives are G. A. Porcheddu of Milan and J. Gerstenecker of Munich.

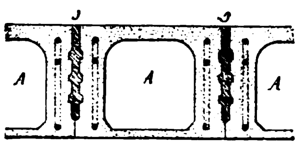
The *floors* are said to be very cheap and are easily constructed. The various types are shown in Fig. 175. The beams are made on a slab of cement in a shed; they are at the same time formed by a machine which cuts and smooths them, so that they have a width of 9·8 inches, and the form shown section (*B*). After setting they are conveyed to the site of the work, and placed in position against one another, the hollow joints being filled with cement grout.

FLOORS OF HOLLOW BEAMS—SIEGWERT SYSTEM.

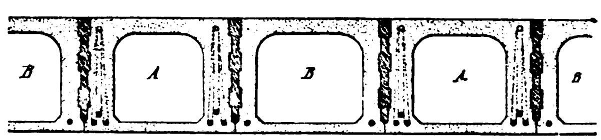
(A)
Plan of a Floor.



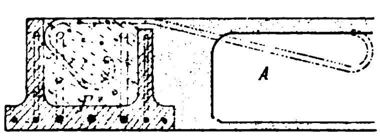
(B)
Section of Beam.



(D)
Combination of Hollow Beams and Beams with only the bottom Slab Reinforced.



(C)
Section of Reinforced Hollow Beams.



(E)
Combination of Hollow Beams and Double Beams with only the bottom Slab Reinforced.

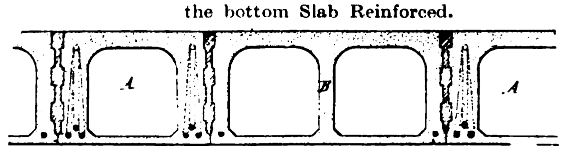
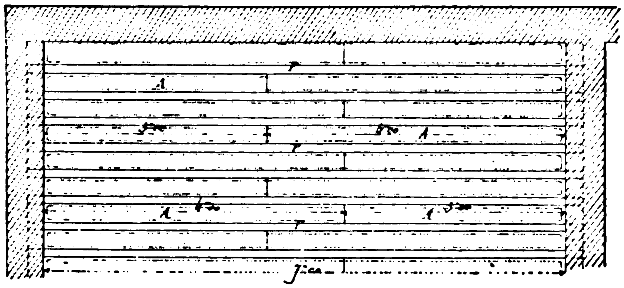
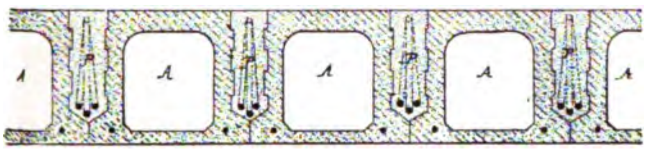


FIG. 175

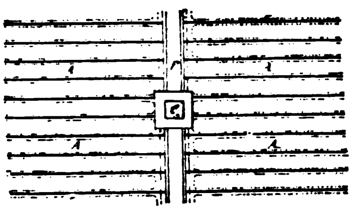
(F)
Floor of 23 Feet Span.



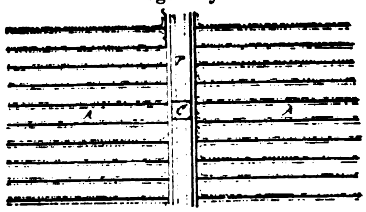
(G)
Section of Floor 23 Feet Span.



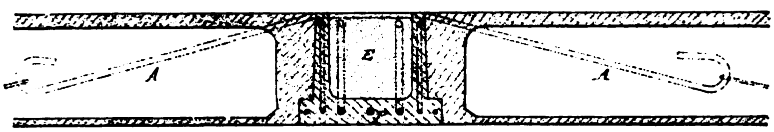
(H)
Hidden Hollow Stringer dividing large Bays.



(I)
Visible Hollow Stringer dividing large Bays.



(J)
Section of a Hidden Hollow Stringer.



(K)
Section of a Visible Hol'ow Stringer.

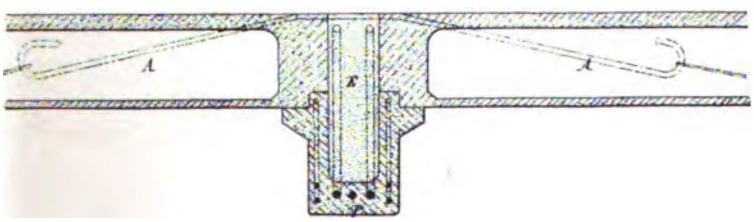


FIG. 175

The plan (A) represents an ordinary floor, for which the beams A are shown, section (B); the beams P must be specially reinforced, as they support a part of the floor; they have the shape shown, section (C), being in the form of troughs, which are filled with concrete when in place.

In the case of very small spans light beams only are used, such as those shown, sections (*D*) and (*E*); their sides are thinner and they have a reinforcement only along the bottom slab.

The plan (*F*) and section (*G*) show the arrangement adopted for a floor of 23 feet span.

As beams of this length cannot be conveniently transported, lengths of 9·48 and 13·12 feet, formed as shown *A* section (G), are employed, which are placed end to end. The neighbouring pairs break joint with each other, and the longitudinal spaces *P* are of sufficient width to form beams strong enough for the span of 23 feet. In these spaces the reinforcing rods are placed, after which the troughs are filled in with concrete in well rammed layers, in the same manner as ordinary beams.

The plans (*H*) and (*I*) and sections (*J*) and (*K*) show the method adopted when the span exceeds about 26 feet; the floor is then divided into two bays by trough-shaped beams or stringers, which support the hollow beams of the usual form. The stringers rest on columns, and may be made so as not to show, as in section (*J*), or projecting below the floor as in section (*K*).

These floors cost between 7½*d*. and 1*s*. 1*d*. per square foot, according to the span and load. They have the advantage of doing away with the necessity of falsework, and also may be brought into use very soon after being constructed, excepting in the case shown in section (*G*) and plan (*F*), where the concrete forming the beam must be given time to thoroughly set.

Stolte System

This system, which is constructed by the firm of Deutscher Cementbau-Gesellschaft Paul Stolte, of Berlin, consists of reinforced hollow blocks moulded in advance, of which floors are constructed having spans up to 8·20 feet. These blocks are 9·84 inches wide and from 3·15 to 3·94 inches deep, and have longitudinal hollow spaces, between which upright flat bars are embedded, as shown (Fig. 176).

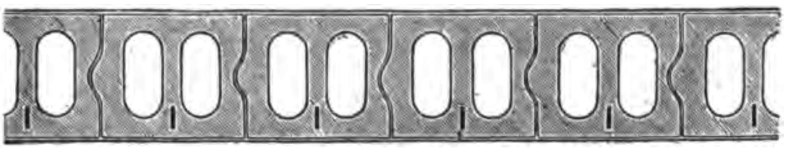


FIG. 176

The blocks are laid across between rolled joists, which form the beams for the floor, no scaffolding being required. The joints between the blocks are made with cement mortar. Timber beams are sometimes used instead of rolled joists for supporting the slabs. The blocks are laid either on the top or bottom flanges when rolled joists are used.

The Thacher System

This system is used for *arched bridges*. The reinforcement is formed of a double series of flat bars rolled with projections like rivet-heads or of rods flattened out alternately in planes at right angles to one another. One series of these are placed near the intrados, and the other near the extrados, of the arch. The object of the projections on the bars is to prevent any tendency of slipping through the concrete.

This system is constructed by the Concrete-Steel Engineering Company, Park Row Buildings, New York.

De Vallière System

M. E. de Vallière, having constructed in reinforced concrete for some years, has lately introduced a system of his own, which is constructed by the firm of de Vallière, Simon and C[ie], Place de la Cathédrale, Lausanne.

The chief difference to other systems is in the transverse reinforcements, which are formed in long lengths, bent up and down. The main rods are passed through

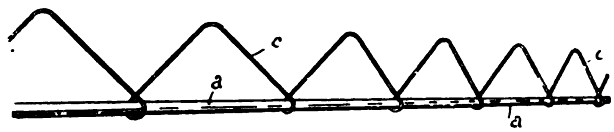


Fig. 177

these, and they are then pulled out to any spacing that may be required. Fig. 177 shows this arrangement.

The *floor slabs* are reinforced with a series of rods along the bottom, running across from beam to beam; these are held to the upper surface of the concrete by transverse reinforcement, as shown (Fig. 178).

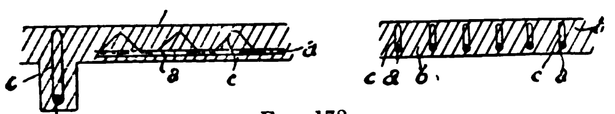


Fig. 178

Freely supported *beams* are reinforced only along the bottom, but if built in bent-up rods are added, either one or more being used in each set.

Fig. 179 shows a beam with one rod, and the disposition of the transverse reinforcements.

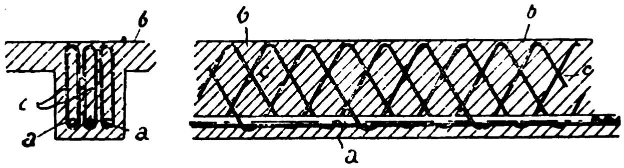


Fig. 179

Figs. 180 are similar views of a beam with three rods. The construction is monolithic, and the beams are designed as of T-section.

The methods of calculation employed by M. de Vallière are those recommended by Professor Ritter, of Zurich [p. 360]. Steel only is used for the reinforcements. This firm have constructed reservoirs, gasometer tanks, aqueducts and bridges, as well as floors and covers to reservoirs. The largest covered reservoirs as yet constructed were two for the town of Cully, having each a capacity of 441,500 gallons.

Fig. 181 shows the section of a bridge for a pipe of 19·7 inches diameter, carrying water from the high

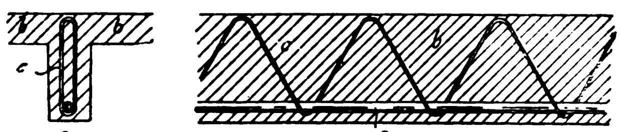


Fig. 180

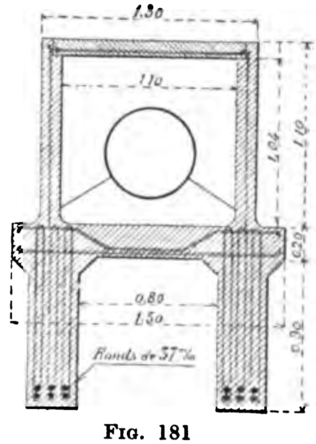


Fig. 181

country to Lausanne, across the Bay of Clarens. This bridge consists of 4 spans, one of which is 52½ feet. The whole length of the bridge is 137·4 feet, the width being 4·92 feet. The figure is a cross-section of the 52½-foot span.

Visintini System.

This form of construction has been recently brought out by M. Fray Visintini, of 12, Gartenstrasse, Zürich. All the portions of a structure in this system are

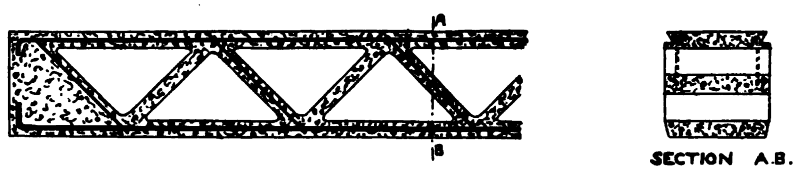


FIG. 182

made in advance. The *floors or roofs* are formed of a series of pieces of lattice construction, as shown in Fig. 182, laid side by side, with the joints run in with grout—these constitute the whole floors when the spans are small, but, when large spans are necessary, intermediate beams of a similar lattice type are employed on which the floor pieces are laid. The roof pieces are sloped down at the eaves and have narrow vertical ribs along the edges throughout their whole length, which raise the joints slightly above the general level. The use of the lattice form enables a considerable saving in weight to be obtained, the concrete which is eliminated between the latticings being considered as serving no useful purpose. The latticings sloping towards the centre have reinforcing bars embedded in them, these bars being hooked round the main upper and lower rods, but the latticings sloping towards the supports are not reinforced since they will act only in compression.

Figs. 183 show the construction of *columns* on this system. These support sills

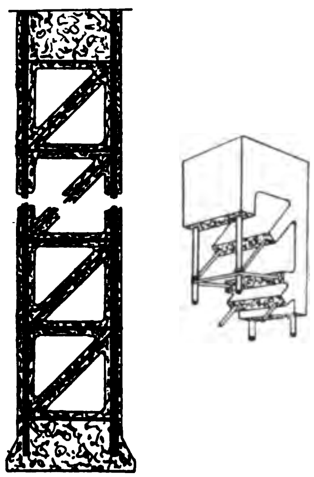


FIG. 183

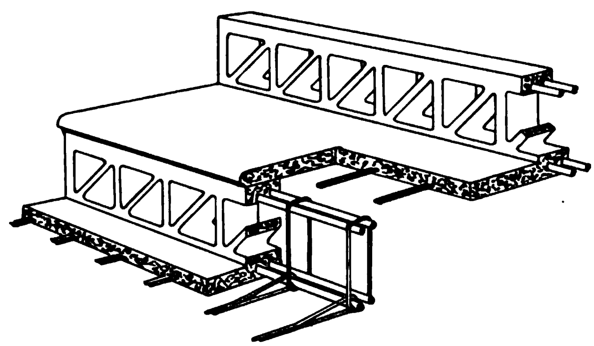


FIG. 184

of lattice construction usually of a similar form as the floor pieces and beams, but in these the latticings near the centre are all reinforced. Sometimes the sills have latticings similar to those of the columns.

Fig. 184 shows the method adopted for the construction of *stairways*, the risers being built into the wall either at one or both ends. The wires forming stirrups of the risers are continued to form the reinforcements of the treads.

For *wall* construction a series of the columns are placed side by side.

Besides the advantages pertaining generally to the moulding of the parts in advance, this system has a special advantage due to the form of the pieces which lends itself to the attachment of the lifting gear, and the handling while setting in position.

The disadvantages are those obtaining in all pieces moulded in advance, when compared to monolithic construction, in which the separate parts can be thoroughly tied together as they are brought up.

This system besides being employed for buildings is also well adapted for the construction of light footbridges. For floor spans between 6·56 and 19·78 feet to bear a load of 51 pounds per square foot. The width of the floor pieces is generally 8 inches and their depth varies as a rule between 6 and 8¼ inches. The upper slab of the piece has a thickness of from 1 to 1·38 inches, the lower slab from 1 to 1·8 inches, and the latticing from 0·60 to 0·80 inches. The diameter of the upper and lattice rods is usually—0·157 inches, while the bottom rods vary from 0·275 to 0·67 inches.

The proportions used for the mortar of which the pieces are constructed is 1 of cement to 3 of sand.

The system is represented in England by Mr. C. H. Reynolds, A.M.I.C.E., A.M.I.M.E., Clevelands, Manor Road, Teddington.

Walser-Gérard System

This system originated in Switzerland, but has recently been employed largely in Italy by M. Maciachini, who took up the Italian patents.

Beams and Floors.—The beams have always a double reinforcement, generally unsymmetrical. The number of top rods is always one in excess of those at the bottom in order that the transverse wire reinforcement may be bent round both series as shown (Fig. 185). The lower rods of the secondary beams pass over those of the main beams. For light loads only one rod is placed at the bottom, and two at the top, while when the main beams require special strength two series of rods are used in the bottom, the longitudinal reinforcements of the secondary beams passing between them.

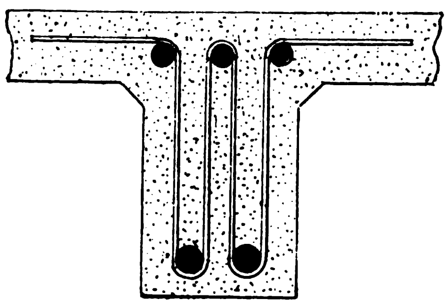


Fig. 185

The floor slab is usually reinforced with rods perpendicular to the direction of the beams; passing under their upper reinforcements. These are not shown in Fig. 185.

Occasionally the upper rod is added to the floor reinforcement, only extending a short distance on each side of the beam. In this case the floor rods and the ends of the stirrups are tied together with wire wrapping.

When the beams are heavily loaded, the transverse reinforcements are placed as shown (Figs. 186, 187), the centre stirrup being inclined in order to better resist the shearing stresses.

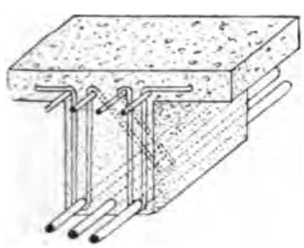


Fig. 186

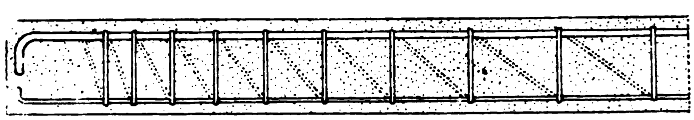


Fig. 187

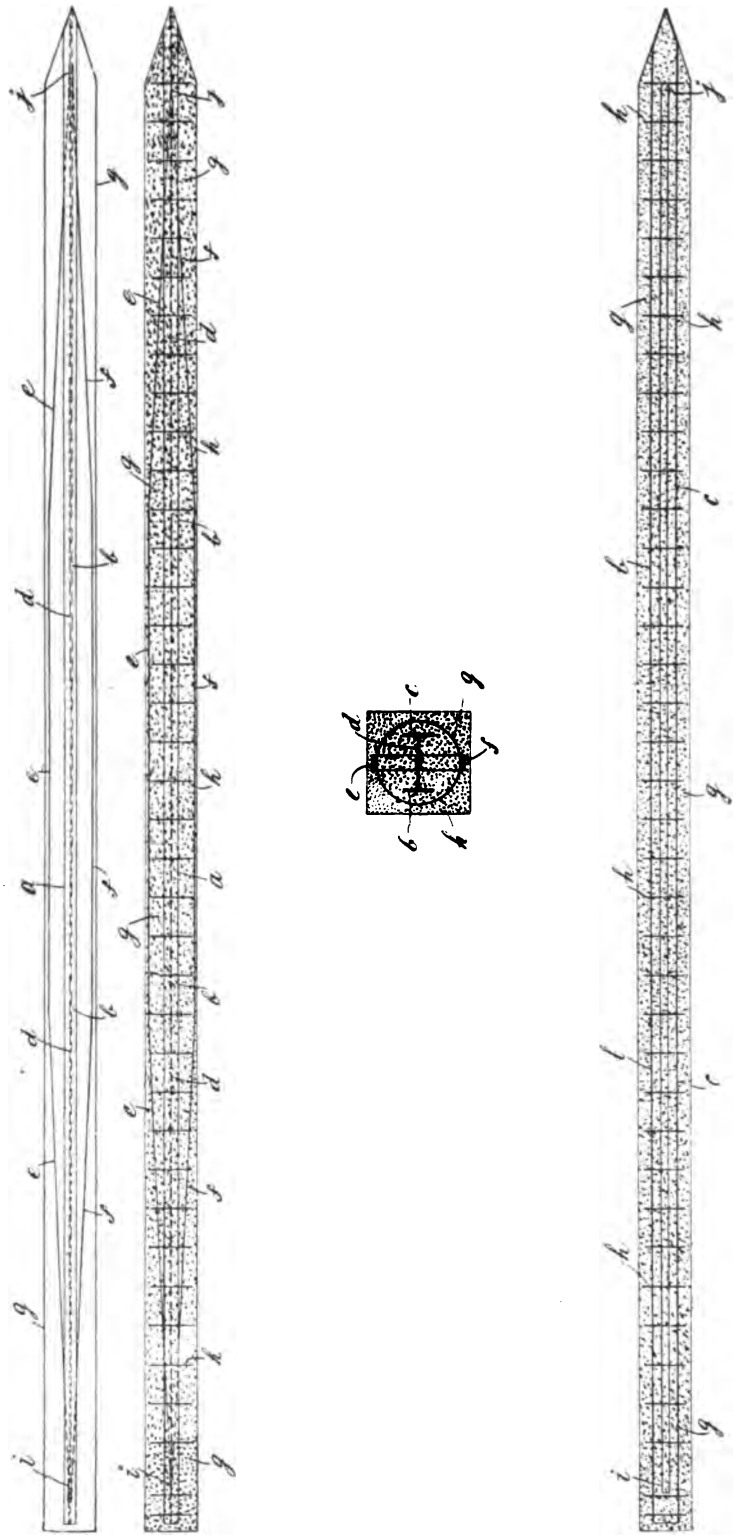


Fig. 188

If the reinforcements of the floor slab run parallel to the beams, short rods are placed across below the upper beam reinforcements, and rest on the neighbouring floor rods. It will be seen that in this system all the reinforcements are excellently arranged so as to give mutual support.

The Williams System

This system has been recently patented by Mr. A. E. Williams, A.M.I.C.E., A.M.I.M.E., of Dagenham Docks, Essex.

Mr. Williams has for some time taken considerable interest in reinforced concrete construction, having added to the wharfage at Dagenham Docks by the erection of a reinforced concrete jetty of considerable magnitude. This jetty and an engine and boiler-house were constructed in 1901 on the Hennebique system. Mr. Williams' system is particularly adapted to the construction of foundations and jetties, although it can be used for complete buildings, and other general work.

The *pile* on the Williams system is reinforced with a rolled joist, the point being formed by cutting away the web for a short distance and forging the flanges to a point. A pile shoe of the ordinary form may be used, but this is sometimes omitted. The concrete is further strengthened by means of flat steel hoops placed fairly close together.

When bending is to be feared, the simple rolled joists does not give sufficient stiffness in the direction normal to the web, and consequently Mr. Williams adds two flat steel bars, one on either side, as shown (Fig. 188). These are attached to the web at the ends, and bent out in the form of a truss, or may be curved if desired. Several holes are drilled through the web of the joists, and tubes placed through them to form holes in the pile for use in transport or pitching. These piles, 14 by 14 inches, can be sold at 4*s*. per cubic foot. The *capping to the piles* when used as a foundation to support a decking or other superstructure are frequently reinforced with single rolled joists, the connexions with the piles being made as shown (Figs. 189, 190, 191).

Fig. 189 is a sectional plan showing the junction between a pile and cross-capping. After driving, the top of the pile is broken away for some distance, exposing the top of the rolled joist. The rolled joists of the capping are then placed in position, leaving a slight clearance between their ends and the flanges of the pile reinforcement. Flat bars, as shown, are next put in and bent at their ends to obtain a good hold in the concrete. These are secured to the capping joists by round rods (*b*), which are passed through holes in the webs of the joists and through the flat bars. The whole combination is then embedded in concrete.

Figs. 190 and 191 show a further method of forming the connexion between the capping and the piles when the sills only run in one direction. Fig. 190 is a longitudinal section and Fig. 191 a cross-section of the capping.

In this case the concrete of the pile is broken away exposing a short length of the joist, the top of which has been machined true before being put in place.

The rolled joists forming the reinforcement to the capping are placed bearing on the machined surface, and two straps of flat iron are placed over them, one on each side of the pile reinforcement as shown, The concrete is then filled in, tying the whole together.

Figs. 190 and 191 also show the reinforcement adopted by Mr. Williams for his *deckings* and *floors*.

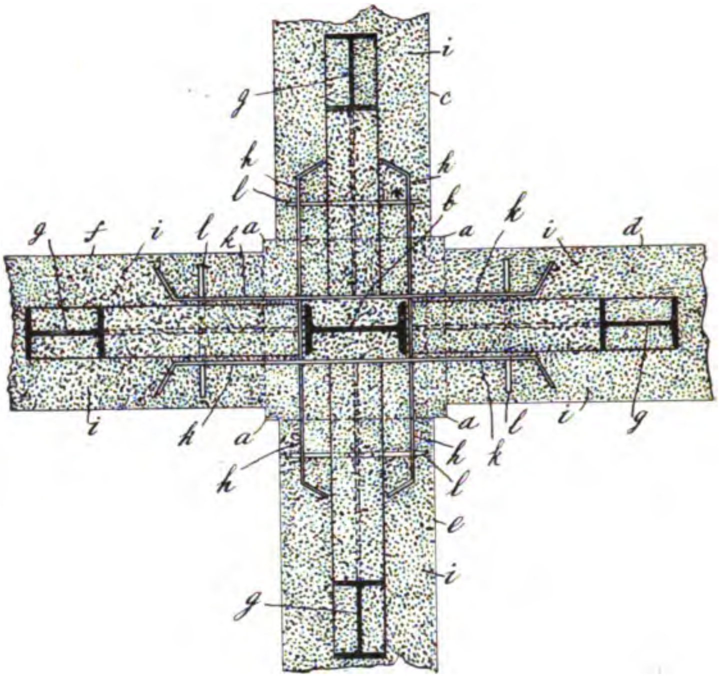


Fig. 189

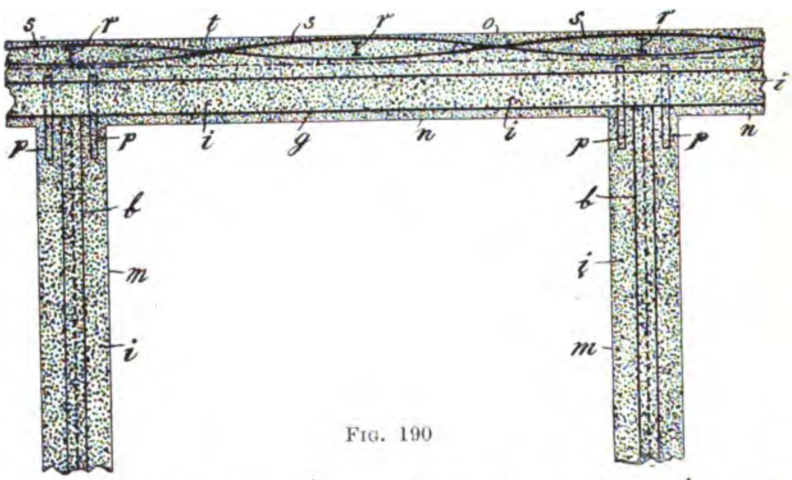


Fig. 190

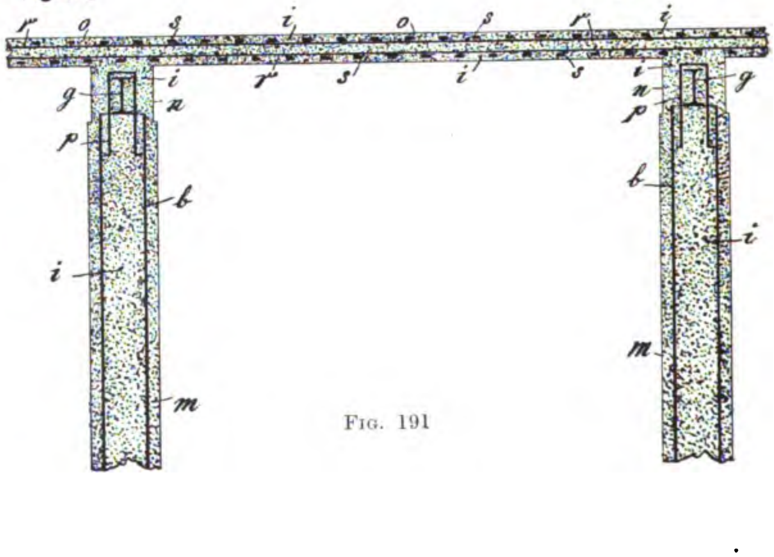


Fig. 191

The main bars are formed of small rolled joists, and the transverse reinforcements of flat bars bent over and under the joists, as shown.

An arrangement for *beams* and *sills* of long span is shown in Fig. 192. Here, instead of a single rolled joist, a built-up reinforcement is used, formed of straight angle irons or other sections along the top and bottom, connected by two pair of inclined ties of flat bars on edge, or other suitable section, which are riveted or bolted to them. Further bars are added along the top and bottom of the beam from the terminations of the inclined bars, as shown at (v). These are connected to the other reinforcements by the same bolts or rivets which secure the inclined and main bars together.

A spirally wound hooping of wire about 0·20 inches diameter, with a pitch of about 3 inches, is sometimes added, as shown at the left end of the beam. These would of course extend throughout the whole length.

Fig. 193 shows a beam in which there are more than one set of inclined bars on each side of the centre. The hooping which is sometimes added is shown at the centre in this figure, but would of course extend throughout the whole length of the beam.

The *columns* on this system would be reinforced in the same manner as the piles.

Mr. Williams has carried out an interesting series of tests on beams reinforced on the several methods (shown in Figs. 192 and 193), and also with no inclined ties, but only the wire hooping. For these tests he used small rolled joists for the reinforcing sections. The results show that the beams reinforced with longitudinal and inclined joists with a hooping of 0·20 inch diameter wire with a pitch of 3 inches have considerably the greatest resistance.

The form of reinforcement which comes next to this in efficiency is that of the longitudinals with inclined ties, but without the hooping, and the least efficient method appears to be that using longitudinals with wire hooping, but with no inclined ties.

The difference between the two last methods is not as great as would be expected if ordinary transverse reinforcements, placed comparatively far apart, had been used instead of the hoops, clearly indicating the advantages of hooping. This advantage would be the greater, since symmetrical upper and lower reinforcements were used, as it would enable the concrete to follow the deformation of the metal on the compression side, as will be explained later.

This is exactly what would be expected, and it is further interesting to note that in the later case the failure showed distinct signs of shearing, whereas with both the beams with the inclined bars there was no indication that the failure was in any measure due to shearing.

52. Wünsch System

This system was introduced by the firm of Robert Wünsch, of Budapest, Hungary, in 1892.

The *floors* are either flat or arched, the reinforcing sections being T-irons in both types. The flat floors have inverted T-irons embedded in the floor and ceiling slabs.

The floor slabs rest on the upper flanges of rolled joists, and the inverted T-

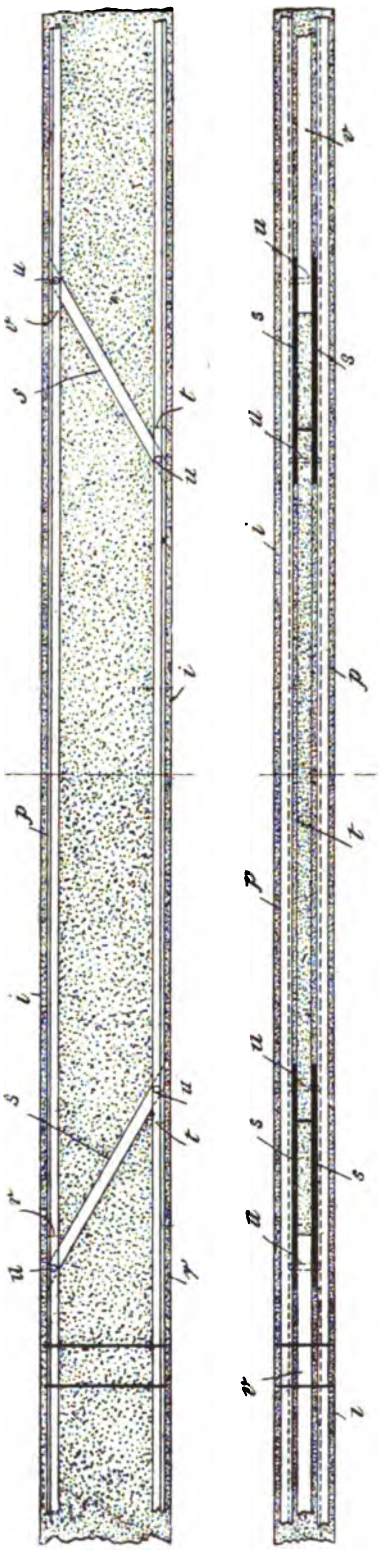


FIG. 192

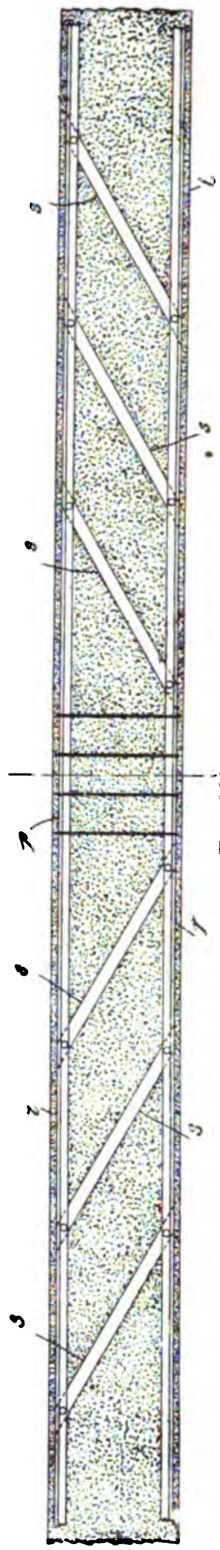


FIG. 193

irons of the *ceiling slabs* rest on their bottom flanges, as shown (Fig. 194). The space between the two slabs is left hollow, and may be used for carrying pipes, wires, &c.

In another type of *flat floors* the ceiling slab is not reinforced, but is formed of

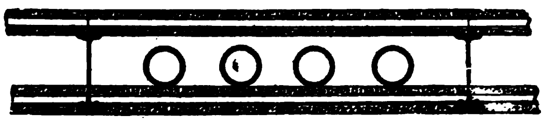


Fig. 194

plaster slabs secured to timbers, which rest on the lower flanges of the supporting joists. The flat floors are only used for small spans; for large spans an arched form of construction is employed, such as that shown (Fig. 195).

The *arches* have a flat extrados, and are usually sprung between rolled joists acting as beams. The T-section reinforcement at the extrados is horizontal and upright, that at the intrados is curved and inverted; the webs of the two reinforcements are riveted together at the crown. The T-sections are also riveted to the flanges

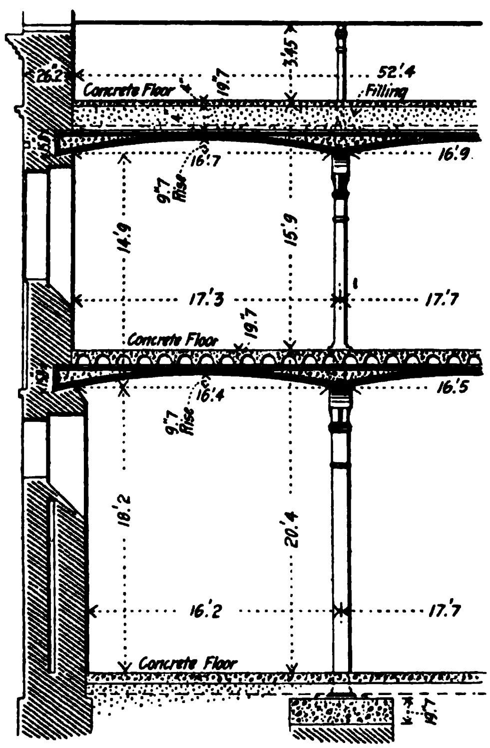


Fig. 195

of the supporting joists. At the walls the two reinforcements are connected by vertical angle irons riveted to them.

Arched bridges up to 83 feet span with a rise of $\frac{1}{10}$ the span have been constructed on the Wünsch system. For large bridges bulb tees are sometimes used. The reinforcements are spaced from 1·64 to 1·97 feet apart.

Part III

MATERIALS

General Remarks

The value of constructing in reinforced concrete is that the materials are used to the best advantage, and consequently the dimensions can be reduced to a minimum; but it is incumbent upon us, while reducing the dimensions, to exercise the greatest care in the selection of the materials to be employed, and since there is a saving in the quantity we can afford to pay more attention to the quality.

Though the materials, wrought iron or steel and the ingredients of concrete, are all common and easily procurable, yet they should be rigidly inspected to ensure that the quality of the work is uniform. Their various properties can be readily tested so that it is possible to know the strength and other qualities of the materials available, and to design the structure accordingly.

Where poor materials have to be used the coefficients employed in the calculations must be in proportion, but where possible only materials of the best quality should be allowed, even when additional expenditure must be incurred in obtaining them, and workmanship of a similar standard should be demanded.

As it is most important that the concrete should be of the same strength throughout, the materials used in making it must be of uniform quality.

The water used in mixing must be clean, and care should be exercised to make certain of this, as many waters, though polluted, appear clean, and yet may have a very marked effect on the concrete gauged with them.[1]

The wrought iron and steel employed are generally of the ordinary commercial sections and quality; only in very few instances are special sections and materials used. They should be procured from a firm of good standing, and their strength, elasticity and ductility guaranteed.

The timber for the falsework must be carefully selected, of good quality and not liable to twist or shake, as a slight failure of the supporting timber while the concrete is setting, though it may not even be perceptible, may damage the structure to such an extent as to cause a failure.

CONCRETE

Matrix

GENERAL REMARKS.—Perhaps the most important factor in a structure of reinforced concrete is the quality of the matrix, which must always be cement, as limes do not give sufficient strength for the small dimensions of such a structure.

As a general rule, a finely-ground slow-setting cement is used, but some con-

[1] The harm which may result from insufficient care in this point is shown by an actual instance where a large quantity of concrete was absolutely spoilt in consequence of some chemical refuse being turned into a clear mountain stream some distance above the place where water was being abstracted for a reinforced concrete structure.

structors employ quick setting cements, especially for pipes, reservoirs and similar purposes.

For reinforced concrete work cements of doubtful quality should in no case be employed, and for this reason natural cements must be avoided, as their behaviour is very uncertain, and they are more likely to be uneven in quality than artificial cements in which the ingredients can be proportioned with exactness.

The use of slag cements cannot be recommended unless they are burnt after the mixing of the ground ingredients. They require careful testing before their employment is decided upon. They are weaker as a rule than Portland cement, and far from uniform in their character, depending as they do entirely on the chemical constituents of the slag.

Since slags are composed mainly of alumina and silica and are wanting in lime, the finely ground slag is mixed in the requisite proportions with slaked lime, and the two are then ground together so as to become thoroughly mixed. The materials being mixed after burning have not the same chemical combinations as in Portland cement, and they usually contain a higher percentage of alumina and sulphides, and at times an excess of magnesia. Sometimes the ground slag and lime are burnt after mixing, in which case the cement is of a much better character and frequently as good as the best Portland cements.

La Société Pavin de Lafarge give the following chemical analysis of their slag cement made at Vitry-le-François—

Silica	25·20	per cent.
Alumina and iron	17·65	,, ,,
Lime	48·40	,, ,,
Magnesia	2·20	,, ,,
Residue and Loss	6·45	,, ,,

The mean results of tests of this cement are shown in Table VII, opposite, to which have been added results given by MM. Bergner and Guillerme in their book, *Cement Armé*.

In his *Masonry Construction*, Mr. Baker says : " Slag cements contain excess of sulphides and are therefore unfit for use in air, particularly a very dry atmosphere, although under water they may give satisfactory results."

The best known slow setting cement is that manufactured under the name of " Portland," and it is advisable that this should be exclusively used for structures in reinforced concrete.

Specification Requirements for Portland Cement

As time goes on, there is no doubt that, as is the case with structural ironwork, reinforced concrete works will be placed in the hands of well known and reputable firms who are specialists, and will probably have testing rooms with all necessary appliances where qualified inspectors approved by the engineer or architect may make the required tests. The engineer or architect will, however, design the structure and draw up the specification, and it must be his duty to see that proper materials are employed.

For small works elaborate instruments for testing are sometimes not provided for economy's sake, but, as a general rule, this form of construction is carried out by firms who make it a speciality, and who undertake the testing of the materials, guaranteeing that only those of the best qualities are supplied.

It is therefore deemed advisable to briefly detail the specification requirements for all the materials used, and to indicate the tests which should be adopted.

TABLE VII

Authority	Place where Made	Percentage of Residue	Meshes of Sieve per Lineal Inch	Tensile Strength Pounds per Square Inch									Compressive Strength Pounds per Square Inch								
				Neat					1 to 3				Neat					1 to 3			
				Days					Days				Days					Days			
				2	7	28	42	84	7	28	42	84	2	7	28	42	84	7	28	42	84
Pavin de Lafarge	Vitry-le-François	15	178		284	455		483	213	355		370		2560	3124		4972	1706	3413		3550
MM. Bergner and Guillerme*				256 to 356	398 to 484	568 to 710		710 to 854					2200 to 2620	3410 to 6110	4770 to 6100		4600 to 7350				
MM. Berner and Guillerme	Cleveland .	19	127		350	436			240	334	360	396									
	Cleveland .	21	,,		404	485	502		216	366	396	427									
	Newcastle.	16	,,		561	700			298	420	456										
	Cleveland .	12	,,		483	611	645		289	405	430	450									

* Good sample.

The qualities for which the cement should be carefully and frequently tested are—

1. Coolness.
2. Fineness of grinding.
3. Specific gravity.
4. Constancy of volume.
5. Time of setting.
6. Chemical composition.
7. Strength.
   (a) Cohesive strength.
   (b) Adhesive strength.

1. Coolness.—The cement should under no circumstances be allowed to be used fresh, but should be spread on delivery in thin layers, not exceeding 12 inches in thickness, in a rainproof shed, and turned over at least once a week either by tilting floors, or in some other manner, and should not be used until it will pass the following test—

After mixing for between two and five minutes with 22½ per cent. of water there must not be more than 6° F. rise in temperature in one hour.

It cannot be too strongly insisted on that the cement must be properly air-slaked before use. This will cause the work to be more costly, but is well repaid by the immunity from expansion in the work which an otherwise sound cement often experiences from being used when insufficiently cool. Jets of steam have been employed for "slaking" cements calcined by the rotary process, which are frequently very quick setting, and good results have been obtained by their use.

On small works the expense entailed by the provision of a shed for this purpose may seem out of all proportion to the cost of the work, but as reinforced concrete comes more and more into use, recognized firms will doubtless have large sheds in which cement is stocked, so that it can be delivered in a proper condition to any small works where a special cement shed is not justified.

2. Fineness of Grinding.—The importance of fine grinding need not be enlarged upon, as it is well known that it has a very great influence on the properties of the cement. Providing all other details are in order, the cementitious value of a cement depends directly upon the amount of impalpable powder which it contains.

After being gently shaken for five minutes the cement must not leave more than 15 per cent. residue on a 180 × 180 sieve, the size of the wire being 0·0018 inches diameter, or No. 47½ British Standard Wire Gauge; it must all pass a 76 × 76 sieve, the size of the wire being 0·0044, or No. 41 British Standard Wire Gauge.[1]

For this test 100 parts of the cement by weight are placed in the sieve, and after shaking for five minutes the *residue* is weighed. It would be unfair to judge by weighing what is caught beneath the sieve, as some may be lost in the air. This test is much stricter than most British specifications at the present time, but a well-ground cement should easily pass it, and such a standard is quite usual in German specifications.

3. Specific Gravity.—The test most generally adopted in this country is that of the weight per strike bushel. The specific gravity is, however, a far better and surer indication of thoroughness of burning than the weight per strike bushel.

[1] These sizes of wire are according to Continental and American practice, in which the diameter of the wire is one half the size of the opening.

The manner of filling the bushel measure greatly influences the weight, while by taking the specific gravity there is no chance of error if ordinary care is exercised.

It is impossible to obtain a heavy weight per bushel with a finely ground cement, as the two are opposed to one another. The finer the cement the lighter it will be, as there will be more air in the interstices.

The specific gravity must not be less than 3·15 *with fresh cement, or* 3·08 *after* 24 *hours' aeration in a ¼-inch layer, and shall not be more than* 3·25 *for fresh cements.*

Between the extremes of 60° and 80°F. this test is not affected by temperature, providing it remains constant throughout the test.

The simplest apparatus for determining the specific gravity is Schuman's specific gravity bottle or other volumeter graduated to cubic centimetres with decimal sub-divisions.

The liquid employed for filling is generally oil of turpentine or petroleum, since with water a chemical change would take place. The turpentine or petroleum must be perfectly free from water, which can be secured by allowing the liquid to stand over good quicklime or similar dehydrating agent, which will not be acted on by the liquid itself. A large bottle of this can be prepared and used as required, being kept in a tank of water.

The cement is dried for about twenty minutes on a metal plate. The number of grammes required is then weighed out and poured carefully into the volumeter, (which has been previously filled to the proper height with the liquid) through a funnel, so that none sticks to the sides, the bottom of the funnel being kept just above the liquid.

The volume of the displacement is then read in decimals of cubic centimetres. The specific gravity will be the number of *grammes* of cement used divided by the displacement in cubic centimetres. For Schuman's apparatus 100 grammes of cement are used.

Messrs. Stanger and Blount's flask is perhaps more easy to use than Schuman's, as the long neck of the latter makes it somewhat unwieldy, and it requires great care to get the cement down the neck without sticking to the sides. Messrs. Stanger and Blount's apparatus is a small flask with a graduated neck. The body of the flask has a capacity of 64 cubic centimetres.

The graduations on the neck are marked from 14 to 17 cubic centimetres with decimal subdivisions. Fifty cubic centimetres of the liquid are placed in the flask, being measured in a pipette, or other measure, which is easily procured. The liquid is poured into the flask through a funnel if a pipette is not employed for measuring, so that the sides of the neck may be kept dry. Fifty grammes of cement are then gradually added through a funnel and the displacement read.

The liquid introduced being 50 cubic centimetres and the known capacity of 64 cubic centimetres, there remains 14 cubic centimetres to be displaced before the first graduation, marked 14, is reached.

The specific gravity will be $\dfrac{50}{\text{displacement in cubic centimetres}}$.

The liquid must be brought to its original temperature before the displacements are read, which is easily done by placing the apparatus for a short time in the same water on which the stock bottle is kept. Mr. Baker, in his *Masonry Construction*, says that a change of 1° centigrade in the turpentine between the readings of the volumeter will make a difference of 0·08 in the resulting specific gravity.

Keat's specific gravity bottle is another apparatus of a different type which may be employed for this purpose.

It is in the form of two bulbs with a narrow neck, between which a ring (*b*) is etched; another ring (*a*) is also etched round the neck at the top. The bottle is filled through a funnel with the oil of turpentine or other suitable liquid to the ring (*b*). It is then weighed in a balance, after which the cement is gradually added through a funnel until the liquid rises to the ring (*a*). The bottle is then re-weighed, which will give the weight of the cement added. If the bottle is marked 1,000 grains, which means that the portion between (*a*) and (*b*) has a capacity of 1,000 grains of water at 60°F, the weight of the added material is the specific gravity. If it is marked 500 grains, the specific gravity is the weight of the added cement divided by 2. The weight must, of course, be in grains.

4. CONSTANCY OF VOLUME.—Although constancy of volume when the cement has set is important for all works in concrete, it is even more important when reinforced concrete is the material to be employed.

After aeration of the cement for 24 hours in a layer ¼ inch thick, a pat about 3 inches diameter, ½ inch thick at the centre and reduced to fine edges, mixed for between 2 and 5 minutes on a non-absorbent surface with 22½ per cent. of water, shall be placed on a glass plate, and allowed to set under a damp cloth for 24 hours; it shall then be placed in cold water, which shall be raised to boiling point and kept boiling for 3 hours. There shall be no signs of warping or cracking. Should the pat leave the glass, this shall not be taken as a sign that the cement is unsuitable. The temperature while the cement is being gauged and while it is setting shall be between 52° and 72°F. This test is undoubtedly severe, but in view of the importance of the works, every possible precaution should be taken to ensure that all cement used is of undoubted soundness, and all cements manufactured by scientific methods should be able without difficulty to satisfactorily pass it.

5. TIME OF SETTING.—This is a property which varies greatly, and appears at present to be to some extent beyond the maker's control. A cement may be perfectly sound and of good quality which takes its initial set within five minutes of the commencement of gauging and its final set within twenty minutes, but such cements will generally set more slowly after sufficient air slaking.[1]

It is well known that the time of setting within wide limits is no criterion of the quality of a Portland cement. For certain purposes it is necessary to obtain either a slow or quick-setting cement according to circumstances, and in such cases it is necessary to restrict the times of setting within well-defined limits.

For ordinary purposes a cement for which the initial set takes place in not less than ten minutes after the commencement of gauging and the final set in not more than five and a half hours, will be sufficiently slow to allow of it being mixed satisfactorily for concrete, particularly as an excess of water is generally used in gauging.

For reinforced concrete, however, it is usually advisable to use a cement which is specially slow setting, in order that the separate layers may become thoroughly incorporated.

After 24 hours' aeration in a layer ¼ inch thick the cement shall be mixed on a non-absorbent surface for not less than 2 minutes nor more than 5 minutes with

[1] The Associated Portland Cement Manufacturers have recently patented a process, by which rotary ground cement is treated with a steam jet while passing through the grinding mills. This process appears to be very effectual in slowing down the setting to any reasonable requirements.

22½ per cent. of water into a paste, and be tested by the Vicat or other suitable apparatus. The initial set shall not take place under 20 minutes, nor the final set under 5 or over 10 hours, the paste being covered with a damp cloth between the testing with the needle. The temperature during this test shall not vary beyond the extremes of 63° and 67° F.

For this test the Vicat needle apparatus may be employed, in which case the paste is placed in a brass cylinder 4 centimetres high and 8 centimetres in diameter, with a needle 1 millimetre square suspended above and weighted to 300 grammes. This needle is lowered to the top of the paste and let drop. When it ceases to penetrate to the bottom of the cylinder the initial set has taken place, and when it makes no impression on the cement the time of final set is recorded. The tests are frequently made in this country with a needle $\frac{1}{16}$th of an inch square weighted to 2½ pounds.[1]

Another method, adopted for this test in America, is to make two pats on glass plates 3 inches in diameter and ½ inch at the centre reducing to thin edges, one being used for the initial and one for the final set. The test for the initial set is made by applying a $\frac{1}{12}$ inch diameter needle weighted to a quarter of a pound. If any indentation is made the cement has not commenced to set. For the final set a needle of $\frac{1}{24}$ inch diameter and loaded to 1 pound is applied. There must be no indentation when the final set has taken place.

Particular emphasis should be laid on the initial set, as many cements that are slow in the final set commence to set quickly and then harden more slowly, until the final set may take as long as the best slow-setting cements. The time of setting varies with the temperature, being slower as it decreases, hence the necessity of defining more restricted limits for this test than for others.

6. Chemical Composition.—A chemical analysis of the cement employed should be carefully made, more particularly for the proportions of magnesia and sulphuric acid, and the proportion of soluble silicia and alumina to the lime. The chemical analysis is specially important when the cement is required for works exposed to the action of sea water, in which case the percentage of sulphate of lime and of magnesia should be limited.

Sulphate of lime or gypsum is used frequently to cause a naturally quick-setting cement to set slowly and to increase the short time tensile strength. (The addition of 1 to 2 per cent. of gypsum will alter the time of setting from a few minutes to several hours). An excess of lime is frequently used with the gypsum to hide it temporarily. The addition of gypsum will also enable a cement to pass the boiling test for constancy of volume. Excess of sulphides is indicated by brownish or yellowish blotches on pats exposed to air, and by a greenish fracture in pats kept in water.

A cement that has been adulterated with gypsum will again become quick setting if mixed with a solution of carbonate of soda.

The chemical composition can of course only be determined by a properly qualified analytical chemist accustomed to testing cements, but there is no doubt that it should be known to the engineer, since it is one of the most important guides to the character of a cement.

[1] Care should be taken that sufficient paste is made to more than fill the mould, so that after shaking down, the "scum," or finest particles of cement that rise to the surface, may be struck off with the surplus paste, leaving a true and perfect surface on which to make the impressions. If a "scum" is allowed to remain on the surface, the results will not be reliable.

The chemical composition must be as follows—

Lime	from	58	to	63	per cent.
Silica	,,	21	,,	24	,, ,,
Alumina	,,	6	,,	8	,, ,,
Ferric oxide	,,	3	,,	4·5	,, ,,
Sulphuric anhydride	not more than			1·5	,, ,,
Magnesia	,, ,,		,,	1·25	,, ,,
Insoluble residue	,, ,,		,,	1	,, ,,
Alkalies (not including loss in analysis)	,, ,,		,,	1·5	,, ,,

7. Tests for Strength.—Although these are the tests most generally specified, they are perhaps the least important as specification requirements, since if the other tests mentioned above are complied with, a cement can hardly fail to give good results for cohesion and adhesion. These latter tests, however, especially that for adhesion, add to our knowledge of the properties of the cement, and give an assurance, if properly carried out and interpreted, that the resistance of the concrete will be sufficient to bear the imposed stresses.

It is very usual to find it specified that the average result from a set of briquettes shall be taken. This, however, is not always the case. For acceptance tests *the maximum* result should be allowed, since a cement cannot have less strength than the maximum obtained, and the premature failure of the other briquettes is due to defective manipulation.

For experimental purposes when making tests for the effects of different ingredients, for the methods of mixing, for the effects of circumstances, etc., on mortar or cements, it is *the average* value which is undoubtedly the proper one to take. In tests for resistances, where the values obtained will be employed in the valuations of co-efficients for the calculations or other similar purposes *the lowest* result must be taken for obvious reasons.

The tests for strength should be always carried out by a thoroughly experienced man, since the results depend largely on the proper manipulation. The briquettes almost universally adopted at the present day have a breaking section of one square inch area, and the records are given at once in pounds per square inch.

The testing machine which must be on a firm base to avoid vibrations, should apply the stress automatically at a rate of 100 pounds in 15 seconds. It is very necessary that the table on which the briquettes are moulded and left to set should be perfectly steady, as any vibration during the process of setting has a marked effect on the strength of a cement or mortar.

The briquette moulds should be placed on sheets of damp blotting-paper.

(*a*) Cohesive Strength.—*The cement shall be mixed for not less than* 2 *nor more than* 5 *minutes on a non-absorbent surface with* 22½ *per cent. of water; it shall then be placed in moulds resting on a metal or similar surface, which has been slightly greased with mineral oil, the cement paste being well pressed into the moulds with the fingers or a spatula, but not rammed. The briquettes shall then be left under a damp cloth for* 24 *hours after which the moulds shall be removed and the briquettes placed in cold water until tested. The maximum tensile strength obtained from each set of briquettes, which must be at least three in number, must be as follows—*

2	*days after gauging,*	*at least*	230	*pounds per square inch.*
4	,, ,, ,,	,,	350	,, ,, ,, ,,
7	,, ,, ,,	,,	450	,, ,, ,, ,,
28	,, ,, ,,	,,	580	,, ,, ,, ,,

There must be a rise in strength of at least 50 *pounds per square inch between each period. The stress to be applied at the rate of* 100 *pounds per* 15 *seconds.*

The temperature of the testing room and the water shall be kept between the limits of 52° and 72° F.

The table on which the briquettes are made and left to set must be free from vibration.

(*b*) Adhesive Strength.—*The mortar shall be mixed for not less than* 2 *minutes and not more than* 5 *minutes on a non-absorbent surface in the proportions by weight of* 1 *of cement to* 3 *of perfectly dry Leighton Buzzard or other standard sand, which has passed a* 20 × 20 *sieve and been retained on a* 30 × 30 *sieve;* 8 *per cent. of water to the weight of cement and sand together shall be used in mixing. This mortar shall then be rammed (but not unduly) into moulds, resting on a metal or similar surface, which has been slightly greased with mineral oil. The briquettes shall then be left under a damp cloth for* 48 *hours, after which the moulds shall be removed and the briquettes placed in cold water until tested. The maximum tensile strength obtained from each set of briquettes, which must be at least three in number, must be as follows—*

7 days after moulding, at least 180 *pounds per square inch.*
28 ,, ,, ,, ,, 270 ,, ,, ,, ,,

There must also be a rise in strength of at least 50 *pounds per square inch between* each *period. The stress to be applied at the rate of* 100 *pounds per* 15 *seconds.*

The temperature of the testing room and the water shall be kept between the limits of 52° *and* 72° *F.*

The table on which the briquettes are made and left to set must be free from vibration.

When making the adhesive tests for acceptance, it is well also to make similar tests, using the sand employed on the works. This will be a test for the quality of sand, and also gives data for the resistance of the concrete used. The results of these tests should approximate those where standard sand is used if that employed on the works is of good quality, such as is required for reinforced concrete.

Quick-Setting Cements.—Quick-setting cements are generally weaker than those which are slow setting. They are usually natural cements, and are therefore somewhat uncertain in their chemical composition and uneven in quality. They should not be air slaked.

For works where impermeability is not necessary, these cements are sometimes used for rendering the surface and for making good, if this is required, but it is better to use one that is slow setting for this purpose, as quick-setting cements are more liable to cause cracking of the surface. They may, however, be used for rendering purposes where impermeability is a desideratum, since in such cases the atmospheric conditions are not so variable. It is better, however, to use slow-setting cement even in this case if possible. They are employed by several firms for the construction of pipes and of sewers and reservoirs. This is particularly the case in those systems where special sections of reinforcements are adopted, such as those of Bonna and Bordenave, and where the mortar is poured into the moulds in the form of a grout.

Slow-setting cements are, as a rule, more expensive than those which set quickly, will not bear so much water, and require more moulds on account of the time which must elapse before removal.

In the Bordenave system the mortar is always made of quick-setting cements for the construction of pipes and reservoir walls. For the Bonna system mixtures of slow and quick setting cements are employed. In both cases those that set slowly are used for the floors and roofs of reservoirs and for rendering the surface where required.

The tests for these cements may be conducted in the same manner as described for those that are slow setting, but they will require more water for gauging.

They should be finely ground, leaving not more than 15 per cent. residue on a 120 × 120 sieve, the size of the wires being 0·0028 inches diameter, or No. 45 British Imperial Standard Wire Gauge.

Their specific gravity will vary between 2·70 and 3·00. This will necessitate 45 grammes of cement being used for Stanger and Blount's flask instead of 50 grammes.

For the test for constancy of volume, pats mixed with 30 per cent. of water, 3 inches diameter and ¼ inch thick at the centre, reduced to fine edges, placed in water after setting, should show no signs of disintegration or warping after immersion for 7 days.

The initial set should take place between 3 and 20 minutes after moulding, and *the final set* in not more than 1 hour.

As regards chemical constituents, quick-setting natural cements should contain enough silica to completely silicate the lime. The stones from which they are obtained should have a large proportion of iron and alumina, especially iron, compared with the lime, and the alumina should not combine with the lime. The high alumina in quick-setting cements renders them unfit for permanent use in sea water. For natural cements the stones are broken very small and burnt at a low temperature.

The strength of quick-setting cements varies considerably. Baker, in his *Masonry Construction,* gives the following values in pounds per square inch for the tensile strength of natural cements.

TABLE VIII

Times before Testing	Neat	Cement to 2 Sand
24 hours (in water after setting)	100	—
7 days (1 day in air, 6 days in water)	200	125
28 days (1 day in air, 27 days in water)	300	200

Faija's specification for quick setting Portland cement gives the following values in pounds per square inch for neat cement placed in water when set.

TABLE IX

Time of Test after Moulding	Tensile Strength
3 days	176.
7 days	400, with a rise of 20 to 25% of that of 3 days.

The strengths for natural cements should be at least those given by Mr. Baker.

The amount of water required for gauging will be about 30 per cent. for neat cement and 15 per cent. for 1 of cement to 2 of sand.

These cements will not bear more than 2 of sand to 1 of cement. Only one briquette should be gauged and moulded at a time.

Quick-setting Portland cements are stronger than natural cements and will sometimes approximate to the specification requirements given for slow-setting cements.

Aggregates

General Remarks.—The selection of the aggregates for reinforced concrete works, is a matter which necessitates great care. Those which are easily obtainable in the locality very naturally suggest themselves in the first instance, and if in every way suitable will be used, but they must be carefully tested as to their properties, and not employed if found unsuitable, unless their quality is taken into account in making the calculations. It must always be borne in mind that a weak aggregate will make a weak concrete, and that adding to the proportion of cement used will not strengthen the mixture.

Sand.—Only quite clean sand should be used, and for this reason it should preferably be washed through the screen decided upon for its size of grain. Any sand which may have come in contact with alkaline or acid solutions must not be used.

The question of the best sizes of sand grains has received a good deal of attention.

The conclusions of M. Feret,[1] Chief of the Laboratory of the Ponts et Chaussées at Boulogne, drawn from an extensive series of experiments, are perhaps the most useful on this subject.

The French commission for the standardization of methods of testing give the name of sand to mixtures in which the grains will pass a sieve of 5 millimetres × 5 millimetres mesh (about $\frac{1}{5} \times \frac{1}{5}$ inches).

They designate as large grains those between 5 and 2 millimetres; as medium grains, 2 and 0·5 millimetres; and as fine grains, those less than 0·5 millimetres, or about $\frac{1}{50}$ of an inch; and term the proportions of the several grains which a sand may contain its granular composition.

M. Feret formed the following conclusions on the effect of the granular composition of the sand on the resistances of mortars.

For mortars containing the same weight of matrix to the same weight of sand—

1. The resistance is less for the mortars with regular grains, and becomes less and less as the grains become smaller.

2. The resistance increases as the sand becomes more mixed, the maximum resistance being obtained for sands in which there are no medium grains, the proportion of large grains being double the fine grains which must include the matrix itself.

It appears from M. Feret's experiments that it is necessary to know the granular composition of the sand used, and to only use those which are the best obtainable, and that it is never advisable to use sands composed of fine grains only, although the employment of a sand of large and fine grains is advantageous.

M. Feret also found that for sands of the same granular composition, but of different nature, the volume of voids become less as the grains are more rounded.

Some experiments made by Herr R. Dykerhoff on the effect of finely ground sand on the strength of mortars show that there is an increase of strength when finely ground sand is added to a sand of normal grains. They also show that cal-

[1] M. Feret's experiments and deductions will be more fully treated under the heading of the proportioning of ingredients.

careous sand is chemically wholly inactive when used for making mortar. Table X shows the results of these experiments.

The sand and marble were ground to pass a sieve closely approximating to our 180 × 180 mesh, and was therefore finer than the cement.

TABLE X

Proportions of Ingredients by Weight					Tensile Strength in Pounds per Square Inch			Weight of 5 Briquettes
Cement	Normal Sand	Rhine Sand	Ground Sand	Ground Marble	7 Days	28 Days	90 Days	
1	4	—	—	—	221	249	309	786
1	4	—	½	—	256	307	366	825
1	4	—	—	½	255	307	372	820
1	—	4½	—	—	—	—	362	805
1	—	4	—	—	—	—	426	827

The Rhine sand is an ordinary river sand of mixed grains.

These tests show that the density and strength of mortars are influenced by the granular composition of the sand.

In marine structures and those to resist water pressure the density of the concrete as well as the strength is of great importance, and therefore in such structures a sand containing the smallest possible percentage of voids should be employed.

Some further tests which emphasize the same point were published in the "*Annales des Ponts et Chaussées*," vol. 2, 1890, the results of which are given in Table XI.

TABLE XI

Size of Sand	Percentage of Water	Tensile Strength in Kilogrammes per Square Centimetre					
		3 Days	7 Days	28 Days	3 Months	1 Year	2 Years
1	15	1·97	3·83	6·56	8·79	10·73	12·14
2	17	1·29	3·87	7·49	10·45	11·35	13·14
3	22	1·00	2·50	5·24	9·27	11·11	11·24
4	27	0·77	2·22	4·90	6·90	8·32	10·36
5	33	0·39	1·94	4·17	6·74	8·21	9·06
Mixture of above sand	22	1·45	3·29	7·39	10·49	13·17	14·16

All these tests were made with the same proportions of ingredients. Sand No. 1 was very coarse, and No. 5 was a "blown sand"; Nos. 2, 3, and 4 were graduated between these.

The tests show that the strength increases with the size of grain, as M. Feret has pointed out, if even grained sands are used; but that the mixture of all the sizes or a sand of varying granular composition will make a stronger mortar than any one sand used alone. The mixture of fine and coarse sand only was, unfortunately, not tested. This, according to M. Feret, would have given the best result.

Mr Baker, in his *Masonry Construction*, gives the following tables showing the effect of the size of grain of sands in cement mortars.

TABLE XII

EFFECT OF FINENESS OF SAND UPON 1 to 2 CEMENT MORTAR.

Size of Sand		Tensile Strength Pounds per Square Inch at the following times after Mixing				
Passed Meshes per Lineal Inch	Retained Meshes per Lineal Inch	After 7 Days	1 Month	3 Months	6 Months	1 Year
4	8	243	442	539	470	668
8	16	269	345	473	512	572
16	20	186	250	313	397	392
20	30	211	281	322	402	440
30	50	149	205	238	275	318
50	75	122	214	260	275	308
75	100	98	153	211	208	253
100	—	98	155	161	229	271

He notes that the sand on line 4 had greater range of sizes and consequently fewer voids than that for line 3, which is the probable reason for its giving better results. And that if this is the true reason, the sands for the upper portion of the table are relatively better than they appear, as those in the bottom portion have greater range of sizes.

TABLE XIII

TENSILE STRENGTHS OF 3 TO 1 CEMENT MORTAR WITH NATURAL SANDS, ALL OF WHICH APPEARED TO BE OF THE SAME CHARACTER.

Percentage of Weight Retained on Sieves with Varying Number of Meshes per Lineal Inch								Percentage by Weight Passing Sieve with 100 Meshes per Lineal Inch	Tensile Strength Pounds per Square Inch
4	8	16	20	30	50	75	100		
0	26	21	16	11	9	8	7	2	700
0	29	29	13	10	12	5	1	1	447
0	22	21	11	17	20	8	1	1	370
0	13	15	10	19	33	6	1	1	341
0	9	10	6	11	45	15	2	1	332
0	13	15	7	8	38	15	4	1	309
0	0	0	0	1	6	69	23	2	246
0	0	0	0	0	0	0	6	94	200
0	0	0	2	3	15	45	30	5	189

These tables show that care is necessary when selecting a sand.

Shingle and Broken Stone.—There is some ambiguity about the terms used to imply a gravel with sand and a gravel with the sand screened out of it. For the purposes of this book *the name of "shingle" is given to a gravel with the sand screened out and "gravel" to the mixture of sand and stones.*

For reinforced concrete the shingle or broken stone where the concrete surrounds any reinforcement ought never to be larger than that which has passed a ½-inch screen, where the reinforcements are of small sectional area placed close together or where the distance from the reinforcement to the surface of the piece is small.

Where larger sections are employed the stones may with advantage be of larger size, but should not exceed those which have passed through a 1¼-inch screen, and even in this case a small aggregate is better near the reinforcement in order that good contact may be obtained, especially in the re-entrant angles of profile sections.

The sand must in all cases be screened out of the gravel in order that the materials may be properly proportioned. The proportion of sand in gravel varies considerably, and it is extremely difficult to assess what are the true proportions. It is therefore advisable to incur the extra expense of screening when the concrete is to be used in a reinforced structure, where the accuracy of the proportions is of great importance.

The shingle must be perfectly clean, and if it has come in contact with alkaline or acid solutions must in no case be used.

The broken stone should be of a hard close-grained quality, clean and free from argillaceous or organic matter. Broken slag, unless free from sulphur, must not be used.

M. Feret found that all other things being equal the resistance of a concrete increased with the size of the broken stone, but for reinforced concrete practical considerations limit us in this direction. Broken stone will make a stronger concrete than shingle, all other considerations being the same. Mr. Baker, in his *Masonry Construction*, gives the results of a series of experiments on this question made by the city of Washington, from which it appears the mean result obtained with Portland cement was that a concrete made from shingle was only 93 per cent. of the strength of a broken stone concrete. A series of French experiments also quoted by Mr. Baker show that the crushing strength for shingle Portland cement concrete was only 79 per cent. of that for broken stone concrete.

Ashes and Coke Breeze.—Concretes of furnace ashes or coke breeze are lighter than those of shingle or broken stone. It has also, what is in certain cases an advantage, the faculty of being penetrable by nails, and can be cut or chipped when required. Its porosity causes it to be a bad conductor of heat and sound, and it is, therefore, particularly adapted for fireproof and house construction. This property also allows it to absorb the humidity of the air better than concrete of shingle or broken stone.

Great care is necessary in the selection of ashes for reinforced concrete, as only those which have been thoroughly burnt ought to be used. Concrete of these materials is frequently employed for floors reinforced with "expanded metal," and also in the Roebling and Matrai systems, in the latter being employed merely as a filling over the concrete of harder materials which surround the reinforcements.

When these aggregates are used the concrete should be mixed with plenty of water and never rammed, for the obvious reason that the breeze or ashes will become crushed by the ramming, and the particles will not be thoroughly surrounded by the mortar.

The sulphur and other impurities in the aggregate may have a deleterious effect on the reinforcement, but this need not be feared if the concrete is mixed wet so that a protective coating is formed on the metal. Pumice or volcanic tuffs are also used in some countries as aggregates. They make very good concrete for use in similar cases to those in which coke breeze or ashes could be employed. Reinforced slabs of these materials can only be feebly reinforced on account of their small resistance to compression, and must necessarily be of small span.

Proportions of Ingredients

1. **General Remarks.**—The proportions of ingredients will naturally depend on the nature of the work. A large proportion of cement must be used in works to withstand the action of water in order to ensure impermeability. It is also advisable to use a rich concrete for those parts of a structure acting under direct compression, since the greater the resistance of the concrete the greater will be the economy of the reinforcement, as will be explained later (p. 288).

It is, however, well known that the less the proportion of cement the less will be the liability to change of volume, the expansion and contraction being due entirely to the cement. It would therefore appear that, with the exception of some special cases, it is advisable to use somewhat less proportion of cement for parts of a structure under tensile or bending than for those under compressive stresses.

Some constructors employ concrete of varying strength for different parts of the same piece, altering the proportions according to the stresses which have to be resisted. When this method is adopted there is a tendency to form lines of cleavage at the junction of the concrete where the proportions of ingredients have been altered. On that account it is better practice to mix all the concrete for one piece in the same proportions to avoid the danger of forming these lines of cleavage.

Sand and cement alone are usually employed for slabs and for beams of small depth, but for thicker work concrete of shingle or broken stone is advisable. For thin slabs the proportions should not be less than 1 of cement to 2 of sand, when the best materials are used; with materials of worse quality, 1 to 1½ or even 1 to 1 ought to be employed.

To resist the action of liquids the use of richer concrete not only increases the impermeability, but also the resistance to chemical action. The thickness of the piece may also be reduced which causes the setting to be quicker and the structure lighter, both of which are very desirable properties for pipes or sewers moulded in advance, where the moulds are a considerable item of the cost and where lightness is very essential for the purposes of transport.

Some constructors use different proportions of cement for the concrete at the front and back of reservoir walls, where the thickness required for impermeability may be considered as about ½ inch, to an inch, and the remainder of the thickness is necessitated for the purposes of strength. They employ a rich mixture of cement and sand only for the internal face, while for the back-work a poorer concrete is employed, containing shingle or broken stone in addition to the sand. Others use a rich concrete of sand and cement only around the reinforcement, and reduce the proportion of cement on either side, sometimes adding shingle or broken stone.

The impervious portion of the concrete must always be of rich mortar. The proportions, where both materials are measured by volume, should be 1 to 2 or 1 to 1½. In some cases 1 to 1 is used for the facing of reservoirs; Mr. Newman in his *Notes on Concrete* states that 1 to 1½ will resist about 75 feet head of water.

Broken stone or shingle which has passed a ½-inch screen with 50 per cent. of its volume of 1 to 1½ mortar will make a very dense concrete; such as may be used for impervious works. For sea works a large proportion of cement should be used, and the concrete must be as free from voids as possible.

Until experiments have been made on the action of the salts in sea water on the metal in reinforced concrete, it will be better to use only fresh water in gauging

TABLE XIV

Name of Constructor or Author	Type of Works.	Proportions by Weight in Pounds of Cement to Volume of Aggregates in Cubic Yards				Proportions in Cubic Feet of Aggregate per bag of 224 pounds of Cement.			Proportions by Volume Cement being taken as 85 pounds per Cubic Foot				Remarks
		Cement Pounds	Sand Cubic Yards	Gravel Cubic Yards	Stone Cubic Yds.	Sand Cubic Feet	Gravel Cubic Feet	Stone Cubic Feet	Cement	Sand	Gravel	Stone	
Ed Coignet.	Thick slabs with small resistance.	420	—	1	—	—	14·4	—	1	—	5·40	—	
,,	Ordinary Work.	505	—	1	—	—	11·97	—	1	—	4·50	—	Maximum of shingle ½ cubic yard or shingle not more in quantity than the sand.
,,	Arches, floors and walls not directly acted on by water.	670 to 755	—	1	—	—	9·02 to 8·00	—	1	—	3·46 to 3·00	—	
,,	For resisting water pressure.	1,170 to 1340,	—	1	—	—	5·17 to 4·51	—	1	—	1·97 to 1·72	—	
,,	Ditto, ditto, with rendered face.	670to84	—	1	—	—	9·02 to 7·2	—	1	—	3·46 to 2·8	—	
Pavin de Laforge	Conduits exposed to the air and under no water pressure.	510 to 590	—	1	—	—	11·86 to 10·26	—	1	—	4·50 to 3·86	—	
,,	Large span arches, etc.	590 to 670	—	1	—	—	10·26 to 9·02	—	1	—	3·86 to 3·46	—	
,,	Sea works.	750 to 1,000	—	1	—	—	8·06 to 6·05	—	1	—	3·00 to 2·30	—	
,,	Interior slabs 2 cms. or 0·79 inches thick.	1,340	—	1	—	—	4·51	—	1	—	1·72	—	
,,	Exterior slabs 3 cms or 1·18 inches thick.	1,340 to 1,670	—	1	—	—	4·51 to 3·62	—	1	—	1·72 to 1·38	—	
,,	Bridge floor slabs 5 cms. or 2·0 inches thick.	1,500 to 1,840	—	1	—	—	4·03 to 3·28	—	1	—	1·53 to 1·25	—	
,,	Pipes, reservoirs and sewers under water pressure.	1,340 to 2,000	—	1	—	—	4·51 to 3·02	—	1	—	1·72 to 1·15	—	
,,	Mixture containing 1 volume of mortar (in proportion of 670 lb. of cement per cubic yard of sand, or 1 bag to 9·02 cubic feet of sand) to 2 volumes of broken stone for 1 cubic yard in place.	335	0·5	—	1	9·02	—	18·04	1	3·42	—	6·84	Per Cubic Yard of Concrete Cement cubic feet 3·94; Sand cubic feet 13·50; Stone cubic feet 27
,,	Mixture containing 1 volume of mortar (in proportion of 840 lb. of cement per cubic yard of sand or 1 bag of cement to 7·26 cubic feet of sand) to 3 volumes of broken stone for 1 cubic yard in place.	500	0·60	—	0·90	7·26	—	9·68	1	2·75	—	4·13	Per Cubic Yard of Concrete Cement cubic feet 5·88; Sand cubic feet 16·20; Stone cubic feet 24·30
Monier	Beams and floors	680 to 756	—	1	—	8·89 to 8·00	—	—	1	—	3·46 to 3·00	—	Proportions very generally employed by constructors.
,,	Thin floors.	*765	—	1	—	8	—	—	1	—	3·00	—	

,,	Arches, walls and thicker floors.	*570 510	— —	1 1	— —	—	10·8 11·80	—	1 1	— 1·9	4 4½	— —	
Hennebique	Old gauging for beams & floors	580	0·47	—	1	4·8	—	10·18	1	0·92	—	4·1	
,,	Now gauging for ditto.	500	0·15 to 0·25	—	1	1·81 to 3·5	—	12·10	1	—	—	4·8	Average proportions by volume
,,	Sea works.	840 to 1,000	0·15 to 0·25	—	1	1·08 to 0·91	—	7·2	1	0·56	—	2·8	,, ,, ,,
						to 1·8 to 1·51		to 6·05	1	0·46		2·3	,, ,, ,,
,,	Reservoirs, etc., body of wall	1,260	—	1	—	4·8	—	—	1	—	1·8	—	
,,	Reservoirs etc., inner face.	2,500	—	1	—	2·42	—	—	1	—	0·92	—	
Bousserow&Garric	Floors and columns.	720	0·712	—	1	6·05	—	8·35	1	2·40	—	3·2	
,, ,,	Beams.	840	0·712	—	1	5·21	—	7·19	1	2·05	—	2·8	
"Expandod Metal" Company	Floors.	*570	—	1	—	—	10·8	—	1	—	4	—	Aggregate to pass ⅜ in. mesh and to have about 30% of sand
Bonna	Pipes.	960 Quick setting 280 Mod. slow setting	—	1	—	—	—	—	1	—	1·8	—	Ordinary weather.
,,	,,	680 Quick setting 560 Mod. slow setting		1	—	—	—	—	1	—	1·8	—	In great heat.
Société de Ciment de la Port de France.	Pipes.	840 Quick setting	0·63	—	1	4·54	—	7·26	1	1·82	—	2·8	Water 75 to 82·5 % of weight of cement.
Bernager	Pipes.	930 Quick setting	0·55	—	1	3·57	—	—	1	1·48	—	2·6	Water 75 to 82·5% of weight of cement.
Ellinger constructor of De Valliere system in America.	Beams and floors.	*453 —	0·60	—	1	8·02	—	13·36	1	3	—	5	Stone ranging from 1 to 1¼ in. across.
Ransome	Beams.	*765	—	1	—	—	8	—	1	—	3	—	Not larger than a pea.
,,	Arches and heavy work.	*382	0·33	—	1	5·27	—	15·81	1	2	—	6	
Melan	Arches.	*570	0·5	—	1	5·4	—	10·8	1	2	—	4	Sometimes two proportions are used, the arch proper being richer than the rest.
Thacher	Arches.	*570	0·5	—	1	5·4	—	10·8	1	2	—	4	
,,	Abutments.	*382	0·5	—	1	7·90	—	15·81	1	3	—	6	
Newman, *Notes on Concrete.*	Poorest mixture for impermeability.	*1,147	1	—		5·27	—	—	1	2	—	—	
,,	To resist pressure of 75 feet head of water.	*1,530	1	—	—	3·95	—	—	1	1½	—	—	Concrete can be made very dense if stones are used which have passed ½ in. screen mixed with half their volume of this mortar.
Advocated by M. Considère	Hooped concrete compression pieces.	1,000	0·30	—	0·90	1·8	—	5·45	1	0·69	—	2·07	Makes 1 cubic yard of concrete shingle or broken stone between ⅛ and 1 in., sand less than ⅛ in.

* Proportions given by volume only. Proportions by weight of cement deduced, assuming the weight of cement as 85 pounds per cubic foot.

NOTE.—The constructors who use gravel vary the proportion of sand from 50 to 25 per cent. of the total volume of aggregate.

for sea work, as although for ordinary concrete work sea water does not appear to have any ill effect, it is possible that the contained salts might have an injurious action on the metal. If it is feasible, the proportioning of the ingredients should be done by weight of cement to volume of sand and stone, as is the custom in France.

There is always a doubt as to the amount of the cement that is measured in the gauge boxes, as this depends entirely on the manner of filling, and although the error is in most cases on the side of excess, it is undoubtedly fairer and more accurate to proportion the cement by weight. This is very easily done if the cement is weighed as it is bagged up after air slaking, the proportions being taken as various volumes of sand and stone to the bag of cement weighing 2 cwts.

The proportions employed by the various constructors vary considerably, as will be seen by Table XIV. The proportions in this table are given in volumes and weights of cement per cubic yard of aggregates. A column is also given showing the number of cubic feet of aggregate per bag of cement weighing 2 cwts. If the bags contain less than 224 pounds of cement, these proportions must be altered accordingly.

Proportions of Ingredients for Mortars.—It is generally agreed that whatever method is used for the proportioning of the cement and sand it is impossible under ordinary conditions to obtain a mortar entirely free from voids. The voids in the sand vary considerably from apparently insignificant causes, the humidity playing an important part in the percentage of voids.

Mr. Gillette states that as a rule 20 per cent of the voids in the sand are left unfilled if the sand is unscreened.[1]

La Société Pavin de Lafarge, in their pamphlet on the subject of limes and cements, state, as the result of numerous experiments, that by adding to a sand which has been weighed dry 2 per cent. of its volume of water, the weight of a unit volume of sand diminishes 20 per cent. of its initial value, which shows that the space between the grains becomes changed by the addition of a cement paste.

MM. Pavin de Lafarge came to the conclusion "that all methods of proportioning based on the measurements of the voids before mixing must be rejected." The same conclusion is come to by others when considering the same subject.

The experiments carried out by M. Feret, Chief of the Laboratory of the Ponts et Chaussées at Boulogne, and the conclusions he has deduced from them, are perhaps the most valuable on the subject of the proportioning of mortars.

M. Feret studied the question in a new way. He takes c, s, e and v as the absolute volume of cement, sand, water and voids contained in the unit volume of mortar under consideration, so that $1 = c + s + e + v$. He calls $c + s$ the compactness of the mortar, being the sum of the solid elements. He further studied the variations of resistance of mortars in which the four elements c, s, e and v were varied. The mortars tested were "plastic mortars," so as to approach as nearly as possible those used in actual work. The test pieces were broken by compression, as the compressive resistances are more regular and less dependent on the faults in mixing, etc., than the tensile.

M. Feret found at first that for a series of mortars, made with the same cement and inert sands, the resistances to compression after the same time had elapsed

[1] *Engineering News*, vol. xlvi. No. 23.

since mixing and under the same conditions are always a function of the ratio, $\frac{c}{e \times v}$ or $\frac{c}{1-(c+s)}$ whatever the nature and size of the sand grains, and the proportions of the elements, sand and water, in the mortar. In all cases he found that $P=K\left[\frac{c}{1-(c+s)}\right]^2$. P being the resistance, and K a co-efficient which varies with each sort of cement or lime, and with the conditions of storage of the specimen.

The study of resistances therefore became the study of the variation of the compactness $(c+s)$ of mortars, the resistances having been found not merely to increase with c, but with $(c+s)$.

As this compactness will vary not only with the nature of the sand but also with that of the matrix, M. Feret made a series of experiments on mortars made of the same cement, then repeated them, varying only the cement, and found that the above laws were wholly independent of the nature of the matrix. The number of experiments was considerable, and the laws which were deduced were not arrived at without full consideration by M. Feret of the influence on the compactness of mortars, of the water used in mixing, of the sand, and also of the matrix.

The effect of the sand has been already studied, the maximum compactness being obtained for sands in which there are no medium grains, the proportion of the large grains being double the fine grains, which must include the matrix. It follows therefore, that the compactness of a mortar does not change when a portion of the matrix is replaced by an equal volume of fine sand. The resistance is, of course, greater as the proportion of cement increases, since it varies as $\left[\frac{c}{1-(c+s)}\right]^2$. The influence of the amount of water will be treated later.

The following method may be used for determining the proportions after M. Feret's conclusions :—

1. The specific gravity of the sand must be determined with care ; this can be done by placing in a graduated flask, such as Schuman's or Stanger and Blount's, a determined weight of sand, say 50 grammes, after having first poured in the proper amount of water. Having made certain that all the air is expelled, the specific gravity of the sand will be $\frac{50}{\text{displacement of water in cubic centimetres.}}$

2. The specific gravity of the cement must be obtained if not already known.

3. A litre of sand must be weighed, and its weight recorded.

4. A certain number of litres of sand must be mixed intimately with a determined weight of the matrix (being that of the proportion which is to be tested), the least possible amount of water being used to obtain a plastic mortar, the water being measured in a glass graduated to give the weight.

5. A cube of 10 cm. sides or a litre measure is filled with the mortar and lightly pressed in with the fingers.

6. The remaining mortar is weighed.

Now if k is the weight of the matrix per litre of sand,

p the weight of a litre of sand,
w the specific gravity of the sand,
w' the specific gravity of the matrix,
x the number of litres of sand employed,
q the weight of the water added,
v the weight of the mortar remaining,

M the total weight of the gauged mortar before using,

Then $M = px + kx + q$.

The weight of the sand in the mortar in the 10 cm. cube will be—

$px\left(1 - \frac{v}{M}\right) = P$, and $\frac{P}{p}$ = the number of litres of sand in the 10 cm. cube

The weight of the matrix in the cube will be—

$$kx\left(1 - \frac{v}{M}\right) = Q$$

The volumes corresponding to P and Q will be—

$\frac{P}{w} = V$, and $\frac{Q}{w'} = W$. V and W being the absolute volumes of sand and matrix.

And as the volumes of the cube and the volume under consideration are both 1 litre, it will suffice to add these two absolute volumes, V and W, together, to obtain the compactness.

A litre being taken as the unit of volume, we must take a kilogramme as the unit of weight. Other units could be taken, but the litre is a very convenient measure for experimental purposes.

MM. Pavin de Lafarge, from the results of numerous experiments which they have carried out, advise the rejection of all sand of which the compactness is not at least 0·700 for cement mortars.

It is certain that for a given sand the resistance will increase with the proportion of cement, while the compactness remains very nearly constant when different proportions of cement are used. The compactness will, however, naturally vary somewhat according to the force with which the mortar is pressed into the cube; it is necessary therefore to try the experiment several times for each gauging to arrive at a mean value.

MM. Pavin de Lafarge gives the following table of safe resistances to crushing of 5 cm. cubes for various gaugings and compactness, which are converted into pounds per square inch.

TABLE XV

Compactness	Ages Months	Proportions			
		224 Pounds of Cement to 11·97 Cubic Feet of Sand About 1 to 5	224 Pounds Cement to 10·34 Cubic Feet of Sand About 1 to 4	224 Pounds of Cement to 9·02 Cubic Feet of Sand About 1 to 3½	224 Pounds of Cement to 8·00 Cubic Feet of Sand About 1 to 3
		Pounds per sq. in.	Pounds per sq. in.	Pounds per sq. in.	Pounds per sq. in.
0·700	1	—	242	355	569
	2	—	412	497	853
	3	—	569	640	1,138
0·750	1	427	569	710	—
	2	497	710	995	—
	3	782	924	1,422	—

It appears evident that we cannot proportion mortar from the percentage of voids in the sand; we may, however, decide on the proportions, and test the various sands to find the one that gives us the greatest compactness; we can also find the amount of cement and sand which are necessary to make a unit volume of mortar.

Proportions of Ingredients for Concrete. — M. Feret found that the formula which he deduced in the case of mortars applied also to concretes in laboratorial tests made with small broken stone passing a ring of 0·79 inches diameter and retained on a ring of 0·38 inches diameter.

It appeared evident in all cases that with equal compactness concretes should be stronger as they contain more matrix, and with the same richness their resistance increases with their compactness. As in the case of mortars the compactness of concretes does not change when a portion of the matrix is replaced by an equal volume of fine sand. Also, when the sand is not too large, the compactness of the concrete differs very slightly for small variations in the proportions of the mortar, if the volume of the mortar remains the same and is mixed with the same quantity of stone.

According to M. Feret, the concrete has the greatest compactness with a fine matrix, a sand with uniform grains, which must be large in proportion to the cement grains and small in proportion to the stone, and the stone should be as large as possible.

The following are the results obtained from M. Feret's experiments—

1. The compactness of concrete increases with the size of the stone.
2. It varies inversely as the proportion of fine grains in the mortar.
3. It decreases generally as the proportion of the mortar to the same volume of stone increases.

It appears advisable *to increase as much as possible the size of the broken stone.* (In reinforced concrete we are limited, by practical considerations, in this direction.) *To diminish the proportion of the fine grains in the mortar by using a rich mortar made with large grained sand. To use the least possible amount of mortar that will thoroughly surround the stones.*

It is a well established fact that a concrete made of a rich mortar and broken stone or shingle will always be stronger than the mortar by itself. Mr. Baker, in his *Masonry Construction*, gives the following table deduced from the results obtained by Dr. R. Dykerhoff in a series of tests on cubes of mortar and shingle concrete twenty-eight days after moulding.

TABLE XVI

Proportions			Crushing Strength Pounds per Square Inch	Strength of Concrete in terms of that of Mortar
Portland Cement	Sand	Stone		
1	2	0	2,158	100
1	2	3	2,783	129
1	2	5	2,414	126
1	3	0	1,406	100
1	3	5	1,661	114
1	3	6½	1,534	109
1	4	0	1,068	100
1	4	5	1,291	121
1	4	8½	1,221	114

Broken stone concrete is probably comparatively stronger than shingle concrete. Mr. Baker also mentions the results of tests on cubes of concrete and mortar under the direction of General Q. A. Gillmore, U.S.A., showing that concrete mixed in proportions of 1 of cement, 3 of sand, and 6 of broken stone was 15 per cent. stronger than the 1 to 3 mortar by itself.

In another series of tests a concrete composed of 1 of cement, 1½ of sand, and 6 of broken stone was only 5 per cent. weaker than the mortar tested alone.

The most important elements in the strength of a concrete are the quality of the cement, the proper proportioning of the mortar to the voids in the stone, and the richness of the mortar.

Table XVII, which is an extract from Mr. Baker's book, shows the value of the correct proportioning.

TABLE XVII

Proportions			Crushing Strength			
			7 Days		28 Days	
Portland Cement	Sand	Broken Stone	Pounds per Square Inch	Relative	Pounds per Square Inch	Relative
1	2	2	494	0·60	565	0·81
1	2	3	611	0·75	555	0·80
1	2	4	819	1·00	613	0·88
1	2	5	581	0·71	680	0·97
1	2	6	500	0·61	698	1·00
1	3	3	333	—	205	0·53
1	3	4	—	—	366	0·95
1	3	5	—	—	386	1·00
1	3	6	—	—	357	0·92

The test cubes were 12 × 12 × 12 inches, and the water was 20 per cent. of the weight of the cement and sand.

The remarkable falling off of the 1 : 2 : 4 cube at 28 days is unaccountable, and in all probability was due to some defect in manipulation. The great excess of strength shown by this mixture at 7 days indicates that it should be the strongest mixture after 28 days.

The increase of strength by the employment of a richer mortar is shown by some experiments, mentioned by Mr. Baker, carried out at Watertown Arsenal, U.S.A., on broken stone concrete in which the voids in the stone were practically filled with mortar. The cubes, 12 × 12 × 12 inches, were tested 600 days after moulding, and were kept in water during the interval.

TABLE XVIII

Composition		No. of Cubes Tested	Average Crushing Strength Pounds per Square Inch	Remarks
Cement	Sand			
1	1	3	4,467	The results of the individual tests agreed well among themselves.
1	2	6	3,731	
1	3	6	2,553	
1	4	6	2,015	
1	5	2	1,796	
1	6	1	1,365	

The best method to adopt for the proper proportioning of the ingredients in concrete is still somewhat in dispute, but it appears reasonable to consider, not the cement, sand and stone each separately, but, since the strength of a concrete depends on the strength of the mortar employed, to decide first on the proportions of ingredients for the mortar, and then to determine the proportion of the mortar to the stone which will give the best results. It has been shown that with a mortar of the same richness there is a certain proportion of stone that will give the greatest resistance; this appears to be that in which the voids in the stone will be just filled with the mortar, each stone thus being surrounded, but no excess of mortar allowed. Such a concrete will be stronger than one which has either less or more mortar.

This fact is pointed out by M. Feret as the deduction arrived at from his extensive experiments; it is also shown by Tables XVI and XVII. Mr. Baker gives a table showing the results of a series of experiments made in order to determine the proportion of mortar required to fill the voids in a broken stone. The proportions for the mortar were 1 of cement to 2 of sand, both measured loose. The broken stone was such that—

30 per cent. was retained on a 1-inch screen.
51 ,, ,, ,, ,, on a screen with 5 meshes per lineal inch.
19 ,, ,, ,, ,, ,, ,, 20 ,, ,, ,,

and contained only 28 per cent. of voids when rammed.

From this table it appears that about 40 per cent. excess of mortar will be required over the voids in the broken stone to form a rammed concrete absolutely free from voids, and that with an amount of mortar equal to the voids there are still 7 per cent. of voids in the concrete.

Mr. Baker concludes that this increase in volume may be considered as a maximum, since the mortar was "dry" and the stone unscreened. With moderately wet mortar and the same stone the voids remaining were only about half this, or 3½ per cent., and with moist mortar and stone ranging from 2 inches to 1 inch, there was no increase in volume when the amount of mortar used was the same as the volume of voids.

Mr. Gillette points out that a large excess of mortar will be required when unscreened stone is used, and Mr. Newman, in his *Notes on Concrete*, states that with stones that have passed a ½-inch screen the mortar should be 50 per cent. of the volume of the stone to make an impervious concrete.

As has been pointed out before, it is advisable to use screened stone or shingle for reinforced concrete, in which case the sizes will range as a general rule from ¼ to ¾ inch; such stone or shingle will have from 40 to 45 per cent. of voids when rammed, according as there is much or little variation in size, the effect of ramming being to decrease the volume of the stone from about 5 to 8 per cent.

To obtain the proper proportions—

1. Find the percentage of decrease of volume of the stone by ramming. (This may be taken in most cases as 5 per cent., being a sufficiently near approximation.)
2. Find the percentage of voids in the broken stone or shingle.
3. Decide on the percentage of mortar in excess of voids that shall be employed. (For such stone as is used in reinforced concrete this need not be more than 5 per cent.)
4. Find the weight of cement and volume of sand in a unit volume of mortar as described (p. 137).

5. Find from (1) the volume of stone when rammed to the unit volume loose, and by adding to this the extra volume caused by the percentage of mortar which is in excess of the voids, the volume of the concrete produced by a unit volume of loose stone will be obtained. (In this case the rammed stone is not supposed to alter its volume by the addition of mortar which will exactly fill the voids, since with such stone as used for reinforced concrete this will closely approximate the real conditions.)

6. The volume of loose stone to form a unit volume of concrete is then obtained by dividing 1 by the volume of concrete found above, and the volume of rammed stone will be less than this quantity by the percentage found in (1).

7. Find the volume of mortar which will be required for this volume of rammed stone, from which may be determined the amount of cement and sand required for a unit volume of concrete.

As an example: Suppose—

1. That the stone decreases 5 per cent. of its volume by ramming.
2. That it has 45 per cent. of voids when rammed.
3. That we use 5 per cent. of mortar in excess of the voids in the stone.
4. That 770 pounds of cement and 25·5 cubic feet of sand are found to make 1 cubic yard of mortar.
5. As the stone decreases 5 per cent. by ramming, 1 cubic yard of loose stone will make 0·95 cubic yards when rammed, and will have 45 per cent. of voids, therefore—

$$0{\cdot}95 + (0{\cdot}95 \times 0{\cdot}45 \times 0{\cdot}05) = 0{\cdot}97 \text{ cubic yards of concrete.}$$

6. As 0·97 cubic yards of concrete are made from 1 cubic yard of loose stone, we shall require $\frac{1}{0{\cdot}97} = 1{\cdot}03$ cubic yards of loose stone to make 1 cubic yard of concrete. The volume of the stone when rammed will be $1{\cdot}03 \times 0{\cdot}95 = 0{\cdot}98$ cubic yards.
7. The mortar is 5 per cent in excess of the voids in the rammed stone; we have therefore for the volume of mortar required to make 1 cubic yard of concrete $0{\cdot}98 \times 0{\cdot}45 \times 1{\cdot}05 = 0{\cdot}48$ cubic yards. But there are 770 pounds of cement and 25·5 cubic feet of sand in 1 cubic yard of mortar; we must therefore have $770 \times 0{\cdot}48 = 370$ pounds of cement and $25{\cdot}5 \times 0{\cdot}48 = 12{\cdot}24$ cubic feet of loose sand to make one cubic yard of concrete.

We have, then, for 1 cubic yard of concrete—

TABLE XIX

Cement		Sand Loose Cubic Feet	Stone Loose Cubic Feet	Proportions by Volume			Proportions in Cubic Feet of Aggregate per Bag of 224 Pounds of Cement	
Weight Pounds	Cubic Feet at 85 lbs. per Cubic Foot			Cement	Sand	Stone	Sand	Stone
370	4·35	12·24	27·81	1	2·8	6·4	7·41	16·84

Table XX has been compiled from details of tests made by Mr. Q. C. Sabin, Assistant U.S. Engineer, to determine the amount of cement and sand required to make a cubic yard of mortar, but this table can only be taken as an approximation, since the amounts depend on the nature of the sand and the cement.

TABLE XX

Proportions of Mortar		Approximate Weight of Cement required to form Cubic Yard of Mortar Pounds	Approximate Number of Cubic Feet required to make 1 Cubic Yard of Mortar.	
Portland Cement	Sand		Portland Cement Loose	Sand Loose
1	1	1,445	17	17
1	2	935	11	22
1	3	680	8	24
1	4	530	6¼	25

The sand used for these tests had 37½ per cent. of voids when loose. Sufficient water was employed to make moisture flush to the surface when the mortar was struck with the back of a shovel.

Amount of Water to be used in Mixing.—There is much difference of opinion as to the quantity of water that should be allowed for mixing concrete.

Some constructors mix very dry and trust that ramming will make a homogeneous concrete, and one which will form a protective coating on the reinforcements. In this case the mortar used in the concrete has the appearance of a damp sand before being mixed with the stone and rammed into place.

Others use a fairly wet mixture, as less ramming is required, and the protection of the reinforcements is more assured.

"Dry" concrete well rammed is deemed by its advocates to have less voids than wet concrete, but this must greatly depend on the amount of ramming. On the other hand, there is a risk in using "dry" concrete of not having sufficient water to enable the concrete to set properly throughout.

Perhaps the most general opinion amongst constructors at the present time is that very little water should be used, and the concrete thoroughly rammed.

M. Considère advocates the use of "dry" concrete well rammed, and M. Christophe, in his book on *Béton Armé*, strongly advises its employment, holding that it is stronger than "wet" concrete, and that voids are formed when excess of water is used.

On the other hand, many constructors, M. Hennebique, M. Cottançin and M. Chaudy amongst them, use a fairly wet mixture, and numerous experiments show that no loss of strength results from the use of moderately wet concrete, and some clearly prove it to be stronger than "dry."

From some extensive experiments made by Mr. G. W. Rafter and published in the New York State Engineer's Report, it was found that with an excess of water, so that the concrete was jelly-like under the ram, it was only 12 per cent. weaker than vigorously rammed concrete with the least possible amount of water.

It is certain that the employment of moderately wet concrete better ensures the protection of the reinforcements and cheapens the production, both on account of the smaller amount of ramming required and the consequent need for less rigid falsework.

It seems that in a general way the amount of water may vary within certain limits, both too wet and too dry concretes being dangerous.

In certain cases it may be better to use "wet" and in others "dry," but there must always be sufficient water to hydrate all the cement. Where the work is easily got at, and the ramming can be thoroughly effected around the reinforcements, a "dry" concrete is probably the better, but care is necessary that it must not err on the side of excessive dryness. The broken stone should always be thoroughly well wetted before gauging, so as not to absorb the water from the mortar. There is also danger with the use of "dry" concrete of forming lines of cleavage between the successive layers.

Where the piece is of small dimensions and the space around the reinforcements not easily got at for ramming, it is advisable to employ a "wet" concrete, but in this case care must be taken to avoid the use of excessive water, and a certain amount of working and ramming should be done to eliminate the air and to prevent voids being left.

In some cases where pipes, etc., are formed by running grout of quick setting cement into moulds, ramming is entirely dispensed with.

In the Wayss and other systems very dry concrete is used, and the ramming is thoroughly done. The results obtained are wonderfully satisfactory, but water is cheaper than ramming, and so long as too much water is not used it does not seem that the strength is affected in any marked degree.

M. Hennibique until quite recently mixed his concrete very dry, but he now uses more water with less ramming.

Mr. Thacher in his specification states that no more water shall be used than the concrete will bear without quaking during ramming.

Table XXI gives the results of some special tests showing the effect on the strength of 3 to 1 cement mortar by varying the percentage of water. The briquettes were left in the moulds for 48 hours, and then placed in water until tested.

TABLE XXI

Percentage of Water by Weight to Sand and Cement	Tested at 1 month		Tested at 3 months		Tested at 6 months	
	Average Tensile Strength of 3 Briquettes Pounds per Square Inch	Comparison with that mixed with 10 per cent of Water	Average Tensile Strength of 3 Briquettes Pounds per Square Inch	Comparison with that mixed with 10 per cent. of Water	Average Tensile Strength of 3 Briquettes Pounds per Square Inch	Comparison with that mixed with 10 per cent. of Water
5	191	76·9	249	94·7	263	88·8
6¼	193	78·0	242	92·1	283	95·6
7½	198	80·2	256	97·4	326	110·1
8¾	210	85·2	270	102·6	330	111·5
10	247	100·0	263	100·0	296	100·0
12½	223	90·3	243	92·0	286	96·6
15	216	87·5	236	89·7	270	91·2
17½	182	73·6	226	85·9	265	89·5
20	180	72·9	220	83·6	251	84·8

These were, of course, laboratorial tests, the ramming being very thorough. They show that with water between 7 and 12½ per cent. there is no great variation of strength, but that percentages between 7 and 10 have the advantage after three months. At the end of one month percentages between 8¾ and 15 appear to give the best results. With less thorough ramming it is certain that the lower percentages would in a great measure lose their advantage, and it should be remembered that in practice we must not count too much on the ramming.

From his experiments on mortars and concretes, M. Feret forms the following conclusions with respect to the quantity of water used in gauging—

1. The quantity of water to use in making a plastic mortar increases proportionately to the nature and weight of the cement and the fineness of the sand, a portion of the water being necessary for hydrating the cement, and another for wetting the grains of sand.

2. That the compactness of a mortar gauged with increasing quantities of water varies, following different laws, according to the granular composition of the sand. With sand containing many fine grains it diminishes constantly with the addition of water; with other compositions it reaches a maximum for a determined proportion of water, generally a little less than that which corresponds to the plastic condition.

From numerous experiments by different authorities it has been found that from 4 to 4½ gallons of water per cubic foot of cement used is about the best proportion for a sound concrete. This would be from about 45 to 50 per cent. by weight of water to the cement employed.

The French Government, after extensive trials, specify the following tests for the proper consistency—

1. Consistency should not change if the mortar be gauged for an additional three minutes after an initial five minutes.

2. If a small quantity of mortar be dropped from a trowel, the trowel ought to be left perfectly clean.

3. A little mortar worked gently in the hands should be easily moulded into a ball, on the surface of which water should appear.

4. When the above ball is dropped from a height of 20 inches on to a hard surface, it must retain its rounded shape without cracking.

For making these tests it is best to mix the mortar with a minimum of water at the commencement and add further water until the desired consistency is attained, from which the amount required for a unit of cement is easily assessed.

Methods of Mixing

Machine Mixing.—The mixing of concrete should be done by machinery, where the works are of sufficient magnitude to allow of the cost of the plant, as there is more certainty that it is all evenly and thoroughly done than when mixed by hand.

In this case the materials should first be thoroughly mixed " dry," after which the proper amount of water is added, and the mixing continued until the concrete is uniform.

There are many machines made by different firms for this purpose, all of which will give satisfactory results.

Where there is sufficient height the " gravity " mixers are very suitable and portable. These have a hopper at the top, into which the materials are placed, and in the body are a series of sloping shelves, which throw the materials from side to side as they gravitate down, water being added some little distance from the top.

The rotary machines are of various kinds. In some a drum is used with internal projections; in others the materials are carried along a closed trough by a spiral conveyor. No machine that has a grinding action should be allowed, as this damages the particles of sand in the mortar made by them.

Hand Mixing.—When hand mixing is employed, which must be the case, except on large works, as machinery will not be economical where only small quantities are required at a time; the sand and cement should be first thoroughly incorporated in a "dry" state by turning over and over until well mixed. The water must then be added through a rose, and the turning continued until a uniform mortar is made. The stones, which have been previously wetted, should be spread on the top of the mortar, and the mixture turned over and over until all the stones are covered with mortar.

The least number of turnings allowed should be two with dry cement and sand, and two when the water has been added, the turning while the water is being added not being counted as one of the wet turnings. After the stones have been spread on the top of the mortar, the mixture must be turned over at least three times. The number of turnings should not be specified, however, except perhaps as a minimum, since the sufficiency of mixing depends greatly on the manner in which the operation is performed. The shovel must be twisted in the hand while turning to obtain good results.

Great care must be taken that the concrete is well and evenly mixed, as the strength of a reinforced structure depends greatly on the evenness of the concrete employed. It is needless to add that all concreting materials must be deposited and mixed on clean, close-boarded stages of sufficient size.

Reinforcing Metal

General Remarks.—Up till comparatively recently wrought iron has been considered by most constructors as the best material for reinforcements. Steel is, however, coming more into use for this purpose, and in some cases is undoubtedly the better material to employ, but wrought iron possesses all the qualities required in most instances, and is frequently cheaper. Where special rolled sections are used, such as those of the Bonna or Bordenave systems, mild steel is always employed; "expanded metal" is also manufactured of mild steel. When ordinary mixtures of concrete are employed it will be found that wrought iron is the most economical metal to use unless steel is equally cheap.

Where strong concretes are employed, as for structures to resist water pressure, such as pipes, reservoirs and sea works, the use of steel is generally more economical than that of wrought iron, as a higher unit resistance can be allowed for the reinforcement. The same applies to columns and pieces under direct compression, where it is usually advisable to use strong concrete and steel reinforcements. Sometimes for beams and like structures the portion of the concrete in compression is mixed with a greater proportion of cement, in order to obtain greater resistance on the compression side of the neutral axis. In such a case it will be cheaper to employ steel as the reinforcement, but it is not good practice to vary the proportions in the concrete for one piece. The use of wrought iron or mild steel is generally a question of economy, and except in special cases it may be taken as a general rule that it is better to use whichever may be cheapest at the time, and when steel is at a lower price than wrought iron it is always the better material to use.

Where welding is necessary, wrought iron is safer than steel.

Wrought Iron.—The iron most frequently employed for reinforcing purposes is in the form of round rods of commercial quality and sizes, but sometimes flat or square bars or profile sections are used. *The iron should be good malleable iron, which will bear* 48,000 *pounds per square inch before fracture, with an elongation of* 10 *per cent. in a length of* 8 *inches. A bar of this iron should bend when cold* 180° *round a curve whose diameter is twice the thickness or diameter of the bar without sign of failure.* The unit resistance of iron bars becomes greater as they decrease in size if they are not annealed after rolling, but this is seldom counted on except in the cases where wires are employed.

In deciding on the size of reinforcing bars to suit a particular case, it is well to remember that extras are charged as a rule for sizes of ordinary bars less than ½ an inch diameter when round, or 1 × $\frac{5}{16}$ inches or ¾ × ½ inches when flat. At the present time about 5*s*. per ton is charged extra on per $\frac{1}{16}$ inch between ½ inch and ¼ inch diameter rods, and below ¼ inch diameter the additional cost increases more rapidly until $\frac{3}{16}$ inch rods may cost £4 or so extra per ton. The additional cost for small flats varies from about 10*s*. per ton for the larger sizes to about £8 per ton for the smallest. Small angles and tees below about 2½ × 2½ × ¼ inches are also charged for at additional rates of from 10*s*. to £3 per ton.

Steel.—*The steel should be of a mild quality, and be capable of bearing not less than* 60,000 *and not more than* 70,000 *pounds per square inch at breaking, with an elongation of not less than* 20 *per cent. in a length of* 8 *inches. A bar should bend when cold through an angle of* 180°, *and close down upon itself without cracking.*

As in the case of iron, the unit resistance increases as the size diminishes if the bars are not annealed after rolling. The same remarks as to extra cost of small sizes apply to steel as have been already made on the subject of wrought iron.

Part IV

PRACTICAL CONSTRUCTION

Methods Employed in the Erection of Reinforced Concrete Structures

Generally speaking there are three methods adopted in the erection or manufacture of reinforced concrete—

1. The pieces may be moulded in advance, either out of doors or in sheds, and be placed in position after they have set sufficiently.

2. The whole or part of the structure may be moulded in place on timber falsework, the falsework being struck when the concrete has hardened sufficiently for this to be effected.

3. The reinforcement may be erected or partially erected first, and serve as an aid in the support of the moulds.

Any one of these systems may be employed alone or in combination with others.

The advisability of good timbering cannot be too strongly insisted on for reinforced concrete work, as the stability of the whole structure, especially when all parts are moulded together, depends largely on the strength and rigidity of the falsework. Here, as in all other cases where timbering is required, good methods and a somewhat large outlay at the commencement will be found the greatest real economy.

Moulds

Treatment of Moulds.—The moulds for reinforced concrete work must be stiff and strong enough to bear the weight of the concrete, the ramming, etc., and; when supported, the weight of the men and materials, without bulging or vibration. Vibration has a serious effect on the setting of concrete, and in some cases it has been held to be the cause of the complete failure of structures.

Moulds should be of as simple a character as possible and lend themselves to easy supporting, so that they may be put together, taken to pieces, erected and removed with the minimum amount of labour; standard sized pieces being employed in building them up which can be used again and again or easily altered to suit different requirements. They are generally made of timber which should be carefully selected, and none used that is liable to warp or twist. Much trouble will be avoided if the planks forming the moulds are put together in such a manner that if they swell under the action of the moisture no deformation will occur. In the Ransome system the moulds are frequently made of planks splayed along one of their edges to allow the closing up of the joints caused by the swelling without fear of warping. The planks are placed tight against one another producing a continuous surface; if the humidity swells the timber, the bevelled edge of one plank slides over the square edge of the next, the thickness of the planking is not altered to any great degree, and scarcely any deformation is produced (Fig. 196).

FIG. 196

With square edged planks and open joints, which is the form usually employed, the openings must be stopped so that the water from the concrete may not drain away carrying some of the cement with it. M. Hennebique now usually makes his moulds with open joints and lines them with coarse canvas, which absorbs the excess of water.

When no covering is used the inner faces are generally planed, and should be as smooth as possible in order that little making good may be necessary. The concrete must not be allowed to adhere to the moulds, as, when this occurs, the removal is rendered difficult, the surface of the concrete is damaged, and the moulds require to be scraped before being used again. There are several methods for preventing this adherence; the inner surfaces being either brushed over with mineral oil (fatty oils act upon the concrete and must in no case be employed for greasing the moulds), or they are lined with paper, canvas or jute; sometimes where a superior finish is desired the boards are lined with sheet iron, zinc or plaster of paris, but this is seldom done except where the same moulds can be re-used many times or where in the case of arches or beams there are many spans of the same size. In the Koenen system, where these materials are always employed to cover the centreings for the arches, etc., little or no dressing is required after the falsework is removed.

For large span arches a layer of puddle has been employed over rough lagging, the puddle being covered with thick paper, and this method might well be applied for other mouldings.

Special surfaces are sometimes used as in the case of the imitation stone facing often employed in the Ransome system. These are formed by securing V-shaped strips to the moulds, striking before the concrete is quite set, and finishing the face either with a light pick or a chisel according to the surface required (Fig. 197).

FIG. 197

It is important that the sizes of beams, columns and other parts should be the same where feasible, as this will effect much economy in the moulds. Where sheet iron or zinc are used for coverings or linings, there is a distinct saving of cost in this respect, but zinc has been found in some cases to have a chemical action on the concrete and it is therefore better to use sheet iron.

When pieces are moulded before erection the moulds are greatly simplified, as they can be supported throughout their whole length, and therefore require less resistance in themselves since they are not subjected to bending action. Iron moulds can be economically used if many pieces of one size and shape are required.

For thin slabs, a piece of sheet iron or a block of stone or concrete covered with thick paper or canvas may be used with no casing. Moulds of reinforced concrete have been employed covered with plaster of paris to prevent adherence.

Falsework

General Remarks.—The falsework used for reinforced concrete structures is so intimately connected to the moulds when the structure is formed in place that the methods adopted in making and supporting the mould must be described together. The previous remarks on moulds apply both to those employed by themselves or as part of the necessary falsework of a structure moulded in place. The usual systems of falsework applied to buildings will be touched on first, and a brief description of that of a special nature for other works described afterwards.

Columns. — The boxes for square columns are formed in three ways — 1. They are brought up entirely as the work proceeds. 2. Three of the sides are formed before the moulding is commenced, the fourth side being brought up as the concrete is deposited. 3. All four sides are formed to the full height, and all the concrete put in from the top. In the first case the sides of the boxes are formed of horizontal boards and held in position by four uprights at the angles, which in their turn are framed together by horizontal cross-pieces on their outer faces. Triangular strips are placed up the inside corners of the boxes to form the chamfers.

When this method is employed, the concrete can be worked in from all sides, which is a great advantage. In the second case the three sides are formed of vertical planks; these extend to the bottom of the splayed portion where the columns are connected to the principal beams. These planks are held together by strips of board nailed to the sides at about three feet intervals, and also in the Hennebique system by special clamps, shown in Fig. 198. In this clamp the lower horizontal

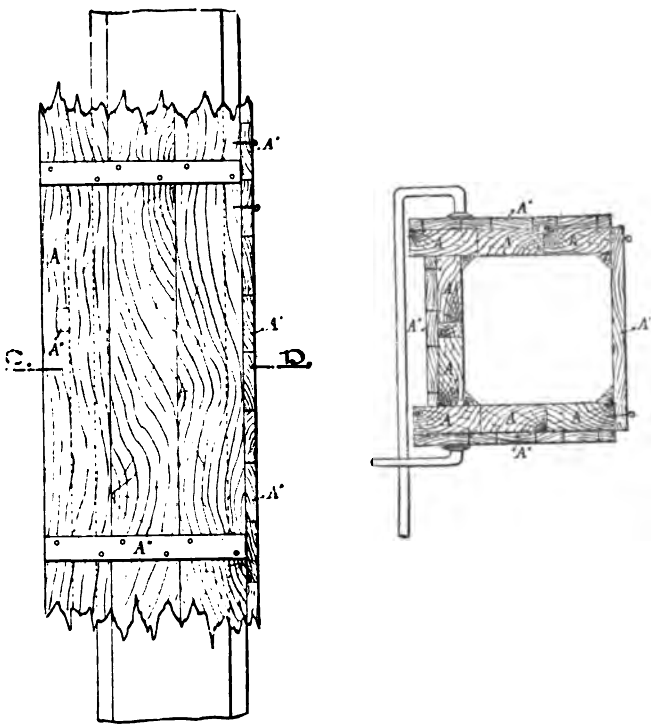


Fig. 198

piece slides on the vertical piece. The clamp is placed over the box, and the horizontal piece is knocked up tight with a hammer by hitting its shorter end; this gives it a tilt, in consequence of which the greater the pressure on its clipping end the tighter it holds. The loosening is effected by knocking back the shorter end. Some nails with large heads may also be driven partially in so that they can be easily withdrawn. Triangular strips of wood are placed in the corners to form the chamfers. The remaining side of the box is brought up with boards placed horizontally, and secured to the sides by large-headed nails, which are left slightly projecting. When the concrete is set the nails are withdrawn and the boxes removed.

When this method is adopted the concrete can only be put in and worked from one side, and consequently it is more difficult to obtain proper consolidation.

Fig. 199 shows the ground floor columns for the Southampton Cold Storage, constructed in this manner on the Hennebique System.

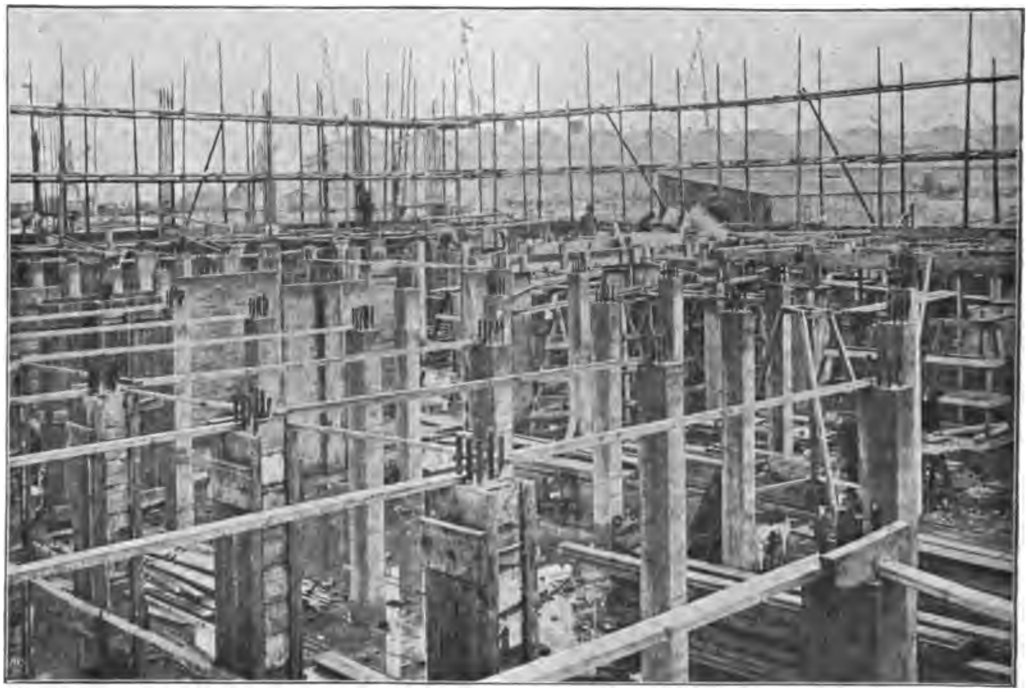


FIG. 199

In the third case the boxes are usually formed of horizontal boards, held by upright planks and cross strips. The concrete must be mixed very wet, and the proper filling of the moulds and surrounding of the reinforcement left to gravity. This is not good practice except for special cases, as there can be no certainty that the concrete is homogeneous and that it has the resistance for which its demensions were calculated, nor is there any assurance that the reinforcement is properly surrounded and protected.

It is usual to leave the boxes on columns moulded by the first two methods for about twenty-four hours after the moulding is complete, but when the third method is adopted a longer time must be allowed unless a quick-setting cement is used.

For round columns, special boxes or drums in two halves are employed. If there is a moulding on the column, this may be formed by a series of bevelled or curved strips, which are shaped to fit round the column and placed on the top of one another to form the moulding.

Walls and Partitions.—The moulds for walls and partitions are usually much the same as those for columns. Sometimes the box is formed by two timber sides for its entire height, after the reinforcement of network is in place and has been fastened to the neighbouring walls and beams. The concrete is then mixed wet and poured in, the network being shaken to make it go down and settle as much as possible. This method has the same drawbacks as that of forming the boxes for columns to their full height before commencing the concreting.

In some systems, partitions are at times moulded against one flat surface on which the concrete is spread in layers, in the same manner as described later for pipes (p. 198). This method is economical in the matter of timbering.

Walls and partitions of the Hennebique and similar systems are constructed between two timber shutters, one being made to its full height before the concreting is commenced, while the other is brought up as the work proceeds. The men working from this side can well reach the concrete for the purposes of ramming, and can also place the longitudinal reinforcements in position as required.

In the Ransome system the two timber partitions are brought up as the work proceeds, the planks forming them being held in place by vertical uprights. These are placed where possible at the framings of bays, so that they may be tied together by cross pieces through the wall; when this is impossible the uprights are connected by strips of wood at intervals, which are removed as the work comes up to them.

Messrs. Ransome now use a movable frame for their walls constructed of pairs of slotted standards one on either side of the site of the wall, and held together by bolts passing through the slots. These standards, which hold the moulding boards in position, are arranged to slide upwards on the outer faces of the boards as the wall progresses, and can be made to conform with any breaks or projections in the line of the building. Fig. 200 is a sketch of one of these standards. Fig. 201

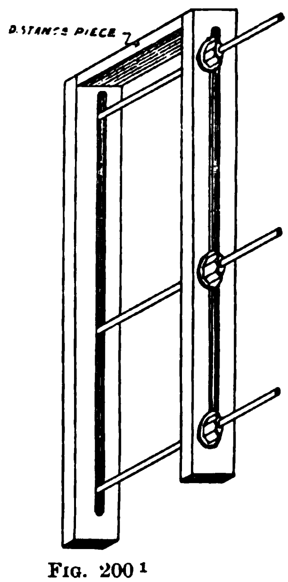


FIG. 200[1]

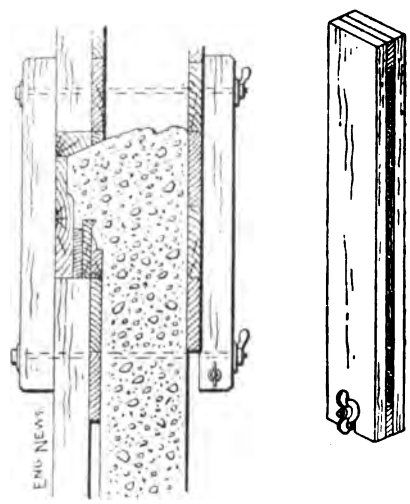


FIG. 201

[1] The top bolt in this figure should have been shown at a lower level.

clearly indicates the method adopted for the formation of mouldings. The right hand view shows the manner in which the slotted standards are generally made.

When the concrete has been brought up to the top of the standards, the hand nuts are slakened in one pair of uprights at a time, and the standards pushed up as much as desired, which movement depends on the nature of the work. When the limit of movement allowed by the slots is reached, the bottom bolt, which now bears against the lower end of the slot, is withdrawn, and placed at the top, and so on. The boards as they are left free by the upward movement of the standards are re-used above those already in place. The holes left by the withdrawal of the bolts are filled in after the moulding is complete.

This apparatus is patented by Messrs. Ransome, but its cost is very small, being only about £32 for a building 100 feet long and 50 feet wide with plain walls.

The bolts used to hold wall moulds in position can be knocked out as soon as the concrete has set sufficiently to remove the moulds, but a better plan is to make the bolts in three pieces connected by square sleeve nuts, from 1 to 2 inches within the wall. When the forms are removed the ends of the bolt are easily unscrewed and withdrawn, the central portion being left in and the holes stopped with mortar.

Hollow walls are usually formed with a timber core, which can be removed as the work sets; the last portion, however, must be left in.

Chimney flues are frequently moulded on stiff paper or cardboard pipes filled with gravel or sand, which is allowed to escape when the concrete has sufficiently hardened.

For exterior walls, thin slabs of reinforced concrete with projecting pieces of hoop iron have been employed by M. Hennebique as moulds, each having a concrete projection which forms a gauge for the thickness of the work, the exterior faces of the slabs being finished in such a way as to form an ornamental and effective face to the wall. For M. Hennebique's country house at Bourg-la-Reine [Frontispiece] this outer face is formed by embedding flat flints in the concrete at varying distances apart, the effect obtained being very pleasing. The slabs used for this house were only about 1¼ inches thick, and about 2 feet long by 10 inches high. They were jointed with neatly finished sunk mortar joints. The reinforcing rods for the wall were inserted in the mass of concrete behind them, but they themselves were reinforced with thick wire for the purpose of ensuring their safe transport.

Beams and Solid Floors.—After the columns and walls which support the beams have been moulded, the boxes for the principal beams are generally formed to a level just below the undersides of the secondary beams, the splayed top of the columns being formed at the same time, as this is in reality a deepening of the principal beams near the supports, and consequently must be moulded with them.

The bottoms of these boxes are formed of one width and length of timber if possible, the length terminating at the sides of the columns or walls as the case may be. The length of the sides of the boxes extends from side to side of the building; these overlap the bottom so that the supporting props may be held between them. The sides are held in position by iron clamps similar to those described for the column moulds (Fig. 198), these are placed under the bottom and clip the sides together. Care must be taken that the top of the sides are at a level just below the bottom of the secondary beams, as the sides of the boxes for these will rest upon their top edges. The chamfers on the bottoms of the beams are formed by small triangular strips, lightly nailed in place. The bottoms of the boxes are held up with slightly inclined props, supported on wedges from the ground or lower floor.

The principal beams are now moulded to the level of the underside of the secondary beams, after which the boxes for the latter are formed, the sides being placed so as to just cover the top of the planks forming the bottoms, which are supported on timber cleats nailed to the side timbers of the principal beam boxes. The sides of the secondary beam boxes rest on those of the principal beam moulds and are held together by clamps, and pieces of board nailed to their top edges, which act as distance pieces. The level of the top of the sides of the secondary and the addition to the sides of the principal beams is made such as will allow for the thickness of the planks that will be employed for the centreing of the floor slab. The planks forming the sides of the principal beams are held together by pieces of board nailed to their sides, and the added planks have distance strips nailed across from side to side of the mould.

Props having been placed under the centres of the secondary beam boxes, the moulding of both the primary and secondary beams is brought up. Great care should be exercised in the alignment of the secondary beams, their positions being marked on the moulds for the primary beams before the erection of their boxes is commenced.

Fig. 202 shows the first floor of the Southampton Cold Storage. The beams having been completed, the men are preparing the floor centreing.

FIG. 202

When the moulding of the beams is completed, the floor is commenced as soon as possible, being formed on a close boarded centre, the planks for which are supported on the tops of the sides of the principal beam boxes and run parallel to the secondary beams, the edges of the two outside planks of each bay being even with the inside of the secondary beam boxes,and the ends of all the planks being in the same position in respect to the inside of the moulds for the principal beams. Some transoms are placed in each bay to support the floor centreing, being held up by cleats nailed to the sides of the secondary beam boxes.

Fig. (203) indicates the procedure adopted as described above.

With falsework of this description the boxes for the beams must remain in position

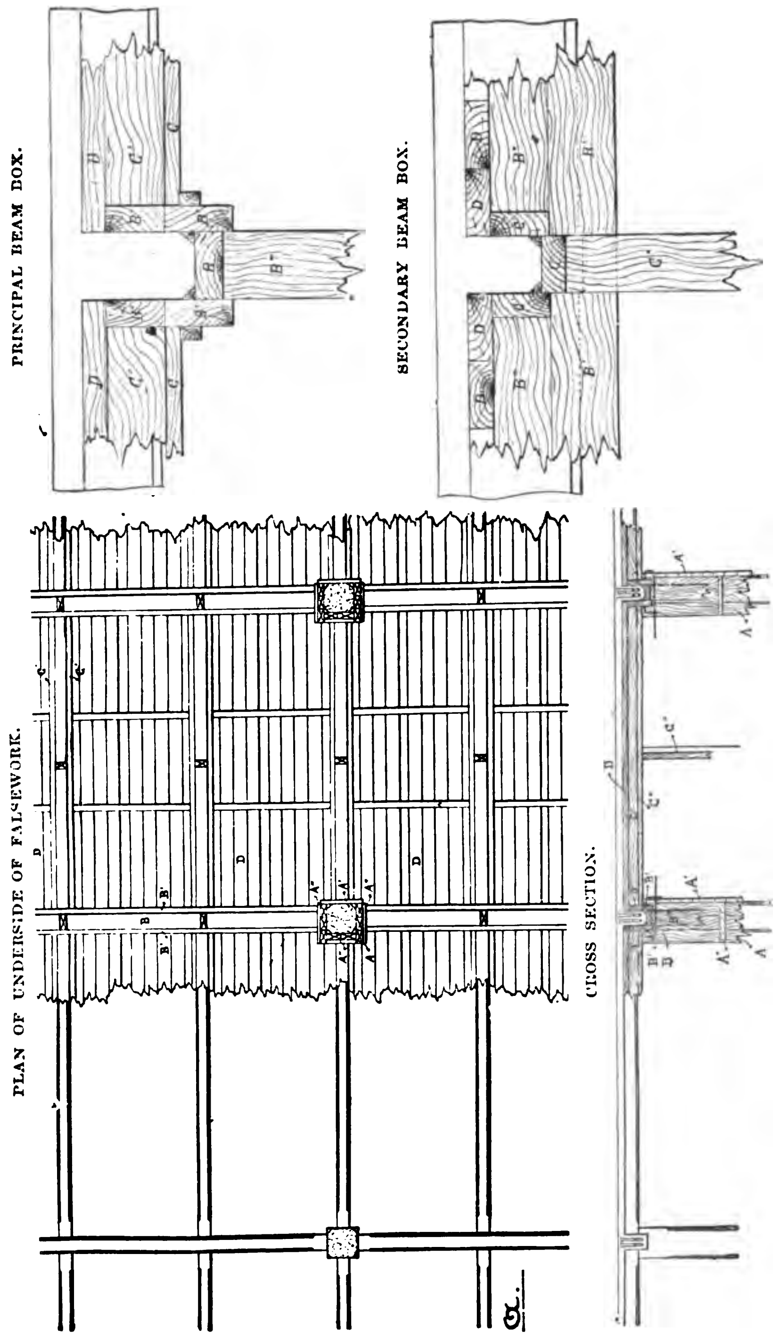


Fig. 203

while the floor is being constructed. To obviate this inconvenience the tops of the sides of the beam moulds are sometimes made level with the underside of the floor, and when the concrete of the beams has sufficiently set, the sides of their boxes are removed, leaving only the bottoms and their supporting props in place; planks are then placed along each side of the secondary beams (in many cases the sides of the beam box are used for this purpose) and secured by clamps and inclined props, their tops being kept at such a level from the underside of the floor as to allow for the thickness of the planks of the floor centreing and also for the transoms which span the distance between the secondary beams and support the centreing. These transoms are sometimes placed on the skew to avoid cutting, and are spaced about five feet apart.

The platform for moulding the floor is usually laid from each side, the space at the centre being seldom of a plank width, is filled in by a piece of thin 1-inch or ¾-inch board with splayed edges, supported on wooden blocks resting on the transoms. Fig. 204 shows this method of forming the floor centreing.

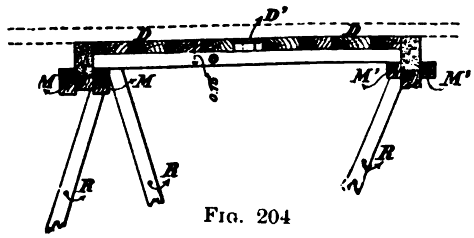


FIG. 204

As a rule the bays of a floor are of the same size, and it is only necessary to have three or four of the centreings which may be used over and over again, but before the centreing is struck one of the planks near the centre of the bay must be well propped, and this must be left in for at least a month when the rest of the centreing is removed.

Fig. 205 is a view taken of the Great Western Railway Warehouse at the Royal Albert Docks in the course of construction.

FIG. 205

A method frequently adopted to economize the use of timber for the purposes of centreing is to mould the floor on a thin slab of reinforced concrete which is supported on the beams and becomes a part of the floor, but this method is more generally employed for double floors with hidden beams, described later. The supporting slab in such a case must not be included in the calculated thickness of the floor.

The striking of the floor centreing, etc., should not be commenced until the concrete is thoroughly hard, which will be about a week after it is deposited. The sides of the beam boxes may however be removed sooner, as this will aid the setting. The bottoms of these may be removed from three to four days after the floor or other slab is complete.

Messrs. Pavin de Larfarge leave their moulds in position for seven or eight days at least, and let the supports remain under the beams as long as possible.

The supports under the centres of the beams and floor slabs should be left in position for three or four weeks as a precautionary measure.

The leaving off of the concreting while the moulds and centreing are being put into place is a serious disadvantage, as the concrete must have lines of cleavage in consequence of the time which elapses between the periods of concreting, however carefully the old surface may be prepared before recommencing. The stirrups or similar reinforcements, however, serve to tie these layers together, and should be employed for this purpose, though they may not be required to resist the ordinary shearing stresses.

When a stoppage must be made for the night, if moulding the bottoms of the principal beams, the place for leaving off must be over a column, as here the lower portion is in compression. When the secondary beams and upper portions of the principal beams are being formed, the secondary beams must be left off on each side of the principal beams and the principal beam moulded throughout at one time. The portion of the span near the supports should be chosen, as the floor slab as a general rule takes up all the compression at the centre. As regards the proper place to leave off when constructing a floor, it must be at the centres of the spans of the beams, as here the floor acts in compression.

There is no doubt that the putting in of the concrete in layers at different time

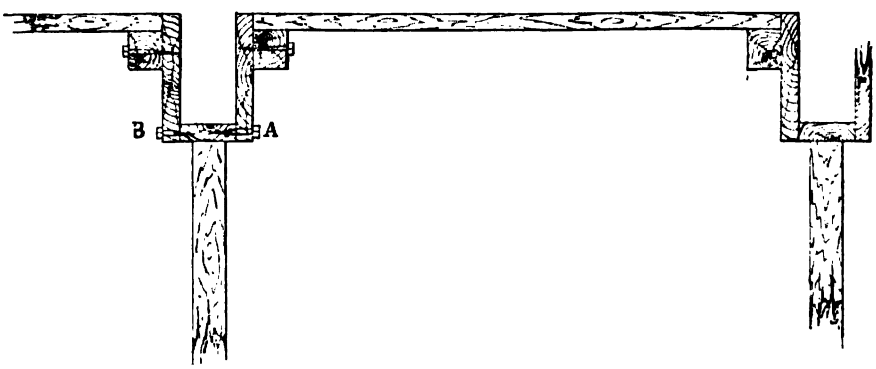


FIG. 206

must tend to cause weakness, and it is a question whether the loss of economy by forming all the falsework together and moulding everything at one time, as is done by some constructors, will not be repaid by the gain in strength. On the other hand, the whole area of a monolithic floor must generally be moulded in portions, for it is hardly possible to form the whole in one day, if it is of any size, and consequently the concreting must be discontinued somewhere. And although we

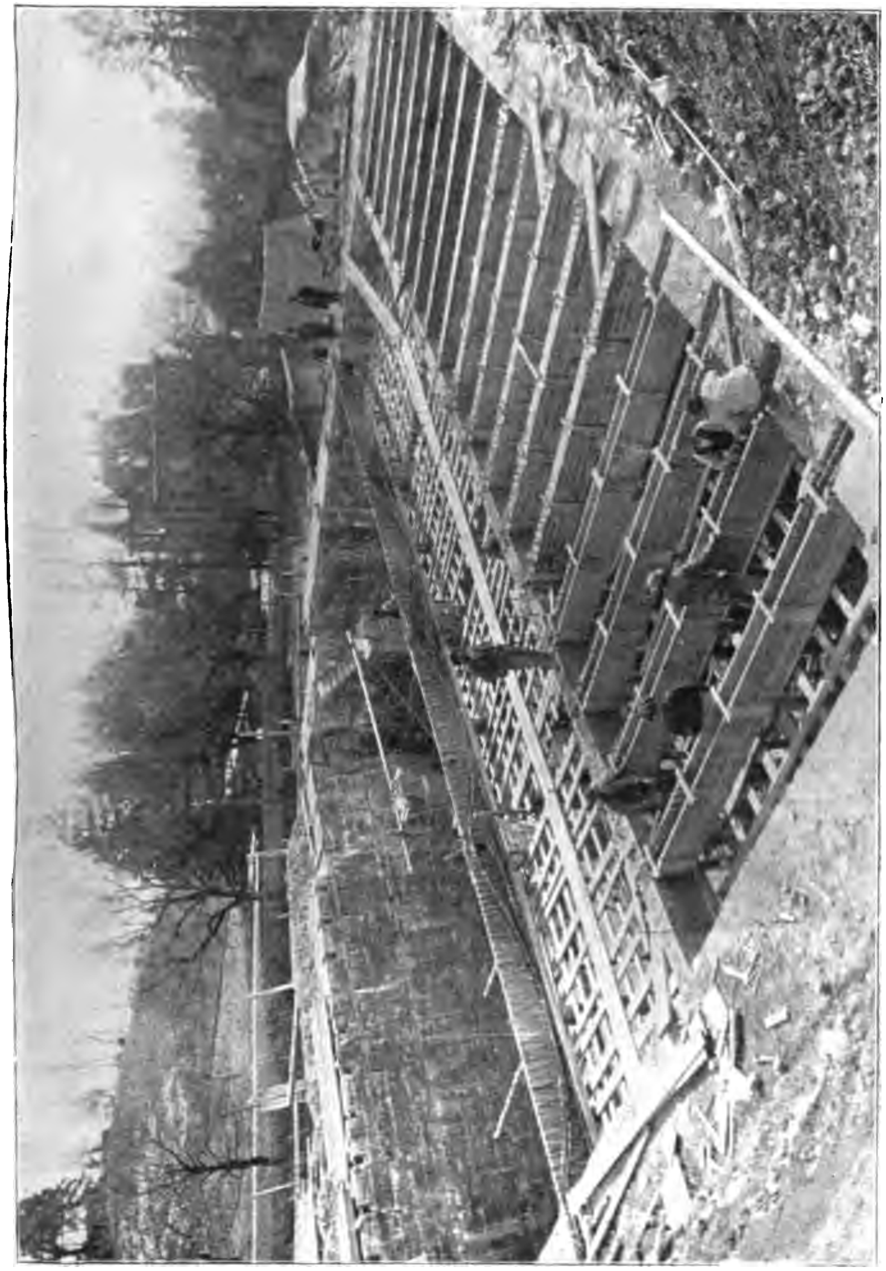


FIG. 207.

gain an increase of strength in the vertical direction if we mould all the depth at once, we shall still have a lack of continuity in the area, and even with this method there must be a pause while the reinforcement is being put into place.

When the falsework is erected altogether the ramming cannot be so effectively performed, and the working of the concrete will not be so well done around the reinforcement on account of the depth of the beam boxes.

Fig. 206 shows the method adopted by MM. Pavin de Lafarge, who erect all the falsework before commencing the concreting, and Fig. 207 shows the construction of the covering for the Reservoir de Montalegre, on the Vallêrie et Simon system for the town of Lausanne, the principal beams being constructed first, portions being left out where the secondary beams will join them. The boxing for the beams is made complete before the concreting is commenced.

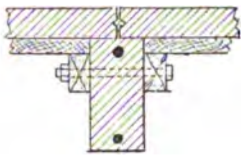


FIG. 208

M. Coignet passes bolts through the sides of his beam boxes; these support the floor slab, being held up by the bolts which are retained in position by the concrete of the beams (Fig. 208).

In the Ransome system, the floors are frequently divided up into a series of small equal bays by beams of the same size intersecting at right angles. This enables the falsework to be of a more perfect form, without very much affecting the economy. Inverted boxes of timber are used, of the same size and shape as the bays of the flooring, the floor slab being formed on the tops and the sides act as sides of the beam moulds, the distance between the adjacent boxes being the width of the beam, and their depths equal to that of the beams plus the thickness of the timbers which form the bottoms of the beam moulds. These are supported by strips fixed across from bottom to bottom of the neighbouring boxes.

The boxes themselves are supported by strips of wood, nailed to the sides of the column moulds where they abut on these, and by some props at other places. Each box is made in two halves by a diagonal joint which is left slightly open, made good by sliding boards, so that the two parts may be slid past one another, and that there may be some lateral play when erecting and striking.

The beams on this system have a slight taper, being of a smaller width at the bottom than at the top, which also permits the setting and striking of the falsework to be more easily performed; the planks forming the tops of the inverted boxes are of the splayed type shown in Fig. 196. This type of floor is

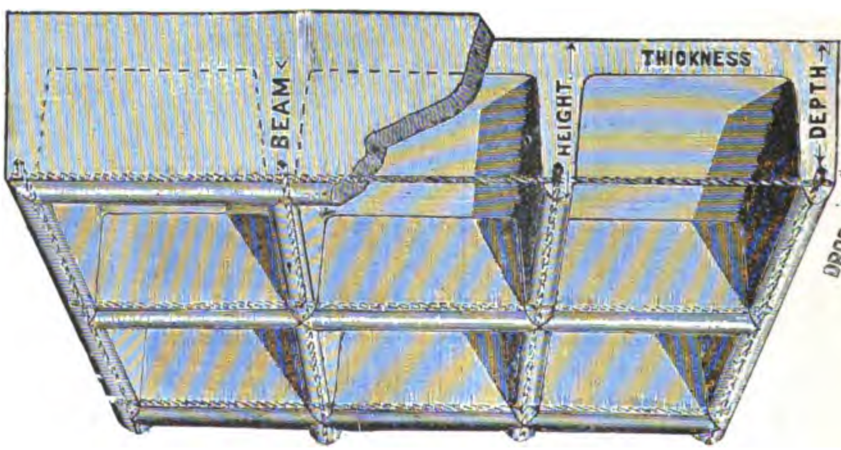


FIG. 209

shown in Fig. 209, from which the arrangement of the falsework will be clearly understood.

Hollow Floors.—Where flat or arched monolithic floors are formed with beams or ribs, and a ceiling slab below, so that the beams are completely or partially hidden, a hollow floor is the result, and a special manner of moulding is rendered necessary.

The method adopted by M. Cottançin will be described under "Moulding in Advance," as he frequently moulds both the ribs and ceiling slabs in advance, and only forms the floor in place.

M. Hennebique also moulds the ceiling and floor slabs before placing them in position. The bottoms of the principal and secondary beams are at the same level in this case, planks being first propped up to form the bottoms of the beam boxes, the ceiling slabs being placed so as to rest on these (Fig. 210). The sides of the boxes are then placed in position, being formed of planks cut on the splay on the top edge as the beams are widened out at the top to receive the floor slabs (*Fig.* 211). The sides are usually formed of two planks, those at the top being placed outside in order that the splay of the beams may be continued beyond the thickness of one plank. These top planks are held in position by strips of iron (*a b*, Fig. 211) about 18 inches apart, passing through the top of the beams,

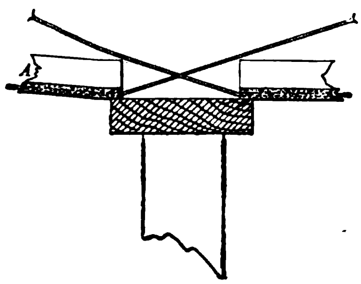


FIG. 210

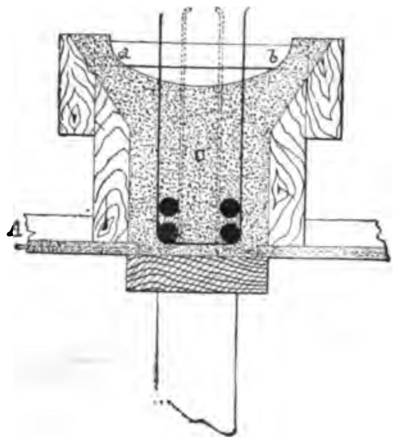


FIG. 211

the top planks being left unsplayed at these places. The beams are moulded to the underside of the floor slab, and when the concrete has sufficiently set the sides of the boxes are removed and the floor slabs placed in position. In some cases the floor slabs are moulded *in situ*, but this method is not so economical in the matter of falsework and some timber must be left in. A thin slab of reinforced concrete supported on the beams is also sometimes employed as a centreing and forms part of the floor when complete.

Similar systems to that of M. Hennebique employ very much the same methods.

M. Coularou constructs the double floors of his system (Fig. 55) at one time. A platform is established below the ceiling. The sides of the beams are held by lateral boarding, and the space left between the ceiling and floor is filled with cinder concrete. Sometimes, however, the space is left and the moulding effected by means of tubes or arches of concrete to fit the whole space or simply the curved tops of the beams and the underside of the floor slab.

MM. Pavin de Lafarge always mould their ceiling slab in place at the same time as the beams, the floor slab being made in advance. Fig. 212 shows the method adopted. Other systems form the ceiling on a network suspended from

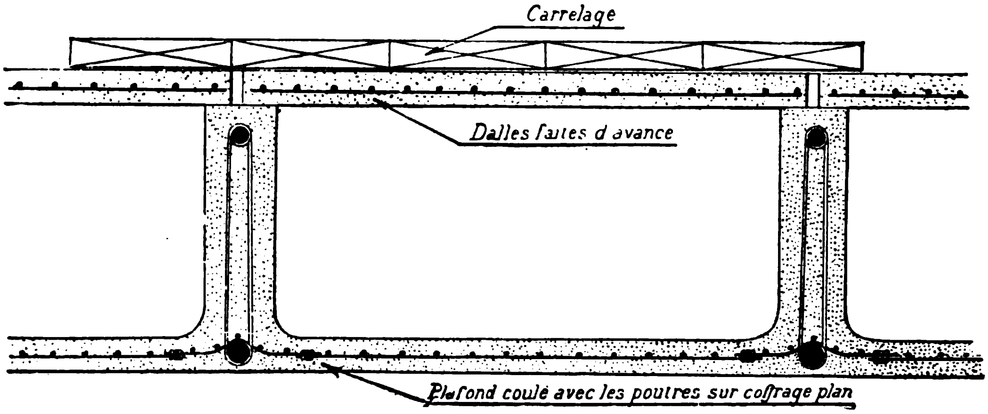


FIG. 212

the floor slab, and forming a lathing for the ceiling plaster—the same method is also adopted frequently by the constructors that employ rolled joists as beams.

The methods of constructing the falsework that have been described are those for ordinary buildings; but there are of course many instances where special requirements have to be dealt with necessitating peculiar treatment.

Fig. 213 is reproduced from a photograph showing the construction of a

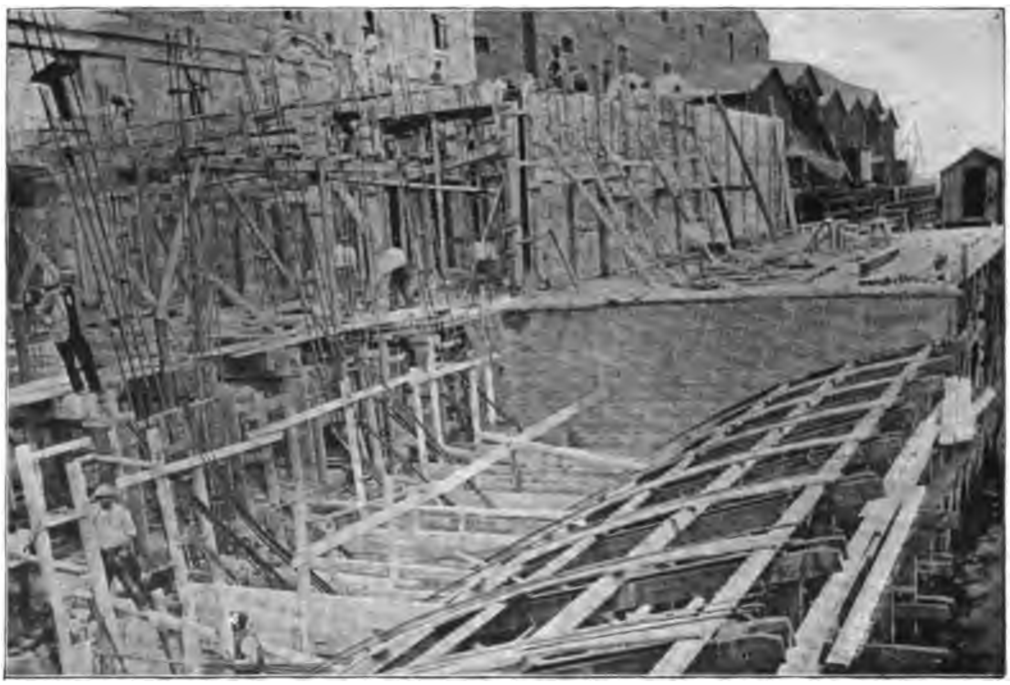


FIG. 213

cantilever quay at Chantenay, France, by M. Hennebique. There being insufficiency of room at the back of this quay for the anchorage of the cantilevers, they were tied into the warehouse at the back of the wharf, which was built at the same time. The whole quay in reality depends for its stability on the reinforced concrete warehouse behind it, and the moulding of the quay and the building had to be arranged so that they might be brought up together. Fig. 214 shows the

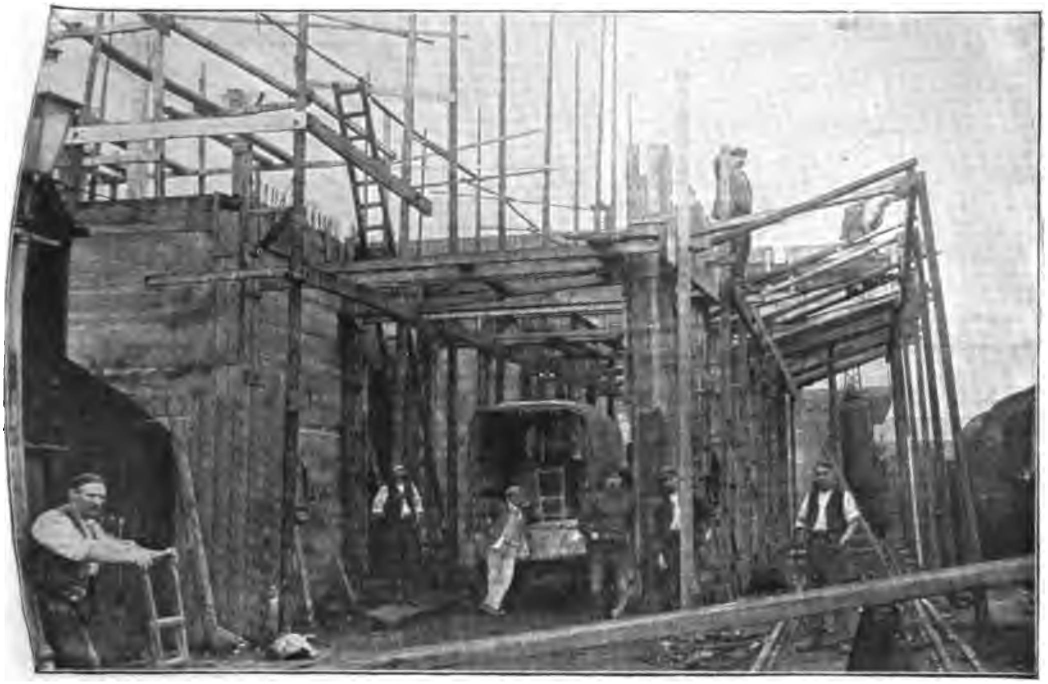


FIG. 214

arrangement for moulding the cantilevers for the Great Western Railway grain warehouse at Plymouth. These are only two cases where special treatment was necessary, and there are, of course, many others of similar importance.

Straight Bridges, Subways, etc.—The falsework for straight bridges is very similar to that already described for floors. It is however sometimes necessary to support these from above in order that a clear headway may be kept during the construction. Baulks of timber or, when the span is considerable, framed trusses, are in this case employed to support the moulds and centreing by the means of iron suspension rods with cross pieces, for supporting the moulds, attached to their lower ends by means of nuts. The rods are drawn out after the work is complete and the holes filled in. The sides of the beam boxes are brought up as the work proceeds.

Arches Without Ribs.—In ordinary cases when the arch is of the same form throughout the whole width, as in the Monier and similar systems, there is nothing special in the centreing, excepting that the two faces are formed against upright planks secured to the outer edges. Fig. 215 shows a form of centreing used for the construction of a Melan arch, and Fig. 216 another Melan centreing, in which special means were adopted to support the boulder stones which formed the face of the arch and projected from 6 to 8 inches ; 8 × 12 inch stringers being hung in stirrups (Fig. 217) attached to the projection of the caps of the uprights for this purpose.

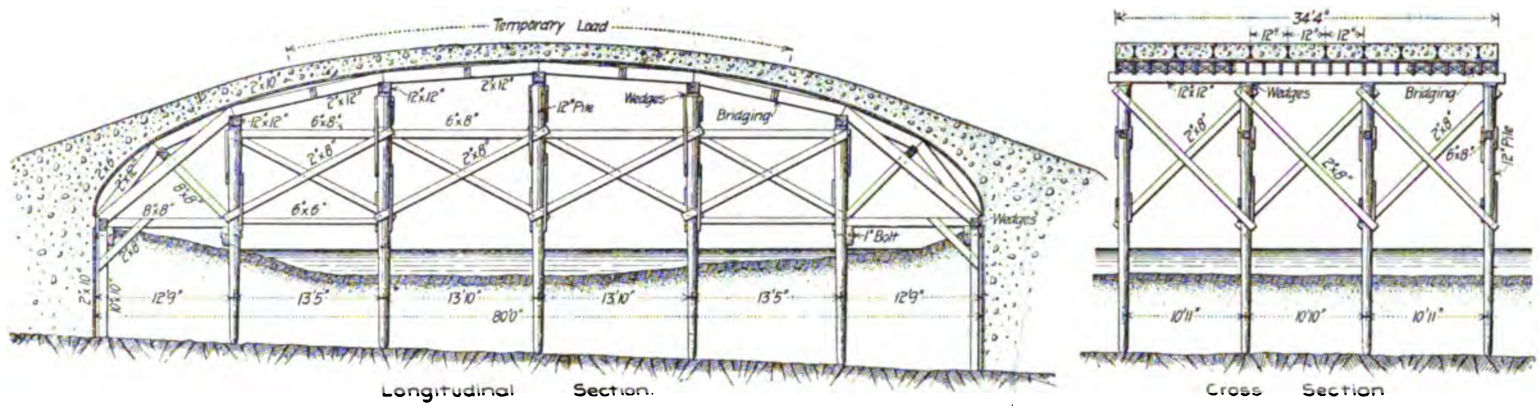


Fig. 215

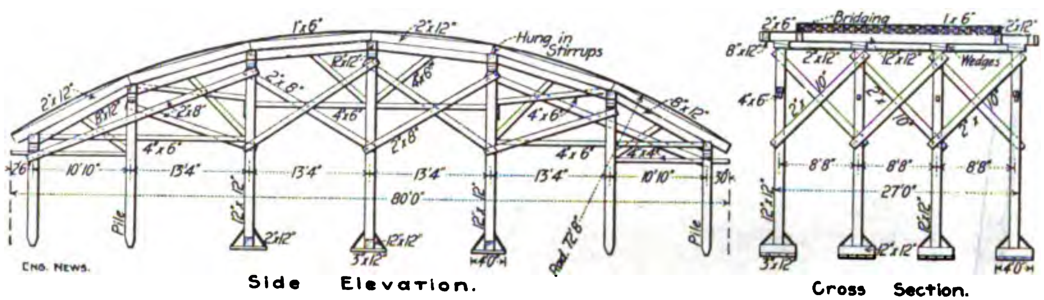


Fig. 216

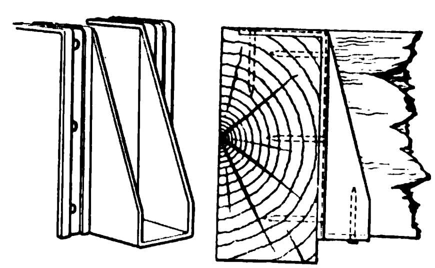


Fig. 217

In elliptical arches, where the curve is steep at the springings, a series of planks are placed in position as the work proceeds to form the extrados.

Arches with Ribs.—For bridges formed of a series of arched ribs supporting a decking, the method of falsework is very similar to that described for floors, excepting that the boxes for the ribs are curved instead of straight, and are supported on ordinary bridge centreing. If the boxes are deep the sides are brought up as the work proceeds.

The concrete of the ribs is generally allowed to harden somewhat, before the decking is commenced, which permits the use of lighter supporting centreing, as only sufficient strength is necessary to safely support the ribs until the concrete has set. Fig. 218 shows the centreing for the Chatellerault Bridge, France,

FIG. 218

constructed in this manner by M. Hennebique, the side arches of which were 136 feet span and the centre span 164 feet.

Hollow Arches with Ribs.—M. Hennebique moulds the intrados and extrados on slabs of reinforced concrete, moulded on the ground and left in place when the concreting is finished, as part of the thickness. The slabs to support the intrados are first propped up in position, and this portion formed *in situ*, spaces being left for the ribs, by means of longitudinal strips of wood having notches made in their bottom edges to allow the rods of the intrados, which are left projecting, to pass through them.

After forming the intrados, these strips are taken out and the rib boxes formed, the sides being held in position by cross-braced frames, extending from rib to rib. The end posts of these frames are left in position, after the sides of the rib boxes have been removed, to support the slabs on which the extrados will be formed, and are cut off to the proper height for this purpose. One side of the rib box is put in for its full height, but the other is brought up as the work proceeds. The sides of these boxes are used again and again, being struck as soon as the concrete has set sufficiently to allow of their removal.

Some of the props which support the slabs on which the extrados is formed are got out before the bay is completed, but the remainder have to be abandoned.

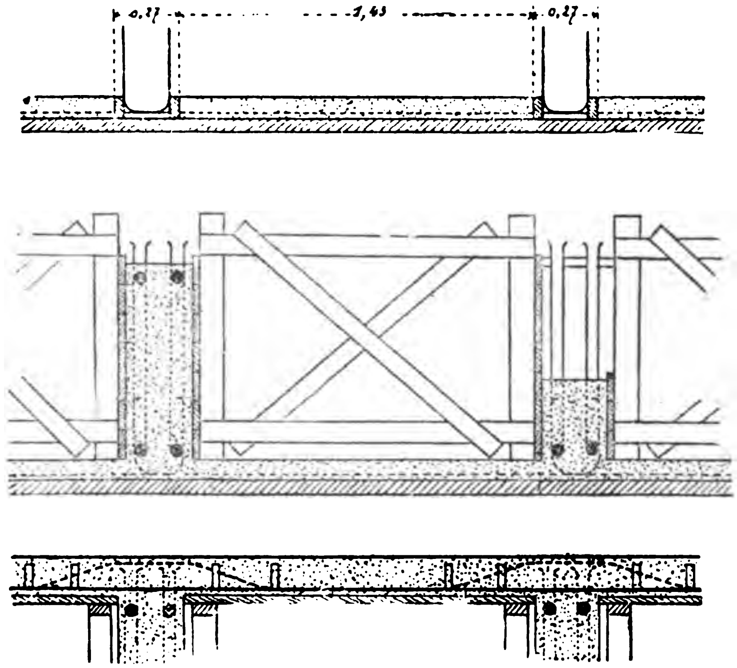


FIG. 219

With this method there is a slight danger that the props which must be left in may cause local strains when a rolling load passes over them. Fig. 219 shows this form of falsework, and Fig. 220 shows the form of arch when completed.

Methods Adopted when the Reinforcements are Employed to Support the

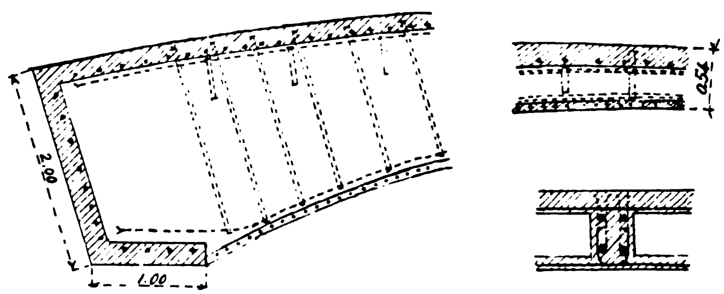


FIG. 220

Moulds.—When the reinforcing skeleton is built up first and sustains itself in position, it has, very commonly, sufficient rigidity and strength to enable it to support the moulds either entirely or partially.

There are three distinct advantages accrueing from such a disposition—

1. Economy of falsework ;

2. Avoidance of heavy supporting timbers or trusses for subways, straight bridges, or other like cases where the headway must not be obstructed ;
3. The advantage of more freedom around and under the work.

With this method, however, the reinforcement being loaded before the concrete is in place, must take initial strains, which will remain after the concrete is set, and cause initial stresses in the bars and concrete by their tendency to reassume their original form.

The reinforcement has to bear the weight of itself, the concrete, and the moulds, while the concrete is setting, whereas when the moulds are externally supported the supports take all the load until the concrete has set sufficiently to take its share of the stresses. The initial stress in the concrete, caused by the tendency of the reinforcement to regain its normal state, will be compressive. This is an advantage on the tensile side, but will reduce the compressive resistance.

A further disadvantage is that there must always be a slight vibration of the reinforcement during the working in of the concrete, caused by the ramming and also the movements of men and materials, which must somewhat seriously affect the "adherence" between the concrete and metal. Small vibrations of the reinforcement while the concrete is setting may almost entirely destroy its "adhesion" to the metal as well as having a prejudicial effect on its own cohesion.

This method when employed for walls and similar parts has not the drawbacks mentioned above in such a degree as when used for beams and other pieces subjected to bending. It is frequently used for these structures with comparatively light reinforcements, the shuttering being tied to the reinforcement with wire ties.

Where heavy sections of reinforcement, such as rolled joists, tees, etc., are used in floors, deckings, etc., the centreing for the slab is hung from the reinforcement by means of special hooks, since with this method of reinforcement the sections are well able to support the load put on them during the moulding.

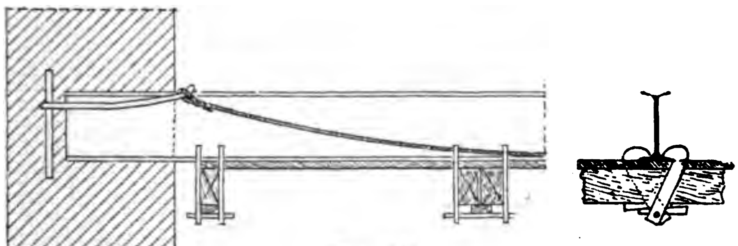


FIG. 221

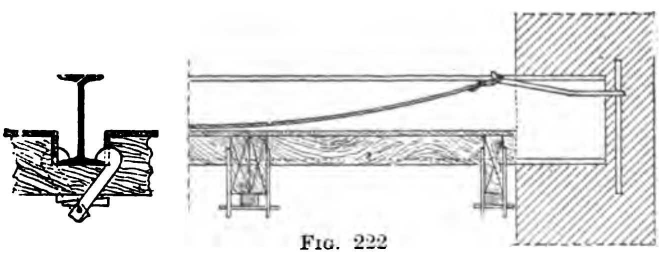
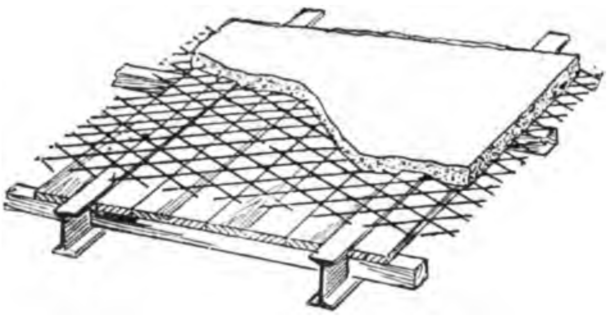


FIG. 222

Figs. 221 and 222 show this form of attachment, being the method adopted by M. Mattrai. Fig. 221 is that used for a floor, the underside of which is level with the bottom of the beam, Fig. 222 being the method adopted when the bottom of the beam appears below the floor.

Floors Supported by Rolled Joists.—The employment of rolled joists for beams effects an economy in erection, and they are used by several constructors mainly for this reason. So far as the resistance of the stresses is concerned, however, this method is extremely uneconomical, partially by reason of the excess of metal in the web and partially in consequence of the fact that the compressive resistance of the concrete is not brought into play.



Expanded Metal Tension Bond; Coring laid on centering. and Concrete Floor partly laid.

FIG. 223

When the floor slab rests on the top flange, the timber staging on which it is moulded is supported from the lower flange, as shown in Fig. 223, which represents the method employed for floors reinforced with "expanded metal." Should the underside of the slab be of an arched form, the bearers would be curved to the necessary radius. When the slab is supported on the bottom flange the bearers must be suspended by special hook attachments, such as those already described and shown in Figs. 221 and 222. Fig. 224 shows the method adopted when constructing Melan arched floors.

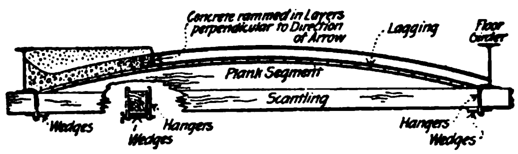


FIG. 224

When secondary arched ribs are employed for a floor on the "Golding system" (expanded metal) the arrangement of the falsework is shown (Fig. 225). The bearing timbers for the floor centreing are supported by cleats nailed to the sides of the boxes for the arched ribs.

A great number of methods are employed to facilitate the erection and striking of staging of this kind, and for adapting the bearers to varying spans. The bearers are frequently made of pieces of iron with an arrangement of some sort to allow of the length of the span being altered to suit various requirements, and a distinct saving in timber is effected by the use of such bearers.

One method adopted is to form the bearer of two bars, placed side by side, running past one another in rings, and having pivoted hooks at their outer extremities, which rest on the bottom flanges of adjacent joists (Fig. 226). The sliding of the rings is provided for by strips of metal held on the top of the bars by clips, these strips being thicker than the metal of the rings. To strike this form of centre the rings are knocked apart until the bars are free from each other, they then drop by turning on the hook pivots and the staging falls away. This form of bearer can also be employed when the beams are of reinforced concrete.

Arch Centreing Partially Supported from the Reinforcement.—For arches on the Melan system, where curved rolled joists or small built up girders are employed for the reinforcement, these joists or girders are frequently used to partially support the centreing.

The lagging is close boarded and rests on longitudinal ribs, of timbers built up of segments curved at their top and straight on their bottom edges; at their junctions there are cross bearers on wedges or jacks supported on braced uprights

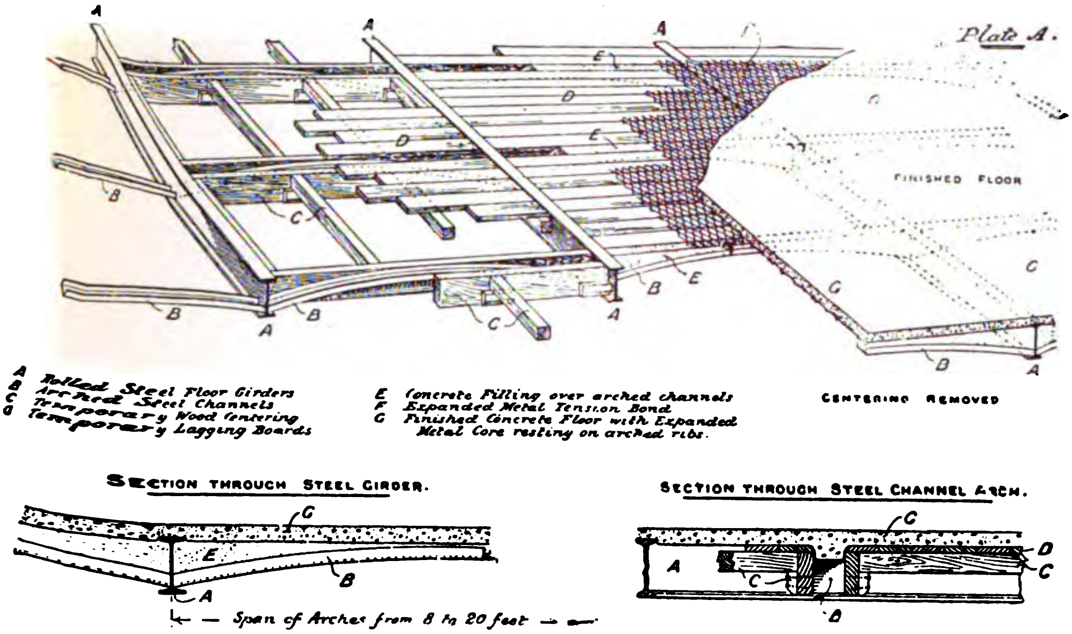


FIG. 225

in the same manner as ordinary centreing. Half-way between the cross bearers are other transverse pieces, supported by a hanging stirrup arrangement from the

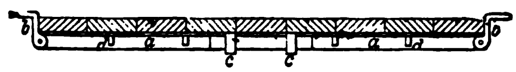


FIG. 226

reinforcing sections (Fig. 227). In this way, about half the load, while the moulding is taking place, is put on the braced uprights, and the other half on the reinforcing sections.

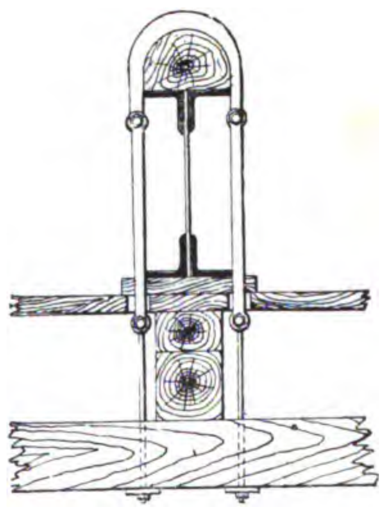


FIG. 227

This method makes the necessary falsework much lighter than that required when heavy sections of reinforcement are not employed, and the use of large sections was adopted by M. Melan partly because of the ease and cheapness in erection.

The weight of the concrete tends to make the reinforcing ribs deflect. When this occurs the wedges or jacks are loosened or tightened up until the bearing is approximately even on all the supports. Holes are left around the stirrups when putting the concrete in place in order that they may be got out when the centreing is struck; they are filled in after the falsework has been removed.

Reinforcements and Special Networks Employed to form the Moulds.—In some cases the reinforcement itself forms the mould. In the Möller system the flat suspension bars are used in this way as the bottom of the mould for the fish-bellied beams (Figs. 132 and 133). In the "expanded metal" system, the channels forming the secondary beams are employed as the forms on which they are moulded (Fig. 225).

The Roebling, Donath and Rabitz systems employ a fine wire mesh on which to mould arched and flat floors (Figs. 167 to 173 and Fig. 65).

In all these cases the metal left exposed is either covered with a layer of mortar after the moulding has been completed, or surrounded by a close mesh wire netting or a light section of "expanded metal," which act as lathing, to which the protective coating of mortar will adhere.

Iron columns and joists are also surrounded with "expanded metal" or ordinary network on which mortar is placed (Figs. 73, 167 and 168), but this is hardly a case of reinforced concrete. Concrete columns are also sometimes formed inside a mould of iron network or "expanded metal."

It is difficult, and in fact often impossible, to ram the concrete when these methods are adopted, as the fine mesh will become deformed under ramming. It may be added that the mortar which protects the exposed surfaces of flat metal cannot be made to properly adhere to the metal, and does not offer any resistance to the imposed stresses.

Large Sewers, Tunnels, and Culverts.—These are moulded *in situ*; the walls between special shuttering and the arches on centreing similar to that employed for ordinary brickwork, masonry, or concrete.

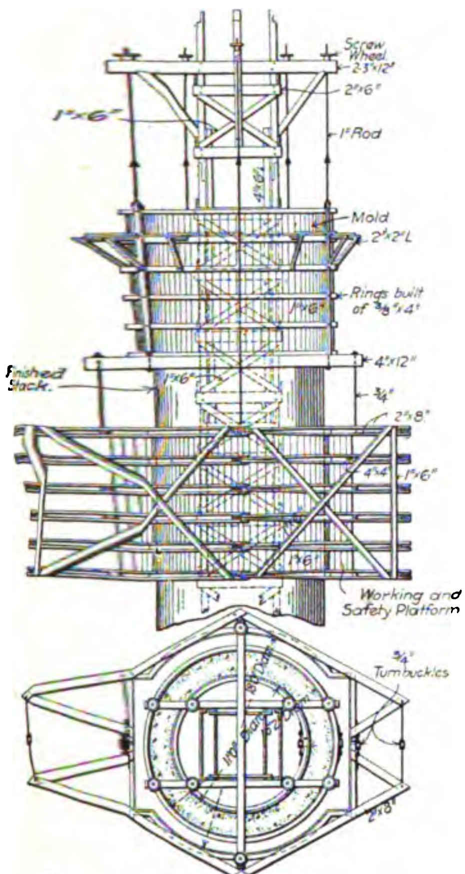
Chimney Shaft Construction.—It may be interesting to briefly describe the falsework and moulds employed by Messrs. Ransome for the construction of a reinforced concrete chimney 165 feet high, for the Pacific Electric Railway Company at Los Angeles,[1] a description of which is given (page 426).

Figs. 228, 229 and 230 show the details of the special moulds and falsework. The walls were carried up with vertical cells, having a small opening through the division walls to allow proper circulation of the air. These cells were moulded by the use of special core moulds, the interior and exterior of the shaft being formed between a core and shell, all of which were lifted as the work progressed.

Fig. 228 shows the method of falsework consisting of a tower about 6½ feet square, built up on the centre line inside the chimney, and kept a little in advance of the construction of the concrete. It had four 4 × 6 inch corner posts, with 2 × 10 inch horizontal braces 5 feet apart, and 6 × 1 inch cross bracing in each of the panels. Across the upper horizontal pieces two pairs of transverse beams 16 feet long were placed, which projected 5 feet beyond the sides of the tower and supported the moulds. A 20-foot beam at right angles to these was set across them at the centre to support the sides of the lower working platform, which was also suspended from the base timbers of the outer mould. The inner and outer moulds were suspended from the cross beams by four vertical rods with long threads at the upper ends, engaging with screw wheels bearing on the beams, enabling them to be lifted or lowered.

Horizontal planks were supported inside the chimney to provide working platforms for the men who deposited and rammed in the concrete, and a light working

[1] From description published in *Engineering Record*, April 11, 1903.



Fixed and Movable Falsework for building Stack:

FIG. 228

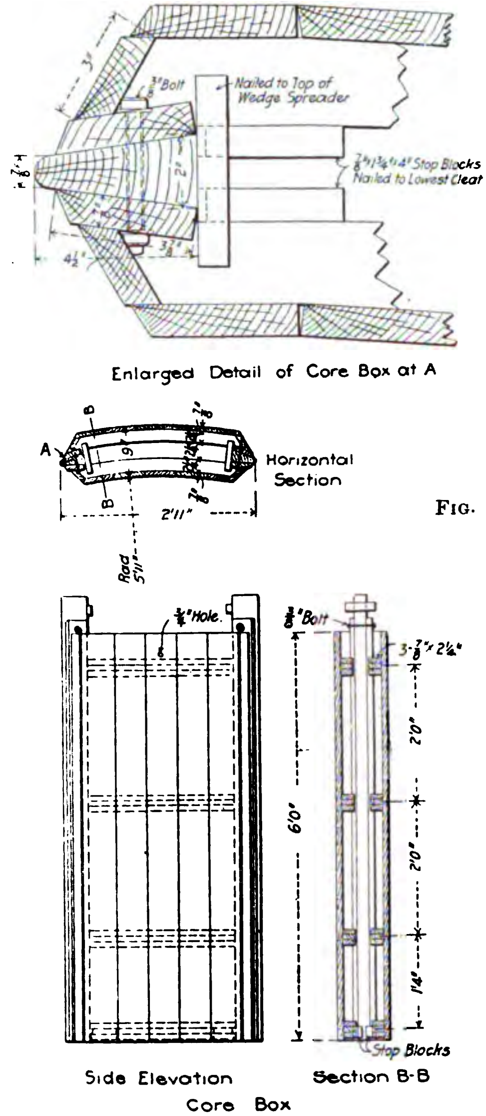
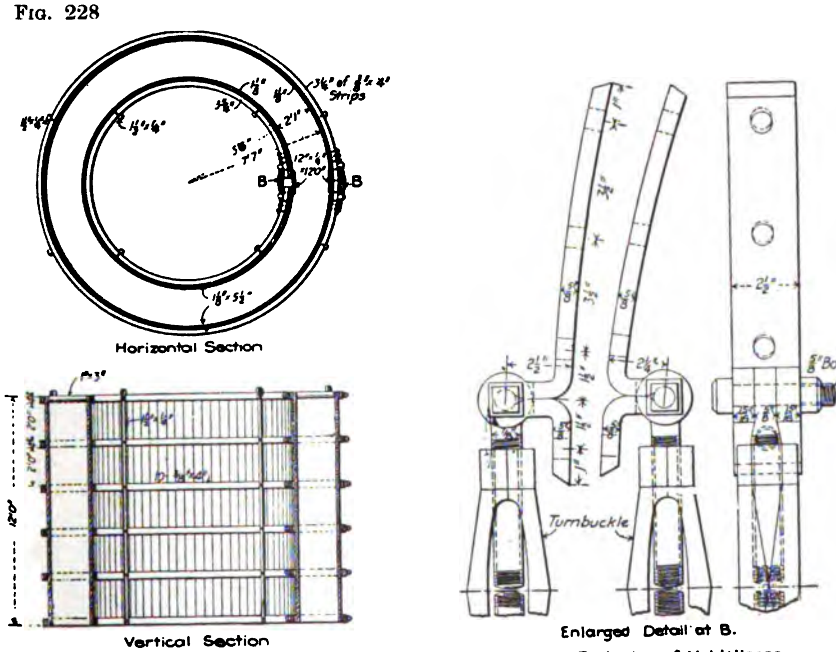


FIG. 230



Movable Mold for Shaft

FIG. 229

Adjustment Fastening of Mold Hoops

platform was bracketed out near the top of the outside mould. The lower suspended six-sided platform was not used for the chimney at Los Angeles, but was employed for the workmen who finish the outer surface after the moulding.

To prevent removing the cross-beams and in order to facilitate the extension of the falsework tower, a telescopic section 24 feet long was built inside, and at each extension was first moved up to the required height, the bearings of the projecting beams being transferred to it. Another section of the main tower was then built up, which received the moulds and platforms.

The vertical wooden staves for the formation of the main core and shell moulds (Fig. 229) had both their edges bevelled to an angle of 10° so as to be in contact on the face next the concrete, and were hooped together by circular bands, 5 inches thick, built up of ⅜-inch strips, 4 inches wide, and connected together by special jaw bands (shown enlarged in Fig. 229), bolted to them at their ends, and engaging pairs of sleeve nuts by which they could be securely tightened. The core boxes for the spaces between the inner and outer shells (Fig. 230) were formed in separate halves, with the opposite faces at the ends bevelled to correspond with a centre wedge, shown clearly in the enlarged detail (Fig. 230). This wedge was held in place during the moulding by a bolt engaging the side strips beyond the ends of the core box. To collapse the mould the wedges were driven inwards. Vertical wooden strips half an inch thick were lightly nailed to the wedges of the core boxes in adjacent cells to form the opening through the division walls.

All the work of construction was done from the inside of the chimney, and all the materials were lifted by electric hoists. When moving the moulds, which was generally done each morning, the hoops of the outer mould being first slackened and the mould raised so as to still remain on about two feet of the finished work, the hoops were then tightened up so as to securely clasp the concrete. The inner core and the cores for the cells were then lifted, but kept slightly below the outer mould. The core boxes for the cells were lifted by means of iron pins, which were put through the holes shown on the upper portion. After these pins were in position the core was collapsed by striking in the wedges, and these were withdrawn, the sides lifted, and the whole reassembled for the next lift.

The outer moulds were lifted first, as they usually required scraping. which was done after they had been reset. If the inner core required scraping it was raised before the outer mould. The cores and moulds were smeared with petroline at each operation of lifting.

Treatment of Reinforcing Metal

Reinforcements Employed for Ordinary Buildings.—In many systems of a similar character to the Hennibique, Ransome, etc., the reinforcement is merely placed in position, the longitudinal and transverse bars not being tied together in any way, except in the cases where distance pieces are required, as for columns, piles, etc.

Other systems of the same type as the Monier, Pavin de Lafarge, etc., tie their rods together with annealed wires.

Again, others, like the Cottançin, Roebling, and Rabitz, weave the reinforcements.

Where the metal is not tied or woven much labour is saved, as the bars require no further treatment than that of being carefully placed in position and having the concrete well rammed around them, care being taken to retain them in their proper place during this operation. Where, however, tying or weaving has to be done, skilled men must be selected for this work, as the greatest care is required in its execution.

M. Cottançin sometimes weaves his networks on frames with a series of holes along the sides and ends, in which pegs are placed for the wires to be passed round. The spacing of the mesh is controlled by omitting pegs when the spacing is increased. The spacing of the wires is not kept to the exact distance throughout the whole span, but they are left in irregular lines as woven, regularity of spacing not being considered necessary so long as there are the requisite number of strands in each division of the length or breadth of the piece. Fig. 45 shows the appearance of the network when complete.

Where woven networks are employed they are frequently made in advance and wrapped into rolls, being simply spread out when in place.

M. Mattrai employs steel wires and cables of varying sizes for his floor reinforcements, and also as a supplementary reinforcement to his beams. Where the Melan I-beams for floor reinforcements bear against the supporting joists, wedges are inserted as shown (Fig. 231).

Column reinforcements generally consist of vertical rods with wire or flat iron

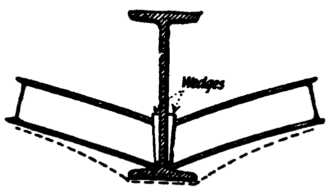


Fig. 231

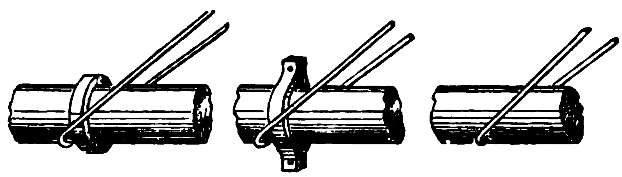


Fig. 232

cross-ties to hold them together. The vertical rods bear on an iron plate which is embedded in the foundation block.

M. Cottançin employs wires threaded through specially made perforated bricks as reinforcements for his walls and columns.

For beams or slabs of the Hennebique and similar systems, the rods are cut to their proper lengths, notched and opened at the ends, and bent to the required forms. The stirrups of hoop iron, wire, or the transverse reinforcements of various kinds are employed by many constructors to supply resistance to the shearing stresses, and tie the portions of the beams and slabs together, are cut to length, and bent to the necessary forms before being placed in the work. In some cases, as in the Vallerie et Simon system, the wire for these reinforcements is bent up and down for long lengths and is pulled out to the required spacing on the work. The distance pieces for piles, columns, lintels, etc., are also formed into the shapes required before being placed in the work.

In some systems, as in the Coignet, Pavin de Lafarge, and others, the transverse reinforcements are tied or wound round to the longitudinal rods, not merely passing under them as in other systems.

Other constructors prevent any relative movement of the longitudinal and transverse reinforcements by some method which prevents any sliding. M. Piketty either places the transverse reinforcements in small notches in the longitudinal rods or behind rings shrunk or clamped on to these bars (Fig. 232).

In yet other systems, the transverse reinforcements are formed by the longitudinal rods being bent up (as Figs. 163 and 192 and 193), or by inclined bars connected to the longitudinal reinforcements. The Locher system is of this type (Figs. 110 and 111).

In the Degon system the transverse reinforcements are bent up and down and connected in various ways, as shown (Figs. 57, 58 and 59).

The Maciachini reinforcement forms a complete hooping of the concrete, and is so placed to prevent the failure by swelling of the concrete in compression. This form of reinforcement is shown (Fig. 112).

M. Coularou employs hooked rods for transverse reinforcements (Fig. 54).

M. Chaudy employs a transverse reinforcement for his floors and wall slabs in the form of a square toothed rack (Fig. 33), and a similar type, but curved in the form of a series of U's (Fig. 59) is used in the Degon system.

In the Chaudy system angles back to back are used, and in some cases employed as the longitudinal reinforcements for beams, in which case vertical flat bars are placed at certain intervals and riveted between the angle irons.

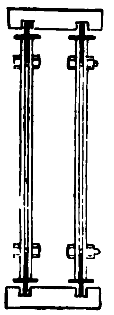


Fig. 233

M. Bonna and others employ built-up sections which have a great amount of rigidity in themselves. Fig. 233 shows the Bonna method of reinforcing beams with a double reinforcement.

For slabs of the Donath system, small I-sections, and for those of the Müeller system, flat bar longitudinals are connected by flat iron V-cross bracing as shown (Figs. 65 and 146).

Hoops of flat or other section iron or steel are employed at the springings of domes, to take up the lateral thrust on the supporting walls or columns.

Special Reinforcements

Expanded Metal.—The "expanded metal" used for reinforced concrete work is made from very mild steel with an ultimate resistance of 48,000 pounds per square inch, and an elongation of 21 per cent. in a length of eight inches. It is manufactured from flat plates of thicknesses varying from ¼ inch to about ⅛ inch, and when expanded the usual meshes are from 6 inches to 3 inches in width, but for lathing and similar purposes thinner plates are used and smaller meshes made, the plates varying in thickness down to 24 B.W.G., and the expansion being as little as twice the original width of the plate.

The manufacture is performed by placing the sheets vertically resting on their edge. They are then slotted and pulled out at one operation. After being slotted they are drawn out laterally, so that the width of the finished sheet is in reality produced from the height of the original plate when placed with its edge downwards. There is no waste of material or loss of weight.

The expansion effected varies from about six to twelve times the original width of the plate. No alteration is however made in the length, the strands being consequently somewhat stretched. A portion is left uncut so that a strong "selvedge" edge is formed. It has been found that the ultimate strength is increased from 48,000 to about 63,000 pounds per square inch through the operation of expanding.

The cost of the sheets ordinarily employed varies at the present time from about 2*s*. 7*d*. per square yard for the smaller sizes to 9½*d*. per square yard for the larger.

Ransome Twisted Bars.—The Ransome bars are of square section in either steel or iron, but usually of steel; they are twisted when cold in a lathe, one end being attached to the chuck and the other to the fixed head.

Tests made by Mr. Ransome show that the operation of twisting changes the properties of the metal considerably. For a metal with an ultimate resistance of 56,000 pounds per square inch and a limit of elasticity of about 34,000 pounds per square inch, the ultimate resistance was raised to 80,000 pounds per square inch by twisting to about ten spirals per lineal yard. In another series of experiments it was found that for an ordinary wrought iron bar $1\frac{3}{16} \times 1\frac{3}{16}$ inches there was a gain in strength of from 3 to 24 per cent. with $1\frac{1}{4}$ to 20 turns per lineal yard, and a metal of a superior quality gave 53 per cent. gain of resistance for 20 turns per lineal yard. The cost of twisting does not exceed about 4*s*. 3*d*. per ton.

Habrick Bars.—In the Habrick system flat bars are employed which are twisted when hot by special rolling mills.

Thacher Bars.—These are rolled so as to have projections alternately in planes at right angles to one another, or sometimes with projections like rivet heads.

Johnson Bars.—These are square bars rolled with a series of projections extending across the whole width of each side.

Kahn Reinforcements.—Mr. Kahn forms his longitudinal and transverse reinforcements out of one bar of a diamond section with projecting flat wings as described (p. 70).

Donath Bars.—Specially formed bars of sheet iron in the shape of an S are employed in this system.

Reinforcement for Hooped Columns.—M. Considère makes some remarks on the subject of the reinforcement for hooped compression pieces, deduced from his experiments, of which it is proposed to give an outline. The hooping should be well tied together and must not open under pressure.

The experiments referred to on page 242, show that the hoopings must be close together to give the concrete a satisfactory coefficient of elasticity and column resistance. A small irregularity of spacing will not impair the crushing resistance, but too much irregularity must be avoided, and if one of the hoopings were left out the stability might be seriously endangered. For elasticity and column resistance the regularity of spacing need not then be absolutely perfect.

The deformation measured over sufficient lengths depends on the average spacing for the modification of column resistance. It appears that separate hoops or spiral reinforcements may be employed, the spirals being formed of rods or wires of as great length as can be procured.

If independent hoops are used they must be securely fastened, in order that there may be no possibility of their opening under internal pressure, and also that they may not show appreciable deformation at the connection. The usual way to secure the hoopings is to overlap the ends and trust to the locking produced by the "adhesive" resistance of the metal and concrete due to the overlap. Such "adhesive" resistance causes considerable stresses near the circumference of the piece, where the pressure and tendency to flexure require all the strength obtainable.

It is also evident that the regularity of spacing of independent hoops must rest entirely on the care of the workmen, on which it is unwise to place too much reliance. M. Considère therefore considers that spiral reinforcements are to be preferred, and points out that drawn wires of great resistance can be obtained up to half an inch diameter, and steel rods five-eighths of an inch diameter can be obtained in rolls up to 150 feet or so in length; also that the largest rods that can be required for the hooping are sold in rolls at least 80 to 100 feet long. With such length 10 to 14 spirals may be made continuous.

To insure the proper transmission of the tension from one rod to another it will suffice to embed the first spiral of one rod between the last spirals of the preceding, and turn both the ends into the centre of the piece. This will cause the additional stresses produced by the "adhesion" to be taken up in the central portion where there is a considerable excess of strength.

With independent hoops this method of procedure, which proves very satisfactory in the case of spirals, cannot be employed, since the central portion becomes overcrowded with the ends of the hoops which have been bent in. When the spirals have been made for any piece the pitch should be found at which the skeleton will stand vertically by reason of its elasticity if left to itself. The spirals once made and thus checked cannot alter to any great extent while being put into place and while the concrete is added.

Continuous tubes for various reasons detailed by M. Considère are not as efficient for reinforcing hooped pieces as wire or rod spirals.

M. Considère concludes his remarks by saying that the objects to be aimed at are *to give the concrete a high ductility and to increase at the same time to a high degree its limits of elasticity and crushing as well as column resistance. The hooping must form close loops and have the least number of joints. These conditions lead to the adoption of helical spirals combined with longitudinal rods forming a continuous skeleton which will resist efficiently the transverse swelling of the concrete.*

The spacing of the spirals should be from $\frac{1}{7}$ to $\frac{1}{10}$ the diameter of the hooping when longitudinal rods are also employed.

Pipe and Reservoir Reinforcements.[1]—The *Bordenave reinforcement* for pipes is formed by machinery and is wound helically through rolls, which can be set any required size. After it has been wound it is placed on a core, adjustable by means of toggles, and pulled out till the required pitch is obtained.

The longitudinals are placed inside or outside the spirals according as the pressure will be from within or without, and tied to them in the proper positions with pieces of wire. Fig. 234 shows the type sections for Bordenave pipes, and Table XXII gives the details of the reinforcements.

The circular reinforcements *for elevated reservoirs* in this system are also curved by machinery, being passed through a series of rolls, which form them to the required sweep. This is effected in place by a special portable machine. The circular reinforcements are placed at varying distances apart in the height of the walls. The reinforcement for the bottom of the reservoir is first formed, with straight rods crossing one another and tied together at their intersections, and also tied to the first circular reinforcement of the wall.

After the reinforcement of the bottom is in position, vertical timber uprights are placed about five feet apart on the inside of a circumference, which is set out on the bottom; these uprights being firmly fixed to form a rigid template or gauge. The elevator carrying the curving machinery is then set up outside the circumference; its frame is formed of four vertical posts tied together by horizontal and cross bracing, and extends higher than the depth required for the reservoir.

A frame is suspended between the posts, and slides on them by means of rollers. One side of the frame is made in the form of a table on which the machinery for

[1] A description of the construction of some small circular reservoirs of reinforced brickwork in India is given in the Appendix.

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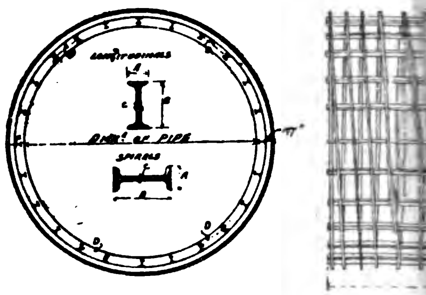

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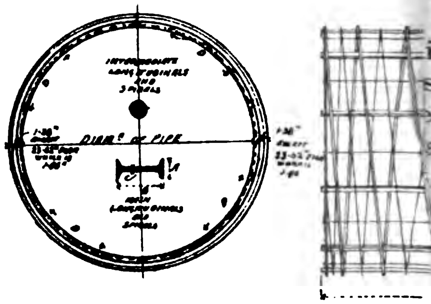

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Internal Diameters of Pipes Inches	Form of Section	A Inches	B Inches	
23·62	I	0·196	0·47	0·(
21·65	I	0·196	0·47	0·
19·70	I	0·196	0·47	0·0
15·75	I	0·196	0·47	0·0

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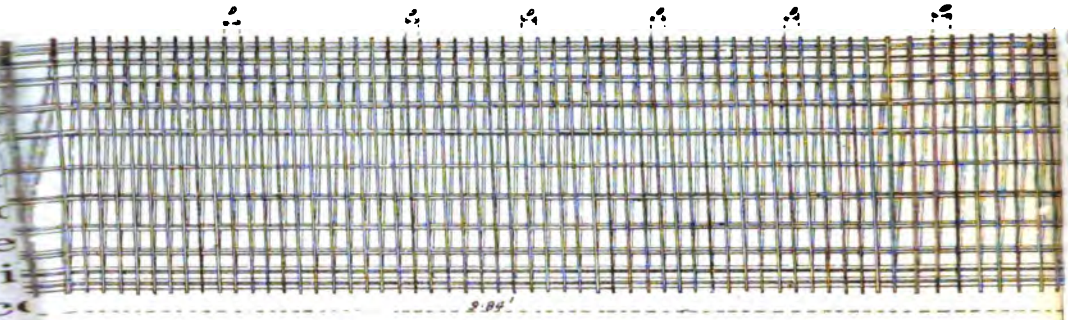
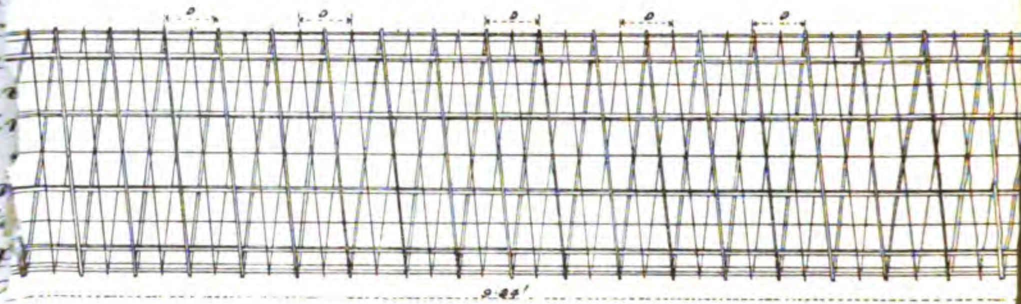




REINFORCEM

FIG.

TABLE XX

SHOWING DIMENSIONS, ETC., FOR BORDE

For Head of 100 Feet									
inals				Spirals					
es	D Inches	Sect. Area Square Inches	Weight of Bar lbs.	Form of Section	A Inches	B Inches	C Inches	D Inches	Sect. Ar Square Inches
55	3·31	0·042	0·142	I	0·22	0·55	0·059	1·69	0·051
55	3·31	0·042	0·142	I	0·22	0·55	0·059	1·85	0·051
55	3·35	0·042	0·142	I	0·22	0·55	0·059	2·04	0·051
5	3·39	0·042	0·142	I	0·197	0·47	0·055	2·28	0·042

curving the circular reinforcements is carried, together with the guide, and the winder supporting the I-bar which is to be curved. At the top of the frame a self-sustaining differential pulley block and tackle are fixed, the hook of which engages rings on three rods from which the movable frame is suspended. This frame is lowered level with the bottom of the reservoir, and the first bar is curved round the uprights of the template; it is then fixed to these with hook nails.

The apparatus is now lifted by the pulley to the height desired for the spacing of the circular reinforcements; a second bar is curved round and fastened to the template uprights like the first, and these operations are continued till all the circular reinforcements are in place. The hook nails are fixed on the uprights, at the necessary distances apart, before the curving is begun, forming the supports to the bar as it is curved. These hook nails besides supporting the bars, serve as a guide for the spacing.

After the movable frame has been lifted about 3½ feet a platform is suspended below, by four vertical hooked rods, on which two men can stand to work the bar curving machine, the frame and suspended platform being lifted together. The last circular bars having been placed at the top of the reservoir wall, with somewhat less spacing, to take the thrust of the roof, the vertical bars are placed in position inside the circular ones and tied to them with wire ties. When the skeleton is complete the hook nails fastening the circular reinforcements to the template uprights are withdrawn, and the concrete run into place as described later. The reinforcement for the spherical roof is formed by a skeleton of circular or spiral bars, resting on slightly curved radial bars, the alternate ones being stopped as the spacing becomes small.

The circular and radial bars are tied together at their intersections and also to the circular reinforcements at the top of the walls, and the radial bars are bent down in to the wall.

The *Pavin de Lafarge* pipe reinforcements are constructed of longitudinal bars, with circumferential bars wound round them; these, if of small enough dimensions, are in the form of spirals, but if of large dimensions a series of circular hoops is employed, the extremities of each hoop being welded together. The reinforcements for horse-shoe conduits and sewers are made in much the same way.

For circular reservoirs above ground level in this system a rigid reinforcement of flats or channel irons is employed, formed of uprights and circular rings spaced farther and farther apart as the height from the bottom increases.

The uprights are bent in at the bottom and top to form the floor and roof radial reinforcements, their upper ends are secured to a central plate, or, if the diameter is large, to circular rings from which further bars are continued to a central plate; alternate bars are stopped where the spacing becomes too small (Fig. 161). Bolts are used for fastening the skeleton together temporarily, and are subsequently replaced by rivets.

Between the circular and upright reinforcements of the main skeleton, a mesh of circular and vertical round rods is placed, the circular rings being spaced at varying distances apart according to the pressure.

For very small reservoirs as those for wine, which hold only from about 2,000 to 2,500 gallons, the same method is adopted, excepting that the circular reinforcements both primary and secondary are evenly spaced.

The *Monier* and similar reinforcements for pipes are formed of longitudinal and spiral rods tied together at their intersections. For reservoirs, silos, etc., similar reinforcements are adopted, but the circular rods are in the form of welded hoops. The rods are of varying sizes, and as a rule spaced evenly, the sizes being mixed according to the resistance required, the rods employed usually varying between about 1 inch and $\frac{3}{8}$ths of an inch. diameter.

M. Coignet for the pipes constructed on his system employs hoops of rods, both ends of which are hooked. A longitudinal rod is passed through all the hooks,

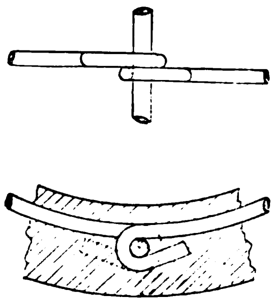


Fig 235

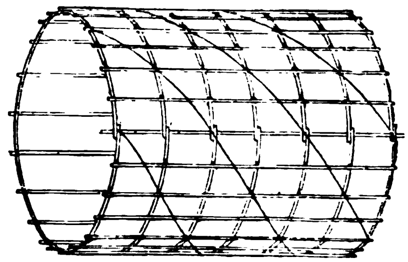


Fig. 236

and securely holds the ends of the hooped rods in place (Fig. 235). Further longitudinal rods are also tied at proper intervals to the hooping, and in addition half spirals of wire are wound round the skeleton (Fig. 236). M. Coignet employs a system of electric welding for the hoop reinforcements, and also for the reinforcements of his arches.

In the *Degon* system the circular reinforcements are formed of rods bent in the wave form similar to the slab transverse reinforcements, the longitudinal rods are placed in the outer hollows. When a double reinforcement is

Fig. 237

employed, alternate circular rods are bent reverse ways and the longitudinal rods are placed as before, or the bends of the two series of rods are brought close together and the longitudinal rods placed between them.

The Cottançin pipe and reservoir reinforcements are formed of a basket work of wire, the circular wires being spaced according to the pressure. To secure impermeability, M. Cottançin embeds a special tube between two shells of reinforced concrete. The tube is formed of a strip of thin metal wound spirally, the overlappings being made good with wire gauze or vegetable fibre and some glutinous mixture. This he covers with gutta-percha and protects with canvas (Fig. 52).

The pipes on the *Bonna* system are formed of his cross-section bars, either with spiral or hooped circular reinforcements.

When hoops are employed the bars are cut to the proper lengths, bent into hoops, and their ends fastened together with a riveted cover joint. The hoops are forced over a cylindrical core to make them truly circular, then put into a kind of frame (Fig. 237) formed of uprights supporting two longitudinal bars, one at the top and one at the bottom. These bars have notches cut in them at the proper distances apart to receive the hoops.

After the hoops have been fixed in position, the longitudinal bars are added

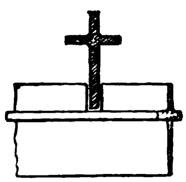


FIG. 238

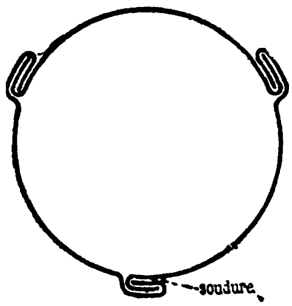


FIG. 239

at proper distances apart, having been previously notched out to fit on to the hoops (Fig. 238). When the longitudinal bars are in place they are tied to the hoops. For high pressures two series of reinforcing skeletons are employed, with a sheet iron or steel tube between them. These tubes are also used inside pipes with single reinforcements.

The tubes are generally formed in three pieces, which are secured together by being bent at their edges to form clips, in the same manner as sheet lead or zinc are joined, the joints being closed with solder (Fig. 239). The processes of cutting to length, forming the folds at the edges, bending to the required radius, and clasping the folds together, are effected by special machinery.

A layer of thin sheet lead, bitumen or caoutchouc are sometimes employed embedded in the concrete to obtain water tightness.

When the double series of bars with a tube between are employed, the longitudinal bars for the inner series are formed on a collapsible template constructed of circular ribs notched to receive the longitudinal bars, which are made with indents to receive the circular bars; these having been previously cut to the proper lengths, bent to the required radius, etc., are placed in position around the longitudinals, and are tied to them with thin wire. The tubes are now slipped over the inner skeleton and the outer reinforcement is formed on the tube.

The arrangement of the reinforcement for two reservoirs for the town of Locle, each containing 221,000 gallons, on the *Vallerie et Simon* system, is shown in

Fig. 240. The circular rods having been bent round a series of uprights, the intermediate verticals were placed in position and tied to the hoops.

For large rectangular underground reservoirs, the methods adopted are very similar to those described above for ordinary buildings.

The reinforcements for large sewers, subways, conduits, and similar works are of the same character as described for pipes. They are generally built up *in situ in the form of longitudinals and hoops*, or the bottoms, side walls and arches are formed of separate series of rods. Sometimes, however, the whole skeleton is made outside the trench and put in place in lengths.

Pile Reinforcements.—These are generally much the same as those for

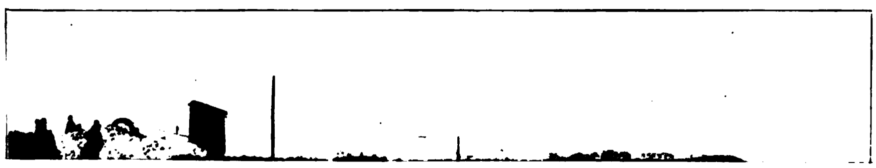


Fig. 240

columns. M. Hennebique for his hollow piles employs forked rods to hold the longitudinal rods apart, as well as the wire cross-pieces for holding them together.

Mr. A. E. Williams, of Dagenham Docks, constructs a pile with a rolled joist reinforcement and a bent flat bar to strengthen the sides parallel to the web. He also places loops in the concrete at frequent intervals (Fig. 188).

The reinforcement of the piles of the Rechtem, Vering and Dopking system are built up of rolled joists framed together. Those of the Armoured Concrete Construction Company have angle irons at each corner, connected by straps at frequent intervals and having a special diaphragm of wire bent in the form of a spring intermediate with the straps (Fig. 145).

Reinforcements for Large Span Arches.—These are generally much the same as those employed for beams and floors.

M. Hennibique for his arches employs hooked rods as shown (Fig. 241) for his transverse reinforcement in the vertical plane. These are hooked round cross rods placed under the main longitudinal bottom rods, and at the upper ends are bent over similar cross rods which lie above the upper main longitudinals. These transverse rods are not hooked at both ends, as the form of the upper portion lends itself more easily to adjustment, since by the time the top longitudinals are reached the transverse reinforcements are firmly fixed in the concrete.

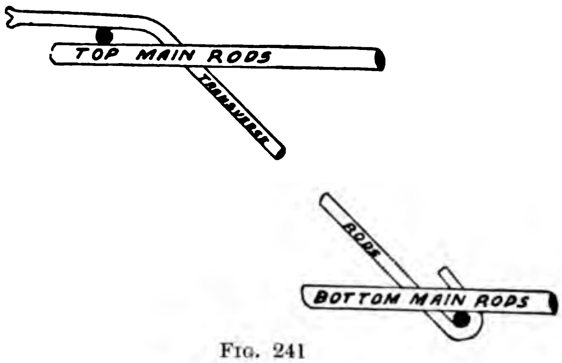


FIG. 241

Some systems, such as the Melan, employ rolled joists or built up girders as arch reinforcements, which method lends itself conveniently to the formation of hinged connexions, such as those shown (Fig. 242), which shows reinforcing ribs for a three-hinged arch bridge at Steyr (Austria). The twisted bars of the Ransome system are connected together by union .sleeves of a pitch to conform to the twist of the bars.

Page 180—See slip at page 197 as to the hollow pile here mentioned not being Mr. Hennebique's but that of Mr. L. G. Mouchel.

C.F.M.

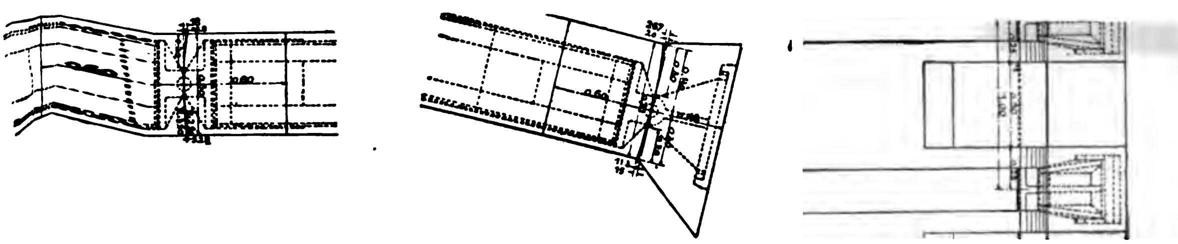


FIG. 242

Large Sewers, Subways and Conduits of Horse-shoe Section.—For these structures it is usual to place the reinforcement in position before building up the moulds, and in some systems the reinforcement is of such a character that it can be used for supporting the falsework. In others, when the skeleton is in the form of a light network, as in the Monier and similar systems, no support can be obtained from it. In these cases a method frequently adopted is to place irons of a V or channel shape longitudinally along the bottom of the walls to receive the lower ends of the bars of the circumferential reinforcement, which are curved to the shape of the arch and side walls before being put into place. These are let down into the trench, and are supported at their tops by longitudinal timber bars notched at the proper distances apart to receive them. The longitudinal bars are then put into place, and are fastened to the circumferential bars either by wire ties, bolts or rivets, according to their section.

The putting on of the longitudinal rods is commmenced at the bottom of the side walls, and the network is completed to the springing of the arch. Shutters are then placed in position, and the concrete is worked in and rammed around the reinforcement between the sides of the trench and the shutters. When the side walls are finished centres for the arch are set, the longitudinal rods of the arch are put in place, and the network packed up off the lagging with small flat pieces of stone; after which the concrete is put in, in the same manner as for ordinary concrete arches, and well rammed and worked round the reinforcement, the extrados being smoothed off with a trowel.

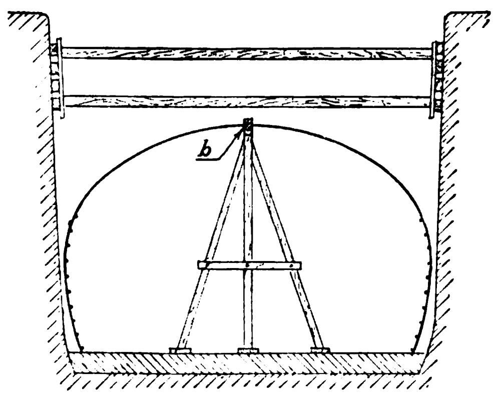


FIG. 243

When the reinforcement is sufficiently rigid *M. Coignet* supports the whole from a notched timber longitudinal supported on single uprights at both ends of a length (Fig. 243).

Large Circular Sewers, Subways or Conduits.—These are formed much in the same way as those of a horse-shoe shape. *M. Coignet*, however, employs a special manner of erection. A narrow block of concrete is first inserted at the invert for the distance of a length, and rigid frames of timber *G H I J* (Figs. 244 and 245) are erected at certain distances apart having rolled joist cross-pieces

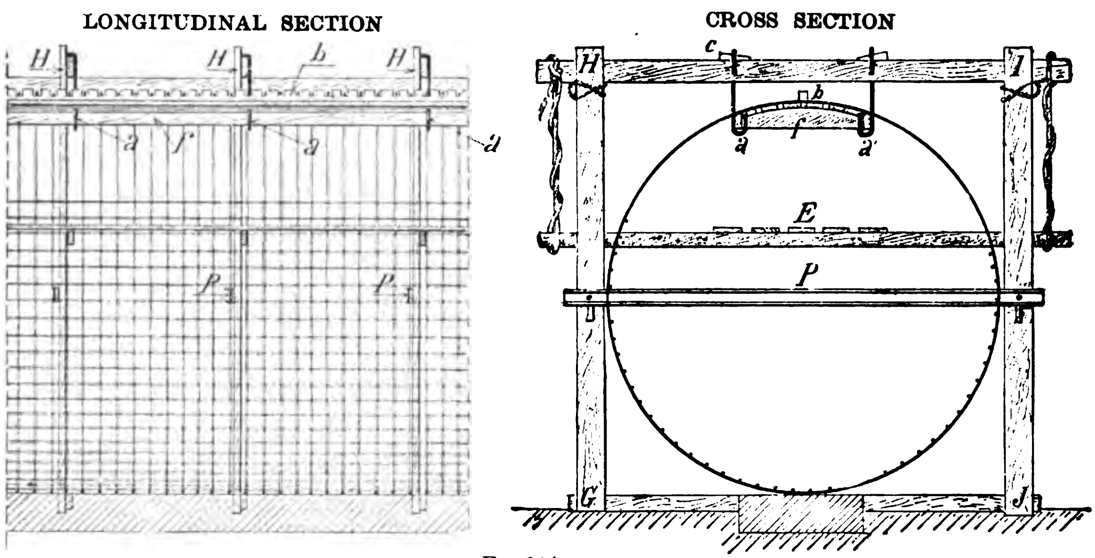


FIG. 244

(*P*) at about the centre of the height for supporting the ribs for the arch; these are also used for hanging a scaffold for the workmen when constructing the bottom half, the scaffold for the arch being hung from the projecting ends of the top cross-piece of the frame.

A narrow centre (*f*) (Fig.244), about 3 feet wide and 12 to 14 feet long, is hung by hooked rods (*a*) from the top cross-pieces on the top of the laggings. At the middle of this centre a notched piece of timber (*b*) is fixed, the notches being spaced the distance apart of the circular reinforcing hoops. These reinforcements, which have been previously bent either by hand or machine to the required radius and tied, welded, or otherwise fastened together at the ends, are placed in the notches, and the narrow centre is then hung in its proper position. The pieces (*Q*) (Fig. 245) are now secured to the frames, and the laggings for the bottom portion placed upon them.

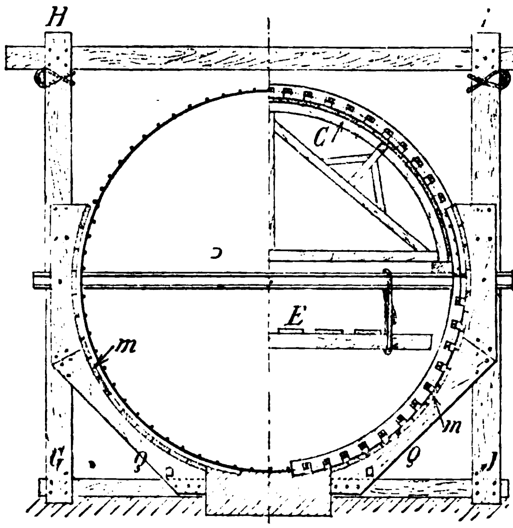


FIG. 245

The longitudinal bars for the bottom portion are put into place, being guided for position by curved strips which are notched at the proper distances apart, these strips (*N*) (Fig. 246) are made the same depth as the required thickness for the concrete, the notches being deep enough to allow the longitudinal bars to take their proper position. When the longitudinal rods for the bottom portion are in place the mortar, mixed stiff, is thrown against the laggings and smoothed off by screeding.

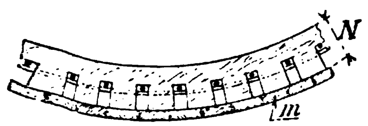


FIG. 246

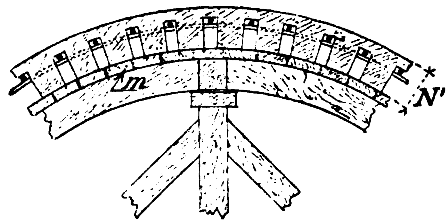


FIG. 247

When the bottom portion is finished and the concrete has set, the hooked rods (*a*) (Fig. 244) are lowered and the short centre released and removed to the next length, being replaced by a framed arch centreing (*C*) (Fig. 245), which is lagged and supports curved and notched strips (*N*) (Fig. 247). The arch is then formed in the same way as described for the lower portion and the extrados screeded up.

Moulding in Place

Transport of Material.—Though in reinforced concrete work the quantity of material to be hauled is less than that for structures of plain concrete, still if the system of transport has been carefully laid out and the general management of the different gangs is properly watched, considerable economy can be effected.

The arrangements for the circulation of the concrete, should be independent of the moulds and centreing, and be supported by separate scaffolding if possible. Means must be allowed to enable the wagons, etc., conveying the concrete to pass one another, and arrangements made so that all parts of the work may be efficiently and promptly served.

In the United States aerial cableways are frequently employed and are an excellent arrangement, as the conveyors are always out of the way of the staging for the men and materials, and cannot cause any vibration to the moulding falsework. Messrs. Ransome have patented an apparatus for transport purposes, consisting of a series of hoisting buckets, with a travelling crane, which is moved around the walls on their upright slotted standards mentioned above. A man stationed on the wall receives and empties the buckets as they are hoisted and rams the contents into place. No scaffolding whatever is required about the wall when this apparatus is used in conjunction with the movable frames (Figs. 200 and 201).

Such an arrangement is undoubtedly economical and facilitates rapid construction. There is, however, considerable danger that the travelling crane, though of light make, will cause considerable vibration and thereby damage the concrete, since it is supported by the frames holding the wall moulding boards.

On a bridge constructed by Mr. Thacher a trough conveyor 60 feet long was employed. It was provided with trap doors, through which the concrete was deposited where required.

General Remarks on the Bringing up of the Work.—The methods adopted for putting the concrete and reinforcements in place vary somewhat in the several systems at present in vogue. Sometimes the metallic skeleton is built up first by the aid of special light scaffolding, being held together by a few bolts, rivets or other fastenings, and supports the forms in which the concrete is moulded, and in some cases is itself employed to form some portion of the moulds.

The concrete may be poured into the moulds in a semi-liquid state, which method is adopted when the whole of the forms are built up to the full height at one time. It is, however, generally practicable to ram it in fairly dry when the moulds are kept just in advance of the work, and this is better practice if it can be accomplished. In other cases falsework and moulds are first erected, the reinforcement is then put in place and secured in position, after which the concrete is added. This manner of treatment is frequently adopted where the reinforcing skeleton is made up of pieces fastened together, but is not sufficiently rigid to enable it to support any external load unaided.

When this method is employed the concrete can be put in without intermission, and hence the liability to the formation of lines of cleavage is avoided. Where, however, a reinforcement is placed in position before the concrete on its underside is in, there is, in some cases, considerable difficulty in properly ramming the concrete around it, so as to fill in all the spaces below the metal.

Perhaps the most general custom is to build up the moulds as the work proceeds and to fill in the concrete as they are brought up, putting the reinforcement in position as required. This method is generally employed by the systems in which the reinforcing sections are not secured together and can be placed separately, and in those where a network must be held in position by the concrete, as is the case with Cottançin floors.

The disadvantage of the stoppages necessary in the concreting operations during the time required to place the reinforcements, and while the moulds are being raised, has been referred to (p. 158). Care must also be taken to retain the metal in its proper place while the concrete is being filled in and rammed.

The concrete should be put in as soon as possible after mixing, and where the surface has been left for any time before a fresh layer is added this surface must be left as rough as possible and should be thoroughly cleaned and washed over with neat cement of the consistency of cream, a layer of mortar being spread on before further concrete is added. The work is rendered easier by reason of the moulds being built up as required, and the workmen can be better supervised. In the Monier and other systems all the above methods are adopted to suit different cases, but the latter is the one most generally used by M. Hennebique and in the Ransome and similar systems.

The falsework should be left in place as long as possible, except in special cases, as for instance the sides of beam boxes, which may be removed as soon as the concrete has set sufficiently to be self-supporting. The floor centreings should be left in position for ten days or a fortnight.

A detailed description is given below of the work as conducted in filling the moulds which are built up as the work proceeds, being that adopted by M. Hennebique; other similar systems employ much the same methods.

Forming Ordinary Floors, Walls, Etc.—The concrete is formed in layers varying in thickness from one to two inches in beams and about three-quarters of an inch for floors and slabs generally. It is deposited in layers of about double these thicknesses, which are reduced by ramming. The first layer having been formed, the straight rods are put in position, with the stirrups placed at their proper intervals; these are held upright by a little concrete placed around them.

If the stirrups are very high, they are held by cross-pieces of wood lightly nailed across the top of the moulds. The next layer of concrete is then put in and rammed carefully, special precautions being taken not to displace the rods or stirrups and to keep these latter open. The stirrups should bear all round the bottom half of the rods, and great care in supervision is necessary to insure this being the case, as the ramming tends to lift the rods away from the stirrups.

When the concrete has been brought up to the level of the underside of the bent rods, these are placed in position and the concreting continued until the beam is complete, the tops of the stirrups appearing above the surface of the concrete as these terminate just below the surface of the floor. The centreing for the slab is next erected, after which the straight and bent rods, and stirrups for this are laid in position. The first layer of concrete is now put on, and the rods etc., lifted by it above the centreing, further layers are then added, great care being taken that the stirrups remain in contact with the rods.

Sometimes the first layer of concrete is put on before the reinforcements are placed in position, but the former method is the best, since it enables the concreting to be carried on without pause. Fig. 248 shows the floor of the Cattle Wharf, Prince's Jetty, Liverpool, during construction.

Special rammers are used of varying sizes (*a*) (Fig. 249), being formed of square or rectangular pieces of cast iron, with a protruding socket in the centre of their upper surfaces, into which a wooden handle is placed of the length required. The cast iron head is usually made about $\frac{3}{4}$ of an inch thick, and the sizes being about $6\frac{1}{2} \times 6\frac{1}{2}$, $6\frac{1}{2} \times 2\frac{1}{2}$, and $2\frac{1}{2} \times 2\frac{1}{2}$ inches. The socket is generally about 12 inches long.

Other rammers as (*b*) and (*c*) (Fig. 249) are also employed. These forms of rammer are not used when working the concrete between the branches of the stirrups, or for ramming around the reinforcements. The instruments used for these purposes are of wrought iron shaped as (*d*) (Fig. 249), of lengths up to 6 feet, the diameter of the iron being about $\frac{3}{4}$ of an inch. A head about 4 inches long and $\frac{3}{4}$ of an inch deep at the point is formed at both ends, one being forged down to a thickness of about $\frac{1}{2}$ an

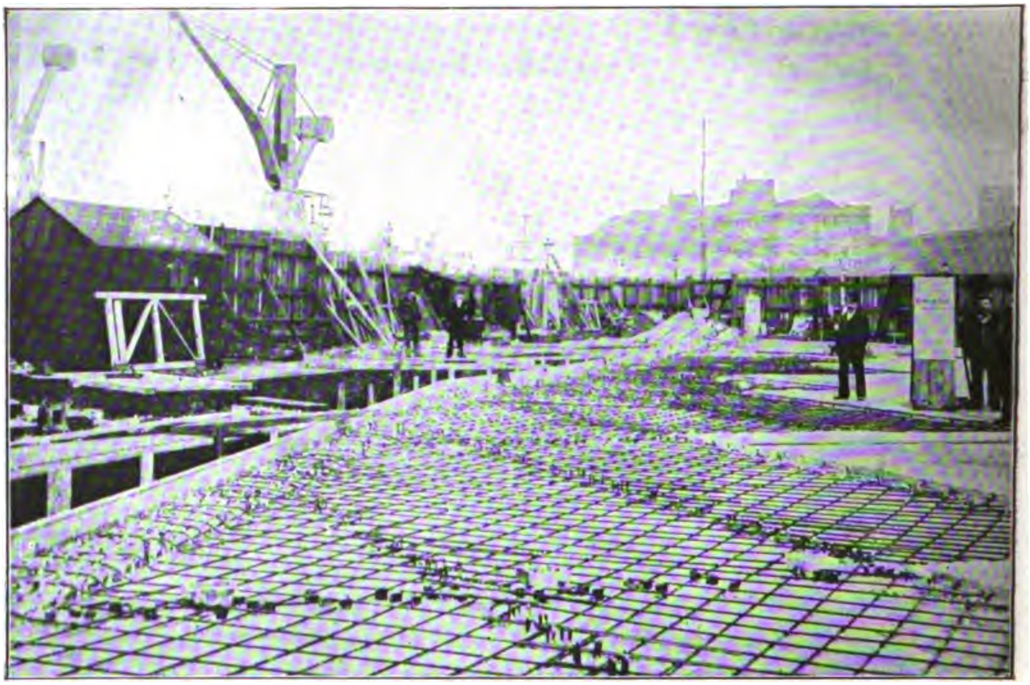


Fig. 248

inch for a depth of $\frac{3}{4}$ of an inch, the other is jumped up to a thickness of about an inch or more for a depth of $2\frac{1}{2}$ inches, being the full depth of the head.

These are the principal forms used for beams, columns, walls, and similar cases where the moulds are box-shaped. For floors and slabs a beater is employed, with a long curved handle (*e*) (Fig. 249), this is made entirely of wood, the head being about 16 × 16 inches, and about $1\frac{1}{4}$ inches thick at the centre, curved off all ways to about $\frac{3}{4}$ of an inch at the edges. The total length of this beater from the outer edge of the head to the end of the handle may be about $5\frac{1}{2}$ to 6 feet, and the perpendicular depth from the underside of the head to the end of the handle approximates $2\frac{1}{2}$ feet. Sometimes a simple rod of iron is used for working the concrete around the reinforcement in walls and columns.

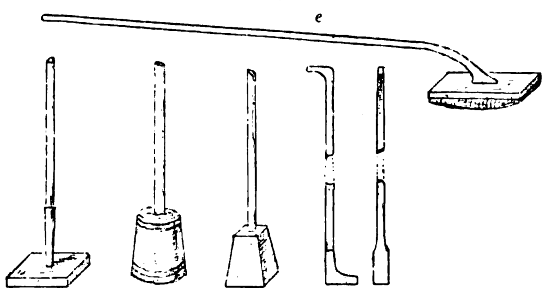


Fig. 249[1]

In columns and walls, the upright reinforcements are placed in position before the concreting is commenced, and are held in place by temporary stays. The cross pieces usually of wire, are slipped down from the top a good many at a time, and are left behind

[1] The implements are lettered from left to right (*d*) having two views and (*e*) being that along the top of the figure.

in the concrete as necessary. The longitudinal rods in walls are placed in position when the concrete has been brought up to the level at which they are required.

It is very important that the concrete should be well rammed and worked round the reinforcements. It should not be too dry or too wet, as if too dry it

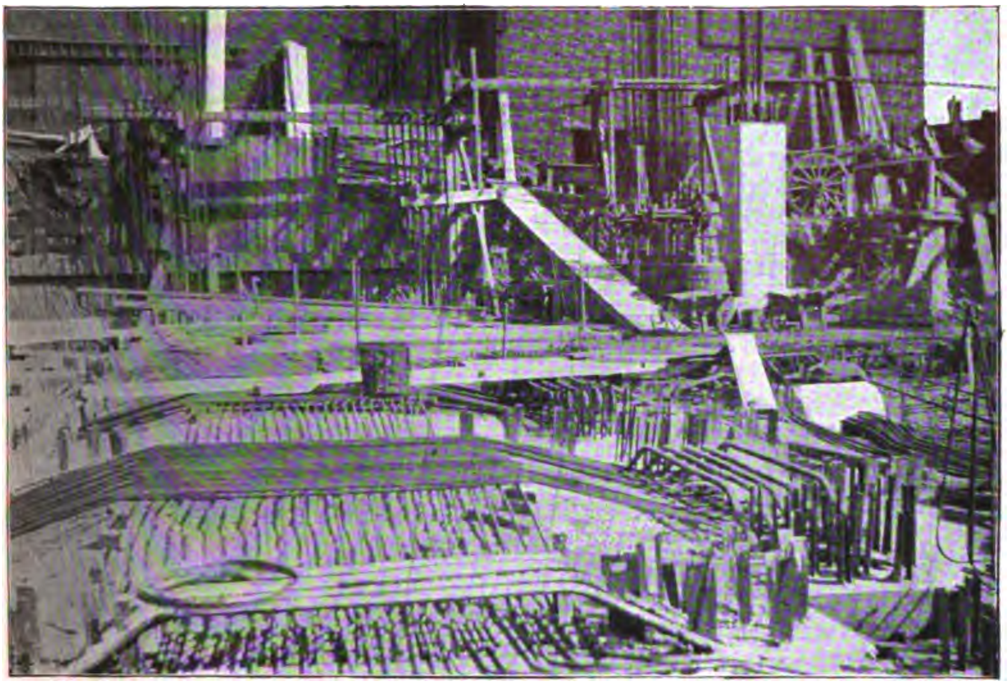


Fig. 250

will not ram properly. If moisture does not show on the surface while ramming, the concrete is not wet enough, and must be further wetted, but such wetting should be avoided as far as possible.

Fig. 250 shows the general sill to distribute the weight of the building for the Co-operative Wholesale Society's warehouse at Newcastle.

Fig. 251

In the Mattrai form of construction, which differs considerably from other methods, all the reinforcements are put in place before any concrete is deposited. Fig. 251 shows a Mattrai floor in course of construction.

Hollow Floor and Wall Construction.—*M. Hennebique* sometimes moulds both the floor and ceiling slabs in advance.

After the planks forming the bottom of the beam boxes are in position, the ceiling slabs are lifted and placed on these, having been made of such a size that they have a bearing of about three-quarters of an inch all round. The reinforcing rods in these slabs are left projecting for such a distance as will allow their ends, when bent up round the main beam rods, to reach nearly to the upper surface of the floor. After the first layer of concrete has been put in the straight rods of the beams are placed in position, and those projecting from the ceiling slab are bent up into a vertical position, each series round the rods in the beam nearest to the slab from which they project. The stirrups are also put in place, and in this case embrace all the rods, with one leg on each side of the beam (Fig. 252).

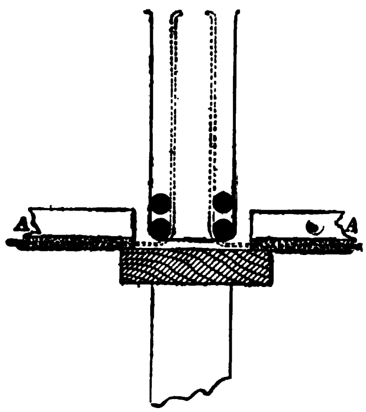


FIG. 252

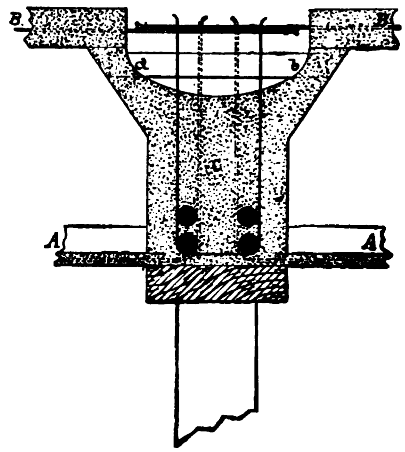


FIG. 253

The sides of the beam boxes having been formed as described (page 161) the concrete is carried up in the usual way, the bent rods being put in when the filling is brought up to their level. The beam is moulded to the underside of the floor slab, the upper surface being formed concave (Fig. 253), so that the stirrups and rods from the ceiling slab project. When the concrete has set sufficiently, the sides of the boxes are removed, the floor slabs placed in position, with their reinforcing rods projecting and crossing one another over the concavities left in the top surface of the beams. This hollow is then filled in with concrete, which unites the stirrups, vertical rods from the ceiling slabs and the horizontal rods of the floor slabs. The portion which forms the floor slabs above the hollows is lastly carefully smoothed off, so that the slabs may be continuous. When the floor slabs are moulded in situ more centreing is necessary, the moulding being performed as described for solid floors (page 185).

In the *Pavin de Lafarge* system the ceiling slabs are moulded at the same time as the beams, the reinforcing rods of the slabs being tied to short rods carried over the main beam reinforcing rods with annealed wire. The floor slabs are moulded in advance, and frequently receive a covering of brick, wood, or other paving (Fig. 254). A small space is left over each beam in this case to allow for any expansion.

M. Coignet forms his ceiling slabs and beams together, the rods of the ceiling slab being continuous and bent up so as to pass over the reinforcing rods of the beams.

M. Coularou simply passes the ceiling reinforcements over the longitudinal beam rods and into the adjoining ceiling slab, giving a good overlap.

Hollow Walls are generally formed in bays with cross ribs of concrete extending the whole height. *M. Hennebique* carries the floor slabs through to the outer slab of the hollow walls. The reinforcements are temporarily held in position by framed timber templates.

5. **Methods of Construction of Reservoirs and Similar Works.**—Some methods employed in bringing up the concrete for reservoirs, silos and similar

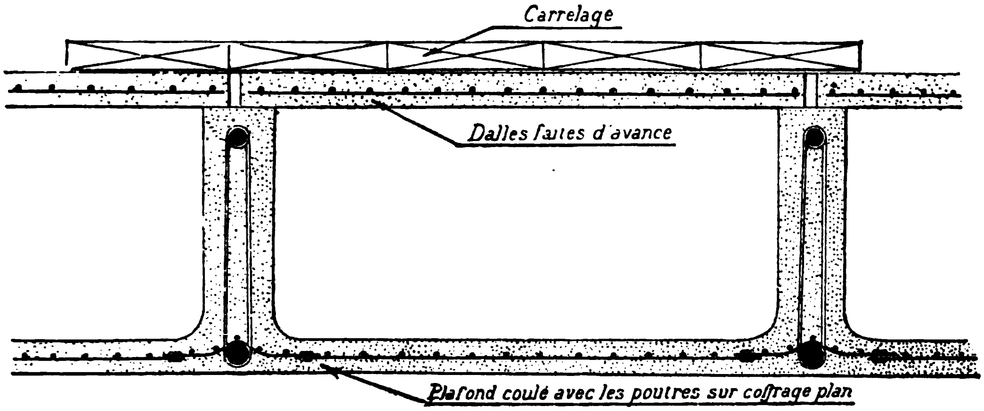


FIG. 254

structures are shown (Figs. 255 and 256). Fig. 255 is a view of a grain silo at Nogent-sur-Seine, in course of construction, showing the reinforcements and the falsework. The sides were moulded between small shutters, held in position by bolts passing through the concrete. These holes are filled in after the shutters have been removed.

Fig. 256 shows the construction of a gasometer-tank for the town of Geneva, which had an internal diameter of 106·6 feet and a depth of 24·6 feet. The walls in this case were formed between a boxing ofboards held in position by wood strips, nailed across the top, and bolts at the bottom, the latter being removed as the work proceeded and the holes filled in. This gasometer-tank was built by *MM. de Vallerie et Simon*, and was completed in 80 days from the commencement of excavation.

Methods of Constructing Arches.—Arches of the Monier or similar types are generally constructed in the following manner.

After the centreing is in position, a diagram of the bars is drawn on the lagging. The longitudinal and transverse bars are placed in position forming a mesh. A very usual custom is to first lay out transverse bars from 18 inches to 3 feet apart, being some multiple of the spacing. These are fastened lightly to the lagging, the longitudinal bars, curved to the proper sweep, are then placed in position, and are tied to the transverse bars. Lastly the remainder of the transverse bars are put in place, and are tied to every other longitudinal bar. All the bars are as long as possible, and the joints in the longitudinals are placed where there is no fear of tensile stresses. When lengthening is necessary the bars are overlapped for a length of about 24 times their diameter, and are tied together by a wrapping of wire. When the longitudinal bars require lengthening the overlapping for the several reinforcements should break joint, those for neighbouring bars being as remote as possible from one another.

When the network has been formed, it is raised to its proper distance off the lagging and supported there by small flat stones; the concrete is then put in under and round the network, being carefully rammed round the bars; it is put on, in from

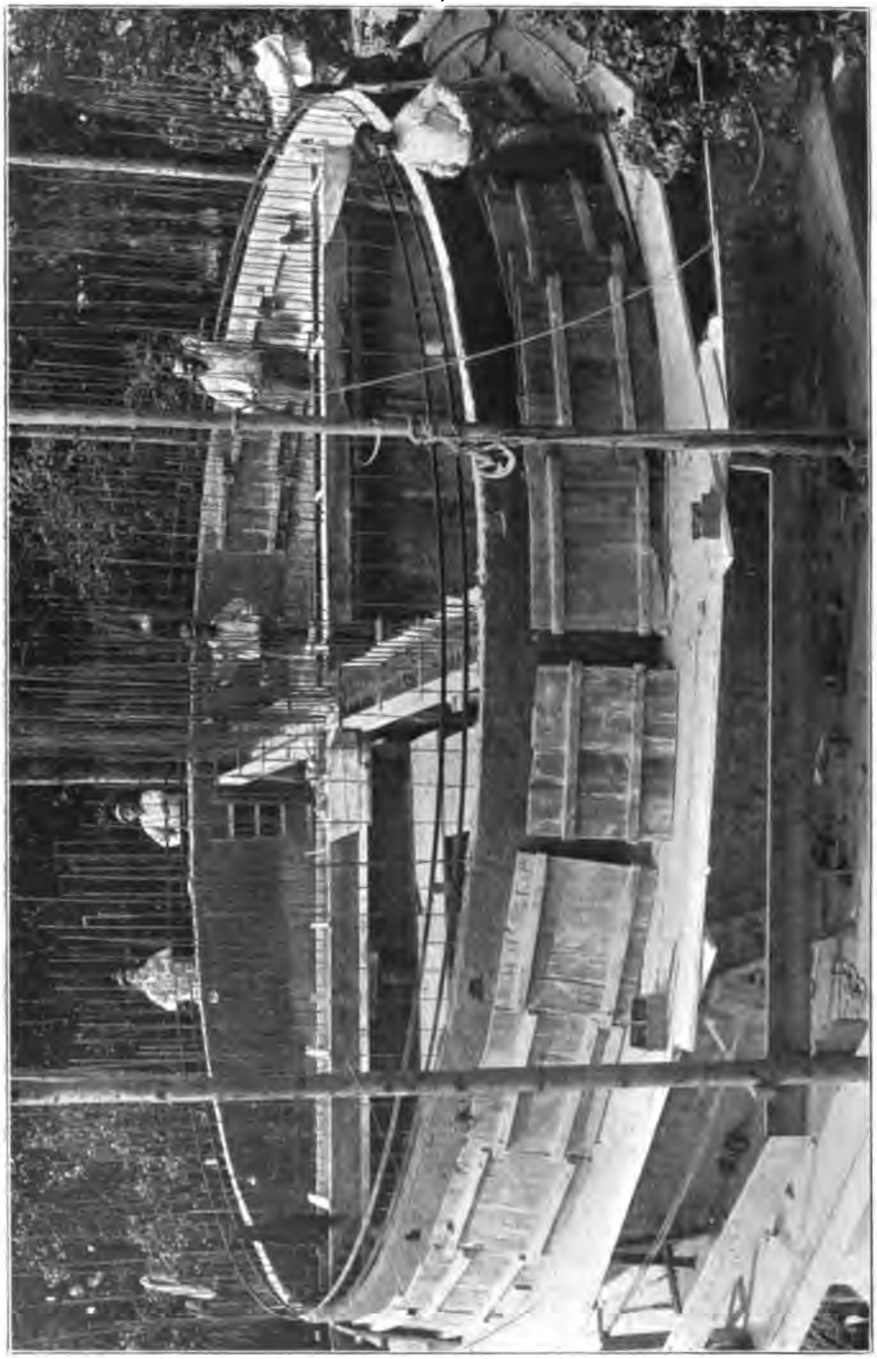


FIG. 255

4 to 6 inch layers, being well consolidated with iron rammers and wooden beaters. The concreting is commenced at the springings and haunches at the same time, to

avoid any lifting of the centres, and is carried on each way from these places, so that the settlement may be as even as possible.

The arch is closed at about the same time at the crown and places on each side between the springings and haunches. Sometimes, in large arches, the concreting is commenced only at the springings and worked up to meet at the crown. In this case

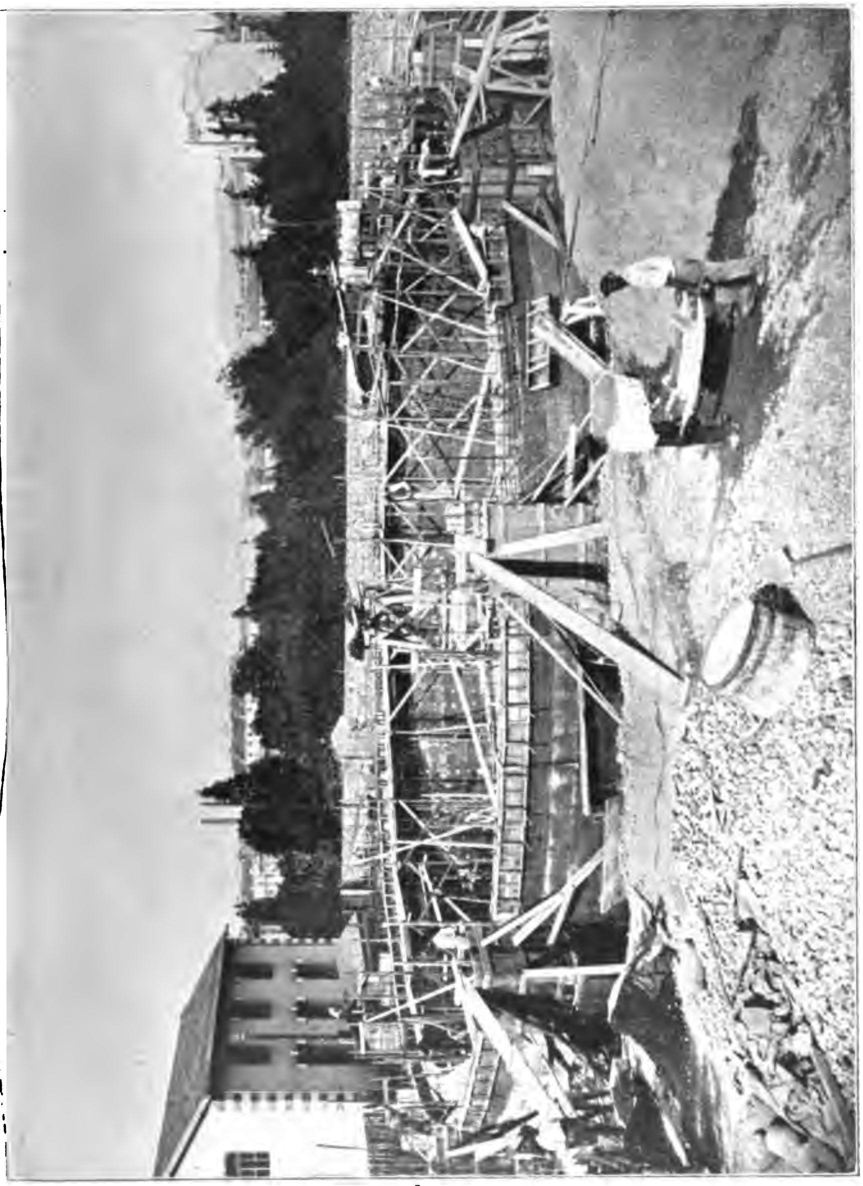


Fig. 256

the filling is only done to the level of the extrados network, which is then put on, having been previously made on the ground, after which the concreting is completed. The vertical timbers which are placed on the centres to form the faces of the arch are cut off to the form of the extrados, and act as screeds for forming the surface.

If the concreting is done in longitudinal sections, as is necessary in the case of large arches, intermediate screeds are used. The extrados is formed by means of straight edges in the usual way, and is sometimes further smoothed with a trowel or float. The surface is then protected by a covering of bags or about six inches of sand, which are constantly moistened so that the concrete may be kept damp until it has completely set.

Where, as in the case of elliptic arches, a boxing must be employed to form the extrados near the springings, the concrete is placed in transverse layers from the springings, being well rammed around the reinforcements and against the centreing and extrados coverings, the latter being brought up as the work proceeds. In this case the transverse bars of the network must be put on as the concrete is filled in.

Arches with longitudinal reinforcements only are treated in very much the same way as described for arches reinforced with networks. It is always advisable to use transverse reinforcements in both directions in arches of large span, as the arch acts mostly under compression, and therefore the concrete requires lateral support against swelling.

Arches Reinforced with Heavy Sections.—In these arches the reinforcements are first put into place by the aid of special scaffolding, and are frequently employed to partially support the centreing. The concreting is effected in much the same manner as described above, the centres being loaded in such a way that their deformation is avoided. The reinforcements having a certain rigidity, the ramming can be effected tangentially to the centreing, and the layers brought up in such a manner that they form as it were voussoirs across the whole width of the arch.

Temporary strips of wood may be inserted across the arch and the concrete worked in up to these; they are then removed and the next block commenced.

In the Melan and similar systems the arch is divided into longitudinal sections by the rolled joists or other reinforcements; the concreting is therefore carried on

Fig. 257

section by section each being begun at the springings; can be completed to the crown in a comparatively short space of time without break. Fig. 257 shows a Melan arch ready for the concreting.

Where large sections of reinforcement are employed, great care must be exercised in ramming the concrete around the joists or other reinforcements, so that they may be properly surrounded, and special care is necessary in working in the concrete into the *re-entrant* angles of the top flanges. The main sections should be held together laterally.

Arches with Ribs.—These are formed in the same manner as described above for beams and floors (page 185), the concrete being placed in layers, and the reinforcing sections are put in position as the work proceeds. For hollow ribbed arches of the Hennebique system the reinforcements and concrete are put in place as in the case of hollow floors (page 188), excepting that the intrados and extrados are moulded *in situ* on special slabs of reinforced concrete, which are left in.

Large Sewers, Subways and Conduits.—The methods employed in forming these have been already described under "Treatment of Reinforcements," as the building up of the skeleton and putting in of the concrete are intimately connected, and cannot well be described separately (*vide* pages 181 to 183).

Tunnels.—These can be constructed in a similar manner to large subways or conduits excepting that the arch is closed as a tunnel arch with block laggings.

Striking of Centres.—The centres are generally left up until the concrete has set sufficiently to have attained almost its full strength, i.e. for about a month, but some constructors strike them as soon as possible, so that they may be re-used. The striking of the centres too soon is likely to cause considerable settlement in the arch, and consequently an initial straining of the materials. It has, however, been pointed out that the centres cannot take up the same form as the arch, under climatic influences, since under the influence of heat the arch will expand while the centreing contracts.

The lifting of the arch from the centreing may in some measure be prevented by keeping the surface damp ; but the centreing being protected from the damp will certainly have a tendency to contract, which in time would cause an automatic slacking. It may be well, however, to keep the centreing wet by playing a hose on it at intervals for about two weeks after the work is complete, though this is not good for the preservation of the timber.

The time allowed after an arch is complete before the centreing is struck varies from two weeks to two months. It is probably the best practice to allow the centres to remain in position for about six weeks after the moulding is complete ; but the planks against the faces of the arch should be struck as soon as possible, as this will help the setting. The portions of the centreing near the springings are sometimes struck a fortnight or so before the rest so as to allow the arch to take part in any settlement of the abutments, but if this is done there must be an open joint left in the spandril walls.

Moulding in Advance

General Remarks.—The pieces moulded in advance should not be put into position until sufficient time (a month at least) has elapsed for them to have attained very nearly their full strength, as in all probability when placed in position they can be conveniently used to support the falsework and materials required for parts of the structure. They must be left in their moulds until sufficiently set (usually from about three days to a week, depending on the nature of the piece), after which the boxes may be removed and re-used, while the finished piece is left to harden either in the air, as is generally the case, or in water, damp sand or sawdust. Hardening in the air produces initial tensile stresses in the concrete, while hardening in water puts the concrete into compression (p. 252). M. Considère points out that the hardening in water is advantageous for pieces which will be subjected to direct compression, when they are hooped (*vide* p. 247).

The staging on which the moulding is performed must be rigid and free from vibrations.

Moulding *in situ* should be universally adopted for important parts which are dependent on one another, as when moulded separately, in advance, there can be no intimate connexion with the other parts of the structure. When pieces are moulded in advance great care is necessary when moving, lifting and placing them in position, so that they may not be damaged in any way.

All thin slabs should be made with special care, and the concrete used must not be mixed any weaker than 2 to 1 when the very best materials are used; with materials of worse quality 1½ to 1, or even 1 to 1, are employed.

When the mortar is set, i.e. about a day after finishing, the surface should be well wetted and covered with a layer of wet sand, which must be kept moist for about a fortnight, after which it may be removed and the slab placed in position. These remarks apply equally to slabs moulded in place.

Parts of Buildings Moulded in Advance.—Many advantages are gained by moulding the parts for building in advance.

1. It is more economical to mould the beams, slabs, etc., in a shed than on falsework.
2. The moulding can be carried on in all weathers, causing a considerable saving in time.
3. There is more freedom and space where no falsework or very little is employed than with the mass of props, boxes, and staging rendered necessary when moulding *in situ*.
4. The erection can go on without intermission.
5. The structure will bear its final loading immediately the parts are in place.
6. The building up of the structure is greatly facilitated in consequence of the beams, slabs, etc., merely requiring lifting and placing in position.
7. The pieces can be tested, if required, before being placed in the work.

Moulding in advance may be recommended for pieces of small dimensions, such as slabs, and floors like those of the *Siegwart*, *Stolte*, and *Visintini* systems (pages 106, 108 and 110); on the ground of economy, especially so when there are a number of pieces of a similar size, for which the same moulds can be re-used. Where ceiling slabs of reinforced concrete are used, it is evidently an advantage to mould them before erection, since they can then be employed to mould the floor upon.

M. Cottançin employs this method with his double floors. The ceiling slabs are propped up off projections on the beams so as to support the floor slabs whilst being moulded, and are then lowered again on to the projections after the floor has set (Fig. 258). Fig. 259 shows this method during the construction of a floor. When the beams and floor slabs are constructed to act separately, as in those cases where the floor slabs merely rest on the beams, both the floor slabs and beams may be moulded in advance with economy and

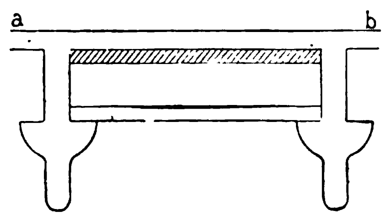


FIG. 258

without fear of any detrimental effect on the structure, the calculations having been made without any reference to a united action. In this case the slabs should be carefully bedded on the beams, so as to have an even bearing. If the surfaces of the beams are irregular they should be made level before the slabs are placed in position.

When beams, or ribs are moulded in advance and placed bodily in position, they are generally made of light scantling for ease in handling, and placed near together. In some cases, intersecting ribs are moulded in advance, with the object of saving the expense and trouble of falsework, the intersections being made good when they are in position. The ribs are lifted into place three or four weeks after moulding, and are then used to support the falsework on which the floors are moulded.

The Cottançin and Coignet systems employ this method. In the *Cottançin*

FIG. 259

system the network of the beams is left projecting beyond the concrete at the upper edge, and the floor network is interlaced with it; the two are considered to act together when the floor is complete. Fig. 260 shows such a floor during construction. In the *Coignet* system, the transverse reinforcements of the beams are carried into the floor slab. In the *Bonna* system, floors are sometimes constructed in the same manner.

When this method is employed, the beams when in position, must be well stayed against lateral movements, they must also have sufficient strength to enable them to support unaided the imposed loads during the mouldings of the floor.

This method of construction is undoubtedly economical, but there is more difficulty in ensuring the proper connexion between the beams or ribs and the floor slab than in the case of monolithic construction; the upper portions of the beams are moulded after the lower portion has completely set, and there is consequently every reason to doubt whether the two portions are ever properly united, though the reinforcements being interwoven or carried from the beam into the slab will certainly tie them together.

It appears doubtful practice to mould the beams in advance when they are intended and calculated to act as one with the floor slab in resisting the imposed stresses, although the systems which adopt this method certainly obtain fair results.

In the *Hennebique* system, in which the whole main structure is moulded in place, it is customary to make in advance the lintels, sills, lesser uprights and window frames of the façade, also thin partitions and similar pieces which do not affect the stability of the structure, and to build them in bodily as the work proceeds (Fig. 87). *M. Coignet* also employs this method.

Fig. 200

Theory shows that when each is considered by itself the rectangular beam is more economical than the T-beam, but it loses this advantage in practice, since the T-beam employs the depth of the floor slab for a certain width on each side for resisting the compressive stresses, whereas the rectangular beam, merely supporting the floor slab, gains no resistance from it, and must be made deeper to obtain the requisite compressive resistance. For a true balance of economy, only the portion of the T-beam below the floor slab should be compared with a rectangular beam.

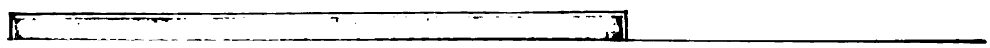
Piles.—Piles may be moulded either vertically or horizontally, but by vertical moulding better results may be obtained. The piles act under direct compression, and consequently it is better that the layers should be normal to the direction of the pressure, the ramming being performed in the direction in which the load will act.

Horizontal moulding, on the other hand, is the more simple and economical method, and may always be employed for sheet piling where the stress is applied transversely; it is frequently used with good results for bearing piles, with a great saving in cost.

The vertical moulds for piles should be erected near one another, and held by a rack or framework of timber. Fig. 261 shows the pile rack for the Southampton cold storage constructed by the *Hennebique* system. The moulds are similar to those described for columns (page 198) except that the triangular strips are not placed in the corners, as no chamfers are required.

When the concrete has set, usually about one week after moulding, the nails which hold the horizontal boards in position are withdrawn, and this side is removed, the remaining sides eased, and the pile withdrawn by a crane and deposited lying flat on the ground, or stacked with others until required for the work. Piles may be driven about one month after moulding, but it is advisable, if possible, to leave them to harden for a somewhat longer period.

M. Hennebique now constructs hollow piles, the hollow spaces being formed



The vertical moulding is done from various stages placed at such a distance apart that a man can always conveniently work in the concrete and ram it well around the reinforcements. Horizontal moulding is performed in the same manner as already described for beams.

The *Hennebique and similar piles* have vertical reinforcing rods. These are bent together at the bottom of the pile, which is formed in an iron shoe, similar to those used for timber piles. Several of the wire cross-ties for holding the rods together are slipped down from the top at one time, and are left behind in the concrete as they are required.

If one series of rods have not sufficient length further rods are added, the connexion being formed by simply overlapping them for a length of about twenty-four times their diameter. A better method, however, would be to cut off the ends of the rods truly square and abut them against one another, passing a sleeve of iron tubing over the joint, since this will enable the rods to act as one. The tops of the reinforcing rods are usually covered by about $1\frac{1}{2}$ inches of concrete.

Mr. A. E. Williams, of Dagenham Docks, forms the point of his piles by cutting away the web of the joist and forging the flanges together; he moulds horizontally.

The piles of the *Rechten, Vereng* and *Dopking* system, in which the reinforcement consists of rolled joists framed together, are usually moulded horizontally, but, in this case, whether horizontal or vertical moulds are employed, it is equally difficult to well ram the concrete around the reinforcement.

Pipes, Sewers, etc.—Pipes and similar structures are moulded in several different ways, slow-setting cement being employed generally, except where the mortar is poured into the moulds, when quick-setting cements are used. Pipes are moulded, before being laid, up to a diameter of about 6½ feet and sometimes as much as 7½ feet, and lengths up to 10 or 12 feet, but when larger conduits are required they are moulded in place. The thickness of shell seldom exceeds 2¾ to 3¼ inches and pipes up to about 9 inches diameter are usually from 1½ to 2 inches thick.

The *Monier* pipes and sewers, if of sufficient thickness to allow a rammer to be worked, are moulded vertically, with an outside casing and a collapsible core. If too thin to be moulded in this way, they are formed in the following manner:—

The reinforceing network is placed on a collapsible core, of the size of the internal diameter of the pipe. The concrete, which is mixed stiff, is then thrown hard against the core, and passing through the reinforcement forms a layer behind, the network being shaken during the process.

When the first layer has partly set, a second layer is added in the same manner, excepting that the shaking of the network is discontinued. This process goes on until the required thickness is attained. Each layer is about three-eighths of an inch thick. The inside and outside are then finished off with thin layers floated on.

MM. Pavin de Lafarge also employ much the same method for constructing sewers.

The moulding of pipes is performed horizontally by *Herren Wayss & Co.*, who have invented a special machine for this purpose. Their pipes are formed on a rotating drum, of sheet iron, or wood covered with zinc, receiving the mortar through a hopper on its upper surface; this is spread over the surface of the drum by rollers. A special arrangement is employed to prevent the mortar from breaking away from the lower portion of the drum during rotation. One coat of mortar is put on the drum, after which the reinforcement of network is wound round, having been previously formed, then a further layer of mortar is added, and the process continued until the desired thickness is attained.

Pipes are often moulded vertically, by pouring in the mortar of the consistence of a stiff grout. No ramming can be done in this case, and the proper surrounding of the reinforcement can only be effected by the mortar penetrating every crevice under the action of gravity. Long rods are however used in some instances to aid the flow of the grout. When special sections of reinforcement are used the spiral form aids the penetration of the mortar very materially, as the air is not so likely to become imprisoned in the re-entrant angles.

When this method is adopted the sides of the moulds should be frequently struck with wooden mallets during the running in, to free the air and consolidate the grout. This method is more simple and economical than that where the mortar is put in in layers and rammed; it also is more likely to make the pipes thoroughly water-tight, although there seems no cause to fear the results in the case of fairly dry concrete, well rammed. The bottom portion of the pipe receives more consolidation than the upper portion. There does not appear, however, to be any difference in the powers of resistance to leakage at the two ends of the pipe on this account.

A quick-setting cement has less resistance than one that is slow setting, but for such structures as pipes, the concrete acts almost entirely as a medium for distributing the stresses, which are taken up by the reinforcement; it only acts directly in a slight measure in resisting the tendency to bending in a longitudinal direction between the circular reinforcements.

The *Bordenave* and *Bonna systems* are perhaps the chief ones which employ this method. *MM. Pavin de Lafarge* also mould their pipes in this manner.

It may be interesting to describe in more detail the methods of moulding employed by M. Bordenave and M. Bonna.

In the *Bordenave system* a covered stage, on which the mortar is gauged and the necessary work done, is mounted on a framework on wheels, the underside of the stage being at such a height above the ground that it clears the full length of a pipe. At one end there is a hole, over which a framed tower is erected carrying pulley blocks, on which the cores of the moulds are lifted and lowered, the tower being of such a height that when the core is lifted to the full height, its bottom will just clear the top of the stage. This apparatus is called a "pondeuse" (laying hen), and runs on a tramroad encircling the cement and other necessary sheds.

A ring having the thickness and internal diameter of the required pipe is placed on the ground, and the reinforcing skeleton, which has been previously made as described (page 176), is placed vertically on this ring in its proper position. The core, which is made collapsible, so that its diameter may be varied by means of toggles, and to facilitate its removal, is then lowered into position and opened to the desired diameter. The shell, which is made to open and shut, is then suspended, open, from the stage, and is taken down and put into place, closed and fastened.

The mould thus formed is then filled with grout, through a special funnel fixed to the stage. Before the mortar is quite set the funnel is lifted away, and the top of the pipe is formed by hand. When the grout has sufficiently set, the shell is opened and the core slackened, and both are removed, the pipe being left standing and the "pondeuse" moved to cast the next pipe. The finished pipes remain in place until the apparatus comes round to within a short distance of them, when they are lifted by a crane on to trolleys and taken to a stacking ground. Fig. 262 shows the methods adopted.

M. Bonna moulds his pipes in much the same manner, the reinforcement being placed on a ring of wood, the core of sheet iron, whose diameter can be varied, is placed inside, and a steel shell outside, which is in two halves held together by hoops. If the interior of the pipe is to be lined with a sheeting of steel (which is done in this system when the pipes are to be under considerable head) the steel lining sometimes acts as the core on which the pipe is moulded.

Each mould is filled at one time through two telescopic spouts, fixed to a decking, carried as in the Bordenave system, on a framework attached to a rolling carriage running on a tram road. A funnel-shaped collar is placed on the mould to facilitate the filling. The reinforcing skeleton is held in position by cleats, to ensure its being kept truly in place. During the running in of the grout the mould is hit with wooden mallets, to get the air out and properly consolidate the grout. The moulds are struck twenty minutes after moulding, the pipes being finished with a thin rendering of the surface, and are removed and stacked after standing for two days.

Methods of Connecting the Lengths of Piping.—Pipes are usually connected together in the trench by collars of reinforced concrete, formed in the same manner as the pipes, their inside diameter being slightly larger than the outside diameter of the pipes themselves. The collar is threaded on to the pipe already in the trench, the next pipe is then put in, the opening between the two

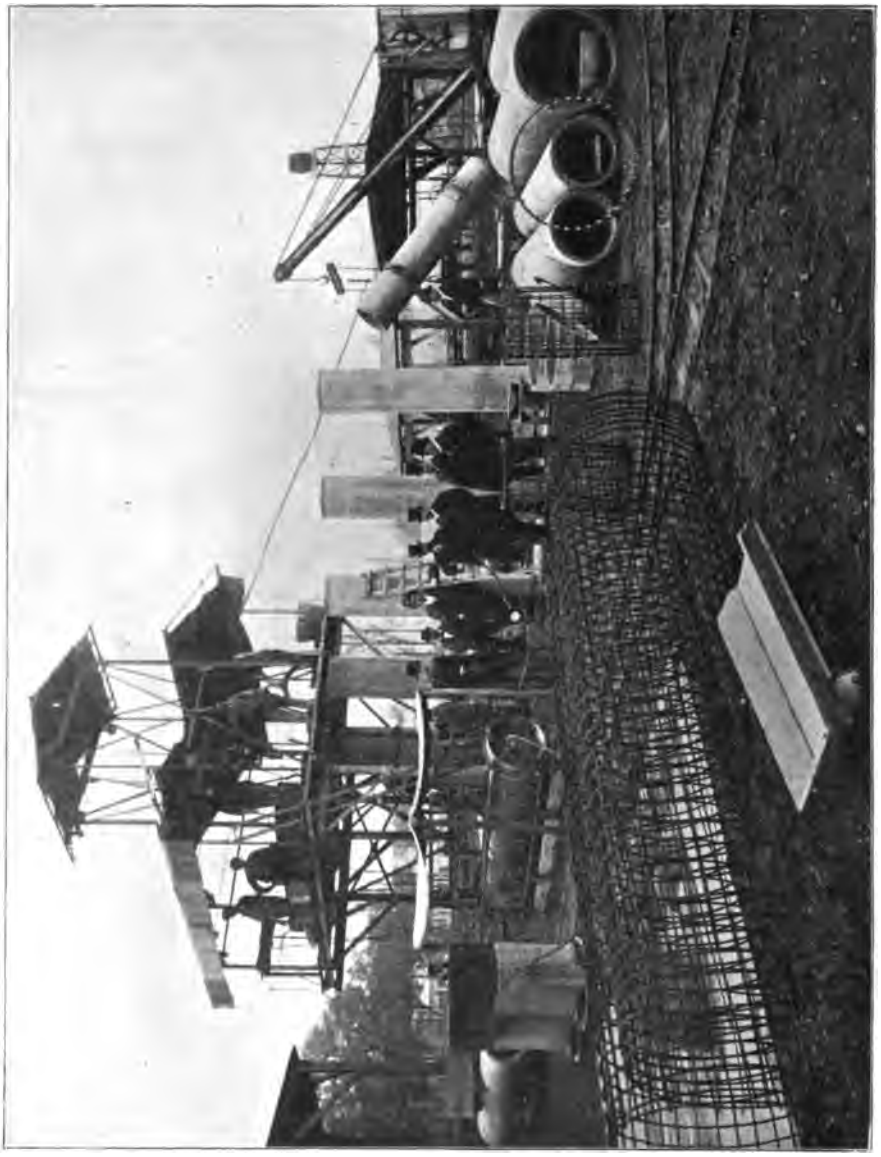


FIG. 262

is first filled with a cement stopping, and then the collar is slipped over the joint; the ends of the collar are then stopped with mortar, and the annular space between the collar and the pipes is filled in with grout through holes left in the collar for this purpose. Sometimes joints are formed with indiarubber rings to allow for expansion.

MM. Pavin de Lafarge make an expansion joint formed by a plain collar and two angle collars of iron; the angle collars are secured by bolts, and the joint is made by two rings of indiarubber, which are pressed between the angle rings and the plain collar when tightening up (Fig. 263).

M. Bonna's usual method of jointing his steel lined pipes is shown in Figs. 264

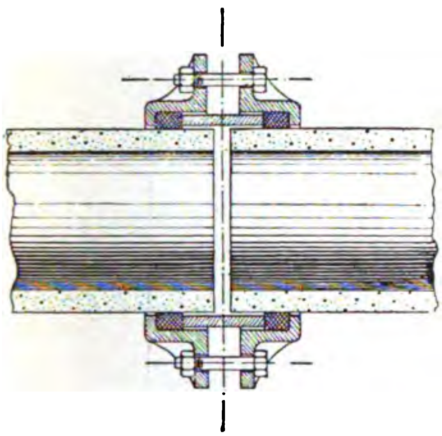


Fig. 263

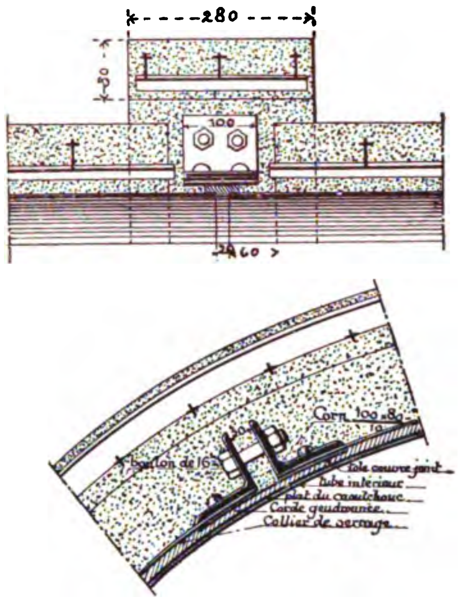


Fig. 264

and 265. A ring of steel in two halves connected by bolts is placed over one or two indiarubber rings, placed on the steel lining, which is left projecting from the ends of the pipes. The bolts are tightened up, compressing the rubber rings, thereby ensuring the complete water-tightness of the joint. The reinforced concrete collar is placed over the joint which is made good in the usual way.

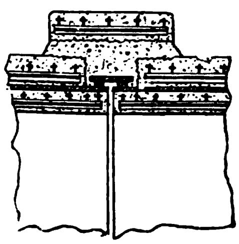


Fig. 265

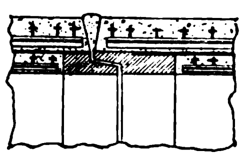


Fig. 266

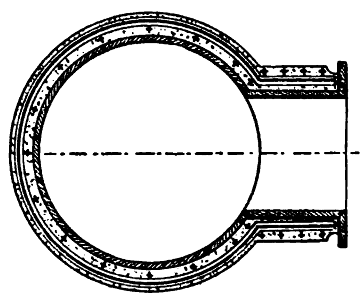


Fig. 267

Another form of joint employed by M. Bonna is shown in Fig. 266. Metal rings formed as shown are welded to the ends of the steel tube. The pipes are then brought together, the joint being made by a layer of red and white lead or cement mortar.

Branch pieces are formed in one piece with the main pipes. Fig. 267 shows a branch on the Bonna system, and sufficiently indicates the methods adopted. If the branch is continued in reinforced concrete the end is finished plain, and the joint with the following pipe made with a collar in the usual way.

M. Bonna makes the service connections by breaking away the concrete where required, and forming a flat surface. On this he places a service cock, which is secured to the pipe by means of special strap boats. Other systems employ much the same method. The joint between the pipe and the saddle is made with either a sheet of leather, indiarubber, or asbestos.

Methods Employed for Driving Piles.—Hollow tube piles and caissons of the *Monier and similar systems* are sunk by a water jet in the usual way, or in the case

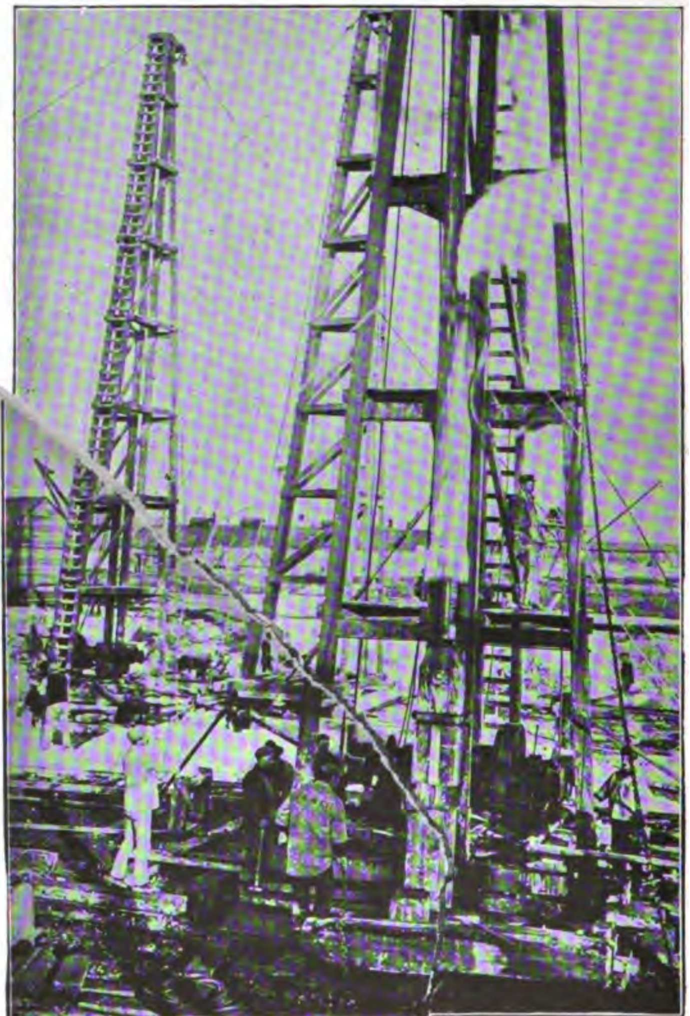


FIG. 268

of large caissons by excavations in the interior, the top being weighted. Solid or hollow piles of the *Hennebique* system and the piles of the *Dopking and other systems* are driven by an *ordinary* ringing machine or pile driver in exactly the same manner as ordinary timber piles; the monkey, however, may be heavier than for timber piles.

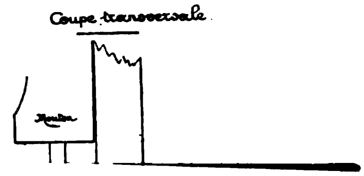
Fig. 268 is from a photograph showing the driving of 14 × 14 inch Hennebique piles 43 feet long at Plymouth.

The force of the blow should be uniformly distributed over the head of the pile, so that there may be no shattering of the concrete by unequal shocks.

In the *Dopking* system an oak block is placed on the top of the pile. Another method is to place blocks of soft wood on the top of the pile with a wooden dolly over them, having a hollow iron shoe, which is filled with old rope.

M. Hennebique places an iron hood with a spherical or partially domed head on the top of the pile, in which a bag of sawdust is placed. Sometimes the hood is filled with sand or sawdust and is caulked at the bottom with clay and yarn. In some cases he merely employs a wooden dolly between the monkey and the pile head.

Another method which is employed is to have a collar of steel plate round the pile head, surrounded by three straps riveted on at the top, bottom and middle. The collar is slightly larger inside than the pile head, and is secured by hard wood wedges; it projects about two-thirds of its height above the head. Alternate layers of sawdust and shavings are placed in the collar, which when compressed occupy about half the space between the top of the collar and the head of the pile. The shavings are



Page 202—See slip at page 197 as to the hollow pile here mentioned not being Mr. Hennebique's but that of Mr. L. G. Mouchel.

C.F.M.

the original concrete. ...
in the calculations.

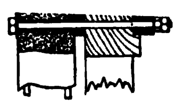


Fig. 269

It is very seldom that such special moulds are used as to leave the face in a condition which requires no further attention. Generally when the moulds are struck the surface of the concrete is somewhat rough, and shows the plank markings, or the impression of the canvas, paper, etc., which was used to line the moulds. For some buildings, such as factories, etc. where a pleasing appearance is not necessary, no further work is done to the surfaces. In most cases, however, the exposed faces are covered with a coating of varying thickness depending on the roughness of the concrete, and in some degree on the amount of ornamental finish required.

Any heavy ornamental moulding should be roughly formed in the original concrete so that the finishing layer may be as thin as possible, since the mean thickness of this coat ought not to exceed half an inch, and it is well to keep it down to a quarter of an inch if possible. Where however the moulds can be removed before the concrete has set, the facing layer may be increased with advantage. The rendering of the face should be put on directly the moulds are struck, and consolidated as much as possible by employing a wood or cork float for applying it.

The mortar, except when impermeability is necessary, should be gauged in the same proportions as the mortar of the concrete on the main work, and not be richer than this, as the richer it is the more it will expand and contract under atmospheric influences, and the face and back work will expand or contract unevenly.

The sand used must not have very large grains unless some special finish requiring these is decided upon. Messrs. Cubitt (Builders, Grays Inn Road) have obtained an excellent facing by employing portland stone chippings as an aggregate; the finish has the appearance of portland stone, but is much more durable.

The facing mortar is often coloured to avoid a patchy appearance; when colouring matter is employed, great care should be exercised in order that a perfectly even colour is obtained, as a badly coloured surface is worse than one of the ordinary mortar. Bridges are frequently finished in this way, the facing layer being sometimes worked in against moulds covered with sheet iron, zinc, or plaster of paris as the work is brought up, a piece of sheet iron being placed between the face mortar and ordinary work and pulled up gently before they set to obtain a good bond. The facing is of course made thicker in this case.

Where sheets are not used between the two mixings, the face work is often brought up with the back work by spreading it on the moulds before the ordinary concrete is put on, care being taken not to force the stones through to the face.

The limit of colouring matter added to the cement ought not to exceed about 10 per cent. by volume, and should be kept below this if possible. Yellow and black are the most usual colours employed. A coating of shellac is frequently applied to plaster of paris moulds to obtain a smooth and non-adhesive surface.

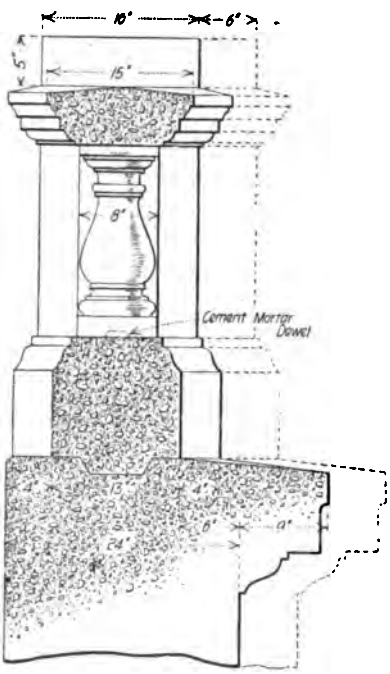


Fig. 270

The faces of walls in which the outer coating is brought up as the work proceeds are often rubbed over with a cork float, with a small amount of mortar after the moulds have been struck, which fills in any air holes and leaves an even face.

Parts of the work where no great strength is necessary, such as parapets, etc., of bridges, are frequently moulded on the ground and lifted and set in position after they have completely set. Fig. 270 shows a balustrade parapet of this type. Bridges and other work can be faced with brick or stone, put on after the concrete is moulded, or used instead of shuttering, being built up as the concrete is put in, with sufficient number of ties extending well into the concrete in the usual way.

The Ransome system employ an imitation stone face by placing V-shaped wood strips on their moulds, striking as soon as possible and then either picking over the surface or dressing it with a chisel according to the finish required (Fig. 197). The concrete used at the face is coloured to match any stone, and is often formed of sand obtained by crushing the stone which it is desired to imitate.

A method sometimes employed in forming a surface to a reinforced concrete structure is to splash on a thick grout with a large brush. This leaves a slight roughness which will not show hair cracks, and frequently looks better than a surface which is quite smooth.

Slow setting or a mixture of quick and slow-setting cements are frequently used for facing, with the object of an expeditious finish, but it is better to always use slow-setting cements, since, for even variation under atmospheric changes, the cement of the facing and that of the main work should be the same.

The network for Monier arches is sometimes placed directly on the lagging and the concrete put in above it. After the centre is struck the exposed network is covered with layers of mortar of varying proportions, commencing with 3 to 1 and terminating at the intrados with neat cement. For obvious reasons this method is not to be recommended. A joint of cleavage is left at the underside of the network, neat cement is absolutely sure to expand and contract badly with atmospheric changes, and consequently will show cracks; and further, the layers of different proportions all vary in their expansion and contraction, and are almost certain to separate more or less from each other.

Where a very good face is required it is best to make the moulds with special care, and to line them with sheet iron, zinc, or plaster of paris covered with shellac.

For facing reservoirs and similar structures, a thicker layer is used than $\frac{1}{4}$ to $\frac{1}{2}$ inch as recommended above, and this is mixed in richer proportions, in some cases being as rich as 1 to 1. After the facing is completed the water is let in and allowed to remain for a week or a fortnight, as this will prevent any cracking of the surface during setting, and the subsequent variations in temperature and humidity are very slight.

For slabs M. Hennebique covers the surface with a layer of about three-quarters of an inch, mixed twice as strong as the rest of the slab. He puts this on directly after the last layer of the ordinary gauging, so that the bond shall be perfect.

Where it is possible any exposed surface, such as that of bridge decking, roofs, linings to slopes, pavement slabs, etc., should be covered when completed with a layer of sacks or some other suitable material for as long a period as possible, the covering being kept damp. This will prevent the shelling up of the surface. When concrete is brought to a surface by straight edges working on screeds or by floating or similar procedure, it is well to sprinkle sand on the surface as it is formed, the tendency being for the cement to rise to the top of the slab, and this surface is sure to peel off unless sand is mixed with the cement while it is being formed.

Hinges for Arches

There are several methods employed for forming the hinges in reinforced concrete arches.

When heavy sections of reinforcements are employed, as the rolled joists or built up girders of the Melan system, these lend themselves to the formation of hinge joints similar to those used in metallic bridges (Fig 242). In this case concrete at the joint is often formed as shown in Fig. 271, but may equally well be finished off square at each side of the joint, a small space being left allowing for the play of

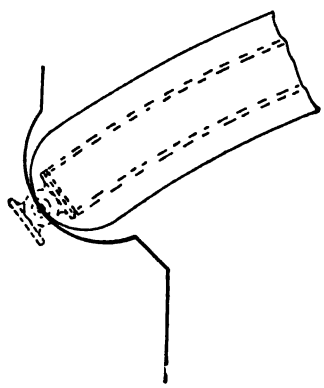


FIG. 271

the hinge. The hinge, when small reinforcing sections are used, may be formed after the manner shown in Fig. 271, the two curved surfaces of different radii forming a hinge in themselves. This method was introduced by Herr Köpke. The concrete may also be finished off on each side at the joint in iron shoes, which are united by a hinge.

Another very usual method of forming the hinges for reinforced concrete arches is to use narrow lead plates about three-quarters of an inch thick, the joints where the lead is inserted being carefully scraped out and left open.

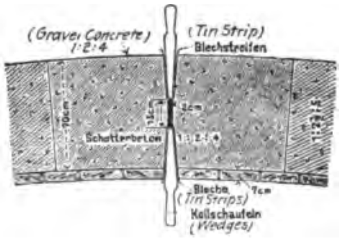


Fig. 272

An excellent method used for closing a hinged concrete arch near Chingen, Würtemberg, is shown in Fig. 272, which explains itself. The longitudinal wedge-shaped mould boards were removed 24 hours after the concrete was put into place. Herr Von Leibrand in 1891 published the results of some experiments on lead to discover its fitness for hinges of masonry arches, an extract of this publication being given in the *Engineering News*. Herr Von Leibrand found that cast lead cubes with 3·12 square inch sides and a density of 11·3 supported a pressure of 711 pounds per square inch for 26 hours without showing any signs of lateral yielding; under a pressure of 1,023 pounds per square inch they commenced to yield slowly.

Increasing the load every ten minutes till a total of 4,260 pounds per square inch on the original surface was reached, the yielding increased, but not rapidly. At this load the lead had an area of 3·89 square inches, making the unit load 3,285 instead of 4,260 pounds per square inch.

Increasing the load further to 12,780 pounds per square inch on the original surface the yielding increased rapidly, and the unit load only increased from 3,285 to 4,180 pounds per square inch.

This behaviour of lead cubes indicates a property of great importance for the purpose of arch hinges, since if the curve of pressures approaches the edge of the lead plates, and the pressure exceeds the resistance of the lead to compression, it yields at the point acted upon, and the pressure per unit surface will consequently be reduced.

Herr Von Leibrand advises that blocks which have a great resistance should be inserted adjoining the lead hinge plate, such as basalt, granite, or hard sandstone, when the work is of large dimensions. In ordinary cases a resistance of 10,650 to 11,360 pounds per square inch is sufficient, but this was for ordinary arches; the resistance for reinforced concrete need not be so great.

For spans from 50 to 130 feet lead fillets are employed, three-quarters of an inch in thickness, and of such a width that the maximum pressure will be about 1,700 pounds per square inch.

For arches of from 130 to 170 feet span Herr Von Leibrand has employed hinges like those used for metal arches, the concrete being received by hinged shoes, as described above.

Many advantages are gained by forming an arch with hinges.

1. The stresses on an unhinged arch are indeterminable, whereas when hinges are used they can be accurately found.

2. Changes of temperature do not alter the stresses.

3. Premature striking of the centreing has less effect, and any sinking of the centre while moulding can be rectified without any dangerous straining.

The thrust in hinged arches is undoubtedly greater than when no hinges are inserted, but this is compensated for by the more accurate determination of the stresses and the nullifying of temperature effects. Arches of small span will be more economical if they are constructed without hinges, but there seems no doubt that hinges should be used for large span arches. It must be borne in mind, however, that hinges of any sort soon become practically useless if they are not properly attended to. Metal hinges become clogged and rusted, and those formed with open joints become choked up with débris. The three-hinged type is undoubtedly the best to adopt.

Part V

EXPERIMENTAL RESEARCH AND DATA DEDUCED THEREFROM

Before discussing the theoretical calculations for structures in reinforced concrete, it is well to review the experimental researches which have been carried out on this material and the results which have been obtained from them. Unfortunately, it cannot be said that we have a thorough knowledge of the various properties of reinforced concrete.

When properly combined with metal, concrete appears to gain properties which do not exist in the material when by itself, and although much has been done by the various experimenters in recent years to increase our knowledge on the subject of the elastic behaviour of reinforced concrete, we are still very far from having a true perception of the characteristics of the composite material.

It may be that we are wrong from the commencement in attempting to treat it after the manner of structural ironwork, and that although the proper allowances for the elastic properties of the dual material is an advancement on the empirical formulae at first employed, and used by many constructors at the present time, yet we may be entirely wrong in our method of treatment.

The molecular theory, i.e. the prevention of molecular deformation by supplying resistances of the reverse kind to the stresses on small particles, may prove to be the true method of treatment for a composite material such as concrete and metal. This theory is the basis of the Cottançin construction which certainly produces good results and very light structures, and M. Considère's latest researches on the subject of hooped concrete are somewhat on these lines.

Where, however, as is the case in most of the systems at present employed, comparatively large sections of metal are used for the purpose of directly resisting the stresses, we must treat the subject on the usual lines and use formulae based on the direct elastic resistances of the materials and the deformations which are produced in its various applications.

Before we can establish formulae such as these it will be necessary to study the properties of the materials as shown by carefully conducted experiments with the object of obtaining the necessary data on which to base our calculations. We must know—

1. The coefficients of elasticity in tension and compression of the two materials, and the effect of these on the distribution of the resistance.

2. The stresses in tension and compression and shearing which they will safely bear.

3. The frictional or "adhesive" resistance of the metal and concrete, how the stresses are transmitted between the two materials, and the effect of this upon the deformation.

4. The initial stresses set up due to the hardening of the concrete and permanent deformations and their effect on the resistance.

It is fortunate that as far as concerns the elongation or contraction due to changes of temperature, we may consider both materials as acting together.

Professor William D. Pence, of Purdue University, U.S.A., made a series of experiments to find the coefficient of expansion of portland cement concrete. The results of these experiments are given in Table XXIII.

TABLE XXIII

Proportions of Concrete	No. of Tests	Maximum	Minimum	Average
1:2:4 Broken stone	7	0·0000057	0·0000052	0·0000055
1:2:4 Shingle	5	0·0000055	0·0000052	0·00000535

The coefficients of expansion of wrought iron may be taken as 0·0000068, and that of steel 0·0000067.

From the above it appears that for a change of temperature of 70° Fahr. (about the maximum in England) the greatest difference of deformation for the same length of broken stone concrete and wrought iron would be 0·000091 of their length, and for shingle concrete and wrought iron the difference would be 0·000098.

Taking the highest of these values, the difference of deformation of the concrete and iron for 70° Fahr. change of temperature would only be 0·117 inches for a length of 100 feet.

According to M. Bouniceau, the expansion of portland cement concrete for 1° centigrade is 0·0000143, and that of iron is 0·0000145. M. Coignet gives for 1° centigrade 0·0000130 to 0·0000148 for the metal and 0·0000135 for concrete.

The Resistance of Concrete

The resistance of concrete is very variable, as it is influenced by many circumstances and conditions, of which the chief are—

1. The proportion of ingredients.
2. The quality of ingredients.
3. The amount of water used.
4. The method and amount of mixing.
5. The amount of consolidation effected.
6. The form of the piece.
7. The atmospheric conditions while the process of hardening is in progress.
8. The time since moulding.
9. The manner of application of the load.

Beyond these conditions, which affect the compressive and tensile strength alike, it has also been observed that the tensile strength is more variable than the compressive strength, which is probably due to the bringing into play of the "adhesion" between the cement and the aggregate.

For important works special tests may be made on the concrete which will be used, but, as a general rule. it will be sufficient to use coefficients derived from the results obtained by the various experimenters who have made a study of the subject.

Resistance to Compression

Mr. Baker, in *Masonry Construction*,[1] gives a table showing that the resistance of concrete to crushing varies very nearly with the amount and quality of the cement used, provided the mortar is not more than is necessary to properly fill the voids in the stone.

TABLE XXIV

RELATIVE TO STRENGTHS OF CONCRETES TESTED 600 DAYS AFTER MOULDING.

Proportions of the Mortar used with the same Stone		Proportion of Cement Relative	Crushing Strength		Relative Strength of Last Column
Cement	Sand		Actual	Derived from Plotting Results and taking the Mean Line	
1	1	1·00	4,467	5,000	1·00
1	2	0·67	3,731	3,300	0·66
1	3	0·50	2,553	2,500	0·50
1	4	0·40	2,015	2,000	0·40
1	5	0·33	1,796	1,600	0·32

TABLE XXV

SHOWING COMPARATIVE STRENGTHS OF MORTARS MIXED IN VARIOUS PROPORTIONS
[Compare Columns 4 and 12.]

Proportions of Mortar						Tensile strength in pounds per square inch at various periods, each figure average of three briquettes			Comparative strengths at 3 months	Comparative strengths at 6 months	Comparative strengths. Average of 3 and 6 months periods
Cement.	Sand.	Percentage of cement to whole volume.	Percentage of cement compared with 4 to 1.	Percentage of water to weight of cement only.	Percentage of water to weight of cement and sand.	1 month	3 months	6 months			
1	2	3	4	5	6	7	8	9	10	11	12
Neat		100	500	20		431	660	723	275	258	267
1	1	50	250	25	12½	350	513	530	209	189	200
1	2	33	165	30	10	323	433	430	180	154	167
1	3	25	125	40	10	265	290	360	121	129	125
1	4	20	100	40	8	236	240	280	100	100	100

Table XXV shows the results of special tests made on cement mortars and indicates the same relation as that shown in Table XXIV.

Table XVIII on page 140 shows the crushing resistance obtained for concrete in various proportions, and Tables XXVIII and XXIX below give the results of other experiments. We may safely conclude that for concrete such as employed for reinforced structures in the proportions of about 510 pounds of cement per cubic yard of sand and shingle, i.e. about 1 : 2 : 4 or 1 : 3 carefully made and well rammed into place, the compressive resistance will be at least 2,150 pounds per square inch one month after moulding, and will attain a strength of at least 2,600 pounds per square inch in three months.

[1] Published by Willey & Sons, New York.

As a rule structures are loaded within a month or six weeks after completion. We may therefore consider that the resistance to crushing will be from 2,150 to 2,200 pounds per square inch.

For richer or poorer concretes the resistances will vary approximately with the relative amounts of cement and sand used, as shown in Tables XXIV and XXV. M. Considère found that insufficiency of ramming reduced the resistances to compression and also to tension in a marked degree. He concludes that the compressive resistance is 2,560 pounds per square inch, but considers it prudent to allow only 2,130 pounds per square inch.

Resistance to Tension

Professor Hatt gives the following table of strengths of 3 to 1, cement mortars. The cement used was slow setting and finely ground, 98 per cent. passing a sieve of 100 meshes per lineal inch; it also passed the boiling water test for constancy of volume.

The sand was clean, sharp pit sand; 84 per cent. was retained on a 30 sieve, and 20 per cent. on a 20 sieve.

TABLE XXVI

Time after Moulding	Strengths in Pounds per Square Inch		Ratio of Compressive to Tensile Strengths
	Compressive	Tensile	
7 days	3,145	412	7·65
28 days	5,860	505	11·60
3 months	8,005	626	12·78

These strengths appear to be somewhat high, but the table is more especially useful in determining the ratios of the compressive and tensile strengths. It is generally admitted that the tensile resistance of concrete is from $\frac{1}{10}$ to $\frac{1}{12}$ that in compression.

The tensile resistances after a month or six weeks will therefore be about from 215 to 220 pounds per square inch, and after three months about 260 pounds per square inch. M. Considère found the tensile resistance of concrete to be 215 pounds per square inch.

Professor Hatt allows as much as 300 pounds per square inch in tension, and Herr Sanders found in some experiments tensile resistances as high as from 310 to 450 after one month and 400 to 510 after three months for concrete in the proportion of 1 cement to 2 of sand and 2 of shingle.

Elasticity of Concrete under Compression

Of all the experiments made for the study of the deformation of concrete under direct compression, those of Professor Bach, of Stuttgard, are perhaps the most valuable and well known. Unfortunately many of the other authorities who have carried out experiments on this subject have taken no account of the permanent sets of the concrete under gradually increasing loads.

The coefficient of elasticity of concrete is not a practically constant quantity like that of iron and steel, but has only an instantaneous value, which varies with the load for the same concrete. Concrete also differs from the structural metals in that it takes permanent sets under very light loads, and if these permanent sets are not deducted from the total deformation under gradually increasing loads, we do not get the true elastic deformation.

To obtain accurate results the load must be applied and removed until there is no permanent deformation each time an addition is made to the pressure, note being taken of the permanent set after each loading, which must be deducted from the total deformation to obtain the true elastic shortening. The elastic behaviour of concrete during successive loadings and unloadings is shown in Fig. 273.

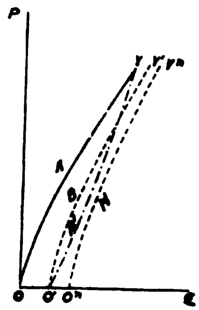


FIG. 273

When a load is gradually applied the shortening of the piece, which is at first small, increases more and more rapidly as the load increases and the curve A plotted with the applied loads as the ordinates and the shortenings as abscissae becomes less and less concave as the pressure increases. On the load being gradually removed, the curve A′ takes a convex form and shows a permanent set O O′ on the horizontal axis. On again applying the same load, the curve of deformation, starting from the new origin, is still concave, but its extremity Y′ is not very much beyond its previous position Y. On unloading again the origin is slightly moved to O″. With several applications and removals of the same load the origin is moved further and further, to the right, but the increases become less and less until there is no permanent set. The permanent set of concrete appears then to be in a great measure taken up by the first loading, and for subsequent applications of the same load it acts more and more nearly as a perfectly elastic material.

Professor Bach's experiments were conducted on cylinders 9·8 inches diameter and 39·4 inches high, of various proportions of ingredients, and at from three to four months after moulding. The deformations were measured on a length of 29·5 inches. The load (114 pounds per square inch) was applied and removed again and again until there was no further permanent set, and the total contraction remained constant for three applications of the load.

The experiment was then repeated in the same manner with double the former load, and continued; increasing the load by 114 pounds per square inch, each time applying and removing the pressure as before. The permanent set, elastic deformation and total contraction were thus determined for the various loads.

The results were plotted and the curves obtained appeared to be divided into three parts: the first portion was a concave curve, the second and longest portion was nearly straight, and the third had a sharp concave curve to the point of rupture. These three curves had no abrupt changes, but each portion ran gradually into the following. The curve of deformation thus found agrees very closely with that obtained by MM. Souleyre and Anglade, of the Ponts et Chaussées, from some experiments made at Constantine, excepting that from their experiments the junctions between the three portions of the curve are more marked.

The curve obtained by MM. Souleyre and Anglade is shown in Fig. 274.

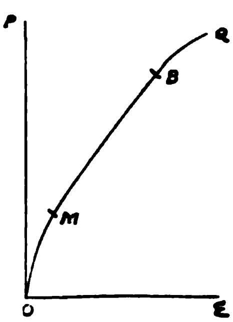


FIG. 274

It was noticed that the three portions of the curve followed closely the swelling of the test pieces under the load.

Professor Bach deduced the following equation from his experiments—

$$\lambda = \frac{1}{E_p} p^n \qquad (1),$$

which is the equation for the compressive deformation curve of concrete for the loads usually allowed in reinforced concrete construction;

λ being the elastic deformation per unit length,

p the pressure per unit surface,

And E_p and n coefficients which depend on the quality of the concrete.

It will be observed that this equation is very different for that employed for structural metals based on Hooke's law, where $\lambda = \frac{p}{E}$ (2), where E is constant.

An equation of the same kind can, however, be employed in the form of

$$\lambda = \frac{c}{E_c} \qquad (3),$$

Where c is the pressure per unit surface and is the same as p and

$$E_c = \frac{E_p}{p^{(n-1)}} \qquad (4).$$

Equation (4) will give the values of E_c or the modulus of elasticity of the concrete in compression, on which may be based the coefficients to be employed in the calculations.

Table XXVII. gives the results obtained by Professor Bach from his experiments, and the deduced values for E_c. The figures in the last series of columns must be multiplied by 10^6 to obtain the values for E_c.

TABLE XXVII

Proportions of Ingredients in Test Pieces					Value given by Professor Bach for Formula $\lambda = \frac{1}{E_p} p^n$		Values of $c = p$ in Pounds per Square Inch									
							114	228	342	456	570	684	798	912	1026	1140
Cement	Danube Sand	Egginge Sand	Broken Stone	Danube Shingle	n	E_p Pounds per Square Inch	Corresponding values of $\frac{E_c}{10^6}$ in Pounds per Square Inch from Formula $E_c = \frac{E_p}{p^{(n-1)}}$									
1					1·09	$4·63 \times 10^6$	2·94	2·77	2·67	2·60	2·53	2·50	2·47	2·44	2·42	2·39
1	1·5				1·11	$6·79 \times 10^6$	4·02	3·73	3·57	3·45	3·36	3·31	3·26	3·20	3·16	3·13
1	3				1·15	$6·69 \times 10^6$	3·28	2·96	2·79	2·66	2·56	2·50	2·44	2·40	2·36	2·32
1	4·5				1·17	$5·13 \times 10^6$	2·29	2·03	1·89	1·80	1·72	1·68	1·63	1·61	1·58	1·55
1	2·5			5	1·14	$6·16 \times 10^6$	3·17	2·87	2·71	2·62	2·52	2·46	2·42	2·37	2·33	2·30
1		2·5	5		1·16	$9·96 \times 10^6$	4·66	4·17	3·91	3·74	3·59	3·49	3·41	3·35	3·28	3·23
1	5			6	1·14	$5·79 \times 10^6$	2·97	2·70	2·54	2·45	2·36	2·32	2·27	2·23	2·19	2·20
1	3		6		1·16	$8·28 \times 10^6$	3·87	3·47	3·26	3·11	2·99	2·91	2·83	2·77	2·73	2·68
1	5			10	1·16	$4·73 \times 10^6$	2·22	1·98	1·85	1·78	1·70	1·66	1·62	1·58	1·57	1·56
1		5	10		1·20	$8·90 \times 10^6$	3·44	3·00	2·76	2·62	2·49	2·40	2·33	2·27	2·22	2·20

For the same quality of ingredients, the coefficient of elasticity increases with the proportion of sand, attaining its maximum value when the mortar is in the proportion of 1 of cement to 1½ of sand; it then diminishes as the proportion of sand increases, until it again reaches its value for neat cement with the proportions of 1 of cement to 3 of sand.

The coefficient n of Bach's equation increases generally as the proportion of aggregates increases, being 1·09 for neat cement, 1·11 to 1·17 for mortars, and from 1·14 to 1·20 for concrete.

The curves (Fig. 275) have been plotted for the values of E_c for mortars and concretes of the proportions usually adopted in structures of reinforced concrete.

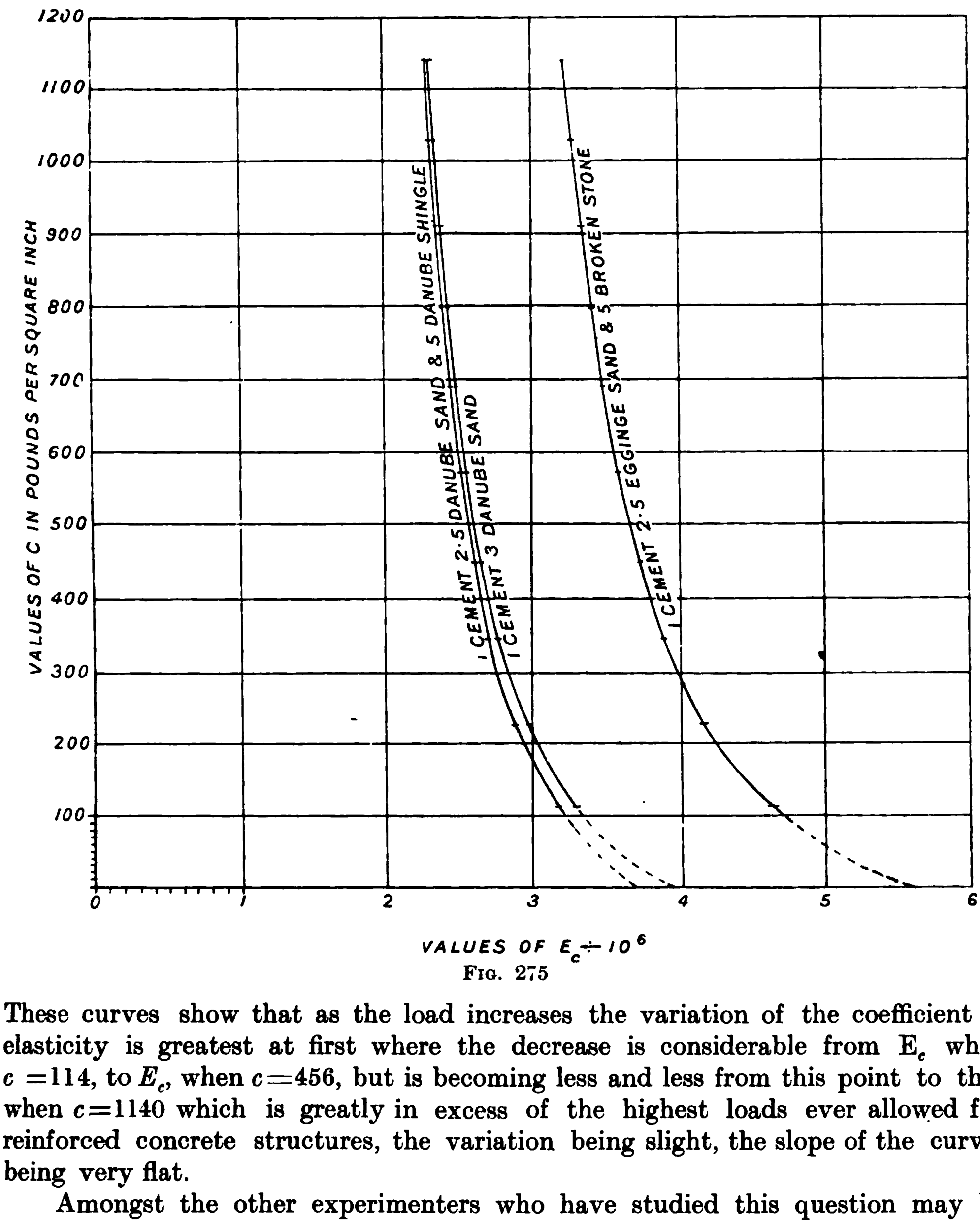


FIG. 275

These curves show that as the load increases the variation of the coefficient of elasticity is greatest at first where the decrease is considerable from E_c when $c = 114$, to E_c, when $c = 456$, but is becoming less and less from this point to that when $c = 1140$ which is greatly in excess of the highest loads ever allowed for reinforced concrete structures, the variation being slight, the slope of the curves being very flat.

Amongst the other experimenters who have studied this question may be mentioned M. Considère, MM. Coignet and de Tedesco, M. Durand Claye and Professor Hatt, of Purdue University, U.S.A. A series of experiments have also been carried out at the Watertown Arsenal, U.S.A. Unfortunately, however, most of these authorities have neglected the deduction of the permanent set for each load, and consequently the results obtained are very diverse.

M. Considère found that for a concrete in proportions of 510 pounds of cement half a cubic yard of sand and half a cubic yard of fine shingle the coefficient of elasticity varies between $2{\cdot}84 \times 10^6$ and $3{\cdot}70 \times 10^6$, but thinks it prudent to allow $2{\cdot}70 \times 10^6$.

The Watertown Arsenal tests were conducted on concrete made under service conditions with coarse sand and broken stone with the least possible amount of water. The curve obtained for the stress strain diagram was a continuous curve, and the value of E_c depending on the portion considered. The load was released at regular intervals and the total set observed, the value of E_c being obtained by dividing the stress by the shortening obtained, by deducting the previous set from the total contraction due to the stress.

Professor Hatt, in a paper read before the Indiana Engineering Society,[1] gave the following table showing the results of these tests.

TABLE XXVIII

Proportions of the Concrete			Age	E_c	Stress where Measured Pounds per Square Inch	Crushing Stress Pounds per Square Inch
Cement	Sand	Stone				
1	2	4	9 days	$1{\cdot}66 \times 10^6$	1,000	1,944
1	2	4	3 months	$3{\cdot}46 \times 10^6$	1,000	2,200
1	2	4	6 months	$4{\cdot}50 \times 10^6$	1,000	3,500
1	3	6	9 days	$1{\cdot}95 \times 10^6$	1,000	2,308
1	3	6	3 months	$3{\cdot}75 \times 10^6$	1,000	2,500
1	3	6	6 months	$2{\cdot}81 \times 10^6$	1,000	3,500

These values do not agree well with those of Professor Bach, the discordance being due to the Watertown Arsenal tests making no account of the permanent set.

In a later paper, read before the American Section of the International Association for Testing Materials, Professor Hatt gives the following as the results obtained from a series of experiments on broken stone, gravel, and cinder concrete, showing that the values of E_c for gravel and cinder concrete are less than those for broken stone concrete.

TABLE XXIX

Proportions of the Concrete					Age. Days	E_c	Stress where Measured Pounds per Square Inch	Crushing Stress Pounds per Square Inch
Cement	Sand	Broken Stone	Gravel	Cinders				
1	2	4			9	$4{\cdot}70 \times 10^6$	750	2,880
1	2	4			9	$3{\cdot}94 \times 10^6$	1,500	
1	2	4			14	$4{\cdot}34 \times 10^6$	750	2·575
1	2	4			14	$3{\cdot}68 \times 10^6$	1,500	
1	2			4	9	$5{\cdot}58 \times 10^5$		495
1	2			4	9	$5{\cdot}53 \times 10^5$		595
1	2			4	7	$6{\cdot}30 \times 10^5$		416
1			5		6	$2{\cdot}09 \times 10^6$		1,185

[1] An extract of which appeared in the *Engineering Record*, May 18, 1902.

The test pieces were cylindrical, 8 inches diameter and 12 inches high. The materials for the mortar were first mixed dry and then again with water, after which the stones or cinders were added, and the whole thoroughly mixed, the consistency being fairly dry. For the gravel concrete the sand was not screened out.

Ratio between the Co-efficients of Elasticity of Concrete in Compression and those of Iron and Steel

This ratio of $\frac{E_l}{E_c}=m$ is the factor which is employed in the calculations of reinforced concrete structures, and fortunately its value has little effect on the calculations within fairly wide limits. M. Christophe shows that in a piece under direct compression with 1 per cent. of reinforcement, a variation of 50 per cent. in the value of m only alters the results by $3\frac{1}{2}$ per cent., and increasing the percentage of reinforcement to 5 per cent., the results are still only altered by 13 per cent. MM. Boussiron and Garric have also demonstrated that with variations in the value of $\frac{E_l}{E_c}$, 50 per cent. above and below 10, the moments of resistance for a slab 4 inches thick with 1 per cent. of reinforcement along the bottom only vary 16 per cent. and 12 per cent. respectively.

The coefficients of elasticity of iron and steel are the same in compression and tension, and have practically constant values if the limit of elasticity is not exceeded.

The value of E_l for wrought iron may be taken as $28{\cdot}4 \times 10^6$, and that of steel as $31{\cdot}3 \times 10^6$ or 10 per cent. greater than for wrought iron.

From Professor Bach's experiments and the deductions therefrom the value $\frac{E_l}{E_c}=m$ for wrought iron and the mortars and concretes of diagram (Fig. 275) varies between 7·7 and 5 when $c=1$, until at the pressure of 1,000 pounds per square inch allowed for reinforced concrete its least value will be from 12·0 to 8·6.

The Watertown Arsenal tests give the value of m at three months as about 8, and at nine days about 16. M. Considère's conclusions give the value of m as from 7·7 to 10·5. When steel is used as a reinforcement the values from Professor Bach's experiments will be from 8·5 to 5·6 when $c=1$, to 13·3 to 9·5 when c attains the pressure of 1,000 pounds per square inch.

We may safely assume the ratio m to be 10. The value of c at which E_c is taken being the maximum allowed value for each case considered.

If for any work it should be desired to obtain a more accurate value, this can be done by making a series of experiments on the concrete that will be used, but the effect on the calculations of a slight error in the value of m will hardly warrant the trouble.

Elasticity of Concrete under Tension

The elastic behaviour of concrete under tensile stresses is more variable than that under compression. Experiments on this quality have been made by M. Considère, MM. Souleyre and Anglade, Herr Hartig, Herren Grut and Neilsen, Professor Hatt, and others.

The conclusions of the various experimenters vary very considerably however, some holding that the curve of elastic deformation under tensile stresses is very similar to that for compression, and that Professor Bach's equation may be used with different values for E_p and n; others hold that the variation of the coefficient of elasticity in tension may be neglected by reason of its comparative smallness. and that there is no point which can be taken as the limiting stress; while yet others agree that for small stresses the coefficient of elasticity is practically invariable, but hold that as the stress becomes greater there is a period of great increments of elongation, the slope of the elastic curve becoming very flat. It is extremely probable that since the tensile strength is very variable, the deformation will be variable also.

M. Christophe points out that M. Hartig's experiments tend to show that the coefficients of elasticity in compression and tension are practically the same for small loads, and that for a short distance on each side of the neutral axis of a beam of concrete the coefficients have nearly the same value. This state of affairs does not appear to proceed very far, the curves of deformation under the same gradually increasing stresses in compression and tension will consequently commence with a uniform slope, but gradually vary from one another. This is in all probability the true state of the case, and many persons hold this view.

M. Considère in his tests on reinforced prisms under direct tension, described before the Congress of 1900, found a very similar behaviour to that already described for the elastic deformation of concrete under direct compression. Taking the elongations for the abscissae and the stresses for the ordinates, the latter increased rapidly during a period corresponding to an extension of 0·1 thousands, after which they increased very slowly up to an elongation which amounts in some cases to 2·00 thousands.

In gradually taking off the load, after extending the test up to B (Fig. 276), the curve $B\ C$ of the deformation approaches a straight line, except at the two ends, the permanent deformation being $O\ C$. In reloading until E and again taking off the load the curves $C\ D\ E$ and $E\ F\ G$ are obtained, $E\ F\ G$ like $B\ C$, is nearly straight and inclined at a less angle the further the deformation is pushed.

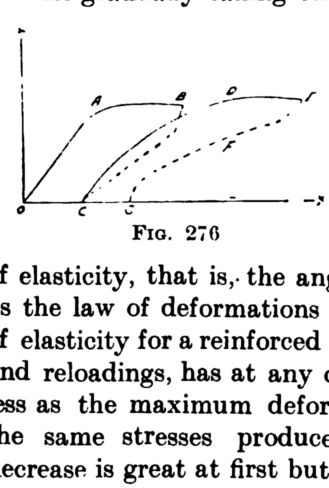


FIG. 276

The true value of the instantaneous coefficient of elasticity, that is, the angle of the tangent to the curve at any point, will give us the law of deformations of reinforced concrete. The instantaneous coefficient of elasticity for a reinforced piece having been submitted to successive unloadings and reloadings, has at any one of the operations a nearly constant value, which is less as the maximum deformation has been extended further. The repetition of the same stresses produces a decrease of the coefficient of elasticity, which decrease is great at first but tends to become nil.

In submitting the piece after the first testing to a series of loadings and unloadings without exceeding the maximum reached during the first test, the concrete behaves like a new material which possesses a coefficient of elasticity less than that of untested concrete, but hereafter invariable, though there is a slight tendency to diminution when the stresses are very frequently repeated.

M. Considère found that the amount of water used in mixing the concrete had a considerable influence on the coefficient of elasticity of the concrete which, with an excess of water, may drop considerably, the resistance decreasing at the same time but in a less degree.

The insufficiency of ramming appeared to reduce the coefficient of elasticity and the resistance in an equal measure. Herren Grut and Neilsen[1] found the coefficient of elasticity of 3 to 1 mortar 8 weeks old to be $3{\cdot}67 \times 106$.

M. de Joly,[2] in some experiments carried out for Le Service français des phares et balises, found that for mortars of 1,000 pounds of cement per cubic yard of sand, and for concretes of 840 pounds of cement to ½ a cubic yard of sand, and the same quantity of pea shingle, the coefficient of elasticity in tension was practically constant for specimens 1 month old, being $3{\cdot}00 \times 10^6$, and that it increases somewhat with the age.

The following table shows the results of Professor Hatt's experiments on 1 : 2 : 4 broken stone concrete.

TABLE XXX

Age. Days	E_t	Elongation at Rupture, 1 part in	Strength in Pounds per Square Inch
35	$2{\cdot}7 \times 10^6$	11,660	300
33	$2{\cdot}4 \times 10^6$	8,750	305
28	$1{\cdot}4 \times 10^6$	4,400	360
26	$1{\cdot}9 \times 10^6$	7,700	280
Average	$2{\cdot}1 \times 10^6$	7,000	311

Limiting Stress of Concrete in Compression

It is very doubtful whether concrete has a limiting stress of the same nature as the elastic limit of the structural metals. Professor Bach's curve of deformation shows no break to which it is possible to point as the limit of elasticity, if such a term may be used for a material of the nature of concrete. MM. Souleyre and Anglade's curve, on the contrary, clearly shows such a critical point at the break when the straight portion terminates and the curve B Q (Fig. 274) begins. This point is at a little more than ½ the ultimate resistance. The critical limit of concrete doubtless varies considerably according to the quality.

M. Christophe, however, points out that from experiments which have been made, there appears to be a limit below which the same load may be repeatedly applied and removed without increasing the total deformation, and he considers this limit of permanent resistance to be as a general rule about two-thirds of the breaking load in compression.

We have concluded (p. 212), that the crushing resistance for such concretes as are usually employed will be from 2,150 to 2,200 pounds per square inch within a month or six weeks after moulding, and that it will rise to 2,600 pounds per square inch in a period of three months. The safe limit may consequently be taken as ⅔ of these stresses. We may therefore assume that the safe limit of resistance in the first case will be from 1,430 to 1,470 pounds per square inch, and that for the second case 1,735 pounds per square inch.

Safe Compressive Stresses to be allowed in Calculations

To obtain the value to be given to the safe compressive stresses under various conditions, we may form some conclusions from the various experiments and the behaviour of existing structures.

The safe stress *under direct compression* is less than that under flexure, it being always found that the resistance to the direct stresses in plain concrete beams is

[1] *Ingeniøren*, Copenhagen, March 7th, 1896.

[2] *Annales des Ponts et Chaussées*, 7th Série, Tome 16, 1898.

much greater than that in direct compression, which result is also obtained in all beams of whatever material.

A very usual allowance in empirical formulae for the concrete under direct compression is 360 pounds per square inch or thereabouts, but these formulae take no account of the elasticity of the materials, and the real stresses may range from this figure to as much as from 700 to 1,000 pounds per square inch.

The official allowance in Prussia for bridges of concrete in the proportions of 1 : 2½ : 5 to 1 : 3 : 6, is from 285 to 500 pounds per square inch ; and M. Christophe mentions the case of a large span arch in Würtemburg with hinges at the key and springings where the pressure reaches 525 pounds per square inch.

In the United States, 500 pounds per square inch compressive stress is frequently allowed for concrete bridges.

In concrete, reinforced as is usually the case with longitudinal rods and cross ties, the strength of the concrete is in some measure increased by the resistance against swelling which is exerted by the cross ties.

If we take the safe limit as 1,450 pounds per square inch, which is the mean value after a month to six weeks from moulding as given in the last paragraph, and if we further allow a factor of safety of 3·5, which is ample on the safe limiting stress, we find that the safe compressive resistance becomes 415 pounds per square inch. The values obtained from experiments on large pieces are considerably higher than this. the values obtained from the experiments by M. Gary and the Austrian Commission on Arches, which are detailed later (p. 239), being as high as about 2,510 to 3,490 pounds per square inch at failure.

Taking a safe limit as ⅔ of the final resistance and a factor of safety of 3.5 as before, we get as safe stresses 504 and 665 pounds per square inch respectively.

M. Christophe allows two values in his calculations, one for steady loads and one for cases where the pieces are subjected to shocks, vibrations, or flexure, or where the loading is uncertain.

In the first case he allows 590 and in the second 360 pounds per square inch. In special cases where there is a certainty of a steady load and the structure is unimportant, it is safe to allow the higher limit, but these cases are few and can be dealt with specially by making such allowance.

For general purposes it seems that we may with perfect safety allow 400 *pounds per square inch for pieces under direct compression, reinforced with longitudinal rods and cross ties in the usual way.* The special treatment of "hooped concrete," where much higher limits may be allowed, will be dealt with separately.

The safe allowance for the concrete in compression *in pieces subjected to bending.* such as beams and slabs, will be considerably more than the above.

In structures designed by empirical formulae allowing from 360 to 425 pounds per square inch, the real compressive stresses, making allowance for the elasticity of the materials, will amount to as much as 800 pounds per square inch, and sometimes more than this, and in beams with a double reinforcement where the sectional area of both reinforcements are calculated as for ordinary iron beams, and the resistance of the concrete neglected, if the top reinforcement is ever stressed to an amount approximating the allowed stress, the concrete will be subjected to a stress of 1,000 pounds per square inch or more. It is generally found in tests to failure that the first signs of rupture occur either at the centre of the span by the stretching of the reinforcement, or near the supports under shearing, and that in a properly proportioned piece the flaking on the compression side does not occur until the final rupture.

It appears, then, that a properly designed and constructed piece of reinforced concrete subjected to bending, if it is so reinforced that it offers the necessary resistance to the shearing stresses, will not fail until the limit of elasticity of the reinforcement in tension has been exceeded, when the concrete in compression must be under considerable stress.

M. Considère adopts 2,130 pounds per square inch as the ultimate compressive stress of concrete, and allows a factor of safety of 2·5 for ordinary cases and 2·0 when a considerable period has elapsed before bringing the piece into use. He therefore obtains 910 and 1,065 pounds per square inch respectively for the safe stresses in his formula.

M. Christophe adopts for his maximum and minimum compressive resistance of the concrete, for pieces subjected to bending, values 20 per cent. higher than those allowed in direct compression members, and therefore obtains 710 and 430 pounds per square inch respectively for the two limits for pieces under a steady load or light moving load where the structure is not important and for pieces subjected to vibrations and shocks.

Taking everything into consideration, it appears that we may safely adopt a value of 500 *pounds per square inch for the resistance of concrete to compression in pieces subjected to bending, for general purposes.* The allowance for special cases must of necessity be left to the judgment of the designer.

It must be remembered that these safe resistances only apply to concretes in the usual proportions of about 1 : 2 : 4, or thereabouts, of which—

1. The portland cement is of good quality, sound and slow setting.
2. The aggregate composed of clean, sharp sand or crushed stone, if of undoubted quality, and shingle or broken stone which has passed screens of the proper size (usually from ½ to ¾ inch).
3. The concrete is properly and carefully mixed with a moderate amount of clean water, care being taken that it is not too dry or too wet.
4. The concrete is well rammed into position in thin layers.
5. The falsework is rigid and firm, and that the necessary work does not cause any vibration.
6. All the work is thoroughly supervised and carefully done.
7. The structure is not loaded until at least a month after completion.

It cannot be too much insisted on that if reinforced structures are to be designed on theoretical lines, the greatest care must be taken in the construction, and only thoroughly reliable men employed. The working stresses allow a large margin of safety, but if the work is carelessly carried out or scamped in any way, good results cannot be expected and will certainly not be obtained.

The blame in such a case must rest entirely on the manner of carrying out of the work, and not on the material or the design of the structure.

Splendid results have been obtained from reinforced concrete structures scientifically designed, but it is useless to design an economical structure if the materials employed for the construction are bad, or the work carelessly carried out. If the work cannot be well done it is better to construct in some material which can be properly treated, since the strength of reinforced concrete, perhaps more than any other structural material, depends upon its proper manipulation.

For weaker or stronger concretes the safe resistance may be varied according to Table XXIV, page 211.

For concretes of coke, breeze, or furnace ashes the resistance may be taken as about ⅓ of that for broken stone or shingle, and may be even less.

In specially important cases tests should be made on the concrete that will be used in the work.

Limiting Stress of Concrete in Tension

The resistance of concrete in tension is usually altogether neglected at the present day in making the calculations for reinforced concrete structures. It is, however, certain that the concrete offers some resistance to tensile stresses, and many authorities consider that this resistance should not be neglected. M. Considère's experiments on reinforced concrete pieces seem to indicate that the tensile resistance of the concrete is by no means negligible, and that its resistance is retained at its maximum for comparatively great deformations, and much discussion has arisen on this subject. Herr Sanders, of the Amsterdamsche Fabrieken van Cementizer Werken takes the tensile resistance of the concrete into consideration in his calculations for beams, etc., having made a series of experiments which show that some such resistance must exist. Professor Hatt in America strongly advocates that allowance should be made for this resistance, and other authors take cognizance of it in their formulae.

It is perhaps best, until the further experiments have been carried out on this property, to neglect the tensile resistance of the concrete, as the error will not be great and is on the side of safety, and also it can offer no resistance where cracked.

As has been pointed out before, we are still in a state of uncertainty as to all the valuable properties of the rational combination of concrete and iron or steel, and although it will probably be found that the composite material when properly combined has more resistance than we are at present aware of, yet, until we have a more definite knowledge of its real characteristics, it is well to remain on the side of safety. We have taken the tensile resistance of concrete as being from 215 to 220 pounds per square inch a month or six weeks from the time of moulding and 260 pounds per square inch after a period of three months (p. 212). Allowing the same factor of $\frac{2}{3}$,[1] the safe limits will be from 144 to 150 and 175 pounds per square inch respectively. M. Considère allows 170 pounds per square inch, and other authors from 140 upwards.

Resistance of Concrete to Shearing

The resistance of concrete to shearing has been studied by M. Feret, Director of the Laboratory of the Ponts et Chaussées at Boulogne, Herr Bauschinger and M. Mesager, Director of the Laboratory of the School of the Ponts et Chaussées. The figures which have been obtained vary considerably, in consequence of the difficulty of obtaining rupture by shearing alone.

M. Feret concludes that the ultimate shearing resistance is proportional to that for compression, and obtains the relation that the shearing resistance is from 0·16 to 0·20 of the compressive strength ; this would give us, taking 2,175 pounds per square inch, (the mean compressive resistance at from four to six weeks) a shearing strength of from 350 to 435 pounds per square inch, and at a period of three months from 415 to 520 pounds per square inch.

In a paper presented at the 1901 Budapest Congress, M. Considère gives the value of the resistance of concrete to shearing deduced from M. Mesnager's[2] experiments as from 20 to 30 per cent. higher than the tensile resistance; this gives, taking the values (on page 212) from 260 to 285 pounds per square inch as the mean at a period

[1] M. de Joly, from a series of experiments conducted for Le Service français des phares et balises, believes the elastic limit of concrete in tension is not less than $\frac{2}{3}$ the ultimate resistance.

[2] Directeur de l'Ecole des Ponts et Chaussées, Paris.

from four to six weeks, and 310 to 340 pounds per square inch at three months, which are considerably below those found by M. Feret. Many authors assume that the resistance of concrete to shearing is less than its resistance to tension, and consequently give it a much lower value, but this assumption appears to be erroneous. M. Considère quotes an experiment in which a hollow iron cylinder 7½ inches diameter was filled with neat cement paste, and placed in a rock to be acted on by the force of the waves, this force bent the cylinder to a radius of about 21½ inches; it was then sawn in two on the plane of the bent axis; the cement showed only a few sliding surfaces, between which uninjured pieces were found showing a deformation by sliding of the fibres over one another of at least 1 in 5. He further remarks that it is well known that the stretching of reinforced concrete without breaking far exceeds what would be supposed, showing that concrete will bear considerable shearing stresses when it is reinforced.

From tests it is shown that the shearing, or relative displacement of points in the reinforcement and in the surrounding concrete separated by the small distance of 0·2 to 0·3 inches is extremely small, so long as the stresses do not exceed certain values which correspond with the elastic limit of the reinforcement. After this limit is passed they will increase rapidly, and an increase of load from $\frac{1}{10}$ to $\frac{1}{5}$ at most, the displacement has the relatively great value of $\frac{1}{100}$ to $\frac{3}{100}$ as shown by diagram (Fig. 277).

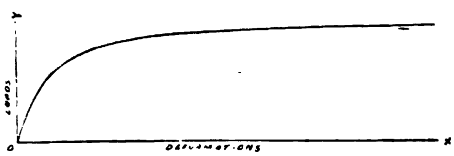


FIG. 277

M. Considère points out further that there is a great resemblance between this curve and that for tension, but the values for shearing are far greater than those for the tensile deformation curve, probably 10 to 15 times as much.

The stresses per square unit of contact surface which tend to cause the relative movement of a point on the reinforcement, the displacements of which were taken as the abscissa in Fig. 277, are not at all proportional to the loads imposed which are represented by the ordinates. On the other hand, the stresses per unit surface vary as we consider the rings of concrete further and further away from the reinforcement, the measured shearing deformation being thus a complex resultant of several slidings which occur in the fibres submitted to the varying stresses.

Safe Shearing Resistance of Concrete

In calculating the resistance to shearing stresses in a structure of reinforced concrete, most constructors do not take any account of the resistance of the concrete itself, but in many instances the pieces have no special reinforcement for the resistance of these stresses, and it is well to inquire into what resistance the concrete will offer.

M. Christophe allows 35 and 21 pounds per square inch as the safe maximum and minimum resistances of concrete to shearing, but he assumes that the shearing is less than the tensile resistance, and that the ultimate resistance to shearing is 128 pounds per square inch; whereas both M. Feret and M. Considère show it to be greater, being as much as from 350 to 435 pounds per square inch, according to M. Feret.

It appears, then, that we may safely assume a *shearing resistance for the concrete of ⅛ the direct compressive resistance, or* 50 *pounds per square inch.*

Adherence or Frictional Resistance to Sliding of the Reinforcement

The property generally referred to as "adherence" of the concrete to the metal is probably only slightly due to any direct adherence, and would perhaps be better defined as a frictional resistance due to the setting of the concrete, the outer portions of which harden first, causing the concrete around the reinforcement to become compressed, and so clasp the reinforcement tightly.

There is very little real adherence to clean iron, as can be easily proved by the simple experiment of moulding some concrete and placing a piece of square iron on the top, lightly pressing it into the concrete with the fingers. When the concrete has set it will be found that there is very little difficulty in removing the iron. If the piece of square iron is pressed well into the concrete, there will be considerable difficulty in its removal, but it will be found that this is caused by the holding of the sides by the concrete, and that the bottom comes away quite easily and is perfectly clean, the concrete being left with a smooth bottom surface. Also, when a reinforced structure is being demolished the iron comes away fairly easily when the concrete is broken away from it on three sides.

To whatever cause this property is due, however, it is one which is of the greatest importance in works of reinforced concrete, the resistance obtained in many of the systems now in use depend on it almost entirely.

The latest tests that have been carried out for determining this resistance show that the load which produces the first loosening of the reinforcement is not proportional to the surface of adherence, and it seems very probable that the metal does not commence to slide through the concrete until its limit of elasticity is passed and it commences to become reduced in diameter.

Herr Bauschinger and M. de Joly from a series of experiments concluded that the "adherence" of concrete to iron or steel rods was from 570 to 710 pounds per square inch of surface. These results are perhaps those most frequently quoted by the various authors. They show the "adherence" as being due to the contact surface, and sufficiently great to allow the fear of a failure in this respect being neglected when making the calculations.

MM. Coignet and de Tedesco made experiments on round rods of 0·63, 0·79 and 1·30 inches diameter embedded 3·94, 7·87 and 11·81 inches in blocks of concrete mixed in the proportions of 920 pounds of cement per cubic yard of sand. The metal was allowed to project about three-quarters of an inch above the block. which had a hole in its base larger than the size of the rod, so as to allow of its being pushed down. The tests were made only six days after moulding, so it may be safely concluded that the results are low. They obtained values of from 285 to 355 pounds per square inch of surface, and from the conditions of the experiments they are of the opinion that the higher value may be taken as a minimum under practical conditions.

Some later experiments conducted by M. de Joly for Le Service français des phares et balises[1] tend to show that the sliding of the reinforcements is due to their contraction when the limit of elasticity is passed, and they are of the opinion that the real failure is probably due to a shearing of the concrete itself.

The metal used for this series of experiments was in the form of round iron rods of two qualities, and of from 1·18 and 1·42 inches in diameter. These were placed to a depth of 23·6 inches in holes made in stone blocks and grouted in with neat portland cement paste. The tests were made after one month, during which time the blocks remained in air.

[1] *Annales des Ponts et Chaussées*, 7th Série, Tome 16, 1898.

The force required to draw out the rods when referred to the unit of surface was very variable, and appeared to vary with the diameter of the rods, being respectively 542, 586 and 686 pounds per square inch of surface in one series of experiments and 286, 442 and 503 in another. On, however, referring the force required to produce the first movement to the sectional area of the rods, the load per unit was found to be sensibly constant for the same quality of metal and to correspond closely with the elastic limit, being 34,130 and 46,650 pounds per square inch, for metals having elastic limits of 34,130 and 45,500 respectively.

M. de Joly further states that generally a layer of cement remained on the rods after being drawn out, pointing to the probability of the failure being due to the shearing of the concrete itself. This points to the necessity of inquiry into the tendency to shearing of the concrete around the reinforcements. That a failure is more likely to occur from the shearing of the concrete than the sliding of the reinforcement is a fact which is daily becoming more recognised.

M. Feret's experiments are most interesting, as showing the conditions which affect the "adherence" of the metal and concrete. He concludes that the form of section of the reinforcement has a great influence on the "adherence" by reason of the contraction and expansion of the concrete while setting, which causes an initial tendency to disunion, which varies with the form of reinforcing section and its position in the concrete. M. Feret's experiments were conducted on rods of the ordinary types used in practice, 0·79 inches in diameter and 3·94 inches long, embedded 2¾ inches in cubes of concrete. He found that—

1. The "adhesive" resistance varied at different points along the length of the rods.
2. It was greater with a rough than with a smooth surface.
3. It increased with the proportion of cement up to a certain limit.
4. It varied very slightly with the quality of slow-setting portland cement.
5. It increased somewhat with the fineness of grinding.
6. It was less with quick setting than with slow-setting portland cement.
7. It was very variable for slag cements.
8. It increased for mortars with the size of the sand grains up to a certain point.
9. It was less for concretes than for mortars.
10. It increased rapidly with the proportion of water used in mixing, and reached its maximum for a somewhat sloppy concrete the consistency of which was wetter than that which gives the maximum resistance to compressed and tensile stresses.
11. It increased with the age of the concrete.

Professor Hatt made tests to determine the "adhesive" resistance of concrete and iron. The values he obtained are given in Table XXXI, and are those for the ultimate resistance with reference to the surface of the rods nominally in contact with the concrete. The tests were made by drawing out the rods.

TABLE XXXI

Diameter of Rod in Inches	Age of Specimen in Days	Depth of Rod in Concrete Inches	"Adhesion" in Pounds per Square Inch of Surface in Contact		
			Maximum	Minimum	Average
$\frac{7}{16}$	32	72	735	470	636
$\frac{5}{8}$	35	76	780	714	756

Professor Hatt remarks that the sliding friction after the first movement was from 60 to 70 per cent. of the "adhesion." On breaking open the concrete it was found that the contact between the mortar and iron was not universal, and was irregularly distributed over the surface of the rods.

M. Considère has also carried out some experiments on this property of reinforced concrete. He found the "adherence" to be 256 pounds per square inch of surface for rolled iron rods of 0·24 inches diameter, the surfaces of which were the same as that of the rods used in practice, and the proportions of the concrete 510 pounds of cement per half a cubic yard of sand and half a cubic yard of fine shingle, the specimens being stored in air. With concrete of 720 pounds of cement to the same quantity of sand and shingle and with reinforcements of wire 0·17 inches diameter, slightly rusted, the specimen being stored in water, the slipping resistance varied from 327 to 500 pounds per square inch of surface.

M. Considère found that for concrete exposed to the air the amount of water used in mixing had a great influence, too dry concrete "adhering" badly. An excess of water giving the concrete the necessary fluidity for filling up the voids around the reinforcements produced the best results.

M. Considère, however, considers that this advantage of wet concrete is counterbalanced by a notable diminution of tensile and compressive resistance. He points out that the above resistances are rather less than those given by the experiments of Herr Bauschinger and M. de Joly (570 to 710 pounds per square inch of surface), but that the reinforcements of pieces under bending are surrounded by concrete which has to stand tensile stresses generally beyond the elastic limit, whereas the experiments of Herr Bauschinger and M. de Joly were conducted on rods sealed in concrete blocks, in which the stresses besides those due to the slipping of the reinforcements were comparatively unimportant.

M. Considère further states that it is quite natural that the resistance against slipping and shearing stresses should be more in bars tested for these stresses alone than in the parts of reinforced prisms of which the concrete exerts a part of its cohesive resistance in the elongation beyond its elastic limit.

From the results of the experiments detailed above it may be concluded that the resistance against sliding of the reinforcements are least for a metal with a polished or slightly greasy surface, and greatest for one slightly rusted. It has been found that the protection of the metal is better effected when the original surface is slightly rusted (p. 11). It appears, therefore, that taking all considerations into account, it is well to use bars which are slightly rusted since in this case true adhesion comes into play and the metal is more efficiently protected.

Many experimenters have taken great trouble to obtain a smooth polished surface for the metal, and some authors advocate a thorough cleaning of the reinforcements before embedding them in the concrete. This procedure is entirely the reverse to the best practice, and also will add considerable and unnecessary expense to the work. With a metal slightly rusted there need be no fear of failure due to want of sliding resistance until the reinforcement becomes strained beyond its elastic limit, a state of deformation which should of course never be approached in practice.

With special reinforcements such as the Habrick, Ransome, Thacher and Johnson bars any sliding through the concrete is absolutely prevented, but they offer no resistance to the shearing of the concrete around the reinforcement.

EXPERIMENTAL RESEARCH AND DEDUCED DATA

Safe Resistances to Compression, Tension and Shear of the Metal Employed as Reinforcement

1. **Compression.**—As will be seen later when discussing the calculations, the compressive resistances are governed by the allowed resistance of the concrete under direct compression; and the allowances for the resistance of the reinforcement in tension and the concrete in compression when bending is under consideration.

For pieces under direct compression, therefore, it is economically advisable, as a general rule, to use a rich concrete, since taking m or $\frac{E_f}{E_c}$ as equal to 10, the direct compressive resistance of the metal will be 10 times that allowed for the concrete.

2. **Tension.**—The resistance of the metal in tension, in pieces subjected to direct tension or bending, has a great effect on the stability of the piece, as, if too high stresses are allowed, there is danger of the concrete cracking under tensile strain. The reinforcement will also commence to slide through the concrete when its limit of elasticity is passed. It is therefore advisable to keep the working stress of the tensile reinforcement well below its elastic limit. The usual working stresses allowed are from 10,000 to 15,000 pounds per square inch for wrought iron, and from 15,000 to 22,000 pounds per square inch for steel.

M. Christophe allows 9,000 and 13,500 pounds per square inch as the minimum and maximum for wrought iron, and for steel 14,000 to 21,000 pounds per square inch.

Taking all matters into consideration we may safely allow—

For wrought iron . 10,000 *pounds per square inch,*
And for steel. . . 15,000 ,, ,, ,,

Round rods of small sectional area have without doubt greater resistance than plates, but the extra strength is not great except for very small sections. Some reinforcements, not of the ordinary commercial sections, have by their special treatment higher resistances.

When a metal is stretched cold to a stress beyond its elastic limit it acquires new properties which give higher values for its elasticity and strength, but these are lost on annealing. The final breaking of cold-drawn metal is more sudden and its resistance to shocks less than for ordinary qualities.

Iron or steel wires have greater relative resistance, as the diameter is smaller. There will be an increase of strength of 50 per cent. for wires of No. 10 British standard wire gauge, and the resistance will be double that for ordinary iron when the gauge is No. 19. Small rolled sections, such as those employed by MM. Bordenave and Bonna, also acquire greater resistance through rolling.

In the bars of the Ransome system the cold twisting strengthens them more or less according to the amount of twisting. The increase of their resistance permits a higher value to be used for their working stresses. Special tests should be made on sections for which it is desired to allow extra resistances, as the quality of the metal has a great influence on the increase of strength. "Expanded metal" is greatly strengthened by the treatment it undergoes in being expanded.

3. **Shearing.**—The safe shearing stress for the metal employed for resisting shearing stresses, such as stirrups or other transverse reinforcements, may be taken as $\frac{4}{5}$ of the tensile resistance allowed for the metal, being—

For wrought iron $\frac{4}{5} \times 10{,}000 = 8{,}000$ *pounds per square inch,*
And for steel . $\frac{4}{5} \times 15{,}000 = 12{,}000$,, ,, ,,

If unannealed wire is used the working stresses may be increased, being in the same proportion to the tensile stresses.

M. Considère's Tests on Concrete Prisms under Bending

M. Considère made a great number of experiments, amongst which those quoted below are perhaps the most interesting. These experiments were carried out on plain and reinforced concrete prisms, having the dimensions shown in Fig. 278.

The concrete was made in the proportions of 583 pounds of cement to 1 cubic yard of good quartz sea-sand, gauged with 37 per cent. of water to the weight of

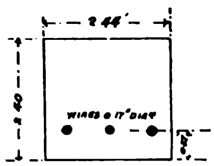


FIG. 278

cement. The prisms were carefully made, and well rammed. Some of the reinforced prisms contained three unannealed wires 0·17 inches diameter at a distance of 0·27 inches from the surface, others 17 unannealed wires of 0·075 inches diameter, and yet others with one rolled iron rod of 0·30 inches diameter. The instruments for measuring the deformations were very scientific and exact.

The plain concrete prism broke after supporting for some minutes a bending moment of 996 inch-pounds, producing a contraction of the outer fibres under compression of 0·131 thousandths of the original length, and an elongation of the extreme fibres under tension of 0·201 thousandths. At the moment of rupture the elongation of the surface under tension was 0·266 thousandths.

The results obtained from the prisms reinforced with the 0·17 inch and 0·075 inch wires were almost absolutely identical, and those for the prism reinforced with the rolled iron rod 0·30 inches diameter only differed by reason of the smaller elasticity of rolled iron and the greater distance of the rod from the surface of the prism, rendered necessary in consequence of its greater diameter.

The results obtained from the prism reinforced with the three wires of 0·17 inches diameter were as follows—

It bore a bending moment of 6,815 inch-pounds without producing rupture, the elongation of the outer fibres in tension, without cracking, being 1·98 thousandths of the original length, being nearly 20 times as great as that of similar plain mortars in direct tension, which only elongate 0·10 thousandths without breaking, or about 8 times that under bending as shown by the elongation of 0·226 thousandths for the plain prism.

Special tests made on plain concrete under direct tension showed that the concrete broke after an elongation of about 0·10 thousandths. (M. Considère and others have made many experiments on this difference between the results obtained for the elongation under direct tension and under bending, and it has been found that the elongation under bending is, at a mean, about 2½ times as great as that under simple tension.)

The prism was subsequently unloaded and reloaded 139,052 times, the bending moment varying from 2,996 to 4,815 inch-pounds, or from 44 to 71 per cent. of the maximum. These loadings caused a permanent elongation of from 0·545 to 1·27 thousandths. After these tests the surface of the prism at the most stretched portion was intact except for two very small cracks of 0·08 and 0·16 inches long.

Pieces of the prism having a section of $0{\cdot}60 \times 0{\cdot}47$ inches, and length of from 3·15 to 7·87 inches, were then sawn out at the bottom, in the part where the elongation was most, and the fact that these could be handled showed that the concrete was not broken. These small pieces were tested by bending, and were found to still possess considerable resistance, in fact very nearly as much as if they had not been tested before and cut away from the reinforcements. M. Considère calculated the stresses supported by the concrete of the reinforced prism by considering that the total resistance is equal to the sum of the partial resistances of the reinforcement and the concrete, and admitting the hypothesis of the conservation of plain sections, i.e. that the cross sections of a beam remain plain surfaces during bending. The coefficient of elasticity for the metal in tension was found by testing identical wires in simple tension.

The extension of the reinforcement was calculated from the extension and contraction of the outer fibres of the concrete on the two opposite faces.

According to Hooke's law, if p is the stress per square inch, ϵ the total elongation of the reinforcement, l its original length, and E_f the coefficient of elasticity of the metal—

$$p = \frac{E_f\,\epsilon}{l} \quad (1).$$

The position of the neutral axis was obtained from the elongation and contraction of the outer fibres (Fig. 279).

The resisting moment of the reinforcement being the stress p derived from equation (1), multiplied by the area of the reinforcement and by the lever arm

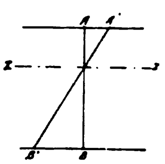


Fig. 279

between the axis of the reinforcement, and the centre of action of the compressive resistance of the concrete in the upper portion of the prism (a portion of which will form a couple with the stress in the reinforcement), the distance of this centre of action from the neutral axis M. Considère takes as ⅔ the distance from the upper surface to the neutral axis. The resistance due to the concrete in tension was obtained by deducting the resisting moment of the reinforcement, and portion of the concrete in compression, found as above, from the total bending moment of the prism. This gave the resisting moment of the concrete in tension, which forms a further couple with a portion of the compressive resistance of the concrete. The lever arm of the couple in this case being ⅔ the depth of the prism, from this the coefficient of elasticity of the concrete in tension was calculated.

M. Considère gives a table of the results obtained, which has been left in metric units, as it is only employed for comparative purposes.

TABLE XXXII

Bending Moments Supported by the Prism	Distance of the Neutral Axis from the Surface		Elongation per Metre		Value of E for the Iron	Tensile Stress in the Iron		Lever Arm of this Tension	Moments Produced by the Iron	Moments Produced by the Concrete in Tension
	Compressed	Stretched	Measured for the Concrete	Calculated for the Iron		Per Square m.m.	Total			
1	2	3	4	5	6	7	8	9	10	11
kgm.	m.m.	m.m.	m.m.	m.m.		kgm.	kgm.	m.	kgm.	kgm.
5·18	28·7	32·3	0·038	0·031	$2{\cdot}17 \times 10^9$	0·67	28	0·0450	1·28	3·90
11·48	28·7	32·3	0·092	0·075	$2{\cdot}17 \times 10^9$	1·63	69	0·0450	3·12	8·38
19·88	28·7	32·3	0·186	0·145	$2{\cdot}17 \times 10^9$	3·15	134	0·0450	6·03	13·85
30·38	27·4	33·6	0·424	0·337	$2{\cdot}15 \times 10^9$	7·35	309	0·0450	13·90	16·48
40·88	25·5	35·1	0·775	0·620	$2{\cdot}11 \times 10^9$	13·10	558	0·0445	24·83	16·05
49·28	25·3	35·7	1·050	0·840	$2{\cdot}10 \times 10^9$	17·60	750	0·0442	33·15	16·13
63·98	24·4	36·7	1·520	1·280	$2{\cdot}06 \times 10^9$	25·34	1,079	0·044	47·48	16·50
78·68	24·4	36·6	1·980	1·600	$2{\cdot}00 \times 10^9$	32·00	1,363	0·044	59·97	18·71

It will be noticed that the fourth and last figures in column 11 are excessive; similar results appeared in other tests of a like nature, and M. Considère formed the opinion that they must be due to a slight longitudinal slipping of the reinforcements.

Neglecting these irregularities, which are of small importance, it will be seen that the resisting moment, produced by the concrete in tension, at first increases rapidly and regularly, then more and more slowly to the value of 16 kilogrammetres, after which it remains practically constant until the extended face of the concrete attains an elongation of 1·98 thousandths.

The figures in column 11, and the fact that the tension surface of the prism remained practically intact, show that *reinforced concrete will bear very much greater elongation than has been thought without cracking, and under these elongations will still retain its maximum resistance.* M. Considère considers this phenomenon as similar to that noticed in metals, and points out that when a rod of mild steel is tested in tension the elongation is at first uniform, and increases up to about 18 to 22 per cent., then the rod becomes more and more reduced in area at one place, where the rupture will take place after a local elongation of 200 to 300 per cent. The rupture occurs at the moment of deformation when the augmentation of the resistance per unit of area due to the local elongation is insufficient to compensate for the contraction of the sectional area.

In bars of steel subjected to flexure, this contraction is not produced, since the swelling of the compression fibres compensates for the drawing out of those under tension, and also because the fibres under tension do not all arrive at once to the critical elongation.

The test of the reinforced prism shows further that although the elongation of concrete under bending is more than its elongation under direct tension, it still falls far short of that when the concrete acts together with the metal of the reinforcement.

It therefore appears that under bending the aid given by the less stretched fibres and those which are compressed does not suffice to bring into play the whole ductility of the concrete, which however shows itself when it acts with a metal the limit of elasticity of which considerably surpasses that of the concrete, and which comes to the aid of the weak sections, preventing their premature deformation, and allowing all the sections to take the maximum elongation of which the material is capable.

M. Considère constructs his stress-strain curve from the particulars given in Table XXXII, from which the molecular properties of the concrete can be obtained, i.e. the coefficient of elasticity and the stress in tension and compression, when the concrete suffers successive deformations.

In this curve the elongations and contractions are positive or negative, and plotted as abscissae, and the tensile and compressive stresses are the positive and negative ordinates. The coefficient of elasticity for small stresses, which may be supposed to be the same for compression and tension, gives the inclination to the x axis, of the tangent at the origin of the curves. The stress-strain curve obtained takes the form shown in Fig. 280. The curve on the tension side commences with a curve from O to M, and terminates with a straight line M, N parallel to the axis x. The curve for the reinforcements would of course be a straight line as long as the limit of elasticity was not exceeded.

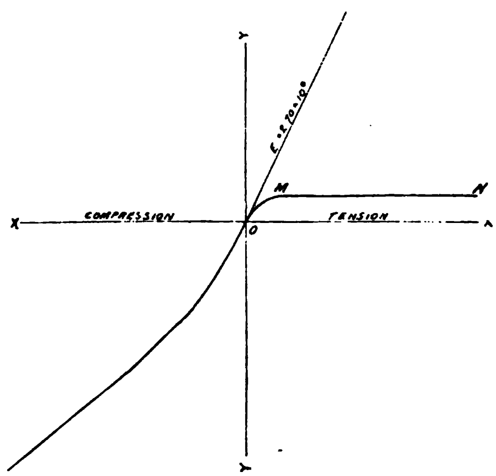


FIG. 280

From the curves (Fig. 280) the stress at any point in the depth of a beam can be determined. The first portion of the curve for tensions gives information about the coefficient of elasticity of the concrete, but when the curve changes its direction it is an indication that the concrete has attained its "elastic limit," and the corresponding abscissa gives the stress at this point. Finally, the curve is parallel to that which would be obtained if the concrete offered no tensile resistance, and the distance between the curve and the parallel line through the origin is proportional to the resisting stress of the concrete.

M. Considère obtains the following figures by allowing the coefficients, for the concrete, which he considers prudent to take for the purposes of design, i.e.—

Coefficient of elasticity, $2{\cdot}70 \times 10^6$.

Maximum tensile resistance, 170 pounds per square inch.

Maximum compressive resistance, 2,130 pounds per square inch.

TABLE XXXIII[1]

Elongations and Contractions	Maximum Tensile Stress Pounds per Square Inch	Maximum Compressive Stress Pounds per Square Inch
0·04	107	107
0·10	156	256
0·25	170	569
0·50	170	924
1·00	170	1,493
1·50	170	2,130

[1] Some later experiments made in the autumn of 1903 for the French Commission on re-

It may be remarked that other authors do not agree with M. Considère that the concrete in tension will still act with its maximum resistance after the "limit of elasticity" is passed, and yet others hold this opinion, Herr Sanders, Herr von Emperger, and Professor Hatt amongst the number. M. Christophe, however, protests strongly on this point.

It is however certain that M. Considère's experiments point to some such conclusion, and although it is perhaps prudent at the present time, and until there is more unanimity on this point, to neglect such resistance, it appears extremely probable that concrete *when reinforced* obtains some such property.

M. Christophe's Experiments on Reinforced Pieces under Bending

M. Christophe, in his book *Le Béton Armé*, describes a series of experiments which he conducted carefully on three sheet piles of the Hennebique system under bending. The piles were 15·75 × 5·91 inches by 18 feet long, with a semi-circular recess at either edge 2·75 inches diameter. The reinforcement consisted of three rods ½ inch diameter, 4·57 inches apart along each of the wider sides, with upright and transverse cross-pieces formed of annealed wire 0·157 inches diameter, the sets being 9·84 inches apart; the area of the reinforcing rods being 0·7 per cent. of the area of the concrete.

The proportions of the concrete were 661 pounds of cement to 0·57 cubic yards of sand and 1·14 cubic yards of shingle and broken stone, or about 1 : 2½ : 5.

The piles were 2½ months old when tested, and were freely supported, lying on their wider sides with a span of 13·12 feet. The load was applied by bags of sand of 110 pounds weight distributed over half the span.

The first pile broke under a distributed load of 4,872 pounds, including the weight of the pile itself, and 220 pounds concentrated at the centre. The other two failed under a distributed load of 4,872 pounds. The bending moment was therefore 95,857 inch-pounds, which gives as the value for μ (unit coefficient of resistance or $\frac{M}{b\,d^2}$) 186. The pile was considered as being truly rectangular. The first cracks in the pile first tested were observed on the tension surface under a uniformly distributed load of 2,530 pounds, not including the weight of the pile itself, but they closed on the removal of the load.

The two other piles showed very small and hardly perceptible cracks under a distributed load of 1,320 pounds, but these did not assume any importance until the distributed load was 2,530 pounds, and a further load near each support of 330 pounds had been applied. The weight of the pile itself, amounting to 1,234 pounds, has not been added to any of these loads.

The measured local deformations were very regular for the first pile, and very irregular for the other two. It was apparent that these last suffered from some fault in construction, although their final resistance was not much inferior to the first.

M. Christophe draws the conclusion that the reinforcements compensated for the non-homogenic nature of the concrete, giving to the pieces of reinforced concrete a fairly regular resistance.

inforced concrete under the direction of M. Mesnager on reinforced prisms under direct tension, showed an elongation of the concrete of $\frac{1\cdot35}{1000}$ before failure, and that after the maximum resistance of the concrete had been reached it remained constant until rupture took place. These experiments confirm M. Considère's conclusions, but when remarking on them M. Considère points out that owing to the danger of the formation of initial cracks or other imperfections, the tensile resistance of the concrete must be neglected in practical work. He insists, however, on the necessity of taking it into account when calculating the deformations.

It will be noticed that this is very much the same conclusion as that arrived at by M. Considère.

M. Christophe points out that these experiments, in which the local deformations were very carefully measured, show that the solidity in reinforced concrete prevents premature rupture; at one section the concrete may offer its full resistance and the reinforcement be very slightly stressed, while in another the concrete may become cracked, but the rods take up the stresses and pass them over the crack. He selects the results given by the test of the first pile for the section at the centre of the span, to verify the principle demonstrated by M. Considère, that the concrete in tension, after passing its "elastic limit," still retains its maximum resistance.

The deformations were only measured by two instruments 4·09 inches apart on a vertical line, and consequently do not suffice to study the action of the interior forces, if the hypothesis of the conservation of plane sections during bending is not allowed. M. Christophe therefore assumes that this hypothesis is true, so that the line $A\,B$ takes the position $A'\,B'$ after deformation (Fig. 281). On this assumption the contraction of the upper reinforcement is proportional to the length $C\,C'$, and the extension of the lower reinforcement to the length $D\,D'$.

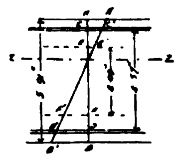


FIG. 281

The two points E and F at which the deformation were measured taking up the positions E' and F'; with a uniformly distributed load of 1,320 pounds the mean deformations measured were—

$E\,E' = -\ 0{\cdot}04$ thousandths of the original length,

$F\,F' = +\ 0{\cdot}24$ thousandths of the original length,

from which the position of the neutral axis is obtained, the distance $E\,F$ being 4·09 inches.

The maximum deformations of the concrete were—

Compression $A\,A' = -\ 0{\cdot}10$ thousandths.

Tension $B\,B' = +\ 0{\cdot}30$ thousandths.

The deformations of the rods were calculated as—

Contraction of top rods. . . $= 0{\cdot}06$ thousandths.

Elongation of bottom rods. . $= 0{\cdot}26$ thousandths.

M. Christophe assumes the coefficient of elasticity of iron as $28{\cdot}44 \times 10^6$, and from the above figures deduces from Hooke's law that the bottom rods offered a resistance of $28{\cdot}44 \times 10^6 \times 0{\cdot}00026$, or 7,394 pounds per square inch, or a total stress of 4,576 pounds for the three rods.

The resistance of the top rods was 1,706 pounds per square inch, or a total resistance of 1,056 pounds for the three rods. When taking moments of the internal forces acting at their respective centres of application, the exact point for the concrete is not known, as this depends on the law of distribution of the forces, or the stress-strain curve, on which subject there is some difference of opinion, some authors thinking that it is of a parabolic form, and others assuming it to be rectilinear.

M. Christophe is of the opinion that the centre of application cannot vary much, and that the straight line stress-strain curve will approximate very closely to the truth. He therefore takes the point of application for the compressive resistance of concrete as at a height of $\frac{2}{3}\,O\,A$, and supposes the coefficient of elasticity to remain constant for the variation of stress on the height of the section; he also neglects the recesses in the concrete, and considers the section as rectangular, which is very nearly the case. The moments are taken about the centre of action of the stresses in the concrete under compression.

They are therefore as below—

For the lower rods $4{,}576 \times 4{\cdot}74 =$ $+\ 21{,}690$ inch pounds.
For the upper rods $1{,}056 \times 0{\cdot}17 =$ $-\ 179$ inch pounds.

Total moment of resistance of the reinforcements $= 21{,}511$ inch pounds.

The bending moment $= \dfrac{1{,}320 \times 13{\cdot}12 \times 12}{8} = 25{,}977$ inch pounds.

The concrete in compression having no moment, it follows that the concrete in tension must have a resisting moment of 4,466 inch pounds. The measurement of the deformations showed that the maximum elongation for this concrete ($B\,B'$) was 0·30 thousandths..

M. Christophe points out that this extension, together with the initial extension produced by the weight of the pile itself, is more than the maximum extension of a test piece subjected to simple tension, but is less than the extensions of the prisms experimented on by M. Considère (p. 228), and therefore the concrete must have passed its "elastic limit" and be offering its maximum resistance if M. Considère's conclusions are correct.

From M. Considère's results M. Christophe assumes as a close approximation that the tension in the concrete may be considered as uniform throughout the total height $O\,B$. On this supposition the tensile resistance of the concrete will be $\dfrac{4{,}446}{3{\cdot}20} = 1{,}396$ pounds, which gives a unit resistance of $\dfrac{1{,}396}{(15{\cdot}75 \times 4{\cdot}41) - [(2{\cdot}75)^2 \times 0{\cdot}78]}$ or 22 pounds per square inch.

But M. Christophe points out that the total tension of the concrete under the total load due to the exterior loading and the weight of the pile itself will probably be somewhat more than double the amount.

He goes on to say that the concrete of which the pile is made should be offering its maximum unit resistance of 170 pounds per square inch, according to M. Considère, and draws the conclusion that the application of the hypothesis followed by M. Considère for the interpretation of the results of his experiments is reduced to a "material impossibility." He remarks however that if the truth of the hypotheses which gave the value of 4,466 inch pounds for the moment of resistance of the concrete in tension is admitted, it would be naturally concluded that it would have cracked for a great part of the distance $O\,B$, but the pile showed no signs of cracks; yet in spite of this the experiment does not allow him to generalize the property pointed out by M. Considère.

These diverse results, obtained by two such eminent authorities as M. Considère and M. Christophe, show that we shall not be right in allowing the conclusions arrived at by M. Considère for use in our calculations until they are more firmly established. The diverse results may be due to the assumptions in both cases of a straight line stress-strain curve in compression, and the conclusions arrived at by both these authorities certainly warrant the assumption of a parabolic stress-strain curve in compression.

It is also certain that the concrete in tension will bear a much greater elongation when reinforced than when by itself, and there is a reasonable probability that, after passing its "elastic limit," it still offers some resistance, although the absolute amount must be still considered doubtful.

Professor Hatt's Experiments of Reinforced Concrete Beams

Professor Hatt, in a paper read before the American section of the International Association for Testing Materials,[1] describes some tests carried out by himself on plain and reinforced concrete beams 8 inches square, freely supported 80 inches between supports, the reinforcements being of wrought iron and the loading applied at the centre of the span. Load deflection curves were plotted from the results of the tests. The following is a tabulated list of results :—

TABLE XXXIV

Proportion of Ingredients of Concrete	Nature of Piece: Reinforcement	Nature of Piece: Distance of Reinforcement from Underside Inches	Age when Tested Days	Condition: Original	Condition: After Test	Point where Curve commences to turn from Straight Line: Load lbs.	Point where Curve commences to turn from Straight Line: Deflection Inches	First Crack: Load lbs.	First Crack: Deflection Inches	Failure: Load lbs.	Failure: Deflection Inches
1:2:4 Stones	None		28	Normal	Crack 4 in. from centre, weight 480 lb.	1,200	0·001			2,400	0·003
„	„		28	„	Crack at centre, weight 485 lb.	1,200	0·004			2,200	0·0145
„	1%	2	25	„	Crack 2 in. from centre.	2,500	0·026	5,000	0·140	5,500	0·163
„	„	„	28	„	Crack at centre	2,000	0·020	5,000	0·136	6,250	0·186
„	2%	„	25	„	„	2,000	0·020	5,750	0·109	10,250	0·278
„	„	„	27	Rather Wet.	„	2,500	0·030	6,500	0·133	10,250	0·278
„	1%	1	25	Quivery.	„	2,500	0·016	7,250	0·138	7,500	0·145
„	„	„	23	„	„	2,500	0·025	6,500	0·150	7,300	0·177
„	2%	„	27	Very wet.	Crack 6 in. from centre.	3,000	0·024	10,000	0·174	12,000	0·247
„	„	„	25	Rather dry.	Cracks 6 in. from centre.	3,000	0·026	10,000	0·158	11,800	0·208
„	1%	1½	30	Quivery.	Two cracks 2 in. each side of centre.	2,000	0·020	4,000	0·087	6,500	0·176
„	„	„	30	Normal.	Cracks 3 in. from centre.	2,000	0·017	5,500	0·138	6,400	0·170
„	None.		7	Rather dry	Tested at ½ span.	1,400	0·004			3,400	0·008
„	„		8	Normal.	Weight 480 lb.	2,000	0·0014			1,500	0·018
„	1%	2	7	Rather dry.	Cracks 4 in. from centre, weight 500 lb.	500	0·007	4,250	0·160	6,000	0·248
„	„	„	9	Normal.	Cracks at centre.	1,000	0·012	3,500	0·101	5,500	0·280
1:3:5 Cinder	2%	1	17	Normal.	Cracks 3 in. from centre. Failure later in compression.	1,000	0·026	5,000	0·268	5,250	0·288
„	„	„	17	Too dry.	Cracks 4 in. from centre.	500	0·014	2,000	0·068	2,300	0·082
„	None.		14	Normal.	Crack at centre.					600	0·032
„	„		11	„	Tested at ½ span.	205	0·003			1,100	0·019
„	„		11	„	„ „	205	0·002			1,100	0·013
1:5 Gravel	3%	1	6	„	Failed suddenly by horizontal shearing along reinforcement at end of beam.	4,000	0·036	10,000	0·124	11,500	0·160

Professor Hatt remarks, with reference to the experiments on broken stone concrete beams about one month old, that the curve of deflections was nearly a straight line until a load of from 1,500 to 3,000 pounds had been applied. At higher loads the deflection increased more rapidly, but the curve again became a straight line, the deflection increasing uniformly with the load until at a load of from 4,000 to 10,000 pounds a crack occurs at the lower surface of the concrete. Beyond this point the deflection still increased uniformly with the load until the iron reinforce-

[1] Published in *Engineering Record*, June 28, 1902.

ment reached its elastic limit, at which time the deflection increased rapidly without any corresponding increase of load. Fig. 282 shows typical load-deflection curves, also the effect of percentage and disposition of reinforcement; and Fig. 283 shows the effects of age in the reinforced concrete beams.

It will be seen from the diagram (Fig. 282) that 1 per cent. of reinforcement

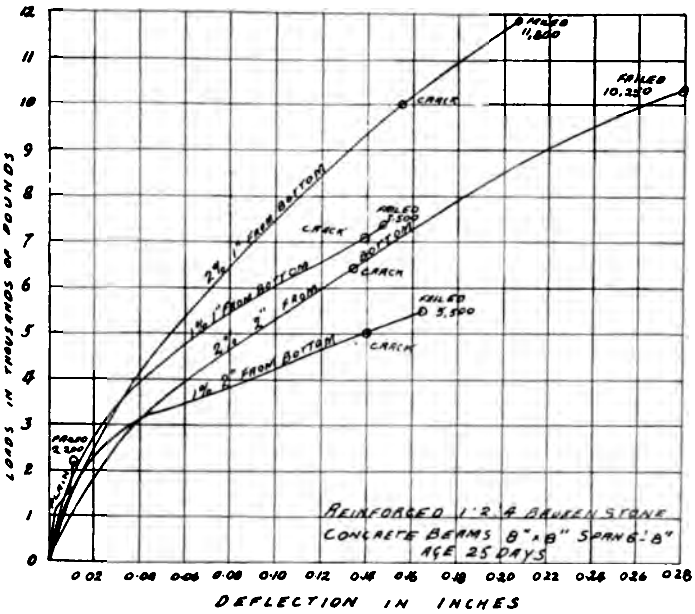


FIG. 282

one inch from the bottom gives an increase of 34 per cent. to the strength of a plain concrete beam. Tables XXXV and XXXVI show the comparative strength and flexibility of the beams.

TABLE XXXV

1 : 2 : 4 BROKEN CONCRETE BEAMS PLAIN AND WITH VARIOUS PERCENTAGES OF REINFORCEMENT.

	Plain Concrete	Reinforcement 1 inch from the Bottom		Reinforcement 2 inches from the Bottom	
		1%	2%	1%	2%
Strength, pounds .	2,300	7,200	10,000	5,000	6,000
Flexibility, inches .	0·01	0·14	0·16	0·13	0·12

TABLE XXXVI

	1:2:5 Cinder Concrete		1:2:4 Broken Stone Concrete	
	Plain	Reinforced with 2% 1 inch from Bottom	Plain	Reinforced with 2% 1 inch from Bottom
Strength, pounds	600	5,000	1,800	10,000
Flexibility, inches	0·023	0·26	0·016	0·16

Professor Hatt remarks that in none of the broken stone concrete beams was there any indication that the maximum compressive strength of the concrete was reached at the load at which the reinforcement failed, but that, if steel reinforcement had been used, the compressive strength of the concrete might have been developed. The reinforcement in no case pulled out of the concrete and the stones were broken across at the section of rupture.

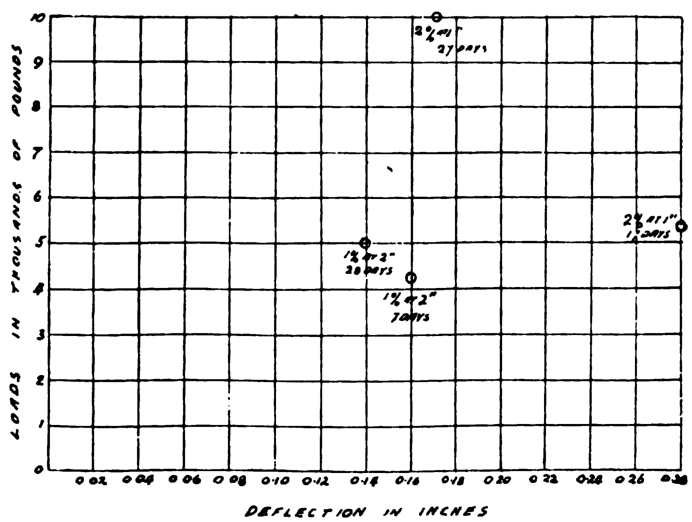


FIG. 283

Professor Hatt also made some tests on direct tension and compression which have been mentioned with reference to the moduli of elasticity of concrete. He also found that the average elongations of the test pieces at rupture were—

For plain concrete . .	1 in 7,000.
For reinforced concrete .	1 in 1,140.

and forms the same opinion as M. Considère that the effect of the reinforcement is to distribute the maximum elongation over the entire length of the piece, whereas in the case of plain concrete the maximum elongation is confined to the fractured section.

Herr Sanders' Experiments on Reinforced Concrete Beams

Herr Sanders, of the Amsterdamsche Fabrieken van Cement-ijzer Werken, made a very interesting series of experiments on the behaviour of reinforced concrete beams, which clearly show that the concrete in tension will suffer great deformation without cracking, and also indicate that at the same time it offers considerable resistance.

The tests were made on beams of mortar and concrete in the proportions of of 1 : 2, 1 : 2 : 2, 1 : 3, and 1 : 3 : 3 with ratios of reinforcement to the total area of the piece of $\frac{1}{80}$, $\frac{1}{70}$, $\frac{1}{60}$, $\frac{1}{50}$, and $\frac{1}{40}$, all the reinforcements being 0·40 inches from the lower surface, the side under tension only being reinforced. The pieces had widths of 4·6, 6·48, 6·73, and 7·48 inches, the depth of 4 inches being the same in every case. The tests were made at various periods after moulding, the usual periods being in the neighbourhood of 30 days and 92 days.

In thirty-two cases out of forty tests no cracks showed on the tension surface before the final rupture, although the finish was very smooth and perfect, and the deflection varied between 9 and 16·5 millimetres in a span of 2 metres. In ten others the final rupture occurred immediately after showing cracks and with no further increase of load. The remaining eight of the pieces failed first by flaking on the compression side, due apparently to an excess of tensile reinforcement. The failure in ten cases was due in a great measure to shearing, this being shown by the formation of the cracks.

Tests of Reinforced Concrete in Direct Tension by Le Service française des phares et balises

The French service of lighthouses and beacons made a series of experiments on prisms 4·72 × 6·53 inches 3·94 feet long between the enlarged heads for the clips of the testing machine. The prisms were of neat cement, mortar of 1,000 pounds of cement per cubic yard of sand and concrete of 840 pounds of cement to ½ a cubic yard of sand and ½ a cubic yard of pea shingle. The area of these prisms was 31 square inches. The elongation was measured on a length of 3·28 feet. Round reinforcing rods were placed in various positions.

M. de Joly has published a description and study of these tests[1]: he found that for a piece with a reinforcement of one rod at the centre, the measured elongation at a maximum was 0·074 while the calculated deformation was 0·067, on the supposition that the reinforcement, and the surrounding concrete, elongate the same amount, and with a coefficient of elasticity for the concrete found by special experiments with similar mixtures. When reinforcements are used near the outer surface the stretching is less than the theoretic value, and the elongation decreases as the rods are farther and farther removed from the centre of the piece; the calculated elongation being 0·059 whereas the measured deformations varied between 0·0562 and 0·051. The load was not increased to breaking, and the observations were made with about ⅓ the breaking load.

M. de. Joly considers that the differences shown between experimental and theoretical research can be explained by the fact of the load being applied through clips, causing an uneven distribution; the load being greater at the outer edges than at the centre of the prism due to the form of head, the deformation was therefore greater at the sides where the greater stresses occur, than at the axis of the prism where they are least, and consequently the axial reinforcement relieved the concrete less than the reinforcements at the outer edges.

M. Christophe does not think this explanation sufficient, but believes that, besides this unequal stretching of the outside and central portions of the piece, the metal also lags behind, forming cone-shaped depressions in the surrounding concrete (Fig. 284). He explains this by stating that the difference between the observations and the theoretical reasoning proves the unequal stretching of the

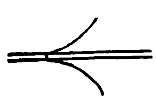


Fig. 284

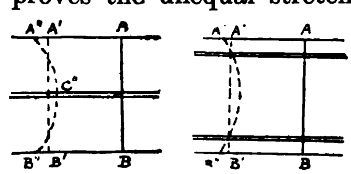


Fig. 285

reinforcement and surrounding concrete, for, if the metal is supposed to follow up completely the elongation of the concrete, the real stretching measured at the exterior of the piece A A″, B B″ (Figs. 285) must in all cases be greater than the

[1] *Annales des Ponts et Chaussées*, 7th Série, Tome 16, 1898.

theoretic elongation A A′, B B′; because the sum of the tensions produced by the elongation which brings the section A B into its theoretic position A' B' is equal to the sum of the tensions produced by the real stretching bringing the section into the curved position A'' C'' B''; this curve must therefore cut the straight line.

If, however, the real elongation is less than that arrived at theoretically, it is clearly necessary that an elastic movement of the metal, in the concrete, be combined with the unequal stretching of the fibres. Adopting this view of the case, and supposing that all the fibres in a test piece of plain concrete stretch uniformly, M. Cristophe points out that with the axial reinforcement the apparent elongation at the exterior would be greater than the theoretical stretching (Fig. 286), while with two reinforcements one at each side, as shown in Fig. 287, it would be less, and that the measured deformation would become less and less as the reinforcements approached the outer edges.

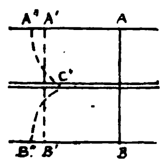


FIG. 286

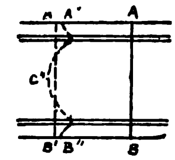


FIG. 287

Many other authors have also come to the conclusion that the deformation due to the manner of application of the load, and that due to the elastic movement of the reinforcements in the concrete, act together, and are inseparably connected.

Professor Brik says on the subject, in a paper in the *Oesterr. Wochenschr. f. d. seffenth Bairdienst*[1]: " owing to the great difference in the value, of the elastic moduli of steel and concrete, shearing stresses are induced near the steel. These stresses cause longitudinal displacements of the concrete around the embedded steel, which will take place even when sliding is prevented by the adhesive resistance. The original plane section thus becomes a warped surface with a funnel-shaped depression around the steel " . . . " This explains the fact that measurements of elongations taken during tests show an advance movement of the surrounding concrete relatively to the elongation of the steel."

Experiments on Reinforced Concrete under Direct Compression

Professor Gary, of the Polytechnical School, Charlottenburg, has tested a column reinforced on the Hennebique system, 9·84 inches square and 10·56 feet high. The proportions of the concrete were 1 of cement to 4 of gravel, and the column was reinforced by four rods of 1·18 inches diameter with cross-pieces of 3·15 × 0·12 inch plates 19·7 inches apart. The sectional area of the rods was 4·5 per cent. of the concrete.

The test was made three months after moulding, the column failing under a load of 3,640 pounds per square inch by a flaking off of the concrete between the cross-pieces, due to the swelling of the column. The cross-ties did not show any signs of failure, but the vertical rods were bent between them. This test shows that a column reinforced in the usual way with longitudinal rods and cross-ties some distance apart will always fail first by the swelling of the concrete between the ties, when there is no flexure.

The Commission on Arches of the Society of Austrian Engineers and Architects carried out tests on thirteen blocks, some of which were 15·75 inch cubes and others 9·7 inches square by 3·28 feet high. The concrete was mixed in the proportions of 1 of cement to 3½ of gravel, and the reinforcements were on the Wayss system, being formed of a series of small diameter vertical rods, placed near the outer surface of

[1] An extract of which appeared in the *Engineering Record*, Aug. 23, 1901.

the concrete, surrounded by wire hoops at frequent intervals; the tests were made at periods between six weeks and six months after moulding. The total amount of metal in the vertical rods was about 1 per cent. of the cubical contents of the block. As in the last case, no failure could occur by flexure.

The least load to produce rupture was 3,839 pounds per square inch, and the first signs of failure showed themselves by a bending of the vertical rods and a shelling off of the exterior concrete; the final rupture being produced by the interior portion of the concrete cracking vertically.

These tests indicate that the wire hooping is a good method of reinforcement, but that the hoops should be closer together than is usual at the present time.

M. Considère's Experiments on Hooped Concrete[1]

M. Considère has published in *Le Genie Civil,* November and December, 1902, and January, 1903,[2] a series of articles on the subject of hooped compression members which are very interesting and instructive. After pointing out the advantages that are gained by the prevention of the swelling under compression, and the inadequacy of vertical reinforcements as generally employed, he gives a description of his experiments, and draws certain conclusions therefrom.

The first series of tests were on small prisms of mortar 1·6 inches diameter, hooped with fine wire. The deformations were not measured, and the results given in Table XXXVII can therefore only be used to verify the resistance to crushing or comparison with the assumptions to be made.

TABLE XXXVII

Weight of cement per cubic yard of sand in pounds	675	675	675	675	730
Age when tested in days	8	14	22	23	100
Ratio of volume of iron to volume of concrete	0·02	0·03	0·04	0·02	0·034
Resistance to crushing in pounds per square inch of total section	4,870	6,540	7,360	4,930	10,500
Ditto ditto of concrete not reinforced	569	711	853	853	2,420
Increase of resistance due to the hooping	4,301	5,829	6,507	4,077	8,080
Product of the ratio of iron to concrete by 78,200 lbs.	1,564	2,346	3,128	1,564	2,688
Ratio of the values of the last two lines	2·7	2·5	2·1	2·6	3·0

M. Considère remarks that the wire employed for the hooping was cold drawn, and had not a very definite elastic limit; from its curve of deformation, however, 78,200 pounds per square inch appeared the proper value, and is taken as such. It is multiplied by the ratio of iron to concrete in the last line but one, giving the compressive resistance which the same amount of metal would give if it were employed as longitudinal reinforcements instead of hoops. The last line gives the coefficient of efficiency to resist compression of the metal used as hooping, as against the same quantity employed in the form of longitudinal rods.

The last prism was the only one which had time to set to almost, if not quite, its maximum strength. This prism gave a resistance of 10,500 pounds per square inch, with a volume of metal 0·034 of the total volume of the cylinder, no longitudinal reinforcements being used.

The hooped concrete had a density of 2·4, and that of iron is 7·8; the ratio of

[1] A very interesting test to destruction of a parabolic "bowstring" bridge of reinforced concrete, with a hooped compression boom, is described in the Appendix.

[2] An extract of these papers appeared in the *Engineering Record,* Dec. 20th and 27th, 1902, and January 10th and 17th, 1903.

the densities of the two materials is therefore 3·2. To compute the resistance of an iron bar of the same weight per square inch of section, the resistance of the cylinder (10,500) must be multiplied by 3·2, giving a resistance of 33,600 pounds per square inch.

The resistance of a riveted iron column weakened by the holes will not be more than from 36,000 to 39,000 pounds per square inch, taking the total sectional area. It may therefore be said that a cylinder of reinforced concrete of the nature of that shown in the last column of Table XXXVII, with the small proportion of 0·034 of reinforcement to the volume of the concrete, has very nearly as much resistance as that of a riveted iron column of the same weight. The effects of the hooping are, however, less advantageous with respect to the coefficient of elasticity, and consequently to the resistance against yielding by flexure.

To study the question of the resistance to flexure, experiments were made on long members, and their deformations measured.

A series of experiments were made on thirty-eight prisms of octagonal section 5·9 inches diameter, made by M. Hennebique of concrete in the proportions of 660 and 1,302 pounds of cement to half a cubic yard of sand and 1 cubic yard of shingle. Some of the prisms were of plain concrete, and others had various forms of reinforcement; some had a length of 1·64 feet for testing the resistance to direct crushing, and others were 4·25 feet long for the study of the elasticity and ductility of the hooped concrete. Table XXXVIII and Fig. 288 give the results of these experiments, which comprised 1,200 observations, divided into six groups so as to bring the useful information into a small compass.

TABLE XXXVIII

Group	Nature of Prism and Reinforcement	Behaviour
1	Plain Concrete. 1·64 feet long	Crushed under a load of 1,050 lb. per square inch.
2	Reinforced with helical spirals having a diameter averaging 5·5 inches. The wire employed was ¼-inch diameter cold drawn. Spirals placed 1·18 inches centre to centre. 1·64 feet long.	Crushed under a load of 5,120 lb. per square inch of total section. First failure, 1,730 lb. per square inch.
3	Reinforced with helical spirals having a diameter averaging 5·5 inches. The wire employed was 0·17 inches diameter cold drawn. Spirals placed 0·59 inches centre to centre. 1·64 feet long.	Stood without crushing a pressure of 5,400 b. per square inch per total total section. First failure 2,480 lb. per square inch.
4	Reinforced in same manner as group 2, with the addition of 8 longitudinal wires ¼ inch diameter in contact with the inside of the spirals. 4·25 feet long.	Failed as a long column under a pressure of 4,550 lb. per square inch.
5	Reinforced in same manner as group 3, with addition of longitudinal wires as group 4. 4·25 feet long.	Failed as a long column under a pressure of 5,400 lb. per square inch.
6	Reinforced with eight longitudinal wires 0·35 inches diameter, tied together by belts of iron wire 0·17 inches diameter spaced 3·15 inches apart. 4·25 feet long.	Crushed under a load of 2,420 lb. per square inch.

Fig. 288 is a diagram of deformation showing the behaviour of the six prisms. The figures against each line are the proportion of the volume of reinforcement to the total volume of the prism, being in the first figure for the longitudinals and in the second for the spirals.

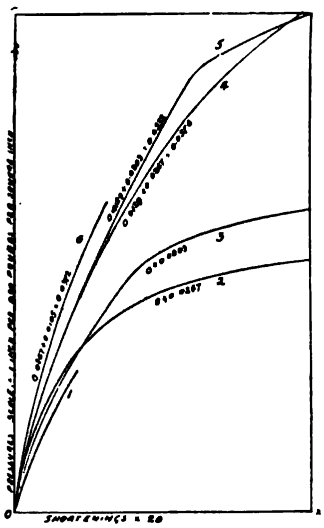


FIG. 288

Group No. 6 approaches closely the manner of reinforcement usually adopted at the present time.

General Properties of Ordinary Reinforced and Hooped Compression Members.—The following phenomena were observed during the above experiments, but no special instruments were used for measurements.

Group No. 1 broke suddenly without warning, and the failure of *Group No.* 6 was almost as sudden, the breaking load for the latter only exceeding that producing the first cracks by 7 per cent., the reinforcing rods bending outwards between the belts and the concrete becoming crushed. (This agrees well with Professor Gary's experiments (p. 239) showing that concrete in compression when either un-reinforced or reinforced with longitudinal rods will break suddenly with a very slight deformation.)

Groups 2, 3, 4 *and* 5 behaved at the commencement of the loading like the other prisms, showing only very small deformations under light loads, but this quasi-elastic period was not terminated by the sudden failure of the specimen. The shortening was observed to increase rapidly, and cracks appeared in the concrete covering the hoops, which gradually increased in size.

In *Group No.* 2, cracks appeared under the comparatively light load of 1,730 pounds per square inch, and soon after the concrete began to flake off, and finally failed between the spirals, which were 1·18 inches apart. There was, however, nothing in the failure to show that the metal had reached its elastic limit, the final failure being due to the swelling of the concrete. In the case of *Group No.* 3 the cracks did not appear till the pressure amounted to 2,480 pounds per square inch, and the deformation being 0·355 per cent. The flaking off of the exterior concrete commenced later than in *Group No.* 2 under a pressure of 5,400 pounds per square inch. No fracture of the concrete was observed, and the prism did not fail. *Groups Nos.* 4 *and* 5 showed cracks under a pressure of 2,900 and 3,360 pounds per square inch, the network formed by the spirals and longitudinals appearing to have completely resisted the lateral failure of the concrete.

The results obtained from the study of the long hooped prisms indicate that the failure takes place only after deformations as great as 3 per cent. of the original length.

The conclusions to be drawn from this series of experiments are that *concrete not reinforced, or reinforced by longitudinal rods only, even when tied together by cross-ties spaced much nearer than is usual in present day practice, will break by swelling under small deformations and without warning; while hooped concrete sustains without crushing considerably heavier loads, and only fails a long time after cracks in the surface and exaggerated deformation have given warning of the danger.*

M. Considère remarks that with spirals spaced not more than $\frac{1}{5}$ of the diameter of the turns apart, resistances were obtained which were independent of the spacing, but that on the other hand, as regards the appearance of cracks in the outer layer, the elasticity and the resistance to flexure, better results were obtained as the spirals were closer together, and that the best results were found when longitudinal reinforcements were used against the inside of the spirals.

These facts, and others, lead to the adoption of *a spacing of the spirals of from $\frac{1}{7}$ to $\frac{1}{10}$ of the diameter of the turns, and the use of longitudinal reinforcing rods.* Experiments on prisms of quite different dimensions have proved that the above ratio holds true, almost independently of the absolute values of the dimensions.

Ductility of Hooped Concrete.—Numerous experiments proved that in hooped prisms, bent under great pressures, the concrete did not break, and maintained its cohesion.

A hooped prism of concrete in the proportions of 840 pounds of cement per cubic yard of gravel, was subjcted to a pressure of 7,940 pounds per square inch of the original section. The prism became very bent, the greatest deflection of 0.4 inches in a length of 13 inches, the curvature being more accentuated in the central portions, the least radius being about 2 feet. Very few riveted metallic pieces would stand this flexure without failure. The stretched fibres showed no transverse cracks, so cannot have suffered much from the extension. The computed shortening of the compression fibres gave the enormous figure of 17 per cent.

The hooping and longitudinal reinforcements were afterwards removed, and the remaining concrete, 4·25 feet in length, bore handling without breaking, and required 55 pounds applied at the centre of a 3·61 foot span to break it by bending.

One of the halves of a prism which had been less deformed but had supported a mean pressure of 7,940 pounds per square inch was broken by bending, and showed a calculated tensile resistance of 250 pounds per square inch, which differs very little from the initial tensile strength of the concrete.

Other hooped prisms after being similarly tested in compression after the removal of their reinforcements, were subjected to a further direct compression. One with the same proportions for the concrete as the last-mentioned only bent under a pressure of 6,970 pounds per square inch with a shortening of 0·6 per cent., and after the removal of the reinforcements possessed an average compressive resistance of over 1,420 pounds per square inch ; its maximum resistance greatly exceeding this figure.

Another, of which the concrete was proportioned with 630 pounds of cement per cubic yard of gravel, withstood a pressure of 10,270 pounds per square inch with a shortening of 2·4 per cent. on an average and 2·8 per cent. on the most stressed side. The inside cylinder after removing the spirals sustained a pressure of 9,700 pounds per square inch on an area of 10½ square inches.

The above tests show *that hooped concrete will withstand considerable shortening without becoming disintegrated and retains a great portion of its original resistance.* It seems that it is safe to deduce from this *that within the limits of the small deformations of actual practice, the resistance of hooped concrete can be considered constant after it has reached its maximum.*

This result is very much the same as that obtained by M. Considère for tension on reinforced concrete.

The Elastic Behaviour of Hooped Concrete.—M. Considère made a great number of experiments of the elastic behaviour of hooped concrete. Tables XXXVIII and XXXIX give the details of some of the most interesting. All the prisms were octagonal, the diameter being 6 inches and the length 4·25 feet.

TABLE XXXIX

DETAILS OF THE COMPOSITION OF THE TEST PIECES

No. of Prism	Proportion of Cement per Cubic Yard of Gravel Pounds	Spirals		Longitudinal Rods	
		Diameter Inches	Spacing Inches	Number	Diameter Inches
7	840	0·25	0·79	8	0·3125
8	,,	,,	,,	,,	,,
9	,,	,,	,,	20	0·276
10	420	,,	,,	8	,,

Similar results might have been expected from these tests, but the facts proved otherwise. With *Prism No.* 7, for pressures below 2,845 pounds per square inch, the coefficient of elasticity proved to be $7{\cdot}11 \times 10^6$, while for Prism No. 8, which was identical, its value was only $2{\cdot}85 \times 10^6$. This difference was due to the quantity of water used in mixing the concrete, which was excessive for Prism No. 8.

M. Considère points out that the first lesson to be learnt from these experiments is the irregularity of the concrete which may arise if proper supervision is not observed. The " elastic limit " and the resistance against crushing were almost independent of the amount of water and varied very much according to the proportion of cement used, while the co-efficient of elasticity is greatly influenced by the amount of water, but hardly at all by the proportion of cement. The curves of deformation follow closely those obtained for plain concrete. During loading and unloading the deformations show a permanent set, which increased if the same load was repeated, but in a less and less degree and rapidly approaching its final limit. A reduction in the temporary deformation is obtained during the loadings and unloadings subsequent to the first, which appreciably increases the coefficient of elasticity, a result also very similar to that obtained by tests on plain concrete.

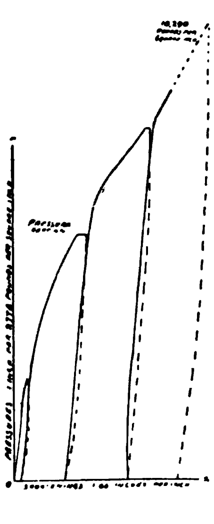


FIG. 289

A more important result is however observed from the curve of deformations. It will be noticed (Fig. 289) that the curves turn their concave side to the axis of pressures, whereas it is turned the opposite way in the curves for the first application of the load.[1] (This is more clearly seen in Fig. 288.) It follows that *the coefficient of elasticity which is represented by the inclination of the tangent to the curve of deformation increases with the pressure in the unloading and reloading instead of decreasing with an increase of the load as under the first application.*

The flexure of a column is to be feared under high pressures. *It is therefore unfortunate that the coefficient of elasticity, which is directly proportional to the column resistance, decreases with the increase of the pressure under the first application of the load ; on the other hand, it is especially fortunate that hooped concrete, after having been subjected to a*

[1] The curve for the first loading in Fig. 289 appears concave, but is really almost a straight line. The curves (Fig. 288), which are all for first loadings are more pronounced, as the elongations are plotted to a larger scale than those of Fig. 289.

TABLE XXXIX

Effects of Repeated Loading and Unloading

FIRST LOADING AND UNLOADING

Pressure, Pounds per Square Inch	1,053	1,850	2.375	3,330	3,825	4,490	3,825	3,330	2,575	1,850	1,053	0
Shortening in inches, No. 7 . .	0·0047	0·0095	0·0130	0·0158	0·0260	0·0480	0·0480	0·0378				0·0146
,, ,, No. 8 . .	0·0173	0·0276	0·0350	0·0437	0·0567	0·0658	0·0658	0·0615		0·0496	0·0480	0·0248
,, ,, No. 9 . .	0·0118	0·0205	0·0264	0·0350	0·0437	0·0445	0·0445	0·400	0·0354	0·0335		0·0102

SECOND LOADING

Pressure, Pounds per Square Inch	1,850	3,300	4,490	4,890	5,150	5,420	5,600	6,210	6,740	
Shortening in inches, No. 7 . .		0·0449	0·0540	0·0645	0·0662	0·0685	0·0827	0·1356	0·1940	Flexure at 6,958 pounds per square inch.
,, ,, No. 8 . .	0·0540	0·0642	0·0745	0·0772	0·0934	0·0945				Flexure at 6,335 pounds per square inch.
,, ,, No. 9 . .	3·0335	0·0406	0·0540	0·0582	0·0630	0·0765	0·0875			Flexure at 7,940 pounds per square inch.

FIRST LOADING AND UNLOADING OF PRISM NO. 10

Pressure, Pounds per Square Inch	526	1,053	1,990	2,375	2,815	3,300	3,565	3,300	3,040	2,815	2,375	1,990	1,053	526
Shortening in inches . . .	0·0146	0·0236	0·0276	0·0323	0·0437	0·0567	0·0831	0·0827	0·0807	0·0787	0·0768	0·0748	0·0615	0·0540

SECOND LOADING OF PRISM NO. 10

Pressure, Pounds per Square Inch	0	526	1,053	1,990	2,815	3,040	3,300	3,570	3,980	4,350	4,750	5,150	5,285
Shortening in inches	0·0350	0·0524	0·0599	0·0717	0·0827	0·0831	0·0842	0·0905	0·0945	0·0984	0·1260	0·1693	0·2047

first loading, has a coefficient of elasticity which increases with the pressure, a fact which has to M. Considère's knowledge never been observed for other materials.

A special and carefully conducted experiment was made to place these facts more clearly.

Fig. 289 shows the curves of deformation obtained, and Table XL gives the resulting coefficients of elasticity.

TABLE XL

Pressures, Pounds per Square Inch		855	1,990	4,270	6,830
Coefficients of elasticity	First loading . .	$2 \cdot 18 \times 10^6$	$0 \cdot 85 \times 10^6$	$0 \cdot 38 \times 10^6$	$0 \cdot 28 \times 10^6$
	Unloadings and reloadings	$1 \cdot 99 \times 10^6$	[1] $4 \cdot 91 \times 10^6$	$2 \cdot 42 \times 10^6$	$3 \cdot 27 \times 10^6$

After the high pressure of 10,290 pounds per square inch had been applied it was taken off, and it was found that the coefficient of elasticity was as high as after the application of the lightest pressures.

For the sake of simplicity only the average of the two last operations is given in Table XL.

Reviewing the results of these experiments, it may be stated that, *the application of a first pressure on a hooped prism, no matter how high that pressure may be, has the effect of raising its "elastic limit" up to that pressure. The coefficient of elasticity, which is developed by the hooped concrete under all the variations of the pressures between the lowest and the previously applied load, is higher than the highest coefficient of elasticity which the prism had prior to the first load and which held only for a low pressure. The increase in the coefficient of elasticity of the tested concrete after the first load had been applied compared with that before, is so much the more the poorer the concrete was made and the lower its quality.*

Hooped concrete does not show in this respect any likeness to reinforced concrete under tension, where the coefficient of elasticity decreases considerably after appreciable deformations, and the more so the greater these deformations have been.[2]

Elasticity and Resistance of the Concrete in Hooped Members.—To find the real effect of the hooping on the prisms it would be best to make identical prisms with and without hooping, but this is rendered impossible by the nature of the materials.

Two methods were used for this purpose by M. Considère—1. By preparation of as nearly identical prisms as possible, and correcting their differences. 2. By testing the prism with its spirals, and then again after their removal.

1. The following points were noticed in the case of the identical prisms. At the commencement of the loading the spirals are not really tight on the concrete, as the contraction during the setting in air decreases the diameter of the core. The effect of the hooping is shown more quickly in the subsequent unloadings and reloadings after the first load which produces the proper contact.

The load required to produce proper contact between the spirals and the concrete is about 220 pounds per square inch.

[1] This coefficient of $4 \cdot 91 \times 10^6$ is converted from M. Considère's figure which is probably a misprint and should be $2 \cdot 06 \times 10^6$.

[2] M. Considère refers here to tension members reinforced with straight rods in the ordinary way. It is probable, however, that different results might be obtained if the tension member were reinforced in such a way that the tensile stress on the metal would cause the concrete to become compressed, as is the case in some degree with "expanded metal."

This explains the increase of the coefficient of elasticity due to the hooping, which is equal to 90 per cent. of that which would be obtained by an equal weight of longitudinal reinforcing rods during the first loading, and *double* that which would be obtained from using these during the subsequent loadings and unloadings,

The conclusions derived by M. Considère from these later experiments were that *the hoops only begin to be seriously stressed under the first application of the load on prisms hardened in air when the longitudinal rods have already passed their elastic limit and are almost at their ultimate strength, and consequently can offer no further resistance.* M. Considère further remarks that it is easy to understand why the action of the hoops extends through a wider range than that of the longitudinal rods. The elongation of the spirals is caused by the swelling of the concrete, which is comparatively small, and varies between 0·3 and 0·4 of the longitudinal shortening. This explains the great deformation which can take place in hooped concrete without injury to either the concrete or the metal.

With pieces hardened in water the concrete will expand, putting an initial tension on both the hoops and the longitudinal rods, the hoops being stressed still higher on the application of the load, while the longitudinal rods must have their initial tension overcome before they can be called upon to take up the compression.

Whereas a first or test loading is required for hooped pieces hardened in air to cause them to act efficiently under subsequent loadings, and giving them a higher coefficient of elasticity, it is probable that similar results would be obtained without the test loading if the hooped pieces could be kept in water, or moist air, during a period of time before exposing them to the air.

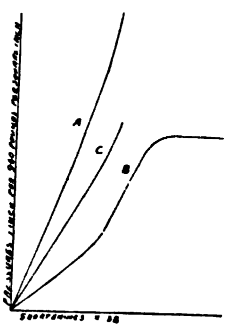


FIG. 290

2. The elasticity and resistance of the same core of concrete, first hooped, and then after the removal of the hooping, were compared, and the results are shown in Fig. 290.

The curve A shows by its ordinates the resistance offered by the hooped core, the curve B shows the resistance after the removal of the hooping, and the curve C has for ordinates the difference of the ordinates of the curves A and B, showing the increase in the resistance due to the hooping. These experiments showed that *concrete which has been loaded when hooped has, after the removal of the spirals, a resistance which after attaining a certain amount remains constant, notwithstanding the increase of the deformation, at any rate within wide limits ; and that the previous compression, while hooped, gives the concrete, besides a greater ductility, an increase of resistance of about* 50 per cent.

M. Considère has made further experiments on hooped concrete prisms, which completely bear out his former results, and may be compared with those mentioned previously. He constructed an octagonal prism 19·69 inches long, the diameter being 4·33 inches.

The proportions of the concrete were 1,000 pounds of portland cement to 0·90 cubic yards of shingle to pass a 1 inch ring and 0·30 cubic yards of ¼ inch sand. spirals were of iron wire 0·17 inches diameter rolled round a cylinder of 3·77 inches diameter, with a pitch of 0·71 inches. There were also 8 longitudinals of the same sized wire. The total section area of the prism was 15·5 square inches, and the area of the circle enclosed by the spirals 11·16 square inches. The proportions of the volume of the reinforcement to the *total* volume of the prism was 0·035, that of the spirals being 0·024 and that of the longitudinals 0·011.

The first sign of failure by the peeling off of the outer shell occurred under a load of 3,410 pounds per square inch of *total* section. And the final failure by bending occurred at a load of 12,691 pounds per square inch *on the section surrounded by the spirals ;* no absolute breaking occurred.

The shortening of the prism under this load was $\frac{24}{1000}$, while at a load of 2,858 pounds per square inch (i.e. before the outer shell peeled off) the shortening was $\frac{1 \cdot 2}{1000}$. A similar prism without reinforcement failed under a load of 2,337 pounds per square inch.

A second prism with the same proportions of concrete, but of larger dimensions, has also been tested by M. Considère. The length and shape were the same, but the *total* sectional area amounted to 131 square inches, and that of the cylinder enwrapped by the spirals to 88·7 square inches. The spiral reinforcement consisted of 0·39 inch diameter iron wire wound round a circle of 10·63 inches diameter with a pitch of 1·46 inches. There were also eight longitudinals of 0·59 inches diameter iron wire. The proportion of the volume of reinforcement to the *total* volume of the prism was 0·0387, that of the spirals being 0·022, and that of the longitudinals 0·0167.

The prism failed by the breaking of one of the spirals under a load of 9,271 pounds per square inch *of the section enclosed by the spirals.* The shortening of the prism at failure was $\frac{42}{1000}$.

M. Considère points out that the resistance of 0·39 inch wire would be less per millimetre than that for a 0·17 inch wire.

M. Considère has further experimented on hooped hollow columns, and, as might be expected, has found that hooped pieces must be solid, or at any rate not have hollows of any considerable size.

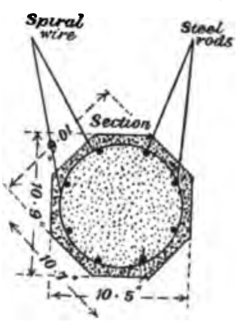


Fig. 291

Mr. W. Dunn, with the co-operation of Messrs. Cubitt and Co., has made and tested a hooped column such as recommended by M. Considère, but, whereas the columns tested by M. Considère were of small dimensions, Mr. Dunn's column was one such as would be employed in practice. This renders his experiment of the very greatest interest, and great credit is due to him for taking up the subject in such a practical manner.

The column was of octagonal section of the dimensions shown (Fig 291) and 10 feet in height. The reinforcement consisted of eight vertical wires ¼ inch diameter extending for the full height of the column. These were bound round spirally with ⅛-inch wire, the length of the wire employed for the spiral being 227 feet. The proportion of longitudinals is therefore 0·00419, and that of the spirals 0·00296 of the total volume of the concrete. The wire was first wound closely round a drum and was then allowed to spring out to the exact pitch of 1¼ inches, being $\frac{1}{7 \cdot 28}$ of the diameter of the spiral (M. Considère recommends $\frac{1}{7}$ to $\frac{1}{10}$), the wire worker having no difficulty in arranging this. Both sets of wire were of mild steel, and the verticals and spirals were bound together with fine wire at the crossings to keep them in position while the column was moulded. The diameter of the spiral was 9 inches. The moulding was done vertically, in 6-inch layers, and rammed with a wooden rammer 8 inches in diameter.

The concrete was mixed in the proportions of 1 of portland cement to 2 of Leighton Buzzard sand and 2½ of pea shingle, or about 865 pounds of portland cement to 0·83 cubic yards of sand and 1 cubic yard of shingle.

The column was kept in the mould for five days, and was then removed and stored in wet sawdust for a further period of 56 days. It was then tested horizontally in a machine by Messrs. Kirkaldy, the effect of the bending due to its own weight being counteracted by a weight, equal to half the weight of the column, acting upwards at the centre.

The gradual shortening of the column under an increasing load is shown in the

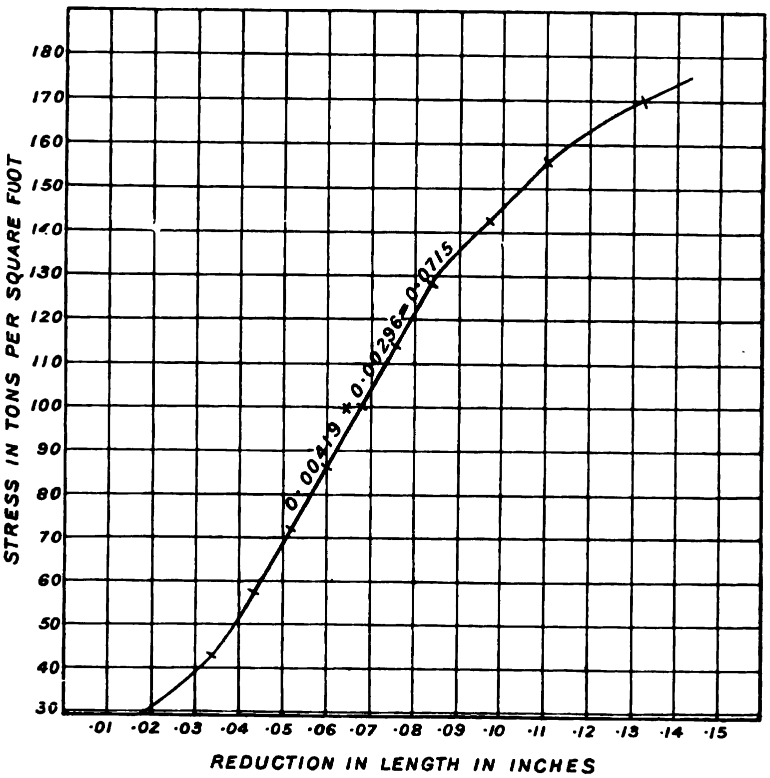


Fig. 292

diagram (Fig. 292). The stresses being those per square foot of section within the spiral windings, the portion outside not being reinforced does not add materially to the resistance, and is only necessary as a protective coating. This diagram shows that, as in the case of M. Considère's tests, there is a great shortening at first under light load due to the particles taking a permanent set amongst themselves and to the concrete taking up its bearing against the spirals. The proportion of the volumes of the reinforcements with respect to the *total* volume of the concrete is written on the curve. It will be seen that the proportion of reinforcement is much less than in M. Considère's tests Fig. 288). This accounts for the difference in resistance.

At about 780 pounds per square inch the shortenings became regular, i.e. the stress and strain weie proportional. This continued up to about 2,020 pounds per square inch, when the outer casing of the concrete outside the spirals began to crack (it will be noticed that this is about the resistance under direct compression for concrete not reinforced). The shortening then increased more rapidly than the load. When failure occurred, at 2,815 pounds per square inch, it was local and near one end where the outer casing flaked off (*vide* Fig. 293). The spiral wire broke in two places showing the characteristic reduction of area at the point of fracture and the longitudinal wires bulged outwards at the point of fracture. The concrete at this place proved exceedingly friable, and easily rubbed off with the fingers, the sand seeming to be not as sharp as required in the best concrete work.

FIG. 293

Two sample 12-inch cubes of the concrete of the same material and mixing as that used for the column, and of the same age, were tested at the same time, and bore 1,310 and 2,050 pounds per square inch respectively.

The failure of Mr. Dunn's hooped column was gradual, and even after the failure the two portions of the column were sufficiently held together by the eight longitudinal wires to permit of the column being slung out of the machine as one piece, by means of a rope sling round its centre. Fig. 293 shows a view after failure. The column was made by men having no special experience in reinforced concrete construction, and under no specially skilled supervision.

Mr. Dunn suggests, as the safe load to put on such a column, $\frac{1}{3}$ of the load at which the failure first began, i.e. $\frac{2020}{3}$ or 673 pounds per square inch, which would amount to a total safe load of 19 tons, or about the same load as we should use on a 6×5 inch rolled joist weighing $24\frac{1}{2}$ pounds per foot and used as a stancheon 10 feet long.

It seems, however, that we might consider the safe load as $\frac{2815}{3} = 938$ pounds per square inch, as the failure by flaking off of the outer coating is no indication of the strength of the column, and a load of 938 pounds per square inch could be applied without any fear of such scaling. This load might be applied at the end of two months with complete confidence, and the margin of safety would increase with the age.

It will be noticed that the diagram (Fig. 292) is very similar to those obtained by M. Considère, Fig. 289 showing a concave curve at the commencement. Considère found this result for loadings and unloadings after the first, but not the first loading. The coefficient of elasticity appears to have a constant value between pressures of 780 and 2,020 pounds per square inch, i.e. $3 \cdot 18 \times 10^6$.

The volume of reinforcements in Mr. Dunn's column was only 0·00715 of the *total* volume of the concrete, whereas, it will be seen on reference to M. Considère's experiments that the proportions used in his series of tests for one loading were from 0·0332 to 0·0376 (*vide* Fig. 288), and from 0·035 to 0·0387 in the later ones. In Mr. Dunn's experiment the volume of spirals was 0·00296 of the volume of the concrete, and in M. Considère's this proportion varied between 0·0203 and 0·0267 in the first series of experiments, and from 0·022 to 0·024 in the later experiments. This difference in the percentage of reinforcement accounts for the fact that Mr. Dunn's column failed under a load of 2,815 pounds per square inch. and M. Considère's at loads of from 4,550 to 5,400 pounds per square inch in his first experiments, and from 9,271 to 12,691 for his later experiments; Mr. Dunn's columns failing by the spiral wires breaking. This variation in results points to the conclusion that the area of both reinforcements should be more than allowed by Mr. Dunn.

The following table shows the proportions of concrete used in the various tests. The comparative poorness of Mr. Dunn's concrete will also partly account for the lower results obtained in his experiment.

TABLE XLI

	Portland Cement, Pounds	Sand, Cubic Yards	Shingle Cubic Yards
M. Considère's first tests	1,320	0·5	1
" later "	1,110	0·33	1
Mr. Dunn's test	865	0·83	1

It will also be noticed that in Professor Gary's experiments on columns reinforced with longitudinal rods and cross-pieces, the failure occurred at a load of 3,640 pounds per square inch; but in this case the area of reinforcement *in the vertical rods* was 0·045 that of the concrete and the test was made three months after moulding, whereas Mr. Dunn's column was tested after a period of two months.

In the experiments by the Commission on Arches on columns reinforced by vertical rods with hoops at frequent intervals, the load at failure was 3,829 pounds per square inch, but the proportion of metal in the vertical rods alone was 0·01 of the volume of the concrete, and to this the proportion of the hoops should be added for making a comparison.

Change of Volume while Setting due to Hygrometrical Conditions

M. Considère has made a valuable series of experiments on the behaviour of plain and reinforced pieces setting in water and in air.

The variations in the length of the test pieces were carefully measured by a micrometer, indicating to $\frac{1}{100}$ of a millimetre. Table XLII gives the results obtained from four prisms while setting under water.

The mortar was mixed in the proportions of 920 pounds of cement to a cubic yard of silicious sand, and the reinforcement consisted of an iron rod 0·4 inches diameter. The size of the prisms was 2·36 × 0·98 inches by 23·6 inches in length.

TABLE XLII

BEHAVIOUR OF PRISMS WHILE SETTING UNDER WATER

Elongations in $\frac{1}{100,000}$ of the original length

Number of days after moulding		1	2	3	4	5	6	7	14	21	28	35	42	49	56	63
Neat Cement	Plain . .	7	15	21	27	32	37	41	59	69	73	75	77	78	78	79
	Reinforced	2	3	4	5	6	8	9	13	16	18	20	21	22	22	22
Mortar	Plain . .	3	10	13	15	17	18	19	20	22	24	26	27	27	27	28
	Reinforced	2	2	2	3	3	3	4	4	4	4	5	5	5	5	6

It was found that cement not reinforced extends about $\frac{1}{1000}$ of the original length in one year and $\frac{1 \cdot 5}{1000}$ to $\frac{2}{1000}$ in two to three years. The extensions of the mortar appears to be about one-third those of neat cement.

The mean calculated stresses produced by the elongation in the reinforced neat cement were about 6,256 pounds per square inch *tensile stress in the reinforcement* and 360 pounds per square inch *compressive stress in the cement,* with maximum stresses at the centre of the prism of 7,821 and 455 pounds per square inch respectively. For the mortar the mean *tensile stress in the reinforcement* due to the elongation was 1,706 pounds per square inch, and the mean *compressive stress in the concrete* was 100 pounds per square inch, with a probable maximum at the centre of the prism of about 130.

Varying the proportion of metal to the concrete *the extension of the reinforcement increased with the decrease in the percentage of metal* to the total area of concrete. Prisms setting in air instead of expanding like those hardening under water will contract, but according to a less regular law.

TABLE XLIII

BEHAVIOUR OF PRISMS WHILE SETTING IN AIR.

Contractions in $\frac{1}{100,000}$ of the original length

Number of days after moulding		1	2	3	4	5	6	7	14	21	28	35	42	49	56	63
Neat Cement	Plain . .	60	58	57	58	60	64	70	95	110	118	123	128	130	131	132
	Reinforced	6	9	12	14	16	17	20	22	23	24	25	25	25	25	25
Mortar	Plain . .	22	21	20	21	22	26	29	38	42	44	45	47	47	49	50
	Reinforced	4	6	7	8	9	9	9	9	10	10	10	10	10	10	10

Both the plain prisms showed a considerable contraction on the first day, after this it practically ceased during three days, and was even replaced by a slight expansion. Finally the contraction recommenced at a decreasing speed, and far less than during the first day. This stop in the contraction at the end of the first day coincided with the period of drying of the prisms, and the temperature of the mixture, which had been below that of the air, gradually approached it. The expansion due to the variation of temperature probably had a predominant influence, which concealed the contraction due to the hardening.

The maximum contraction for the neat cement not reinforced appeared to be $\frac{1\cdot5}{1000}$ to $\frac{2}{1000}$ of the original length in two or three years. The reinforced cement and mortar contracted according to a regular and continuous law.

The following stresses were calculated :—

In the neat cement prism *the reinforcement suffered a mean compressive stress* of 7,110 pounds per square inch with a maximum of about 8,890, and *the cement a mean tensile stress* of 410 pounds per square inch with a maximum of about 510. In the mortar prism the *compressure* stress in the reinforcement had a mean value of 2,845 pounds per square inch, and the mean *tensile* stress in the concrete was 155 pounds per square inch.

From the results of these and other similar tests, M. Considère comes to the conclusion that *the initial tensions* developed in the concrete of a prism during setting in air by the action of reinforcements of sufficient sectional area reaches very nearly the ultimate resistance of similar pieces of plain concrete at the same age, and this is the reason for the regular contraction of the reinforced prisms. M. Considère remarks that the concrete in a reinforced piece is therefore extended during the setting in air beyond its "elastic limit," and it must support a tensional stress equal to its maximum resistance at the same age, as the resistance of the concrete remains practically constant after the "elastic limit" is passed.

The contraction of the metal, and the tension in the cement or mortar which produced it increased considerably during only twenty-eight days. The stop of the increase coincided with the formation of *transverse* cracks in the neat cement prism.

The mortar prism showed no cracks visible to the eye, but it was impossible to tell whether or not there existed any capillary cracks.

Deductions from the General Behaviour of Pieces Reinforced with Straight Reinforcements only under Flexure

For comparative purposes it is necessary to know certain facts as regards the behaviour of reinforced pieces.

1. The proportion of the ingredients of the concrete.
2. The quality of the materials employed.
3. The quantity of water used in mixing the concrete.
4. The atmospheric conditions during the hardening.
5. The amount of ramming.
6. The nature and form of reinforcement.
7. The method of support and loading.
8. The size and shape of the piece.

And any other factors which may affect the observations.

To test reinforced concrete in a proper scientific manner is consequently not feasible in practical tests for acceptance purposes or practical operation. As a rule sufficient information regarding the practical tests on beams and slabs is not published to allow the deduction of any very definite information.

More or less laboratorial experiments have, however, been made on slabs and rectangular and T-beams, freely supported at the ends and under uniformly distributed and concentrated central loads. It has been found from these that as the load is gradually increased the deflection varies more and more rapidly, and its curve is very much the same as that found for the deformations under simple compression by Professor Bach and MM. Souleyre and Anglade (Fig. 274), *vide* Fig. 282, showing the results of Professor Hatt's experiments. The permanent deflection for small loads is imperceptible, but increases faster than the increase of the load; with alternative loading and unloading the curve of deflections again takes the same form as that for the deformations under simple compression (Fig. 274).

The portion of the piece under tensile stresses in nearly every case shows the first indications of yielding; the cracks which appear at first sometimes extend as far as the centre of the beam, but close up so as to be imperceptible on removing the load. These small cracks which sometimes appear to have been formed prior to the loading do not seem to have any effect on the stability of the piece, and it has been found that test pieces which have become cracked accidentally before being experimented upon behave exactly the same as they would have done had they been intact.

M. Considère, in speaking of the influence of hair cracks in the concrete, says: "It is well known that in properly constructed reinforced concrete buildings not exposed to excessive heat, cracks, invisible under a magnifying glass, will sometimes occur before or during the usual acceptance tests. On the other hand, laboratorial experiments prove that the cracks resulting from a sufficient increase of load beyond that usual in practice have no noticeable influence on the progress of the deformation as long as they are not visible to the naked eye; and this could be foreseen, as a crack can only influence a very small portion of the total length. We may conclude from the above that the deformations which occur at practical tests of constructions are not influenced in any appreciable measure by the invisible cracks that might pre-exist or occur during these tests."

As the load on a piece is increased the cracks become more numerous and pronounced, and at last penetrate into the upper portion of the piece, the final failure being caused by the widening of one of these cracks when the reinforcement slips and bends; a flaking also occurs at this period at the compression surface, the concrete becoming detached in horizontal layers.

It may then be concluded that under normal circumstances in pieces reinforced with the usual reinforcements for the direct resistance of the stresses—

1. The initial cracks in the concrete acting in tension have no effect on the ultimate resistance of the piece under bending.
2. The tendency to failure is on the tension side of the piece.
3. The cracks first formed on the tension side gradually extend into the compression side under relatively heavy loading, which indicates a rise in the position of the neutral axis as the load increases.
4. As the failure proceeds there is a slight slipping of the reinforcement through the concrete due to the contraction of area of the metal in tension, unless it is of such a character that will specially resist the slipping.

5. The reinforcement in tension fails under bending when the crack producing failure opens.

6. The last portion to fail is the surface of the concrete under compressive stresses, which flakes off in horizontal layers.

7. In some cases the first failure may be due to failure by shearing near the supports which is indicated by curved cracks varying from the horizontal to the vertical.

8. The strength of reinforced concrete under bending increases with age, but not in the same degree as its resistance to direct compression, clearly indicating that the resistance to compression is not the ruling factor in the failure under bending.

Behaviour of Reinforced Slabs [1]

Many experiments have been carried out on slabs reinforced in various manners and of various proportions of concrete. The general results of these tests, were as follows—

The failure of slabs reinforced by ordinary rods or wires is not similar to that of "expanded metal" slabs, or such as are provided with an adequate resistance to any slipping of the reinforcements.

In slabs with rod reinforcements the failure commences by the cracking of the concrete on the tensile side and a slipping of the reinforcements through the concrete caused by a local stretching of the rods and consequent diminution of sectional area, which occurs only after the metal is stressed beyond its elastic limit. The reinforcement does not break and the slab still retains some elastic properties, the deflection decreasing on the removal of the load.

In slabs reinforced with special bars to resist sliding, "expanded metal" or similar reinforcement the metal generally fails at the same time as the concrete; with a large percentage of metal, however the reinforcement does not break at once, but in every case the final rupture follows closely on the appearance of the first cracks. It is also found that the meshes of the "expanded metal" close up under severe loads (Fig. 294), but that it does not slip through the concrete. Fig. 294 shows the effect of the tensile stress due to bending on "expanded

A FIG. 294 *B*

metal" embedded near the tensile surface of a concrete slab. *A* is a view of the metal before it is put into a slab, and *B* shows the meshes as closed up after the slab has been tested under bending.

[1] M. Christophe has given in his work, *Le Beton Armé* a very complete treatment of the results obtained from the tests to failure of reinforced slabs, beams and arches. The following paragraphs have been in a large measure drawn from his work, together with the figures Nos. 295, 297 to 301.

There is more stretching of "expanded metal" previous to rupture than of an ordinary rod or wire reinforcement. A reinforcement in the nature of a woven mesh such as the Cottançin behaves in a somewhat similar manner to "expanded metal," but not so pronounced.

The shearing stresses have only a very slight effect on the behaviour of slabs, causing the cracks to have a slight inclination. The fracture of slabs is, however, never due to shearing alone.

The resistance to bending varies with the breadth and square of the depth as is the case for ordinary beams. M. Christophe compares the various experiments by the coefficient μ which expresses the relation of the bending moment to the width and depth of the slab [$M = \mu bd^2$].

He finds the following values for μ from the experiments carried out on Monier slabs—

TABLE XLIV

Experimenter	Proportions of Ingredients		Age Months	Percentage of Metal	Values of μ, Inch Pound Units
	Cement	Sand			
Herr Nothenius . .	1	3	1	0·4	142
,, ,, . .	,,	,,	,,	0·6	170 to 270
,, ,, . .	,,	,,	,,	1·0	256 to 284
,, ,, . .	,,	,,	,,	1·3	355 to 383
,, ,, . .	,,	,,	,,	2·1	341 to 398
Herr Bauschinger. .	,,	,,	3	1·0 to 1·45	420 to 613
Herren Hanisch & Spitzer	1	3½	9	0·8	295
,, ,, ,,	,,	,,	,,	1·6	461

Table XLV gives the values for μ for the slabs reinforced with expanded metal tested by Messrs. Fowler and Baker.

TABLE XLV

Proportions of Ingredients				Age. Days	Percentage of Metal	Span Feet	Values of μ Inch pound Units
Cement	Sand	Thames Shingle	Water				
1	2	1	½	77	0·76	3·5	408
1	2	1	½	77	,,	6·5	334
1	2	1	½	63	,,	3·5	327
1	2	1	½	63	,,	6·5	334
1	2	1	½	77	0·45	3·5	376
1	2	1	½	77	,,	6·5	303
1	2	1	½	63	,,	3·5	232
1	2	1	½	63	,,	6·5	303

It will be seen that the values of μ are higher for similar percentage of metal in slabs reinforced with "expanded metal" than in slabs with Monier reinforcement.

The failure is always gradual with a Monier slab, whereas the metal fails with the concrete in a slab reinforced with 0·45 per cent. of "expanded metal"—with 0·76 per cent. of "expanded metal" the reinforcement does not fail and the slab behaves more like a Monier slab. It is probable that the increase in resistance of an "expanded metal" slab is due to some extent to the tendency to compress the concrete by the closing of the meshes.

The Behaviour of Rectangular Beams

The failure of rectangular beams is as a general rule very similar to that already described for slabs ; Fig. 295 shows such a failure.

The first portion to fail will almost always be the concrete under tensile stress, which will crack at the point where the bending moment is greatest, these cracks

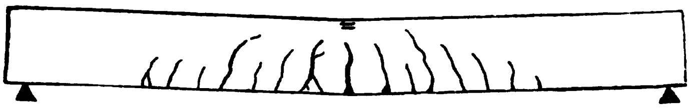


FIG. 295

will gradually develop until they penetrate into the portion under compression showing a raising of the neutral axis.

The last portion to fail is the surface under compression which will flake up, as shown, when the reinforcement bends, due to the opening of the cracks.

Since for any given bending moment the resistance to bending varies inversely as the breadth multiplied by the square of the depth, while the shearing resistance varies inversely as the breadth multiplied by the depth or the unit moment of resistance $\mu = \frac{M}{bd^2}$ and the unit shearing resistance $\kappa = \frac{K}{bd}$, it follows that in rectangular beams the first failure is very seldom due to shearing, especially when bent-up rods are employed. The shearing stress, however, has an influence on the behaviour, as shown by the inclination of the cracks which is produced by the action of the shearing stresses. When beams are too narrow the concrete will shear along the reinforcement, especially when only straight bars are used. The reinforcements seldom, if ever, fail by breaking ; they will, however, bend when the surrounding concrete becomes detached, and will also slip through the concrete as the final failure approaches if they are not specially formed to resist such sliding.

The Behaviour of T-Beams

If a T-beam is properly designed, and the leg has a sufficient width, the failure will be of the same nature as that already described for rectangular beams. In the case of T-beams, however, the bending resistance varies inversely as the width across the table (B, Fig. 296), multiplied by the square of the depth for a given

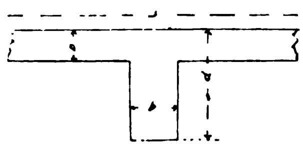


FIG. 296

bending moment, whereas the shearing resistance varies inversely as the breadth of the rib (b, Fig. 296) multiplied by the depth, or $\mu = \frac{M}{Bd^2}$ and $\kappa = \frac{K}{bd}$; conse-

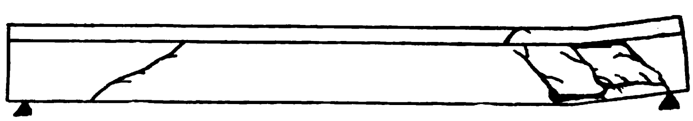


FIG. 297

quently there is more probability of a failure due mainly to shearing in the case of a T-beam than of a rectangular beam.

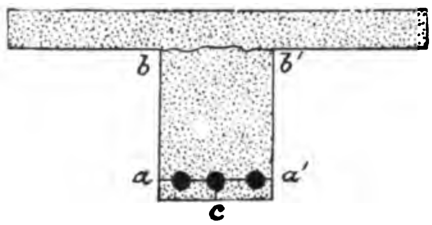


FIG. 298

Such a failure is shown, Figs. 297 and 298.

It appears, then, that it is specially necessary to inquire into the shearing resistance of T-beams, and to avoid reducing too greatly the width of the leg.

Values of μ for Beams

Table XLVI gives some values of μ obtained from some recent tests to failure carried out by Professor Gaetano Lanza, of the Massachusetts Institute of Technology, on freely supported 8 x 12 inch reinforced concrete beams, made of concrete in the proportions of 1 : 3 : 6 mixed with from $6\frac{1}{2}$ to 7 per cent. of water, reinforced with square bars along the bottom. To these have been added the values obtained from Professor Hatt's experiments (Table XXXV).

TABLE XLVI

Experimenter	Percentage of Metal to Area of Concrete	Distance of Reinforcements from Bottom Surface. Inches	Age when Tested. Days	Value of μ in Inch-Pound Units
Prof. Gaetano Lanza .	0·13	2	57	256
,, ,, ,, .	2·0	2	53	309
,, ,, ,, .	2·4	2	43	322
,, ,, ,, .	3·2	2	57	404
Prof. Hatt	1·0	$1\frac{1}{2}$	30	254
,, ,, ...	1·0	2	28	244
,, ,, ...	2·0	1	27	469
,, ,, ...	2·0	2	27	400

Professor Hatt's beams were not of good proportions, since they were of the same breadth as depth, but they appear to have given good results notwithstanding.

M. Christophe gives the values shown in Table XLVII as approximately true for freely supported beams one month old, and made of concrete mixed in the usual proportions, reinforced with straight tensile rods only.

TABLE XLVII

Percentage of Metal to Area *bd* in the case of Rectangular, and *Bd* in the case of T Beams	Value of μ in Inch Pound Units
0·2	57 to 142
0·5	170 to 200
0·8	285
2·0	568
3·5	995

The Behaviour of Reinforced Concrete Arches

The resistance of the abutments has a marked effect on the behaviour of arches. The failure of those with a curved extrados is generally due to an opening out of the span, caused by the abutments yielding, and that of an arch with a flat extrados by the top of the abutment breaking away and allowing the arch to drop at the centre, unless the reinforcements of the extrados are continued into the abutment and penetrate into it for some distance.

The failure is generally very similar to that of beams in the case of an arch with a curved extrados, the first cracks occuring at the intrados, and a flaking of the extrados taking place just before the final rupture.

In the case of arches with a flat extrados, or those with a curved intrados when

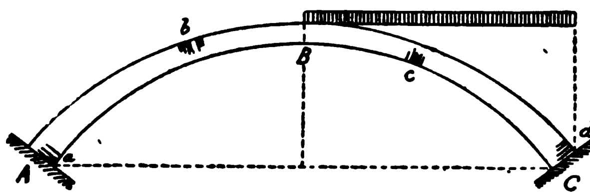


FIG. 299

the abutments remain firm, the first failure will occur at the place where the greatest tensile stress occurs, whether it be at the intrados or extrados.

Figs. 299 and 300 are given by M. Christophe, and show manner of failure which may be expected under a uniformly distributed load over half the span, which is

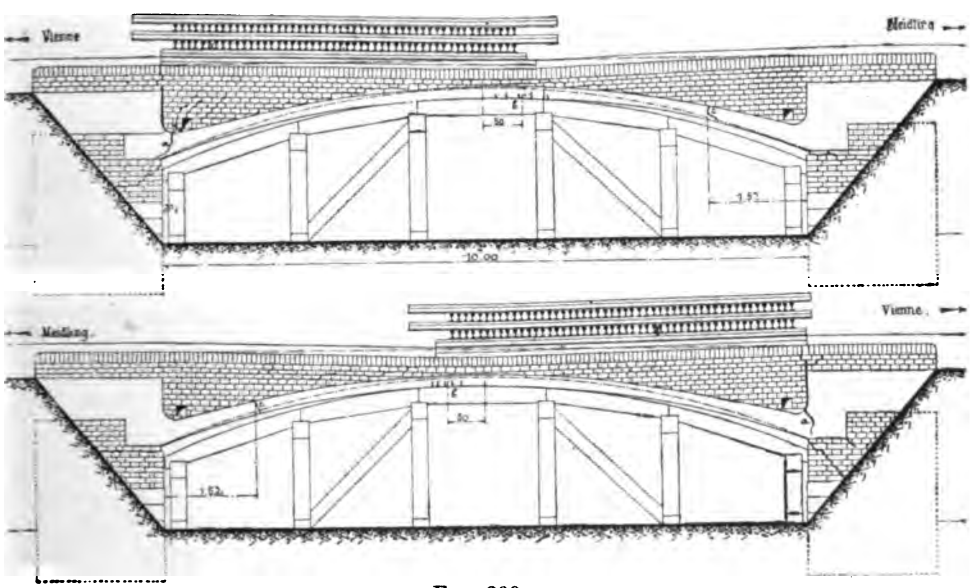


FIG. 300

the disposition which causes the greatest deviation of the curve of pressures from the neutral surface. The first signs of failure in the bridge shown in Fig. 300 was

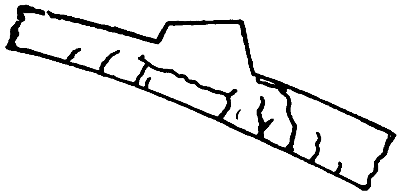


FIG. 301

the formation of cracks at (*a*); those at (*b*) only appeared just prior to the final failure.

The only case in which cracks due to shearing are at all likely to occur in arches is under the piers of spandril arches, being entirely local and due to the concentrated loading at these points. Such a failure is shown in Fig. 301.

Deductions which may be Drawn from the Observed Behaviour of Arches

It is evident that the object to keep in view when designing reinforced arches is to place the reinforcement so that aid may be given to those parts where there is a tendency to failure. In arches with a curved extrados it appears well to have a reinforcement throughout the whole length of the intrados, and a further reinforcement at the extrados from the abutments to the joint of rupture, or for about ¼ the span on either side. For arches with open spandrils a special reinforcement should be provided under the piers of the small roadway arches to resist the local tendency to shear.

For those with a flat extrados it would appear advisable to place reinforcements in the abutments or to continue the reinforcement at the extrados of the arch well down into the abutments, so as to tie the upper to the lower portions.

As reinforced arches act mostly under compression, the main reinforcements should be tied well together both in a vertical and horizontal sense, to resist as much as possible the swelling of the concrete. The position of the line of resistance under varying positions of the loading must be studied, as also the effect of the variations of temperature in the case of unhinged arches, as the portions of the arch where tensile stresses will occur are different according to the varying conditions. For arches with a flat extrados it is generally advisable to have a reinforcement throughout the whole span near both the intrados and extrados.

The Counteraction of the Stresses produced in Reinforced Concrete by Inducing Strains of an Opposite Kind

Up to quite recent times very little has been said as to the probability of the treatment of reinforced structures in any different manner than the usual one of directly resisting the imposed stresses.

From a study of the general behaviour of this material, however, one cannot avoid coming to the conclusion that the present general method of treatment may not be the best, and that instead of resisting the stresses by placing bars or wires in such positions that they may take up the direct stresses, greater strength will be obtained by so placing the reinforcement, that instead of directly resisting the imposed stresses, they may always act in tension, the concrete being compressed, whatever the nature of the main stresses may be. In a piece or the portion of a piece under tension so placing the metal that it will induce compressive strains in the concrete, while the metal itself acts in tension, will obtain the best results from both materials.

Similarly, the metal being placed, when the piece or portion of the piece is under compression, so that the swelling of the concrete may cause tensile stresses to be induced in the reinforcement. It appears evident that if these conditions are realized we must obtain a great resistance from a piece, as, both the materials used are acting under the best conditions.

M. Considère's experiments on hooped compression members must force this fact upon us, and the great resistance obtained from structures in which some such actions exist can only be explained by this point of view. It is to be hoped that scientific experiments may be carried out on these lines, as up to the present time there is not sufficient information on which to base any definite statement, and until such information is supplied we must not be too hasty in our assumptions.

Recapitulation of Data

We may safely assume the following :—

1. *That the ratio* $\frac{E_t}{E_c} = m$ *will be* 10. *The value of c at which* E_c *is taken being the maximum allowed value for the case considered.*

2. *That for general purposes we may allow* 400 *pounds per square inch for the resistance of the concrete under direct compression when reinforced with longitudinal bars and cross ties in the usual way.*

3. *That for general purposes we may allow* 500 *pounds per square inch for the maximum resistance of the concrete to compression in pieces subjected to bending.*

4. *That the shearing resistance of concrete is* 50 *pounds per square inch.*

5. *That as a general rule there will be no slipping of the reinforcements through the concrete, but that the tendency to shearing of the concrete around the reinforcement should be enquired into and provided for if found necessary.*

6. *That the following values may be allowed for the resistance of the reinforcements :—*

	For Wrought Iron *Pounds per square inch*	*For Steel* *Pounds per square inch*
In tension and compression	10,000	15,000
In shear	8,000	12,000

PART VI

CALCULATIONS

I

NECESSARY HYPOTHESES

General Remarks

It is necessary to assume certain hypotheses before entering upon the calculations for obtaining the dimensions of reinforced concrete pieces, and unfortunately it is by no means certain that these are absolutely true. The most elaborate theories based on all the elastic properties of the materials must commence from these hypotheses, and in consequence their precision and minuteness is to a great extent nullified at the outset.

It is better, however, to employ formulae which have a scientific basis than to entirely rely on the empirical equations of practical constructors, although these may, and do, give good results when intelligently employed by those who have had many years' experience in this form of construction, and are thoroughly acquainted with all practical considerations.

It is not suggested that any theory employed is the absolutely correct one, but when the results of practical experience are kept in view it is possible to obtain theories based on scientific principles, which are approximately correct, and will enable dimensions to be calculated, which may with perfect safety be used in designing a structure.

It must, however, be borne in mind that all theories, even the most elaborate, are only approximate, being based on the best information that can be obtained on the subject. We are consequently justified in assuming hypotheses on which to base our calculations which may not be absolutely correct. It is proposed to study these briefly, and to point out wherein they may be inexact.

That the Applied Forces are Perpendicular to the Neutral Surface of Pieces Subjected to Bending

This is a general hypothesis which is assumed for all beams and girders, and though it is clearly incorrect, in consequence of the deflection of the piece, it is certainly sufficiently near the truth for practical purposes for cases of simple bending. It cannot, of course, be applied to arches which are not subjected to simple flexure.

That each Fibre be Supposed to Act by Itself, not being Effected by the Contiguous Fibres

This supposes that each fibre will be elongated or contracted by the stress applied to it, as if it were alone. It is in all probability scientifically inaccurate to suppose that such is the case for beams, since there is a "striction," as M. Considère has pointed out, between adjacent fibres which modifies the deformations, allowing the fibres to sustain greater deformation under bending than they would bear under direct stresses in a testing machine. This effect is the probable cause of the divergence of the theoretical and practical strength of rectangular homogeneous beams, the breaking load on a cast iron beam being from two to three times the calculated one, due to this lateral adhesion between the fibres which resists the tendency to longitudinal shearing. The admission of this hypothesis is on the side of safety, and may be allowed therefore for the calculations of reinforced concrete pieces subjected to bending on account of the simplification it allows.

That there is Always a Solid Contact between the Reinforcement and the surrounding Concrete

It is generally assumed that the concrete and the metal act together, i.e. that the concrete follows the deformations of the reinforcements. This again greatly simplifies the calculations, since it follows that the stresses in the concrete and reinforcement are to one another as their coefficients of elasticity.

It is, however, very doubtful if such is really the case. Unfortunately, as M. Christophe points out in his remarks on the experiments of the French service of lighthouses and beacons on pieces under direct tension (p. 238), the results given can be explained by a lagging behind of the concrete surrounding the reinforcing sections, and it is extremely probable that some such deformation exists in pieces of reinforced concrete.

M. Harel de La Nöe believes that the concrete, where in contact with the iron, forms such a cup-like depression, but not until the limit of elasticity of the metal has been passed the slipping of the reinforcement being caused by the contraction in the section of the metal after it has been stressed beyond its elastic limit, or by a shearing of the concrete.

It is certain that when approaching rupture there is a failure in the proper adherence of the concrete and iron, whether the breaking is due to a shearing action near the supports or a failure in tension near the centre of the span. But pieces of reinforced concrete are not calculated for the ultimate strength, and therefore this does not really concern us.

It must be admitted, however, that there is an extreme possibility that the reinforcement and the concrete in contact with it are not equally stretched or compressed, and that the theory of elasticity cannot be applied with absolute truth to a heterogeneous material, as it can be to the structural metals. The concrete in itself is a heterogeneous material, and is subject to small voids and cracks even when carefully made; it has also a very different molecular construction to that of the metal and the sudden change from the comparatively large grained concrete to the metal, cannot but have some effect on the deformations.

The great shearing stresses where the concrete comes in contact with the metal must also have a tendency to cause a displacement of the reinforcement in the concrete. There appears then every likelihood of a difference of deformation in the reinforcement and surrounding concrete, and that to consider no such difference as existing is an incorrect assumption, but fortunately such unequal deformation has a very small effect on the accuracy of the calculations where the reinforcements are formed of a series of bars of small dimensions placed near each other and at regular intervals, and we may assume this as a sufficiently near approximation to the truth for practical purposes in such cases. When large sections such as angles, tees, joists or bars of large diameter are employed spaced far apart, this unequal deformation will, however, have greater effect and when such reinforcements are employed they should be well tied together ; if this is not done a large factor of safety must be used to compensate for the local inequality in the deformations.

M. Christophe has remarked that *the object to be aimed at in reinforced concrete construction is to make the material as homogeneous as possible*—a piece of advice which it is well to bear in mind.

Where the shearing stresses are small, there is not so much reason for avoiding the use of large sections placed far apart, and consequently this method of reinforcement may be employed for arches. In the Melan and Wünsch systems arches are constructed with such sections with success, and are calculated on the hypothesis of the equal deformations of the reinforcement and surrounding concrete.

That the Cross Sections remain Plane Surfaces during Loading

This hypothesis of the conservation of plane sections is perhaps the most important of those used for the calculations for a piece subjected to bending, since it furnishes what is practically the starting point; that two sections originally parallel will, after the load has been applied, rotate about the neutral plane and remain true planes.

This hypothesis rests mainly on the two previously considered, and if they are granted we must also allow their effect on the conservation of plane sections to be neglected.

It has been pointed out wherein these former hypotheses are incorrect, and it will be clearly seen that they have a great influence upon the behaviour of the sections of a piece under bending stresses. Also the strains in reinforced concrete under direct compression are effected by the manner of application of the load, which, as M. Christophe points out, is always concentrated on a more or less small area, causing this hypothesis to be in fault, at any rate in the neighbourhood of the point of application, it being impossible in practice to make perfectly certain as to the true distribution of the load.

When dealing with a piece subjected to bending, the employment of this hypothesis is due to the simplification it makes in the calculations. It has been proved to be true to a certain extent by experiments on rectangular *metallic* pieces, but the existence of shearing strains must tend to cause the sections to take a curved form, since the paths of the combined direct and shearing stresses follow curved lines in a beam.

When reinforced concrete is considered, it is certain that the elastic properties of concrete are entirely different to those of metals, and the results obtained from experiments where this hypothesis has been employed certainly do not prove it to be a true one.

Professor Brik makes the following remarks on the subject—" Even in homogeneous beams the longitudinal shearing stresses cause the sections to be curved surfaces instead of the assumed plane, and this is still more pronounced in the case of concrete beams. The modulus of elasticity of concrete varies with the stresses on it, and concrete beams will on this account alone show a deformation of their cross sections under the action of ordinary bending stresses. This deformation will be still further increased by the shearing stresses. If the beam is reinforced by embedded metal, owing to the great difference in the values of the elastic moduli of the metal and concrete, shearing stresses are induced near the metal. These stresses cause longitudinal displacements of the concrete mass around the reinforcements, which take place even when sliding is prevented by the adhesive resistance. The original plane section thus becomes a warped surface with a funnel-shaped depression around the reinforcements. This deformation will be added to the deformations mentioned above. This explains the fact that measurements of elongations taken during tests show an advancement of the surrounding concrete relatively to the elongation of the metal.

"The above causes result in important errors in the common static computations, so that the later should be considered as approximations only. The effects of these errors are especially considerable in the results of deflections found by the ordinary computations. Owing to the longitudinal displacement the deflections will be considerably greater than those given by the calculations.

"It will be seen from what has been pointed out that although we may continue to use the hypothesis of the conservation of plane sections for the sake of the simplicity attained by its employment, we must bear in mind that the results obtained are only approximate.

"These results will be sufficiently correct for practical purposes where the reinforcement is formed of a series of rods of small diameter placed near each other and at regular intervals."[1]

It therefore appears that we must treat this hypothesis with suspicion until carefully conducted experiments have been carried out and proved it to be correct. We may, however, retain it for simplicity's sake, but it should always be borne in mind that at best it is only an approximation of the real state of the case.

That there are no Initial Stresses set up due to Expansion or Contraction during the Setting of the Concrete, Changes of Temperature, or the Permanent Deformations

This hypothesis has been generally assumed, but M. Considère's valuable experiments on the variation in volume due to the setting of concrete (p. 252) show that it is an incorrect assumption. To recapitulate :—M. Considère shows that for pieces which harden under water, tensile stresses are developed in the reinforcements and compressive stresses in the concrete, and that, when the hardening takes place in the air, compressive stresses are set up in the reinforcements and tensile stresses in the concrete. He comes to the conclusion that the initial tensions developed in the concrete during the setting in air will very nearly reach the ultimate resistance in tension of similar pieces of plain concrete at the same age.

If the resistance of the concrete in tension is neglected, there does not seem much cause for fear on this account, since, if a reinforcement is employed in compression, the nature of the case prevents it being stressed to anything like its elastic limits, since the unit stress must be limited, as will appear later. The initial tension in the concrete subjected to compression will add to its resistance. These initial stresses

[1] From a paper, an extract of which was published in the *Engineering Record*, Aug. 23, 1901.

may therefore be neglected when assigning the safe resistances to be allowed for the reinforcement and the concrete.

M. Considère also believes that the concrete in tension, when reinforced, still offers its maximum resistance after being elongated beyond the amount which similar plain concrete will bear without rupture under direct tension. This is a point which requires further proof, and it certainly did not appear to be the case in M. Christophe's experiment (described p. 232)

The changes of temperature have also an effect on pieces of reinforced concrete, and specially so in the case of unhinged arches, where the changes in temperature materially add to the stresses, and have to be considered when designing such a structure.

Where the upper and lower surfaces of a straight piece are subjected to very different temperatures, it is advisable to place a light reinforcement in the form of a mesh near the surface exposed to the heat, to prevent cracking of the concrete. For exposed roofs and the platforms of bridges this is a very necessary precaution.

As far as the permanent deformations are concerned, M. Harel de La Nöe considers that the great elongation which concrete can sustain when reinforced is due to the reaction caused by the permanent deformation of the concrete. It is a known fact that permanent deformations occur in concrete under small loads; M. Harel de La Nöe points out that these cause initial stresses in reinforced concrete: he considers two fibres in solid contact, one of metal and the other of concrete (Fig. 302), and further that the common section A is removed to B by a tensile stress.

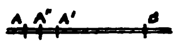


FIG. 302

When the stress is removed the metal fibre would return to A, while that of the concrete would only move to A' if it were alone; the permanent deformation of the concrete acting alone would be AA'; but if the two fibres remain in solid contact the common section will move to a position intermediate between A and A'—i.e. to A'' such that the tension produced in the reinforcement by the deformation AA'' equals the compression in the concrete due to the deformation $A'A''$. It results that after having subjected a piece of reinforced concrete to bending, on removing the load the concrete in tension is compressed, and when a second loading is applied it will commence by returning to the state of elastic equilibrium at A′ before being subjected to any tensile stress. It is rendered then, on account of the first loading, capable of a greater resistance to tension.

M. Harel de la Nöe further states that a piece of reinforced concrete is often observed to rise up during the interval between two loadings. This contra-flexure, which causes tensile stresses in the upper portion, shows the existence of a self-contained tensile stress in the reinforcement.

With respect to the subject of initial stresses, M. Considère remarks that it is a well-known fact that pieces of reinforced concrete, with a tensile reinforcement only, when exposed to the air, show a gradual deflection which increases during a period of a year to eighteen months. The contraction produced by the hardening takes place freely in the unreinforced fibres, which will be under compressive stresses, whereas the deformations are reduced at the opposite face by the resistance of the reinforcement; these unequal contractions of the two faces producing deflections in the same direction. Besides this, during the long duration of the hardening the concrete possesses in a lessening degree the facility of yielding under the loads. From these two causes curvatures result which may be confounded with those due to the loads on completely hardened beams.

These deflections are less than one-fifth and more than one-tenth of the total deflection under the load, and it would be an error to attribute them entirely to the alteration of the elasticity of the stretched concrete, for they will have been increased by the permanent shortening of the compressed non-reinforced fibres, and by a slight slipping of the reinforcements through the concrete.

Measurements of the length of the reinforcements in pieces subjected to repeated loads show that these, after the tests, returned to their original length, and consequently that they caused no sensible initial tensions or compressions in the pieces. Thus the concrete was free from tensile or compressive stresses.

M. Considère therefore cannot agree with M. Harel de La Nöe that the action of the reinforcements on the stretched concrete would create dangerous internal stresses, and that, contraflexions and the prospect of cracks in the fibres subjected to compression would result. He considers that if there are cracks they have quite another cause. The compressed fibres having to stand, during the test, pressures which alter their elasticity, they may get permanent shortenings which prevent their taking their original length which the unaltered elastic force of the reinforcements on the opposite face tends to impose upon them.

All things considered it appears that the internal stresses caused by the discordance of deformations can only have a very slight effect on the resistance of the piece, and that probably on the side of safety. They may therefore be neglected in the calculations.

The Effect of the Elastic Behaviour of the Concrete on the Resistance it Offers to the Imposed Stresses

There is still a great deal of difference of opinion on this matter, and consequently it will be necessary to discuss it in detail.

The variation in the resistance of the concrete in successive fibres above and below the neutral axis depends in the first place upon the hypothesis of the conservation of plane sections. If the sections do not remain true planes during flexure, the deformations will vary irregularly, and consequently even if the coefficient of elasticity were constant, the stresses would still vary in an irregular manner. For the purposes of this inquiry, however, the sections may be supposed to remain true planes.

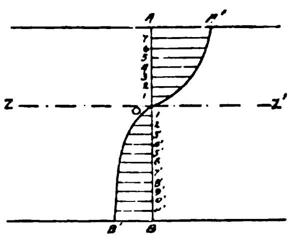


Fig. 303

On this supposition the strains in the successive fibres above or below the neutral axis vary regularly, and therefore, if we suppose ZOZ' (Fig. 303) to represent the neutral axis of a beam and AOB a vertical section, we may set off the compressive and tensile strains on OA and OB for each fibre, such as 01, 02, 03, etc., for the compression side, and 01′, 02′, 03′, etc., for the side under tension. With a modulus of elasticity which varies with the stress, such as that of concrete, we shall get a stress-strain curve somewhat of the form $A'OB'$, always supposing that the cross sections remain true planes during the bending of the beam. The portion of the curve from O to B' cannot be properly determined as up to the present the ratio which ought to exist between the coefficients of elasticity for like stresses in compression and tension has not been able to be determined by experiment.

A stress-strain curve such as this is assumed by Herr Sanders, of the

"Amsterdamsche fabriek van ciment ijzerwerken," except that he makes a break in the curves at the neutral axis (Fig. 304). To obtain the curves OA' and OB' he employs Herr Bach's equation for the curve of deformation in direct compression $\lambda = \frac{1}{E_p} p^n$, from which he obtains both the compression and tension curves by giving different values to the constants E_p and n in tension and compression. The formulae arrived at from the adoption of this curve for tensions and compressions in the concrete is very complicated, and Herr Sanders gives a simplified form of curve for practical purposes formed by two straight lines OA'' and OB'' which cut the curves OA' and OB' at points situated at two-thirds the distance from the neutral axis to the surfaces of the piece.

Herr Spitzer, chief engineer for the firm of G. A. Wayss, at Vienna, and Professor Lütken have also proposed similar stress-strain curves to those of Herr Sanders considering the curves as parabolas of the second degree.

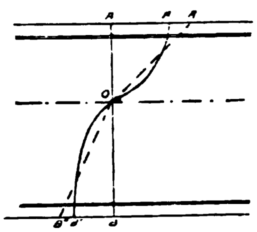


Fig. 304

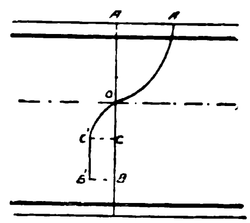


Fig. 305

Professor Hatt in America uses a somewhat similar hypothesis (Fig. 305). He considers three phases in the resistance of a piece of reinforced concrete subjected to bending, and advocates the use of steel as a reinforcement. The first, until a somewhat indefinite point C is reached in the curve of resistance at which it takes a bend, the stress at this point Professor Hatt takes as 750 pounds per square inch for the concrete in compression, and 300 pounds per square inch for that in tension, the values for $\frac{E_c}{E_t}$ and $\frac{E_f}{E_t}$ being 2 and 12 respectively. E_c and E_t being the coefficients of elasticity of the concrete in compression and in tension respectively.

The second phase takes us to the point B when the concrete cracks in tension, the stresses for this point being taken as 1,500 and 300 pounds per square inch for the concrete in compression and tension respectively. The maximum elongation of the concrete being considered as $\frac{1}{1000}$, and the values of $\frac{E_c}{E_t}$ and $\frac{E_f}{E_t}$ as 12 and 90 respectively.

The third phase is taken to the point of rupture, where the stress in the reinforcement is taken as 36,000 pounds per square inch, or the elastic limit of the steel, which he found was reached before the concrete failed in compression.

He uses the parabola as the stress-strain curves in compression and tension for his first and second phases, an assumption which he considers justified in the case of the compressional stress-strain curves from the examination of a large number of tests made at the Watertown Arsenal. The results of his examination are given in Table XLVIII.

TABLE XLVIII[1]

Mixture	Age	Ratio of Area of Stress Strain Diagram to Area of Surrounding Rectangle	Ratio of Moment of Stress Strain Diagram to Moment Area of Surrounding Rectangle
1 : 3 : 6	4 months	0·753	0·605
1 : 2 : 4	4 months	0·601	0·482
1 : 2 : 4	4 months	0·340	0·478
1 : 2 : 4	3 months	0·698	0·560
1 : 3 : 6	9 days	0·658	0·400
1 : 3 : 6	1 month	0·671	0·407
1 : 3 : 6	3 months	0·694	0·413
1 : 3 : 6	6 months	0·716	0·425
1 : 2 : 4	9 days	0·639	0·392
1 : 2 : 4	1 month	0·773	0·437
1 : 2 : 4	3 months	0·720	0·415
1 : 2 : 4	6 months	0·722	0·427
Average of above . . .		0·657	0·453
Theoretical value from parabola .		0·666	0·417

In a previous study of the subject Professor Hatt assumed stress-strain curves as shown in Fig. 306, in which the curve for tensions is formed after the deductions of M. Considère, mentioned later, except that the triangle *OCD* is added for the sake of simplicity. For his third phase Professor Hatt considers the concrete in tension as offering no resistance, all the tensile resistance of the piece being that due to the reinforcement.

The equations found on these suppositions allow no factor of safety, as the stresses are taken at their maximum values.

Like M. Considère's supposition, this assumption in the second phase has the advantage of taking into account the resistance offered by the reinforcement without having to consider the concrete in tension as having cracked, and also does not assume the concrete in tension as being stressed beyond the stresses

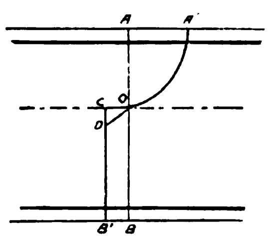


Fig. 306

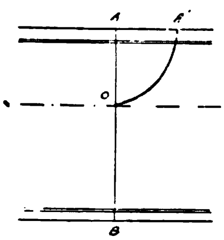


Fig. 307

usually allowed—at the same time it is not advisable to assume any resistance as being offered by the concrete in tension.

Professor Ritter has also proposed the stress-strain curve to compression as a parabola, but he neglects the resistance in tension (Fig. 307).

If the coefficient of elasticity has the same constant value in compression and tension, the stress-strain diagrams would be represented by similar triangles and the neutral axis would be at the centre of the beam (Fig. 308). Such is assumed to be the case by several authors, who have proposed formulae for reinforced concrete based on the theory of elasticity. This supposition renders the calculations very

[1] *Engineering Record*, May 10, 1902.

simple, since it is only necessary to consider the reinforcement, in the cross section, as being replaced by an area of concrete equal to that of the metal multiplied by the ratio of the coefficient of elasticity assumed for the concrete, to that of the metal, or by $m = \frac{E_f}{E_c}$, and to consider this additional concrete as acting at the same distance from the neutral axis as the reinforcement. The calculations being conducted as if the piece were entirely of concrete. After the stresses in the concrete in compression and tension are found, the stress in the reinforcement will be that of the concrete at the same distance from the neutral axis multiplied by m.

The concrete in this case is supposed to have a uniform modulus of elasticity not only in compression, but in tension also, which is certainly a very erroneous assumption except perhaps under very light loads. There is no doubt that in reality the fibres in tension stretch far more rapidly than those in compression contract, and consequently the "elastic limit" is sooner reached in tension than in compression; the neutral axis will, therefore, always be nearer the top than the bottom of a beam

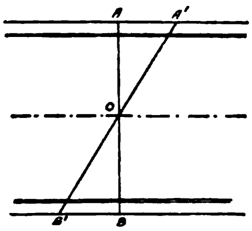


FIG. 308

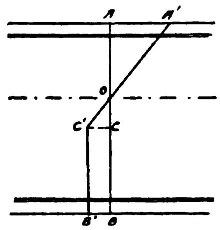


FIG. 309

of plain concrete, and the stress-strain curve in tension will have the form shown in Fig. 303, and, while the stress-strain curve in compression remains a straight line, that in tension will approach more and more nearly to a perpendicular to the neutral axis. If the concrete cracks in tension it can have no tensile resistance, and as small hair cracks are formed in many cases before any load is applied, it is not advisable to consider any tensile resistance in the concrete.

The value of the coefficient of elasticity in tension certainly does not remain constant for stresses higher than about 130 pounds per square inch, which are almost invariably exceeded in practice; this hypothesis can, therefore, in any case only be true for very small loads.

Amongst those who have assumed this stress-strain curve are MM. Lefort, Résal and de Mayas, Ingénieurs des Ponts et Chaussées; Professor Neumann, of Austria; and many others.

Another supposition is that the coefficient of elasticity has a different value in compression than in tension, but that in each case the value is constant, the result being that the stress-strain curves take the form of those assumed by Herr Sanders, for practical use shown dotted in Fig. 304. Professor Melan, of Zurich, assumes this hypothesis; it is an improvement on the last assumption, but, while the limiting stress in tension is reduced, the line OB' should not be straight for the same reason as given above, and on this supposition the concrete in tension must never be allowed to crack.

M. Considère considers the stress-strain curve as being in the form shown in Fig. 309. The coefficient of elasticity is considered as constant on the compression side, and to have the same constant value of $2 \cdot 7 \times 10^6$ up to a stress of 170 pounds per square inch on the tension side; after this stress is attained the coefficient of elasticity becomes nil, and the line $C' B'$ is parallel to AB, the concrete still being supposed to have a resistance of 170 pounds per square inch. M. Piketty has assumed a similar stress-strain curve for some of his calculations.

M. Harel de La Nöe also uses this form for his stress-strain curve, and assumes for the stress CC' 142 pounds per square inch, and takes for his coefficient of elasticity of the concrete $4{\cdot}66 \times 10^6$. This method has the advantage of taking account of the considerable aid offered by the reinforcements without having to consider the concrete in tension as having become cracked, and at the same time not assuming the concrete as being stressed beyond the stresses usually allowed in practice.

M. Considère, however, considers that the concrete in tension may be taken into account as adding to the resistance, whereas it is doubtful if the concrete in tension can safely be always considered as being intact at all points. M. Christophe's experiments on Hennebique sheet piles under flexure (p. 232) appear to indicate that the resistance of the concrete in tension is negligable, the want of resistance being probably due to invisible cracks. There is also no doubt that cracks do occur in the tensile portion of the concrete under stresses which certainly are not sufficient to exceed the limit of elasticity of the reinforcement. M. de Joly found that the test pieces for the experiments described (p. 238), after having been kept for 3 or 4 months, showed numerous cracks which were not visible directly after the tests. He considers it certain that these occurred during the tests, but were not visible until the variations in the atmospheric conditions caused them to open.

It cannot be doubted that practical constructors have obtained an instinctive knowledge of the real properties of reinforced concrete, and they in their empirical calculations invariably leave the concrete in tension out of account in the resistance of the piece. M. Christophe points out that the concrete is often found to be cracked even before any load is applied. M. Considère himself also mentions this fact. In this condition it can offer no resistance in tension. If a crack does occur, we must necessarily take this section as the one for which to make the calculations of the resistance. M. Considère's studies have, however, considerable value, and there is no doubt that his hypothesis must be used when calculating the deformation of the entire piece.

M. Considère points out that a beam of concrete of the usual proportions employed in practice, which has become cracked, does not give a moment of resistance much lower than a beam which is intact except when the percentage of reinforcement is small. He states that there is no loss of resistance when the area of reinforcement is 3 per cent. of the area of the concrete, and only becomes 12 per cent. with 2 per cent. of reinforcement, and 36 per cent. with 1 per cent. of reinforcement.

If the calculations are made on the assumption that the concrete offers no resistance in tension, the only conditions considered are those for a section which has become cracked, and the stresses and the deformations so calculated are not applicable to the portions of the solid between two cracks.

It may be granted, then, that if it is desired to calculate the total deformations, as, when the deflection of the piece is in question, we must consider that the concrete offers some resistance in tension. M. Considère himself admits that the supposition that the concrete in tension offers no resistance gives satisfactory results in calculation of the dimensions, but insists very rightly that it cannot be applied for the calculation of the deformations, and, from consideration of the limits of load which may be realised during actual working, finds that the difference between the reality and the results obtained from the different hypotheses have the values given in table XLIX.

TABLE XLIX.

Hypotheses	Percentage of Difference from Reality	
	Elongation of Reinforcement	Contraction of Concrete
That the concrete in tension retains its coefficient of elasticity during the complete test	45	50
That the concrete in tension does not have any action in the resistance	110	25
That the concrete in tension remains perfectly elastic so long as the extension does not exceed the value which causes rupture in plane concrete, but that when the limit is exceeded it cracks	100	20
That the concrete in tension remains perfectly elastic as long as its extension does not exceed the value which causes rupture in plain concrete, and that after this limit is reached the coefficient of elasticity becomes nil, but the concrete can still offer its maximum resistance	5	5

Professor Von Thullie goes very elaborately into the manner of resistance of a heterogeneous material. He divides the resistance offered by a solid of reinforced concrete to bending into two stages. During the first stage the concrete resists equally in tension and compression, while in the second he considers that it has become cracked in tension, and therefore offers no resistance to tensile stresses. He at first assumed the coefficient of elasticity in compression as constant throughout both phases, but later he altered the value for the second phase and obtained the stress-strain curve shown in Fig. 310; the tensile stress CC' being the limiting resistance of the concrete in tension, which M. Thullie takes as 285 pounds per square inch, or, if freedom from all risk is required, 215 pounds per square inch.

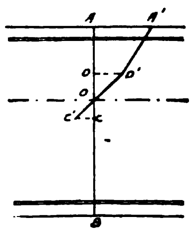


FIG. 310

The value of the tensile resistance of the concrete is very small and is neglected in the calculations. In compression the coefficient of elasticity has a constant value of $2{\cdot}84 \times 10^6$ for stresses up to 710 pounds per square inch at DD', after which it is taken as half its former value or $1{\cdot}42 \times 10^6$.

This stress-strain curve is certainly better than those on the same lines which consider only one phase, as it does give some value to the reinforcement during the second stage. On the tension side, nevertheless, it is extremely probable that, for the higher stresses allowed, the curve would not remain a straight line.

Prof. Von Thullie also allows in the second stage, which he employs for determining the section of reinforcement, a resistance for wrought iron of about 50,000 pounds per square inch, and for concrete in compression from 1,780 to 2,840 pounds per square inch. The stress-strain curve for the concrete will certainly not remain a straight line for such stresses, as the deformation of the section due to the metal having passed its elastic limit will prevent this, even if the coefficient of elasticity in compression were to remain constant for stresses increasing to such amounts. If for this calculation stresses such as are usually allowed were assumed, there would be no great objection to the stress-strain curve in compression being considered as a straight line.

M. Thullie admits that the complications of the formulae on the supposition of the break in the stress-strain curve in compression are hardly justified, and that the results on this supposition are not very different from those obtained when considering the line $OD'A'$ as straight, the assumption he made at the commencement of his studies.

Professor Ostenfeld's method is very similar to that of Professor Von Thullie, except that instead of the stress-strain curve for the concrete in tension being a straight line up to a stress very near to the ultimate resistance, he has obtained from experiments carried out by Herren Grut and Nielsen on the elasticity of concrete in tension, a stress-strain curve formed by two straight lines OC' and $C'B'$ Fig. 311, the stress CC' being taken as 115 pounds per square inch. The coefficient of elasticity for the portion of the curve OC' he takes as $3{\cdot}55 \times 10^6$, being the same as that for the compression side, which he considers constant throughout both phases, not having two values as in Prof. Von Thullie's method. The coefficient for the portion $C'B'$ is taken as $0{\cdot}995 \times 10^6$, the limiting stress BB′ being 200 to 230 pounds per square inch.

If the resistance of the concrete in tension is allowed, this method is

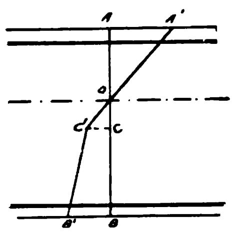


FIG. 311

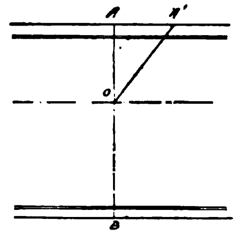


FIG. 312

undoubtedly more correct than Prof. Von Thullie's, but not so correct as M. Considère's.

M. Christophe adopts a straight line stress-strain curve in compression, with a coefficient of elasticity of $2{\cdot}84 \times 10^6$, and allows no resistance of the concrete in tension; his stress-strain curve is therefore that shown in Fig. 312. He assumes two limits of resistance of the concrete in compression of 425 and 710 pounds per square inch according to the method of loading, and allows in a similar way 8,520 and 14,220 pounds per square inch for the reinforcement of wrought iron, and 12,780 and 21,330 if of steel.

There is no doubt that the stress-strain curve of the concrete in compression is not a straight line if the hypothesis of the conservation of plane sections is allowed, but that it closely approximates to a parabolic arc, as shown by table XLVIII.

The assumption of a straight line stress-strain curve doubtless errs on the side of safety, if proper coefficients are employed in the formulae derived from this supposition, but the first failure of a well designed beam is seldom caused by the insufficient resistance to compression, being either due to a failure under tension near the centre of the span or to shearing stresses near the abutments. It appears evident, therefore, that from some cause at present not known, the resistance to compression of reinforced concrete pieces subjected to flexure is comparatively great. The extra resistance caused by the errors of the hypotheses Nos. 2 and 5 below (pp. 263 and 266) on the side of safety, probably accounts partially for the excess of strength in compression, and there may also be some effect such as M. Considère has found for the concrete in tension, the evident insufficiency of tensile resistance making this more noticeable in tension than in compression.

In any case it seems that we may with perfect safety allow that the stress-strain curve of the concrete in compression is parabolic, while assuming the conservation of plane sections to be a true hypothesis, although from the present state of our knowledge neither of these hypotheses is strictly true. It is probable that some resistance is offered by the concrete in tension, although it may be better to ignore it in the calculations, as is done by most authors and all practical constructors, since if cracks occur the concrete in tension can offer no resistance at the crack.

Although much has been accomplished recently to extend our knowledge of the properties of reinforced concrete, we are unfortunately still uncertain as to their true significance, and while it is certain that it is better to err on the side of safety, still there is no reason why we should not take advantage of the facts which have been demonstrated under practical working conditions, and assume an hypothesis which is as likely to be true as any other, and allows us to take some advantage of the practical fact that the beam of reinforced concrete has ample resistance in compression. We shall therefore assume that the stress-strain curve in compression is parabolic (being that shown in Fig. 306), but that the concrete offers no resistance in tension, and beyond this that the maximum tensile strain must not exceed $\frac{1}{1000}$, the value allowed by M. Considère, being $\frac{1 \cdot 5}{1000}$.

Hypotheses Allowed

From the review of the various hypotheses it appears that we can safely assume as a basis from which our calculations may be derived—

1. *That the applied forces are perpendicular to the neutral surface in pieces subjected to simple bending, but not in arches or structures under complex bending.*

2. *That each fibre acts by itself, not being affected by the contiguous fibres.*

3. *That if the reinforcements are disposed with a view of obtaining sufficient homogeneity, we may consider the reinforcements as always being in solid contact with the surrounding concrete, and therefore that both the metal and surrounding concrete are equally deformed.*

4. *That the sections remain true planes during bending.*

5. *That there are no initial stresses, or that with an increasing load commencing from nothing the stresses in the different fibres will also commence from nil.*

6. *That the coefficient of elasticity of the concrete does not remain constant in compression, but that the stress-strain curve is parabolic.*

7. *That the elongation of the concrete in tension must not exceed* $\frac{1}{1000}$, *and that no tensile resistance is offered by the concrete with the reserve that the latter assumption is not adopted for the calculation of the deformations.*

REINFORCED CONCRETE

II.

LOADS, BENDING MOMENTS, SHEARING FORCES, ETC.

General Remarks.—The loads may be applied in practice in many different ways, each one of which requires special treatment, but this must necessarily be left in great measure to the discretion of the designer.

For instance, if the load on a column is greater on one side than another, this fact must be taken into consideration in the calculations for the design, as in addition to the direct pressure there will be a bending moment, and the piece must be treated very much in the same way as an arch, where there is usually a direct thrust and a bending moment. The bending moment in this case is not to be confused with the column flexure, which takes place in a long column even under direct pressure.

Many other cases requiring special treatment will be met with in practice, but the general lines laid down will serve as a guide in the design in all instances.

In the case of pieces subjected to transverse loading, the position and manner of distribution of the load has a great effect on the position and amount of the maximum bending moments and shearing stresses.

Many excellent text-books have been published which deal very thoroughly with this question, and it is only proposed to go very briefly into the subject, the remarks that will be made only covering a few cases which may not be generally well known and referring especially to reinforced concrete.

WEIGHT OF REINFORCED CONCRETE

It is very necessary to assess fairly closely the weight of concrete when reinforced, as in many cases the weight of the structure itself is a very large percentage of the total load. The weight must, of course, depend upon the proportion of ingredients and nature and amount of reinforcement. In the proportions generally adopted for reinforced works the mortar or concrete will have about the following weights.

Mortar of cement and sand	120 to 130 pounds per cubic foot.
Concrete of shingle or broken stone . . .	130 to 140 ,, ,, ,, ,,
Concrete of coke breeze or furnace ashes . .	70 to 80 ,, ,, ,, ,,

If we assume a concrete weighing 140 pounds per cubic foot and the reinforcement of wrought iron at 480 or of steel at 490 pounds per cubic foot, we must add for each 1% of reinforcement $\frac{480-140}{100}$ or 3·4 pounds per cubic foot for wrought iron, and similarly 3·5 pounds per cubic foot for steel.

The amount of reinforcement seldom exceeds from 3 to 4 per cent. *We may therefore safely assume the maximum weight of reinforced concrete of shingle or broken stone to be* 155 *pounds per cubic foot.*

In France the allowance is generally 2,500 kilogrammes per cubic metre, the English equivalent being 155·5 pounds per cubic foot.

LOADS ON COLUMNS AND PILES

Columns.—The loads on columns will be the reactions of the beams which they support. The reactions for ordinary cases are well known.

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CALCULATIONS

The reactions in the case of beams extending over many spans and freely supported at the ends are given below for handy reference although they may be found in most of the well-known text-books dealing with the subject. The form of table is an excellent one and is adopted from Du Bois' *Graphical Statics.*

TABLE L.

Number of spans									
I	$\frac{1}{2}$	$\frac{1}{2}$							
II	$\frac{3}{8}$	$\frac{10}{8}$	$\frac{3}{8}$						
III	$\frac{4}{10}$	$\frac{11}{10}$	$\frac{11}{10}$	$\frac{4}{10}$					
IV	$\frac{11}{28}$	$\frac{32}{28}$	$\frac{26}{28}$	$\frac{32}{28}$	$\frac{11}{28}$				
V	$\frac{15}{38}$	$\frac{43}{38}$	$\frac{37}{38}$	$\frac{37}{38}$	$\frac{43}{38}$	$\frac{15}{38}$			
VI	$\frac{41}{104}$	$\frac{118}{104}$	$\frac{100}{104}$	$\frac{106}{104}$	$\frac{100}{104}$	$\frac{118}{104}$	$\frac{41}{104}$		
VII	$\frac{58}{142}$	$\frac{161}{142}$	$\frac{137}{142}$	$\frac{143}{142}$	$\frac{143}{142}$	$\frac{137}{142}$	$\frac{161}{142}$	$\frac{58}{142}$	
VIII	$\frac{153}{388}$	$\frac{440}{388}$	$\frac{374}{388}$	$\frac{392}{388}$	$\frac{386}{388}$	$\frac{392}{388}$	$\frac{374}{388}$	$\frac{440}{388}$	$\frac{153}{388}$

The numbers in the diamonds, representing the supports must be multiplied by wL to give the reactions.

Piles.—For the load which a pile will have to bear, to be used for calculating the dimensions of the reinforcement, we must take the greatest load which it can have to support during the operation of driving.

This will be $P = Q \times 8\sqrt{\text{fall of ram in feet}}$ [1]

(Q) being the weight of the ram.

The piles must be well guided to avoid flexure.

For the bearing power of piles after they have been driven we may use Rutter's formula, often spoken of as the Dutch formula.

This formula stands—

$$P = \frac{h}{e} \cdot \frac{Q^2}{Q+q} + Q + q \qquad [2]$$

where—

P is the load in pounds.

Q the weight of the monkey in pounds.

q the weight of the pile in pounds.

h the height of fall in inches.

e the last penetration in inches, which may generally be taken as from about 0·25 to 0·10 inches according to the nature of the ground; in ordinary conditions 0·25 is considered sufficient.

The value found for P is the resistance to further penetration and must be reduced for the safe bearing power—a factor of safety of 10 is very generally allowed.

BENDING MOMENTS AND SHEARING FORCES

General Remarks.—In the following discussion L represents the span, W a concentrated load, w a distributed load per unit length, M_C and M_A the bending moments at the centre of the span and over the supports respectively. The units most convenient to adopt are the inch and the pound, the width of strip considered in the case of slabs being one foot. The load is, however, frequently taken as per foot run and the span in feet, the bending moment being multiplied by 12 to bring it to inch pounds.

In the case of a piece supported freely at the ends, the bending moment and shearing forces under ordinary distributed and concentrated loads are well known and need no further remarks.

Piece Built in at Both Ends.—When a piece of uniform section is perfectly built in at the ends, the points of contra-flexure are at distances of 0·21 L and 0·25 L from each support for a uniformly distributed, and a central concentrated load respectively.

There are two maximum bending moments—

For a uniformly distributed load.	For a load concentrated at centre.
$M_C = +\frac{wL^2}{24}$	$M_C = +\frac{WL}{8}$
$M_A = -\frac{wL^2}{12}$	$M_A = -\frac{WL}{8}$

The + *sign* representing those moments which produce a *downward deflection*, and the − *sign* those which produce an *upward deflection*. *These signs will always be used in a similar manner when treating of bending moments.*

The maximum shearing forces are the same as for a freely supported piece, being alike at both ends.

For a uniformly distributed load.	For a load concentrated at centre.
$K = \frac{wL}{2}$ [3]	$K = \frac{W}{2}$ [4]

The shearing force in both cases being nil at the centre of the span.

In reinforced concrete structures the building in can never be considered as perfect, and consequently the above values for the maximum bending moments are incorrect.

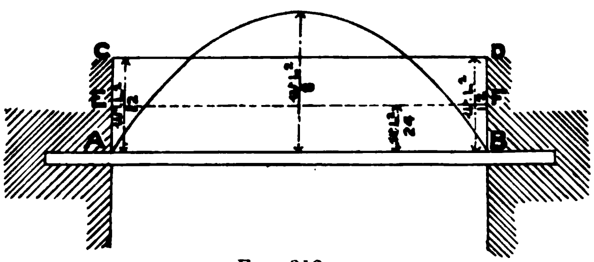


Fig. 313

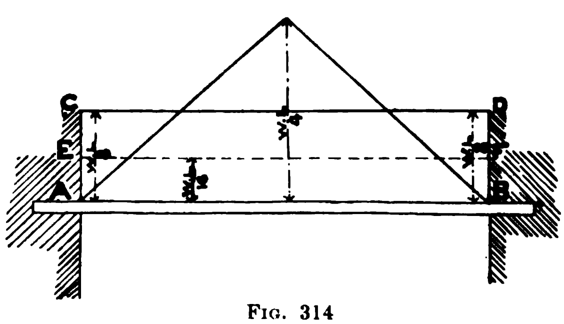


Fig. 314

If we construct the diagrams of bending moments for a freely supported beam as in Figs. 313 and 314, for a uniformly distributed and a centrally concentrated load, the closing line situated in the positions CD is that for a perfectly built-in beam of uniform section. Reinforced concrete beams are never perfectly built in and are seldom of uniform section; we must therefore place the closing line, according to judgment, in some position either horizontally or inclined within the parallelogram $ABCD$.

The exact position will in

reality vary under different circumstances and must be left to the discretion of the designer.

For a beam of uniform *strength* the bending moments are the same as those for a beam of uniform *section* for a centrally concentrated load, but under a uniformly distributed load that at the support is $\frac{3}{32}wL^2$, and that at the centre $\frac{1}{32}wL^2$.

A reinforced concrete beam is neither of uniform section nor of uniform strength when, as is usually the case, it is constructed with no compressive reinforcement and with bent bars. For perfect building in, therefore, the closing line would take some intermediate position.

Many constructors assume the maximum bending moment for a uniformly distributed load as $M = +\frac{wL^2}{10}$ at the centre of the span.

If this value is admitted we must remember that there is still, a reflex action at the supports as can be seen by the diagram (Fig. 313) which will be

$$-\left(\frac{1}{8}-\frac{1}{10}\right) wL^2 \text{ or } -\frac{1}{40}wL^2,$$

Similarly, if we consider the value of M_C for a concentrated central load to be $M_C = +\frac{WL}{6}$ we find that the bending moment over the supports will be

$$M_A = -\frac{WL}{12}$$

but these values are not a sufficient allowance for this bending moment.

If the line EF is placed midway between AB and CD, we get the mean values for the bending moments, between those for a freely supported and those for a perfectly built in beam. It will generally be safe to assume this position, unless circumstances lead the designer to prefer some other and the closing line EF will be assumed throughout the present treatment of the subject.

In which case we get—

For a uniformly distributed load.

$$M_C = +\frac{wL^2}{12} \quad [5]$$

$$M_A = -\frac{wL^2}{24} \quad [6]$$

For a load concentrated at centre.

$$M_C^1 = +\frac{3WL}{16} \quad [7]$$

$$M_A = -\frac{WL}{16} \quad [8]$$

As shown by the dotted closing lines in Figs 313 and 314.

These are the values which are recommended for use in the treatment of pieces of reinforced concrete, subject to the designer's judgment.

Pieces Supported at one End and Fixed at the Other.—If a piece is rigidly fixed at one end and supported at the other the point of contraflexure is at a distance of $0{\cdot}267L$ from the fixed end for a uniformly distributed load, the distance being $0{\cdot}33L$ for a central concentrated load.

The maximum bending moments for a uniformly distributed load are at a distance of 0·367 from the free end and over the fixed support, being $M_{max.} = +\frac{9}{128}wL^2$ and $M_A = -\frac{wL^2}{8}$ respectively.

For a load concentrated at the centre the bending moments are—

$$M_{max.} = +\frac{5}{32}WL \text{ and } M_A = -\frac{3}{16}WL$$

The values for the shearing forces are as follows—

For a uniformly distributed load. For a load concentrated at centre.

At the fixed support $K_{max.} = \frac{5}{8}wL$ $\qquad K_{max.} = \frac{11}{16}W.$

At the free support $K_{max.} = \frac{3}{8}wL$ $\qquad K_{max.} = \frac{5}{16}W.$

If we take the means as before, we get—

For a uniformly distributed load. For a load concentrated at the centre.

$$M_{max.} = +0{\cdot}097wL^2 \quad [9] \qquad M_{max.} = +0{\cdot}203WL \quad [11].$$

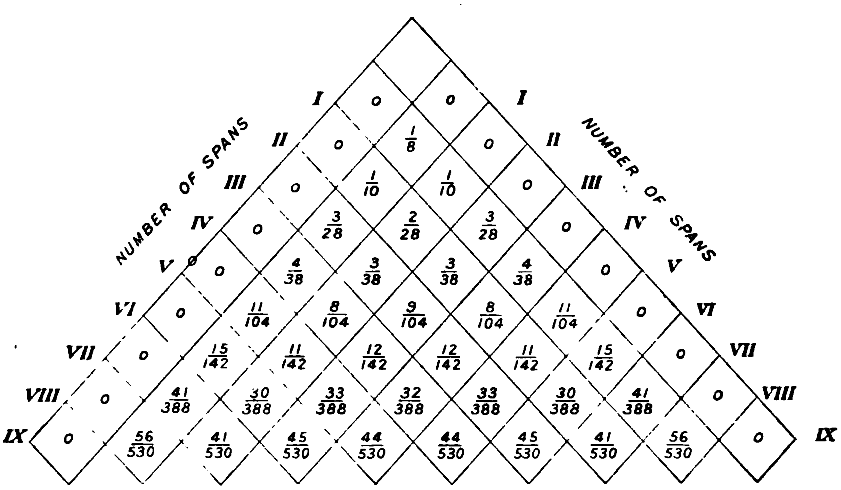
$$M_A = -\frac{1}{16}wL^2 \quad [10] \qquad M_A = -\frac{3}{32}WL \quad [12]$$

At fixed support $K_{max.} = \frac{9}{16}wL$ [13] $\qquad K_{max.} = \frac{19}{32}W$ [15]

At free support $K_{max.} = \frac{7}{16}wL$ [14] $\qquad K_{max.} = \frac{13}{32}W$ [16].

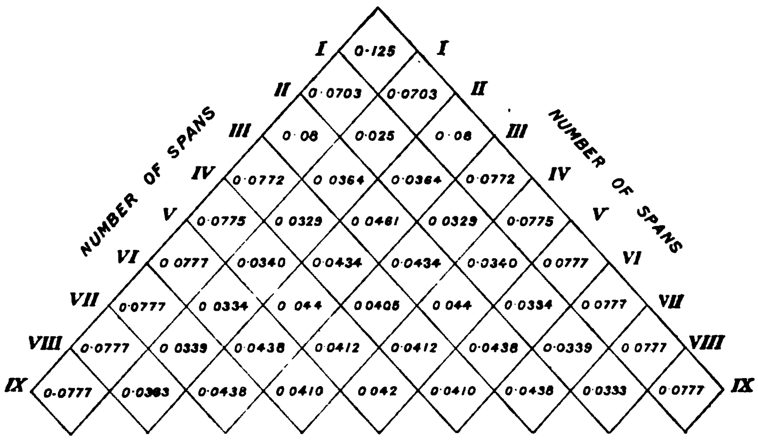
A Piece Extending over Several Supports.—This is the only case where we may consider the building in as perfect over the intermediate supports. Tables LI. and LII. give the bending moments at the supports and the maximum bending moments on the intervening spans for a continuous beam of uniform section freely supported at the ends. The maximum bending moment on the spans will not be at the centre, but will be very near it except in the cases of the side spans. When the beam is built in at the ends, there would be an alteration of the bending moments throughout the whole of the outer spans.

LI.



The numbers in the diamonds, representing the supports, must be multiplied by wL^2 to give the bending moments.

LII.



The numbers in the diamonds representing the spans, must be multiplied by wL^2 to give the bending moments.

By drawing a diagram of bending moments for a free span and erecting the bending moment at the first support given in Table LI. the closing line can be drawn according to the conditions obtaining at the abutment, and from this line the maximum bending moment on the side span may be determined. But usually in practice it will be sufficient if we assume the bending moment at the abutments to be the same as for a single-span built-in beam, i.e. $-\frac{1}{24}wL^2$ in every case, and neglect the alteration of the maximum bending moment on the span itself.

A Load of a Constantly Increasing Intensity from Nil at one End of the Piece to (w) at the Other.—It may be well to mention, before concluding the discussion on bending moments and shearing stresses on beams, the special cases where the load increases from one end to the other of the piece as it applies to rectangular reservoirs above ground level with floor and a roof and also to rectangular silos.

If the piece were freely supported, the maximum bending moment would be at a distance of $0{\cdot}577L$ from the end where the load intensity is nil, and would have a value of

$$M_{maxi} = 0{\cdot}064wL^2.$$

With rigid building in the maximum bending moment on the beam will be at a section at a distance of $0{\cdot}447L$ from the heavily loaded end, and the points of contraflexure will have distances of $0{\cdot}08L$ and $0{\cdot}73L$ from the same end.

The bending moments will be approximately—

At the lightly loaded support $M_{AL} = -0{\cdot}037wL^2$.
At the heavily loaded support $M_{AH} = -0{\cdot}023wL^2$.
At the section at a distance of $0{\cdot}553L$ from the end where the load is nil $M_S = +0{\cdot}033wL^2$.

In the case of an elevated reservoir with floor and roof with a length of wall comparatively great with respect to the depth $w = 62{\cdot}5L$ in pounds per square foot on a strip a foot wide.

As the building in cannot be considered perfect, it is advisable to allow the means between the two cases for the values of the bending moments.

We have therefore in such a case the approximate value of—

For a covered elevated reservoir.

$$M_{AL} = -\ 0{\cdot}02\,wL^2 \quad [17] \qquad M_{AL} = -1{\cdot}25L^3 \quad [20]$$
$$M_{AH} = -0{\cdot}0125wL^2 \quad [18] \qquad M_{AH} = -0{\cdot}78L^3 \quad [21]$$
$$M_S = +0{\cdot}05wL^2 \quad [19] \qquad M_S = +3{\cdot}125L^3 \quad [22]$$

(Equations 20, 21 and 22, will be in foot-pounds on a strip one foot wide, L being in feet must be multiplied by 12 to reduce to inch pounds on a strip one foot wide.)

The shearing forces may be taken as the same as for a freely supported piece, being maximum near the heavily loaded support where it has the value

$$K = \frac{wL}{3} \qquad [23].$$

Slabs.—With a slab either built in or freely supported at the four edges, supposing (B) the smaller span and (L) the longer, the bending moments given for beams must be multiplied by $\frac{L^4}{L^4+B^4}$ where (B) is the span of the beam, and when (L) is the span of the beam the coefficient becomes $\frac{B^4}{B^4+L^4}$

The greatest bending moment will be that for the *shorter span.* In this case the coefficient has the following values—

When L is infinite	$\frac{L^4}{L^4+B^4}$	becomes	1
When $L=B$	,,	,,	$\frac{1}{2}$
When $L=2B$	,,	,,	0·94

When reinforced concrete is under consideration, the stability parallel to the longer side must be provided for with reinforcing bars, and the necessary sections for these bars must be obtained for both spans.

The co-efficient for the *longer span* will have value as below—

When L is infinite	$\frac{B^4}{B^4+L^4}$	becomes	0
When $L=B$	,,	,,	$\frac{1}{2}$
When $L=2B$	,,	,,	0·06

We have, then, for a uniformly distributed load on a built-in slab of one span—using the values of M_C and M_A given for beams on equations [5] and [6],

For the *shorter span.*

$$M_C = +\frac{wB^2}{12} \times \frac{L^4}{L^4+B^4} \qquad [24]$$

$$M_A = -\frac{wB^2}{24} \times \frac{L^4}{L^4+B^4} \qquad [25]$$

For the *longer span.*

$$M_C = +\frac{wL^2}{12} \times \frac{B^4}{B^4+L^4} \qquad [26]$$

$$M_A = -\frac{wL^2}{24} \times \frac{B^4}{B^4+L^4} \qquad [27]$$

For a square slab.

$$M_C = +\frac{1}{24}wL^2 \qquad [28]$$

$$M_A = -\frac{1}{48}wL^2 \qquad [29]$$

For freely supported slabs the bending moments at the centre will be—

For the *shorter span*.

$$M_C = \frac{wB^2}{8} \times \frac{L^4}{L^4 + B^4} \quad [30]$$

For the *longer span*.

$$M_C = \frac{wL^2}{8} \times \frac{B^4}{B^4 + L^4} \quad [31]$$

When the slab passes over several supports.

The values given in Tables LI. and LII. may be used, remembering that for the shorter span the tabular coefficients must be multiplied by $w\,B^2$ and $\frac{L^4}{L^4 + B^4}$, while for the longer span they must be multiplied by $w\,L^2$ and $\frac{B^4}{B^4 + L^4}$.

When using these values, however, it must be remembered that the slab is not absolutely fixed at the supports by reason of the deflection of the *beams*, but this deflection is so slight that for practical purposes we may neglect it.

The subject of the bending moments on slabs is not very thoroughly dealt with in any English book on applied mechanics commonly in use. Mr. W. Dunn, however, in an article published in the *Journal of the Royal Institute of British Architects*, May 26, 1900, has gone very carefully into this. An extract from his paper is given as a footnote, with the hope that it may be interesting.

For a flat square plate supported on all edges and uniformly loaded, the bending moment on the plate is given as half that due to the same loading on the same plate supported as a beam at two opposite edges only.

This assumes that the opposite sides act independently; or that, while a small square particle in the centre of a simple beam is under bending stress on two opposite faces only, in the plate there are equal stresses on all four faces.

The maximum bending moment is: for the beam $\frac{\mathbf{W}\,l}{8}$*; for the plate one half or $\frac{\mathbf{W}\,l}{16}$.

The maximum bending moment for the beam fixed at ends is $\frac{\mathbf{W}l}{12}$; the maximum bending moment for the plate fixed at all edges is as before, one half or $\frac{\mathbf{W}\,l}{24}$.

The maximum bending stress on the oblong rectangular plate uniformly loaded may be obtained as follows: The maximum stress is on the particle in the centre of the plate, and the tendency is to split along the long axis, which we shall call L. The short axis we shall call B.

Consider first one elemental strip B in the centre of the length as a simple beam of the span B, of unit width; of a depth equal to the thickness of the plate and under an uniform load equal to w_B per lineal unit. The deflection of this simple beam in the centre, by the ordinary formula, would be

$$\Delta_B = w_B B^4 \frac{5}{384\,E\,I}.$$

Consider next the similar elemental strip L, in the centre of the width, as a simple beam of span L, of unit width, and of a depth equal to the thickness of the plate. This simple beam bears an uniform load equal to w_L per lineal unit. Its deflection would be

$$\Delta_L = w_L L^4 \frac{5}{384\,E\,I}.$$

Now as these two elemental strips are part of the plate, the deflection at the centre of each beam must be equal, and as the modulus of elasticity E and the moment of inertia I are by our hypothesis the same for both beams, it follows that

$$w_B B^4 \frac{5}{384\,E\,I} = w_L L^4 \frac{5}{384\,E\,I}, \text{ or } w_B B^4 = w_L L^4.$$

* *For explanation of the symbols used in this note, vide page* 286.

Now the *total* load W per square unit is equal to the sum of these two.

$$W = w_B + w_L.$$

In other words, of the total load per square unit W, one portion w_B is borne by the sides and one portion w_L is borne by the ends.

We proceed as follows—

$$w_B B^4 = w_L L^4.$$

Adding $w_B L^4$ to both sides we have—

$$w_B B^4 + w_B L^4 = w_L L^4 + w_B L^4.$$
$$w_B (B^4 + L^4) = L^4 (w_L + w_B).$$
$$w_B (B^4 + L^4) = L^4 W.$$
$$\therefore \frac{L^4 W}{(B^4 + L^4)} = w_B.$$

We have here w_B in terms of the total load per square unit, the width and the length.

Now the bending moment of the elemental strip or beam B is, by the ordinary formula,

$$\text{Bending moment of } B = \frac{w_B B^2}{8}.$$

Substituting the last value of w_B, this becomes

$$= \frac{W L^4 B^2}{8 (B^4 + L^4)}.$$

These formulae involve the fourth power of numbers and are troublesome to use. I have therefore prepared a diagram by the aid of which the bending moments in rectangular plates uniformly loaded and supported on all edges may be obtained as readily as the bending moments on simple beams.

To find the bending moments from this diagram,

1. Divide the length of the plate by the breadth.
2. Trace the vertical line corresponding to this result from the scale at the foot of the diagram to intersection with the curved line.
3. Read from the scale at the left-hand side the value of the horizontal through the point of intersection.
4. Multiply the total load on the plate by the breadth of the plate, and by this number and divide by eight. This will give the bending moment (in terms of the units employed, i.e. inch-lb., foot-lb., foot-cwt., etc., as the case may be).

Now $\frac{\mathbf{W} B}{8}$ is the bending moment on the plate acting as a simple beam of span B not supported at the ends L, and the diagram shows clearly that as the length becomes greater in proportion to the breadth of the plate, the strength approximates more to that of a similar plate supported at the sides only. The bending moment of a square plate is ·5 of the bending moment of a slab of similar size and under similar load supported at two sides only. But the bending moment of a plate whose length is twice its breadth, is ·941 of the bending moment of a similar slab supported at the sides only. Accordingly, when the length becomes twice the breadth, it is an error on the side of safety to calculate the strength of such a plate as that of a simple beam supported at the sides only.

If we now consider the case of a plate fixed at all edges and not merely supported, we should find the deflection Δ_B of an elemental strip B under an uniform load w_B per lineal unit, according to the usual formula for the deflection of a beam fixed at the ends, to be

$$\Delta_B = \frac{w_B B^4}{384 E I}.$$

From this we find as before,

$$w_B = \frac{L^4 W}{(B^4 + L^4)},$$

and as the bending moment for an elemental strip B considered as a beam fixed at the ends is—

$$\text{Bending moment} = \frac{w_B B^2}{12},$$

on substituting the value of w_B from the previous equation, we have

Maximum bending moment on the elemental strip $= \frac{W L^4 B^2}{12 (B^4 + L^4)}$.

The diagram may be used to find the bending moment in this case also, taking $\frac{\mathbf{W} B}{12}$ or $\frac{\mathbf{W} B}{24}$ as the multiplier of the vertical scale,

VALUES OF $\frac{L^4}{L^4+B^4}$

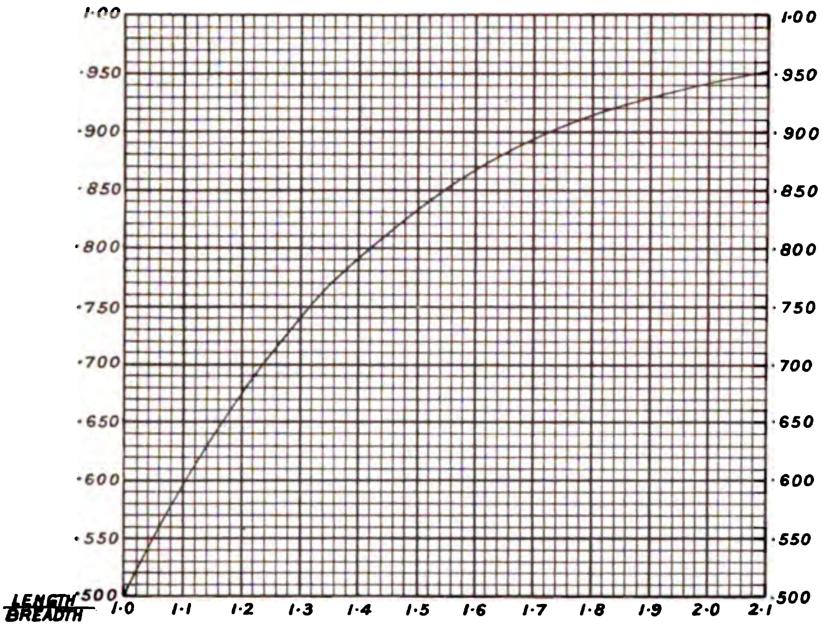


Fig. 315

VALUES OF $\frac{B^4}{B^4+L^4}$

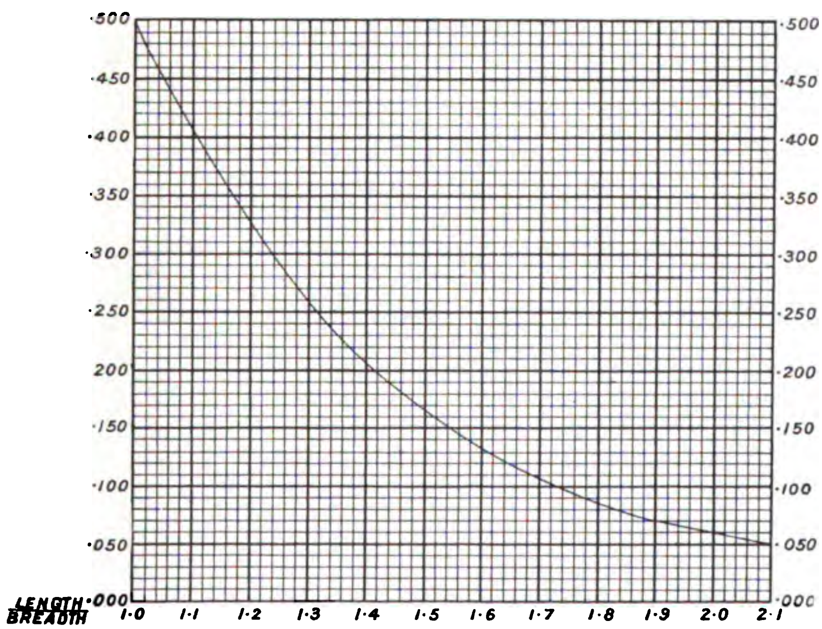


Fig. 316

It may be well to refer for a moment to the question of fixed ends.

Perfect fixing is practically only obtainable in the case where the concrete is continuous over the tops of the supporting beams or walls. In such a case the maximum stress occurs at the edges and not at the centres of the slabs. This may account for the cracking over the beams in slabs of concrete imperfectly fixed.

W = total uniformly distributed load on a beam or rectangular plate.
W = load per square unit uniformly distributed.
$\left.\begin{matrix} w_B \\ w_L \end{matrix}\right\}$ as described in body of paper.
l = length of any beam.
E = modulus of elasticity.
I = moment of inertia of a beam.
B = breadth of rectangular plate.
L = length of rectangular plate.

NOTE.—Any system of units may be used. Thus W w_B, w_L and E may be in pounds per square inch, and **W** in pounds; B, L may be in inches, and I in quadric inches: in this case the bending moments will be found in inch-pounds.

Mr. Dunn's diagram is given (Fig. 315) and a similar diagram for the values of $\frac{B^4}{B^4+L^4}$ is given in Fig. 316. These diagrams will prove very useful by reason of the simplification of the necessary calculations for the bending moments on slabs.

The shearing forces may without much error be deduced by allowing the same coefficients of reduction as used for the bending moments.

For a uniformly distributed load the shearing forces will have the following values close to the support at the middle of the longer side—

$$K=\frac{1}{2}wB\times\frac{L^4}{L^4+B^4} \qquad [32]$$

and at the middle of the shorter support—

$$K=\frac{1}{2}wL\times\frac{B^4}{B^4+L^4} \qquad [33]$$

For a square slab—

$$K=\frac{1}{4}wL \qquad [34]$$

WIND PRESSURE

General Remarks.—The wind pressure will vary somewhat according to the structure. It would be greater on a plain wall exposed on the front and rear face than on a building with four sides, on account of the vacuum which is produced behind a thin structure by the force of the wind.

As a rule, in the case of reinforced concrete the structure is not of a thin nature compared with its length, and we may take from 30 to 40 pounds per square foot as the maximum pressure in any but very exposed situations, or where the configuration of the land tends to concentrate the wind on the structure under consideration.

In such situations the limit should be increased to 50 or 55 pounds per square foot.

For Isolated Erections such as chimneys, telegraph poles, towers, and structures of a similar nature the treatment must be that for a piece subject to combined direct stress and bending, for which the formulae are given (p. 399).

The pressure of the wind is reduced when it acts on inclined surfaces ; for a *spherical surface* the area of the vertical axial plane must be multiplied by 0·41, for a structure *circular in plan* the area of the vertical axial plane must be reduced by multiplying by 0·50, for an *octagonal plan* the multiplier will be 0·56, and for an *hexagonal plan* it will be probably about 0·66. As a general rule the centre of action of the wind pressure will be at the centre of gravity of the area of the plane on the vertical axis above the section under consideration. In the case of telegraph poles there will be two centres of pressure, one for the pole itself (the bending moment due to the force acting on this is small), and the other that due to the force on the wires. The centre of gravity for the piece under consideration may be found by calculation, or by cutting out a piece of cardboard or paper of uniform thickness, and suspending it consecutively from two corners and drawing vertical lines from the points of suspension—where these lines cross is the centre of gravity of the piece. Having found the centre of pressure, the bending moment will be the total force due to the wind pressure multiplied by the height from the section under consideration to the centre of pressure ; the total force and the centre of pressure being those for the area above the section in question. It will be noticed when reference is made to the formulae for pieces subjected to direct stress and bending combined, that only solid rectangular pieces have been considered. When the piece is hollow, as in the case of a chimney, or of other than a rectangular cross-section, the formulae will require slight alteration, but the same reasoning will apply. In such a case, however, it will greatly simplify the formulae if we consider the stress-strain curve of the concrete in compression as a straight line, and the result will be nearly the same as if we were to consider it as parabolic, and the error will be on the side of safety.

With a straight line stress-strain curve the altered formulae will be easily deduced in the same manner by which the equations for a solid piece have been derived (pp. 397 to 406). The neutral line will still be at the centre of the piece, but, for a hollow section, the resistance of the two portions of concrete, in the case where no tensile stresses are exerted, and the one portion where these stresses are induced, will each act at its own centre of gravity.

When a Building of the Usual Kind is under consideration it is generally unnecessary to inquire into its stability against wind pressure, as this is amply provided for. The end walls will in this case act as cantilevers, and the floors and roof as deep girders, connecting the exposed face to them, in a perfectly rigid manner.

In the case of buildings with many windows or rectangular openings in the end faces, it will be necessary to provide special reinforcements, at the junctions of the vertical and horizontal framings, to strengthen these parts against any turning effect produced by the wind pressure on the exposed face. This is usually done by placing inclined rods at the angles.

REINFORCED CONCRETE

III

FORMULAE

DIRECT COMPRESSION—PIECES WITH LONGITUDINAL REINFORCEMENTS

Pieces Reinforced with Small Sections.—A transverse section of the piece is considered. This is displaced in a direction parallel to itself by the application of a load P.

We have therefore—

$$P = c\Delta + f\omega \quad [1].$$

As the displacement of the concrete and reinforcements are the same, we have the relation—

$$\frac{c}{E_c} = \frac{f}{E_f} \text{ or } f = c\frac{E_f}{E_c} \quad [2].$$

$$\text{And we have allowed that } m = \frac{E_f}{E_c} = 10 \quad [3].$$

We have $f = mc$ [4],[1]
and $P = c\,(\Delta + m\omega)$ [5].

When it is required to check a piece already designed, c is first found from [5], after which the value for f is obtained from [4].

If it is necessary to design a column or other piece under direct compression and to be reinforced with small sections, we know the value of P, and we decide on the working stress of the concrete.

We obtain the value of f or the working stress of the reinforcement from [4].

Sometimes the sectional area of the piece has been decided on from practical considerations, in which case we can find the sectional area of the reinforcements from equation [5], which may be written—

$$m\omega = \frac{P}{c} - \Delta \quad [6].$$

If we do not know the value of Δ, we must give a value to $\frac{\omega}{\Delta}$, i.e. the percentage of reinforcement. Suppose $\frac{\omega}{\Delta} = \psi$ and p being the unit stress the piece would bear if it were homogeneous,

$$p = \frac{P}{\Delta} \quad [7].$$

Taking $m = 10$ we have from equation [5]

$$p = c\,(1 + 10\,\psi) \quad [8].$$

The sectional area of the reinforcements is found by—

$$\omega = \psi\,\Delta \quad [9].$$

Allowing a working unit stress on the concrete of 400 pounds per square inch, we get from equation [8] the values of p given in table on next page.

[1] It will be seen that for pieces under direct compression the stronger the concrete the greater will be the economy due to the presence of the reinforcements.

TABLE LIII [1]

Percentage of Reinforcement	Values of p for c = 400
½	420
1	440
2	480
3	520
4	560
5	600

For long columns of a length of more than 20 diameters it is necessary to check the strength for the resistance to flexure. For this purpose we may employ Euler's formulae, where the pressure $P = \pi^2 \dfrac{IE}{(kL)^2}$ [10].

Here I is the moment of inertia of the column.

E the coefficient of elasticity.

L the total length.

k a coefficient depending on the manner of fixing the ends of the column.

In the case of a rectangular column of reinforced concrete

$$I = \frac{bd^3}{12} + m\omega y^2 \quad [11],$$

y being the distance of the centres of the bars from the axial plane of the column.

E in equation [10] may, when this value of I is inserted, be taken as the coefficient of elasticity of the concrete, or $2{\cdot}85 \times 10^6$.

Equation [10] will therefore take the form of

$$P = \pi^2 \frac{\left(\frac{bd^3}{12} + 10\,\omega y^2\right) 2{\cdot}85 \times 10^6}{(kL)^2} \quad [12].$$

The values of P derived from this equation are the *greatest loads* the column will bear without breaking, and therefore we must use a factor of safety to obtain the *safe loads*. This factor should not be less than 4. The following table gives the values of k, and the equation for the *safe load* corresponding to the several methods of fixing the ends of the column, the factor of safety allowed being 4.

TABLE LIV

Methods of Fixing the Ends of the Column	Values of k	Equations for the Safe Loads, allowing a Factor of Safety of 6
Both ends rounded	1	$2{\cdot}48 \dfrac{\left(\frac{bd^3}{12} + 10\,\omega y^2\right) 2{\cdot}85 \times 10^6}{L^2}$
Both ends fixed	$\dfrac{1}{2}$	$9{\cdot}87 \dfrac{\left(\frac{bd^3}{12} + 10\omega y^2\right) 2{\cdot}85 \times 10^6}{L^2}$
One end fixed and the other rounded	$\dfrac{1}{\sqrt{2}}$	$4{\cdot}94 \dfrac{\left(\frac{bd^3}{12} + 10\omega y^2\right) 2{\cdot}85 \times 10^6}{L^2}$
One end fixed and the other free	2	$0{\cdot}62 \dfrac{\left(\frac{bd^3}{12} + 10\omega y^2\right) 2{\cdot}85 \times 10^6}{L^2}$

[1] The values for intermediate percentages can be found by interpolation, since the curve of the values of p is a straight line.

If a higher factor of safety is decided upon the co-efficients must be reduced in proportion.

For square columns $b = d$ and the expression $\frac{bd^3}{12}$ in the above equations becomes $\frac{d^4}{12}$.

For circular columns the value of I will be $(0{\cdot}0491\, d^4 + 10\, \omega\, y^2)$ where d is the diameter of the column.

This value replaces the expression $\left(\frac{bd^3}{12} + 10\omega y^2\right)$ in the above equations.

In practice b is seldom less than $\frac{1}{20}L$ *for rectangular columns, nor d less than* $\frac{1}{20}L$ *for circular columns, in which case there is no need to calculate the safety against flexure.*

Pieces Reinforced with Large Sections. — When the reinforcements are of large sectional areas they will take up a considerable percentage of the area of the whole piece, and it is necessary to take into consideration this reduction of area.

The area of the concrete will therefore be $(\Delta - \omega)$, and equation [1] becomes

$$P = c\,(\Delta - \omega) + f\omega \quad [13].$$

We have also in place of equation [5]

$$P = c\,\{\,\Delta + (m-1)\omega\} \quad [14].$$

The relation in equation [4] remaining the same, or

$$f = mc \quad [15].$$

When it is required to check a piece already designed, c is found from equation [14], after which f can be deduced from equation [15], the value of m being 10. In designing a structure with this system of reinforcement, as before, we know P and decide on the unit stress c, the value of f may be deduced from equation [15].

If the section of the piece has been decided upon we can determine the value of ω from equation [14], which can be written

$$(m-1)\,\omega = \frac{P}{C} - \Delta \quad [16].$$

If we do not know the sectional area of the piece we must, as before, give a value to $\psi = \frac{\omega}{\Delta}$, and taking p as the unit stress the piece would bear if it were homogeneous, we have in place of equation [8]—

$$p = c\,(1 + 9\,\psi) \quad [17].$$

The sectional area of the reinforcements is given as before by

$$\omega = \psi\,\Delta \quad [18].$$

Allowing a working unit stress on the concrete of 400 pounds per square inch, we get from equation [17] the values of p given in Table LV.

To check the column for resistance to flexure we again use Euler's formulae which in this case may be written—

$$P = \frac{\pi^2 \{(I_c - I_f) + mI_f\}}{(kL)^2} E_c \quad [19].$$

Where I_c is the moment of inertia of the whole section of the column I_f, the least moment of inertia of the reinforcement with reference to the axial plane of the column, and E_c the co-efficient of elasticity for the concrete. I_c, as before, will have the following values—

Square column $\frac{d^4}{12}$.

Rectangular column $\frac{bd^3}{12}$.

Round column $0{\cdot}0491d^4$, where d is the diameter.

TABLE LV [1]

Percentage of Reinforcement	Values of p for c = 400
2·5	490
3	508
4	544
5	580
6	616
7	652
8	688
9	724
10	760

Table LVI. below gives the moments of inertia of several different sections about their own axis. To this must be added the area of the section multiplied by the square of the distance of its centre of gravity from the axial plane of the column for the value of I_f. For several reinforcements I_f may have different values for each reinforcement, in which case these values must be added together for the value of the moment of inertia of the total metal, I_f being replaced by ΣI_f.

The equations for the *safe load* given in Table LIV. will stand, if the expression $\left(\frac{bd^3}{12} + 10\omega y^2\right)$ be replaced by $(I_c + 9I_f)$.

When the load acts on the column with any eccentricity, the calculations must be made as for a piece subjected to compound flexure (p.p. 397 to 406), the distance of the point of action from the axis of the column being the same in all sections.

DIRECT COMPRESSION—PIECES IN WHICH LONGITUDINALS AND HOOPING ARE EMPLOYED.[2]

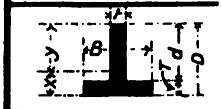
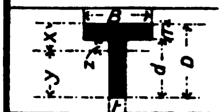
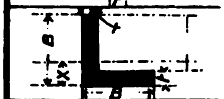
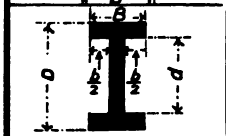
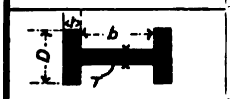
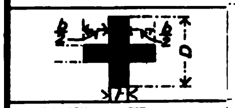
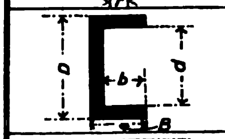
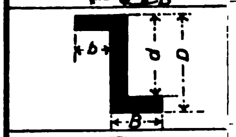
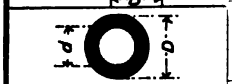
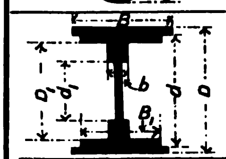
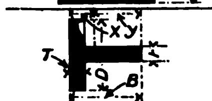
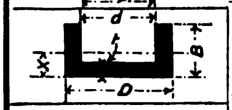
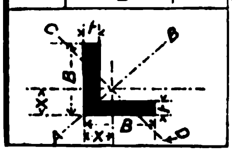
General Remarks.—This form of construction for pieces under direct compression has not been employed to any great extent up to the present.

The introduction of this improved method of construction is mainly due to M. Considère, who published the results of his researches and the conclusions arrived at in *Le Génie Civil* in December, 1902, and January, 1903, an extensive

[1] The values for intermediate percentages can be found by interpolation, since the curve of the values of p is straight line.

[2] M. Considère has patented his method of hooping for compression members.

TABLE LVI

FORM OF SECTION	MOMENT OF INERTIA AND DISTANCE TO NEUTRAL AXIS
	$x = \left\{\frac{BT^2}{2} + td\left(\frac{d}{2} + T\right)\right\} \div (BT + td)$ $I = \frac{BT^3}{12} + BT\left(x - \frac{T}{2}\right)^2 + \frac{td^3}{12} + td\left(D - \frac{d}{2} - x\right)^2$
	$y = \frac{d}{2} + \frac{\frac{1}{2}BDT}{BT + dt}$ $I = \frac{B(x^3 - z^3) + t(y^3 + z^3)}{3}$
	$x = \frac{B^2 + (B - t)t}{4B - 2t}$ $I = (B - t)\frac{t^3}{12} + (B - t)t\left(x - \frac{t}{2}\right)^2 + \frac{tB^3}{12} + Bt\left(\frac{B}{2} - x\right)^2$
	$I = \frac{BD^3 - bd^3}{12}$
	$I = \frac{2tD^3 + bT^3}{12}$
	$I = \frac{td^3 + bt^3}{12}$
	$I \doteqdot \frac{BD^3 - bd^3}{12}$
	$I = \frac{BD^3 - 8b(d - \frac{1}{2}D)^3}{12}$
	$I = \frac{\pi}{64}(D^4 - d^4)$
	$I = \frac{B(D^3 - d^3) + B_1(d^3 - D_1^3) + b(D_1^3 - d_1^3) + td_1^3}{12}$ I APPROX $= \frac{A_T A_C}{A_T + A_C} D$ A_T = area tension flange A_C = area compression flange D = mean effective depth
	$I = \frac{TD^3}{12} + \frac{(B - T)t^3}{12}$
	$x = \frac{\frac{Dt^2}{2} + \frac{(D - d)B^2}{2}}{td + (D - d)B}$ $I = \frac{dt^3}{12} + dt\left(x - \frac{t}{2}\right)^2 + \frac{D - d}{12}B^3 + B(D - d)\left(\frac{B}{2} - x\right)^2$
	$x = \frac{B^2 + (B - t)t}{4B - 2t}$ I max. :– axis $AB = \frac{B^4}{24} - \frac{(B - 2t)^4}{24} + t^2\left(\frac{B - t}{2}\right)^2$ I min. :– axis $CD = \frac{B^4 - (B - 2t)^4}{2} + 2B^2\left(\frac{B}{3} - x\right)^2 - 2(B - 2t)^2\left(\frac{B + t}{3} - x\right)^2 + 2t^2\left(\frac{B}{2} - x + \frac{t}{6}\right)^2$

extract of which appeared in the *Engineering Record*, Vol. 46, Nos. 25 and 26, and Vol. 47, Nos. 2-4. The following discussion has been obtained from these two publications—

M. Considère's experiments on "Hooped Concrete" have already been discussed (page 240). They covered the various points which had to be known to determine the values of the coefficient of elasticity and the compressive resistances of pieces reinforced by hoops and longitudinals.

Very little was said in respect to the elastic limit, it only being shown that it can be raised at will to any amount desired, even up to the compressive resistance by testing the piece under a sufficient test-load.

The value of the elastic limit is therefore unimportant for structures made of pieces which have been subjected to test loads exceeding the working load (page 246). For those structures however which are moulded in place, the elastic limit is of importance and must receive attention.

The elastic limit under a first load evidently depends on that of the concrete which is reached before that of the metal. The shortening of the concrete is much increased under higher pressures, and hence the practical elastic limit is reached when the shortening attains 0·8 to 0·13 per cent., according to the nature of the concrete.

Experiments have shown that the resistance of concrete in hooped pieces exceeds by about 50 per cent. that of the same concrete not reinforced, but it is prudent to neglect this fact. The further fact will also be neglected; that the longitudinal rods undergo, before the failure of hooped pieces, considerable contractions which are much greater than the elastic shortenings of the metal, and which will therefore produce resistances much above the elastic limit.

Neglecting these two facts, therefore, the following rules are obtained—

Coefficient of Elasticity of Hooped Concrete. — 1: — For the first load the coefficient of elasticity of a hooped piece is equal to the sum of the coefficients of the concrete, of the longitudinal rods, and of the imaginary longitudinals whose volume may be assumed as 90 per cent. of that of the hoops (page 247). 2:— For pressures less than a previous load, the coefficient of elasticity will be equal to the sum of the coefficients of the concrete as increased by the first load of the existing longitudinal rods, and of imaginary longitudinals whose volume may be assumed as *double* that of the hooping (page 247).

Elastic Limit.—The elastic limit of a hooped piece for a first load is equal to the natural elastic limit of the concrete increased by the resistance of the reinforcement as found for a shortening of 0·08 to 0·13 per cent., and computed on the basis indicated above for the coefficients of elasticity under a first load.

Every load has the effect of making the final elastic limit practically equal to the pressure due to that load (page 246).

Compressive Resistance.—The compressive resistance of a hooped piece *exceeds* the sum of the following three elements—1. The compressive resistance of the concrete without reinforcement. 2. The compressive resistance of the longitudinal rods stressed to their elastic limit. 3. The compressive resistance which would have been produced by the imaginary longitudinals at the elastic limit of the hooping metal, the volume of the imaginary longitudinals being taken as 2·4 times that of the hooping (page 240).[1]

[1] This can be expressed algebraically in the form—

$$P—\Delta c + w_L f_L + 2{\cdot}4 w_H f_H \qquad [19a].$$

Where w_L. w_H f_L and f_H refer to the sectional areas and elastic limits of the longitudinal and hooping reinforcements respectively. This value of P must be reduced by a factor of safety (vide p. 296.)

Resistance to Flexure.—It is not sufficient for a piece under direct compression to have adequate crushing strength, it must also resist the lateral flexure.

Euler's formula gives the resistance p of a member whose coefficient of elasticity is E, the length between hinges L, and the least radius of gyration r or $\sqrt{\dfrac{\text{I}}{\text{sectional area}}}$

$$p=\pi^2 \frac{r^2E}{L^2} \quad [20].$$

M. Considère states that this formula is exact only for very long pieces of little resistance, and does not agree with the results obtained on columns of diameters usually met with in practice. In his report to the Congress on "Methods of Construction" of 1889, and to the French Commission on Methods of Testing, 1902, M. Considère showed that Euler's formula is exact for iron columns of any dimensions if for the coefficient of elasticity, that value is introduced which it has when the column is under flexure, and not that corresponding to a light load.

Such an interpretation does not allow of the direct solution for p, since the formula contains a value for E which is itself dependent on p; but the formula may be written in the form

$$\frac{r}{L}=\frac{1}{\pi}\sqrt{\frac{p}{E}} \quad [21],$$

and in this form can be used to determine the value which shall be given to the ratio $\frac{L}{r}$ in order that the column resistance shall have the value p. It is therefore only necessary to introduce varying values of p with their corresponding coefficients of elasticity, from which a table of the values of $\frac{L}{r}$ can be made.

In applying Euler's formula to concrete it is also necessary to remember that the formula is based on the assumption that a loaded column has an indefinitely small curvature, and is in equilibrium under the action of the pressures which pass through the centres of gravity of its ends. There must therefore be equilibrium between the moment of resistance due to the bending and the bending moment caused by the load, which is equal to the product of the load by the initial deflection.

The moment of resistance consists of two parts—

1. The *increases in pressure* caused in the fibres on the concave side of the piece, by the increased shortening due to flexure.

2. The *decreases in pressure* of the fibres on the convex side whose shortenings are reduced by the bending. The coefficient of elasticity for (1) is the coefficient of the material under a first load, if the column has not been loaded before, and for (2) the coefficient for an unloading or decreasing load. We have therefore two coefficients in place of the one in Euler's formula.

These two coefficients are almost equal under light loads, and under increase of the load they diverge until the difference becomes very great when the elastic limit is exceeded.

Denoting the ratio of the smaller to the greater coefficient by n, and the distance from the neutral axis to the extreme fibre having the less coefficient of elasticity by x, and the total depth of the section being taken as unity. If the two components of the couple are equal we have $nx^2 = (1-x)^2$ or $x = \dfrac{1}{1+\sqrt{n}}$ [22].

The distance between the two forces of the couple being always $\frac{2}{3}$, the total width being unity, independent of the value of x, their moment is proportional to either of them, for example to $nx^2 = \dfrac{1}{\left(1+\frac{1}{\sqrt{n}}\right)^2}$ [23].

It results from equation [22] that—

When	$n =$	1	0·50	0·25	0·09
	$x =$	0·5	0·586	0·667	0·77
and therefore	$x^2 =$	0·25	0·344	0·445	0·593
and from equation [23]	$n\,x^2 =$	0·25	0·172	0·111	0·053
or moments are proportional to		1	0·688	0·444	0·21

since when the coefficients of elasticity are equal, that value for E must be used in Euler's formulae.

When n is near to unity the average value of the co-efficients of elasticity can be introduced into Euler's formula without using equation [23]. We can also obtain instructive figures without making the table mentioned above.

M. Considère found by plotting the results of the experiments on prisms 7, 8 and 9, Table XXXIX. (page 245), that for the pressure of 3,270 pounds per square inch the coefficients of elasticity varied between $2{\cdot}845 \times 10^6$ and $3{\cdot}57 \times 10^6$ under the first loading, and between $5{\cdot}69 \times 10^6$ and $7{\cdot}11 \times 10^6$ during the unloading.

If the lowest values of these coefficients be taken, i.e. $2{\cdot}845 \times 10^6$ and $5{\cdot}69 \times 10^6$, $n = 0{\cdot}50$—the value of E for insertion in equation [21] will therefore be $5{\cdot}69 \times 10^6 \times 0{\cdot}688 = 3{\cdot}91 \times 10^6$. The value of $\dfrac{r}{L}$ thus obtained from [21] is 0·0091. For cylindrical pieces this value corresponds to $L = 27$ diameters.

In other words, in order that a hooped concrete column should have under a first load a column resistance of 3,270 pounds per square inch, the length of the column from centre to centre of " hinges " must not exceed 27 diameters.

Higher values than 3,270 pounds per square inch can be obtained under the *first* load only by appreciably reducing the length of the column, since above this *value* the coefficient of elasticity rapidly decreases. It should however be remembered that the resistance also increases rapidly with a decrease in length, being proportional to $\left(\dfrac{r}{L}\right)^2$.

Much higher resistances can be found for members which have been preliminarily subjected to a sufficient test load.

Thus for a load of 6,400 per square inch an average coefficient of elasticity exceeding $4{\cdot}83 \times 10^6$ can be expected, and the Euler formula shows that to obtain the above high value the greatest length of a circular column must not exceed 22 diameters.

Round ended columns are rarely met with in practice, and the lengths could therefore somewhat exceed the limits named. The effective lengths with various end connexions are given by Mr. Claxton Fidler as L, if the column has rounded or hinged ends, $\frac{6}{10}L$ if the column has fixed ends, and $\frac{8}{10}L$ if the column has one end fixed and the other rounded, L being in each case the total length of the column.

The values above of 3,270 pounds per square inch for a first load and 6,400 pounds per square inch after a preliminary test load, are the limits of resistance, and consequently a margin of safety must be left for working loads.

Factor of Safety for Hooped Pieces.—M. Considère is of the opinion that logically taking all the properties of hooped concrete into consideration, a factor of safety of 2 to 2·5 would not be unreasonable But hooped concrete being a novel method of construction, he proposes for the present a *factor of safety of* 3 *to* 3·5.

M. Considère points out that in reinforced concrete structures subjected to bending the factor of safety often only amounts to 2, and still the structure shows no signs of yielding; the iron being stressed in tension to between 11,000 and 14,000 pounds per square inch, and sometimes still more, whereas the elastic limit is only 23,000 to 25,000 pounds per square inch. He therefore considers that taking into account the much greater reliability of hooped concrete, this provision of a factor of safety of 3 to 3·5 is very much on the side of safety.

The concrete in pieces moulded in place with these factors of safety is only stressed to from 935 to 1,090 pounds per square inch, and in pieces which have been subjected to a test load the stresses in the concrete will be from 1,850 to 2,135 pounds per square inch.

The proportions used for the concrete in the tests from which the above values were taken was 1,000 pounds of cement to 0·9 cubic yards of shingle of sizes between $\frac{1}{5}$ of an inch and 1 inch, and 0·3 cubic yards of sand screened through a $\frac{1}{5}$-inch mesh.

M. Considère recommends these proportions for hooped concrete pieces in consequence of the great increase in resistance of concrete mixed in these proportions over that with 500 pounds of cement, which is shown by the results of the experiments on prisms 7, 8 and 9, Table XXXIX., of which the concrete was of the former proportions, and of prism 10 in the same table, of which the concrete was of the proportions of 500 pounds of cement to 0·9 of shingle and 0·3 sand.

Proposed Formulae for Hooped Concrete Pieces under Direct Compression.—The results derived by M. Considère from his experiments on hooped compression pieces do not give any method for the calculation of the diameter of the hooping wires, neither do they take into account the area of the longitudinal reinforcement which is necessary for the distribution of the stresses, due to the swelling of the concrete, onto the spiral windings. M. Considère only states that the compressive resistance of a hooped piece is made up of the resistance of the concrete together with that of the longitudinals stressed to their elastic limit and resistance of an imaginary amount of metal as longitudinals of 2·4 times the area of the hooping wire.

From this statement it might be thought that if the diameter of the hooping wire was increased to any amount, we might obtain a resistance from it of the same amount as would be derived from longitudinals of 2·4 times its area. It is evident, however, that such would not be the case, and it would be surely better to consider the hoopings only so far as they resist the swelling of the concrete, giving it the power of following up the deformations of the longitudinal reinforcements, without failing.[1]

We may obtain data which will give us the necessary sectional area of the

[1] M. Considère's later experiments on hooped members (vide appendix) show that the resistance given by his formula is true within very wide limits.

hoopings and longitudinals from such experiments as those of Mr. Dunn, where the piece was tested until the hooping wire failed.

We have from Mr. Dunn's experiment on a hooped concrete column (p. 250), that the failure occurred under a load of 2,815 pounds per square inch on a column wound with a spiral hooping of 0·127 inch diameter wire, the diameter of the hooped core being 9 inches and the pitch 1¼ inches.

From this result we may arrive at approximate data from which to calculate the dimensions for the wire in hooped pieces.

The steel wire used for the spiral winding of Mr. Dunn's column was tested by Kirkaldy, and its ultimate strength, from an average of six tests, was found to be 1,026 pounds.

In Mr. Dunn's experiment one of the spirals was broken before the column failed.

The spirals were spaced 1¼ inches apart and consequently the hoop tension on an imaginary circumferential hoop 1 inch wide would be $\frac{1026}{1 \cdot 25} = 821$ pounds.

This was caused by the internal pressure exerted by the concrete when swelling.

We have therefore from the usual formula $T = \frac{qd}{2}$, where T is the hoop tension on a strip one inch wide, q the internal pressure per square inch, and d the diameter in inches.

The diameter of Mr. Dunn's column was 9 inches, therefore $q = \frac{2 \times 821}{9} = 182$ pounds per square inch.

Now, the direct compressive stress on the column was, as stated above, 2,805 pounds per square inch.

We have the general formula for stability under two sets of forces acting on planes at right angles to each other—

$$\frac{P}{q} = \frac{1 + \operatorname{Sin} \phi}{1 - \operatorname{Sin} \phi},$$

where p is the direct downward pressure, q the pressure exerted normally to the sides, and ϕ the angle of stability. In the present case, $p = 2805$ and $q = 182$ pounds per square inch. We have, therefore—

$$\frac{2815}{182} = \frac{1 + \operatorname{Sin} \phi}{1 - \operatorname{Sin} \phi} \quad [24],$$

from which we get—

$$\operatorname{Sin} \phi = 0 \cdot 8781, \text{ or } \phi = 61^\circ - 25' \quad [25].$$

Now, if $\frac{P}{n} = q$—

$$n = \frac{1 + \operatorname{Sin} \phi}{1 - \operatorname{Sin} \phi} = \frac{2815}{182} = 15 \cdot 46 \quad [26].$$

These values may be safely considered as sufficiently accurate for a fairly good concrete, such as that employed by Mr. Dunn, which would be obtained, using ordinary care, but inferior to that which would be employed by specialists in this form of construction.

The value of $n = 15 \cdot 46$ would probably err considerably on the side of safety if the concrete were proportioned, mixed and deposited with special care, such as should always be taken when constructing in reinforced concrete.

Now for a hooped piece under a direct steady load. We may assume the value of p to be 2,000 pounds per square inch, which would give a value of 20,000 pounds per square inch for the steel longitudinal reinforcements (when the piece would be subjected to vibrations these values should be 1,500 and 15,000 respectively).

We have found the value of n to be 15·46 and $\frac{P}{n}=q=\frac{2000}{15\cdot 46}=130=$ the internal lateral pressure per square inch. Now $\frac{qd}{2}=$ the hoop tension, where d is the diameter of the hooped core in inches.

We get therefore $\frac{130d}{2}=65d=$ the hoop tension per inch width, but we may space the hooping wires $\frac{d}{8}$ inches apart. Therefore $65d\times\frac{d}{8}=\frac{8}{d^2}=$ hoop tension in the hooping wires. If we consider the safe stress on the steel hoopings as 25,000 pounds per square inch, since it will be drawn wire, the sectional area of the wire will be $\frac{8d^2}{25{,}000}$ or the diameter of the wire—

$$\delta=\sqrt{\frac{8d^2}{25{,}000\times 0\cdot 7854}}=0\cdot 02 \qquad [27].$$

The diameter of the distribution rods may be calculated as follows.

We have found that $q=$ the internal lateral pressure $=130$ pounds per square inch. Now we have a spacing for the spirals as $\frac{d}{8}$, which is the span for the longitudinals. These may be considered as having fixed ends. If we have eight longitudinals the distance between these will be $\frac{\pi d}{8}$.

We have, therefore, for the maximum bending moment—

$$M=\frac{130\times\frac{\pi d}{8}\times\frac{d^2}{64}}{12} \qquad [28]:$$

We have also the well-known formula—

$$M=\frac{fI}{y} \qquad [29].$$

I for circular rods $=0\cdot 0491(\delta_1)^4$ $y=\frac{1}{2}\delta_1$ the diameter of the rods; being δ_1
f may be taken as 20,000 pounds per square inch, since the rods are of small diameter.

Equating [28] and [29].

We have $1964(\delta_1)^3=0\cdot 0665d^3$ or $\delta_1=0\cdot 032d$ [30].

The value of $\delta=0\cdot 02d$ gives a diameter for the hooping wire about 42 per cent. greater than that used by Mr. Dunn, and about one-half that employed by M. Considère. With such a reinforcement we might load a column up to 2,000 or 1,500 pounds on the square inch of sectional area inside the spiral windings, according to the nature of the loading. We might also add to the area of the longitudinal reinforcements and consider such additional area as giving a resistance of 20,000 or 15,000 pounds per square inch according to the nature of the loading.

We should therefore obtain the formulae for a steady load—

$$P=2{,}000\,\Delta+20{,}000\,\omega \qquad [29],$$

when the structure was subject to vibrations—

$$P = 1{,}500\ \Delta + 15{,}000\ \omega \quad [30].$$

in which Δ is the area of the concrete within the spiral winding and ω the additional area of reinforcement.

LONGITUDINAL DIRECT STRESSES IN PIECES SUBJECTED TO BENDING.[1]

Rectangular Pieces with a single system of Reinforcement of Small Sectional Area and Depth compared with that of the whole piece near the Tensile Surface (Fig. 317)

The reinforcement being of this character, we may consider the stresses in the metal as of uniform intensity over the whole area, and that they act at the centre of gravity of the section.

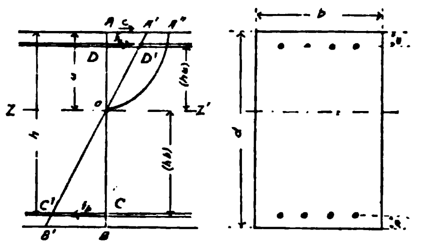


FIG. 317

The compressive resistance is in this case that due to the concrete above the neutral axis ZZ′, and the tensile resistance is supposed to be supplied by the lower reinforcement only. The compressive resistance is therefore represented by the area of the parabolic figure $A\,O\,A'' \times b$, since $A\,A''$ represents the maximum compressive resistance of the concrete, and the area $A\,O\,A''$ is $\frac{2}{3}$ of the surrounding rectangle, the above expression becomes $\frac{2}{3}\,cub$.

The tensile resistance is represented by ωf.

Since the compressive and tensile resistances must be equal, we have

$$\tfrac{2}{3}\,cub = \omega f \quad [1].$$

From the hypothesis of the conservation of plane sections—

$$\frac{AA'}{OA} = \frac{CC'}{OC}.\quad \text{But } AA' : CC' :: \frac{c}{E_c} : \frac{f}{E_f}.$$

Therefore $\dfrac{c}{u\,E_c} = \dfrac{f}{(hb)\,E_f}$ [2].

Substituting m for the ratio $\dfrac{E_f}{E_c}$, we get $f = cm\,\dfrac{(hb)}{u}$ [3].

Also the bending moment is equal to the moment of resistance of the concrete in compression and the reinforcement in tension.

The stress-strain curve of the concrete being parabolic, the centre of action of the stresses is at a point $\frac{3}{8}$ of the height of $O\,A$ from O.

[1] The manner of treatment adopted for the calculations of pieces subjected to bending is in the main that used by M, Christophe in his book *Le Béton Armé*, as by the use of this method the final formulae for finding the depth of the piece and the sectional area of the reinforcement are very much simplified. The difference between the formulae in this work and those deduced by M. Christophe lies chiefly in the difference of the stress-strain curve of the concrete in compression, which is considered here as parabolic.

We have therefore $M = \frac{5}{8}u \times \frac{2}{3}\ cub + (hb)\ \omega f$

$$= \frac{5}{12}cu^2b + (hb)\ \omega f \qquad [4].$$

Substituting the value of f from equation [3] in [1] and [4] we have—

$$\frac{2}{3}u^2 b - m\omega\,(hb) = 0 \qquad [5],$$

$$\text{and } M = \frac{c}{u}\left\{\frac{5}{12}u^3 b + m\omega\,(hb)^2\right\} \qquad [6].$$

We have also the relation $(hb) = (h - u)$ [7].

Substituting this value in [5] we get

$$\frac{2}{3}u^2b - m\omega\,(h - u) = 0,$$

from which

$$u = -\frac{3}{4}m\frac{\omega}{b} + \sqrt{\frac{9}{16}\frac{m^2\omega^2}{b^2} + \frac{3}{2}\frac{m\omega h}{b}} \qquad [8],$$

and substituting equations [5] and [7], in equation [6] we get

$$M = \frac{cub}{12}(8h - 3u) \qquad [9].$$

If it is required that a structure already designed shall be checked, the value of (u) is obtained from equation [8], (h) is known, and the values of $(u)$and (h) substituted in equation [9], give the value for (c); that of (f) being found by replacing (c), (hb) and (u) by their values in equation [3].

The values for designing a structure may be found as follows—

Take $m = 10$, $u = \gamma d$, $\omega = \psi\, bd$ and $M = \mu\, bd^2$, γ, ψ and μ being coefficients.

It has been found that good proportions for the value of h in respect to d, so that a sufficient covering of concrete may be allowed, are—

$$\text{For Slabs, } h = \frac{5}{6}d \text{ and } \beta = \frac{1}{6}d.$$

$$\text{For Beams, } h = \frac{9}{10}d \text{ and } \beta = \frac{1}{10}d.$$

Slabs.—Considering first the case of slabs, which includes any thin pieces subjected to bending, such as floors, roofs, thin walls, etc., equation [8] will become by the substitution of the above values—

$$u = \frac{15}{2}\psi d\left(-1 + \sqrt{1 + \frac{2}{9\psi}}\right)$$

$$\text{or } \gamma = \frac{u}{d} = \frac{15}{2}\psi\left(-1 + \sqrt{1 + \frac{2}{9\psi}}\right) \qquad [10].$$

From equation [7] we get

$$(hb) = \frac{d}{6}(5 - 6\gamma) \qquad [11].$$

Substituting for m, u, M and ω, and the value of $(h\,b)$ from equation [11] in equation [9] we get

$$\mu = \frac{c\gamma}{36}(20 - 9\gamma) \qquad [12].$$

From equation [3] we obtain

$$c=\frac{3f\gamma}{5(5-6\gamma)} \qquad [13],$$

and substituting this value in [12] we get for the value of μ in respect of f

$$\mu=\frac{f\gamma^2}{60(5-6\gamma)}(20-9\gamma) \qquad [14].$$

From the results of the experiments and observations of M. Considère and others on the behaviour of reinforced concrete beams, we may safely assume that the concrete will elongate at least $\frac{1}{1000}$ before cracking when subjected to bending strains (the elongation allowed by M. Considère is $\frac{1 \cdot 5}{1000}$, and in his experiments he measured $\frac{2}{1000}$). The elongation of the reinforcement under these conditions will be

$$\lambda=\frac{(hb)}{(hb)+\beta}\times\frac{1}{1000}=\frac{(5-6\gamma)}{(6-6\gamma)}\times\frac{1}{1000},$$

and the stress in the reinforcement must not be greater than

$$f_{max}=\frac{(5-6\gamma)}{(6-6\gamma)}\times\frac{1}{1000}\times E_f \qquad [15].$$

This equation enables us to assure the safety of the concrete in tension.

Except when a very large ratio of metal to concrete is employed, the neutral axis always remains above the centre of depth of a slab.

We will suppose that the neutral axis is at the centre of the depth, in which case $\gamma=\frac{1}{2}$ and equation [15] becomes

For wrought iron $f_{max}=\frac{2}{3\times10^3}\times 28{\cdot}45\times10^6 = 18{,}966$ pounds per square inch.

For steel $f_{max}=\frac{2}{3\times10^3}\times 31{\cdot}3\times10^6 = 20{,}866$ pounds per square inch.

Both of which are considerably higher stress intensities than are usually allowed in the calculations. This, combined with the fact that the value of γ is not at all likely to be anything approaching $\frac{1}{2}$, shows that it is unnecessary to inquire into the tensile strain in the concrete, except in very special cases

From equation [10] the values of (γ) for the various percentages of reinforcement have been calculated, and the diagram (Fig. 318) has been plotted, from which intermediate values can be obtained. It will be observed that the value of (γ) only depends on the percentage of reinforcement.

Similarly the values of (μ) in respect to (c) and (f) have been calculated for the various percentages of wrought iron and steel reinforcement from equations [12] and [14] by inserting the values of (γ) already obtained, and the diagram (Fig. 319) has been plotted, from which the value of (μ) for any percentage of reinforcement can be found. It will be seen that the curves of (μ) in respect to the concrete and reinforcement intersect one another, showing the percentage of reinforcement for which both (c) and (f) are acting at their maximum allowed resistance.

For percentages to the left of these points of intersection the value of (μ) in

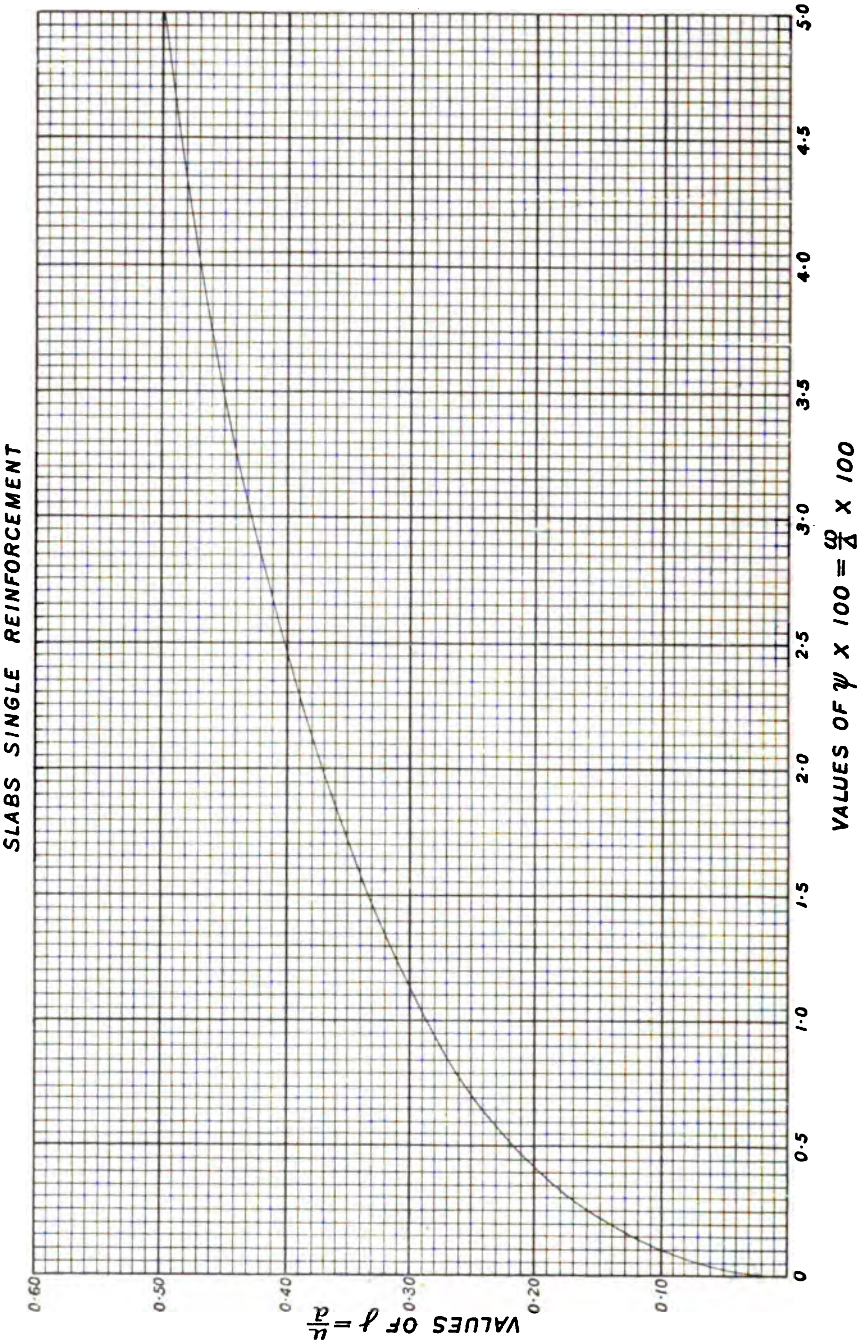


DIAGRAM SHOWING POSITION OF NEUTRAL AXIS FOR VARIOUS PERCENTAGES OF METAL.

SLABS SINGLE REINFORCEMENT

Fig. 318

respect to (f) must be taken, and for those to the right the values of (μ) in respect to (c). The curves for the proper values of (μ) have therefore a cusp at these points. These curves are shown in full lines, and the portions that are not required in dotted lines. It will be seen that higher percentages of reinforcement than that for maximum economy are very uneconomical with respect to the reinforcement.

To find the values of (ψ), (γ), and (μ) for the most economical section we have from equation [1] substituting the values of $\omega = \psi bd$ and $u = \gamma d$

$$\psi = \frac{2c\gamma}{3f} \qquad [16].$$

From equation [13] we obtain

$$\gamma = \frac{25c}{3(f+10c)} \qquad [17].$$

Substituting equation [17] in equation [16] we get

$$\psi = \frac{50c^2}{9f\,(f+10c)} \qquad [18].$$

Equation [18] gives the economic value of (ψ) and [17] that of (γ), which substituted in [12] or [14] will give the economic value of (μ).

These values, with others for μ, γ, and ψ, corresponding to Figs. 318 and 319, are given in Table LVII., together with the value for expressions $\frac{\gamma}{36}(20-9\gamma)$ and $\frac{\gamma^2}{60\,(5-6\gamma)}(20-9\gamma)$, which will greatly simplify the calculation of similar tables for different values of (c) and (f).

The economic values for (ψ), (γ) and (μ) were obtained from equations [18], [17], and [12], or [14].

The values of (c), and (f) for wrought iron and steel are taken as 500, 10,000 and 15,000 pounds per square inch respectively.

TABLE LVII

VALUES OF μ AND γ FOR SLABS WITH SINGLE SYSTEM OF REINFORCEMENT.

Percentage of Reinforcement	$\psi = \frac{\omega}{\Delta}$	$\gamma = \frac{u}{d}$	$\frac{\gamma}{36}(20-9\gamma)$	$\frac{j^2}{60(5-6\gamma)}(20-9\gamma)$	μ: $c=500$	$f=10,000$	$f=15,000$
0·25	0·0025	0·159	0·0817	0·0019	—	19·0	28·5
0·47	**0·0047**	**0·208**	—	—	**[51·9]**	—	**51·9**
0·5	0·005	0·215	0·1084	0·00375	[54·2]	37·5	~~56·8~~
0·75	0·0075	0·255	0·1239	0·00551	[61·9]	55·1	~~82·6~~
0·91	**0·0091**	**0·278**	—	—	**67·6**	**67·6**	—
1	0·01	0·285	0·1394	0·00721	69·7	~~72·1~~	~~108·2~~
1·5	0·015	0·335	0·1580	0·01063	79·0	~~106·3~~	~~159·4~~
2	0·02	0·372	0·1712	0·01386	85·6	~~138·6~~	~~209·9~~
2·5	0·025	0·400	0·1819	0·01700	90·9	~~170·0~~	~~255·0~~
3	0·03	0·427	0·1912	0·02015	95·5	~~201·5~~	~~302·2~~
4	0·04	0·468	0·2052	0·02629	102·6	~~262·9~~	~~394·3~~
5	0·05	0·499	0·2171	0·03100	108·5	~~310·0~~	~~465·0~~

The figures through which a line has been drawn are those which cannot be used, those in brackets only apply to a piece with steel reinforcement, and those in heavy type are for the economical section.

DIAGRAM SHOWING UNIT BENDING MOMENT FOR VARIOUS PERCENTAGES OF REINFORC

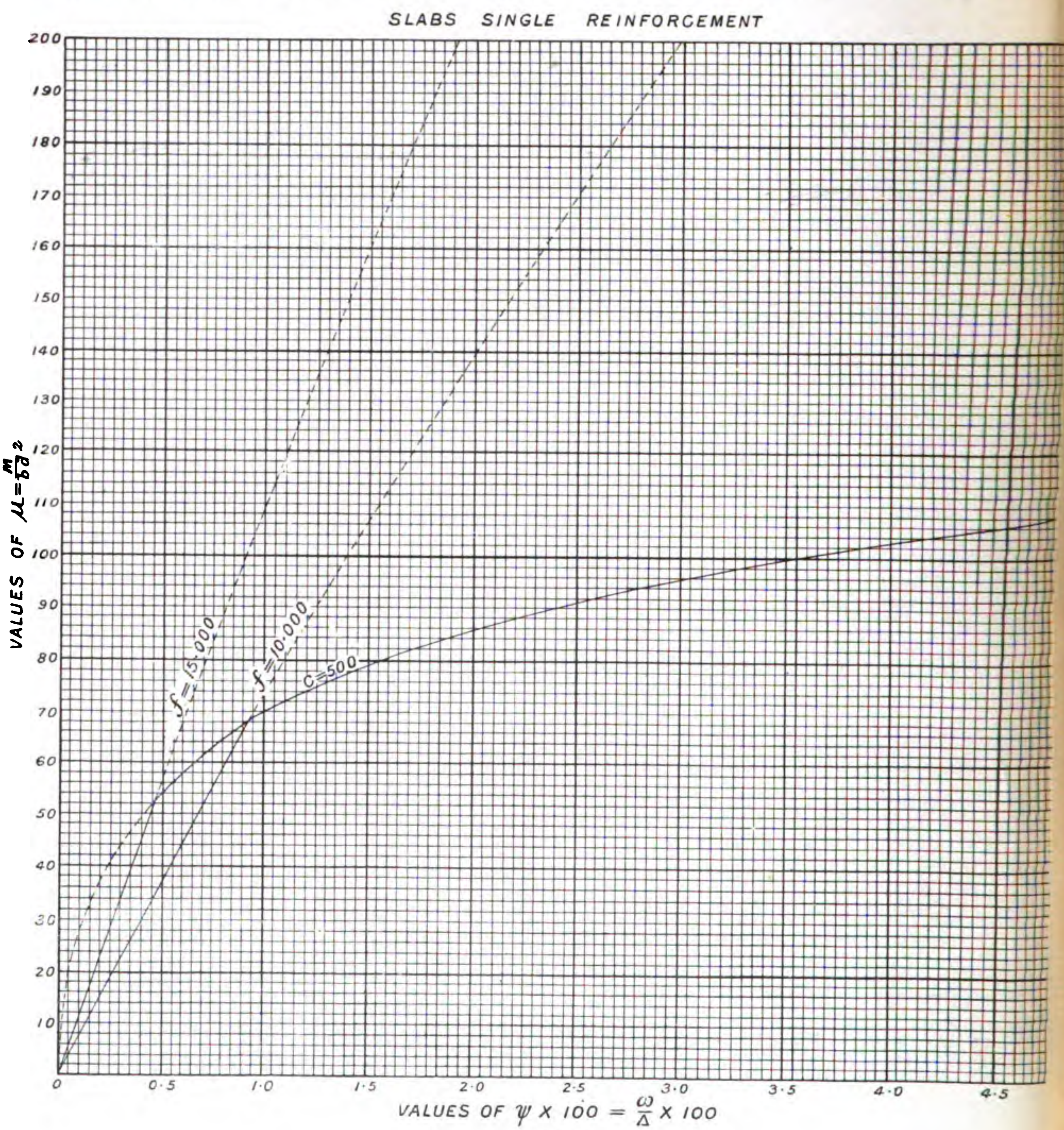


FIG. 319

CALCULATIONS

Having obtained the value of (μ), the thickness of the piece is given by the equation $d=\sqrt{\frac{M}{\mu b}}$ and for slabs (b) may be 12 inches *if the load is taken in pounds per square foot, M being, of course, calculated in inch-pounds.*

The area of the reinforcement is obtained from the equation $\omega = \psi bd$, and is distributed into as many bars or wires as may be considered necessary in the width of 12 inches.

Taking the economical values for (μ) we get

For a wrought iron reinforcement $d = 0{\cdot}035\sqrt{M}$ and $\psi = 0{\cdot}0091$.

For a steel reinforcement $d = 0{\cdot}04\sqrt{M}$ and $\psi = 0{\cdot}0047$.

From these economical values we can plot curves giving the depth of slab necessary for any bending moment on a width of 12 inches. This has been done, and the diagram obtained is given, Fig. 320.

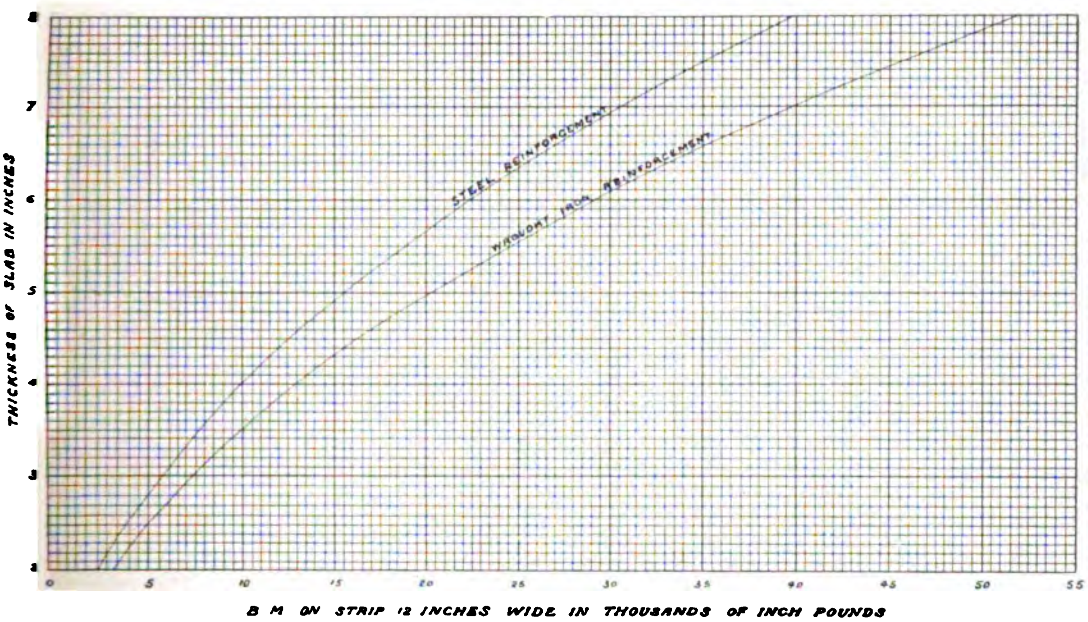


Fig. 320

It will be seen that for a steel reinforcement the depth of the slab, for any given bending moment, will be greater than when a wrought-iron reinforcement is employed, but the percentage of reinforcement is much less for steel than wrought iron.

Beams.—For beams we take $h=\frac{9}{10}d$ in place of $h=\frac{5}{6}d$, all the other values remaining as before.

Therefore, for beams and similar structures subjected to bending, equation [10] becomes

$$\gamma = \frac{15}{2}\psi\left(-1+\sqrt{1 \times \frac{6}{25\psi}}\right) \qquad [19],$$

X

and equation [11] takes the form—

$$(hb)=\frac{d}{10}\,(9-10\gamma) \qquad [20].$$

DIAGRAM SHOWING POSITION OF NEUTRAL AXIS FOR VARIOUS PERCENTAGES OF REINFORCEMENT

BEAMS SINGLE REINFORCEMENT.

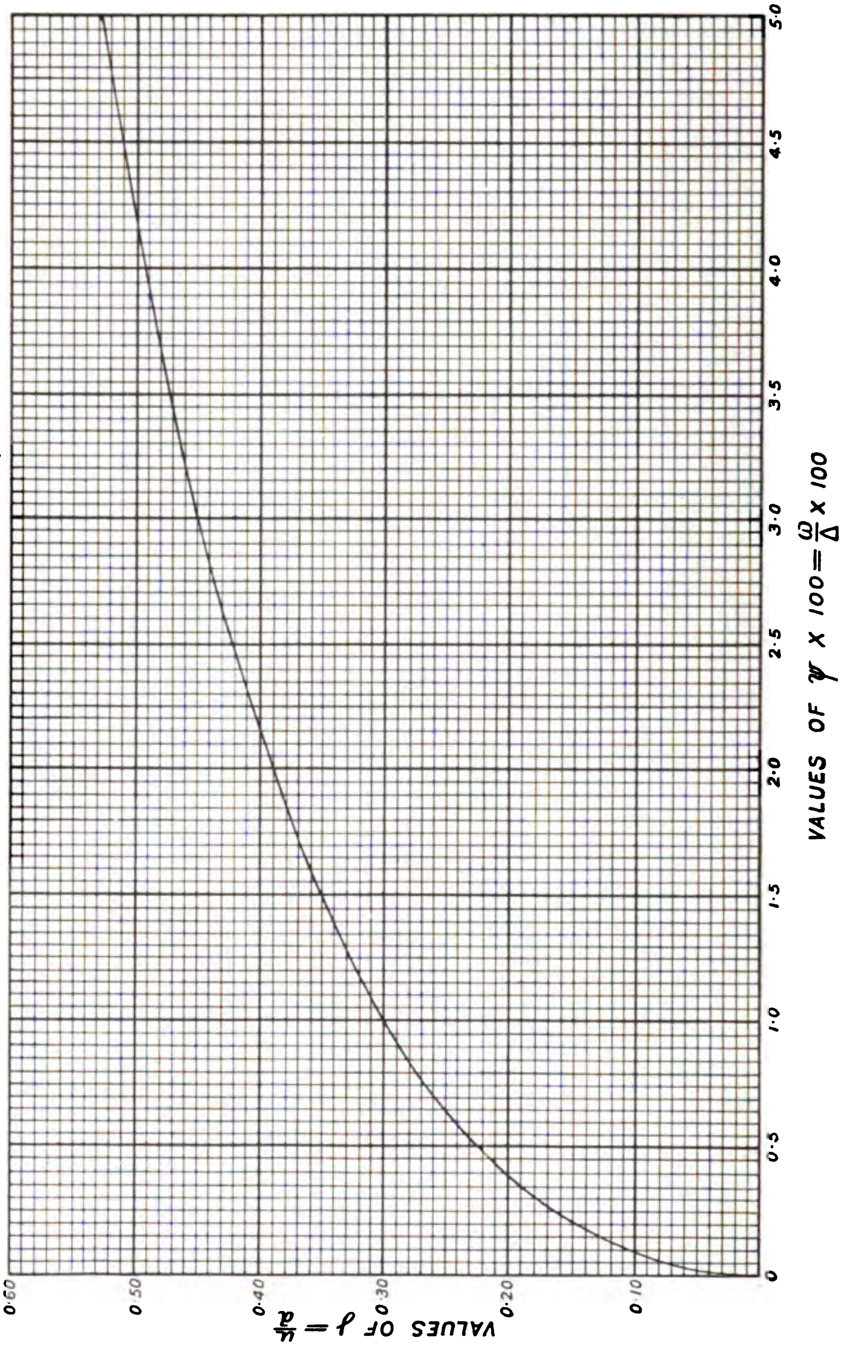


Fig. 321

DIAGRAM SHOWING UNIT BENDING MOMENT FOR VARIOUS PERCENTAGES OF REINFORCEMENT

BEAMS SINGLE REINFORCEMENT

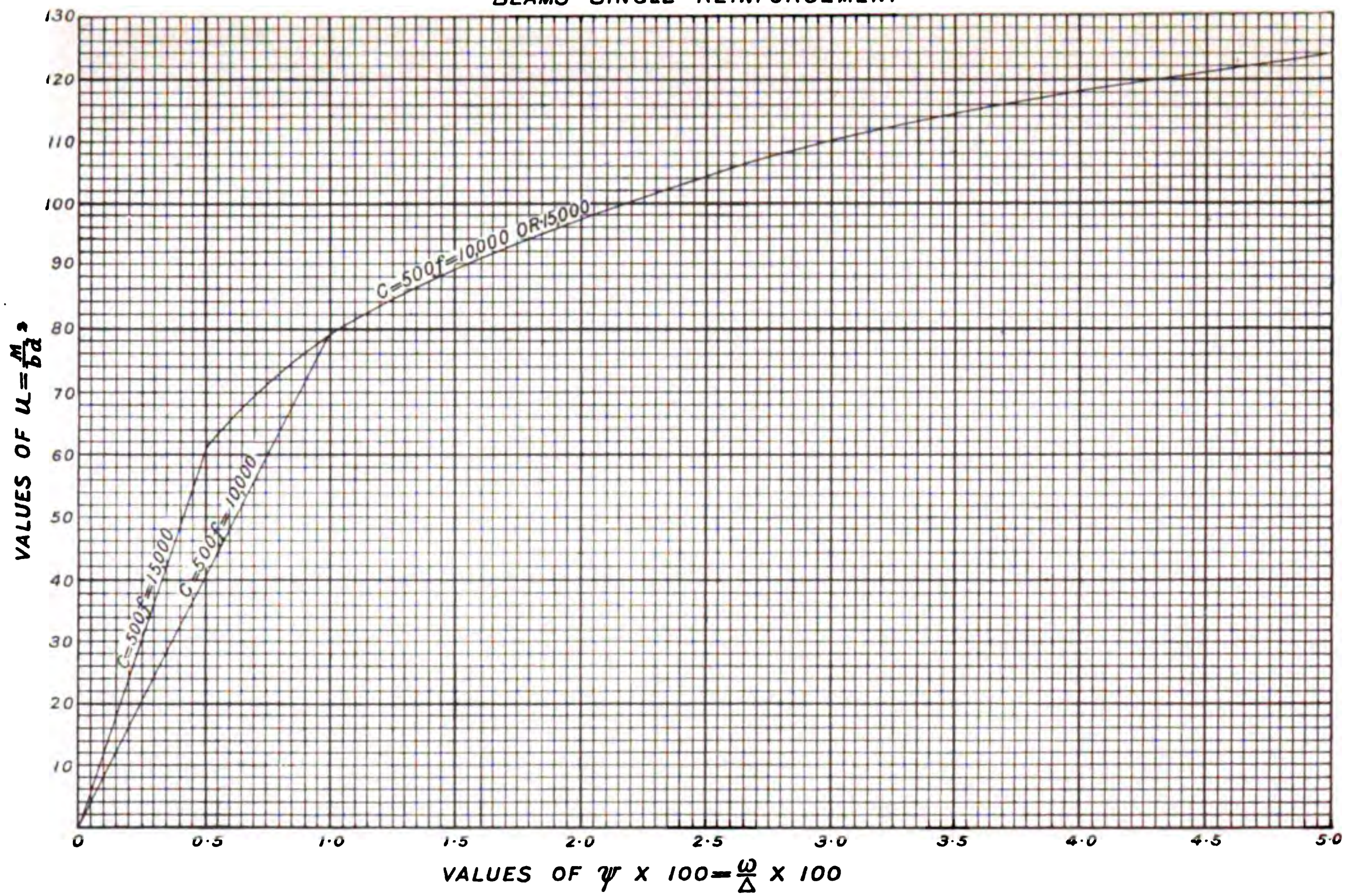


Fig 322

Equation [12] will be

$$\mu=\frac{c\gamma}{120}(72-30\gamma) \qquad [21],$$

and equation [13]

$$c=\frac{f\gamma}{(9-10\gamma)} \qquad [22].$$

Substituting equation [22] in equation [21] we get

$$\mu=\frac{f\gamma^2}{120(9-10\gamma)}(72-30\gamma) \qquad [23].$$

Equation [15] for assuring the safety of the concrete in tension becomes—

$$f_{max}=\frac{(9-10\gamma)}{(10-10\gamma)}\times\frac{1}{1000}\times E_f \qquad [24].$$

With $\gamma=\frac{1}{2}$ we get—

For wrought iron $f_{max}=\frac{4}{5\times10^3}\times 28{\cdot}45\times10^6=22{,}760$ pounds per square inch.

And for steel . . . $f_{max}=\frac{4}{5\times10^3}\times 31{\cdot}3\times10^6=25{\cdot}040$,, ,, ,,

Values considerably higher than those obtained in the case of slabs and far in excess of any allowance in practice.

The value of (γ) does not become as great as ½ except for large percentages, and we may, therefore, neglect the inquiry into the tensile strain of the concrete except in special cases.

The values of (γ) obtained from equation [19] have been plotted on the diagram (Fig. 321), and those of (μ) from equations [21] and [23], substituting the value of (γ), are given by the curves in Fig. 322, the proper values only being used in this case.

To find the values of ψ, γ and μ for the most economical section, equation [16] will be the same as before

$$\psi=\frac{2c\gamma}{3f} \qquad [25],$$

equation [17] becomes

$$\gamma=\frac{9c}{(f+10c)} \qquad [26],$$

and equation [18] will be

$$\psi=\frac{6c^2}{f(f+10c)} \qquad [27],$$

As before, [27] gives the economic value for (ψ) and [26] that of (γ), which substituted in [21] or [23] will give the value for (μ).

Table LVIII. gives the values for ψ, γ, and μ, corresponding to Figs. 321 and 322, and those for the expressions—

$$\frac{\gamma}{120}(72-30\gamma) \text{ and } \frac{\gamma^2}{120(9-10\gamma)}(72-30\gamma)$$

for various percentages of reinforcement.

TABLE LVIII

VALUES OF μ AND γ FOR BEAMS WITH SINGLE SYSTEM OF REINFORCEMENT

Percentage of Reinforcement	$\psi = \frac{\omega}{\Delta}$	$\gamma = \frac{u}{d}$	$\frac{\gamma}{120}(72 - 30\gamma)$	$\frac{\gamma^2}{120(\psi - 10\gamma)}(72 - 30\gamma)$	μ	
					$c=500$ $f=10{,}000$	$c=500$ $f=15{,}000$
0·25	0·0025	0·166	0·092	0·00207	20·7	31·0
0·5	**0·005**	**0·225**	0·122	0·00405	40·5	**61·0**
0·75	0·0075	0·266	0·141	0·00595	59·5	70·5
1·0	**0·01**	**0·300**	0·157	0·00787	**78·7**	78·7
1·5	0·015	0·351	0·178	0·01156	89·0	89·0
2·0	0·02	0·390	0·193	0·01501	96·5	96·5
2·5	0·025	0·422	0·208	0·01839	104·0	104·0
3·0	0·03	0·450	0·222	0·02194	111·0	111·0
4·0	0·04	0·492	0·235	0·02833	117·5	117·5
5·0	0·05	0·528	0·247	0·03504	123·5	123·5

In this case the economical values for wrought iron and steel reinforcements happen to be for percentages of reinforcement of 1 and 0·5 respectively.

To obtain the thickness or depth of the piece we have the equation $M = \mu bd^2$, but we must know the ratio $\frac{b}{d}$ before we can solve it. This ratio is frequently taken as $\frac{2}{3}$, and this is a good proportion. Some constructors make their beams much too narrow; the reinforcements must be sufficiently far apart for the concrete between them to be able to transmit the stresses. The distance from *outside to outside* of neighbouring longitudinals should never be less than the sum of their diameters or widths.

This method cannot be applied directly, for until we find (b) and (d) we cannot know the value of (ω).

Taking the diagonal of a beam of iron, steel or wood as constant, the most economic ratio of breadth to depth is $\frac{5}{7}$, which is $\frac{1}{21}$ in excess of the value $\frac{2}{3}$ given above.

We may, therefore, take $\frac{b}{d}$ as $\frac{2}{3}$, from which

$$d^3 = \frac{3M}{2\mu} \qquad [28],$$

$$\text{or } \log. d = \frac{0{\cdot}176091 + \log. M - \log. \mu}{3} \qquad [29]$$

We may also give different values to (b) and calculate a table for the values of d to correspond for varying values of M.

Sometimes the depth is settled beforehand by the conditions relating to the structure, in which case the breadth must be calculated from the equation—

$$b = \frac{M}{\mu d^2} \qquad [30]$$

The area of the reinforcement is obtained from the equation $\omega = \psi\, bd$, and is divided up into the number of rods or other reinforcement desired.

Taking the economical values for (μ) equation [29] becomes

For a wrought iron reinforcement, $\log. d = \frac{\log. M - 1{\cdot}719884}{3}$ and $\psi = 0{\cdot}01$.

For a steel reinforcement, $\log. d = \frac{\log. M - 1{\cdot}609239}{3}$ and $\psi = 0{\cdot}005$.

Rectangular Pieces with a Double System of Reinforcement of Small Sectional Area and Depth compared with that of the whole Piece, there being a reinforcement near both the compressive and tensile surfaces (Fig. 323)—

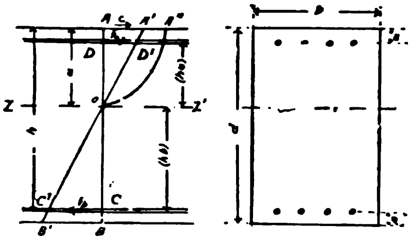


FIG. 323

As in the last case, we may consider the stresses in the metal as of uniform intensity over the whole area, and that they act at the centre of gravity of the section.

In this instance the compressive resistance is that of the concrete above the neutral axis ZZ′ added to that of the compressive reinforcement, the tensile resistance being supplied by the tensile reinforcement alone.

The compressive resistance is therefore represented by the area of the parabolic figure AOA″ $\times\ b + \omega_u \times f_u$. Since AA″ represents the maximum compressive resistance of the concrete, and the area AOA″ equals $\frac{2}{3}$ of the surrounding rectangle, the above expression becomes $\frac{2}{3}cub + \omega_u f_u$.

The tensile resistance is represented by $\omega_b f_b$. The compressive and tensile resistances must be equal, we have then

$$\frac{2}{3}cub + \omega_u f_u = \omega_b f_b \qquad [31]$$

From the hypotheses of the conservation of plane sections, we have—

$$\frac{AA'}{OA} = \frac{CC'}{OC} = \frac{DD'}{OD}. \quad \text{But } AA' : CC' : DD' :: \frac{c}{E_c} : \frac{f_b}{E_f} : \frac{f_u}{E_f},$$

$$\text{or, } \frac{c}{uE_c} = \frac{f_b}{(hb)E_f} = \frac{f_u}{(hu)E_f} \qquad [32].$$

If we take m as the ratio $\frac{E_f}{E_c}$, we get—

$$f_b = cm\frac{(hb)}{u} \qquad [33],$$

$$f_u = cm\frac{(hu)}{u} \qquad [34]$$

$$\text{and } f_u = f_b\frac{(hu)}{(hb)} \qquad [35].$$

Also the bending moment is equal to the moment of resistance of the concrete and compressive reinforcement in compression, and the tensile reinforcement in tension. And as the stress-strain curve of the concrete in compression is parabolic we have—

$$M = \frac{5}{12}cu^2b + (hu)\omega_u f_u + (hb)\omega_b f_b \qquad [36].$$

Substituting the values of f_b and f_u from equations [33] and [34] in equations [31] and [36] we have

$$\frac{2}{3}u^2b + m\{\omega_u(hu) - \omega_b(hb)\} = 0 \qquad [37];$$

$$\text{and } M=\frac{c}{u}\left[\frac{5}{12}u^3b+m\{\omega_u(hu)^2+\omega_b(hb)^2\}\right] \qquad [38]$$

We have also

$$(hb)=(h-u) \qquad [39]$$

$$\text{and } (hu)=(u-a) \qquad [40].$$

Substituting these values in equation [37]—

$$u=-\frac{3}{4}\frac{m(\omega_u+\omega_b)}{b}+\sqrt{\frac{9}{16}\frac{m^2(\omega_u+\omega_b)^2}{b^2}+\frac{3}{2}\frac{m(\omega_u a+\omega_b h)}{b}} \qquad [41].$$

If it is required to check a structure already designed (u) is found from equation [41] which will give the values of (hu) and (hb). These values substituted in equation [38] give the resistance of the concrete (c), after which the values for (f_b) and (f_u) follow from equations [33] and [34].

To obtain the values necessary for designing a structure, we take, as before, $m=10$, $u=\gamma d$, $\omega_b=\psi bd$, and $M=\mu bd^2$, and the same proportions for the value of h being—

$$\text{For Slabs } h=\frac{5}{6}d \text{ and } a=\beta=\frac{1}{6}d.$$

$$\text{For Beams } h=\frac{9}{10}d \text{ and } a=\beta=\frac{1}{10}d.$$

It will be observed that in the case of a double reinforcement we take only the percentage of the *tensile* reinforcement for the purposes of calculation, using the equation $\omega_b=\psi\, bd$. When a double reinforcement is under consideration we must consider different ratios of $\frac{\omega_u}{\omega_b}$, as it is necessary to know this ratio, if the problem is to be solved.

The following ratios are taken as they will cover most requirements—

$$\omega_u=\tfrac{1}{4}\omega_b,\ \omega_u=\tfrac{1}{2}\omega_b,\ \omega_u=\tfrac{3}{4}\omega_b \text{ and } \omega_u=\omega_b.$$

Slabs.—Considering first the case of slabs. Substituting the values for m, u, ω_b, μ, h and a, and replacing ω_u by its value in respect of ω_b, we get from equation [41]—

If $\omega_u=\frac{1}{4}\omega_b$

$$\gamma=\frac{75}{8}\psi\left(-1+\sqrt{1+\frac{56}{375\psi}}\right) \qquad [42].$$

If $\omega_u=\frac{1}{2}\omega_b$

$$\gamma=\frac{45}{4}\psi\left(-1+\sqrt{1+\frac{44}{405\psi}}\right) \qquad [43].$$

If $\omega_u=\frac{3}{4}\omega_b$

$$\gamma=\frac{105}{8}\psi\left(-1+\sqrt{1+\frac{184}{2205\psi}}\right) \qquad [44].$$

If $\omega_u=\omega_b$

$$\gamma=15\psi\left(-1+\sqrt{1+\frac{1}{15\psi}}\right) \qquad [45].$$

Substituting the values of $u=\gamma d$, $h=\frac{5}{6}d$ and $a=\frac{1}{6}d$ in equations [39] and [40] we get—

$$(hb)=\frac{d}{6}(5-6\gamma) \qquad [46]$$

$$(hu)=\frac{d}{6}(6\gamma-1) \qquad [47].$$

Substituting these values, with $M=\mu bd^2$ and $u=\gamma d$ in equation [38] we get

$$\mu=\frac{c}{\gamma bd}\left[\frac{5}{12}\gamma^3 bd+\frac{5}{3}\left\{(\omega_u(6\gamma-1)^2+\omega_b(5-6\gamma)^2\right\}\right] \qquad [48].$$

Replacing ω_u by its value in respect to ω_b, and further replacing ω_b by ψbd we get for the values of (μ) for different ratios of ω_u to ω_b.—

When $\omega_u=\frac{1}{4}\omega_b$

$$\mu=\frac{5c}{12\gamma}\left[\gamma^3+\frac{1}{6}\psi\left\{(6\gamma-1)^2+4(5-6\gamma)^2\right\}\right] \qquad [49].$$

When $\omega_u=\frac{1}{2}\omega_b$

$$\mu=\frac{5c}{12\gamma}\left[\gamma^3+\frac{1}{3}\psi\left\{(6\gamma-1)^2+2(5-6\gamma)^2\right\}\right] \qquad [50].$$

When $\omega_u=\frac{3}{4}\omega_b$

$$\mu=\frac{5c}{12\gamma}\left[\gamma^3+\frac{1}{6}\psi\left\{3(6\gamma-1)^2+4(5-6\gamma)^2\right\}\right] \qquad [51].$$

When $\omega_u=\omega_b$

$$\mu=\frac{5c}{12\gamma}\left[\gamma^3+\frac{2}{3}\psi\left\{(6\gamma-1)^2+(5-6\gamma)^2\right\}\right] \qquad [52].$$

From equation [33] $f_b=cm\frac{(hb)}{u}$ we get by substitution for the values of (m), (hb) and (u)—

$$f_b=\frac{5c}{3\gamma}(5-6\gamma) \qquad [53].$$

And from [34]

$$f_u=f_b\frac{(6\gamma-1)}{(5-6\gamma)} \qquad [54].$$

From equation (53) we get—

$$c=\frac{3\gamma}{5f_b(5-6\gamma)} \qquad [55].$$

Substituting this value for [c] in equations [49] to [52] we get—

When $\omega_u=\frac{1}{4}\omega_b$

$$\mu=\frac{f_b}{4(5-6\gamma)}\left[\gamma^3+\frac{1}{6}\psi\left\{(6\gamma-1)^2+4(5-6\gamma)^2\right\}\right] \qquad [56].$$

When $\omega_u=\frac{1}{2}\omega_b$

$$\mu=\frac{f_b}{4(5-6\gamma)}\left[\gamma^3+\frac{1}{3}\psi\left\{(6\gamma-1)^2+2(5-6\gamma)^2\right\}\right] \qquad [57].$$

When $\omega_u=\frac{3}{4}\omega_b$

$$\mu=\frac{f_b}{4(5-6\gamma)}\left[\gamma^3+\frac{1}{6}\psi\left\{3(6\gamma-1)^2+4(5-6\gamma)^2\right\}\right] \qquad [58].$$

When $\omega_u = \omega_b$

$$\mu = \frac{f_b}{4(5-6\gamma)}\left[\gamma^3 + \frac{2}{3}\psi\left\{(6\gamma-1)^2 + (5-6\gamma)^2\right\}\right] \qquad [59].$$

The equation for assuring the safety of the concrete in tension is the same as that for a single reinforcement—

$$f_{b\,max} \frac{(5-6\gamma)}{(6-6\gamma)} \times \frac{1}{1000} \times E_f \qquad [60].$$

The values of (γ) and (μ) for the various ratios of compressive and tensile reinforcement for different percentages of area of the *tensile* reinforcement to the total area of the piece have been plotted on diagrams (Figs. 324 to 331).

To find the values for ψ, γ, and μ for the most economical sections, i.e. those where (c) and (f_b) have their maximum allowed values :—

We have from equation [31]

$$\frac{2}{3} cub = \omega_b f_b - \omega_u f_u$$

by substituting for (f_u) its value from equation [54], and replacing (ω_u) by its value with respect to (ω_b) and finally replacing (ω_b) by $\psi\, bd$.

When $\omega_u = \frac{1}{4}\omega_b$

$$\psi = \frac{8c\gamma}{9f_b}\left\{\frac{(5-6\gamma)}{(7-10\gamma)}\right\} \qquad [61].$$

When $\omega_u = \frac{1}{2}\omega_b$

$$\psi = \frac{4c\gamma}{3f_b}\left\{\frac{(5-6\gamma)}{(11-18\gamma)}\right\} \qquad [62].$$

When $\omega_u = \frac{3}{4}\omega_b$

$$\psi = \frac{8c\gamma}{3f_b}\left\{\frac{5-6\gamma)}{(23-42\gamma)}\right\} \qquad [63].$$

When $\omega_u = \omega_b$

$$\psi = \frac{c\gamma}{9f_b}\left\{\frac{(5-6\gamma)}{(1-2\gamma)}\right\} \qquad [64].$$

From equation [53] we get

$$\gamma = \frac{25c}{3(f_b + 10c)} \qquad [65].$$

It will be seen that (γ) is only dependent on the values of (f_b) and (c), and therefore is the same for all ratios of (ω_u) to (ω_b). Substituting the values of (γ) from [65] in equation [61] to [64], we get—

When $\omega_u = \frac{1}{4}\omega_b$

$$\psi = \frac{500c^2}{9(f_b + 10c)(21f_b - 40c)} \qquad [66].$$

When $\omega_u = \frac{1}{2}\omega_b$

$$\psi = \frac{500c^2}{9(f_b + 10c)(11f_b - 40c)} \qquad [67].$$

DIAGRAM SHOWING POSITION OF NEUTRAL AXIS FOR VARIOUS PERCENTAGES OF TENSILE REINFORCEMENT

SLABS $W_u = 1/4\ W_b$

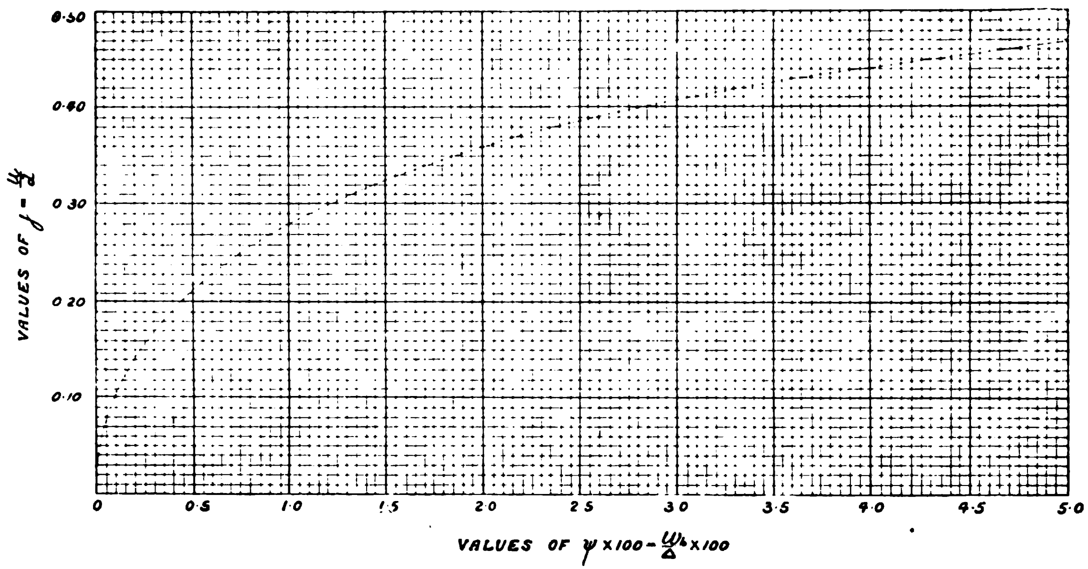


FIG. 324

DIAGRAM SHOWING POSITION OF NEUTRAL AXIS FOR VARIOUS PERCENTAGES OF TENSILE REINFORCEMENT

SLABS $\omega_u = \frac{1}{2}\,\omega_b$

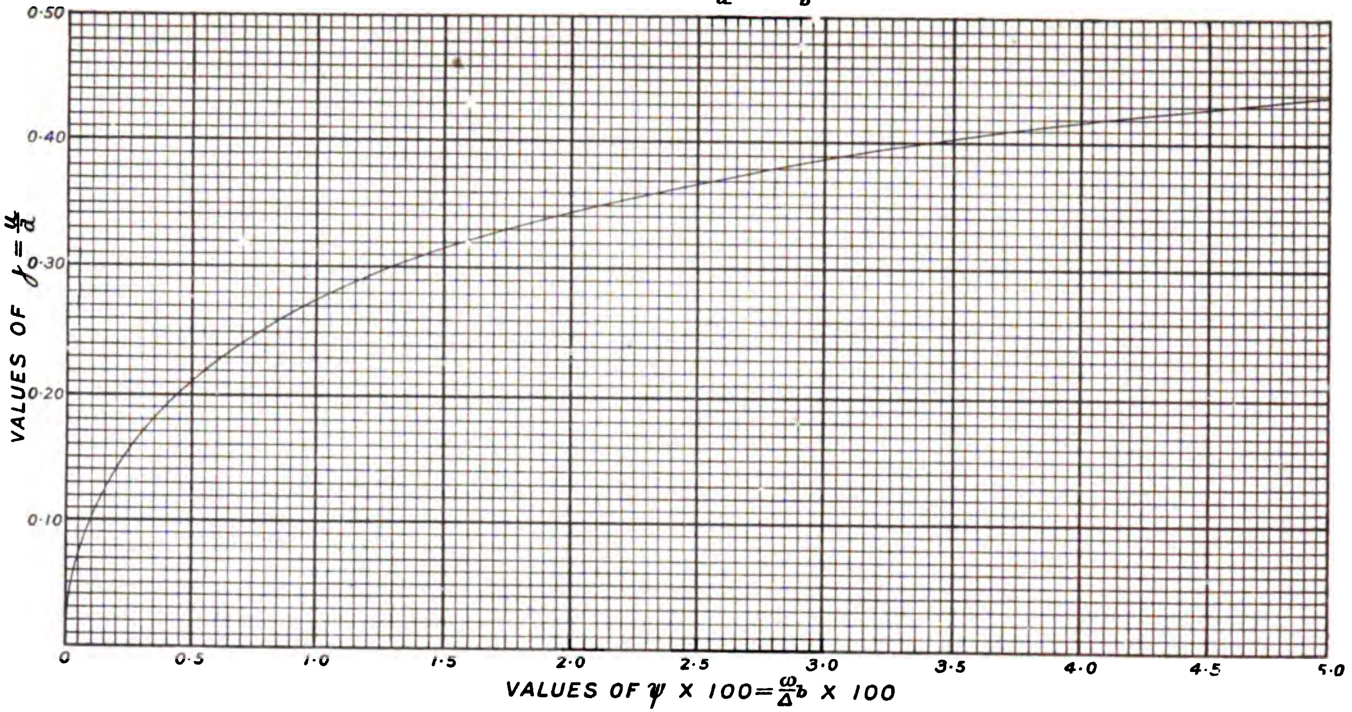


FIG. 325

DIAGRAM SHOWING POSITION OF NEUTRAL AXIS FOR VARIOUS PERCENTAGES OF TENSILE REINFORCEMENT

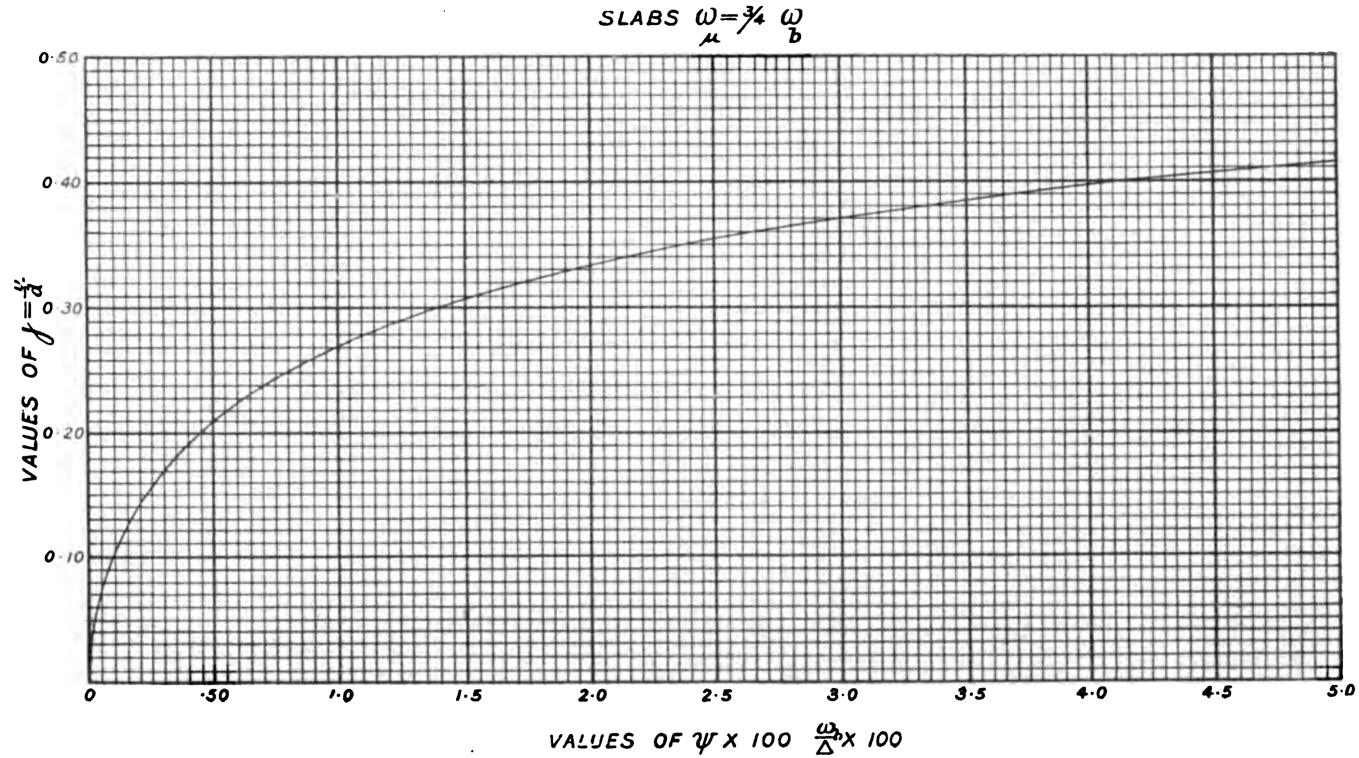


Fig. 326

DIAGRAM SHOWING POSITION OF NEUTRAL AXIS FOR VARIOUS PERCENTAGES OF TENSILE REINFORCEMENT

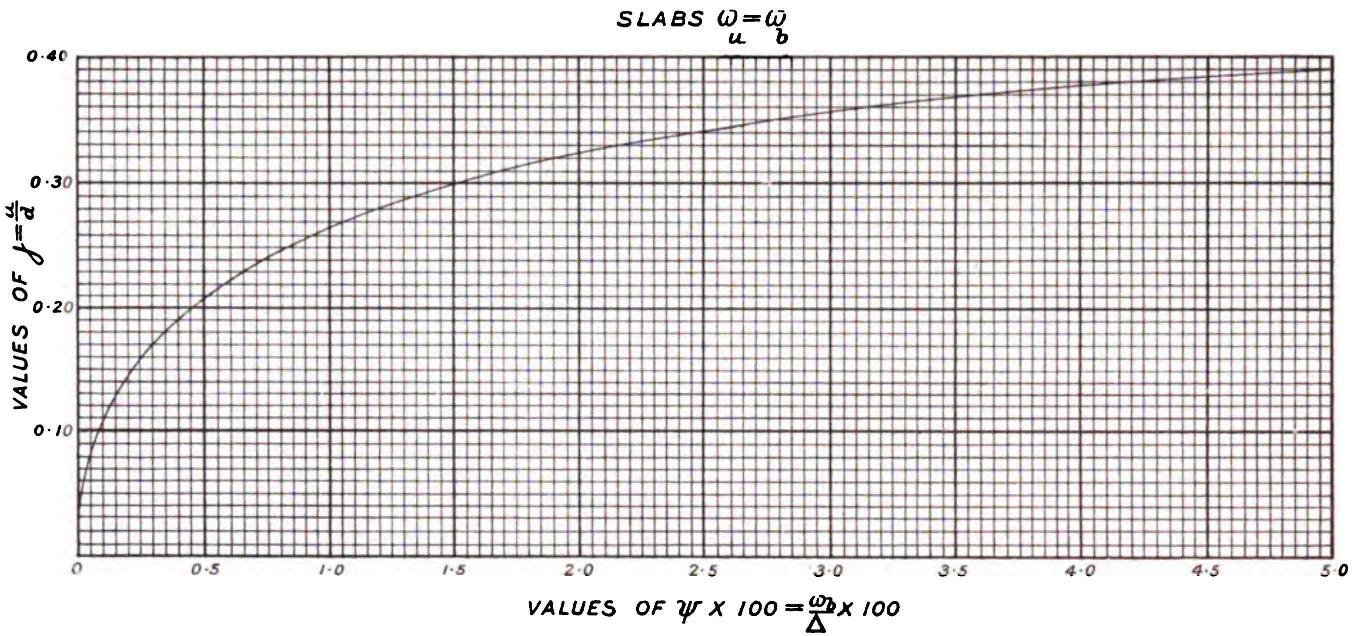


Fig. 327

DIAGRAM SHOWING UNIT BENDING MOMENT FOR VARIOUS PERCENTAGES OF TENSILE REINFORCEMENT

SLABS $\omega_u = \frac{1}{4}\,\omega_s$

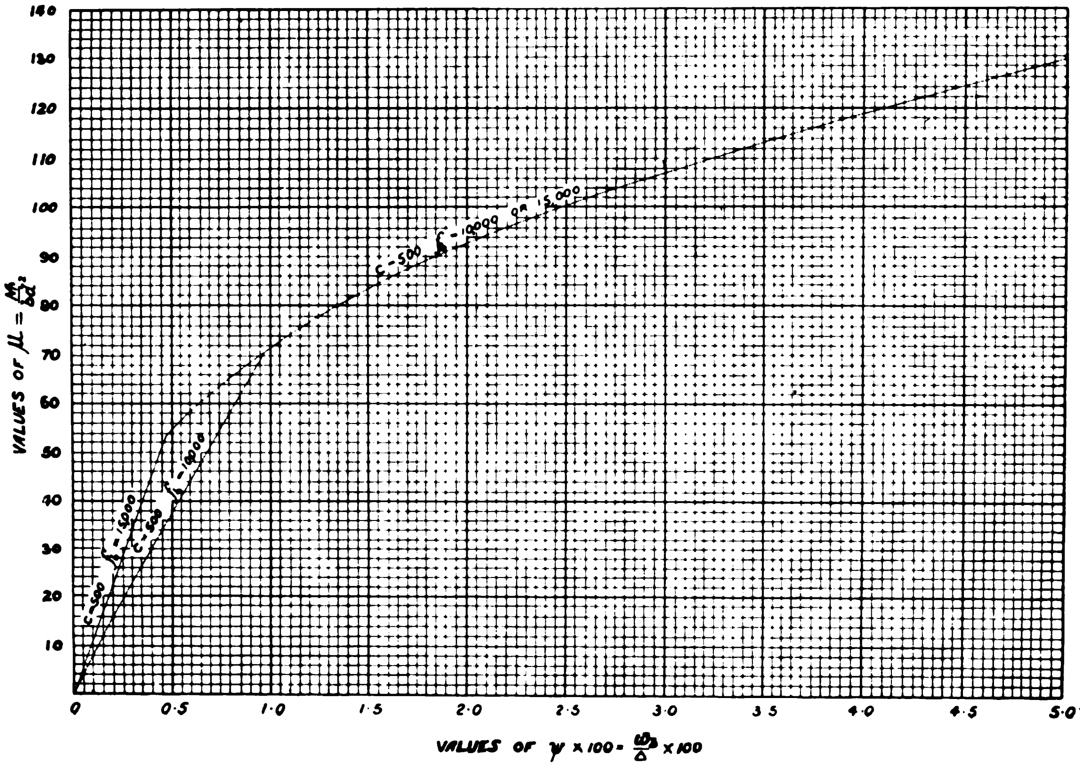


FIG. 328

DIAGRAM SHOWING UNIT BENDING MOMENT FOR VARIOUS PERCENTAGES OF TENSILE, REINFORCEMENT

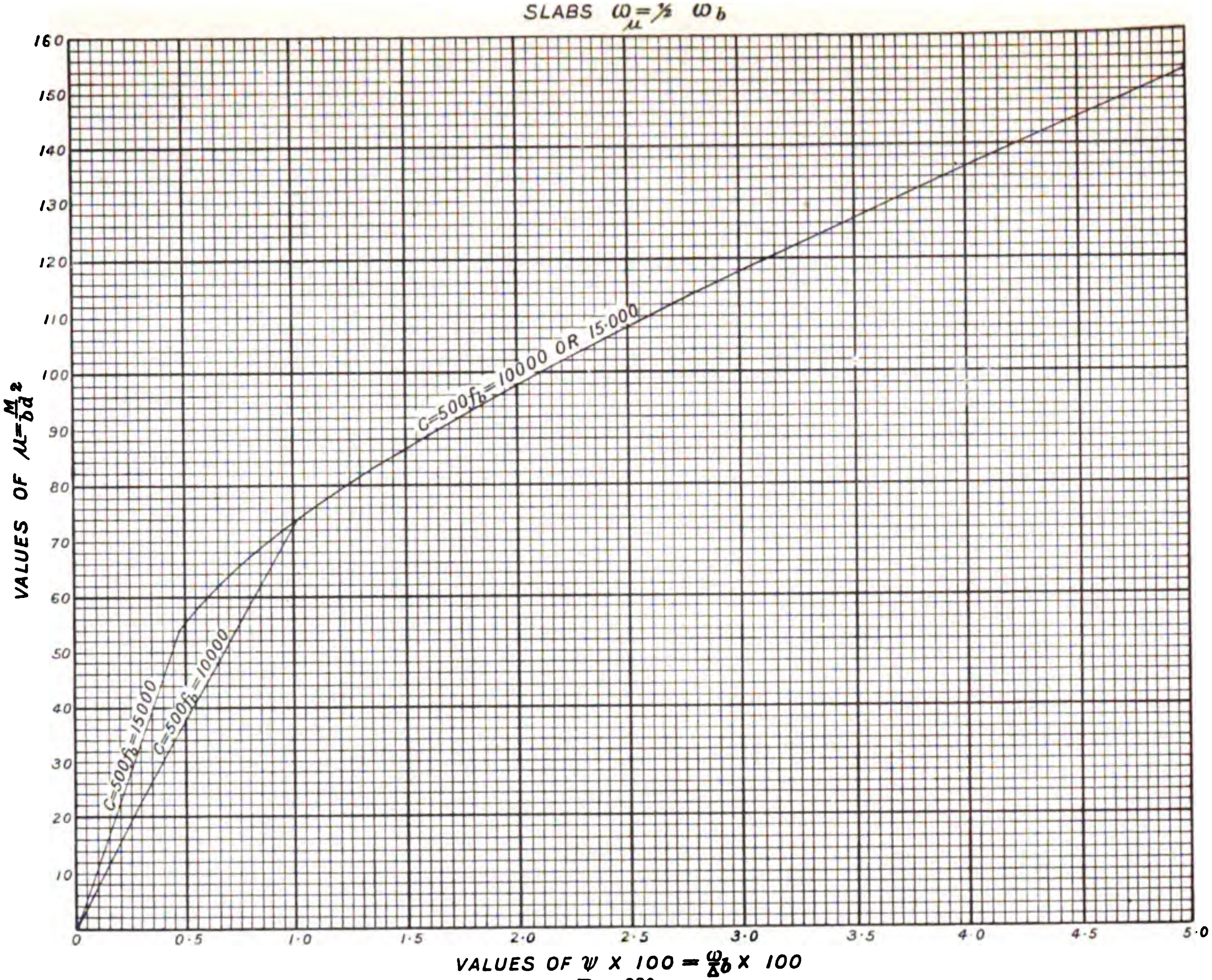
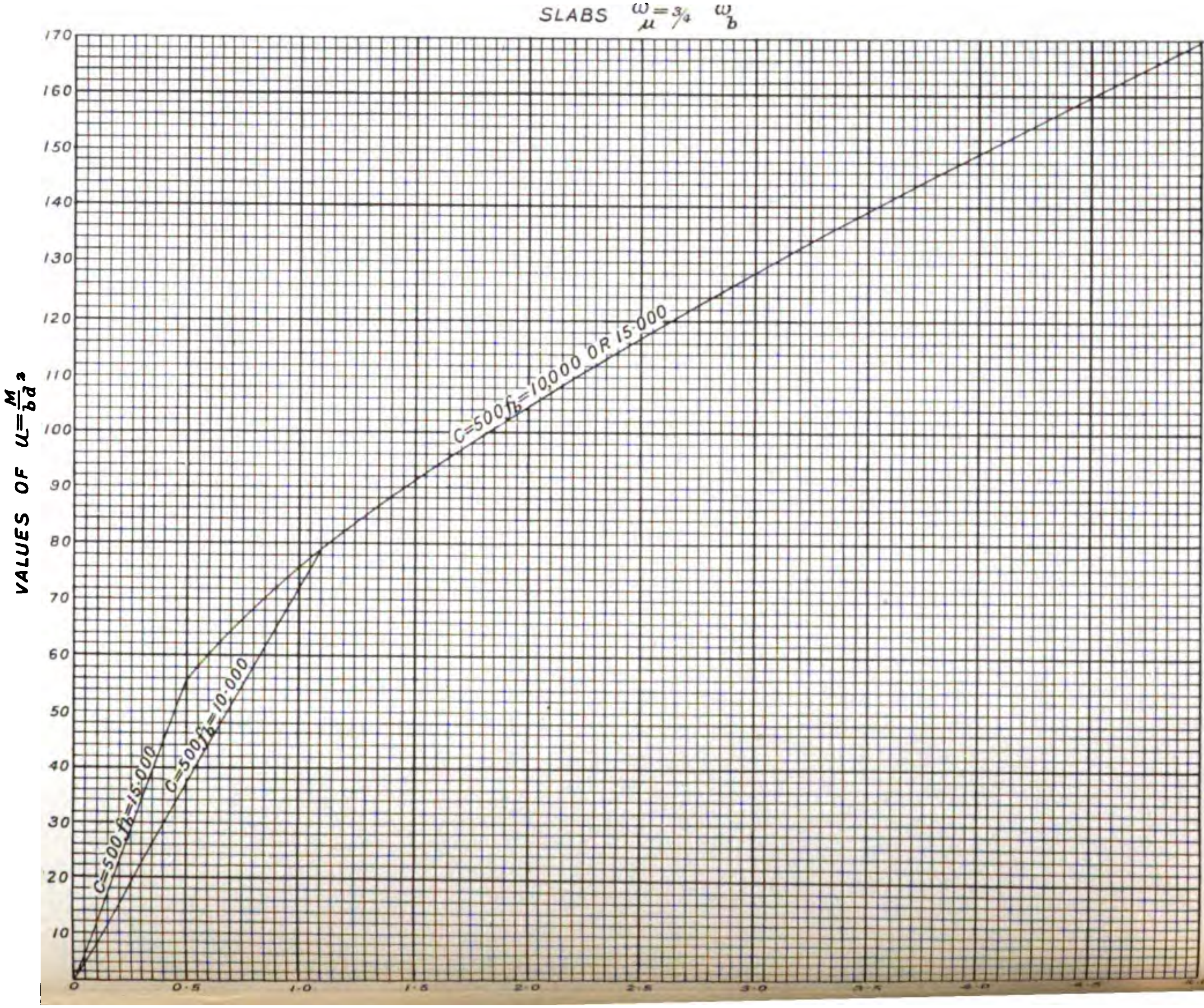


Fig. 329

DIAGRAM SHOWING UNIT BENDING MOMENT FOR VARIOUS PERCENTAGES OF TENSILE REINFORCEMENT



M SHOWING UNIT BENDING MOMENT FOR VARIOUS PERCENTAGES OF TENSILE REINFORCEMENT

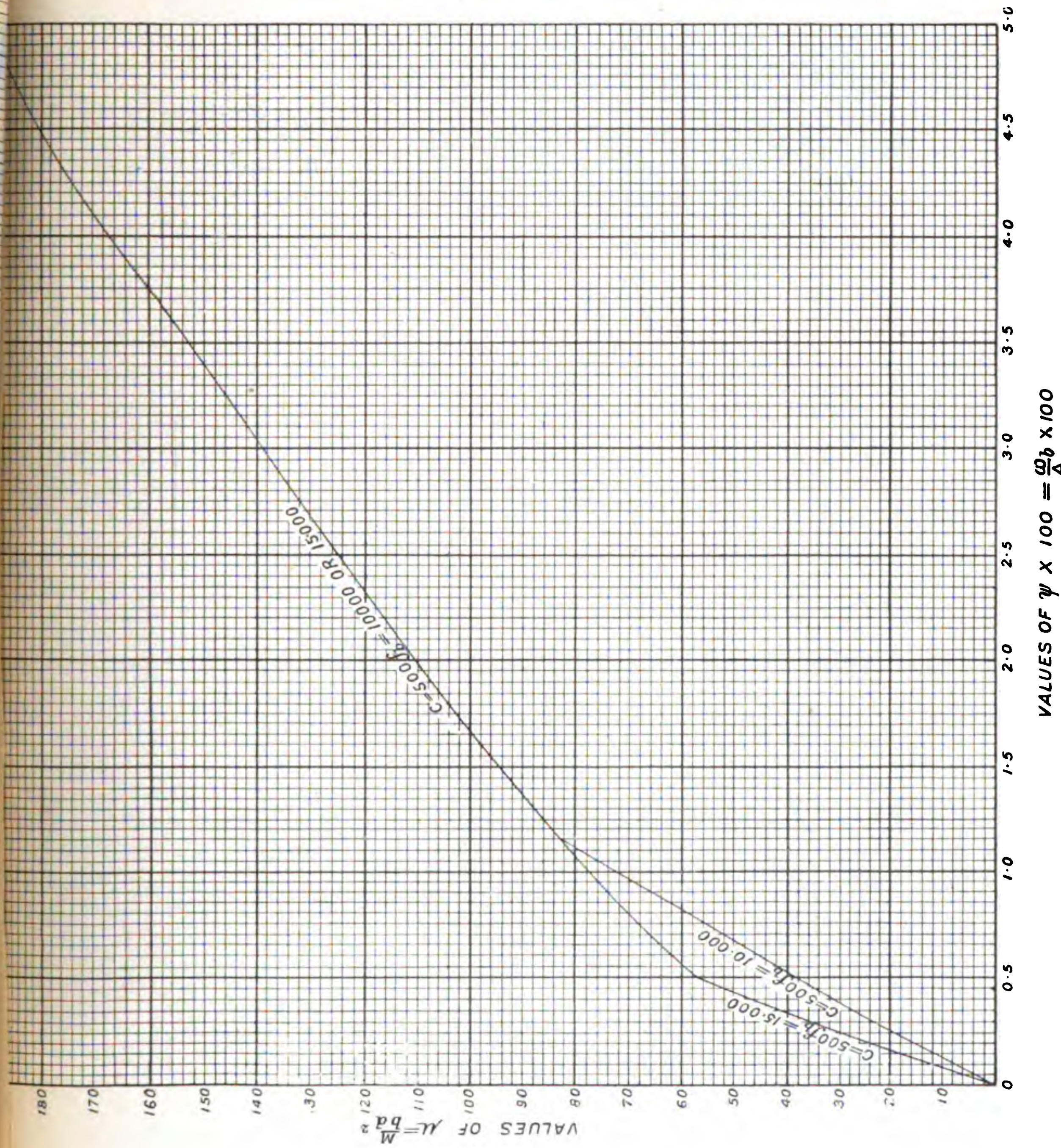


Fig. 331

Y

When $\omega_u = \frac{3}{4}\omega_b$

$$\psi = \frac{1000c^2}{9(f_b + 10c)(23f_b - 120c)} \qquad [68].$$

When $\omega_u = \omega_b$

$$\psi = \frac{250c^2}{9(f_b + 10c)(6f_b - 40c)} \qquad [69].$$

Equations [66] to [69] give the economic values for (ψ) and equation [65] that for (γ), which is the same in all cases. The value of (γ) substituted in equations [49] to [52], or [56] to [59], will give the economic values for μ. Tables LIX to LXII give the values for (ψ), (γ) and (μ), corresponding to Figs. 324 to 331, and those for the expressions which multiplied by any values allowed for (c) and (f_b) will give the values of (μ).

The economic values are printed in heavy type.

TABLE LIX

VALUES OF μ AND γ FOR SLABS WITH A DOUBLE REINFORCEMENT WHEN $\omega_u = \frac{1}{4}\omega_b$

Percentages of *Tensile* Reinforcement to Total Area of Piece	$\psi = \frac{\omega_b}{\Delta}$	$\gamma = \frac{u}{d}$	Value of $\frac{\mu}{c}$	Value of $\frac{\mu}{f_b}$	μ c=500 f_b=10,000	μ c=500 f_b=15,000
0·25	0·0025	0·159	0·0820	0·001934	19·3	29·0
0·47	**0·0047**	**0·208**	—	—	—	**53·2**
0·50	0·005	0·213	0·1091	0·003748	37·5	54·5
0·75	0·0075	0·251	0·1272	0·005426	54·3	63·6
0·97	**0·0097**	**0·278**	—	—	**70·5**	—
1·0	0·01	0·280	0·1429	0·007231	71·5	71·5
1·5	0·015	0·324	0·1665	0·010590	83·2	83·2
2·0	0·02	0·357	0·1850	0·013875	92·5	92·5
2·5	0·025	0·384	0·2003	0·017114	100·1	100·1
3·0	0·03	0·407	0·2134	0·02036	106·7	106·7
4·0	0·04	0·440	0·2385	0·02668	119·3	119·3
5·0	0·05	0·467	0·2583	0·03261	129·2	129·2

TABLE LX

VALUES OF μ AND γ FOR SLABS WITH DOUBLE REINFORCEMENT WHEN $\omega_u = \frac{1}{2}\omega_b$

Percentage of *Tensile* Reinforcement to Total Area of Piece	$\psi = \frac{\omega_b}{\Delta}$	$\gamma = \frac{u}{d}$	Value of $\frac{\mu}{c}$	Value of $\frac{\mu}{f_b}$	μ c=500 f_b=10,000	μ c=500 f_b=15,000
0·25	0·0025	0·159	0·0836	0·001971	19·7	29·6
0·48	**0·0048**	**0·208**	—	—	—	**54·1**
0·5	0·005	0·211	0·1114	0·003773	37·7	55·7
0·75	0·0075	0·247	0·1304	0·005496	55·0	65·2
1·0	0·01	0·274	0·1463	0·007167	71·7	73·2
1·03	**0·0103**	**0·278**	—	—	**74·4**	—
1·5	0·015	0·315	0·1742	0·010589	87·1	87·1
2·0	0·02	0·344	0·1962	0·013855	98·1	98·1
2·5	0·025	0·368	0·2159	0·017068	107·9	107·9
3·0	0·03	0·388	0·2346	0·020442	117·3	117·3
4·0	0·04	0·418	0·2712	0·027370	135·6	135·6
5·0	0·05	0·438	0·3073	0·034055	153·6	153·6

TABLE LXI

VALUES OF μ AND γ FOR SLABS WITH DOUBLE REINFORCEMENT WHEN $\omega_u = \frac{3}{4}\omega_b$

Percentage of *Tensile* Reinforcement to Total Area of Piece	$\psi = \frac{\omega_b}{\Delta}$	$\gamma = \frac{u}{d}$	Value of $\frac{\mu}{c}$	Value of $\frac{\mu}{f_b}$	μ	
					$c=500$ $f_b=10{,}000$	$c=500$ $f_b=15{,}000$
0·25	0·0025	0·159	0·0820	0·001934	19·3	29·0
0·49	**0·0049**	**0·208**	—	—	—	**55·1**
0·5	0·005	0·210	0 1111	0·003754	37·5	55·6
0·75	0·0075	0·244	0·1327	0·005495	54·9	66·4
1·0	0·01	0·269	0·1513	0·007209	72·1	75·7
1·09	**0·0109**	**0·278**	—	—	**78·4**	—
1·5	0·015	0·307	0·1815	0·010595	90·8	90·8
2·0	0·02	0·333	0·2085	0·013885	104·3	104·3
2·5	0·025	0·354	0·2328	0·017189	116·4	116·4
3·0	0·03	0·374	0·2530	0·020601	126·5	126·5
4·0	0·04	0·388	0·2986	0·026414	149·3	149·3
5·0	0·05	0·413	0·3396	0·033356	169·8	169·8

TABLE LXII

VALUES OF μ AND γ FOR SLABS WITH DOUBLE REINFORCEMENT WHEN $\omega_u = \omega_b$

Percentage of *Tensile* Reinforcement to Total Area of Piece	$\psi = \frac{\omega_b}{\Delta}$	$\gamma = \frac{u}{d}$	Value of $\frac{\mu}{c}$	Value of $\frac{\mu}{f_b}$	μ	
					$c=500$ $f_b=10{,}000$	$c=500$ $f_b=15{,}000$
0·25	0·0025	0·160	0·08242	0·001981	19·8	29·7
0·5	**0·005**	**0·208**	**0·11392**	0·003796	38·0	**56·9**
0·75	0·0075	0·241	0·13822	0·005429	54·3	69·1
1·0	0·01	0·266	0·15502	0·007267	72·7	77·5
1·16	**0·0116**	**0·278**	—	—	**82·4**	—
1·5	0·015	0·299	0·18932	0·010594	94·7	94·7
2·0	0·02	0·324	0·21871	0·013913	109·4	109·4
2·5	0·025	0·344	0·24968	0·016968	124·8	124·8
3·0	0·03	0·355	0·27542	0·020468	137·7	137·7
4·0	0·04	0·378	0·33108	0·026992	165·5	165·5
5·0	0·05	0·390	0·38876	0·034216	194·4	194·4

Having obtained the value of (μ), the thickness of the piece is given by the equation $d=\sqrt{\frac{M}{\mu b}}$ (b) ; being 12 inches *if the load is taken in pounds per square foot, M being, of course, calculated in inch-pounds.* The area of the lower reinforcement is obtained from equations $\omega_b = \psi bd$ and ω_u by its ratio to ω_b, the metal being distributed in as many bars or wires as may be considered necessary.

Beams.

In the case of beams where the value of $h=\frac{9}{10}d$ and $a=\beta=\frac{1}{10}d$. Substituting the values for m, u, ω_b, μ, h *and* a and replacing ω_u by its value in respect to ω_b, we get from equation [41]—

If $\omega_u = \frac{3}{4}\omega_b$

$$\gamma = \frac{75}{8}\psi\left(-1+\sqrt{1+\frac{888}{5625\psi}}\right) \quad [70].$$

If $\omega_u = \frac{1}{2}\omega_b$

$$\gamma = \frac{90}{8}\psi\left(-1+\sqrt{1+\frac{228}{2025\psi}}\right) \quad [71].$$

If $\omega_u = \frac{3}{4}\omega_b$

$$\gamma = \frac{105}{8}\psi\left(-1+\sqrt{1+\frac{936}{11025\psi}}\right) \quad [72].$$

If $\omega_u = \omega_b$

$$\gamma = 15\psi\left(-1+\sqrt{1+\frac{1}{15\psi}}\right) \quad [73].$$

Substituting the values of $u = \gamma d$, $h = \frac{9}{10}d$ and $a = \frac{1}{10}d$ in equations [39] and [40], we get—

$$(hb) = \frac{d}{10}(9-10\gamma) \quad [74].$$

$$(hu) = \frac{d}{10}(10\gamma-1) \quad [75].$$

Substituting these values with $M = \mu bd^2$ and $u = \gamma d$ in equation [38], we get—

$$\mu = \frac{c}{\gamma bd}\left[\frac{5}{12}\gamma^3 bd + \frac{1}{10}\left\{\omega_u(10\gamma-1)^2 + \omega_b(9-10\gamma)^2\right\}\right] \quad [76].$$

Replacing ω_u by its value in respect to ω_b and further replacing ω_b by ψbd, we get for the values of (μ) for different ratios of ω_u to ω_b—

When $\omega_u = \frac{1}{4}\omega_b$

$$\mu = \frac{c}{\gamma}\left[\frac{5}{12}\gamma^3 + \frac{1}{40}\psi\left\{(10\gamma-1)^2 + 4(9-10\gamma)^2\right\}\right] \quad [77].$$

When $\omega_u = \frac{1}{2}\omega_b$

$$\mu = \frac{c}{\gamma}\left[\frac{5}{12}\gamma^3 + \frac{1}{20}\psi\left\{(10\gamma-1)^2 + 2(9-10\gamma)^2\right\}\right] \quad [78].$$

When $\omega_u = \frac{3}{4}\omega_b$

$$\mu = \frac{c}{\gamma}\left[\frac{5}{12}\gamma^3 + \frac{1}{40}\psi\left\{3(10\gamma-1)^2 + 4(9-10\gamma)^2\right\}\right] \quad [79].$$

When $\omega_u = \omega_b$

$$\mu = \frac{c}{\gamma}\left[\frac{5}{12}\gamma^3 + \frac{1}{10}\psi\left\{(10\gamma-1)^2 + (9-10\gamma)^2\right\}\right] \quad [80].$$

From equation [33] $f_b = cm\frac{(hb)}{u}$ we get by substituting for the values of m, (hb)

and *u*,

$$f_b = \frac{c}{\gamma}(9-10\gamma) \quad [81];$$

and from equation [34]

$$f_u = f_b\frac{(10\gamma-1)}{(9-10\gamma)} \quad [82].$$

From equation [81] we get—

$$c = \frac{\gamma}{f_b(9-10\gamma)} \quad [83].$$

Substituting this value for c in equations [77] to [80] we get—

When $\omega_u = \frac{1}{4}\omega_b$

$$\mu = \frac{f_b}{(9-10\gamma)}\left[\frac{5}{12}\gamma^3 + \frac{1}{40}\psi\left\{(10\gamma-1)^2 + 4(9-10\gamma)^2\right\}\right] \quad [84].$$

When $\omega_u = \frac{1}{2}\omega_b$

$$\mu = \frac{f_b}{(9-10\gamma)}\left[\frac{5}{12}\gamma^3 + \frac{1}{20}\psi\left\{(10\gamma-1)^2 + 2(9-10\gamma)^2\right\}\right] \quad [85].$$

When $\omega_u = \frac{3}{4}\omega_b$

$$\mu = \frac{f_b}{(9+10\gamma)}\left[\frac{5}{12}\gamma^3 + \frac{1}{40}\psi\left\{3(10\gamma+1)^2 + 4(9-10\gamma)^2\right\}\right] \quad [86].$$

When $\omega_u = \omega_b$

$$\mu = \frac{f_b}{(9-10\gamma)}\left[\frac{5}{12}\gamma^3 + \frac{1}{10}\psi\left\{(10\gamma-1)^2 + (9-10\gamma)^2\right\}\right] \quad [87].$$

The equation for assuring the safety of the concrete in tension is the same as that for a single reinforcement.

$$f_{b_{\max}} = \frac{(9-10\gamma)}{(10-10\gamma)} \times \frac{1}{1000} \times E_t \quad [88].$$

The values of (γ) and (μ) for the various ratios of compressive and tensile reinforcement for different percentages of area of the *tensile* reinforcement to the total area of the piece have been plotted on diagrams (Figs 332 to 339). To find the values for (ψ), (γ) and (μ) for the most economical sections, i.e. those where (c) and (f_b) have their maximum allowed values, we have from equation [31] $\frac{2}{3}cub = \omega_b f_b - \omega_u f_u$, by substituting for ($f_u$) its value from equation [82], and replacing (ω_u) by its value with respect to (ω_b), and finally replacing ω_b by ψbd.

$$\text{When } \omega_u = \tfrac{1}{4}\omega_b \quad \psi = \frac{8c\gamma}{3f_b}\left\{\frac{(9-10\gamma)}{(37-50\gamma)}\right\} \quad [89].$$

DIAGRAM SHOWING POSITION OF NEUTRAL AXIS FOR VARIOUS PERCENTAGES OF TENSILE REINFORCEMENT

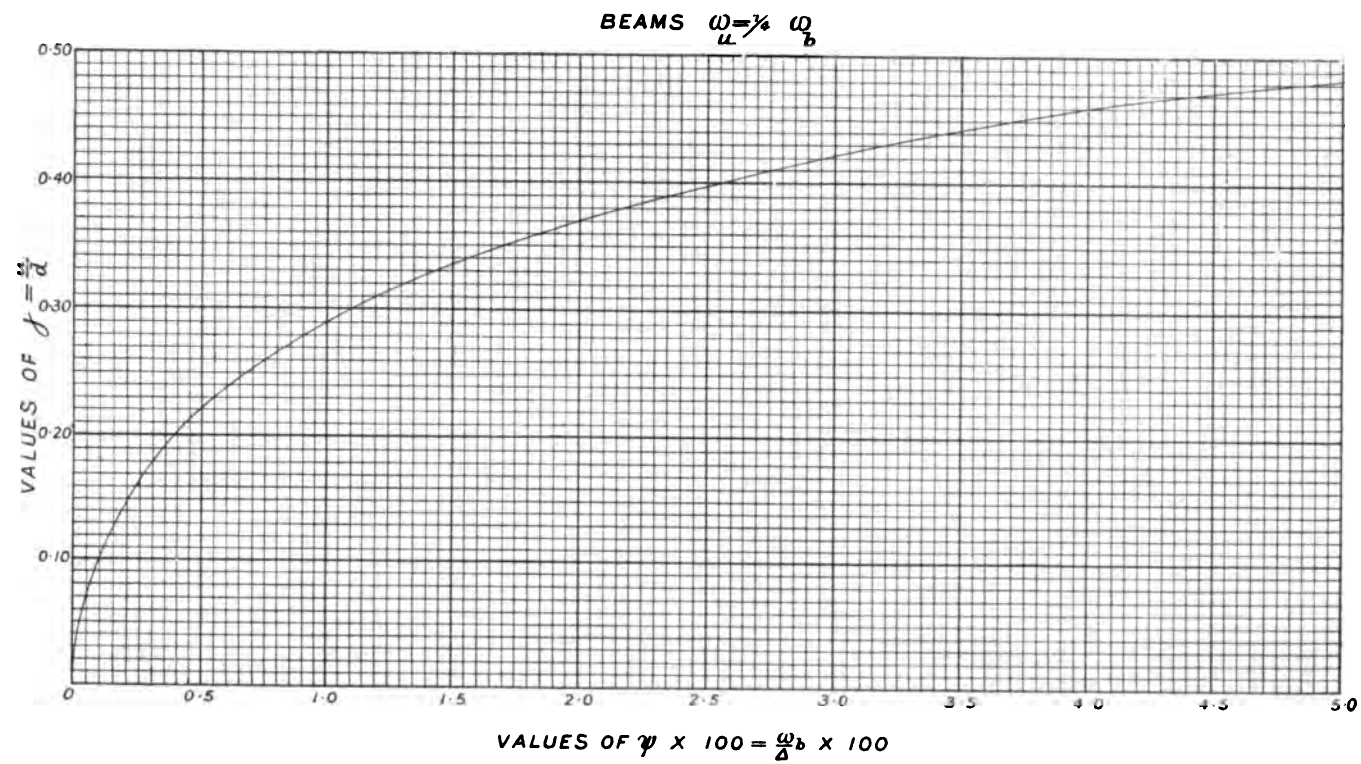


FIG. 332

DIAGRAM SHOWING POSITION OF NEUTRAL AXIS FOR VARIOUS PERCENTAGES OF TENSILE REINFORCEMENT

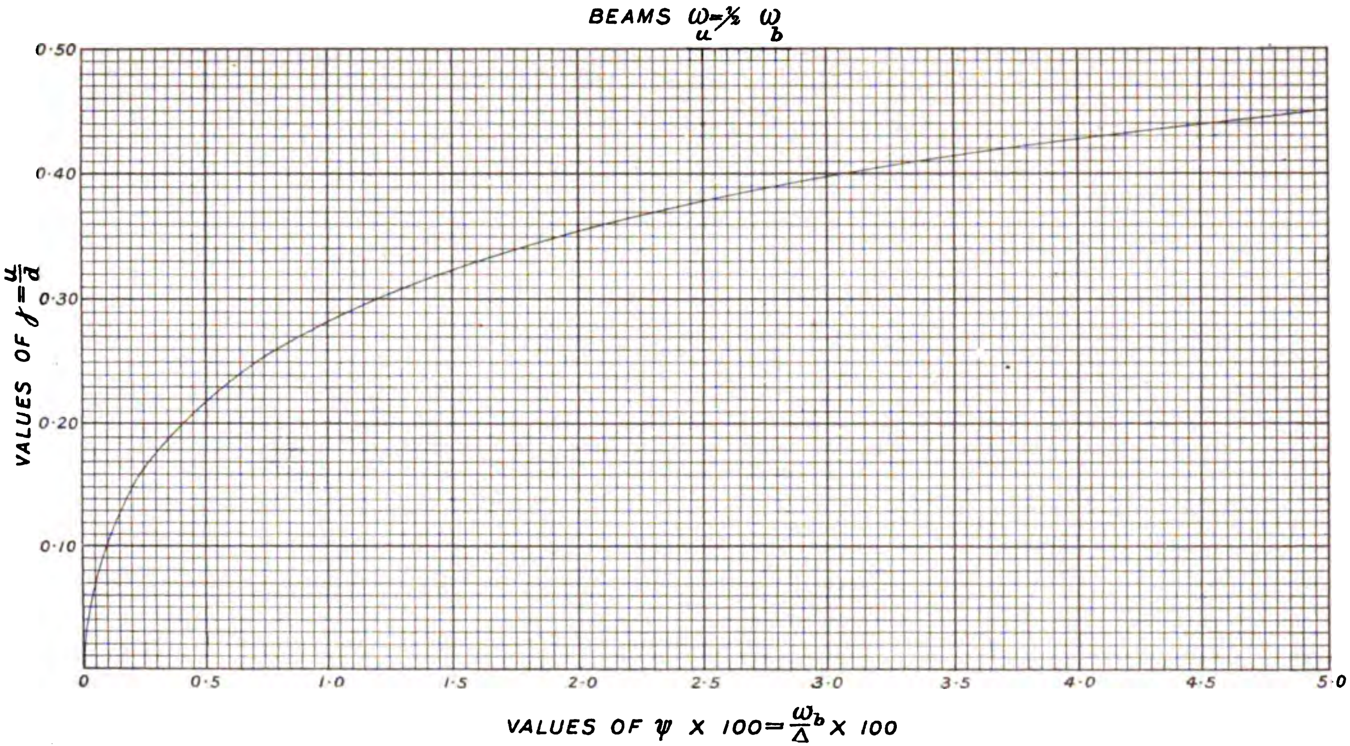


FIG. 333

DIAGRAM SHOWING POSITION OF NEUTRAL AXIS FOR VARIOUS PERCENTAGES OF TENSILE REINFORCEMENT

BEAMS $\omega_u = \frac{7}{4}\,\omega_b$

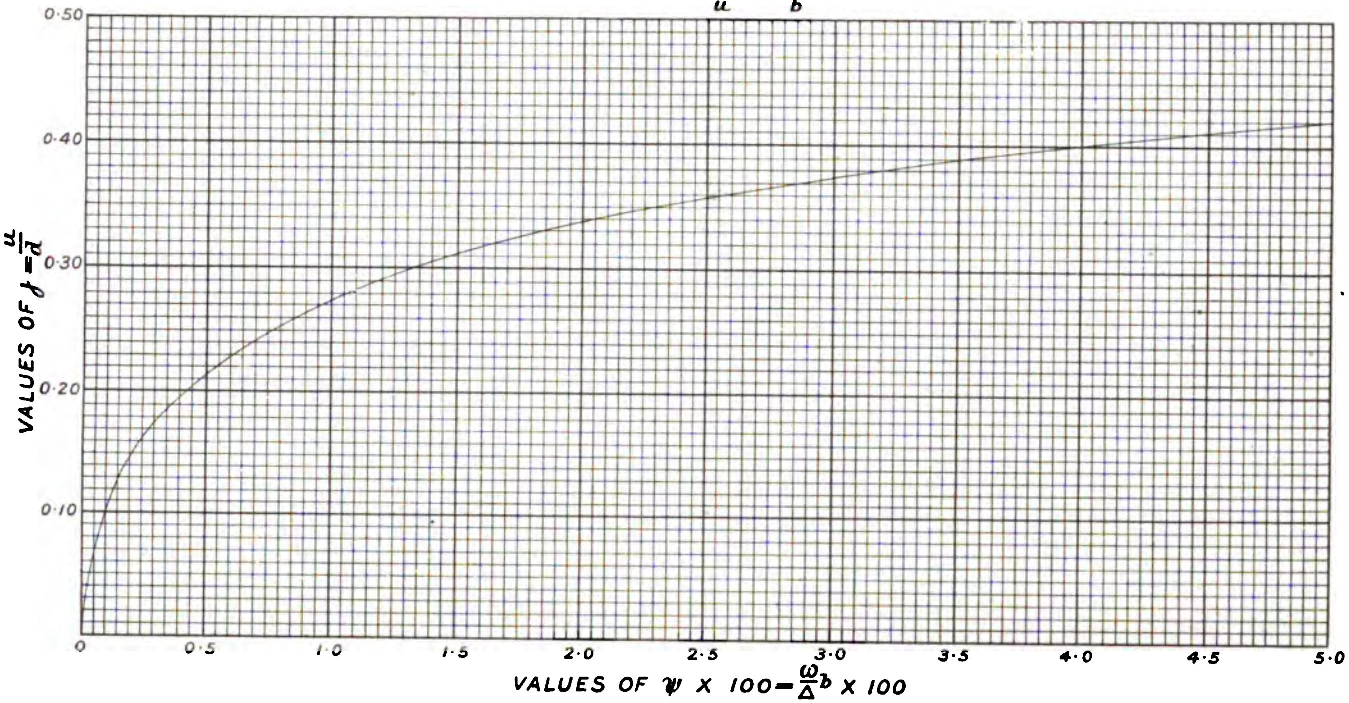


FIG. 334

DIAGRAM SHOWING POSITION OF NEUTRAL AXIS FOR VARIOUS PERCENTAGES OF TENSILE REINFORCEMENT

BEAMS $\omega_u = \omega_b$

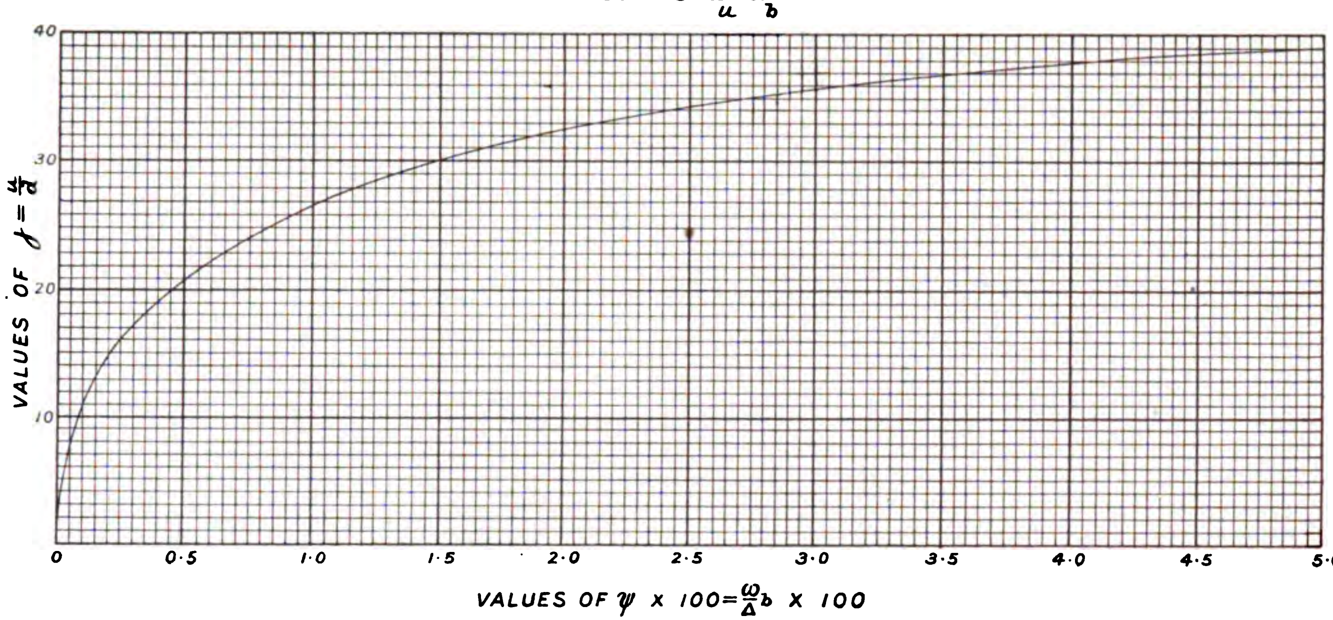


FIG. 335

DIAGRAM SHOWING UNIT BENDING MOMENT FOR VARIOUS PERCENTAGES OF TENSILE REINFORCEMENT

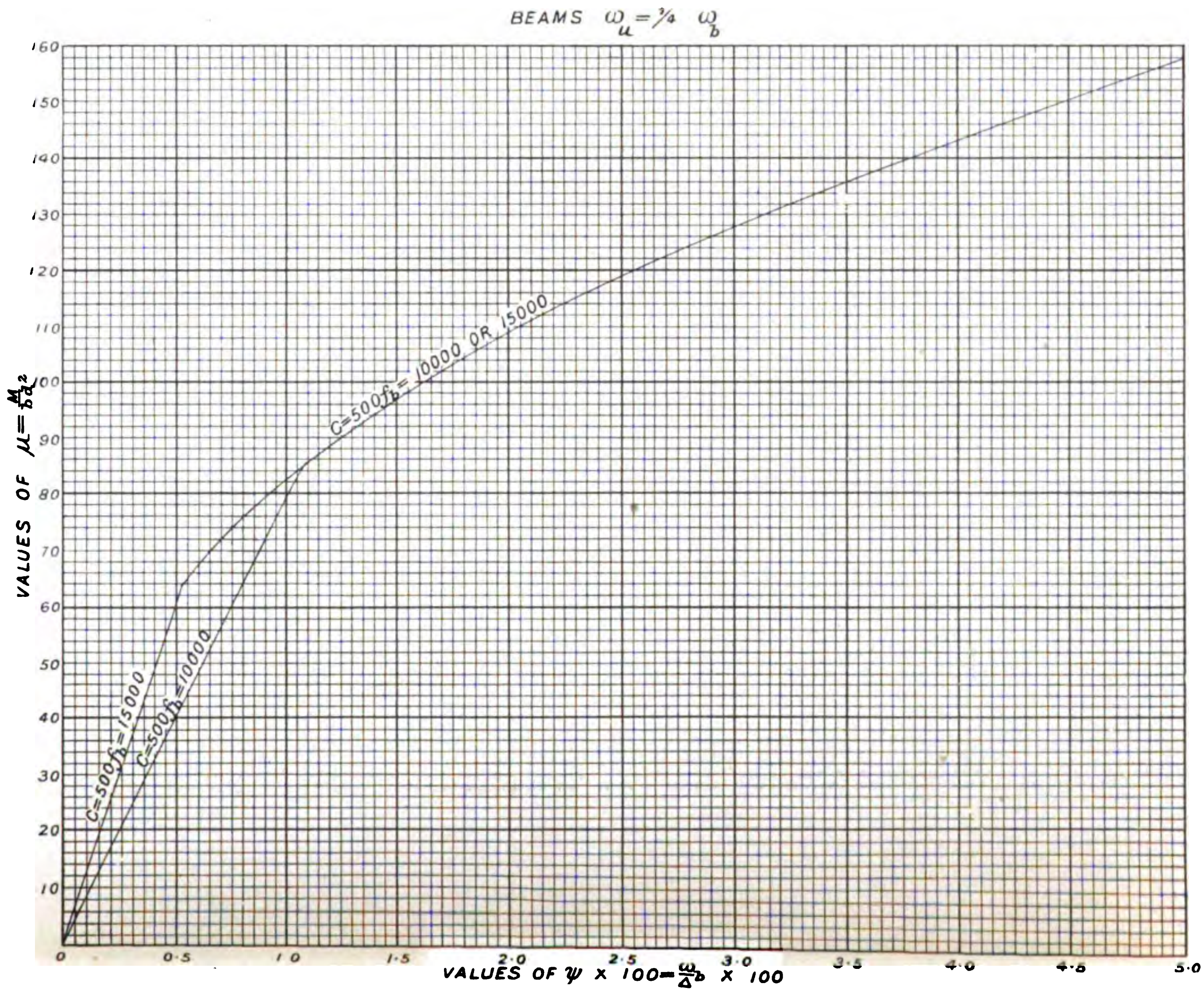


FIG. 336

[DI]AGRAM SHOWING UNIT BENDING MOMENT FOR VARIOUS PERCENTAGES OF TENSILE REINFORCEMENT

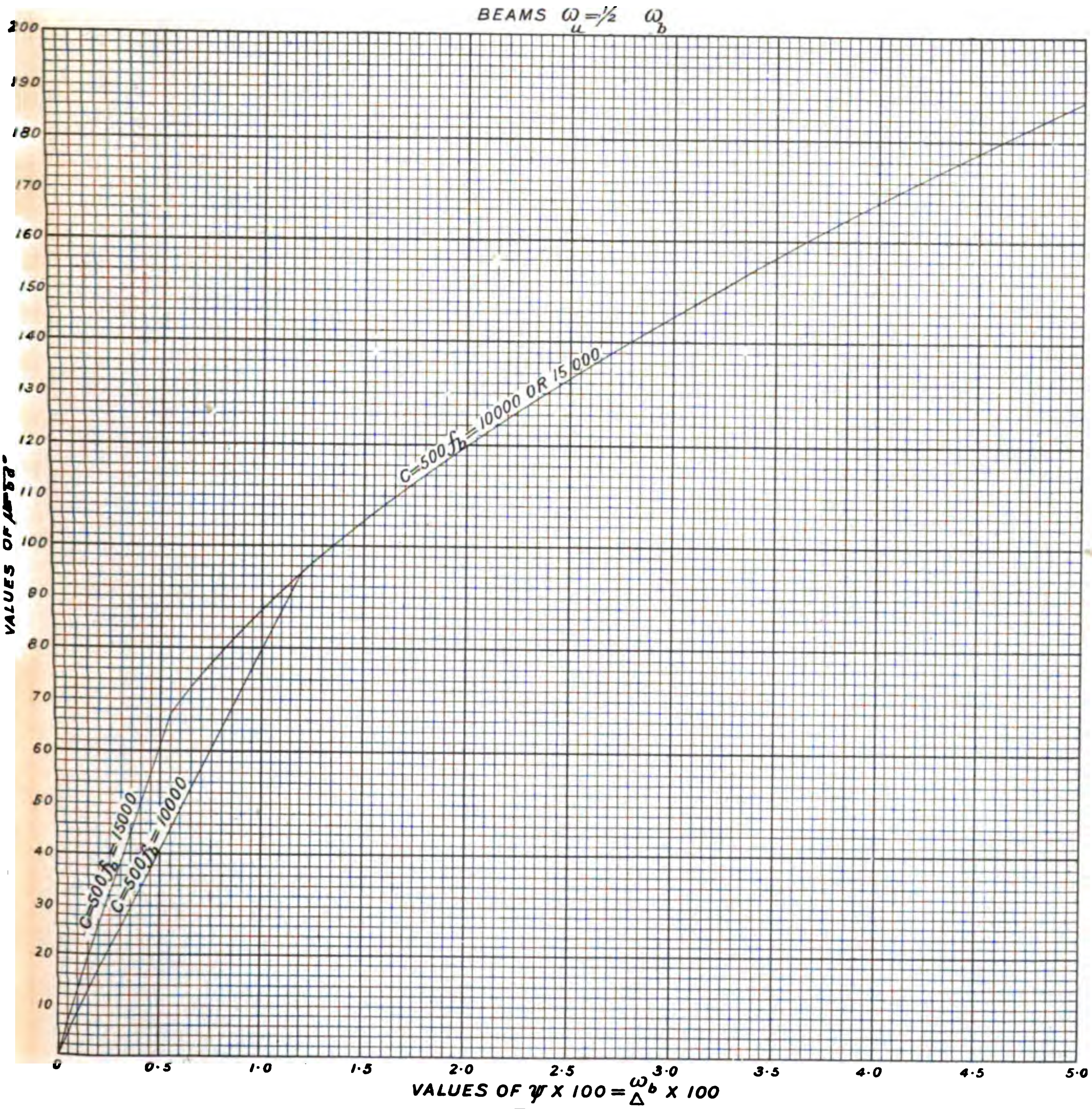


FIG. 337

DIAGRAM SHOWING UNIT BENDING MOMENT FOR VARIOUS PERCENTAGES OF TENSILE REINFORCEMENT

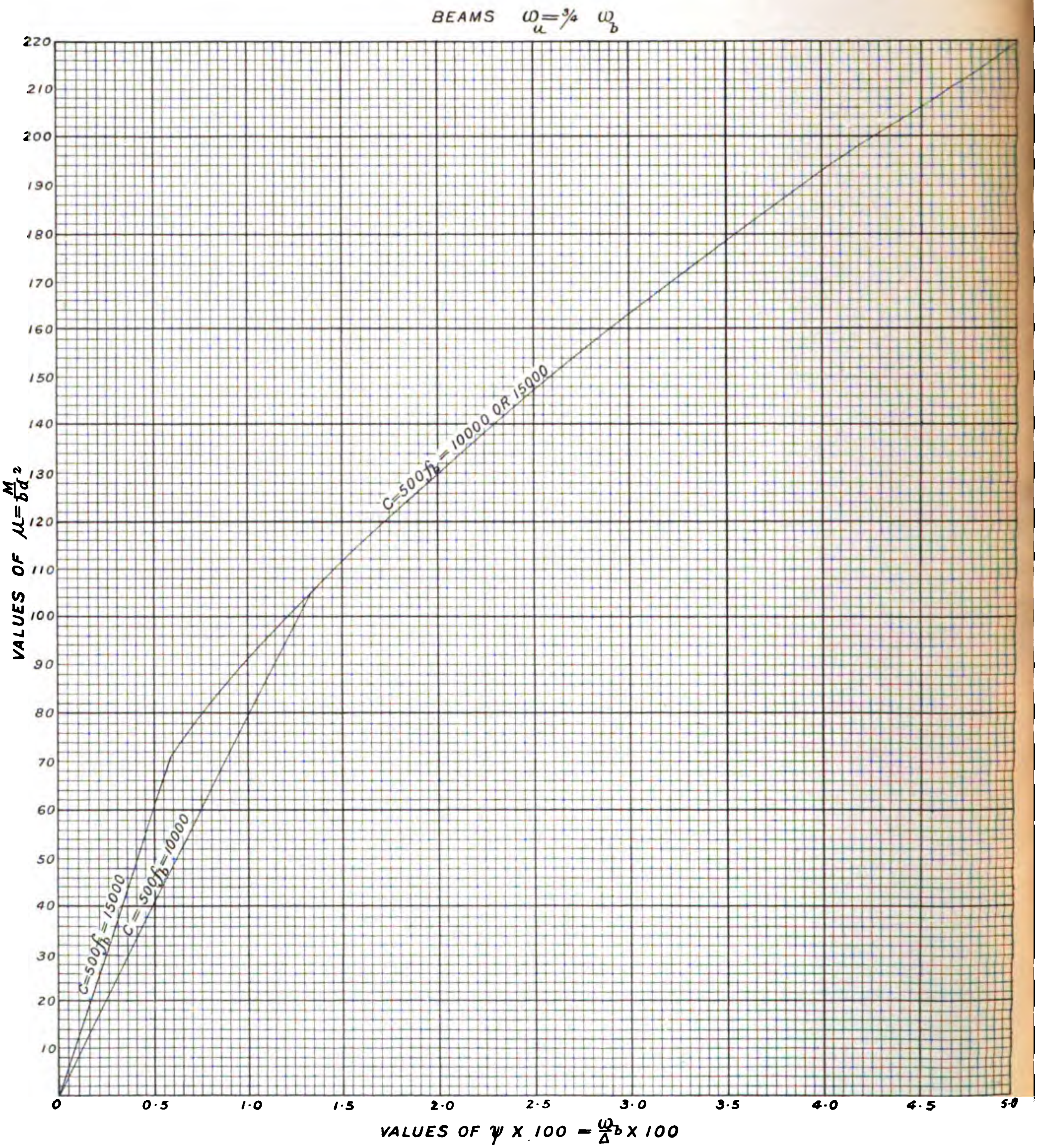


FIG. 338

RAM SHOWING UNIT BENDING MOMENT FOR VARIOUS PERCENTAGES OF TENSILE REINFORCEMENT

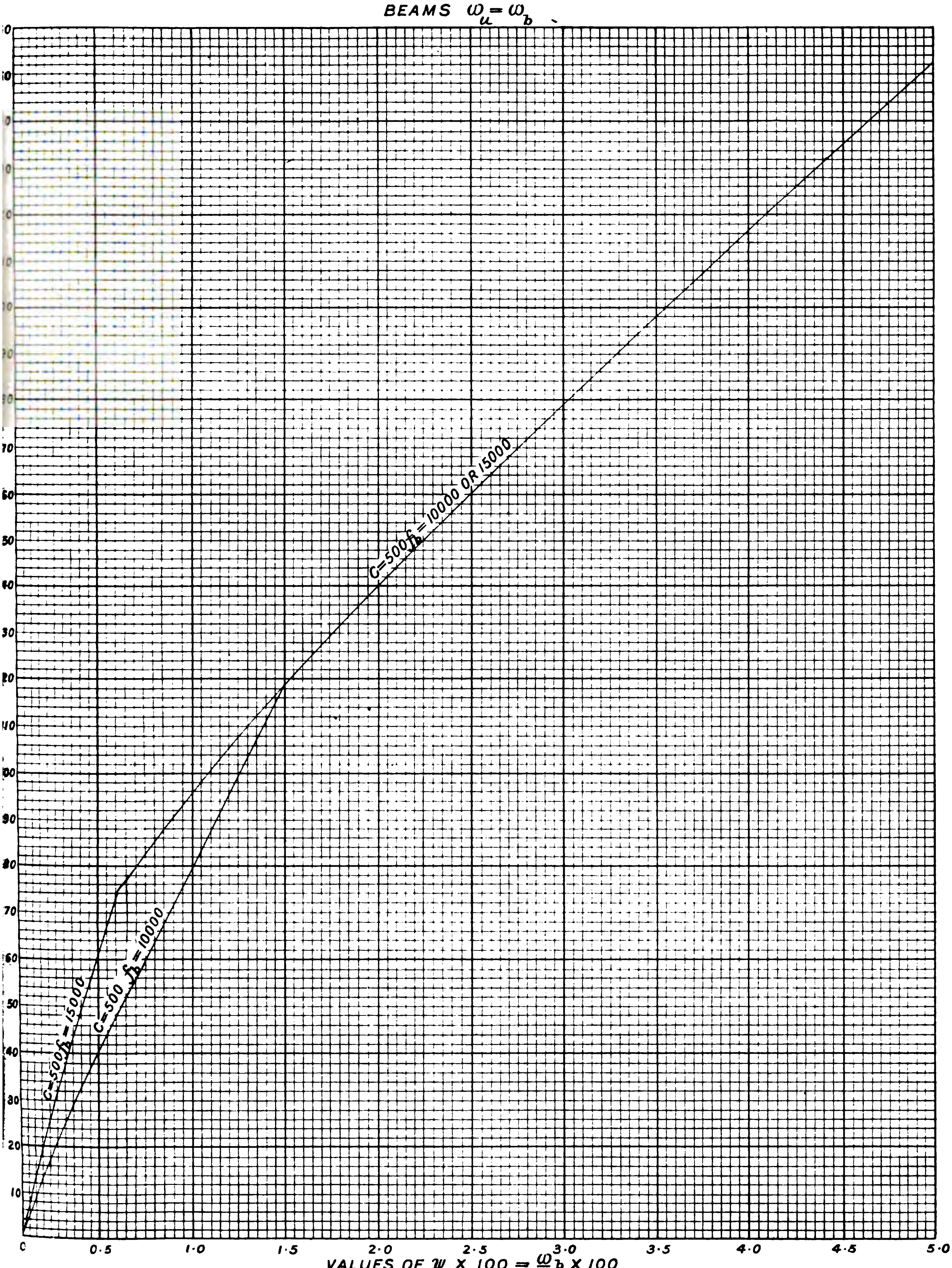


Fig. 339

$$\text{When } \omega_u = \tfrac{1}{2}\omega_b \quad \psi = \frac{4c\gamma}{3f_b}\left\{\frac{(9-10\gamma)}{(19-30\gamma)}\right\} \quad [90].$$

$$\text{When } \omega_u = \tfrac{3}{4}\omega_b \quad \psi = \frac{8c\gamma}{3f_b}\left\{\frac{(9-10\gamma)}{(39-70\gamma)}\right\} \quad [91].$$

$$\text{When } \omega_u = \omega_b \quad \psi = \frac{c\gamma}{15f_b}\left\{\frac{(9-10\gamma)}{(1-2\gamma)}\right\} \quad [92].$$

From equation [81] we get—

$$\gamma = \frac{9c}{(f_b + 10c)} \quad [93].$$

As γ is only dependent on the values of (f_b) and (c) it is the same for all ratios of ω_u to ω_b.

Substituting the value of (γ) from equation [93] in equations [89] to [92] we get—

$$\text{When } \omega_u = \tfrac{1}{4}\omega_b \quad \psi = \frac{216c^2}{(f_b + 10c)(37f_b - 80c)} \quad [94].$$

$$\text{When } \omega_u = \tfrac{1}{2}\omega_b \quad \psi = \frac{108c^2}{(f_b + 10c)(19f_b - 80c)} \quad [95].$$

$$\text{When } \omega_u = \tfrac{3}{4}\omega_b \quad \psi = \frac{216c^2}{3(f_b + 10c)(13f_b - 80c)} \quad [96].$$

$$\text{When } \omega_u = \omega_b \quad \psi = \frac{27c^2}{5(f_b + 10c)(f_b - 8c)} \quad [97].$$

Equations [94] to [97] give the economic values for (ψ) and equation [93] that for (γ) which is the same for all cases, and the value of (γ) substituted in equations [77] to [80] or [84] to [87] will give the economic values for (μ).

Tables LXIII to LXVI give the values for ψ, γ and μ, corresponding to Figs. 332 to 339, and those for the expressions which, multiplied by any values allowed for (c) and (f_b), will give the values of (μ). The economic values are printed in heavy type.

TABLE LXIII

VALUES OF μ AND γ FOR BEAMS WITH DOUBLE REINFORCEMENT WHEN $\omega_u = \frac{1}{4}\omega_b$

Percentage of *Tensile* Reinforcement to Total Area of Piece	$\psi = \frac{\omega_b}{\Delta}$	$\gamma = \frac{u}{d}$	Value of $\frac{\mu}{c}$	Value of $\frac{\mu}{f_b}$	μ $c = 500$, $f_b = 10{,}000$	μ $c = 500$, $f_b = 15{,}000$
0·25	0·0025	0·164	0·0930	0·002073	20·7	31·1
0·5	0·005	0·220	0·1259	0·004075	40·7	61·1
0·524	**0·00524**	**0·225**	—	—	—	**63·9**
0·75	0·0075	0·259	0·1483	0·005991	59·9	74·1
1·0	0·01	0·289	0·1669	0·007894	78·9	83·4
1·09	**0·0109**	**0·300**	—	—	**85·6**	—
1·5	0·015	0·335	0·1960	0·011621	98·0	98·0
2·0	0·02	0·371	0·2181	0·015298	109·1	109·1
2·5	0·025	0·398	0·2384	0·018894	119·2	119·2
3·0	0·03	0·421	0·2557	0·022479	127·8	127·8
4·0	0·04	0·460	0·2846	0·029754	142·3	142·3
5·0	0·05	0·482	0·3158	0·036407	157·9	157·9

TABLE LXIV

VALUES OF μ AND γ FOR BEAMS WITH DOUBLE REINFORCEMENT WHEN $\omega_u = \frac{1}{2}\omega_b$

Percentage of *Tensile* Reinforcement to Total Area of Piece	$\psi = \frac{\omega b}{\Delta}$	$\gamma = \frac{u}{d}$	Value of $\frac{\mu}{c}$	Value of $\frac{\mu}{f_b}$	μ	
					$c=500$ $f_b=10{,}000$	$c=500$ $f_b=15{,}000$
0·25	0·0025	0·163	0·0947	0·002094	20·9	31·5
0·5	0·005	0·216	0·1293	0·004083	40·8	61·2
0·55	**0·0055**	**0·225**	—	—	—	**67·1**
0·75	0·0075	0·253	0·1540	0·006025	60·2	77·0
1·0	0·01	0·281	0·1749	0·007938	79·4	87·5
1·2	**0·012**	**0·300**	—	—	**94·5**	—
1·5	0·015	0·322	0·2104	0·011717	105·2	105·2
2·0	0·02	0·353	0·2395	0·015454	119·7	119·7
2·5	0·025	0·377	0·2659	0·019172	132·9	132·9
3·0	0·03	0·398	0·2895	0·022145	144·8	144·8
4·0	0·04	0·427	0·3356	0·030289	167·8	167·8
5·0	0·05	0·450	0·3774	0·037745	187·7	187·7

TABLE LXV

VALUES OF μ AND γ FOR BEAMS WITH DOUBLE REINFORCEMENT WHEN $\omega_u = \frac{3}{4}\omega_b$

Percentage of *Tensile* Reinforcement to Total Area of Piece	$\psi = \frac{\omega b}{\Delta}$	$\gamma = \frac{u}{d}$	Value of $\frac{\mu}{c}$	Value of $\frac{\mu}{f_b}$	μ	
					$c=500$ $f_b=10{,}000$	$c=500$ $f_b=15{,}000$
0·25	0·0025	0·162	0·0951	0·002072	20·7	31·1
0·5	0·005	0·212	0·1327	0·004088	40·9	61·3
0·581	**0·00581**	**0·225**	—	—	—	**70·7**
0·75	0·0075	0·247	0·1592	0·006018	60·2	79·6
1·0	0·01	0·273	0·1820	0·007964	79·6	91·0
1·33	**0·0133**	**0·300**	—	—	**104·8**	—
1·5	0·015	0·311	0·2236	0·011809	111·8	111·8
2·0	0·02	0·338	0·2596	0·015612	129·8	129·8
2·5	0·025	0·357	0·2941	0·019340	147·0	147·0
3·0	0·03	0·374	0·3254	0·023134	162·7	162·7
4·0	0·04	0·399	0·3852	0·030688	192·6	192·6
5·0	0·05	0·420	0·4393	0·038431	219·6	219·6

TABLE LXVI

VALUES OF μ AND γ FOR BEAMS WITH DOUBLE REINFORCEMENT WHEN $\omega_u = \omega_b$

Percentage of *Tensile* Reinforcement to Total Area of Piece	$\psi = \frac{\omega b}{\Delta}$	$\gamma = \frac{u}{d}$	Value of $\frac{\mu}{c}$	Value of $\frac{\mu}{f_b}$	μ	
					$c=500$ $f_b=10{,}000$	$c=500$ $f_b=15{,}000$
0·25	0·0025	0·160	0·0966	0·002090	20·9	31·3
0·5	0·005	0·208	0·1357	0·003979	39·8	59·7
0·613	**0·00613**	**0·225**	—	—	—	**74·4**
0·75	0·0075	0·241	0·1655	0·006005	60·0	82·7
1·0	0·01	0·266	0·1911	0·008019	80·2	95·6
1·5	**0·015**	**0·300**	0·2373	0·011866	**118·6**	118·6
2·0	0·02	0·324	0·2796	0·015726	139·8	139·8
2·5	0·025	0·344	0·3181	0·019610	159·1	159·1
3·0	0·03	0·355	0·3584	0·023347	179·2	179·2
4·0	0·04	0·378	0·4298	0·031116	214·9	214·9
5·0	0·05	0·390	0·5043	0·038573	252·2	252·2

It may be mentioned here, with respect to the values for h of $\frac{1}{8}d$ for slabs and $\frac{1}{10}d$ for beams, that there must never be less than half an inch covering of concrete below or above the metal. If it is found, when the piece is designed, that the covering is less than half an inch, the depth must be increased so as to secure this minimum.

In practice the nearest fraction of an inch above the value of d obtained by the calculation will be taken as the depth, and it will seldom be found that the covering is too little.

The size of a beam with double reinforcement is found according to equations [29] or [30].

The area of the lower reinforcement is obtained from the equation $\omega_b = \psi bd$, and that of (ω_u) by its relation to (ω_b), the metal being distributed into the requisite number of rods or other reinforcements.

Tables LXVII and LXVIII show the economic values of (d) and (ψ) for various proportions of reinforcement; the values for single reinforcements are added to make tables complete.

TABLE LXVII

ECONOMICAL VALUES OF (d) AND (ψ) FOR SLABS

Reinforcement		d	$\frac{\omega_b}{\Delta} = \psi$
Nature	Value of ω_u		
Iron	0	$0{\cdot}0351\sqrt{M}$	0·0091
,,	$\frac{1}{4}\omega_b$	$0{\cdot}0344\sqrt{M}$	0·0097
,,	$\frac{1}{2}\omega_b$	$0{\cdot}0337\sqrt{M}$	0·0103
,,	$\frac{3}{4}\omega_b$	$0{\cdot}0325\sqrt{M}$	0·0109
,,	ω_b	$0{\cdot}0317\sqrt{M}$	0·0116
Steel	0	$0{\cdot}0400\sqrt{M}$	0·0047
,,	$\frac{1}{4}\omega_b$	$0{\cdot}0395\sqrt{M}$	0·0047
,,	$\frac{1}{2}\omega_b$	$0{\cdot}0390\sqrt{M}$	0·0048
,,	$\frac{3}{4}\omega_b$	$0{\cdot}0385\sqrt{M}$	0·0049
,,	ω_b	$0{\cdot}0380\sqrt{M}$	0·0050

It can be clearly seen from the above table that a double reinforcement is very uneconomical, for whereas for a single reinforcement of iron the ratio of the area of reinforcement to the total area of the piece is 0·0091 and the depth $0{\cdot}0351\sqrt{M}$.

For a double reinforcement where compressive and tensile reinforcements have the same area the ratio of the total area of reinforcement to the total area of the piece is 0·0232 and the depth is $0{\cdot}0317\sqrt{M}$, the reinforcement being increased by 60·8 per cent., while the depth is only reduced by 9·7 per cent.

Many of the constructors who employ a double symmetrical reinforcement, calculate the sizes for resisting all the stresses, neglecting the resistance of the concrete, and frequently use the reinforcement to partially support the falsework, which is not good practice.

The concrete must support certain stresses, however the reinforcement may have been calculated, and the assumption that the reinforcement will take all the stresses cannot be true. If the concrete is defective for any reason, and cannot bear the strain to which it is subjected, the whole piece must fail, as the metal skeleton is not sufficiently rigid to bear the imposed loads unaided, in spite of its having been calculated as resisting all the direct stresses.

TABLE LXVIII

ECONOMICAL VALUES OF (d) AND (ψ) FOR BEAMS.

Reinforcement		Log. d	$\frac{\omega_b}{\Delta} = \psi$
Nature	Value of ω_u		
Iron	0	$\frac{\text{Log. } M - 1{\cdot}719884}{3}$	0·01
,,	$\frac{1}{4}\omega_b$	$\frac{\text{Log. } M - 1{\cdot}756383}{3}$	0·0109
,,	$\frac{1}{2}\omega_b$	$\frac{\text{Log. } M - 1{\cdot}799341}{3}$	0·012
,,	$\frac{3}{4}\omega$	$\frac{\text{Log. } M - 1{\cdot}844270}{3}$	0·0133
,,	ω	$\frac{\text{Log. } M - 1{\cdot}897994}{3}$	0·015
Steel	0	$\frac{\text{Log. } M - 1{\cdot}609239}{3}$	0·005
,,	$\frac{1}{4}\omega_b$	$\frac{\text{Log. } M - 1{\cdot}629410}{3}$	0·00524
,,	$\frac{1}{2}\omega$	$\frac{\text{Log. } M - 1{\cdot}650632}{3}$	0·0055
,,	$\frac{3}{4}\omega_b$	$\frac{\text{Log. } M - 1{\cdot}673328}{3}$	0·00581
,,	ω_b	$\frac{\text{Log. } M - 1{\cdot}695482}{3}$	0·00613

When the bending may be on either side, as in the division walls of reservoirs and like structures, a double reinforcement is of course essential, but generally speaking it is uneconomical, and should only be employed when absolutely necessary. Bent reinforcements are, however, useful, as they resist the shearing stresses and supply the tensile resistance over and near the supports. With moving loads the top reinforcement should extend about a quarter the span on either side of the supports, which may be reduced to one-eighth, if the piece does not extend over several supports. The straight bottom rods must, of course, extend throughout the whole span and pass well into or over the supports. When bent reinforcements are employed it is frequently necessary to add further straight reinforcements at the top over and near the supports, as the bent bars in themselves may not be sufficient to resist the tensile stresses.

T-Shaped Beams

Before we can enter upon the calculations for this type of beam (perhaps the most common form of reinforced concrete construction), we must study the question of the width across the table, or the width of the slab which may be considered to act with the beam proper.

Some constructors allow a width for their T-beams of the distance from centre to centre of the beams proper, but, if this assumption is allowed, we must add the compressive stresses found for the slab acting by itself to those found for the concrete as part of the T-beams.

It is an entirely erroneous assumption to consider that a monolithic structure is properly designed when the depth of the slab is first calculated under the assumption that it acts by itself and subsequently the T-beams calculated as being of a width of the total span, the slabs and beams being assumed to take the maximum allowed compressive stresses.

It can be clearly seen that in such a case the compression surface of the slab has in reality at the centre of the span to bear double its proper stress, if only one series of beams are employed, and if the floor is formed of principal and secondary beams forming squares on plan, the slab will be stressed to three times its assumed safe resistance in compression. The real distribution of the stresses in a monolithic construction is indeterminate. It is therefore advisable to assume the width of a T-beam as some fraction of the span, and allow the maximum compressive stresses in the calculations both for beams and slabs.

The slab being partially built in, the lower portion near the supports will be under compression when the slab is considered by itself, and the contraflexure will be about a quarter the span from the supports. We may safely assume, then, that the width of a T-beam is one-half the distance from centre to centre of the beams, and allow the maximum compressive stress on the concrete in our calculations both for the slab and the T-beams.

Some constructors only allow one-third of the distance from centre to centre of beams as the width of the T-beams, but this value is unnecessarily small.

T-shaped Beams with a Tensile Reinforcement only of small Sectional Area and depth compared to those of the Whole Piece (Fig. 340).

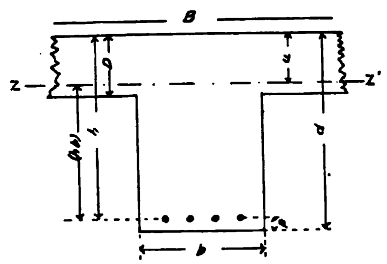


FIG. 340

When the T-beam has the slab at the top, the whole width of the slab and sometimes a portion of the rib act in compression for the whole span in freely supported beams, and for the central portion of the span when the beam is built in or rests on intermediate supports.

Taking (B) as the width of the slab and supposing the neutral axis to coincide with or be above the underside of the slab, all the equations [1] to [9] stand, if we substitute (B) for (b) in these equations.

Also all the equations [19] to [27] will stand, substituting as before (B) for (b) and *noting that in this case the percentage of reinforcement and the value for* (μ) *are taken as those of a beam of breadth* (B) *and depth* (d).

The width (B) has always been decided on from practical considerations, being one-half of the distance from centre to centre of the beams. We can therefore obtain the value for (d) directly from the equation—

$$d=\sqrt{\frac{M}{\mu B}} \quad [98].$$

If the neutral axis coincides with the underside of the slab (u) becomes equal to (D). The thickness of the slab has already been calculated from the equations for rectangular sections.

For the purposes of checking the dimensions of a T-beam already designed we find (u) from equation [8]; substituting (B) for (b) if this value is the same or less than (D), we proceed to check the dimensions as described for rectangular beams.

If, however, (u) is found to be greater than (D), we must use the equations [101] [100] and [102] below.

If in designing a T-beam the value for (u) obtained for the economic section is greater than (D), we may either take a less economic value or work by trial and error from the following formulæ.

When the neutral axis is below the underside of the slab, equation [5] becomes

$$\frac{2}{3}\left[u^2B-\left\{(u-D)^2(B-b)\right\}\right]-m\omega\,(hb)=0, \quad [99],$$

and equation [6]—

$$M=\frac{c}{u}\left[\frac{5}{12}\left\{u^3B-(u-D)^3(B-b)\right\}+m\omega(hb)^2\right] \quad [100]$$

also equation [8] becomes from equation [99]—

$$u=-\frac{1}{4b}\left\{4D(B-b)+3m\omega\right\}+$$

$$\sqrt{\frac{1}{16b^2}\left\{4D(B-b)+3(m\omega)\right\}^2+\frac{1}{2b}\left\{2D^2(B-b)+3m\omega h\right\}} \quad [101].$$

As in the case of rectangular beams—

$$f=cm\frac{(hb)}{u} \quad [102].$$

If it is decided to design a T-beam with tensile reinforcement only when (u) has been found to be greater than (D).

Equation [100] may be written—

$$M=\frac{c}{u}\left[\frac{5}{12}u(u^2B)-\frac{5}{12}(u-D)(u-D)^2(B-b)+m\omega(hb)(hb)\right] \quad [103];$$

but from equation [99]—

$$m\omega(hb)=\frac{2}{3}u^2B-\frac{2}{3}(u-D)^2(B-b) \quad [104].$$

Substituting [104] in [103] and replacing (hb) by its value $(h-u)$ we get—

$$M=\frac{c}{u}\left[u^2B\left\{\frac{5u}{12}+\frac{2h-2u}{3}\right\}-\left\{(u-D)^2(B-b)\right\}\left\{\frac{5u-5D}{12}+\frac{2h-2u}{3}\right\}\right]$$

$$\text{or } M=\frac{c}{12u}\left[u^2B(8h-3u)-(u-D)^2(B-b)(8h-3u-5D)\right] \quad [105].$$

Generally speaking it will be found that the T-beam will be well proportioned. if (b) is made $\frac{1}{12}$ the distance from centre to centre of the beams or—

$$b=\frac{B}{6} \quad [106].$$

Substituting this value for (b) in equation [105] we get—

$$M=\frac{cB}{12u}\left[u^2(8h-3u)-\frac{5}{6}(u-D)^2(8h-3u-5D)\right] \quad [107].$$

From equation [102] substituting for (hb) its value $(h-u)$ and replacing m by its value of 10, we get—

$$u=\frac{10ch}{f+10c} \quad [108].$$

Giving (c) and (f) their maximum values, and by trying different dimensions for (h), various values for (u) can be found from equation [108], which, inserted in equation [107], will prove whether the right value for (h) has been chosen. Trial must be made in this way until we arrive at the true values.

When this has been accomplished (ω) follows from equation [104], which may be written—replacing (hb) by $(h-u)$ and (m) by 10—

$$\omega=\frac{\{u^2B-(u-D)^2(B-b)\}}{15(h-u)} \quad [109],$$

and by replacing b by $\frac{B}{6}$

$$\omega=\frac{B\{6u^2-5(u-D)^2\}}{90(h-u)} \quad [110].$$

If it is found that (b) is too small for practical purposes, a slight increase in its value affects the results very little and is on the side of safety. The width (b) should be made big enough to allow the ramming around the reinforcement to be thoroughly effective. The width (b) is governed by the resistance to shearing which it must be large enough to supply; the value of $\frac{B}{6}$ will be ample for this (vide also page 309).

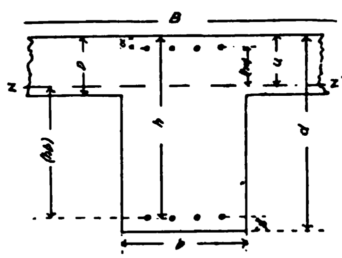


FIG. 341

In T-beams the neutral axis is always well above the centre of the depth, and therefore it is unnecessary to inquire into the tensile strain of the concrete, but, if required, this can easily be done on the lines laid down for rectangular beams.

T-shaped Beams with Double Reinforcement of Small Sectional Area and Depth Compared with that of the Piece (Fig. 341).

If the neutral axis is above or coincides with the underside of the slab, equations [31] to [41] apply if we substitute (B) for (b). Also all equations [70] to [97] substituting (B) for (b) *and remembering that the percentages of reinforcement and value for μ are taken as those for a beam of width (B) depth (d)*, and (d) is found directly from equation [98]—

$$d=\sqrt{\frac{M}{\mu B}}$$

When checking the dimensions for a beam already designed we find (u) from equation [41] substituting (B) for (b). If this value is the same or less than D, we check the dimensions as described for rectangular beams. If (u) is found to be greater than (D) we must use the equations given below.

If in designing a T-beam the value of (u) obtained for the economic section is greater than (D), we may take a less economical value or work by trial and error from the following formulæ—

When the neutral axis is below the underside of the slab, equation [37] becomes

$$\frac{2}{3}u^2B-\frac{2}{3}\left\{(u-D)^2(B-b)\right\}+m\left\{\omega_u(hu)-\omega_b(hb)\right\}=0 \quad [111],$$

and equation [38]—

$$M=\frac{c}{u}\left[\frac{5}{12}\left\{u^3B-(u-D)^3(B-b)\right\}+m\left\{\omega_u(hu)^2+\omega_b(hb)^2\right\}\right] \quad [112].$$

From equation [111]—

$$u=-\frac{1}{4b}\left\{4D(B-b)+3m(\omega_u+\omega_b)\right\}+$$

$$\sqrt{\frac{1}{16b^2}\left\{4D(B-b)+3m(\omega_u+\omega_b)\right\}^2+\frac{1}{2b}\left\{2D^2(B-b)+3m(\omega_u a+\omega_b h)\right\}} \quad [113].$$

Substituting $\left(\frac{B}{6}\right)$ for (b), $\frac{1}{10}d$ for a, $\frac{9}{10}d$ for h, $\frac{9d-10u}{10}$ for (hb), $\frac{10u-d}{10}$ for (hu), and 10 for m we get—

$$u=-\frac{5}{B}\left\{DB+9(\omega_u+\omega_b)\right\}+$$

$$\sqrt{\frac{25}{B^2}\left\{DB+9(\omega_u+\omega_b)\right\}^2+\frac{1}{B}\left\{5D^2B+9d(\omega_u+9\omega_b)\right\}} \quad [114],$$

$$\text{and } M=\frac{c}{u}\left[\frac{5B}{72}\left\{6u^3-5(u-D)^3\right\}+\frac{1}{10}\left\{\omega_u(10u-d)^2+\omega_b(9d-10u)^2\right\}\right] \quad [115].$$

f_u and f_b are always found from equations [33] and [34].

Inverted T-Beams (Fig. 342).—

The inverted T-beam is very seldom employed, but the ends of a built in upright T-beam will act as if the T-beam were inverted, the portion in tension being at the top and that under compression at the bottom. In this case there is generally a double reinforcement formed either by bending up some of the tensile reinforcing rods or by placing separate reinforcement near the upper surface over the supports and for some distance on each side.

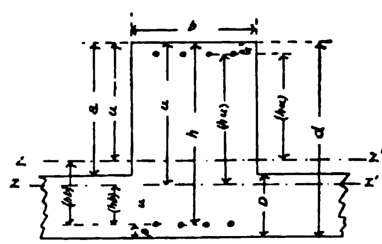


FIG. 342

If the neutral axis is above or coincides with the upper surface of the slab, then the equations are the same as those already given for rectangular beams, *the width of the beam being (b) in this instance.*

If, however, the neutral axis is below the upper surface of the slab, the equations become—

$$\frac{2}{3}\left\{u^2b+(u-a)^2(B-b)\right\}+m\left\{\omega_u(hu)-\omega_b(hb)\right\}=0 \quad [116],$$

$$\text{and } M=\frac{c}{u}\left[\frac{5}{12}\left\{u^3b+(u-a)^3(B-b)\right\}+m\left\{\omega_u(hu)^2+\omega_b(hb)^2\right\}\right] \quad [117].$$

From [116] we get—

$$u=\frac{1}{4B}\left\{4\mathrm{a}(B-b)-3m(\omega_u+\omega_b)\right\}+$$

$$\sqrt{\frac{1}{16B}\left\{4\mathrm{a}(B-b)-3m(\omega_u\quad\omega_b)\right\}^2-\frac{1}{2B}\left\{2\mathrm{a}^2(B-b)-3m(\omega_u a+\omega_b h)\right\}}\quad[118].$$

Substituting the several values as in the last case we get—

$$u=\frac{5}{6B}\left\{\mathrm{a}B-9(\omega_u+\omega_b)\right\}+$$

$$\sqrt{\frac{25}{36B^2}\left\{\mathrm{a}B-9(\omega_u+\omega_b)\right\}^2-\frac{1}{2B}\left\{5\mathrm{a}^2B-9d(\omega_u+9\omega_b)\right\}}\quad[119],$$

$$\text{and } M=\frac{c}{u}\left[\frac{5B}{72}\left\{u^3+5(u-\mathrm{a})^3\right\}+\frac{1}{10}\left\{\omega_u(10u-d)^2+\omega_b(9d-10u)^2\right\}\right]\quad[120]$$

f_u and f_b are found from equations [33] and [34].

If the reinforcement should be only along the bottom (a case very seldom met with in practice, since it can only occur when the whole beam is inverted or all the rods are brought up near the supports), the above equations will be like equation [99]—

$$\frac{2}{3}\left\{u^2b+(u-\mathrm{a})^2(B-b)\right\}-m\omega(hb)=0\,[121].$$

From which

$$u=\frac{1}{4B}\left\{4\mathrm{a}(B-b)-3m\omega\right\}+$$

$$\sqrt{\frac{1}{16B}\left\{4\mathrm{a}(B-b)-3m\omega\right\}^2-\frac{1}{2B}\left\{2\mathrm{a}^2(B-b)-3m\omega h\right\}}\quad[122],$$

and, like equation [105]—

$$M=\frac{c}{12u}\left[u^2b(8h-3u)+(u-\mathrm{a})^2(B-b)(8h-3u-5\mathrm{a})\right]\quad[123].$$

The value of u for the most economical section may be found from equation [108].

We can also replace b by $\frac{B}{6}$ in equation [123], giving

$$M=\frac{cB}{72u}\left[u^2(8h-3u)+5(u-\mathrm{a})^2(8h-3u-5\mathrm{a})\right]\quad[124].$$

f is found from equation [102]; and from equation [121] replacing (hb) by $(h-u)$, m by 10, and b by $\frac{B}{6}$

$$\omega=B\frac{\{u^2+5(u-\mathrm{a})^2\}}{90(h-u)}\quad[125].$$

All the above equations may be simplified by making $\mathrm{a}=\frac{3}{2}b$ or $\frac{1}{4}B$, which is a good proportion. By substituting $\frac{1}{4}B$ for a in equations [124] and [125] we eliminate (a), and can then work in the same manner as indicated for upright T-beams with single reinforcements.

For inverted T-beams it is more necessary to inquire into the strain of the concrete tension; as (u) in this instance is nearly always below the centre of the depth, f or f_b (as the case may be) divided by E_f will give the strain at the level of the reinforcement.

Denoting this by (s), we have—

$$s\frac{\{(hb)+\beta\}}{(hb)} \text{ must be less than } \frac{1}{1000} \quad [126].$$

It is universally the practice to increase the depth of a T-beam at the supports by a curved or splayed corbelling, which has the effect of lowering the neutral axis and so preventing the cracking of the concrete from excessive strain. It also provides more material for taking up the compressive stresses.

In the usual case where an inverted T-beam is employed, i.e. at the supports of an upright T-beam, the piece has been already designed and only requires verifying for the stresses at the supports.

Pieces with Reinforcements of Large Sectional Area. (Fig. 343.)—In this case the reinforcement has in itself a resistance against bending, and neither the depth nor the sectional area can be neglected when compared to that of the whole piece.

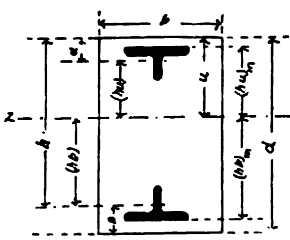


FIG. 343

The values f_u and f_b will therefore be considered as the mean and f_{um} and f_{bm} as the maximum resistances of the sections; (hu) and (hb) being the distances from the neutral axis to the centres of gravity of the metal sections; and $(hu)_m$ and $(hb)_m$ the distances from the neutral axis to the outer fibres of the reinforcements and further i_u and i_b will be the moments of inertia of the reinforcements about their own centres of gravity.

The compressive reinforcement takes up an appreciable area compared with that of the whole piece, which must be deducted from the area of the concrete in compression. The resistance of the concrete displaced by this reinforcement may be supposed to act at its centre of gravity.

The equation of a parabola is $y^2=4\,ax$, and in this case, for the extreme fibre of the concrete of the piece (x) is equal to (c) and (y) is equal to (u). We have therefore for the extreme fibre $4a=\frac{u^2}{c}$. The general equation becomes—

$$y^2=\frac{u^2}{c}x.$$

y at the centre of gravity of the upper reinforcement is equal to (hu). We have therefore for the value of x—

$x=\frac{(hu)^2}{u^2}c$ = the stress on the area of concrete replaced by the upper reinforcement.

Equation [31] becomes—

$$\frac{2}{3}cub - \frac{(hu)^2}{u^2}c\,\omega_u + \omega_u f_u = \omega_b f_b \qquad [127].$$

For the calculations of the moment of resistance of the piece, the moment of each reinforcement acting by itself must be included, and for the compressive reinforcement the moment of the concrete which it replaces must be deducted. Therefore we have instead of equation [36]—

$$M = \frac{5}{12}cu^2b - \frac{(hu)^3c\,\omega_u}{u^2} - \frac{c(hu)i_u}{u^2} + f_u\omega_u(hu) + \frac{f_u\,i_u}{(hu)} + f_b\,\omega_b(hb) + \frac{f_b\,i_b}{(hb)} \qquad [128].$$

We have also

$$f_u = cm\frac{(hu)}{u} \qquad [129]$$

$$\text{and } f_b = cm\frac{(hb)}{u} \qquad [130],$$

which are the same as before.

Substituting these values in equations [127] and [128] we get—

$$\frac{2}{3}u^3b - (hu)\,\omega_u\Big\{(hu) - mu\Big\} - m(hb)u\,\omega_b = 0 \qquad [131],$$

and

$$M = \frac{c}{u}\left[\frac{5}{12}u^3b - \Big\{(hu)^2\,\omega_u + i_u\Big\}\Big\{\frac{(hu)}{u} - m\Big\} + m\Big\{(hb)^2\,\omega_b + i_b\Big\}\right. \qquad [132].$$

The maximum stresses on the reinforcements are given by—

$$f_{um} = cm\frac{(hu)_m}{u} \qquad [133],$$

$$\text{and } f_{bm} = cm\frac{(hb)_m}{u} \qquad [134].$$

The relations

$$(hu) = (u - a) \qquad [135],$$

$$\text{and } (hb) = (h - u) \qquad [136]$$

remain as before.

If there is only a single tensile reinforcement, the equations become—

$$f = cm\frac{(hb)}{u} \qquad [137],$$

$$\frac{2}{3}u^2b - m(hb)\,\omega = 0 \qquad [138],$$

$$M = \frac{c}{u}\Big\{\frac{5}{12}u^3b + m\Big\{(hb)^2\,\omega + i\Big\} \qquad [139],$$

$$f_{max} = cm\frac{(hb)_m}{u} \qquad [140],$$

$$\text{and } (hb) = (h - u) \qquad [141].$$

The values of the moments of inertia for various sections are given in Table LVI (page 292).

The equations for T-beams reinforced with large sections follow from the above, being compiled in exactly the same manner.

The employment of reinforcements of large sectional area is not to be recommended, as the best practice in reinforced concrete construction for pieces subjected to simple bending is to keep the metal as far as possible from the neutral axis of the piece, where it acts at its best advantage. The design of reinforced concrete is similar to that of metal girders in this respect.

The rolled joist is uneconomical as regards the use of the metal, its economy being a purely practical consideration. The same remark applies to a plate-web girder, true economy of material being only obtained when the web is of the open type, being only sufficient to take up the shearing stresses. With reinforced concrete the practical economy of the rolled joist and plate web does not apply, as the cost of construction is very nearly the same whether we use large sections or small.

In the case of slabs, the tests and experiments which have been carried out prove that the best type is that when small sections of reinforcement are employed fairly close together; and the fact that but few constructors employ large reinforcing sections in beams, is a sufficient proof that small sections are the most economical for these.

For arches which act mainly in compression, large sections may be employed with economy, and many constructors, Melan and Wünsch among the number, employ this type with success. For arches large sections have also the advantage of readily adapting themselves to hinged connexions.

When large reinforcing sections are employed it is the usual practice to use them for the purpose of supporting or partially supporting the falsework, and in consequence they gain an extensive practical advantage; but it cannot be considered good practice to support the falsework by the aid of the reinforcement, as, when this is done, there must be a certain amount of initial strain and there is more risk to vibration. These remarks do not, of course, refer to the use of rolled joists as beams, this form of construction not being reinforced concrete in the true sense of the term.

SHEARING STRESSES IN PIECES SUBJECTED TO BENDING

General Treatment.—For the purpose of examining into the calculations for the resistance to the shearing stresses, the relation between the shearing force at any section and the bending moment on a beam will be first considered.

The following reasoning will apply to any section of a beam or cantilever, but, for the purposes of the study of this relationship, a section to the left of the centre of the span of a beam will be taken.

Now, the vertical shearing force (K) at any such section is equal to the reaction (R_L) of the left support, less the load or loads on the beam to the left of the section under consideration.

The bending moment (M_1) at the same section is equal to the reaction (R_L) multiplied by the distance (x) of the section from the support, less the load or loads to the left of the section, each multiplied by its distance from the section.

These equations may be stated—

$$K = R_L - \Sigma W,$$

$$\text{and } M_1 = xR_L - \Sigma(Wz),$$

where (z) is the distance of each load (W) from the section.

Let the bending moment be now taken a small distance (δx) to the right of the former section. If there is no load on this length—

$$M_2 = R_L(x + \delta x) - \Sigma\{W(z + \delta x)\},$$

which may be written—

$$M_2 = R_L x - \Sigma(Wz) + (R_L - \Sigma W)\delta x.$$

But $M_1 = R_L - \Sigma(Wz)$ and $K = R_L - \Sigma W$. Consequently—

$$M_2 - M_1 = K\delta x,$$

$$\text{or } K = \frac{M_2 - M_1}{\delta x} \quad [1],$$

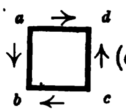
or $K =$ the tangent to the curve of bending moments or the rate of change of the bending moment.

If there is a load on the length (δx), the bending moment (M_2) is increased by ($K\delta x$) less the load on the length (δx) multiplied by its leverage about the first section, but this lever arm must always be less than (δx), and therefore by making (δx) small enough we may neglect the effect of the increment of load.

Consequently, equation [1] stands for all cases.

Besides the vertical shear on a piece under flexure, there are also longitudinal shearing resistances, and these two sets of stresses balance one another.

The horizontal shearing resistances are those which add the increments to the direct resistance of the fibres. If there were not shearing stress between the fibres they would simply slide on one another and exert no stresses to resist bending. The vertical shearing force is resisted by the sum of the horizontal shearing resistances in all the fibres of the section.

On each small square particle 

($abcd$) in a beam there are horizontal and vertical forces acting, which to be in equilibrium must be equal to one another. The shearing resistances that it is necessary to provide in a beam are in reality these horizontal ones which prevent the tendency of the fibres to slide over one another and so cause the failure of the beam, in consequence of their inability to exert the necessary resistance.

For the purposes of inquiry into the resistance to shearing stresses the stress-strain curve of the concrete in compression will be considered as a straight line, although for the direct longitudinal stresses it has been supposed to be parabolic. The sectional area of the concrete in a beam is always arrived at by considering the longitudinal stresses, and at most the resistance to shearing is merely taken as that of the section already decided upon. When transverse reinforcements are employed the resistance of the concrete may be neglected altogether, since it is difficult to decide on the proportion of the stress resisted by it, and also it does not effect the economy to any great extent if the transverse reinforcement alone is considered as resisting the shearing stresses.

These reinforcements have also the practical advantage of tying together the several layers of the beam, more especially in the case where some time may elapse between the deposition of the several layers; it is therefore immaterial if their sectional area is somewhat in excess of its proper theoretical value. Considering the stress-strain curve as a straight line greatly simplifies the formulae, and since the slight error is on the side of safety, one is justified in making use of this supposition.

If therefore this curve is considered as a straight line, it is clear that the added increment of the longitudinal stresses in a beam being represented by the equal triangles OEF and OGH (Fig. 344) the longitudinal shearing stress on any horizontal plane must be equal to the sum of the increments above the plane in question. The effect is shown graphically in Fig. 345, where $E'O'G$ represents the vertical plane as a section upon which there are acting shearing stresses at each point in its height, which shearing stresses are represented by the ordinates from $E'O'G'$ to the curve $E'O''G'$. It will be noticed that these increase from the top surface of the beam until the neutral axis is reached, when the direct stresses change sign, and consequently the increments become minus. The greatest shearing stress being at $O'O''$, the curve $E'O''G'$ is a parabola, and the greatest shearing stress is consequently $\frac{3}{2}$ the average.

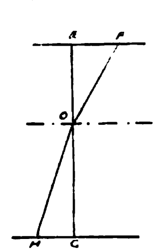


FIG. 344

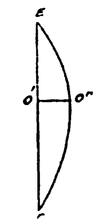


FIG. 345

Now, the average is—

$$k_{aver.} = \frac{K}{bd}$$

and therefore the maximum will be—

$$k_{max.} = \frac{3K}{2bd}$$

Now, if we consider two vertical sections AB and $A'B'$ of a slice of unit width in a piece under flexure (Fig 346) a very small distance δx apart, and take $c_{y,}$ as the difference between the longitudinal direct compressive stresses on AB and $A'B'$ at a distance y' from the neutral fibre and c' as the difference between the maximum stresses at the outer fibre—$c_{y,} : c' :: y' : u'$, or

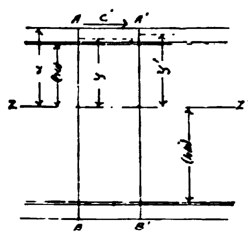


FIG. 346

$$c_{y,} = \frac{c'y'}{u} \qquad [2],$$

and as

$$c' = \frac{u\dfrac{(M_2 - M_1)}{\delta x}}{I} \qquad [3].$$

Where $\dfrac{M_1 - M_2}{\delta x}$ is the increment of the bending moment on the portion of δx of the beam and I is the moment of inertia of the beam, we have—

$$c_{y,} = \frac{\dfrac{(M_2 - M_1)y'}{\delta x}}{I} \qquad [4],$$

But as we have seen that—

$$\left(\frac{M_2 - M_1}{\delta x}\right) = K.$$

Therefore—

$$c_{y'} = \frac{Ky'}{I} \qquad [5].$$

If we consider $c_{y'}$ as the mean stress acting on a portion of the section included between the top of the beam and a plane at a distance y from the neutral axis—

$$y' = \frac{u-y}{2} + y = \frac{u+y}{2} \qquad [6].$$

As the shearing stress at any plane is equal to the total longitudinal stresses above that plane, we have for the shearing stress at y—

$$k_y = c_{y'}(u-y) \quad [7].$$

Substituting from equations [5] and [6] in equation [7], we get for the shearing stress at y—

$$k_y = \frac{K}{2I} \quad (u^2 - y^2) \quad [8].$$

This is the general equation for the shearing stresses at a plane at any distance y from the neutral axis.

When we employ a reinforcement this general equation will only apply for any planes between the top of the beam and the first reinforcement.

Slabs or Rectangular Beam with a Single Reinforcement of Small Sectional Area, and small depth in respect to the total depth of the piece.

In this case the shearing stress at any plane above the neutral axis is given by the general equation [8]. It is a maximum at the neutral axis where $y=o$ and

$$k_{max.} = \frac{K}{2I} u^2 \qquad [9].$$

If the stress-strain curve of the concrete is a straight line, equation [5] (page 300) for the direct stresses becomes $\frac{1}{2} u^2 b = m\omega(hb)$. We may therefore write equation [9] as

$$k_{max.} = \frac{K}{Ib} m \, \omega \, (hb) \qquad [10].$$

In the portion of the beam under tension the resistance of the concrete is neglected; the shearing stress therefore does not alter until the lower reinforcement is reached, at the axis of which we obtain—

$$k_{(hb)} = \frac{K}{Ib} m \, \omega (hb) - \frac{K}{Ib} m \, \omega (hb) = 0. \quad [10a].$$

The value of I may be found from equations similar to those used for the direct stresses, but differing in that the stress-strain curve of the concrete in compression is now considered as a straight line.

Equation [5] (page 300) will be $\frac{1}{2} u^2 b - m\omega(hb) = 0 \qquad [11].$

Equation [6] (page 300) $M = \frac{c}{u}\left\{\frac{1}{3}u^2 b + m\omega (hb)^2\right\}$ [12].

Equation [7] (page 300) remains $(hb) = (h - u)$ [13].

Substituting [13] and [11], in [12] we get—

$$M = \frac{c}{u}\left\{\frac{1}{6}u^2 b(3h - u)\right\}.$$

and from the well-known general formulae $M = \frac{f_1}{y}I$—

$$I = \frac{1}{6}u^2 b\,(3h - u) \qquad [14].$$

Substituting this value for I in equation [9], we get—

$$k_{max.} = \frac{K}{b\left(h - \frac{u}{3}\right)} \qquad [15].$$

Slabs or Rectangular Beam with Double Reinforcement of Small Sectional Area and small depth in respect to the total depth of the beam.

Here the general equation [8] applies until we reach the first reinforcement at its centre of gravity $y = (hu)$ and if there were no reinforcement—

$$k_{(hu)} \text{ would} = \frac{K}{2I}\left\{u^2 - (hu)^2\right\} \qquad [16].$$

At the centre of this reinforcement (if we consider the whole resistance of the metal to act at its axis) the direct longitudinal stress rapidly increases, and we have the shearing stress—

$$k_{fu} = \frac{K}{2I}\left\{u^2 - (hu)^2\right\} + \frac{K}{Ib}m\,(hu)\,\omega_u,$$

or—

$$k_{fu} = \frac{K}{Ib}\left[\frac{1}{2}\left\{u^2 - (hu)^2\right\}b + m(hu)\omega_u\right] \qquad [17].$$

Below this reinforcement the shearing stress increases as the distance from the neutral axis diminishes, and we have for any plane below the first reinforcement till the neutral axis is reached—

$$k_y = \frac{K}{Ib}\left\{\frac{1}{2}(u^2 - y^2)\,b + m(hu)\omega_u\right\} \qquad [18].$$

At the neutral axis $y = o$, and the shearing stress becomes—

$$k_{max.} = \frac{K}{Ib}\left\{\frac{1}{2}u^2 b + m(hu)\omega_u\right\} \qquad [19].$$

As the stress-strain curve for the concrete in compression is supposed to be a straight line, the relation in equation [37] (page 310) for direct stresses becomes—

$$\frac{1}{2}u^2b + m\,(hu)\,\omega_u = m\,(hb)\,\omega_b.$$

We may therefore write equation [19]—

$$k_{max.} = \frac{K}{Ib}\,m\,(hb)\,\omega_b \qquad [20],$$

which is the same as [10] for single reinforcement.

In the portion of the beam under tension, the resistance of the concrete being neglected, equation [20] will stand until the bottom reinforcement is reached; *at the axis of this reinforcement the shearing stress becomes nil*, as in the case of single reinforced pieces.

Equation [38] (page 311) for direct stresses becomes—

$$M = \frac{c}{u}\left[\frac{1}{3}u^3\;b + m\left\{\omega_u(hu)^2 + \omega_b(hb)^2\right\}\right],$$

from which, using the general formula $M = \frac{f}{y}I$—

$$I = \frac{1}{3}u^3 b + m\left\{\omega_u(hu)^2 + \omega_b(hb)^2\right\} \qquad [21].$$

This is the value for I which must be used in the above equations.

T-Beams with a Tensile Reinforcement only of small sectional area and depth compared with those of the whole piece.

For the shearing stresses in these beams equation [10] stands—

$$k_{\,max.} = \frac{K}{Ib}\,m\,(hb)\,\omega \quad [22].$$

If the neutral axis falls inside or at the lower surface of the slab, the value or If will be that for a rectangular beam of width B—

$$\text{or } I = \frac{1}{6}\,u^2\,B\,(3h - u) \quad [23].$$

If the neutral axis is below the slab, equation [103] (page 339) for direct stresses when the stress strain curve for the concrete is a straight line will be—

$$M = \frac{c}{u}\left[\frac{1}{3}u\,(u^2B) - \frac{1}{3}\,(u-D)\,(u-D)^2\,(B-b) + m\,\omega\,(hb)(hb)\right]$$

and equation [104] (page 339)—

$$m\,\omega\,(hb) = \frac{1}{2}\,u^2\,B - \frac{1}{2}(u-D)^2\,(B-b).$$

Substituting this in the above and replacing (hb) by its value $(h-u)$ we get—

$$M=\frac{c}{u}\left[\frac{1}{6}u^2B(3h-u)-\frac{1}{6}(u-D)^2(B-b)\ (3h-u-2D)\right]$$

From which

$$I=\frac{1}{6}u^2B(3h-u)-\frac{1}{6}(u-D)^2(B-b)(3h-u-2D) \quad [24].$$

Therefore in the first case $k_{max}=\dfrac{K}{b\left(h-\dfrac{u}{3}\right)}$ [25],

which is the same as for rectangular beams; and in the second case we must use equation [22], retaining the value b in the denominator, as if we have sufficient strength in the portion of the rib below the neutral axis we are certain of having ample resistance in the slab. The value of I will be that given in equation [24].

T-Beams with Double Reinforcement of small sectional area and depth compared with those of the whole piece.

The value of I in this case, when the stress-strain curve of the concrete is a straight line, will be—

$$I=\frac{1}{3}u^3B+m\left\{\omega_u(hu)^2+\omega_b(hb)^2\right\} \quad [26],$$

if the neutral axis falls within or at the lower surface of the slab.

If it falls below, this equation becomes—

$$I=\frac{1}{3}u^3B-\frac{1}{3}(u-D)^3(B-b)+m\left\{\omega_u(hu)^2+\omega_b(hb)^2\right\} \quad [27].$$

Equation [20] stands—

$$k_{max.}=\frac{K}{Ib}m(hb)\omega_b \quad [28].$$

The values for I being those given in equations [26] and [27], and the value b being retained when the neutral axis falls within the rib for the same reason as given for T-beams with single reinforcements.

Inverted T-Beams with Double Reinforcement of small sectional area and depth compared with those of the whole piece.

For the rib in this case the equations are the same as those for a rectangular beam of width b.

If the neutral axis falls within the slab, the shearing stress at this plane will be

$$k_{max.}=\frac{K}{IB}m(hb)\omega,$$

but this is not the place where failure is to be feared. The weakest plane is that just above the surface of the slab where the width of the beam is only b. Here we have a shearing stress of

$$k_{max} = \frac{K}{Ib}\left\{ m\,(hb)\,\omega_b - \frac{1}{2}(u-a)^2 b \right\} \quad [29].$$

For equation [20]—

$$k_y = \frac{K}{Ib}\left\{ \frac{1}{2}(u^2 - y^2)\,b + m\,(hu)\,\omega_u \right\}$$

becomes in this case

$$k_{max} = \frac{K}{Ib}\left[\frac{1}{2} b \left\{ (u^2 - (u-a)^2 \right\} - \frac{1}{2}(u-a)^2 (B-b) + m\,(hu)\,\omega_u \right],$$

but equation [116] (page 341) for direct stresses, when the stress-strain curve of the concrete is a straight line, will be—

$$\frac{1}{2}\left\{ u^2 b + (u-a)^2 (B-b) \right\} + m\,\omega_u\,(hu) = m\omega_b\,(hb),$$

which substituted in the previous equation gives equation [29].

Rectangular Beams with Reinforcements of Large Sectional Area and Depth. The shearing stress below the first reinforcement becomes in this case—

$$k_y = \frac{K}{Ib}\left\{ \frac{1}{2}(u^2 - y^2)\,b - (hu)\,\omega_u + m\,(hu)\,\omega_u \right\} \quad [30],$$

as we must deduct the stress due to the concrete replaced by the upper reinforcement.

At the neutral axis $y = o$ we get therefore—

$$k_{max.} = \frac{K}{Ib}\left\{ \frac{1}{2} u^2 b - (hu)\,\omega_u + m\,(hu)\,\omega_u \right\} \quad [31].$$

But equation [127] (page 344) for direct stresses when the stress-strain curve for the concrete is a straight line becomes

$$\frac{1}{2} cub - \frac{(hu)}{u} c\,\omega_u + f_u\,\omega_u = f_b\,\omega_b \quad [32].$$

Replacing f_u and f_b by the irrespective values $cm\,\frac{(hu)}{u}$ and $cm\,\frac{(hb)}{u}$ we get in place of equation [131] (page 344) for direct stresses

$$\frac{1}{2} u^2 b - (hu)\,\omega_u + m\,(hu)\,\omega_u = m(hb)\,\omega_b \; [33].$$

Substituting equation [33] in [31], we get as in the case for reinforcements of small sectional area—

$$k_{\,max} = \frac{K}{Ib}\, m\,(hb)\,\omega_b \quad [34].$$

With the stress-strain curve of the concrete as a straight line, equation [132] (page 344) for direct stresses will become—

$$M = \frac{c}{u}\left[\frac{1}{3}u^3 b + (m-1)\left\{\omega_u (hu)^2 + i_u\right\} + m\left\{\omega_b (hb)^2 + i_b\right\}\right] \quad [35],$$

from which we get—

$$I = \frac{1}{3}u^3 b + (m-1)\left\{\omega_u (hu)^2 + i_u\right\} + m\left\{\omega_b (hb)^2 + i_b\right\} \quad [36].$$

This is the value of I to be used in equation [34]. *For single reinforcement* equation [34] stands and the expression for the moment of inertia will be the same as [36] leaving out the terms containing i_u and ω_u. We get therefore—

$$k_{max} = \frac{K}{Ib} m (hb) \omega_b \quad [37],$$

$$\text{and } I = \frac{1}{3}u^3 b + m\left\{\omega_b (hb)^2 + i_b\right\} \quad [38].$$

Alteration of Shearing Stresses due to an Inclined Reinforcement.— When an inclined reinforcement is employed, if its area is ω_f and the direct tensile resistance is f, and ϕ is the angle it makes with the horizontal (Fig. 347),

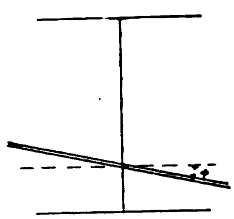


FIG. 347

The total direct resistance $= f\omega_f$ [39].

The resistance to vertical shearing $= f\omega_f \, Sin\, \phi$ [40].

The vertical shearing force is therefore reduced by the employment of this reinforcement by the amount given by equation [40].

Therefore the shearing force at any point cut by the reinforcement will be—

$$K' = K - f\,\omega_f \, Sin\, \phi \quad [41].$$

K' must then be substituted for K in the former equations when a bent reinforcement is employed.

With a freely supported beam and a curved reinforcement which will closely approximate to a parabolic curve, the shearing force will be nil as in the case of bow-string metallic bridges if the load is uniformly distributed and not a moving load.

Shearing Stresses on Vertical Reinforcements in the Vertical Plane.— When there are special vertical reinforcements to resist shearing, each plane in the depth of the beam between the lower reinforcement and a plane close to the top is traversed by the reinforcement; the shearing stress on the plane is consequently in reality resisted partly by the concrete and partly by the vertical reinforcement, but, for reasons already pointed out (page 346), the resistance of the concrete will be neglected.

Considering therefore that the concrete does not assist the metal in resisting the shearing stresses, we need only find the total shearing force acting on the portion of the beam between the centres of two neighbouring transverse reinforcements, and by substituting this value for K in the preceding formulae we obtain the maximum shearing stress. This stress divided by the sectional area of the metal gives the unit resistance of the reinforcement which must be below its limit of resistance to shearing.

If we consider a length of a beam to the left of the centre between a section (A) at a distance of (x) from the left support and a section (B) at a distance of (y) from the left support, and use the following symbols—

$R_L =$ reaction at left support,

$W =$ any load to the left of section (A),

$W_1 =$ any load between the sections (A) and (B),

(The W's being of any intensity),

$z =$ the distances of the loads (W) from the section (A),

and $z_1 =$ the distances of the loads (W_1) from the section (B),

We have for the shearing force at section (A)—

$$K_1 = R_L - \Sigma W,$$

and for the bending moment at section (A)—

$$M_1 = R_L x - \Sigma(Wz).$$

The mean value of the *decrease* of the shearing force on the length $(y-x)$ between the sections (A) and (B) will be—

$$\frac{\Sigma(W_1 z_1)}{(y-x)}.$$

We have therefore for the mean value of the total shearing force on the length $(y-x)$ between the sections (A) and (B)—

$$K_m = R_L - \Sigma W - \frac{\Sigma(W_1 z_1)}{(y-x)}.$$

The total shearing force on the length $(y-x)$ will therefore be—

$$K = R_L (y-x) - \Sigma\{W(y-x)\} - \Sigma(W_1 z_1).$$

Now, the bending moment at section (B) will be—

$$M_2 = R_L y - \Sigma(Wz) - \Sigma\{W(y-x)\} - \Sigma(W_1 z_1).$$

and the increase of the bending moment over the length $(y-x)$ will be—

$$M_2 - M_1 = R_L(y-x) - \Sigma\{W(y-x)\} - \Sigma(W_1 z_1) = K.$$

We obtain then for the maximum resistance of the metal—

$$k_s = \frac{(M_2 - M_1)}{I\,\omega_s} m\,(hb)\,\omega_b \qquad [42],[1]$$

$M_2 - M_1$ being the increment of the bending moment on the length of the beam under consideration.

In this case we take for a single reinforcement—

$$I = \frac{1}{3} u^3 b + m\,\omega\,(hb)^2 \qquad [43],$$

The other values for I remaining the same.

[1] Vide equation [20] page 350.

Equation [42] will replace the equations for the shearing stress on the concrete which are no longer applicable.

Equation [42] may be written—

$$w_s = \frac{(M_2 - M_1)}{Ik_s} m(hb)\omega_b \qquad [44],$$

from which by inserting the several values we can find ω_s

When a bent reinforcement is also employed, the expression $(M_2 - M_1)$ must be replaced by

$$(M_2 - M_1) - f\,\omega_f\,x\,Sin\,\phi \qquad [45],$$

when x is the length of the beam under consideration, f the direct tensile resistance of the bent reinforcement, and ϕ its angle of inclination to the horizontal at the point where transverse reinforcement cuts it.

The vertical reinforcements are usually made of the same sectional area throughout the span, and are spaced farther and farther apart as they approach

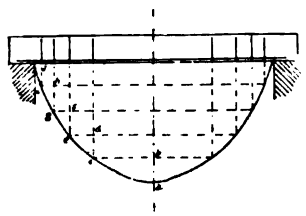


FIG. 348[1]

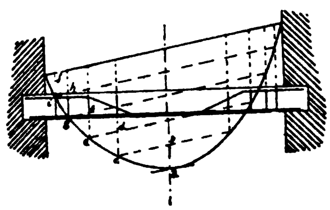


FIG. 349

the centre of the beam, so that the expression $(M_2 - M_1)$ remains constant. The proper spacing may be found graphically as shown in Figs 348 and 349.

The diagram of bending moments is plotted and the maximum ordinate drawn which is divided into equal parts; through each of the points so found a line is drawn parallel to the closing line of the diagram. Where these lines cut the curve verticals are drawn which give the centres of the vertical reinforcements.

At any section of the beam the bending moment is represented by the ordinate to the curve. By dividing the maximum ordinate into equal parts it can be clearly seen that the increment of bending moment for each distance from centre to centre of the neighbouring transverse reinforcements is the same as they are represented by *ab*, *cd*, *ef* etc.

The area for the reinforcement is found for that nearest to the support, the increment of bending moment being in this case the bending moment at that reinforcement.

Equation [44] may also be used for a system of inclined shearing reinforcements sloping towards the centre of the span, but in this case the value of $(M_2 - M_1)$ must be that on the length of the beam included between the upper extremities of the inclined reinforcements, and the area ω_s must be that on the horizontal plane cutting the reinforcements.

[1] The bent up rods in this figure should have been shown with the top bends at the supports.

The Shearing Stresses in the Concrete Surrounding the Reinforcements.—To obtain this we must know the total perimeter of the reinforcement. If we take χ_u as the perimeter of the compressive reinforcement and χ_b that of the tensile, we obtain for the shearing stress around the compressive reinforcement—

$$k_u = \frac{K}{I\chi_u} m\ (hu)\ \omega_u \qquad [46]$$

and for that around the tensile reinforcement—

$$k_b = \frac{K}{I\chi_b} m\ (hb)\ \omega_b \qquad [47]$$

vide equations [10A.] and that prior to [17] pages 348 and 349.

These formulae are not modified by the presence of transverse reinforcements, unless these are secured to the longitudinals and special forms of twisted or corrugated bars offer no resistance to such shearing stresses.

VARIOUS METHODS OF CALCULATION FOR PIECES SUBJECTED TO BENDING.

General Remarks.—Many different methods of calculation, both semi-empirical and scientific, have been proposed by various authorities. It is not intended to give all these in detail, as their interest is chiefly academical. Those only have been selected that are largely employed or have a particular interest. Those of M. Hennebique and Professor Ritter are constantly employed. Those recommended by M. Christophe are used by some firms. M. Considère's formulae are interesting in that they take account of the tensile resistance of the concrete and are also those which should be employed for calculating the deformations, since in this case the concrete in tension cannot be neglected.

The Hennebique Method of Calculation.—M. Hennebique assumes the weight of reinforced concrete as being 140 pounds per cubic foot, the resistance of the concrete in compression as 360 and sometimes 410 pounds per square inch, and the resistance of the iron in tension is taken as 14,220 pounds per square inch, while for steel he allows from 17,065 to 21,330 pounds.

The bending moment on built-in slabs is arrived at by a peculiar method. If w is the weight of the piece itself per lineal unit, and w_1 the uniformly distributed load per lineal unit, the bending moment for a built-in beam is taken as

$$\frac{(w + w_1)\,L^2}{10},$$

a very customary allowance. The slabs being supported on all edges, the bending moments are considered in both directions. If L is the longer span and B the shorter span, there are supposed to be two bending moments—

$$M_L = \frac{(w + w_1)\,L^2}{10}, \quad \text{and } M_B = \frac{(w + w_1)\,B^2}{10},$$

and the maximum on the slab is considered as the mean of these two, or—

$$M = \frac{M_L + M_B}{2}.$$

The disposition of the reinforcements, crossing one another and being continuous over the supporting beams, is assumed to considerably diminish this bending moment. It is therefore reduced to one-third, and the value for use in the calculations is taken as $\frac{M}{3}$.

This method is simple, but the truth of the reasoning is not clear.

For finding the depth of a slab or rectangular beam and the area of the reinforcement, M. Hennebique considers all the concrete above the neutral axis as equally resisting the compressive stress. Assuming the same symbols as those used previously, M. Hennebique uses the equation—

$$c\,u\,b \times \frac{u}{2} = \frac{M}{2},$$

$$\text{from which } u^2 = \frac{M}{cb} \qquad [1].$$

This equation gives the value of u, or the distance of the neutral axis from the surface under greatest compressive stress if we know b.

In the case of slabs, b is the unit width. The value of (hb) is then found from the equation

$$(hb) = (h - u) \qquad [2],$$

h being supposed to be known.

And, finally, the area of the reinforcement which resists the remaining half of the bending moment is given by the equation—

$$f\,\omega\,(hb) = \frac{M}{2},$$

$$\text{from which } \omega = \frac{M}{2f\,(hb)} \qquad [3].$$

Inserting M. Hennebique's values for c and f for a wrought iron reinforcement, we get —

$$u = 0{\cdot}053\sqrt{\frac{M}{b}} \qquad [4],$$

$$\text{and } \omega = \frac{M}{28440\left(h - 0{\cdot}053\sqrt{\frac{M}{b}}\right)} \; [5].$$

For thin slabs M. Hennebique assumes a value of h not less than $h = 1{\cdot}5u$, which substituted in [2] and [5] with the value of u from [4] and $b = 12$

$$h = 0{\cdot}023\sqrt{M} \qquad [6],$$

$$\text{and } \omega = 0{\cdot}0045\sqrt{M} \qquad [7].$$

For a rectangular beam or piece of considerable depth M. Hennebique does not exceed for the value of h

$$h = 2{\cdot}5u.$$

The errors in this method of calculation are evident. In the first place the concrete in compression does not resist the stresses equally throughout its whole depth, and secondly instead of equating the resultant compression to the resultant tension M. Hennebique equates the moments of these resultants.

The T-beams of the Hennebique system are designed in the following manner. The depth of the floor slab is found by the previous formulae, the width of the table is assumed to be the whole distance from centre to centre of the beams, *i.e.* the slab is supposed to resist the compressive stresses on the beam for half the distance between the centres of the beams on either side. M. Hennebique further assumes that the slab alone resists the compressive stresses, and that it resists them equal through the whole of its depth. He therefore obtains the equation

$$cBD \times \left(u - \frac{D}{2}\right) = \frac{M}{2},$$

$$\text{from which } u = \frac{M}{2cBD} + \frac{D}{2} \qquad [8],$$

which gives a value for u, or the distance of the neutral axis from the surface under greatest compressive stress.

He then gives a value to h in the neighbourhood of 2·5 according to judgment, and obtains the equation—

$$(hb) = (h - u) \qquad [9]$$

The sectional area of the reinforcement is found from equation [3]—

$$\omega = \frac{M}{2f\,(hb)} \qquad [3].$$

The errors of this method of calculaton are similar to those already detailed for slabs. There is a further error in assuming the whole thickness of the slab as always resisting the compressive stresses, since cases occur when the neutral axis lies within this thickness. On the other hand it frequently happens that a portion of the beam itself acts in compression, but in this case the extra resistance may be placed against the error in assuming the stresses as equally resisted throughout the whole depth of the slab.

When compression rods are added to the tensile reinforcements these rods are considered to resist to their maximum allowed stress—no allowance being made for the fact that they cannot act under more stress than the deformation of the concrete allows, and that if they do act with their maximum resistance the concrete must be stressed to about one-tenth of that value in consequence of the difference of the coefficients of elasticity of the two materials.

M. Hennebique either assumes the position of the neutral axis or the sectional area of the tensile reinforcement. Whichever of these two values is not assumed is found by equation [3], on the assumption that the reinforcement in tension resists half the bending moment.

This equation being written—

$$\omega = \frac{\mathrm{M}}{2f\,(h-u)} \qquad [10],$$

and the value of h being assumed as before. The other half of the bending moment is then equated to the compressive resisting moment by the equation—

$$\frac{M}{2} = cBD \times \left(u - \frac{D}{2}\right) + f_u\,\omega_u \times (u - a) \qquad [11].$$

The Calculations for the Stirrups are effected in the following Way.—The bent rods are first supposed to resist half the maximum shearing stress at the supports. The stirrups are then designed to resist the other half. From this supposition the following equation is obtained—

$$\sigma = \frac{K}{2\,k_m} \quad [12].$$

This gives the sectional area of the stirrups for a length of the beam equal to the distance between the centres of tension and compression. In a slab or rectangular beam this distance will be $\left(h - \frac{u}{2}\right)$, and for a T-beam $\left(h - \frac{D}{2}\right)$

If the distance between the two first sets of stirrups near the supports differs from this value, the sectional area is altered proportionately.

The sectional area of the stirrups is divided up into the number of stirrups in the width of the beam, each having two branches. The resistances of the metal to shearing are taken as from 8,530 to 9,955 pounds per square inch for wrought iron and 11,375 pounds per square inch for steel. This method of calculation is not always adhered to, however, as sometimes the strength of the stirrups is increased as a matter of precaution.

The above method of calculation for the area of the transverse reinforcements is almost universally adopted by the constructors who use them, excepting that very frequently the resistance of the bent bars is neglected.

The stirrups remain the same sectional area throughout the whole length of the beam, but the distance apart of the sets is increased as the centre of the span is approached.

Calculations generally adopted in Systems that employ a Symmetrical Reinforcement.—The reinforcement in these systems is usually constructed so as to have a certain resistance before the concrete is added, in order that it may be able to assist in the support of the falsework—MM. Pavin de Lafarge, M. Bonna and M. Chaudy are amongst those who adopt this method. The area of the reinforcements are calculated in the same manner as the flanges of an ordinary girder, and the compressive resistance of the concrete is neglected.

The equation therefore becomes—if h_1 is the distance from centre to centre of the upper and lower members and ω is the sectional area of *each* series of reinforcements, and f is the unit resistance, which is the same in compression and tension—

$$M = h_1\,\omega\,f.$$

This method of calculation does not take into account the true elastic behaviour of the reinforced concrete beam. The concrete in compression cannot in reality be neglected, and will, as a matter of fact, resist a great proportion of the compressive stresses, the different co-efficients of elasticity of the two materials preventing the reinforcement in compression from acting up to its maximum allowed resistance until the concrete has suffered very considerable deformation. The neutral axis of the piece is also considered to be at the centre of the depth, whereas, in reality, it always takes up a position nearer to the compressive than the tensile surface unless the percentage of tensile reinforcement is very large.

It may be asked why we cannot neglect the deformation of the concrete in compression in the same manner as we neglect the portion in tension, and that, if the concrete in tension allows the reinforcement to act up to its limiting stress, by having the property, when combined with the reinforcement, of submitting to great elongations without cracking, the same property may also exist in the concrete under compression.

If the compressive side of the beam were reinforced by hooping, in the manner described when discussing hooped columns (page 297), the concrete would gain some such property, since the swelling is prevented, and great deformations have been obtained without rupture, in experiments carried out on pieces with this type of reinforcement.

Concrete in compression has not been found to have the property of being able to undergo excessive deformation when in combination with longitudinal rods only.

Professor Ritter's Calculations for Pieces subjected to Bending.[1]—These are used by many constructors at the present day, amongst which Herren Wayss and Fratag and MM. de Vallierie et Simon may be mentioned. The Government of Switzerland insist on all reinforced concrete structures being checked by these formulae, which render their adoption in Switzerland almost a necessity.

Herr Ritter considers that the maximum resistance of the concrete in compression may be taken as 430 pounds per square inch for a piece subjected to vibrations or where the loading is uncertain ; but that for pieces under steady assessable loads this resistance may be increased to from 500 to 570 pounds per square inch. He takes the value of $m=\frac{E_f}{E_c}$ as 10. The tensional resistance of the concrete he neglects in his calculations, while admitting that when combined with the reinforcement it will bear from 430 to 570 pounds per square inch with safety. The tensile resistance of the wrought iron reinforcement he places at from 14,200 to 17,100 pounds per square inch.

Herr Ritter proceeds by considering that if it is desired to find the stresses produced by a given bending moment M on a given cross-section, the area of the reinforcement must be multiplied by m and the increased area regarded as of concrete. Calculations must be then made to find the centre of gravity and moment of inertia of the supposed enlarged section thus obtained.

Then if (u) is the distance of the centre of gravity from the surface under greatest compressive stress and (y) its distance from the centre of gravity of the reinforcement, the maximum compression of the concrete is $c=\frac{uM}{I}$ according to the generally accepted expression $M=\frac{fI}{y}$, and the stress on the reinforcement $f=10\frac{yM}{I}$.

If σ is the sectional area of the piece supposed to be entirely formed of concrete,

ω the area of the reinforcement,

d the total depth of the piece,

b the breadth of the piece,

h_1 the distance between the centres of action of the compressive and tensile stresses,

β the distance of the centre of gravity of the reinforcement from the surface under tensile stress.

[1] *Schweizerische Bauzeitung*, Nos. 5, 6 and 7, 1899.

S the static moment of the section with respect to the compressive surface,
I_T the moment of inertia of the section with respect to the compressive surface,
u the distance of the neutral axis of the piece from its compressive surface,
I the moment of inertia of the section about its centre of gravity,
F the total tensile stress, we have, for a slab or rectangular beam,

$$\sigma = bd + 10\omega \qquad [1]$$

$$S = db \times \frac{d}{2} + 10\omega(d-\beta) = \frac{d^2b}{2} + 10\omega(d-\beta) \qquad [2].$$

$$I_T = \left(db \times \frac{d}{2}\right)\frac{2}{3}d + 10\,\omega(d-\beta)\ (d-\beta) = \frac{d^3b}{3} + 10\,\omega(d-\beta)^2 \qquad [3].$$

$$u = \frac{S}{\sigma} \qquad [4].$$

$$I = I_T - (\sigma \times u^2) \qquad [5].$$

$$c = \frac{Mu}{I} \qquad [6].$$

And maximum unit tensile stress in the concrete $= t = \dfrac{M\,(d-u)}{I}$ **[7].**

$$F = 10\frac{M\,\{d-(u+\beta)\}}{I} \qquad [8].$$

For a T-beam with a width of table $=B$ and a depth of slab $=D$, we get—

$$\sigma = db + (B-b)\,D + 10\,\omega \qquad [9].$$

$$S = \frac{d^2b}{2} + (B-b)\frac{D^2}{2} + 10\,\omega(d-\beta) \qquad [10].$$

$$I_T = \frac{d^3b}{3} + (B-b)\frac{D^3}{3} + 10\,\omega\ (d-\beta)^2 \qquad [11].$$

$$u = \frac{S}{\sigma} \qquad [12].$$

$$I = I_T - (\sigma \times u^2) \qquad [13].$$

$$c = \frac{Mu}{I} \qquad [14],$$

and $t = \dfrac{M\,(d-u)}{I}$ **[15].**

$$F = 10\,\frac{M\{d-(u+\beta)\}}{I} \qquad [16].$$

The total stresses found by [8] and [16] are, however, not the true ones on the reinforcement, as a resistance is allowed to be exerted by the concrete in tension. According to the assumption that the concrete exerts no tensile stress, the total tensile stresses must be taken up by the reinforcement, so that the neutral axis retains the position found above.

To find the unit stress on the reinforcement under these conditions, Professor Ritter proceeds as follows. The centre of pressure of the concrete in compression

line in a plane at a distance of $\frac{1}{3}u$ from the compressive surface. The centre of action of the tensile stresses in the reinforcement may, without appreciable error, be considered as the centre of gravity of the bars.

We therefore get the equation for the distance between the centres of action of the compressive and tensile stresses as—

$$h_1 = d - (\tfrac{1}{3}u + \beta) \qquad [17].$$

and consequently the true total stress in the reinforcement is given by the equation—

$$F_1 = \frac{M}{h_1} \qquad [18],$$

$$\text{and the unit stress } f = \frac{F_1}{\omega} \qquad [19],$$

and the stress c in the concrete may be checked by the equation—

$$\frac{1}{2}\,cub = F_1, \text{ or } c = \frac{2F_1}{ub} \qquad [20].$$

When the dimensions of the piece are not known, Professor Ritter proposes the following method of procedure. The neutral axis is assumed at first to be in the centre of the depth. The depth of the piece is then calculated from the usual formula—

$$M = \frac{1}{6}\,cbd^2,$$

$$\text{or } d = \sqrt{\frac{6M}{cb}} \qquad [21].$$

If b is taken as $\frac{2}{3}d$, this formula becomes for different values of c—

When $c = 400$	450	500
$d = \sqrt[3]{\frac{M}{45}}$	$\sqrt[3]{\frac{M}{50}}$	$\sqrt[3]{\frac{M}{56}}$.

The area of the reinforcement is also found for a preliminary trial by the equation—

$$\omega f\left\{d - \left(\frac{d}{6} + \beta\right)\right\} = M \text{ or } \omega = \frac{M}{f\,(\frac{5}{6}d - \beta)} \qquad [22].$$

The value of β is assumed as some ratio of d.

Messrs. Wayss and Fratag use the value $\beta = \frac{d}{12}$, but this is too small for slabs, and should in that case be increased to $\beta = \frac{d}{6}$. These values of d and ω are inserted in the above formulae as first trials.

Professor Ritter believes that the stress-strain curve in compression is parabolic, but does not consider the difference of the values of c and f sufficient to make very appreciable difference. He also admits the tensile resistance of the concrete, but does not consider it worth while to take account of it.

A particular value is claimed for this method of calculation, in that the stress on the concrete in tension is checked, and, in consequence, the danger of assuming too small a width for the leg of a T-beam is avoided.

M. Considère's Calculations.—Using the previous symbols and adding the following—

t = maximum tensile stress in the concrete ;

$(d\,x)$ distance from the tensile surface to neutral axis,

(du) distance from the tensile surface to the centre of gravity of the tensile reinforcement.

and u being ratios.

M. Considère's formulae take the following form for a single reinforcement—

Equilibrium is attained when

$$bt\,(dx) + f\,\psi\,db - d\,(1-x)\,b\,\frac{c}{2} = 0,$$

$\frac{c}{2}$ being the mean compression in the concrete.

We may write

$$2\,tx + 2\,f\,\psi - (1-x)\,c = 0.$$

The coefficient of elasticity is considered constant through both tension and compression, he obtains, therefore, from the hypothesis of the conservation of plane sections—

$$c = \frac{f}{m} \times \frac{1-x}{x-u} \qquad [1].$$

The equation of equilibrium therefore becomes—

$$2\,tx + 2\,f\,\psi - \frac{(1-x)^2}{(x-u)} \times \frac{f}{m} = 0,$$

$$\text{or } (2m\,t-1)\,x^2 + 2\,(mt\,\psi + t - mtu)\,x - (2m\,\psi\,u + 1)\,t = 0 \qquad [2].$$

The bending moment finally is expressed by the equation—

$$M = bd\,tx\left[\frac{dx}{2} + \frac{2}{3}(d-dx)\right] + dbf\,\psi\left[(dx-du) + \frac{2}{3}(d-dx)\right]$$

$$= bd^2\left[\frac{tx^2}{2} + \frac{2tx}{3} - \frac{2tx^2}{3} + f\,\psi\left(x - u - \frac{2}{3}x + \frac{2}{3}\right)\right]$$

$$= bd^2\left[tx\,\frac{4-x}{6} + f\,\psi\left(\frac{x-3u+2}{3}\right)\right]$$

$$M = \frac{1}{6}\,bd^2\,[tx\,(4-x) + 2\,f\,\psi\,(x-3\,u+2)] \qquad [3].$$

If ψ, t, u and m are known, equation [2] gives the position of the neutral axis; equation [1] gives the ratio of $\frac{c}{f}$, and finally equation [3] gives the moment of resistance of the piece.

M. Considère assumes the value of u as $\frac{1}{12}$, from which the equations become

$$c = \frac{12f}{m} \times \frac{1-x}{12x-1} \qquad [4],$$

$$12\,(2\,mi-1)\,x^2+2\,(12\,mf\,\psi+12\,f-mt)\,x-(2\,m\,\psi+12)\,t=0 \quad [5],$$

$$\text{and } M=\frac{bd^2}{3}\,[2\,tx\,(4-x)+f\,\psi\,(4\,x+7)]. \quad [6]$$

The values of t, f and m are found by experimenting on the chosen materials, and from these the values of c and $\frac{M}{bd^2}$ are calculated for the most unfavourable case, which is when the metal has reached its elastic limit. Next, the proportion of the metal should be varied by giving different values to ψ, and for every case the equations resolved, giving c, x, and $\frac{M}{bd^2}=\mu$,

M. Considère has calculated the values of c, x and $\mu=\frac{M}{bd^2}$ in this manner, and has tabulated his results. These are given in Table LXIX, together with the comparative costs given by M. Considère.

TABLE LXIX

Nature of materials: Metal	Nature of materials: Concrete; pounds of cement to one cubic yard of gravel	Elastic limit of metal; pounds per square inch	Ultimate resistance of Concrete, in pounds per square inch: Tension	Ultimate resistance of Concrete, in pounds per square inch: Compression	Proportion of area of metal to total area of piece ψ	Ratio of $\frac{E_f}{E_c}=m$	Value of x, or the ratio of distance of neutral axis from surface under tension, to total depth of the piece	Maximum compressive stress on the concrete: pounds per square inch	Unit moment of resistance μ inch pound; units	Cost of test piece per cubic foot; shillings	Proportional numbers for expense per inch pound of unit bending moment. Column 11 divided by Column 10 and multiplied by 100,000
1	2	3	4	5	6	7	8	9	10	11	12
Iron	505	22,750	170	2,133	0·01	7	0·57	1,521	223	1·46	654
,,	,,	,,	,,	,,	0·02	6·5	0·49	2,033	372	1·90	511
,,	,,	,,	,,	,,	0·03	6	0·42	2,654	512	2·35	459
,,	1,347	,,	426	5,119	0·01	10	0·57	2,744	307	1·90	619
,,	,,	,,	,,	,,	0·03	9	0·46	3,285	593	2·80	472
,,	,,	,,	,,	,,	0·04	8·7	0·42	3,754	734	3·25	443
Steel	,,	42,660	,,	,,	0·01	10	0·60	3,214	465	1·95	419
,,	,,	,,	,,	,,	0·02	8·5	0·51	4,451	741	2·44	329

M. Considère advises the mixture of concrete in the proportions of 505 pounds of cement to 1 cubic yard of sand and shingle (equivalent to 300 kilos of cement per cubic metre of sand and shingle), and that the ratio of the area of metal to the total area of the piece should not exceed 0·0217 in beams subjected to only steady loads, and 0·008 for pieces subjected to vibrations of moving loads. He also recommends the use of wrought iron when there will be excessive vibrations, and of steel when there is danger of reactions due to the building in a monolithic construction. He further advises the use of steel with a concrete rich in cement for structures where impermeability is required.

M. Considère very rightly points out that for tests of a structure the measuring of the *deformations* should be substituted for the usual practice of measuring the *deflections*.

It is very difficult to determine the deflection which a piece should have under a stated load, whereas the deformations can be readily calculated for any required load.

For the calculation of the *deformations* the tensile resistance of the concrete must be taken account of, and therefore, when such calculations are required, we may in every case use M. Considère's formulae without appreciable error; the error due to the fact that the stress-strain curve is assumed to be a straight line will certainly have the effect of making the calculated deformations more than they should be, but not in a dangerous degree.

M. Considère's Calculations for the Deformations of Pieces with Single Reinforcement.—For these calculations we have—

The sum of the tensions = the sum of the compressions,

$$\text{or } tx - \frac{t^2}{2c}(1-x) + \psi f = \frac{c}{2}(1-x) \qquad [1].^1$$

The proportion of tension in the reinforcements to the compression of the extreme fibres of concrete is equal to the product of the proportions of the distances to the neutral axis by the proportion of the co-efficients of elasticity of the two materials—

$$\text{or } \frac{c}{f} = \frac{1}{m}\frac{(1-x)}{(x-u)} \qquad [2].^1$$

The bending moment of the reinforced piece is produced by the tensions of the reinforcement and those of the concrete and the compressions of the concrete—

$$\text{or } M = bd^2\left[\frac{tx^2}{2} - \frac{t^3}{6c^2}(1-x)^2 + \psi f(x-u) + \frac{c}{3})1-x)^2\right] \qquad [3],^1$$

t and m being known either by direct experiment or from former results.

The value of f from [2] is substituted in [1]—

$$\text{giving } tx - \frac{t^2}{2c}(1-x) + m\psi c\left(\frac{x-u}{1-x}\right) = \frac{c}{2}(1-x) \qquad [4].^1$$

Now by choosing arbitrary values for c the curve of deformation for the typical concrete will give the value for E_c, and therefore for m; corresponding to this pressure or m may also be given a value (say 10). Then, by substituting the values for c, t, m and u (which is known) in equation [4] the value of x is found, and by substituting the values of c, t, m, x and u in equations [1] and [3] f and M are found.

By giving different values to c within the possible limits corresponding groups of the values of c, f and M are obtained, each of which will satisfy the equations and will characterize one of the possible states of equilibrium of the beam.

Now the lengthening ϵ_f of the metal $= \frac{f}{E_f}$ and the shortening ϵ_c of the concrete fibres under the greatest compression $= \frac{c}{E_c}$.

[1] When the area of the concrete is measured in square centimetres and the area of the reinforcement in square millimetres, the expressions ψf and $\frac{1}{m}\frac{(1-x)}{(x-u)}$ must be multiplied by 100.

The co-efficient of elasticity of the reinforcement is known. That for the concrete is equal to that of the iron divided by the value of m corresponding to that found or used for introduction into equation [4].

In order that the results thus obtained may be applied to any prism having the same ratio of reinforcement ψ, whatever the values of d and b may be, it is sufficient to consider the unit moment $\mu = \frac{M}{bd^2}$.

The groups of μ, ϵ_c and ϵ_f, each of which will be realized when the test load causes the absolute moment M and the moment μ, referred to a square section of a unit side.

These formulae are only used when the limit of elasticity in the extreme tensional fibres of the concrete has been passed. For smaller loads the coefficient of elasticity retains its normal value and the deformations must be calculated by the ordinary formulae, taking into account, for the calculation of the moments of inertia the difference of the perfect coefficients of elasticity of the reinforcement and the concrete.

M. Considère found that the difference between the actual measurements of the deformations and the calculated deformations by the use of equations [1] to [4], was 5 per cent. both for the concrete and reinforcement.

M. Considère's Method for the Treatment of Deformation of Pieces with a Double Reinforcement.—If we consider the section of a prism subjected to a unit moment μ, the neutral axis will have a position which is determined by the equations [1] to [4]. Adding to this prism a supplementary section S_t of tensile reinforcement, and another S_c of compressive reinforcement, and if, as in the previous calculations, we take (hu) and (hb) as the distances of the compressive and tensile reinforcements from the neutral axis. The resisting stresses caused by these reinforcements will be equal if the sections of the respective reinforcements satisfy the equation $S_t\,(hb) = S_c\,(hu)$. The equilibrium will not be disturbed if, at the same time that the additional reinforcements are added, the moment of the external forces is increased by a quantity equal to the resisting moment which gives the tensile stress of S_t and the compressive stress of S_c.

Now, as the supplementary bottom reinforcement is at the same distance from the neutral axis as the single reinforcement, it will undergo the same elongation, and consequently have the same unit tension f given by the formulae [1] to [4]. The supplementary resisting moment is therefore

$$S_t f\left\{(hu) + (hb)\right\}.$$

So nothing is altered in the unit stresses and in the deformations of a piece with single reinforcement, if a double reinforcement be added to it, the sections of which are in the ratio of $\frac{(hb)}{(hu)}$, *and if the bending moment is at the same time increased by* $S_t f\,\left\{(hu) + (hb)\right\}$.

M. Christophe's Calculations.—M. Christophe's formulae are very similar to those developed above, the difference being in the assumption of a straight line stress-strain curve in compression by M. Christophe, whereas a parabolic curve is adopted in the formulae here recommended. The difference appears in the area of the stress-strain diagram for the concrete in compression and the position of the centre of action of the compressive stresses. M. Christophe uses two sets of unit stresses according to the nature of the loading, being 710 and 430 pounds per square inch for concrete in compression; 13,500 and 9,000 pounds per square inch for wrought iron and 21,000 and 14,000 for steel reinforcements.

Comparison of Different Formulae.—It may be interesting to give a table showing the results obtained from the application of different formulae. The piece taken for consideration is a slab 12 inches wide with a single system of reinforcement under a bending moment

equal to when $M=\frac{wL^2}{12}$, as recommended in this work, $= 34{,}011$ inch-pounds,

and when $M=\frac{wL^2}{10}$, as allowed by M. Hennebique, $=40{,}813$ inch pounds.

VARIOUS RESULTS OBTAINED BY THE USE OF DIFFERENT FORMULAE

I Nature of Reinforcement	II As Recommended in this Work for Economical Reinforcement	Christophe's Formulae Economical Reinforcement		Hennebique		Ritter's Formulae with same Dimensions as taken in Column II
		III Lower Limit	IV Higher Limit	V Calculated	VI With Depth of Slab and Position of Reinforcement as per Column II	
Wrought Iron	$d=6{\cdot}45''$ $\omega=0{\cdot}7040\square''$	$d=7{\cdot}74\square''$ $\omega=0{\cdot}644\square''$	$d=6{\cdot}27''$ $\omega=0{\cdot}522\square''$	$h=4{\cdot}646''$ $d=$ say $5{\cdot}65''$ $\omega=0{\cdot}91\square''$	$h=5{\cdot}38''$, $d=6{\cdot}45''$ } assumed $u=3{\cdot}07''$ $\omega=0{\cdot}660\square''$	$d=6{\cdot}45''$, $h=5{\cdot}38''$, $\omega=0{\cdot}704\square''$ } assumed $c=390$ $f=11{,}368$
				Steel at 17,065 pounds per square inch		
Steel	$d=7{\cdot}37''$ $\omega=0{\cdot}416\square''$	$d=8{\cdot}85''$ $\omega=0{\cdot}368\square''$	$d=7''$ $\omega=0{\cdot}292\square''$	$h=4{\cdot}6''$ $d=$ say $5{\cdot}5''$ $\omega=0{\cdot}780\square''$	$h=6{\cdot}14''$, $d=7{\cdot}37''$ } assumed $u=3{\cdot}07''$ $\omega=0{\cdot}430\square''$	$d=7{\cdot}37''$, $h=6{\cdot}14''$, $\omega=0{\cdot}416\square''$ } assumed $c=308$ $f=16{,}804$

It appears that the Hennebique calculations give somewhat less area of *wrought iron* reinforcement and about the same area of *steel* reinforcement as the formulae recommended and developed above.

It is interesting to see the results of checking by Professor Ritter's method. The stress on the concrete is found to be considerably lower than the 500 pounds per square inch which has been allowed, and the stresses on the reinforcements are slightly higher than the values assumed for the recommended formulae, but are well within the limits allowed by Professor Ritter.

A PROPOSED METHOD OF REINFORCEMENT AND CALCULATION.

General Remarks.—It would appear from the experiments and tests which have been carried out that, for beams, the best results should be obtained, if a hooping were provided along the whole length of the piece, which would prevent the swelling of the concrete under compressive stresses, and some form of reinforcement inserted on the tensile side, which would induce compressive stresses in the concrete.

In a piece subjected to bending the internal pressure due to the swelling of the concrete will not be equally distributed, as it is in a column under direct compression; the piece must also be rectangular instead of circular in section, and consequently a circular hooping is unattainable.

For finding the size of the wire for the purposes of hooping, we will therefore assume the diameter as being half the depth of the piece, since although the rectangular hooping would not be so effectual in resisting the swelling as if it were circular, the fact that the internal stresses are not equally distributed, considered by itself, would justify our assuming a diameter less than $\frac{d}{2}$ for the portion under compression. There can be no doubt that such a rectangular hooping, would effectually resist the swelling, since the tendency to swell laterally would cause the hooping wires to resist the upward thrust of the concrete, and vice versa.

We have allowed a safe stress of 15,000 pounds per square inch on steel reinforcements, which value will give us a compressive stress on the concrete of 1,500 pounds per square inch, as the concrete being hooped can follow completely the deformation of the reinforcement.

We therefore obtain for the value of $\frac{p}{n}=q=\frac{1500}{15{\cdot}41}=97$ pounds per square inch (vide the treatment of hooped pieces under direct compression, page 296.) and we get $\frac{d}{4}\times 97 = 24\,d =$ hoop tension per inch width ; d being in inches,

The spacing of the hoops may be taken as $\frac{d}{10}$.

Therefore $24\,d \times \frac{d}{10} = 2{\cdot}4\,d^2 =$ hoop tension in the wire.

Consequently the diameter of the wire becomes—

$$\delta = \sqrt{\frac{2{\cdot}4\,d^2}{25{,}000 \times 0{\cdot}7854}} = 0{\cdot}012\,d.$$

This gives the diameter for the wire at the section where the greatest bending moment occurs, with a spacing of $\frac{d}{10}$ inches.

It appears from the calculation for the longitudinals for hooped pieces under direct compression (p. 297) that we may safely assume the diameter of the distribution rods to be $1\frac{1}{2}$ times the diameter of the hooping, or $0{\cdot}018d$. They should not be spaced more than three inches apart.

The area calculated for the longitudinal reinforcements in compression, where they replace a distribution rod must always be additional to the area of the distribution rod as found for a diameter of $0{\cdot}018d$.

The distribution rods along the tensile surface of a beam may, however, be included in the area of metal resisting the direct tensile stresses, since these would tend to counteract any lateral bending of the distribution rods.

The diameter and the spacing of the hooping wires have been found as described for the section at which the maximum bending moment occurs, but since the bending moment decreases while the dimensions of the piece remain constant, we know that the internal stress, and consequently the hoop tension, will diminish as the bending moment becomes less.

This decrease of the hoop tension may be allowed for, either by decreasing the diameter or the spacing of the hooping wires. The diameter must remain the same for practical reasons : we must therefore increase the spacing, as the bending moment diminishes.

The variation in the spacing may be obtained in the following manner.

Draw the beam to scale, and plot the curve of bending moments according to the method and amount of loading.

If the piece is freely supported, divide the portions of the span between the section where the maximum bending moment occurs and the supports into any desired number of parts (say ten), and erect perpendiculars to the curve of bending moments.

Then, if y_{max} is the length of the maximum ordinate and y_n that of any other, the spacing at the ordinate in question will be $\frac{d}{10} \times \frac{y_{max}}{y_n}$.

If the spacings at the several points on the span are thus found, we can arrange the distance between the hoopings, so that it gradually increases from point to point towards the supports from the section at which the maximum bending moment occurs. The spacing need not be absolutely accurate, but should be as correct as practicable, without causing undue expense.

If the load on the beam is uniformly distributed, it is of course unnecessary to plot the curve of bending moments, since the ordinates will be those of a parabola, and the ratio of their lengths to that of the central ordinate can be found in Molesworth's (or other) *Engineering Pocket Book*.

When the beam is built in, partially or entirely, at the ends, the closing line must be drawn according to judgment or on the lines laid down (page 280).

The beam is designed for the maximum bending moment, which may occur at the supports or on the span. The spacing is $\frac{d}{10}$ at the section where the greatest bending moment occurs, and that at other points is found as described above, the ordinates being of course taken from the closing line of the curve of bending moments, which in this case represents the beam.

With a reinforcement such as this, the concrete would suffer greater shortenings without fear of failure, and could be compressed to such an extent that the full resistance might be obtained from the reinforcing bars under compression.

We may therefore safely consider that the concrete in compression will bear a stress $\frac{1}{10}$ that allowed for the reinforcing metal, since the shortenings of the concrete would follow those of the metal, certainly up to the maximum allowed stresses in the steel.

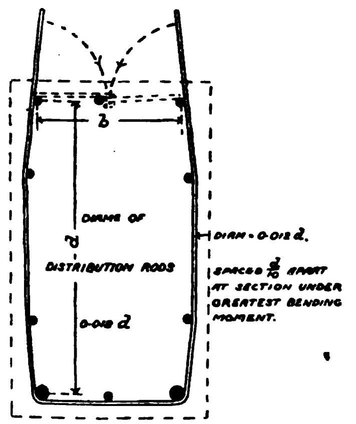


Fig 350

A steel reinforcement would be used as most economical in a case such as this, and we may allow, as before, a maximum stress in the steel both in tension and compression of 15,000 pounds per square inch.

The depth of the beam would be considered as that between the upper and lower hoopings, as this would be the effective depth ; but a further amount of concrete must be placed outside this, covering the reinforcement by half an inch as a minimum.

Since the reinforcing bars are placed against the hooping in the manner shown (Fig. 350), we may without serious error assume the distance between their axes as the effective depth (d) of the piece.

Lastly, since it is uncertain what effect the hooping may have on the stress-strain curve of the concrete, we will assume that this curve is a straight line.

To calculate the sizes or resistance of a piece, we may proceed by considering the neutral axis as being at the centre of the depth, as this must be its position for maximum economy.

Rectangular Beams

When a reinforcement is only used along the tensile surface, we have, using the same symbols as before—

$$\tfrac{1}{2}\ cub = \omega_b\ f_b \quad [1].$$

but we have $c = 1{,}500$ $f = 15{,}000$ and $u = \frac{1}{2}d$.

We get therefore—

$$375\,db = 15{,}000\ \omega_b.$$

Replacing ω_b by $\psi\ bd$—

$$375db = 15{,}000\ \psi\ bd,$$

$$\text{or } \psi = 0{\cdot}025 \quad [2].$$

This gives us the ratio of the area of reinforcement to the area of the piece (db).

We have, further, for the value of M—

$$M = \tfrac{1}{3}\ cu^2b + \omega_b\,f_b\,(d-u) \quad [3].$$

Substituting the values as before, and inserting the value of ψ from equation [2], we get

$$M = 125d^2b + 187{\cdot}5d^2b,$$

$$M = 312{\cdot}5d^2b \quad [4].$$

From equation [4] we can find the depth of the piece required for any bending moment by taking for b any ratio of d we may desire.

For this system of reinforcement $b = \frac{1}{2}d$ is a good proportion, since it slightly increases the depth, and therefore the stiffness ; but this ratio may be varied from this value to $b = \frac{3}{4}d$ as desired.

The sectional area of reinforcement will be always $0{\cdot}025\ bd$, or 2·5 per cent., this being the economic proportion.

If a double system of reinforcement is to be employed,

we may proceed as follows. We have the equation—

$$\tfrac{1}{2}\ cub + \omega_u\,f_u = \omega_b\,f_b \quad [5].$$

Again making $u = \frac{1}{2}d$, as this gives the economic position of the neutral axis, and inserting the values as before, we get—

$$375db = 15{,}000\ (\omega_b - \omega_u).$$

Now, if we make $\omega_b = \psi\,bd$ and $\omega_u = \lambda\psi\,bd$, we get $375\,db = 15{,}000\,\psi\,db\,(1-\lambda)$

$$\text{Or } \lambda = 1 - \frac{375}{15{,}000\,\psi}.$$

$$\text{Therefore } \lambda\psi = \psi - 0{\cdot}025 \quad [6].$$

Which is what might be expected from the value of ψ in equation [2].

We have also the equation—

$$M = \tfrac{1}{3}cu^2b + \omega_u\,f_u\,.u + \omega_b\,f_b\,(d-u) \quad [7],$$

from which we get by substituting the several values, including that for $\lambda\psi$ from equation [6]—

$$M = d^2b\,(15{,}000\,\psi - 62{\cdot}5) \quad [8].$$

Equation [8] gives the depth of beam for any bending moment and percentage of area of bottom reinforcement, but it will be seen that if double reinforcement is to be used the value of ψ must be greater than 0·025. Probably the best ratio of breadth to depth will be $\frac{1}{2}$, in which case equation [8] becomes—

$$d=\sqrt[3]{\frac{2M}{15{,}000\,\psi-62{\cdot}5}} \quad [9].$$

If the depth is assumed, and also the ratio of b to d, equation [4] will show whether a double reinforcement is necessary, and if such is the case equation [9] will give the value of ψ for the assumed depth, and equation [6] will give the value $\lambda\psi$ for the upper reinforcement.

The sectional areas thus determined for the reinforcing bars must be divided up into the requisite number of bars which will replace several of the distributing rods, and consequently the area of $0{\cdot}018d$ must be added to each of the longitudinal *compression* bars to provide for the area necessary for distributing the pressure over the hooping.

T-Beams

In the case of T-beams, the whole depth of the slab would always assist in resisting the compressive stresses.

For a single reinforcement we should have, in place of equation [1]—

$$\frac{c}{2}\left[B\,\frac{d}{2}-(B-b)\left(\frac{d}{2}-D\right)\right]=\omega_b\,f_b \quad [10],$$

$$\text{or} \quad \frac{c}{4}\left\{Bd-(B-b)\,(d-2D)\right\}=\omega_b\,f_b.$$

Inserting the several values, we have—

$$375\,\{\,Bd-(B-b)\,(d-2D)\}=15{,}000\,\psi\,bd \quad [11],$$

where *the sectional area of the reinforcement is referred to the area of the rectangle bd.*

As before, we should assume B to equal half the distance from centre to centre of the beams, and b as $\frac{1}{6}B$, from which we get—

$$\psi=\frac{0{\cdot}25D+0{\cdot}025d}{d} \quad [12],$$

$$\text{or} \quad \omega=b(0{\cdot}25\,D+0{\cdot}025d) \quad [13].$$

For the value of M, we have—

$$M=\frac{c}{3}\left[B\,\left(\frac{d}{2}\right)^2-(B-b)\left(\frac{d-2D}{2}\right)^2\right]+\omega_b\,f_b\,\frac{d}{2} \quad [14],$$

from which, inserting the values as before, we get—

$$M=125\,[Bd^2-(B-b)\,(d-2D)^2]+7{,}500\,\psi\,bd^2,$$

and replacing B by $6b$, we get—

$$M=125\,bd^2+2{,}500\,bd\,D-2{,}500\,bD^2+7{,}500\,\psi\,bd^2,$$

and inserting the value of ψ from [12], we get—

$$M=b\,(312{\cdot}5\,d^2+4{,}375\,dD-2{,}500\,D^2) \quad [15].$$

The value of D will have been obtained previously and b equals $\frac{1}{12}$ the distance from centre to centre of the beams. We can therefore obtain the value of d from [15], and we may afterwards obtain the sectional area of reinforcement from equation [13].

When a double longitudinal reinforcement is used, we shall have similarly, as for the case of rectangular beams—

$$\lambda\psi = \psi - \frac{0{\cdot}25\,D + 0{\cdot}025\,d}{d} \qquad [16],$$

and in this case—

$$M = 125\,bd^2 + 2{,}500\,bdD - 2{,}500\,bD^2 + 7{,}500\,\psi\,bd^2 + 7{,}500\,\lambda\psi\,bd^2.$$

Inserting the value of $\lambda\psi$ from [16], we get—

$$M = b\,(625\,dD - 62{\cdot}5\,d^2 - 2{,}500\,D^2 + 15{,}000\,\psi\,d^2) \qquad [17],$$

from which the value of d can be obtained for any bending moment with various percentages of reinforcement since the distance between the beam centres gives a value to b. If the depth is assumed, equation [15] will show if a double reïnforcement is necessary, and if such is the case equation [17] will give the value of ψ for the assumed depth, and equation [16] will give $\lambda\psi$ or the percentage of the compressive reinforcement.

In the case of T-beams the concrete of the floor slab, which would not be hooped, would be stressed somewhat highly in compression in the neighbourhood of the beams and towards the centre of their span, but when considered apart from the beam proper, the upper portion of the slab would be subjected to tensile stresses at these places, and these would counteract the excessive compressive stresses due to its action at part of the T-beam. The section (Fig. 350) gives the details of the reinforcements. The bottom longitudinal reinforcements would be bent up near the ends of the beams, leaving the lower surface at a distance from the supports of $\frac{1}{4}$ the span and reaching the upper surface at the edge of the supports. They would then be carried horizontally over the supports, and for a distance of $\frac{1}{8}$ the span into the adjoining beam. At the walls short additional rods would be added at the top projecting into the beam for a distance of $\frac{1}{8}$ the span. If the beam were freely supported there would of course be no necessity for the prolongation or extra rods.

For the portion of the span, after the longitudinals have left the lower surface, rods with a diameter of $0{\cdot}018d$ would be put in to replace them, and the ends of these would either be bent up into the heart of the beam or continued along the main longitudinal rods and tied to them with wire wrapping. The hooping would be bent out at the sides, as shown in the figure, to allow the longitudinals when bent up to pass the distribution rods.

The lower main reinforcements and the bottom and side distribution rods would be tied, every here and there, to the hoopings, so that they would remain in place while the concrete is being deposited and rammed.

To complete the reinforcement, for the purpose of inducing compressive stresses in the concrete on the tensile side of the neutral axis, sheets of " expanded metal " with selvidge edges on both sides and of a width equal to b (Fig. 350), and having the longest diagonal of the meshes in the direction of the length of the beam, would be placed about two inches apart in the depth of the beam, the lowest sheet resting on the bottom rods. The metal for these sheets would be thinner as the neutral axis was approached.

Any diamond-meshed reinforcement would equally well serve this purpose.

These reinforcements would be placed in the bottom portion for the whole length of the span, and in a built-in beam along the top portion over the supports, and for about an eighth of the span from the supports.

For moderate loads the distribution rods would be the only direct longitudinal reinforcement employed, and for lighter loads no rods would be used along the bottom.

If it were necessary to provide further reinforcements to resist shearing, small diameter rods would be inserted in the vertical plane, being placed further and further apart from the supports towards the centre ; these would be inclined at an angle of 45° at the supports, and approach nearer and nearer to the vertical as the centre was approached. They would be passed through the meshes of the expanded metal or similar reinforcement and be bent so as to pass along for a short distance below the bottom sheet, and above the top sheet in built-in beams.

Neither the mesh nor the shearing reinforcements are shown in Fig. 350.

The ends of the hooping wires would be left unclosed at the top until the concrete was brought up slightly above their underside ; they would then be bent over, while the concrete was soft and, in the case of a rectangular beam, hooked together as shown.

In a T-beam the ends would be left longer and be bent so as to pass side by side, projecting well into the floor slab, where they would be hooked over rods placed parallel to the beams or through the meshes of the woven or expanded metal reinforcement. They would thus reinforce the upper portion of the floor slab where most necessary.

For the slabs, expanded metal or some diamond-meshed reinforcement would be used, and in order that the greatest efficiency might be obtained the length of the mesh would not always run in the same direction, as is usually the case, but the metal would be placed so that the length of the mesh was always as nearly as possible in the direction of the maximum tensile stress.

If a beam such as has been described were to be treated by measuring the deformations under increasing loads, it is practically certain that its behaviour would indicate the safety of the assumptions made in the proposed method of calculation, and that the concrete on the compression side would be found to safely take up the deformations necessary to permit the reinforcements to act at their maximum allowed resistance.

The resistance of the concrete on the tensile side has been neglected, as before, but it is almost certain that it would offer considerable resistance when reinforced as suggested.

It must be understood that this method has not been practically tested, and although it is believed that it is the rational form of reinforcement, and that the resistance obtained by its employment will far exceed that of any method at present in vogue ; still *it should be subjected to practical tests before the absolute truth of the reasoning is allowed.*

PIPES, CIRCULAR RESERVOIRS AND SIMILAR STRUCTURES

When Under Internal Pressure.—The direct tension on the shell of a pipe or elevated circular reservoir or silo is given by the usual formulae—

$$T=\tfrac{1}{2}\,p\,\delta \quad [1],$$

where δ is the internal diameter, and p the unit pressure. If p is in pounds per square inch—

$$p=0{\cdot}34\,H_w,$$

H_w being the head of water in feet.

For a reservoir or water pipe, therefore—

$$T=0{\cdot}175\,H_w\,\delta \quad [2],$$

T being in pounds per square inch, H_w the head in feet, and δ the internal diameter in inches.

As the resistance of the concrete is neglected in tension we must have—

$$T=f\,\omega \quad [3],$$

being the allowed unit stress in the reinforcement, and ω the sectional area of metal in the hooping reinforcement for the length of the structure taken.

If we take a length (l) in inches, all units being in inches and pounds, we get therefore from [1] and [3]—

$$\omega=\frac{lp\,\delta}{2f} \quad [4],$$

and from [2] and [3]—

$$\omega=\frac{0{\cdot}175\,H_w\,\delta\,l}{f} \quad [5],$$

from which we find the total sectional area of the hooping reinforcement for the length under consideration, which must be divided up into a suitable number of hoops or spirals.

For calculating the sectional area of the longitudinal bars, the portion of the shell between two adjacent hooping bars must be considered, this portion being treated as a slab built in at the ends, and of a span equal to the distance (L) between the hooping bars. For practical purposes the slab may be considered as flat between the two adjacent longitudinals.

As the shell and the longitudinals are continuous, we may consider the slab as securely fixed at the ends, and therefore the bending moments will be—

$$M_A=-\frac{1}{12}\,w\,L^2 \quad [6],$$

and

$$M_C=\frac{1}{24}\,w\,L^2 \quad [7].$$

At the hooping bars the concrete is in compression at the exterior of the shell, and the interior is in tension, while the reverse is the case at the centre of the span between the two hooping bars.

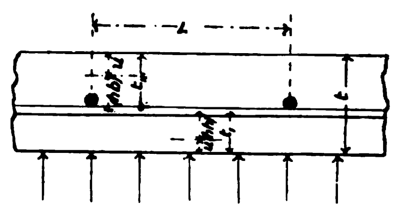


FIG. 351

The longitudinal bars bear against the inside of the hoopings ; it is therefore necessary to know the distances $t_{\prime}$ and $t_{\prime\prime}$ (Fig. 351), or the axes of the longitudinals from the surfaces of the supposed slab. This will give the position of the hooping reinforcement in the thickness of the shell. We also require the sectional area of the longitudinals.

The thickness (t) of the shell is always decided upon from practical considerations, and in a great measure follows that which has been found good in previous examples (*vide* page 198).

If we suppose the width under consideration (b) to be 12 inches, we already know the span L and the load ω (being the pressure on the strip 12 inches wide). We can consequently assess value for M_A and M_C in equations [6] and [7]. Further, we have—

$$t_{\prime\prime}=(t-t_{\prime}) \quad [8]$$

From equations [8] and [9] for rectangular pieces with single reinforcements [page 300].

$$u=-\frac{3}{4}m\frac{\omega}{b}+\sqrt{\frac{9}{16}\frac{m^2\omega^2}{b}+\frac{3}{2}\frac{m\,\omega\,h}{b}},$$

and

$$M=\frac{cub}{12}(8h-3u).$$

We get by substituting the above values together with $c = 500$ and $m = 10$—

$$u=\frac{5}{8}\,\omega\left(-1+\sqrt{1+\frac{32}{5}h}\right) \quad [9],$$

and—

$$u=\frac{3}{16}h+\sqrt{\frac{9}{256}h^2-\frac{M}{1500}} \quad [10].$$

Therefore we have—

$$\frac{5}{8}\,\omega\left(-1+\sqrt{1+\frac{32}{5}h}\right)=\frac{3}{16}h+\sqrt{\frac{9}{256}h^2-\frac{M}{1{,}500}},$$

or —

$$\frac{5}{8}\,\omega=\frac{\frac{3}{16}h+\sqrt{\frac{9}{256}h^2-\frac{M}{1{,}500}}}{\left(-1+\sqrt{1+\frac{32}{5}h}\right)} \quad [11].$$

Now ω must have the same value whether we are assessing the values of $t_{\prime\prime}$ or $t_{\prime}$. Consequently, inserting the several values we have

$$\frac{t_{\prime}\left(1+\sqrt{1-\frac{16\,wL^2}{10{,}125t_{\prime}^{\,2}}}\right)}{\left(-1+\sqrt{1+\frac{32}{5}t_{\prime}}\right)}=\frac{(t-t_{\prime})\left(1+\sqrt{1-\frac{8\,wL^2}{10{,}125\,(t-t_{\prime})^2}}\right)}{\left(-1+\sqrt{1+\frac{32}{5}(t-t_{\prime})}\right)} \quad [12],$$

and by inserting the values of $w\,L^2$ and t in the above, we obtain an equation for $t_{\prime}$.

The best manner to solve this equation will be to insert several values for t' and find the values for each side of the equation.

By plotting these on squared paper for the several values of $t_{\prime}$ a point where their respective curves coincide will easily be found, which gives the proper value for $t_{/}$.

Having found $t_{/}$, equation [10] will give the value for u, if we substitute $t_{/}$ for h. Having found u, equation [9] will give the value of ω if we again substitute $t_{/}$ for h.

The sectional area ω of the reinforcement is divided up into the number of bars which may be desired in the width of 12 inches.

Having decided on the size of the reinforcements the values of $t_{/}$ or $t_{//}$ will give the position of the hooping reinforcement in the thickness of the shell.

In the case of a pipe which has to bear transport, and handling while being deposited in the trench, it is well to somewhat increase the sizes of bars found by calculation, for the same reason that we always increase the theoretical thickness of a cast-iron pipe. This provision is, however, of less relative importance in the case of a reinforced concrete pipe, on account of the thickness of shell and nature of the reinforcement.

Many of the practical constructors only calculate for the hooping reinforcement and select a size for the longitudinals from practical experience without any calculation. If this course is adopted the hoopings should be placed at the centre of the thickness of the shell.

The hooping bars may be spirally wound or in the form of hoops, there being no *theoretical* advantage in the employment of either form, but there is a *practical* advantage in a spiral reinforcement, in that there are fewer joints, and such a form is usually employed for small sections. Where the reinforcement is built up first of rolled I, L, T or cross sections, and the concrete is poured into the moulds, the spiral form of hooping reinforcement allows the air to escape more easily, as it cannot become imprisoned, and the concrete consequently surrounds the reinforcement more perfectly.

The longitudinals should be always secured to the circular reinforcement, as this helps to keep the latter in position. Where large pipes are used, and a double reinforcement is adopted, each set of longitudinals must of course be placed inside the hooping reinforcement to which it is attached.

The above method of treatment applies to pipes, circular reservoirs or tanks, silos, and similar structures, the only difference being that the pressure in pipes is uniform, while that in reservoirs, silos, etc., varies with the height. In the latter cases it is usual to consider the pressure as uniform over heights of 12 to 18 inches (on the Continent they usually take heights of 40 or 50 centimetres), and vary the sections as the depth decreases.

When under External Pressure.—In this case, as the piece is in compression, we may allow for the resistance of the concrete.

As before, we have the general formula—

$$P = \tfrac{1}{2}\, p\, \delta \quad [13],$$

δ being the external diameter in this instance, P being the direct compressive stress on the shell. The method of treatment is the same as for the determination of the pieces under direct compression (p. 288).

Taking for the value of $\psi = \frac{\omega}{\Delta} = \frac{\omega}{lt}$ [14],

where ω is the sectional area of hooping reinforcement in a length (l) of the piece, and (t) is the thickness of the shell. We assume the limiting unit stress (c) on the concrete from which the area of the hooping reinforcement (ω) is deduced as shown in pages 288 and 290, either by assuming a thickness of shell or a value for (ψ). The unit stress on the concrete for ordinary proportions may be taken as 400 pounds per square inch, but if a richer mixture is used a higher stress may be allowed ; if, on the other hand, quick-setting cement is employed, this unit stress must be reduced.

The sectional area found for the hooping reinforcement must be divided up into a certain number of bars, which will fix their sectional area and spacing.

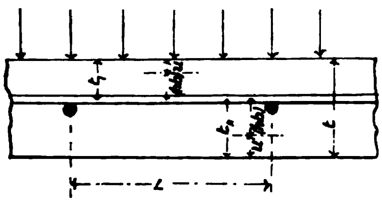


FIG. 352

The calculation of the longitudinal bars is made in exactly the same manner as for structures under internal pressure, the longitudinals in this case being on the *outside* of the circular bars (Fig. 352). The tensile and compressive stresses are the reverse to those of a piece under internal pressure. The general remarks which have been made on the manner of treatment, etc., apply equally to pieces under external, as to those under internal, pressure.

SMALL SPAN ARCHES

Arches with Uniformly Distributed Load, and Considered as Parabolic.—For small span arches, such as those used for floors, the arch may be considered as parabolic and the load as uniformly distributed. The curve of pressures is therefore parabolic.

If (w) is load per square unit, (L) the span, and (y) the rise, (H) being the horizontal thrust, we have—

$$Hy=\frac{wL}{2}\times\frac{L}{4}$$

or

$$H=\frac{wL^2}{8y} \quad [1].$$

If we call the reaction at the springings (R_s)—

$$\text{Then } R_s=\sqrt{\left(\frac{wL^2}{2}\right)+H^2}$$

or

$$R_s=\frac{wL^2}{8y}\sqrt{1+\frac{16y^2}{L^2}} \quad [2].$$

Both the horizontal thrust and the reaction at the springings will act at the neutral surface of the arch.

Herren Wayss and Fratag proceed as follows. The maximum compression being at the springing—

$$(d-\omega)\,c+\omega f=\frac{wL^2}{8y}\sqrt{1+\frac{16y^2}{L^2}},$$

making $\omega=\psi\,\Delta$, and considering the width of the piece as unity, $\omega=\psi\,d$.

$$\text{Then } d=\frac{\frac{wL^2}{8y}\sqrt{\frac{1+16y^2}{L^2}}}{c+\psi(f-c)}.$$

taking the ratio of $\frac{L}{y}$ as 10, $d = 1{\cdot}35\, wL \times \frac{1}{c + \psi\,(f - c)}$ [a]

and $\omega = \psi\, d$. [b]

Segmental Arches with Uniformly Distributed Loads.—The following method is used by Herren Wayss and Fratag (Monier system) for arches with single reinforcements near the intrados under a uniformly distributed load. The bending moment is considered as varying as $\frac{v}{L}$. It may therefore be assumed that the parabolic line of resistance varies approximately an equal amount above and below the segmental arch line, so that the algebraical sum of the areas included between these two lines, which represent the bending moment, is nil.

It follows that the area of the segment of the parabola having a rise (y) is equal to that of the segment of the circle, having the rise (v) (Fig. 353).

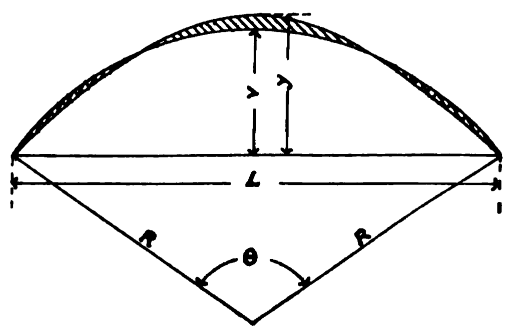


FIG. 353

We get therefore, if (θ) is the central angle of the curve of the arch in circular measure, and (R) the radius of the neutral line, and (L) the span—

$$\frac{2}{3}Ly = \frac{R^2}{2}\theta - \frac{L}{2}\,(R - v) \qquad [3].$$

If there are A degrees in the angle θ, we have—

$$Sin\ A = 2\ Cos\ \frac{A}{2}\ Sin\ \frac{A}{2}$$

$$= 2\frac{(R - v)}{R} \times \frac{L}{2R} = \frac{(R - v)L}{R^2} \qquad [4],$$

and we have also—

$$R^2 = \frac{L^2}{4} + (R - v)^2,$$

from which—

$$2\,Rv = \frac{L^2}{4} + v^2$$

$$R = \frac{L^2}{8\,v} + \frac{v}{2} \qquad [5].$$

Substituting [5] in [4], we get—

$$Sin\ A = \frac{\left(\frac{L^2}{8v} - \frac{v}{2}\right)}{\left(\frac{L^2}{8v} + \frac{v}{2}\right)^2} L \qquad [6].$$

$$\text{Now } \theta = \frac{A}{57{\cdot}295}, \text{ and } A = Sin^{-1} \frac{\left(\frac{L^2}{8v} - \frac{v}{2}\right)}{\left(\frac{L^2}{8v} + \frac{v}{2}\right)^2} L,$$

Therefore—

$$\theta = Sin^{-1}\frac{\left(\frac{L^2}{8v}-\frac{v}{2}\right)}{\left(\frac{L^2}{8v}+\frac{v}{2}\right)^2}L \div 57{\cdot}295 \quad [7].$$

Substituting [5] and [7] in [3], we get—

$$\frac{2}{3}Ly = \frac{1}{2}\left(\frac{L^2}{8v}+\frac{v}{2}\right)^2 \times \left\{Sin^{-1}\frac{\left(\frac{L^2}{8v}-\frac{v}{2}\right)}{\left(\frac{L^2}{8v}+\frac{v}{2}\right)^2}L \div 57{\cdot}295\right\} - \frac{L}{2}\left(\frac{L^2}{8v}-\frac{v}{2}\right)$$

$$\frac{2}{3}Ly = \frac{L^2}{128}\left(\frac{L}{v}+\frac{4\ v}{L}\right)^2 \times \left\{Sin^{-1}\frac{\left(\frac{L^2}{8v}-\frac{v}{2}\right)}{\left(\frac{L^2}{8v}+\frac{v}{2}\right)^2} \div 57{\cdot}295\right\} - \frac{L^2}{16}\left(\frac{L}{v}-\frac{4v}{L}\right)$$

Dividing through by $\frac{2}{3}L^2$, we get—

$$\frac{y}{L} = \frac{3}{256}\left(\frac{L}{v}+\frac{4v}{L}\right)^2 \times \left\{Sin^{-1}\frac{\left(\frac{L^2}{8v}-\frac{v}{2}\right)}{\left(\frac{L^2}{8v}+\frac{v}{2}\right)^2} \div 57{\cdot}295\right\} - \frac{3}{32}\left(\frac{L}{v}-\frac{4v}{L}\right) \quad [8].$$

Inserting the values assumed for $\frac{L}{v}$ (this value is usually taken as $\frac{L}{v}=10$), we get—

$$\frac{y}{L} = \frac{3}{256}(10+0{\cdot}4)^2 \times \left\{Sin^{-1}\frac{8\,(10-0{\cdot}4)}{(10+0{\cdot}4)^2} \div 57{\cdot}295\right\} - \frac{3}{32}(10-0{\cdot}4),$$

$$\text{or} \quad \frac{y}{L} = 1{\cdot}2675\,(Sin^{-1}0{\cdot}71 \div 57{\cdot}295) - 0{\cdot}9$$

$$Sin^{-1}0{\cdot}71 = 45{\cdot}233°.$$

Therefore we have—

$$\frac{y}{L} = 0{\cdot}1013 \quad [9],$$

$$\text{and } (y-v) = L\,(0{\cdot}1013 - 0{\cdot}1) = 0{\cdot}0013\,L \quad [10].$$

The maximum bending moment, if H is the horizontal thrust, is—

$$M\ max = H\,(y-v) \quad [11].$$

From equation [1]—

$$H = \frac{wL^2}{8y} = \frac{wL}{8} \times \frac{L}{y} \quad [12].$$

We have, therefore, inserting [12] in [11]—

$$M\ max = \frac{wL}{8} \times (y-v) \div \frac{y}{L} \quad [13].$$

If, as before. we assume $\frac{L}{v}=10$, substituting the values from [9] and [10], we get therefore at the crown—

$$M\ max=0{\cdot}0016\,wL^2 \quad [14],$$

and also a horizontal thrust as in equation [12], which becomes, for the case where $\frac{L}{v}=10$—

$$H=1{\cdot}234\,wL \quad [15].$$

The thrust at the springings has the value (derived from equation [2])—

$$R=1{\cdot}33\,wL \quad [16].$$

In the Monier (Wayss and Fratag) system the further method of procedure is as follows—

If M R is the moment of resistance, Δ the sectional area of the piece, and σ the greatest stress—

$$\sigma=0{\cdot}0016\,\frac{wL^2}{\frac{\mathrm{M\,R}}{\sigma}}+1{\cdot}234\frac{wL}{\Delta} \quad [17].$$

For slabs, Herren Wayss and Fratag proceed as follows, considering the neutral axis as at the centre of the depth, and the centre of the reinforcement as one-twelfth the depth from the lower surface—

$$f\,\omega=\frac{cd}{4} \quad [18],$$

$$\text{and } M\,R=\frac{cd}{4}\times\frac{3}{4}d=0{\cdot}1875\,cd^2 \quad [19].$$

From [18] and [19]—

$$d=2{\cdot}31\sqrt{\frac{M}{c}} \quad [20],$$

$$\text{and } \omega=0{\cdot}25\,d\times\frac{c}{f} \quad [21].$$

Replacing $M\,R$ by its value $0{\cdot}1875\,cd^2$. And where the width of the piece is taken as unity—

$$\Delta=d \quad [22].$$

Substituting the values from [19] and [22] in [17]—

$$\sigma=0{\cdot}0085\,\frac{wL^2}{d^2}+1{\cdot}234\,\frac{wL}{d} \quad [23],$$

making σ = safe compressive stress in the concrete (c), equation [23] becomes—

$$d^2-\frac{1{\cdot}234\,wL}{c}d=0{\cdot}0085\frac{wL^2}{c},$$

and solving the quadratic—

$$d=\frac{0{\cdot}617\,wL}{c}+\sqrt{0{\cdot}0085\frac{wL^2}{c}+\frac{0{\cdot}38\,w^2L^2}{c^2}}$$

$$d=\frac{wL}{c}\left[0{\cdot}617+\sqrt{0{\cdot}38+0{\cdot}0085\frac{c}{w}}\right] \quad [24].$$[1]

The sectional area of the reinforcement is taken as for slabs, though it is slightly in excess of the requirements, or from equation [21]—

$$\omega=0{\cdot}25\frac{c}{f}d \quad [25].$$

Method for Arches Loaded over Half the Span and considered Parabolic.—Another method employed for the calculations for arches is to consider the neutral line of the arch as parabolic, which is approximately true when the rise is small as compared with the span.

The dead load is supposed to be uniformly distributed. The live load is assumed to cover only half the span, as this loading causes the greatest bending moment. The curve of pressures for the dead load follows the curve of the arch, and that for the live load considered alone is supposed to pass through the neutral surface curve of the arch at the crown and springings. This is the same as assuming hinges at these places. In this case the thrust at the crown becomes—

$$H=\frac{L^2}{16v}(w+2p) \quad [26],$$

and that at the springings—

$$R=\frac{L^2}{16v}(w+2p)\sqrt{\frac{4v^2}{L^2}+1} \quad [27].$$

The dead load produces no bending moment as it is uniformly distributed, and the curve of the arch assumed to be parabolic.

The maximum bending moment due to the live load only is produced at a section a quarter the length of the span from the springings, tending to cause a downward deflection on the loaded side, and an upward deflection on the unloaded side.

The ordinate of the parabolic pressure curve at the section $\frac{1}{4}L$ from the springing is $\frac{3}{4}v$. And the vertical component of the thrust at the springing on the unloaded side due to the live load only is (taking it as the reaction of a girder) $\frac{wL}{8}$.

The horizontal thrust due to the live load only $=\frac{wL^2}{16v}$.

Taking moments, we get—

$$M\ max=\frac{wL^2}{16v}\times\frac{3}{4}v-\frac{wL}{8}\times\frac{L}{4}$$

$$M\ max=\frac{wL^2}{64} \quad [28].$$

These equations will apply to any arch hinged at the crown and springings, if the weight of the arch and roadway can be considered as uniformly distributed, which is seldom the case in practice.[2]

[1] This is the well-known "Monier" formula for arches, and as can be readily seen there are several erroneous assumptions made in deducing it.

[2] M. Godard's formula for arches is given p. 97.

LARGE SPAN ARCHES AND OTHER PIECES SUBJECTED TO DIRECT STRESS AND BENDING COMBINED

General Remarks.—In treating the question of pieces subjected to both direct and bending stresses the first essential is to know the position of the curve of pressures[1] through the piece and the magnitude of the resulting pressures at different sections. When we have found the curve of pressures and its position on an *arch ring*, we may consider the force lines forming the pressure curve as acting at the vertical load lines. We have therefore at each of these sections a force R acting in the direction of the pressure curve at this point (Fig. 354). The effect of this force is not altered if we imagine two forces equal to R as acting at the neutral surface of the arch in opposite directions parallel to its line of action. This is the same as substituting for R a thrust at the neutral surface, and a couple with a lever arm equal to the radial distance from the neutral surface to the pressure curve.

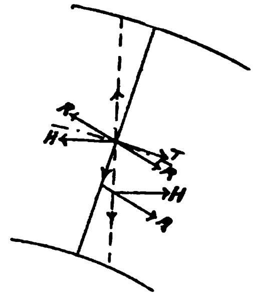


FIG. 354

The thrust at the neutral surface may be resolved into components *tangential and normal to the neutral* surface; the normal component only produces shearing, and is always small and is consequently negligible. The tangential component is the *direct* thrust, which we may call T.

The forces R and R of the couple producing the bending moment may also be resolved into *vertical and horizontal components.*

The vertical components act in opposite directions and therefore balance one another, and we have left a couple of horizontal forces with a lever arm of the vertical distance between the neutral surface and the pressure curve.

The horizontal force of the couple is the *horizontal* thrust, and is the same for all sections.

We have therefore the general equation for the bending moment $M = H \times t$, where t is the vertical distance from the neutral surface to the pressure curve, and varies at each section considered.

Now it will be seen that if at any section we were to resolve the forces R and R of the couple into components normal and tangential to the neutral surface of the arch instead of the components acting in vertical and horizontal directions, the normal components would balance each other, and we should be left with a couple of forces of the same magnitude and acting in the same direction as T, with a lever arm equal to the distance between the neutral surface and the pressure curve measured on the *radial* line of the arch, and this couple would produce a moment equal to $T \times$ the radial distance from the neutral surface to the pressure curve $= H \times t = M$.

We therefore get the relation $\frac{M}{T} =$ *radial* distance from the neutral surface to the pressure curve. In the case of columns or other pieces that are not curved there will be only one plane of reference in place of the radial and vertical planes of arches, also T will be the direct vertical thrust and there will be

[1] The reasoning used in finding the position of the pressure curve, etc., follows closely that employed by Prof. William Cain in his "Elastic Arches" and "Steel Concrete Arches" published by W. Van Nostrand Company.

no expression similar to H. The relation $\frac{M}{T}$ will be equal to the *horizontal* distance from the neutral surface to the pressure curve.

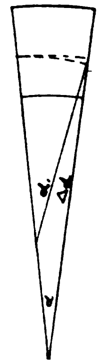


FIG. 355

Effect of the Bending Moments on an Arch Ring.—Consider a very small slice of an arch (Fig. 355) of a length Δs along the neutral surface, and having a central angle a. The direct thrust T cannot cause any change of curvature, but under the action of the bending moment M we may suppose the central angle changed to a, the curvature being increased if R acts below the neutral surface (as then the greatest compression is at the intrados) and decreased when R acts above the neutral surface—the angle $a_1 = a + \Delta a$.

$$\text{Therefore } \Delta a = (a_1 - a) \quad [1],$$

and Δa is the change of inclination of the tangents to the curve due to the change of curvature, as is clearly seen by the exaggerated case (Fig. 356).

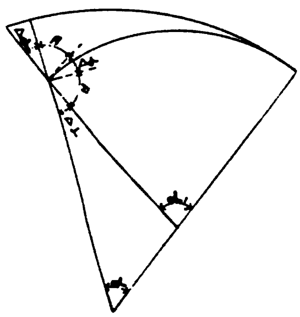


FIG. 356

If we consider the bending moment as plus when it is left handed, then Δa is plus when M is plus or R acts above the neutral surface, and Δa is minus when M is minus, or R acts below the neutral surface.

If we call (v) the distance of any fibre of area (a) from the neutral surface, (v) being plus for fibres above and minus for fibres below the neutral surface, as the length of the arc is very small, it may be considered always as the arc of a circle and the axis of a fibre in the same plane as concentric with it.

Therefore the length of a fibre before flexure is $(\Delta s + va)$, and after flexure it becomes $(\Delta s + va_1)$.

The change of length is $v\ (a_1 - a)$, or from equation [1]—

$$\text{The change of length} = v\ \Delta a \quad [2].$$

If the unit stress of the concrete is (c), and of the metal is (f), since the stress on any fibre $= \dfrac{\text{elongation of fibre,}}{\text{original length of fibre}} \times$ co-efficient of elasticity, the stress on a fibre of concrete is—

$$ca = \frac{v\ \Delta a}{\Delta s + v\ a}\ \text{a}\ E_c \quad [3],$$

and on a fibre of the metal—

$$fa = \frac{v\Delta a}{\Delta s + v\ a}\ \text{a}\ E_f \quad [4].$$

The co-efficient of elasticity of the concrete is here assumed to have a constant value.

The $(\Delta s + va)$ in the denominators may be replaced by Δs, without appreciable error.

The sum of all the stresses (due to flexure only) acting on the entire section is therefore—

$$\Sigma\ \text{ca} + \Sigma\ f\text{a} = \frac{E_c\ \Delta a}{\Delta s}\ \Sigma\ (v\text{a}) + \frac{E_f\ \Delta a}{\Delta s}\ \Sigma\ (v\text{a}) \quad [5].$$

The moment of the stress on any fibre about the neutral surface must be (a c v) or (a f v) according as the fibre is of concrete or metal. Therefore the total bending moment $=$ total resisting moment will be— Σ a c v $+$ Σ a f v.

Then from equation [5]—

$$M = E_c \frac{\Delta a}{\Delta s} \Sigma (v^2 \text{ a}) + E_f \frac{\Delta a}{\Delta s} \Sigma (v^2 \text{ a}) \quad [6],$$

but Σ (v^2 a) is the moment of inertia of the concrete or metal. We have therefore—

$$M = E_c \frac{\Delta a}{\Delta_s} I_c + E_f \frac{\Delta a}{\Delta_s} I_f,$$

$$\text{or } M = \frac{\Delta a}{\Delta_s} [E_c I_c + E \ I_f],$$

$$\text{Therefore } \Delta a = \frac{M \Delta s}{E_c [I_c + mI_f]} \quad [7].$$

We must now assume for the purpose of the graphical treatment *that for an appreciable length Δs, several feet for instance, Δa is given by equation* [7], *provided that M is taken as constant and equal to the value corresponding to that at the mid point of the length, or $\frac{1}{2}$ Δs distant from either end, I_c and I_f being also taken there.* This assumption is very nearly true.

As the total change in the inclination of the end tangents for a length s is the sum of all the infinitesimal changes for the part of the arch under consideration, or

$$\Sigma \left(\frac{M \Delta s}{E_c [I_c + m I_f]} \right);$$

Δs being very small, the above assumption means that this expression is equal approximately to

$$\theta = \frac{M_c s}{E_c [I_c + m I_f]} \quad [8],$$

where $s = \Sigma \ \Delta \ s$ and M_c is the moment at the middle of s, I_c and I_f being also taken there.

If a, b, c (Fig. 357) represents the neutral surface line of an unstrained arch, and (s) a length of the neutral line whose centre is b. When the arch is loaded the neutral line changes shape, and the change of the inclination of the end tangents to the neutral arc s is given by equation [8], where M and E_c are constant, and M, I_c and I_f are taken at b.

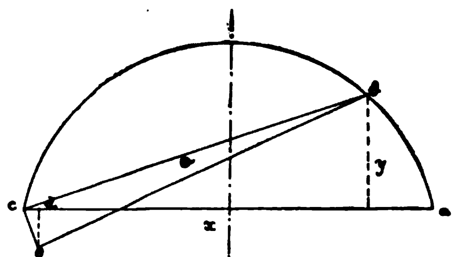


FIG. 357

Suppose the end c to be temporarily free, then the bending on s alone will cause a rotation of the arc b c about b equal to θ, so that the line b c will rotate through an infinitesimal distance c e, taken as perpendicular to b c.

Taking c as origin, and c a as the axis of x and the axis of y as vertical, and calling the co-ordinates of b, x and y, further drawing e d perpendicular to c a; then from similarity of triangles—

$$c\,d \ : \ c\,e \ : \ : \ y \ : \ b\,c, \quad \text{or } cd = \frac{ce}{bc} y \text{ and } \frac{ce}{bc} = \theta.$$

Therefore $c\,d = y\,\theta$ [9],

similarly $d\,e = x\,\theta$ [10].

This assumes that if M, I_c, I_f, x and y are all taken at the mid point of the arc s as a sort of average, the horizontal and vertical deflections of c, due to s, are given with a sufficiently close approximation by the above equations.

The total horizontal and vertical displacements of c due to the bending of all the portions of the arch are then given by $\Sigma\ (y\ \theta)$ and $\Sigma\ (x\ \theta)$.

Further, if the tangent at (a) moves through at small angle β we have a vertical deflection at (c), due to it, of $\beta.\overline{ac}$ the horizontal displacement being *nil*. We have therefore from equation [8]

$$\Sigma \frac{M.s.y}{E_c\,(I_c + m\,I_f)} \quad [11]$$

$$\text{and } \Sigma \frac{M.s.x}{E_c\,(I_c + m\,I_f)} + \beta.\overline{ac} \quad [12].$$

The total *change* of inclination of the tangents at (a) and (c) is similarly—

$$\Sigma\,\theta = \frac{M.s}{E_c\,(I_c + m\,I_f)}. \quad [13]$$

For the purpose of the graphical treatment to follow s and $E_c\ (I_c\ +\ m\ I_f)$ are considered as being constant : s is very nearly so since reinforced concrete arches have small rise compared to the span, and the span will be divided into equal parts. We are forced to consider $E_c\ (I_c\ +\ m\ I_f)$ as constant, since we do not know the area of the reinforcement, or whether tensile stresses will be induced. The location of the pressure curve on this assumption will be sufficiently close for practical purposes, but the allowable stresses must be taken lower than would be necessary if we could know the values of I_c and I_f, partly in consequence of the above assumption, but in a greater measure for the reason that until we know these values we cannot take the temperature stresses into consideration for arches not hinged at the crown and springings.

We have then the following conditions—

When the arch is hinged only at the springings, the span is invariable. Therefore—

$$\Sigma\ (M\ y)\ =\ 0 \quad [14].$$

The vertical deflection of c with respect to a is zero, but β will have a value ; therefore $\Sigma\ (M\ x)$ cannot be zero.

When the arch is hinged only at the crown, the horizontal movement of one half must be equal to the other at the crown, but will be in opposite directions. If A and B are the springings, and C the central hinge—

$$\Sigma_C^A\ (My) - \Sigma_B^C\ (My) \quad [15].$$

and since the vertical deflection at the crown must be the same for each half—

$$\Sigma_C^A\ (Mx) = \Sigma_B^C\ (Mx) \quad [16].$$

The origin in this case being taken at the crown.

When the arch is continuous, having no hinges, we get—

$$E\ M\ =\ 0 \quad [17].$$

$$E\ (M\ y)\ =\ 0 \quad [18].$$

$$E\ (M\ x)\ =\ 0 \quad [19].$$

When the arches have been designed from the location of the pressure curves as detailed below, and by the use of the formulae given for pieces subjected to direct stress and bending combined, they may, if considered advisable, be checked by the method given by Professor Cain in his *Steel Concrete Arches*, in which he divides the neutral surface curve into varying spaces, so as to make $\frac{s}{E_c[I_c + mI_t]}$ constant.

PRESSURE CURVE.

General Remarks.—The general principle employed for finding the true pressure curve on any arch due to the loading, and methods of fixing, whether hinged or otherwise, is stated as follows by Professor Cain, in his *Elastic Arches*.[1]

"If in any arch the equilibrium polygon (due to the weights) be constructed which has the same horizontal thrust as the arch actually exerts ; and if its closing line be drawn from consideration of the conditions imposed by the supports, etc. ; and if, furthermore, the neutral surface curve of the arch itself be regarded as another equilibrium polygon due to some systems of loading not given, and its closing line be also found from the same considerations respecting supports, etc. ; then when these two polygons are placed so that these closing lines coincide and their areas partially cover each other, the ordinates intercepted between these two polygons are proportional to the real bending moments acting in the arch."

We have also, as a principle of the equilibrium polygon, that if the ordinates have to be altered in a given ratio, the pole distance is altered in the inverse ratio. This simply means that if the slope of the lines in the diagram of forces is to be altered, the vertical forces remaining the same, it is necessary to increase the pole distance for a flatter slope, and decrease it for a steeper slope.

If the springings are at different levels we have, instead of a horizontal thrust, a thrust parallel to the line joining the springings. In the discussion to follow, it is always assumed that the springings are at the same level.

The methods employed to find the position of the curve of pressures assume that the elastic resistance of the arch is the same throughout, which greatly simplifies the working.

This is unfortunately very seldom the true state of the case, and consequently, *except in the case of three-hinged arches*, we must assume lower safe stresses than for ordinary bending in consequence.

When the Arch is Hinged at the Crown and Springings (Fig. 358).—The location of the pressure curve in this case is a simple matter, as it must pass through the hinges.

Draw in the neutral surface curve a_{21}, a_{20}, a_{19} . . . a_1 by joining points taken at the centre of different sections (the dotted line Fig. 358). Divide the span $A\ B$ into an equal number of equal parts (20 in the example, and this number is the least that should be taken), and drop perpendiculars P_1, P_2, P_3, etc. . . . P_{20} from the centre of each part.

Suppose the live load as extending over half the span (the right half in the example (Fig 358). Find the total load on each part, including the weight of the arch

[1] The method employed by Professor Cain is partly followed in the paragraphs dealing with the location of the pressure curve, by his permission. For a full description of his method, the reader is referred to *Elastic Arches* and *Concrete Steel Arches*, published by Van Nostraud & Co., 23, Murray Street, New York, at 50 cents. each.

PRESSURE CURVE FOR A THREE-HINGED ARCH

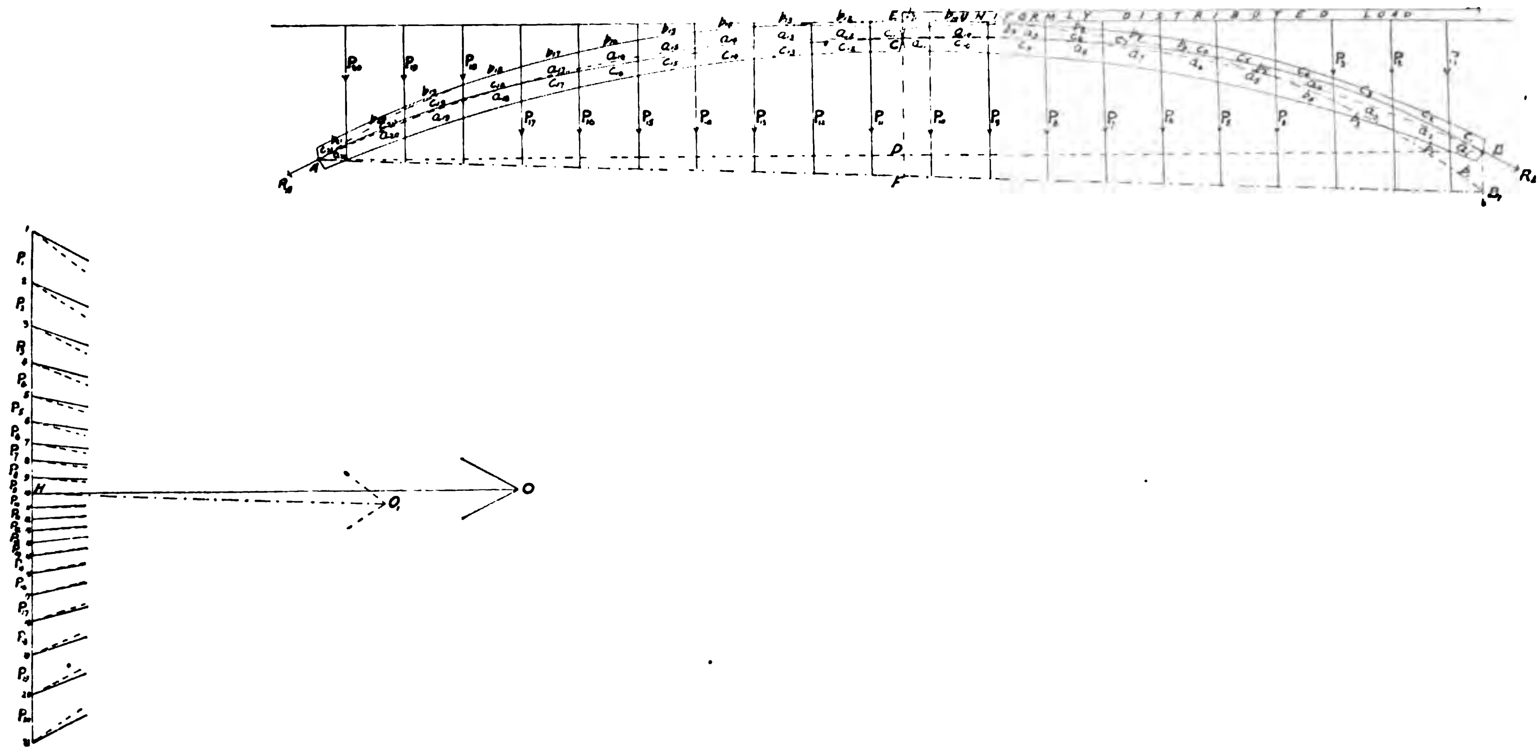


FIG. 358

and spandril filling, etc. These loads will be supposed to act on the lines P_1, P_2, P_3, etc. Plot these to a scale of loads on a vertical line 1, 2, 3 . . . 21 ; starting from 1 make 1, 2 $= P_1$, 2, 3 $= P_2$, 3, 4 $= P_3$, etc., etc.

Take any point O_l as a trial pole, and join 1, O_l, 2, O_l, 3, O_l. . . . 21,O_l. Starting from A draw b_{21} parallel to $\overline{21,O_l}$ till it cuts P_{20} ; from this point draw b_{20} parallel to $\overline{20,O_l}$ till it cuts P_{19}, and so on till b_1 parallel to 1·O_l cuts the vertical through B at B_l.

Join AB_l, this is the closing line to the *trial* curve of pressures.

From O_l draw O_lH parallel to B_lA, cutting $\overline{1, 21}$ in H. From H draw $H\,O$ horizontally to the right, making $HO = HO_l \times \frac{EF}{CD}$, for the curve of pressures must pass through A, C and B. O will be the true pole, and $O\,H$ measured to the scale of loads will be the horizontal thrust.

To locate the true pressure curve we may either join 1, 2, 3, etc. to O, and in the same way as before draw in the curve c_{21}, c_{20}, c_{19} . . . c_1, or we may plot from the line $A\,B$ the ordinates from the line $A\,B_l$ to the pressure curve b_{21}, b_{20} . . . b_1, each altered by being multiplied by $\frac{CD}{EF}$. The curve c_{21}, c_{20}, c_{19} . . . c_1 (shown in full line Fig. 358) is the true pressure curve, and R_A and R_B give the directions of the thrusts at the hinges A and B. The magnitudes are given by the lines $\overline{O \cdot 21}$ and $\overline{O \cdot 1}$ respectively measured to the scale of loads.

Similarly all the other lines in the diagram of forces such as $\overline{O \cdot 2}$, $\overline{O \cdot 3}$, etc., measured to the scale of loads, give the magnitude of the thrusts at the vertical load lines P_1, P_2, P_3, etc.

When the left half of the arch is covered with the live load, the curve of pressures on the left half span (Fig. 358) will apply to the right half, and vice versa. It will be seen that as the load passes over the bridge the stresses at the extrados and intrados over parts of the arch, will be reversed, which shows the necessity for double reinforcements. These remarks apply to all arches whether hinged or not.

Arch Hinged at the Springings only (Fig. 359).—When an arch is hinged at the springings only the curve of pressures must pass through these points. We plot the neutral surface curve, divide the span, and draw a trial pressure curve as before. In this case it is better for the sake of clearness to draw this curve below the arch.

We may therefore suppose that the trial pressure curve b_{21}, b_{20}, b_{19} . . . b_1 has been drawn with a closing line A_lB_l (Fig. 359), since the closing line for the trial pressure curve, treating the curve as that for a girder, must pass through the ends in consequence of the hinges.

Now we may *suppose* the ordinates on the lines P_1 P_2 P_3, etc., between A_lB_l and the *trial* pressure curve b_{21}, b_{20}, b_{19}, etc. (which we will call the ordinates of the type y_b) as laid off from $A\,B$, to give a polygon passing through A and B. This need not be actually done.

Now, if equilibrium is to exist, we have the condition (equation [14] page 385), that $\Sigma\,(M.y) = 0$, but $M = Ht$, where H is constant. Consequently $\Sigma\,(t.y) = 0$.

We will call the ordinates between $A\,B$ and the curve a_{21}, a_{20}, a_{19} . . . a, the ordinates of the type (y). The condition given above means, then, that—

$$\Sigma\,(y_b - y)\,y = 0 \quad [1].$$

PRESSURE CURVE FOR AN ARCH HINGED AT THE SPRINGINGS

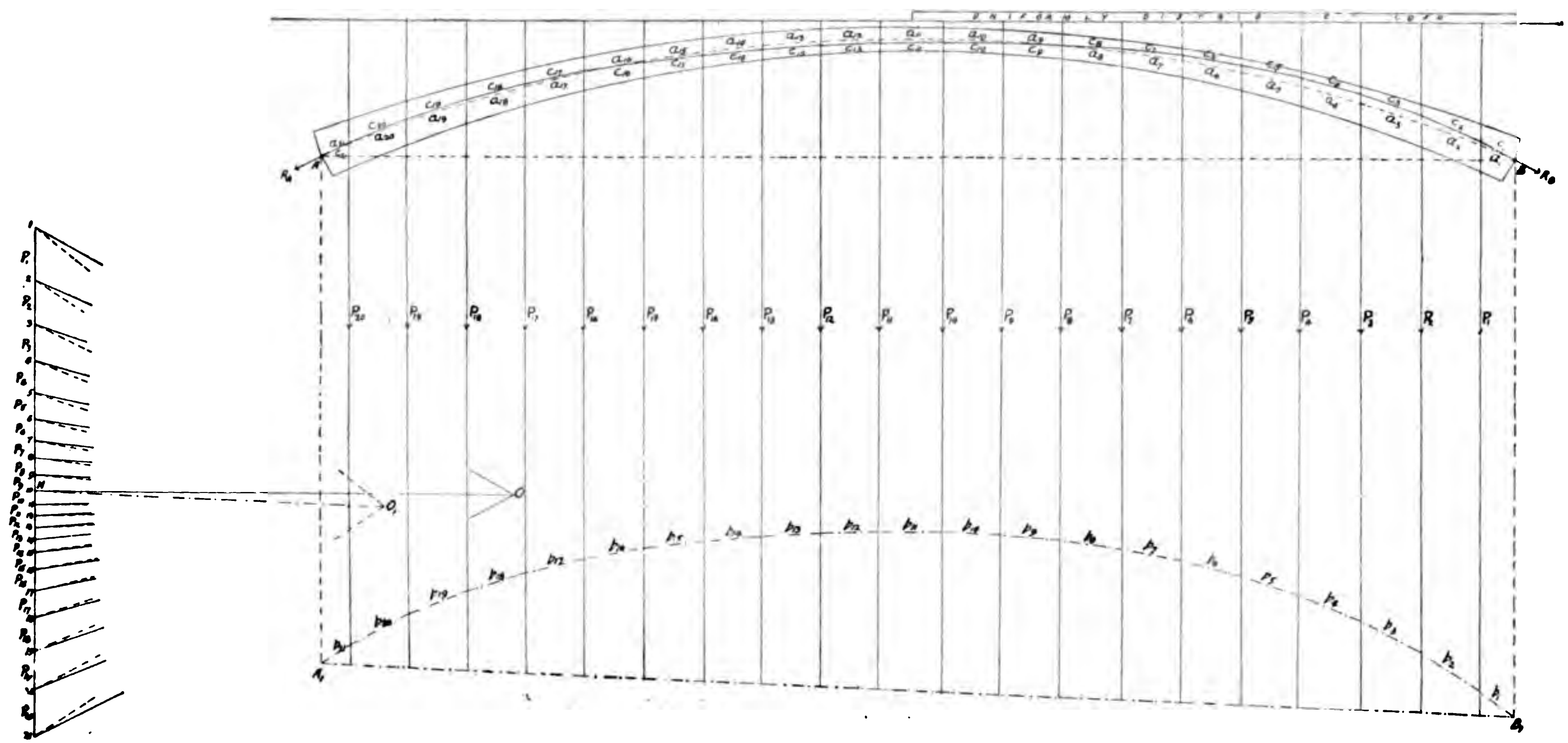


FIG. 359

The ordinates of the type $(y_b - y)$ varying in sign according as y_b or y is the greater.

Equation [1] may be written—

$$\Sigma\, y_b\, y = \Sigma\, y^2 \quad [2].$$

If the equality of equation [2] does not hold, all the ordinates of the type y_b must be altered in the ratio of $\frac{\Sigma\, y^2}{\Sigma\, y_b\, y}$ to locate the points on the true pressure curve, or, generally, $y_c = y_b \times \frac{\Sigma\, y^2}{\Sigma\, y^b y}$.

By plotting the varying values of y_c thus found from $A\ B$ on the lines P_1, P_2, P_3, etc., the true pressure curve $c_1, c_2, c_3 \ldots c_{21}$ is located.

To obtain the horizontal thrust and to be able to draw the true diagram of forces, we draw from the trial pole O_l a line parallel to $A_l\ B_l$ to cut the force line 1,21 in H, and from H draw a horizontal line $H\ O$, making $HO = H\ O_l\ \frac{\Sigma\, y_b\, y}{\Sigma\, y^2}$.

O will be the true pole, and $H\ O$, measured to the scale of loads, will be the horizontal thrust. By drawing lines from 1, 2, 3, etc. to O, the magnitude of the pressures at any section are similarly found. The differences between the ordinates y and y_c give the arms of the bending moments, which are considered as positive when the curve (c) is above the curve (a), and negative when (c) passes below (a). The tendency to tensile stress is at the intrados for a positive, and at the extrados for a negative bending moment.

Arch Hinged at the Crown only (Fig. 360).—In this case the pressure curve must pass through the hinge, and at this point the vertical displacement must be the same for both halves of the arch, and the horizontal displacements equal but of opposite sign.

Divide the span into an equal number of equal parts and proceed to draw the trial pressure curve $b_{21}, b_{20}, b_{19} \cdots b_1$ as before (Fig. 360). Drop a vertical line from the hinge C to cut the curve (b) at C_l. Now the closing line of the force polygon treated as that for a girder, with the condition (equation [16] page 385) that $\Sigma_C^A\,(Mx), = \Sigma_B^C\,(Mx)$ must pass through this point, and be so placed that the sum of the ordinates from the closing line to the pressure curve measured on the lines $P_1, P_2, P_3 \ldots P_{10}$, on one side of C_l is equal to the sum of the similar ordinates measured on the lines $P_{21}, P_{20} \ldots P_{11}$ on the other side of C_l.

If we consider the ordinates as forces to any scale the above equality will be true when their resultant passes through the point C_l.

Draw a trial closing line (closing line giving x) and plot the ordinates from this line to the curve (b) on a vertical line in the same manner as the forces P_1, P_2, P_3, etc., were plotted, and choosing any pole, O_x draw an equilibrium polygon. Now produce the first and last lines till they meet and drop a vertical.

This vertical is the line of action of their resultant. If we find that this line is at a distance (x) on one side of the central vertical, we must slightly *lower* this end of the closing line of the *trial* pressure curve and raise the other end by the same amount so that the new *trial* closing line still passes through C_l. Say we move the ends a distance (z); we must find the position of the resultant of the ordinates from the new *trial* closing line to the curve (b) in the same manner as described above.

PRESSURE CURVE FOR AN ARCH HINGED AT THE CROWN

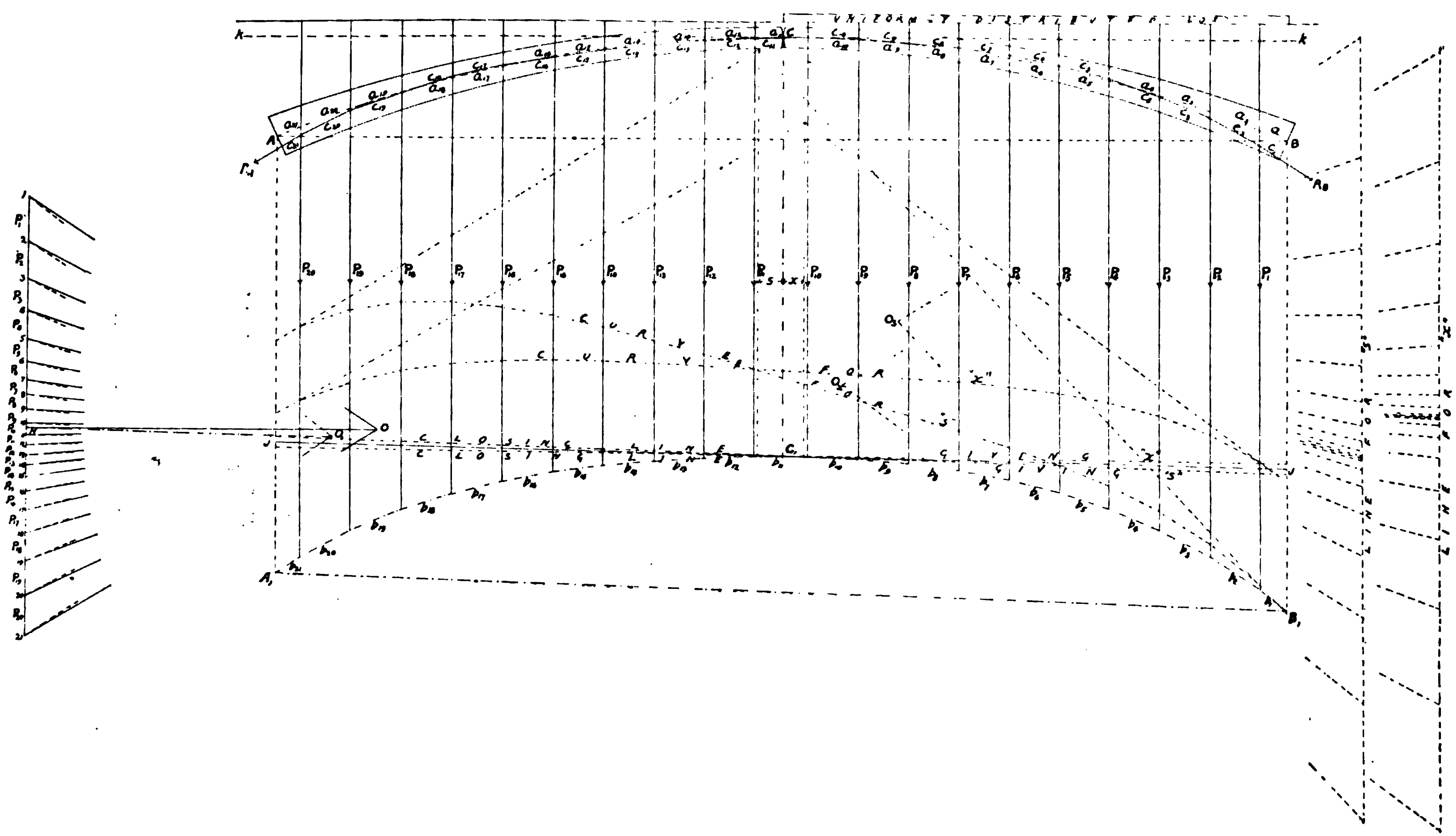


Fig. 360

Suppose, now, the line of action of the resultant falls at a distance (s) from the vertical through C_l and on the opposite side to the former resultant. If we now *raise* the end of the closing line, *which was lowered before* by an amount equal to $z \times \frac{s}{x+s}$, and *lower* the other end by the same amount, we shall obtain the position of the true closing line. If on the second trial the resultant had fallen again on the same side of the vertical through C_l, we should have to *lower* that end of the trial closing line a further distance equal to $z \times \frac{s}{x-s}$. We have now obtained a curve of pressures with its true closing line according to the conditions affecting the arch: we will call this closing line jj.

The closing line $k\,k$ for the neutral surface curve (a) treated as curve of pressures for some unknown system of loads must also pass through the hinge, and since the curve is symmetrical it will be a horizontal line, and can be drawn in at once. Now, *suppose* the ordinates on the lines P_1, P_2, P_3, etc., from $j\,j$ to the curve b_{21} . . b_{20} . . . b_1 [ordinates of the type (jb)] as laid off on the arch from the closing line k k. (This need not be actually done; since, as will be seen, we can take off the necessary ordinates from the bottom figure).

If we consider the ordinates from A B to the neutral surface curve a_1, a_2, a_3 . . . a_{21} as of type (y) and those from the closing line $k\,k$ to the neutral surface curve as of the type ($k\,a$). And further considering the ordinates intercepted between the curves (a) and (b) [supposing the ordinates ($j\,b$) to be laid off from (k k)] as of the type ($a\,b$).

From equation [15] page 385, and since the ordinates of the type (ab) are proportional to the bending moments,

$$\text{or } \Sigma_C^A[(ab)\,y] - \Sigma_C^B\,[(a\,b)\,y] \quad [1].$$

This may be expressed—

$$\Sigma_C^A\,[(ka)\,y] - \Sigma_C^A\,[(kb)\,y] = \Sigma_B^C\,[(kb)\,y] - \Sigma_B^C\,[\,(ka)\,y] \quad [2].$$

$$\text{or } \Sigma_B^A\,[(ka)\,y] = \Sigma_B^A\,[(kb)\,y] \quad [3],$$

$$\text{but } \Sigma_B^A\,[(kb)\,y] = \Sigma_{B_l}^{A_l}\,[(j\,b)\,y].$$

We must therefore have—

$$\Sigma_B^A\,[(ka)\,y] = \Sigma_{B_l}^{A_l}[(jb)\,y] \quad [4],$$

$$\Sigma_B^A[(ka)\,y] \text{ is constant.}$$

Therefore, if the equality [4] does not hold we must alter the ordinates of the type ($j\,b$) in the ratio of $\frac{\Sigma\,[(ka)\,y]}{\Sigma\,[(jb)\,y]}$, or the true ordinates to be laid off from kk will be found by the general equation—

$$kc = j\,b \times \frac{\Sigma\,[(ka)\,y]}{\Sigma[(j\,b)y]}.$$

To obtain the diagrams of forces with the true pole distance, or horizontal thrust, we must draw from O_l a line parallel to the closing line $j\,j$ to cut the vertical

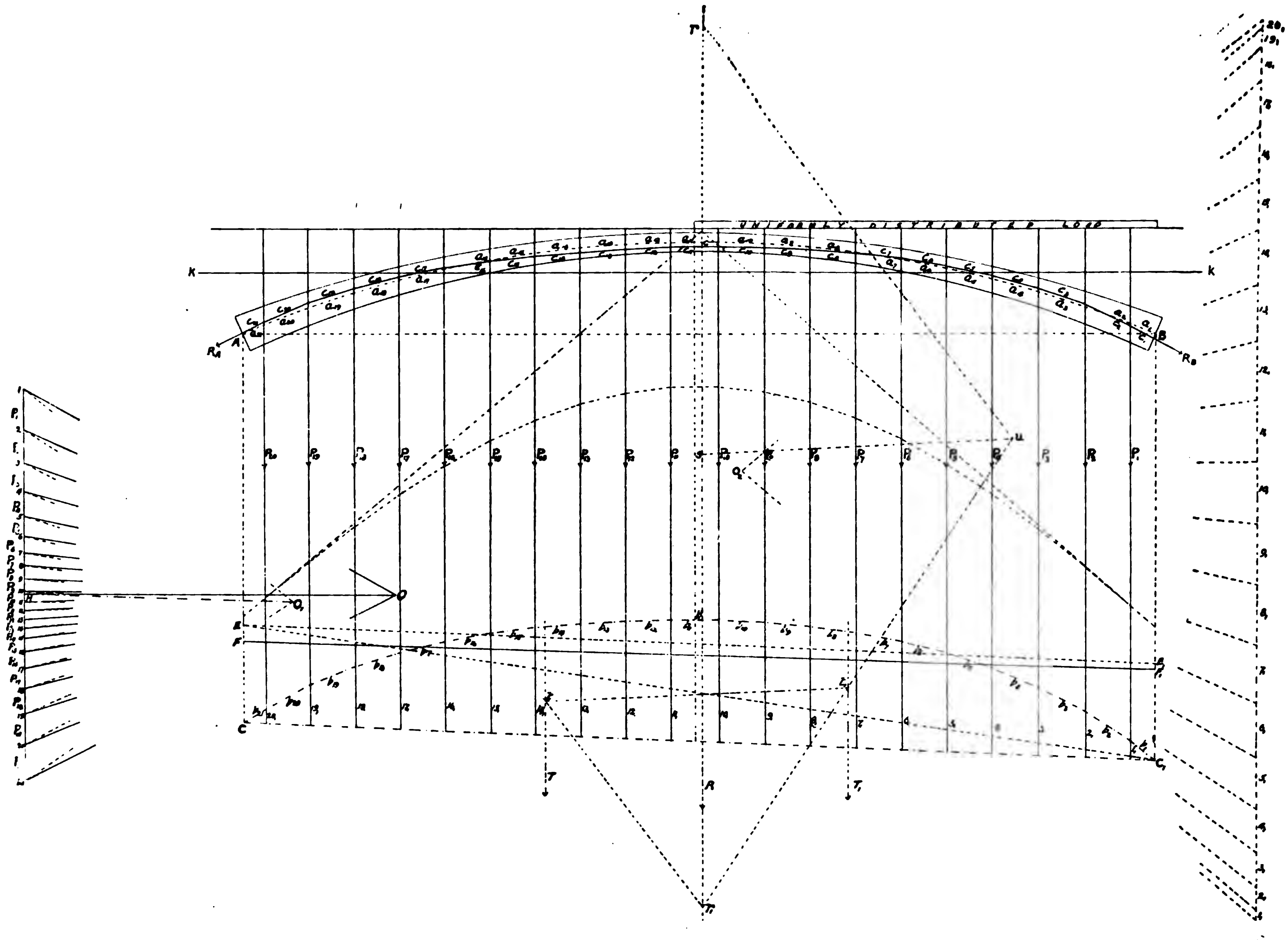


FIG. 361

force line at H, and from H draw the horizontal line $H\,O$, making $H\,O = H\,O_{,} \times \frac{\Sigma\,[(j\,b)\;y]}{\Sigma\,[(ka)\,y]}$ = The true horizontal thrust to the scale of loads, and by joining $1{\cdot}O$, $2{\cdot}O$, $3{\cdot}O$, etc., we obtain the true diagram of forces.

The true pressure curves may be located by plotting the ordinates of type $k\,c$ on the lines P_1, P_2, P_3, and the reactions at the springings may be drawn in parallel to the lines $O{\cdot}1$ and $O{\cdot}21$ of the force diagram.

The difference between the ordinates $(k\,a)$ and $(k\,c)$ will give the arm of the bending moment on each section.

When the Arch is Continuous, having no Hinges [Fig. 361].—Draw the equilibrium polygon as before by dividing the span into an equal number of equal parts and proceeding as already described. Also draw in the neutral surface curve $a_1, a_2, a_3 \;.\;.\;.\; a_{21}$ through the centres of gravity of each section.

We must find a closing line to the equilibrium polygon, $b_1, b_2, b_3 \;.\;.\;.\; b_{21}$, considered as that for a simple girder, so that $\Sigma\,(Mx) = o$ (equation [19] page 385), or the algebraical sum of the ordinates from the closing line to the curve, measured on the vertical force lines P_1, P_2, $P_3 \;.\;.\;.\; P_{20}$, is *nil*, the ordinates being taken as positive when above the curve and negative when below. Join $CC_{,}$ and draw a trial closing line $EE_{,}$.

Now the above requirement is the same as stating that the sum of the ordinates of the type $C\,E$ must be equal to the sum of the ordinates of the type $C\,b$. Therefore if we treat these ordinates as forces, their resultant must coincide and be equal in magnitude.

Plot the *halves* of the ordinates of the type $C\,b$ or $1_{,}, 2_{,}, 3_{,} \;.\;.\;.\; 20_{,}$, measured to any scale of distance, on a vertical line. All the ordinates are halved, since the same result is obtained and space is saved in the diagrams.

Choose any pole O_2 and draw the equilibrium polygon. Produce the first and last lines till they meet and drop a vertical from this point. This vertical is the line of action of the resultant of $1_{,}$, $2_{,}$, $3_{,}$, etc., treated as forces. The sum of their half-lengths will give the magnitude. Let $r\,r_{,}$ represent this value to the same scale as the ordinates were measured.

Now join $E\,C_{,}$, dividing the ordinates of the type $C\,E$ into two portions, those in the triangle $C\,E\,C_{,}$ and those in the triangle $C_{,}E_{,}E$.

Again regarding these ordinates as forces, the resultant T of those in the triangle $C\,E\,C_{,}$ will act at a point on $C\,C_{,}$ one-third of its length from C. Similarly, the resultant $T_{,}$ of the ordinates in the triangle $C\,E_{,}E$ will act at a point on $C\,C_{,}$ one-third of its length from $C_{,}$. Drop vertical lines through. These will be the lines of action of T and $T_{,}$. [These positions only hold in the case where the span is divided into equal parts. If the arc were divided into equal parts, the positions of the lines of action of T and $T_{,}$ would have to be found in the same manner as the position of R has been found.]

We have now the position and magnitude of R and the positions of T and $T_{,}$, and we require the true magnitude of T and $T_{,}$ so that their resultant may be equal to R.

Having laid off $r\,r_{,}$ equal to R, draw from any point u, $u\,r$ and $u\,r_{,}$. Now from any point (say $r_{,}$) on $r\,r_{,}$ draw $r_{,}\,t$ parallel to $u\,r$, cutting the line of action of T at t. Also draw $r_{,}t_{,}$ parallel to $u\,r_{,}$ cutting the line of action $T_{,}$ at $t_{,}$. Join $t\,t_{,}$, and draw $u\,s$ parallel to $t\,t_{,}$, cutting $r\,r_{,}$ at s.

If the resultant of T and $T_{,}$ is to be equal to R, T must be equal to $s\,r$ and $T_{,}$

must be equal to $s\,r_{,}$. Now it is clear that the position of T and $T_{,}$ is not altered by revolving the line $E\,C_{,}$ about $C_{,}$ to take up some position $F\,C_{,}$, since all the ordinates are altered in the same proportion. Similarly their positions are not altered by subsequently revolving $F\,E_{,}$ about F to take up some position $F\,F_{,}$. We may therefore, in this manner, alter the magnitudes of T and $T_{,}$ at will.

If T or the sum of the *half*-ordinates between $E\,C_{,}$ and $C\,C_{,}$ measured to the same scale of distance as was adopted for the ordinates $1_{,}$, $2_{,}$, $3_{,}$, etc., is not equal to $s\ r_{,}$, we must alter the position of E to F, so that $C\,E : C\,F :: T : rs$, or make $C\,F = \frac{C\,E \times r\,s}{T.}$

Similarly, if $T_{,}$ or the sum of the *half*-ordinates between $E\,C_{,}$ and $E\,E_{,}$ measured to the previous scale of distance, is not equal to $r_{,}\,s$, we must alter the position of $E_{,}$ to $F_{,}$, so that $C_{,}E_{,} : C_{,}F_{,} :: T_{,} : r_{,}s$ or make $C_{,}F_{,} = \frac{C_{,}E_{,} \times r_{,}s}{T_{,}.}$

Now $F\,F_{,}$ will be the true closing line for the equilibrium polygon (b), and may be tested to see if the sums of the ordinates from the curve $b_1, b_2, b_3 \cdots b_{21}$ above and below $F\,F_{,}$ are equal.

The closing line for the neutral surface curve (a) treated as an equilibrium polygon, due to some unknown form of loading, must now be determined.

Since the curve is symmetrical this line will be horizontal. If $k\,k$ is to be the closing line, we have the condition that the algebraical sum of the ordinates of the type ($k\,a$) must be *nil*, the ordinates being taken as positive when above $k\,k$ and negative when below. It is therefore necessary to place the horizontal line $k\,k$ at a distance from $A\,B$ equal to the mean length of the ordinates [of the type (y)] from the line $A\,B$ to the neutral surface curve (a).

If, then, we add the lengths of the ordinates of the type (y) on the force lines P_1, P_2, P_3, etc., and divide by their number, we find the distance [say (e)] from $A\,B$ at which the horizontal closing line $k\,k$ is to be drawn.

Now if we imagine the closing line $F\,F_{,}$ to be placed so as to coincide with $k\,k_{,}$, we must have the condition (from equation [18] page 385, and since the ordinates of the type ($a\,b$) must be proportional to the bending moments at the various sections) that the summation of the products of the ordinates of the type ($a\,b$) and those of the type (y) must be *nil*.

This is the same as saying that—

$$\Sigma\,[(ka) \times y] \text{ should equal } \Sigma\,[(Fb) \times y] \quad [1].$$

Now, $\Sigma\,[(ka) \times y]$ may be written $\Sigma\,(y-e)\,y$ or $\Sigma\,y^2 - e\,\Sigma\,y$. And since the neutral surface curve is symmetrical these sums need only be found for half the arch and the total multiplied by two for the whole arch.

$\Sigma\,[(F\,b) \times y]$ must be found for the whole of the curve $b_1, b_2, b_3 \ldots b_{21}$, but the working can be simplified by working in the following manner : $\Sigma\,[(F\,b) \times y] = [(F\,b)_{10} + (F\,b)_{11}]\,y_{10} + [(F\,b)_9 + (F\,b)_{12}]\,y_9 + [(F\,b)_8 + (F\,b)_{13}]\,y_8 + \ldots + [(Fb)_5 + (F\,b)_{16}]\,y_5 - [(F\,b)_4 + (F\,b)_{17}]\,y_4 - \ldots - [(F\,b)_1 + (F\,b)_{20}]\,y_1$.

If the equality—

$$\Sigma\,y^2 - e\,\Sigma\,y = \Sigma\,[(F\,b)\,y] \quad [2]$$

does not hold, we must alter all the ordinates of the type ($F\,b$) in the ratio of $\frac{\Sigma\,y^2 - e\,\Sigma\,y}{\Sigma\,[(F\,b)\ y]}$, or to locate the true pressure curve, $c_1\ c_2, c_3, \ldots {}_{21}$, we must lay off from $k\ k$ ordinates $(k\ c)$, for which the general equation is—

$$(kc) = Fb \times \frac{\Sigma y^2 - e\Sigma y}{\Sigma [(F b)\ y]} \quad [3].$$

To obtain the diagram of forces with the true pole distance or horizontal thrust we must draw from $O_{\prime}$ a line parallel to the closing line $F\ F_{\prime\prime}$ cutting the force line 1,21 at H, and from H draw a horizontal line $H O$, making $HO = HO_{\prime} \times \frac{\Sigma [(Fb)\ y}{\Sigma y^2 - e\Sigma y}$ the true horizontal thrust to the scale of loads. By joining $O{\cdot}1$, $O{\cdot}2$, $O{\cdot}3$, etc., we obtain the true diagram of forces. The reactions at the springings may be drawn in on the pressure curve (c) parallel to the lines $O{\cdot}1$ and $O{\cdot}21$ of the force diagram. The pressure curve c may be drawn in from the diagram of forces by finding the ordinate $(k\ c)$ at the crown, and so finding a point on the pressure curve the ordinate

$$(kc)_C = Fb \times \frac{\Sigma y^2 - e\Sigma y}{\Sigma [(Fb)_C y]} \quad [4].$$

Professor Cain's Method.—Professor Cain employs a slightly different method for locating the true pressure curve dividing the *arc* of the neutral surface curve into parts and dropping verticals from the *centre* of each division. The dead and live loads on the arch are then found *between* those verticals, with the exception of the crown, where two loads are found, each midway between the vertical and the centre of the span, and at the springings where the load is outside the end verticals.

These loads (the P's) may be supposed to act at the mid points between the verticals when the neutral surface curve is divided into *equal* parts.

From the centres of action of the *(P's)* further verticals are dropped for the purpose of drawing in the trial curve of equilibrium, which is plotted from the diagram of forces obtained from the *(P's)*.

After the trial curve of equilibrium has been drawn using the (P) verticals, the ordinates employed for finding the true closing lines for the trial curve of equilibrium and for locating the true pressure curve on the arch ring are those on the verticals from the mid points of the original divisions of the neutral surface curve.

The (P) verticals are shown dotted in Fig. 362, and the verticals used for the ordinates are shown in full lines.

The true horizontal thrust or pole distance is found as already described, and gives the bending moments directly.

The thrusts at each point on the pressure curve are found from the direction and magnitude of their ray on the diagram of forces, drawn with the true pole distance, which passes through the respective points on the pressure curve "c."

Professor Cain points out that by the use of this method true points on the pressure curve are found, whereas those got by using the load verticals as ordinates are always the points furthest removed from the curve obtained by dividing the arc into infinitesimal divisions.

For arches with a small rise of from $\frac{1}{8}$ to $\frac{1}{10}$ the span (such as those generally adopted when reinforced concrete is used), the error resulting from the division of the *span* into equal parts in place of the *arc*, and taking the vertical load lines at the centre points of these divisions and using these lines as the ordinates for locating the pressure curve, is very small.

When, however, the ratio of the rise to the span is greater, the neutral surface curve must be divided into equal parts, as the pressure curve will be more abrupt in its changes of inclination. The method described above should therefore be employed. In this case, since the horizontal distances apart of the ordinates are not all the same, in consequence of the division of the *arc* in place of the *span*, the lines of action of the "*T*" resultants in the treatment for a continuous arch must be found in the same manner as the resultant "*R*."

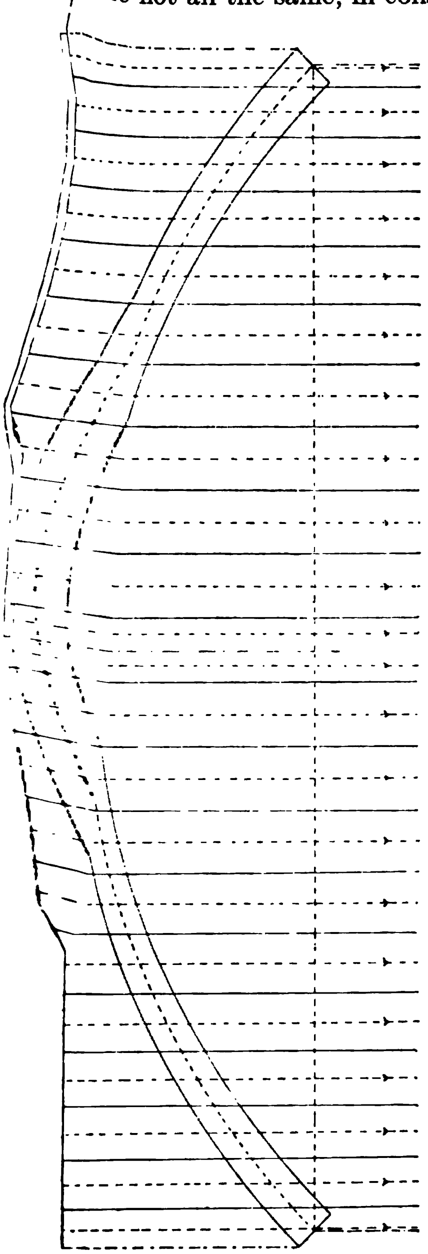


Fig. 362

All the other cases are treated in exactly the same manner as already described, except that the ordinates are taken on the full lines (Fig. 362) dropped from the mid points of the arc divisions instead of on the load lines.

Temperature Stresses—It is advisable to check the structure as designed to see if it has sufficient resistance to withstand the stresses induced by changes of temperature. A change in temperature above or below the normal or the temperature at which the arch was finished produces a virtual change of span.

If we suppose the greatest deviations above and below the mean temperature as $+t°$ and $-t°$, a fall in temperature tends to shorten the span and a rise to lengthen it, but a change of span is resisted by the abutments, except in the case of a three-hinged arch, where there are no stresses due to temperature. It will therefore be seen that a rise of temperature will cause compressive strains and a fall tensile strains.

The stresses due to a rise or fall of temperature always act along the closing line of the neutral surface of the arch considered as a force polygon. It will not alter the effect if we replace this force (say H_t) by a force in the same direction and magnitude acting at the springings or the extremities of the neutral surface curve and a couple formed of forces of the same magnitude and direction with a lever arm equal to the distance of the closing line from the line joining the extremities of the neutral surface curve, or (*e*) page 395. If we denote the span of the neutral surface curve by L, the expansion and the contraction due to a change of one degree of temperature by $\epsilon = 0{\cdot}000006$ (page 210) we get $L\,\epsilon\,t°$ as the change in length of the span. As the arch may be considered symmetrical we need only consider one-half, as the stresses in the other half due to change of temperature will be exactly the same.

We have now the condition that—

$$\frac{L \epsilon t^\circ}{2} = \frac{H_t}{E_c[I_c + mI_f]} \Sigma (ka) \times y.$$

$$\text{Therefore, } L \epsilon t^\circ = \frac{2 H_t}{E_c[I_c + m I_f]} \Sigma\ (ka)\, y. \quad [1].$$

Where I_c and I_f are the respective moments of inertia of the concrete and reinforcement, E_c = co-efficient of elasticity of the concrete, and $m = \frac{E_f}{E_c} = 10$. E_c may be taken as $2 \cdot 8 \times 10^6$ pounds per square inch.

From equation [1] we get—

$$H_t = \frac{E_c[I_c + m\ I_f] \times L \epsilon t^\circ}{2 \Sigma\ (ka)\, y} \quad [2].$$

The ordinates of the type (ka) for a continuous arch $=(e-y)$.

We know the values of $E_c\ [I_c + m\ \ I_f]$ and of $L \epsilon t^\circ$. The summation $\Sigma\ [(ka) \times y] = \Sigma\ y^2\ - e\ \Sigma\ y$. We have only to find the values for the ordinates of the type y to find the value of $\Sigma\ y^2 - e\ \Sigma\ y$. It is well to divide the half span into an equal number of as many equal parts as can be conveniently done for temperature stresses, say, 20 to 30.

Having found H_t we can find the moment produced by the temperature changes at any point along the arch ring and add this to the moment found previously, checking by the formulæ for direct stress and bending combined, allowing the usual resistances for the materials. The moments due to temperature have a different sign as the line $k\ k$ is above or below the neutral surface curve.

When the arch is hinged at the springings or crown only, when hinged at the springings H_t acts along the line joining the hinges and when hinged at the crown it acts along the line parallel to the line joining the ends of the neutral surface curve and passing through the hinge at the crown, and the value of H_t and its moments being altered accordingly.

In the case of the arch hinged at the springings the summation in the denominator of equation [2] becomes therefore $\Sigma\ (y^2)$; and in the case of an arch hinged at the crown, the summation becomes Σe (e in this case being y_{max}.), or $y_{max} \times$ the number of parts taken on the span line.

CALCULATIONS

CALCULATIONS FOR PIECES SUBJECTED TO BENDING AND DIRECT STRESSES COMBINED.

General Remarks.—Pieces subjected to stresses of this nature are usually arches, although other pieces may be also stressed in the same way, as, for instance, columns under eccentric loading and compression pieces under the effects of wind pressure.

There are two cases which must be considered—

1. When only one kind of stresses are produced, as in the case of a column with a load bearing slightly out of the centre, or an arch in which the line of pressures lies well within the arch ring.

2. When the piece is stressed in both tension and compression.

The usual cases met with in practice are those in which the main stresses are compressive, and in the following such a disposition will be assumed.

When only Compressive Stresses are produced and the Reinforcement is of small Sectional area and Depth and at both sides of the piece. In this case we have a thrust T acting at the neutral surface and a bending moment M. Under the effect of these the section $A\ B$ (Fig. 363) may be supposed to take up position $A_2\ B_3$, while $A\ A_1$ and $B\ B_1$ represent the minimum and maximum compressive stresses in the concrete c_1 and c_2.

As seen from Fig. 363—

$$\text{The thrust } T=(db)\left\{c_1+\tfrac{2}{3}(c_2-c_1)\right\}+f_b\,\omega_b+f_u\,\omega_u$$

$$T=(db)\,\tfrac{2}{3}(c_2+c_1)+f_b\,\omega_b+f_u\,\omega_u \quad [1].$$

$$\text{The bending moment } M=\tfrac{2}{3}(c_2-c_1)\,d\times\left(\tfrac{1}{2}-\tfrac{3}{8}\right)d\times b+f_u\,\omega_u\,(\nu\,u)+f_b\,\omega_b\,(\nu\,b),$$

$$\text{or } M=\tfrac{1}{12}(c_2-c_1)\,d^2\,b+f_u\,\omega_u\,(\nu\,u)-f_b\,\omega_b\,(\nu\,b) \quad [2].$$

To find the values of f_u and f_b, it will be seen from Fig. 363 that the hypothesis of the conservation of plane sections gives us—

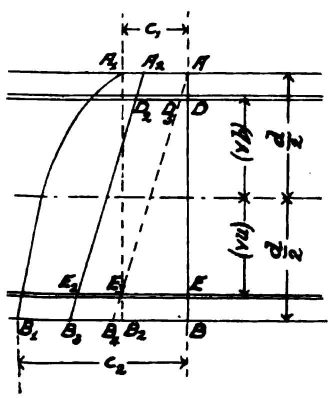


FIG. 363

$$\text{Since } DD_3=\frac{BB_4\times AD}{AB},$$

$$DD_2=AA_2+\frac{BB_4\times AD}{AB},$$

$$\text{but } DD_2:AA_2:BB_4=\frac{f_b}{E_f}:\frac{c_1}{E_c}:\frac{c_2-c_1}{E_c},$$

$$\text{also } AD=\frac{d}{2}-(\nu b) \text{ and } AB=d,$$

we have then—

$$f_b=m\left\{\frac{(c_2+c_1)}{2}-(c_2-c_1)\frac{(\nu b)}{d}\right\} \quad [3],$$

$$\text{and similarly } f_u=m\left\{\frac{c_2+c_1}{2}+(c_2-c_1)\frac{(\nu u)}{d}\right\} \quad [4].$$

Substituting [3] and [4] in [1] and [2] we get—

$$T=\frac{c_2+c_1}{6}\left\{4\,db+3\,m\,(\omega_b+\omega_u)\right\}+(c_2-c_1)\frac{m}{d}\left\{\omega_u\,(\nu u)-\omega_b(\nu b)\right\} \quad [5],$$

$$\text{and } M=(c_2-c_1)\left[\frac{1}{12}d^2b+\frac{m}{d}\left\{\omega_u\,(\nu u)^2+\omega_b\,(\nu b)^2\right\}\right]+\frac{(c_2+c_1)}{2}\,m\left\{\omega_u\,(\nu u)-\omega_b\,(\nu b)\right\} \quad [6].$$

If the Reinforcement is symmetrical,[1] $\omega_u = \omega_b$ and $(\nu u) = (\nu b)$. We get then—

$$T = \frac{c_2 + c_1}{3}\left\{2db + 3m\omega\right\} \quad [7],$$

$$\text{and } M = (c_2 - c_1)\left\{\frac{1}{12}d^2 b + \frac{2m}{d}\omega\nu^2\right\} \quad [8],$$

when $c_1 = 0$, i.e. at the limit when the whole piece is in compression—

$$\frac{M}{T} = \frac{1}{4d}\cdot\frac{d^3 b + 24m\omega\nu^2}{2db + 3m\omega} \quad [9],$$

with an unsymmetrical reinforcement—

$$\text{The limit of } \frac{M}{T} = \frac{1}{2}\cdot\frac{4d^3 b + 12m\left\{\omega_u(\nu u)^2 + \omega_b(\nu b)^2\right\} + 6md\left\{\omega_u(\nu u) - \omega_b(\nu b)\right\}}{4d^2 b + 3md(\omega_b + \omega_u) + 6m\left\{\omega_u(\nu u) - \omega_b(\nu b)\right\}} \quad [10].$$

When Tensile Stresses are Induced, the Reinforcement being of small Sectional Area and Depth, and at both sides of the piece.

When this disposition occurs, we shall have (Fig. 364)—

$$T = \frac{2}{3}cub + \omega_u f_u - \omega_b f_b \quad [11],$$

$$M = \frac{2}{3}cub\left(\frac{d}{2} - \frac{3}{8}u\right) + f_u\omega_u(\nu u) + f_b\omega_b(\nu b) \quad [12],$$

and, as in the case of piece subjected to simple bending—

$$f_u = cm\frac{(hu)}{u} \quad [13],$$

$$f_b = cm\frac{(hb)}{u} \quad [14].$$

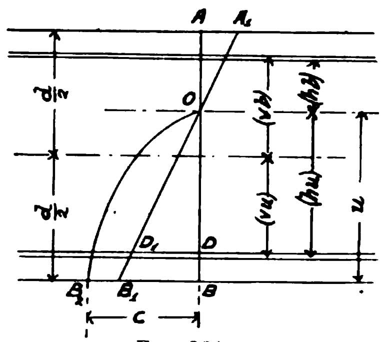


FIG. 364

Substituting [13] and [14] in [11] and [12]—

$$T = \frac{c}{u}\left[\frac{2}{3}u^2 b + m\left\{\omega_u(hu) - \omega_b(hb)\right\}\right] \quad [15],$$

$$M = \frac{c}{u}\left[\frac{2}{3}u^2 b\left(\frac{d}{2} - \frac{3}{8}u\right) + m\left\{\omega_u(hu)(\nu u) + \omega_b(hb)(\nu b)\right\}\right] \quad [16],$$

with a symmetrical reinforcement—

$$\omega_u = \omega_b \text{ and } (\nu u) = (\nu b).$$

We get therefore—

$$T = \frac{c}{u}\left[\frac{2}{3}u^2 b + m\omega\left\{(hu) - (hb)\right\}\right] \quad [17].$$

And as $(hb) + (hu) = 2\nu$—

$$M = \frac{c}{u}\left\{\frac{2}{3}u^2 b\left(\frac{d}{2} - \frac{3}{8}u\right) + 2m\omega\nu^2\right\} \quad [18].$$

When the Reinforcement is only on One Side.

If the curve of pressures remains always on one side of the neutral surface of an arch, one reinforcement may be sufficient.

[1] This is the usual disposition adopted for pieces of th's kind.

We have then for the case where the whole arch is in compression, by retaining the reinforcement under the greatest compression and eliminating that which tends towards tension—

$$T = \frac{(c_2+c_1)}{6}\left(4+3\,m\,\omega_u\right) + (c_2-c_1)\,\frac{m}{d}\,\omega_u\,(\nu u) \quad [19],$$

$$M = (c_2-c_1)\left\{\frac{1}{12}\,d^2b + \frac{m}{d}\,\omega_u\,(\nu u)^2\right\} + \frac{c_2+c_1}{2}\,m\,\omega_u\,(\nu u) \quad [20],$$

$$\text{and } f = m\left\{\frac{(c_2+c_1)}{2} + (c_2-c_1)\,\frac{(\nu u)}{d}\right\} \quad [21].$$

And for the case when part of the arch is in tension, retaining the reinforcement on this side—

$$T = \frac{c}{u}\left\{\frac{2}{3}\,u^2b - \omega_b\,(\nu\,b)\right\} \quad [22],$$

$$M = \frac{c}{u}\left\{\frac{2}{3}\,u^2b\left(\frac{d}{2}-\frac{3}{8}\,u\right) + m\,\omega_b(hb)\,(\nu b)\right\} \quad [23],$$

$$\text{and } f = cm\,\frac{(hb)}{u} \quad [24].$$

In all these formulae the curve of pressures is supposed to be passing below the neutral surface of the piece as shown in the figures, but the same formulae will of course apply if it passes above, the piece being considered as reversed.

Use of the above formulae. When we require to use these formulae to check a piece of given dimensions we proceed in the same manner as for pieces under simple bending. *When the reinforcement is symmetrical,*

Taking $m = 10 =_T \frac{T}{bd}\,\mu = \frac{M}{bd^2}\,\psi = \frac{2\omega}{bd}$, 2ω being in this case *the whole area* of the two reinforcements, we have also

$$\nu = \frac{d}{2} - \frac{d}{10} = 0{\cdot}4d \text{ and } \gamma = \frac{u}{d}.$$

We first try whether the piece is subject to tension by using the formula [9], which, substituting the above values, becomes—

$$\frac{M}{T} = \frac{d}{4}\cdot\frac{1+19{\cdot}2\,\psi}{2+15\,\psi} \quad [25].$$

If $\frac{M}{T}$ is equal to or less than $\frac{d}{4}\cdot\frac{1+19{\cdot}2\,\psi}{2+15\,\psi}$ we know that there will be no tensile stresses, and we proceed to find the values of c and f under these conditions.

We have from [7] and [8]—

$$c_2 + c_1 = \frac{3\,T}{2\,db + 3\,m\,\omega},$$

$$\text{and } c_2 - c_1 = \frac{M}{\frac{1}{12}\,d^2b + \frac{2m}{d}\,\omega\,\nu^2}.$$

Inserting the values given above, we get—

$$c_2 + c_1 = \frac{3\,\tau}{2 + 15\,\psi},$$

$$\text{and } c_2 - c_1 = \frac{12\,\mu}{1 + 19{\cdot}2\,\psi}.$$

From which we get—

$$c_{\text{max.}} = \frac{1}{2}\left\{\frac{3\,\tau}{2 + 15\,\psi} + \frac{12\,\mu}{1 + 19{\cdot}2\,\psi}\right\} \quad [26],$$

$$\text{and } c_{\text{min.}} = \frac{1}{2}\left\{\frac{3\,\tau}{2 + 15\,\psi} - \frac{12\,\mu}{1 + 19{\cdot}2\,\psi}\right\} \quad [27].$$

By inserting these values of $c_{\max}$ and $c_{\min}$ in [3] and [4], and as the reinforcement is symmetrical, replacing $(\nu\,b)$ and $(\nu\,u)$ by ν, we find the values of f_b and f_u.

If we find from equation [25] that tensile stresses will be produced, we must find the value of u $(hb) = \frac{d}{10}(9 - 10\,\gamma)$ and $(hu) = \frac{d}{10}(10\,\gamma - 1)$.

Here $(hb) - (hu) = d\,(2\,\gamma - 1)$, and we have from equations [17] and [18]—

$$T = \frac{c}{u}\left[\frac{2}{3}\,u^2 b + m\,\omega\left\{(hu) - (hb)\right\}\right],$$

$$\text{and } M = \frac{c}{u}\left[\frac{2}{3}\,u^2 b\left(\frac{d}{2} - \frac{3}{8}\,u\right) + 2\,m\,\omega\,\nu^2\right].$$

Inserting the several values, these equations reduce to—

$$c = \frac{3\,\tau\gamma}{2\,\gamma^2 + 15\psi(2\,\gamma - 28)} \quad [28],$$

$$c = \frac{12\,\gamma\mu}{\gamma^2\,(4 - 3\,\gamma) + 3\,\psi} \quad [29].$$

These two values of c must equal one another. We therefore obtain—

$$\gamma^3 - \frac{4}{3}\,\gamma^2\left(1 - \frac{2\,\mu}{\tau}\right) + 40\,\gamma\,\frac{\psi\mu}{\tau} = \psi\left(1 + 20\,\frac{\mu}{\tau}\right) \quad [30],$$

from which the value of γ may be found by transposing the equation so that both sides contain γ and plot, on squared paper, values for each side for several values of γ where the curves intersect gives the proper value of γ.

Having found the value of γ, that of c is deduced from [28] or [29], and from [13] and [14] (page 400) we have—

$$f_b = cm\,\frac{(hb)}{u},$$

$$\text{and } f_u = cm\,\frac{(hu)}{u},$$

which, inserting the values above, reduce to—

$$f_b = c\,\frac{(9 - 10\,\gamma)}{\gamma} \quad [31],$$

$$\text{and } f_u = c\frac{(10\gamma - 1)}{\gamma} \quad [32].$$

The value of $c_{max.}$ should not be greater than 400 pounds per square inch.

If it is required to determine the dimensions for a piece having only the values of M and T, we must at first suppose it to be entirely in compression and assume the limiting compression in the concrete ($c_{max.} = c_2$). We have then, always supposing a symmetrical reinforcement, the values of M and T, the width (b) (generally taken as 12 inches) the value of $\nu = 0{\cdot}4d$ and $m = 10$, and we wish to determine (d) and (ω).

For determining these it is necessary to further assume a value of $\psi = \frac{2\omega}{bd}$, as otherwise we have not sufficient data. It is also possible to assume a depth (d); this is frequently done when arches are under consideration, in which case the calculation for (ω) is a simple matter, but the percentage of reinforcement is the usual value assumed for other pieces.

We have then from equations [7] and [8] (p. 400),

$$c_{max.} = \frac{1}{2}\left\{\frac{3T}{2db + 3m\omega} + \frac{M}{\frac{1}{12}d^2b + \frac{2m}{d}\omega\nu^2}\right\},$$

which becomes, by inserting the values which we have assumed—

$$c_{max.} = \frac{1}{2bd^2}\left\{\frac{3Td}{2 + 15\psi} + \frac{12M}{(1 + 19{\cdot}2\psi)}\right\},$$

or

$$d^2 - \frac{d}{2bc}\left\{\frac{3T}{2 + 15\psi}\right\} = \frac{1}{2bc}\left\{\frac{12M}{1 + 19{\cdot}2\psi}\right\}$$

$$d = \frac{3T}{4bc(2 + 15\psi)} + \sqrt{\frac{9T^2}{16b^2c^2(2 + 15\psi)_2} + \frac{\cdot 12M}{2bc(1 + 19{\cdot}2\psi)}},$$

$$d = \frac{3}{4bc}\left[\frac{T}{2 + 15\psi} + \sqrt{\frac{T^2}{(2 + 15\psi)^2} + \frac{32bc}{3} \times \frac{M}{(1 + 19{\cdot}2\psi)}}\right] \quad [33].$$

which gives (d). We have also

$$\omega = \frac{\psi bd}{2} \quad [34].$$

From equation [25] (page 401) the value of (d) must be such that

$$\frac{M}{T} \text{ is equal to or less than } \frac{d}{4} \cdot \frac{1 + 19{\cdot}2\psi}{2 + 15\psi}.$$

If this relation is not satisfied at first, we may reduce the percentage (ψ) which increases the depth (d), but, if it still remains unsatisfied, we must consider the case where tensile stresses exist.

On this supposition we may give (c) its maximum value (400 when ordinary mixtures are used) and (f_b) a value of $10c$. We can find (u) from the equation—

$$f_b = cm\frac{(hb)}{u},$$

where

$$(hb) = \frac{9d - 10u}{10} \text{ and } m = 10,$$

which gives

$$u = \frac{9d}{10 + \frac{f_b}{c}} \quad [35].$$

But in this case since the value of (f_b) will be equal to $10c$, consequently (u) must be equal to $\frac{9}{20}d$.

We must now find the value of (u) for different values of (d).

We also get from equations [17] and [18] (page 400)—

$$T = \frac{c}{u}\left[\frac{2}{3}u^2b + m\,\omega\,\{(hu) - (hb)\}\right],$$

and

$$M = \frac{c}{u}\left[\frac{2}{3}u^2b\left(\frac{d}{2} - \frac{3}{8}u\right) + 2\,m\,\omega\,v^2\right],$$

and we have further $(hb) = \frac{9d - 10u}{10}$ and $(hu) = \frac{10u - d}{10}$,

which gives

$$\omega = u\frac{\left(\frac{2}{3}ub - \frac{T}{c}\right)}{10\,(d - 2u)} \quad [36],$$

and

$$\omega = u\frac{\left[\frac{M}{c} - \frac{2}{3}ub\left(\frac{d}{2} - \frac{3}{8}u\right)\right]}{3{\cdot}2d^2} \quad [37].$$

Equations [36] and [37] must give the same values for (ω), and the value of (d) must be altered until they are the same.

But as we have the relation $u = \frac{9}{20}d$, equation [36] shows that $\frac{T}{c}$ must be less than $\frac{2}{3}ub$.

Also equation [37] gives the relation—

$$\frac{M}{c} > \frac{2}{3}ub\left(\frac{d}{2} - \frac{3}{8}u\right).$$

$$\text{And as } \frac{T}{c} < \frac{2}{3}ub \text{ or } T < \frac{2}{3}cb \times u,$$

replacing (u) by its value from equation [35],

$$T \text{ must be less than } \frac{2}{3}cb \times \frac{9d}{10 + \frac{f_b}{c}} \quad [38],$$

and as $\frac{T}{c} < \frac{2}{3} ub$ and $\frac{M}{c} > \frac{2}{3} ub \left(\frac{d}{2} - \frac{3}{8}\right)$,

$$\frac{M}{T} \text{ must be greater than } \left(\frac{d}{2} - \frac{3}{8}u\right),$$

or replacing u by its value from equation [35]

$$\frac{M}{T} > \frac{d}{2} - \frac{3}{8} \cdot \frac{9d}{10 + \frac{f_b}{c}} \qquad [39].$$

If these conditions [38] and [39] cannot be realized it is impossible to obtain the maximum resistances from the materials, we can only assume a limiting stress for one of the two materials (usually of the reinforcement), and not attempt to obtain the maximum for the other. This being the case, a value for (d) is assumed, and different values of (u) are tried in equation [35], which may be written in the form—

$$c = \frac{uf_b}{9d - 10u} \qquad [40].$$

The various values for (c) are tried in equations [36] and [37] until the same value for (ω) is obtained.

Pieces with Reinforcements of Large Sectional Area.—When reinforcements of large sectional area are employed the reinforcements are always symmetrical. In this case, however, both the depth and the sectional area of the reinforcements must be taken into account, and the resistance of the concrete which is replaced by the reinforcements when in compression must be deducted. As before, (f_u) and (f_b) will be taken as the mean resistance of the reinforcing sections, and (f_{um}) and (f_{bm}) as their maximum resistance, (i) representing their moment of inertia about their centre of gravity.

When only Compressive Stresses are Produced (Fig. 365). We get—

$$T = \frac{2}{3}(c_1 + c_2)(db - 2\omega) + \omega(f_b + f_u) \qquad [1].$$

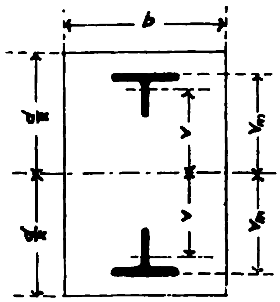


FIG. 365

Now, considering the stress on the portions of concrete replaced by the reinforcements, as the equation of a parabola is $y^2 = 4ax$, and in this case for the extreme fibre of the concrete under the greatest compression $x = (c_2 - c_1)$ and $y = d$, we have—

$$4a = \frac{d^2}{(c_2 - c_1)},$$

and the general equation becomes—

$$y^2 = \frac{d^2}{(c_2 - c_1)} x,$$

y at the centre of gravity of the reinforcement under greatest compression $=$ $\left(\frac{d - 2v}{2}\right)$, therefore $x = \frac{\left(\frac{d-2v}{2}\right)^2 \times (c_2 - c_1)}{d^2}$ = the stress on the area of concrete replaced by the reinforcement under greatest compression.

Similarly the stress on the area of concrete replaced by the reinforcement under least compression $= \dfrac{\left(\dfrac{d+2\nu}{2}\right)^2 \times (c_2-c_1)}{d^2}$.

We get therefore—

$$M=\frac{1}{12}(c_2-c_1)d^2b-\omega\left\{\frac{\left(\dfrac{d-2\nu}{2}\right)^3}{d^2}(c_2-c_1)+\frac{\left(\dfrac{d+2\nu}{2}\right)^3}{d^2}(c_2-c_1)\right\}$$

$$-(c_2-c_1)\frac{\dfrac{(d-2\nu)}{2}}{d^2}-(c_2-c_1)\frac{\dfrac{(d+2\nu)}{2}}{d^2}+\omega\nu(f_u-f_b)+\frac{i}{\nu}(f_u-f_b),$$

which reduces to—

$$M=(c_2-c_1)\left[\frac{1}{12}d^2b-\frac{1}{d^2}\left[\omega\left\{\left(\frac{d-2\nu}{2}\right)^3+\left(\frac{d+2\nu}{2}\right)^3\right\}\right]\right]+\omega\nu(f_u-f_b)+\frac{i}{\nu}(f_u-f_b) \quad [2].$$

And we have also—

$$f_b=m\left\{\frac{(c_2+c_1)}{2}-(c_2-c_1)\frac{\nu}{d}\right\} \quad [3],$$

$$f_u=m\left\{\frac{(c_2+c_1)}{2}+(c_2-c_1)\frac{\nu}{d}\right\} \quad [4],$$

$$f_{bm}=m\left\{\frac{(c_2+c_1)}{2}-(c_2-c_1)\frac{\nu_m}{d}\right\} \quad [5],$$

$$f_{um}=m\left\{\frac{(c_2+c_1)}{2}+(c_2-c_1)\frac{\nu_m}{d}\right\} \quad [6].$$

Replacing f_b and f_u in equations [1] and [2] by these values, we get—

$$T=(c_1+c_2)\left\{\frac{2}{3}db+\omega\left(m-\frac{4}{3}\right)\right\} \quad [7],$$

and—

$$M=(c_2-c_1)\left[\frac{1}{12}d^2b-\frac{1}{d^2}\left[\frac{\omega}{8}\left\{(d-2\nu)^3+(d+2\nu)^3\right\}+d\right]+2m\left\{\omega\frac{\nu^2}{d}+id\right\}\right] \quad [8].$$

When Tensile Stresses are Produced (Fig. 366)

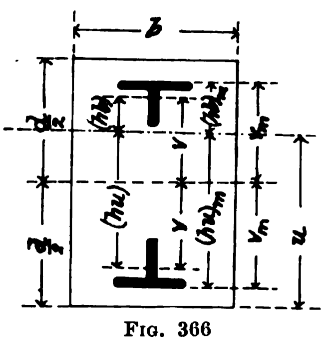


Fig. 366

The equations in this case become—

$$T=\frac{2}{3}c(ub-\omega)+\omega(f_u-f_b) \quad [9],$$

the stress at the centre of gravity of the compressive reinforcement –

$$x=\frac{(hu)^2}{u^2}c,$$

therefore—

$$M=\frac{2}{3}cub\left(\frac{d}{2}-\frac{3}{8}u\right)-\frac{(hu)^3}{u^2}c-\frac{(hu)i}{u^2}c+\omega\nu(f_b+f_u)+\frac{i}{u}(f_b+f_u) \quad [10].$$

And we have further—

$$f_b=cm\frac{(hb)}{u} \quad [11],$$

$$f_u=cm\frac{(hu)}{u} \quad [12],$$

$$f_{bm}=cm\frac{(hb)_m}{u} \quad [13],$$

$$f_{um}=cm\frac{(hu)_m}{u} \quad [14],$$

Substituting we get $T=\frac{c}{u}\left[\frac{2}{3}u(ub-\omega)+m\,\omega\left\{(hu)-(hb)\right\}\right]$ [15],

and $\mathrm{M}=\frac{c}{u}\left[\frac{2}{3}u^2b\left(\frac{d}{2}-\frac{3}{8}u\right)-\frac{1}{u}\left\{(hu)^3+(hu)i\right\}+m\left\{(hb)+(hu)\right\}\left\{\omega\nu+\frac{i}{u}\right\}\right]$ [16].

PROFESSOR MELAN'S SEMI-EMPIRICAL CALCULATIONS FOR ARCHES

It may be interesting to give the semi-empirical method adopted by Prof. Melan in the calculations for arches on his system.

The deformation of an elastic arch due to the bending moment M is proportional to $\frac{1}{EI}$, where E is the coefficient of elasticity and I the moment of inertia of the section.

Prof. Melan neglects the influence of the tangential stresses T, and assumes that the load which acts on a section of the arch is divided between the concrete and metal in the proportion of the expressions $E_c I_c$ and $E_f I_f$, where the c and f refer to the concrete and reinforcement respectively.

He therefore considers the ratio of the load on the concrete to that on the reinforcement as $\frac{E_c I_c}{E_f I_f}=a$, or replacing these by their values and taking $\frac{E_f}{E_c}$ as 10—

$$a=\frac{bd^3}{120\,I_f}.$$

The concrete then supports a portion of the load equal to $\beta=\frac{a}{1+a}$, and the reinforcement a portion $\beta_1=\frac{1}{1+a}$. If T and M are taken for a unit width of the arch—

$$c=\beta\left(\frac{T}{d}+\frac{6\,M}{d^2}\right),$$

$$\text{and } f=\beta_1\left(\frac{T}{\omega}+\frac{M}{S}\right)\,b,$$

where S is the modulus of section of the reinforcement.

To find the values of T and M—if L is the span, v the rise, p the dead load, and ω the live load per unit of surface. Professor Melan calculates as follows, under a uniformly distributed load.

The total load causes a thrust at the crown—

$$T = \frac{1}{8}\frac{(p+w)L^2}{v},$$

and the moments are at the crown $M_C = \frac{1}{3}T(v_1 - v)$,

and at the springings $M_A = -\frac{2}{3}T(v_1 - v)$,

where
$$v_1 = v + \frac{45}{4}\frac{E_c\frac{bd^3}{12} + E_f I_f}{(E_c bd + E_f \omega)v} = v + \frac{15}{16}\cdot\frac{bd^3 + 120 I_f}{(bd + 10\,\omega)v}.$$

If the load only extends over half the span, the thrust at the crown

$$T_1 = \frac{1}{16}(2p + w)\frac{L^2}{v}.$$

If the haunches have sufficient rigidity, Prof. Melan considers the dangerous sections are at the springing of the loaded side and at a point in the unloaded half of the arch situated about $\frac{3}{16}L$ from the crown.

For the last section the bending moment will be—

$$M_S = -\left[\frac{9}{1024}wL^2 + \frac{2}{3}T_1(v_1 - v)\right],$$

and at the springing—

$$M_A = -\left[\frac{1}{64}wL^2 + \frac{2}{3}T_1(v_1 - v)\right].$$

Further, on account of the relatively great spacing of the rolled joists or girders of the Melan system, it is necessary to also consider the bending of the arch transversely between the joists.

Professor Melan supposes that, in this direction, the arch forms a slab of a span equal to the distance between the joists for distributing on them their portion of the total load. He calculates the slab of plain concrete as a piece of ordinary section built in at the supports, and limiting the stress as 460 pounds per square inch. This condition gives him a relation between the thickness of the arch and the spacing of the joists.

SPHERICAL AND CONICAL COVERINGS

The graphical method of treatment for obtaining the stress of domes of thin shells, given below, is very simple and direct, and is based on a paper by Mr. W. Dunn in the March number of the "Transactions of the Royal Institute of British Architects."

A, Fig. 367, (p. 410) shows the half-section of the dome of uniform thickness and of uniform weight per square foot, which is hemispherical and of a material capable of resisting tensile and compressive stresses.

Assume the centre line as representing to some scale yet to be determined the weight of the dome, or of some section of it. Divide this line into any number of convenient parts (for convenience, sixteen equal parts have been taken), and mark the divisions $1_{\prime}$, $2_{\prime}$, $3_{\prime}$. Draw through $1_{\prime}$, $2_{\prime}$, $3_{\prime}$, etc., horizontal lines to cut the section of the dome in 1, 2, 3, etc.

Then as the area of any segment of a sphere equals $c\ h$, where c is the circumference of the sphere and h is the height of the segment, any length such as $\overline{2_{\prime}3_{\prime}}$, or $\overline{5_{\prime}6_{\prime}}$ on the centre vertical, measured on the same scale upon which that vertical represents the whole weight of the dome or the chosen portion of it, will give the weight of the segment, $\overline{2,3}$, or $\overline{5,6}$ of the dome or chosen portion.

As the thickness of the dome is inconsiderable, the pressure may be considered as uniformly distributed over any section such as 1, and therefore tangential to the surface. Draw through 1 a line $\overline{1,a}$ tangential to the surface (at right angles to the radius $\overline{1_{\prime},16_{\prime}}$), and through o' produce ao' indefinitely. Through o' and $1_{\prime}$ draw $\overline{o't}$ and $\overline{1_{\prime}t}$ to intersect in t^1, we have the triangle of forces holding that part $\overline{o'1}$ of the dome in equilibrium; $\overline{o'1_{\prime}}$ is the weight of it (being the load on point 1) the line $1_{\prime}t$ the direct compression uniformly distributed over the horizontal section, and the line $\overline{o't^1}$ the radial thrust, all measured to the same scale (not yet determined) as $o'16_{\prime}$.

If the point 1 were taken very near the crown, the load would be very small, and the horizontal thrust and direct stress would also be reduced; at the crown itself there is no stress. This is one of the material points of difference between the dome and the arch, which latter construction has always a thrust at the crown.

Proceeding to section 2, we form the stress diagram giving the four-sided figure $1_{\prime}$ $2_{\prime}$ t_2t_1; $\overline{1_{\prime}2_{\prime}}$ being the weight of the segment, $\overline{2_{\prime}t^2}$ the direct stress upon the lower section, $\overline{t^2t^1}$ giving the radial thrust. Proceeding similarly for the remainder of the figure, the direct thrusts cut the horizontal lines further and further from the load line until we reach 6, when the intersections begin to approach the load line again, making the polygon of forces for each section similar to the four-sided figure *abcd* (Fig. 367A).

The diagram of stresses may take another form (*B*, Fig. 367). Set off the load line as before, and through o'' draw lines parallel to those tangential to the surface of the dome at the various points to intersect with the horizontal lines. Through these intersections draw the curve from o'' to $16_{\prime\prime}$. Then we have the stress diagram but of form x, y, x, o'' following the vertical, then the horizontal, and then the inclined line; the radial thrust above the point where the curve turns to the load line again and the radial tensions below that point being given by the differences $\overline{x,y}$.

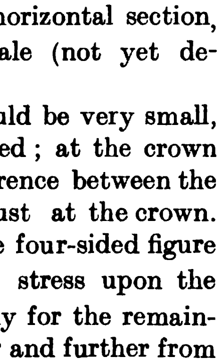


FIG. 367A.

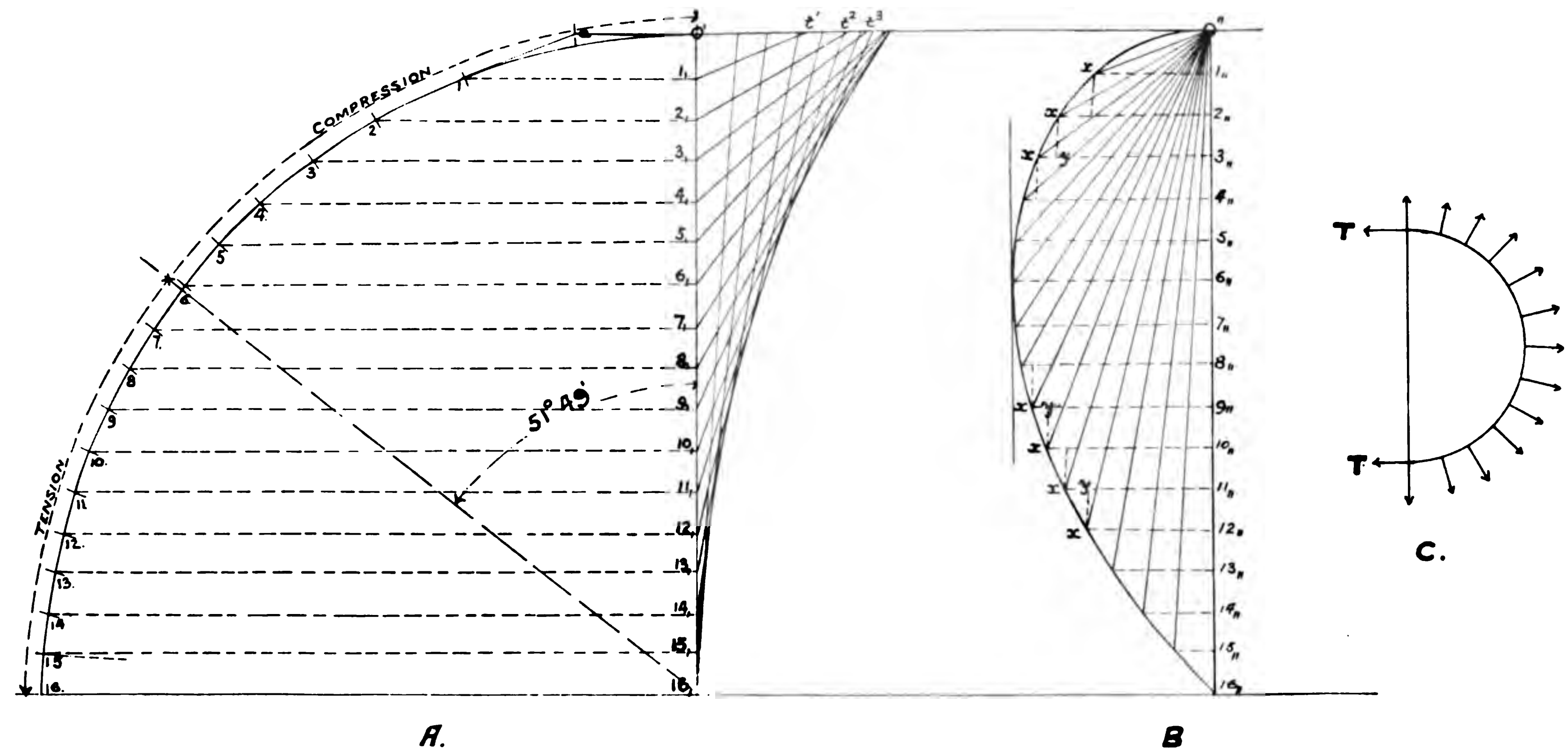


FIG. 307

These thrusts and tensions, or the $\overline{x,y's}$, must be resolved into equivalent thrusts or tensions acting at right angles to the plane of the paper—that is, actually upon the vertical faces of the section.

Let $\overline{o''16_{''}}$ represent the total weight of the dome, then any $\overline{x,y}$ shows the total radial thrust upon the corresponding section, which we shall call R. Being uniformly distributed its intensity per unit of circumference equals

$$\frac{\mathrm{R}}{\text{units in the circumference}},$$

just as the intensity of pressure on a column equals the total pressure divided by the units of area in the column.

In any circular ring under uniform normal pressure (C, Fig. 367), as in a cylinder holding water, the resultant tension or compression T (which we call hoop tension) equals the intensity of the radial pressure multiplied by the radius, that is—

$$\frac{R \times \text{radius}}{\text{circumference}} = \text{hoop tension} = T.$$

This $\frac{\text{radius}}{\text{circumference}}$ is a constant quantity for any circle, and equals $\frac{1}{6{\cdot}2832}$ nearly, so that $\frac{R}{6{\cdot}2832} = T$.

When, therefore, $\overline{o''16_{''}}$ represents the total weight of the dome, we must divide each horizontal $\overline{x,y}$ by 6·2832 for the hoop tension.

Suppose we take $\overline{o''16_{''}}$ to represent $\frac{1}{6{\cdot}2832}$ of the weight of the dome, then we shall not require to divide the $\overline{x,y's}$, as they will each equal the hoop tension at that point; i.e. if we take, not the weight of 360° but $\frac{360}{6{\cdot}2832}$, or 57·3° of the dome, the horizontal $\overline{x,y's}$ give the hoop tension or compression directly.

Form such a scale that $\overline{o''16_{''}}$ measures the weight of 57·3°; the lengths $\overline{xo''}$ measured to that scale give the total compression on a horizontal plane on a segment of 57·3° of the dome (this segment is in plan).

As the length of an arc of 57·3° equals its radius we have only to divide the lengths $\overline{x,o''}$ by the radius at the corresponding points to get the intensity of pressure on the horizontal section.

At the joint of no hoop thrust or hoop tension the maximum horizontal thrust is caused as is clearly seen in (B, Fig. 367). Consequently if the dome were to spring from this joint the provision to prevent the supports spreading would be the maximum obtainable for the dome under consideration. This joint is frequently called the joint of rupture; it is situated at a height above the springing line of $\frac{1}{2}(\sqrt{5}-1)$ radius, or about 51°·49′ from the vertical. Above that joint the dome tends to collapse inwards; below it tends to spread outwards.

This graphical treatment will apply to any special covering.

The weight of the covering will have to be estimated in the following manner. Above any horizontal section the weight of the dome will be $w \times 2\,\pi\,r\,v$, where r is the radius to which the dome is struck, v the rise, and w the weight per unit area.

This will give the total weight or the length of the line $\overline{o\cdot 16_{\prime}}$.

The weights of the various portions will be given as before by horizontal lines through the covering, dividing $\dot{o}\cdot 16_{\prime}$ into parts, these parts being proportional to portions of the covering.

The processes given for A and B, Fig. 367 will still apply, save only that the line similar to $o\cdot 16'$ will not be the radius to which the segment of the sphere is struck : we shall still have the value of the hoop tension given by $\frac{R \times \text{radius}}{\text{circumference}} = \frac{R}{6\cdot 2832}$.
In the method shown in B the curve will not close on to the line $\overline{o''16_{\prime\prime}}$ but will show a horizontal thrust at the springing which would have to be resisted by a metal ring (if we suppose a spherical covering springing from 10 (Fig. 367) $\overline{10_{\prime\prime}\ x}$ would give this thrust).

The same methods can be used for conical coverings, since it happens that the weights above any horizontal section are proportional to the vertical distances from the apex of the cone to that section.

In a cone the greatest horizontal thrust is at the base if the abutments do not yield, or if there is a reinforcement to tie in the base. If the abutments yield the cone itself, if capable of doing so, also supplies the necessary resistance to twisting open.

These results are only correct for *thin* coverings of material capable of resisting tension and compression, of true spherical or conical section, and of uniform weight per unit of surface. Such a material as reinforced concrete may be assumed to satisfy these conditions. If we put an eye to the dome (i.e. omit the central upper part, say the part above the line $\overline{1.1_{\prime}}$), then the horizontal line o', t^1, t^2, t^3, etc., is lowered to the level of $\overline{1.1_{\prime}}$, and the position of the joint of rupture is also lowered. If a heavy load, such as a lantern, were put at 1, then the horizontal line o', t^1, t^2, t^3, etc., would be raised, increasing the part under hoop tension and diminishing the part under hoop compression. If the section is varied and becomes pointed, or of any other curvature, there is also a change in the position of this joint. [1]

[1] M. Godard's formula for domes is given p. 97.

LIST OF SYMBOLS USED IN THE

W = concentrated load.
w = distributed load per lineal unit.
P = pressure or load.
p = pressure per square unit.
K = shearing force.
k = shearing stress per square unit.
c = maximum compressive stress per square unit on the concrete subjected to direct stress or simple bending.
c_1 = minimum compressive stress per square unit on the concrete in pieces subjected to direct stress and bending combined.
c_2 = maximum compressive stress per square unit on the concrete in pieces subjected to direct stress and bending combined.
t = maximum tensile stress per square unit on concrete.
f = stress per square unit on the metal.
f^a = stress per square unit on the compressive reinforcement.
f^b = stress per square unit on the tensile reinforcement.
M = bending moment = moment of resistance.
T = Tangential stress, i.e. thrust normal to the radius in arches, hoop tension or hoop compression.
H = Horizontal thrust in pieces subjected to direct stress and bending combined.
H_w = head of water.
R = Reaction at the springings of an arch.
R_B = " " " "
R_A = " " " "
Δ = sectional area of the whole piece.
ω = sectional area of the metal.
ω = sectional area of the compressive reinforcement.
ω_b = sectional area of the tensile reinforcement.
ω_u = sectional area of transverse shearing reinforcement.
d = total depth of a piece subjected to bending.
D = depth of the floor slab in T beams.
b = breadth of a rectangular piece subjected to bending or of the rib of a T beam.
B = breadth of the floor slab in a T beam.
R = radius.
δ = diameter.
χ_u = perimeter of compressive reinforcement.
χ_b = " tensile "
L = span.
l = any length.
v = rise of arch or dome.
u = distance of the neutral axis in any section from the surface subjected to the greatest compressive stress.
g = distance from the upper surface of the floor slab to the top of the rib in inverted T beams.
h = the maximum distance from the centre of gravity of any tensile reinforcement to that surface of any piece subjected to the greatest compressive stress.
(hu) = distance from the centre of gravity of the compressive reinforcement to the neutral axis.
(hb) = distance from the centre of gravity of the tensile reinforcement to the neutral axis.
$(hu)_m$ = distance from the extreme edge of the compressive reinforcement when it is of large sectional area, to the neutral axis.

$(hb)_m$ = di
me
ax
ν = distan
me
dir
(νu) = dist
act
tra
ing
(νb) = dist.
act
the
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$(\nu u)_m$ = dis
un
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cor
$(\nu b)_m$ = dis
un
neu
ber
sec
E_c = coeff
E_f = coeff
$m = \frac{E_f}{E_c}$
I = momen
i_u = mome
ow
i_b = mome
ow
a = distanc
sur
str
β = distanc
sur
pre
θ = angle
A = angle
ϕ = angle
$\gamma = \frac{u}{d}$.
$\mu = \frac{M}{bd^2}$
$\kappa = \frac{K}{bd}$
$\tau = \frac{T}{bd}$
$\psi = \frac{\Delta}{\omega}$ or
ϵ = elongat
$t°$ = degree
$\left.\begin{matrix}x\\y\end{matrix}\right\}$ = co-or
Σ = sign of
$\pi = 3{\cdot}1416.$

Part VII

SOME STRUCTURES WHICH HAVE BEEN ERECTED IN REINFORCED CONCRETE

I BUILDINGS

Foundation of Reinforced Concrete Piling

Fig. 368 shows the general arrangement of reinforced concrete piling employed to support the wall columns for the ten-storey steel-framed addition to the Hallenbeck Building at Park and Pearl Streets, New York.[1]

The clusters of piles are in two longitudinal rows, about 16 feet apart, centre to centre, and support a series of rolled joists, running transversely. The piles are 28 feet long and 12 inches

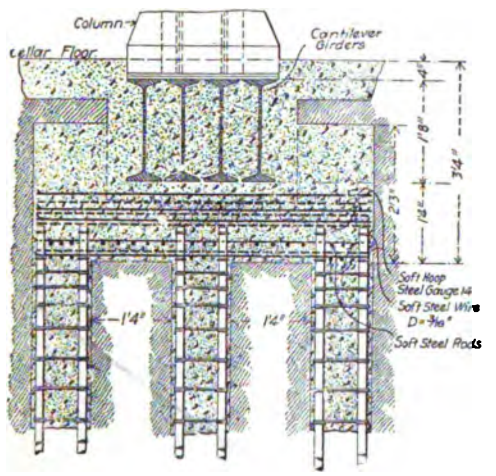


FIG. 368

square, made of Portland cement concrete, in the proportion of 1 : 2 : 4, the stone being ½-inch crushed trap rock.

The reinforcement consists of four $1\frac{7}{16}$-inch vertical mild steel rods, tied together on all sides by horizontal hooked wires, 5 inches apart, the rods being bent in at the bottom, and bearing on a cast-iron shoe. The concrete extends 2 inches above the top of the rods, but 5 inches is cut away after driving, to leave the rods projecting, so that they may penetrate into the

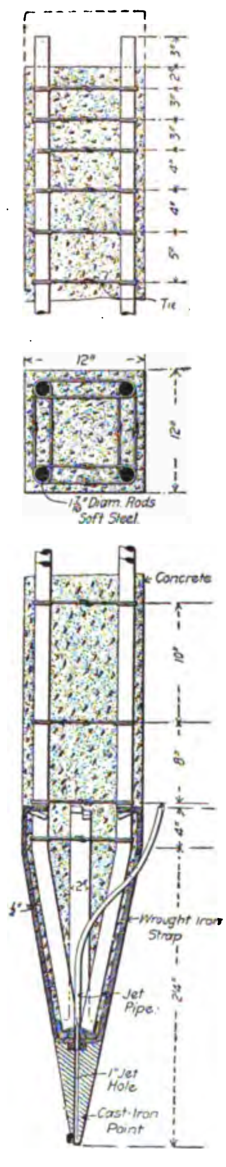


FIG. 369

[1] Described in the *Engineering Record*, April 11, 1903.

caping, which is 16 inches thick, reinforced by longitudinal and transverse rods, as shown.

Some of the piles were made with a central jet pipe, as shown in Fig. 369, which gives the details of the construction.

Melan Floors and Deckings

The floor on the *Melan system* for a spinning factory at Tetschen, Austria,[1] is shown in Fig. 370. The area of the floor is 40,000 square feet, and it is loaded

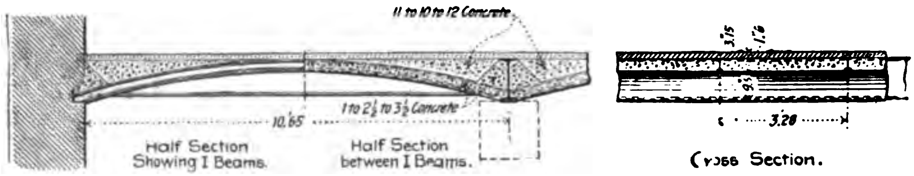


FIG. 370

with heavy machinery. The curved rolled joists forming the floor reinforcement are supported on longitudinal joists of larger section, and the whole is embedded in concrete. The arched joists are 3 inches deep, and are spaced 3·28 feet apart, centre to centre, their span being 10·65 feet, and the rise 9·5 inches. The depth of the concrete in the arch is 3·15 inches, and is mixed in the proportions of 1 to 2½ to 3½, and is carried over the longitudinal joists as shown. The filling above the arch proper is of 1 to 3 to 7 concrete, and not 11 to 10 to 12, as shown.

A gang of fourteen men laid from 700 to 800 square feet of floor in one ten-hour day. Four men were employed placing and fastening the joists and fixing the centres, one man for the cement, and one for the sand, four mixing the concrete, two carrying the concrete, and two ramming into place. The amount of concrete laid per man per hour was 2 cubic feet.

Fig. 371 shows the decking on the *Melan system* for a plate girder bridge at

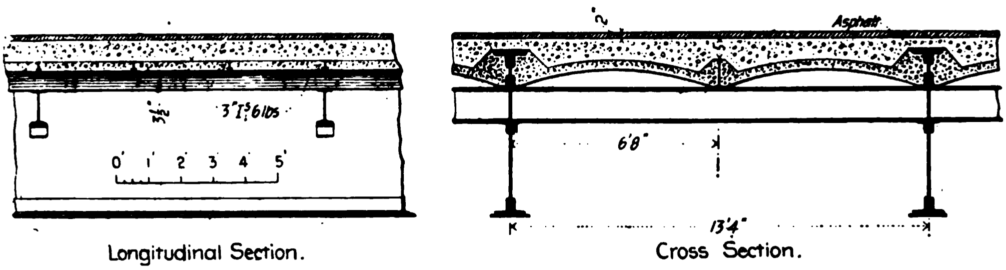


FIG 371.

Budapest, Hungary, to carry a wheel load of 6 tons. The span of the arches is 6 feet 8 inches, and the rise 6 inches. The concrete is 3½ inches thick, reinforced with 3-inch curved rolled joists, weighing 6 pounds per lineal foot, and spaced 3 feet centres.

The concrete cost 5½*d.* per square foot, and the steel work 2*d.*, the total cost being 7½*d.* per square foot.

A similar decking with 3⅛-inch joists, 40 inches apart, was loaded with 2,400 pounds per square foot without injury.

[1] Description in *Engineering News*, April 12, 1894.

SOME STRUCTURES ERECTED IN REINFORCED CONCRETE

Floor of Brewery at Budapest

The floors of a brewery at Budapest, Hungary,[1] on the *Wünsch system*, is shown in Fig. 372. The first floor has a load of 240 pounds, and the second floor 80 pounds per square foot. The surface of the floors is 174 feet by 52½ feet.

Two rows of iron columns support the floors, 14 feet apart longitudinally and 17 feet 7 inches transversely, the distance of the side rows from the walls at the bottom being 16 feet 2 inches.

The arches running longitudinally from column to column are 15¾ inches wide, 14 feet span, and 1·4 feet rise, and are reinforced by four 2·16 × 2·16 × 0·28 inch angle irons, two of which are curved to form the bottom chord, and two are straight, forming the top chord. The transverse arched floors are carried on these, being reinforced with 1 × 1 × 0·15 inch T-irons placed in a similar way, spaced 7·9 inches apart for the first and 13 inches for the second floor.

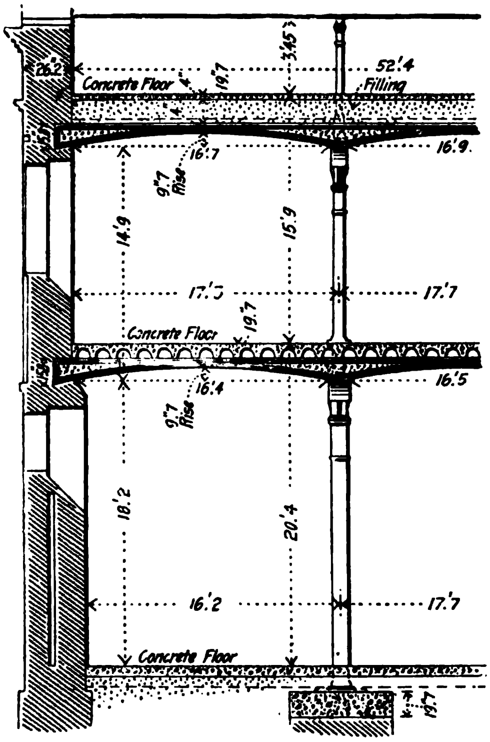


FIG. 372

The concrete was made in the proportions of 1 of cement, 2½ of sand, and 3½ of broken stone. The total area of floor was 8,500 square feet, and it was completed in four working days. The centres were left in for one week after the concreting was completed. The weight of the floor was 320 pounds per square foot, the weight of the wrought iron reinforcements 65,000 pounds, and the volume of concrete 798 cubic yards.

Floors and Stairway for the Petit Palais des Beaux Arts, Paris Exhibition, 1900

The floors employed for the Petit Palais des Beaux Arts, Paris Exhibition of 1900,[2] are shown in Figs. 373 and 374. They were constructed by *M. Hennebique*, two types being employed.

Fig. 373 shows the flat floor with beams reinforced with three straight and three bent rods, of about 1 inch diameter. The floor slab itself is reinforced with alternate straight and bent rods of 0·47 inch diameter.

The proportions used for the concrete were 550 pounds of Portland cement,

[1] Description in *Engineering News*, April 12, 1894.
[2] Described in *Engineering News*, November 10, 1898.

422½ quarts of sand, and 898 quarts of gravel. The centres were struck eight days and the load applied thirty days after completion of the work.

The arched floors shown in Fig. 374 have a span of 20 feet and a rise of only 9 inches. The thickness at the crown is 3½, and at the springing 12½ inches.

The reinforcement consists of two sets of longitudinal rods, 0·67 inch diameter and 3·28 feet apart, one set being horizontal and placed about an inch below the extrados, those in adjacent arches overlapping on the piers. The other set is bent to the curve of the intrados, and is placed ¾ inch from this surface. These rods are carried about 7 inches on to the piers.

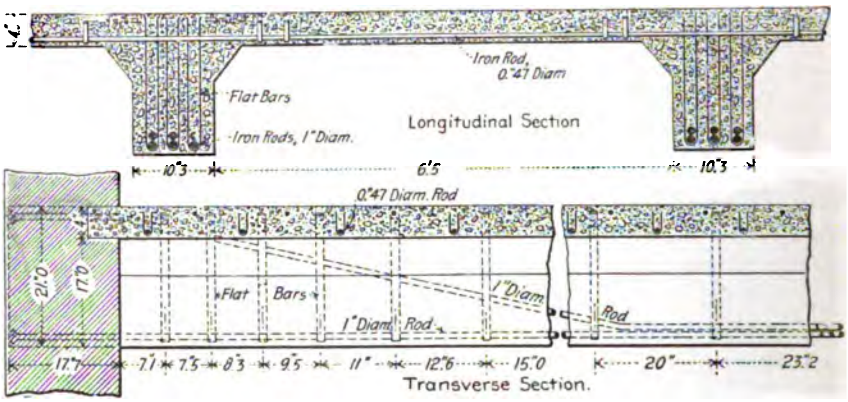


FIG. 373

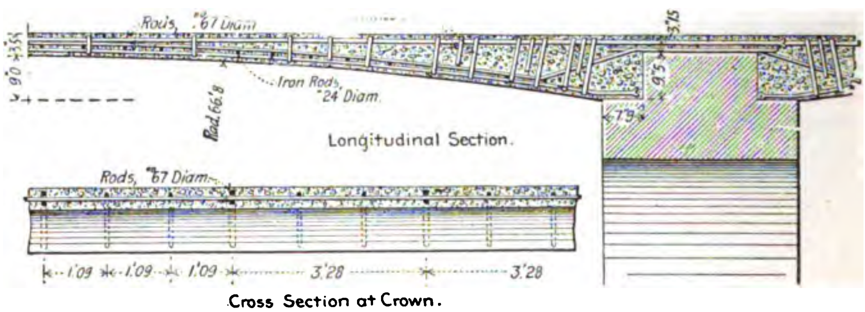


FIG. 374

Both sets of rods have stirrups of about 1¼ inches by 14 B.S.W.G. hoop iron passed round them, the ends terminating near the opposite face of the concrete.

On the curved set of rods a series of transverse rods, 0·24 inch diameter, are placed 8 inches apart. These rods are 25 feet long, the overlap of consecutive rods being 2·3 feet.

A further series of bent longitudinal rods, 0·24 inch diameter, about 13 inches apart, are placed on these, their ends being carried horizontally over the piers, where they overlap similar rods from the neighbouring spans. The load was 740 pounds per square foot, and the floors cost 4*s*. 4½*d*. per square foot.

Fig. 375 shows the large spiral stairway in this building before the steps were put on. This stairway in one half-turn rises from the ground to the first floor, the difference of level being 16·4 feet, and is self-supporting for this distance. At the top it is connected to a circular gallery, which is cantilevered out from the surrounding walls.

The inclined slab on which the steps are formed is 6·23 feet wide and 3·94 inches thick. It is carried by two spiral stringers, 5·91 inches wide and 19·69 inches deep, reinforced with three pairs of 1·18 inches diameter rods, one pair being

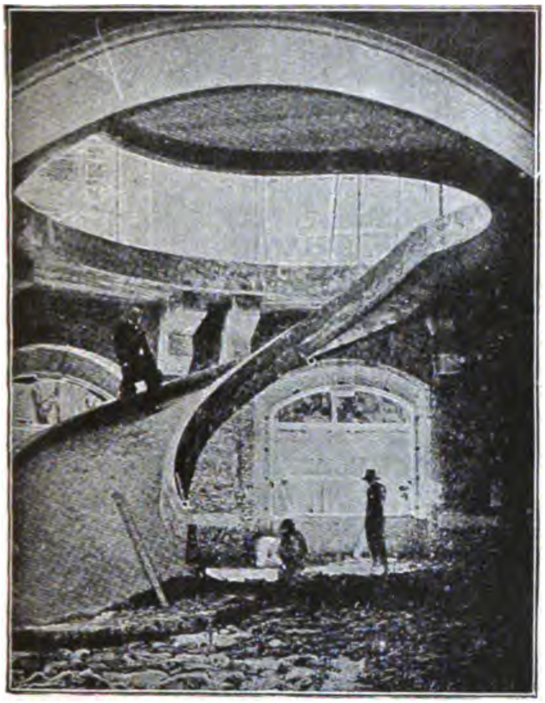


FIG. 375

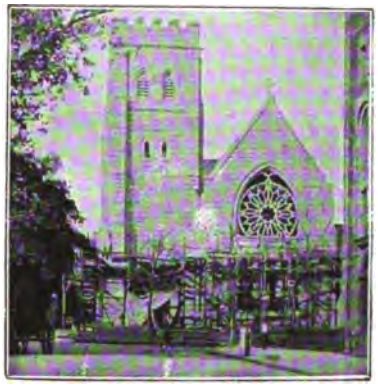


FIG. 376

at the top, another at the bottom, and the third pair being bent. The slab is reinforced in the same manner as that employed for floors. The gallery forming a landing for the stairway has a width of 6·56 feet.

St. James' Church at Brooklyn—(Fig. 376)

This very fine building is constructed entirely of reinforced concrete on the *Ransome system*. The body of the church covers an area of 138 × 89 feet, and is 59 feet high to the top of the walls. The tower covers an area of 23 × 23 feet, and has a height of 112 feet. The thickness of the walls is only 15¾ inches, and their outer faces are treated so as to imitate stone.

The concrete employed on this structure was mixed in the proportion of 1 of cement to 10 of crushed limestone, to pass a 1-inch ring.

Turret Stairway, Lange Voorhout, at La Haye

The turret containing a winding stairway (shown in Fig. 377) was constructed by the *Amsterdamische fabrick van cimént-ijzen werken*, in 1895, of reinforced concrete. It has a height of 39·4 feet, the thickness of wall being only 1·6 inches.

M. Hennebique's House in Rue Danton, Paris—(Fig. 378)

This house is an excellent example of what may be effected by the use of reinforced concrete. It is an extremely fine building, and cannot fail to attract

FIG. 377

FIG. 378

the notice of any who pass it. The whole edifice is constructed completely of reinforced concrete, and the mouldings and general finish leave nothing to be desired. It was erected by *M. Hennebique* in 1900.

The thickness of all the walls is reduced to a minimum, that of the face being only 7 inches.

The plan of the site is a right-angled triangle with a rounded apex, the building standing at the corner of two streets meeting at an acute angle. Part of the house is used by M. Hennebique for offices, and part for his private apartments. After seeing this house, it is impossible to doubt that reinforced concrete can be used with excellent effect for the construction of buildings of great architectural beauty.

SOME STRUCTURES ERECTED IN REINFORCED CONCRETE

Reinforced Concrete Residence at New York

Figs. 379 to 385 show the details of the construction of a five-storied residence with a basement in Fortieth Street, New York, built for Mr. W. C. Sheldon[1] on the *Hennebique system.*

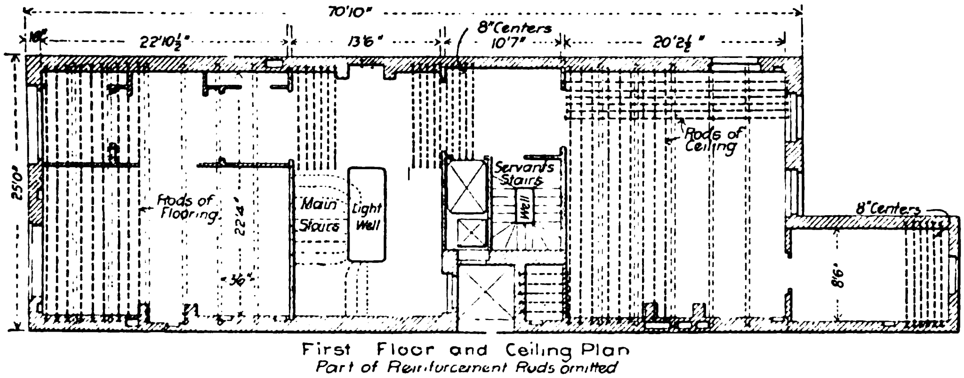


FIG. 379

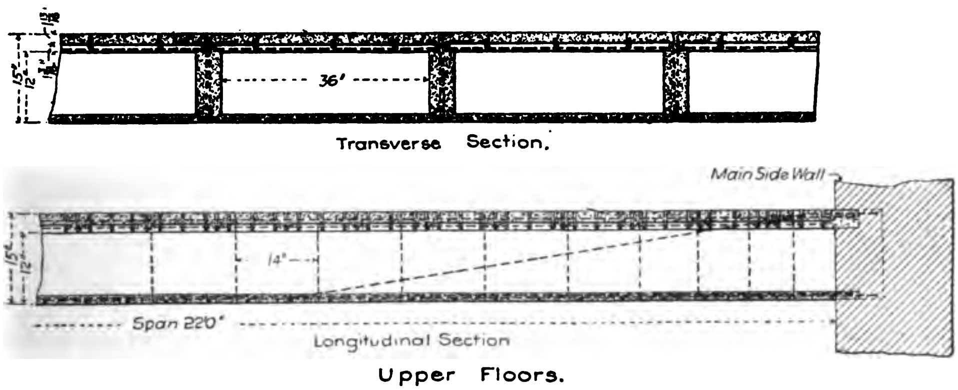
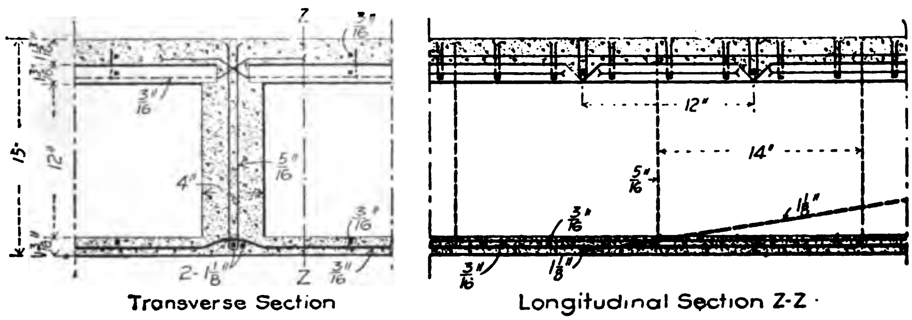


FIG. 380



DETAIL OF CENTRE OF FLOORBEAM.

FIG. 381

Fig. 379 is a plan of the first floor, to which the second and third floors are very similar.

The floors are of the hollow type, with hidden beams, Fig. 380. The beams are about 36 inches apart in the clear, except near the stair and light shafts.

[1] Described in the *Engineering Record,* January 31, 1903.

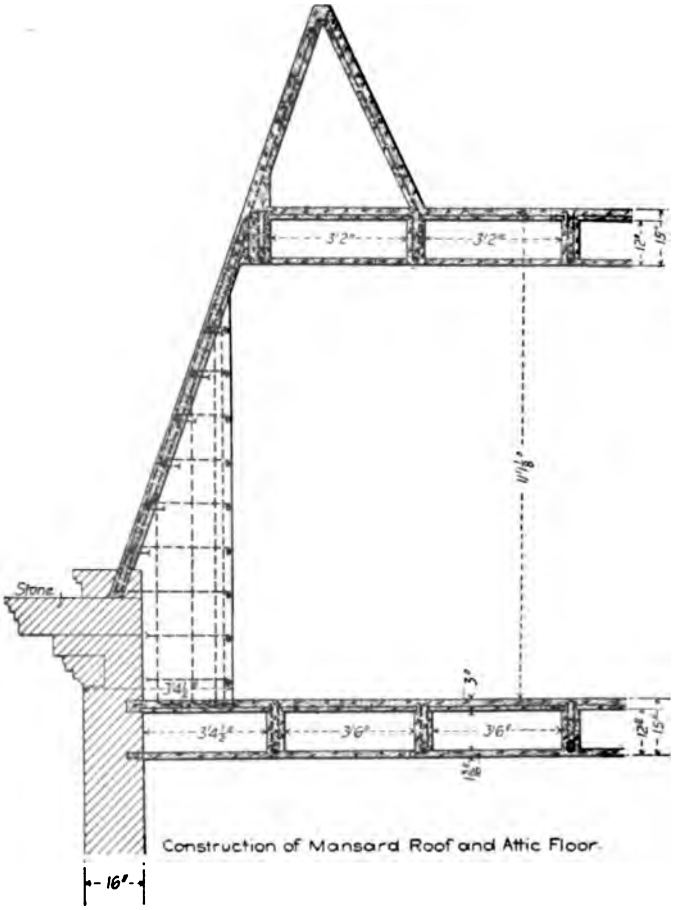


Fig. 382

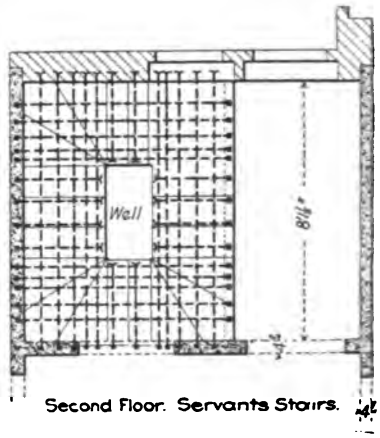


Fig. 383

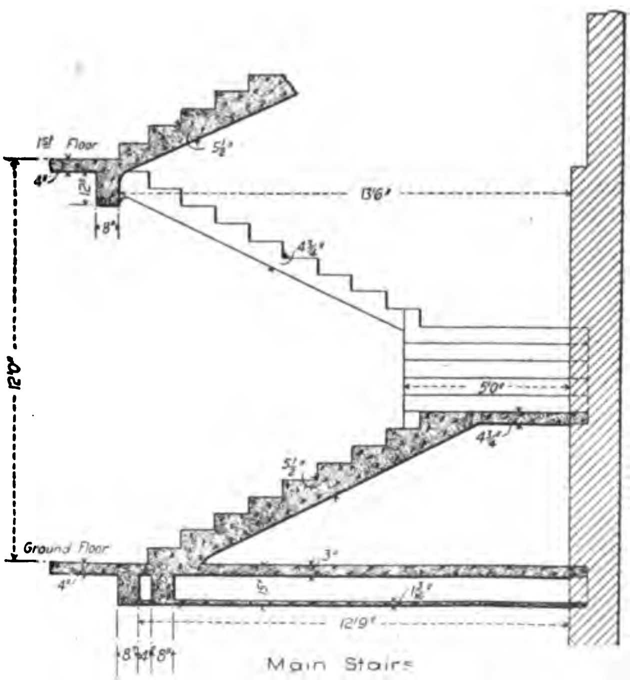


Fig. 384

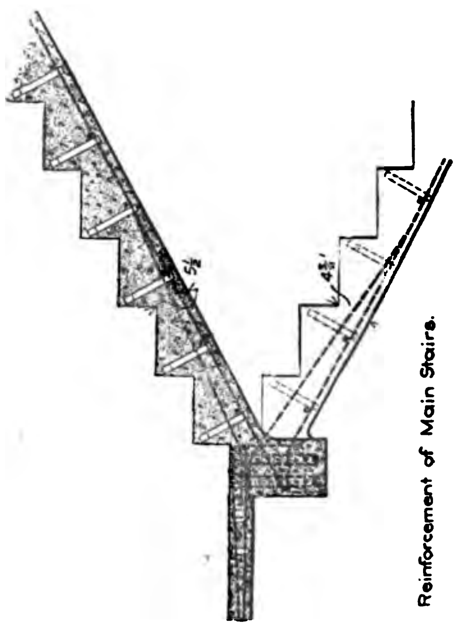


Fig. 385

The reinforcements consist of two $1\frac{1}{8}$-inch rods, one bent and the other straight. The floor slab is $1\frac{13}{16}$ inches thick, with V-shaped ribs $1\frac{13}{16}$ inches deep and about 12 inches apart, as shown in Fig. 381. The reinforcement is formed of $\frac{3}{16}$-inch rods in both directions.

The ceilings slabs are $1\frac{3}{16}$ inches thick, and are reinforced with longitudinal and transverse rods $\frac{3}{16}$ inches diameter, the former 12 inches and the latter 7 inches apart. The stirrups throughout are formed of $\frac{5}{16}$-inch rods. Fig. 380 shows a transverse and longitudinal section of the floors, and Fig. 381 gives the details of the reinforcements, etc.

The roof (Fig. 382) of the Mansard type, projecting at the top to form a parapet, is of reinforced concrete covered with asphalte bricks welded together. It is formed with vertical webs which form buttresses. The construction is clearly shown in the figure.

The stairs are formed of reinforced concrete, with inclined plane surfaces on the underside. The servants' stairs turn with winders around three sides of a narrow well, and have reinforcements in the walls and under the steps, as shown in Fig. 383. The main stairs are in half flights with intermediate landings, and are supported at the top and bottom on extra strong floor beams, shown in Fig. 384. The method of reinforcement of the main stairs is given in Fig. 385.

The main walls are of brick, and carry the floors, except for two single storey reinforced concrete columns in the basement. The partition walls are of reinforced concrete 4 inches thick.

The Palais de l'Électricité at the Paris Exhibition of 1900—(Fig. 386)

This beautiful structure, originally designed and carried out in reinforced concrete by *M. Coignet* as the "Chateau d'Eau," was a notable feature of the Exhibition, being much admired. It is one of the few buildings which are still left standing. The main cove or grotto is 147·6 feet high and 82 feet wide across the front.

Fig. 386

It is supported by a series of longitudinal partitions, about 4 inches thick, formed of curved ribs, with a mean span of 33 feet. These are supported by the columns, walls and groined arches of the galleries and staircases below.

The vertical wall of the grotto, which is semi-circular in plan, has a height of about 33 feet, and is only 4 inches thick, stiffened by a series of vertical 8 x 8 inch ribs. The semi-dome forming the roof of the grotto is formed of a slab 2·36 inches thick, carried by arched ribs running parallel to the face, and connected by curved cross ribs. The arched face is formed of a double semi-arch of two

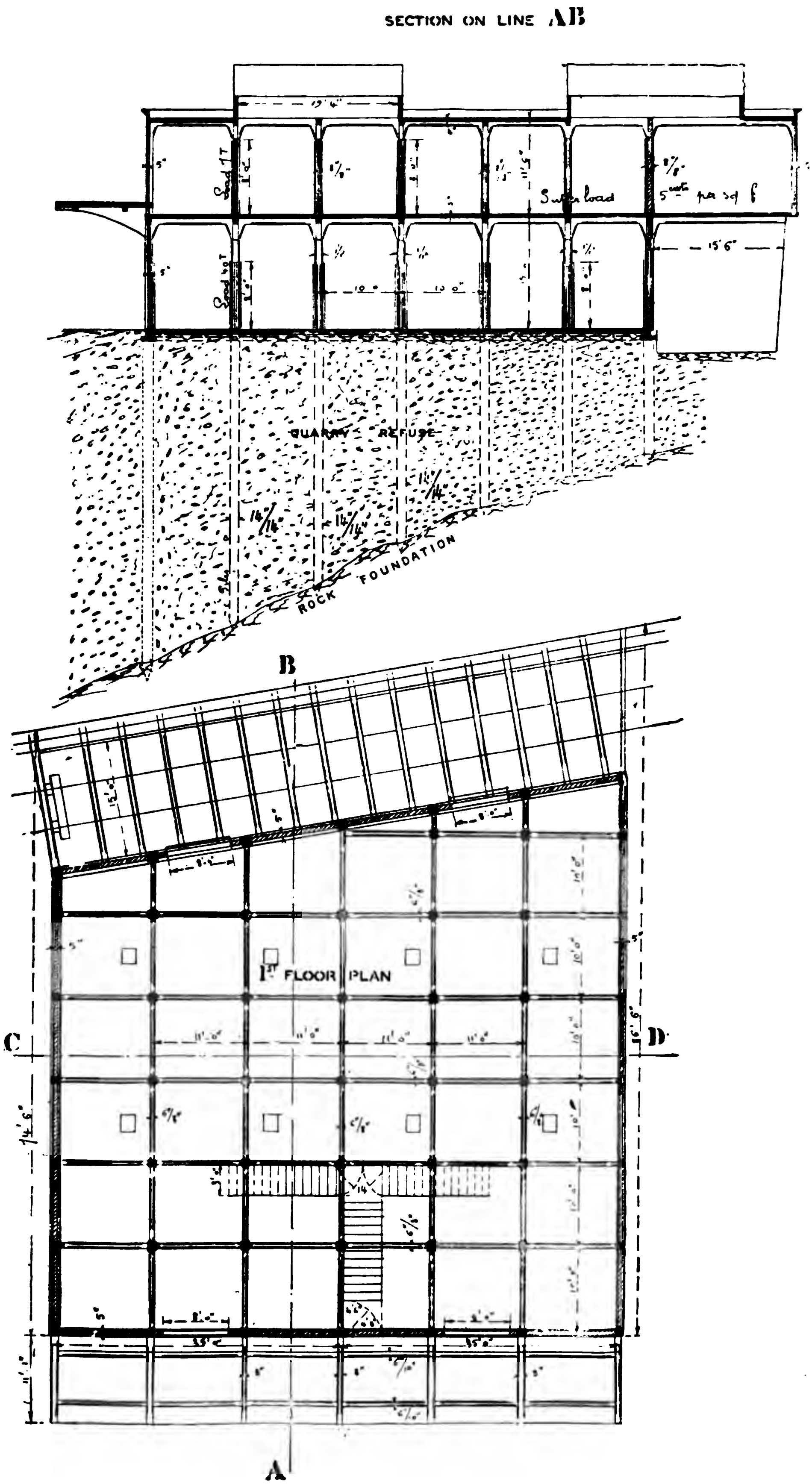

SECTION ON LINE AB
QUARRY REFUSE
ROCK FOUNDATION
1st FLOOR PLAN
A
B
C
D

Fig. 387

ribs 82 feet span, with cross connexions. The ends of the cross ribs carrying the domed slab abut against these ribs. The very fine ornamental work was of course merely roughly formed, being finished by special artificers.

Great Western Railway Grain Warehouse, Plymouth

The plan and section of this warehouse, built on the *Hennebique system,* are shown in Fig. 387. The frontage on the west wharf is 70 feet, and the maximum depth 80 feet. It has a ground floor area of 4,200 square feet, the floor being constructed of a depth of 6 inches of plain concrete with 1 inch of cement rendering, and is laid on the old rubble filling of the quay. This is the only part of the building not reinforced. The first floor has an area of 5,500 square feet, and is 5 inches thick, resting on beams supported on 12 × 12-inch reinforced columns, 10 and 11½ feet apart, which are continuous with the 30-foot reinforced concrete piles 14 × 14 inches. The load provided for is 5 cwt. per square foot.

The flat reinforced roof has an area of 5,500 square feet, and is 4 inches thick. It rests on reinforced beams, supported on 8 × 8-inch columns, 10 and 11½ feet apart, which are continuous with the 12 × 12-inch columns carrying the first floor.

The verandah has an area of 1,800 square feet, and is constructed along the whole front of the warehouse. Its floor is reinforced and 5 inches thick, resting

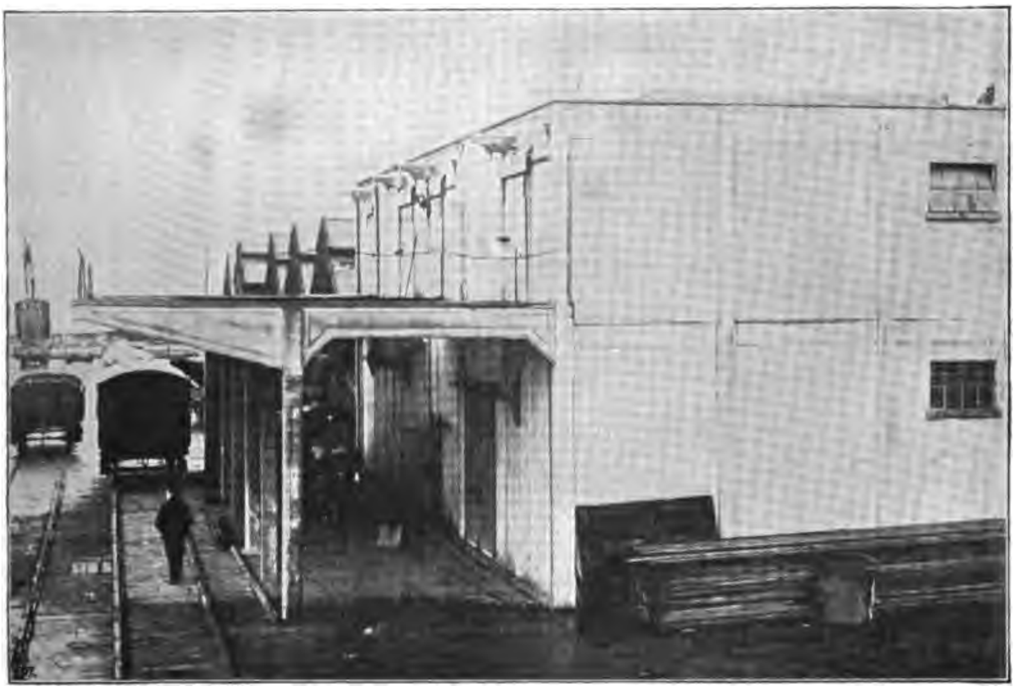


FIG. 388

on beams supported on seven 12 × 12-inch columns, continuous with reinforced piles driven down to the rock. The centres of these columns are 14 feet 7 inches from the face of the building, and the body of the verandah projects 26 feet; the difference of 11 feet 5 inches is carried on cantilevers projecting from the columns.

Fig. 388 is a reproduction from a photograph of the finished building. The

four reinforced catsheads, built in the front wall of the warehouse, are each designed to lift a ton, and were tested to 30 cwt. without any apparent effort. The first floor, roof, and cantilevered portion of the verandah were tested considerably above the calculated load, and showed very slight deflections.

This building cost 4*d*. per cubic foot of air space, exclusive of the pile foundations.

Warehouse for the Co-operative Wholesale Society at Newcastle-on-Tyne [1]

This building, which is 120 feet high, is constructed entirely of reinforced concrete, on the Hennebique system. The site on which the warehouse had to be constructed was very bad for the purposes of foundations, being of made ground for a depth of 18 feet, below which was 18 feet of silt and quicksand, 10 feet of soft clay, 5 feet of hard clay, and 10 feet of silt, sand and gravel. The whole building was therefore practically floated on a raft of reinforced concrete, of which Figs. 389 and 390 show the construction.

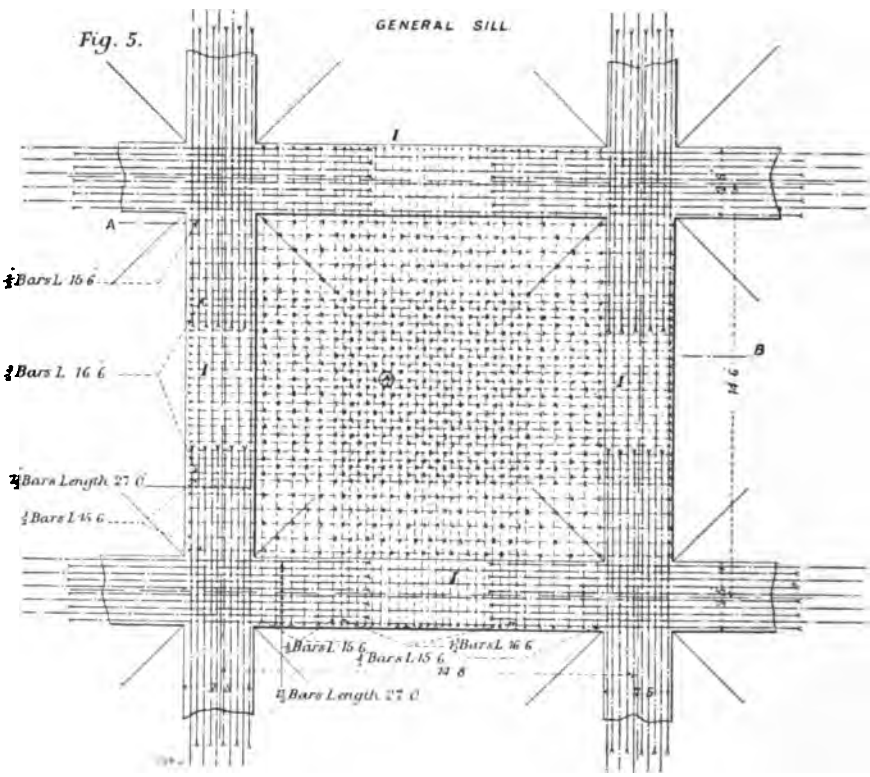


Fig. 389

A settlement of 3½ inches at the front and 3 inches at the back took place between the dates of the completion of the general cill and the construction of the first floor, after which no further settlement occurred.

[1] Description in *Engineering*, April 17, 1903.

The walls of the basement are 18 inches thick, and have to withstand the thrust of the surrounding ground. The walls at the ground floor are 12 inches thick, and they reduce to 4 inches at the level of the roof. The section of the basement columns is 29 × 29 inches.

FIG. 390

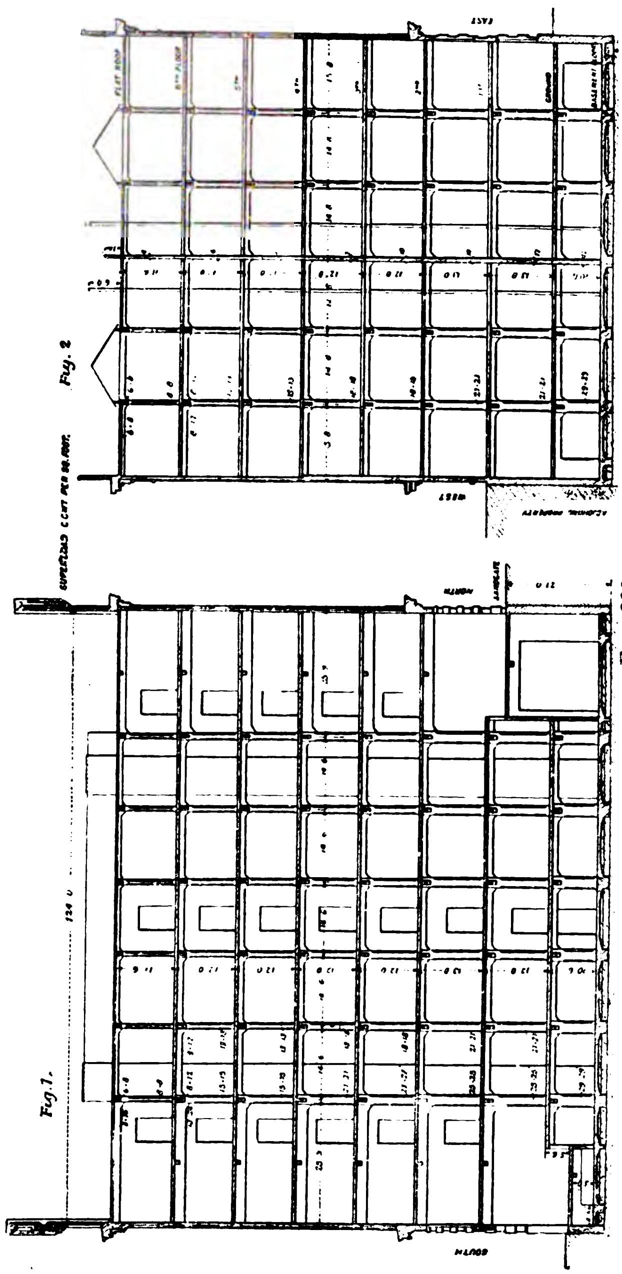


FIG. 391

The floor beams are generally 7 inches wide by 12 inches deep to the underside of the floor slab, which is 7 inches thick, the span of the beams being 14½ feet. Longitudinal and transverse sections of the building are shown in Fig. 391.

The floors are constructed to carry 684 pounds per square foot, and have been tested under a load of 996 pounds per square foot, the load being applied gradually. Great credit is due to the Newcastle Corporation for permitting their building bye-laws to be suspended in favour of this building.

Chimney Stack at Los Angelos

Fig. 392 shows the details of a reinforced concrete chimney shaft, constructed on the *Ransome* system for the Pacific Electric Railway Company at Los Angelos, California.[1]

The figures show clearly the construction of the shaft, which consists of two concentric walls, separated by an air space of 11 to 16 inches, increasing in width towards the top. The outer shell above the shoulder is 9, 6 and 5 inches thick respectively up to the cap, in sections of about equal height, the inner shell being 5, 4½ and 4 inches thick respectively from bottom to top, in corresponding sections. The inner shell terminates 4 feet below the cap, and is free to elongate independently of the outer shell.

At intervals of 30 inches measured around the shaft the air space is contracted, for lengths of 6 inches, to a width of 2½ inches ; at every 5 feet in height this is again reduced to three-quarters of an inch by the introduction of a concrete brick in the wall. This arrangement allows the outer shell to oscillate three-quarters of an inch without bringing any pressure on the inner shell.

The reinforcement is of Ransome cold twisted steel bars, the horizontal reinforcement consisting of ¼-inch bars placed at intervals, averaging 18 inches in the inner and 24 inches in the outer shell. The vertical reinforcement of the outer shell is formed of ¾-inch bars placed 12 inches apart in the lower third of the stack above the flues, 2 feet apart in the middle section, and 4 feet in the top section. In the inner shell ¼-inch bars are used, spaced 3 feet apart.

The upper 4 feet above the ornamental cap is 2 inches thick, being reinforced with expanded metal. The ornamental cap is formed of 28 blocks moulded on the ground. These blocks are hollow, and are formed by a shell of 2 inches of concrete, with stiffening cross-partitions, the whole being reinforced with ¼-inch twisted steel bars.

The proportions for the concrete were 1 : 2 : 4 for the inner and 1 : 2 : 6 for the outer shell, the stone being broken to 1 inch or less. The foundation is reinforced with steel rails, placed 12 inches apart, in two layers crossing one another. The chimney weighs about 1,280 tons, and the distributed load on its base is less than 2 tons per square foot.

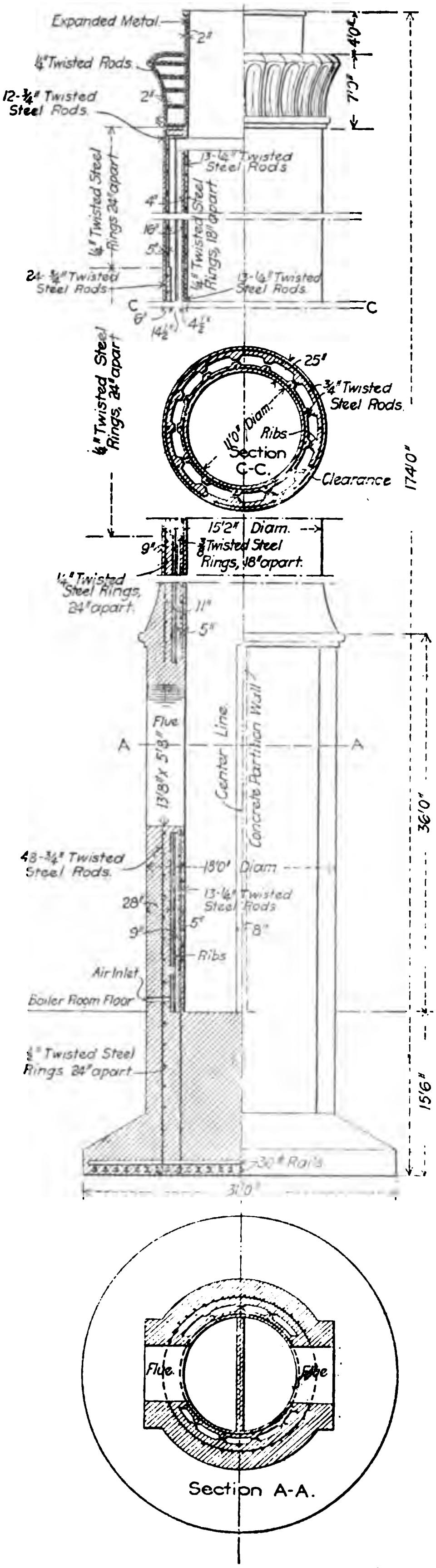
The vertical reinforcing bars were spliced together with sleeve nuts. The circular bars which are overlapped from 12 to 18 inches at their ends, were bent into the proper shape and deposited on the concrete as it was filled in.

The twists on the bars were as follows—

¼-inch bars	10 to 12 per lineal foot
⅜-inch bars	7 to 8 ,, ,, ,,
½-inch bars	5 to 6 ,, ,, ,,
¾-inch bars	2½ to 3 ,, ,, ,,

The concrete was mixed wet enough to run slowly down a slope of 1 to 2 when shot from a wheelbarrow.

[1] Described in the *Engineering Record*, April 11, 1903.



Revised Design of Stack

FIG. 92

The Wesleyan Church of St. Sidwell's, Exeter [1]

Although this structure is not in accordance with the building bye-laws of the city, it was allowed to be built after full consideration of the Streets Committee, who showed a commendable appreciation of the scope of this form of construction.

The system adopted is that of *M. Cottançin*, the foundations, walls, and columns being of steel-cored brickwork, and the floors, with their stiffening ribs, being formed of concrete, with woven steel networks embedded.

The foundations are of the cellular type, with intersecting thin walls, and are only 3 feet deep, although the soil is not all that can be desired.

These cellular compartments, after being well filled in with earth, are covered with a 2-inch slab of concrete, the network of the slab being tied to the wires projecting from the walls of the foundations.

The church is square on plan at the ground level, but becomes octagonal at the level of the gallery, and is to be covered by an octagonal dome supporting a heavy lantern at a height of 80 feet from the level of the ground.

The area covered at ground level is 5,040 square feet.

The walls are hollow, and are formed of two thicknesses of 3-inch steel-cored brickwork, with cross walls tying the two together, and extending the full height of the building. The hollow space is to be used for heating and ventilating purposes, and will keep the building at an even temperature in winter and summer.

A sloping gallery 14 feet wide surrounds the whole building, with the exception of the choir side, and is octagonal in plan. It is entirely self-supporting, with the aid of the walls at the back, there being no columns below or rods above. The inclined floor of concrete, with an embedded network, is reinforced by intersecting steel-cored ribs, some of brickwork and some of concrete, which are in this case placed above the floor slab. The main ribs run diagonally to points in the walls, where their network is attached to that of the wall, and have a secondary system intersecting and tied to them. All the rib networks are interwoven with the floor slab, so that the whole act together. Fig. 393 is a view of this gallery during construction. The stairs leading to the gallery at either end are also formed of steel-cored concrete, and assist in its support, being in fact the chief support at the ends.

The front of the gallery is formed of concrete with an embedded steel network, and forms a light beam resting on the stiffening ribs, and helping to distribute the load on the main ribs and the staircases.

The dome is constructed in two thicknesses, with stiffening ribs between.

The inner thickness is 3 inches, and is built of steel-cored brickwork, while the outer layer is formed of concrete 2 inches thick, with a woven steel network embedded.

The steel coring to the roof is tied to that of the walls, making the reinforcement continuous throughout the whole building. No timber is employed anywhere for structural purposes, the whole edifice being entirely fireproof. All lead, zinc, or other metal for flashings, flats or gutters is absolutely dispensed with, the flats and gutters being all formed of concrete, reinforced where necessary. The floors, gallery and staircases are lined with ordinary joinery for the sake of

[1] Described in the *Contract Journal*, December 31, 1902.

comfort and appearance, but this and the necessary seating, etc., are the only parts where timber is employed.

The total cost of this church is about £3,000, and all the materials and labour are being obtained locally.

FIG. 393.

The building is being constructed on the designs and under the supervision of Messrs. Commin & Coles, architects, of Exeter.

Church of Saint Jean de Montmartre, Paris

This church was erected by M. A. de Baudot, architect to the Dioceses of Paris, on the *Cottançin system* for the parish of Montmartre.

This is a very poor quarter, where a very large church had to be built with very little expenditure.

The boldness of the design has secured for this edifice the nickname of "The Folly of the Century."

The outside walls are 115 feet in height, and are only 4½ inches thick, with steel wires interlaced through holes in the bricks and along the bed joints.

A portion of one side, 29½ feet in length, is entirely unsupported, and maintains the height of wall of 115 feet without any girder, the wall being self-supporting, while on another side there is an unsupported length of 65 feet, of which the height is 38·37 feet.

The church has two floors, the upper floor being supported on reinforced brick columns $39\frac{1}{2}$ feet apart, and only $17\frac{3}{4} \times 17\frac{3}{4}$ inches square.

The floor is only 2 inches thick, with 14 × 2-inch reinforcing ribs (beams).

Fig. 394 gives a view of the lower floor of this church, showing the columns supporting the upper floor and the domed roof at the end.

FIG 394

The whole structure is reinforced on the Cottançin system, which secures an extremely light edifice combined with great strength.

Fig. 395 shows a portion of the flat domes with which the church is covered, with the stiffening ribs. The domes are formed of two 2-inch layers of concrete with the ribs between, the whole being reinforced with Cottançin networks.

This church is a truly wonderful example of the scope of reinforced brickwork

FIG. 395

and concrete, and is well worth a visit to any one interested in this type of construction. The total cost was £14,000.

Château de Orfraisiére

All the floors and roofs throughout this extensive château are of reinforced concrete, on the *Cottançin system.* It was built for MM. Robert and Henri de Wendel, who, although they are well known experts in ordinary metallic construction, gave the preference to the employment of this system of reinforced concrete for the building of their château.

Fig. 396 shows the stables roof, which has a span of 56 feet, with the small rise of 16 inches. Both the roof slab and ribs are only 2 inches thick, the depth of the ribs being 14 inches. The intersecting ribs, besides forming an extremely stiff and strong roof, produce an appearance which is everything that could be desired from an aesthetical point of view.

A further interesting feature at this château is the dome, which is 125 feet above the ground level, and 52 feet in diameter, being supported on 18 × 18-inch columns of steel-cored brickwork.

MM. Plé Frère's Shoe Factory, Paris

The lower part of this building is of ordinary brickwork, with a roof of concrete 2 inches thick, reinforced with Cottançin network.

The whole upper portion is built of reinforced brickwork and concrete on the *Cottançin system.*

The walls are one brick thick, and have a core of steel wires. These support the central glazed roof, of which the rafters are all of steel-cored concrete 14 × 2

FIG. 396

FIG. 397

inches, with a rise of 14 inches, the span being 46 feet ; the glazing bars and purlins are also of reinforced concrete.

The roof has an extremely light appearance, which is obtained by the use of a minimum amount of material. At the end of the building is an engine room of reinforced concrete, with a tank holding a depth of three feet of water over the whole area of the roof. The chimney stack is also of cored brickwork.

The total cost of the reinforced portions was £1,680.

Fig. 397 shows the interior of the factory, which is very light and airy.

House in Avenue Rapp, Paris

The whole of the interior and back portions of this house, as well as the front

Fig. 398.

from the first floor level, are built entirely of reinforced concrete, brickwork and stoneware, on the *Cottançin system.*

It was designed by M. Lavirotte, a well-known Paris architect, and was one of the six houses specially chosen for reward by the "Concours des Façades" of Paris in the year 1902.

The façade above the second floor is of reinforced glazed ceramic ware, the balcony being self-supporting, but from the level of the street to that of the second floor ordinary masonry has been employed, this being the only part which is not reinforced. Fig. 398 is an illustration showing the façade.

The walls facing the courtyard at the back are constructed of two thicknesses of 2¼-inch steel-cored brickwork, tied together by similarly reinforced cross walls; all the internal walls, including those of the staircase and lift shaft, being built of reinforced brickwork, those for the staircase and lift shaft having a single thickness of 4½ inches, while the main partition walls are in two thicknesses of 2¼ inches, built in a similar manner to the external walls.

The staircase is also formed of concrete reinforced on the Cottançin system.

There are no projections for flues, these being accommodated in the hollow walls, which are also used for heating and ventilating purposes, the saving of floor space, by the suppression of the usual chimneybreasts, being about 60 square feet.

The floors are formed of a 2-inch reinforced slab, with the Cottançin intersecting stiffening ribs 2 inches thick, all the reinforcing network being interwoven and tied to the wires of the walls.

The roofs and flats are also formed of reinforced concrete, no lead or zinc work whatever being employed. The tiles are attached to the roof reinforcements by wire ties, which prevents their becoming loose and falling off.

The cost of this house was £5,000, or about 20 per cent. under that for the ordinary form of construction.

Nicolaieff Lighthouse [1]

This lighthouse has recently been constructed of reinforced concrete for the Russian Government to light the canal which connects the town of Nicolaieff to the Black Sea

It is about 110 feet high above the base to the floor level of the upper chamber, where the diameter is about 6½ feet. The sides have a slight batter from the top to a section about 40 feet above the base, where the batter increases and becomes concave, so as to obtain a diameter of 20·7 feet at the bottom. The footing is formed of a cup-shaped concrete block 8 feet deep, with a horizontal surface 28·2 feet diameter. The side walls of the foundation are battered on the outer face and enclose a circular space 19·7 feet diameter, which is filled in with earth and covered with a reinforced concrete slab forming the bottom floor of the lighthouse.

The inside of the shaft is made without divisions, floors or bracing, and only contains a spiral staircase reaching to an upper cylindrical chamber of 14·8 feet inside diameter and 9·8 feet high, which projects beyond the walls of the shaft, the floor being supported on reinforced cantilevers. On the top of this chamber

[1] This structure was described in *Le Genie Civil* and also in *The Engineering Record,* January 23, 1904.

there is a domed lantern about 6·8 feet diameter and 13 feet high. The concrete was mixed in the proportions of 661 pounds of Portland cement to 14 cubic feet of coarse sand and 28 cubic feet of washed shingle. The reinforcements of the superstructure consisted of circular and vertical steel reinforcements tied together at their intersections. The diameter of the circular rods was $\frac{3}{4}$ of an inch, and they were spaced 12 inches apart. The vertical rods had a diameter of $\frac{7}{8}$ of an inch, and were spaced about 9 inches apart.

At the junctions of the circular and vertical rods, they were spliced with wire wrapping.

The foundation walls are reinforced with rods, near the vertical and battered faces connected by horizontal ties. The slab forming the bottom floor is 30 inches deep, and is reinforced with $\frac{1}{4}$-inch horizontal rods crossing one another at right angles near both the upper and lower surfaces. The corbels supporting the upper floor were moulded with the side walls and the floor slab, and have their undersides curved to form mouldings. They are reinforced with horizontal top rods and inclined bottom rods, which are secured to the vertical wall reinforcements by wire wrappings; the upper and lower rods are also connected together by vertical and inclined transverse reinforcements, which are tied to the main rods.

The lighthouse was calculated to resist a wind pressure of 55 pounds per square inch.

It is stated that the reinforced structure cost 40 per cent. less than a corresponding lighthouse built entirely of brick or metal; also that the brick lighthouse would have weighed 1,365 tons against 460 tons, which is the weight of the structure as erected.

Fig. 399

Columns and floors for a Warehouse at Roubaix Courcoing, constructed on the *Coularou system.*

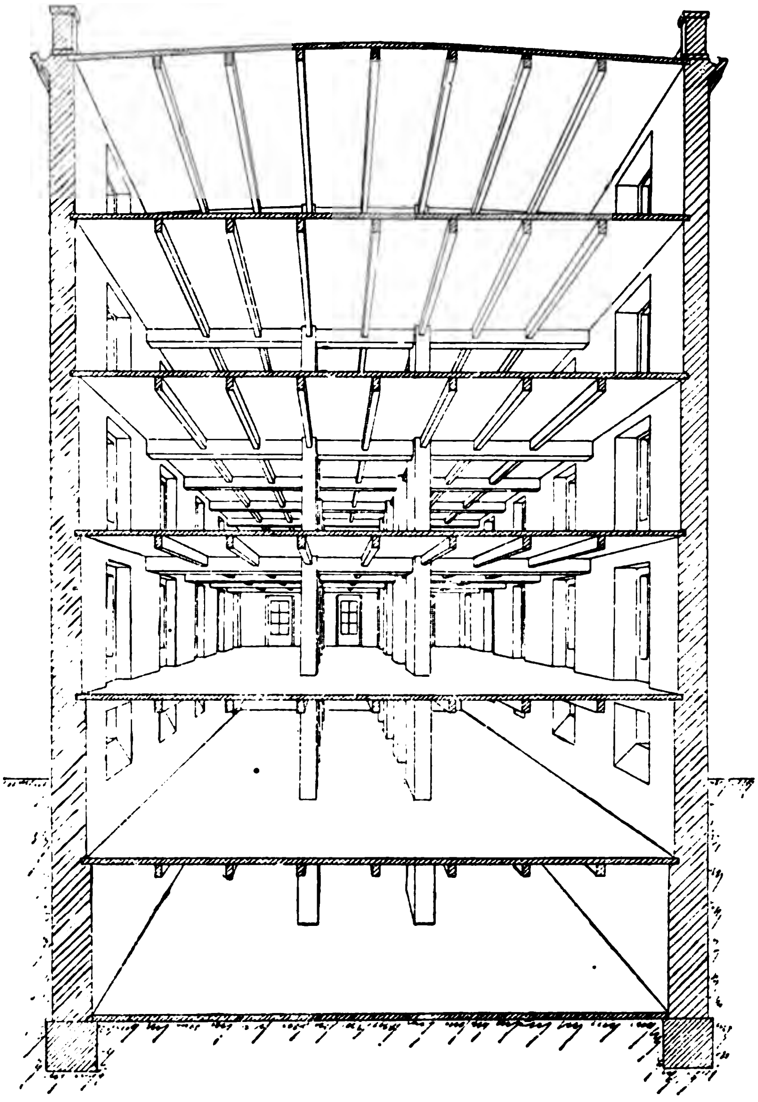


Fig. 400

Warehouse for Perrin David et Cie., Lausanne. The columns, floors, and roofs are of reinforced concrete on the *Vallerie-Simon system.* Load, 246 pounds per square foot. The top floor carries machinery.

SOME STRUCTURES ERECTED IN REINFORCED CONCRETE

Fig. 401

Flour Mill and Grain Silos at Brest (France), constructed of reinforced concrete on the *Hennebique system*. The foundations being of soft ground, all these buildings were built off a general raft of reinforced concrete.

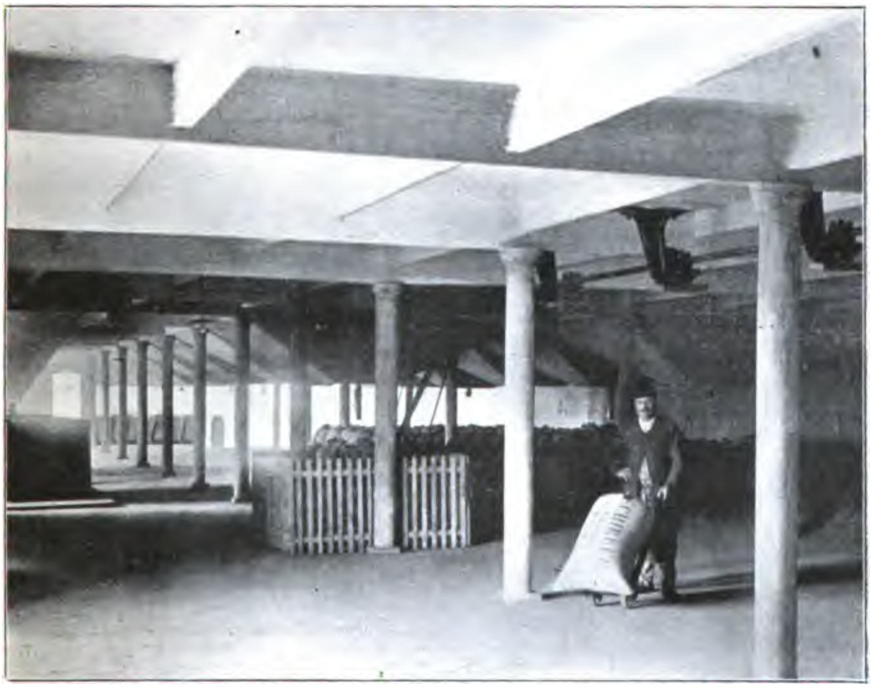


Fig. 402

Floors and Silos constructed on the *Demay system* at Rheims for M. Schirber.

Fig. 403

Cattle Market and Abattoirs at Alexandria. Built entirely of reinforced concrete on the *Hennebique system.* The illustration shows the wonderfully light and cool appearance of the buildings.

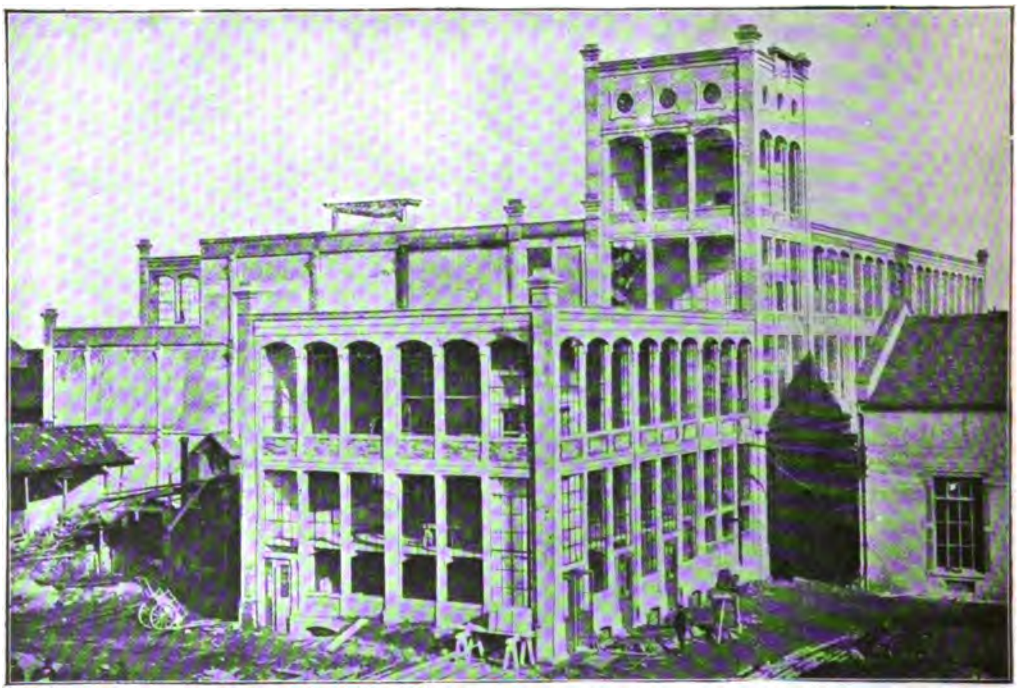


Fig. 404

Spinning Mill at Mullhouse, entirely built of reinforced concrete on the *Hennebique system.*

FIG. 406

Signal Cabin at Joinville, France. Built entirely of reinforced concrete on the *Hennebique system*. The signal shown in the illustration is supported by the building itself on a platform projecting from the face, and carried by two reinforced concrete cantilevers.

FIG. 405

Theatre at Lille, built entirely of reinforced concrete on the *Hennebique system*. This is an example of the manner in which reinforced concrete adapts itself to ornamental uses. Most of the enrichments were roughly cast as the work was brought up, and completed subsequently at a very small cost.

II. RESERVOIRS AND SILOS [1]

Covered Reservoir for the Town of Vimoutiers (Orne), France

This reservoir is half below and half above ground, and was constructed on the Monier system by the *Sociétié de la Plaine de Saint Denis* (*Chaudy*). It is circular, with an internal diameter of 29½ feet, and a depth of water of 13 feet 1½ inches, and has a domed roof with 3·28 feet rise, supported on six radial arched ribs, meeting on a round column at the centre of the reservoir. The wall is 5·92 inches thick, and is reinforced with vertical rods 0·275 inch diameter at the centre of the wall, spaced 3·94 inches centre to centre, and bent round into the floor and roof slabs, circular horizontal rods being placed outside them 3·66 inches apart, and having the following diameters, commencing at the bottom—

10 rcds	0·67 inch diameter	8 rods . . .	0·39 inch diameter
10 „	0·59 „ „	5 „ . . .	0·35 „ „
9 „	0·47 „ „	2 „ . . .	0·83 „ „

at the top.

The floor is 3·94 inches thick, and is reinforced with a network of 0·236-inch rods, with a mesh of 3·94 inches.

The roof slab is 3·74 inches thick, with a network of 0·236-inch rods and a 2·36-inch mesh. The arched ribs supporting this slab have a thickness of 3·94 inches, and a depth below the roof slab of 6·9 inches. They are reinforced by an 0·98-inch diameter rod along both the top and bottom, with stirrups of 0·275 rods bent completely over the longitudinal rod at the top and terminating in the floor slab, and spaced 3·94 inches apart in the two outer quarters of the span, and 11·82 inches apart in the central half of the span.

The central column has a diameter of 19·7 inches at the bottom and 15·75 inches at the top, and is reinforced with 8 vertical rods, 0·79 inch diameter, bent out at the top and bottom, prevented from bulging out by four rings to which they are tied, and a spiral winding of wire.

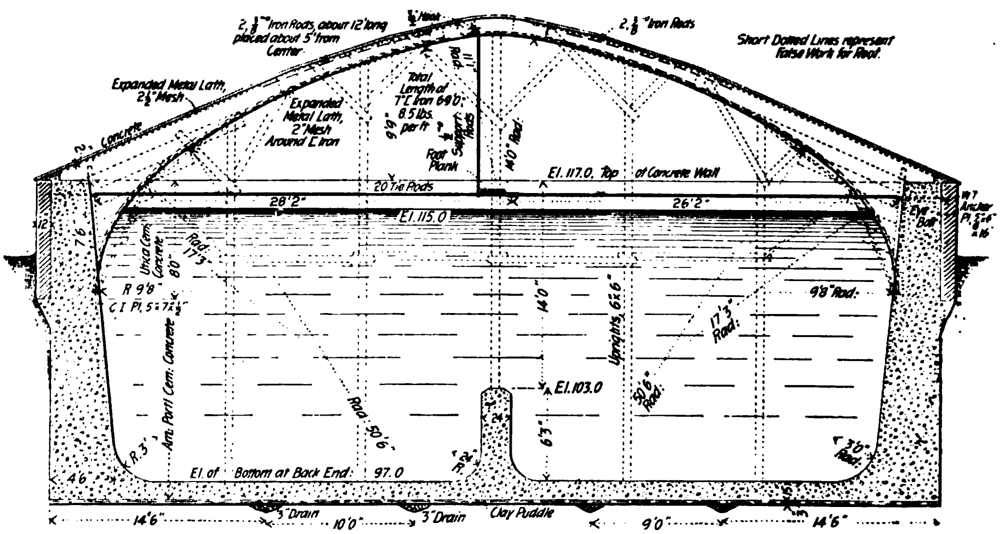


FIG. 407

[1] A description of some circular reservoirs constructed by Major Stokes-Roberts, R.E., in India, is given in Appendix III.

SOME STRUCTURES ERECTED IN REINFORCED CONCRETE

Reservoir at Rocheford, Illinois, U.S.A.

Fig. 407 shows the reservoir, the roof of which is reinforced with *expanded metal* on the *Golding system*. The dotted lines show the arrangement of the false-work.

Fig. 408 is an enlarged detail showing the method of construction. The channel ribs were placed 7 feet apart, centre to centre, and their ends rest on ½-inch cast-iron plates 7½ feet below the top of the wall. The rods, 1⅛ inches diameter, with tightening screws, were passed through the concrete near each rib, one foot below the top of the wall, and are riveted to 16 × 6 × ⅝-inch plates.

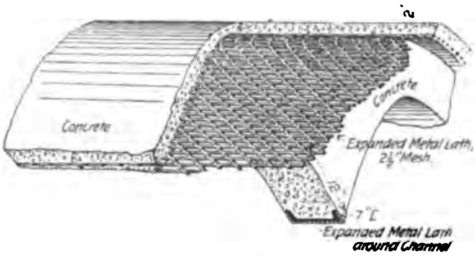


FIG. 408

The concrete for the roof was mixed in the proportions of 1 of Portland cement to 2 of sand and 5 of shingle. The thickness of the roof slab is 2 inches, with a finishing coat, ½ inch thick, of 1 to 2½ mortar.

The cost of the roof was about £411.

Covered Service Reservoirs at Seraing (Belgium)

These three underground reservoirs were constructed in 1898 by *M. Hennebique*. Two of the reservoirs contain 22,000, and the other 176,000 gallons. Details of the largest are shown in Fig. 409.

The side walls are reinforced with vertical rods on both faces, each set being spaced 7·87 inches, centre to centre, and formed of rods 0·55 inch diameter, with hoop-iron stirrups connecting them with the opposite face. These rods are bent over at the top and bottom, so as to penetrate into the floor and roof slabs from 1·97 to 2·23 feet. The division wall is reinforced in the same manner, excepting that each series of rods are spaced 9·84 inches, centre to centre, and are bent opposite ways at the top and bottom (*vide* Coupe, L. M.) All the walls are 5·91 inches thick.

The six columns, spaced 13 feet centres to support the roof, are 7·87 inches square, and are reinforced with four rods 0·945 inch diameter, held together by crossties of wire 1·64 feet apart. The bottom ends of the vertical rods abut against a plate 11·8 inches square and 0·236 inch thick, which is embedded in the floor slab 7·87 inches thick.

The floor slab is only reinforced under the columns, where flat bars are used of lengths varying from 5·91 to 2·62 feet long, placed in six layers, each formed of two series of bars crossing one another at right angles, as shown in the elevation of the column.

The roof is supported on main and secondary beams, spaced 13 feet and 5·45 feet centres. The main beams are 7·87 inches wide, and 13·78 inches deep to the underside of the floor slab, and have a reinforcement of two bent and two straight rods 1·18 inches diameter, with the usual hoop-iron stirrups. The secondary beams have a width of 6·3 inches and a depth of 11·81 inches, and are similarly reinforced with rods 0·98 inch diameter. The roof slabs are 4·72 inches thick, and are reinforced with longitudinal rods of 0·59 inch diameter at the top and bottom and stirrups connecting the bottom rods to the upper surface of the slab.

The valve chamber above the reservoirs is also constructed of reinforced concrete, the thickness of wall being only 2·36 inches.

The cost of the three reservoirs, including all fittings, was £1,436.

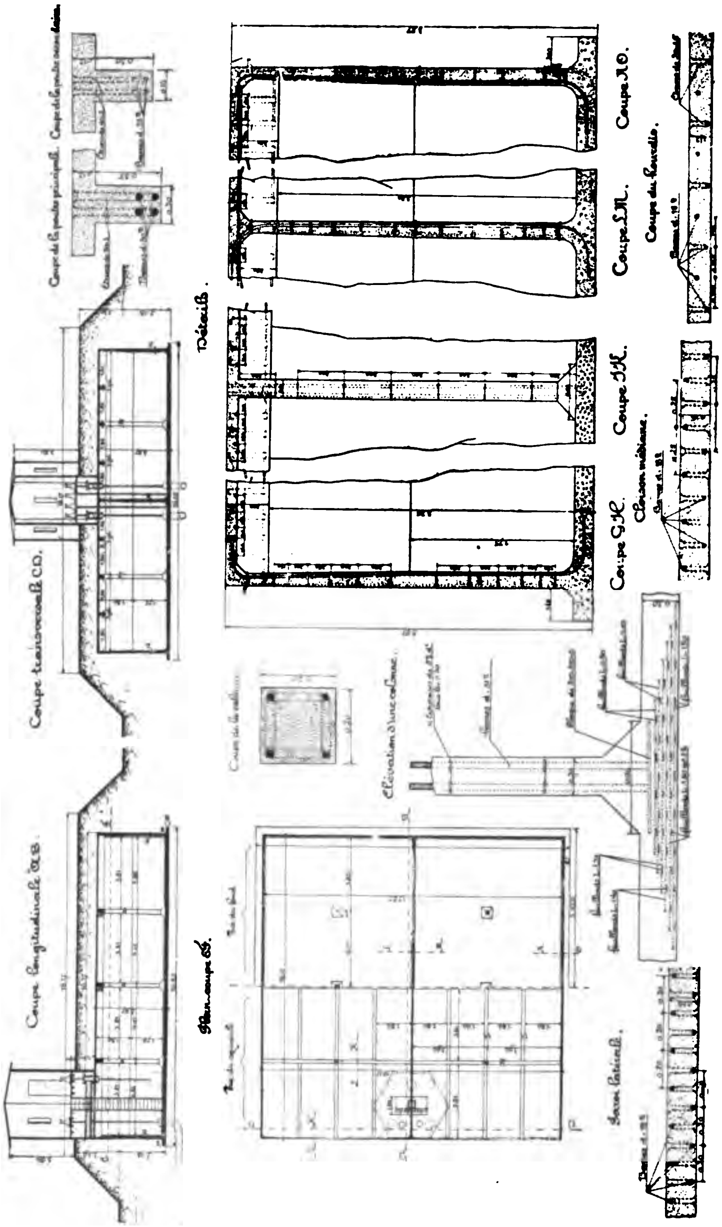


FIG. 409

Fort Revere Tower, near Boston, U.S.A.[1]

This tower, containing a standpipe 50 feet high, has recently been completed. The tower and pipe are both constructed of reinforced concrete, on the *Hennebique system*. The original design was for a masonry tower and steel standpipe, but as the tender sent in by the Hennebique firm was 30 per cent. lower than any other, it

[1] *Engineering Record*, August 22, 1903.

was decided to allow them to do the work. The tower is octagonal, and is formed of reinforced concrete columns, the panels being filled in with brickwork. The piers and brick fillings rest on a moulded base of reinforced concrete, about 12 feet in height.

The distance between the opposite walls of the tower is 25 feet, while the internal diameter of the standpipe is 20 feet. The space between the pipe and the tower allows the insertion of a winding stairway of reinforced concrete, leading to an observatory floor below the roof. This floor is 2 feet in the clear above the top of the standpipe, and is formed of reinforced concrete beams and floor slabs. The piers forming the angles of the tower are each reinforced with six vertical rods, ¾-inch diameter. The observatory floor is 3 inches thick, supported on two intersecting beams 6 inches wide and 12 inches deep.

The wall of the standpipe is 6 inches thick at the bottom, where it has to withstand 50 feet head of water, and 3 inches thick at the top. The bottom is 3 inches thick, and the junction between the walls and the floor is thickened out, the inside being formed at an angle of 45°. Both the wall and floor are coated inside with a watertight finish 1 inch thick, of 1 to 1 Portland cement mortar. The wall is reinforced with two sets of circular and vertical rods; the upright rods are 2 inches apart transversely, and are staggered, the rods in each set being spaced about 16 inches apart circumferentially.

The two sets of horizontal hoops each encircle one of the sets of vertical rods; they are made of ½-inch rods with welded joints in the lower two-thirds of the height, and of ⅜-inch rods with wire-wound lap joints for the upper one-third.

The vertical spacing of the hoops increases from the bottom upwards. For the ½-inch hoops there are 23 :—1¾-inch, 41 :— 2-inch, 34 :—2½-inch, 22 :— 3-inch, 13 :—3½-inch, and 23 :—3¾-inch spaces. For the ⅜-inch hoops there are 9 :—3-inch, 6 :—3½-inch, and 6 :—3¾-inch spaces, the inner and outer hoops at each level up to this height being in the same horizontal plane. For the remaining 16 feet the two sets of hoops are staggered, the vertical distances between the successive inner and outer hoops increasing from 2 inches to 7½ inches; that is, the hoops on each set are spaced from 4 to 15 inches apart.

The bottom of the standpipe is reinforced with two sets of ¼-inch rods, spaced 4 inches, centre to centre, and crossing one another at right angles. The rods are bent up at their ends, extending into the wall for a height of about 12 inches. The junction of the wall with the bottom is further reinforced with a set of ⅜-inch rods, extending about 20 inches radially into the floor slab, and extending to a height of about 24 inches in the wall; these are placed in the centre of the thicknesses of the wall and floor, and are bent at angles of 135°, passing near the inner face at the thickened angle between the two. A hoop of ⅜-inch rod bears against their underside at the centre of the angle, and they are also anchored into the corner by stirrups of 1 × ¼-inch steel, 7 inches long, and spaced 8 inches apart.

Covered Reservoir above Ground at Boulogne-sur-Seine

This reservoir, containing 66,200 gallons, an illustration of which is shown (Fig. 410), was constructed by *M. J. Bordenave* for La Cie Générale des Eaux, 1892.

The reinforcement consists of Bordenave I-bars, the spacing of the spirals varying for each 1·64 feet in height, except near the top, where they are placed closer together to take the thrust of the roof.

The details as to sizes of the bars and thicknesses of the concrete are clearly shown in the illustration, and need no further description.

This reservoir is a good example of such structures as erected by M. Bordenave.

The inlet pipe of cast iron, 16·93 inches diameter, is near the centre, as are

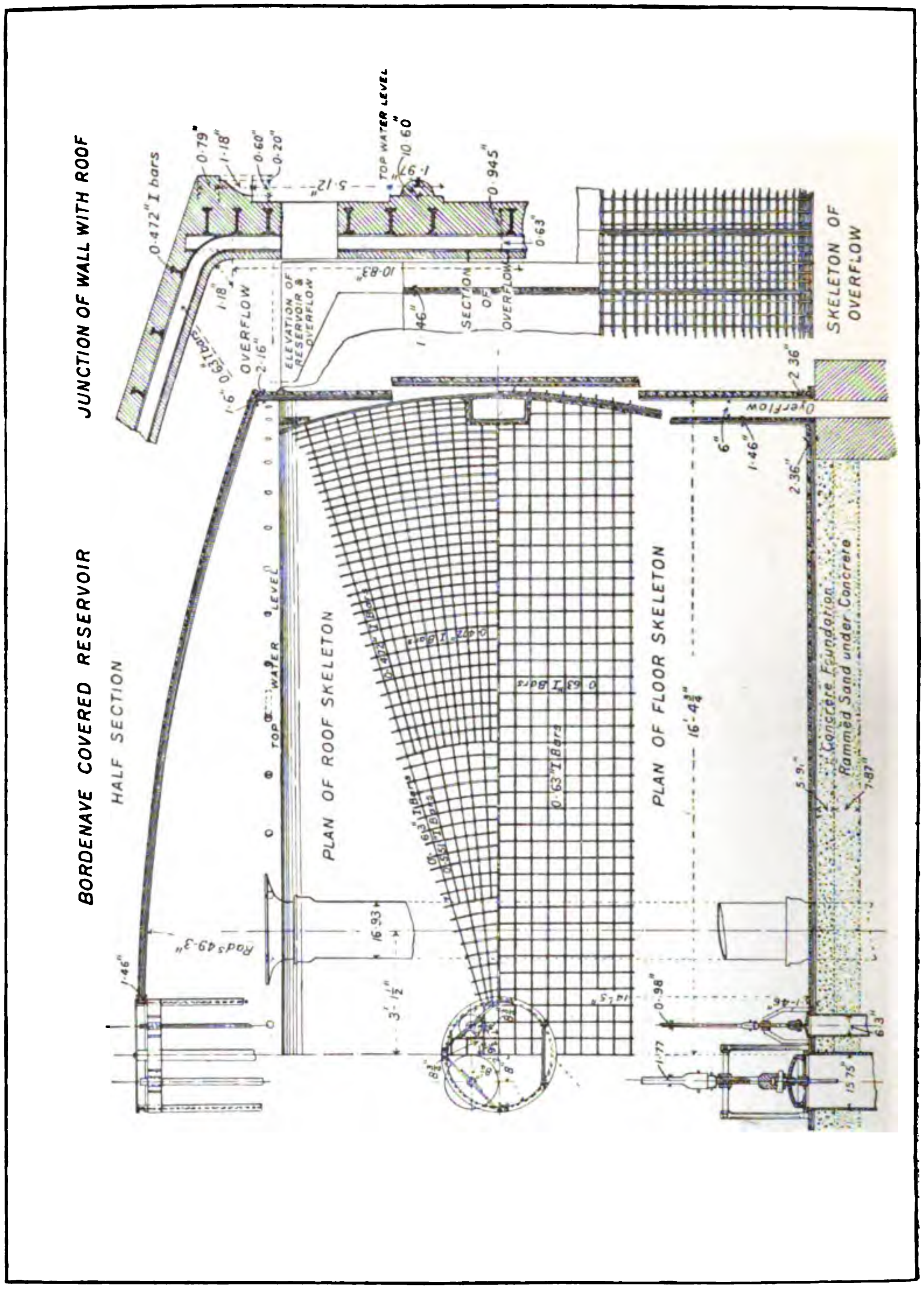


Fig. 410.

the cast-iron outlet and wash-out pipes of 15·75 and 6·3 inches diameter respectively. The overflow is placed at the side, and is formed of reinforced concrete. The arrangement of the outlet and wash-out valves are shown in the illustration, and are similar to those which might be employed for reservoirs of ordinary construction. The ventilation is effected by a series of circular openings left in the concrete just above top-water line.

Circular Covered Reservoir at Memours (France)

This reservoir, which has a capacity of 441,500 gallons, is an excellent example of the *Cottançin* method of constructing service reservoirs.

The floor foundations are of the cellular type of brickwork, with steel wire cores. These are only about 14 inches deep, the open spaces being filled in with well-rammed earth before the floor slab is added, the steel wire basket-work for which is interwoven with the wires of the intersecting foundation walls.

The outer wall is only 4½ inches thick, the columns being 14 × 14 inches square. The steel wire coring is carried up continuously from the foundations through the wall and columns, and is interwoven with the reinforcements of the groined roof, which is formed of arched ribs of reinforced concrete and brickwork, intersecting one another to form the groining. These ribs are 14 inches deep, and have a rise of 19·7 inches, while the thickness of both the roof slab and arched ribs is only 2 inches, their respective steel wire networks being interlaced, so that the whole roof acts together. Fig. 411 is a view of the roof during construction.

The cost of this reservoir was £756.

A similar form of construction is used for gasholder tanks, except that in this case, since there is no roof, the outer wall is reinforced with vertical and horizontal steel-cored brick ribs, the network of which is tied to that of the main wall.

FIG. 411

FIG. 412

Interior of a reinforced concrete Reservoir at Lausanne, built on the *Hennebique system*. Capacity, 2,640,000 gallons.

FIG. 413

Elevated Reservoir on the *Coularou system*, constructed near Canes (Gard).

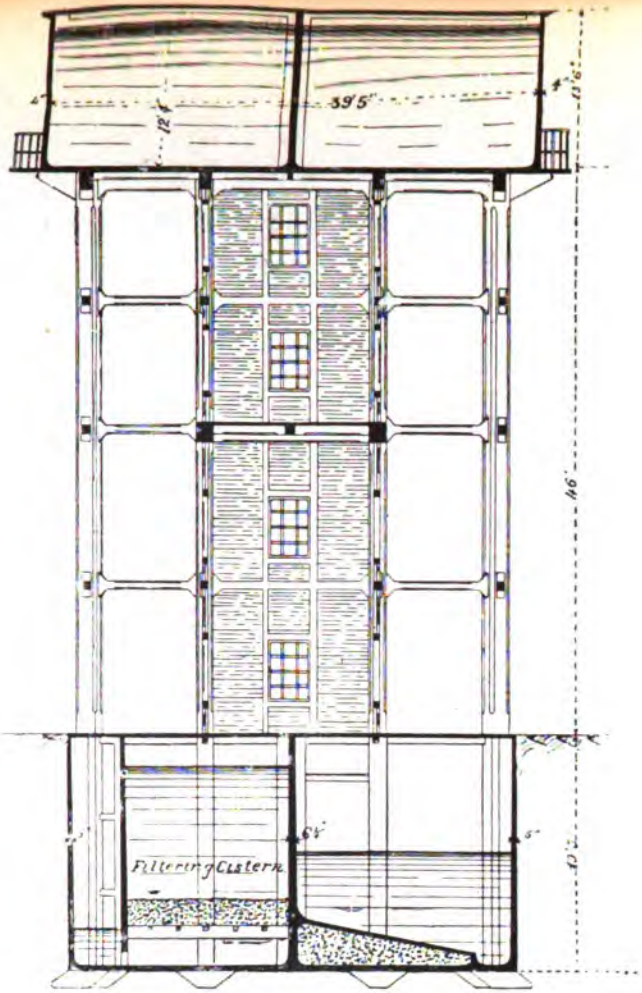


FIG. 415

Elevated Reservoir at Bellancourt, near Paris, on *Hennebique system.* Capacity, 120,000 gallons.

FIG. 414

Reinforced Water Tank on the *Hennebique system* at Bournemouth; 15,000 gallons capacity; diameter, 20 ft. 10 in.; height of tank, 10 ft.; thickness of sides, top portion, 4 in., bottom portion, 5 in.; thickness of bottom, 5 in.; supporting columns 35 ft. high, section 18 × 18 in.

FIG. 416

Elevated Reservoir on *Hennebique system* at St. Mariel, France. Capacity, 60,000 gallons.

Cement Bins at Chicago

Figs. 417 and 418 show the plan and section of a set of four cylindrical cement bins of reinforced concrete on the *Monier system*, lately built for the Illinois Steel Company in South Chicago.[1] The total capacity of the bins is about 1,150 tons. The scale on Fig. 418 applies to the plan also. The base is formed of a layer of concrete 3 feet thick, with a netting of ⅜-inch round steel rods, with a 9-inch mesh embedded about 5 inches from the bottom, the rods being tied together at their intersections with No. 18 wire (U.S. standard) 0·05 inch diameter.

A series of piers rise from this bed, being 12½ feet high and 22 inches wide; those near the outer circumference of the bins are 3 feet 5 inches long, but near the central portion of the structure they are from 6 feet 8 inches to 7 feet 10 inches long.

[1] Described in *Engineering News*, December 11, 1902.

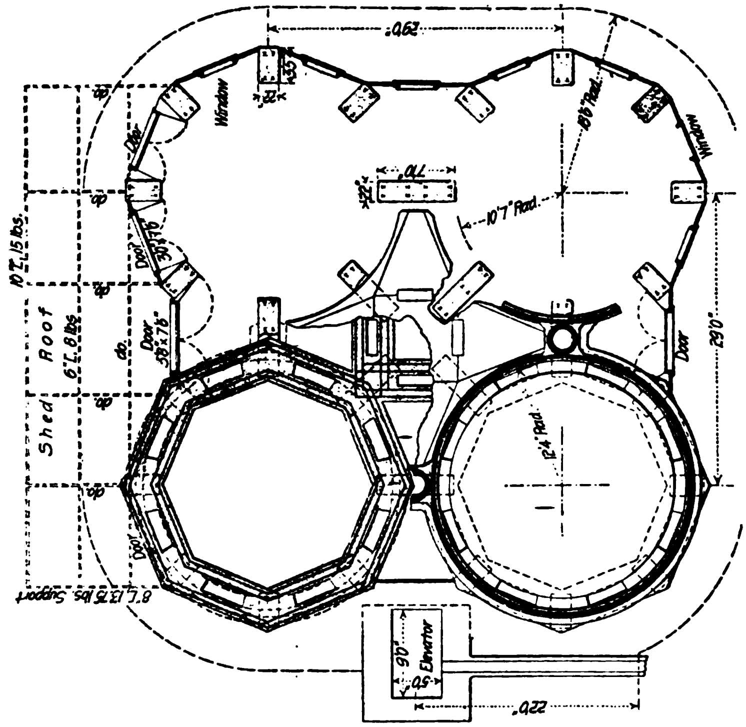
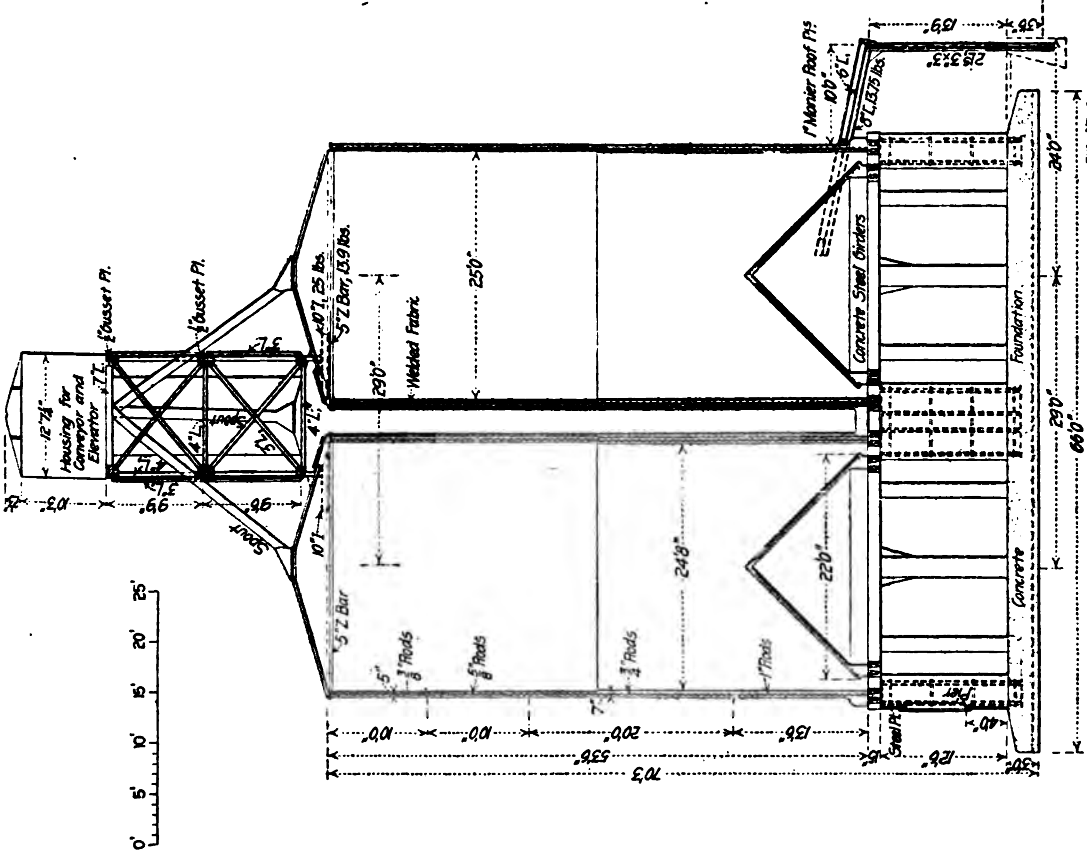


Fig. 417

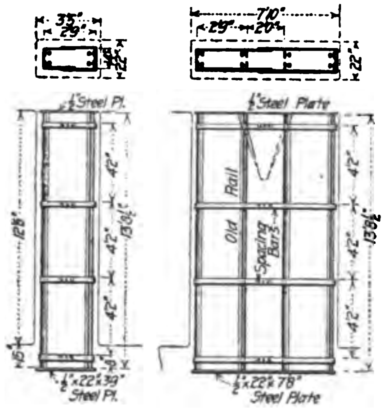


Vertical Section.
Fig. 416

G G

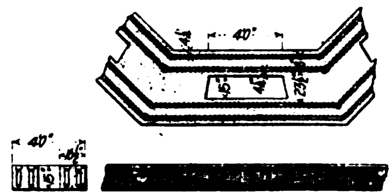
The small piers have a reinforcement of four steel rails connected by spacing bars riveted to them, and resting on ½-inch steel plates embedded about 15 inches below the surface of the floor slab, and their upper ends abut against similar plates. The larger piers have a reinforcement of six or eight rails. Details of the piers are shown in Fig. 419. The tops of the piers are connected by 4 feet by 15-inch reinforced concrete girders on caps, with vertical openings at intervals for the discharge spouts.

Each cap is reinforced by four horizontal straight rods near the top and four rods bent in the form of a truss, with sheets of wire netting, on each side of each pair of rods. Fig. 420 shows the details of the caps. The bins rest on this system of capping, their walls being 7 inches thick in the lower part and 5 inches thick at the top.



Detail of Piers.

FIG. 419

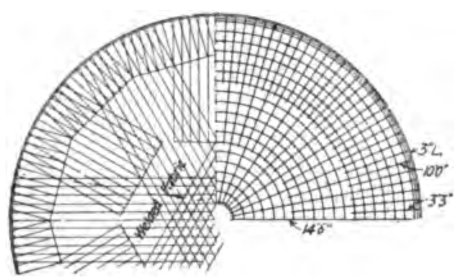


Detail of Girder.

FIG. 420

The reinforcements consist of a continuous network of No. 9 wires (0·156 inch diameter) electrically welded at their intersections, forming meshes 1 inch by 4 inches around the network, and alternately inside and outside are placed a series of rods 1 inch in diameter near the base and varying, as shown in Fig. 417, to ⅜-inch diameter at the top ; these rods are placed 4 inches apart and are tied to the netting by wires.

The top of each bin is finished by a ring of 5-inch Z-bar. The conical roofs are 2 inches thick, with a manhole at the edge and an opening at the top for the spout. The bottom of the bins are made conical, with eight discharge openings in each in the annular space between the base of the cones and the sides of the bins. The openings are about 15 inches by 48 inches, and each serves two spouts leading to conveyors. The conical bottoms are 4 inches thick, reinforced with rods and netting, as shown in Fig. 421. Between each of the four cylindrical bins is a 30-inch cylindrical shaft, and a further bin is constructed in the central portion between the cylindrical bins, details of which are shown in Fig. 422.



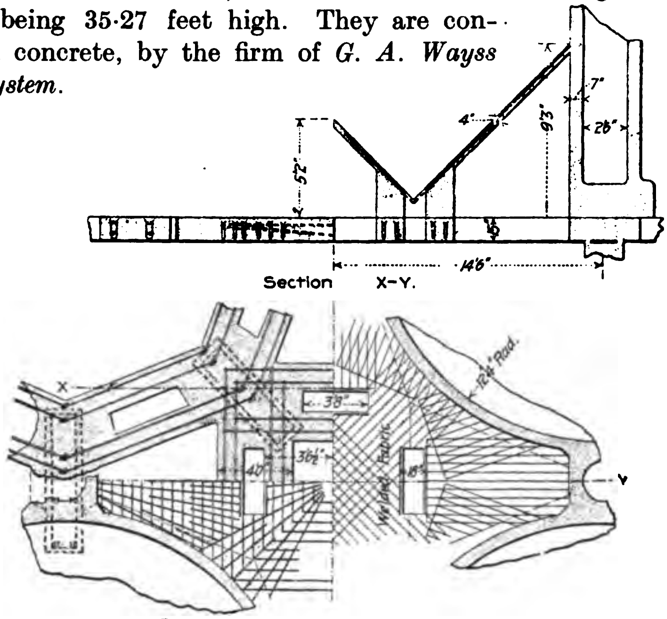
Detail of Conical Tank Bottom

FIG. 421

The concrete for the foundation slab and piers was formed of 1 part of Portland cement to 3 parts of coarse sand and 4 parts of broken stone. That for the bins was composed of 1 part of Portland cement to 3½ of sand, no stone being employed. The concrete was mixed moderately wet, and lightly rammed in wooden moulds.

Grain Silos at Obéramstadt (Germany)

This is a very large structure of thirteen silos, six of which have a total height of 44·78 feet, the remainder being 35·27 feet high. They are constructed entirely of reinforced concrete, by the firm of *G. A. Wayss and Fraytag*, on the *Monier system*.



Detail of Central Tank Bottom.
FIG. 422

The walls have a thickness of 8·46 inches at the bottom and 4·72 inches at the top, and rest on columns 19·69 × 15·75 inches, reinforced with nine rods 0·86 inch diameter. The feet of the columns rest on a reinforced slab of 9·84 feet wide and 9·84 inches thick, which runs round the whole structure, tying the bases of the columns together.

The bottoms of the silos are formed inclining both ways from the centre, and are 7·87 inches thick, and rest on reinforced beams which run from column to column. These beams are reinforced with four rods, two of 0·87 inch and two of 0·71 inch, placed along the bottom.

The reinforcement of the walls consists of pairs of vertical rods 0·59 inch diameter, one at each side of the wall, and horizontal rods of diameters between 0·71 and 0·27 inch diameter, and varying according to the pressure, the several sizes being intermixed as required to obtain the proper sectional area of metal for the portion of the height under consideration. The tops of the walls are stiffened by a 9·84 × 9·84 capping, reinforced with four rods of 0·47 inch diameter, one at each corner.

The whole series of silos only cost £675, or about half the cost of a similar structure in ordinary masonry. Fig. 423 shows the general arrangement, and Fig. 424 the details.

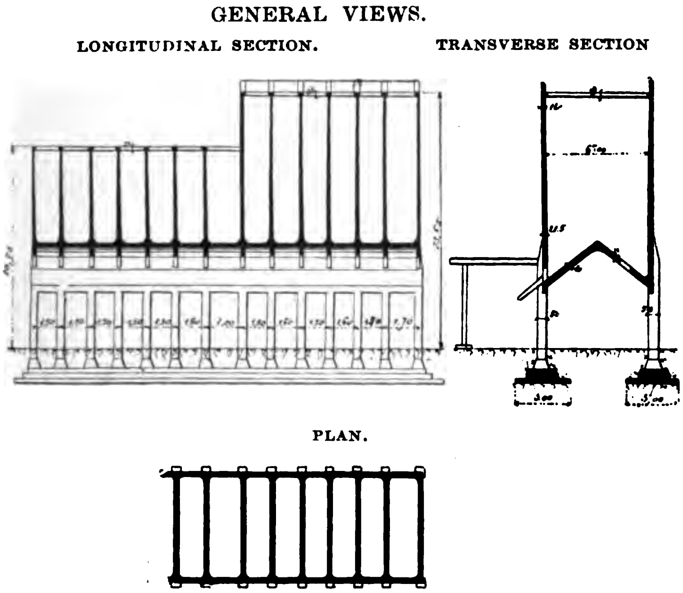


FIG. 423

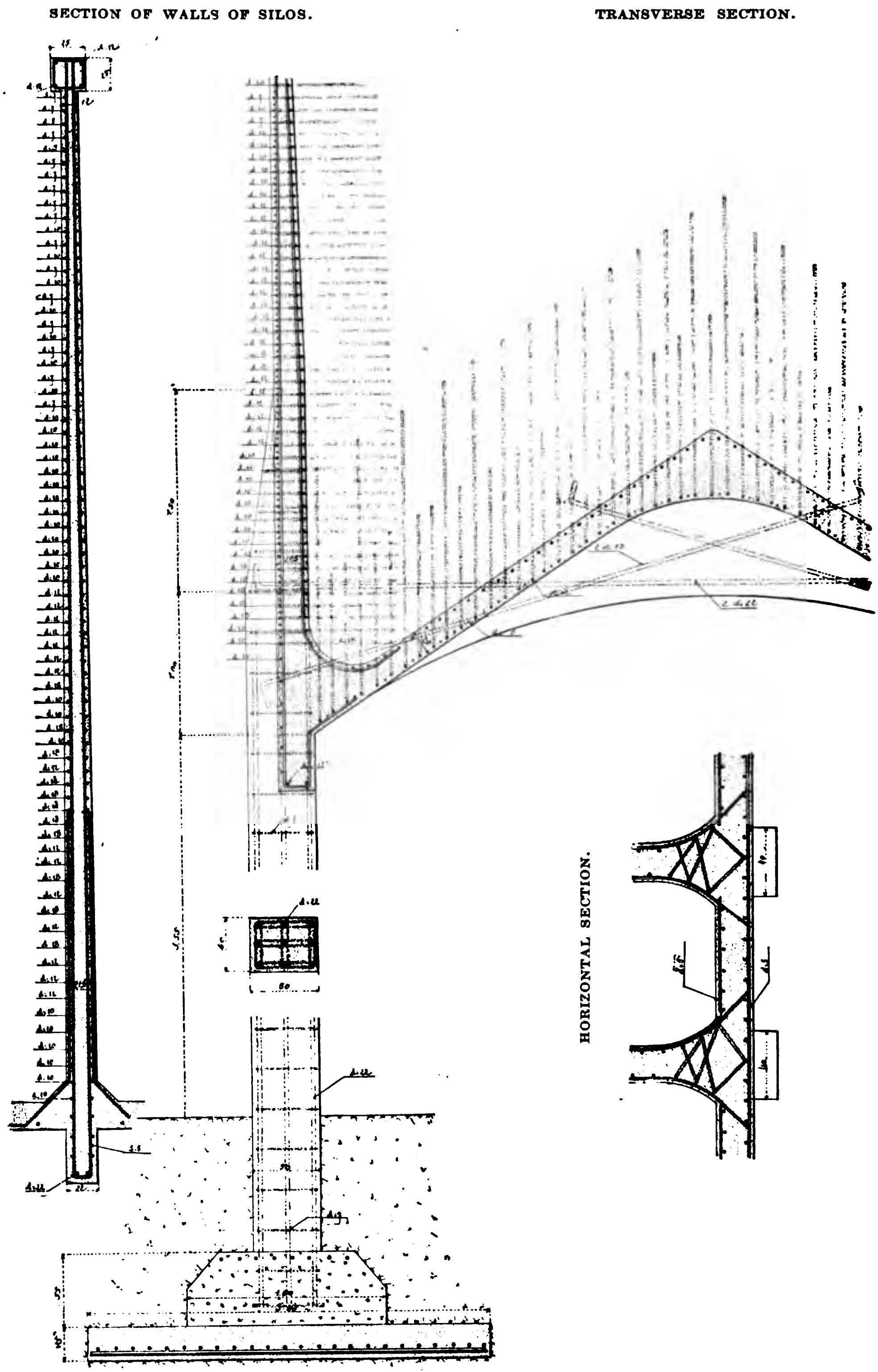

SECTION OF WALLS OF SILOS.
TRANSVERSE SECTION.
HORIZONTAL SECTION.

Fig. 424

III. AQUEDUCTS, PIPE LINES AND SEWERS.

The Aqueduct carrying Water to the Simplon Tunnel Works

This aqueduct carries the water from the Rhone to the northern face of the new tunnel for use in ventilating and mining. The canal is 1·86 miles long, with a fall of 6·35 feet per mile. A view of a portion of the length is given in Fig. 425. The

FIG. 425

canal is roofed over, and is carried on supports 16·4 feet apart. The details are shown in Fig. 426.

The conduit is 6·23 × 6·23 feet inside, and has side walls 3·94 inches thick;

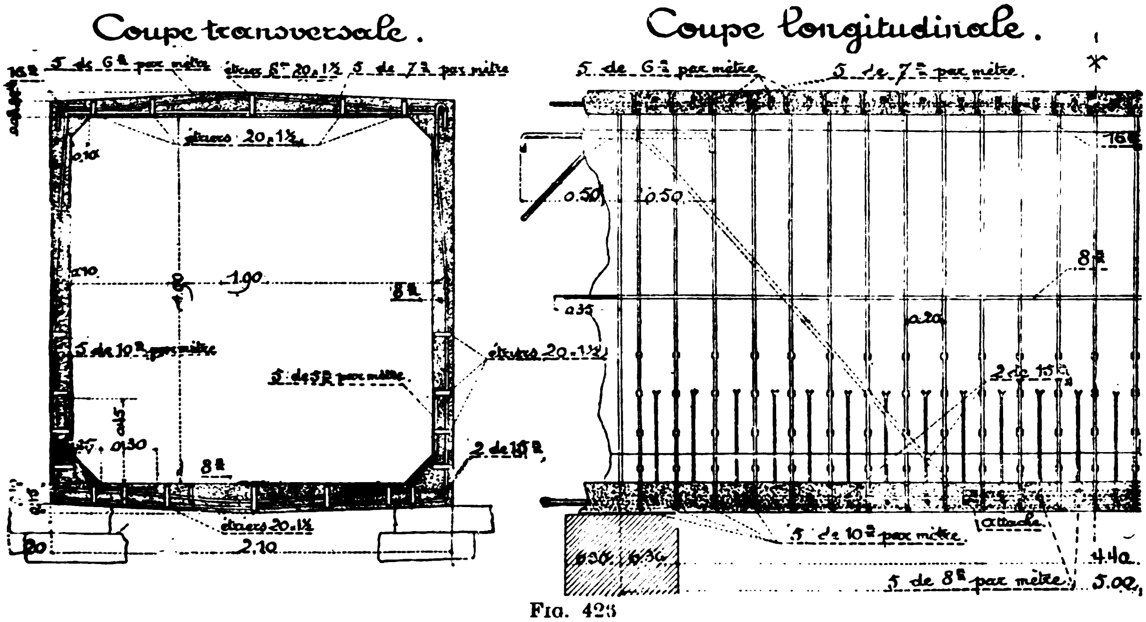


FIG. 426

the bottom having a thickness of 3·94 inches at the sides, and being thickened out towards the centre, where it attains a thickness of 5·91 inches. The roof is formed in a similar manner, having a thickness of 3·15 inches at the sides and 4·72 inches at the centre. The vertical reinforcing rods in the walls are 0·394 inch diameter, and are spaced 7·87 inches apart. There are also three longitudinal rods at the top, bottom and centre, the one at the top being 0·63 inch, that at the centre 0·315 inch, and that at the bottom 0·59 inch in diameter. There is also a trussed rod 0·59 inch diameter passing along the bottom at the centre of the span, and being bent up to the top at the sides.

The reinforcement of the bottom is made up of two sets of rods of 0·394 inch and 0·315 inch diameter, both sets being spaced 7·87 inches apart; the 0·394 inch rods are bent up and form the side reinforcements, and those of 0·315 inch diameter are also bent up and extend 1·48 feet into the sides. There are also two longitudinal rods of 0·315 inch diameter, spaced evenly on each side of the centre.

The top is reinforced with two sets of rods, each spaced 7·87 inches apart, one being 0·275 and the other 0·236 inch in diameter. All the rods have stirrups passing round them, these being formed of 0·787 × 0·038 inch hoop iron.

The general arrangement of the reinforcements is clearly shown in the details.

The canal is supported on reinforced concrete columns with a mean height of 13 to 19·5 feet, and was constructed on the *Hennebique system*. The loading used in the calculations was an exterior load of 61 pounds per square foot, and a head of water inside of 1·31 feet above the top of the side walls.

FIG. 427

Canal of reinforced concrete on the *Hennebique system* at Frenvilliers, Switzerland.

When constructing the canal, expansion joints open to the interior were left over the supports ; these joints were filled after the first contraction of the cement, some few being covered externally with a sheet of flexible metal.

The canal is always full of water, and therefore does not suffer from changes of temperature. Any small fissures are very soon filled up by the silt carried down by the water, and leakage is thereby prevented. The cost was about £3 13*s*. per yard run.

Pipe Line, 2,300 yards long, for the Hydro Electric Works of Champ (Isére)

This conduit was constructed by the firm of Rossignol and Delamarche, of Grenoble, and is designed to resist a head of 65·6 feet of water.

The invert of the pipe proper is surrounded with a layer of lime concrete, the cross-section of which has an extrados in the form of a polygonal figure of five sides, the bottom being 5·58 and the other sides 4·23 feet across.

The least thickness of this outer layer was 10 inches, and the greatest (at the bottom) about 2 feet. The diameter of the pipe is 10·82 feet, and the thickness of shell from 7·9 to 9·8 inches.

The reinforcement consists of hoops formed of iron rods from 0·43 to 0·89 inch diameter, and longitudinals of rods from 0·24 to 0·48 inch diameter. The hoops were spaced 4 inches apart, and the longitudinals 4½ inches. The length of the rods was approximately 37 feet, and the hoops were constructed on the site of the works, three men forming about 3,000 hoops per day, or sufficient to construct a length of 984 feet.

The joints of the hoops were welded, the rods being jumped up so that the joint was somewhat thicker than the rest. Six forges were kept constantly at work, each welding 500 hoops per day. The semi-circular layer of lime concrete was first formed ; when this was in place five ribs of 2 inches depth were carefully formed running parallel to its axis on which the hoops were placed. When the hoops were in position the longitudinals were placed inside them, at the proper distance apart, and the two tied together with iron wire.

The mould for constructing the pipes was formed of—

1. An extensible core.
2. An outer shell in two halves.
3. An end cap in two pieces, one inside and one outside the reinforcement.

The core and shell were held together by distance pieces and bolts, and their spacing was ruled by the thickness of the cap, which was 7·9 inches.

The length of the mould was 14·76 feet, which allowed a length of 13·12 feet to be constructed without moving the falsework, which was heavy and not easily handled. The cement mortar forming the shell of the pipe was run in.

The Duplication of the Inverted Syphons Carrying Water to Brussels

In September, 1902, the Compagnie Intercommunale des Eaux of Brussels commenced the duplication of several of their syphons, using reinforced concrete pipes. The first portion, a length of about 2½ miles, was carried out by *M. Bonna*, and completed in August, 1903. The pipes laid under this contract have in a large measure been in use for over a year, and have given every satisfaction.

The second portion of the work, which was commenced in April, 1904, has been let to *M. Bordenave*, and comprises a total length of about 2 miles.

The concrete for all the pipes is mixed in the proportions of 1,180 pounds of quick-setting cement of the Porte de France to 1 cubic yard of clean gravelly sand.

The ultimate tensile strength of the steel is 56,880 to 63,990 pounds per square inch, with a mean elongation of 20 per cent., and a minimum of 18 per cent.

The steel has to be bent both ways, hot and cold, without any sign of cracking.

The tubes inserted between two reinforcing skeletons in the Bonna pipes, when the head exceeds 65·6 feet, are of steel $\frac{1}{25}$ of an inch thick.

The pipes measure 13·12 feet when laid, and the variation in thickness of shell is not allowed to exceed 0·17 inch.

The length of the collars at the joints is 11·8 inches.

The size of the spirals is calculated, allowing a safe stress of 11,376 pounds per square inch on the steel.

Tables LXIX and LXX give the details of the various syphons—

TABLE LXIX

SYPHONS CONSTRUCTED BY M. BONNA

Description of Syphon	Diameter Inches	Length, Yards		Maximum Heads Feet
		With Steel Tubes Inserted Head over 65·6 ft.	Without Steel Tubes Head under 65·6 ft.	
Syphon de la Thyle .	27·56	88	339	75·5
Syphon de Rg-d'Her .	23·62	131	481	101·7
Syphon de la Dyle .	29·53	1170	1232	137·8
Syphon du Cala . .	31·54	175	905	88·6
Total lengths		1564	2957	

TABLE LXX

SYPHONS BEING CONSTRUCTED BY M. BORDENAVE

Description of Syphon	Diameter Inches	Length Yards	Maximum Heads Feet
Syphon de Tongrinne . .	43·3	609	21·3
Syphon de Sombreffe . .	43·3	372	31·2
Syphon de Marbais . .	27·6	344	14·1
Syphon de Waterloo . .	39·4	1091	23·0
Syphon de la Grande Espinette	39·4	703	17·1
Syphon de la Forêt de Soignes	39·4	395	18·4
Total length		3514	

FIG. 428

Pipe Line on the *Bordenave system.*

Argenteuil Tunnel for the Sewage System of Paris

This tunnel is a part of the main drainage system to the sewage farm at Achères. From the pumping station of Colombes to the top of the Argenteuil hill the sewage is carried in pressure mains; from this point, after crossing the Seine, the two conduits of 5·91 feet diameter are laid in a tunnel of the form shown in Fig. 429.

This subway was constructed by *M. Coignet*, and reinforced after the Monier system. The invert is of plain concrete, and slopes each way to a rectangular drain, as shown. The span of the tunnel is 16·92 feet, and the height from invert to crown 10·95 feet. The arch is elliptical, with a rise of 6·56 feet, and the curve of the arch is continued for the side walls, making the section of the tunnel of the horseshoe shape. The thickness of the concrete in the arch and side walls is only 3·54 inches, strengthened at the haunches, as shown in the figure.

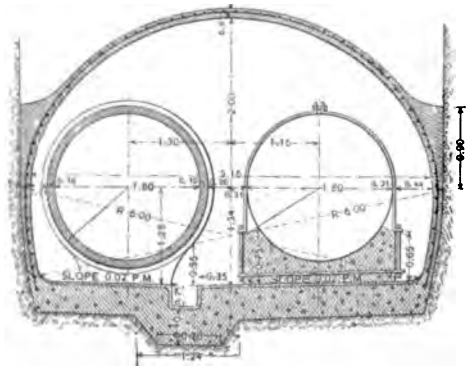


FIG. 429

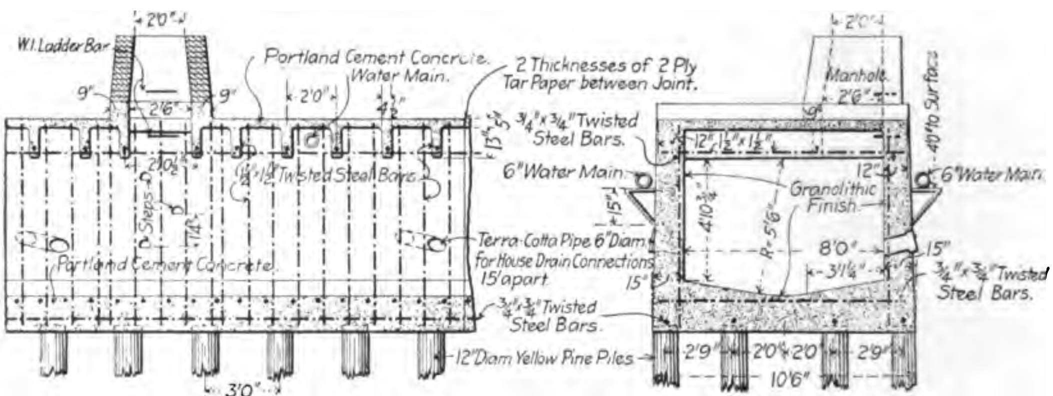
The reinforcement consists of two series of steel rods 4·33 inches apart, the set of rods following the curvature of the arch and side walls are 0·67 inch diameter, and are continuous throughout their entire length; the ends rest in 1·97 × 0·98 inch channel steels. The longitudinals are 0·748 inch diameter, and are placed inside the curved rods from the bottom of the side walls to a distance of 3·28 feet above the springings of the arch. For the remainder of the arch they are placed on the outside.

The curved rods are placed at the centre of the thickness of the concrete.

One of the pipe lines through the tunnel is of steel. The other is of reinforced concrete, 5·9 feet diameter, and 3·94 inches thick, was constructed by *M. Bonna*, and is described in detail elsewhere (p. 33).

Sewer at Philadelphia, U.S.A.

Fig. 430 shows the longitudinal and transverse sections of a reinforced concrete sewer, on the *Ransome* system, constructed in McKean Street, Philadelphia.[1] The special form of section was rendered necessary in consequence of the small cover over the roof.



Longitudinal Section. Transverse Section.

FIG. 430

The roof consists of a series of beams 13 inches deep and 4 inches thick, spaced 2 feet, centre to centre, and reinforced by one straight bar near their underside. The

[1] Described in the *Engineering Record*, April 12, 1902.

concrete was in the proportions of 1 : 2 : 5. The sand ranged in size up to ¼ inch, and the broken stone from ¼ inch to 1½ inches.

The floor and side walls were reinforced as shown, the concrete being in the proportions of 1 : 3 : 6.

The granolithic finish was formed of 1 part of Portland cement to 1 of sand and 1 of granolithic grit. That for the side walls was placed in a layer 1 inch thick, on the face of the moulds, in advance of the main work, and the stones in the body of the concrete were carefully kept back from the face. The granolithic finish to the invert was made 2 inches in thickness. When the surfaces had hardened they were brushed over with a grout of 1 of Portland cement to 1 of sand. Ransome square twisted steel bars were used throughout for the reinforcements.

Sewer at Harrisburg, U.S.A.[1]

Fig. 431 shows a 5-foot intercepting sewer, reinforced with "expanded metal,"

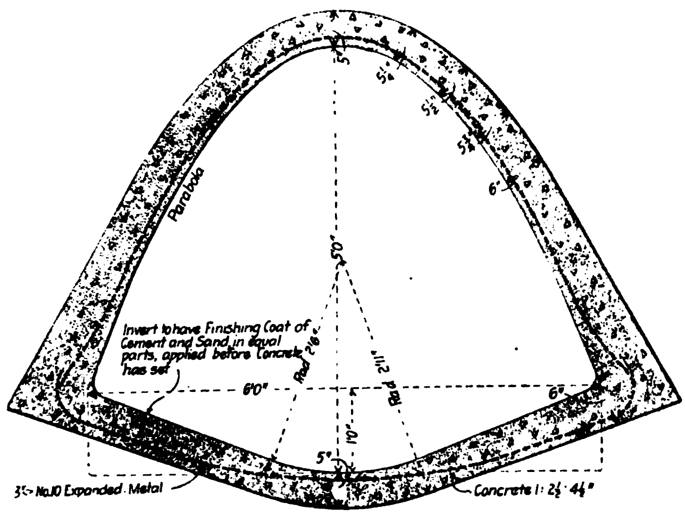


FIG. 431

constructed at Harrisburg, Pa., U.S.A., where a 14-foot by 8-foot inlet regulating chamber and a 5-foot by 3½-foot oval silt basin are also constructed of concrete reinforced with a 3-inch mesh of No. 10 expanded metal, and also a 4-foot sewer of the same form of section as shown.

The lengths of the 5-foot and 4-foot sewers were 385 and 7,635 feet respectively.

Sewer for the City of Clevedon, U.S.A.[2]

This sewer is 3½ miles long and 13½ feet internal diameter, and is being built of reinforced concrete at a cost of about £230,500.

A length of about two miles is 35 to 44 feet below the surface, and is only 17 feet in the clear away from the centre line of the Lake Shore and Michigan Southern Railway. This section is being constructed in an open trench of the cross-section shown (Fig. 432).

[1] Described in the *Engineering Record*, October 11, 1902.
[2] Described in the *Engineering Record*, August 29, 1903.

The invert is not reinforced, but the side walls have two staggered rows of $2 \times \frac{1}{2}$-inch steel bars 15 inches apart centre to centre, built in so as to project into the arch ring. After the centreing is in position, two further rows of similar bars, curved as shown in Fig. 432, are placed in position and bolted to the bars projecting from the invert. To these bars are bolted eight lines of horizontal $1\frac{1}{2} \times \frac{1}{2}$-inch steel bars placed longitudinally.

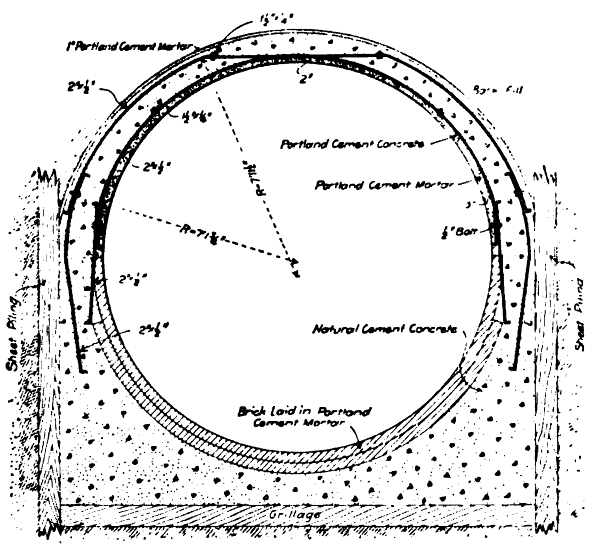


Fig. 432.

Portland cement mortar 3 inches thick is then laid on the lagging embedding the inner row of bars, and forming a finished surface for the soffit of the arch, the lagging having been covered with building paper water-proofed with parafine.

Before this layer of mortar is set, the concrete is rammed in between it and the sheet piling to a height of 18 inches above the springing line, and the remainder of the concrete rammed in against the mortar without the use of outside moulds. The mortar is of sufficient thickness to prevent any of the stones in the concrete penetrating to the soffit.

The arch concrete is generally mixed in the proportions of 1 of Portland cement to 3 of sand and $7\frac{1}{2}$ of broken stone, screened through a $1\frac{1}{2}$-inch mesh, but, where the voids in the stone exceed 40 per cent., the proportions are 1 to 3 to 6. The back filling is commenced as soon as the concrete is 6 to 12 hours old, but the centres are not removed until two weeks have elapsed.

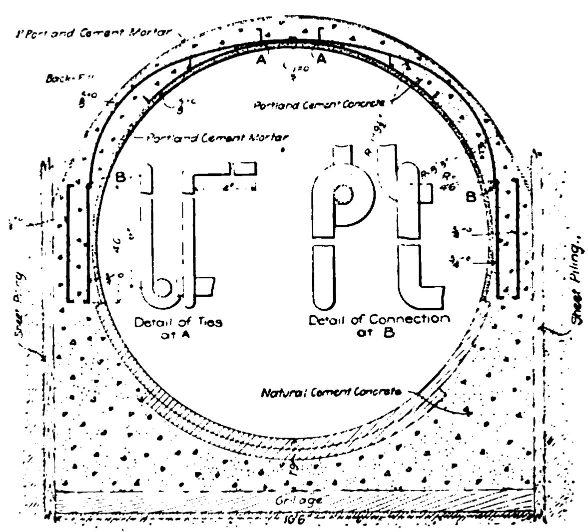


Fig. 433.

The Gilbert Street section, as shown (Fig. 433), is about 2,000 feet long, and from 24 to 30 feet below the surface.

For this section round rods are used for the reinforcement, and are hooked together, as shown in the details (Fig. 433) instead of being bolted, and are placed in a different manner, as shown. The primary rods only are anchored to those projecting from the side walls, the anchoring being at a distance of about 2 feet above the springing line.

These rods are calculated to take the tensile stresses, but are insufficient at the crown, where a secondary system is placed alternately with the main rods, being curved to follow the curve of the intrados, with their ends bent out radially to obtain a hold on the concrete. These secondary rods extend for about 40° on each side of the vertical centre line.

Both sets of rods are ⅝ inch diameter, and are spaced 6 inches apart at the crown.

On these rods are placed ½-inch longitudinals, which are tied to them at the intersections with wire.

At *A* (Fig. 433), vertical hooked and bent rods are placed to tie the curved rods to the concrete at the crown. The reinforcements, which are embedded in the side walls are ¾ inch diameter, the outer series being placed to counteract any tendency to spreading of the completed sewer and to prevent the side walls being forced inwards during construction.

The construction of this sewer is on the system of *Mr. Walter C. Parmley*, M. Am. Soc. C.E., who is Assistant City Engineer, and who is in charge of the sewers.

The average contract price for the sewer of reinforced concrete on the Parmley system is about £13 per lineal foot, whereas for construction in ordinary brickwork the tenders were about £15 15*s*. per lineal foot.

About 3½ miles of other sewers are being constructed in Cleveland on the same system, varying from 5½ to 12½ feet internal diameter, at a cost of about £113,143.

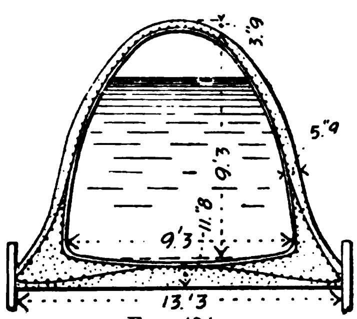


FIG. 434

Sewer constructed at Hamburg on the *Monier system*.

IV. BRIDGES[1]

Cantilever Bridge at Boskoop, Holland, for the Post and Telegraph Department

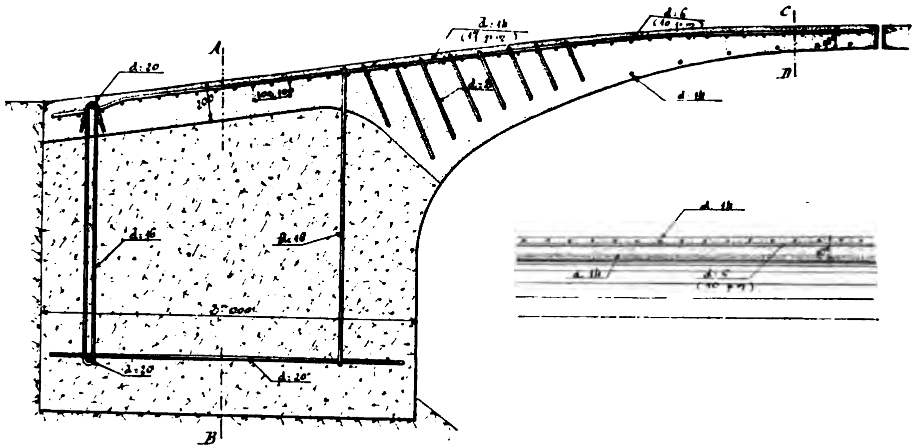
This bridge, which was built by the *Amsterdamsch Fabrick van Ciment Ijzer Werken*, who construct on the Monier system, is in two halves, separated by two channel irons, as shown in Fig. 435. These channels irons very nearly touch each other in reality, the opening being enlarged in the drawing to show it more clearly. This method was adopted in consequence of the bad nature of the foundations. By forming the bridge of two cantilevers the thrust was taken off the abutments, which, from the nature of the case, were the most important part of the bridge. The span is only 16·4 feet, and the arch is elliptical.

The reinforcement of the cantilevers consists of 0·55 inch diameter longitudinal rods 2·23 inches apart and 0·24 inch diameter transverse rods 3·28 inches apart

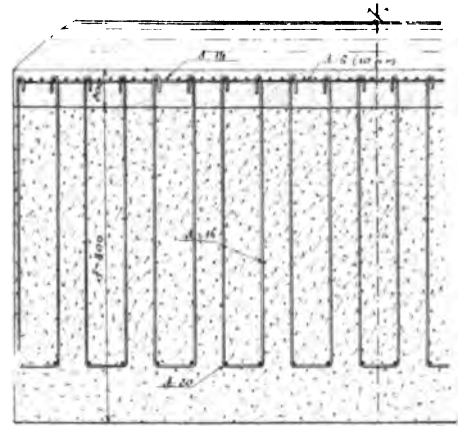
[1] A description of some arched bridges erected by Major Stokes-Roberts, R.E., in India is given in Appendix III.

BRIDGE AT BOSKOOP (HOLLAND).

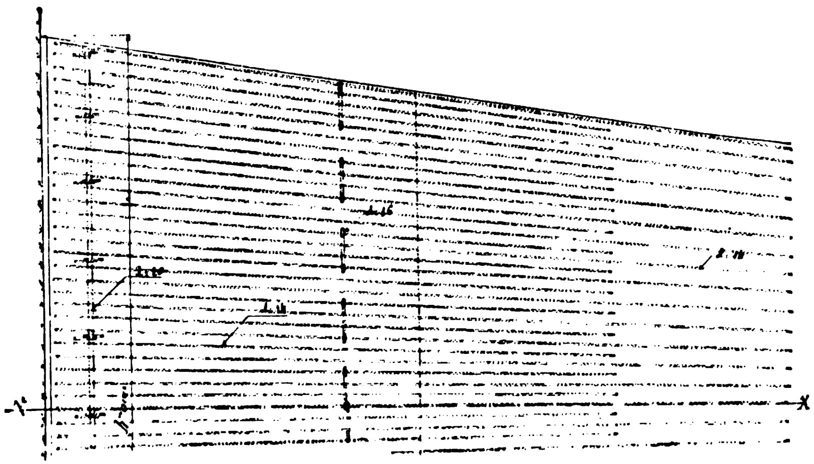
HALF LONGITUDINAL SECTION.



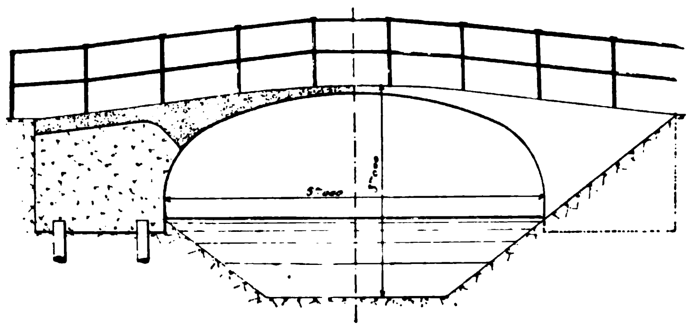
HALF SECTION A B.



HALF PLAN OF UPPER LONGITUDINAL REINFORCEMENT.



HALF TRANSVERSE SECTION.



HALF SECTION HALF PLAN.

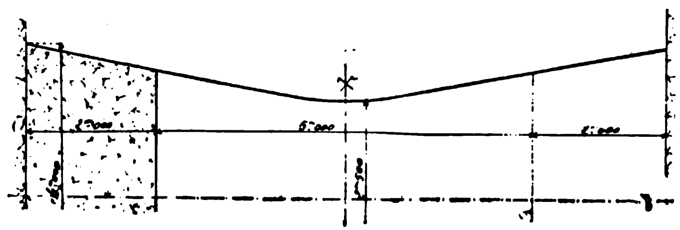


FIG. 435

near the extrados, and a series of 0·55 inch diameter transverse rods near the intrados.

The longitudinal rods of the extrados are tied to the lower series by hooked rods of 0·31 inch diameter. The longitudinal rods are tied down to the lower portion of the abutment near the face by a series of bent and hooked rods of 0·63 inch diameter, each of which is passed round two 0·79 inch rods embedded near the bottom of the concrete. The back portion of the abutment is also tied to the arch concrete in a very similar manner. Fig. 435 shows the details of the construction.

Bridge at Chatellerault, France

This bridge, of which a view is shown (Fig. 436), is constructed of reinforced

FIG. 436

concrete on the *Hennebique system*. The total length of the bridge is 443 feet, and it is built with three arches. The two side spans are 131 feet, with a rise of 13 feet 2 inches, and the central arch has a span of 164 feet and a rise of 15 feet 9 inches. Each span is composed of four ribs of reinforced concrete, 1 foot 7 inches wide, those for the side spans having a depth of 17·3 inches at the crown, and 31·5 inches at the springing, and those for the centre span depths of 21·6 and 35 inches respectively. These are connected by a continuous slab of reinforced concrete 13·8 and 9·8 inches thick for the centre span and side spans respectively, these thicknesses being included in the depths of the ribs as given above. The decking is 25 feet 3 inches wide, made up of a roadway 16 feet 5 inches in width and two footpaths 4 feet 11 inches wide, having a thickness of 7·9 inches at the centre, and 4·7 inches at the sides, the roadway decking is carried by longitudinal and transverse beams having a width of 7·9 inches, these

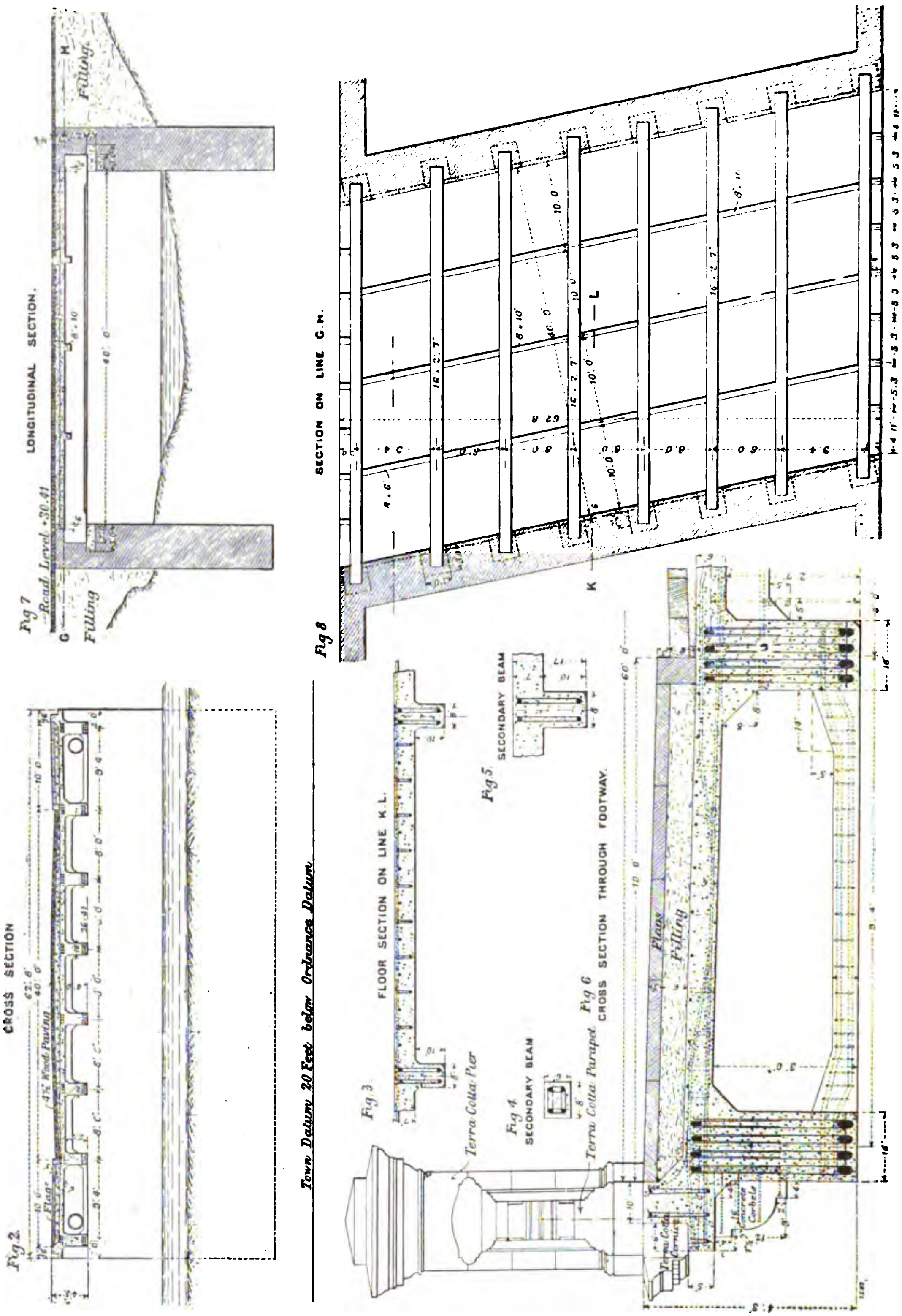

CROSS SECTION
Fig 2
4½ Wood Paving
LONGITUDINAL SECTION
Fig 7
Road Level +30.41
Filling
Town Datum 20 Feet below Ordnance Datum
Fig 8
SECTION ON LINE G.M.
Fig 3
FLOOR SECTION ON LINE K.L.
Fig 4
SECONDARY BEAM
Fig 5
SECONDARY BEAM
Fig 6
CROSS SECTION THROUGH FOOTWAY
Terra Cotta Pier
Terra Cotta Parapet
Flags
Filling

FIG. 437

running longitudinally and transversely, being 11·8 and 9·8 inches deep respectively from the underside of the decking. These beams are supported at their intersections by 7·9 inch columns rising from the arched ribs.

The footpaths are carried for a width of 3 feet 5 inches on cantilevers, projecting from the face of the bridge. The bridge was calculated to bear a rolling load of two files of two-axled carts, weighing 16 tons each, the footpaths bearing at the same time a dead load of 100 pounds per square foot.

The concreting of the arches was commenced on August 15, 1899, and the bridge was completed on November 5 of the same year, the centreing being removed on December 5; after which a series of severe tests were carried out, with very satisfactory results.

Bridge over the Sutton Drain at Hull [1]

This bridge is constructed on the *Hennebique system*, and is the first road bridge of reinforced concrete in England. The method of construction is shown in Fig. 437.

The bridge is slightly on the skew, the square span being 40 feet; it has a width of 60 feet between the parapets, made up of a 40-foot road and two 10-foot footpaths.

The load to be carried was four wagons on the bridge at one time, each carrying 25 tons on two axles 8 feet apart. There are eight main beams 16 inches wide, and 2 feet 7 inches deep to the underside of the decking, which is 6 inches thick. These beams are reinforced with 4 straight and 4 bent rods, $1\frac{3}{4}$ inches diameter along the bottom, and two sets of 4 straight rods $1\frac{1}{4}$ inches diameter along the top.

The cross beams are 8 inches wide and 10 inches deep to the underside of the floor, and are reinforced with two straight rods, along the top and bottom. The cross-beams are only placed under the roadway. Under the footways there are three cross-beams, 8 inches wide and 6 inches deep, connecting the bottoms of the main beams and reinforced with four straight rods tied together with wire ties. These beams are constructed for the purpose of carrying water and gas pipes across the bridge.

The parapets and string courses, which are of terra-cotta, are supported by a projecting ledge of reinforced concrete, carried on corbels.

The cost of the superstructure is stated by the Engineer (Mr. A. E. White, Borough Engineer of Hull), to have been only about half that for steel girders and decking.

Quay Debilly Bridge, Paris

This bridge was constructed for the Paris Exhibition of 1900, being a continuation of the Pont d'Jena over the Seine, giving direct access from the Champ de Mars to the Trocadero. It has a span of 46 feet, and is constructed, together with the abutment walls, entirely of reinforced concrete on the *Hennebique system*.

Fig. 438 shows the details of this bridge and its abutments. The total width across is 98 feet 5 inches, which is made up of a roadway 26 feet 3 inches, and two footpaths each 36 feet 1 inch wide.

The decking is 7 inches thick at the centre of the roadway, reducing to

[1] Description in *Engineering*, January 2, 1903.

4·92 inches at the curbs. The footpaths are raised 3·94 inches above the roadway, and have a thickness of 4·72 inches. This decking is carried on 12 reinforced ribs of 46 feet span and 2 feet rise, having a curved intrados and flat extrados,

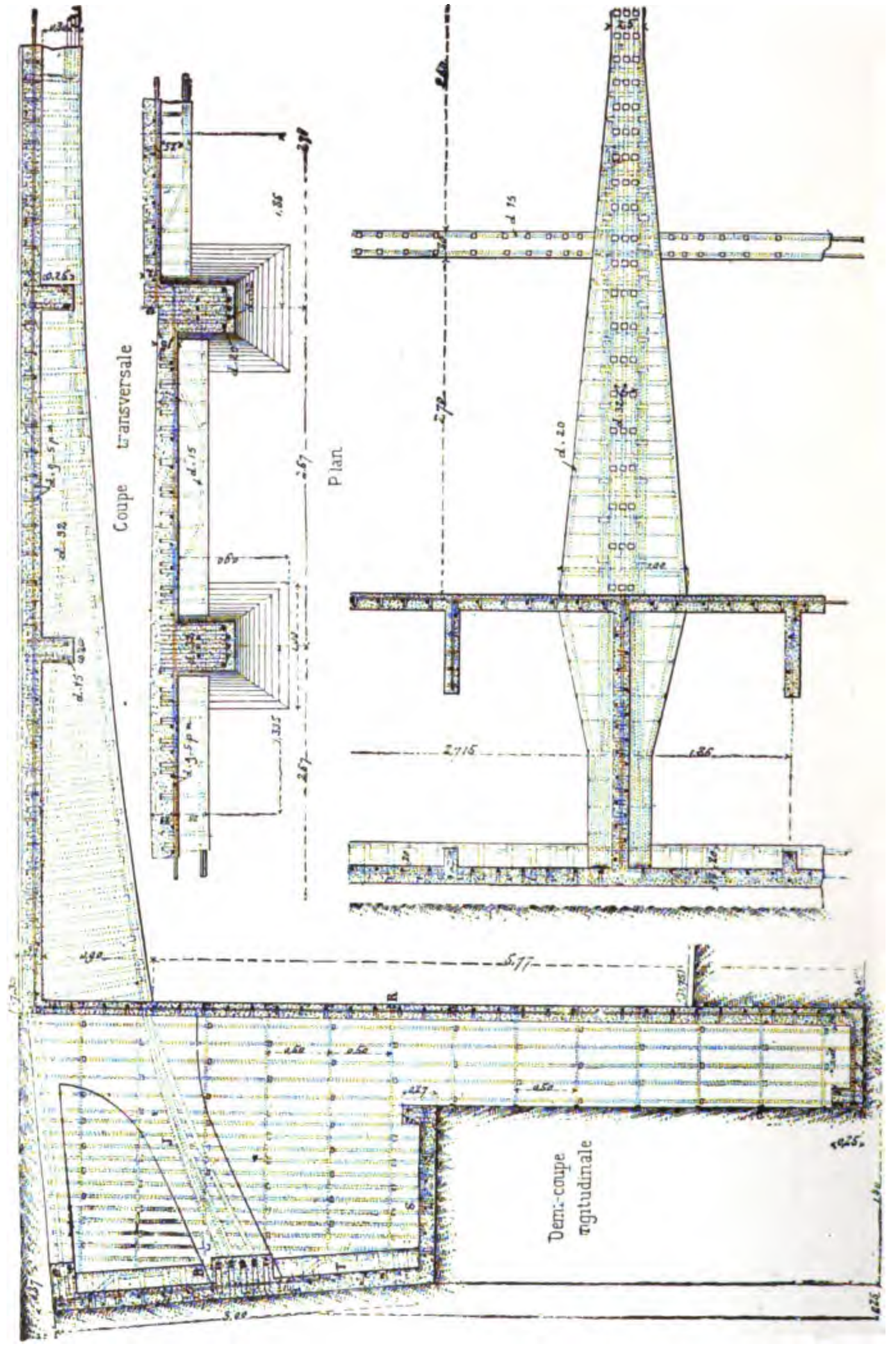


FIG. 438

their depth at the centre being 11·8 inches and 35·4 inches at the springings, spaced 8 feet and 8 feet 11 inches centre to centre, the three at the centre having the 8-foot spacing. They remain of an even width of 9·8 inches for some distance on each side of the centre, and then taper out on both sides till their width at the springing is 3·28 feet, except in the case of the two outside ribs, which only taper on the one side.

The curved ribs are reinforced with three straight and three bent up rods 1·26 inches diameter, and two straight rods 0·79 inch diameter. The cross beams carrying the decking have two bent and two straight rods of 0·59 inch diameter. There are four of these beams in the length of the bridge, having a width of 7·87 inches, and a depth of 9·84 inches to the underside of the decking. The decking and footways are reinforced with rods crossing at right angles 0·35 inch diameter, and spaced 7·9 inches apart.

The ribs are carried back, beyond the face wall of the abutment, being anchored to a slab of reinforced concrete which has an L-shaped section, and is placed some distance behind the face wall. A vertical diaphragm wall is carried down on the centre line of each rib, tying the face and back L-shaped walls of the abutment together.

The face wall of the abutment is carried down to a lower level than the arch or wall, and is also L-shaped, but turned the opposite way. Besides the diaphragm walls mentioned above, there are an intermediate series of narrow ribs carried down to the bottom slab of the face wall. The form of the abutment is clearly shown in Fig. 438. The arched ribs are connected by four rows of cross-beams 7·9 by 9·8 inches (the depth being measured to the underside of the decking). These are spaced 9 feet 2 inches centre to centre, which assist in supporting the decking.

The bridge is calculated to bear a uniformly distributed load of 123 pounds per square foot.

The view (Fig. 439) shows the extreme lightness and graceful appearance of the structure.

FIG. 439

Decking of Bridge over Lincoln Park Lagoon, Chicago

Fig. 440 shows the decking of the cantilever bridge across Lincoln Park Lagoon, Chicago,[1] constructed by Mr. M. F. McCarthy. The decking is constructed of a series of longitudinal arches, 9½ inches span and 8½ inches rise; the total depth of the decking is 13 inches, the top 1 inch being formed of granitoid dressing; the width of the concrete between the consecutive arches is 2½ inches; the decking is carried on cross girders spaced 18 feet centre to centre.

The reinforcement of the decking consists of two groups of four steel wires 0·172 inch diameter, or No. 8 U.S. standard wire gauge. These are placed at the centre of the ribs of the arched deckings, as shown in the figure. One group is bent down over the tops of the cross girders, and is carried along the bottom of the decking, with about half an inch covering of concrete. The other group is sloped down from the top of the cross girders to the centre of the span of the decking, and is there attached to the lower group.

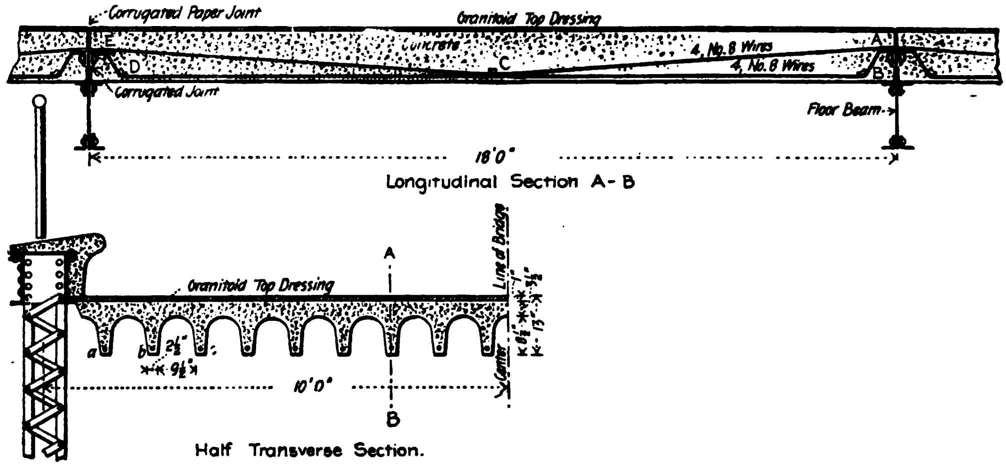


Fig. 440

A joint of corrugated paper is made over each cross girder, and also on each side of the web, to allow for expansion and contraction. The top of the cross girders are about 6 inches above the lower surface of the decking. The cost of this decking was about 1*s*. 0½*d*. per square foot.

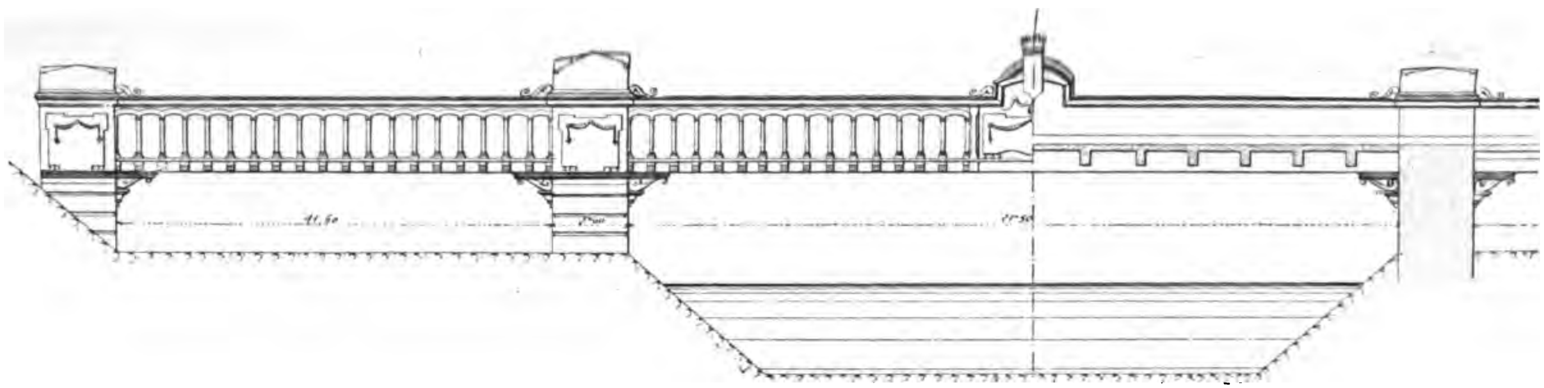
Skew Bridge at Milan, supported by Reinforcing the Parapets

This bridge was constructed by *M. Maciachini on the Walser-Gerard system.* It has a centre span of 83·66 feet and two side spans of 34·78 feet, measured on the skew, the skew angle being 65°. The piers are 6·56 feet thick, and are also of reinforced concrete. The width of the bridge between parapets is 22·96 feet, including two footpaths 3·28 feet wide. The main beams, which also form the parapets, are 1·97 inches thick and 6·56 feet deep. The height above the footpaths is 3·28 feet.

The beams for the 83·66 feet span (shown in Fig. 442) had not sufficient area of concrete to resist the compressive stresses, and had therefore to be reinforced with 22 rods of 1·89 inches diameter. The bottom reinforcement consists of

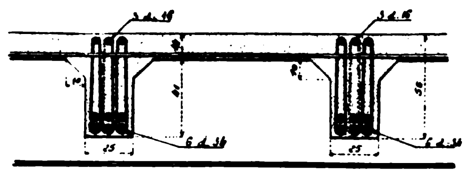
[1] Described in the *Engineering News*, April 4, 1895.

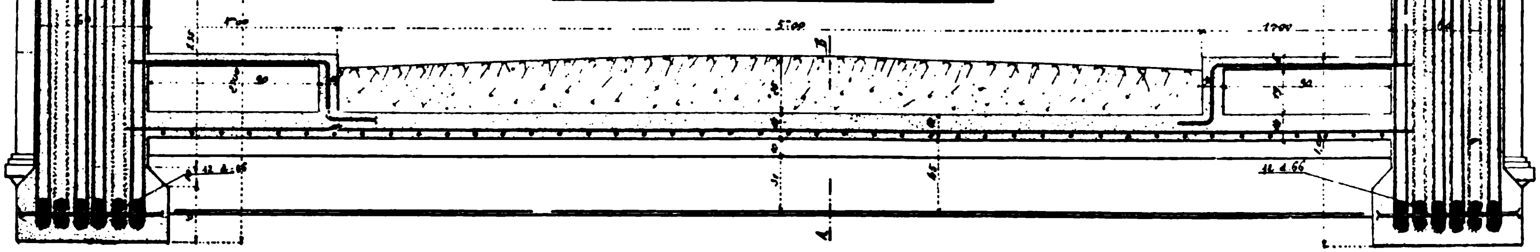
Elevation.



Bridge at Milan. System Walser-Gerard.
Fig. 441

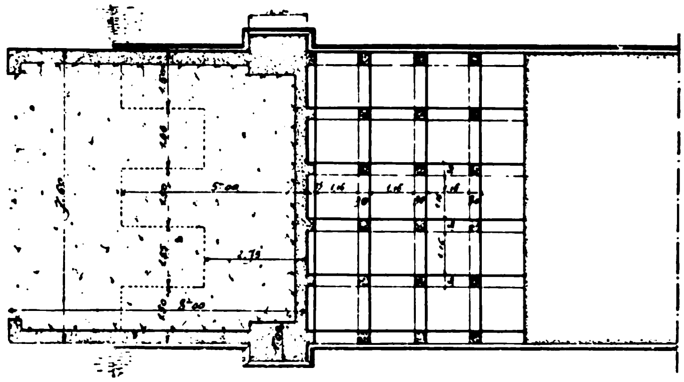
Section AB



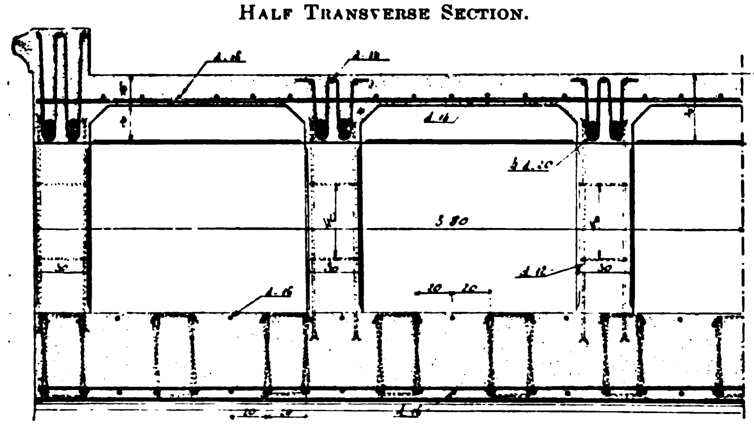


Transverse Section of Centre Span.
Fig. 442

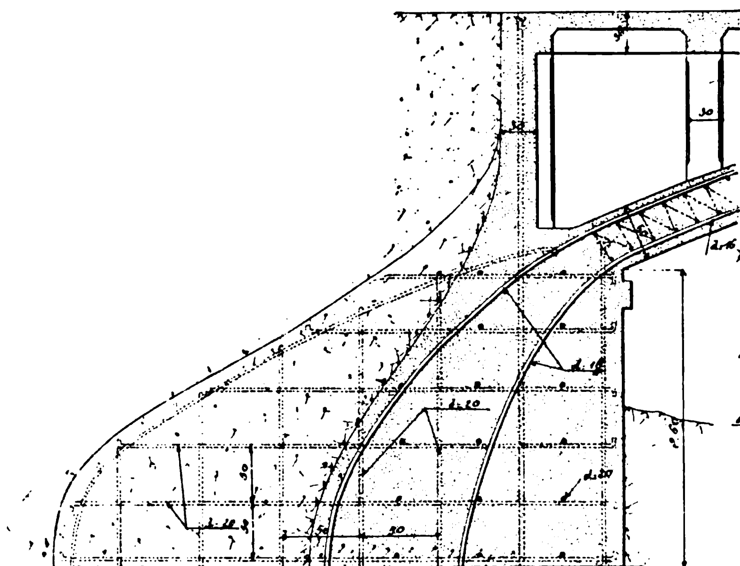
Half Plan.



Half Transverse Section.



Detail of Abutment.



Elevation.

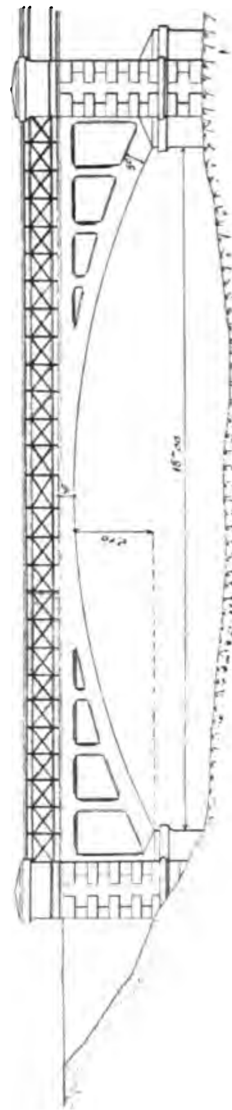
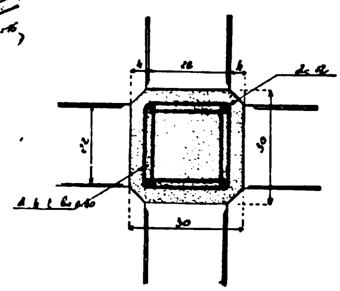
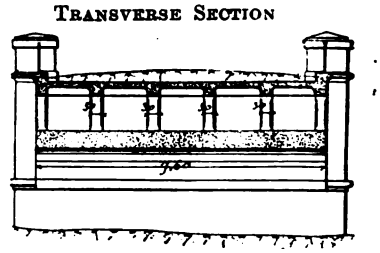


Fig. 443

Section of Columns.



Transverse Section



twelve rods of 2·59 inches diameter. A series of small rods are placed transversely between each set of longitudinal rods.

The decking, which is 5·5 inches thick, is carried by cross-beams 9·84 inches wide, and projecting 16·14 inches below the decking, with which they are monolithic. There are seven of these beams for the side spans and thirteen for the centre span. These cross-beams are reinforced with three rods of 0·63 inch diameter at the top, and six rods of 1·34 inches diameter along the bottom. The decking is reinforced with longitudinal and transverse rods. A general view of the bridge is shown in Fig. 441, and the details of the cross beams, decking and centre span main beams in Fig. 442.

Bridge over the River Bormida, at Altare (Italy)

This bridge was constructed in 1901 *by M. Maciachini on the Walser-Gerard system*, and replaced one of the spans of an old timber bridge. It is formed of a segmental arch of 59 feet span and 6·9 feet rise, and 24·93 feet wide, having a depth of 11·8 inches at the crown and 19·7 inches at the springings. The decking is supported from the arch by means of columns 3·81 feet centre to centre both ways, having a section of 11·9 × 11·9 inches. The columns are connected at the top by longitudinal and transverse beams, 8·7 inches wide and 7·9 inches deep below the decking, which has a thickness of 6·3 inches. The arch is reinforced with 0·63 inch diameter rods near the intrados and extrados, spaced 7·9 inches centre to centre, and tied together as shown in Fig. 443. In the half transverse section and the detail of the abutment a series of transverse rods also 0·63 inch diameter are placed over the bottom series of longitudinal rods. The longitudinal rods are carried down almost to the bottom of the abutment and pier. The abutment is 9 feet thick, with three counterforts, extending the thickness to 16·4 feet, and having a width of 4·92 feet.

The system of reinforcement is shown in the figure, being formed of vertical, transverse, and longitudinal rods 0·79 inch diameter and spaced 19·7 inches apart, the longitudinal rods being dispensed with in the counterforts. The column reinforcement consists of four vertical rods 0·47 inch diameter, tied together by wire ties 15·75 inches apart, the vertical rods being carried well into the arch, and beams. The beams are reinforced with four 0·79 inch rods along the bottom and three 0·55 inch rods along the top, connected by transverse wires as shown. The longitudinal rods forming the decking reinforcement are 0·63 inch diameter, and the transverse rods 0·55 inch diameter.

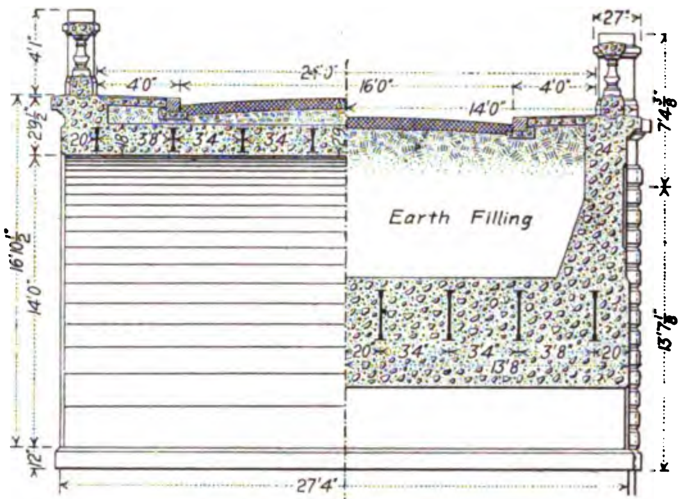
The total cost of the bridge was £660.

Bridge over Rock Creek, National Park, Columbia District

Figs. 444 and 445 show the details of a reinforced concrete bridge over Rock Creek, in the National Park, Columbia District,[1] constructed on the *Melan system*. It is a five-centre arch, with a span of 80 feet and a rise of 14 feet, the width between the parapets being 24 feet. The thickness of the arch at the crown is 18 inches, while at the springings it is increased to 7¾ feet; the arch being thickened out very slowly to 22 feet each side of the centre, after which it increases rapidly.

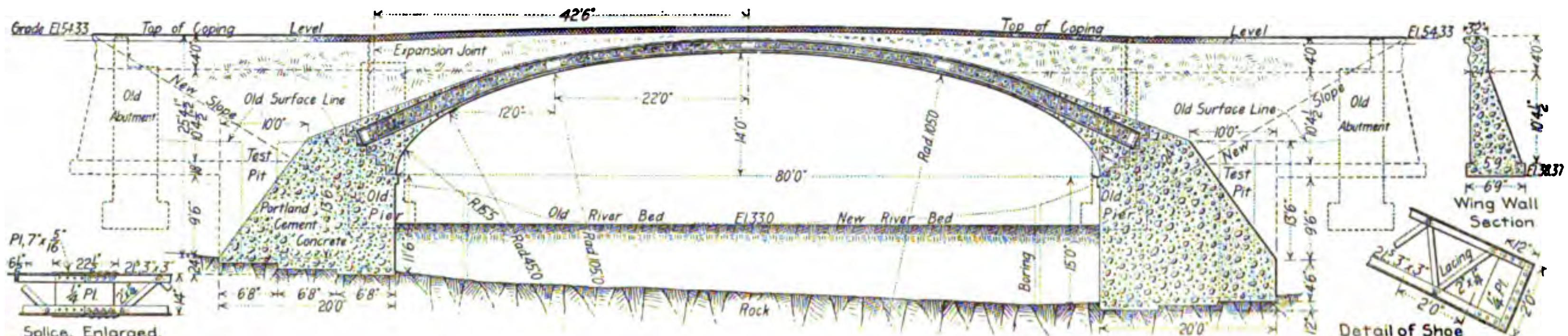
The reinforcement consists of ten steel lattice girders, 3 feet 4 inches apart centre to centre, the girders being 14 inches deep at the crown, remaining of this depth for a distance of 22 feet each side of the centre, and then increasing until their depth at the springing is 2 feet. The girders are made up of top and bottom

[1] Described in *Engineering News*, October 31, 1901.



Half Section at Crown. | Half Section near Abutment.

Fig. 444



Longitudinal Section.

Fig. 445

chords, of two 3 × 3-inch angles, connected by 2 × ¼-inch lattice bars. The girders are each in four lengths connected by ¼-inch gusset web plates, and $\frac{5}{16}$-inch flange covers, each being 22½ inches long. The ends of the girders are formed of two 3 × 3-inch angles and a ¼-inch gusset plate 12 inches long. The girders follow the curve of the intrados up to a point 34 feet on each side from the centre, from which point they are tangential to their former curvature, being nearer the extrados than the intrados at the springing.

The arch is not solidly connected to the abutment, but a stepped joint is made to allow a small amount of play. The facing, balustraded parapet, string, etc., are all formed with cement mortar.

The stiffeners shown in Fig. 446 were placed in the spandril walls, and a ½-inch expansion joint is left between the spandril and wing walls and also at points 20 feet on each side of the crown. The stiffeners were formed as shown in the figure.

The facing to the bridge was formed of coloured mortar, one part of lamp-black being used to twelve parts of cement, the proportions of the mortar being

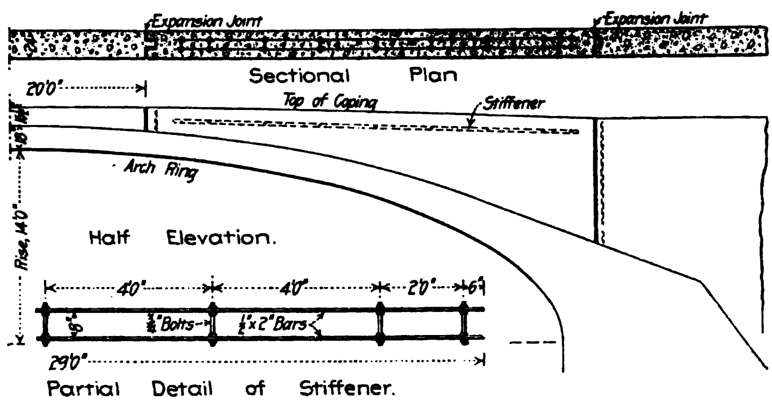


FIG. 446

one of cement to two of sand, which had passed a ⅛-inch mesh. The facing averaged ¾ inch thick, and was carried up with the concrete on the straight work; sheet-iron plates were used ⅛ inch thick, 10 inches deep, and 5 feet long, kept ¾ inch back from the face of the form by temporary wood strips. The mortar was placed between the plates and the form, after which the concrete was put in position and rammed; the plates were raised as the work was brought up, 4 inches hold being always left below the top of each layer until the next was complete, or until the end of the day's work or completion of the wall. Where these sheets could not be employed the mortar was spread on the surface of the forms before the concrete was put in, care being taken not to ram the concrete through the face layer.

The forms for exposed surfaces were tongued and grooved and planed; they were coated with boiled linseed oil. After the forms had been removed, the surfaces were floated with a cork float, using a little mortar to fill the air-holes, etc. The mortar used was in proportions of one of cement to two of sand, and the floating was done immediately after the removal of the forms. Lastly, the whole structure was washed over with grout, all the mortar and grout being coloured as described above. The efflorescence was cleaned off with diluted hydrochloric acid, four or five parts of water being used to one of acid. This solution was used with ordinary scrubbing brushes, and water was played on the work, while the acid cleaning was being performed to prevent any penetration.

The centre was struck after 31 days by lowering it ¼ inch at the crown, but the arch followed the lagging. On the second day the lowering was continued for a further ¼ of an inch when the lagging was free. The centre was removed on the third day. The arch deflected ⅜ inch at the crown, but subsequently regained its original position.

The bridge was commenced at the end of October, 1900, and completed in the middle of April, 1901, the total cost being £4,416.

About 5 cubic feet of concrete can be laid per man per day of 10 hours, when constructing bridges on the Melan system.

Bridge over Elbow Creek, Hyde Park, on the Hudson

The *Melan* bridge crossing Elbow Creek, on Mr. Vanderbilt's private estate of Hyde Park, on the Hudson,[1] is shown in Fig. 447. This bridge is in an improved

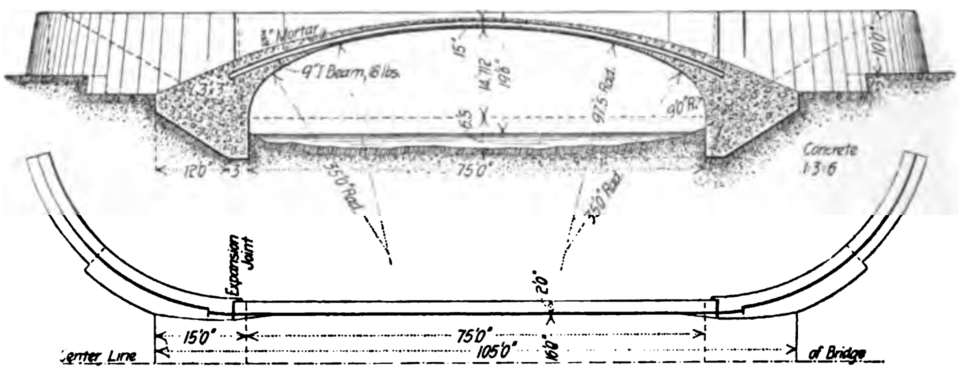


Fig. 447

part of the grounds, and is finished with a Portland cement mortar face with moulded balustraded parapets. It has a span of 75 feet and a rise of 14 feet 8 inches, and is 20 feet wide, including parapets. The curve of the intrados is five centred.

The arch ring is 15 inches thick at the crown, increasing slowly towards the springings, having a thickness of 21 inches at points 22·5 feet from the centre. The thickness from these points increases more rapidly to the springings.

The reinforcement is formed of 9-inch rolled steel joists, weighing 18 pounds to the lineal foot. The abutments, spandrils and wing walls are formed of concrete in the proportion of 1 of Portland cement to 2 of sand and 6 of broken stone. Near the springings the proportions are gradually increased to 1 to 2 to 4, which is the gauging used for the arch.

The moulding of the arch was commenced simultaneously from both springings for the full width of the bridge, and the work was continued night and day until the arch ring was complete and the spandrils carried up to the string under the parapet. Expansion joints were provided between the spandrils and wing walls,

[1] Description in *Engineering News*, November 10, 1898.

to allow freedom of action to the arch. The exposed faces were all finished with about 1½ inches of mortar, deposited at the same time as the main concrete, and

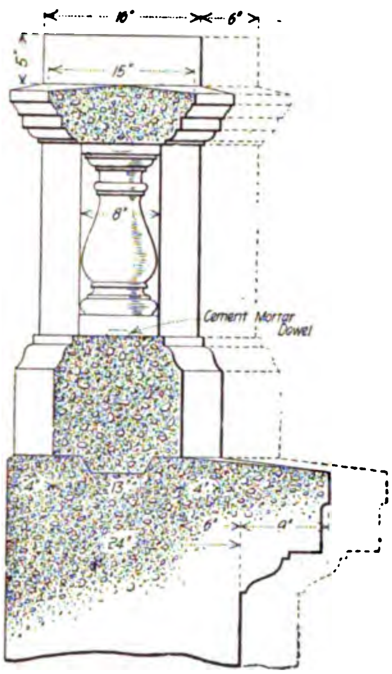


FIG. 448

the moulds were all covered with a layer of plaster of Paris, which was oiled before the concrete was deposited. Fig. 448 shows a detail of the balustraded parapets.

Rock Creek Bridge, Washington

Fig. 449 shows the Rock Creek boulder-faced bridge at Washington, U.S.A., designed by Captain Lansing H. Beach, on the *Melan system.*[1]

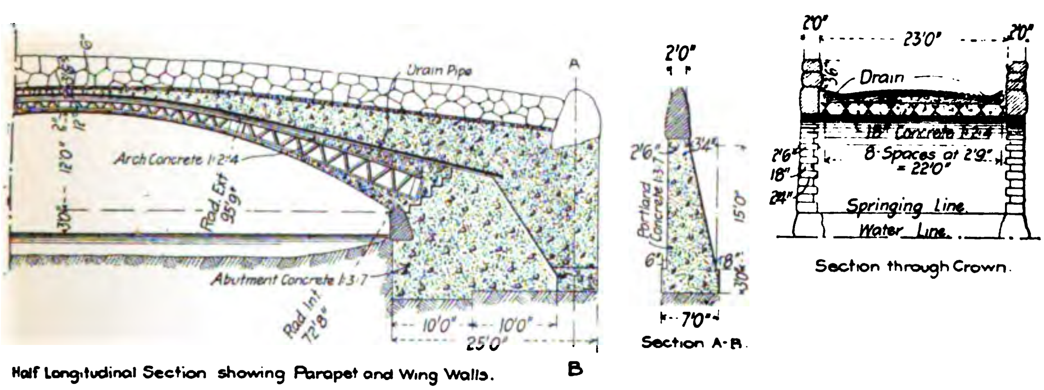


FIG. 449

The arch is segmental, 80 feet span and 12 feet rise, the width of the roadway being 23 feet. The boulder face of each stone projects at least 2 inches, and not

[1] Description published in *Engineering Record.*

more than 15 inches, beyond the neat lines of the bridge, the face stones being set in 1 to 2 cement mortar. The back of the stones were plastered with at least ½ inch of mortar before the concrete was rammed against them.

The arch stones were from 3 to 4 feet deep, 1½ to 3 feet wide, and 1½ to 2 feet long, exclusive of projections. Each stone was attached to the adjacent reinforcing girder by a ⅝ × ½-inch steel clamp, cemented for at least 2 inches in a hole in the stone. The outside reinforcing girders were bound together before the concrete was deposited around them with ⅝-inch wire ropes. The concrete for the arch was mixed in the proportion of 1 to 2 to 4, and that for the abutments and spandril filling at 1 to 3 to 7. The thickness of the arch at the crown was 18 inches. The reinforcing girders were of the open lattice type, being eight in number, spaced 2⅝ feet apart centre to centre.

The total cost of the bridge was £3,595.

Bridge over the Ocher, at Brunswick

This is a very good example of the *Möller* type of bridge construction. The face beams are straight, with a small curve at the abutments, and are reinforced. They carry the parapet and part of the footways. The remaining beams are of the *Möller* fish-belly type, and are spaced 3·28 feet centre to centre. Those supporting the roadway have somewhat greater depth than those carrying the footways. The span of this bridge is 75·46 feet and the total width 27·88 feet, of which the roadway is 16·4 feet, each footway 5·32 feet. The reinforcement of the decking consists of rolled joist, and the parabolic reinforcement of the beams of flat iron bars.

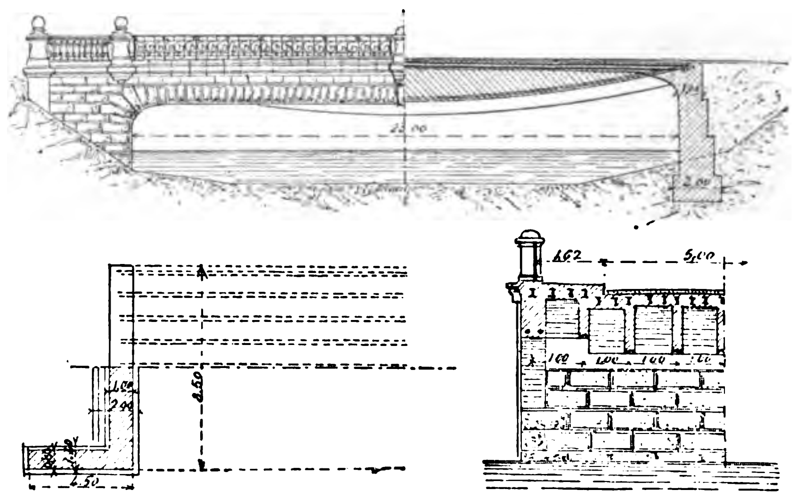


FIG. 450

Footbridge of the Railway at Kreinsen

This is another bridge on the *Möller system*. It consists of five spans, four of which are 37·4 feet and one 40·68 feet. The width of the decking is 4·92 feet, and it is supported in each span by two fish-bellied beams, the reinforcement consisting of flat iron bars 5·91 × 0·55 inches, with six cross pieces of angle iron. Angle irons 2·36 × 2·36 × 0·39 inches are embedded transversely in the concrete of the floor slab, and carry the uprights of the railing.

The bridge is supported on five iron piers and one of concrete, and is calculated for a load of 80 pounds per square foot.

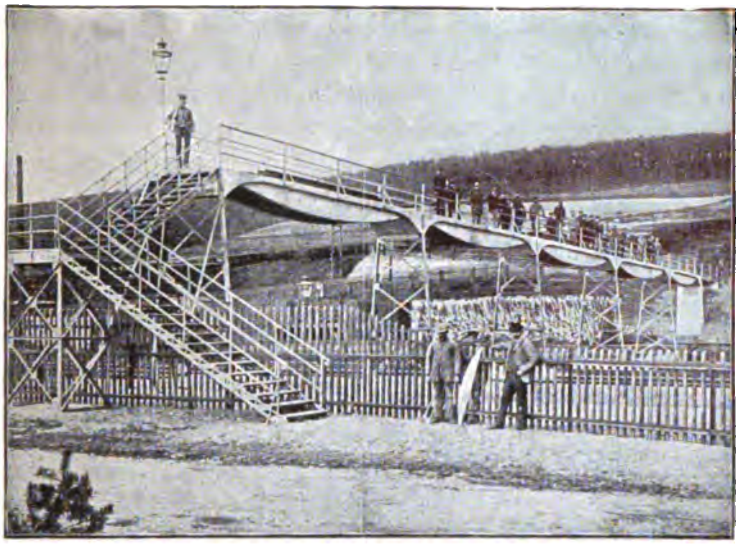


FIG. 451

Footbridge over the Saarbrücken Railway at Neunkirchen

This bridge (Fig. 452) was constructed by the *Société des Constructions Monier* (Wayss). It has a span of 45·93 feet and a rise of 14·76 feet, with a thickness at the crown of 5·9 inches and a width of 6·56 feet. It was designed for a load of 100 pounds per square foot. On one side the decking is level and supported by two flat arches, with a thin pier resting on the abutment of the main arch. The

FIG. 452

other side of the bridge is formed with steps, to a landing on the hollow abutment. These steps are continued to the ground level on an arched stairway, as shown in the illustration. The whole bridge and stairway is of reinforced concrete.

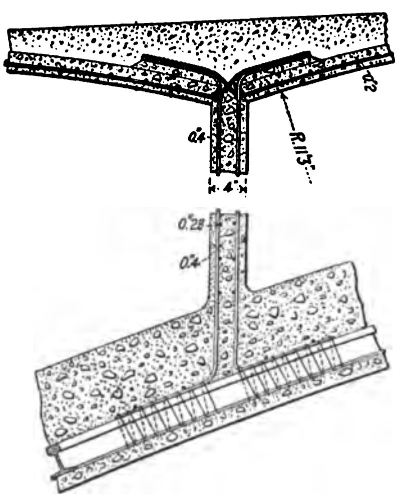
Footbridge near Copenhagen [1]

Fig. 453 shows the method of construction of a footbridge over a railway near Copenhagen.

The span is 71·7 feet and the rise 8·45 feet; the depth of the main arch is 9·8 inches at the crown and 14·2 inches at the springings; the width between parapets being 10·3 feet. The reinforcement consists of five parallel ribs of bent rails weighing 18·8 pounds per lineal foot, 2·5 feet apart centres, and joined together longitudinally.

The footway is carried on Monier arches, 2 inches deep and 7·35 foot span, the radius of the intrados being 11·25 feet. These are supported on piers 4 inches thick, extending over the full width of the bridge, and supported from the main arch. The reinforcement of these arches is formed of a network of wires, 0·2 inch diameter, with a 4-inch mesh. In each pier there are two nettings, formed of 0·4 inch square vertical rods and 0·275 inch horizontal wires. These networks are secured to the iron rails at the foot of the piers by bending back the vertical rods and tying them to the rails by a lapping of annealed wire. At the top the vertical rods are again bent back, and the longitudinal arch rods are carried forward and bent up, the two sets being tied together by a wire wrapping. The concrete has the proportions of 1 of Portland cement to 3 parts of gravel for the arches and piers, and 1 of cement to 4 of sand and 7 of shingle for the filling over the Monier arches and in the abutments.

This bridge was built in the spring of 1879, and cost about £400.



SECTION.

FIG. 453

Over Bridge for a Parish Road

This bridge, a view of which is shown (Fig. 454), has an extremely light and pleasing appearance. It was constructed by *M. Piketty*, under the direction of M. Harel de la Nöe. It has three bays of 14·76 feet, each of the piers being formed of two columns, 13·78 inches square, connected at the top by beams 3·94 inches deep below the floor slab. The side spans have a slight inclination upwards towards the centre span.

The main decking, which is 10·24 inches thick at the sides and 11·8 inches at the centre, and has a width of 7·87 feet, is supported by the cross-beams over the columns and by two longitudinal beams, one on each side. The underside of the longitudinal beams is level with the bottom of the decking, and their total depth is 17·32 inches.

From the top of these beams, slabs 3·15 inches thick project for a distance of 3·5 feet, being carried at their extremities by longitudinal beams 6·3 inches deep from the underside of the slabs and 3·94 inches wide, having a projection corbelled out from them for a distance of 6·69 inches, and having a moulded face. These beams rest on cantilevers, projecting from the columns.

[1] Described in *Engineering News*, July 21, 1898.

The footways are 2·62 feet wide, and the remainder of the total projection of 3·5 feet is taken up by the parapet. The whole bridge is of reinforced concrete, the bottom reinforcement of the floor slab, longitudinal main beams, and beams over the piers, consisting of steel rails, there being five longitudinal rails in the floor slab and two in each of the beams. The upper reinforcement of the beams and slabs is formed of iron rods, the top reinforcement of the cross-beams being near the upper surface of the decking, part of which in reality forms the beam.

The small longitudinal beams carrying the footway and parapet are reinforced with four iron rods, two at the top and two at the bottom. All the reinforcements are connected vertically by transverse wires. The columns are reinforced by four iron rods with wire cross-ties.

The abutments are formed of reinforced sills, 13·78 inches wide and 21·65 inches deep, carried on each side by piers 4·92 feet long, and 23·6 inches wide, which are carried up to form the pilaster finish at the end of the bridge.

Fig. 454

Bridge at Bangor, Maine, U.S.A.[1]

This bridge, constructed on the *Ransome system*, has two arches, one of 46 feet 8 inches span and 8 feet 7 inches rise, the thickness at the crown and springings being 11 and 30 inches respectively. The other arch has a span of 38 feet 2 inches and a rise of 7 feet 6 inches, with a thickness at the crown of 10 inches.

The reinforcement consists of pairs of cold twisted steel bars, three-quarters of an inch square, placed two inches from the intrados, and extrados and spaced 14 inches centre to centre transversely. Where the bars require lengthening, they are joined together by a union, which screws on to the twists of the bars. The upper bars were held in position by vertical transverse boards, each board being removed as the concreting reached it.

The concrete of the arch was in the proportion of 1 of Portland cement to 2 of sand, and 4 of gravel, of all sizes from 2½ inches to the size of a pea.

[1] Description in *Engineering News*, March 20, 1902.

Bridge over the River Caudal, at Micres (Spain)

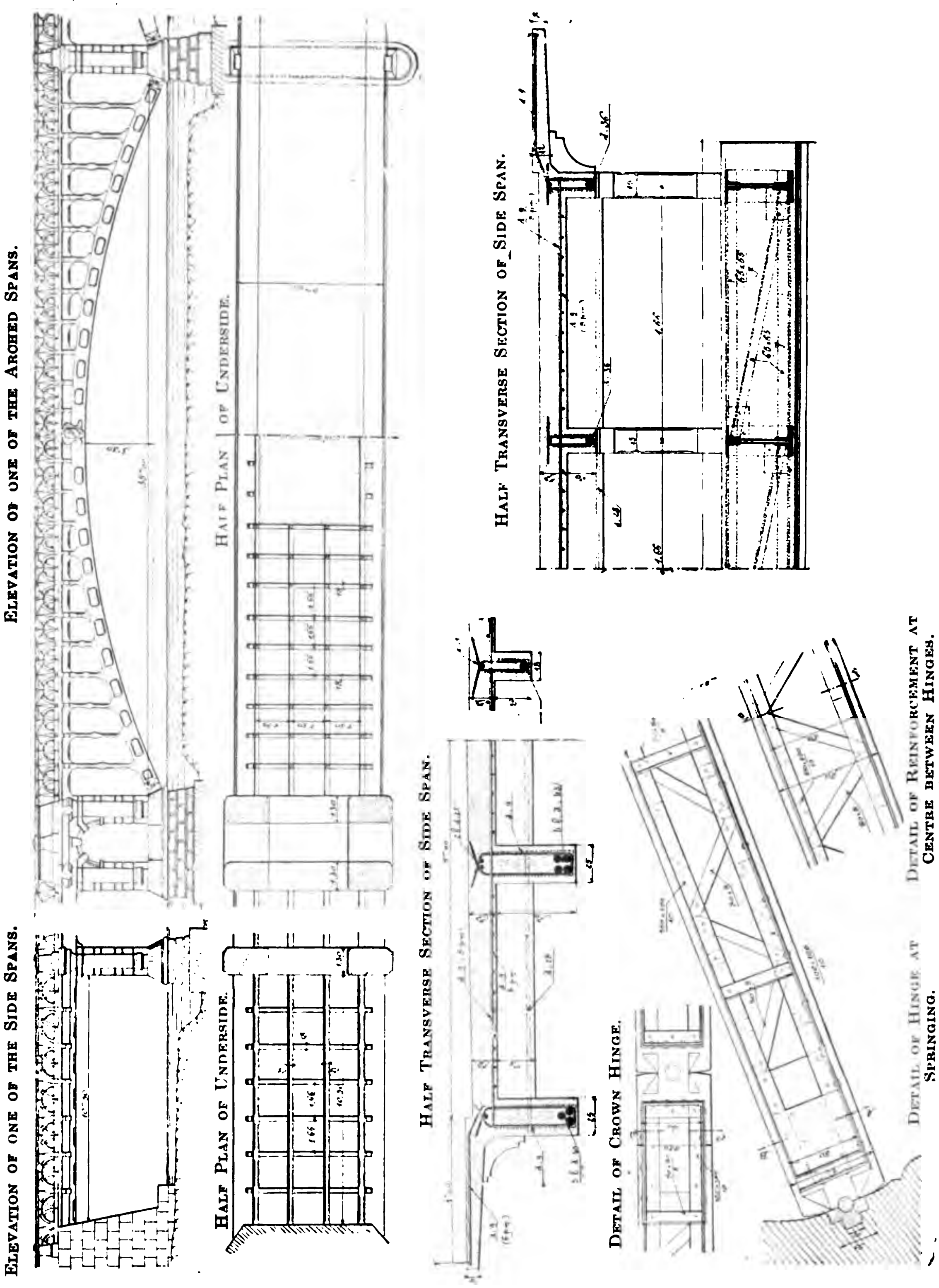


Fig. 455

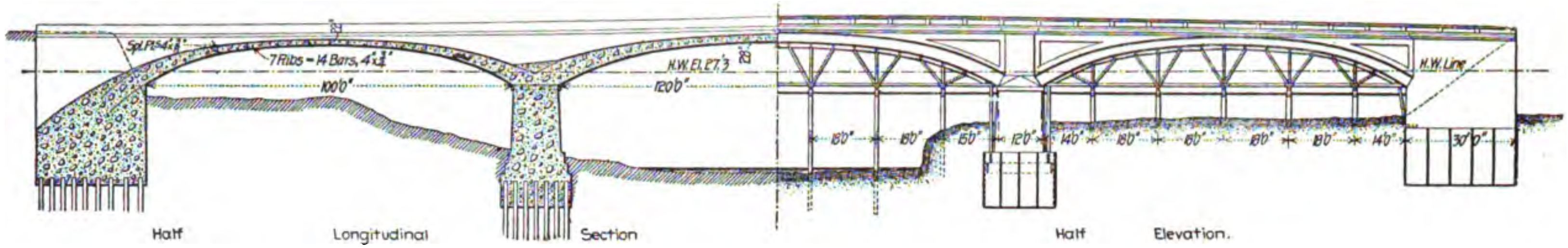


Fig. 456

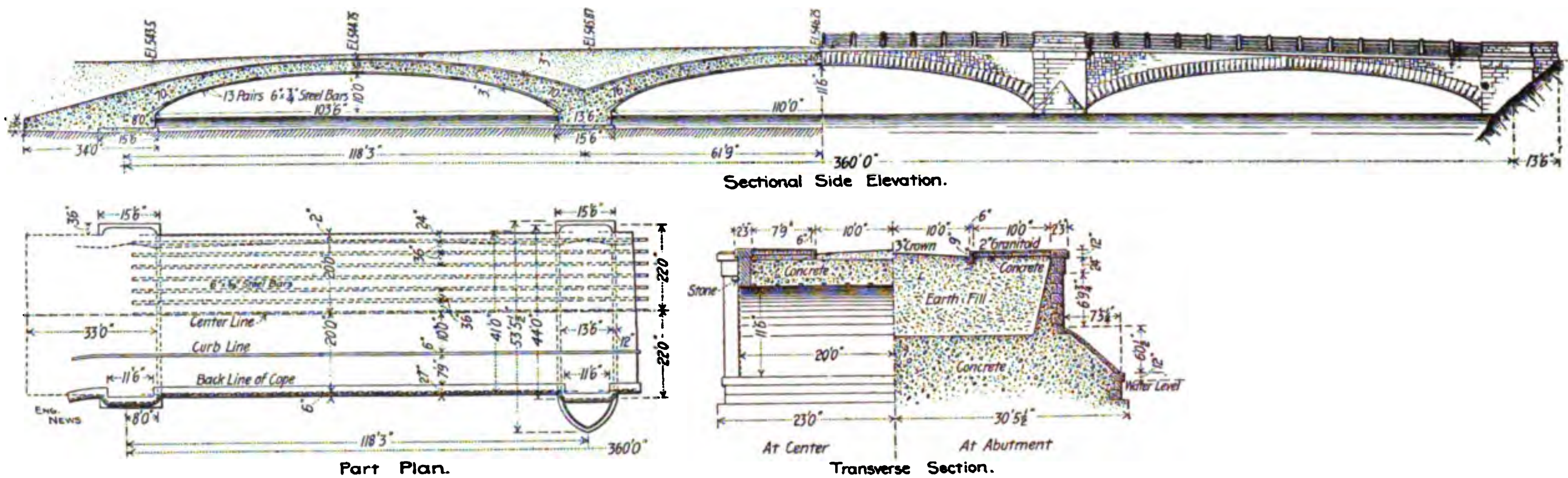


Fig. 457

The concreting was done in longitudinal strips, commencing at the springings and closing at the crown.

Short half-inch twisted steel bars were left projecting out from each day's work to tie the arch together transversely. The footways were carried on longitudinal walls 6 inches thick under the curbs, and similar walls 8 inches thick at the face of the arches. The walls are of concrete in the proportions of 1 to 3 to 6, reinforced with twisted steel bars, and have buttresses 8 feet apart. To tie these walls to the arch ring, half-inch twisted steel bars were left projecting radially from the arch concrete. The soffit, face of the arch, coping and footways have a granolithic finish, the spandril walls being dressed with a picked face.

The road was only closed for traffic for sixteen days, and there was no vibration in the bubble of a level placed on the bridge, and no measurable deflection when the bridge was tested with a rolling load of fifty tons, which was taken across the bridge at a trot.

Bridge over the River Caudal, at Micres (Spain)

This bridge (Fig. 455) is situated in the centre of a town rich in monuments and decorations, which necessitated an ornamental structure. It was constructed on the *Ribéra system*, and combines in a most striking manner a beauty and lightness of design which is probably unequalled in any bridge constructed up to the present time of reinforced concrete.

The total length is 361 feet, made up of two three-hinged arch spans of 114·8 feet span and 11·48 feet rise, and three spans of 34·45 feet. The width of the bridge is 22·96 feet, made up of a roadway 16·40 feet wide and two cantilevered footpaths 3·28 feet wide. The decking of the arched spans are supported on longitudinal and transverse beams, those running longitudinally being 5·9 inches wide, with a depth of 7·9 inches below the floorslab, and are spaced 4·92 feet centre to centre, while those running transversely are 7·1 inches wide and 9·8 inches deep, and are spaced 5·45 feet centre to centre. The beams are supported where they cross on $7{\cdot}1 \times 5{\cdot}9$-inch columns, which rest on the arch, having a thickness of 17 inches at the crown, 27·5 inches at the haunches, and 23·6 inches at the springings.

The reinforcement of the arches consists of four longitudinal arched lattice girders, formed of top and bottom chords, made up of two $3{\cdot}94 \times 3{\cdot}94 \times 0{\cdot}39$-inch angles, back to back, and connected by $3{\cdot}15 \times 3{\cdot}15 \times 0{\cdot}315$-inch radial angles and $3{\cdot}15 \times 0{\cdot}315$-inch cross-bracing.

The girders have a depth of 15·75 inches at the crown, 23·6 inches at the centre between hinges, and 19·7 inches at the springing, and are connected transversely. 5·45 feet apart, with horizontal ties at the top and bottom, and a cross-bracing, all of $2{\cdot}56 \times 2{\cdot}56 \times 0{\cdot}275$ inch angles.

The arrangement of the hinges is clearly shown in the figure. The column reinforcements consist of four rods, one at each corner. The longitudinal beams are reinforced with a 1·42-inch diameter rod at the bottom, and a 0·98-inch diameter rod at the top, and the transverse beams with a 1·10-inch rod at the bottom and a 0·98-inch rod at the top, the longitudinal reinforcements being tied together by transverse wires.

The decking throughout the whole length of the bridge is 6·7 inches thick, and is reinforced along the bottom with longitudinal and transverse rods 0·35 inches diameter, spaced 4·92 inches centre to centre both ways.

The side spans consist of four longitudinal beams 9·84 inches wide, 27·56 inches deep below the decking, and spaced 4·92 feet centre to centre. These are connected by transverse beams identical with those in the arched portion.

The reinforcement of the outer longitudinal beams consists of three rods of 1·57 inches diameter and one top rod of 0·98 inch diameter, connected by a transverse reinforcement of 0·35-inch rod, which size is used throughout for the transverse reinforcement of the beams. The two interior longitudinal beams have four 1·69-inch rods at the bottom, and two 0·98-inch rods at the top, connected by transverse reinforcements.

The projecting footpaths are 4·72 inches thick where they leave the main decking, and 3·15 inches thick at the extremity, and are reinforced with 0·35-inch diameter rods, 6·6 inches apart near their upper surface. The bridge was calculated for a uniformly distributed load of 82 pounds per square foot and a rolling load of two-wheeled carts weighing 8 tons.

The cost of the whole structure was £6,067 ; while the estimated cost of a steel bridge was over £8,000 or 43 per cent. more.

The Bridge over the Jacaguas River, Porto Rico [1]

This bridge constructed on the *Thacher system* is shown in Fig. 456. The centre opening has a span of 120 feet, with a rise of 12 feet. The two end spans are 100 feet, with a rise of 11·28 feet. The width between parapets is 18 feet. The reinforcement consisted of Thacher bars, with projections like rivet heads, to prevent sliding.

In each arch there were seven ribs, each consisting of two 4 × ¾-inch bars, placed near the extrados and intrados, connected transversely by 4 inch × ⅜-inch plates. The concrete for the arches was mixed in the proportions of 1 of Portland cement, to 2 of sand and 4 of shingle, to pass a 1¼-inch ring, and that for the abutment, piers and spandrils was 1 to 3 to 6 of shingle to pass a 2-inch ring.

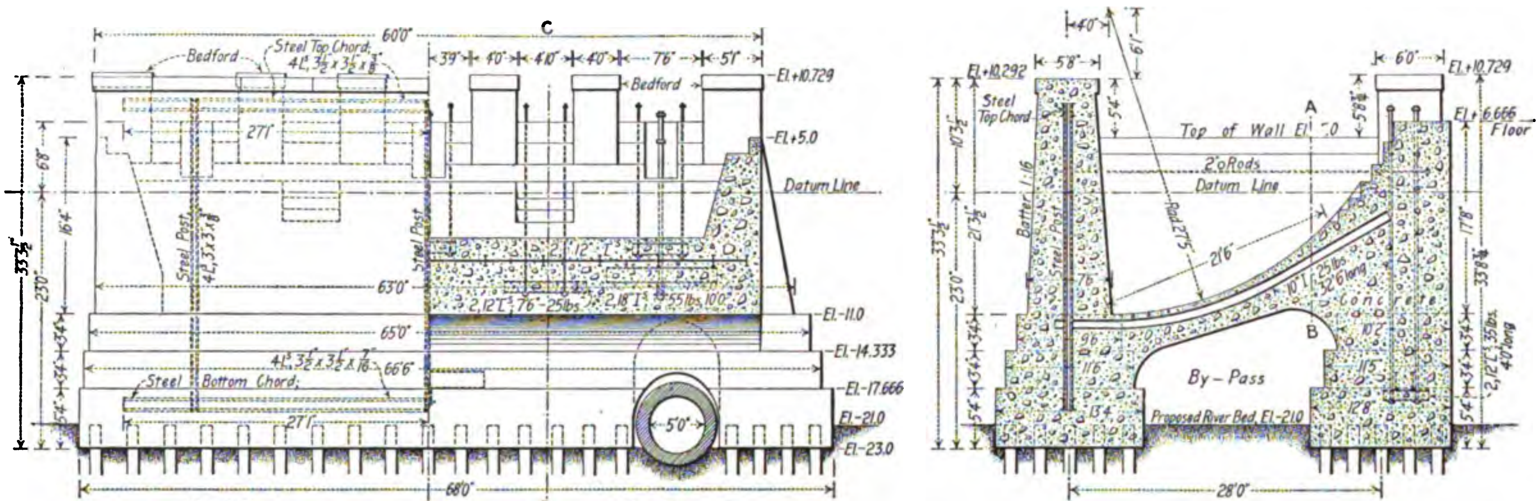
The bridge was faced throughout with mortar made of screenings from a stone crusher. It was found that when the screenings were freed from dust the mortar was 12 per cent. stronger than that made from clean, sharp standard sand, and when they contained from 22 to 30 per cent. of fine dust the strength of the mortar was nearly the same as when standard sand was used. The arch concrete was laid in three longitudinal sections, that at the centre being 9, and those at the side 5½ feet wide. The bridge was commenced in February, 1900, and finished in March, 1901.

The centres for the centre span were lowered forty days after completion, and there was a deflection of ⅝ inch, increasing to ⅞ inch after one month. The centres for the east span were lowered thirty days after completion, the deflection being ¾ inch, increasing to 1½ inches in two months. The spans were tested with a load of from 21·5 to 23·5 tons, concentrated at the centre, the greatest deflection of the centre span under this load being 0·015 inches.

Bridge between Mainland and Green Island, Niagara Falls [2]

Fig. 457 shows the bridge constructed of reinforced concrete between the mainland and Green Island, Niagara Falls. The side spans are three centre arches, 103½ feet span and 10 feet rise, with an arch thickness of 38 inches at the centre,

[1] Described in *Engineering News*, August 1, 1901.
[2] Described in *Engineering News*, December 6, 1900.



Half Front Elevation.

Half Vertical Section A-B.

Vertical Section C-D.

Fig. 458

increasing to 70 inches at the springings. The centre span is also a three-centre arch having a span of 110 feet, and a rise of 11 feet, the thickness of the arch being 40 inches at the centre, increasing to 76 inches at the springings. The width of the bridge between parapets is 40 feet.

The reinforcement consists of two sets of thirteen 6 inch × $\frac{3}{4}$-inch steel bars, three feet apart, the outside ones being two feet from the faces of the arch. One set of bars is placed three inches below the extrados, commencing about 8 feet from the face of the abutments, and following the curve of the extrados, the bars from the neighbouring spans almost meeting over the piers. The other set is placed three inches above the intrados, and follows the curve of the arch almost to the springing, from which point they are continued tangential to their former curvature. The bridge is faced with limestone, the arch stones being tooled, and the spandrils random rock faced.

The concrete for the arches was in the proportions of 1 to 2 to 4 of broken stone and shingle, passing a $1\frac{1}{4}$-inch ring, and including all crushed stone above quarter inch. That for the abutments, piers and spandrils was 1 to 3 to 6 of broken stone and shingle, passing a two-inch ring and retained on a quarter-inch, with plums in the piers and abutments not less than $1\frac{1}{2}$ cubic feet. The concrete for the arches was commenced at the springings, and laid in longitudinal sections, wide enough to enclose at least two of the steel bars.

The cost of the bridge was about £21,250.

Abutments and Piers for Clybourne Place Bridge, Chicago

Fig. 458 shows the abutments and piers for the Clybourne Place drawbridge

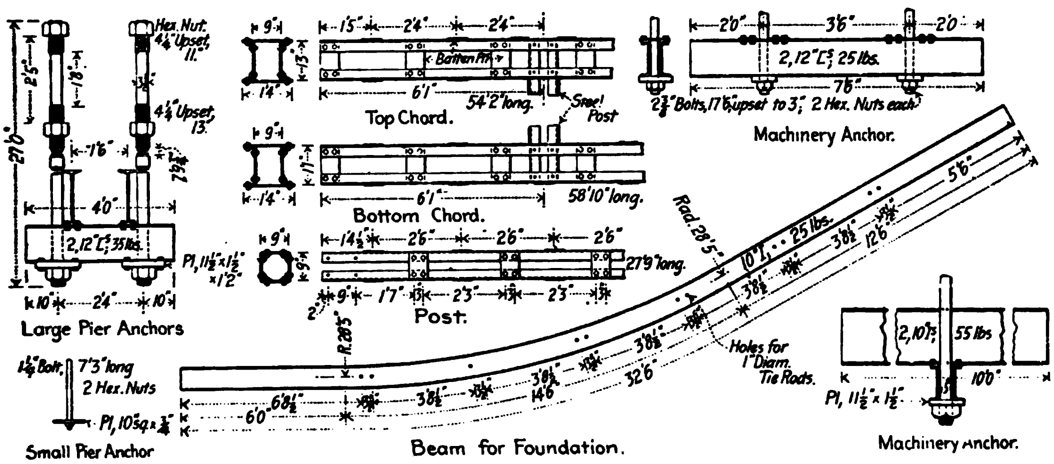


FIG. 459

over the Chicago River.[1] The main pier is 60 feet long at coping and 73 ft. 8 in. at the bottoms with the upstream end battered $2\frac{1}{2}$ to 1 and the downstream end vertical, both faces having a batter of 1 in 16. A bye-pass is formed through each abutment, as shown. Both the piers and abutments are reinforced with a steel framework, the details of which are shown in Fig. 459.

[1] Description published in the *Engineering News*, Volume xlv., No. 3.

The top and bottom chords are of open box section 9×9 inches inside and 16×13 in. over the four $3 \times 3 \times \frac{7}{8}$-in. flange angles, which are connected by vertical and horizontal batten plates. These chords are connected by three vertical posts of square section, 9×9 in., formed of four $3 \times 3 \times \frac{3}{8}$ in. angles, with the flanges inwards, and 9-in. batten plates. These posts are 27 ft. 9 in. long.

Rolled joists 10 in. deep, and weighing 25 pounds per lineal foot, are built into the bottom of the concrete; these are $32\frac{1}{2}$ ft. long, and have the centre portion bent to a curve of 28 ft. 5 in. radius. They are placed 5 ft. 2 in. apart except in the outer portion, where this distance is reduced to 3 ft. The joists are connected transversely by 1-in. rods 4 ft. apart.

The concrete was put in in 6-in. layers, and mixed in the proportions of 1 of Portland cement to 3 of sand and 5 of $1\frac{1}{2}$-in. broken stone. Sufficient water was used to make the concrete quake under the rams. The sides of the piers, tail-pits, and exposed faces were faced with a 2-in. coat of 1 to 1 mortar applied before the concrete had quite set.

The work was designed and carried out by Mr. Ed. Wilmann, City Engineer of Chicago.

FIG. 460

Bridge for the town of Perpignan. Span, 49·2 ft.; width, 32·8 ft.; to carry a rolling load consisting of a roller weighing 28 tons, constructed on the *Bousseron and Garric system.*

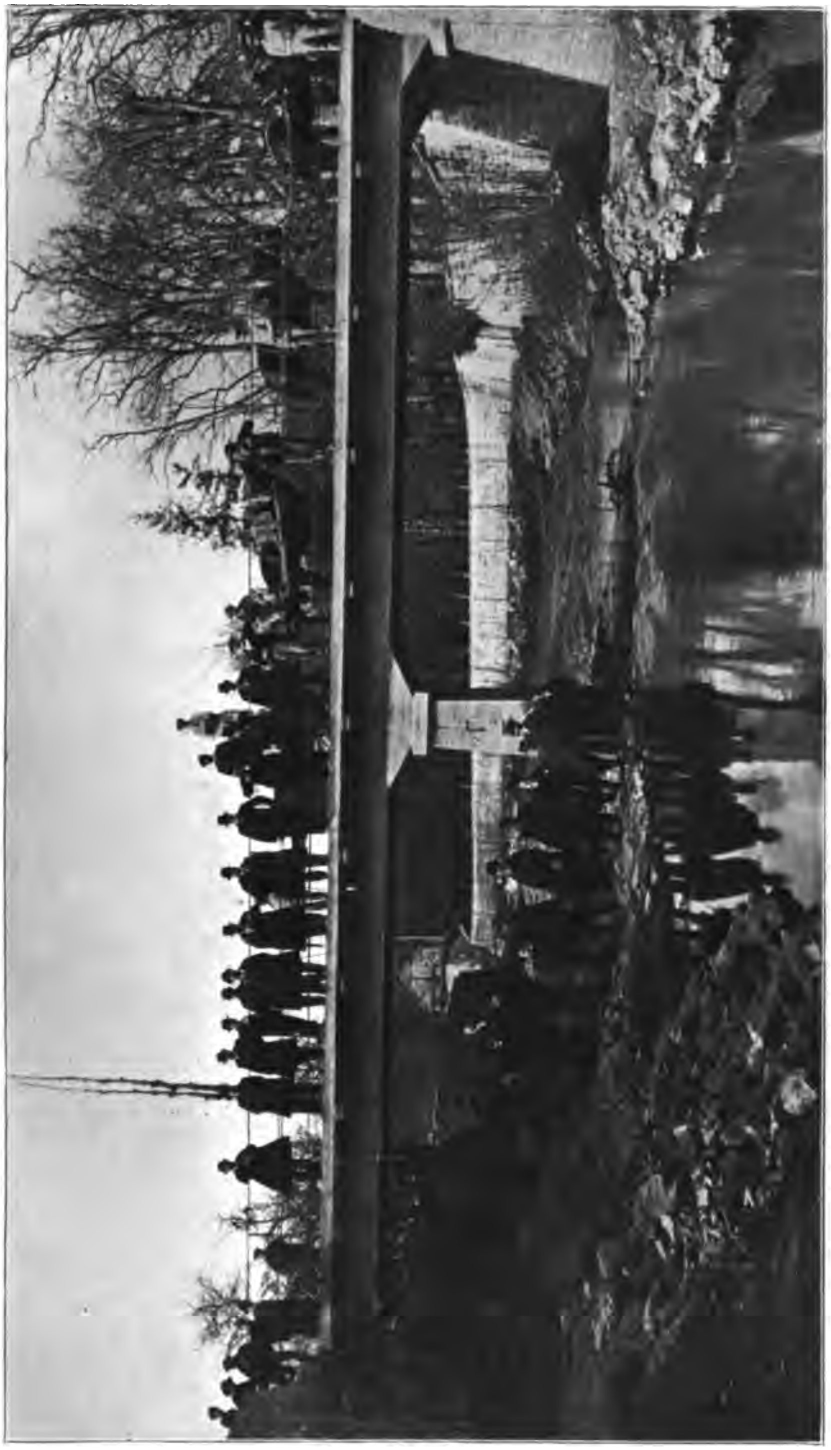


Fig. 461

Bridge over the Ouche at Plombiére-lés-Dijon, constructed on the *Coularou system*. Span, 68·24 ft. Road, 9·84 ft. wide, with two footways 2·46 ft. wide, supported on corbels. Rolling load, 818 pounds, tested to 1,227 pounds per square foot.

Fig. 462

Skew Bridge on main road at La Maladiere, Lausanne, Switzerland, constructed on the *Hennebique system*. Span, 49 ft. 10 in.; depth of beams, 3½ ft.; thickness of decking, 7 in. Tested with moving load of 18 tons, with a deflection of $\frac{1}{13000}$ of the span.

SOME STRUCTURES ERECTED IN REINFORCED CONCRETE

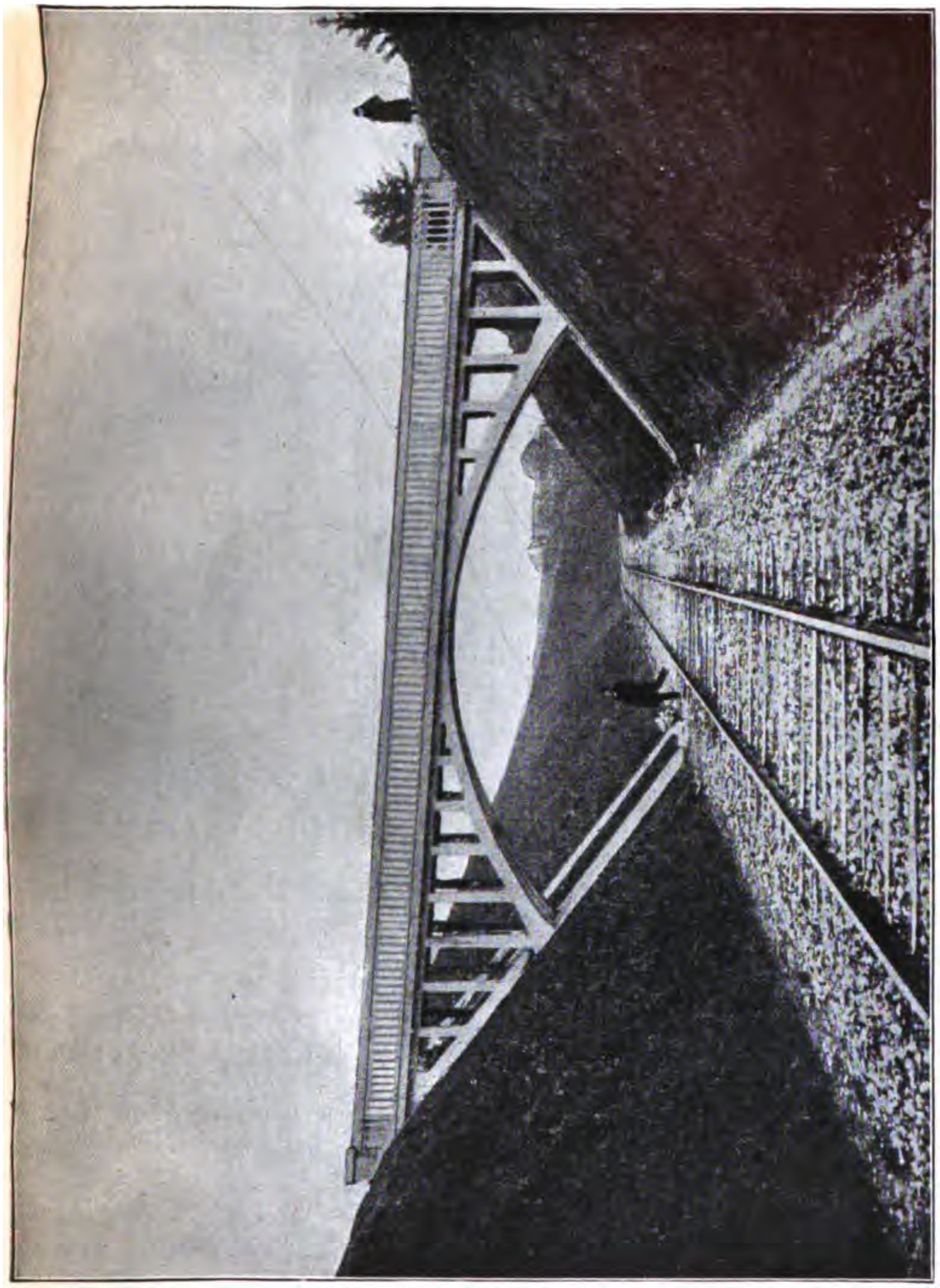


FIG. 463

Bridge over the Eastern Railway of France, between Provens and Easterney. Span, 49 ft. 3 in; width, 6 ft. 7 in. Bridge is formed of two ribs 5 ft. 5 in. apart. The ribs are 12 × 12 in. at springings, and 8 × 8 in. at centre. The decking, 5 in. thick, is carried on 8 × 8 in. beams, supported from the ribs by 8 × 8 in. piers. The whole bridge is of reinforced concrete, on the *Hennebique system.*

FIG. 464

Footbridge over the Western Railway of Franco at L'Orient. Constructed of reinforced concrete, on the *Hennebique system.*

SOME STRUCTURES ERECTED IN REINFORCED CONCRETE

Fig. 465

Boulder-faced reinforced concrete bridge at Hyde Park, New York, constructed on the *Melan system.*

FIG. 466

Ybbs Bridge (Austria), constructed by Herren Wayss & Co. Span, 144¾ ft.

FIG. 467

Bridge on the *Ribéra system* constructed over the Golbardo River in Spain. Its span is nearly 100 feet, and the construction is clearly shown in the illustration.

FIG. 468

Reinforced Concrete Bridge at Ponta Ajourée a Kropini (Austria). Span, 65·6 feet.

V. MISCELLANEOUS

Reinforced Concrete Footway

The reinforced concrete footway, constructed by the *Metalloid Sidewalk Co.*, St. Louis, U.S.A.,[1] is shown in Fig. 469. The slabs are constructed in sheds, where the work is carefully supervised.

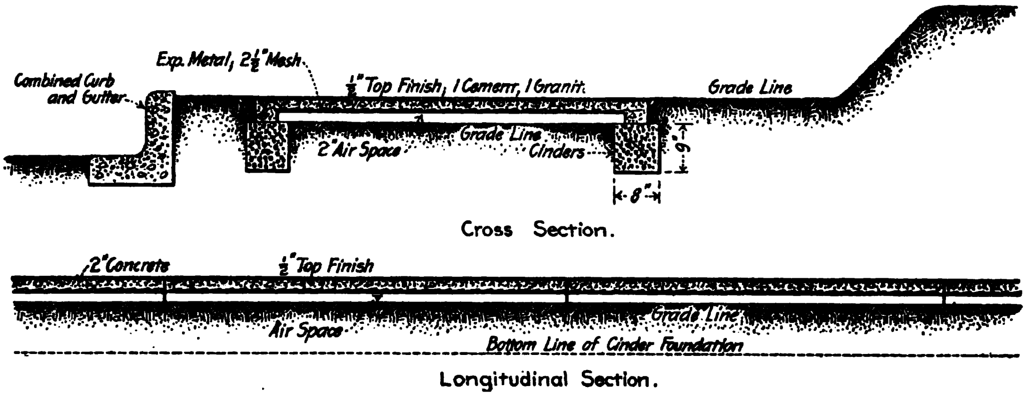


Fig. 469

The reinforcement consists of 2½-in. mesh "expanded metal" near the bottom surface, which prevents cracking or breaking, and permits poorer concrete and bigger slabs to be used than in the case of ordinary pavement construction. The concrete is mixed in the proportions of 1 of Portland cement to 5 of limestone chippings containing sufficient fine material to replace the sand.

The slabs span the whole width of the footpath, and have a rib at each side, resting on beds of cinders or gravel, laid in trenches, and leaving a 2-in. air-space under the pavement. The thickness of the slab proper is 2 ins. and that of the ribs 4 ins., and the top is finished with a ½-in. layer of Portland cement and crushed granite gauged 1 to 1.

Another form is made of concrete mixed in the proportions of 1 of Portland cement to 5 of cinders, with a ½-in. top finish of 1 of Portland cement to 2 of sifted granite or other hard stone.

One great advantage of this method of pavement construction is that of the ready removal of the slabs to allow of excavations being made under the footpath, while there is also a saving in consequence of the elimination of under dressing. These pavements are said to be 33 per cent. cheaper than those of the ordinary form of construction with a cinder or gravel foundation 4 to 8 ins. thick, 3 to 4 ins. of concrete, and half an inch top finish. The paving can be sold at 6*d*. per square foot, and give a profit of 30 per cent.

Vault-Light Slabs

Figs. 470 and 471 show a method adopted by *Aberthaw Construction Company*, 8, Beacon Street, Boston, U.S.A., for constructing vault lights.[2]

[1] Description in *Engineering News*, November 1, 1900.
[2] Description in *Engineering News*, September 12, 1901.

The slab is supported (as shown in Fig. 470) on rolled joists, the main joists being 5 ft. centre to centre and the secondary joists 4 ft. 2 in., an expansion joint being left between the slabs over the main joists. Fig. 471 shows an enlarged detail of the slab, which is formed of tapered glass lenses, 2⅝ in. diameter at the top and 3⅝ in. centre to centre, let into a slab of reinforced concrete. This slab is reinforced by ¼ to $\frac{3}{16}$ in. *Ransome* twisted steel bars, placed just above the bottom surface of the ribs between the glass lenses. The lenses are held by three

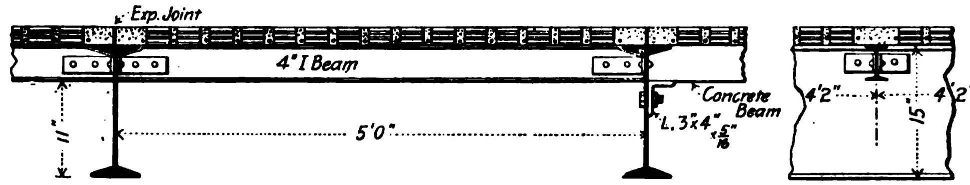


FIG. 470

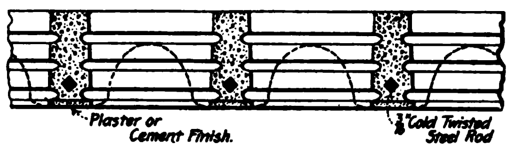


FIG. 471

circular indents in the concrete, and their thickness is the same as that of the slab at the edge, but they are formed with a cup-shaped hollow on their undersides. To form the slab, the lenses are first placed in position and the concrete is moulded around them.

One of these slabs, with ¼-in. reinforcing bars, was tested with a load of 11,882 pounds, distributed over a 8½-in. circular disc, or an area of 0·394 square feet, at the centre of the slab, the load being applied 914 pounds at a time.

With a load of 5,484 pounds there was a deflection of $\frac{5}{16}$ in., and the concrete began to crack; the cracking continued until the load amounted to 9,140 pounds, when the deflection measured $\frac{49}{64}$ in., and the lenses began to crack. The breakage continued until the load of 11,882 pounds was reached, when a general crushing of the concrete and glass took place, and the deflection amounted to $1\frac{41}{64}$ ins. After the load was removed the permanent deflection measured 1¼ ins. With the load of 11,882 pounds the rods remained intact, and the slab continued to support the load.

Bank Protections for the Ghent to Terneuzen Canal, Belgium

Fig. 472 shows cross and longitudinal sections of the sheet piling and pitching for the protection of the banks of this canal. The piling is constructed on the *Hennebique system*, the details of which are given in Figs 473 and 474.

The protection of the lower portion of the banks, which is almost vertical, is formed by a continuous series of reinforced sheet piles (Fig. 473). These are held back at their heads by bars, anchored to piles, driven 24·6 feet behind them. The caping of the sheet piling and the walings for the anchor piles were also con-

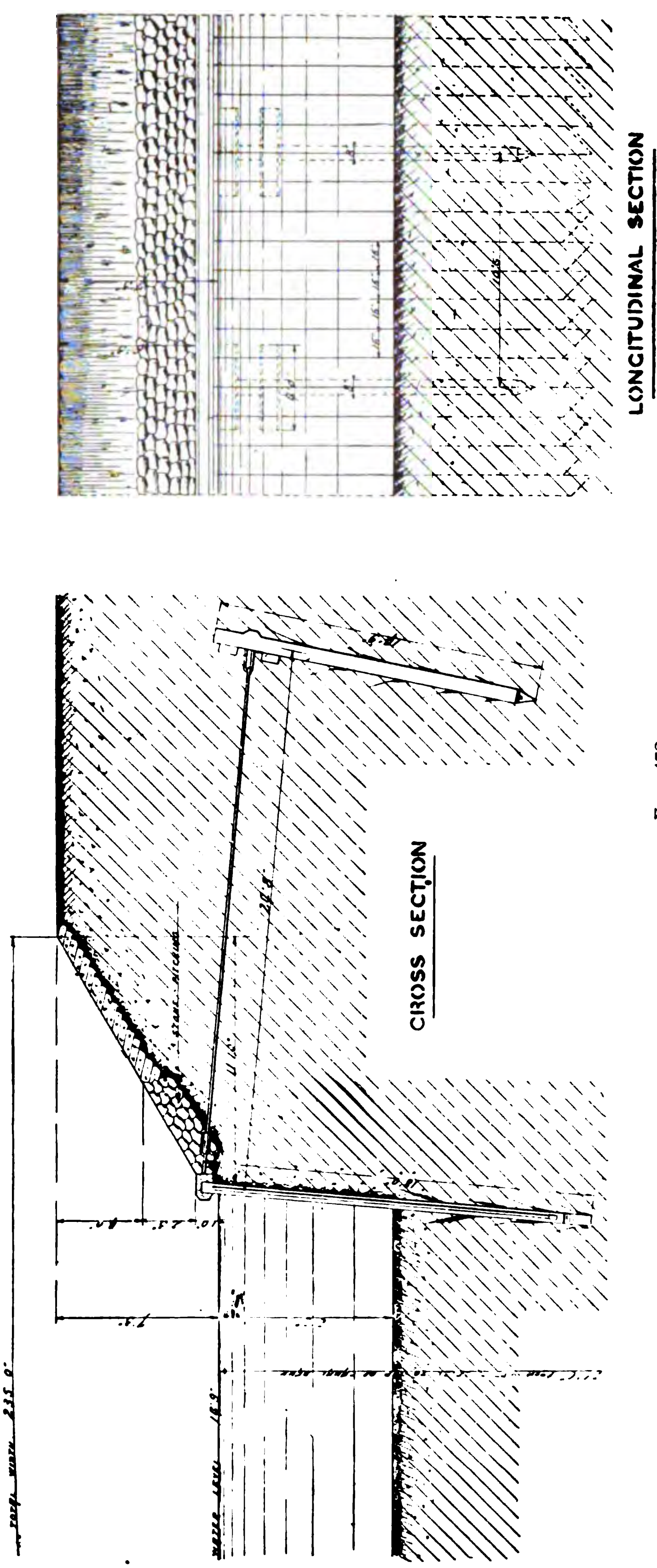


Fig. 472

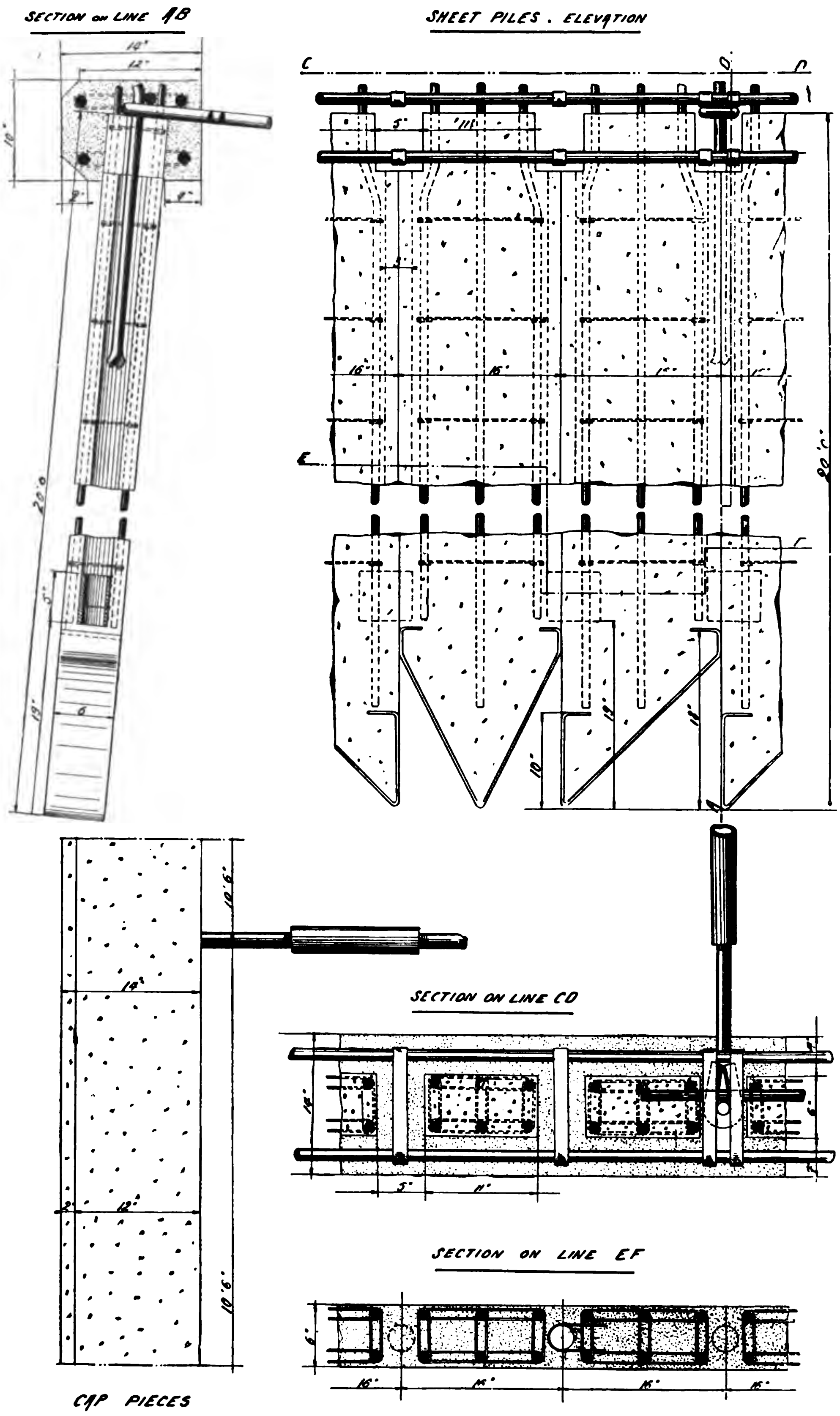

SECTION ON LINE AB
SHEET PILES . ELEVATION
SECTION ON LINE CD
SECTION ON LINE EF
CAP PIECES

Fig. 473

constructed of reinforced concrete. The anchor piles with their walings are shown in Fig. 474. Above the sheet-piling, the bank sloped back, and is pitched with stone, the pitching being held up by the top of the sheet piling.

This method of bank protection is exactly similar to the usual method employed in timber sheet piling, but the reinforced concrete is practically everlasting, whereas timber piles decay in time.

The cost of this work was greater than if timber had been employed, being about £6 11*s*. 6*d*. per lineal yard.

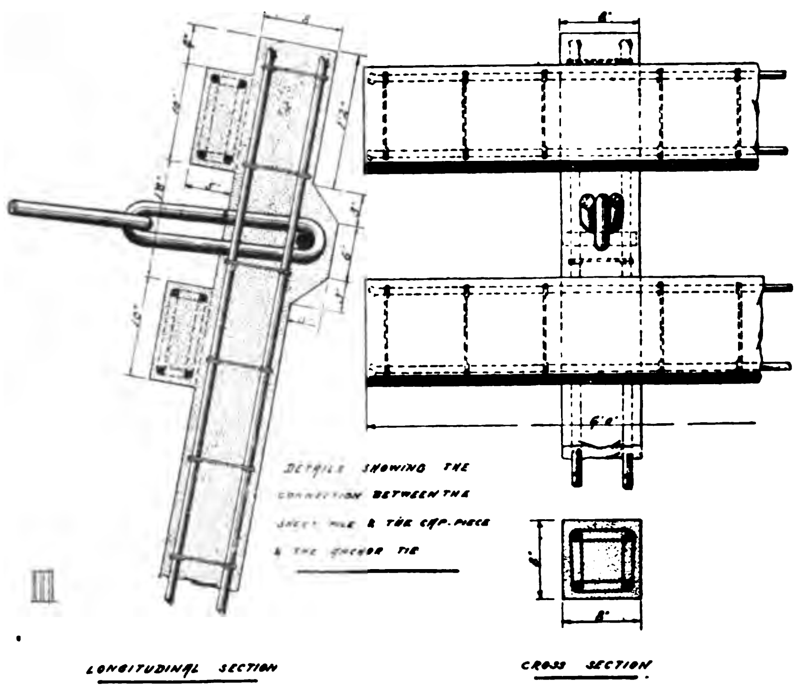


FIG. 474

Slope Protection on the Bank of the Wentowkanal at Marienthal

This slope protection was constructed by *C. Rabitz*, of Berlin.

The reinforced covering is held down by anchor irons driven into the ground, and having screwed points. The width measured on the slope is 11·77 feet, and the inclination is 1 to 1. The concrete slab is 7·87 inches thick, and is reinforced by frames of flat bars, 1·97 × 0·24 inches, placed upright except where they cross and are connected. The spacing of the flat bars of the framing is 11·48 by 10·95 feet. The anchor irons are clipped round these bars, and are spaced as shown in Fig. 477. A diagonal network of round rods 0·24 inch diameter was placed on the framing, and tied to it at the intersections (as shown Fig. 476), the mesh of the network being 6·35 inches.

A finish to the protection is made by carrying it back into the bank for a distance of 1·97 feet at the top, and forming a small projection along the face. This protection is somewhat thick, and the reinforcement is heavier than that generally adopted for similar works. It is constructed for a length of 148 yards, and is continuous for the whole distance, having no expansion joints.

The concrete has been found to crack under variations of temperature, pointing to the advisability of having these joints. In a bank protection on the *Möller system* at Kiel, expansion joints were left every 65½ feet, a strip of bituminous sheeting being placed below and another in the joint itself. This protection was only 2·36 inches thick.

A *Möller* slope lining, which is somewhat similar to that described above, costs from about 1*s*. 11*d*. to about 3*s*. 1*d*. per square yard.

CROSS SECTION OF SLOPE PROTECTION.

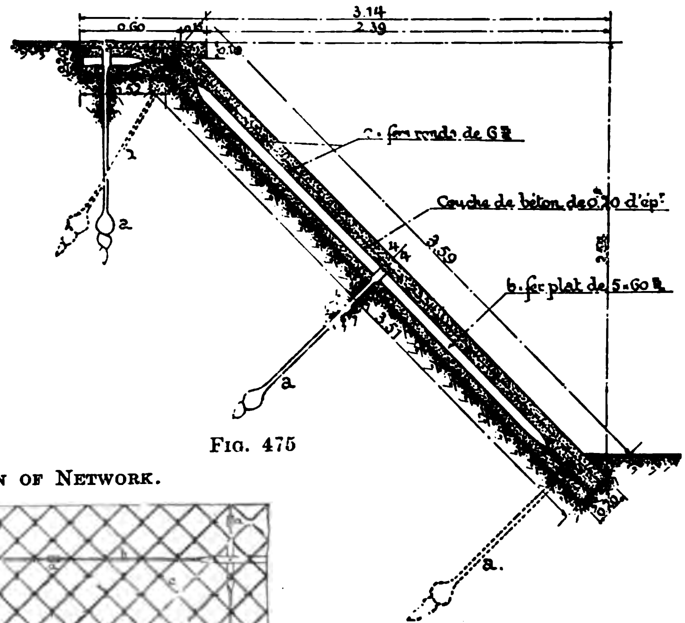
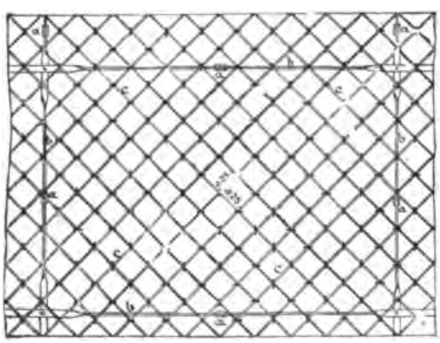


FIG. 475

PLAN OF NETWORK.



SPACING OF ANCHORS AND FRAMINGS.

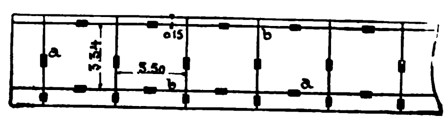


FIG. 476

FIG. 477

Railway Sleepers (Sarda System).

There have been many ideas for reinforced concrete sleepers, but it is believed that these are the only kind that have been used up to the present time. They are reinforced with "expanded metal," placed vertically, and tied together transversely, as shown.

The illustrations show the various methods of attachment for flanged and double headed rails. It will be seen that the bolts can be removed with the greatest ease when required.

The sleepers have a width of 9·45 inches at the bottom, and 8·66 inches at the top, and are 5·9 inches deep under the rails, reducing to 3·94 inches at the centre and ends.

Figs. 478 and 479 show the method employed when flanged rails are used, Fig. 478 being the general view and Fig. 479 the details. Fig. 480 is a plan and elevation

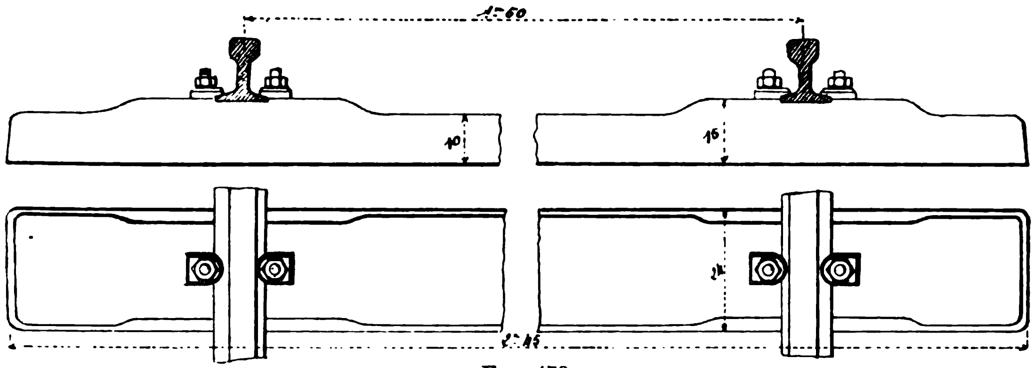


FIG. 478

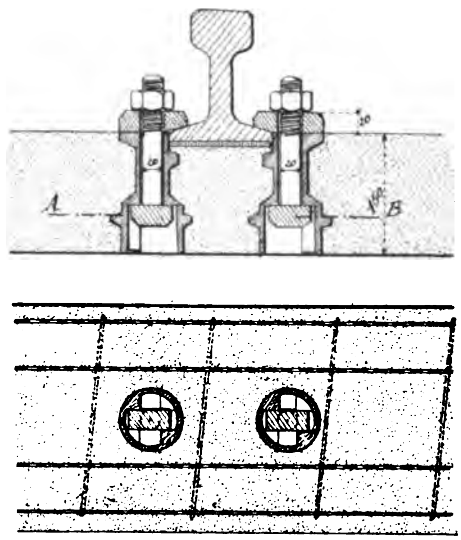


FIG. 479

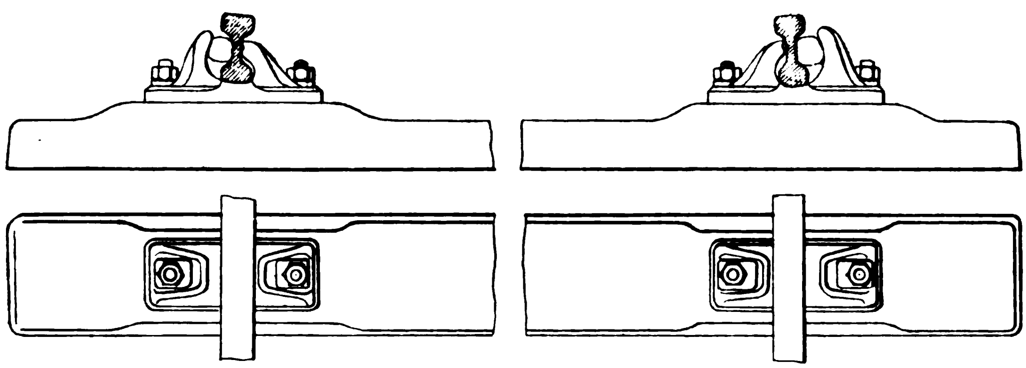


FIG. 480

showing one form of chair with two bolt fastenings, details of which are shown in Figs. 481 and 482. Figs. 483 are an elevation, plan, and sectional plan, showing

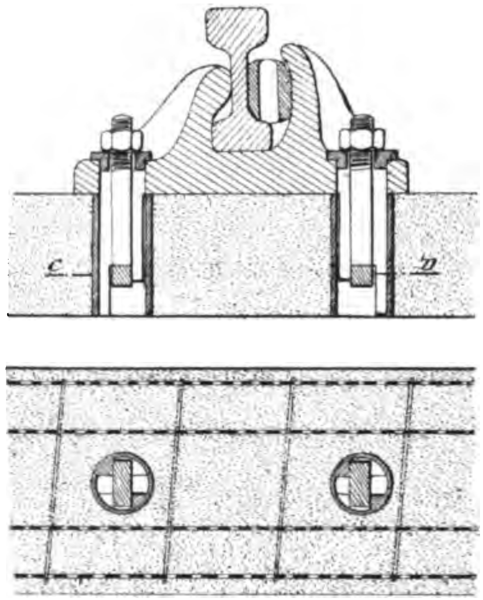


FIG. 481

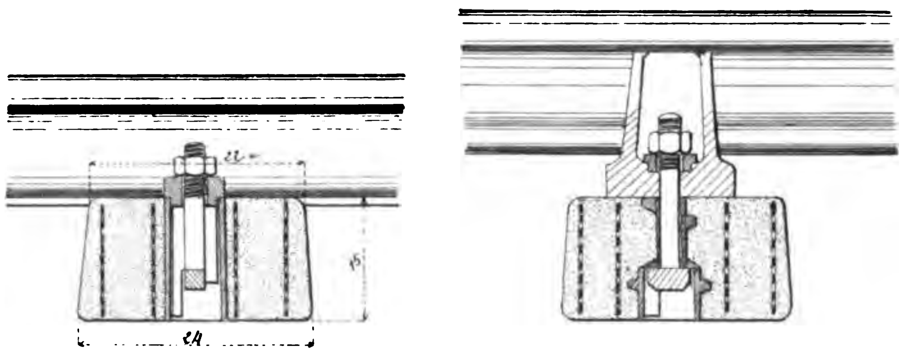
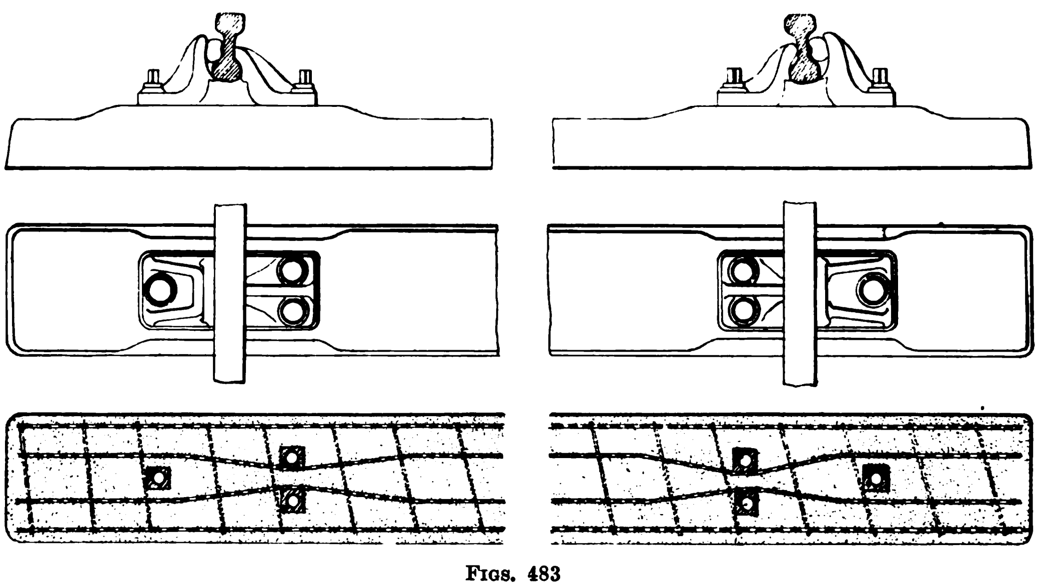


FIG. 482



FIGS. 483

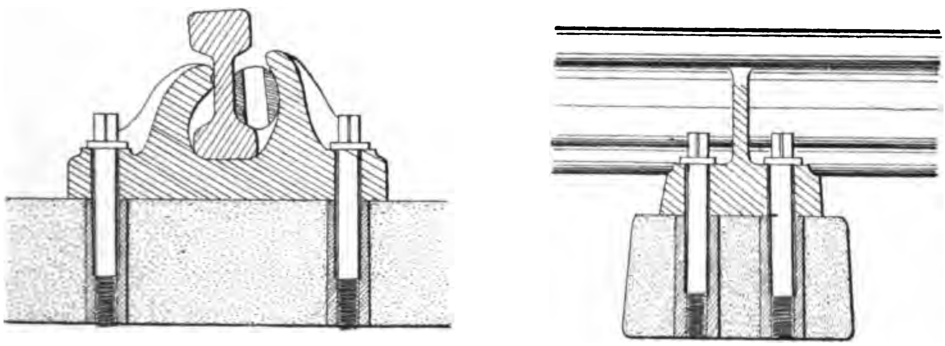
the reinforcement of a sleeper with chairs, having three coach screw attachments. The details of this arrangement are shown in Figs. 484.

The sleepers were first employed in May, 1900, at Bordeaux, on the French State Railways, after which they were adopted by the Tramway Company at Perpignan, and then by the Northern Railway Company of Spain.

In April, 1903, 200 of these sleepers were put down for the express line of the French State Railways, near Bordeaux.

The life of timber sleepers is from four to five years, for those of iron about ten years, whereas it is probable that reinforced concrete sleepers would last for fifty years or more.

The price of the Sarda sleepers varies from about 8*s*. 6*d*. to 9*s*. 3*d*. per sleeper having a length of about 7 feet 4 inches.



FIGS. 484

Troughing for Electric Cables.

The Steel Core Concrete Company, of 22, Laurence Pountney Lane, E.C., construct a troughing of concrete reinforced with "expanded metal" for use with the solid system of electric cable laying, the troughing being straight, curved, or in the form of T-pieces. The troughs are formed of cement-mortar or concrete moulded on a core of "expanded metal" lathing, with light steel rods attached along the top edges, and pieces of sheet metal fixed at the ends to form the metallic facing of the socket and spigot ends of the lengths of trough. The walls of the troughing are from five-eighths to three-quarters of an inch thick, the sockets and spigots being moulded in the thickness of the walls, so that when laid the troughing is of even width from end to end.

With this arrangement the steel core is entirely within the concrete, while there is a metallic contact throughout, the metallic conductor being enclosed and insulated by its covering of cement-mortar or concrete.

The troughs are filled in bitumen or pitch surrounding the cables, and are covered either with a covering, the steel core of which is in contact with strips of metal extending outside the concrete and bent downwards so as to clip the metal endpieces of the sockets of the troughing ; or loose sheets of "expanded metal" lathing are laid on the bitumen or pitch filling before the tile, brick or concrete slab covers are put in place. These are placed with their ends touching, and are connected to the metallic end pieces of the troughing by a thin ribbon of steel or lead inserted between the metal faces of the spigot and socket, the ribbon being bent over or hooked into the covering sheet of "expanded metal."

The troughs are very strong, extremely light, and the " earth sheath " forms a most perfect protection against electrolytic currents or breakdowns. For high-tension work the socket and spigot ends of successive lengths of troughing can be rigidly attached to each other by bolts passing through the metal end pieces, and the loose covering sheets of " expanded metal " may terminate in a metal strip secured by the same bolts.

FIG. 485

Retaining Wall, constructed on the *Bousseron and Garric system*, for the Paris and Orleans Railway Company. Length, 1,312 ft. ; mean height, 13·12 ft.

FIG. 486

Cantilever Quay on the River Loire, at Nantes (France). Constructed of reinforced concrete, on the *Hennebique system*, the cantilevers being tied back to the Warehouse behind, which is also built of reinforced concrete. The platform is 27 feet wide, and overhangs the river by 24 feet 7 inches. It is calculated to carry two cranes of 20 tons, a goods train with locomotive, and in addition a load of 250 pounds per square foot.

FIG. 487

Jetty at Woolston, Southampton, constructed of reinforced concrete, on the *Hennebique system*, calculated to carry a moving load of 5 cwts. per square foot, with a crane to lift 35 tons at the end. The foundations were made strong enough to support a crane lifting 60 tons, in case the latter should eventually be substituted.

APPENDIX

I

M. CONSIDÈRE'S TEST TO DESTRUCTION OF THE PONT D'IVRY.[1]

General Remarks.

This test was carried out prior to the erection of a reinforced concrete bridge of several spans, on the bowstring girder type, designed by M. Considère, the largest span of which is to be 196·8 feet.

The trial bridge had a span of 65·6 feet, or one-third the maximum span of the proposed bridge, and was comprised of two parabolic bowstring reinforced concrete girders connected by wind-bracing, and a decking supported on beams. The advantages accruing to this type of construction are evident.

1. The girders being of the parabolic bowstring type, the intensity of stress is the same

Fig. 488

throughout the whole length of each member, which enables the maximum economy of material to be obtained.

2. The stresses are almost entirely direct in all the members, with the exception of the floor beams and decking.

3. The use of reinforced concrete enables the full resistance to be obtained from the metal, there being no deductions required in the tension members due to rivet holes, etc.

4. The employment of this type of bridge, when constructing in reinforced concrete, reduces the weight of the structure to the minimum obtainable.

5. Since all the metal is efficiently protected, little or no maintenance will be required.

The decision to make a test to failure on a bridge of similar design to that proposed,

[1] M. Considère has patented this method of hooped construction.

and of practical dimensions, was a wise one, since such a test affords information as to the margin of security of the structure, and shows where the tendency to failure will occur.

THE CONSTRUCTION OF THE TEST BRIDGE.

A view of the trial bridge as constructed is shown in Fig. 488 and the general arrangement and sizes of the several members and the methods designed for forming the connexions between the latticings and the main booms in Fig. 489. The central panel was counter-braced. The junction of the top and bottom booms was effected by securing iron plates to the ends of the tension bars by passing these rods through holes in the plates and jumping up their ends, the thrust of the parabolic compression boom being taken by these plates.

The tension rods were calculated to take the whole of the tensile stresses, and the concrete for these members consequently required less care in consolidating, and was put in fairly wet, so as to better protect the metal. The spacing of the rods in the bottom boom is only governed by the size of the jumped-up heads. The top boom is reinforced with longitudinal rods and a spiral hooping.

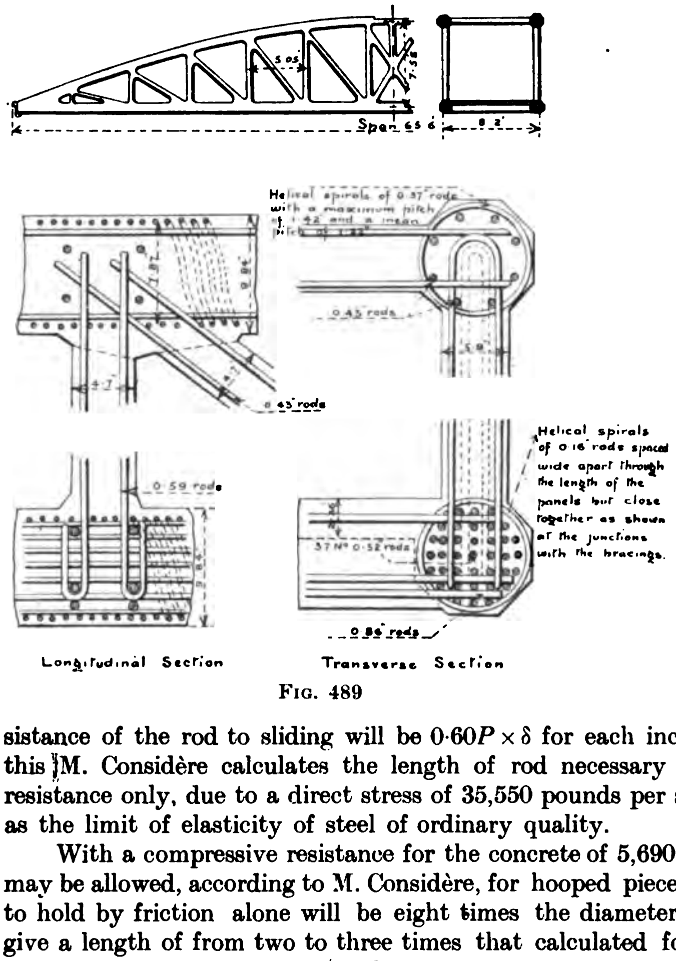


FIG. 489

The length of bar necessary to hold itself, by frictional resistance only, can be easily calculated for the attachment of the rods of the lattice bracing to the top boom. M. Considère allows a coefficient of resistance for the axial plane of a rod of from 0·30 to 0·50 the total stress, and takes the lower limit for his calculations. As the pressure acts on both sides, the total value is 0·60, so that, if P is the force acting on the piece in pounds and δ is the diameter of the rod, the resistance of the rod to sliding will be $0{\cdot}60P \times \delta$ for each inch of length of the rod. From this M. Considère calculates the length of rod necessary to resist sliding by frictional resistance only, due to a direct stress of 35,550 pounds per square inch, which he assumes as the limit of elasticity of steel of ordinary quality.

With a compressive resistance for the concrete of 5,690 pounds per square inch, which may be allowed, according to M. Considère, for hooped pieces, the length of rod necessary to hold by friction alone will be eight times the diameter. It is consequently easy to give a length of from two to three times that calculated for the bars, and M. Considère further forms each pair of reinforcements out of one rod bent over at the top; it will therefore be seen that the assemblage of the rods of the latticings with the top boom is a simple matter.

For the connexion of the rods of the latticings with the bottom boom it is not advisable to count on the "adhesive," or frictional resistance, since this boom is in tension. M. Considère therefore passes the rods of the decking and its supporting beams between the main longitudinals of the bottom boom, and hooks the rods of the web bracing around these; and further, to guard against any displacement of the main longitudinals by the pull of the rods of the bracing, he encircles them with a light spiral winding similar to that of the hooped compression boom. These spirals are spaced far apart, through the length of the panel, but approach one another closely at the joint.

M. Considère considers the proper attachment of the rods of the bracing to those of the bottom boom as a point requiring special attention, and this made the test to failure the more necessary.

It was decided that the size of the top boom should be reduced for the central bay from 9·84 to 7·87 inches and a stronger spiral reinforcement used, the hooping being formed of steel rods 0·433 inch diameter enrolled on a core 6·7 inches diameter, with a spacing of 1·18 inches. In reality, the diameter of the hooping rods was 0·429 inch, and the diameter of the core was 6·3 inches, the spacing of the spirals being 0·94 inch as a mean and 1·38 inches as a maximum.

To guard against any bending of this smaller section, an upright was placed, as shown in the figures, at the centre of the bay. This support was found to produce flexure instead of resisting it, and would not be necessary in a practical design.

The arrangement of the connexions between the rods of the bracing and the bottom boom were carried out exactly as designed and shown in Fig. 489, but, on account of a vexatious misunderstanding, the semi-circles which were to connect the rods in the upper boom were left out in most of the connexions, and consequently the reinforcements of the uprights and diagonals were formed of independent rods hooked at their ends, and these penetrated into the hooped members for varying distances, sometimes of very small length. Further, the rods of the upright where the failure occurred were found to be placed on one side of the piece so as to be close against the lower portion of the spiral hoopings, and consequently they put upon these a considerable extra stress, which hastened the failure, although the connexion formed by the joint was still perfectly secure.

Wind-bracings of transverse pieces and diagonals were placed, connecting the top booms at all joints excepting those of the two extreme panels.

The decking was supported on beams placed at each connexion of the latticings and the bottom boom.

By making the test bridge in exact proportions to the sizes calculated for the larger proposed span, a structure is produced which would not really be suitable for spanning a 65·6 foot opening, as it is necessary to reduce the sizes of the members and the spacing of the spirals to such amounts as to render the ramming of the concrete very difficult.

It was not, however, a question of establishing a type of bridge which would be suitable for a span of 65·6 feet, but to test the dispositions which could be adopted in structures of large span, and consequently the weight of the test bridge itself would be less than that of a suitable bridge for the span of 65·6 feet.

As concerned the tension members, it is necessary to increase as much as possible the proportions of metal to the embedding concrete, to employ a steel of great resistance, and to cause them to act under high stresses.

M. Considère points out that if the numerous metal bars forming the reinforcement of the tension booms were not in the exact directions of the forces which should act upon them, due to their sag, one would fear that they could not straighten themselves out at the moment of the application of the loads without producing deformations and perhaps cracks in the concrete which envelopes them. To prevent this inconvenience, it is sufficient to apply in advance a sufficient portion of the forces which the rods will have to support when in working, and not to put in the concrete until they have taken the directions that they must have eventually.

To produce these tensions, the following method was adopted. Instead of forming the false work of inclined timbers, and tie beams to prevent their spreading, the inclined timbers only were used with their feet attached to the steel plates held by the rods of the bottom boom. A double advantage is gained by this disposition, for, besides giving the necessary tension to the rods of the bottom boom, an economy of false work is also effected.

When the false work for the two girders had thus been established and well braced together, templates were suspended from them, which gave the proper spacing of the reinforcements of the bottom booms and governed the height, so as to give to these members a rectilineal form The boxes for moulding the several pieces in the usual way were then attached to the false work. The uprights and diagonals were moulded first, then the hooped compression booms and the wind-bracing, after which the decking was formed.

In the case of the Pont d'Ivry, the weight was found to be insufficient to straighten out the rods of the bottom boom, on account of the small span (although this would easily have been realized with large spans); a load of five tons of rails was consequently suspended from the falsework.

It would, of course, be unnecessary to take these precautions for an actual bridge of 65·6 feet span; it would suffice to reduce the percentage of metal and at the same time increase the weight of the girders, which in this case would not cause any inconvenience.

For practical purposes it is impossible to use unannealed steel wires for the hooping, since the spirals must have diameters more than 0·2 inch, which is the largest diameter produced by drawing.

The hoopings of the trial bridge were formed of ordinary mild steel rods having an ultimate resistance of 58,018 pounds per square inch and a limit of elasticity of 37,114 pounds per square inch for ordinary sizes, and for small sizes a resistance of 55,060 and an elastic limit of 34,044 pounds per square inch. The spirals were in two sets, each having double the designed spacing, the one set being wound between the other, and the ends of each being finished off about the centre of the length of the other winding.

The ramming had to be done in a direction normal to the axis of the impression boom, which might be supposed to produce a worse result than when done in the direction of the axis; but, in spite of this, the behaviour was perfectly satisfactory. On account of the small size of the members, a shingle passing a screen of from 0·47 to 0·59 inch mesh was used instead of that passing a one-inch mesh, such as is customary in such work. For the hooped pieces a mixture of 1,339 pounds of cement was used to 0·8 cubic yard of shingle and 0·4 cubic yard of sand, which gives 1·28 cubic yards of concrete. This would mean 1,045 pounds of cement per cubic yard of concrete in place. M. Considère points out that since the larger the size of the shingle the smaller will be the percentage of voids, if 1-inch shingle were used it would only be necessary to use 1,170 pounds of cement to 1·2 cubic yards of sand and shingle, or 820 pounds of cement per cubic yard of concrete in place. The cement was reduced to 1,003 pounds for the same quantities of aggregates for the uprights and diagonals of the bracing, and the same weight, but using sand only for the aggregate, was employed for the bottom boom. For the decking and wind-bracing the weight of cement was further reduced to 502 pounds.

Description of the Test.

The test of the bridge was carried out under the supervision of M. Considère, M. Mesnager (Chef du Service des Laboratoires des Ponts et Chaussées), M. Mercier (Principal Conducteur du Laboratoire de l'Ecole des Ponts et Chaussées), and M. Caillebotte. The construction of the bridge was finished on July 28, 1903, and the testing carried out on November 11, 12, and 13.

As a commencement, a load was applied in such a manner as to stress the reinforcements of the diagonals nearest to the centre to 14,220 pounds per square inch. For this purpose 12·1 tons of rails, 14·76 feet long, were first placed on the eastern half of the length of the bridge, laid longitudinally, to distribute the load in such a manner that the resistance of the decking was assured and the production of excessive bending strains prevented in the bottom booms, such as would be produced by loads placed between the uprights. After this, further rails were laid transversely throughout the panels of this half, and in this manner the necessary loading of 34 tons was completed. Under this loading there was no trace of fatigue at any of the joints.

Before proceeding further, it is well to state the sectional area of the compression boom and the stresses which were put on the concrete for each 10 tons of uniformly distributed load. The following Table gives this information for the ordinary size of the boom and for the reduced section at the central panel.

TABLE LXXI

Nature of Member	Usual Type		Centre Bay	
	Total Section	Core within Spirals	Total Section	Core within Spirals
Form of section	Octagon	Circular	Octagon	Circular
Diameter of inscribed circle in inches	9·84	7·87	7·87	6·3
Designation	C	C_c	R	R_c
Area of cross-section, square inches	80·3	48·7	51·6	31·2
Stress in pounds per square inch produced by a load of 10 tons	149	246	231	390

M. Considère points out that the ratio of the weight of the structure itself to the load it has to support varies within wide limits according to the span, and the only method to employ in drawing conclusions from a test, which may be applicable to all cases, is to refer the results to the total load, including the weight of the structure itself. The total weight of the trial bridge was approximately 25 tons, and this load must consequently be added to the super load of rails to arrive at the total stresses on the members.

With a super load of 60 *tons of rails, or a total load of* 85 *tons,*
we have $C=1{,}267$, $C_c=2{,}091$, $R=1{,}963$, and $R_c=3{,}315$.

Under this load, which is that for which the bridge was designed, there was no damage, although it was as heavy a stress as can be allowed on pieces with light hooping in the case of the small section R_c. After this load had been on for 12 hours, the deflection at the centre was a little over $\frac{1}{2000}$ of the span, being more than is usual for reinforced concrete structures of the ordinary type, but the concrete was stressed considerably above the safe allowance in ordinary cases, and, as M. Considère has shown by his laboratorial experiments, the hooping does not increase the coefficient of elasticity to such an extent as the resistance.

The deformations were measured by Manet-Rabut apparati. These are given for the various loadings in Table LXXII, together with the deflections. During a prolonged application of the super load of 60 tons the apparatii A and B (Fig. 492) showed a contraction of $\frac{19}{1000000}$, and the apparatii C and D $\frac{20}{1000000}$.

M. Considère points out that a boom of mild steel stressed to 11,376 pounds per square inch of the total sectional area, not deducting rivet holes, would suffer a contraction approximating $\frac{40}{1000000}$. If $\frac{60}{85}$ of this figure is taken for comparison, with the deformation of the hooped concrete booms, while the total load on the bridge has increased from 25 to 85 tons, this deformation becomes $\frac{28}{1000000}$, which is about half again as much as those of $\frac{19}{1000000}$ and $\frac{20}{1000000}$ observed for the hooped concrete booms. The comparison will be even more favourable to the reinforced concrete bridge if the mean of the contractions of the top booms and the elongations of the bottom booms of $\frac{14\cdot5}{1000000}$ is taken, since it is then found that the deformation of the reinforced concrete bridge was about $\frac{6}{10}$ of that of a steel bridge of the same form.

It is also to be noted that since the proportion of the weight of the structure itself to the moving load is much higher for a reinforced concrete than for a steel bridge of the same type, it will suffer far less deformation during the passage of the load.

FIG. 490.

Taking all these factors into account, one may safely say that the deformations produced by sudden loads on bridges such as that tested by M. Considère are half those of a similar metallic structure.

With a super load of 180 *tons of rails, or a total load of* 205 *tons.*

The bridge showed no signs of failure before the application of this load, so it is needless to refer specially to the intermediate loadings. At this load we have—

$$C = 3{,}055,\ C_c = 5{,}044,\ R = 4{,}735,\ R_c = 7{,}995.$$

The first signs of failure occurred when the loading reached this amount of three times that for which the bridge was calculated. Hardly perceptible hair cracks, less open than those often noticed in beams reinforced in the ordinary manner under the working loading, appeared in the tension booms about a third of the span from the supports. Cracks also appeared in the concrete surrounding the hoopings of the top boom at the connexions I and II (Fig. 492) of the northern girder, and II of the southern girder. These cracks inclined at a small angle to the horizontal at their upper and lower ends, but had a steeper slope throughout the central portion of the boom of the height of vertical faces of the octagonal member. At the same time two of the four piers which supported the bridge and rested on a raft of reinforced concrete settled somewhat. The piers to settle were one at either end supporting different girders, giving a twist to the bridge, which was partially rectified by hollowing out the ground under the piers that had not settled.

With a super load of 200 *tons, or a total load of* 225 *tons.*

At this load $C = 3{,}353,\ C_c = 5{,}535,\ R = 5{,}200,\ R_c = 8{,}775.$

The above mentioned phenomena increased under this load, and also the casing on the central portion of the compression boom commenced to shell off, leaving the hooping exposed. This occurred under a stress of 5,200 pounds per square inch, and was purely superficial.

It was not until the super load amounted to from 220 to 240 tons, or the total load to from 245 to 265 tons, that the casing on the ordinary section of the boom commenced to scale, the stresses being higher than 3,654 and 5,972 pounds per square inch on the total section and the hooped core respectively.

With a super load of 241 *tons, or a total load of* 266 *tons.*

$$C = 3{,}963,\ C_c = 6{,}544,\ R = 6{,}145,\ R = 10{,}223.$$

This load produced the final failure. Five spirals, at the joint I (Fig. 492) of the northern beam, broke, showing the characteristic reduction of area. The concrete, thus left unsupported, crushed and sheared obliquely, as shown (Fig. 490).

This failure produced a distinct report, followed almost simultaneously by that caused by the failure by bending of the compression boom in the two panels of the southern beam near the supports on the east side. The second failure, which was evidently caused by the first, produced a considerable local bending in the boom between the first and second joints, accompanied by cracks perpendicular to the axis. The concrete between these cracks appeared to have retained its resistance, and would without doubt have sustained considerably higher stresses than those to which the member was subjected.

The tension booms were almost intact, and, in spite of the considerable flexure and the shocks to which they had been subjected, they only showed very slight cracks.

The damage was much greater in the latticings, which were reinforced in the ordinary manner. The uprights and diagonals of the first two panels were entirely broken out and only retained portions of the concrete on the rods, which were considerably bent (Fig. 492).

On examination of the portions where the failures occurred, it was found that the compression boom was practically undamaged, except at the joint I (Fig. 492) of the northern girder, and throughout four-fifths of the length the casing of concrete was intact. At the reduced central portion no further damage had occurred beyond the scaling off of the outer layer already described, and this portion did not fail, in spite of the severe shock to which it was subjected at the final rupture.

The reinforcements of the upright where the failure occurred, instead of being united by a symmetrical bend of large radius well embedded in the hooped core, were bent to a radius of only 2·55 inches, and penetrated not more than 3·54 inches into the core. They were also unsymmetrical, both the rods of one pair being at one side of the piece and one of the branches almost parallel to the neighbouring spirals of the boom, being not more than 1·18 inches away from them at several points.

FIG. 491

In spite of this serious defect, the reinforcements had not slipped in the concrete, but the tension they exerted, acting almost directly on the spirals, hastened the failure.

M. Considère points out that it is probable that the tension of these reinforcements added quite 9,954 pounds per square inch to the normal stress on the hoopings, or more than a quarter the elastic limit of the metal.

Although all the joints between the latticings and the top boom were badly constructed, only three out of twenty-four showed any signs of failure, and out of these only one failed by the breaking of the spirals, and that under a very heavy loading.

This shows that, in spite of the unfortunate mistakes in making these connexions, they all withstood very high stresses. It may also be pointed out that the shelling off of the external layer of concrete always considerably precedes the final failure, and gives ample warning.

M. Considère further points out that it is easy to give the joints a larger margin of security by reducing the spacing of the spirals where these occur.

Minor Faults in Construction.

M. Considère mentions the following minor faults in construction of the test bridge, which did not have any effect on the final resistance.

1. The hooped core having a much greater resistance and coefficient of elasticity than the outer casing of concrete, it is specially important that this envelope should be of uniform thickness. It was found, however, that the spirals were absolutely tangential to the outer surface of the concrete at certain places, while at others the covering was 1·57 inches thick. The result of this was that undulations were produced in the hooped core, such that in a length of 10·24 inches of the core of the central panel, an initial deflection of 0·275 inch was measured. M. Considère points out that, as the inconvenience of an eccentricity of the hoopings is inversely proportional to the diameter, there is nothing to fear on this account in an actual construction, and that to ensure an even covering it is only necessary to pack the hoopings off from the sides of their moulds by pieces of concrete of practically the same thickness at several places along and around them before putting in the concrete.

2. The axes of the boom members meeting at the same joint should evidently cut one another at the point of junction of the corresponding bars of the latticings. Such was not the case in the trial bridge, and considerable errors were noticed due to this.

3. The spacing of the spirals was fairly regular as a rule in the ordinary sized top boom, but, on account of the small diameter of the core and the use of larger rods, the spacing for the central portion was not uniform, and varied between the relatively wide limits of 0·945 and 1·417 inches. This appeared to have no bad effect.

4. The extremities of some of the spirals were not bent inwards along the diameter of the boom, as was intended, several of them only penetrated from 1·97 to 2·36 inches into the core; there was, however, no bad effect, thanks to the considerable frictional resistance of the highly compressed concrete and the precaution which was taken in alternating the joints of the spirals.

5. The hooping of the compression booms should have been prolonged up to the end plates, being slightly deformed so as to pass between the rods of the tension boom. It was forgotten to insert these last windings prior to the putting in place of the longitudinal rods of the tension boom and the forming of their heads, so as to retain the plates in position. To remedy this error separate hoopings were placed at the ends, which were given a considerable excess of strength to compensate for the lack of continuity.

In spite of this defect, the ends of the girders showed no signs of damage.

Resistance

M. Considère concluded from his laboratorial experiments (page 293) that the resistance of a hooped piece was made up of—

1. The resistance of the concrete when not hooped.
2. The resistance of the longitudinal reinforcements stressed to their elastic limit.
3. The resistance of imaginary longitudinals having at least 2·4 times the sectional area of the spiral reinforcement and stressed to its elastic limit.

Employing this method of calculation for the resistance of the hooped members of the Pont d'Ivry, M. Considère considers, very rightly, that the percentage of reinforcement must be referred to the hooped core, since the outer shell has no effect on the ultimate resistance, if, as is usually the case, it flakes off some time before the final rupture.

On this assumption he finds that the spirals are 4·6 or 4·04 per cent. of the volume of the concrete according as their pitch is taken as the mean of 1·22 or as the maximum of 1·42 inches. The percentage of the longitudinals was 2·42.

The resistance of the piece for the mean spacing of the spirals is therefore made up of—

The resistance of the spirals $= 0{\cdot}046 \times 2{\cdot}4 \times 37{,}114 = 4{,}097$ pounds per sq. in.
,, ,, longitudinals $= 0{\cdot}0242 \times 37{,}114 =$ 898 ,, ,,
,, ,, concrete = 2,560 ,, ,,

The total resistance being $= 7{,}555$ pounds per sq. in.

Making a similar calculation for the maximum spacing of the spirals, the resistance is found to be 7,000 pound per square inch.

The resistance of the concrete is assumed, but will be verified after the bridge is destroyed.

The absolute pressure to which the ordinary section of boom was subjected did not exceed 6,540 pounds per square inch, and consequently the test does not absolutely establish the truth of the method of calculation proposed by M. Considère.

It is important, however, to note that throughout a portion of four-fifths of the total length of the boom of ordinary section no failure occurred, and, what is still more remarkable, the concrete forming the casing did not peel off, which will always occur some time before the final failure, as shown by the behaviour of the reduced section which scaled at a pressure of 7,882 pounds per square inch on the hooped core, and stood without failure the enormous pressure of 10,223 pounds per square inch.

For the portion of the beam of reduced sectional area the percentage of spiral reinforcements was 9·75 or 6·5 according as the mean pitch of 0·94 or the maximum of 1·42 inches is taken.

The percentage of longitudinal reinforcements was 3·8 of the hooped core.

Taking the mean spacing of the spirals and proceeding as before, we get—

The resistance of the spirals $= 0{\cdot}0975 \times 2{\cdot}4 \times 33{,}844 \times 7{,}929$ pounds per sq. in.
,, ,, longitudinals $= 0{\cdot}038 \times 36{,}970 =$ 1,405 ,, ,,
,, ,, concrete = 2,560 ,, ,,

The total resistance being $= 11{,}894$ pounds per sq. in.

Making a similar calculation for the maximum spacing of the spirals, the resistance is found to be 9,250 pounds per square inch.

The piece withstood a pressure of 10,223 pounds per square inch, and must have resisted more than this amount in consequence of the sudden shock due to the failure of the bridge. It is probable therefore that the real resistance lies between the two figures found as above, or may even be greater than that calculated from the mean spacing. This result, therefore, appears to indicate the truth of M. Considère's method of calculation.

Coefficient of Elasticity.

From the deformations of the hooped boom the coefficients of elasticity are found to be those given in Table LXXII showing a coefficient becoming smaller as the pressure increases, as in the case of the laboratorial experiments under a first loading.

TABLE LXXII

Portion of Ordinary Section		Central Portion of Reduced Section	
Coefficient	Pressures between which Coefficient was taken Pounds per Square Inch	Coefficient	Pressures between which Coefficient was taken Pounds per Square Inch
$4{\cdot}52 \times 10^6$	885 to 1268	$6{\cdot}95 \times 10^6$	580 to 1970
$3{\cdot}85 \times 10^6$	1268 to 2463	$6{\cdot}11 \times 10^6$	1970 to 3825
$2{\cdot}84 \times 10^6$	2463 to 3060	$2{\cdot}10 \times 10^6$	3825 to 4750

The pressures are referred to the whole section of concrete, including the outer casing.

APPENDIX

Ductility.

The sudden failure of the bridge being due to the breaking of the spirals at a joint, the effects of the remarkable ductility of hooped concrete were not observable. At the time of rupture, however, some of the portions were under considerable flexure, which produced tensile cracks in the concrete.

Since it was not known to any certainty beforehand what the behaviour of the reduced section would be, it was deemed advisable, as has been mentioned previously, to support its centre by a short upright from the lattice bracing of the central panel, but from the information gained from the measured deformations, it was found that this support, instead of aiding the hooped member in resisting bending, was in itself a cause of flexure. The compressional stresses produced in the member, caused the upper ends of the lattice bracings to approach one another, and consequently tended to elevate the two ends of the member; the central upright acted as a tie, holding down the centre, and therefore resisted the general raising, producing a flexure shown by the instruments measuring the deformations (*vide* Instrument B, Table LXXIII). The concrete of the upright and the upper ends of the latticings was cut away to lessen this effect (*vide* Fig. 490), but this was done too late to have any appreciable effect. On plotting the deformations, M. Considère found that under the final loading the upper longitudinals of this hooped piece were subjected to contractions four times as great as those of the lower longitudinals.

It is interesting to note that under this flexure the hooped core of the central reduced section withstood the pressure of 10,223 pounds per square inch, which would certainly have produced the premature failure of less ductile materials.

Symptoms Announcing the Fatigue of Hooped Concrete, and Possibility of doing away with the inconvenience due to the Shelling off of the Outer Layer.

Before the hooped core has attained the limit of safe resistance, the outer shell will scale off. It was also noticed in the test of the Pont d'Ivry, that noticeable cracks occurred at several of the joints under a load of 180 tons, or three-fourths the final load of 241 tons. The difference between the resistance of the hooped core and the outer shell was more in the case of the ordinary section than in that of the reduced section at the central panel. This shelling of the outer envelope giving notice of the approach of failure is a very advantageous property of hooped concrete.

After the outer layer has flaked off it can be replaced at very small expense, and in consequence of the hooped core having taken a permanent set, the new envelope will not shell off so soon on the reloading. It is also possible to give further resistance to the hooped core by putting on a supplementary winding of steel wire. If the new winding is introduced at a connexion, the concrete of the bracing is broken through, the new winding put on and the concrete pieced up. The destruction of the concrete has no influence on the resistance of the bracing, since these act in tension.

Compared with other materials, hooped concrete has a distinct advantage due to the notice of approaching failure. It is true that if the cross ties of a compression piece, reinforced with longitudinal rods only, are placed near enough together, the behaviour of the piece is somewhat similar to that of hooped concrete, but the difference between the first scaling and the final rupture is never very great in such a case.

In the case of a well designed metal structure, the final failure is generally due to the metal having passed its elastic limit, of which there is no warning. It may also be mentioned that in the case of pieces of reinforced concrete subjected to bending, where the failure will be due to tensile stresses, considerable deformations, due to the excessive elongation of the reinforcements, will occur long before the loading becomes at all dangerous. This property will often allow the necessary precautions to be taken to avoid failure. This is a distinct advantage over metallic construction, in which the failure by tension will occur suddenly in the parts pierced by rivet holes.

The Uncertainty of the Behaviour of the Material due to Bad Workmanship.

M. Considère remarks, in referring to this disadvantage, that whereas engineers, who are unacquainted with the use of reinforced concrete, recognize generally its good qualities and the advantages which are realized by its employment, they fear the danger of bad workmanship in its construction.

If, however, the statistics of works in this material are studied, of which hundreds of millions have been constructed (over twenty million structures have been erected by the Hennebique system alone), there do not appear to have been a greater proportion of accidents in works of this material than in any other kind of construction, and as a rule such failures as have occurred were due to premature striking of the falsework or some indiscretion during construction, and even against these eventualities the use of hooped concrete gives a great measure of security.

The accidents which occur in reinforced concrete under compression are the direct consequence of the slipping of the reinforcements through the concrete, or to the crushing of the concrete. In hooped concrete the tendency to the slipping of the reinforcements is much reduced, due to the lateral compression caused by the hooping, and the crushing of the concrete is absolutely prevented.

The adherence was not brought into play for the tension rods of the bottom boom of the Pont d'Ivry, since they were held by the end plates; nor was it requisitioned in the connexions of the bracings with the tension boom, since the rods of the bracing were all hooked securely to the main reinforcements of the floor beams, which were themselves held by being passed between the rods of the bottom boom.

The frictional resistance of the hooped concrete of the top boom on the rods of the bracing would have been sufficient of itself, but as an extra precaution these were formed with bends at the top.

To judge the effect of the use of an inferior quality of concrete on the crushing resistance of hooped members, we may refer to the calculations made on page 516. If the resistance due to the concrete itself is reduced one-half, the total resistance is only reduced from 7,555 to 6,275 pounds per square inch in the ordinary sized boom, and from 11,894 to 10,614 for the portion of reduced section, which still leaves a large margin of security.

The coefficient of elasticity, when inferior concrete is employed, will be diminished in a greater measure than the resistance, and this will reduce the resistance to flexure of long pieces, but, for structures similar to the Pont d'Ivry, where the unsupported lengths are comparatively small, this will not affect the security. M. Considère calculates that with the initial coefficient of elasticity reduced to the improbably low figure of $2{\cdot}13 \times 10^6$ and remaining at this amount, a piece of hooped concrete with a length of ten diameters would support a pressure of 14,220 pounds per square inch without flexure.

The Resistances Obtained.

Since his laboratorial experiments on hooped concrete (page 240), M. Considère has followed up the application of this method of reinforcement to diverse works, and bearing in mind the importance of the amount of dead load in structures of large span, he specially studied the possibility of constructing members with great resistance, using mild steel of commercial quality for the spirals instead of drawn wire.

It was for this purpose that he substituted a piece, in which the spirals have a volume of 6 per cent. that of the concrete, in the central panel of the Pont d'Ivry. The resistances obtained without failure for this piece have been mentioned above, and M. Considère intends to make further experiments on pieces with a larger percentage of reinforcement.

Factor of Safety.

A coefficient of security is necessary for several reasons.

APPENDIX

1. Certain precautions apply to all constructions of whatever material, the most important being the probability of errors in the calculations and the possibility of an increase of the loading above that for which the structure was designed.

2. It is always necessary to provide against possible bad workmanship, but it has been shown that there is not much to fear on this account in structures of hooped concrete, since a very large portion of the resistance is due to the spirals. It is easy to check the disposition of these before the putting in of the concrete and their resistance is very uniform, since they are bent cold, and are not subjected to any forging, welding, etc., prior to being put into place.

3. In all structures there is always a possibility of an uneven settlement of the supports and secondary stresses caused by the rigidity of the connexions. These effects are resisted by the ductility of the material, and it is here that metal structures gain an advantage over those constructed of ordinary masonry.

It has been shown that hooped concrete possesses the property of ductility in a marked degree, and M. Considère mentions a further case in which a hooped prism 5·91 inches diameter suffered without breaking a curvature of 1·96 feet radius.

It is evident, therefore, that as far as concerns the ductility, there is no reason for adopting a greater factor of safety than is allowed for a similar metallic member.

4. It is necessary to take into account the effects which may be produced in a structure due to dynamical stresses and vibrations. As far as the first of these is concerned, hooped concrete has an immense advantage over metal in consequence of its relative weight, under compressive stresses of about 1,280 pounds per square inch. The advantage diminishes as the unit stresses increase, but the relative resistance is never in favour of the metal structure, since the weight of the concrete will probably always be in excess of that of the metal.

As regards the resistance to vibrations. M. Considère mentions the case of the sleepers of reinforced concrete on the Dreux line, which have been subjected to the passage of trains for five years, and have retained all their qualities. He also mentions the frequent use of reinforced concrete, with complete success, for the foundations for heavy machinery and the supports for shafting, which are subjected to very great and incessant vibrations.

5. In the determination of the factor of safety, it is especially necessary to take into account the effects of climatic conditions on the proposed structure. As far as these effects are concerned, it is well known that the metal in reinforced concrete structures is absolutely protected from rusting, a deterioration which must be particularly provided for in the calculations for metallic structures.

Taking all matters into consideration, M. Considère sees no reason for employing a larger factor of safety for hooped concrete than for ordinary metallic structures of a similar nature. The French regulations of 1891 allow the stresses of 12,086 and 16,352 pounds per square inch on ordinary iron and mild steel respectively in bridges of over 98·4 feet span. These figures correspond to a factor of safety of 2 to 2·30 on the limits of elasticity of the two materials.

The allowed stresses are lower for small works, but the security is no greater in consequence of the special dangers which menace light structures.

It seems probable that in a few years the same factors of safety will be allowed for hooped concrete as for metallic structures. And these may be accepted at the present time for tension members such as those of the Pont d'Ivry, in which the reinforcements were firmly secured at the ends and were continuous throughout the whole length of the piece. M. Considère considers that it is necessary to use more prudence for the present in the case of hooped compression members, and proposes to limit the pressure on the members with 6 per cent. of spiral and 2·5 per cent. of longitudinal reinforcement to 2,135 pounds per square inch on the total sectional area of the piece. This figure corresponds to a factor of safety greater than 3·7, since the reduced section on the Pont d'Ivry supported 10,223 pounds per square inch on the hooped core, which corresponds to a pressure of 7,820 pounds per square inch on the total section if there had not been an excess in the thickness of the concrete forming the casing.

With an allowed stress of 2,135 pounds per square inch the factor of safety will be 2·4 with respect to the pressure of 5,200 pounds per square inch of the total section being the stress under which the scaling took place without in any way affecting the stability of the piece.

M. Considère is of the opinion that according to circumstances and particularly taking into account the slightness of the members and the spans of the structures of which they form a part, one may vary the percentage of metal between 2·5 and 6·0 per cent. for the spirals and between 1·5 and 2·5 per cent. for the longitudinal rods, and allow pressures on the concrete between 1,280 and 2,135 pounds per square inch.

The percentage of metal in the tension members will vary in the same proportion.

Conclusions.

M. Considère considers that the following conclusions are justified.

1. The resistance of the metal when used as spiral hooping is at least 2·4 times as great as when it is used as ordinary longitudinal reinforcement, which permits the attainment of great resistance at little cost. There appears to be no further information required in this direction.

2. The ductility of hooped concrete makes it capable of resisting great deformations without inconvenience, and consequently it will easily resist settlements and secondary stresses.

3. Hooped concrete possesses the well known qualities of all reinforced concrete, great rigidity of connexions, resistance to shock, rapid extinction of vibrations.

4. The danger of bad workmanship is reduced to a minimum by the use of hooped concrete.

5. The approach of failure is announced some time before it will take place by the scaling off of the outer casing, and such scaling is in no degree detrimental to the resistance.

6. It is easy to add to the resistance of a hooped member in an existing structure, without putting it temporarily out of use, since this may be done by wrapping on new spirals of drawn iron or steel wire around the existing core and replacing the outer casing of concrete.

7. The employment of continuous bars or drawn wire of iron or hard steel for the reinforcement of tension pieces gives a great resistance for these members with the minimum of weight.

8. The connexion between the bars of the web bracing with those of the main booms is perfectly easy.

9. Ordinary reinforced or hooped concrete is always heavier than framed metallic construction for any given structure.

10. Structures formed of reinforced concrete do not lend themselves to launching, nor to erection by building out, and consequently necessitate the employment of expensive falsework.

11. Hooped concrete has an advantage over other kinds of construction in respect to the cost of material and labour, if it is compared with pieces capable of supporting the same load.

12. The durability of structures formed of ordinary reinforced or hooped rich concrete is practically infinite, and its maintenance a minimum.

The disadvantage (9), may be reduced in a great measure by the increase of the percentage and resistance of the reinforcement.

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Applied Super Loads	...nt	Instrument I	Instrument K
Commencement of loading		mm.	mm.
12·1 ton	50	+ 0·0350	+ 0·0450
	75	+ 0·1525	+ 0·1775
34 ,,	50	+ 0·1550	+ 0·1825
	00	+ 0·1700	+ 0·2625
60 ,,	00	+ 0·1700	+ 0·2800
40 ,,	25	+ 0·1450	+ 0·1925
60 ,,	75	+ 0·1775	+ 0·2725
	50	+ 0·2075	+ 0·2375
80 ,,			
	50	+ 0·1950	+ 0·2450
100 ,,	75	+ 0·2250	+ 0·3075
120 ,,	00	+ 0·2825	+ 0·4775
140 ,,	25	+ 0·3150	+ 0·5475
	00	+ 0·3975	+ 0·6575
160 ,,			
	25	+ 0·3975	+ 0·7000
180 ,, (1)	25	+ 0·4250	+ 0·8100
(2)	...nts removed.		+ 0·5875
200 ,,			+ 0·7000
220 ,,			
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APPENDIX

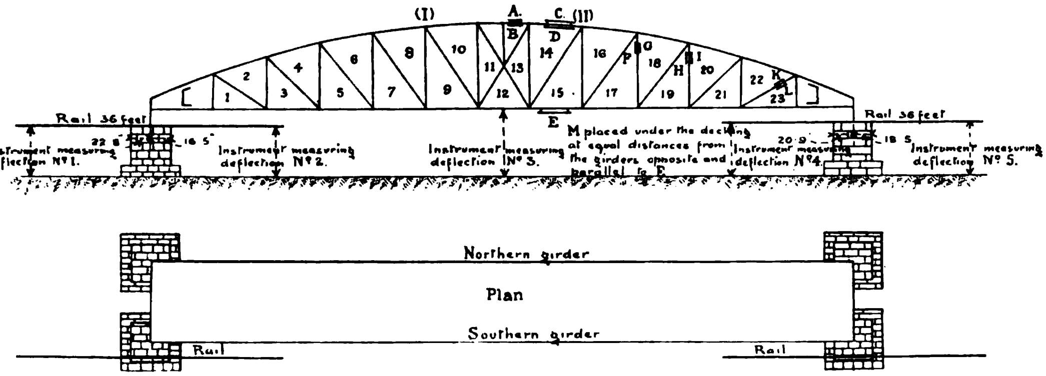

FIG. 492

DISPOSITION OF THE MANET-RABUT INSTRUMENTS

	A	B	C	D	E	F	G	H	I	J	K	L	M
Lengths intercepted between the middle of the fastenings receiving the Manet-Rabut instrument in metres	0·20	0·20	1·00	1·00	1·00	0·20	0·20	0·20	0·20	0·20	0·20	0·20	1·00
The mean distance of the branch uniting the two parts of the Manet-Rabut instruments with reference to the extreme fibre, in centimetres	6·0	7·5	6·0	6·2	6·2	5·5	6·0	6·0	6·0	5·5	6·0	6·2	—

NOTE.—The installation of instrument B leaves something to be desired.

The fastenings were not in line with one another, it was necessary to use wedges of metal to make up the disparity and to make it possible to put the instrument in position.

The metallic pieces connecting the two parts of the Manet-Rabut instruments were protected against the sudden variations of temperature by means of a paper wrapping. Thermometers placed in contact with two of these branches (instruments C and E) supply the means of ascertaining the variations of temperature.

APPENDIX II

Prevention of Leakage in Concrete Structures containing Liquids.

It has been found that a mixture of soft soap or caustic potash and alum incorporated with the mortar of a protective layer or rendering will secure the complete impermeability of concrete structures. The protective layer may be brought up against the forms at the same time as the concrete is deposited, or may be applied as a rendering.

The proportions used may be 2 pounds of soft soap and 12 pounds of alum with 30 gallons of water per cubic yard of mortar; or 2 pounds of caustic potash and 5 pounds of powdered alum to 10 quarts of water, making a standard solution of which 3¾ quarts may be used with each mixing, consisting of 2 bags (448 pounds) of cement, and twice its volume of sand.

It appears that the use of these mixtures produces no effect on the strength of the concrete, for although the soft soap would slightly reduce the strength, and if used in large quantities would cause checking and cracking, the alum alone would tend to increase the strength. These two substances also enter into a chemical combination, which prevents any individual action on the concrete.

This method of producing impermeability was put forward by Mr. W. C. Hawley, of the Pennsylvania Water Company, in a paper published in the *Journal of the New England Water Works Association,* and read at the meeting held on December 9, 1903, an abridgment being published in the *Engineering Record* of December 12, 1903.

APPENDIX III

Works Constructed by Major Stokes-Roberts, R.E., in India.

Major Stokes-Roberts has been instrumental in introducing the use of reinforced concrete and brickwork for Government purposes in India. He is of the opinion that a simple and reliable system of reinforcement and calculation is essential in that country, since the recognized firms cannot be employed to do the work, which must be entrusted to native labourers.

It is hoped that the tables and diagrams previously given for slabs and beams will be found useful in simplifying the necessary calculations for these pieces, since by their employment any beam or slab may be designed without any difficulty, and by the use of only a few simple formulæ, when once the bending moment on the piece has been obtained.

Major Stokes-Roberts has elaborated a system for the construction of reinforced tanks, of which a description is given below. A short description is also given of two arched bridges constructed by him to carry foot traffic and a line of 1½-feet gauge tramways.

APPENDIX

Reinforced Tanks, as constructed by Major Stokes-Roberts.

The provision and erection of centreing forms one of the main items of expenditure for tanks with concrete walls, and with a view to avoiding this, Major Stokes-Roberts, has designed a new type of reinforced brickwork, which has been successfully used for small reservoirs up to 23 feet in internal diameter and 10 feet in depth. The structure consists essentially of a brickwork cylinder, encircled with metal hoops, which take all the tensile stresses produced by the water pressure. The cylindrical walls so far constructed have been 9 inches thick, but for small diameters where economy of space or materials is necessary, no doubt 4½-inch brickwork could be used. During the construction of the cylinder a number of wires are built in radially and left projecting from the face of the brickwork (Fig. 493),[1] with the following purposes in view :—

Fig. 493.

Fig. 494.

(i.) To form a series of brackets on which the horizontal metal rings are supported, whilst being put together, as described, in the specification below.

(ii.) To ensure the correct vertical spacing of the horizontal rings, and to clip them tightly to the cylinder. The rings for the walls of reinforced concrete tanks have a tendency to get displaced during the ramming of the concrete, unless tightly secured to the vertical distributing bars, but this is entirely obviated in the system now described.

(iii.) To ensure that a space is left between the exterior of the cylinder and the backs of the rings, in order that the cement mortar which is afterwards put on may be in complete contact with the ironwork, and so save it from deterioration.

(iv.) To give additional strength on the principle of the Monier distributing bars, where the rings are placed some distance apart vertically.

If the specification is carefully followed there will be no difficulty at all in making these tanks water-tight, and although the reinforcement may appear somewhat complicated in the drawings, in actual practice it will be found that it can be completed rapidly and more easily than a Monier grill. With the exception of the small additional rates for covering the extra trouble in keeping the brickwork truly cylindrical, for building in the radial wires, and for fixing the reinforcing rings, there is no expenditure to meet, beyond the cost of the iron, and the ordinary local rates for brickwork, concrete and plastering, so that in most localities this form of construction should prove decidedly economical in comparison either with plain masonry of a cross-section giving the same resistance to the water pressure, or with cast and wrought iron. In comparing with the latter, the capitalized cost of maintenance and the question of durability must be taken into consideration.

[1] Fig. 493 shows radial wires in a tank of 23 feet internal diameter.

An elevated tank can also be constructed on this system, with the addition of a reinforced concrete bottom. In this case the centreing for the bottom, if made horizontal, is easily put up, and at small cost. These tanks can be finished off with slabbed or monolithic roofs of reinforced concrete, but the domical covering shown in Fig. 494 is perhaps neater in appearance.

Should at any time increased depth and storage capacity be required, the necessary extra reinforcement can be provided without difficulty, whereas iron or masonry tanks are only suitable for the depth of water for which designed.

The following specification was drawn up for the construction of a 5,000 gallons tank, and is applicable, with slight modifications to increased depths and diameters.

SPECIFICATION.

1. A brickwork cylinder of 12½ ft. internal diameter and 6½ ft. in height will be built in cement 9 in. thick, the greatest care being taken that all joints are properly filled in. The mortar will consist of three parts clean sharp sand to one of cement.

2. The bricks, instead of being soaked in water in the usual manner, will be immersed before use in a thin cement grout.

3. Pieces of old telegraph wire will be built in radially at one foot intervals, as shown

FIG. 495.

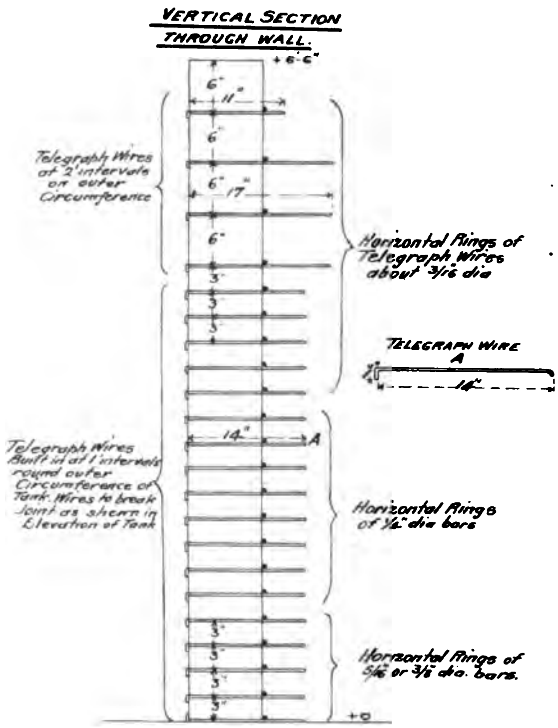

FIG. 496.

in Figs. 495 and 496, and breaking joint vertically, as in Fig. 493, and in dotted and full lines, Fig. 495.

The circumference of the tank being 44 ft., forty-four of these wires will be required for each of the lower horizontal joints (3 in. vertical intervals), and twenty-two wires for each of the four upper joints (6 in. vertical intervals).

4. Great care will be taken to make the brickwork truly cylindrical, and as the work rises, a layer of cement plaster (1 cement, 3 sand) left rough and not exceeding ¼ in. thick, will be given to the *inner and outer* faces of the wall.

5. On completion of the brickwork, the internal water-tight coat of plaster (1 cement, 2 sand), $\frac{1}{4}$ in. thick (making $\frac{1}{2}$ in. in all with the $\frac{1}{4}$ in. already applied), will be put on, and both it and the external face will be kept thoroughly wet.

6. When the brickwork has well set, and the radial wires are secure, the horizontal iron rings for encircling the cylinder can be made up in straight lengths on the ground, shown in Fig. 497, each bar being hooked at the end (bent cold); care being taken that

Fig. 497.

the end hooks are either both in the horizontal plane as shown at *X.Y.* (Fig. 497), or both in the vertical plane as *O.P.*

The total length of ring as laid out on the ground should be the *actual* circumference of the brickwork (say 44 ft.).

7. The bottom ring will then be laid out on the horizontal brackets formed by the bottom radial wires, which will be at once turned up, as shown at *A* (Fig. 498), and tapped gently into position with a piece of wood.

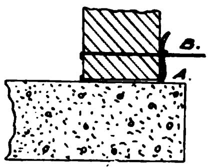

Fig. 498.

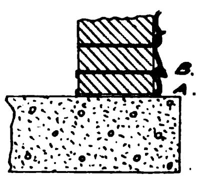

Fig. 499.

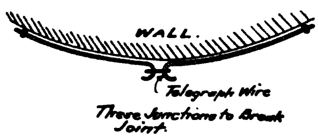

Fig. 500.

8. The second ring can then be placed on the second layer of radial wires, which will be turned upwards as before.

The joint shown (Fig. 500) should break joint with the similar junction of the ring below.

9. Joint *B* can now be completed, as shown in Fig. 499, by turning down the projecting end of wire shown above *B* in Fig. 498.

10. The lower horizontal rings are thus clipped to the wall at distances of 6 in. throughout their length, and kept away from the wall by a distance equal to the diameter of the telegraph wire.

If care has been taken with the clip joints it will be found that the horizontal rings are rigidly held in position, and no tightening up at all will be required.

The hooks *X.Y.* (Fig. 497) can then be secured with two complete turns of telegraph wire (thus giving a cross-section of four wires at the centre of joint) as in Fig. 500.

11. On completion of the external network the whole of the ironwork will be carefully brushed over with cement grout, and immediately afterwards a layer of cement plaster (1 cement, 3 sand) sufficiently thick to well cover all the joints in the bars, will be laid on fairly dry, and may be consolidated by being well tapped into position with small wooden beaters, commencing from the bottom. A total of about $1\frac{1}{4}$ in. thickness of plaster (including the original $\frac{1}{4}$ in.) should suffice. [Fig. 501 shows the tank with the reinforcement, the ironwork on the left side having been brushed over with grout.]

12. The thrust rail or girdle for the dome shown in Fig. 502 will then be put on, and the construction of the latter (vide separate specification) can be proceeded with.

13. The cornices and ornamentation will be the final operation.

Fig. 501.

14. When the dome has thoroughly set the centreing will be removed through the manhole.

Note.—The only calculations necessary are for the external horizontal rings, which are calculated exactly as for the rings of concrete pipes or reservoirs. The vertical spacing of the rings will in the case of brickwork be 3 inches or multiples thereof to suit the horizontal joints; but if stone is used for the cylinder, the depth of the lower courses should not exceed in depth the distance which is considered suitable as a maximum spacing for the rings.

The radial wires for tanks up to 23 feet in internal diameter have so far been built in at 1-foot intervals, but there is no objection to this distance being doubled, especially for larger tanks where the telegraph wire could be replaced by ¼-inch soft iron bars.

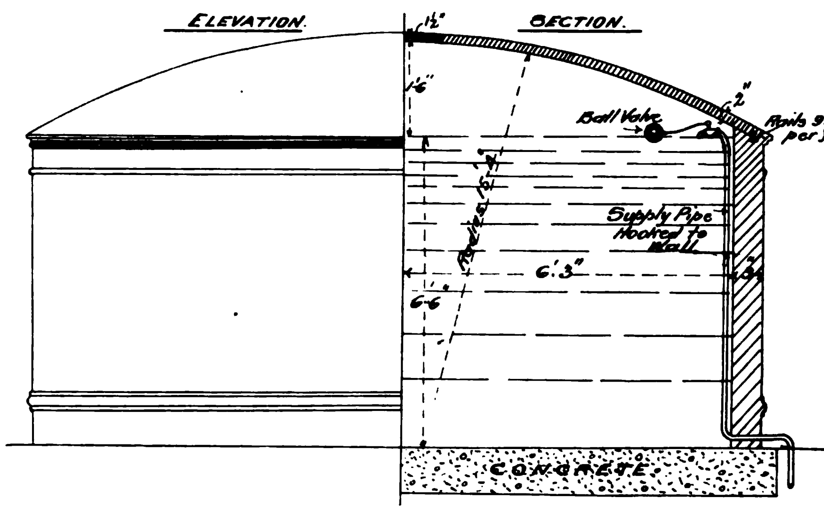

Fig. 502.

Specification for Dome for Reinforced Brickwork Tank of 12 ft. 6 in. Internal Diameter, and 5,000 Gallons Capacity.

1. The thrust due to the dome will be taken up by an old tram rail weighing 9 pounds to the yard bent to a circle of 13 feet 3 inches internal diameter (Figs. 502 and 503).

2. This rail will be placed in position on the centre of the wall, and the mud centreing resting on supports in sand boxes will then be put up, a good proportion of sand being used

in the upper surface of the mud plaster (the completed centreing with rail in position is shown in Fig. 503, and the sand box supporting the props in Fig. 504).

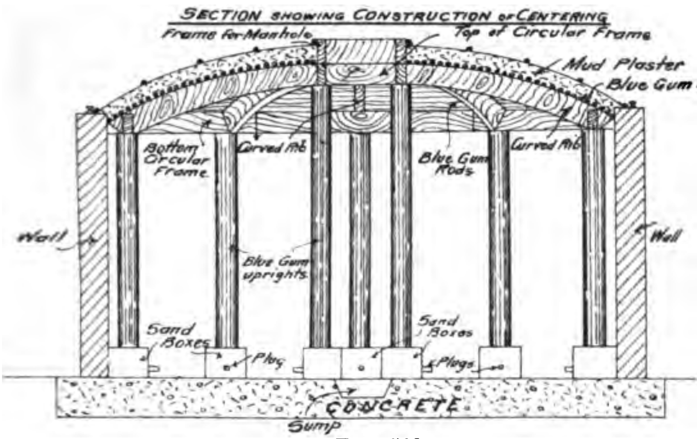

Fig. 503.

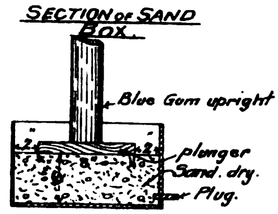

Fig. 504.

3. The ring ($R1$) (Figs. 505 and 506) for the manhole opening will now be laid on the top of the dome, and hooked to it will be eight radial rods (telegraph wire), which will be secured to the thrust rail by fine wire. The length of these radial rods is 5 feet 6 inches *on plan.*

4. As soon as the eight radials are secured in position the remaining rings $R2$, $R3$, $R4$, and $R5$, will be placed on top of them, and tied in place with thin wire. $R5$ will rest on the lower flange of the thrust rail.

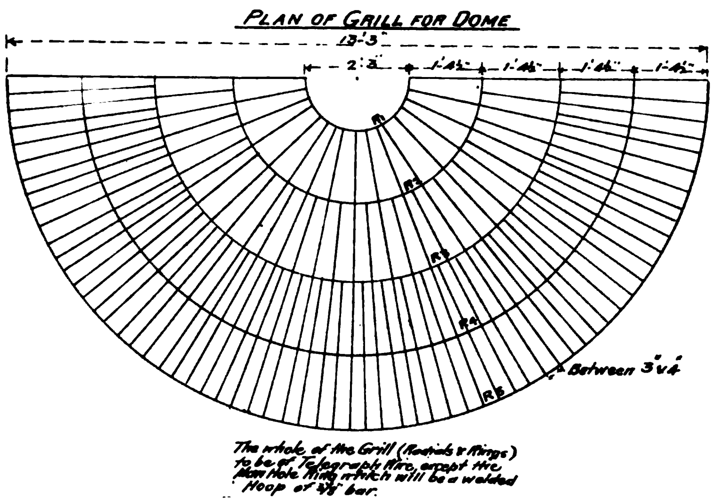

Fig. 505.

5. The remaining radials (Fig. 505) will then be hooked on to the ring from which they originate, as shown in Fig. 506, and will be tied at intervals with wire as necessary.

6. The dome will be 2 in. thick at thrust rail, and 1½ in. next the manhole.

The mortar will be laid in thin layers, well rammed, commencing above the wall and working upwards.

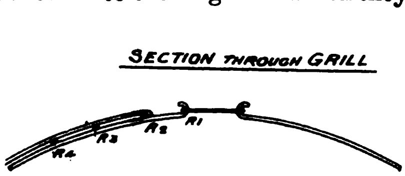

Fig. 506.

7. The centreing will be arranged as in Fig. 503, and when the cement has set thoroughly the plugs will be removed from the sand boxes (Fig. 504), so that the centreing may gradually and uniformly subside.

This specification was written for Indian practice, and any suitable timber would be employed for the centreing instead of blue gum.

The cost of the tank containing 5,000 gallons with a spherical covering was £34, and that of the 25,000 gallons tank with a flat roof (23 feet internal diameter and 10 feet deep) was £73, but these costs apply to Indian practice and to the particular district, and will vary somewhat with the local prices of materials of labour.

Bridges Erected by Major Stokes-Roberts.

The two bridges shown in Figs. 507 and 508 were constructed in 1901, and are interest ing, as it is believed that they were the first to be constructed of reinforced concrete in India. They were designed as foot-bridges and for the passage of 1½-feet gauge tramways, and are both on the skew.

In each case the square span is 30 feet, the thickness at the crown being 3 inches, and that at the haunches 5 inches. In order to give a large waterway and to keep down to the gradient of the tram-line, the bridge shown in Fig. 507 was given a rise of only one-twelfth

Fig. 507

span, and consisted of a slab (Fig. 509), on which the brick superstructure was afterwards erected. As the superstructure showed a tendency to crack under the vibrations set up by the traffic and the vibrations caused by a large pulverizer which was working close at hand, extra rigidity was given to the spandril filling, by forming this of concrete in place of earth filling, as originally intended, the upper horizontal surface being reinforced with light bars.

The abutments of both bridges consisted of ordinary lime concrete about 6 × 6 × 3 feet resting on the banks of the stream, and protected from undercutting by 3-inch reinforced concrete apron walls, which can be seen in Figs. 507 and 508.

The reinforcement for the bridge shown in Fig. 507 consists of a 3-inch mesh network near the intrados, formed of ⅝ and ½-inch round longitudinals (these being the only sections obtainable at the time), with ¼-inch square transverse bars.

APPENDIX

For the bridge shown in Fig. 508 the 3-in. network is formed of $\frac{1}{4}$-inch square longitudinals with transverse rods of $\frac{3}{16}$-inch diameter turned up into the concrete of the spandril walls at intervals.

Both the arches are constructed of 1 to 3 cement mortar.

The spandril walls of the bridge shown in Fig. 508 are monolithic with the arch, giving the structure great rigidity, and the passage of a trolley loaded to $4\frac{1}{2}$ tons at various speeds caused no visible deformation at the crown or quarter spans.

Fig. 510 shows the extreme simplicity of the centreing, which rested on slack blocks, and was lowered about one month after the completion of the bridges.

The joints in the longitudinals consist of a simple overlap of twenty-five diameters, tied in three places with wrappings of No. 20 standard wire gauge, as shown in Fig. 511.

FIG. 508

FIG. 509

M M

Fig. 510

Fig. 511

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Butler & Tanner,
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