RAILROAD CONSTRUCTION.

THEORY AND PRACTICE.

A TEXT-BOOK FOR THE USE OF STUDENTS IN COLLEGES AND TECHNICAL SCHOOLS.

BY

WALTER LORING WEBB, C.E.,

Associate Member American Society of Civil Engineers; sometime Assistant Professor of Civil Engineering in the University of Pennsylvania; etc.

SECOND EDITION, REVISED AND ENLARGED.

FIRST THOUSAND.

NEW YORK:
JOHN WILEY & SONS.
LONDON: CHAPMAN & HALL, LIMITED.
1903.

PRESS OF
BRAUNWORTH & CO.
BOOKBINDERS AND PRINTERS
BROOKLYN, N. Y.

PREFACE TO FIRST EDITION.

THE preparation of this book was begun several years ago, when much of the subject-matter treated was not to be found in print, or was scattered through many books and pamphlets, and was hence unavailable for student use. Portions of the book have already been printed by the mimeograph process or have been used as lecture-notes, and hence have been subjected to the refining process of class-room use.

The author would call special attention to the following features:

a. Transition curves; the multiform-compound-curve method is used, which has been followed by many railroads in this country; the particular curves here developed have the great advantage of being exceedingly simple, and although the method is not theoretically exact, it is demonstrable that the differences are so small that they may safely be neglected.

b. A system of earthwork computations by means of a slide-rule (which accompanies the volume) which enables one to compute readily the volume of the most complicated earthwork forms with an accuracy only limited by the precision of the cross-sectioning.

c. The "mass curve" in earthwork; the theory and use of this very valuable process.

d. Tables I, II, III, and IV have been computed *ab novo*. Tables I and II were checked (after computation) with other tables, which are generally considered as standard, and all discrepancies were further examined. They are believed to be perfect.

e. Tables V, VI, VII, and IX have been borrowed, by permission, from "Ludlow's Mathematical Tables." It is believed that five-place tables give as accurate results as actual field practice requires. Tables VIII and X have been compiled to conform with Ludlow's system.

The author wishes to acknowledge his indebtedness to Mr. Chas. A. Sims, civil engineer and railroad contractor, for reading and revising the portions relating to the cost of earthwork.

Since the book is written primarily for students of railroad engineering in technical institutions, the author has assumed the usual previous preparation in algebra, geometry, and trigonometry.

WALTER LORING WEBB.

UNIVERSITY OF PENNSYLVANIA,
PHILADELPHIA,
Jan. 1, 1900.

PREFACE TO SECOND EDITION.

Since the issue of the first edition the author has conferred with many noted educators in civil engineering, among them the late Professors E. A. Fuertes and J. B. Johnson, regarding the most desirable size of page for this book. The inconvenience of the octavo edition for field-work was found to be limiting its use. It was therefore decided to recast the whole work and reduce the page from "octavo" to "pocket-book" size. Advantage was then taken of the opportunity to revise freely and to add new matter. The original text has now been almost doubled by the addition of several chapters on structures, train resistance, rolling stock, etc., and also several chapters giving the fundamental principles of the economics of railroad location. Those who are familiar with the late Mr. Wellington's masterpiece, "The Economic Theory of Railway Location," will readily appreciate the author's indebtedness to that work. But while the same general method has been followed, the author has taken advantage of the classification of operating expenses adopted by the Interstate Commerce Commission, has used the figures published by them (which were unavailable when Mr. Wellington wrote), and has developed the theory on an independent basis, with the exception of a few minor details. Those who deny the utility of such methods of computation are referred to §§ 367, 426, and elsewhere for a practical discussion of that subject.

The author's primary aim has been to produce a "text-book for students," and the subject-matter has therefore been cut down to that which may properly be required of students in the time usually allotted to railroad work in a civil-engineering curriculum. On this account no extended discussion has been given to the multitudinous forms of various railroad devices in the chapters on structures. The aim has been to teach the principles and to guide the students into proper methods of investigation.

January, 1903.

TABLE OF CONTENTS.

CHAPTER I.

RAILROAD SURVEYS.

CHAPTER II.

ALIGNMENT.

CHAPTER III.

EARTHWORK.

CHAPTER IV.

TRESTLES.

CHAPTER V.

TUNNELS.

CHAPTER VI.

CULVERTS AND MINOR BRIDGES.

CHAPTER VII.

BALLAST.

CHAPTER VIII.

TIES AND OTHER FORMS OF RAIL SUPPORT.

CHAPTER IX.

RAILS.

CHAPTER X.

RAIL-FASTENINGS.

CHAPTER XI.

SWITCHES AND CROSSINGS.

CHAPTER XII.

MISCELLANEOUS STRUCTURES AND BUILDINGS.

CHAPTER XIII.

YARDS AND TERMINALS.

CHAPTER XIV.

BLOCK SIGNALING.

CHAPTER XV.

ROLLING STOCK.

CHAPTER XVI.

TRAIN RESISTANCE.

CHAPTER XVII.

COST OF RAILROADS.

PART II.

RAILROAD ECONOMICS.

CHAPTER XVIII.

INTRODUCTION.

CHAPTER XIX.

THE PROMOTION OF RAILROAD PROJECTS.

CHAPTER XX.

OPERATING EXPENSES.

CHAPTER XXI.

DISTANCE.

CHAPTER XXII.

CURVATURE.

CHAPTER XXIII.

GRADE.

CHAPTER XXIV.

THE IMPROVEMENT OF OLD LINES.

RAILROAD CONSTRUCTION.

CHAPTER I.

RAILROAD SURVEYS.

THE proper conduct of railroad surveys presupposes an adequate knowledge of almost the whole subject of railroad engineering, and particularly of some of the complicated questions of Railroad Economics, which are not generally studied except at the latter part of a course in railroad engineering, if at all. This chapter will therefore be chiefly devoted to methods of instrumental work, and the problem of choosing a general route will be considered only as it is influenced by the topography or by the application of those elementary principles of Railroad Economics which are self-evident or which may be accepted by the student until he has had an opportunity of studying those principles in detail

RECONNOISSANCE SURVEYS.

1. **Character of a reconnoissance survey.** A reconnoissance survey is a very hasty examination of a belt of country to determine which of all possible or suggested routes is the most promising and best worthy of a more detailed survey. It is essentially very rough and rapid. It aims to discover those salient features which instantly stamp one route as distinctly superior to another and so narrow the choice to routes which are so nearly equal in value that a more detailed survey is necessary to decide between them.

2. **Selection of a general route.** The general question of running a railroad between two towns is usually a financial rather

than an engineering question. Financial considerations usually determine that a road *must* pass through certain more or less important towns between its termini. When a railroad runs through a thickly settled and very flat country, where, from a topographical standpoint, the road may be run by any desired route, the "right-of-way agent" sometimes has a greater influence in locating the road than the engineer. But such modifications of alignment, on account of business considerations, are foreign to the engineer's side of the subject, and it will be hereafter assumed that topography alone determines the location of the line. The consideration of those larger questions combining finance and engineering (such as passing by a town on account of the necessary introduction of heavy grades in order to reach it) will be taken up in Chap. XIX, *et seq.*

3. **Valley route.** This is perhaps the simplest problem. If the two towns to be connected lie in the same valley, it is frequently only necessary to run a line which shall have a nearly uniform grade. The reconnoissance problem consists largely in determining the difference of elevation of the two termini of this division and the approximate horizontal distance so that the proper grade may be chosen. If there is a large river running through the valley, the road will probably remain on one side or the other throughout the whole distance, and both banks should be examined by the reconnoissance party to determine which is preferable. If the river may be easily bridged, both banks may be alternately used, especially when better alignment is thereby secured. A river valley has usually a steeper slope in the upper part than in the lower part. A uniform grade throughout the valley will therefore require that the road climbs up the side slopes in the lower part of the valley. In case the "ruling grade" * for the whole road is as great as or greater than the steepest natural valley slope, more freedom may be used in adopting that alignment which has the least cost—regardless of grade. The natural slope of large rivers is almost invariably so low that grade has no influence in determining the choice of location. When bridging is necessary, the river banks should be examined for suitable locations for abutments

* The *ruling grade* may here be loosely defined as the maximum grade which is permissible. This definition is not strictly true, as may be seen later when studying Railroad Economics, but it may here serve the purpose.

and piers. If the soil is soft and treacherous, much difficulty may be experienced and the choice of route may be largely determined by the difficulty of bridging the river except at certain favorable places.

4. **Cross-country route.** A cross-country route always has one or more summits to be crossed. The problem becomes more complex on account of the greater number of possible solutions and the difficulty of properly weighing the advantages and disadvantages of each. The general aim should be to choose the lowest summits and the highest stream crossings, provided that by so doing the grades between these determining points shall be as low as possible and shall not be greater than the ruling grade of the road. Nearly all railroads combine cross-country and valley routes to some extent. Usually the steepest natural slopes are to be found on the cross-country routes, and also the greatest difficulty in securing a low through grade. An approximate determination of the ruling grade is usually made during the reconnoissance. If the ruling grade has been previously decided on by other considerations, the leading feature of the reconnoissance survey will be the determination of a general route along which it will be possible to survey a line whose maximum grade shall not exceed the ruling grade.

5. **Mountain route.** The streams of a mountainous region frequently have a slope exceeding the desired ruling grade. In such cases there is no possibility of securing the desired grade by following the streams. The penetration of such a region may only be accomplished by "development"—accompanied perhaps by tunneling. "Development" consists in deliberately increasing the length of the road between two extremes of elevation so that the rate of grade shall be as low as desired. The usual method of accomplishing this is to take advantage of some convenient formation of the ground to introduce some lateral deviation. The methods may be somewhat classified as follows:

(a) Running the line up a convenient lateral valley, turning a sharp curve and working back up the opposite slope. As shown in Fig. 1, the considerable rise between A and B was surmounted by starting off in a very different direction from the general direction of the road; then, when about one-half of the desired rise had been obtained, the line crossed the valley and continued the climb along the opposite slope. **(b)** *Switch-*

back. On the steep side-hill *BCD* (Fig. 1) a very considerable gain in elevation was accomplished by the switchback *CD*. The gain in elevation from *B* to *D* is very great. On the other hand, the speed must always be slow; there are two complete stoppages of the train for each run; all trains must run backward from *C* to *D*. (c) *Bridge spiral.* When a valley is so narrow at some point that a bridge or viaduct of reasonable length can span the valley at a considerable elevation above the

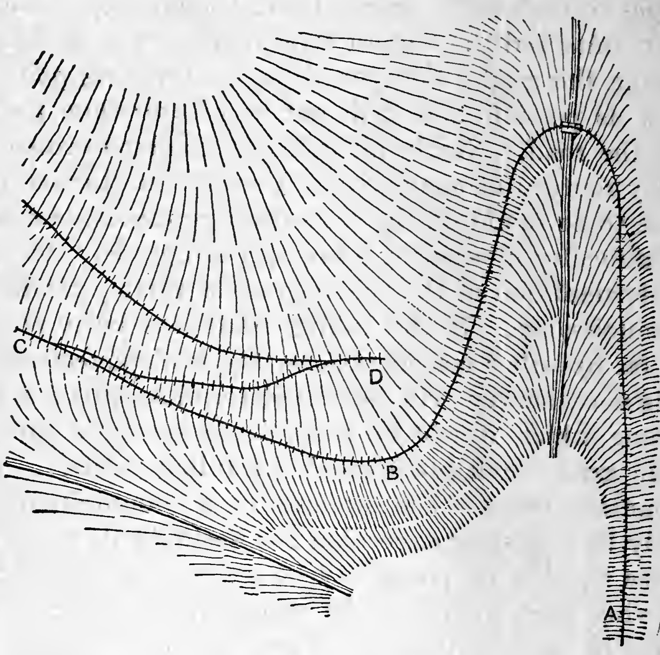
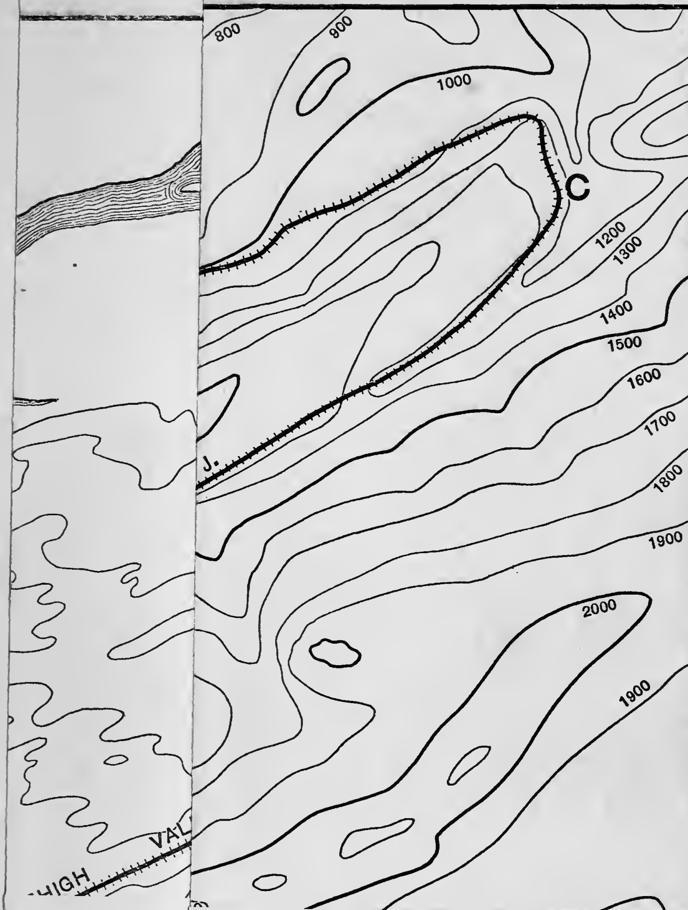


Fig. 1.

bottom of the valley, a bridge spiral may be desirable. In Fig. 2 the line ascends the stream valley past *A*, crosses the stream at *B*, works back to the narrow place at *C*, and there crosses itself, having gained perhaps 100 feet in elevation. (d) *Tunnel spiral* (Fig. 3). This is the reverse of the previous plan. It implies a thin steep ridge, so thin at some place that a tunnel through it will not be excessively long. Switchbacks and spirals are sometimes necessary in mountainous countries, but they should not be considered as normal types of construction. A region must be very difficult if these devices cannot be avoided.

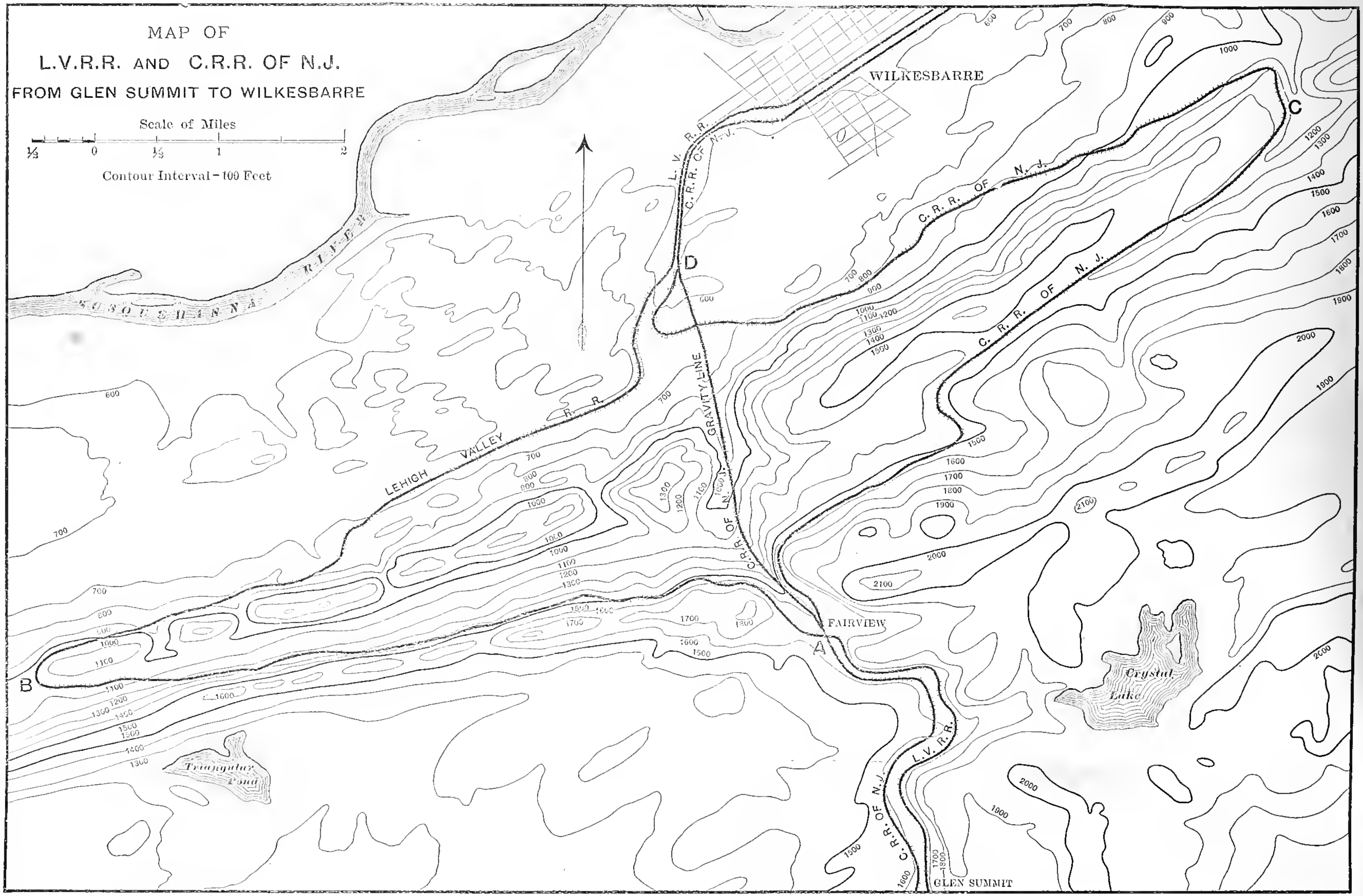
On Plate I are shown three separate ways (as actually constructed) of running a railroad between two points a little over three miles apart and having a difference of elevation of nearly

PLATE I.

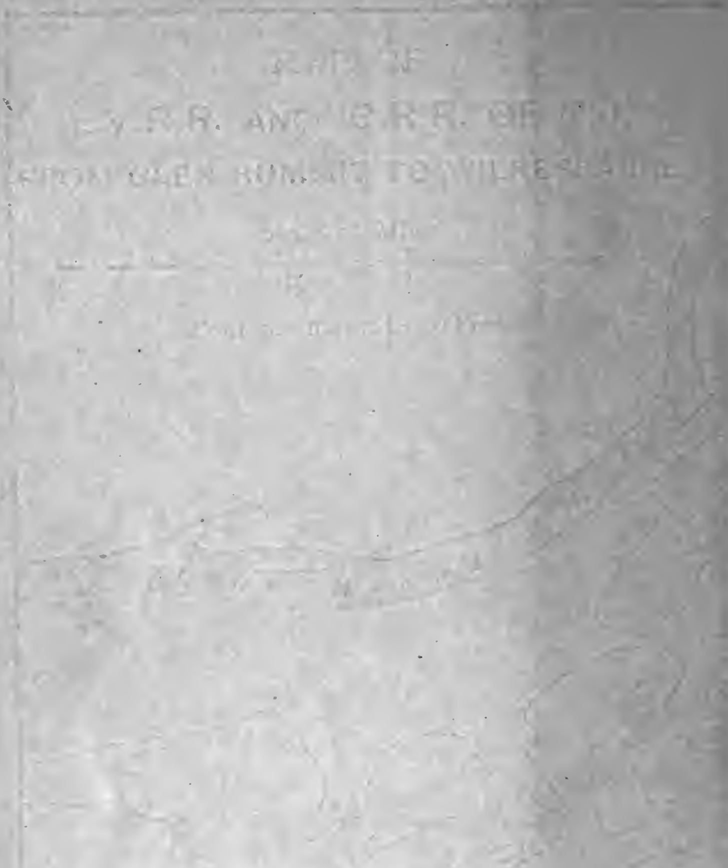


800
800
1000
C
1200
1300
1400
1500
1600
1700
1800
1900
2000
1900
J.
VAL
HIGH

ra



(To face page 4.)



1100 feet. At *A* the Central R. R. of New Jersey runs *under* the Lehigh Valley R. R. and soon turns off to the northeast for about six miles, then doubles back, reaching *D*, a fall of about 1050 feet with a track distance of about 12.7 miles. The L. V. R. R. at *A* runs to the westward for six to seven miles,

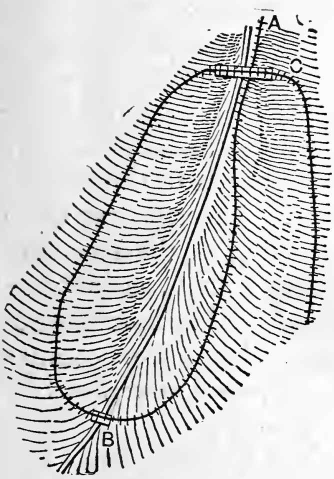


Fig. 2.

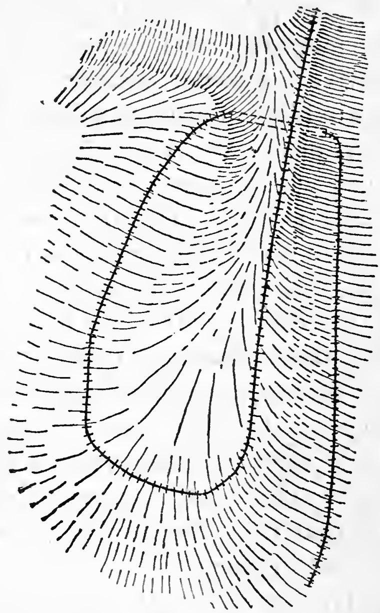


Fig. 3.

then turns back until the roads are again close together at *D*. The track distance is about 14 miles and the drop a little greater, since at *A* the L. V. R. R. crosses *over* the other, while at *D* they are at practically the same level. From *B* to *C* the distance is over eleven miles. From *A* directly down to *D* the C. R. R. of N. J. runs a "gravity" road, used exclusively for freight, on which cars alone are hauled by cable. The main-line routes are remarkable examples of sheer "development." Even as constructed the L. V. R. R. has a grade of about 95 feet per mile, and this grade has proved so excessive for freight work that the company has constructed a cut-off (not shown on the map) which leaves the main line at *A*, nearly parallels the C. R. R. to *C*, and then running in a northeasterly direction again joins the main line beyond Wilkesbarre. The grade is thereby cut down to 65 feet per mile.

Rack railways and cable roads, although types of mountain railroad construction, will not be here considered.

6. Existing maps. The maps of the U. S. Geological Survey are exceedingly valuable as far as they have been completed. So far as topographical considerations are concerned, they almost dispense with the necessity for the reconnoissance and "first preliminary" surveys. Some of the State Survey maps will give practically the same information. County and township maps can often be used for considerable information as to the relative *horizontal* position of governing points, and even some approximate data regarding elevations may be obtained by a study of the streams. Of course such information will not dispense with surveys, but will assist in so planning them as to obtain the best information with the least work. When the relative horizontal positions of points are reliably indicated on a map, the reconnoissance may be reduced to the determination of the relative elevations of the governing points of the route.

7. Determination of relative elevations. A recent description of European methods includes spirit-leveling in the reconnoissance work. This may be due to the fact that, as indicated above, previous topographical surveys have rendered unnecessary the "exploratory" survey which is required in a new country, and that their reconnoissance really corresponds more nearly to our preliminary.

The perfection to which barometrical methods have been brought has rendered it possible to determine differences of elevation with sufficient accuracy for reconnoissance purposes by the combined use of a mercurial and an aneroid barometer. The mercurial barometer should be kept at "headquarters," and readings should be taken on it at such frequent intervals that any fluctuation is noted, and throughout the period that observations with the aneroid are taken in the field. At each observation there should also be recorded the time, the reading of the attached thermometer, and the temperature of the external air. For uniformity, the mercurial readings should then be "reduced to 32° F." The form of notes for the mercurial barometer readings should be as follows:

Time.	Merc. Barom.	Attached Therm.	Reduction to 32° F.	External Therm.	Corrected reading.
7:00 A. M.	29.872	72°	—.117	73°	29.755
:15	.866	73.5	.121	75	.745
:30	.858	75	.125	76	.733
:45	.850	76	.127	77	.723

The corrections in column 4 are derived from Table XI by interpolation.

Before starting out, a reading of the aneroid should be taken at headquarters coincident with a reading of the mercurial. The difference is one value of the correction to the aneroid. As soon as the aneroid is brought back another comparison of readings should be made. Even though there has been considerable rise or fall of pressure in the interval, the *difference* in readings (the correction) should be substantially the same provided the aneroid is a good instrument. If the difference of elevation is excessive (as when climbing a high mountain) even the best aneroid will "lag" and not recover its normal reading for several hours, but this does not apply to such differences of elevation as are met with in railroad work. The best aneroids read directly to $\frac{1}{100}$ of an inch of mercury and may be estimated to $\frac{1}{1000}$ of an inch—which corresponds to about 0.9 foot difference of elevation. In the field there should be read, at each point whose elevation is desired, the aneroid, the time, and the temperature. These readings, corrected by the mean value of the correction between the aneroid and the mercurial, should then be combined with the reading of the mercurial (interpolated if necessary) for the times of the aneroid observations and the difference of elevation obtained. The field notes for the aneroid should be taken as shown in the first four columns of the tabular form. The "corrected aneroid" readings of column 5 are found by correcting the readings of column 3 by the mean difference between the mercurial and aneroid when compared at morning and night. Column 6 is a copy of the "corrected readings" from the office notes, interpolated when necessary for the proper time. Column 7 is similarly obtained. Col. 8 is obtained from cols. 4 and 5, and col. 9 from cols. 6 and 7, with the aid of Table XII. The correction for temperature (col. 11), which is generally small unless the difference of elevation is large, is obtained with the aid of Table XIII. The elevations in Table XII are elevations above an assumed datum plane, where under the given atmospheric conditions the mercurial reading would be 30″. Of course the position of this assumed plane changes with varying atmospheric conditions and so the elevations are to be considered as *relative* and their difference taken. [See the author's "Problems in the Use and Adjustment of Engineering In-

(Left-hand page of Notes.)

Time.	Place.	Aneroid.	Therm.	Corr. Aner.	Corr. Merc.
7:00	Office	29.628	73°	....	29.755
7:10	Δ0	29.662	72°	29.789	29.748
7:30	saddle-back	29.374	63°	29.501	29.733
7:50	river cross.	29.548	70°	29.675	29.720

struments," Prob. 22.] Important points should be observed more than once if possible. Such duplicate observations will be found to give surprisingly concordant results even when a general fluctuation of atmospheric pressure so modifies the tabulated readings that an agreement is not at first apparent. Variations of pressure produced by high winds, thunder-storms, etc., will generally vitiate possible accuracy by this method. By "headquarters" is meant any place whose elevation above any given datum is known and where the mercurial may be placed and observed while observations within a range of several miles are made with the aneroid. If necessary, the elevation of a new headquarters may be determined by the above method, but there should be if possible several independent observations whose accordance will give a fair idea of their accuracy.

The above method should be neither slighted nor used for more than it is worth. When properly used, the errors are compensating rather than cumulative. When used, for example, to determine that a pass B is 260 feet higher than a determined bridge crossing at A which is six miles distant, and that another pass C is 310 feet higher than A and is ten miles distant, the figures, even with all necessary allowances for inaccuracy, will give an engineer a good idea as to the choice of route especially as affected by ruling grade. There is no comparison between the time and labor involved in obtaining the above information by barometric and by spirit-leveling methods, and *for reconnoissance purposes* the added accuracy of the spirit-leveling method is hardly worth its cost.

8. Horizontal measurements, bearings, etc. When there is no map which may be depended on, or when only a skeleton map is obtainable, a rapid survey, sufficiently accurate for the purpose, may be made by using a pocket compass for bearings and a telemeter, odometer, or pedometer for distances. The telemeter [stadia] is more accurate, but it requires a definite clear

(Right-hand page of Notes.)

Temp. at headqu.	Approx. field read.	Approx. headq. read.	Diff.	Corr. for temp.	Diff. elev.
—	—	—	—	—	—
75°	192	230	− 38	−(+ 2)	− 40
76	457	244	+213	+(+10)	+223
77	297	256	+ 41	+(+ 2)	+ 43

sight from station to station, which may be difficult through a wooded country. The odometer, which records the revolutions of a wheel of known circumference, may be used even in rough and wooded country, and the results may be depended on to a small percentage. The pedometer (or pace-measurer) depends for its accuracy on the actual movement of the mechanism for each pace and on the uniformity of the pacing. Its results are necessarily rough and approximate, but it may be used to fill in some intermediate points in a large skeleton map. A hand-level is also useful in determining the relative elevation of various topographical features which may have some bearing on the proper location of the road.

9. **Importance of a good reconnoissance.** The foregoing instruments and methods should be considered only as aids in exercising an educated common sense, without which a proper location cannot be made. The reconnoissance survey should command the best talent and the greatest experience available. If the general route is properly chosen, a comparatively low order of engineering skill can fill in a location which will prove a paying railroad property; but if the general route is so chosen that the ruling grades are high and the business obtained is small and subject to competition, no amount of perfection in detailed alignment or roadbed construction can make the road a profitable investment.

PRELIMINARY SURVEYS.

10. **Character of survey.** A preliminary railroad survey is properly a topographical survey of a belt of country which has been selected during the reconnoissance and within which it is estimated that the located line will lie. The width of this belt will depend on the character of the country. When a railroad is to follow a river having very steep banks the choice of location is sometimes limited at places to a very few feet of width

and the belt to be surveyed may be correspondingly narrowed. In very flat country the desired width may be only limited by the ability to survey points with sufficient accuracy at a considerable distance from what may be called the "backbone line" of the survey.

11. Cross-section method. This is the only feasible method in a wooded country, and is employed by many for all kinds of country. The *backbone* line is surveyed either by observing magnetic bearings with a compass or by carrying forward

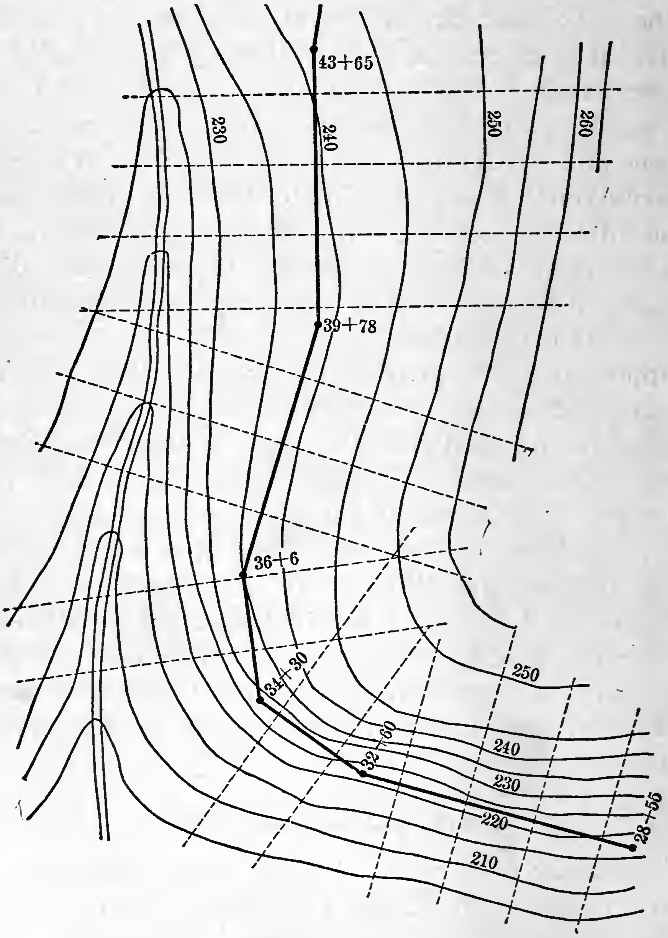


FIG. 4.

absolute azimuths with a transit. The compass method has the disadvantages of limited accuracy and the possibility of considerable local error owing to local attraction. On the other hand there are the advantages of greater simplicity, no necessity for a back rodman, and the fact that the errors are purely local and not cumulative, and may be so limited, with care, that they will cause no vital error in the subsequent location survey. The transit method is essentially more accurate, but is liable to be more laborious and troublesome. If a large tree is encountered, either it must be cut down or a troublesome operation of offsetting must be used. If the compass is employed under these circumstances, it need only be set up on the far side of the tree and the former bearing produced. An error in reading a transit azimuth will be carried on throughout the survey. An error of only five minutes of arc will cause an offset of nearly eight feet in a mile. Large azimuth errors may, however, be avoided by immediately checking each new azimuth with a needle reading. It is advisable to obtain true azimuth at the beginning of the survey by an observation on the sun or Polaris, and to check the azimuths every few miles by azimuth observations. Distances along the backbone line should be measured with a chain or steel tape and stakes set every 100 feet. When a course ends at a substation, as is usually the case, the remaining portion of the 100 feet should be measured along the next course. The level party should immediately obtain the elevations (to the nearest tenth of a foot) of all stations, and also of the lowest points of all streams crossed and even of dry gullies which would require culverts.

12. Cross-sectioning. It is usually desirable to obtain contours at five-foot intervals This may readily be done by the use of a Locke level (which should be held on top of a simple five-foot stick), a tape, and a rod ten feet in length graduated to feet and tenths. The method of use may perhaps be best explained by an example. Let Fig. 5 represent a section *perpendicular* to the survey line—such a section as would be made by the dotted lines in Fig. 4. *C* represents the station point. Its elevation as determined by the level is, say, 158.3 above datum. When the Locke level on its five-foot rod is placed at *C*, the level has an elevation of 163.3. Therefore when a point is found (as at *a*) where the level will read 3.3 on the rod, that point has an elevation of 160.0 and its distance from the center gives the position of the 160-foot contour. Leaving the long rod at that point (*a*), carry the level to some point (*b*) such that the level will sight at the *top* of the rod. *b* is then on the 165-

foot contour, and the *horizontal* distance *ab* added to the horizontal distance *ac* gives the position of that contour from the center. The contours on the lower side are found similarly. The first rod reading will be 8.3, giving the 155-foot contour.

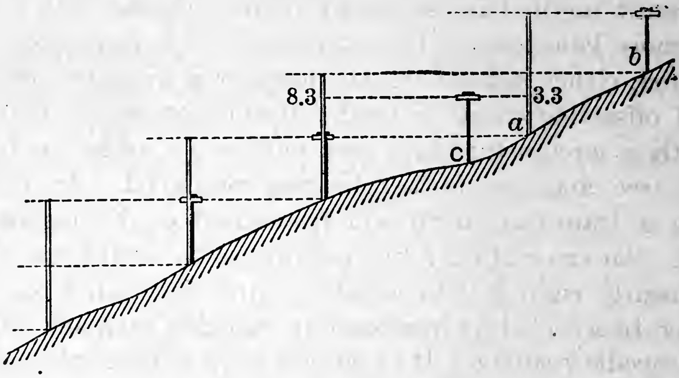


FIG. 5.

Plot the results in a note-book which is ruled in quarter-inch squares, using a scale of 100 feet per inch in both directions. Plot the work UP the page; then when looking ahead along the line, the work is properly oriented. When a contour crosses

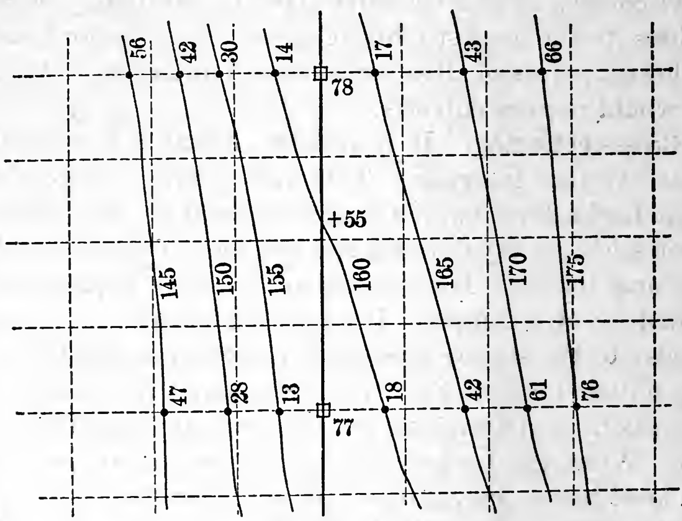


FIG. 6.

the survey line, the place of crossing may be similarly determined. If the ground flattens out so that five-foot contours are very far apart, the absolute elevations of points at even fiftyfoot distances from the center should be determined. The method is exceedingly rapid. Whatever error or inaccuracy occurs is confined in its effect to the one station where it occurs. The work being thus plotted in the field, unusually irregular topography may be plotted with greater certainty and no great error can occur without detection. It would even be possible by this method to detect a gross error that might have been made by the level party

13. Stadia method. This method is best adapted to fairly open country where a "shot" to any desired point may be taken without clearing. The *backbone* survey line is the same as in the previous method except that each course is limited to the practicable length of a stadia sight. The distance between stations should be checked by foresight and backsight—also the vertical angle. Azimuths should be checked by the needle. Considering the vital importance of leveling on a railroad survey it might be considered desirable to run a line of levels over the stadia stations in order that the leveling may be as precise as possible; but when it is considered that a preliminary survey is a somewhat hasty survey of a route that *may* be abandoned, and that the errors of leveling by the stadia method (which are compensating) may be so minimized that no proposed route would be abandoned on account of such small error, and that the effect of such an error may be easily neutralized by a slight change in the location, it may be seen that excessive care in the leveling of the preliminary survey is hardly justifiable.

Since the students taking this work are assumed to be familiar with the methods of stadia topographical surveys, this part of the subject will not be further elaborated.

14. "First" and "Second" preliminary surveys. Some engineers advocate two preliminary surveys. When this is done, the first is a very rapid survey, made perhaps with a compass, and is only a better grade of reconnoissance. Its aim is to rapidly develop the facts which will decide for or against any proposed route, so that if a route is found to be unfavorable another more or less modified route may be adopted without having wasted considerable time in the survey of useless details. By this time the student should have grasped the fundamental idea that both the reconnoissance and preliminary surveys are not surveys of *lines* but of *areas*; that their aim is to survey only those topographical features which would have a determining influence on any railroad line which might be constructed through that particular territory, and that the value of a locating engineer is largely measured by his ability to recognize those determining influences with the least amount of work from his surveying corps. Frequently too little time is spent on the comparative study of preliminary lines. A line will be hastily decided on after very little study; it will then be surveyed with minute detail and estimates carefully worked up, and the claims of any other suggested route will then be handicapped, if not disregarded, owing to an unwillingness to discredit and throw away a large amount of detailed surveying. The cost of two or three extra preliminary surveys (*at critical sections* and not over the whole line) is utterly insignificant compared with the probable improvement in the "operating value" of a line located after such a comparative study of preliminary lines.

LOCATION SURVEYS.

15. "Paper location." When the preliminary survey has been plotted to a scale of 200 feet per inch and the contours drawn in, a study may be made for the location survey. Disregarding for the present the effect on location of transition curves, the alignment may be said to consist of straight lines (or "tangents") and circular curves. The "paper location" therefore consists in plotting on the preliminary map a succession of straight lines which are tangent to the circular curves connecting them. The determining points should first be considered. Such points are the termini of the road, the lowest practicable point over a summit, a river-crossing, etc. So far as is possible, having due regard to other considerations, the road should be a "surface" road, i.e., the cut and fill should be made as small as possible. The maximum permissible grade must also have been determined and duly considered. The method of location differs radically according as the lines joining the determining points have a very low grade or have a grade that approaches the maximum permissible. With very low natural grades it is only necessary to strike a proper balance between the requirements for easy alignment and the avoidance of excessive earthwork. When the grade between two determined points approaches the maximum, a study of the location may be begun by finding a strictly surface line which will connect those points with a line at the given grade. For example, suppose the required grade is 1.6% and that the contours are drawn at 5-foot intervals. It will require 312 feet of 1.6% grade to rise 5 feet. Set a pair of dividers at 312 feet and step off this interval on successive contours. This line will in general be very irregular, but in an easy country it may lie fairly close to the proper location line, and even in difficult country such a surface line will assist greatly in selecting a suitable location. When the larger part of the line will evidently consist of tangents, the tangents should be first located and should then be connected by suitable curves. When the curves predominate, as they generally will in mountainous country, and particularly when the line is purposely lengthened in order to reduce the grade, the curves should be plotted first and the tangents may then be drawn connecting them. Considering the ease with which such lines may be drawn on the preliminary map, it is frequently advisable, after making such a paper location, to begin all over, draw a new line over some specially difficult section and compare results. Profiles of such lines may be readily drawn by noting their intersection with each contour crossed. Drawing on each profile the required grade line will furnish an approximate idea of the *comparative* amount of earthwork required. After deciding on the paper location, the length of each tangent, the central angle (see § 21), and the radius of each curve should be measured as accurately as possible. Since a slight error made in such measurements, taken from a map with a scale of 200 feet per inch, would by accumulation cause serious discrepancies between the plotted location and the location as afterward surveyed in the field, frequent tie lines and angles should be determined between the plotted location line and the preliminary line, and the location should be altered, as may prove necessary, by changing the length of a tangent or changing the central angle or radius of a curve, so that the agreement of the check-points will be sufficiently close. The errors of an inaccurate preliminary survey may thus be easily neutralized (see § 33). When the preliminary line has been properly run, its "backbone" line will lie very near the location line and will probably cross it at frequent intervals, thus rendering it easy to obtain short and numerous tie lines.

16. **Surveying methods.** A transit should be used for alignment, and only precise work is allowable. The transit stations

should be centered with tacks and should be tied to witness-stakes, which should be located outside of the range of the earth-work, so that they will neither be dug up nor covered up. All original property lines lying within the limits of the right of way should be surveyed with reference to the location line, so that the right-of-way agent may have a proper basis for settlement. When the property lines do not extend far outside of the required right of way they are frequently surveyed completely.

The leveler usually reads the target to the nearest thousandth of a foot on turning-points and bench-marks, but reads to the nearest tenth of a foot for the elevation of the ground at stations. Considering that $\frac{1}{1000}$ of a foot has an angular value of only 7

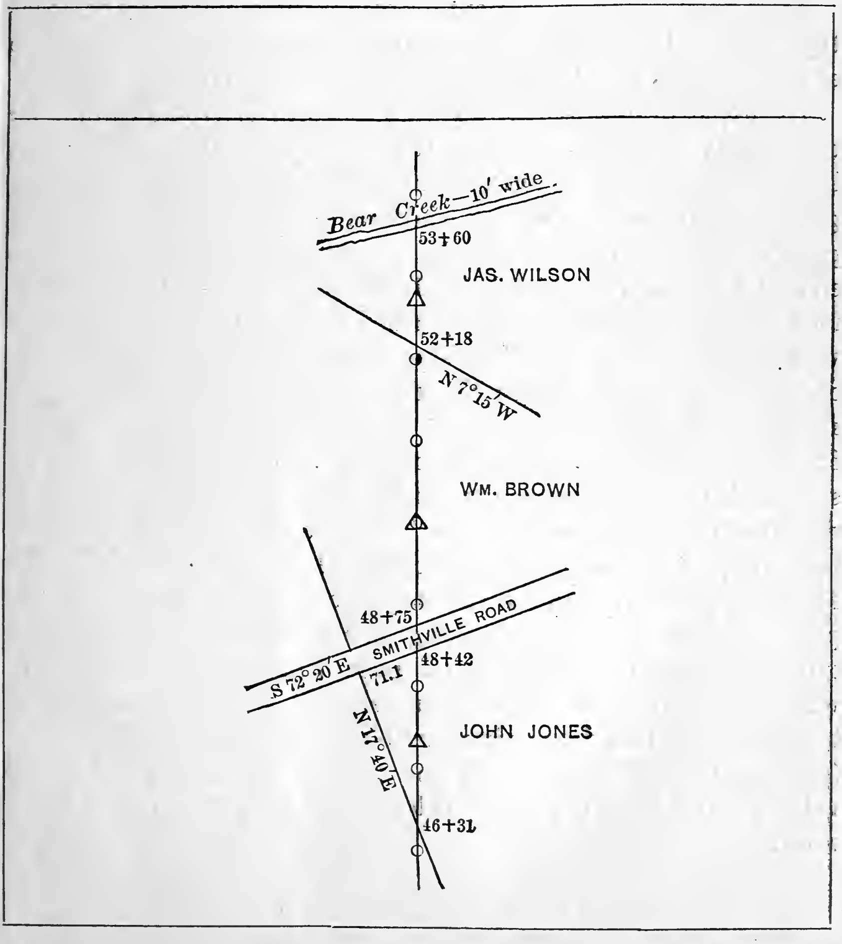
FORM OF NOTES.

[Left-hand page.]

Sta.	Alignment.	Vernier.	Tangential Deflection.	Calculated Bearing.	Needle.
54					
53 ⊙ +72.2	P.T.	9° 11′	18° 22′	N 54° 48′ E	N 62° 15′ E
52	[3° 24′ curve to right for 18° 22′; tang. dist., 272.5]	7 57			
51		6 15			
⊙ 50		4 33			
49		2 51			
48		1 09			
⊙ +32 47	P.C.	0°			
46				N 36° 26′ E	N 44° 0′ E

seconds at a distance of 300 feet, and that one division of a level-bubble is usually about 30 seconds, it may be seen that it is a useless refinement to read to thousandths unless corresponding care is taken in the use of the level. The leveler should also locate his bench-marks outside of the range of earthwork. A knob of rock protruding from the ground affords an excellent mark. A large nail, driven in the roots of a tree, which is not to be disturbed, is also a good mark. These marks should be clearly described in the note-book. The leveler should obtain the elevation of the ground at all station-points; also at all sudden breaks in the profile line, determining also the distance of these breaks from the previous even station. This will in-

[Right-hand page.]



clude the position and elevation of all streams, and even dry gullies, which are crossed

Measurements should preferably be made with a steel tape, care being taken on steep ground to insure horizontal measurements. Stakes are set each 100 feet, and also at the beginning and end of all curves. Transit-points (sometimes called "plugs" or "hubs") should be driven flush with the ground, and a "witness-stake," having the "number" of the station, should be set three feet to the right. For example, the witness-stake might have on one side "137 + 69.92," and on the other side "P C 4° R," which would signify that the transit hub is 69.92 feet beyond station 137, or 13769.92 feet from the beginning of the line, and also that it is the "point of curve" of a "4° curve" which turns to the *right.*

Alignment. The alignment is evidently a part of the location survey, but, on account of the magnitude and importance of the subject, it will be treated in a separate chapter.

17. Form of Notes. Although the Form of Notes cannot be thoroughly understood until after curves are studied, it is here introduced as being the most convenient place. The right-hand page should have a sketch showing all roads, streams, and property lines crossed with the bearings of those lines. This should be drawn to a scale of 100 feet per inch—the quarter-inch squares which are usually ruled in note-books giving convenient 25-foot spaces. This sketch will always be more or less distorted on curves, since the center line is always shown as *straight* regardless of curves. The station points ("Sta." in first column, left-hand page) should be placed opposite to their sketched positions, which means that even stations will be recorded on every *fourth* line. This allows three intermediate lines for substations, which is ordinarily more than sufficient. The notes should read UP the page, so that the sketch will be properly oriented when looking ahead along the line The other columns on the left-hand page will be self-explanatory when the subject of curves is understood. If the "calculated bearings" are based on azimuthal observations, their agreement (or *constant* difference) with the needle readings will form a valuable check on the curve calculations and the instrumental work.

CHAPTER II.

ALIGNMENT.

In this chapter the alignment of the *center line* only of a pair of rails is considered. When a railroad is crossing a summit in the grade line, although the horizontal projection of the alignment may be straight, the vertical projection will consist of two sloping lines joined by a curve. When a curve is on a grade, the center line is really a spiral, a curve of double curvature, although its horizontal projection is a circle. The center line therefore consists of straight lines and curves of single and double curvature. The simplest method of treating them is to consider their horizontal and vertical projections separately. In treating simple, compound, and transition curves, only the horizontal projections of those curves will be considered.

SIMPLE CURVES.

18. Designation of curves. A curve may be designated either by its radius or by the angle subtended by a chord of unit length. Such an angle is known as the " degree of curve " and is indicated by D. Since the curves that are practically used have very long radii, it is generally impracticable to make any use of the actual center, and the curve is located without reference to it. If AB in Fig. 4 represents a unit chord (C) of a curve of radius R, then by the above definition the angle AOB equals D. Then

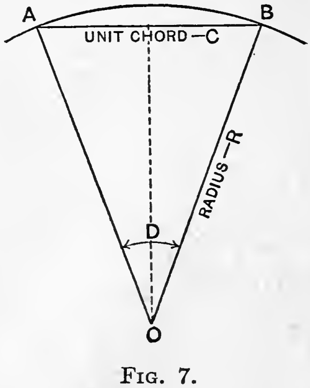


FIG. 7.

$$AO \sin \tfrac{1}{2}D = \tfrac{1}{2}AB = \tfrac{1}{2}C.$$

$$\therefore \quad R = \frac{\frac{1}{2}C}{\sin \frac{1}{2}D}, \qquad \qquad (1)$$

or, by inversion,

$$\sin \tfrac{1}{2}D = \frac{C}{2R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The unit chord is variously taken throughout the world as 100 feet, 66 feet, and 20 meters. In the United States 100 feet is invariably used as the unit chord length, and throughout this work it will be so considered. Table I has been computed on this basis. It gives the radius, with its logarithm, of all curves from a 0° 01′ curve up to a 10° curve, varying by single minutes. The sharper curves, which are seldom used, are given with larger intervals.

An approximate value of R may be readily found from the following simple rule, which should be memorized:

$$R = \frac{5730}{D}.$$

Although such values are not mathematically correct, since R does not strictly vary inversely as D, yet the resulting value is within a tenth of one per cent for all commonly used values of R, and is sufficiently close for many purposes, as will be shown later.

19. Length of a subchord. Since it is impracticable to measure along a curved arc, curves are always measured by laying off 100-foot chord lengths. This means that the actual arc is always a little longer than the chord. It also means that a *subchord* (a chord shorter than the unit length) will be a little longer than the ratio of the angles subtended would call for. The truth of this may be seen without calculation by noting that two equal subchords, each subtending the angle $\frac{1}{2}D$, will evidently be slightly longer than 50 feet each. If c be the length of a subchord subtending the angle d, then, as in Eq. 2,

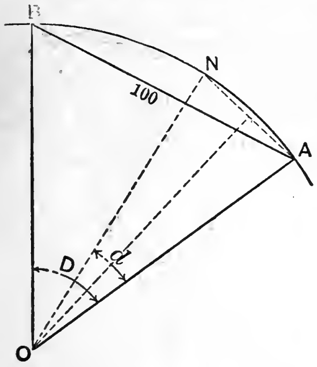


Fig. 8.

$$\sin \tfrac{1}{2}d = \frac{c}{2R},$$

or, by inversion,

$$c = 2R \sin \tfrac{1}{2}d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The *nominal length* of a subchord $= 100\dfrac{d}{D}$ For example, a *nominal* subchord of 40 feet will subtend an angle of $\frac{40}{100}$ of $D°$; its *true length* will be slightly more than 40 feet, and may be computed by Eq. 3. The *difference* between the nominal and true lengths is maximum when the subchord is about 57 feet long, but with the low degrees of curvature ordinarily used the difference may be neglected. With a 10° curve and a nominal chord length of 60 feet, the true length is 60.049 feet. Very sharp curves should be laid off with 50-foot or even 25-foot chords (nominal length). In such cases especially the true lengths of these subchords should be computed and used instead of the nominal lengths.

20. Length of a curve. The length of a curve is always indicated by the quotient of $100\Delta \div D$. If the quotient of $\Delta \div D$ is a whole number, the length as thus indicated is the true length—*measured in 100-foot chord lengths.* If it is an odd number or if the curve begins and ends with a subchord (even though $\Delta \div D$ is a whole number), theoretical accuracy requires that the *true* subchord lengths shall be used, although the difference may prove insignificant. The length of the arc (or the mean length of the two rails) is therefore always in excess of the length as given above. Ordinarily the amount of this excess is of no practical importance. It simply adds an insignificant amount to the length of rail required.

Example. Required the nominal and true lengths of a 3° 45′ curve having a central angle of 17° 25′. First reduce the degrees and minutes to decimals of a degree. $(100 \times 17° 25') \div 3° 45' = 1741.667 \div 3.75 = 464.444$. The curve has four 100-foot chords and a nominal chord of 64.444 The true chord should be 64.451. The actual arc is

$$17°.4167 \times \frac{\pi}{180°} \times R = 464.527$$

The excess is therefore $464.527 - 464.451 = 0.076$ foot.

21. Elements of a curve. Considering the line as running from A toward B, the beginning of the curve, at A, is called the *point of curve* (PC). The other end of the *curve*, at B, is called the *point of tangency* (*PT*). The intersection of the tangents is called the *vertex* (*V*). The angle made by the tangents at *V*, which equals the angle made by the radii to the extremities of the curve, is called the *central angle* (Δ). *AV* and *BV*, the two equal tangents from the vertex to the *PC* and *PT*, are called the *tangent distances* (*T*). The chord *AB* is called the *long chord* (*LC*). The intercept *HG* from the middle of the long chord to the middle of the arc is called the *middle ordinate* (*M*). That part of the secant *GV* from the middle of the arc to the vertex is called the *external distance* (*E*). From the figure it is very easy to derive the following frequently used relations:

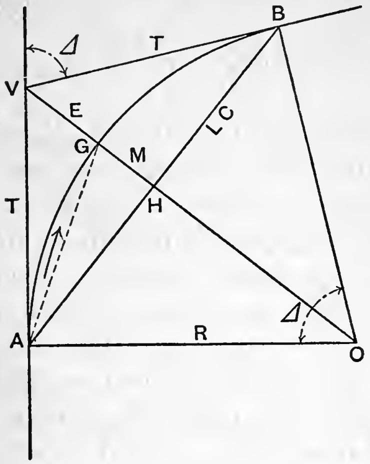


FIG. 9.

$$T = R \tan \tfrac{1}{2}\Delta \quad \ldots \ldots \quad (4)$$
$$LC = 2R \sin \tfrac{1}{2}\Delta \quad \ldots \ldots \quad (5)$$
$$M = R \operatorname{vers} \tfrac{1}{2}\Delta \quad \ldots \ldots \quad (6)$$
$$E = R \operatorname{exsec} \tfrac{1}{2}\Delta \quad \ldots \ldots \quad (7)$$

22. Relation between *T*, *E*, and Δ. Join *A* and *G* in Fig. 9. The angle $VAG = \frac{1}{4}\Delta$, since it is measured by one half of the arc *AG* between the secant and tangent. $AGO = 90° - \frac{1}{4}\Delta$.

$$AV : VG :: \sin AGV : \sin VAG;$$
$$\sin AGV = \sin AGO = \cos \tfrac{1}{4}\Delta;$$
$$T : E :: \cos \tfrac{1}{4}\Delta : \sin \tfrac{1}{4}\Delta;$$
$$T = E \cot \tfrac{1}{4}\Delta. \quad \ldots \ldots \quad (8)$$

The same relation may be obtained by dividing Eq. 4 by Eq. 7, since $\tan a \div \operatorname{exsec} a = \cot \frac{1}{2}a$.

23. Elements of a 1° curve. From Eqs. 1 to 8 it is seen that the elements of a curve vary directly as *R*. It is also seen to be very nearly true that *R* varies inversely as *D*. If the elements of a 1° curve for various central angles are calculated and tabulated, the elements of a curve of *D*° curvature may be approximately found by dividing by *D* the corresponding elements of a 1° curve having the same central angle. For small central angles and low degrees of curvature the errors involved by the approximation are insignificant, and even for larger angles the errors are so small that *for many purposes* they may be disregarded

In Table II is given the value of the tangent distances, external distances, and long chords for a 1° curve for various central angles The student should familiarize himself with the degree of approximation involved by solving a large number of cases under various conditions by the exact and by the approximate methods, in order that he may know when the approximate method is sufficiently exact for the intended purpose. The approximate method also gives a ready check on the exact method.

24. Exercises. (*a*) What is the tangent distance of a 4° 20′ curve having a central angle of 18° 24′?

(*b*) Given a 3° 30′ curve and a central angle of 16° 20′, how far will the curve pass from the vertex? [Use Eq. 7.]

(*c*) An 18° curve is to be laid off using 25-foot (nominal) chord lengths. What is the true length of the subchords?

(*d*) Given two tangents making a central angle of 15° 24′. It is desired to connect these tangents by a curve which shall pass 16.2 feet from their intersection. How far down the tangent will the curve begin and what will be its radius? (Use Eq. 8 and then use Eq 4 inverted.)

25. Curve location by deflections. The angle between a secant and a tangent (or between two secants intersecting on an arc) is measured by one half of the intercepted arc. Beginning at the *PC* (*A* in Fig. 10), if the first chord is to be a full chord we may deflect an angle *VAa* ($=\frac{1}{2}D$), and the point *a*, which is 100 feet from *A*, is a point on the curve. For the next station, *b*, deflect an *additional* angle *bAa* ($=\frac{1}{2}D$) and, with one end of the tape at *a*, swing the other end until the 100-foot point is on the line *Ab*. The point *b* is then on the curve. If the final chord *cB* is a subchord, its *additional deflection* ($\frac{1}{2}d$) is something less than $\frac{1}{2}D$. The last *deflection* (*BAV*) is

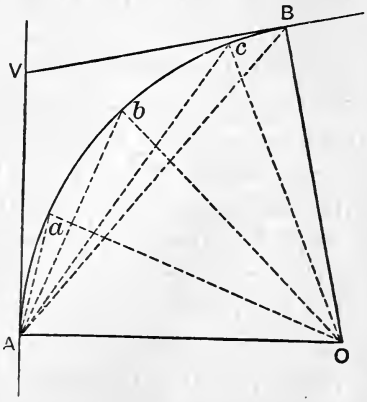


FIG. 10.

of course $\frac{1}{2}\Delta$. It is particularly important, when a curve begins or ends with a subchord and the deflections are odd quantities, that the last additional deflection should be carefully computed and added to the previous deflection, to check the mathematical work by the agreement of this last computed deflection with $\frac{1}{2}\Delta$.

Example. Given a 3° 24′ curve having a central angle of 18° 22′ and beginning at sta. 47+32, to compute the deflections. The nominal length of curve is $18° 22' \div 3° 24' = 18.367 \div 3.40 = 5.402$ stations or 540.2 feet. The curve therefore ends at sta. 52+72.2. The deflection for sta. 48 is $\frac{68}{100} \times \frac{1}{2}(3° 24') = 0.68 \times 1°.7 = 1°.156 = 1° 09'$ nearly. For each additional 100 feet it is 1° 42′ additional. The final additional deflection for the final subchord of 72.2 feet is

$$\frac{72.2}{100} \times \frac{1}{2}(3° 24') = 1°.2274 = 1° 14' \text{ nearly.}$$

The deflections are

P. C . . . Sta.	47+32	0°
	48	$0° + 1° 09' = 1° 09'$
	49	$1° 09' + 1° 42' = 2° 51'$
	50	$2° 51' + 1° 42' = 4° 33'$
	51	$4° 33' + 1° 42' = 6° 15'$
	52	$6° 15' + 1° 42' = 7° 57'$
P. T.	52+72.2	$7° 57' + 1° 14' = 9° 11'$

As a check $9° 11' = \frac{1}{2}(18° 22') = \frac{1}{2}\Delta$. (See the Form of Notes in § 17.)

26. Instrumental work. It is generally impracticable to locate more than 500 to 600 feet of a curve from one station. Obstructions will sometimes require that the transit be moved up every 200 or 300 feet. There are two methods of setting off the angles when the transit has been moved up from the *PC*.

(a) The transit may be sighted at the previous transit station with a reading on the plates equal to the deflection angle from that station to the station occupied, but with the angle set off on the *other* side of 0°, so that when the telescope is turned to 0° it will sight along the tangent at the station occupied. Plunging the telescope, the forward stations may be set off by deflecting the proper deflections from the tangent at the station occupied

This is a very common method and, when the degree of curvature is an even number of degrees and when the transit is only set at even stations, there is but little objection to it. But the degree of curvature is sometimes an odd quantity, and the exigencies of difficult location frequently require that substations be occupied as transit stations. Method (*a*) will then require the recalculation of all deflections for each new station occupied. The mathematical work is largely increased and the probability of error is very greatly increased and not so easily detected. Method (*b*) is just as simple as method (*a*) even for the most simple cases, and for the more difficult cases just referred to the superiority is very great.

(**b**) Calculate the deflection for each station and substation throughout the curve as though the whole curve were to be located from the *PC*. The computations may thus be completed and *checked* (as above) before beginning the instrumental work. If it unexpectedly becomes necessary to introduce a substation at any point, its deflection from the *PC* may be readily interpolated. The stations actually set from the *PC* are located as usual. RULE. **When the transit is set on any forward station, backsight to ANY previous station with the plates set at the deflection angle for the station sighted at. Plunge the telescope and sight at any forward station with the deflection angle originally computed for that station.** When the plates read the deflection angle for the station occupied, the telescope is sighting along the tangent at that station—which is the method of getting the forward tangent when occupying the *PT*. Even though the station occupied is an unexpected substation, when the instrument is properly oriented at that station, the angle reading for *any* station, forward or back, is that originally computed for it from the *PC*. In difficult work, where there are obstructions, a valuable check on the accuracy may be found by sighting backward at *any* visible station and noting whether

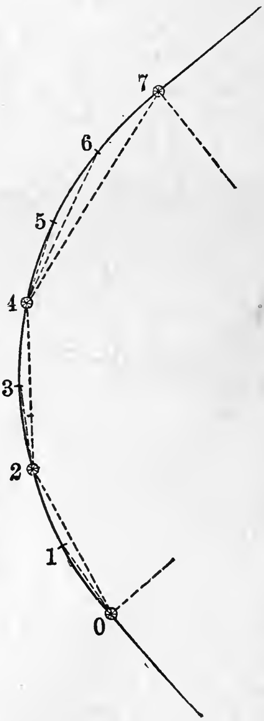


FIG. 11.

its deflection agrees with that originally computed. As a numerical illustration, assume a 4° curve, with 28° curvature, with stations 0, 2, 4, and 7 occupied. After setting stations 1 and 2, set up the transit at sta. 2 and backsight to sta. 0 with the deflection for sta. 0, which is 0°. The reading on sta. 1 is 2°; when the reading is 4° the telescope is tangent to the curve, and when sighting at 3 and 4 the deflections will be 6° and 8°. Occupy 4; sight to 2 with a reading of 4°. When the reading is 8° the telescope is tangent to the curve and, by plunging the telescope, 5, 6, and 7 may be located with the originally computed deflections of 10°, 12°, and 14°. When occupying 7 a backsight may be taken to any visible station with the plates reading the deflection for that station; then when

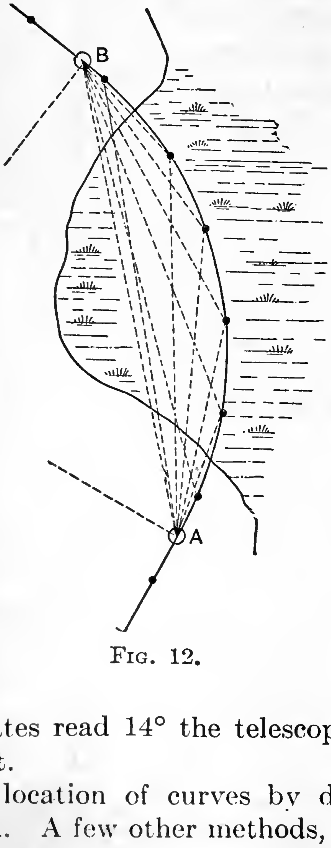


Fig. 12.

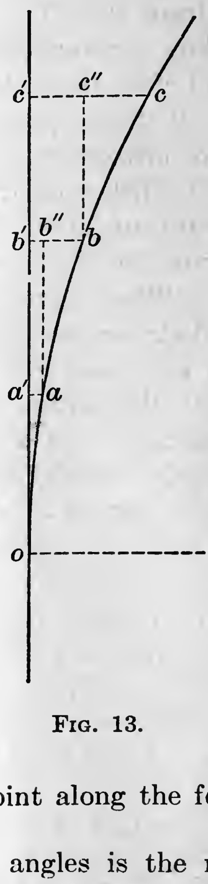


Fig. 13.

the plates read 14° the telescope will point along the forward tangent.

The location of curves by deflection angles is the normal method. A few other methods, to be described, should be considered as exceptional.

27. Curve location by two transits. A curve might be located more or less on a swamp where accurate chaining would be exceedingly difficult if not impossible. The long chord AB (Fig. 12) may be determined by triangulation or otherwise, and the elements of the curve computed, including (possibly) subchords at each end. The deflection from A and B to each point may be computed. A rodman may then be sent (by whatever means) to locate long stakes at points determined by the simultaneous sightings of the two transits.

28. Curve location by tangential offsets. When a curve is very flat and no transit is at hand the following method may be used (see Fig. 13): Produce the back tangent as far forward as necessary. Compute the ordinates Oa', Ob', Oc', etc., and the abscissæ $a'a$, $b'b$, $c'c$, etc. If Oa is a full station (100 feet), then

$$\left.\begin{aligned}
Oa' &= Oa' &&= 100 \cos \tfrac{1}{2}D, \text{ also} = R \sin D;\\
Ob' &= Oa' + a'b' &&= 100 \cos \tfrac{1}{2}D + 100 \cos \tfrac{3}{2}D,\\
&&&\qquad \text{also} = R \sin 2D;\\
Oc' &= Oa' + a'b' + b'c' &&= 100(\cos \tfrac{1}{2}D + \cos \tfrac{3}{2}D + \cos \tfrac{5}{2}D),\\
&&&\qquad \text{also} = R \sin 3D;
\end{aligned}\right\} \quad (9)$$

etc.

$$\left.\begin{aligned}
a'a &= &&100 \sin \tfrac{1}{2}D, \text{ also} = R \operatorname{vers} D;\\
b'b &= a'a + b''b &&= 100 \sin \tfrac{1}{2}D + 100 \sin \tfrac{3}{2}D,\\
&&&\qquad \text{also} = R \operatorname{vers} 2D;\\
c'c &= b'b + c''c &&= 100(\sin \tfrac{1}{2}D + \sin \tfrac{3}{2}D + \sin \tfrac{5}{2}D),\\
&&&\qquad \text{also} = R \operatorname{vers} 3D;
\end{aligned}\right\} \quad (10)$$

etc.

The functions $\frac{1}{2}D$, $\frac{3}{2}D$, etc., may be more conveniently used *without* logarithms, by adding the several *natural* trigonometrical functions and pointing off two decimal places. It may also be noted that Ob' (for example) is one half of the long chord for four stations; also that $b'b$ is the middle ordinate for four stations. If the engineer is provided with tables giving the long chords and middle ordinates for various degrees of curvature, these quantities may be taken (perhaps by interpolation) from such tables.

If the curve begins or ends at a substation, the angles and terms will be correspondingly altered. The modifications may be readily deduced on the same principles as above, and should be worked out as an exercise by the student.

In Table II are given the long chords for a 1° curve for various values of Δ. Dividing the value as given by the degree of the curve, we have an approximate value which is amply close for low degrees of curvature, especially for laying out curves without a transit. For example, given a 4° 30′ curve, required the ordinate Oc'. This is evidently one half of a chord of six stations, with $\Delta = 27°$. Dividing 2675.1 (which is the long chord of a 1° curve with $\Delta = 27°$) by 4.5 we have 594.47; one half of this is the required ordinate, $Oc' = 297.23$. The exact value is 297.31, an excess of .08, or less than .03 of 1%. The true values are always slightly in excess of the value as computed from Table II.

Exercise. A 3° 40′ curve begins at sta. 18+70 and runs to sta. 23+60. Required the tangential offsets and their corresponding ordinates. The first ordinate $= 30 \cos \frac{1}{2}(\frac{30}{100} \times 3° 40') = 30 \times .99995 = 29.9985$; the offset $= 30 \sin 0° 33' = 30 \times .0096 = 0.288$. For the second full station (sta. 20) the ordinate $= \frac{1}{2}$ long chord for $\Delta = 2(1° 06' + 3° 40')$ with $D = 3° 40'$. Dividing 476.12, from Table II, by $3\frac{2}{3}$, we have 129.85. Otherwise, by Eq. 9, the ordinate $= 30 \times \cos 0° 33' + 100 \cos (1° 06' + 1° 50') = 30.00 + 99.87 = 129.87$. The offset for sta. $20 = 30 \sin 0° 33' + 100 \sin (1° 06' + 1° 50') = 0.288 + 5.12 = 5.41$. Work out similarly the ordinates and offsets for sta. 21, 22, 23, and 23+60.

29. Curve location by middle ordinates. Take first the simpler case when the curve begins at an even station. If we consider (in Fig. 14) the curve produced back to z, the chord $za = 2 \times 100 \cos \frac{1}{2}D$, $A'a = 100 \cos \frac{1}{2}D$, and $A'A = am = zn = 100 \sin \frac{1}{2}D$. Set off AA' perpendicular to the tangent and $A'a$ parallel to the tangent. $AA' = aa' = bb' = cc'$, etc. $= 100 \sin \frac{1}{2}D$. Set off aa' perpendicular to $a'A$. Produce Aa' until $a'b = A'a$, thus determining b. Succeeding points of the curve may thus be determined indefinitely.

Suppose the curve begins with a subchord. As before $ra = Am' = c' \cos \frac{1}{2}d'$, and $rA = am' = c' \sin \frac{1}{2}d'$. Also $sz = An' = c'' \cos \frac{1}{2}d''$, and $sA = zn' = c'' \sin \frac{1}{2}d''$, in which $(d' + d'') = D$. The points z and a being determined on the ground, aa' may be computed and set off as before and the curve continued in full stations. A subchord at the end of the curve may be located by a similar process.

30. **Curve location by offsets from the long chord.** (Fig. 16.) Consider at once the general case in which the curve commences with a subchord (curvature, d'), continues with one or more full

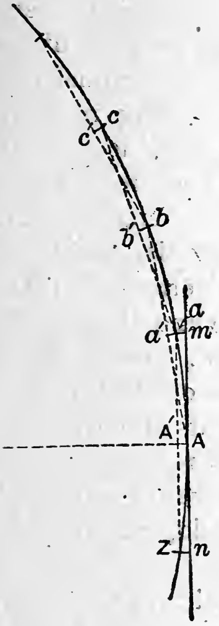


FIG. 14.

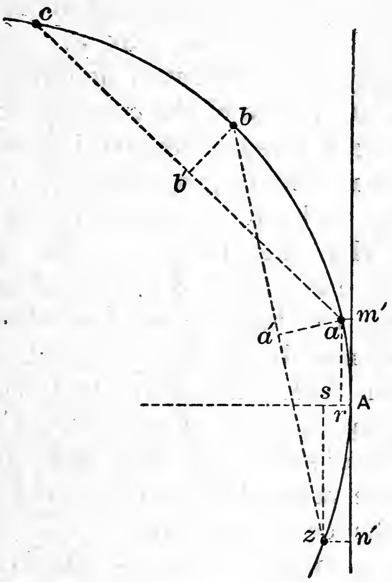


FIG. 15.

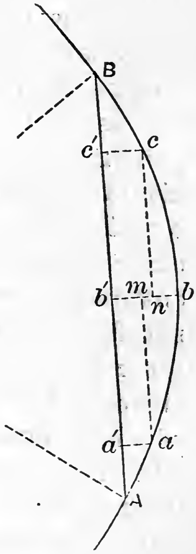


FIG. 16.

chords (curvature of each, D), and ends with a subchord with curvature d''. The numerical work consists in computing first AB, then the various abscissæ and ordinates. $AB = 2R \sin \frac{1}{2}\Delta$.

$$\left.\begin{aligned}
Aa' &= Aa' &&= c' \cos \tfrac{1}{2}(\Delta - d');\\
Ab' &= Aa' + a'b' &&= c' \cos \tfrac{1}{2}(\Delta - d') + 100 \cos \tfrac{1}{2}(\Delta - 2d' - D);\\
Ac' &= Aa' + a'b' + b'c' &&= c' \cos \tfrac{1}{2}(\Delta - d') + 100 \cos \tfrac{1}{2}(\Delta - 2d' - D)\\
& && \qquad + 100 \cos \tfrac{1}{2}(\Delta - 2d'' - D);\\
\text{also}\\
&= AB - Bc' &&= 2R \sin \tfrac{1}{2}\Delta - c'' \cos \tfrac{1}{2}(\Delta - d'').
\end{aligned}\right\} \quad (11)$$

$$\left.\begin{aligned}
a'a &= a'a &&= c' \sin \tfrac{1}{2}(\Delta - d');\\
b'b &= a'a + mb &&= c' \sin \tfrac{1}{2}(\Delta - d') + 100 \sin \tfrac{1}{2}(\Delta - 2d' - D);\\
c'c &= b'b - nb &&= c' \sin \tfrac{1}{2}(\Delta - d') + 100 \sin \tfrac{1}{2}(\Delta - 2d' - D)\\
& && \qquad - 100 \sin \tfrac{1}{2}(\Delta - 2d'' - D);\\
\text{also} & &&= c'' \sin \tfrac{1}{2}(\Delta - d'').
\end{aligned}\right\} \quad (12)$$

The above formulæ are considerably simplified when the curve begins and ends at even stations. When the curve is very long a regular law becomes very apparent in the formation of all terms between the first and last. There are too few terms in the above equations to show the law.

31. Use and value of the above methods. The chief value of the above methods lies in the possibility of doing the work without a transit. The same principles are sometimes employed, even when a transit is used, when obstacles prevent the use of the normal method (see § 32, *c*). If the terminal tangents have already been accurately determined, these methods are useful to locate points of the curve when rigid accuracy is not essential. Track foremen frequently use such methods to lay out unimportant sidings, especially when the engineer and his transit are not at hand. Location by tangential offsets (or by offsets from the long chord) is to be preferred when the curve is flat (i.e., has a small central angle Δ) and there is no obstruction along the tangent, or long chord. Location by middle ordinates may be employed regardless of the length of the curve, and in cases when both the tangents and the long chord are obstructed. The above methods are but samples of a large number of similar methods which have been devised. The choice of the particular method to be adopted must be determined by the local conditions.

32. Obstacles to location. In this section will be given only a few of the principles involved in this class of problems, with illustrations. The engineer must decide, in each case, which is the best method to use. It is frequently advisable to devise a special solution for some particular case.

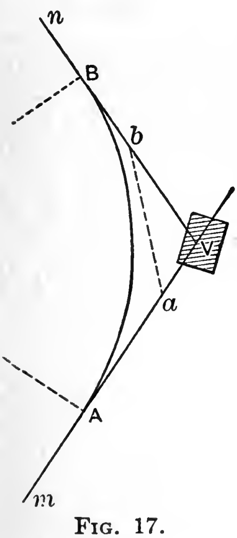


Fig. 17.

a. When the vertex is inaccessible. As shown in § 26, it is not absolutely essential that the vertex of a curve should be located on the ground. But it is very evident that the angle between the terminal tangents is determined with far less probable error if it is measured by a single measurement at the vertex rather than as the result of numerous angle measurements along the curve, involving several positions of the transit and comparatively short sights. Sometimes the location of the tangents is already determined on the ground (as by *bn* and *am*, Fig. 17), and it is required to join the tangents by a curve of given radius. *Method.* Measure *ab* and the angles *Vba* and *baV*. Δ is the sum of these angles. The distances *bV* and *aV* are computable from the above data. Given Δ and *R*, the tangent distances are computable, and then *Bb* and *aA* are found by subtracting *bV* and *aV* from the tangent distances. The curve may then be run from *A*, and the work may be checked by noting whether the curve as run ends at *B*—previously located from *b*.

Example. Assume $ab = 546\,82$; angle $a = 15°\,18'$; angle $b = 18°\,22'$; $D = 3°\,40'$; required *aA* and *bB*.

$\Delta = 15°\,18' + 18°\,22' = 33°\,40'$

Eq. (4)	R $(3°\,40')$..............	$3.1939\bar{2}$
	$\tan \frac{1}{2}\Delta = \tan 16°\,50'$............	9.48080
	$T = 472.85$..............	$2.6747\bar{2}$
$aV = ab \dfrac{\sin 18°\,22'}{\sin 33°\,40'}$	ab.........................	$2.7378\bar{4}$
	$\log \sin 18°\,22'$..............	$9.4984\bar{4}$
	co-log $\sin 33°\,40'$..............	0.25621
	$aV = 310.81$................	2.49250
	$AV = 472.85$	
	$aA = 162.04$	
$bV = ab \dfrac{\sin 15°\,18'}{\sin 33°\,40'}$	ab.........................	$2.7378\bar{4}$
	$\log \sin 15°\,18'$..............	$9.4213\bar{9}$
	co-log $\sin 33°\,40'$..............	0.25621
	$bV = 260.29$..............	2.41545
	$BV = 472.85$	
	$bB = 212.56$	

b. When the point of curve (or point of tangency) is inaccessible. At some distance (*As*, Fig 18) an unobstructed line *pn* may be run parallel with *AV*. $nv = py = As = R \text{ vers } a.$

$$\therefore \text{ vers } a = As \div R.$$

$$ns = ps = R \sin a.$$

At y, which is at a distance ps back from the *computed* position of A, make an offset sA to p. Run pn parallel to the tangent. A tangent to the curve at n makes an angle of a with np. From n the curve is run in as usual

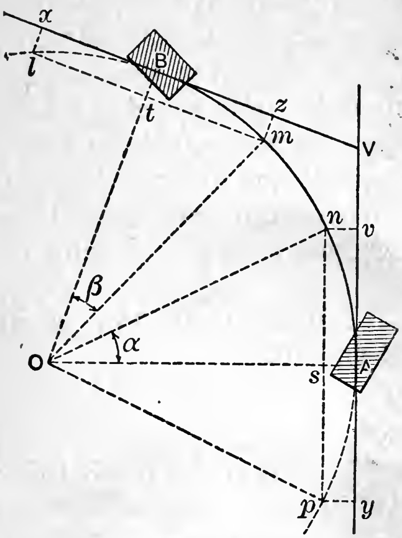


FIG. 18.

If the point of tangency is obstructed, a similar process, somewhat reversed, may be used. β is that portion of Δ still to be laid off when m is reached. $tm = tl = R \sin \beta$. $mz = tB = lx = R$ vers β.

c. When the central part of the curve is obstructed. a is the central angle between two points of the curve between which a chord may be run. a may equal *any* angle, but it is preferable that a should be a multiple of D, the degree of curve, and that the points m and n should be on even stations. $mn = 2R \sin \frac{1}{2}a$. A point s may be located by an offset ks from the chord mn by a similar method to that outlined in § 30.

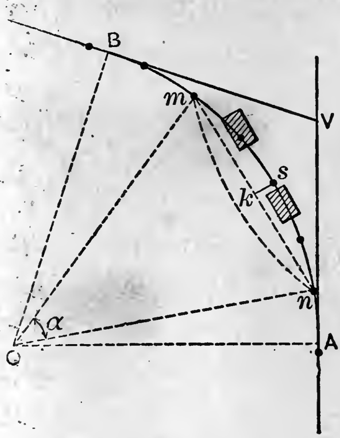


FIG. 19.

The device of introducing the dotted curve mn having the same radius of curvature as the other, although neither necessary nor advisable in the case shown in Fig. 19, is sometimes the best method of surveying around an obstacle. The offset from any point on the dotted curve to the corresponding point on the true curve is twice the "ordinate to the long chord," as computed in § 30.

33. Modifications of location. The following methods may be used in allowing for the discrepancies between the "paper location" based on a more or less rough preliminary survey and the more accurate instrumental location. (See § 15.) They are also frequently used in locating new parallel tracks and modifying old tracks.

a. To move the forward tangent parallel to itself a distance x, the point of curve (A) remaining fixed. (Fig. 20.)

$$V'h = B'r = x'.$$

$$VV' = \frac{V'h}{\sin hVV'} = \frac{x}{\sin \Delta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$AV' = AV + VV'.$$

The triangle BmB' is isosceles and $Bm = B'm$.

$$R' - R = O'O = mB = \frac{B'r}{\text{vers } B'mB} = \frac{x'}{\text{vers } \Delta}.$$

$$\therefore \; R' = R + \frac{x'}{\text{vers } \Delta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The solution is very similar in case the tangent is moved inward to $V''B''$. Note that this method necessarily changes the

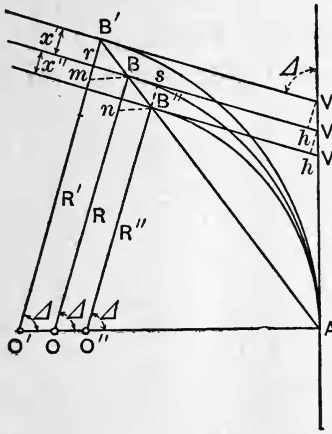


Fig. 20.

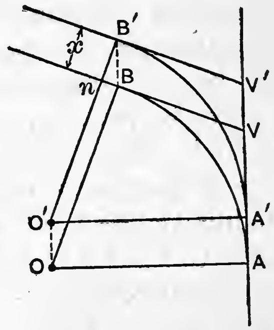


Fig. 21.

radius. If the radius is not to be changed, the point of curve must be altered as follows:

b. To move the forward tangent parallel to itself a distance x, the radius being unchanged. (Fig. 21.) In this case the whole curve is moved bodily a distance $OO' = AA' = VV' = BB'$, and moved parallel to the first tangent AV

$$BB' = \frac{B'n}{\sin nBB'} = \frac{x}{\sin \Delta} = AA'. \quad . \quad . \quad . \quad (15)$$

c. To change the direction of the forward tangent at the point of tangency. (Fig. 22.) This problem involves a change (a) in the central angle and also requires a new radius. An error in the determination of the central angle furnishes an occasion for its use.

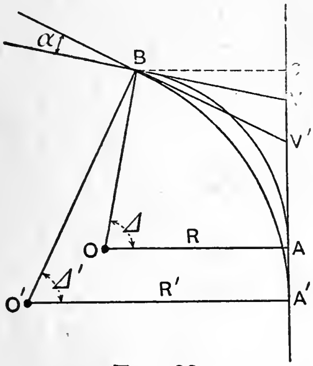


FIG. 22.

R, Δ, a, AV, and BV are known.

$$\Delta' = \Delta - a.$$

$Bs = R$ vers Δ. $\quad Bs = R'$ vers Δ'.

$$\therefore \; R' = R \frac{\text{vers } \Delta}{\text{vers } (\Delta - a)}. \quad . \quad (16)$$

$As = R \sin \Delta$. $\quad A's = R' \sin \Delta'$.

$$\therefore \; AA' = A's - As = R' \sin \Delta' - R \sin \Delta. \quad . \quad . \quad (17)$$

The above solutions are given to illustrate a large class of problems which are constantly arising. All of the ordinary problems can be solved by the application of elementary geometry and trigonometry.

34. Limitations in location. It may be required to run a curve that shall join two given tangents and also pass through a given point The point (P, Fig. 23) is assumed to be determined by its distance (VP) from the vertex and by the angle $AVP = \beta$.

It is required to determine the radius (R) and the tangent distance (AV). Δ is known.

$$PVG = \tfrac{1}{2}(180° - \Delta) - \beta$$
$$= 90° - (\tfrac{1}{2}\Delta + \beta).$$
$$PP' = 2VP \sin PVG$$
$$= 2VP \cos (\tfrac{1}{2}\Delta + \beta).$$
$$PSV = \tfrac{1}{2}\Delta.$$
$$\therefore \; SP = VP \frac{\sin \beta}{\sin \frac{1}{2}\Delta}.$$

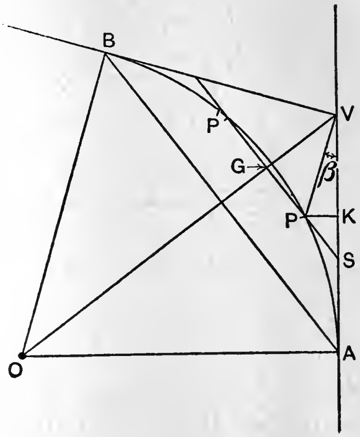


FIG. 23.

$$AS=\sqrt{SP\times SP'}=\sqrt{SP(SP+PP')}$$

$$=\sqrt{VP\frac{\sin\beta}{\sin\frac{1}{2}\Delta}\left[VP\frac{\sin\beta}{\sin\frac{1}{2}\Delta}+2VP\cos\left(\tfrac{1}{2}\Delta+\beta\right)\right]}$$

$$=VP\sqrt{\frac{\sin^2\beta}{\sin^2\frac{1}{2}\Delta}+\frac{2\sin\beta\cos\left(\frac{1}{2}\Delta+\beta\right)}{\sin\frac{1}{2}\Delta}}.$$

$$SV=VP\frac{\sin\left(\frac{1}{2}\Delta+\beta\right)}{\sin\frac{1}{2}\Delta}.$$

$$AV=AS+SV$$

$$=\frac{VP}{\sin\frac{1}{2}\Delta}\left[\sin\left(\tfrac{1}{2}\Delta+\beta\right)+\sqrt{\sin^2\beta+2\sin\beta\sin\tfrac{1}{2}\Delta\cos\left(\tfrac{1}{2}\Delta+\beta\right)}\right]. \quad (18)$$

$$R=AV\cot\tfrac{1}{2}\Delta.$$

In the special case in which P is on the median line OV, $\beta=90°-\frac{1}{2}\Delta$, and $(\frac{1}{2}\Delta+\beta)=90°$. Eq. 18 then reduces to

$$AV=\frac{VP}{\sin\frac{1}{2}\Delta}(1+\cos\tfrac{1}{2}\Delta)=VP\cot\tfrac{1}{4}\Delta,$$

as might have been immediately derived from Eq. 8.

In case the point P is given by the offset PK and by the distance VK, the triangle PKV may be readily solved, giving the distance VP and the angle β, and the remainder of the solution will be as above.

35. Determination of the curvature of existing track. (a) *Using a transit.* Set up the transit at any point in the center of the track. Measure in each direction 100 feet to points also in the center of the track. Sight on one point with the plates at 0°. Plunge the telescope and sight at the other point. The angle between the chords equals the degree of curvature.

(b) *Using a tape and string.* Stretch a string (say 50 feet long) between two points on the inside of the head of the outer rail. Measure the ordinate (x) between the *middle* of the string and the head of the rail. Then

$$R=\frac{\text{chord}^2}{8x}\text{(very nearly)}. \quad . \quad . \quad . \quad . \quad (19)$$

For, in Fig. 24, since the triangles AOE and ADC are similar,

$AO : AE :: AD : DC$ or $R = \frac{1}{2}\overline{AD}^2 \div x$. When, as is usual, the arc is very short compared with the radius, $AD = \frac{1}{2}AB$, very nearly. Making this substitution we have Eq. 19. With a chord of 50 feet and a 10° curve, the resulting difference in x is .0025 of an inch—far within the possible accuracy of such a method. The above method gives the radius of the inner head of the outer rail. It should be diminished by $\frac{1}{2}g$ for the radius of the center of the track. With easy curvature, however, this will not affect the result by more than one or two tenths of one per cent.

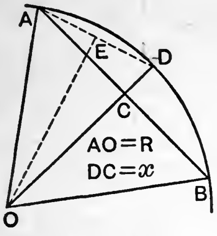


FIG. 24.

The inversion of this formula gives the required middle ordinate for a rail on a given curve. For example, the middle ordinate of a 30-foot rail, bent for a 6° curve, is

$$x = 900 \div (8 \times 955) = .118 \text{ foot} = 1.4 \text{ inches}.$$

Another much used rule is to require the foreman to have a string, knotted at the center, of such length that the middle ordinate, measured in inches, equals the degree of curve. To find that length, substitute (in Eq. 19) $5730 \div D$ for R and $D \div 12$ for x. Solving for *chord*, we obtain *chord* = 61.8 feet. The rule is not theoretically exact, but, considering the uncertain stretching of the string, the error is insignificant. In fact, the distance usually given is 62 feet, which is close enough for all purposes for which such a method should be used.

36. Problems. A systematic method of setting down the solution of a problem simplifies the work. Logarithms should always be used, and *all* the work should be so set down that a revision of the work to find a supposed error may be readily done. The value of such systematic work will become more apparent as the problems become more complicated. The two solutions given below will illustrate such work.

a. Given a 3° curve beginning at Sta. 27 + 60 and running to Sta. 32 + 45. Compute the ordinates and offsets used in locating the curve by tangential offsets.

b. With the same data as above, compute the distances to locate the curve by offsets from the long chord.

c. Assume that in Fig. 17 *ab* is measured as 217.6 feet, the angle $abV=17° 42'$, and the angle $baV=21° 14'$. Join the tangents by a 4° 30′ curve. Determine bB and aA.

d. Assume that in a case similar to Fig. 18 it was noted that a distance (As) equal to 12 feet would clear the building. Assume that $\Delta=38° 20'$ and that $D=4° 40'$. Required the value of a and the position of n. *Solution:*

vers $a=As \div R$	$As=12$	$\log=1.07918$
	R (for 4° 40′ curve)	$\log=3.0892\bar{3}$
	$a=8° 01'$	$\log \text{vers } a=7.9899\bar{4}$
$ns=R \sin a$		$\log \sin a=9.1444\bar{5}$
		$\log R=3.0892\bar{3}$
	$ns=171.27$	$\log=2.23369$

e. Assume that the forward tangent of a 3° 20′ curve having a central angle of 16° 50′ must be moved 3.62 feet *inward*, without altering the $P.C.$ Required the change in radius.

f. Given two tangents making an angle of 36° 18′. It is required to pass a curve through a point 93.2 feet from the vertex, the line from the vertex to the point making an angle of 42° 21′ with the tangent. Required the radius and tangent distance. *Solution:* Applying Eq. 18, we have

	2	$\log=0.30103$
	$\beta=42° 21'$	$\log \sin=9.82844$
	$\frac{1}{2}\Delta=18° 09'$	$\log \sin=9.4934\bar{6}$
	$(\frac{1}{2}\Delta+\beta)=60° 30'$	$\log \cos=9.69234$
	.20667	$9.3152\bar{7}$
$\log \sin^2 \beta=9.65688$....	.45382	
$2\|9.81987$....	.66049	
$9.9099\bar{3}$....	.81271	
nat. sin 60° 30′....	$.870\bar{3}$	
	$1.683\bar{0}$............	$\log=0.22610$
$VP=93.2$....................		$\log=1.9694\bar{1}$
		$2.1955\bar{1}$
		$\log \sin \frac{1}{2}\Delta=9.4934\bar{6}$
Tang. dist. $AV=503.36$..........		$\log=2.70205$
		$\log \cot \frac{1}{2}\Delta=10.48437$
$R=1536.1$..................		$\log=3.18642$
$D=3° 44'$		

COMPOUND CURVES.

37. Nature and use. Compound curves are formed by a succession of two or more simple curves of different curvature. The curves must have a common tangent at the point of compound curvature (*P.C.C.*). In mountainous regions there is frequently a necessity for compound curves having several changes of curvature. Such curves may be located separately as a succession of simple curves, but a combination of two simple curves has special properties which are worth investigating and utilizing. In the following demonstrations R_2 always represents the *longer* radius and R_1 the *shorter*, no matter which succeeds the other. T_1 is the tangent adjacent to the curve of shorter radius (R_1), and is invariably the shorter tangent. Δ_1 is the central angle of the curve of radius R_1, but it may be greater or less than Δ_2

38. Mutual relations of the parts of a compound curve having two branches. In Fig. 25, AC and CB are the two branches of

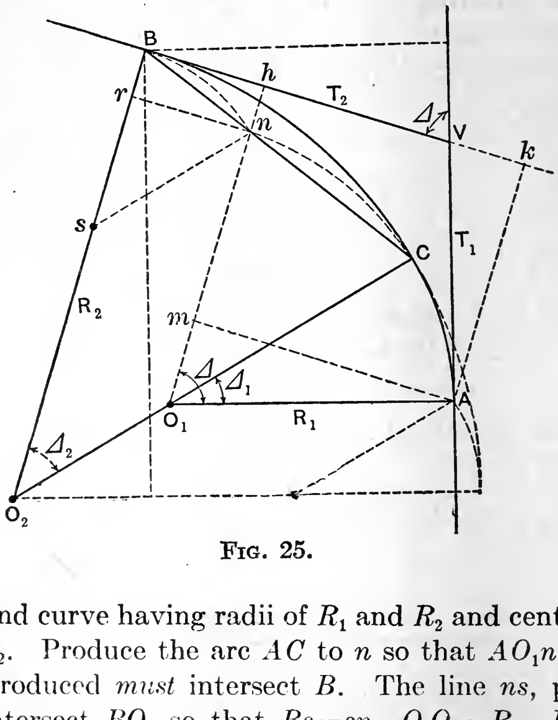


Fig. 25.

the compound curve having radii of R_1 and R_2 and central angles of Δ_1 and Δ_2. Produce the arc AC to n so that $AO_1n = \Delta$. The chord Cn produced *must* intersect B. The line ns, parallel to CO_2, will intersect BO_2 so that $Bs = sn = O_2O_1 = R_2 - R_1$. Draw Am perpendicular to O_1n. It will be parallel to hk.

$$Br = sn \text{ vers } Bsn = (R_2 - R_1) \text{ vers } \Delta_2;$$
$$mn = AO_1 \text{ vers } AO_1n = R_1 \text{ vers } \Delta;$$
$$Ak = AV \sin AVk = T_1 \sin \Delta;$$
$$Ak = hm = mn + nh = mn + Br.$$
$$\therefore\ T_1 \sin \Delta = R_1 \text{ vers } \Delta + (R_2 - R_1) \text{ vers } \Delta_2. \quad . \quad . \quad (20)$$

Similarly it may be shown that

$$T_2 \sin \Delta = R_2 \text{ vers } \Delta - (R_2 - R_1) \text{ vers } \Delta_1. \quad . \quad . \quad (21)$$

The mutual relations of the elements of compound curves may be solved by these two equations. For example, assume the tangents as fixed (Δ therefore known) and that a curve of given radius R_1 shall start from a given point at a distance T_1 from the vertex, and that the curve shall continue through a given angle Δ_1. Required the other parts of the curve. From Eq. 20 we have

$$R_2 - R_1 = \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{\text{vers } \Delta_2}.$$

$$\therefore\ R_2 = R_1 + \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{\text{vers } (\Delta - \Delta_1)}. \quad . \quad . \quad . \quad . \quad . \quad (22)$$

T_2 may then be obtained from Eq. 21.

As another problem, given the location of the two tangents, with the two tangent distances (thereby locating the *PC* and *PT*), and the central angle of each curve; required the two radii. Solving Eq. 20 for R_1, we have

$$R_1 = \frac{T_1 \sin \Delta - R_2 \text{ vers } \Delta_2}{\text{vers } \Delta - \text{vers } \Delta_2}.$$

Similarly from Eq. 21 we may derive

$$R_1 = \frac{T_2 \sin \Delta - R_2(\text{vers } \Delta - \text{vers } \Delta_1)}{\text{vers } \Delta_1}.$$

Equating these, reducing, and solving for R_2, we have

$$R_2 = \frac{T_1 \sin \Delta \text{ vers } \Delta_1 - T_2 \sin \Delta \text{ (vers } \Delta - \text{vers } \Delta_2)}{\text{vers } \Delta_2 \text{ vers } \Delta_1 - (\text{vers } \Delta - \text{vers } \Delta_1)(\text{vers } \Delta - \text{vers } \Delta_2)}. \quad (23)$$

Although the various elements may be chosen as above with considerable freedom, there are limitations. For example, in Eq. 22, since R_2 is always greater than R_1, the term to be added to R_1 must be essentially positive—i.e., $T_1 \sin \Delta$ must be greater than $R_1 \text{ vers } \Delta$. This means that $T_1 > R_1 \dfrac{\text{vers } \Delta}{\sin \Delta}$, or that

$T_1 > R_1 \tan \frac{1}{2}\Delta$, or that T_1 is greater than the corresponding tangent on a simple curve. Similarly it may be shown that T_2 is *less* than $R_2 \tan \frac{1}{2}\Delta$ or *less* than the corresponding tangent on a simple curve. Nevertheless T_2 is always greater than T_1. In the limiting case when $R_2 = R_1$, $T_2 = T_1$, and $\Delta_2 = \Delta_1$.

39. Modifications of location. Some of these modifications may be solved by the methods used for simple curves. For example:

a. It is desired to move the tangent *VB*, Fig. 26, parallel to itself to *V'B'*. Run a new curve from the *P.C.C.* which shall reach the new tangent at *B'*, where the chord of the old curve

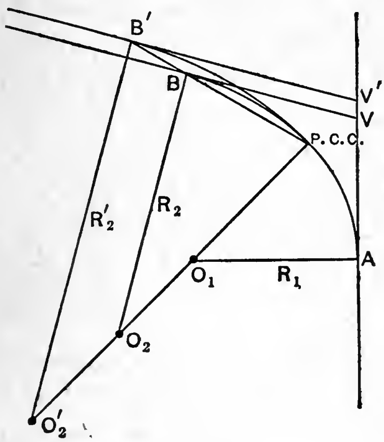


FIG. 26.

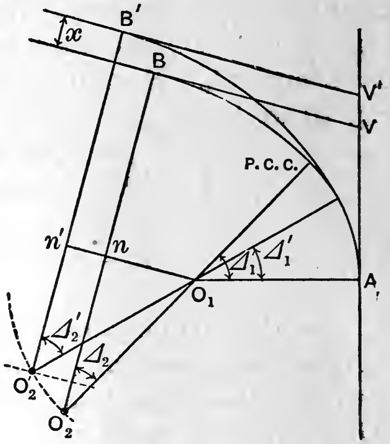


FIG. 27.

intersects the new tangent. The solution is almost identical with that in § 33, *a*.

b. Assume that it is desired to change the forward tangent (as above) but to retain the same radius. In Fig. 27

$$(R_2 - R_1) \cos \Delta_2 = O_2 n;$$
$$(R_2 - R_1) \cos \Delta_2' = O_2' n'.$$
$$x = O_2 n - O_2' n' = (R_2 - R_1)(\cos \Delta_2 - \cos \Delta_2').$$

$$\cos \Delta_2' = \cos \Delta_2 - \frac{x}{R_2 - R_1}. \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The *P.C.C.* is moved *backward* along the sharper curve an angular distance of $\Delta_2' - \Delta_2 = \Delta_1 - \Delta_1'$.

In case the tangent is moved inward rather than outward, the solution will apply by transposing Δ_2 and Δ_2'. Then we shall have

$$\cos \Delta_2' = \cos \Delta_2 + \frac{x}{R_2 - R_1}. \quad . \quad . \quad . \quad . \quad . \quad (25)$$

The *P.C.C.* is then moved *forward.*

c. Assume the same case as (**b**) except that the larger radius comes first and that the tangent adjacent to the smaller radius is moved. In Fig. 28

$(R_2-R_1)\cos \Delta_1 = O_1n;$

$(R_2-R_1)\cos \Delta_1' = O_1'n'.$

$x = O_1'n' - O_1n$

$= (R_2-R_1)(\cos \Delta_1' - \cos \Delta_1).$

$$\cos \Delta_1' = \cos \Delta_1 + \frac{x}{R_2-R_1}. \qquad (26)$$

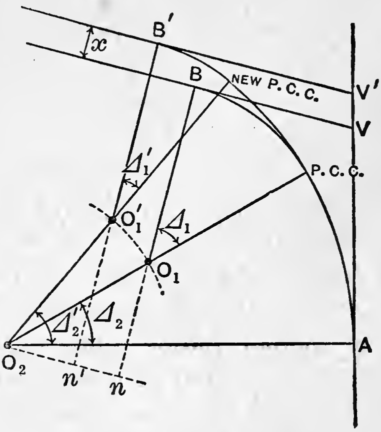


FIG. 28.

The *P.C.C.* is moved *forward* along the easier curve an angular distance of $\Delta_1' - \Delta_1 = \Delta_2 - \Delta_2'$.

In case the tangent is moved *inward*, transpose as before and we have

$$\cos \Delta_1' = \cos \Delta_1 - \frac{x}{R_2-R_1}. \quad . \quad . \quad . \quad . \quad . \quad (27)$$

The *P.C.C.* is moved *backward*.

d. Assume that the radius of one curve is to be altered without changing either tangent. Assume conditions as in Fig. 29.

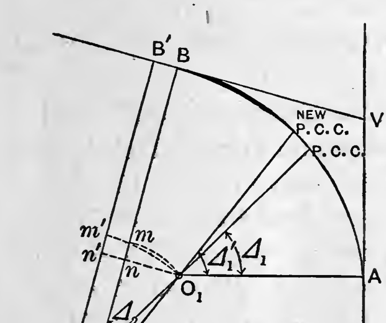


FIG. 29.

For the diagrammatic solution assume that R_2 is to be increased by O_2S. Then, since R_2' must pass through O_1 and extend beyond O_1 a distance O_1S, the locus of the new center must lie on the arc drawn about O_1 as center and with OS as radius. The locus of O_2' is also given by a line $O_2'p$ parallel to BV and at a distance of R_2' (equal to $S \ldots P.C.C.$) from it. The new center is therefore at the intersection O_2'. An arc with radius R_2' will therefore be tangent at B' and tangent to the old curve *produced* at NEW *P.C.C.* Draw O_1n' perpendicular to O_2B.

With O_2 as center draw the arc O_1m, and with O_2' as center draw the arc O_1m'. $mB = m'B' = R_1$.

$$\therefore\ mn = m'n' = (R_2' - R_1)\ \text{vers}\ \Delta_2' = (R_2 - R_1)\ \text{vers}\ \Delta_2.$$

$$\therefore\ \text{vers}\ \Delta_2' = \frac{(R_2 - R_1)}{(R_2' - R_1)}\ \text{vers}\ \Delta_2. \quad . \quad . \quad . \quad . \quad (28)$$

$$O_1n = (R_2 - R_1)\sin\Delta_2;$$

$$O_1n' = (R_2' - R_1)\sin\Delta_2'.$$

$$BB' = O_1n' - O_1n = (R_2' - R_1)\sin\Delta_2' - (R_2 - R_1)\sin\Delta_2. \qquad (29)$$

This problem may be further modified by assuming that the radius of the curve is decreased rather than increased, or that the smaller radius follows the larger. The solution is similar and is suggested as a profitable exercise.

It might also be assumed that, instead of making a given change in the radius R_2, a given change BB' is to be made. Δ_2' and R_2' are required. Eliminate R_2' from Eqs. 28 and 29 and solve the resulting equation for Δ_2'. Then determine R_2' by a suitable inversion of either Eq. 28 or 29.

As in §§ 32 and 33, the above problems are but a few, although perhaps the most common, of the problems the engineer may meet with in compound curves. All of the ordinary problems may be solved by these and similar methods.

40. Problems. *a.* Assume that the two tangents of a compound curve are to be 348 feet and 624 feet, and that $\Delta_1 = 22° 16'$ and $\Delta_2 = 28° 20'$. Required the radii.

[*Ans.* $R_1 = 326.92$; $R_2 = 1574.85$.]

b. A line crosses a valley by a compound curve which is first a 6° curve for 46° 30′ and then a 9° 30′ curve for 84° 16′. It is afterward decided that the last tangent should be 6 feet farther up the hill. What are the required changes? [*Note.* The second tangent is evidently moved *outward.* The solution corresponds to that in the first part of § 39, *c.* The *P.C.C.* is moved forward 16.39 feet. If it is desired to know how far the *P.T.* is moved in the direction of the tangent (i.e., the *projection* of BB', Fig. 28, on $V'B'$), it may be found by observing that it is equal to $nn' = (R_2 - R_1)(\sin\Delta_1 - \sin\Delta_1')$. In this case it equals 0.65 foot, which is very small because Δ_1 is nearly 90°. The value of Δ_2 (46° 30′) is not used, since the solution is independent of the value of Δ_2. The student should learn to recognize which quantities are mutually related and therefore essential to a solution, and which are independent and non-essential.]

TRANSITION CURVES.

41. Superelevation of the outer rail on curves. When a mass is moved in a circular path it requires a centripetal force to keep it moving in that path. By the principles of mechanics we know that this force equals $Gv^2 \div gR$, in which G is the weight, v the velocity in feet per second, g the acceleration of gravity in feet per second in a second, and R the radius of curvature. If the two rails of a curved track were laid on a level (transversely), this centripetal force could only be furnished by the pressure of the wheel-flanges against the rails. As this is very objectionable, the outer rail is elevated so that the reaction of the rails against the wheels shall contain a horizontal component equal to the required centripetal force. In Fig. 30, if ob represents the reaction, oc will represent the weight G, and ao will represent the required centripetal force. From similar triangles we may write $sn : sm :: ao : oc$. Call $g = 32.17$. Call $R = 5730 \div D$, which is sufficiently accurate for this purpose (see § 19). Call $v = 5280V \div 3600$, in which V is the velocity in miles per hour. mn is the distance between rail centers, which, for an 80-lb. rail and standard gauge, is 4.916 feet sm is slightly less than this. As an average value we may call it 4.900, which is its exact value when the superelevation is $4\frac{3}{4}$ inches. Calling $sn = e$, we have

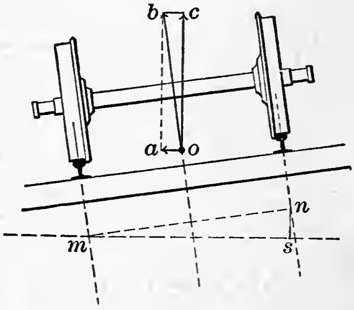


FIG. 30.

$$e = sm\frac{ao}{oc} = 4.9\frac{Gv^2}{gR}\frac{1}{G} = \frac{4.9 \times 5280^2 V^2 D}{32.17 \times 3600^2 \times 5730}.$$

$$e = .0000572V^2D. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

It should be noticed that, according to this formula, the required superelevation varies as the *square* of the velocity, which means that a change of velocity of only 10% would call for a change of superelevation of 21%. Since the velocities of trains over any road are extremely variable, it is impossible to adopt any superelevation which will fit all velocities even approximately. The above fact also shows why any over-refinement in the calculations is useless and why the above approximations, which are really small, are amply justifiable. For example, the above formula contains the approximation that $R = 5730 \div D$. In the extreme case of a 10° curve the error involved would be about 1%. A change of about $\frac{1}{2}$ of 1% in the velocity, or say from 40 to 40.2 miles per hour, would mean as much. The error in e due to the assumed constant value of sm is never more than a very small fraction of 1%. The rail-laying is not done closer than this. The following tabular form is based on Eq. (30):

SUPERELEVATION OF THE OUTER RAIL (IN FEET) FOR VARIOUS VELOCITIES AND DEGREES OF CURVATURE.

Velocity in Miles per Hour.	Degree of Curve.									
	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
30	.05	.10	.15	.20	.26	.31	.36	.41	.46	.51
40	.09	.18	.27	.37	.46	.55	.64	.73	.82	
50	.14	.29	.43	.57	.71	.86				
60	.20	.41	.62	.82						

42. **Practical rules for superelevation.** A much used rule for superelevation is to "elevate one half an inch for each degree of curvature." The rule is rational in that e in Eq. 30 varies directly as D. The above rule therefore agrees with Eq. 30 when V is about 27 miles per hour. However applicable the rule may have been in the days of low velocities, the elevation thus computed is too small now. The rule to elevate one inch for each degree of curvature is also used and is precisely similar in its nature to the above rule. It agrees with Eq. 30 when the velocity is about 38 miles per hour, which is more nearly the average speed of trains.

Another (and better) rule is to "elevate for the speed of the fastest trains." This rule is further justified by the fact that a four-wheeled truck, having two parallel axles, will always tend to run to the outer rail and will require considerable flange pressure to guide it along the curve. The effect of an excess of superelevation on the slower trains will only be to relieve this flange pressure somewhat. This rule is coupled with the limitation that the elevation should never exceed a limit of six inches—sometimes eight inches. This limitation implies that locomotive engineers must reduce the speed of fast trains around sharp curves until the speed does not exceed that for which the actual superelevation used is suitable. The heavy line in the tabular form (§ 41) shows the six-inch limitation.

Some roads furnish their track foremen with a list of the superelevations to be used on each curve in their sections. This method has the advantage that each location may be separately studied, and the proper velocity, as affected by local conditions (*e.g.*, proximity to a stopping-place for all trains), may be determined and applied.

Another method is to allow the foremen to determine the superelevation for each curve by a simple measurement taken at the curve. The rule is developed as follows: By an inversion of Eq. 19 we have

$$x = chord^2 \div 8R. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Putting x equal to e in Eq. 30 and solving for "*chord*," we have

$$\begin{aligned} chord^2 &= .0000572V^2DSR \\ &= 2.621V^2. \\ chord &= 1.62V. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32) \end{aligned}$$

To apply the rule, assume that 50 miles per hour is fixed as the velocity from which the superelevation is to be computed. Then $1.62V = 1.62 \times 50 = 81$ feet, which is the distance given to the trackmen. Stretch a tape (or even a string) with a length of 81 feet between two points on the inside head of the outer rail of the outer head of the inner rail. The ordinate at the middle point then equals the superelevation. The values of this chord length for varying velocities are given in the accompanying tabular form.

Velocity in miles per hour...	20	25	30	35	40	45	50	55	60
Chord length in feet........	32.4	40.5	48.6	56.7	64.8	72.9	81.0	89.1	97.2

The following tabular form shows the standard (at one time) on the N. Y., N. H. & H. R. R. It should be noted that the elevations do not increase proportionately with the radius, and that they are higher for descending grades than for level or ascending grades. This is on the basis that the velocity on curves and on ascending grades will be less than on descending grades. For example, the superelevation for a 0° 30′ curve on a descending grade corresponds to a velocity of about 54 miles per hour, while for a 4° curve on a level or ascending grade the superelevation corresponds to a velocity of only about 38 miles per hour.

TABLE OF THE SUPERELEVATION OF THE OUTER RAIL ON CURVES.

N. Y., N. H. & H. R. R.

Degree of curve.	Level or ascending grade.	Descending grade.
	inches.	inches.
0° 30′	$0\frac{3}{4}$	1
1 00	$1\frac{1}{2}$	$1\frac{3}{4}$
1 15	$1\frac{3}{4}$	2
1 30	2	$2\frac{1}{4}$
1 45	$2\frac{1}{4}$	$2\frac{1}{2}$
2 00	$2\frac{3}{8}$	$2\frac{3}{4}$
2 15	$2\frac{5}{8}$	3
2 30	$2\frac{7}{8}$	$3\frac{1}{4}$
2 45	3	$3\frac{3}{8}$
3 00	$3\frac{1}{8}$	$3\frac{5}{8}$
3 15	$3\frac{3}{8}$	$3\frac{7}{8}$
3 30	$3\frac{5}{8}$	4
3 45	$3\frac{7}{8}$	$4\frac{1}{8}$
4 00	4	$4\frac{1}{2}$

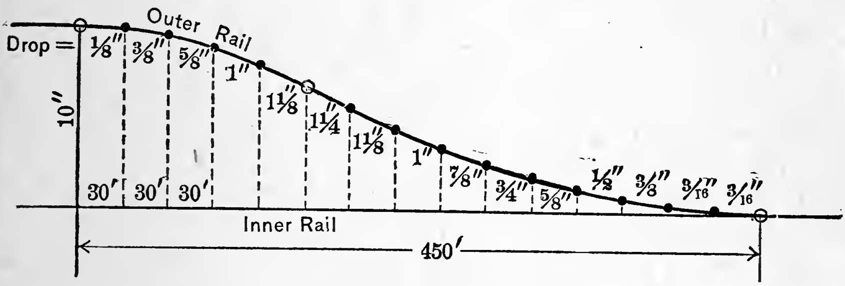
43. Transition from level to inclined track. On curves the track is inclined transversely; on tangents it is level. The transition from one condition to the other must be made gradually. If there is no transition curve, there must be either inclined track on the tangent or insufficiently inclined track on the curve or both. Sometimes the full superelevation is continued through the total length of the curve and the "run-off" (having a length of 100 to 400 feet) is located entirely on the tangents at each end. In other practice it is located partly on the tangent and partly on the curve. Whatever the method, the superelevation is correct at only one point of the run-off. At all other points it is too great or too small. This (and other causes) produces objectionable lurches and resistances when entering and leaving curves. The object of transition curves is to obviate these resistances.

On the Lehigh Valley R. R. the run-off is made in the form of a reversed vertical curve, as shown in the accompanying figure. According to this system the length of run-off varies from 120 feet, for a superelevation of one inch, to 450 feet, for a superelevation of ten inches. Such a superelevation as ten inches is very unusual practice, but is successfully operated on that road. The curve is concave upward for two-thirds of its length and then reverses so that it is convex upward.

TABLE FOR RUN-OFF OF ELEVATION OF OUTER RAIL OF CURVES.

Drop in inches for each 30-foot rail commencing at theoretical point of curve.

Elevation.	$\frac{1}{8}''$	$\frac{1}{4}''$	$\frac{3}{8}''$	$\frac{1}{2}''$	$\frac{5}{8}''$	$\frac{3}{4}''$	$\frac{7}{8}''$	1″	$1\frac{1}{8}''$	$1\frac{1}{4}''$	$1\frac{1}{8}''$	1″	$\frac{7}{8}''$	$\frac{3}{4}''$	$\frac{5}{8}''$	$\frac{1}{2}''$	$\frac{3}{8}''$	$\frac{1}{4}''$	$\frac{3}{16}''$	$\frac{1}{8}''$	$\frac{1}{16}''$	Total.
1″	..	30	30	..	..	..	..	..	...	...	...	..	..	..	..	..	..	30	...	30	...	120
2″	..	30	..	..	..	30	..	..	...	...	...	..	..	..	..	30	30	..	...	30	...	150
3″	..	30	..	..	..	30	..	..	...	...	...	..	30	..	30	..	30	..	...	30	...	180
4″	..	30	..	30	..	..	30	..	...	...	...	..	30	..	30	30	..	30	...	30	...	240
5″	..	30	..	30	..	..	..	30	...	...	...	30	..	30	30	30	..	30	...	30	...	270
6″	..	30	..	30	..	..	30	..	30	...	...	30	..	30	30	30	..	30	...	30	...	300
7″	..	30	..	30	..	..	30	..	30	...	30	30	..	30	30	..	30	30	...	30	...	330
8″	..	30	..	30	..	30	..	..	30	30	30	30	..	30	30	..	30	..	30	..	30	360
9″	30	..	..	30	..	30	..	30	30	...	30	30	30	30	30	30	30	..	30	..	30	420
10″	30	..	30	..	30	..	..	30	30	30	30	30	30	30	30	30	30	..	30	..	30	450



The figure (and also the lower line of the tabulated form) shows the drop for each thirty-foot rail length. For shorter lengths of run-off, the drop for each 30 feet is shown by the corresponding lines in the tabular form. Note in each horizontal line that the sum of the drops, under which 30 is found, equals the total superelevation as found in the first column. For example, for 4 inches superelevation, length of curve 240 feet, the successive drops are $\frac{1}{4}''$, $\frac{1}{2}''$, $\frac{7}{8}''$, $\frac{7}{8}''$, $\frac{5}{8}''$, $\frac{1}{2}''$, $\frac{1}{4}''$, and $\frac{1}{8}''$ whose sum is 4 inches. Possibly the more convenient form would be to indicate for each 30-foot point the actual superelevation of the outer rail, which would be for the above case (running from the tangent to the curve) $\frac{1}{8}''$, $\frac{3}{8}''$, $\frac{7}{8}''$, $1\frac{1}{2}''$, $2\frac{3}{8}''$, $3\frac{1}{4}''$, $3\frac{3}{4}''$, 4″.

44. Fundamental principle of transition curves. If a curve has variable curvature, beginning at the tangent with a curve of infinite radius, and the curvature gradually sharpens until it equals the curvature of the required simple curve and there becomes tangent to it, the superelevation of such a transition curve may begin at zero at the tangent, gradually increase to the required superelevation for the simple curve, and yet have at every point the superelevation required by the curvature at that point. Since in Eq. (30) e is directly proportional to D, the required curve must be one in which the degree of curve increases directly as the distance along the curve. The mathematical development of such a curve is quite complicated. It has, however, been developed, and tables have been computed for its use, by Prof. C. L. Crandall. The following method has the advantage of great simplicity, while its agreement with the true transition curve is as close as need be, as will be shown.

45. Multiform compound curves. If the transition curve commences with a very flat curve and at regular even chord lengths compounds into a curve of sharper curvature until the desired curvature is reached, the increase in curvature at each chord point being uniform, it is plain that such a curve is a close approximation to the true spiral, especially since the rails as laid will *gradually* change their curvature rather than maintain a uniform curvature throughout each chord length and then abruptly change the curvature at the chord points. Such a curve, *as actually laid*, will be a much closer approximation to the true curve than the multiform compound curve by which it is set out. There will actually be a *gradual* increase in curvature which increases directly as the length of the curve.

46. Required length of spiral. The required length of spiral evidently depends on the amount of superelevation to be gained, and also depends somewhat on the speed. If the spiral is laid off in 25-foot chord lengths, with the first chord subtending a 1° curve, the second a 2° curve, etc., the fifth chord will subtend a 5° curve, and the increase from this last chord to a 6° curve is the same as the uniform increase of curvature between the chords. The same spiral extended would run on to a 12° curve in $(12-1)25=275$ feet. The last chord of a spiral should have a smaller degree of curvature than the simple curve to which it is joined. If the curves are very sharp, such as are used in street work and even in suburban trolley work, an increase in degree of curvature of 1° per 25 feet will not be sufficiently rapid, as such a rate would require too long curves. 2°, 10°, or even 20° increase per 25 feet may be necessary, but then the chords should be reduced to 5 feet. Such a rapid rate of increase is justified by the necessary reduction in speed. On the other hand, very high speed will make a lower rate of increase desirable, and therefore a spiral whose degree of curvature increases only 0° 30′ per 25 feet may be used. Such a spiral would require a length of 375 feet to run on to an 8° curve, which is inconveniently long, but it might be used to run on to a 4° curve, where its length would be only 175 feet. Three spirals have been developed in Table IV, each with chords of 25 feet, the rate of increase in the degree of curvature being 0° 30′, 1° and 2° per chord. One of these will be suitable for any curvature found on ordinary steam-railroads.

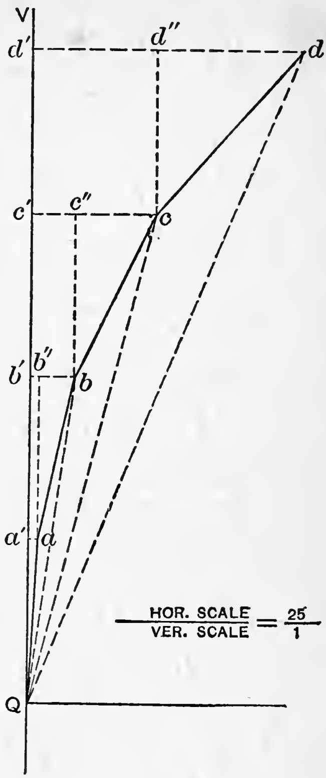


FIG. 31.

47. To find the ordinates of a 1°-per-25-feet spiral. Since the first chord subtends a 1° curve, its central angle is 0° 15′ and the angle aQV (Fig. 31) is 7′ 30″. The tangent at a makes an angle of 15′ with VQ. The angle between the chord ba and the tangent at a is $\frac{1}{2}(30') = 15'$, and the angle $bab'' = \frac{1}{2}(30') + 15' = 30'$. Similarly

$$\text{the angle } cbc'' = \tfrac{1}{2}(45') + 30' + 15' = 67'\,30'' = 1^\circ\,07'\,30'',$$

and the angle $dcd'' = 2^\circ\,0'$.

The ordinate $aa' = 25 \sin 7'\,30''$, and

$$\begin{aligned} Qa' &= 25 \cos 7'\,30''. \\ Qb' &= Qa' + a'b' \\ &= Qa' + ab'' \\ &= 25\ (\cos 7'\,30'' + \cos 30'). \\ bb' &= b'b'' + bb'' \\ &= 25\ (\sin 7'\,30'' + \sin 30'). \end{aligned}$$

Similarly the ordinates of c, d, etc., may be obtained.

48. To find the deflections from any point of the spiral. $aQV = 7' \, 30''$. Tan $bQV = bb' \div Qb'$; tan $cQV = cc' \div Qc'$; etc. Thus we are enabled to find the deflection angles from the tangent at Q to any point of the spiral.

The tangent to the curve at c (Fig. 32) makes an angle of

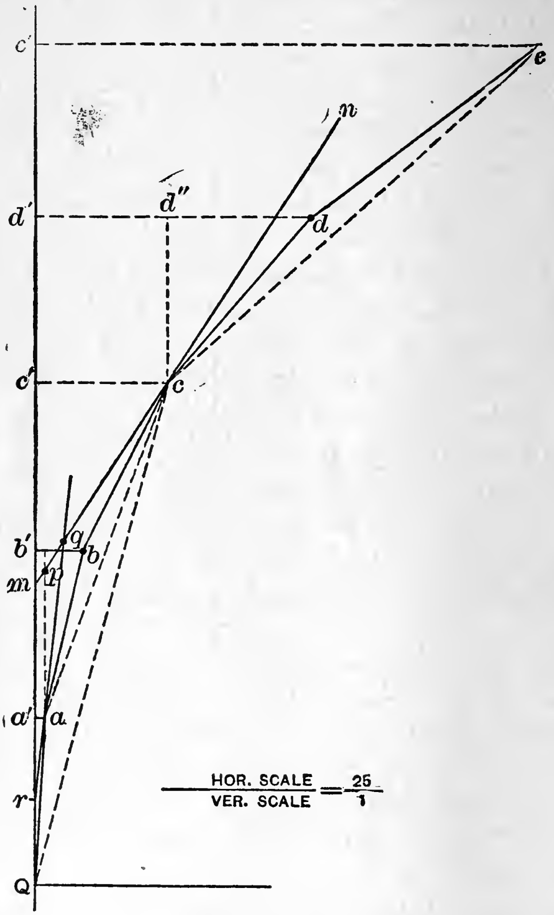


Fig. 32.

$1° \, 30'$ with QV, or $cmV = 1° \, 30'$. $Qcm = cmV - cQm$. The value of cQm is known from previous work. The deflection from c to Q then becomes known.

$acm = cmV - cap = cmV - caq - qap$. caq is the deflection angle to c from the tangent at a and will have been previously computed numerically. $qap = 15'$. acm therefore becomes known.

$$bcm = \tfrac{1}{2} \text{ of } 45' = 22' \, 30'';$$
$$dcn = \tfrac{1}{2} \text{ of } 60' = 30'.$$

$ecn = ecd'' - ncd''$, $ncd'' = cmV$, $\tan\ ecd'' = (ee' - d''d') \div c'e'$, all of which are known from the previous work.

By this method the deflections from the tangent at any point

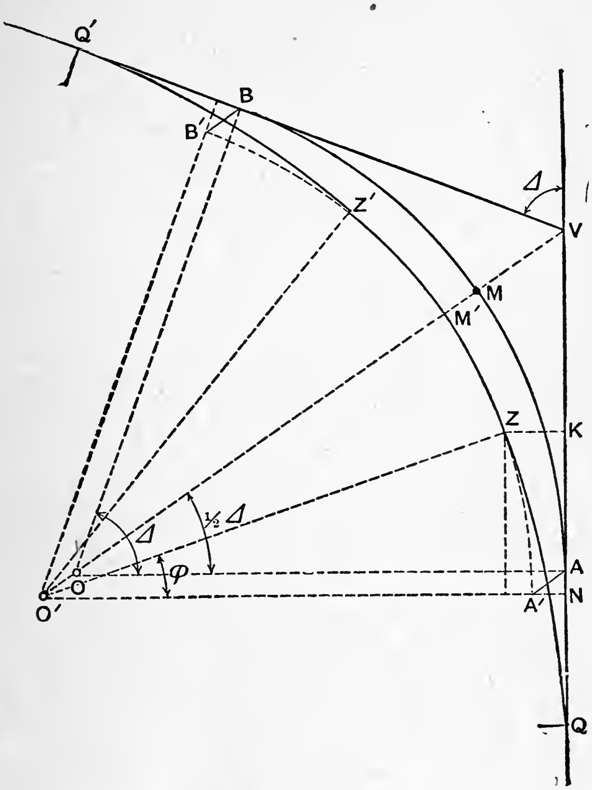


FIG. 33.

of the curve to any other point are determinable. These values are compiled in Table IV. The corresponding values of these angles when the increase in the degree of curvature per chord length is 30′, and when it is 2°, are also given in Table IV.

49. Connection of spiral with circular curve and with tangent. See Fig. 33.* Let AV and BV be the tangents to be connected

* The student should at once appreciate the fact of the necessary distortion of the figure. The distance MM' in Fig. 33 is perhaps 100 times its real proportional value.

by a $D°$ curve, having a suitable spiral at each end. If no spirals were to be used, the problem would be solved as in simple curves giving the curve AMB Introducing the spiral has the effect of throwing the curve away from the vertex a distance MM' and reducing the central angle of the $D°$ curve by 2ϕ. Continuing the curve beyond Z and Z' to A' and B', we will have $AA'=BB'=MM'$. $ZK=$the x ordinate and is therefore known. Call $MM'=m$. $A'N=x-R \text{ vers } \phi$. Then

$$m=MM'=AA'=\frac{A'N}{\cos \frac{1}{2}\Delta}=\frac{x-R \text{ vers } \phi}{\cos \frac{1}{2}\Delta}. \quad \ldots \quad (33)$$

$$\begin{aligned} NA &= AA' \sin \tfrac{1}{2}\Delta=(x-R \text{ vers } \phi) \tan \tfrac{1}{2}\Delta. \\ VQ &= QK-KN+NA+AV \\ &= y-R \sin \phi+(x-R \text{ vers } \phi) \tan \tfrac{1}{2}\Delta+R \tan \tfrac{1}{2}\Delta \\ &= y-R \sin \phi+x \tan \tfrac{1}{2}\Delta+R \cos \phi \tan \tfrac{1}{2}\Delta. \quad \ldots \quad (34) \end{aligned}$$

When $A'N$ has already been computed, it may be more convenient to write

$$VQ=y+R(\tan \tfrac{1}{2}\Delta-\sin \phi)+A'N \tan \tfrac{1}{2}\Delta. \quad \ldots \quad (35)$$

$$\begin{aligned} VM' &= VM+MM' \\ &= R \text{ exsec } \tfrac{1}{2}\Delta+\frac{x}{\cos \frac{1}{2}\Delta}-\frac{R \text{ vers } \phi}{\cos \frac{1}{2}\Delta}. \quad \ldots \quad (36) \end{aligned}$$

$$\begin{aligned} AQ &= VQ-AV \\ &= y-R \sin \phi+(x-R \text{ vers } \phi) \tan \tfrac{1}{2}\Delta. \quad \ldots \quad (37) \end{aligned}$$

Example. To join two tangents making an angle of 34° 20′ by a 5° 40′ curve and suitable spirals. Use 1°-per-25-feet spirals with five chords. Then $\phi=3° 45'$, $x=2.999$, $\frac{1}{2}\Delta=17° 10'$, and $y=124.942$.

[Eq. 33]			R	3.00497
			vers ϕ	7.33063
		2.166		0.33560
		$x=$ 2.999		
		$A'N=$ 0.833		$9.9206\bar{4}$
			$\cos \frac{1}{2}\Delta$	9.98021
	$m=MM'=AA'$	$=$ 0.872		$9.9404\bar{3}$
[Eq. 36]			R	3.00497
			exsec $\frac{1}{2}\Delta$	8.66863
		$VM=$47.164		$1.6736\bar{0}$
		$m=$ 0.872		
		$VM'=$48.036		
[Eq. 35]	$y=$124.942	nat. $\tan \frac{1}{2}\Delta=$ .30891		
		nat. $\sin \phi=$ .06540		
		.24351		$9.3865\bar{1}$
			R	3.00497
	246.314			$2.3914\bar{8}$
		[See above]	$A'N$	9.92064
			$\tan \frac{1}{2}\Delta$	9.48984
	0.257		AN	$9.4104\bar{8}$
	$VQ=$371.513			
[Eq. 37]			R	3.00497
			$\tan \frac{1}{2}\Delta$	9.48984
	312.471		AV	2.49481
	$AQ=$ 59.042			

50. Field-work. When the spiral is designed during the original location, the tangent distance VQ should be computed and the point Q located. It is hardly necessary to locate all of the points of the spiral until the track is to be laid. The extremities should be located, and as there will usually be one and perhaps two full station points on the spiral, these should also be located. Z may be located by setting off $QK=y$ and $KZ=x$, or else by the tabular deflection for Z from Q and the distance ZQ, which is the long chord. Setting up the instrument at Z and sighting back at Q with the proper deflection, the tangent at Z may be found and the circular curve located as usual, its central angle being $\Delta-2\phi$. A similar operation will locate Q' from Z'.

To locate points on the spiral. Set up at Q, with the plates reading 0° when the telescope sights along VQ. Set off from Q the deflections given in Table IV for the instrument at Q, using a chord length of 25 feet, the process being like the method for simple curves except that the deflections are irregular. If a full station-point occurs within the spiral, interpolate between the deflections for the adjacent spiral-points. For example, a spiral begins at Sta. 56+15. Sta. 57 comes 10 feet beyond the third spiral point. The deflection for the third point is 35′ 0″; for the fourth it is 56′ 15″. $\frac{10}{25}$ of the difference (21′ 15″) is 8′ 30″; the deflection for Sta. 57 is therefore 43′ 30″. This method is not theoretically accurate, but the error is small. Arriving at Z, the forward alignment may be obtained by sighting back at Q (or at any other point) with the given deflection

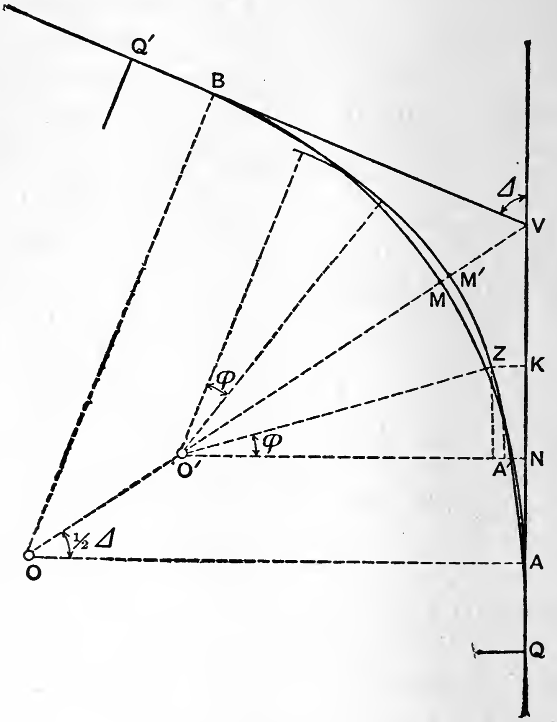


FIG. 34.

for that point from the station occupied. Then when the plates read 0° the telescope will be tangent to the spiral and to the succeeding curve. All rear points should be checked from Z. If it is necessary to occupy an intermediate station, use the deflections given for that station, orienting as just explained for Z, checking the back points and locating all forward points up to Z if possible.

After the center curve has been located and Z' is reached, the other spiral must be located but *in reverse order*, i.e., the sharp curvature of the spiral is at Z' and the curvature decreases toward Q'.

51. To replace a simple curve by a curve with spirals. This *may* be done by the method of § 49, but it involves shifting the whole track a distance m, which in the given example equals 0.87 foot. Besides this the track is appreciably shortened, which would require rail-cutting. But the track may be kept at practically the same length and the lateral deviation from the old track may be made very small by slightly sharpening the curvature of the old track, moving the new curve so that it is wholly or partially *outside* of the old curve, the remainder of it with the spirals being *inside* of the old curve. It is found by experience that a decrease in radius of from 1% to 5% will answer the purpose. The larger the central angle the less the change. The solution is as indicated in Fig. 34.

$$
\begin{aligned}
O'N &= R' \cos \phi + x. \\
O'V &= O'N \sec \tfrac{1}{2}\Delta \\
&= R' \cos \phi \sec \tfrac{1}{2}\Delta + x \sec \tfrac{1}{2}\Delta. \\
m &= MM' = MV - M'V \\
&= R \operatorname{exsec} \tfrac{1}{2}\Delta - (O'V - R') \\
&= R \operatorname{exsec} \tfrac{1}{2}\Delta - R' \cos \phi \sec \tfrac{1}{2}\Delta - x \sec \tfrac{1}{2}\Delta + R'. \quad . \quad . \quad . \quad . \qquad (38) \\
AQ &= QK - KN + NV - VA \\
&= y - R' \sin \phi + (R' \cos \phi + x) \tan \tfrac{1}{2}\Delta - R \tan \tfrac{1}{2}\Delta \\
&= y - R' \sin \phi + R' \cos \phi \tan \tfrac{1}{2}\Delta - (R - x) \tan \tfrac{1}{2}\Delta. \quad . \quad . \quad . \qquad (39)
\end{aligned}
$$

The length of the old curve from Q to $Q' = 2AQ + 100\dfrac{\Delta}{D}$.

The length of the new curve from Q to $Q' = 2L + 100\dfrac{\Delta - 2\phi}{D'}$,

in which L is the length of each spiral.

Example. Suppose the old curve is a 7° 30′ curve with a central angle of 38° 40′. As a trial, compute the relative length of a new 8° curve with spirals of seven chords. $\phi = 7° 0'$; $\tfrac{1}{2}\Delta = 19° 20'$; R (for the 7° 30′ curve) $= 764.489$; R' (for the 8° curve) $= 716.779$; $x = 7.628$.

[Eq. 38]			R	2.88337
			exsec $\frac{1}{2}\Delta$	8.77642
	45.687			1.65979
	$R'=$716.779			
	762.466		R'	2.85538
			cos ϕ	9.99675
			sec $\frac{1}{2}\Delta$	0.02521
		753.953		2.87734
			x	0.88241
			sec $\frac{1}{2}\Delta$	0.02521
		8.084		0.90762
	762.037	762.037		
	$m=$ 0.429			
[Eq. 39]	$y=$174.722		R'	2.85538
			sin ϕ	9.08589
		87.353		1.94128
			R'	2.85538
			cos ϕ	9.99675
			tan $\frac{1}{2}\Delta$	9.54512
	249.606			2.39725
			$R=$764.489	
			$x=$ 7.628	
			756.861	2.87901
			tan $\frac{1}{2}\Delta$	9.54512
		265.543		2.42413
	424.328	352.896		
	352.896			
	$AQ=$71.432			

The length of the old curve from Q to Q' is

$$100\frac{\Delta}{D} = 100\frac{38.667}{7.5} = \quad\ldots\ldots\quad 515.556$$

$$2AQ = 2 \times 71.432 = \quad\ldots\ldots\quad 142.864$$

$$658.420$$

$$\text{New curve: } 100\frac{\Delta - 2\phi}{D'} = 100\frac{38.667 - 14.000}{8.0} = 308.333$$

$$2L = 2 \times 175 = 350.000$$

$$658.333 \qquad 658.333$$

$$\text{Difference in length} = 0.087$$

Considering that this difference may be divided among 22 joints (using 30-foot rails) no rail-cutting would be necessary. If the difference is too large, a slight variation in the value of the new radius R' will reduce the difference as much as necessary. A truer comparison of the lengths would be found by comparing the lengths of the arcs.

52. Application of transition curves to compound curves. Since compound curves are only employed when the location is limited by local conditions, the elements of the compound curve should be determined (as in §§ 38 and 39) regardless of the transition curves, depending on the fact that the lateral shifting of the curve when transition curves are introduced is very small. If the limitations are very close, an estimated allowance may be made for them.

Methods have been devised for inserting transition curves between the branches of a compound curve, but the device is

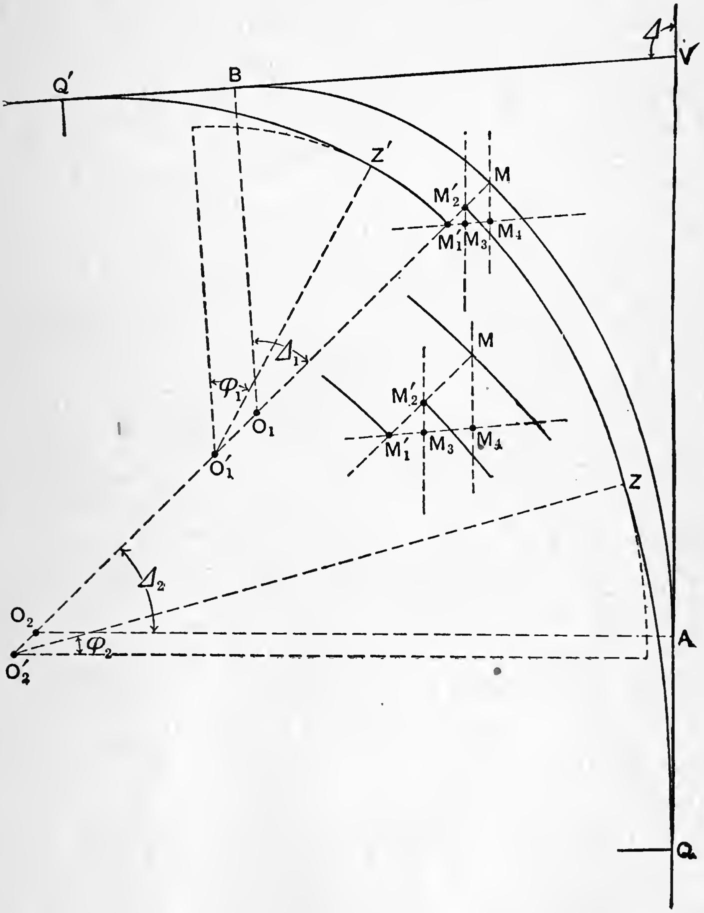


FIG. 35.

complicated and usually needless, since when the train is once on a curve the wheels press against the outer rail steadily and a change in curvature will not produce a serious jar even though the superelevation is temporarily a little more or less than it should be.

If the easier curve of the compound curve is less than 3° or 4°, there may be no need for a transition curve off from that branch. This problem then has two cases according as transition curves are used at both ends or at one end only.

a. *With transition curves at both ends.* Adopting the method of § 49, calling $\Delta_1=\frac{1}{2}\Delta$, we may compute $m_1=MM_1'$. Similarly, calling $\Delta_2=\frac{1}{2}\Delta$, we may compute $m_2=MM_2'$. But M_1' and M_2' must be made to coincide. This may be done by moving the curve $Z'M_1'$ and its transition curve parallel to $Q'V$ a distance $M_1'M_3$, and the other curve parallel to QV a distance $M_2'M_3$. In the triangle $M_1'M_3M_2'$, the angle at $M_1'=90°-\Delta_1$, the angle at $M_2'=90°-\Delta_2$, and the angle at $M_3=\Delta$.

$$\left.\begin{aligned}\text{Then}\quad M_1'M_3&=M_1'M_2'\frac{\sin(90°-\Delta_2)}{\sin\Delta}=(m_1-m_2)\frac{\cos\Delta_2}{\sin\Delta}.\\ \text{Similarly}\quad M_2'M_3&=M_1'M_2'\frac{\sin(90°-\Delta_1)}{\sin\Delta}=(m_1-m_2)\frac{\cos\Delta_1}{\sin\Delta}.\end{aligned}\right\}\quad(40)$$

b. *With a transition curve on the sharper curve only.* Compute $m_1=MM_1'$ as before; then move the curve Z_1M_1' parallel to $Q'V$ a distance of

$$M_1'M_4=m_1\frac{\cos\Delta_2}{\sin\Delta}.\quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

The simple curve MA is moved parallel to VA a distance of

$$MM_4=m_1\frac{\cos\Delta_1}{\sin\Delta}.\quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

If Δ_1 and Δ_2 are both small, $M_1'M_4$ and MM_4 may be more than m_1, but the lateral deviation of the new curve from the old will always be less than m_1.

53. To replace a compound curve by a curve with spirals. The solution is somewhat analogous to that of § 51. Compute m_1 for the sharper branch of the curve, placing $\Delta_1=\frac{1}{2}\Delta$ in Eq. 38. Since m_1 and m_2 for the two branches of the curve must be identical, a value for R_2' must be found which will satisfy the determined value of $m_2=m_1$. Solving Eq. 38 for R', we obtain

$$R'=\frac{R\,\text{vers}\,\frac{1}{2}\Delta-m\cos\frac{1}{2}\Delta-x}{\cos\phi-\cos\frac{1}{2}\Delta}.\quad . \quad . \quad . \quad . \quad . \quad (43)$$

Substituting in this equation the known value of m_1 $(=m_2)$ and calling $R'=R_2'$, $R=R_2$, and $\Delta_2=\frac{1}{2}\Delta$, solve for R_2'. Obtain the value of AQ for each branch of the curve separately by Eq. 39, and compare the lengths of the old and new lines.

Example. Assume a compound curve with $D_1=8°$, $D_2=4°$, $\Delta_1=36°$, and $\Delta_2=32°$. Use 1°-per-25-feet spirals; $\phi_1=7°\ 0'$; $\phi_2=1°\ 30'$. Assume that the sharper curve is sharpened from 8° 0′ to 8° 12′.

[Eq. 38]			R_1	$2.8553\bar{8}$
			exsec 36°	9.37303
	169.209	. . . .		2.22842
	$R_1'=699.326$			
	868.535		R_1'	2.84468
			$\cos\phi_1$	9.99675
			$\sec\Delta_1$	0.09204
		857.970		2.93347
			x_1	0.88241
			$\sec\Delta_1$	0.09204
		9.429		0.97445
	867.399	867.399		
	$m_1=$ 1.136			
[Eq. 43]			R_2	3.15615
			vers 32°	$9.1817\bar{0}$
	217.700	. . . .		$2.3378\bar{5}$
			$m_1=1.136$	0.05538
			cos 32°	9.92842
		0.963		9.98380
		$x_2=0.763$		
	1.726	1.726		
	215.974	. . . .		2.33440
			nat. cos ϕ = .99966	
			nat. cos Δ_2 = .84805	
			.15161	9.18073
	$R_2'=1424.54$	[4° 1′ 22″]		3.15367
[Eq. 39]	$y_1=174.722$		R_1'	2.84468
			$\sin\phi_1$	$9.0858\bar{9}$
		85.226		$1.9305\bar{7}$
			R_1'	2.84468
			$\cos\phi_1$	9.99675
			$\tan\frac{1}{2}\Delta$ $[\Delta_1=36°]$	9.86126
	504.302	. . . .		2.70269
			$R_1=716.779$	
			$x_1=$ 7.628	
			709.151	$2.8507\bar{4}$
			$\tan\frac{1}{2}\Delta$	9.86126
	679.024			
	600.461	515.235		2.71200
	$AQ_1=$ 78.563	600.461		

[Eq. 39]		R_2'	$3.1536\bar{7}$
$y_2 =$ 74.994		$\sin\phi_2$	8.41792
	37.290	. . . .	$1.5715\bar{9}$
		R_2'	3.15367
		$\cos\phi_2$	9.99985
		$\tan\frac{1}{2}\Delta(\Delta_2=32°)$	9.79579
	889.843	. . . .	$2.9493\bar{1}$
		$R_2 =$ 1432.69	
		$x_2 =$ 0.76	
		1431.93	3.15592
		$\tan\frac{1}{2}\Delta$	9.79579
	894.770	. . . .	2.95171
964.837	932.060		
932.060			
$AQ_2 =$ 32.777			

For the length of the old track we have :

$$100\frac{\Delta_1}{D_1} = 100\frac{36°}{8°} = 450.$$

$$100\frac{\Delta_2}{D_2} = 100\frac{32°}{4°} = 800.$$

$$AQ_1 = 78.563$$
$$AQ_2 = 32.777$$
$$1361.340$$

For the length of the new track we have:

$100\frac{\Delta_1-\phi_1}{D_1'} = 100\frac{29°}{8°.20} =$		353.659
$100\frac{\Delta_2-\phi_2}{D_2'} = 100\frac{30°.5}{4°.023} =$		758.140
Spiral on 8° 12′ curve		175.000
" " 4° 01′ 22″ "		75.
Length of new track	=	1361.799
" " old "	=	1361.340
Excess in length of new track	=	0.459 feet.

Since the new track is slightly longer than the old, it shows that the new track runs too far *outside* the old track at the *P.C.C.* On the other hand the offset m is only 1.136. The maximum amount by which the new track comes *inside* of the old track at two points, presumably not far from Z' and Z, is very difficult to determine exactly. Since it is desirable that the maximum offsets (inside and outside) should be made as nearly equal as possible, this feature should not be sacrificed to an effort to make the two lines of precisely equal length so that the rails need not be cut. Therefore, if it is found that the offsets inside the old track are nearly equal to m (1.136), the above figures should stand. Otherwise m may be diminished (and the above excess in length of track diminished) by *increasing* R_1' very slightly and making the necessary consequent changes.

VERTICAL CURVES.

54. Necessity for their use. Whenever there is a change in the rate of grade, it is necessary to eliminate the angle that would be formed at the point of change and to connect the two grades by a curve. This is especially necessary at a sag between two grades, since the shock caused by abruptly forcing an upward motion to a rapidly moving heavy train is very severe both to the track and to the rolling stock. The necessity for vertical curves was even greater in the days when link couplers were in universal use and the "slack" in a long train was very great. Under such circumstances, when a train was moving down a heavy grade the cars would crowd ahead against the engine. Reaching the sag, the engine would begin to pull out, rapidly taking out the slack. Six inches of slack on each car would amount to several feet on a long train, and the resulting jerk on the couplers, especially those near the rear of the train, has frequently resulted in broken couplers or even derailments. A vertical curve will practically eliminate this danger if the curve is made long enough, but the rapidly increasing adoption of close spring couplers and air-brakes, even for freight trains, is obviating the necessity for such very long curves.

55. Required length. Theoretically the length should depend on the change in the rate of grade and on the length of the longest train on the road. A sharp change in the rate of grade requires a long curve; a long train requires a long curve; but since the longest trains are found on roads with light grades and small changes of grade, the required length is thus somewhat equalized. It has been claimed that a total curve length equal to one-third of the train length for each *tenth* of a per cent of change of rate of grade will *certainly* prevent the rear of the train from crowding against the cars in front, but such a length is admittedly excessive. Half of this length is probably ample and one-fourth of it is probably safe. Therefore, we may say, taking the even fraction $\frac{1}{10}$ rather than $\frac{1}{12}$,

length of vertical curve = (length of longest train) × (change of rate of grade in per cent).

For example, assume a change of rate of grade of 2%; assume that the longest train will be about 720 feet. Then, by the above rule, the length of curve should be $720 \times 2 = 1440$ feet. Such rules are seldom if ever applied except in the most approximate way. On many roads a uniform length of only 400 feet is adopted for all vertical curves. The required length over a hump is certainly much less than that through a sag. Added length increases the amount of earthwork required both in cuts and fills, but the resulting saving in operating expenses will always justify a considerable increase.

56. Form of curve. In Fig. 36 assume that A and C, equi-

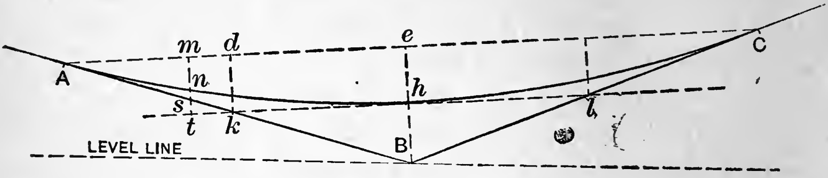


FIG. 36.

distant from B, are the extremities of the vertical curve. Bisect AC at e; draw Be and bisect it at h. Bisect AB and BC at k and l. The line kl will pass through h. A parabola may be drawn with its vertex at h which will be tangent to AB and BC at A and C. It may readily be shown* from the properties of a parabola that if an ordinate be drawn at *any* point (as at n) we will have

$$sn : eh \text{ (or } hB) : : \overline{Am}^2 : \overline{Ae}^2,$$

or

$$sn = eh\frac{\overline{Am}^2}{\overline{Ae}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

In Fig. 36 the grades are necessarily exaggerated enormously. With the proportions found in practice we may assume that ordinates (such as mt, eB, etc.) are perpendicular to either grade, as may suit our convenience, without any appreciable error. In the numerical case given below, the variation of these ordinates from the vertical is 0° 07′, while the effect of this variation on the calculations in this case (as in the most extreme cases) is absolutely inappreciable. It may easily be shown that the angle CAB = half the *algebraic difference* of the rates of grade. Call the difference, expressed in per cent of grade, r; then $CAB = \frac{1}{2}r$. Let l = length (in "stations" of 100 feet) of the line AC, which is practically equal to the horizontal

* See note at foot of p. 63.

measurement. Since the angle CAB is one-half the total change of grade at B, it follows that $Be = \frac{1}{2}l \times \frac{1}{2}r$ Therefore

$$Bh = \tfrac{1}{8}lr. \quad \ldots \ldots \ldots \quad (44a)$$

Since Bh (or eh) are constant for any one curve, the correction sn at any point (see Eq. 44) equals a constant times Am^2.

57. **Numerical example.** Assume that B is located at Sta. 16+20; that the curve is to be 1200 feet long; that the grade of AB is -0.8%, and of $BC + 1.2\%$; also that the elevation of B above the datum plane is 162.6. Then the algebraic difference of the grades, r, $=1.2-(-0.8)=2.0$; $l=12$. $Bh=\frac{1}{8}lr = \frac{1}{8}\times 12\times 2 = 3.0$. A is at Sta. 10+20 and its elevation is $162.6+(6\times 0.8)=167.4$; C is at Sta. 22+20 and its elevation is $162.6+(6\times 1.2)=169.8$. The elevation of Sta. 11 is found by adding sn to the elevation of s on the straight grade line. The constant $(eh \div \overline{Ae}^2)$ equals in this case $3.0 \div 600^2 = \frac{1}{120000}$. Therefore the curve elevations are

A,	Sta. 10+20,	$162.6+(6.00\times 0.8)$	$=167.40$
	11	$167.4-(\ .80\times 0.8)+\frac{1}{120000}\ 80^2$	$=166.81$
	12	$167.4-(1.80\times 0.8)+\frac{1}{120000}\ 180^2$	$=166.23$
	13	$167.4-(2.80\times 0.8)+\frac{1}{120000}\ 280^2$	$=165.81$
	14	$167.4-(3.80\times 0.8)+\frac{1}{120000}\ 380^2$	$=165.56$
	15	$167.4-(4.80\times 0.8)+\frac{1}{120000}\ 480^2$	$=165.48$
	16	$167.4-(5.80\times 0.8)+\frac{1}{120000}\ 580^2$	$=165.56$
B,	16+20,	$162.6+3.0$	$=165.60$
	17	$169.8-(5.20\times 1.2)+\frac{1}{120000}\ 520^2$	$=165.81$
	18	$169.8-(4.20\times 1.2)+\frac{1}{120000}\ 420^2$	$=166.23$
	19	$169.8-(3.20\times 1.2)+\frac{1}{120000}\ 320^2$	$=166.81$
	20	$169.8-(2.20\times 1.2)+\frac{1}{120000}\ 220^2$	$=167.56$
	21	$169.8-(1.20\times 1.2)+\frac{1}{120000}\ 120^2$	$=168.48$
	22	$169.8-(\ .20\times 1.2)+\frac{1}{120000}\ 20^2$	$=169.56$
C,	22+20,	$162.6+(6.00\times 1.2)$	$=169.80$

DEMONSTRATION OF EQ. 44.

The general equation of a parabola passing through the point n (Fig. 36) may be written

$$y^2 + y_n^2 = 2p(x + x_n);$$

from which

$$x_n = \frac{y^2}{2p} + \frac{y_n^2}{2p} - x.$$

When $x = x_A$, $y = y_A$, and we have

$$x_n = \frac{y_A^2}{2p} + \frac{y_n^2}{2p} - x_A.$$

The general equation of a tangent passing through the point A may be written

$$yy_A = p(x + x_A),$$

from which
$$x = \frac{yy_A}{p} - x_A.$$

When $x = x_s$, $y = y_s[= y_n]$, and we have

$$x_s = \frac{y_n y_A}{p} - x_A.$$

$$\overline{sn} = x_n - x_s = \frac{y_A^2 + y_n^2 - 2y_n y_A}{2p}$$

$$= \frac{(y_A - y_n)^2}{2p} = \frac{\overline{Am}^2}{2p},$$

$$2p = \frac{y_A^2}{x_A} = \frac{\overline{Ae}^2}{\overline{eh}}.$$

$$\therefore \quad \overline{sn} = \overline{eh}\frac{\overline{Am}^2}{\overline{Ae}^2}.$$

This proves the general proposition that if secants are drawn parallel to the axis of x, intersecting a parabola and a tangent to it, the intercepts between the tangent and the parabola are proportional to the square of the distances (measured parallel to y) from the tangent point.

CHAPTER III.

EARTHWORK.

FORM OF EXCAVATIONS AND EMBANKMENTS.

58. Usual form of cross-section in cut or fill. The normal form of cross-section in cut is as shown in Fig. 37, in which $e \ldots g$ represents the natural surface of the ground, no matter

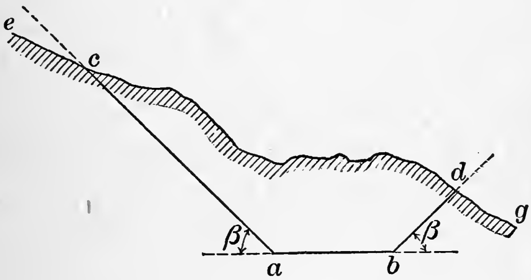


FIG. 37.

how irregular; ab represents the position and width of the required roadbed; ac and bd represent the "side slopes" which begin at a and b and which intersect the natural surface at such

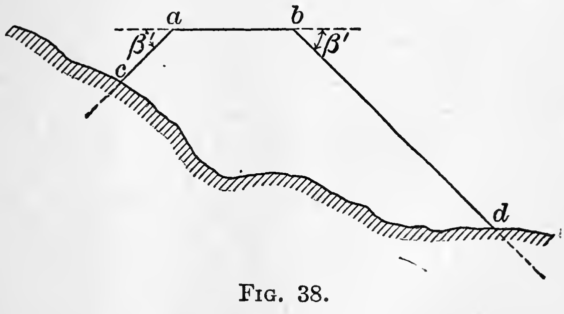


FIG. 38.

points (c and d) as will be determined by the required slope angle (β).

The normal section in fill is as shown in Fig. 38. The points c and d are likewise determined by the intersection of the required side slopes with the natural surface. In case the required roadbed (*ab* in Fig. 39) intersects the natural surface, both cut

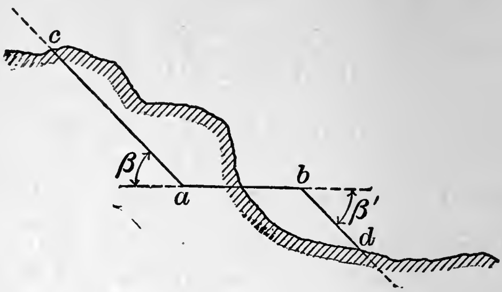


FIG. 39.

and fill are required, and the points *c* and *d* are determined as before. Note that β and β' are not necessarily equal. Their proper values will be discussed later.

59. Terminal pyramids and wedges. Fig. 40 illustrates the general form of cross-sections when there is a transition from cut to fill. *a . . . g* represents the grade line of the road which

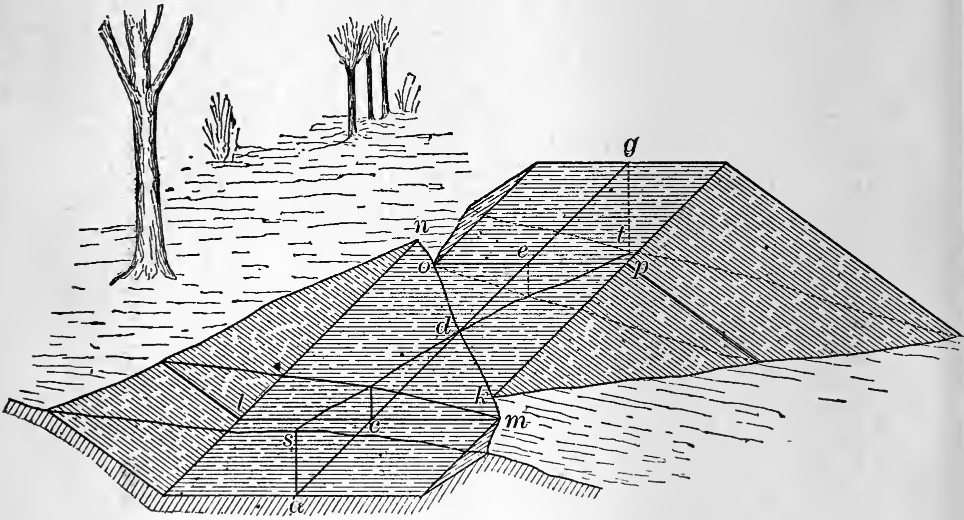


FIG. 40.

passes from cut to fill at *d*. *sdt* represents the surface profile. A cross-section taken at the point where either side of the roadbed *first* cuts the surface (the point *m* in this case) will usually be triangular if the ground is regular. A similar cross-section should be taken at *o*, where the other side of the roadbed cuts the surface. In general the earthwork of cut and fill terminates in two pyramids. In Fig. 40 the pyramid vertices are at *n* and *k*, and the bases are *lhm* and *opq*. The roadbed is generally wider in cut than in fill, and therefore the section *lhm* and the altitude *ln* are generally greater than the section *opq* and the altitude *pk*. When the line of intersection of the roadbed and natural surface (*nodkm*) becomes perpendicular to the axis of the roadbed (*ag*) the pyramids become wedges whose bases are the nearest convenient cross-sections.

60. Slopes. a. Cuttings. The required slopes for cuttings vary from perpendicular cuts, which may be used in hard rock which will not disintegrate by exposure, to a slope of perhaps 4 horizontal to 1 vertical in a soft material like quicksand or in a clayey soil which flows easily when saturated. For earthy materials a slope of 1 : 1 is the maximum allowable, and even this should only be used for firm material not easily affected by saturation. A slope of $1\frac{1}{2}$ horizontal to 1 vertical is a safer slope for average earthwork. It is a frequent blunder that slopes in cuts are made too steep, and it results in excessive work in clearing out from the ditches the material that slides down, at a much higher cost per yard than it would have cost to take it out at first, to say nothing of the danger of accidents from possible landslides.

b. Embankments. The slopes of an embankment vary from 1 : 1 to 1.5 : 1. A rock fill will stand at 1 : 1, and if some care is taken to form the larger pieces on the outside into a rough dry wall, a much steeper slope can be allowed. This method is sometimes a necessity in steep side-hill work. Earthwork embankments generally require a slope of $1\frac{1}{2}$ to 1. If made steeper at first, it generally results in the edges giving way, requiring repairs until the ultimate slope is nearly or quite $1\frac{1}{2}$: 1. The difficulty of incorporating the added material with the old embankment and preventing its sliding off frequently makes these repairs disproportionately costly.

61. Compound sections. When the cut consists partly of earth and partly of rock, a compound cross-section must be made. If borings have been made so that the contour of the rock surface is accurately known, then the true cross-section may be determined. The rock and earth should be calculated separately, and this will require an accurate knowledge of where the rock "runs out"—a difficult matter when it must be determined by boring. During construction the center part of the earth cut would be taken out first and the cut widened until a sufficient width of rock surface had been exposed so that the rock cut would have its proper width and side slopes. Then the earth slopes could be cut down at the proper angle. A "berm" of about three feet is usually left on the edges of the rock cut as

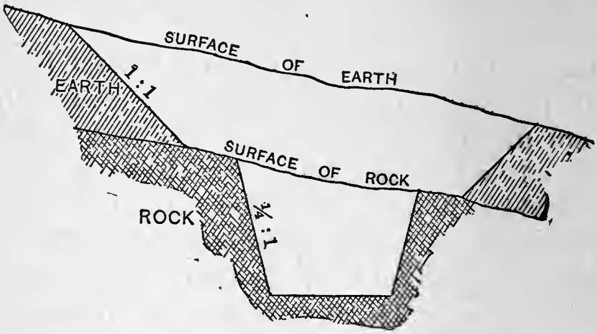


FIG. 41.

a margin of safety against a possible sliding of the earth slopes. After the work is done, the amount of excavation that has been made is readily computable, but accurate preliminary estimates are difficult. The area of the cross-section of earth in the figure must be determined by a method similar to that developed for borrow-pits (see § 89).

62. Width of roadbed. Owing to the large and often disproportionate addition to volume of cut or fill caused by the addition of even one foot to the width of roadbed, there is a natural tendency to reduce the width until embankments become unsafe and cuts are too narrow for proper drainage. The cost of maintenance of roadbed is so largely dependent on the drainage of the roadbed that there is true economy in making an ample allowance for it. The practice of some of the leading railroads of the country in this respect is given in the following table, in which are also given some data belonging more properly to the subject of superstructure.

It may be noted from the table that the average width for an *earthwork* cut, single track, is about 24.7 feet, with a minimum of 19 feet 2 inches. The widths of fills, single track, average over 18 feet, with numerous minimums of 16 feet. The widths for double track may be found by adding the distance between track centers, which is usually 13 feet.

WIDTH OF ROADBED FOR SINGLE AND DOUBLE TRACK—SLOPE RATIOS—DISTANCES BETWEEN TRACK CENTERS.

Road.	Single Track.		Double Track.		Slope Ratios.		Distance Between Track Centers.
	Cut.	Fill.	Cut.	Fill.	Cut.	Fill.	
A., T. & Santa Fé	28′ earth	20			1 : 1	1.5 : 1	
	22′ rock				$\frac{1}{4}$: 1		
Chicago, Burlington & Quincy	14+(2×5)*	16	28+(2×5)	30	1.5 : 1	1.5 : 1	14′
Chicago, Milwaukee & St. Paul	18+(2×6)	20 to 24	31+(2×6)	33 to 37	1.5 : 1	1.5 : 1	13′
C., C., C. & St. Louis	20+(2×4)	20	33+(2×4)	33	1.5 : 1	1.5 : 1	13′
Illinois Central	32.5	18			1.5 : 1	1.5 : 1	
Erie	20′ 8½″	20′ 8½″	33′ 8½″	33′ 8½″	1.5 : 1	1.5 : 1	13′
Lehigh Valley	14+(2×3.5)	16	27+(2×3.5)	30	1 : 1	1.5 : 1	13′
Lake Shore & Michigan Southern			33+(2×7.25)	32	1.5 : 1	1.5 : 1	13′
Louisville & Nashville	13+(2×4.5)	16			1 : 1	1.5 : 1	
Michigan Central			(33+2×2.5)	33	1.5 : 1	1.5 : 1	13′
N. Y., N. H. & H.			30	30	1.5 : 1	1.5 : 1	12′
Norfolk & Western	21′ 2″ earth	17′ 2″	34′ 2″ earth	30′ 2′	1.5 : 1	1.5 : 1	13′
	16′ rock		29′ rock		$\frac{1}{4}$: 1 $\frac{1}{2}$: 1		13′
Pennsylvania	19′ 2″ light traffic	19′ 2″					
	27′ 2″ heavy "	19′ 2″	31′ 4″+(2×4)	31′ 4″	1.5 : 1	1.5 : 1	12′ 2″
Union Pacific	14+(2×3.5)	16			1 : 1	1.5 : 1	

* (2×5) signifies two ditches each 5 feet wide; the following cases should be interpreted similarly.

63. Form of subgrade. The stability of the roadbed depends largely on preventing the ballast and subsoil from becoming saturated with water The ballast must be porous so that it will not retain water, and the subsoil must be so constructed that it will readily drain off the rain-water that soaks through the ballast. This is accomplished by giving the subsoil a curved form, convex upward, or a surface made up of two or three planes, the two outer planes having a slope of about 1:24 (sometimes more and sometimes less, depending on the soil) and the middle plane, if three are used, being level. When a circular form is used, a crowning of 6 inches in a total width of 17 or 18 feet is generally used. Occasionally the subgrade is made level, especially in rock-cuts, but if the subsoil is previously compressed by rolling, as required on the N. Y. C. & H. R. R. R., or if the subsoil is drained by tile drains laid underneath the ditches, the necessity for slopes is not so great. Rock cuts are generally required to be excavated to one foot below subgrade and then filled up again to subgrade with the same material, if it is suitable.

64. Ditches. "The stability of the track depends upon the strength and permanence of the roadbed and structures upon which it rests; whatever will protect them from damage or prevent premature decay should be carefully observed. The worst enemy is WATER, and the further it can be kept away from the track, or the sooner it can be diverted from it, the better the track will be protected. Cold is damaging only by reason of the water which it freezes; therefore the first and most important provision for good track is drainage." (Rules of the Road Department, Illinois Central R. R.)

The form of ditch generally prescribed has a flat bottom 12″ to 24″ wide and with sides having a minimum slope, except in rock-work, of 1 : 1, more generally 1.5 : 1 and sometimes 2 : 1. Sometimes the ditches are made V-shaped, which is objectionable unless the slopes are low The best form is evidently that which will cause the greatest flow for a given slope, and this will evidently be the form in which the ratio of area to wetted perimeter is the largest. The semicircle fulfills this condition better than any other form, but the nearly vertical sides would be difficult to maintain. (See Fig. 42.) A ditch, with a flat bottom and such

FIG. 42.

slopes as the soil requires, which approximates to the circular form will therefore be the best.

When the flow will probably be large and at times rapid it will be advisable to pave the ditches with stone, especially if the soil is easily washed away. Six-inch tile drains, placed 2′ under the ditches, are prescribed on some roads. (See Fig. 43.) No better method could be devised to insure a dry subsoil. The ditches through cuts should be led off at the end of the cut so that the adjacent embankment will not be injured.

Wherever there is danger that the drainage from the land above a cut will drain down into the cut, a ditch should be made near the edge of the cut to intercept this drainage, and this ditch should be continued, and paved if necessary, to a point where the outflow will be harmless. Neglect of these simple and inexpensive precautions frequently causes the soil to be loosened on the shoulders of the slopes during the progress of a heavy rain, and results in a landslide which will cost more to repair than the ditches which would have prevented it for all time.

Ditches should be formed along the bases of embankments; they facilitate the drainage of water from the embankment, and may prevent a costly slip and disintegration of the embankment.

65. Effect of sodding the slopes, etc. Engineers are unanimously in favor of rounding off the shoulders and toes of embankments and slopes, sodding the slopes, paving the ditches, and providing tile drains for subsurface drainage, all to be put in during original construction. (See Fig. 43.) Some of the highest grade specifications call for the removal of the top layer of vegetable soil from cuts and from under proposed fills to some convenient place, from which it may be afterwards spread on the slopes, thus facilitating the formation of sod from grass-seed. But while engineers favor these measures and their economic value may be readily demonstrated, it is generally impossible to obtain the authorization of such specifications from railroad directors and promoters. The addition to the original cost of the roadbed is considerable, but is by no means as great as the capitalized value of the extra cost of maintenance resulting from the usual practice. Fig. 43 is a copy of designs * presented at a convention of the American Society of Civil Engineers by Mr. D. J. Whittemore, Past President of the Society and Chief Engineer of the Chi., Mil. & St. Paul

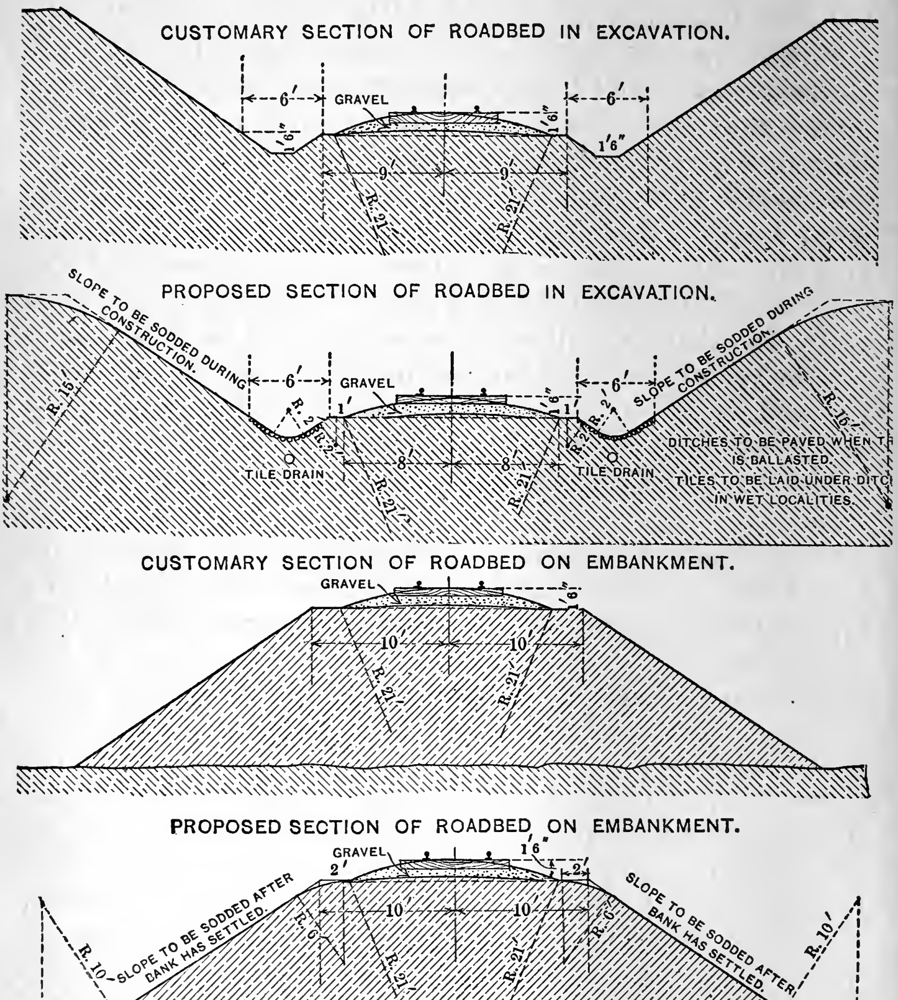


FIG. 43.—"WHITTEMORE ON RAILWAY EXCAVATION AND EMBANKMENTS" Trans. Am. Soc. C. E., Sept. 1894.

R. R. The "customary sections" represent what is, with some variations of detail, the practice of many railroads. The "pro-

* Trans. Am. Soc. Civil Eng., Sept. 1894.

posed sections" elicited unanimous approval. They should be adopted when not prohibited by financial considerations.

EARTHWORK SURVEYS.

66. Relation of actual volume to the numerical result. It should be realized at the outset that the accuracy of the result of computations of the volume of any given mass of earthwork has but little relation to the accuracy of the mere numerical work. The process of obtaining the volume consists of two distinct parts. In the first place it is assumed that the volume of the earthwork may be represented by a more or less complicated geometrical form, and then, secondly, the volume of such a geometrical form is computed. A desire for simplicity (or a frank willingness to accept approximate results) will often cause the cross-section men to assume that the volume may be represented by a very simple geometrical form which is really only a very rough approximation to the true volume. In such a case, it is only a waste of time to compute the volume with minute numerical accuracy. One of the first lessons to be learned is that economy of time and effort requires that the accuracy of the numerical work should be kept proportional to the accuracy of the cross-sectioning work, and also that the accuracy of both should be proportional to the use to be made of the results. The subject is discussed further in § 94.

67. Prismoids. To compute the volume of earthwork, it is necessary to assume that it has some geometric form whose volume is readily determinable. The general method is to consider the volume as consisting of a series of *prismoids*, which are solids having parallel plane ends and bounded by surfaces which may be formed by lines moving continuously along the edges of the bases These surfaces may also be considered as the surfaces generated by lines moving along the edges joining the corresponding points of the bases, these edges being the directrices, and the lines being always parallel to either base, which is a plane director. The surfaces thus developed may or may not be planes. The volume of such a prismoid is readily determinable (as explained in § 70 *et seq.*), while its definition is so very general that it may be applied to very rough ground. The "two plane ends" are sections perpendicular to the axis of the road. The roadbed and side slopes (also plane) form three of the side surfaces. The only approximation lies in the degree of accuracy with which the plane (or warped) surfaces coincide with the actual surface of the ground between these two sections. This accuracy will depend (*a*) on the number of points which are taken in each cross-section and the accuracy with which the lines joining these points coincide with the actual cross-sections; (*b*) on the skill shown in selecting places for the cross-sections so that the warped surfaces shall coincide as nearly as possible with the surface of the ground. In fairly smooth country, cross-sections every 100 feet, placed at the even stations, are sufficiently accurate, and such a method simplifies the computations greatly; but in rough country cross-sections must be interpolated as the surface demands. As will be explained later, carelessness or lack of judgment in cross-sectioning will introduce errors of such magnitude that all refinements in the computations are utterly wasted.

68. Cross-sectioning. The process of cross-sectioning consists in determining at any place the intersection by a vertical plane of the prism of earth lying between the roadbed, the side slopes, and the natural surface. The intersection with the road-

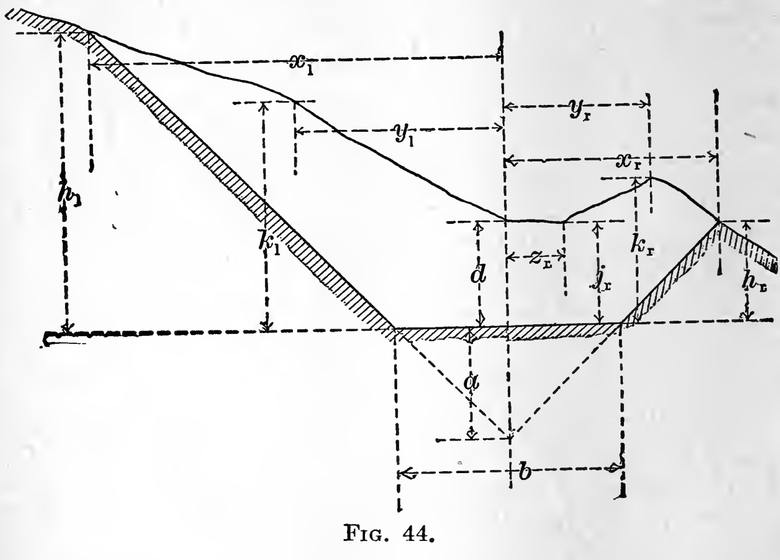


FIG. 44.

bed and side slopes gives three straight lines. The intersection with the natural surface is in general an irregular line. On smooth regular ground or when approximate results are acceptable this line is assumed to be straight. According to the irregularity of the ground and the accuracy desired more and more "intermediate points" are taken.

The distance (d in Fig. 44) of the roadbed below (or above) the natural surface at the center is known or determined from the profile or by the computed establishment of the grade line. The distances out from the center of all "breaks" are determined with a tape. To determine the elevations for a cut, set up a level at any convenient point so that the line of sight is higher than any point of the cross-section, and take a rod reading on the center point. This rod reading added to d gives the height of the instrument (H. I.) above the roadbed. Subtracting from H. I. the rod reading at any "break" gives the height of that point above the roadbed (h_l, k_l, h_r, etc.). This is true for all cases in excavation. For fill, the rod reading at center minus d equals the H. I., which may be positive or negative. When negative, add to the "H. I." the rod readings of the intermediate points to get their depths below "grade"; when positive, subtract the "H. I." from the rod readings.

The heights or depths of these intermediate points above or below grade need only be taken to the nearest tenth of a foot, and the distances out from the center will frequently be sufficiently exact when taken to the nearest foot. The roughness of the surface of farming land or woodland generally renders useless any attempt to compute the volume with any greater accuracy than these figures would imply unless the form of the ridges and hollows is especially well defined. The position of the slope-stake points is considered in the next section. Additional discussion regarding cross-sectioning is found in § 82.

69. Position of slope-stakes. The slope-stakes are set at the intersection of the required side slopes with the natural surface, which depends on the center cut or fill (d). The distance of the slope-stake from the center for the lower side is $x=\frac{1}{2}b+s(d+y)$; for the up-hill side it is $x'=\frac{1}{2}b+s(d-y')$. s is the "slope ratio" for the side slopes, the ratio of horizontal to vertical. In the above equation both x and y are unknown. Therefore some position must be found by trial which will satisfy the equation. As a preliminary, the value of x for the point $a=\frac{1}{2}b+sd$, which is the value of x for *level* cross-sections. In the case of fills on sloping ground the value of x on the *down-hill* side is *greater* than this; on the *up-hill* side it is *less*. The difference in distance is s times the difference of elevation. Take a numerical case corresponding with Fig. 45. The rod reading on c is 2.9; $d=4.2$; therefore the telescope is $4.2-2.9=1.3$ *below* grade. $s=1.5:1$, $b=16$. Hence for the point a (or for level ground) $x=\frac{1}{2}\times 16+1.5\times 4.2=14.3$. At a distance out of 14.3 the ground is seen to be about 3 feet lower, which will not only require $1.5\times 3=4.5$ more, but enough additional distance so that the added distance shall be 1.5 times the additional drop. As a first trial the rod may be held at 24 feet out and a reading of, say, 8.3 is obtained. $8.3+1.3=9.6$, the depth of the point below grade. The point on the slope line (n) which has this depth below grade is at a distance from the center

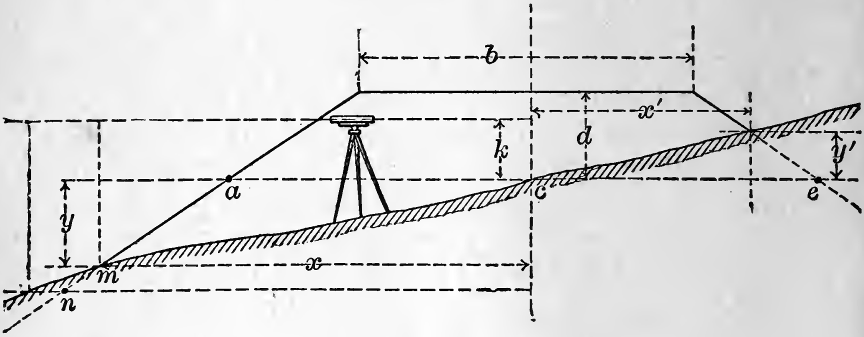


FIG. 45.

$x=8+1.5\times 9.6=22.4$. The point on the surface (s) having that depth is 24 feet out. Therefore the true point (m) is nearer the center. A second trial at 20.5 feet out gives a rod reading of, say, 7.1 or a depth of 8.4 below grade. This corresponds to a distance out of 20.6. Since the natural soil (especially in farming lands or woods) is generally so rough that a difference of elevation of a tenth or so may be readily found by slightly varying the location of the rod (even though the distance from the center is the same), it is useless to attempt too much refinement, and so in a case like the above the combination of 8.4 below grade and 20.6 out from center may be taken to indicate the proper position of the slope-stake. This is usually indicated in the form of a fraction, the distance out being the denominator and the height above (or below) grade being the numerator; the fact of *cut* or *fill* may be indicated by C or F. Ordinarily a second trial will be sufficient to determine with sufficient accuracy the true position of the slope-stake. Experienced men will frequently estimate the required distance out to within a few tenths at the first trial. The left-hand pages of the note-book should have the station number, surface elevation, grade elevation, center cut or fill, and rate of grade. The right-hand pages should be divided in the center and show the distances out and heights above grade of all points, as is illustrated in § 84. The notes should read UP the page, so that when looking ahead along the line the figures are in their proper relative position. The "fractions" farthest from the center line represent the slope-stake points.

69a. Setting slope-stakes by means of "automatic" slope-stake rods. The equipment consists of a specially graduated tape and a specially constructed rod. The tape may readily be prepared by marking on the back side of an ordinary 50-foot tape which is graduated to feet and tenths. Mark "0" at "$\frac{1}{2}b$" from the tape-ring. The same tape may be used for several values of "$\frac{1}{2}b$" by placing the *zero* at the maximum distance $\frac{1}{2}b$ from the ring. Then graduate from the zero backward, at true scale, to the ring. When $\frac{1}{2}b$ is less than this maximum, the tape will not be used clear to the ring. In general, the tape must be so held that the zero is always $\frac{1}{2}b$ from the center stake. Mark off "feet" and "tenths" on a scale proportionate to the slope ratio. For example, with the usual slope ratio of 1.5 : 1 each "foot" would measure 18 inches and each "tenth" in proportion.

The rod, 10 feet long, is shod at each end and has an endless tape passing within the shoes at each end and over pulleys—to reduce friction. The tape should be graduated in feet and tenths, from 0 to 20 feet—the 0 and 20 coinciding. By moving the tape so that 0 is at the bottom of the rod—or (practically) so that the 1-foot mark on the tape is one foot above the bottom of the shoe, an index mark may be placed on the back of the rod (say at 15—on the tape) and this readily indicates when the tape is "set at zero."

The method of use may best be explained from the figure and from the explicit rules as stated. The proof is given for two assumed positions of the level.

(1) Set up the level so that it is higher than the "center" and (if possible) higher than both slope-stakes, but not more than a rod-length higher. On very steep ground this may be impossible and each slope-stake must be set by separate positions of the level.

(2) Set the rod-tape at zero (i.e., so that the 15-foot mark on the *back* is at the index mark).

(3) Hold the rod at the center-stake (B) and note the reading (n_1 or n_2). Consider n to be always plus; consider d to be plus for cut and minus for fill.

(4) *Raise* the tape on the *face* side of the rod $(n+d)$. Applied literally (and algebraically), when the level is *below* the roadbed (only possible for fill), $(n+d)=(n_2+(-d_f))=n_2-d_f$. This being numerically negative, the tape is *lowered* (d_f-n_2). With level at (1), for fill, $(n+d)=(n_1+(-d_f))=(n_1-d_f)$; this being positive, the tape is raised. With level at (1), for cut, the tape is raised (n_1+d_c). In every case the effect is the same as if the telescope were set at the elevation of the roadbed.

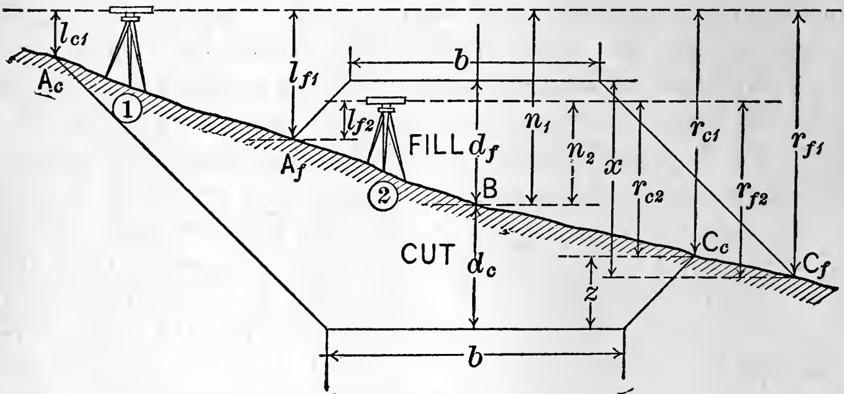


FIG. 45*a*.

(5) With the special distance-tape, so held that its zero is $\frac{1}{2}b$ from the center, carry the rod out until the rod reading equals the reading indicated by the tape. Since in cut the tape is raised $(n+d)$, the zero of the rod-tape is always higher than the level (unless the rod is held at or below the elevation of the roadbed—which is only possible on side-hill work), and the reading at either slope-stake is necessarily *negative*. The reading for slope-stakes in fill is always positive.

(6) Record the rod-tape reading as the numerator of a fraction and the *actual distance* out (read directly from the *other* side of the distance-tape) as the denominator of the fraction.

Proof. Fill. Level at (1). Tape is raised (n_1-d_f). When rod is held at C_f, the rod reading is $+x$, which $=r_{f1}-(n_1-d_f)$. But the reading on the back side of the distance-tape is also x.

Fill. Level at (2). Tape is raised (n_2-d_f), i.e., it is *lowered* (d_f-n_2). When rod is held at C_f, the rod reading is $+x$, which similarly $= r_{f_2} - (n_2 - d_f) = r_{f_2} + (d_f - n_2)$. Distance-tape as before.

Cut Level at (1). Tape is raised $(n_1 + d_c)$. When rod is held at C_c the rod reading is $-z$, which $= r_{c_1} - (n_1 + d_c)$, i.e., $z = (n_1 + d_c) - r_{c_1}$. The distance-tape will read z.

Side-hill work. It is easily demonstrated that the method, when followed literally, may be applied to side-hill work, although there is considerable chance for confusion and error, when, as is usual, $\frac{1}{2}b$ and the slope ratio are different for cut and for fill.

The method appears complicated at first, but it becomes mechanical and a time-saver when thoroughly learned. The advantages are especially great when the ground is fairly level transversely, but decrease when the difference of elevation of the center and the slope-stake is more than the rod length. By setting the rod-tape "at zero," the rod may always be used as an ordinary level rod and the regular method adopted, as in § 69. Many engineers who have thoroughly tested these rods are enthusiastic in their praise as a time-saver.

COMPUTATION OF VOLUME.

70. Prismoidal formula. Let Fig. 46 represent a triangular prismoid. The two triangles forming the ends lie in *parallel* planes, but since the angles of one triangle are not equal to the corresponding angles of the other triangle, at least two of the surfaces must be *warped*. If a section, parallel to the bases, is

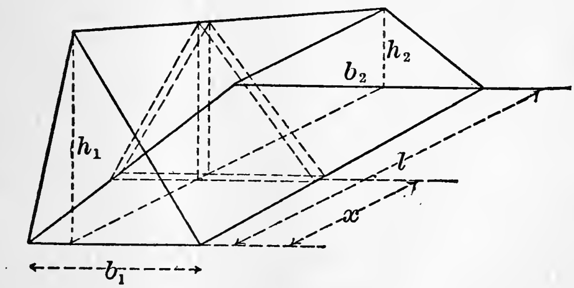


FIG. 46.

made at any point at a distance x from one end, the area of the section will evidently be

$$A_x = \tfrac{1}{2} b_x h_x = \tfrac{1}{2}\left[b_1 + (b_2 - b_1)\frac{x}{l}\right]\left[h_1 + (h_2 - h_1)\frac{x}{l}\right].$$

The volume of a section of infinitesimal length will be $A_x dx$, and the total volume of the prismoid will be *

$$\int_0^l A_x dx = \tfrac{1}{2}\int_0^l \left[b_1 + (b_2 - b_1)\frac{x}{l}\right]\left[h_1 + (h_2 - h_1)\frac{x}{l}\right] dx$$

$$= \tfrac{1}{2}\left[b_1 h_1 x + (b_2 - b_1)h_1\frac{x^2}{2l} + b_1(h_2 - h_1)\frac{x^2}{2l}\right.$$

$$\left. + (b_2 - b_1)(h_2 - h_1)\frac{x^3}{3l^2}\right]_0^l$$

$$= \tfrac{1}{2}\left\{b_1 h_1 l + [(b_2 - b_1)h_1 + b_1(h_2 - h_1)]\frac{l}{2} + (b_2 - b_1)(h_2 - h_1)\frac{l}{3}\right\}$$

$$= \frac{l}{2}[\tfrac{1}{3}b_1 h_1 + \tfrac{1}{6}b_1 h_2 + \tfrac{1}{6}b_2 h_1 + \tfrac{1}{3}b_2 h_2]$$

$$= \frac{l}{6}[\tfrac{1}{2}b_1 h_1 + \tfrac{1}{2}b_1(h_1 + h_2) + \tfrac{1}{2}b_2(h_1 + h_2) + \tfrac{1}{2}b_2 h_2]$$

$$= \frac{l}{6}\left[\tfrac{1}{2}b_1 h_1 + 4\left(\frac{1}{2}\cdot\frac{b_1 + b_2}{2}\cdot\frac{h_1 + h_2}{2}\right) + \tfrac{1}{2}b_2 h_2\right]$$

$$= \frac{l}{6}[A_1 + 4A_m + A_2], \qquad (45)$$

in which A_1, A_2, and A_m are the areas respectively of the two bases and of the middle section. Note that A_m is *not* the *mean* of A_1 and A_2, although it does not necessarily differ very greatly from it.

The above proof is absolutely independent of the values, absolute or relative, of b_1, b_2, h_1, or h_2. For example, h_2 may be zero and the second base reduces to a line and the prismoid becomes wedge-shaped; or b_2 and h_2 may both vanish, the second base becoming a point and the prismoid reduces to a pyramid. Since every prismoid (as defined in § 67) may be reduced to a combination of triangular prismoids, wedges, and pyramids, and

* Students unfamiliar with the Integral Calculus may take for granted the fundamental formulæ that $\int dx = x$, that $\int x dx = \frac{1}{2}x^2$, and that $\int x^2 dx = \frac{1}{3}x^3$; also that in integrating between the limits of l and 0 (zero), the value of the integral may be found by simply substituting l for x after integration.

since the formula is true for any one of them individually, it is true for all collectively; therefore it may be stated that *

The volume of a prismoid equals one sixth of the perpendicular distance between the bases multiplied by the sum of the areas of the two bases plus four times the area of the middle section.

While it is always possible to compute the volume of any prismoid by the above method, it becomes an extremely complicated and tedious operation to compute the true value of the middle section if the end sections are complicated in form. It therefore becomes a simpler operation to compute volumes by approximate formulæ and apply, if necessary, a correction. The most common methods are as follows:

71. Averaging end areas. The volume of the triangular prismoid (Fig. 46), computed by averaging end areas, is $\frac{l}{2}[\frac{1}{2}b_1h_1+\frac{1}{2}b_2h_2]$. Subtracting this from the true volume (as given in the equation above Eq. 45), we obtain the correction

$$\frac{l}{12}[(b_1-b_2)(h_2-h_1)]. \quad . \quad . \quad . \quad . \quad . \quad (46)$$

This shows that if either the h's or b's are equal, the correction vanishes; it also shows that if the bases are roughly similar and b varies roughly with h (which *usually* occurs, as will be seen later), the correction will be *negative*, which means that the method of averaging end areas *usually* gives *too large* results.

72. Middle areas. Sometimes the middle area is computed and the volume is assumed to be equal to the length times the middle area. This will equal $\frac{l}{2}\times\frac{b_1+b_2}{2}\times\frac{h_1+h_2}{2}$. Subtracting this from the true volume, we obtain the correction

$$\frac{l}{24}(b_1-b_2)(h_1-h_2). \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

As before, the form of the correction shows that if either the h's or b's are equal, the correction vanishes; also under the *usual* conditions, as before, the correction is *positive* and only one-half as large as by averaging end areas. Ordinarily the labor involved in the above method is no less than that of applying the exact prismoidal formula.

* The student should note that the derivation of equation (45) does not complete the proof, but that the statements in the following paragraph are logically necessary for a general proof.

73. Two-level ground. When *approximate* computations of earthwork are sufficiently exact the field-work may be materially reduced by observing simply the center cut (or fill) and the natural slope α, measured with a clinometer. The area of such a section (see Fig. 48) equals

$$\tfrac{1}{2}(a+d)(x_l+x_r)-\frac{ab}{2}.$$

But

$$x_l \tan\beta = a+d+x_l\tan\alpha,$$

from which $$x_l=\frac{a+d}{\tan\beta-\tan\alpha}.$$

Similarly, $$x_r=\frac{a+d}{\tan\beta+\tan\alpha}.$$

Substituting,

$$\text{Area}=(a+d)^2\frac{\tan\beta}{\tan^2\beta-\tan^2\alpha}-\frac{ab}{2}. \quad . \quad . \quad . \quad (48)$$

The values a, $\tan\beta$, $\tan^2\beta$ are constant for all sections, so that it requires but little work to find the area of any section.

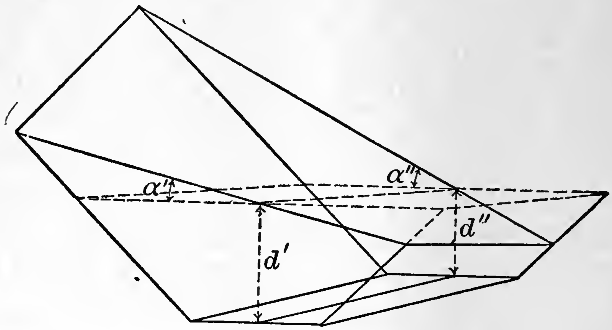


Fig. 47.

As this method of cross-sectioning implies considerable approximation, it is generally a useless refinement to attempt to com-

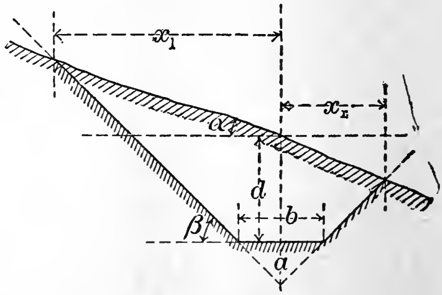


Fig. 48.

pute the volume with any greater accuracy than that obtained by averaging end areas. It may be noted that it may be easily proved that the correction to be applied is of the same form as that found in § 71 and equals

$$\frac{l}{12}[(x_l'+x_r')-(x_l''+x_r'')]\,[(d''+a)-(d'+a)],$$

which reduces to

$$\text{Corr.} = \frac{l}{6}\left\{\left[(a+d')\frac{\tan\beta}{\tan^2\beta-\tan^2\alpha'}-(a+d'')\frac{\tan\beta}{\tan^2\beta-\tan^2\alpha''}\right][d''-d']\right\}. \quad (49)$$

When $d''=d'$ the correction vanishes. This shows that when the center heights are equal there is no correction—regardless of the slope. If the slope is uniform throughout, the form of the correction is simplified and is invariably *negative*. Under the usual conditions the correction is *negative*, i.e., the method *generally* gives *too large* results.

74. Level sections. When the country is very level or when only approximate preliminary results are required, it is sometimes assumed that the cross-sections are level. The method of level sections is capable of easy and rapid computation. The area may be written as

$$(a+d)^2 s - \frac{ab}{2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (50)$$

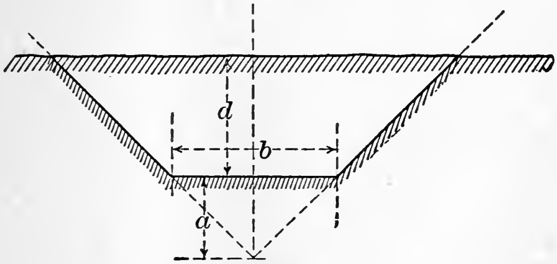


FIG. 49.

This also follows from Eq. 48 when $a=0$ and $\tan\beta=\frac{1}{s}$. s here represents the "slope ratio," i.e., the ratio of the horizontal projection of the slope to the vertical. A table is very readily formed giving the area in square feet of a section of given depth and for any given width of roadbed and ratio of side-slopes.

The area may also be readily determined (as illustrated in the following example) without the use of such a table; a table of squares will facilitate the work. Assuming the cross-sections at equal distances ($=l$) apart, the total approximate volume for any distance will be

$$\frac{l}{2}[A_0+2(A_1+A_2+\ldots A_{n-1})+A_n]. \quad . \quad . \quad . \quad (51)$$

The prismoidal correction may be directly derived from Eq. 46 as $\frac{l}{12}[2(a+d')s-2(a+d'')s][(a+d'')-(a+d')]$, which reduces to

$$-\frac{ls}{6}(d'-d'')^2 \quad \text{or} \quad -\frac{l}{12}\frac{b}{a}(d'-d'')^2. \quad . \quad . \quad (52)$$

This may also be derived from Eq. 49, since $\alpha=0$, $\tan\alpha=0$, and $\tan\beta=2a\div b$ This correction is *always* negative, showing that the method of averaging end areas, when the sections are level, always gives too large results. The prismoidal correction for any one prismoid is therefore a constant times the *square* of a difference. The squares are always positive whether the differences are positive or negative. The correction therefore becomes

$$-\frac{l}{12}\frac{b}{a}\Sigma(d'\sim d'')^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

75. Numerical example: level sections. Given the following center heights for the same number of consecutive stations 100 feet apart; width of roadbed 18 feet; slope $1\frac{1}{2}$ to 1.

The products in the fifth column may be obtained very readily and with sufficient accuracy by the use of the slide-rule described in § 79. The products should be considered as $(a+d)(a+d)\div\frac{1}{s}$. In this problem $s=1\frac{1}{2}$, $\frac{1}{s}=.6667$. To apply the rule to the first case above, place 6667 on scale B over 89 on scale A, then opposite 89 on scale B will be found 118.8 on scale A. The position of the decimal point will be evident from an approximate mental solution of the problem.

Sta.	Center Height.	$a+d$	$(a+d)^2$	$(a+d)^2 s$	Areas.	$d' \sim d''$	$(d' \sim d'')^2$
17	2.9	8.9	79.21	118.81	118.81		
18	4.7	10.7	114.49	171.74	$\times 2 =$ 343.48	1.8	3.24
19	6.8	12.8	163.84	245.76	$\times 2 =$ 491.52	2.1	4.41
20	11.7	17.7	313.29	469.93	$\times 2 =$ 939.86	4.9	24.01
21	4.2	10.2	104.04	156.06	$\times 2 =$ 312.12	7.5	56.25
22	1.6	7.6	57.76	86.64	86.64	2.6	6.76
					2292.43		94.67

$$\frac{ab}{2}=\frac{6\times 18}{2}=54 \qquad 10\times 54 = \underline{540}$$

$$1752.43$$

$$\frac{1752.43\times 100}{2\times 27}=3245 \text{ cub. yards} \quad = \text{approx. vol.}$$

$$\text{Corr.} = -\frac{100\times 18}{12\times 6\times 27}\times 94.67 \quad = -91 \text{ cub. yds.}$$

$$3245-91=3154 \text{ cub. yds.} \quad = \text{exact volume.}$$

The above demonstration of the correction to be applied to the approximate volume, found by averaging end areas, is introduced mainly to give an idea of the amount of that correction. Absolutely level sections are practically unknown, and the error involved in assuming any given sections as truly level will ordinarily be greater than the computed correction. If greater accuracy is required, more points should be obtained in the cross-sectioning, which will generally show that the sections are not truly level.

76. Equivalent sections. When sections are very irregular the following method may be used, especially if great accuracy is not required. The sections are plotted to scale and then a uniform slope line is obtained by stretching a thread so that the undulations are averaged and an *equivalent section* is obtained. The *center depth* (d) and the *slope angle* (α) of this line can be obtained from the drawing, but it is more convenient to measure the distances (x_l and x_r) from the center. The area may then be obtained independent of the center depth as follows: Let s = the slope ratio of the side slopes $= \cot \beta = \frac{b}{2a}$. (See Fig. 50.)

Then the

$$\text{Area} = \frac{1}{2}\left(\frac{x_l+x_r}{s}\right)(x_l+x_r) - \frac{x_r}{s}\frac{x_r}{2} - \frac{x_l}{s}\frac{x_l}{2} - \frac{ab}{2}$$

$$= \frac{x_l x_r}{s} - \frac{ab}{2}. \qquad \qquad (54)$$

The true volume, according to the prismoidal formula, of a length of the road measured in this way will be

$$\frac{l}{6}\left[\frac{x_l'x_r'}{s}-\frac{ab}{2}+4\left(\frac{x_l'+x_l''}{2}\,\frac{x_r'+x_r''}{2}\,\frac{1}{s}-\frac{ab}{2}\right)+\frac{x_l''x_r''}{s}-\frac{ab}{2}\right].$$

If computed by averaging end areas, the approximate volume will be

$$\frac{l}{2}\left[\frac{x_l'x_r'}{s}-\frac{ab}{2}+\frac{x_l''x_r''}{s}-\frac{ab}{2}\right].$$

Subtracting this result from the true volume, we obtain as the correction

$$\text{Correction}=\frac{l}{6s}(x_l''-x_l')(x_r'-x_r''). \quad . \quad . \quad (55)$$

This shows that if the side distances to either the right or left are equal at adjacent stations the correction is *zero*, and also that if the difference is small the correction is also small and very probably within the limit of accuracy obtainable by that method of cross-sectioning. In fact, as has already been shown in the latter part of § 75, it will usually be a useless

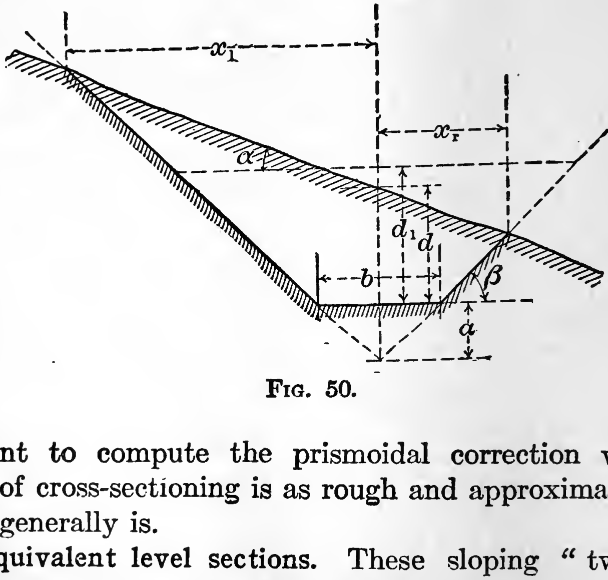


FIG. 50.

refinement to compute the prismoidal correction when the method of cross-sectioning is as rough and approximate as this method generally is.

77. Equivalent level sections. These sloping "two-level" sections are sometimes transformed into "level sections of equal area," and the volume computed by the method of level sections (§ 74). But the true volume of a prismoid with sloping ends does not agree with that of a prismoid with equivalent bases and level ends except under special conditions, and when this method is used a correction must be applied if accuracy is desired, although, as intimated before, the assumption that the sections have uniform slopes will frequently introduce greater inaccuracies than that of this method of computation. The following demonstration is therefore given to show the scope and limitations of the errors involved in this much used method.

In Fig. 50, let d_1 be the center height which gives an equivalent level section. The area will equal $(a+d_1)^2s-\frac{ab}{2}$, which must equal the area given in § 76, $\frac{x_l x_r}{s}-\frac{ab}{2}$. $s=\frac{b}{2a}$.

$$\therefore \quad (a+d_1)^2 s=\frac{x_l x_r}{s},$$

or
$$a+d_1=\frac{\sqrt{x_l x_r}}{s}. \qquad \qquad (56)$$

To obtain d_1 directly from notes, given in terms of d and α, we may substitute the values of x_l and x_r given in § 73, which gives

$$a+d_1=(a+d)\frac{\tan\beta}{\sqrt{\tan^2\beta-\tan^2\alpha}}=\frac{a+d}{\sqrt{1-s^2\tan^2\alpha}}. \quad . \quad (57)$$

The *true* volume of the equivalent section may be represented by

$$\frac{ls}{6}\left[(a+d_1')^2+4\left(\frac{a+d_1'}{2}+\frac{a+d_1''}{2}\right)^2+(a+d_1'')^2\right].$$

From this there should be subtracted the volume of the "grade prism" under the roadbed to obtain the volume of the cut that would be actually excavated, but in the following comparison, as well as in other similar comparisons elsewhere made, the volume of the grade prism invariably cancels out, and so for the sake of simplicity it will be disregarded. This expression for volume may be transposed to

$$\frac{ls}{6}\left[\frac{x_l'x_r'}{s^2}+4\left(\frac{\sqrt{x_l'x_r'}}{2s}+\frac{\sqrt{x_l''x_r''}}{2s}\right)^2+\frac{x_l''x_r''}{s^2}\right].$$

The true volume of the prismoid with sloping ends is (see § 76)

$$\frac{l}{6}\left[\frac{x_l'x_r'}{s}+4\left(\left(\frac{x_l'+x_l''}{2}\right)\left(\frac{x_r'+x_r''}{2}\right)\frac{1}{s}\right)+\frac{x_l''x_r''}{s}\right].$$

The difference of the two volumes

$$=\frac{l}{6s}\left(x_l'x_r'+x_l''x_r'+x_l'x_r''+x_l''x_r''-x_l'x_r'\right.$$
$$\left.-2\sqrt{x_l'x_r'x_l''x_r''}-x_l''x_r''\right)$$

$$=\frac{l}{6s}\left(\sqrt{x_l'x_r''}-\sqrt{x_l''x_r'}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

This shows that "equivalent level sections" do *not* in general give the true volume, there being an exception when $x_l'x_r''=x_l''x_r'$. This condition is fulfilled when the slope is uniform, i.e., when $a'=a''$. When this is nearly so the error is evidently not large. On the other hand, if the slopes are inclined in opposite directions the error may be very considerable, particularly if the angles of slope are also large.

78. Three-level sections. The next method of cross-sectioning in the order of complexity, and therefore in the order of

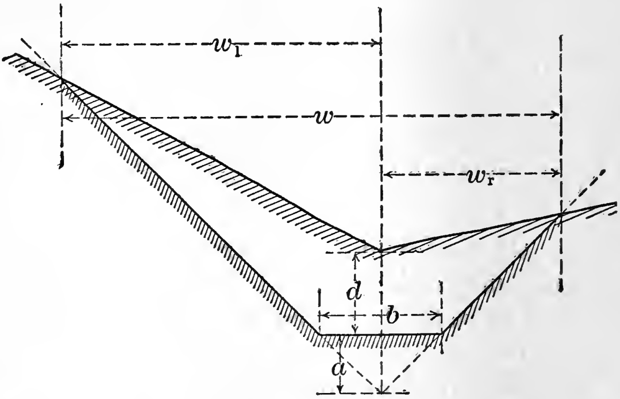


FIG. 51.

accuracy, is the method of three-level sections. The area of the section is $\frac{1}{2}(a+d)(w_r+w_l)-\frac{ab}{2}$, which may be written

$\frac{1}{2}(a+d)w-\frac{ab}{2}$, in which $w=w_r+w_l$ If the volume is computed by averaging end areas, it will equal

$$\frac{l}{4}[(a+d')w'-ab+(a+d'')w''-ab]. \quad . \quad . \quad . \quad (59)$$

If we divide by 27 to reduce to cubic yards, we have, when $l=100$,

Vol $(\ldots\ldots)=\frac{25}{27}(a+d')w'-\frac{25}{27}ab+\frac{25}{27}(a+d'')w''-\frac{25}{27}ab$

For the next section

Vol $(\ldots\ldots)=\frac{25}{27}(a+d'')w''-\frac{25}{27}ab+\frac{25}{27}(a+d''')w'''-\frac{25}{27}ab$

For a partial station length compute as usual and multiply result by $\frac{\text{length in feet}}{100}$ The prismoidal correction may be obtained by applying Eq 46 to each side in turn For the left side we have

$$\frac{l}{12}[(a+d')-(a+d'')](w_l''-w_l'), \text{ which equals}$$

$$\frac{l}{12}(d'-d'')(w_l''-w_l').$$

For the right side we have, similarly,

$$\frac{l}{12}(d'-d'')(w_r''-w_r').$$

The total correction therefore equals

$$\frac{l}{12}(d'-d'')[(w_l''+w_r'')-(w_l'+w_r')]$$

$$=\frac{l}{12}(d'-d'')(w''-w').$$

Reduced to cubic yards, and with $l=100$,

$$\text{Pris. Corr.}=\frac{25}{81}(d'-d'')(w''-w'). \quad . \quad . \quad . \quad (60)$$

When this result is compared with that given in Eq. 55 there is an apparent inconsistency. If two-level ground is considered as but a special case of three-level ground, it would seem as if the same laws should apply. If, in Eq. 55, $x_r'=x_r''$, and x_l'' is different from x_l', the equation reduces to zero; but in this case d' would also be different from d''; and since $x_l'+x_r'$ would $=w'$, and $x_l''+x_r''=w''$ in Eq. 60, $w''-w'$ would not equal zero and the correction would be some finite quantity and not zero. The explanation lies in the difference in the form and volume of the prismoids, according to the method of the formation of the warped surfaces. If the surface is supposed to be generated by the locus of a line moving parallel to the ends as plane directors and along two *straight* lines lying *in* the side slopes, then $x_l^{\text{mid.}}$ will equal $\frac{1}{2}(x_l'+x_l'')$, and $x_r^{\text{mid.}}$ will equal $\frac{1}{2}(x_r'+x_r'')$, but the profile of the center line will *not* be straight and $d^{\text{mid.}}$ will *not* equal $\frac{1}{2}(d'+d'')$. On the other hand, if the surfaces be generated by *two* lines moving parallel to the ends as plane directors and along a *straight* center line and straight side lines lying in the slopes, a warped surface will be generated each side of the center line, which will have uniform slopes on each side of the center at the two ends and *nowhere else*. This shows that when the upper surface of earthwork is warped (as it generally is), two-level ground should not be considered as a special case of three-level ground. This discussion, however, is only valuable to explain an apparent inconsistency and error. The method of two-level ground should only be used when such refinements as are here discussed are of no importance as affecting the accuracy.

An example is given on the opposite page to illustrate the method of three-level sections.

In the first column of yards

$$210=\tfrac{25}{27}(a+d)w=\tfrac{25}{27}\times7.3\times31.1;$$
507, 734, etc., are found similarly;
$$595=210-61+507-61;$$
$$448=\tfrac{40}{100}(507-61+734-61);$$
$$602=\tfrac{60}{100}(734-61+392-61);$$
$$449=392-61+179-61.$$

For the prismoidal correction,

$$-20=\tfrac{25}{81}(d'-d'')(w''-w')=\tfrac{25}{81}(2.6-8.1)(42.8-31.1)$$
$$=\tfrac{25}{81}(-5.5)(+11.7).$$

For the next line, $-3=\frac{40}{100}[\frac{25}{81}(-2.6)(+8.7)]$, and similarly for the rest. The "F" in the columns of center heights, as well

Station.	Center.	Left.	Right.	$a+d$	w	Yards.		$d'-d''$	$w''-w'$	Pris. Corr.	$x_l \sim x_r$	$\frac{V(x_l \sim x_r)}{3R}$	Curv. Corr.*
17	$2.6F$	$\frac{10.6F}{22.9}$	$\frac{0.8F}{8.2}$	7.3	31.1	210					14.7	+1	
18	$8.1F$	$\frac{15.8F}{30.7}$	$\frac{3.4F}{12.1}$	12.8	42.8	507	595	−5.5	+11.7	−20	18.6	+3	+4
+ 40	$10.7F$	$\frac{20.2F}{37.3}$	$\frac{4.8F}{14.2}$	15.4	51.5	734	448	−2.6	+ 8.7	− 3	23.1	+6	+4
19	$6.4F$	$\frac{14.0F}{28.0}$	$\frac{2.1F}{10.1}$	11.1	38.1	392	602	+4.3	−13.4	−11	17.9	+2	+5
20	$3.7F$	$\frac{5.8F}{15.7}$	$\frac{0.2F}{7.3}$	8.4	23.0	179	449	+2.7	−15.1	−13	8.4	+1	+3
										−47			+16

Roadbed, 14′ wide in fill.
Slope $1\frac{1}{2}$ to 1.

$a = \frac{b}{2s} = \frac{14}{3} = 4.7;$

$\frac{25}{27}ab = 61.$

Approx. Vol. = 2094
Pris. corr. = 47
True Vol. = 2047 (disregarding curv. corr.)*

*For the derivation of the curvation correction, see § 93.

as in the columns of "right" and "left," are inserted to indicate *fill* for all those points. Cut would be indicated by "*C*."

79. **Computation of products.** The quantities $\frac{25}{27}(a+d)w$ and $\frac{25}{27}ab$ represent in each case the product of two variable terms and a constant. These products are sometimes obtained from tables which are calculated for all ordinary ranges of the variable terms as arguments. A similar table computed for $\frac{25}{81}(d'-d'')(w''-w')$ will assist similarly in computing the prismoidal correction. Prof. Charles L. Crandall, of Cornell University, is believed to be the first to prepare such a set of tables, which were first published in 1886 in "Tables for the Computation of Railway and Other Earthwork." Another easy method of obtaining these products is by the use of a slide-rule. A slide-rule has been designed by the author to accompany this volume.* It is designed particularly for this special work, although it may be utilized for many other purposes for which slide-rules are valuable. To illustrate its use, suppose $(a+d)=28.2$, and $w=62.4$; then

$$\frac{25}{27}(a+d)w=\frac{28.2\times 62.4}{1.08}.$$

Set 108 (which, being a constant of frequent use, is specially marked) on the sliding scale (B) opposite 282 on the other scale (A), and then opposite 624 on scale B will be found 1629 on scale A, the 162 being read directly and the 9 read by estimation. Although strict rules may be followed for pointing off the final result, it only requires a very simple mental calculation to know that the result must be 1629 rather than 162.9 or 16290. For products less than 1000 cubic yards the result may be read directly from the scale; for products between 1000 and 5000 the result may be read directly to the nearest 10

* The first edition of this book was octavo, and a pasteboard slide-rule, especially marked, accompanied each volume. Cutting down the size of the pages to "pocket size" prevents the incorporation of the rule with the present edition. Any slide-rule with a logarithmic unit 22½ inches long will do equally well provided that the 108 mark is specially distinguished for ready use in computing the volume and that the 324 mark is similarly distinguished for use in computing the prismoidal correction.

yards, and the tenths of a division estimated. Between 5000 and 10000 yards the result may be read directly to the nearest 20 yards, and the fraction estimated; but prisms of such volume will never be found as simple triangular prisms—at least, an assumption that any mass of ground was as regular as this would probably involve more error than would occur from faulty estimation of fractional parts. Facilities for reading as high as 10000 cubic yards would not have been put on the scale except for the necessity of finding such products as $\frac{25}{27}(9.1\times 9.5)$, for example. This product would be read off from the same part of the rule as $\frac{25}{27}(91\times 95)$. In the first case the product (80.0) could be read directly to the nearest .2 of a cubic yard, which is unnecessarily accurate. In the other case, the product (8004) could only be obtained by estimating $\frac{4}{20}$ of a division.

The computation for the prismoidal correction may be made similarly except that the divisor is 3.24 instead of 1.08. For example, $\frac{25}{81}(5.5\times 11.7)=\frac{5.5\times 11.7}{3.24}$. Set the 324 on scale B (also specially marked like 108) opposite 55 on scale A, and proceed as before.

80. Five-level sections. Sometimes the elevations over each edge of the roadbed are observed when cross-sectioning. These are distinctively termed "five-level sections." If the center, the slope-stakes, and *one* intermediate point on each side (*not* necessarily over the edge of the roadbed) are observed, it is termed an "irregular section." The field-work of cross-sectioning five-level sections is no less than for irregular sections with one intermediate point; the computations, although capable of peculiar treatment on account of the location of the intermediate point, are no easier, and in some respects more laborious; the cross-sections obtained will not in general represent the actual cross-sections as truly as when there is perfect freedom in locating the intermediate point; as it is generally inadvisable or unnecessary to employ five-level sections throughout the length of a road, the change from one method to another adds a possible element of inaccuracy and loses the advantage of uniformity of method, particularly in the notes and *form* of computations. On these accounts the method will not be further developed, except to note that this case, as well as any other, may be solved by dividing the whole prismoid into triangular prismoids, computing the volume by averaging end areas, and computing the prismoidal correction by adding the computed corrections for each elementary triangular prismoid.

81. Irregular sections. In cross-sectioning irregular sections, the distance from the center and the elevation above "grade" of every "break" in the cross-section must be observed. The area of the irregular section may be obtained by computing the area of the trapezoids (*five*, in Fig. 44) and subtracting the two external triangles. For Fig. 44 the area would be

$$\frac{h_l+k_l}{2}(x_l-y_l)+\frac{k_l+d}{2}y_l+\frac{d+j_r}{2}z_r+\frac{j_r+k_r}{2}(y_r-z_r)$$
$$+\frac{k_r+h_r}{2}(x_r-y_r)-\frac{h_l}{2}\left(x_l-\frac{b}{2}\right)-\frac{h_r}{2}\left(x_r-\frac{b}{2}\right).$$

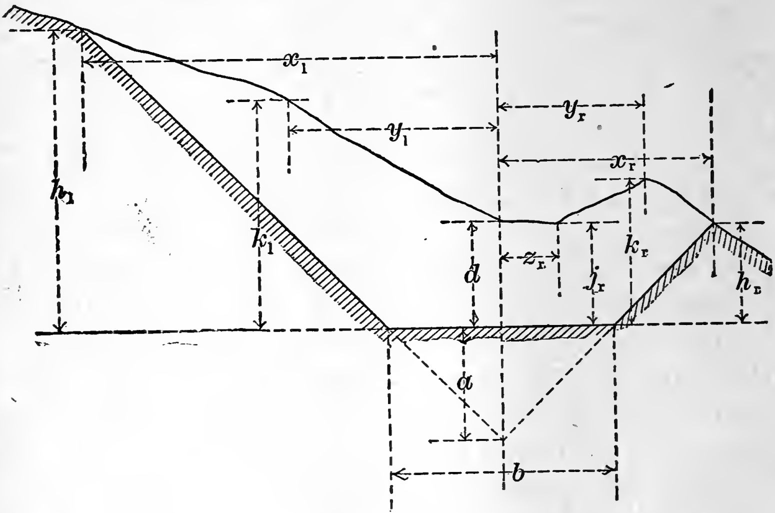


FIG. 44.

Expanding this and collecting terms, of which many will cancel, we obtain

$$\text{AREA}=\frac{1}{2}\Big[x_lk_l+y_l(d-h_l)+x_rk_r+y_r(j_r-h_r)$$
$$+z_r(d-k_r)+\frac{b}{2}(h_l+h_r)\Big]. \quad . \quad . \quad . \quad (61)$$

An examination of this formula will show a perfect regularity in its formation which will enable one to write out a similar formula for any section, no matter how irregular or how many points there are, without any of the preliminary work. The formula may be expressed in words as follows:

Area equals one-half the sum of products obtained as follows:

the distance to each slope-stake times the height above grade of the point next inside the slope-stake;

the distance to each intermediate point in turn times the height of the point just inside minus the height of the point just outside;

finally, one-half the width of the roadbed times the sum of the slope-stake heights.

If one of the sides is perfectly regular from center to slope-stake, it is easy to show that the rule holds literally good. The "point next inside the slope-stake" in this case is the center; the intermediate terms for that side vanish. The *last term* must always be used. The rule holds good for three-level sections, in which case there are three terms, which may be reduced to two. Since these two terms are both variable quantities for each cross-section, the special method, given in § 78, in which one term $\left(\frac{ab}{2}\right)$ is a constant for all sections, is preferable. In the general method, each intermediate "break" adds another term.

82. Volume of an irregular prismoid. If there is a break at one cross-section which is not represented at the next, the ridge (or hollow) implied by that break is supposed to "vanish" at the next section. In fact, the volume will not be correctly represented unless a cross-section is taken at the point where the ridge or hollow "vanishes" or "runs out." To obtain the true prismoidal correction it is necessary to observe on the ground the place where a break in an adjacent section, which is not represented in the section being taken, runs out. For example, in Fig. 52, the break on the left of section A'', at a distance of y_l'' from the center, is observed to run out in section A' at a distance of y_l' from the center. The volume of the prismoid, computed by the prismoidal formula as in § 70, will involve the midsection, to obtain the dimension of which would require a laborious computation. A simpler process is to compute the volume by averaging end areas as in § 81 and apply a prismoidal correction. To do this write out an expression for each end area similar to that given in Eq. 61. The sum of these areas times $\frac{l}{2}$ gives the approximate volume. As before, for partial station lengths, multiply the result by $\frac{\text{length in feet}}{100}$. There will be no constant subtractive term, $\frac{25}{27}ab$, as in § 78. The *true* prismoidal correction may be computed, as in § 83, or the following approximate method may be used: Consider the irregular section to be three-level ground *for the purpose of*

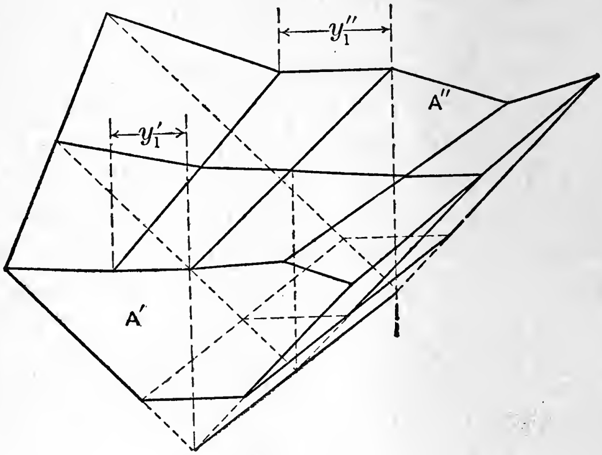


FIG. 52.

computing the correction only. This has the advantage of less labor in computation than the use of the true prismoidal correction, and although the error involved may be considerable in individual sections, the error is as likely to be positive as negative, and in the long run the error will not be large and generally will be much less than would result by the neglect of any prismoidal correction.

83. True prismoidal correction for irregular prismoids. As intimated in § 82, each cross-section should be assumed to have the same number of sides as the adjacent cross-section when computing the prismoidal correction. This being done, it permits the division of the whole prismoid into elementary triangular prismoids, the dimensions of the bases of which being given in each case by a vertical distance above grade line and by the horizontal distance between two adjacent breaks. The summation of the prismoidal corrections for each of the elementary triangular prismoids will give the true prismoidal correction. Assuming for an example the cross-section of Fig. 44, with a cross-section of the same number of sides, and with dimensions similarly indicated, for the other end, the prismoidal correction becomes (see Eq. 46)

$$\frac{l}{12}\Bigg[(h_l'-h_l'')[(x_l''-y_l'')-(x_l'-y_l')]+(k_l'-k_l'')[(x_l''-y_l'')-(x_l'-y_l')]$$
$$+(k_l'-k_l'')(y_l''-y_l')+(d'-d'')(y_l''-y_l')+(d'-d'')(z_r''-z_r')$$
$$+(j_r'-j_r'')(z_r''-z_r')+(j_r'-j_r'')[(y_r''-z_r'')-(y_r'-z_r')]$$
$$+(k_r'-k_r'')[(y_r''-z_r'')-(y_r'-z_r')]$$
$$+(k_r'-k_r'')[(x_r''-y_r'')-(x_r'-y_r')]+(h_r'-h_r'')[(x''-y_r'')-(x_r'-y_r')]$$
$$-(h_l'-h_l'')\left[\left(x_l''-\frac{b}{2}\right)-\left(x_l'-\frac{b}{2}\right)\right]$$
$$-(h_r'-h_r'')\left[\left(x_r''-\frac{b}{2}\right)-\left(x_r'-\frac{b}{2}\right)\right]\Bigg]$$

Expanding this and collecting terms, of which many will cancel, we obtain

$$\text{Pris.Corr}=\frac{l}{12}\Big[(x_l''-x_l')(k_l'-k_l'')+(y_l''-y_l')[(d'-h_l')-(d''-h_l'')]$$
$$+(x_r''-x_r')(k_r'-k_r'')+(y_r''-y_r')[(j_r'-h_r')-(j_r''-h_r'')]$$
$$+(z_r''-z_r')[(d'-k_r')-(d''-k_r'')]\Big]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

By comparing this equation with Eq. 61 a remarkable coincidence in the law of formation may be seen, which enables this formula to be written by mere inspection and to be applied numerically with a minimum of labor from the computations for end areas, as will be shown (§ 84) by a numerical example. For each term in Eq. 61, as, for example, $y_r(j_r-h_r)$, there is a correction term in Eq. 62 of the form

$$(y_r''-y_r')[(j_r'-h_r')-(j_r''-h_r'')].$$

Each one of these terms [y_r'', y_r', $(j_r'-h_r')$, and $(j_r''-h_r'')$] has been previously used in finding the end areas and has its place in the computation sheet. The summation of the products of these differences times a constant gives the total true prismoidal correction in cubic yards for the whole prismoid considered.

The *constant* is the same as that computed in § 78, i.e., $\frac{25}{81}$.

84. Numerical example; irregular sections; volume with true prismoidal correction. (See page 98.)

Roadbed 18 feet wide in cut; slope $1\frac{1}{2}$ to 1.

Sta.	Center $\left\{\begin{matrix}\text{cut}\\\text{or}\\\text{fill.}\end{matrix}\right.$	Left.			Right.	
19	0.6c	$\frac{3.6c}{14.4}$	$\left(\frac{2.3c}{8.2}\right)$	$\left(\frac{1.8c}{6.0}\right)$	$\frac{0.1c}{4.2}$	$\frac{0.4c}{9.6}$
18	2.3c	$\frac{4.2c}{15.3}$	$\frac{6.8c}{8.4}$	$\frac{3.2c}{5.2}$	$\left(\frac{1.9c}{3.6}\right)$	$\frac{1.2c}{10.8}$
17	7.6c	$\frac{8.2c}{21.3}$	$\frac{10.2c}{17.4}$	$\frac{8.0c}{6.1}$	$\left(\frac{5.8c}{8.0}\right)$	$\frac{4.2c}{15.3}$
+42	10.2c	$\frac{12.2c}{27.3}$	$\left(\frac{12.3c}{22.0}\right)$	$\frac{12.6c}{8.2}$	$\frac{6.2c}{7.5}$	$\frac{8.4c}{21.6}$
16	6.8c	$\frac{8.9c}{22.4}$		$\frac{7.6c}{12.0}$	$\frac{3.2c}{4.1}$	$\frac{2.6c}{12.9}$

The figures in the bracket $\left(\frac{12.3c}{22.0}\right)$ mean that it was noted in the field that the break, indicated at Sta. 17 as being 17.4 to the left, ran out at Sta. 16+42 at 22.0 to the left. By interpolation between 8.2 and 27.3 the height of this point is *computed* as 12.3. The quantities in the other brackets are obtained similarly. These quantities are only used when the computation of the true prismoidal correction is desired. They are not needed in computing the volume by averaging end areas, nor are they used at all if the prismoidal correction is to be obtained by assuming (*for this purpose*) the ground to be *three-level* ground.

In the tabular form on page 99 the figures within the braces ($\overbrace{\quad\quad}$) are NOT used in computing the volume, but are only used to obtain the *differences* of widths or heights with which to compute the *true* prismoidal correction. It may be noted, as a check, that the volume, computed from these figures in the braces, is the same as that computed from the other figures The figures within each brace (or bracket) constitute a group which must be used in connection with a group which has the same number of points, on the same side of the center, in the *next* cross-section, previous or succeeding. In the column of "Yards" under "True pris. corr.," we have, for example, $(-5) = \frac{42}{100}(-7+0-8+3)$.

85. Volume of irregular prismoid, with approximate prismoidal correction. If the prismoidal correction is obtained approxi-

VOLUME OF IRREGULAR PRISMOID, WITH TRUE PRISMOIDAL CORRECTION.

Sta.	Width.	Height.	Yards.		True pris. corr.		
					$w''-w'$	$h'-h''$	Yards.
16	L[22.4	7.6	158				
	12.0	— 2.1	— 23				
	12.9]R	3.2	40				
	4.1]R	4.2	16				
	9.0	11.5	96				
+42	L[27.3	12.6	319		+4.9	—5.0	−7
	8.2	— 2.0	— 15		−3.8	−0.1	0
	L{27.3	12.3					
	22.0	0.4					
	8.2	— 2.1					
	21.6]R	6.2	124		+8.7	−3.0	−8
	7.5]R	1.8	13		+3.4	+2.4	+3
	9.0	20.6	172	378			(−5)
17	L[21.3	10.2	201		−6.0	+2.1	−4
	17.4	— 0.2	— 3		−4.6	+0.6	−1
	6.1	— 2.6	— 14		—2.1	+0.5	0
	15.3}R	5.8			—6.3	+0.4	−1
	8.0}R	3.4			+0.5	−1.6	0
	15.3]R	7.6	107				
	9.0	12.4	103	584			(−3)
18	L[15.3	6.8	95		−6.0	+3.4	−6
	8.4	— 1.0	— 7		−9.0	+0.8	−2
	5.2	— 4.5	— 22		−0.9	+1.9	−1
	10.8]R	2.3	23		−4.5	+5.3	−7
	10.8}R	1.9					
	3.6}R	1.1					
	9.0	5.4	45	528			(−16)
19	L[14.4	0.6	8				
	L{14.4	2.3			−0.9	+4.5	−1
	8.2	— 1.8			−0.2	+0.8	0
	6.0	— 1.7			+0.8	−2.8	−1
	9.6]R	0.1	1		−1.2	+1.8	−1
	4.2]R	0.2	1		+0.6	+0.9	0
	9.0	4.0	33	177			(−3)

Approx. vol.	= 1667	−27
True pris. corr	= −27	
True volume	= 1640 cubic yards	

mately, by the method outlined in § 82, the process will be as shown in the tabular form on page 100. Not only is the numerical work considerably less than the exact method, but the discrepancy in cubic yards is almost insignificant.

86. Illustration of value of approximate rules. The tabulation on page 100 shows that when the volume of an irregular prismoid is computed by averaging end areas and is corrected

VOLUME OF IRREGULAR PRISMOID, WITH APPROXIMATE PRISMOIDAL CORRECTION.

Sta.	W'th	H'ght	Yards.		Cen. Height.	Total width	$d'-d''$	$w''-w'$	Approx. pris.corr.
	22.4	7.6	158		+6.8	35.3			
	12.0	−2.1	−23						
16	12.9	3.2	40						
	4.1	4.2	16						
	9.0	11.5	96						
	27.3	12.6	319		+10.2	48.9	−3.4	+13.6	−14
	8.2	−2.0	−15						
+42	21.6	6.2	124						
	7.5	1.8	13						
	9.0	20.6	172	378					(−6)
	21.3	10.2	201		+ 7.6	36.6	+2.6	−12.3	−10
	17.4	−0.2	− 3						
17	6.1	−2.6	−14						
	15.3	7.6	107						
	9.0	12.4	103	584					(−6)
	15.3	6.8	95		+ 2.3	26.1	+5.3	−10.5	−17
	8.4	−1.0	− 7						
18	5.2	−4.5	−22						
	10.8	2.3	23						
	9.0	5.4	45	528					(−17)
	14.4	0.6	8		+ 0.6	24.0	+1.7	−2.1	−1
19	9.6	0.1	1						
	4.2	0.2	1						
	9.0	4.0	33	177					(−1)

Approx. volume = 1667 −30
Approx. pris. corr. = −30

Corrected volume = 1637 cubic yards

by considering the ground as three-level ground (*for the purposes of the correction only*), the error for the different sections

Sections.	True volume.	Approx. vol. by averaging end areas.	Difference or *true* pris. corr.	Approx. pris. corr. on basis of three-level ground.	Error.	Approx. vol. computed from center and side heights *only*.	Error.
16......16+42	373	378	− 5	− 6	−1	396	−23
16+42..17	581	584	− 3	− 6	−3	577	+ 4
17......18	512	528	−16	−17	−1	463	+49
18......19	174	177	− 3	− 1	+2	147	+27
	1640	1667	−27	−30	−3	1583	+57

is sometimes positive and sometimes negative, and in this case amounts to only 3 yards in 1640—less than $\frac{1}{5}$ of 1%. If the prismoidal correction had been neglected, the error would have been 27 yards—nearly 2%. The approximate results are here *too large* for each section—as is usually the case. If points between the center and slope-stakes are omitted and the volume computed as if the ground were *three-level* ground, the error is quite large in individual sections, but the errors are both positive and negative and therefore compensating.

87. Cross-sectioning irregular sections. The prismoids considered have *straight* lines joining corresponding points in the two cross-sections. The center line must be straight between two cross-sections. If a ridge or valley is found lying diagonally across the roadbed, a cross-section *must* be interpolated at the lowest (or highest) point of the profile. Therefore a "break" at any section cannot be said to run out at the other section on the *opposite* side of the center. It must run out on the *same* side of the center or possibly *at* the center. Very frequently complicated cross-sectioning may be avoided by computing the volume, by some special method, of a mound or hollow when the ground is comparatively regular except for the irregularity referred to.

88. Side-hill work. When the natural slope cuts the roadbed there is a necessity for both cut and fill at the same cross-section.

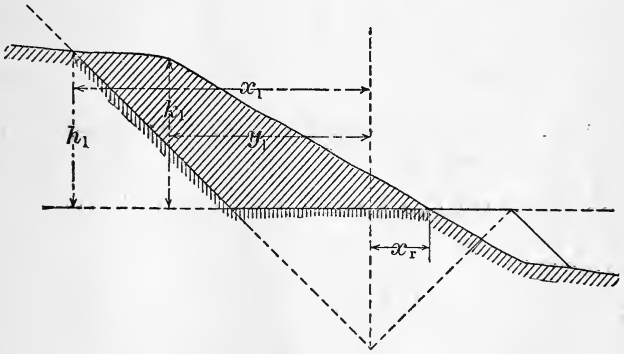


FIG. 53.

When this occurs the cross-sections of both cut and fill are often so nearly triangular that they may be considered as such without great error, and the volumes may be computed separately as triangular prismoids without adopting the more elaborate form of computation so necessary for complicated irregular sections. When the ground is too irregular for this the best plan is to follow the uniform system. In computing the cut, as in Fig. 53, the left side would be as usual; there would be a small center cut and an ordinate of zero at a short distance to the right of the center. Then, *ignoring the fill*, and applying Eq. 61 strictly, we have two terms for the left side, one for the right, and the term involving $\frac{1}{2}b$, which will be $\frac{1}{2}bh_l$ in this case, since $h_r=0$, and the equation becomes

$$\text{Area}=\tfrac{1}{2}[x_lk_l+y_l(d-h_l)+x_rd+\tfrac{1}{2}bh_l].$$

The area for fill may also be computed by a strict application of Eq. 61, but for Fig. 54 all distances for the left side are zero and the elevation for the first point out is zero. d also must be

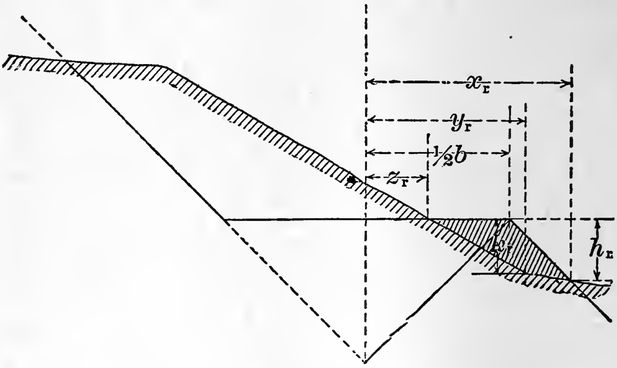


Fig. 54.

considered as zero. Following the rule, § 81, literally, the equation becomes

$$\text{Area}_{(\text{Fill})}=\tfrac{1}{2}[x_rk_r+y_r(o-h_r)+z_r(o-k_r)+\tfrac{1}{2}b(o+h_r)],$$

which reduces to

$$\tfrac{1}{2}[x_rk_r-y_rh_r-z_rk_r+\tfrac{1}{2}bh_r].$$

(*Note* that x_r, h_r, etc., have different significations and values in this and in the preceding paragraphs.) The "terminal pyramids" illustrated in Fig. 40 are instances of side-hill work for very short distances. Since side-hill work always implies *both* cut and fill at the same cross-section, whenever either the cut or fill disappears and the earthwork becomes wholly cut or wholly fill, that point marks the end of the "side-hill work," and a cross-section should be taken at this point.

89. Borrow-pits. The cross-sections of borrow-pits will vary not only on account of the undulations of the surface of the

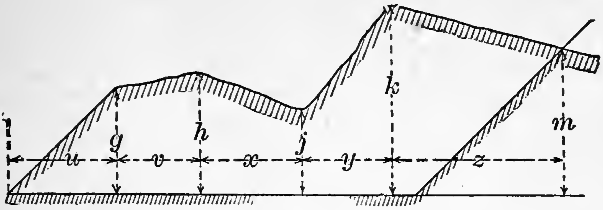


FIG. 55.

ground, but also on the sides, according to whether they are made by widening a convenient cut (as illustrated in Fig. 55) or simply by digging a pit. The sides should always be properly sloped and the cutting made cleanly, so as to avoid unsightly roughness. If the slope ratio on the right-hand side (Fig. 55) is s, the area of the triangle is $\frac{1}{2}sm^2$. The area of the section is $\frac{1}{2}[ug+(g+h)v+(h+j)x+(j+k)y+(k+m)z-sm^2]$. If all the horizontal measurements were referred to one side as an origin, a formula similar to Eq. 61 could readily be developed, but little or no advantage would be gained on account of any simplicity of computation. Since the *exact* volume of the earth borrowed is frequently necessary, the prismoidal correction should be computed; and since such a section as Fig. 55 does not even approximate to a three-level section, the method suggested in § 82 cannot be employed. It will then be necessary to employ the exact method, § 83, by dividing the volume into triangular prismoids and taking the summation of their corrections, found according to the general method of § 71.

90. Correction for curvature. The volume of a solid, generated by revolving a plane area about an axis lying in the plane but outside of the area, equals the product of the given area times the length of the path of the center of gravity of the area. If the centers of gravity of all cross-sections lie in the center of the road, where the length of the road is measured, there is absolutely no necessary correction for curvature. If all the cross-sections in any given length were exactly the same and therefore had the same eccentricity, the correction for curvature would be very readily computed according to the above principle. But when both the areas and the eccentricities vary from point to point, as is generally the case, a theoretically exact solution is quite complex, both in its derivation and application. Suppose, for simplicity, a curved section of the road, of uniform cross-sections and with the center of gravity of every cross-section at the same distance e from the center line of the road. The length of the path of the center of gravity will be to the length of the center line as $R \pm e : R$. Therefore we have *True vol.*: *nominal vol.* $:: R \pm e : R$. $\therefore$ *True vol.* $= lA\frac{R \pm e}{R}$ for a volume of uniform area and eccentricity. For any other area and eccentricity we have, similarly, *True vol.'* $= lA'\frac{R \pm e'}{R}$. This shows that the effect of curvature is the same as increasing (or diminishing) the area by a quantity depending on the area and eccentricity, the increased (or diminished) area being found by multiplying the actual area by the ratio $\frac{R \pm e}{R}$. This being independent of the value of l, it is true for infinitesimal lengths. If the eccentricity is assumed to vary uniformly between two sections, the *equivalent area* of a cross-section located midway between the two end cross-sections would be $A_m \frac{\left(R \pm \frac{e' + e''}{2}\right)}{R}$.

Therefore the volume of a solid which, when straight, would be $\frac{l}{6}(A' + 4A_m + A'')$, would then become

$$\textit{True vol.} = \frac{l}{6R}\left[A'(R \pm e') + 4\,A_m\left(R \pm \frac{e' + e''}{2}\right) + A''(R \pm e'')\right].$$

Subtracting the nominal volume (the true volume when the prismoid is straight), the

$$\textit{Correction} = \pm\frac{l}{6R}\left[(A' + 2A_m)e' + (2A_m + A'')e''\right]. \quad . \quad (63)$$

Another demonstration of the same result is given by Prof. C. L. Crandall in his "Tables for the Computation of Railway and other Earthwork," in which is obtained by calculus methods the summation of elementary volumes having variable areas with variable eccentricities. The exact application of Eq. 63 requires that A_m be known, which requires laborious computations, but no error worth considering is involved if the equation is written approximately

$$Curv.\,corr. = \frac{l}{2R}(A'e' + A''e''), \quad . \quad . \quad . \quad . \quad (64)$$

which is the equation generally used. The approximation consists in assuming that the difference between A' and A_m equals the difference between A_m and A'' but with opposite sign. The error due to the approximation is always utterly insignificant.

91. Eccentricity of the center of gravity. The determination of the true positions of the centers of gravity of a long series of irregular cross-sections would be a very laborious operation, but fortunately it is generally sufficiently accurate to consider the cross-sections as three-level ground, or, for side-hill work, to

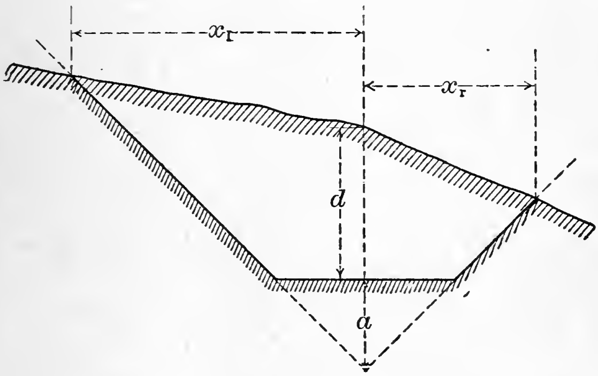


FIG. 56.

be triangular, *for the purpose of this correction.* The eccentricity of the cross-section of Fig. 56 (including the grade triangle) may be written

$$e = \frac{\frac{(a+d)x_l}{2}\frac{x_l}{3} - \frac{(a+d)x_r}{2}\frac{x_r}{3}}{\frac{(a+d)x_l}{2} + \frac{(a+d)x_r}{2}} = \frac{1}{3}\frac{x_l^2 - x_r^2}{x_l + x_r} = \frac{1}{3}(x_l - x_r). \quad . \quad (65)$$

The side toward x_l being considered positive in the above demonstration, if $x_r > x_l$, e would be negative, i.e., the center of gravity would be on the right side. Therefore, for three-level ground, the correction for curvature (see Eq. 64) may be written

$$Correction = \frac{l}{6R}[A'(x_l' - x_r') + A''(x_l'' - x_r'')].$$

Since the approximate volume of the prismoid is

$$\frac{l}{2}(A + A') = \frac{l}{2}A' + \frac{l}{2}A'' = V' + V'',$$

in which V' and V'' represent the number of cubic yards corresponding to the area at each station, we may write

$$Corr.\ in\ cub.\ yds. = \frac{1}{3R}[V'(x_l' - x_r') + V''(x_l'' - x_r'')]. \quad . \ (66)$$

It should be noted that the value of e, derived in Eq. 65, is the eccentricity of the whole area including the triangle under the roadbed. The eccentricity of the true area is greater than this and equals

$$e \times \frac{\text{true area} + \frac{1}{2}ab}{\text{true area}} = e_1.$$

The required quantity ($A'e'$ of Eq. 64) equals *true area* $\times e_1$ which equals (*true area* $+ \frac{1}{2}ab) \times e$. Since the value of e is very simple, while the value of e_1 would, in general, be a complex quantity, it is easier to use the simple value of Eq. 65 and add $\frac{1}{2}ab$ to the area. Therefore, in the case of three-level ground the subtractive term $\frac{25}{27}ab$ (§ 78) should *not* be subtracted in computing this correction. For irregular ground, when computed by the method given in §§ 81 and 82, which does not involve the grade triangle, a term $\frac{25}{27}ab$ must be *added* at every station when computing the quantities V' and V'' for Eq. 66.

It should be noted that the factor $1 \div 3R$, which is constant for the length of the curve, may be computed with all necessary accuracy and without resorting to tables by remembering that

$$R = \frac{5730}{\text{degree of curve}}.$$

Since it is useless to attempt the computation of railroad earthwork closer than the nearest cubic yard, it will frequently be possible to write out all curvature corrections by a simple mental process upon a mere inspection of the computation sheet. Eq. 66 shows that the correction for each station is of the form $\frac{V(x_l-x_r)}{3R}$. $3R$ is generally a large quantity—for a 6° curve it is 2865. (x_l-x_r) is generally small. It may frequently be seen by inspection that the product $V(x_l-x_r)$ is roughly twice or three times $3R$, or perhaps less than half of $3R$, so that the corrective term for that station may be written 2, 3, or 0 cubic yards, the fraction being disregarded. For much larger absolute amounts the correction must be computed with a correspondingly closer percentage of accuracy.

The algebraic sign of the curvature correction is best determined by noting that the center of gravity of the cross-section is on the right or left side of the center according as x_r is greater or less than x_l, and that the correction is *positive* if the center of gravity is on the *outside* of the curve, and *negative* if on the *inside*.

It is frequently found that x_l is uniformly greater (or uniformly less) than x_r throughout the length of the curve. Then the curvature correction for each station is uniformly positive or negative. But in irregular ground the center of gravity is apt

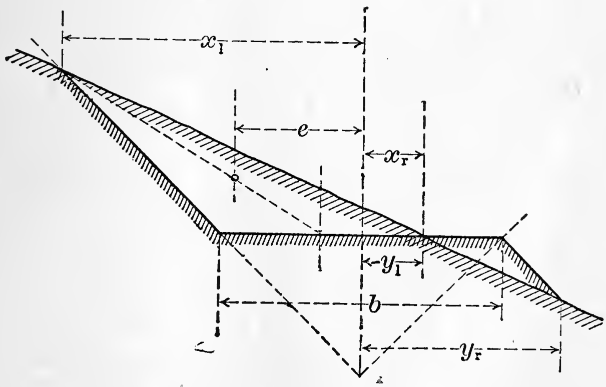


FIG. 57.

to be irregularly on the outside or on the inside of the curve, and the curvature correction will be correspondingly positive or negative. If the curve is to the *right*, the correction will be positive or negative according as (x_l-x_r) is positive or negative; if the curve is to the *left*, the correction will be positive or negative according as $(x_r - x_l)$ is positive or negative. Therefore when computing curves to the *right* use the form $(x_l - x_r)$ in Eqs. 66 and 68; when computing curves to the *left* use the form $(x_r - x_l)$ in these equations; the algebraic sign of the correction will then be strictly in accordance with the results thus obtained.

92. Center of gravity of side-hill sections. In computing the correction for side-hill work the cross-section would be treated as triangular unless the error involved would evidently be too great to be disregarded. The center of gravity of the triangle lies on the line joining the vertex with the middle of the base and at $\frac{1}{3}$ of the length of this line from the base. It is therefore equal to the distance from the center to the foot of this line plus $\frac{1}{3}$ of its horizontal projection. Therefore

$$e = \left[\frac{b}{2} - \frac{1}{2}\left(\frac{b}{2} + x_r\right)\right] + \frac{1}{3}\left[x_l - \left(\frac{b}{2} - \frac{1}{2}\left(\frac{b}{2} + x_r\right)\right)\right]$$

$$= \frac{b}{4} - \frac{x_r}{2} + \frac{x_l}{3} - \frac{b}{12} + \frac{x_r}{6}$$

$$= \frac{b}{6} + \frac{x_l}{3} - \frac{x_r}{3}$$

$$= \frac{1}{3}\left[\frac{b}{2} + (x_l - x_r)\right] \quad \ldots \ldots \ldots \ldots \quad (67)$$

By the same process as that used in § 91 the correction equation may be written

$$\text{Corr. in cub. yds.} = \frac{1}{3R}\left[V'\left(\frac{b}{2} + (x_l' - x_r')\right) + V''\left(\frac{b}{2} + (x_l'' - x_r'')\right)\right]. \quad (68)$$

It should be noted that since the grade triangle is not used in this computation the volume of the grade prism is *not* involved in computing the quantities V' and V''.

The eccentricities of cross-sections in side-hill work are *never* zero, and are frequently quite large. The total volume is generally quite small. It follows that the correction for curvature is generally a vastly larger proportion of the total volume than in ordinary three-level or irregular sections.

If the triangle is wholly to one side of the center, Eq. 67 can still be used. For example, to compute the eccentricity of the triangle of fill, Fig. 57, denote the two distances to the slope-stakes by y_r and $-y_l$ (note the minus sign). Applying Eq. 67 literally (noting that $\frac{b}{2}$ must here be considered as negative in order to make the notation consistent) we obtain

$$e=\frac{1}{3}\left[-\frac{b}{2}+(-y_l-y_r)\right],$$

which reduces to

$$e=-\frac{1}{3}\left[\frac{b}{2}+y_l+y_r\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

As the algebraic signs tend to create confusion in these formulæ, it is more simple to remember that for a triangle lying on *both* sides of the center e is always numerically equal to $\frac{1}{3}\left[\frac{b}{2}+(x_l\sim x_r)\right]$, and for a triangle entirely on one side, e is numerically equal to $\frac{1}{3}\left[\frac{b}{2}\right.$ + the numerical *sum* of the two distances out]. The algebraic sign of e is readily determinable as in § 91.

93. Example of curvature correction. Assume that the fill in § 78 occurred on a 6° curve to the *right*. $\frac{1}{3R}=\frac{1}{2865}$. The quantities 210, 507, etc., represent the quantities V', V'', etc., since they include in each case the 61 cubic yards due to the grade prism. Then

$$\frac{V(x_l\sim x_r)}{3R}=\frac{210(22.9-8.2)}{2865}=\frac{3101.7}{2865}=+1.$$

The sign is plus, since the center of gravity of the cross-section is on the left side of the center and the road curves to the right, thus making the true volume larger. For Sta. 18 the correction, computed similarly, is $+3$, and the correction for the whole section is $1+3=4$. For Sta. $18+40$ the correction is computed as 6 yards. Therefore, for the 40 feet, the correction is $\frac{40}{100}(3+6)=3.6$, which is called 4. Computing the others similarly we obtain a total correction of $+16$ cubic yards.

94. Accuracy of earthwork computations. The preceding methods give the *precise volume* (except where approximations are distinctly admitted) of the prismoids which are *supposed* to represent the volume of the earthwork. To appreciate the accuracy necessary in cross-sectioning to obtain a given accuracy in volume, consider that a fifteen-foot length of the cross-section, which is assumed to be straight, really sags 0.1 foot, so that the cross-section is in error by a triangle 15 feet wide and 0.1 foot high. This sag 0.1 foot high would hardly be detected by the eye, but in a length of 100 feet in each direction it would make an error of volume of 1.4 cubic yards in *each* of the two prismoids, assuming that the sections at the other ends were perfect. If the cross-sections at both ends of a prismoid were in error by this same amount, the volume of that prismoid would be in error by 2.8 cubic yards if the errors of area were both plus or both minus. If one were plus and one minus, the errors would neutralize each other, and it is the compensating character of these errors which permits any confidence in the results as obtained by the usual methods of cross-sectioning. It demonstrates the utter futility of attempting any closer accuracy than the nearest cubic yard. It will thus be seen that if an error really exists at *any* cross-section it involves the prismoids on *both* sides of the section, even though all the other cross-sections are perfect. As a further illustration, suppose that cross-sections were taken by the method of slope angle and center depth (§ 73), and that a cross-section, assumed as uniform, sags 0.4 foot in a width of 20 feet. Assume an equal error (of same sign) at the other end of a 100-foot section. The error of volume for that one prismoid is 38 cubic yards.

The computations further assume that the warped surface, passing through the end sections, coincides with the surface of the ground. Suppose that the cross-sectioning had been done with mathematical perfection; and, to assume a simple case, suppose a sag of 0.5 foot between the sections, which causes an error equal to the volume of a pyramid having a base of 20 feet (in each cross-section) times 100 feet (between the cross-sections) and a height of 0.5 foot. The volume of this pyramid is $\frac{1}{3}(20 \times 100) \times 0.5 = 333$ cub. ft. $= 12$ cub. yds. And yet this sag or hump of 6 inches would generally be utterly unnoticed, or at least disregarded.

When the ground is very rough and broken it is sometimes practically impossible, even with frequent cross-sections, to locate warped surfaces which will closely coincide with all the sudden irregularities of the ground. In such cases the computations are necessarily more or less approximate and dependence must be placed on the compensating character of the errors.

95. **Approximate computations from profiles.** As a means of comparing the relative amounts of earthwork on two or more proposed routes which have been surveyed by preliminary surveys, it will usually be sufficiently accurate to compare the *areas* of cutting (assuming that the cut and fill are approximately balanced) as shown by the several profiles. The errors involved may be large in individual cases and for certain small sections, but fortunately the errors (in comparing two lines) will be largely compensated. The errors are much larger on side-hill work than when the cross-sections are comparatively level. The errors become large when the depth of cut or fill is very great. If the lines compared have the same general character as to the slope of the cross-sections, the proportion of side-hill work, and the average depth of cut or fill, the error involved in considering their relative volumes of cutting to be as the relative areas of cutting on the profiles (obtained perhaps by a planimeter) will probably be small. If the volume in each case is computed by assuming the sections as *level*, with a depth equal to the center cut, the error involved will depend only on the amount of side-hill work and the degree of the slope. If these features are about the same on the two lines compared, the error involved is still less.

FORMATION OF EMBANKMENTS.

96. **Shrinkage of earthwork.** The evidence on this subject as to the amount of shrinkage is very conflicting, a fact which is probably due to the following causes:

1. The various kinds of earthy material act very differently as respects shrinkage. There has been but little uniformity in the *classification of earths* in the tests and experiments that have been made.

2. Very much depends on the *method* of forming an embankment (as will be shown later). Different reports have been based on different methods—often without mention of the method.

3. An embankment requires considerable *time* to shrink to its final volume, and therefore much depends on the time elapsed between construction and the measurement of what is supposed to be the settled volume.

P. J. Flynn quotes some experiments (*Eng. News*, May 1, 1886) made in India, in which pits were dug having volumes of 400 to 600 cubic feet. The material, when piled into an embankment, measured largely in excess of the original measurement—as is the universal experience. The pits were refilled with the same material. As the rains, very heavy in India, settled the material in the pits, more was added to keep the pits full. Even after the rainy season was over, there was in every case material in excess. This would seem to indicate a permanent *expansion*, although it is possible that the observations were not continued for a sufficient time to determine the final settled volume.

On the contrary, notes made by Mr. Elwood Morris many years ago on the behavior of embankments of several thousand cubic yards, formed in layers by carts and scrapers, one winter intervening between commencement and completion, showed in each case a permanent *contraction* averaging about 10%.

All authorities agree that rockwork *expands* permanently when formed into an embankment, but the percentages of expansion given by different authorities differ even more than with earth—varying from 8 to 90%. Of course this very large range in the coefficient is due to differences in the character of the rock. The softer the rock and the closer its similarity to earth, the less will be its expansion. On account of the conflicting statements made, and particularly on account of the influence of methods of work, but little confidence can be felt in any given coefficient, especially when given to a fraction of a per cent, but the consensus of American practice seems to average about as follows:

Permanent contraction of earth.	about 10%
" expansion of rock.	40 to 60%

These values for rock should be materially reduced, according to judgment, when the rock is soft and liable to disintegrate. The hardest rocks, loosely piled, may occasionally give even higher results. The following is given by several authors as the permanent contraction of several grades of earth:

Gravel or sand	about	8%
Clay	"	10%
Loam	"	12%
Loose vegetable surface soil	"	15%

It may be noticed from the above table that the harder and cleaner the material the less is the contraction. Perfectly clean gravel or sand would not probably change volume appreciably. The above coefficients of shrinkage and expansion may be used to form the following convenient table.

Material.	To make 1000 cubic yards of embankment will require	1000 cubic yards measured in excavation will make
Gravel or sand	1087 cubic yards	920 cubic yards
Clay	1111 " "	900 " "
Loam	1136 " "	880 " "
Loose vegetable soil	1176 " "	850 " "
Rock, large pieces	714 " "	1400 " "
" small "	625 " "	1600 " "
	measured in excavation	of embankment.

97. Allowance for shrinkage. On account of the initial expansion and subsequent contraction of earth, it becomes necessary to form embankments higher than their required ultimate form in order to allow for the subsequent shrinkage. As the shrinkage appears to be all vertical (practically), the embankment must be formed as shown in Fig. 58. The effect of shrink-

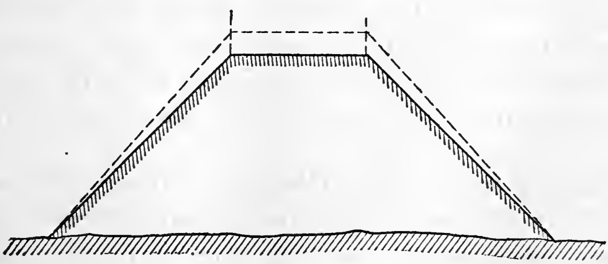


FIG. 58.

age should not be confounded with that of slipping of the sides, which is especially apt to occur if the embankment is subjected to heavy rains very soon after being formed, and also when the embankments are originally steep. It is often difficult to form an embankment at a slope of 1 : 1 which will not slip more or less before it hardens.

Very high embankments shrink a greater percentage than lower ones. Various rules giving the relation between shrinkage and height have been suggested, but they vary as badly as the suggested coefficients of contraction, probably for the same causes. As the fact is unquestionable, however, the extra height of the embankment must be varied somewhat as in Fig. 59, which represents a longitudinal section of an embankment.

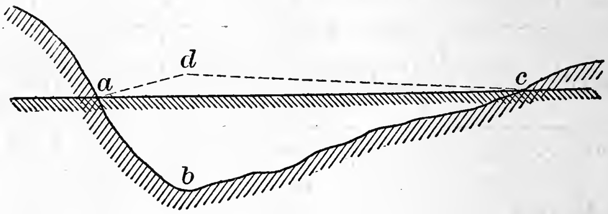


Fig. 59.

As considerable time generally elapses between the completion of the embankment and the actual running of trains, the grade *ad* will generally be nearly flattened down to its ultimate form before traffic commences, but such grades are occasionally objectionable if added to what is already a ruling grade. With some kinds of soil the time required for complete settlement may be as much as two or three years, but, even in such cases, it is probable that one-half of the settlement will take place during the first six months. The engineer should therefore require the contractor to make all fills about 8 to 15% (according to the material) higher than the profiles call for, in order that subsequent shrinkage may not reduce it to less than the required volume.

98. Methods of forming embankments. When the method is not otherwise objectionable, a high embankment can be formed very cheaply (assuming that carts or wheelbarrows are used) by dumping over the end and building to the full height (or even higher, to allow for shrinkage) as the embankment proceeds. This allows more time for shrinkage, saves nearly all the cost of spreading (see Item 4, § 111), and reduces the cost of roadways

(Item 5). Of course this method is especially applicable when the material comes from a place as high as or higher than grade, so that no up-hill hauling is required.

Another method is to spread it in layers two or three feet thick (see Fig. 60), which are made concave upwards to avoid

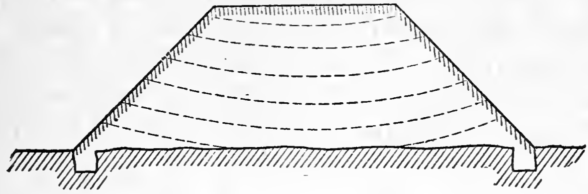


Fig. 60.

possible sliding on each other. Spreading in layers has the advantage of partially ramming each layer, so that the subsequent shrinkage is very small. Sometimes small trenches are dug along the lines of the toes of the embankment. This will frequently prevent the sliding of a large mass of the embankment, which will then require extensive and costly repairs, to say nothing of possible accidents if the sliding occurs after the road is in operation. Incidentally these trenches will be of value in draining the subsoil. When circumstances require an embankment on a hillside, it is advisable to cut out "steps" to prevent a possible sliding of the whole embankment. Merely ploughing the side-hill will often be a cheaper and sufficiently effective method.

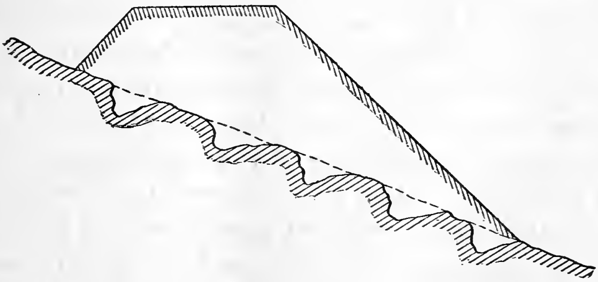


Fig. 61.

Occasionally the formation of a very high and long embankment may be most easily and cheaply accomplished by building a trestle to grade and opening the road. Earth can then be procured where most convenient, perhaps several miles away, loaded on cars with a steam-shovel, hauled by the trainload, and dumped from the cars with a patent unloader. On such a large scale, the cost per yard would be very much less than by ordinary methods—enough less sometimes to more than pay for the temporary trestle, besides allowing the road to be opened for traffic very much earlier, which is often a matter of prime financial importance. It may also obviate the necessity for extensive borrow-pits in the immediate neighborhood of the heavy fill and also utilize material which would otherwise be wasted.

COMPUTATION OF HAUL.

99. Nature of subject. As will be shown later when analyzing the cost of earthwork, the most variable item of cost is that depending on the distance hauled. As it is manifestly impracticable to calculate the exact distance to which every individual cartload of earth has been moved, it becomes necessary to devise a means which will give at least an equivalent of the haulage of all the earth moved. Evidently the *average* haul for any mass of earth moved is equal to the distance from the center of gravity of the excavation to the center of gravity of the embankment formed by the excavated material. As a rough approximation the center of gravity of a cut (or fill) may sometimes be considered to coincide with the center of gravity of that part of the profile representing it, but the error is frequently very large. The center of gravity may be determined by various methods, but the method of the "mass diagram" accomplishes the same ultimate purpose (the determination of the haul) with all-sufficient accuracy and also furnishes other valuable information.

100. Mass diagram. In Fig. 62 let $A'B' \ldots G'$ represent a profile and grade line drawn to the usual scales. Assume A' to be a point past which no earthwork will be hauled. Such a point is determined by natural conditions, as, for example, a river crossing, or one end of a long level stretch along which no grading is to be done except the formation of a low embankment from the material excavated from ample drainage ditches on each side. Above the profile draw an indefinite horizontal line (ACn in Fig. 62), which may be called the "zero line." Above every station point in the profile draw an ordinate (above or below the zero line) which will represent the algebraic sum of the cubic yards of cut and fill (calling cut + and fill −) from the point A' to the point considered. The computations of these ordinates should first be made in tabular form as shown below. In doing this shrinkage must be allowed for by considering how much embankment would actually be made by so many cubic yards of excavation of such material. For example, it will be found that 1000 cubic yards of sand or gravel, measured in place (see § 97), will make about 920 cubic yards of embankment; therefore all cuttings in sand or gravel should be discounted in about this proportion. Excavations in rock should be increased in the proper ratio. In short, all excavations should be valued according to the amount of *settled* embankment that could be made from them. Place in the first column a list of the stations; in the second column, the number of cubic yards of cut or fill between each station and the preceding station; in the third and fourth columns, the kind of material and the proper shrinkage factor; in the fifth column, a repetition of the quantities in cubic yards, except that the excavations are diminished (or increased, in the case of rock) to the number of cubic yards of settled embankment which may be made from them. In the sixth column place the *algebraic sum* of the quantities in the fifth column (calling cuts + and fills −) from the starting-point to the station considered. These algebraic sums at each station will be the ordinates, drawn to some scale, of the mass curve. The scale to be used will depend somewhat on whether

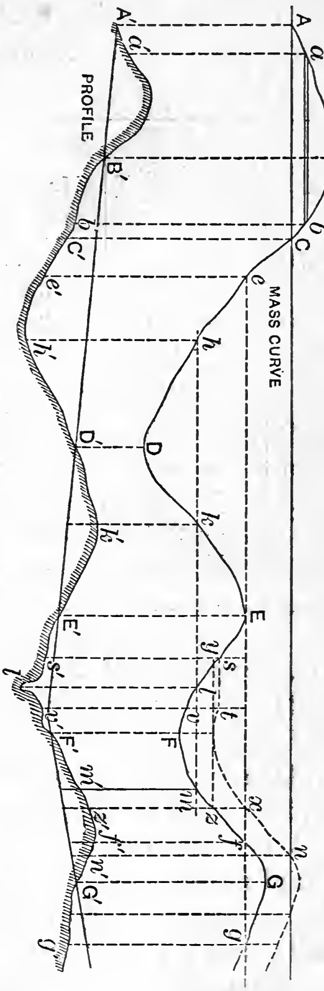


FIG. 62.—MASS DIAGRAM.

the work is heavy or light, but for ordinary cases a scale of 5000 cubic yards per inch may be used. Drawing these ordinates to scale, a curve *A*, *B*, . . . *G* may be obtained by joining the extremities of the ordinates.

Sta.	Yards { cut + / fill −	Material.	Shrinkage factor.	Yards, reduced for shrinkage.	Ordinate in mass curve.
46 + 70	. . . .	. . . .	. . . .	. . . .	0
47	+ 195	Clayey soil	− 10 per cent	+ 175	+ 175
48	+ 1792	" "	− 10 "	+ 1613	+ 1788
+ 60	+ 614	" "	− 10 "	+ 553	+ 2341
49	− 143	. . . .	. . . .	− 143	+ 2198
50	− 906	. . . .	. . . .	− 906	+ 1292
51	− 1985	. . . .	. . . .	− 1985	− 693
52	− 1721	. . . .	. . . .	− 1721	− 2414
+ 30	− 112	. . . .	. . . .	− 112	− 2526
53	+ 177	Hard rock	+ 60 per cent	+ 283	− 2243
+ 70	+ 180	" "	+ 60 "	+ 289	− 1954
54	− 52	. . . .	. . . .	− 52	− 2006
+ 42	− 71	. . . .	. . . .	− 71	− 2077
55	+ 276	Clayey soil	− 10 per cent	+ 249	− 1828
56	+ 1242	" "	− 10 "	+ 1118	− 710
57	+ 1302	" "	− 10 "	+ 1172	+ 462

101. Properties of the mass curve.

1. The curve will be rising while over cuts and falling while over fills.

2. A tangent to the curve will be horizontal (as at *B*, *D*, *E*, *F*, and *G*) when passing from cut to fill or from fill to cut.

3 When the curve is *below* the "zero line" it shows that material must be drawn *backward* (to the left); and *vice versa*, when the curve is *above* the zero line it shows that material must be drawn *forward* (to the right).

4. When the curve crosses the zero line (as at *A* and *C*) it shows (in this instance) that the cut between *A′* and *B′* will just provide the material required for the fill between *B′* and *C′*, and that no material should be hauled past *C′*, or, in general, past any intersection of the mass curve and the zero line.

5. If any horizontal line be drawn (as *ab*), it indicates that the cut and fill between *a′* and *b′* will just balance.

6. When the center of gravity of a given volume of material is to be moved a given distance, it makes no difference (at least theoretically) how far each individual load may be hauled or how any individual load may be disposed of. The summation

of the products of each load times the distance hauled will be a constant, whatever the method, and will equal the total volume times the movement of the center of gravity. The *average haul*, which is the movement of the center of gravity, will therefore equal the summation of these products divided by the total volume. If we draw two horizontal parallel lines at an infinitesimal distance dx apart, as at ab, the small increment of cut dx at a' will fill the corresponding increment of fill at b', and this material must be hauled the distance ab. Therefore the product of ab and dx, which is the product of distance times volume, is represented by the area of the infinitesimal rectangle at ab, and the total area ABC represents the summation of volume times distance for all the earth movement between A' and C'. This summation of products divided by the total volume gives the average haul.

7. The horizontal line, tangent at E and cutting the curve at e, f, and g, shows that the cut and fill between e' and E' will just balance, and that a *possible* method of hauling (whether desirable or not) would be to "borrow" earth for the fill between C' and e', use the material between D' and E' for the fill between e' and D', and similarly balance cut and fill between E' and f' and also between f' and g'.

8. Similarly the horizontal line $hklm$ may be drawn cutting the curve, which will show another *possible* method of hauling. According to this plan, the fill between C' and h' would be made by borrowing; the cut and fill between h' and k' would balance; also that between k' and l' and between l' and m'. Since the area $ehDkE$ represents the measure of haul for the earth between e' and E', and the other areas measure the corresponding hauls similarly, it is evident that the sum of the areas $ehDkE$ and $ElFmf$, which is the measure of haul of all the material between e' and f', is largely in excess of the sum of the areas hDk, kEl, and lFm, plus the somewhat uncertain measures of haul due to borrowing material for $e'h'$ and wasting the material between m' and f'. Therefore to make the measure of haul a minimum a line should be drawn which will make the sum of the areas between it and the mass curve a minimum. Of course this is not necessarily the cheapest plan, as it implies more or less borrowing and wasting of material, which *may* cost more than the amount saved in haul. The comparison of the two methods is quite simple, however. Since the amount

of fill between e' and h' is represented by the *difference* of the ordinates at e and h, and similarly for m' and f', it follows that the amount to be borrowed between e' and h' will exactly equal the amount wasted between m' and f'. By the first of the above methods the haul is excessive, but is definitely known from the mass diagram, and all of the material is utilized; by the second method the haul is reduced to about one-half, but there is a known quantity in cubic yards wasted at one place and the same quantity borrowed at another. The length of haul necessary for the borrowed material would need to be ascertained; also the haul necessary to waste the other material at a place where it would be unobjectionable. Frequently this is best done by widening an embankment beyond its necessary width. The computation of the relative cost of the above methods will be discussed later (§ 116).

9. Suppose that it were deemed best, after drawing the mass curve, to introduce a trestle between s' and v', thus saving an amount in fill equal to tv. If such had been the original design, the mass curve would have been a straight horizontal line between s and t and would continue as a curve which would be at all points a distance tv above the curve $vFmzfGg$. If the line Ef is to be used as a zero line, its intersection with the new curve at x will show that the material between E' and z' will just balance if the trestle is used, and that the amount of haul will be measured by the area between the line Ex and the broken line $Estx$. The same computed result may be obtained without drawing the auxiliary curve txn . . . by drawing the horizontal line zy at a distance $xz(=tv)$ below Ex. The amount of the haul can then be obtained by adding the triangular area between Es and the horizontal line Ex, the rectangle between st and Ex, and the irregular area between vFz and y . . . z (which last is evidently equal to the area between tx and E . . . x). The disposal of the material at the right of z' would then be governed by the indications of the profile and mass diagram which would be found at the right of g'. In fact it is difficult to decide with the best of judgment as to the proper disposal of material without having a mass diagram extending to a considerable distance each side of that part of the road under immediate consideration.

102. Area of the mass curve. The area may be computed most readily by means of a planimeter, which is capable with reasonable care of measuring such areas with as great accuracy as is necessary for this work. If no such instrument is obtainable, the area may be obtained by an application of "Simpson's rule." The ordinates will usually be spaced 100 feet apart. Select an *even* number of such spaces, leaving, if necessary, one or more triangles or trapezoids at the ends for separate and independent computation. Let $y_0 \ldots y_n$ be the ordinates, i.e., the number of cubic yards at each station of the mass curve, or the figures of "column six" referred to in § 100. Let the uniform distance between ordinates (=100 feet) be called 1, i.e., one *station*. Then the units of the resulting area will be cubic yards hauled one station. Then the

$$\text{Area} = \tfrac{1}{3}[y_0 + 4(y_1 + y_3 + \ldots y_{(n-1)}) + 2(y_2 + y_4 + \ldots y_{(n-2)}) + y_n]. \qquad (70)$$

When an ordinate occurs at a substation, the best plan is to ignore it at first and calculate the area as above. Then, if the difference involved is too great to be neglected, calculate the area of the triangle having the extremity of the ordinate at the substation as an apex, and the extremities of the ordinates at the adjacent stations as the ends of the base. This may be done by finding the ordinate at the substation that would be a proportional between the ordinates at the adjacent full stations. Subtract this from the real ordinate (or *vice versa*) and multiply the difference by $\frac{1}{2} \times 1$. An inspection will often show that the correction thus obtained would be too small to be worthy of consideration. If there is more than one substation between two full stations, the corrective area will consist of two triangles and one or more trapezoids which may be similarly computed, if necessary.

When the zero line (Fig. 62) is shifted to eE, the drop from AC (produced) to E is known in the same units, cubic yards. This constant may be subtracted from the numbers ("column 6," § 100) representing the ordinates, and will thus give, without any scaling from the diagram, the exact value of the modified ordinates.

103. Value of the mass diagram. The great value of the mass diagram lies in the readiness with which different plans for the disposal of material may be examined and compared. When the mass curve is once drawn, it will generally require only a shifting of the horizontal line to show the disposal of the material by any proposed method. The mass diagram also shows the extreme length of haul that will be required by any proposed method of disposal of material. This brings into consideration the "limit of profitable haul," which will be fully discussed in § 116. For the present it may be said that with each method of carrying material there is some limit beyond which the expense of hauling will exceed the loss resulting from borrowing and wasting. With wheelbarrows and scrapers the limit of profitable haul is comparatively short, with carts and tram-cars it is much longer, while with locomotives and cars it may be several miles. If, in Fig. 62, *eE* or *Ef* exceeds the limit of profitable haul, it shows at once that some such line as *hklm* should be drawn and the material disposed of accordingly.

104. **Changing the grade line.** The formation of the mass curve and the resulting plans as to the disposal of material are based on the mutual relations of the grade line and the surface profile and the amounts of cut and fill which are thereby implied. If the grade line is altered, every cross-section is altered, the amount of cut and fill is altered, and the mass curve is also changed. At the farther limit of the actual change of the grade line the revised mass curve will have (in general) a different ordinate from the previous ordinate at that point. From that point on, the revised mass curve will be parallel to its former position, and the revised curve may be treated similarly to the case previously mentioned in which a trestle was introduced. Since it involves tedious calculations to determine accurately how much the volume of earthwork is altered by a change in grade line, especially through irregular country, the effect on the mass curve of a change in the grade line cannot therefore be readily determined except in an approximate way. Raising the grade line will evidently increase the fills and diminish the cuts, and *vice versa*. Therefore if the mass curve indicated, for example, either an excessively long haul or the necessity for borrowing material (implying a fill) and wasting material farther on (implying a cut), it would be *possible* to diminish the fill (and hence the amount of material to be borrowed) by lowering the grade line near that place, and diminish the cut (and hence the amount of material to be wasted) by raising the grade line at or near the place farther on. Whether the advantage thus gained would compensate for the possibly injurious effect of these changes on the grade line would require patient investigation. But the method outlined shows how the mass

curve might be used to indicate a possible change in grade line which might be demonstrated to be profitable.

105. Limit of free haul. It is sometimes specified in contracts for earthwork that *all* material shall be entitled to free haul up to some specified limit, say 500 or 1000 feet, and that all material drawn farther than that shall be entitled to an allowance on the *excess* of distance. It is manifestly impracticable to measure the excess for each load, as much so as to measure the actual haul of each load The mass diagram also solves this problem very readily. Let Fig. 63 represent a pro-

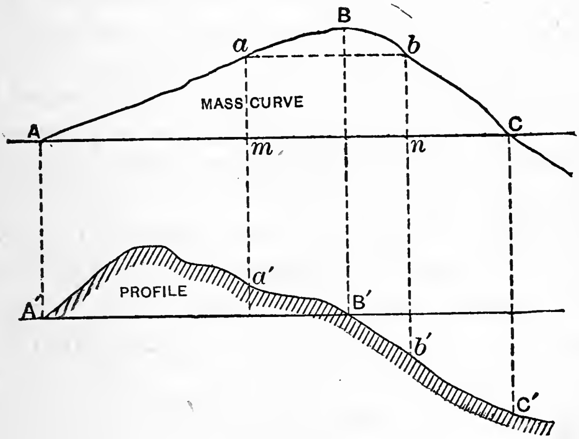


FIG. 63.

file and mass diagram of about 2000 feet of road, and suppose that 800 feet is taken as the limit of free haul. Find two points, a and b, in the mass curve *which are on the same horizontal line* and which are 800 feet apart. Project these points down to a' and b'. Then the cut and fill between a' and b' will just balance, and the cut between A' and a' will be needed for the fill between b' and C'. In the mass curve, the area between the horizontal line ab and the curve aBb represents the haulage of the material between a' and b', which is all free. The rectangle $abmn$ represents the haulage of the material in the cut $A'a'$ across the 800 feet from a' to b'. This is also free. The sum of the two areas Aam and bnC represents the haulage entitled to an allowance, since it is the summation of the products of cubic yards times the *excess* of distance hauled.

If the amount of cut and fill was symmetrical about the point

B', the mass curve would be a symmetrical curve about the vertical line through B, and the two limiting lines of free haul would be placed symmetrically about B and B'. In general there is no such symmetry, and frequently the difference is considerable The area $aBbnm$ will be materially changed according as the two vertical lines am and bn, always 800 feet apart, are shifted to the right or left. It is easy to show that the area $aBbnm$ is a *maximum* when ab is horizontal. The minimum value would be obtained either when m reached A or n reached C, depending on the exact form of the curve. Since the position for the minimum value is manifestly unfair, the best definite value obtainable is the maximum, which must be obtained as above described. Since $aBbnm$ is made maximum, the remainder of the area, which is the allowance for overhaul, becomes a minimum. The areas Aam and bCn may be obtained as in § 102. If the whole area $AaBbCA$ has been previously computed, it may be more convenient to compute the area $aBbnm$ and subtract it from the total area.

Since the intersections of the mass curve and the "zero line" mark limits past which no material is drawn, it follows that there will be no allowance for overhaul except where the distance between consecutive intersections of the zero line and mass curve exceeds the limit of free haul.

Frequently all allowances for overhaul are disregarded: the profiles, estimates of quantities, and the required disposal of material are shown to bidding contractors, and they must then make their own allowances and bid accordingly. This method has the advantage of avoiding possible disputes as to the amount of the overhaul allowance, and is popular with railroad companies on this account. On the other hand the facility with which different plans for the disposal of material may be studied and compared by the mass-curve method facilitates the adoption of the most economical plan, and the elimination of uncertainty will frequently lead to a safe reduction of the bid, and so the method is valuable to both the railroad company and the contractor.

ELEMENTS OF THE COST OF EARTHWORK.

(The following analysis of the cost of earthwork follows the general method given in the well-known papers published by

Ellwood Morris, C.E., in the Journal of the Franklin Institute in September and October, 1841. Numerous corroborative data have been obtained from various other sources, and also figures on methods not then in vogue.)

106. General divisions of the subject. The variations in the cost of earthwork are caused by the greatly varying conditions under which the work is done, chief among which is character of material, method of carriage, and length of haul. Any general system of computation must therefore differentiate the total cost into such elementary items that all differences due to variations in conditions may be allowed for. The variations due to character of material will be allowed for by an estimate on loose light sandy soil, and also an estimate on the heaviest soils, such as stiff clay and hard-pan. These represent the extremes (excluding rock, which will be treated separately), and the cost of intermediate grades must be estimated by interpolating between the extreme values. The general divisions of the subject will be: *

1. Loosening.
2. Loading.
3. Hauling.
4. Spreading.
5. Keeping roadways in order.
6. Repairs, wear, depreciation, and interest on cost of plant.
7. Superintendence and incidentals.
8. Contractor's profit.

By making the estimates on the basis of $1 per day for the cost of common labor, it is a simple matter to revise the estimates according to the local price of labor by multiplying the final estimates of cost by the price of labor in dollars per day.

107. Item 1. LOOSENING. (a) **Ploughs.** Very light sandy soils can frequently be shovelled without any previous loosening, but it is generally economical, even with very light material, to use a plough. Morris quotes, as the results of experiments, that a three-horse plough would loosen from 250 to 800 cubic yards of earth per day, which at a valuation of $5 per day would make the cost per yard vary from 2 cents to 0.6 cent. Trautwine estimates the cost on the basis of two men handling a two-horse plough at a total cost of $3.87 per day, being $1 each for

---

* Trautwine.

the men, 75 c. for each horse, and an allowance of 37 c. for the plough, harness, etc. From 200 to 600 cubic yards is estimated as a fair day's work, which makes a cost of 1.9 c. to 0.65 c. per yard, which is substantially the same estimate as above. Extremely heavy soils have sometimes been loosened by means of special ploughs operated by traction-engines.

(b) **Picks.** When picks are used for loosening the earth, as is frequently necessary and as is often done when ploughing would perhaps be really cheaper, an estimate * for a fair day's work is from 14 to 60 cubic yards, the 14 yards being the estimate for stiff clay or cemented gravel, and the 60 yards the estimate for the lightest soil that would require loosening. At $1 per day this means about 7 c. to 1.7 c. per cubic yard, which is about three times the cost of ploughing. Five feet of the face is estimated † as the least width along the face of a bank that should be allowed to enable each laborer to work with freedom and hence economically.

(c) **Blasting.** Although some of the softer shaly rocks may be loosened with a pick for about 15 to 20 c. per yard, yet rock in general, frozen earth, and sometimes even compact clay are most economically loosened by blasting. The subject of blasting will be taken up later, §§ 117-123.

(d) **Steam-shovels.** The items of loosening and loading merge together with this method, which will therefore be treated in the next section.

108. Item 2. Loading. (a) Hand-shovelling. Much depends on proper management, so that the shovellers need not wait unduly either for material or carts. With the best of management considerable time is thus lost, and yet the intervals of rest need not be considered as entirely lost, as it enables the men to work, while actually loading, at a rate which it would be physically impossible for them to maintain for ten hours. Seven shovellers are sometimes allowed for each cart; otherwise there should be five, two on each side and one in the rear. Economy requires that the number of loads per cart per day should be made as large as possible, and it is therefore wise to employ as many shovellers as can work without mutual interference and without wasting time in waiting for material or carts. The figures obtainable for the cost of this item are unsatisfactory on

* Trautwine. † Hurst.

account of their large disagreements. The following are quoted as the number of cubic yards that can be loaded into a cart by an average laborer in a working day of ten hours, the lower estimate referring to heavy soils, and the higher to light sandy soils: 10 to 14 cubic yards (Morris), 12 to 17 cubic yards (Haskoll), 18 to 22 cubic yards (Hurst), 17 to 24 cubic yards (Trautwine), 16 to 48 cubic yards (Ancelin). As these estimates are generally claimed to be based on actual experience, the discrepancies are probably due to differences of management. If the average of 15 to 25 cubic yards be accepted, it means, on the basis of $1 per day, 6.7 c. to 4 c. per cubic yard. These estimates apply only to earth. *Rockwork* costs more, not only because it is harder to handle, but because a cubic yard of solid rock, measured in place, occupies about 1.8 cubic yards when broken up, while a cubic yard of earth will occupy about 1.2 cubic yards. Rockwork will therefore require about 50% more loads to haul a given volume, *measured in place*, than will the same nominal volume of earthwork. The above authorities give estimates for loading rock varying from 6.9 c. to 10 c. per cubic yard. The above estimates apply only to the loading of carts or cars with shovels or by hand (loading masses of rock). The cost of loading wheelbarrows and the cost of scraper work will be treated under the item of hauling.

(b) **Steam-shovels.*** Whenever the magnitude of the work will warrant it there is great economy in the use of steam-shovels. These have a "bucket" or "dipper" on the end of a long beam, the bucket having a capacity varying from $\frac{1}{2}$ to $2\frac{1}{2}$ cubic yards. Steam-shovels handle all kinds of material from the softest earth to shale rock, earthy material containing large boulders, tree-stumps, etc. The capacity of the larger sizes is about 3000 cubic yards in 10 hours. They perform all the work of loosening and loading. Their economical working requires that the material shall be hauled away as fast as it can be loaded, which usually means that cars on a track, hauled by horses or mules, or still better by a locomotive, shall be used. The expenses for a steam-shovel, costing about $5000, will average about $1000 per month. Of this the engineer will get $100; the

* For a thorough treatment of the capabilities, cost, and management of steam-shovels the reader is referred to "Steam-shovels and Steam-shovel Work," by E. A. Hermann. D. Van Nostrand Co., New York.

fireman $50; the cranesman $90; repairs perhaps $250 to $300; coal, from 15 to 25 tons, cost very variable on account of expensive hauling; water, a very uncertain amount, sometimes costing $100 per month; about five laborers and a foreman, the laborers getting $1.25 per day and the foreman $2.50 per day, which will amount to $227.50 per month. This gang of laborers is employed in shifting the shovel when necessary, taking up and relaying tracks for the cars, shifting loaded and unloaded cars, etc. In shovelling through a deep cut, the shovel is operated so as to undermine the upper parts of the cut, which then fall down within reach of the shovel, thus increasing the amount of material handled for each new position of the shovel. If the material is too tough to fall down by its own weight, it is sometimes found economical to employ a gang of men to loosen it or even blast it rather than shift the shovel so frequently. Non-condensing engines of 50 horse-power use so much water that the cost of water-supply becomes a serious matter if water is not readily obtainable. The lack of water facilities will often justify the construction of a pipe line from some distant source and the installation of a steam-pump. Hence the seemingly large estimate of $100 per month for water-supply, although under favorable circumstances the cost may almost vanish. The larger steam-shovels will consume nearly a ton of coal per day of 10 hours. The expense of hauling this coal from the nearest railroad or canal to the location of the cut is often a very serious item of expense and may easily double the cost per ton. Some steam-shovels have been constructed to be operated by electricity obtained from a plant perhaps several miles away. Such a method is especially advantageous when fuel and water are difficult to obtain.

109. Item 3. HAULING. The cost of hauling depends on the number of round trips per day that can be made by each vehicle employed. As the cost of each vehicle is practically the same whether it makes many trips or few, it becomes important that the number of trips should be made a maximum, and to that end there should be as little delay as possible in loading and unloading. Therefore devices for facilitating the passage of the vehicles have a real money value.

(a) Carts. The average speed of a horse hauling a two-wheeled cart has been found to be 200 feet per minute, a little slower when hauling the load and a little faster when returning empty. This figure has been repeatedly verified. It means an allowance of one minute for each 100 feet (or "station") of "lead—the lead being the distance the earth is hauled." The time lost in loading, dumping, waiting to load, etc., has been found to average 4 minutes per load. Representing the number of stations (100 feet) of lead by s, the number of loads handled in 10 hours (600 minutes) would be $600 \div (s+4)$. The number of loads per cubic yard, measured in the bank, is differentiated by Morris into three classes, viz.:

3 loads per cubic yard in descending hauling;
$3\frac{1}{2}$ " " " " " level hauling; and
4 " " " " " ascending hauling.

Attempts have been made to estimate the effect of the grade of the roadway by a theoretical consideration of its rate, and of the comparative strength of a horse on a level and on various grades. While such computations are always practicable on a railway (even on a temporary construction track), the traction on a temporary earth roadway is always very large and so very variable that any refinements are useless. On railroad earthwork the hauling is generally nearly level or it is *descending*—forming embankments on low ground with material from cuts in high ground. The only common exception occurs when an embankment is formed from borrow-pits on low ground. One method of allowing for ascending grade is to add to the horizontal distance 14 times the difference of elevation for work with carts and 24 times the difference of elevation for work with wheelbarrows, and use that as the lead. For example, using carts, if the lead is 300 feet and there is a difference of elevation of 20 feet, the lead would be considered equivalent to $300+(14\times 20)=580$ feet on a level.

Trautwine assumes the average load for all classes of work to be $\frac{1}{3}$ cubic yard, which figure is justified by large experience. Using one figure for all classes of work simplifies the calculations and gives the number of cubic yards carried per day of 10 hours equal to $\frac{600}{3(s+4)}$. Dividing the cost of a cart per day by the number of cubic yards carried gives the cost of hauling per yard. In computing the cost of a cart per day, Trautwine refers to the practice of having one driver manage four carts, thus making a charge of 25 c. per day for each cart for the driver.

75 c. is allowed for the horse, which is supposed to be the total cost, including that for Sundays and rainy days. 25 c. more is allowed for the cart, harness, repairs, etc., thus making a total cost of \$1.25 per day. Some contractors employ a greater number of drivers and expect each to assist in loading. There is found to be no saving in total cost per yard, while the chances of loafing are perhaps greater. Morris instances five actual cases in which the cost of the cart (reduced to the basis of \$1 per day for labor) varied from \$1.37 to \$1.48. The items of these costs were not given.

Since the time required for loading loose rock is greater than for earthwork, less loads will be hauled per day. The time allowance for loading, etc., is estimated by Trautwine as 6 minutes instead of 4 as for earth. Considering the great expansion of rock when broken up (see § 97), one cubic yard of solid rock, measured in place, would furnish the equivalent of five loads of earthwork of $\frac{1}{3}$ cubic yard. Therefore, on the basis of five loads per cubic yard, the number of cubic yards handled per day per cart would be $\frac{600}{5(s+6)}$.

$$\text{Cost per yard in cents} = \frac{125 \times 5(s+6)}{600}. \quad . \quad . \quad (71)$$

(b) Wagons. For longer leads (i.e., from $\frac{1}{3}$ to $\frac{2}{3}$ of a mile) wagons drawn by two horses have been found most economical. The wagons have bottoms of loose thick narrow boards and are unloaded very easily and quickly by lifting the individual boards and breaking up the continuity of the bottom, thus depositing the load directly underneath the wagon. The capacity is about one cubic yard. The cost may be estimated on the same principles as that for carts.

(c) Wheelbarrows. According to Trautwine, the speed of moving wheelbarrows may be considered the same as for carts, 200 feet per minute; the time spent in loading and dumping is $1\frac{1}{4}$ minutes, and in addition about $\frac{1}{10}$ of the time is wasted in short rests, adjusting the wheeling planks, etc. On the basis of \$1 per day for labor, an allowance of 5 c. for the barrow, and 14 loads per cubic yard, the cost of hauling per cubic yard (computed on the same principles as above) will be

$$\frac{105 \times 14(s+1.25)}{600 \times 0.9} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

For rockwork the number of loads per cubic yard is estimated as 24, and the time spent in loading, etc., estimated at 1.6 minutes instead of 1.25 minutes, which makes the estimate

$$\text{Cost per cubic yard} = \frac{105 \times 24(s+1.6)}{600 \times 0.9}. \quad . \quad . \quad (73)$$

(d) Scrapers.* Scrapers, or scoops, are especially useful in canal work, and also for railroad work when a low embankment is to be formed from borrow-pits at the sides, when the distance does not exceed 100 feet, nor the vertical height 15 feet. The slope should not exceed 1.5 to 1. Under these conditions scraper work is cheaper than any other method. Scooping may be done all in one direction, in which case two half-turns are made for each load moved; or it may be done in both directions (from both sides on to a bank, or, in canal work, from the center to each bank), in which case one load is hauled to each half-turn. The capacity of the scoops (the "drag" variety) is $\frac{1}{10}$ cubic yard; the time lost in loading, unloading, and all other ways per load (except in turning) will average $\frac{2}{3}$ minute; the time lost in each half-turn (semi-circle) is $\frac{1}{3}$ minute; the speed of the horses may be estimated as 70 feet of *lead* per minute, the lead being here considered as the *sum* of the vertical and horizontal distances, and the estimate including the time of going and returning. If a represents the sum of the horizontal and vertical distances, the number of cubic yards handled per day of 10 hours by "side-scooping" will be

$$0.1\left(\frac{600}{\frac{a}{70}+1\frac{1}{3}}\right), \quad \text{which equals} \quad \frac{4200}{a+93\frac{1}{3}}.$$

For "double-scooping" the formula becomes

$$0.1\left(\frac{600}{\frac{a}{70}+1}\right), \quad \text{which equals} \quad \frac{4200}{a+70}.$$

Dividing the cost of a scraper per day (estimated at \$2.75) by the number of yards handled per day gives the average cost per yard.

* Condensed from Journ. Franklin Inst., Oct. 1841, by Morris.

Except in very loose sandy soil it is best to plough the earth first, which will cost *about* 1 c. per yard. (See § 107.) Drag-scrapers are now made chiefly of steel, and their capacity is more nearly 0.15 cubic yard. Wheeled scrapers, having a capacity of about 0.5 cubic yard, are frequently used with even greater economy and for greater distances, as they are cheaper than carts up to 250 or 300 feet of lead. Both drag- and wheel-scrapers are best operated in gangs of perhaps 10, using extra or "snap" teams to help load, and a few extra men to help in loading and unloading. The average cost of one scraper per day may thus be easily calculated and the average number of cubic yards handled per day computed as above, from which the cost per yard may be estimated.

(e) **Cars and horses.** The items of cost by this method are (*a*) charge for horses employed, (*b*) charge for men employed strictly in hauling, (*c*) charge for shifting rails when necessary, (*d*) repairs, depreciation, and interest on cost of cars and track. Part of this cost should strictly be classified under items 5 and 6, mentioned in § 106, but it is perhaps more convenient to estimate them as follows:

The traction of a car on rails is so very small and constant that grade resistance constitutes a very large part of the total resistance if the grade is 1% or more. For all ordinary grades it is sufficiently accurate to say that the grade resistance is to the gross weight as the rise is to the distance. If the distance is supposed to be measured along the slope, the proportion is strictly true; i.e., on a 1% grade the grade resistance is 1 lb. per 100 of weight or 20 lbs. per ton. If the resistance on a level at the usual velocity is $\frac{1}{120}$, a grade of 1 : 120 (0.83%) will exactly double it. If the material is hauled *down* a grade of 1 : 120, the cars will run by gravity after being started. The work of hauling will then consist practically of hauling the empty cars up the grade. The grade resistance depends only on the rate of grade and the weight, but the tractive resistance will be *greater per ton of weight* for the unloaded than for the loaded cars. The tractive power of a horse is less on a grade than on a level, not only because the horse raises his own weight in addition to the load, but is anatomically less capable of pulling on a grade than on a level. In general it will be possible to plan the work so that loaded cars need not be hauled *up* a grade, unless an embankment is to be formed from a low borrow-pit, in which case another method would probably be advisable. These computations are chiefly utilized in designing the method of work—the proportion of horses to cars. An example may be quoted from English practice (Hurst), in which the cars had a capacity of $3\frac{1}{3}$ cubic yards, weighing 30 cwt. empty. Two horses took five "wagons" $\frac{3}{4}$ of a mile on a level railroad and made 15 journeys per day of 10 hours, i.e., they handled 250 yards per day. In addition to those on the "straight road," another horse was employed to make up the train of loaded wagons. With a short lead the straight-road horses were employed for this purpose. In the above example the number of men required to handle these cars, shift the tracks, etc., is not given, and so the exact cost of the above work cannot be analyzed. It may be noticed that the two horses travelled $22\frac{1}{2}$ miles per day, drawing in one direction a load, including the weight of the cars, of about 57,300 lbs., or 28.65 net tons. Allowing $\frac{1}{120}$ as the necessary tractive force, it would require a pull of 477.5 lbs., or 239 lbs. for each horse. With a velocity of 220 feet per minute this would amount to $1\frac{1}{2}$ horse-power per horse, exerted for only a short time, however, and allowing considerable time for rest and for drawing only the empty cars. The cars generally used in this country have a capacity of $1\frac{1}{2}$ cubic yards and cost about \$65 apiece. Besides the shovellers and dumping-gang, several men and a foreman will be required to keep the track in order and to make the constant shifts that are necessary. Two trains are generally used, one of which is loaded while the other is run to the dump. Some passing-place is necessary, but this is generally provided by having a switch at the cut and running the trains on each track alternately. This insures a train of cars always at the cut to keep the shovellers employed. The cost of hauling per cubic yard can only be computed when the number of laborers, cars, and horses employed are known, and these will depend on the lead, on the character of the excavation, on the grade, if any, etc., and must be so proportioned that the shovellers need not wait for cars to fill, nor the dumping-gang for material to handle, nor the horses and drivers for cars to haul. Much skill is necessary to keep a large force in smooth running order.

(f) **Cars and locomotives.** 30-lb. rails are the lightest that should be used for this work, and 35- or 40-lb. rails are better. One or two narrow-gauge locomotives (depending on the length of haul), costing about \$2500 each, will be necessary to handle two trains of about 15 cars each, the cars having a capacity of about 2 cubic yards and costing about \$100 each. Some cars can be obtained as low as \$70. A force of about five mea nnd a foreman will be required to shift the tracks. The track-shifters, except the foreman, may be common laborers. The dumping-gang will require about seven men. Even when the material is all taken *down* grade the grades may be too steep for the safe hauling of loaded cars down the grade, or for hauling empty cars up the grade. Under such circumstances temporary trestles are necessary to reduce the grade. When these are used, the uprights and bracing are left in the embankment—only the stringers being removed. This is largely a necessity, but is partially compensated by the fact that the trestle forms a core to the embankment which prevents lateral shifting during settlement. The average speed of the trains may be taken as 10 miles per hour or 5 miles of lead per hour. The time lost in loading and unloading is estimated (Trautwine) as 9 minutes or .15 of an hour. The number of trips per day of 10 hours will equal $\dfrac{10}{\frac{1}{5}\text{ (miles of lead)} + .15}$ or $\dfrac{50}{\text{(miles of lead)} + .75}$. Of course this quotient *must* be a whole number. Knowing the number of trains and their capacity, the total number of cubic yards handled is known, which, divided into the total daily cost of the trains, will give the cost of hauling per yard. The daily cost of a train will include

(*a*) Wages of engineer, who frequently fires his own engine;

(*b*) Fuel, about $\frac{1}{4}$ to 1 ton of bituminous coal, depending on work done;

(*c*) Water, a very variable item, frequently costing \$3 to \$5 per day;

(*d*) Repairs, variable, frequently at rate of 50 to 60% per year;

(*e*) Interest on cost and depreciation, 16 to 40%.

To these must be added, to obtain the total cost of the haui,

(*f*) Wages of the gang employed in shifting track.

110. Choice of method of haul dependent on distance. In light side-hill work in which material need not be moved more than 12 or 15 feet, i.e., moved *laterally* across the roadbed, the earth may be moved most cheaply by mere shovelling. Beyond 12 feet scrapers are more economical. At about 100 feet drag-scrapers and wheelbarrows are equally economical. Between 100 and 200 feet wheelbarrows are generally cheaper than either carts or drag-scrapers, but wheeled scrapers are always cheaper than wheelbarrows. Beyond 500 feet two-wheeled carts become the most economical up to about 1700 feet; then four-wheeled wagons become more economical up to 3500 feet. Beyond this cars on rails, drawn by horses or by locomotives, become cheaper. The economy of cars on rails becomes evident for distances as small as 300 feet provided the volume of the excavation will justify the outlay. Locomotives will always be cheaper than horses and mules providing the work to be done is of sufficient magnitude to justify the purchase of the necessary plant and risk the loss in selling the plant ultimately as second-hand equipment, or keeping the plant on hand and idle for an indefinite period waiting for other work. Horses will not be economical for distances much over a mile. For greater distances locomotives are more economical, but the question of "limit of profitable haul" (§ 116) must be closely studied, as the circumstances are certainly not common when it is advisable to haul material much over a mile.

111. Item 4. **SPREADING.** The cost of spreading varies with the method employed in dumping the load. When the earth is tipped over the edge of an embankment there is little if any necessary work. Trautwine allows about $\frac{1}{4}$ c. per cubic yard for keeping the dumping-places clear and in order. This would represent the wages of one man at $1 per day attending to the unloading of 1200 two-wheeled carts each carrying $\frac{1}{3}$ cubic yard. 1200 carts in 10 hours would mean an average of two per minute, which implies more rapid and efficient work than may be depended on. The allowance is probably too small. When the material is dumped in layers some levelling is required, for which Trautwine allows 50 to 100 cubic yards as a fair day's work, costing from 1 to 2 cents per cubic yard. The cost of spreading will not ordinarily exceed this and is frequently nothing—all depending on the method of unloading. It should be noted that Mr. Morris's examples and computations (Jour. Franklin Inst., Sept. 1841) disregard altogether any special charge for this item.

112. Item 5. **KEEPING ROADWAYS IN ORDER.** This feature is important as a measure of true economy, whatever the system of transportation, but it is often neglected. A petty saving in

such matters will cost many times as much in increased labor in hauling and loss of time. With some methods of haul the cost is best combined with that of other items.

(a) **Wheelbarrows.** Wheelbarrows should generally be run on planks laid on the ground. The adjusting and shifting of these planks is done by the wheelers, and the time for it is allowed for in the 10% allowance for "short rests, adjusting the wheeling plank, etc." The actual cost of the planks must be added, but it would evidently be a very small addition per cubic yard in a large contract. When the wheelbarrows are run on planks placed on "horses" or on trestles the cost is very appreciable; but the method is frequently used with great economy. The variations in the requirements render any general estimate of such cost impracticable.

(b) **Carts and wagons.** The cost of keeping roadways in order for carts and wagons is sometimes estimated merely as so much per cubic yard, but it is evidently a function of the *lead.* The work consists in draining off puddles, filling up ruts, picking up loose stones that may have fallen off the loads, and in general doing everything that will reduce the traction as much as possible. Temporary inclines, built to avoid excessive grade at some one point, are often measures of true economy. Trautwine suggests $\frac{1}{10}$ c. per cubic yard per 100 feet of lead for earthwork and $\frac{2}{10}$ c. for rockwork, as an estimate for this item when carts are used.

(c) **Cars.** When cars are used a shifting-gang, consisting of a foreman and several men (say five), are constantly employed in shifting the track so that the material may be loaded and unloaded where it is desired. The average cost of this item may be estimated by dividing the total daily cost of this gang by the number of cubic yards handled in one day.

113. Item 6. REPAIRS, WEAR, DEPRECIATION, AND INTEREST ON COST OF PLANT. The amount of this item evidently depends upon the character of the soil—the harder the soil the worse the wear and depreciation. The *interest on cost* depends on the current borrowing value of money. The estimate for this item has already been included in the allowances for horses, carts, ploughs, harness, wheelbarrows, steam-shovels, etc. Trautwine estimates $\frac{1}{4}$ c. per cubic yard for picks and shovels. Depreciation is generally a large percentage of the cost of earth-working tools, the life of all being limited to a few years, and of many tools to a few months.

114. Item 7. SUPERINTENDENCE AND INCIDENTALS. The incidentals include water-carriers, trimming cuts to grade, digging the side ditches, trimming up the sides of borrow-pits to prevent their becoming unsightly, etc. These last operations yield but little earth and cost far more than the price paid per cubic yard. Morris allows 1 c. per cubic yard for this item; Trautwine allows $1\frac{3}{4}$ to 2 c. for it; while others combine items 6 and 7 and call them 5% of the total cost, which method has the merit of making the cost of items 6 and 7 a function of the character of soil and length of lead.

115. Item 8. CONTRACTOR'S PROFIT. This is usually estimated at from 6 to 15%, according to the sharpness of the competition and the possible uncertainty as to true cost owing to unfavorable circumstances. The contractor's real profit may vary considerably from this. He often pays clerks, boards and lodges the laborers in shanties built for the purpose, or keeps a supply-store, and has various other items both of profit and expense. His profit is largely dependent on skill in so handling the men that all can work effectively without interference or delays in waiting for others. An unusual season of bad weather will often affect the cost very seriously. It is a common occurrence to find that two contractors may be working on the same kind of material and under precisely similar conditions and at the same price, and yet one may be making money and the other losing it—all on account of difference of management.

116. Limit of profitable haul. As intimated in §§ 103 and 110, there is with every method of haul a limit of distance beyond which the expense for excessive hauling will exceed the loss resulting from borrowing and wasting. This distance is somewhat dependent on local conditions, thus requiring an independent solution for each particular case, but the general principles involved will be about as follows: Assume that it has been determined, as in Fig. 62, that the cut and fill will exactly balance between two points, as between e and x, assuming that, as indicated in § 101 (9), a trestle has been introduced between s and t, thus altering the mass curve to $Estxn$ ... Since there is a balance between A' and C', the material for the fill between C' and e' must be obtained either by "borrowing" in the immediate neighborhood or by transportation from the excavation between z' and n'. If cut and fill have been approximately balanced in the selection of grade line, as is ordinarily done, borrowing material for the fill $C'e'$ implies a wastage of material at the cut $z'n'$. To compare the two methods, we may place against the plan of borrowing and wasting, (*a*) cost, if any, of extra right of way that may be needed from which to obtain earth for the fill $C'e'$; (*b*) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the borrow-pit and of the fill, and the other expenses incidental to borrowing M cubic yards for the fill $C'e'$; (*c*) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the cut $z'n'$ and of the spoil-bank, and the other expenses incidental to wasting M cubic yards at the cut $z'n'$; (*d*) cost, if any, of land needed for the spoil-bank. The cost of the other plan will be the cost of loosening, loading, hauling (the hauling being represented by the trapezoidal figure *Cexn*), and the other expenses incidental to making the fill $C'e'$ with the material from the cut $z'n'$, the amount of material being M cubic yards, which is represented in the figure by the vertical ordinate from e to the line Cn. The difference between these costs will be the cost, if any, of land for borrow-pit and spoil-bank *plus* the cost of loosening, loading, etc. (except hauling and roadways) of M cubic yards, *minus* the difference in cost of the excessive haul from Ce to xn and the comparatively short hauls from borrow-pit and to spoil-bank.

As an illustration, taking some of the estimates previously given for operating with average material, the cost of all items, except hauling and roadways, would be about as follows: loosening, with plough, 1.2 c., loading 5.0 c., spreading 1.5 c., wear, depreciation, etc., .25 c., superintendence, etc., 1.5 c.; total 8.95 c. Suppose that the haul for both borrowing and wasting averages 100 feet or 1 station. Then the cost of haul per yard, using carts, would be (§ 109, *a*) $[125\times3(1+4)]\div600=3.125$ c. The cost of roadways would be about 0.1 c. per yard, making a total of 3.225 c. per cubic yard. Assume $M=10000$ cubic yards and the area $Cexn=180000$ yards-stations or the equivalent of 10000 yards hauled 1800 feet. This haul would cost $[125\times3(18+4)]\div600=13.75$ c. per cubic yard. The cost of roadways will be $18\times.1$ or 1.8 c., making a total of 15.55 c. for hauling and roadways. The difference of cost of hauling and roadways will be $15.55-(2\times3.225)=9.10$ c. per yard or \$910 for the 10000 yards. Offsetting this is the cost of loosening, etc., 10000 yards, at 8.95 c., costing \$895. These figures may be better compared as follows:

Long Haul.	Loosening, etc.,	10000	yards,		@ 8.95 c.	\$ 895.
	Hauling,	10000	"		@ 15.55 c.	1555.
						\$2450.
Borrowing and Wasting.	Loosening, etc.,	10000	yards	(borrowed),	@ 8.95 c.	\$895.
	" "	10000	"	(wasted),	@ 8.95 c.	895.
	Hauling, etc.,	10000	"	(borrowed),	@ 3.225 c.	322.50
	" "	10000	"	(wasted),	@ 3.225 c.	322.50
						\$2435.00

These costs are practically balanced, but no allowance has been made for right of way. If any considerable amount had to be paid for that, it would decide this particular case in favor of the long haul. This shows that *under these conditions* 1800 feet is *about* the limit of profitable haul, the land costing nothing extra.

BLASTING.

117. Explosives. The effect of blasting is due to the extremely rapid expansion of a gas which is developed by the decomposition of a very small amount of solid matter. Blasting compounds may be divided into two general classes, (*a*) slow-burning and (*b*) detonating. Gunpowder is a type of the slow-burning compounds. These are generally ignited by heat; the ignition proceeds from grain to grain; the heat and pressure produced are comparatively low. Nitro-glycerine is a type of the detonating compounds. They are exploded by a shock which *instantaneously* explodes the whole mass. The heat and pressure developed are far in excess of that produced by the explosion of powder. Nitro-glycerine is so easily exploded that it is very dangerous to handle. It was discovered that if the nitro-glycerine was absorbed by a spongy material like infusorial earth, it was much less liable to explode, while its power when actually exploded was practically equal to that of the amount of pure nitro-glycerine contained in the dynamite, which is the name given to the mixture of nitro-glycerine and infusorial earth. Nitro-glycerine is expensive; many other explosive chemical compounds which properly belong to the slow-burning class are comparatively cheap. It has been conclusively demonstrated that a mixture of nitro-glycerine and some of the cheaper chemicals has a greater explosive force than the sum of the strengths of the component parts when exploded separately. Whatever the reason, the fact seems established. The reason is possibly that the explosion of the nitro-glycerine is sufficiently powerful to produce a *detonation* of the other chemicals, which is impossible to produce by ordinary means, and that this explosion caused by detonation is more powerful than an ordinary explosion. The majority of the explosive compounds and "powders" on the market are of this character—a mixture of 20 to 60 per cent. of nitro-glycerine with variable proportions of one or more of a great variety of explosive chemicals.

The choice of the explosive depends on the character of the rock. A hard brittle rock is most effectively blasted by a detonating compound. The rapidity with which the full force of the explosive is developed has a shattering effect on a brittle substance. On the contrary, some of the softer tougher rocks and indurated clays are but little affected by dynamite. The result is but little more than an enlargement of the blast-hole. Quarrying must generally be done with blasting-powder, as the quicker explosives are too shattering. Although the results obtained by various experimenters are very variable, it may be said that pure nitro-glycerine is eight times as powerful as black powder, dynamite (75% nitro-glycerine) six times, and gun-cotton four to six times as powerful. For open work where time is not particularly valuable, black powder is by far the cheapest, but in tunnel-headings, whose progress determines the progress of the whole work, dynamite is so much more effective and so expedites the work that its use becomes economical.

118. Drilling. Although many very complicated forms of drill-bars have been devised, the best form (with slight modifications to suit circumstances) is as shown in Fig. 64, (*a*) and (*b*). The width should flare at the bottom (*a*) about 15 to 30%. For hard rock the curve of the edge should be somewhat flatter and for soft rock somewhat more curved than shown, Fig. 64, (*a*). Sometimes the angle of the two faces is varied from that given, Fig. 64, (*b*), and occasionally the edge is purposely blunted so as to give a crushing rather than a cutting effect. The drills will require sharpening for each 6 to 18 inches depth of hole, and will require a new edge to be worked every 2 to 4 days.

For drilling vertical holes the *churn-drill* is the most economical. The drill-bar is of iron, about 6 to 8 feet long, $1\frac{1}{4}''$ in diameter, weighs about 25 to 30 lbs., and is shod with a piece of steel welded on. The bar is lifted a few inches between each blow, turned partially around, and allowed to fall, the impact doing the work. From 5 to 15 feet of holes, depending on the character of the rock, is a fair day's work—10 hours. In very soft rocks even more than this may be done. This method is

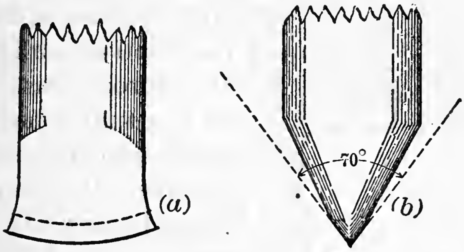
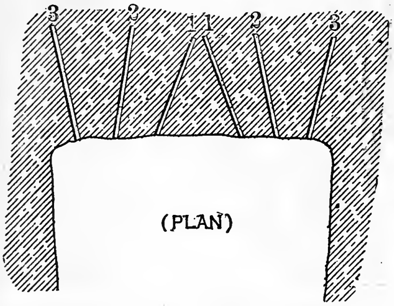


FIG. 64.

inapplicable for inclined holes or even for vertical holes in confined places, such as tunnel-headings. For such places the only practical *hand* method is to use hammers. This may be done by light drills and light hammers (one-man work), or by heavier drills held by one man and struck by one or two men with heavy hammers. The conclusion of an exhaustive investigation as to the relative economy of light or heavy hammers is that the light-hammer method is more economical for the softer rocks, the heavy-hammer method is more economical for the harder rocks, but that the light-hammer method is always more expeditious and hence to be preferred when time is important.

The subject of machine rock-drills is too vast to be treated here. The method is only practicable when the amount of work to be done is large, and especially when time is valuable. The machines are generally operated by compressed air for tunnel-work, thus doing the additional service of supplying fresh air to the tunnel-headings where it is most needed. The cost per foot of hole drilled is quite variable, but is usually somewhat less than that of hand-drilling—sometimes but a small fraction of it.

119. Position and direction of drill-holes. As the cost of drilling holes is the largest single item in the total cost of blasting, it is necessary that skill and judgment should be used in so locating the holes that the blasts will be most effective. The greatest effect of a blast will evidently be in the direction of the "line of least resistance." In a strictly homogeneous material this will be the shortest line from the center of the explosive to the surface. The variations in homogeneity on account of laminations and seams require that each case shall be judged according to experience. In open-pit blasting it is generally easy to obtain two and sometimes three exposed faces to the rock, making it a simple matter to drill holes so that a blast will do effective work. When a solid face of rock must be broken into, as in a tunnel-heading, the work is necessarily ineffectual and expensive. A conical or wedge-shaped mass will first be blown out by simultaneous blasts in the holes marked 1, Fig. 65; blasts in the holes marked 2 and 3 will then complete the cross-section of the heading. A great saving in cost may often be secured by skilfully taking advantage of seams, breaks, and irregularities. When the work is economically done there is but little noise or throwing of rock, a covering of old timbers and branches of trees generally sufficing to confine the smaller pieces which would otherwise fly up.



DRILL HOLES IN TUNNEL HEADING

FIG. 65.

120. Amount of explosive. The amount of explosive required varies as the cube of the line of least resistance. The best results are obtained when the line of least resistance is $\frac{3}{4}$ of the depth of the hole; also when the powder fills about $\frac{1}{3}$ of the hole. For average rock the amount of powder required is as follows:

Line of least resistance............	2 ft.	4 ft.	6 ft.	8 ft.
Weight of powder................	$\frac{1}{4}$ lb.	2 lbs.	$6\frac{3}{4}$ lbs.	16 lbs.

Strict compliance with all of the above conditions would require that the diameter of the hole should vary for every case. While this is impracticable, there should evidently be some variation in the size of the hole, depending on the work to be done. For example, a 1″ hole, drilled 2′ 8″ deep, with its line of least resistance 2′, and loaded with $\frac{1}{4}$ lb. of powder, would be filled to a depth of $9\frac{1}{2}''$, which is nearly $\frac{1}{3}$ of the depth. A $3''$ hole, drilled $8'$ deep, with its line of least resistance $6'$, and loaded with $6\frac{3}{4}$ lbs. of powder, would be filled to a depth of over $28''$, which is also nearly $\frac{1}{3}$ of the depth. One pound of blasting-powder will occupy about 28 cubic inches. Quarrying necessitates the use of numerous and sometimes repeated light charges of powder, as a heavy blast or a powerful explosive like dynamite is apt to shatter the rock. This requires more powder to the cubic yard than blasting for mere excavation, which may usually be done by the use of $\frac{1}{4}$ to $\frac{1}{3}$ lb. of powder per cubic yard of easy open blasting. On account of the great resistance offered by rock when blasted in headings in tunnels, the powder used per cubic yard will run up to 2, 4, and even 6 lbs. per cubic yard. As before stated, nitro-glycerine is about eight times (and dynamite about six times) as powerful as the same *weight* of powder.

121. Tamping. Blasting-powder and the slow-burning explosives require thorough tamping. Clay is probably the best, but sand and fine powdered rock are also used. Wooden plugs, inverted expansive cones, etc., are periodically reinvented by enthusiastic inventors, only to be discarded for the simpler methods. Owing to the extreme rapidity of the development of the force of a nitro-glycerine or dynamite explosion, tamping is not so essential with these explosives, although it unquestionably adds to their effectiveness. Blasting under water has been effectively accomplished by merely pouring nitro-glycerine into the drilled holes through a tube and then exploding the charge without any tamping except that furnished by the superincumbent water. It has been found that air-spaces about a charge make a material reduction in the effectiveness of the explosion. It is therefore necessary to carefully ram the explosive into a solid mass. Of course the liquid nitro-glycerine needs no ramming, but dynamite should be rammed with a *wooden* rammer. Iron should be carefully avoided in ramming gunpowder. A copper bar is generally used.

122. Exploding the charge. Black powder is generally exploded by means of a fuse which is essentially a cord in which there is a thin vein of gunpowder, the cord being protected by tar, extra linings of hemp, cotton, or even gutta-percha. The fuse is inserted into the middle of the charge, and the tamping carefully packed around it so that it will not be injured. To produce the detonation required to explode nitro-glycerine and dynamite, there must be an initial explosion of some easily ignited explosive. This is generally accomplished by means of caps containing fulminating-powder which are exploded by electricity. The electricity (in one class of caps) heats a very fine platinum wire to redness, thereby igniting the sensitive powder, or (in another class) a spark is caused to jump through the powder between the ends of two wires suitably separated. Dynamite can also be exploded by using a small cartridge of gunpowder which is itself exploded by an ordinary fuse.

123. **Cost.** Trautwine estimates the cost of blasting (for mere excavation) as averaging 45 cents per cubic yard, falling as low as 30 cents for easy but *brittle* rock, and running up to 60 cents and even $1 when the cutting is shallow, the rock especially tough, and the strata unfavorably placed. Soft tough rock frequently requires more powder than harder brittle rock.

124. **Classification of excavated material.** The classification of excavated material is a fruitful source of dispute between contractors and railroad companies, owing mainly to the fact that the variation between the softest earth and the hardest rock is so gradual that it is very difficult to describe distinctions between different classifications which are unmistakable and indisputable. The classification frequently used is (*a*) earth, (*b*) loose rock, and (*c*) solid rock. As blasting is frequently used to loosen "loose rock" and even "earth" (if it is frozen), the fact that blasting is employed cannot be used as a criterion, especially as this would (if allowed) lead to unnecessary blasting for the sake of classifying material as rock.

Earth. This includes clay, sand, gravel, loam, decomposed rock and slate, boulders or loose stones not greater than 1 cubic foot (3 cubic feet, P. R. R.), and sometimes even "hard-pan." In general it will signify material which *can* be loosened by a plough with two horses, or with which one picker can keep one shoveller busy.

Loose rock. This includes boulders and loose stones of more than one cubic foot and less than one cubic yard; stratified rock, not more than six inches thick, separated by a stratum of clay; also all material (not classified as earth) which may be loosened by pick or bar and which "*can* be quarried without blasting, although blasting may occasionally be resorted to."

Solid rock includes all rock found in masses of over one cubic yard which cannot be removed except by blasting.

It is generally specified that the engineer of the railroad company shall be the judge of the classification of the material, but frequently an appeal is taken from his decisions to the courts.

125. **Specifications for earthwork.** The following specifications, issued by the Norfolk and Western R. R., represent the average requirements. It should be remembered that very strict specifications invariably increase the cost of the work, and frequently add to the cost more than is gained by improved quality of work.

1. The grading will be estimated and paid for by the cubic yard, and will include clearing and grubbing, and all open excavations, channels, and embankments required for the formation of the roadbed, and for turnouts and sidings; cutting all ditches or drains about or contiguous to the road; digging the foundation-pits of all culverts, bridges, or walls; reconstructing turnpikes or common roads in cases where they are destroyed or interfered with; changing the course or channel of streams; and all other excavations or embankments connected with or incident to the construction of said Railroad.

2. All grading, except where otherwise specified, whether for cuts or fills, will be measured in the excavations and will be classified under the following heads, viz.: Solid Rock, Loose Rock, Hard-pan, and Earth.

SOLID ROCK shall include all rock occurring in masses which, in the judgment of the said Engineer Maintenance of Way, may be best removed by blasting.

LOOSE ROCK shall include all kinds of shale, soapstone, and other rock which, in the judgment of the said Engineer Maintenance of Way, can be removed by pick and bar, and is soft and loose enough to be removed without blasting, although blasting may be occasionally resorted to; also, detached stone of less than one (1) cubic yard and more than one (1) cubic foot.

HARD-PAN shall consist of tough indurated clay or cemented gravel, which requires blasting or other equally expensive means for its removal, or which cannot be ploughed with less than four horses and a railroad plough, or which requires two pickers to a shoveller, the said Engineer Maintenance of Way to be the judge of these conditions.

EARTH shall include all material of an earthy nature, of whatever name or character, not unquestionably loose rock or hardpan as above defined.

POWDER. The use of powder in cuts will not be considered as a reason for any other classification than earth, unless the material in the cut is clearly other than earth under the above specifications.

3. Earth, gravel, and other materials taken from the excavations, except when otherwise directed by the said Engineer Maintenance of Way or his assistant, shall be deposited in the adjacent embankment; the cost of removing and depositing which, when the distance necessary to be hauled is not more than sixteen hundred (1600) feet, shall be included in the price paid for the excavation.

4. EXTRA HAUL will be estimated and paid for as follows: whenever material from excavations is necessarily hauled a greater distance than sixteen hundred (1600) feet, there shall be paid in addition to the price of excavation the price of extra haul per 100 feet, or part thereof, after the first 1600 feet; the necessary haul to be determined in each case by the said Engineer Maintenance of Way or his assistant, from the profile and cross-sections, and the estimates to be in accordance therewith.

5. All embankments shall be made in layers of such thickness and carried on in such manner as the said Engineer Maintenance of Way or his assistant may prescribe, the stone and heavy materials being placed in slopes and top. And in completing the fills to the proper grade such additional heights and fulness of slope shall be given them, to provide for their settlement, as the said Engineer Maintenance of Way, or his assistant, may direct. Embankments about masonry shall be built at such times and in such manner and of such materials as the said Engineer Maintenance of Way or his assistant may direct.

6. In procuring materials for embankments from without the line of the road, and in wasting materials from cuttings, the place and manner of doing it shall in each case be indicated by the Engineer Maintenance of Way or his assistant; and care must be taken to injure or disfigure the land as little as possible. Borrow-pits and spoil-banks must be left by the Contractor in regular and sightly shape.

7. The lands of the said Railroad Company shall be cleared to the extent required by the said Engineer Maintenance of

Way, or his assistant, of all trees, brushes, logs, and other perishable materials, which shall be destroyed by burning or deposited in heaps as the said Engineer Maintenance of Way, or his assistant, may direct. Large trees must be cut not more than two and one-half ($2\frac{1}{2}$) feet from the ground, and under embankments less than four (4) feet high they shall be cut close to the ground. All small trees and bushes shall be cut close to the ground.

8. Clearing shall be estimated and paid for by the acre or fraction of an acre.

9. All stumps, roots, logs, and other obstructions shall be grubbed out, and removed from all places where embankments occur less than two (2) feet in height; also, from all places where excavations occur and from such other places as the said Engineer Maintenance of Way or his assistant may direct.

10. Grubbing shall be estimated and paid for by the acre or fraction of an acre.

11. Contractors, when directed by the said Engineer Maintenance of Way or his assistant in charge of the work, will deposit on the side of the road, or at such convenient points as may be designated, any stone, rock, or other materials that they may excavate; and all materials excavated and deposited as above, together with all timber removed from the line of the road, will be considered the property of the Railroad Company, and the Contractors upon the respective sections will be responsible for its safe-keeping until removed by said Railroad Company, or until their work is finished.

12. Contractors will be accountable for the maintenance of safe and convenient places wherever public or private roads are in any way interfered with by them during the progress of the work. They will also be responsible for fences thrown down, and for gates and bars left open, and for all damages occasioned thereby.

13. Temporary bridges and trestles, erected to facilitate the progress of the work, in case of delays at masonry structures from any cause, or for other reasons, will be at the expense of the Contractor.

14. The line of road or the gradients may be changed in any manner, and at any time, if the said Engineer Maintenance of Way or his assistant shall consider such a change necessary or expedient; but no claim for an increase in prices of excavation

or embankment on the part of the Contractor will be allowed or considered unless made in writing before the work on that part of the section where the alteration has been made shall have been commenced. The said Engineer Maintenance of Way or his assistant may also, on the conditions last recited, increase or diminish the length of any section for the purpose of more nearly equalizing or balancing the excavations and embankments, or for any other reason.

15. The roadbed will be graded as directed by the said Engineer Maintenance of Way or his assistant, and in conformity with such breadths, depths, and slopes of cutting and filling as he may prescribe from time to time, and no part of the work will be finally accepted until it is properly completed and dressed off at the required grade.

CHAPTER IV.

TRESTLES.

126. Extent of use. Trestles constitute from 1 to 3% of the length of the average railroad. It was estimated in 1889 that there was then about 2400 miles of single-track railway trestle in the United States, divided among 150,000 structures and estimated to cost about \$75,000,000. The annual charge for maintenance, estimated at $\frac{1}{8}$ of the cost, therefore amounted to about \$9,500,000 and necessitated the annual use of perhaps 300,000,000 ft. B. M. of timber. The corresponding figures at the present time must be somewhat in excess of this. The magnitude of this use, which is causing the rapid disappearance of forests, has resulted in endeavors to limit the use of timber for this purpose. Trestles may be considered as justifiable under the following conditions:

a. Permanent trestles.

1. Those of *extreme* height—then called viaducts and frequently constructed of iron or steel, as the Kinzua viaduct, 302 ft. high.

2. Those across waterways—*e.g.*, that across Lake Pontchartrain, near New Orleans, 22 miles long.

3. Those across swamps of soft deep mud, or across a river-bottom, liable to occasional overflow.

b. Temporary trestles.

1. To open the road for traffic as quickly as possible—often a reason of great financial importance.

2. To quickly replace a more elaborate structure, destroyed by accident, on a road already in operation, so that the interruption to traffic shall be a minimum.

3. To form an earth embankment with earth brought from a distant point by the train-load, when such a measure would cost less than to borrow earth in the immediate neighborhood.

4. To bridge an opening temporarily and thus allow time to learn the regimen of a stream in order to better proportion the

size of the waterway and also to facilitate bringing *suitable* stone for masonry from a distance. In a new country there is always the double danger of either building a culvert too small, requiring expensive reconstruction, perhaps after a disastrous washout, or else wasting money by constructing the culvert unnecessarily large. Much masonry has been built of a very poor quality of stone because it could be conveniently obtained and because good stone was unobtainable except at a prohibitive cost for transportation. Opening the road for traffic by the use of temporary trestles obviates both of these difficulties.

127. Trestles vs. embankments. Low embankments are very much cheaper than low trestles both in first cost and maintenance. Very high embankments are very expensive to construct, but cost comparatively little to maintain. A trestle of equal height may cost much less to construct, but will be expensive to maintain—perhaps $\frac{1}{8}$ of its cost per year. To determine the height beyond which it will be cheaper to maintain a trestle rather than build an embankment, it will be necessary to allow for the cost of maintenance. The height will also depend on the relative cost of timber, labor, and earthwork. At the present average values, it will be found that for less heights than 25 feet the *first cost* of an embankment will *generally* be less than that of a trestle; this implies that a permanent trestle should never be constructed with a height less than 25 feet except for the reasons given in § 126. The height at which a permanent trestle is certainly cheaper than earthwork is more uncertain. A high grade line joining two hills will invariably imply at least a culvert if an embankment is used. If the culvert is built of masonry, the cost of the embankment will be so increased that the height at which a trestle becomes economical will be materially reduced. The cost of an embankment increases much more rapidly than the height—with very high embankments more nearly as the square of the height—while the cost of trestles does not increase as rapidly as the height. Although local circumstances may modify the application of any set rules, it is probably seldom that it will be cheaper to build an embankment 40 or 50 feet high than to permanently maintain a wooden trestle of that height. A steel viaduct would probably be the best solution of such a case. These are frequently used for permanent structures, especially when very high. The cost of maintenance is much less than that of wood, which makes the use of iron or steel preferable for permanent trestles unless wood is abnormally cheap. Neither the cost nor the construction of iron or steel trestles will be considered in this chapter.

128. Two principal types. There are two principal types of wooden trestles—pile trestles and framed trestles. The great objection to pile trestles is the rapid rotting of the portion of the pile which is underground, and the difficulty of renewal. The maximum height of pile trestles is about 30 feet, and even this height is seldom reached. Framed trestles have been constructed to a height of considerably over 100 feet They are frequently built in such a manner that any injured piece may be readily taken out and renewed without interfering with traffic. Trestles consist of two parts—the supports called "bents," and the stringers and floor system. As the stringers and floor system are the same for both pile and framed trestles, the "bents" are all that need be considered separately.

PILE TRESTLES.

129. Pile bents. A pile bent consists generally of four piles driven into the ground deep enough to afford not only sufficient vertical resistance but also lateral resistance. On top of these piles is placed a horizontal "cap." The caps are fastened to the tops of the piles by methods illustrated in Fig. 66. The method of fastening shown in each case should not be considered as applicable only to the particular type of pile bent used to illustrate it. Fig. 66 (*a* and *d*) illustrates a mortise-joint with a hardwood pin about $1\frac{1}{4}''$ in diameter. The hole for the pin should be bored separately through the cap and the mortise, and the hole through the cap should be at a slightly higher level than that through the mortise, so that the cap will be drawn down tight when the pin is driven. Occasionally an iron dowel (an iron pin about $1\frac{1}{2}''$ in diameter and about $6''$ long) is inserted partly in the cap and partly in the pile. The use of drift-bolts, shown in Fig. 66 (*b*), is cheaper in first cost, but renders repairs and renewals very troublesome and expensive. "Split caps," shown in Fig. 66 (*c*), are formed by bolting two half-size strips on each side of a tenon on top of the pile. Repairs are very easily and cheaply made without interference with the traffic and without injuring other pieces of the bent. The smaller pieces are more easily obtainable in a sound condition; the decay of one does not affect the other, and the first cost is but little if any greater than the method of using a single piece. For further discussion, see § 136.

For very light traffic and for a height of about 5 feet three vertical piles will suffice, as shown in Fig. 66 (*a*). Up to a height

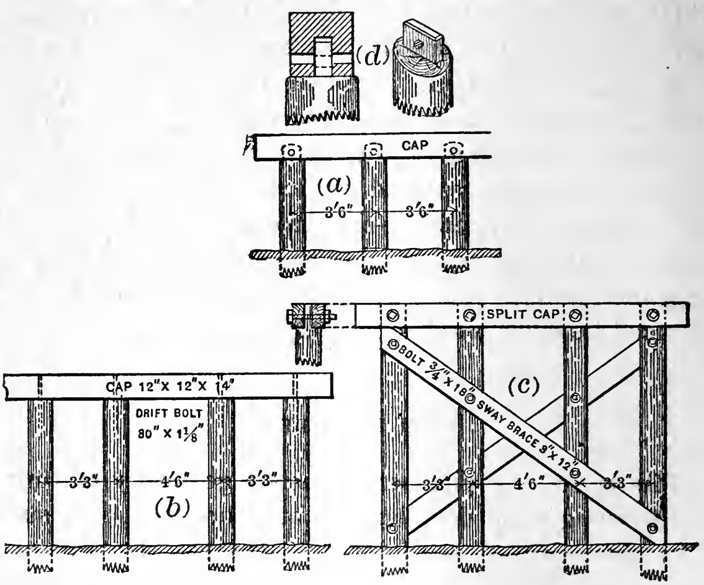


FIG. 66.

of 8 or 10 feet four piles may be used without sway-bracing, as in Fig 66 (*b*), if the piles have a good bearing. For heights greater than 10 feet sway-bracing is generally necessary. The outside piles are frequently driven with a batter varying from 1 : 12 to 1 : 4.

Piles are made, if possible, from timber obtained in the vicinity of the work. Durability is the great requisite rather than strength, for almost any timber is strong enough (except as noted below) and will be suitable if it will resist rapid decay. The following list is quoted as being in the order of preference on account of durability:

1. Red cedar	5. White pine	9. White oak	12. Black oak
2. Red cypress	6. Redwood	10. Post-oak	13. Hemlock
3. Pitch-pine	7. Elm	11. Red oak	14. Tamarac
4. Yellow pine	8. Spruce		

Red-cedar piles are said to have an average life of 27 years with a possible maximum of 50 years, but the timber is rather weak, and if exposed in a river to flowing ice or driftwood is apt to be injured. Under these circumstances oak is preferable, although its life may be only 13 to 18 years.

130. Methods of driving piles. The following are the principal methods of driving piles:

a. A hammer weighing 2000 to 3000 lbs. or more, sliding in guides, is drawn up by horse-power or a portable engine, and allowed to fall *freely*.

b. The same as above except that the hammer does not fall freely, but drags the rope and revolving drum as it falls and is thus quite materially retarded. The mechanism is a little more simple, but is less effective, and is sometimes made deliberately deceptive by a contractor by retarding the blow, in order to apparently indicate the requisite resistance on the part of the pile.

The above methods have the advantage that the mechanism is cheap and can be transported into a new country with comparative ease, but the work done is somewhat ineffective and costly compared with some of the more elaborate methods given below.

c. Gunpowder pile-drivers, which automatically explode a cartridge every time the hammer falls. The explosion not only forces the pile down, but throws up the hammer for the next blow. For a given height of fall the effect is therefore doubled. It has been shown by experience, however, that when it is attempted to use such a pile-driver rapidly the mechanism becomes so heated that the cartridges explode prematurely, and the method has therefore been abandoned.

d. Steam pile-drivers, in which the hammer is operated directly by steam. The hammer falls freely a height of about 40 inches and is raised again by steam. The effectiveness is largely due to the rapidity of the blows, which does not allow time between the blows for the ground to settle around the pile and increase the resistance, which does happen when the blows are infrequent. "The hammer-cylinder weighs 5500 lbs., and with 60 to 75 lbs. of steam gives 75 to 80 blows per minute. With 41 blows a large unpointed pile was driven 35 feet into a hard clay bottom in half a minute." Such a driver would cost about $800.

The above four methods are those usual for dry earth. In very soft wet or sandy soils, where an unlimited supply of water is available, the *water-jet* is sometimes employed. A pipe is fastened along the side of the pile and extends to the pile-point. If water is forced through the pipe, it loosens the sand around the point and, rising along the sides, decreases the side resistance so that the pile sinks by its own weight, aided perhaps by extra weights loaded on. This loading may be accomplished by connecting the top of the pile and the pile-driver by a block and tackle so that a portion of the weight of the pile-driver is continually thrown on the pile.

Excessive driving frequently fractures the pile below the surface and thereby greatly weakens its bearing power. To prevent excessive "brooming" of the top of the pile, owing to the action of the hammer, the top should be protected by an iron ring fitted to the top of the pile. The "brooming" not only renders the driving ineffective and hence uneconomical, but vitiates the value of any test of the bearing power of the pile by noting the sinking due to a given weight falling a given distance. If the pile is so soft that brooming is unavoidable, the top should be adzed off frequently, and especially should it be done just before the final blows which are to test its bearing-power.

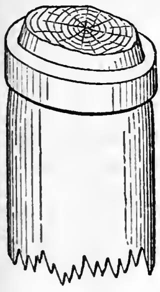


FIG. 67.

In a new country judgment and experience will be required to decide intelligently whether to employ a simple drop-hammer machine, operated by horse-power and easily transported but uneconomical in operation, or a more complicated machine working cheaply and effectively after being transported at greater expense.

131. Pile-driving formulæ. If R = the resistance of a pile, and s the set of the pile during the last blow, w the weight of the pile-hammer, and h the fall during the last blow, then we may state the approximate relation that $Rs = wh$, or $R = \frac{wh}{s}$. This is the basic principle of all rational formulæ, but the maximum weight which a pile will sustain after it has been driven some time is by no means equal to the resistance of the pile during the last blow. There are also many other modifying elements which have been variously allowed for in the many proposed formulæ. The formulæ range from the extreme of empirical simplicity to very complicated attempts to allow properly for all modifying causes. As the simplest rule, specifications sometimes require that the piles shall be driven until the pile will not sink more than 5 inches under five consecutive blows of a 2000-lb. hammer falling 25 feet. The "*Engineering News* formula" * gives the safe load as $\frac{2wh}{s+1}$, in which $w=$ weight of hammer, $h=$fall in *feet*, $s=$set of pile in *inches* under the last blow. This formula is derived from the above basic formula by calling the safe load $\frac{1}{6}$ of the final resistance, and by adding (arbitrarily) 1 to the final set (s) as a compensation for the extra resistance caused by the settling of earth around the pile between each blow. This formula is used only for ordinary hammer-driving. When the piles are driven by a steam pile-driver the formula becomes safe load$=\frac{2wh}{s+0.1}$. For the "gunpowder pile-driver," since the explosion of the cartridge drives the pile in with the same force with which it throws the hammer upward, the effect is *twice* that of the fall of the hammer, and the formula becomes safe load$=\frac{4wh}{s+0.1}$. In these last two formulæ the constant in the denominator is changed from $s+1$ to $s+0.1$. The constant (1.0 or 0.1) is supposed to allow, as before stated, for the effect of the extra resistance caused by the earth settling around the pile between each blow. The more rapid the blows the less the opportunity to settle and the less the proper value of the constant.

The above formulæ have been given on account of their simplicity and their practical agreement with experience. Many other formulæ have been proposed, the majority of which are more complicated and attempt to take into account the weight of the pile, resistance of the guides, etc. While these elements, as well as many others, have their influence, their effect is so overshadowed by the indeterminable effect of other elements—as, for example, the effect of the settlement of earth around the pile between blows—that it is useless to attempt to employ anything but a purely empirical formula.

Examples. 1. A pile was driven with an ordinary hammer weighing 2500 pounds until the sinking under five consecutive blows was $15\frac{1}{2}$ inches. The fall of the hammer during the last

---

* *Engineering News*, Nov. 17, 1892.

blows was 24 feet. What was the safe bearing power of the pile?

$$\frac{2wh}{s+1}=\frac{2\times2500\times24}{(\frac{1}{5}\times15.5)+1}=\frac{120000}{4.1}=29300 \text{ pounds.}$$

2. Piles are being driven into a firm soil with a steam pile-driver until they show a *safe* bearing power of 20 tons. The hammer weighs 5500 pounds and its fall is 40 inches. What should be the sinking under the final blow?

$$40000=\frac{2wh}{s+0.1}=\frac{2\times5500\times3.33}{s+0.1},$$

$$=\frac{36667}{40000}-0.1=.81 \text{ inch.}$$

132. Pile-points and pile-shoes. Piles are generally sharpened to a blunt point. If the pile is liable to strike boulders, sunken logs, or other obstructions which are liable to turn the point, it is necessary to protect the point by some form of shoe. Several forms in cast iron have been used, also a wrought-iron shoe, having four "straps" radiating from the apex, the straps being nailed on to the pile, as shown in Fig. 68 (*b*). The cast-iron form shown in Fig. 68 (*a*) has a base cast around a drift-bolt. The recess on the top of the base receives the bottom of the pile and prevents a tendency to split the bottom of the pile or to force the shoe off laterally.

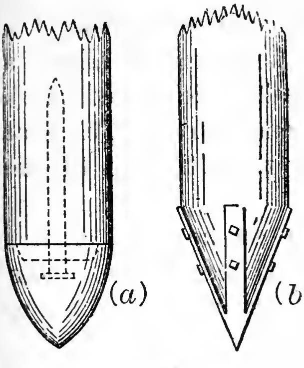


FIG. 68.

133. Details of design. No theoretical calculations of the strength of pile bents need be attempted on account of the extreme complication of the theoretical strains, the uncertainty as to the real strength of the timber used, the variability of that strength with time, and the insignificance of the economy that would be possible even if exact sizes could be computed. The piles are generally required to be not less than 10″ or 12″ in diameter at the large end. The P. R. R. requires that they shall be "not less than 14 and 7 inches in diameter at butt and small end respectively, exclusive of bark, which must be removed." The removal of the bark is generally required in good work. Soft *durable* woods, such as are mentioned in § 129, are best for the piles, but the caps are generally made of oak or yellow pine. The caps are generally 14 feet long (for single track) with a cross-section 12″×12″ or 12″×14″. "Split caps" would consist of two pieces 6″×12″. The sway-braces, never used for less heights than 6′, are made of 3″×12″ timber, and are spiked on with $\frac{3}{8}$″ spikes 8″ long. The floor system will be the same as that described later for framed trestles.

134. **Cost of pile trestles.** The cost, per linear foot, of piling depends on the method of driving, the scarcity of suitable timber, the price of labor, the length of the piles, and the amount of shifting of the pile-driver required. The cost of soft-wood piles varies from 8 to 15 c. per lineal foot, and the cost of oak piles varies from 10 to 30 c. per foot according to the length, the longer piles costing more per foot. The cost of driving will average about $2.50 per pile, or 7.5 to 10 c. per lineal foot. Since the cost of shifting the pile-driver is quite an item in the total cost, the cost of driving a long pile would be *less* per foot than for a short pile, but on the other hand the cost of the pile is *greater* per foot, which tends to make the total cost per foot constant. Specifications generally say that the piling will be paid for per lineal foot of piling *left in the work*. The wastage of the tops of piles sawed off is always something, and is frequently very large. Sometimes a small amount per foot of piling sawed off is allowed the contractor as compensation for his loss. This reduces the contractor's risk and possibly reduces his bid by an equal or greater amount than the extra amount actually paid him.

FRAMED TRESTLES.

135. **Typical design.** A typical design for a framed trestle bent is given in Fig. 69. This represents, with slight variations of detail, the plan according to which a large part of the framed trestle bents of the country have been built—i.e., of those less than 20 or 30 feet in height, not requiring multiple story construction.

136. **Joints.** (a) The **mortise-and-tenon joint** is illustrated in

Fig. 69 and also in Fig. 66 (*a*). The tenon should be about

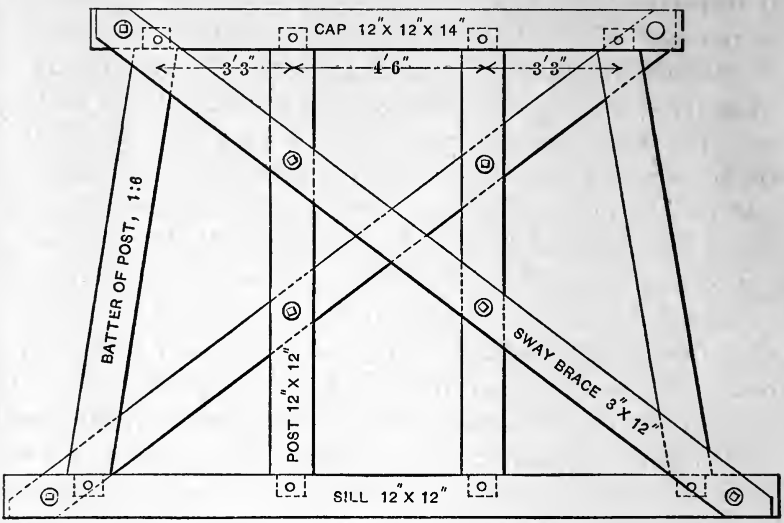


FIG. 69.

3″ thick, 8″ wide, and 5½″ long. The mortise should be cut a little deeper than the tenon. "Drip-holes" from the mortise to the outside will assist in draining off water that may accumulate in the joint and thus prevent the rapid decay that would otherwise ensue. These joints are very troublesome if a single post decays and requires renewal. It is generally required that the mortise and tenon should be thoroughly daubed with paint before putting them together. This will tend to make the joint water-tight and prevent decay from the accumulation and retention of water in the joint.

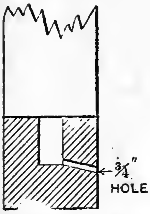


FIG. 70.

(b) **The plaster joint.** This joint is made by bolting and spiking a 3″×12″ plank on both sides of the joint. The cap and sill should be notched to receive the posts. Repairs are greatly facilitated by the use of these joints. This method has been used by the Delaware and Hudson Canal Co. [R. R.].

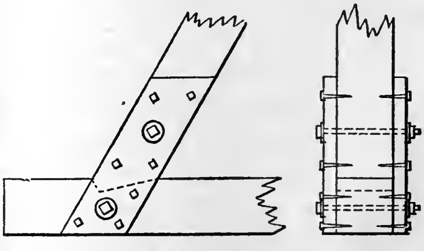


FIG. 71.

(c) **Iron plates.** An iron plate of the form shown in Fig. 72

(*b*) is bent and used as shown in Fig. 72 (*a*). Bolts passing through the bolt-holes shown secure the plates to the timbers and make a strong joint which may be readily loosened for repairs. By slight modifications in the design the method may be used for inclined posts and complicated joints.

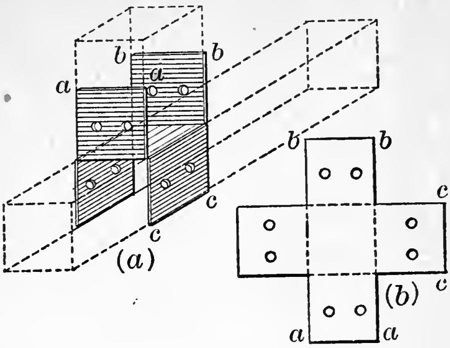


Fig. 72.

(d) Split caps and sills. These are described in § 129. Their advantages apply with even greater force to framed trestles.

(e) Dowels and drift-bolts. These joints facilitate cheap and rapid construction, but renewals and repairs are very difficult, it being almost impossible to extract a drift-bolt, which has been driven its full length, without splitting open the pieces containing it. Notwithstanding this objection they are extensively used, especially for temporary work which is not expected to be used long enough to need repairs.

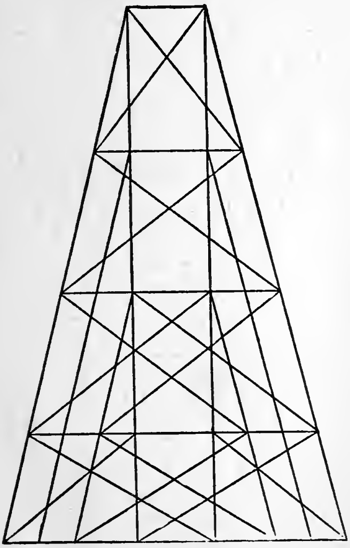


Fig. 73.

137. Multiple-story construction. Single-story framed trestle bents are used for heights up to 18 or 20 feet and exceptionally up to 30 feet. For greater heights some such construction as is illustrated in a skeleton design in Fig. 73 is used. By using split sills between each story and separate vertical and batter posts in each story, any piece may readily be removed and renewed if necessary. The height of these stories varies, in different designs, from 15 to 25 and even 30 feet. In some designs the structure of each story is independent of the stories above and below. This greatly facilitates both the original construction and subsequent repairs.

In other designs the verticals and batter-posts are made continuous through two consecutive stories. The structure is somewhat stiffer, but is much more difficult to repair.

Since the bents of any trestle are usually of variable height and those heights are not always an even multiple of the uniform height desired for the stories, it becomes necessary to make the

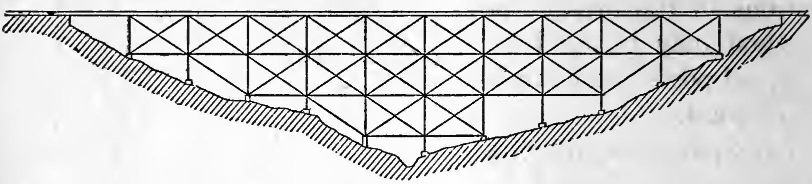


FIG. 74.

upper stories of uniform height and let the odd amount go to the lowest story, as shown in Figs. 73 and 74.

138. **Span.** The shorter the span the greater the number of trestle bents; the longer the span the greater the required strength of the stringers supporting the floor. Economy demands the adoption of a span that shall make the sum of these require-

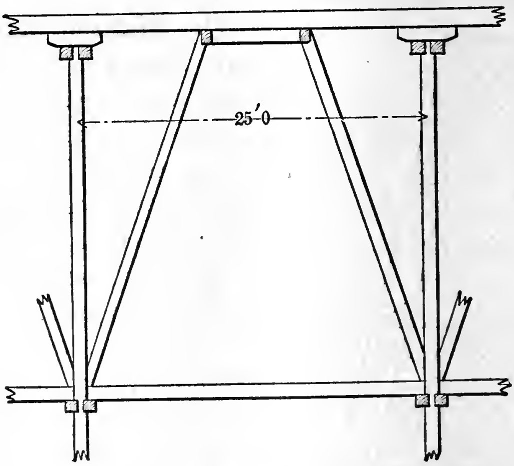


FIG. 75.

ments a minimum. The higher the trestle the greater the cost of each bent, and the greater the span that would be justifiable. Nearly all trestles have bents of variable height, but the advantage of employing uniform standard sizes is so great that many roads use the same span and sizes of timber not only for the panels of any given trestle, but also for all trestles regardless of height. The spans generally used vary from 10 to 16 feet. The Norfolk and Western R. R. uses a span of 12′ 6″ for all single-story trestles, and a span of 25′ for all multiple-story trestles. The stringers are the same in both cases, but when the span is 25 feet, knee-braces are run from the sill of the first story below to near the middle of each set of stringers. These knee-braces are connected at the top by a "straining-beam" on which the stringers rest, thus supporting the stringer in the center and virtually reducing the span about one-half.

139. Foundations. (a) **Piles.** Piles are frequently used as a foundation, as in Fig. 76, particularly in soft ground, and also for temporary structures. These foundations are cheap, quickly constructed, and are particularly valuable when it is financially necessary to open the road for traffic as soon as possible and with the least expenditure of money; but there is the disadvantage of inevitable decay within a few years unless the piles are chemically treated, as will be discussed later. Chemical treatment, however, increases the cost so that such a foundation would often cost more than a foundation of stone. A pile should be driven under each post as shown in Fig. 76.

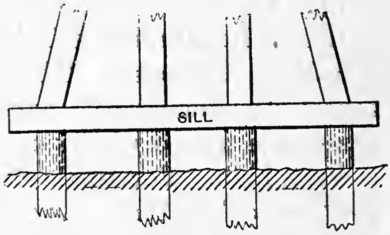


FIG. 76.

(b) **Mud-sills.** Fig. 77 illustrates the use of mud-sills as built by the Louisville and Nashville R. R. Eight blocks 12″×12″×6′ are used under each bent. When the ground is very soft, two additional timbers (12″×12″×length of bent-sill), as shown by the dotted lines, are placed underneath. The number required evidently depends on the nature of the ground.

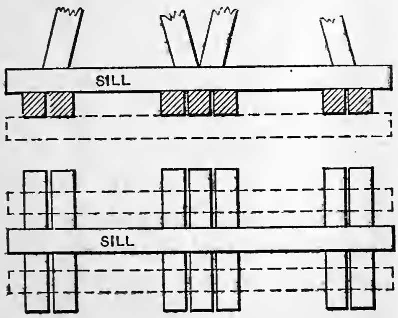


FIG. 77.

(c) **Stone foundations.** Stone foundations are the best and the most expensive. For very high trestles the Norfolk and

Western R. R. employs foundations as shown in Fig. 78, the walls being 4 feet thick. When the height of the trestle is 72 feet or less (the plans requiring for 72′ in height a foundation-wall 39′ 6″ long) the foundation is made continuous. The sill

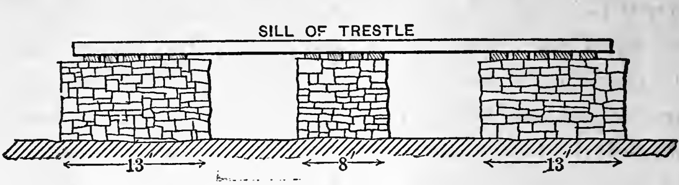


FIG. 78.

of the trestle should rest on several short lengths of 3″×12″ plank laid transverse to the sill on top of the wall.

140. Longitudinal bracing. This is required to give the structure longitudinal stiffness and also to reduce the columnar length of the posts. This bracing generally consists of horizontal "waling-strips" and diagonal braces. Sometimes the braces are placed wholly on the outside posts unless the trestle is very high. For single-story trestles the P. R. R. employs the "laced" system, i.e., a line of posts joining the cap of one bent with the sill of the next, and the sill of that bent with the cap of the next. Some plans employ braces forming an × in alternate panels. Connecting these braces in the center more than doubles their columnar strength. Diagonal braces, when bolted to posts, should be fastened to them as near the ends of the posts as possible. The sizes employed vary largely, depending on the clear length and on whether they are expected to act by tension or compression. 3″×12″ planks are often used when the design would require tensile strength only, and 8″×8″ posts are often used when compression may be expected.

141. Lateral bracing. Several of the more recent designs of trestles employ diagonal lateral bracing between the caps of adjacent bents. It adds greatly to the stiffness of the trestle and better maintains its alignment. 6″×6″ posts, forming an × and connected at the center, will answer the purpose.

142. Abutments. When suitable stone for masonry is at hand and a suitable subsoil for a foundation is obtainable without too much excavation, a masonry abutment will be the best. Such an abutment would probably be used when masonry footings for trestle bents were employed (§ 139, *c*).

Another method is to construct a "crib" of 10″×12″ timber, laid horizontally, drift-bolted together, securely braced and embedded into the ground. Except for temporary construction such a method is generally objectionable on account of rapid decay.

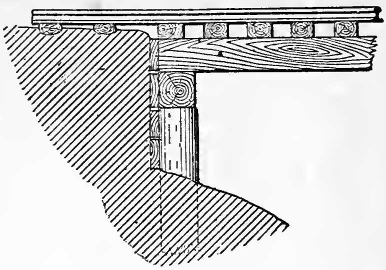


Fig. 79.

Another method, used most commonly for pile trestles, and for framed trestles having pile foundations (§ 139, *a*), is to use a pile bent at such a place that the natural surface on the *up-hill* side is not far below the cap, and the thrust of the material, filled in to bring the surface to grade, is insignificant. 3″×12″ planks are placed behind the piles, cap, and stringers to retain the filled material.

FLOOR SYSTEMS.

143. Stringers. The general practice is to use two, three, and even four stringers under each rail. Sometimes a stringer is placed under each guard-rail. Generally the stringers are made of two panel lengths and laid so that the joints alternate. A few roads use stringers of only one panel length, but this practice is strongly condemned by many engineers. The stringers should be separated to allow a circulation of air around them and prevent the decay which would occur if they were placed close together. This is sometimes done by means of 2″ planks, 6′ to 8′ long, which are placed over each trestle bent. Several bolts, passing through all the stringers forming a group and through the separators, bind them all into one solid construction. Cast-iron "spools" or washers, varying from 4″ to $\frac{3}{4}$″ in length (or thickness), are sometimes strung on each bolt so as to separate the stringers. Sometimes washers are used between the separating planks and the stringers, the object of the separating planks then being to bind the stringers, especially abutting stringers, and increase their stiffness.

The most common size for stringers is 8″×16″. The Pennsylvania Railroad varies the width, depth, and number of stringers under each rail according to the clear span. It may be noticed that, assuming a uniform load per running foot, both the pressure per square inch at the ends of the stringers (the caps having a width of 12″) and also the stress due to transverse strain are kept *approximately* constant for the variable gross load on these varying spans.

Clear span.	No. of pieces under each rail.	Width.	Depth.
10 feet	2	8 inches	15 inches
12 "	2	8 "	16 "
14 "	2	10 "	17 "
16 "	3	8 "	17 "

144. **Corbels.** A corbel (in trestle-work) is a stick of timber (perhaps two placed side by side), about 3′ to 6′ long, placed underneath and along the stringers and resting on the cap. There are strong prejudices for and against their use, and a corresponding diversity in practice. They are bolted to the stringers and thus stiffen the joint. They certainly reduce the objectionable crushing of the fibers at each end of the stringer, but if the corbel is no wider than the stringers, as is generally the case, the area of pressure between the corbels and the cap is

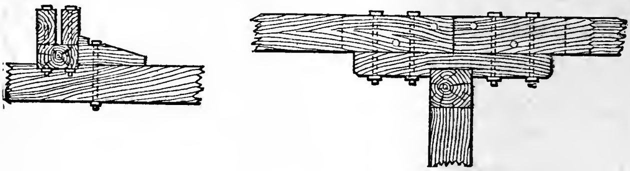


FIG. 80.

no greater and the pressure per square inch on the cap is no less than the pressure on the cap if no corbels were used. If the corbels and cap are made of hard wood, as is recommended by some, the danger of crushing is lessened, but the extra cost and the frequent scarcity of hard wood, and also the extra cost and labor of using corbels, may often neutralize the advantages obtained by their use.

145. **Guard-rails.** These are frequently made of 5″×8″ stuff, notched 1″ for each tie. The sizes vary up to 8″×8″, and the depth of notch from $\frac{3}{4}''$ to $1\frac{1}{2}''$. They are generally bolted to every third or fourth tie. It is frequently specified that they shall be made of oak, white pine, or yellow pine. The joints are made over a tie, by halving each piece, as illustrated in Fig. 81. The joints on opposite sides of the trestle should be "stag-

gered." Some roads fasten every tie to the guard-rail, using a bolt, a spike, or a lag-screw.

Guard-rails were originally used with the idea of preventing the wheels of a derailed truck from running off the ends of the ties. But it has been found that an outer guard-rail alone (without an inner guard-rail) becomes an actual element of danger, since it has frequently happened that a derailed wheel has caught on the outer guard-rail, thus causing the truck to slew around

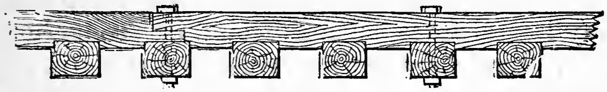


FIG. 81.

and so produce a dangerous accident. The true function of the *outside* guard-rail is thus changed to that of a tie-spacer, which keeps the ties from spreading when a derailment occurs. The inside guard-rail generally consists of an ordinary steel rail spiked about 10 inches inside of the running rail. These inner guard-rails should be bent inward to a point in the center of the track about 50 feet beyond the end of the bridge or trestle. If the inner guard-rails are placed with a clear space of 10 inches inside the running rail, the outer guard-rails should be *at least* 6′ 10″ apart. They are generally much farther apart than this.

146. Ties on trestles. If a car is derailed on a bridge or trestle, the heavily loaded wheels are apt to force their way between the ties by displacing them unless the ties are closely spaced and fastened. The clear space between ties is generally equal to or less than their width. Occasionally it is a little more than their width. $6'' \times 8''$ ties, spaced 14″ to 16″ from center to center, are most frequently used. The length varies from 9′ to 12′ for single track. They are generally notched $\frac{1}{2}''$ deep on the under side where they rest on the stringers. Oak ties are generally required even when cheaper ties are used on the other sections of the road. Usually every third or fourth tie is bolted to the stringers. When stringers are placed underneath the guard-rails, bolts are run from the top of the guard-rail to the under side of the stringer. The guard-rails thus hold down the whole system of ties, and no direct fastening of the ties to the stringers is needed.

147. Superelevation of the outer rail on curves. The location of curves on trestles should be avoided if possible, especially when the trestle is high. Serious additional strains are introduced especially when the curvature is sharp or the speed high. Since such curves are sometimes practically unavoidable, it is necessary to design the trestle accordingly. If a train is stopped on a curved trestle, the action of the train on the trestle is evidently vertical. If the train is moving with a considerable velocity, the resultant of the weight and the centrifugal action is a force somewhat inclined from the vertical. Both of these conditions may be expected to exist at times. If the *axis* of the system of posts is vertical (as illustrated in methods *a*, *b*, *c*, *d*, and *e*), any lateral force, such as would be produced by a moving train, will tend to rack the trestle bent. If the stringers are set vertically, a centrifugal force likewise tends to tip them sidewise. If the axis of the system of posts (or of the stringers) is inclined so as to coincide with the pressure of the train on the trestle when the train is moving at its normal velocity, there is no tendency to rack the trestle when the train is moving at that velocity, but there will be a tendency to rack the trestle or twist the stringers when the train is stationary. Since a moving train is usually the normal condition of affairs, as well as the condition which produces the maximum stress, an inclined axis is evidently preferable from a theoretical standpoint; but whatever design is adopted, the trestle should evidently be sufficiently cross-braced for either a moving or a stationary load, and any proposed design must be studied as to the effect of *both* of these conditions. Some of the various methods of securing the requisite superelevation may be described as follows:

(a) Framing the outer posts longer than the inner posts, so that the cap is inclined at the proper angle; axis of posts vertical. (Fig. 82.) The method requires more work in framing the trestle, but simplifies subsequent track-laying and maintenance, unless it should be found that the superelevation adopted is unsuitable, in which case it could be corrected by one of the other methods given below. The stringers tend to twist when the train is stationary.

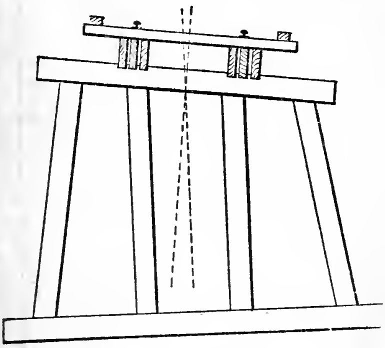


FIG. 82.

(b) Notching the cap so that the stringers are at a different

elevation. (Fig. 83.) This weakens the cap and requires that all ties shall be notched to a bevelled surface to fit the stringers, which also weakens the ties. A centrifugal force will tend to twist the stringers and rack the trestle.

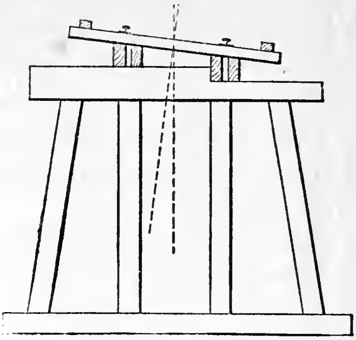


FIG. 83.

(c) **Placing wedges underneath the ties at each stringer.** These wedges are fastened with two bolts. Two or more wedges will be required for each tie. The additional number of pieces required for a long curve will be immense, and the work of inspection and keeping the nuts tight will greatly increase the cost of maintenance.

(d) **Placing a wedge under the outer rail at each tie.** This requires but one extra piece per tie. There is no need of a wedge under the inner tie in order to make the rail normal to the tread. The resulting inward inclination is substantially that produced by some forms of rail-chairs or tie-plates. The spikes (a little longer than usual) are driven through the wedge into the tie. Sometimes "lag-screws" are used instead of spikes. If experience proves that the superelevation is too much or too little, it may be changed by this method with less work than by any other.

(e) **Corbels of different heights.** When corbels are used (see § 144) the required inclination of the floor system may be obtained by varying the depth of the corbels.

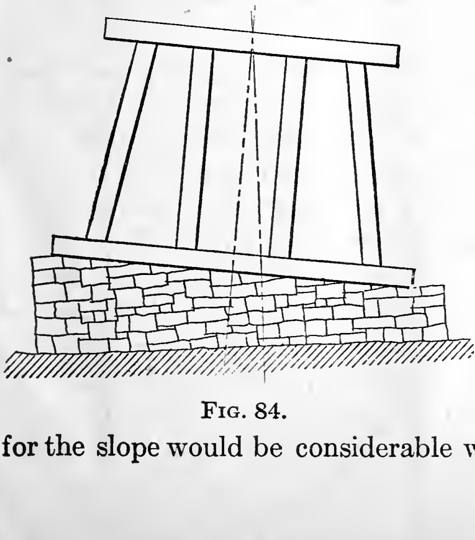


FIG. 84.

(f) **Tipping the whole trestle.** This is done by placing the trestle on an inclined foundation. If very much inclined, the trestle bent must be secured against the possibility of slipping sidewise, for the slope would be considerable with a sharp curve, and the vibration of a moving train would reduce the coefficient of friction to a comparatively small quantity.

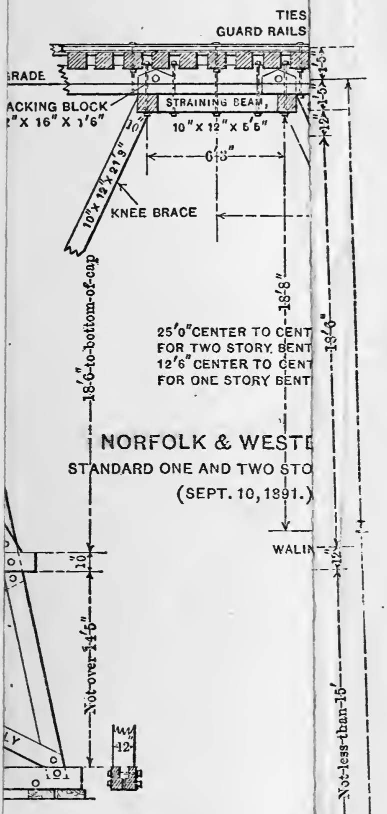
(g) **Framing the outer posts longer.** This case is identical with case (*a*) except that the axis of the system of posts is inclined, as in case (*f*), but the sill is horizontal.

The above-described plans will suggest a great variety of methods which are possible and which differ from the above only in minor details.

148. Protection from fire. Trestles are peculiarly subject to fire, from passing locomotives, which may not only destroy the trestle, but perhaps cause a terrible disaster. This danger is sometimes reduced by placing a strip of galvanized iron along the top of each set of stringers and also along the tops of the caps. Still greater protection was given on a long trestle on the Louisville and Nashville R. R. by making a solid flooring of timber, covered with a layer of ballast on which the ties and rails were laid as usual.

Barrels of water should be provided and kept near all trestles, and on very long trestles barrels of water should be placed every two or three hundred feet along its length. A place for the barrels may be provided by using a few ties which have an extra length of about four feet, thus forming a small platform, which should be surrounded by a railing. The track-walker should be held accountable for the maintenance of a supply of water in these barrels, renewals being frequently necessary on account of evaporation. Such platforms should also be provided as REFUGE-BAYS for track-walkers and trackmen working on the trestle. On very long trestles such a platform is sometimes provided with sufficient capacity for a hand-car.

149. Timber. Any strong durable timber may be used when the choice is limited, but oak, pine, or cypress are preferred when obtainable. When all of these are readily obtainable, the various parts of the trestle will be constructed of different kinds of wood—the stringers of long-leaf pine, the posts and braces of pine or red cypress, and the caps, sills, and corbels (if used) of white oak. The use of oak (or a similar hard wood) for caps, sills, and corbels is desirable because of its greater strength in resisting crushing across the grain, which is the critical test for these parts. There is no physiological basis to the objection, sometimes made, that different species of timber, in contact with each other, will rot quicker than if only one



PLUMB POSTS, SINGLE BE

" " IN LOWER STORY, DOUBLE

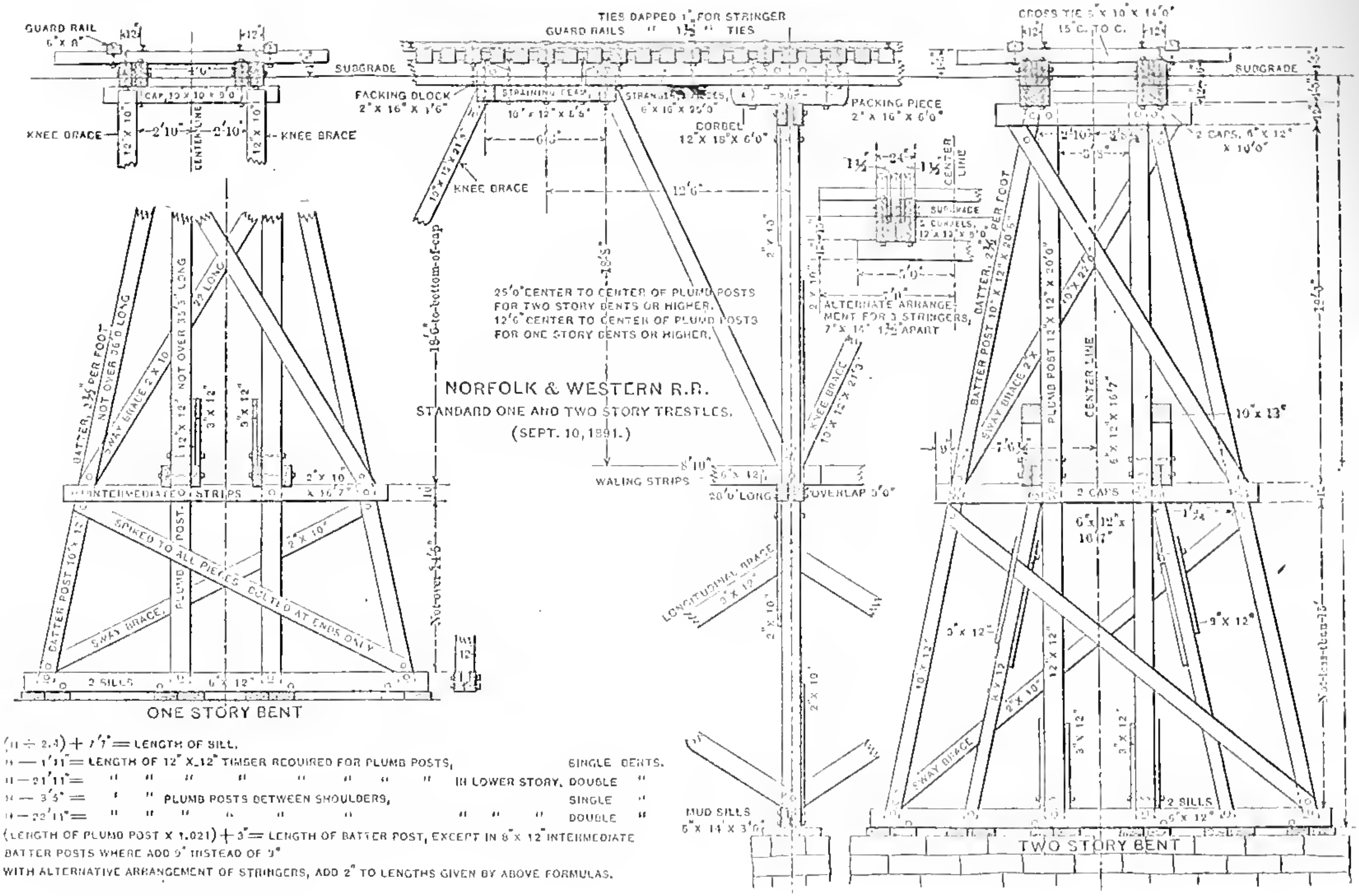
DERS, SINGLE

" " " DOUBLE

TTER POST, EXCEPT IN 8" X 12" INTERMEDIATE

TO LENGTHS GIVEN BY ABOVE FORMULAS.

PLATE II.



(H ÷ 2.4) + 1'7" = LENGTH OF SILL.
H — 1'11" = LENGTH OF 12" X 12" TIMBER REQUIRED FOR PLUMB POSTS, SINGLE BENTS.
H — 21'11" = " " " " " " " " IN LOWER STORY, DOUBLE "
H — 3'5" = " " PLUMB POSTS BETWEEN SHOULDERS, SINGLE "
H — 22'11" = " " " " " " " " " DOUBLE "
(LENGTH OF PLUMB POST X 1.021) + 3" = LENGTH OF BATTER POST, EXCEPT IN 8" X 12" INTERMEDIATE BATTER POSTS WHERE ADD 9" INSTEAD OF 3"
WITH ALTERNATIVE ARRANGEMENT OF STRINGERS, ADD 2" TO LENGTHS GIVEN BY ABOVE FORMULAS.

(To face page 168.)

kind of timber is used. When a very extensive trestle is to be built at a place where suitable growing timber is at hand but there is no convenient sawmill, it will pay to transport a portable sawmill and engine and cut up the timber as desired.

150. Cost of framed timber trestles. The cost varies widely on account of the great variation in the cost of timber. When a railroad is first penetrating a new and undeveloped region, the cost of timber is frequently small, and when it is obtainable from the company's right-of-way the only expense is felling and sawing. The work per M, B. M., is small, considering that a single stick 12″×12″×25′ contains 300 feet, B. M., and that sometimes a few hours' work, worth less than $1, will finish all the work required on it. Smaller pieces will of course require more work per foot, B. M. Long-leaf pine can be purchased from the mills at from $8 to $12 per M feet, B. M., according to the dimensions. To this must be added the freight and labor of erection. The cartage from the nearest railroad to the trestle may often be a considerable item. Wrought iron will cost about 3 c. per pound and cast iron 2 c., although the prices are often lower than these. The amount of iron used depends on the detailed design, but, as an average, will amount to $1.50 to $2 per 1000 feet, B. M., of timber. A large part of the trestling of the country has been built at a contract price of about $30 per 1000 feet, B. M., erected. While the cost will frequently rise to $40 and even $50 when timber is scarce, it will drop to $13 (cost quoted) when timber is cheap.

DESIGN OF WOODEN TRESTLES.

151. Common practice. A great deal of trestling has been constructed without any rational design except that custom and experience have shown that certain sizes and designs are *probably* safe. This method has resulted occasionally in failures but more frequently in a very large waste of timber. Many railroads employ a uniform size for all posts, caps, and sills, and a uniform size for stringers, all regardless of the height or span of the trestle. For repair work there are practical reasons favoring this. "To attempt to run a large lot of sizes would be more wasteful in the end than to maintain a few stock sizes only. Lumber can be bought more cheaply by giving a general order for 'the run of the mill for the season,' or 'a cargo lot,' specifying approximate percentages of standard stringer size, of 12×12-inch stuff, 10×10-inch stuff, etc., and a liberal proportion of 3- or 4-inch plank, all lengths thrown in. The 12×12-inch stuff, etc., is ordered all lengths, from a certain specified length up. In case of a wreck, washout, burn-out, or sudden call for a trestle to be completed in a stated time, it is much more economical and practical to order a certain number of carloads of 'trestle stuff' to the ground and there to select piece after piece as fast as needed, dependent only upon the length of stick required. When there is time to make the necessary surveys of the ground and calculations of strength, and to wait for a special bill of timber to be cut and delivered, the use of different sizes for posts in a structure would be warranted to a certain extent." * For new construction, when there is generally sufficient time to design and order the proper sizes, such wastefulness is less excusable, and under any conditions it is both safer and more economical to prepare *standard designs* which can be made applicable to varying conditions and which will at the same time utilize as much of the strength of the timber as can be depended on. In the following sections will be given the elements of the preparation of such standard designs, which will utilize uniform sizes with as little waste of timber as possible. It is *not* to be understood that special designs should be made for each individual trestle.

152. Required elements of strength. The *stringers* of trestles are subject to transverse strains, to crushing across the grain at the ends, and to shearing along the neutral axis. The strength of the timber must therefore be computed for all these kinds of stress. *Caps* and *sills* will fail, if at all, by crushing across the grain; although subject to other forms of stress, these could hardly cause failure in the sizes usually employed. There is an apparent exception to this: if piles are improperly driven and an uneven settlement subsequently occurs, it may have the effect of transferring practically all of the weight to two or three piles, while the *cap* is subjected to a severe transverse strain which may cause its failure. Since such action is caused generally by avoidable errors of construction it may be considered as abnormal, and since such a failure will generally occur by a *gradual* settlement, all danger may be avoided by reasonable

---

* From "Economical Designing of Timber Trestle Bridges."

care in inspection. *Posts* must be tested for their columnar strength. These parts form the bulk of the trestle and are the parts which can be definitely designed from known stresses. The stresses in the bracing are more indefinite, depending on indeterminate forces, since the inclined posts take up an unknown proportion of the lateral stresses, and the design of the bracing may be left to what experience has shown to be safe, without involving any large waste of timber.

153. Strength of timber. Until recently tests of the strength of timber have generally been made by testing small, selected, well-seasoned sticks of "clear stuff," free from knots or imperfections. Such tests would give results so much higher than the vaguely known strength of large unseasoned "commercial" timber that very large factors of safety were recommended—factors so large as to detract from any confidence in the whole theoretical design. Recently the U. S. Government has been making a thoroughly scientific test of the strength of full-size timber under various conditions as to seasoning, etc. The work has been so extensive and thorough as to render possible the economical designing of timber structures.

One important result of the investigation is the determination of the great influence of the moisture in the timber and the law of its effect on the strength. It has been also shown that timber soaked with water has substantially the same strength as green timber, even though the timber had once been thoroughly seasoned. Since trestles are exposed to the weather they should be designed on the basis of using green timber. It has been shown that the strength of green timber is very regularly about 55 to 60% of the strength of timber in which the moisture is 12% of the dry weight, 12% being the proportion of moisture usually found in timber that is protected from the weather but not heated, as, *e.g.*, the timber in a barn. Since the moduli of rupture have all been reduced to this standard of moisture (12%), if we take *one-eighth* of the rupture values, it still allows a factor of safety of about five, even on green timber. On page 172 there are quoted the values taken from the U. S. Government reports on the strength of timber, the tests probably being the most thorough and reliable that were ever made.

On page 173 are given the "average safe allowable working unit stresses in pounds per square inch," as recommended by the committee on "Strength of Bridge and Trestle Timbers," the work being done under the auspices of the Association of Railway Superintendents of Bridges and Buildings. The report was presented at their fifth annual convention, held in New Orleans, in October, 1895.

Moduli of rupture for various timbers. [12% moisture.]

(Condensed from U. S. Forestry Circular, No. 15.)

No.	Species.	Weight per cubic foot.	Cross-bending. Ultimate Strength.	Cross-bending. Modulus of Elasticity.	Crushing endwise.	Crushing across the grain.	Shearing along the grain.
1	Long-leaf pine....	38	12 600	2 070 000	8000	1180	700
2	Cuban "	39	13 600	2 370 000	8700	1220	700
3	Short-leaf "	32	10 100	1 680 000	6500	960	700
4	Loblolly "	33	11 300	2 050 000	7400	1150	700
5	White "	24	7 900	1 390 000	5400	700	400
6	Red "	31	9 100	1 620 000	6700	1000	500
7	Spruce "	39	10 000	1 640 000	7300	1200	800
8	Bald cypress......	29	7 900	1 290 000	6000	800	500
9	White cedar.......	23	6 300	910 000	5200	700	400
10	Douglas spruce....	32	7 900	1 680 000	5700	800	500
11	White oak.......	50	13 100	2 090 000	8500	2200	1000
12	Overcup "	46	11 300	1 620 000	7300	1900	1000
13	Post "	50	12 300	2 030 000	7100	3000	1100
14	Cow "	46	11 500	1 610 000	7400	1900	900
15	Red "	45	11 400	1 970 000	7200	2300	1100
16	Texan "	46	13 100	1 860 000	8100	2000	900
19	Willow "	45	10 400	1 750 000	7200	1600	900
20	Spanish "	46	12 000	1 930 000	7700	1800	900
21	Shagbark hickory..	51	16 000	2 390 000	9500	2700	1100
27	Pignut " ..	56	18 700	2 730 000	10900	3200	1200
28	White elm........	34	10 300	1 540 000	6500	1200	800
29	Cedar "	46	13 500	1 700 000	8000	2100	1300
30	White ash........	39	10 800	1 640 000	7200	1900	1100

154. Loading. As shown in § 138, the span of trestles is always small, is generally 14 feet, and is never greater than 18 feet except when supported by knee-braces. The greatest load that will ever come on any one span will be the concentrated loading of the drivers of a consolidation locomotive. With spans of 14 feet or less it is impossible for even the four pairs of drivers to be on the same span at once. The weight of the rails, ties, and guard-rails should be added to obtain the total load on the stringers, and the weight of these, plus the weight of the stringers, should be added to obtain the pressure on the caps or corbels.

AVERAGE SAFE ALLOWABLE WORKING UNIT STRESSES IN POUNDS, PER SQUARE INCH. RECOMMENDED BY THE COMMITTEE ON "STRENGTH OF BRIDGE AND TRESTLE TIMBERS." (ASSOCIATION OF RAILWAY SUPERINTENDENTS OF BRIDGES AND BUILDINGS: FIFTH ANNUAL CONVENTION, NEW ORLEANS, OCTOBER, 1895.)

Kind of timber.	Tension.		Compression.			Transverse.		Shearing.	
			With grain.						
	With grain.	Across grain.	End bearing.	Column under 15 diameters.	Across grain.	Extreme fiber stress.	Modulus of elasticity.	With grain.	Across grain.
Factor of safety....	Ten.	Ten.	Five.	Five.	Four.	Six.	Two.	Four.	Four.
White oak	1000	200	1400	900	500	1000	550 000	200	1000
White pine	700	50	1100	700	200	700	500 000	100	500
Southern, long-leaf, or Georgia yellow pine	1200	60	1600	1000	350	1200	850 000	150	1250
Douglas, Oregon, and Washington fir or pine; Yellow fir	1200	....	1600	1200	300	1100	700 000	150	....
Douglas, Oregon, and Washington fir or pine; Red fir	1000	....	....	....	....	800	....	....	....
Northern or short-leaf yellow pine	900	50	1200	800	250	1000	600 000	100	1000
Red pine	900	50	1200	800	200	800	600 000	....	....
Norway pine	800	....	1200	800	200	700	600 000	....	....
Canadian (Ottawa) white pine	1000	....	....	1000	....	....	....	100	....
Canadian (Ontario) red pine	1000	....	....	1000	....	800	700 000	100	....
Spruce and Eastern fir	800	50	1200	800	200	700	600 000	100	750
Hemlock	600	....	....	800	150	600	450 000	100	600
Cypress	600	....	1200	800	200	800	450 000	....	....
Cedar	800	....	1200	800	200	800	350 000	....	400
Chestnut	900	....	....	1000	250	800	500 000	150	400
California redwood	700	....	....	800	200	750	350 000	100	....
California spruce	....	....	....	800	....	800	600 000	....	....

This dead load is almost insignificant compared with the live load and may be included with it. The weight of rails, ties, etc., may be estimated at 200 pounds per foot. To obtain the weight on the caps the weight of the stringers must be added, which depends on the design and on the weight per cubic foot of the wood employed. But as the weight of the stringers is comparatively small, a considerable percentage of variation in weight will have but an insignificant effect on the result. Disregarding all refinements as to actual dimensions, the ordinary maximum loading for standard-gauge railroads may be taken as that due to four pairs of driving-axles, spaced 5′ 0″ apart and giving a pressure of 25000 pounds per axle. This should be increased to 40000 pounds per axle (same spacing) for the heaviest traffic. On the basis of 25000 pounds per axle the following results have been computed:

STRESSES ON VARIOUS SPANS DUE TO MOVING LOADS OF 25000 POUNDS, SPACED 5′ 0″ APART.

Span in feet.	Max. moment, ft. lbs.	Max. shear.	Max. load on one cap.
10	65 000	38 500	52 100
12	103 600	45 000	62 700
14	142 400	49 600	74 200
16	181 400	54 725	85 700
18	220 600	60 100	97 900

Although the dead load does not vary in proportion to the live load, yet, considering the very small influence of the dead load, there will be no appreciable error in assuming the corresponding values, for a load of 40000 lbs. per axle, to be $\frac{40}{25}$ of those given in the above tabulation.

155. Factors of safety. The most valuable result of the government tests is the knowledge that under given moisture conditions the strength of various species of sound timber is not the variable uncertain quantity it was once supposed to be, but that its strength can be relied on to a comparatively close percentage. This confidence in values permits the employment of lower factors of safety than have heretofore been permissible. Stresses, which when excessive would result in immediate destruction, such as cross-breaking and columnar stresses, should be allowed a higher factor of safety—say 6 or 8 for green timber. Other stresses, such as crushing across the grain and shearing along the neutral axis, which will be apparent to inspection before it is dangerous, may be allowed lower factors—say 3 to 5.

156. Design of stringers. The strength of rectangular beams of equal width varies as the square of the depth; therefore deep beams are the strongest. On the other hand, when any cross-sectional dimension of timber much exceeds 12″ the cost is much higher per M, B. M., and it is correspondingly difficult to obtain thoroughly sound sticks, free from wind-shakes, etc. Wind-shakes especially affect the shearing strength. Also, if the required transverse strength is obtained by using high narrow stringers, the area of pressure between the stringers and the cap may become so small as to induce crushing across the grain. This is a very common defect in trestle design. As already indicated in § 138, the span should vary roughly with the average height of the trestle, the longer spans being employed when the trestle bents are very high, although it is usual to employ the same span throughout any one trestle.

To illustrate, if we select a span of 14 feet, the load on one cap will be 74200 lbs. If the stringers and cap are made of long-leaf yellow pine, which require the closely determined value of 1180 lbs. per square inch to produce a crushing amounting to 3% of the height on timber with 12% moisture, we may use 200 lbs. per square inch as a safe pressure even for green timber; this will require 371 square inches of surface. If the cap is 12″ wide, this will require a width of 31 inches, or say 2 stringers under each rail, each 8 inches wide. For rectangular beams

$$\text{Moment} = \tfrac{1}{6}R'bh^2.$$

Using for R' the safe value 1275 lbs. per square inch, we have

$$142400 \times 12 = \tfrac{1}{6} \times 1275 \times 32 \times h^2,$$

from which $h = 15''.9$. If desired, the width may be increased to 9″ and the depth correspondingly reduced, which will give similarly $h = 14''.9$, or say 15″. This shows that two beams, 9″×15″, under each rail will stand the transverse bending and have more than enough area for crushing.

The shear per square inch will equal

$$\frac{3}{2}\,\frac{\text{total shear}}{\text{cross-section}} = \frac{3}{2}\,\frac{49600}{4 \times 9 \times 15} = 138 \text{ lbs. per sq. inch,}$$

which is a safe value, although it should preferably be less Hence the above combination of dimensions will answer.

The deflection should be computed to see if it exceeds the somewhat arbitrary standard of $\frac{1}{200}$ of the span. The deflection for *uniform loading* is

$$\Delta = \frac{5Wl^3}{32bh^3E},$$

in which l = length in inches;

W = total load, assumed as uniform;
E = modulus of elasticity, given as 2,070,000 lbs.

per sq. in. for long-leaf pine, 12% dry, and assumed to be 1,200,000 for green timber. Then

$$\Delta = \frac{5 \times 72800 \times 168^3}{32 \times 36 \times 15^3 \times 1200000} = 0''.37$$

$$\tfrac{1}{200} \times 168'' = 0''.84,$$

so that the calculated deflection is well within the limit. Of course the loading is not strictly uniform, but even with a liberal allowance the deflection is still safe.

For the heaviest practice (40000 lbs. per axle) these stringer dimensions must be correspondingly increased.

157. Design of posts. Four posts are generally used for single-track work. The inner posts are usually braced by the cross-braces, so that their columnar strength is largely increased; but as they are apt to get more than their share of work, the advantage is compensated and they should be treated as unsupported columns for the total distance between cap and sill in simple bents, or for the height of stories in multiple-story construction. The caps and sills are assumed to have a width of 12″. It facilitates the application of bracing to have the columns of the same width and vary the other dimension as required.

Unfortunately the experimental work of the U. S. Government on timber testing has not yet progressed far enough to establish unquestionably a general relation between the strength of long columns and the crushing strength of short blocks. The following formula has been suggested, but it cannot be considered as established:

$$f = F \times \frac{700 + 15c}{700 + 15c + c^2}, \quad \text{in which}$$

f = allowable working stress per sq. in for long columns;
F = " " " " " " " short blocks;
$c = \frac{l}{d}$;
l = length of column in inches;
d = least cross-sectional dimensions in inches.

Enough work has been done to give great reliability to the two following formulæ for white pine and yellow pine, quoted from Johnson's "Materials of Construction," p. 684:

$$\text{Working load per sq. in.} = p = 1000 - \frac{1}{4}\left(\frac{l}{h}\right)^2, \text{ long-leaf pine;}$$

$$\text{" " " " "} = p = 600 - \frac{1}{8}\left(\frac{l}{h}\right)^2, \text{ white pine;}$$

in which l = length of column in inches, a[n]
h = least cross-sectional dimension in inches.

The frequent practice is to use $12'' \times 12''$ posts for all trestles. If we substitute in the above formula $l = 20' = 240''$ and $h = 12''$ we have $p = 1000 - \frac{1}{4}(\frac{240}{12})^2 = 900$ lbs.

$900 \times 144 = 129600$ lbs., the *working load* for *each* post. This is more than the total load on one trestle bent and illustrates the usual great waste of timber. Making the post $8'' \times 12''$ and calculating similarly, we have $p = 775$, and the working load per column is $775 \times 96 = 74400$ lbs. As considerable must be allowed for "weathering," which destroys the strength of the outer layers of the wood, and also for the dynamic effect of the live load, $8'' \times 12''$ may not be too great, but it is certainly a safe dimension. $12'' \times 6''$ would possibly prove amply safe in practice. One method of allowing for weathering is to disregard the outer half-inch on all sides of the post, i.e., to calculate the strength of a post one inch smaller in each dimension than the post actually employed. On this basis an $8'' \times 12'' \times 20'$

post, computed as a $7'' \times 11'$ post, would have a *safe* columnar strength of 706 lbs. per square inch. With an area of 77 square inches, this gives a working load of 54362 lbs. for *each post*, or 217448 lbs. for the four posts. Considering that 74200 lbs. is the maximum load on one cap (14 feet span), the great excess of strength is apparent.

158. Design of caps and sills. The stresses in caps and sills are very indefinite, except as to crushing across the grain. As the stringers are placed almost directly over the inner posts, and as the sills are supported just under the posts, the transverse stresses are almost insignificant. In the above case four posts have an area of $4 \times 12'' \times 8'' = 384$ sq. in. The total load, 74200 lbs., will then give a pressure of 193 pounds per square inch, which is within the allowable limit. This one feature might require the use of $8'' \times 12''$ posts rather than $6'' \times 12''$ posts, for the smaller posts, although probably strong enough as posts, would produce an objectionably high pressure.

159. Bracing. Although some idea of the stresses in the bracing could be found from certain assumptions as to wind-pressure, etc., yet it would probably not be found wise to decrease, for the sake of economy, the dimensions which practice has shown to be sufficient for the work. The economy that would be possible would be too insignificant to justify any risk. Therefore the usual dimensions, given in §§ 139 and 140, should be employed.

CHAPTER V.

TUNNELS.

SURVEYING.

160. Surface surveys. As tunnels are always dug from each end and frequently from one or more intermediate shafts, it is necessary that an accurate surface survey should be made between the two ends. As the natural surface in a locality where a tunnel is necessary is almost invariably very steep and rough, it requires the employment of unusually refined methods of work to avoid inaccuracies. It is usual to run a line on the surface that will be at every point vertically over the center line of the tunnel. Tunnels are generally made straight unless curves are absolutely necessary, as curves add greatly to the cost. Fig. 85 represents roughly a longitudinal section of the

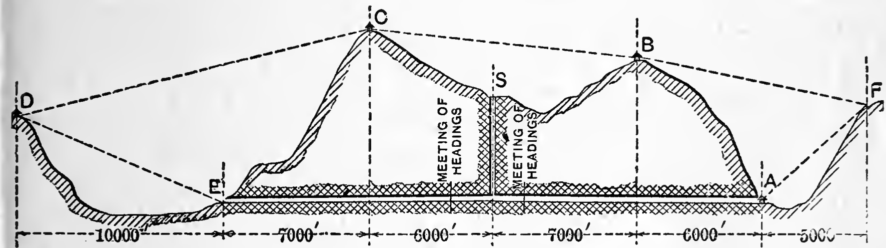


FIG. 85.—SKETCH OF SECTION OF THE HOOSAC TUNNEL.

Hoosac Tunnel. Permanent stations were located at *A*, *B*, *C*, *D*, *E*, and *F*, and stone houses were built at *A*, *B*, *C*, and *D*. These were located with ordinary field transits at first, and then all the points were placed as nearly as possible in one vertical plane by repeated trials and minute corrections, using a very large specially constructed transit. The stations *D* and *F* were necessary because *E* and *A* were invisible from *C* and *B*. The alignment at *A* and *E* having been determined with great accu racy, the true alignment was easily carried into the tunnel.

The relative elevations of A and E were determined with great accuracy. Steep slopes render necessary many settings of the level per unit of horizontal distance and require that the work be unusually accurate to obtain even fair accuracy per unit of distance. The levels are usually re-run many times until the probable error is a very small quantity

The exact horizontal distance between the two ends of the tunnel must also be known, especially if the tunnel is on a grade. The usual steep slopes and rough topography likewise render accurate horizontal measurements very difficult. Frequently when the slope is steep the measurement is best obtained by measuring along the slope and allowing for grade. This may be very accurately done by employing two tripods (level or transit tripods serve the purpose very well), setting them up slightly less than one tape-length apart and measuring between horizontal needles set in wooden blocks inserted in the top of each tripod. The elevation of each needle is also observed. The true horizontal distance between two successive positions of the needles then equals the square root of the difference of the squares of the inclined distance and the difference of elevation. Such measurements will probably be more accurate than those made by attempting to hold the tape horizontal and plumbing down with plumb-bobs, because (1) it is practically difficult to hold both ends of the tape truly horizontal; (2) on steep slopes it is impossible to hold the down-hill end of a 100-foot tape (or even a 25-foot length) on a level with the other end, and the great increase in the number of applications of the unit of measurement very greatly increases the probable error of the whole measurement; (3) the vibrations of a plumb-bob introduce a large probability of error in transferring the measurement from the elevated end of the tape to the ground, and the increased number of such applications of the unit of measurement still further increases the probable error.

161. Surveying down a shaft. If a shaft is sunk, as at S, Fig 85, and it is desired to dig out the tunnel in both directions from the foot of the shaft so as to meet the headings from the outside, it is necessary to know, when at the bottom of the shaft, the elevation, alignment, and horizontal distance from each end of the tunnel.

The *elevation* is generally carried down a shaft by means of a steel tape. This method involves the least number of applications of the unit of measurement and greatly increases the accuracy of the final result.

The *horizontal distance from each end* may be easily transferred down the shaft by means of a plumb-bob, using some of the precautions described in the next paragraph.

To transfer the *alignment* from the surface to the bottom of a shaft requires the highest skill because the shaft is always small, and to produce a line perhaps several thousand feet long in a direction given by two points 6 or 8 feet apart requires that the two points must be determined with extreme accuracy. The eminently successful method adopted in the Hoosac Tunnel will be briefly described: Two beams were securely fastened across the top of the shaft (1030 feet deep), the beams being placed transversely to the direction of the tunnel and as far apart as possible and yet allow plumb-lines, hung from the intersection of each beam with the tunnel center line, to swing freely at the bottom of the shaft. These intersections of the beams with the center line were determined by averaging the results of a large number of careful observations for alignment. Two fine parallel wires, spaced about $\frac{1}{16}''$ apart, were then stretched between the beams so that the center line of the tunnel bisected at all points the space between the wires. Plumb-bobs, weighing 15 pounds, were suspended by fine wires beside each cross-beam, the wires passing between the two parallel alignment wires and bisecting the space. The plumb-bobs were allowed to swing in pails of water at the bottom. Drafts of air up the shaft required the construction of boxes surrounding the wires. Even these precautions did not suffice to absolutely prevent vibration of the wire at the bottom through a very small arc. The mean point of these vibrations in each case was then located on a rigid cross-beam suitably placed at the bottom of the shaft and at about the level of the roof of the tunnel. Short plumb-lines were then suspended from these points whenever desired; a transit was set (by trial) so that its line of collimation passed through both plumb-lines and the line at the bottom could thus be prolonged.

Some recent experience in the "Tamarack" shaft, 4250 feet deep, shows that the accuracy of the results may be affected by air-currents to an unsuspected extent. Two 50-lb. cast-iron plumb-bobs were suspended with No. 24 piano-wire in this shaft. The carefully measured distances between the wires at top and bottom were 16.32 and 16.43 feet respectively. After considerable experimenting to determine the cause of the variation, it was finally concluded that air-currents were alone responsible. The variation of the bobs from a true vertical plane passing through the wires at the top was of course an unknown quantity, but since the variation in *one* direction amounted to 0.11 foot, the accuracy in other directions was very questionable. This shows that a careful comparative measurement between the wires at top and bottom should always be made as a test of their parallelism.

162. Underground surveys. Survey marks are frequently placed on the timbering, but they are apt to prove unreliable on account of the shifting of the timbering due to settlement of the surrounding material. They should never be placed at the bottom of the tunnel on account of the danger of being disturbed or covered up. Frequently holes are drilled in the roof and filled with wooden plugs in which a hook is screwed exactly on line. Although this is probably the safest method, even these plugs are not always undisturbed, as the material, unless very hard, will often settle slightly as the excavation proceeds. When a tunnel is perfectly straight and not too long, alignment-points may be given as frequently as desired from permanent stations located *outside* the tunnel where they are not liable to disturbance. This has been accomplished by running the alignment through the upper part of the cross-section, at one side of the center, where it is out of the way of the piles of masonry material, débris, etc., which are so apt to choke up the lower part of the cross-section. The position of this line relative to the cross-section being fixed, the alignment of any required point of the cross-section is readily found by means of a light frame or template with a fixed target located where this line would intersect the frame when properly placed. A level-bubble on the frame will assist in setting the frame in its proper position.

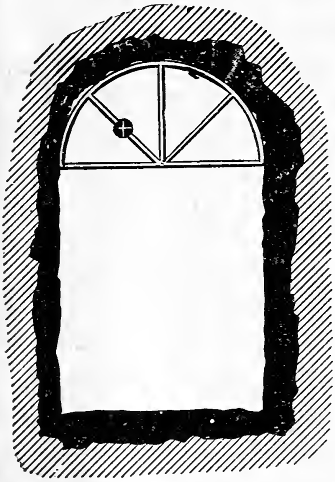


FIG. 86.

In all tunnel surveying the cross-wires must be illuminated by a lantern, and the object sighted at must also be illuminated. A powerful dark-lantern with the opening covered with *ground glass* has been found useful. This may be used to illuminate a plumb-bob string or a very fine rod, or to place behind a brass plate having a narrow slit in it, the axis of the slit and plate being coincident with the plumb-bob string by which it is hung.

On account of the interference to the surveying caused by the work of construction and also by the smoke and dust in the air resulting from the blasting, it is generally necessary to make the surveys at times when construction is temporarily suspended.

163. Accuracy of tunnel surveying. Apart from the very natural desire to do surveying which shall check well, there is an important financial side to accurate tunnel surveying. If the survey lines do not meet as desired when the headings come together, it may be found necessary, if the error is of appreciable size, to introduce a slight curve, perhaps even a reversed curve, into the alignment, and it is even conceivable that the tunnel section would need to be enlarged somewhat to allow for these curves. The cost of these changes and the perpetual annoyance due to an enforced and undesirable alteration of the original design will justify a considerable increase in the expenses of the survey. Considering that the cost of surveys is usually but a small fraction of the total cost of the work, an increase of 10 or even 20% in the cost of the surveys will mean an insignificant addition to the total cost and frequently, if not generally, it will result in a saving of many times the increased cost. The accuracy actually attained in two noted American tunnels is given as follows: The Musconetcong tunnel is about 5000 feet long, bored through a mountain 400 feet high. The error of alignment at the meeting of the headings was 0′.04, error of levels 0′.015, error of distance 0′.52. The Hoosac tunnel is over 25000 feet long. The heading from the east end met the heading from the central shaft at a point 11274 feet from the east end and 1563 feet from the shaft. The error in alignment was $\frac{5}{16}$ of an inch, that of levels "a few hundredths," error of distance "trifling." The alignment, corrected at the shaft, was carried on through and met the heading from the west end at a point 10138 feet from the west end and 2056 feet from the shaft. Here the error of alignment was $\frac{9}{16}''$ and that of levels 0.134 ft.

DESIGN.

164. **Cross-sections.** Nearly all tunnels have cross-sections peculiar to themselves—all varying at least in the details. The *general* form of a great many tunnels is that of a rectangle surmounted by a semi-circle or semi-ellipse. In very soft material an inverted arch is necessary along the bottom. In such cases the sides will generally be arched instead of vertical. The sides are frequently battered. With very long tunnels, several forms of cross-section will often be used in the same tunnel, owing to differences in the material encountered. In solid rock, which will not disintegrate upon exposure, no lining is required, and the cross-section will be the irregular section left by the blasting, the only requirement being that no rock shall be left within the required cross-sectional figure. Farther on, in the same tunnel, when passing through some very soft treacherous material, it may be necessary to put in a full arch lining—top, sides, and bottom—which will be nearly circular in cross-section. For an illustration of this see Figs. 87 and 88.

The width of tunnels varies as greatly as the designs. Single-track tunnels generally have a width of 15 to 16 feet. Occasionally they have been built 14 feet wide, and even less, and also up to 18 feet, especially when on curves. 24 to 26 feet is the most common width for double track. Many double-track tunnels are only 22 feet wide, and some are 28 feet wide. The heights are generally 19 feet for single track and 20 to 22 feet for double track. The variations from these figures are considerable. The lower limits depend on the cross-section of the rolling stock, with an indefinite allowance for clearance and ventilation. Cross-sections which coincide too closely with what is absolutely required for clearance are objectionable, because any slight settlement of the lining which would otherwise be harmless would then become troublesome and even dangerous. Figs. 87, 88, and 89 * show some typical cross-sections.

165. **Grade.** A grade of at least 0 2% is needed for drainage. If the tunnel is at the summit of two grades, the tunnel grade should be practically level, with an allowance for drainage, the

* Drinker's "Tunneling."

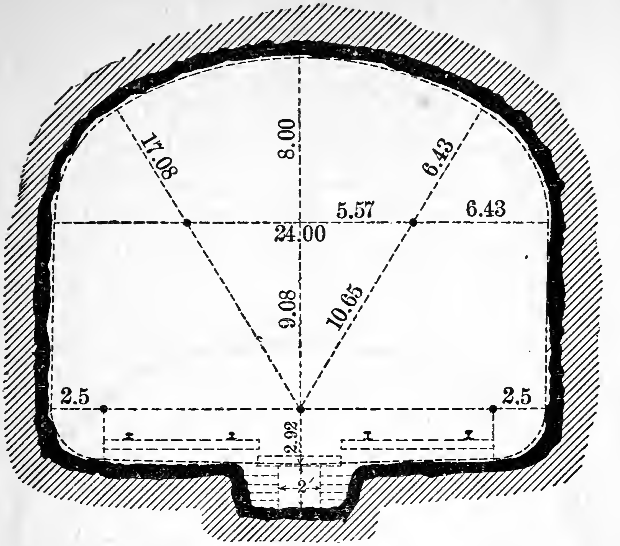


FIG. 87.—HOOSAC TUNNEL. SECTION THROUGH SOLID ROCK.

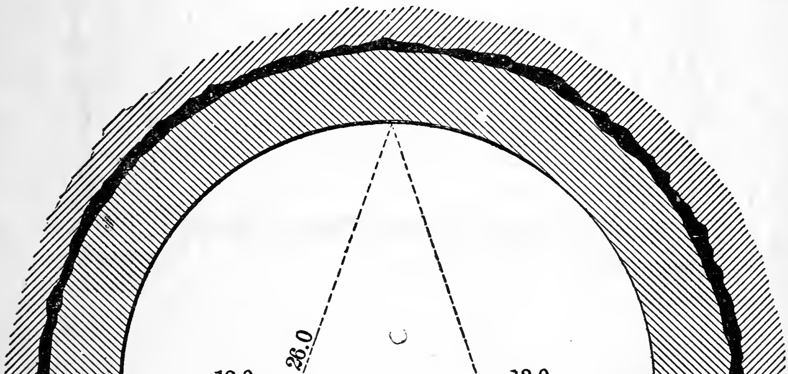


FIG. 88.—HOOSAC TUNNEL. SECTION THROUGH SOFT GROUND.

actual summit being perhaps in the center so as to drain both ways. When the tunnel forms part of a long ascending grade, it is advisable to reduce the grade through the tunnel unless the tunnel is very short. The additional atmospheric resistance and the decreased adhesion of the driver wheels on the damp rails in a tunnel will cause an engine to work very hard and still more rapidly vitiate the atmosphere until the accumulation of poisonous gases becomes a source of actual danger to the engineer and

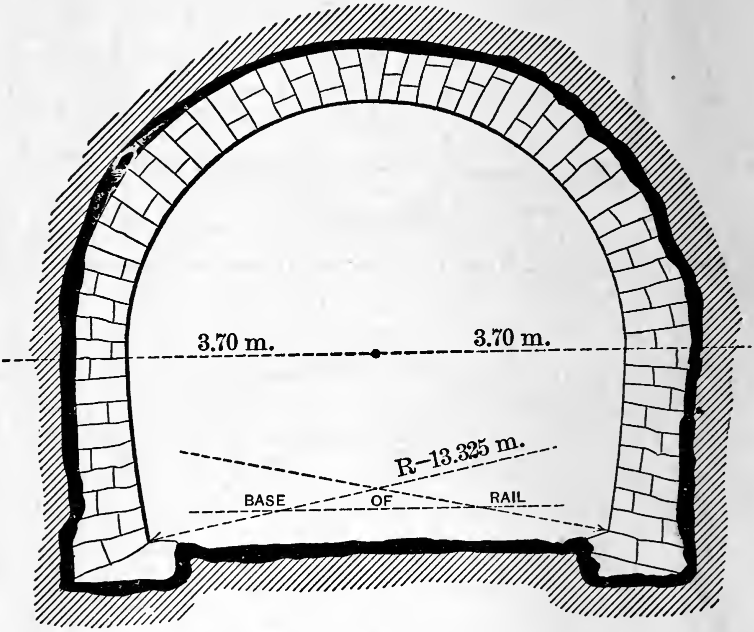


FIG. 89.—ST. CLOUD TUNNEL.

fireman of the locomotive and of extreme discomfort to the passengers. If the nominal ruling grade of the road were maintained through a tunnel, the maximum resistance would be found in the tunnel. This would probably cause trains to stall there, which would be objectionable and perhaps dangerous.

166. Lining. It is a characteristic of many kinds of rock and of all earthy material that, although they may be self-sustaining when first exposed to the atmosphere, they rapidly disintegrate and require that the top and perhaps the sides and even the bottom shall be lined to prevent caving in. In this country, when timber is cheap, it is occasionally framed as an arch and used as the *permanent* lining, but masonry is always to be preferred. Frequently the cross-section is made extra large so that a masonry lining may subsequently be placed inside the wooden lining and thus postpone a large expense until the road is better able to pay for the work. In very soft unstable material, like quicksand, an arch of cut stone voussoirs may be necessary to withstand the pressure. A good quality of brick is occasionally used for lining, as they are easily handled and make good masonry if the pressure is not excessive. Only the best of cement mortar should be used, economy in this feature being the worst of folly. Of course the excavation must include the outside line of the lining. Any excavation which is made outside of this line (by the fall of earth or loose rock or by excessive blasting) must be refilled with stone well packed in. Occasionally it is necessary to fill these spaces with concrete. Of course it is not necessary that the lining be uniform throughout the tunnel.

167. Shafts. Shafts are variously made with square, rectangular, elliptical, and circular cross-sections. The rectangular

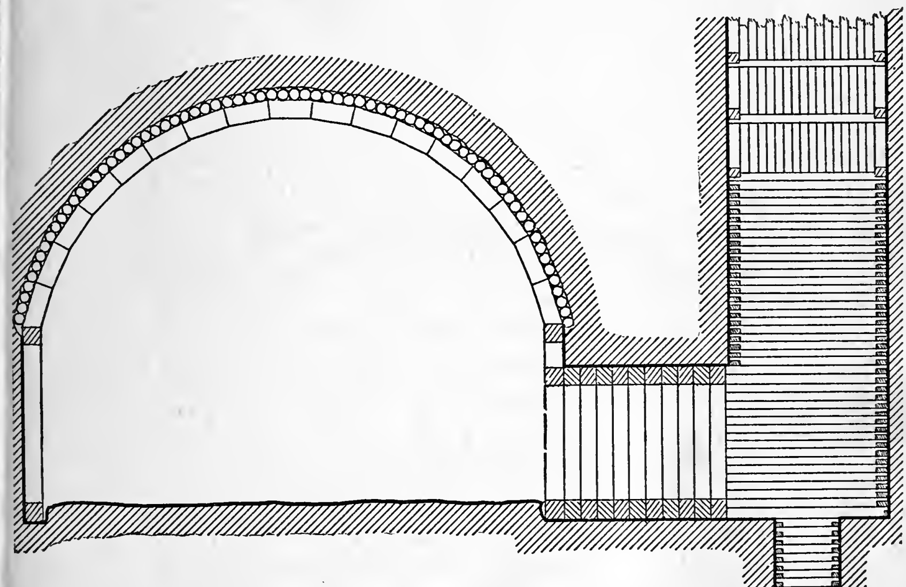


FIG. 90.—CONNECTION WITH SHAFT, CHURCH HILL TUNNEL.

cross-section, with the longer axis parallel with the tunnel, is most usually employed. Generally the shaft is directly over the center of the tunnel, but that always implies a complicated connection between the linings of the tunnel and shaft, provided such linings are necessary. It is easier to sink a shaft near to one side of the tunnel and make an opening through the nearly vertical side of the tunnel. Such a method was employed in the Church Hill Tunnel, illustrated in Fig. 90.* Fig. 91 † shows a cross-section for a large main shaft. Many shafts have been built with the idea of being left open permanently for ventilation and have therefore been elaborately lined with masonry.

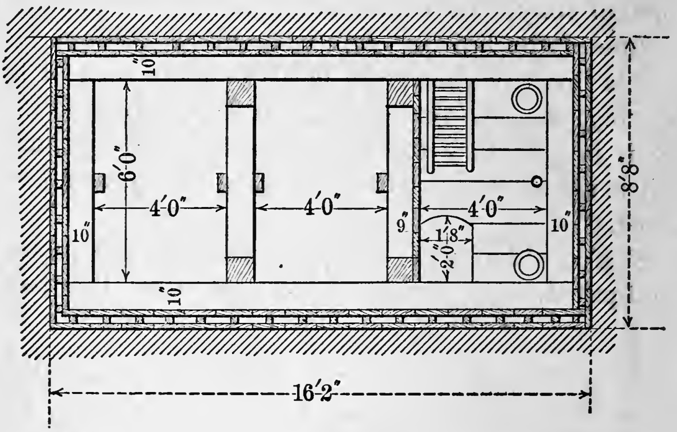


Fig. 91.—Cross-section. Large Main Shaft.

The general consensus of opinion now appears to be that shafts are worse than useless for ventilation; that the quick passage of a train through the tunnel is the most effective ventilator; and that shafts only tend to produce cross-currents and are ineffective to clear the air. In consequence, many of these elaborately lined shafts have been permanently closed, and the more recent practice is to close up a shaft as soon as the tunnel is completed. Shafts always form drainage-wells for the material they pass through, and sometimes to such an extent that it is a serious matter to dispose of the water that collects at the bottom, requiring the construction of large and expensive drains.

168. Drains. A tunnel will almost invariably strike veins of water which will promptly begin to drain into the tunnel and not only cause considerable trouble and expense during construction, but necessitate the provision of permanent drains for its perpetual disposal. These drains must frequently be so large as

* Drinker's "Tunneling."

† Rziha, "Lehrbuch der Gesammten Tunnelbaukunst."

to appreciably increase the required cross-section of the tunnel. Generally a small open gutter on each side will suffice for this purpose, but in double-track tunnels a large covered drain is often built between the tracks. It is sometimes necessary to thoroughly grout the outside of the lining so that water will not force its way through the masonry and perhaps injure it, but may freely drain down the sides and pass through openings in the side walls near their base into the gutters.

CONSTRUCTION.

169. Headings. The methods of all tunnel excavation depend on the general principle that all earthy material, except the softest of liquid mud and quicksand, will be self-sustaining over a greater or less area and for a greater or less time after excavation is made, and the work consists in excavating some material and immediately propping up the exposed surface by timbering and poling-boards. The excavation of the cross-section begins with cutting out a "heading," which is a small horizontal drift whose breast is constantly kept 15 feet or more in advance of the full cross-sectional excavation. In solid self-sustaining rock, which will not decompose upon exposure to air, it becomes simply a matter of excavating the rock with the least possible expenditure of time and energy. In soft ground the heading must be heavily timbered, and as the heading is gradually enlarged the timbering must be gradually extended and perhaps replaced, according to some regular system, so that when the full cross-section has been excavated it is supported by such timbering as is intended for it. The heading is sometimes made on the center line near the top; with other plans, on the center line near the bottom; and sometimes two simultaneous headings are run in the two lower corners. Headings near the bottom serve the purpose of draining the material above it and facilitating the excavation. The simplest case of heading timbering is that shown in Fig. 92, in which cross-timbers are placed at intervals just under the roof, set in notches cut in the side walls and supporting poling-boards which sus-

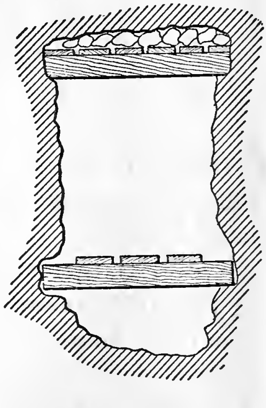


Fig. 92.

tain whatever pressure may come on them. Cross-timbers near the bottom support a flooring on which vehicles for transporting material may be run and under which the drainage may freely escape. As the necessity for timbering becomes greater, side timbers and even bottom timbers must be added, these timbers supporting poling-boards, and even the breast of the heading must be protected by boards suitably braced,

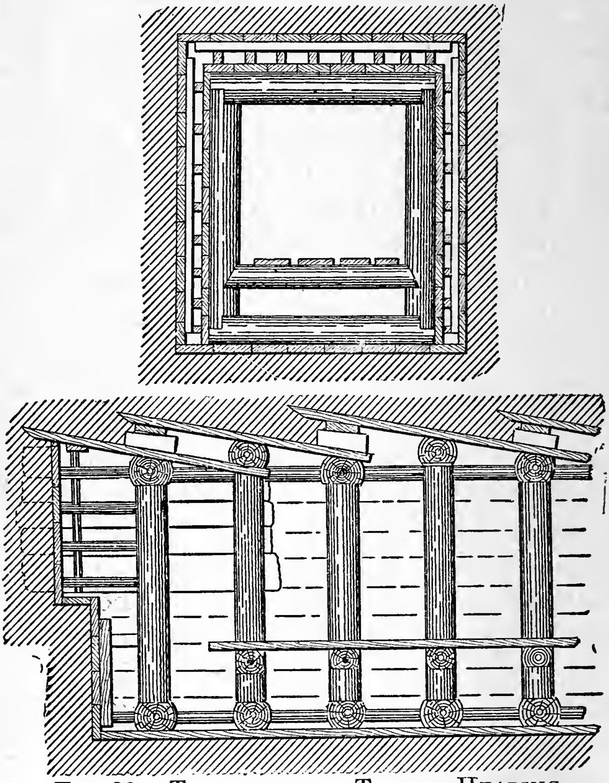


FIG. 93.—TIMBERING FOR TUNNEL HEADING.

as shown in Fig. 93. The supporting timbers are framed into collars in such a manner that added pressure only increases their rigidity.

170. Enlargement. Enlargement is accomplished by removing the poling-boards, one at a time, excavating a greater or less amount of material, and immediately supporting the exposed material with poling-boards suitably braced. (See Figs. 93 and 94.) This work being systematically done, space is thereby obtained in which the framing for the full cross-section may be gradually introduced. The framing is constructed with a cross-section so large that the masonry lining may be constructed within it.

171. Distinctive features of various methods of construction. There are six general systems, known as the English, German, Belgian, French, Austrian, and American. They are so named

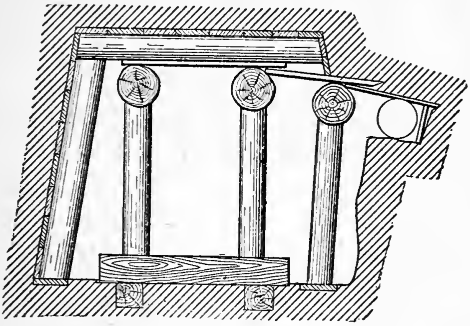


Fig. 94.

from the origin of the methods, although their use is not confined to the countries named. Fig. 95 shows by numbers (1 to 5) the order of the excavation within the cross-sections. The English, Austrian, and American systems are alike in excavating the entire cross-section before beginning the construction of the masonry lining. The German method leaves a solid core (5) until practically the whole of the lining is complete. This has the disadvantage of extremely cramped quarters for work, poor ventilation, etc. The Belgian and French methods agree in excavating the upper part of the section, building the arch at once, and supporting it temporarily until the side walls are built. The Belgian method then takes out the core (3), removes very short sections of the sides (4) immediately underpinning the arch with short sections of the side walls and thus gradually constructing the whole side wall. The French method digs out the sides (3), supporting the arch temporarily with timbers and then replacing the timbers with masonry; the core (4) is taken out last. The French method has the same disadvantage as the German—working in a cramped space. The Belgian and French systems have the disadvantage that the arch, supported temporarily on timber, is very apt to be strained and cracked by the slight settlement that so frequently occurs in soft material. The English, Austrian, and American methods differ mainly in the design of the timbering. The English support the roof by lines of very heavy *longitudinal* timbers which are supported at comparatively wide intervals by a heavy framework occupying the

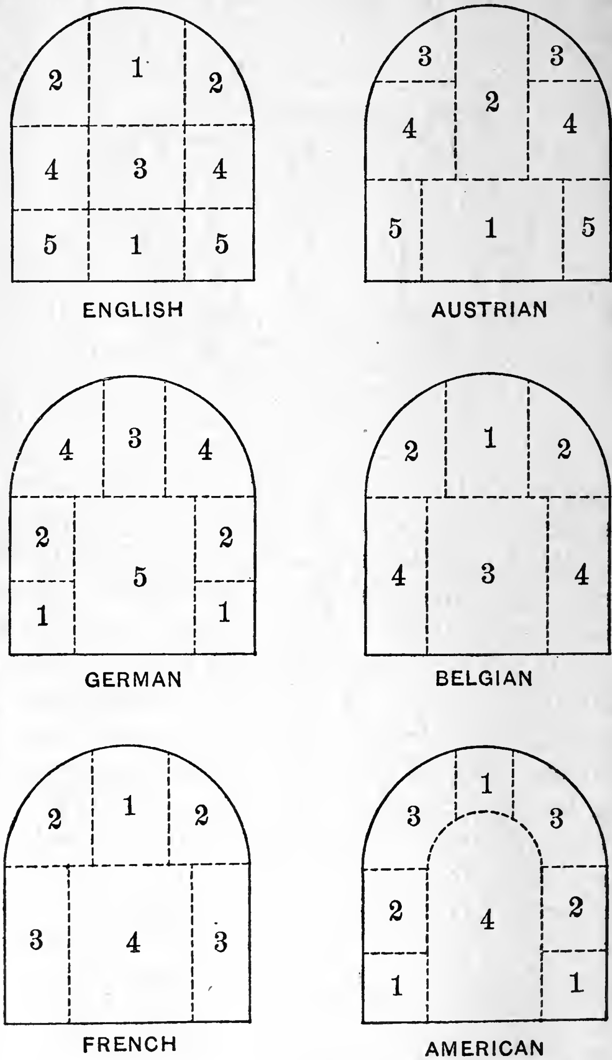


Fig. 95.—Order of Working by the Various Systems.

whole cross-section. The Austrian system uses such frequent cross-frames of timber-work that poling-boards will suffice to support the material between the frames. The American system agrees with the Austrian in using frequent cross-frames supporting poling-boards, but differs from it in that the "cross-frames" consist simply of arches of 3 to 15 wooden voussoirs, the voussoirs being blocks of 12″×12″ timber about 2 to 8 feet long and cut with joints normal to the arch. These arches are put together on a centering which is removed as soon as the arch is keyed up and thus immediately opens up the full cross-section, so that the center core (4) may be immediately dug out and the masonry constructed in a large open space. The American system has been used successfully in very soft ground, but its advantages are greater in loose rock, when it is much cheaper than the other methods which employ more timber. Fig. 90 and Plate III illustrate the use of the American system. Fig. 90 shows the wooden arch in place. The masonry arch may be placed when convenient, since it is *possible* to lay the track and commence traffic as soon as the wooden arch is in place. The student is referred to Drinker's "Tunneling" and to Rziha's "Lehrbuch der Gesammten Tunnelbaukunst" for numerous illustrations of European methods of tunnel timbering.

172. **Ventilation during construction.** Tunnels of any great length must be artificially ventilated during construction. If the excavated material is rock so that blasting is necessary, the need for ventilation becomes still more imperative. The invention of compressed-air drills simultaneously solved two difficulties. It introduced a motive power which is unobjectionable in its application (as gas would be), and it also furnished at the same time a supply of just what is needed—pure air. If no blasting is done (and sometimes even when there is blasting), air must be supplied by direct pumping. The cooling effect of the sudden expansion of compressed air only reduces the otherwise objectionably high temperature sometimes found in tunnels. Since pure air is being continually pumped in, the foul air is thereby forced out.

173. **Excavation for the portals.** Under normal conditions there is always a greater or less amount of open cut preceding and following a tunnel. Since all tunnel methods depend (to some slight degree at least) on the capacity of the exposed material to act as an arch, there is implied a considerable thickness of material above the tunnel. This thickness is reduced to nearly zero over the tunnel portals and therefore requires special treatment, particularly when the material is very soft. Fig. 96 *

* Rziha, "Lehrbuch der Gesammten Tunnelbaukunst."

illustrates one method of breaking into the ground at a portal. The loose stones are piled on the framing to give stability to the framing by their weight and also to retain the earth on the

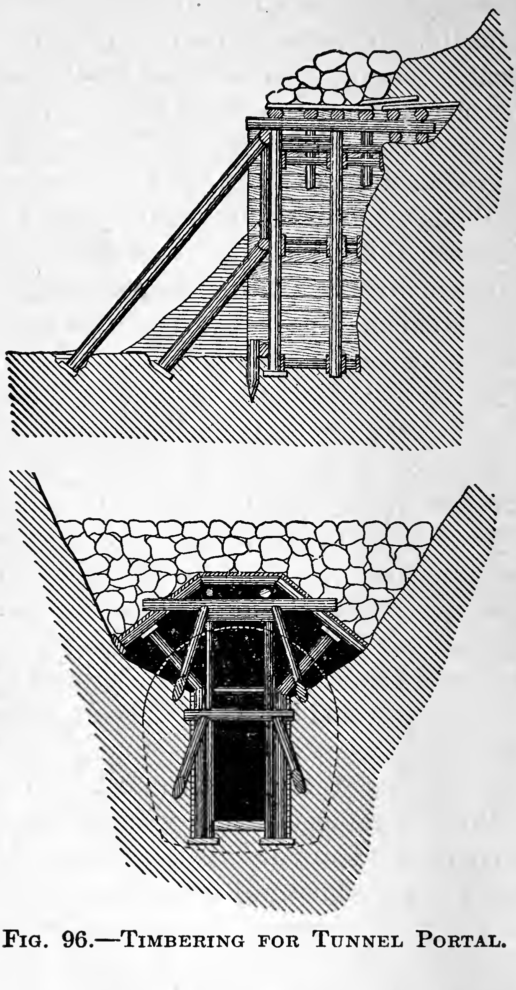
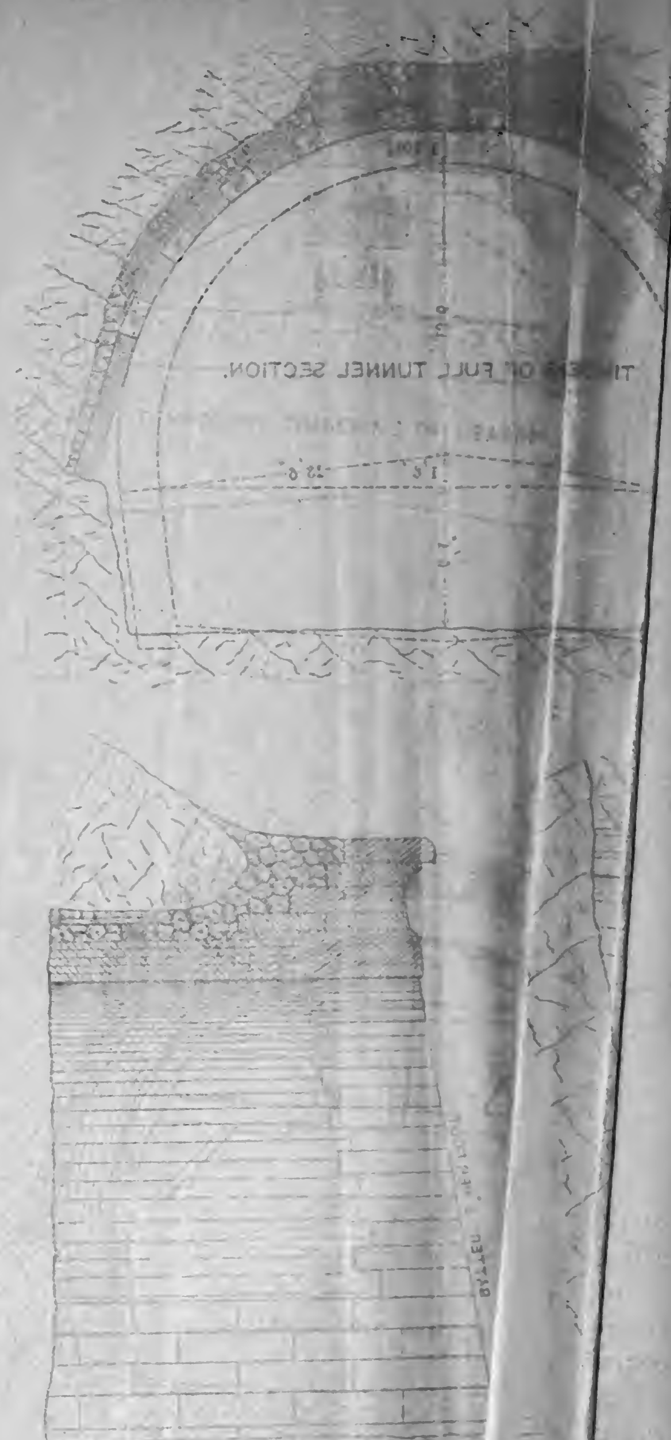


FIG. 96.—TIMBERING FOR TUNNEL PORTAL.

slope above. Another method is to sink a temporary shaft to the tunnel near the portal; immediately enlarge to the full size and build the masonry lining; then work back to the portal. This method is more costly, but is preferable in very treacherous ground, it being less liable to cause landslides of the surface material.

174. Tunnels vs. open cuts. In cases in which an open cut rather than a tunnel is a possibility the ultimate consideration is generally that of first cost combined with other financial con-


OF FULL TUNNEL SECTION.

siderations and annual maintenance charges directly or indirectly connected with it. Even when an open cut may be constructed at the same cost as a tunnel (or perhaps a little cheaper) the tunnel may be preferable under the following conditions:

1. When the soil indicates that the open cut would be liable to landslides.

2. When the open cut would be subject to excessive snow-drifts or avalanches.

3. When land is especially costly or it is desired to run under existing costly or valuable buildings or monuments. When running through cities, tunnels are sometimes constructed as open cuts and then arched over.

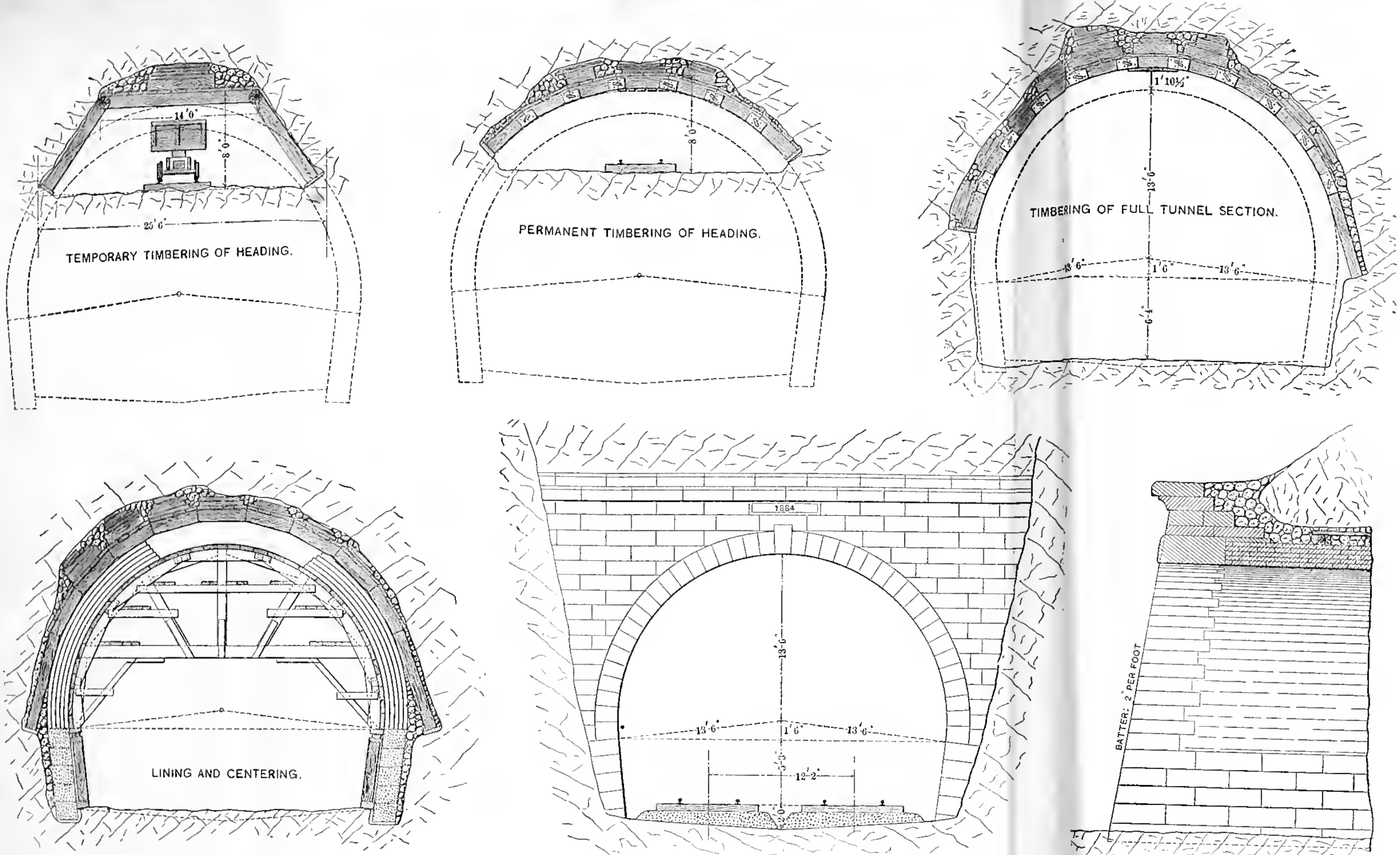
These cases apply to tunnels *vs.* open cuts when the alignment is fixed by other considerations than the mere topography. The broader question of excavating tunnels to avoid excessive grades or to save distance or curvature, and similar problems, are hardly susceptible of general analysis except as questions of railway economics and must be treated individually.

175. **Cost of tunneling.** The cost of any construction which involves such uncertainties as tunneling is very variable. It depends on the material encountered, the amount and kind of timbering required, on the size of the cross-section, on the price of labor, and especially on the *reconstruction* that *may* be necessary on account of mishaps.

Headings generally cost \$4 to \$5 per cubic yard for excavation, while the remainder of the cross-section in the same tunnel may cost about half as much. The average cost of a large number of tunnels in this country may be seen from the following table:*

Material.	Cost per cubic yard.				Cost per lineal foot.	
	Excavation.		Masonry.			
	Single.	Double.	Single.	Double.	Single.	Double.
Hard rock.....	\$5.89	\$5.45	\$12.00	\$8.25	\$69.76	\$142.82
Loose rock.....	3.12	3.48	9.07	10.41	80.61	119.26
Soft ground....	3.62	4.64	15.00	10.50	135.31	174.42

* Figures derived from Drinker's "Tunneling."



Elevation of Portal.
Phœnixville Tunnel P. S. V. R. R.

Longitudinal Section of Portal.

(*To face page* 195.)

A considerable variation from these figures may be found in individual cases, due sometimes to unusual skill (or the lack of it) in prosecuting the work, but the figures will generally be sufficiently accurate for preliminary estimates or for the comparison of two proposed routes.

CHAPTER VI.

CULVERTS AND MINOR BRIDGES.

176. Definition and object. Although a variable percentage of the rain falling on any section of country soaks into the ground and does not immediately reappear, yet a very large percentage flows over the surface, always seeking and following the lowest channels. The roadbed of a railroad is constantly intersecting these channels, which frequently are normally dry. In order to prevent injury to railroad embankments by the impounding of such rainfall, it is necessary to construct waterways through the embankment through which such rainflow may freely pass. Such waterways, called culverts, are also applicable for the bridging of very small although perennial streams, and therefore in this work the term culvert will be applied to all water-channels passing through a railroad embankment which are not of sufficient magnitude to require a special structural design, such as is necessary for a large masonry arch or a truss bridge.

177. Elements of the design. A well-designed culvert must afford such free passage to the water that it will not "back up" over the adjoining land nor cause any injury to the embankment or culvert. The ability of the culvert to discharge freely all the water that comes to it evidently depends chiefly on the area of the waterway, but also on the form, length, slope, and materials of construction of the culvert and the nature of the approach and outfall. When the embankment is very low and the amount of water to be discharged very great, it sometimes becomes necessary to allow the water to discharge "under a head," i e., with the surface of the water above the top of the culvert. Safety then requires a much stronger construction than would otherwise be necessary to avoid injury to the culvert or embankment by washing. The necessity for such construction should be avoided if possible.

AREA OF THE WATERWAY.

178. Elements involved. The determination of the required area of the waterway involves such a multiplicity of indeterminate elements that any close determination of its value from purely theoretical considerations is a practical impossibility. The principal elements involved are:

a. Rainfall. The real test of the culvert is its capacity to discharge without injury the flow resulting from the extraordinary rainfalls and "cloud bursts" that may occur once in many years. Therefore, while a knowledge of the average annual rainfall is of very little value, a record of the maximum rainfall during heavy storms for a long term of years may give a relative idea of the maximum demand on the culvert.

b. Area of watershed. This signifies the total area of country draining into the channel considered. When the drainage area is very small it is sometimes included within the area surveyed by the preliminary survey. When larger it is frequently possible to obtain its area from other maps with a percentage of accuracy sufficient for the purpose. Sometimes a special survey for the purpose is considered justifiable.

c. Character of soil and vegetation. This has a large influence on the rapidity with which the rainflow from a given area will reach the culvert. If the soil is hard and impermeable and the vegetation scant, a heavy rain will run off suddenly, taxing the capacity of the culvert for a short time, while a spongy soil and dense vegetation will retard the flow, making it more nearly uniform and the maximum flow at any one time much less.

d. Shape and slope of watershed. If the watershed is very long and narrow (other things being equal), the water from the remoter parts will require so much longer time to reach the culvert that the flow will be comparatively uniform, especially when the slope of the whole watershed is very low. When the slope of the remoter portions is quite steep it may result in the nearly simultaneous arrival of a storm-flow from all parts of the watershed, thus taxing the capacity of the culvert.

e. Effect of design of culvert. The principles of hydraulics show that the slope of the culvert, its length, the form of the cross-section, the nature of the surface, and the form of the approach and discharge all have a considerable influence on the area of cross-section required to discharge a given volume of water in a given time, but unfortunately the combined hydraulic effect of these various details is still a very uncertain quantity.

179. Methods of computation of area. There are three possible methods of computation.

(a) **Theoretical.** As shown above it is a practical impossibility to estimate correctly the combined effect of the great multiplicity of elements which influence the final result. The nearest approach to it is to estimate by the use of empirical formulæ the amount of water which will be presented at the upper end of the culvert in a given time and then to compute, from the principles of hydraulics, the rate of flow through a culvert of given construction, but (as shown in § 178, *e*) such methods are still very unreliable, owing to lack of experimental knowledge. This method has *apparently* greater scientific accuracy than other methods, but a little study will show that the elements of uncertainty are as great and the final result no more reliable. The method is most reliable for streams of uniform flow, but it is under these conditions that method (*c*) is most useful. The theoretical method will not therefore be considered further.

(b) **Empirical.** As illustrated in § 180, some formulæ make the area of waterway a function of the drainage area, the formula being affected by a coefficient the value of which is estimated between limits according to the judgment Assuming that the formulæ are sound, their use only narrows the limits of error, the final determination depending on experience and judgment.

(c) **From observation.** This method, considered by far the best for permanent work, consists in observing the high-water marks on contracted channel-openings which are on the same stream and as near as possible to the proposed culvert. If the country is new and there are no such openings, the wisest plan is to bridge the opening by a temporary structure in wood which has an ample waterway (see § 126, *b*, 4) and carefully observe all high-water marks on that opening during the 6 to 10 years which is ordinarily the minimum life of such a structure. As shown later, such observations may be utilized for a close computation of the required waterway. Method (*b*) may be utilized for an approximate calculation for the required area for the temporary structure, using a value which is intentionally excessive, so that a permanent structure of sufficient capacity may subsequently be constructed *within* the temporary structure.

180. Empirical formulæ. Two of the best known empirical formulæ for area of the waterway are the following:

(a) Myer's formula:

Area of waterway in square feet $= C \times \sqrt{\text{drainage area in acres}}$, where C is a coefficient varying from 1 for flat country to 4 for mountainous country and rocky ground. As an illustration, if the drainage area is 100 acres, the waterway area should be from 10 to 40 square feet, according to the value of the coefficient chosen. It should be noted that this formula does not regard the great variations in rainfall in various parts of the world nor the design of the culvert, and also that the final result depends largely on the choice of the coefficient.

(b) Talbot's formula:

Area of waterway in square feet $= C \times \sqrt[4]{(\text{drainage area in acres})^3}$. "For steep and rocky ground C varies from $\frac{2}{3}$ to 1. For rolling agricultural country subject to floods at times of melting snow, and with the length of the valley three or four times its width, C is about $\frac{1}{3}$; and if the stream is longer in proportion to the area, decrease C. In districts not affected by accumulated snow, and where the length of the valley is several times the width, $\frac{1}{5}$ or $\frac{1}{6}$, or even less, may be used. C should be increased for steep side slopes, especially if the upper part of the valley has a much greater fall than the channel at the culvert." * As an illustration, if the drainage area is 100 acres the area of waterway should be $C \times 31.6$. The area should then vary from 5 to 31 square feet, according to the character of the country. Like the previous estimate, the result depends on the choice of a coefficient and disregards local variations in rainfall, except as they may be arbitrarily allowed for in choosing the coefficient.

181. Value of empirical formulæ. The fact that these formulæ, as well as many others of similar nature that have been suggested, depend so largely upon the choice of the coefficient shows that they are valuable "more as a guide to the judgment than as a working rule," as Prof. Talbot explicitly declares in

* Prof. A. N. Talbot, "Selected Papers of the Civil Engineers' Club of the Univ. of Illinois."

commenting on his own formula. In short, they are chiefly valuable in indicating a probable maximum and minimum between which the true result probably lies.

182. Results based on Observation. As already indicated in § 179, observation of the stream in question gives the most reliable results. If the country is new and no records of the flow of the stream during heavy storms has been taken, even the life of a temporary wooden structure may not be long enough to include one of the unusually severe storms which must be allowed for, but there will usually be some high-water mark which will indicate how much opening will be required. The following quotation illustrates this: "A tidal estuary may generally be safely narrowed considerably from the extreme water lines if stone revetments are used to protect the bank from wash. Above the true estuary, where the stream cuts through the marsh, we generally find nearly vertical banks, and we are safe if the faces of abutments are placed even with the banks. In level sections of the country, where the current is sluggish, it is usually safe to encroach somewhat on the general width of the stream, but in rapid streams among the hills the width that the stream has cut for itself through the soil should not be lessened, and in ravines carrying mountain torrents the openings must be left very much larger than the ordinary appearance of the banks of the stream would seem to make necessary." *

As an illustration of an observation of a storm-flow through a temporary trestle, the following is quoted: "Having the flood height and velocity, it is an easy matter to determine the volume of water to be taken care of. I have one ten-bent pile trestle 135 feet long and 24 feet high over a spring branch that ordinarily runs about six cubic inches per second. Last summer during one of our heavy rainstorms (four inches in less than three hours) I visited this place and found by float observations the surface velocity at the highest stage to be 1.9 feet per second. I made a high-water mark, and after the flood-water receded found the width of stream to be 12 feet and an average depth of $2\frac{3}{4}$ feet. This, with a surface velocity of 1.9 feet per second, would give approximately a discharge of 50

---

* J. P. Snow, Boston & Maine Railway. From Report to Association of Railway Superintendents of Bridges and Buildings. 1897.

cubic feet, or 375 gallons, per second. Having this information it is easy to determine size of opening required." *

183. Degree of accuracy required. The advantages resulting from the use of standard designs for culverts (as well as other structures) have led to the adoption of a comparatively small number of designs. The practical use made of a computation of required waterway area is to determine which one of several standard designs will most nearly fulfill the requirements. For example, if a 24-inch iron pipe, having an area of 3.14 square feet, is considered to be a little small, the next size (30-inch) would be adopted; but a 30-inch pipe has an area of 4.92 square feet, which is 56% larger. A similar result, except that the percentage of difference might not be quite so marked, will be found by comparing the areas of consecutive standard designs for stone box culverts.

The advisability of designing a culvert to withstand any storm-flow that may *ever* occur is considered doubtful. Several years ago a record-breaking storm in New England carried away a very large number of bridges, etc., hitherto supposed to be safe. It was not afterward considered that the design of those bridges was faulty, because the extra cost of constructing bridges capable of withstanding such a flood, added to interest for a long period of years, would be enormously greater than the cost of repairing the damages of such a storm once or twice in a century. Of course the element of danger has some weight, but not enough to justify a great additional expenditure, for common prudence would prompt unusual precautions during or immediately after such an extraordinary storm.

PIPE CULVERTS.

184. Advantages. Pipe culverts, made of cast iron or earthenware, are very durable, readily constructed, moderately cheap, will pass a larger volume of water in proportion to the area than many other designs on account of the smoothness of the surface, and (when using iron pipe) may be used very close to the track when a low opening of large capacity is required. Another advantage lies in the ease with which they may be inserted through a somewhat larger opening that has been

* A. J. Kelley, Kansas City Belt Railway. From Report to Association of Railway Superintendents of Bridges and Buildings. 1897.

temporarily lined with wood, without disturbing the roadbed or track.

185. Construction. Permanency requires that the foundation shall be firm and secure against being washed out. To accomplish this, the soil of the trench should be hollowed out to fit the lower half of the pipe, making suitable recesses for the bells. In very soft treacherous soil a foundation-block of concrete is sometimes placed under each joint, or even throughout the whole length. When pipes are laid through a slightly larger timber culvert great care should be taken that the pipes are properly supported, so that there will be no settling nor development of unusual strains when the timber finally decays and gives way. To prevent the washing away of material around the pipe the ends should be protected by a bulkhead. This is best constructed of masonry (see Fig. 97), although wood is sometimes used for cheap and minor constructions. The joints should be calked, especially when the culvert is liable to run full or when the outflow is impeded and the culvert is liable to be partly or wholly filled during freezing weather. The cost of a calking of clay or even hydraulic cement is insignificant compared with the value of the additional safety afforded. When the grade of the pipe is perfectly uniform, a very low rate of grade will suffice to drain a pipe culvert, but since some unevenness of grade is inevitable through uneven settlement or imperfect construction, a grade of 1 in 20 should preferably be required, although much less is often used. The length of a pipe culvert is approximately determined as follows:

$$\textit{Length} = 2s\,(\textit{depth of embankment to top of pipe}) + (\textit{width of roadbed}),$$

in which s is the slope ratio (horizontal to vertical) of the banks. In practice an even number of lengths will be used which will most nearly agree with this formula.

186. Iron-pipe culverts. Simple cast-iron pipes are used in sizes from 12″ to 48″ diameter. These are usually made in lengths of 12 feet with a few lengths of 6 feet, so that any required length may be more nearly obtained. The lightest pipes made are sufficiently strong for the purpose, and even those which would be rejected because of incapacity to withstand pressure may be utilized for this work. In Fig. 97 are shown the standard plans used on the C. C. C. & St. L. Ry., which may be considered as typical plans.

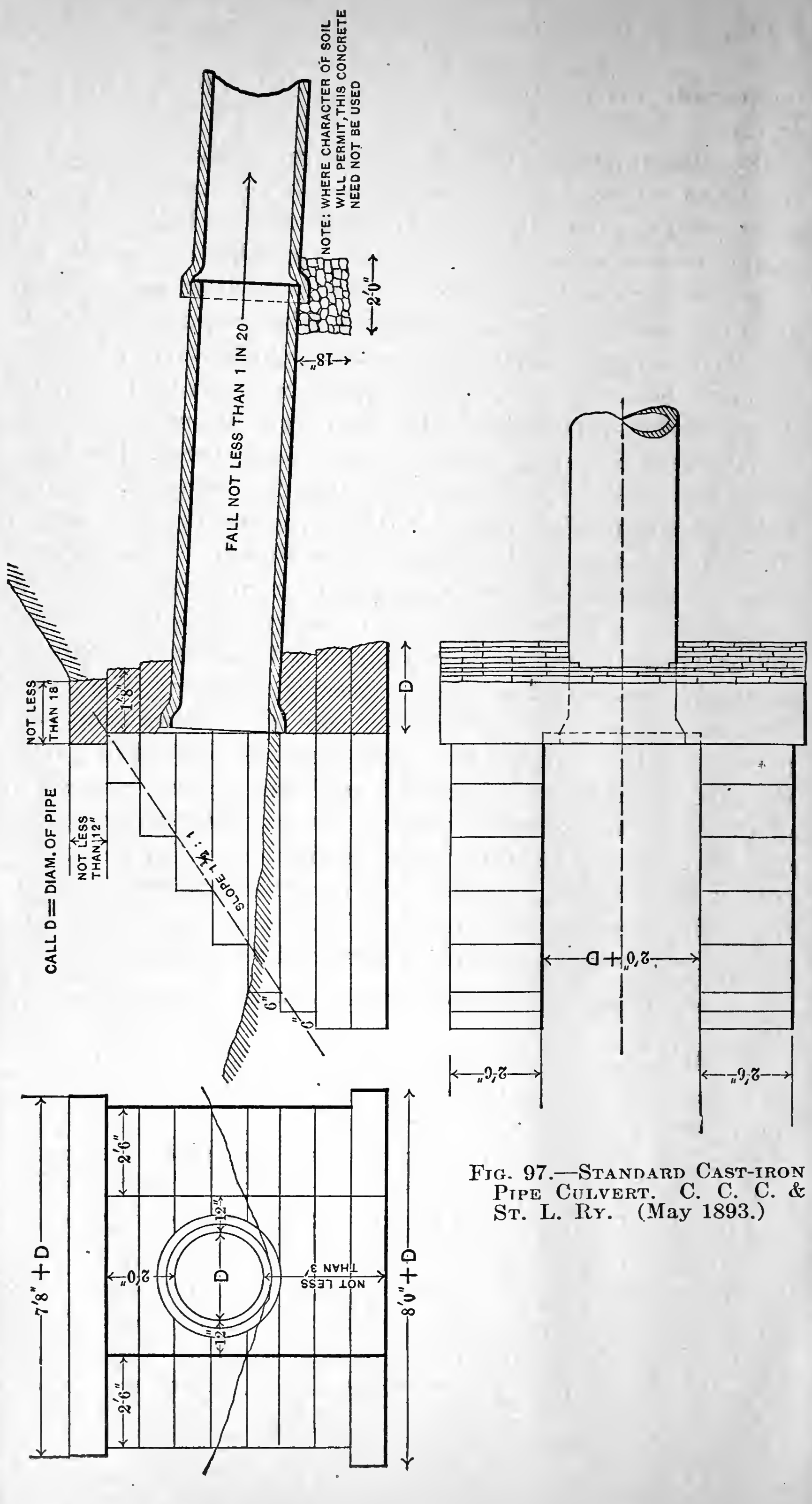


FIG. 97.—STANDARD CAST-IRON PIPE CULVERT. C. C. C. & ST. L. RY. (May 1893.)

Pipes formed of cast-iron segments have been used up to 12 feet diameter. The shell is then made comparatively thin, but is stiffened by ribs and flanges on the outside. The segments break joints and are bolted together through the flanges. The joints are made tight by the use of a tarred rope, together with neat cement.

187. Tile-pipe culverts. The pipes used for this purpose vary from 12″ to 24″ in diameter. When a larger capacity is required two or more pipes may be laid side by side, but in such a case another design might be preferable. It is frequently specified that "double-strength" or "extra-heavy" pipe shall be used, evidently with the idea that the stresses on a culvert-pipe are greater than on a sewer-pipe. But it has been conclusively demonstrated that, no matter how deep the embankment, the pressure cannot exceed a somewhat uncertain maximum, also that the greatest danger consists in placing the pipe so near the ties that shocks may be directly transferred to the pipe without the cushioning effect of the earth and ballast. When the pipes are well bedded in *clear* earth and there is a

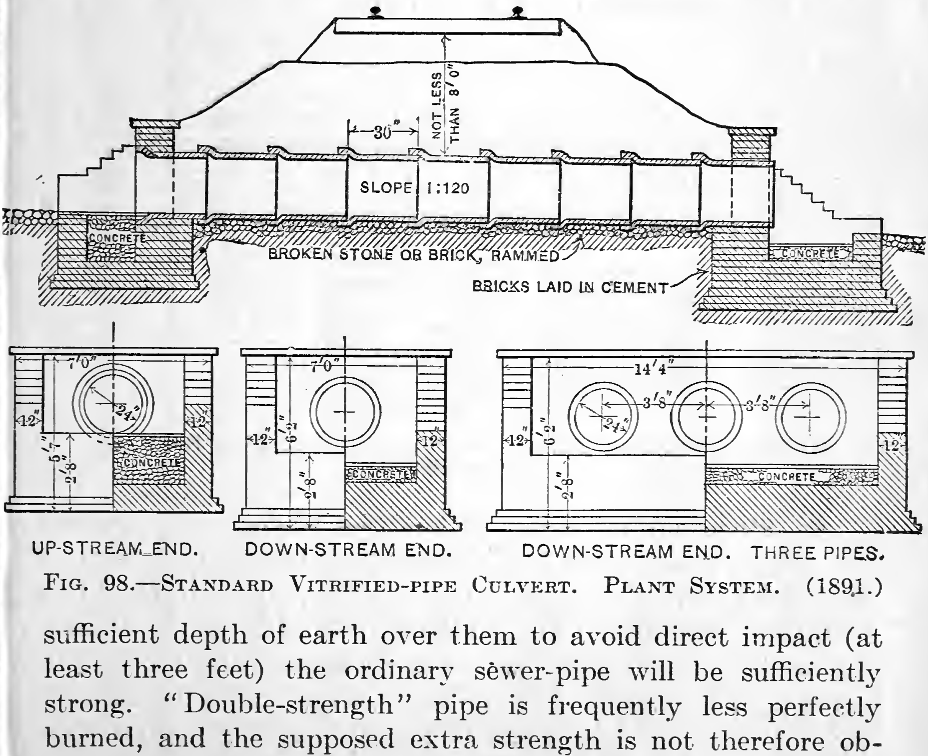


FIG. 98.—STANDARD VITRIFIED-PIPE CULVERT. PLANT SYSTEM. (1891.)

sufficient depth of earth over them to avoid direct impact (at least three feet) the ordinary sewer-pipe will be sufficiently strong. "Double-strength" pipe is frequently less perfectly burned, and the supposed extra strength is not therefore obtained. In Fig. 98 are shown the standard plans for vitrified-pipe culverts as used on the "Plant system." Tile pipe is much cheaper than iron pipe, but is made in much shorter lengths and requires much more work in laying and especially to obtain a uniform grade.

BOX CULVERTS.

188. Wooden box culverts. This form serves the purpose of a cheap temporary construction which allows the use of a ballasted roadbed. As in all temporary constructions, the area should be made considerably larger than the calculated area §§ 179–182), not only for safety but also in order that, if the smaller area is demonstrated to be sufficiently large, the permanent construction (probably pipe) may be placed inside without disturbing the embankment. All designs agree in using heavy timbers (12″×12″, 10″×12″, or 8″×12″) for the side walls, cross-timbers for the roof, every fifth or sixth timber being notched down so as to take up the thrust of the side walls, and planks for the flooring. Fig. 99 shows some of the standard designs as used by the C., M. & St. P. Ry.

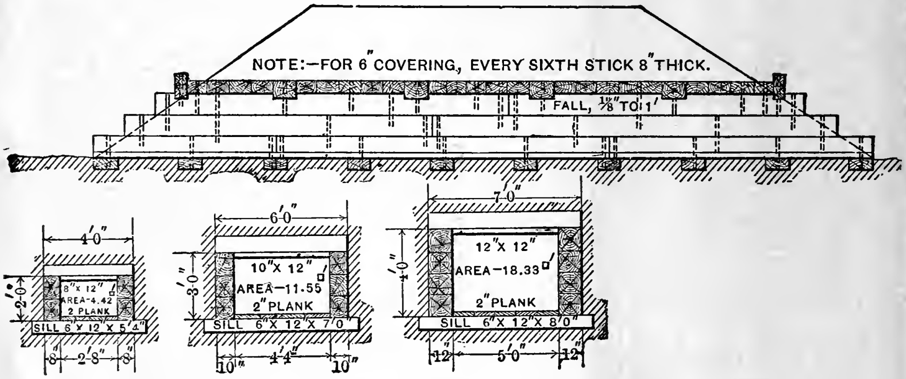


FIG. 99.—STANDARD TIMBER BOX CULVERT. C., M. & ST. P. RY. (Feb. 1889.)

189. Stone box culverts. In localities where a *good* quality of stone is cheap, stone box culverts are the cheapest form of permanent construction for culverts of medium capacity, but their use is decreasing owing to the frequent difficulty in obtaining really suitable stone within a reasonable distance of the culvert. The clear span of the cover-stones varies from 2 to 4 feet. The required thickness of the cover-stones is sometimes calculated by the theory of transverse strains on the basis of certain assumptions of loading—as a function of the height of the embankment and the unit strength of the stone used. Such a method is simply another illustration of a class of calculations which look very precise and beautiful, but which are worse than useless (because misleading) on account of the hopeless uncer-

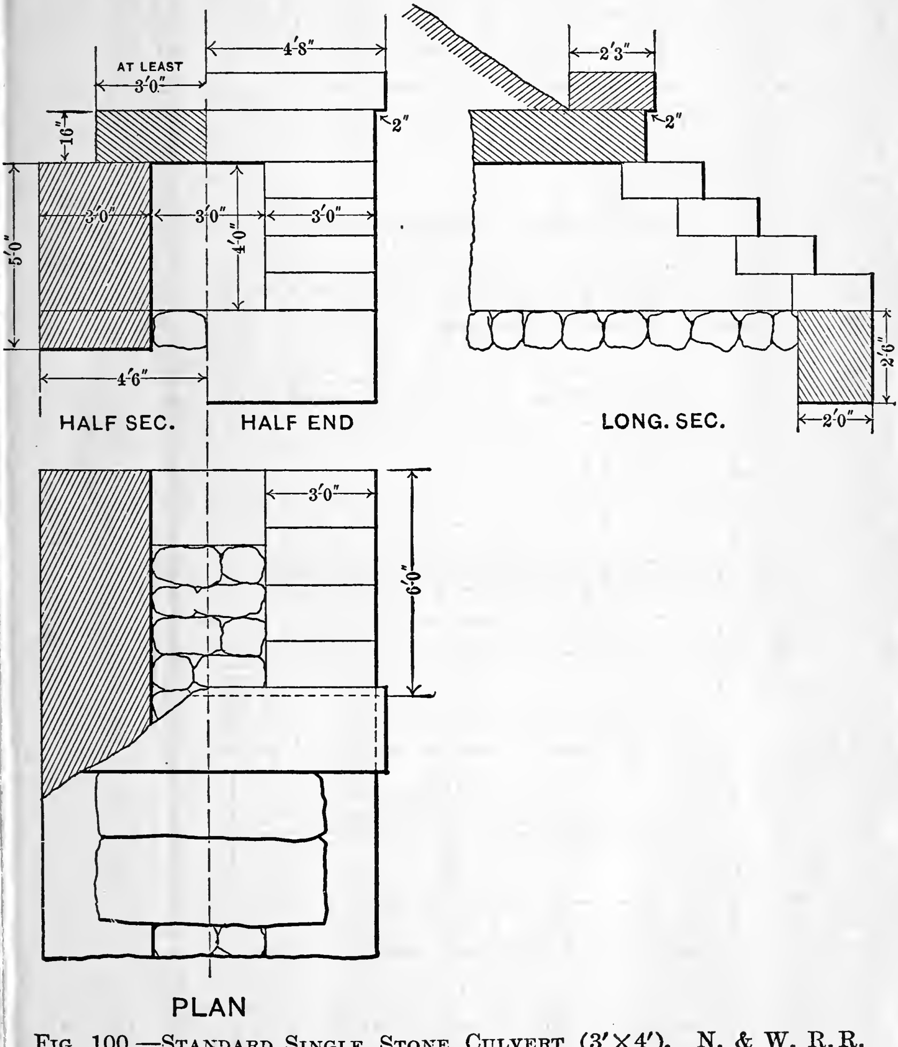


FIG. 100.—STANDARD SINGLE STONE CULVERT (3'×4'). N. & W. R.R. (1890.)

tainty as to the true value of certain quantities which must be used in the computations In the first place the true value of the unit tensile strength of stone is such an uncertain and variable quantity that calculations based on any assumed value for it are of small reliability. In the second place the weight of the prism of earth lying directly above the stone, plus an allowance for live load, is by no means a measure of the load on the stone nor of the forces that tend to fracture it. All earthwork will tend to

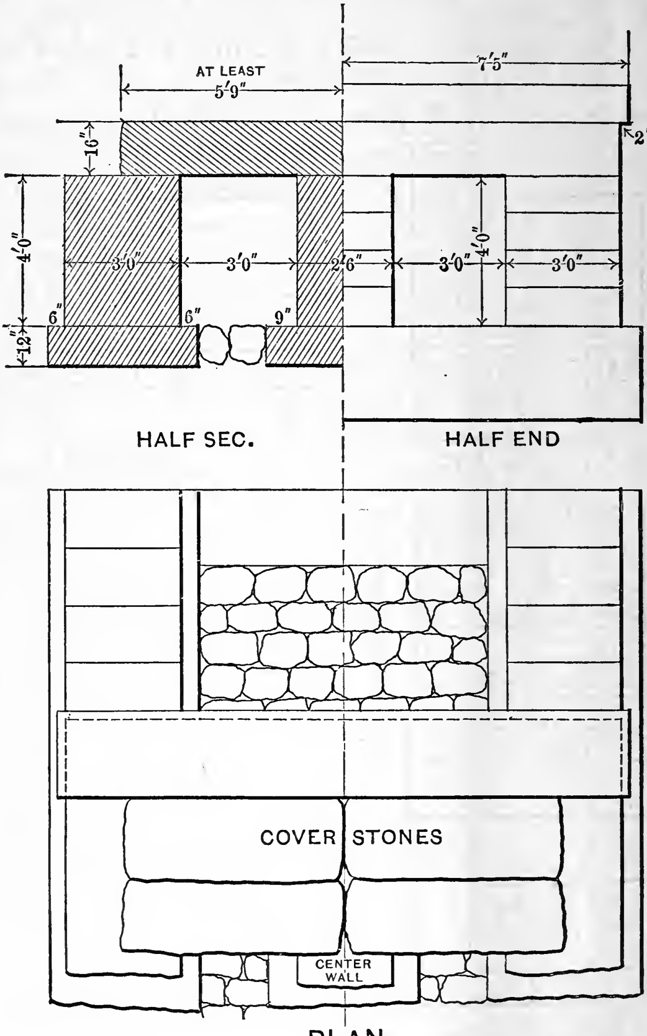


FIG. 100*a*.—STANDARD DOUBLE STONE CULVERT (3'×4'). N. & W. R. R. (1890.)

form an arch above any cavity and thus relieve an uncertain and probably variable proportion of the pressure that might otherwise exist. The higher the embankment the *less* the *pro-*

portionate loading, until at some uncertain height an increase in height will not increase the load on the cover-stones. The effect of frost is likewise large, but uncertain and not computable. The usual practice is therefore to make the thickness such as experience has shown to be safe with a good quality of stone, i.e., about 10 or 12 inches for 2 feet span and up to 16 or 18 inches for 4 feet span. The side walls should be carried down deep enough to prevent their being undermined by scour or heaved by frost. The use of cement mortar is also an important feature of first-class work, especially when there is a rapid scouring current or a liability that the culvert will run under a head. In Figs. 100 and 100a are shown standard plans for single and double stone box culverts as used on the Norfolk and Western R. R.

190. Old-rail culverts. It sometimes happens (although very rarely) that it is necessary to bring the grade line within 3 or 4 feet of the bottom of a stream and yet allow an area of 10 or 12 square feet. A single large pipe of sufficient area could not be used in this case. The use of several smaller pipes side by side would be both expensive and inefficient. For similar reasons neither wooden nor stone box culverts could be used. In such cases, as well as in many others where the head-room is not so limited, the plan illustrated in Fig. 101 is a very satisfactory

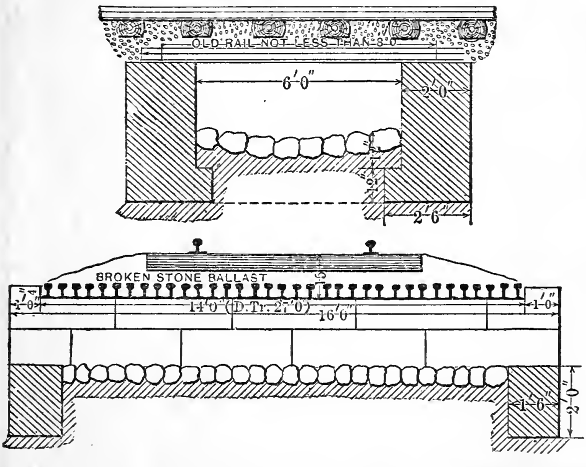


FIG. 101.—STANDARD OLD-RAIL CULVERT. N. & W. R.R. (1895.)

solution of the problem. The old rails, having a length of 8 or 9 feet, are laid close together across a 6-foot opening. Sometimes the rails are held together by long bolts passing through the webs of the rails. In the plan shown the rails are confined by low end walls on each abutment. This plan requires only 15 inches between the base of the rail and the top of the culvert channel. It also gives a continuous ballasted roadbed.

ARCH CULVERTS.

191. Influence of design on flow. The variations in the design of arch culverts have a very marked influence on the cost and efficiency. To combine the least cost with the greatest efficiency, due weight should be given to the following elements: (*a*) amount of masonry. (*b*) the simplicity of the constructive work, (*c*) the design of the wing walls, (*d*) the design of the junction of the wing walls with the barrel and faces of the arch, and (*e*) the safety and permanency of the construction. These elements are more or less antagonistic to each other, and the defects of most designs are due to a lack of proper proportion in the design of these opposing interests. The simplest construction (satisfying elements *b* and *e*) is the straight barrel arch between two parallel vertical head walls, as sketched in Fig. 102, *a*. From a hydraulic standpoint the design is poor, as the water eddies around the corners, causing a great resistance which decreases the flow. Fig. 102, *b*, shows a much better de-

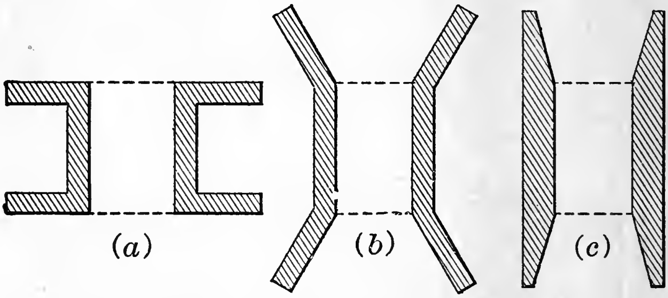


FIG. 102.—TYPES OF CULVERTS.

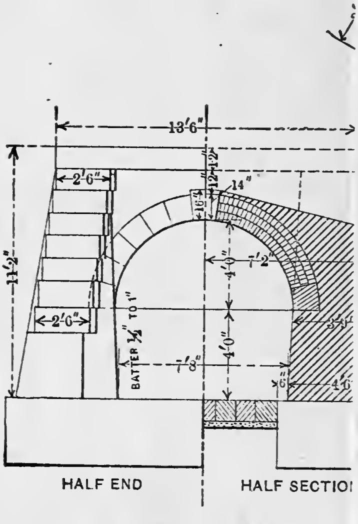
sign in many respects, but much depends on the details of the design as indicated in elements (*b*) and (*d*). As a general thing a good hydraulic design requires complicated and expensive masonry construction, i.e., elements (*b*) and (*d*) are opposed. Design 102, *c*, is sometimes inapplicable because the water is

STANDARD ARCH

8 FEET SPA

NORFOLK & WES

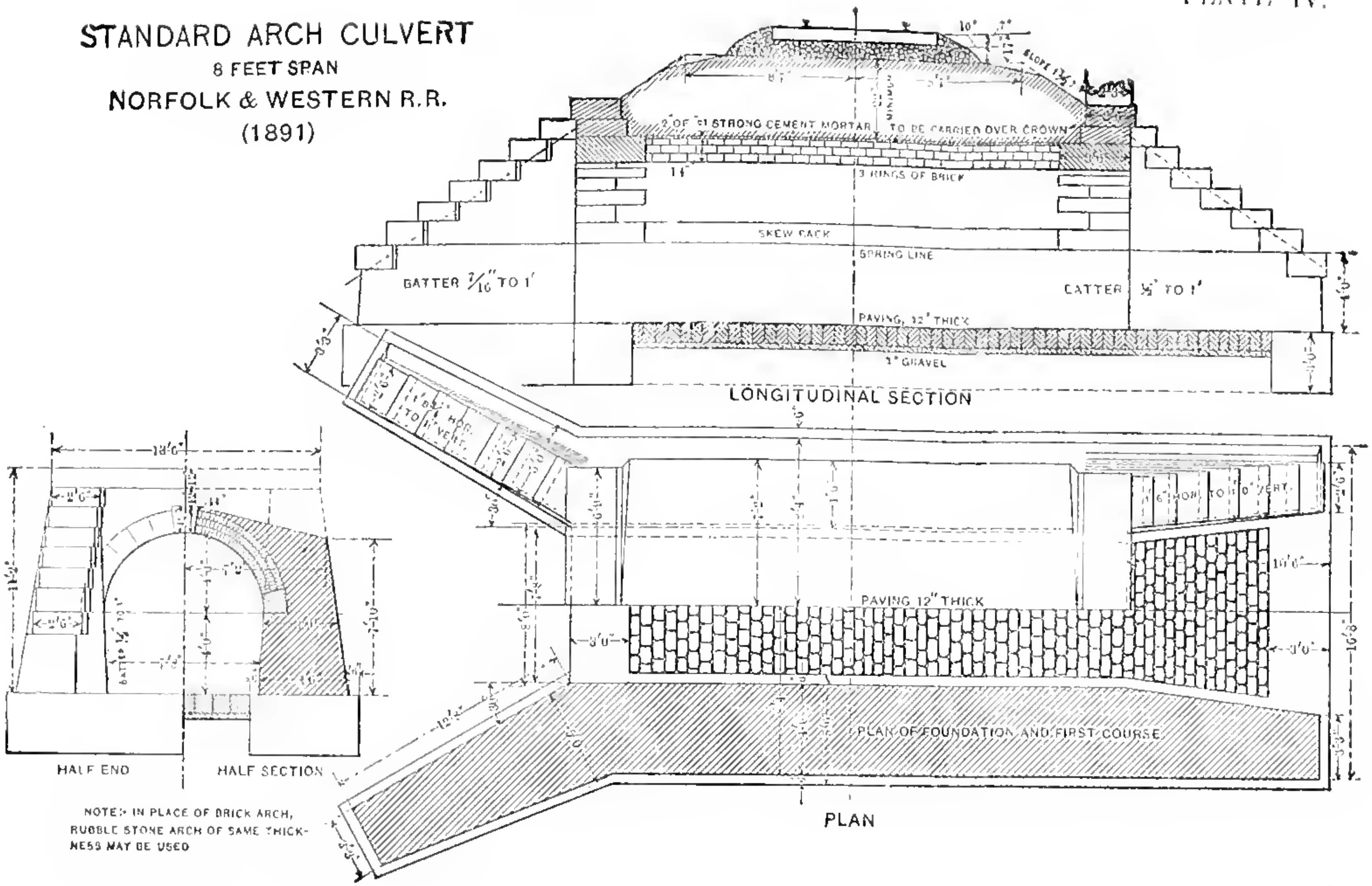
(1891)



NOTE:- IN PLACE OF BRICK ARCH, RUBBLE STONE ARCH OF SAME THICKNESS MAY BE USED

(*To face page* 211.)

STANDARD ARCH CULVERT
8 FEET SPAN
NORFOLK & WESTERN R.R.
(1891)



(To face page 211.)

liable to work in behind the masonry during floods and perhaps cause scour. This design uses less masonry than (*a*) or (*b*).

192. Example of arch culvert design. In Plate IV is shown the design for an 8-foot arch culvert according to the standard of the Norfolk and Western R. R. Note that the plan uses the flaring wing walls (Fig. 102, *b*) on the up-stream side (thus protecting the abutments from scour) and straight wing walls (similar to Fig. 102, *c*) on the down-stream end. This economizes masonry and also simplifies the constructive work. Note also the simplicity of the junction of the wing walls with the barrel of the arch, there being no re-entrant angles below the springing line of the arch. The design here shown is but one of a set of designs for arches varying in span from 6′ to 30′.

MINOR OPENINGS.

193. Cattle-guards. (a) Pit guards. Cattle-guards will be considered under the head of minor openings, since the old-

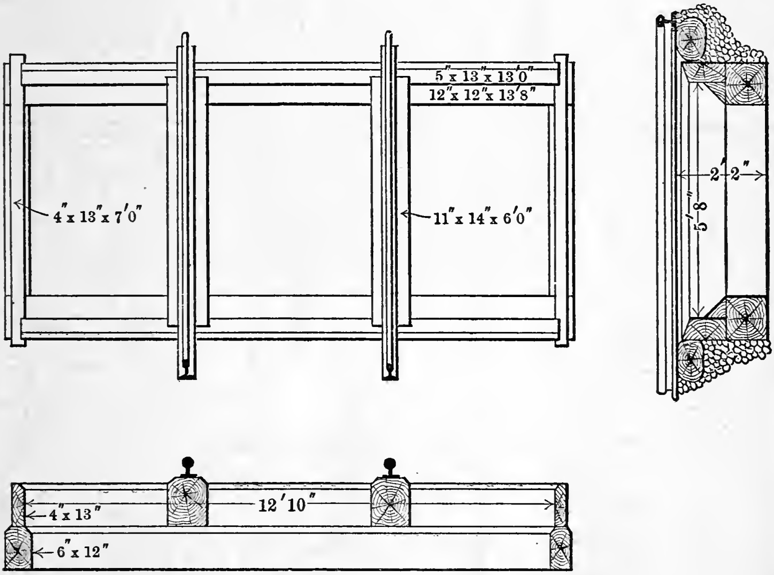


FIG. 103.—PIT CATTLE-GUARDS. P. R. R.

fashioned plan of pit guards, which are even now defended and preferred by some railroad men, requires a break in the continuity of the roadbed. A pit about three feet deep, five feet long, and as wide as the width of the roadbed, is walled up with stone (sometimes with wood), and the rails are supported on heavy timbers laid longitudinally with the rails. The break in the continuity of the roadbed produces a disturbance in the elastic wave running through the rails, the effect of which is noticeable at high velocities. The greatest objection, however, lies in the dangerous consequences of a derailment or a failure of the timbers owing to unobserved decay or destruction by fire—caused perhaps by sparks and cinders from passing locomotives. The very insignificance of the structure often leads to careless inspection. But if a single pair of wheels gets off the rails and drops into the pit, a costly wreck is inevitable. The (once) standard design for such a structure on the Pennsylvania R.R. is shown in Fig. 103.

(b) **Surface cattle-guards.** These are fastened on top of the ties; the continuity of the roadbed is absolutely unbroken and

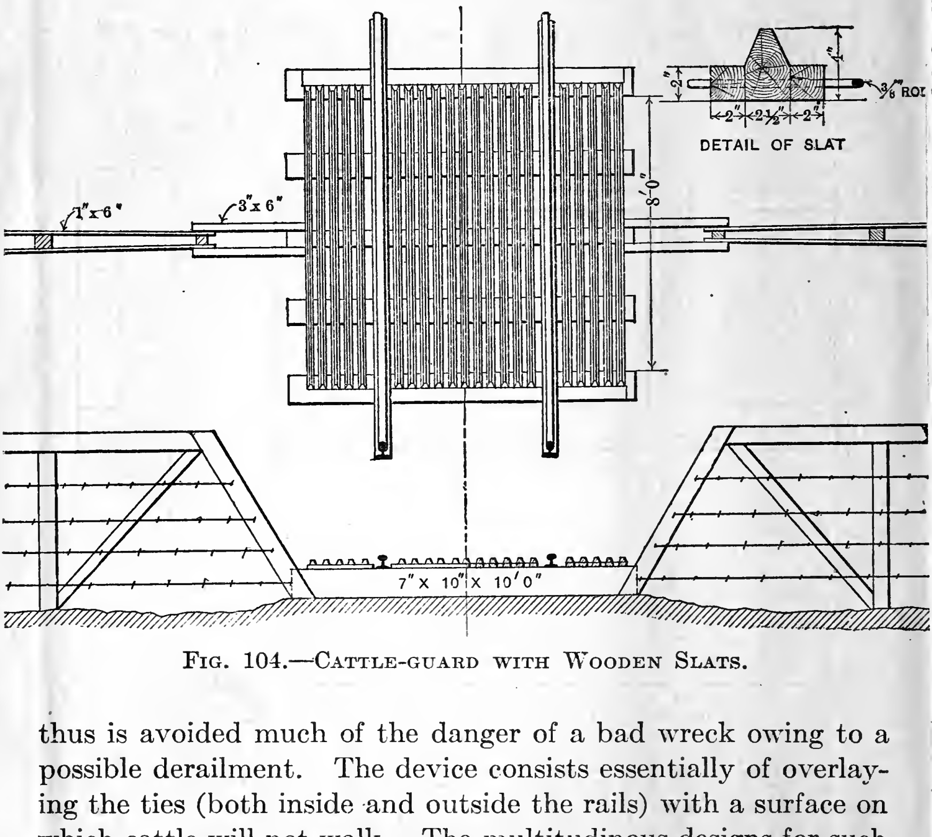


FIG. 104.—CATTLE-GUARD WITH WOODEN SLATS.

thus is avoided much of the danger of a bad wreck owing to a possible derailment. The device consists essentially of overlaying the ties (both inside and outside the rails) with a surface on which cattle will not walk. The multitudinous designs for such a surface are variously effective in this respect. An objection, which is often urged indiscriminately against all such designs, is the liability that a brake-chain which may happen to be dragging *may* catch in the rough bars which are used. The bars are sometimes "home-made," of wood, as shown in Fig. 104. Iron or steel bars are made as shown in Fig. 105. The general construction is the same as for the wooden bars. The metal bars have far greater durabliity, and it is claimed that they are more effective in discouraging cattle from attempting to cross.

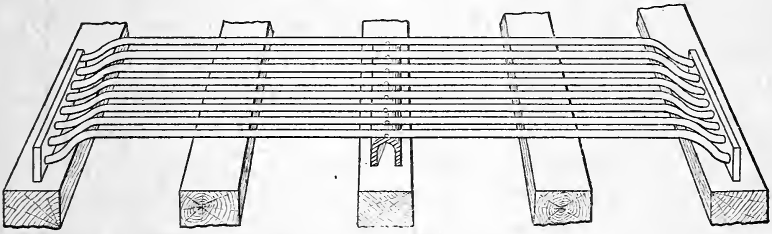
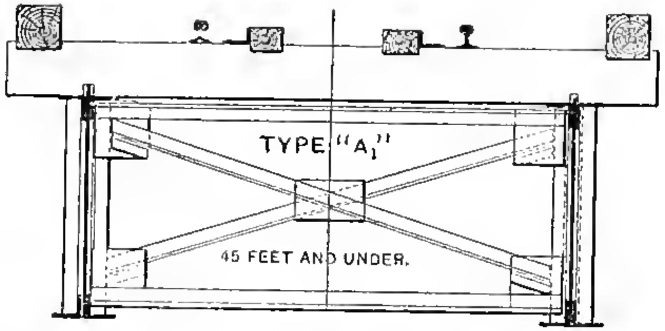


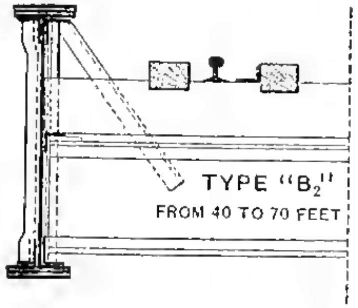
FIG. 105.—MERRILL-STEVENS STEEL CATTLE-GUARD.

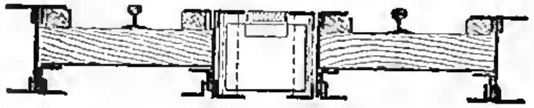
194. **Cattle-passes.** Frequently when a railroad crosses a farm on an embankment, cutting the farm into two parts, the railroad company is obliged to agree to make a passageway through the embankment sufficient for the passage of cattle and perhaps even farm-wagons. If the embankment is high enough so that a stone arch is practicable, the initial cost is the only great objection to such a construction; but if an open wooden structure is necessary, all the objections against the old-fashioned cattle-guards apply with equal force here. The avoidance of a grade crossing which would otherwise be necessary is one of the great compensations for the expense of the construction and maintenance of these structures. The construction is sometimes made by placing two pile trestle bents about 6 to 8 feet apart, supporting the rails by stringers in the usual way, the special feature of this construction being that the embankments are filled in behind the trestle bents, and the thrust of the embankments is mutually taken up through the stringers, which are notched at the ends or otherwise constructed so that they may take up such a thrust. The designs for old-rail culverts and arch culverts are also utilized for cattle-passes when suitable and convenient, as well as the designs illustrated in the following section.

195. **Standard stringer and I-beam bridges.** The advantages of standard designs apply even to the covering of short spans

with wooden stringers or with I beams—especially since the methods do not require much vertical space between the rails and the upper side of the clear opening, a feature which is often of prime importance. These designs are chiefly used for culverts or cattle-passes and for crossing *over* highways—providing such a narrow opening would be tolerated. The plans all imply stone abutments, or at least abutments of sufficient stability to withstand all thrust of the embankments. Some of the designs are illustrated in Plate V. The preparation of these standard designs should be attacked by the same general methods as already illustrated in § 156. When computing the required transverse strength, due allowance should be made for lateral bracing, which should be amply provided for. Note particularly the methods of bracing illustrated in Plate V. The designs calling for iron (or steel) stringers may be classed as permanent constructions, which are cheap, safe, easily inspected and maintained, and therefore a desirable method of construction.





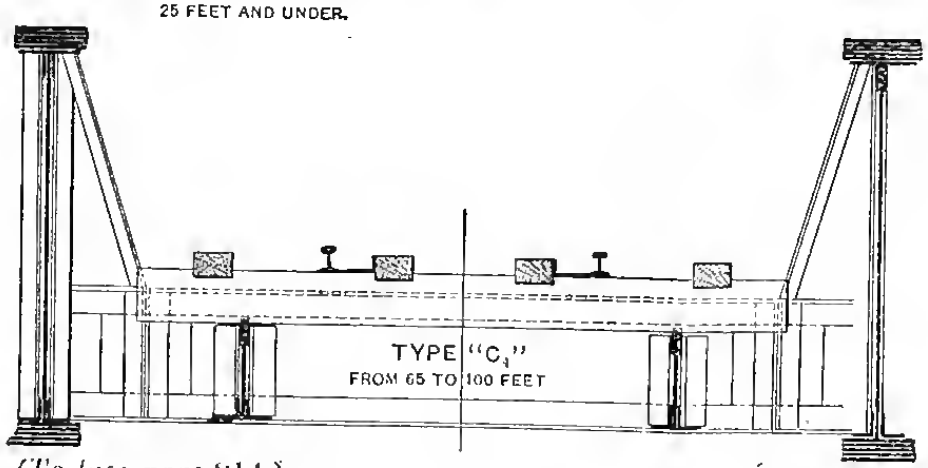


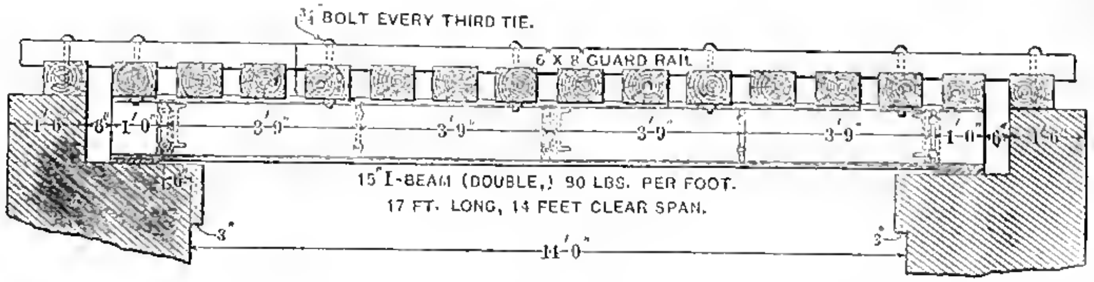
TYPE "E" GIRDER
25 FEET AND UNDER.

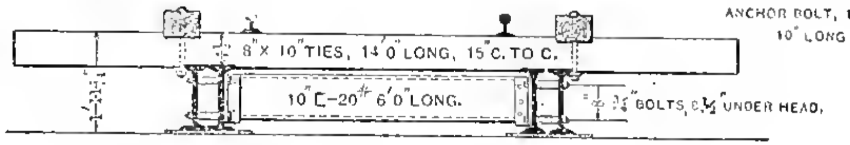
TYPES OF PLATE GIRDER BRIDGES.

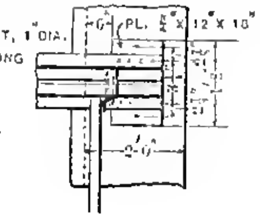
C. M. & ST. P. RY.

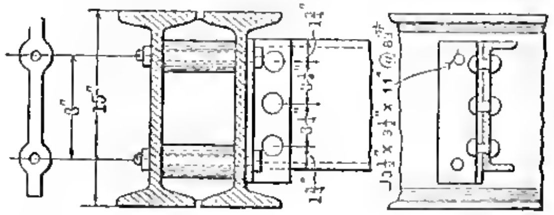
(DEC. 1895.)









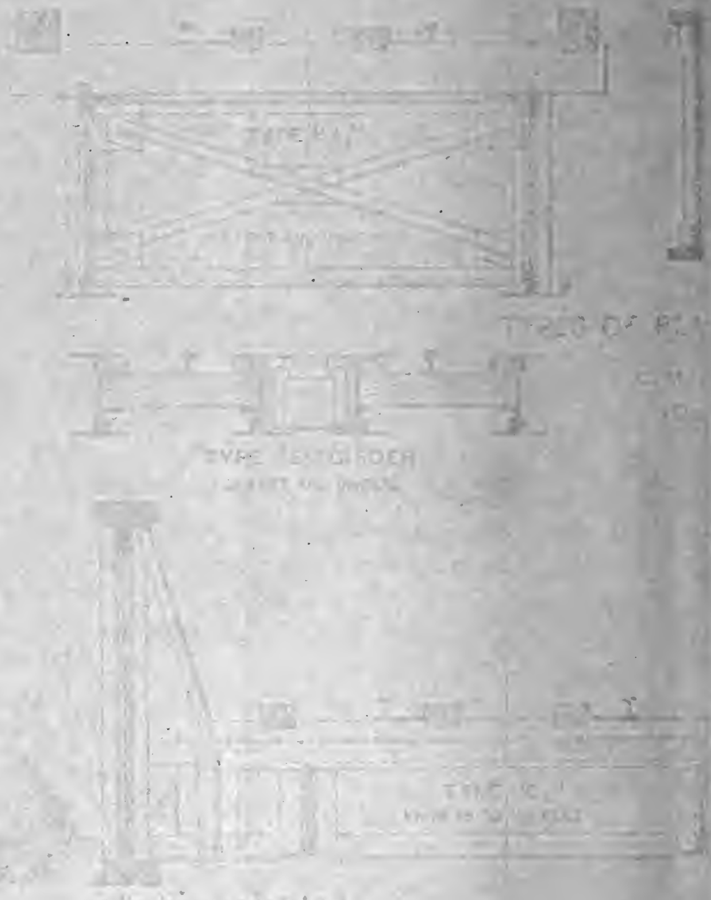


STANDARD I-BRIDGES—14-FT. SPAN.

NORFOLK AND WESTERN R.R.

(1891.)

(To face page 214.)



CHAPTER VII.

BALLAST.

196. Purpose and requirements. "The object of the ballast is to transfer the applied load over a large surface; to hold the timber work in place horizontally; to carry off the rain-water from the superstructure and to prevent freezing up in winter; to afford means of keeping the ties truly up to the grade line; and to give elasticity to the roadbed." This extremely condensed statement is a description of an ideally perfect ballast. The value of any given kind of ballast is proportional to the extent to which it fulfills these requirements. The ideally perfect ballast is not necessarily the most economical ballast for all roads. Light traffic generally justifies something cheaper, but a very common error is to use a very cheap ballast when a small additional expenditure would procure a much better ballast which would be much more economical in the long run.

197. Materials. The materials most commonly employed are gravel and broken stone. Burnt clay, cinders, shells, and small coal are occasionally used as ballast when they are especially cheap and convenient or when better kinds are especially expensive. Although it is hardly correct to speak of the natural soil as ballast, yet many miles of cheap railways are "ballasted" with the natural soil, which is then called "mud ballast."

Mud ballast. When the natural soil is gravelly so that rain will drain through it quickly, it will make a fair roadbed for light traffic, but for heavy traffic, and for the greater part of the length of most roads, the natural soil is a very poor material for ballast; for, no matter how suitable the soil might be along limited sections of the road, it would practically never happen that the soil would be uniformly good throughout the whole length of the road. Considering that a heavy rain will in one day spoil the results of weeks of patient "surfacing" with mud ballast, it is seldom economical to use "mud" if there is a gravel-bed or other source of ballast anywhere on the line of the road.

Cinders. The advantages consist in the excellent facilities for drainage, ease of handling, and cheapness—after the road is in operation One disadvantage is excessive dust in dry weather Cinders are considered preferable to gravel in yards.

Slag. When slag is readily obtainable it furnishes an excellent ballast, free from dust and perfect in drainage qualities Some kinds of slag are objectionable on account of their deleterious chemical effect on the ties and spikes—especially on metallic ties.

Shells, small coal, etc. These comparatively inferior kinds of ballast are used for light traffic when they are especially cheap and convenient. They are extremely dusty in dry weather, break up into very fine dust, and are but little better than mud.

Gravel. This is the most common form of ballast which may be called good ballast. In 1885, the Roadmasters Association of America voted in favor of gravel ballast as against rock ballast. Although not so stated, this action was perhaps due to a conviction of its real economy for the *average* railroad of this country, which may be called a "light traffic" road. Gravel should preferably be screened over a screen having a $\frac{1}{2}''$ mesh, so as to screen out all dirt and the finest stones. Generally a railroad will be able to find at some point along its line a "gravel-pit" affording a suitable supply. This may be dug out with a steam-shovel, screened if necessary, and sent out over the line by the train-load at a comparatively small cost.

Rock or broken stone. Rock ballast is generally specified to be such as will pass through a $1\frac{1}{2}''$ (or $2''$) ring. Although preferably broken by hand, machine-broken stone is much cheaper. It is most easily handled with forks. This also has the effect of screening out the dirt and fine chips which would interfere with effectual drainage. Rock ballast is more expensive in first cost, and also more troublesome to handle, than any other kind, but under heavy traffic will keep in surface better and will require less work for maintainance after the ties have become thoroughly bedded. For roads with very light traffic, running few trains, at comparatively low velocities, the advantages of rock ballast over other kinds are not so pronounced. For such roads rock ballast is an expensive luxury. The amount of traffic which will justify the use of rock ballast will depend on the cost of obtaining ballast of the various kinds.

198. Cross-sections. A depth of 12″ under the tie is generally required on the best roads, but for light traffic this is sometimes reduced to 6″ and even less. The width is generally 1 to 2 feet less than the width of the roadbed proper—excluding ditches. If the ballast has an average width of 10 feet (12 feet at bottom and 8 feet at top) and an average depth of 15 inches (including that placed between the ties), it will require 2444 cubic yards per mile of track. The P R. R. estimates 2500 cubic yards of gravel and 2800 cubic yards of stone ballast per mile of single track. On account of the requirements of drainage the best form of cross-section depends on the kind of ballast used.

Mud ballast. Since the great objection to mud ballast lies in its liability to become soft by soaking up the rain that falls, it becomes necessary that it should be drained as quickly and readily as its nature will permit Fig. 106 shows a typical cross-

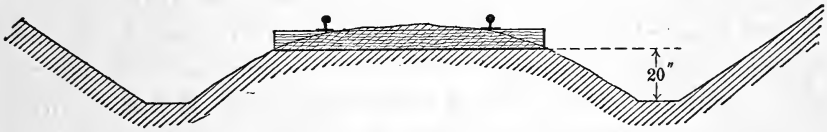


FIG. 106.—"MUD" BALLAST.

section for mud ballast It should be crowned 2″ above the top of the tie at the center, thence sloped so as to leave a slight clearance under the rail between the ties, thence sloping down to the bottom of the tie at each end and continuing to slope down to the ditch (in cut), which should be 18″ or 20″ below the bottom of the tie.

Gravel, cinders, slag, etc. The subgrade is crowned 6″ or 8″ in the center, as shown in Fig. 107 The ballast is crowned

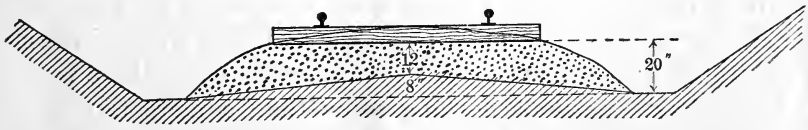


FIG. 107.—GRAVEL BALLAST.

to the top of the tie in the center, but is sloped down to the bottom of the tie at each end This is necessary (and more especially so with mud ballast) to prevent a possible accumulation and settlement of water at the ends of the tie, which would readily soak into the end fibers and produce decay.

Broken stone. Stone ballast is shouldered out beyond the ends of the ties so as to afford greater lateral binding. The space between the ties is filled up level with the tops. The

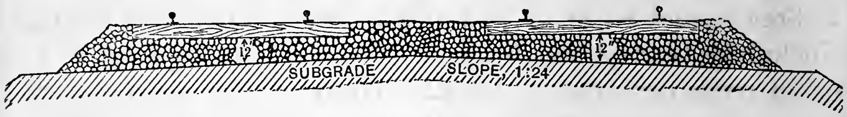


FIG. 108.—BROKEN STONE BALLAST.

perfect drainage of stone ballast permits this to be done without any danger of causing decay of the ties by the accumulation and retention of water.

199. **Methods of laying ballast.** The cheapest method of laying ballast on new roads is to lay ties and rails directly on the prepared subgrade and run a construction train over the track to distribute the ballast. Then the track is lifted up until sufficient ballast is worked under the ties and the track is properly surfaced. This method, although cheap, is apt to injure the rails by causing bends and kinks, due to the passage of loaded construction trains when the ties are very unevenly and roughly supported, and the method is therefore condemned and prohibited in some specifications The best method is to draw in carts (or on a contractor's temporary track) the ballast that is required under the level of the *bottom* of the ties. Spread this ballast carefully to the required surface Then lay the ties and rails, which will then have a very fair surface and uniform support. A construction train can then be run on the rails and distribute sufficient additional ballast to pack around and between the ties and make the required cross-section.

The necessity for constructing some lines at an absolute minimum of cost and of opening them for traffic as soon as possible has often led to the policy of starting traffic when there is little or no ballast—perhaps nothing more than a mere tamping of the natural soil under the ties. When this is done ballast may subsequently be drawn where required by the train-load on flat cars and unloaded at a minimum of cost by means of a "plough" The plough has the same width as the cars and is guided either by a ridge along the center of each car or by short posts set up at the sides of the cars It is drawn from one end of the train to the other by means of a cable. The cable is sometimes operated by means of a small hoisting-engine carried on a car at one end of the train. Sometimes the locomotive is detached temporarily from the train and is run ahead with the cable attached to it.

200. **Cost.** The cost of ballast *in the track* is quite a variable item for different roads, since it depends (*a*) on the first cost of the material as it comes to the road, (*b*) on the distance from the source of supply to the place where it is used, and (*c*) on the method of handling. The first cost of cinder or slag is frequently insignificant A gravel-pit may cost nothing except the price of a little additional land beyond the usual limits of the right of way. Broken stone will usually cost $1 or more per cubic yard If suitable stone is obtainable on the company's land, the cost of blasting and breaking should be somewhat less than this The cost of loading the ballast on to trains will be small (per cubic yard) if it is handled with steam-shovels—as in the case of gravel taken from a gravel-pit Hand-shovelling will cost more. The cost of hauling will depend on the distance hauled, and also, to a considerable extent, on the limitations on the operation of the train due to the necessity of keeping out of the way of regular trains There is often a needless waste in this way. The "mud train" is considered a pariah and entitled to no rights whatever, regardless of the large daily cost of such a train and of the necessary gang of men. The cost of broken stone ballast *in the track* is estimated at $1 25 per cubic yard. The cost of gravel ballast is estimated at 60 c. per cubic yard in the track. The cost of placing and tamping gravel ballast is estimated at 20 c. to 24 c. per cubic yard, for cinders 12 c. to 15 c. per cubic yard. The cost of loading gravel on cars, using a steam-shovel, is estimated at 6 c. to 10 c. per cubic yard.*

* Report Roadmasters Association, 1885.

CHAPTER VIII.

TIES,

AND OTHER FORMS OF RAIL SUPPORT.

201. Various methods of supporting rails. It is necessary that the rails shall be sufficiently supported and braced, so that the gauge shall be kept constant and that the rails shall not be subjected to excessive transverse stress. It is also preferable that the rail support shall be neither rigid (as if on solid rock) nor too yielding, but shall have a *uniform* elasticity throughout. These requirements are more or less fulfilled by the following methods.

(a) **Longitudinals.** Supporting the rails throughout their entire length. This method is very seldom used in this country except occasionally on bridges and in terminals when the longitudinals are supported on cross-ties. In § 224 will be described a system of rails, used to some extent in Europe, having such broad bases that they are self-supporting on the ballast and are only connected by tie-rods to maintain the gauge.

(b) **Cast-iron "bowls" or "pots."** These are castings resembling large inverted bowls or pots, having suitable chairs on top for holding and supporting the rails, and tied together with tie-rods. They will be described more fully later (§ 223).

(c) **Cross-ties of metal or wood.** These will be discussed in the following sections.

202. Economics of ties. The true cost of ties depends on the relative total cost of maintenance for long periods of time. The first cost of the ties delivered to the road is but one item in the economics of the question. Cheap ties require frequent renewals, which cost for the *labor* of each renewal practically the same whether the tie is of oak or of hemlock. Cheap ties make a poor roadbed which will require more track labor to keep even in tolerable condition. The roadbed will require to be disturbed so frequently on account of renewals that the ties never get an opportunity to get settled and to form a smooth roadbed for any length of time. Irregularity in width, thickness, or length of ties is especially detrimental in causing the ballast to act and wear unevenly. The life of ties has thus a more or less direct influence on the life of the rails, on the wear of rolling stock, and on the speed of trains. These last items are not so readily reducible to dollars and cents, but when it can be shown that the total cost, for a long period of time, of several renewals of cheap ties, with all the extra track labor involved, is as great as or greater than that of a few renewals of durable ties, then there is no question as to the real economy. In the following discussions of the merits of untreated ties (either cheap or costly), chemically treated ties, or metal ties, the true question is therefore of the ultimate cost of maintaining any particular kind of ties for an indefinite period, the cost including the first cost of the ties, the labor of placing them and maintaining them to surface, and the somewhat uncertain (but not therefore non-existent) effect of frequent renewals on repairs of rolling stock, on possible speed, etc.

WOODEN TIES.

203. Choice of wood. This naturally depends, for any particular section of country, on the supply of wood which is most readily available. The woods most commonly used, especially in this country, are oak and pine, oak being the most durable and generally the most expensive. Redwood is used very extensively in California and proves to be extremely durable, so far as decay is concerned, but it is very soft and is much injured by "rail-cutting." This defect is being partly remedied by the use of tie-plates, as will be explained later. Cedar, chestnut, hemlock, and tamarack are frequently used in this country. In tropical countries very durable ties are frequently obtained from the hard woods peculiar to those countries. According to a bulletin of the U. S. Department of Agriculture issued some years ago, the proportions of the various kinds used in the United States are about as follows:

Oak	60%	Chestnut	5%	Cypress	2%
Pine	20	Hemlock and Tamarack	3	Various	1
Cedar	6	Redwood	3	Total	100%

The limitations of timber supply have somewhat diminished the use of oak and increased the use of the softer woods in recent years.

204. Durability. The durability of ties depends on the climate; the drainage of the ballast; the volume, weight, and speed of the traffic; the curvature, if any; the use of tie-plates; the time of year of cutting the timber; the age of the timber and the degree of its seasoning before placing in the track; the nature of the soil in which the timber is grown; and, chiefly, on the species of wood employed. The variability in these items will account for the discrepancies in the reports on the life of various woods used for ties.

White oak is credited with a life of 5 to 12 years, depending principally on the traffic. It is both hard and durable, the hardness enabling it to withstand the cutting tendency of the rail-flanges, and the durability enabling it to resist decay. *Pine* and *redwood* resist decay very well, but are so soft that they are badly cut by the rail-flanges and do not hold the spikes very well, necessitating frequent respiking. Since the spikes must be driven within certain very limited areas on the face of each tie, it does not require many spike-holes to "spike-kill" the tie. On sharp curves, especially with heavy traffic, the wheel-flange pressure produces a side pressure on the rail tending to overturn it, which tendency is resisted by the spike, aided sometimes by rail-braces. Whenever the pressure becomes too great the spike will yield somewhat and will be slightly withdrawn. The resistance is then somewhat less and the spike is soon so loose that it must be redriven in a new hole. If this occurs very often, the tie may need to be replaced long before any decay has set in. When the traffic is very light, the wood very durable, and the climate favorable, ties have been known to last 25 years.

205. Dimensions. The usual dimensions for the best roads (standard gauge) are 8′ to 8′ 6″ long, 6″ to 7″ thick, and 8″ to 10″ wide on top and bottom (if they are hewed) or 8″ to 9″ wide if they are sawed. For cheap roads and light traffic the length is shortened sometimes to 7′ and the cross-section also reduced. On the other hand a very few roads use ties 9′ long.

Two objections are urged against sawed ties: first, that the grain is torn by the saw, leaving a woolly surface which induces decay; and secondly, that, since timber is not perfectly straight-grained, some of the fibers are cut obliquely, exposing their ends, which are thus liable to decay. The use of a "planer-saw" obviates the first difficulty. Chemical treatment of ties obviates both of these difficulties. Sawed ties are more convenient to handle, are a necessity on bridges and trestles, and it is even claimed, although against commonly received opinion, that actual trial has demonstrated that they are more durable than hewed ties.

206. Spacing. The spacing is usually 14 to 16 ties to a 30-foot rail. This number is sometimes reduced to 12 and even 10, and on the other hand occasionally increased to 18 or 20 by employing narrower ties. There is no economy in reducing the number of ties very much, since for any required stiffness of track it is more economical to increase the number of supports than to increase the weight of the rail. The decreasing cost of rails and the increasing cost of ties have materially changed the relation between number of ties and weight of rail to produce a given stiffness at minimum cost, but many roads have found it economical to employ a large number of ties rather than increase the weight of the rail. On the other hand there is a practical limit to the number that may be employed, on account of the necessary space between the ties that is required for proper tamping. This width is ordinarily about twice the width of the tie. At this rate, with light ties 6″ wide and with 12″ clear space, there would be 20 ties per 30-foot rail, or 3520 per mile. The smaller ties can generally be bought much cheaper (proportionately) than the larger sizes, and hence the economy.

Track instructions to foremen generally require that the spacing of ties shall *not* be uniform along the length of any rail. Since the joint is generally the weakest part of the rail structure, the joint requires more support than the center of the rail. Therefore the ties are placed with but 8″ or 10″ clear space between them at the joints, this applying to 3 or 4 ties at each joint; the remaining ties, required for each rail length, are equally spaced along the remaining distance.

207. Specifications. The specifications for ties are apt to include the items of size, kind of wood, and method of construction, besides other minor directions about time of cutting, seasoning, delivery, quality of timber, etc.

(a) **Size.** The particular size or sizes required will be somewhat as indicated in § 205.

(b) **Kind of wood.** When the kind or kinds of wood are specified, the most suitable kinds that are available in that section of country are usually required.

(c) **Method of construction.** It is generally specified that the ties shall be hewed on two sides; that the two faces thus made shall be parallel planes and that the bark shall be removed. It is sometimes required that the ends shall be sawed off square; that the timber shall be cut in the winter (when the sap is down); and that the ties shall be seasoned for six months These last specifications are not required or lived up to as much as their importance deserves. It is sometimes required that the ties shall be delivered on the right of way, neatly piled in rows, the alternate rows at right angles, piled if possible on ground not lower than the rails and at least seven feet away from them, the lower row of ties resting on two ties which are themselves supported so as to be clear of the ground.

(d) **Quality of timber.** The usual specifications for sound timber are required, except that they are not so rigid as for a better class of timber work The ties must be sound, reasonably straight-grained, and not very crooked—one test being that a line joining the center of one end with the center of the middle shall not pass outside of the other end. Splits or shakes, especially if severe, should cause rejection.

Specifications sometimes require that the ties shall be cut from single trees, making what is known as "pole ties" and definitely condemning those which are cut or split from larger trunks, giving two "slab ties" or four "quarter ties" for each cross-section, as is illustrated in Fig. 109. Even if pole ties are better, their exclusive use means the rapid destruction of forests of young trees.

FIG. 109.—METHODS OF CUTTING TIES.

208. **Regulations for laying and renewing ties.** The regulations issued by railroad companies to their track foremen will generally include the following, in addition to directions regarding dimensions, spacing, and specifications given in §§ 204–207. When hewn ties of somewhat variable size are used, as is frequently the case, the largest and best are to be selected for use as joint ties. If the upper surface of a tie is found to be warped (contrary to the usual specifications) so that one or both rails do

not get a full bearing across the whole width of the tie, it must be adzed to a true surface along its whole length and not merely notched for a rail-seat. When respiking is necessary and spikes have been pulled out, the holes should be immediately plugged with "wooden spikes," which are supplied to the foreman for that express purpose, so as to fill up the holes and prevent the decay which would otherwise take place when the hole becomes filled with rain-water. Ties should always be laid at right angles to the rails and never obliquely Minute regulations to prevent premature rejection and renewal of ties are frequently made. It is generally required that the requisitions for renewals shall be made by the actual count of the individual ties to be renewed instead of by any wholesale estimates. It is unwise to have ties of widely variable size, hardness, or durability adjacent to each other in the track, for the uniform elasticity, so necessary for smooth riding, will be unobtainable under those circumstances.

209. **Cost of ties.** When railroads can obtain ties cut by farmers from woodlands in the immediate neighborhood, the price will frequently be as low as 20 c for the smaller sizes, running up to 50 c for the larger sizes and better qualities, especially when the timber is not very plentiful Sometimes if a railroad cannot procure suitable ties from its immediate neighborhood, it will find that adjacent railroads control all adjacent sources of supply for their own use and that ties can only be procured from a considerable distance, with a considerable added cost for transportation. First-class oak ties cost about 75 to 80 c. and frequently much more Hemlock ties can generally be obtained for 35 c. or less.

PRESERVATIVE PROCESSES FOR WOODEN TIES.

210. **General principle.** Wood has a fibrous cellular structure, the cells being filled with sap or air. The woody fiber is but little subject to decay unless the sap undergoes fermentation. Preservative processes generally aim at removing as much of the water and sap as possible and filling up the pores of the wood with an antiseptic compound The most common methods (except one) all agree in this general process and only differ in the method employed to get rid of the sap and in the antiseptic chemical with which the fibers are filled One valuable feature of these processes lies in the fact that the softer cheaper woods

(such as hemlock and pine) are more readily treated than are the harder woods and yet will produce practically as good a tie as a treated hard-wood tie and a very much better tie than an untreated hard-wood tie. The various processes will be briefly described, taking up first the process which is fundamentally different from the others, viz., vulcanizing.

211. **Vulcanizing.** The process consists in heating the timber to a temperature of 300° to 500° F. in a cylinder, the air being under a pressure of 100 to 175 lbs. per square inch. By this process the albumen in the sap is coagulated, the water evaporated, and the pores are partially closed by the coagulation of the albumen. It is claimed that the heat sterilizes the wood and produces chemical changes in the wood which give it an antiseptic character. It has been very extensively used on the elevated lines of New York City, and it is claimed to give perfect satisfaction. The treatment has cost that road 25 c. per tie.

212. **Creosoting.** This porcess consists in impregnating the wood with *wood-creosote* or with *dead oil of coal-tar. Wood-creosote* is one of the products of the destructive distillation of wood—usually long-leaf pine. *Dead oil of coal-tar* is a product of the distillation of coal-tar at a temperature between 480° and 760° F. It would require about 35 to 50 pounds of creosote to completely fill the pores of a cubic foot of wood But it would be impossible to force such an amount into the wood, nor is it necessary or desirable. About 10 pounds per cubic foot, or about 35 pounds per tie, is all that is necessary. For piling placed in salt water about 18 to 20 pounds per cubic foot is used, and the timber is then perfectly protected against the ravages of the *teredo navalis.* To do the work, long cylinders, which may be opened at the ends, are necessary. Usually the timbers are run in and out on iron carriages running on rails fastened to braces on the inside of the cylinder. When the load has been run in, the ends of the cylinder are fastened on. The water and air in the pores of the wood are first drawn out by subjecting the wood alternately to steam-pressure and to the action of a vacuum-pump. This is continued for several hours. Then, after one of the vacuum periods, the cylinder is filled with creosote oil at a temperature of about 170° F. The pumps are kept at work until the pressure is about 80 to 100 pounds per square inch, and is maintained at this pressure from one to two hours according to the size of the timber. The oil is then

withdrawn, the cylinders opened, the train pulled out and another load made up in 40 to 60 minutes. The average time required for treating a load is about 18 or 20 hours, the absorption about 10 or 11 pounds of oil per cubic foot, and the cost (1894) from $12.50 to $14.50 per thousand feet B. M.

213. Burnettizing (chloride-of-zinc process). This process is very similar to the creosoting process except that the chemical is chloride of zinc, and that the chemical is not heated before use. The preliminary treatment of the wood to alternate vacuum and pressure is not continued for quite so long a period as in the creosoting process. Care must be taken, in using this process, that the ties are of as uniform quality as possible, for seasoned ties will absorb much more zinc chloride than unseasoned (in the same time), and the product will lack uniformity unless the seasoning is uniform. The A., T. & S. Fé R. R. has works of its own at which ties are treated by this process at a cost of about 25 c. per tie. The Southern Pacific R. R. also has works for burnettizing ties at a cost of 9.5 to 12 c per tie. The zinc-chloride solution used in these works contains only 1.7% of zinc chloride instead of over 3% as used in the Santa Fé works, which perhaps accounts partially for the great difference in cost per tie. One great objection to burnettized ties is the fact that the chemical is somewhat easily washed out, when the wood again becomes subject to decay. Another objection, which is more forcible with respect to timber subject to great stresses, as in trestles, than to ties, is the fact that when the solution of zinc chloride is made strong (over 3%) the timber is made very brittle and its strength is reduced. The reduction in strength has been shown by tests to amount to $\frac{1}{4}$ to $\frac{1}{10}$ of the ultimate strength, and that the elastic limit has been reduced by about $\frac{1}{7}$.

214. Kyanizing (bichloride-of-mercury or corrosive-sublimate process). This is a process of "steeping." It requires a much longer time than the previously described processes, but does not require such an expensive plant. Wooden tanks of sufficient size for the timber are all that is necessary. The corrosive sublimate is first made into a concentrated solution of one part of chemical to six parts of *hot* water. When used in the tanks this solution is weakened to 1 part in 100 or 150. The wood will absorb about 5 to 6.5 pounds of the bichloride per 100 cubic feet, or about one pound for each 4 to 6 ties. The timber is allowed to soak in the tanks for several days, the general rule being about one day for each inch of least thickness and one day over—which means seven days for six-inch ties, or thirteen (to fifteen) days for 12″ timber (least dimension). The process is somewhat objectionable on account of the chemical being such a virulent poison, workmen sometimes being sickened by the fumes arising from the tanks. On the Baden railway (Germany) kyanized ties last 20 to 30 years. On this railway the wood is always air-dried for two weeks after impregnation and before being used, which is thought to have an important effect on its durability. The solubility of the chemical and the liability of the chemical washing out and leaving the wood unprotected is an element of weakness in the method.

215. Wellhouse (or zinc-tannin) process. The last two methods described (as well as some others employing similar chemicals) are open to the objection that since the wood is impregnated with an aqueous solution, it is liable to be washed out very rapidly if the wood is placed under water, and will even disappear, although more slowly, under the action of moisture and rain. Several processes have been proposed or patented to prevent this. Many of them belong to one class, of which the Wellhouse process is a sample. By these processes the timber is successively subjected to the action of two chemicals, each individually soluble in water, and hence readily impregnating the timber, but the chemicals when brought in contact form insoluble compounds which cannot be washed out of the wood-cells. By the Wellhouse process, the wood is first impregnated with a solution of chloride of zinc and glue, and is then subjected to a bath of tannin under pressure. The glue and tannin combine to form an insoluble leathery compound in the cells, which will prevent the zinc chloride from being washed out. It is being used by the A., T. &. S. Fé R. R., their works being located at Las Vegas, New Mexico, and also by the Union Pacific R. R. at their works at Laramie, Wyo. In 1897 Mr. J. M. Meade, a resident engineer on the A., T. & S. Fé, exhibited to the Roadmasters Association of America a piece of a tie treated by this process which had been taken from the tracks after nearly 13 years' service. The tie was selected at random, was taken out for the sole purpose of having a specimen, and was still in sound condition and capable of serving many years longer. The cost of the treatment was then quoted as 13 c. per tie.

It was claimed that the treatment trebled the life of the tie besides adding to its spike-holding power.

216. Cost of treating. The cost of treating ties by the various methods has been estimated as follows *—assuming that the plant was of sufficient capacity to do the work economically: creosoting, 25 c. per tie; vulcanizing, 25 c. per tie; burnettizing (chloride of zinc), 8.25 c. per tie; kyanizing (steeping in corrosive sublimate), 14.6 c. per tie; Wellhouse process (chloride of zinc and tannin), 11.25 c. per tie. These estimates are only for the net cost at the works and do not include the cost of hauling the ties to and from the works, which may mean 5 to 10 c. per tie. Some of these processes have been installed on cars which are transported over the road and operated where most convenient.

217. Economics of treated ties. The fact that treated ties are not universally adopted is due to the argument that the added life of the tie is not worth the extra cost. If ties can be bought for 25 c., and cost 25 c. for treatment, and the treatment only doubles their life, there is apparently but little gained except the work of placing the extra tie in the track, which is more or less offset by the interest on 25 c. for the life of the untreated tie, and the larger initial outlay makes a stronger impression on the mind than the computed ultimate economy. But when ties cost 75 c. and treatment costs only 25 c., or perhaps less, then the economy is more apparent and unquestionable. But this analysis may be made more closely. As shown in § 202, the disturbance of the roadbed on account of frequent renewals of untreated ties is a disadvantage which would justify an appreciable expenditure to avoid, although it is very difficult to closely estimate its true value. The annual cost of a system of ties may be considered as the sum of (*a*) the interest on the first cost, (*b*) the annual sinking fund that would buy a new tie at the end of its life, and (*c*) the average annual cost of maintenance for the life of the tie, which includes the cost of laying and the considerable amount of subsequent tamping that must be done until the tie is fairly settled in the roadbed, besides the regular trackwork on the tie, which is practically constant. This last item is difficult to compute, but it is easy to see that, since

* Bull. No. 9, U. S. Dept. of Agric., Div. of Forestry. App. No. 1, by Henry Flad.

the cost of laying the tie and the subsequent tamping to obtain proper settlement is the same for all ties (of similar form), the *average* annual charge on the longer-lived tie would be much less. In the following comparison item (*c*) is disregarded, simply remembering that the advantage is with the longer-lived tie.

	Untreated tie.	Treated tie.
Original cost	40 cents	65 cents
Life (assumed at)	7 years	14 years
Item (*a*)—interest on first cost @ 4%	1.6 cents	2.6 cents
" (*b*)—sinking fund @ 4%	5.1 "	3.6 "
" (*c*)—(considered here as balanced)	——	——
Average annual cost (except item (*c*))	6.7 cents	6.2 cents

On this basis treated ties will cost 0.5 cent *less* per annum *besides* the advantage of item (*c*) and the still more indefinite advantages resulting from smoother running of trains, less wear and tear on rolling stock, etc., due to less disturbance of the roadbed.

In Europe, where wood is expensive, untreated ties are seldom used, as the treatment is always considered to be worth more than it costs. The rapid destruction of the forests of timber in this country is having the effect of increasing the price, so that it will not be long before treated ties (or metal ties) will be economical for a large majority of the railroads of the country.

(*Note added in* 1902.) Some modifications of the above processes have been devised in recent years, among them being the

Creo-resinate process		—creosote, resin, and formaldehyde;
Water-creosote	"	—emulsion of creosote and water;
Zinc-creosote	"	—emulsion of creosote and zinc-chloride;
Allardyce	"	—injection of chloride of zinc followed by creosote;
Hasselmann	"	—boiling in sulphates of iron, copper, etc.

The Atchison, Topeka and Santa Fé R. R. has compiled a record of treated pine ties removed in 1897, '98, '99, and 1900, showing that the *average* life of the ties removed had been about 11 years. On the Chicago, Rock Island and Pacific R. R., the average life of a very large number of treated hemlock and tamarack ties was found to be 10.57 years. Of one lot of 21850 ties, 12% still remained in the track after 15 years' exposure.

It has been demonstrated that much depends on the minor details of the process—whatever it may be. As an illustration, an examination of a batch of ties, treated by the zinc-creosote process, showed 84% in service after 13 years' exposure; another batch, treated by another contractor by the same process (nominally), showed 50% worthless after a service of six years.

METAL TIES.

218. Extent of use. In 1894 * there were nearly 35000 miles of "metal track" in various parts of the world. Of this total, there were 3645 miles of "longitudinals" (see § 224), found exclusively in Europe, nearly all of it being in Germany. There were over 12000 miles of "bowls and plates" (see § 223), found almost entirely in British India and in the Argentine Republic. The remainder, over 18000 miles, was laid with metal cross-ties of various designs. There were over 8000 miles of metal cross-ties in Germany alone, about 1500 miles in the rest of Europe, over 6000 miles in British India, nearly 1000 miles in the rest of Asia, and about 1500 miles more in various other parts of the world. Several railroads in this country have tried various designs of these ties, but their use has never passed the experimental stage. These 35000 miles represent about 9% of the total railroad mileage of the world—nearly 400000 miles. They represent about 17.6% of the total railroad mileage, exclusive of the United States and Canada, where they are not used at all, except experimentally. In the four years from 1890 to 1894 the use of metal track increased from less than 25000 miles to nearly 35000 miles. This increase was practically equal to the total increase in railroad mileage during that time, exclusive of the increase in the United States and Canada. This indicates a large growth in the percentage of metal track to total mileage, and therefore an increased appreciation of the advantages to be derived from their use.

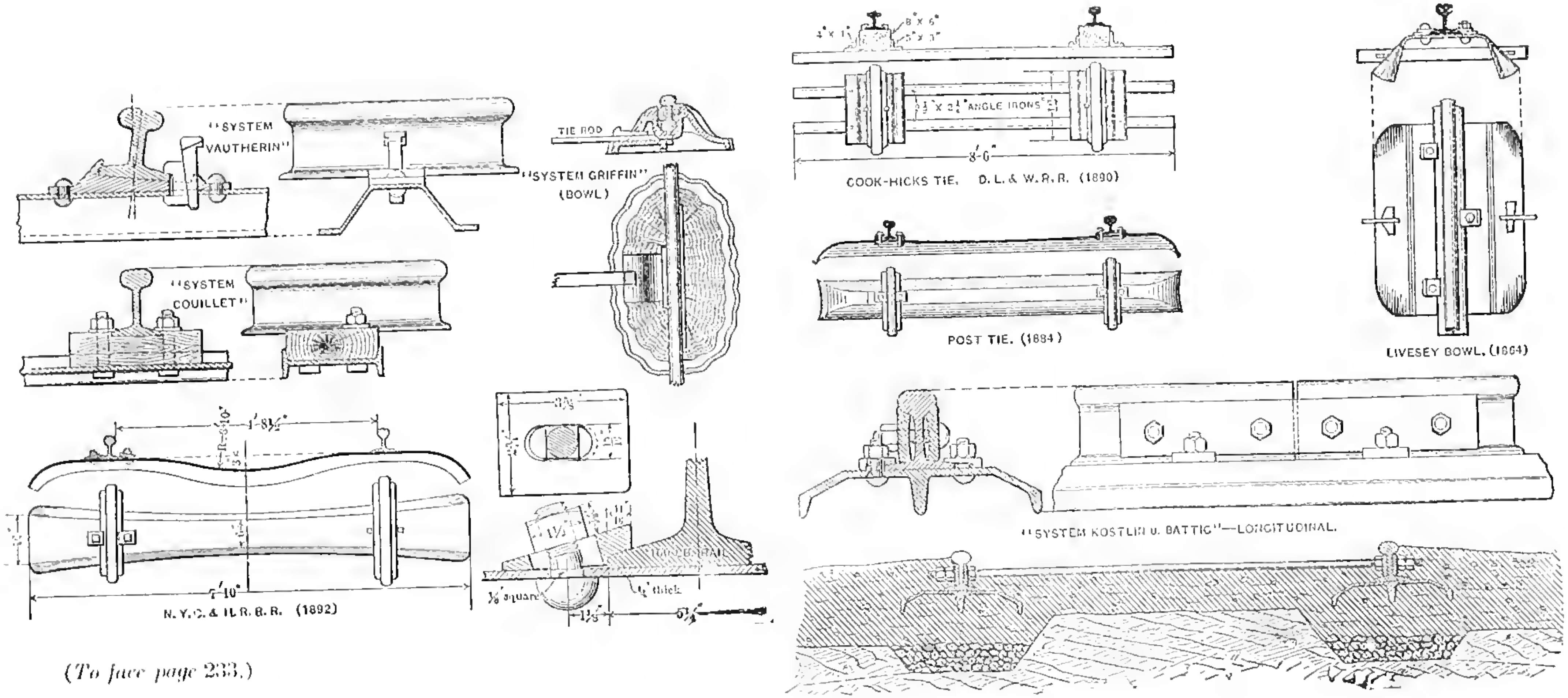
219. Durability. The durability of metal ties is still far from being a settled question, due largely to the fact that the best form for such ties is not yet determined, and that a large part of the apparent failures in metal ties have been evidently due to defective design. Those in favor of them estimate the life as from 30 to 50 years. The opponents place it at not more

* Bulletin No. 9, U. S. Dept. of Agriculture, Div. of Forestry.

than 20 years, or perhaps as long as the best of wooden ties. Unlike the wooden tie, however, which deteriorates as much with time as with usage, the life of a metal tie is more largely a function of the traffic. The life of a well-designed metal tie has been estimated at 150000 to 200000 trains; for 20 trains per day, or say 6000 per year, this would mean from 25 to 33 years. 20 trains per day on a *single* track is a much larger number than will be found on the majority of railroads. Metal ties are found to be subject to rust, especially when in damp localities, such as tunnels; but on the other hand it is in such confined localities, where renewals are troublesome, that it is especially desirable to employ the best and longest-lived ties. Paint, tar, etc., have been tried as a protection against rust, but the efficacy of such protection is as yet uncertain, the conditions preventing any renewal of the protection—such as may be done by repainting a bridge, for example. Failures in metal cross-ties have been largely due to cracks which begin at a corner of one of the *square* holes which are generally *punched* through the tie, the holes being made for the bolts by which the rails are fastened to the tie. The holes are generally *punched* because it is cheaper. Reaming the holes after punching is thought to be a safeguard against this frequent cause of failure. Another method is to round the corners of the square punch with a radius of about $\frac{1}{8}''$. If a crack has already started, the spread of the crack may be prevented by drilling a small hole at the end of it.

220. Form and dimensions of metal cross-ties. Since stability in the ballast is an essential quality for a tie, this must be accomplished either by turning down the end of the tie or by having some form of lug extending downward from one or more points of the tie. The ties are sometimes depressed in the center (see Plate VI, N. Y. C. & H. R. R. R. tie) to allow for a thick covering of ballast on top in order to increase its stability in the ballast. This form requires that the ties should be sufficiently well tamped to prevent a tendency to bend out straight, thus widening the gauge. Many designs of ties are objectionable because they cannot be placed in the track without disturbing adjacent ties. The failure of many metal cross-ties, otherwise of good design, may be ascribed to too light weight. Those weighing much less than 100 pounds have proved too light. From 100 to 130 pounds weight is being used satisfactorily on German railroads. The general outside dimensions are about

PLATE VI.



(*To face page 233.*)

METAL TIES.

the same as for wooden ties, except as to thickness. The metal is generally from $\frac{1}{4}''$ to $\frac{3}{8}''$ thick. They are, of course, only made of wrought iron or steel, cast iron being used only for "bowls" or "plates" (see § 223). The details of construction of some of the most commonly used ties may be seen by a study of Plate VI.

221. **Fastenings.** The devices for fastening the rails to the ties should be such that the gauge may be widened if desired on curves, also that the gauge can be made true regardless of slight inaccuracies in the manufacture of the ties, and also that shims may be placed under the rail if necessary during cold weather when the tie is frozen into the ballast and cannot be easily disturbed. Some methods of fastening require that the base of the rail be placed against a lug which is riveted to the tie or which forms a part of it. This has the advantage of reducing the number of pieces, but is apt to have one or more of the disadvantages named above. Metal keys or wooden wedges are sometimes used, but the majority of designs employ some form of bolted clamp. The form adopted for the experimental ties used by the N. Y. C. & H. R. R. R. (see Plate VI) is especially ingenious in the method used to vary the gauge or allow for inaccuracies of manufacture. Plate VI shows some of the methods of fastening adopted on the principal types of ties.

222. **Cost.** The cost of metal cross-ties in Germany averages about 1.6 c. per pound or about $1.60 for a 100-lb. tie. The ties manufactured for the N. Y. C. & H. R. R. R. in 1892 weighed about 100 lbs. and cost $2.50 per tie, but if they had been made in larger quantities and with the present price of steel the cost would possibly have been much lower. The item of freight from the place of manufacture to the place where used is no inconsiderable item of cost with some roads. Metal cross-ties have been used by some street railroads in this country. Those used on the Terre Haute Street Railway weigh 60 pounds and cost about 66 c. for the tie, or 74 c. per tie with the fastenings.

---

223. **Bowls or plates.** As mentioned before, over 12000 miles of railway, chiefly in British India and in the Argentine Republic, are laid with this form of track. It consists essentially of large cast-iron inverted "bowls" laid at intervals under each rail and opposite each other, the opposite bowls being tied together with tie-rods. A suitable chair is riveted or bolted on to the top of each bowl so as to properly hold the rail. Being

made of cast iron, they are not so subject to corrosion as steel or wrought iron. They have the advantage that when old and worn out their scrap value is from 60% to 80% of their initial cost, while the scrap value of a steel or wrought-iron tie is practically nothing. Failure generally occurs from breakage, the failures from this cause in India being about 0.4% per annum. They weigh about 250 lbs. apiece and are therefore quite expensive in first cost and transportation charges. There are miles of them in India which have already lasted 25 years and are still in a serviceable condition. Some illustrations of this form of tie are shown in Plate VI.

224. Longitudinals.* This form, the use of which is confined almost exclusively to Germany, is being gradually replaced on many lines by metal cross-ties. The system generally consists of a compound rail of several parts, the upper bearing rail being very light and supported throughout its length by other rails, which are suitably tied together with tie-rods so as to maintain the proper gauge, and which have a sufficiently broad base to be properly supported in the ballast. One great objection to this method of construction is the difficulty of obtaining proper drainage especially on grades, the drainage having a tendency to follow along the lines of the rails. The construction is much more complicated on sharp curves and at frogs and switches. Another fundamentally different form of longitudinal is the Haarman compound "self-bearing" rail, having a base 12″ wide and a height of 8″, the alternate sections breaking joints so as to form a practically continuous rail.

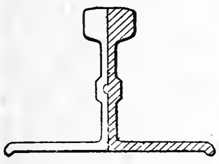


Fig. 110.

Some of the other forms of longitudinals are illustrated in Plate VI.

For a very complete discussion of the subject of metal ties, see the "Report on the Substitution of Metal for Wood in Railroad Ties" by E. E. Russell Tratman, it being Bulletin No. 4, Forestry Division of the U. S. Dept. of Agriculture.

* Although the discussion of longitudinals might be considered to belong more properly to the subject of Rails, yet the essential idea of all designs must necessarily be the *support* of a rail-head on which the rolling stock may run, and therefore this form, unused in this country, will be briefly described here.

CHAPTER IX.

RAILS.

225. Early forms. The first rails ever laid were wooden stringers which were used on very short tram-roads around coal-mines. As the necessity for a more durable rail increased, owing chiefly to the invention of the locomotive as a motive power, there were invented successively the cast-iron "fish-belly" rail and various forms of wrought-iron strap rails which finally developed into the T rail used in this country and the double-headed rail, supported by chairs, used so extensively in England. The cast-iron rails were cast in lengths of about 3 feet and were supported in iron chairs which were sometimes set upon stone piers. A great deal of the first railroad track of this country was laid with longitudinal stringers of wood placed upon cross-ties, the inner edge of the stringers being protected by wrought-iron straps. The "bridge" rails were first rolled in this country in 1844. The "pear" section was an approach to the present form, but was very defective on account of the difficulty of designing a good form of joint. The "Stevens" section was designed in 1830 by Col. Robert L. Stevens, Chief Engineer of the Camden and Amboy Railroad; although quite defective in its proportions, according to the present knowledge of the requirements, it is essentially the present form. In 1836, Charles Vignoles invented essentially the same form in England; this form is therefore known throughout England and Europe as the Vignoles rail.

226. Present standard forms. The larger part of modern railroad track is laid with rails which are either "T" rails or the double-headed or "bull-headed" rails which are carried in chairs. The double-headed rail was designed with a symmetrical form with the idea that after one head had been worn out by traffic the rail could be reversed, and that its life would be practically doubled. Experience has shown that the wear of the rail in the chairs is very great; so much so that when one head has been worn out by traffic the whole rail is generally useless.

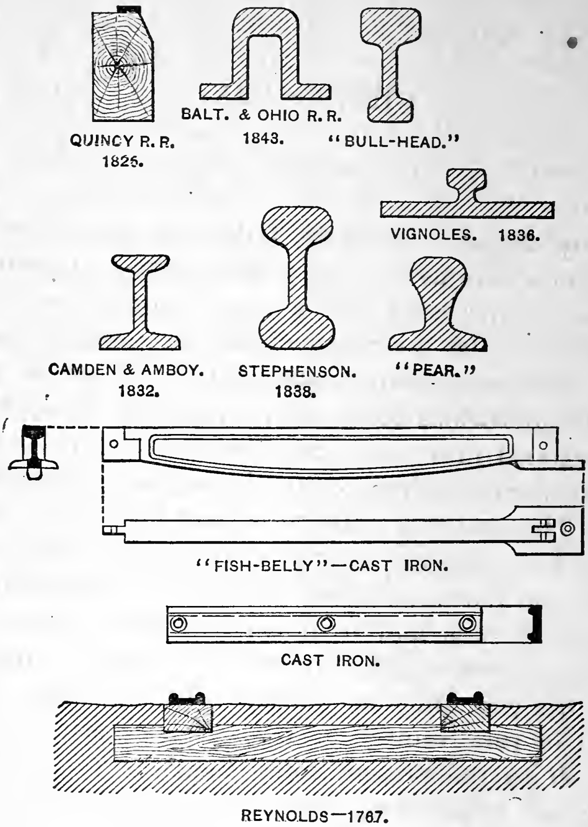


Fig. 111.—Early Forms of Rails.

If the rail is turned over, the worn places, caused by the chairs, make a rough track and the rail appears to be more brittle and subject to fracture, possibly due to the crystallization that may have occurred during the previous usage and to the reversal of stresses in the fibers. Whatever the explanation, experience has demonstrated the *fact*. The "bull-headed" rail has the lower head only large enough to properly hold the wooden keys with which the rail is secured to the chairs (see Fig. 112) and furnish the necessary strength. The use of these rails requires the use of two cast-iron chairs for each tie. It is claimed that such track is better for heavy and fast traffic, but it is more

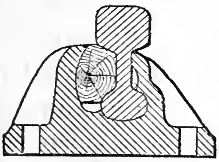


Fig. 112. — Bull-headed Rail and Chair.

expensive to build and maintain. It is the standard form of track in England and some parts of Europe.

Until a few years ago there was a very great multiplicity in the designs of "T" rails as used in this country, nearly every prominent railroad having its own special design, which perhaps differed from that of some other road by only a very minute and insignificant detail, but which nevertheless would require a complete new set of rolls for rolling. This certainly must have had a very appreciable effect on the cost of rails. In 1893, the American Society of Civil Engineers, after a very exhaustive investigation of the subject, extending over several years, having obtained the opinions of the best experts of the country, adopted a series of sections which have been very extensively adopted by the railroads of this country. Instead of having the rail sections for various weights to be geometrically similar figures, certain dimensions are made constant, regardless of the weight. It was decided that the metal should be distributed through the section in the proportions of—head 42%, web 21%, and flange 37%. The top of the head should have a radius of

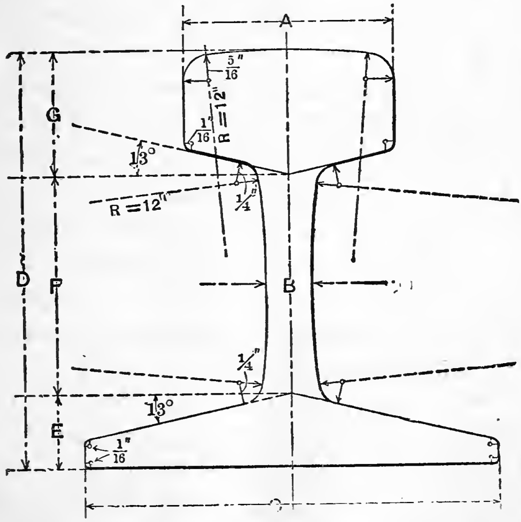


FIG. 113.—AM. SOC. C. E. STANDARD RAIL SECTION.

12″; the top corner radius of head should be $\frac{5}{16}''$; the lower corner radius of head should be $\frac{1}{16}''$; the corners of the flanges, $\frac{1}{16}''$ radius; side radius of web, 12″; top and bottom radii of web corners, $\frac{1}{4}''$; and angles with the horizontal of the under side of the head and the top of the flange, 13°. The sides of the head are vertical.

The height of the rail (*D*) and the width of the base (*C*) are always made equal to each other.

	Weight per Yard.												
	40	45	50	55	60	65	70	75	80	85	90	95	100
A	$1\frac{7}{8}''$	$2''$	$2\frac{1}{8}''$	$2\frac{1}{4}''$	$2\frac{3}{8}''$	$2\frac{13}{32}''$	$2\frac{7}{16}''$	$2\frac{15}{32}''$	$2\frac{1}{2}''$	$2\frac{9}{16}''$	$2\frac{5}{8}''$	$2\frac{11}{16}''$	$2\frac{3}{4}''$
B	$\frac{25}{64}$	$\frac{27}{64}$	$\frac{7}{16}$	$\frac{15}{32}$	$\frac{31}{64}$	$\frac{1}{2}$	$\frac{33}{64}$	$\frac{17}{32}$	$\frac{35}{64}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$
C & D	$3\frac{1}{2}$	$3\frac{11}{16}$	$3\frac{7}{8}$	$4\frac{1}{16}$	$4\frac{1}{4}$	$4\frac{7}{16}$	$4\frac{5}{8}$	$4\frac{13}{16}$	5	$5\frac{3}{16}$	$5\frac{3}{8}$	$5\frac{9}{16}$	$5\frac{3}{4}$
E	$\frac{5}{8}$	$\frac{21}{32}$	$\frac{11}{16}$	$\frac{23}{32}$	$\frac{49}{64}$	$\frac{25}{32}$	$\frac{13}{16}$	$\frac{27}{32}$	$\frac{7}{8}$	$\frac{57}{64}$	$\frac{59}{64}$	$\frac{15}{16}$	$\frac{31}{32}$
F	$1\frac{55}{64}$	$1\frac{31}{32}$	$2\frac{1}{16}$	$2\frac{11}{64}$	$2\frac{17}{64}$	$2\frac{3}{8}$	$2\frac{15}{32}$	$2\frac{35}{64}$	$2\frac{5}{8}$	$2\frac{3}{4}$	$2\frac{55}{64}$	$2\frac{63}{64}$	$3\frac{5}{64}$
G	$1\frac{1}{64}$	$1\frac{1}{16}$	$1\frac{1}{8}$	$1\frac{11}{64}$	$1\frac{7}{32}$	$1\frac{9}{32}$	$1\frac{11}{32}$	$1\frac{27}{64}$	$1\frac{1}{2}$	$1\frac{25}{64}$	$1\frac{19}{32}$	$1\frac{41}{64}$	$1\frac{45}{64}$

The chief features of disagreement among railroad men relate to the radius of the upper corner of the head and the slope of the side of the head. The radius ($\frac{5}{16}''$) adopted for the upper corner (constant for all weights) is a little more than is advocated by those in favor of "sharp corners" who often use a radius of $\frac{1}{4}''$. On the other hand it is much less than is advocated by those who consider that it should be nearly equal to (or even greater than) the larger radius universally adopted for the corner of the wheel-flange. The discussion turns on the relative rapidity of rail wear and the wear of the wheel-flanges as affected by the relation of the form of the wheel-tread to that of the rail. It is argued that sharp rail corners wear the wheel-flanges so as to produce sharp flanges, which are liable to cause derailment at switches and also to require that the tires of engine-drivers must be more frequently turned down to their true form. On the other hand it is generally believed that rail wear is much less rapid while the area of contact between the rail and wheel-flange is small, and that when the rail has worn down, as it invariably does, to nearly the same form as the wheel-flange, the rail wears away very quickly.

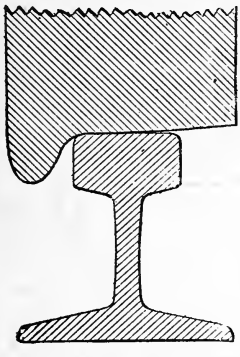


FIG. 114.—RELATION OF RAIL TO WHEEL-TREAD.

227. Weight for various kinds of traffic. The heaviest rails in regular use weigh 100 lbs. per yard, and even these are only used on some of the heaviest traffic sections of such roads as the

N. Y. Central, the Pennsylvania, the N. Y., N. H. & H., and a few others. Probably the larger part of the mileage of the country is laid with 60- to 75-lb. rails—considering the fact that "the larger part of the mileage" consists of comparatively light-traffic roads and may exclude all the heavy trunk lines. Very light-traffic roads are sometimes laid with 56-lb. rails. Roads with fairly heavy traffic generally use 75- to 85-lb. rails, especially when grades are heavy and there is much and sharp curvature. The tendency on all roads is toward an increase in the weight, rendered necessary on account of the increase in the weight and capacity of rolling stock, and due also to the fact that the price of rails has been so reduced that it is both better and cheaper to obtain a more solid and durable track by increasing the weight of the rail rather than by attempting to support a weak rail by an excessive number of ties or by excessive track labor in tamping. It should be remembered that in buying rails the mere weight is, in one sense, of no importance. The important thing to consider is the STRENGTH and the STIFFNESS. If we assume that all weights of rails have *similar* cross-section, (which is nearly although not exactly true), then, since for beams of similar cross-sections the *strength* varies as the *cube* of the homologous dimensions and the *stiffness* as the *fourth powers* while the area (and therefore the weight per unit of length) only varies as the *square*, it follows that the stiffness varies as the square of the weight, and the strength as the $\frac{3}{2}$ power of the weight. Since for ordinary variations of weight the price per ton is the same, adding (say) 10% to the weight (and cost) adds 21% to the stiffness and over 15% to the strength. As another illustration, using an 80-lb. rail instead of a 75-lb. rail adds only $6\frac{2}{3}$% to the cost, but adds about 14% to the stiffness and nearly 11% to the strength. This shows why heavier rails are more economical and are being adopted even when they are not absolutely needed on account of heavier rolling stock. The stiffness, strength, and consequent durability are increased in a much greater ratio than the cost.

228. Effect of stiffness on traction. A very important but generally unconsidered feature of a stiff rail is its effect on tractive force. An extreme illustration of this principle is seen when a vehicle is drawn over a soft sandy road. The constant compression of the sand in front of the wheel has virtually the same effect on traction as drawing the wheel up a grade whose steepness depends on the radius of the wheel and the depth of the rut. On the other hand, if a wheel, made of perfectly elastic material, is rolled over a surface which, while supported with absolute rigidity, is also perfectly elastic, there would be a forward component, caused by the expanding of the compressed metal just behind the center of contact, which would just balance the backward component. If the rail was supported throughout its length by an absolutely rigid support, the high elasticity of the wheel-tires and rails would reduce this form of resistance to an insignificant quantity, but the ballast and even the ties are comparatively inelastic. When a weak rail yields, the ballast is more or less compressed or displaced, and even though the elasticity of the rail brings it back to nearly its former place, the work done in compressing an inelastic material is wholly lost. The effect of this on the fuel account is certainly very considerable and yet is frequently entirely overlooked. It is practically impossible to compute the saving in tractive power, and therefore in cost of fuel, resulting from a given increase in the weight and stiffness of the rail, since the yielding of the rail is so dependent on the spacing of the ties, the tamping, etc. But it is not difficult to perceive in a general way that such an economy is possible and that it should not be neglected in considering the value of stiffness in rails.

229. **Length of rails.** The standard length of rails with most railroads is 30 feet. In recent years many roads have been trying 45-foot and even 60-foot rails. The argument in favor of longer rails is chiefly that of the reduction in track-joints, which are costly to construct and to maintain and are a fruitful source of accidents. Mr. Morrison of the Lehigh Valley R. R.* declares that, as a result of extensive experience with 45-foot rails on that road, he finds that they are much less expensive to handle, and that, being so long, they can be laid around sharp curves without being curved in a machine, as is necessary with the shorter rails. The great objection to longer rails lies in the difficulty in allowing for the expansion, which will require, in the coldest weather, an opening at the joint of nearly $\frac{3}{4}''$ for a 60-foot rail. The Pennsylvania R. R. and the Norfolk and Western R. R. each have a considerable mileage laid with 60-foot rails.

---

* Report, Roadmasters Association, 1895.

230. Expansion of rails. Steel expands at the rate of .0000065 of its length per degree Fahrenheit. The extreme range of temperature to which any rail will be subjected will be about 160°, or say from −20° F. to +140° F. With the above coefficient and a rail length of 60 feet the expansion would be 0.0624 foot, or about ¾ inch. But it is doubtful whether there would ever be such a range of motion even if there were such a range of temperature. Mr. A. Torrey, chief engineer of the Mich. Cent. R. R., experimented with a section over 500 feet long, which, although not a single rail, was made "continuous" by rigid splicing, and he found that there was no appreciable additional contraction of the rail at any temperature below +20° F. The reason is not clear, but the *fact* is undeniable.

The heavy girder rails, used by the street railroads of the country, are bonded together with perfectly tight rigid joints which do not permit expansion. If the rails are laid at a temperature of 60° F. and the temperature sinks to 0°, the rails have a *tendency* to contract .00039 of their length. If this tendency is resisted by the friction of the pavement in which the rails are buried, it only results in a tension amounting to .00039 of the modulus of elasticity, or say 10920 pounds per square inch, assuming 28 000 000 as the modulus of elasticity. This stress is not dangerous and may be permitted. If the temperature rises to 120° F., a tendency to expansion and buckling will take place, which will be resisted as before by the pavement, and a compression of 10920 pounds per square inch will be induced, which will likewise be harmless. The range of temperature of rails which are buried in pavement is much less than when they are entirely above the ground and will probably never reach the above extremes. Rails supported on ties which are only held in place by ballast must be allowed to expand and contract almost freely, as the ballast cannot be depended on to resist the distortion induced by any considerable range of temperature, especially on curves.

231. Rules for allowing for temperature. Track regulations generally require that the track foremen shall use iron (*not* wooden) shims for placing between the ends of the rails while splicing them. The thickness of these shims should vary with the temperature. Some roads use such approximate rules as the following: "The proper thickness for coldest weather is $\frac{5}{16}$ of an inch; during spring and fall use ⅛ of an inch, and in the very hottest weather $\frac{1}{16}$ of an inch should be allowed." This is on the basis of a 30-foot rail. When a more accurate adjustment than this is desired, it may be done by assuming some very high temperature (120° to 150° F.) as a maximum, when the joints should be *tight;* then compute in tabular form the spacing for each temperature, varying by 20°, allowing 0''.0468 (almost exactly $\frac{3}{64}''$) for each 20° change. Such a tabular form would be about as follows (rail length 30 feet):

Temperature....	150°	130°	110°	90°	70°	50°	30°	10°	−10°	−30°
Rail opening....	0	$\frac{3}{64}''$	$\frac{3}{32}''$	$\frac{9}{64}''$	$\frac{3}{16}''$	$\frac{15}{64}''$	$\frac{9}{32}''$	$\frac{21}{64}''$	$\frac{3}{8}''$	$\frac{27}{64}''$

One practical difficulty in the way of great refinement in this work is the determination of the real temperature of the rail when it is laid. A rail lying in the hot sun has a very much higher temperature than the air. The temperature of the rail cannot be obtained even by exposing a thermometer directly to the sun, although such a result might be the best that is easily obtainable. On a cloudy or rainy day the rail has practically the same temperature as the air; therefore on such days there need be no such trouble.

232. Chemical composition. About 98 to 99.5% of the composition of steel rails is iron, but the value of the rail, as a rail, is almost wholly dependent upon the large number of other chemical elements which are, or may be, present in very small amounts. The manager of a steel-rail mill once declared that their aim was to produce rails having in them—

Carbon........................	0.32 to 0.40%
Silicon........................	0.04 to 0.06%
Phosphorus....................	0.09 to 0.105%
Manganese.....................	1.00 to 1.50%

The analysis of 32 specimens of rails on the Chic., Mil. & St. Paul R. R. showed variations as follows:

Carbon........................	0.211 to 0.52%
Silicon........................	0.013 to 0.256%
Phosphorus....................	0.055 to 0.181%
Manganese.....................	0.35 to 1.63%

These quantities have the same general relative proportions as the rail-mill standard given above, the differences lying mainly in the broadening of the limits. Increasing the percentage of carbon by even a few hundredths of one per cent makes the rail harder, but likewise more brittle. If a track is well ballasted and not subject to heaving by frost, a harder and more brittle rail may be used without excessive danger of breakage, and such a rail will wear much longer than a softer tougher rail, although the softer tougher rail may be the better rail for a road having a less perfect roadbed.

A small but objectionable percentage of sulphur is sometimes found in rails, and very delicate analysis will often show the presence, in very minute quantities, of several other chemical elements. The use of a very small quantity of nickel or aluminum has often been suggested as a means of producing a more durable rail. The added cost and the uncertainty of the amount of advantage to be gained has hitherto prevented the practical use or manufacture of such rails.

233. Testing. Chemical and mechanical testing are both necessary for a thorough determination of the value of a rail. The chemical testing has for its main object the determination of those minute quantities of chemical elements which have such a marked influence on the rail for good or bad. The mechanical testing consists of the usual tests for elastic limit, ultimate strength, and elongation at rupture, determined from pieces cut out of the rail, besides a "drop test." The drop test consists in dropping a weight of 2000 lbs. from a height of 16 to 20 feet on to the center of a rail which is supported on abutments. placed three or four feet apart. The number of blows required to produce rupture or to produce a permanent set of specified magnitude gives a measure of the strength and toughness of the rail.

233a. Proposed standard specifications for steel rails. The following specifications for steel rails are those proposed by a committee of the American Railway Engineering and Maintenance of Way Association in March, 1902:

1. (*a*) Steel may be made by the Bessemer or open-hearth process.

(*b*) The entire process of manufacture and testing shall be in accordance with the best standard current practice, and special care shall be taken to conform to the following instructions:

(*c*) Ingots shall be kept in a vertical position in pit-heating furnaces.

(*d*) No bled ingots shall be used.

(*e*) Sufficient material shall be discarded from the top of the ingots to insure sound rails.

CHEMICAL PROPERTIES.

2. Rails of the various weights per yard specified below shall conform to the following limits in chemical composition:

	50 to 59+ lbs. per cent.	60 to 69+ lbs. per cent.	70 to 79+ lbs. per cent.	80 to 89+ lbs. per cent.	90 to 100 lbs. per cent.
Carbon	0.35–0.45	0.38–0.48	0.40–0.50	0.43–0.53	0.45–0.55
Phosphorus shall not exceed	0.10	0.10	0.10	0.10	0.10
Silicon shall not exceed	0.20	0.20	0.20	0.20	0.20
Manganese	0.70–1.00	0.70–1.00	0.75–1.05	0.80–1.10	0.80–1.10

PHYSICAL PROPERTIES.

3. One drop test shall be made on a piece of rail not more than 6 feet long, selected from every fifth blow of steel. The test-piece shall be taken from the top of the ingot. The rail shall be placed head upwards on the supports and the various sections shall be subjected to the following impact tests:

Weight of Rail in Pounds per Yard.	Height of Drop in Feet.
45 to and including 55	15
More than 55 " " " 65	16
" " 65 " " " 75	17
" " 75 " " " 85	18
" " 85 " " " 100	19

If any rail break when subjected to the drop test two additional tests will be made of other rails from the same blow of steel, and if either of these latter tests fail, all the rails of the blow which they represent will be rejected; but if both of these additional test-pieces meet the requirements all the rails of the blow which they represent will be accepted. If the rails from the tested blow shall be rejected for failure to meet the requirements of the drop test, as above specified, two other rails will be subjected to the same tests, one from the blow next preceding and one from the blow next succeeding, the rejected blow. In case the first test taken from the preceding or succeeding blow shall fail two additional tests shall be taken from the same blow of steel, the acceptance or rejection of which shall also be determined as specified above, and if the rails of the preceding or succeeding, blow shall be rejected, similar tests may be taken from the previous or following blows, as the case may be, until the entire group of five blows is tested, if necessary. The acceptance or rejection of all rails from any blow will depend upon the results of the tests thereof.

HEAT TREATMENT.

The number of passes and speed of train shall be so regulated that on leaving the rolls at the final pass the temperature of the rail will not exceed that which requires a shrinkage allowance at the hot saws of 6 inches for 85-lb. and $6\frac{1}{8}$ inches for 100-lb. rails, and no artificial means of cooling the rails shall be used between the finishing pass and the hot saws.

TEST-PIECES AND METHODS OF TESTING.

4. The drop-test machine shall have a tup of 2000 lbs. weight, the striking face of which shall have a radius of not more than 5 inches, and the test rail shall be placed head upwards on solid supports 3 feet apart. The anvil-block shall weigh at least 20000 lbs., and the support shall be a part of, or firmly secured to, the anvil.

5. The manufacturer shall furnish the inspector, daily, with carbon determinations of each blow, and a complete chemical analysis every 24 hours, representing the average of the other elements contained in the steel. These analyses shall be made on drillings taken from a small test ingot.

FINISH.

6. Unless otherwise specified the section of rail shall be the American standard, recommended by the American Society of Civil Engineers, and shall conform, as accurately as possible, to the templet furnished by the railroad company, consistent with paragraph No. 7, relative to the specified weight. A vari-

ation in height of $\frac{1}{64}$ inch less and $\frac{1}{32}$ inch greater than the specified height will be permitted. A perfect fit of the splice-bars, however, shall be maintained at all times.

7. The weight of the rails shall be maintained as nearly as possible, after complying with paragraph No. 6, to that specified in contract. A variation of one-half of one per cent for an entire order will be allowed. Rails shall be accepted and paid for according to actual weights.

8. The standard length of rails shall be 33 feet. Ten per cent of the entire order will be accepted in shorter lengths, varying by even feet down to 27 feet. A variation of $\frac{1}{4}$ inch in length from that specified will be allowed.

9. Circular holes for splice-bars shall be drilled in accordance with the specifications of the purchaser. The holes shall accurately conform to the drawing and dimensions furnished in every respect, and must be free from burrs.

10. Rails shall be straightened while cold, smooth on head, sawed square at ends, and, prior to shipment, shall have the burr, occasioned by the saw-cutting, removed, and the ends made clean. No. 1 rails shall be free from injurious defects and flaws of all kinds.

BRANDING.

11. The name of the maker, the month and year of manufacture shall be rolled in raised letters on the side of the web, and the number of the blow shall be stamped on each rail

INSPECTION.

12. The inspector representing the purchaser shall have all reasonable facilities afforded to him by the manufacturer to satisfy him that the finished material is furnished in accordance with these specifications. All tests and inspections shall be made at the place of manufacture, prior to shipment.

NO. 2 RAILS.

13. Rails that possess any injurious physical defects, or which for any other cause are not suitable for first quality, or No. 1 rails, shall be considered as No. 2 rails, provided, however, that rails which contain any physical defects which seriously impair their strength shall be rejected. The ends of all No. 2 rails shall be painted in order to distinguish them.

234. Rail wear on tangents. When the wheel loads on a rail are abnormally heavy, and particularly when the rail has but little carbon and is unusually soft, the concentrated pressure on the rail is frequently greater than the elastic limit, and the metal "flows" so that the head, although greatly abraded, will spread somewhat outside of its original lines, as shown in Fig. 115. The rail wear that occurs on tangents is almost exclusively on top. Statistics show that the rate of rail wear on tangents decreases as the rails are more worn. Tests of a large number of rails on tangents have shown a rail wear averaging nearly one pound per yard per 10 000 000 tons of traffic. There is about 33 pounds of metal in one yard of the head of an 80-lb. rail. As an extreme value this may be worn down one-half, thus giving a tonnage of 165 000 000 tons for the life of the rail. Other estimates bring the tonnage down to 125 000 000 tons. Since the locomotive is considered to be responsible for one-half (and possibly more) of the damage done to the rail, it is found that the rate of wear on roads with shorter trains is more rapid in proportion to the tonnage, and it is therefore thought that the life of a rail should be expressed in terms of the number of trains. This has been estimated at 300 000 to 500 000 trains.

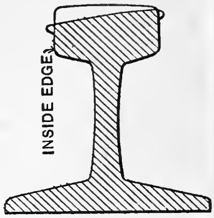


FIG. 115.

235. Rail wear on curves. On curves the maximum rail wear occurs on the inner side of the head of the outer rail, giving a worn form somewhat as shown in Fig. 116. The dotted line shows the nature and progress of the rail wear on the inner rail of a curve. Since the pressure on the outer rail is somewhat lateral rather than vertical, the "flow" does not take place to the same extent, if at all, on the outside, and whatever flow would take place on the inside is immediately worn off by the wheel-flange. Unlike the wear on tangents, the wear on curves is at a greater rate as the rail becomes more worn.

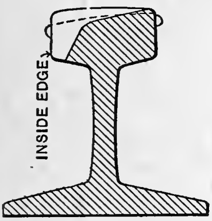


FIG. 116.

The inside rail on curves wears chiefly on top, the same as on a tangent, except that the wear is much greater owing to the longitudinal slipping of the wheels on the rail, and the lateral slipping that must occur when a rigid four-wheeled truck is guided around a curve. The outside rail is subjected to a greater or less proportion of the longitudinal slipping, likewise to the lateral slipping, and, worst of all, to the grinding action of the flange of the wheel, which grinds off the side of the head.

The results of some very elaborate tests, made by Mr. A M. Wellington, on the Atlantic and Great Western R. R., on the wear of rails, seem to show that the rail wear on curves may be expressed by the formula: "Total wear of rails on a d degree curve in pounds per yard per 10 000 000 tons duty $= 1 + 0.03d^2$." "It is not pretended that this formula is strictly correct even in theory, but several theoretical considerations indicate that it may be nearly so." According to this formula the average rail wear on a 6° curve will be about twice the rail wear on a tangent. While this is approximately true, the various causes modifying the rate of rail wear (length of trains, age and quality of rails, etc.) will result in numerous and large variations from the above formula, which should only be taken as indicating an approximate law.

236. Cost of rails. In 1873 the cost of steel rails was about \$120 per ton, and the cost of iron rails about \$70 per ton Although the steel rails were at once recognized as superior to iron rails on account of more uniform wear, they were an expensive luxury. The manufacture of steel rails by the Bessemer process created a revolution in prices, and they have steadily dropped in price until, during the last few years, steel rails have been manufactured and sold for \$22 per ton. At such prices there is no longer any demand for iron rails, since the cost of manufacturing them is substantially the same as that of steel rails, while their durability is unquestionably inferior to that of steel rails.

CHAPTER X.

RAIL-FASTENINGS.

RAIL-JOINTS

237. Theoretical requirements for a perfect joint. A perfect rail-joint is one that has the *same strength* and *stiffness*—no more and no less—as the rails which it joins, and which will not interfere with the regular and uniform spacing of ties. It should also be reasonably cheap both in first cost and in cost of maintenance. Since the action of heavy loads on an elastic rail is to cause a wave of translation in front of each wheel, any change in the stiffness or elasticity of the rail structure will cause more or less of a shock, which must be taken up and resisted by the joint. The greater the change in stiffness the greater the shock, and the greater the destructive action of the shock. The perfect rail-joint must keep both rail-ends truly in line both laterally and vertically, so that the flange or tread of the wheel need not jump or change its direction of motion suddenly in passing from one rail to the other. A consideration of all the above requirements will show that only a perfect welding of rail-ends would produce a joint of uniform strength and stiffness which would give a uniform elastic wave ahead of each wheel. As welding is impracticable for ordinary railroad work (see § 230), some other contrivance is necessary which will approach this ideal as closely as may be.

238. Efficiency of the ordinary angle-bar. Throughout the middle portion of a rail the rail acts as a continuous girder. If we consider for simplicity that the ties are unyielding, the deflection of such a continuous girder between the ties will be but one-fourth of the deflection that would be found if the rail were cut half-way between the ties and an equal concentrated load were divided equally between the two unconnected ends. The maximum stress for the continuous girder would be but one-half of that in the cantilevers. Joining these ends with rail-joints will give the ordinary "suspended" joint. In order to maintain uniform strength and stiffness the angle-bars must supply the deficiency. These theoretical relations are modified to an unknown extent by the unknown and variable yielding of the ties. From some experiments made by the Association of Engineers of Maintenance of Way of the P. R. R.* the following deductions were made:

1. The capacity of a "suspended" joint is greater than that of a "supported" joint—whether supported on one or three ties. (See § 240.)

2. That (with the particular patterns tested) the angle-bars alone can carry only 53 to 56% of a concentrated load placed on a joint.

3. That the capacity of the whole joint (angle-bars and rail) is only 52.4% of the strength of the unbroken rail.

4. That the ineffectiveness of the angle-bar is due chiefly to a deficiency in compressive resistance.

Although it has been universally recognized that the angle-bar is not a perfect form of joint, its simplicity, cheapness, and reliability have caused its almost universal adoption. Within a very few years other forms (to be described later) have been adopted on trial sections and have been more and more extended, until their present use is very large. The present time (1900) is evidently a transition period, and it is quite probable that within a very few years the now common angle-plate will be as unknown in standard practice as the old-fashioned "fish-plate" is at the present time.

239. Effect of rail gap at joints. It has been found that the jar at a joint is due almost entirely to the *deflection* of the joint and scarcely at all to the small gap required for expansion. This gap causes a drop equal to the versed sine of the arc having a chord equal to the gap and a radius equal to the radius of the wheel. Taking the extreme case (for a 30-foot rail) of a $\frac{3}{8}''$ gap and a 33″ freight-car wheel, the drop is about $\frac{1}{1000}''$. In order to test how much the jarring at a joint is due to a gap between the rails, the experiment was tried of cutting shallow notches in the top of an otherwise solid rail and running a locomotive and an inspection car over them. The resulting jarring was practically imperceptible and not comparable to the jar produced at joints. Notwithstanding this fact, many plans have

* Roadmasters Association of America—Reports for 1897.

been tried for avoiding this gap. The most of these plans consist essentially of some form of compound rail, the sections breaking joints. (Of course the design of the compound rail has also several other objects in view.) In Fig. 117 are shown a

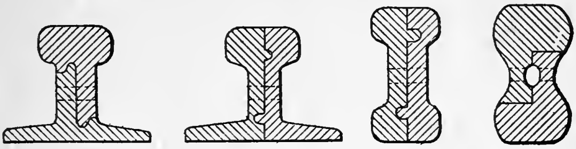


FIG. 117.—COMPOUND RAIL SECTIONS.

few of the very many designs which have been proposed. These designs have invariably been abandoned after trial. Another plan, which has been extensively tried on the Lehigh Valley R. R., is the use of mitered joints. The advantages gained by their use are as yet doubtful, while the added expense is unquestionable. The "Roadmasters Association of America" in 1895 adopted a resolution recommending mitered joints for double track, but their use does not seem to be growing.

240. "Supported," "suspended," and "bridge" joints. In a supported joint the ends of the rails are on a tie. If the angle-plates are short, the joint is entirely supported on one tie; if very long, it may be possible to place three ties under one angle-bar and thus the joint is virtually supported on three ties rather than one. In a suspended joint the ends of the rails are midway between two ties and the joint is supported by the two. There have always been advocates of both methods, but suspended joints are more generally used than supported joints. The opponents of three-tie joints claim that either the middle tie will be too strongly tamped, thus making it a supported joint, or that, if the middle tie is weakest, the joint becomes a very long (and therefore weak) suspended joint between the outer joint-ties, or that possibly one of the outer joint-ties gives way, thus breaking the angle-plate at the joint. Another objection which is urged is that unless the bars are very long (say 44 inches, as used on the Mich. Cent. R. R.) the ties are too close for proper tamping. The best answer to these objections is the successful use of these joints on several heavy-traffic roads

"Bridge"-joints are similar to suspended joints in that the joint is supported on two ties, but there is the important difference that the bridge joint supports the rail from *underneath* and there is no transverse stress in the rail, whereas the suspended joint requires the combined transverse strength of both angle-bars and rail. A serious objection to bridge-joints lies in the fact of their considerable thickness between the rail base and the tie. When joints are placed "staggered" rather than "opposite" (as is now the invariable standard practice), the ties supporting a bridge-joint must either be notched down, thus weakening the tie and promoting decay at the cut, or else the tie must be laid on a slope and the joint and the opposite rail do not get a fair bearing.

241. Failures of rail-joints. It has been observed on double-track roads that the maximum rail wear occurs a few inches beyond the rail gap at the joint in the direction of the traffic. On single-track roads the maximum rail wear is found a few inches *each* side of the joint rather than at the extreme ends of the rail, thus showing that the rail end deflects down under the wheel until (with fast trains especially) the wheel actually jumps the space and strikes the rail a few inches beyond the joint, the impact producing excessive wear. This action, which is called the "drop," is apt to cause the first tie beyond the joint to become depressed, and unless this tie is carefully watched and maintained at its proper level, the stresses in the angle-bar may actually become reversed and the bar may break at the top. The angle-bars of a suspended joint are normally in compression at the top. The mere reversal of the stresses would cause the bars

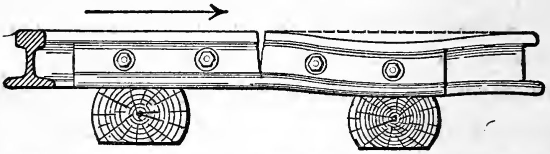


FIG. 118.—EFFECT OF "WHEEL DROP" (EXAGGERATED).

to give way with a less stress than if the stress were always the same in kind. A supported joint, and especially a three-tie joint (see § 240), is apt to be broken in the same manner.

242. Standard angle-bars. An angle-bar must be so made as to closely fit the rails. The great multiplicity in the designs of rails (referred to in Chapter IX) results in nearly as great variety in the detailed dimensions of the angle-bars. The sections here illustrated must be considered only as types of the variable forms necessary for each different shape of rail. The absolutely essential features required for a fit are (1) the angles of the upper and lower surfaces of the bar where they fit against the rail, and (2) the height of the bar. The bolt-holes in the bar and rail must also correspond. The holes in the angle-plates are elongated or made oval, so that the track-bolts, which are

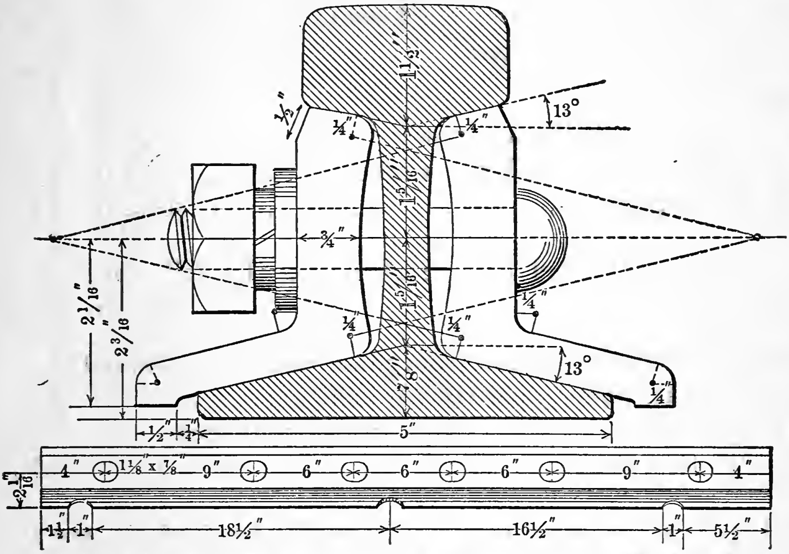


FIG. 119.—STANDARD ANGLE-BAR—80-LB. RAIL. M. C. R.R.

made of corresponding shape immediately under the head, will not be turned by jarring or vibration. The holes in the rails are made of larger diameter (by about $\frac{1}{4}''$) than the bolts, so as to allow the rail to expand with temperature.

243. **Later designs of rail-joints.** In Plate VII are shown various designs which are competing for adoption. The most prominent of these (judging from the discussion in the convention of the Roadmasters Association of America in 1897) are the "Continuous" and the "Weber." Each of them has been very extensively adopted, and where used are universally preferred to angle-plates. Nearly all the later designs embody more or less directly the principle of the bridge-joint, i.e., support the rail from underneath. An experience of several years will be required to demonstrate which form of joint best satisfies the somewhat opposed requirements of minimum cost (both initial and for maintenance) and minimum wear of rails and rolling stock.

243a. Proposed specifications for steel splice-bars. The following specifications for steel splice-bars were proposed in 1900 by Committee No. 1, American Section, International Association for Testing Materials.

1. Steel for splice-bars may be made by the Bessemer or open-hearth process.

2. Steel for splice-bars shall conform to the following limits in chemical composition:

	Per cent.
Carbon shall not exceed.	0.15
Phosphorus shall not exceed.	0.10
Manganese.	0.30 to 0.60

3. Splice-bar steel shall conform to the following physical qualities:

Tensile strength, pounds per square inch.	54000 to 64000
Yield point, pounds per square inch.	32000
Elongation, per cent in eight inches shall not be less than.	25

4. (*a*) A test specimen cut from the head of the splice-bar shall bend 180° flat on itself without fracture on the outside of the bent portion.

(*b*) If preferred the bending test may be made on an unpunched splice-bar, which, if necessary, shall be first flattened and shall then be bent 180° flat on itself without fracture on the outside of the bent portion.

5. A test specimen of 8-inch gauged length, cut from the head of the splice-bar, shall be used to determine the physical properties specified in paragraph No. 3.

6. One tensile specimen shall be taken from the rolled splice-bars of each blow or melt, but in case this develops flaws, or breaks outside of the middle third of its gauged length, it may be discarded and another test specimen submitted therefor.

7. One test specimen cut from the head of the splice-bar shall be taken from a rolled bar of each blow or melt, or if preferred the bending test may be made on an unpunched splice-bar, which, if necessary, shall be flattened before testing. The bending test may be made by pressure or by blows.

8. For the purposes of this specification, the yield point shall

be determined by the careful observation of the drop of the beam or halt in the gauge of the testing machine.

9. In order to determine if the material conforms to the chemical limitations prescribed in paragraph No. 2 herein, analysis shall be made of drillings taken from a small test ingot.

10. All splice-bars shall be smoothly rolled and true to templet. The bars shall be sheared accurately to length and free from fins or cracks, and shall perfectly fit the rails for which they are intended. The punching and notching shall accurately conform in every respect to the drawing and dimensions furnished.

11. The name of the maker and the year of manufacture shall be rolled in raised letters on the side of the splice-bar.

12. The inspector representing the purchaser shall have all reasonable facilities afforded to him by the manufacturer, to satisfy him that the finished material is furnished in accordance with these specifications. All tests and inspections shall be made at the place of manufacture, prior to shipment.

TIE-PLATES.

244. Advantages. (*a*) As already indicated in § 204, the life of a soft-wood tie is very much reduced by "rail-cutting" and "spike-killing," such ties frequently requiring renewal long before any serious decay has set in. It has been practically demonstrated that the "rail-cutting" is not due to the mere pressure of the rail on the tie, even with a maximum load on the rail, but is due to the impact resulting from vibration and to the longitudinal working of the rail. It has been proved that this rail-cutting is practically prevented by the use of tie-plates. (*b*) On curves there is a tendency to overturn the outer rail due to the lateral pressure on the side of the head. This produces a concentrated pressure of the outer edge of the base on the tie which produces rail-cutting and also draws the inner spikes. Formerly the only method of guarding against this was by the use of "rail-braces," one pattern of which is shown in Fig. 120. But it has been found that tie-plates serve the purpose even better, and rail-braces have been abandoned where tie-plates are used. (*c*) Driving spikes through holes in the plate enables the spikes on *each* side of the rail to mutually support each other, no matter in which (lateral) direction the rail may tend to move, and this probably accounts in large measure for the added stability obtained by the use of tie-plates. (*d*) The wear in spikes, called "necking," caused by the vertical vibration of the rail against them, is very greatly reduced. (*e*) The cost is very small compared with the value of the added life of the tie, the large reduction in the work of track main-

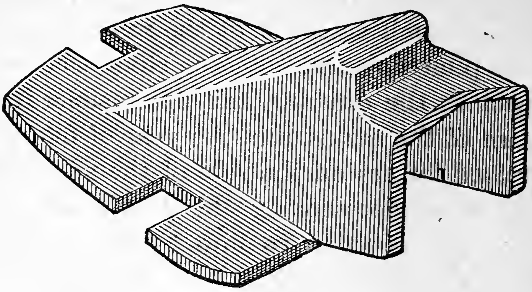
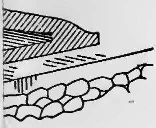


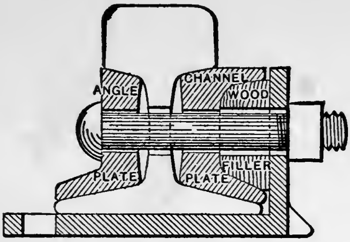
FIG. 120.

tenance, and the smoother running on the better track which is obtained. It has been estimated that by the use of tie-plates the life of hard-wood ties is increased from one to three years, and the life of soft-wood ties is increased from three to six years. From the very nature of the case, the value of tie-plates is greater when they are used to protect soft ties.

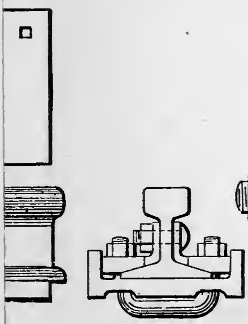
245. Elements of the design. The earliest forms of tie-plates were flat on the bottom, but it was soon found that they would work loose, allow sand and dirt to get between the rail and the plate and also between the plate and the tie, which would cause excessive wear. Such plates are also apt to produce an objectionable rattle Another fault of the earlier designs was the use of plates so thin that they would buckle. The latest designs have flanges or "teeth" formed on the lower surface which penetrate the tie about $\frac{3}{4}''$ to $1\frac{3}{8}''$. Opinion is still divided on the question of whether these teeth should run with the grain or across the grain. If the flanges run with the grain, they generally extend the whole length of the tie-plate—as in the Wolhaupter design. If the grain is to be cut crosswise, several teeth about 1″ wide will be used—as in the Goldie design.

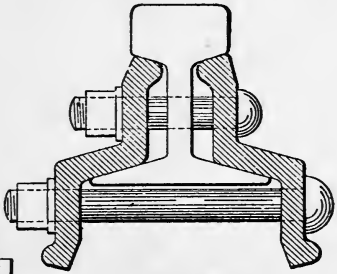
It is a very important feature that the spike-holes should be so punched that the spikes will fit closely to the base of the rail. Otherwise a lateral motion of the rail will be permitted which will defeat one of the main objects of the use of the plate.



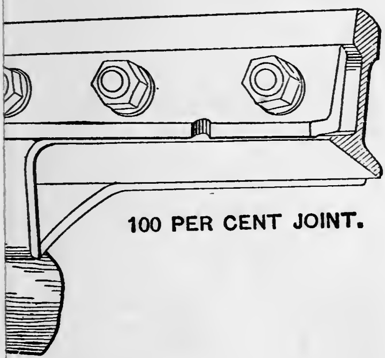


WEBER RAIL JOINT.

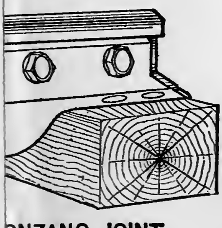




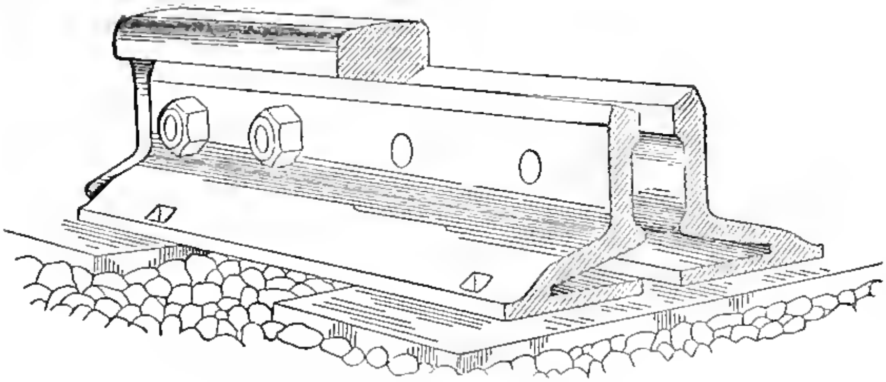
CLOUD JOINT.



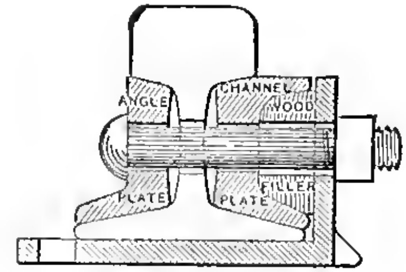
100 PER CENT JOINT.



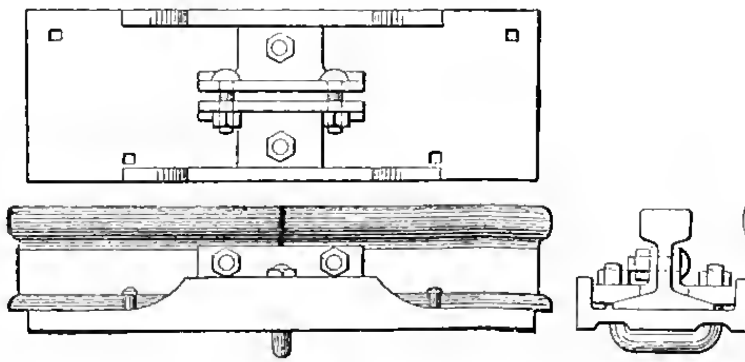
ONZANO JOINT.



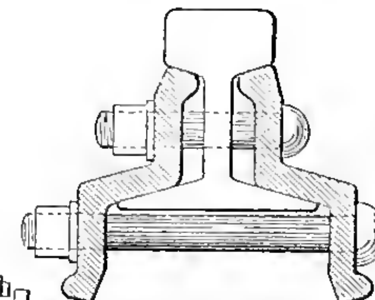
CONTINUOUS RAIL JOINT.



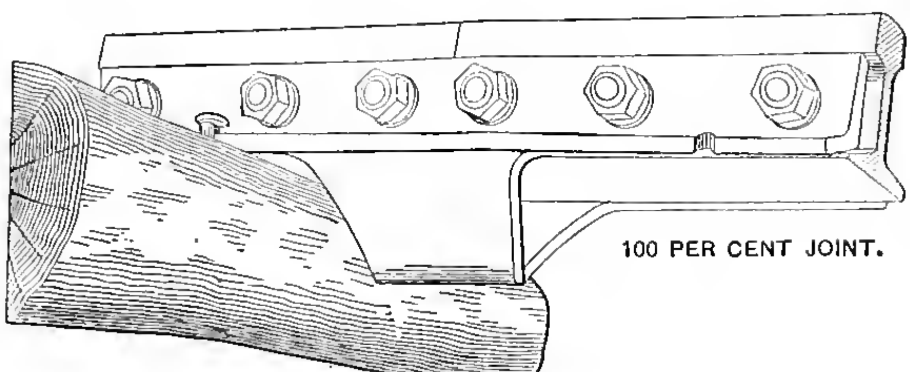
WEBER RAIL JOINT.



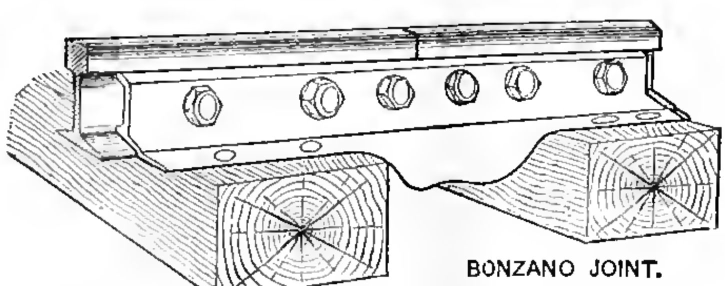
FISHER BRIDGE JOINT.



CLOUD JOINT.



100 PER CENT JOINT.



BONZANO JOINT.

(*To face page* 256.)

CONTINUOUS RAIL JOINT

[illegible] BRIDGE JOINT

Another unsettled detail is the use of "shoulders" on the upper surface. On the one hand it is claimed that the use of shoulders relieves the spikes of side pressure from the rail and prevents "necking." On the other hand it is claimed that if the

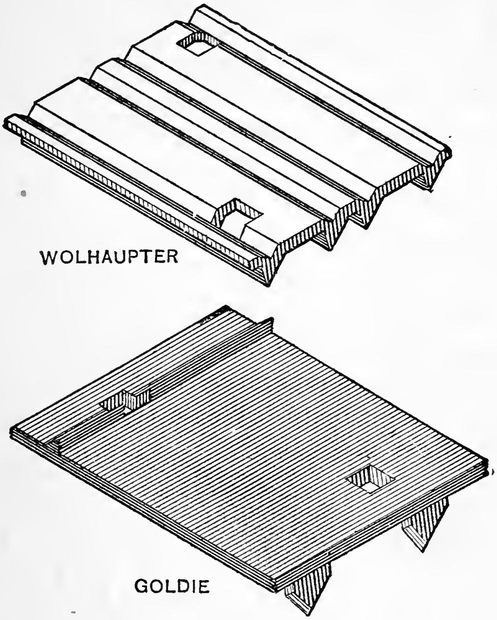


FIG. 121.—TIE-PLATES.

the plain plate is once properly set with new spikes (at least with spikes not already necked) the spikes will not neck appreciably, and that, as the shouldered plates cost more, the additional expenditure is unnecessary.

The above designs should be studied with reference to the manner in which they fulfill the requirements which have been already stated. As in the case of rail-joints, the best forms of tie-plates are of comparatively recent design, and experience with them is still insufficient to determine beyond all question which designs are the best.

246. Method of setting. A very important detail in the process of setting the tie-plates on the ties is that the flanges or teeth should penetrate the tie as far as desired when the plates are first put in position. It requires considerable force to press the teeth into a tie. In a few cases trackmen have depended on the easy process of waiting for passing trains to force the teeth down. Until the teeth are down the spikes cannot be driven home, and this apparently cheap and easy process results in loose spikes and rails. If the trackmen neglect even temporarily to tighten these spikes, it will become impossible to make them tight ultimately. The plates are generally pounded into place with a 10- to 16-pound sledge-hammer. A very good method was adopted once during the construction of a bridge when a pile-driver was at hand. The bridge-ties were placed under the pile-hammer. The plates, accurately set to gauge, were then forced in by a blow from the 3000-lb. hammer falling 2 or 3 feet

SPIKES.

247. Requirements. The rails must be held to the ties by a fastening which will not only give sufficient resistance, but which will retain its capacity for resistance. It must also be cheap and easily applied. The ordinary track-spike fulfills the last requirements, but has comparatively small resisting power, compared with screws or bolts. Worse than all, the tendency to

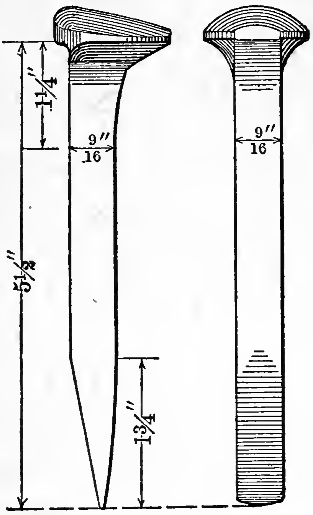


FIG. 122.

FIG. 123.

vertical vibration in the rail produces a series of upward pulls on the spike that soon loosens it. When motion has once begun the capacity for resistance is greatly reduced, and but little more vibration is required to pull the spike out so much that redriving is necessary. Driving the spike to place again in the same hole is of small value except as a very temporary expedient, as its holding power is then very small. Redriving the spikes in new holes very soon "spike-kills" the tie. Many plans have been devised to increase the holding power of spikes, such as making them jagged, twisting the spike, swelling the spike at about the center of its length, etc. But it has been easily demonstrated that the fibers of the wood are generally so crushed and torn by driving such spikes that their holding power is less than that of the plain spike.

The ordinary spike (see Fig. 122) is made with a square cross-section which is uniform through the middle of its length, the lower $1\frac{3}{4}''$ tapering down to a chisel edge, the upper part swelling out to the head. The Goldie spike (see Fig. 123) aims to improve this form by reducing to a minimum the destruction of the fibers. To this end, the sides are made smooth, the edges are clean-cut, and the point, instead of being chisel-shaped, is ground down to a pyramidal form. Such fiber-cutting as occurs is thus accomplished without much crushing, and the fibers are thus pressed away from the spike and slightly downward. Any tendency to draw the spike will therefore cause the fibers to press still harder on the spike and thus increase the resistance.

248. Driving. The holding power of a spike depends largely on how it is driven. If the blows are eccentric and irregular in direction, the hole will be somewhat enlarged and the holding power largely decreased. The spikes on each side of the rail in any one tie should not be directly opposite, but should be staggered Placing them directly opposite will tend to split the tie, or at least decrease the holding power of the spikes. The direction of staggering should be reversed in the two pairs of spikes in any one tie (see Fig. 124). This will tend to prevent any twisting of the tie in the ballast, which would otherwise loosen the rail from the tie.

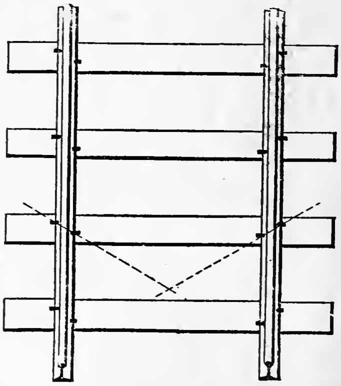


FIG. 124.—SPIKE-DRIVING.

249. Screws and bolts. The use of these abroad is very extensive, but their use in this country has not passed the experimental stage. The screws are "wood"-screws (see Fig. 125), having large square heads, which are screwed down with a track-wrench. Holes, having the same diameter as the *base* of the screw-heads, should be first bored into the tie, at exactly the right position and at the proper angle with the vertical. A light wooden frame is sometimes used to guide the auger at the proper angle. Sometimes the large head of the screw bears directly against the base of the rail, as with the ordinary spike. Other designs employ a plate, made to fit the rail on one side, bearing on the tie on the other side, and through which the screw passes. These screws cost much more than the spikes and require more work to put in place, but their holding power is much greater and the work of track maintenance is very much less. Screw-bolts, passing entirely through the tie, having the head at the bottom of the tie and the nut on the upper side, are also used abroad. These are quite difficult to replace, requiring that the ballast be dug out beneath the tie, but on the other hand the

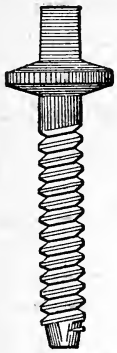


FIG. 125.

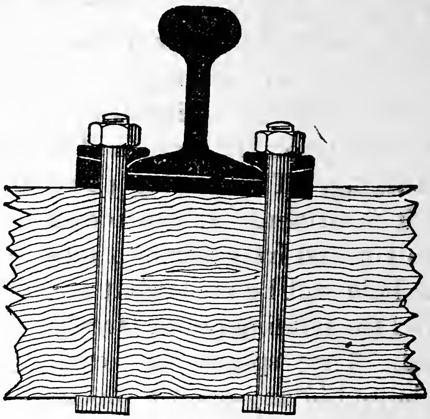


FIG. 126.

occasions for replacing such a bolt are comparatively rare, as their durability is very great. The use of screws or bolts increases the life of the tie by the avoidance of "spike-killing." It is capable of demonstration that the reduced cost of maintenance and the resulting improvement in track would much more than repay the added cost of screws and bolts, but it seems impossible to induce railroad directors to authorize a large and immediate additional expenditure to make an annual saving whose value, although unquestionably considerable, cannot be exactly computed.

250. "Wooden spikes." Among the regulations for track-laying given in § 208, mention was made of wooden "spikes," or plugs, which are used to fill up the holes when spikes are withdrawn. The value of the policy of filling up these holes is unquestionable, since the expense is insignificant compared with the loss due to the quick and certain decay of the tie if these holes are allowed to fill with water and remain so. But the method of making these plugs is variable. On some roads they are "hand-made" by the trackmen out of otherwise useless scraps of lumber, the work being done at odd moments. This policy, while apparently cheap, is not necessarily so, for the hand-made plugs are irregular in size and therefore more or less inefficient. It is also quite probable that if the trackmen are required to make their own plugs, they would spend time on these very cheap articles which could be more profitably employed otherwise. Since the holes made by the spikes are larger at the top than they are near the bottom, the plugs should *not* be of uniform cross-section but should be slightly wedge-shaped. The "Goldie tie-plug" (see Fig. 127) has been designed to fill these requirements. Being machine-made, they are uniform in size; they are of a shape which will best fit the hole; they can be furnished of any desired wood, and at a cost which makes it a wasteful economy to attempt to cut them by hand.

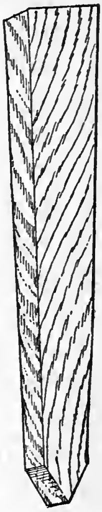


FIG. 127.

TRACK-BOLTS AND NUT-LOCKS.

251. Essential requirements. The track-bolts must have sufficient strength and must be screwed up tight enough to hold the angle-plates against the rail with sufficient force to develop the full transverse strength of the angle-bars. On the other hand the bolts should not be screwed so tight that slipping may not take place when the rail expands or contracts with temperature. It would be impossible to screw the bolts tight enough to prevent slipping during the contraction due to a considerable fall of temperature on a straight track, but when the track is curved, or when expansion takes place, it is conceivable that the resistance of the ties in the ballast to lateral motion may be less than the resistance at the joint. A test to determine this resistance was made by Mr. A. Torrey, chief engineer of the Mich. Cent. R. R., using 80-lb. rails and ordinary angle-bars, the bolts being screwed up as usual. It required a force of about 31000 to

35000 lbs. to start the joint, which would be equivalent to the stress induced by a change of temperature of about 22°. But if the central angle of any given curve is small, a comparatively small lateral component will be sufficient to resist a compression of even 35000 lbs. in the rails. Therefore there will ordinarily be no trouble about having the joints screwed too tight. The vibration caused by the passage of a train reduces the resistance to slipping. This vibration also facilitates an objectionable feature, viz., loosening of the nuts of the track-bolts. The bolt is readily prevented from turning by giving it a form which is *not* circular immediately under the head and making corresponding holes in the angle-plate. Square holes would answer the purpose, except that the square corners in the holes in the angle-plates would increase the danger of fracture of the plates. Therefore the holes (and also the bolts, under the head) are made of an oval form, or perhaps a square form with rounded corners, avoiding angles in the outline.

The nut-locks should be simple and cheap, should have a life at least as long as the bolt, should be effective, and should not lose their effectiveness with age. Many of the designs that have been tried have been failures in one or more of these particulars as will be described in detail below.

252. Design of track-bolts. In Fig. 128 is shown a common design of track-bolt. In its general form this represents the bolt used on nearly all roads, being used not only with the common angle-plates, but also with many of the improved designs of rail-joints. The variations are chiefly a general increase in size to correspond with the increased weight of rails, besides variations in detail dimensions which are frequently unimportant. The diameter is usually $\frac{3}{4}''$ to $\frac{7}{8}''$; 1″ bolts are sometimes used for the heaviest sections of rails. As to length, the bolt should not extend more than $\frac{1}{2}''$ outside of the nut when it is screwed up. If it extends farther than this it is liable to be

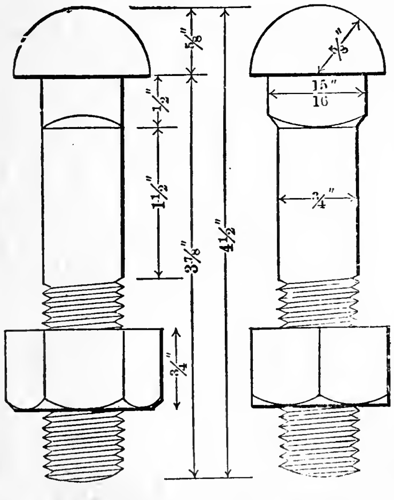


FIG. 128.—TRACK-BOLT.

broken off by a possible derailment at that point. The lengths used vary from $3\frac{1}{4}''$, which may be used with 60-lb. rails, to 5″, which is required with 100-lb. rails. The length required depends somewhat on the type of nut-lock used.

253. Design of nut-locks. The designs for nut-locks may be divided into three classes: (*a*) those depending entirely on an elastic washer which absorbs the vibration which might otherwise induce turning; (*b*) those which jam the threads of the bolt and nut so that, when screwed up, the frictional resistance is too great to be overcome by vibration; (*c*) the "positive" nut-locks—those which mechanically hold the nut from turning. Some of the designs combine these principles to some extent. The "vulcanized fiber" nut-lock is an example of the first class. It consists essentially of a rubber washer which is protected by an iron ring. When first placed this lock is effective, but the rubber soon hardens and loses its elasticity and it is then ineffective and worthless. Another illustration of class (*a*) is the use of wooden blocks, generally 1″ to 2″ oak, which extend the entire length of the angle-bar, a single piece forming the washer for the four or six bolts of a joint. This form is cheap, but the wood soon shrinks, loses its elasticity, or decays so that it soon becomes worthless, and it requires constant adjustment to keep it in even tolerable condition. The "Verona" nut-lock is another illustration of class (*a*) which also combines some of the positive elements of class (*c*). It is made of tempered steel and, as shown in Fig. 129, is warped and has sharp edges or points. The warped form furnishes the element of elastic pressure when the nut is screwed up. The steel being harder than the iron of the angle-bar or of the nut, it bites into them, owing to the great pressure that must exist when the washer is squeezed nearly flat, and thus prevents any *backward* movement, although forward movement (or tightening the bolt) is not interfered with. The "National" nut-lock is a type of the second class (*b*), in which, like the "Harvey" nut-lock, the nut and lock are combined in one piece. With six-bolt angle-bars and 30-foot rails, this means a saving of 2112 pieces on each mile of single track. The "National" nuts are open on one side. The hole is drilled and the thread is cut slightly smaller than the bolt, so that when the nut is screwed up it is forced slightly open and therefore presses on the threads of the bolt with such force that vibration cannot jar it loose. Unlike the "National" nut, the "Harvey" nut is solid, but the form of the thread is progressively varied so that the thread pinches the thread of the bolt and the frictional resistance to turning is too great to be affected by vibration.

The "Jones" nut-lock, belonging to class (c), is a type of a nut-lock that does not depend on elasticity or jamming of screw-threads. It is made of a thin flexible plate, the square part of

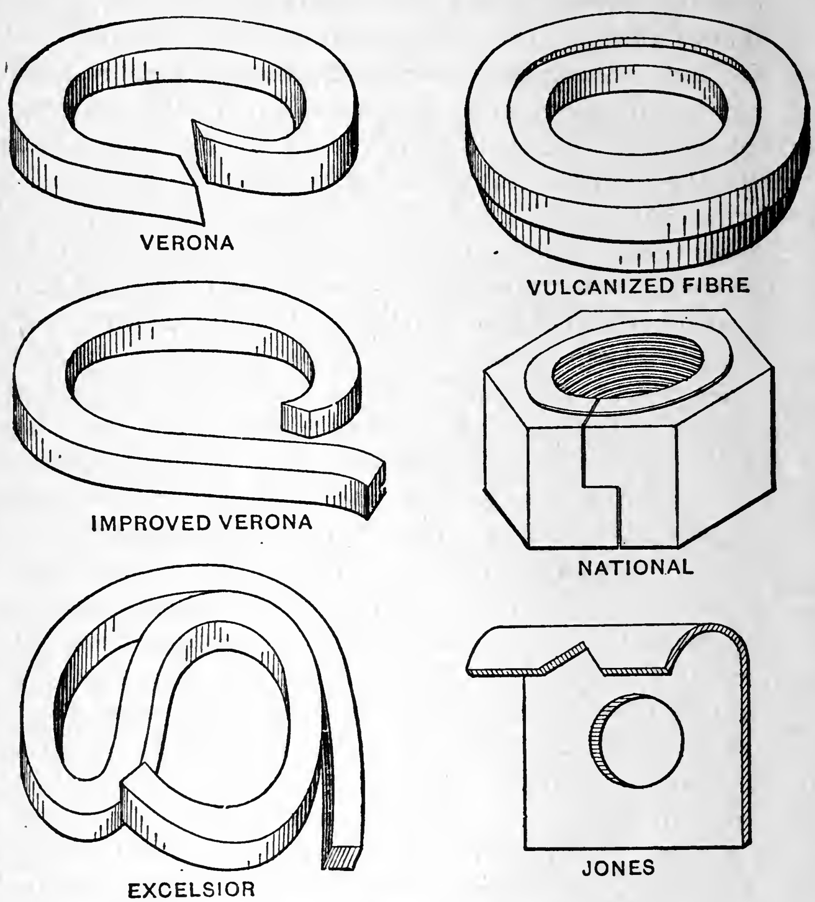


FIG. 129.—TYPES OF NUT-LOCKS.

which is so large that it will not turn after being placed on the bolt. After the nut is screwed up, the thin plate is bent over so that the re-entrant angle of the plate engages the corner of the nut and thus mechanically prevents any turning. The metal is supposed to be sufficiently tough to endure without fracture as many bendings of the plate as will ever be desired. Nut-locks of class (c) are not in common use.

CHAPTER XI.

SWITCHES AND CROSSINGS.

SWITCH CONSTRUCTION.

254. **Essential elements of a switch.** Flanges of some sort are a necessity to prevent car-wheels from running off from the rails on which they may be moving. But the flanges, although a neccssity, are also a source of complication in that they require some special mechanism which will, when desired, guide the wheels out from the controlling influence of the main-line rails. This must either be done by raising the wheels high enough so that the flanges may pass *over* the rails, or by breaking the continuity of the rails in such a way that channels or "flange spaces" are formed *through* the rails. An ordinary stub-switch breaks the continuity of the main-line rails in three places, two of them at the switch-block and one at the frog. The Wharton switch avoids two of these breaks by so placing inclined planes that the wheels, rolling on their flanges, will surmount these inclines until they are a little higher than the rails. Then the wheels on the side toward which the switch runs are guided over and across the main rail on that side This rise being accomplished in a short distance, it becomes impracticable to operate these switches except at slow speeds, as any sudden change in the path of the center of gravity of a car causes very destructive jars both to the switch and to the rolling stock. The other general method makes a break in one main rail (or both) at the switch-block. In both methods the wheels are led to one side by means of the "lead rails," and finally one line of wheels passes *through* the main rail on that side by means of a "frog." There are some designs by which even this break in the main rail is avoided, the wheels being led *over* the main rail by means of a short *movable* rail which is on occasion placed across the main rail, but such designs have not come into general use.

255. **Frogs.** Frogs are provided with two channel-ways or "flange spaces" through which the flanges of the wheels move.

Each channel cuts out a parallelogram from the tread area. Since the wheel-tread is always wider than the rail, the wing rails will support the wheel not only across the space cut out by the channel, but also until the tread has passed the point of the frog and can obtain a broad area of contact on the tongue of the frog. This is the theoretical idea, but it is very imperfectly

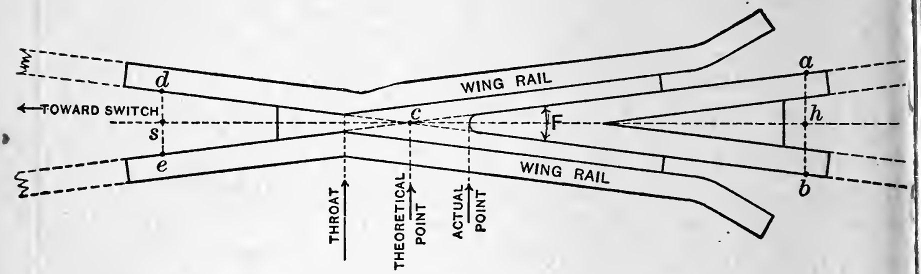


FIG. 130.—DIAGRAMMATIC DESIGN OF FROG.

realized. The wing rails are sometimes subjected to excessive wear owing to "hollow treads" on the wheels—owing also to the frog being so flexible that the point "ducks" when the wheel approaches it. On the other hand the sharp point of the frog will sometimes cause destructive wear on the tread of the wheel. Therefore the tongue of the frog is not carried out to the sharp theoretical point, but is purposely somewhat blunted. But the break which these channels make in the continuity of the tread area becomes extremely objectionable at high speeds, being mutually destructive to the rolling stock and to the frog. The jarring has been materially reduced by the device of "spring frogs"—to be described later. Frogs were originally made of cast iron—then of cast iron with wearing parts of cast steel, which were fitted into suitable notches in the cast iron. This form proved extremely heavy and devoid of that elasticity of track which is necessary for the safety of rolling stock and track at high speeds. The present universal practice is to build the frog up of pieces of rails which are cut or bent as required. These pieces of rails (at least four) are sometimes assembled by riveting them to a flat plate, but this method is now but little used, except for very light work. The usual practice is now chiefly divided between "bolted" and "keyed" frogs. In each case the space between the rails, except a sufficient flange-way, is filled with a cast-iron filler and the whole assemblage of parts

[illegible]

is suitably bolted or clamped together, as is illustrated in Plate VIII. The operation of a spring-rail frog is evident from the figure. Since a siding is usually operated at slow speed, while the main track may be operated at fast speed, a spring-rail frog will be so set that the tread is continuous for the main track and broken for the siding. This also means that the spring-rail will only be moved by trains moving at a (presumably) slow speed on to the siding. For the fast trains on the main line such a frog is substantially a "fixed" frog and has a tread which is practically continuous.

256. To find the frog number. The frog number (n) equals the ratio of the distance of any part on the tongue of the frog from the theoretical point of the frog divided by the width of the tongue at that point, i.e. $=hc \div ab$ (Fig. 130). This value may be directly measured by applying any convenient unit of measure (even a knife, a short pencil, etc.) to some point of the tongue where the width just equals the unit of measure, and then noting how many times the unit of measure is contained in the distance from that place to the theoretical point. But since c, the theoretical point, is not so readily determinable with exactitude, it being the imaginary intersection of the gauge lines, it may be more accurate to measure de, ab, and hs; then n, the frog number, $=hs \div (ab+de)$. If the frog angle be called F, then

$$n = hc \div ab = hs \div (ab+de) = \tfrac{1}{2} \cot \tfrac{1}{2}F;$$

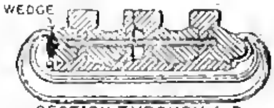
i.e., $$\cot \tfrac{1}{2}F = 2n.$$

257. Stub switches. The use of these, although once nearly universal, has been practically abandoned as turnouts from *main track* except for the poorest and cheapest roads. In some States their use on main track is prohibited by law. They have the sole merit of cheapness with adaptability to the circumstances of very light traffic operated at slow speed when a considerable element of danger may be tolerated for the sake of economy. The rails from A to B (see Fig. 131*) are not fastened

* The student should at once appreciate that in Fig. 131, as well as in nearly all the remaining figures in this chapter, it becomes necessary to use excessively large frog angles, short radii, and a very wide gauge in order to illustrate the desired principles with figures which are sufficiently small for the page. In fact, the proportions used in the figures are such that serious mechanical difficulties would be encountered if they were used. These difficulties are here ignored because they can be neglected in the proportions used in practice.

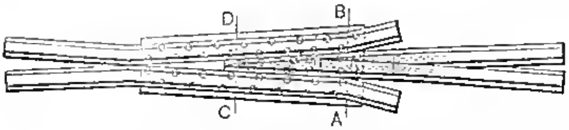
PLATE VIII.

ELLIOT STEEL CLAMP FROG.



SECTION THROUGH A-B

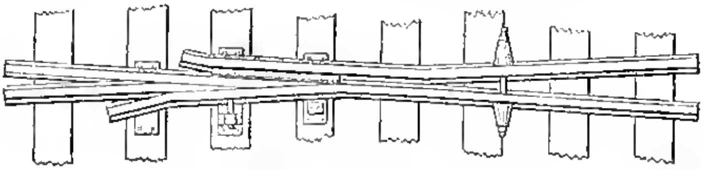
WEIR BOLTED STIFF FROG.



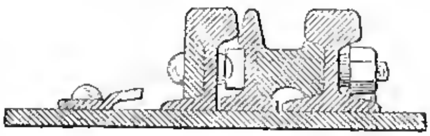
SECTION THROUGH C-D.

SECTION THROUGH A-B.

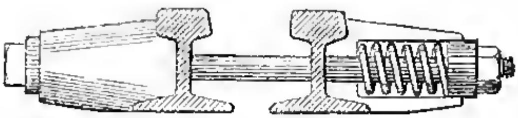
ELLIOT PLATE RIVETED FROG.



WEIR SPRING-RAIL FROG.



SECTION THROUGH PLATE AT POINT.



SECTION THROUGH SPRING-HOUSING.

(To face page 267.)

to the ties; they are fastened to each other by tie-rods which keep them at the proper gauge; at and back of B they are

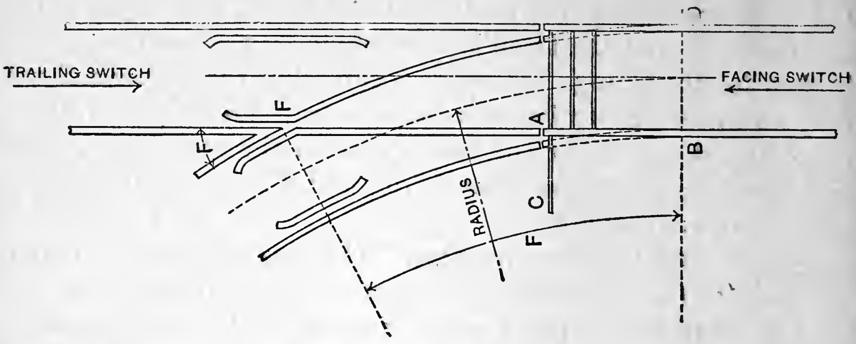


FIG. 131.—STUB SWITCH.

securely spiked to the ties, and at A they are kept in place by the connecting bar (C) fastened to the switch-stand. One great objection to the switch is that, in its usual form, when operated as a trailing switch, a derailment is inevitable if the switch is misplaced. The very least damage resulting from such a derailment must include the bending or breaking of the tie-rods of the switch-rail. Several devices have been invented to obviate this objection, some of which succeed very well mechanically, although their added cost precludes any economy in the total cost of the switch. Another objection to the switch is the looseness of construction which makes the switches objectionable at high speeds. The gap of the rails at the head-block is always considerable, and is sometimes as much as two inches. A driving-

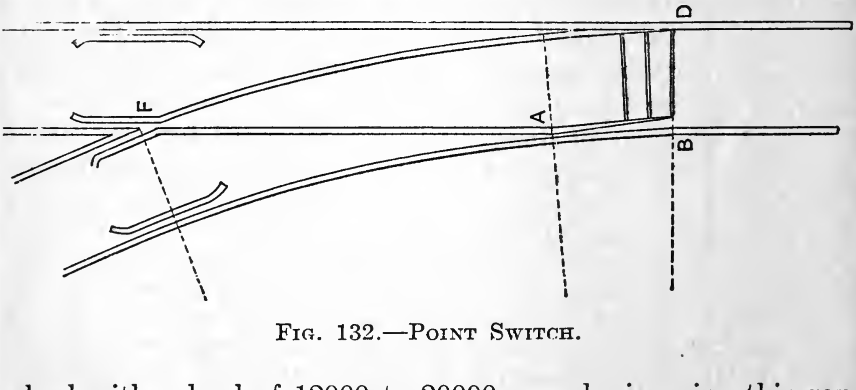


FIG. 132.—POINT SWITCH.

wheel with a load of 12000 to 20000 pounds, jumping this gap with any considerable velocity, will do immense damage to the farther rail end, besides producing such a stress in the construction that a breakage is rendered quite likely, and such a breakage might have very serious consequences.

258. Point switches. The essential principle of a point switch is illustrated in Fig. 132. As is shown, one main rail and also one of the switch-rails is unbroken and immovable. The other main rail (from A to F) and the corresponding portion of the other lead rail are substantially the same as in a stub switch. A portion of the main rail (AB) and an equal length of the opposite lead rail (usually 15 to 24 feet long) are fastened together by tie-rods. The end at A is jointed as usual and the other end is pointed, both sides being trimmed down so that the feather edge at B includes the web of the rail. In order to retain in it as much strength as possible, the point-rail is raised so that it rests on the base of the stock-rail, one side of the base of the point-rail being entirely cut away. As may be seen in Fig. 133, although the influence of the point of the rail in moving the wheel-flange away from the stock-rail is really zero at that point, yet the rail has all the strength of the web and about one-half that of the base—a very fair angle-iron. The planing runs back in *straight* lines, until at about six or seven feet back from the point the full width of the head is obtained. The full width of the base will only be obtained at about 13 feet from the point. An 80-lb. rail is 5 inches wide at the base. Allowing $\frac{3}{4}''$ more for a spike between the rails, this gives $5\frac{3}{4}''$ as the minimum width between rail centers at the joint. The minimum angle of the switch-point (using a 15-foot point-rail) is therefore the angle whose tangent is $\frac{5.75}{15\times 12} =$ .03914, which is the tangent of 1° 50′. Switch-rails are sometimes used with a length of 24 feet, which reduces the angle of the switch-point to 1° 09′.

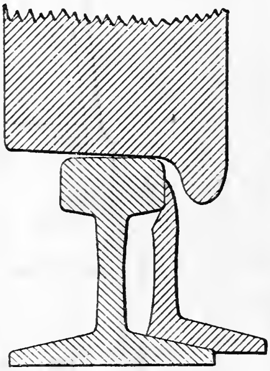


FIG. 133.

---

259. Switch-stands. The simplest and cheapest form is the "ground lever," which has no target. The radius of the circle described by the connecting-rod pin is precisely one-half the throw. From the nature of the motion the device is practically self-locking in either position, padlocks being only used to prevent malicious tampering. The numerous designs of upright stands are always combined with targets, one design of which is

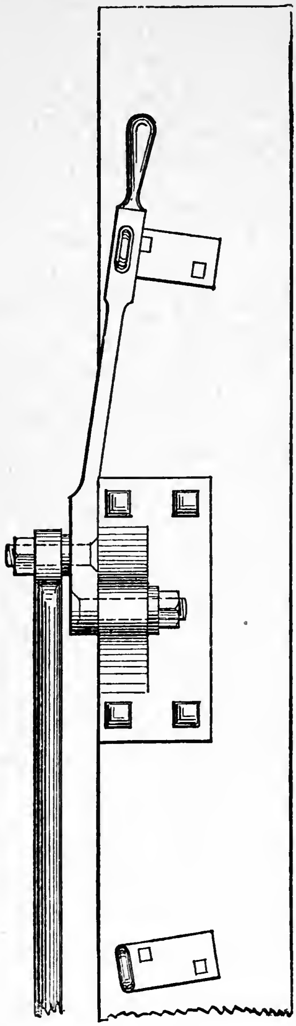


FIG. 134.—GROUND LEVER FOR THROWING A SWITCH.

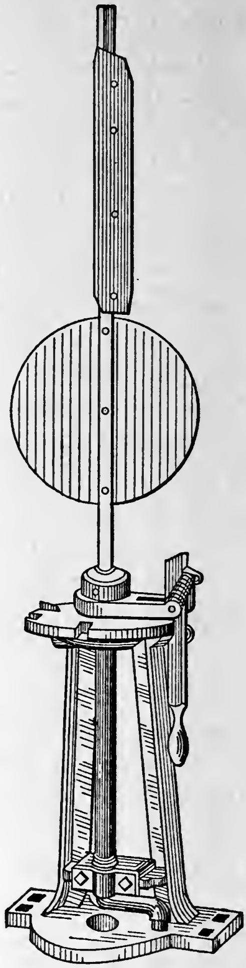


FIG. 135.

illustrated in Fig. 135. When the road is equipped with interlocking signals, the switch-throw mechanism forms a part of the design

260. Tie-rods. These are fastened to the webs of the rails by means of lugs which are bolted on, there being usually a hinge-joint between the rod and the lug. Four such tie-rods are generally necessary. The first rod is sometimes made without hinges, which gives additional stiffness to the comparatively weak rail-points. The old-fashioned tie-rod, having jaws fitting the base of the rail, was almost universally used in the days of stub switches. One great inconvenience in their use lies in the fact that they must be slipped on, one by one, over the *free* ends of the switch-rails. Sometimes the lugs are fastened to the rail-webs by rivets instead of bolts.

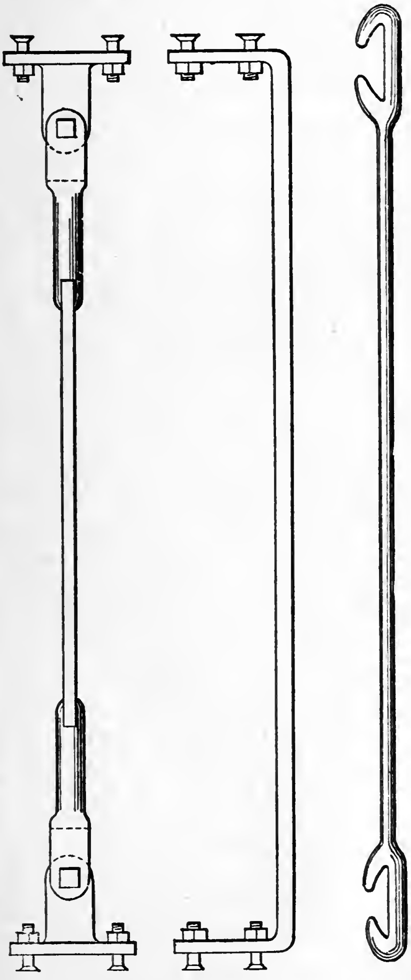


FIG. 136.—FORMS OF TIE-RODS.

261. Guard-rails. As shown in Figs. 131 and 132, guard-rails are used on both the main and switch tracks opposite the frog-point. Their function is not only to prevent the possibility of the wheel-flanges passing on the wrong side of the frog-point, but also to save the side of the frog-tongue from excessive wear. The necessity for their use may be realized by noting the apparent wear usually found on the side of the head of the guard-rail. The flange-way space between the heads of the guard-rail and wheel-rail therefore becomes a definite quantity and should equal about two inches. Since this is less than the space between the heads of ordinary (say 80-pound) rails when placed base to base, to say nothing of the $\frac{3}{4}''$ necessary for spikes, it becomes necessary to cut the flange of the guard-rail. The length of the rail is made from 10 to 15 feet, the ends being bent as shown in Fig. 132, so as to prevent the possibility of the end of the rail being struck by a wheel-flange.

MATHEMATICAL DESIGN OF SWITCHES.

In all of the following demonstrations regarding switches, turnouts, and crossovers, the lines are assumed to represent the *gauge-lines*—i.e., the lines of the *inside* of the head of the rails.

262. Design with circular lead-rails. The simplest method is to consider that the lead-rails curve out from the main track-rails by arcs of circles which are tangent to the main rails and which extend to the frog-point F. The simple curve from D to F is of such radius that $(r+\frac{1}{2}g)$ vers $F = g$, in which F = the frog angle, g = gauge, L = the "lead" (BF), and r = the radius of the *center* of the switch-rails.

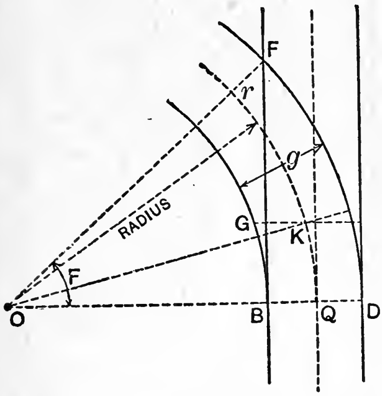


FIG. 137.

$$\therefore\ r+\tfrac{1}{2}g=\frac{g}{\text{vers } F}. \qquad (74)$$

Also, $BF \div BD = \cot \frac{1}{2}F$; $BD = g$; $BF = L$.

$$\therefore\ L = g \cot \tfrac{1}{2}F. \qquad (75)$$

Also, $$L = (r+\tfrac{1}{2}g) \sin F; \qquad (76)$$

$$QT = 2r \sin \tfrac{1}{2}F. \qquad (77)$$

These formulæ involve the angle F. As shown in Table III, the angles (F) are always odd quantities, and their trigonometric functions are somewhat troublesome to obtain closely with ordinary tables. The formulæ may be simplified by substituting the frog-number n, from the relation that $n=\frac{1}{2}\cot\frac{1}{2}F$. Since

$$r-\tfrac{1}{2}g=L\cot F \quad \text{and} \quad r+\tfrac{1}{2}g=L\operatorname{cosec} F,$$

then
$$\begin{aligned} r&=\tfrac{1}{2}L(\cot F+\operatorname{cosec} F)\\ &=\tfrac{1}{2}g\cot\tfrac{1}{2}F(\cot F+\operatorname{cosec} F)\\ &=\tfrac{1}{2}g\cot^2\tfrac{1}{2}F, \quad \text{since } (\cot a+\operatorname{cosec} a)=\cot\tfrac{1}{2}a\\ &=2gn^2. \qquad (78)\end{aligned}$$

Also, $L=2gn$, ... (79)

from which $r=n\times L$. . . . . (80)

These extremely simple relations may obviate altogether the necessity for tables, since they involve only the frog-number and the gauge. On account of the great simplicity of these rules, they are frequently used as they are, regardless of the fact that the curve is never a uniform simple curve from switch-block to frog. In the first place there is a considerable length of the gauge-line within the frog, which is straight unless it is purposely curved to the proper curve while being manufactured, which is seldom if ever done—except for the very large-angled frogs used for street-railway work, etc. It is also doubtful whether the switch-rails (BA, Fig. 131) are bent to the computed curve when the rails are set for the switch. The switch-rails of point switches are *straight*, thus introducing a stretch of straight track which is about one-fifth of the total length of the lead-rails. The effect of these modifications on the length and radius of the lead-rails will be developed and discussed in the next four sections.

The throw (t) of a stub switch depends on the weight of the rail, or rather on the width of its base. The throw must be at least $\frac{3}{4}''$ more than that width. The head-block should therefore be placed at such a distance from the heel of the switch (B) that the versed sine of the arc equals the throw. These points *must* be opposite on the two rails, but the points on the two rails where these relations are exactly true will not be opposite. Therefore, instead of considering either of the two radii $(r+\frac{1}{2}g)$ and $(r-\frac{1}{2}g)$, the mean radius r is used. Then (see Fig. 137)

$$\operatorname{vers} KOQ=t\div r,$$

and the length of the switch-rails is

$$QK=r\sin KOQ. \qquad (81)$$

These relations develop another disadvantage in the use of a stub switch. The required value of BG, using a No. 10 frog and 80-pound rail, is 30.1 feet—slightly more than a full rail length. It would be unsafe to leave so much of the track unspiked from the ties. Whether this is obviated by spiking down a portion of the switch-rails (virtually shortening the lead) or by moving the switch-block nearer the heel of the switch (shortening the switch-rails), but still maintaining the required throw, the theoretical accuracy of the curve is hopelessly lost.

263. Effect of straight frog-rails. A portion of the ends of the rails of a frog are free and *may* be bent to conform to the switch-rail curve, but there is a considerable portion which is fitted to the cast-iron filler, and this portion is always straight. Call the length of this straight portion back from the frog-point f ($=FH$, Fig. 138). Then we have

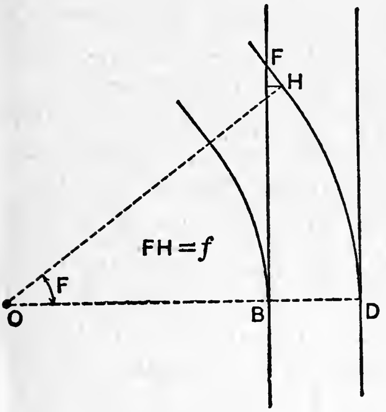


FIG. 138.

$$r+\tfrac{1}{2}g=(g-f\sin F)\div\operatorname{vers} F$$
$$=\frac{g}{\operatorname{vers} F}-f\cot\tfrac{1}{2}F$$
$$=\frac{g}{\operatorname{vers} F}-2fn. \quad . \quad (82)$$

$$BF=L=(g-f\sin F)\cot\tfrac{1}{2}F+f\cos F$$
$$=2gn-f\sin F\cot\tfrac{1}{2}F+f\cos F$$
$$=2gn-f(1+\cos F)+f\cos F$$
$$=2gn-f. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

Since $r-\frac{1}{2}g=(L-f\sec F)\cot F$, and

$$r+\tfrac{1}{2}g=(L-f\cos F)\operatorname{cosec} F,$$

$$r=\tfrac{1}{2}L(\cot F+\operatorname{cosec} F)-\tfrac{1}{2}f\sec F\cot F-\tfrac{1}{2}f\cos F\operatorname{cosec} F$$

$$=Ln-\tfrac{1}{2}f\left(\frac{1+\cos f}{\sin f}\right).$$

$$r=Ln-\tfrac{1}{2}f\cot\tfrac{1}{2}F$$
$$=Ln-fn. \quad \text{Then from (83)}$$
$$r=2gn^2-2fn. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

264. Effect of straight point-rails. The "point switches," now so generally used, have *straight* switch-rails. This requires an *angle* in the alignment rather than turning off by a tangential curve. The angle is, however, very small (between 1° and 2°), and the disadvantages of this angle are small compared with the very great advantages of the device.

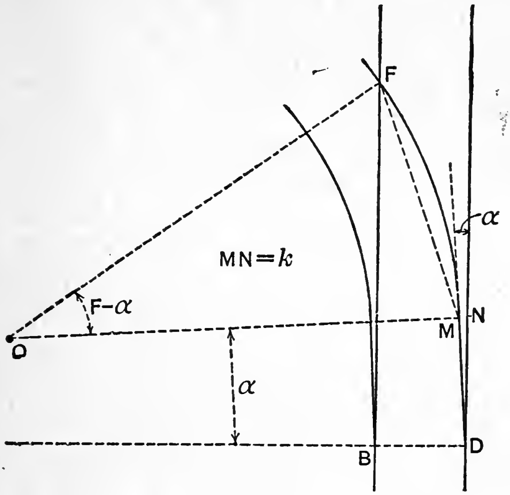


Fig. 139.

$$FM = \frac{g-k}{\sin \frac{1}{2}(F+a)};$$

$$r + \tfrac{1}{2}g = \frac{FM}{2 \sin \frac{1}{2}(F-a)}$$

$$= \frac{g-k}{2 \sin \frac{1}{2}(F+a) \sin \frac{1}{2}(F-a)}$$

$$= \frac{g-k}{\cos a - \cos F}. \qquad (85)$$

$$BF = L = FM \cos \tfrac{1}{2}(F+a) + DN$$
$$= (g-k) \cot \tfrac{1}{2}(F+a) + DN. \qquad (86)$$

265. **Combined effect of straight frog-rails and straight point-rails.** It becomes necessary in this case to find a curve which shall be tangent to both the point-rail and the frog-rail. The curve therefore begins at M, its tangent making an angle of a (usually 1° 50′) with the main rail, and runs to H. The central

angle of the curve is therefore $(F-a)$. The angle of the chord HM with the main rails is therefore

$$\tfrac{1}{2}(F-a)+a=\tfrac{1}{2}(F+a);$$

$$HM=\frac{g-f\sin F-k}{\sin\frac{1}{2}(F+a)};$$

$$r+\tfrac{1}{2}g=\frac{HM}{2\sin\frac{1}{2}(F-a)}$$

$$=\frac{g-f\sin F-k}{2\sin\frac{1}{2}(F+a)\sin\frac{1}{2}(F-a)}$$

$$=\frac{g-f\sin F-k}{\cos a-\cos F}; \quad\quad (87)$$

$$ST=2r\sin\tfrac{1}{2}(F-a). \quad\quad (88)$$

$$BF=L=HM\cos\tfrac{1}{2}(F+a)+f\cos F+DN$$
$$=(g-f\sin F-k)\cot\tfrac{1}{2}(F+a)+f\cos F+DN. \quad\quad (89)$$

It may be more simple, if $(r+\frac{1}{2}g)$ has already been computed, to write

$$L=2(r+\tfrac{1}{2}g)\sin\tfrac{1}{2}(F-a)\cos\tfrac{1}{2}(F+a)+f\cos F+DN$$
$$=(r+\tfrac{1}{2}g)(\sin F-\sin a)+f\cos F+DN. \quad\quad (90)$$

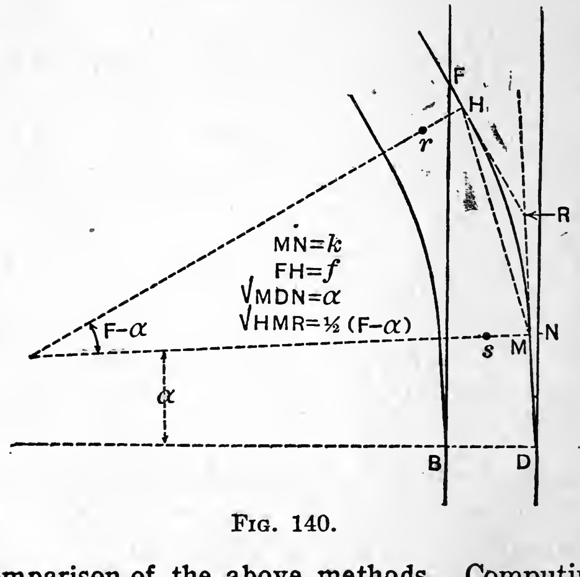


FIG. 140.

266. Comparison of the above methods. Computing values for r and L by the various methods, on the uniform basis of a

No. 9 frog, standard gauge 4′ 8½″, $f=3'.37$, $k=5\frac{3}{4}''=0'.479$, $DN=15'\ 0''$, and $a=1°\ 50'$, we may tabulate the comparative results:

	§ 262. Simple circle. Curved frog-rail. Curved switch-rail.	§ 263. Straight frog-rail. Curved switch-rail.	§ 264. Curved frog-rail. Straight switch-rail.	§ 265. Straight frog-rail. Straight switch-rail.
r	762.75	702.00	747.48	681.16
Deg. of curve	7° 31′	8° 10′	7° 40′	8° 25′
L	84.75	81.37	74.00	72.13

This shows that the effect of using straight frog-rails and straight switch-rails is to sharpen the curve and shorten the lead in each case separately, and that the combined effect is still greater. The effect of the straight switch-rails is especially marked in reducing the length of lead, and therefore Eq. 78 to 80, although having the advantage of extreme simplicity, cannot be used for point-switches without material error. The effect of the straight frog-rail is less, and since it can be materially reduced by bending the free end of the frog-rails, the influence of this feature is frequently ignored, the frog-rails are assumed to be curved, and Eq. 85 and 86 are used. (See § 276 for a further discussion of this point.)

267. Dimensions for a turnout from the OUTER side of a curved

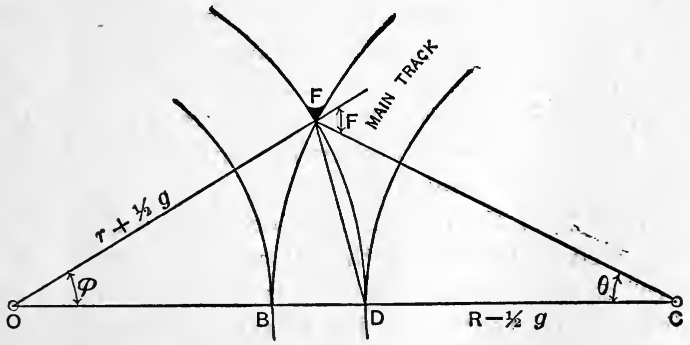


FIG. 141.

track. In this demonstration the switch-rails will be considered as uniformly circular from the switch-points to the frog-point.

In the triangle FCD (Fig. 141) we have

$(FC+CD):(FC-CD)::\tan\frac{1}{2}(FDC+DFC):\tan\frac{1}{2}(FDC-DFC)$;

but $\qquad \frac{1}{2}(FDC+DFC)=90°-\frac{1}{2}\theta$

and $\qquad \frac{1}{2}(FDC-DFC)=\frac{1}{2}F.$

Also, $\qquad FC+CD=2R \quad$ and $\quad FC-CD=g$;

$$\therefore \quad 2R : g :: \cot\tfrac{1}{2}\theta : \tan\tfrac{1}{2}F$$

$$:: \cot\tfrac{1}{2}F : \tan\tfrac{1}{2}\theta;$$

$$\therefore \quad \tan\tfrac{1}{2}\theta=\frac{gn}{R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

Also, $\qquad OF : FC :: \sin\theta : \sin\phi; \qquad$ but $\quad \phi=(F-\theta)$;

then $$r+\tfrac{1}{2}g=(R+\tfrac{1}{2}g)\frac{\sin\theta}{\sin(F-\theta)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

$$BF=L=2(R+\tfrac{1}{2}g)\sin\tfrac{1}{2}\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (93)$$

If the curvature of the main track is very sharp or the frog angle unusually small, F may be less than θ; in which case the center O will be on the same side of the main track as C. Eq. 92 will become (by calling $r=-r$ and changing the signs)

$$(r-\tfrac{1}{2}g)=(R+\tfrac{1}{2}g)\frac{\sin\theta}{\sin(\theta-F)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (94)$$

If we call d the degree of curve corresponding to the radius r, and D the degree of curve corresponding to the radius R, also d' the degree of curve of a turnout from a straight track (the frog angle F being the same), it may be shown that $d=d'-D$ (very nearly). To illustrate we will take three cases, a number 6 frog (very blunt), a number 9 frog (very commonly used), and a number 12 frog (unusually sharp). Suppose $D=4°\ 0'$; also $D=10°\ 0'$; $g=4'\ 8\frac{1}{2}''=4'.708$.

A brief study of the tabular form on p. 279 will show that the error involved in the use of the approximate rule for ordinary curves (4° or less) and for the usual frogs (about No. 9) is really insignificant, and that, even for sharper curves (10° or more), or for very blunt frogs, the error would never cause damage, considering the lower probable speed. In the most unfavorable case noted above the change in radius is about 1%. On account of the closeness of the approximation the method is frequently used. The remarkable agreement of the computed values of L

Frog number.	$D=4°$.				"L" for *straight* track.
	d	$d'-D$	Error.	L	
6	12° 54′ 20″	12° 57′ 52″	0° 03′ 32″	56.57	56.50
9	3 30 27	3 31 04	0 0 37	84.85	84.75
12	0 13 33	0 13 36	0 0 03	112.72	113.00

Frog number.	$D=10°$.				"L" for *straight* track.
	d	$d'-D$	Error.	L	
6	6° 53′ 24″	6° 57′ 52″	0° 04′ 28″	56.66	56.50
9	2 27 54	2 28 56	0 01 02	84.86	84.75
12	5 44 26	5 46 24	0 01 58	112.91	113.00

with the corresponding values for a straight main track (the lead rails circular throughout) shows that the error is insignificant in using the more easily computed values.

268. Dimensions for a turnout from the INNER side of a curved track. (Lead rails circular throughout.) From Fig. 142 we have, from the triangle DFC,

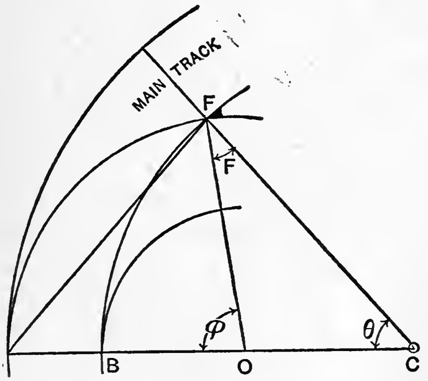


FIG. 142.

$$DF+FC:DF-FC::\tan\tfrac{1}{2}(DFC+FDC):\tan\tfrac{1}{2}(DFC-FDC);$$

but $\qquad \frac{1}{2}(DFC+FDC)=90°-\frac{1}{2}\theta$

and $\qquad \frac{1}{2}(DFC-FDC)=\frac{1}{2}F;$

$$\therefore\quad 2R:g::\cot\tfrac{1}{2}\theta:\tan\tfrac{1}{2}F$$

$$::\cot\tfrac{1}{2}F\tan\tfrac{1}{2}\theta;$$

$$\therefore\quad \tan\tfrac{1}{2}\theta=\frac{gn}{R}. \qquad (95)$$

From OFC, $OF : FC :: \sin\theta : (F+\theta)$.

$$(r+\tfrac{1}{2}g)=(R-\tfrac{1}{2}g)\frac{\sin\theta}{\sin(F+\theta)}. \quad . \quad . \quad . \quad . \quad . \quad (96)$$

$$L=BF=2(R-\tfrac{1}{2}g)\sin\tfrac{1}{2}\theta. \quad . \quad . \quad . \quad (97)$$

As in § 267, it may be readily shown that the degree of the turnout (d) is *nearly* the *sum* of the degree of the main track (D) and the degree (d') of a turnout from a straight track when the frog angle is the same. The discrepancy in this case is somewhat greater than in the other, especially when the curvature of the main track is sharp. If the frog angle is also large, the curvature of the turnout is excessively sharp. If the frog angle is very small, the liability to derailment is great. Turnouts to the inside of a curved track should therefore be avoided, unless the curvature of the main track is small.

269. Double turnout from a straight track. In Fig. 143 the frogs F_l and F_r are generally made equal. Then, if there are

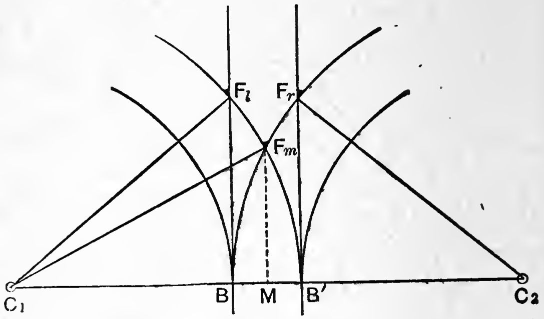


FIG. 143.

uniform curves from B' to F_l and from B to F_r, the required value of F_m is obtained from

$$\text{vers}\ \tfrac{1}{2}F_m=\frac{g}{2(r+\tfrac{1}{2}g)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

r being found from Eq. 78, in which n is the frog number of F_l or F_r.

$$MF_m=r\tan\tfrac{1}{2}F_m;$$

but since $n_m=\tfrac{1}{2}\cot\tfrac{1}{2}F_m$,

$$MF_m=\frac{r}{2n_m}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (99)$$

Since $\text{vers } F_l = \frac{g}{(r+\frac{1}{2}g)}$,

$$\text{vers } \tfrac{1}{2}F_m = \tfrac{1}{2} \text{ vers } F_l. \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

Also, since $(C_1F_m)^2 = (MF_m)^2 + (C_1M)^2$, we have

$$(r+\tfrac{1}{2}g)^2 = \left(\frac{r}{2n_m}\right)^2 + r^2;$$

$$r^2 + rg + \tfrac{1}{4}g^2 = \frac{r^2}{4n_m{}^2} + r^2.$$

Simplifying and substituting, $r = 2gn^2$, we have

$$2g^2n^2 + \tfrac{1}{4}g^2 = \frac{4g^2n^4}{4n_m{}^2};$$

$$n_m{}^2 = \frac{n^4}{2n^2 + \frac{1}{4}}.$$

Dropping the $\frac{1}{4}$, which is always insignificant in comparison with $2n^2$, we have

$$n_m = \frac{n}{\sqrt{2}} = n \times .707 \text{(approx.)}. \quad . \quad . \quad . \quad . \quad . \quad (101)$$

Frogs are usually made with angles corresponding to integral values of n, or sometimes in "half" sizes, *e.g.* 6, $6\frac{1}{2}$, 7, $7\frac{1}{2}$, etc. If No. $8\frac{1}{2}$ frogs are used for F_l and F_r, the exact frog number for F_m is 6.01. This is so nearly 6 that a No. 6 frog may be used without sensible inaccuracy. Numbers 7 and 10 are a less perfect combination. If sharp frogs must be used, $8\frac{1}{2}$ and 12 form a very good combination.

If it becomes necessary to use other frogs because the right combination is unobtainable, it may be done by compounding the curve at the middle frog. F_l and F_r should be greater than $\frac{1}{2}F_m$. If equal to $\frac{1}{2}F_m$, the rails would be straight from the middle frog to the outer frogs. In Fig. 144, $\theta_1 = F_l - \frac{1}{2}F_m$.

Drawing the *chord* $\overline{F_lF_m}$,

$$\overline{KF_lF_m} = F_l - \tfrac{1}{2}\theta_1 = F_l - \tfrac{1}{2}F_l + \tfrac{1}{4}F_m = \tfrac{1}{2}(F_l + \tfrac{1}{2}F_m);$$

$$\overline{F_lF_m} = \frac{\overline{KF_m}}{\sin \overline{KF_lF_m}} = \frac{g}{2 \sin \frac{1}{2}(F_l + \frac{1}{2}F_m)}; \quad . \quad . \quad (102)$$

$$\overline{KF_l} = \overline{KF_m} \cot \overline{KF_lF_m} = \tfrac{1}{2}g \cot \tfrac{1}{2}(F_l + \tfrac{1}{2}F_m); \quad . \quad (103)$$

$$(r_1 + \tfrac{1}{2}g) = \frac{\overline{F_lF_m}}{2 \sin \frac{1}{2}\theta} = \frac{g}{4 \sin \frac{1}{2}(F_l + \frac{1}{2}F_m) \sin \frac{1}{2}(F_l - \frac{1}{2}F_m)}$$

$$= \frac{\frac{1}{2}g}{\cos \frac{1}{2}F_m - \cos F_l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (104)$$

If three frogs, all different, *must* be used, the largest may be selected as F_m; the radius of the lead rails may be found by an inversion of Eq. 98; F_m may be located in the center of the tracks by Eq. 99; then each of the smaller frogs may be located

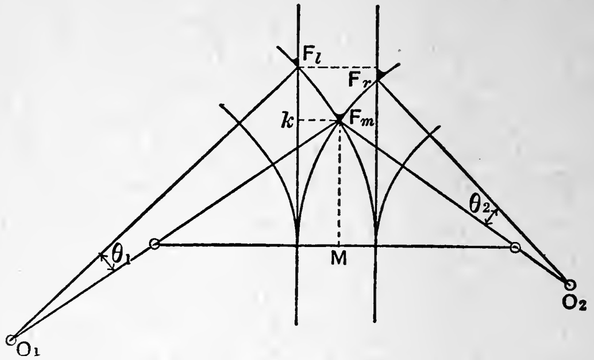


FIG. 144.

by separate applications of Eq. 102 or 103, the radius being determined by Eq. 104.

270. Two turnouts on the same side. In Fig. 145, let O_1 bisect O_2D. Then $(r_1+\frac{1}{2}g)=\frac{1}{2}(r_2+\frac{1}{2}g)$; also, $O_1O_2=O_1F_l$ and $F_r=F_l$.

$$\text{vers } F_m=\frac{g}{r'+\frac{1}{2}g}=\frac{2g}{r+\frac{1}{2}g}; \quad . \quad . \quad . \quad . \quad . \quad (105)$$

$$BF_m=(r'+\tfrac{1}{2}g)\sin F_m. \quad . \quad . \quad . \quad . \quad . \quad (106)$$

It may readily be shown that the relative values of F_r, F_l, and F_m are almost identical with those given in § 269; as may be

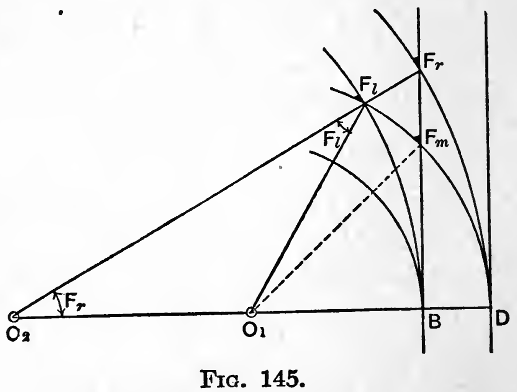


FIG. 145.

apparent when it is considered that the middle switch may be regarded simply as a curved main track, and that, as developed in § 267, the dimensions of turnouts are nearly the same whether the main track is straight or slightly curved.

271. Connecting curve from a straight track. The "connecting curve" is the track lying between the frog and the side track where it becomes parallel to the main track (FS in Fig. 146 or 147). Call d the distance between track centers. The angle $FO_1R = F$ (see Fig. 146). Call r' the radius of the connecting curve. Then

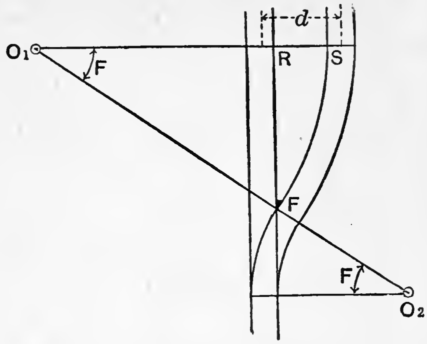


Fig. 146.

$$(r' - \tfrac{1}{2}g) = \frac{d-g}{\text{vers } F}; \quad . \quad (107)$$

$$FR = (r' - \tfrac{1}{2}g) \sin F. \quad . \quad . \quad . \quad . \quad . \quad . \quad (108)$$

If it is considered that the distance FR consumes too much track room it may be shortened by the method indicated in Fig. 151.

272. Connecting curve from a curved track to the OUTSIDE. When the main track is curved, the required quantities are the

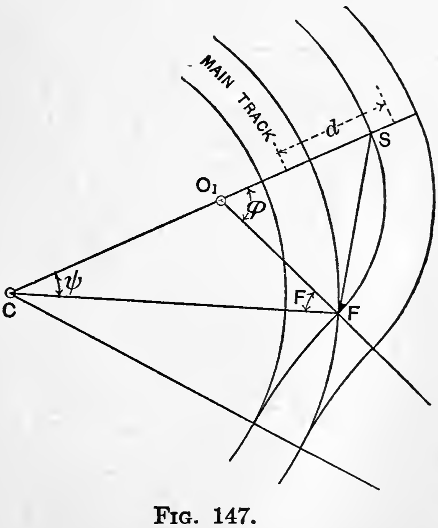


Fig. 147.

radius r of the connecting curve from F to S, Fig. 147, and its length or central angle. In the triangle CSF

$$CS + CF : CS - CF :: \tan \tfrac{1}{2}(CFS + CSF) : \tan \tfrac{1}{2}(CFS - CSF);$$

but $\frac{1}{2}(CFS+CSF)=90-\frac{1}{2}\psi$; and, since the triangle O_1SF is isosceles, $\frac{1}{2}(CFS-CSF)=\frac{1}{2}F$;

$$\therefore \quad 2R+d:d-g::\cot\tfrac{1}{2}\psi:\tan\tfrac{1}{2}F$$
$$::\cot\tfrac{1}{2}F:\tan\tfrac{1}{2}\psi;$$

$$\therefore \quad \tan\tfrac{1}{2}\psi=\frac{2n(d-g)}{2R+d}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (109)$$

From the triangle CO_1F we may derive

$$r-\tfrac{1}{2}g:R+\tfrac{1}{2}g::\sin\psi:\sin(F+\psi);$$

$$r-\tfrac{1}{2}g=(R+\tfrac{1}{2}g)\frac{\sin\psi}{\sin(F+\psi)}. \quad . \quad . \quad . \quad . \quad . \quad (110)$$

Also $$FS=2(r-\tfrac{1}{2}g)\sin\tfrac{1}{2}(F+\psi). \quad . \quad . \quad . \quad . \quad . \quad (111)$$

273. Connecting curve from a curved track to the INSIDE.

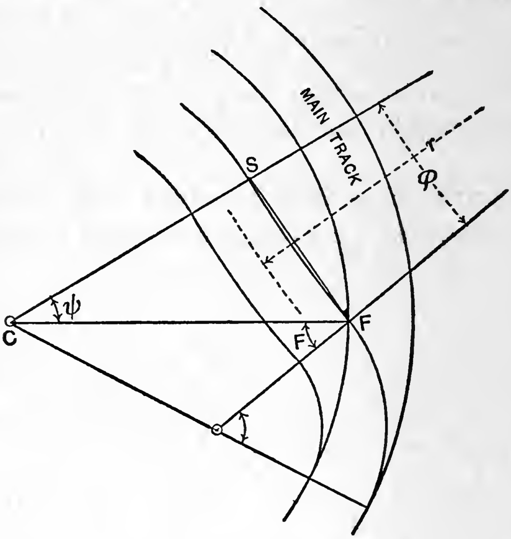


FIG. 148.

As above, it may readily be deduced from the triangle CFS (see Fig. 148) that

$$(2R-d):(d-g)::\cot\tfrac{1}{2}\psi:\tan\tfrac{1}{2}F,$$

and finally that

$$\tan\tfrac{1}{2}\psi=\frac{2n(d-g)}{2R-d}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (112)$$

Similarly we may derive (as in Eq. 110)

$$(r-\tfrac{1}{2}g)=(R-\tfrac{1}{2}g)\frac{\sin\psi}{\sin(F-\psi)}. \quad . \quad . \quad . \quad . \quad (113)$$

Also $$FS = 2(r - \tfrac{1}{2}g) \sin \tfrac{1}{2}(F - \psi). \quad . \quad . \quad . \quad . \quad (114)$$

Two other cases are possible. (*a*) r may increase until it becomes infinite (see Fig. 149), then $F = \psi$. In such a case we may write, by substituting in Eq. 112,

$$2R - d = 4n^2(d - g). \quad . \quad (115)$$

This equation shows the value of R, which renders this case possible with the given values of n, d, and g. (*b*) ψ may be greater than F. As before (see Fig. 150)

$$2R - d : d - g :: \cot \tfrac{1}{2}\psi : \tan \tfrac{1}{2}F;$$

$$\tan \tfrac{1}{2}\psi = \frac{2n(d-g)}{2R-d},$$

the same as Eq. 112, but

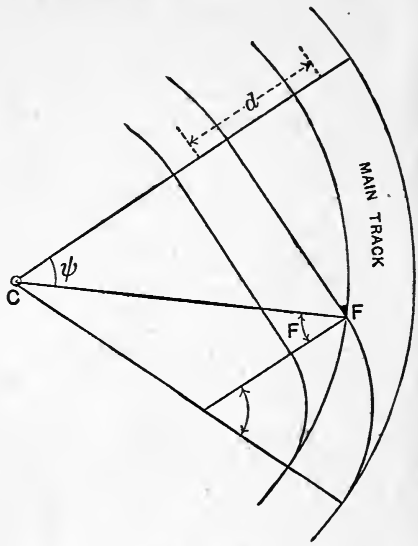


FIG. 149.

$$r + \tfrac{1}{2}g = (R - \tfrac{1}{2}g)\frac{\sin \psi}{\sin(\psi - F)}. \quad . \quad . \quad . \quad . \quad (116)$$

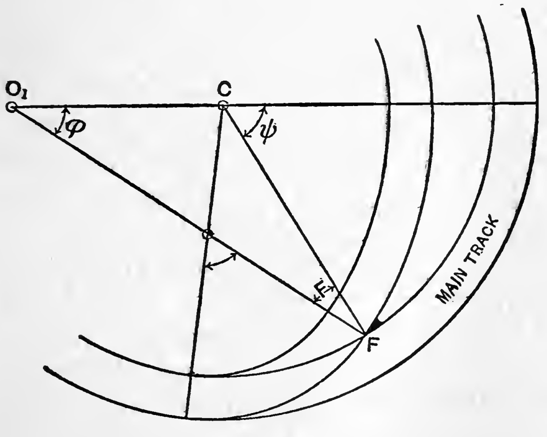


FIG. 150.

Problem. To find the dimensions of a connecting curve running to the INSIDE of a curved main track; number 9 frog, 4° 30′ curve, $d = 13'$, $g = 4'\ 8\tfrac{1}{2}''$.

Solution.

Eq. 112.

$$d = 13.000$$
$$g = 4.708$$
$$(d-g) = 8.292$$

$$R = 1273.6$$
$$2R = 2547.2$$
$$2R - d = 2534.2$$
$$\log(2R-d) = 3.40384$$

$$\log 2n = 1.2552\bar{7}$$
$$\log(d-g) = .91866$$
$$\text{co-log}\,(2R-d) = 6.59616$$
$$\log\tan\tfrac{1}{2}\Psi = 8.7700\bar{9}$$
$$\tfrac{1}{2}\Psi = 3°\ 22'\ 14''$$
$$\Psi = 6°\ 44'\ 28''$$
$$F = 6°\ 21'\ 35''$$
$$(\Psi - F) = 0°\ 22'\ 53'$$

Eq. 116.

$$R = 1273.6$$
$$\tfrac{1}{2}g = 2.35$$
$$(R - \tfrac{1}{2}g) = 1271.25$$
$$(\Psi - F) = 1373'',\ \log = 3.13767$$
$$4.68557$$
$$\log\sin(\Psi - F) = 7.82324$$

$$\log(R - \tfrac{1}{2}g) = 3.10423$$
$$\log\sin\Psi = 9.0696\bar{0}$$
$$\text{co-log}\sin(\Psi - F) = 2.17676$$
$$(r + \tfrac{1}{2}g) = 22418.0\ \ldots 4.3505\bar{9}$$
$$r = 22415.6$$
$$\underline{d = 0°\ 15'}$$

Eq. 114.

$$\tfrac{1}{2}(\Psi - F) = 686.''5\ \ldots 2.83664$$
$$4.6855\bar{7}$$
$$\sin\tfrac{1}{2}(\Psi - F) = 7.52221$$

$$2 \ldots 0.30103$$
$$(r - \tfrac{1}{2}g) = 22413.3 \ldots 4.3505\bar{0}$$
$$\sin\tfrac{1}{2}(\Psi - F) \ldots 7.5222\bar{1}$$
$$\underline{FS = 149.19} \ldots\ldots 2.17375$$

274. Crossover between two parallel straight tracks. (See Fig. 151.) The turnouts are as usual. The crossover track may be straight, as shown by the full lines, or it may be a reversed curve, as shown by the dotted lines. The reversed curve shortens the total length of track required, but is somewhat objectionable. The first method requires that both frogs must be equal. The second method permits unequal frogs, although equal frogs are preferable. The length of straight crossover track is F_1T.

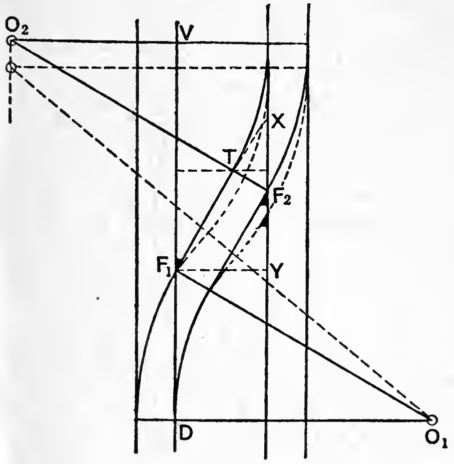


FIG. 151.

$$F_1T \sin F_1 + g \cos F_1 = d - g;$$

$$F_1T = \frac{d-g}{\sin F_1} - g \cot F_1. \quad \ldots\ldots \quad (117)$$

The total distance along the track may be derived as follows:

$$DV = 2DF_1 + F_2Y = 2DF_1 + XY - XF_2;$$
$$XY = (d-g)\cot F_1; \qquad XF_2 = g \div \sin F_2;$$
$$\therefore\ DV = 2DF_1 + (d-g)\cot F_1 - \frac{g}{\sin F_2}. \quad . \quad . \quad . \quad . \quad (118)$$

If a reversed curve with equal frogs is used, we have

$$\text{vers}\ \theta = \frac{d}{2r}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (119)$$

also

$$DQ = 2r \sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (120)$$

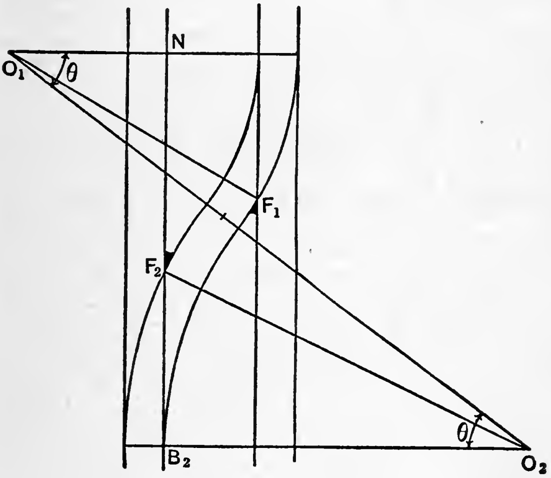


Fig. 152.

If the frogs are unequal, we will have (see Fig. 152)

$$r_2 \text{ vers}\ \theta + r_1 \text{ vers}\ \theta = d;$$

$$\therefore\ \text{vers}\ \theta = \frac{d}{r_1 + r_2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (121)$$

also the distance along the track

$$B_2N = (r_1 + r_2)\sin\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (122)$$

Problem. A crossover is to be placed between two parallel straight tracks, 12′ 2″ between centers, using a No. 8 and a No. 9

frog, and with a reversed curve between the frogs. Required the total distance between switch-points (the distance B_2N in Fig. 152).

Solution. If straight point rails and straight frog rails are used, the radii, r_1 and r_2, taken from the middle section of Table III, are 527.91 and 681.16.

Eq. 122. $$\text{vers } \theta = \frac{d}{r_1+r_2}$$

$r_1 = 527.91$ $\quad$ $d = 12'\ 2'' = 12.16,$ $\quad$ $\log = 1.08517$

$r_2 = 681.16$ $\quad$ $\log(r_1+r_2) = 3.08245$

$r_1+r_2 = 1209.07$ $\quad$ $\log \text{vers } \theta = 8.00272$

$\theta = 8°\ 08'\ 06''$

Eq. 122. $\quad$ $\log(r_1+r_2) = 3.08245$

$\log \sin \theta = 9.15077$

$B_2N = 171.09$ $\quad$ $\log 171.09 = 2.23322$

The length of the curve from $B_2 = 100(\theta \div d) = 100(8°\ 08'\ 06'' \div 8°\ 25') = 96.65$. The length of the other curve is $100(8°\ 08'\ 06' \div$

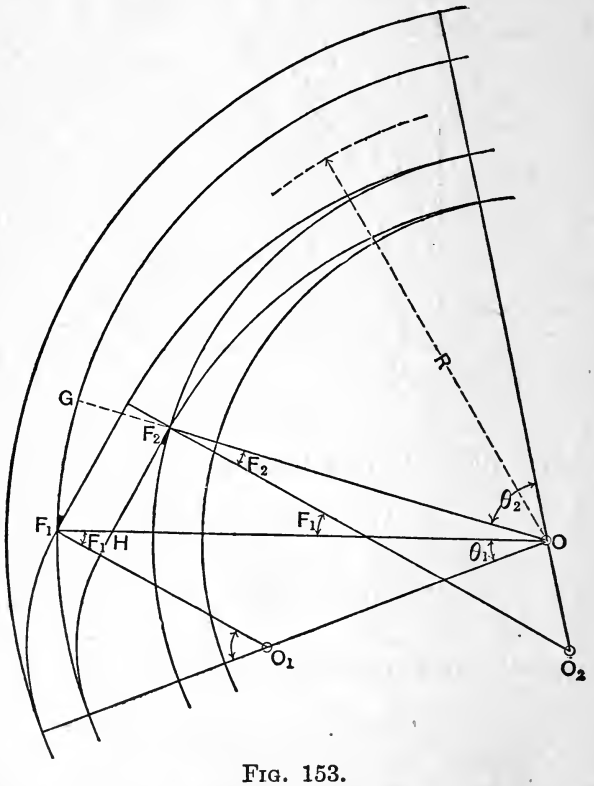


FIG. 153.

$10°\ 52') = 74.86$. As a check, $96.65 + 74.86 = 171.51$, which is slightly in excess of 171.09, as it should be.

275. Crossover between two parallel curved tracks. (a) Using a straight connecting curve. This solution has limitations. If one frog (F_1) is chosen, F_2 becomes determined, being a function of F_1. If F_1 is less than some limit, depending on the width (d) between the parallel tracks, this solution becomes impossible. In Fig. 153 assume F_1 as known. Then $F_1H = g \sec F_1$. In the triangle HOF_2 we have

$$\sin HF_2O : \sin F_2HO :: HO : F_2O;$$

$$\sin F_2HO = \cos F_1; \quad HF_2O = 90° + F_2;$$

$$\therefore \quad \sin HF_2O = \cos F_2.$$

$$HO = R + \tfrac{1}{2}d - \tfrac{1}{2}g - g \sec F_1; \quad F_2O = R - \tfrac{1}{2}d + \tfrac{1}{2}g;$$

$$\therefore \cos F_2 = \cos F_1 \frac{R + \frac{1}{2}d - \frac{1}{2}g - g \sec F_1}{R - \frac{1}{2}d + \frac{1}{2}g}. \quad . \quad . \quad . \quad . \quad . \quad (123)$$

Knowing F_2, θ_2 is determinable from Eq. 91. Fig. 153 shows the case where θ_2 is greater than F_2. Fig. 154 shows the case where it is less. The demonstration of Eq. 123 is applicable to

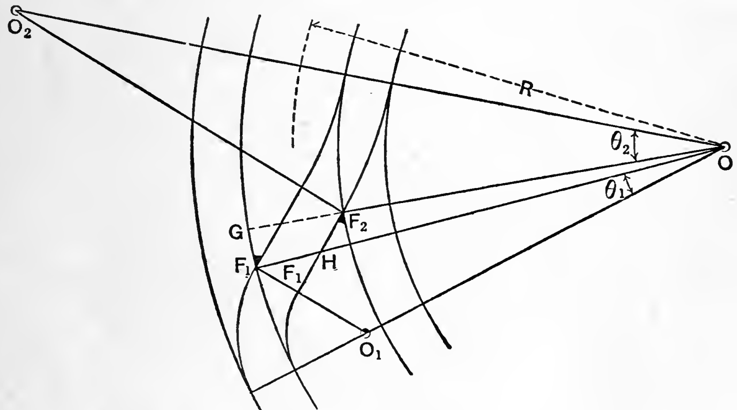


FIG. 154.

both figures The relative position of the frogs F_1 and F_2 may be determined as follows, the solution being applicable to both Figs. 153 and 154:

$$HOF_2 = 180° - (90° - F_1) - (90° + F_2) = F_1 - F_2.$$

Then

$$GF_1 = 2(R + \tfrac{1}{2}d - \tfrac{1}{2}g) \sin \tfrac{1}{2}(F_1 - F_2). \quad . \quad . \quad . \quad . \quad . \quad (124)$$

Since F_2 comes out *any* angle, its value will not be in general that of an even frog number, and it will therefore need to be made to order.

(b) Continuing the switch-rail curves until they meet as a reversed curve. In this case F_1 and F_2 may be chosen at pleasure (within limitations), and they will of course be of regular sizes and equal or unequal as desired. F_1 and F_2 being known, θ_1 and θ_2 are computed by Eq. 95 and 91. In the triangle OO_1O_2 (see Fig. 155)

$$\text{vers}\ \psi = \frac{2(S-OO_2)(S-OO_1)}{(OO_2)(OO_1)},$$

in which $S = \frac{1}{2}(OO_1 + OO_2 + O_1O_2)$;

but

$$OO_1 = R + \tfrac{1}{2}d - r_1,$$
$$OO_2 = R - \tfrac{1}{2}d - r_2,$$
$$O_1O_2 = r_1 + r_2;$$
$$\therefore \quad S = \tfrac{1}{2}(2R + 2r_2) = R + r_2;$$
$$S - OO_2 = R + r_2 - R + \tfrac{1}{2}d - r_2 = \tfrac{1}{2}d;$$
$$S - OO_1 = R + r_2 - R - \tfrac{1}{2}d + r_1 = r_1 + r_2 - \tfrac{1}{2}d;$$

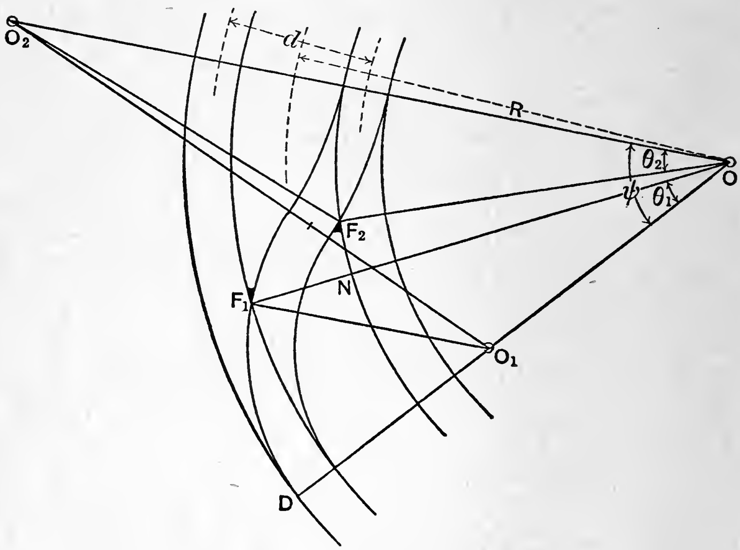


FIG. 155.

$$\text{vers}\ \psi = \frac{d(r_1 + r_2 - \frac{1}{2}d)}{(R - \frac{1}{2}d + r_2)(R + \frac{1}{2}d - r_1)}; \quad . \quad . \quad . \quad . \quad (125)$$

$$\sin OO_2O_1 = \sin \psi \frac{OO_1}{O_1O_2} = \sin \psi \frac{R + \frac{1}{2}d - r_1}{r_1 + r_2}; \quad . \quad . \quad . \quad (126)$$

$$O_2O_1D = \psi + O_1O_2O; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (127)$$

$$NF_2 = 2(R - \tfrac{1}{2}d + \tfrac{1}{2}g) \sin \tfrac{1}{2}(\psi - \theta_1 - \theta_2). \quad . \quad . \quad (128)$$

Although the above method introduces a reversed curve, yet it uses up less track than the first method and permits the use of ordinary frogs rather than those having some special angle which must be made to order.

Problem. Required the dimensions of a crossover on a 4° 30′ curve when the distance between track centers is 13 feet. The frog for the outer main track (F_1 in Fig. 155) is No. 9; F_2 is No. 7. Then $R=1273.6$; R_1, for the inner main track, $=1280.1$; $D_1=4° 29'$; $R_2=1267.1$; $D_2=4° 31'$; $r_1=$radius for $(d_1+D_1)°$ curve $=$ radius for $(8° 25' + 4° 29')$ curve $=445.09$; $r_2=$radius for $(d_2-D_2)°$ curve $=$ radius for $(14° 27' - 4° 31')$ curve $=577.53$. (See §§ 267–268.)

Eq. 125.	$d=13$;	$\log=1.11394$
$r_1+r_2-\frac{1}{2}d=1016.12$;		$\log=3.00694$
$R-\frac{1}{2}d+r_2=1844.63$;	$\log=3.26586$;	co-log$=6.73414$
$R+\frac{1}{2}d-r_1=835.01$;	$\log=2.92169$;	co-log$=7.07831$
	$\Psi=7° 30' 35''$	$\log \text{vers}\ \Psi=7.93334$
Eq. 126.		$\log\sin\Psi=9.11626$
		$\log(R+\frac{1}{2}d-r_1)=2.92169$
$r_1+r_2=1022.62$;	$\log=3.00971$;	co-log$=6.99028$
	$OO_2O_1=6° 07' 34''$;	$\sin OO_2O_1=9.02823$
Eq. 127.	$O_2O_1D=7° 30' 35''+6° 07' 34''=13° 38' 09''$	

Lead from switch point No. 1 up to $F_1=72.20$; $\theta_1=\frac{72.20}{100}d'$, where d' corresponds to the radius $(R+\frac{1}{2}d-\frac{1}{2}g)$ or 1277.75; $d'=4° 29'$; $\theta_1=3° 14'$.

Lead from switch point No. 2 up to $F_2=61.65$; $\theta_2=\frac{61.65}{100}d''$, where d'' corresponds to the radius $(R-\frac{1}{2}d+\frac{1}{2}g)$ or 1269.45; $d''=4° 31'$; $\theta_2=2° 47'$.

Eq. 128.

$$2;\ \log=0.30103$$
$$R-\tfrac{1}{2}d+\tfrac{1}{2}g=1269.45;\ \log=3.10361$$
$$\tfrac{1}{2}(\psi-\theta_1-\theta_2)=0° 44' 48'';\ \log\sin=8.11497$$
$$NF_2=33.08. \qquad \log 33.08=1.51961$$

Length of curve with radius $r_1=100\frac{13° 38' 09''}{12° 54'}=105.70$;

" " " " " $r_2=100\frac{6° 07' 34''}{9° 56'}=\ 61.67$;

Total length of curve between switch points $=167.37$.

As a check, the sum of the two *leads* and NF_2 equals (72.20+

61.65+33.08) = 166.93, which is a little *less* than the length of the curve, as it should be.

Note that the point of reversed curve is placed .02′ (= ¼″) beyond the frog point F_2. If the computations had apparently indicated the point of reversed curve coming between the frog point and the switch point, it would have shown the impracticability of the combination of No. 7 and No. 9 frogs with this particular degree of curve, gauge of track, and distance between track centers. If both frogs were made No 9, the total length of track between switch points would be increased to over 188 feet and the point of reversed curve would be nearly at the middle point. This shows that the frog numbers should be nearly equal, but also shows that there is some choice "within limitations."

276. Practical rules for switch-laying. A consideration of the previous sections will show that the formulæ are comparatively simple when the lead rails are assumed as circular; that they become complicated, even for turnouts from a straight main track, when the effect of straight frog and point rails is allowed for, and that they become hopelessly complicated when allowing for this effect on turnouts from a curved main track. It is also shown (§ 267) that the length of the lead is practically

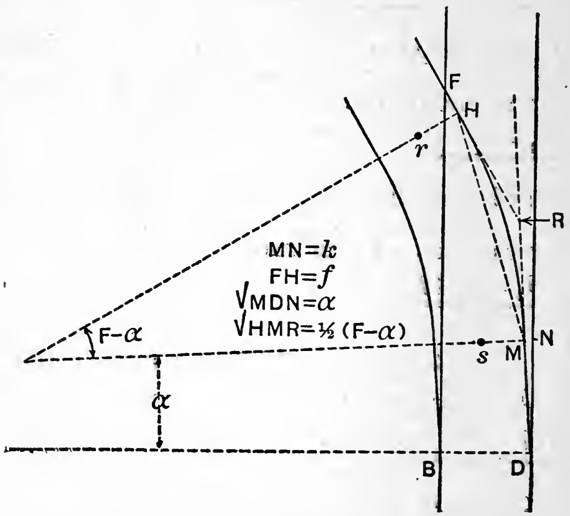


FIG. 140.

the same whether the main track is straight or is curved with such curves as are commonly used, and that the degree of curve of the lead rails from a curved main track may be found with close approximation by mere addition or subtraction. From this it may be assumed that if the length of lead (L) and the radius of the lead rails (r) are computed from Eq. 87 and 90 for various frog angles, the same leads may be used for curved main track; also, that the degree of curve of the lead rails may be found by addition or subtraction, as indicated in § 267, and that the approximations involved will not be of practical detriment. In accordance with this plan Table III has been computed from Eq. 87, 88, and 90. The *leads* there given may be used for all main tracks, straight or curved. The table gives the degree of curve of the lead rails for *straight* main track; for a turnout to the *inside*, *add* the degree of curve of the main track; for a turnout to the *outside*, *subtract* it.

If the position of the switch-block is definitely determined, then the rails must be cut accordingly; but when some freedom is allowable (which never need exceed 15 feet and may require but a few inches), one rail-cutting may be avoided. Mark on the rails at B, F, and D (see Fig. 140); measure off the length of the switch-rails DN; offset $\frac{1}{2}g+k$ from N for the point S. The point H may be located (temporarily) by measuring along the rail a distance $FH(=f)$ and then swinging out a distance of $f \div n$ (n being the frog number). $HT=\frac{1}{2}g$ and is measured at right angles to FH. Points for track centers between S and T may be laid off by a transit or by the use of a string and tape. Substituting in Eq. 31 the value of R and of *chord* ($=ST$), we may compute $x(=db)$. Locate the middle point d and the quarter points a'' and c''. Then $a''a$ and $c''c$ each equal three-fourths of db. Theoretically this gives a parabola rather than a circle, but the difference for all practical cases is too small for measurement.

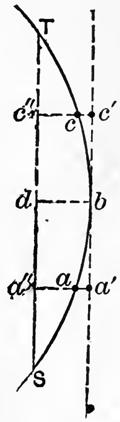


FIG. 156.

Example. Given a main track on a 4° curve; a turnout to the outside, using a number 9 frog; gauge $4'\ 8\frac{1}{2}''$; $f=3'.37$; $k=5\frac{3}{4}''$; $DN=15'\ 0''$ and $a=1°\ 50'$. Then for a *straight* track r would equal 631.16 [$d=9°\ 05'$]. For this curved track d will be nearly $(9°\ 05'-4°)=5°\ 05'$, or r will be 1131.2. L for the *straight* track would be 72.20; but since the lead is slightly increased (see § 267) when the turnout is on the outside of a curve, L may here be called 72.5. $FH=f=3'.37$; $f \div n = 3.37 \div 9 = 0'.375 = 4''.5$. H, T, and S may be located as described above. ST may be measured on the ground, or it may be computed from Eq. 88, giving the value of 53.80 feet for straight track. Since it is slightly more for a turnout to the outside of a curve, it may be called 54.0. Then $x = db = \frac{54.5}{8 \times 1131.2} = 0.322$ foot, and aa'' and $cc'' = 0.24$ foot.

CROSSINGS.

277. Two straight tracks. When two straight tracks cross each other, four frogs are necessary, the angles of two of them

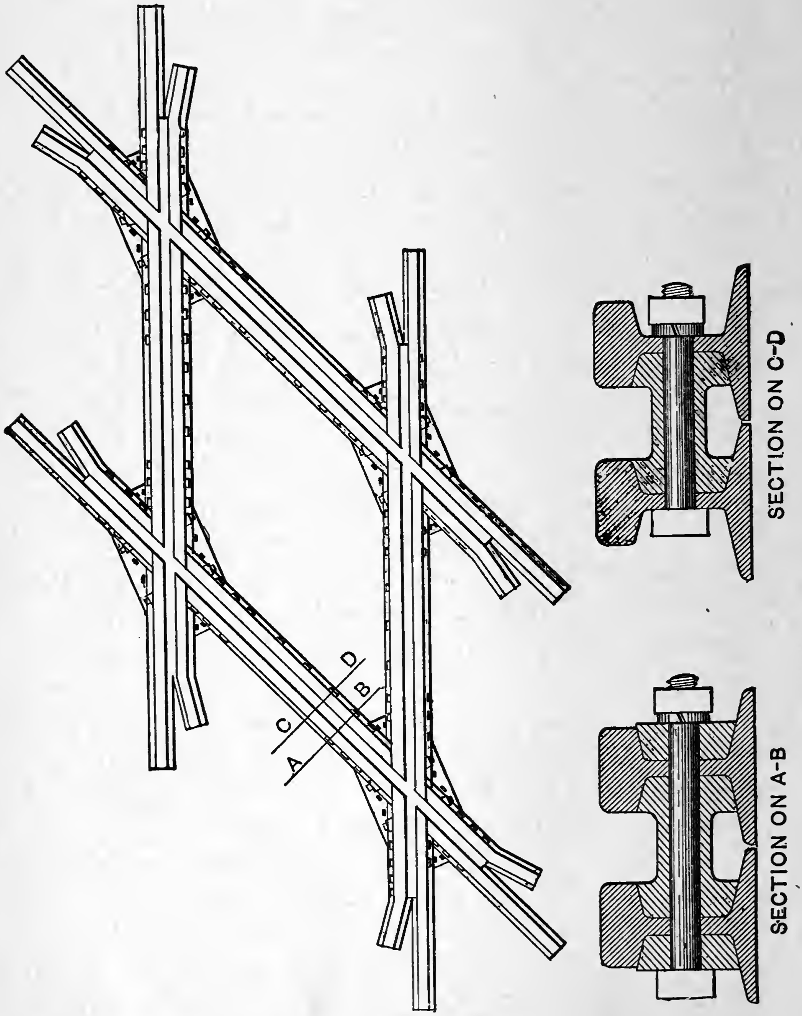


FIG. 157.—CROSSING.

being supplementary to the angles of the other. Since such crossings are sometimes operated at high speeds, they should be very strongly constructed, and the angles should preferably be 90° or as near that as possible. The frogs will not in general be "stock" frogs of an even number, especially if the angles are large, but must be made to order with the required angles as measured. In Fig. 157 are shown the details of such a crossing. Note the fillers, bolts, and guard-rails.

278. One straight and one curved track. Structurally the crossing is about the same as above, but the frog angles are all unequal. In Fig. 158, R is known, and the angle M, made by the center lines of the tracks at their point of intersection, is also known.

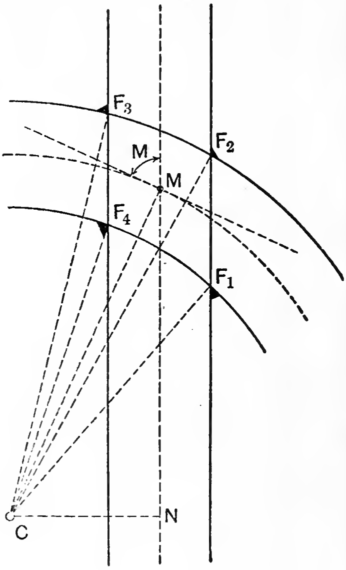


Fig. 158.

$$M=NCM. \quad NC=R\cos M.$$

$$\left.\begin{aligned}
(R-\tfrac{1}{2}g)\cos F_1 &= NC+\tfrac{1}{2}g;\\
\therefore\ \cos F_1 &= \frac{R\cos M+\tfrac{1}{2}g}{R-\tfrac{1}{2}g}.\\
\text{Similarly}\qquad&\\
\cos F_2 &= \frac{R\cos M+\tfrac{1}{2}g}{R+\tfrac{1}{2}g},\\
\cos F_3 &= \frac{R\cos M-\tfrac{1}{2}g}{R+\tfrac{1}{2}g},\\
\cos F_4 &= \frac{R\cos M-\tfrac{1}{2}g}{R-\tfrac{1}{2}g}.
\end{aligned}\right\}\quad(129)$$

$$\left.\begin{aligned}
F_3F_4 &= (R+\tfrac{1}{2}g)\sin F_3-(R-\tfrac{1}{2}g)\sin F_4;\\
HF_4 &= (R-\tfrac{1}{2}g)(\sin F_4-\sin F_1).
\end{aligned}\right\}\quad . \quad . \quad (130)$$

279. Two curved tracks. The four frogs are unequal, and the angle of each must be computed. The radii R_1 and R_2 are known; also the angle M. r_1, r_2, r_3, and r_4 are therefore known by adding or subtracting $\frac{1}{2}g$, but the lines are so indicated for brevity. Call the angle $MC_1C_2=C_1$, the angle $MC_2C_1=C_2$, and the line $C_1C_2=c$. Then

$$\tfrac{1}{2}(C_1+C_2)=90°-\tfrac{1}{2}M$$

and

$$\tan\tfrac{1}{2}(C_1-C_2)=\cot\tfrac{1}{2}M\frac{R_2-R_1}{R_2+R_1}. \quad . \quad . \quad . \quad . \quad (131)$$

C_1 and C_2 then become known and

$$c=C_1C_2=R_2\frac{\sin M}{\sin C_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (132)$$

In the triangle $F_1C_1C_2$, call $\frac{1}{2}(c+r_1+r_4)=s_1$; $s_2=\frac{1}{2}(c+r_2+r_4)$;

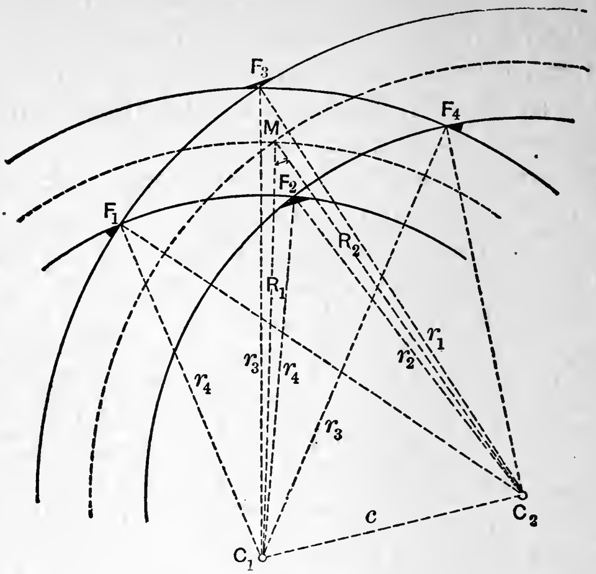


FIG. 159.

$s_3=\frac{1}{2}(c+r_1+r_3)$; and $s_4=\frac{1}{2}(c+r_2+r_3)$. Then, by formula 29, Table XXX,

$$\left.\begin{aligned} &\text{vers } F_1=\frac{2(s_1-r_1)(s_1-r_4)}{r_1r_4}. \\ \text{Similarly}\qquad &\text{vers } F_2=\frac{2(s_2-r_2)(s_2-r_4)}{r_2r_4}, \\ &\text{vers } F_3=\frac{2(s_3-r_1)(s_3-r_3)}{r_1r_3}, \\ &\text{vers } F_4=\frac{2(s_4-r_2)(s_4-r_3)}{r_2r_3}. \end{aligned}\right\} \quad \ldots \quad (133)$$

$$\sin C_1C_2F_4=\sin F_4\frac{r_3}{c};$$

$$\sin C_1C_2F_2=\sin F_2\frac{r_4}{c};$$

$$\therefore \quad F_2C_2F_4=C_1C_2F_4-C_1C_2F_2, \quad \ldots \quad (134)$$

$$\sin F_1C_1C_2=\sin F_1\frac{r_1}{c};$$

$$\sin F_2C_1C_2=\sin F_2\frac{r_2}{c},$$

$$\therefore \quad F_1C_1F_2=F_1C_1C_2-F_2C_1C_2; \quad \ldots \quad (135)$$

from which the chords F_1F_2 and F_2F_4 are readily computed.

F_1F_2 and F_2F_4 are nearly equal. When the tracks are straight and the gauges equal, the quadrilateral is equilateral.

Problem. Required the frog angles and dimensions for a crossing of two curves ($D_1=4°$; $D_2=3°$) when the angle of their tangents at the point of intersection $=62°\ 28'$ (the angle M in Fig. 159).

Solution

$$R_1=1432.7;\quad R_2=1910.1;$$
$$r_1=R_2+\tfrac{1}{2}g=1910.1+2.35=1912.45;$$
$$r_2=R_2-\tfrac{1}{2}g=1910.1-2.35=1907.75;$$
$$r_3=R_1+\tfrac{1}{2}g=1432.7+2.35=1435.05;$$
$$r_4=R_1-\tfrac{1}{2}g=1432.7-2.35=1430.35.$$

Eq. 131. $\log\cot\tfrac{1}{2}M=0.21723$

$R_2-R_1=477.4$; $\log=2.67888$

$R_2+R_1=3342.8$; $\log=3.52411$; co-log$=\underline{6.47589}$

$\tfrac{1}{2}(C_1-C_2)=13°\ 15'\ 07''$; $\tan 13°\ 15'\ 07''=9.37200$

$\tfrac{1}{2}(C_1+C_2)=\underline{58°\ 46'}$ $\quad[\tfrac{1}{2}(C_1+C_2)=90°-\tfrac{1}{2}M]$

$C_1=72°\ 01'\ 07''$

$C_2=45°\ 30'\ 53''$

Eq. 132. $\log R_2=3.28105$

$\log\sin M=9.9477\bar{9}$

$\log\sin C_1=9.97825$; co-log$=\underline{0.02175}$

$\underline{c=C_1C_2=1780.7;}$ $\qquad\log C_1C_2=3.2505\bar{9}$

Eq. 133.

$c=1780.7$	$c=1780.7$	$c=1780.7$	$c=1780.7$
$r_1=1912.45$	$r_2=1907.75$	$r_1=1912.45$	$r_2=1907.75$
$r_4=\underline{1430.35}$	$r_4=\underline{1430.35}$	$r_3=\underline{1435.05}$	$r_3=\underline{1435.05}$
$2\lfloor 5123.50$	$2\lfloor 5118.80$	$2\lfloor 5128.20$	$2\lfloor 5123.50$
$s_1=2561.75$	$s_2=2559.40$	$s_3=2564.10$	$s_4=2561.75$
$s_1-r_1=649.30$	$s_2-r_2=651.65$	$s_3-r_1=651.65$	$s_4-r_2=654.00$
$s_1-r_4=1131.40$	$-r_4=1129.05$	$s_3-r_3=1129.05$	$s_4-r_3=1126.70$

$\log 2=0.30103$

(s_1-r_1); $\log 649.30=2.8124\bar{4}$

(s_1-r_4); $\log 1131.40=3.0536\bar{1}$

$r_1=1912.45$; $\log=3.28159$; co-log$=6.71841$

$r_4=1430.35$; $\log=3.15544$; co-log$=\underline{6.84456}$

$\underline{F_1=62°\ 25'\ 31''};$ $\qquad\log\text{vers } 62°\ 25'\ 31''=9.73006$

$\log 2=0.30103$

(s_2-r_2); $\log 651.65=2.8140\bar{1}$

(s_2-r_4); $\log 1129.05=3.0527\bar{1}$

$r_2=1907.75$; $\log=3.28052$; co-log$=6.71948$

$r_4=1430.35$; $\log=3.15544$; co-log$=\underline{6.84456}$

$\underline{F_2=62°\ 33'\ 55''};$ $\qquad\log\text{vers } 62°\ 33'\ 55''=9.73180$

	$\log 2 = 0.3010\bar{3}$
	$(s_3 - r_1);\quad \log 651.65 = 2.8140\bar{1}$
	$(s_3 - r_3);\quad \log 1129.05 = 3.0527\bar{1}$
$r_1 = 1912.45;\quad \log = 3.28159;$	$\text{co-log} = 6.71841$
$r_3 = 1435.05;\quad \log = 3.1568\bar{6};$	$\text{co-log} = 6.8431\bar{3}$
$\underline{F_3 = 62° \ 21' \ 57'';}$	$\log \text{vers } 62° \ 21' \ 57'' = 9.7293\bar{0}$
	$\log 2 = 0.30103$
	$(s_4 - r_2);\quad \log 654.00 = 2.81558$
	$(s_4 - r_3);\quad \log 1126.70 = 3.05181$
$r_2 = 1907.75;\quad \log = 3.28052;$	$\text{co-log} = 6.71948$
$r_3 = 1435.05;\quad \log = 3.1568\bar{6};$	$\text{co-log} = 6.8431\bar{3}$
$\underline{F_4 = 62° \ 30' \ 14'';}$	$\log \text{vers } 62° \ 30' \ 14'' = 9.7310\bar{3}$

As a check, the *mean* of the frog angles $= 62° \ 27' \ 54''$, which is within $6''$ of the value of M.

Eq. 134.	$\log \sin F_4 = 9.9479\bar{4}$
	$\log r_3 = 3.1568\bar{6}$
$\log c = 3.2505\bar{9};$	$\text{co-log } c = 6.7494\bar{0}$
$C_1C_2F_4 = 45° \ 37' \ 51'';$	$\sin C_1C_2F_4 = 9.8542\bar{1}$
	$\log \sin F_2 = 9.9481\bar{8}$
	$\log r_4 = 3.15544$
	$\text{co-log } c = 6.7494\bar{0}$
$C_1C_2F_2 = 45° \ 28' \ 17'';$	$\sin C_1C_2F_2 = 9.85303$
$\underline{F_2C_2F_4} = 45° \ 37' \ 51'' - 45° \ 28' \ 17'' = \underline{0° \ 09' \ 34''}.$	
	$\log 2 = 0.30103$
	$\log r_2 = 3.28052$
$\frac{1}{2}(0° \ 09' \ 34'') = 0° \ 04' \ 47'';$	$\log \sin = \begin{cases} 4.6855\bar{7} \\ 2.45788 \end{cases}$
$\underline{F_2F_4 = 5.309};$	$\log F_2F_4 = 0.7250\bar{0}$
Eq. 135.	$\sin F_1 = 9.9476\bar{3}$
	$\log r_1 = 3.28159$
	$\text{co-log } c = 6.7494\bar{0}$
$F_1C_1C_2 = 72° \ 10' \ 22'';$	$\sin F_1C_1C_2 = 9.97863$
	$\sin F_2 = 9.9481\bar{8}$
	$\log r_2 = 3.28052$
	$\text{co-log } c = 6.7494\bar{0}$
$F_2C_1C_2 = 71° \ 57' \ 38'';$	$\sin F_2C_1C_2 = 9.97811$
$\underline{F_1C_1F_2} = 72° \ 10' \ 22'' - 71° \ 57' \ 38'' = \underline{0° \ 12' \ 44''}.$	
	$\log 2 = 0.30103$
	$\log r_4 = 3.15544$
$\frac{1}{2}(0° \ 12' \ 44'') = 0° \ 06' \ 22'';$	$\log \sin = \begin{cases} 4.6855\bar{7} \\ 2.5820\bar{6} \end{cases}$
$\underline{F_1F_2 = 5.298};$	$\log F_1F_2 = 0.72411$

As a check, F_2F_4 and F_1F_2 are very nearly equal, as they should be.

CHAPTER XII.

MISCELLANEOUS STRUCTURES AND BUILDINGS.

WATER-STATIONS AND WATER-SUPPLY.

280. **Location.** The water-tank on the tender of a locomotive has a capacity of from 2500 to 5000 gallons—sometimes less, rarely very much more. The consumption of water is very variable, and will correspond very closely with the work done by the engine. On a long down grade it is very small; on a ruling grade going up it may amount to 150 gallons per mile in exceptional cases, although 60 to 100 gallons would be a more usual figure. Nominally a locomotive could run 40 miles or more on one tankful, but it would be impracticable to separate the water-stations by such an interval. On roads of the smallest traffic, 15 to 20 miles should be the maximum interval between stations; 10 miles is a more common interval on heavy traffic-roads. But these intervals are varied according to circumstances. In the early history of some of the Pacific railroads it was necessary to attach one or more tank-cars to each train in order to maintain the supply for the engine over stretches of 100 miles and over where there was no water. Since then water-stations have been obtained at great expense by boring artesian wells. The individual locations depend largely on the facility with which a sufficient supply of suitable water may be obtained. Streams intersecting the railroad are sometimes utilized, but if such a stream passes through a limestone region the water is apt to be too hard for use in the boilers. More frequently wells are dug or bored. When the local supply at some determined point is unsuitable, and yet it is necessary to locate a water-station there, it may be found justifiable to pipe the water several miles. The construction of municipal water-works at suitable places along the line has led to the frequent utilization of such supplies. In such cases the railroad is generally the largest single consumer and obtains the most favorable rates. When possible, water-stations are located at regular stopping points and at division termini.

281. Required qualities of water. Chemically pure water is unknown except as a laboratory product. The water supplied by wells, springs, etc., is always more or less charged with calcium and magnesium carbonates and sulphates, as well as other impurities. The evaporation of water in a boiler precipitates these impurities to the lower surfaces of the boiler, where they sometimes become incrusted and are difficult to remove. The protection of the iron or steel of a boiler from the fierce heat of the fire depends on the presence of water on the other side of the surface, which will absorb the heat and prevent the metal from assuming an excessively high temperature. If the water side of the metal becomes covered or incrusted with a deposit of chemicals, the conduction of heat to the water is much less free, the metal will become more heated and its deterioration or destruction will be much more rapid. An especially common effect is the production of leaks around the joints between tubes and tube-sheets and the joints in the boiler-plates. Such injury can only be prevented by the application of one (or both) of two general methods—(*a*) the frequent cleaning of the boilers and (*b*) the chemical purification of the water before its introduction into the boiler. Although "manholes" and "handholes" are made in boilers, it is physically impossible to clean out every corner of the inside of a boiler where deposits will form and where they are especially objectionable—on the tube-sheets. Such a cleaning is troublesome and expensive.

Chemical purification is generally accomplished by treating the water before it enters the boiler. The reagents chiefly employed are quicklime and sodium carbonate. Lime precipitates the bicarbonate of lime and magnesia. Sodium carbonate gives, by double decomposition in the presence of sulphate of lime, carbonate of lime, which precipitates, and soluble sulphate of soda, which is non-incrustant. When this is done in a purifying tank, the purified water is drawn off from the top of the tank and supplied pure to the engines. The precipitants are drawn off from the settling-basin at the bottom of the tank. This purification, which makes no pretense of being chemically perfect, may be accomplished for a few cents per 1000 gallons. It is used much more extensively in Europe than in this country, the Southern Pacific being the only railroad which has employed such methods on a large scale. Reliance is frequently placed on the employment of a "non-incrustant" which is introduced directly into the boiler. When no incrustation takes place the accumulation of precipitant and mud in the bottom of the boiler may be largely removed by mere "blowing off" or by washing out with a hose.

American practice may therefore be summarized as follows: (*a*) Employing as pure water as possible; (*b*) cleaning out boilers by "blowing off" or by washing out with a hose or by physical scraping at more or less frequent intervals or when other repairs are being made; (*c*) the occasional employment of non-incrustants; (*d*) the occasional chemical treatment of water before it enters the tender-tank.

282. Tanks. Whatever the source, the water must be led or pumped into tanks which are supported on frames so that the bottoms of the tanks are about 12 feet above the rails. Wooden

TABLE XIV.—CAPACITY OF CYLINDRICAL WATER-TANKS IN UNITED STATES STANDARD GALLONS OF 231 CUBIC INCHES.

Height in feet.	Diameter of tank in feet.							
	10	12	14	16	18	20	22	24
6	3525	5076	6909	9024	11421	14101	17062	20305
7	4113	5922	8061	10528	13325	16451	19905	23689
8	4700	6768	9212	12032	15229	18801	22749	27073
9	5288	7614	10364	13536	17132	21151	25592	30457
10	5875	8460	11515	15041	19036	23501	28436	33841
11	6463	9306	12667	16545	20939	25851	31280	37225
12	7050	10152	13819	18049	22843	28201	34123	40609
13	7638	10998	14970	19553	24746	30551	36967	43994
14	8225	11844	16122	21057	26650	32901	39810	47378
15	8813	12690	17273	22561	28554	35251	42654	50762
16	9400	13536	18425	24065	30457	37601	45498	54146
17	9988	14383	19576	25569	32361	39951	48341	57530
18	10575	15229	20728	27073	34264	42301	51185	60914
19	11163	16075	21879	28577	36168	44652	54028	64298
20	11750	16921	23031	30081	38071	47002	56872	67682
21	12338	17767	24182	31585	39975	49352	59716	71067
22	12925	18613	25334	33089	41879	51702	62559	74451
23	13513	19459	26485	34593	43782	54052	65403	77835
24	14101	20305	27637	36097	45686	56402	68246	81219
25	14688	21151	28789	37601	47589	58752	71090	84603

tanks having a diameter of 24 feet, 16 feet high, and with a capacity of over 50,000 gallons are frequently employed. Iron or steel tanks are also used.

In Table XIV is shown the capacity of cylindrical water-tanks in United States standard gallons of 231 cubic inches. From this table the dimensions of a tank of any desired capacity may readily be found. Two or more tanks are sometimes used rather than construct one of excessive size. The smaller sizes shown in the table are of course too small for ordinary use, but that part of the table was filled out for its possible convenience otherwise. On single-track roads where all engines use one track the tank may be placed 8′ 5″ from the track center; this gives sufficient clearance and yet permits the use of a single swinging pipe which will reach from the bottom of the tank to the tender manhole. In Fig. 160 is illustrated

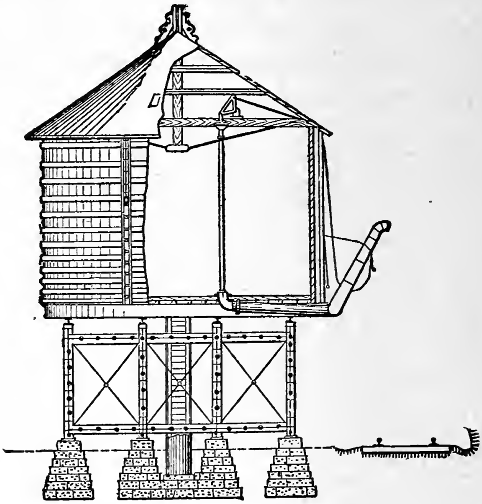


FIG. 160.—WATER-TANK.

one form of wooden tank. They are preferably manufactured by those who make a special business of it and who by the use of special machinery can insure tight joints. When it is inconvenient to place the tank near the track, or when there is a double track, a "stand-pipe" becomes necessary. See § 285. One of the most difficult and troublesome problems is to prevent freezing, particularly in the valves and pipes. Not only are the pipes carefully covered but fires must be maintained during cold weather. When the pumping is accomplished by means of a steam-pump, supplied from a steam-boiler in the pump-house under the tank, coils of steam-pipe may be employed to heat the water or to heat the pipes. Partial protection may be obtained by means of a double roof and double bottom, the spaces being filled with sawdust or some other non-conductor of heat.

283. Pumping. The pumping is done most reliably with steam-pumps or gas-engines, although hot-air engines, windmills, and even man-power are occasionally employed. Economy of operation requires that the water-stations shall be so located that each tank shall be used regularly and that each pump shall be regularly operated for maintaining the water-supply. On the other hand, the pump should not be *required* to regularly work at night to maintain the supply and should have an excess capacity of say 25%. When a tank is but little used, it will still require the labor of an attendant, and his time will be largely wasted unless he can be utilized for other labor about the station. In recent years gasoline has been extensively employed as a fuel for the pumping-engines. The chief advantages of its use lies in the extreme simplicity of the mechanism and the very slight attention it requires, which permits their being operated by station-agents and others, who are paid $10 per month extra, instead of paying a regular pumper $35 per month. "Screenings," "slack coal," etc., are used as fuel for steam-pumps and may frequently be delivered at the pump-house at a cost not exceeding 30 cents per ton, but even at that price the cost of pumping per thousand gallons, although dependent on the horizontal and vertical distances to the source of supply and to the tank, will generally run at 2 cents to 6 cents per 1000 gallons. In many cases where steam plants have been replaced by gasoline plants, the cost of pumping per 1000 gallons has been reduced to one third or even one fourth of the cost of steam pumping. Of course the cost, using windmills, is reduced to the mere maintenance of the machinery, but the unreliability of wind as a motive power and the possibility of its failure to supply water when it is imperatively needed has made this form of motive power unpopular. (See report to Ninth Annual Convention of the Association of Railway Superintendents of Bridges and Buildings, Oct. 1899.)

284. Track tanks. These are chiefly required as one of the means of avoiding delays during fast-train service. A trough, made of steel plate, is placed between the rails on a stretch of *perfectly level* track. A scoop on the end of a pipe is lowered from under the tender into the tank while the train is in motion. The rapid motion scoops up the water, which then flows into the tender tank. The following brief description of an installation on the Baltimore & Ohio Railroad between Baltimore and

Philadelphia will answer as a general description of the method. The trough is made of $\frac{3}{16}''$ steel plate, 19″ wide, 6″ deep, and has a length of 1200 feet. There is riveted on each side a line of $1\frac{1}{8}'' \times 2'' \times \frac{1}{4}''$ angle bars. These angle bars rest on the ties. Ordinary track spikes hold these angle bars to the ties, but permit expansion as with rails. The tanks are firmly anchored at the center, the ends being free to expand or contract. The plates are 15 feet long and are riveted with $\frac{7}{16}''$ rivets, 20 rivets per joint. At each end is an inclined plane 13′ 8″ long. If the fireman should neglect to raise the scoop before the end of the tank is reached, the inclined plane will raise it automatically and a catch will hold it raised. Water is supplied to the tanks by a No. 9 Blake pump having a capacity of 260 gallons per minute. During cold weather, freezing is prevented by injecting into the side of the tanks, at intervals of 45 feet, jets of steam, which come through $\frac{1}{8}''$ holes. Two boilers of 80 and 95 H.P. are required for pumping and to keep the water from freezing. During warm weather an upright 25 H.P. boiler suffices for the pumping. The cost of installation was about $10,000 to $11,000, the cost of maintenance being about $132.50 per month.

285. Stand-pipes. These are usually manufactured by those who make a specialty of such track accessories, and who can ordinarily be trusted to furnish a correctly designed article. In Fig. 161 is shown a form manufactured by the Sheffield Car Co. Attention is called to the position of the valve and to the device for holding the arm parallel to the track when not in use so that it will not be struck by a passing train. When a stand pipe is located between parallel tracks, the strict requirements of clearance demand that the tracks shall be bowed outward slightly. If the tracks were originally straight, they may be shoved over by the trackmen, the shifting gradually running out at about 100 feet each side of the stand-pipe. If the tracks were originally curved, a slight change in radius will suffice to give the necessary extra distance between the tracks.

BUILDINGS.

286. Station platforms. These are most commonly made of planks at minor stations. Concrete is used in better-class work, also paving brick. An estimate of the cost of a platform of paving brick laid at Topeka, Kan., was $4.89 per 100 square feet when laid flat and $7.24 per 100 square feet when laid on edge. The curbing cost 36 cents per linear foot. Cinders, curbed by timbers

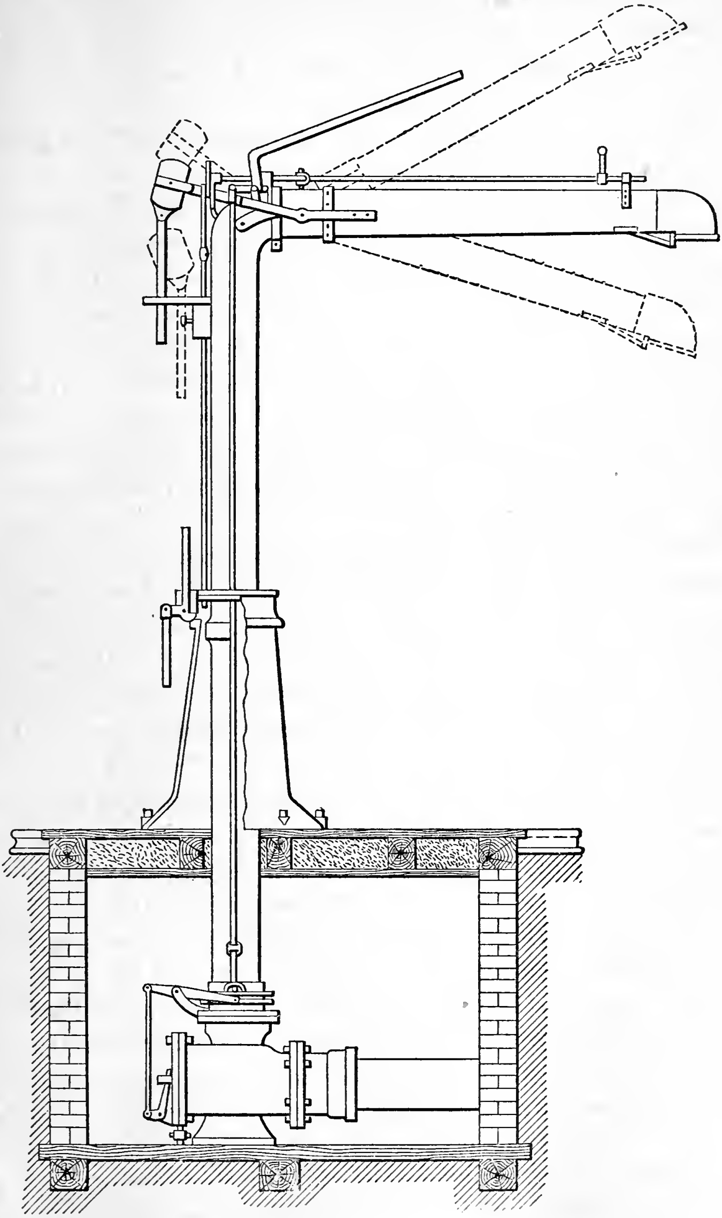


FIG. 161.—STAND-PIPE.

or stone, bound by iron rods, make a cheap and fairly durable platform, but in wet weather the cinders will be tracked into the stations and cars. Three inches of crushed stone on a cinder foundation is considered to be still better, after it is once thoroughly packed, than a cinder surface.

Elevation.—The elevation of the platform with respect to the rail has long been a fruitful source of discussion. Some roads make the platforms on a level with the top of the rail, others 3″ above, others still higher. As a matter of convenience to the passengers, the majority find it easier to enter the car from a high platform, but experience proves that accidents are more numerous with the higher platforms, unless steps are discarded altogether and the cars are entered from level platforms, as is done on elevated roads. As a railroad must generally pay damages to the stumbling passenger, they prefer to build the lower platform. Convenience requires that the rise from the platform to the lowest step should not be greater than the rise of the car steps. This rise is variable, but with the figures usually employed the application of the rule will make the platform 5″ to 15″ above the rail.

Position with respect to tracks.—Low platforms are generally built to the ends of the ties, or, if at the level of the top of the rail, are built to the rail head. Car steps usually extend 4′ 6″ from the track center and are 14″ to 24″ above the rail. The platform must have plenty of clearance, and when the platform is high its edge is generally required to be 5′ 6″ from the track center.

287. Minor stations. For a complete discussion of the design of stations of all kinds, including the details, the student is referred to "Buildings and Structures of American Railroads," by Walter G. Berg, now Chief Engineer of the Lehigh Valley Railroad. The subject is too large for adequate discussion here, but a few fundamental principles will be referred to.

Rooms required. An office and waiting-room is the minimum. A baggage-room, toilet-rooms, and express office are successively added as the business increases. In the Southern States a separate waiting-room for colored people is generally provided. It used to be common to have separate waiting-rooms for men and women. Experience proved that the men's waiting-room became a lounging place and smoking-room for loafers, and now large single waiting-rooms are more common even in the more pretentious designs, smoking being excluded. The office usually has a bay window, so that a more extended view of the track is obtainable. The women's toilet-room is entered from the waiting-room. The men's toilet-room, although built immediately adjoining the other in order to simplify the plumbing, is entered from outdoors. Old-fashioned designs built the station as a residence for the station-agent; later designs have very generally abandoned this idea. "Combination" stations (passenger and freight) are frequently built for small local stations, but their use seems to be decreasing and there is now a tendency to handle the freight business in a separate building.

288. Section-houses. These are houses built along the right-of-way by the railroad company as residences for the trackmen. The liability of a wreck or washout at any time and at any part of the road, as well as the convenience of these houses for ordinary track labor, makes it all but essential that the trackmen should live on the right-of-way of the road, so that they may be easily called on for emergency service at any time of day or night. This is especially true when the road passes through a thinly settled section, where it would be difficult if not impossible to obtain suitable boarding-places. It is in no sense an extravagance for a railroad to build such houses. Even from the direct financial standpoint the expense is compensated by the corresponding reduction in wages, which are thus paid partly in free house rent. And the value of having men on hand for emergencies will often repay the cost in a single night. Where the country is thickly settled the need for such houses is not so great, and railroads will utilize or perhaps build any sort of suitable house, but on Southern or Western roads, where the need for such houses is greater, standard plans have been studied with great care, so as to obtain a maximum of durability, usefulness, comfort, and economy of construction. (See Berg's Buildings, etc., noted above.) On Northwestern roads, protection against cold and rain or snow is the chief characteristic; on Southern roads good ventilation and durability must be chiefly considered. Such houses may be divided into two general classes—(*a*) those which are intended for trackmen only and which may be built with great simplicity, the only essential requirements being a living-room and a dormitory, and (*b*) those which are intended for families, the houses being then distinguished as "dwelling-houses for employees.

289. Engine-houses. Small engine-houses are usually built rectangular in plan. Their minimum length should be somewhat greater than that of the longest engine on the road. They may be built to accommodate two engines on one track, but then they should be arranged to be entered at either end, so that neither engine must wait for the other. In width there may be as many tracks as desired, but if the demand for stalls is large, it will probably be preferable to build a "roundhouse." Rectangular engine-houses are usually entered by a series of parallel tracks switching off from one or more main tracks, no turn-table being necessary. If a turn-table is placed outside (because one

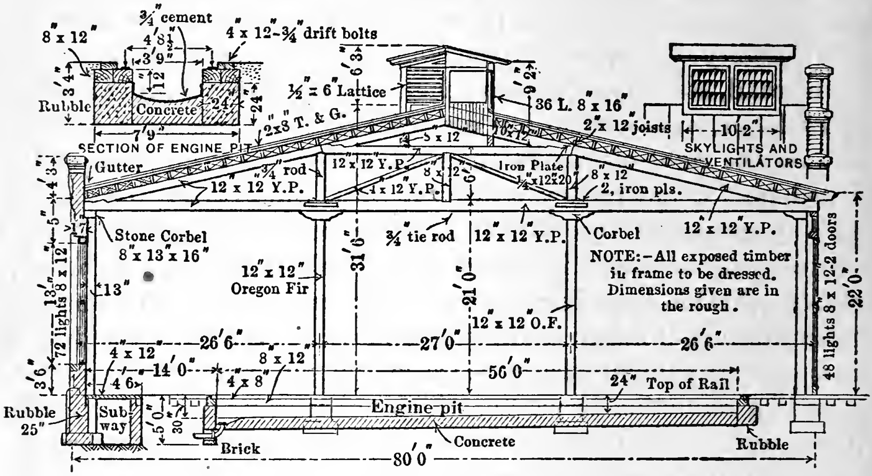


FIG. 162.—ENGINE-HOUSE.

is needed at that part of the road) enough track should be allowed between the house and the turn-table so that engines may be quickly removed from the engine-house in case of fire without depending on the turn-table to get them out of danger.

Roundhouses. The plan of these is generally polygonal rather than circular. The straight walls are easier to build; the construction is more simple, and the general purpose is equally well served. They may be built as a part of a circle or a complete circle, a passageway being allowed, so that there are two entrances instead of one. When space is very limited a roundhouse with turn-table will accommodate more engines in proportion to the space required (including the approaches) than a rectangular house. The enlarged space on the outer side of each segment of a roundhouse furnishes the extra space which is needed for the minor repairs which are usually made in a roundhouse. One disadvantage is that supervision is not quite so easy or effective as in rectangular houses. Of course such houses are used not only for storing and cleaning engines, but also for minor repairs which do not require the engine to be sent to the shops for a general overhauling.

Construction. The outer walls are usually of brick. The inner walls consist almost entirely of doors and the piers between them, although there is usually a low wall from the top of the door frames to the roof line, which usually slopes outward so as to turn rain-water away from the central space.

Roofs. Many roofs have been built of slate with iron truss framing, with the idea of maximum durability. The slate is good, but experience shows that the iron framing deteriorates very rapidly from the action of the gases of combustion of the engines which must be "fired" in the houses before starting. Roof frames are therefore preferably made of wood.

Floors. These are variously constructed of cinders, wood, brick, and concrete. Brick has been found to be the best material. Anything short of brick is a poor economy; concrete is very good if properly done but is somewhat needlessly expensive.

Ventilation. This is a troublesome and expensive matter. The general plan is to have "smoke-jacks" which drop down over the stack of each engine as it reaches its precise place in its stall and which will carry away all smoke and gas. Such a movable stack is most easily constructed of thin metal—say galvanized iron—but these will be corroded by the gases of combustion in two or three years. Vitrified pipe, cast iron, expanded metal and cement, and even plain wood painted with "fireproof" paint, have been variously tried, but all methods have their unsatisfactory features. (For an extended discussion of roundhouse floors and ventilation see the Proc. Assoc. of Railway Supts. of Bridges and Buildings for 1898, pp. 112–135.)

SNOW STRUCTURES.

290. **Snow-fences.** Snow structures are of two distinct kinds—fences and sheds. A snow-fence implies drifting snow—snow carried by wind—and aims to cause all drifting snow to be deposited away from the track. Some designs actually succeed in making the wind an agent for clearing snow from the track where it has naturally fallen. A snow-fence is placed at right angles to the prevailing direction of the wind and 50 to 100 feet away from the tracks. When the road line is at right angles to

the prevailing wind, the right-of-way fence may be built as a snow-fence—high and with tight boarding. Hedges have sometimes been planted to serve this purpose. When the prevailing wind is oblique, the snow fences must be built in sections where they will serve the best purpose. The fences act as wind breakers, suddenly lowering the velocity of the wind and causing the snow carried by the wind to be deposited along the fence. Portable fences are frequently used, which are placed (by permission of the adjoining property owners) outside of the right-of-way. If a drift forms to the height of the portable fence the fence may be replaced on the top of the drift, where it may act as before, forming a still higher drift. When the prevailing wind runs along the track line, snow-fences built in short sections on the sides will cause snow to deposit around them while it scours its way along the track line, actually clearing it. Such a method is in successful operation at some places on the White Mountain and Concord divisions of the Boston & Maine Railroad. Snow-fences, in connection with a moderate amount of shoveling and plowing, suffice to keep the tracks clear on railroads not troubled with avalanches. In such cases snow-sheds are the only alternative.

291. Snow-sheds. These are structures which will actually keep the tracks clear from snow regardless of its depth outside. Fortunately they are only necessary in the comparatively rare situations where the snowfall is excessive and where the snow is liable to slide down steep mountain slopes in avalanches. These avalanches frequently bring down with them rocks, trees, and earth, which would otherwise choke up the road-bed and render it in a moment utterly impassable for weeks to come. The sheds are usually built of $12'' \times 12''$ timber framed in about the same manner as trestle timbering; the "bents" are sometimes placed as close as 5 feet, and even this has proved insufficient to withstand the force of avalanches. The sheds are therefore so designed that the avalanche will be *deflected* over them instead of spending its force against them. Although these sheds are only used in especially exposed places, yet their length is frequently very great and they are liable to destruction by fire. To confine such a fire to a limited section, "fire-breaks" are made—i.e., the shed is discontinued for a length of perhaps 100 feet. Then, to protect that section of track, a V-shaped deflector will be placed on the uphill side which will deflect all descending material so that it passes over the sheds. Solid crib work is largely used for these structures. Fortunately suitable timber for such construction is usually plentiful and cheap where these structures are necessary. Sufficient ventilation is obtained by longitudinal openings along one side immediately under the roof. "Summer" tracks are usually built outside the sheds to avoid the discomfort of passing through these semi-tunnels in pleasant weather. The fundamental elements in the design of such structures is shown in Fig. 163, which illustrates some of the sheds used on the Canadian Pacific Railroad.

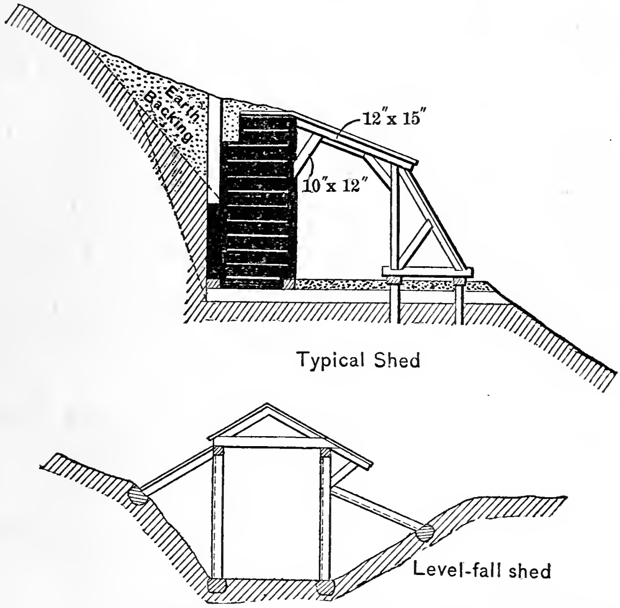


FIG. 163.—SNOW-SHEDS—CANADIAN PACIFIC RAILROAD.

292. **Turn-tables.** The essential feature of a turn-table is a carriage of sufficient size and strength to carry a locomotive, the carriage turning on a pivot of sufficient size to carry such a load. The carriage revolves in a circular pit whose top has the same general level as the surrounding tracks. The carriages were formerly made largely of wood; very many of those still in use are of cast iron. Structural steel is now universally employed for all modern work and since the construction of the carriage and the pivot is a special problem in structures, no further attention will here be paid to the subject, except to that part which the railroad engineer must work out

—laying out the site and preparing the foundation. The *minimum* length of such a carriage (and therefore the diameter of the pit) is evidently the length over all of the longest engine and tender in use on the road. Usually 60-foot turn-tables will suffice for an ordinary road, and for light-traffic roads employing small engines, 50 feet or even less may be sufficient. Many of the heavier freight engines of recent make have a total length of about 65 feet; therefore 70-foot turn-tables are a better standard for heavy-traffic roads. A retaining-wall should be built around the pit. The stability of this wall immediately under the tracks should be especially considered. The most important feature is the stability of the foundation of the pivot, which must sustain a concentrated pressure, more or less eccentric, of perhaps 150 tons. When firm soil or rock may be easily reached, this need give no trouble, but in a soft, treacherous soil a foundation of concrete or piling may be necessary. If the soil is very porous, it may be depended on to carry away all rain-water which may fall into the pit before the foundations are affected, but when the soil is tenacious it may be necessary to drain the subsoil thoroughly and carry off immediately all surface drainage by means of subsoil pipes which have a suitable outfall.

The location of the turn-table in the yard is a part of the general subject of "Yards," and will be considered in the next chapter.

CHAPTER XIII.

YARDS AND TERMINALS.

293. Value of proper design. A large part of the total cost of handling traffic, particularly freight, is that incurred at terminals and stations. In illustration of this, consider the relative total cost of handling a car-load of coal and a car-load (of equal weight) of mixed merchandise. The coal will be loaded in bulk on the cars at the mines, where land is comparatively cheap, and the cars grouped into a train without regard to order, since they are (usually) uniform in structure, loading, and contents. When the terminal or local station is reached they are run on tracks occupying property which is usually much cheaper than the site of the terminal tracks and freight-houses; they are unloaded by gravity into pockets or machine conveyors and the empty cars are rapidly hauled by the train-load out of the way. On the other hand, the merchandise is loaded by hand on the car from a freight-house occupying a central and valuable location, the car is hauled out into a yard occupying valuable ground, is drilled over the yard tracks for a considerable aggregate mileage before starting for its destination, where the same process is repeated in inverse order. In either case the terminal expenses are evidently a large percentage of the total cost and, once loaded, it makes but little difference just how far the car is hauled to the other terminal. But the very evident increase in terminal charges for general merchandise over those for coal (large as they are) gives a better idea of the magnitude of terminal charges.

Many yards are the result of growth, adding a few tracks at a time, without much evidence of any original plan. In such cases the yard is apt to be very inefficient, requiring a much larger aggregate of drilling to accomplish desired results, requiring much more *time* and hence blocking traffic and finally adding greatly to the cost of terminal service, although the fact of its being a needless addition to cost may be unsuspected or not fully appreciated. An unwillingness or inability to spend money for the necessary changes, and the difficulty of making the changes while the yard is being used, only prolong the bad state of affairs and an inefficient makeshift is frequently adopted. Assume that an improvement in the design of the yard will permit a saving of the use of one switching engine, or for example, that the work may be accomplished with three switching engines instead of four. Assuming a daily cost of $25, we have in 313 working days an annual saving of $7825, which, capitalized at 5%, gives $156,500, enough to reconstruct any ordinary yard.*

294. Divisions of the subject. The subject naturally divides itself into three heads—(*a*) Yards for receiving, classifying, and distributing freight cars, called more briefly freight yards; (*b*) yards and conveniences for the care of engines, such as ash tracks, turn-tables, coal-chutes, sand-houses, water-tanks, or water stand-pipes, etc., and (*c*) passenger terminals.

FREIGHT YARDS.

295. General principles. It should be recognized at the start that at many places an ideally perfect yard is impossible, or at least impracticable, generally because ground of the required shape or area is practically unobtainable. But there are some general principles which may and should be followed in every yard and other ideals which should be approached as nearly as possible. Nevertheless every yard is an independent problem. Before taking up the design of freight yards, it is first necessary to consider the general object of such yards and the general principles by which the object is accomplished. These may be briefly stated as follows:

1. A yard is a device, a machine, by which incoming cars are sorted and classified—some sent to warehouses for unloading, some sent to connecting railroads, some made up for local distribution along the road, some sent for repairs, and, in short a device by which all cars are sent *through* and *out* of the yard as quickly as possible.

2. Except when a road's business is decreasing, or when its equipment is greater than its needs and its cars must be stored, efficiency of management is indicated by the rapidity with which the passage of cars *through* the yard is accomplished.

3. When a yard is the terminal of a "division," the freight

* Estimate of Mr. H. G. Hetzler, C., B. & Q. Ry.

trains will be pulled into a "receiving track" and the engine and caboose detached. The caboose will be run on to a "caboose track," which should be conveniently near, and the engine is run off to the engine yard. If the train is a "through" train and no change is to be made in its make-up, it will only need to wait for another engine and perhaps another caboose. If the cars are to be distributed, they will be drawn off by a switching engine to the "classification yard."

4. The design of a yard is best studied by first picking out the ladder tracks and the through tracks which lead from one division of the yard to another. These are tracks which must always be kept open for the passage of trains, in contradistinction to the tracks on which cars may be left standing, even though it is only for a few moments, while drilling is being done. Such a set of tracks, which may be called the skeleton of the yard, is shown by heavy lines in Fig. 164. Each line indicates a pair of rails. The tracks of the storage yards are shown by the lighter lines.

5. There is a distinct advantage in having all storage tracks double-ended—except "team tracks." Team tracks are those which have spaces for the accommodation of teams, so that loading or unloading may be done directly between the cars and teams. To avoid the necessity of teams passing over the tracks, these are best placed on the outskirts of the yard and consist of short stub-sidings arranged in pairs. But storage tracks should have an outlet at each end so as to reduce the amount of drilling neces sary to reach a car which may be at the extreme end of a long string of cars. This is done usually by means of two "ladder" tracks, parallel to each other, which thus make the storage tracks between them of equal length.

6. The equality of length of these storage tracks is a point insisted on by many, but on the other hand, trains are not always of uniform length even on any one division. Loaded trains and trains of empties will vary greatly in length, and the various styles and weights of freight engines employed necessitate other variations in the weights and lengths of trains hauled. With storage tracks of somewhat variable length a larger percentage of track length may be utilized, there will be less hauling over a useless length of track, and (assuming that the plot of ground available for yard purposes has equally favorable conditions for yard design) more business may be handled in a yard of given area.

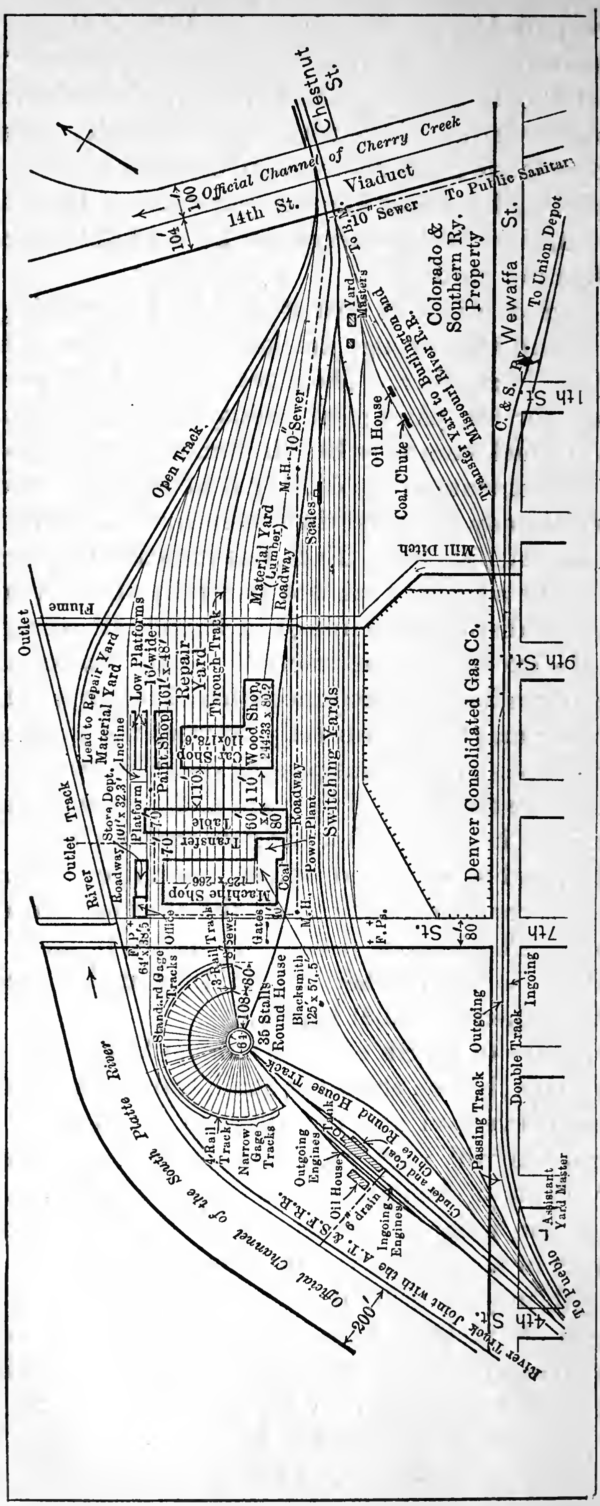


FIG. 164.—PLAN OF THE NEW SHOPS AND YARDS OF THE COLORADO & SOUTHERN RAILWAY AT DENVER.

7 Yards are preferably built so that the tracks have a grade of 0.5%—sometimes a little more than this—in the direction of the traffic through the yard. This grade, which will overcome a tractive resistance of 10 pounds per ton, will permit cars to be started down the ladder tracks by a mere push from the switching engine. They are then switched on to the desired storage track and run down that track by gravity until stopped at the desired place by a brakeman riding on the cars

8. Although not absolutely necessary, there is an advantage in having all frog numbers and switch dimensions uniform. No. 7 frogs are most commonly used. Sharper-angled frogs make easier riding, less resistance and less chance of derailment, but on the other hand require longer leads and more space. No. 6 and even No. 5 frogs are sometimes used on account of economy of space, but they have the disadvantages of greater tractive resistance, greater wear and tear on track and rolling stock, and greater danger of derailment.

296. Relation of yard to main tracks. Safety requires that there should be no connection between the yard tracks and the main tracks except at each end of the yard, where the switches should be amply protected by signals. Sometimes the main tracks run through the yard, making practically two yards—one for the traffic in either direction—but this either requires a double layout of tracks and houses (such as ash tracks, coal-chutes, sand-houses, etc.), or a very objectionable amount of crossing of the main-line tracks. The preferable method is to have the main line tracks entirely on the outside of the yard. A method which is in one respect still better is to spread the main tracks so that they run on each side of the yard. In this case there is never any necessity to cross one main track to pass from the yard to the other main track; a train may pass from the yard to either main track and still leave the other main track free and open. The ideal arrangement is that by which some of the tracks cross *over* or *under* all opposing tracks. By this means all connections between the yard and the main tracks may be by "trailing" switches; that is, trains will run on to the main track in the direction of motion on that main track. Of course all this applies only to *double* main track.

An important element of yard design is to have a few tracks immediately adjoining the main tracks and separate from the yard proper on which outgoing trains may await their orders to take

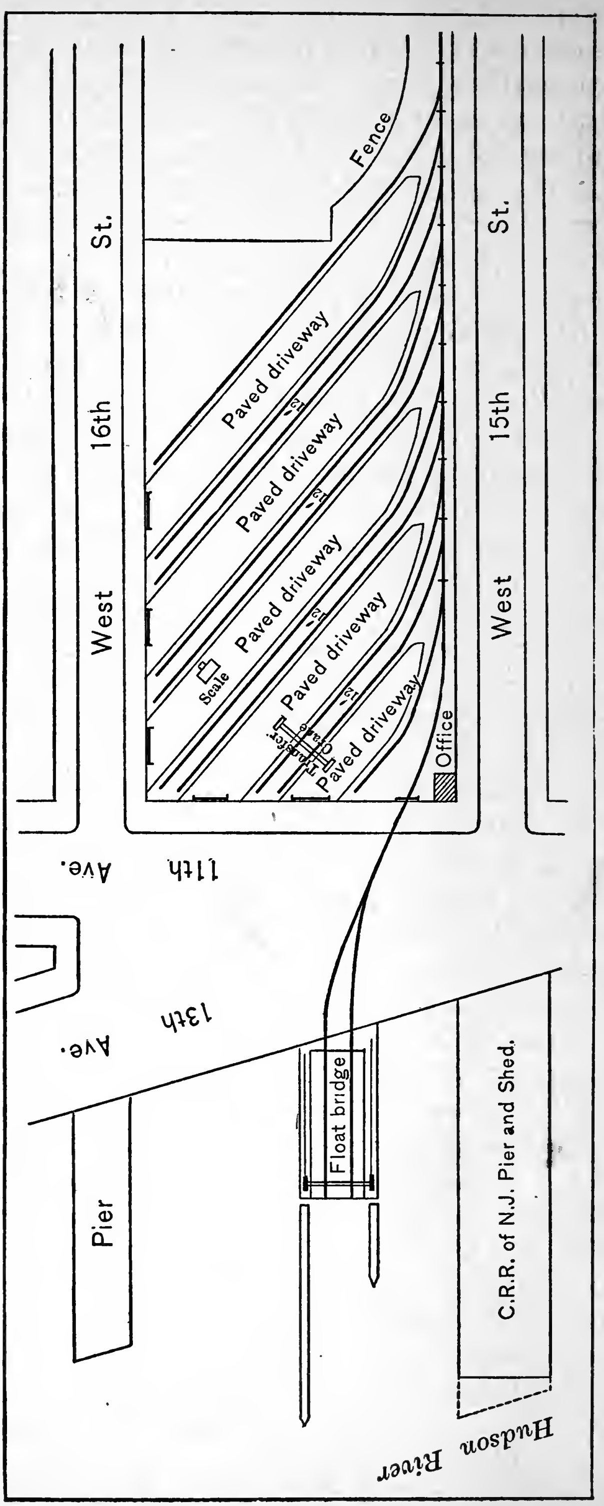


FIG. 165.—MINOR FREIGHT YARD.

the main track. When the orders come, they may start at once without any delay, without interfering with any yard operations, and they are not occupying tracks which may form part of the system needed for switching.

297. Minor freight yards. The term here refers to the sub-stations, only found in the largest cities, to which cars will be sent to save in the amount of necessary team hauling and also to relieve a congestion of such loading and unloading at the main freight terminal. The cars are brought to these yards sometimes on floats (as is done so extensively at various points around New York Harbor), or they are run down on a long siding running perhaps through the city streets. But the essential feature of these yards is the maximum utilization of every square foot of yard space, which is always very valuable and which is frequently of such an inconvenient shape that a great ingenuity is required to obtain good results. There is generally a temptation to use excessively sharp curves. When the radii are greater then 150 feet no especial trouble is encountered. Curves with radius as short as 50 feet have been used in some yards. On such curves the long cars now generally used make a sharper angle with each other than that for which the couplers were designed and special coupler-bars become necessary. The two general methods of construction are (*a*) a series of parallel team tracks (as previously described and as illustrated further in Fig. 165), and (*b*) the "loop system," as is illustrated in Fig. 166.

298. Transfer cranes. These are almost an essential feature for yards doing a large business. The transportation of built-up girders, castings for excessively heavy machinery, etc., which weigh five to thirty tons and even more, creates a necessity for machinery which will easily transfer the loads from the car to the truck and *vice versa*. An ordinary "gin-pole" will serve the purpose for loads which do not much exceed five tons. A fixed framework, covering a span long enough for a car track and a team space, with a trolley traveling along the upper chord, is the next design in the order of cost and convenience. Increasing the span so that it covers two car tracks and two team spaces will very materially increase the capacity. Making the frame movable so that it travels on tracks which are parallel to the car tracks, giving the frame a longitudinal motion equal to two or three car lengths, and finally operating the raising and traveling mechanism by power, the facility for rapidly disposing of

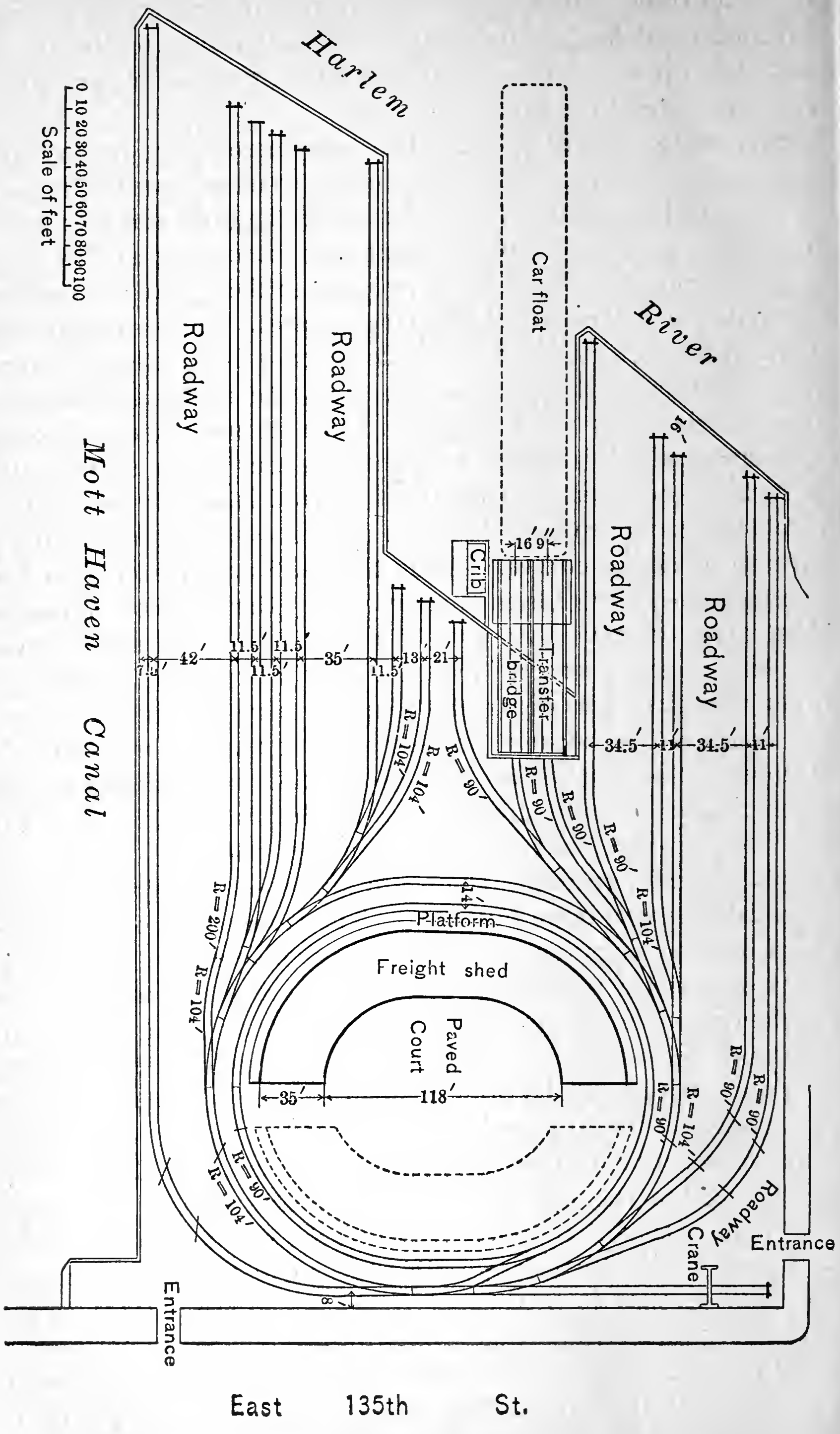


FIG. 166.—MINOR FREIGHT YARD ON A HARBOR FRONT.

heavy articles of freight is greatly increased. Of course only a very small proportion of freight requires such handling, and the business of a yard must be large or perhaps of a special character to justify and pay for the installation of such a mechanism. Figs. 165 and 166 each indicate a transfer crane, evidently of the fixed type.

299. Track scales. The location of these should be on one of the receiving tracks near the entrance to the yard, but not on the main track. It is always best to have a "dead track" over the scales—i.e., a track which has one rail on the solid side wall of the scale pit and the other supported at short intervals by posts which come up through the scale platform and yet do not touch it. These rails and the regular scale rails switch into one track by means of point rails a few feet beyond each end of the scales. The switches should be normally set so that all trains will use the dead track, unless the scales are to be operated. It has been found possible in a gravity yard to weigh a train with very little loss of time by running each car slowly by gravity over the scales and weighing them as they pass over.

ENGINE YARDS.

300. General principles. Engine yards must contain all the tracks, buildings, structures, and facilities which are necessary for the maintenance, care, and storage of locomotives and for providing them with all needed supplies. The supplies are fuel, water, sand, oil, waste, tallow, etc. Ash-pits are generally necessary for the prompt and economical disposition of ashes; engine-houses are necessary for the storage of engines and as a place where minor repairs can be quickly made. A turn-table is another all but essential requirement. The arrangement of all these facilities in an engine yard should properly depend on the form of the yard. In general they should be grouped together and should be as near as possible to the place where through engines drop the trains just brought in and where they couple on to assembled outgoing trains, so that all unnecessary running light may be avoided. In Figs. 164 and 167 are shown two designs which should be studied with reference to the relative arrangement of the yard facilities.

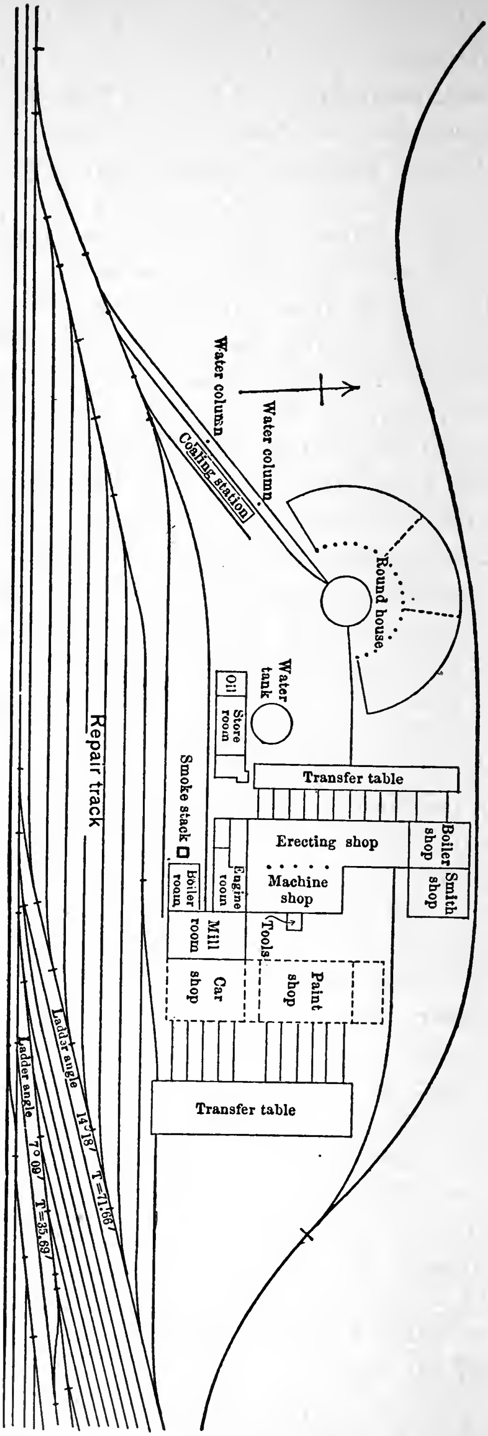


FIG. 167.—ENGINE YARD AND SHOPS, URBANA, ILL.

PASSENGER TERMINALS.

(Passenger terminals are one of the logical subdivisions of this chapter, but their construction does not concern one engineer in a thousand. The local conditions attending their construction are so varied that each case is a special problem in itself—a problem which demands in many respects the services of the architect rather than the engineer. The student who wishes to pursue this subject is referred to an admirable chapter in "Buildings and Structures of American Railroads," by Walter G. Berg, Chief Engineer of the Lehigh Valley Railroad.)

CHAPTER XIV.

BLOCK SIGNALING.

GENERAL PRINCIPLES.

301. Two fundamental systems. The growth of systems of block signaling has been enormous within the last few years—both in the amount of it and in the development of greater perfection of detail. The development has been along two general lines: (*a*) the *manual*, in which every change of signal is the result of some definite action on the part of some signalman, but in which every action is so controlled or limited or subject to the inspection of others that a mistake is nearly, if not quite, impossible; (*b*) the *automatic*, in which the signals are operated by mechanism, which cannot set a wrong signal as long as the mechanism is maintained in proper order. The fundamental principles of the two systems will be briefly outlined, after which the chief details of the most common systems will be pointed out.

302. Manual systems. Any railroad which has a telegraph line and an operator at all regular stations may (and generally does) operate its trains according to the fundamental principles of the manual block system even though it makes no claim to a block-signal system. The basic idea of such a system is that after a train has passed a given telegraph- or signal-station, no other train will be permitted to follow it into that "block" until word is telegraphed from the next station ahead that the first train has passed out of that block. With a double-track road the operation is very simple; trains may be run at short intervals with long blocks; with an average speed of 30 miles per hour and blocks 5 miles long, trains *could* be run on a ten minute interval (nearly). A road with any such traffic would, of course, have much shorter blocks, and, practically, they would need to be considerably shorter.

With a single-track road the operation is much more complex, since the operator must keep himself informed of the movements of the trains in *both* directions. The ratio of length of block to train interval would be only one half (and practically much less than half) what it could be with a double-track road When such a system is adhered to rigidly, it is called an *absolute block* system. But when operating on this system, a delay of one train will necessarily delay every other train that follows closely after. A portion, if not all, of the delay to subsequent trains may be avoided, although at some loss of safety, by a system of permissive blocking. By this system an operator may give to a succeeding train a "clearance card" which permits it to pass into the next block, but at a reduced speed and with the train under such control that it may be stopped on very short notice, especially near curves. One element of the danger of this system is the *discretionary* power with which it invests the signalmen, a discretion which may be wrongfully exercised. A modification (which is a fruitful source of collisions on single-track roads) is to order two trains to enter a block approaching each other, and with instructions to pass each other at a passing siding at which there is no telegraph-station. When the instructions are properly made out and literally obeyed, there is no trouble, but every thousandth or ten thousandth time there is a mistake in the orders, or a misunderstanding or disobedience, and a collision is the result. The telegraph line, a code of rules, a corps of operators, and signals under the immediate control of the operators, are all that is absolutely needed for the simple manual system.

303. Development of the manual system. One great difficulty with the simple system just described is that each operator is practically independent of others except as he may receive general or specific orders from a train-dispatcher at the division headquarters. Such difficulties are somewhat overcome by a very rigid system of rules requiring the signalmen at each station to keep the adjacent signalmen or the train-dispatcher informed of the movements of all trains past their own stations. When these rules (which are too extensive for quotation here) are strictly observed, there is but little danger of accident, and a neglect by any one to observe any rule will generally be apparent to at least one other man. Nevertheless the safety of trains depends on *each* signalman doing his duty, and a little carelessness or forgetfulness on the part of any one man may cause an accident. The signaling between stations *may* be done by ordinary telegraphic messages or by telephone, but is frequently done by electric bells, according to a code of signals, since these may be readily learned by men who would have more difficulty in learning the Morse code.

In order to have the signalmen mutually control each other, the "controlled manual" system has been devised. The first successful system of this kind which was brought into extensive use is the "Sykes" system, of which a brief description is as follows: Each signal is worked by a lever; the lever is locked by a latch, operated by an electro-magnet, which, with other necessary apparatus, is inclosed in a box. When a signal is set at danger, the latch falls and locks the lever, which cannot be again set free until the electro-magnet raises the latch. The magnet is energized only by a current, the circuit of which is closed by a "plunger" at the *next* station ahead; just above the plunger is an "indicator," also operated by the current, which displays the words *clear* or *blocked*. (There are variations on this detail.) When a train arrives at a block station (A), the signalman should have previously signaled to the station *ahead* (B) for permission to free the signal. The man ahead (B) pushes in the "plunger" on his instrument (assuming that the previous train has already passed him), which electrically opens the lock on the lever at the previous station (A). The signal at A can then be set at "safety." As soon as the train has passed A the signal at A must be set at "danger." A further development is a device by which the mere passage of the train over the track for a few feet beyond the signal will automatically throw the signal to "danger." After the signal once goes to danger, it is automatically locked and cannot be released except by the man in advance (B), who will not do so until the train has passed him. The "indicator" on B's instrument shows "blocked" when A's signal goes to danger after the train has passed A, and B's plunger is then locked, so that he cannot release A's signal while a train is in the block. As soon as the train has passed A, B should prepare to get his signals ready by signaling ahead to C, so that if the block between B and C is not obstructed, B may have his signals at "safety" so that the train may pass B without pausing. The student should note the great advance in safety made by the Sykes system; a signal cannot be set free except by the combined action of two men, one the man who actually operates the signal and the other the man at the station ahead, who frees the signal electrically and who by his action certifies that the block immediately ahead of the train is clear.

A still further development makes the system still more "automatic" (as described later), and causes the signal to fall to danger or to be kept locked at danger, if even a single pair of wheels comes on the rails of a block, or if a switch leading from a main track is opened.

304. **Permissive blocking.** "Absolute" blocking renders accidents due to collisions almost impossible unless an engineer runs by an adverse signal. The signal mechanism is usually so designed that, if it gets out of order, it will inevitably fall to "danger," i.e., as described later, the signal-board is counterbalanced by a weight which is much heavier. If the wire breaks, the counterweight will fall and the board will assume the horizontal position, which always indicates "danger." But it sometimes happens that when a train arrives at a signal-station, the signalman is unable to set the signal at safety. This may be because the previous train has broken down somewhere in the next block, or because a switch has been left open, or a rail has become broken, or there is a defect of some kind in the electrical connections. In such cases, in order to avoid an indefinite blocking of the whole traffic of the road, the signalman may give the engineer a "caution-card" or a "clearance card," which authorizes him to proceed slowly and with his train under complete control into the block and through it if possible. If he arrives at the next station without meeting any obstruction it merely indicates a defective condition of the mechanism, which will, of course, be promptly remedied. Usually the next section will be found clear, and the train may proceed as usual. On roads where the "controlled manual" system has received its highest development, the rules for permissive blocking are so rigid that there is but little danger in the practice, unless there is an absolute disobedience of orders.

305. **Automatic systems.** By the very nature of the case, such systems can only be used to indicate to the engineers of trains something with reference to the passage of previous trains. The complicated shifting of switches and signals which is required in the operation of yards and terminals can only be accomplished by "manual" methods, and the only automatic features of these methods consist in the mechanical checks

(electric and otherwise), which will prevent wrong combinations of signals. But for long stretches of the road, where it is only required to separate trains by at least one block length, an automatic system is generally considered to be more reliable. As expressed forcibly by a railroad manager, "an automatic system does not go to sleep, get drunk, become insane, or tell lies when there is any trouble." The same cannot always be said of the employés of the manual system.

The basic idea of all such systems is that when a train passes a signal-station (A), the signal automatically assumes the "danger" position. This may be accomplished electrically, pneumatically, or even by a direct mechanism. When the train reaches the end of the block at B and passes into the next block the signal at B will be set at danger and the signal at A will be set at safety. The lengths of the blocks are usually so great that the only practicable method of controlling from B the mechanism at A is by electricity, although the actual motive power at A may be pneumatic or mechanical. At one time the current from A to B was carried on ordinary wires. This method has the very positive advantage of reliability, definite resistance to the current, and small probability of short-circuiting or other derangement. But now all such systems use the rails for a track circuit and this makes it possible to detect the presence of a single pair of wheels on the track anywhere in the block, or an open switch, or a broken rail. Any such circumstances, as well as a defect in the mechanism, will break or short-circuit the current and will cause the signal to be set at danger. To prevent an indefinite blocking of traffic owing to a signal persistently indicating danger, most roads employing such a system have a rule substantially as follows: When a train finds a signal at danger, after waiting one minute (or more, depending on the rules), it may proceed slowly, expecting to find an obstruction of some sort; if it reaches the next block without finding any obstruction and finds the next signal clear, it may proceed as usual, but must promptly report the case to the superintendent. Further details regarding these methods will be given later. See § 310.

306. "Distant" signals. The close running of trains that is required on heavy-traffic roads, especially where several branches combine to enter a common terminal, necessitates the use of very short blocks. A heavy train running at high speed an hardly make a "service" stop in less than 2000 feet, while he curves of a road (or other obstructions) frequently make t difficult to locate a signal so that it can be seen more than a ew hundred feet away. It would therefore be impracticable o maintain the speed now used with heavy trains if the engi- neer had no foreknowledge of the condition in which he will ind a signal until he arrives within a short distance of it. To overcome this difficulty the "distant" signal was devised. This s ...ced about 1800 or 2000 feet from the "home" signal, and ...erlocked with it so that it gives the *same* signal. The dis- t... signal is frequently placed on the same pole as the home ...l of the previous block. When the engineer finds the "...nt signal "clear," it indicates that the succeeding home ...l is also clear, and that he may proceed at full speed and ...expect to be stopped at the next signal; for the distant ...al cannot be cleared until the succeeding home signal is ...red, which cannot be done until the block succeeding that ...ear. A clear distant signal therefore indicates a clear track ... two succeeding blocks. When the engineer finds the distant ...al blocked, he need not stop (providing the home signal is ...r). It simply indicates that he must be prepared to stop ...he next home signal and must reduce speed if necessary. ...nay happen that by the time he reaches the succeeding home ...al it has already been cleared, and he may proceed without ...pping. This device facilitates the rapid running of trains, ...h no loss of safety, and yet with but a moderate addition to ... signaling plant.

...07. **"Advance" signals.** It sometimes becomes necessary ... locate a signal a few hundred feet short of a regular passen- ...-station. A train might be halted at such a signal because ... was not cleared from the signal-station ahead—perhaps a ...le or two ahead. For convenience, an "advance" signal ...ay be erected immediately beyond the passenger-station. The train will then be permitted to enter the block as far as the advance signal and may deliver its passengers at the station. The advance signal is interlocked with the home signal back of it, and cannot be cleared until the home signal is cleared and the entire block ahead is clear. In one sense it adds another block, but the signal is entirely controlled from the signal station back of it.

MECHANICAL DETAILS.

308. Signals. The primitive signal is a mere cloth flag. A better signal is obtained when the flag is suspended in a suitable place from a fixed horizontal support, the flag weighted at the bottom, and so arranged that it may be drawn up and out of sight by a cord which is run back to the operator's office. The next step is the substitution of painted wood or sheet metal for the cloth flag, and from this it is but a step to the standard semaphore on a pole, as is illustrated in Fig. 168. The simple flag, operated for convenience with a cord, is the signal employed on thousands of miles of road, where they perhaps make no claim to a block-signal system, and yet where the trains are run according to the fundamental rules of the simple manual block method.

Semaphore boards. These are about 5 feet long, 8 inches wide at one end, and tapered to about 6 inches wide at the hinge end. The boards are fastened to a casting which has a ring to hold a red glass which may be swung over the face of a lantern, so as to indicate a red signal. "Distant" signal-boards usually have their ends notched or pointed; the "home" signal-boards are square ended. The boards are always to the *right* of the hinge when a train is approaching them. The "home" signals are generally painted red and the "distant" signals green, although these colors are not invariable. The backs of the boards are painted white. Therefore any signal-board which appears on the *left* side of its hinge will also appear *white*, and is a signal for traffic in the opposite direction, and is therefore of no concern to an engineman.

Poles and bridges. When the signals are set on poles, they are generally placed on the right-hand side of the track. When there are several tracks, four or more, a bridge is frequently built and then each signal is over its own track. When switches run off from a main track, there may be several signal-boards over one track. The upper one is the signal for the main track and the lower ones for the several switches. In Fig. 169 is shown a "bridge" with its various signal-boards controlling the several tracks and the switches running off from them.

"Banjo" signals. This name is given to a form of signal, illustrated in Fig. 170, in which the indication is taken from the

(*To face page* 330.)

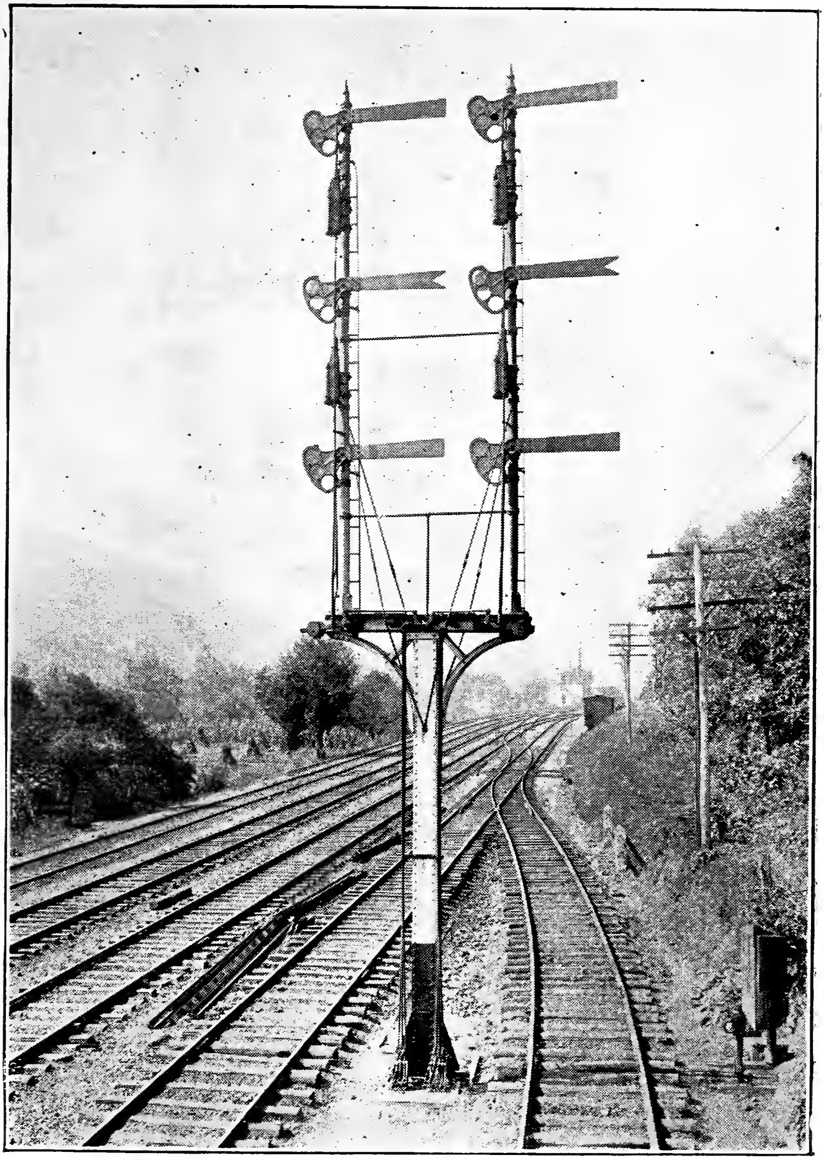


FIG. 168.—SEMAPHORES.

(*To face page 330.*)

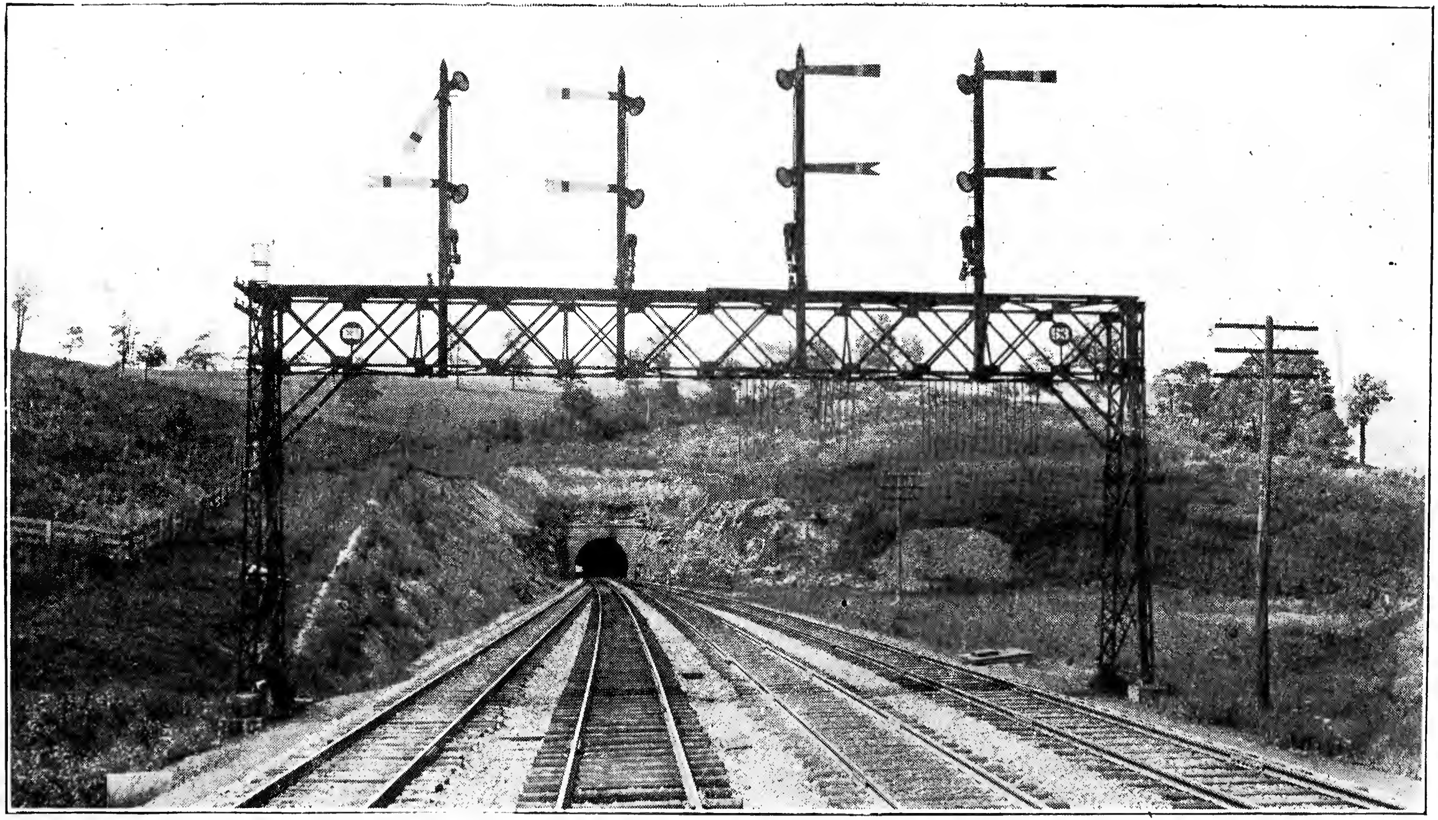


Fig. 169.—Signal Bridge.

(*To face page* 330.)

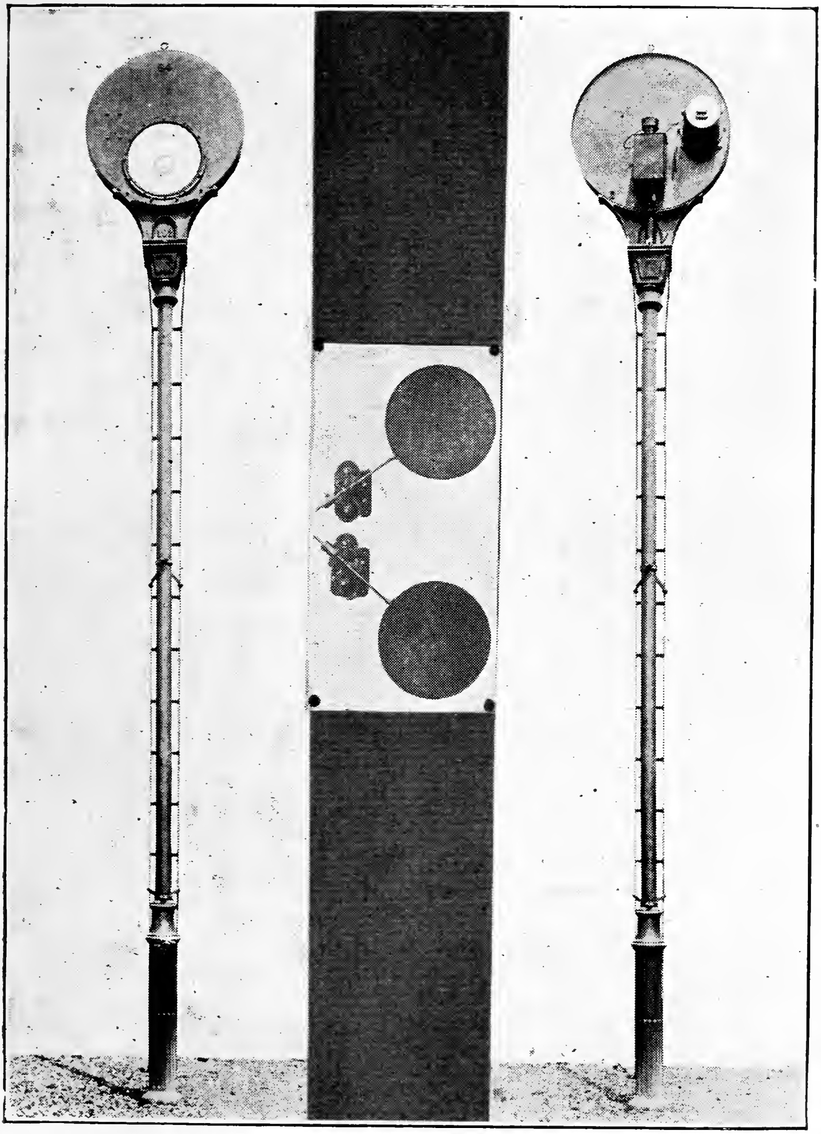


Fig. 170.—" Banjo " Signals.

color of a round disk inclosed with glass. This is the distinctive signal of the Hall Signal Company, and is also used by the Union Switch and Signal Company. The great argument in their favor is that they may be worked by an electric current of low voltage, which is therefore easily controlled; that the mechanism is entirely inside of a case, is therefore very light, and is not exposed to the weather. The argument urged against them is that it is a signal of *color* rather than *form* or *position*, and that in foggy weather the signal cannot be seen so easily; also that unsuspected color-blindness on the part of the engineman may lead to an accident. Notwithstanding these objections, this form of signal is used on thousands of miles of line in this country.

309. **Wires and pipes.** Signals are usually operated by levers in a signal-cabin, the levers being very similar to the reversing-lever of a locomotive. The distance from the levers to the signals is, of course, very variable, but it is sometimes 2000 feet. The connecting-link for the most distant signals is usually No. 9 wire; for nearer signals and for all switches operated from the cabin it may be 1-inch pipe. When not too long, one pipe will serve for both motions, forward and back. When wires are used, it is sometimes so designed (in the cheaper systems) that one wire serves for one motion, gravity being depended on for the other, but now all good systems require two wires for each signal.

Compensators. Variations of temperature of a material with as high a coefficient as iron will cause very appreciable difference of length in a distance of several hundred feet, and a dangerous lack of adjustment is the result. To illustrate: A fall of 60° F. will change the length of 1000 feet of wire by

$$1000 \times 60 \times .0000065 = 0.39 \text{ foot} = 4.68 \text{ inches.}$$

A much less change than this will necessitate a readjustment of length, unless automatic compensators are used. A compensator for pipes is very readily made on the principle illustrated in Fig. 171. The problem is to preserve the distance between a and d constant regardless of the temperature. Place the compensator half-way between a and d, or so that $ab = cd$. A fall of temperature contracts ab to ab'. Moving b to b' will cause c to move to c', in which $bb' = cc'$. But cd has also shortened to $c'd$; therefore d remains fixed in position. To avoid too great angular motion, one such compensator should be used for each 500 feet. If a line 1000 feet long is to be provided for, two compensators would be used, 250 feet from each end. Note that in operating through a compensator the *direction* of motion changes; i.e., if *a* moves to the right, *d* moves to the left, or if there is compression in *ab* there is tension in *cd*, and

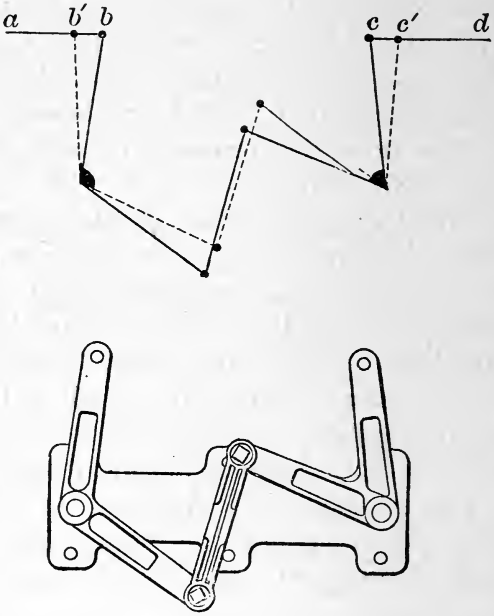


Fig. 171.—Standard Pipe Compensator.

vice versa. Therefore this form of compensator can only be used with pipes which will withstand compression. It has seemed impracticable to design an equally satisfactory compensator for wires, although there are several designs on the market.

Guides around curves and angles. When wires are required to pass around curves of large angle, pulleys are used, and a length of chain is substituted for the wire. For pipes, when the curve is easy the pipes are slightly bent and are guided through pulleys. When the angle is sharper, "angles" are used. The operation of these details is self-evident from an inspection of Fig. 172.

310. Track circuit for automatic signaling. The several systems of automatic signaling differ in the minor details, but nearly all of them agree in the following particulars. A current of low potential is run from a battery at one end of a section through one line of rails to the other end of the section, then through a relay, and then back to the battery through the other

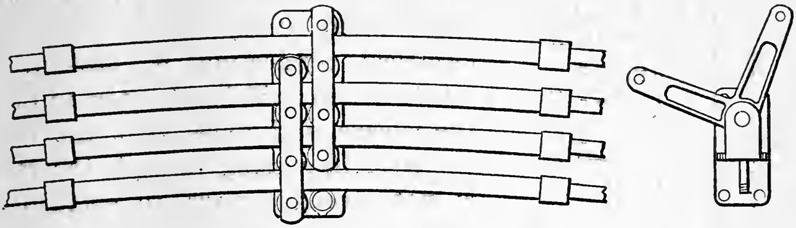


FIG. 172.—DEFLECTING-RODS.

line of rails. To avoid the excessive resistance which would occur at rail joints which may become badly rusted, a wire suitably attached to the rails is run around each joint. In order to insulate the rails of one section from the rails at either end and yet maintain the rails structurally continuous, the ends of the rails at these dividing points are separated by an insulator and the joint pieces are either made of wood or have some insulating material placed between the rails and the ordinary metal joint. The bolts must also be insulated. When the relay is energized by a current, it closes a local circuit at the signal-station, which will set the signal there at "safety." The resistance of the relay is such that it requires nearly the whole current to work it and to keep the local circuit closed. Therefore, when there is any considerable loss of current from one rail to the other, the relay will not be sufficiently energized, the local circuit will be broken, and the signal will automatically fall to danger. This diversion of current from one rail to the other before the current reaches the relay may be caused in several ways: the presence of a pair of wheels on the rails anywhere in the section will do it; also the breakage of a rail; also the opening of a switch anywhere in the section; also the presence of a pair of wheels on a siding between the "fouling point" and the switch. (The "fouling point" of a siding is that point where the rails first commence to approach the main track.) In Fig. 173 is shown all of the above details, as well as some others. At *A*, *B*, and the "fouling point" are shown the insulated joints. The batteries and signals are arranged for train motion to the *right*. When a train has passed the points near A, where the wires leave the rails for the relay, the current from the "track battery" at B will pass through the wheels and axles, and although no electrical connection is broken, so much current will be shunted through the wheels and axles that the weak current still passing through the relay is not strong enough to energize it against its spring and the "signal-magnet" circuit is broken, and the signal A goes to "danger." At the turnout the rails between the fouling point and the switch are so connected (and insulated) that a pair of wheels on these rails will produce the same effect as a pair on the main track. This is to guard against the effect of a car standing too near the switch, even though it is not on the main track. When the train passes B, if there is no other interruption of the current, the track battery at B again lenergizes the relay at A, the signal-magnet circuit at A is closed, and the signal is drawn to "safety."

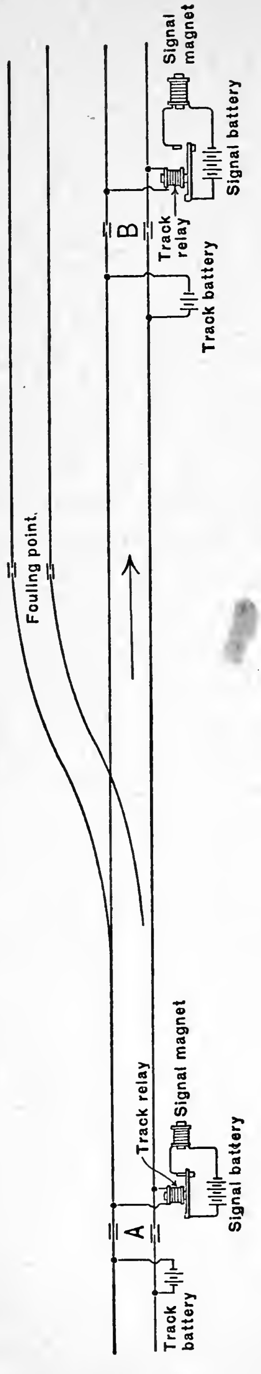


Fig. 173.

(The present edition has omitted several subdivisions of this general subject, notably the "staff system," used chiefly in England, and all discussions of "interlocking" which is an essential feature of the operation of large terminal yards. A future edition may supply these deficiencies, although an exhaustive treatment of the subject of Signaling would require a separate volume.)

CHAPTER XV.

ROLLING-STOCK.

(It is perhaps needless to say that the following chapter is in no sense a course in the design of locomotives and cars. Its chief idea is to give the student the elements of the construction of those vehicles which are to use the track which he may design—to point out the mutual actions and reactions of vehicle against track and to show the effect on track wear of variations in the design of rolling-stock. The most of the matter given has a direct practical bearing on track-work, and it is considered that all of it is so closely related to his work that the civil engineer may study it with profit.)

WHEELS AND RAILS.

311. Effect of rigidly attaching wheels to their axles. The wheels of railroad rolling-stock are invariably secured rigidly to the axles, which therefore revolve with the wheels. The chief reason for this is to avoid excessive wear between the axles and the wheels.

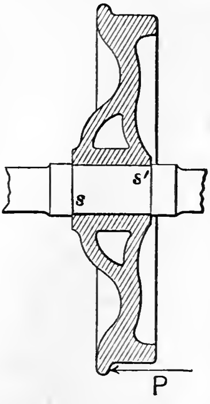


FIG. 174.

Any axle must always be somewhat loose in its journals. A sidewise force P (see Fig. 174) acting against the circumference of the wheel will produce a much greater pressure on the axle at S and S', and if the wheel moves on the axle, the wear at S and S' will be excessive. But when the axle is fitted to the wheel with a "forced fit" and does not revolve, the mere *pressure* produced at S is harmless. When two wheels are fitted tight to an axle, as in Fig. 175, and the axle revolves in the journals aa, a sidewise pressure of the rail against the wheel flange will only produce a slight and harmless increase of the journal pressure Q, although at Q there is sliding contact. Twisting action in the journals is thus practically avoided, since a small pressure at the journal-boxes at each end of the axle suffices to keep the axle truly in line.

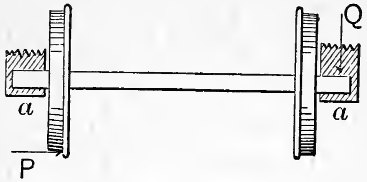


FIG. 175.

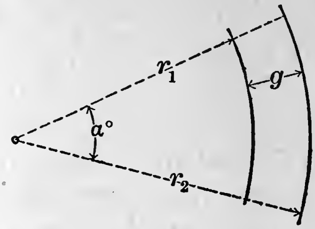


FIG. 176.

On the other hand, when the wheels are rigidly attached to their axles, both wheels must turn together, and when rounding curves, the inner rail being shorter than the outer rail, one wheel must slip by an amount equal to that difference of length. The amount of this slip is readily computable:

$$\text{Longitudinal slip} = \frac{2\pi a°}{360°}(r_2 - r_1) = \frac{2\pi g}{360°}a° = Ca°, \quad . \quad (136)$$

in which C is a constant for any one gauge, and $g =$ the track gauge $= (r_2 - r_1)$. For standard gauge (4.708) the slip is .08218 foot per degree of *central angle.* This shows that the longitudinal slipping around any curve of any given central angle will be *independent of the degree of the curve.* The constant (.08218) here given is really somewhat too small, since the true gauge that should be considered is the distance between the lines of tread on the rails. This distance is a somewhat indeterminate and variable quantity, and probably averages 4.90 feet, which would increase the constant to .086. The slipping may occur by the inner wheel slipping ahead or the outer wheel slipping back, or by both wheels slipping. The total slipping will be constant in any case. The slipping not only consumes power, but wears both the wheels and the rail. But even these disadvantages are not sufficient to offset the advantages resulting from rigid wheels and axles.

312. Effect of parallel axles. Trucks are made with two or three parallel axles (except as noted later), in order that the axles shall mutually guide each other and be kept approximately perpendicular to the rails. If the curvature is very sharp and the wheel-base comparatively long (as is notably the case on street railways at street corners), the front and rear wheels

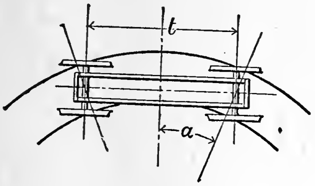


FIG. 177.

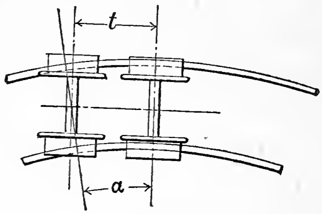


FIG. 178.

will stand at the same angle (a) with the track, as shown in Fig. 177. But it has been noticed that for ordinary degrees of curvature, the rear wheels stand radial to the curve (see Fig. 178), and for steam railroad work this is the normal case. When the two parallel axles are on a curve (as shown), the wheels tend to run in a straight line. In order that they shall run on a curve they must slip laterally. The principle is illustrated in an exaggerated form in Fig. 179. The wheel *tends* to roll from a toward b. Therefore in passing along the track from a to c it must actually slip laterally an amount bc which equals $ac \sin a$.

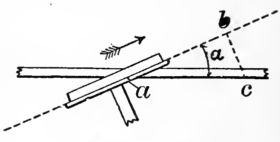


FIG. 179.

Let t=length of the wheel-base (Figs. 177 and 178); r=radius of curve; then for the first case (Fig. 177), $\sin a = t \div 2r$; for the second and usual case (Fig. 178), $\sin a = t \div r$; for t=5 feet and r=radius of a 1° curve, $a = 0° \ 03'$ for the second case. a varies (practically) as the degree of curve. The lateral slipping *per unit* of distance traveled therefore equals $\sin a$. As an illustration, given a 5-foot wheel-base on a 5° curve, $a = 0° \ 15'$, $\sin a = .00436$, and for each 100 feet traveled along the curve the lateral slip of the front wheels would be 0.436 foot. There would be no lateral slipping of the rear wheels, assuming that the rear axle maintained itself radial.

From the above it might be inferred that the flanges of the forward wheels will have much greater wear than those of the rear wheels. Since cars are drawn in both directions about equally, no difference in flange wear due to this cause will occur, but locomotives (except switching-engines) run forward almost exclusively, and the excess wear of the front wheels of the pilot- and tender-trucks is plainly observable.

For a given curve the angle *a* (and the accompanying resistance) is evidently greater the greater the distance between the axles. On the other hand, if the two axles are very close together, there will be a tendency for the truck to twist and the wheels to become jammed, especially if there is considerable play in the gauge. The flange friction would be greater and would perhaps exceed the saving in lateral slipping. A general rule is that the axles should never be closer together than the gauge.

Although the slipping per unit of length along the curve varies directly as the degree of curvature, the length of curve necessary to pass between two tangents is inversely as the degree of curve, and the total slipping between the two tangents is independent of the degree of curve. Therefore when a train passes between two tangents, the total slipping of the wheels on the rails, longitudinal and lateral, is a quantity which depends **only** on the **central angle** and is **independent** of the **radius** or **degree of curve.**

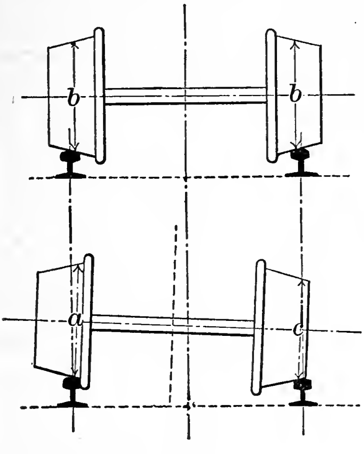


Fig. 180.

313. Effect of coning wheels. The wheels are always set on the axle so that there is some "play" or chance for lateral motion between the wheel-flanges and the rail. The treads of the wheel are also "coned." This coning and play of gauge are shown in an exaggerated form in Fig. 180. When the wheels are on a tangent, although there will be occasional oscillations from side to side, the normal position will be the symmetrical position in which the circles of tread *bb* are equal. When centrifugal force throws the wheel-flange against the rail, the circle of tread *a* is larger than *b*, and much larger than *c*; therefore the wheels will tend to roll in a circle whose radius equals the slant height of a cone whose elements would pass through the unequal circles *a* and *c*. If this radius equaled the radius of the track, and if the axle were free to assume a radial position, the wheels would roll freely on the rails without any slipping or flange pressure. Under such ideal conditions, coning would be a valuable device, but it is impracticable to have all axles radial, and the radius of curvature of the track is an extremely variable quantity. It has been demonstrated that with parallel axles the influence of coning diminishes as the distance between the axle increases, and that the effect is practically inappreciable when the axles are spaced as they are on locomotives and car-trucks. The coning actually used is very slight (see Chapter XV, § 332) and has a different object. It is so slight that even if the axles were radial it would only prevent the slipping on a very light curve—say a 1° curve.

314. **Effect of flanging locomotive driving-wheels.** If all the wheels of all locomotives were flanged it would be practically impossible to run some of the longer types around sharp curves. The track-gauge is always widened on curves, and especially on sharp curves, but the widening would need to be excessive to permit a consolidation locomotive to pass around an 8° or 10° curve if all the drivers were flanged. The action of the wheels on a curve is illustrated in Figs. 181, 182, and 184. All small truck-wheels are flanged. The rear drivers are always flanged and four-driver engines usually have all the drivers flanged. Consolidation engines have only the front and rear drivers flanged. Mogul and ten-wheel engines have one pair of drivers blank. On Mogul engines it is always the middle pair. On ten-wheel engines, when used on a road having sharp curves, it is preferable to flange the front and rear driving-wheels and use a "swing bolster" (see § 315); when the curvature is easy, the middle and rear drivers may be flanged and the truck made with a rigid center. The blank drivers have the same total width as the other drivers and of course a much wider tread, which enables these drivers to remain on the rail, even though the curvature is so sharp that the tread overhangs the rail considerably.

315. **Action of a locomotive pilot-truck.** The purpose of the pilot-truck is to guide the front end of a locomotive around a curve and to relieve the otherwise excessive flange pressure that would be exerted against the driver-flanges. There are two classes of pilot-trucks—(a) those having fixed centers and (b) those having shifting centers. This second class is again subdivided into two classes, which are radically different in their action—(b_1) four-wheeled trucks having two parallel axles

and (b_2) two-wheeled trucks which are guided by a "radius-bar." The action of the four-wheeled fixed-centered truck (a) is shown in Fig. 181. Since the center of the truck is forced

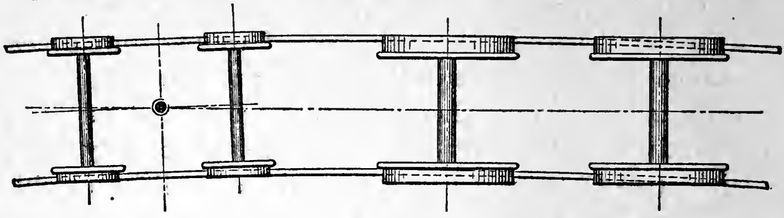


FIG. 181.—FIXED CENTER PILOT-TRUCK.

to be in the center of the track, the front drivers are drawn away from the outer rail. The rear outer driver tends to roll *away* from the outer rail rather than toward it, and so the effect

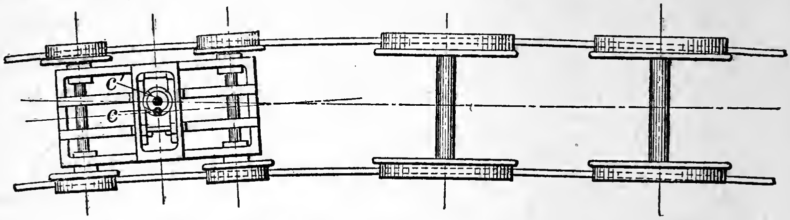


FIG. 182.—FOUR-WHEELED TRUCK—SHIFTING CENTER.

of the truck is to relieve the driver-flanges of any excessive pressure due to curvature. The only exception to this is the case where the curvature is sharp. Then the front inner driver may be pressed against the *inner* rail, as indicated in Fig. 181. This limits the use of this type of wheel-base on the sharper curves.

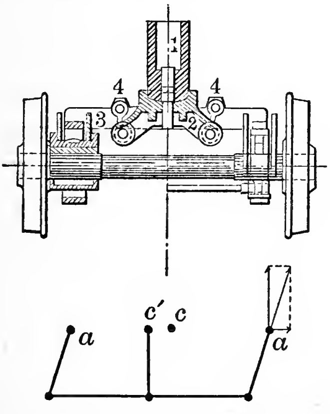


FIG. 183.—ACTION OF SHIFTING CENTER.

The next type—(b_1) four-wheeled trucks with shifting centers—is much more flexible on sharp curvature; it likewise draws the front drivers away from the outer rail. The relative position of the wheels is shown in Fig. 182, in which c' represents the position of center-pin and c the displaced truck center. The structure and action of the truck is shown in Fig. 183. The "center-pin" (1) is supported on the "truck-bolster" (2), which is hung by the "links" (4) from the "cross-ties" (3). The links are therefore in tension and when the wheels are forced to one side by the rails the *links* are inclined and the front of the engine is drawn inward by a force equal to the weight on the bolster times the tangent of the angle of inclination of the links. This assumes that all links are vertical when the truck is in the center. Frequently the opposite links are normally inclined to each other, which somewhat complicates the above simple relation of the forces, although the general principle remains identical.

The two-wheeled pilot-truck with shifting center is illustrated in Fig. 184. The figure shows the facility with which

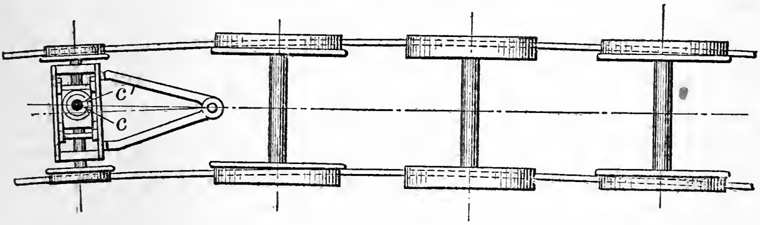


FIG. 184.—TWO-WHEELED TRUCK—SHIFTING CENTER.

an engine with long wheel-base may be made to pass around a comparatively sharp curve by omitting the flanges from the middle drivers and using this form of pilot-truck. As in the previous case, the eccentricity of the center of the truck relative to the center-pin induces a centripetal force which draws the front of the engine inward. But the swing-truck is not the only source of such a force. If the "radius-bar pin" were placed at O' (see Fig. 185), the truck-axle would be radial. But the radius-bar is always made somewhat shorter than this, and the pin is placed at O, a considerable distance ahead of O', thus creating a tendency for the truck to run toward the inner rail and draw the front of the locomotive in that direction. This tendency will be objectionably great if the radius-bar is made too short, as has been practically demonstrated in cases when the radius-bar has been subsequently lengthened with a resulting improvement in the running of the engine.

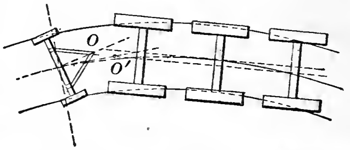


FIG. 185.—ACTION OF TWO-WHEELED TRUCK.

LOCOMOTIVES.

GENERAL STRUCTURE.

316. Frame. The frame or skeleton of a locomotive consists chiefly of a collection of forged wrought-iron bars, as shown in Figs. 186 and 187. These bars are connected at the

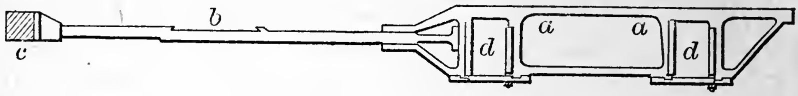


FIG. 186.—ENGINE-FRAME.

front end by the "bumper" (c), which is usually made of wood. A little further back they are rigidly connected at bb by the cylinders and boiler-saddle. The boilers rest on the frames at $aaaa$ by means of "pads," which are bolted to the fire-box, but which permit a free expansion of the boiler along the frame. This expansion is sometimes as much as $\frac{5}{16}''$. On a "consolidation" engine (frame shown in Fig. 187) it is frequently

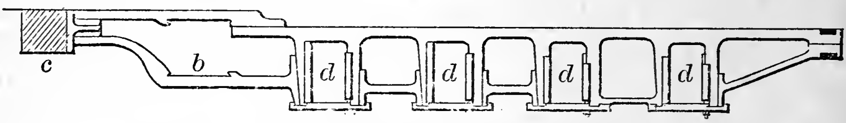


FIG. 187.—ENGINE-FRAME—CONSOLIDATION TYPE.

necessary to use vertical swing-levers about 12″ long instead of "pads." The swinging of the levers permit all necessary expansion. At the back the frames are rigidly connected by the iron "foot-plate." The driving-axles pass through the "jaws" $dddd$, which hold the axle-boxes. The frame-bars have a width (in plan) of 3″ to 4″. The depth (at a) is about the same. Fig. 186 shows a frame for an "American" type of locomotive; Fig. 187 shows a frame for a "Consolidation" type (see § 323).

317. Boiler. A boiler is a mechanism for transferring the latent heat of fuel to water, so that the water is transformed from cold water into high-pressure steam, which by its expansion will perform work. The efficiency of the boiler depends largely on its ability to do its work rapidly and to reduce to a minimum the waste of heat through radiation. The boiler contains a fire-box (see Fig. 188), in which the fuel is burned. The gases of consumption pass from the fire-box through the numerous boiler-tubes into the "smoke-box" S and out through the smoke-stack. The fire-box consists of an inner and outer shell separated by a layer of water about 3″ thick. The exposure of water-surface to the influence of the fire is thus very complete. The efficiency of this transferal of heat is somewhat indicated by the fact that, although the temperature of the gases in the fire-box is probably from 3000° to 4000° F., the temperature in the smoke-box is generally reduced to 500° to

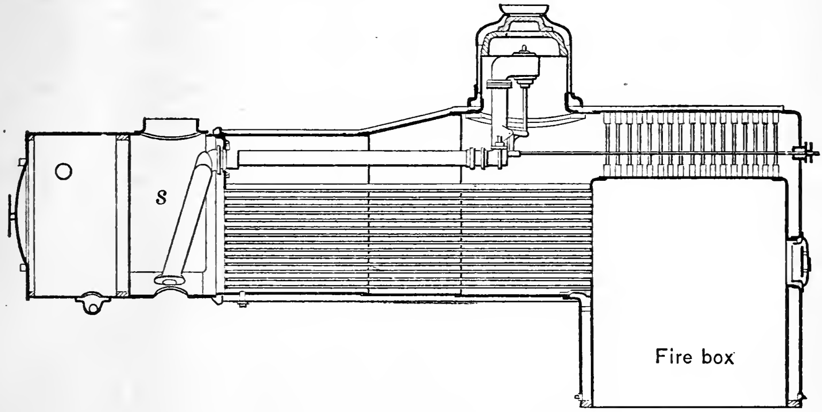


FIG. 188.—LOCOMOTIVE-BOILER.

600° F. If the steam pressure is 180 lbs., the temperature of the water is about 380° F., and, considering that heat will not pass from the gas to the water unless the gas is hotter than the water, the water evidently absorbs a large part of the theoretical maximum. Nevertheless gases at a temperature of 600° F. pass out of the smoke-stack and such heat is utterly wasted.

The tubes vary from $1\frac{3}{4}''$ to $2''$, inside diameter, with a thickness of about $0''.10$ to $0''.12$. The aggregate cross-sectional area of the tubes should be about one eighth of the grate area. The number will vary from 140 to 250. They are made as long as possible, but the length is virtually determined by the type and length of engine.

318. Fire-box. The fire-box is surrounded by water on the four sides and the top, but since the water is subjected to the boiler pressure, the plates, which are about $\frac{5}{16}''$ thick, must be stayed to prevent the fire-box from collapsing. This is easily accomplished over the larger part of the fire-box surface by having the outside boiler-plates parallel to the fire-box plates and separated from them by a space of about 3″. The plates are then mutually held by "stay-bolts." See Fig. 189. These are about $\frac{7}{8}''$ in diameter and spaced 4″ to $4\frac{1}{2}''$. The $\frac{3}{16}''$ hole, drilled $1\frac{1}{4}''$ deep, indicated in the figure, will allow the escape of steam if the bolt breaks just behind the plate, and thus calls attention to the break. The stay-bolts are turned down to a diameter equal to that at the root of the screw-threads. This method of supporting the fire-box sheets is used for the two sides, the entire rear, and for the front of the fire-box up to the boiler-barrel. The "furnace tube-sheet"—the upper part of the front of the fire-box—is stayed by the tubes. But the top of the fire-box is troublesome. It must always be covered with water so that it will not be "burned" by the intense heat. It must therefore be nearly, if not quite, flat. There are three general methods of accomplishing this.

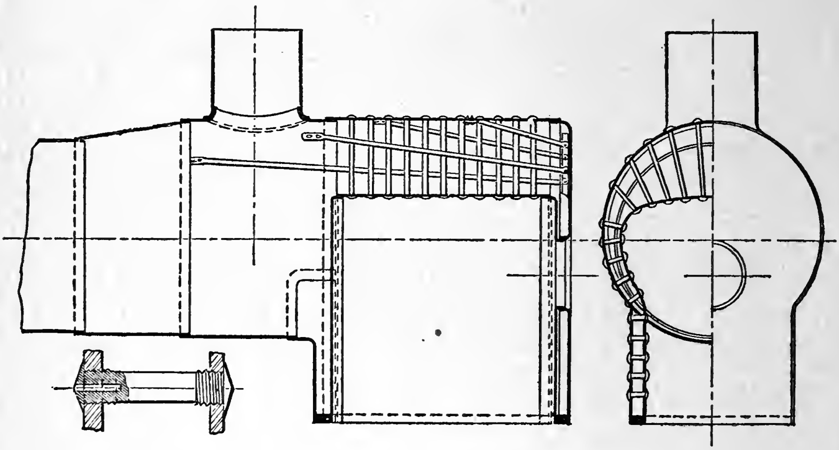


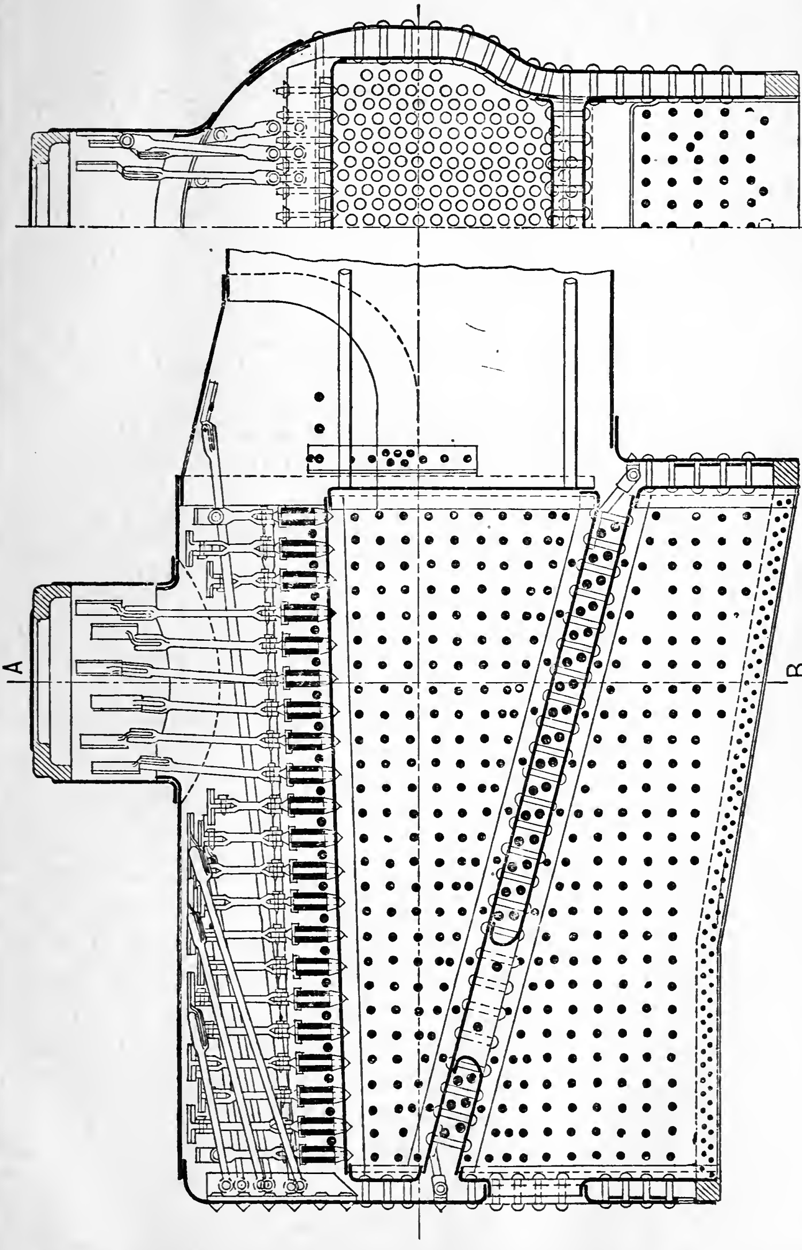
FIG. 189. FIG. 190.

(a) **Radial stays.** This construction is indicated in Fig. 190. Incidentally there is also shown the diagonal braces for resisting the pressure on the back end of the boiler above the fire-box. It may be seen that the stays are not perpendicular to either the crown-sheet or the boiler-plate. This is objectionable and is obviated by the other methods.

(b) **Crown-bars.** These bars are in pairs, rest on the side furnace-plates, and are further supported by stays. See Fig. 191.

(c) **Belpaire fire-box.** The boiler above the fire-box is rectangular, with rounded corners. The stays therefore are perpendicular to the plates. See Fig. 192.

Fire-brick arches. These are used, as shown in Fig. 193, to force all the gases to circulate through the upper part of the fire-box. Perfect combustion requires that all the carbon shall be turned into carbon dioxide, and this is facilitated by the forced circulation.



Half-section through *AB*.

FIG. 191.—BOILER SHOWING CROWN-BARS AND WATER-TABLE.

Water-tables. The same object is attained by using a water-table instead of a brick arch—as shown in Fig. 191. But it has the further advantages of giving additional heating-surface and avoiding the continual expense of maintaining the bricks. One

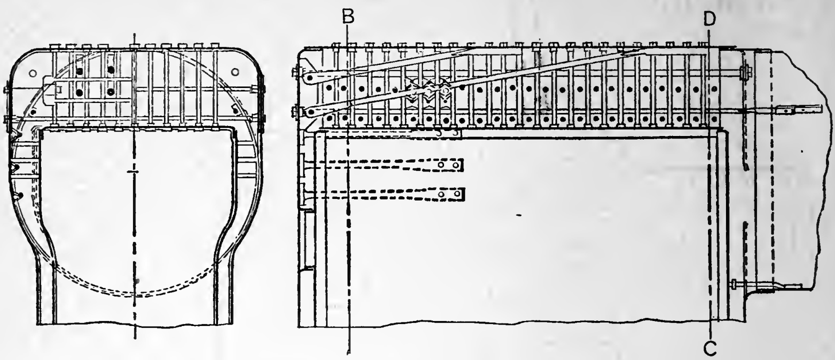


FIG. 192.—"BELPAIRE" FIRE-BOX.
Half-section through AB. Half-section through CD.

feature of the design is the use of a number of steam-jets which force air into the fire-box and assist the combustion.

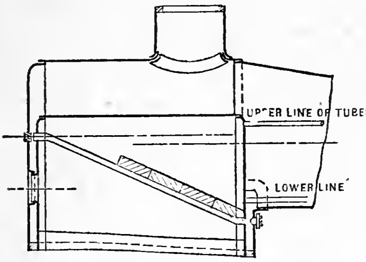


FIG. 193.—FIRE-BRICK ARCH.

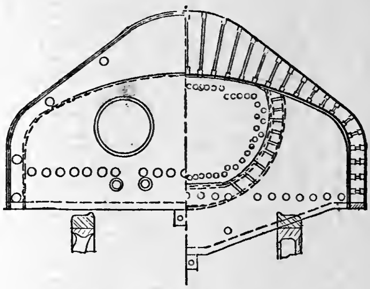


FIG. 194.—WOOTTEN FIRE-BOX.

Area. Fire-boxes are usually limited in width to the practicable width between the wheels—thus giving a net inside width of about 3 feet and a maximum length of 10 to 11 feet—this being about the maximum distance over which the firemen can properly control the fire. About 37 square feet is the maximum area obtainable except when the "Wootten" fire-box is used—illustrated in Fig. 194. Here the grate is raised above the driving-wheels and has (in the case shown) a width of 8′ $0\frac{1}{8}$″. The fire-box area is over 76 square feet. Note that *two* furnace-doors are used.

9. Coal consumption. No form of steam-boiler (except a boiler for a steam fire-engine) requires as rapid production of steam, considering the size of the boiler and fire-box, as a locomotive. The combustion of coal per square foot of grate per hour for stationary boilers averages about 15 to 25 lbs. and seldom exceeds that amount. An ordinary maximum for a locomotive is 125 lbs. of coal per square foot of grate-area per hour, and in some recent practice 220 lbs. have been used. Of course such excessive amounts are wasteful of coal, because a considerable percentage of the coal will be blown out of the smoke-stack unconsumed, the draft necessary for such rapid consumption being very great. The only justification of such rapid and wasteful coal consumption is the necessity for rapid production of steam. The best quality of coal is capable of evaporating about 14 lbs. of water per pound of coal, i.e., change it from water at 212° to steam at 212°; the heat required to change water at ordinary temperatures to steam at ordinary working pressure is (roughly) about 20% more. From 6 to 9 lbs. of water per pound of coal is the average performance of ordinary locomotives, the efficiency being less with the higher rates of combustion. Some careful tests of locomotive coal consumption gave the following figures: when the consumption of coal was 50 lbs. per square foot of grate-area per hour, the rate of evaporation was 8 lbs. of water per pound of coal. When the rate of coal consumption was raised to 180, the evaporation dropped to 5 lbs. of water per pound of coal. It has been demonstrated that the efficiency of the boiler is largely increased by an increased length of boiler-tubes. The actual consumption of coal per mile is of course an exceedingly variable quantity, depending on the size and type of the engine and also on the work it is doing—whether climbing a heavy grade with its maximum train-load or running easily over a level or down grade. A test of a 50-ton engine, running without any train at about 20 to 25 miles per hour, showed an average consumption of 21 lbs. of coal per mile. Statistics of the Pennsylvania Rail road show a large increase (as might be expected, considering the growth in size of engines and weight of trains) in the average number of pounds of coal burned per *train*-mile—some of the figures being 55 lbs. in 1863, 72 lbs. in 1872, and nearly 84 lbs. in 1883. Figures are published showing an average consumption of about 10 lbs. of coal per passenger-car mile, and 4 to 5 lbs. per freight-car mile. But these figures are always obtained by dividing the total consumption per train-mile by the number of cars, the coal due to the weight of the engine being thrown in Wellington developed a rule, based on the actual performance of a very large number of passenger-trains, that the number of pounds of coal per mile $=21.1+6\ 74$ times the number of passenger-cars. The amount of coal assigned to the engine agrees remarkably with the test noted above For freight-trains the amount assigned to the engine should be much greater (since the engine is much heavier), and that assigned to the individual cars much less, although the great increase in freight-car weights in recent years has caused an increase in the coal required per car.

320. Heating-surface. The rapid production of steam requires that the hot gases shall have a large heating-surface to which they can impart their heat. From 50 to 75 square feet of heating-surface is usually designed for each square foot of grate-area. A more recently used rule is that there should be from 60 to 70 square feet of *tube* heating-surface per square foot of grate-area for bituminous coal. 40 or 50 to 1 is more desirable for anthracite coal Almost the whole surface of the fire-box has water behind it, and hence constitutes heating-surface. Although this surface forms but a small part of the total (nominally), it is really the most effective portion, since the difference of temperature of the gases of combustion and the water is here a maximum, and the flow of heat is therefore the most rapid. The heating-surface of the tubes varies from 85 to 93% of the total, or about 7 to 15 times the heating-surface in the fire-box. Sometimes the heating-surface is as much as 2300 square feet, but usually it is less than 2000, even for engines which must produce steam rapidly.

Some of the most recent locomotives have greatly exceeded these figures One just constructed for the New York Central and Hudson River Railroad has the following figures: heating-surface, 3500 sq. ft ; grate-area, 50 sq ft ; cylinders, $21'' \times 26''$; total weight, 176000 lbs ; weight on drivers, 95000 lbs.; drivers, 79″ diameter; with 85% of the boiler pressure, it developed an adhesion of 24700 lbs., which represented a factor of adhesion of $\frac{1}{3.85}$

Another rule used by designers is that the engine should have 1 sq ft of heating-surface for each 50 of 60 lbs of weight, efficiency being indicated by a low weight. For the above engine the ratio is 53

321. Loss of efficiency in steam pressure. The effective work done by the piston is never equal to the theoretical energy contained in the steam withdrawn from the boiler. This is due chiefly to the following causes:

(*a*) The steam is "**wire-drawn**," i.e., the pressure in the cylinder is seldom more than 85 to 90% of the boiler pressure. This is due largely to the fact that the steam-ports are so small that the steam cannot get into the cylinder fast enough to exert its full pressure. It is often purposely wire-drawn by partially closing the throttle, so that the steam may be used less rapidly.

(*b*) **Entrained water.** Steam is always drawn from a dome placed over the boiler so that the steam shall be as far above the water-surface as possible, and shall be as dry as possible. In spite of this the steam is not perfectly dry and carries with it water at a temperature of, say, 361°, and pressure of 140 lbs per square inch. When the pressure falls during the expansion and exhaust, this hot water turns into steam and absorbs the necessary heat from the hot cylinder-walls. This heat is then carried out by the exhaust and wasted.

(*c*) The **back pressure** of the exhaust-steam, which depends on the form of the exhaust-passages, etc. This amounts to from 2 to 20% of the power developed.

(*d*) **Clearance-spaces.** When cutting off at full stroke this waste is considerable (7 to 9%), but when the steam is used expansively the steam in these clearance-spaces expands and so its power is not wholly lost.

(*e*) **Radiation.** In spite of all possible care in jacketing the cylinders, some heat is lost by radiation.

(*f*) **Radiation into the exhaust-steam.** This is somewhat analogous to (*b*). Steam enters the cylinder at a temperature of, say, 361°; the walls of the cylinder are much cooler, say 250°; some heat is used in raising the temperature of the cylinder-walls; some steam is vaporized in so doing; when the exhaust is opened the temperature and pressure fall; the heat temporarily absorbed by the cylinder-walls is reabsorbed by the exhaust-steam, re-evaporating the vapor previously formed, and thus a certain portion of heat-energy goes **through** the cylinder without doing any useful work. With an early cut-off the loss due to this cause is very great.

The sum of all these losses is exceedingly variable. They are usually less at lower speeds. The loss in *initial pressure* (the difference between boiler pressure and the cylinder pressure at the beginning of the stroke) is frequently over 20%, but this is not all a net loss With an early cut-off the average cylinder pressure for the whole stroke is but a small part of the boiler pressure, yet the horse-power developed may be as great as, or greater than that developed at a lower speed, later cut-off, and higher average pressure

322. Tractive power The work done by the two cylinders during a complete revolution of the drivers evidently = area of pistons × average steam pressure × stroke × 2 × 2. The resistance overcome evidently = tractive force at circumference of drivers times distance traveled by drivers (which is the circumference of the drivers) Therefore

$$\text{Tractive force} = \left\{ \frac{\text{area pistons} \times \text{average steam pressure} \times \text{stroke} \times 2 \times 2.}{\text{circumference of drivers}} \right.$$

Dividing numerator and denominator by π (3 1415), we have

$$\text{Tractive force} = \left\{ \frac{(\text{diam piston})^2 \times \text{average steam pressure} \times \text{stroke}}{\text{diameter of driver}} \right., \quad . \quad (137)$$

which is the usual rule Although the rule is generally stated in this form, there are several deductions In the first place the net effective area of the piston is less than the nominal on account of the area of the piston-rod. The ratio of the areas of the piston-rod and piston varies, but the effect of this reduction is usually from 1.3 to 1 7% No allowance has been made for friction—of the piston, piston-rod, cross-head, and the various bearings This would make a still further reduction of several per cent. Nevertheless the above simple rule is used, because, as will be shown, no great accuracy can be utilized.

The tractive force is limited by the adhesion between the drivers and the rails, and this is a function of the weight on the drivers. Under the most favorable conditions this has been tested to amount to one-third the weight on the drivers, but such a ratio cannot be depended on Wellington used the ratio one-fourth The Baldwin Locomotive Works in their "Locomotive Data" give tables and diagrams based on $\frac{1}{4}$, $\frac{9}{40}$, and $\frac{1}{5}$ adhesion. As low a value as $\frac{1}{6}$ or even $\frac{1}{7}$ is occasionally used, but such a low rate of adhesion would only be found when the rails were abnormally slippery. In a well-designed locomotive the tractive force, as computed above, and the tractive adhesion should be made about equal. The uncertainty in the coefficient of adhesion shows the futility of any refinement in the computation of the tractive force.

It is only at very slow speeds that an engine can utilize all of its tractive force. When running at a high speed, the utmost horse-power that the engine can develop will only produce a draw-bar pull, which is but a small part of the possible tractive force. Power is the product of force times velocity. If the power is constant and the velocity increases, the force must decrease. This fact is well shown in the figures of some tests of a locomotive. The dimensions were as follows: cylinders, $18'' \times 24''$; drivers, 68″; weight on drivers, 60000 lbs.; heating-surface, 1458 sq. ft.; grate-area, 17 sq. ft. During one test the average cylinder pressure was 83.3 lbs. (boiler pressure, 145); 14-inch cut-off and throttle $\frac{3}{4}$ open). By the above formula (137),

$$\text{Tractive force} = \frac{18^2 \times 83.3 \times 24}{68} = 9525 \text{ lbs.}$$

At $\frac{1}{4}$ adhesion the tractive force was 15000 lbs; even at $\frac{1}{5}$ adhesion, it would be 12000 lbs. This shows that at the speed of this test (26.3 in. per hour) scarcely more than $\frac{2}{3}$ of the tractive power was utilized. A still more marked case, shown by another test with the same engine, taken when the speed was 53.4 miles per hour, indicated an average cylinder pressure of 37.2 lbs., the throttle being $\frac{1}{8}$ open and the valves cutting off at 8″. In this case the tractive power, computed as before, equals 4254 lbs., about $\frac{1}{14}$ of the weight on the drivers and about $\frac{1}{3}$ of the tractive force which is possible at slow speeds. In the first case, the tractive power (9525) times the speed in feet per second (38.57) divided by 550 gives the indicated horse-power, 668. In the second case, although the tractive forces developed was so much less, the speed was much greater and the horse-power was about the same, 606.

The above figures illustrate some of the foregoing statements regarding loss of efficiency. In both cases the steam was wire-drawn. The boiler pressure was 145 lbs., but when the throttle was only $\frac{3}{4}$ open and the steam was cut-off at 14″ (24″ stroke) the average steam pressure in the cylinder was reduced to 83.3 lbs. With the throttle but $\frac{1}{3}$ open and the valves cutting off at 8″ ($\frac{1}{3}$ of the stroke), the average pressure was cut down to 37.2 lbs.—about $\frac{1}{4}$ of the boiler pressure. Note that the heating-surface per square foot of grate-area ($1458 \div 17 = 86$) is very large (see § 320). Note also that the horse-power developed divided by the grate-area (17) gives 39 and 36 H.P. per square foot of grate-area. This is exceptionally large—25 or 30 being a more common figure.

The maximum tractive power is required when a train is starting, and fortunately it is at low velocities that the maximum tractive force can be developed. The motion of the piston is so slow that there is but little reduction of steam pressure, and the valves are generally placed to cut off at full stroke. For the above engine, with 145 lbs. boiler pressure, the absolute maximum of tractive force is $\frac{18^2 \times 145 \times 24}{68} =$ 16581 lbs. Of course, this maximum would never be reached unless the boiler pressure were increased. A common rule is to consider that the average effective cylinder pressure for slow speed and full stroke will be 80% of the boiler pressure. This would reduce the tractive force to the (nominal) value of 13265 lbs., and the corresponding cylinder pressure would be 116 lbs. per square inch. With an effective cylinder pressure of about 131 lbs. the tractive power is 15000 lbs., which is $\frac{1}{4}$ of the total weight on the drivers. This illustrates the general rule, stated above, that the cylinders, drivers, and boiler pressure should be so proportioned that the maximum tractive force should about equal the maximum adhesion which could be obtained.

As another numerical example, the dimensions of a recently constructed heavy consolidation engine are quoted. The cylinders are 24″×32″; diameter of drivers, 54″; total weight of engine and tender, 391400 lbs.; weight of engine, 250300 lbs.; weight on drivers, 225200 lbs.; capacity of tender, 7500 gallons; the boiler has 406 tubes, $2\frac{1}{4}$″ in diameter and 15′ long; fire-box, 132″×$40\frac{1}{4}$″; heating-surface of tubes, 3564 sq. ft.; of fire-box, 241 sq. ft.—total, 3805 sq. ft.; boiler pressure, 220 lbs. per square inch. Applying Eq. 132, we may compute 75093 lbs. as the absolute maximum of tractive power. In fact this is an unattainable limit, for reasons before stated. The tractive force is given as 63000, which corresponds to an effective cylinder pressure of about 185 lbs., about 84% of the boiler pressure. This tractive force is 28% of the weight on the drivers, a tractive ratio of 1:3.6.

RUNNING GEAR.

323. **Types of running gear.** (a) **"American."** This was once the almost universal type for both passenger and freight service. It is still very commonly used for passenger service, but it is not the best form for heavy freight work.

(b) **"Columbia."** Four drivers, one pair of pilot-truck wheels and one pair of trailing wheels behind the drivers. The low trailing wheels permit a desirable enlargement of the fire-box. This is a recent type, used exclusively for passenger service.

(c) **"Atlantic."** Similar to *b* except that the pilot-truck has four wheels instead of two.

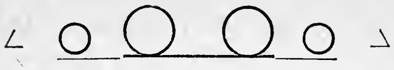
(d) **"Mogul."** These are used for both passenger and freight service, but are not well adapted for either high speed or great tractive power.

(e) **"Ten-wheel."** Similar to *d* except that the pilot-truck has four wheels instead of two. The use is similar to that of *d*.

(f) **"Consolidation."** The present standard for freight service. It permits great tractive power without excessive concentrated loads on the track.

(g) **Switching-engines.** These have four or six (and exceptionally even eight or ten) drivers and no truck-wheels. They are only adapted for slow speed when a maximum of tractive power is needed. Sometimes the water-tank and even a small fuel-box is loaded on. Since fuel is always near at hand for a yard-engine, the fuel-box need not be large.

(h) **"Double-enders."** As explained in § 315, truck-wheels are needed in front of the drivers to guide them around curves. If an ordinary engine is run backward, the flanges of the rear drivers will become badly worn, and if the speed is high, the danger of derailment is considerable. In suburban service,



when the runs are short, it is preferable to run the engines forward and backward, rather than turn them at each end of the route. Therefore a pilot-truck is placed at each end.

(i) **"Miscellaneous types."** Almost every conceivable combination of drivers and truck-wheels has been used. The "Mastodon" is similar to the "Consolidation" except that the pilot-truck has four wheels instead of two. The "Decapod" has ten driving-wheels. The "Forney" (named after the inventor) has been very extensively used on elevated roads. The weight of the boiler and machinery is carried on four driving-wheels; the engine-frame is extended so as to include a small tank and fuel-box, the weight of which is chiefly supported by a truck of two or four wheels. They run best when running "backward," i.e., tender first.

324. Equalizing-levers. The ideal condition of track, from the standpoint of smooth running of the rolling stock, is that the rails should always lie in a plane surface. While this condition is theoretically possible on tangents, it is unobtainable on curves, and especially on the approaches to curves when the outer rail is being raised. Even on tangents it is impossible to *maintain* a perfect surface, no matter how perfectly the track may have been laid. In consequence of this, the points of contact of the wheels of a locomotive, or even of a four-wheeled truck, will not ordinarily lie in one plane. The rougher and more defective the track, the worse the condition in this respect. Since the frame of a locomotive is practically rigid, if the frame rests on the driver-axles through the medium of springs at each axle-bearing, the compression of the springs (and hence the pressure of the drivers on the rail) will be variable if the bearing-points of the drivers are not in one plane This variable pressure affects the tractive power and severely strains the frame. Applying the principle that a tripod will stand on an even surface, a mechanism is employed which *virtually* supports the locomotive on three points, of which one is usually the center-bearing of the forward truck. On each side the pressure is so distributed among the drivers that even if a driver rises or falls with reference to the others, the load carried by each driver is unaltered, and that side of the engine rises or falls by one nth of the rise or fall of the single driver, where n represents the number of wheels. The principle involved is shown in an exaggerated form in Fig. 195. In the diagram, MN represents the normal position of the frame when the wheels are on line. The frame is supported by the hangers at a, c, f, and h. ab, de, and gh are horizontal levers vibrating about the points H, K, and L, which are supported by the axles. While it is *possible* with such a system of levers to make MN assume a position not parallel with its natural position, yet, by an extension of the principle that a beam balance loaded with equal weights will always be horizontal, the effect of raising or lowering a wheel will be to move MN parallel to itself.

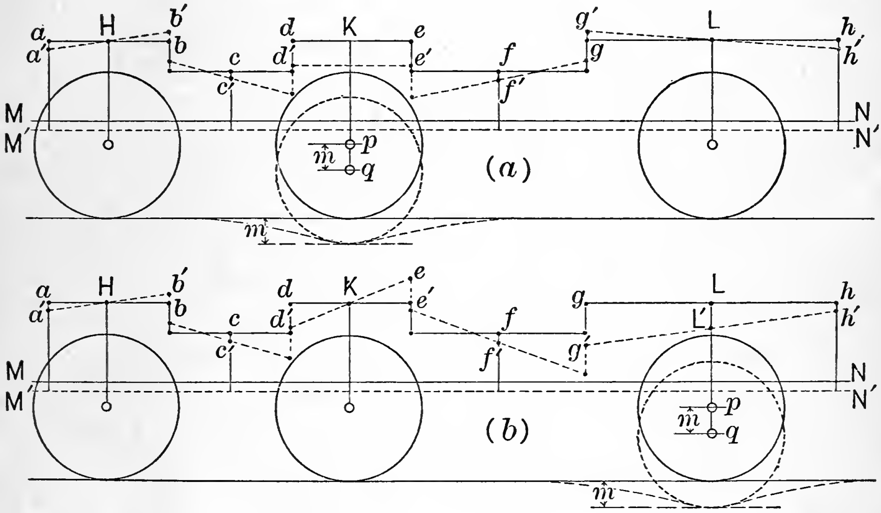


FIG. 195.—ACTION OF EQUALIZING-LEVERS.

It only remains to determine *how much* is the motion of MN relative to the rise or drop of the wheel.

The dotted lines represent the positions of the wheels and levers when one wheel drops into a depression. The wheel center drops from p to q, a distance m. L drops to L', a distance m (see Fig. 195, b); M drops to M', an unknown distance x; therefore $aa'=x$; $bb'=x$; $cc'=x$; $dd'=3x=ee'$; $ff'=x$; $\therefore gg'=5x$; $hh'=x$; $LL'=\frac{1}{2}(gg'+hh')=\frac{1}{2}(6x)=m$; $\therefore x=\frac{1}{3}m$; i.e., MN drops, parallel to itself, $1/n$ as much as the wheel drops, where n is the number of wheels. The resultant effect caused by the simultaneous motion of two wheels with reference to the third is evidently the algebraic sum of the effects of each wheel taken separately.

The practical benefits of this device are therefore as follows:

(*a*) When any driver reaches a rough place in the track, a high place or a low place, the stress in all the various hangers and levers is unchanged.

(*b*) The motion of the frame (represented by the bar MN in Fig. 195) is but $1/n$ of the motion of the wheel, and the jar and vibration caused by a roughness in the track is correspondingly reduced.

The details of applying these principles are varied, but in general it is done as follows;

(a) **American and ten-wheeled types.** Drivers on each side form a system. The center-bearing pilot-truck is the third point of support. The method is illustrated in Fig. 196.

(b) **Mogul and consolidation types.** The front pair of drivers is connected with the two-wheeled pilot-truck (as illustrated in Fig. 197) to form one system. The remaining drivers on each side are each formed into a system

The device of equalizers is an American invention. Until recently it has not been used on foreign locomotives. The necessity for its use becomes less as the track is maintained with greater perfection and is more free from sharp curves. A locomotive not equipped with this device would deteriorate very rapidly on the comparatively rough tracks which are usually found on light-traffic roads. It is still an open question to what extent the neglect of this device is responsible for the statistical fact that average freight-train loads on foreign trains are less in proportion to the weight on the drivers than is the case with American practice. The recent increasing use of this device on foreign heavy freight locomotives is perhaps an acknowledgment of this principle.

325. **Counterbalancing.** At very high velocities the centrifugal force developed by the weight of the rotating parts becomes a quantity which cannot be safely neglected. These rotating parts include the crank-pin, the crank-pin boss, the side rod, and that part of the weight of the connecting-rod which may be considered as rotating about the center of the crank-driver. As a numerical illustration, a driving-wheel 62″ in diameter, running 60 miles per hour, will revolve 325 times per minute. The weights are:

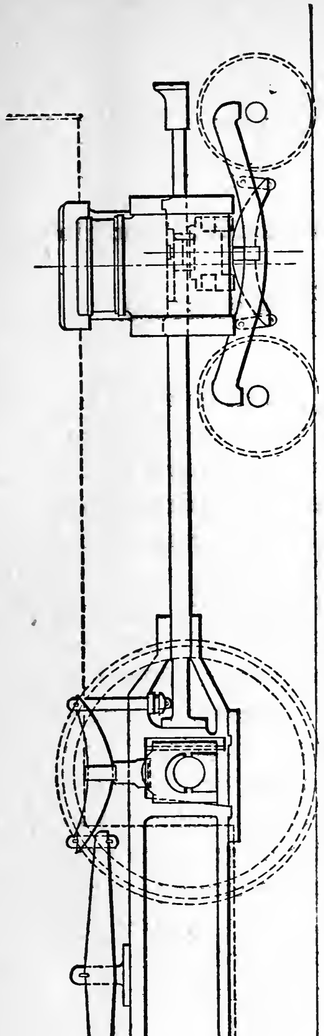


FIG. 196.—EQUALIZING-LEVERS FOR "AMERICAN" TYPE.

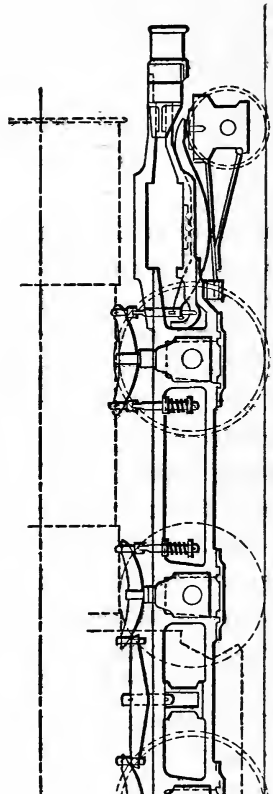


FIG. 197.—EQUALIZING-LEVERS FOR "MOGUL" TYPE.

Crank-pin	110	lbs.
" boss	150	"
One-half side rod	240	"
Back end of connecting-rod	190	"
Total	690	lbs.

If the stroke is 24″, the radius of rotation is 12″, or 1 foot. Then

$$\frac{Gv^2}{gr}=\frac{690\times4\pi^2 1^2\times325^2}{32.2\times1\times60^2}=24821 \text{ lbs.},$$

which is half as much again as the weight on a driver, 16000 lbs. Therefore if *no* counterbalancing were used, the pressure between the drivers and the rail would always be less (at any velocity) when the crank-pin was at its highest point. At a velocity of about 48 miles per hour the pressure would become zero, and at higher velocities the wheel would actually be thrown from the rail. As an additional objection, when the crank-pin was at the lowest point, the rail pressure would be increased (velocity 60 miles per hour) from 16000 lbs. to nearly 41000 lbs., an objectionably high pressure. These injurious effects are neutralized by "counterbalancing." Since all of the above-mentioned weights can be considered as concentrated at the center of the crank-pin, if a sufficient weight is so placed in the drivers that the center of gravity of the eccentric weight is diametrically opposite to the crank-pin, this centrifugal force can be wholly balanced. This is done by filling up a portion of the space between the spokes. If the center of gravity of the counterbalancing weight is 20″ from the center, then, since the crank-pin radius is 12″, the required weight would be $690\times\frac{12}{20}=414$ lbs.

In addition to the effect of these revolving parts there is the effect of the sudden acceleration and retardation of the reciprocating parts. In the engine above considered the weights of these reciprocating parts will be:

Front end of connecting-rod	150	lbs.
Cross-head	174	"
Piston and piston-rod	300	"
Total	624	lbs.

Assume as before that the reciprocating parts may be considered as concentrated at one point, the point P of the diagram in Fig. 198. Since the motion of P is horizontal only,

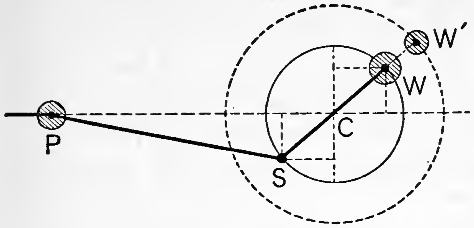


FIG. 198.—ACTION OF COUNTERBALANCE.

the force required to overcome its inertia at any point will exactly equal the *horizontal component* of the force required to overcome the inertia of an *equal* weight at S, revolving in a circular path. Then evidently the horizontal component of the force required to keep W in the circular path will exactly balance the force required to overcome the inertia of P. Of course $W=P$. But a smaller weight W', whose weight is inversely proportional to its radius of rotation, will evidently accomplish the same result. In the above numerical case, if the center of gravity of the counterweights is 20″ from the center, the required weight to completely counterbalance the reciprocating parts would be $624\times\frac{12}{20}=374.4$ lbs. This counterweight need not be all placed on the driver carrying the main crank-pin, but can be (and is) distributed among all the drivers. Suppose it were divided between the two drivers in the above case. At 60 miles per hour such a counterweight would produce an additional pressure of 11211 lbs. when the counterweight was down, or a lifting force of the same amount when the counterweight was up. Although this is not sufficient to lift the driver from the rail, it would produce an objectionably high pressure on the rail (over 27000 lbs.), thus inducing just what it was desired to avoid on account of the eccentric rotating parts. Therefore a compromise must be made. Only a portion (one half to three fourths) of the weight of the reciprocating parts is balanced. Since the effect of the rotating weights is to cause variable pressure on the rail, while the effect of the reciprocating parts is to cause a horizontal wobbling or "nosing" of the locomotive, it is impossible to balance both. Enough counterweight is introduced to partially neutralize the effect of the reciprocating parts, still leaving some tendency to horizontal wobbling, while the counterweights which were introduced to reduce the wobbling cause some variation of pressure. By using hollow piston-rods of steel, ribbed cross-heads, and connecting- and side-rods with an I section, the weight of the reciprocating parts may be greatly lessened without reducing their strength, and with a decrease in weight the effect of the unbalanced reciprocating parts and of the "excess balance" (that used to balance the reciprocating parts) is largely reduced.

Current practice is somewhat variable on three features:

(*a*) The proportion of the weight of the connecting-rod which should be considered as revolving weight.

(*b*) The proportion of the total reciprocating weight that should be balanced.

(*c*) The distribution among the drivers of the counterweight to balance the reciprocating parts.

An exact theoretical analysis of (*a*) shows that it is a function of the weights and dimensions of the reciprocating parts. The weight which may be considered as revolving equals *

$$W_1\left(\frac{r^2+k^2-rd\left(1+\frac{r}{l}\right)}{l^2-r^2}\right)+W_2\frac{r^2}{l^2-r^2},$$

in which r=radius of the crank, l=length of connecting-rod, k=distance of center of gyration from wrist-pin, d=distance of center of gravity from wrist-pin, W_1=weight of connecting-rod in pounds, and W_2=weight of piston, piston-rod, and cross-head in pounds; all dimensions in feet. An application of this formula will show that for the dimensions of usual practice, from 51 to 57% of the weight of the connecting-rod should be considered as revolving weight.

The principal rules which have been formulated for counter-balancing may be stated as follows:

1. Each wheel should be balanced correctly for the revolving parts connected with it.

2. *In addition,* introduce counterbalance sufficient for 50% of the weight of the reciprocating parts for ordinary engines, increasing this to 75% when the reciprocating parts are excessively heavy (as in compound locomotives) or when the engine

* R. A. Parke, in R. R. Gazette, Feb. 23, 1894.

is light and unable to withstand much lateral strain or when the wheel-base is short.

3. Consider the weight of the connecting-rod as $\frac{1}{2}$ revolving and $\frac{1}{2}$ reciprocating when it is over 8 feet long; when shorter than 8 feet, consider $\frac{6}{10}$ of the weight as revolving and $\frac{4}{10}$ as reciprocating.

4. The part of the weight of the connecting-rod considered as revolving should be entirely balanced in the crank-driver wheel.

5. The "excess balance" should be divided equally among the drivers.

6. Place the counterbalance as near the rim of the wheel as possible and also as near the outside of the wheel as possible in order that the center of gravity shall be as near as possible opposite the center of gravity of the rods, etc., which are all outside of even the plane of the face of the wheel.

In Fig. 199 is shown a section of a locomotive driver with the cavities in the casting for the accommodation of the lead which is used for the counterbalance weight. Incidentally several other features and dimensions are shown in the illustration.

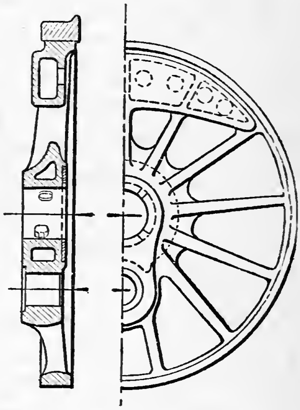


Fig. 199.—Section of Locomotive-driver.

326. Mutual relations of the boiler power, tractive power, and cylinder power for various types. The design of a locomotive includes three *distinct* features which are varied in their mutual relations according to the work which the engine is expected to do.

(a) **The boiler power.** This is limited by the rate at which steam may be generated in a boiler of admissible size and weight. Engines which are designed to haul very fast trains which are comparatively light must be equipped with very large grates and heating surfaces so that steam may be developed with great rapidity in order to keep up with the very rapid consumption. Engines for very heavy freight work are run at very much lower velocity and at a lower piston speed in spite of the fact that more strokes are required to cover a given distance and the demand on the boiler for *rapid* steam production is not as great as with high-speed passenger-engines. The capacity of a boiler to produce steam is therefore limited by the limiting weight of the general type of engine required. Although improvements may be and have been made in the design of fire-boxes so as to increase the steam-producing capacity without adding proportionately to the weight, yet there is a more or less definite limit to the boiler power of an engine of given weight.

(b) **The tractive power.** This is a function of the weight on the drivers. The absolute limit of tractive adhesion between a steel-tired wheel and a steel rail is about one third of the pressure, but not more than one fourth of the weight on the drivers can be depended on for adhesion and wet rails will often reduce this to one fifth and even less. The tractive power is therefore absolutely limited by the practicable weight of the engine. In some designs, when the maximum tractive power is desired, not only is the entire weight of the boiler and running gear thrown on the drivers, but even the tank and fuel-box are loaded on. Such designs are generally employed in switching-engines (or on engines designed for use on abnormally heavy mountain grades) in which the maximum tractive power is required, but in which there is no great tax on the boiler for *rapid* steam production (the speed being always very low), and the boiler and fire-box, which furnish the great bulk of the weight of an engine, are therefore comparatively light, and the requisite weight for traction must, therefore, be obtained by loading the drivers as much as possible. On the other hand, engines of the highest speed cannot possibly produce steam fast enough to maintain the required speed unless the load be cut down to a comparatively small amount. The tractive power required for this comparatively small load will be but a small part of the weight of the engine, and therefore engines of this class have but a small proportion of their weight on the drivers; generally have but two driving-axles and sometimes but one.

(c) **Cylinder power.** The running gear forms a mechanism which is simply a means of transforming the energy of the boiler into tractive force and its power is unlimited, within the practical conditions of the problem. The power of the running gear depends on the steam pressure, on the area of the piston, on the diameter of the drivers, and on the ratio of crank-pin radius to wheel radius, or of stroke to driver diameter. It is always possible to increase one or more of these elements by a relatively small increase of expenditure until the cylinders are able to make the drivers slip, assuming a sufficiently great resistance. Since the power of the engine is limited by the power of its weakest feature, and since the running gear is the most easily controlled feature, the power of the running gear (or the "cylinder power") is always made somewhat excessive on all well-designed engines. It indicates a badly designed engine if it is stalled and unable to move its drivers, the steam pressure being normal. If it is attempted to use a freight-engine on *fast* passenger service, it will probably fail to attain the desired speed on account of the steam pressure falling. The tractive power and cylinder power are superabundant, but the boiler cannot make steam as fast as it is needed for high speed, especially when the drivers are small. The practical result would be a comparatively low speed kept up with a forced fire. If it is attempted to use a high-speed passenger-engine on heavy freight service, the logical result is a slipping of the drivers until the load is reduced. The boiler power and cylinder power are ample, but the weight on the drivers is so small that the tractive power is only sufficient to draw a comparatively small load.

These relations between boiler, cylinder, and tractive power are illustrated in the following comparative figures referring to a fast passenger-engine, a heavy freight-engine, and a switching-engine. The weights of the passenger- and freight-engines are about the same, but the passenger-engine has only 72% of

Kind.	Cylinders.	Total W ght.	Wt. on Driv'rs	Heating Surface, sq. ft.	Grate area sq. ft.	Steam Pressure in Boiler.	$\frac{\text{Stroke.}}{\text{Diam. Driver.}}$
Fast passenger.	19″×24″	126700	81500	1831.8	26.2	180	$\frac{24}{78} = .31$
Heavy freight.	20″×24″	128700	112600	1498.3	31.5	140	$\frac{24}{50} = .48$
Switcher......	19″×24″	109000	109000	1498.0	22.8	160	$\frac{24}{50} = .48$

the tractive power of the freight. But the passenger-engine has 22% more heating-surface and can generate steam much faster; it makes less than two thirds as many strokes in covering a given distance, but it runs at perhaps twice the speed and probably consumes steam much faster. The switch-engine is lighter in total weight, but the tractive power is nearly as great as the freight and much greater than the passenger-engine. While the heating-surfaces of the freight- and switching-engines are practically identical, the grate area of the switcher is much less; its speed is always low and there is but little necessity for rapid steam development.

While these figures show the general tendency for the relative proportions, and in this respect may be considered as typical, there are large variations. The recent enormous increase in the dead weight of passenger-trains has necessitated greater tractive power. This has been provided sometimes by using "Mogul" and "ten-wheel" engines, which were originally designed for freight work. On the other hand, the demand for fast freight service, and the possibility of safely operating such trains by the use of air-brakes, has required that heavy freight-engines shall be run at comparatively high speeds, and that requires the rapid production of steam, large grate areas, and heating surfaces. But in spite of these variations, the normal standard for passenger service is a four-driver engine carrying about two thirds of the weight of the engine on the drivers, which are very large; the normal standard for freight work is the "consolidation," with perhaps 90% of the weight on the drivers, which are small, but which must have the pony truck for such speed as it uses; and finally the normal standard for switching service has all the weight on the drivers and has comparatively low steam-producing capacity.

327. Life of locomotives. The life of locomotives (as a whole) may be taken as about 800000 miles or about 22 to 24 years. While its life should be and is considered as the period between its construction and its final consignment to the scrap pile, parts of the locomotive may have been renewed more than once. The boiler and fire-box are especially subject to renewal. The mileage life is much longer than formerly. This is due partly to better design and partly to the custom of drawing the fires less frequently and thereby avoiding some of the destructive strains caused by extreme alterations of heat and cold. Recent statistics give the average annual mileage on twenty-three leading roads to be 41000 miles.

CARS.

328. **Capacity and size of cars.** The capacity of freight-cars has been enormously increased of late years. About thirty years ago the usual live-load capacity for a box-car was about 20000 lbs. In 1893 the standard box-car, gondola-cars, etc., of the Pennsylvania Railroad on exhibition at the Chicago Exposition, had a live-load capacity of 60000 lbs. and a dead weight of 30000 to 33000 lbs With a full load, the weight on each wheel is nearly 12000 lbs , which equals or exceeds the load usually placed on the drivers of ordinary locomotives. But now cars with a live-load capacity of 80000 lbs. are almost standard, 100000-lb. cars are very common, and even larger cars are made for special service. (See Fig. 200.)

The limitation of the carrying capacity for some kinds of freight depends somewhat on the amount of live load that can be carried within given dimensions; for the cross-section of a car is limited to the extreme dimensions which may be safely run through the tunnels and through bridges as at present constructed, and the length is somewhat limited by the difficulty of properly supporting an excessively heavy load, distributed over an unusually long span, by a structure which is subjected to excessive jar, concussion, compression, and tension. The cross-sectional limit seems to have been scarcely reached yet, except, perhaps, in the case of furniture and carriage-cars, whose load per cubic foot is not great. The usual width of freight-cars is about 8 to 9 feet, while parlor-cars and sleepers are generally 10 feet wide and sometimes 11 feet. The highest point of a train is usually the smoke-stack of the locomotive which is generally 14 feet above the rails and occasionally over 15 feet. A sleeping-car usually has the highest point of the car about 14 feet above the rails. Box-cars are usually about 8 feet high (above the sills), with a total height of about 11′ 3″. Refrigerator-cars are usually about 9′ high and furniture-cars about 10′ above the sills, the truck adding about 3′ 3″. The usual length is 34 feet, but 35 to 40 feet is not uncommon. Passenger-cars (day coaches) are usually 50 feet long, exclusive of the end platforms and weigh 45000 to 50000 lbs. Sixty passengers at 150 pounds apiece (a high average) will only add 9000 lbs. to the weight. A parlor-car or sleeper is generally about 65 feet long exclusive of the platforms, which add about 6′ 6″. The weight is anywhere from 60000 to 80000 lbs.

The weight of the 25 or 30 passengers it *may* carry is hardly worth considering in comparison.

329. Stresses to which car-frames are subjected. A car is structurally a truss, supported at points at some distance from the ends and subjected to transverse stress. There is, therefore, a change of flexure at two points between the trucks. Besides this stress the floor is subjected to compression when the cars are suddenly stopped and to tension when in ordinary motion, the tension being greater as the train resistance is greater and as the car is nearer the engine. The tension is sometimes relieved by means of continuous drawbars (see § 331), but this affords no relief against impact during compression, which is really more destructive. The shocks, jars, and sudden strains to which the car-frames are subjected are very much harder on them than the mere static strains due to their maximum loads if the loads were quiescent. Consequently any calculations based on the static loads are practically valueless, except as a very rough guide, and previous experience must be relied on in designing car bodies. As evidence of the increasing demand for strength in car-frames, it has been recently observed that freight-cars, built some years ago and built almost entirely of wood, are requiring repairs of wooden parts which have been *crushed* in service, the wood being perfectly sound as regards decay.

330. The use of metal. The use of metal in car construction is very rapidly increasing.. The demand for greater strength in car-frames has grown until the wooden framing has become so heavy that it is found possible to make steel frames and trucks at a small additional cost, the steel frames being twice as strong and yet reducing the dead weight of the car about 5000 lbs., a consideration of no small value, especially on roads having heavy grades. Another reason for the increasing use of metal is the great reduction in the price of rolled or pressed steel, while the cost of wood is possibly higher than before. The advocates of the use of steel advise steel floors, sides, etc. For box-cars a wooden floor has advantages. For ore and coal-cars an all-metal construction has advantages. (Fig. 201.) In Germany, where steel frames have been almost exclusively in use for many years, they have not yet been able to determine the normal age limit of such frames; none have yet *worn* out. The life is estimated at 50 to 80 years.

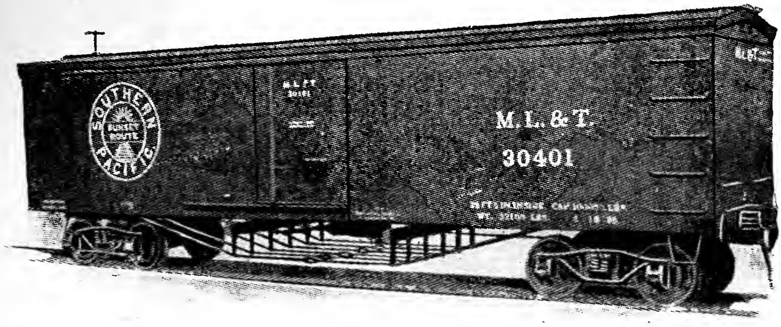


FIG. 200.—100,000-LB. BOX CAR.

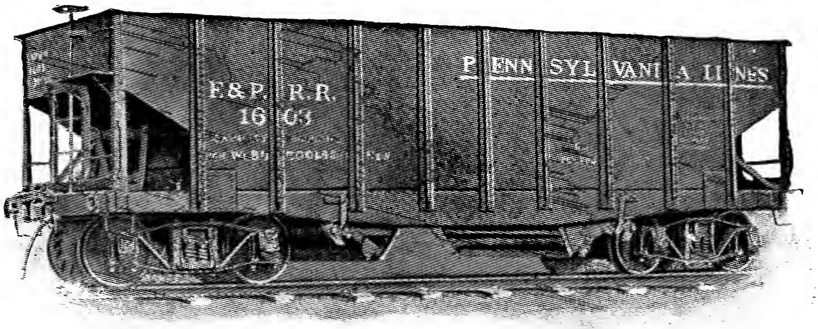


FIG. 201.—STEEL COAL CAR.

FIG. 202.—WOODEN BOX CAR; STEEL FRAME.

(*To face page* 366.)

Brake-beams are also best made of metal rather than wood, as was formerly done. Metal brake-beams are generally used on cars having air-brakes, as a wooden beam must be excessively large and heavy in order to have sufficient rigidity.

Truck-frames (see Fig. 203), which were formerly made principally of wood, are now largely made of pressed steel. It makes a reduction in weight of about 3000 lbs. per car. The increased durability is still an uncertain quantity.

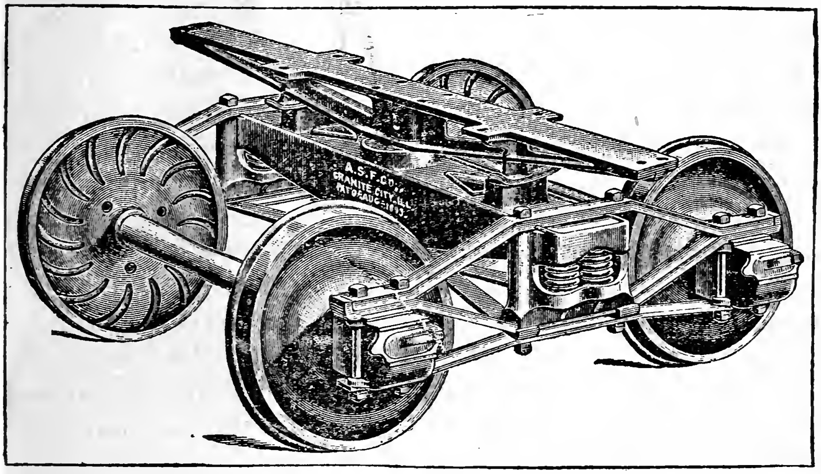


FIG. 203.

331. **Draft-gear.** These are of necessity made with springs for all passenger- and freight-cars. Coal-jimmies are often fastened together by links dropped over hooks, but the larger coal-cars require springs to absorb the shocks. There is a considerable theoretical advantage in "continuous draft-gear," i.e. having a rod (or pair of rods) running continuously from end to end of the car so that there shall be no tensile stress on the car-body itself. But there are several objections in practice. (*a*) The draft-rod, if there is but one, should be in the center line of the car, i.e. pass through the two truck-centers and the king-pins, which is impracticable. This difficulty is sometimes obviated in an objectionable way by running the draw-bar *above* the truck-center. A better method is to use a pair of rods. (*b*) The rod is of no value during *compression*, and it is the compression a car receives by minor collisions during switching which produces maximum injury to the car-body and the draft-gear. (*c*) The rod is much more liable to injury and requires much more expensive repairs when injured.

The older method is to bolt the beams holding the draft-gear to the under side of the car-body. This form is objectionable owing to the fact that the push and pull, being transmitted through the car-body, act eccentrically, tend to loosen the draft-beams from the car-body, and in case of a violent collision have been known to actually buckle the car-body upward (the cars being "flats"). The fastening of the draft-gear to the car-body has been made more secure by using cast-iron keys, then still more so by running the beams back to the "transoms" (the heavy cross-beams which support the car and transfer its weight to the trucks), then by making a *double* center sill extending through the length of the car. Another device is to run the draft-gear *through* the end sill and then the line of push and pull running through the car-frame instead of under it, the car-frame can furnish its maximum resistance.

332. Gauge of wheels and form of wheel-tread. In Fig. 204 is shown the standard adopted by the Master Car Builders' Association at their twentieth annual convention. Note the normal position of the gauge-line on the wheel-tread. In Fig. 114, p. 238, the relation of rail to wheel-tread is shown on a smaller scale. It should be noted that there is no definite position where the wheel-flange is absolutely "chock-a-block" against the rail. As the pressure increases the wheel mounts a little higher on the rail until a point is soon reached when the resistance is too great for it to mount still higher. By this means is avoided the shock of unyielding impact when the car sways from side to side. When the gauge between the inner faces of the wheels is greater or less than the limits given in the figure, the interchange rules of the Master Car Builders' Association authorize a road to refuse to accept a car from another road for transportation. At junction points of railroads inspectors are detailed to see that this rule (as well as many others) is complied with in respect to all cars offered for transfer.

TRAIN-BRAKES.

333. Introduction. Owing to the very general misapprehension that exists regarding the nature and intensity of the action of brakes, a complete analysis of the problem is considered justifiable. This misapprehension is illustrated by the common notion (and even practice) that the effectiveness of

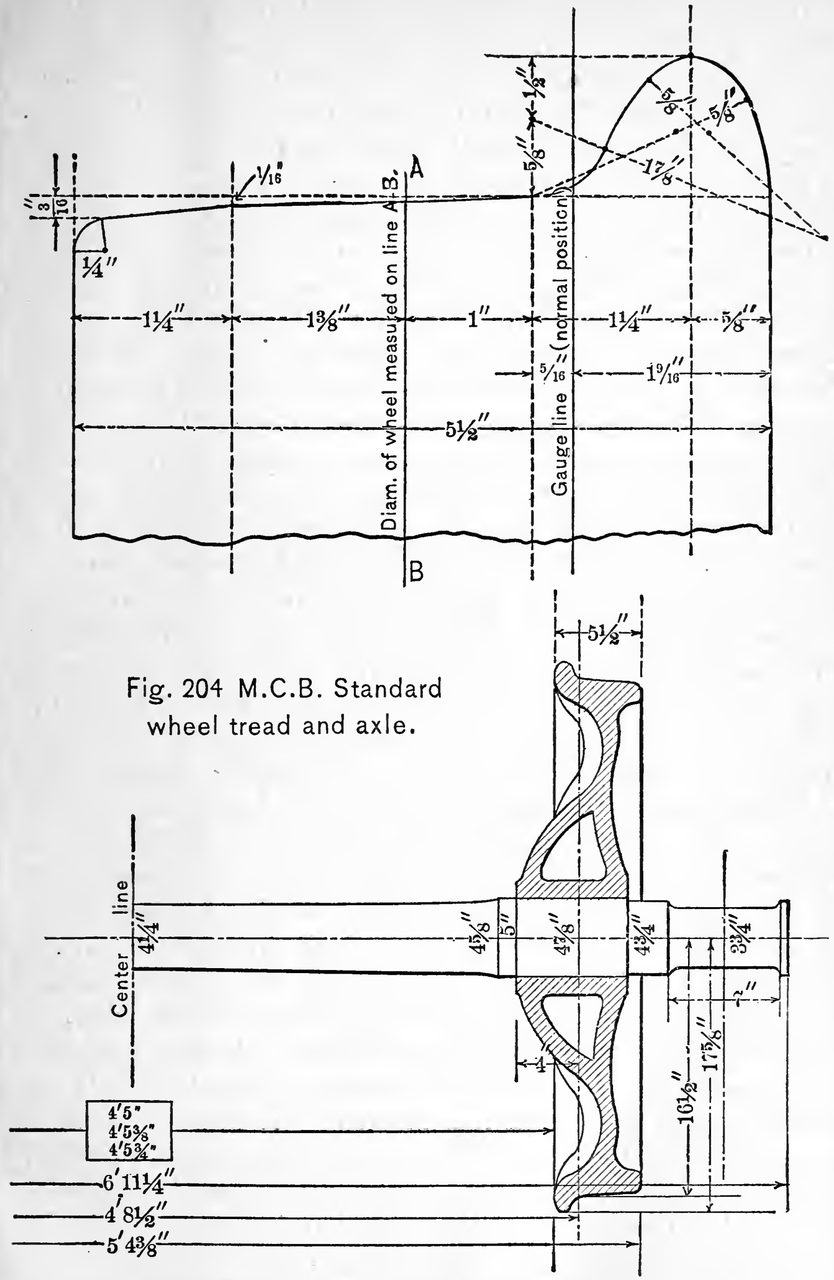


Fig. 204.—M. C. B. Standard Wheel-tread and Axle.

braking a car is proportional to the brake pressure, and therefore a brakeman is frequently seen using a bar to obtain a greater leverage on the brake-wheel and using his utmost strength to obtain the maximum pull on the brake-chain while the car is skidding along with locked wheels.

When a vehicle is moving on a track with a considerable velocity, the mass of the vehicle possesses kinetic energy of translation and the wheels possess kinetic energy of rotation. To stop the vehicle, this energy must be destroyed. The rotary kinetic energy will vary from about 4 to 8% of the kinetic energy of translation, according to the car loading (see § 347). On steam railroads brake action is obtained by pressing brake-shoes against car-wheel treads. As the brake-shoe pressure increases, the brake-shoes retard with increasing force the rotary action of the wheels. As long as the wheels do not slip or "skid" on the rails, the adhesion of the rails forces them to rotate with a circumferential velocity equal to the train velocity. The retarding action of the brake-shoe checks first the rotative kinetic energy (which is small), and the remainder develops a *tendency* for the wheel to slip on the rail. Since the rotative kinetic energy is such a small percentage of the total, it will hereafter be ignored, except as specifically stated, and it will be assumed for simplicity that the only work of the brakes is to overcome the kinetic energy of translation. The possible effect of grade in assisting or preventing retardation, and the effect of all other track resistances, is also ignored. The amount of the developed force which retards the train movement is limited to the possible adhesion or *static* friction between the wheel and the rail. When the friction between the brake-shoe and the wheel exceeds the adhesion between the wheel and the rail, the wheel skids, and then the friction between the wheel and the rail at once drops to a much less quantity. It must therefore be remembered at the outset that the retarding action of brake-shoes on wheels as a means of stopping a train is absolutely limited by the possible static friction between the braked wheels and the rails.

334. Laws of friction as applied to this problem. Much of the misapprehension regarding this problem arises from a very common and widespread misstatement of the general laws of friction. It is frequently stated that friction is independent of the velocity and of the unit of pressure. The first of these so-called laws is not even approximately true. A very exhaustive series of tests were made by Capt. Douglas Galton on the Brighton Railway in England in 1878 and 1879, and by M. George Marié on the Paris and Lyons Railway in 1879, with trains which were specially fitted with train-brakes and with dynagraphs of various kinds to measure the action of the brakes. Experience proved that variations in the condition of the rails (wet or dry), and numerous irregularities incident to measuring the forces acting on a heavy body moving with a high velocity, were such as to give somewhat discordant results, even when the conditions were made as nearly identical as possible. But the tests were carried so far and so persistently that the general laws stated below were demonstrated beyond question, and even the numerical constants were determined as closely as they may be practically utilized. These laws may be briefly stated as follows:

(*a*) The coefficient of friction between cast-iron brake-blocks and steel tires is about .3 when the wheels are "just moving"; it drops to about .16 when the velocity is about 30 miles per hour, and is less than .10 when the velocity is 60 miles per hour. These figures fluctuate considerably with the condition of the rails, wet or dry.

(*b*) The coefficient of friction is greatest when the brakes are first applied; it then reduces very rapidly, decreasing nearly one third after the brakes have been applied 10 seconds, and dropping to nearly one half in the course of 20 seconds. Although the general truth of this law was established beyond question, the tests to demonstrate the law of the variation of friction with time of application were too few to determine accurately the numerical constants.

(*c*) The friction of skidded wheels on rails is always very much less than the adhesion when the wheel is rolling on the rail—sometimes less than one third as much.

(*d*) An analysis of the tests all pointed to a law that the friction developed does *not* increase as rapidly as the *intensity* of pressure increases, but this may hardly be considered as an established law.

(*e*) The adhesion between the wheel and the rail appears to be independent of velocity. The adhesion here means the force that must be developed before the wheel will slip on the rail.

The practical effect of these laws is shown by the following observed phenomena:

(*a*) When the brakes are first applied (the velocity being very high), a brake pressure far in excess of the weight on the wheel (even three or four times as much) may be applied without skidding the wheel. This is partly due to the fact that the wheel has a very high rotative kinetic energy (which varies as the square of the velocity, and which must be overcome first), but it is chiefly due to the fact that the coefficient of friction at the higher velocity is very small (at 60 miles per hour it is about .07), while the adhesion between the wheel and the rail is independent of the velocity.

(*b*) As the velocity decreases the brake pressure must be decreased or the wheels will skid. Although the friction decreases with the time required to stop and increases with the reduction of speed, and these two effects tend to neutralize each other, yet unless the stop is very slow, the increase in friction due to reduction of speed is much greater than the decrease due to time, and therefore the brake pressure must not be greater than the weight on the wheel, unless momentarily while the speed is still very high.

(*c*) The adhesion between wheels and rails varies from .20 to .25 and over when the rail is dry. When wet and slippery it may fall to .18 or even .15. The use of sand will always raise it above .20, and on a dry rail, when the sand is not blown away by wind, it may raise it to .35 or even .40.

(*d*) Experiments were made with an automatic valve by which the brake-shoe pressure against the wheel should be reduced as the friction increased, but since (1) the essential requirement is that the friction produced by the brake-shoes shall not exceed the adhesion between rail and wheel, and since (2) the rail-wheel adhesion is a very variable quantity, depending on whether the rail is wet or dry, it has been found impracticable to use such a valve, and that the best plan is to leave it to the engineer to vary the pressure, if necessary, by the use of the brake-valve.

MECHANISM OF BRAKES.

335. Hand-brakes. The old style of brakes consists of brake-shoes of some type which are pressed against the wheel-treads by means of a brake-beam, which is operated by means of a hand-windlass and chain operating a set of levers. It is desirable that brakes shall not be set so tightly that the wheels shall be locked, and then slide over the track, producing flat places on them, which are very destructive to the rolling-stock and track afterward, on account of the impact occasioned at each revolution. With air-brakes the maximum pressure of the brake-shoes can be quite carefully regulated, and they are so designed that the maximum pressure exerted by any pair of brake-shoes on the wheels of any axle shall not exceed a certain per cent. of the weight carried by that axle when the car is *empty*, 90% being the figure usually adopted for passenger-cars and 70% for freight-cars. Consider the case of a freight-car of 100000 lbs. capacity, weighing 33100 lbs., or 8275 lbs. on an axle, and equipped with a hand-brake which operates the levers and brake-beams, which are sketched in Fig. 205. The dead weight on an axle is 8275 lbs.; 70% of

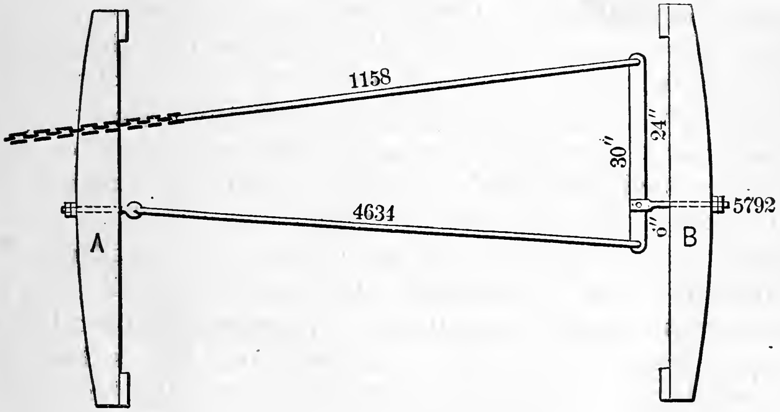


FIG. 205.—SKETCH OF MECHANISM OF HAND-BRAKE.

this is 5792 lbs., which is the maximum allowable pressure per brake-beam, or 2896 lbs. per brake-shoe. With the dimensions shown, such a pressure will be produced by a pull of about 1158 lbs. on the brake-chain. The power gained by the brake-wheel is not equal to the ratio of the brake-wheel diameter to the diameter of the shaft, about which the brake-chain winds, which is about 16 to $1\frac{1}{2}$. The ratio of the circumference of the brake-wheel to the length of chain wound up by one complete turn would be a closer figure. The loss of efficiency in such a clumsy mechanism also reduces the effective ratio. Assuming the *effective* ratio as 6:1 it would require a pull of 193 lbs. at the circumference of the brake-wheel to exert 1158 lbs. pull on the brake-chain, or 5792 lbs. pressure on the wheels at B, and even this will not lock the wheels when the car is empty, much less when it is loaded. Note that the pressures at A and B are unequal. This is somewhat objectionable, but it is unavoidable with this simple form of brake-beam. More complicated forms to avoid this are sometimes used. Hand-brakes are, of course, cheapest in first cost, and even with the best of automatic brakes, additional mechanism to operate the brakes by hand in an emergency is always provided, but their slow operation when a quick stop is desired makes it exceedingly dangerous to attempt to run a train at high speed unless some automatic brake directly under the control of the engineer is at hand. The great increase in the average velocity of trains during recent years has only been rendered possible by the invention of automatic brakes.

336. "Straight" air-brakes. The essential constructive features of this form of brake are (1) an air-pump on the engine, operated by steam, which compresses air into a reservoir on the engine; (2) a "brake-pipe" running from the reservoir to the rear of the engine and pipes running under each car, the pipes having flexible connections at the ends of the cars and engine; (3) a cylinder and piston under each car which operates the brakes by a system of levers, the cylinder being connected to the brake-pipe. The reservoir on the engine holds compressed air at about 45 lbs. pressure. To operate the brakes, a valve on the engine is opened which allows the compressed air to flow from the reservoir through the brake-pipe to each cylinder, moving the piston, which thereby moves the levers and applies the brakes. The *defects* of this system are many: (1) With a long train, considerable time is required for the air to flow from the reservoir on the engine to the rear cars, and for an emergency-stop even this delay would often be fatal; (2) if the train breaks in two, the rear portion is not provided with power for operating the brakes, and a dangerous collision would often be the result; (3) if an air-pipe coupling bursts under any car, the whole system becomes absolutely helpless, and as such a thing might happen during some emergency, the accident would then be especially fatal.

This form of brake has almost, if not entirely, passed out of use. It is here briefly described in order to show the logical development of the form which is now in almost universal use, the automatic.

337. **Automatic air-brakes.** The above defects have been overcome by a method which may be briefly stated as follows: A reservoir for compressed air is placed under each car and the tender; whenever the pressure in these reservoirs is reduced for any reason, it is automatically replenished from the main reservoir on the engine; whenever the pressure in the brake-pipe is reduced for **any** cause (opening a valve at any point of its length, parting of the train, or bursting of a pipe or coupler), valves are automatically moved under each car to operate the piston and put on the brakes. *All* the brakes on the train are thus applied almost simultaneously. If the train breaks in two, *both* sections will at once have *all* the brakes applied automatically; if a coupling or pipe bursts, the brakes are at once applied and attention is thereby attracted to the defect; if an emergency should arise, such that the conductor desires to stop the train instantly without even taking time to signal to the engineer, he can do so by opening a valve placed on each car, which admits air to the train-pipe, which will set the brakes on the whole train, and the engineer, being able to discover instantly what had occurred, would shut off steam and do whatever else was necessary to stop the train as quickly as possible. The most important and essential detail of this system is the "automatic triple valve" placed under each car. Quoting from the Westinghouse Air-brake Company's Instruction Book, "A moderate reduction of air pressure in the train-pipe causes the greater pressure remaining stored in the auxiliary reservoir to force the piston of the triple valve and its slide-valve to a position which will allow the air in the auxiliary reservoir to pass directly into the brake-cylinder and apply the brake. A sudden or violent reduction of the air in the train-pipe produces the same effect, and in addition causes supplemental valves in the triple valve to be opened, permitting the pressure from the train-pipe to also enter the brake-cylinder, augmenting the pressure derived from the auxiliary reservoir about 20%, producing practically instantaneous action of the brakes to their highest efficiency throughout the entire train. When the pressure in the brake-pipe is again restored to an

amount in excess of that remaining in the auxiliary reservoir, the piston- and slide-valves are forced in the opposite direction to their normal position, opening communication from the train-pipe to the auxiliary reservoir, and permitting the air in the brake-cylinder to escape to the atmosphere, thus releasing the brakes. If the engineer wishes to apply the brake, he moves the handle of the engineer's brake-valve to the right, which first closes a port, retaining the pressure in the main reservoir, and then permits a portion of the air in the train-pipe to escape. To release the brakes, he moves the handle to the extreme left, which allows the air in the main reservoir to flow freely into the brake-pipe, restoring the pressure therein."

338. Tests to measure the efficiency of brakes. Let v represent the velocity of a train in feet per second; W, its weight; F, the retarding force due to the brakes; d, the distance in feet required to make a stop; and g, the acceleration of gravity (32.16 feet per square second); then the kinetic energy possessed by the train (disregarding for the present the rotative kinetic energy of the wheels) $=\frac{Wv^2}{2g}$. The work done in stopping the train $=Fd$. $\therefore Fd=\frac{Wv^2}{2g}$. The ratio of the retarding force to the weight,

$$\frac{F}{W}=\frac{v^2}{2gd}=.0155\frac{v^2}{d}.$$

In order to compare tests made under varying conditions, the ratio $F\div W$ should be corrected for the effect of grade (+ or −), if any, and also for the proportion of the weight of the train which is on *braked* wheels. For example, a train weighed 146076 lbs., the proportion on braked wheels was 67%, speed 60 feet per second, length of stop 450 feet, track level. Substituting these values in the above formula, we find $(F\div W)$ $=.124$. This value is really unduly favorable, since the ordinary track resistance helps to stop the train. This has a value of from 6 to 20 lbs. per ton, averaging say 10 lbs. per ton during the stop, or .005 of the weight. Since the effect of this is small and is nearly constant for all trains, it may be ignored in comparative tests. The grade in this case was level, and therefore grade had no effect. But since only 67% of the weight was on braked wheels, the ratio, on the basis of *all* the wheels braked, or of the weight reduced to that actually on the braked wheels, is $0.124 \div .67 = 0.185$. This was called a "good" stop, although as high a ratio as 0.200 has been obtained.

339. Brake-shoes. Brake-shoes were formerly made of wrought iron, but when it was discovered that cast-iron shoes would answer the purpose, the use of wrought-iron shoes was abandoned, since the cast-iron shoes are so much cheaper. A cheap practice is to form the brake-shoe and its head in one piece, which is cheaper in first cost, but when the wearing-surface is too far gone for further use, the whole casting must be renewed. The "Christie" shoe, adopted by the Master Car Builders' Association as standard, has a separate shoe which is fastened to the head by means of a wrought-iron key. The shoe is beveled $\frac{1}{4}''$ in a width of $3\frac{3}{8}''$ to fit the coned wheel. This is a greater bevel than the standard coning of a car-wheel. It is perhaps done to allow for some bending of the brake-beam and also so that the maximum pressure (and wear) should come on the outside of the tread, rather than next to the flange, where it might tend to produce sharp flanges. By concentrating the brake-shoe wear on the outer side of the tread, the wear on the tread is more nearly equalized, since the rail wears the wheel-tread chiefly near the flange. This same idea is developed still further in the "flange-shoes," which have a curved form to fit the wheel-flange and which bear on the wheel on the flange and on the outside of the tread. It is claimed that by this means the standard form of the tread is better preserved than when the wear is entirely on the tread. The Congdon brake-shoe is one of a type in which wrought-iron pieces are inserted in the face of a cast-iron shoe. It is claimed that these increase the life of the shoe.

CHAPTER XVI.

TRAIN RESISTANCE.

340. Classification of the various forms. The various resistances which must be overcome by the power of the locomotive may be classified as follows:

(*a*) *Resistances internal to the locomotive,* which include friction of the valve-gear, piston- and connecting-rods, journal friction of the drivers; also all the loss due to radiation, condensation, friction of the steam in the passages, etc. In short, these resistances are the sum-total of the losses by which the power at the circumference of the drivers is less than the power developed by the boiler.

(*b*) *Velocity resistances,* which include the atmospheric resistances on the ends and sides; oscillation and concussion resistances, due to uneven track, etc.

(*c*) *Wheel resistances,* which include the rolling friction between the wheels and the rails of *all* the wheels (including the drivers); also the journal friction of all the axles, except those of the drivers.

(*d*) *Grade and curve resistances,* which include those resistances which are due to grade and to curves, and which are not found on a straight and level track.

(*e*) *Brake resistances.* As shown later, brakes consume power and to the extent of their use increase the energy to be developed by the locomotive.

(*f*) *Inertia resistances.* The resistance due to inertia is not generally considered as a train resistance because the energy which is stored up in the train as kinetic energy may be utilized in overcoming future resistances. But in a discussion of the demands on the tractive power of the engine, one of the chief items is the energy required to *rapidly* give to a starting train its normal velocity. This is especially true of suburban trains, which must acquire speed very quickly in order that their general average speed between termini may be even reasonably fast.

341. **Resistances internal to the locomotive.** These are resistances which do not tax the adhesion of the drivers to the rails, and hence are frequently considered as not being a part of the train resistance properly so called. If the engine were considered as lifted from the rails and made to drive a belt placed around the drivers, then all the power that reached the belt would be the power that is ordinarily available for adhesion, while the remainder would be that consumed internally by the engine. The power developed by an engine may be obtained by taking indicator diagrams which show the actual steam pressure in a cylinder at any part of a stroke. From such a diagram the average steam pressure is easily obtained, and this average pressure, multiplied by the length of the stroke and by the net area of the piston, gives the energy developed by one half-stroke of one piston. Four times this product divided by 550 times the time in seconds required for one stroke gives the "indicated horse-power" Even this calculation gives merely the power behind the piston, which is several per cent. greater than the power which reaches the circumference of the drivers, owing to the friction of the piston, piston-rod, cross-head, connecting-rod bearings, and driving-wheel journals. (See § 322, Chapter XV.) By measuring the amount of water used and turned into steam, and by noting the boiler pressure, the energy possessed by the steam used is readily computed. The indicator diagrams will show the amount of steam that has been effective in producing power at the cylinders. The steam accounted for by the diagrams will ordinarily amount to 80 or 85% of the steam developed by the boiler, and the other 15 or 20% represents the loss of energy due to radiation, condensation, etc. From actual tests it has been found that the power consumed by an engine running light is about 11% of that required by the engine when working hard in express freight service. But since the engine resistances (friction, etc.) are increased when it is pulling a load, it was estimated, after allowing for this fact, that about 15 or 16% of the power developed by the pistons was consumed by the engine, leaving about 84 to 85% for the train.

342. **Velocity resistances.** (*a*) *Atmospheric.* This consists of the head and tail resistances and the side resistance. The head

and tail resistances are nearly constant for all trains of given velocity, varying but slightly with the varying cross-sections of engines and cars. The side resistance varies with the length of the train and the character of the cars, box-cars or flats, etc. Vestibuling cars has a considerable effect in reducing this side resistance by preventing much of the eddying of air-currents between the cars, although this is one of the least of the advantages of vestibuling. Atmospheric resistance is generally assumed to vary as the square of the velocity, and although this may be nearly true, it has been experimentally demonstrated to be at least inaccurate. The head resistance is generally assumed to vary as the area of the cross-section, but this has been definitely demonstrated to be very far from true. A freight-train composed partly of flat-cars and partly of box-cars will encounter considerably more atmospheric resistance than one made exclusively of either kind, other things being equal. The definite information on this subject is very unsatisfactory, but this is possibly due to the fact that it is of little practical importance to know just how much such resistance amounts to.

(*b*) *Oscillatory and concussive.* These resistances are considered to vary as the square of the velocity. Probably this is nearly, if not quite, correct on the general principle that such resistances are a succession of impacts and the force of impacts varies as the square of the velocity. These impacts are due to the defects of the track, and even though it were possible to make a precise determination of the amount of this resistance in any particular case, the value obtained would only be true for that particular piece of track and for the particular degree of excellence or defect which the track *then* possessed. The general improvement of track maintenance during late years has had a large influence in increasing the possible train-load by decreasing the train resistance. The expenditure of money to improve track will give a road a large advantage over a competing road with a poorer track, by reducing train resistance, and thus reducing the cost of handling traffic.

343. Wheel resistances. (*a*) *Rolling friction of the wheels.* To determine experimentally the rolling friction of wheels, apart from all journal friction, is a very difficult matter and has never been satisfactorily accomplished. Theory as well as practice shows that the higher and the more perfect the elasticity of the wheel and the surface, the less will be the rolling friction. But the determination, if made, would be of theoretical interest only.

The combined effect of rolling friction and journal friction is determinable with comparative ease. From the nature of the case no great reduction of the rolling friction by any device is possible. It is only a very insignificant part of the total train resistance.

(*b*) *Journal friction of the axles.* This form of resistance has been studied quite extensively by means of the measurement of the force required to turn an axle in its bearings under various conditions of pressure, speed, extent of lubrication, and temperature. The following laws have been fairly well established: (1) The coefficient of friction increases as the pressure diminishes; (2) it is higher at very slow speeds, gradually diminishing to a minimum at a speed corresponding to a train velocity of about 10 miles per hour, then slowly increasing with the speed; it is very dependent on the perfection of the lubrication, it being reduced to one sixth or one tenth, when the axle is lubricated by a bath of oil rather than by a mere pad or wad of waste on one side of the journal; (3) it is much lower at higher temperature, and *vice versa.* The practical effect of these laws is shown by the observed facts that (1) loaded cars have a less resistance per ton than unloaded cars, the figures being (for speeds of about 10 to 20 miles per hour):

For passenger- and loaded freight-cars...	4 lbs. per ton
" empty freight-cars................	6 " " "
" street-cars.......................	10 " " "
" freight-trucks without load........	14 " " "

(2) When starting a train, the resistances are about 20 lbs. per ton, notwithstanding the fact that the velocity resistances are practically zero; at about 2 miles per hour it will drop to 10 lbs. per ton and above 10 miles per hour it may drop to 4 lbs. per ton if the cars are in good condition. (3) The resistance could probably be materially lowered if some practicable form of journal-box could be devised which would give a more perfect lubrication. (4) It is observed that freight-train loads must be cut down in winter by about 10 or 15% of the loads that the same engine can haul over the same track in summer. This is due partly to the extra roughness and inelasticity of the

track in winter, and partly to increased radiation from the engine wasting some energy, but this will not account for all of the loss, and the effect, which is probably due largely to the lower temperature of the journal-boxes, is very marked and costly. It has been suggested that a jacketing of the journal-boxes, which would prevent rapid radiation of heat and enable them to retain some of the heat developed by friction, would result in a saving amply repaying the cost of the device.

Roller journals for cars have been frequently suggested, and experiments have been made with them. It is found that they are very effective at low velocities, greatly reducing the starting resistance, which is very high with the ordinary forms of journals. But the advantages disappear as the velocity increases. The advantages also decrease as the load is increased, so that with heavily loaded cars the gain is small. The excess of cost for construction and maintenance has been found to be more than the gain from power saved.

344. Grade resistance. The amount of this may be computed with mathematical exactness. Assume that the ball or cylinder (see Fig. 206) is being drawn up the plane. If W

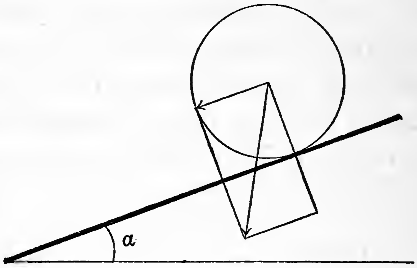


FIG. 206.

is the weight, N the normal pressure against the rail, and G the force required to hold it or to draw it up the plane with uniform velocity, the rolling resistances being considered zero or considered as provided for by other forces, then

$$G:W:h:d, \quad \text{or} \quad G=\frac{Wh}{d};$$

but for all ordinary railroad grades, $d=c$ to within a tenth of 1%, i.e., $G=\frac{Wh}{c}=W\times$rate of grade. In order that the student may appreciate the exact amount of this approximation the percentage of slope distance to its horizontal projection is given in the following tabular form:

Grade in per cent.	1	2	3	4	5
$\frac{\text{Slope dist.}}{\text{hor. dist.}} \times 100$........	100.005	100.020	100.045	100.080	100.125
Grade in per cent.	**6**	**7**	**8**	**9**	**10**
$\frac{\text{Slope dist.}}{\text{hor. dist.}} \times 100$........	100.180	100.245	100.319	100.404	100.499

This shows also the error on various grades of measuring with the tape on the ground rather than held horizontally. Since almost all railroad grades are less than 2% (where the error is but .02 of 1%), and anything in excess of 4% is unheard of for normal construction, the error in the approximation is generally too small for practical consideration.

If the rate of grade is 1 : 100, $G=W\times\frac{1}{100}$, i.e., $G=20$ lbs. per ton; ∴ for *any* per cent. of grade, $G=(20\times\text{per cent. of grade})$ pounds per ton. When moving up a grade this force G is to be overcome in addition to all the other resistances. When moving down a grade, the force G assists the motion and may be more than sufficient to move the train at its highest allowable velocity. The force required to move a train on a level track at ordinary freight-train speeds (say 20 miles per hour) is about 7 lbs. per ton. A down grade of $\frac{7}{20}$ of 1% will furnish the same power; therefore on a down grade of 0.35%, a freight-train would move indefinitely at about 20 miles per hour. If the grade were higher and the train were allowed to gain speed freely, the speed would increase until the resistance at that speed would equal W times the rate of grade, when the velocity would become uniform and remain so as long as the conditions were constant. If this speed was higher than a safe permissible speed, brakes must be applied and power wasted. The fact that one terminal of a road is considerably higher than the other does not necessarily imply that the extra power needed to overcome the difference of elevation is a total waste of energy, especially if the maximum grades are so low that brakes will never need to be applied to reduce a dangerously high velocity, for although more power must be used in ascending the grades, there is a considerable saving of power in descending the grades. The amount of this saving will be discussed more fully in Chapter XXIII.

345. Curve resistance. Some of the principal laws will be here given without elaboration. A more detailed discussion will be given in Chapter XXII.

(*a*) While the total curve resistance increases as the degree of curve increases, the resistance *per degree of curve* is much greater for easy curves than for sharp curves; *e.g.*, the resistance on the excessively sharp curves (radius 90 feet) of the elevated roads of New York City is very much less *per degree of curve* than that on curves of 1° to 5°. (*b*) Curve resistance increases with the velocity. (*c*) The total resistance on a curve depends on the central angle rather than on the radius; i.e., two curves of the same central angle but of different radius would cause about the same total curve resistance. This is partly explained by the fact that the longitudinal slipping will be the same in each case. (See § 311, Chapter XV.) In each case also the trucks must be twisted around and the wheels slipped laterally on the rails by the same amount Δ°. (See § 312, Chapter XV.)

346. Brake resistances. If a down grade is excessively steep so that brakes must be applied to prevent the train acquiring a dangerous velocity, the energy consumed is hopelessly lost without any compensation. When trains are required to make frequent stops and yet maintain a high average speed, considerable power is consumed by the application of brakes in stopping. All the energy which is thus turned into heat is hopelessly lost, and in addition a very considerable amount of steam is drawn from the boiler to operate the air-brakes, which consume the power already developed. It can be easily demonstrated that engines drawing trains in suburban service, making frequent stops, and yet developing high speed between stops, will consume a very large proportion of the total power developed by the use of brakes. Note the double loss. The brakes consume power already developed and stored in the train as kinetic or potential energy, while the operation of the brakes requires additional steam power from the engine.

347. Inertia resistance. The two forms of train resistance which under some circumstances are the greatest resistances to be overcome by the engine are the grade and inertia resistances, and fortunately both of these resistances may be computed with mathematical precision. The problem may be stated as follows: What constant force P (in addition to the forces required to overcome the various frictional resistances, etc.) will be required to impart to a body a velocity of v feet per second in a distance of s feet? The required number of foot-pounds of energy is evidently Ps. But this work imparts a kinetic energy which may be expressed by $\frac{Wv^2}{2g}$. Equating these values, we have $Ps=\frac{Wv^2}{2g}$, or

$$P=\frac{Wv^2}{2gs}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (138)$$

The force required to increase the velocity from v_1 to v_2 may likewise be stated as $P=\frac{W}{2gs}(v_2^2-v_1^2)$. Substituting in the formula the values $W=2000$ lbs. (one ton), $g=32.16$, and $s=5280$ feet (one mile), we have

$$P=.00588(v_2^2-v_1^2).$$

Multiplying by $(5280\div3600)^2$ to change the unit of velocity to miles per hour, we have

$$P=.01267(V_2^2-V_1^2).$$

But this formula must be modified on account of the rotative kinetic energy which must be imparted to the wheels of the cars. The precise additional percentage depends on the particular design of the cars and their loading and also on the design of the locomotive. Consider as an example a box-car, 60000 lbs. capacity, weighing 33000 lbs. The wheels have a diameter of 36″ and their radius of gyration is about 13″. Each wheel weighs 700 lbs. The rotative kinetic energy of each wheel is 4877 ft.-lbs. when the velocity is 20 miles per hour, and for the eight wheels it is 39016 ft.-lbs. For greater precision (really needless) we may add 192 ft.-lbs. as the rotative kinetic energy of the axles. When the car is fully loaded (weight 93000 lbs.) the kinetic energy of translation is 1,244,340 ft.-lbs.; when empty (weight 33000 lbs.) the energy is 441540 ft.-lbs. The rotative kinetic energy thus adds (for this particular car) 3.15% (when the car is loaded) and 8.9% (when the car is empty) to the kinetic energy of translation. The kinetic energy which is similarly added, owing to the rotation of the wheels and axles of the locomotive, might be similarly computed. For one type of locomotive it has been figured at about 8%. The variations in design, and particularly the fluctuations of loading, render useless any great precision in these computations. For a train of "empties" the figure would be high, probably 8 to 9%; for a fully loaded train it will not much exceed 3%. Wellington considered that 6% is a good average value to use (actually used 6.14% for "ease of computation"), but considering (*a*) the increasing proportion of live load to dead load in modern car design, (*b*) the greater care now used to make up *full* train-loads, and (*c*) the fact that *full* train-loads are the critical loads, it would appear that 5% is a better average for the conditions of modern practice. Even this figure allows something for the higher percentage for the locomotive and something for a few empties in the train. Therefore, adding 5% to the coefficient in the above equation, we have the true equation

$$P = .0133(V_2^2 - V_1^2), \quad . \quad . \quad . \quad . \quad . \quad . \quad (139)$$

in which V_2 and V_1 are the higher and lower velocities respectively in *miles per hour*, and P is the force required *per ton* to impart that difference of velocity in a distance of *one mile* If more convenient, the formula may be used thus:

$$P_1 = \frac{70.224}{s}(V_2^2 - V_1^2), \quad . \quad . \quad . \quad . \quad . \quad (140)$$

in which s is the distance in feet and P_1 is the corresponding force.

As a numerical illustration, the force required per ton to impart a kinetic energy due to a velocity of 20 miles per hour in a distance of 1000 feet will equal

$$P_1 = \frac{70.224(400-0)}{1000} = 28 \text{ lbs.},$$

which is the equivalent (see § 344) of a 1.4% grade. Since the velocity enters the formula as V^2, while the distance enters only in the first power, it follows that it will require *four* times the force to produce twice the velocity in the same distance, or that with the *same* force it will require four times the distance to attain twice the velocity.

As another numerical illustration, if a train is to increase its speed from 15 miles per hour to 60 miles per hour in a distance of 2000 feet, the force required (in addition to all the other resistances) will be

$$P_1 = \frac{70.224(3600-225)}{2000} = 118.50 \text{ lbs. per ton.}$$

This is equivalent to a 5.9% grade and shows at once that it would be impossible unless there were a very heavy down grade, or that the train was very light and the engine very powerful.

348. Formulæ for train resistance. These are generally given in one of the forms

$$\left.\begin{array}{l} R = fV + c \ldots\ldots\ (a) \\ R = fV^2 + c \ldots\ldots\ (b) \end{array}\right\}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (141)$$

in which R is the resistance per ton, f is a coefficient to be determined, V is the velocity in miles per hour, and c is a constant, also to be determined. These formulæ disregard grade and curve resistances, inertia resistance, and the active resistance (or assistance) of *wind*, as distinct from mere atmospheric resistance. In short, they are supposed to give the resistance of a train moving at a uniform velocity over a straight and level track, there being no appreciable wind. Both formulæ are empirical, since the resistances do not vary either directly or as the square of the velocity. Some resistances vary nearly as the square and some nearly as the first power.

The quantity c represents the journal friction and rolling friction, and these are assumed to be constant, although careful tests of journal friction show that its variation with velocity is irregular (see § 343). This shows that such simple formulæ must always be inaccurate, but some formulæ have been suggested, having either of these general forms, which agree very closely with the results of actual tests.

(*a*) *Searles's formula*,

$$R = 4.82 + .00536V^2 + \frac{.00048V^2 \text{ (wt. of eng. and tender)}^2}{\text{gross weight of train}} \qquad (142)$$

in which R = total resistance in pounds per ton and V is the velocity in miles per hour. This formula does not take account of any difference in the form of the train (whether box-cars or flats), which would have a great influence on the atmospheric resistance; neither does it take into account the relation of length to weight, or whether the cars are loaded or empty. Nevertheless the results agree very closely with the determinations of actual train tests. If the resistance is computed according to this formula for a given class of engine (*e.g.*, a heavy consolidation), and for various lengths of train, it is found that the resistance *per ton of the gross weight of the train* is much less when the train is long, and for a train of ordinary length the resistance hardly increases as fast as the velocity until the velocity is great.

According to this formula, a heavy consolidation engine drawing forty loaded freight-cars would have to overcome a resistance of about 8.2 lbs. per ton of the gross weight of the train at a velocity of 20 miles per hour. At a velocity of 10 miles per hour this resistance drops to about 5.7 lbs. per ton. And so the value of 8 lbs. per ton, used by Wellington in his computations of the total power of locomotives on grades, may be considered a safe figure, especially as the velocity at critical places may be assumed to be reduced as much as necessary.

(*b*) *Wellington's formulæ*,

$$R=\left\{\begin{array}{ll} 3.9+.0065V^2+\dfrac{.57V^2}{W} \ldots & \text{for loaded flat-cars} \\[2ex] 3.9+.0075V^2+\dfrac{.64V^2}{W} \ldots & \text{for loaded box-cars} \\[2ex] 6.0+.0083V^2+\dfrac{.57V^2}{W} \ldots & \text{for empty flat-cars} \\[2ex] 6.0+.0106V^2+\dfrac{.64V^2}{W} \ldots & \text{for empty box-cars} \end{array}\right\}. \qquad (143)$$

Notice in these formulæ the additional journal resistance (indicated by the constant term) for unloaded cars. The last term evidently indicates the atmospheric resistance. The middle term allows for the oscillatory resistances. Assuming the constant term and the coefficients to have been correctly determined, these formulæ should be better than Searles's, since a choice of formulæ can be made depending on the conditions. A train consisting partly of box-cars and partly of flat-cars will have a higher resistance than is shown by any of the above formulæ (and *not* a mean value), on account of the increased atmospheric resistance acting on the irregular form of the train.

(*c*) *Engineering News formula,*

$$R=\frac{V}{4}+2. \qquad \qquad (144)$$

This formula belongs to class (*a*), Eq. 141. Its very simplicity makes it valuable for general use, but like the succeeding formula, it does not take account of variations in the form of the train, which have a very material influence on the train resistance.

(*d*) *D. K. Clark's formula,*

$$\left.\begin{array}{ll} R=\frac{V^2}{171}+8 \ldots\ldots\ldots\ldots & \text{(for tons of 2240 lbs.)} \\ R=.00522V^2+7.14 \ldots\ldots & \text{(for tons of 2000 lbs.)} \end{array}\right\} \qquad (145)$$

This is a very old formula, and is mentioned because all of Clark's formulæ carry much weight. But in this case the formula is quite defective. The constant term (7.14) representing the journal and rolling friction is too large and thus the formula gives too large a resistance at low velocities; the coefficient of V^2(.00522) is less than in the other formulæ, and so at very high velocities the figures would be less than those given by Searles's or Wellington's formulæ, and less than the results of actual tests. For mean velocities the figures accord fairly well with those given by the other formulæ and by actual tests.

(*e*) *Baldwin Locomotive Works formula.* The Baldwin Locomotive Works have adopted a formula of their own as the result of the experience they have been able to accumulate. It is stated

$$R=\frac{V}{6}+3. \qquad \qquad (146)$$

It is claimed that this formula agrees well with actual tests, and in fact is based on the results of tests, but it evidently cannot allow for known variations in the length or character of the train. As a general formula for locomotives which are to pull *any* kind of a load, the formula is of more value for practical use than Searles's or Wellington's.

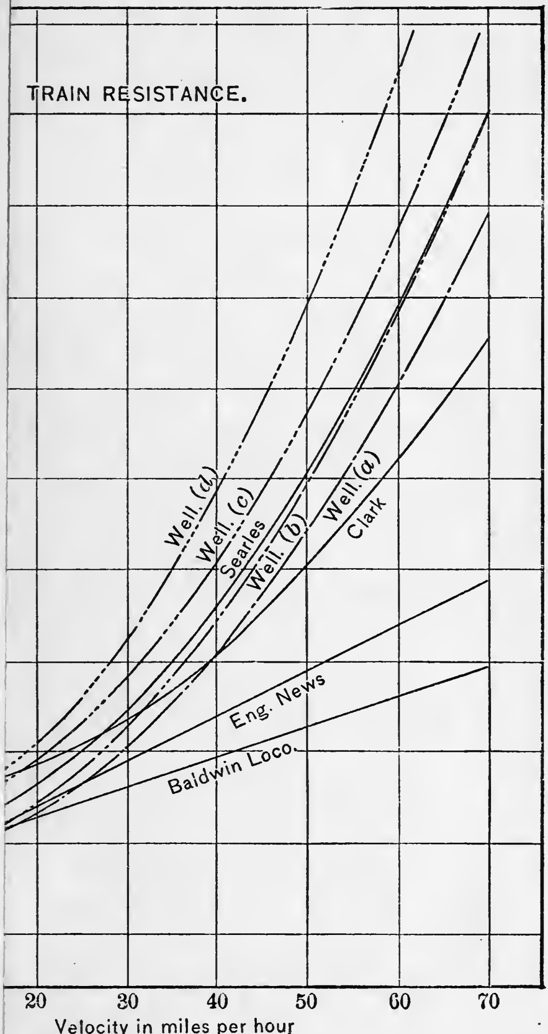
In Plate IX is shown graphically the resistance per ton of four trains according to these five formulæ. For purposes of comparison of the formulæ, the weight of engine and total weight of cars is made the same for the four trains. The resistance would therefore be the same by formulæ (*a*), (*c*), (*d*), and (*e*). The differences would only appear when applying Wellington's formula. Assume the following as train-loads:

(*a*)	Engine, 64 tons;	loaded	flat-cars,	648 tons
(*b*)	" 64 "	"	box- "	648 "
(*c*)	" 64 "	empty	flat- "	648 "
(*d*)	" 64 "	"	box- "	648 "

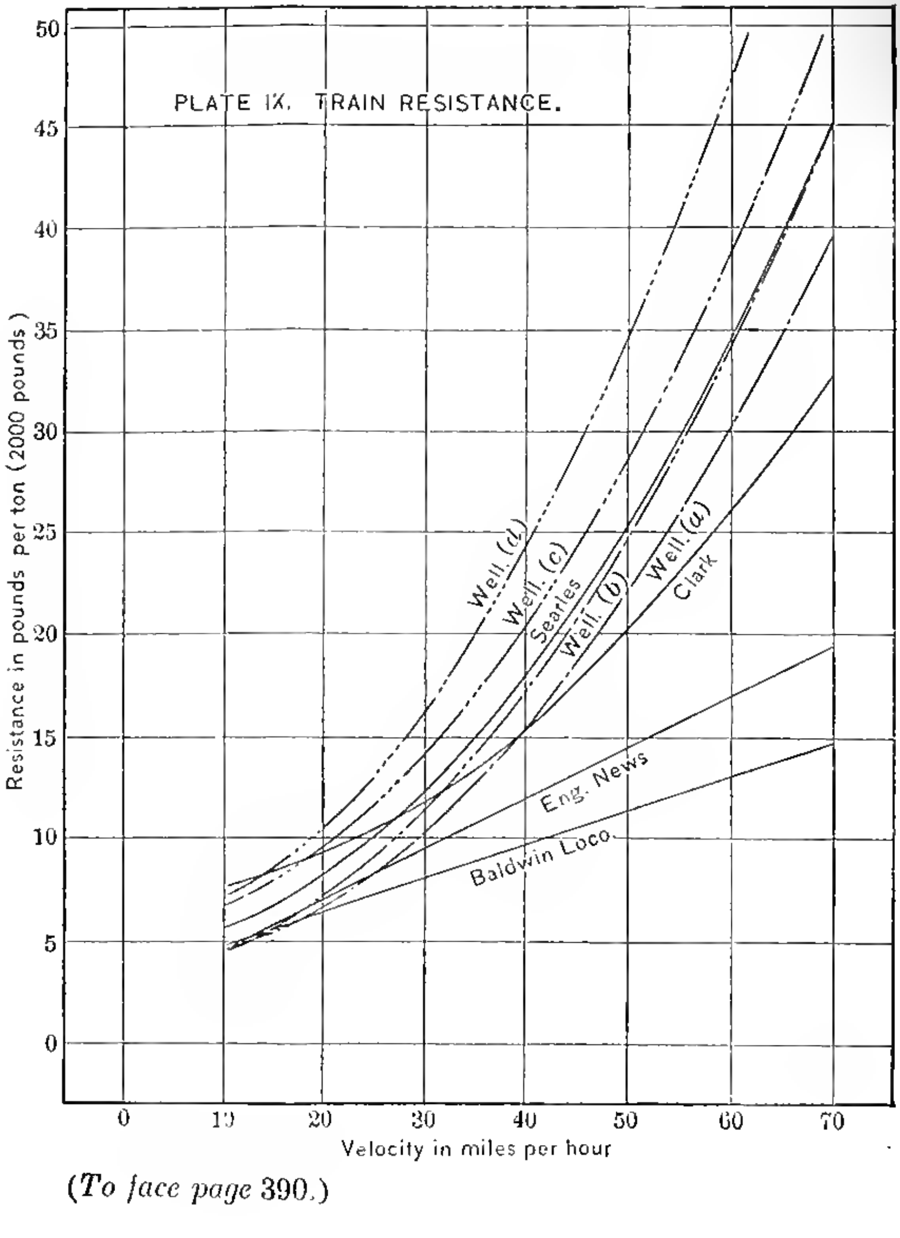
When applying any of these formulæ, due allowance must be made for grade and curve resistances, inertia resistances, and the possible retarding influence of a high wind must be considered if it is a question of the power of a locomotive of given type to draw a given load up a given grade.

349. Dynamometer tests. These are made by putting a "dynamometer-car" between the engine and the cars to be tested. Suitable mechanism makes an automatic record of the force which is transmitted through the dynamometer at any instant, and also a record of the velocity at any instant. One of the practical difficulties is the accurate determination of the velocity at any instant when the velocity is fluctuating. When the velocity is decreasing, the kinetic energy of the train is being turned into work and the force transmitted through the dynamometer is less than the amount of the resistance which is actually being overcome. On the other hand, when the velocity is increasing, the dynamometer indicates a larger force than that required to overcome the resistances, but the excess force is being stored up in the train as kinetic energy. Grade has a similar effect, and the force indicated by the dynamometer may be greater or less than that required at the given velocity on a level by the force which is derived from, or is turned into, potential energy. Therefore the resistance indicated by the dynamometer of a train will not be that on a level track at uniform velocity, unless the track is actually level and the velocity really uniform.

Dynamometer tests under other circumstances are therefore of no value unless it is possible to determine the true velocity at any instant and its rate of change, and also to determine the grade. Of course, the grade is easily found. An


TRAIN RESISTANCE.
Well. (d)
Well. (c)
Searles
Well. (b)
Well. (a)
Clark
Eng. News
Baldwin Loco.
20
30
40
50
60
70
Velocity in miles per hour

90.)



(*To face page* 390.)


Resistance in pounds per ton (2000 pounds)
0
5
10
15
20
25
30
35
40
45
50
0
10

[To face page

allowance for an increase or decrease of kinetic or potential energy must therefore be made before it is possible to know how much force is being spent on the ordinary resistances.

350. Gravity or "drop" tests. Dynamometer tests require the use of a dynamometer which is capable of measuring a force of several thousands of pounds, and which therefore cannot determine such values with a close percentage of accuracy, especially if the force is small. A drop test utilizes the force of gravity which may be measured with mathematical accuracy. The general method is to select a stretch of track which has a uniform grade of about 0.7% and which is preferably straight for two or three miles. On such a grade cars with running gear in good condition may be started by a push. The velocity will gradually increase until at some velocity, depending on the resistances encountered, the cars will move uniformly. The only work requiring extreme care with this method is the determination of the velocity. If the velocity is fluctuating, as it is during the time when it is of the greatest importance to know the velocity, it is not sufficient to determine the time rquired to run some long measured distance, for the average velocity thus obtained would probably differ

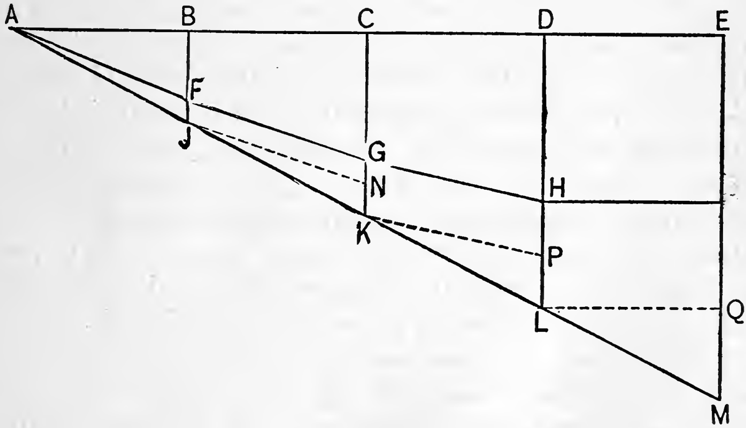


Fig. 207.—Loss in Velocity-head.

considerably from the velocity at the beginning and end of that space. If the train consists of five cars or more, the velocity may be determined electrically (as described by Wellington in his "Economic Location," etc., p. 793 *et seq.*) from the automatic record made on a chronograph of the passage of the first wheel and the last, the chronograph also recording automatically the ticks of a clock beating seconds. From this the exact time of the passage of the first and last wheels of the train of cars may be determined to the tenth or twentieth of a second.

Velocity-head. From theoretical mechanics we know that if a body descends through any path by the action of gravity, and is unaffected by friction, its velocity at any point in the direction of the path of motion is $V=\sqrt{2gh}$. If the body is retarded by resistances, its velocity at any point will be less than this. If AM, Fig. 207, represents any grade (exaggerated of course), then BJ, CK, etc., represent the actual fall at any point. Let BF represent the fall h_1, determined from $h_1=\frac{v_1^2}{2g}$, in which v_1 is the actual observed velocity at J. Then $JF=$ the velocity-head consumed by the resistances between A and J. If the train continues to K, the corresponding h_2 is CG; the remaining fall GK consists of GN ($=JF$, which is the velocity-head lost back of J) and NK, the velocity-head lost between J and K. At some velocity (V_n) on any grade, the velocity will not further increase and the line $AFGHI$ will then be horizontal and at a distance $(h_n)=EI$ below $A \ldots E$. The grade AM is the "grade of repose" for that velocity (V_n); i.e., it is the grade that would just permit the train to move indefinitely at the velocity V_n. The broken line $AFGHI$ should really be a curve, and the grade of repose at any joint is the angle between AM and the tangent to that curve at the given point. The "grade of repose" by its definition gives the total resistance of the train at the particular velocity, or multiplying the grade of repose in per cent. by 20 gives the pounds per ton of resistance. Thus being able to determine the total resistance in pounds per ton at any velocity, the variation of total resistance with velocity may be determined, and then by varying the resistances, using different kinds of cars, empty and loaded, box-cars and flats, the resistances of the different kinds at various velocities may be determined.

CHAPTER XVII

COST OF RAILROADS.

351. General considerations. Although there are many elements in the cost of railroads which are roughly constant per mile of road, yet the published reports of the cost of railroads differ very widely. The variation in the figures is due to several causes. (*a*) Economy requires that a road shall be operated and placed on an earning basis as soon as possible. Therefore the reported cost of a road during the first few years of its existence is somewhat less than that reported later. This is well illustrated when a long series of consecutive reports from an old-established road is available; nearly every year there will be shown an addition to the previous figures. And this is as it should be. The magnificent road-beds of some old roads cannot be the creation of a single season. It takes many years to produce such settled perfect structures. (*b*) A large part of the variation is due to a neglect to charge up "permanent improvements" as additions to the cost of the road. For the first few years of the life of a road a great deal of work is done which is in reality a completion of the work of construction, and yet the cost of it is buried under the item "maintenance of way." For example, a long wooden trestle is replaced by an earth embankment and a culvert. Since the original trestle is to be considered a temporary structure, the *excess* of the cost of the permanent structure over that of the temporary structure should evidently be considered as an addition to the cost of the road. But if the filling-in was done slowly, a few train-loads at a time, and the work scattered over many years, the cost of operating the "mud-train" has perhaps been buried under "maintenance" charges. (*c*) The reports from which many of the following figures were taken have not always analyzed the items of cost with the same detail as has been here attempted, and to that is probably due many of the variations and apparent discrepancies.

The various items of cost will be classified as follows:

1. Preliminary financiering.
2. Surveys and engineering expenses.
3. Land and land damages.
4. Clearing and grubbing.
5. Earthwork.
6. Bridges, trestles, and culverts
7. Trackwork.
8. Buildings and miscellaneous structures.
9. Interest on construction.
10. Telegraph line.

352. Item 1. Preliminary Financiering. The cost of this preliminary work is exceedingly variable. The work includes the clerical and legal work of organization, printing, engraving of stocks and bonds, and (sometimes the most expensive of all) the securing of a charter. This sometimes requires special legislative enactments, or may sometimes be secured from a State railroad commission. It has been estimated that about 2% of the railway capital of Great Britain has been spent in Parliamentary expenses over the charters. These expenses are usually but a small percentage of the total cost of the enterprise, but for important lines the gross cost is large, while the amount of money thus spent by organizations which have never succeeded in constructing their roads is sometimes enormous.

353. Item 2. Surveys and Engineering Expenses. The comparison of a large number of itemized reports on the cost of construction shows that the cost of the "engineering" will average about 2% of the total cost of construction. This includes the cost of surveys and the cost of laying out and superintending the constructive work. The cost of mere surveying up to the time when construction actually commences has been variously quoted at \$60, \$75, and even \$150 per mile. In exceptional cases the surveying for a few miles through some gorge might cost many times this amount, but \$150 per mile may be considered an ordinary maximum for difficult country. On the other hand, much construction has been done over the western prairies after hasty surveys costing not much over \$10 per mile.

354. Item 3. Land and Land Damages. The cost of this item varies from the extreme, in which not only the land for right-of-way but also grants of public land adjoining the road are given to the corporation as a subsidy, to the other extreme, where the right-of-way can only be obtained at exorbitant prices. The width required is variable, depending on the width that may be needed for deep cuts or high fills, or the extra land required for yards, stations, etc. A strip of land 1 mile long and 8.25 feet wide contains precisely 1 acre. An average width of 4 rods (66 feet), therefore, requires 8 acres per mile. On the Boston & Albany Railroad the expenditure assigned to "land and land damages" averages over $25000 per mile. Of course this includes some especially expensive land for terminals and stations in large cities. Less than $300 per mile was assigned to this item by an unimportant 18-mile road.

355. **Item 4. CLEARING AND GRUBBING.** The cost of this may vary from zero to 100% for miles at a time, but as an average figure it may be taken as about 3 acres per mile at a cost of say $50 per acre. The possibility of obtaining valuable timber, which may be utilized for trestles, ties, or otherwise, and the value of which may not only repay the cost of clearing and grubbing, but also some of the cost of the land, should not be forgotten.

356. **Item 5. EARTHWORK.** This item also includes rockwork. The methods of estimating the cost of earthwork and rockwork have been discussed in Chapter III. The percentage of this item to the total cost is very variable. On a western prairie it might not be more than 5 to 10%. On a road through the mountains it will run up to 20 or 25%, and even more. The item also includes tunneling, which on some roads is a heavy item.

357. **Item 6. BRIDGES, TRESTLES, AND CULVERTS.** This item will usually amount to 5 or 6% of the total cost of the road. In special cases, where extensive trestling is necessary, or several large bridges are required, the percentage will be much higher. On the other hand, a road whose route avoids the watercourses may have very little except minor culverts. On the Boston & Albany the cost is given as $5860 per mile; on the Adirondack Railroad, $2845 per mile. Considering their relative character (double and single track), these figures are relatively what we might expect.

358. Item 7. TRACKWORK. This item will be considered as including everything above subgrade, except as otherwise itemized.

(a) Ballast. With an average width, for single track, of 10 feet and an average of 15 inches, 2444 cubic yards of ballast will be required. The Pennsylvania Railroad estimate is 2500 yards of gravel per mile of single track. At an estimate of 60 c. per yard, this costs $1500 per mile. Broken-stone ballast must be filled out over the ends of the ties and therefore more is required; 2800 cubic yards of broken stone at $1.25 per yard in place will cost $3500 per mile.

(b) Ties. Ties cost anywhere from 80 c. down to 35 c. and even 25 c. At an average figure of 50 c., 2640 ties per mile will cost $1320 per mile of single track. The cheaper ties are usually smaller and more must be used per mile, and this tends to compensate the difference in cost.

The following tabular form is convenient for reference:

TABLE XV.—NUMBER OF CROSS TIES PER MILE.

Spacing center to center.	Number per 30′ rail.	Number per mile.
18 inches	20	3520
20 "	18	3168
21 "	$17\frac{1}{7}$	3017
22.5 "	16	2816
24 "	15	2640
25.71"	14	2464
27 "	$13\frac{1}{3}$	2347
30 "	12	2112

(c) Rails. The total weight of the rails used per mile may best be seen by the tabular form.

A convenient and useful rule to remember is that the number of *long* tons (2240 lbs.) per mile of single track equals the weight of the rail per yard times $\frac{11}{7}$. The rule is exact. For example, there are 3520 yards of rail in a mile of single track; at 70 lbs. per yard this equals 246400 lbs., or 110 long tons (exactly); but $70 \times \frac{11}{7} = 110$.

Any calculation of the required weight of rail for a given weight of rolling-stock necessarily depends on the assumptions which are made regarding the support which the rails receive from the ties. This depends not only on the width and spacing

TABLE XVI.—TONS PER MILE (WITH COST) OF RAILS OF VARIOUS WEIGHTS.

Weight in lbs. per yd.	Tons (2240 lb.) per mile of single track.	Cost at $26 per ton.	Cost at $30 per ton.	Weight in lbs. per yd.	Tons (2240 lb.) per mile of single track.	Cost at $26 per ton.	Cost at $30 per ton.
8	12.571	$326.86	$377.14	65	102.143	$2655.71	$3064.29
10	15.714	408.57	471.43	66	103.714	2696.57	3111.43
12	18.857	490.29	565.71	67	105.286	2737.43	3158.59
14	22.000	572.00	660.00	68	106.857	2778.29	3205.79
16	25.143	653.71	754.20	70	110.000	2860.00	3300.00
20	31.429	817.14	942.86	71	111.571	2900.86	3347.14
25	39.286	1021.43	1178.57	72	113.143	2941.71	3394.29
30	47.143	1225.71	1414.29	73	114.714	2982.57	3441.43
35	55.000	1430.00	1650.00	75	117.857	3064.29	3535.71
40	62.857	1634.29	1885.71	78	122.571	3186.86	3677.14
45	70.714	1838.57	2121.43	80	125.714	3268.57	3771.43
48	75.429	1961.14	2262.86	82	128.857	3350.29	3865.71
50	78.571	2042.86	2357.14	85	133.571	3472.86	4007.14
52	81.714	2124.57	2451.43	88	138.286	3595.43	4148.57
56	88.000	2288.00	2640.00	90	141.429	3677.14	4242.86
57	89.571	2328.86	2687.14	92	144.571	3758.86	4337.14
60	94.286	2451.43	2828.57	95	149.286	3881.43	4478.57
61	95.857	2492.29	2875.71	98	154.000	4004.00	4620.00
63	99.000	2574.00	2970.00	100	157.143	4085.71	4714.29

About two per cent. (2%) extra should be allowed for waste in cutting.

of the ties (which are determinable), but also on the support which the ties receive from the ballast, which is not only very uncertain but variable. No general rule can therefore claim any degree of precision, but the following is given by the Baldwin Locomotive Works: **"Each ten pounds weight per yard of ordinary steel rail, properly supported by cross-ties (not less than 14 per 30-foot rail), is capable of sustaining a safe load per wheel of 2240 pounds."** For example, a consolidation locomotive with 112600 lbs. on 8 drivers has a load of 14075 lbs. per wheel. This divided by 2240 gives 6.28. According to the rule, the rails for such a locomotive should weigh at least 62.8 lbs. per yard.

(d) Splice-bars, track-bolts, and spikes. These are usually sold by the pound, except the patented forms of rail-joints, which are sold by the pair. In any case they are subject to market fluctuations in price. As an approximate value the following prices are quoted: Splice-bars, 1.35 c. per pound; track-bolts, 2.4 c.; spikes, 1.75 c. The weight of the splice-bars will depend on the precise pattern adopted—its cross-section and length. For a 45-lb. rail an angle-bar whose original weight in the rolled section is 6.3 lbs. per foot might be used. A pair 21 inches long would weigh 21.5 lbs. For a 70-lb. rail an angle-bar section weighing 9 to 12 lbs. per yard would be used. A pair of the 10-lb. section, with the long 44-inch 6-hole bar, used by the Michigan Central Railroad, would weigh about 70 lbs. Angle-bars suitable for a 100-lb. rail will weigh about 14 to 16 lbs. per foot. The following tables will be useful for reference.

TABLE XVII.—SPLICE-BARS AND BOLTS PER MILE OF TRACK.

Length of rail.	Number of *pairs* of splice-bars.	Number of bolts required.	
		4-hole splice.	6-hole splice.
24 feet	440	1760	2640
25 "	422	1688	2532
26 "	406	1624	2436
27 "	391	1564	2346
28 "	377	1508	2262
30 "	352	1408	2112
33 "	320	1880	1920

TABLE XVIII.—RAILROAD SPIKES.

Size measured under head.	Average number per keg of 200 pounds	Ties 24″ between centers, 4 spikes per tie, number per Mile.		Suitable weight of rail.
		Pounds.	Kegs.	
$5\frac{1}{2}'' \times \frac{9}{16}''$	375	5632	28.16	45 to 100
$5'' \times \frac{9}{16}''$	400	5280	26.40	40 " 56
$5'' \times \frac{1}{2}''$	450	4692	23.46	40

TABLE XIX. TRACK-BOLTS.

Average number in a keg of 200 pounds.

Size of bolt.	Square nut.	Hexagonal nut.	Suitable rail.
$3'' \times \frac{3}{4}''$	250	270	50 pounds and upward.
$3\frac{1}{4}'' \times \frac{3}{4}''$	243	261	
$3\frac{1}{2}'' \times \frac{3}{4}''$	236	253	
$3\frac{3}{4}'' \times \frac{3}{4}''$	229	244	
$4'' \times \frac{3}{4}''$	222	236	
$3\frac{1}{2}'' \times \frac{7}{8}''$	170	180	
$3\frac{3}{4}'' \times \frac{7}{8}''$	165	175	
$4'' \times \frac{7}{8}''$	161	170	
$4\frac{1}{4}'' \times \frac{7}{8}''$	157	165	
$4\frac{1}{2}'' \times \frac{7}{8}''$	153	160	

(e) Track-laying. Much depends on the force of men employed and the use of systematic methods; $528 per mile is the estimate employed by the Pennsylvania Railroad. $500 per mile is the estimate given in § 362.

359. Item 8. BUILDINGS AND MISCELLANEOUS STRUCTURES. Except for rough and preliminary estimates, these items must be individually estimated according to the circumstances. The subitems include depots, engine-houses, repair-shops, water-stations, section- and tool-houses, besides a large variety of smaller buildings. The structures include turn-tables, cattle-guards, fencing, road-crossings, overhead bridges, etc. The detailed estimate, given in § 362, illustrates the cost of these smaller items.

360. Item 9. INTEREST ON CONSTRUCTION. The amount of capital that must be spent on a railroad before it has begun to earn anything is so very large that the interest on the cost during the period of construction is a very considerable item. The amount that must be charged to this head depends on the current rate of money on the time required for construction and on the ability of the capitalists to retain their capital where it will be earning something until it is actually needed to pay the company's obligations. Of course, it is not necessary to have the entire capital needed for construction on hand when construction commences. Assuming money to be worth 6%, that the work of construction will require one year, that the money may be retained where it will earn something for an average period of six months after construction commences, or, in other words, it will be out of circulation six months before the road is opened for traffic and begins to earn its way, then we may charge 3% on the total cost of construction.

361. Item 10. TELEGRAPH LINES. This evidently depends on the scale of the road and the magnitude of the business to be operated. In the following estimate it is given as $200 per mile, which evidently is intended to apply to the business of a small road.

362. Detailed estimate of the cost of a line of road. The following estimate was given in the *Engineering News* of Dec. 27, 1900, of the cost of the Duluth, St. Cloud, Glencoe & Mankato Railroad, 157.2 miles long.

The estimate is exactly as copied from the *Engineering News*. There are some numerical discrepancies. Item 26 should evidently be based on the sum of the first 25 items, and item 27 on the sum of the first 26. The figures in parentheses ( ) are deduced from the figures given.

1.	Right-of-way: 1905.3 acres (12.12 acres per mile) @ $100 per acre		$190530
2.	Clearing and grubbing. 144 acres (0.916 acre per mile) @ $50 per acre		7200
3.	Earth excavation. 1907590 cu. yds. (12135 cu. yds. per mile) @ 15 c.		286138
4.	Rock excavation. 5100 cu. yds. (32.44 cu. yds. per mile) @ 80 c.		4080
5.	Wooden-box culverts. 508300 ft. B.M. @ $30 per M.	$15249	
	Iron-pipe culverts. 879840 lbs. @ 3c. per lb.	26395	41644
6.	Pile trestling. 4600 lin. ft. @ 35 c. per lin. ft.	1610	
	Timber trestling. 509300 ft. B.M. @ $30 per M.	15279	16889
7.	Bridge masonry: 5520 cu. yds. @ $8 per cu. yd.	44160	
	Bridges, iron, 100 spans, 2000000 lbs. @ 4 c. per lb.	80000	124160
8.	Cattle-guards		8750
9.	Ties (2640 per mile). 419813 (159.02 M.) @ 35 c.		146935
10.	Rails (70 lbs. per yd.): 110 tons per mile, 17492.2 tons (159.02 M.) @$26		384797
11.	Rail sidings (per yd.): 110 tons per mile, 3300 tons (30 M.) @ $26		85800
12.	Switch timbers and ties		3300
13.	Spikes: 5920 lbs. per mile, 1107040 (187 M.) @ 1.75. c. per lb.		19373
14.	Splice-bars. 2635776 lbs. @ 1.35 c. per lb.		35583
15.	Track-bolts (2 to joint (?)): 188458.3 lbs. @ 2.4 c. per lb.		4520
16.	Track-laying. 187.2 miles @ $500 per mile		93600
17.	Ballasting. 2152 cu. yds. per mile, 402854 (187.2 M.) @ 60 c.		241712
18.	Turn-out and switch furnishings		6450
19.	Road-crossings, 68040 ft. B.M. @ $30 per M.		2041
20.	Section and tool-houses, 16 @ $800		12800
21.	Water-stations		15000
22.	Turn-tables, 6 @ $800		4800
23.	Depots, grounds, and repair-shops		78000
24.	Terminal grounds and special land damages		150000
25.	Fencing, 314 miles ($150 per mile)		47100
26.	Engineering and office expenses (5% of $1984458)		99222
27.	Interest on construction (3% of $2083680)		62510
28.	Rolling-stock ($5000 per mile)		786000
29.	Telegraph line: 157 miles @ $200 per mile		31400
			$3060340

Average cost per mile ready for operation, $19467.

Approximate cost of 130 miles from St. Cloud to Duluth, estimated at $23000 per mile.

Approximate cost of entire line from Albert Lea to Duluth, 287.2 miles, $6050340 ($21060 per mile).

PART II.

RAILROAD ECONOMICS.

CHAPTER XVIII.

INTRODUCTION.

363. The magnitude of railroad business. The gross earnings of railroads for the year ending June 30, 1899, were over $1,300,-000,000. This is greater than the combined value of all the gold, silver, iron, wheat, and corn produced by the country. The following figures (to the nearest million of dollars) gives the value of various crops for 1899, according to the current U. S. Yearbook of Agriculture:

Gold	71	Oats	198
Silver	33	Hay	412
Iron	245	Coal	256
Wheat	320	Copper	104
Corn	629	Lead	19
Total			2287

About 929000 persons (about one eightieth of the population) were directly employed by the roads for a compensation of about $523,000,000. Probably 3,000,000 to 4,000,000 people were supported by this. Beside all these, probably 5,000,000 employés were kept busy in occupations which are a more or less direct result of railroads, *e.g.*, locomotive- and car-shops, rail-mills, etc. We may therefore estimate that perhaps 20,000,000 people (or, say, one fourth of our population) are supported by railroads or by occupations which owe their chief existence to railroads.

The "number of passengers carried 1 mile" was 14,591,327,613. Calling the population of the United States 75,000,000 for round numbers, it means an average ride of 195 miles for every man, woman, and child.

The "tons carried 1 mile" were 123,667,257,153, or nearly 1650 ton-miles per inhabitant. The payments made to the railroads averaged over $17 per inhabitant.

Turning to a dark side of the picture, we find that the traffic was carried on at a cost of 7123 killed and 44620 injured. This averages one killed every hour and a quarter and one injured every twelve minutes. Of these large numbers, the "passengers" comprised but 239 and 3442 respectively. The remainder were employés and "others," the "others" consisting largely of "trespassers."

The actual bona-fide cost of the railroads of the country cannot be accurately computed (as will be shown later), but the capital, as represented by stocks and bonds, represents $11,033,954,898, or about $147 per inhabitant. This is roughly about one sixth of the total national wealth.

The above figures may give some idea of the magnitude of the interests involved in the operation of railroads. No single business in the country approaches it in capital involved, earnings, number of people affected, or effect on other business.

364. Cost of transportation. The importance of railroads may be also indicated by their power of creating cheap transportation. Less than one hundred years ago local famine and overabundant harvests within a radius of a few miles were not unknown. When the transportation of goods depended on actual porterage by human beings, as has been the case but recently in the Klondike, the transportation of 100 lbs. 20 miles might be considered an average day's labor. At $1 per day, this equals $1 per ton-mile. In 1899 the railroads transported freight at an *average* cost to the public of 0.724 c. per ton per mile, and the feeding of Europe with wheat from Manitoba has become a commercial possibility. In 1899 passengers paid an average charge of 1.925 c. per mile, and a trip of 1000 miles inside of 24 hours is now common.

365. Study of railroad economics—its nature and limitations. The multiplicity of the elements involved in most problems in railroad construction preclude the possibility of a solution which is demonstrably perfect. Barring out the comparatively few cases in this country where it is difficult to obtain *any* practicable location, it may be said that a comparatively low order of talent will suffice to locate anywhere a railroad over which it is physically possible to run trains. It may be very badly located for obtaining business, the ruling grades may be excessive, the alignment may be very bad, and the road may be a hopeless financial failure, and yet trains *can* be run. Among the infinite number of possible locations of the road, the engineer must determine the route which will give the best railroad property for the least expenditure of money—the road whose earning capacity is so great that after paying the operating expenses and interests on the bonds, the surplus available for dividends or improvements is a maximum.

An unfortunate part of the problem is that even the blunders are not always readily apparent nor their magnitude. A defective dam or bridge will give way and every one realizes the failure, but a badly located railroad affects chiefly the finances of the enterprise by a series of leaks which are only perceptible and demonstrable by an expert, and even he can only say that certain changes would probably have a certain financial value.

366. Outline of the engineer's duties. The engineer must realize at the outset the nature and value of the conflicting interests which are involved in variable amount in each possible route.

(a) **The maximum of business must be obtained,** and yet it *may* happen that some of the business may only be obtained by an extravagant expenditure in building the line or by building a line very expensive to operate.

(b) **The ruling grades should be kept low,** and yet this *may* require a sacrifice in business obtained and also *may* cost more than it is worth.

(c) **The alignment should be made as favorable as possible;** favorable alignment reduces the future operating expenses, but it may require a very large immediate outlay.

(d) **The total cost** must be kept within the amount at which the earnings will make it a profitable investment.

(e) **The road must be completed and operated** until the "normal" traffic is obtained and the road is self-supporting without exhausting the capital obtainable by the projectors; for no matter how valuable the property may ultimately become, the projectors will lose nearly, if not quite, all they have invested if they lose control of the enterprise before it becomes a paying investment.

Each new route suggested makes a new combination of the above conflicting elements. The engineer must select a route by first eliminating all lines which are manifestly impracticable and then gradually narrowing the choice to the best routes whose advantages are so nearly equal that a closer detailed comparison is necessary.

The ruling grade and the details of alignment have a large influence on the operating expenses. A large part of this course of instruction therefore consists of a study of operating expenses under average normal conditions, and then a study of the effect on operating expenses of given changes in the alignment.

367. Justification of such methods of computation. It may be argued that the data on which these computations are based are so unreliable (because variable and to some extent non-computable) that no dependence can be based on the conclusions. This is true to the extent that it is useless to claim great precision in the computation of the value of any proposed change of alignment. Suppose, for example, it is computed that a given improvement in alignment will reduce the operating expenses of 20 trains per day by $1000 per year. Suppose the change in alignment may be made for $5000, which may be obtained at 5% interest. Even with large allowances for inaccuracy in the computation of the value, $1000, it evidently will be better to incur an additional interest charge of $250 than increase the annual operating expenses by $1000. Moreover, since traffic is almost sure to increase (and interest charges are generally decreasing), the advantage of the improvement will only increase as time passes. On the other hand, if the improvement cannot be made except by an expenditure of, say, $50000, the change would evidently be unjustifiable. When the interest on the first cost is practically equal to the annual operating value of the proposed improvement, there is evidently but little choice; no great harm can result from either decision, and the decision frequently will depend on the willingness to increase the total amount invested in the enterprise.

To express the above question more generally, in every computation of the operating value of a proposed improvement, it may always be shown that the true value lies somewhere between some maximum and some minimum. Closer calculations and more reliable data will narrow the range between these extreme values. According as the interest on the cost of the proposed improvement is greater or less than the mean of these limits, we may judge of its advisability. The range of the limits shows the uncertainty. If it lies outside of the limits there is no uncertainty, assuming that the limits have been properly determined. If well within the limits, either decision will answer unless other considerations determine the question. And so, although it is not often possible to obtain precise values, we may generally reach a conclusion which is unquestionable. Even under the most unfavorable circumstances, the computations, when made with the assistance of all the broad common sense and experience that can be brought to bear, will point to a decision which is much better than mere "judgment," which is responsible for very many glaring and costly railroad blunders. In short, Railroad Economics means the application of systematic methods of work *plus* experience and judgment, rather than a dependence on judgment unsystematically formed. It makes no pretense to furnishing mechanical rules by which all railroad problems may be solved by any one, but it does give a general method of applying principles by which an engineer of experience and judgment can apply his knowledge to better advantage. To the engineer of limited experience the methods are invaluable; without such methods of work his opinions are practically worthless; with them his conclusions are frequently more sound than the unsystematically formed judgments of a man with a glittering record. But the engineer of great experience may use these methods to form the best opinions which are obtainable, for he can apply his experience to make any necessary local modifications in the method of solution. The dangers lie in the extremes, either recklessly applying a rule on the basis of insufficient data to an unwarrantable extent, or, disgusted with such evident unreliability, neglecting altogether such systematic methods of work.

CHAPTER XIX.

THE PROMOTION OF RAILROAD PROJECTS.

368. Method of formation of railroad corporations. Many business enterprises, especially the smaller ones, are financed entirely by the use of money which is put into them directly in the form of stock or mere partnership interest. A railroad enterprise is frequently floated with a comparatively small financial expenditure on the part of the original promoters. The promoters become convinced that a railroad between *A* and *B*, passing through the intermediate towns of *C* and *D*, with others of less importance, will be a paying investment. They organize a company, have surveys made, obtain a charter, and then, being still better able (on account of the additional information obtained) to exploit the financial advantages of their scheme, they issue a prospectus and invite subscriptions to bonds. Sometimes a portion of these bonds are guaranteed, principal and interest, or perhaps the principal alone, by townships or by the national government. The cost of this preliminary work, although large in gross amount if the road is extensive, is yet but an insignificant proportion of the total amount involved. The proportionate amount that *can* be raised by means of bonds varies with the circumstances. In the early history of railroad building, when a road was projected into a new country where the traffic possibilities were great and there was absolutely no competition, the financial success of the enterprise would seem so assured that no difficulty would be experienced in raising from the sale of bonds all the money necessary to construct and equip the road. But the promoters (or stockholders) must furnish all money for the preliminary expenses, and must make up all deficiencies between the proceeds of the sale of the bonds and the capital needed for construction.

"In theory, stocks represent the property of the responsible owners of the road, and bonds are an encumbrance on that property. According to this theory, a railroad enterprise should begin with an issue of stock somewhere near the value of the property to be created and no more bonds should be issued than are absolutely necessary to complete the enterprise. Now it is not denied that there are instances in which this theory is followed out. In New England, for example, as well as in some of the Southern States, there are a few roads represented wholly by stock or very lightly mortgaged. But this theory does not conform to the general history of railway construction in the United States, nor is it supported by the figures that appear in the summary. The truth is, railroads are built on borrowed capital, and the amount of stock that is issued represents in the majority of cases the difference between the actual cost of the undertaking and the confidence of the public expressed by the amount of bonds it is willing to absorb in the ultimate success of the venture." *

"The same general law obtains and has always obtained throughout the world, that such properties (as railways) are always built on borrowed money up to the limit of what is regarded as the positive and certain minimum value. The risk only—the dubious margin which is dependent upon sagacity, skill, and good management—is assumed and held by the company proper who control and manage the property." †

369. The two classes of financial interests—the security and profits of each. From the above it may be seen that stocks, bonds, car-trust obligations, and even current liabilities represent railroad capital. The issue of the bonds "was one means of collecting the capital necessary to create the property against which the mortgage lies." The variation between these interests lies chiefly in the security and profits of each. The current liabilities are either discharged or, as frequently happens, they accumulate until they are funded and thus become a definite part of the railroad capital.

The growth of this tendency is shown in the following tabular form:

The bonded interest has greater security than the stock, but less profit. The interest on the bonds must be paid before any money can be disbursed as dividends. If the bond interest

* Henry C. Adams, Statistician, U. S. Int. Con. Commission.

† A. M. Wellington, Economic Theory of Railway Location.

Railroads in the United States.	June 30, 1888.		June 30, 1898.	
	Amount, millions.	Per cent.	Amount, millions.	Per cent.
Stocks.	3864	47.5	5311	44.6
Funded debt	3869	47.6	5510	46.3
Current liabilities, etc.	396	4.9	1087	9.1

is not paid, a receivership, and perhaps a foreclosure and sale of the road, is a probability, and in such case the stockholder's interests are frequently wiped out altogether. The bondholder's real profit is frequently very different from his nominal profit. He sometimes buys the bonds at a very considerable discount, which modifies the rate which the interest received bears to the amount really invested. Even the bondholder's security may suffer if his mortgage is a second (or fifth) mortgage, and the foreclosure sale fails to net sufficient to satisfy all previous claims.

On the other hand, the stockholder, who may have paid in but a small proportion of his subscription, *may*, if the venture is successful, receive a dividend which equals 50 or 100% of the money actually paid in, or, as before stated, his entire holdings may be entirely wiped out by a foreclosure sale. When the road is a great success and the dividends very large, additional issues of stock are generally made, which are distributed to the stockholders in proportion to their holdings, either gratuitously or at rates which give the stockholders a large advantage over outsiders. This is the process known as "watering." While it may sometimes be considered as a legitimate "salting down" of profits, it is frequently a cover for dishonest manipulation of the money market.

For the twelve years between 1887 and 1899 about *two thirds* of all the railroad stock in the United States paid *no* dividends, while of those that paid dividends the average rate varied from 4.96 to 5.74%. The year from June 30, 1898, to June 30, 1899, was the most prosperous year of the group, and yet nearly 60% of all railroad stock paid *no* dividend, and the average rate paid by those which paid at all was 4.96%. The total amount distributed in dividends was greater than ever before, but the average rate is the least of the above group because many roads, which had passed their dividends for many previous years, distinguished themselves by declaring a dividend, even though small. During that same period but 13.35% of the stock paid over 6% interest. The total dividends paid amounted to but 2.01% of all the capital stock, while investments ordinarily are expected to yield from 4 to 6% (or more) according to the risk. Of course the effect of "watering" stock is to decrease the nominal rate of dividends, but there is no dodging the fact that, watered or not, even in that year of "good times," about 60% of all the stock paid *no* dividends. Unfortunately there are no accurate statistics showing how much of the stock of railroads represents actual paid-in capital and how much is "water." The great complication of railroad finances and the dishonest manipulation to which the finances of some railroads have been subjected would render such a computation practically worthless and hopelessly unreliable now.

During the year ending June 30, 1898 (which may in general be considered as a sample), 15.82% of the funded debt paid no interest. About one third of the funded debt paid between 4 and 5% interest, which is about the average which is paid.

The income from railroads (both interest on bonds and dividends on stock) may be shown graphically by diagrams, such as are given in the annual reports of the Interstate Commerce Commission. They show that while railroad investments are occasionally very profitable, the average return is less than that of ordinary investments *to the investors*. The *indirect* value of railroads in building up a section of country is almost incalculable and is worth many times the cost of the roads. It is a discouraging fact that very few railroads (old enough to have a history) have escaped the experience of a receivership, with the usual financial loss to the then stockholders. But there is probably not a railroad in existence which, however much a financial failure in itself, has not profited the community more than its cost.

370. The small margin between profit and loss to projectors. When a railroad is built entirely from the funds furnished by its promoters (or from the sale of stock) it will generally be a paying investment, although the rate of payment may be very small. The percentage of receipts that is demanded for actual operating expenses is usually about 67%. The remainder will usually pay a reasonable interest on the total capital involved. But the operating expenses are frequently 90 and even 100% of the gross receipts. In such cases even the bondholders do not get their due and the stockholders have absolutely nothing. Therefore the stockholder's interest is very speculative. A comparatively small change in the business done (as is illustrated numerically in § 372) will not only wipe out altogether the dividend—taken from the last small percentage of the total receipts and which may equal 50% or more of the capital stock *actually paid in*—but it may even endanger the bondholders' security and cause them to foreclose their mortgage. In such a case the stockholders' interest is usually entirely lost. It does not alter the essential character of the above-stated relations that the stockholders sometimes protect themselves somewhat by buying bonds. By so doing they simply decrease their risk and also decrease the possible profit that might result from the investment of a given total amount of capital.

371. **Extent to which a railroad is a monopoly.** It is a popular fallacy that a railroad, when not subject to the direct competition of another road, has an absolute monopoly—that it controls "all the traffic there is" and that its income will be practically independent of the facilities afforded to the public. The growth of railroad traffic, like the use of the so-called necessities or luxuries of life, depends entirely on the supply and the cost (in money or effort) to obtain it. A large part of railroad traffic belongs to the unnecessary class—such as traveling for pleasure. Such traffic is very largely affected by mere matters of convenience, such as well-built stations, convenient terminals, smooth track, etc. The freight traffic is very largely dependent on the possibility of delivering manufactured articles or produce at the markets so that the *total* cost of production and transportation shall not exceed the total cost in that same market of similar articles obtained elsewhere. The creation of facilities so that a factory or mine may successfully compete with other factories or mines will develop such traffic. The receipts from such a traffic may render it possible to still further develop facilities which will in return encourage further business. On the other hand, even the partial withdrawal of such facilities may render it impossible for the factory or mine to compete successfully with rivals; the traffic furnished by them is completely cut off and the railroad (and indirectly the whole community) suffers correspondingly. The "strictly necessary" traffic is thus so small that few railroads could pay

their operating expenses from it. The dividends of a road come from the last comparatively small percentage of its revenue, and such revenue comes from the "unnecessary" traffic which must be coaxed and which is so easily affected by apparently insignificant "conveniences."

372. Profit resulting from an increase in business done; loss resulting from a decrease. In a subsequent chapter it will be shown that a large portion of the operating expenses are independent of small fluctuations in the business done and that the operating expenses are roughly two thirds of the gross revenue. Assume that by changes in the alignment the business obtained has been increased (or diminished) 10%. Assume for simplicity that the operating expenses on the revised track are the same as on the route originally planned; also that the cost of the track is the same and hence the fixed charges are assumed to be constant for all the cases considered. Assume the fixed charges to be 28%. The additional business, when carried in cars otherwise but partly filled will hardly increase the operating expenses by a measurable amount. When extra cars or extra trains are required, the cost will increase up to about 60% of the average cost per train mile. We may say that 10% increase may in general be carried at a rate of 40% of the average cost of the traffic. A reduction of 10% in traffic may be assumed to reduce expenses a similar amount. The effect of the change in business will therefore be as follows:

	Business increased 10%.	Business decreased 10%.
Operating exp. = 67	67(1 + 10% × 40%) = 69.68	67(1 − 10% × 40%) = 64.32
Fixed charges = 28	.... 28.00	.... 28.00
95	97.68	92.32
Total income... 100	Income............ 110.00	Income............ 90.00
Available for dividends....... 5	Available for dividends........... 12.32	Deficit............. 2.32

In the one case the increase in business, which may often be obtained by judicious changes in the alignment or even by better management without changing the alignment, more than doubles the amount available for dividends. In the other case the profits are gone, and there is an absolute deficit. The above is a numerical illustration of the argument, previously stated, of the small margin between profit and loss to the original projectors.

373. Estimation of probable volume of traffic and of probable growth. Since traffic and traffic facilities are mutually interdependent and since a large part of the normal traffic is merely potential until the road is built, it follows that the traffic of a road will not attain its normal volume until a considerable time after it is opened for operation. But the estimation even of this normal volume is a very uncertain problem. The estimate may be approached in three ways:

1st. The actual gross revenue derived by all the railroads in that section of the country (as determined by State or U. S. Gov. reports) may be divided by the total population of the section and thus the average annual expenditure per head of population may be determined. A determination of this value for each one of a series of years will give an idea of the normal rate of growth of the traffic. Multiplying this annual contribution by the population which may be considered as tributary gives *a valuation* of the possible traffic. Such an estimate is unreliable (*a*) because the *average* annual contribution may not fit that particular locality, (*b*) because it is very difficult to correctly estimate the number of the true tributary population especially when other railroads encroach more or less into the territory. Since a rough value of this sort may be readily determined, it has its value as a check, if for nothing else.

2d. The actual revenue obtained by some road whose circumstances are as nearly as possible identical with the road to be considered may be computed. The weak point consists in the assumption that the character of the two roads is identical or in incorrectly estimating the allowance to be made for observed differences. The method of course has its value as a check.

3d. A laborious calculation may be made from an actual study of the route—determining the possible output of all factories, mines, etc., the amount of farm produce and of lumber that might be shipped, with an estimate of probable passenger traffic based on that of like towns similarly situated. This method is the best when it is properly done, but there is always the danger of leaving out sources of income—both existent and that to be developed by traffic facilities, or, on the other hand, of overestimating the value of expected traffic. In the following tabular form are shown the population, gross receipts, receipts per head of population, mileage, earnings per mile of line operated, and mileage per 10,000 of population for the whole United States. It should be noted that the values are only *averages*, that individual variations are large, and that only a very rough dependence may be placed on them as applied to any particular case.

Year.	Population (estimated).	Gross receipts.	Receipts per head of population.	Mileage.†	Earnings per mile of line operated.	Mileage per 10,000 population.‡
1888...	60,100,000	$910,621,220	$15.15	136,884	$6653	24.94
1889...	61,450,000	964,816,129	15.81	153,385	6290	25.67
1890...	*62,801,571	1051,877,632	16.75	156,404	6725	26.05
1891...	64,150,000	1096,761,395	17.10	161,275	6801	26.28
1892...	65,500,000	1171,407,343	17.89	162,397	7213	26.19
1893...	68,850,000	1220,751,874	18.26	169,780	7190	26.40
1894...	68,200,000	1073,361,797	15.74	175,691	6109	26.20
1895...	69,550,000	1075,371,462	15.46	177,746	6050	25.97
1896...	70,900,000	1150,169,376	16.22	181,983	6320	25.78
1897...	72,350,000	1122,089,773	15.53	183,284	6122	25.53
1898...	73,600,000	1247,325,621	16.95	184,648	6755	25.32
1899...	74,950,000	1313,610,118	17.53	187,535	7005	25.25
1900...	*76,295,220	1480,673,054	19.41	190,406	7776	24.96

* Actual.

† Excludes a small percentage not reporting "gross receipts."

‡ Actual mileage.

The probable growth in traffic, after the traffic has once attained its normal volume, is a small but almost certain quantity. In the above tabular form this is indicated by the gradual growth in "receipts per head of population" from 1888 to 1893. Then the sudden drop due to the panic of 1893 is clearly indicated, and also the gradual growth in the last few years. Even in England, where the population has been nearly stationary for many years, the growth though small is unmistakable. On the other hand the growth in some of the Western States has been very large. For example, the gross earnings per head of population in the State of Iowa increased from $1.42 in 1862 to $10.00 in 1870, and to $19.46 in 1884.

There will seldom be any justification in building to accommodate a larger business than what is "in sight." Even if it could be anticipated with certainty that a large increase in business would come in ten years, there are many reasons why it would be unwise to build on a scale larger than that required for the business to be immediately handled. Even though it may cost more in the future to provide the added accommodations (*e.g.* larger terminals, engine-houses, etc.), the extra expense will be nearly if not quite offset by the interest saved by avoiding the larger outlay for a period of years which may often prove much longer than was expected. A still more important reason is the avoidance of uselessly sinking money at a time when every cent may be needed to insure the success of the enterprise as a whole.

374. Probable number of trains per day. Increase with growth of traffic. The number of passenger trains per day cannot be determined by dividing the total number of passengers estimated to be carried per day by the capacity of the cars that can be hauled by one engine. There are many small railroads, running three or four passenger trains per day each way, which do not carry as many passengers all told as are carried on one heavy train of a trunk line. But because the bulk of the passenger traffic, especially on such light-traffic roads, is "unnecessary" traffic (see § 371) and must be encouraged and coaxed, the trains must be run much more frequently than mere capacity requires. The minimum number of passenger trains per day on even the lightest-traffic road should be two. These need not necessarily be passenger trains exclusively. They may be mixed trains.

The number required for freight service may be kept more nearly according to the actual tonnage to be moved. At least one local freight will be required, and this is apt to be considerably within the capacity of the engine. Some very light-traffic roads have little else than local freight to handle, and on such there is less chance of economical management. Roads with heavy traffic can load up each engine quite accurately according to its hauling capacity and the resulting economy is great. Fluctuations in traffic are readily allowed for by adding on or dropping off one or more trains. Passenger trains must be run on regular schedule, full or empty. Freight trains are run by train-despatcher's orders. A few freight trains per day may be run on a nominal schedule, but all others will be run as extras. The criterion for an increase in the number of passenger trains is impossible to define by set rules. Since it should always come before it is absolutely demanded by the train capacity being overtaxed, it may be said in general terms that a train should be added when it is believed that the consequent increase in facilities will cause an increase in traffic the value of which will equal or exceed the added expense of the extra train.

375. **Effect on traffic of an increase in facilities.** The term facilities here includes everything which facilitates the transport of articles from the door of the producer to the door of the consumer. As pointed out before, in many cases of freight transport, the reduction of facilities below a certain point will mean the entire loss of such traffic owing to local inability to successfully compete with more favored localities. Sometimes owing to a lack of facilities a railroad company feels compelled to pay the cartage or to make a corresponding reduction on what would normally be the freight rate. In competitive freight business such a method of procedure is a virtual necessity in order to retain even a respectable share of the business. Even though the railroad has no direct competitor, it must if possible enable its customers to meet their competitors on even terms. In passenger business the effect of facilities is perhaps even more marked. The pleasure travel will be largely cut down if not destroyed. It is on record that a railroad company once ordered the manager of a station restaurant to largely increase the attractions at that restaurant (as a method of attracting traffic) and agreed to pay the expected resulting loss. The net result was not only a large increase in railroad business (as was expected), but even an increase in the profits of the restaurant.

376. **Loss caused by inconvenient terminals and by stations far removed from business centers.** This is but a special case of the subject discussed just in the preceding paragraph. The competition once existing between the West Shore and the New York Central was hopeless for the West Shore from the start. The possession of a terminal at the Grand Central Station gave the New York Central an advantage over the West Shore with its inconvenient terminal at Weehawken which could not be compensated by any obtainable advantage by the West Shore. This is especially true of the passenger business. The through freight business passing through or terminating at New York is handled so generally by means of floats that the disadvantage in this respect is not so great. The

enormous expenditure (roughly $10,000,000) made by the Pennsylvania R. R., on the Broad Street Station (and its approaches) in Philadelphia, a large part of which was made in crossing the Schuylkill River and running to City Hall Square, rather than retain their terminal in West Philadelphia, is an illustration of the policy of a great road on such a question. The fact that the original plan and expenditure has been very largely increased since the first construction proves that the management has not only approved the original large outlay, but saw the wisdom of making a very large increase in the expenditure.

The construction of great terminals is comparatively infrequent and seldom concerns the majority of engineers. But an engineer has frequently to consider the question of the location of a way station with reference to the business center of the town. The following points may (or may not) have to be considered, and the real question consists in striking a proper balance between conflicting considerations.

(1) During the early history of a railroad enterprise it is especially needful to avoid or at least postpone all expenditures which are not demonstrably justifiable.

(2) The ideal place for a railroad station is a location immediately contiguous to the business center of the town. The location of the station even one fourth of a mile from this may result in a loss of business. Increase this distance to one mile and the loss is very serious. Increase it to five miles and the loss approaches 100%.

(3) The cost of the ideal location and the necessary right of way may be a very large sum of money for the new enterprise. On the other hand the increase in property values and in the general prosperity of the town, caused by the railroad itself, will so enhance the value of a more convenient location that its cost at some future time will generally be extravagant if not absolutely prohibitory. The original location is therefore under ordinary conditions a finality.

(4) To some extent the railroad will cause a movement of the business center toward it, especially in the establishment of new business, factories, etc., but the disadvantages caused to business already established is permanent.

(5) In any attempt to compute the loss resulting from a location at a given distance from the business center it must be

recognized that each problem is distinct in itself and that any change or growth in the business of the town changes the amount of this loss.

The argument for locating the station at some distance from the center of the town may be based on (*a*) the cost of right of way, thus involving the question of a large initial outlay, (*b*) the cost of very expensive construction (*e.g.* bridges), again involving a large initial outlay, (*c*) the avoidance of excessive grade into and out of the town. It sometimes happens that a railroad is following a line which would naturally cause it to pass at a considerable elevation above (rarely below) the town. In this case there is to be considered not only the possible greater initial cost, but the even more important increase in operating cost due to the introduction of a very heavy grade. To study such a case, compute the annual increase in operating expenses due to the additional grade, curvature, and distance; add to this the annual interest on the increased initial cost (if any) and compare this sum with the estimated annual loss due to the inconvenient location. The estimation of the increase in operating expenses is discussed in a subsequent chapter. The loss of business due to inconvenient location can only be guessed at. Wellington says that at a distance of one mile the loss would average 25%, with upper and lower limits of 10 and 40%, depending on the keenness of the competition and other modifying circumstances. For each additional mile reduce 25% of the preceding value. While such estimates are grossly approximate, yet with the aid of sound judgment they are better than nothing and may be used to check gross errors.

377. General principles which should govern the expenditure of money for railroad purposes. It will be shown later that the elimination of grade, curvature, and distance have a positive money value; that the reduction of ruling grade is of far greater value; that the creation of facilities for the handling of a large traffic is of the highest importance and yet the added cost of these improvements is sometimes a large percentage of the cost of *some road* over which it would be physically possible to run trains between the termini.

The subsequent chapters will be largely devoted to a discussion of the value of these details, but the general principles governing the expenditure of money for such purposes may be stated as follows:

1. No money should be spent (beyond the unavoidable minimum) unless it may be shown that the addition is in itself a profitable investment. The additional sum may not wreck the enterprise and it may add something to the value of the road, but unless it adds more than the improvement costs it is not justifiable.

2. If it may be positively demonstrated that an improvement will be more valuable to the road than its cost, it should certainly be made even if the required capital is obtained with difficulty. This is all the more necessary if the neglect to do so will permanently hamper the road with an operating disadvantage which will only grow worse as the traffic increases.

3. This last principle has two exceptions: (*a*) the cost of the improvement may wreck the whole enterprise and cause a total loss to the original investors. For, unless the original promoters can build the road and operate it until its stock has a market value and the road is beyond immediate danger of a receivership, they are apt to lose the most if not all of their investment; (*b*) an improvement which is very costly although unquestionably wise may often be postponed by means of a cheap temporary construction. Cases in point are found at many of the changes of alignment of the Pennsylvania R. R., the N. Y., N. H. & H. R. R., and many others. While some of the cases indicate faulty original construction, at many of the places the original construction was wise, considering the then scanty traffic, and now the improvement is wise considering the great traffic.

CHAPTER XX.

OPERATING EXPENSES.

378. Distribution of gross revenue. When a railroad comprises but one single property, owned and operated by itself, the distribution of the gross revenue is a comparatively simple matter. The operating expenses then absorb about two thirds of the gross revenue; the fixed charges (chiefly the interest on the bonds) require about 25 or 30% more, leaving perhaps 3 to 8% (more or less) available for dividends. A recent report on the Fitchburg R. R. shows the following:

Operating expenses	$5,083,571	69.1%
Fixed charges	1,567,640	21.3%
Available for dividends, surplus, or permanent improvements	708,259	9.6%
Total revenue	$7,359,470	100.0%

But the financial statements of a large majority of the railroad corporations are by no means so simple. The great consolidations and reorganizations of recent years have been effected by an exceedingly complicated system of leases and sub-leases, purchases, "mergers," etc., whose forms are various. Railroads in their corporate capacity frequently own stocks and bonds of other corporations (railroad properties and otherwise) and receive, as part of their income, the dividends (or bond interest) from the investments.

In consequence of this complication, the U. S. Interstate Commerce Commission presents a "condensed income account" of which the following is a sample (1899):

Gross earnings from operation (received by station-agents, etc)	$1,313,610,118
Less operating expenses (fuel, wages, etc.)	856,968,999
Income from operation	456,641,119
Income from other sources (lease of road, stocks, bonds, etc.)	148,713,983
Total income	605,355,102

Total deductions from income (interest, rents for lease of road, taxes, etc.)	441 200,289
Net income	164,154,813
Total dividends (including "other payments")	111,089,936
Surplus from operations	53,064,877

In the above account an item of *income* (*e.g.* lease of road) reported by one road will be reported as a "deduction from income" by the road which leases the other.

The above statement may be reduced to an income account of all the railways *considered as one system.* We then have

Operating expenses	$856,968,999	
Salaries and maintenance of leased lines (really operating expenses, but considered above as fixed charges against the leasing lines)	595,192	
	857,564,191	64.1%
Net interest and taxes	295,098,014	22.0%
Available for dividends, adjustments, and improvements	186,992,909	13.9%
	1,339,655,114	100.0%
Gross earnings from operation	1,313,610,118	
Clear income from investments (i.e., the balance of intercorporate payments and receipts on corporate investments)	26,044,996	
	1.339,655,114	

Of the $186,992,909, the amount disbursed as dividends to outside stockholders (besides that paid to railroads in their corporate capacity) was $94,273,796. This left a balance of $92,719,113 "available for adjustments and improvements." Of this part was spent in permanent improvements, part was advanced to cover deficits in the operation of weak lines and more than half was left as "surplus," *i.e.* working capital.

The percentages of the gross revenue which are devoted to operating expenses, fixed charges, and dividends are not *necessarily* an indication of creditable management or the reverse. Causes utterly beyond the control of the management, such as the local price of coal, may abnormally increase certain items of expense, while ruinous competition may cut down the gross revenue so that little or nothing is left for dividends. A favorable location will sometimes make a road prosperous in spite of bad management. On the other hand, the highest grade of skill will fail to keep some roads out of the hands of a receiver.

379. Fourfold distribution of operating expenses. The distribution of operating expenses here used is copied from the method of the Interstate Commerce Commission. The aim is to divide the expenses into groups which are as mutually independent and distinct as possible—although, as will be seen later, a change in one item of expense will variously affect other items. The groups are:

	Average value.
1. Maintenance of way and structures. . . . . The values for five years have an extreme range of about 1.2%. The subdivisions of this group and of the others will be given later.	20.662%
2. Maintenance of equipment. . . . . Extreme range of 1.834%. The tendency has been for this item to grow larger, not only in absolute amount but in percentage of total expenditure.	16.892%
3. Conducting transportation. . . . . This item has been growing *relatively* less. During (and immediately after) the panic of 1893, the maintenance of way and of equipment was made as small as possible, which made the cost of conducting transportation *relatively* larger. During the recent more prosperous years deficiencies of equipment have been made up, making this item relatively less.	57.793%
4. General expenses. . . . . A nearly constant item.	4.653%
	100.000%

The above percentages represent the averages given by the reports for the five years from 1895 to 1899 inclusive.

380. Operating expenses per train-mile. The reports of the U. S. Interstate Commerce Commission give the average cost per train-mile for every railroad in the United States. Although there are wide variations in these values, it is remarkable that the very large majority of roads give values which agree to within a small range, and that within this range are found not only the great trunk lines with their enormous train mileage, but also roads with very light traffic.

In the following tabular form is shown a statement taken from the report for 1898 of ten of the longest railroads in the United States and, in comparison with them, a corresponding statement regarding ten more roads selected *at random*, except in the respect that each had a mileage of less than 100 miles. Although the extreme variations are greater, yet there is no very marked difference in the general values for operating expenses per train-mile, or in the ratio of expenses to earnings. The averages for the ten long roads agree fairly well with the averages for the whole country, but there would be no trouble (as is shown by some of the individual cases) in finding another group of ten short roads giving either greater or less average values than those given. And yet the tendency to uniform values, regardless of the mileage, is very striking.

No. in report.		Mileage.	Operating expenses per train-mile.	Ratio $\frac{\text{exp.}}{\text{earn.}}$.
	Whole United States	186,396	0.956	65.58
71	Canadian Pacific	6,568	0.854	58.31
1465	C., M. & St. P.	6,191	0.883	58.94
1443	C., B., & Q.	5,860	0.881	60.87
1879	Southern Pacific	5,426	1.320	58.70
1142	Southern	5,232	0.809	65.32
1436	Chicago & Northwestern	5,086	0.885	63.35
1405	A., T. & S. F	4,565	0.917	67.59
1560	Northern Pacific	4,524	1.177	46.81
1495	Great Northern	3,860	1.101	46.97
1264	Illinois Central	3,807	.764	63.56
	Average of ten	....	0.969	59.04
7	Bennington & Rutland	59	0.582	71.42
105	Mont. & Wells R.	44	0.828	83.96
167	Balto. & Del. Bay	45	1.098	102.83
234	Cent. N. Y. & W.	63	0.454	91.17
888	Man. & N. E.	99	0.739	54.49
1074	Farmv. & Powh.	93	0.781	76.22
1284	Lex. & East.	94	0.975	68.46
1540	Manistique	60	1.162	69.01
1812	Wh. & Bl. River Val.	64	.799	53.08
1979	No. Pac. Coast	88	.769	66.58
	Average of ten	....	0.819	73.72

The constancy of the average cost per train mile for several years past may be noted from the following tabular form.

The enforced economies after the panic of 1893 are well shown. The reduction generally took the form of a lowering of the standards of maintenance of way and of maintenance of

Year.	Average cost per train-mile.
1890	96.006
1891	95.707
1892	96.580
1893	97.272
1894	93.478
1895	91.829
1896	93.838
1897	92.918
1898	95.635
1899	98.390
	95.165

equipment. The marked advance from 1897 to 1898 and to 1899 was largely caused by the necessity for restoring the roads to proper condition, replenishing worn-out equipment and providing additional equipment to handle the greatly increased volume of business.

In looking over the list, it may be noted that the cases where the operating expenses per train-mile and the ratio of expenses to earnings vary very greatly from the average are almost invariably those of the very small roads or of "junction roads" where the operating conditions are abnormal. For example, one little road, with a total length of 13 miles and total annual operating expenses of $5342, spent but 22¼c. per train-mile, which precisely exhausted its earnings. As another abnormal case, a road 44 miles long spent $3.81 per train-mile, which was nearly *fourteen* times its earnings. In another case a road 13 miles long earned $7.76 per train-mile and spent $6.03 (78%) on operating expenses, but the fixed charges were abnormal and the earnings were less than half the sum of the operating expenses and fixed charges. The *normal* case, even for the small road, is that the cost per train-mile and the ratio of operating expenses to earnings will agree fairly well with the average, and when there is a marked difference it is generally due to some abnormal conditions of expenses or of earning capacity.

381. Reasons for uniformity in expenses per train-mile. The chief reason is that, although on the heavy-traffic road everything is kept up on a finer scale, better roadbed, heavier rails, better rolling stock, more employees, better buildings, stations, and terminals, etc., yet the number of trains is so much greater that the divisor is just enough larger to make the average cost about constant. This is but a general statement of a fact which will be discussed in detail under the different items of expense.

382. Detailed classification of expenses with ratios to the total expense. The Interstate Commerce Commission now publishes each year a classification with detailed summation for the cost of each item. These summations are made up from reports furnished by railroads which have (in the reports already made) represented about 94% of the total traffic handled. In the annexed tabular form (Table XX) are shown the percentages which each item bears to the total. The character of the changes from year to year in these ratios is very instructive and will be considered in the detailed discussion of the items which will follow.

Table XX is copied from the Interstate Commerce Commission report for 1899, pp. 88–90.

383. Elements of the cost (with variations and tendencies) of the various items. The I. C. C. report for the year ending June 30, 1895, was the first to include the distribution of expenses according to the present classification. The number of reports since then are too few to be of much value in determining the tendency to variation of the several items, and similar calculations made in previous years have by no means an equal reliability. Nevertheless the items as given are reliable and may be utilized, as far as any such computations are to be depended on, in estimating future expenses. A great deal of very interesting and instructive information may be derived from a study of the variations of these items, but the chief purpose of this discussion is to point out those elements of the cost of operating trains which may be affected by such changes of location as an engineer is able to make. There are some items of expense with which the engineer has not the slightest concern—nor will they be altered by any change in alignment or constructive detail which he may make. In the following discussion such items will be passed over with a brief discussion of the sub-items included.

MAINTENANCE OF WAY.

384. Item 1. REPAIRS OF ROADWAY. The item of repairs of roadway is very large—about half of the total cost of main-

TABLE XX. SUMMARY SHOWING CLASSIFICATION OF OPERATING EXPENSES OF ALL RAILROADS IN THE UNITED STATES FOR THE YEAR ENDING JUNE 30, 1899, AND PROPORTION OF EACH CLASS TO TOTAL, CLASSIFIED FOR THE YEARS ENDING JUNE 30, 1899 TO 1895.

Item.	Amount.	Per cent.				
	1899.	1899.	1898.*	1897.†	1896.‡	1895.§
Maintenance of way and structure:						
1. Repairs of roadway	$87,307,140	10.720	10.643	10.644	10.738	10.235
2. Renewals of rails	10,767,381	1.322	1.391	1.546	1.444	1.499
3. Renewals of ties	23,623,325	2.901	3.232	3.357	3.028	2.918
4. Repairs and renewals of bridges and culverts	19,335,860	2.374	2.512	2.472	2.265	2.268
5. Repairs and renewals of fences, road crossings, signs, and cattle-guards	3,968,408	.487	.537	.509	.561	.520
6. Repairs and renewals of buildings and fixtures	17,762,120	2.181	1.957	1.745	1.794	1.648
7. Repairs and renewals of docks and wharves	2,070,098	.254	.245	.231	.270	.235
8. Repairs and renewals of telegraph	1,153,408	.142	.137	.126	.135	.134
9. Stationery and printing	208 775	.026	.025	.024	.027	.033
10. Other expenses	3,628,539	.446	.349	.318	.372	.304
Total	$169,825,054	20.853	21.028	20.972	20.634	19.824
Maintenance of equipment:						
11. Superintendence	$5,147,586	.632	.656	.667	.666	.629
12. Repairs and renewals of locomotives	50,555,264	6.208	5.887	5.663	5.978	5.660
13. Repairs and renewals of passenger cars	17,623,134	2.164	2.188	2.265	2.216	2.211
14. Repairs and renewals of freight cars	57,320,521	7.038	7.210	6.376	7.193	6.008
15. Repairs and renewals of work cars	1,708,416	.210	.159	.140	.145	.121
16. Repairs and renewals of marine equipment	2,012,478	.247	.242	.215	.173	.167
17. Repairs and renewals of shop machinery and tools	4,167,798	.512	.486	.478	.520	.455
18. Stationery and printing	329,261	.040	.038	.039	.040	.041
19. Other expenses	4,429,987	.544	.493	.509	.460	.469
Total	$143,294,445	17.595	17.359	16.352	17.391	15.761

* Based on $766,332,900, excluding $51,640,376 unclassified.
† Based on $692,491,637 excluding $60,033,127 unclassified.
‡ Based on $721,730,766, excluding $51,258,278 unclassified.
§ Based on $675,228,640, excluding $50,491,775 unclassified.

Item.	Amount.	Per cent.				
	1899.	1899.	1898.*	1897.†	1896.‡	1895.§
Conducting transportation:						
20. Superintendence	$14,392,691	1.767	1.744	1.845	1.731	1.718
21. Engine and round-house men	78,913,978	9.690	9.645	9.922	9.733	9.917
22. Fuel for locomotives	77,187,344	9.478	9.457	9.392	9.669	10.408
23. Water-supply for locomotives	5,038,615	.619	.646	.677	.691	.724
24. Oil, tallow, and waste for locomotives	2,922,095	.359	.355	.374	.379	.413
25. Other supplies for locomotives	1,438,054	.177	.156	.160	.229	.200
26. Train service	61,756,607	7.583	7.660	7.589	7.784	7.993
27. Train supplies and expenses	12,439,675	1.527	1.525	1.508	1.556	1.526
28. Switchmen, flagmen, and watchmen	33,791,383	4.149	4.080	4.171	4.120	4.160
29. Telegraph expenses	15,525,232	1.906	1.907	2.000	1.978	2.078
30. Station service	61,160,732	7.510	7.758	8.002	7.710	8.112
31. Station supplies	5,664,045	.696	.692	.751	.794	.876
32. Switching charges, balance	2,895 893	.356	.331	327	.356	.344
33. Car-mileage, balance	16,367,903	2.010	2.107	2.203	2.054	2.096
34. Hire of equipment, balance	2,993,088	.368	.342	.249	.322	.384
35. Loss and damage	5,976,082	.734	.706	.692	.775	.784
36. Injuries to persons	7,116,212	.874	.882	.874	.840	.945
37. Clearing wrecks	1,197,901	.147	.130	.123	.126	.130
38. Operating marine equipment	7,065,668	.868	.958	.934	.826	.848
39. Advertising	3,569,073	.438	.417	.428	.417	.429
40. Outside agencies	14,507,499	1.781	1.762	1.727	1.567	1.623
41. Commissions	1,580,909	.194	.181	.158	.168	.154
42. Stockyards and elevators	936,939	.115	.155	.140	.126	.116
43. Rents for tracks, yards, and terminals	15,482,170	1.901	1.914	1.953	1.746	1.727
44. Rents for buildings and other property	3,967 353	.487	.475	.501	.502	.497
45. Stationery and printing	5,107,066	.627	.583	.633	.616	.589
46. Other expenses	5,456,377	.670	.624	.587	.547	.669
Total	$464,450,584	57.031	57.194	57.920	57.362	59.460
General expenses.						
47. Salaries of general officers	$9,535,486	1.171	1.165	1.235	1.213	1.333
48. Salaries of clerks and attendants	10,864,401	1.334	1.334	1.368	1.409	1.385
49. General office expenses and supplies	2,373,912	.292	.285	.301	.311	.315
50. Insurance	3,032,888	.372	.395	.436	.446	.487
51. Law expenses	5,783,372	.710	.655	.791	.725	.806
52. Stationery and printing (general offices)	1,390,670	.171	.176	.163	.165	.172
53. Other expenses	3,838,987	.471	.409	.462	.344	.457
Total	$36,819,716	4.521	4.419	4.756	4.613	4.955
Recapitulation of expenses						
54. Maintenance of way and structures	$169,825,054	20.853	21.028	20.972	20.634	19.824
55. Maintenance of equipment	143,294,445	17.595	17.359	16.352	17.391	15.761
56. Conducting transportation	464,450,584	57.031	57.194	57.920	57.362	59.460
57. General expenses	36,819,716	4.521	4.419	4.756	4.613	4.955
Grand total ‖	$814,389,799	100.000	100.000	100.000	100.000	100.000

‖ Excludes $42,579,197 unclassified.

(To face page 425.)

cost about constant. This is but a general statement of a fact which will be discussed in detail under the different items of expense.

382. Detailed classification of expenses with ratios to the total expense. The Interstate Commerce Commission now publishes each year a classification with detailed summation for the cost of each item. These summations are made up from reports furnished by railroads which have (in the reports already made) represented about 94% of the total traffic handled. In the annexed tabular form (Table XX) are shown the percentages which each item bears to the total. The character of the changes from year to year in these ratios is very instructive and will be considered in the detailed discussion of the items which will follow.

Table XX is copied from the Interstate Commerce Commission report for 1899, pp. 88–90.

383. Elements of the cost (with variations and tendencies) of the various items. The I. C. C. report for the year ending June 30, 1895, was the first to include the distribution of expenses according to the present classification. The number of reports since then are too few to be of much value in determining the tendency to variation of the several items, and similar calculations made in previous years have by no means an equal reliability. Nevertheless the items as given are reliable and may be utilized, as far as any such computations are to be depended on, in estimating future expenses. A great deal of very interesting and instructive information may be derived from a study of the variations of these items, but the chief purpose of this discussion is to point out those elements of the cost of operating trains which may be affected by such changes of location as an engineer is able to make. There are some items of expense with which the engineer has not the slightest concern—nor will they be altered by any change in alignment or constructive detail which he may make. In the following discussion such items will be passed over with a brief discussion of the sub-items included.

MAINTENANCE OF WAY.

384. Item 1. Repairs of Roadway. The item of repairs of roadway is very large—about half of the total cost of main-

tenance of way and structures. It includes the cost of frogs, switches, switch-stands, and interlocking signals. The distribution and laying of ties and rails, ballasting and tamping track, ditching, weeding, widening and protecting banks, the maintenance of snow-fences, dikes, and retaining walls, are also included. In short, any expense of maintaining the roadbed in condition which cannot be definitely assigned to one of the next few items will generally belong to this item—except perhaps those of item 10 (*q.v.*). The larger part of such items of expense is labor, and the variations will largely depend on the fluctuations in the wages of trackmen. Formerly these were much higher than now. About fifteen years ago they had dropped to what Wellington considered to be a permanent average of $1.25 per day. In 1893 it had dropped to $1.22, then in 1897 and 1898 to $1.16. In 1899 it was raised to $1.18.

In 1899 the *average* cost of this item per mile of main track was about $480, but this figure, after all, is of but little value because, for the reason already given in general in §381, it will be found that the cost for any road varies almost exactly as the train-mileage and will average very closely to 11c. per train-mile, whether the traffic be heavy or light.

385. Item 2. Renewal of Rails: This item may be considered as having been withdrawn from the previous item simply because it is one of the largest of the single items and because its cost is very readily determined. It includes the cost of the rails, their inspection, and their delivery (but not their distribution). The item shows a large percentage of variation, the figures (percentage of total expenses) being 1.322, 1.391, 1.546, 1.444, and 1.499 by the last five reports. The drop from 1.546 in 1897 to 1.391 in 1898 was just 10%. These fluctuations are due first to that considerable fluctuation in the price of rails which railroads can hardly expect to escape, and secondly to variations in the standard of maintenance caused first by hard times, which are then followed by unusual expenditures in good times, or by the expenditures absolutely essential to restore the track to its former condition. The item includes all rails wherever used, whether on main track, siding, repair track, gravel track, on wharves or coal-docks, and even includes guard-rails. But it does *not* include any rail attachments such as joints, frogs, switches, etc. The rate of rail wear under various conditions has already been discussed in Chapter IX.

386. Item 3. RENEWAL OF TIES. As with the previous item, this item is simply a detachment from the general item, repairs of roadway. As with rails, the cost of laying and distributing the ties is not included, but the cost of tie-plates and tie-plugs, also chemical treatment for preservation, if such is used, is included in this item. While the cost will vary considerably between different roads on account of first cost, kind of wood, climate, etc., the item for any one road for a period of years cannot vary greatly, unless there is a marked change in the standard of maintenance. The actual cost of such work has already been discussed in Chapter VIII.

387. Item 4. REPAIRS AND RENEWALS OF BRIDGES AND CULVERTS. This item includes not only the maintenance cost of all bridges, trestles, viaducts, and culverts, but of all piers, abutments, riprapping, etc., necessary to maintain them, and even the cost of operating drawbridges. The locating engineer is not concerned with this item, except as he may consider that some distance which is to be added (or cut out) has the average number of culverts and bridges. With culverts and small bridges there would be little or no error in such an assumption, but if there were any large bridges on the portion of track under discussion, they would need special consideration.

388. Items 5 to 10. REPAIRS AND RENEWALS OF FENCES, ROAD CROSSINGS, AND CATTLE-GUARDS—OF BUILDINGS AND FIXTURES—OF DOCKS AND WHARVES—OF TELEGRAPH PLANT; STATIONERY AND PRINTING; AND "OTHER EXPENSES." These items in the aggregate amount to but 3% of the average cost per train-mile. The fluctuations have so small an effect on the average cost per train-mile that they may be neglected. In item 5 are included not only those things which are specifically mentioned, but also those structures which in general are not directly affected by the running of trains. For example, "road crossings" include not only the maintenance of highway crossings at grade, but also overhead highway crossings and whatever a railroad may have to pay for the maintenance of a bridge by which another railroad crosses it. On the other hand, the maintenance of a bridge by which a railroad crosses another road (highway or railroad) is charged to bridges. The effect (if any) of these items on any changes in construction which an engineer may make will be specifically discussed in the succeeding chapters.

MAINTENANCE OF EQUIPMENT.

389. Item 11. **SUPERINTENDENCE.** This item includes those fixed charges in superintendence which do not fluctuate with small variations in business done. It includes the salaries of superintendent of motive power, master mechanic, master car-builder, foremen, etc., but does not include that of road foremen of engines nor enginemen. In a general way the item is proportional to the general scale of business of the road, but does not fluctuate with it.

390. Item 12. **REPAIRS AND RENEWALS OF LOCOMOTIVES.** This item must be studied by the locating engineer in order to determine the effect on locomotive repairs and renewals of an addition to distance (considered in Chapter XXI), the effect (chiefly in wheel wear) of a reduction in curvature (considered in Chapter XXII), or the effect of grade (considered in Chapter XXIII). In studying the effect of grade, the policy of adopting heavier locomotives and the effect of this on this item must also be considered. This item includes the expenses of work whose effect is supposed to last for an indefinite period. It does not include the expense of cleaning out boilers, packing cylinders, etc., which occurs regularly and which is charged to item 21, round-house men. It does include all current repairs, general overhauling, and even the replacement of old and worn-out locomotives by new ones to the extent of keeping up the original standard and number. Of course additions beyond this must be considered as so much increase in the original capital investment. As a locomotive becomes older the *annual* repair charge becomes a larger percentage on the first cost, and it may become as much as one fourth and even one third of the first cost. When a locomotive is in this condition it is usually consigned to the scrap-pile; the annual cost for maintenance becomes too large an item for its annual mileage. The effect on expenses of increasing the weight of engines is too complicated a problem to admit of precise solution, but certain elements of it may be readily computed. While the cost of repairs is greater for the heavier engines, the increase is only about one half as fast as the increase in weight—some of the sub-items not being increased at all.

391. Items 13, 14, 15. REPAIRS AND RENEWALS OF PASSENGER CARS, OF FREIGHT CARS, AND OF WORK CARS. As with engine repairs, the item excludes consumable supplies (oil, waste, illuminating oil or gas, ice, etc.), but includes in general all items necessary to maintain the cars up to the full standard of condition and number, and even to replace old worn-out cars by new. When, as is frequently the case with both cars and locomotives, the new rolling stock is larger, better, and of a higher standard than that which is replaced, the difference in cost should be added to capital investment. The chief concern of the locating engineer regarding this item is the effect on car repairs of additional distance, of variations in curvature (affecting wheel wear chiefly), and of grade (affecting the draft-gear and general wear and tear). These items will be considered under their proper heads in the following chapters.

392. Items 16, 17, 18, and 19. REPAIRS AND RENEWALS OF MARINE EQUIPMENT—OF SHOP MACHINERY AND TOOLS; STATIONERY AND PRINTING; OTHER EXPENSES. The location of the road along the line has no connection with the maintenance of marine equipment. The maintenance of shop machinery and tools can only be affected as the work of repairs of rolling stock fluctuates, and of course in a much smaller ratio. No change which an engineer can effect will have any appreciable influence on this item.

The other items are too small and have too little connection with location to be here discussed except as it may be considered that they vary with train mileage, which an engineer may influence (see Chapter XXIII, Grades).

CONDUCTING TRANSPORTATION.

393. Item 20. SUPERINTENDENCE. As with item 11, this item is not subject to minor fluctuations in business, but only varies with changes in the general scale of the business of the road.

394. Item 21. ENGINE AND ROUND-HOUSE MEN. This item includes the wages of engineers, firemen, and also all men employed around the engine-houses except those who are making such repairs as should be charged to maintenance of equipment (item 12). The item is a large one, but is only affected by one class of change of location—a difference in length of line. The wages of the round-house men constitute but a small percentage of this item, and the wages of the enginemen vary almost directly as the mileage. On very short roads, where the number of round trips which may properly constitute a day's work is definitely limited and on which there is but little night or Sunday work, the wages may be practically by the day, and a variation in length of several hundred feet or even a few miles in the length of the road may make practically no difference in the wages paid. But on the larger roads, operated by divisions, on which (especially in freight work) there is no distinction of day or night, week day or Sunday, the varying length of divisions is equalized by calling them $1\frac{1}{3}$ or $1\frac{1}{4}$ runs, a "run" usually being considered as about 100 miles. The enginemen are then paid according to the number of runs made per month. The effect on this item of variations in distance is discussed more fully in Chapter XXI.

395. Item 22. **FUEL FOR LOCOMOTIVES.** The item includes the entire cost of the fuel until it is placed in the engine-tender. The cost therefore includes not only the first cost at the point of delivery to the road, but also the expense of hauling it over the road from the point of delivery to the various coaling stations and the cost of operating the coal-pockets from which it is loaded on to the tenders. Although the cost is fairly regular for any one road, it is exceedingly variable for different roads. Roads running through the coal regions can often obtain their coal for eighty or ninety cents per ton. Other roads far removed from the coal-mines have been compelled to pay six dollars per ton. In the three succeeding chapters there will be considered in detail the effect on fuel consumption of variations in location. It will be shown that fuel consumption is quite largely independent of distance and the number of cars hauled.

396. Items 23, 24, and 25. **WATER-SUPPLY; OIL, TALLOW, AND WASTE; OTHER SUPPLIES FOR LOCOMOTIVES.** The cost of the water-supply is quite largely a fixed charge except where it is supplied by municipalities at meter rates. The consumption of all these supplies will vary nearly as the engine-mileage.

397. Item 26. **TRAIN SERVICE.** This item is one of the largest single items and includes in general the wages of all the train-hands except the enginemen. As with enginemen, they are paid according to the number of runs. The item is therefore of importance to the locating engineer from the one

standpoint of distance, and even then only when the variation in distance which is considered will affect the classification of the run and therefore the rate of pay for that run.

398. Item 27. TRAIN SUPPLIES AND EXPENSES. These include the large list of consumable supplies such as lubricating oil, illuminating oil or gas, ice, fuel for heating, cleaning materials, etc., which are used on the cars, and not on the locomotives. The consumption of some of these articles is chiefly a matter of time;—in other cases it is a function of the mileage.

399. Items 28, 29, 30, and 31. SWITCHMEN, FLAGMEN, AND WATCHMEN; TELEGRAPH EXPENSES; STATION SERVICE; AND STATION SUPPLIES. These items will be proportional to the general scale of business of the road, but are independent of small fluctuations in business. The main items are obvious from the titles. Many sub-items, which are very small or are of occasional or accidental occurrence, are also included under these items for lack of a better classification.

400. Items 32, 33, and 34. SWITCHING CHARGES—BALANCE; CAR MILEAGE—BALANCE; HIRE OF EQUIPMENT. The first of these is a charge paid by a road to other corporations for switching done for the road. The locating engineer is not concerned with this item.

CAR MILEAGE. This is a charge paid by a road for the use of the cars (chiefly freight cars) of another road. To save the rehandling of freight at junctions the policy of running freight cars on to foreign roads is very extensively adopted. Since the foreign road receives (ultimately) its mileage proportion of the freight charge, it justly pays the home road a rate which is supposed to represent the value of the use of a freight car for so many miles. The foreign road then loads up the freight car with freight consigned to some point on the home road and sends it back, again paying mileage for the distance traveled on the foreign road, a proportional freight charge having been received for that service. By a clearing-house arrangement the various roads settle their debit and credit accounts with each other by the payment of a balance. Such is the simple theory. In practice the cars are not sent back to the home road at once, but wander off according to the local demand. As long as strict account is kept of the movements of every car and the home road is paid a charge which really covers the value of such service, no harm is done the home road except that sometimes, when business has suddenly increased, the home road cannot get enough cars to handle its business. The value of a car is then abnormally above its ordinary value and the home road suffers for lack of the rolling stock which belongs to it. The charge being paid according to mileage, any variations of distance have a direct bearing on this item.

HIRE OF EQUIPMENT. This may refer to locomotives or cars which are hired for a special service, or, on very poor roads, it may refer to equipment, which is hired rather than purchased. The locating engineer has no concern with this item.

401. Items 35, 36, and 37. LOSS AND DAMAGE; INJURIES TO PERSONS; CLEARING WRECKS. These expenses are fortuitous and bear no absolute relation to road-mileage or train-mileage. While they depend largely on the standards of discipline on the road, even the best of roads have to pay some small proportion of their earnings to these items. The possible relation between curvature and accidents is discussed in Chapter XXII, but otherwise the locating engineer has no concern with these items.

402. Items 38 to 53. All of the remaining items (for a list of which see § 382) are of no concern to the locating engineer. They are either general expenses (such as taxes) or are special items (such as the operation of marine equipment) which will not be changed by variations in distance, curvature, or grades which a locating engineer may make. They will not therefore be further discussed.

CHAPTER XXI.

DISTANCE.

403. Relation of distance to rates and expenses. Rates are usually based on distance traveled, on the apparent hypotheses that each additional mile of distance adds its proportional amount not only to the service rendered but also to the expense of rendering it. Neither hypothesis is true. The value of the service of transporting a passenger or a ton of freight from *A* to *B* is a more or less uncertain gross amount depending on the necessities of the case and independent of the exact distance. Except for that very small part of passenger traffic which is undertaken for the mere pleasure of traveling, the general object to be attained in either passenger or freight traffic is the transportation from *A* to *B*, however it is attained. A mile greater distance does not improve the service rendered; in fact, it consumes valuable time of the passengers and perhaps deteriorates the freight. From the standpoint of service rendered, the railroad which adopts a more costly construction and thereby saves a mile or more in the route between two places is thereby fairly entitled to additional compensation rather than have it cut down as it would be by a strict mileage rate. The actual value of the service rendered may therefore vary from an insignificant amount which is less than any reasonable charge (which therefore discourages such traffic) and its value in cases of necessity—a value which can hardly be measured in money. If the passenger charge between New York and Philadelphia were raised to $5, $10, or even $20, there would still be some passengers who would pay it and go, because *to them* it would be worth $5, $10, or $20, or even more. Therefore, when they pay $2.50 they are not paying what the service is worth to them. The service rendered cannot therefore be made a measure of the charge, nor is the service rendered proportional to the miles of distance.

The idea that the cost of transportation is proportional to the distance is much more prevalent and is in some respects more justifiable, but it is still far from true. This is especially true of passenger service. The cost of transporting a single passenger is but little more than the cost of printing his ticket. Once aboard the train, it makes but little difference to the railroad whether he travels one mile or a hundred. Of course there are certain very large expenses due to the passenger traffic which must be paid for by a tariff which is rightfully demanded, but such expenses have no relation to the cost of an additional mile or so of distance inserted between stations. The same is true to a slightly less degree of the freight traffic. As shown later, the items of expense in the total cost of a train-mile, which are directly affected by a small increase in distance, are but a small proportion of the total cost.

404. The conditions other than distance that affect the cost; reasons why rates are usually based on distance. Curvature and minor grades have a considerable influence on the cost of transportation, as will be shown in detail in the next two chapters, but they are never considered in making rates. Ruling grades have a very large influence on the cost, but they are likewise disregarded in making rates. An accurate measure of the effect of these elements is difficult and complicated and would not be appreciated by the general public. Mere distance is easily calculated; the public is satisfied with such a method of calculation; and the railroads therefore adopt a tariff which pays expenses and profits even though the charges are not in accordance with the expenses or the service rendered.

An addition to the length of the line may (and generally does) involve curvature and grade as well as added distance. In this chapter is considered merely the effect of the added distance. The effect of grade and curvature must be considered separately, according to the methods outlined in succeeding chapters. The additional length considered is likewise assumed *not* to affect the business done nor the number of stations, but that it is a mere addition to length of track.

EFFECT OF DISTANCE ON OPERATING EXPENSES.

405. Effect of slight changes in distance on maintenance of way. With a few unimportant exceptions all the items of expense under maintenance of way and structures (see § 407) will be increased directly as any increase in distance. This must certainly be true for items 1, 2, 3, and 5, which alone comprise about three fourths of the total expense for maintenance of way. If we assume that the proposed change of length involves no difference in the number of bridges, culverts, buildings, and fixtures, docks and wharves, we may consider items 4, 6, and 7 to be unaffected. This will generally be true for small changes in length, measured in feet. For larger differences, measured in miles, items 4 and 6 will vary nearly as the distance. The same may be said of items 9 and 10. The cost of maintaining the telegraph line will probably be increased about 60% of the unit cost. The effect of changes in distance on these various items of maintenance of way (as well as the other items of expense of a train-mile) will be tabulated in § 408.

406. Effect on maintenance of equipment. The relation between an increase in length of line and the expenses of items 11, 15, 17, 18, and 19 are quite indefinite. In some respects they would be unaffected by slight changes of distance. From other points of view there is no reason why the expenses should not be considered proportionate to the distance. For example, the added track will probably require as much work from the construction train as any other part of the road and is therefore responsible for as much of the "repairs and renewals of work-cars"—item 15. Fortunately all of these items are so small, even in the aggregate, that little error will be involved by either decision. It will therefore be assumed that these items are affected 100% for large additions in distance and but 50% for small additions.

Item 16 is evidently unaffected.

Item 12. Locomotives deteriorate (1) with age; (2) by expansion and contraction, especially of the fire-boxes, when fires are drawn and relighted; (3) on account of the strains due to stopping and starting; (4) the strains and wear of wheels due to curved track; (5) the additional stresses due to grade and change of grade; and (6) on account of the work of pulling on a straight level track. Observe that the first five causes have no direct relation to an addition of mere distance (the possible curvature or grade incident to the additional distance being a separate matter). How much of the total deterioration is due to the last cause? Wellington attacks this problem as follows: the records of engine-repair shops readily furnish the proportionate cost of the repairs of boiler, running-gear, etc. An estimate is then made of the effect of each cause on each item. For example, the boiler is responsible for 20% of the repairs and renewals. Of this 7% (say one third) is assigned to "terminal service, getting up steam, making up trains," 4% to curvature and grades, 2% to "stopping and starting at way stations," and the other 7% to "distance on tangent between stations." The other items are treated similarly. Wellington says, "As this [subdivision of expenses] has been done with great care to get the best attainable authority for each (which it would occupy too much space to give in detail), the margin for possible error is not great enough to be of moment, although no absolute exactness can be claimed for it." His final estimate is that distance is responsible for 42% of the total cost of repairs and renewals. This value will therefore be used for all additional distances, great or small.

Items 13 and 14. The causes of deterioration of both passenger and freight cars may be classified exactly as above—omitting merely cause 2—the expansion and contraction due to firing. Considering that a large part of the repairs of freight cars is due to the draft-gear and brakes, which are affected chiefly by the heavy strains due to stopping and starting and to grades, while the repairs of wheels are largely due to the wear of wheels on curves, it is not surprising that he allows only 36% of the cost of repairs and renewals of freight cars to be due to straight distance. He made no direct estimate for passenger-cars, but points out the fact that the maintenance of the seats, furniture, and ornamentation make up much more than half the cost of passenger-car repairs. A large part of such deterioration is due to age and the weather, although that of the seats is largely a function of passenger wear and therefore of distance traveled. Although the items of deterioration in passenger cars is very different from those of freight cars, yet if a similar calculation is made for passenger cars it will be found that the final figure is substantially the same as for freight cars and will here be so regarded.

407. Effect on conducting transportation. Item 20. This is evidently unaffected by small or even considerable additions to distance.

Item 21. Theoretically, train wages should vary as mileage. On the larger roads, where, especially in the freight service, there is little or no distinction of day or night, week-day or Sunday, it is practically impossible to hire the trainmen to work between certain definite hours of the day and pay them accordingly, as is done with factory employees. As explained in Chapter XX, § 394, the system usually adopted of paying trainmen is such that small changes of distance (measured in feet) would not affect train wages. The wages of round-house men would not be affected under any conditions, and those of the enginemen and of the trainmen (item 26) would not generally be affected unless the change of distance is very great—perhaps ten miles. Since items 21 and 26 are both very large, it will not do to ignore this item or to average it. The pay of round-house men is about 7% of item 21. We may therefore say that if the change in distance is so great that trains wages will be affected, item 21 will be affected 93% and item 26 will be affected 100%. For shorter changes of distance they will be unaffected.

Item 22. A surprisingly large percentage of the fuel consumed is not utilized in drawing a train along the road. Part of this loss is due to firing up, part is wasted when the engine is standing still, which is a large part of the total time. The policy of banking fires instead of drawing them reduces the injury resulting from great fluctuations in temperature, but the total coal consumed is about the same and we may therefore consider that almost a fireboxful of coal is wasted whether the fires are banked or drawn. The amount thus wasted (or at least *not* utilized in direct hauling) has been estimated at 5 to 10% *of the whole consumption.* Experiments* have shown that an engine standing idle in a yard, protected from wind, well jacketed, etc., will require from 25 to 32 lbs. of coal per hour simply to keep up steam. It has been found that the fastest express trains will lose *one fourth* of their total time between termini in stops, and freight trains on a single-track road will generally spend four hours per day on sidings. The waste of coal from this cause is estimated at 3 to 6% of the total consumption. The energy consumed in stopping and starting is very great. A train running 30 miles per hour has enough kinetic energy to move it on a level straight track more than two miles. Every time a train running at 30 miles per hour is stopped, enough energy is consumed by the brakes to run

* Wellington, Economic Theory, p. 207.

it from one to two miles. When starting, it will require an equal amount of work to restore that velocity, *in addition* to the ordinary resistances. It has been shown that on the Manhattan Elevated Railroad, where stops will average every three eighths of a mile, this cause alone will account for the consumption of nearly *three fourths* of the fuel. Of course on ordinary railroads the proportion is not nearly so great, but it is probably as much as 10 to 20% as an average figure. For a through express train making but few stops the figure would be small, except for the effect of "slow-downs." For suburban trains the proportion would be abnormally high. The fuel required to overcome the added resistances due to curvature and grade are of course exceedingly variable, depending on the particular alignment of the road considered. An approach to the truth may be made by considering the average curvature per mile for the roads of the United States and the average grades, and computing, by the methods given in subsequent chapters, the extra fuel consumed on account of such average conditions, and these items will apparently be responsible for 3% due to curvature and about 15% due to grades. Summarizing the above we have:

Firing	5 to	10%	
Wasted while still	3 "	6%	
Stopping and starting	10 "	20%	
Average curvature	3 "	3%	
Average grade	15 "	15%	
	36	54	
Direct hauling	64 "	46	Average, 55%
	100	100	

This shows that the addition of mere straight level distance would not increase the consumption of fuel more than 55% of the average consumption per mile.

Items 23, 24, and 25. If water is paid for by meter, the cost is strictly according to consumption, which would vary almost according to the number of engine-miles. When supplied from the company's own plant, as is usually the case, a slight increase will not appreciably affect the cost. Nothing is wasted during firing or while the engine is still. The use is therefore more nearly as the mileage, and the cost for an additional mile may be considered as 50% of its average cost per train-mile. Items 24 and 25 will be considered similarly. Fortunately these items, whose variation with additional distance is somewhat obscure and variable, only aggregate a little over 1% of the cost of a train-mile and therefore a considerable percentage of error is of little or no importance.

Item 26. (See comments on item 21.)

Item 27. This item, as well as many other small items that follow, will be irregularly affected by a small increase in distance. It would appear equally wrong to say that they would be unaffected or to say that they will vary directly as the mileage. 50% will be allowed.

Item 28. The necessity for flagmen and watchmen varies in general as the mileage. An addition in distance is less apt to increase the number of switchmen. 50% of this item will be added.

Item 29. Telegraph expenses include the wages of operators (unaffected), and the special expenses due to offices and telegraph stations and to operating the line—the maintenance of the line being charged to item 8. This item will be but little affected, if at all, by additional distance, but 20% will be allowed. Items 30, 31, 32, and 34 are unaffected. Items 33, 35, 36, and 37 are affected 100%. Items 38 to 46 are unaffected.

The "general expenses" (items 46 to 53) will be unaffected.

408. Estimate of total effect on expenses of small changes in distance (measured in feet); estimate for distances measured in miles. According to the accompanying compilation the cost of operating additional distance will be about 35% of the average cost per train-mile when the additional distance is small, but will be about 56% if the additional distance is several miles. The figures may also be considered as the saving in the operating expenses resulting from a shortening of the line.

The average cost of a train-mile during the years from 1890 to 1899 varied from 91.8c. to 98.4c., with an average value of 95.2c. On this basis the above figures become 33.2 and 53.3 cents per train-mile respectively. Some trains run 365 days per year, others but 313. The tendency is toward the larger figure and it will therefore be used in these calculations. The added cost per daily train per year for *each foot* of distance is

$$\frac{33.2 \times 365 \times 2}{5280} = 4.59\text{c.}$$

When the distance is measured by miles the added cost per daily train per year for *each mile* of distance is:

$$53.3 \times 365 \times 2 = \$389.$$

TABLE XXI.—EFFECT ON OPERATING EXPENSES OF GREAT (AND SMALL) CHANGES IN DISTANCE.

Item No.	Normal average.	Per cent affected.		Cost per mile.	
		Great.	Small.	Great.	Small.
1	10.596	100	100	10.60	10.60
2	1.440	100	100	1.32	1.32
3	3.093	100	100	3.09	3.09
4	2.378	100	0	2.38	0
5	.523	100	100	.52	.52
6	1.865	30	0	.56	0
7	.247	0	0	0	0
8	.135	60	60	.08	.08
9	.027	100	0	.03	0
10	.358	100	0	.36	0
	20.662	....	....	18.94	15.61
11	.650	50	0	.32	0
12	5.879	42	42	2.47	2.47
13	2.209	36	36	.80	.80
14	6.765	36	36	2.44	2.44
15	.155	100	50	.15	.08
16	.209	0	0	0	0
17	.490	100	50	.49	.25
18	.040	100	50	.04	.02
19	.495	100	50	.50	.25
	16.892	....	....	7.21	6.31
20	1.761	0	0	0	0
21	9.781	93	0	9.10	0
22	9.681	55	55	5.32	5.32
23	.671	50	50	.34	.34
24	.276	50	50	.19	.19
25	.184	50	50	.09	.09
	22.454	....	....	15.04	5.94

Item No.	Normal average.	Per cent affected.		Cost per mile.	
		Great.	Small.	Great.	Small.
	22.454			15.04	5.94
26	7.722	100	0	7.72	0
27	1.528	50	50	.76	.76
28	4.136	50	50	2.07	2.07
29	1.974	20	20	.39	.39
30	7.818	0	0	0	0
31	.762	0	0	0	0
32	.345	0	0	0	0
33	2.094	100	100	2.09	2.09
34	.333	0	0	0	0
35	.738	100	100	.74	.74
36	.883	100	100	.88	.88
37	.131	100	100	.13	.13
38	.887				
39	.426				
40	1.692				
41	.171				
42	.130	0	0	0	0
43	1.848				
44	.492				
45	.610				
46	.619				
	57.793	....	....	29.82	13.00
47					
48					
49					
50	4.653	0	0	0	0
51					
52					
53					
	100.000	....	....	55.97	34.92

Light-traffic roads are more apt to run their trains on week days only, and a corresponding reduction should be made in these cases.

Regarding the accuracy of the above computations, it should be noted that the most uncertain items are generally the smallest, and that even the largest variations that can reasonably be made of the above figures will not very greatly alter the final result. A numerical illustration of the value of saving distance will be given later.

EFFECT OF DISTANCE ON RECEIPTS.

409. Classification of traffic. There are various methods of classifying traffic, according to the use it is intended to make of the classification. The method here adopted will have reference to its competitive or non-competitive character and also to the method of division of the receipts on through traffic. Traffic may be classified first as "through" and "local"—through traffic being that traveling over two (or more) lines, no matter how short or non-competitive it may be; "local" traffic is that confined entirely to one road. A fivefold classification is however necessary—which is:

A. Non-competitive local—on one road with no choice of routes.

B. Non-competitive through—on two (or more) roads, but with no choice.

C. Competitive local—a choice of two (or more) routes, but the entire haul may be made on the home road.

D. Competitive through—direct competition between two or more routes each passing over two or more lines.

E. Semi-competitive through—a non-competitive haul on the home road and a competitive haul on foreign roads.

There are other possible combinations, but they all reduce to one of the above forms so far as their essential effect is concerned.

410. Method of division of through rates between the roads run over. Through rates are divided between the roads run over in proportion to the mileage. There may be terminal charges and possibly other more or less arbitrary deductions to be taken from the total amount received, but when the final division is made the remainder is divided according to the mileage. On account of this method of division and also because non-competitive rates are always fixed according to the distance, there results the unusual feature that, unlike curvature and grade, there is a compensating advantage in increased distance, which applies to all the above kinds of traffic except one (competitive local), and that the compensation is sometimes sufficient to make the added distance an actual source of profit. It has just been proved that the cost of hauling a train an additional mile is only 35 to 56% of the average cost. Therefore in all non-competitive business (local or through) where the rate is according to the distance, there is an actual profit in all such added distance. In competitive local business, in which the rate is fixed by competition and has practically no relation to distance, any additional distance is dead loss. In competitive through business the profit or loss depends on the distances involved. This may best be demonstrated by examples.

411. Effect of a change in the length of the home road on its receipts from through competitive traffic. Suppose the home road is 100 miles long and the foreign road is 150 miles long. Then the home road will receive $\frac{100}{100+150}=40\%$ of the through rate.

Suppose the home road is lengthened 5 miles; then it will receive $\frac{105}{105+150}=41.176\%$ of the through rate. The traffic being competitive, the rate will be a fixed quantity regardless of this change of distance. By the first plan the rate received is 0.4% per mile; adding 5 miles, the rate for the original 100 miles may be considered *the same* as before; and that the additional 5 miles receive 1.176%, or 0.235% per mile. This is 59% of the original rate per mile, and since this is more than the cost per mile for the additional distance (see § 408), the added distance is evidently *in this case* a source of distinct profit. On the other hand, if the line is shortened 5 miles, it may be similarly shown that not only are the receipts lessened, but that the saving in operating expenses by the shorter distance is less than the reduction in receipts.

A second example will be considered to illustrate another phase. Suppose the home road is 200 miles long and the foreign road is 50 miles long. In this case the home road will receive $\frac{200}{200+50}=80\%$ of the through rate. Suppose the home road is lengthened 5 miles; then it will receive $\frac{205}{205+50}=80.392\%$ of the through rate. By the first plan the rate received is 0.400% per mile; adding 5 miles, there is a surplus of 0.392, or 0.0784 per mile, which is but 19.6% of the original rate.

At this rate the extra distance evidently is not profitable, although it is not a dead loss—there is some compensation.

412. The most advantageous conditions for roads forming part of a through competitive route. From the above it may be seen that when a road is but a short link in a long competitive through route, an addition to its length will increase its receipts and increase them more than the addition to the operating expenses.

As the proportionate length of the home road increases the less will this advantage become, until at some proportion an increase in distance will just pay for itself. As the proportionate length grows greater the advantage becomes a disadvantage until, when the competitive haul is entirely on the home road, any increase in distance becomes a net loss without any compensation. It is therefore advantageous for a road to be a short link in a long competitive route; an increase in that link will be financially advantageous; if the total length is less than that of the competing line, the advantage is still greater, for then the rate received per mile will be greater.

413. Effect of the variations in the length of haul and the classes of the business actually done. The above distances refer to particular lengths of haul and are not necessarily the total lengths of the road. Each station on the road has traffic relations with an indefinite number of traffic points all over the country. The traffic between each station on the road and any other station in the country between which traffic may pass therefore furnishes a new combination, the effect of which will be an element in the total effect of a change of distance. In consequence of this, any *exact* solution of such a problem becomes impracticable, but a sufficiently accurate solution for all practical purposes is frequently obtainable. For it frequently happens that the great bulk of a road's business is non-competitive, or, on the other hand, it may be competitive-through, and that the proportion of one or more definite kinds of traffic is so large as to overshadow the other miscellaneous traffic. In such cases an approximate but sufficiently accurate solution is possible.

414. General conclusions regarding a change in distance. (a) In *all* non-competitive business (local and through) the added distance is actually profitable. Sometimes practically all of the business of the road is non-competitive; a considerable proportion of it is always non-competitive.

(b) When the competitive local business is very large and the competitive through business has a very large average home haul compared with the foreign haul, the added distance is a source of loss. Such situations are unusual and are generally confined to trunk lines.

(c) The above may be still further condensed to the general conclusion that there is always *some* compensation for the added cost of operating an added length of line and that it frequently is a source of actual profit.

(d) There is, however, a limitation which should not be lost sight of. The above argument may be carried to the logical conclusion that, if added distance is profitable, the engineer should purposely lengthen the line. But added distance means added operating expenses. A sufficient tariff to meet these is a tax on the community—a tax which more or less discourages traffic. It is contrary to public policy to burden a community with an avoidable expense. But, on the other hand, a railroad is not a charitable organization, but a money-making enterprise, and cannot be expected to unduly load up its first cost in order that subsequent operating expenses may be unduly cheapened and the tariff unduly lowered. A common reason for increased distance is the saving of the first cost of a very expensive although shorter line.

(e) Finally, although there is a considerable and uncompensated loss resulting from curvature and grade which will justify a considerable expenditure to avoid them, there is by no means as much justification to incur additional expenditure to avoid distance. Of course needless lengthening should be avoided. A moderate expenditure to shorten the line may be justifiable, but large expenditures to decrease distance are never justifiable except when the great bulk of the traffic is exceedingly heavy and is competitive.

415. Justification of decreasing distance to save time. It should be recalled that the changes which an engineer may make which are physically or financially possible will ordinarily have but little effect on the time required for a trip. The time which can thus be saved will have practically no value for the freight business—at least any value which would justify changing the route. When there is a large directly competitive passenger traffic between two cities (*e.g.* New York to Philadelphia) a difference of even 10 minutes in the time required for a run might have considerable financial importance, but such cases are comparatively rare. It may therefore be concluded that the value of the time saved by shortening distance will not ordinarily be a justification for increased expense to accomplish it.

416. Effect of change of distance on the business done. The above discussion is based on the assumption that the business done is unaffected by any proposed change in distance. If a proposed reduction in distance involves a loss of business obtained, it is almost certainly unwise. But if by increasing the distance the original cost of the road is decreased (because the construction is of less expensive character), if the receipts are greater, and are increased still more by an increase in business done, then the change is probably wise. While it is almost impossible in a subject of such complexity to give a general rule, the following is generally safe: Adopt a route of such length that the annual traffic per mile of line is a maximum. This statement may be improved by allowing the element of original cost to enter and say, adopt a route of such length that the annual traffic per mile of line divided by the average cost per mile is a maximum. Even in the above the operating cost per mile, as affected by the curvature and grades on the various routes, does not enter, but any attempt to formulate a general rule which would allow for variable operating expenses would evidently be too complicated for practical application.

CHAPTER XXII.

CURVATURE.

417. General objections to curvature. In the popular mind curvature is one of the most objectionable features of railroad alignment. The cause of this is plain. The objectionable qualities are on the surface, and are apparent to the non-technical mind. They may be itemized as follows:

1. Curvature increases **operating expenses** by increasing (**a**) the required tractive force, (**b**) the wear and tear of roadbed and track, (**c**) the wear and tear of equipment, and (**d**) the required number of track-walkers and watchmen.

2. It may affect the **operation of trains** (**a**) by limiting the length of trains, and (**b**) by preventing the use of the heaviest types of engines.

3. It may affect **travel** (**a**) by the difficulty of making time, (**b**) on account of rough riding, and (**c**) on account of the apprehension of danger.

4. There is actually an increased **danger** of collision, derailment, or other form of accident.

Some of these objections are quite definite and their true value may be computed. Others are more general and vague and are usually exaggerated. These objections will be discussed in inverse order.

418. Financial value of the danger of accident due to curvature. At the outset it should be realized that in general the problem is *not* one of curvature *vs.* no curvature, but simply sharp curvature *vs.* easier curvature (the central angle remaining the same), or a greater or less percentage of elimination of the degrees of central angle. A straight road between termini is in general a financial (if not a physical) impossibility. The practical question is then, how much is the financial value of such diminution of danger that may result from such eliminations of curvature as an engineer is able to make?

In the year 1898 there were 2228 railroad accidents reported by the *Railroad Gazette*, whose lists of all accidents worth reporting are very complete. Of these a very large proportion clearly had no relation whatever to curvature. But suppose we assume that 50% (or 1114 accidents) were directly caused by curvature. Since there are approximately 200,000 curves on the railroads of the country, there was on the average an accident for every 179 curves during the year. Therefore we may say, according to the theory of probabilities, that the chances are even that an accident may happen on any particular curve in 179 years. This assumes all curves to be equally dangerous, which is not true, but we may temporarily consider it to be true. If, at the time of the construction of the road, $1.00 were placed at compound interest at 5% for 179 years, it would produce in that time $620.89 for each dollar saved, wherewith to pay all damages, while the amount necessary to eliminate that curvature, even if it were possible, would probably be several thousand dollars. The number of passengers carried one mile for one killed in 1898–99 was 61,051,580. If a passenger were to ride continuously at the rate of sixty miles per hour, day and night, year after year, he would need to ride for more than 116 years before he had covered such a mileage, and even then the probabilities of his death being due to curvature or to such a reduction of curvature as an engineer might accomplish are very small. Of course particular curves are often, for special reasons, a source of danger and justify the employment of special watchmen. They would also justify very large expenditures for thier elimination if possible. But as a general proposition it is evidently impossible to assign a definite money value to the danger of a serious accident happening on a particular curve which has no special elements of danger.

Another element of safety on curved track is that trait of human nature to exercise greater care where the danger is more apparent. Many accidents are on record which have been caused by a carelessness of locomotive engineers on a *straight* track when the extra watchfulness usually observed on a curved track would have avoided them.

419. Effect of curvature on travel. (a) Difficulty in making time. The growing use of transition curves has largely eliminated the necessity for reducing speed on curves, and even when the speed is reduced it is done so easily and quickly by means of air-brakes that but little time is lost. If two parallel lines were competing sharply for passenger traffic, the handicap of sharp curvature on one road and easy curvature on the other might have a considerable financial value, but ordinarily the *mere reduction* of time due to sharp curvature will not have any computable financial value.

(b) **On account of rough riding.** Again, this is much reduced by the use of transition curves. Some roads suffer from a general reputation for crookedness, but in such cases the excessive curvature is practically unavoidable. This cause probably does have some effect in influencing competitive passenger traffic.

(c) **On account of the apprehension of danger.** This doubtless has its influence in deterring travel. The amount of its influence is hardly computable. When the track is in good condition and transition curves are used so that the riding is smooth, even the apprehension of danger will largely disappear.

Travel is doubtless more or less affected by curvature, but it is impossible to say how much. Nevertheless the engineer should not ordinarily give this item any financial weight whatsoever. Freight traffic (two thirds of the total) is unaffected by it. It chiefly affects that limited class of sharply competitive passenger traffic—a traffic of which most roads have not a trace.

420. Effect on operation of trains. (a) Limiting the length of trains. When curvature actually limits the length of trains, as is sometimes true, the objection is valid and serious. But this can generally be avoided. If a curve occurs on a ruling grade without a reduction of the grade sufficient to compensate for the curvature, then the resistance on that curve will be a maximum and that curve will limit the trains to even a less weight than that which may be hauled on the ruling grade. In such cases the unquestionably correct policy is to "compensate for curvature," as explained later (see §§ 427, 428), and not allow such an objection to exist. It is *possible* for curvature to limit the length of trains even without the effect of grade. On the Hudson River R. R. the total net fall from Albany to New York is so small that it has practically no influence in determining grade. On the other hand, a considerable portion of the route follows a steep rocky river bank which is so crooked that much curvature is unavoidable and very sharp curvature can only be avoided by very large expenditure. As a consequence sharp curvature has been used and the resistance on the curves is far greater than that of any fluctuations of grade which it was necessary to use. Or, at least, a comparatively small expenditure would suffice to cut down any grade so that its resistance would be less than that of some curve which could not be avoided except at an enormous cost. And as a result, since the length of trains is really limited by curvature, minor grades of 0.3 to 0.5% have been freely introduced which might be removed at comparatively small expense The above case is very unusual. Low grades are usually associated with generally level country where curvature is easily avoided—as in the Camden and Atlantic R. R. Even in the extreme case of the Hudson River road the maximum curvature is only equivalent to a comparatively low ruling grade.

(b) **Preventing the use of the heaviest types of engines.** The validity of this objection depends somewhat on the degree of curvature and the detailed construction of the engine. While some types of engines might have difficulty on curves of extremely short radius, yet the objection is ordinarily invalid. This will best be appreciated when it is recalled that the "Consolidation" type was originally designed for use on the sharp curvature of the mountain divisions of the Lehigh Valley R. R., and that the type has been found so satisfactory that it has been extensively employed elsewhere. It should also be remembered that during the Civil War an immense traffic daily passed over a hastily constructed trestle near Petersburg, Va., the track having a radius of 50 feet. As a result of a test made at Renovo on the Philadelphia and Erie R. R. by Mr. Isaac Dripps, Gen. Mast. Mech., in 1875,* it was claimed that a Consolidation engine encountered less resistance per ton than one of the "American" type. Whether the test was strictly reliable or not, it certainly demonstrated that there was no trouble in using these heavy engines on very sharp curvature, and we may therefore consider that, except in the most extreme cases, this objection has no force whatsoever.

---

* Seventh An. Rep. Am. Mast. Mech. Assn.

EFFECT OF CURVATURE ON OPERATING EXPENSES.

421. Relation of radius of curvature and of degrees of central angle to operating expenses. The smallest consideration will show that the sharper the curvature the greater will be the tractive force required, also the greater per unit of track length will be the rail wear and the general wear and tear on roadbed and rolling stock. But it would be inconvenient to use a relation between operating expenses and radius of curvature, because even when such a relation was found there would be two elements to consider in each problem—the radius and the length of the curve. The method which will be here developed cannot claim to be strictly accurate or even strictly logical, but, as will be shown later, the most uncertain elements of the computation have but a small influence on the final result, and the method is in general the only possible method of solution. The outline of the method is as follows:

(1) For reasons given in detail later, it is found that the expenses, wear, etc., on the track from A to B will be substantially the same whether by the route M or N. The wear, etc.,

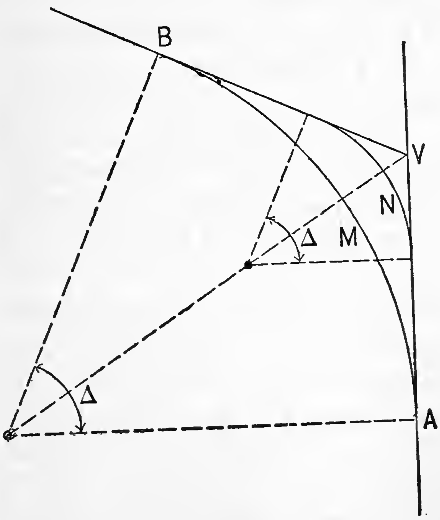


FIG. 208.

per foot at N is of course greater, but the length of curve is less. Therefore the effect of the curvature depends on the degrees of central angle Δ and is independent of the radius.

(2) At what degree of curvature is the total train resistance double its value on a tangent? Probably no one figure would be exact for all conditions. Train resistance varies with the velocity and with the various conditions of train loading even on a tangent, and it is by no means certain (or even probable) that the ratio would be exactly the same for all conditions. As an average figure we may say that a train running at average velocity on a 10° curve will encounter a resistance due to curvature of about 10 lbs. per ton, which is the average resistance found on a level tangent. On a 10° curve therefore the resistance is doubled.

(3) A train-mile costs about so much—approximately \$1.00. Doubling the tractive resistance will increase certain items of expenditure about so much. Their combined value is so much per cent of the cost of a train-mile. A mile of continuous 10° curve contains 528° of central angle. A mile of such track would add so much per cent to the average train-mile expenses, and each degree of central angle is responsible for $\frac{1}{528}$ of this increase. Since the increase is irrespective of radius and depends only on the degrees of central angle, we therefore say that each degree of central angle of a curve will add so much to the average operating expenses of a train-mile.

The "cost per train-mile" considered above should be considered as the cost of a mile of level tangent. If we for a moment consider that all the railroads of the country were made absolutely straight and level, it is apparent that the average cost per train-mile instead of being about 95c. would be somewhat less. The percentage should therefore be applied to this reduced value, but the net effect of this change would evidently be small.

422. Effect of curvature on maintenance of way. A very large proportion of the items of expense in a train-mile are absolutely unaffected by curvature. It will therefore simplify matters somewhat if we at once throw out all the unaffected items. Of the items of maintenance of way and structure all but the first three will be thrown out. Item 4 will be somewhat affected when bridges or trestles occur on a curve. But when it is considered what a very small percentage of this small item (2.378%) could be ascribed to curvature, since the very large majority of bridges and trestles are purposely made straight, and since culverts, etc., are not affected, we may evidently ignore any variation in the item.

Item 1. Repairs of Roadway. A very large proportion of the sub-items are absolutely unaffected. The care of embankments and slopes, ditching, weeding, etc., are evidently unaffected. The track labor on rails and ties and the work of surfacing will evidently be somewhat increased and yet it is very seldom that the length of a track section would be decreased simply on account of excessive curvature. But 528° per mile is an excessive amount of curvature. The average for the whole country is about 30° per mile, and there are very few instances of that amount of curvature (528°) in the length of a single mile. As before intimated, it is reasonable to assume that the *extra* work *per foot* on a 20° curve would be 10 times the extra work on a 2° curve, which verifies the general statement that the extra cost varies as the degrees of central angle. Considering how much of this item is independent of curvature and how little even the track labor is affected, it is possibly overstating the case to allow 25% increase for 528° of curvature in one mile.

Item 2. Renewals of Rails. Wellington says that some experiments made by himself and others made by Dr. Dudley agree in indicating that the rail wear on tangents may be considered as 1 lb. per yard per 10,000,000 tons duty, while the extra wear on curves would be $\frac{1}{2}$ lb. per degree of curve per 10,000,000 tons duty. Therefore on a 10° curve the *extra* rail wear would be five times as great as on a tangent and the *increase* would therefore be 500%. On iron rails and on inferior steel rails the wear on tangents would be larger proportionally, and this is probably the reason for Wellington's adopting an average increase of but 300%, and this same figure will be adopted.

Item 3. Renewals of Ties. Curvature affects ties by increasing the "rail cutting" and on account of the more frequent respiking, which "spike-kills" the ties even before they have decayed. Wellington estimates that a tie which will last nine years on a tangent will last but six years on a 10° curve. He adds 50% for tie renewals. He considers the decrease in tie life to be proportional to the degree of curve and therefore again verifies the general statement made above regarding the expense of curvature.

423. Effect of curvature on maintenance of equipment. Items 11, 16, 18, and 19 will be considered as unaffected.

Item 12. Repairs and Renewals of Locomotives. Curvature affects locomotive repairs by increasing very largely the wear on tires and wheels, and also the wear and strain due to the additional power required. Wellington neglected the last cause since the resistance due to curvature is so small compared to that due to even a moderate grade. He further considered that only 30% of the items of engine repairs are affected at all by curvature, and that the effect of curvature and grades on these is only $\frac{1}{3}$ or 10%, and that curvature is responsible for 60% of that, or, finally, that only 6% of engine repairs are caused by curvature as it exists. He then computed that the actual average curvature of railroads (about 30° per mile) is but $\frac{1}{20}$ of the 600° (instead of 528°) in his standard mile. Therefore he said that 600° of curvature would increase engine repairs by $20 \times 6\%$, or 120%. He acknowledges that the reasoning is not conclusive. It apparently is weak in this respect: the resistance, and also the wear, is less per degree of curve on the sharper curves than on the easier. On this account, and also because 528° of curvature is considered the standard, rather than 600°, the estimate will be cut down to 100%. (Another method of computation will be substituted for this as soon as possible.)

Items 13, 14, and 15. For similar reasons the estimates for these items will be made 100%. The effect of curvature will apply to all cars about equally.

Item 17. The repairs and renewals of shop machinery and tools will not be increased more than 50% per mile for the additional repairs required of the above equipment.

424. Effect of curvature on conducting transportation. We may at once throw out all items except 22, 23, 24, and 25, a small part of 28, and possibly 35, 36, and 37. This last group has already been discussed in § 418; the aggregate of the three items is but 1.752%; curvature is responsible for only a small proportion of the item, and the reduction which an engineer is able to effect would be so small that we may neglect it.

Item 28 is somewhat analogous to the above. Curvature does not affect a large part of the item, but an extreme case of curvature will occasionally require an extra watchman. Considering, however, that curvature does *not* in general require watchmen, and that such cases are the unusual cases in mountainous regions where the curvature is unavoidable and not materially reducible, it would evidently be wrong to charge curvature in general with such an item, although there would be justification for it in individual cases. It will therefore be ignored.

Items 22, 23, 24, and 25. In § 407, Chapter XXI, the proportion of fuel assigned to direct hauling on a tangent is computed as amounting to about 55%. Since this direct resistance is assumed to be exactly doubled, we will charge 55% for fuel. There will evidently be no error worth considering in allowing the same proportionate amount as the charge for water, oil, waste, etc.

"General expenses," items 47 to 53, are of course unaffected.

425. Estimate of total effect per degree of central angle. Compiling the above estimates we have the following tabulation:

TABLE XXII.—EFFECT ON OPERATING EXPENSES OF CHANGES IN CURVATURE.

Item No.	Normal average.	Per cent affected.	Cost per mile, per cent.
1	10.596	25	2.649
2	1.440	300	4.320
3	3.093	50	1.546
4 } 10 }	5.533	0	0
	20.662	....	8.515
11	.650	0	0
12	5.879	100	5.879
13	2.209	100	2.209
14	6.765	100	6.765
15	.155	100	.155
16	.209	0	0
17	.490	50	.245
18	.040	0	0
19	.495	0	0
	16.892	....	15.253
20	1.761	0	0
21	9.781	0	0
22	9.681	55	5.335
23	.671	55	.369
24	.376	55	.207
25	.184	55	.101
26 } 46 }	35.340	0	0
	57.794	....	6.012
47 } 53 }	4.653	0	0
	100.000	....	29.780

According to it, 528° of curvature in one mile would increase the expenses of each train passing over it by 29.78% of the average cost of a train-mile, and according to the general principles laid down in § 421, 1° of central angle of any curve, no matter what the radius, will increase the expenses by $\frac{1}{528}$ of 29.78%, or .0564% per degree. Therefore the cost per year per daily train each way is (at 95 c. per train-mile)

$$95 \times .0564\% \times 2 \times 365 = 39.11 \text{ c.}$$

As a simple illustration (a more extended one will be given later), suppose that by using greater freedom with regard to earthwork the crooked line sketched may be reduced to the simple curve shown and a curvature of, say, 110° may be reduced to, say, 60°.

Note that since the extreme tangents are identical, the saving in central angle results from the elimination of the reversed

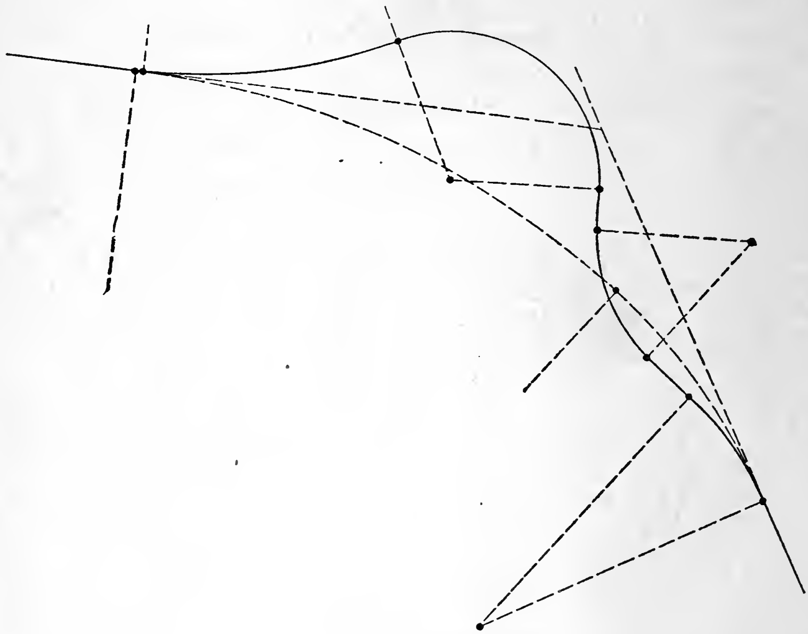


FIG. 209.

curvature and of that part of the direct curvature necessary to balance the reversed curvature. Assume that there are six daily trains each way. Then the annual saving is

$$50° \times .3911 \times 6 = \$117.33,$$

which at 5% would justify an expenditure of $2346.60. If the extra cost of construction does not exceed this, the improvement is justifiable, and is made all the more so if the probabilities are great that the future traffic will largely exceed six trains per day. At the same time the warning regarding "discounting the future" with respect to expected traffic should not be neglected. The possible effect of change of distance has not been referred to in the above problem. In any case it is a distinct problem. According to the above sketch, the difference in distance is probably very slight, and considering the compensating character of extra distance, such small differences may usually be disregarded. The possible effect of change of grade will be discussed in the next chapter. Assuming that there is no difference to be considered on account of either grade or distance, the question hinges on the advisability of spending $2346.60 for the improvement.

426. Reliability and value of the above estimate. It should be realized at the outset that no extreme accuracy is claimed for the above estimate. The effect of curvature is somewhat variable as well as uncertain, but such estimates have this great value. Vary the estimates of individual items as you please (within reason), and the final result is still about the same and may be used to guide the judgment. As an illustration, suppose that the item of renewals of rails is assumed to be affected 400% rather than 300%, the justifiable expenditure to avoid the curvature in the above case may similarly be computed as $2460, an increase of less than 5%. But, after all, the real question is not whether the improvement is worth $2346 or $2460. The extra work involved may perhaps be done for $500 or it may require $10000. The above general method furnishes a criterion which, while not accurate, is so much better than a reliance on vague judgment that it should not be ignored.

COMPENSATION FOR CURVATURE.

427. Reasons for compensation. The effect of curvature on a grade is to increase the resistance by an amount which is equivalent to a material addition to that grade. On minor grades the addition is of little importance, but when the grade is nearly or quite the ruling grade of the road, then the additional resistance induced by a curve will make that curve a place of maximum resistance and the real maximum will be a "virtual grade" somewhat higher than the nominal maximum. If, in Fig. 210,

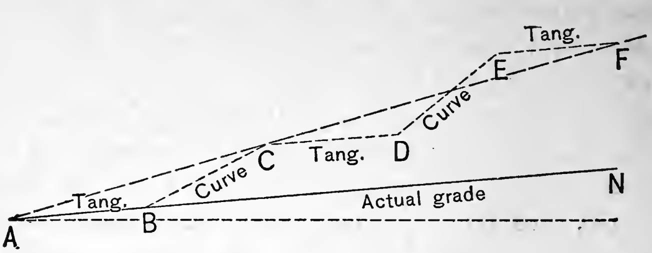


FIG. 210.

AN represents an actual *uniform* grade consisting of tangents and curves, the "virtual grade" on curves at *BC* and *DE* may be represented by *BC* and *DE*. If *BC* and *DE* are very long, or if a stop becomes necessary on the curve, then the full disadvantage of the curve becomes developed. If the whole grade may be operated without stoppage, then, as elaborated further in the next chapter, the whole grade may be operated as if equal to the average grade, *AF*, which is better than *BC*, although much worse than *AN*. The process of "compensation" consists in reducing the grade on every curve by such an amount that the actual resistance on each curve, due to both curvature and grade, shall precisely equal the resistance on the tangent. The practical effect of such reduction is that the "virtual" grade is kept constant, while the nominal grade fluctuates.

One effect of this is that (see Fig. 211) instead of accomplish-

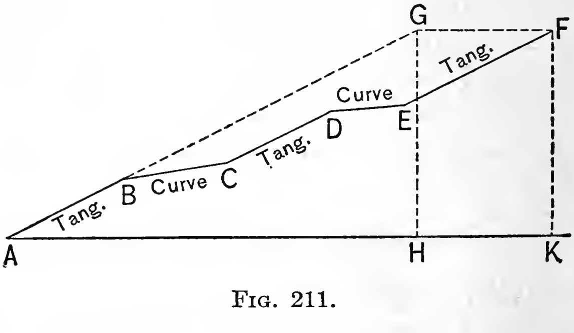


FIG. 211.

ing the vertical rise from *A* to *G* (i.e., *HG*) in the horizontal distance *AH*, it requires the horizontal distance *AK*. Such an addition to the horizontal distance can usually be obtained by proper development, and it should always be done on a ruling grade. Of course it is possible that it will cost more to accomplish this than it is worth, but the engineer should be sure of this before allowing this virtual increase of the grade.

European engineers early realized the significance of unreduced curvature and the folly of laying out a uniform ruling grade regardless of the curvature encountered. Curve compensation is now quite generally allowed for in this country, but thousands of miles have been laid out without any compensation. A very common limitation of curvature and grade has been the alliterative figures 6° curvature and 60 feet per mile of grade, either singly or in combination. Assuming that the resistance on a 6° curve is equivalent to a 0.3% grade (15.84 feet per mile), then a 6° curve occurring on a 60-foot grade would develop more resistance than a 75-foot grade on a tangent. The "mountain cut-off" of the Lehigh Valley Railroad near Wilkesbarre is a fine example of a heavy grade compensated for curvature, and yet so laid out that the virtual grade is uniform from bottom to top, a distance of several miles.

428. The proper rate of compensation. This evidently is the rate of grade of which the resistance just equals the resistance due to the curve. But such resistance is variable. It is greater as the velocity is lower; it is generally about 2 lbs. per ton (equivalent to a 0.1% grade) per degree of curve when starting a train. On this account, the compensation for a curve which occurs at a known stopping-place for the heaviest trains should be 0.1% per degree of curve. The resistance is not even strictly proportional to the degree of curvature, although it is usually considered to be so. In fact most formulæ for curve resistance are based on such a relation. But if the experimentally determined resistances for low curvatures are applied to the excessive curvature of the New York Elevated road, for example, the rules become ridiculous. On this account the compensation *per degree of curve* may be made less on a sharp curve than on an easy curve. The compensation actually required for very fast trains is less than for slow trains, say 0.02 or 0.03% per degree of curve; but since the comparatively slow and heavy freight trains are the trains which are chiefly limited by ruling grade, the compensation must be made with respect to those trains. From 0.04 to 0.05% per degree is the rate of compensation most usually employed for average conditions. Curves which occur *below* a known stopping-place for *all* trains need not be compensated, for the extra resistance of the curve will be simply utilized in place of brakes to stop the train. If a curve occurs just *above* a stopping-place, it is very serious and should be amply compensated. Of course the down-grade traffic need not be considered.

It sometimes happens that the ordinary rate of compensation will consume so much of the vertical height (especially if the curvature is excessive) that a steeper through grade must be adopted than was first computed, and then the trains might stall on the tangents rather than on the curves. In such cases a slight reduction in the rate of compensation might be justifiable. The proper rate of compensation can therefore be estimated from the following rules:

(1) On the upper side of a stopping-place for the heaviest trains compensate 0.10% per degree of curve.

(2) On the lower side of such a stopping-place do not compensate at all.

(3) Ordinarily compensate about 0.05% per degree of curve.

(4) Reduce this rate to 0.04% or even 0.03% per degree of curve if the grade on tangents must be increased to reach the required summit.

(5) Reduce the rate somewhat for curvature above 8° or 10°.

(6) Curves on minor grades need not be compensated.

429. The limitations of maximum curvature. What is the maximum degree of curvature which should be allowed on any road? The true answer is probably that there is no definite limit. It has been shown that sharp curvature does not prevent the use of the heaviest types of engines, and although a sharp curve unquestionably increases operating expenses, the increase is but one of degree with hardly any definite limit. The general character of the country and the gross capital available (or the probable earnings) are generally the true criterions.

A portion of the road from Denver to Leadville, Col., is an example of the necessity of considering sharp curvature. The traffic that might be expected on the line was so meagre and yet the general character of the country was so forbidding that a road built according to the usual standards would have cost very much more than the traffic could possibly pay for. The line as adopted cost about $20,000 per mile, and yet in a stretch of 11.2 miles there are about 127 curves. One is a 25° 20′ curve, twenty-four are 24° curves, twenty-five are 20° curves, and seventy-two are sharper than 10°. If 10° had been made the limit (a rather high limit according to usual ideas), it is probable that the line would have been found impracticable (except with prohibitive grades) unless four or five times as much per mile had been spent on it, and this would have ruined the project financially.

For many years the main-line traffic of the Baltimore and Ohio R. R. has passed over a 300-foot curve (19° 10′) and a 400-foot curve (14° 22′) at Harper's Ferry. A few years ago some reduction was made in this by means of a tunnel, but the fact that such a road thought it wise to construct and operate such curves (and such illustrations on the heaviest-traffic roads are quite common) shows how foolish it is for an engineer to sacrifice money or (which is much more common) sacrifice gradients in order to reduce the *rate* of curvature on a road which at its best is but a second- or third-class road.

Of course such belittling of the effects of curvature may be (and sometimes is) carried to an extreme and cause an engineer to fail to give to curvature its due consideration. Degrees of central angle should always be reduced by all the ingenuity of the engineer, and should only be limited by the general relation between the financial and topographical conditions of the problem. Easy curvature is in general better than sharp curvature and should be adopted when it may be done at a small financial sacrifice, especially since it reduces distance generally and may even cut down the initial cost of that section of the road. But large financial expenditures are rarely, if ever, justifiable where the net result is a mere increase in radius without a reduction in central angle. An analysis of the changes which have been so extensively made during late years on the Penn. R. R. and the N. Y., N. H. & H. R. R. will show invariably a reduction of distance, or of central angle, or both, and perhaps incidentally an increase in radius of curvature. There are but few, if any, cases where the sole object to be attained by the improvement is a mere increase in radius.

CHAPTER XXIII.

GRADE.

430. Two distinct effects of grade. The effects of grade on train expenses are of two distinct kinds; one possible effect is very costly and should be limited even at considerable expenditure; the other is of comparatively little importance, its cost being slight. As long as the length of the train is not limited, the occurrence of a grade on a road simply means that the engine is required to develop so many foot-pounds of work in raising the train so many feet of vertical height. For example, if a freight train weighing 600 tons (1,200,000 lbs.) climbs a hill 50 feet high, the engine performs an *additional* work of creating 60,000,000 foot-pounds of potential energy. If this height is surmounted in 2 miles and in 6 minutes of actual time (20 miles per hour), the *extra* work is 10,000,000 foot-pounds per minute, or about 303 horse-power. But the disadvantages of such a rise are always largely compensated. Except for the fact that one terminus of a road is generally higher than the other, every up grade is followed, more or less directly, by a down grade which is operated partly by the potential energy acquired during the previous climb. But when we consider the trains running in both directions even the difference of elevation of the termini is largely neutralized. If we could eliminate frictional resistances and particularly the use of brakes, the *net* effect of minor grades on the operation of minor grades in both directions would be zero. Whatever was lost on any up grade would be regained on a succeeding down grade, or at any rate on the return trip. On the very lowest grades (the limits of which are defined later) we may consider this to be literally true, viz., that nothing is lost by their presence; whatever is temporarily lost in climbing them is either immediately regained on a subsequent light down grade or is regained on the return trip. If a stop is required at the bottom of a sag, there is a net and uncompensated loss of energy.

On the other hand, if the length of trains is limited by the grade, it will require more trains to handle a given traffic. The receipts from the traffic are a definite sum. The cost of handling it will be nearly in proportion to the number of trains. Anticipating a more complete discussion, it may be said as an example that increasing the ruling grade from 1.20% (63.36 feet per mile) to 1.55% (81.84 feet per mile—an increase of about 18.5 feet per mile) will be sufficient to increase the required number of trains for a given gross traffic about 25%, i.e., five trains will be required to handle the traffic which four trains would have handled before at a cost slightly more than four-fifths as much. The effect of this on dividends may readily be imagined.

431. Application to the movement of trains of the laws of accelerated motion. When a train starts from rest and acquires its normal velocity, it overcomes not only the usual tangent resistances (and perhaps curve and grade resistances), but it also performs work in storing into the train a vast fund of kinetic energy. This work is not lost, for every foot-pound of such energy may later be utilized in overcoming resistances, provided it is not wasted by the action of train-brakes. If for a moment we consider that a train runs without any friction, then, when running at a velocity of v feet per second, it possesses a kinetic energy which would raise it to a height h feet, when $h=\frac{v^2}{2g}$, in which g is the acceleration of gravity $=32.16$. Assuming that the engine is exerting just enough energy to overcome the frictional resistances, the train would climb a grade until the train was raised h feet above the point where its velocity was v. When it had climbed a height h' (less than h) it would have a velocity $v_1=\sqrt{2g(h-h')}$. As a numerical illustration, assume $v=30$ miles per hour $=44$ feet per second. Then $h=\frac{v^2}{2g}=30.1$ feet, and assuming that the engine was exerting just enough force to overcome the rolling resistances on a level, the kinetic energy in the train would carry it for two miles up a grade of 15 feet per mile, or half a mile up a grade of 60 feet per mile. When the train had climbed 20 feet, there would still be 10.1 feet left and its velocity would be $v_1=\sqrt{2g(10.1)}=25.49$ feet per second $=17.4$ miles per hour. These figures, however, must be slightly modified on account of the weight and the revolving action of the wheels, which form a considerable percentage of the total weight of the train. When train velocity is being acquired, part of the work done is spent in imparting the energy of rotation to the driving-wheels and various truck-wheels of the train. Since these wheels run on the rails and must turn as the train moves, their rotative kinetic energy is just as effective—as far as it goes—in becoming transformed back into useful work. The proportion of this energy to the total kinetic energy has already been demonstrated (see Chapter XVI, § 347). The value of this correction is variable, but an average value of 5% has been adopted for use in the accompanying tabular form (Table XXIII), in which is given the corrected "velocity head" corresponding to various velocities in miles per hour. The table is computed from the following formula:

$$\text{Velocity head} = \frac{v^2 \text{ in ft. per sec.}}{64.32} = \frac{1.4667v \text{ in m. per h.}}{64.32} = 0.03344v^2$$

adding 5% for the rotative kinetic energy of the wheels, $0.00167v^2$

The corrected velocity head therefore equals $0.03511v^2$

Part of the figures of Table XXIII were obtained by interpolation and the final *hundredth* may be in error by one unit, but it may readily be shown that the final hundredth is of no practicable importance. It is also true that the chief use made of this table is with velocities much less than 50 miles per hour. Corresponding figures may be obtained for higher velocities, if desired, by multiplying the figure for *half* the velocity by *four*.

432. Construction of a virtual profile. The following simple demonstration will be made on the basis that the ordinary tractive resistances and also the tractive force of the locomotive are independent of velocity. For a considerable range of velocity which includes the most common freight-train velocities this assumption is so nearly correct that the method will give an approximately correct result, but for higher velocities and for more accurate results a more complicated method (given later) must be used. The following demonstration will serve well as a preliminary to the more accurate method. It may best be illustrated by considering a simple numerical example.

Assuming that a train is passing A (see Fig. 212), running at 30 miles per hour. Assume that the throttle is not changed or any brakes applied, but that the engine continues to exert the

TABLE XXIII.—VELOCITY HEAD (REPRESENTING THE KINETIC ENERGY) OF TRAINS MOVING AT VARIOUS VELOCITIES.

Vel. mi. hr.	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
10	3.51	3.58	3.65	3.72	3.79	3.87	3.95	4.02	4.10	4.17
11	4.25	4.33	4.41	4.49	4.57	4.65	4.73	4.81	4.89	4.97
12	5.06	5.15	5.23	5.32	5.41	5.50	5.58	5.67	5.75	5.84
13	5.93	6.02	6.12	6.21	6.31	6.40	6.50	6.59	6.69	6.78
14	6.88	6.98	7.08	7.19	7.29	7.39	7.49	7.60	7.70	7.80
15	7.90	8.00	8.11	8.22	8.33	8.44	8.55	8.66	8.77	8.88
16	8.99	9.10	9.21	9.32	9.43	9.55	9.67	9.79	9.91	10.03
17	10.15	10.27	10.39	10.51	10.63	10.75	10.87	10.99	11.12	11.25
18	11.38	11.50	11.63	11.76	11.89	12.02	12.15	12.28	12.41	12.55
19	12.68	12.81	12.95	13.08	13.22	13.35	13.49	13.63	13.77	13.91
20	14.05	14.19	14.33	14.47	14.61	14.75	14.89	15.04	15.19	15.34
21	15.49	15.64	15.79	15.94	16.09	16.24	16.39	16.54	16.69	16.84
22	17.00	17.15	17.30	17.46	17.62	17.78	17.94	18.10	18.26	18.42
23	18.58	18.74	18.90	19.06	19.22	19.38	19.55	19.72	19.89	20.06
24	20.23	20.40	20.57	20.74	20.91	21.08	21.25	21.42	21.59	21.77
25	21.95	22.12	22.30	22.48	22.66	22.84	23.02	23.20	23.38	23.56
26	23.74	23.92	24.10	24.28	24.46	24.65	24.84	25.03	25 22	25.41
27	25.60	25.79	25.98	26.17	26.36	26.55	26.74	26.93	27.13	27.33
28	27.53	27.73	27.93	28.13	28.33	28.53	28.73	28.93	29.13	29.33
29	29.53	29.73	29.93	30.13	30.34	30.55	30.76	30.97	31.18	31.39
30	31.60	31.81	32.02	32.23	32.44	32.65	32.86	33.08	33.30	33.52
31	33.74	33.96	34.18	34.40	34.62	34.84	35.06	35.28	35.50	35.72
32	35.95	36.17	36.39	36.62	36.85	37.08	37.31	37.54	37.77	38.00
33	38.23	38.46	38.69	38.92	39.15	39.38	39.62	39.86	40.10	40.34
34	40.58	40.82	41.06	41.30	41.54	41.78	42.02	42.26	42.51	42.76
35	43.01	43.26	43.51	43.76	44.01	44.26	44.51	44.76	45.01	45.26
36	45.51	45.76	46.01	46.26	46.52	46.78	47.04	47.30	47.56	47.82
37	48.08	48.34	48.60	48.86	49.12	49.38	49.64	49.91	50.18	50.45
38	50.72	50.99	51.26	51.53	51.80	52.07	52.34	52.61	52.88	53.15
39	53.42	53.69	53.96	54.23	54.51	54.79	55.07	55.35	55.63	55.91
40	56.19	56.47	56.75	57.03	57.31	57.59	57.87	58.16	58.45	58.74
41	59.03	59.32	59.61	59.90	60.19	60.48	60.77	61.06	61.35	61.64
42	61.94	62.23	62.52	62.82	63.12	63.42	63.72	64.02	64.32	64.62
43	64.92	65.22	65.52	65.82	66.12	66.43	66.74	67.05	67.36	67.67
44	67.98	68.29	68.60	68.91	69.22	69.53	69.84	70.15	70.46	70.78
45	71.10	71.42	71.74	72.06	72.38	72.70	73.02	73.34	73.66	73.98
46	74.30	74.62	74.94	75.26	75.59	75.92	76.25	76.58	76.91	77.24
47	77.57	77.90	78.23	78.56	78.89	79.22	79.55	79.89	80.23	80.57
48	80.91	81.25	81.59	81.93	82.27	82.61	82.95	83 29	83.63	83.97
49	84.32	84.66	85.00	85.34	85.69	86.04	86.39	86.74	87.09	87.44
50	87.79	88.14	88.49	88.85	89.20	89.55	89.91	90.26	90.61	90.97

same draw-bar pull. At *A* its "velocity head" is that due to 30 miles per hour, or 31.60 feet. At *B* it has gained 40 feet more, and its velocity is that due to a velocity head of 71.60 feet, or slightly over 45 miles per hour. At *B′* its velocity is again 30 miles per hour and velocity head 31.60 feet. At *C* the velocity head is but 6.60 feet and the velocity about 13.7 miles per hour.

As the train runs from C to D its velocity increases to 30 miles at C' and to over 45 miles per hour at D. At E the velocity again becomes 30 miles per hour. Although there will be some slight modifications of the above figures in actual practice, yet the above is not a fanciful theoretical sketch. Thousands of just such undulations of grade are daily operated in such a way, without disturbing the throttle or applying brakes, and the draw-bar pull, if measured by a dynamometer, would be found to be practically constant. Of course the above case assumes that

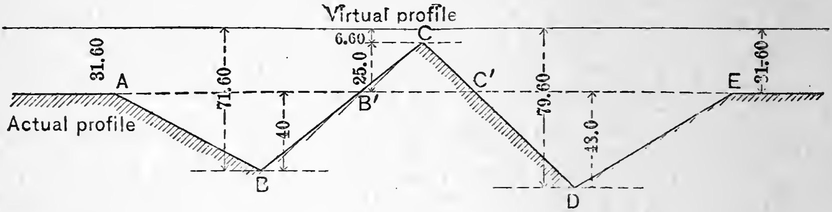


FIG. 212.

there are no stoppages and that the speed through the sags is not so great that safety requires the application of brakes. Observe that the "virtual profile" is here a straight line—as it always is when the draw-bar pull is constant. The virtual profile (in this case as well as in every other case, illustrations of which will follow) is found by adding to the actual profile at any point an ordinate which represents the "velocity head" due to the velocity of the train at that point.

As another case, assume that a train is climbing the grade AE and exerting a pull just sufficient to maintain a constant velocity up that grade. Then $A'B'$ (parallel to AB) is the virtual profile, AA' representing the velocity head. A stop being required at C, steam is shut off and brakes are applied at B, and the velocity head BB' reduces to zero at C. The train

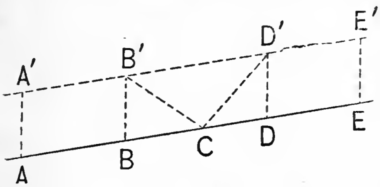


FIG. 213.

starts from C, and at D attains a velocity corresponding to the ordinate DD'. At D the throttle may be slightly closed so that the velocity will be uniform and the virtual grade is $D'E'$, parallel to DE.

From the above it may be seen that a virtual profile has the following properties:

(a) When the velocity is *uniform*, the virtual profile is parallel with the actual.

(*b*) When the velocity is increasing the profiles are separating; when decreasing the profiles are approaching.

(*c*) When the velocity is zero the profiles coincide.

433. **Use, value, and possible misuse.** The essential feature respecting grades is the demand on the locomotive. From the foregoing it may readily be seen that the ruling grade of a road is not necessarily the steepest nominal grade. When a grade may be operated by momentum, i.e., when every train has an opportunity to take "a run at the hill," it may become a very harmless grade and not limit the length of trains, while another grade, actually much less, which occurs at a stopping-place for the heaviest trains, will require such extra exertion to get trains started that it may be the worst place on the road. Therefore the true way to consider the value of the grade at any critical place on the road is to construct a virtual profile for that section of the road. The required length of such a profile is variable, but in general may be said to be limited by points on each side of the critical section at which the velocity is definite, as at a stopping-place (velocity zero), or a long heavy grade where it is the minimum permissible, say 10 or 15 miles per hour.

Since the velocities of different trains vary, each train will have its own virtual profile at any particular place. The fast passenger trains are generally unaffected, practically. The requirement of high average speed necessitates the use of powerful engines, and grades which would stall a heavy freight will only cause a momentary and harmless reduction of speed of the fast passenger train.

A possible misuse of virtual profiles lies in the chance that a station or railroad grade crossing may be subsequently located on a heavy grade that was designed to be operated by momentum. But this should not be used as an argument against the employment of a virtual profile. The virtual profile shows the *actual state of the case* and only points out the necessity, if an unexpected requirement for a full stoppage of trains at a critical point has developed, of changing the location (if a station), or of changing the grade by regrading or by using an overhead crossing. Examples of such modifications are given in Chapter XXIV, The Improvement of Old Lines.

434. **Undulatory grades. Advantages.** Money can generally be saved by adopting an actual profile which is not strictly uniform—the matter of compensation for curvature being here

ignored. Its effect on the operation of trains is harmless provided the sag or hump is not too great. In Fig. 214 the undulatory grade may actually be operated as a uniform grade AG. The sag at C must be considered as a sag, even though BC is actually an up grade. But the engine is supposed to be working

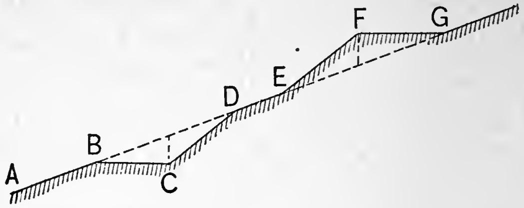


FIG. 214.

hard enough to carry a train at uniform velocity up a grade AG. Therefore it *gains* in velocity from B to C, and from C to D loses an equal amount. It may even be proven that the *time* required to pass the sag will be slightly less than the time required to run the uniform grade.

Disadvantages. The hump at F is dangerous in that, if the velocity at E is not equal to that corresponding to the extra velocity-head ordinate at F, the train will be stalled before reaching F. In practice there should be considerable margin. Any train should have a velocity of at least 10 miles per hour in passing any summit. This corresponds to a velocity head of 3.51 feet. An extra heavy head wind, slippery rails, etc., may use up any smaller margin and stall the train. If the grade AG is a ruling grade, then *no* hump should be allowed under any circumstances. For the heaviest trains are supposed to be so made up that the engine will *just* haul them up the ruling grades—of course with some margin for safety. Any increase of this grade, however short, would probably stall the train.

Safe limits. It is quite possible to have a sag so deep that it is not safe to allow freight trains to rush through them without the use of brakes. The use of brakes of course adds a distinct element of cost. To illustrate: If a freight train is running at a velocity of 20 miles per hour (velocity head 14.05 feet) and encounters a sag of 25 feet, the velocity head at the bottom of the sag will be 39.05 feet, which corresponds to a velocity of 33.3 miles per hour. This approaches the limit of safe speed for freight trains, and certainly passes the limit for trains not equipped with air-brakes and automatic couplers.

The term "safe limits" as used here, refers to the limits within which a freight train may be safely operated without the application of brakes or varying the work of the engine. Of course much greater undulations are frequently necessary and are safely operated, but it should be remembered that they add a distinct element to the cost of operating trains and that they must not be considered as harmless or that they should be introduced unless really necessary.

MINOR GRADES.

435. **Basis of cost of minor grades.** The basis of the computation of this least objectionable form of grade is as follows: The resistance encountered by a train on a level straight track is somewhat variable, depending on the velocity and the number and character of the cars, but for average velocities we may consider that 10 lbs. per ton is a reasonable figure. This value agrees fairly well with the results of some dynamometer tests made by Mr. P. H. Dudley, using a passenger train of 313 tons running at about 50 miles per hour. It also agrees with Searles's formulæ (based on experiments) for the resistance of a freight train with 40 cars running 25 miles per hour. Ten pounds per ton is the grade resistance of a 0.5% grade, or that of 26.4 feet per mile. On the above basis, a 0.5% grade will just double the tractive resistance on a level straight track. We may compute, as in the previous chapter, the cost of doubling the tractive resistance for one mile. But since the extra resistance is due to lifting the train through 26.4 feet of elevation, we may divide the extra cost of a mile of 0.5% grade by 26.4 and we have the cost of *one foot* of difference of elevation, and then (disregarding the limiting effect of grades) we may say that this cost of one foot of difference of elevation will be independent of the rate of grade. There are, however, limitations to this general proposition which will be developed in the next section.

436. **Classification of minor grades.** These are classified with reference to their effect on the operation of trains. In the first class are grades which may be operated without changing the work of the engine and which have practically no other effect than a harmless fluctuation of the velocity. But a grade which belongs to this class when considering a fast passenger train will belong to another class when considering a slow but heavy

freight train. And since it is the slow heavy freight trains which must be chiefly considered, a grade will usually be classified with respect to them. The limit of class A (the harmless class) therefore depends on the maximum allowable speed. The effect of a sag on speed will depend on the vertical feet of drop rather than on the rate of grade, for with the engine working as usual on even a light down grade a train would soon exceed permissible speed. Assume that a freight train runs at an average speed of 15 miles per hour with a minimum of 10 miles and a permissible maximum of 30 miles per hour. Assume that a train runs up the grade at A with a uniform velocity of 15 miles per hour, i.e., the engine is working so that the velocity *would* be uniform to C. How much sag (BB') can there be without the speed exceeding 30 miles per hour?

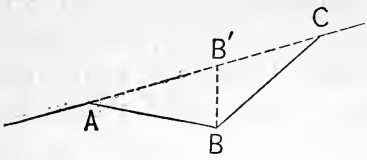


FIG. 215.

Velocity head for 30 miles per hour............	31.60
" " " 15 " " "	7.90
The drop BB' will therefore be................	23.70

While each case must be figured by itself, considering the probable velocity of approach and the maximum permissible velocity, we may say that a sag of *about* 24 feet will ordinarily mark the limit of this class. With a higher velocity of approach even this limit will be much reduced.

The classification therefore applies to sags and humps and to the vertical feet of drop or climb which are involved, rather than to grade *per se*. The practical application of these principles is necessarily confined to humps or sags which are possibly removable and does not apply to the long grades which are essential to connect predetermined points of the route—grades which are irreducible except by development and which must be studied as ruling grades (see §§ 440–445).

The application therefore consists in the comparative study of two proposed grades, noting the relative energy required to operate them and the probable cost. The depth in feet saved would be the maximum difference between the grades, and the classification will depend on the necessary method of operating the trains.

The next classification (B) applies to drops so deep that steam must be shut off when descending the grade, while the work required of the engine when ascending the opposite grade is correspondingly increased. The loss is not so serious as in the next case, but the inability of the engine to work continuously may result in a failure to accumulate sufficient kinetic energy to carry the train over a succeeding summit.

The third class (C) includes the grades so long that brakes must be applied to prevent excessive velocity. The loss involved is very heavy; the brakes require power for their application, they wear the brake-shoes and wheel-tires, they destroy kinetic or potential energy which had previously been created, while the tax on the locomotive on the corresponding ascending grade is very great. The ascending grade may or may not be a ruling grade.

437. **Effect on operating expenses.** As in Chapter XXII we may at once throw out a large proportion of the items of expense of an average train-mile. In "maintenance of way and structures" items 4 to 10 are evidently unaffected.

Item 1. REPAIRS OF ROADWAY. It is very plain that a large proportion of the sub-items are absolutely unaffected by minor grades. In fact it is a little difficult to ascribe any definite increase to any sub-item. The rail wear is somewhat increased and this will have some effect on the trackwork, but on the other hand the increased grade sometimes results in better drainage and therefore less work to keep the track in condition. Wellington allows 5% increase as a "liberal estimate" for class C, and no increase for the other classes.

Item 2. RENEWALS OF RAILS. Observations of rail wear on heavy grades show that it is much greater than on level tangents. But usually such heavy grades are operated by shorter trains or with the help of pusher engines, and the proportion of engine tonnage to the total is much greater than is ordinarily the case And since an engine has much greater effect on rail wear than cars, particularly on account of the use of sand, an excess of engine tonnage would have a marked effect. But such circumstances would inevitably accompany ruling grades and not minor grades. Nevertheless the effect of the use of sand on up grades and the possible skidding of wheels on down grades will wear the rails somewhat. Even the possible slipping of the drivers, although sand is not used, will wear the rails. Wellington allows 10% increase for class *C* and 5% for class *B*.

Item 3. RENEWALS OF TIES. The added wear of ties might be considered proportional to that of the rails except that, as in the case of the roadbed in general, the better drainage secured by the grade will tend to increase the life of the ties. Wellington makes the estimate the same as for item 1, 5% for class *C* and no increase for the other classes.

Maintenance of equipment. Items 11, 16, 17, 18, and 19 are evidently unaffected. **Items 12 to 15.** The chief sub-items of increase will evidently be the repairs and renewals of wheels and brake-shoes both for locomotives and cars. In the case of cars the draw-bar is apt to suffer from severe alternate compression and extension due to push and pull. The locomotive mechanism will suffer somewhat from the extra demands on it, and the boiler on account of the intermittent character of the demands on it. It would seem as if such effects would be quite large, but an examination of the comparative records of engine and car repairs on mountain divisions and on comparatively level divisions shows no such difference as might be expected. On this account Wellington cuts down these items to 4% for class *C* and 1% for class *B*.

Conducting transportation. As in Chapter XXI, § 407, since the resistance is assumed to be doubled, we may take the same figure (55%) as the cost of the fuel for climbing the 26.4 feet. But the total cost of both the rise and fall is to be considered. In class *B*, although steam is shut off, heat (and fuel) is wasted by mere radiation. This has been estimated (Chapter XXI, § 407) as about 5%. Therefore we may allow 60% for class *B*. For class *C* we must allow in addition the energy spent in applying brakes, which we may assume as 5% more, making 65%. Items 23, 24, and 25 may be estimated similarly. The other items under this head as well as General Expenses are evidently unaffected.

438. Estimate of the cost of one foot of change of elevation. Collecting these estimates, we have the accompanying tabular form, showing that the percentage of increase for operating grades of class *B* or class *C* will be 6.77% and 8.56%, respectively. On the basis of an average cost of 95 c. per train-mile, the additional cost for the 26.4 feet in one mile would be 6.43 c. and 8.13 c., or 0.24 c. and 0.31 c. per foot. For each train per day each way per year the value per foot of difference of elevation is:

$$\text{For class } B\text{: } 2 \times 365 \times \$0.0024 = \$1.75;$$
$$\text{`` \quad ``} \quad C\text{: } 2 \times 365 \times \$0.0031 = \$2.26.$$

It will frequently happen that a grade must be considered as belonging to class C for heavy freight trains, and that it belongs to class B or even class A for other trains. If no Sunday trains are run, 313 should be used instead of 365 as a multiplier in the above equations.

439. Operating value of the removal of a hump in a grade. As a simple illustration of the above, suppose that the irregular grade $ABCD$ may be cut down to the uniform grade AD, either by direct lowering of the track or by a modification of alignment. If a freight train, running to the *left*, passes C at a velocity of 15 miles per hour (velocity head, 7.90 feet) and drops

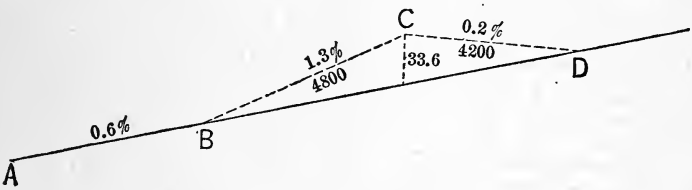


FIG. 216.

down to B (a vertical fall of 62.40 feet) without shutting off steam, its velocity head at B would be 70.30 feet and its velocity 44.8 miles per hour, which is inadmissible. Therefore the hump certainly does not belong to class A, for freight trains. Suppose that steam is shut off when passing C. Then, if we consider the average value of 0.5% as the grade which is equivalent to the normal rolling resistances, we may consider the 1.3% grade as an 0.8% grade, down which the train passes without resistance. An 0.8% grade for 4800 feet would be a drop of 38.40 feet, and the velocity head at B would be $7.90+38.40=46.30$ feet, and the velocity would be 36.3 miles per hour, which may or may not be considered as inadmissible for freight trains. According to the decision the grade belongs to class C or to class B. For trains moving to the *right*, or up grade, there is no definite criterion, but a grade of 1.3% is a severe tax on a locomotive, even if it is not a ruling grade. Ignoring its possible

TABLE XXIV.—EFFECT ON OPERATING EXPENSES OF CHANGES IN GRADE.

No.	Item (abbreviated).*	Normal average.	Class B.		Class C.	
			Per cent affected	Cost per mile	Per cent affected	Cost per mile
1	Roadway.	10.596	0	0	5	.53
2	Rails.	1.440	5	.07	10	.14
3	Ties.	3.093	0	.07	5	.15
4–10	Bridges, buildings, etc.	5.533	0	0	0	0
	Maintenance of way.	20.662	. . . .	.07	. . . .	.82
11	Superintendence.	.650	0	0	0	0
12	Repairs locomotives . .	5.879	1	.06	4	.23
13	Repairs pass. cars	2.209	1	.02	4	.09
14	Repairs freight cars. . .	6.765	1	.07	4	.27
15	Repairs work cars	.155	1	.00	4	.06
16	Marine equipment	.209	0	0	0	0
17	Shops.	.490	0	0	0	0
18	Stat. and printing	.040	0	0	0	0
19	Other expenses	.495	0	0	0	0
	Main. of equip.	16.892	. . . .	.15	. . . .	.65
20	Superintendence.	1.761	0	0	0	0
21	Enginemen.	9.781	0	0	0	0
22	Fuel.	9.681	60	5.81	65	6.29
23	Water.	.671	60	.40	65	.44
24	Oil, etc.	.376	60	.23	65	.24
25	Other supplies.	.184	60	.11	65	.12
26–46	Train service, station service, etc.	35.340	0	0	0	0
	Conducting transp . .	57.794	. . . .	6.55	. . . .	7.09
47–53	General expenses	4.653	0	0	0	0
		100.000	. . . .	6.77	. . . .	8.56

* For full title of item see Table XX.

limiting effect (which is a separate matter), the value of the 33.6 feet is evidently

$$33.6 \times \$2.26 = \$75.94 \text{ per daily train for class } C$$

and

$$33.6 \times \$1.75 = \$58.80 \text{ `` `` `` `` `` } B.$$

Assuming that there are six daily trains each way for which the grade would be classified as *C*, and four others which could operate the grade as a "*B*" grade, the total annual cost would be

$$6 \times \$75.94 = \$455.64$$
$$4 \times \$58.80 = \$235.20$$
$$\overline{\quad \$690.84}$$

This annual cost, capitalized at 5%, equals $13817, which is the justifiable expenditure to avoid the hump. Assuming that the cut would involve 300000 cubic yards, at 20 c. per cubic yard, it would cost $60000 to make the through cut. On the above basis the cut would not be justifiable, but a small part of such cutting would so reduce the hump that it would not belong to class *C* for any trains, and it might even be reduced to class *A*. Of course other solutions are possible. A slightly different route may be chosen from *B* to *D*, involving a different distance, different curvature, and a marked reduction in the hump. The effect of all such changes must be combined and their *net* effect determined.

RULING GRADES.

440. Definition. Ruling grades are those which limit the weight of the train of cars which may be hauled by one engine. The subject of "pusher grades" will be considered later. For the present it will suffice to say that on all well-designed roads the large majority of the grades on any one division are kept below some limit which is considered the ruling grade. If a heavier grade is absolutely necessary no special expense will be made to keep it below a rate where the resistance is twice (or possibly three times) the resistance on the ruling grade, and then the trains can be hauled unbroken up these few special grades with the help of one (or two) pusher engines. So far as limitation of train length is concerned, these pusher grades are no worse than the regular ruling grades and, except for the expense of operating the pusher engines (which is a separate matter), they are not appreciably more expensive than any ruling grade. As before stated, the engineer cannot alter very greatly the general level of the road when the general route has been decided on. He may remove sags or humps, or he may lower the natural grade of the route by development in order to bring the grade within the adopted limit of ruling grade. The financial value of removing sags and humps has been considered. It now remains to determine the financial relation between the lowest permissible ruling grade and the money which may profitably be spent to secure it.

441. Choice of ruling grade. It is of course impracticable for an engine to drop off or pick up cars according to the grades which may be encountered along the line. A train load is made up at one terminus of a division and must run to the other terminus. Excluding from consideration any short but steep grades which may *always* be operated by momentum, and also all pusher grades, the maximum grade on that division is the ruling grade.

It will evidently be economy to reduce the few grades which naturally would be a little higher than the great majority of others until such a large amount of grade is at some uniform limit that a reduction at all these places would cost more than it is worth. The precise determination of this limit is practically impossible, but an approximate value may be at once determined from a general survey of the route. The distance apart of the termini of the division into their difference of elevation is a first trial figure for the rate of the grade. If a grade even approximately uniform is impossible owing to the elevations of predetermined intermediate points, the worst place may be selected and the natural grade of that part of the route determined. If this grade is much steeper than the general run of the natural grades, it may be policy to reduce it by development or to boldly plan to operate that place as a pusher grade. The choice of possible grades thus has large limitations, and it justifies very close study to determine the best combination of grades and pusher grades. When the choice has narrowed down to two limits, the lower of which may be obtained by the expenditure of a definite extra sum, the choice may be readily computed, as will be developed.

442. Maximum train load on any grade. The tractive power of a locomotive has been discussed in Chap. XV, § 322. The *net* train load which may be placed behind any engine is the difference between the weight of the engine itself and the *gross* load which can be handled under the given circumstances, with a given weight on the drivers. Since the design of locomotives is so variable, it is impracticable to show in tabular form the power of all kinds of locomotives on all grades. In Table XXV are given the tractive powers of locomotives of a wide range of types and weights and with various ratios of adhesion. They may be accepted as typical figures and will serve to compute the effect of variations of grade on train load. In Table XXVI is given the total train resistance in pounds per ton for various grades and for various values of track resistances. By a combination

TABLE XXV.—TRACTIVE POWER OF VARIOUS TYPES OF LOCOMOTIVES AT VARIOUS RATES OF ADHESION.

Kind.	Gauge.	Total weight of engine and tender.		Weight of engine only.	Weight on drivers.	Tractive power when rate of adhesion is		
		lbs.	tons.			$\frac{1}{4}$.	$\frac{9}{40}$	$\frac{1}{5}$
American...	Nar.	49,000	24.5	32,000	22,000	5,500	4,950	4,400
"	"	80,000	40	49,000	32,000	8,000	7,200	6,400
Mogul......	"	81,000	40.5	51,000	42,000	10,500	9,450	8,400
10-wheel...	"	87,000	43.5	55,000	42,000	10,500	9,450	8,400
Consol......	"	62,000	31	39,000	34,000	8,500	7.650	6,800
"	"	144,000	72	94,000	84,000	21,000	18,900	16,800
American...	Stand.	104,000	52	62,000	40,000	10,000	9,000	8,000
"Chautau"..	"	314,600	157.3	190,600	99,400	24,850	22,365	19,880
Mogul......	"	206,000	103	126,000	106,400	26,600	23,940	21,280
10-wheel ...	"	276,000	138	176,510	127,010	31,752	28,577	25,402
Consol......	"	214,000	107	120,000	106,000	26,500	23,850	21,200
"	"	324,800	162.4	204,800	181,200	45,300	40,770	36,240

of these two tables the net train load on any grade under given conditions may be quickly determined For example, an ordinary consolidation engine having a weight of 106000 pounds on the drivers (see Table XXV) will have a tractive force of 26500 pounds under fair conditions of track, when the adhesion ratio is $\frac{1}{4}$. When climbing slowly up a grade of 1.30% the tractive resistance will be about 32 pounds per ton if the rolling-stock and track are fair—assuming a tractive resistance on a level of 6 pounds per ton. Dividing 26500 by 32 we have 828 tons, the gross train load. Subtracting 107 tons, the weight of the engine and tender in working order, we have 721 tons, the *net* load. Incidentally we may note that, cutting down the grade to 0.90% (a reduction of only 21.12 feet per mile), the resistance per ton is reduced to 24 pounds and the gross train load is increased to 1104 tons and the net load to 997 tons—an increase of about 38%.

As another numerical example, consider a contractor's locomotive (not referred to in Table XXV), a light four-wheel-connected-tank narrow-gauge engine, with a total weight of 12000 pounds, all on the drivers. On the rough temporary track used by contractors the tractive ratio may be as low as $\frac{1}{5}$. The tractive adhesion should therefore be taken as 2400 pounds. Assume that the grade when hauling "empties" is 4.7% and that the tractive resistance on such a track *on a level* is 10 pounds per ton. By Table XXVI, the total train resistance is therefore (by interpolation) 104 pounds per ton. $2400 \div 104 = 23$ tons; subtracting the weight of the engine we have 17 tons, the net load of empty cars—perhaps twenty cars weighing 1700 pounds per car.

In general, and to compute accurately the train load under conditions not exactly given in the tables, the maximum train load may be computed according to the following rule:

The maximum load behind an engine on any grade may be found by multiplying the weight on the drivers by the ratio of adhesion and dividing this by the sum of the grade and tractive resistances per ton; this gives the gross load, from which the weight of the engine and tender must be subtracted to find the net load.

443. **Proportion of the traffic affected by the ruling grade.** Some very light traffic roads are not so fortunate as to have a traffic which will be largely affected by the rate of the ruling grade. When passenger traffic is light, and when, for the sake of encouraging traffic, more frequent trains are run than are required from the standpoint of engine capacity, it may happen that no passenger trains are really limited by any grade on the road—i.e., an extra passenger car *could* be added if needed. The maximum grade then has no worse effect (for passenger trains) than to cause a harmless reduction of speed at a few points. The local freight business is frequently affected in practically the same way. All coal, mineral, or timber roads are affected by the rate of ruling grade as far as such traffic is concerned. Likewise the through business in general merchandise, especially of the heavy traffic roads, will generally be affected by the rate of ruling grade. Therefore in computing the effect of ruling grade, the total number of trains on the road should not ordinarily be considered, but only the trains to which cars are added, until the limit of the hauling power of the engine on the ruling grades is reached.

444. **Financial value of increasing the train load.** The gross receipts for transporting a given amount of freight is a definite sum regardless of the number of train loads. The cost of a train mile is practically constant. If it were exactly so, the saving in operating expenses would be strictly proportional to the number of trains saved. How will the cost per train

TABLE XXVI.—TOTAL TRAIN RESISTANCE PER TON (OF 2000 POUNDS) ON VARIOUS GRADES.

Grade.		When tractive resistance *on a level* in pounds per ton is					Grade.		When tractive resistance *on a level* in pounds per ton is				
Rate per cent.	Feet per mile.	6	7	8	9	10	Rate per cent.	Feet per mile.	6	7	8	9	10
0.00	0.00	6	7	8	9	10	2.00	105.60	46	47	48	49	50
.05	2.64	7	8	9	10	11	.05	108.24	47	48	49	50	51
.10	5.28	8	9	10	11	12	.10	110.88	48	49	50	51	52
.15	7.92	9	10	11	12	13	.15	113.52	49	50	51	52	53
.20	10.56	10	11	12	13	14	.20	116.16	50	51	52	53	54
0.25	13.20	11	12	13	14	15	2.25	118.80	51	52	53	54	55
.30	15.84	12	13	14	15	16	.30	121.44	52	53	54	55	56
.35	18.48	13	14	15	16	17	.35	124.08	53	54	55	56	57
.40	21.12	14	15	16	17	18	.40	126.72	54	55	56	57	58
.45	23.76	15	16	17	18	19	.45	129.36	55	56	57	58	59
0.50	26.40	16	17	18	19	20	2.50	132.00	56	57	58	59	60
.55	29.04	17	18	19	20	21	.55	134.64	57	58	59	60	61
.60	31.68	18	19	20	21	22	.60	137.28	58	59	60	61	62
.65	34.32	19	20	21	22	23	.65	139.92	59	60	61	62	63
.70	36.96	20	21	22	23	24	.70	142.56	60	61	62	63	64
0.75	39.60	21	22	23	24	25	2.75	145.20	61	62	63	64	65
.80	42.24	22	23	24	25	26	.80	147.84	62	63	64	65	66
.85	44.88	23	24	25	26	27	.85	150.48	63	64	65	66	67
.90	47.52	24	25	26	27	28	.90	153.12	64	65	66	67	68
0.95	50.16	25	26	27	28	29	.95	155.76	65	66	67	68	69
1.00	52.80	26	27	28	29	30	3.00	158.40	66	67	68	69	70
.05	55.44	27	28	29	30	31	.05	161.04	67	68	69	70	71
.10	58.08	28	29	30	31	32	.10	163.68	68	69	70	71	72
.15	60.72	29	30	31	32	33	.15	166.32	69	70	71	72	73
.20	63.36	30	31	32	33	34	.20	168.96	70	71	72	73	74
1.25	66.00	31	32	33	34	35	3.25	171.60	71	72	73	74	75
.30	68.64	32	33	34	35	36	.30	174.24	72	73	74	75	76
.35	71.28	33	34	35	36	37	.35	176.88	73	74	75	76	77
.40	73.92	34	35	36	37	38	.40	179.52	74	75	76	77	78
.45	76.56	35	36	37	38	39	.45	182.16	75	76	77	78	79
1.50	79.20	36	37	38	39	40	3.50	184.80	76	77	78	79	80
.55	81.84	37	38	39	40	41	4.00	211.20	86	87	88	89	90
.60	84.48	38	39	40	41	42	4.50	237.60	96	97	98	99	100
.65	87.12	39	40	41	42	43	5.00	264.00	106	107	108	109	110
.70	89.76	40	41	42	43	44	5.50	290.40	116	117	118	119	120
1.75	92.40	41	42	43	44	45	6.00	316.80	126	127	128	129	130
.80	95.04	42	43	44	45	46	6.50	343.20	136	137	138	139	140
.85	97.68	43	44	45	46	47	7.00	369.60	146	147	148	149	150
.90	100.32	44	45	46	47	48	8.00	422.40	166	167	168	169	170
1.95	102.96	45	46	47	48	49	9.00	475.20	186	187	188	189	190
2.00	105.60	46	47	48	49	50	10.00	528.00	206	207	208	209	210

mile vary when by a reduction in ruling grade more cars are handled in one train than before? First, compute the effect of increasing the train load so that one less engine will handle the traffic, or, for example, that an engine can haul 11 cars instead of 10 or 44 instead of 40—that 10 engines will do the work for which 11 engines would be required with the steeper grade. What will be the relative cost of running 10 heavy trains rather than 11 lighter trains, or, rather, what will be the extra cost of the extra engine?

Since the gross traffic to be handled is assumed to be the same, the number of cars required to handle it will also be the same whatever the number of trains, and the effect of those cars on the wear and tear of track, etc., will evidently be constant. The locomotive, on account of the greater concentration of loading of the driver wheels, damages the track (in proportion to its tonnage) much more than the cars. It has been estimated that the locomotive is responsible for one half of the track wear Such an estimate is verified by the wear of rails on steep tracks around coal-mines where standard cars are hauled by cables. If we assume that 50% of Items 2 and 3 and of that part of Item 1 which varies with tonnage is due to the locomotives, then the *extra* expense caused by the extra engine will be 50% of Items 2 and 3 and 50% of 25% of Item 1. The other items of maintenance of way are unaffected except that truss bridges, trestles, and the maintenance of a few buildings will be slightly affected by the extra locomotive. But the actual effect is quite indefinite and is evidently very small.

Maintenance of equipment: Engine repairs will evidently be affected according to the mileage. Throughout the ruling grade of the road (by whichever system of grades) the engines (assumed of uniform style) are working at their utmost capacity. On the lighter grades and level sections the engines will have easier work when the cars are fewer and this will have a tendency to reduce engine repairs. Suppose that by decreasing the number of cars 10% on the easy grades the engine repairs on each engine are reduced 2%. There is little or no justification for estimating the reduction to be more than this. Then on the ten engines the saving is 20% of the average charge for 1 engine. Suppose that by decreasing the number of cars 20% on the easy grades the engine repairs are reduced 4%, on the five engines they are reduced 20% again. In either case the net added cost due to the extra engine would be but 80% of the average cost While the above estimate is but a guess, yet it is very evident that the extra cost for this item is but little less than the normal charge.

Car repairs will be reduced by a decrease in the number of cars per train. The average draw-bar pull will be less, the wear and tear due to stoppage and starting will be less. This is the one item in which an increased number of trains for the same tonnage is an actual advantage. The saving per car is evidently greater when 4 trains are increased to 5 than when 10 trains are increased to 11; but the saving per train added on is constant. Wellington estimates the saving to be 10%. His basis of calculation is somewhat different, but it reduces to the same thing. The estimate applies chiefly to Item 14 and to Item 13 in so far as passenger trains are affected by ruling grade. The other items of maintenance of equipment are but little, if any, affected.

Conducting transportation. Items 20, 21, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 45, and 46 may be considered as varying according to the train mileage. While some of them seem to have but little direct connection with train mileage, yet if a road increases its traffic from 10 trains a day to 20 trains a day all of these items seem to increase in due proportion.

Fuel, etc., for locomotives (Items 22–25) will increase nearly as the engine mileage. In either case the engines work to the limit of their capacity on the ruling grades. In either case the loss of heat due to radiation is the same. But the engines with the lighter trains work a little easier on the light or level grades. By the same course of reasoning as was given regarding engine repairs the fuel saving from the normal requirement for the extra engine will be about the same no matter whether there is an addition of one engine in 5 or 10. The saving in fuel will be assumed at 25% of the normal consumption, or rather that the use of the extra engine adds 75% of the normal charge for fuel. The same estimate applies to items 23, 24, and 25.

Car mileage is unaffected. Items 38 to 44 will be considered as unaffected, also the general expenses.

445. Operating value of a reduction in the rate of the ruling grade. Collecting the above estimates, we have Table XXVII. To this must be added something for the capital cost of the extra engine. Assume that it costs $10000 and that its mileage life is 800000 miles. This makes an average charge of 1.25 c. per mile. Of course the cost of operation, maintenance, and repairs is included in the tabulated expense. 54.82% of 95 c. = 52.08 c. Adding 1.25 c., we have 53.33 c.

TABLE XXVII.—COST OF AN ADDITIONAL TRAIN TO HANDLE A GIVEN TRAFFIC.

No.	Item (abbreviated).	Normal average.	Per cent affected.	Cost per cent.
1	Roadway	10.596	12.5	1.32
2	Rails	1.440	50	.72
3	Ties	3.093	50	1.55
4–10	Bridges, buildings, etc	5.533	0	0
	Maintenance of way	20.662	....	3.59
11	Superintendence	.650	0	0
12	Repairs of locomotives	5.879	80	4.70
13	Repairs of passenger-cars	2.209	5	.11
14	Repairs of freight-cars	6.765	10	.68
15–19	Miscellaneous	1.389	0	0
	Maintenance of equipment	16.892	....	3.91
20	Superintendence	1.761	100	1.76
21	Enginemen	9.781	100	9.78
22–25	Fuel, etc	10.912	75	8.18
26–32	Train service, etc	24.285	100	24.28
33	Car mileage	2.094	0	0
34–37	Damages, etc	2.085	100	2.09
38–44	Miscellaneous	5.646	0	0
45–46	Stationery, etc	1.229	100	1.23
	Conducting transportation	57.793	....	47.32
47–53	General expenses	4.653	0	0
		100.000	....	54.82

As a practical application of the above figures, assume that on a constructed and operated road the ruling grade on a 100-mile division is 1.6%; the actual traffic affected by ruling grade is 8 daily trains with a net load of 552 tons or 4416 tons. It is found that with an expenditure of $400000 the ruling grade may be reduced to 1.2%. Will it pay? At 1.2% grade the net load behind an 80-ton consolidation engine, with 48 tons on the drivers, adhesion $\frac{1}{4}$, is 720 tons. The traffic (4416 tons) may therefore be hauled by 6 engines, the balance, less than 100 tons, being taken care of by lighter trains not affected by the ruling grade. Since the *additional* cost of the engine drawing lighter trains is 53 c. per mile, the saving by reducing from 8 engines to 6 is that due to 2 engines. The annual saving is therefore $2 \times \$0.5333 \times 100 \times 365 = \$38930.90$, which capitalized at $5\% = \$778618$. This shows that if the improvement can be accomplished for \$400000 it is worth while.

As in other similar problems, it must be reiterated that although there are some more or less uncertain elements in the above estimates, yet with a considerable margin for error in individual items the value of the whole improvement will not be very greatly altered and the estimate will be infinitely better than an indefinite reliance on vague "judgment." Of course certain items in the above estimates are somewhat variable and should be altered to fit the particular case to be computed.

PUSHER GRADES.

446. General principles underlying the use of pusher engines. On nearly all roads there are some grades which are greatly in excess of the general average rate of grade and these heavy grades cannot usually be materially reduced without an expenditure which is excessive and beyond the financial capacity of the road. If no pusher engines are used, the length of all heavy trains is limited by these grades. The financial value of the reduction of such ruling grades has already been shown. But in the operation of pusher grades there is incurred the additional cost of pusher-engine service, for a pusher engine must run *twice* over the grade for each train which is assisted. It is possible for this additional expense to equal or even exceed the advantage to be gained. In any case it means the adoption of the lesser of two evils, or the adoption of the more economical method. A simple example will illustrate the point. Assume that at one point on the road there is a grade of 1.9% which is five miles long. Assume that all other grades are less than 0.92%. If pushers are not to be used the net capacity of a 107-ton consolidation engine with 53 tons on the drivers, assuming $\frac{9}{40}$ adhesion and 6 pounds per ton for normal resistance, will be 435 tons, and that will be the maximum weight of train allowable. By using pusher engines on this one 5-mile grade the train load is at once doubled and the number of trains cut down one half. This double load, 870 tons, can easily be hauled by one engine up the 0.92% grades. As a rough comparison, free from details and allowances, we may say:

(a) 10 trains per day over a 100-mile division, 435 tons net per train, will require 1000 engine miles daily.

(b) 5 trains per day handling the same traffic, 870 tons net per train, with $2\times5\times5$ pusher-engine miles, will require $(5\times100)+(2\times5\times5)=550$ engine miles daily. There is thus a large saving in the number of engine miles and also in the number of the engines required for the work. Moreover, the engines are working to the limit of their capacity for a much larger proportion of the time, and their work is therefore more economically done. The work of overcoming the normal resistances of so many loaded cars over so many miles of track and of lifting so many tons up the gross differences of elevation of predetermined points of the line is approximately the same whatever the exact route, and if the grades are so made that fewer engines working more constantly can accomplish the work as well as more engines which are not hard worked for a considerable proportion of the time, the economy is very apparent and unquestionable. Wellington expresses it concisely: "It is a truth of the first importance that the objection to high gradients is not the work which the engines have to do on them, but it is the work which they do *not* do when they thunder over the track with a light train behind them, from end to end of a division, in order that the needed power may be at hand at a few scattered points where alone it is needed."

447. Balance of grades for pusher service. In the above illustration the "through" grade and the "pusher" grade are "balanced" for the use of one equal pusher. It is therefore evident that if some intermediate grade (such as 1.4%) were permitted, it could only be operated by (*a*) making it the ruling grade and cutting down all train loads from 870 tons to 594 tons, or (*b*) operating it as a pusher grade, although with a loss of economy, since two engines would have much more power than necessary. The proper plan in such a case would be to strive to reduce the 1.4% grade to 0.92%, or, if that seemed impracticable, to attempt to get an operating advantage at the expense of an increase of the 1.4% grade to anything short of 1.9%. For the increase in *rate* of grade would cost almost nothing, and some advantage might be obtained which would practically compensate for the introduction of a pusher grade. *Another possible* solution would be to operate the 1.9% with two pushers, adopt a corresponding grade for use with one pusher and a corresponding ruling grade for through trains. With the above data these three grades would be 1.90% 1.27%, and 0.54%, obtained as follows:

Tractive power of three engines $= 106000 \times \frac{9}{40} \times 3 = 71550$ pounds.

Resistance on 1.9% grade $= 6 + (20 \times 1.9) = 44$ lbs. per ton.

$71550 \div 44 = 1626$ = gross load in tons.

$1626 - (3 \times 107) = 1305$ = net load in tons.

$1305 + (2 \times 107) = 1519$ = gross load on the one-pusher grade.

Tractive power of two engines = 47700 lbs.

$47700 \div 1519 = 31.40$ = possible tractive force in lbs. per ton.

$(31.40 - 6) \div 20 = 1.27\%$ = permissible grade for one pusher.

$1305 + 107 = 1412$ = gross load on the through grade.

Tractive power of one engine = 23850 lbs.

$23350 \div 1412 = 16.89$ = possible tractive force in lbs. per ton.

$(16.89 - 6) \div 20 = 0.54\%$ = permissible through grade.

It should be realized that, assuming the accuracy of the normal resistance (6 lbs.) and the normal adhesion ($\frac{9}{40}$) and with the use of 107-ton locomotives with 53 tons on the drivers, the above figures are *precisely* what is required for hauling with one, two, and three engines. Other types of engines, other values for resistance and adhesion will vary considerably the gross load in tons which may be hauled up those grades, but starting with 0.54% as a through grade, the corresponding values for one and for two pushers would vary but slightly from those given. To show the tendency of these variations, the corresponding values have been computed as follows:

Adhesion.	Resistance per ton.	Load on drivers.	Through grade.	One-pusher grade.	Two-pusher grade.
$\frac{9}{40}$	6 lbs.	53 tons.	0.54%	1.27%	1.90%
$\frac{9}{40}$	7 "	53 "	.54%	1.31%	1.96%
$\frac{1}{4}$	6 "	53 "	.54%	1.28%	1.93%
$\frac{1}{5}$	6 "	53 "	.54%	1.26%	1.86%
$\frac{1}{5}$	7 "	53 "	.54%	1.29%	1.92%

The above form shows that *increasing* the resistance per ton and *decreasing* the adhesion have opposite effects on altering the ratio of these grades, and as a storm, for example, would increase the resistance and decrease the adhesion, the changes in the ratio would be compensating although the absolute reduction in train load might be considerable.

In Table XXVIII is shown a series of "balanced" grades on which a given *net* train load may be operated by means of one or two pusher engines. For example, assuming a track resistance of 6 pounds per ton, a consolidation engine of the type shown in the table can haul a train weighing 977 tons (exclusive of the engine) up a grade of 0.80%. If this is the maximum *through* grade, pusher grades as high as 1.70% for one pusher, or 2.46% for two pushers, may be introduced and the same *net* load may be hauled up these grades.

The ratios of pusher grade to through grade, as given in Table XXVIII, are exactly true only for the conditions named as to weight and type of engine, ratio of adhesion, and normal track resistance. But a little comparative study of the two halves of Table XXVIII and of the tabular form given on page 483 will show that although the net load which can be hauled on any grade varies considerably with the normal track resistance and also with the ratio of adhesion, yet the *ratios* of through to pusher grade, for either one or two pushers, varies but slightly with ordinary changes in these conditions. Therefore when the precise conditions are unknown or variable, the figures of Table XXVIII may be considered as applicable to any ordinary practice, especially for preliminary computations. For final calculations on any proposed ruling grade and pusher grade, the whole problem should be worked out on the principles outlined above and on the basis of the best data obtainable.

Problem: If the *through* ruling grade for the road has been established at 1.12%, what pusher grades are permissible? *Answer:* Interpolating in Table XXVIII, we may employ a grade of 2.22% if the track and road-bed are to be such that a tractive resistance of 6 pounds per ton can be expected. With a poorer track, the normal resistance assumed as 8 pounds per ton, the rate is raised to 2.27%. The *increase* in rate of pusher grade with increase of resistance is due to the fact that the net load hauled is less—so much less that on the pusher grade a larger part of the adhesion is available to overcome a grade resistance.

448. Operation of pusher engines. The maximum efficiency in operating pusher engines is obtained when the pusher engine is kept constantly at work, and this is facilitated when the pusher grade is as long as possible, i.e., when the heavy grades and the great bulk of the difference of elevation to be surmounted is

TABLE XXVIII.—BALANCED GRADES FOR ONE, TWO, AND THREE ENGINES.

Basis.—Through and pusher engines alike; consolidation type; total weight, 107 tons; weight on drivers, 53 tons; adhesion, $\frac{9}{40}$, giving a tractive force for each engine of 23850 lbs.; normal track resistance, 6 (also 8) lbs. per ton.

Through grade.	Track resistance, 6 lbs.			Track resistance, 8 lbs.		
	Net load for one engine in tons (2000 lbs.).	Corresponding pusher grade for *same net load.* One pusher.	Two pushers.	*Net* load for one engine in tons (2000 lbs.).	Corresponding pusher grade for *same net load.* One pusher.	Two pushers.
Level.	3868 tons	0.28%	0.55%	2874 tons	0.37%	0.72%
0.10%	2874 "	0.47%	0.82%	2278 "	0.56%	0.98%
0.20%	2278 "	0.66%	1.08%	1880 "	0.74%	1.23%
0.30%	1880 "	0.84%	1.33%	1596 "	0.92%	1.47%
0.40%	1596 "	1.02%	1.57%	1384 "	1.09%	1.70%
0.50%	1384 "	1.19%	1.80%	1218 "	1.27%	1.92%
0.60%	1218 "	1.37%	2.02%	1085 "	1.44%	2.14%
0.70%	1085 "	1.54%	2.24%	977 "	1.60%	2.36%
0.80%	977 "	1.70%	2.46%	887 "	1.77%	2.56%
0.90%	887 "	1.87%	2.66%	810 "	1.93%	2.76%
1.00%	810 "	2.03%	2.86%	745 "	2.09%	2.96%
1.10%	745 "	2.19%	3.06%	688 "	2.24%	3.15%
1.20%	688 "	2.34%	3.25%	638 "	2.40%	3.33%
1.30%	638 "	2.50%	3.43%	594 "	2.55%	3.51%
1.40%	594 "	2.65%	3.61%	555 "	2.70%	3.68%
1.50%	555 "	2.80%	3.78%	521 "	2.85%	3.85%
1.60%	521 "	2.95%	3.95%	489 "	2.99%	4.02%
1.70%	489 "	3.09%	4.12%	461 "	3.13%	4.17%
1.80%	461 "	3.23%	4.27%	435 "	3.27%	4.33%
1.90%	435 "	3.37%	4.43%	411 "	3.42%	4.49%
2.00%	411 "	3.52%	4.59%	390 "	3.55%	4.63%
2.10%	390 "	3.65%	4.73%	370 "	3.68%	4.78%
2.20%	370 "	3.78%	4.88%	352 "	3.81%	4.92%
2.30%	352 "	3.91%	5.02%	335 "	3.94%	5.05%
2.40%	335 "	4.04%	5.15%	319 "	4.07%	5.19%
2.50%	319 "	4.17%	5.29%	304 "	4.20%	5.32%

at one place. For example, a pusher grade of three miles followed by a comparatively level stretch of three miles and then by another pusher grade of two miles cannot all be operated as cheaply as a continuous pusher grade of five miles. Either the two grades must be operated as a continuous grade of eight miles (sixteen pusher miles per trip) or else as two short pusher grades, in which case there would be a very great loss of time and a difficulty in so arranging the schedules that a train need not wait for a pusher or the pushers need not waste too much time in idleness waiting for trains. If the level stretch were imperative, the two grades would probably be operated as one, but an effort should be made to bring the grades together. It is not necessary to bring the trains to a stop to uncouple the pusher engine, but a stop is generally made for coupling on, and the actual cost in loss of energy and in wear and tear of stopping and starting a heavy train is as great as the cost of running an engine light for several miles.

There are two ways in which it is *possible* to economize in the use of pusher engines. (*a*) When the traffic of a road is so very light that a pusher engine will not be kept reasonably busy on the pusher grade it *may* be worth while to place a siding long enough for the longest trains both at top and bottom of the pusher grade and then take up the train in sections. Perhaps the worst objection to this method is the time lost while the engine runs the extra mileage, but with such very light traffic roads a little time more or less is of small consequence. On light traffic roads this method of surmounting a heavy grade will be occasionally adopted even if pushers are never used. If the traffic is fluctuating, the method has the advantage of only requiring such operation when it is needed and avoiding the purchase and operation of a pusher engine which has but little to do and which might be idle for a considerable proportion of the year. (*b*) The second possible method of economizing is only practicable when a pusher grade begins or ends at or near a station yard where switching-engines are required. In such cases there is a possible economy in utilizing the switching-engines as pushers, especially when the work in each class is small, and thus obtain a greater useful mileage. But such cases are special and generally imply small traffic.

A telegraph-station at top and bottom of a pusher grade is generally indispensable to effective and safe operation.

449. **Length of a pusher grade.** The virtual length of the pusher grade, as indicated by the mileage of the pusher engine, is always somewhat in excess of the true length of the grade as shown on the profile, and sometimes the excess length is very great. If a station is located on a lower grade within a mile or so of the top or bottom of a pusher grade, it will ordinarily be advisable to couple or uncouple at or near the station, since the telegraph-station, switching, and signaling may be

more economically operated at a regular station. If the extra engine is coupled on ahead of the through engine (as is sometimes required by law for passenger trains) the uncoupling at the top of the grade may be accomplished by running the assistant engine ahead at greater speed after it is uncoupled, and, after running it on a siding, clearing the track for the train. But this requires considerable extra track at the top of the grade. Therefore, when estimating the length of the pusher grade, the most desirable position for the terminal sidings must be studied and the length determined accordingly rather than by measuring the mere length of the grade on the profile. Of course these odd distances are always *excess;* the coupling or uncoupling should not be done while on the grade.

450. Cost of pusher-engine service. The cost evidently depends partly on the mileage run, while some items are wholly independent of the mileage. A pusher engine, when working on grades where the conditions are fairly favorable, will accomplish a mileage of 100 to 125 miles per day, and this is about equal to that of an ordinary freight engine. Therefore such items as wages which are independent of mileage will be assumed to cost as much per mile as they do for ordinary train service. If the mileage is less than this, an extra allowance should be made.

The effect of a pusher engine on maintenance of way may be considered to be the same as that produced by an additional engine, as developed in § 444. The same allowance (3.59%) will therefore be made. The cost of repairs and renewals of locomotives may be estimated the same as for other engines. Wages for engine and round-house men will be the same. There is certainly no ground for considering that the cost of fuel and other engine supplies can be materially less than the usual figures. On the return trip down the grade the engine runs almost without steam (after getting started), but, on the other hand, the engine works hard when climbing up the grade. The cost of switchmen, etc., and telegraph expenses (Items 28 and 29) will evidently add their full quota. Collecting these items, we have 36.27% or 34.46 c. for each mile run. Adding, as in § 445, 1.25 c. as interest charge on the cost of the engine, we have 35.71 c. Then each mile of the incline will cost twice this or 71.42 c. for a round trip, or $71.42 \times 365 = \$260$ per year per mile of incline per daily train needing assistance.

TABLE XXIX.—ITEMS OF THE COST PER MILE OF A PUSHER ENGINE.

No.	Items.	Normal average.	Per cent affected.	Cost per engine mile, per cent.
1	Repairs of roadway..........	10.596	12.5	1.32
2	Renewals of rails............	1.440	50	.72
3	Renewals of ties.............	3.093	50	1.55
12	Repairs of locomotives.......	5.879	100	5.88
21	Enginemen..................	9.781	100	9.78
22–25	Engine supplies.............	10.912	100	10.91
28	Switchmen, etc..............	4.136	100	4.14
29	Telegraph..................	1.974	100	1.97
				36.27

451. Numerical comparison of pusher and through grades. In § 445 the computation was made of the desirability of reducing a 1.6% ruling grade to a 1.2% grade. Suppose it is found that by keeping the 1.6% grades as pusher grades having a total length of 20 miles on a 100 mile division, the other grades may be reduced to a grade not exceeding 0.713% (the corresponding through grade) for an expenditure of \$200000. Will it pay? The saving by cutting down trains from 8 to 4, computed as before, would be (see § 445) $4\times\$0.5333\times100\times365=\$77862$. But this saving is only accomplished by the employment of pushers making four round trips over 20 miles of pusher grades at a cost of $4\times20\times\$260=\$20800$.

The net annual saving is therefore \$57062, which when capitalized at $5\%=\$1,141,240$.

The above estimate probably has this defect. The total daily pusher-engine mileage is but $2\times4\times20=160$, scarcely work enough for two pushers. Unless the pusher grades were bunched into two groups of about 10 miles each, two pusher engines could not do the work. If the number of trains was much larger, then the above method of calculation would be more exact even though the 20 miles of pusher grade was divided among four or five different grades. Therefore with the above data the annual cost of the pusher service would probably be much more—perhaps twice as much—and the annual saving about \$36000, which would justify an expenditure of \$720000. But even this would very amply justify the assumed expenditure of \$200000 which would accomplish this result.

The above computation is but an illustration of the general truth which has been previously stated. In spite of the uncertainties and the variations of many items in the above estimates it will generally be possible to make a computation which will show unquestionably, as in the above instance, what is the best and the most economical method of procedure. When the capitalized valuations of both methods are so nearly equal that a proper choice is more difficult, the question will frequently be determined by the relative ease of raising additional capital.

BALANCE OF GRADES FOR UNEQUAL TRAFFIC.

452. **Nature of the subject.** It sometimes happens, as when a road runs into a mountainous country for the purpose of hauling therefrom the natural products of lumber or minerals, that the heavy grades are all in one direction—that the whole line consists of a more or less unbroken climb having perhaps a few comparatively level stretches, but no down grade (except possibly a slight sag) in the direction of the general up grade. With such lines this present topic has no concern. But the majority of railroads have termini at nearly the same level (500 feet in 500 miles has no practical effect on grade) and consist of up and down grades in nearly equal amounts and rates. The general rate of ruling grade is determined by the character of the country and the character and financial backing of the road to be built. It is always possible to reduce the grade at some point by "development" or in general by the expenditure of more money. It has been tacitly assumed in the previous discussions that when the ruling grade has been determined all grades in either direction are cut down to that limit. If the traffic in both directions were the same this would be the proper policy and sometimes is so. But it has developed, especially on the great east and west trunk lines, that the *weight* of the eastbound freight traffic is enormously greater than that of the westbound—that westbound trains consist very largely of "empties" and that an engine which could haul twenty loaded cars up a given grade in eastbound traffic could haul the same cars empty up a much higher grade when running west. As an illustration of the large disproportion which may exist, the eastbound ton-mileage on the P. R. R. between the years 1851 and 1885 was 3.7 times the westbound ton-mileage. Between the years 1876 and 1880 the ratio rose to more than 4.5 to 1.

On such a basis it is as important and necessary to obtain, say, a 0.6% ruling grade against the eastbound traffic as to have, say, a 1.0% grade against the westbound traffic. This is the basis of the following discussion. It now remains to estimate the probable ratio of the traffic in the two directions and from that to determine the proper "balance" of the opposite ruling grades.

453. Computation of the theoretical balance. Assume first, for simplicity, that the exact business in either direction is accurately known. A little thought will show the truth of the following statements.

1. The locomotive and passenger-car traffic in both directions is equal.

2. Except as a road may carry emigrants, the passenger traffic in both directions is equal. Of course there are innumerable individual instances in which the return trip is made by another route, but it is seldom if ever that there is any marked tendency to uniformity in this. Considering that a car load of, say, 50 passengers at 150 pounds apiece weigh but 7500 pounds, which is $\frac{1}{6}$ of the 45000 pounds which the car may weigh, even a considerable variation in the number of passengers will not appreciably affect the hauling of cars on grades. On parlor-cars and sleepers the ratio of live load to dead load (say 20 passengers, 3000 pounds, and the car, 75000 pounds) is even more insignificant. The effect of passenger traffic on balance of grades may therefore be disregarded.

3. Empty cars have a greater resistance *per ton* than loaded cars. Therefore in computing the hauling capacity of a locomotive hauling so many tons of "empties," a larger figure must be used for the ordinary tractive resistances—say two pounds per ton greater.

4. Owing to greater or less imperfections of management a small percentage of cars will run empty or but partly full in the direction of greatest traffic.

5. Freight having great bulk and weight (such as grain, lumber, coal, etc.) is run from the rural districts toward the cities and manufacturing districts.

6. The return traffic—manufactured products—although worth as much or more, do not weigh as much.

As a simple numerical illustration assume that the weight of the cars is 40% and the live load 60% of the total load when the cars are "full"—although not loaded to their absolute limit of capacity. Assume that the relative weight of live load to be hauled in the other direction is but $\frac{1}{3}$. Then the gross train load (exclusive of the locomotive) is $40+(\frac{1}{3}\times 60)=60\%$ of the load in the other direction. Assume that the grade against the heaviest traffic is 0.9%. An 80-ton engine with 48 tons on the drivers, $\frac{1}{4}$ adhesion, normal tractive resistance 6 pounds per ton, will haul a train of 920 tons up that grade. Of this load the cars are assumed to weigh 40%, or 368 tons, and the live load 552 tons. On the return trip the weight of the cars with their load is but $920\times 60\%=552$ tons, or with the engine 632 tons. This could be hauled up a 1.60% grade, assuming that the resistance was the same per ton. But $\frac{2}{3}$ of the return cars must be figured as empty; they make an added resistance of 2 pounds per ton; these cars weigh $\frac{2}{3}$ of 368 tons, or 245.33 tons. The balance of the train weighs $632-245.33=386.67$ tons. Then we have

$$\begin{aligned} 245.33\times 8 &= 1962.67 \\ 386.67\times 6 &= 2320. \\ \hline &\ 4282.67 \text{ pounds,} \end{aligned}$$

which is the tractive force required for rolling resistances, etc. Subtracting this from the total adhesion, 24000 pounds, we have left 19717.33 pounds available for grade, or 31.2 pounds per ton, which corresponds to a 1.56% grade, which is the proper balance of grade *under the above conditions.*

454. Computation of relative traffic. Some of the principal elements have already been referred to, but in addition the following facts should be considered.

(a) The greatest disparity in traffic occurs through the handling of large amounts of coal, lumber, iron ore, grain, etc. On roads which handle but little of these articles or on which for local reasons coal is hauled one way and large shipments of grain the other way the disparity will be less and will perhaps be insignificant.

(b) A marked change in the development of the country may, and often does, cause a marked difference in the disparity of traffic. The heaviest traffic (in mere weight) is always toward manufacturing regions and away from agricultural regions. But when a region, from being purely agricultural or mineral, becomes largely manufacturing, or when a manufacturing region develops an industry which will cause a growth of heavy freight traffic from it, a marked change in the relative freight movement will be the result.

(c) Very great fluctuations in the relative traffic may be expected for prolonged intervals.

(d) An estimate of the relative traffic may be formed by the same general method used in computing the total traffic of the road (see § 373, Chap. XIX) or by noting the relative traffic on existing roads which may be assumed to have practically the same traffic as the proposed road will obtain.

CHAPTER XXIV.

THE IMPROVEMENT OF OLD LINES.

455. **Classification of improvements.** The improvements here considered are only those of alignment—horizontal and vertical. Strictly there is no definite limit, either in kind or magnitude, to the improvements which may be made. But since a railroad cannot ordinarily obtain money, even for improvements, to an amount greater than some small proportion of the previously invested capital, it becomes doubly necessary to expend such money to the greatest possible advantage. It has been previously shown that securing additional business and increasing the train load are the two most important factors in decreasing dividends. After these, and of far less importance, come reductions of curvature, reductions of distance (frequently of doubtful policy, see Chap. XXI, § 414), and elimination of sags and humps. These various improvements will be briefly discussed.

(a) **Securing additional business.** It is not often possible by any small modification of alignment to materially increase the business of a road. The cases which do occur are usually those in which a gross error of judgment was committed during the original construction. For instance, in the early history of railroad construction many roads were largely aided by the towns through which the road passed, part of the money necessary for construction being raised by the sale of bonds, which were assumed or guaranteed and subsequently paid by the towns. Such aid was often demanded and exacted by the promoters. Instances are not unknown where a failure to come to an agreement has caused the promoters to deliberately pass by the town at a distance of some miles, to the mutual disadvantage of the road and the town. If the town subsequently grew in spite of this disadvantage, the *annual* loss of business might readily amount to more than the original sum in dispute.

Such an instance would be a legitimate opportunity for study of the advisability of a re-location.

As another instance (the original location being justifiable) a railroad might have been located along the bank of a considerable river too wide to be crossed except at considerable expense. When originally constructed the enterprise would not justify the two extra bridges needed to reach the town. A growth in prosperity and in the business obtainable might subsequently make such extra expense a profitable investment.

(b) **Increasing the train load.** On account of its importance this will be separately considered in § 458 *et seq.*

(c) **Reductions in curvature and distance and the elimination of sags and humps.** The financial value of these improvements has already been discussed in Chapters XXI, XXII, and XXIII. Such improvements are constantly being made by all progressive roads. The need for such changes occurs in some cases because the original location was very faulty, the revised location being no more expensive than the original, and in other cases because the original location was the best that was then financially possible and because the present expanded business will justify a change.

(d) **Changing the location of stations or of passing sidings.** The station may sometimes be re-located so as to bring it nearer to the business center and thus increase the business done. But the principal reasons for re-locating stations or passing sidings is that starting trains may have an easier grade on which to overcome the additional resistances of starting. Such changes will be discussed in detail in § 460.

456. Advantages of re-locations. There are certain undoubted advantages possessed by the engineer who is endeavoring to improve an old line.

(*a*) The gross traffic to be handled is definitely known.

(*b*) The actual cost per train-mile for that road (which *may* differ very greatly from the average) is also known, and therefore the value of the proposed improvement can be more accurately determined.

(*c*) The actual performance of such locomotives as are used on the road may be studied at leisure and more reliable data may be obtained for the computations.

457. Disadvantages of re-locations. The disadvantages are generally more apparent and frequently appear practically insuperable—more so than they prove to be on closer inspection. (*a*) It frequently means the abandonment of a greater or less length of old line and the construction of new line. At first thought it might seem as if a change of line such as would permit an increase of train-load of 50 or perhaps 100% could never be obtained, or at least that it could not be done except at an impracticable expense. On the contrary a change of 10% of the old line is frequently all that is necessary to reduce the grades so that the train-loads hauled by one engine may be nearly if not quite doubled. And when it is considered that the cost of a road to sub-grade is generally not more than one-third of the total cost of construction and equipment per mile, it becomes plain that an expenditure of but a small percentage of the original outlay, expended where it will do the most good, will often suffice to increase enormously the earning capacity.

(*b*) One of the most difficult matters is to convince the financial backers of the road that the proposed improvement will be justifiable. The cause is simple. The disadvantages of the original construction lie in the large increase of certain items of expense which are necessary to handle a given traffic. And yet the fact that the expenditures are larger than they need be are only apparent to the expert, and the fact that a saving may be made is considered to be largely a matter of opinion until it is demonstrated by actual trial. On the other hand the cost of the proposed changes is definite, and the very fact that the road has been uneconomically worked and is in a poor financial condition makes it difficult to obtain money for improvements.

(*c*) The legal right to abandon a section of operated line and thus reduce the value of some adjoining property has sometimes been successfully attacked. A common instance would be that of a factory which was located adjoining the right of way for convenience of transportation facilities. The abandonment of that section of the right of way would probably be fatal to the successful operation of the factory. The objection may be largely eliminated by the maintenance of the old right of way as a long siding (although the business of the factory might not be worth it), but it is not always so easy of solution, and this phase of the question must always be considered.

REDUCTION OF VIRTUAL GRADE.

458. Obtaining data for computations. As developed in the last chapter (§§ 432–434) the real object to be attained is the reduction of the *virtual grade*. The method of comparing grades under various assumed conditions was there discussed. When the road is still "on paper" some such method is all that is possible; but when the road is in actual operation the virtual grade of the road at various critical points, with the rolling stock actually in use, may be determined by a simple test and the effect of a proposed change may be reliably computed. Bearing in mind the general principle that the virtual grade line is the locus of points determined by adding to the actual grade profile ordinates equal to the velocity head of the train, it only becomes necessary to measure the velocity at various points. Since the velocity is *not* usually uniform, its precise determination at any instant is almost impossible, but it will generally be found to be sufficiently precise to assume the velocity to be uniform for a short distance, and then observe the time required to pass that short space. Suppose that an ordinary watch is used and the time taken to the nearest second. At 30 miles per hour, the velocity is 44 feet per second. To obtain the time to within 1%, the time would need to be 100 seconds and the space 4400 feet. But with variable velocity there would be too great error in assuming the velocity as uniform for 4400 feet or for the time of 100 seconds. Using a stop-watch registering fifths of a second, a 1% accuracy would require but 20 seconds and a space of 880 feet, at 30 miles per hour. Wellington suggests that the space be made 293 feet 4 inches, or $\frac{1}{18}$ of a mile; then the speed in miles per hour equals $200 \div s$, in which s is the time in seconds required to traverse the 293′ 4″. For instance, suppose the time required to pass the interval is 12.5 seconds. $\frac{1}{18}$ mile in 12.5 seconds = one mile in 225 seconds, or 16 miles per hour. But likewise $200 \div 12.5 = 16$, the required velocity. The following features should be noted when obtaining data for the computations:

(*a*) All critical grades on the road should be located and their profiles obtained—by a survey if necessary.

(*b*) At the bottom and top of all long grades (and perhaps at intermediate points if the grades are very long) spaces of known length (preferably $293\frac{1}{3}$ feet) should be measured off and marked by flags, painted boards, or any other serviceable targets.

(*c*) Provided with a stop-watch marking fifths of seconds the observer should ride on the trains affected by these grades and note the exact interval of time required to pass these spaces. If the space is $293\frac{1}{3}$ feet, the velocity in miles per hour $=200\div$ interval in seconds. In general,

$$V = \frac{\text{distance in feet} \times 3600}{\text{time in seconds} \times 5280}. \quad . \quad . \quad . \quad . \quad (147)$$

(*d*) Since these critical grades are those which require the greatest tax on the power of the locomotive, the conditions under which the locomotive is working must be known—i.e., the steam pressure, point of cut-off, and position of the throttle. Economy of coal consumption as well as efficient working at high speeds requires that steam be used expansively (using an early cut-off), and even that the throttle be partly closed; but when an engine is slowly climbing up a maximum grade with a full load it is not exerting its maximum tractive power unless it has its maximum steam pressure, wide-open throttle, and is cutting off nearly at full stroke. These data must therefore be obtained so as to know whether the engine is developing at a critical place all the tractive force of which it is capable. The condition of the track (wet and slippery or dry) and the approximate direction and force of the wind should be noted with sufficient accuracy to judge whether the test has been made under ordinary conditions rather than under conditions which are exceptionally favorable or unfavorable.

(*e*) The train-loading should be obtained as closely as possible. Of course the dead weight of the cars is easily found, and the records of the freight department will usually give the live load with all sufficient accuracy.

459. Use of the data obtained. A very brief inspection of the results, freed from refined calculations or uncertainties, will demonstrate the following truths:

(*a*) If, on a uniform grade, the velocity increases, it shows that, under those conditions of engine working, the load is less than the engine can handle on that grade

(*b*) If the velocity decreases, it shows that the load is greater than the engine can handle on an indefinite length of such grade. It shows that such a grade is being operated by momentum. From the rate of decrease of velocity the maximum practicable length of such a grade (starting with a given velocity) may be easily computed.

(*c*) By combining results under different conditions of grade but with practically the same engine working, the tractive power of the engine may be determined (according to the principles previously demonstrated) for any grade and velocity. For example: On an examination of the profile of a division of a road the maximum grade was found to be 1.62% (85.54 feet per mile). At the bottom and near the top of this grade two lengths of 293′ 4″ are laid off. The distance between the centers of these lengths is 6000 feet. A freight train moving up the grade is timed at $9\frac{2}{5}$ seconds on the lower stretch and $7\frac{3}{5}$ seconds on the upper. These times correspond to $\frac{200}{9.4}$ and $\frac{200}{7.6}$ or 21.3 and 26.3 miles per hour respectively. It is at once observed that the velocity has increased and that the engine could draw even a heavier load up such a grade for an indefinite distance. How much heavier might the load be?

For simplicity we will assume that the conditions were normal, neither exceptionally favorable nor unfavorable, and that the engine was worked to its maximum capacity. The engine is a "consolidation" weighing 128700 pounds, with 112600 pounds on the drivers. The train-load behind the engine consists of ten loaded cars weighing 465 tons and eleven empties weighing 183 tons, thus making a total train-weight of 712 tons. Applying Eq. 140, we find that the *additional* force which the engine has actually exerted per ton in increasing the velocity from 21.3 to 26.3 miles per hour in a distance of 6000 feet is

$$P=\frac{70.224}{6000}(26.3^2-21.3^2)=2.78 \text{ pounds } \textit{per ton}$$

The grade resistance on a 1.62% grade is 32.4 pounds per ton. With an average train resistance of seven pounds per ton the total necessary pull for uniform velocity would be 39.4 pounds. But the engine is actually exerting an additional pull of 2.78 pounds per ton. Evidently its total load might be increased proportionately, i.e., the total train-load might equal

$$712\times\frac{2.78+39.4}{39.4}=762 \text{ tons.}$$

This shows that 50 tons additional might have been loaded on, say, three more empties or one loaded car and one empty. An overload of a few tons would easily be made up by a very slight reduction in the velocity.

The above calculation should of course be considered simply as a "single observation." The performance of the same engine on the same grade (as well as on many other grades) on succeeding days should also be noted. It may readily happen that variations in the condition of the track or of the handling of the engine may make considerable variation in the results of the several calculations, but when the work is properly done it is always possible to draw definite and very positive deductions.

460. **Reducing the starting grade at stations.** The resistance to starting a train is augmented from two causes: (*a*) the tractive resistances are usually about 20 pounds per ton instead of, say, 6 pounds, and (*b*) the inertia resistance must be overcome. The inertia resistance of a freight train (see § 347) which is expected to attain a velocity of 15 miles per hour in a distance of 1000 feet is (see Eq. 140)

$P=\frac{70.224}{1000}(15^2-0)=15.8$ pounds per ton, which is the equivalent of a 0.79% grade. Adding this to a grade which nearly or quite equals the ruling grade, it virtually creates a new and higher ruling grade. Of course that additional force can be greatly reduced at the expense of slower acceleration, but even this cannot be done indefinitely, and an acceleration to only 15 miles per hour in 1000 feet is as slow as should be allowed for. With perhaps 14 pounds per ton additional tractive resistance, we have about 30 pounds per ton additional—equiva-

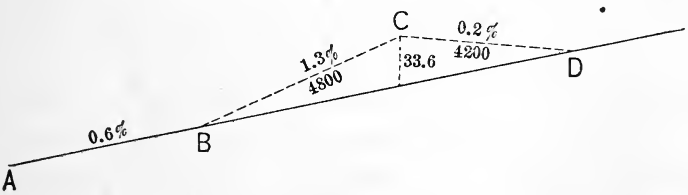


Fig. 217.

lent to a 1.5% grade. Instances are known where it has proven wise to create a *hump* (in what was otherwise a uniform grade) at a station. The effect of this on high-speed passenger trains moving *up* the grade would be merely to reduce their speed very slightly. No harm is done to trains moving *down* the grade. Freight trains moving *up* the grade and intending to stop at the station will merely have their velocity reduced as they approach the station and will actually save part of the wear and tear otherwise resulting from applying brakes. When the trains start they are assisted by the short down grade, just where they need assistance most. Even if the grade CD is still an up grade, the pull required at starting is *less* than that required on the uniform grade by an amount equal to 20 times the difference of the grade in per cent.

APPENDIX.

THE ADJUSTMENTS OF INSTRUMENTS.

THE accuracy of instrumental work may be vitiated by any one of a large number of inaccuracies in the geometrical relations of the parts of the instruments. Some of these relations are so apt to be altered by ordinary usage of the instrument that the makers have provided adjusting-screws so that the inaccuracies may be readily corrected. There are other possible defects, which, however, will seldom be found to exist, provided the instrument was properly made and has never been subjected to treatment sufficiently rough to distort it. Such defects, when found, can only be corrected by a competent instrument-maker or repairer.

A WARNING is necessary to those who would test the accuracy of instruments, and especially to those whose experience in such work is small. Lack of skill in handling an instrument will often indicate an apparent error of adjustment when the real error is very different or perhaps non-existent. It is always a safe plan when testing an adjustment to note the amount of the apparent error; then, beginning anew, make another independent determination of the amount of the error. When two or more *perfectly independent* determinations of such an error are made it will generally be found that they differ by an appreciable amount. The *differences* may be due in variable measure to careless inaccurate manipulation and to instrumental *defects* which are wholly independent of the particular test being made. Such careful determinations of the amounts of the errors are generally advisable in view of the next paragraph.

DO NOT DISTURB THE ADJUSTING-SCREWS ANY MORE THAN NECESSARY. Although metals are apparently rigid, they are really elastic and yielding. If some parts of a complicated mechanism, which is held together largely by friction, are subjected to greater internal stresses than other parts of the mechanism, the jarring resulting from handling will frequently cause a slight readjustment in the parts which will tend to more nearly equalize the internal stresses. Such action frequently occurs with the adjusting mechanism of instruments. One screw may be strained more than others. The friction of parts may prevent the opposing screw from *immediately* taking up an equal stress. Perhaps the adjustment appears perfect under these conditions Jarring diminishes the friction between the parts, and the unequal stresses tend to equalize. A motion takes place which, although microscopically minute, is sufficient to indicate an error of adjustment. A readjustment made by unskillful hands may not make the final adjustment any more perfect. The frequent shifting of adjusting-screws wears them badly, and when the screws are worn it is still more difficult to keep them from moving enough to vitiate the adjustments. It is therefore preferable in many cases to refrain from disturbing the adjusting-screws, especially as the accuracy of the work done is not *necessarily* affected by errors of adjustment, as may be illustrated:

(*a*) Certain operations are *absolutely* unaffected by certain errors of adjustment.

(*b*) Certain operations are so slightly affected by certain *small* errors of adjustment that their effect may properly be neglected.

(*c*) Certain errors of adjustment may be readily allowed for and neutralized so that no error results from the use of the unadjusted instrument. Illustrations of all these cases will be given under their proper heads.

ADJUSTMENTS OF THE TRANSIT.

1. *To have the plate-bubbles in the center of the tubes when the axis is vertical.* Clamp the upper plate and, with the lower clamp loose, swing the instrument so that the plate-bubbles are parallel to the lines of opposite leveling-screws. Level up until both bubbles are central. Swing the instrument 180°. If the bubbles again settle at the center, the adjustment is perfect. If either bubble does not settle in the center, move the leveling-screws until the bubble is *half-way* back to the center. Then, before touching the adjusting-screws, note carefully the position of the bubbles and observe whether the bubbles always settle at the *same* place in the tube, no matter to what position the in-

strument may be rotated. When the instrument is so leveled, the axis is truly vertical and the discrepancies between this constant position of the bubbles and the centers of the tubes measure the errors of adjustment. By means of the adjusting-screws bring each bubble to the center of the tube. If this is done so skillfully that the true level of the instrument is not disturbed, the bubbles should settle in the center for all positions of the instrument. Under unskillful hands, two or more such trials may be necessary.

When the plates are not horizontal, the measured angle is greater than the true horizontal angle by the difference between the measured angle and its projection on a horizontal plane. When this angle of inclination is small, the difference is insignificant. Therefore when the plate-bubbles are *very nearly* in adjustment, the error of measurement of horizontal angles may be far within the lowest unit of measurement used. A *small* error of adjustment of the plate-bubble *perpendicular* to the telescope will affect the horizontal angles by only a small proportion of the error, which will be perhaps imperceptible. Vertical angles will be affected by the same insignificant amount. A *small* error of adjustment of the plate-bubble *parallel* to the telescope will affect horizontal angles very slightly, but will affect vertical angles by the full amount of the error.

All error due to unadjusted plate-bubbles may be avoided by noting in what positions in the tubes the bubbles will remain fixed for all positions of azimuth and then keeping the bubbles adjusted to these positions, for the axis is then truly vertical. It will often save time to work in this way temporarily rather than to stop to make the adjustments. This should especially be done when accurate vertical angles are required.

When the bubbles are truly adjusted, they should remain stationary regardless of whether the telescope is revolved with the upper plate loose and the lower plate clamped or whether the whole instrument is revolved, the plates being clamped together. If there is any appreciable difference, it shows that the two vertical axes or "centers" of the plates are not concentric. This may be due to cheap and faulty construction or to the excessive wear that may be sometimes observed in an old instrument originally well made. In either case it can only be corrected by a maker.

2. *To make the revolving axis of the telescope perpendicular to the vertical axis of the instrument.* This is best tested by using a long plumb-line, so placed that the telescope must be pointed upward at an angle of about 45° to sight at the top of the plumb-line and downward about the same amount, if possible, to sight at the lower end. The vertical axis of the transit must be made truly vertical. Sight at the upper part of the line, clamping the horizontal plates. Swing the telescope down and see if the cross-wire again bisects the cord. If so, the adjustment is *probably* perfect (a conceivable exception will be

noted later); if not, raise or lower one end of the axis by means of the adjusting-screws, placed at the top of one of the standards, until the cross-wire will bisect the cord both at top and bottom. The plumb-bob may be steadied, if necessary, by hanging it in a pail of water. As many telescopes cannot be focused on an object nearer than 6 or 8 feet from the telescope, this method requires a long plumb-line swung from a high point, which may be inconvenient.

Another method is to set up the instrument about 10 feet from a high wall. After leveling, sight at some convenient mark high up on the wall. Swing the telescope down and make a mark (when working alone some convenient natural mark may generally be found) low down on the wall. Plunge the telescope and revolve the instrument about its vertical axis and again sight at the upper mark. Swing down to the lower mark. If the wire again bisects it, the adjustment is perfect. If not, fix a point *half-way* between the two positions of the lower mark. The plane of this point, the upper point, and the center of the instrument is truly vertical. Adjust the axis to these upper and lower points as when using the plumb-line.

3. *To make the line of collimation perpendicular to the revolving axis of the telescope.* With the instrument level and the telescope nearly horizontal point at some well-defined point at a distance of 200 feet or more. Plunge the telescope and establish a point in the opposite direction. Turn the whole instrument about the vertical axis until it again points at the first mark. Again plunge to "direct position" (*i.e.*, with the level-tube *under* the telescope). If the vertical cross-wire again points at the second mark, the adjustment is perfect. If not, the error is *one-fourth* of the distance between the two positions of the second mark. Loosen the capstan screw on one side of the telescope and tighten it on the other side until the vertical wire is set at the one-fourth mark. Turn the whole instrument by means of the tangent screw until the vertical wire is *midway* between the two positions of the second mark. Plunge the telescope. If the adjusting has been skillfully done, the cross-wire should come exactly to the first mark. As an "erecting eyepiece" reinverts an image already inverted, the ring carrying the cross-wires must be moved in the *same* direction as the *apparent* error in order to correct that error.

The necessity for the third adjustment lies principally in the practice of producing a line by plunging the telescope, but when this is required to be done with great accuracy it is always better to obtain the forward point by reversion (as described above for making the test) and take the *mean* of the two forward points. Horizontal and vertical angles are practically unaffected by *small* errors of this adjustment, unless, in the case of horizontal angles, the vertical angles to the points observed are very different.

Unnecessary motion of the adjusting-screws may sometimes be avoided by carefully establishing the forward point on line by repeated reversions of the instrument, and thus determining by repeated trials the exact amount of the error. *Differences* in the amount of error determined would be evidence of inaccuracy in manipulating the instrument, and would show that an adjustment based on the first trial would *probably* prove unsatisfactory.

The 2d and 3d adjustments are mutually dependent. If either adjustment is badly out, the other adjustment cannot be made except as follows:

(*a*) The second adjustment can be made regardless of the third when the lines to the high point and the low point make *equal* angles with the horizontal.

(*b*) The third adjustment can be made regardless of the second when the front and rear points are *on a level* with the instrument.

When both of these requirements are *nearly* fulfilled, and especially when the error of either adjustment is small, no trouble will be found in perfecting either adjustment on account of a small error in the other adjustment.

If the test for the second adjustment is made by means of the plumb-line and the vertical cross-wire intersects the line at all points as the telescope is raised or lowered, it not only demonstrates at once the accuracy of that adjustment, but also shows that the third adjustment is either perfect or has so small an error that it does not affect the second.

4. *To have the bubble of the telescope-level in the center of the tube when the line of collimation is horizontal.* The line of collimation should coincide with the optical axis of the telescope. If the object-glass and eyepiece have been properly centered, the previous adjustment will have brought the vertical cross-wire to the center of the field of view. The horizontal cross-wire should also be brought to the center of the field of view, and the bubble should be adjusted to it.

a. Peg method. Set up the transit at one end of a nearly level stretch of about 300 feet. Clamp the telescope with its bubble in the center. Drive a stake vertically under the eyepiece of the transit, and another about 300 feet away. Observe the height of the center of the eyepiece (the telescope being level) above the stake (calling it a); observe the reading of the rod when held on the other stake (calling it b); take the instrument to the other stake and set it up so that the eyepiece is vertically over the stake, observing the height, c; take a reading on the first stake, calling it d. If this adjustment is perfect, then

$$a-d=b-c,$$

$$\text{or} \qquad (a-d)-(b-c)=0.$$

$$\text{Call} \qquad (a-d)-(b-c)=2m.$$

When m is positive, the line points downward;
" m " negative, " " " upward.

To adjust: if the line points *up*, sight the horizontal cross-wire (by moving the vertical tangent screw) at a point which is m lower, then adjust the bubble so that it is in the center.

By taking several independent values for a, b, c, and d, a mean value for m is obtained, which is more reliable and which may save much unnecessary working of the adjusting-screws.

b. Using an auxiliary level. When a carefully adjusted level is at hand, this adjustment may sometimes be more easily made by setting up the transit and level, so that their lines of collimation are as nearly as possible at the same height. If a point may be found which is half a mile or more away and which is on the horizontal cross-wire of the level, the horizontal cross-wire of the transit may be pointed directly at it, and the bubble adjusted accordingly. Any slight difference in the heights of the lines of collimation of the transit and level (say $\frac{1}{4}''$) may almost be disregarded at a distance of $\frac{1}{2}$ mile or more, or, if the difference of level would have an appreciable effect, even this may be practically eliminated by making an estimated allowance when sighting at the distant point. Or, if a distant point is not available, a level-rod with target may be used at a distance of (say) 300 feet, making allowance for the carefully determined difference of elevation of the two lines of collimation.

5. *Zero of vertical circle.* When the line of collimation is truly horizontal and the vertical axis is truly vertical, the reading of the vertical circle should be 0°. If the arc is adjustable, it should be brought to 0°. If it is not adjustable, the *index error* should be observed, so that it may be applied to all readings of vertical angles.

ADJUSTMENTS OF THE WYE LEVEL.

1. *To make the line of collimation coincide with the center of the rings.* Point the intersection of the cross-wires at some

well-defined point which is at a considerable distance. The instrument need not be level, which allows much greater liberty in choosing a convenient point. The vertical axis should be clamped, and the clips over the wyes should be loosened and raised. Rotate the telescope in the wyes. The intersection of the cross-wires should be continually on the point. If it is not, it requires adjustment. Rotate the telescope 180° and adjust *one-half* of the error by means of the capstan-headed screws that move the cross-wire ring. It should be remembered that, with an erecting telescope, on account of the inversion of the image, the ring should be moved in the direction of the *apparent* error. Adjust the other half of the error with the leveling-screws. Then rotate the telescope 90° from its usual position, sight accurately at the point, and then rotate 180° from that position and adjust any error as before. It may require several trials, but it is necessary to adjust the ring until the intersection of the cross-wires will remain on the point for any position of rotation.

If such a test is made on a very distant point and again on a point only 10 or 15 feet from the instrument, the adjustment may be found correct for one point and incorrect for the other. This indicates that the object-slide is improperly centered. Usually this defect can only be corrected by an instrument-maker. If the difference is very small it may be ignored, but the adjustment should then be made on a point which is at about the mean distance for usual practice—say 150 feet.

If the whole image appears to shift as the telescope is rotated, it indicates that the eyepiece is improperly adjusted. This defect is likewise usually corrected only by the maker. It does not interfere with instrumental accuracy, but it usually causes the intersection of the cross-wires to be eccentric with the field of view.

2. *To make the axis of the level-tube parallel to the line of collimation.* Raise the clips as far as possible. Swing the level so that it is parallel to a pair of opposite leveling-screws and clamp it. Bring the bubble to the middle of the tube by means of the leveling-screws. Take the telescope out of the wyes and replace it end for end, using *extreme care* that the wyes are not jarred by the action. If the bubble does not come to the center, correct *one-half* of the error by the vertical adjusting-screws at one end of the bubble. Correct the other half by the leveling-screws. Test the work by again changing the telescope end for end in the wyes.

Care should be taken while making this adjustment to see that the level-tube is vertically under the telescope. With the bubble in the center of the tube, rotate the telescope in the wyes for a considerable angle each side of the vertical. If the first half of the adjustment has been made and the bubble moves, it shows that the axis of the wyes and the axis of the level-tube are not in the same vertical plane although both have been made horizontal. By moving one end of the level-tube *sidewise* by means of the horizontal screws at one end of the tube, the two axes may be brought into the same plane. As this adjustment is liable to disturb the other, both should be alternately tested until both requirements are complied with.

By these methods the axis of the bubble is made parallel to the axis of the wyes; and as this has been made parallel to the lines of collimation by means of the previous adjustment, the axis of the bubble is therefore parallel to the line of collimation.

3. *To make the line of collimation perpendicular to the vertical axis.* Level up so that the instrument is approximately level over both sets of leveling-screws. Then, after leveling carefully over one pair of screws, revolve the telescope 180°. If it is not level, adjust half of the error by means of the capstan-headed screw under one of the wyes, and the other half by the leveling-screws. Reverse again as a test.

When the first two adjustments have been accurately made, good leveling may always be done by bringing the bubble to the center by means of the leveling-screws, at every sight if necessary, even if the third adjustment is not made. Of course this third adjustment should be made as a matter of convenience, so that the line of collimation may be always level no matter in what direction it may be pointed, but it is not *necessary* to stop work to make this adjustment every time it is found to be defective.

ADJUSTMENTS OF THE DUMPY LEVEL.

1. *To make the axis of the level-tube perpendicular to the vertical axis.* Level up so that the instrument is approximately level over both sets of leveling-screws. Then, after leveling carefully over one pair of screws, revolve the telescope 180°. If it is not level, adjust *one-half* of the error by means of the adjusting-screws at one end of the bubble, and the other half by means of the leveling-screws. Reverse again as a test.

2. *To make the line of collimation perpendicular to the vertical axis.* The method of adjustment is identical with that for the transit (No. 4, p. 505) except that the cross-wire must be

adjusted to agree with the level-bubble rather than *vice versa*, as is the case with the corresponding adjustment of the transit; i.e., with the level-bubble in the center, raise or lower the horizontal cross-wire until it points at the mark known to be on a level with the center of the instrument.

If the instrument has been well made and has not been distorted by rough usage, the cross-wires will intersect at the center of the field of view when adjusted as described. If they do not, it indicates an error which ordinarily can only be corrected by an instrument-maker. The error may be due to any one of several causes, which are

(*a*) faulty centering of object-slide;

(*b*) faulty centering of eyepiece;

(*c*) distortion of instrument so that the geometric axis of the telescope is not perpendicular to the vertical axis. If the error is only just perceptible, it will not probably cause any error in the work.

EXPLANATORY NOTE ON THE USE OF THE TABLES.

The logarithms here given are "five-place," but the last figure sometimes has a special mark over it (*e.g.*, $\bar{6}$) which indicates that one-half a unit in the last place should be *added.* For example

the value	includes all values between
·69586	·6958575000 + and ·6958624999...
$·6958\bar{6}$	·6958625000 + and ·6958674999...

The maximum error in any one value therefore does not exceed one-quarter of a fifth-place unit.

When adding or subtracting such logarithms allow a half-unit for such a sign. For example

·69586	·69586	$·6958\bar{6}$
·10841	$·1084\bar{1}$	$·1084\bar{1}$
$·1294\bar{7}$	$·1294\bar{7}$	$·1294\bar{7}$
$·9337\bar{4}$	·93375	$·9337\bar{5}$

All other logarithmic operations are performed as usual and are supposed to be understood by the student.

TABLE I.—RADII OF CURVES.

Deg	0°		1°		2°		3°		Deg
Min	Radius.	Log *R*	Radius.	Log *R*	Radius.	Log *R*	Radius.	Log *R*	Min
0	∞	∞	5729.6	3.75813	2864.9	3.45711	1910.1	3.28105	0
1	343775	5.53627	5635.7	.75095	2841.3	.45351	1899.5	.27864	1
2	171887	5.23524	5544.8	.74389	2818.0	.44993	1889.1	.27625	2
3	114592	5.05915	5456.8	.73694	2795.1	.44639	1878.8	.27387	3
4	85944	4.93421	5371.6	.73010	2772.5	.44287	1868.6	.27151	4
5	68755	4.83730	5288.9	.72336	2750.4	.43939	1858.5	.26915	5
6	57296	4.75812	5208.8	3.71673	2728.5	3.43593	1848.5	3.26681	6
7	49111	.69117	5131.0	.71020	2707.0	.43249	1838.6	.26448	7
8	42972	.63318	5055.6	.70377	2685.9	.42909	1828.8	.26217	8
9	38197	.58203	4982.3	.69743	2665.1	.42571	1819.1	.25986	9
10	34377	.53627	4911.2	.69118	2644.6	.42235	1809.6	.25757	10
11	31252	4.49488	4842.0	3.68502	2624.4	3.41903	1800.1	3.25529	11
12	28648	.45709	4774.7	.67895	2604.5	.41572	1790.7	.25303	12
13	26444	.42233	4709.3	.67296	2584.9	.41245	1781.5	.25077	13
14	24555	.39014	4645.7	.66705	2565.6	.40919	1772.3	.24853	14
15	22918	.36018	4583.8	.66122	2546.6	.40597	1763.2	.24629	15
16	21486	4.33215	4523.4	3.65547	2527.9	3.40276	1754.2	3.24407	16
17	20222	.30582	4464.7	.64979	2509.5	.39958	1745.3	.24186	17
18	19099	.28100	4407.5	.64419	2491.3	.39642	1736.5	.23967	18
19	18093	.25752	4351.7	.63865	2473.4	.39329	1727.8	.23748	19
20	17189	.23524	4297.3	.63319	2455.7	.39017	1719.1	.23530	20
21	16370	4.21405	4244.2	3.62780	2438.3	3.38708	1710.6	3.23314	21
22	15626	.19385	4192.5	.62247	2421.1	.38401	1702.1	.23098	22
23	14947	.17454	4142.0	.61720	2404.2	.38097	1693.7	.22884	23
24	14324	.15606	4092.7	.61200	2387.5	.37794	1685.4	.22670	24
25	13751	.13833	4044.5	.60686	2371.0	.37494	1677.2	.22458	25
26	13222	4.12130	3997.5	3.60178	2354.8	3.37195	1669.1	3.22247	26
27	12732	.10491	3951.5	.59676	2338.8	.36899	1661.0	.22037	27
28	12278	.08911	3906.6	.59180	2323.0	.36604	1653.0	.21827	28
29	11854	.07387	3862.7	.58689	2307.4	.36312	1645.1	.21619	29
30	11459	.05915	3819.8	.58204	2292.0	.36021	1637.3	.21412	30
31	11090	4.04491	3777.9	3.57724	2276.8	3.35733	1629.5	3.21206	31
32	10743	.03112	3736.8	.57250	2261.9	.35446	1621.8	.21000	32
33	10417	.01776	3696.6	.56780	2247.1	.35162	1614.2	.20796	33
34	10111	4.00479	3657.3	.56316	2232.5	.34879	1606.7	.20593	34
35	9822.2	3.99221	3618.8	.55856	2218.1	.34598	1599.2	.20390	35
36	9549.3	3.97997	3581.1	3.55401	2203.9	3.34318	1591.8	3.20189	36
37	9291.3	.96807	3544.2	.54951	2189.8	.34041	1584.5	.19988	37
38	9046.7	.95649	3508.0	.54506	2176.0	.33765	1577.2	.19789	38
39	8814.8	.94521	3472.6	.54065	2162.3	.33491	1570.0	.19590	39
40	8594.4	.93421	3437.9	.53629	2148.8	.33219	1562.9	.19392	40
41	8384.8	3.92349	3403.8	3.53197	2135.4	3.32949	1555.8	3.19195	41
42	8185.2	.91302	3370.5	.52769	2122.3	.32680	1548.8	.18999	42
43	7994.8	.90281	3337.7	.52345	2109.2	.32412	1541.9	.18804	43
44	7813.1	.89282	3305.7	.51925	2096.4	.32147	1535.0	.18610	44
45	7639.5	.88306	3274.2	.51510	2083.7	.31883	1528.2	.18417	45
46	7473.4	3.87352	3243.3	3.51098	2071.1	3.31621	1521.4	3.18224	46
47	7314.4	.86418	3213.0	.50691	2058.7	.31360	1514.7	.18032	47
48	7162.0	.85503	3183.2	.50287	2046.5	.31101	1508.1	.17842	48
49	7015.9	.84608	3154.0	.49886	2034.4	.30843	1501.5	.17652	49
50	6875.6	.83731	3125.4	.49490	2022.4	.30587	1495.0	.17462	50
51	6740.7	3.82871	3097.2	3.49097	2010.6	3.30332	1488.5	3.17274	51
52	6611.1	.82027	3069.6	.48707	1998.9	.30079	1482.1	.17087	52
53	6486.4	.81200	3042.4	.48321	1987.3	.29827	1475.7	.16900	53
54	6366.3	.80388	3015.7	.47939	1975.9	.29577	1469.4	.16714	54
55	6250.5	.79591	2989.5	.47559	1964.6	.29328	1463.2	.16529	55
56	6138.9	3.78809	2963.7	3.47183	1953.5	3.29081	1457.0	3.16344	56
57	6031.2	.78040	2938.4	.46811	1942.4	.28835	1450.8	.16161	57
58	5927.2	.77285	2913.5	.46441	1931.5	.28590	1444.7	.15978	58
59	5826.8	.76542	2889.0	.46075	1920.7	.28347	1438.7	.15796	59
60	5729.6	.75813	2864.9	.45711	1910.1	.28105	1432.7	.15615	60

Deg	4°		5°		6°		7°		Deg
Min	Radius.	Log R	Radius.	Log R	Radius.	Log R	Radius.	Log R	Min
0	1432.7	3.15615	1146.3	3.05929	955.37	2.98017	819.02	2.91329	0
1	1426.7	.15434	1142.5	.05784	952.72	.97896	817.08	.91226	1
2	1420.8	.15255	1138.7	.05640	950.09	.97776	815.14	.91123	2
3	1415.0	.15076	1134.9	.05497	947.48	.97657	813.22	.91021	3
4	1409.2	.14897	1131.2	.05354	944.88	.97537	811.30	.90918	4
5	1403.5	.14720	1127.5	.05211	942.29	.97418	809.40	.90816	5
6	1397.8	3.14543	1123.8	3.05069	939.72	2.97300	807.50	2.90714	6
7	1392.1	.14367	1120.2	.04928	937.16	.97181	805.61	.90612	7
8	1386.5	.14191	1116.5	.04787	934.62	.97063	803.73	.90511	8
9	1380.9	.14017	1112.9	.04646	932.09	.96945	801.86	.90410	9
10	1375.4	.13843	1109.3	.04506	929.57	.96828	800.00	.90309	10
11	1369.9	3.13669	1105.8	3.04366	927.07	2.96711	798.14	2.90208	11
12	1364.5	.13497	1102.2	.04227	924.58	.96594	796.30	.90107	12
13	1359.1	.13325	1098.7	.04088	922.10	.96478	794.46	.90007	13
14	1353.8	.13154	1095.2	.03949	919.64	.96361	792.63	.89907	14
15	1348.4	.12983	1091.7	.03811	917.19	.96246	790.81	.89807	15
16	1343.2	3.12813	1088.3	3.03674	914.75	2.96130	789.00	2.89708	16
17	1338.0	.12644	1084.8	.03537	912.33	.96015	787.20	.89608	17
18	1332.8	.12475	1081.4	.03400	909.92	.95900	785.41	.89509	18
19	1327.6	.12307	1078.1	.03264	907.52	.95785	783.62	.89410	19
20	1322.5	.12140	1074.7	.03128	905.13	.95671	781.84	.89312	20
21	1317.5	3.11974	1071.3	3.02992	902.76	2.95557	780.07	2.89213	21
22	1312.4	.11808	1068.0	.02857	900.40	.95443	778.31	.89115	22
23	1307.4	.11642	1064.7	.02723	898.05	.95330	776.55	.89017	23
24	1302.5	.11477	1061.4	.02589	895.71	.95217	774.81	.88919	24
25	1297.6	.11313	1058.2	.02455	893.39	.95104	773.07	.88821	25
26	1292.7	3.11150	1054.9	3.02322	891.08	2.94991	771.34	2.88724	26
27	1287.9	.10987	1051.7	.02189	888.78	.94879	769.61	.88627	27
28	1283.1	.10825	1048.5	.02056	886.49	.94767	767.90	.88530	28
29	1278.3	.10663	1045.3	.01924	884.21	.94655	766.19	.88433	29
30	1273.6	.10502	1042.1	.01792	881.95	.94544	764.49	.88337	30
31	1268.9	3.10341	1039.0	3.01661	879.69	2.94433	762.80	2.88241	31
32	1264.2	.10182	1035.9	.01530	877.45	.94322	761.11	.88145	32
33	1259.6	.10022	1032.8	.01400	875.22	.94212	759.43	.88049	33
34	1255.0	.09864	1029.7	.01270	873.00	.94101	757.76	.87953	34
35	1250.4	.09705	1026.6	.01140	870.80	.93991	756.10	.87858	35
36	1245.9	3.09548	1023.5	3.01010	868.60	2.93882	754.44	2.87762	36
37	1241.4	.09391	1020.5	.00882	866.41	.93772	752.80	.87668	37
38	1236.9	.09234	1017.5	.00753	864.24	.93663	751.16	.87573	38
39	1232.5	.09079	1014.5	.00625	862.07	.93554	749.52	.87478	39
40	1228.1	.08923	1011.5	.00497	859.92	.93446	747.89	.87384	40
41	1223.7	3.08769	1008.6	3.00370	857.78	2.93337	746.27	2.87290	41
42	1219.4	.08614	1005.6	.00242	855.65	.93229	744.66	.87196	42
43	1215.1	.08461	1002.7	3.00116	853.53	.93122	743.06	.87102	43
44	1210.8	.08308	999.76	2.99989	851.42	.93014	741.46	.87008	44
45	1206.6	.08155	996.87	.99863	849.32	.92907	739.86	.86915	45
46	1202.4	3.08003	993.99	2.99738	847.23	2.92800	738.28	2.86822	46
47	1198.2	.07852	991.13	.99613	845.15	.92693	736.70	.86729	47
48	1194.0	.07701	988.28	.99488	843.08	.92587	735.13	.86636	48
49	1189.9	.07550	985.45	.99363	841.02	.92480	733.56	.86544	49
50	1185.8	.07400	982.64	.99239	838.97	.92374	732.01	.86451	50
51	1181.7	3.07251	979.84	2.99115	836.93	2.92269	730.45	2.86359	51
52	1177.7	.07102	977.06	.98992	834.90	.92163	728.91	.86267	52
53	1173.6	.06954	974.29	.98869	832.89	.92058	727.37	.86175	53
54	1169.7	.06806	971.54	.98746	830.88	.91953	725.84	.86084	54
55	1165.7	.06658	968.81	.98624	828.88	.91849	724.31	.85992	55
56	1161.8	3.06511	966.09	2.98501	826.89	2.91744	722.79	2.85901	56
57	1157.9	.06365	963.39	.98380	824.91	.91640	721.28	.85810	57
58	1154.0	.06219	960.70	.98258	822.93	.91536	719.77	.85719	58
59	1150.1	.06074	958.03	.98137	820.97	.91433	718.27	.85629	59
60	1146.3	.05929	955.37	.98017	819.02	.91329	716.78	.85538	60

Deg.	8°		9°		10°		11°		Deg.
Min.	Radius.	Log *R*	Radius.	Log *R*	Radius.	Log *R*	Radius.	Log *R*	Min.
0	716.78	2.85538	637.27	2.80432	573.69	2.75867	521.67	2.71739	0
1	715.29	.85448	636.10	.80352	572.73	.75795	520.88	.71674	1
2	713.81	.85358	634.93	.80272	571.78	.75723	520.10	.71608	2
3	712.34	.85268	633.76	.80192	570.84	.75651	519.32	.71543	3
4	710.87	.85178	632.60	.80113	569.90	.75579	518.54	.71478	4
5	709.40	.85089	631.44	.80033	568.96	.75508	517.76	.71413	5
6	707.95	2.85000	630.29	2.79954	568.02	2.75436	516.99	2.71348	6
7	706.49	.84911	629.14	.79874	567.09	.75365	516.21	.71283	7
8	705.05	.84822	627.99	.79795	566.16	.75293	515.44	.71218	8
9	703.61	.84733	626.85	.79716	565.23	.75222	514.68	.71153	9
10	702.17	.84644	625.71	.79637	564.31	.75151	513.91	.71088	10
11	700.75	2.84556	624.58	2.79558	563.38	2.75080	513.15	2.71024	11
12	699.33	.84468	623.45	.79480	562.47	.75009	512.38	.70959	12
13	697.91	.84380	622.32	.79401	561.55	.74939	511.63	.70895	13
14	696.50	.84292	621.20	.79323	560.64	.74868	510.87	.70831	14
15	695.09	.84204	620.09	.79245	559.73	.74798	510.11	.70767	15
16	693.70	2.84117	618.97	2.79167	558.82	2.74727	509.36	2.70702	16
17	692.30	.84029	617.87	.79089	557.92	.74657	508.61	.70638	17
18	690.91	.83942	616.76	.79011	557.02	.74587	507.86	.70575	18
19	689.53	.83855	615.66	.78934	556.12	.74517	507.12	.70511	19
20	688.16	.83768	614.56	.78856	555.23	.74447	506.38	.70447	20
21	686.78	2.83682	613.47	2.78779	554.34	2.74377	505.64	2.70383	21
22	685.42	.83595	612.38	.78702	553.45	.74307	504.90	.70320	22
23	684.06	.83509	611.30	.78625	552.56	.74238	504.16	.70257	23
24	682.70	.83423	610.21	.78548	551.68	.74168	503.42	.70193	24
25	681.35	.83337	609.14	.78471	550.80	.74099	502.69	.70130	25
26	680.01	2.83251	608.06	2.78395	549.92	2.74030	501.96	2.70067	26
27	678.67	.83166	606.99	.78318	549.05	.73961	501.23	.70004	27
28	677.34	.83080	605.93	.78242	548.17	.73892	500.51	.69941	28
29	676.01	.82995	604.86	.78165	547.30	.73823	499.78	.69878	29
30	674.69	.82910	603.80	.78089	546.44	.73754	499.06	.69815	30
31	673.37	2.82825	602.75	2.78013	545.57	2.73685	498.34	2.69752	31
32	672.06	.82740	601.70	.77938	544.71	.73617	497.62	.69690	32
33	670.75	.82656	600.65	.77862	543.86	.73548	496.91	.69627	33
34	669.45	.82571	599.61	.77786	543.00	.73480	496.19	.69565	34
35	668.15	.82487	598.57	.77711	542.15	.73412	495.48	.69503	35
36	666.86	2.82403	597.53	2.77636	541.30	2.73343	494.77	2.69440	36
37	665.57	.82319	596.50	.77561	540.45	.73275	494.07	.69378	37
38	664.29	.82235	595.47	.77486	539.61	.73207	493.36	.69316	38
39	663.01	.82152	594.44	.77411	538.76	.73140	492.66	.69254	39
40	661.74	.82068	593.42	.77336	537.92	.73072	491.96	.69192	40
41	660.47	2.81985	592.40	2.77261	537.09	2.73004	491.26	2.69131	41
42	659.21	.81902	591.38	.77187	536.25	.72937	490.56	.69069	42
43	657.95	.81819	590.37	.77112	535.42	.72869	489.86	.69007	43
44	656.69	.81736	589.36	.77038	534.59	.72802	489.17	.68946	44
45	655.45	.81653	588.36	.76964	533.77	.72735	488.48	.68884	45
46	654.20	2.81571	587.36	2.76890	532.94	2.72668	487.79	2.68823	46
47	652.96	.81489	586.36	.76816	532.12	.72601	487.10	.68762	47
48	651.73	.81406	585.36	.76742	531.30	.72534	486.42	.68701	48
49	650.50	.81324	584.37	.76669	530.49	.72467	485.73	.68640	49
50	649.27	.81243	583.38	.76595	529.67	.72401	485.05	.68579	50
51	648.05	2.81161	582.40	2.76522	528.86	2.72334	484.37	2.68518	51
52	646.84	.81079	581.42	.76449	528.05	.72267	483.69	.68457	52
53	645.63	.80998	580.44	.76376	527.25	.72201	483.02	.68396	53
54	644.42	.80917	579.47	.76303	526.44	.72135	482.34	.68335	54
55	643.22	.80836	578.49	.76230	525.64	.72069	481.67	.68275	55
56	642.02	2.80755	577.53	2.76157	524.84	2.72003	481.00	2.68214	56
57	640.83	.80674	576.56	.76084	524.05	.71937	480.33	.68154	57
58	639.64	.80593	575.60	.76012	523.25	.71871	479.67	.68094	58
59	638.45	.80513	574.64	.75939	522.46	.71805	479.00	.68033	59
60	637.27	.80432	573.69	.75867	521.67	.71739	478.34	.67973	60

Deg.	Radius.	Log R	Deg.	Radius.	Log R	Deg.	Radius.	Log R	Deg.	Radius	Log R
12°	478.34	2.67973	14°	410.28	2.61307	16°	359.26	2.55541	21°	274.37	2.43833
2	477.02	.67853	2	409.31	.61205	5	357.42	.55317	10	272.23	.43494
4	475.71	.67734	4	408.34	.61102	10	355.59	.55094	20	270.13	.43157
6	474.40	.67614	6	407.38	.61000	15	353.77	.54872	30	268.06	.42823
8	473.10	.67495	8	406.42	.60898	20	351.98	.54652	40	266.02	.42492
10	471.81	2.67376	10	405.47	2.60796	25	350.21	.54432	50	264.02	.42163
12	470.53	.67258	12	404.53	.60694	30	348.45	2.54214	22°	262.04	2.41837
14	469.25	.67140	14	403.58	.60593	35	346.71	.53997	10	260.10	.41513
16	467.98	.67022	16	402.65	.60492	40	344.99	.53780	20	258.18	.41192
18	466.72	.66905	18	401.71	.60391	45	343.29	.53565	30	256.29	.40873
20	465.46	2.66788	20	400.78	2.60291	50	341.60	.53351	40	254.43	.40557
22	464.21	.66671	22	399.86	.60190	55	339.93	.53138	50	252.60	.40243
24	462.97	.66555	24	398.94	.60090	17°	338.27	2.52927	23°	250.79	2.39931
26	461.73	.66439	26	398.02	.59990	5	336.64	.52716	10	249.01	.39622
28	460.50	.66323	28	397.11	.59891	10	335.01	.52506	20	247.26	.39315
30	459.28	2.66207	30	396.20	2.59791	15	333.41	.52297	30	245.53	.39010
32	458.06	.66092	32	395.30	.59692	20	331.82	.52090	40	243.82	.38707
34	456.85	.65977	34	394.40	.59593	25	330.24	.51883	50	242.14	.38407
36	455.65	.65863	36	393.50	.59494	30	328.68	2.51677	24°	240.49	2.38109
38	454.45	.65748	38	392.61	.59396	35	327.13	.51472	10	238.85	.37813
40	453.26	2.65634	40	391.72	2.59298	40	325.60	.51269	20	237.24	.37519
42	452.07	.65521	42	390.84	.59199	45	324.09	.51066	30	235.65	.37227
44	450.89	.65407	44	389.96	.59102	50	322.59	.50864	40	234.08	.36937
46	449.72	.65294	46	389.08	.59004	55	321.10	.50663	50	232.54	.36649
48	448.56	.65181	48	388.21	.58907	18°	319.62	2.50464	25°	231.01	2.36363
50	447.40	2.65069	50	387.34	2.58809	5	318.16	.50265	30	226.55	.35517
52	446.24	.64957	52	386.48	.58713	10	316.71	.50067	26°	222.27	.34688
54	445.09	.64845	54	385.62	.58616	15	315.28	.49869	30	218.15	.33875
56	443.95	.64733	56	384.77	.58519	20	313.86	.49673	27°	214.18	2.33078
58	442.81	.64622	58	383.91	.58423	25	312.45	.49478	30	210.36	.32296
13°	441.68	2.64511	15°	383.06	2.58327	30	311.06	2.49284	28°	206.68	.31529
2	440.56	.64400	2	382.22	.58231	35	309.67	.49090	30	203.13	.30776
4	439.44	.64290	4	381.38	.58135	40	308.30	.48898	29°	199.70	2.30037
6	438.33	.64180	6	380.54	.58040	45	306.95	.48706	30	196.38	.29310
8	437.22	.64070	8	379.71	.57945	50	305.60	.48515	30°	193.19	.28597
10	436.12	2.63960	10	378.88	2.57850	55	304.27	.48325	30	190.09	.27896
12	435.02	.63851	12	378.05	.57755	19°	302.94	2.48136	31°	187.10	2.27207
14	433.93	.63742	14	377.23	.57661	5	301.63	.47948	32	181.40	.25863
16	432.84	.63633	16	376.41	.57566	10	300.33	.47760	33	176.05	.24563
18	431.76	.63524	18	375.60	.57472	15	299.04	.47573	34	171.02	.23303
20	430.69	2.63416	20	374.79	2.57378	20	297.77	.47388	35	166.28	.22083
22	429.62	.63308	22	373.98	.57284	25	296.50	.47203	36	161.80	2.20899
24	428.56	.63201	24	373.17	.57191	30	295.25	2.47018	37	157.58	.19749
26	427.50	.63093	26	372.37	.57097	35	294.00	.46835	38	153.58	.18633
28	426.44	.62986	28	371.57	.57004	40	292.77	.46652	39	149.79	.17547
30	425.40	2.62879	30	370.78	2.56911	45	291.55	.46471	40	146.19	.16492
32	424.35	.62773	32	369.99	.56819	50	290.33	.46289	41	142.77	2.15464
34	423.32	.62666	34	369.20	.56726	55	289.13	.46109	42	139.52	.14464
36	422.28	.62560	36	368.42	.56634	20°	287.94	2.45930	43	136.43	.13489
38	421.26	.62454	38	367.64	.56542	5	286.76	.45751	44	133.47	.12539
40	420.23	2.62349	40	366.86	2.56450	10	285.58	.45573	45	130.66	.11613
42	419.22	.62243	42	366.09	.56358	15	284.42	.45396	46	127.97	2.10709
44	418.20	.62138	44	365.31	.56266	20	283.27	.45219	47	125.39	.09827
46	417.19	.62034	46	364.55	.56175	25	282.12	.45044	48	122.93	.08965
48	416.19	.61929	48	363.78	.56084	30	280.99	2.44869	49	120.57	.08124
50	415.19	2.61825	50	363.02	2.55993	35	279.86	.44694	50	118.31	.07302
52	414.20	.61721	52	362.26	.55902	40	278.75	.44521	52	114.06	2.05713
54	413.21	.61617	54	361.51	.55812	45	277.64	.44348	54	110.13	.04192
56	412.23	.61514	56	360.76	.55721	50	276.54	.44176	56	106.50	.02736
58	411.25	.61410	58	360.01	.55631	55	275.45	.44004	58	103.13	.01340
14°	410.28	2.61307	16°	359.26	2.55541	21°	274.37	2.43833	60	100.00	2.00000

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS FOR A 1° CURVE.

Δ	Tang. *T.*	Ext. Dist. *E.*	Long Chord *LC.*	Δ	Tang. *T.*	Ext. Dist. *E.*	Long Chord *LC.*	Δ	Tang. *T.*	Ext. Dist. *E.*	Long Chord *LC.*
1°	50.00	0.218	100.00	11°	551.70	26.500	1098.3	21°	1061.9	97.58	2088.3
10′	58.34	0.297	116.67	10	560.11	27.313	1114.9	10	1070.6	99.15	2104.7
20	66.67	0.388	133.33	20	568.53	28.137	1131.5	20	1079.2	100.75	2121.1
30	75.01	0.491	150.00	30	576.95	28.974	1148.1	30	1087.8	102.35	2137.4
40	83.34	0.606	166.66	40	585.36	29.824	1164.7	40	1096.4	103.97	2153.8
50	91.68	0.733	183.33	50	593.79	30.686	1181.2	50	1105.1	105.60	2170.2
2°	100.01	0.873	199.99	12°	602.21	31.561	1197.8	22°	1113.7	107.24	2186.5
10	108.35	1.024	216.66	10	610.64	32.447	1214.4	10	1122.4	108.90	2202.9
20	116.68	1.188	233.32	20	619.07	33.347	1231.0	20	1131.0	110.57	2219.2
30	125.02	1.364	249.98	30	627.50	34.259	1247.5	30	1139.7	112.25	2235.6
40	133.36	1.552	266.65	40	635.93	35.183	1264.1	40	1148.4	113.95	2251.9
50	141.70	1.752	283.31	50	644.37	36.120	1280.7	50	1157.0	115.66	2268.3
3°	150.04	1.964	299.97	13°	652.81	37.069	1297.2	23°	1165.7	117.38	2284.6
10	158.38	2.188	316.63	10	661.25	38.031	1313.8	10	1174.4	119.12	2301.0
20	166.72	2.425	333.29	20	669.70	39.006	1330.3	20	1183.1	120.87	2317.3
30	175.06	2.674	349.95	30	678.15	39.993	1346.9	30	1191.8	122.63	2333.6
40	183.40	2.934	366.61	40	686.60	40.992	1363.4	40	1200.5	124.41	2349.9
50	191.74	3.207	383.27	50	695.06	42.004	1380.0	50	1209.2	126.20	2366.2
4°	200.08	3.492	399.92	14°	703.51	43.029	1396.5	24°	1217.9	128.00	2382.5
10	208.43	3.790	416.58	10	711.97	44.066	1413.1	10	1226.6	129.82	2398.8
20	216.77	4.099	433.24	20	720.44	45.116	1429.6	20	1235.3	131.65	2415.1
30	225.12	4.421	449.89	30	728.90	46.178	1446.2	30	1244.0	133.50	2431.4
40	233.47	4.755	466.54	40	737.37	47.253	1462.7	40	1252.8	135.36	2447.7
50	241.81	5.100	483.20	50	745.85	48.341	1479.2	50	1261.5	137.23	2464.0
5°	250.16	5.459	499.85	15°	754.32	49.441	1495.7	25°	1270.2	139.11	2480.2
10	258.51	5.829	516.50	10	762.80	50.554	1512.3	10	1279.0	141.01	2496.5
20	266.86	6.211	533.15	20	771.29	51.679	1528.8	20	1287.7	142.93	2512.8
30	275.21	6.606	549.80	30	779.77	52.818	1545.3	30	1296.5	144.85	2529.0
40	283.57	7.013	566.44	40	788.26	53.969	1561.8	40	1305.8	146.79	2545.3
50	291.92	7.432	583.09	50	796.75	55.132	1578.3	50	1314.0	148.75	2561.5
6°	300.28	7.863	599.73	16°	805.25	56.309	1594.8	26°	1322.8	150.71	2577.8
10	308.64	8.307	616.38	10	813.75	57.498	1611.3	10	1331.6	152.69	2594.0
20	316.99	8.762	633.02	20	822.25	58.699	1627.8	20	1340.4	154.69	2610.3
30	325.35	9.230	649.66	30	830.76	59.914	1644.3	30	1349.2	156.70	2626.5
40	333.71	9.710	666.30	40	839.27	61.141	1660.8	40	1358.0	158.72	2642.7
50	342.08	10.202	682.94	50	847.78	62.381	1677.3	50	1366.8	160.76	2658.9
7°	350.44	10.707	699.57	17°	856.30	63.634	1693.8	27°	1375.6	162.81	2675.1
10	358.81	11.224	716.21	10	864.82	64.900	1710.3	10	1384.4	164.87	2691.3
20	367.17	11.753	732.84	20	873.35	66.178	1726.8	20	1393.2	166.95	2707.5
30	375.54	12.294	749.47	30	881.88	67.470	1743.2	30	1402.0	169.04	2723.7
40	383.91	12.847	766.10	40	890.41	68.774	1759.7	40	1410.9	171.15	2739.9
50	392.28	13.413	782.73	50	898.95	70.091	1776.2	50	1419.7	173.27	2756.1
8°	400.66	13.991	799.36	18°	907.49	71.421	1792.6	28°	1428.6	175.41	2772.3
10	409.03	14.582	815.99	10	916.03	72.764	1809.1	10	1437.4	177.55	2788.4
20	417.41	15.184	832.61	20	924.58	74.119	1825.5	20	1446.3	179.72	2804.6
30	425.79	15.799	849.23	30	933.13	75.488	1842.0	30	1455.1	181.89	2820.7
40	434.17	16.426	865.85	40	941.69	76.869	1858.4	40	1464.0	184.08	2836.9
50	442.55	17.066	882.47	50	950.25	78.264	1874.9	50	1472.9	186.29	2853.0
9°	450.93	17.717	899.09	19°	958.81	79.671	1891.3	29°	1481.8	188.51	2869.2
10	459.32	18.381	915.70	10	967.38	81.092	1907.8	10	1490.7	190.74	2885.3
20	467.71	19.058	932.31	20	975.96	82.525	1924.2	20	1499.6	192.99	2901.4
30	476.10	19.746	948.92	30	984.53	83.972	1940.6	30	1508.5	195.25	2917.6
40	484.49	20.447	965.53	40	993.12	85.431	1957.1	40	1517.4	197.53	2933.7
50	492.88	21.161	982.14	50	1001.70	86.904	1973.5	50	1526.3	199.82	2949.8
10°	501.28	21.886	998.74	20°	1010.29	88.389	1989.9	30°	1535.3	202.12	2965.9
10	509.68	22.624	1015.35	10	1018.89	89.888	2006.3	10	1544.2	204.44	2982.0
20	518.08	23.375	1031.95	20	1027.49	91.399	2022.7	20	1553.1	206.77	2998.1
30	526.48	24.138	1048.54	30	1036.09	92.924	2039.1	30	1562.1	209.12	3014.2
40	534.89	24.913	1065.14	40	1044.70	94.462	2055.5	40	1571.0	211.48	3030.2
50	543.29	25.700	1081.73	50	1053.31	96.013	2071.9	50	1580.0	213.86	3046.3
11°	551.70	26.500	1098.33	21°	1061.93	97.577	2088.3	31°	1589.0	216.25	3062.4

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS FOR A 1° CURVE.

Δ	Tang. *T*.	Ext. Dist. *E*.	Long Chord *LC*.	Δ	Tang. *T*.	Ext. Dist. *E*.	Long Chord *LC*.	Δ	Tang. *T*.	Ext. Dist. *E*.	Long Chord *LC*.
31°	1589·0	216·25	3062·4	41°	2142·2	387·38	4013·1	51°	2732·9	618·39	4933·4
10	1598·0	218·66	3078·4	10	2151·7	390·71	4028·7	10	2743·1	622·81	4948·4
20	1606·9	221·08	3094·5	20	2161·2	394·06	4044·3	20	2753·4	627·24	4963·4
30	1615·9	223·51	3110·5	30	2170·8	397·43	4059·9	30	2763·7	631·69	4978·4
40	1624·9	225·96	3126·6	40	2180·3	400·82	4075·5	40	2773·9	636·16	4993·4
50	1633·9	228·42	3142·6	50	2189·9	404·22	4091·1	50	2784·2	640·66	5008·4
32°	1643·0	230·90	3158·6	42°	2199·4	407·64	4106·6	52°	2794·5	645·17	5023·4
10	1652·0	233·39	3174·6	10	2209·0	411·07	4122·2	10	2804·9	649·70	5038·4
20	1661·0	235·90	3190·6	20	2218·6	414·52	4137·7	20	2815·2	654·25	5053·4
30	1670·0	238·43	3206·6	30	2228·1	417·99	4153·3	30	2825·6	658·83	5068·3
40	1679·1	240·96	3222·6	40	2237·7	421·48	4168·8	40	2835·9	663·42	5083·3
50	1688·1	243·52	3238·6	50	2247·3	424·98	4184·3	50	2846·3	668·03	5098·2
33°	1697·2	246·08	3254·6	43°	2257·0	428·50	4199·8	53°	2856·7	672·66	5113·1
10	1706·3	248·66	3270·6	10	2266·6	432·04	4215·3	10	2867·1	677·32	5128·0
20	1715·3	251·26	3286·6	20	2276·2	435·59	4230·8	20	2877·5	681·99	5142·9
30	1724·4	253·87	3302·5	30	2285·9	439·16	4246·3	30	2888·0	686·68	5157·8
40	1733·5	256·50	3318·5	40	2295·6	442·75	4261·8	40	2898·4	691·40	5172·7
50	1742·6	259·14	3334·4	50	2305·2	446·35	4277·3	50	2908·9	696·13	5187·6
34°	1751·7	261·80	3350·4	44°	2314·9	449·98	4292·7	54°	2919·4	700·89	5202·4
10	1760·8	264·47	3366·3	10	2324·6	453·62	4308·2	10	2929·9	705·66	5217·3
20	1770·0	267·16	3382·2	20	2334·3	457·27	4323·6	20	2940·4	710·46	5232·1
30	1779·1	269·86	3398·2	30	2344·1	460·95	4339·0	30	2951·0	715·28	5246·9
40	1788·2	272·58	3414·1	40	2353·8	464·64	4354·5	40	2961·5	720·11	5261·7
50	1797·4	275·31	3430·0	50	2363·5	468·35	4369·9	50	2972·1	724·97	5276·5
35°	1806·6	278·05	3445·9	45°	2373·3	472·08	4385·3	55°	2982·7	729·85	5291·3
10	1815·7	280·82	3461·8	10	2383·1	475·82	4400·7	10	2993·3	734·76	5306·1
20	1824·9	283·60	3477·7	20	2392·8	479·59	4416·1	20	3003·9	739·68	5320·9
30	1834·1	286·39	3493·5	30	2402·6	483·37	4431·4	30	3014·5	744·62	5335·6
40	1843·3	289·20	3509·4	40	2412·4	487·16	4446·8	40	3025·2	749·59	5350·4
50	1852·5	292·02	3525·3	50	2422·3	490·98	4462·2	50	3035·8	754·57	5365·1
36°	1861·7	294·86	3541·1	46°	2432·1	494·82	4477·5	56°	3046·5	759·58	5379·8
10	1870·9	297·72	3557·0	10	2441·9	498·67	4492·8	10	3057·2	764·61	5394·5
20	1880·1	300·59	3572·8	20	2451·8	502·54	4508·2	20	3067·9	769·66	5409·2
30	1889·4	303·47	3588·6	30	2461·7	506·42	4523·5	30	3078·7	774·73	5423·9
40	1898·6	306·37	3604·5	40	2471·5	510·33	4538·8	40	3089·4	779·83	5438·6
50	1907·9	309·29	3620·3	50	2481·4	514·25	4554·1	50	3100·2	784·94	5453·3
37°	1917·1	312·22	3636·1	47°	2491·3	518·20	4569·4	57°	3110·9	790·08	5467·9
10	1926·4	315·17	3651·9	10	2501·2	522·16	4584·7	10	3121·7	795·24	5482·5
20	1935·7	318·13	3667·7	20	2511·2	526·13	4599·9	20	3132·6	800·42	5497·2
30	1945·0	321·11	3683·5	30	2521·1	530·13	4615·2	30	3143·4	805·62	5511·8
40	1954·3	324·11	3699·3	40	2531·1	534·15	4630·4	40	3154·2	810·85	5526·4
50	1963·6	327·12	3715·0	50	2541·0	538·18	4645·7	50	3165·1	816·10	5541·0
38°	1972·9	330·15	3730·8	48°	2551·0	542·23	4660·9	58°	3176·0	821·37	5555·6
10	1982·2	333·19	3746·5	10	2561·0	546·30	4676·1	10	3186·9	826·66	5570·2
20	1991·5	336·25	3762·3	20	2571·0	550·39	4691·3	20	3197·8	831·98	5584·7
30	2000·9	339·32	3778·0	30	2581·0	554·50	4706·5	30	3208·8	837·31	5599·3
40	2010·2	342·41	3793·8	40	2591·1	558·63	4721·7	40	3219·7	842·67	5613·8
50	2019·6	345·52	3809·5	50	2601·1	562·77	4736·9	50	3230·7	848·06	5628·3
39°	2029·0	348·64	3825·2	49°	2611·2	566·94	4752·1	59°	3241·7	853·46	5642·8
10	2038·4	351·78	3840·9	10	2621·2	571·12	4767·3	10	3252·7	858·89	5657·3
20	2047·8	354·94	3856·6	20	2631·3	575·32	4782·4	20	3263·7	864·34	5671·8
30	2057·2	358·11	3872·3	30	2641·4	579·54	4797·5	30	3274·8	869·82	5686·3
40	2066·6	361·29	3888·0	40	2651·5	583·78	4812·7	40	3285·8	875·32	5700·8
50	2076·0	364·50	3903·6	50	2661·6	588·04	4827·8	50	3296·9	880·84	5715·2
40°	2085·4	367·72	3919·3	50°	2671·8	592·32	4842·9	60°	3308·0	886·38	5729·7
10	2094·9	370·95	3935·0	10	2681·9	596·62	4858·0	10	3319·1	891·95	5744·1
20	2104·3	374·20	3950·6	20	2692·1	600·93	4873·1	20	3330·3	897·54	5758·5
30	2113·8	377·47	3966·3	30	2702·3	605·27	4888·2	30	3341·4	903·15	5772·9
40	2123·3	380·76	3981·9	40	2712·5	609·62	4903·2	40	3352·6	908·79	5787·3
50	2132·7	384·06	3997·5	50	2722·7	614·00	4918·3	50	3363·8	914·45	5801·7
41°	2142·2	387·38	4013·1	51°	2732·9	618·39	4933·4	61°	3375·0	920·14	5816·0

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS FOR A 1° CURVE.

Δ	Tang. *T.*	Ext. Dist. *E.*	Long Chord *LC.*	Δ	Tang. *T.*	Ext. Dist. *E.*	Long Chord *LC.*	Δ	Tang. *T.*	Ext. Dist. *E.*	Long Chord *LC.*
61°	3375.0	920.14	5816.0	71°	4086.9	1308.2	6654.4	81°	4893.6	1805.3	7442.2
10′	3386.3	925.85	5830.4	10	4099.5	1315.5	6668.0	10	4908.0	1814.7	7454.9
20	3397.5	931.58	5844.7	20	4112.1	1322.9	6681.6	20	4922.5	1824.1	7467.5
30	3408.8	937.34	5859.1	30	4124.8	1330.3	6695.1	30	4937.0	1833.6	7480.2
40	3420.1	943.12	5873.4	40	4137.4	1337.7	6708.6	40	4951.5	1843.1	7492.8
50	3431.4	948.92	5887.7	50	4150.1	1345.1	6722.1	50	4966.1	1852.6	7505.4
62°	3442.7	954.75	5902.0	72°	4162.8	1352.6	6735.6	82°	4980.7	1862.2	7518.0
10	3454.1	960.60	5916.3	10	4175.6	1360.1	6749.1	10	4995.4	1871.8	7530.5
20	3465.4	966.48	5930.5	20	4188.4	1367.6	6762.5	20	5010.0	1881.5	7543.1
30	3476.8	972.39	5944.8	30	4201.2	1375.2	6776.0	30	5024.8	1891.2	7555.6
40	3488.2	978.31	5959.0	40	4214.0	1382.8	6789.4	40	5039.5	1900.9	7568.2
50	3499.7	984.27	5973.3	50	4226.8	1390.4	6802.8	50	5054.3	1910.7	7580.7
63°	3511.1	990.24	5987.5	73°	4239.7	1398.0	6816.3	83°	5069.2	1920.5	7593.2
10	3522.6	996.24	6001.7	10	4252.6	1405.7	6829.6	10	5084.0	1930.4	7605.6
20	3534.1	1002.3	6015.9	20	4265.6	1413.5	6843.0	20	5099.0	1940.3	7618.1
30	3545.6	1008.3	6030.0	30	4278.5	1421.2	6856.4	30	5113.9	1950.3	7630.5
40	3557.2	1014.4	6044.2	40	4291.5	1429.0	6869.7	40	5128.9	1960.2	7643.0
50	3568.7	1020.5	6058.4	50	4304.6	1436.8	6883.1	50	5143.9	1970.3	7655.4
64°	3580.3	1026.6	6072.5	74°	4317.6	1444.6	6896.4	84°	5159.0	1980.4	7667.8
10	3591.9	1032.8	6086.6	10	4330.7	1452.5	6909.7	10	5174.1	1990.5	7680.1
20	3603.5	1039.0	6100.7	20	4343.8	1460.4	6923.0	20	5189.3	2000.6	7692.5
30	3615.1	1045.2	6114.8	30	4356.9	1468.4	6936.2	30	5204.4	2010.8	7704.9
40	3626.8	1051.4	6128.9	40	4370.1	1476.4	6949.5	40	5219.7	2021.1	7717.2
50	3638.5	1057.7	6143.0	50	4383.3	1484.4	6962.8	50	5234.9	2031.4	7729.5
65°	3650.2	1063.9	6157.1	75°	4396.5	1492.4	6976.0	85°	5250.3	2041.7	7741.8
10	3661.9	1070.2	6171.1	10	4409.8	1500.5	6989.2	10	5265.6	2052.1	7754.1
20	3673.7	1076.6	6185.2	20	4423.1	1508.6	7002.4	20	5281.0	2062.5	7766.3
30	3685.4	1082.9	6199.2	30	4436.4	1516.7	7015.6	30	5296.4	2073.0	7778.6
40	3697.2	1089.3	6213.2	40	4449.7	1524.9	7028.8	40	5311.9	2083.5	7790.8
50	3709.0	1095.7	6227.2	50	4463.1	1533.1	7041.9	50	5327.4	2094.1	7803.0
66°	3720.9	1102.2	6241.2	76°	4476.5	1541.4	7055.0	86°	5343.0	2104.7	7815.2
10	3732.7	1108.6	6255.2	10	4489.9	1549.7	7068.2	10	5358.6	2115.3	7827.4
20	3744.6	1115.1	6269.1	20	4503.4	1558.0	7081.3	20	5374.2	2126.0	7839.6
30	3756.5	1121.7	6283.1	30	4516.9	1566.3	7094.4	30	5389.9	2136.7	7851.7
40	3768.5	1128.2	6297.0	40	4530.4	1574.7	7107.5	40	5405.6	2147.5	7863.8
50	3780.4	1134.8	6310.9	50	4544.0	1583.1	7120.5	50	5421.4	2158.4	7876.0
67°	3792.4	1141.4	6324.8	77°	4557.6	1591.6	7133.6	87°	5437.2	2169.2	7888.1
10	3804.4	1148.0	6338.7	10	4571.2	1600.1	7146.6	10	5453.1	2180.2	7900.1
20	3816.4	1154.7	6352.6	20	4584.8	1608.6	7159.6	20	5469.0	2191.1	7912.2
30	3828.4	1161.3	6366.4	30	4598.5	1617.1	7172.6	30	5484.9	2202.2	7924.3
40	3840.5	1168.1	6380.3	40	4612.2	1625.7	7185.6	40	5500.9	2213.2	7936.3
50	3852.6	1174.8	6394.1	50	4626.0	1634.4	7198.6	50	5517.0	2224.3	7948.3
68°	3864.7	1181.6	6408.0	78°	4639.8	1643.0	7211.6	88°	5533.1	2235.5	7960.3
10	3876.8	1188.4	6421.8	10	4653.6	1651.7	7224.5	10	5549.2	2246.7	7972.3
20	3889.0	1195.2	6435.6	20	4667.4	1660.5	7237.4	20	5565.4	2258.0	7984.2
30	3901.2	1202.0	6449.4	30	4681.3	1669.2	7250.4	30	5581.6	2269.3	7996.2
40	3913.4	1208.9	6463.1	40	4695.2	1678.1	7263.3	40	5597.8	2280.6	8008.1
50	3925.6	1215.8	6476.9	50	4709.2	1686.9	7276.1	50	5614.2	2292.0	8020.0
69°	3937.9	1222.7	6490.6	79°	4723.2	1695.8	7289.0	89°	5630.5	2303.5	8031.9
10	3950.2	1229.7	6504.4	10	4737.2	1704.7	7301.9	10	5646.9	2315.0	8043.8
20	3962.5	1236.7	6518.1	20	4751.2	1713.7	7314.7	20	5663.4	2326.6	8055.7
30	3974.8	1243.7	6531.8	30	4765.3	1722.7	7327.5	30	5679.9	2338.2	8067.5
40	3987.2	1250.8	6545.5	40	4779.4	1731.7	7340.3	40	5696.4	2349.8	8079.3
50	3999.5	1257.9	6559.1	50	4793.6	1740.8	7353.1	50	5713.0	2361.5	8091.2
70°	4011.9	1265.0	6572.8	80°	4808.7	1749.9	7365.9	90°	5729.7	2373.3	8103.0
10	4024.4	1272.1	6586.4	10	4822.0	1759.0	7378.7	10	5746.3	2385.1	8114.7
20	4036.8	1279.3	6600.1	20	4836.2	1768.2	7391.4	20	5763.1	2397.0	8126.5
30	4049.3	1286.5	6613.7	30	4850.5	1777.4	7404.1	30	5779.9	2408.9	8138.2
40	4061.8	1293.7	6627.3	40	4864.8	1786.7	7416.8	40	5796.7	2420.9	8150.0
50	4074.4	1300.9	6640.9	50	4879.2	1796.0	7429.5	50	5813.6	2432.9	8161.7
71°	4086.9	1308.2	6654.4	81°	4893.6	1805.3	7442.2	91°	5830.5	2444.9	8173.4

TABLE III.—SWITCH LEADS AND DISTANCES.

LEAD-RAILS CIRCULAR THROUGHOUT; GAUGE 4′ 8½″. See § 262.

Frog No. (n).	Frog Angle (F).	Lead (L) (Eq. 79).	Chord (QT) (Eq. 77).	Radius of Lead-rails (r, Eq. 78).	Log r.	Degree of Curve (d).	Frog No. (n).
4	14° 15′ 00″	37·67	37·38	150·67	2·17801	38° 46′	4
4·5	12 40 59	42·37	42·12	190·69	·28032	30 24	4·5
5	11 25 16	47·08	46·85	235·42	·37183	24 32	5
5·5	10 23 20	51·79	51·58	284·85	·45462	20 13	5·5
6	9 31 38	56·50	56·30	339·00	·53020	16 58	6
6·5	8 47 51	61·21	61·03	397·85	·59972	14 26	6·5
7	8 10 16	65·92	65·75	461·42	·66409	12 26	7
7·5	7 37 41	70·62	70·47	529·69	·72402	10 50	7·5
8	7 09 10	75·33	75·19	602·67	·78007	9 31	8
8·5	6 43 59	80·04	79·90	680·36	·83273	8 26	8·5
9	6 21 35	84·75	84·62	762·75	·88238	7 31	9
9·5	6 01 32	89·46	89·33	849·85	·92934	6 45	9·5
10	5 43 29	94·17	94·05	941·67	2·97389	6 05	10
10·5	5 27 09	98·87	98·76	1038·19	3·01627	5 32	10·5
11	5 12 18	103·58	103·47	1139·42	·05668	5 02	11
11·5	4 58 45	108·29	108·19	1245·36	·09529	4 36	11·5
12	4 46 19	113·00	112·90	1356·00	3·13226	4 14	12

TURNOUTS WITH STRAIGHT POINT-RAILS AND STRAIGHT FROG-RAILS; GAUGE 4′ 8½″. See § 265.

Frog No. (n).	Switch Point Angle (a).	L'gth of Switch Point (DN).	L'gth of Str'g't Frog-rail (f).	Lead (L) (Eq. 90).	Chord (ST) (Eq. 88).	Radius of Lead-rails (r, Eq. 87).	Log r.	Degree of Curve (d).	Frog No. (n).
4	3° 40′	7·5	1·50	32·20	23·09	125·21	2·09764	47° 05′	4
4·5	3 40	7·5	1·69	34·29	25·03	159·25	·20208	36 36	4·5
5	2 45	10·0	1·87	41·85	29·88	197·65	·29589	29 22	5
5·5	2 45	10·0	2·06	44·16	32·03	240·44	·38100	24 00	5·5
6	1 50	15·0	2·25	56·00	38·66	288·09	·45953	19 59	6
6·5	1 50	15·0	2·44	58·84	41·34	340·19	·53172	16 54	6·5
7	1 50	15·0	2·62	61·65	43·98	397·65	·59950	14 27	7
7·5	1 50	15·0	2·81	64·36	46·50	460·00	·66276	12 29	7·5
8	1 50	15·0	3·00	67·04	48·99	527·91	·72256	10 52	8
8·5	1 50	15·0	3·19	69·60	51·38	600·94	·77883	9 33	8·5
9	1 50	15·0	3·37	72 20	53·80	681·16	·83325	8 25	9
9·5	1 50	15·0	3·56	74·70	56·11	767·11	·88486	7 28	9·5
10	1 50	15·0	3·75	77·04	58·28	858·14	·93356	6 41	10
10·5	1 50	15·0	3·94	79·51	60·57	959·00	2·98182	5 59	10·5
11	1 50	15·0	4·12	81·82	62·69	1065·52	3·02756	5 23	11
11·5	1 50	15·0	4·31	84·09	64·78	1180·16	3·07194	4 51	11·5
12	1 50	15·0	4·50	86·16	66·67	1299·93	3·11392	4 24	12

TRIGONOMETRICAL FUNCTIONS OF THE FROG ANGLES (F).

Frog No. (n).	Frog Angle (F).	Nat. sin F.	Nat. cos F.	Log sin F.	Log cos F.	Log cot F.	Log vers F.	Frog No. (n).
4	14° 15′ 00″	·24615	·96923	9·39120	9·98642	10·59522	8·48811	4
4·5	12 40 49	·21951	·97561	·34145	·98927	·64782	·38721	4·5
5	11 25 16	·19802	·98020	·29670	·99131	·69461	·29670	5
5·5	10 23 20	·18033	·98360	·25606	·99282	·73675	·21467	5·5
6	9 31 38	·16552	·98621	·21884	·99397	·77513	·13966	6
6·5	8 47 51	·15294	·98823	·18453	·99486	·81033	·07058	6·5
7	8 10 16	·14213	·98985	·15268	·99557	·84288	8·00655	7
7·5	7 37 41	·13274	·99115	·12301	·99614	·87313	7·94691	7·5
8	7 09 10	·12452	·99222	·09522	·99660	·90138	·89110	8
8·5	6 43 59	·11724	·99310	·06909	·99699	·92790	·83864	8·5
9	6 21 35	·11077	·99385	·04442	·99732	·95289	·78915	9
9·5	6 01 32	·10497	·99448	9·02107	·99759	·97652	·74232	9·5
10	5 43 29	·09975	·99501	8·99891	·99783	10·99892	·69788	10
10·5	5 27 09	·09502	·99548	·97781	·99803	11·02021	·65560	10·5
11	5 12 18	·09072	·99588	·95770	·99820	·04050	·61528	11
11·5	4 58 45	·08679	·99623	·93848	·99836	·05987	·57676	11·5
12	4 46 19	·08319	·99653	8·92007	9·99849	11·07842	7·53986	12

Point.	Total Central Angle φ	Nat. sin φ	Nat. cos φ	Log sin φ	Log cos φ	Log vers φ	x	Log x	y
1	0° 07′ 30″	.0022	one	7.3387$\bar{8}$	10.00000	4.3765$\bar{4}$	0.027	8.4356$\bar{8}$	25.000
2	0 22 30	.006$\bar{5}$	one	7.8159$\bar{0}$	9.99999	5.3307$\bar{8}$	0.136	9.1346$\bar{5}$	50.000
3	0 45 00	.0131	.9999	8.1169$\bar{2}$	9.9999$\bar{6}$	5.93284	0.382	9.5818$\bar{1}$	74.999
4	1 15 00	.0218	.999$\bar{7}$	8.3387$\bar{5}$	9.9998$\bar{9}$	6.37653	0.818	9.9128$\bar{0}$	99.995
5	1 52 30	.0327	.999$\bar{4}$	8.51480	9.9997$\bar{6}$	6.72869	1.500	0.1760$\bar{2}$	124.985
6	2 37 30	.0458	.998$\bar{9}$	8.6608$\bar{5}$	9.9995$\bar{4}$	7.02091	2.481	0.3946$\bar{7}$	149.966
7	3 30 00	.061$\bar{0}$	.998$\bar{1}$	8.7856$\bar{7}$	9.99919	7.2707$\bar{2}$	3.817	0.58171	174.930
8	4 30 00	.078$\bar{4}$	.9969	8.8946$\bar{4}$	9.99866	7.4889$\bar{2}$	5.561	0.74514	199.870
9	5 37 30	.0980	.9952	8.9913$\bar{0}$	9.9979$\bar{0}$	7.68262	7.792	0.89164	224.772
10	6 52 30	.1197	.9928	9.0781$\bar{0}$	9.9968$\bar{6}$	7.85675	10.489	1.0206$\bar{1}$	249.623

0° 30′-per-25-feet spiral.

—sighting at ——	Deflections from the tangent at the point occupied when the instrument is at—										
	Q	1	2	3	4	5	6	7	8	9	10
Q	0° 0′ 0″	0° 3′ 45″	0° 13′ 07″	0° 27′ 30″	0° 46′ 52″	1° 11′ 15″	1° 40′ 37″	2° 15′ 00″	2° 54′ 22″	3° 38′ 45″	4° 28′ 10″
1	0 3 45	0 0 00	0 07 30	0 20 37	0 38 45	1 01 52	1 30 00	2 03 07	2 41 15	3 24 22	4 12 30
2	0 9 22	0 7 30	0 00 00	0 11 15	0 28 07	0 50 00	1 16 52	1 48 45	2 25 37	3 07 30	3 54 22
3	0 17 30	0 16 52	0 11 15	0 00 00	0 15 00	0 35 37	1 01 15	1 31 52	2 07 30	2 48 07	3 33 45
4	0 28 07	0 28 45	0 24 22	0 15 00	0 00 00	0 18 45	0 43 07	1 12 30	1 46 52	2 26 15	3 10 37
5	0 41 15	0 43 07	0 40 00	0 31 52	0 18 45	0 00 00	0 22 30	0 50 37	1 23 45	2 01 52	2 45 00
6	0 56 52	1 00 00	0 58 07	0 51 15	0 39 22	0 22 30	0 00 00	0 26 15	0 58 07	1 35 00	2 16 52
7	1 15 00	1 19 22	1 18 45	1 13 07	1 02 30	0 46 52	0 26 15	0 00 00	0 30 00	1 05 37	1 46 15
8	1 35 37	1 41 15	1 41 52	1 37 30	1 28 07	1 13 45	0 54 22	0 30 00	0 00 00	0 33 45	1 13 07
9	1 58 45	2 05 37	2 07 30	2 04 22	1 56 15	1 43 07	1 25 00	1 01 52	0 33 45	0 00 00	0 37 30
10	2 24 20	2 32 30	2 35 37	2 33 45	2 26 52	2 15 00	1 58 07	1 36 15	1 09 22	0 37 30	0 00 00

TABLE IV.—ELEMENTS OF TRANSITION CURVES.

Point.	Total Central Angle ϕ	Nat. sin ϕ	Nat. cos ϕ	Log sin ϕ	Log cos ϕ	Log vers ϕ	x	Log x	y
1	0° 15′	.0043	one	7.63981	9.99999	4.97860	.055	8.73672	25.000
2	0 45	.0131	.9999	8.11692	9.99996	5.93284	.273	9.43616	49.999
3	1 30	.0262	.9996	8.41792	9.99985	6.53488	.763	9.88252	74.994
4	2 30	.0436	.9990	8.63968	9.99958	6.97853	1.636	0.21378	99.979
5	3 45	.0654	.9978	8.81560	9.99907	7.33063	2.999	0.47697	124.942
6	5 15	.0915	.9958	8.96143	9.99817	7.62274	4.900	0.69548	149.865
7	7 00	.1218	.9925	9.08589	9.99675	7.87238	7.628	0.88241	174.722
8	9 00	.1564	.9877	9.19433	9.99462	8.09031	11.107	1.04559	199.479
9	11 15	.1951	.9808	9.29023	9.99157	8.28363	15.502	1.19039	224.090
10	13 45	.2377	.9713	9.37600	9.98737	8.45724	20.913	1.32041	248.497

1°-per-25-feet spiral.

—sighting at —	Deflections from the tangent at the point occupied when the instrument is at—										
	Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Q	0° 0′ 0″	0° 07′ 30″	0° 26′ 15″	0° 55′ 0″	1° 33′ 45″	2° 22′ 30″	3° 21′ 15″	4° 30′ 00″	5° 48′ 47″	7° 17′ 34″	8° 56′ 22″
1	0 07 30	0 00 00	0 15 00	0 41 15	1 17 30	2 03 45	3 00 00	4 06 15	5 22 30	6 48 47	8 25 05
2	0 18 45	0 15 00	0 00 00	0 22 30	0 56 15	1 40 00	2 33 45	3 37 30	4 51 15	6 15 02	7 48 49
3	0 35 00	0 33 45	0 22 30	0 00 00	0 30 00	1 11 15	2 02 30	3 03 45	4 15 00	5 36 15	7 07 32
4	0 56 15	0 57 30	0 48 45	0 30 00	0 00 00	0 37 30	1 26 15	2 25 00	3 33 45	4 52 30	6 21 17
5	1 22 30	1 26 15	1 20 00	1 03 45	0 37 30	0 00 00	0 45 00	1 41 15	2 47 30	4 03 45	5 30 00
6	1 53 45	2 00 00	1 56 15	1 42 30	1 18 45	0 45 00	0 00 00	0 52 30	1 56 15	3 10 00	4 33 45
7	2 30 00	2 38 45	2 37 30	2 26 15	2 05 00	1 33 45	0 52 30	0 00 00	1 00 00	2 11 15	3 32 30
8	3 11 13	3 22 30	3 23 45	3 15 00	2 56 15	2 27 30	1 48 45	1 00 00	0 00 00	1 07 30	2 26 15
9	3 57 26	4 11 13	4 14 58	4 08 45	3 52 30	3 26 15	2 50 00	2 03 45	1 07 30	0 00 00	1 15 00
10	4 48 38	5 04 55	5 11 11	5 07 28	4 53 43	4 30 00	3 56 15	3 12 30	2 18 45	1 15 00	0 00 00

Point.	Total Central Angle φ	Nat. sin φ	Nat. cos φ	Log sin φ	Log cos φ	Log vers φ	x	Log x	y
1	0° 30′	.008$\bar{7}$	.999$\bar{9}$	7.94084	9.9999$\bar{8}$	5.58066	0.109	9.0377$\bar{4}$	25.000
2	1 30	.026$\bar{2}$	.999$\bar{6}$	8.41792	9.9998$\bar{5}$	6.53488	0.545	9.7367$\bar{0}$	49.996
3	3 00	.052$\bar{3}$	.998$\bar{6}$	8.7188$\bar{0}$	9.9994$\bar{0}$	7.13687	1.527	0.18388	74.977
4	5 00	.087$\bar{1}$	.9962	8.9402$\bar{9}$	8.9983$\bar{4}$	7.58039	3.271	0.5146$\bar{8}$	99.916
5	7 30	.130$\bar{5}$	.991$\bar{4}$	9.1157$\bar{0}$	9.9962$\bar{7}$	7.9322$\bar{2}$	5.992	0.7776$\bar{2}$	124.767
6	10 30	.182$\bar{2}$	.983$\bar{2}$	9.2606$\bar{3}$	9.9926$\bar{6}$	8.22389	9.903	0.9957$\bar{9}$	149.459
7	14 00	.2419	.970$\bar{3}$	9.3836$\bar{7}$	9.9369$\bar{0}$	8.4728$\bar{2}$	15.208	1.1820$\bar{7}$	173.890
8	18 00	.3090	.951$\bar{0}$	9.48998	9.9782$\bar{0}$	8.6896$\bar{9}$	22.099	1.34437	197.922
9	22 30	.382$\bar{7}$	.9239	9.5828$\bar{4}$	9.9656$\bar{1}$	8.8815$\bar{0}$	30.752	1.48787	221.376
10	27 30	.461$\bar{7}$	.8870	9.6644$\bar{0}$	9.94793	9.0530$\bar{3}$	41.317	1.61613	244.034

2°-per-25-feet spiral.

—sighting at—	Deflections from the tangent at the point occupied when the instrument is at—										
	Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Q	0° 00′ 00″	0° 15′ 00″	0° 52′ 30″	1° 50′ 00″	3° 07′ 30″	4° 45′ 00″	6° 42′ 30″	9° 00′ 05″	11° 37′ 45″	14° 35′ 30″	17° 53′ 25″
1	0 15 00	0 00 00	0 30 00	1 22 30	2 35 00	4 07 30	6 00 01	8 12 34	10 45 10	13 37 52	16 50 42
2	0 37 30	0 30 00	0 00 00	0 45 00	1 52 30	3 20 00	5 07 30	7 15 02	9 42 36	12 30 15	15 38 00
3	1 10 00	1 07 30	0 45 00	0 00 00	1 00 00	2 22 30	4 05 00	6 07 30	8 30 04	11 12 39	14 15 20
4	1 52 30	1 55 00	1 37 30	1 00 00	0 00 00	1 15 00	2 52 30	4 50 00	7 07 30	9 45 05	12 42 42
5	2 45 00	2 52 30	2 40 00	2 07 30	1 15 00	0 00 00	1 30 00	3 22 30	5 35 00	8 07 30	11 00 06
6	3 47 28	3 59 59	3 52 30	3 25 00	2 37 30	1 30 00	0 00 00	1 45 00	3 52 30	6 20 00	9 07 30
7	4 59 53	5 17 26	5 14 58	4 52 30	4 10 00	3 07 30	1 45 00	0 00 00	2 00 00	4 22 30	7 05 00
8	6 22 15	6 44 50	6 47 24	6 29 56	5 52 30	4 55 00	3 37 30	2 00 00	0 00 00	2 15 00	4 52 30
9	7 54 30	8 22 08	8 29 45	8 17 21	7 44 55	6 52 30	5 40 00	4 07 30	2 15 00	0 00 00	2 30 00
10	9 36 35	10 09 17	10 22 00	10 14 40	9 47 18	8 59 54	7 52 30	6 25 00	4 37 30	2 30 00	0 00 00

N.	0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	216	260	303	346	389
101	432	475	518	561	604	646	689	732	775	817
102	860	902	945	987	*030	*072	*114	*157	*199	*241
103	01 283	326	368	410	452	494	536	578	619	661
104	703	745	787	828	870	911	953	994	*036	*077
105	02 119	160	201	243	284	325	366	407	448	489
106	530	571	612	653	694	735	775	816	857	898
107	938	979	*019	*060	*100	*141	*181	*221	*262	*302
108	03 342	382	422	463	503	543	583	623	663	703
109	742	782	822	862	901	941	981	*020	*060	*100
110	04 139	178	218	257	297	336	375	415	454	493
111	532	571	610	649	688	727	766	805	844	883
112	922	960	999	*038	*076	*115	*154	*192	*231	*269
113	05 308	346	384	423	461	499	538	576	614	652
114	690	728	766	804	842	880	918	956	994	*032
115	06 070	107	145	183	220	258	296	333	371	408
116	446	483	520	558	595	632	670	707	744	781
117	818	855	893	930	967	*004	*040	*077	*114	*151
118	07 188	225	261	298	335	372	408	445	481	518
119	554	591	627	664	700	737	773	809	845	882
120	918	954	990	*026	*062	*098	*134	*170	*206	*242
121	08 278	314	350	386	422	457	493	529	564	600
122	636	671	707	742	778	813	849	884	920	955
123	990	*026	*061	*096	*131	*166	*202	*237	*272	*307
124	09 342	377	412	447	482	517	552	586	621	656
125	691	725	760	795	830	864	899	933	968	*002
126	10 037	071	106	140	174	209	243	277	312	346
127	380	414	448	483	517	551	585	619	653	687
128	721	755	789	822	856	890	924	958	991	*025
129	11 059	092	126	160	193	227	260	294	327	361
130	394	427	461	494	528	561	594	627	661	694
131	727	760	793	826	859	892	925	958	991	*024
132	12 057	090	123	156	189	221	254	287	320	352
133	385	418	450	483	515	548	580	613	645	678
134	710	743	775	807	840	872	904	937	969	*001
135	13 033	065	097	130	162	194	226	258	290	322
136	354	386	417	449	481	513	545	577	608	640
137	672	703	735	767	798	830	862	893	925	956
138	988	*019	*051	*082	*113	*145	*176	*207	*239	*270
139	14 301	332	364	395	426	457	488	519	550	582
140	613	644	675	706	736	767	798	829	860	891
141	922	952	983	*014	*045	*075	*106	*137	*167	*198
142	15 229	259	290	320	351	381	412	442	473	503
143	533	564	594	624	655	685	715	745	776	806
144	836	866	896	926	956	987	*017	*047	*077	*107
145	16 137	166	196	226	256	286	316	346	376	405
146	435	465	494	524	554	584	613	643	672	702
147	731	761	791	820	849	879	908	938	967	997
148	17 026	055	085	114	143	172	202	231	260	289
149	318	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	753	782	811	840	869
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	$4\bar{3}$	43	42	41
.1	4.3	4.3	4.2	4.1
.2	8.7	8.6	8.4	8.2
.3	13.0	12.9	12.6	12.3
.4	17.4	17.2	16.8	16.4
.5	21.7	21.5	21.0	20.5
.6	26.1	25.8	25.2	24.6
.7	30.4	30.1	29.4	28.7
.8	34.8	34.4	33.6	32.8
.9	39.1	38.7	37.8	36.9

	$4\bar{0}$	40	39	38
.1	4.0	4.0	3.9	3.8
.2	8.1	8.0	7.8	7.6
.3	12.1	12.0	11.7	11.4
.4	16.2	16.0	15.6	15.2
.5	20.2	20.0	19.5	19.0
.6	24.3	24.0	23.4	22.8
.7	28.3	28.0	27.3	26.6
.8	32.4	32.0	31.2	30.4
.9	36.4	36.0	35.1	34.2

	$3\bar{7}$	37	36	35
.1	3.7	3.7	3.6	3.5
.2	7.5	7.4	7.2	7.0
.3	11.2	11.1	10.8	10.5
.4	15.0	14.8	14.4	14.0
.5	18.7	18.5	18.0	17.5
.6	22.5	22.2	21.6	21.0
.7	26.2	25.9	25.2	24.5
.8	30.0	29.6	28.8	28.0
.9	33.7	33.3	32.4	31.5

	$3\bar{4}$	34	33	32
.1	3.4	3.4	3.3	3.2
.2	6.9	6.8	6.6	6.4
.3	10.3	10.2	9.9	9.6
.4	13.8	13.6	13.2	12.8
.5	17.2	17.0	16.5	16.0
.6	20.7	20.4	19.8	19.2
.7	24.1	23.8	23.1	22.4
.8	27.6	27.2	26.4	25.6
.9	31.0	30.6	29.7	28.8

	$3\bar{1}$	31	30	29
.1	3.1	3.1	3.0	2.9
.2	6.3	6.2	6.0	5.8
.3	9.4	9.3	9.0	8.7
.4	12.6	12.4	12.0	11.6
.5	15.7	15.5	15.0	14.5
.6	18.9	18.6	18.0	17.4
.7	22.0	21.7	21.0	20.3
.8	25.2	24.8	24.0	23.2
.9	28.3	27.9	27.0	26.1

N.	0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	753	782	811	840	869
151	897	926	955	984	*012	*041	*070	*098	*127	*156
152	18 184	213	241	270	298	327	355	384	412	440
153	469	497	526	554	582	611	639	667	695	724
154	752	780	808	836	864	893	921	949	977	*005
155	19 033	061	089	117	145	173	201	229	256	284
156	312	340	368	396	423	451	479	507	534	562
157	590	617	645	673	700	728	755	783	810	838
158	865	893	920	948	975	*003	*030	*057	*085	*112
159	20 139	167	194	221	249	276	303	330	357	385
160	412	439	466	493	520	547	574	601	628	655
161	682	709	736	763	790	817	844	871	898	924
162	951	978	*005	*032	*058	*085	*112	*139	*165	*192
163	21 219	245	272	298	325	352	378	405	431	458
164	484	511	537	564	590	616	643	669	695	722
165	748	774	801	827	853	880	906	932	958	984
166	22 011	037	063	089	115	141	167	193	219	245
167	271	297	323	349	375	401	427	453	479	505
168	531	557	582	608	634	660	686	711	737	763
169	788	814	840	865	891	917	942	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
171	299	325	350	375	401	426	451	477	502	527
172	553	578	603	628	653	679	704	729	754	779
173	804	829	855	880	905	930	955	980	*005	*030
174	24 055	080	105	129	154	179	204	229	254	279
175	304	328	353	378	403	427	452	477	502	526
176	551	576	600	625	650	674	699	723	748	773
177	797	822	846	871	895	920	944	968	993	*017
178	25 042	066	091	115	139	164	188	212	237	261
179	285	309	334	358	382	406	430	455	479	503
180	527	551	575	599	623	647	672	696	720	744
181	768	792	816	840	863	887	911	935	959	983
182	26 007	031	055	078	102	126	150	174	197	221
183	245	269	292	316	340	363	387	411	434	458
184	482	505	529	552	576	599	623	646	670	693
185	717	740	764	787	811	834	858	881	904	928
186	951	974	998	*021	*044	*068	*091	*114	*137	*161
187	27 184	207	230	254	277	300	323	346	369	392
188	416	439	462	485	508	531	554	577	600	623
189	646	669	692	715	738	761	784	806	829	852
190	875	898	921	944	966	989	*012	*035	*058	*080
191	28 103	126	149	171	194	217	239	262	285	307
192	330	352	375	398	420	443	465	488	510	533
193	555	578	600	623	645	668	690	713	735	758
194	780	802	825	847	869	892	914	936	959	981
195	29 003	025	048	070	092	114	137	159	181	203
196	225	248	270	292	314	336	358	380	402	424
197	446	468	490	512	534	556	578	600	622	644
198	666	688	710	732	754	776	798	820	841	863
199	885	907	929	950	972	994	*016	*038	*059	*081
200	30 103	124	146	168	190	211	233	254	276	298
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	29	28	27
.1	2.9	2.8	2.7
.2	5.8	5.6	5.4
.3	8.7	8.4	8.1
.4	11.6	11.2	10.8
.5	14.5	14.0	13.5
.6	17.4	16.8	16.2
.7	20.3	19.6	18.9
.8	23.2	22.4	21.6
.9	26.1	25.2	24.3

	$2\bar{6}$	26
.1	2.6	2.6
.2	5.3	5.2
.3	7.9	7.8
.4	10.6	10.4
.5	13.2	13.0
.6	15.9	15.6
.7	18.5	18.2
.8	21.2	20.8
.9	23.8	23.4

	$2\bar{5}$	25	24
.1	2.5	2.5	2.4
.2	5.1	5.0	4.8
.3	7.6	7.5	7.2
.4	10.2	10.0	9.6
.5	12.7	12.5	12.0
.6	15.3	15.0	14.4
.7	17.8	17.5	16.8
.8	20.4	20.0	19.2
.9	22.9	22.5	21.6

	$2\bar{3}$	23
.1	2.3	2.3
.2	4.7	4.6
.3	7.0	6.9
.4	9.4	9.2
.5	11.7	11.5
.6	14.1	13.8
.7	16.4	16.1
.8	18.8	18.4
.9	21.1	20.7

	$2\bar{2}$	22	$2\bar{1}$
.1	2.2	2.2	2.1
.2	4.5	4.4	4.3
.3	6.7	6.6	6.4
.4	9.0	8.8	8.6
.5	11.2	11.0	10.7
.6	13.5	13.2	12.9
.7	15.7	15.4	15.0
.8	18.0	17.6	17.2
.9	20.2	19.8	19.3

P. P.

N.		0	1	2	3	4	5	6	7	8	9
200	30	103	124	146	168	190	211	233	254	276	298
201		319	341	363	384	406	427	449	470	492	513
202		535	556	578	599	621	642	664	685	707	728
203		749	771	792	813	835	856	878	899	920	941
204		963	984	*005	*027	*048	*069	*090	*112	*133	*154
205	31	175	196	217	239	260	281	302	323	344	365
206		386	408	429	450	471	492	513	534	555	576
207		597	618	639	660	681	702	722	743	764	785
208		806	827	848	869	890	910	931	952	973	994
209	32	014	035	056	077	097	118	139	160	180	201
210		222	242	263	284	304	325	346	366	387	407
211		428	449	469	490	510	531	551	572	592	613
212		633	654	674	695	715	736	756	776	797	817
213		838	858	878	899	919	940	960	980	*001	*021
214	33	041	061	082	102	122	142	163	183	203	223
215		244	264	284	304	324	344	365	385	405	425
216		445	465	485	505	525	546	566	586	606	626
217		646	666	686	706	726	746	766	786	806	825
218		845	865	885	905	925	945	965	985	*004	*024
219	34	044	064	084	104	123	143	163	183	203	222
220		242	262	281	301	321	341	360	380	400	419
221		439	459	478	498	518	537	557	576	596	615
222		635	655	674	694	713	733	752	772	791	811
223		830	850	869	889	908	928	947	966	986	*005
224	35	025	044	063	083	102	121	141	160	179	199
225		218	237	257	276	295	314	334	353	372	391
226		411	430	449	468	487	507	526	545	564	583
227		602	621	641	660	679	698	717	736	755	774
228		793	812	831	850	869	888	907	926	945	964
229		983	*002	*021	*040	*059	*078	*097	*116	*135	*154
230	36	173	191	210	229	248	267	286	305	323	342
231		361	380	399	417	436	455	474	492	511	530
232		549	567	586	605	623	642	661	679	698	717
233		735	754	773	791	810	828	847	866	884	903
234		921	940	958	977	996	*014	*033	*051	*070	*088
235	37	107	125	143	162	180	199	217	236	254	273
236		291	309	328	346	364	383	401	420	438	456
237		475	493	511	530	548	566	584	603	621	639
238		657	676	694	712	730	749	767	785	803	821
239		840	858	876	894	912	930	948	967	985	*003
240	38	021	039	057	075	093	111	129	147	165	183
241		201	219	237	255	273	291	309	327	345	363
242		381	399	417	435	453	471	489	507	525	543
243		560	578	596	614	632	650	667	685	703	721
244		739	757	774	792	810	828	845	863	881	899
245		916	934	952	970	987	*005	*023	*040	*058	*076
246	39	093	111	129	146	164	181	199	217	234	252
247		269	287	305	322	340	357	375	392	410	427
248		445	462	480	497	515	532	550	567	585	602
249		620	637	655	672	689	707	724	742	759	776
250		794	811	828	846	863	881	898	915	933	950
N.		0	1	2	3	4	5	6	7	8	9

P. P.

	22	21		20̄	20		19̄	19		18̄	18		17̄	17
.1	2.2	2.1	.1	2.0	2.0	.1	1.9	1.9	.1	1.8	1.8	.1	1.7	1.7
.2	4.4	4.2	.2	4.1	4.0	.2	3.9	3.8	.2	3.7	3.6	.2	3.5	3.4
.3	6.6	6.3	.3	6.1	6.0	.3	5.8	5.7	.3	5.5	5.4	.3	5.2	5.1
.4	8.8	8.4	.4	8.2	8.0	.4	7.8	7.6	.4	7.4	7.2	.4	7.0	6.8
.5	11.0	10.5	.5	10.2	10.0	.5	9.7	9.5	.5	9.2	9.0	.5	8.7	8.5
.6	13.2	12.6	.6	12.3	12.0	.6	11.7	11.4	.6	11.1	10.8	.6	10.5	10.2
.7	15.4	14.7	.7	14.3	14.0	.7	13.6	13.3	.7	12.9	12.6	.7	12.2	11.9
.8	17.6	16.8	.8	16.4	16.0	.8	15.6	15.2	.8	14.8	14.4	.8	14.0	13.6
.9	19.8	18.9	.9	18.4	18.0	.9	17.5	17.1	.9	16.6	16.2	.9	15.7	15.3

N.	0	1	2	3	4	5	6	7	8	9
250	39 794	811	828	846	863	881	898	915	933	950
251	967	984	*002	*019	*036	*054	*071	*088	*105	*123
252	40 140	157	174	191	209	226	243	260	277	295
253	312	329	346	363	380	398	415	432	449	466
254	483	500	517	534	551	569	586	603	620	637
255	654	671	688	705	722	739	756	773	790	807
256	824	841	858	875	892	908	925	942	959	976
257	993	*010	*027	*044	*061	*077	*094	*111	*128	*145
258	41 162	179	195	212	229	246	263	279	296	313
259	330	346	363	380	397	413	430	447	464	480
260	497	514	530	547	564	581	597	614	631	647
261	664	680	697	714	730	747	764	780	797	813
262	830	846	863	880	896	913	929	946	962	979
263	995	*012	*028	*045	*061	*078	*094	*111	*127	*144
264	42 160	177	193	209	226	242	259	275	292	308
265	324	341	357	373	390	406	423	439	455	472
266	488	504	521	537	553	569	586	602	618	635
267	651	667	683	700	716	732	748	765	781	797
268	813	829	846	862	878	894	910	927	943	959
269	975	991	*007	*023	*040	*056	*072	*088	*104	*120
270	43 136	152	168	184	200	216	233	249	265	281
271	297	313	329	345	361	377	393	409	425	441
272	457	473	489	505	520	536	552	568	584	600
273	616	632	648	664	680	695	711	727	743	759
274	775	791	806	822	838	854	870	886	901	917
275	933	949	965	980	996	*012	*028	*043	*059	*075
276	44 091	106	122	138	154	169	185	201	216	232
277	248	263	279	295	310	326	342	357	373	389
278	404	420	435	451	467	482	498	513	529	545
279	560	576	591	607	622	638	653	669	685	700
280	716	731	747	762	778	793	809	824	839	855
281	870	886	901	917	932	948	963	978	994	*009
282	45 025	040	055	071	086	102	117	132	148	163
283	178	194	209	224	240	255	270	286	301	316
284	332	347	362	377	393	408	423	438	454	469
285	484	499	515	530	545	560	576	591	606	621
286	636	652	667	682	697	712	727	743	758	773
287	788	803	818	833	848	864	879	894	909	924
288	939	954	969	984	999	*014	*029	*044	*059	*075
289	46 090	105	120	135	150	165	180	195	210	225
290	240	255	269	284	299	314	329	344	359	374
291	389	404	419	434	449	464	479	493	508	523
292	538	553	568	583	597	612	627	642	657	672
293	687	701	716	731	746	761	775	790	805	820
294	834	849	864	879	894	908	923	938	952	967
295	982	997	*011	*026	*041	*055	*070	*085	*100	*114
296	47 129	144	158	173	188	202	217	232	246	261
297	275	290	305	319	334	348	363	378	392	407
298	421	436	451	465	480	494	509	523	538	552
299	567	581	596	610	625	639	654	668	683	697
300	712	726	741	755	770	784	799	813	828	842
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	17	17
.1	1.7	1.7
.2	3.5	3.4
.3	5.2	5.1
.4	7.0	6.8
.5	8.7	8.5
.6	10.5	10.2
.7	12.2	11.9
.8	14.0	13.6
.9	15.7	15.3

	16	16
.1	1.6	1.6
.2	3.3	3.2
.3	4.9	4.8
.4	6.6	6.4
.5	8.2	8.0
.6	9.9	9.6
.7	11.5	11.2
.8	13.2	12.8
.9	14.8	14.4

	15	15
.1	1.5	1.5
.2	3.1	3.0
.3	4.6	4.5
.4	6.2	6.0
.5	7.7	7.5
.6	9.3	9.0
.7	10.8	10.5
.8	12.4	12.0
.9	13.9	13.5

	14	14
.1	1.4	1.4
.2	2.9	2.8
.3	4.3	4.2
.4	5.8	5.6
.5	7.2	7.0
.6	8.7	8.4
.7	10.1	9.8
.8	11.6	11.2
.9	13.0	12.6

P. P.

N.	0	1	2	3	4	5	6	7	8	9
300	47 712	726	741	755	770	784	799	813	828	842
301	856	871	885	900	914	928	943	957	972	986
302	48 000	015	029	044	058	072	087	101	115	130
303	144	158	173	187	201	216	230	244	259	273
304	287	301	316	330	344	358	373	387	401	415
305	430	444	458	472	487	501	515	529	543	558
306	572	586	600	614	629	643	657	671	685	699
307	714	728	742	756	770	784	798	812	827	841
308	855	869	883	897	911	925	939	953	967	982
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49 136	150	164	178	192	206	220	234	248	262
311	276	290	304	318	332	346	359	373	387	401
312	415	429	443	457	471	485	499	513	526	540
313	554	568	582	596	610	624	637	651	665	679
314	693	707	720	734	748	762	776	789	803	817
315	831	845	858	872	886	900	913	927	941	955
316	968	982	996	*010	*023	*037	*051	*065	*078	*092
317	50 106	119	133	147	160	174	188	201	215	229
318	242	256	270	283	297	311	324	338	352	365
319	379	392	406	420	433	447	460	474	488	501
320	515	528	542	555	569	583	596	610	623	637
321	650	664	677	691	704	718	731	745	758	772
322	785	799	812	826	839	853	866	880	893	907
323	920	933	947	960	974	987	*001	*014	*027	*041
324	51 054	068	081	094	108	121	135	148	161	175
325	188	201	215	228	242	255	268	282	295	308
326	322	335	348	361	375	388	401	415	428	441
327	455	468	481	494	508	521	534	547	561	574
328	587	600	614	627	640	653	667	680	693	706
329	719	733	746	759	772	785	798	812	825	838
330	851	864	877	891	904	917	930	943	956	969
331	983	996	*009	*022	*035	*048	*061	*074	*087	*100
332	52 114	127	140	153	166	179	192	205	218	231
333	244	257	270	283	296	309	322	335	348	361
334	374	387	400	413	426	439	452	465	478	491
335	504	517	530	543	556	569	582	595	608	621
336	634	647	660	672	685	698	711	724	737	750
337	763	776	789	801	814	827	840	853	866	879
338	891	904	917	930	943	956	968	981	994	*007
339	53 020	033	045	058	071	084	097	109	122	135
340	148	160	173	186	199	211	224	237	250	262
341	275	288	301	313	326	339	352	364	377	390
342	402	415	428	440	453	466	478	491	504	516
343	529	542	554	567	580	592	605	618	630	643
344	656	668	681	693	706	719	731	744	756	769
345	782	794	807	819	832	845	857	870	882	895
346	907	920	932	945	958	970	983	995	*008	*020
347	54 033	045	058	070	083	095	108	120	133	145
348	158	170	183	195	208	220	232	245	257	270
349	282	295	307	320	332	344	357	369	382	394
350	407	419	431	444	456	469	481	493	506	518
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	$\bar{14}$	14
.1	1.4	1.4
.2	2.9	2.8
.3	4.3	4.2
.4	5.8	5.6
.5	7.2	7.0
.6	8.7	8.4
.7	10.1	9.8
.8	11.6	11.2
.9	13.0	12.6

	$\bar{13}$	13
.1	1.3	1.3
.2	2.7	2.6
.3	4.0	3.9
.4	5.4	5.2
.5	6.7	6.5
.6	8.1	7.8
.7	9.4	9.1
.8	10.8	10.4
.9	12.1	11.7

	$\bar{12}$	12
.1	1.2	1.2
.2	2.5	2.4
.3	3.7	3.6
.4	5.0	4.8
.5	6.2	6.0
.6	7.5	7.2
.7	8.7	8.4
.8	10.0	9.6
.9	11.2	10.8

N.	0	1	2	3	4	5	6	7	8	9	P. P.
350	54 407	419	431̄	444	456̄	469	481	493̄	506	518̄	
											12̄
351	530̄	543̲	555̄	568	580̲	592̄	605	617̲	629̄	642	.1 1·2̄
352	654̲	666̄	679	691̄	703̄	716	728	740̲	753	765	.2 2·5̲
353	777̄	790̲	802	814̲	826̄	839	851	863̄	876̲	888	.3 3·7̄
354	900̄	912̄	925	937̲	949̄	961	974	986̲	998̄	*010̄	.4 5·0̲
355	55 023	035	047̄	059̄	071	084	096	108̄	120̄	133̲	.5 6·2̄
356	145	157	169̄	181̄	194	206	218	230̄	242̄	254̲	.6 7·5̲
357	267	279̲	291	303̲	315	327̄	340	352	364	376̲	.7 8·7̄
358	388̄	400̄	412̄	424̲	437	449	461	473	485	497̄	.8 10·0̲
359	509̄	521̄	533̄	545̄	558	570	582	594	606	618	.9 11·2̄
360	630̄	642̄	654̄	666̄	678̄	690̄	702̄	714̄	726̄	738̄	
											12
361	750̄	762̄	775	787	799	811	823	835	847̲	859̲	.1 1·2
362	871̲	883̲	895	907̲	919	931̲	943̲	955̲	966̄	978̄	.2 2·4
363	990̄	*002̄	*014̄	*026̄	*038̄	*050̄	*062̄	*074̲	*086̄	*098̲	.3 3·6
364	56 110̲	122	134	146	158	170̲	181̄	193̄	205̲	217̄	.4 4·8
365	229̄	241	253	265̲	277̲	288̄	300̄	312̄	324̄	336	.5 6·0
366	348	360̲	372̲	383̄	395̄	407̄	419̲	431̲	443	455̲	.6 7·2
367	466̄	478̲	490̄	502	514	525̲	537̄	549̲	561	573	.7 8·4
368	585̲	596̄	608̄	620	632̲	643̄	655̄	667̄	679̲	691̲	.8 9·6
369	702̄	714̄	726	738	749̄	761̄	773	785	796̄	808̄	.9 10·8
370	820	832	843̄	855̄	867	879	890̄	902̄	914	925̄	
											11̄
371	937̄	949	961̲	972̄	984	996̲	*007̄	*019̄	*031̲	*042̄	.1 1·1̄
372	57 054̄	066	077̄	089̄	101̲	112̄	124̲	136	147̄	159̲	.2 2·3̲
373	171	182̄	194̲	206	217̄	229	240̄	252̄	264	275̄	.3 3·4̲
374	287	299̲	310̄	322	333̄	345	357̲	368̄	380̲	391̄	.4 4·6̲
375	403	414̲	426̄	438̲	449̄	461̲	472̄	484̲	495̄	507̲	.5 5·7̄
376	519	530̄	542	553̄	565	576̄	588	599̲	611	622̄	.6 6·9̲
377	634	645̄	657	668̄	680	691̄	703	714̲	726	737̄	.7 8·0̄
378	749	760̄	772	783̄	795̲	806̄	818	829̄	841̲	852̄	.8 9·2̲
379	864	875̄	887	898̄	909̄	921	932̄	944	955̄	967	.9 10·3̄
380	978̄	990	*001	*012̄	*024	*035̄	*047	*058̄	*069̄	*081	
											11
381	58 092̄	104̲	115̄	126̄	138	149̄	161̲	172	183̄	195̲	.1 1·1
382	206̄	217̄	229̲	240̄	252	263̲	274̄	286	297̲	308̄	.2 2·2
383	320	331̲	342̄	354	365̲	376̄	388	399	410̲	422	.3 3·3
384	433	444̲	455̄	467	478̄	489̄	501̲	512	523̄	535̲	.4 4·4
385	546̲	557̄	568̄	580̲	591̲	602̄	613̄	625̲	636̲	647̄	.5 5·5
386	658̄	670̲	681̲	692̄	703̄	715	726̲	737̄	748̄	760	.6 6·6
387	771	782̄	793̄	804̲	816	827	838̄	849̲	861̲	872	.7 7·7
388	883	894̄	905̄	916̄	928̲	939	950̄	961̄	972̄	984̲	.8 8·8
389	995	*006	*017̄	*028̄	*039̄	*050̄	*062	*073	*084	*095̄	.9 9·9
390	59 106̄	117̄	128̄	140	151	162	173	184̄	195̄	206̄	
											10̄
391	217̄	229̲	240	251	262	273	284̄	295̄	306̄	317̄	.1 1·0̄
392	328̄	339̲	351̲	362	373	384̲	395̲	406	417̲	428̲	.2 2·1̲
393	439̄	450̄	461̲	472̄	483̄	494̲	505̄	516̄	527̄	538̄	.3 3·1̄
394	549̄	560̄	571̲	582̄	593̄	604̲	615̄	626̄	637̄	648̄	.4 4·2̲
395	659̄	670̲	681̲	692̄	703̄	714̲	725̄	736	747̄	758̄	.5 5·2̄
396	769̄	780̄	791̄	802̄	813̄	824̲	835̲	846̲	857̲	868̲	.6 6·3̲
397	879̲	890	901	912	923	933̄	944̄	955̲	966̄	977̄	.7 7·3̄
398	988̄	999	*010	*021	*032	*043̲	*053̄	*064̲	*075̄	*086̄	.8 8·4̲
399	60 097̄	108	119	130	141	151̄	162̄	173̄	184̄	195	.9 9·4̄
400	206	217	227̄	238̄	249̄	260̄	271	282	293	303̄	
N.	0	1	2	3	4	5	6	7	8	9	P. P.

N.	0	1	2	3	4	5	6	7	8	9
400	60 206	217	227	238	249	260	271	282	293	303
401	314	325	336	347	357	368	379	390	401	412
402	422	433	444	455	466	476	487	498	509	519
403	530	541	552	563	573	584	595	606	616	627
404	638	649	659	670	681	692	702	713	724	735
405	745	756	767	777	788	799	810	820	831	842
406	852	863	874	884	895	906	916	927	938	949
407	959	970	981	991	*002	*013	*023	*034	*044	*055
408	61 066	076	087	098	108	119	130	140	151	161
409	172	183	193	204	215	225	236	246	257	268
410	278	289	299	310	320	331	342	352	363	373
411	384	394	405	416	426	437	447	458	468	479
412	489	500	511	521	532	542	553	563	574	584
413	595	605	616	626	637	647	658	668	679	689
414	700	710	721	731	742	752	763	773	784	794
415	805	815	825	836	846	857	867	878	888	899
416	909	920	930	940	951	961	972	982	993	*003
417	62 013	024	034	045	055	065	076	086	097	107
418	117	128	138	149	159	169	180	190	200	211
419	221	232	242	252	263	273	283	294	304	314
420	325	335	345	356	366	376	387	397	407	418
421	428	438	449	459	469	480	490	500	510	521
422	531	541	552	562	572	582	593	603	613	624
423	634	644	654	665	675	685	695	706	716	726
424	736	747	757	767	777	788	798	808	818	828
425	839	849	859	869	879	890	900	910	920	931
426	941	951	961	971	981	992	*002	*012	*022	*032
427	63 043	053	063	073	083	093	104	114	124	134
428	144	154	164	175	185	195	205	215	225	235
429	245	256	266	276	286	296	306	316	326	336
430	347	357	367	377	387	397	407	417	427	437
431	447	458	468	478	488	498	508	518	528	538
432	548	558	568	578	588	598	608	618	628	639
433	649	659	669	679	689	699	709	719	729	739
434	749	759	769	779	789	799	809	819	829	839
435	849	859	869	879	889	899	909	919	928	938
436	948	958	968	978	988	998	*008	*018	*028	*038
437	64 048	058	068	078	088	098	107	117	127	137
438	147	157	167	177	187	197	207	217	226	236
439	246	256	266	276	286	296	306	315	325	335
440	345	355	365	375	384	394	404	414	424	434
441	444	453	463	473	483	493	503	512	522	532
442	542	552	562	571	581	591	601	611	621	630
443	640	650	660	670	679	689	699	709	718	728
444	738	748	758	767	777	787	797	806	816	826
445	836	846	855	865	875	885	894	904	914	923
446	933	943	953	962	972	982	992	*001	*011	*021
447	65 031	040	050	060	069	079	089	098	108	118
448	128	137	147	157	166	176	186	195	205	215
449	224	234	244	253	263	273	282	292	302	311
450	321	331	340	350	360	369	379	389	398	408
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	11
.1	1.1
.2	2.2
.3	3.3
.4	4.4
.5	5.5
.6	6.6
.7	7.7
.8	8.8
.9	9.9

	$1\bar{0}$
.1	1.0
.2	2.1
.3	3.1
.4	4.2
.5	5.2
.6	6.3
.7	7.3
.8	8.4
.9	9.4

	10
.1	1.0
.2	2.0
.3	3.0
.4	4.0
.5	5.0
.6	6.0
.7	7.0
.8	8.0
.9	9.0

	$\bar{9}$
.1	0.9
.2	1.9
.3	2.8
.4	3.8
.5	4.7
.6	5.7
.7	6.6
.8	7.6
.9	8.5

N.	0	1	2	3	4	5	6	7	8	9
450	65 321	331	340	350	360	369	379	389	398	408
451	417	427	437	446	456	466	475	485	494	504
452	514	523	533	542	552	562	571	581	590	600
453	610	619	629	638	648	657	667	677	686	696
454	705	715	724	734	744	753	763	772	782	791
455	801	810	820	830	839	849	858	868	877	887
456	896	906	915	925	934	944	953	963	972	982
457	991	*001	*010	*020	*029	*039	*048	*058	*067	*077
458	66 086	096	105	115	124	134	143	153	162	172
459	181	190	200	209	219	228	238	247	257	266
460	276	285	294	304	313	323	332	342	351	360
461	370	379	389	398	408	417	426	436	445	455
462	464	473	483	492	502	511	520	530	539	548
463	558	567	577	586	595	605	614	623	633	642
464	652	661	670	680	689	698	708	717	726	736
465	745	754	764	773	782	792	801	810	820	829
466	838	848	857	866	876	885	894	904	913	922
467	931	941	950	959	969	978	987	996	*006	*015
468	67 024	034	043	052	061	071	080	089	099	108
469	117	126	136	145	154	163	173	182	191	200
470	210	219	228	237	246	256	265	274	283	293
471	302	311	320	329	339	348	357	366	376	385
472	394	403	412	422	431	440	449	458	467	477
473	486	495	504	513	523	532	541	550	559	568
474	578	587	596	605	614	623	633	642	651	660
475	669	678	687	697	706	715	724	733	742	751
476	760	770	779	788	797	806	815	824	833	842
477	852	861	870	879	888	897	906	915	924	933
478	943	952	961	970	979	988	997	*006	*015	*024
479	68 033	042	051	060	070	079	088	097	106	115
480	124	133	142	151	160	169	178	187	196	205
481	214	223	232	241	250	259	268	277	286	295
482	304	313	322	331	340	349	358	367	376	385
483	394	403	412	421	430	439	448	457	466	475
484	484	493	502	511	520	529	538	547	556	565
485	574	583	592	601	610	619	628	637	646	654
486	663	672	681	690	699	708	717	726	735	744
487	753	762	770	779	788	797	806	815	824	833
488	842	851	860	868	877	886	895	904	913	922
489	931	940	948	957	966	975	984	993	*002	*010
490	69 019	028	037	046	055	064	073	081	090	099
491	108	117	126	134	143	152	161	170	179	187
492	196	205	214	223	232	240	249	258	267	276
493	284	293	302	311	320	328	337	346	355	364
494	372	381	390	399	408	416	425	434	443	451
495	460	469	478	487	495	504	513	522	530	539
496	548	557	565	574	583	592	600	609	618	627
497	635	644	653	662	670	679	688	697	705	714
498	723	731	740	749	758	766	775	784	792	801
499	810	819	827	836	845	853	862	871	879	888
500	897	905	914	923	931	940	949	958	966	975
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	10	9	9	8
.1	1.0	0.9	0.9	0.8
.2	2.0	1.9	1.8	1.7
.3	3.0	2.8	2.7	2.5
.4	4.0	3.8	3.6	3.4
.5	5.0	4.7	4.5	4.2
.6	6.0	5.7	5.4	5.1
.7	7.0	6.6	6.3	5.9
.8	8.0	7.6	7.2	6.8
.9	9.0	8.5	8.1	7.6

P. P.

N.	0	1	2	3	4	5	6	7	8	9
500	69 897	905	914	923	931	940	949	958	966	975
501	984	992	*001	*010	*018	*027	*036	*044	*053	*061
502	70 070	079	087	096	105	113	122	131	139	148
503	157	165	174	182	191	200	208	217	226	234
504	243	251	260	269	277	286	294	303	312	320
505	329	337	346	355	363	372	380	389	398	406
506	415	423	432	441	449	458	466	475	483	492
507	501	509	518	526	535	543	552	560	569	578
508	586	595	603	612	620	629	637	646	654	663
509	672	680	689	697	706	714	723	731	740	748
510	757	765	774	782	791	799	808	816	825	833
511	842	850	859	867	876	884	893	901	910	918
512	927	935	944	952	961	969	978	986	995	*003
513	71 011	020	028	037	045	054	062	071	079	088
514	096	105	113	121	130	138	147	155	164	172
515	180	189	197	206	214	223	231	239	248	256
516	265	273	282	290	298	307	315	324	332	340
517	349	357	366	374	382	391	399	408	416	424
518	433	441	449	458	466	475	483	491	500	508
519	516	525	533	542	550	558	567	575	583	592
520	600	608	617	625	633	642	650	659	667	675
521	684	692	700	709	717	725	734	742	750	758
522	767	775	783	792	800	808	817	825	833	842
523	850	858	867	875	883	891	900	908	916	925
524	933	941	949	958	966	974	983	991	999	*007
525	72 016	024	032	040	049	057	065	074	082	090
526	098	107	115	123	131	140	148	156	164	173
527	181	189	197	206	214	222	230	238	247	255
528	263	271	280	288	296	304	312	321	329	337
529	345	354	362	370	378	386	395	403	411	419
530	427	436	444	452	460	468	476	485	493	501
531	509	517	526	534	542	550	558	566	575	583
532	591	599	607	615	624	632	640	648	656	664
533	672	681	689	697	705	713	721	729	738	746
534	754	762	770	778	786	795	803	811	819	827
535	835	843	851	859	868	876	884	892	900	908
536	916	924	932	941	949	957	965	973	981	989
537	997	*005	*013	*021	*030	*038	*046	*054	*062	*070
538	73 078	086	094	102	110	118	126	134	143	151
539	159	167	175	183	191	199	207	215	223	231
540	239	247	255	263	271	279	287	295	303	311
541	319	328	336	344	352	360	368	376	384	392
542	400	408	416	424	432	440	448	456	464	472
543	480	488	496	504	512	520	528	536	544	552
544	560	568	576	584	592	600	608	615	623	631
545	639	647	655	663	671	679	687	695	703	711
546	719	727	735	743	751	759	767	775	783	791
547	798	806	814	822	830	838	846	854	862	870
548	878	886	894	902	909	917	925	933	941	949
549	957	965	973	981	989	997	*004	*012	*020	*028
550	74 036	044	052	060	068	075	083	091	099	107
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	9
·1	0·9
·2	1·8
·3	2·7
·4	3·6
·5	4·5
·6	5·4
·7	6·3
·8	7·2
·9	8·1

	$\bar{8}$
·1	0·8
·2	1·7
·3	2·5
·4	3·4
·5	4·2
·6	5·1
·7	5·9
·8	6·8
·9	7·6

	8
·1	0·8
·2	1·6
·3	2·4
·4	3·2
·5	4·0
·6	4·8
·7	5·6
·8	6·4
·9	7·2

	$\bar{7}$
·1	0·7
·2	1·5
·3	2·2
·4	3·0
·5	3·7
·6	4·5
·7	5·2
·8	6·0
·9	6·7

N.	0	1	2	3	4	5	6	7	8	9
550	74 036̄	044	052	060	068	075̄	083̄	091̄	099̄	107̄
551	115	123	131̲	139̲	146̄	154̄	162̄	170̄	178	186̲
552	194̲	202	209̄	217̄	225̄	233	241̲	249̲	257̲	264̄
553	272̄	280	288̲	296̲	304̲	312	319̄	327̄	335̄	343̲
554	351̲	359	366̄	374̄	382̄	390̲	398	406	413̄	421̄
555	429̄	437̲	445	453	460̄	468̲	476̲	484	492	499̄
556	507̄	515̄	523	531	538̄	546̄	554̄	562	570	577̄
557	585̄	593̄	601̄	609	616̄	624̄	632̲	640	648	655̄
558	663̄	671	679̲	687̲	694̄	702̄	710	718	725̄	733̄
559	741̲	749	756̄	764̄	772	780	788	795̄	803̄	811
560	819	826̄	834̄	842	850	857̄	865̄	873	881	888̄
561	896̄	904̲	912	919̄	927̲	935	942̄	950̄	958̲	966
562	973̄	981̄	989̲	997	*004̄	*012̄	*020	*027̄	*035̄	*043
563	75 051	058̄	066̄	074	081̄	089̄	097	105	112̄	120
564	128	135̄	143̄	151	158̄	166̄	174	182̲	189̄	197
565	205̲	212̄	220	228	235̄	243̄	251̲	258̄	266̄	274̲
566	281̄	289̄	297̲	304̄	312̄	320̲	327̄	335̄	343̲	350̄
567	358̄	366̲	373̄	381̄	389̲	396̄	404̲	412̲	419̄	427̲
568	435	442̄	450̲	458	465̲	473̲	480̄	488̲	496	503̄
569	511	519	526̄	534	541̄	549̄	557	564̄	572	580
570	587̄	595	602̄	610̄	618	625̄	633	641	648̄	656
571	663̄	671	679	686̄	694	701̄	709	717̲	724̄	732
572	739̄	747	755̲	762̄	770	777̄	785	792̄	800̄	808
573	815̄	823	830̄	838	846̲	853̄	861̲	868̄	876̲	883̄
574	891	899̲	906̄	914̲	921̄	929̲	936̄	944̲	951̄	959̲
575	967	974̄	982̲	989̄	997̲	*004̄	*012	*019̄	*027̲	*034̄
576	76 042̲	050	057̄	065	072̄	080	087̄	095	102̄	110̲
577	117̄	125	132̄	140̲	147̄	155̲	162̄	170̲	178	185̄
578	193	200̄	208	215̄	223	230̄	238	245̲	253	260̄
579	268	275̄	283	290̄	298	305̄	313	320̄	328	335̄
580	343	350̄	358	365̄	372̄	380	387̄	395	402̄	410
581	417̄	425	432̄	440̲	447̄	455̲	462̄	470̲	477̄	485̲
582	492̄	500̲	507	514̄	522̲	529̄	537̲	544̄	552̲	559̄
583	567̲	574̄	582	589̲	596̄	604̲	611̄	619̲	626̄	634
584	641̄	648̄	656̲	663̄	671̲	678̄	686	693̄	700̄	708̲
585	715̄	723	730̄	738	745̄	752̄	760	767̄	775	782̄
586	790	797	804̲	812	819̄	827	834	841̄	849	856̄
587	864̲	871	878̄	886	893̄	901̲	908	915̄	923	930̄
588	937̄	945	952̄	960	967̄	974̄	982	989̄	997	*004
589	77 011̄	019	026̄	033̄	041	048̄	055̄	063	070̄	078
590	085	092̄	100	107̄	114̄	122	129̄	136̄	144	151̄
591	158̄	166̲	173̄	181	188̲	195̄	203	210̲	217̄	225
592	232	239̄	247	254̲	261̄	269	276̲	283̄	291	298
593	305̄	313	320̲	327̄	335	342	349̄	356̄	364	371̄
594	378̄	386	393̄	400̄	408	415	422̄	430	437	444̄
595	451̄	459	466̄	473̄	481	488	495̄	503̲	510	517̄
596	524̄	532	539	546̄	554̲	561	568̄	575̄	583̲	590
597	597̄	604̄	612	619̄	626̄	634	641	648̄	655̄	663̲
598	670̲	677̄	684̄	692	699̲	706̄	713̄	721̲	728	735̄
599	742̄	750	757	764̄	771̄	779	786	793̄	800̄	808
600	815	822̄	829̄	837	844	851̄	853̄	866	873	880
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	8
.1	0.8
.2	1.6
.3	2.4
.4	3.2
.5	4.0
.6	4.8
.7	5.6
.8	6.4
.9	7.2

	7̄
.1	0.7̄
.2	1.5̲
.3	2.2̄
.4	3.0
.5	3.7̄
.6	4.5̲
.7	5.2̄
.8	6.0̲
.9	6.7̄

	7
.1	0.7
.2	1.4
.3	2.1
.4	2.8
.5	3.5
.6	4.2
.7	4.9
.8	5.6
.9	6.3

P. P.

N.	0	1	2	3	4	5	6	7	8	9
600	77 815	822	829	837	844	851	858	866	873	880
601	887	894	902	909	916	923	931	938	945	952
602	959	967	974	981	988	995	*003	*010	*017	*024
603	78 031	039	046	053	060	067	075	082	089	096
604	103	111	118	125	132	139	147	154	161	168
605	175	182	190	197	204	211	218	226	233	240
606	247	254	261	269	276	283	290	297	304	311
607	319	326	333	340	347	354	362	369	376	383
608	390	397	404	412	419	426	433	440	447	454
609	461	469	476	483	490	497	504	511	518	526
610	533	540	547	554	561	568	575	583	590	597
611	604	611	618	625	632	639	646	654	661	668
612	675	682	689	696	703	710	717	725	732	739
613	746	753	760	767	774	781	788	795	802	810
614	817	824	831	838	845	852	859	866	873	880
615	887	894	901	908	915	923	930	937	944	951
616	958	965	972	979	986	993	*000	*007	*014	*021
617	79 028	035	042	049	056	063	070	078	085	092
618	099	106	113	120	127	134	141	148	155	162
619	169	176	183	190	197	204	211	218	225	232
620	239	246	253	260	267	274	281	288	295	302
621	309	316	323	330	337	344	351	358	365	372
622	379	386	393	400	407	414	421	428	435	442
623	449	456	462	469	476	483	490	497	504	511
624	518	525	532	539	546	553	560	567	574	581
625	588	595	602	609	616	622	629	636	643	650
626	657	664	671	678	685	692	699	706	713	720
627	727	733	740	747	754	761	768	775	782	789
628	796	803	810	816	823	830	837	844	851	858
629	865	872	879	886	892	899	906	913	920	927
630	934	941	948	954	961	968	975	982	989	996
631	80 003	010	016	023	030	037	044	051	058	065
632	071	078	085	092	099	106	113	120	126	133
633	140	147	154	161	168	174	181	188	195	202
634	209	216	222	229	236	243	250	257	263	270
635	277	284	291	298	304	311	318	325	332	339
636	345	352	359	366	373	380	386	393	400	407
637	414	421	427	434	441	448	455	461	468	475
638	482	489	495	502	509	516	523	529	536	543
639	550	557	563	570	577	584	591	597	604	611
640	618	625	631	638	645	652	658	665	672	679
641	686	692	699	706	713	719	726	733	740	746
642	753	760	767	774	780	787	794	801	807	814
643	821	828	834	841	848	855	861	868	875	882
644	888	895	902	909	915	922	929	936	942	949
645	956	962	969	976	983	989	996	*003	*010	*016
646	81 023	030	036	043	050	057	063	070	077	083
647	090	097	104	110	117	124	130	137	144	151
648	157	164	171	177	184	191	197	204	211	218
649	224	231	238	244	251	258	264	271	278	284
650	291	298	304	311	318	324	331	338	345	351
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	7
.1	0·7
.2	1·5
.3	2·2
.4	3·0
.5	3·7
.6	4·5
.7	5·2
.8	6·0
.9	6·7

	7
.1	0·7
.2	1·4
.3	2·1
.4	2·8
.5	3·5
.6	4·2
.7	4·9
.8	5·6
.9	6·3

	6
.1	0·6
.2	1·3
.3	1·9
.4	2·6
.5	3·2
.6	3·9
.7	4·5
.8	5·2
.9	5·8

P. P.

N.		0	1	2	3	4	5	6	7	8	9
650	81	291	298	304	311	318	324	331	338	345	351
651		358	365	371	378	385	391	398	405	411	418
652		425	431	438	444	451	458	464	471	478	484
653		491	498	504	511	518	524	531	538	544	551
654		558	564	571	577	584	591	597	604	611	617
655		624	631	637	644	650	657	664	670	677	684
656		690	697	703	710	717	723	730	736	743	750
657		756	763	770	776	783	789	796	803	809	816
658		822	829	836	842	849	855	862	869	875	882
659		888	895	901	908	915	921	928	934	941	948
660		954	961	967	974	980	987	994	*000	*007	*013
661	82	020	026	033	040	046	053	059	066	072	079
662		086	092	099	105	112	118	125	131	138	145
663		151	158	164	171	177	184	190	197	203	210
664		217	223	230	236	243	249	256	262	269	275
665		282	288	295	302	308	315	321	328	334	341
666		347	354	360	367	373	380	386	393	399	406
667		412	419	425	432	438	445	451	458	464	471
668		477	484	490	497	503	510	516	523	529	536
669		542	549	555	562	568	575	581	588	594	601
670		607	614	620	627	633	640	646	653	659	666
671		672	678	685	691	698	704	711	717	724	730
672		737	743	750	756	763	769	775	782	788	795
673		801	808	814	821	827	834	840	846	853	859
674		866	872	879	885	892	898	904	911	917	924
675		930	937	943	949	956	962	969	975	982	988
676		994	*001	*007	*014	*020	*027	*033	*039	*046	*052
677	83	059	065	071	078	084	091	097	103	110	116
678		123	129	136	142	148	155	161	168	174	180
679		187	193	200	206	212	219	225	231	238	244
680		251	257	263	270	276	283	289	295	302	308
681		314	321	327	334	340	346	353	359	365	372
682		378	385	391	397	404	410	416	423	429	435
683		442	448	455	461	467	474	480	486	493	499
684		505	512	518	524	531	537	543	550	556	562
685		569	575	581	588	594	600	607	613	619	626
686		632	638	645	651	657	664	670	676	683	689
687		695	702	708	714	721	727	733	740	746	752
688		759	765	771	778	784	790	796	803	809	815
689		822	828	834	841	847	853	859	866	872	878
690		885	891	897	904	910	916	922	929	935	941
691		948	954	960	966	973	979	985	992	998	*004
692	84	010	017	023	029	035	042	048	054	061	067
693		073	079	086	092	098	104	111	117	123	129
694		136	142	148	154	161	167	173	179	186	192
695		198	204	211	217	223	229	236	242	248	254
696		261	267	273	279	286	292	298	304	311	317
697		323	329	335	342	348	354	360	367	373	379
698		385	392	398	404	410	416	423	429	435	441
699		447	454	460	466	472	479	485	491	497	503
700		510	516	522	528	534	541	547	553	559	565
N.		0	1	2	3	4	5	6	7	8	9

P. P.

	7
.1	0.7
.2	1.4
.3	2.1
.4	2.8
.5	3.5
.6	4.2
.7	4.9
.8	5.6
.9	6.3

	6
.1	0.6
.2	1.3
.3	1.9
.4	2.6
.5	3.2
.6	3.9
.7	4.5
.8	5.2
.9	5.8

	6
.1	0.6
.2	1.2
.3	1.8
.4	2.4
.5	3.0
.6	3.6
.7	4.2
.8	4.8
.9	5.4

P. P.

N.	0	1	2	3	4	5	6	7	8	9
700	84 510	516	522	528	534	541	547	553	559	565
701	572	578	584	590	596	603	609	615	621	627
702	633	640	646	652	658	664	671	677	683	689
703	695	701	708	714	720	726	732	739	745	751
704	757	763	769	776	782	788	794	800	806	813
705	819	825	831	837	843	849	856	862	868	874
706	880	886	893	899	905	911	917	923	929	936
707	942	948	954	960	966	972	979	985	991	997
708	85 003	009	015	021	028	034	040	046	052	058
709	064	070	077	083	089	095	101	107	113	119
710	126	132	138	144	150	156	162	168	174	181
711	187	193	199	205	211	217	223	229	236	242
712	248	254	260	266	272	278	284	290	297	303
713	309	315	321	327	333	339	345	351	357	363
714	370	376	382	388	394	400	406	412	418	424
715	430	436	443	449	455	461	467	473	479	485
716	491	497	503	509	515	521	527	533	540	546
717	552	558	564	570	576	582	588	594	600	606
718	612	618	624	630	636	642	648	655	661	667
719	673	679	685	691	697	703	709	715	721	727
720	733	739	745	751	757	763	769	775	781	787
721	793	799	805	811	817	823	829	835	841	847
722	853	859	865	872	878	884	890	896	902	908
723	914	920	926	932	938	944	950	956	962	968
724	974	980	986	992	998	*004	*010	*016	*022	*028
725	86 034	040	046	052	058	063	069	075	081	087
726	093	099	105	111	117	123	129	135	141	147
727	153	159	165	171	177	183	189	195	201	207
728	213	219	225	231	237	243	249	255	261	267
729	273	278	284	290	296	302	308	314	320	326
730	332	338	344	350	356	362	368	374	380	386
731	391	397	403	409	415	421	427	433	439	445
732	451	457	463	469	475	481	486	492	498	504
733	510	516	522	528	534	540	546	552	558	563
734	569	575	581	587	593	599	605	611	617	623
735	628	634	640	646	652	658	664	670	676	682
736	688	693	699	705	711	717	723	729	735	741
737	746	752	758	764	770	776	782	788	794	800
738	805	811	817	823	829	835	841	847	852	858
739	864	870	876	882	888	894	899	905	911	917
740	923	929	935	941	946	952	958	964	970	976
741	982	987	993	999	*005	*011	*017	*023	*028	*034
742	87 040	046	052	058	064	069	075	081	087	093
743	099	104	110	116	122	128	134	140	145	151
744	157	163	169	175	180	186	192	198	204	210
745	215	221	227	233	239	245	250	256	262	268
746	274	279	285	291	297	303	309	314	320	326
747	332	338	343	349	355	361	367	372	378	384
748	390	396	402	407	413	419	425	431	436	442
749	448	454	460	465	471	477	483	489	494	500
750	506	512	517	523	529	535	541	546	552	558
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.

	$\bar{6}$
·1	0·6
·2	1·3
·3	1·9
·4	2·6
·5	3·2
·6	3·9
·7	4·5
·8	5·2
·9	5·8

	6
·1	0·6
·2	1·2
·3	1·8
·4	2·4
·5	3·0
·6	3·6
·7	4·2
·8	4·8
·9	5·4

	$\bar{5}$
·1	0·5
·2	1·1
·3	1·6
·4	2·2
·5	2·7
·6	3·3
·7	3·8
·8	4·4
·9	4·9

P. P.

N.	0	1	2	3	4	5	6	7	8	9
750	87 506	512	517	523	529	535	541	546	552	558
751	564	570	575	581	587	593	598	604	610	616
752	622	627	633	639	645	650	656	662	668	673
753	679	685	691	697	702	708	714	720	725	731
754	737	743	748	754	760	766	771	777	783	789
755	794	800	806	812	817	823	829	835	840	846
756	852	858	863	869	875	881	886	892	898	904
757	909	915	921	927	932	938	944	949	955	961
758	967	972	978	984	990	995	*001	*007	*012	*018
759	88 024	030	035	041	047	053	058	064	070	075
760	081	087	093	098	104	110	115	121	127	133
761	138	144	150	155	161	167	172	178	184	190
762	195	201	207	212	218	224	229	235	241	247
763	252	258	264	269	275	281	286	292	298	303
764	309	315	320	326	332	337	343	349	355	360
765	366	372	377	383	389	394	400	406	411	417
766	423	428	434	440	445	451	457	462	468	474
767	479	485	491	496	502	508	513	519	525	530
768	536	542	547	553	558	564	570	575	581	587
769	592	598	604	609	615	621	626	632	638	643
770	649	654	660	666	671	677	683	688	694	700
771	705	711	716	722	728	733	739	745	750	756
772	761	767	773	778	784	790	795	801	806	812
773	818	823	829	835	840	846	851	857	863	868
774	874	879	885	891	896	902	907	913	919	924
775	930	936	941	947	952	958	964	969	975	980
776	986	992	997	*003	*008	*014	*019	*025	*031	*036
777	89 042	047	053	059	064	070	075	081	087	092
778	098	103	109	114	120	126	131	137	142	148
779	153	159	165	170	176	181	187	193	198	204
780	209	215	220	226	231	237	243	248	254	259
781	265	270	276	282	287	293	298	304	309	315
782	320	326	332	337	343	348	354	359	365	370
783	376	381	387	393	398	404	409	415	420	426
784	431	437	442	448	454	459	465	470	476	481
785	487	492	498	503	509	514	520	525	531	536
786	542	548	553	559	564	570	575	581	586	592
787	597	603	608	614	619	625	630	636	641	647
788	652	658	663	669	674	680	685	691	696	702
789	707	713	718	724	729	735	740	746	751	757
790	762	768	773	779	784	790	795	801	806	812
791	817	823	828	834	839	845	850	856	861	867
792	872	878	883	889	894	900	905	911	916	922
793	927	933	938	943	949	954	960	965	971	976
794	982	987	993	998	*004	*009	*015	*020	*026	*031
795	90 036	042	047	053	058	064	069	075	080	086
796	091	097	102	107	113	118	124	129	135	140
797	146	151	156	162	167	173	178	184	189	195
798	200	205	211	216	222	227	233	238	244	249
799	254	260	265	271	276	282	287	292	298	303
800	309	314	320	325	330	336	341	347	352	358
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	6
.1	0.6
.2	1.2
.3	1.8
.4	2.4
.5	3.0
.6	3.6
.7	4.2
.8	4.8
.9	5.4

	5
.1	0.5
.2	1.1
.3	1.6
.4	2.2
.5	2.7
.6	3.3
.7	3.8
.8	4.4
.9	4.9

	5
.1	0.5
.2	1.0
.3	1.5
.4	2.0
.5	2.5
.6	3.0
.7	3.5
.8	4.0
.9	4.5

P. P.

N.	0	1	2	3	4	5	6	7	8	9
800	90 309	314	320	325	330	336	341	347	352	358
801	363	368	374	379	385	390	396	401	406	412
802	417	423	428	433	439	444	450	455	460	466
803	471	477	482	488	493	498	504	509	515	520
804	525	531	536	542	547	552	558	563	569	574
805	579	585	590	596	601	606	612	617	622	628
806	633	639	644	649	655	660	666	671	676	682
807	687	692	698	703	709	714	719	725	730	736
808	741	746	752	757	762	768	773	778	784	789
809	795	800	805	811	816	821	827	832	838	843
810	848	854	859	864	870	875	880	886	891	896
811	902	907	913	918	923	929	934	939	945	950
812	955	961	966	971	977	982	987	993	998	*003
813	91 009	014	019	025	030	036	041	046	052	057
814	062	068	073	078	084	089	094	100	105	110
815	116	121	126	131	137	142	147	153	158	163
816	169	174	179	185	190	195	201	206	211	217
817	222	227	233	238	243	249	254	259	264	270
818	275	280	286	291	296	302	307	312	318	323
819	328	333	339	344	349	355	360	365	371	376
820	381	386	392	397	402	408	413	418	423	429
821	434	439	445	450	455	461	466	471	476	482
822	487	492	497	503	508	513	519	524	529	534
823	540	545	550	556	561	566	571	577	582	587
824	592	598	603	608	614	619	624	629	635	640
825	645	650	656	661	666	671	677	682	687	692
826	698	703	708	714	719	724	729	735	740	745
827	750	756	761	766	771	777	782	787	792	798
828	803	808	813	819	824	829	834	839	845	850
829	855	860	866	871	876	881	887	892	897	902
830	908	913	918	923	928	934	939	944	949	955
831	960	965	970	976	981	986	991	996	*002	*007
832	92 012	017	023	028	033	038	043	049	054	059
833	064	069	075	080	085	090	096	101	106	111
834	116	122	127	132	137	142	148	153	158	163
835	168	174	179	184	189	194	200	205	210	215
836	220	226	231	236	241	246	252	257	262	267
837	272	277	283	288	293	298	303	309	314	319
838	324	329	335	340	345	350	355	360	366	371
839	376	381	386	391	397	402	407	412	417	423
840	428	433	438	443	448	454	459	464	469	474
841	479	485	490	495	500	505	510	515	521	526
842	531	536	541	546	552	557	562	567	572	577
843	583	588	593	598	603	608	613	619	624	629
844	634	639	644	649	655	660	665	670	675	680
845	685	691	696	701	706	711	716	721	727	732
846	737	742	747	752	757	762	768	773	778	783
847	788	793	798	803	809	814	819	824	829	834
848	839	844	850	855	860	865	870	875	880	885
849	891	896	901	906	911	916	921	926	931	937
850	942	947	952	957	962	967	972	977	982	988
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	$\bar{5}$
.1	0.5
.2	1.1
.3	1.6
.4	2.2
.5	2.7
.6	3.3
.7	3.8
.8	4.4
.9	4.9

	5
.1	0.5
.2	1.0
.3	1.5
.4	2.0
.5	2.5
.6	3.0
.7	3.5
.8	4.0
.9	4.5

P. P.

N.	0	1	2	3	4	5	6	7	8	9
850	92 942	947	952	957	962	967	972	977	982	988
851	993	998	*003	*008	*013	*018	*023	*028	*034	*039
852	93 044	049	054	059	064	069	074	079	084	090
853	095	100	105	110	115	120	125	130	135	140
854	146	151	156	161	166	171	176	181	186	191
855	196	201	207	212	217	222	227	232	237	242
856	247	252	257	262	267	272	278	283	288	293
857	298	303	308	313	318	323	328	333	338	343
858	348	354	359	364	369	374	379	384	389	394
859	399	404	409	414	419	424	429	434	439	445
860	450	455	460	465	470	475	480	485	490	495
861	500	505	510	515	520	525	530	535	540	545
862	550	556	561	566	571	576	581	586	591	596
863	601	606	611	616	621	626	631	636	641	646
864	651	656	661	666	671	676	681	686	691	696
865	701	706	711	716	721	726	731	736	742	747
866	752	757	762	767	772	777	782	787	792	797
867	802	807	812	817	822	827	832	837	842	847
868	852	857	862	867	872	877	882	887	892	897
869	902	907	912	917	922	927	932	937	942	947
870	952	957	962	967	972	977	982	987	992	997
871	94 002	007	012	017	022	026	031	036	041	046
872	051	056	061	066	071	076	081	086	091	096
873	101	106	111	116	121	126	131	136	141	146
874	151	156	161	166	171	176	181	186	191	196
875	201	206	210	215	220	225	230	235	240	245
876	250	255	260	265	270	275	280	285	290	295
877	300	305	310	315	320	324	329	334	339	344
878	349	354	359	364	369	374	379	384	389	394
879	399	404	409	413	418	423	428	433	438	443
880	448	453	458	463	468	473	478	483	487	492
881	497	502	507	512	517	522	527	532	537	542
882	547	552	556	561	566	571	576	581	586	591
883	596	601	606	611	615	620	625	630	635	640
884	645	650	655	660	665	670	674	679	684	689
885	694	699	704	709	714	719	724	728	733	738
886	743	748	753	758	763	768	773	777	782	787
887	792	797	802	807	812	817	821	826	831	836
888	841	846	851	856	861	865	870	875	880	885
889	890	895	900	905	909	914	919	924	929	934
890	939	944	949	953	958	963	968	973	978	983
891	988	992	997	*002	*007	*012	*017	*022	*026	031
892	95 036	041	046	051	056	061	065	070	075	080
893	085	090	095	099	104	109	114	119	124	129
894	134	138	143	148	153	158	163	167	172	177
895	182	187	192	197	201	206	211	216	221	226
896	231	235	240	245	250	255	260	264	269	274
897	279	284	289	294	298	303	308	313	318	323
898	327	332	337	342	347	352	356	361	366	371
899	376	381	385	390	395	400	405	410	414	419
900	424	429	434	438	443	448	453	458	463	467
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	5
.1	0.5
.2	1.1
.3	1.6
.4	2.2
.5	2.7
.6	3.3
.7	3.8
.8	4.4
.9	4.9

	5
.1	0.5
.2	1.0
.3	1.5
.4	2.0
.5	2.5
.6	3.0
.7	3.5
.8	4.0
.9	4.5

	4
.1	0.4
.2	0.9
.3	1.3
.4	1.8
.5	2.2
.6	2.7
.7	3.1
.8	3.6
.9	4.0

P. P.

N.	0	1	2	3	4	5	6	7	8	9	P. P.
900	95 424	429	434	438	443	448	453	458	463	467	
901	472	477	482	487	492	496	501	506	511	516	
902	520	525	530	535	540	544	549	554	559	564	
903	569	573	578	583	588	593	597	602	607	612	
904	617	621	626	631	636	641	645	650	655	660	
905	665	669	674	679	684	689	693	698	703	708	
906	713	717	722	727	732	737	741	746	751	756	
907	760	765	770	775	780	784	789	794	799	804	
908	808	813	818	823	827	832	837	842	847	851	
909	856	861	866	870	875	880	885	890	894	899	
910	904	909	913	918	923	928	933	937	942	947	5
911	952	956	961	966	971	975	980	985	990	994	·1 0·5
912	999	*004	*009	*014	*018	*023	*028	*033	*037	*042	·2 1·0
913	96 047	052	056	061	066	071	075	080	085	090	·3 1·5
914	094	099	104	109	113	118	123	128	132	137	·4 2·0
915	142	147	151	156	161	166	170	175	180	185	·5 2·5
916	189	194	199	204	208	213	218	222	227	232	·6 3·0
917	237	241	246	251	256	260	265	270	275	279	·7 3·5
918	284	289	293	298	303	308	312	317	322	327	·8 4·0
919	331	336	341	345	350	355	360	364	369	374	·9 4·5
920	379	383	388	393	397	402	407	412	416	421	
921	426	430	435	440	445	449	454	459	463	468	
922	473	478	482	487	492	496	501	506	511	515	
923	520	525	529	534	539	543	548	553	558	562	
924	567	572	576	581	586	590	595	600	605	609	
925	614	619	623	628	633	637	642	647	651	656	
926	661	666	670	675	680	684	689	694	698	703	
927	708	712	717	722	726	731	736	741	745	750	
928	755	759	764	769	773	778	783	787	792	797	
929	801	806	811	815	820	825	829	834	839	843	
930	848	853	857	862	867	871	876	881	885	890	4
931	895	899	904	909	913	918	923	927	932	937	·1 0·4
932	941	946	951	955	960	965	969	974	979	983	·2 0·9
933	988	993	997	*002	*007	*011	*016	*020	*025	*030	·3 1·3
934	97 034	039	044	048	053	058	062	067	072	076	·4 1·8
935	081	086	090	095	099	104	109	113	118	123	·5 2·2
936	127	132	137	141	146	151	155	160	164	169	·6 2·7
937	174	178	183	188	192	197	202	206	211	215	·7 3·1
938	220	225	229	234	239	243	248	252	257	262	·8 3·6
939	266	271	276	280	285	289	294	299	303	308	·9 4·0
940	313	317	322	326	331	336	340	345	349	354	
941	359	363	368	373	377	382	386	391	396	400	
942	405	409	414	419	423	428	432	437	442	446	
943	451	456	460	465	469	474	479	483	488	492	
944	497	502	506	511	515	520	525	529	534	538	
945	543	548	552	557	561	566	570	575	580	584	
946	589	593	598	603	607	612	616	621	626	630	
947	635	639	644	649	653	658	662	667	671	676	
948	681	685	690	694	699	703	708	713	717	722	
949	726	731	736	740	745	749	754	758	763	768	
950	772	777	781	786	790	795	800	804	809	813	
N.	0	1	2	3	4	5	6	7	8	9	P. P.

N.	0	1	2	3	4	5	6	7	8	9
950	97 77$\bar{2}$	777	78$\bar{1}$	786	79$\bar{0}$	795	800	80$\bar{4}$	809	81$\bar{3}$
951	818	82$\bar{2}$	827	83$\bar{1}$	83$\bar{6}$	841	84$\bar{5}$	850	85$\bar{4}$	859
952	86$\bar{3}$	86$\bar{8}$	873	87$\bar{7}$	882	88$\bar{6}$	891	89$\bar{5}$	900	90$\bar{4}$
953	90$\bar{9}$	914	91$\bar{8}$	923	92$\bar{7}$	932	93$\bar{6}$	941	94$\bar{5}$	95$\bar{0}$
954	955	95$\bar{9}$	964	96$\bar{8}$	973	97$\bar{7}$	982	98$\bar{6}$	991	996
955	98 00$\bar{0}$	005	00$\bar{9}$	014	01$\bar{8}$	023	02$\bar{7}$	032	03$\bar{6}$	041
956	046	05$\bar{0}$	055	05$\bar{9}$	064	06$\bar{8}$	073	07$\bar{7}$	082	08$\bar{6}$
957	091	09$\bar{5}$	10$\bar{0}$	105	10$\bar{9}$	114	11$\bar{8}$	123	12$\bar{7}$	132
958	13$\bar{6}$	141	14$\bar{5}$	150	15$\bar{4}$	159	16$\bar{3}$	16$\bar{8}$	173	17$\bar{7}$
959	182	18$\bar{6}$	191	19$\bar{5}$	200	20$\bar{4}$	209	21$\bar{3}$	218	22$\bar{2}$
960	227	23$\bar{1}$	236	24$\bar{0}$	245	24$\bar{9}$	25$\bar{4}$	259	26$\bar{3}$	268
961	27$\bar{2}$	277	28$\bar{1}$	286	29$\bar{0}$	295	29$\bar{9}$	304	30$\bar{8}$	313
962	31$\bar{7}$	322	32$\bar{6}$	331	33$\bar{5}$	340	34$\bar{4}$	349	35$\bar{3}$	358
963	36$\bar{2}$	367	37$\bar{1}$	376	38$\bar{0}$	385	38$\bar{9}$	394	39$\bar{8}$	403
964	40$\bar{7}$	412	41$\bar{6}$	421	42$\bar{5}$	430	43$\bar{4}$	439	44$\bar{3}$	448
965	45$\bar{2}$	457	46$\bar{1}$	466	47$\bar{0}$	475	47$\bar{9}$	484	48$\bar{8}$	493
966	49$\bar{7}$	502	50$\bar{6}$	511	51$\bar{5}$	520	52$\bar{4}$	529	53$\bar{3}$	538
967	54$\bar{2}$	547	55$\bar{1}$	556	56$\bar{0}$	565	56$\bar{9}$	574	57$\bar{8}$	583
968	58$\bar{7}$	592	59$\bar{6}$	601	60$\bar{5}$	610	61$\bar{4}$	619	62$\bar{3}$	628
969	63$\bar{2}$	637	64$\bar{1}$	646	65$\bar{0}$	655	65$\bar{9}$	66$\bar{3}$	668	67$\bar{2}$
970	677	68$\bar{1}$	686	69$\bar{0}$	695	69$\bar{9}$	704	70$\bar{8}$	713	71$\bar{7}$
971	722	72$\bar{6}$	731	73$\bar{5}$	740	74$\bar{4}$	749	753	75$\bar{7}$	762
972	76$\bar{6}$	771	77$\bar{5}$	780	78$\bar{4}$	789	79$\bar{3}$	798	80$\bar{2}$	807
973	81$\bar{1}$	81$\bar{5}$	820	82$\bar{4}$	829	83$\bar{3}$	838	84$\bar{2}$	847	85$\bar{1}$
974	856	86$\bar{0}$	865	86$\bar{9}$	87$\bar{3}$	878	88$\bar{2}$	887	89$\bar{1}$	896
975	90$\bar{0}$	905	90$\bar{9}$	914	91$\bar{8}$	92$\bar{2}$	927	93$\bar{1}$	936	94$\bar{0}$
976	945	94$\bar{9}$	954	95$\bar{8}$	963	967	97$\bar{1}$	976	98$\bar{0}$	985
977	98$\bar{9}$	994	99$\bar{8}$	*003	*007	*01$\bar{1}$	*016	*02$\bar{0}$	*025	*02$\bar{9}$
978	99 034	03$\bar{8}$	043	047	05$\bar{1}$	056	06$\bar{0}$	065	06$\bar{9}$	074
979	07$\bar{8}$	08$\bar{2}$	087	09$\bar{1}$	096	10$\bar{0}$	105	10$\bar{9}$	11$\bar{3}$	118
980	12$\bar{2}$	127	13$\bar{1}$	136	14$\bar{0}$	145	149	15$\bar{3}$	158	16$\bar{2}$
981	167	17$\bar{1}$	176	180	18$\bar{4}$	189	19$\bar{3}$	198	20$\bar{2}$	20$\bar{6}$
982	211	21$\bar{5}$	220	22$\bar{4}$	229	23$\bar{3}$	23$\bar{7}$	242	24$\bar{6}$	251
983	25$\bar{5}$	260	264	26$\bar{8}$	273	27$\bar{7}$	282	28$\bar{6}$	29$\bar{0}$	295
984	29$\bar{9}$	304	30$\bar{8}$	31$\bar{2}$	317	32$\bar{1}$	326	33$\bar{0}$	335	339
985	34$\bar{3}$	348	35$\bar{2}$	357	36$\bar{1}$	36$\bar{5}$	370	37$\bar{4}$	379	38$\bar{3}$
986	38$\bar{7}$	392	39$\bar{6}$	401	40$\bar{5}$	40$\bar{9}$	414	41$\bar{8}$	423	42$\bar{7}$
987	43$\bar{1}$	436	44$\bar{0}$	445	44$\bar{9}$	45$\bar{3}$	458	46$\bar{2}$	467	47$\bar{1}$
988	47$\bar{5}$	480	48$\bar{4}$	489	49$\bar{3}$	49$\bar{7}$	502	50$\bar{6}$	511	515
989	51$\bar{9}$	524	52$\bar{8}$	533	537	54$\bar{1}$	546	55$\bar{0}$	55$\bar{4}$	559
990	56$\bar{3}$	568	57$\bar{2}$	57$\bar{6}$	581	58$\bar{5}$	590	594	59$\bar{8}$	603
991	60$\bar{7}$	61$\bar{1}$	616	62$\bar{0}$	625	62$\bar{9}$	63$\bar{3}$	638	64$\bar{2}$	647
992	651	65$\bar{5}$	660	66$\bar{4}$	66$\bar{8}$	673	67$\bar{7}$	682	686	69$\bar{0}$
993	695	69$\bar{9}$	70$\bar{3}$	708	71$\bar{2}$	717	721	72$\bar{5}$	730	73$\bar{4}$
994	73$\bar{8}$	743	74$\bar{7}$	75$\bar{1}$	756	76$\bar{0}$	765	769	77$\bar{3}$	778
995	78$\bar{2}$	78$\bar{6}$	791	79$\bar{5}$	800	804	80$\bar{8}$	813	817	82$\bar{1}$
996	826	83$\bar{0}$	83$\bar{4}$	839	84$\bar{3}$	84$\bar{7}$	852	85$\bar{6}$	861	865
997	86$\bar{9}$	874	878	88$\bar{2}$	887	89$\bar{1}$	89$\bar{5}$	900	90$\bar{4}$	90$\bar{8}$
998	913	91$\bar{7}$	922	926	93$\bar{0}$	935	939	94$\bar{3}$	948	952
999	95$\bar{6}$	961	965	96$\bar{9}$	974	97$\bar{8}$	98$\bar{2}$	987	99$\bar{1}$	99$\bar{5}$
1000	000 000	00$\bar{4}$	00$\bar{8}$	013	01$\bar{7}$	02$\bar{1}$	026	03$\bar{0}$	03$\bar{4}$	039
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.

	5
.1	0.5
.2	1.0
.3	1.5
.4	2.0
.5	2.5
.6	3.0
.7	3.5
.8	4.0
.9	4.5

	$\bar{4}$
.1	0.$\bar{4}$
.2	0.9
.3	1.$\bar{3}$
.4	1.8
.5	2.$\bar{2}$
.6	2.7
.7	3.$\bar{1}$
.8	3.6
.9	4.$\bar{0}$

	4
.1	0.4
.2	0.8
.3	1.2
.4	1.6
.5	2.0
.6	2.4
.7	2.8
.8	3.2
.9	3.6

P. P.

N.	0		1	2	3	4	5	6	7	8	9
1000	000	000	043̄	087	130̄	173̄	217	260̄	304	347̄	390̄
01		434̲	477̄	521̲	564̲	607̄	651̲	694̄	737̄	781̲	824̄
02		867̄	911	954̲	997̄	*041	*084̲	*127̄	*171	*214̲	*257̄
03	001	301̲	344	387̄	431̲	474̲	517̄	560̄	604̲	647̲	690̄
04		733̄	777̲	820̲	863̲	906̄	950	993̲	*036̄	*079̲	*123
05	002	166	209̄	252̲	295̲	339̲	382	425̄	468̄	511̄	555̲
06		598̲	641̲	684̲	727̄	770̄	814	857	900̲	943̲	986̲
07	003	029̲	072̲	115̲	159	202	245	288	331̄	374̄	417̄
08		460̄	503̄	546̄	590̲	633̲	676̲	719̲	762̲	805̲	848̲
09		891	934	977	*020̄	*063̄	*106̄	*149̄	*192̄	*235̄	*278̄
1010	004	321̄	364̄	407̄	450̄	493̄	536̄	579̄	622̄	665	708
11		751̲	794̲	837̲	880	923	966	*009	*051̄	*094̲	*137̄
12	005	180̄	223̲	266̄	309	352	395	438̲	481̲	523̲	566̄
13		609̄	652̄	695̲	738̲	781	824	866̄	909̲	952̲	995̲
14	006	038	081	123̲	166̲	209	252	295̲	337̄	380̄	423̄
15		466̲	509̲	551̄	594̄	637̲	680̲	722̄	765̄	808̲	851
16		893̄	936̲	979̲	*022	*064̲	*107̲	*150	*193	*235̲	*278
17	007	321	363̲	406̄	449̲	491̲	534̄	577̲	620̲	662̄	705̲
18		748	790̄	833̲	875̄	918̲	961	*003̄	*046̲	*089	131̲
19	008	174	217	259̄	302	344̄	387	430	472̄	515	557̄
1020		600	642̄	685̄	728	770̄	813	855̄	898	940̄	983
21	009	025̄	068̲	111	153̲	196	238̲	281	323̲	366̲	408̄
22		451̲	493̄	536̲	578̄	621̲	663̄	706̲	748̲	790̄	833̲
23		875̄	918̲	960̄	*003	*045̲	*088	*130̲	*172̲	*215	*257̲
24	010	300	342̄	385̲	427	469̲	512̲	554̄	596̲	639̲	681̄
25		724̲	766̲	808̄	851̲	893̲	935̄	978̲	*020̲	*062̄	*105
26	011	147̲	189̲	232	274̲	316̲	359	401̄	443̲	486̲	528
27		570̄	612̲	655̲	697̄	739̄	782̲	824̲	866̲	908̄	951
28		993̲	*035̲	*077̄	*120	*162	*204̲	*246̲	*288̲	*331	*373
29	012	415̄	457̄	500	542	584	626̄	668̄	710̄	753	795
1030		837	879̄	921̄	963̄	*006	*048	*090	*132̄	174̄	216̄
31	013	258̲	301	343	385	427	469	511̄	553̄	595̄	637̲
32		679̲	722̲	764̲	806̲	848̲	890̲	932̲	974̲	*016̲	*058̲
33	014	100̄	142̄	184̲	226̄	268̄	310̄	352̄	394̲	436̄	478̄
34		520̄	562̄	604̲	646̄	688̄	730̄	772̄	814̄	856̄	898̄
35		940̄	982̄	*024̲	*066̄	*108̲	*150̄	*192	*234	*276	*318
36	015	360	401̄	443̄	485̲	527̄	569̄	611	653	695̲	737̲
37		779̲	820̄	862̄	904̄	946̲	988̲	*030̲	*072	*113̄	155̲
38	016	197̄	239̲	281	323	364̄	406̲	448̄	490	532	573̲
39		615̄	657̄	699	741	782̄	824̄	866̄	908	950	991̄
1040	017	033̄	075	117	158̄	200̄	242	284	325̄	367̄	409
41		450̲	492̄	534	576̲	617̄	659̄	701̲	742̄	784̄	826̲
42		867̲	909̄	951̲	992̄	*034̄	*076̲	*117̄	*159̲	*201̲	*242̄
43	018	284̲	326	367̄	409̲	451	492̄	534	575̲	617̄	659̲
44		700̲	742	783̲	825̄	867̲	908̄	950̲	991̄	*033̲	*074̄
45	019	116̲	158	199̲	241	282̄	324	365̲	407	448̲	490
46		531̄	573	614̲	656	697̄	739	780̄	822	863̲	905
47		946̲	988̲	*029̄	*071̲	*112̄	*154̲	*195̄	*237̲	*278̲	*320
48	020	361̄	402̄	444	485̄	527	568̲	610	651̄	692̄	734
49		775̄	817	858̄	899̄	941	982̄	*024	*065	*106̄	*148
1050	021	189̄	230̄	272	313̄	354̄	396	437̄	478̄	520	561̄
N.	0		1	2	3	4	5	6	7	8	9

P. P.

	43̄	43
.1	4.3̄	4.3
.2	8.7̲	8.6
.3	13.0̄	12.9
.4	17.4̲	17.2
.5	21.7̄	21.5
.6	26.1̲	25.8
.7	30.4̄	30.1
.8	34.8̲	34.4
.9	39.1̄	38.7

	42̄	42
.1	4.2̄	4.2
.2	8.5̲	8.4
.3	12.7̄	12.6
.4	17.0̲	16.8
.5	21.2̄	21.0
.6	25.5̲	25.2
.7	29.7̄	29.4
.8	34.0̲	33.6
.9	38.2̄	37.8

	41̄	41
.1	4.1̄	4.1
.2	8.3̲	8.2
.3	12.4̄	12.3
.4	16.6̲	16.4
.5	20.7̄	20.5
.6	24.9̲	24.6
.7	29.0̄	28.7
.8	33.2̲	32.8
.9	37.3̄	36.9

P. P.

N.	0		1	2	3	4	5	6	7	8	9	P.P.	
1050	021	189	230	272	313	354	396	437	478	520	561		**41**
51		602	644	685	726	768	809	850	892	933	974	.1	4.1
52	022	015	057	098	139	181	222	263	304	346	387	.2	8.3
53		428	469	511	552	593	634	676	717	758	799	.3	12.4
54		840	882	923	964	*005	*046	*088	*129	*170	*211	.4	16.6
55	023	252	293	335	376	417	458	499	540	581	623	.5	20.7
56		664	705	746	787	828	869	910	951	993	*034	.6	24.9
57	024	075	116	157	198	239	280	321	362	403	444	.7	29.0
58		485	526	568	609	650	691	732	773	814	855	.8	33.2
59		896	937	978	*019	*060	*101	*142	*183	*224	*265	.9	37.3
1060	025	306	347	388	429	469	510	551	592	633	674		**41**
61		715	756	797	838	879	920	961	*002	*042	*083	.1	4.1
62	026	124	165	206	247	288	329	370	410	451	492	.2	8.2
63		533	574	615	656	696	737	778	819	860	901	.3	12.3
64		941	982	*023	*064	*105	*145	*186	*227	*268	*309	.4	16.4
65	027	349	390	431	472	512	553	594	635	675	716	.5	20.5
66		757	798	838	879	920	961	*001	*042	*083	*123	.6	24.6
67	028	164	205	246	286	327	368	408	449	490	530	.7	28.7
68		571	612	652	693	734	774	815	856	896	937	.8	32.8
69		977	*018	*059	*099	*140	*181	*221	*262	*302	*343	.9	36.9
1070	029	384	424	465	505	546	586	627	668	708	749		**40**
71		789	830	870	911	951	992	*032	*073	*114	*154	.1	4.0
72	030	195	235	276	316	357	397	438	478	519	559	.2	8.1
73		599	640	680	721	761	802	842	883	923	964	.3	12.1
74	031	004	044	085	125	166	206	247	287	327	368	.4	16.2
75		408	449	489	529	570	610	651	691	731	772	.5	20.2
76		812	852	893	933	973	*014	*054	*094	*135	*175	.6	24.3
77	032	215	256	296	336	377	417	457	498	538	578	.7	28.3
78		619	659	699	739	780	820	860	900	941	981	.8	32.4
79	033	021	061	102	142	182	222	263	303	343	383	.9	36.4
1080		424	464	504	544	584	625	665	705	745	785		**40**
81		825	866	906	946	986	*026	*066	*107	147	187	.1	4.0
82	034	227	267	307	347	388	428	468	508	548	588	.2	8.0
83		628	668	708	748	789	829	869	909	949	989	.3	12.0
84	035	029	069	109	149	189	229	269	309	349	389	.4	16.0
85		429	470	510	550	590	630	670	710	750	790	.5	20.0
86		830	870	910	950	990	*029	*069	*109	*149	*189	.6	24.0
87	036	229	269	309	349	389	429	469	509	549	589	.7	28.0
88		629	669	708	748	788	828	868	908	948	988	.8	32.0
89	037	028	068	107	147	187	227	267	307	347	386	.9	36.0
1090		426	466	506	546	586	625	665	705	745	785		**39**
91		825	864	904	944	984	*023	*063	*103	143	183	.1	3.9
92	038	222	262	302	342	381	421	461	501	540	580	.2	7.9
93		620	660	699	739	779	819	858	898	938	977	.3	11.8
94	039	017	057	096	136	176	216	255	295	335	374	.4	15.8
95		414	454	493	533	572	612	652	691	731	771	.5	19.7
96		810	850	890	929	969	*008	*048	*088	*127	*167	.6	23.7
97	040	206	246	286	325	365	404	444	483	523	563	.7	27.6
98		602	642	681	721	760	800	839	879	918	958	.8	31.6
99		997	*037	*076	*116	*155	*195	*234	*274	*313	*353	.9	35.5
1100	041	392	432	471	511	550	590	629	669	708	748		
N.	0		1	2	3	4	5	6	7	8	9	P.P.	

Log sin ϕ = log ϕ'' + S. log ϕ'' = log sin ϕ + S'.
Log tan ϕ = log ϕ'' + T. log ϕ'' = log tan ϕ + T'.

0°

″	′	S	T	Log. Sin.	S′	T′	Log. Tan.
0	**0**	4·685 5$\bar{7}$	5$\bar{7}$	— ∞	5·314 4$\bar{2}$	4$\bar{2}$	— ∞
60	1	5$\bar{7}$	5$\bar{7}$	6·46 37$\bar{2}$	4$\bar{2}$	4$\bar{2}$	6·46 37$\bar{2}$
120	2	5$\bar{7}$	5$\bar{7}$	·76 47$\bar{5}$	4$\bar{2}$	4$\bar{2}$	·76 47$\bar{5}$
180	3	5$\bar{7}$	5$\bar{7}$	·94 08$\bar{4}$	4$\bar{2}$	4$\bar{2}$	·94 08$\bar{4}$
240	4	5$\bar{7}$	5$\bar{7}$	7·06 57$\bar{8}$	4$\bar{2}$	4$\bar{2}$	7·06 57$\bar{8}$
300	5	4·685 5$\bar{7}$	5$\bar{7}$	7·16 26$\bar{9}$	5·314 4$\bar{2}$	4$\bar{2}$	7·16 26$\bar{9}$
360	6	5$\bar{7}$	5$\bar{7}$	·24 18$\bar{7}$	4$\bar{2}$	4$\bar{2}$	·24 188
420	7	5$\bar{7}$	5$\bar{7}$	·30 88$\bar{2}$	4$\bar{2}$	4$\bar{2}$	·30 88$\bar{2}$
480	8	5$\bar{7}$	5$\bar{7}$	·36 68$\bar{1}$	4$\bar{2}$	4$\bar{2}$	·36 68$\bar{1}$
540	9	5$\bar{7}$	5$\bar{7}$	·41 797	4$\bar{2}$	4$\bar{2}$	·41 797
600	**10**	4·685 5$\bar{7}$	5$\bar{7}$	7·46 37$\bar{2}$	5·314 4$\bar{2}$	4$\bar{2}$	7·46 37$\bar{2}$
660	11	5$\bar{7}$	5$\bar{7}$	·50 51$\bar{2}$	4$\bar{2}$	4$\bar{2}$	·50 512
720	12	5$\bar{7}$	5$\bar{7}$	·54 29$\bar{0}$	4$\bar{2}$	4$\bar{2}$	·54 291
780	13	5$\bar{7}$	5$\bar{7}$	·57 76$\bar{7}$	4$\bar{2}$	4$\bar{2}$	·57 76$\bar{7}$
840	14	5$\bar{7}$	5$\bar{7}$	·60 98$\bar{5}$	4$\bar{2}$	4$\bar{2}$	·60 98$\bar{5}$
900	15	4·685 5$\bar{7}$	58	7·63 98$\bar{1}$	5·314 4$\bar{2}$	42	7·63 982
960	16	5$\bar{7}$	58	·66 78$\bar{4}$	4$\bar{2}$	42	·66 785
1020	17	5$\bar{7}$	58	·69 41$\bar{7}$	4$\bar{2}$	42	·69 41$\bar{8}$
1080	18	5$\bar{7}$	58	·71 89$\bar{9}$	4$\bar{2}$	42	·71 90$\bar{0}$
1140	19	5$\bar{7}$	58	·74 248	4$\bar{2}$	42	·74 24$\bar{8}$
1200	**20**	4·685 57	58	7·76 47$\bar{5}$	5·314 43	42	7·76 476
1260	21	57	58	·78 59$\bar{4}$	43	42	·78 59$\bar{5}$
1320	22	57	58	·80 61$\bar{4}$	43	42	·80 61$\bar{5}$
1380	23	57	58	·82 54$\bar{5}$	43	42	·82 54$\bar{6}$
1440	24	57	58	·84 39$\bar{3}$	43	42	·84 39$\bar{4}$
1500	25	4·685 57	5$\bar{8}$	7·86 16$\bar{6}$	5·314 43	4$\bar{1}$	7·86 16$\bar{7}$
1560	26	57	5$\bar{8}$	·87 86$\bar{9}$	43	4$\bar{1}$	·87 871
1620	27	57	5$\bar{8}$	·89 50$\bar{8}$	43	4$\bar{1}$	·89 51$\bar{0}$
1680	28	57	5$\bar{8}$	·91 088	43	4$\bar{1}$	·91 08$\bar{9}$
1740	29	57	5$\bar{8}$	·92 612	43	4$\bar{1}$	·92 61$\bar{3}$
1800	**30**	4·685 57	5$\bar{8}$	7·94 084	5·314 43	4$\bar{1}$	7·94 086
1860	31	57	5$\bar{8}$	·95 508	43	4$\bar{1}$	·95 510
1920	32	57	5$\bar{8}$	·96 88$\bar{7}$	43	4$\bar{1}$	·96 88$\bar{9}$
1980	33	57	59	·98 22$\bar{3}$	43	41	·98 22$\bar{5}$
2040	34	57	59	·99 520	43	41	·99 522
2100	35	4·685 5$\bar{6}$	59	8·00 77$\bar{8}$	5·314 4$\bar{3}$	41	8·00 78$\bar{1}$
2160	36	5$\bar{6}$	59	·02 002	4$\bar{3}$	41	·02 00$\bar{4}$
2220	37	5$\bar{6}$	59	·03 192	4$\bar{3}$	4$\bar{1}$	·03 19$\bar{4}$
2280	38	5$\bar{6}$	5$\bar{9}$	·04 350	4$\bar{3}$	4$\bar{0}$	·04 35$\bar{2}$
2340	39	5$\bar{6}$	5$\bar{9}$	·05 478	4$\bar{3}$	4$\bar{0}$	·05 481
2400	**40**	4·685 5$\bar{6}$	5$\bar{9}$	8·06 57$\bar{7}$	5·314 4$\bar{3}$	4$\bar{0}$	8·06 58$\bar{0}$
2460	41	5$\bar{6}$	5$\bar{9}$	·07 65$\bar{0}$	4$\bar{3}$	4$\bar{0}$	·07 65$\bar{3}$
2520	42	5$\bar{6}$	5$\bar{9}$	·08 69$\bar{6}$	4$\bar{3}$	4$\bar{0}$	·08 69$\bar{9}$
2580	43	5$\bar{6}$	60	·09 71$\bar{8}$	4$\bar{3}$	40	·09 72$\bar{1}$
2640	44	5$\bar{6}$	60	·10 71$\bar{6}$	4$\bar{3}$	40	·10 72$\bar{0}$
2700	45	4·685 56	60	8·11 69$\bar{2}$	5·314 44	40	8·11 69$\bar{6}$
2760	46	56	60	·12 647	44	40	·12 651
2820	47	56	6$\bar{0}$	·13 58$\bar{1}$	44	4$\bar{0}$	·13 58$\bar{5}$
2880	48	56	6$\bar{0}$	·14 49$\bar{5}$	44	3$\bar{9}$	·14 49$\bar{9}$
2940	49	56	6$\bar{0}$	·15 39$\bar{0}$	44	3$\bar{9}$	·15 39$\bar{5}$
3000	**50**	4·685 56	6$\bar{0}$	8·16 268	5·314 44	3$\bar{9}$	8·16 27$\bar{2}$
3060	51	56	6$\bar{0}$	·17 12$\bar{8}$	44	3$\bar{9}$	·17 13$\bar{3}$
3120	52	56	61	·17 97$\bar{1}$	44	39	·17 97$\bar{6}$
3180	53	5$\bar{6}$	61	·18 79$\bar{8}$	4$\bar{4}$	39	·18 80$\bar{3}$
3240	54	5$\bar{5}$	61	·19 610	4$\bar{4}$	39	·19 61$\bar{5}$
3300	55	4·685 5$\bar{5}$	6$\bar{1}$	8·20 40$\bar{7}$	5·314 4$\bar{4}$	3$\bar{9}$	8·20 41$\bar{2}$
3360	56	5$\bar{5}$	6$\bar{1}$	·21 18$\bar{9}$	4$\bar{4}$	3$\bar{8}$	·21 19$\bar{5}$
3420	57	5$\bar{5}$	6$\bar{1}$	·21 95$\bar{8}$	4$\bar{4}$	3$\bar{8}$	·21 96$\bar{4}$
3480	58	5$\bar{5}$	6$\bar{1}$	·22 71$\bar{3}$	4$\bar{4}$	3$\bar{8}$	·22 71$\bar{9}$
3540	59	5$\bar{5}$	62	·23 45$\bar{5}$	4$\bar{4}$	38	·23 462

Log sin ϕ = log ϕ'' + S. Log tan ϕ = log ϕ'' + T.

1°

log ϕ'' = log sin ϕ + S'. log ϕ'' = log tan ϕ + T'.

″	′	S	T	Log. Sin.	S′	T′	Log. Tan.
3600	**0**	4·685 5$\overline{5}$	62	8·24 18$\overline{5}$	5·314 4$\overline{4}$	38	8·24 192
3660	1	55	62	·24 90$\underline{3}$	45	38	·24 910
3720	2	55	6$\underline{2}$	·25 60$\overline{9}$	45	3$\underline{8}$	·25 61$\overline{6}$
3780	3	55	6$\overline{2}$	·26 304	45	3$\overline{7}$	·26 31$\overline{1}$
3840	4	55	6$\overline{2}$	·26 988	45	3$\overline{7}$	·26 99$\overline{5}$
3900	5	4·685 55	6$\overline{2}$	8·27 66$\overline{1}$	5·314 45	3$\overline{7}$	8·27 66$\underline{9}$
3960	6	5$\underline{5}$	63	·28 32$\overline{4}$	4$\underline{5}$	37	·28 33$\overline{2}$
4020	7	5$\overline{4}$	63	·28 97$\overline{7}$	4$\overline{5}$	37	·28 98$\overline{5}$
4080	8	5$\overline{4}$	6$\underline{3}$	·29 62$\overline{0}$	4$\overline{5}$	3$\underline{7}$	·29 62$\underline{9}$
4140	9	5$\overline{4}$	6$\overline{3}$	·30 25$\overline{4}$	4$\overline{5}$	3$\overline{6}$	·30 26$\overline{3}$
4200	**10**	4·685 5$\overline{4}$	6$\overline{3}$	8·30 87$\overline{9}$	5·314 4$\overline{5}$	3$\overline{6}$	8·30 88$\overline{8}$
4260	11	5$\overline{4}$	6$\overline{3}$	·31 49$\overline{5}$	4$\underline{5}$	3$\overline{6}$	·31 50$\overline{4}$
4320	12	5$\overline{4}$	64	·32 10$\overline{2}$	4$\overline{5}$	36	·32 11$\underline{2}$
4380	13	54	64	·32 70$\overline{1}$	46	36	·32 71$\overline{1}$
4440	14	54	64	·33 29$\overline{2}$	46	36	·33 30$\overline{2}$
4500	15	4·685 54	6$\overline{4}$	8·33 87$\overline{5}$	5·314 46	3$\overline{5}$	8·33 88$\overline{5}$
4560	16	54	6$\overline{4}$	·34 45$\overline{0}$	46	3$\overline{5}$	·34 461
4620	17	54	65	·35 01$\underline{8}$	46	35	·35 02$\underline{9}$
4680	18	5$\underline{4}$	65	·35 57$\overline{8}$	4$\underline{6}$	35	·35 58$\overline{9}$
4740	19	5$\overline{3}$	65	·36 13$\overline{1}$	4$\overline{6}$	35	·36 143
4800	**20**	4·685 5$\overline{3}$	6$\overline{5}$	8·36 67$\overline{7}$	5·314 4$\overline{6}$	3$\overline{4}$	8·36 68$\overline{9}$
4860	21	5$\overline{3}$	6$\overline{5}$	·37 217	4$\overline{6}$	3$\overline{4}$	·37 229
4920	22	5$\overline{3}$	6$\overline{5}$	·37 750	4$\overline{6}$	3$\overline{4}$	·37 762
4980	23	5$\overline{3}$	66	·38 276	4$\overline{6}$	34	·38 289
5040	24	53	66	·38 796	47	34	·38 809
5100	25	4·685 53	6$\overline{6}$	8·39 310	5·314 47	3$\overline{3}$	8·39 32$\overline{3}$
5160	26	53	6$\overline{6}$	·39 818	47	3$\overline{3}$	·39 83$\overline{1}$
5220	27	5$\underline{3}$	67	·40 320	4$\underline{7}$	33	·40 33$\underline{4}$
5280	28	5$\overline{2}$	67	·40 816	4$\overline{7}$	33	·40 83$\overline{0}$
5340	29	5$\overline{2}$	67	·41 307	4$\overline{7}$	33	·41 32$\overline{1}$
5400	**30**	4·685 5$\overline{2}$	6$\overline{7}$	8·41 79$\underline{2}$	5·314 4$\overline{7}$	3$\overline{2}$	8·41 807
5460	31	5$\overline{2}$	6$\overline{7}$	·42 27$\overline{1}$	4$\overline{7}$	3$\overline{2}$	·42 287
5520	32	5$\overline{2}$	68	·42 74$\underline{6}$	4$\overline{7}$	32	·42 76$\underline{2}$
5580	33	52	6$\underline{8}$	·43 21$\overline{5}$	48	3$\underline{2}$	·43 23$\overline{1}$
5640	34	52	6$\overline{8}$	·43 680	48	3$\overline{1}$	·43 696
5700	35	4·685 52	6$\overline{8}$	8·44 13$\overline{9}$	5·314 48	3$\overline{1}$	8·44 156
5760	36	5$\underline{2}$	69	·44 594	4$\underline{8}$	31	·44 61$\underline{1}$
5820	37	5$\overline{1}$	6$\underline{9}$	·45 04$\underline{4}$	4$\overline{8}$	3$\underline{1}$	·45 06$\overline{1}$
5880	38	5$\overline{1}$	6$\overline{9}$	·45 48$\overline{9}$	4$\overline{8}$	3$\overline{0}$	·45 507
5940	39	5$\overline{1}$	6$\overline{9}$	·45 930	4$\overline{8}$	3$\overline{0}$	·45 948
6000	**40**	4·685 5$\overline{1}$	6$\overline{9}$	8·46 36$\overline{6}$	5·314 4$\overline{8}$	3$\overline{0}$	8·46 385
6060	41	51	70	·46 79$\overline{8}$	49	30	·46 81$\underline{7}$
6120	42	51	7$\underline{0}$	·47 22$\overline{6}$	49	3$\underline{0}$	·47 24$\overline{5}$
6180	43	51	7$\overline{0}$	·47 65$\underline{0}$	49	2$\overline{9}$	·47 66$\overline{9}$
6240	44	51	7$\overline{0}$	·48 06$\overline{9}$	49	2$\overline{9}$	·48 089
6300	45	4·685 5$\overline{0}$	7$\underline{1}$	8·48 48$\underline{5}$	5·314 4$\overline{9}$	2$\underline{9}$	8·48 505
6360	46	5$\overline{0}$	7$\overline{1}$	·48 89$\overline{6}$	4$\overline{9}$	2$\overline{8}$	·48 917
6420	47	5$\overline{0}$	7$\overline{1}$	·49 304	4$\overline{9}$	2$\overline{8}$	·49 32$\underline{5}$
6480	48	5$\overline{0}$	72	·49 708	4$\overline{9}$	28	·49 72$\overline{9}$
6540	49	50	72	·50 108	50	28	·50 130
6600	**50**	4·685 50	7$\overline{2}$	8·50 50$\overline{4}$	5·314 50	2$\overline{7}$	8·50 52$\overline{6}$
6660	51	50	7$\overline{2}$	·50 89$\overline{7}$	50	2$\overline{7}$	·50 920
6720	52	5$\underline{0}$	73	·51 28$\underline{6}$	5$\underline{0}$	27	·51 310
6780	53	4$\overline{9}$	7$\underline{3}$	·51 67$\overline{2}$	5$\overline{0}$	2$\underline{7}$	·51 696
6840	54	4$\overline{9}$	7$\overline{3}$	·52 055	5$\overline{0}$	2$\overline{6}$	·52 079
6900	55	4·685 4$\overline{9}$	7$\overline{3}$	8·52 43$\overline{4}$	5·314 5$\overline{0}$	2$\overline{6}$	8·52 45$\overline{8}$
6960	56	49	7$\underline{4}$	·52 810	51	2$\underline{6}$	·52 835
7020	57	49	7$\overline{4}$	·53 18$\underline{3}$	51	2$\overline{5}$	·53 208
7080	58	49	7$\overline{4}$	·53 55$\overline{2}$	51	2$\overline{5}$	·53 57$\overline{8}$
7140	59	49	75	·53 91$\overline{8}$	51	25	·53 94$\overline{4}$

Log sin ϕ = log ϕ'' + S.
Log tan ϕ = log ϕ'' + T.

2°

log ϕ'' = log sin ϕ + S'.
log ϕ'' = log tan ϕ + T'.

″	′	S	T	Log. Sin.	S′	T′	Log. Tan.
7200	0	4.685 48	75	8.54 282	5.314 51	25	8.54 308
7260	1	48	75	.54 642	51	24	.54 669
7320	2	48	75	.54 999	51	24	.55 027
7380	3	48	76	.55 354	52	24	.55 381
7440	4	48	76	.55 705	52	23	.55 733
7500	5	4.685 48	76	8.56 054	5.314 52	23	8.56 083
7560	6	48	77	.56 400	52	23	.56 429
7620	7	47	77	.56 743	52	22	.56 772
7680	8	47	77	.57 083	52	22	.57 113
7740	9	47	78	.57 421	52	22	.57 452
7800	10	4.685 47	78	8.57 756	5.314 53	22	8.57 787
7860	11	47	78	.58 089	53	21	.58 121
7920	12	47	79	.58 419	53	21	.58 451
7980	13	46	79	.58 747	53	21	.58 779
8040	14	46	79	.59 072	53	20	.59 105
8100	15	4.685 46	80	8.59 395	5.314 53	20	8.59 428
8160	16	46	80	.59 715	54	20	.59 749
8220	17	46	80	.60 033	54	19	.60 067
8280	18	46	81	.60 349	54	19	.60 384
8340	19	45	81	.60 662	54	19	.60 698
8400	20	4.685 45	81	8.60 973	5.314 54	18	8.61 009
8460	21	45	82	.61 282	54	18	.61 319
8520	22	45	82	.61 589	55	18	.61 626
8580	23	45	82	.61 893	55	17	.61 931
8640	24	45	83	.62 196	55	17	.62 234
8700	25	4.685 44	83	8.62 496	5.314 55	16	8.62 535
8760	26	44	83	.62 795	55	16	.62 834
8820	27	44	84	.63 091	55	16	.63 131
8880	28	44	84	.63 385	56	15	.63 425
8940	29	44	84	.63 677	56	15	.63 718
9000	30	4.685 43	85	8.63 968	5.314 56	15	8.64 009
9060	31	43	85	.64 256	56	14	.64 298
9120	32	43	86	.64 543	56	14	.64 585
9180	33	43	86	.64 827	57	14	.64 870
9240	34	43	86	.65 110	57	13	.65 153
9300	35	4.685 43	87	8.65 391	5.314 57	13	8.65 435
9360	36	42	87	.65 670	57	12	.65 715
9420	37	42	87	.65 947	57	12	.65 993
9480	38	42	88	.66 223	58	12	.66 269
9540	39	42	88	.66 497	58	11	.66 543
9600	40	4.685 42	89	8.66 769	5.314 58	11	8.66 816
9660	41	41	89	.67 039	58	10	.67 087
9720	42	41	89	.67 308	58	10	.67 356
9780	43	41	90	.67 575	59	10	.67 624
9840	44	41	90	.67 840	59	09	.67 890
9900	45	4.685 41	91	8.68 104	5.314 59	09	8.68 154
9960	46	40	91	.68 366	59	08	.68 417
10020	47	40	91	.68 627	59	08	.68 678
10080	48	40	92	.68 886	60	08	.68 938
10140	49	40	92	.69 144	60	07	.69 196
10200	50	4.685 40	93	8.69 400	5.314 60	07	8.69 453
10260	51	39	93	.69 654	60	06	.69 708
10320	52	39	93	.69 907	60	06	.69 961
10380	53	39	94	.70 159	61	06	.70 214
10440	54	39	94	.70 409	61	05	.70 464
10500	55	4.685 38	95	8.70 657	5.314 61	05	8.70 714
10560	56	38	95	.70 905	61	04	.70 962
10620	57	38	96	.71 150	61	04	.71 208
10680	58	38	96	.71 395	62	03	.71 453
10740	59	38	97	.71 638	62	03	.71 697

′	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
0	− ∞		− ∞		+ ∞	0.00 000	60
1	6.46 372	30103	6.46 372	30103	3.53 627	0.00 000	59
2	6.76 475	17609	6.76 475	17609	3.23 524	0.00 000	58
3	6.94 084	12494	6.94 084	12494	3.05 915	0.00 000	57
4	7.06 578	9691	7.06 578	9691	2.93 421	0.00 000	56
5	7.16 269	7918	7.16 269	7918	2.83 730	0.00 000	55
6	7.24 187	6695	7.24 188	6694	2.75 812	0.00 000	54
7	7.30 882	5799	7.30 882	5799	2.69 117	0.00 000	53
8	7.36 681	5115	7.36 681	5115	2.63 318	0.00 000	52
9	7.41 797	4575	7.41 797	4575	2.58 203	0.00 000	51
10	7.46 372	4139	7.46 372	4139	2.53 627	0.00 000	50
11	7.50 512	3778	7.50 512	3779	2.49 488	0.00 000	49
12	7.54 290	3476	7.54 291	3476	2.45 709	9.99 999	48
13	7.57 767	3218	7.57 767	3218	2.42 233	9.99 999	47
14	7.60 985	2996	7.60 985	2996	2.39 014	9.99 999	46
15	7.63 981	2803	7.63 982	2803	2.36 018	9.99 999	45
16	7.66 784	2633	7.66 785	2633	2.33 215	9.99 999	44
17	7.69 417	2482	7.69 418	2482	2.30 582	9.99 999	43
18	7.71 899	2348	7.71 900	2348	2.28 099	9.99 999	42
19	7.74 248	2227	7.74 248	2227	2.25 751	9.99 999	41
20	7.76 475	2119	7.76 476	2119	2.23 524	9.99 999	40
21	7.78 594	2020	7.78 595	2020	2.21 405	9.99 999	39
22	7.80 614	1930	7.80 615	1930	2.19 384	9.99 999	38
23	7.82 545	1848	7.82 546	1848	2.17 454	9.99 999	37
24	7.84 393	1772	7.84 394	1773	2.15 605	9.99 999	36
25	7.86 166	1703	7.86 167	1703	2.13 832	9.99 999	35
26	7.87 869	1639	7.87 871	1639	2.12 129	9.99 999	34
27	7.89 508	1579	7.89 510	1579	2.10 490	9.99 998	33
28	7.91 088	1524	7.91 089	1524	2.08 910	9.99 998	32
29	7.92 612	1472	7.92 613	1472	2.07 386	9.99 998	31
30	7.94 084	1424	7.94 086	1424	2.05 914	9.99 998	30
31	7.95 508	1379	7.95 510	1379	2.04 490	9.99 998	29
32	7.96 887	1336	7.96 889	1336	2.03 111	9.99 998	28
33	7.98 223	1296	7.98 225	1296	2.01 774	9.99 998	27
34	7.99 520	1258	7.99 522	1259	2.00 478	9.99 998	26
35	8.00 778	1223	8.00 781	1223	1.99 219	9.99 997	25
36	8.02 002	1190	8.02 004	1190	1.97 995	9.99 997	24
37	8.03 192	1158	8.03 194	1158	1.96 805	9.99 997	23
38	8.04 350	1128	8.04 352	1128	1.95 647	9.99 997	22
39	8.05 478	1099	8.05 481	1099	1.94 519	9.99 997	21
40	8.06 577	1072	8.06 580	1072	1.93 419	9.99 997	20
41	8.07 650	1046	8.07 653	1046	1.92 347	9.99 997	19
42	8.08 696	1022	8.08 699	1022	1.91 300	9.99 997	18
43	8.09 718	998	8.09 721	999	1.90 278	9.99 996	17
44	8.10 716	976	8.10 720	976	1.89 279	9.99 996	16
45	8.11 692	954	8.11 696	954	1.88 303	9.99 996	15
46	8.12 647	934	8.12 651	934	1.87 349	9.99 996	14
47	8.13 581	914	8.13 585	914	1.86 415	9.99 996	13
48	8.14 495	895	8.14 499	895	1.85 500	9.99 996	12
49	8.15 390	877	8.15 395	877	1.84 605	9.99 995	11
50	8.16 268	860	8.16 272	860	1.83 727	9.99 995	10
51	8.17 128	843	8.17 133	843	1.82 867	9.99 995	9
52	8.17 971	827	8.17 976	827	1.82 023	9.99 995	8
53	8.18 798	811	8.18 803	812	1.81 196	9.99 995	7
54	8.19 610	797	8.19 615	797	1.80 384	9.99 994	6
55	8.20 407	782	8.20 412	783	1.79 587	9.99 994	5
56	8.21 189	768	8.21 195	768	1.78 804	9.99 994	4
57	8.21 958	755	8.21 964	755	1.78 036	9.99 994	3
58	8.22 713	742	8.22 719	742	1.77 280	9.99 994	2
59	8.23 455	730	8.23 462	730	1.76 538	9.99 993	1
60	8.24 185		8.24 192		1.75 808	9.99 993	0
	Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

′	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
0	8.24 185	718	8.24 192	718	1.75 808	9.99 993	60
1	8.24 903	706	8.24 910	706	1.75 090	9.99 993	59
2	8.25 609	694	8.25 616	695	1.74 383	9.99 993	58
3	8.26 304	684	8.26 311	684	1.73 688	9.99 992	57
4	8.26 988	673	8.26 995	673	1.73 004	9.99 992	56
5	8.27 661	663	8.27 669	663	1.72 331	9.99 992	55
6	8.28 324	653	8.28 332	653	1.71 667	9.99 992	54
7	8.28 977	643	8.28 985	643	1.71 014	9.99 992	53
8	8.29 620	634	8.29 629	634	1.70 371	9.99 991	52
9	8.30 254	625	8.30 263	625	1.69 736	9.99 991	51
10	8.30 879	616	8.30 888	616	1.69 111	9.99 991	50
11	8.31 495	607	8.31 504	607	1.68 495	9.99 990	49
12	8.32 102	599	8.32 112	599	1.67 888	9.99 990	48
13	8.32 701	591	8.32 711	591	1.67 288	9.99 990	47
14	8.33 292	583	8.33 302	583	1.66 697	9.99 990	46
15	8.33 875	575	8.33 885	575	1.66 114	9.99 989	45
16	8.34 450	567	8.34 461	568	1.65 539	9.99 989	44
17	8.35 018	560	8.35 029	560	1.64 971	9.99 989	43
18	8.35 578	553	8.35 589	553	1.64 410	9.99 989	42
19	8.36 131	546	8.36 143	546	1.63 857	9.99 988	41
20	8.36 677	539	8.36 689	539	1.63 310	9.99 988	40
21	8.37 217	533	8.37 229	533	1.62 771	9.99 988	39
22	8.37 750	526	8.37 762	527	1.62 238	9.99 987	38
23	8.38 276	520	8.38 289	520	1.61 711	9.99 987	37
24	8.38 796	514	8.38 809	514	1.61 191	9.99 987	36
25	8.39 310	508	8.39 323	508	1.60 676	9.99 986	35
26	8.39 818	502	8.39 831	502	1.60 168	9.99 986	34
27	8.40 320	496	8.40 334	496	1.59 666	9.99 986	33
28	8.40 816	491	8.40 830	491	1.59 169	9.99 986	32
29	8.41 307	485	8.41 321	485	1.58 678	9.99 985	31
30	8.41 792	479	8.41 807	480	1.58 193	9.99 985	30
31	8.42 271	474	8.42 287	475	1.57 713	9.99 985	29
32	8.42 746	469	8.42 762	469	1.57 238	9.99 984	28
33	8.43 215	464	8.43 231	464	1.56 768	9.99 984	27
34	8.43 680	459	8.43 696	460	1.56 304	9.99 984	26
35	8.44 139	454	8.44 156	455	1.55 844	9.99 983	25
36	8.44 594	450	8.44 611	450	1.55 389	9.99 983	24
37	8.45 044	445	8.45 061	445	1.54 938	9.99 982	23
38	8.45 489	440	8.45 507	441	1.54 493	9.99 982	22
39	8.45 930	436	8.45 948	437	1.54 052	9.99 982	21
40	8.46 366	432	8.46 385	432	1.53 615	9.99 981	20
41	8.46 798	428	8.46 817	428	1.53 183	9.99 981	19
42	8.47 226	423	8.47 245	424	1.52 754	9.99 981	18
43	8.47 650	419	8.47 669	419	1.52 330	9.99 980	17
44	8.48 069	415	8.48 089	416	1.51 911	9.99 980	16
45	8.48 485	411	8.48 505	412	1.51 495	9.99 979	15
46	8.48 896	407	8.48 917	408	1.51 083	9.99 979	14
47	8.49 304	404	8.49 325	404	1.50 675	9.99 979	13
48	8.49 708	400	8.49 729	400	1.50 270	9.99 978	12
49	8.50 108	396	8.50 130	396	1.49 870	9.99 978	11
50	8.50 504	393	8.50 526	393	1.49 473	9.99 978	10
51	8.50 897	389	8.50 920	390	1.49 080	9.99 977	9
52	8.51 286	386	8.51 310	386	1.48 690	9.99 977	8
53	8.51 672	382	8.51 696	383	1.48 304	9.99 976	7
54	8.52 055	379	8.52 079	379	1.47 921	9.99 976	6
55	8.52 434	375	8.52 458	376	1.47 541	9.99 975	5
56	8.52 810	373	8.52 835	373	1.47 165	9.99 975	4
57	8.53 183	369	8.53 208	370	1.46 792	9.99 975	3
58	8.53 552	366	8.53 578	366	1.46 422	9.99 974	2
59	8.53 918	363	8.53 944	364	1.46 055	9.99 974	1
60	8.54 282		8.54 308		1.45 691	9.99 973	0
	Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

′	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
0	8.54 282	360	8.54 308	360	1.45 691	9.99 973	60
1	8.54 642	357	8.54 669	358	1.45 331	9.99 973	59
2	8.54 999	354	8.55 027	354	1.44 973	9.99 972	58
3	8.55 354	351	8.55 381	352	1.44 618	9.99 972	57
4	8.55 705	348	8.55 733	349	1.44 266	9.99 971	56
5	8.56 054	346	8.56 083	346	1.43 917	9.99 971	55
6	8.56 400	343	8.56 429	343	1.43 571	9.99 971	54
7	8.56 743	340	8.56 772	341	1.43 227	9.99 970	53
8	8.57 083	338	8.57 113	338	1.42 886	9.99 970	52
9	8.57 421	335	8.57 452	335	1.42 548	9.99 969	51
10	8.57 756	332	8.57 787	333	1.42 212	9.99 969	50
11	8.58 039	330	8.58 121	330	1.41 879	9.99 968	49
12	8.58 419	327	8.58 451	328	1.41 548	9.99 968	48
13	8.58 747	325	8.58 779	325	1.41 220	9.99 967	47
14	8.59 072	323	8.59 105	323	1.40 895	9.99 967	46
15	8.59 395	320	8.59 428	320	1.40 571	9.99 966	45
16	8.59 715	318	8.59 749	318	1.40 251	9.99 966	44
17	8.60 033	316	8.60 067	316	1.39 932	9.99 965	43
18	8.60 349	313	8.60 384	314	1.39 616	9.99 965	42
19	8.60 662	311	8.60 698	311	1.39 302	9.99 964	41
20	8.60 973	309	8.61 009	309	1.38 990	9.99 964	40
21	8.61 282	306	8.61 319	307	1.38 681	9.99 963	39
22	8.61 589	304	8.61 626	305	1.38 374	9.99 963	38
23	8.61 893	302	8.61 931	303	1.38 068	9.99 962	37
24	8.62 196	300	8.62 234	300	1.37 765	9.99 962	36
25	8.62 496	298	8.62 535	299	1.37 465	9.99 961	35
26	8.62 795	296	8.62 834	297	1.37 166	9.99 961	34
27	8.63 091	294	8.63 131	294	1.36 869	9.99 960	33
28	8.63 385	292	8.63 425	293	1.36 574	9.99 959	32
29	8.63 677	290	8.63 718	291	1.36 281	9.99 959	31
30	8.63 968	288	8.64 009	288	1.35 990	9.99 958	30
31	8.64 256	286	8.64 298	287	1.35 702	9.99 958	29
32	8.64 543	284	8.64 585	285	1.35 414	9.99 957	28
33	8.64 827	282	8.64 870	283	1.35 129	9.99 957	27
34	8.65 110	281	8.65 153	281	1.34 846	9.99 956	26
35	8.65 391	279	8.65 435	280	1.34 565	9.99 956	25
36	8.65 670	277	8.65 715	278	1.34 285	9.99 955	24
37	8.65 947	275	8.65 993	276	1.34 007	9.99 954	23
38	8.66 223	274	8.66 269	274	1.33 731	9.99 954	22
39	8.66 497	272	8.66 543	272	1.33 456	9.99 953	21
40	8.66 769	270	8.66 816	271	1.33 184	9.99 953	20
41	8.67 039	268	8.67 087	269	1.32 913	9.99 952	19
42	8.67 308	267	8.67 356	267	1.32 643	9.99 952	18
43	8.67 575	265	8.67 624	266	1.32 376	9.99 951	17
44	8.67 840	264	8.67 890	264	1.32 110	9.99 950	16
45	8.68 104	262	8.68 154	262	1.31 845	9.99 950	15
46	8.68 366	260	8.68 417	261	1.31 583	9.99 949	14
47	8.68 627	259	8.68 678	259	1.31 321	9.99 948	13
48	8.68 886	257	8.68 938	258	1.31 062	9.99 948	12
49	8.69 144	256	8.69 196	256	1.30 803	9.99 947	11
50	8.69 400	254	8.69 453	255	1.30 547	9.99 947	10
51	8.69 654	253	8.69 708	253	1.30 292	9.99 946	9
52	8.69 907	251	8.69 961	252	1.30 038	9.99 945	8
53	8.70 159	250	8.70 214	250	1.29 786	9.99 945	7
54	8.70 409	248	8.70 464	249	1.29 535	9.99 944	6
55	8.70 657	247	8.70 714	248	1.29 286	9.99 943	5
56	8.70 905	245	8.70 962	246	1.29 038	9.99 943	4
57	8.71 150	244	8.71 208	245	1.28 791	9.99 942	3
58	8.71 395	243	8.71 453	243	1.28 546	9.99 942	2
59	8.71 638	241	8.71 697	242	1.28 303	9.99 941	1
60	8.71 880		8.71 939		1.28 060	9.99 940	0
	Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	
0	8.71 880	240	8.71 939	241	1.28 060	9.99 940	60
1	8.72 120	239	8.72 180	240	1.27 819	9.99 940	59
2	8.72 359	237	8.72 420	238	1.27 579	9.99 939	58
3	8.72 597	236	8.72 659	237	1.27 341	9.99 938	57
4	8.72 833	235	8.72 896	235	1.27 104	9.99 938	56
5	8.73 069	233	8.73 131	235	1.26 868	9.99 937	55
6	8.73 302	233	8.73 366	233	1.26 633	9.99 936	54
7	8.73 535	231	8.73 599	232	1.26 400	9.99 935	53
8	8.73 766	230	8.73 831	231	1.26 168	9.99 935	52
9	8.73 997	229	8.74 062	229	1.25 937	9.99 934	51
10	8.74 226	227	8.74 292	228	1.25 708	9.99 933	50
11	8.74 453	226	8.74 520	227	1.25 479	9.99 933	49
12	8.74 680	225	8.74 748	226	1.25 252	9.99 932	48
13	8.74 905	224	8.74 974	225	1.25 026	9.99 931	47
14	8.75 129	223	8.75 199	223	1.24 801	9.99 931	46
15	8.75 353	221	8.75 422	223	1.24 577	9.99 930	45
16	8.75 574	221	8.75 645	221	1.24 354	9.99 929	44
17	8.75 795	219	8.75 867	220	1.24 133	9.99 928	43
18	8.76 015	218	8.76 087	219	1.23 913	9.99 928	42
19	8.76 233	217	8.76 306	218	1.23 693	9.99 927	41
20	8.76 451	216	8.76 524	217	1.23 475	9.99 926	40
21	8.76 667	215	8.76 741	216	1.23 258	9.99 925	39
22	8.76 883	214	8.76 958	214	1.23 042	9.99 925	38
23	8.77 097	213	8.77 172	214	1.22 827	9.99 924	37
24	8.77 310	212	8.77 386	213	1.22 613	9.99 923	36
25	8.77 522	211	8.77 599	212	1.22 400	9.99 922	35
26	8.77 733	210	8.77 811	210	1.22 188	9.99 922	34
27	8.77 943	209	8.78 022	210	1.21 978	9.99 921	33
28	8.78 152	208	8.78 232	209	1.21 768	9.99 920	32
29	8.78 360	207	8.78 441	207	1.21 559	9.99 919	31
30	8.78 567	206	8.78 648	207	1.21 351	9.99 919	30
31	8.78 773	205	8.78 855	206	1.21 144	9.99 918	29
32	8.78 978	204	8.79 061	204	1.20 938	9.99 917	28
33	8.79 183	203	8.79 266	204	1.20 734	9.99 916	27
34	8.79 386	202	8.79 470	203	1.20 530	9.99 916	26
35	8.79 588	201	8.79 673	202	1.20 327	9.99 915	25
36	8.79 789	200	8.79 875	201	1.20 125	9.99 914	24
37	8.79 989	199	8.80 076	200	1.19 923	9.99 913	23
38	8.80 189	198	8.80 276	199	1.19 723	9.99 912	22
39	8.80 387	197	8.80 476	198	1.19 524	9.99 912	21
40	8.80 585	197	8.80 674	197	1.19 326	9.99 911	20
41	8.80 782	195	8.80 871	197	1.19 128	9.99 910	19
42	8.80 977	195	8.81 068	195	1.18 931	9.99 909	18
43	8.81 172	194	8.81 264	195	1.18 736	9.99 908	17
44	8.81 366	193	8.81 459	194	1.18 541	9.99 907	16
45	8.81 560	192	8.81 653	193	1.18 347	9.99 907	15
46	8.81 752	191	8.81 846	192	1.18 154	9.99 906	14
47	8.81 943	191	8.82 038	191	1.17 961	9.99 905	13
48	8.82 134	189	8.82 230	190	1.17 770	9.99 904	12
49	8.82 324	189	8.82 420	190	1.17 579	9.99 903	11
50	8.82 513	188	8.82 610	188	1.17 389	9.99 902	10
51	8.82 701	187	8.82 799	188	1.17 201	9.99 902	9
52	8.82 888	186	8.82 987	187	1.17 012	9.99 901	8
53	8.83 075	185	8.83 175	186	1.16 825	9.99 900	7
54	8.83 260	185	8.83 361	185	1.16 638	9.99 899	6
55	8.83 445	184	8.83 547	185	1.16 453	9.99 898	5
56	8.83 629	183	8.83 732	184	1.16 268	9.99 897	4
57	8.83 813	182	8.83 916	183	1.16 083	9.99 896	3
58	8.83 995	182	8.84 100	182	1.15 900	9.99 896	2
59	8.84 177	181	8.84 282	182	1.15 717	9.99 895	1
60	8.84 358		8.84 464		1.15 535	9.99 894	0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	′

P. P.

	330	320	310	300
6	33.0	32.0	31.0	30.0
7	38.5	37.3	36.1	35.0
8	44.0	42.6	41.3	40.0
9	49.5	48.0	46.5	45.0
10	55.0	53.3	51.6	50.0
20	110.0	106.6	103.3	100.0
30	165.0	160.0	155.0	150.0
40	220.0	213.3	206.6	200.0
50	275.0	266.6	258.3	250.0

	290	280	270	260
6	29.0	28.0	27.0	26.0
7	33.8	32.6	31.5	30.3
8	38.6	37.3	36.0	34.6
9	43.5	42.0	40.5	39.0
10	48.3	46.6	45.0	43.3
20	96.6	93.3	90.0	86.6
30	145.0	140.0	135.0	130.0
40	193.3	186.6	180.0	173.3
50	241.6	233.3	225.0	216.6

	250	240	230	220
6	25.0	24.0	23.0	22.0
7	29.1	28.0	26.8	25.6
8	33.3	32.0	30.6	29.3
9	37.5	36.0	34.5	33.0
10	41.6	40.0	38.3	36.6
20	83.3	80.0	76.6	73.3
30	125.0	120.0	115.0	110.0
40	166.6	160.0	153.3	146.6
50	208.3	200.0	191.6	183.3

	210	200	190	180
6	21.0	20.0	19.0	18.0
7	24.5	23.3	22.1	21.0
8	28.0	26.6	25.3	24.0
9	31.5	30.0	28.5	27.0
10	35.0	33.3	31.6	30.0
20	70.0	66.6	63.3	60.0
30	105.0	100.0	95.0	90.0
40	140.0	133.3	126.6	120.0
50	175.0	166.6	158.3	150.0

	$\bar{9}$	9	8	7	6	5
6	0.9	0.9	0.8	0.7	0.6	0.5
7	1.1	1.0	0.9	0.8	0.7	0.6
8	1.2	1.2	1.0	0.9	0.8	0.6
9	1.4	1.3	1.2	1.0	0.9	0.7
10	1.6	1.5	1.3	1.1	1.0	0.8
20	3.1	3.0	2.6	2.3	2.0	1.6
30	4.7	4.5	4.0	3.5	3.0	2.5
40	6.3	6.0	5.3	4.6	4.0	3.3
50	7.9	7.5	6.6	5.8	5.0	4.1

	$\bar{4}$	4	3	2	1	$\bar{0}$
6	0.4	0.4	0.3	0.2	0.1	0.0
7	0.5	0.4	0.3	0.2	0.1	0.0
8	0.6	0.5	0.4	0.2	0.1	0.0
9	0.7	0.6	0.4	0.3	0.1	0.1
10	0.7	0.6	0.5	0.3	0.1	0.1
20	1.5	1.3	1.0	0.6	0.3	0.1
30	2.2	2.0	1.5	1.0	0.5	0.2
40	3.0	2.6	2.0	1.3	0.6	0.3
50	3.7	3.3	2.5	1.6	0.8	0.4

P. P.

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	
0	8.84 358	180	8.84 464	181	1.15 535	9.99 894	60
1	8.84 538	180	8.84 645	180	1.15 354	9.99 893	59
2	8.84 718	178	8.84 826	179	1.15 174	9.99 892	58
3	8.84 897	178	8.85 005	179	1.14 994	9.99 891	57
4	8.85 075	177	8.85 184	178	1.14 815	9.99 890	56
5	8.85 252	176	8.85 363	177	1.14 637	9.99 889	55
6	8.85 429	176	8.85 540	176	1.14 459	9.99 888	54
7	8.85 605	175	8.85 717	176	1.14 283	9.99 888	53
8	8.85 780	174	8.85 893	175	1.14 107	9.99 887	52
9	8.85 954	174	8.86 068	175	1.13 931	9.99 886	51
10	8.86 128	173	8.86 243	174	1.13 756	9.99 885	50
11	8.86 301	172	8.86 417	173	1.13 582	9.99 884	49
12	8.86 474	171	8.86 590	172	1.13 409	9.99 883	48
13	8.86 645	171	8.86 763	172	1.13 237	9.99 882	47
14	8.86 816	170	8.86 935	171	1.13 065	9.99 881	46
15	8.86 987	169	8.87 106	170	1.12 893	9.99 880	45
16	8.87 156	169	8.87 277	170	1.12 723	9.99 879	44
17	8.87 325	168	8.87 447	169	1.12 553	9.99 878	43
18	8.87 494	167	8.87 616	169	1.12 384	9.99 877	42
19	8.87 661	167	8.87 785	168	1.12 215	9.99 876	41
20	8.87 828	166	8.87 953	167	1.12 047	9.99 875	40
21	8.87 995	165	8.88 120	167	1.11 880	9.99 874	39
22	8.88 160	165	8.88 287	166	1.11 713	9.99 874	38
23	8.88 326	164	8.88 453	165	1.11 547	9.99 873	37
24	8.88 490	163	8.88 618	165	1.11 381	9.99 872	36
25	8.88 654	163	8.88 783	164	1.11 216	9.99 871	35
26	8.88 817	162	8.88 947	163	1.11 052	9.99 870	34
27	8.88 980	162	8.89 111	163	1.10 889	9.99 869	33
28	8.89 142	161	8.89 274	162	1.10 726	9.99 868	32
29	8.89 303	161	8.89 436	162	1.10 563	9.99 867	31
30	8.89 464	160	8.89 598	161	1.10 401	9.99 866	30
31	8.89 624	159	8.89 759	161	1.10 240	9.99 865	29
32	8.89 784	159	8.89 920	160	1.10 079	9.99 864	28
33	8.89 943	158	8.90 080	159	1.09 919	9.99 863	27
34	8.90 101	158	8.90 240	158	1.09 760	9.99 862	26
35	8.90 259	157	8.90 398	158	1.09 601	9.99 861	25
36	8.90 417	156	8.90 557	157	1.09 443	9.99 860	24
37	8.90 573	156	8.90 714	157	1.09 285	9.99 859	23
38	8.90 729	156	8.90 872	156	1.09 128	9.99 858	22
39	8.90 885	155	8.91 028	156	1.08 971	9.99 857	21
40	8.91 040	154	8.91 184	155	1.08 815	9.99 856	20
41	8.91 195	154	8.91 340	155	1.08 660	9.99 855	19
42	8.91 349	153	8.91 495	154	1.08 505	9.99 853	18
43	8.91 502	153	8.91 649	154	1.08 350	9.99 852	17
44	8.91 655	152	8.91 803	153	1.08 196	9.99 851	16
45	8.91 807	151	8.91 957	152	1.08 043	9.99 850	15
46	8.91 959	151	8.92 109	152	1.07 890	9.99 849	14
47	8.92 110	150	8.92 262	151	1.07 738	9.99 848	13
48	8.92 261	150	8.92 413	151	1.07 586	9.99 847	12
49	8.92 411	150	8.92 565	150	1.07 435	9.99 846	11
50	8.92 561	149	8.92 715	150	1.07 284	9.99 845	10
51	8.92 710	148	8.92 866	149	1.07 134	9.99 844	9
52	8.92 858	148	8.93 015	149	1.06 984	9.99 843	8
53	8.93 007	147	8.93 164	149	1.06 835	9.99 842	7
54	8.93 154	147	8.93 313	148	1.06 686	9.99 841	6
55	8.93 301	146	8.93 461	148	1.06 538	9.99 840	5
56	8.93 448	146	8.93 609	147	1.06 390	9.99 839	4
57	8.93 594	146	8.93 756	146	1.06 243	9.99 837	3
58	8.93 740	145	8.93 903	146	1.06 097	9.99 836	2
59	8.93 885	144	8.94 049	145	1.05 950	9.99 835	1
60	8.94 029		8.94 195		1.05 805	9.99 834	0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	′

P. P.

	181	180	178	176
6	18.1	18.0	17.8	17.6
7	21.1	21.0	20.7	20.5
8	24.1	24.0	23.7	23.4
9	27.1	27.0	26.7	26.4
10	30.1	30.0	29.6	29.3
20	60.3	60.0	59.3	58.6
30	90.5	90.0	89.0	88.0
40	120.6	120.0	118.6	117.3
50	150.8	150.0	148.3	146.6

	174	172	170	168
6	17.4	17.2	17.0	16.8
7	20.3	20.0	19.8	19.6
8	23.2	22.9	22.6	22.4
9	26.1	25.8	25.5	25.2
10	29.0	28.6	28.3	28.0
20	58.0	57.3	56.6	56.0
30	87.0	86.0	85.0	84.0
40	116.0	114.6	113.3	112.0
50	145.0	143.3	141.6	140.0

	166	164	162	160
6	16.6	16.4	16.2	16.0
7	19.3	19.1	18.9	18.6
8	22.1	21.8	21.6	21.3
9	24.9	24.6	24.3	24.0
10	27.6	27.3	27.0	26.6
20	55.3	54.6	54.0	53.3
30	83.0	82.0	81.0	80.0
40	110.6	109.3	108.0	106.6
50	138.3	136.6	135.0	133.3

	158	156	154	152
6	15.8	15.6	15.4	15.2
7	18.4	18.2	17.9	17.7
8	21.0	20.8	20.5	20.2
9	23.7	23.4	23.1	22.8
10	26.3	26.0	25.6	25.3
20	52.6	52.0	51.3	50.6
30	79.0	78.0	77.0	76.0
40	105.3	104.0	102.6	101.3
50	131.6	130.0	128.3	126.6

	150	149	148	147
6	15.0	14.9	14.8	14.7
7	17.5	17.4	17.2	17.1
8	20.0	19.8	19.7	19.6
9	22.5	22.3	22.2	22.0
10	25.0	24.8	24.6	24.5
20	50.0	49.6	49.3	49.0
30	75.0	74.5	74.0	73.5
40	100.0	99.3	98.6	98.0
50	125.0	124.1	123.3	122.5

	146	145	1	1	0
6	14.6	14.5	0.1	0.1	0.0
7	17.0	16.9	0.2	0.1	0.0
8	19.4	19.3	0.2	0.1	0.0
9	21.9	21.7	0.2	0.1	0.1
10	24.3	24.1	0.2	0.1	0.1
20	48.6	48.3	0.5	0.3	0.1
30	73.0	72.5	0.7	0.5	0.2
40	97.3	96.6	1.0	0.6	0.3
50	121.6	120.8	1.2	0.8	0.4

P. P.

TABLE VII.—LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS.

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	
0	8.94 029	144	8.94 195	145	1.05 805	9.99 834	60
1	8.94 174	143	8.94 340	144	1.05 659	9.99 833	59
2	8.94 317	143	8.94 485	144	1.05 515	9.99 832	58
3	8.94 460	143	8.94 629	144	1.05 370	9.99 831	57
4	8.94 603	142	8.94 773	143	1.05 226	9.99 830	56
5	8.94 745	142	8.94 917	142	1.05 083	9.99 829	55
6	8.94 887	141	8.95 059	142	1.04 940	9.99 827	54
7	8.95 028	141	8.95 202	142	1.04 798	9.99 826	53
8	8.95 169	140	8.95 344	141	1.04 656	9.99 825	52
9	8.95 310	140	8.95 485	141	1.04 514	9.99 824	51
10	8.95 450	139	8.95 626	141	1.04 373	9.99 823	50
11	8.95 589	139	8.95 767	140	1.04 232	9.99 822	49
12	8.95 728	138	8.95 907	140	1.04 092	9.99 821	48
13	8.95 867	138	8.96 047	139	1.03 952	9.99 819	47
14	8.96 005	138	8.96 186	139	1.03 813	9.99 818	46
15	8.96 143	137	8.96 325	138	1.03 674	9.99 817	45
16	8.96 280	137	8.96 464	138	1.03 536	9.99 816	44
17	8.96 417	136	8.96 602	137	1.03 398	9.99 815	43
18	8.96 553	136	8.96 739	137	1.03 260	9.99 814	42
19	8.96 689	135	8.96 876	137	1.03 123	9.99 813	41
20	8.96 825	135	8.97 013	136	1.02 986	9.99 811	40
21	8.96 960	134	8.97 149	136	1.02 850	9.99 810	39
22	8.97 094	134	8.97 285	135	1.02 714	9.99 809	38
23	8.97 229	134	8.97 421	135	1.02 579	9.99 808	37
24	8.97 363	133	8.97 556	134	1.02 444	9.99 807	36
25	8.97 496	133	8.97 690	134	1.02 309	9.99 805	35
26	8.97 629	132	8.97 825	133	1.02 175	9.99 804	34
27	8.97 762	132	8.97 958	133	1.02 041	9.99 803	33
28	8.97 894	132	8.98 092	133	1.01 908	9.99 802	32
29	8.98 026	131	8.98 225	132	1.01 775	9.99 801	31
30	8.98 157	131	8.98 357	132	1.01 642	9.99 799	30
31	8.98 288	130	8.98 490	131	1.01 510	9.99 798	29
32	8.98 419	130	8.98 621	131	1.01 378	9.99 797	28
33	8.98 549	130	8.98 753	131	1.01 247	9.99 796	27
34	8.98 679	129	8.98 884	131	1.01 116	9.99 794	26
35	8.98 808	129	8.99 015	130	1.00 985	9.99 793	25
36	8.98 937	128	8.99 145	130	1.00 855	9.99 792	24
37	8.99 066	128	8.99 275	129	1.00 725	9.99 791	23
38	8.99 194	127	8.99 404	129	1.00 595	9.99 789	22
39	8.99 322	127	8.99 533	129	1.00 466	9.99 788	21
40	8.99 449	127	8.99 662	128	1.00 337	9.99 787	20
41	8.99 577	126	8.99 791	128	1.00 209	9.99 786	19
42	8.99 703	126	8.99 919	127	1.00 081	9.99 784	18
43	8.99 830	126	9.00 046	127	0.99 953	9.99 783	17
44	8.99 956	125	9.00 174	126	0.99 826	9.99 782	16
45	9.00 081	125	9.00 300	126	0.99 699	9.99 781	15
46	9.00 207	125	9.00 427	126	0.99 573	9.99 779	14
47	9.00 332	124	9.00 553	125	0.99 446	9.99 778	13
48	9.00 456	124	9.00 679	125	0.99 321	9.99 777	12
49	9.00 580	124	9.00 804	125	0.99 195	9.99 776	11
50	9.00 704	123	9.00 930	124	0.99 070	9.99 774	10
51	9.00 828	123	9.01 054	124	0.98 945	9.99 773	9
52	9.00 951	122	9.01 179	124	0.98 821	9.99 772	8
53	9.01 073	122	9.01 303	124	0.98 697	9.99 770	7
54	9.01 196	122	9.01 427	123	0.98 573	9.99 769	6
55	9.01 318	122	9.01 550	123	0.98 450	9.99 768	5
56	9.01 440	121	9.01 673	123	0.98 327	9.99 766	4
57	9.01 561	121	9.01 796	122	0.98 204	9.99 765	3
58	9.01 682	120	9.01 918	122	0.98 081	9.99 764	2
59	9.01 803	120	9.02 040	121	0.97 959	9.99 763	1
60	9.01 923		9.02 162		0.97 838	9.99 761	0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	′

P. P.

	145	144	143	142	141
6	14.5	14.4	14.3	14.2	14.1
7	16.9	16.8	16.7	16.5	16.4
8	19.3	19.2	19.0	18.9	18.8
9	21.7	21.6	21.4	21.3	21.1
10	24.1	24.0	23.8	23.6	23.5
20	48.3	48.0	47.6	47.3	47.0
30	72.5	72.0	71.5	71.0	70.5
40	96.6	96.0	95.3	94.6	94.0
50	120.8	120.0	119.1	118.3	117.5

	140	139	138	137	136
6	14.0	13.9	13.8	13.7	13.6
7	16.3	16.2	16.1	16.0	15.8
8	18.6	18.5	18.4	18.2	18.1
9	21.0	20.8	20.7	20.5	20.4
10	23.3	23.1	23.0	22.8	22.6
20	46.6	46.3	46.0	45.6	45.3
30	70.0	69.5	69.0	68.5	68.0
40	93.3	92.6	92.0	91.3	90.6
50	116.6	115.8	115.0	114.1	113.3

	135	134	133	132
6	13.5	13.4	13.3	13.2
7	15.7	15.6	15.5	15.4
8	18.0	17.8	17.7	17.6
9	20.2	20.1	19.9	19.8
10	22.5	22.3	22.1	22.0
20	45.0	44.6	44.3	44.0
30	67.5	67.0	66.5	66.0
40	90.0	89.3	88.6	88.0
50	112.5	111.6	110.8	110.0

	131	130	129	128
6	13.1	13.0	12.9	12.8
7	15.3	15.1	15.0	14.9
8	17.4	17.3	17.2	17.0
9	19.6	19.5	19.3	19.2
10	21.8	21.6	21.5	21.3
20	43.6	43.3	43.0	42.6
30	65.5	65.0	64.5	64.0
40	87.3	86.6	86.0	85.3
50	109.1	108.3	107.5	106.6

	127	126	125	124	123
6	12.7	12.6	12.5	12.4	12.3
7	14.8	14.7	14.6	14.4	14.3
8	16.9	16.8	16.6	16.5	16.4
9	19.0	18.9	18.7	18.5	18.4
10	21.1	21.0	20.8	20.6	20.5
20	42.3	42.0	41.6	41.3	41.0
30	63.5	63.0	62.5	62.0	61.5
40	84.6	84.0	83.3	82.6	82.0
50	105.8	105.0	104.1	103.3	102.5

	122	121	120	$\bar{1}$	1	$\bar{0}$
6	12.2	12.1	12.0	0.1	0.1	0.0
7	14.2	14.1	14.0	0.2	0.1	0.0
8	16.2	16.1	16.0	0.2	0.1	0.0
9	18.3	18.1	18.0	0.2	0.1	0.1
10	20.3	20.1	20.0	0.2	0.1	0.1
20	40.6	40.3	40.0	0.5	0.3	0.1
30	61.0	60.5	60.0	0.7	0.5	0.2
40	81.3	80.6	80.0	1.0	0.6	0.3
50	101.6	100.8	100.0	1.2	0.8	0.4

P. P.

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	
0	9.01 923	120	9.02 162	121	0.97 838	9.99 761	60
1	9.02 043	119	9.02 283	121	0.97 716	9.99 760	59
2	9.02 163	119	9.02 404	120	0.97 595	9.99 759	58
3	9.02 282	119	9.02 525	120	0.97 475	9.99 757	57
4	9.02 401	119	9.02 645	120	0.97 354	9.99 756	56
5	9.02 520	118	9.02 765	119	0.97 234	9.99 754	55
6	9.02 638	118	9.02 885	119	0.97 115	9.99 753	54
7	9.02 756	118	9.03 004	119	0.96 995	9.99 752	53
8	9.02 874	117	9.03 123	119	0.96 876	9.99 750	52
9	9.02 992	117	9.03 242	118	0.96 757	9.99 749	51
10	9.03 109	116	9.03 361	118	0.96 639	9.99 748	50
11	9.03 225	116	9.03 479	118	0.96 521	9.99 746	49
12	9.03 342	116	9.03 597	117	0.96 403	9.99 745	48
13	9.03 458	116	9.03 714	117	0.96 285	9.99 744	47
14	9.03 574	115	9.03 831	117	0.96 168	9.99 742	46
15	9.03 689	115	9.03 948	116	0.96 051	9.99 741	45
16	9.03 805	114	9.04 065	116	0.95 935	9.99 739	44
17	9.03 919	114	9.04 181	116	0.95 818	9.99 738	43
18	9.04 034	114	9.04 297	115	0.95 702	9.99 737	42
19	9.04 148	114	9.04 413	115	0.95 587	9.99 735	41
20	9.04 262	113	9.04 528	115	0.95 471	9.99 734	40
21	9.04 376	113	9.04 643	114	0.95 356	9.99 732	39
22	9.04 489	113	9.04 758	114	0.95 242	9.99 731	38
23	9.04 602	113	9.04 872	114	0.95 127	9.99 730	37
24	9.04 715	112	9.04 987	114	0.95 013	9.99 728	36
25	9.04 828	112	9.05 101	113	0.94 899	9.99 727	35
26	9.04 940	112	9.05 214	113	0.94 785	9.99 725	34
27	9.05 052	111	9.05 327	113	0.94 672	9.99 724	33
28	9.05 163	111	9.05 440	113	0.94 559	9.99 723	32
29	9.05 275	111	9.05 553	112	0.94 446	9.99 721	31
30	9.05 386	110	9.05 666	112	0.94 334	9.99 720	30
31	9.05 496	110	9.05 778	112	0.94 222	9.99 718	29
32	9.05 607	110	9.05 890	111	0.94 110	9.99 717	28
33	9.05 717	110	9.06 001	111	0.93 998	9.99 715	27
34	9.05 827	109	9.06 113	111	0.93 887	9.99 714	26
35	9.05 936	109	9.06 224	111	0.93 776	9.99 712	25
36	9.06 046	109	9.06 335	110	0.93 665	9.99 711	24
37	9.06 155	109	9.06 445	110	0.93 554	9.99 710	23
38	9.06 264	108	9.06 555	110	0.93 444	9.99 708	22
39	9.06 372	108	9.06 665	109	0.93 334	9.99 707	21
40	9.06 480	108	9.06 775	109	0.93 225	9.99 705	20
41	9.06 588	107	9.06 884	109	0.93 115	9.99 704	19
42	9.06 696	107	9.06 994	109	0.93 006	9.99 702	18
43	9.06 803	107	9.07 102	109	0.92 897	9.99 701	17
44	9.06 910	107	9.07 211	108	0.92 788	9.99 699	16
45	9.07 017	106	9.07 319	108	0.92 680	9.99 698	15
46	9.07 124	106	9.07 428	107	0.92 572	9.99 696	14
47	9.07 230	106	9.07 535	107	0.92 464	9.99 695	13
48	9.07 336	106	9.07 643	107	0.92 357	9.99 693	12
49	9.07 442	105	9.07 750	107	0.92 249	9.99 692	11
50	9.07 548	105	9.07 857	107	0.92 142	9.99 690	10
51	9.07 653	105	9.07 964	106	0.92 035	9.99 689	9
52	9.07 758	104	9.08 071	106	0.91 929	9.99 687	8
53	9.07 863	104	9.08 177	106	0.91 822	9.99 686	7
54	9.07 967	104	9.08 283	105	0.91 716	9.99 684	6
55	9.08 072	104	9.08 389	105	0.91 611	9.99 683	5
56	9.08 176	103	9.08 494	105	0.91 505	9.99 681	4
57	9.08 279	103	9.08 600	105	0.91 400	9.99 679	3
58	9.08 383	103	9.08 705	105	0.91 295	9.99 678	2
59	9.08 486	103	9.08 810	104	0.91 190	9.99 676	1
60	9.08 589		9.08 914		0.91 085	9.99 675	0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	′

P. P.

	121	121	120	119	118
6	12.1	12.1	12.0	11.9	11.8
7	14.2	14.1	14.0	13.9	13.7
8	16.2	16.1	16.0	15.8	15.7
9	18.2	18.1	18.0	17.8	17.7
10	20.2	20.1	20.0	19.8	19.6
20	40.5	40.3	40.0	39.6	39.3
30	60.7	60.5	60.0	59.5	59.0
40	81.0	80.6	80.0	79.3	78.6
50	101.2	100.8	100.0	99.1	98.3

	117	117	116	115
6	11.7	11.7	11.6	11.5
7	13.7	13.6	13.5	13.4
8	15.6	15.6	15.4	15.3
9	17.6	17.5	17.4	17.2
10	19.6	19.5	19.3	19.1
20	39.1	39.0	38.6	38.3
30	58.7	58.5	58.0	57.5
40	78.3	78.0	77.3	76.6
50	97.9	97.5	96.6	95.8

	114	114	113	112	111
6	11.4	11.4	11.3	11.2	11.1
7	13.3	13.3	13.2	13.0	12.9
8	15.2	15.2	15.0	14.9	14.8
9	17.2	17.1	16.9	16.8	16.6
10	19.1	19.0	18.8	18.6	18.5
20	38.1	38.0	37.6	37.3	37.0
30	57.2	57.0	56.5	56.0	55.5
40	76.3	76.0	75.3	74.6	74.0
50	95.4	95.0	94.1	93.3	92.5

	110	110	109	108
6	11.0	11.0	10.9	10.8
7	12.9	12.8	12.7	12.6
8	14.7	14.6	14.5	14.4
9	16.6	16.5	16.3	16.2
10	18.4	18.3	18.1	18.0
20	36.8	36.6	36.3	36.0
30	55.2	55.0	54.5	54.0
40	73.6	73.3	72.6	72.0
50	92.1	91.6	90.8	90.0

	107	107	106	105	104
6	10.7	10.7	10.6	10.5	10.4
7	12.5	12.5	12.3	12.2	12.1
8	14.3	14.2	14.1	14.0	13.8
9	16.1	16.0	15.9	15.7	15.6
10	17.9	17.8	17.6	17.5	17.3
20	35.8	35.6	35.3	35.0	34.6
30	53.7	53.5	53.0	52.5	52.0
40	71.6	71.3	70.6	70.0	69.3
50	89.6	89.1	88.3	87.5	86.6

	103	103	2	1	1
6	10.3	10.3	0.2	0.1	0.1
7	12.1	12.0	0.2	0.2	0.1
8	13.8	13.7	0.2	0.2	0.1
9	15.5	15.4	0.3	0.2	0.1
10	17.2	17.1	0.3	0.2	0.1
20	34.5	34.3	0.6	0.5	0.3
30	51.7	51.5	1.0	0.7	0.5
40	69.0	68.6	1.3	1.0	0.6
50	86.2	85.8	1.6	1.2	0.8

P. P.

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	
0	9.08 589	102	9.08 914	104	0.91 085	9.99 675	60
1	9.08 692	102	9.09 018	104	0.90 981	9.99 673	59
2	9.08 794	102	9.09 123	103	0.90 877	9.99 672	58
3	9.08 897	102	9.09 226	103	0.90 773	9.99 670	57
4	9.08 999	102	9.09 330	103	0.90 670	9.99 669	56
5	9.09 101	101	9.09 433	103	0.90 566	9.99 667	55
6	9.09 202	101	9.09 536	103	0.90 463	9.99 665	54
7	9.09 303	101	9.09 639	102	0.90 360	9.99 664	53
8	9.09 404	101	9.09 742	102	0.90 258	9.99 662	52
9	9.09 505	100	9.09 844	102	0.90 155	9.99 661	51
10	9.09 606	100	9.09 947	101	0.90 053	9.99 659	50
11	9.09 706	100	9.10 048	102	0.89 951	9.99 658	49
12	9.09 806	100	9.10 150	101	0.89 849	9.99 656	48
13	9.09 906	99	9.10 252	101	0.89 748	9.99 654	47
14	9.10 006	99	9.10 353	101	0.89 647	9.99 653	46
15	9.10 105	99	9.10 454	101	0.89 546	9.99 651	45
16	9.10 205	98	9.10 555	100	0.89 445	9.99 650	44
17	9.10 303	99	9.10 655	100	0.89 344	9.99 648	43
18	9.10 402	98	9.10 756	100	0.89 244	9.99 646	42
19	9.10 501	98	9.10 856	100	0.89 144	9.99 645	41
20	9.10 599	98	9.10 956	99	0.89 044	9.99 643	40
21	9.10 697	97	9.11 055	99	0.88 944	9.99 641	39
22	9.10 795	97	9.11 155	99	0.88 845	9.99 640	38
23	9.10 892	97	9.11 254	99	0.88 745	9.99 638	37
24	9.10 990	97	9.11 353	98	0.88 646	9.99 637	36
25	9.11 087	96	9.11 452	98	0.88 548	9.99 635	35
26	9.11 184	97	9.11 550	98	0.88 449	9.99 633	34
27	9.11 281	96	9.11 649	98	0.88 351	9.99 632	33
28	9.11 377	96	9.11 747	98	0.88 253	9.99 630	32
29	9.11 473	95	9.11 845	98	0.88 155	9.99 628	31
30	9.11 570	95	9.11 943	97	0.88 057	9.99 627	30
31	9.11 665	96	9.12 040	97	0.87 959	9.99 625	29
32	9.11 761	95	9.12 137	97	0.87 862	9.99 623	28
33	9.11 856	95	9.12 235	96	0.87 765	9.99 622	27
34	9.11 952	95	9.12 331	97	0.87 668	9.99 620	26
35	9.12 047	94	9.12 428	96	0.87 571	9.99 618	25
36	9.12 141	94	9.12 525	96	0.87 475	9.99 617	24
37	9.12 236	94	9.12 621	96	0.87 379	9.99 615	23
38	9.12 330	94	9.12 717	96	0.87 283	9.99 613	22
39	9.12 425	93	9.12 813	95	0.87 187	9.99 611	21
40	9.12 518	94	9.12 908	95	0.87 091	9.99 610	20
41	9.12 612	93	9.13 004	95	0.86 996	9.99 608	19
42	9.12 706	93	9.13 099	95	0.86 900	9.99 606	18
43	9.12 799	93	9.13 194	95	0.86 805	9.99 605	17
44	9.12 892	93	9.13 289	94	0.86 710	9.99 603	16
45	9.12 985	92	9.13 384	94	0.86 616	9.99 601	15
46	9.13 078	92	9.13 478	94	0.86 521	9.99 600	14
47	9.13 170	92	9.13 572	94	0.86 427	9.99 598	13
48	9.13 263	92	9.13 666	94	0.86 333	9.99 596	12
49	9.13 355	92	9.13 760	93	0.86 239	9.99 594	11
50	9.13 447	91	9.13 854	93	0.86 146	9.99 593	10
51	9.13 538	92	9.13 947	93	0.86 052	9.99 591	9
52	9.13 630	91	9.14 041	93	0.85 959	9.99 589	8
53	9.13 721	91	9.14 134	93	0.85 866	9.99 587	7
54	9.13 813	90	9.14 227	92	0.85 773	9.99 586	6
55	9.13 903	91	9.14 319	92	0.85 680	9.99 584	5
56	9.13 994	90	9.14 412	92	0.85 588	9.99 582	4
57	9.14 085	90	9.14 504	92	0.85 495	9.99 580	3
58	9.14 175	90	9.14 596	92	0.85 403	9.99 579	2
59	9.14 265	90	9.14 688	92	0.85 311	9.99 577	1
60	9.14 355		9.14 780		0.85 219	9.99 575	0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	′

P. P.

	104	103	102	101
6	10·4	10·3	10·2	10·1
7	12·1	12·0	11·9	11·8
8	13·8	13·7	13·6	13·4
9	15·6	15·4	15·3	15·1
10	17·3	17·1	17·0	16·8
20	34·6	34·3	34·0	33·6
30	52·0	51·5	51·0	50·5
40	69·3	68·6	68·0	67·3
50	86·6	85·8	85·0	84·1

	100	100	99	98
6	10·0	10·0	9·9	9·8
7	11·7	11·6	11·5	11·4
8	13·4	13·3	13·2	13·0
9	15·1	15·0	14·8	14·7
10	16·7	16·6	16·5	16·3
20	33·5	33·3	33·0	32·6
30	50·2	50·0	49·5	49·0
40	67·0	66·6	66·0	65·3
50	83·7	83·3	82·5	81·6

	97	97	96	95
6	9·7	9·7	9·6	9·5
7	11·4	11·3	11·2	11·1
8	13·0	12·9	12·8	12·6
9	14·6	14·5	14·4	14·2
10	16·2	16·1	16·0	15·8
20	32·5	32·3	32·0	31·6
30	48·7	48·5	48·0	47·5
40	65·0	64·6	64·0	63·3
50	81·2	80·8	80·0	79·1

	94	94	93	92
6	9·4	9·4	9·3	9·2
7	11·0	10·9	10·8	10·7
8	12·6	12·5	12·4	12·2
9	14·2	14·1	13·9	13·8
10	15·7	15·6	15·5	15·3
20	31·5	31·3	31·0	30·6
30	47·2	47·0	46·5	46·0
40	63·0	62·6	62·0	61·3
50	78·7	78·3	77·5	76·6

	91	91	90	2	1
6	9·1	9·1	9·0	0·2	0·1
7	10·7	10·6	10·5	0·2	0·2
8	12·2	12·1	12·0	0·2	0·2
9	13·7	13·6	13·5	0·3	0·2
10	15·2	15·1	15·0	0·3	0·2
20	30·5	30·3	30·0	0·6	0·5
30	45·7	45·5	45·0	1·0	0·7
40	61·0	60·6	60·0	1·3	1·0
50	76·2	75·8	75·0	1·6	1·2

P. P.

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	
0	9.14 355	90	9.14 780	91	0.85 219	9.99 575	60
1	9.14 445	89	9.14 872	91	0.85 128	9.99 573	59
2	9.14 535	89	9.14 963	91	0.85 037	9.99 571	58
3	9.14 624	89	9.15 054	91	0.84 945	9.99 570	57
4	9.14 713	89	9.15 145	91	0.84 854	9.99 568	56
5	9.14 802	89	9.15 236	90	0.84 763	9.99 566	55
6	9.14 891	88	9.15 327	90	0.84 673	9.99 564	54
7	9.14 980	88	9.15 417	90	0.84 582	9.99 563	53
8	9.15 068	88	9.15 507	90	0.84 492	9.99 561	52
9	9.15 157	88	9.15 598	89	0.84 402	9.99 559	51
10	9.15 245	88	9.15 687	90	0.84 312	9.99 557	50
11	9.15 333	88	9.15 777	89	0.84 222	9.99 555	49
12	9.15 421	87	9.15 867	89	0.84 133	9.99 553	48
13	9.15 508	87	9.15 956	89	0.84 043	9.99 552	47
14	9.15 595	87	9.16 045	89	0.83 954	9.99 550	46
15	9.15 683	87	9.16 134	89	0.83 865	9.99 548	45
16	9.15 770	87	9.16 223	89	0.83 776	9.99 546	44
17	9.15 857	86	9.16 312	88	0.83 687	9.99 544	43
18	9.15 943	86	9.16 401	88	0.83 599	9.99 542	42
19	9.16 030	86	9.16 489	88	0.83 511	9.99 541	41
20	9.16 116	86	9.16 577	88	0.83 422	9.99 539	40
21	9.16 202	86	9.16 665	87	0.83 334	9.99 537	39
22	9.16 288	86	9.16 753	88	0.83 247	9.99 535	38
23	9.16 374	85	9.16 841	87	0.83 159	9.99 533	37
24	9.16 460	85	9.16 928	87	0.83 071	9.99 531	36
25	9.16 545	85	9.17 015	87	0.82 984	9.99 529	35
26	9.16 630	86	9.17 103	87	0.82 897	9.99 528	34
27	9.16 716	85	9.17 190	86	0.82 810	9.99 526	33
28	9.16 801	84	9.17 276	87	0.82 723	9.99 524	32
29	9.16 885	84	9.17 363	86	0.82 636	9.99 522	31
30	9.16 970	84	9.17 450	86	0.82 550	9.99 520	30
31	9.17 054	84	9.17 536	86	0.82 464	9.99 518	29
32	9.17 139	84	9.17 622	86	0.82 377	9.99 516	28
33	9.17 223	84	9.17 708	85	0.82 291	9.99 514	27
34	9.17 307	84	9.17 794	86	0.82 206	9.99 512	26
35	9.17 391	83	9.17 880	85	0.82 120	9.99 511	25
36	9.17 474	83	9.17 965	85	0.82 034	9.99 509	24
37	9.17 558	83	9.18 051	85	0.81 949	9.99 507	23
38	9.17 641	83	9.18 136	85	0.81 864	9.99 505	22
39	9.17 724	83	9.18 221	85	0.81 779	9.99 503	21
40	9.17 807	83	9.18 306	84	0.81 694	9.99 501	20
41	9.17 890	82	9.18 390	84	0.81 609	9.99 499	19
42	9.17 972	82	9.18 475	84	0.81 525	9.99 497	18
43	9.18 055	82	9.18 559	84	0.81 440	9.99 495	17
44	9.18 137	82	9.18 644	84	0.81 356	9.99 493	16
45	9.18 219	82	9.18 728	84	0.81 272	9.99 491	15
46	9.18 301	82	9.18 812	84	0.81 188	9.99 489	14
47	9.18 383	81	9.18 896	83	0.81 104	9.99 487	13
48	9.18 465	81	9.18 979	83	0.81 020	9.99 485	12
49	9.18 546	81	9.19 063	83	0.80 937	9.99 484	11
50	9.18 628	81	9.19 146	83	0.80 854	9.99 482	10
51	9.18 709	81	9.19 229	83	0.80 770	9.99 480	9
52	9.18 790	80	9.19 312	83	0.80 687	9.99 478	8
53	9.18 871	81	9.19 395	82	0.80 604	9.99 476	7
54	9.18 952	80	9.19 478	82	0.80 522	9.99 474	6
55	9.19 032	80	9.19 560	82	0.80 439	9.99 472	5
56	9.19 113	80	9.19 643	82	0.80 357	9.99 470	4
57	9.19 193	80	9.19 725	82	0.80 274	9.99 468	3
58	9.19 273	80	9.19 807	82	0.80 192	9.99 466	2
59	9.19 353	79	9.19 889	82	0.80 110	9.99 464	1
60	9.19 433		9.19 971		0.80 028	9.99 462	0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	′

P. P.

	91	91	90	89
6	9.1	9.1	9.0	8.9
7	10.7	10.6	10.5	10.4
8	12.2	12.1	12.0	11.8
9	13.7	13.6	13.5	13.3
10	15.2	15.1	15.0	14.8
20	30.5	30.3	30.0	29.6
30	45.7	45.5	45.0	44.5
40	61.0	60.6	60.0	59.3
50	76.2	75.8	75.0	74.1

	88	88	87	86
6	8.8	8.8	8.7	8.6
7	10.3	10.2	10.1	10.0
8	11.8	11.7	11.6	11.4
9	13.3	13.2	13.0	12.9
10	14.7	14.6	14.5	14.3
20	29.5	29.3	29.0	28.6
30	44.2	44.0	43.5	43.0
40	59.0	58.6	58.0	57.3
50	73.7	73.3	72.5	71.6

	85	85	84	83
6	8.5	8.5	8.4	8.3
7	10.0	9.9	9.8	9.7
8	11.4	11.3	11.2	11.0
9	12.8	12.7	12.6	12.4
10	14.2	14.1	14.0	13.8
20	28.5	28.3	28.0	27.6
30	42.7	42.5	42.0	41.5
40	57.0	56.6	56.0	55.3
50	71.2	70.8	70.0	69.1

	82	82	81	80
6	8.2	8.2	8.1	8.0
7	9.6	9.5	9.4	9.3
8	11.0	10.9	10.8	10.6
9	12.4	12.3	12.1	12.0
10	13.7	13.6	13.5	13.3
20	27.5	27.3	27.0	26.6
30	41.2	41.0	40.5	40.0
40	55.0	54.6	54.0	53.3
50	68.7	68.3	67.5	66.6

	79	2	1
6	7.9	0.2	0.1
7	9.3	0.2	0.2
8	10.6	0.2	0.2
9	11.9	0.3	0.2
10	13.2	0.3	0.2
20	26.5	0.6	0.5
30	39.7	1.0	0.7
40	53.0	1.3	1.0
50	66.2	1.6	1.2

P. P.

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	
0	9.19 433	80	9.19 971	81	0.80 028	9.99 462	60
1	9.19 513	79	9.20 053	81	0.79 947	9.99 460	59
2	9.19 592	79	9.20 134	81	0.79 865	9.99 458	58
3	9.19 672	79	9.20 216	81	0.79 784	9.99 456	57
4	9.19 751	79	9.20 297	81	0.79 703	9.99 454	56
5	9.19 830	79	9.20 378	81	0.79 622	9.99 452	55
6	9.19 909	79	9.20 459	81	0.79 541	9.99 450	54
7	9.19 988	78	9.20 540	80	0.79 460	9.99 448	53
8	9.20 066	78	9.20 620	81	0.79 379	9.99 446	52
9	9.20 145	78	9.20 701	80	0.79 298	9.99 444	51
10	9.20 223	78	9.20 781	80	0.79 218	9.99 442	50
11	9.20 301	78	9.20 862	80	0.79 138	9.99 440	49
12	9.20 379	78	9.20 942	80	0.79 058	9.99 437	48
13	9.20 457	78	9.21 022	80	0.78 978	9.99 435	47
14	9.20 535	77	9.21 102	79	0.78 898	9.99 433	46
15	9.20 613	77	9.21 181	79	0.78 818	9.99 431	45
16	9.20 690	77	9.21 261	79	0.78 739	9.99 429	44
17	9.20 768	77	9.21 340	79	0.78 659	9.99 427	43
18	9.20 845	77	9.21 420	79	0.78 580	9.99 425	42
19	9.20 922	77	9.21 499	79	0.78 501	9.99 423	41
20	9.20 999	77	9.21 578	79	0.78 422	9.99 421	40
21	9.21 076	76	9.21 657	78	0.78 343	9.99 419	39
22	9.21 152	76	9.21 735	78	0.78 264	9.99 417	38
23	9.21 229	76	9.21 814	78	0.78 186	9.99 415	37
24	9.21 305	76	9.21 892	78	0.78 107	9.99 413	36
25	9.21 382	76	9.21 971	78	0.78 029	9.99 411	35
26	9.21 458	76	9.22 049	78	0.77 951	9.99 408	34
27	9.21 534	75	9.22 127	78	0.77 873	9.99 406	33
28	9.21 609	76	9.22 205	78	0.77 795	9.99 404	32
29	9.21 685	75	9.22 283	77	0.77 717	9.99 402	31
30	9.21 761	75	9.22 360	77	0.77 639	9.99 400	30
31	9.21 836	75	9.22 438	77	0.77 562	9.99 398	29
32	9.21 911	75	9.22 515	77	0.77 484	9.99 396	28
33	9.21 987	75	9.22 593	77	0.77 407	9.99 394	27
34	9.22 062	74	9.22 670	77	0.77 330	9.99 392	26
35	9.22 136	75	9.22 747	77	0.77 253	9.99 389	25
36	9.22 211	74	9.22 824	76	0.77 176	9.99 387	24
37	9.22 286	74	9.22 900	76	0.77 099	9.99 385	23
38	9.22 360	74	9.22 977	76	0.77 022	9.99 383	22
39	9.22 435	74	9.23 054	76	0.76 946	9.99 381	21
40	9.22 509	74	9.23 130	76	0.76 870	9.99 379	20
41	9.22 583	74	9.23 206	76	0.76 793	9.99 377	19
42	9.22 657	73	9.23 282	76	0.76 717	9.99 374	18
43	9.22 731	74	9.23 358	76	0.76 641	9.99 372	17
44	9.22 805	73	9.23 434	76	0.76 565	9.99 370	16
45	9.22 878	73	9.23 510	75	0.76 489	9.99 368	15
46	9.22 952	73	9.23 586	75	0.76 414	9.99 366	14
47	9.23 025	73	9.23 661	75	0.76 338	9.99 364	13
48	9.23 098	73	9.23 737	75	0.76 263	9.99 361	12
49	9.23 171	73	9.23 812	75	0.76 188	9.99 359	11
50	9.23 244	72	9.23 887	75	0.76 113	9.99 357	10
51	9.23 317	73	9.23 962	75	0.76 038	9.99 355	9
52	9.23 390	72	9.24 037	75	0.75 963	9.99 353	8
53	9.23 462	72	9.24 112	74	0.75 888	9.99 350	7
54	9.23 535	72	9.24 186	74	0.75 813	9.99 348	6
55	9.23 607	72	9.24 261	74	0.75 739	9.99 346	5
56	9.23 679	72	9.24 335	74	0.75 664	9.99 344	4
57	9.23 751	72	9.24 409	74	0.75 590	9.99 342	3
58	9.23 823	72	9.24 484	74	0.75 516	9.99 339	2
59	9.23 895	71	9.24 558	74	0.75 442	9.99 337	1
60	9.23 967		9.24 632		0.75 368	9.99 335	0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	′

P. P.

	81	81	80	79
6	8.1	8.1	8.0	7.9
7	9.5	9.4	9.3	9.2
8	10.8	10.8	10.6	10.5
9	12.2	12.1	12.0	11.8
10	13.6	13.5	13.3	13.1
20	27.1	27.0	26.6	26.3
30	40.7	40.5	40.0	39.5
40	54.3	54.0	53.3	52.6
50	67.9	67.5	66.6	65.8

	78	78	77
6	7.8	7.8	7.7
7	9.1	9.1	9.0
8	10.4	10.4	10.2
9	11.8	11.7	11.5
10	13.1	13.0	12.8
20	26.1	26.0	25.6
30	39.2	39.0	38.5
40	52.3	52.0	51.3
50	65.4	65.0	64.1

	76	76	75	74
6	7.6	7.6	7.5	7.4
7	8.9	8.8	8.7	8.6
8	10.2	10.1	10.0	9.8
9	11.5	11.4	11.2	11.1
10	12.7	12.6	12.5	12.3
20	25.5	25.3	25.0	24.6
30	38.2	38.0	37.5	37.0
40	51.0	50.6	50.0	49.3
50	63.7	63.3	62.5	61.6

	73	73	72
6	7.3	7.3	7.2
7	8.6	8.5	8.4
8	9.8	9.7	9.6
9	11.0	10.9	10.8
10	12.2	12.1	12.0
20	24.5	24.3	24.0
30	36.7	36.5	36.0
40	49.0	48.6	48.0
50	61.2	60.8	60.0

	71	71	2	2
6	7.1	7.1	0.2	0.2
7	8.3	8.3	0.3	0.2
8	9.5	9.4	0.3	0.2
9	10.7	10.6	0.4	0.3
10	11.9	11.8	0.4	0.3
20	23.8	23.6	0.8	0.6
30	35.7	35.5	1.2	1.0
40	47.6	47.3	1.6	1.3
50	59.6	59.1	2.1	1.6

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	
0	9.23 967	71	9.24 632	73	0.75 368	9.99 335	60
1	9.24 038	71	9.24 705	74	0.75 294	9.99 333	59
2	9.24 110	71	9.24 779	73	0.75 220	9.99 330	58
3	9.24 181	71	9.24 853	73	0.75 147	9.99 328	57
4	9.24 252	71	9.24 926	73	0.75 073	9.99 326	56
5	9.24 323	71	9.25 000	73	0.75 000	9.99 324	55
6	9.24 394	71	9.25 073	73	0.74 927	9.99 321	54
7	9.24 465	71	9.25 146	73	0.74 854	9.99 319	53
8	9.24 536	70	9.25 219	73	0.74 781	9.99 317	52
9	9.24 607	70	9.25 292	73	0.74 708	9.99 315	51
10	9.24 677	70	9.25 365	72	0.74 635	9.99 312	50
11	9.24 748	70	9.25 437	72	0.74 562	9.99 310	49
12	9.24 818	70	9.25 510	72	0.74 490	9.99 308	48
13	9.24 888	70	9.25 582	72	0.74 417	9.99 306	47
14	9.24 958	69	9.25 654	72.	0.74 345	9.99 303	46
15	9.25 028	70	9.25 727	72	0.74 273	9.99 301	45
16	9.25 098	69	9.25 799	72	0.74 201	9.99 299	44
17	9.25 167	70	9.25 871	72	0.74 129	9.99 296	43
18	9.25 237	69	9.25 943	71	0.74 057	9.99 294	42
19	9.25 306	69	9.26 014	72	0.73 985	9.99 292	41
20	9.25 376	69	9.26 086	71	0.73 913	9.99 290	40
21	9.25 445	69	9.26 158	71	0.73 842	9.99 287	39
22	9.25 514	69	9.26 229	71	0.73 771	9.99 285	38
23	9.25 583	69	9.26 300	71	0.73 699	9.99 283	37
24	9.25 652	68	9.26 371	71	0.73 628	9.99 280	36
25	9.25 721	69	9.26 443	71	0.73 557	9.99 278	35
26	9.25 790	68	9.26 514	70	0.73 486	9.99 276	34
27	9.25 858	68	9.26 584	71	0.73 415	9.99 273	33
28	9.25 927	68	9.26 655	70	0.73 344	9.99 271	32
29	9.25 995	68	9.26 726	70	0.73 274	9.99 269	31
30	9.26 063	68	9.26 796	70	0.73 203	9.99 266	30
31	9.26 131	68	9.26 867	70	0.73 133	9.99 264	29
32	9.26 199	68	9.26 937	70	0.73 062	9.99 262	28
33	9.26 267	67	9.27 007	70	0.72 992	9.99 259	27
34	9.26 335	67	9.27 078	70	0.72 922	9.99 257	26
35	9.26 402	68	9.27 148	70	0.72 852	9.99 255	25
36	9.26 470	67	9.27 218	69	0.72 782	9.99 252	24
37	9.26 537	67	9.27 287	70	0.72 712	9.99 250	23
38	9.26 605	67	9.27 357	69	0.72 642	9.99 248	22
39	9.26 672	67	9.27 427	69	0.72 573	9.99 245	21
40	9.26 739	67	9.27 496	69	0.72 503	9.99 243	20
41	9.26 806	67	9.27 566	69	0.72 434	9.99 240	19
42	9.26 873	66	9.27 635	69	0.72 365	9.99 238	18
43	9.26 940	67	9.27 704	69	0.72 295	9.99 236	17
44	9.27 007	66	9.27 773	69	0.72 226	9.99 233	16
45	9.27 073	66	9.27 842	69	0.72 157	9.99 231	15
46	9.27 140	66	9.27 911	68	0.72 088	9.99 228	14
47	9.27 206	66	9.27 980	69	0.72 020	9.99 226	13
48	9.27 272	66	9.28 049	68	0.71 951	9.99 224	12
49	9.27 339	66	9.28 117	68	0.71 882	9.99 221	11
50	9.27 405	66	9.28 186	68	0.71 814	9.99 219	10
51	9.27 471	65	9.28 254	68	0.71 746	9.99 216	9
52	9.27 536	66	9.28 322	68	0.71 677	9.99 214	8
53	9.27 602	65	9.28 390	68	0.71 609	9.99 212	7
54	9.27 668	65	9.28 459	68	0.71 541	9.99 209	6
55	9.27 733	65	9.28 527	67	0.71 473	9.99 207	5
56	9.27 799	65	9.28 594	68	0.71 405	9.99 204	4
57	9.27 864	65	9.28 662	67	0.71 337	9.99 202	3
58	9.27 929	65	9.28 730	67	0.71 270	9.99 199	2
59	9.27 995	65	9.28 797	67	0.71 202	9.99 197	1
60	9.28 060		9.28 865		0.71 135	9.99 194	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	74	$7\bar{3}$	73
6	7.4	7.3	7.3
7	8.6	8.6	8.5
8	9.8	9.8	9.7
9	11.1	11.0	10.9
10	12.3	12.2	12.1
20	24.6	24.5	24.3
30	37.0	36.7	36.5
40	49.3	49.0	48.6
50	61.6	61.2	60.8

	$7\bar{2}$	72	$7\bar{1}$	71
6	7.2	7.2	7.1	7.1
7	8.4	8.4	8.3	8.3
8	9.6	9.6	9.5	9.4
9	10.9	10.8	10.7	10.6
10	12.1	12.0	11.9	11.8
20	24.1	24.0	23.8	23.6
30	36.2	36.0	35.7	35.5
40	48.3	48.0	47.6	47.3
50	60.4	60.0	59.6	59.1

	$7\bar{0}$	70	$6\bar{9}$	69
6	7.0	7.0	6.9	6.9
7	8.2	8.1	8.1	8.0
8	9.4	9.3	9.2	9.2
9	10.6	10.5	10.4	10.3
10	11.7	11.6	11.6	11.5
20	23.5	23.3	23.1	23.0
30	35.2	35.0	34.7	34.5
40	47.0	46.6	46.3	46.0
50	58.7	58.3	57.9	57.5

	$6\bar{8}$	68	$6\bar{7}$	67
6	6.8	6.8	6.7	6.7
7	8.0	7.9	7.9	7.8
8	9.1	9.0	9.0	8.9
9	10.3	10.2	10.1	10.0
10	11.4	11.3	11.2	11.1
20	22.8	22.6	22.5	22.3
30	34.2	34.0	33.7	33.5
40	45.6	45.3	45.0	44.6
50	57.1	56.6	56.2	55.8

	$6\bar{6}$	66	$6\bar{5}$	65
6	6.6	6.6	6.5	6.5
7	7.7	7.7	7.6	7.6
8	8.8	8.8	8.7	8.6
9	10.0	9.9	9.8	9.7
10	11.1	11.0	10.9	10.8
20	22.1	22.0	21.8	21.6
30	33.2	33.0	32.7	32.5
40	44.3	44.0	43.6	43.3
50	55.4	55.0	54.6	54.1

	$\bar{2}$	2
6	0.2	0.2
7	0.3	0.2
8	0.3	0.2
9	0.4	0.3
10	0.4	0.3
20	0.8	0.6
30	1.2	1.0
40	1.6	1.3
50	2.1	1.6

P. P.

Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	
9.28 060	65	9.28 865	67	0.71 135	9.99 194	60
9.28 125	64	9.28 932	67	0.71 067	9.99 192	59
9.28 189	65	9.29 000	67	0.71 000	9.99 189	58
9.28 254	64	9.29 067	67	0.70 933	9.99 187	57
9.28 319	64	9.29 134	67	0.70 866	9.99 185	56
9.28 383	64	9.29 201	66	0.70 798	9.99 182	55
9.28 448	64	9.29 268	67	0.70 732	9.99 180	54
9.28 512	64	9.29 335	66	0.70 665	9.99 177	53
9.28 576	64	9.29 401	67	0.70 598	9.99 175	52
9.28 641	64	9.29 468	66	0.70 531	9.99 172	51
9.28 705	64	9.29 535	66	0.70 465	9.99 170	50
9.28 769	63	9.29 601	66	0.70 398	9.99 167	49
9.28 832	64	9.29 667	66	0.70 332	9.99 165	48
9.28 896	63	9.29 734	66	0.70 266	9.99 162	47
9.28 960	63	9.29 800	66	0.70 200	9.99 160	46
9.29 023	63	9.29 866	66	0.70 134	9.99 157	45
9.29 087	63	9.29 932	66	0.70 068	9.99 155	44
9.29 150	63	9.29 998	66	0.70 002	9.99 152	43
9.29 213	63	9.30 064	65	0.69 936	9.99 150	42
9.29 277	63	9.30 129	65	0.69 870	9.99 147	41
9.29 340	63	9.30 195	65	0.69 805	9.99 145	40
9.29 403	63	9.30 260	65	0.69 739	9.99 142	39
9.29 466	62	9.30 326	65	0.69 674	9.99 139	38
9.29 528	63	9.30 391	65	0.69 608	9.99 137	37
9.29 591	62	9.30 456	65	0.69 543	9.99 134	36
9.29 654	62	9.30 522	65	0.69 478	9.99 132	35
9.29 716	62	9.30 587	65	0.69 413	9.99 129	34
9.29 779	62	9.30 652	65	0.69 348	9.99 127	33
9.29 841	62	9.30 717	64	0.69 283	9.99 124	32
9.29 903	62	9.30 781	65	0.69 218	9.99 122	31
9.29 965	62	9.30 846	64	0.69 153	9.99 119	30
9.30 027	62	9.30 911	64	0.69 089	9.99 116	29
9.30 089	62	9.30 975	64	0.69 024	9.99 114	28
9.30 151	61	9.31 040	64	0.68 960	9.99 111	27
9.30 213	62	9.31 104	64	0.68 896	9.99 109	26
9.30 275	61	9.31 168	64	0.68 831	9.99 106	25
9.30 336	61	9.31 232	64	0.68 767	9.99 104	24
9.30 398	61	9.31 297	64	0.68 703	9.99 101	23
9.30 459	61	9.31 361	63	0.68 639	9.99 098	22
9.30 520	61	9.31 424	64	0.68 575	9.99 096	21
9.30 582	61	9.31 488	64	0.68 511	9.99 093	20
9.30 643	61	9.31 552	63	0.68 447	9.99 091	19
9.30 704	61	9.31 616	63	0.68 384	9.99 088	18
9.30 765	61	9.31 679	63	0.68 320	9.99 085	17
9.30 826	60	9.31 743	63	0.68 257	9.99 083	16
9.30 886	61	9.31 806	63	0.68 193	9.99 080	15
9.30 947	60	9.31 869	63	0.68 130	9.99 077	14
9.31 008	60	9.31 933	63	0.68 067	9.99 075	13
9.31 068	60	9.31 996	63	0.68 004	9.99 072	12
9.31 129	60	9.32 059	63	0.67 941	9.99 069	11
9.31 189	60	9.32 122	63	0.67 878	9.99 067	10
9.31 249	60	9.32 185	63	0.67 815	9.99 064	9
9.31 309	60	9.32 248	62	0.67 752	9.99 062	8
9.31 370	59	9.32 310	63	0.67 689	9.99 059	7
9.31 429	60	9.32 373	62	0.67 626	9.99 056	6
9.31 489	60	9.32 436	62	0.67 564	9.99 054	5
9.31 549	59	9.32 498	62	0.67 501	9.99 051	4
9.31 609	60	9.32 560	62	0.67 439	9.99 048	3
9.31 669	59	9.32 623	62	0.67 377	9.99 046	2
9.31 728	59	9.32 685	62	0.67 314	9.99 043	1
9.31 788		9.32 747		0.67 252	9.99 040	0
Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	'

P. P.

	$6\bar{7}$	67
6	6.7	6.7
7	7.9	7.8
8	9.0	8.9
9	10.1	10.0
10	11.2	11.1
20	22.5	22.3
30	33.7	33.5
40	45.0	44.6
50	56.2	55.8

	$6\bar{6}$	66	$6\bar{5}$	65
6	6.6	6.6	6.5	6.5
7	7.7	7.7	7.6	7.6
8	8.8	8.8	8.7	8.6
8	10.0	9.9	9.8	9.7
10	11.1	11.0	10.9	10.8
20	22.1	22.0	21.8	21.6
30	33.2	33.0	32.7	32.5
40	44.3	44.0	43.6	43.3
50	55.4	55.0	54.6	54.1

	$6\bar{4}$	64	$6\bar{3}$	63
6	6.4	6.4	6.3	6.3
7	7.5	7.4	7.4	7.3
8	8.6	8.5	8.4	8.4
9	9.7	9.6	9.5	9.4
10	10.7	10.6	10.6	10.5
20	21.5	21.3	21.1	21.0
30	32.2	32.0	31.7	31.5
40	43.0	42.6	42.3	42.0
50	53.7	53.3	52.9	52.5

	$6\bar{2}$	62	$6\bar{1}$	61
6	6.2	6.2	6.1	6.1
7	7.3	7.2	7.2	7.1
8	8.3	8.2	8.2	8.1
9	9.4	9.3	9.2	9.1
10	10.4	10.3	10.2	10.1
20	20.8	20.6	20.5	20.3
30	31.2	31.0	30.7	30.5
40	41.6	41.3	41.0	40.6
50	52.1	51.6	51.2	50.8

	$6\bar{0}$	60	$5\bar{9}$
6	6.0	6.0	5.9
7	7.0	7.0	6.9
8	8.0	8.0	7.9
9	9.1	9.0	8.9
10	10.1	10.0	9.9
20	20.1	20.0	19.8
30	30.2	30.0	29.7
40	40.3	40.0	39.6
50	50.4	50.0	49.6

	3	$\bar{2}$	2
6	0.3	0.2	0.2
7	0.3	0.3	0.2
8	0.4	0.3	0.2
9	0.4	0.4	0.3
10	0.5	0.4	0.3
20	1.0	0.8	0.6
30	1.5	1.2	1.0
40	2.0	1.6	1.3
50	2.5	2.1	1.6

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	
0	9.31 788	59	9.32 747	62	0.67 253	9.99 040	60
1	9.31 847	59	9.32 809	62	0.67 190	9.99 038	59
2	9.31 906	59	9.32 871	62	0.67 128	9.99 035	58
3	9.31 966	59	9.32 933	62	0.67 066	9.99 032	57
4	9.32 025	59	9.32 995	61	0.67 004	9.99 029	56
5	9.32 084	59	9.33 057	61	0.66 943	9.99 027	55
6	9.32 143	59	9.33 118	62	0.66 881	9.99 024	54
7	9.32 202	58	9.33 180	61	0.66 819	9.99 021	53
8	9.32 260	59	9.33 242	61	0.66 758	9.99 019	52
9	9.32 319	58	9.33 303	61	0.66 696	9.99 016	51
10	9.32 378	58	9.33 364	61	0.66 635	9.99 013	50
11	9.32 436	58	9.33 426	61	0.66 574	9.99 010	49
12	9.32 495	58	9.33 487	61	0.66 513	9.99 008	48
13	9.32 553	58	9.33 548	61	0.66 452	9.99 005	47
14	9.32 611	58	9.33 609	60	0.66 390	9.99 002	46
15	9.32 670	58	9.33 670	61	0.66 330	9.98 999	45
16	9.32 728	58	9.33 731	61	0.66 269	9.98 997	44
17	9.32 786	58	9.33 792	60	0.66 208	9.98 994	43
18	9.32 844	58	9.33 852	61	0.66 147	9.98 991	42
19	9.32 902	58	9.33 913	60	0.66 086	9.98 988	41
20	9.32 960	57	9.33 974	60	0.66 026	9.98 986	40
21	9.33 017	58	9.34 034	60	0.65 965	9.98 983	39
22	9.33 075	57	9.34 095	60	0.65 905	9.98 980	38
23	9.33 133	57	9.34 155	60	0.65 845	9.98 977	37
24	9.33 190	57	9.34 215	60	0.65 784	9.98 975	36
25	9.33 248	57	9.34 275	60	0.65 724	9.98 972	35
26	9.33 305	57	9.34 336	60	0.65 664	9.98 969	34
27	9.33 362	57	9.34 396	60	0.65 604	9.98 966	33
28	9.33 419	57	9.34 456	59	0.65 544	9.98 963	32
29	9.33 476	57	9.34 515	60	0.65 484	9.98 961	31
30	9.33 533	57	9.34 575	60	0.65 424	9.98 958	30
31	9.33 590	57	9.34 635	59	0.65 364	9.98 955	29
32	9.33 647	57	9.34 695	59	0.65 305	9.98 952	28
33	9.33 704	56	9.34 754	59	0.65 245	9.98 949	27
34	9.33 761	56	9.34 814	59	0.65 186	9.98 947	26
35	9.33 817	56	9.34 873	59	0.65 126	9.98 944	25
36	9.33 874	56	9.34 933	59	0.65 067	9.98 941	24
37	9.33 930	56	9.34 992	59	0.65 008	9.98 938	23
38	9.33 987	56	9.35 051	59	0.64 948	9.98 935	22
39	9.34 043	56	9.35 110	59	0.64 889	9.98 933	21
40	9.34 099	56	9.35 169	59	0.64 830	9.98 930	20
41	9.34 156	56	9.35 228	59	0.64 771	9.98 927	19
42	9.34 212	56	9.35 287	59	0.64 712	9.98 924	18
43	9.34 268	56	9.35 346	59	0.64 653	9.98 921	17
44	9.34 324	55	9.35 405	58	0.64 594	9.98 918	16
45	9.34 379	56	9.35 464	58	0.64 536	9.98 915	15
46	9.34 435	55	9.35 522	59	0.64 477	9.98 913	14
47	9.34 491	56	9.35 581	58	0.64 418	9.98 910	13
48	9.34 547	55	9.35 640	58	0.64 360	9.98 907	12
49	9.34 602	55	9.35 698	58	0.64 302	9.98 904	11
50	9.34 658	55	9.35 756	58	0.64 243	9.98 901	10
51	9.34 713	55	9.35 815	58	0.64 185	9.98 898	9
52	9.34 768	55	9.35 873	58	0.64 127	9.98 895	8
53	9.34 824	55	9.35 931	58	0.64 068	9.98 892	7
54	9.34 879	55	9.35 989	58	0.64 010	9.98 890	6
55	9.34 934	55	9.36 047	58	0.63 952	9.98 887	5
56	9.34 989	55	9.36 105	57	0.63 894	9.98 884	4
57	9.35 044	54	9.36 163	58	0.63 837	9.98 881	3
58	9.35 099	55	9.36 221	57	0.63 779	9.98 878	2
59	9.35 154	55	9.36 278	58	0.63 721	9.98 875	1
60	9.35 209		9.36 336		0.63 663	9.98 872	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	62	61	61
6	6.2	6.1	6.1
7	7.2	7.2	7.1
8	8.2	8.2	8.1
9	9.3	9.2	9.1
10	10.3	10.2	10.1
20	20.6	20.5	20.3
30	31.0	30.7	30.5
40	41.3	41.0	40.6
50	51.6	51.2	50.8

	60	60	59	59
6	6.0	6.0	5.9	5.9
7	7.0	7.0	6.9	6.9
8	8.0	8.0	7.9	7.8
9	9.1	9.0	8.9	8.8
10	10.1	10.0	9.9	9.8
20	20.1	20.0	19.8	19.6
30	30.2	30.0	29.7	29.5
40	40.3	40.0	39.6	39.3
50	50.4	50.0	49.6	49.1

	58	58	57	57
6	5.8	5.8	5.7	5.7
7	6.8	6.7	6.7	6.6
8	7.8	7.7	7.6	7.6
9	8.8	8.7	8.6	8.5
10	9.7	9.6	9.6	9.5
20	19.5	19.3	19.1	19.0
30	29.2	29.0	28.7	28.5
40	39.0	38.6	38.3	38.0
50	48.7	48.3	47.9	47.5

	56	56	55	55
6	5.6	5.6	5.5	5.5
7	6.6	6.5	6.5	6.4
8	7.5	7.4	7.4	7.3
9	8.5	8.4	8.3	8.2
10	9.4	9.3	9.2	9.1
20	18.8	18.6	18.5	18.3
30	28.2	28.0	27.7	27.5
40	37.6	37.3	37.0	36.6
50	47.1	46.6	46.2	45.8

	54	3	2
6	5.4	0.3	0.2
7	6.3	0.3	0.3
8	7.2	0.4	0.3
9	8.2	0.4	0.4
10	9.1	0.5	0.4
20	18.1	1.0	0.8
30	27.2	1.5	1.2
40	36.3	2.0	1.6
50	45.4	2.5	2.1

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	
0	9.35 209	54	9.36 336	57	0.63 663	9.98 872	60
1	9.35 263	54	9.36 394	57	0.63 606	9.98 869	59
2	9.35 318	54	9.36 451	57	0.63 548	9.98 866	58
3	9.35 372	54	9.36 509	57	0.63 491	9.98 863	57
4	9.35 427	54	9.36 566	57	0.63 433	9.98 860	56
5	9.35 481	54	9.36 623	57	0.63 376	9.98 858	55
6	9.35 536	54	9.36 681	57	0.63 319	9.98 855	54
7	9.35 590	54	9.36 738	57	0.63 262	9.98 852	53
8	9.35 644	54	9.36 795	57	0.63 204	9.98 849	52
9	9.35 698	54	9.36 852	57	0.63 147	9.98 846	51
10	9.35 752	54	9.36 909	57	0.63 090	9.98 843	50
11	9.35 806	54	9.36 966	56	0.63 033	9.98 840	49
12	9.35 860	53	9.37 023	57	0.62 977	9.98 837	48
13	9.35 914	54	9.37 080	56	0.62 920	9.98 834	47
14	9.35 968	53	9.37 136	57	0.62 863	9.98 831	46
15	9.36 021	53	9.37 193	56	0.62 806	9.98 828	45
16	9.36 075	53	9.37 250	56	0.62 750	9.98 825	44
17	9.36 128	53	9.37 306	56	0.62 693	9.98 822	43
18	9.36 182	53	9.37 363	56	0.62 637	9.98 819	42
19	9.36 235	53	9.37 419	56	0.62 580	9.98 816	41
20	9.36 289	53	9.37 475	56	0.62 524	9.98 813	40
21	9.36 342	53	9.37 532	56	0.62 468	9.98 810	39
22	9.36 395	53	9.37 588	56	0.62 412	9.98 807	38
23	9.36 448	53	9.37 644	56	0.62 356	9.98 804	37
24	9.36 501	53	9.37 700	56	0.62 299	9.98 801	36
25	9.36 554	53	9.37 756	55	0.62 243	9.98 798	35
26	9.36 607	53	9.37 812	56	0.62 188	9.98 795	34
27	9.36 660	52	9.37 868	56	0.62 132	9.98 792	33
28	9.36 713	53	9.37 924	55	0.62 076	9.98 789	32
29	9.36 766	52	9.37 979	56	0.62 020	9.98 786	31
30	9.36 818	52	9.38 035	55	0.61 964	9.98 783	30
31	9.36 871	52	9.38 091	55	0.61 909	9.98 780	29
32	9.36 923	52	9.38 146	55	0.61 853	9.98 777	28
33	9.36 976	52	9.38 202	55	0.61 798	9.98 774	27
34	9.37 028	52	9.38 257	55	0.61 742	9.98 771	26
35	9.37 081	52	9.38 313	55	0.61 687	9.98 768	25
36	9.37 133	52	9.38 368	55	0.61 632	9.98 765	24
37	9.37 185	52	9.38 423	55	0.61 576	9.98 762	23
38	9.37 237	52	9.38 478	55	0.61 521	9.98 759	22
39	9.37 289	52	9.38 533	55	0.61 466	9.98 755	21
40	9.37 341	52	9.38 589	55	0.61 411	9.98 752	20
41	9.37 393	51	9.38 644	54	0.61 356	9.98 749	19
42	9.37 445	52	9.38 698	55	0.61 301	9.98 746	18
43	9.37 497	51	9.38 753	55	0.61 246	9.98 743	17
44	9.37 548	52	9.38 808	54	0.61 191	9.98 740	16
45	9.37 600	51	9.38 863	55	0.61 137	9.98 737	15
46	9.37 652	51	9.38 918	54	0.61 082	9.98 734	14
47	9.37 703	51	9.38 972	54	0.61 027	9.98 731	13
48	9.37 755	51	9.39 027	54	0.60 973	9.98 728	12
49	9.37 806	51	9.39 081	54	0.60 918	9.98 725	11
50	9.37 857	51	9.39 136	54	0.60 864	9.98 721	10
51	9.37 909	51	9.39 190	54	0.60 809	9.98 718	9
52	9.37 960	51	9.39 244	54	0.60 755	9.98 715	8
53	9.38 011	51	9.39 299	54	0.60 701	9.98 712	7
54	9.38 062	51	9.39 353	54	0.60 647	9.98 709	6
55	9.38 113	51	9.39 407	54	0.60 592	9.98 706	5
56	9.38 164	51	9.39 461	54	0.60 538	9.98 703	4
57	9.38 215	50	9.39 515	54	0.60 484	9.98 700	3
58	9.38 266	51	9.39 569	54	0.60 430	9.98 696	2
59	9.38 317	51	9.39 623	54	0.60 376	9.98 693	1
60	9.38 367	50	9.39 677	53	0.60 323	9.98 690	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	57	57	56	56
6	5.7	5.7	5.6	5.6
7	6.7	6.6	6.6	6.5
8	7.6	7.6	7.5	7.4
9	8.6	8.5	8.5	8.4
10	9.6	9.5	9.4	9.3
20	19.1	19.0	18.8	18.6
30	28.7	28.5	28.2	28.0
40	38.3	38.0	37.6	37.3
50	47.9	47.5	47.1	46.6

	55	55	54	54
6	5.5	5.5	5.4	5.4
7	6.5	6.4	6.3	6.3
8	7.4	7.3	7.2	7.2
9	8.3	8.2	8.2	8.1
10	9.2	9.1	9.1	9.0
20	18.5	18.3	18.1	18.0
30	27.7	27.5	27.2	27.0
40	37.0	36.6	36.3	36.0
50	46.2	45.8	45.4	45.0

	53	53	52	52
6	5.3	5.3	5.2	5.2
7	6.2	6.2	6.1	6.0
8	7.1	7.0	7.0	6.9
9	8.0	7.9	7.9	7.8
10	8.9	8.8	8.7	8.6
20	17.8	17.6	17.5	17.3
30	26.7	26.5	26.2	26.0
40	35.6	35.3	35.0	34.6
50	44.6	44.1	43.7	43.3

	51	51	50
6	5.1	5.1	5.0
7	6.0	5.9	5.9
8	6.8	6.8	6.7
9	7.7	7.6	7.6
10	8.6	8.5	8.4
20	17.1	17.0	16.8
30	25.7	25.5	25.2
40	34.3	34.0	33.6
50	42.9	42.5	42.1

	3	3	2
6	0.3	0.3	0.2
7	0.4	0.3	0.3
8	0.4	0.4	0.3
9	0.5	0.4	0.4
10	0.6	0.5	0.4
20	1.1	1.0	0.8
30	1.7	1.5	1.2
40	2.3	2.0	1.6
50	2.9	2.5	2.1

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.38 367	50	9.39 677	54	0.60 323	9.98 690	3	60
1	9.38 418	50	9.39 731	53	0.60 269	9.98 687	3	59
2	9.38 468	50	9.39 784	54	0.60 215	9.98 684	3	58
3	9.38 519	50	9.39 838	53	0.60 161	9.98 681	3	57
4	9.38 569	50	9.39 892	53	0.60 108	9.98 678	3	56
5	9.38 620	50	9.39 945	53	0.60 054	9.98 674	3	55
6	9.38 670	50	9.39 999	53	0.60 001	9.98 671	3	54
7	9.38 720	50	9.40 052	53	0.59 947	9.98 668	3	53
8	9.38 771	50	9.40 106	53	0.59 894	9.98 665	3	52
9	9.38 821	50	9.40 159	53	0.59 841	9.98 662	3	51
10	9.38 871	50	9.40 212	53	0.59 787	9.98 658	3	50
11	9.38 921	50	9.40 265	53	0.59 734	9.98 655	3	49
12	9.38 971	50	9.40 318	53	0.59 681	9.98 652	3	48
13	9.39 021	50	9.40 372	53	0.59 628	9.98 649	3	47
14	9.39 071	49	9.40 425	53	0.59 575	9.98 646	3	46
15	9.39 120	50	9.40 478	53	0.59 522	9.98 642	3	45
16	9.39 170	49	9.40 531	52	0.59 469	9.98 639	3	44
17	9.39 220	49	9.40 583	53	0.59 416	9.98 636	3	43
18	9.39 269	49	9.40 636	52	0.59 363	9.98 633	3	42
19	9.39 319	49	9.40 689	53	0.59 311	9.98 630	3	41
20	9.39 368	49	9.40 742	52	0.59 258	9.98 626	3	40
21	9.39 418	49	9.40 794	52	0.59 205	9.98 623	3	39
22	9.39 467	49	9.40 847	52	0.59 153	9.98 620	3	38
23	9.39 516	49	9.40 899	52	0.59 100	9.98 617	3	37
24	9.39 566	49	9.40 952	52	0.59 048	9.98 613	3	36
25	9.39 615	49	9.41 004	52	0.58 995	9.98 610	3	35
26	9.39 664	49	9.41 057	52	0.58 943	9.98 607	3	34
27	9.39 713	49	9.41 109	52	0.58 891	9.98 604	3	33
28	9.39 762	49	9.41 161	52	0.58 838	9.98 600	3	32
29	9.39 811	49	9.41 213	52	0.58 786	9.98 597	3	31
30	9.39 860	49	9.41 266	52	0.58 734	9.98 594	3	30
31	9.39 909	48	9.41 318	52	0.58 682	9.98 591	3	29
32	9.39 957	48	9.41 370	52	0.58 630	9.98 587	3	28
33	9.40 006	49	9.41 422	52	0.58 578	9.98 584	3	27
34	9.40 055	48	9.41 474	51	0.58 526	9.98 581	3	26
35	9.40 103	48	9.41 525	52	0.58 474	9.98 578	3	25
36	9.40 152	48	9.41 577	52	0.58 422	9.98 574	3	24
37	9.40 200	48	9.41 629	51	0.58 370	9.98 571	3	23
38	9.40 249	48	9.41 681	51	0.58 319	9.98 568	3	22
39	9.40 297	48	9.41 732	51	0.58 267	9.98 564	3	21
40	9.40 345	48	9.41 784	52	0.58 216	9.98 561	3	20
41	9.40 394	48	9.41 836	51	0.58 164	9.98 558	3	19
42	9.40 442	48	9.41 887	51	0.58 112	9.98 554	3	18
43	9.40 490	48	9.41 938	51	0.58 061	9.98 551	3	17
44	9.40 538	48	9.41 990	51	0.58 010	9.98 548	3	16
45	9.40 586	48	9.42 041	51	0.57 958	9.98 544	3	15
46	9.40 634	48	9.42 092	51	0.57 907	9.98 541	3	14
47	9.40 682	48	9.42 144	51	0.57 856	9.98 538	3	13
48	9.40 730	47	9.42 195	51	0.57 805	9.98 534	3	12
49	9.40 777	48	9.42 246	51	0.57 753	9.98 531	3	11
50	9.40 825	47	9.42 297	51	0.57 702	9.98 528	3	10
51	9.40 873	47	9.42 348	51	0.57 651	9.98 524	3	9
52	9.40 920	47	9.42 399	51	0.57 600	9.98 521	3	8
53	9.40 968	47	9.42 450	50	0.57 549	9.98 518	3	7
54	9.41 015	47	9.42 501	51	0.57 499	9.98 514	3	6
55	9.41 063	47	9.42 552	50	0.57 448	9.98 511	3	5
56	9.41 110	47	9.42 602	51	0.57 397	9.98 508	3	4
57	9.41 158	47	9.42 653	50	0.57 346	9.98 504	3	3
58	9.41 205	47	9.42 704	50	0.57 296	9.98 501	3	2
59	9.41 252	47	9.42 754	50	0.57 245	9.98 498	3	1
60	9.41 299		9.42 805		0.57 195	9.98 494		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	54	53	53
6	5.4	5.3	5.3
7	6.3	6.2	6.2
8	7.2	7.1	7.0
9	8.1	8.0	7.9
10	9.0	8.9	8.8
20	18.0	17.8	17.6
30	27.0	26.7	26.5
40	36.0	35.6	35.3
50	45.0	44.6	44.1

	52	52	51	51
6	5.2	5.2	5.1	5.1
7	6.1	6.0	6.0	5.9
8	7.0	6.9	6.8	6.8
9	7.9	7.8	7.7	7.6
10	8.7	8.6	8.6	8.5
20	17.5	17.3	17.1	17.0
30	26.2	26.0	25.7	25.5
40	35.0	34.6	34.3	34.0
50	43.7	43.3	42.9	42.5

	50	50	49	49
6	5.0	5.0	4.9	4.9
7	5.9	5.8	5.8	5.7
8	6.7	6.6	6.6	6.5
9	7.6	7.5	7.4	7.3
10	8.4	8.3	8.2	8.1
20	16.8	16.6	16.5	16.3
30	25.2	25.0	24.7	24.5
40	33.6	33.3	33.0	32.6
50	42.1	41.6	41.2	40.8

	48	48	47	47
6	4.8	4.8	4.7	4.7
7	5.6	5.6	5.5	5.5
8	6.4	6.4	6.3	6.2
9	7.3	7.2	7.1	7.0
10	8.1	8.0	7.9	7.8
20	16.1	16.0	15.8	15.6
30	24.2	24.0	23.7	23.5
40	32.3	32.0	31.6	31.3
50	40.4	40.0	39.6	39.1

	3	3
6	0.3	0.3
7	0.4	0.3
8	0.4	0.4
9	0.5	0.4
10	0.6	0.5
20	1.1	1.0
30	1.7	1.5
40	2.3	2.0
50	2.9	2.5

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.41 299	47	9.42 805	51	0.57 195	9.98 494	3	**60**
1	9.41 346	47	9.42 856	50	0.57 144	9.98 491	3	59
2	9.41 394	47	9.42 906	50	0.57 094	9.98 487	3	58
3	9.41 441	47	9.42 956	50	0.57 043	9.98 484	3	57
4	9.41 488	46	9.43 007	50	0.56 993	9.98 481	3	56
5	9.41 534	47	9.43 057	50	0.56 942	9.98 477	3	55
6	9.41 581	47	9.43 107	50	0.56 892	9.98 474	3	54
7	9.41 628	46	9.43 157	50	0.56 842	9.98 470	3	53
8	9.41 675	46	9.43 208	50	0.56 792	9.98 467	3	52
9	9.41 721	47	9.43 258	50	0.56 742	9.98 464	3	51
10	9.41 768	46	9.43 308	50	0.56 692	9.98 460	3	**50**
11	9.41 815	46	9.43 358	50	0.56 642	9.98 457	3	49
12	9.41 861	46	9.43 408	50	0.56 592	9.98 453	3	48
13	9.41 908	46	9.43 458	50	0.56 542	9.98 450	3	47
14	9.41 954	46	9.43 508	49	0.56 492	9.98 446	3	46
15	9.42 000	46	9.43 557	50	0.56 442	9.98 443	3	45
16	9.42 047	46	9.43 607	49	0.56 392	9.98 439	3	44
17	9.42 093	46	9.43 657	49	0.56 343	9.98 436	3	43
18	9.42 139	46	9.43 706	50	0.56 293	9.98 433	3	42
19	9.42 185	46	9.43 756	49	0.56 243	9.98 429	3	41
20	9.42 232	46	9.43 806	49	0.56 194	9.98 426	3	**40**
21	9.42 278	46	9.43 855	49	0.56 144	9.98 422	3	39
22	9.42 324	45	9.43 905	49	0.56 095	9.98 419	3	38
23	9.42 369	46	9.43 954	49	0.56 045	9.98 415	3	37
24	9.42 415	46	9.44 003	49	0.55 996	9.98 412	3	36
25	9.42 461	46	9.44 053	49	0.55 947	9.98 408	3	35
26	9.42 507	45	9.44 102	49	0.55 898	9.98 405	3	34
27	9.42 553	45	9.44 151	49	0.55 848	9.98 401	3	33
28	9.42 598	46	9.44 200	49	0.55 799	9.98 398	3	32
29	9.42 644	45	9.44 249	49	0.55 750	9.98 394	3	31
30	9.42 690	45	9.44 299	49	0.55 701	9.98 391	3	**30**
31	9.42 735	45	9.44 348	49	0.55 652	9.98 387	3	29
32	9.42 781	45	9.44 397	49	0.55 603	9.98 384	3	28
33	9.42 826	45	9.44 446	48	0.55 554	9.98 380	3	27
34	9.42 871	45	9.44 494	49	0.55 505	9.98 377	3	26
35	9.42 917	45	9.44 543	49	0.55 456	9.98 373	3	25
36	9.42 962	45	9.44 592	48	0.55 407	9.98 370	3	24
37	9.43 007	45	9.44 641	49	0.55 359	9.98 366	3	23
38	9.43 052	45	9.44 690	48	0.55 310	9.98 363	3	22
39	9.43 098	45	9.44 738	48	0.55 261	9.98 359	3	21
40	9.43 143	45	9.44 787	48	0.55 213	9.98 356	3	**20**
41	9.43 188	45	9.44 835	48	0.55 164	9.98 352	4	19
42	9.43 233	45	9.44 884	48	0.55 116	9.98 348	3	18
43	9.43 278	44	9.44 932	48	0.55 067	9.98 345	3	17
44	9.43 322	45	9.44 981	48	0.55 019	9.98 341	3	16
45	9.43 367	44	9.45 029	48	0.54 970	9.98 338	3	15
46	9.43 412	45	9.45 077	48	0.54 922	9.98 334	3	14
47	9.43 457	44	9.45 126	48	0.54 874	9.98 331	3	13
48	9.43 501	44	9.45 174	48	0.54 825	9.98 327	3	12
49	9.43 546	45	9.45 222	48	0.54 777	9.98 324	4	11
50	9.43 591	44	9.45 270	48	0.54 729	9.98 320	3	**10**
51	9.43 635	44	9.45 318	48	0.54 681	9.98 316	3	9
52	9.43 680	44	9.45 367	48	0.54 633	9.98 313	3	8
53	9.43 724	44	9.45 415	48	0.54 585	9.98 309	3	7
54	9.43 768	44	9.45 463	47	0.54 537	9.98 306	4	6
55	9.43 813	44	9.45 510	48	0.54 489	9.98 302	3	5
56	9.43 857	44	9.45 558	48	0.54 441	9.98 298	3	4
57	9.43 901	44	9.45 606	47	0.54 393	9.98 295	3	3
58	9.43 945	44	9.45 654	48	0.54 346	9.98 291	3	2
59	9.43 989	44	9.45 702	47	0.54 298	9.98 288	4	1
60	9.44 034		9.45 749		0.54 250	9.98 284		**0**
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	$\overline{50}$	50
6	5.0	5.0
7	5.9	5.8
8	6.7	6.6
9	7.6	7.5
10	8.4	8.3
20	16.8	16.6
30	25.2	25.0
40	33.6	33.3
50	42.1	41.6

	$\overline{49}$	49	$\overline{48}$	48
6	4.9	4.9	4.8	4.8
7	5.8	5.7	5.6	5.6
8	6.6	6.5	6.4	6.4
9	7.4	7.3	7.3	7.2
10	8.2	8.1	8.1	8.0
20	16.5	16.3	16.1	16.0
30	24.7	24.5	24.2	24.0
40	33.0	32.6	32.3	32.0
50	41.2	40.8	40.4	40.0

	$\overline{47}$	47	$\overline{46}$	46
6	4.7	4.7	4.6	4.6
7	5.5	5.5	5.4	5.3
8	6.3	6.2	6.2	6.1
9	7.1	7.0	7.0	6.9
10	7.9	7.8	7.7	7.6
20	15.8	15.6	15.5	15.3
30	23.7	23.5	23.2	23.0
40	31.6	31.3	31.0	30.6
50	39.6	39.1	38.7	38.3

	$\overline{45}$	45	$\overline{44}$	44
6	4.5	4.5	4.4	4.4
7	5.3	5.2	5.2	5.1
8	6.0	6.0	5.9	5.8
9	6.8	6.7	6.7	6.6
10	7.6	7.5	7.4	7.3
20	15.1	15.0	14.8	14.6
30	22.7	22.5	22.2	22.0
40	30.3	30.0	29.6	29.3
50	37.9	37.5	37.1	36.6

	4	$\overline{3}$	3
6	0.4	0.3	0.3
7	0.4	0.4	0.3
8	0.5	0.4	0.4
9	0.6	0.5	0.4
10	0.6	0.6	0.5
20	1.3	1.1	1.0
30	2.0	1.7	1.5
40	2.6	2.3	2.0
50	3.3	2.9	2.5

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.44 034	44	9.45 749	48	0.54 250	9.98 284	3	60
1	9.44 078	44	9.45 797	47	0.54 202	9.98 280	3	59
2	9.44 122	44	9.45 845	47	0.54 155	9.98 277	3	58
3	9.44 166	43	9.45 892	47	0.54 107	9.98 273	4	57
4	9.44 209	44	9.45 940	47	0.54 060	9.98 269	3	56
5	9.44 253	44	9.45 987	47	0.54 012	9.98 266	3	55
6	9.44 297	43	9.46 035	47	0.53 965	9.98 262	4	54
7	9.44 341	43	9.46 082	47	0.53 917	9.98 258	3	53
8	9.44 384	44	9.46 129	47	0.53 870	9.98 255	3	52
9	9.44 428	43	9.46 177	47	0.53 823	9.98 251	4	51
10	9.44 472	43	9.46 224	47	0.53 776	9.98 247	3	50
11	9.44 515	43	9.46 271	47	0.53 728	9.98 244	3	49
12	9.44 559	43	9.46 318	47	0.53 681	9.98 240	4	48
13	9.44 602	43	9.46 366	47	0.53 634	9.98 236	3	47
14	9.44 646	43	9.46 413	47	0.53 587	9.98 233	3	46
15	9.44 689	43	9.46 460	47	0.53 540	9.98 229	4	45
16	9.44 732	43	9.46 507	47	0.53 493	9.98 225	3	44
17	9.44 776	43	9.46 554	47	0.53 446	9.98 222	3	43
18	9.44 819	43	9.46 601	46	0.53 399	9.98 218	4	42
19	9.44 862	43	9.46 647	47	0.53 352	9.98 214	3	41
20	9.44 905	43	9.46 694	47	0.53 305	9.98 211	4	40
21	9.44 948	43	9.46 741	46	0.53 258	9.98 207	3	39
22	9.44 991	43	9.46 788	46	0.53 212	9.98 203	3	38
23	9.45 034	43	9.46 834	47	0.53 165	9.98 200	4	37
24	9.45 077	43	9.46 881	46	0.53 118	9.98 196	3	36
25	9.45 120	42	9.46 928	46	0.53 072	9.98 192	4	35
26	9.45 163	43	9.46 974	46	0.53 025	9.98 188	3	34
27	9.45 206	43	9.47 021	46	0.52 979	9.98 185	4	33
28	9.45 249	42	9.47 067	46	0.52 932	9.98 181	3	32
29	9.45 291	42	9.47 114	46	0.52 886	9.98 177	4	31
30	9.45 334	43	9.47 160	46	0.52 839	9.98 173	3	30
31	9.45 377	42	9.47 207	46	0.52 793	9.98 170	4	29
32	9.45 419	42	9.47 253	46	0.52 747	9.98 166	3	28
33	9.45 462	42	9.47 299	46	0.52 700	9.98 162	4	27
34	9.45 504	42	9.47 345	46	0.52 654	9.98 158	3	26
35	9.45 547	42	9.47 392	46	0.52 608	9.98 155	4	25
36	9.45 589	42	9.47 438	46	0.52 562	9.98 151	3	24
37	9.45 631	42	9.47 484	46	0.52 516	9.98 147	4	23
38	9.45 674	42	9.47 530	46	0.52 469	9.98 143	3	22
39	9.45 716	42	9.47 576	46	0.52 423	9.98 140	4	21
40	9.45 758	42	9.47 622	46	0.52 377	9.98 136	3	20
41	9.45 800	42	9.47 668	45	0.52 331	9.98 132	4	19
42	9.45 842	42	9.47 714	46	0.52 286	9.98 128	4	18
43	9.45 885	42	9.47 760	46	0.52 240	9.98 124	3	17
44	9.45 927	42	9.47 806	45	0.52 194	9.98 121	4	16
45	9.45 969	42	9.47 851	46	0.52 148	9.98 117	3	15
46	9.46 011	41	9.47 897	45	0.52 102	9.98 113	4	14
47	9.46 052	42	9.47 943	46	0.52 057	9.98 109	4	13
48	9.46 094	42	9.47 989	45	0.52 011	9.98 105	3	12
49	9.46 136	41	9.48 034	45	0.51 965	9.98 102	4	11
50	9.46 178	42	9.48 080	45	0.51 920	9.98 098	4	10
51	9.46 220	41	9.48 125	45	0.51 874	9.98 094	3	9
52	9.46 261	41	9.48 171	45	0.51 829	9.98 090	4	8
53	9.46 303	42	9.48 216	45	0.51 783	9.98 086	4	7
54	9.46 345	41	9.48 262	45	0.51 738	9.98 082	3	6
55	9.46 386	41	9.48 307	45	0.51 692	9.98 079	4	5
56	9.46 428	41	9.48 353	45	0.51 647	9.98 075	4	4
57	9.46 469	41	9.48 398	45	0.51 602	9.98 071	3	3
58	9.46 511	41	9.48 443	45	0.51 556	9.98 067	4	2
59	9.46 552	41	9.48 488	45	0.51 511	9.98 063	4	1
60	9.46 593		9.48 534		0.51 466	9.98 059		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	48	$\overline{47}$	47
6	4.8	4.7	4.7
7	5.6	5.5	5.5
8	6.4	6.3	6.2
9	7.2	7.1	7.0
10	8.0	7.9	7.8
20	16.0	15.8	15.6
30	24.0	23.7	23.5
40	32.0	31.6	31.3
50	40.0	39.6	39.1

	$\overline{46}$	46	$\overline{45}$	45
6	4.6	4.6	4.5	4.5
7	5.4	5.3	5.3	5.2
8	6.2	6.1	6.0	6.0
9	7.0	6.9	6.8	6.7
10	7.7	7.6	7.6	7.5
20	15.5	15.3	15.1	15.0
30	23.2	23.0	22.7	22.5
40	31.0	30.6	30.3	30.0
50	38.7	38.3	37.9	37.5

	44	$\overline{43}$	43
6	4.4	4.3	4.3
7	5.1	5.1	5.0
8	5.8	5.8	5.7
9	6.6	6.5	6.4
10	7.3	7.2	7.1
20	14.6	14.5	14.3
30	22.0	21.7	21.5
40	29.3	29.0	28.6
50	36.6	36.2	35.8

	$\overline{42}$	42	$\overline{41}$	41
6	4.2	4.2	4.1	4.1
7	4.9	4.9	4.8	4.8
8	5.6	5.6	5.5	5.4
9	6.4	6.3	6.2	6.1
10	7.1	7.0	6.9	6.8
20	14.1	14.0	13.8	13.6
30	21.2	21.0	20.7	20.5
40	28.3	28.0	27.6	27.3
50	35.4	35.0	34.6	34.1

	4	$\overline{3}$
6	0.4	0.3
7	0.4	0.4
8	0.5	0.4
9	0.6	0.5
10	0.6	0.6
20	1.3	1.1
30	2.0	1.7
40	2.6	2.3
50	3.3	2.9

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.46 593	41	9.48 534	45	0.51 466	9.98 059	3	60
1	9.46 635	41	9.48 579	45	0.51 421	9.98 056	4	59
2	9.46 676	41	9.48 624	45	0.51 376	9.98 052	4	58
3	9.46 717	41	9.48 669	45	0.51 330	9.98 048	4	57
4	9.46 758	41	9.48 714	45	0.51 285	9.98 044	3	56
5	9.46 799	41	9.48 759	45	0.51 240	9.98 040	4	55
6	9.46 840	41	9.48 804	44	0.51 195	9.98 036	4	54
7	9.46 881	41	9.48 849	45	0.51 151	9.98 032	4	53
8	9.46 922	41	9.48 894	45	0.51 106	9.98 028	4	52
9	9.46 963	41	9.48 939	45	0.51 061	9.98 024	3	51
10	9.47 004	41	9.48 984	44	0.51 016	9.98 021	4	50
11	9.47 045	41	9.49 028	45	0.50 971	9.98 017	4	49
12	9.47 086	40	9.49 073	44	0.50 926	9.98 013	4	48
13	9.47 127	41	9.49 118	44	0.50 882	9.98 009	4	47
14	9.47 168	40	9.49 162	45	0.50 837	9.98 005	4	46
15	9.47 208	40	9.49 207	44	0.50 792	9.98 001	3	45
16	9.47 249	41	9.49 252	44	0.50 748	9.97 997	4	44
17	9.47 290	40	9.49 296	44	0.50 703	9.97 993	4	43
18	9.47 330	40	9.49 341	44	0.50 659	9.97 989	4	42
19	9.47 371	40	9.49 385	44	0.50 614	9.97 985	4	41
20	9.47 411	40	9.49 430	44	0.50 570	9.97 981	4	40
21	9.47 452	40	9.49 474	44	0.50 525	9.97 977	4	39
22	9.47 492	40	9.49 518	44	0.50 481	9.97 973	4	38
23	9.47 532	40	9.49 563	44	0.50 437	9.97 969	3	37
24	9.47 573	40	9.49 607	44	0.50 392	9.97 966	4	36
25	9.47 613	40	9.49 651	44	0.50 348	9.97 962	4	35
26	9.47 653	40	9.49 695	44	0.50 304	9.97 958	4	34
27	9.47 694	40	9.49 740	44	0.50 260	9.97 954	4	33
28	9.47 734	40	9.49 784	44	0.50 216	9.97 950	4	32
29	9.47 774	40	9.49 828	44	0.50 172	9.97 946	4	31
30	9.47 814	40	9.49 872	44	0.50 128	9.97 942	4	30
31	9.47 854	40	9.49 916	44	0.50 083	9.97 938	4	29
32	9.47 894	40	9.49 960	43	0.50 039	9.97 934	4	28
33	9.47 934	40	9.50 004	44	0.49 996	9.97 930	4	27
34	9.47 974	40	9.50 048	44	0.49 952	9.97 926	4	26
35	9.48 014	40	9.50 092	44	0.49 908	9.97 922	4	25
36	9.48 054	39	9.50 136	43	0.49 864	9.97 918	4	24
37	9.48 093	40	9.50 179	44	0.49 820	9.97 914	4	23
38	9.48 133	39	9.50 223	43	0.49 776	9.97 910	4	22
39	9.48 173	40	9.50 267	44	0.49 733	9.97 906	4	21
40	9.48 213	39	9.50 311	43	0.49 689	9.97 902	4	20
41	9.48 252	39	9.50 354	43	0.49 645	9.97 898	4	19
42	9.48 292	39	9.50 398	44	0.49 602	9.97 894	4	18
43	9.48 331	39	9.50 442	43	0.49 558	9.97 890	4	17
44	9.48 371	39	9.50 485	43	0.49 514	9.97 886	4	16
45	9.48 410	39	9.50 529	43	0.49 471	9.97 881	4	15
46	9.48 450	39	9.50 572	43	0.49 427	9.97 877	4	14
47	9.48 489	39	9.50 616	43	0.49 384	9.97 873	4	13
48	9.48 529	39	9.50 659	43	0.49 340	9.97 869	4	12
49	9.48 568	39	9.50 702	43	0.49 297	9.97 865	4	11
50	9.48 607	39	9.50 746	43	0.49 254	9.97 861	4	10
51	9.48 646	39	9.50 789	43	0.49 210	9.97 857	4	9
52	9.48 686	39	9.50 832	43	0.49 167	9.97 853	4	8
53	9.48 725	39	9.50 876	43	0.49 124	9.97 849	4	7
54	9.48 764	39	9.50 919	43	0.49 081	9.97 845	4	6
55	9.48 803	39	9.50 962	43	0.49 038	9.97 841	4	5
56	9.48 842	39	9.51 005	43	0.48 994	9.97 837	4	4
57	9.48 881	39	9.51 048	43	0.48 951	9.97 833	4	3
58	9.48 920	39	9.51 091	43	0.48 908	9.97 829	4	2
59	9.48 959	38	9.51 134	43	0.48 865	9.97 824	4	1
60	9.48 998		9.51 177		0.48 822	9.97 820		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	45	45	44	44
6	4.5	4.5	4.4	4.4
7	5.3	5.2	5.2	5.1
8	6.0	6.0	5.9	5.8
9	6.8	6.7	6.7	6.6
10	7.6	7.5	7.4	7.3
20	15.1	15.0	14.8	14.6
30	22.7	22.5	22.2	22.0
40	30.3	30.0	29.6	29.3
50	37.9	37.5	37.1	36.6

	43	43
6	4.3	4.3
7	5.1	5.0
8	5.8	5.7
9	6.5	6.4
10	7.2	7.1
20	14.5	14.3
30	21.7	21.5
40	29.0	28.6
50	36.2	35.8

	41	41	40	40
6	4.1	4.1	4.0	4.0
7	4.8	4.8	4.7	4.6
8	5.5	5.4	5.4	5.3
9	6.2	6.1	6.1	6.0
10	6.9	6.8	6.7	6.6
20	13.8	13.6	13.5	13.3
30	20.7	20.5	20.2	20.0
40	27.6	27.3	27.0	26.6
50	34.6	34.1	33.7	33.3

	39	39	38
6	3.9	3.9	3.8
7	4.6	4.5	4.5
8	5.2	5.2	5.1
9	5.9	5.8	5.8
10	6.6	6.5	6.4
20	13.1	13.0	12.8
30	19.7	19.5	19.2
40	26.3	26.0	25.6
50	32.9	32.5	32.1

	4	4	3
6	0.4	0.4	0.3
7	0.5	0.4	0.4
8	0.6	0.5	0.4
9	0.7	0.6	0.5
10	0.7	0.6	0.6
20	1.5	1.3	1.1
30	2.2	2.0	1.7
40	3.0	2.6	2.3
50	3.7	3.3	2.9

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.48 998	39	9.51 177	43	0.48 822	9.97 820	4	60
1	9.49 037	39	9.51 220	43	0.48 779	9.97 816	4	59
2	9.49 076	38	9.51 263	43	0.48 736	9.97 812	4	58
3	9.49 114	39	9.51 306	43	0.48 693	9.97 808	4	57
4	9.49 153	38	9.51 349	42	0.48 650	9.97 804	4	56
5	9.49 192	39	9.51 392	43	0.48 608	9.97 800	4	55
6	9.49 231	38	9.51 435	42	0.48 565	9.97 796	4	54
7	9.49 269	38	9.51 477	43	0.48 522	9.97 792	4	53
8	9.49 308	38	9.51 520	42	0.48 479	9.97 787	4	52
9	9.49 346	38	9.51 563	42	0.48 437	9.97 783	4	51
10	9.49 385	38	9.51 605	43	0.48 394	9.97 779	4	50
11	9.49 423	38	9.51 648	42	0.48 351	9.97 775	4	49
12	9.49 462	38	9.51 691	42	0.48 309	9.97 771	4	48
13	9.49 500	38	9.51 733	42	0.48 266	9.97 767	4	47
14	9.49 539	38	9.51 776	42	0.48 224	9.97 763	4	46
15	9.49 577	38	9.51 818	42	0.48 181	9.97 758	4	45
16	9.49 615	38	9.51 861	42	0.48 139	9.97 754	4	44
17	9.49 653	38	9.51 903	42	0.48 096	9.97 750	4	43
18	9.49 692	38	9.51 946	42	0.48 054	9.97 746	4	42
19	9.49 730	38	9.51 988	42	0.48 012	9.97 742	4	41
20	9.49 768	38	9.52 030	42	0.47 969	9.97 737	4	40
21	9.49 806	38	9.52 073	42	0.47 927	9.97 733	4	39
22	9.49 844	38	9.52 115	42	0.47 885	9.97 729	4	38
23	9.49 882	38	9.52 157	42	0.47 842	9.97 725	4	37
24	9.49 920	38	9.52 199	42	0.47 800	9.97 721	4	36
25	9.49 958	38	9.52 241	42	0.47 758	9.97 716	4	35
26	9.49 996	37	9.52 284	42	0.47 716	9.97 712	4	34
27	9.50 034	38	9.52 326	42	0.47 674	9.97 708	4	33
28	9.50 072	38	9.52 368	42	0.47 632	9.97 704	4	32
29	9.50 110	37	9.52 410	42	0.47 590	9.97 700	4	31
30	9.50 147	38	9.52 452	42	0.47 548	9.97 695	4	30
31	9.50 185	37	9.52 494	42	0.47 506	9.97 691	4	29
32	9.50 223	37	9.52 536	42	0.47 464	9.97 687	4	28
33	9.50 260	38	9.52 578	41	0.47 422	9.97 683	4	27
34	9.50 298	37	9.52 619	42	0.47 380	9.97 678	4	26
35	9.50 336	37	9.52 661	42	0.47 338	9.97 674	4	25
36	9.50 373	37	9.52 703	41	0.47 296	9.97 670	4	24
37	9.50 411	37	9.52 745	42	0.47 255	9.97 666	4	23
38	9.50 448	37	9.52 787	41	0.47 213	9.97 661	4	22
39	9.50 486	37	9.52 828	41	0.47 171	9.97 657	4	21
40	9.50 523	37	9.52 870	42	0.47 130	9.97 653	4	20
41	9.50 561	37	9.52 912	41	0.47 088	9.97 649	4	19
42	9.50 598	37	9.52 953	41	0.47 046	9.97 644	4	18
43	9.50 635	37	9.52 995	41	0.47 005	9.97 640	4	17
44	9.50 672	37	9.53 036	41	0.46 963	9.97 636	4	16
45	9.50 710	37	9.53 078	41	0.46 922	9.97 632	4	15
46	9.50 747	37	9.53 119	41	0.46 880	9.97 627	4	14
47	9.50 784	37	9.53 161	41	0.46 839	9.97 623	4	13
48	9.50 821	37	9.53 202	41	0.46 797	9.97 619	4	12
49	9.50 858	37	9.53 244	41	0.46 756	9.97 614	4	11
50	9.50 895	37	9.53 285	41	0.46 714	9.97 610	4	10
51	9.50 932	37	9.53 326	41	0.46 673	9.97 606	4	9
52	9.50 969	37	9.53 368	41	0.46 632	9.97 601	4	8
53	9.51 006	37	9.53 409	41	0.46 591	9.97 597	4	7
54	9.51 043	37	9.53 450	41	0.46 549	9.97 593	4	6
55	9.51 080	36	9.53 491	41	0.46 508	9.97 588	4	5
56	9.51 117	37	9.53 533	41	0.46 467	9.97 584	4	4
57	9.51 154	36	9.53 574	41	0.46 426	9.97 580	4	3
58	9.51 190	37	9.53 615	41	0.46 385	9.97 575	4	2
59	9.51 227	36	9.53 656	41	0.46 344	9.97 571	4	1
60	9.51 264		9.53 697		0.46 303	9.97 567		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	43	$\overline{42}$	42
6	4.3	4.2	4.2
7	5.0	4.9	4.9
8	5.7	5.6	5.6
9	6.4	6.4	6.3
10	7.1	7.1	7.0
20	14.3	14.1	14.0
30	21.5	21.2	21.0
40	28.6	28.3	28.0
50	35.8	35.4	35.0

	$\overline{41}$	41
6	4.1	4.1
7	4.8	4.8
8	5.5	5.4
9	6.2	6.1
10	6.9	6.8
20	13.8	13.6
30	20.7	20.5
40	27.6	27.3
50	34.6	34.1

	39	$\overline{38}$	38
6	3.9	3.8	3.8
7	4.5	4.5	4.4
8	5.2	5.1	5.0
9	5.8	5.8	5.7
10	6.5	6.4	6.3
20	13.0	12.8	12.6
30	19.5	19.2	19.0
40	26.0	25.6	25.3
50	32.5	32.1	31.6

	$\overline{37}$	37	$\overline{36}$
6	3.7	3.7	3.6
7	4.4	4.3	4.2
8	5.0	4.9	4.8
9	5.6	5.5	5.5
10	6.2	6.1	6.1
20	12.5	12.3	12.1
30	18.7	18.5	18.2
40	25.0	24.6	24.3
50	31.2	30.8	30.4

	$\overline{4}$	4
6	0.4	0.4
7	0.5	0.4
8	0.6	0.5
9	0.7	0.6
10	0.7	0.6
20	1.5	1.3
30	2.2	2.0
40	3.0	2.6
50	3.7	3.3

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.51 264	37	9.53 697	41	0.46 303	9.97 567	4	60
1	9.51 301	36	9.53 738	41	0.46 262	9.97 562	4	59
2	9.51 337	36	9.53 779	41	0.46 221	9.97 558	4	58
3	9.51 374	36	9.53 820	41	0.46 180	9.97 554	4	57
4	9.51 410	36	9.53 861	41	0.46 139	9.97 549	4	56
5	9.51 447	36	9.53 902	41	0.46 098	9.97 545	4	55
6	9.51 483	36	9.53 943	40	0.46 057	9.97 541	4	54
7	9.51 520	36	9.53 983	41	0.46 016	9.97 536	4	53
8	9.51 556	36	9.54 024	41	0.45 975	9.97 532	4	52
9	9.51 593	36	9.54 065	40	0.45 934	9.97 527	4	51
10	9.51 629	36	9.54 106	41	0.45 894	9.97 523	4	50
11	9.51 665	36	9.54 147	40	0.45 853	9.97 519	4	49
12	9.51 702	36	9.54 187	40	0.45 812	9.97 514	4	48
13	9.51 738	36	9.54 228	41	0.45 772	9.97 510	4	47
14	9.51 774	36	9.54 269	40	0.45 731	9.97 505	4	46
15	9.51 810	36	9.54 309	40	0.45 690	9.97 501	4	45
16	9.51 847	36	9.54 350	40	0.45 650	9.97 497	4	44
17	9.51 883	36	9.54 390	40	0.45 609	9.97 492	4	43
18	9.51 919	36	9.54 431	40	0.45 569	9.97 488	4	42
19	9.51 955	36	9.54 471	40	0.45 528	9.97 483	4	41
20	9.51 991	36	9.54 512	40	0.45 488	9.97 479	4	40
21	9.52 027	36	9.54 552	40	0.45 447	9.97 475	4	39
22	9.52 063	36	9.54 593	40	0.45 407	9.97 470	4	38
23	9.52 099	36	9.54 633	40	0.45 367	9.97 466	4	37
24	9.52 135	35	9.54 673	40	0.45 326	9.97 461	4	36
25	9.52 170	36	9.54 714	40	0.45 286	9.97 457	4	35
26	9.52 206	36	9.54 754	40	0.45 246	9.97 452	4	34
27	9.52 242	35	9.54 794	40	0.45 205	9.97 448	4	33
28	9.52 278	36	9.54 834	40	0.45 165	9.97 443	4	32
29	9.52 314	35	9.54 874	40	0.45 125	9.97 439	4	31
30	9.52 349	35	9.54 915	40	0.45 085	9.97 434	4	30
31	9.52 385	36	9.54 955	40	0.45 045	9.97 430	4	29
32	9.52 421	35	9.54 995	40	0.45 005	9.97 425	4	28
33	9.52 456	35	9.55 035	40	0.44 965	9.97 421	4	27
34	9.52 492	35	9.55 075	40	0.44 925	9.97 416	4	26
35	9.52 527	35	9.55 115	39	0.44 884	9.97 412	4	25
36	9.52 563	35	9.55 155	40	0.44 845	9.97 407	4	24
37	9.52 598	35	9.55 195	40	0.44 805	9.97 403	4	23
38	9.52 634	35	9.55 235	40	0.44 765	9.97 398	4	22
39	9.52 669	35	9.55 275	40	0.44 725	9.97 394	4	21
40	9.52 704	35	9.55 315	40	0.44 685	9.97 389	4	20
41	9.52 740	35	9.55 355	39	0.44 645	9.97 385	4	19
42	9.52 775	35	9.55 394	40	0.44 605	9.97 380	4	18
43	9.52 810	35	9.55 434	39	0.44 565	9.97 376	4	17
44	9.52 846	35	9.55 474	40	0.44 526	9.97 371	4	16
45	9.52 881	35	9.55 514	39	0.44 486	9.97 367	4	15
46	9.52 916	35	9.55 553	40	0.44 446	9.97 362	4	14
47	9.52 951	35	9.55 593	39	0.44 406	9.97 358	4	13
48	9.52 986	35	9.55 633	39	0.44 367	9.97 353	4	12
49	9.53 021	35	9.55 672	39	0.44 327	9.97 349	4	11
50	9.53 056	35	9.55 712	39	0.44 288	9.97 344	4	10
51	9.53 091	35	9.55 751	40	0.44 248	9.97 340	5	9
52	9.53 126	35	9.55 791	39	0.44 208	9.97 335	4	8
53	9.53 161	35	9.55 831	39	0.44 169	9.97 330	4	7
54	9.53 196	34	9.55 870	39	0.44 129	9.97 326	4	6
55	9.53 231	35	9.55 909	39	0.44 090	9.97 321	4	5
56	9.53 266	35	9.55 949	39	0.44 051	9.97 317	4	4
57	9.53 301	34	9.55 988	39	0.44 011	9.97 312	4	3
58	9.53 335	35	9.56 028	39	0.43 972	9.97 308	5	2
59	9.53 370	34	9.56 067	39	0.43 932	9.97 303	4	1
60	9.53 405		9.56 106		0.43 893	9.97 298		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	41	$\overline{40}$	40
6	4.1	4.0	4.0
7	4.8	4.7	4.6
8	5.4	5.4	5.3
9	6.1	6.1	6.0
10	6.8	6.7	6.6
20	13.6	13.5	13.3
30	20.5	20.2	20.0
40	27.3	27.0	26.6
50	34.1	33.7	33.3

	$\overline{39}$	39
6	3.9	3.9
7	4.6	4.5
8	5.2	5.2
9	5.9	5.8
10	6.6	6.5
20	13.1	13.0
30	19.7	19.5
40	26.3	26.0
50	32.9	32.5

	37	$\overline{36}$	36
6	3.7	3.6	3.6
7	4.3	4.2	4.2
8	4.9	4.8	4.8
9	5.5	5.5	5.4
10	6.1	6.1	6.0
20	12.3	12.1	12.0
30	18.5	18.2	18.0
40	24.6	24.3	24.0
50	30.8	30.4	30.0

	$\overline{35}$	35	$\overline{34}$
6	3.5	3.5	3.4
7	4.1	4.1	4.0
8	4.7	4.6	4.6
9	5.3	5.2	5.2
10	5.9	5.8	5.7
20	11.8	11.6	11.5
30	17.7	17.5	17.2
40	23.6	23.3	23.0
50	29.6	29.1	28.7

	5	$\overline{4}$	4
6	0.5	0.4	0.4
7	0.6	0.5	0.4
8	0.6	0.6	0.5
9	0.7	0.7	0.6
10	0.8	0.7	0.6
20	1.6	1.5	1.3
30	2.5	2.2	2.0
40	3.3	3.0	2.6
50	4.1	3.7	3.3

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.53 405	35	9.56 106	39	0.43 893	9.97 298	4	60
1	9.53 440	34	9.56 146	39	0.43 854	9.97 294	4	59
2	9.53 474	34	9.56 185	39	0.43 815	9.97 289	4	58
3	9.53 509	35	9.56 224	39	0.43 775	9.97 285	5	57
4	9.53 544	34	9.56 263	39	0.43 736	9.97 280	4	56
5	9.53 578	34	9.56 303	39	0.43 697	9.97 275	4	55
6	9.53 613	34	9.56 342	39	0.43 658	9.97 271	4	54
7	9.53 647	34	9.56 381	39	0.43 619	9.97 266	5	53
8	9.53 682	34	9.56 420	39	0.43 580	9.97 261	4	52
9	9.53 716	34	9.56 459	39	0.43 540	9.97 257	4	51
10	9.53 750	34	9.56 498	39	0.43 501	9.97 252	4	50
11	9.53 785	34	9.56 537	39	0.43 462	9.97 248	5	49
12	9.53 819	34	9.56 576	39	0.43 423	9.97 243	4	48
13	9.53 854	34	9.56 615	38	0.43 384	9.97 238	4	47
14	9.53 888	34	9.56 654	39	0.43 346	9.97 234	5	46
15	9.53 922	34	9.56 693	39	0.43 307	9.97 229	4	45
16	9.53 956	34	9.56 732	39	0.43 268	9.97 224	4	44
17	9.53 990	34	9.56 771	39	0.43 229	9.97 220	5	43
18	9.54 025	34	9.56 810	38	0.43 190	9.97 215	4	42
19	9.54 059	34	9.56 848	39	0.43 151	9.97 210	4	41
20	9.54 093	34	9.56 887	38	0.43 112	9.97 206	5	40
21	9.54 127	34	9.56 926	39	0.43 074	9.97 201	4	39
22	9.54 161	34	9.56 965	38	0.43 035	9.97 196	5	38
23	9.54 195	34	9.57 003	38	0.42 996	9.97 191	4	37
24	9.54 229	33	9.57 042	39	0.42 958	9.97 187	4	36
25	9.54 263	34	9.57 081	38	0.42 919	9.97 182	5	35
26	9.54 297	34	9.57 119	38	0.42 880	9.97 177	4	34
27	9.54 331	34	9.57 158	38	0.42 842	9.97 173	5	33
28	9.54 365	33	9.57 196	38	0.42 803	9.97 168	4	32
29	9.54 398	34	9.57 235	39	0.42 765	9.97 163	4	31
30	9.54 432	34	9.57 274	38	0.42 726	9.97 159	5	30
31	9.54 466	33	9.57 312	38	0.42 687	9.97 154	4	29
32	9.54 500	34	9.57 350	38	0.42 649	9.97 149	5	28
33	9.54 534	33	9.57 389	38	0.42 611	9.97 144	4	27
34	9.54 567	33	9.57 427	38	0.42 572	9.97 140	5	26
35	9.54 601	33	9.57 466	38	0.42 534	9.97 135	4	25
36	9.54 634	34	9.57 504	38	0.42 495	9.97 130	5	24
37	9.54 668	33	9.57 542	38	0.42 457	9.97 125	4	23
38	9.54 702	33	9.57 581	38	0.42 419	9.97 121	5	22
39	9.54 735	33	9.57 619	38	0.42 380	9.97 116	4	21
40	9.54 769	33	9.57 657	38	0.42 342	9.97 111	5	20
41	9.54 802	33	9.57 696	38	0.42 304	9.97 106	4	19
42	9.54 836	33	9.57 734	38	0.42 266	9.97 102	5	18
43	9.54 869	33	9.57 772	38	0.42 227	9.97 097	5	17
44	9.54 902	33	9.57 810	38	0.42 189	9.97 092	4	16
45	9.54 936	33	9.57 848	38	0.42 151	9.97 087	5	15
46	9.54 969	33	9.57 886	38	0.42 113	9.97 082	4	14
47	9.55 002	33	9.57 925	38	0.42 075	9.97 078	5	13
48	9.55 036	33	9.57 963	38	0.42 037	9.97 073	4	12
49	9.55 069	33	9.58 001	38	0.41 999	9.97 068	5	11
50	9.55 102	33	9.58 039	38	0.41 961	9.97 063	5	10
51	9.55 135	33	9.58 077	38	0.41 923	9.97 058	4	9
52	9.55 168	33	9.58 115	38	0.41 885	9.97 054	5	8
53	9.55 202	33	9.58 153	37	0.41 847	9.97 049	5	7
54	9.55 235	33	9.58 190	38	0.41 809	9.97 044	4	6
55	9.55 268	33	9.58 228	38	0.41 771	9.97 039	5	5
56	9.55 301	33	9.58 266	38	0.41 733	9.97 034	5	4
57	9.55 334	33	9.58 304	37	0.41 695	9.97 029	4	3
58	9.55 367	33	9.58 342	38	0.41 658	9.97 025	5	2
59	9.55 400	33	9.58 380	37	0.41 620	9.97 020	5	1
60	9.55 433		9.58 417		0.41 582	9.97 015		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	39	39
6	3.9	3.9
7	4.6	4.5
8	5.2	5.2
9	5.9	5.8
10	6.6	6.5
20	13.1	13.0
30	19.7	19.5
40	26.3	26.0
50	32.9	32.5

	38	38	37
6	3.8	3.8	3.7
7	4.5	4.4	4.4
8	5.1	5.0	5.0
9	5.8	5.7	5.6
10	6.4	6.3	6.2
20	12.8	12.6	12.5
30	19.2	19.0	18.7
40	25.6	25.3	25.0
50	32.1	31.6	31.2

	35	34	34
6	3.5	3.4	3.4
7	4.1	4.0	3.9
8	4.6	4.6	4.5
9	5.2	5.2	5.1
10	5.8	5.7	5.6
20	11.6	11.5	11.3
30	17.5	17.2	17.0
40	23.3	23.0	22.6
50	29.1	28.7	28.3

	33	33
6	3.3	3.3
7	3.9	3.8
8	4.4	4.4
9	5.0	4.9
10	5.6	5.5
20	11.1	11.0
30	16.7	16.5
40	22.3	22.0
50	27.9	27.5

	5	4
6	0.5	0.4
7	0.6	0.5
8	0.6	0.6
9	0.7	0.7
10	0.8	0.7
20	1.6	1.5
30	2.5	2.2
40	3.3	3.0
50	4.1	3.7

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.55 433	33	9.58 417	38	0.41 582	9.97 015	4	60
1	9.55 466	32	9.58 455	37	0.41 544	9.97 010	5	59
2	9.55 498	33	9.58 493	38	0.41 507	9.97 005	5	58
3	9.55 531	33	9.58 531	37	0.41 469	9.97 000	5	57
4	9.55 564	32	9.58 568	37	0.41 431	9.96 995	4	56
5	9.55 597	33	9.58 606	38	0.41 394	9.96 991	5	55
6	9.55 630	32	9.58 644	37	0.41 356	9.96 986	5	54
7	9.55 662	33	9.58 681	37	0.41 318	9.96 981	5	53
8	9.55 695	32	9.58 719	37	0.41 281	9.96 976	4	52
9	9.55 728	32	9.58 756	37	0.41 243	9.96 971	5	51
10	9.55 760	32	9.58 794	37	0.41 206	9.96 966	5	50
11	9.55 793	33	9.58 831	37	0.41 168	9.96 961	5	49
12	9.55 826	32	9.58 869	37	0.41 131	9.96 956	4	48
13	9.55 858	32	9.58 906	37	0.41 093	9.96 952	5	47
14	9.55 891	32	9.58 944	37	0.41 056	9.96 947	5	46
15	9.55 923	32	9.58 981	37	0.41 018	9.96 942	5	45
16	9.55 956	32	9.59 019	37	0.40 981	9.96 937	5	44
17	9.55 988	32	9.59 056	37	0.40 944	9.96 932	5	43
18	9.56 020	32	9.59 093	37	0.40 906	9.96 927	4	42
19	9.56 053	32	9.59 131	37	0.40 869	9.96 922	5	41
20	9.56 085	32	9.59 168	37	0.40 832	9.96 917	5	40
21	9.56 118	32	9.59 205	37	0.40 794	9.96 912	5	39
22	9.56 150	32	9.59 242	37	0.40 757	9.96 907	5	38
23	9.56 182	32	9.59 280	37	0.40 720	9.96 902	5	37
24	9.56 214	32	9.59 317	37	0.40 683	9.96 897	5	36
25	9.56 247	32	9.59 354	37	0.40 646	9.96 892	5	35
26	9.56 279	32	9.59 391	37	0.40 608	9.96 887	5	34
27	9.56 311	32	9.59 428	37	0.40 571	9.96 882	5	33
28	9.56 343	32	9.59 465	37	0.40 534	9.96 877	4	32
29	9.56 375	32	9.59 502	37	0.40 497	9.96 873	5	31
30	9.56 407	32	9.59 540	37	0.40 460	9.96 868	5	30
31	9.56 439	32	9.59 577	37	0.40 423	9.96 863	5	29
32	9.56 471	32	9.59 614	37	0.40 386	9.96 858	5	28
33	9.56 503	32	9.59 651	37	0.40 349	9.96 853	5	27
34	9.56 535	32	9.59 688	36	0.40 312	9.96 848	5	26
35	9.56 567	32	9.59 724	37	0.40 275	9.96 843	5	25
36	9.56 599	32	9.59 761	37	0.40 238	9.96 838	5	24
37	9.56 631	31	9.59 798	37	0.40 201	9.96 833	5	23
38	9.56 663	32	9.59 835	36	0.40 164	9.96 828	5	22
39	9.56 695	32	9.59 872	37	0.40 128	9.96 823	5	21
40	9.56 727	31	9.59 909	37	0.40 091	9.96 818	5	20
41	9.56 758	32	9.59 946	36	0.40 054	9.96 813	5	19
42	9.56 790	31	9.59 982	37	0.40 017	9.96 808	5	18
43	9.56 822	32	9.60 019	36	0.39 980	9.96 802	5	17
44	9.56 854	31	9.60 056	37	0.39 944	9.96 797	5	16
45	9.56 885	31	9.60 093	36	0.39 907	9.96 792	5	15
46	9.56 917	32	9.60 129	37	0.39 870	9.96 787	5	14
47	9.56 949	31	9.60 166	36	0.39 833	9.96 782	5	13
48	9.56 980	31	9.60 203	36	0.39 797	9.96 777	5	12
49	9.57 012	31	9.60 239	36	0.39 760	9.96 772	5	11
50	9.57 043	31	9.60 276	36	0.39 724	9.96 767	5	10
51	9.57 075	31	9.60 312	37	0.39 687	9.96 762	5	9
52	9.57 106	31	9.60 349	36	0.39 650	9.96 757	5	8
53	9.57 138	31	9.60 386	35	0.39 614	9.96 752	5	7
54	9.57 169	31	9.60 422	36	0.39 577	9.96 747	5	6
55	9.57 201	31	9.60 459	36	0.39 541	9.96 742	5	5
56	9.57 232	31	9.60 495	36	0.39 504	9.96 737	5	4
57	9.57 263	31	9.60 531	36	0.39 468	9.96 732	5	3
58	9.57 295	31	9.60 568	36	0.39 432	9.96 727	5	2
59	9.57 326	31	9.60 604	36	0.39 395	9.96 721	5	1
60	9.57 357		9.60 641		0.39 359	9.96 716		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	38	37	37
6	3.8	3.7	3.7
7	4.4	4.4	4.3
8	5.0	5.0	4.9
9	5.7	5.6	5.5
10	6.3	6.2	6.1
20	12.6	12.5	12.3
30	19.0	18.7	18.5
40	25.3	25.0	24.6
50	31.6	31.2	30.8

	36	36
6	3.6	3.6
7	4.2	4.2
8	4.8	4.8
9	5.5	5.4
10	6.1	6.0
20	12.1	12.0
30	18.2	18.0
40	24.3	24.0
50	30.4	30.0

	33	32	32
6	3.3	3.2	3.2
8	3.8	3.8	3.7
7	4.4	4.3	4.2
9	4.9	4.9	4.8
10	5.5	5.4	5.3
20	11.0	10.8	10.6
30	16.5	16.2	16.0
40	22.0	21.6	21.3
50	27.5	27.1	26.6

	31	31
6	3.1	3.1
7	3.7	3.6
8	4.2	4.1
9	4.7	4.6
10	5.2	5.1
20	10.5	10.3
30	15.7	15.5
40	21.0	20.6
50	26.2	25.8

	5	5	4
6	0.5	0.5	0.4
7	0.6	0.6	0.5
8	0.7	0.6	0.6
9	0.8	0.7	0.7
10	0.9	0.8	0.7
20	1.8	1.6	1.5
30	2.7	2.5	2.2
40	3.6	3.3	3.0
50	4.6	4.1	3.7

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.57 357	31	9.60 641	36	0.39 359	9.96 716	5	60
1	9.57 389	31	9.60 677	36	0.39 322	9.96 711	5	59
2	9.57 420	31	9.60 713	36	0.39 286	9.96 706	5	58
3	9.57 451	31	9.60 750	36	0.39 250	9.96 701	5	57
4	9.57 482	31	9.60 786	36	0.39 213	9.96 696	5	56
5	9.57 513	31	9.60 822	36	0.39 177	9.96 691	5	55
6	9.57 544	31	9.60 859	36	0.39 141	9.96 686	5	54
7	9.57 576	31	9.60 895	36	0.39 105	9.96 681	5	53
8	9.57 607	31	9.60 931	36	0.39 069	9.96 675	5	52
9	9.57 638	31	9.60 967	36	0.39 032	9.96 670	5	51
10	9.57 669	31	9.61 003	36	0.38 996	9.96 665	5	50
11	9.57 700	31	9.61 039	36	0.38 960	9.96 660	5	49
12	9.57 731	31	9.61 076	36	0.38 924	9.96 655	5	48
13	9.57 762	30	9.61 112	36	0.38 888	9.96 650	5	47
14	9.57 792	31	9.61 148	36	0.38 852	9.96 644	5	46
15	9.57 823	31	9.61 184	36	0.38 816	9.96 639	5	45
16	9.57 854	31	9.61 220	36	0.38 780	9.96 634	5	44
17	9.57 885	30	9.61 256	36	0.38 744	9.96 629	5	43
18	9.57 916	31	9.61 292	36	0.38 708	9.96 624	5	42
19	9.57 947	30	9.61 328	36	0.38 672	9.96 619	5	41
20	9.57 977	31	9.61 364	36	0.38 636	9.96 613	5	40
21	9.58 008	30	9.61 400	36	0.38 600	9.96 608	5	39
22	9.58 039	31	9.61 436	36	0.38 564	9.96 603	5	38
23	9.58 070	30	9.61 472	35	0.38 528	9.96 598	5	37
24	9.58 100	30	9.61 507	36	0.38 492	9.96 593	5	36
25	9.58 131	31	9.61 543	36	0.38 456	9.96 587	5	35
26	9.58 162	30	7.61 579	35	0.38 420	9.96 582	5	34
27	9.58 192	30	9.61 615	36	0.38 385	9.96 577	5	33
28	9.58 223	30	9.61 651	35	0.38 349	9.96 572	5	32
29	9.58 253	30	9.61 686	36	0.38 313	9.96 567	5	31
30	9.58 284	30	9.61 722	35	0.38 277	9.96 561	5	30
31	9.58 314	30	9.61 758	36	0.38 242	9.96 556	5	29
32	9.58 345	30	9.61 794	35	0.38 206	9.96 551	5	28
33	9.58 375	30	9.61 829	35	0.38 170	9.96 546	5	27
34	9.58 406	30	9.61 865	36	0.38 135	9.96 540	5	26
35	9.58 436	30	9.61 901	35	0.38 099	9.96 535	5	25
36	9.58 466	30	9.61 936	35	0.38 063	9.96 530	5	24
37	9.58 497	30	9.61 972	35	0.38 028	9.96 525	5	23
38	9.58 527	30	9.62 007	35	0.37 992	9.96 519	5	22
39	9.58 557	30	9.62 043	35	0.37 957	9.96 514	5	21
40	9.58 587	30	9.62 078	35	0.37 921	9.96 509	5	20
41	9.58 618	30	9.62 114	35	0.37 886	9.96 503	5	19
42	9.58 648	30	9.62 149	35	0.37 850	9.96 498	5	18
43	9.58 678	30	9.62 185	35	0.37 815	9.96 493	5	17
44	9.58 708	30	9.62 220	35	0.37 779	9.96 488	5	16
45	9.58 738	30	9.62 256	35	0.37 744	9.96 482	5	15
46	9.58 769	30	9.62 291	35	0.37 708	9.96 477	5	14
47	9.58 799	30	9.62 327	35	0.37 673	9.96 472	5	13
48	9.58 829	30	9.62 362	35	0.37 637	9.96 466	5	12
49	9.58 859	30	9.62 397	35	0.37 602	9.96 461	5	11
50	9.58 889	30	9.62 433	35	0.37 567	9.96 456	5	10
51	9.58 919	30	9.62 468	35	0.37 531	9.96 450	5	9
52	9.58 949	30	9.62 503	35	0.37 496	9.96 445	5	8
53	9.58 979	30	9.62 539	35	0.37 461	9.96 440	5	7
54	9.59 009	29	9.62 574	35	0.37 426	9.96 434	5	6
55	9.59 038	30	9.62 609	35	0.37 390	9.96 429	5	5
56	9.59 068	30	9.62 644	35	0.37 355	9.96 424	5	4
57	9.59 098	29	9.62 679	35	0.37 320	9.96 418	5	3
58	9.59 128	30	9.62 715	35	0.37 285	9.96 413	5	2
59	9.59 158	30	9.62 750	35	0.37 250	9.96 408	5	1
60	9.59 188		9.62 785		0.37 215	9.96 402		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	$\overline{36}$	36
6	3.6	3.6
7	4.2	4.2
8	4.8	4.8
9	5.5	5.4
10	6.1	6.0
20	12.1	12.0
30	18.2	18.0
40	24.3	24.0
50	30.4	30.0

	$\overline{35}$	35
6	3.5	3.5
7	4.1	4.1
8	4.7	4.6
9	5.3	5.2
10	5.9	5.8
20	11.8	11.6
30	17.7	17.5
40	23.6	23.3
50	29.6	29.1

	$\overline{31}$	31
6	3.1	3.1
7	3.7	3.6
8	4.2	4.1
9	4.7	4.6
10	5.2	5.1
20	10.5	10.3
30	15.7	15.5
40	21.0	20.6
50	26.2	25.8

	$\overline{30}$	30	$\overline{29}$
6	3.0	3.0	2.9
7	3.5	3.5	3.4
8	4.0	4.0	3.9
9	4.6	4.5	4.4
10	5.1	5.0	4.9
20	10.1	10.0	9.8
30	15.2	15.0	14.7
40	20.3	20.0	19.6
50	25.4	25.0	24.6

	$\overline{5}$	5
6	0.5	0.5
7	0.6	0.6
8	0.7	0.6
9	0.8	0.7
10	0.9	0.8
20	1.8	1.6
30	2.7	2.5
40	3.6	3.3
50	4.6	4.1

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.59 188	29	9.62 785	35	0.37 215	9.96 402	5	60
1	9.59 217	30	9.62 820	35	0.37 179	9.96 397	5	59
2	9.59 247	29	9.62 855	35	0.37 144	9.96 392	5	58
3	9.59 277	29	9.62 890	35	0.37 109	9.96 386	5	57
4	9.59 306	30	9.62 925	35	0.37 074	9.96 381	5	56
5	9.59 336	29	9.62 960	35	0.37 039	9.96 375	5	55
6	9.59 366	29	9.62 995	35	0.37 004	9.96 370	5	54
7	9.59 395	29	9.63 030	35	0.36 969	9.96 365	5	53
8	9.59 425	29	9.63 065	35	0.36 934	9.96 359	5	52
9	9.59 454	29	9.63 100	35	0.36 899	9.96 354	5	51
10	9.59 484	29	9.63 135	35	0.36 864	9.96 349	5	50
11	9.59 513	29	9.63 170	35	0.36 829	9.96 343	5	49
12	9.59 543	29	9.63 205	34	0.36 794	9.96 338	5	48
13	9.59 572	29	9.63 240	35	0.36 760	9.96 332	5	47
14	9.59 602	29	9.63 275	35	0.36 725	9.96 327	5	46
15	9.59 631	29	9.63 310	34	0.36 690	9.96 321	5	45
16	9.59 661	29	9.63 344	35	0.36 655	9.96 316	5	44
17	9.59 690	29	9.63 379	35	0.36 620	9.96 311	5	43
18	9.59 719	29	9.63 414	34	0.36 585	9.96 305	5	42
19	9.59 749	29	9.63 449	35	0.36 551	9.96 300	5	41
20	9.59 778	29	9.63 484	34	0.36 516	9.96 294	5	40
21	9.59 807	29	9.63 518	34	0.36 481	9.96 289	5	39
22	9.59 837	29	9.63 553	35	0.36 447	9.96 283	5	38
23	9.59 866	29	9.63 588	34	0.36 412	9.96 278	5	37
24	9.59 895	29	9.63 622	34	0.36 377	9.96 272	5	36
25	9.59 924	29	9.63 657	35	0.36 343	9.96 267	5	35
26	9.59 953	29	9.63 692	34	0.36 308	9.96 261	5	34
27	9.59 982	29	9.63 726	34	0.36 273	9.96 256	5	33
28	9.60 012	29	9.63 761	34	0.36 239	9.96 251	5	32
29	9.60 041	29	9.63 795	34	0.36 204	9.96 245	5	31
30	9.60 070	29	9.63 830	34	0.36 170	9.96 240	5	30
31	9.60 099	29	9.63 864	34	0.36 135	9.96 234	5	29
32	9.60 128	29	9.63 899	34	0.36 101	9.98 229	5	28
33	9.60 157	29	9.63 933	34	0.36 066	9.96 223	5	27
34	9.60 186	29	9.63 968	34	0.36 032	9.96 218	5	26
35	9.60 215	29	9.64 002	34	0.35 997	9.96 212	6	25
36	9.60 244	29	9.64 037	34	0.35 963	9.96 206	5	24
37	9.60 273	28	9.64 071	34	0.35 928	9.96 201	5	23
38	9.60 301	29	9.64 106	34	0.35 894	9.96 195	5	22
39	9.60 330	29	9.64 140	34	0.35 859	9.96 190	5	21
40	9.60 359	28	9.64 174	34	0.35 825	9.96 184	5	20
41	9.60 388	29	9.64 209	34	0.35 791	9.96 179	5	19
42	9.60 417	28	9.64 243	34	0.35 756	9.96 173	5	18
43	9.60 445	29	9.64 277	34	0.35 722	9.96 168	5	17
44	9.60 474	28	9.64 312	34	0.35 688	9.96 162	5	16
45	9.60 503	29	9.64 346	34	0.35 653	9.96 157	5	15
46	9.60 532	28	9.64 380	34	0.35 619	9.96 151	5	14
47	9.60 560	28	9.64 415	34	0.35 585	9.96 146	6	13
48	9.60 589	29	9.64 449	34	0.35 551	9.96 140	5	12
49	9.60 618	28	9.64 483	34	0.35 517	9.96 134	5	11
50	9.60 646	28	9.64 517	34	0.35 482	9.96 129	5	10
51	9.60 675	28	9.64 551	34	0.35 448	9.96 123	5	9
52	9.60 703	28	9.64 585	34	0.35 414	9.96 118	5	8
53	9.60 732	28	9.64 620	34	0.35 380	9.96 112	6	7
54	9.60 760	28	9.64 654	34	0.35 346	9.96 106	5	6
55	9.60 789	28	9.64 688	34	0.35 312	9.96 101	5	5
56	9.60 817	28	9.64 722	34	0.35 278	9.96 095	5	4
57	9.60 846	28	9.64 756	34	0.35 244	9.96 090	5	3
58	9.60 874	28	9.64 790	34	0.35 209	9.96 084	6	2
59	9.60 903	28	9.64 824	34	0.35 175	9.96 078	5	1
60	9.60 931		9.64 858		0.35 141	9.96 073		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	$\overline{35}$	35
6	3.5	3.5
7	4.1	4.1
8	4.7	4.6
9	5.3	5.2
10	5.9	5.8
20	11.8	11.6
30	17.7	17.5
40	23.6	23.3
50	29.6	29.1

	$\overline{34}$	34
6	3.4	3.4
7	4.0	3.9
8	4.6	4.5
9	5.2	5.1
10	5.7	5.6
20	11.5	11.3
30	17.2	17.0
40	23.0	22.6
50	28.7	28.3

	30
6	3.0
7	3.5
8	4.0
9	4.5
10	5.0
20	10.0
30	15.0
40	20.0
50	25.0

	$\overline{29}$	29	$\overline{28}$
6	2.9	2.9	2.8
7	3.4	3.4	3.3
8	3.9	3.8	3.8
9	4.4	4.3	4.3
10	4.9	4.8	4.7
20	8.8	9.6	9.5
30	14.7	14.5	14.2
40	19.6	19.3	19.0
50	24.6	24.1	23.7

	6	$\overline{5}$	5
6	0.6	0.5	0.5
7	0.7	0.6	0.6
8	0.8	0.7	0.6
9	0.9	0.8	0.7
10	1.0	0.9	0.8
20	2.0	1.8	1.6
30	3.0	2.7	2.5
40	4.0	3.6	3.3
50	5.0	4.6	4.1

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.60 931	28	9.64 858	34	0.35 141	9.96 073	5	60
1	9.60 959	28	9.64 892	34	0.35 107	9.96 067	5	59
2	9.60 988	28	9.64 926	33	0.35 073	9.96 062	6	58
3	9.61 016	28	9.64 960	34	0.35 040	9.96 056	5	57
4	9.61 044	28	9.64 994	34	0.35 006	9.96 050	5	56
5	9.61 073	28	9.65 028	34	0.34 972	9.96 045	6	55
6	9.61 101	28	9.65 062	34	0.34 938	9.96 039	5	54
7	9.61 129	28	9.65 096	33	0.34 904	9.96 033	5	53
8	9.61 157	28	9.65 129	34	0.34 870	9.96 028	6	52
9	9.61 186	28	9.65 163	34	0.34 836	9.96 022	5	51
10	9.61 214	28	9.65 197	33	0.34 802	9.96 016	5	50
11	9.61 242	28	9.65 231	34	0.34 769	9.96 011	6	49
12	9.61 270	28	9.65 265	34	0.34 735	9.96 005	5	48
13	9.61 298	28	9.65 299	33	0.34 701	9.95 999	5	47
14	9.61 326	28	9.65 332	34	0.34 667	9.95 994	6	46
15	9.61 354	28	9.65 366	33	0.34 633	9.95 988	5	45
16	9.61 382	28	9.65 400	33	0.34 600	9.95 982	5	44
17	9.61 410	28	9.65 433	34	0.34 566	9.95 977	6	43
18	9.61 438	28	9.65 467	33	0.34 532	9.95 971	5	42
19	9.61 466	28	9.65 501	34	0.34 499	9.95 965	6	41
20	9.61 494	28	9.65 535	33	0.34 465	9.95 959	5	40
21	9.61 522	27	9.65 568	33	0.34 431	9.95 954	6	39
22	9.61 550	28	9.65 602	33	0.34 398	9.95 948	5	38
23	9.61 578	28	9.65 635	33	0.34 364	9.95 942	5	37
24	9.61 606	28	9.65 669	34	0.34 331	9.95 937	6	36
25	9.61 634	27	9.65 703	33	0.34 297	9.95 931	5	35
26	9.61 661	28	9.65 736	33	0.34 263	9.95 925	6	34
27	9.61 689	27	9.65 770	33	0.34 230	9.95 919	5	33
28	9.61 717	28	9.65 803	33	0.34 196	9.95 914	6	32
29	9.61 745	27	9.65 837	33	0.34 163	9.95 908	5	31
30	9.61 772	28	9.65 870	33	0.34 129	9.95 902	6	30
31	9.61 800	27	9.65 904	33	0.34 096	9.95 896	5	29
32	9.61 828	28	9.65 937	33	0.34 062	9.95 891	6	28
33	9.61 856	27	9.65 971	33	0.34 029	9.95 885	6	27
34	9.61 883	27	9.66 004	33	0.33 996	9.95 879	5	26
35	9.61 911	27	9.66 037	33	0.33 962	9.95 873	6	25
36	9.61 938	27	9.66 071	33	0.33 929	9.95 867	5	24
37	9.61 966	28	9.66 104	33	0.33 895	9.95 862	6	23
38	9.61 994	27	9.66 137	33	0.33 862	9.95 856	5	22
39	9.62 021	27	9.66 171	33	0.33 829	9.95 850	6	21
40	9.62 049	27	9.66 204	33	0.33 795	9.95 844	6	20
41	9.62 076	27	9.66 237	33	0.33 762	9.95 838	5	19
42	9.62 104	27	9.66 271	33	0.33 729	9.95 833	6	18
43	9.62 131	27	9.66 304	33	0.33 696	9.95 827	5	17
44	9.62 158	27	9.66 337	33	0.33 662	9.95 821	6	16
45	9.62 186	27	9.66 370	33	0.33 629	9.95 815	6	15
46	9.62 213	27	9.66 404	33	0.33 596	9.95 809	5	14
47	9.62 241	27	9.66 437	33	0.33 563	9.95 804	6	13
48	9.62 268	27	9.66 470	33	0.33 529	9.95 798	6	12
49	9.62 295	27	9.66 503	33	0.33 496	9.95 792	5	11
50	9.62 323	27	9.66 536	33	0.33 463	9.95 786	6	10
51	9.62 350	27	9.66 570	33	0.33 430	9.95 780	6	9
52	9.62 377	27	9.66 603	33	0.33 397	9.95 774	6	8
53	9.62 404	27	9.66 636	33	0.33 364	9.95 768	5	7
54	9.62 432	27	9.66 669	33	0.33 331	9.95 763	6	6
55	9.62 459	27	9.66 702	33	0.33 298	9.95 757	6	5
56	9.62 486	27	9.66 735	33	0.33 265	9.95 751	6	4
57	9.62 513	27	9.66 768	33	0.33 232	9.95 745	5	3
58	9.62 540	27	9.66 801	33	0.33 198	9.95 739	6	2
59	9.62 567	27	9.66 834	33	0.33 165	9.95 733	6	1
60	9.62 595		9.66 867		0.33 132	9.95 727		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	34	$3\bar{3}$	33
6	3.4	3.3	3.3
7	3.9	3.9	3.8
8	4.5	4.4	4.4
9	5.1	5.0	4.9
10	5.6	5.6	5.5
20	11.3	11.1	11.0
30	17.0	16.7	16.5
40	22.6	22.3	22.0
50	28.3	27.9	27.5

	$2\bar{8}$	28
6	2.8	2.8
7	3.3	3.2
8	3.8	3.7
9	4.3	4.2
10	4.7	4.6
20	9.5	9.3
30	14.2	14.0
40	19.0	18.6
50	23.7	23.3

	$2\bar{7}$	27
6	2.7	2.7
7	3.2	3.1
8	3.6	3.6
9	4.1	4.0
10	4.6	4.5
20	9.1	9.0
30	13.7	13.5
40	18.3	18.0
50	22.9	22.5

	6	$\bar{5}$
6	0.6	0.5
7	0.7	0.6
8	0.8	0.7
9	0.9	0.8
10	1.0	0.9
20	2.0	1.8
30	3.0	2.7
40	4.0	3.6
50	5.0	4.6

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.62 595	27	9.66 867	32	0.33 132	9.95 727	6	60
1	9.62 622	27	9.66 900	33	0.33 100	9.95 721	5	59
2	9.62 649	27	9.66 933	33	0.33 067	9.95 716	6	58
3	9.62 676	27	9.66 966	33	0.33 034	9.95 710	6	57
4	9.62 703	27	9.66 999	33	0.33 001	9.95 704	6	56
5	9.62 730	27	9.67 032	33	0.32 968	9.95 698	6	55
6	9.62 757	27	9.67 065	32	0.32 935	9.95 692	6	54
7	9.62 784	27	9.67 097	33	0.32 902	9.95 686	5	53
8	9.62 811	27	9.67 130	33	0.32 869	9.95 680	6	52
9	9.62 838	26	9.67 163	33	0.32 836	9.95 674	6	51
10	9.62 864	27	9.67 196	32	0.32 803	9.95 668	6	50
11	9.62 891	27	9.67 229	33	0.32 771	9.95 662	6	49
12	9.62 918	27	9.67 262	32	0.32 738	9.95 656	6	48
13	9.62 945	26	9.67 294	33	0.32 705	9.95 650	6	47
14	9.62 972	27	9.67 327	32	0.32 672	9.95 644	6	46
15	9.62 999	26	9.67 360	33	0.32 640	9.95 638	6	45
16	9.63 025	27	9.67 393	32	0.32 607	9.95 632	5	44
17	9.63 052	26	9.67 425	33	0.32 574	9.95 627	6	43
18	9.63 079	27	9.67 458	32	0.32 541	9.95 621	6	42
19	9.63 106	26	9.67 491	32	0.32 509	9.95 615	6	41
20	9.63 132	27	9.67 523	33	0.32 476	9.95 609	6	40
21	9.63 159	26	9.67 556	32	0.32 443	9.95 603	6	39
22	9.63 186	26	9.67 589	32	0.32 411	9.95 597	6	38
23	9.63 212	26	9.67 621	33	0.32 378	9.95 591	6	37
24	9.63 239	27	9.67 654	32	0.32 345	9.95 585	6	36
25	9.63 266	26	9.67 687	32	0.32 313	9.95 579	6	35
26	9.63 292	26	9.67 719	32	0.32 280	9.95 573	6	34
27	9.63 319	26	9.67 752	32	0.32 248	9.95 567	6	33
28	9.63 345	26	9.67 784	32	0.32 215	9.95 561	6	32
29	9.63 372	26	9.67 817	32	0.32 183	9.95 555	6	31
30	9.63 398	26	9.67 849	32	0.32 150	9.95 549	6	30
31	9.63 425	26	9.67 882	32	0.32 118	9.95 543	6	29
32	9.63 451	26	9.67 914	32	0.32 085	9.95 537	6	28
33	9.63 478	26	9.67 947	32	0.32 053	9.95 530	6	27
34	9.63 504	26	9.67 979	32	0.32 020	9.95 524	6	26
35	9.63 530	26	9.68 012	32	0.31 988	9.95 518	6	25
36	9.63 557	26	9.68 044	32	0.31 955	9.95 512	6	24
37	9.63 583	26	9.68 077	32	0.31 923	9.95 506	6	23
38	9.63 609	26	9.68 109	32	0.31 891	9.95 500	6	22
39	9.63 636	26	9.68 141	32	0.31 858	9.95 494	6	21
40	9.63 662	26	9.68 174	32	0.31 826	9.95 488	6	20
41	9.63 688	26	9.68 206	32	0.31 793	9.95 482	6	19
42	9.63 715	26	9.68 238	32	0.31 761	9.95 476	6	18
43	9.63 741	26	9.68 271	32	0.31 729	9.95 470	6	17
44	9.63 767	26	9.68 303	32	0.31 696	9.95 464	6	16
45	9.63 793	26	9.68 335	32	0.31 664	9.95 458	6	15
46	9.63 819	26	9.68 368	32	0.31 632	9.95 452	6	14
47	9.63 846	26	9.68 400	32	0.31 600	9.95 445	6	13
48	9.63 872	26	9.68 432	32	0.31 567	9.95 439	6	12
49	9.63 898	26	9.68 464	32	0.31 535	9.95 433	6	11
50	9.63 924	26	9.68 497	32	0.31 503	9.95 427	6	10
51	9.63 950	26	9.68 529	32	0.31 471	9.95 421	6	9
52	9.63 976	26	9.68 561	32	0.31 439	9.95 415	6	8
53	9.64 002	26	9.68 593	32	0.31 406	9.95 409	6	7
54	9.64 028	26	9.68 625	32	0.31 374	9.95 403	6	6
55	9.64 054	26	9.68 657	32	0.31 342	9.95 397	6	5
56	9.64 080	26	9.68 690	32	0.31 310	9.95 390	6	4
57	9.64 106	26	9.68 722	32	0.31 278	9.95 384	6	3
58	9.64 132	26	9.68 754	32	0.31 246	9.95 378	6	2
59	9.64 158	25	9.68 786	32	0.31 214	9.95 372	6	1
60	9.64 184		9.68 818		0.31 182	9.95 366		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	33	32	32
6	3.3	3.2	3.2
7	3.8	3.8	3.7
8	4.4	4.3	4.2
9	4.9	4.9	4.8
10	5.5	5.4	5.3
20	11.0	10.8	10.6
30	16.5	16.2	16.0
40	22.0	21.6	21.3
50	27.5	27.1	26.6

	27
6	2.7
7	3.1
8	3.6
9	4.0
10	4.5
20	9.0
30	13.5
40	18.0
50	22.5

	26	26	25
6	2.6	2.6	2.5
7	3.1	3.0	3.0
8	3.5	3.4	3.4
9	4.0	3.9	3.8
10	4.4	4.3	4.2
20	8.8	8.6	8.5
30	13.2	13.0	12.7
40	17.6	17.3	17.0
50	22.1	21.6	21.2

	6	6	5
6	0.6	0.6	0.5
7	0.7	0.7	0.6
8	0.8	0.8	0.7
9	1.0	0.9	0.8
10	1.1	1.0	0.9
20	2.1	2.0	1.8
30	3.2	3.0	2.7
40	4.3	4.0	3.6
50	5.4	5.0	4.6

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.64 184	26	9.68 818	32	0.31 182	9.95 366	6	60
1	9.64 210	26	9.68 850	32	0.31 150	9.95 360	6	59
2	9.64 236	26	9.68 882	32	0.31 117	9.95 353	6	58
3	9.64 262	25	9.68 914	32	0.31 085	9.95 347	6	57
4	9.64 287	26	9.68 946	32	0.31 053	9.95 341	6	56
5	9.64 313	26	9.68 978	32	0.31 021	9.95 335	6	55
6	9.64 339	25	9.69 010	32	0.30 989	9.95 329	6	54
7	9.64 365	26	9.69 042	31	0.30 957	9.95 323	6	53
8	9.64 391	25	9.69 074	32	0.30 926	9.95 316	6	52
9	9.64 416	26	9.69 106	32	0.30 894	9.95 310	6	51
10	9.64 442	25	9.69 138	32	0.30 862	9.95 304	6	50
11	9.64 468	25	9.69 170	32	0.30 830	9.95 298	6	49
12	9.64 493	26	9.69 202	32	0.30 798	9.95 292	6	48
13	9.64 519	25	9.69 234	31	0.30 766	9.95 285	6	47
14	9.64 545	25	9.69 265	32	0.30 734	9.95 279	6	46
15	9.64 570	25	9.69 297	32	0.30 702	9.95 273	6	45
16	9.64 596	26	9.69 329	31	0.30 670	9.95 267	6	44
17	9.64 622	25	9.69 361	32	0.30 639	9.95 260	6	43
18	9.64 647	25	9.69 393	32	0.30 607	9.95 254	6	42
19	9.64 673	25	9.69 425	31	0.30 575	9.95 248	6	41
20	9.64 698	25	9.69 456	32	0.30 543	9.95 242	6	40
21	9.64 724	25	9.69 488	31	0.30 511	9.95 235	6	39
22	9.64 749	25	9.69 520	32	0.30 480	9.95 229	6	38
23	9.64 775	25	9.69 552	31	0.30 448	9.95 223	6	37
24	9.64 800	25	9.69 583	32	0.30 416	9.95 217	6	36
25	9.64 826	25	9.69 615	31	0.30 384	9.95 210	6	35
26	9.64 851	25	9.69 647	31	0.30 353	9.95 204	6	34
27	9.64 876	25	9.69 678	32	0.30 321	9.95 198	6	33
28	9.64 902	25	9.69 710	31	0.30 289	9.95 191	6	32
29	9.64 927	25	9.69 742	31	0.30 258	9.95 185	6	31
30	9.64 952	25	9.69 773	32	0.30 226	9.95 179	6	30
31	9.64 978	25	9.69 805	31	0.30 194	9.95 173	6	29
32	9.65 003	25	9.69 837	31	0.30 163	9.95 166	6	28
33	9.65 028	25	9.69 868	31	0.30 131	9.95 160	6	27
34	9.65 054	25	9.69 900	31	0.30 100	9.95 154	6	26
35	9.65 079	25	9.69 931	31	0.30 068	9.95 147	6	25
36	9.65 104	25	9.69 963	31	0.30 037	9.95 141	6	24
37	9.65 129	25	9.69 994	32	0.30 005	9.95 135	6	23
38	9.65 155	25	9.70 026	31	0.29 973	9.95 128	6	22
39	9.65 180	25	9.70 058	31	0.29 942	9.95 122	6	21
40	9.65 205	25	9.70 089	31	0.29 910	9.95 116	6	20
41	9.65 230	25	9.70 121	31	0.29 879	9.95 109	6	19
42	9.65 255	25	9.70 152	31	0.29 847	9.95 103	6	18
43	9.65 280	25	9.70 183	31	0.29 816	9.95 097	6	17
44	9.65 305	25	9.70 215	31	0.29 785	9.95 090	6	16
45	9.65 331	25	9.70 246	31	0.29 753	9.95 084	6	15
46	9.65 356	25	9.70 278	31	0.29 722	9.95 078	6	14
47	9.65 381	25	9.70 309	31	0.29 690	9.95 071	6	13
48	9.65 406	25	9.70 341	31	0.29 659	9.95 065	6	12
49	9.65 431	25	9.70 372	31	0.29 628	9.95 058	6	11
50	9.65 456	25	9.70 403	31	0.29 596	9.95 052	6	10
51	9.65 481	25	9.70 435	31	0.29 565	9.95 046	6	9
52	9.65 506	24	9.70 466	31	0.29 533	9.95 039	6	8
53	9.65 530	25	9.70 497	31	0.29 502	9.95 033	6	7
54	9.65 555	25	9.70 529	31	0.29 471	9.95 026	6	6
55	9.65 580	25	9.70 560	31	0.29 439	9.95 020	6	5
56	9.65 605	24	9.70 591	31	0.29 408	9.95 014	6	4
57	9.65 630	25	9.70 623	31	0.29 377	9.95 007	6	3
58	9.65 655	25	9.70 654	31	0.29 346	9.95 001	6	2
59	9.65 680	24	9.70 685	31	0.29 314	9.94 994	6	1
60	9.65 704		9.70 716		0.29 283	9.94 988		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	32	32
6	3.2	3.2
7	3.8	3.7
8	4.3	4.2
9	4.9	4.8
10	5.4	5.3
20	10.8	10.6
30	16.2	16.0
40	21.6	21.3
50	27.1	26.6

	31	31
6	3.1	3.1
7	3.7	3.6
8	4.2	4.1
9	4.7	4.6
10	5.2	5.1
20	10.5	10.3
30	15.7	15.5
40	21.0	20.6
50	26.2	25.8

	26	25	25
6	2.6	2.5	2.5
7	3.0	3.0	2.9
8	3.4	3.4	3.3
9	3.9	3.8	3.7
10	4.3	4.2	4.1
20	8.6	8.5	8.3
30	13.0	12.7	12.5
40	17.3	17.0	16.6
50	21.6	21.2	20.8

	24	6	6
6	2.4	0.6	0.6
7	2.8	0.7	0.7
8	3.2	0.8	0.8
9	3.7	1.0	0.9
10	4.1	1.1	1.0
20	8.1	2.1	2.0
30	12.2	3.2	3.0
40	16.3	4.3	4.0
50	20.4	5.4	5.0

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot	Log. Cos.	d.	
0	9.65 70$\bar{4}$	25	9.70 71$\bar{6}$	3$\bar{1}$	0.29 28$\bar{3}$	9.94 98$\underline{8}$	$\bar{6}$	**60**
1	9.65 72$\bar{9}$	2$\bar{4}$	9.70 748	31	0.29 252	9.94 98$\bar{1}$	6	59
2	9.65 754	25	9.70 779	3$\underline{1}$	0.29 221	9.94 975	6	58
3	9.65 77$\underline{9}$	2$\bar{4}$	9.70 81$\underline{0}$	3$\bar{1}$	0.29 19$\underline{0}$	9.94 96$\underline{9}$	6	57
4	9.65 80$\bar{3}$	25	9.70 84$\bar{1}$	31	0.29 15$\bar{8}$	9.94 96$\bar{2}$	$\bar{6}$	56
5	9.65 82$\bar{8}$	2$\bar{4}$	9.70 87$\bar{2}$	3$\underline{1}$	0.29 12$\bar{7}$	9.94 95$\underline{6}$	$\bar{6}$	55
6	9.65 853	25	9.70 90$\bar{3}$	3$\bar{1}$	0.29 09$\bar{6}$	9.94 94$\bar{9}$	$\bar{6}$	54
7	9.65 87$\underline{8}$	2$\bar{4}$	9.70 935	31	0.29 065	9.94 94$\underline{3}$	$\bar{6}$	53
8	9.65 90$\bar{2}$	2$\bar{4}$	9.70 966	31	0.29 034	9.94 93$\bar{6}$	6	52
9	9.65 927	2$\bar{4}$	9.70 997	3$\underline{1}$	0.29 003	9.94 930	$\bar{6}$	51
10	9.65 95$\bar{1}$	25	9.71 02$\underline{8}$	3$\bar{1}$	0.28 972	9.94 92$\bar{3}$	$\bar{6}$	**50**
11	9.65 97$\bar{6}$	2$\bar{4}$	9.71 05$\bar{9}$	31	0.28 94$\underline{0}$	9.94 91$\underline{7}$	$\bar{6}$	49
12	9.66 00$\underline{1}$	2$\bar{4}$	9.71 09$\bar{0}$	31	0.28 90$\underline{9}$	9.94 91$\bar{0}$	$\bar{6}$	48
13	9.66 02$\bar{5}$	24	9.71 12$\underline{1}$	31	0.28 87$\underline{8}$	9.94 90$\underline{4}$	6	47
14	9.66 050	2$\bar{4}$	9.71 15$\bar{2}$	31	0.28 84$\bar{7}$	9.94 89$\bar{7}$	$\bar{6}$	46
15	9.66 07$\bar{4}$	2$\bar{4}$	9.71 18$\bar{3}$	31	0.28 81$\bar{6}$	9.94 89$\underline{1}$	$\bar{6}$	45
16	9.66 09$\underline{9}$	2$\bar{4}$	9.71 21$\bar{4}$	31	0.28 78$\bar{5}$	9.94 88$\bar{4}$	$\bar{6}$	44
17	9.66 12$\bar{3}$	2$\bar{4}$	9.71 24$\bar{5}$	31	0.28 75$\bar{4}$	9.94 87$\underline{8}$	$\bar{6}$	43
18	9.66 14$\underline{8}$	24	9.71 27$\bar{6}$	31	0.28 72$\bar{3}$	9.94 87$\bar{1}$	6	42
19	9.66 17$\bar{2}$	2$\bar{4}$	9.71 30$\bar{7}$	31	0.28 69$\bar{2}$	9.94 865	$\bar{6}$	41
20	9.66 19$\underline{7}$	2$\bar{4}$	9.71 33$\underline{8}$	31	0.28 66$\underline{1}$	9.94 85$\bar{8}$	$\bar{6}$	**40**
21	9.66 22$\bar{1}$	2$\bar{4}$	9.71 36$\bar{9}$	31	0.28 63$\bar{0}$	9.94 85$\underline{2}$	$\bar{6}$	39
22	9.66 24$\underline{6}$	24	9.71 40$\bar{0}$	31	0.28 59$\bar{9}$	9.94 84$\bar{5}$	$\bar{6}$	38
23	9.66 27$\underline{0}$	24	9.71 43$\underline{1}$	31	0.28 56$\underline{8}$	9.94 83$\underline{9}$	6	37
24	9.66 29$\bar{4}$	2$\bar{4}$	9.71 46$\bar{2}$	3$\underline{1}$	0.28 53$\bar{7}$	9.94 83$\bar{2}$	$\underline{7}$	36
25	9.66 31$\underline{9}$	2$\bar{4}$	9.71 493	3$\bar{0}$	0.28 50$\bar{6}$	9.94 82$\bar{5}$	$\bar{6}$	35
26	9.66 34$\bar{3}$	2$\bar{4}$	9.71 524	31	0.28 476	9.94 81$\underline{9}$	$\bar{6}$	34
27	9.66 36$\bar{7}$	2$\bar{4}$	9.71 555	31	0.28 445	9.94 81$\bar{2}$	$\bar{6}$	33
28	9.66 39$\underline{2}$	24	9.71 586	31	0.28 414	9.94 80$\underline{6}$	6	32
29	9.66 41$\bar{6}$	2$\bar{4}$	9.71 617	3$\bar{0}$	0.28 383	9.94 79$\bar{9}$	$\bar{6}$	31
30	9.66 44$\bar{0}$	2$\bar{4}$	9.71 64$\bar{7}$	31	0.28 35$\bar{2}$	9.94 79$\underline{3}$	6	**30**
31	9.66 465	2$\bar{4}$	9.71 67$\bar{8}$	3$\underline{1}$	0.28 32$\bar{1}$	9.94 78$\bar{6}$	$\underline{7}$	29
32	9.66 48$\underline{9}$	2$\bar{4}$	9.71 70$\bar{9}$	3$\bar{0}$	0.28 29$\bar{0}$	9.94 77$\bar{9}$	$\bar{6}$	28
33	9.66 51$\bar{3}$	24	9.71 740	31	0.28 260	9.94 77$\underline{3}$	6	27
34	9.66 53$\bar{7}$	2$\bar{4}$	9.71 771	3$\bar{0}$	0.28 229	9.94 76$\bar{6}$	$\bar{6}$	26
35	9.66 56$\bar{1}$	2$\bar{4}$	9.71 80$\underline{1}$	31	0.28 19$\bar{8}$	9.94 76$\underline{0}$	6	25
36	9.66 586	24	9.71 83$\bar{2}$	3$\underline{1}$	0.28 16$\bar{7}$	9.94 75$\bar{3}$	$\underline{7}$	24
37	9.66 610	2$\bar{4}$	9.71 86$\bar{3}$	3$\bar{0}$	0.28 13$\bar{6}$	9.94 74$\bar{6}$	$\bar{6}$	23
38	9.66 63$\underline{4}$	24	9.71 894	31	0.28 106	9.94 74$\underline{0}$	6	22
39	9.66 65$\bar{8}$	24	9.71 925	3$\bar{0}$	0.28 075	9.94 73$\bar{3}$	$\bar{6}$	21
40	9.66 68$\underline{2}$	24	9.71 955	3$\bar{0}$	0.28 04$\bar{4}$	9.94 72$\underline{7}$	6	**20**
41	9.66 70$\bar{6}$	24	9.71 986	3$\underline{1}$	0.28 014	9.94 72$\bar{0}$	$\underline{7}$	19
42	9.66 73$\bar{0}$	24	9.72 01$\underline{7}$	3$\bar{0}$	0.27 98$\underline{3}$	9.94 71$\bar{3}$	$\bar{6}$	18
43	9.66 75$\bar{4}$	24	9.72 04$\bar{7}$	31	0.27 95$\bar{2}$	9.94 70$\underline{7}$	6	17
44	9.66 77$\bar{8}$	24	9.72 07$\bar{8}$	3$\bar{0}$	0.27 92$\bar{1}$	9.94 70$\bar{0}$	$\underline{7}$	16
45	9.66 80$\bar{2}$	24	9.72 10$\underline{9}$	3$\bar{0}$	0.27 89$\underline{1}$	9.94 69$\bar{3}$	$\bar{6}$	15
46	9.66 82$\bar{6}$	24	9.72 13$\bar{9}$	3$\bar{0}$	0.27 86$\bar{0}$	9.94 68$\underline{7}$	$\bar{6}$	14
47	9.66 85$\bar{0}$	24	9.72 170	3$\underline{1}$	0.27 830	9.94 68$\bar{0}$	6	13
48	9.66 87$\bar{4}$	24	9.72 20$\underline{1}$	3$\bar{0}$	0.27 799	9.94 674	7	12
49	9.66 89$\bar{8}$	24	9.72 23$\bar{1}$	3$\bar{0}$	0.27 76$\bar{8}$	9.94 667	$\bar{6}$	11
50	9.66 92$\bar{2}$	24	9.72 26$\underline{2}$	3$\bar{0}$	0.27 73$\underline{8}$	9.94 66$\bar{0}$	6	**10**
51	9.66 94$\bar{6}$	2$\underline{4}$	9.72 29$\bar{2}$	3$\bar{0}$	0.27 70$\bar{7}$	9.94 654	$\underline{7}$	9
52	9.66 97$\bar{0}$	2$\bar{3}$	9.72 323	3$\underline{1}$	0.27 677	9.94 64$\underline{7}$	$\bar{6}$	8
53	9.66 994	24	9.72 35$\underline{4}$	3$\bar{0}$	0.27 64$\underline{6}$	9.94 64$\bar{0}$	7	7
54	9.67 018	24	9.72 38$\bar{4}$	3$\bar{0}$	0.27 61$\bar{5}$	9.94 63$\bar{3}$	$\bar{6}$	6
55	9.67 042	2$\underline{4}$	9.72 41$\underline{5}$	3$\bar{0}$	0.27 58$\underline{5}$	9.94 62$\underline{7}$	6	5
56	9.67 06$\underline{6}$	2$\bar{3}$	9.72 44$\bar{5}$	3$\bar{0}$	0.27 55$\bar{4}$	9.94 62$\bar{0}$	$\underline{7}$	4
57	9.67 08$\bar{9}$	2$\underline{4}$	9.72 47$\underline{6}$	3$\bar{0}$	0.27 52$\underline{4}$	9.94 61$\bar{3}$	$\bar{6}$	3
58	9.67 11$\bar{3}$	2$\bar{3}$	9.72 50$\bar{6}$	3$\bar{0}$	0.27 49$\bar{3}$	9.94 607	7	2
59	9.67 137	24	9.72 537	3$\bar{0}$	0.27 463	9.94 600	$\bar{6}$	1
60	9.67 161		9.72 56$\bar{7}$		0.27 43$\bar{2}$	9.94 59$\bar{3}$		**0**
	Log. Cos.	d.	Log. Cot	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	3$\bar{1}$	**31**	**3$\bar{0}$**
6	3.$\bar{1}$	3.1	3.$\bar{0}$
7	3.7	3.$\underline{6}$	3.$\bar{5}$
8	4.2	4.$\bar{1}$	4.$\bar{0}$
9	4.$\underline{7}$	4.$\bar{6}$	4.6
10	5.$\bar{2}$	5.$\bar{1}$	5.$\underline{1}$
20	10.$\underline{5}$	10.$\bar{3}$	10.$\bar{1}$
30	15.$\bar{7}$	15.$\underline{5}$	15.$\bar{2}$
40	21.$\underline{0}$	20.$\bar{6}$	20.$\bar{3}$
50	26.$\bar{2}$	25.8	25.4

	25
6	2.5
7	2.$\underline{9}$
8	3.$\bar{3}$
9	3.$\bar{7}$
10	4.$\bar{1}$
20	8.$\bar{3}$
30	12.5
40	16.$\bar{6}$
50	20.$\bar{8}$

	2$\bar{4}$	**24**	**2$\bar{3}$**
6	2.$\bar{4}$	2.4	2.$\bar{3}$
7	2.$\bar{8}$	2.8	2.$\bar{7}$
8	3.$\bar{2}$	3.2	3.$\bar{1}$
9	3.7	3.6	3.$\underline{5}$
10	4.$\underline{1}$	4.0	3.$\underline{9}$
20	8.$\bar{1}$	8.0	7.$\bar{8}$
30	12.$\bar{2}$	12.0	11.$\bar{7}$
40	16.$\bar{3}$	16.0	15.$\bar{6}$
50	20.4	20.0	19.6

	7	**$\bar{6}$**	**6**
6	0.7	0.$\bar{6}$	0.6
7	0.$\underline{8}$	0.$\bar{7}$	0.7
8	0.$\bar{9}$	0.$\bar{8}$	0.8
9	1.$\bar{0}$	1.0	0.9
10	1.$\bar{1}$	1.$\underline{1}$	1.0
20	2.$\bar{3}$	2.$\bar{1}$	2.0
30	3.$\underline{5}$	3.$\bar{2}$	3.0
40	4.$\bar{6}$	4.$\bar{3}$	4.0
50	5.$\bar{8}$	5.4	5.0

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.67 161	23	9.72 567	30	0.27 432	9.94 593	6	60
1	9.67 184	24	9.72 598	30	0.27 402	9.94 587	7	59
2	9.67 208	23	9.72 628	30	0.27 371	9.94 580	6	58
3	9.67 232	24	9.72 659	30	0.27 341	9.94 573	7	57
4	9.67 256	23	9.72 689	30	0.27 311	9.94 566	6	56
5	9.67 279	23	9.72 719	30	0.27 280	9.94 560	7	55
6	9.67 303	24	9.72 750	30	0.27 250	9.94 553	6	54
7	9.67 327	23	9.72 780	30	0.27 219	9.94 546	7	53
8	9.67 350	23	9.72 811	30	0.27 189	9.94 539	6	52
9	9.67 374	23	9.72 841	30	0.27 159	9.94 533	7	51
10	9.67 397	24	9.72 871	30	0.27 128	9.94 526	6	50
11	9.67 421	23	9.72 902	30	0.27 098	9.94 519	7	49
12	9.67 445	23	9.72 932	30	0.27 067	9.94 512	6	48
13	9.67 468	23	9.72 962	30	0.27 037	9.94 506	7	47
14	9.67 492	23	9.72 993	30	0.27 007	9.94 499	7	46
15	9.67 515	23	9.73 023	30	0.26 976	9.94 492	6	45
16	9.67 539	23	9.73 053	30	0.26 946	9.94 485	7	44
17	9.67 562	23	9.73 084	30	0.26 916	9.94 478	6	43
18	9.67 586	23	9.73 114	30	0.26 886	9.94 472	7	42
19	9.67 609	23	9.73 144	30	0.26 855	9.94 465	7	41
20	9.67 633	23	9.73 174	30	0.26 825	9.94 458	6	40
21	9.67 656	23	9.73 205	30	0.26 795	9.94 451	7	39
22	9.67 679	23	9.73 235	30	0.26 765	9.94 444	7	38
23	9.67 703	23	9.73 265	30	0.26 734	9.94 437	6	37
24	9.67 726	23	9.73 295	30	0.26 704	9.94 431	7	36
25	9.67 750	23	9.73 325	30	0.26 674	9.94 424	7	35
26	9.67 773	23	9.73 356	30	0.26 644	9.94 417	6	34
27	9.67 796	23	9.73 386	30	0.26 614	9.94 410	7	33
28	9.67 819	23	9.73 416	30	0.26 584	9.94 403	7	32
29	9.67 843	23	9.73 446	30	0.26 553	9.94 396	6	31
30	9.67 866	23	9.73 476	30	0.26 523	9.94 390	7	30
31	9.67 889	23	9.73 506	30	0.26 493	9.94 383	7	29
32	9.67 913	23	9.73 536	30	0.26 463	9.94 376	6	28
33	9.67 936	23	9.73 567	30	0.26 433	9.94 369	7	27
34	9.67 959	23	9.73 597	30	0.26 403	9.94 362	7	26
35	9.67 982	23	9.73 627	30	0.26 373	9.94 355	7	25
36	9.68 005	23	9.73 657	30	0.26 343	9.94 348	7	24
37	9.68 029	23	9.73 687	30	0.26 313	9.94 341	6	23
38	9.68 052	23	9.73 717	30	0.26 283	9.94 335	7	22
39	9.68 075	23	9.73 747	30	0.26 253	9.94 328	7	21
40	9.68 098	23	9.73 777	30	0.26 223	9.94 321	7	20
41	9.68 121	23	9.73 807	30	0.26 193	9.94 314	7	19
42	9.68 144	23	9.73 837	30	0.26 163	9.94 307	6	18
43	9.68 167	23	9.73 867	30	0.26 133	9.94 300	7	17
44	9.68 190	23	9.73 897	30	0.26 103	9.94 293	7	16
45	9.68 213	23	9.73 927	30	0.26 073	9.94 286	7	15
46	9.68 236	23	9.73 957	30	0.26 043	9.94 279	7	14
47	9.68 259	23	9.73 987	30	0.26 013	9.94 272	7	13
48	9.68 282	23	9.74 017	30	0.25 983	9.94 265	7	12
49	9.68 305	23	9.74 047	29	0.25 953	9.94 258	7	11
50	9.68 328	23	9.74 076	30	0.25 923	9.94 251	6	10
51	9.68 351	23	9.74 106	30	0.25 893	9.94 245	7	9
52	9.68 374	22	9.74 136	30	0.25 863	9.94 238	7	8
53	9.68 397	23	9.74 166	29	0.25 833	9.94 231	7	7
54	9.68 420	23	9.74 196	30	0.25 804	9.94 224	7	6
55	9.68 443	23	9.74 226	30	0.25 774	9.94 217	7	5
56	9.68 466	22	9.74 256	30	0.25 744	9.94 210	7	4
57	9.68 488	23	9.74 286	29	0.25 714	9.94 203	7	3
58	9.68 511	23	9.74 315	30	0.25 684	9.94 196	7	2
59	9.68 534	22	9.74 345	29	0.25 654	9.94 189	7	1
60	9.68 557		9.74 375		0.25 625	9.94 182		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	30	30	29
6	3.0	3.0	2.9
7	3.5	3.5	3.4
8	4.0	4.0	3.9
9	4.6	4.5	4.4
10	5.1	5.0	4.9
20	10.1	10.0	9.8
30	15.2	15.0	14.7
40	20.3	20.0	19.6
50	25.4	25.0	24.6

	24
6	2.4
7	2.8
8	3.2
9	3.6
10	4.0
20	8.0
30	12.0
40	16.0
50	20.0

	23	23	22
6	2.3	2.3	2.3
7	2.7	2.7	2.6
8	3.1	3.0	3.0
9	3.5	3.4	3.4
10	3.9	3.8	3.7
20	7.8	7.6	7.5
30	11.7	11.5	11.2
40	15.6	15.3	15.0
50	19.6	19.1	18.7

	7	6
6	0.7	0.6
7	0.8	0.7
8	0.9	0.8
9	1.0	1.0
10	1.1	1.1
20	2.3	2.1
30	3.5	3.2
40	4.6	4.3
50	5.8	5.4

P. P.

'	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	'
0	9.68 557	23	9.74 375	30	0.25 625	9.94 182	7	60
1	9.68 580	22	9.74 405	30	0.25 595	9.94 175	7	59
2	9.68 602	23	9.74 435	29	0.25 565	9.94 168	7	58
3	9.68 625	22	9.74 464	30	0.25 535	9.94 161	7	57
4	9.68 648	23	9.74 494	29	0.25 505	9.94 154	7	56
5	9.68 671	22	9.74 524	30	0.25 476	9.94 147	7	55
6	9.68 693	23	9.74 554	29	0.25 446	9.94 140	7	54
7	9.68 716	22	9.74 583	29	0.25 416	9.94 133	7	53
8	9.68 739	22	9.74 613	30	0.25 387	9.94 126	7	52
9	9.68 761	23	9.74 643	29	0.25 357	9.94 118	7	51
10	9.68 784	22	9.74 672	30	0.25 327	9.94 111	7	50
11	9.68 807	22	9.74 702	29	0.25 297	9.94 104	7	49
12	9.68 829	22	9.74 732	29	0.25 268	9.94 097	7	48
13	9.68 852	22	9.74 761	30	0.25 238	9.94 090	7	47
14	9.68 874	22	9.74 791	29	0.25 208	9.94 083	7	46
15	9.68 897	23	9.74 821	29	0.25 179	9.94 076	7	45
16	9.68 920	22	9.74 850	29	0.25 149	9.94 069	7	44
17	9.68 942	22	9.74 880	29	0.25 120	9.94 062	7	43
18	9.68 965	22	9.74 909	30	0.25 090	9.94 055	7	42
19	9.68 987	22	9.74 939	29	0.25 060	9.94 048	7	41
20	9.69 010	22	9.74 969	29	0.25 031	9.94 041	7	40
21	9.69 032	22	9.74 998	29	0.25 001	9.94 034	7	39
22	9.69 055	22	9.75 028	29	0.24 972	9.94 026	7	38
23	9.69 077	22	9.75 057	29	0.24 942	9.94 019	7	37
24	9.69 099	22	9.75 087	29	0.24 913	9.94 012	7	36
25	9.69 122	22	9.75 116	29	0.24 883	9.94 005	7	35
26	9.69 144	22	9.75 146	29	0.24 854	9.93 998	7	34
27	9.69 167	22	9.75 175	29	0.24 824	9.93 991	7	33
28	9.69 189	22	9.75 205	29	0.24 795	9.93 984	7	32
29	9.69 211	22	9.75 234	29	0.24 765	9.93 977	7	31
30	9.69 234	22	9.75 264	29	0.24 736	9.93 969	7	30
31	9.69 256	22	9.75 293	29	0.24 706	9.93 962	7	29
32	9.69 278	22	9.75 323	29	0.24 677	9.93 955	7	28
33	9.69 301	22	9.75 352	29	0.24 647	9.93 948	7	27
34	9.69 323	22	9.75 382	29	0.24 618	9.93 941	7	26
35	9.69 345	22	9.75 411	29	0.24 588	9.93 934	7	25
36	9.69 367	22	9.75 441	29	0.24 559	9.93 926	7	24
37	9.69 390	22	9.75 470	29	0.24 529	9.93 919	7	23
38	9.69 412	22	9.75 499	29	0.24 500	9.93 912	7	22
39	9.69 434	22	9.75 529	29	0.24 471	9.93 905	7	21
40	9.69 456	22	9.75 558	29	0.24 441	9.93 898	7	20
41	9.69 478	22	9.75 588	29	0.24 412	9.93 891	7	19
42	9.69 500	22	9.75 617	29	0.24 383	9.93 883	7	18
43	9.69 523	22	9.75 646	29	0.24 353	9.93 876	7	17
44	9.69 545	22	9.75 676	29	0.24 324	9.93 869	7	16
45	9.69 567	22	9.75 705	29	0.24 295	9.93 862	7	15
46	9.69 589	22	9.75 734	29	0.24 265	9.93 854	7	14
47	9.69 611	22	9.75 764	29	0.24 236	9.93 847	7	13
48	9.69 633	22	9.75 793	29	0.24 207	9.93 840	7	12
49	9.69 655	22	9.75 822	29	0.24 177	9.93 833	7	11
50	9.69 677	22	9.75 851	29	0.24 148	9.93 826	7	10
51	9.69 699	22	9.75 881	29	0.24 119	9.93 818	7	9
52	9.69 721	22	9.75 910	29	0.24 090	9.93 811	7	8
53	9.69 743	22	9.75 939	29	0.24 060	9.93 804	7	7
54	9.69 765	22	9.75 968	29	0.24 031	9.93 796	7	6
55	9.69 787	22	9.75 998	29	0.24 002	9.93 789	7	5
56	9.69 809	22	9.76 027	29	0.23 973	9.93 782	7	4
57	9.69 831	21	9.76 056	29	0.23 943	9.93 775	7	3
58	9.69 853	22	9.76 085	29	0.23 914	9.93 767	7	2
59	9.69 875	22	9.76 115	29	0.23 885	9.93 760	7	1
60	9.96 897		9.76 144		0.23 856	9.93 753		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	'

P. P.

	30	29	29
6	3.0	2.9	2.9
7	3.5	3.4	3.4
8	4.0	3.9	3.8
9	4.5	4.4	4.3
10	5.0	4.9	4.8
20	10.0	9.8	9.6
30	15.0	14.7	14.5
40	20.0	19.6	19.3
50	25.0	24.6	24.1

	23
6	2.3
7	2.7
8	3.0
9	3.4
10	3.8
20	7.6
30	11.5
40	15.3
50	19.1

	22	22	21
6	2.2	2.2	2.1
7	2.6	2.5	2.5
8	3.0	2.9	2.8
9	3.4	3.3	3.2
10	3.7	3.6	3.6
20	7.5	7.3	7.1
30	11.2	11.0	10.7
40	15.0	14.6	14.3
50	18.7	18.3	17.9

	7	7
6	0.7	0.7
7	0.9	0.8
8	1.0	0.9
9	1.1	1.0
10	1.2	1.1
20	2.5	2.3
30	3.7	3.5
40	5.0	4.6
50	6.2	5.8

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.69 897	22	9.76 144	29	0.23 856	9.93 753	7	60
1	9.69 919	21	9.76 173	29	0.23 827	9.93 746	7	59
2	9.69 940	22	9.76 202	29	0.23 797	9.93 738	7	58
3	9.69 962	22	9.76 231	29	0.23 768	9.93 731	7	57
4	9.69 984	21	9.76 260	29	0.23 739	9.93 724	7	56
5	9.70 006	22	9.76 289	29	0.23 710	9.93 716	7	55
6	9.70 028	22	9.76 319	29	0.23 681	9.93 709	7	54
7	9.70 050	21	9.76 348	29	0.23 652	9.93 702	7	53
8	9.70 071	22	9.76 377	29	0.23 623	9.93 694	7	52
9	9.70 093	21	9.76 406	29	0.23 594	9.93 687	7	51
10	9.70 115	22	9.76 435	29	0.23 565	9.93 680	7	50
11	9.70 137	21	9.76 464	29	0.23 535	9.93 672	7	49
12	9.70 158	21	9.76 493	29	0.23 506	9.93 665	7	48
13	9.70 180	22	9.76 522	29	0.23 477	9.93 658	7	47
14	9.70 202	21	9.76 551	29	0.23 448	9.93 650	7	46
15	9.70 223	21	9.76 580	29	0.23 419	9.93 643	7	45
16	9.70 245	22	9.76 609	29	0.23 390	9.93 635	7	44
17	9.70 267	21	9.76 638	29	0.23 361	9.93 628	7	43
18	9.70 288	21	9.76 667	29	0.23 332	9.93 621	7	42
19	9.70 310	21	9.76 696	29	0.23 303	9.93 613	7	41
20	9.70 331	22	9.76 725	29	0.23 274	9.93 606	7	40
21	9.70 353	21	9.76 754	29	0.23 245	9.93 599	7	39
22	9.70 375	21	9.76 783	29	0.23 216	9.93 591	7	38
23	9.70 396	21	9.76 812	29	0.23 187	9.93 584	7	37
24	9.70 418	21	9.76 841	29	0.23 158	9.93 576	7	36
25	9.70 439	21	9.76 870	28	0.23 129	9.93 569	7	35
26	9.70 461	21	9.76 899	29	0.23 101	9.93 562	7	34
27	9.70 482	21	9.76 928	29	0.23 072	9.93 554	7	33
28	9.70 504	21	9.76 957	29	0.23 043	9.93 547	7	32
29	9.70 525	21	9.76 986	29	0.23 014	9.93 539	7	31
30	9.70 547	21	9.77 015	28	0.22 985	9.93 532	7	30
31	9.70 568	21	9.77 043	29	0.22 956	9.93 524	7	29
32	9.70 590	21	9.77 072	29	0.22 927	9.93 517	7	28
33	9.70 611	21	9.77 101	29	0.22 898	9.93 509	7	27
34	9.70 632	21	9.77 130	28	0.22 869	9.93 502	7	26
35	9.70 654	21	9.77 159	29	0.22 841	9.93 495	7	25
36	9.70 675	21	9.77 188	29	0.22 812	9.93 487	7	24
37	9.70 696	21	9.77 217	28	0.22 783	9.93 480	7	23
38	9.70 718	21	9.77 245	29	0.22 754	9.93 272	7	22
39	9.70 739	21	9.77 274	29	0.22 725	9.93 465	7	21
40	9.70 760	21	9.77 303	28	0.22 696	9.93 457	7	20
41	9.70 782	21	9.77 332	29	0.22 668	9.93 450	7	19
42	9.70 803	21	9.77 361	28	0.22 639	9.93 442	7	18
43	9.70 824	21	9.77 389	29	0.22 610	9.93 435	7	17
44	9.70 846	21	9.77 418	28	0.22 581	9.93 427	7	16
45	9.70 867	21	9.77 447	29	0.22 553	9.93 420	7	15
46	9.70 888	21	9.77 476	28	0.22 524	9.93 412	7	14
47	9.70 909	21	9.77 504	29	0.22 495	9.93 405	7	13
48	9.70 930	21	9.77 533	28	0.22 466	9.93 397	7	12
49	9.70 952	21	9.77 562	29	0.22 438	9.93 390	8	11
50	9.70 973	21	9.77 591	28	0.22 409	9.93 382	7	10
51	9.70 994	21	9.77 619	28	0.22 380	9.93 374	7	9
52	9.71 015	21	9.77 648	29	0.22 352	9.93 367	7	8
53	9.71 036	21	9.77 677	28	0.22 323	9.93 359	7	7
54	9.71 057	21	9.77 705	28	0.22 294	9.93 352	7	6
55	9.71 078	21	9.77 734	29	0.22 266	9.93 344	7	5
56	9.71 099	21	9.77 763	28	0.22 237	9.93 337	7	4
57	9.71 121	21	9.77 791	28	0.22 208	9.93 329	8	3
58	9.71 142	21	9.77 820	29	0.22 180	9.93 321	7	2
59	9.71 163	21	9.77 849	28	0.22 151	9.93 314	7	1
60	9.71 184		9.77 877		0.22 122	9.93 306		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	29	29	28
6	2.9	2.9	2.8
7	3.4	3.4	3.3
8	3.9	3.8	3.8
9	4.4	4.3	4.3
10	4.9	4.8	4.7
20	9.8	9.6	9.5
30	14.7	14.5	14.2
40	19.6	19.3	19.0
50	24.6	24.1	23.7

	22	21	21
6	2.2	2.1	2.1
7	2.5	2.5	2.4
8	2.9	2.8	2.8
9	3.3	3.2	3.1
10	3.6	3.6	3.5
20	7.3	7.1	7.0
30	11.0	10.7	10.5
40	14.6	14.3	14.0
50	18.3	17.9	17.5

	8	7	7
6	0.8	0.7	0.7
7	0.9	0.9	0.8
8	1.0	1.0	0.9
9	1.2	1.1	1.0
10	1.3	1.2	1.1
20	2.6	2.5	2.3
30	4.0	3.7	3.5
40	5.3	5.0	4.6
50	6.6	6.2	5.8

P. P.

′	Log. Sin.	d.	Log. Tan.	c.d.	Log. Cot.	Log. Cos.	d.	
0	9.71 184	21	9.77 877	28	0.22 122	9.93 306	7	60
1	9.71 205	21	9.77 906	28	0.22 094	9.93 299	7	59
2	9.71 226	21	9.77 934	28	0.22 065	9.93 291	7	58
3	9.71 247	21	9.77 963	29	0.22 037	9.93 284	8	57
4	9.71 268	21	9.77 992	28	0.22 008	9.93 276	7	56
5	9.71 289	21	9.78 020	28	0.21 979	9.93 268	7	55
6	9.71 310	21	9.78 049	28	0.21 951	9.93 261	7	54
7	9.71 331	20	9.78 077	28	0.21 922	9.93 253	8	53
8	9.71 351	21	9.78 106	28	0.21 894	9.93 245	7	52
9	9.71 372	21	9.78 134	28	0.21 865	9.93 238	7	51
10	9.71 393	21	9.78 163	28	0.21 837	9.93 230	7	50
11	9.71 414	20	9.78 191	28	0.21 808	9.93 223	8	49
12	9.71 435	21	9.78 220	28	0.21 780	9.93 215	7	48
13	9.71 456	21	9.78 248	28	0.21 751	9.93 207	7	47
14	9.71 477	21	9.78 277	28	0.21 723	9.93 200	8	46
15	9.71 498	20	9.78 305	28	0.21 694	9.93 192	7	45
16	9.71 518	21	9.78 334	28	0.21 666	9.93 184	7	44
17	9.71 539	20	9.78 362	28	0.21 637	9.93 177	8	43
18	9.71 560	21	9.78 391	28	0.21 609	9.93 169	7	42
19	9.71 581	20	9.78 419	28	0.21 580	9.93 161	8	41
20	9.71 601	21	9.78 448	28	0.21 552	9.93 153	7	40
21	9.71 622	20	9.78 476	28	0.21 523	9.93 146	7	39
22	9.71 643	21	9.78 505	28	0.21 495	9.93 138	8	38
23	9.71 664	20	9.78 533	28	0.21 467	9.93 130	7	37
24	9.71 684	21	9.78 561	28	0.21 438	9.93 123	8	36
25	9.71 705	20	9.78 590	28	0.21 410	9.93 115	7	35
26	9.71 726	20	9.78 618	28	0.21 381	9.93 107	7	34
27	9.71 746	21	9.78 647	28	0.21 353	9.93 100	8	33
28	9.71 767	20	9.78 675	28	0.21 325	9.93 092	7	32
29	9.71 788	20	9.78 703	28	0.21 296	9.93 084	8	31
30	9.71 808	20	9.78 732	28	0.21 268	9.93 076	7	30
31	9.71 829	20	9.78 760	28	0.21 239	9.93 069	8	29
32	9.71 849	21	9.78 788	28	0.21 211	9.93 061	7	28
33	9.71 870	20	9.78 817	28	0.21 183	9.93 053	8	27
34	9.71 891	20	9.78 845	28	0.21 154	9.93 045	7	26
35	9.71 911	20	9.78 873	28	0.21 126	9.93 038	8	25
36	9.71 932	20	9.78 902	28	0.21 098	9.93 030	7	24
37	9.71 952	20	9.78 930	28	0.21 070	9.93 022	8	23
38	9.71 973	20	9.78 958	28	0.21 041	9.93 014	8	22
39	9.71 993	20	9.78 987	28	0.21 013	9.93 006	7	21
40	9.72 014	20	9.79 015	28	0.20 985	9.92 999	8	20
41	9.72 034	20	9.79 043	28	0.20 956	9.92 991	7	19
42	9.72 055	20	9.79 071	28	0.20 928	9.92 983	8	18
43	9.72 075	20	9.79 100	28	0.20 900	9.92 975	8	17
44	9.72 096	20	9.79 128	28	0.20 872	9.92 967	7	16
45	9.72 116	20	9.79 156	28	0.20 843	9.92 960	8	15
46	9.72 136	20	9.79 184	28	0.20 815	9.92 952	8	14
47	9.72 157	20	9.79 213	28	0.20 787	9.92 944	7	13
48	9.72 177	20	9.79 241	28	0.20 759	9.92 936	8	12
49	9.72 198	20	9.79 269	28	0.20 731	9.92 928	8	11
50	9.72 218	20	9.79 297	28	0.20 702	9.92 920	7	10
51	9.72 238	20	9.79 325	28	0.20 674	9.92 913	8	9
52	9.72 259	20	9.79 354	28	0.20 646	9.92 905	8	8
53	9.72 279	20	9.79 382	28	0.20 618	9.92 897	7	7
54	9.72 299	20	9.79 410	28	0.20 590	9.92 889	8	6
55	9.72 319	20	9.79 438	28	0.20 561	9.92 881	8	5
56	9.72 340	20	9.79 466	28	0.20 533	9.92 873	8	4
57	9.72 360	20	9.79 494	28	0.20 505	9.92 865	7	3
58	9.72 380	20	9.79 522	28	0.20 477	9.92 858	8	2
59	9.72 400	20	9.79 551	28	0.20 449	9.92 850	8	1
60	9.72 421		9.79 579		0.20 421	9.92 842		0
	Log. Cos.	d.	Log. Cot.	c.d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	29	$\overline{28}$	28
6	2.9	2.8	2.8
7	3.4	3.3	3.2
8	3.8	3.8	3.7
9	4.3	4.3	4.2
10	4.8	4.7	4.6
20	9.6	9.5	9.3
30	14.5	14.2	14.0
40	19.3	19.0	18.6
50	24.1	23.7	23.3

	21	$\overline{20}$	20
6	2.1	2.0	2.0
7	2.4	2.4	2.3
8	2.8	2.7	2.6
9	3.1	3.1	3.0
10	3.5	3.4	3.3
20	7.0	6.8	6.6
30	10.5	10.2	10.0
40	14.0	13.6	13.3
50	17.5	17.1	16.6

	8	$\overline{7}$
6	0.8	0.7
7	0.9	0.9
8	1.0	1.0
9	1.2	1.1
10	1.3	1.2
20	2.6	2.5
30	4.0	3.7
40	5.3	5.0
50	6.6	6.2

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.72 421	20	9.79 579	28	0.20 421	9.92 842	8	60
1	9.72 441	20	9.79 607	28	0.20 393	9.92 834	7	59
2	9.72 461	20	9.79 635	28	0.20 365	9.92 826	8	58
3	9.72 481	20	9.79 663	28	0.20 337	9.92 818	8	57
4	9.72 501	20	9.79 691	28	0.20 308	9.92 810	8	56
5	9.72 522	20	9.79 719	28	0.20 280	9.92 802	8	55
6	9.72 542	20	9.79 747	28	0.20 252	9.92 794	8	54
7	9.72 562	20	9.79 775	28	0.20 224	9.92 786	8	53
8	9.72 582	20	9.79 803	28	0.20 196	9.92 778	7	52
9	9.72 602	20	9.79 831	28	0.20 168	9.92 771	8	51
10	9.72 622	20	9.79 859	28	0.20 140	9.92 763	8	50
11	9.72 642	20	9.79 887	28	0.20 112	9.92 755	8	49
12	9.72 662	20	9.79 915	28	0.20 084	9.92 747	8	48
13	9.72 682	20	9.79 943	28	0.20 056	9.92 739	8	47
14	9.72 702	20	9.79 971	28	0.20 028	9.92 731	8	46
15	9.72 723	20	9.79 999	28	0.20 000	9.92 723	8	45
16	9.72 743	20	9.80 027	28	0.19 972	9.92 715	8	44
17	9.72 763	20	9.80 055	28	0.19 944	9.92 707	8	43
18	9.72 783	19	9.80 083	28	0.19 916	9.92 699	8	42
19	9.72 802	20	9.80 111	28	0.19 888	9.92 691	8	41
20	9.72 822	20	9.80 139	28	0.19 860	9.92 683	8	40
21	9.72 842	20	9.80 167	28	0.19 832	9.92 675	8	39
22	9.72 862	20	9.80 195	28	0.19 804	9.92 667	8	38
23	9.72 882	20	9.80 223	28	0.19 776	9.92 659	8	37
24	9.72 902	20	9.80 251	27	0.19 748	9.92 651	8	36
25	9.72 922	19	9.80 279	28	0.19 721	9.92 643	8	35
26	9.72 942	20	9.80 307	28	0.19 693	9.92 635	8	34
27	9.72 962	20	9.80 335	28	0.19 665	9.92 627	8	33
28	9.72 982	20	9.80 363	28	0.19 637	9.92 619	8	32
29	9.73 002	19	9.80 391	27	0.19 609	9.92 611	8	31
30	9.73 021	20	9.80 418	28	0.19 581	9.92 603	8	30
31	9.73 041	20	9.80 446	28	0.19 553	9.92 595	8	29
32	9.73 061	19	9.80 474	28	0.19 525	9.92 587	8	28
33	9.73 081	20	9.80 502	27	0.19 497	9.92 579	8	27
34	9.73 101	19	9.80 530	28	0.19 470	9.92 570	8	26
35	9.73 120	20	9.80 558	28	0.19 442	9.92 562	8	25
36	9.73 140	19	9.80 586	27	0.19 414	9.92 554	8	24
37	9.73 160	20	9.80 613	28	0.19 386	9.92 546	8	23
38	9.73 180	19	9.80 641	28	0.19 358	9.92 538	8	22
39	9.73 199	20	9.80 669	27	0.19 330	9.92 530	8	21
40	9.73 219	19	9.80 697	28	0.19 303	9.92 522	8	20
41	9.73 239	19	9.80 725	27	0.19 275	9.92 514	8	19
42	9.73 258	20	9.80 752	28	0.19 247	9.92 506	8	18
43	9.73 278	19	9.80 780	28	0.19 219	9.92 498	8	17
44	9.73 298	19	9.80 808	27	0.19 191	9.92 489	8	16
45	9.73 317	20	9.80 836	28	0.19 164	9.92 481	8	15
46	9.73 337	19	9.80 864	27	0.19 136	9.92 473	8	14
47	9.73 357	19	9.80 891	28	0.19 108	9.92 465	8	13
48	9.73 376	19	9.80 919	27	0.19 080	9.92 457	8	12
49	9.73 396	19	9.80 947	28	0.19 053	9.92 449	8	11
50	9.73 415	20	9.80 975	27	0.19 025	9.92 441	8	10
51	9.73 435	19	9.81 002	27	0.18 997	9.92 433	8	9
52	9.73 455	19	9.81 030	28	0.18 970	9.92 424	8	8
53	9.73 474	19	9.81 058	27	0.18 942	9.92 416	8	7
54	9.73 494	19	9.81 085	28	0.18 914	9.92 408	8	6
55	9.73 513	19	9.81 113	27	0.18 886	9.92 400	8	5
56	9.73 533	19	9.81 141	27	0.18 859	9.92 392	8	4
57	9.73 552	19	9.81 168	28	0.18 831	9.92 383	8	3
58	9.73 572	19	9.81 196	27	0.18 803	9.92 375	8	2
59	9.73 591	19	9.81 224	27	0.18 776	9.92 367	8	1
60	9.73 611		9.81 251		0.18 748	9.92 359		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	28	28	27
6	2.8	2.8	2.7
7	3.3	3.2	3.2
8	3.8	3.7	3.6
9	4.3	4.2	4.1
10	4.7	4.6	4.6
20	9.5	9.3	9.1
30	14.2	14.0	13.7
40	19.0	18.6	18.3
50	23.7	23.3	22.9

	20	20	19
6	2.0	2.0	1.9
7	2.4	2.3	2.3
8	2.7	2.6	2.6
9	3.1	3.0	2.9
10	3.4	3.3	3.2
20	6.8	6.6	6.5
30	10.2	10.0	9.7
40	13.6	13.3	13.0
50	17.1	16.6	16.2

	8	8	7
6	0.8	0.8	0.7
7	1.0	0.9	0.9
8	1.1	1.0	1.0
9	1.3	1.2	1.1
10	1.4	1.3	1.2
20	2.8	2.6	2.5
30	4.2	4.0	3.7
40	5.6	5.3	5.0
50	7.1	6.6	6.2

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.73 611	19	9.81 251	28	0.18 748	9.92 359	8	60
1	9.73 630	19	9.81 279	27	0.18 720	9.92 351	8	59
2	9.73 650	19	9.81 307	27	0.18 693	9.92 342	8	58
3	9.73 669	19	9.81 334	28	0.18 665	9.92 334	8	57
4	9.73 688	19	9.81 362	27	0.18 637	9.92 326	8	56
5	9.73 708	19	9.81 390	27	0.18 610	9.92 318	8	55
6	9.73 727	19	9.81 417	27	0.18 582	9.92 310	8	54
7	9.73 746	19	9.81 445	28	0.18 555	9.92 301	8	53
8	9.73 766	19	9.81 473	27	0.18 527	9.92 293	8	52
9	9.73 785	19	9.81 500	27	0.18 499	9.92 285	8	51
10	9.73 805	19	9.81 528	27	0.18 472	9.92 277	8	50
11	9.73 824	19	9.81 555	27	0.18 444	9.92 268	8	49
12	9.73 843	19	9.81 583	27	0.18 417	9.92 260	8	48
13	9.73 862	19	9.81 610	27	0.18 389	9.92 252	8	47
14	9.73 882	19	9.81 638	28	0.18 362	9.92 244	8	46
15	9.73 901	19	9.81 666	27	0.18 334	9.92 235	8	45
16	9.73 920	19	9.81 693	27	0.18 306	9.92 227	8	44
17	9.73 940	19	9.81 721	27	0.18 279	9.92 219	8	43
18	9.73 959	19	9.81 748	27	0.18 251	9.92 210	8	42
19	9.73 978	19	9.81 776	27	0.18 224	9.92 202	8	41
20	9.73 997	19	9.81 803	27	0.18 196	9.92 194	8	40
21	9.74 016	19	9.81 831	27	0.18 169	9.92 185	8	39
22	9.74 036	19	9.81 858	27	0.18 141	9.92 177	8	38
23	9.74 055	19	9.81 886	27	0.18 114	9.92 169	8	37
24	9.74 074	19	9.81 913	27	0.18 086	9.92 160	8	36
25	9.74 093	19	9.81 941	27	0.18 059	9.92 152	8	35
26	9.74 112	19	9.81 968	27	0.18 031	9.92 144	8	34
27	9.74 131	19	9.81 996	27	0.18 004	9.92 135	8	33
28	9.74 151	19	9.82 023	27	0.17 976	9.92 127	8	32
29	9.74 170	19	9.82 051	27	0.17 949	9.92 119	8	31
30	9.74 189	19	9.82 078	27	0.17 921	9.92 110	8	30
31	9.74 208	19	9.82 105	27	0.17 894	9.92 102	8	29
32	9.74 227	19	9.82 133	27	0.17 867	9.92 094	8	28
33	9.74 246	19	9.82 160	27	0.17 839	9.92 085	8	27
34	9.74 265	19	9.82 188	27	0.17 812	9.92 077	8	26
35	9.74 284	19	9.82 215	27	0.17 784	9.92 069	8	25
36	9.74 303	19	9.82 243	27	0.17 757	9.92 060	8	24
37	9.74 322	19	9.82 270	27	0.17 729	9.92 052	8	23
38	9.74 341	18	9.82 297	27	0.17 702	9.92 043	8	22
39	9.74 360	19	9.82 325	27	0.17 675	9.92 035	8	21
40	9.74 379	19	9.82 352	27	0.17 647	9.92 027	8	20
41	9.74 398	19	9.82 380	27	0.17 620	9.92 018	8	19
42	9.74 417	19	9.82 407	27	0.17 593	9.92 010	8	18
43	9.74 436	19	9.82 434	27	0.17 565	9.92 001	8	17
44	9.74 455	19	9.82 462	27	0.17 538	9.91 993	8	16
45	9.74 474	19	9.82 489	27	0.17 510	9.91 984	8	15
46	9.74 493	18	9.82 516	27	0.17 483	9.91 976	8	14
47	9.74 511	19	9.82 544	27	0.17 456	9.91 967	8	13
48	9.74 530	19	9.82 571	27	0.17 428	9.91 959	8	12
49	9.74 549	19	9.82 598	27	0.17 401	9.91 951	8	11
50	9.74 568	18	9.82 626	27	0.17 374	9.91 942	8	10
51	9.74 587	19	9.82 653	27	0.17 347	9.91 934	8	9
52	9.74 606	19	9.82 680	27	0.17 319	9.91 925	8	8
53	9.74 625	18	9.82 708	27	0.17 292	9.91 917	8	7
54	9.74 643	19	9.82 735	27	0.17 265	9.91 908	8	6
55	9.74 662	18	9.82 762	27	0.17 237	9.91 900	8	5
56	9.74 681	19	9.82 789	27	0.17 210	9.91 891	8	4
57	9.74 700	18	9.82 817	27	0.17 183	9.91 883	8	3
58	9.74 718	19	9.82 844	27	0.17 156	9.91 874	8	2
59	9.74 737	18	9.82 871	27	0.17 128	9.91 866	8	1
60	9.74 756		9.82 898		0.17 101	9.91 857		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	28	27	27
6	2.8	2.7	2.7
7	3.2	3.2	3.1
8	3.7	3.6	3.6
9	4.2	4.1	4.0
10	4.6	4.6	4.5
20	9.3	9.1	9.0
30	14.0	13.7	13.5
40	18.6	18.3	18.0
50	23.3	22.9	22.5

	19	19	18
6	1.9	1.9	1.8
7	2.3	2.2	2.1
8	2.6	2.5	2.4
9	2.9	2.8	2.8
10	3.2	3.1	3.1
20	6.5	6.3	6.1
30	9.7	9.5	9.2
40	13.0	12.6	12.3
50	16.2	15.8	15.4

	8	8
6	0.8	0.8
7	1.0	0.9
8	1.1	1.0
9	1.3	1.2
10	1.4	1.3
20	2.8	2.6
30	4.2	4.0
40	5.6	5.3
50	7.1	6.6

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.74 756	19	9.82 898	27	0.17 101	9.91 857	8	60
1	9.74 775	18	9.82 926	27	0.17 074	9.91 849	8	59
2	9.74 793	19	9.82 953	27	0.17 047	9.91 840	8	58
3	9.74 812	18	9.82 980	27	0.17 019	9.91 832	8	57
4	9.74 831	18	9.83 007	27	0.16 992	9.91 823	9	56
5	9.74 849	19	9.83 035	27	0.16 965	9.91 814	8	55
6	9.74 868	18	9.83 062	27	0.16 938	9.91 806	8	54
7	9.74 887	18	9.83 089	27	0.16 910	9.91 797	8	53
8	9.74 905	19	9.83 116	27	0.16 883	9.91 789	8	52
9	9.74 924	18	9.83 143	27	0.16 856	9.91 780	8	51
10	9.74 943	18	9.83 171	27	0.16 829	9.91 772	8	50
11	9.74 961	18	9.83 198	27	0.16 802	9.91 763	8	49
12	9.74 980	18	9.83 225	27	0.16 774	9.91 755	9	48
13	9.74 998	18	9.83 252	27	0.16 747	9.91 746	8	47
14	9.75 017	19	9.83 279	27	0.16 720	9.91 737	8	46
15	9.75 036	18	9.83 307	27	0.16 693	9.91 729	8	45
16	9.75 054	18	9.83 334	27	0.16 666	9.91 720	8	44
17	9.75 073	18	9.83 361	27	0.16 639	9.91 712	9	43
18	9.75 091	18	9.83 388	27	0.16 612	9.91 703	8	42
19	9.75 110	18	9.83 415	27	0.16 584	9.91 694	8	41
20	9.75 128	18	9.83 442	27	0.16 557	9.91 686	8	40
21	9.75 147	18	9.83 469	27	0.16 530	9.91 677	9	39
22	9.75 165	18	9.83 496	27	0.16 503	9.91 668	8	38
23	9.75 184	18	9.83 524	27	0.16 476	9.91 660	8	37
24	9.75 202	18	9.83 551	27	0.16 449	9.91 651	9	36
25	9.75 221	18	9.83 578	27	0.16 422	9.91 642	8	35
26	9.75 239	18	9.83 605	27	0.16 395	9.91 634	8	34
27	9.75 257	18	9.83 632	27	0.16 368	9.91 625	9	33
28	9.75 276	18	9.83 659	27	0.16 340	9.91 616	8	32
29	9.75 294	18	9.83 686	27	0.16 313	9.91 608	8	31
30	9.75 313	18	9.83 713	27	0.16 286	9.91 599	9	30
31	9.75 331	18	9.83 740	27	0.16 259	9.91 590	8	29
32	9.75 349	18	9.83 767	27	0.16 232	9.91 582	8	28
33	9.75 368	18	9.83 794	27	0.16 205	9.91 573	9	27
34	9.75 386	18	9.83 821	27	0.16 178	9.91 564	8	26
35	9.75 404	18	9.83 848	27	0.16 151	9.91 556	9	25
36	9.75 423	18	9.83 875	27	0.16 124	9.91 547	8	24
37	9.75 441	18	9.83 902	27	0.16 097	9.91 538	9	23
38	9.75 459	18	9.83 929	27	0.16 070	9.91 529	8	22
39	9.75 478	18	9.83 957	27	0.16 043	9.91 521	8	21
40	9.75 496	18	9.83 984	27	0.16 016	9.91 512	9	20
41	9.75 514	18	9.84 011	27	0.15 989	9.91 503	8	19
42	9.75 532	18	9.84 038	27	0.15 962	9.91 495	9	18
43	9.75 551	18	9.84 065	26	0.15 935	9.91 486	8	17
44	9.75 569	18	9.84 091	27	0.15 908	9.91 477	9	16
45	9.75 587	18	9.84 118	27	0.15 881	9.91 468	8	15
46	9.75 605	18	9.84 145	27	0.15 854	9.91 460	9	14
47	9.75 623	18	9.84 172	27	0.15 827	9.91 451	9	13
48	9.75 642	18	9.84 199	27	0.15 800	9.91 442	8	12
49	9.75 660	18	9.84 226	27	0.15 773	9.91 433	9	11
50	9.75 678	18	9.84 253	27	0.15 746	9.91 424	8	10
51	9.75 696	18	9.84 280	27	0.15 719	9.91 416	9	9
52	9.75 714	18	9.84 307	27	0.15 692	9.91 407	9	8
53	9.75 732	18	9.84 334	26	0.15 665	9.91 398	8	7
54	9.75 750	18	9.84 361	27	0.15 639	9.91 389	9	6
55	9.75 769	18	9.84 388	27	0.15 612	9.91 380	8	5
56	9.75 787	18	9.84 415	27	0.15 585	9.91 372	9	4
57	9.75 805	18	9.84 442	27	0.15 558	9.91 363	9	3
58	9.75 823	18	9.84 469	27	0.15 531	9.91 354	8	2
59	9.75 841	18	9.84 496	26	0.15 504	9.91 345	9	1
60	9.75 859		9.84 522		0.15 477	9.91 336		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	27	27	26
6	2.7	2.7	2.6
7	3.2	3.1	3.1
8	3.6	3.6	3.5
9	4.1	4.0	4.0
10	4.6	4.5	4.4
20	9.1	9.0	8.8
30	13.7	13.5	13.2
40	18.3	18.0	17.6
50	22.9	22.5	22.1

	19	18	18
6	1.9	1.8	1.8
7	2.2	2.1	2.1
8	2.5	2.4	2.4
9	2.8	2.8	2.7
10	3.1	3.1	3.0
20	6.3	6.1	6.0
30	9.5	9.2	9.0
40	12.6	12.3	12.0
50	15.8	15.4	15.0

	9	8
6	0.9	0.8
7	1.0	1.0
8	1.2	1.1
9	1.3	1.3
10	1.5	1.4
20	3.0	2.8
30	4.5	4.2
40	6.0	5.6
50	7.5	7.1

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.75 859	18	9.84 522	27	0.15 477	9.91 336	9	60
1	9.75 877	18	9.84 549	27	0.15 450	9.91 327	9	59
2	9.75 895	18	9.84 576	27	0.15 423	9.91 318	8	58
3	9.75 913	18	9.84 603	26	0.15 396	9.91 310	9	57
4	9.75 931	18	9.84 630	27	0.15 370	9.91 301	9	56
5	9.75 949	18	9.84 657	27	0.15 343	9.91 292	8	55
6	9.75 967	18	9.84 684	27	0.15 316	9.91 283	9	54
7	9.75 985	18	9.84 711	26	0.15 289	9.91 274	9	53
8	9.76 003	18	9.84 737	27	0.15 262	9.91 265	9	52
9	9.76 021	18	9.84 764	27	0.15 235	9.91 256	9	51
10	9.76 039	18	9.84 791	26	0.15 208	9.91 247	8	50
11	9.76 057	18	9.84 818	27	0.15 182	9.91 239	9	49
12	9.76 075	17	9.84 845	26	0.15 155	9.91 230	9	48
13	9.76 092	18	9.84 871	27	0.15 128	9.91 221	9	47
14	9.76 110	18	9.84 898	27	0.15 101	9.91 212	9	46
15	9.76 128	18	9.84 925	26	0.15 074	9.91 203	9	45
16	9.76 146	17	9.84 952	27	0.15 048	9.91 194	8	44
17	9.76 164	18	9.84 979	26	0.15 021	9.91 185	9	43
18	9.76 182	18	9.85 005	27	0.14 994	9.91 176	9	42
19	9.76 200	17	9.85 032	27	0.14 967	9.91 167	9	41
20	9.76 217	18	9.85 059	26	0.14 940	9.91 158	9	40
21	9.76 235	18	9.85 086	27	0.14 914	9.91 149	9	39
22	9.76 253	17	9.85 113	26	0.14 887	9.91 140	9	38
23	9.76 271	18	9.85 139	27	0.14 860	9.91 131	9	37
24	9.76 289	17	9.85 166	26	0.14 833	9.91 122	9	36
25	9.76 306	18	9.85 193	27	0.14 807	9.91 113	9	35
26	9.76 324	17	9.85 220	26	0.14 780	9.91 104	9	34
27	9.76 342	18	9.85 246	27	0.14 753	9.91 095	9	33
28	9.76 360	17	9.85 273	26	0.14 726	9.91 086	9	32
29	9.76 377	18	9.85 300	27	0.14 700	9.91 077	9	31
30	9.76 395	17	9.85 327	26	0.14 673	9.91 068	9	30
31	9.76 413	18	9.85 353	26	0.14 646	9.91 059	9	29
32	9.76 431	17	9.85 380	27	0.14 620	9.91 050	9	28
33	9.76 448	17	9.85 407	26	0.14 593	9.91 041	9	27
34	9.76 466	18	9.85 433	27	0.14 566	9.91 032	9	26
35	9.76 484	17	9.85 460	26	0.14 539	9.91 023	9	25
36	9.76 501	17	9.85 487	26	0.14 513	9.91 014	9	24
37	9.76 519	17	9.85 513	27	0.14 486	9.91 005	9	23
38	9.76 536	18	9.85 540	26	0.14 459	9.90 996	9	22
39	9.76 554	17	9.85 567	27	0.14 433	9.90 987	9	21
40	9.76 572	17	9.85 594	26	0.14 406	9.90 978	9	20
41	9.76 589	17	9.85 620	26	0.14 379	9.90 969	9	19
42	9.76 607	17	9.85 647	26	0.14 353	9.90 960	9	18
43	9.76 624	18	9.85 673	27	0.14 326	9.90 951	9	17
44	9.76 642	17	9.85 700	26	0.14 299	9.90 942	9	16
45	9.76 660	17	9.85 727	26	0.14 273	9.90 933	9	15
46	9.76 677	17	9.85 753	27	0.14 246	9.90 923	9	14
47	9.76 695	17	9.85 780	26	0.14 219	9.90 914	9	13
48	9.76 712	17	9.85 807	26	0.14 193	9.90 905	9	12
49	9.76 730	17	9.85 833	26	0.14 166	9.90 896	9	11
50	9.76 747	17	9.85 860	27	0.14 140	9.90 887	9	10
51	9.76 765	17	9.85 887	26	0.14 113	9.90 878	9	9
52	9.76 782	17	9.85 913	26	0.14 086	9.90 869	9	8
53	9.76 800	17	9.85 940	26	0.14 060	9.90 860	9	7
54	9.76 817	17	9.85 966	26	0.14 033	9.90 850	9	6
55	9.76 835	17	9.85 993	27	0.14 007	9.90 841	9	5
56	9.76 852	17	9.86 020	26	0.13 980	9.90 832	9	4
57	9.76 869	17	9.86 046	26	0.13 953	9.90 823	9	3
58	9.76 887	17	9.86 073	26	0.13 927	9.90 814	9	2
59	9.76 904	17	9.86 099	26	0.13 900	9.90 805	9	1
60	9.76 922		9.86 126		0.13 874	9.90 796		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	27	26
6	2.7	2.6
7	3.1	3.1
8	3.6	3.5
9	4.0	4.0
10	4.5	4.4
20	9.0	8.8
30	13.5	13.2
40	18.0	17.6
50	22.5	22.1

	18	17	17
6	1.8	1.7	1.7
7	2.1	2.0	2.0
8	2.4	2.3	2.2
9	2.7	2.6	2.5
10	3.0	2.9	2.8
20	6.0	5.8	5.6
30	9.0	8.7	8.5
40	12.0	11.6	11.3
50	15.0	14.6	14.1

	9	9	8
6	0.9	0.9	0.8
7	1.1	1.0	1.0
8	1.2	1.2	1.1
9	1.4	1.3	1.3
10	1.6	1.5	1.4
20	3.1	3.0	2.8
30	4.7	4.5	4.2
40	6.3	6.0	5.6
50	7.9	7.5	7.1

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.76 922	17	9.86 126	26	0.13 874	9.90 796	9	60
1	9.76 939	17	9.86 152	26	0.13 847	9.90 786	9	59
2	9.76 956	17	9.86 179	27	0.13 821	9.90 777	9	58
3	9.76 974	17	9.86 206	26	0.13 794	9.90 768	9	57
4	9.76 991	17	9.86 232	26	0.13 767	9.90 759	9	56
5	9.77 008	17	9.86 259	26	0.13 741	9.90 750	9	55
6	9.77 026	17	9.86 285	26	0.13 714	9.90 740	9	54
7	9.77 043	17	9.86 312	26	0.13 688	9.90 731	9	53
8	9.77 060	17	9.86 338	26	0.13 661	9.90 722	9	52
9	9.77 078	17	9.86 365	26	0.13 635	9.90 713	9	51
10	9.77 095	17	9.86 391	26	0.13 608	9.90 703	9	50
11	9.77 112	17	9.86 418	26	0.13 582	9.90 694	9	49
12	9.77 130	17	9.86 444	26	0.13 555	9.90 685	9	48
13	9.77 147	17	9.86 471	26	0.13 529	9.90 676	9	47
14	9.77 164	17	9.86 497	26	0.13 502	9.90 666	9	46
15	9.77 181	17	9.86 524	26	0.13 476	9.90 657	9	45
16	9.77 198	17	9.86 550	26	0.13 449	9.90 648	9	44
17	9.77 216	17	9.86 577	26	0.13 423	9.90 639	9	43
18	9.77 233	17	9.86 603	26	0.13 396	9.90 629	9	42
19	9.77 250	17	9.86 630	26	0.13 370	9.90 620	9	41
20	9.77 267	17	9.86 656	26	0.13 343	9.90 611	9	40
21	9.77 284	17	9.86 683	26	0.13 317	9.90 602	9	39
22	9.77 302	17	9.86 709	26	0.13 290	9.90 592	9	38
23	9.77 319	17	9.86 736	26	0.13 264	9.90 583	9	37
24	9.77 336	17	9.86 762	26	0.13 237	9.90 574	9	36
25	9.77 353	17	9.86 788	26	0.13 211	9.90 564	9	35
26	9.77 370	17	9.86 815	26	0.13 185	9.90 555	9	34
27	9.77 387	17	9.86 841	26	0.13 158	9.90 546	9	33
28	9.77 404	17	9.86 868	26	0.13 132	9.90 536	9	32
29	9.77 421	17	9.86 894	26	0.13 105	9.90 527	9	31
30	9.77 439	17	9.86 921	26	0.13 079	9.90 518	9	30
31	9.77 456	17	9.86 947	26	0.13 052	9.90 508	9	29
32	9.77 473	17	9.86 973	26	0.13 026	9.90 499	9	28
33	9.77 490	17	9.87 000	26	0.13 000	9.90 490	9	27
34	9.77 507	17	9.87 026	26	0.12 973	9.90 480	9	26
35	9.77 524	17	9.87 053	26	0.12 947	9.90 471	9	25
36	9.77 541	17	9.87 079	26	0.12 920	9.90 461	9	24
37	9.77 558	17	9.87 105	26	0.12 894	9.90 452	9	23
38	9.77 575	17	9.87 132	26	0.12 868	9.90 443	9	22
39	9.77 592	17	9.87 158	26	0.12 841	9.90 433	9	21
40	9.77 609	17	9.87 185	26	0.12 815	9.90 424	9	20
41	9.77 626	17	9.87 211	26	0.12 789	9.90 414	9	19
42	9.77 643	17	9.87 237	26	0.12 762	9.90 405	9	18
43	9.77 660	17	9.87 264	26	0.12 736	9.90 396	9	17
44	9.77 677	16	9.87 290	26	0.12 709	9.90 386	9	16
45	9.77 693	17	9.87 316	26	0.12 683	9.90 377	9	15
46	9.77 710	17	9.87 343	26	0.12 657	9.90 367	9	14
47	9.77 727	17	9.87 369	26	0.12 630	9.90 358	9	13
48	9.77 744	17	9.87 395	26	0.12 604	9.90 348	9	12
49	9.77 761	16	9.87 422	26	0.12 578	9.90 339	9	11
50	9.77 778	17	9.87 448	26	0.12 551	9.90 330	9	10
51	9.77 795	17	9.87 474	26	0.12 525	9.90 320	9	9
52	9.77 812	16	9.87 501	26	0.12 499	9.90 311	9	8
53	9.77 828	17	9.87 527	26	0.12 472	9.90 301	9	7
54	9.77 845	17	9.87 553	26	0.12 446	9.90 292	9	6
55	9.77 862	16	9.87 580	26	0.12 420	9.90 282	9	5
56	9.77 879	17	9.87 606	26	0.12 393	9.90 273	9	4
57	9.77 896	17	9.87 632	26	0.12 367	9.90 263	9	3
58	9.77 913	16	9.87 659	26	0.12 341	9.90 254	9	2
59	9.77 929	17	9.87 685	26	0.12 315	9.90 244	9	1
60	9.77 946		9.87 711		0.12 288	9.90 235		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	27	26̄	26
6	2.7	2.6	2.6
7	3.1	3.1	3.0
8	3.6	3.5	3.4
9	4.0	4.0	3.9
10	4.5	4.4	4.3
20	9.0	8.8	8.6
30	13.5	13.2	13.0
40	18.0	17.6	17.3
50	22.5	22.1	21.6

	17̄	17	16̄
6	1.7	1.7	1.6
7	2.0	2.0	1.9
8	2.3	2.2	2.2
9	2.6	2.5	2.5
10	2.9	2.8	2.7
20	5.8	5.6	5.5
30	8.7	8.5	8.2
40	11.6	11.3	11.0
50	14.6	14.1	13.7

	9̄	9
6	0.9	0.9
7	1.1	1.0
8	1.2	1.2
9	1.4	1.3
10	1.6	1.5
20	3.1	3.0
30	4.7	4.5
40	6.3	6.0
50	7.9	7.5

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.77 946	16	9.87 711	26	0.12 288	9.90 235	9	60
1	9.77 963	17	9.87 737	26	0.12 262	9.90 225	9	59
2	9.77 980	16	9.87 764	26	0.12 236	9.90 216	9	58
3	9.77 996	17	9.87 790	26	0.12 209	9.90 206	10	57
4	9.78 013	16	9.87 816	26	0.12 183	9.90 196	9	56
5	9.78 030	16	9.87 843	26	0.12 157	9.90 187	9	55
6	9.78 046	17	9.87 869	26	0.12 131	9.90 177	9	54
7	9.78 063	16	9.87 895	26	0.12 104	9.90 168	9	53
8	9.78 080	17	9.87 921	26	0.12 078	9.90 158	9	52
9	9.78 097	16	9.87 948	26	0.12 052	9.90 149	9	51
10	9.78 113	16	9.87 974	26	0.12 026	9.90 139	9	50
11	9.78 130	17	9.88 000	26	0.11 999	9.90 130	10	49
12	9.78 147	16	9.88 026	26	0.11 973	9.90 120	9	48
13	9.78 163	16	9.88 053	26	0.11 947	9.90 110	9	47
14	9.78 180	16	9.88 079	26	0.11 921	9.90 101	9	46
15	9.78 196	16	9.88 105	26	0.11 895	9.90 091	9	45
16	9.78 213	17	9.88 131	26	0.11 868	9.90 082	10	44
17	9.78 230	16	9.88 157	26	0.11 842	9.90 072	9	43
18	9.78 246	16	9.88 184	26	0.11 816	9.90 062	9	42
19	9.78 263	16	9.88 210	26	0.11 790	9.90 053	9	41
20	9.78 279	16	9.88 236	26	0.11 763	9.90 043	10	40
21	9.78 296	16	9.88 262	26	0.11 737	9.90 033	9	39
22	9.78 312	16	9.88 288	26	0.11 711	9.90 024	9	38
23	9.78 329	17	9.88 315	26	0.11 685	9.90 014	10	37
24	9.78 346	16	9.88 341	26	0.11 659	9.90 004	9	36
25	9.78 362	16	9.88 367	26	0.11 633	9.89 995	9	35
26	9.78 379	16	9.88 393	26	0.11 606	9.89 985	10	34
27	9.78 395	16	9.88 419	26	0.11 580	9.89 975	9	33
28	9.78 412	16	9.88 445	26	0.11 554	9.89 966	9	32
29	9.78 428	16	9.88 472	26	0.11 528	9.89 956	10	31
30	9.78 444	16	9.88 498	26	0.11 502	9.89 946	9	30
31	9.78 461	16	9.88 524	26	0.11 476	9.89 937	9	29
32	9.78 477	16	9.88 550	26	0.11 449	9.89 927	10	28
33	9.78 494	16	9.88 576	26	0.11 423	9.89 917	9	27
34	9.78 510	16	9.88 602	26	0.11 397	9.89 908	10	26
35	9.78 527	16	9.88 629	26	0.11 371	9.89 898	9	25
36	9.78 543	16	9.88 655	26	0.11 345	9.89 888	10	24
37	9.78 559	16	9.88 681	26	0.11 319	9.89 878	9	23
38	9.78 576	16	9.88 707	26	0.11 293	9.89 869	10	22
39	9.78 592	16	9.88 733	26	0.11 266	9.89 859	9	21
40	9.78 609	16	9.88 759	26	0.11 240	9.89 849	10	20
41	9.78 625	16	9.88 785	26	0.11 214	9.89 839	9	19
42	9.78 641	16	9.88 811	26	0.11 188	9.89 830	10	18
43	9.78 658	16	9.88 838	26	0.11 162	9.89 820	9	17
44	9.78 674	16	9.88 864	26	0.11 136	9.89 810	10	16
45	9.78 690	16	9.88 890	26	0.11 110	9.89 800	9	15
46	9.78 707	16	9.88 916	26	0.11 084	9.89 791	10	14
47	9.78 723	16	9.88 942	26	0.11 058	9.89 781	10	13
48	9.78 739	16	9.88 968	26	0.11 032	9.89 771	9	12
49	9.78 755	16	9.88 994	26	0.11 005	9.89 761	10	11
50	9.78 772	16	9.89 020	26	0.10 979	9.89 751	9	10
51	9.78 788	16	9.89 046	26	0.10 953	9.89 742	10	9
52	9.78 804	16	9.89 072	26	0.10 927	9.89 732	10	8
53	9.78 821	16	9.89 098	26	0.10 901	9.89 722	9	7
54	9.78 837	16	9.89 124	26	0.10 875	9.89 712	10	6
55	9.78 853	16	9.89 150	26	0.10 849	9.89 702	10	5
56	9.78 869	16	9.89 177	26	0.10 823	9.89 692	9	4
57	9.78 885	16	9.89 203	26	0.10 797	9.89 683	10	3
58	9.78 902	16	9.89 229	26	0.10 771	9.89 673	10	2
59	9.78 918	16	9.89 255	26	0.10 745	9.89 663	10	1
60	9.78 934		9.89 281		0.10 719	9.89 653		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	26	26
6	2.6	2.6
7	3.1	3.0
8	3.5	3.4
9	4.0	3.9
10	4.4	4.3
20	8.8	8.6
30	13.2	13.0
40	17.6	17.3
50	22.1	21.6

	17	16	16
6	1.7	1.6	1.6
7	2.0	1.9	1.8
8	2.2	2.2	2.1
9	2.5	2.5	2.4
10	2.8	2.7	2.6
20	5.6	5.5	5.3
30	8.5	8.2	8.0
40	11.3	11.0	10.6
50	14.1	13.7	13.3

	10	9
6	1.0	0.9
7	1.1	1.1
8	1.3	1.2
9	1.5	1.4
10	1.6	1.6
20	3.3	3.1
30	5.0	4.7
40	6.6	6.3
50	8.3	7.9

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.78 934	16	9.89 281	26	0.10 719	9.89 653	9	60
1	9.78 950	16	9.89 307	26	0.10 693	9.89 643	10	59
2	9.78 966	16	9.89 333	26	0.10 667	9.89 633	10	58
3	9.78 982	16	9.89 359	26	0.10 641	9.89 623	10	57
4	9.78 999	16	9.89 385	26	0.10 615	9.89 613	9	56
5	9.79 015	16	9.89 411	26	0.10 589	9.89 604	10	55
6	9.79 031	16	9.89 437	26	0.10 563	9.89 594	10	54
7	9.79 047	16	9.89 463	26	0.10 537	9.89 584	10	53
8	9.79 063	16	9.89 489	26	0.10 511	9.89 574	10	52
9	9.79 079	16	9.89 515	26	0.10 485	9.89 564	10	51
10	9.79 095	16	9.89 541	26	0.10 459	9.89 554	9	50
11	9.79 111	16	9.89 567	26	0.10 433	9.89 544	10	49
12	9.79 127	16	9.89 593	26	0.10 407	9.89 534	10	48
13	9.79 143	16	9.89 619	26	0.10 381	9.89 524	10	47
14	9.79 159	16	9.89 645	26	0.10 355	9.89 514	10	46
15	9.79 175	16	9.89 671	26	0.10 329	9.89 504	10	45
16	9.79 191	16	9.89 697	26	0.10 303	9.69 494	10	44
17	9.79 207	16	9.89 723	26	0.10 277	9.89 484	10	43
18	9.79 223	16	9.89 749	26	0.10 251	9.89 474	10	42
19	9.79 239	16	9.89 775	26	0.10 225	9.89 464	10	41
20	9.79 255	16	9.89 801	26	0.10 199	9.89 454	10	40
21	9.79 271	16	9.89 827	26	0.10 173	9.89 444	10	39
22	9.79 287	16	9.89 853	26	0.10 147	9.89 434	10	38
23	9.79 303	16	9.89 879	26	0.10 121	9.89 424	10	37
24	9.79 319	16	9.89 905	26	0.10 095	9.89 414	10	36
25	9.79 335	16	9.89 931	26	0.10 069	9.89 404	10	35
26	9.79 351	16	9.89 957	25	0.10 043	9.89 394	10	34
27	9.79 367	15	9.89 982	26	0.10 017	9.89 384	10	33
28	9.79 383	16	9.90 008	26	0.09 991	9.89 374	10	32
29	9.79 399	16	9.90 034	26	0.09 965	9.89 364	10	31
30	9.79 415	16	9.90 060	26	0.09 939	9.89 354	10	30
31	9.79 431	15	9.90 086	26	0.09 913	9.89 344	10	29
32	9.79 446	16	9.90 112	26	0.09 887	9.89 334	10	28
33	9.79 462	16	9.90 138	25	0.09 861	9.89 324	10	27
34	9.79 478	15	9.90 164	26	0.09 836	9.89 314	10	26
35	9.79 494	16	9.90 190	26	0.09 810	9.89 304	10	25
36	9.79 510	16	9.90 216	26	0.09 784	9.89 294	10	24
37	9.79 526	15	9.90 242	26	0.09 758	9.89 284	10	23
38	9.79 541	16	9.90 268	26	0.09 732	9.89 274	10	22
39	9.79 557	16	9.90 294	25	0.09 706	9.89 264	10	21
40	9.79 573	15	9.90 319	26	0.09 680	9.89 253	10	20
41	9.79 589	16	9.90 345	26	0.09 654	9.89 243	10	19
42	9.79 605	15	9.90 371	26	0.09 628	9.89 233	10	18
43	9.79 620	16	9.90 397	25	0.09 602	9.89 223	10	17
44	9.79 636	15	9.90 423	26	0.09 577	9.89 213	10	16
45	9.79 652	16	9.90 449	26	0.09 551	9.89 203	10	15
46	9.79 668	15	9.90 475	26	0.09 525	9.89 193	10	14
47	9.79 683	16	9.90 501	25	0.09 499	9.89 182	10	13
48	9.79 699	15	9.90 526	26	0.09 473	9.89 172	10	12
49	9.79 715	15	9.90 552	26	0.09 447	9.89 162	10	11
50	9.79 730	16	9.90 578	26	0.09 421	9.89 152	10	10
51	9.79 746	15	9.90 604	25	0.09 395	9.89 142	10	9
52	9.79 762	15	9.90 630	26	0.09 370	9.89 132	10	8
53	9.79 777	16	9.90 656	26	0.09 344	9.89 121	10	7
54	9.79 793	15	9.90 682	25	0.09 318	9.89 111	10	6
55	9.79 809	15	9.90 707	26	0.09 292	9.89 101	10	5
56	9.79 824	16	9.90 733	26	0.09 266	9.89 091	10	4
57	9.79 840	15	9.90 759	26	0.09 240	9.89 081	10	3
58	9.79 856	15	9.90 785	25	0.09 214	9.89 070	10	2
59	9.79 871	15	9.90 811	26	0.09 189	9.89 060	10	1
60	9.79 887		9.90 837		0.09 163	9.89 050		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	26	25
6	2.6	2.5
7	3.0	3.0
8	3.4	3.4
9	3.9	3.8
10	4.3	4.2
20	8.6	8.5
30	13.0	12.7
40	17.3	17.0
50	21.6	21.2

	16	16	15
6	1.6	1.6	1.5
7	1.9	1.8	1.8
8	2.2	2.1	2.0
9	2.5	2.4	2.3
10	2.7	2.6	2.6
20	5.5	5.3	5.1
30	8.2	8.0	7.7
40	11.0	10.6	10.3
50	13.7	13.3	12.9

	10	10	9
6	1.0	1.0	0.9
7	1.2	1.1	1.1
8	1.4	1.3	1.2
9	1.6	1.5	1.4
10	1.7	1.6	1.6
20	3.5	3.3	3.1
30	5.2	5.0	4.7
40	7.0	6.6	6.3
50	8.7	8.3	7.9

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.79 887	16	9.90 837	26	0.09 163	9.89 050	10	60
1	9.79 903	15	9.90 863	25	0.09 137	9.89 040	10	59
2	9.79 918	15	9.90 888	26	0.09 111	9.89 030	10	58
3	9.79 934	15	9.90 914	25	0.09 085	9.89 019	10	57
4	9.79 949	15	9.90 940	26	0.09 060	9.89 009	10	56
5	9.79 965	15	9.90 966	26	0.09 034	9.88 999	10	55
6	9.79 980	15	9.90 992	25	0.09 008	9.88 989	10	54
7	9.79 996	15	9.91 017	26	0.08 982	9.88 978	10	53
8	9.80 011	15	9.91 043	26	0.08 956	9.88 968	10	52
9	9.80 027	15	9.91 069	25	0.08 930	9.88 958	10	51
10	9.80 042	15	9.91 095	26	0.08 905	9.88 947	10	50
11	9.80 058	15	9.91 121	25	0.08 879	9.88 937	10	49
12	9.80 073	15	9.91 146	26	0.08 853	9.88 927	10	48
13	9.80 089	15	9.91 172	25	0.08 827	9.88 917	10	47
14	9.80 104	15	9.91 198	26	0.08 802	9.88 906	10	46
15	9.80 120	15	9.91 224	26	0.08 776	9.88 896	10	45
16	9.80 135	15	9.91 250	25	0.08 750	9.88 886	10	44
17	9.80 151	15	9.91 275	26	0.08 724	9.88 875	10	43
18	9.80 166	15	9.91 301	25	0.08 698	9.88 865	10	42
19	9.80 182	15	9.91 327	26	0.08 673	9.88 855	10	41
20	9.80 197	15	9.91 353	25	0.08 647	9.88 844	10	40
21	9.80 213	15	9.91 378	26	0.08 621	9.88 834	10	39
22	9.80 228	15	9.91 404	25	0.08 595	9.88 823	10	38
23	9.80 243	15	9.91 430	26	0.08 570	9.88 813	10	37
24	9.80 259	15	9.91 456	25	0.08 544	9.88 803	10	36
25	9.80 274	15	9.91 481	26	0.08 518	9.88 792	10	35
26	9.80 289	15	9.91 507	25	0.08 492	9.88 782	10	34
27	9.80 305	15	9.91 533	26	0.08 467	9.88 772	10	33
28	9.80 320	15	9.91 559	25	0.08 441	9.88 761	10	32
29	9.80 335	15	9.91 584	26	0.08 415	9.88 751	10	31
30	9.80 351	15	9.91 610	25	0.08 389	9.88 740	10	30
31	9.80 366	15	9.91 636	26	0.08 364	9.88 730	10	29
32	9.80 381	15	9.91 662	25	0.08 338	9.88 720	10	28
33	9.80 397	15	9.91 687	26	0.08 312	9.88 709	10	27
34	9.80 412	15	9.91 713	25	0.08 286	9.88 699	10	26
35	9.80 427	15	9.91 739	26	0.08 261	9.88 688	10	25
36	9.80 443	15	9.91 765	25	0.08 235	9.88 678	10	24
37	9.80 458	15	9.91 790	26	0.08 209	9.88 667	10	23
38	9.80 473	15	9.91 816	25	0.08 183	9.88 657	10	22
39	9.80 488	15	9.91 842	25	0.08 158	9.88 646	10	21
40	9.80 504	15	9.91 867	26	0.08 132	9.88 636	10	20
41	9.80 519	15	9.91 893	25	0.08 106	9.88 625	10	19
42	9.80 534	15	9.91 919	26	0.08 081	9.88 615	10	18
43	9.80 549	15	9.91 945	25	0.08 055	9.88 604	10	17
44	9.80 564	15	9.91 970	25	0.08 029	9.88 594	10	16
45	9.80 580	15	9.91 996	26	0.08 004	9.88 583	10	15
46	9.80 595	15	9.92 022	25	0.07 978	9.88 573	10	14
47	9.80 610	15	9.92 047	26	0.07 952	9.88 562	10	13
48	9.80 625	15	9.92 073	25	0.07 926	9.88 552	10	12
49	9.80 640	15	9.92 099	25	0.07 901	9.88 541	10	11
50	9.80 655	15	9.92 124	26	0.07 875	9.88 531	10	10
51	9.80 671	15	9.92 150	25	0.07 849	9.88 520	10	9
52	9.80 686	15	9.92 176	25	0.07 824	9.88 510	10	8
53	9.80 701	15	9.92 201	26	0.07 798	9.88 499	10	7
54	9.80 716	15	9.92 227	25	0.07 772	9.88 489	10	6
55	9.80 731	15	9.92 253	25	0.07 747	9.88 478	11	5
56	9.80 746	15	9.92 278	26	0.07 721	9.88 467	10	4
57	9.80 761	15	9.92 304	25	0.07 695	9.88 457	10	3
58	9.80 776	15	9.92 330	25	0.07 670	9.88 446	10	2
59	9.80 791	15	9.92 355	26	0.07 644	9.88 436	10	1
60	9.80 806		9.92 381		0.07 618	9.88 425		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	26	$2\bar{5}$
6	2.6	2.5
7	3.0	3.0
8	3.4	3.4
9	3.9	3.8
10	4.3	4.2
20	8.6	8.5
30	13.0	12.7
40	17.3	17.0
50	21.6	21.2

	16	$1\bar{5}$	15
6	1.6	1.5	1.5
7	1.8	1.8	1.7
8	2.1	2.0	2.0
9	2.4	2.3	2.2
10	2.6	2.6	2.5
20	5.3	5.1	5.0
30	8.0	7.7	7.5
40	10.6	10.3	10.0
50	13.3	12.9	12.5

	11	$1\bar{0}$	10
6	1.1	1.0	1.0
7	1.3	1.2	1.1
8	1.4	1.4	1.3
9	1.6	1.6	1.5
10	1.8	1.7	1.6
20	3.6	3.5	3.3
30	5.5	5.2	5.0
40	7.3	7.0	6.6
50	9.1	8.7	8.3

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.80 806	15	9.92 381	25	0.07 618	9.88 425	10	60
1	9.80 822	15	9.92 407	25	0.07 593	9.88 415	11	59
2	9.80 837	15	9.92 432	26	0.07 567	9.88 404	10	58
3	9.80 852	15	9.92 458	25	0.07 541	9.88 393	10	57
4	9.80 867	15	9.92 484	25	0.07 516	9.88 383	10	56
5	9.80 882	15	9.92 509	25	0.07 490	9.88 372	11	55
6	9.80 897	15	9.92 535	26	0.07 465	9.88 361	10	54
7	9.80 912	15	9.92 561	25	0.07 439	9.88 351	10	53
8	9.80 927	15	9.92 586	25	0.07 413	9.88 340	11	52
9	9.80 942	15	9.92 612	26	0.07 388	9.88 329	10	51
10	9.80 957	15	9.92 638	25	0.07 362	9.88 319	10	50
11	9.80 972	15	9.92 663	25	0.07 336	9.88 308	11	49
12	9.80 987	14	9.92 689	25	0.07 311	9.88 297	10	48
13	9.81 001	15	9.92 714	26	0.07 285	9.88 287	10	47
14	9.81 016	15	9.92 740	25	0.07 259	9.88 276	11	46
15	9.81 031	15	9.92 766	25	0.07 234	9.88 265	10	45
16	9.81 046	15	9.92 791	25	0.07 208	9.88 255	10	44
17	9.81 061	15	9.92 817	25	0.07 183	9.88 244	11	43
18	9.81 076	14	9.92 842	26	0.07 157	9.88 233	10	42
19	9.81 091	15	9.92 868	25	0.07 131	9.88 223	11	41
20	9.81 106	15	9.92 894	25	0.07 106	9.88 212	10	40
21	9.81 121	15	9.92 919	25	0.07 080	9.88 201	11	39
22	9.81 136	14	9.92 945	26	0.07 055	9.88 190	10	38
23	9.81 150	15	9.92 971	25	0.07 029	9.88 180	11	37
24	9.81 165	15	9.92 996	25	0.07 003	9.88 169	10	36
25	9.81 180	14	9.93 022	25	0.06 978	9.88 158	11	35
26	9.81 195	15	9.93 047	25	0.06 952	9.88 147	10	34
27	9.81 210	15	9.93 073	25	0.06 927	9.88 137	11	33
28	9.81 225	14	9.93 098	26	0.06 901	9.88 126	10	32
29	9.81 239	15	9.93 124	25	0.06 875	9.88 115	11	31
30	9.81 254	14	9.93 150	25	0.06 850	9.88 104	10	30
31	9.81 269	15	9.93 175	25	0.06 824	9.88 094	11	29
32	9.81 284	15	9.93 201	25	0.06 799	9.88 083	11	28
33	9.81 299	14	9.93 226	25	0.06 773	9.88 072	10	27
34	9.81 313	15	9.93 252	26	0.06 748	9.88 061	11	26
35	9.81 328	14	9.93 278	25	0.06 722	9.88 050	11	25
36	9.81 343	15	9.93 303	25	0.06 696	9.88 039	10	24
37	9.81 358	14	9.93 329	25	0.06 671	9.88 029	11	23
38	9.81 372	14	9.93 354	25	0.06 645	9.88 018	11	22
39	9.81 387	15	9.93 380	25	0.06 620	9.88 007	10	21
40	9.81 402	14	9.93 405	25	0.06 594	9.87 996	11	20
41	9.81 416	15	9.93 431	25	0.06 569	9.87 985	11	19
42	9.81 431	14	9.93 456	25	0.06 543	9.87 974	11	18
43	9.81 446	14	9.93 482	26	0.06 518	9.87 963	10	17
44	9.81 460	15	9.93 508	25	0.06 492	9.87 953	11	16
45	9.81 475	14	9.93 533	25	0.06 466	9.87 942	11	15
46	9.81 490	14	9.93 559	25	0.06 441	9.87 931	11	14
47	9.81 504	15	9.93 584	25	0.06 415	9.87 920	10	13
48	9.81 519	14	9.93 610	25	0.06 390	9.87 909	11	12
49	9.81 534	14	9.93 635	25	0.06 364	9.87 898	11	11
50	9.81 548	14	9.93 661	25	0.06 339	9.87 887	11	10
51	9.81 563	15	9.93 686	25	0.06 313	9.87 876	11	9
52	9.81 578	14	9.93 712	25	0.06 288	9.87 865	11	8
53	9.81 592	14	9.93 737	25	0.06 262	9.87 854	10	7
54	9.81 607	14	9.93 763	25	0.06 237	9.87 844	11	6
55	9.81 621	14	9.93 788	25	0.06 211	9.87 833	11	5
56	9.81 636	14	9.93 814	26	0.06 186	9.87 822	11	4
57	9.81 650	14	9.93 840	25	0.06 160	9.87 811	11	3
58	9.81 665	15	9.93 865	25	0.06 134	9.87 800	11	2
59	9.81 680	14	9.93 891	25	0.06 109	9.87 789	11	1
60	9.81 694		9.93 916		0.06 083	9.87 778		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	26	25
6	2.6	2.5
7	3.0	3.0
8	3.4	3.4
9	3.9	3.8
10	4.3	4.2
20	8.6	8.5
30	13.0	12.7
40	17.3	17.0
50	21.6	21.2

	15	15	14
6	1.5	1.5	1.4
7	1.8	1.7	1.7
8	2.0	2.0	1.9
9	2.3	2.2	2.2
10	2.6	2.5	2.4
20	5.1	5.0	4.8
30	7.7	7.5	7.2
40	10.3	10.0	9.6
50	12.9	12.5	12.1

	11	10
6	1.1	1.0
7	1.3	1.2
8	1.4	1.4
9	1.6	1.6
10	1.8	1.7
20	3.6	3.5
30	5.5	5.2
40	7.3	7.0
50	9.1	8.7

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.81 694	14	9.93 916	25	0.06 083	9.87 778	11	60
1	9.81 709	14	9.93 942	25	0.06 058	9.87 767	11	59
2	9.81 723	14	9.93 967	25	0.06 032	9.87 756	11	58
3	9.81 738	14	9.93 993	25	0.06 007	9.87 745	11	57
4	9.81 752	14	9.94 018	25	0.05 981	9.87 734	11	56
5	9.81 767	14	9.94 044	25	0.05 956	9.87 723	11	55
6	9.81 781	14	9.94 069	25	0.05 930	9.87 712	11	54
7	9.81 796	14	9.94 095	25	0.05 905	9.87 701	11	53
8	9.81 810	14	9.94 120	25	0.05 879	9.87 690	11	52
9	9.81 824	14	9.94 146	25	0.05 854	9.87 679	11	51
10	9.81 839	14	9.94 171	25	0.05 828	9.87 668	11	50
11	9.81 853	14	9.94 197	25	0.05 803	9.87 657	11	49
12	9.81 868	14	9.94 222	25	0.05 777	9.87 645	11	48
13	9.81 882	14	9.94 248	25	0.05 752	9.87 634	11	47
14	9.81 897	14	9.94 273	25	0.05 726	9.87 623	11	46
15	9.81 911	14	9.94 299	25	0.05 701	9.87 612	11	45
16	9.81 925	14	9.94 324	25	0.05 675	9.87 601	11	44
17	9.81 940	14	9.94 350	25	0.05 650	9.87 590	11	43
18	9.81 954	14	9.94 375	25	0.05 625	9.87 579	11	42
19	9.81 969	14	9.94 400	25	0.05 599	9.87 568	11	41
20	9.81 983	14	9.94 426	25	0.05 574	9.87 557	11	40
21	9.81 997	14	9.94 451	25	0.05 548	9.87 546	11	39
22	9.82 012	14	9.94 477	25	0.05 523	9.87 535	11	38
23	9.82 026	14	9.94 502	25	0.05 497	9.87 523	11	37
24	9.82 040	14	9.94 528	25	0.05 472	9.87 512	11	36
25	9.82 055	14	9.94 553	25	0.05 446	9.87 501	11	35
26	9.82 069	14	9.94 579	25	0.05 421	9.87 290	11	34
27	9.82 083	14	9.94 604	25	0.05 395	9.87 479	11	33
28	9.82 098	14	9.94 630	25	0.05 370	9.87 468	11	32
29	9.82 112	14	9.94 655	25	0.05 344	9.87 457	11	31
30	9.82 126	14	9.94 681	25	0.05 319	9.87 445	11	30
31	9.82 140	14	9.94 706	25	0.05 293	9.87 434	11	29
32	9.82 155	14	9.94 732	25	0.05 268	9.87 423	11	28
33	9.82 169	14	9.94 757	25	0.05 243	9.87 412	11	27
34	9.82 183	14	9.94 782	25	0.05 217	9.87 401	11	26
35	9.82 197	14	9.94 808	25	0.05 192	9.87 389	11	25
36	9.82 212	14	9.94 833	25	0.05 166	9.87 378	11	24
37	9.82 226	14	9.94 859	25	0.05 141	9.87 367	11	23
38	9.82 240	14	9.94 884	25	0.05 115	9.87 356	11	22
39	9.82 254	14	9.94 910	25	0.05 090	9.87 345	11	21
40	9.82 209	14	9.94 935	25	0.05 064	9.87 333	11	20
41	9.82 283	14	9.94 961	25	0.05 039	9.87 322	11	19
42	9.82 297	14	9.94 986	25	0.05 014	9.87 311	11	18
43	9.82 311	14	9.95 011	25	0.04 988	9.87 300	11	17
44	9.82 325	14	9.95 037	25	0.04 963	9.87 288	11	16
45	9.82 339	14	9.95 062	25	0.04 937	9.87 277	11	15
46	9.82 354	14	9.95 088	25	0.04 912	9.87 266	11	14
47	9.82 368	14	9.95 113	25	0.04 886	9.87 254	11	13
48	9.82 382	14	9.95 139	25	0.04 861	9.87 243	11	12
49	9.82 396	14	9.95 164	25	0.04 836	9.87 232	11	11
50	9.82 410	14	9.95 189	25	0.04 810	9.87 221	11	10
51	9.82 424	14	9.95 215	25	0.04 785	9.87 209	11	9
52	9.82 438	14	9.95 240	25	0.04 759	9.87 198	11	8
53	9.82 452	14	9.95 266	25	0.04 734	9.87 187	11	7
54	9.82 467	14	9.95 291	25	0.04 708	9.87 175	11	6
55	9.82 481	14	9.95 316	25	0.04 683	9.87 164	11	5
56	9.82 495	14	9.95 342	25	0.04 658	9.87 153	11	4
57	9.82 509	14	9.95 367	25	0.04 632	9.87 141	11	3
58	9.82 523	14	9.95 393	25	0.04 607	9.87 130	11	2
59	9.82 537	14	9.95 418	25	0.04 581	9.87 118	11	1
60	9.82 551	14	9.95 443	25	0.04 556	9.87 107	11	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	25	25
6	2.5	2.5
7	3.0	2.9
8	3.4	3.3
9	3.8	3.7
10	4.2	4.1
20	8.5	8.3
30	12.7	12.5
40	17.0	16.6
50	21.2	20.8

	14	14
6	1.4	1.4
7	1.7	1.6
8	1.9	1.8
9	2.2	2.1
10	2.4	2.3
20	4.8	4.6
30	7.2	7.0
40	9.6	9.3
50	12.1	11.6

	11	11
6	1.1	1.1
7	1.3	1.3
8	1.5	1.4
9	1.7	1.6
10	1.9	1.8
20	3.8	3.6
30	5.7	5.5
40	7.6	7.3
50	9.6	9.1

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.82 551	14	9.95 443	25	0.04 556	9.87 107	11	60
1	9.82 565	14	9.95 469	25	0.04 531	9.87 096	11	59
2	9.82 579	14	9.95 494	25	0.04 505	9.87 084	11	58
3	9.82 593	14	9.95 520	25	0.04 480	9.87 073	11	57
4	9.82 607	14	9.95 545	25	0.04 454	9.87 062	11	56
5	9.82 621	14	9.95 571	25	0.04 429	9.87 050	11	55
6	9.82 635	14	9.95 596	25	0.04 404	9.87 039	11	54
7	9.82 649	14	9.95 621	25	0.04 378	9.87 027	11	53
8	9.82 663	14	9.95 647	25	0.04 353	9.87 016	11	52
9	9.82 677	14	9.95 672	25	0.04 327	9.87 004	11	51
10	9.82 691	14	9.95 697	25	0.04 302	9.86 993	11	50
11	9.82 705	14	9.95 723	25	0.04 277	9.86 982	11	49
12	9.82 719	14	9.95 748	25	0.04 251	9.86 970	11	48
13	9.82 733	13	9.95 774	25	0.04 226	9.86 959	11	47
14	9.82 746	14	9.95 799	25	0.04 200	9.86 947	11	46
15	9.82 760	14	9.95 824	25	0.04 175	9.86 936	11	45
16	9.82 774	14	9.95 850	25	0.04 150	9.86 924	11	44
17	9.82 788	14	9.95 875	25	0.04 124	9.86 913	11	43
18	9.82 802	13	9.95 901	25	0.04 099	9.86 901	11	42
19	9.82 816	14	9.95 926	25	0.04 074	9.86 890	11	41
20	9.82 830	14	9.95 951	25	0.04 048	9.86 878	11	40
21	9.82 844	14	9.95 977	25	0.04 023	9.86 867	11	39
22	9.82 858	13	9.96 002	25	0.03 997	9.86 855	11	38
23	9.82 871	14	9.96 027	25	0.03 972	9.86 844	11	37
24	9.82 885	14	9.96 053	25	0.03 947	9.86 832	11	36
25	9.82 899	13	9.96 078	25	0.03 921	9.86 821	11	35
26	9.82 913	14	9.96 104	25	0.03 896	9.86 809	11	34
27	9.82 927	13	9.96 129	25	0.03 871	9.86 798	12	33
28	9.82 940	14	9.96 154	25	0.03 845	9.86 786	11	32
29	9.82 954	14	9.96 180	25	0.03 820	9.86 774	11	31
30	9.82 968	13	9.96 205	25	0.03 795	9.86 763	11	30
31	9.82 982	14	9.96 230	25	0.03 769	9.86 751	11	29
32	9.82 996	13	9.96 256	25	0.03 744	9.86 740	11	28
33	9.83 009	14	9.96 281	25	0.03 718	9.86 728	12	27
34	9.83 023	13	9.96 306	25	0.03 693	9.86 716	11	26
35	9.83 037	14	9.96 332	25	0.03 668	9.86 705	11	25
36	9.83 051	13	9.96 357	25	0.03 642	9.86 693	11	24
37	9.83 064	14	9.96 383	25	0.03 617	9.86 682	11	23
38	9.83 078	13	9.96 408	25	0.03 592	9.86 670	12	22
39	9.83 092	14	9.96 433	25	0.03 566	9.86 658	11	21
40	9.83 106	13	9.96 459	25	0.03 541	9.86 647	11	20
41	9.83 119	13	9.96 484	25	0.03 516	9.86 635	12	19
42	9.83 133	14	9.96 509	25	0.03 490	9.86 623	11	18
43	9.83 147	13	9.96 535	25	0.03 465	9.86 612	11	17
44	9.83 160	13	9.96 560	25	0.03 440	9.86 600	12	16
45	9.83 174	14	9.96 585	25	0.03 414	9.86 588	11	15
46	9.83 188	13	9.96 611	25	0.03 389	9.86 577	11	14
47	9.83 201	13	9.96 636	25	0.03 364	9.86 565	12	13
48	9.83 215	14	9.96 661	25	0.03 338	9.86 553	11	12
49	9.83 229	13	9.96 687	25	0.03 313	9.86 542	12	11
50	9.83 242	13	9.96 712	25	0.03 287	9.86 530	11	10
51	9.83 256	13	9.96 737	25	0.03 262	9.86 518	11	9
52	9.83 269	14	9.96 763	25	0.03 237	9.86 507	12	8
53	9.83 283	13	9.96 788	25	0.03 211	9.86 495	11	7
54	9.83 297	13	9.96 813	25	0.03 186	9.86 483	12	6
55	9.83 310	13	9.96 839	25	0.03 161	9.86 471	11	5
56	9.83 324	13	9.96 864	25	0.03 135	9.86 460	12	4
57	9.83 337	13	9.96 889	25	0.03 110	9.86 448	11	3
58	9.83 351	14	9.96 915	25	0.03 085	9.86 436	12	2
59	9.83 365	13	9.96 940	25	0.03 059	9.86 424	12	1
60	9.83 378		9.96 965		0.03 034	9.86 412		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	25	25
6	2.5	2.5
7	3.0	2.9
8	3.4	3.3
9	3.8	3.7
10	4.2	4.1
20	8.5	8.3
30	12.7	12.5
40	17.0	16.6
50	21.2	20.8

	14	13
6	1.4	1.3
7	1.6	1.6
8	1.8	1.8
9	2.1	2.0
10	2.3	2.2
20	4.6	4.5
30	7.0	6.7
40	9.3	9.0
50	11.6	11.2

	12	11	11
6	1.2	1.1	1.1
7	1.4	1.3	1.3
8	1.6	1.5	1.4
9	1.8	1.7	1.6
10	2.0	1.9	1.8
20	4.0	3.8	3.6
30	6.0	5.7	5.5
40	8.0	7.6	7.3
50	10.0	9.6	9.1

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.83 378	13	9.96 965	25	0.03 034	9.86 412	11	60
1	9.83 392	13	9.96 991	25	0.03 009	9.86 401	12	59
2	9.83 405	13	9.97 016	25	0.02 984	9.86 389	11	58
3	9.83 419	13	9.97 041	25	0.02 958	9.86 377	12	57
4	9.83 432	13	9.97 067	25	0.02 933	9.86 365	11	56
5	9.83 446	13	9.97 092	25	0.02 908	9.86 354	12	55
6	9.83 459	13	9.97 117	25	0.02 882	9.86 342	12	54
7	9.83 473	13	9.97 143	25	0.02 857	9.86 330	11	53
8	9.83 486	13	9.97 168	25	0.02 832	9.86 318	12	52
9	9.83 500	13	9.97 193	25	0.02 806	9.86 306	12	51
10	9.83 513	13	9.97 219	25	0.02 781	9.86 294	12	50
11	9.83 527	13	9.97 244	25	0.02 756	9.86 282	11	49
12	9.83 540	13	9.97 269	25	0.02 730	9.86 271	12	48
13	9.83 554	13	9.97 295	25	0.02 705	9.86 259	12	47
14	9.83 567	13	9.97 320	25	0.02 680	9.86 247	11	46
15	9.83 580	13	9.97 345	25	0.02 654	9.86 235	12	45
16	9.83 594	13	9.97 370	25	0.02 629	9.86 223	12	44
17	9.83 607	13	9.97 396	25	0.02 604	9.86 211	12	43
18	9.83 621	13	9.97 421	25	0.02 578	9.86 199	12	42
19	9.83 634	13	9.97 446	25	0.02 553	9.86 187	11	41
20	9.83 647	13	9.97 472	25	0.02 528	9.86 176	12	40
21	9.83 661	13	9.97 497	25	0.02 502	9.86 164	12	39
22	9.83 674	13	9.97 522	25	0.02 477	9.86 152	12	38
23	9.83 688	13	9.97 548	25	0.02 452	9.86 140	12	37
24	9.83 701	13	9.97 573	25	0.02 427	9.86 128	12	36
25	9.83 714	13	9.97 598	25	0.02 401	9.86 116	12	35
26	9.83 728	13	9.97 624	25	0.02 376	9.86 104	12	34
27	9.83 741	13	9.97 649	25	0.02 351	9.86 092	12	33
28	9.83 754	13	9.97 674	25	0.02 325	9.86 080	12	32
29	9.83 768	13	9.97 699	25	0.02 300	9.86 068	12	31
30	9.83 781	13	9.97 725	25	0.02 275	9.86 056	12	30
31	9.83 794	13	9.97 750	25	0.02 249	9.86 044	12	29
32	9.83 808	13	9.97 775	25	0.02 224	9.86 032	12	28
33	9.83 821	13	9.97 801	25	0.02 199	9.86 020	12	27
34	9.83 834	13	9.97 826	25	0.02 174	9.86 008	12	26
35	9.83 847	13	9.97 851	25	0.02 148	9.85 996	12	25
36	9.83 861	13	9.97 877	25	0.02 123	9.85 984	12	24
37	9.83 874	13	9.97 902	25	0.02 098	9.85 972	12	23
38	9.83 887	13	9.97 927	25	0.02 072	9.85 960	12	22
39	9.83 900	13	9.97 952	25	0.02 047	9.85 948	12	21
40	9.83 914	13	9.97 978	25	0.02 022	9.85 936	12	20
41	9.83 927	13	9.98 003	25	0.01 996	9.85 924	12	19
42	9.83 940	13	9.98 028	25	0.01 971	9.85 912	12	18
43	9.83 953	13	9.98 054	25	0.01 946	9.85 900	12	17
44	9.83 967	13	9.98 079	25	0.01 921	9.85 887	12	16
45	9.83 980	13	9.98 104	25	0.01 895	9.85 875	12	15
46	9.83 993	13	9.98 129	25	0.01 870	9.85 863	12	14
47	9.84 006	13	9.98 155	25	0.01 845	9.85 851	12	13
48	9.84 019	13	9.98 180	25	0.01 819	9.85 839	12	12
49	9.84 033	13	9.98 205	25	0.01 794	9.85 827	12	11
50	9.84 046	13	9.98 231	25	0.01 769	9.85 815	12	10
51	9.84 059	13	9.98 256	25	0.01 744	9.85 803	12	9
52	9.84 072	13	9.98 281	25	0.01 718	9.85 791	12	8
53	9.84 085	13	9.98 306	25	0.01 693	9.85 778	12	7
54	9.84 098	13	9.98 332	25	0.01 668	9.85 766	12	6
55	9.84 111	13	9.98 357	25	0.01 642	9.85 754	12	5
56	9.84 124	13	9.98 382	25	0.01 617	9.85 742	12	4
57	9.84 138	13	9.98 408	25	0.01 592	9.85 730	12	3
58	9.84 151	13	9.98 433	25	0.01 567	9.85 718	12	2
59	9.84 164	13	9.98 458	25	0.01 541	9.85 705	12	1
60	9.84 177		9.98 483		0.01 516	9.85 693		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	25	25
6	2.5	2.5
7	3.0	2.9
8	3.4	3.3
9	3.8	3.7
10	4.2	4.1
20	8.5	8.3
30	12.7	12.5
40	17.0	16.6
50	21.2	20.8

	13	13
6	1.3	1.3
7	1.6	1.5
8	1.8	1.7
9	2.0	1.9
10	2.2	2.1
20	4.5	4.3
30	6.7	6.5
40	9.0	8.6
50	11.2	10.8

	12	12	11
6	1.2	1.2	1.1
7	1.4	1.4	1.3
8	1.6	1.6	1.5
9	1.9	1.8	1.7
10	2.1	2.0	1.9
20	4.1	4.0	3.8
30	6.2	6.0	5.7
40	8.3	8.0	7.6
50	10.4	10.0	9.6

P. P.

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.84 177	13	9.98 483	25	0.01 516	9.85 693	12	60
1	9.84 190	13	9.98 509	25	0.01 491	9.85 681	12	59
2	9.84 203	13	9.98 534	25	0.01 465	9.85 669	12	58
3	9.84 216	13	9.98 559	25	0.01 440	9.85 657	12	57
4	9.84 229	13	9.98 585	25	0.01 415	9.85 644	12	56
5	9.84 242	13	9.98 610	25	0.01 390	9.85 632	12	55
6	9.84 255	13	9.98 635	25	0.01 364	9.85 620	12	54
7	9.84 268	13	9.98 660	25	0.01 339	9.85 608	12	53
8	9.84 281	13	9.98 686	25	0.01 314	9.85 595	12	52
9	9.84 294	13	9.98 711	25	0.01 289	9.85 583	12	51
10	9.84 307	13	9.98 736	25	0.01 263	9.85 571	12	50
11	9.84 320	13	9.98 762	25	0.01 238	9.85 559	12	49
12	9.84 333	13	9.98 787	25	0.01 213	9.85 546	12	48
13	9.84 346	13	9.98 812	25	0.01 187	9.85 534	12	47
14	9.84 359	13	9.98 837	25	0.01 162	9.85 522	12	46
15	9.84 372	13	9.98 863	25	0.01 137	9.85 509	12	45
16	9.84 385	13	9.98 888	25	0.01 112	9.85 497	12	44
17	9.84 398	13	9.98 913	25	0.01 086	9.85 485	12	43
18	9.84 411	13	9.98 938	25	0.01 061	9.85 472	12	42
19	9.84 424	12	9.98 964	25	0.01 036	9.85 460	12	41
20	9.84 437	13	9.98 989	25	0.01 010	9.85 448	12	40
21	9.84 450	13	9.99 014	25	0.00 985	9.85 435	12	39
22	9.84 463	13	9.99 040	25	0.00 960	9.85 423	12	38
23	9.84 476	13	9.99 065	25	0.00 935	9.85 411	12	37
24	9.84 489	13	9.99 090	25	0.00 909	9.85 398	12	36
25	9.84 502	12	9.99 115	25	0.00 884	9.85 386	12	35
26	9.84 514	13	9.99 141	25	0.00 859	9.85 374	12	34
27	9.84 527	13	9.99 166	25	0.00 834	9.85 361	12	33
28	9.84 540	13	9.99 191	25	0.00 808	9.85 349	12	32
29	9.84 553	12	9.99 216	25	0.00 783	9.85 336	12	31
30	9.84 566	13	9.99 242	25	0.00 758	9.85 324	12	30
31	9.84 579	13	9.99 267	25	0.00 733	9.85 312	12	29
32	9.84 592	12	9.99 292	25	0.00 707	9.85 299	12	28
33	9.84 604	13	9.99 318	25	0.00 682	9.85 287	12	27
34	9.84 617	13	9.99 343	25	0.00 657	9.85 274	12	26
35	9.84 630	12	9.99 368	25	0.00 631	9.85 262	12	25
36	9.84 643	13	9.99 393	25	0.00 606	9.85 249	12	24
37	9.84 656	13	9.99 419	25	0.00 581	9.85 237	12	23
38	9.84 669	12	9.99 444	25	0.00 556	9.85 224	12	22
39	9.84 681	13	9.99 469	25	0.00 530	9.85 212	12	21
40	9.84 694	12	9.99 494	25	0.00 505	9.85 199	12	20
41	9.84 707	13	9.99 520	25	0.00 480	9.85 187	12	19
42	9.84 720	12	9.99 545	25	0.00 455	9.85 174	12	18
43	9.84 732	13	9.99 570	25	0.00 429	9.85 162	12	17
44	9.84 745	12	9.99 595	25	0.00 404	9.85 149	12	16
45	9.84 758	13	9.99 621	25	0.00 379	9.85 137	12	15
46	9.84 771	12	9.99 646	25	0.00 353	9.85 124	12	14
47	9.84 783	13	9.99 671	25	0.00 328	9.85 112	12	13
48	9.84 796	12	9.99 697	25	0.00 303	9.85 099	12	12
49	9.84 809	13	9.99 722	25	0.00 278	9.85 087	12	11
50	9.84 822	12	9.99 747	25	0.00 252	9.85 074	12	10
51	9.84 834	12	9.99 772	25	0.00 227	9.85 062	12	9
52	9.84 847	13	9.99 798	25	0.00 202	9.85 049	12	8
53	9.84 860	12	9.99 823	25	0.00 177	9.85 037	13	7
54	9.84 872	12	9.99 848	25	0.00 151	9.85 024	12	6
55	9.84 885	13	9.99 873	25	0.00 126	9.85 011	12	5
56	9.84 898	12	9.99 899	25	0.00 101	9.84 999	12	4
57	9.84 910	12	9.99 924	25	0.00 076	9.84 986	12	3
58	9.84 923	13	9.99 949	25	0.00 050	9.84 974	13	2
59	9.84 936	12	9.99 974	25	0.00 025	9.84 961	12	1
60	9.84 948		0.00 000		0.00 000	9.84 948		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	25	25
6	2.5	2.5
7	3.0	2.9
8	3.4	3.3
9	3.8	3.7
10	4.2	4.1
20	8.5	8.3
30	12.7	12.5
40	17.0	16.6
50	21.2	20.8

	13	13
6	1.3	1.3
7	1.6	1.5
8	1.8	1.7
9	2.0	1.9
10	2.2	2.1
20	4.5	4.3
30	6.7	6.5
40	9.0	8.6
50	11.2	10.8

	12	12
6	1.2	1.2
7	1.4	1.4
8	1.6	1.6
9	1.9	1.8
10	2.1	2.0
20	4.1	4.0
30	6.2	6.0
40	8.3	8.0
50	10.4	10.0

P. P.

TABLE VIII.

LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

	0°				1°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	− ∞		− ∞		6.18271	1435	6.18278	1436	0
1	2.62642	60206	2.62642	60206	.19707	1412	.19714	1412	1
2	3.22848	35218	3.22848	35218	.21119	1389	.21126	1390	2
3	3.58066	24987	3.58066	24987	.22509	1368	.22516	1368	3
4	3.83054	19382	3.83054	19382	.23877	1346	.23884	1347	4
5	4.02436	15836	4.02436	15836	6.25223	1326	6.25231	1326	5
6	.18272	13389	.18272	13389	.26549	1306	.26557	1306	6
7	.31662	11598	.31662	11598	.27856	1286	.27864	1287	7
8	.43260	10230	.43260	10230	.29142	1268	.29151	1268	8
9	.53490	9151	.53491	9151	.30410	1250	.30419	1250	9
10	4.62642	8278	4.62642	8279	6.31660	1232	6.31669	1232	10
11	.70920	7558	.70921	7557	.32892	1214	.32901	1215	11
12	.78478	6953	.78478	6952	.34107	1198	.34116	1198	12
13	.85431	6437	.85431	6437	.35305	1182	.35315	1182	13
14	.91868	5992	.91868	5993	.36487	1166	.36497	1166	14
15	4.97860	5605	4.97861	5605	6.37653	1150	6.37663	1151	15
16	5.03466	5266	5.03466	5266	.38803	1135	.38814	1135	16
17	.08732	4964	.08732	4964	.39938	1121	.39949	1121	17
18	.13696	4696	.13697	4696	.41059	1106	.41070	1106	18
19	.18393	4455	.18393	4456	.42165	1093	.42177	1093	19
20	5.22848	4238	5.22849	4238	6.43258	1078	6.43270	1079	20
21	.27086	4040	.27087	4040	.44337	1066	.44349	1066	21
22	.31126	3861	.31127	3861	.45403	1052	.45415	1053	22
23	.34987	3697	.34988	3697	.46455	1040	.46468	1040	23
24	.38684	3545	.38685	3545	.47496	1028	.47509	1028	24
25	5.42230	3406	5.42231	3407	6.48524	1016	6.48537	1015	25
26	.45636	3278	.45638	3278	.49539	1004	.49553	1004	26
27	.48915	3158	.48916	3159	.50544	992	.50557	993	27
28	.52073	3048	.52075	3048	.51536	981	.51550	982	28
29	.55121	2944	.55123	2945	.52518	970	.52532	970	29
30	5.58066	2848	5.58068	2848	6.53488	960	6.53503	960	30
31	.60914	2757	.60916	2758	.54448	949	.54463	950	31
32	.63672	2672	.63674	2672	.55397	939	.55413	939	32
33	.66344	2593	.66346	2593	.56336	929	.56352	929	33
34	.68937	2518	.68940	2517	.57265	919	.57281	919	34
35	5.71455	2447	5.71457	2447	6.58184	909	6.58201	909	35
36	.73902	2379	.73904	2380	.59093	900	.59110	900	36
37	.76282	2316	.76284	2316	.59993	891	.60011	891	37
38	.78598	2256	.78601	2256	.60884	882	.60902	882	38
39	.80854	2199	.80857	2199	.61766	872	.61784	873	39
40	5.83053	2145	5.83056	2145	6.62639	864	6.62657	864	40
41	.85198	2093	.85201	2093	.63503	855	.63522	856	41
42	.87291	2044	.87295	2043	.64359	847	.64378	848	42
43	.89335	1996	.89338	1997	.65206	839	.65226	839	43
44	.91332	1952	.91335	1952	.66045	831	.66065	831	44
45	5.93284	1909	5.93288	1909	6.66876	823	6.66897	823	45
46	.95193	1868	.95197	1868	.67700	815	.67720	816	46
47	.97061	1829	.97065	1829	.68515	808	.68536	808	47
48	5.98890	1790	5.98894	1791	.69323	800	.69345	800	48
49	6.00680	1755	6.00685	1755	.70124	793	.70145	794	49
50	6.02435	1720	6.02440	1720	6.70917	786	6.70939	786	50
51	.04155	1686	.04160	1687	.71703	779	.71725	779	51
52	.05842	1654	.05847	1654	.72482	772	.72505	772	52
53	.07496	1623	.07501	1623	.73254	765	.73277	765	53
54	.09120	1594	.09125	1594	.74019	758	.74043	759	54
55	6.10714	1565	6.10719	1565	6.74777	752	6.74802	752	55
56	.12279	1537	.12284	1537	.75529	745	.75554	746	56
57	.13816	1511	.13822	1511	.76275	739	.76300	739	57
58	.15327	1484	.15333	1485	.77014	733	.77040	733	58
59	.16811	1460	.16818	1460	.77747	726	.77773	727	59
60	6.18271		6.18278		6.78474		6.78500		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

′	2° Log. Vers.	D	2° Log. Exsec.	D	3° Log. Vers.	D	3° Log. Exsec.	D	′
0	6.78474	721	6.78500	721	7.13687	481	7.13746	481	0
1	.79195	714	.79221	715	.14168	478	.14228	479	1
2	.79909	709	.79937	709	.14646	475	.14707	476	2
3	.80618	703	.80646	703	.15122	473	.15183	474	3
4	.81322	697	.81350	698	.15595	470	.15657	471	4
5	6.82019	692	6.82048	692	7.16066	468	7.16129	469	5
6	.82711	686	.82740	687	.16534	466	.16598	466	6
7	.83398	681	.83427	682	.17000	463	.17064	464	7
8	.84079	676	.84109	676	.17463	460	.17528	461	8
9	.84755	670	.84785	671	.17923	458	.17989	459	9
10	6.85425	665	6.85457	666	7.18382	455	7.18448	456	10
11	.86091	660	.86123	660	.18837	453	.18905	454	11
12	.86751	655	.86783	656	.19291	451	.19359	452	12
13	.87407	650	.87439	651	.19742	448	.19811	449	13
14	.88057	646	.88090	646	.20191	446	.20260	447	14
15	6.88703	641	6.88737	641	7.20637	444	7.20707	445	15
16	.89344	636	.89378	636	.21081	442	.21152	442	16
17	.89980	631	.90015	632	.21523	440	.21595	440	17
18	.90612	627	.90647	628	.21963	437	.22035	438	18
19	.91239	622	.91275	623	.22400	435	.22473	436	19
20	6.91862	618	6.91898	618	7.22836	433	7.22909	434	20
21	.92480	613	.92516	614	.23269	431	.23343	431	21
22	.93093	609	.93131	610	.23700	429	.23775	429	22
23	.93703	605	.93741	605	.24129	426	.24204	427	23
24	.94308	601	.94346	601	.24555	424	.24632	425	24
25	6.94909	597	6.94948	597	7.24980	422	7.25057	423	25
26	.95506	592	.95545	593	.25402	420	.25480	421	26
27	.96099	589	.96139	589	.25823	418	.25902	419	27
28	.96688	584	.96728	585	.26241	416	.26321	417	28
29	.97272	581	.97313	581	.26658	414	.26738	415	29
30	6.97853	577	6.97895	577	7.27072	412	7.27153	413	30
31	.98430	573	.98472	574	.27485	410	.27567	411	31
32	.99004	569	.99046	570	.27895	409	.27978	409	32
33	6.99573	565	6.99616	566	.28304	406	.28387	407	33
34	7.00139	562	7.00182	563	.28711	405	.28795	405	34
35	7.00701	558	7.00745	559	7.29116	402	7.29200	404	35
36	.01259	555	.01304	555	.29518	401	.29604	402	36
37	.01814	551	.01860	552	.29919	399	.30006	400	37
38	.02366	548	.02412	548	.30319	397	.30406	398	38
39	.02914	544	.02960	545	.30716	395	.30804	396	39
40	7.03458	541	7.03505	541	7.31112	393	7.31201	394	40
41	.03999	537	.04047	538	.31505	392	.31595	393	41
42	.04537	534	.04585	535	.31897	390	.31988	391	42
43	.05071	531	.05120	531	.32288	388	.32379	389	43
44	.05603	527	.05652	528	.32676	386	.32768	388	44
45	7.06130	525	7.06180	525	7.33063	385	7.33156	385	45
46	.06655	521	.06706	522	.33448	383	.33542	384	46
47	.07177	518	.07228	519	.33831	382	.33926	382	47
48	.07695	515	.07747	516	.34213	380	.34309	380	48
49	.08211	512	.08263	513	.34593	378	.34689	379	49
50	7.08723	509	7.08776	509	7.34971	377	7.35069	377	50
51	.09232	506	.09286	507	.35348	375	.35446	376	51
52	.09739	503	.09793	503	.35723	373	.35822	374	52
53	.10242	500	.10297	501	.36097	371	.36196	373	53
54	.10743	497	.10798	498	.36468	370	.36569	371	54
55	7.11240	495	7.11297	495	7.36839	368	7.36940	369	55
56	.11735	492	.11792	493	.37207	367	.37310	368	56
57	.12227	489	.12285	490	.37574	366	.37678	366	57
58	.12716	486	.12775	487	.37940	364	.38044	365	58
59	.13203	484	.13262	484	.38304	362	.38409	363	59
60	7.13687		7.13746		7.38667		7.38773		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

	4°				5°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	7.38667	361	7.38773	361	7.58039	289	7.58204	290	0
1	.39028	359	.39134	360	.58328	287	.58494	289	1
2	.39387	358	.39495	359	.58615	287	.58783	288	2
3	.39745	356	.39854	357	.58902	286	.59071	287	3
4	.40102	355	.40211	356	.59188	285	.59358	286	4
5	7.40457	353	7.40567	354	7.59473	284	7.59645	285	5
6	.40810	352	.40922	353	.59758	283	.59930	284	6
7	.41163	350	.41275	352	.60041	282	.60214	283	7
8	.41513	349	.41627	350	.60323	281	.60498	282	8
9	.41863	348	.41977	349	.60604	280	.60780	281	9
10	7.42211	346	7.42326	347	7.60885	279	7.61062	280	10
11	.42557	345	.42673	346	.61164	279	.61342	280	11
12	.42903	343	.43019	345	.61443	277	.61622	279	12
13	.43246	342	.43364	343	.61721	277	.61901	278	13
14	.43589	341	.43708	342	.61998	276	.62179	277	14
15	7.43930	339	7.44050	340	7.62274	275	7.62456	276	15
16	.44270	338	.44390	339	.62549	274	.62733	275	16
17	.44608	337	.44730	338	.62823	273	.63008	274	17
18	.44946	335	.45068	337	.63096	272	.63282	274	18
19	.45281	334	.45405	335	.63369	272	.63556	273	19
20	7.45616	333	7.45740	334	7.63641	270	7.63829	272	20
21	.45949	332	.46075	332	.63911	270	.64101	271	21
22	.46281	330	.46407	332	.64181	269	.64372	270	22
23	.46612	329	.46739	330	.64451	268	.64643	269	23
24	.46941	328	.47070	329	.64719	267	.64912	269	24
25	7.47270	327	7.47399	328	7.64986	266	7.65181	268	25
26	.47597	325	.47727	327	.65253	266	.65449	267	26
27	.47922	324	.48054	325	.65519	265	.65716	266	27
28	.48247	323	.48379	324	.65784	264	.65982	265	28
29	.48570	322	.48703	323	.66048	263	.66247	264	29
30	7.48892	321	7.49026	322	7.66311	263	7.66512	264	30
31	.49213	320	.49348	321	.66574	261	.66776	263	31
32	.49533	318	.49669	319	.66836	261	.67039	262	32
33	.49852	317	.49989	318	.67097	260	.67301	261	33
34	.50169	316	.50307	317	.67357	259	.67562	261	34
35	7.50485	315	7.50624	316	7.67617	258	7.67823	260	35
36	.50800	314	.50941	315	.67875	258	.68083	259	36
37	.51114	313	.51256	313	.68133	257	.68342	258	37
38	.51427	311	.51569	313	.68390	256	.68601	257	38
39	.51739	311	.51882	311	.68647	255	.68858	257	39
40	7.52050	309	7.52194	310	7.68902	255	7.69115	256	40
41	.52359	308	.52504	309	.69157	254	.69371	255	41
42	.52667	307	.52814	308	.69411	253	.69627	254	42
43	.52975	306	.53122	307	.69665	252	.69881	254	43
44	.53281	305	.53429	306	.69917	252	.70135	253	44
45	7.53586	304	7.53735	305	7.70169	251	7.70388	252	45
46	.53890	303	.54041	304	.70421	250	.70641	252	46
47	.54193	302	.54345	303	.70671	250	.70893	251	47
48	.54495	300	.54648	302	.70921	249	.71144	250	48
49	.54796	300	.54950	301	.71170	248	.71394	250	49
50	7.55096	299	7.55251	299	7.71418	247	7.71644	248	50
51	.55395	297	.55550	299	.71666	247	.71892	248	51
52	.55692	297	.55849	298	.71913	246	.72141	247	52
53	.55989	295	.56147	296	.72159	245	.72388	246	53
54	.56285	295	.56444	296	.72404	245	.72635	246	54
55	7.56580	293	7.56740	295	7.72649	244	7.72881	245	55
56	.56873	293	.57035	294	.72893	243	.73126	245	56
57	.57166	292	.57329	292	.73137	242	.73371	244	57
58	.57458	290	.57621	292	.73379	242	.73615	243	58
59	.57749	290	.57913	291	.73621	241	.73859	242	59
60	7.58039		7.58204		7.73863		7.74101		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	360	**350**	**340**
6	36.0	35.0	34.0
7	42.0	40.8	39.6
8	48.0	46.6	45.3
9	54.0	51.5	51.0
10	60.0	58.3	56.6
20	120.0	116.6	113.3
30	180.0	175.0	170.0
40	240.0	233.3	226.6
50	300.0	291.6	283.3
	330	**320**	**310**
6	33.0	32.0	31.0
7	38.5	37.3	36.1
8	44.0	42.6	41.3
9	49.5	48.0	46.5
10	55.0	53.3	51.6
20	110.0	106.6	103.3
30	165.0	160.0	155.0
40	220.0	213.3	206.6
50	275.0	266.6	258.3
	300	**290**	**280**
6	30.0	29.0	28.0
7	35.0	33.8	32.6
8	40.0	38.6	37.3
9	45.0	43.5	42.0
10	50.0	48.3	46.6
20	100.0	96.6	93.3
30	150.0	145.0	140.0
40	200.0	193.3	186.6
50	250.0	241.6	233.3
	270	**260**	**250**
6	27.0	26.0	25.0
7	31.5	30.3	29.1
8	36.0	34.6	33.3
9	40.5	39.0	37.5
10	45.0	43.3	41.6
20	90.0	86.6	83.3
30	135.0	130.0	125.0
40	180.0	173.3	166.6
50	225.0	216.6	208.3
	240	**230**	**220**
6	24.0	23.0	22.0
7	28.0	26.8	25.6
8	32.0	30.6	29.3
9	36.0	34.5	33.0
10	40.0	38.3	36.6
20	80.0	76.6	73.3
30	120.0	115.0	110.0
40	160.0	153.3	146.6
50	200.0	191.6	183.3
	210	**200**	**190**
6	21.0	20.0	19.0
7	24.5	23.3	22.1
8	28.0	26.6	25.3
9	31.5	30.0	28.5
10	35.0	33.3	31.6
20	70.0	66.6	63.3
30	105.0	100.0	95.0
40	140.0	133.3	126.6
50	175.0	166.6	158.3

P. P.

′	6° Lg. Vers.	D	6° Log. Exs.	D	7° Lg. Vers.	D	7° Log. Exs.	D	′
0	7.73863	241	7.7410$\bar{1}$	242	7.8723$\bar{8}$	20$\bar{6}$	7.87563	208	0
1	.74104	240	.7434$\bar{3}$	24$\bar{1}$	.8744$\bar{4}$	20$\bar{5}$	.87771	207	1
2	.74344	239	.74585	241	.8765$\bar{0}$	20$\bar{5}$	.87978	207	2
3	.74583	239	.74826	240	.8785$\bar{5}$	20$\bar{4}$	.88185	20$\bar{6}$	3
4	.74822	238	.75066	239	.88060	20$\bar{4}$	.8839$\bar{1}$	206	4
5	7.75060	23$\bar{7}$	7.75305	239	7.8826$\bar{4}$	204	7.8859$\bar{7}$	205	5
6	.7529$\bar{7}$	23$\bar{6}$	.75544	238	.88468	20$\bar{3}$	.88803	205	6
7	.7553$\bar{4}$	23$\bar{6}$	.7578$\bar{2}$	23$\bar{7}$	.88672	203	.89008	20$\bar{4}$	7
8	.7577$\bar{0}$	23$\bar{5}$	.7601$\bar{9}$	237	.8887$\bar{5}$	20$\bar{2}$	.8921$\bar{2}$	204	8
9	.76006	23$\bar{4}$	.7625$\bar{6}$	236	.8907$\bar{7}$	202	.8941$\bar{6}$	20$\bar{3}$	9
10	7.7624$\bar{0}$	23$\bar{4}$	7.7649$\bar{2}$	23$\bar{5}$	7.8927$\bar{9}$	20$\bar{1}$	7.89620	203	10
11	.7647$\bar{5}$	23$\bar{3}$	.76728	235	.89481	20$\bar{1}$	.8982$\bar{3}$	20$\bar{2}$	11
12	.7670$\bar{8}$	233	.76963	23$\bar{4}$	.8968$\bar{2}$	20$\bar{0}$	.9002$\bar{5}$	20$\bar{2}$	12
13	.7694$\bar{1}$	232	.7719$\bar{7}$	23$\bar{3}$	.8988$\bar{2}$	200	.90228	20$\bar{1}$	13
14	.7717$\bar{3}$	232	.77431	23$\bar{3}$	.9008$\bar{2}$	19$\bar{9}$	.9042$\bar{9}$	201	14
15	7.7740$\bar{5}$	23$\bar{1}$	7.7766$\bar{4}$	232	7.90282	19$\bar{9}$	7.9063$\bar{0}$	20$\bar{1}$	15
16	.7763$\bar{6}$	23$\bar{0}$	.7789$\bar{7}$	23$\bar{1}$	.9048$\bar{1}$	19$\bar{8}$	.9083$\bar{1}$	20$\bar{0}$	16
17	.77867	230	.7812$\bar{8}$	23$\bar{1}$	.9068$\bar{0}$	19$\bar{8}$	.9103$\bar{2}$	19$\bar{9}$	17
18	.78097	229	.78360	23$\bar{0}$	.9087$\bar{8}$	19$\bar{7}$	.9123$\bar{1}$	19$\bar{9}$	18
19	.78326	22$\bar{8}$	.7859$\bar{0}$	230	.91076	19$\bar{7}$	.91431	199	19
20	7.78554	228	7.78820	22$\bar{9}$	7.9127$\bar{3}$	19$\bar{7}$	7.9163$\bar{0}$	19$\bar{8}$	20
21	.78783	227	.79050	229	.9147$\bar{0}$	19$\bar{6}$	.9182$\bar{8}$	19$\bar{8}$	21
22	.79010	22$\bar{7}$	.79279	228	.91667	19$\bar{6}$	.9202$\bar{7}$	19$\bar{7}$	22
23	.7923$\bar{7}$	22$\bar{6}$	.79507	228	.91863	19$\bar{5}$	.9222$\bar{4}$	197	23
24	.7946$\bar{3}$	22$\bar{5}$	.79735	22$\bar{7}$	.9205$\bar{8}$	195	.9242$\bar{1}$	197	24
25	7.79689	22$\bar{5}$	7.7996$\bar{2}$	22$\bar{6}$	7.9225$\bar{3}$	195	7.9261$\bar{8}$	19$\bar{6}$	25
26	.7991$\bar{4}$	22$\bar{4}$	.8018$\bar{8}$	226	.9244$\bar{8}$	194	.9281$\bar{5}$	19$\bar{5}$	26
27	.8013$\bar{8}$	224	.8041$\bar{4}$	225	.9264$\bar{2}$	194	.9301$\bar{0}$	19$\bar{5}$	27
28	.8036$\bar{2}$	22$\bar{3}$	.8063$\bar{9}$	225	.9283$\bar{6}$	193	.93206	195	28
29	.80586	22$\bar{2}$	.8086$\bar{4}$	224	.9302$\bar{9}$	193	.93401	195	29
30	7.8080$\bar{8}$	22$\bar{2}$	7.8108$\bar{8}$	224	7.9322$\bar{2}$	19$\bar{2}$	7.93596	194	30
31	.8103$\bar{1}$	22$\bar{1}$	.8131$\bar{2}$	223	.9341$\bar{5}$	19$\bar{2}$	.93790	194	31
32	.8125$\bar{2}$	221	.8153$\bar{5}$	22$\bar{2}$	.9360$\bar{7}$	19$\bar{1}$	.93984	193	32
33	.8147$\bar{3}$	22$\bar{0}$	.81758	222	.93799	19$\bar{1}$	.94177	193	33
34	.81694	220	.81980	221	.9399$\bar{0}$	19$\bar{0}$	.94370	19$\bar{2}$	34
35	7.81914	219	7.82201	22$\bar{1}$	7.9418$\bar{1}$	19$\bar{0}$	7.9456$\bar{2}$	192	35
36	.82133	219	.8242$\bar{2}$	22$\bar{0}$	.9437$\bar{1}$	19$\bar{0}$	.9475$\bar{4}$	192	36
37	.8235$\bar{2}$	21$\bar{8}$	.8264$\bar{2}$	21$\bar{9}$	.9456$\bar{1}$	18$\bar{9}$	.9494$\bar{6}$	191	37
38	.8257$\bar{0}$	21$\bar{7}$	.82862	219	.94751	189	.9513$\bar{7}$	191	38
39	.82788	217	.83081	219	.94940	18$\bar{9}$	.9532$\bar{8}$	19$\bar{0}$	39
40	7.83005	217	7.8330$\bar{0}$	218	7.9512$\bar{9}$	18$\bar{8}$	7.95519	190	40
41	.83222	216	.8351$\bar{8}$	21$\bar{7}$	.9531$\bar{7}$	18$\bar{7}$	.95709	18$\bar{9}$	41
42	.83438	21$\bar{5}$	.8373$\bar{5}$	21$\bar{7}$	.95505	188	.9589$\bar{8}$	18$\bar{9}$	42
43	.8365$\bar{3}$	215	.8395$\bar{2}$	216	.95693	187	.96088	18$\bar{8}$	43
44	.8386$\bar{8}$	21$\bar{4}$	.84169	216	.95880	18$\bar{6}$	.9627$\bar{6}$	18$\bar{8}$	44
45	7.84083	214	7.8438$\bar{5}$	21$\bar{5}$	7.9606$\bar{6}$	18$\bar{6}$	7.96465	188	45
46	.8429$\bar{7}$	21$\bar{3}$	.8460$\bar{0}$	215	.96253	18$\bar{6}$	.96653	188	46
47	.8451$\bar{0}$	213	.8481$\bar{5}$	21$\bar{4}$	.96439	185	.96841	187	47
48	.8472$\bar{3}$	212	.85030	21$\bar{3}$	.9662$\bar{4}$	185	.97028	187	48
49	.8493$\bar{5}$	212	.8524$\bar{3}$	21$\bar{3}$	.9680$\bar{9}$	18$\bar{4}$	.97215	18$\bar{6}$	49
50	7.8514$\bar{7}$	21$\bar{1}$	7.85457	213	7.96994	18$\bar{4}$	7.9740$\bar{1}$	186	50
51	.85359	211	.85670	21$\bar{2}$	.9717$\bar{8}$	184	.9758$\bar{7}$	18$\bar{5}$	51
52	.85570	210	.8588$\bar{2}$	21$\bar{1}$	.9736$\bar{2}$	18$\bar{3}$	.97773	18$\bar{5}$	52
53	.85780	210	.8609$\bar{4}$	21$\bar{1}$	.97546	183	.9795$\bar{8}$	18$\bar{4}$	53
54	.8599$\bar{0}$	209	.8630$\bar{5}$	211	.97729	183	.98143	18$\bar{4}$	54
55	7.86199	209	7.8651$\bar{6}$	210	7.97912	182	7.9832$\bar{7}$	18$\bar{4}$	55
56	.86408	208	.8672$\bar{6}$	210	.98094	182	.98512	18$\bar{3}$	56
57	.86616	208	.8693$\bar{6}$	20$\bar{9}$	.98276	182	.9869$\bar{5}$	18$\bar{3}$	57
58	.8682$\bar{4}$	20$\bar{7}$	.8714$\bar{6}$	208	.98458	181	.98879	183	58
59	.8703$\bar{1}$	20$\bar{6}$	.8735$\bar{4}$	20$\bar{8}$	.98639	181	.99062	18$\bar{2}$	59
60	7.87238		7.87563		7.98820		7.9924$\bar{4}$		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

′	180	$\bar{9}$	9
6	18.0	0.$\bar{9}$	0.9
7	21.0	1.$\bar{1}$	1.$\bar{0}$
8	24.0	1.$\bar{2}$	1.2
9	27.0	1.$\bar{4}$	1.$\bar{3}$
10	30.0	1.$\bar{6}$	1.5
20	60.0	3.$\bar{1}$	3.0
30	90.0	4.$\bar{7}$	4.5
40	120.0	6.$\bar{3}$	6.0
50	150.0	7.9	7.5

	$\bar{8}$	8	$\bar{7}$
6	0.$\bar{8}$	0.8	0.$\bar{7}$
7	1.$\bar{0}$	0.$\bar{9}$	0.9
8	1.$\bar{1}$	1.$\bar{0}$	1.0
9	1.3	1.$\bar{2}$	1.$\bar{1}$
10	1.$\bar{4}$	1.$\bar{3}$	1.$\bar{2}$
20	2.$\bar{8}$	2.$\bar{6}$	2.$\bar{5}$
30	4.$\bar{2}$	4.0	3.$\bar{7}$
40	5.$\bar{6}$	5.$\bar{3}$	5.$\bar{0}$
50	7.1	6.$\bar{6}$	6.$\bar{2}$

	7	$\bar{6}$	6
6	0.7	0.$\bar{6}$	0.6
7	0.$\bar{8}$	0.$\bar{7}$	0.7
8	0.$\bar{9}$	0.$\bar{8}$	0.8
9	1.$\bar{0}$	1.0	0.9
10	1.$\bar{1}$	1.$\bar{1}$	1.0
20	2.$\bar{3}$	2.$\bar{1}$	2.0
30	3.5	3.$\bar{2}$	3.0
40	4.$\bar{6}$	4.$\bar{3}$	4.0
50	5.8	5.4	5.0

	$\bar{5}$	5	$\bar{4}$	4
6	0.$\bar{5}$	0.5	0.$\bar{4}$	0.$\bar{4}$
7	0.$\bar{6}$	0.6	0.5	0.$\bar{4}$
8	0.$\bar{7}$	0.$\bar{6}$	0.6	0.$\bar{5}$
9	0.8	0.$\bar{7}$	0.$\bar{7}$	0.$\bar{6}$
10	0.9	0.$\bar{8}$	0.$\bar{7}$	0.$\bar{6}$
20	1.$\bar{8}$	1.$\bar{6}$	1.$\bar{5}$	1.3
30	2.$\bar{7}$	2.5	2.$\bar{2}$	2.$\bar{0}$
40	3.$\bar{6}$	3.$\bar{3}$	3.$\bar{0}$	2.$\bar{6}$
50	4.6	4.$\bar{1}$	3.$\bar{7}$	3.$\bar{3}$

	$\bar{3}$	3	$\bar{2}$	2
6	0.$\bar{3}$	0.3	0.$\bar{2}$	0.2
7	0.$\bar{4}$	0.$\bar{3}$	0.$\bar{3}$	0.$\bar{2}$
8	0.$\bar{4}$	0.4	0.$\bar{3}$	0.$\bar{2}$
9	0.5	0.$\bar{4}$	0.4	0.3
10	0.6	0.5	0.$\bar{4}$	0.$\bar{3}$
20	1.$\bar{1}$	1.0	0.$\bar{8}$	0.6
30	1.$\bar{7}$	1.5	1.$\bar{2}$	1.0
40	2.$\bar{3}$	2.0	1.$\bar{6}$	1.$\bar{3}$
50	2.9	2.5	2.1	1.$\bar{6}$

	$\bar{1}$	1	$\bar{0}$
6	0.$\bar{1}$	0.1	0.$\bar{0}$
7	0.2	0.$\bar{1}$	0.$\bar{0}$
8	0.2	0.$\bar{1}$	0.0
9	0.$\bar{2}$	0.$\bar{1}$	0.1
10	0.$\bar{2}$	0.$\bar{1}$	0.$\bar{1}$
20	0.$\bar{5}$	0.$\bar{3}$	0.$\bar{1}$
30	0.$\bar{7}$	0.5	0.$\bar{2}$
40	1.$\bar{0}$	0.$\bar{6}$	0.3
50	1.$\bar{2}$	0.$\bar{8}$	0.4

P. P.

	8°				9°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	7.98820	180	7.99244	182	8.09031	160	8.09569	162	0
1	.99000	180	.99427	182	.09192	160	.09732	162	1
2	.99180	179	.99609	181	.09352	160	.09894	162	2
3	.99360	179	.99790	181	.09512	159	.10056	161	3
4	.99539	179	7.99971	180	.09671	159	.10217	161	4
5	7.99718	178	8.00152	180	8.09830	159	8.10378	161	5
6	7.99897	178	.00332	180	.09989	158	.10539	160	6
7	8.00075	177	.00512	180	.10148	158	.10700	160	7
8	.00253	178	.00692	179	.10306	158	.10860	160	8
9	.00431	177	.00871	179	.10464	157	.11020	160	9
10	8.00608	176	8.01050	178	8.10622	157	8.11180	159	10
11	.00784	176	.01229	178	.10779	157	.11340	159	11
12	.00961	176	.01407	178	.10936	157	.11499	159	12
13	.01137	176	.01585	177	.11093	156	.11658	158	13
14	.01313	175	.01763	177	.11250	156	.11816	158	14
15	8.01488	175	8.01940	177	8.11406	156	8.11975	158	15
16	.01663	175	.02117	176	.11562	155	.12133	158	16
17	.01838	174	.02293	176	.11718	155	.12291	157	17
18	.02012	174	.02469	175	.11873	155	.12448	157	18
19	.02186	173	.02645	175	.12029	155	.12605	157	19
20	8.02359	173	8.02820	175	8.12184	154	8.12762	157	20
21	.02533	173	.02995	175	.12338	154	.12919	156	21
22	.02706	172	.03170	174	.12492	154	.13075	156	22
23	.02878	172	.03345	174	.12647	153	.13232	155	23
24	.03050	172	.03519	173	.12800	153	.13387	156	24
25	8.03222	171	8.03692	173	8.12954	153	8.13543	155	25
26	.03394	171	.03866	173	.13107	153	.13698	155	26
27	.03565	171	.04039	173	.13260	152	.13854	154	27
28	.03736	170	.04212	172	.13413	152	.14008	154	28
29	.03906	170	.04384	172	.13565	152	.14163	154	29
30	8.04076	170	8.04556	171	8.13717	152	8.14317	154	30
31	.04246	169	.04728	171	.13869	151	.14471	153	31
32	.04416	169	.04899	171	.14021	151	.14625	153	32
33	.04585	169	.05070	170	.14172	151	.14778	153	33
34	.04754	168	.05241	170	.14323	151	.14932	153	34
35	8.04922	168	8.05411	170	8.14474	150	8.15085	152	35
36	.05090	168	.05581	170	.14625	150	.15237	152	36
37	.05258	167	.05751	169	.14775	150	.15390	152	37
38	.05426	167	.05921	169	.14925	149	.15542	152	38
39	.05593	167	.06090	169	.15075	150	.15694	152	39
40	8.05760	166	8.06259	168	8.15225	149	8.15846	151	40
41	.05926	166	.06427	168	.15374	149	.15997	151	41
42	.06093	166	.06595	168	.15523	149	.16148	151	42
43	.06259	165	.06763	167	.15672	148	.16299	150	43
44	.06424	165	.06931	167	.15820	148	.16450	150	44
45	8.06589	165	8.07098	167	8.15968	148	8.16600	150	45
46	.06754	165	.07265	166	.16116	148	.16750	150	46
47	.06919	164	.07431	166	.16264	147	.16900	149	47
48	.07083	164	.07598	166	.16412	147	.17050	149	48
49	.07247	164	.07764	165	.16559	147	.17199	149	49
50	8.07411	163	8.07929	165	8.16706	146	8.17349	148	50
51	.07575	163	.08095	165	.16852	146	.17497	149	51
52	.07738	162	.08260	164	.16999	146	.17646	148	52
53	.07900	162	.08424	164	.17145	146	.17795	148	53
54	.08063	162	.08589	164	.17291	145	.17943	148	54
55	8.08225	161	8.08753	163	8.17437	145	8.18091	147	55
56	.08387	162	.08917	164	.17582	145	.18238	147	56
57	.08549	161	.09081	163	.17728	145	.18386	147	57
58	.08710	161	.09244	163	.17873	144	.18533	147	58
59	.08871	160	.09407	162	.18017	144	.18680	146	59
60	8.09031		8.09569		8.18162		8.18827		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	180	170	160
6	18.0	17.0	16.0
7	21.0	19.8	18.6
8	24.0	22.6	21.3
9	27.0	25.5	24.0
10	30.0	28.3	26.6
20	60.0	56.6	53.3
30	90.0	85.0	80.0
40	120.0	113.3	106.6
50	150.0	141.6	133.3

	150	140
6	15.0	14.0
7	17.5	16.3
8	20.0	18.6
9	22.5	21.0
10	25.0	23.3
20	50.0	46.6
30	75.0	70.0
40	100.0	93.3
50	125.0	116.6

	$\bar{9}$	9	$\bar{8}$
6	0.9	0.9	0.8
7	1.1	1.0	1.0
8	1.2	1.2	1.1
9	1.4	1.3	1.3
10	1.6	1.5	1.4
20	3.1	3.0	2.8
30	4.7	4.5	4.2
40	6.3	6.0	5.6
50	7.9	7.5	7.1

	8	$\bar{7}$	7
6	0.8	0.7	0.7
7	0.9	0.9	0.8
8	1.0	1.0	0.9
9	1.2	1.1	1.0
10	1.3	1.2	1.1
20	2.6	2.5	2.3
30	4.0	3.7	3.5
40	5.3	5.0	4.6
50	6.6	6.2	5.8

	$\bar{6}$	6
6	0.6	0.6
7	0.7	0.7
8	0.8	0.8
9	1.0	0.9
10	1.1	1.0
20	2.1	2.0
30	3.2	3.0
40	4.3	4.0
50	5.4	5.0

	$\bar{5}$	5
6	0.5	0.5
7	0.6	0.6
8	0.7	0.6
9	0.8	0.7
10	0.9	0.8
20	1.8	1.6
30	2.7	2.5
40	3.6	3.3
50	4.6	4.1

P. P.

	10°				11°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	8.18162	144	8.18827	146	8.26417	131	8.27223	133	0
1	.18306	144	.18973	146	.26548	131	.27356	133	1
2	.18450	144	.19120	146	.26679	131	.27490	133	2
3	.18594	143	.19266	145	.26810	130	.27623	133	3
4	.18738	143	.19411	145	.26941	130	.27756	133	4
5	8.18881	143	8.19557	145	8.27071	130	8.27889	132	5
6	.19024	142	.19702	145	.27201	130	.28021	132	6
7	.19167	142	.19847	145	.27331	130	.28153	132	7
8	.19309	142	.19992	144	.27461	129	.28286	132	8
9	.19452	142	.20137	144	.27590	129	.28418	132	9
10	8.19594	142	8.20281	144	8.27719	129	8.28550	131	10
11	.19736	142	.20425	144	.27849	128	.28681	131	11
12	.19878	141	.20569	144	.27977	129	.28813	131	12
13	.20019	141	.20713	143	.28106	128	.28944	131	13
14	.20160	141	.20857	143	.28235	128	.29075	131	14
15	8.20301	140	8.21000	143	8.28363	128	8.29206	130	15
16	.20442	140	.21143	143	.28491	128	.29336	130	16
17	.20582	140	.21286	142	.28619	128	.29467	130	17
18	.20723	140	.21428	142	.28747	127	.29597	130	18
19	.20863	140	.21571	142	.28875	127	.29727	130	19
20	8.21003	139	8.21713	142	8.29002	127	8.29857	130	20
21	.21142	139	.21855	141	.29129	127	.29987	129	21
22	.21282	139	.21996	141	.29256	127	.30117	129	22
23	.21421	139	.22138	141	.29383	126	.30246	129	23
24	.21560	138	.22279	141	.29510	126	.30375	129	24
25	8.21698	138	8.22420	140	8.29636	126	8.30504	129	25
26	.21837	138	.22561	140	.29763	126	.30633	128	26
27	.21975	138	.22701	140	.29889	126	.30762	128	27
28	.22113	137	.22842	140	.30015	125	.30890	128	28
29	.22251	138	.22982	140	.30140	125	.31019	128	29
30	8.22389	137	8.23122	140	8.30266	125	8.31147	128	30
31	.22526	137	.23262	139	.30391	125	.31275	127	31
32	.22663	136	.23401	139	.30516	125	.31402	127	32
33	.22800	137	.23540	139	.30642	124	.31530	127	33
34	.22937	136	.23679	139	.30766	124	.31657	127	34
35	8.23073	136	8.23818	138	8.30891	124	8.31785	127	35
36	.23209	136	.23957	138	.31015	124	.31912	127	36
37	.23346	135	.24095	138	.31140	124	.32039	126	37
38	.23481	136	.24234	138	.31264	124	.32165	126	38
39	.23617	135	.24372	137	.31388	123	.32292	126	39
40	8.23752	135	8.24509	138	8.31511	124	8.32418	126	40
41	.23888	135	.24647	137	.31635	123	.32544	126	41
42	.24023	135	.24784	137	.31758	123	.32670	126	42
43	.24158	134	.24922	137	.31882	123	.32796	125	43
44	.24292	134	.25059	136	.32005	123	.32922	125	44
45	8.24426	134	8.25195	136	8.32128	122	8.33047	125	45
46	.24561	134	.25332	136	.32250	122	.33173	125	46
47	.24695	133	.25468	136	.32373	122	.33298	125	47
48	.24828	133	.25604	136	.32495	122	.33423	124	48
49	.24962	133	.25740	136	.32617	122	.33547	125	49
50	8.25095	133	8.25876	135	8.32739	122	8.33672	124	50
51	.25228	133	.26012	135	.32861	121	.33797	124	51
52	.25361	132	.26147	135	.32983	121	.33921	124	52
53	.25494	133	.26282	135	.33104	121	.34045	123	53
54	.25627	132	.26417	134	.33225	121	.34169	124	54
55	8.25759	132	8.26552	134	8.33347	121	8.34293	124	55
56	.25891	132	.26686	134	.33468	120	.34417	123	56
57	.26023	132	.26821	134	.33588	120	.34540	123	57
58	.26155	131	.26955	134	.33709	120	.34663	123	58
59	.26286	131	.27089	134	.33829	120	.34786	123	59
60	8.26417		8.27223		8.33950		8.34909		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	130	120
6	13.0	12.0
7	15.1	14.0
8	17.3	16.0
9	19.5	18.0
10	21.6	20.0
20	43.3	40.0
30	65.0	60.0
40	86.6	80.0
50	108.3	100.0

	4	4	3
6	0.4	0.4	0.3
7	0.5	0.4	0.4
8	0.6	0.5	0.4
9	0.7	0.6	0.5
10	0.7	0.6	0.6
20	1.5	1.3	1.1
30	2.2	2.0	1.7
40	3.0	2.6	2.3
50	3.7	3.3	2.9

	3	2
6	0.3	0.2
7	0.3	0.3
8	0.4	0.3
9	0.4	0.4
10	0.5	0.4
20	1.0	0.8
30	1.5	1.2
40	2.0	1.6
50	2.5	2.1

	2	1
6	0.2	0.1
7	0.2	0.2
8	0.2	0.2
9	0.3	0.2
10	0.3	0.2
20	0.6	0.5
30	1.0	0.7
40	1.3	1.0
50	1.6	1.2

	1	0
6	0.1	0.0
7	0.1	0.0
8	0.1	0.0
9	0.1	0.1
10	0.1	0.1
20	0.3	0.1
30	0.5	0.2
40	0.6	0.3
50	0.8	0.4

P. P.

	12°				13°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	8.33950	120	8.34909	123	8.40875	110	8.42002	113	0
1	.34070	120	.35032	122	.40985	110	.42116	113	1
2	.34190	119	.35155	122	.41096	110	.42229	113	2
3	.34309	120	.35277	122	.41206	110	.42343	113	3
4	.34429	119	.35399	122	.41317	110	.42456	113	4
5	8.34549	119	8.35522	122	8.41427	110	8.42569	113	5
6	.34668	119	.35644	121	.41537	110	.42682	113	6
7	.34787	119	.35765	122	.41647	109	.42795	113	7
8	.34906	119	.35887	121	.41757	110	.42908	112	8
9	.35025	118	.36009	121	.41867	109	.43021	112	9
10	8.35143	118	8.36130	121	8.41976	109	8.43133	112	10
11	.35262	118	.36251	121	.42086	109	.43246	112	11
12	.35380	118	.36372	120	.42195	109	.43358	112	12
13	.35498	118	.36493	121	.42304	109	.43470	112	13
14	.35616	117	.36614	120	.42413	109	.43582	112	14
15	8.35734	118	8.36734	120	8.42522	108	8.43694	111	15
16	.35852	117	.36855	120	.42630	109	.43805	111	16
17	.35969	117	.36975	120	.42739	108	.43917	111	17
18	.36086	117	.37095	120	.42847	108	.44028	111	18
19	.36204	117	.37215	120	.42956	108	.44139	111	19
20	8.36321	116	8.37335	119	8.43064	108	8.44251	111	20
21	.36437	117	.37454	119	.43172	108	.44362	111	21
22	.36554	116	.37574	119	.43280	108	.44473	110	22
23	.36671	116	.37693	119	.43388	107	.44583	110	23
24	.36787	116	.37812	119	.43495	107	.44694	110	24
25	8.36903	116	8.37931	118	8.43603	107	8.44804	110	25
26	.37019	116	.38050	119	.43710	107	.44915	110	26
27	.37135	115	.38169	118	.43817	107	.45025	110	27
28	.37251	115	.38287	118	.43924	107	.45135	109	28
29	.37366	115	.38406	118	.44031	106	.45245	110	29
30	8.37482	115	8.38524	118	8.44138	107	8.45355	110	30
31	.37597	115	.38642	118	.44245	106	.45465	109	31
32	.37712	115	.38760	118	.44351	106	.45574	109	32
33	.37827	115	.38878	117	.44458	106	.45684	109	33
34	.37942	114	.38995	117	.44564	106	.45793	109	34
35	8.38057	114	8.39113	117	8.44670	106	8.45902	109	35
36	.38171	114	.39230	117	.44776	105	.46011	109	36
37	.38286	114	.39347	117	.44882	106	.46120	108	37
38	.38400	114	.39464	117	.44988	105	.46229	109	38
39	.38514	114	.39581	116	.45093	105	.46338	108	39
40	8.38628	113	8.39698	116	8.45199	105	8.46446	108	40
41	.38741	114	.39814	116	.45304	105	.46555	108	41
42	.38855	113	.39931	116	.45409	105	.46663	108	42
43	.38969	113	.40047	116	.45514	105	.46771	108	43
44	.39082	113	.40163	116	.45619	105	.46879	108	44
45	8.39195	113	8.40279	116	8.45724	104	8.46987	107	45
46	.39308	113	.40395	115	.45829	105	.47095	108	46
47	.39421	113	.40511	115	.45934	104	.47203	107	47
48	.39534	112	.40626	115	.46038	104	.47310	107	48
49	.39646	112	.40742	115	.46142	104	.47417	107	49
50	8.39758	112	8.40857	115	8.46247	104	8.47525	107	50
51	.39871	112	.40972	115	.46351	104	.47632	107	51
52	.39983	112	.41087	115	.46455	103	.47739	107	52
53	.40095	112	.41202	114	.46558	104	.47846	106	53
54	.40207	111	.41317	114	.46662	103	.47953	107	54
55	8.40318	111	8.41431	114	8.46766	103	8.48060	106	55
56	.40430	111	.41546	114	.46869	103	.48166	106	56
57	.40541	111	.41660	114	.46972	103	.48273	106	57
58	.40652	111	.41774	114	.47076	103	.48379	106	58
59	.40764	111	.41888	114	.47179	103	.48485	106	59
60	8.40875		8.42002		8.47282		8.48591		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	120	119	118
6	12.0	11.9	11.8
7	14.0	13.9	13.7
8	16.0	15.8	15.7
9	18.0	17.8	17.7
10	20.0	19.8	19.6
20	40.0	39.6	39.3
30	60.0	59.5	59.0
40	80.0	79.3	78.6
50	100.0	99.1	98.3

	117	116	115
6	11.7	11.6	11.5
7	13.6	13.5	13.4
8	15.6	15.4	15.3
9	17.5	17.4	17.2
10	19.5	19.3	19.1
20	39.0	38.6	38.3
30	58.5	58.0	57.5
40	78.0	77.3	76.6
50	97.5	96.6	95.8

	114	113	112
6	11.4	11.3	11.2
7	13.3	13.2	13.0
8	15.2	15.0	14.9
9	17.1	16.0	16.8
10	19.0	18.8	18.6
20	38.0	37.6	37.3
30	57.0	56.5	56.0
40	76.0	75.3	74.6
50	95.0	94.1	93.3

	111	110	109
6	11.1	11.0	10.9
7	12.9	12.8	12.7
8	14.8	14.6	14.5
9	16.6	16.5	16.3
10	18.5	18.3	18.1
20	37.0	36.6	36.3
30	55.5	55.0	54.5
40	74.0	73.3	72.6
50	92.5	91.6	90.8

	108	107	106
6	10.8	10.7	10.6
7	12.6	12.5	12.3
8	14.4	14.2	14.1
9	16.2	16.0	15.9
10	18.0	17.8	17.6
20	36.0	35.6	35.3
30	54.0	53.5	53.0
40	72.0	71.3	70.6
50	90.0	89.1	88.3

	105	104	0
6	10.5	10.4	0.0
7	12.2	12.1	0.0
8	14.0	13.8	0.0
9	15.7	15.6	0.1
10	17.5	17.3	0.1
20	35.0	34.6	0.1
30	52.5	52.0	0.2
40	70.0	69.3	0.3
50	87.5	86.6	0.4

P. P.

′	14° Lg. Vers.	D	14° Log. Exs.	D	15° Lg. Vers.	D	15° Log. Exs.	D	′
0	8.47282	102	8.48591	106	8.53242	96	8.54748	99	0
1	.47384	103	.48697	106	.53338	95	.54847	99	1
2	.47487	102	.48803	105	.53434	96	.54946	99	2
3	.47590	102	.48909	105	.53530	95	.55045	99	3
4	.47692	102	.49014	105	.53625	95	.55144	99	4
5	8.47795	102	8.49120	105	8.53721	95	8.55243	99	5
6	.47897	102	.49225	105	.53816	95	.55342	98	6
7	.47999	102	.49331	105	.53911	95	.55441	98	7
8	.48101	102	.49436	105	.54007	95	.55539	98	8
9	.48203	101	.49541	105	.54102	95	.55638	98	9
10	8.48304	101	8.49646	104	8.54197	94	8.55736	98	10
11	.48406	101	.49750	105	.54291	95	.55834	98	11
12	.48507	101	.49855	104	.54386	94	.55933	98	12
13	.48609	101	.49960	104	.54481	94	.56031	98	13
14	.48710	101	.50064	104	.54575	94	.56129	97	14
15	8.48811	101	8.50168	104	8.54670	94	8.56226	98	15
16	.48912	101	.50273	104	.54764	94	.56324	97	16
17	.49013	100	.50377	104	.54858	94	.56422	97	17
18	.49114	101	.50481	104	.54952	94	.56519	97	18
19	.49215	100	.50585	103	.55046	94	.56617	97	19
20	8.49315	100	8.50688	104	8.55140	93	8.56714	97	20
21	.49415	100	.50792	103	.55234	94	.56812	97	21
22	.49516	100	.50896	103	.55328	93	.56909	97	22
23	.49616	100	.50999	103	.55421	93	.57006	97	23
24	.49716	100	.51102	103	.55515	93	.57103	97	24
25	8.49816	100	8.51205	103	8.55608	93	8.57200	96	25
26	.49916	99	.51309	103	.55701	93	.57296	97	26
27	.50015	100	.51412	102	.55795	93	.57393	96	27
28	.50115	99	.51514	103	.55888	93	.57490	96	28
29	.50215	99	.51617	102	.55981	93	.57586	96	29
30	8.50314	99	8.51720	102	8.56074	92	8.57682	96	30
31	.50413	99	.51822	102	.56166	92	.57779	96	31
32	.50512	99	.51925	102	.56259	93	.57875	96	32
33	.50611	99	.52027	102	.56352	92	.57971	96	33
34	.50710	98	.52129	102	.56444	92	.58067	95	34
35	8.50809	99	8.52231	102	8.56536	92	8.58163	96	35
36	.50908	98	.52333	102	.56629	92	.58259	95	36
37	.51006	98	.52435	101	.56721	92	.58354	96	37
38	.51105	98	.52537	101	.56813	92	.58450	95	38
39	.51203	98	.52638	101	.56905	92	.58546	95	39
40	8.51301	98	8.52740	101	8.56997	92	8.58641	95	40
41	.51399	98	.52841	101	.57089	91	.58736	95	41
42	.51497	98	.52943	101	.57180	91	.58832	95	42
43	.51595	97	.53044	101	.57272	91	.58927	95	43
44	.51693	98	.53145	101	.57363	91	.59022	95	44
45	8.51791	97	8.53246	101	8.57455	91	8.59117	94	45
46	.51888	97	.53347	101	.57546	91	.59211	95	46
47	.51986	97	.53448	100	.57637	91	.59306	94	47
48	.52083	97	.53548	100	.57728	91	.59401	94	48
49	.52180	97	.53649	100	.57819	91	.59495	94	49
50	8.52277	97	8.53749	100	8.57910	90	8.59590	94	50
51	.52374	97	.53850	100	.58001	91	.59684	94	51
52	.52471	96	.53950	100	.58092	90	.59779	94	52
53	.52568	97	.54050	100	.58182	90	.59873	94	53
54	.52665	96	.54150	100	.58273	90	.59967	94	54
55	8.52761	96	8.54250	100	8.58363	90	8.60061	94	55
56	.52858	96	.54350	99	.58453	90	.60155	94	56
57	.52954	96	.54449	100	.58544	90	.60249	93	57
58	.53050	96	.54549	99	.58634	90	.60342	94	58
59	.53146	96	.54649	99	.58724	90	.60436	93	59
60	8.53242		8.54748		8.58814		8.60530		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	103	102	101
6	10.3	10.2	10.1
7	12.0	11.9	11.8
8	13.7	13.6	13.4
9	15.4	15.3	15.1
10	17.1	17.0	16.8
20	34.3	34.0	33.6
30	51.5	51.0	50.5
40	68.6	68.0	67.3
50	85.8	85.0	84.1

	100	99	98
6	10.0	9.9	9.8
7	11.6	11.5	11.4
8	13.3	13.2	13.0
9	15.0	14.8	14.7
10	16.6	16.5	16.3
20	33.3	33.0	32.6
30	50.0	49.5	49.0
40	66.6	66.0	65.3
50	83.3	82.5	82.6

	97	96	95
6	9.7	9.6	9.5
7	11.3	11.2	11.1
8	12.9	12.8	12.6
9	14.5	14.4	14.2
10	16.1	16.0	15.8
20	32.3	32.0	31.6
30	48.5	48.0	47.5
40	64.6	64.0	63.3
50	80.8	80.0	79.1

	94	93	92
6	9.4	9.3	9.2
7	10.9	10.8	10.7
8	12.5	12.4	12.2
9	14.1	13.9	13.8
10	15.6	15.5	15.3
20	31.3	31.0	30.6
30	47.0	46.5	46.0
40	62.6	62.0	61.3
50	78.3	77.5	76.6

	91	90	0
6	9.1	9.0	0.0
7	10.6	10.5	0.0
8	12.1	12.0	0.0
9	13.6	13.5	0.1
10	15.1	15.0	0.1
20	30.3	30.0	0.1
30	45.5	45.0	0.2
40	60.6	60.0	0.3
50	75.8	75.0	0.4

P. P.

	16°				17°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	8.58814	90	8.60530	93	8.64043	84	8.65984	88	0
1	.58904	89	.60623	93	.64128	84	.66072	88	1
2	.58993	90	.60716	93	.64212	84	.66160	88	2
3	.59083	89	.60810	93	.64296	84	.66248	88	3
4	.59173	89	.60903	93	.64381	84	.66336	88	4
5	8.59262	89	8.60996	93	8.64465	84	8.66425	87	5
6	.59351	89	.61089	93	.64549	84	.66512	88	6
7	.59441	89	.61182	92	.64633	84	.66600	88	7
8	.59530	89	.61275	93	.64717	84	.66688	87	8
9	.59619	89	.61368	92	.64801	83	.66776	87	9
10	8.59708	89	8.61460	92	8.64884	83	8.66863	88	10
11	.59797	89	.61553	92	.64968	84	.66951	87	11
12	.59886	88	.61645	92	.65052	83	.67039	87	12
13	.59974	89	.61738	92	.65135	83	.67126	87	13
14	.60063	88	.61830	92	.65218	83	.67213	87	14
15	8.60152	88	8.61922	92	8.65302	83	8.67301	87	15
16	.60240	88	.62014	92	.65385	83	.67388	87	16
17	.60328	88	.62106	92	.65468	83	.67475	87	17
18	.60417	88	.62198	92	.65551	83	.67562	87	18
19	.60505	88	.62290	91	.65634	83	.67649	87	19
20	8.60593	88	8.62382	92	8.65717	83	8.67736	86	20
21	.60681	88	.62474	91	.65800	82	.67822	87	21
22	.60769	88	.62565	91	.65883	82	.67909	86	22
23	.60857	87	.62657	91	.65965	83	.67996	86	23
24	.60944	87	.62748	91	.66048	82	.68082	86	24
25	8.61032	87	8.62840	91	8.66131	82	8.68169	86	25
26	.61119	87	.62931	91	.66213	82	.68255	86	26
27	.61207	87	.63022	91	.66295	82	.68341	86	27
28	.61294	87	.63113	91	.66378	82	.68428	86	28
29	.61381	87	.63204	90	.66460	82	.68514	86	29
30	8.61469	87	8.63295	91	8.66542	82	8.68600	86	30
31	.61556	87	.63386	91	.66624	82	.68686	85	31
32	.61643	87	.63477	90	.66706	82	.68772	86	32
33	.61730	86	.63567	90	.66788	82	.68858	86	33
34	.61816	87	.63658	90	.66870	81	.68944	85	34
35	8.61903	86	8.63748	90	8.66951	81	8.69029	86	35
36	.61990	86	.63839	90	.67033	82	.69115	85	36
37	.62076	86	.63929	90	.67115	81	.69201	85	37
38	.62163	86	.64019	90	.67196	81	.69286	85	38
39	.62249	86	.64109	90	.67277	81	.69372	85	39
40	8.62336	86	8.64199	90	8.67359	81	8.69457	85	40
41	.62422	86	.64289	90	.67440	81	.69542	85	41
42	.62508	86	.64379	90	.67521	81	.69627	85	42
43	.62594	86	.64469	89	.67602	81	.69712	85	43
44	.62680	86	.64559	90	.67683	81	.69798	85	44
45	8.62766	86	8.64649	89	8.67764	81	8.69883	84	45
46	.62852	85	.64738	89	.67845	80	.69967	85	46
47	.62937	85	.64828	89	.67926	81	.70052	85	47
48	.63023	85	.64917	89	.68007	80	.70137	84	48
49	.63108	85	.65006	89	.68087	80	.70222	84	49
50	8.63194	85	8.65096	89	8.68168	80	8.70306	84	50
51	.63279	85	.65185	89	.68248	80	.70391	84	51
52	.63364	85	.65274	89	.68329	80	.70475	84	52
53	.63449	85	.65363	89	.68409	80	.70560	84	53
54	.63534	85	.65452	89	.68489	80	.70644	84	54
55	8.63619	85	8.65541	88	8.68569	80	8.70728	84	55
56	.63704	85	.65629	88	.68650	80	.70813	84	56
57	.63789	84	.65718	89	.68730	80	.70897	84	57
58	.63874	85	.65807	88	.68810	79	.70981	84	58
59	.63959	84	.65895	88	.68889	80	.71065	84	59
60	8.64043		8.65984		8.68969		8.71149		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	93	92	91
6	9.3	9.2	9.1
7	10.8	10.7	10.6
8	12.4	12.2	12.1
9	13.9	13.8	13.6
10	15.5	15.3	15.1
20	31.0	30.6	30.5
30	46.5	46.0	45.5
40	62.0	61.3	60.6
50	77.5	76.6	75.8

	90	89	88
6	9.0	8.9	8.8
7	10.5	10.4	10.2
8	12.0	11.8	11.7
9	13.5	13.3	13.2
10	15.0	14.8	14.6
20	30.0	29.6	29.3
30	45.0	44.5	44.0
40	60.0	59.3	58.6
50	75.0	74.1	73.3

	87	86	85
6	8.7	8.6	8.5
7	10.1	10.0	9.9
8	11.6	11.4	11.3
9	13.0	12.9	12.7
10	14.5	14.3	14.1
20	29.0	28.6	28.3
30	43.5	43.0	42.5
40	58.0	57.3	56.6
50	72.5	71.6	70.8

	84	83	82
6	8.4	8.3	8.2
7	9.8	9.7	9.5
8	11.2	11.0	10.9
9	12.6	12.4	12.3
10	14.0	13.8	13.6
20	28.0	27.6	27.3
30	42.0	41.5	41.0
40	56.0	55.3	54.6
50	70.0	69.1	68.3

	81	80	79
6	8.1	8.0	7.9
7	9.4	9.3	9.2
8	10.8	10.6	10.5
9	12.1	12.0	11.8
10	13.5	13.3	13.1
20	27.0	26.6	26.3
30	40.5	40.0	39.5
40	54.0	53.3	52.6
50	67.5	66.6	65.8

	0
6	0.0
7	0.0
8	0.0
9	0.1
10	0.1
20	0.1
30	0.2
40	0.3
50	0.4

P. P.

′	18° Lg. Vers.	D	18° Log. Exs.	D	19° Lg. Vers.	D	19° Log. Exs.	D	′
0	8.68969	79	8.71149	83	8.73625	75	8.76058	79	0
1	.69049	80	.71232	84	.73700	75	.76137	80	1
2	.69129	79	.71316	83	.73775	75	.76217	79	2
3	.69208	79	.71400	84	.73851	75	.76297	79	3
4	.69288	79	.71484	83	.73926	75	.76376	80	4
5	8.69367	79	8.71567	83	8.74001	75	8.76456	79	5
6	.69446	79	.71651	83	.74076	75	.76536	79	6
7	.69526	79	.71734	83	.74151	75	.76615	79	7
8	.69605	79	.71817	83	.74226	75	.76694	79	8
9	.69684	79	.71901	83	.74301	75	.76774	79	9
10	8.69763	79	8.71984	83	8.74376	74	8.76853	79	10
11	.69842	79	.72067	83	.74451	75	.76932	79	11
12	.69921	78	.72150	83	.74526	74	.77011	79	12
13	.70000	79	.72233	83	.74600	74	.77090	79	13
14	.70079	78	.72316	83	.74675	74	.77169	79	14
15	8.70157	78	8.72399	82	8.74749	74	8.77248	79	15
16	.70236	78	.72481	83	.74824	74	.77327	78	16
17	.70314	78	.72564	82	.74898	74	.77406	79	17
18	.70393	78	.72647	82	.74973	74	.77485	78	18
19	.70471	78	.72729	82	.75047	74	.77563	78	19
20	8.70550	78	8.72812	82	8.75121	74	8.77642	78	20
21	.70628	78	.72894	82	.75195	74	.77720	79	21
22	.70706	78	.72977	82	.75269	74	.77799	78	22
23	.70784	78	.73059	82	.75343	74	.77877	78	23
24	.70862	78	.73141	82	.75417	74	.77956	78	24
25	8.70940	78	8.73223	82	8.75491	73	8.78034	78	25
26	.71018	77	.73306	82	.75565	74	.78112	78	26
27	.71096	78	.73388	82	.75639	73	.78191	78	27
28	.71174	77	.73470	81	.75712	73	.78269	78	28
29	.71251	77	.73551	82	.75786	74	.78347	78	29
30	8.71329	77	8.73633	82	8.75860	73	8.78425	78	30
31	.71406	77	.73715	81	.75933	73	.78503	78	31
32	.71484	77	.73797	81	.76006	73	.78581	78	32
33	.71561	77	.73878	81	.76080	73	.78659	77	33
34	.71639	77	.73960	81	.76153	73	.78736	78	34
35	8.71716	77	8.74041	81	8.76226	73	8.78814	77	35
36	.71793	77	.74123	81	.76300	73	.78892	77	36
37	.71870	77	.74204	81	.76373	73	.78969	77	37
38	.71947	77	.74286	81	.76446	73	.79047	77	38
39	.72024	77	.74367	81	.76519	73	.79124	77	39
40	8.72101	76	8.74448	81	8.76592	72	8.79202	77	40
41	.72178	77	.74529	81	.76664	73	.79279	77	41
42	.72255	76	.74610	81	.76737	72	.79357	77	42
43	.72331	77	.74691	80	.76810	73	.79434	77	43
44	.72408	76	.74772	81	.76883	72	.79511	77	44
45	8.72485	76	8.74853	81	8.76955	72	8.79588	77	45
46	.72561	76	.74934	80	.77028	72	.79665	77	46
47	.72637	76	.75014	80	.77100	72	.79742	77	47
48	.72714	76	.75095	80	.77173	72	.79819	77	48
49	.72790	76	.75175	80	.77245	72	.79896	76	49
50	8.72866	76	8.75256	80	8.77317	72	8.79973	77	50
51	.72942	76	.75336	80	.77390	72	.80050	76	51
52	.73018	76	.75417	80	.77462	72	.80126	77	52
53	.73094	76	.75497	80	.77534	72	.80203	76	53
54	.73170	76	.75577	80	.77606	72	.80280	76	54
55	8.73246	76	8.75658	80	8.77678	72	8.80356	76	55
56	.73322	76	.75738	80	.77750	72	.80433	76	56
57	.73398	75	.75818	80	.77822	71	.80509	76	57
58	.73473	75	.75898	80	.77893	72	.80586	76	58
59	.73549	76	.75978	80	.77965	72	.80662	76	59
		75		80		71		76	
60	8.73625		8.76058		8.78037		8.80738		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	84	83	82
6	8.4	8.3	8.2
7	9.8	9.7	9.5
8	11.2	11.0	10.9
9	12.6	12.4	12.3
10	14.0	13.8	13.6
20	28.0	27.6	27.3
30	42.0	41.5	41.0
40	56.0	55.3	54.6
50	70.0	69.1	68.3

	81	80	79
6	8.1	8.0	7.9
7	9.4	9.3	9.2
8	10.8	10.6	10.5
9	12.1	12.0	11.8
10	13.5	13.3	13.1
20	27.0	26.6	26.3
30	40.5	40.0	39.5
40	54.0	53.3	52.6
50	67.5	66.6	65.3

	78	77	76
6	7.8	7.7	7.6
7	9.1	9.0	8.8
8	10.4	10.2	10.1
9	11.7	11.5	11.4
10	13.0	12.8	12.6
20	26.0	25.6	25.3
30	39.0	38.5	38.0
40	52.0	51.3	50.6
50	65.0	64.1	63.3

	75	74	73
6	7.5	7.4	7.3
7	8.7	8.6	8.5
8	10.0	9.8	9.7
9	11.2	11.1	10.9
10	12.5	12.3	12.1
20	25.0	24.6	24.3
30	37.5	37.0	36.5
40	50.0	49.3	48.6
50	62.5	61.6	60.8

	72	71	0
6	7.2	7.1	0.0
7	8.4	8.3	0.0
8	9.6	9.4	0.0
9	10.8	10.6	0.1
10	12.0	11.8	0.1
20	24.0	23.6	0.1
30	36.0	35.5	0.2
40	48.0	47.3	0.3
50	60.0	59.1	0.4

′	20° Lg. Vers.	D	Log. Exs.	D	21° Lg. Vers.	D	Log. Exs.	D	′
0	8.78037	71	8.80738	76	8.82229	68	8.85214	73	0
1	.78108	71	.80814	76	.82297	68	.85287	73	1
2	.78180	71	.80891	76	.82366	68	.85360	72	2
3	.78251	71	.80967	76	.82434	68	.85433	73	3
4	.78323	71	.81043	76	.82502	67	.85506	73	4
5	8.78394	71	8.81119	76	8.82569	68	8.85579	72	5
6	.78466	71	.81195	76	.82637	68	.85651	73	6
7	.78537	71	.81271	75	.82705	67	.85724	72	7
8	.78608	71	.81346	76	.82773	68	.85797	72	8
9	.78679	71	.81422	75	.82841	67	.85869	72	9
10	8.78750	71	8.81498	75	8.82908	67	8.85942	72	10
11	.78821	71	.81573	76	.82976	67	.86014	72	11
12	.78892	71	.81649	75	.83043	67	.86087	72	12
13	.78963	70	.81725	75	.83111	67	.86159	72	13
14	.79034	71	.81800	75	.83178	67	.86231	72	14
15	8.79105	70	8.81876	75	8.83246	67	8.86304	72	15
16	.79175	71	.81951	75	.83313	67	.86376	72	16
17	.79246	70	.82026	75	.83380	67	.86448	72	17
18	.79317	70	.82102	75	.83447	67	.86520	72	18
19	.79387	70	.82177	75	.83515	67	.86592	72	19
20	8.79458	70	8.82252	75	8.83582	67	8.86664	72	20
21	.79528	70	.82327	75	.83649	67	.86736	72	21
22	.79598	70	.82402	75	.83716	67	.86808	72	22
23	.79669	70	.82477	74	.83783	67	.86880	71	23
24	.79739	70	.82552	75	.83850	66	.86952	72	24
25	8.79809	70	8.82627	75	8.83916	67	8.87024	71	25
26	.79879	70	.82702	74	.83983	66	.87095	72	26
27	.79949	70	.82776	75	.84050	67	.87167	71	27
28	.80019	70	.82851	74	.84117	66	.87239	71	28
29	.80089	70	.82926	74	.84183	66	.87310	71	29
30	8.80159	70	8.83000	74	8.84250	66	8.87382	71	30
31	.80229	69	.83075	74	.84316	66	.87453	71	31
32	.80299	70	.83149	74	.84383	66	.87525	71	32
33	.80369	69	.83224	74	.84449	66	.87596	71	33
34	.80438	69	.83298	74	.84515	66	.87668	71	34
35	8.80508	69	8.83373	74	8.84582	66	8.87739	71	35
36	.80577	69	.83447	74	.84648	66	.87810	71	36
37	.80647	69	.83521	74	.84714	66	.87881	71	37
38	.80716	69	.83595	74	.84780	66	.87953	71	38
39	.80786	69	.83670	74	.84846	66	.88024	71	39
40	8.80855	69	8.83744	74	8.84912	66	8.88095	71	40
41	.80924	69	.83818	74	.84978	66	.88166	71	41
42	.80993	69	.83892	74	.85044	66	.88237	71	42
43	.81063	69	.83966	73	.85110	66	.88308	70	43
44	.81132	69	.84039	74	.85176	65	.88378	71	44
45	8.81201	69	8.84113	73	8.85242	66	8.88449	71	45
46	.81270	69	.84187	74	.85308	65	.88520	70	46
47	.81339	68	.84261	73	.85373	65	.88591	70	47
48	.81407	69	.84334	73	.85439	66	.88661	71	48
49	.81476	68	.84408	73	.85505	65	.88732	70	49
50	8.81545	69	8.84481	73	8.85570	65	8.88803	70	50
51	.81614	68	.84555	73	.85626	65	.88873	70	51
52	.81682	68	.84628	73	.85701	65	.88944	70	52
53	.81751	68	.84702	73	.85766	65	.89014	70	53
54	.81819	68	.84775	73	.85832	65	.89085	70	54
55	8.81888	68	8.84848	73	8.85897	65	8.89155	70	55
56	.81956	68	.84922	73	.85962	65	.89225	70	56
57	.82025	68	.84995	73	.86027	65	.89295	70	57
58	.82093	68	.85068	73	.86092	65	.89366	70	58
59	.82161	68	.85141	73	.86158	65	.89436	70	59
60	8.82229		8.85214		8.86223		8.89506		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	76	75	74
6	7.6	7.5	7.4
7	8.8	8.7	8.6
8	10.1	10.0	9.8
9	11.4	11.2	11.1
10	12.6	12.5	12.3
20	25.3	25.0	21.6
30	38.0	37.5	37.0
40	50.6	50.0	49.3
50	63.3	62.5	61.6

	73	72	71
6	7.3	7.2	7.1
7	8.5	8.4	8.3
8	9.7	9.6	9.4
9	10.9	10.8	10.6
10	12.1	12.0	11.8
20	24.3	24.0	23.6
30	36.5	36.0	35.5
40	48.6	48.0	47.3
50	60.8	60.0	59.1

	70	69	68
6	7.0	6.9	6.8
7	8.1	8.0	7.9
8	9.3	9.2	9.0
9	10.5	10.3	10.2
10	11.6	11.5	11.3
20	23.3	23.0	22.6
30	35.0	34.5	34.0
40	46.6	46.0	45.3
50	58.3	57.5	56.6

	67	66	65
6	6.7	6.6	6.5
7	7.8	7.7	7.6
8	8.9	8.8	8.6
9	10.0	9.9	9.7
10	11.1	11.0	10.8
20	22.3	22.0	21.6
30	33.5	33.0	32.5
40	44.6	44.0	43.3
50	55.8	55.0	54.1

	0
6	0.0
7	0.0
8	0.0
9	0.1
10	0.1
20	0.1
30	0.2
40	0.3
50	0.4

′	22° Lg. Vers.	D	22° Log. Exs.	D	23° Lg. Vers.	D	23° Log. Exs.	D	′
0	8.86223	64	8.89506	70	8.90034	62	8.93631	67	0
1	.86287	65	.89576	70	.90096	62	.93699	67	1
2	.86352	65	.89646	70	.90158	62	.93766	67	2
3	.86417	65	.89716	69	.90220	62	.93833	67	3
4	.86482	64	.89786	70	.90282	62	.93901	67	4
5	8.86547	65	8.89856	70	8.90344	62	8.93968	67	5
6	.86612	64	.89926	69	.90406	61	.94035	67	6
7	.86676	64	.89995	70	.90467	62	.94102	67	7
8	.86741	64	.90065	69	.90529	61	.94170	67	8
9	.86805	64	.90135	70	.90591	61	.94237	67	9
10	8.86870	64	8.90205	69	8.90652	62	8.94304	67	10
11	.86934	64	.90274	69	.90714	61	.94371	67	11
12	.86999	64	.90344	69	.90776	61	.94438	67	12
13	.87063	64	.90413	69	.90837	61	.94505	67	13
14	.87127	64	.90433	69	.90899	61	.94572	66	14
15	8.87192	64	8.90552	69	8.90960	61	8.94638	67	15
16	.87256	64	.90622	69	.91021	61	.94705	66	16
17	.87320	64	.90691	69	.91083	61	.94772	67	17
18	.87384	64	.90760	69	.91144	61	.94839	66	18
19	.87448	64	.90830	69	.91205	61	.94905	66	19
20	8.87512	64	8.90899	69	8.91267	61	8.94972	67	20
21	.87576	64	.90968	69	.91328	61	.95039	66	21
22	.87640	64	.91037	69	.91389	61	.95105	66	22
23	.87704	63	.91106	69	.91450	61	.95172	66	23
24	.87768	64	.91175	69	.91511	61	.95238	66	24
25	8.87832	63	8.91244	69	8.91572	61	8.95305	66	25
26	.87895	64	.91313	68	.91633	61	.95371	66	26
27	.87959	63	.91382	69	.91694	61	.95437	66	27
28	.88023	63	.91451	69	.91755	60	.95504	66	28
29	.88086	63	.91520	68	.91815	61	.95570	66	29
30	8.88150	63	8.91588	69	8.91876	60	8.95636	66	30
31	.88213	63	.91657	68	.91937	60	.95703	66	31
32	.88277	63	.91726	68	.91997	61	.95769	66	32
33	.88340	63	.91794	68	.92058	60	.95835	66	33
34	.88404	63	.91863	69	.92119	60	.95901	66	34
35	8.88467	63	8.91932	68	8.92179	60	8.95967	66	35
36	.88530	63	.92000	68	.92240	60	.96033	66	36
37	.88593	63	.92068	68	.92300	60	.96099	66	37
38	.88656	63	.92137	68	.92361	60	.96165	66	38
39	.88720	63	.92205	68	.92421	60	.96231	66	39
40	8.88783	63	8.92274	68	8.92487	60	8.96297	65	40
41	.88846	63	.92342	68	.92542	60	.96362	66	41
42	.88909	62	.92410	68	.92602	60	.96428	65	42
43	.88971	63	.92478	68	.92662	60	.96494	66	43
44	.89034	63	.92546	68	.92722	60	.96560	65	44
45	8.89097	62	8.92615	68	8.92782	60	8.96625	65	45
46	.89160	63	.92683	68	.92842	60	.96691	66	46
47	.89223	62	.92751	68	.92902	60	.96757	65	47
48	.89285	62	.92819	68	.92962	60	.96822	65	48
49	.89348	63	.92887	68	.93022	60	.96888	65	49
50	8.89411	62	8.92955	67	8.93082	59	8.96953	65	50
51	.89473	62	.93022	68	.93142	60	.97018	66	51
52	.89536	62	.93090	67	.93202	59	.97084	65	52
53	.89598	62	.93158	68	.93261	60	.97149	65	53
54	.89660	62	.93226	67	.93321	59	.97214	65	54
55	8.89723	62	8.93293	68	8.93381	59	8.97280	65	55
56	.89785	62	.93361	67	.93440	60	.97345	65	56
57	.89847	62	.93429	67	.93500	59	.97410	65	57
58	.89910	62	.93496	67	.93560	59	.97475	65	58
59	.89972	62	.93564	67	.93619	59	.97540	65	59
60	8.00034		8.93631		8.93679		8.97606		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	70	69	68
6	7.0	6.9	6.8
7	8.1	8.0	7.9
8	9.3	9.2	9.0
9	10.5	10.3	10.2
10	11.6	11.5	11.3
20	23.3	23.0	22.6
30	35.0	34.5	34.0
40	46.6	46.0	45.3
50	58.3	57.5	56.6

	67	66	65
6	6.7	6.6	6.5
7	7.8	7.7	7.6
8	8.9	8.8	8.6
9	10.0	9.9	9.7
10	11.1	11.0	10.8
20	22.3	22.0	21.6
30	33.5	33.0	32.5
40	44.6	44.0	43.3
50	55.8	55.0	54.1

	64	63	62
6	6.4	6.3	6.2
7	7.4	7.3	7.2
8	8.5	8.4	8.2
9	9.6	9.4	9.3
10	10.6	10.5	10.3
20	21.3	21.0	20.6
30	32.0	31.5	31.0
40	42.6	42.0	41.3
50	53.3	52.5	51.6

	61	60	59
6	6.1	6.0	5.9
7	7.1	7.0	6.9
8	8.1	8.0	7.8
9	9.1	9.0	8.8
10	10.1	10.0	9.8
20	20.3	20.0	19.6
30	30.5	30.0	29.5
40	40.6	40.0	39.3
50	50.8	50.0	49.1

	0
6	0.0
7	0.0
8	0.0
9	0.1
10	0.1
20	0.1
30	0.2
40	0.3
50	0.4

P. P.

	24°				25°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	8.93679	59	8.97606	65	8.97170	57	9.01443	62	0
1	.93738	59	.97671	65	.97227	56	.01505	63	1
2	.93797	59	.97736	65	.97284	57	.01568	62	2
3	.93857	59	.97801	64	.97341	57	.01631	63	3
4	.93916	59	.97865	65	.97398	57	.01694	62	4
5	8.93975	59	8.97930	65	8.97455	56	9.01756	63	5
6	.94034	59	.97995	64	.97511	57	.01819	62	6
7	.94094	59	.98060	65	.97568	56	.01882	62	7
8	.94153	59	.98125	65	.97625	56	.01944	63	8
9	.94212	59	.98190	64	.97681	56	.02007	62	9
10	8.94271	59	8.98254	64	8.97738	57	9.02070	62	10
11	.94330	59	.98319	64	.97795	56	.02132	62	11
12	.94389	59	.98383	65	.97851	56	.02195	62	12
13	.94448	58	.98448	64	.97908	56	.02257	62	13
14	.94506	59	.98513	64	.97964	56	.02319	62	14
15	8.94565	59	8.98577	64	8.98020	56	9.02382	62	15
16	.94624	58	.98642	64	.98077	56	.02444	62	16
17	.94683	59	.98706	64	.98133	56	.02506	62	17
18	.94742	58	.98770	64	.98190	56	.02569	62	18
19	.94800	58	.98835	64	.98246	56	.02631	62	19
20	8.94859	58	8.98899	64	8.98302	56	9.02693	62	20
21	.94917	58	.98963	64	.98358	56	.02755	62	21
22	.94976	58	.99028	64	.98414	56	.02817	62	22
23	.95034	58	.99092	64	.98470	56	.02880	62	23
24	.95093	58	.99156	64	.98527	56	.02942	62	24
25	8.95151	58	8.99220	64	8.98583	56	9.03004	62	25
26	.95210	58	.99284	64	.98639	56	.03066	62	26
27	.95268	58	.99348	64	.98695	55	.03128	62	27
28	.95326	58	.99412	64	.98750	56	.03190	62	28
29	.95384	58	.99476	64	.98806	56	.03252	61	29
30	8.95443	58	8.99540	64	8.98862	56	9.03313	62	30
31	.95501	58	.99604	64	.98918	55	.03375	62	31
32	.95559	58	.99668	64	.98974	56	.03437	61	32
33	.95617	58	.99732	63	.99030	55	.03499	62	33
34	.95675	58	.99796	64	.99085	55	.03561	61	34
35	8.95733	58	8.99860	63	8.99141	56	9.03622	61	35
36	.95791	57	.99923	64	.99197	55	.03684	62	36
37	.95849	58	8.99987	63	.99252	55	.03746	61	37
38	.95907	58	9.00051	63	.99308	55	.03807	61	38
39	.95965	58	.00114	64	.99363	55	.03869	61	39
40	8.96023	57	9.00178	63	8.99419	55	9.03930	61	40
41	.96080	57	.00242	63	.99474	55	.03992	61	41
42	.96138	58	.00305	63	.99529	55	.04053	61	42
43	.96196	57	.00369	63	.99585	55	.04115	61	43
44	.96253	57	.00432	63	.99640	55	.04176	61	44
45	8.96311	57	9.00495	63	8.99695	55	9.04238	61	45
46	.96368	57	.00559	63	.99751	55	.04299	61	46
47	.96426	57	.00622	63	.99806	55	.04360	61	47
48	.96483	57	.00686	63	.99861	55	.04421	61	48
49	.96541	57	.00749	63	.99916	55	.04483	61	49
50	8.96598	57	9.00812	63	8.99971	55	9.04544	61	50
51	.96656	57	.00875	63	9.00026	55	.04605	61	51
52	.96713	57	.00938	63	.00081	55	.04666	61	52
53	.96770	57	.01002	63	.00136	55	.04727	61	53
54	.96827	57	.01065	63	.00191	55	.04788	61	54
55	8.96885	57	9.01128	63	9.00246	55	9.04850	61	55
56	.96942	57	.01191	63	.00301	55	.04911	61	56
57	.96999	57	.01254	63	.00356	54	.04972	61	57
58	.97056	57	.01317	63	.00411	55	.05033	60	58
59	.97113	57	.01380	63	.00466	54	.05093	61	59
60	8.97170		9.01443		9.00520		9.05154		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	65	64	63
6	6.5	6.4	6.3
7	7.6	7.4	7.3
8	8.6	8.5	8.4
9	9.7	9.6	9.4
10	10.8	10.6	10.5
20	21.6	21.3	21.0
30	32.5	32.0	31.5
40	43.3	42.6	42.0
50	54.1	53.3	52.5

	62	61	60
6	6.2	6.1	6.0
7	7.2	7.1	7.0
8	8.2	8.1	8.0
9	9.3	9.1	9.0
10	10.3	10.1	10.0
20	20.6	20.3	20.0
30	31.0	30.5	30.0
40	41.3	40.6	40.0
50	51.6	50.8	50.0

	59	58	57
6	5.9	5.8	5.7
7	6.9	6.7	6.6
8	7.8	7.7	7.6
9	8.8	8.7	8.5
10	9.8	9.6	9.5
20	19.6	19.3	19.0
30	29.5	29.0	28.5
40	39.3	38.6	38.0
50	49.1	48.3	47.5

	56	55	54
6	5.6	5.5	5.4
7	6.5	6.4	6.3
8	7.4	7.3	7.2
9	8.4	8.2	8.1
10	9.3	9.1	9.0
20	18.6	18.3	18.0
30	28.0	27.5	27.0
40	37.3	36.6	36.0
50	46.6	45.8	45.0

	0
6	0.0
7	0.0
8	0.0
9	0.1
10	0.1
20	0.1
30	0.2
40	0.3
50	0.4

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

	26°				27°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.00520	55	9.05154	61	9.03740	52	9.08752	59	0
1	.00575	54	.05215	61	.03792	52	.08811	59	1
2	.00630	54	.05276	60	.03845	53	.08870	59	2
3	.00684	54	.05337	61	.03898	52	.08929	59	3
4	.00739	55	.05398	60	.03950	52	.08988	59	4
5	9.00794	54	9.05458	61	9.04002	52	9.09047	59	5
6	.00848	54	.05519	60	.04055	52	.09106	58	6
7	.00903	54	.05580	60	.04107	52	.09164	59	7
8	.00957	54	.05640	60	.04160	52	.09223	59	8
9	.01011	54	.05701	61	.04212	52	.09282	58	9
10	9.01066	54	9.05762	60	9.04264	52	9.09341	59	10
11	.01120	54	.05822	60	.04317	52	.09400	58	11
12	.01174	54	.05883	60	.04369	52	.09458	59	12
13	.01229	54	.05943	60	.04421	52	.09517	58	13
14	.01283	54	.06004	60	.04473	52	.09576	58	14
15	9.01337	54	9.06064	60	9.04525	52	9.09634	58	15
16	.01391	54	.06124	60	.04577	52	.09693	59	16
17	.01445	54	.06185	60	.04630	52	.09752	58	17
18	.01499	54	.06245	60	.04682	52	.09810	58	18
19	.01554	54	.06305	60	.04734	52	.09869	58	19
20	9.01608	54	9.06366	60	9.04786	51	9.09927	58	20
21	.01662	53	.06426	60	.04837	52	.09986	58	21
22	.01715	54	.06486	60	.04889	52	.10044	58	22
23	.01769	54	.06546	60	.04941	52	.10102	58	23
24	.01823	54	.06606	60	.04993	51	.10161	58	24
25	9.01877	53	9.06667	60	9.05045	52	9.10219	58	25
26	.01931	54	.06727	60	.05097	51	.10278	58	26
27	.01985	53	.06787	60	.05148	52	.10336	58	27
28	.02038	54	.06847	60	.05200	52	.10394	58	28
29	.02092	53	.06907	60	.05252	51	.10452	58	29
30	9.02146	53	9.06967	60	9.05303	52	9.10511	58	30
31	.02199	54	.07027	60	.05355	51	.10569	58	31
32	.02253	53	.07087	59	.05407	51	.10627	58	32
33	.02307	53	.07146	60	.05458	51	.10685	58	33
34	.02360	53	.07206	60	.05510	51	.10743	58	34
35	9.02414	53	9.07266	59	9.05561	51	9.10801	58	35
36	.02467	53	.07326	60	.05613	51	.10859	58	36
37	.02521	53	.07386	59	.05664	51	.10917	58	37
38	.02574	53	.07445	60	.05715	51	.10975	58	38
39	.02627	53	.07505	59	.05767	51	.11033	58	39
40	9.02681	53	9.07565	59	9.05818	51	9.11091	58	40
41	.02734	53	.07624	59	.05869	51	.11149	57	41
42	.02787	53	.07684	59	.05921	51	.11207	58	42
43	.02840	53	.07743	60	.05972	51	.11265	58	43
44	.02894	53	.07803	59	.06023	51	.11323	57	44
45	9.02947	53	9.07863	59	9.06074	51	9.11380	58	45
46	.03000	53	.07922	59	.06125	51	.11438	58	46
47	.03053	53	.07981	59	.06176	51	.11496	57	47
48	.03106	53	.08041	59	.06227	51	.11554	57	48
49	.03159	53	.08100	59	.06279	51	.11611	58	49
50	9.03212	53	9.08160	59	9.06330	50	9.11669	57	50
51	.03265	53	.08219	59	.06380	51	.11727	57	51
52	.03318	53	.08278	59	.06431	51	.11784	57	52
53	.03371	52	.08338	59	.06482	51	.11842	57	53
54	.03423	53	.08397	59	.06533	51	.11899	58	54
55	9.03476	53	9.08456	59	9.06584	50	9.11957	57	55
56	.03529	52	.08515	59	.06635	51	.12015	57	56
57	.03582	52	.08574	59	.06686	50	.12072	57	57
58	.03634	53	.08634	59	.06736	51	.12129	57	58
59	.03687	52	.08693	59	.06787	50	.12187	57	59
60	9.03740		9.08752		9.06838		9.12244		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	61	60	59
6	6.1	6.0	5.9
7	7.1	7.0	6.9
8	8.1	8.0	7.8
9	9.1	9.0	8.8
10	10.1	10.0	9.8
20	20.3	20.0	19.6
30	30.5	30.0	29.5
40	40.6	40.0	39.3
50	50.8	50.0	49.1

	58	57
6	5.8	5.7
7	6.7	6.6
8	7.7	7.6
9	8.7	8.5
10	9.6	9.5
20	19.3	19.0
30	29.0	28.5
40	38.6	38.0
50	48.3	47.5

	55	54
6	5.5	5.4
7	6.4	6.3
8	7.3	7.2
9	8.2	8.1
10	9.1	9.0
20	18.3	18.0
30	27.5	27.0
40	36.6	36.0
50	45.8	45.0

	53	52
6	5.3	5.2
7	6.2	6.0
8	7.0	6.9
9	7.9	7.8
10	8.8	8.6
20	17.6	17.3
30	26.5	26.0
40	35.3	34.6
50	44.1	43.3

	51	0
6	5.1	0.0
7	5.9	0.0
8	6.8	0.0
9	7.6	0.1
10	8.5	0.1
20	17.0	0.1
30	25.8	0.2
40	34.0	0.3
50	42.5	0.4

P. P.

′	28° Lg. Vers.	D	28° Log. Exs.	D	29° Lg. Vers.	D	29° Log. Exs.	D	′
0	9.06838	50	9.12244	57	9.09823	49	9.15641	56	0
1	.06888	51	.12302	57	.09872	48	.15697	55	1
2	.06939	50	.12359	57	.09920	49	.15752	56	2
3	.06990	50	.12416	57	.09969	48	.15808	55	3
4	.07040	50	.12474	57	.10018	49	.15864	56	4
5	9.07091	50	9.12531	57	9.10067	48	9.15920	55	5
6	.07141	50	.12588	57	.10115	48	.15975	56	6
7	.07192	50	.12645	57	.10164	49	.16031	55	7
8	.07242	50	.12703	57	.10213	48	.16087	55	8
9	.07293	50	.12760	57	.10261	48	.16142	56	9
10	9.07343	50	9.12817	57	9.10310	48	9.16198	55	10
11	.97393	50	.12874	57	.10358	48	.16254	55	11
12	.07444	50	.12931	57	.10407	48	.16309	55	12
13	.07494	50	.12988	57	.10455	48	.16365	55	13
14	.07544	50	.13045	57	.10504	48	.16420	55	14
15	9.07594	50	9.13102	57	9.10552	48	9.16476	55	15
16	.07644	50	.13159	57	.10601	48	.16531	55	16
17	.07695	50	.13216	56	.10649	48	.16587	55	17
18	.07745	50	.13273	57	.10697	48	.16642	55	18
19	.07795	50	.13330	57	.10746	48	.16698	55	19
20	9.07845	50	9.13387	57	9.10794	48	9.16753	55	20
21	.07895	50	.13444	56	.10842	48	.16808	55	21
22	.07945	50	.13500	57	.10890	48	.16864	55	22
23	.07995	50	.13557	56	.10939	48	.16919	55	23
24	.08045	50	.13614	57	.10987	48	.16974	55	24
25	9.08095	50	9.13671	56	9.11035	48	9.17029	55	25
26	.08145	50	.13727	57	.11083	48	.17085	55	26
27	.08195	49	.13784	56	.11131	48	.17140	55	27
28	.08244	50	.13841	56	.11179	48	.17195	55	28
29	.08294	49	.13897	57	.11227	48	.17250	55	29
30	9.08344	50	9.13954	56	9.11275	48	9.17305	55	30
31	.08394	49	.14011	56	.11323	48	.17361	55	31
32	.08443	49	.14067	56	.11371	47	.17416	55	32
33	.08493	50	.14124	56	.11419	48	.17471	55	33
34	.08543	49	.14180	56	.11467	48	.17526	55	34
35	9.08592	49	9.14237	56	9.11515	47	9.17581	55	35
36	.08642	49	.14293	56	.11562	48	.17636	55	36
37	.08691	49	.14350	56	.11610	48	.17691	55	37
38	.08741	49	.14406	56	.11658	47	.17746	55	38
39	.08790	49	.14462	56	.11706	48	.17801	55	39
40	9.08840	49	9.14519	56	9.11754	47	9.17856	54	40
41	.08889	49	.14575	56	.11801	47	.17910	55	41
42	.08939	49	.14631	56	.11849	48	.17965	55	42
43	.08988	49	.14688	56	.11897	47	.18020	54	43
44	.09087	49	.14744	56	.11944	47	.18075	55	44
45	9.09087	49	9.14800	56	9.11992	47	9.18130	55	45
46	.09136	49	.14856	56	.12039	47	.18185	54	46
47	.09185	49	.14913	56	.12087	47	.18239	55	47
48	.09234	49	.14969	56	.12134	47	.18294	54	48
49	.09284	49	.15025	56	.12182	47	.18349	54	49
50	9.09333	49	9.15081	56	9.12229	47	9.18403	55	50
51	.09382	49	.15137	56	.12277	47	.18458	54	51
52	.09431	49	.15193	56	.12324	47	.18513	54	52
53	.09480	49	.15249	56	.12371	47	.18567	54	53
54	.09529	49	.15305	56	.12419	47	.18622	54	54
55	9.09578	49	9.15361	56	9.12466	47	9.18676	54	55
56	.09627	49	.15417	56	.12513	47	.18731	55	56
57	.09676	48	.15473	56	.12560	47	.18786	54	57
58	.09725	49	.15529	55	.12608	47	.18840	54	58
59	.09774	49	.15585	56	.12655	47	.18894	54	59
60	9.09823		9.15641		9.12702		9.18949		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	57	57	56
6	5.7	5.7	5.6
7	6.7	6.6	6.6
8	7.6	7.6	7.5
9	8.6	8.5	8.5
10	9.6	9.5	9.4
20	19.1	19.0	18.8
30	28.7	28.5	28.2
40	38.3	38.0	37.6
50	47.9	47.5	47.1

	56	55	55
6	5.6	5.5	5.5
7	6.5	6.5	6.4
8	7.4	7.4	7.3
9	8.4	8.3	8.2
10	9.3	9.2	9.1
20	18.6	18.5	18.3
30	28.0	27.7	27.5
40	37.3	37.0	36.6
50	46.6	46.2	45.8

	54	54
6	5.4	5.4
7	6.3	6.3
8	7.2	7.2
9	8.2	8.1
10	9.1	9.0
20	18.1	18.0
30	27.2	27.0
40	36.3	36.0
50	45.4	45.0

	51	50	50
6	5.1	5.0	5.0
7	5.9	5.9	5.8
8	6.8	6.7	6.6
9	7.6	7.6	7.5
10	8.5	8.4	8.3
20	17.0	16.8	16.6
30	25.5	25.2	25.0
40	34.0	33.6	33.3
50	42.5	42.1	41.6

	49	49	48
6	4.9	4.9	4.8
7	5.8	5.7	5.6
8	6.6	6.5	6.4
9	7.4	7.3	7.3
10	8.2	8.1	8.1
20	16.5	16.3	16.1
30	24.7	24.5	24.2
40	33.0	32.6	32.3
50	41.2	40.8	40.4

	48	47	47
6	4.8	4.7	4.7
7	5.6	5.5	5.5
8	6.4	6.3	6.2
9	7.2	7.1	7.0
10	8.0	7.9	7.8
20	16.0	15.8	15.6
30	24.0	23.7	23.5
40	32.0	31.6	31.3
50	40.0	39.6	39.1

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

′	30° Lg. Vers.	D	30° Log. Exs.	D	31° Lg. Vers.	D	31° Log. Exs.	D	′
0	9.12702	47	9.18949	54	9.15483	45	9.22176	53	0
1	.12749	47	.19003	54	.15528	45	.22229	53	1
2	.12796	47	.19058	54	.15574	45	.22282	53	2
3	.12843	47	.19112	54	.15619	45	.22335	53	3
4	.12890	47	.19167	54	.15665	45	.22388	53	4
5	9.12937	47	9.19221	54	9.15710	45	9.22441	53	5
6	.12984	47	.19275	54	.15755	45	.22494	53	6
7	.13031	47	.19329	54	.15801	45	.22547	53	7
8	.13078	47	.19384	54	.15846	45	.22600	53	8
9	.13125	47	.19438	54	.15891	45	.22653	53	9
10	9.13172	46	9.19492	54	9.15937	45	9.22706	53	10
11	.13219	47	.19546	54	.15982	45	.22759	53	11
12	.13266	47	.19601	54	.16027	45	.22812	53	12
13	.13313	46	.19655	54	.16073	45	.22865	52	13
14	.13359	47	.19709	54	.16118	45	.22918	53	14
15	9.13406	46	9.19763	54	9.16163	45	9.22971	53	15
16	.13453	47	.19817	54	.16208	45	.23024	52	16
17	.13500	46	.19871	54	.16253	45	.23076	53	17
18	.13546	46	.19925	54	.16298	45	.23129	52	18
19	.13593	46	.19979	54	.16343	45	.23182	53	19
20	9.13639	47	9.20033	54	9.16388	45	9.23235	52	20
21	.13686	46	.20087	54	.16434	45	.23287	53	21
22	.13733	46	.20141	54	.16479	44	.23340	52	22
23	.13779	46	.20195	54	.16523	45	.23393	53	23
24	.13826	46	.20249	53	.16568	45	.23446	52	24
25	9.13872	46	9.20303	54	9.16613	45	9.23498	52	25
26	.13919	46	.20357	54	.16658	45	.23551	52	26
27	.13965	46	.20411	54	.16703	45	.23603	52	27
28	.14011	46	.20465	53	.16748	44	.23656	53	28
29	.14058	46	.20518	54	.16793	45	.23709	52	29
30	9.14104	46	9.20572	53	9.16838	44	9.23761	52	30
31	.14151	46	.20626	54	.16882	45	.23814	52	31
32	.14197	46	.20680	53	.16927	44	.23866	52	32
33	.14243	46	.20733	54	.16972	45	.23919	52	33
34	.14289	46	.20787	53	.17017	44	.23971	53	34
35	9.14336	46	9.20841	53	9.17061	44	9.24024	52	35
36	.14382	46	.20894	54	.17106	45	.24076	52	36
37	.14428	46	.20948	53	.17151	44	.24128	52	37
38	.14474	46	.21002	53	.17195	44	.24181	52	38
39	.14520	46	.21055	53	.17240	44	.24233	52	39
40	9.14566	46	9.21109	53	9.17284	44	9.24285	52	40
41	.14612	46	.21162	53	.17329	44	.24138	52	41
42	.14658	46	.21216	53	.17373	44	.24390	52	42
43	.14704	46	.21269	53	.17418	44	.24442	52	43
44	.14750	46	.21323	53	.17462	44	.24495	52	44
45	9.14796	46	9.21376	53	9.17507	44	9.24547	52	45
46	.14842	46	.21430	53	.17551	44	.24599	52	46
47	.14888	46	.21483	53	.17596	44	.24651	52	47
48	.14934	46	.21537	53	.17640	44	.24704	52	48
49	.14980	45	.21590	53	.17684	44	.24756	52	49
50	9.15026	46	9.21643	53	9.17729	44	9.24808	52	50
51	.15071	45	.21697	53	.17773	44	.24860	52	51
52	.15117	46	.21750	53	.17817	44	.24912	52	52
53	.15163	46	.21803	53	.17861	44	.24964	52	53
54	.15209	45	.21857	53	.17906	44	.25016	52	54
55	9.15254	45	9.21910	53	9.17950	44	9.25068	52	55
56	.15300	46	.21963	53	.17994	44	.25120	52	56
57	.15346	45	.22016	53	.18038	44	.25172	52	57
58	.15391	45	.22070	53	.18082	44	.25224	52	58
59	.15437	45	.22123	53	.18126	44	.25276	52	59
60	9.15483	46	9.22176	53	9.18170	44	9.25328	52	60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	$5\bar{4}$	54	$5\bar{3}$
6	5.4	5.4	5.3
7	6.3	6.3	6.2
8	7.2	7.2	7.1
9	8.2	8.1	8.0
10	9.1	9.0	8.9
20	18.1	18.0	17.8
30	27.2	27.0	26.7
40	36.3	36.0	35.6
50	45.4	45.0	44.6

	53	$5\bar{2}$	52
6	5.3	5.2	5.2
7	6.2	6.1	6.0
8	7.0	7.0	6.9
9	7.9	7.9	7.8
10	8.8	8.7	8.6
20	17.6	17.5	17.3
30	26.5	26.2	26.0
40	35.3	35.0	34.6
50	44.1	43.7	43.3

	$4\bar{7}$	47	$4\bar{6}$
6	4.7	4.7	4.6
7	5.5	5.5	5.4
8	6.3	6.2	6.2
9	7.1	7.0	7.0
10	7.9	7.8	7.7
20	15.8	15.6	15.5
30	23.7	23.5	23.2
40	31.6	31.3	31.0
50	39.6	39.1	38.7

	46	$4\bar{5}$	45
6	4.6	4.5	4.5
7	5.3	5.3	5.2
8	6.1	6.0	6.0
9	6.9	6.8	6.7
10	7.6	7.6	7.5
20	15.3	15.1	15.0
30	23.0	22.7	22.5
40	30.6	30.3	30.0
50	38.3	37.9	37.5

	$4\bar{4}$	44
6	4.4	4.4
7	5.2	5.1
8	5.9	5.8
9	6.7	6.6
10	7.4	7.3
20	14.8	14.6
30	22.2	22.0
40	29.6	29.3
50	37.1	36.6

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

	32°				33°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.18170	44	9.25328	52	9.20771	42	9.28412	51	0
1	.18214	44	.25380	52	.20814	42	.28463	51	1
2	.18258	44	.25432	52	.20856	42	.28514	50	2
3	.18302	44	.25484	51	.20899	43	.28564	51	3
4	.18346	44	.25536	52	.20942	42	.28615	51	4
5	9.18390	44	9.25588	52	9.20984	42	9.28666	50	5
6	.18434	44	.25640	52	.21027	42	.28717	51	6
7	.18478	44	.25692	51	.21069	42	.28768	50	7
8	.18522	43	.25743	52	.21112	42	.28818	50	8
9	.18566	44	.25795	51	.21154	42	.28869	51	9
10	9.18610	44	9.25847	52	9.21196	42	9.28920	50	10
11	.18654	43	.25899	51	.21239	42	.28970	50	11
12	.18697	44	.25950	52	.21281	42	.29021	51	12
13	.18741	43	.26002	51	.21324	42	.29072	50	13
14	.18785	44	.26054	51	.21366	42	.29122	51	14
15	9.18829	43	9.26105	52	9.21408	42	9.29173	50	15
16	.18872	43	.26157	51	.21451	42	.29223	50	16
17	.18916	43	.26209	51	.21493	42	.29274	50	17
18	.18959	44	.26260	51	.21535	42	.29324	51	18
19	.19003	43	.26312	52	.21577	42	.29375	50	19
20	9.19047	43	9.26364	51	9.21620	42	9.29426	50	20
21	.19090	43	.26415	51	.21662	42	.29476	50	21
22	.19134	43	.26467	51	.21704	42	.29527	50	22
23	.19177	43	.26518	51	.21746	42	.29577	50	23
24	.19221	43	.26570	51	.21788	42	.29627	50	24
25	9.19264	43	9.26621	51	9.21830	42	9.29678	50	25
26	.19308	43	.26673	51	.21872	42	.29728	50	26
27	.19351	43	.26724	51	.21914	42	.29779	50	27
28	.19395	43	.26776	51	.21956	42	.29829	50	28
29	.19438	43	.26827	51	.21998	42	.29879	50	29
30	9.19481	43	9.26878	51	9.22040	42	9.29930	50	30
31	.19525	43	.26930	51	.22082	42	.29980	50	31
32	.19568	43	.26981	51	.22124	42	.30030	50	32
33	.19611	43	.27032	51	.22166	42	.30081	50	33
34	.19654	43	.27084	51	.22208	42	.30131	50	34
35	9.19698	43	9.27135	51	9.22250	41	9.30181	50	35
36	.19741	43	.27186	51	.22292	42	.30231	50	36
37	.19784	43	.27238	51	.22334	42	.30282	50	37
38	.19827	43	.27289	51	.22376	41	.30332	50	38
39	.19870	43	.27340	51	.22417	42	.30382	50	39
40	9.19914	43	9.27391	51	9.22459	41	9.30432	50	40
41	.19957	43	.27443	51	.22501	42	.30482	50	41
42	.20000	43	.27494	51	.22543	41	.30533	50	42
43	.20043	43	.27545	51	.22584	41	.30583	50	43
44	.20086	43	.27596	51	.22626	42	.30633	50	44
45	9.20129	43	9.27647	51	9.22668	41	9.30683	50	45
46	.20172	43	.27698	51	.22709	41	.30733	50	46
47	.20215	43	.27749	51	.22751	41	.30783	50	47
48	.20258	43	.27800	51	.22792	41	.30833	50	48
49	.20301	42	.27852	51	.22834	41	.30883	50	49
50	9.20343	43	9.27903	51	9.22876	41	9.30933	50	50
51	.20386	43	.27954	51	.22917	41	.30983	50	51
52	.20429	43	.28005	51	.22959	41	.31033	50	52
53	.20472	42	.28056	51	.23000	41	.31083	50	53
54	.20515	43	.28107	50	.23042	41	.31133	49	54
55	9.20558	42	9.28157	51	9.23083	41	9.31183	50	55
56	.20600	43	.28208	51	.23124	41	.31233	50	56
57	.20643	43	.28259	51	.23166	41	.31283	50	57
58	.20686	42	.28310	51	.23207	41	.31333	50	38
59	.20728	43	.28361	50	.23248	41	.31383	49	59
60	9.20771		9.28412		9.23290		9.31432		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	52	$\overline{51}$	51
6	5.2	5.1	5.1
7	6.0	6.0	5.9
8	6.9	6.8	6.8
9	7.8	7.7	7.6
10	8.6	8.6	8.5
20	17.3	17.1	17.0
30	26.0	25.7	25.5
40	34.6	34.3	34.0
50	43.3	42.9	42.5

	$\overline{50}$	50	$\overline{49}$
6	5.0	5.0	4.9
7	5.9	5.8	5.8
8	6.7	6.6	6.6
9	7.6	7.5	7.4
10	8.4	8.3	8.2
20	16.8	16.6	16.5
30	25.2	25.0	24.7
40	33.6	33.3	33.0
50	42.1	41.6	41.2

	44	$\overline{43}$	43
6	4.4	4.3	4.3
7	5.1	5.1	5.0
8	5.8	5.8	5.7
9	6.6	6.5	6.4
10	7.3	7.2	7.1
20	14.6	14.5	14.3
30	22.0	21.7	21.5
40	29.3	29.0	28.6
50	36.6	36.2	35.8

	$\overline{42}$	42	$\overline{41}$
6	4.2	4.2	4.1
7	4.9	4.9	4.8
8	5.6	5.6	5.5
9	6.4	6.3	6.2
10	7.1	7.0	6.9
20	14.1	14.0	13.8
30	21.2	21.0	20.7
40	28.3	28.0	27.6
50	35.4	35.0	34.6

	41
6	4.1
7	4.8
8	5.4
9	6.1
10	6.8
20	13.6
30	20.5
40	27.3
50	34.1

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

′	34° Lg. Vers.	D	34° Log. Exs.	D	35° Lg. Vers.	D	35° Log. Exs.	D	′
0	9.23290	41	9.31432	50	9.25731	40	9.34395	49	0
1	.23331	41	.31482	50	.25771	40	.34444	48	1
2	.23372	41	.31532	49	.25811	40	.34492	49	2
3	.23414	41	.31582	50	.25851	40	.34541	49	3
4	.23455	41	.31632	49	.25891	40	.34590	49	4
5	9.23496	41	9.31681	50	9.25931	40	9.34639	48	5
6	.23537	41	.31731	49	.25971	40	.34688	49	6
7	.23579	41	.31781	50	.26011	39	.34737	48	7
8	.23620	41	.31831	49	.26051	40	.34785	49	8
9	.23661	41	.31880	50	.26091	40	.34834	49	9
10	9.23702	41	9.31930	49	9.26131	40	9.34883	48	10
11	.23743	41	.31980	49	.26171	39	.34932	48	11
12	.23784	41	.32029	49	.26210	40	.34980	49	12
13	.23825	41	.32079	50	.26250	40	.34029	48	13
14	.23866	41	.32129	49	.26290	39	.35078	49	14
15	9.23907	41	9.32178	49	9.26330	40	9.35127	48	15
16	.23948	41	.32228	49	.26370	39	.35175	49	16
17	.23989	41	.32277	49	.26409	40	.35224	48	17
18	.24030	41	.32327	50	.26449	39	.35273	48	18
19	.24071	40	.32377	49	.26489	39	.35321	48	19
20	9.24112	41	9.32426	49	9.26528	40	9.35370	49	20
21	.24153	41	.32476	49	.26568	39	.35419	48	21
22	.24194	41	.32525	49	.26608	39	.35467	48	22
23	.24235	40	.32575	49	.26647	39	.35516	48	23
24	.24275	41	.32624	49	.26687	39	.35564	48	24
25	9.24316	40	9.32673	49	9.26726	40	9.35613	48	25
26	.24357	41	.32723	49	.26766	39	.35661	48	26
27	.24398	40	.32772	49	.26806	39	.35710	48	27
28	.24438	41	.32822	49	.26845	39	.35758	48	28
29	.24479	40	.32871	49	.26885	39	.35807	48	29
30	9.24520	41	9.32920	49	9.26924	39	9.35855	48	30
31	.24561	40	.32970	49	.26964	39	.35904	48	31
32	.24601	40	.33019	49	.27003	39	.35952	48	32
33	.24642	40	.33069	49	.27042	39	.36001	48	33
34	.24682	41	.33118	49	.27082	39	.36049	48	34
35	9.24723	40	9.33167	49	9.27121	39	9.36098	48	35
36	.24764	40	.33216	49	.27161	39	.36146	48	36
37	.24804	40	.33266	49	.27200	39	.36194	48	37
38	.24845	40	.33315	49	.27239	39	.36243	48	38
39	.24885	40	.33364	49	.27278	39	.36291	48	39
40	9.24926	40	9.33413	49	9.27318	39	9.36340	48	40
41	.24966	40	.33463	49	.27357	39	.36388	48	41
42	.25007	40	.33512	49	.27396	39	.36436	48	42
43	.25047	40	.33561	49	.27435	39	.36484	48	43
44	.25087	40	.33610	49	.27475	39	.36533	48	44
45	9.25128	40	9.33659	49	9.27514	39	9.36581	48	45
46	.25168	40	.33708	49	.27553	39	.36629	48	46
47	.25209	40	.33758	49	.27592	39	.36678	48	47
48	.25249	40	.33807	49	.27631	39	.36726	48	48
49	.25289	40	.33856	49	.27670	39	.36774	48	49
50	9.25329	40	9.33905	49	9.27709	39	9.36822	48	50
51	.25370	40	.33954	49	.27749	39	.36870	48	51
52	.25410	40	.34003	49	.27788	39	.36919	48	52
53	.25450	40	.34052	49	.27827	39	.36967	48	53
54	.25490	40	.34101	49	.27866	39	.37015	48	54
55	9.25531	40	9.34150	49	9.27905	39	9.37063	48	55
56	.25571	40	.34199	49	.27944	38	.37111	48	56
57	.25611	40	.34248	49	.27982	39	.37159	48	57
58	.25651	40	.34297	49	.28021	39	.37207	48	58
59	.25691	40	.34346	49	.28060	39	.37255	48	59
60	9.25731		9.34395		9.28099		9.37303		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	50	$\overline{49}$	49
6	5.0	4.9	4.9
7	5.8	5.8	5.7
8	6.6	6.6	6.5
9	7.5	7.4	7.3
10	8.3	8.2	8.1
20	16.6	16.5	16.3
30	25.0	24.7	24.5
40	33.3	33.0	32.6
50	41.6	41.2	40.8

	$\overline{48}$	48
6	4.8	4.8
7	5.6	5.6
8	6.4	6.4
9	7.3	7.2
10	8.1	8.0
20	16.1	16.0
30	24.2	24.0
40	32.3	32.0
50	40.4	40.0

	$\overline{41}$	41
6	4.1	4.1
7	4.8	4.8
8	5.5	5.4
9	6.2	6.1
10	6.9	6.8
20	13.8	13.6
30	20.7	20.5
40	27.6	27.3
50	34.6	34.1

	$\overline{40}$	40
6	4.0	4.0
7	4.7	4.6
8	5.4	5.3
9	6.1	6.0
10	6.7	6.6
20	13.5	13.3
30	20.2	20.0
40	27.0	26.6
50	33.7	33.3

	$\overline{39}$	39
6	3.9	3.9
7	4.6	4.5
8	5.2	5.2
9	5.9	5.8
10	6.6	6.5
20	13.1	13.0
30	19.7	19.5
40	26.3	26.0
50	32.9	32.5

	$\overline{38}$
6	3.8
7	4.5
8	5.1
9	5.8
10	6.4
20	12.8
30	19.2
40	25.6
50	32.1

P. P.

	36°				37°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.2809$\underline{9}$	3$\underline{9}$	9.3730$\bar{3}$	4$\bar{8}$	9.3039$\bar{8}$	3$\bar{7}$	9.4016$\underline{3}$	4$\underline{7}$	0
1	.2813$\bar{8}$	3$\bar{8}$	.37352	48	.30436	3$\underline{8}$	.4021$\bar{0}$	4$\bar{7}$	1
2	.28177	39	.37400	48	.3047$\underline{4}$	3$\underline{7}$	.40258	4$\underline{7}$	2
3	.28816	39	.37448	48	.3051$\bar{1}$	3$\bar{7}$	.4030$\underline{5}$	4$\bar{7}$	3
4	.28255	3$\bar{8}$	.37496	48	.30549	3$\underline{8}$	.4035$\bar{2}$	4$\underline{7}$	4
5	9.2829$\underline{3}$	3$\underline{9}$	9.37544	48	9.3058$\underline{7}$	3$\underline{7}$	9.4039$\bar{9}$	4$\bar{7}$	5
6	.2833$\bar{2}$	3$\bar{8}$	.37592	4$\underline{8}$	.3062$\bar{4}$	3$\bar{7}$	.40447	47	6
7	.28371	3$\underline{9}$	.3764$\underline{0}$	4$\bar{7}$	.30662	3$\underline{8}$	.40494	4$\underline{7}$	7
8	.2841$\underline{0}$	3$\bar{8}$	.3768$\bar{7}$	48	.3070$\underline{0}$	3$\bar{7}$	.4054$\underline{1}$	4$\bar{7}$	8
9	.2844$\bar{8}$	3$\underline{9}$	.3773$\bar{5}$	48	.3073$\bar{7}$	3$\bar{7}$	.4058$\bar{8}$	47	9
10	9.28487	3$\underline{8}$	9.3778$\underline{3}$	48	9.3077$\underline{5}$	3$\underline{7}$	9.4063$\underline{5}$	4$\underline{7}$	10
11	.2852$\underline{6}$	3$\bar{8}$	.3783$\bar{1}$	48	.3081$\bar{2}$	3$\underline{7}$	.4068$\bar{2}$	4$\bar{7}$	11
12	.2856$\underline{4}$	3$\underline{9}$	.3787$\underline{9}$	4$\underline{8}$	.3085$\underline{0}$	3$\underline{7}$	.40730	47	12
13	.2860$\bar{3}$	3$\bar{8}$	.3792$\bar{7}$	4$\bar{7}$	.3088$\bar{7}$	3$\bar{7}$	.40777	47	13
14	.28642	3$\bar{8}$	.37975	48	.30925	3$\bar{7}$	.40824	4$\underline{7}$	14
15	9.28680	3$\bar{8}$	9.38023	48	9.3096$\bar{2}$	3$\underline{7}$	9.4087$\underline{1}$	4$\bar{7}$	15
16	.2871$\underline{9}$	3$\bar{8}$	.38071	4$\underline{8}$	.3100$\underline{0}$	3$\underline{7}$	.4091$\bar{8}$	47	16
17	.2875$\bar{7}$	3$\bar{8}$	.3811$\underline{9}$	4$\bar{7}$	.3103$\bar{7}$	3$\underline{7}$	.4096$\underline{5}$	47	17
18	.28796	39	.3816$\bar{6}$	48	.3107$\underline{5}$	3$\bar{7}$	.4101$\bar{2}$	47	18
19	.28835	3$\bar{8}$	.3821$\underline{4}$	4$\bar{7}$	.3111$\bar{2}$	3$\bar{7}$	.4105$\bar{9}$	47	19
20	9.28873	3$\bar{8}$	9.38262	4$\underline{8}$	9.31150	3$\underline{7}$	9.4110$\underline{6}$	47	20
21	.28912	3$\underline{8}$	.3831$\underline{0}$	4$\bar{7}$	.3118$\underline{7}$	3$\underline{7}$	.4115$\bar{3}$	47	21
22	.2895$\underline{0}$	3$\bar{8}$	.3835$\bar{7}$	4$\underline{8}$	.3122$\bar{4}$	3$\bar{7}$	.4120$\underline{0}$	47	22
23	.2898$\bar{8}$	38	.3840$\bar{5}$	4$\bar{7}$	.31262	37	.4124$\underline{7}$	47	23
24	.29027	3$\bar{8}$	.38453	4$\underline{8}$	.31299	3$\underline{7}$	.4129$\bar{4}$	47	24
25	9.29065	3$\bar{8}$	9.3850$\underline{1}$	4$\bar{7}$	9.3133$\bar{6}$	3$\bar{7}$	9.4134$\underline{1}$	47	25
26	.2910$\underline{4}$	3$\bar{8}$	.3854$\bar{8}$	4$\underline{8}$	.31374	3$\underline{7}$	.4138$\bar{8}$	47	26
27	.2914$\underline{2}$	3$\underline{8}$	.3859$\bar{6}$	4$\bar{7}$	.3141$\underline{1}$	3$\bar{7}$	.4143$\underline{5}$	47	27
28	.2918$\bar{0}$	3$\bar{8}$	.38644	48	.3144$\bar{8}$	37	.4148$\underline{2}$	47	28
29	.29219	3$\bar{8}$	.38692	4$\underline{7}$	.3148$\bar{5}$	3$\bar{7}$	.4152$\bar{9}$	4$\bar{6}$	29
30	9.2925$\underline{7}$	3$\underline{8}$	9.38739	4$\bar{7}$	9.31523	37	9.41576	47	30
31	.2929$\bar{5}$	3$\bar{8}$	.3878$\underline{7}$	4$\bar{7}$	.31560	3$\underline{7}$	.41623	47	31
32	.29334	3$\underline{8}$	.3883$\underline{4}$	4$\underline{8}$	.3159$\underline{7}$	3$\bar{7}$	.41670	4$\underline{7}$	32
33	.2937$\underline{2}$	3$\bar{8}$	.3888$\bar{2}$	4$\bar{7}$	.3163$\underline{4}$	37	.4171$\underline{7}$	4$\bar{6}$	33
34	.2941$\bar{0}$	3$\underline{8}$	.38930	4$\underline{7}$	.3167$\bar{1}$	3$\underline{7}$	.4176$\bar{3}$	47	34
35	9.29448	3$\bar{8}$	9.38977	4$\underline{7}$	9.3170$\bar{8}$	3$\bar{7}$	9.4181$\underline{0}$	4$\underline{7}$	35
36	.29487	38	.3902$\underline{5}$	4$\bar{7}$	.31746	37	.4185$\bar{7}$	4$\bar{6}$	36
37	.29525	3$\underline{8}$	.3907$\underline{2}$	4$\underline{8}$	.31783	37	.41904	47	37
38	.2956$\underline{3}$	3$\bar{8}$	.3912$\bar{0}$	4$\bar{7}$	.31820	37	.41951	47	38
39	.2960$\bar{1}$	38	.39168	4$\bar{7}$	.31857	37	.41998	4$\bar{6}$	39
40	9.2963$\underline{9}$	38	9.3921$\bar{5}$	4$\underline{8}$	9.31894	37	9.4204$\underline{4}$	4$\underline{7}$	40
41	.2967$\bar{7}$	3$\underline{8}$	.3926$\underline{3}$	4$\underline{7}$	.31931	37	.4209$\bar{1}$	4$\bar{6}$	41
42	.2971$\bar{5}$	3$\bar{8}$	.3931$\bar{0}$	4$\underline{7}$	.31968	37	.42138	4$\underline{7}$	42
43	.29754	38	.3935$\underline{8}$	4$\bar{7}$	.32005	37	.4218$\underline{5}$	4$\bar{6}$	43
44	.29792	38	.3940$\bar{5}$	4$\bar{7}$	.32042	37	.4223$\bar{1}$	4$\underline{7}$	44
45	9.29830	38	9.3945$\underline{3}$	4$\underline{7}$	9.32079	37	9.4227$\bar{8}$	4$\bar{6}$	45
46	.29868	38	.3950$\bar{0}$	4$\bar{7}$	.32116	37	.42325	4$\underline{7}$	46
47	.29906	38	.39548	4$\underline{7}$	.32153	37	.4237$\underline{2}$	4$\bar{6}$	47
48	.29944	38	.3959$\underline{5}$	4$\bar{7}$	.32190	37	.4241$\bar{8}$	47	48
49	.29982	3$\underline{8}$	.3964$\bar{2}$	4$\underline{7}$	.32227	3$\bar{6}$	.4246$\bar{5}$	4$\bar{6}$	49
50	9.3002$\underline{0}$	3$\bar{7}$	9.3969$\underline{0}$	4$\underline{7}$	9.3226$\bar{3}$	37	9.4251$\underline{2}$	4$\bar{6}$	50
51	.3005$\bar{7}$	38	.3973$\bar{7}$	4$\bar{7}$	.3230$\underline{0}$	3$\underline{7}$	.4255$\bar{8}$	4$\underline{7}$	51
52	.3009$\underline{5}$	38	.39785	4$\underline{7}$	.3233$\bar{7}$	3$\bar{6}$	.4260$\bar{5}$	4$\underline{6}$	52
53	.3013$\bar{3}$	38	.3983$\underline{2}$	4$\bar{7}$	.32374	37	.4265$\underline{2}$	4$\bar{6}$	53
54	.3017$\bar{1}$	3$\underline{8}$	.3987$\bar{9}$	4$\bar{7}$	.32411	3$\bar{6}$	.4269$\bar{8}$	4$\bar{6}$	54
55	9.3020$\bar{9}$	3$\bar{7}$	9.39927	4$\underline{7}$	9.3244$\bar{7}$	3$\underline{7}$	9.42745	4$\underline{7}$	55
56	.30247	3$\underline{8}$	.3997$\underline{4}$	4$\underline{7}$	.3248$\bar{4}$	3$\bar{6}$	.4279$\underline{2}$	4$\underline{6}$	56
57	.3028$\underline{5}$	3$\bar{7}$	.4002$\bar{1}$	4$\bar{7}$	.32521	3$\underline{7}$	.4283$\bar{8}$	4$\bar{6}$	57
58	.3032$\bar{2}$	38	.40069	47	.3255$\underline{8}$	3$\bar{6}$	.4288$\underline{5}$	4$\bar{6}$	58
59	.3036$\bar{0}$	38	.40116	4$\bar{7}$	.3259$\bar{4}$	37	.4293$\bar{1}$	4$\bar{6}$	59
60	9.3039$\bar{8}$		9.4016$\bar{3}$		9.3263$\bar{1}$		9.42978		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	4$\bar{8}$	48
6	4.$\bar{8}$	4.8
7	5.$\bar{6}$	5.6
8	6.$\bar{4}$	6.4
9	7.3	7.2
10	8.$\underline{1}$	8.0
20	16.$\underline{1}$	16.0
30	24.$\underline{2}$	24.0
40	32.$\bar{3}$	32.0
50	40.4	40.0

	4$\bar{7}$	47
6	4.$\bar{7}$	4.7
7	5.$\underline{5}$	5.$\underline{5}$
8	6.$\bar{3}$	6.$\underline{2}$
9	7.1	7.$\bar{0}$
10	7.$\underline{9}$	7.$\bar{8}$
20	15.$\bar{8}$	15.$\bar{6}$
30	23.$\underline{7}$	23.$\underline{5}$
40	31.$\bar{6}$	31.$\bar{3}$
50	39.6	39.$\underline{1}$

	4$\bar{6}$
6	4.$\bar{6}$
7	5.4
8	6.2
9	7.$\underline{0}$
10	7.$\bar{7}$
20	15.$\underline{5}$
30	23.$\bar{2}$
40	31.$\underline{0}$
50	38.$\bar{7}$

	39	3$\bar{8}$
6	3.$\underline{9}$	3.$\bar{8}$
7	4.$\bar{5}$	4.$\underline{5}$
8	5.$\underline{2}$	5.$\bar{1}$
9	5.$\bar{8}$	5.8
10	6.5	6.$\underline{4}$
20	13.0	12.$\bar{8}$
30	10.5	19.$\bar{2}$
40	26.0	25.$\bar{6}$
50	32.5	32.1

	38	3$\bar{7}$
6	3.$\underline{8}$	3.$\bar{7}$
7	4.$\underline{4}$	4.4
8	5.$\bar{0}$	5.0
9	5.$\underline{7}$	5.$\underline{6}$
10	6.$\bar{3}$	6.$\bar{2}$
20	12.6	12.$\underline{5}$
30	19.$\underline{0}$	18.$\bar{7}$
40	25.$\bar{3}$	25.$\underline{0}$
50	31.$\bar{6}$	31.$\bar{2}$

	37	3$\bar{6}$
6	3.7	3.$\bar{6}$
7	4.$\underline{3}$	4.$\underline{2}$
8	4.$\bar{9}$	4.$\bar{8}$
9	5.$\underline{5}$	5.5
10	6.$\bar{1}$	6.$\underline{1}$
20	12.$\bar{3}$	12.$\underline{1}$
30	18.$\underline{5}$	18.$\bar{2}$
40	24.$\bar{6}$	24.$\bar{3}$
50	30.$\bar{8}$	30.4

P. P.

	38°				39°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.32631	36	9.42978	46	9.34802	35	9.45752	45	0
1	.32668	36	.43024	47	.34837	36	.45797	46	1
2	.32704	37	.43071	46	.34873	35	.45843	46	2
3	.32741	36	.43118	46	.34909	35	.45889	46	3
4	.32778	36	.43164	46	.34944	35	.45935	45	4
5	9.32814	36	9.43211	46	9.34980	36	9.45981	46	5
6	.32851	37	.43257	46	.35016	35	.46027	46	6
7	.32888	36	.43304	46	.35051	35	.46073	45	7
8	.32924	36	.43350	46	.35087	35	.46118	46	8
9	.32961	36	.43396	46	.35122	35	.46164	46	9
10	9.32997	36	9.43443	46	9.35158	35	9.46210	45	10
11	.33034	36	.43489	46	.35193	35	.46256	46	11
12	.33070	36	.43536	46	.35229	35	.46302	45	12
13	.33107	36	.43582	46	.35264	35	.46347	46	13
14	.33143	36	.43629	46	.35300	35	.46393	45	14
15	9.33180	36	9.43675	46	9.35335	35	9.46439	46	15
16	.33216	36	.43721	46	.35370	35	.46485	45	16
17	.33252	36	.43768	46	.35406	35	.46530	46	17
18	.33289	36	.43814	46	.35441	35	.46576	45	18
19	.33325	36	.43861	46	.35477	35	.46622	46	19
20	9.33361	36	9.43907	46	9.35512	35	9.46668	45	20
21	.33398	35	.43953	46	.35547	35	.46713	45	21
22	.33434	36	.43999	46	.35583	35	.46759	46	22
23	.33470	36	.44046	46	.35618	35	.46805	45	23
24	.33507	36	.44092	46	.35653	35	.46850	45	24
25	9.33543	36	9.44138	46	9.35689	35	9.46896	46	25
26	.33579	36	.44185	46	.35724	35	.46942	45	26
27	.33615	36	.44231	46	.35759	35	.46987	45	27
28	.33652	36	.44277	46	.35794	35	.47033	45	28
29	.33688	36	.44323	46	.35829	35	.47078	46	29
30	9.33724	36	9.44370	46	9.35865	35	9.47124	45	30
31	.33760	36	.44416	46	.35900	35	.47170	45	31
32	.33796	36	.44462	46	.35935	35	.47215	45	32
33	.33833	36	.44508	46	.35970	35	.47261	45	33
34	.33869	36	.44554	46	.36005	35	.47306	46	34
35	9.33905	36	9.44601	46	9.36040	35	9.47352	45	35
36	.33941	36	.44647	46	.36076	35	.47398	45	36
37	.33977	36	.44693	46	.36111	35	.47443	45	37
38	.34013	36	.44739	46	.36146	35	.47489	45	38
39	.34049	36	.44785	46	.36181	35	.47534	45	39
40	9.34085	36	9.44831	46	9.36216	35	9.47580	45	40
41	.34121	36	.44877	46	.36251	35	.47625	45	41
42	.34157	36	.44924	46	.36286	35	.47671	45	42
43	.34193	36	.44970	46	.36321	35	.47716	45	43
44	.34229	36	.45016	46	.36356	35	.47762	45	44
45	9.34265	36	9.45062	46	9.36391	35	9.47807	45	45
46	.34301	36	.45108	46	.36426	35	.47852	45	46
47	.34337	36	.45154	46	.36461	34	.47898	45	47
48	.34373	35	.45200	46	.36495	35	.47943	45	48
49	.34408	36	.45246	46	.36530	35	.47989	45	49
50	9.34444	36	9.45292	46	9.36565	35	9.48034	45	50
51	.34480	35	.45338	46	.36600	34	.48080	45	51
52	.34516	36	.45384	46	.36635	35	.48125	45	52
53	.34552	35	.45430	46	.36670	35	.48170	45	53
54	.34587	36	.45476	46	.36705	34	.48216	45	54
55	9.34623	35	9.45522	46	9.36739	35	9.48261	45	55
56	.34659	36	.45568	46	.36774	34	.48306	45	56
57	.34695	35	.45614	46	.36809	35	.48352	45	57
58	.34730	36	.45660	46	.36844	34	.48397	45	58
59	.34766	35	.45706	46	.36878	35	.48442	45	59
60	9.34802		9.45752		9.36913		9.48488		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	47	46
6	4.7	4.6
7	5.5	5.4
8	6.2	6.2
9	7.0	7.0
10	7.8	7.7
20	15.6	15.5
30	23.5	23.2
40	31.3	31.0
50	39.1	38.7

	46	45
6	4.6	4.5
7	5.3	5.3
8	6.1	6.0
9	6.9	6.8
10	7.6	7.6
20	15.3	15.1
30	23.0	22.7
40	30.6	30.3
50	38.3	37.9

	45
6	4.5
7	5.2
8	6.0
9	6.7
10	7.5
20	15.0
30	22.5
40	30.0
50	37.5

	37	36
6	3.7	3.6
7	4.3	4.2
8	4.9	4.8
9	5.5	5.5
10	6.1	6.1
20	12.3	12.1
30	18.5	18.2
40	24.6	24.3
50	30.8	30.4

	36	35
6	3.6	3.5
7	4.2	4.1
8	4.8	4.7
9	5.4	5.3
10	6.0	5.9
20	12.0	11.8
30	18.0	17.7
40	24.0	23.6
50	30.0	29.6

	35	34
6	3.5	3.4
7	4.1	4.0
8	4.6	4.6
9	5.2	5.2
10	5.8	5.7
20	11.6	11.5
30	17.5	17.2
40	23.3	23.0
50	29.1	28.7

P. P.

	40°				41°					
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	
0	9.36913	34	9.48488	45	9.38968	34	9.51190	45	0	
1	.36948	34	.48533	45	.39002	33	.51235	44	1	
2	.36982	35	.48578	45	.39035	34	.51279	45	2	
3	.37017	34	.48624	45	.39069	33	.51324	44	3	
4	.37052	34	.48669	45	.39103	34	.51369	45	4	
5	9.37086	35	9.48714	45	9.39137	33	9.51414	44	5	
6	.37121	34	.48759	45	.39170	33	.51458	45	6	
7	.37156	34	.48805	45	.39204	34	.51503	44	7	
8	.37190	34	.48850	45	.39238	33	.51548	44	8	
9	.37225	34	.48895	45	.39271	33	.51592	45	9	
10	9.37259	34	9.48940	45	9.39305	34	9.51637	44	10	
11	.37294	34	.48986	45	.39339	33	.51682	44	11	
12	.37328	34	.49031	45	.39372	33	.51726	45	12	
13	.37363	34	.49076	45	.39406	33	.51771	44	13	
14	.37397	34	.49121	45	.39439	33	.51816	44	14	
15	9.37432	34	9.49166	45	9.39473	34	9.51860	45	15	
16	.37466	34	.49211	45	.39507	33	.51905	44	16	
17	.37501	34	.49257	45	.39540	33	.51950	44	17	
18	.37535	34	.49302	45	.39574	33	.51994	44	18	
19	.37570	34	.49347	45	.39607	33	.52039	45	19	
20	9.37604	34	9.49392	45	9.39641	33	9.52084	44	20	
21	.37639	34	.49437	45	.39674	33	.52128	44	21	
22	.37673	34	.49482	45	.39708	33	.52173	44	22	
23	.37707	34	.49527	45	.39741	33	.52217	44	23	
24	.37742	34	.49572	45	.39774	33	.52262	44	24	
25	9.37776	34	9.49618	45	9.39808	33	9.52306	45	25	
26	.37810	34	.49663	45	.39841	33	.52351	44	26	
27	.37845	34	.49708	45	.39875	33	.52396	44	27	
28	.37879	34	.49753	45	.39908	33	.52440	44	28	
29	.37913	34	.49798	45	.39941	33	.52485	44	29	
30	9.37947	34	9.49843	45	9.39975	33	9.52529	44	30	
31	.37982	34	.49888	45	.40008	33	.52574	44	31	
32	.38016	34	.49933	45	.40041	33	.52618	44	32	
33	.38050	34	.49978	45	.40075	33	.52663	44	33	
34	.38084	34	.50023	45	.40108	33	.52707	44	34	
35	9.38118	34	9.50068	45	9.40141	33	9.52752	44	35	
36	.38153	34	.50113	45	.40175	33	.52796	44	36	
37	.38187	34	.50158	45	.40208	33	.52841	44	37	
38	.38221	34	.50203	45	.40241	33	.52885	44	38	
39	.38255	34	.50248	45	.40274	33	.52930	44	39	
40	9.38289	34	9.50293	45	9.40307	33	9.52974	44	40	
41	.38323	34	.50338	45	.40341	33	.53018	44	41	
42	.38357	34	.50383	44	.40374	33	.53063	44	42	
43	.38391	34	.50427	45	.40407	33	.53107	44	43	
44	.38425	34	.50472	45	.40440	33	.53152	44	44	
45	9.38459	34	9.50517	45	9.40473	33	9.53196	44	45	
46	.38493	34	.50562	45	.40506	33	.53240	44	46	
47	.38527	34	.50607	44	.40540	33	.53285	44	47	
48	.38561	34	.50652	45	.40573	33	.53329	44	48	
49	.38595	34	.50697	45	.40606	33	.53374	44	49	
50	9.38629	34	9.50742	45	9.40639	33	9.53418	44	50	
51	.38663	34	.50787	44	.40672	33	.53462	44	51	
52	.38697	33	.50831	45	.40705	33	.53507	44	52	
53	.38731	34	.50876	45	.40738	33	.53551	44	53	
54	.38765	34	.50921	44	.40771	33	.53595	44	54	
55	9.38799	34	9.50966	45	9.40804	33	9.53640	44	55	
56	.38833	33	.51011	44	.40837	33	.53684	44	56	
57	.38866	34	.51055	45	.40870	33	.53728	44	57	
58	.38900	33	.51100	45	.40903	33	.53773	44	58	
59	.38934	34	.51145	44	.40936	33	.53817	44	59	
60	9.38968		9.51190		9.40969		9.53861		60	
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′	

P. P.

	$\overline{45}$	45
6	4.5	4.5
7	5.3	5.2
8	6.0	6.0
9	6.8	6.7
10	7.6	7.5
20	15.1	15.0
30	22.7	22.5
40	30.3	30.0
50	37.9	37.5

	$\overline{44}$	44
6	4.4	4.4
7	5.2	5.1
8	5.9	5.8
9	6.7	6.6
10	7.4	7.3
20	14.8	14.6
30	22.2	22.0
40	29.6	29.3
50	37.1	36.6

	35	$\overline{34}$
6	3.5	3.4
7	4.1	4.0
8	4.6	4.6
9	5.2	5.2
10	5.8	5.7
20	11.6	11.5
30	17.5	17.2
40	23.3	23.0
50	29.1	28.7

	34	$\overline{33}$
6	3.4	3.3
7	3.9	3.9
8	4.5	4.4
9	5.1	5.0
10	5.6	5.6
20	11.3	11.1
30	17.0	16.7
40	22.6	22.3
50	28.3	27.9

	33
6	3.3
7	3.8
8	4.4
9	4.9
10	5.5
20	11.0
30	16.5
40	22.0
50	27.5

P. P.

	42°				43°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.40969	32	9.53861	44	9.42918	32	9.56505	43	0
1	.41001	33	.53906	44	.42950	32	.56549	44	1
2	.41034	33	.53950	44	.42982	32	.56593	44	2
3	.41067	33	.53994	44	.43014	32	.56637	43	3
4	.41100	32	.54038	44	.43046	32	.56680	44	4
5	9.41133	33	9.54083	44	9.43078	32	9.56724	44	5
6	.41166	33	.54127	44	.43110	31	.56768	43	6
7	.41199	32	.54171	44	.43142	32	.56812	44	7
8	.41231	33	.54215	44	.43174	32	.56856	43	8
9	.41264	32	.54259	44	.43206	32	.56899	44	9
10	9.41297	33	9.54304	44	9.43238	32	9.56943	44	10
11	.41330	32	.54348	44	.43270	32	.56987	43	11
12	.41362	33	.54392	44	.43302	32	.57031	44	12
13	.41395	32	.54436	44	.43334	31	.57075	43	13
14	.41428	33	.54480	44	.43365	32	.57118	44	14
15	9.41461	32	9.54525	44	9.43397	32	9.57162	43	15
16	.41493	33	.54569	44	.43429	32	.57206	44	16
17	.41526	32	.54613	44	.43461	31	.57250	43	17
18	.41559	32	.54657	44	.43493	32	.57293	44	18
19	.41591	32	.54701	44	.43525	32	.57337	43	19
20	9.41624	33	9.54745	44	9.43557	31	9.57381	43	20
21	.41657	32	.54790	44	.43588	32	.57424	44	21
22	.41689	32	.54834	44	.43620	31	.57468	43	22
23	.41722	32	.54878	44	.43652	32	.57512	44	23
24	.41754	32	.54922	44	.43684	31	.57556	43	24
25	9.41787	32	9.54966	44	9.43715	32	9.57599	43	25
26	.41819	32	.55010	44	.43747	31	.57643	44	26
27	.41852	33	.55054	44	.43779	31	.57687	43	27
28	.41885	32	.55098	44	.43810	32	.57730	43	28
29	.41917	32	.55142	44	.43842	31	.57774	44	29
30	9.41950	32	9.55186	44	9.43874	32	9.57818	43	30
31	.41982	32	.55230	44	.43906	31	.57861	43	31
32	.42014	32	.55275	44	.43937	31	.57905	44	32
33	.42047	32	.55319	44	.43969	31	.57949	43	33
34	.42079	32	.55363	44	.44000	32	.57992	43	34
35	9.42112	32	9.55407	44	9.44032	31	9.58036	43	35
36	.42144	32	.55451	44	.44064	31	.58079	44	36
37	.42177	32	.55495	44	.44095	31	.58123	43	37
38	.42209	32	.55539	44	.44127	31	.58167	43	38
39	.42241	32	.55583	44	.44158	31	.58210	43	39
40	9.42274	32	9.55627	44	9.44190	31	9.58254	43	40
41	.42306	32	.55671	44	.44221	31	.58297	44	41
42	.42338	32	.55715	44	.44253	31	.58341	43	42
43	.42371	32	.55759	44	.44284	31	.58385	43	43
44	.42403	32	.55803	44	.44316	31	.58428	43	44
45	9.42435	32	9.55847	43	9.44347	31	9.58472	43	45
46	.42467	32	.55890	44	.44379	31	.58515	43	46
47	.42500	32	.55934	44	.44410	31	.58559	43	47
48	.42532	32	.55978	44	.44442	31	.58602	43	48
49	.42564	32	.56022	44	.44473	31	.58646	43	49
50	9.42596	32	9.56066	44	9.44504	31	9.58689	43	50
51	.42629	32	.56110	44	.44536	31	.58733	43	51
52	.42661	32	.56154	43	.44567	31	.58776	44	52
53	.42693	32	.56198	44	.44599	31	.58820	43	53
54	.42725	32	.56242	44	.44630	31	.58864	43	54
55	9.42757	32	9.56286	44	9.44661	31	9.58907	43	55
56	.42789	32	.56330	44	.44693	31	.58951	43	56
57	.42822	32	.56374	43	.44724	31	.58994	43	57
58	.42854	32	.56417	44	.44755	31	.59037	43	58
59	.42886	32	.56461	43	.44787	31	.59081	43	59
60	9.42918		9.56505		9.44818		9.59124		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	44	44
6	4.4	4.4
7	5.2	5.1
8	5.9	5.8
9	6.7	6.6
10	7.4	7.3
20	14.8	14.6
30	22.2	22.0
40	29.6	29.3
50	37.1	36.6

	43	43
6	4.3	4.3
7	5.1	5.0
8	5.8	5.7
9	6.5	6.4
10	7.2	7.1
20	14.5	14.3
30	21.7	21.5
40	29.0	28.6
50	36.2	35.8

	33	32
6	3.3	3.2
7	3.8	3.8
8	4.4	4.3
9	4.9	4.9
10	5.5	5.4
20	11.0	10.8
30	16.5	16.2
40	22.0	21.6
50	27.5	27.1

	32	31
6	3.2	3.1
7	3.7	3.7
8	4.2	4.2
9	4.8	4.7
10	5.3	5.2
20	10.6	10.5
30	16.0	15.7
40	21.3	21.0
50	26.6	26.2

	31
6	3.1
7	3.6
8	4.1
9	4.6
10	5.1
20	10.3
30	15.5
40	20.6
50	25.8

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

′	44° Lg. Vers.	D	44° Log. Exs.	D	45° Lg. Vers.	D	45° Log. Exs.	D	′
0	9.44818	31	9.59124	43	9.46671	30	9.61722	43	0
1	.44849	31	.59168	43	.46701	30	.61765	43	1
2	.44880	31	.59211	43	.46732	30	.61808	43	2
3	.44912	31	.59255	43	.46762	30	.61852	43	3
4	.44943	31	.59298	43	.46793	30	.61895	43	4
5	9.44974	31	9.59342	43	9.46823	30	9.61938	43	5
6	.45005	31	.59385	43	.46853	30	.61981	43	6
7	.45036	31	.59429	43	.46884	30	.62024	43	7
8	.45068	31	.59472	43	.46914	30	.62067	43	8
9	.45099	31	.59515	43	.46945	30	.62110	43	9
10	9.45130	31	9.59559	43	9.46975	30	9.62153	43	10
11	.45161	31	.59602	43	.47005	30	.62196	43	11
12	.45192	31	.59646	43	.47036	30	.62239	43	12
13	.45223	31	.59689	43	.47066	30	.62282	43	13
14	.45254	31	.59732	43	.47096	30	.62326	43	14
15	9.45285	31	9.59776	43	9.47127	30	9.62369	43	15
16	.45316	31	.59819	43	.47157	30	.62412	43	16
17	.45348	31	.59863	43	.47187	30	.62455	43	17
18	.45379	31	.59906	43	.47218	30	.62498	43	18
19	.45410	31	.59949	43	.47248	30	.62541	43	19
20	9.45441	31	9.59993	43	9.47278	30	9.62584	43	20
21	.45472	31	.60036	43	.47308	30	.62627	43	21
22	.45503	31	.60079	43	.47339	30	.62670	43	22
23	.45534	31	.60123	43	.47369	30	.62713	43	23
24	.45565	30	.60166	43	.47399	30	.62756	43	24
25	9.45595	31	9.60209	43	9.47429	30	9.62799	43	25
26	.45626	31	.60253	43	.47459	30	.62842	43	26
27	.45657	31	.60296	43	.47490	30	.62885	43	27
28	.45688	31	.60339	43	.47520	30	.62928	43	28
29	.45719	31	.60383	43	.47550	30	.62971	43	29
30	9.45750	30	9.60426	43	9.47580	30	9.63014	43	30
31	.45781	31	.60469	43	.47610	30	.63057	43	31
32	.45812	31	.60512	43	.47640	30	.63100	43	32
33	.45843	30	.60556	43	.47670	30	.63143	43	33
34	.45873	31	.60599	43	.47700	30	.63186	43	34
35	9.45904	31	9.60642	43	9.47731	30	9.63229	43	35
36	.45935	30	.60685	43	.47761	30	.63272	43	36
37	.45966	31	.60729	43	.47791	30	.63315	43	37
38	.45997	30	.60772	43	.47821	30	.63358	43	38
39	.46027	31	.60815	43	.47851	30	.63401	42	39
40	9.46058	30	9.60858	43	9.47881	30	9.63443	43	40
41	.46089	31	.60902	43	.47911	30	.63486	43	41
42	.46120	30	.60945	43	.47941	30	.63529	43	42
43	.46150	31	.60988	43	.47971	30	.63572	43	43
44	.46181	30	.61031	43	.48001	30	.63615	43	44
45	9.46212	30	9.61075	43	9.48031	30	9.63658	42	45
46	.46242	31	.61118	43	.48061	29	.63701	43	46
47	.46273	30	.61161	43	.48090	30	.63744	43	47
48	.46304	30	.61204	43	.48120	30	.63787	43	48
49	.46334	31	.61247	43	.48150	30	.63830	43	49
50	9.46365	30	9.61291	43	9.48180	30	9.63873	42	50
51	.46396	30	.61334	43	.48210	29	.63915	43	51
52	.46426	30	.61377	43	.48240	30	.63958	43	52
53	.46457	30	.61420	43	.48270	30	.64001	43	53
54	.46487	31	.61463	43	.48300	29	.64044	42	54
55	9.46518	30	9.61506	43	9.48329	30	9.64087	43	55
56	.46549	30	.61550	43	.48359	30	.64130	43	56
57	.46579	30	.61593	43	.48389	29	.64173	43	57
58	.46610	30	.61636	43	.48419	30	.64216	42	58
59	.46640	30	.61679	43	.48449	29	.64258	43	59
60	9.46671		9.61722		9.48478		9.64301		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	43	43
6	4.3	4.3
7	5.1	5.0
8	5.8	5.7
9	6.5	6.4
10	7.2	7.1
20	14.5	14.3
30	21.7	21.5
40	29.0	28.6
50	36.2	35.8

	42
6	4.2
7	4.9
8	5.6
9	6.4
10	7.1
20	14.1
30	21.2
40	28.3
50	35.4

	31	31
6	3.1	3.1
7	3.7	3.6
8	4.2	4.1
9	4.7	4.6
10	5.2	5.1
20	10.5	10.3
30	15.7	15.5
40	21.0	20.6
50	26.2	25.8

	30	30
6	3.0	3.0
7	3.5	3.5
8	4.0	4.0
9	4.6	4.5
10	5.1	5.0
20	10.1	10.0
30	15.2	15.0
40	20.3	20.0
50	25.4	25.0

	29
6	2.9
7	3.4
8	3.9
9	4.4
10	4.9
20	9.8
30	14.7
40	19.6
50	24.6

P. P.

	46°				47°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.48478	30	9.64301	43	9.50243	29	9.66864	42	0
1	.48508	29	.64344	42	.50272	29	.66907	43	1
2	.48538	30	.64387	43	.50301	29	.66950	42	2
3	.48568	29	.64430	43	.50330	29	.66992	42	3
4	.48597	30	.64473	42	.50359	29	.67035	42	4
5	9.48627	29	9.64515	43	9.50388	29	9.67077	42	5
6	.48657	29	.64558	43	.50417	29	.67120	42	6
7	.48686	30	.64601	42	.50446	29	.67162	43	7
8	.48716	29	.64644	43	.50475	29	.67205	42	8
9	.48746	29	.64687	42	.50504	29	.67248	42	9
10	9.48775	30	9.64729	43	9.50533	29	9.67290	42	10
11	.48805	29	.64772	43	.50562	29	.67333	42	11
12	.48835	29	.64815	42	.50591	28	.67375	42	12
13	.48864	29	.64858	43	.50619	29	.67418	42	13
14	.48894	29	.64901	42	.50648	29	.67460	42	14
15	9.48923	30	9.64943	43	9.50677	29	9.67503	43	15
16	.48953	29	.64986	42	.50706	28	.67546	42	16
17	.48983	29	.65029	43	.50735	29	.67588	42	17
18	.49012	29	.65072	42	.50764	29	.67631	42	18
19	.49042	29	.65114	43	.50793	28	.67673	42	19
20	9.49071	29	9.65157	42	9.50821	29	9.67716	42	20
21	.49101	29	.65200	43	.50850	29	.67758	42	21
22	.49130	29	.65243	42	.50879	28	.67801	42	22
23	.49160	29	.65285	43	.50908	29	.67843	42	23
24	.49189	29	.65328	42	.50937	28	.67886	42	24
25	9.49219	29	9.65371	43	9.50965	29	9.67928	42	25
26	.49248	29	.65414	42	.50994	28	.67971	42	26
27	.49278	29	.65456	43	.51023	29	.68013	42	27
28	.49307	29	.65499	42	.51052	28	.68056	42	28
29	.49336	29	.65542	43	.51080	29	.68098	42	29
30	9.49300	29	9.65585	42	9.51109	28	9.68141	42	30
31	.49395	29	.65627	43	.51138	29	.68183	42	31
32	.49425	29	.65670	42	.51167	28	.68226	42	32
33	.49454	29	.65713	42	.51195	28	.68268	42	33
34	.49483	29	.65755	43	.51224	29	.68311	42	34
35	9.49513	29	9.65798	42	9.51253	28	9.68353	42	35
36	.49542	29	.65841	43	.51281	28	.68396	42	36
37	.49571	29	.65884	42	.51310	28	.68438	42	37
38	.49601	29	.65926	42	.51338	29	.68481	42	38
39	.49630	29	.65969	43	.51367	28	.68523	42	39
40	9.49059	29	9.66012	42	9.51396	28	9.68566	42	40
41	.49689	29	.66054	42	.51424	28	.68608	42	41
42	.49718	29	.66097	43	.51453	28	.68651	42	42
43	.49747	29	.66140	42	.51481	28	.68693	42	43
44	.49776	29	.66182	42	.51510	29	.68735	42	44
45	9.49806	29	9.66225	43	9.51539	28	9.68778	42	45
46	.49835	29	.66268	42	.51567	28	.68820	42	46
47	.49864	29	.66310	42	.51596	28	.68863	42	47
48	.49893	29	.66353	43	.51624	28	.68905	42	48
49	.49922	29	.66396	42	.51653	28	.68948	42	49
50	9.49952	29	9.66438	42	9.51681	28	9.68990	42	50
51	.49981	29	.66481	42	.51710	28	.69033	42	51
52	.50010	29	.66523	43	.51738	28	.69075	42	52
53	.50039	29	.66566	42	.51767	28	.69117	42	53
54	.50068	29	.66609	42	.51795	28	.69160	42	54
55	9.50097	29	9.66651	42	9.51823	28	9.69202	42	55
56	.50126	29	.66694	43	.51852	28	.69245	42	56
57	.50155	29	.66737	42	.51880	28	.69287	42	57
58	.50185	29	.66779	42	.51909	28	.69330	42	58
59	.50214	29	.66822	42	.51937	28	.69372	42	59
60	9.50243		9.66864		9.51965		9.69414		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	43	42
6	4.3	4.2
7	5.0	4.9
8	5.7	5.6
9	6.4	6.4
10	7.1	7.1
20	14.3	14.1
30	21.5	21.2
40	28.6	28.3
50	35.8	35.4

	42
6	4.2
7	4.9
8	5.6
9	6.3
10	7.0
20	14.0
30	21.0
40	28.0
50	35.0

	30	29
6	3.0	2.9
7	3.5	3.4
8	4.0	3.9
9	4.5	4.4
10	5.0	4.9
20	10.0	9.8
30	15.0	14.7
40	20.0	19.6
50	25.0	24.6

	29	28
6	2.9	2.8
7	3.4	3.3
8	3.8	3.8
9	4.3	4.3
10	4.8	4.7
20	9.6	9.5
30	14.5	14.2
40	19.3	19.0
50	24.1	23.7

	28
6	2.8
7	3.2
8	3.7
9	4.2
10	4.6
20	9.3
30	14.0
40	18.6
50	23.3

P. P.

′	48° Lg. Vers.	D	48° Log. Exs.	D	49° Lg. Vers.	D	49° Log. Exs.	D	′
0	9.51965̄	28̄	9.69414̄	42̄	9.53648̄	27̄	9.71954̲	42̄	0
1	.51994̲	28̄	.69457̲	42̲	.53676	28	.71996̄	42̲	1
2	.52022̄	28	.69499̄	42̄	.53704̲	27̲	.72038̄	42̄	2
3	.52050̄	28̄	.69542	42	.53731̄	27̄	.72081	42	3
4	.52079	28̄	.69584	42̄	.53759	28	.72123	42̄	4
5	9.52107̄	28	9.69626̄	42̄	9.53787̲	27̄	9.72165̲	42̲	5
6	.52135̄	28̄	.69669̲	42̄	.53814̄	27̄	.72207̄	42̄	6
7	.52164̲	28̄	.69711̲	42	.53842	28	.72250	42̲	7
8	.52192̄	28	.69753̄	42̄	.53870̲	27̄	.72292̲	42̄	8
9	.52220̄	28̄	.69796	42̄	.53897̄	27̲	.72334̲	42̲	9
10	9.52249	28	9.69838̄	42̄	9.53925̲	27̄	9.72376̄	42̄	10
11	.52277̲	28	.69881	42	.53952̲	28	.72419	42	11
12	.52305̄	28	.69923̲	42̄	.53980̄	27̄	.72461	42̲	12
13	.52333̄	28̄	.69965̄	42̄	.54008̲	27	.72503̲	42̄	13
14	.52362	28	.70008	42̄	.54035̄	27̄	.72545̄	42̲	14
15	9.52390	28	9.70050̲	42̄	9.54063̲	27̄	9.72587̄	42̄	15
16	.52418̲	28̄	.70092̄	42̄	.54090̄	27̄	.72630	42̲	16
17	.52446̲	28̲	.70135̲	42̄	.54118̲	27̄	.72672̲	42̄	17
18	.52474̄	28̄	.70177̄	42̄	.54145̄	27	.72714̲	42	18
19	.52503	28	.70220	42	.54173	27̄	.72756̄	42̄	19
20	9.52531	28	9.70262̲	42̄	9.54200̄	27̄	9.72799	42	20
21	.52559	28	.70304̄	42̄	.54228̲	27̄	.72841	42̲	21
22	.52587̲	28̄	.70347	42	.54255̄	27̄	.72883̲	42̄	22
23	.52615̄	28	.70389̲	42̄	.54283̲	27	.72925̄	42	23
24	.52643̄	28	.70431̄	42̄	.54310̄	27̄	.72967̄	42̄	24
25	9.52671̲	28	9.70474	42̲	9.54338̲	27̄	9.73010	42	25
26	.52699̄	28̲	.70516̲	42̄	.54365̄	27̄	.73052	42̲	26
27	.52727̄	28̄	.70558̄	42̄	.54393̲	27̄	.73094̲	42̄	27
28	.52756	28	.70601	42	.54420̄	27	.73136̄	42	28
29	.52784̲	28	.70643	42̄	.54448	27̄	.73178̄	42̄	29
30	9.52812	28	9.70685̄	42̄	9.54475̲	27̄	9.73221	42	30
31	.52840	28	.70728	42̲	.54502̄	27̄	.73263	42̲	31
32	.52868	28	.70770̲	42̄	.54530̲	27̄	.73305̲	42̄	32
33	.52896	28	.70812̄	42	.54557̄	27	.73347̄	42	33
34	.52924̲	28	.70854̄	42̄	.54585	27̲	.73389̄	42̲	34
35	9.52952	28	9.70897̲	42̄	9.54612̲	27̄	9.73431̄	42̄	35
36	.52980	28	.70939̲	42̲	.54639̄	27̄	.73474	42̲	36
37	.53008	28	.70981̄	42̄	.54667	27	.73516̲	42̄	37
38	.53036	28	.71024̲	42̄	.54694̲	27̄	.73558̲	42	38
39	.53064̲	28	.71066̄	42̲	.54721̄	27̲	.73600̄	42̲	39
40	9.53092	28̲	9.71108	42̄	9.54748̄	27̄	9.73642̄	42̄	40
41	.53120̲	27̄	.71151	42̲	.54776̲	27̄	.73685	42	41
42	.53147̄	28	.71193̄	42̄	.54803̄	27̄	.73727	42̲	42
43	.53175̄	28	.71235̄	42̄	.54830̄	27̄	.73769̲	42̄	43
44	.53203̄	28̲	.71278	42	.54858	27̲	.73811̄	42	44
45	9.53231̄	27̄	9.71320̲	42̄	9.54885̲	27̄	9.73853̄	42̲	45
46	.53259	28	.71362̄	42̲	.54912̲	27̲	.73895̄	42̄	46
47	.53287	28	.71404̄	42̄	.54939̄	27̄	.73938	42	47
48	.53315	28	.71447	42	.54967	27	.73980	42	48
49	.53343	27̄	.71489	42̄	.54994	27̲	.74022	42̄	49
50	9.53370̄	28	9.71531̄	42̲	9.55021̲	27̄	9.74064̲	42	50
51	.53398̄	28̲	.71573̄	42̄	.55048̄	27̲	.74106̄	42̲	51
52	.53426̄	27̄	.71616̲	42̄	.55075̄	27̄	.74148̄	42̄	52
53	.53454	28	.71658̄	42	.55103	27	.74191	42	53
54	.53482	27̄	.71700̄	42̄	.55130	27	.74233	42̲	54
55	9.53509̄	28̲	9.71743	42̲	9.55157	27̲	9.74275̲	42̄	55
56	.53537̄	27̄	.71785̲	42̄	.55184̲	27̄	.74317̲	42	56
57	.53565	28̲	.71827̄	42̄	.55211̄	27	.74359̄	42̲	57
58	.53593	27̄	.71869̄	42	.55238̄	27	.74401̄	42̄	58
59	.53620̄	28	.71912	42̄	.55265	27	.74444	42	59
60	9.53648̄		9.71954̲		9.55292̄		9.74486		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	42̄	42
6	4.2̄	4.2
7	4.9̄	4.9
8	5.6̄	5.6
9	6.4	6.3
10	7.1̲	7.0
20	14.1̄	14.0
30	21.2̄	21.0
40	28.3̄	28.0
50	35.4	35.0

	28̄	28
6	2.8̄	2.8̲
7	3.3	3.2̄
8	3.8	3.7̄
9	4.3̲	4.2̲
10	4.7̄	4.6̄
20	9.5̲	9.3̄
30	14.2̄	14.0̲
40	19.0	18.6̄
50	23.7̄	23.3

	27̄	27
6	2.7̄	2.7̲
7	3.2̲	3.1̄
8	3.6̄	3.6̲
9	4.1	4.0̄
10	4.6	4.5
20	9.1̄	9.0
30	13.7̄	13.5
40	18.3̄	18.0
50	22.9	22.5

P. P.

	54°				55°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs	D	′
0	9.55292	27	9.74486	42	9.56900	26	9.77012	42	0
1	.55319	27	.74528	42	.56926	26	.77055	42	1
2	.55347	27	.74570	42	.56953	26	.77097	42	2
3	.55374	27	.74612	42	.56979	26	.77139	42	3
4	.55401	27	.74654	42	.57005	26	.77181	42	4
5	9.55428	27	9.74696	42	9.57032	26	9.77223	42	5
6	.55455	27	.74739	42	.57058	26	.77265	42	6
7	.55482	27	.74781	42	.57085	26	.77307	42	7
8	.55509	27	.74823	42	.57111	26	.77349	42	8
9	.55536	27	.74865	42	.57138	26	.77391	42	9
10	9.55563	27	9.74907	42	9.57164	26	9.77433	42	10
11	.55590	27	.74949	42	.57190	26	.77475	42	11
12	.55617	27	.74991	42	.57217	26	.77517	42	12
13	.55644	27	.75033	42	.57243	26	.77560	42	13
14	.55671	27	.75076	42	.57269	26	.77602	42	14
15	9.55698	27	9.75118	42	9.57296	26	9.77644	42	15
16	.55725	26	.75160	42	.57322	26	.77686	42	16
17	.55751	27	.75202	42	.57348	26	.77728	42	17
18	.55778	27	.75244	42	.57375	26	.77770	42	18
19	.55805	27	.75286	42	.57401	26	.77812	42	19
20	9.55832	27	9.75328	42	9.57427	26	9.77854	42	20
21	.55859	26	.75370	42	.57454	26	.77896	42	21
22	.55886	27	.75413	42	.57480	26	.77938	42	22
23	.55913	27	.75455	42	.57506	26	.77980	42	23
24	.55940	26	.75497	42	.57532	26	.78022	42	24
25	9.55906	27	9.75539	42	9.57559	26	9.78064	42	25
26	.55993	27	.75581	42	.57585	26	.78107	42	26
27	.56020	26	.75623	42	.57611	26	.78149	42	27
28	.56047	27	.75665	42	.57637	26	.78191	42	28
29	.56074	27	.75707	42	.57664	26	.78233	42	29
30	9.56101	26	9.75750	42	9.57690	26	9.78275	42	30
31	.56127	27	.75792	42	.57716	26	.78317	42	31
32	.56154	26	.75834	42	.57742	26	.78359	42	32
33	.56181	27	.75876	42	.57768	26	.78401	42	33
34	.56208	26	.75918	42	.57794	26	.78443	42	34
35	9.56234	27	9.75960	42	9.57821	26	9.78485	42	35
36	.56261	26	.76002	42	.57847	26	.78527	42	36
37	.56288	27	.76044	42	.57873	26	.78569	42	37
38	.56315	26	.76086	42	.57899	26	.78611	42	38
39	.56311	26	.76128	42	.57925	26	.78653	42	39
40	9.56368	27	9.76171	42	9.57951	26	9.78696	42	40
41	.56395	26	.76213	42	.57977	26	.78738	42	41
42	.56421	26	.76255	42	.58003	26	.78780	42	42
43	.56448	27	.76297	42	.58029	26	.78822	42	43
44	.56475	26	.76339	42	.58055	26	.78864	42	44
45	9.56501	26	9.76381	42	9.58082	26	9.78906	42	45
46	.56528	26	.76423	42	.58108	26	.78948	42	46
47	.56554	27	.76465	42	.58134	26	.78990	42	47
48	.56581	26	.76507	42	.58160	26	.79032	42	48
49	.56608	26	.76549	42	.58186	26	.79074	42	49
50	9.56634	26	9.76592	42	9.58212	26	9.79116	42	50
51	.56661	26	.76634	42	.58238	26	.79158	42	51
52	.56687	26	.76676	42	.58264	26	.79200	42	52
53	.56714	27	.76718	42	.58290	26	.79242	42	53
54	.56741	26	.76760	42	.58316	26	.79285	42	54
55	9.56767	26	9.76802	42	9.58342	25	9.79327	42	55
56	.56794	26	.76844	42	.58367	26	.79369	42	56
57	.56820	26	.76886	42	.58393	26	.79411	42	57
58	.56847	26	.76928	42	.58419	26	.79453	42	58
59	.56873	26	.76970	42	.58445	26	.79495	42	59
60	9.56900		9.77012		9.58471		9.79537		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	4$\bar{2}$	42
6	4.2	4.2
7	4.9	4.9
8	5.6	5.6
9	6.4	6.3
10	7.1	7.0
20	14.1	14.0
30	21.2	21.0
40	28.3	28.0
50	35.4	35.0

	2$\bar{7}$	27
6	2.7	2.7
7	3.2	3.1
8	3.6	3.6
9	4.1	4.0
10	4.6	4.5
20	9.1	9.0
30	13.7	13.5
40	18.3	18.0
50	22.9	22.5

	2$\bar{6}$	26
6	2.6	2.6
7	3.1	3.0
8	3.5	3.4
9	4.0	3.9
10	4.4	4.3
20	8.8	8.6
30	13.2	13.0
40	17.6	17.3
50	22.1	21.6

	2$\bar{5}$
6	2.5
7	3.0
8	3.4
9	3.8
10	4.2
20	8.5
30	12.7
40	17.0
50	21.2

P. P.

	52°				53°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.58471	26	9.79537	42	9.60008	25	9.82062	42	0
1	.58497	25	.79579	42	.60034	25	.82104	42	1
2	.58523	26	.79621	42	.60059	25	.82146	42	2
3	.58549	26	.79663	42	.60084	25	.82188	42	3
4	.58575	26	.79705	42	.60110	25	.82230	42	4
5	9.58601	25	9.79747	42	9.60135	25	9.82272	42	5
6	.58626	26	.79789	42	.60160	25	.82315	42	6
7	.58652	26	.79831	42	.60185	25	.82357	42	7
8	.58678	25	.79874	42	.60211	25	.82399	42	8
9	.58704	26	.79916	42	.60236	25	.82441	42	9
10	9.58730	25	9.79958	42	9.60261	25	9.82483	42	10
11	.58755	26	.80000	42	.60286	25	.82525	42	11
12	.58781	26	.80042	42	.60312	25	.82567	42	12
13	.58807	25	.80084	42	.60337	25	.82609	42	13
14	.58833	26	.80126	42	.60362	25	.82651	42	14
15	9.58859	25	9.80168	42	9.60387	25	9.82694	42	15
16	.58884	26	.80210	42	.60412	25	.82736	42	16
17	.58910	25	.80252	42	.60438	25	.82778	42	17
18	.58936	26	.80294	42	.60463	25	.82820	42	18
19	.58962	25	.80336	42	.60488	25	.82862	42	19
20	9.58987	25	9.80378	42	9.60513	25	9.82904	42	20
21	.59013	26	.80420	42	.60538	25	.82946	42	21
22	.59039	25	.80463	42	.60563	25	.82988	42	22
23	.59064	26	.80505	42	.60589	25	.83031	42	23
24	.59090	25	.80547	42	.60614	25	.83073	42	24
25	9.59116	25	9.80589	42	9.60639	25	9.83115	42	25
26	.59141	26	.80631	42	.60664	25	.83157	42	26
27	.59167	25	.80673	42	.60689	25	.83199	42	27
28	.59193	25	.80715	42	.60714	25	.83241	42	28
29	.59218	25	.80757	42	.60739	25	.83283	42	29
30	9.59244	26	9.80799	42	9.60764	25	9.83325	42	30
31	.59270	25	.80841	42	.60789	25	.83368	42	31
32	.59295	25	.80883	42	.60814	25	.83410	42	32
33	.59321	25	.80925	42	.60839	25	.83452	42	33
34	.59346	25	.80968	42	.60864	25	.83494	42	34
35	9.59372	25	9.81010	42	9.60889	25	9.83536	42	35
36	.59397	26	.81052	42	.60914	25	.83578	42	36
37	.59423	25	.81094	42	.60939	25	.83620	42	37
38	.59449	25	.81136	42	.60964	25	.83663	42	38
39	.59474	25	.81178	42	.60989	25	.83705	42	39
40	9.59500	25	9.81220	42	9.61014	25	9.83747	42	40
41	.59525	25	.81262	42	.61039	25	.83789	42	41
42	.59551	25	.81304	42	.61064	25	.83831	42	42
43	.59576	25	.81346	42	.61089	25	.83873	42	43
44	.59602	25	.81388	42	.61114	25	.83916	42	44
45	9.59627	25	9.81430	42	9.61139	25	9.83958	42	45
46	.59653	25	.81473	42	.61164	25	.84000	42	46
47	.59678	25	.81515	42	.61189	24	.84042	42	47
48	.59704	25	.81557	42	.61214	25	.84084	42	48
49	.59729	25	.81599	42	.61239	25	.84126	42	49
50	9.59754	25	9.81641	42	9.61264	25	9.84168	42	50
51	.59780	25	.81683	42	.61289	24	.84211	42	51
52	.59805	25	.81725	42	.61313	25	.84253	42	52
53	.59831	25	.81767	42	.61338	25	.84295	42	53
54	.59856	25	.81809	42	.61363	25	.84337	42	54
55	9.59881	25	9.81851	42	9.61388	24	9.84379	42	55
56	.59907	25	.81894	42	.61413	25	.84422	42	56
57	.59932	25	.81936	42	.61438	24	.84464	42	57
58	.59958	25	.81978	42	.61462	25	.84506	42	58
59	.59983	25	.82020	42	.61487	25	.84548	42	59
60	9.60008		9.82062		9.61512		9.84590		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	42	42
6	4.2	4.2
7	4.9	4.9
8	5.6	5.6
9	6.4	6.3
10	7.1	7.0
20	14.1	14.0
30	21.2	21.0
40	28.3	28.0
50	35.4	35.0

	26	25
6	2.6	2.5
7	3.0	3.0
8	3.4	3.4
9	3.9	3.8
10	4.3	4.2
20	8.6	8.5
30	13.0	12.7
40	17.3	17.0
50	21.6	21.2

	25	24
6	2.5	2.4
7	2.9	2.8
8	3.3	3.2
9	3.7	3.7
10	4.1	4.1
20	8.3	8.1
30	12.5	12.2
40	16.6	16.3
50	20.8	20.4

P. P.

'	54° Lg. Vers.	D	54° Log. Exs.	D	55° Lg. Vers.	D	55° Log. Exs.	D	'
0	9.61512	24	9.84590	42	9.62984	24	9.87125	42	0
1	.61537	25	.84632	42	.63008	24	.87167	42	1
2	.61562	24	.84675	42	.63032	24	.87209	42	2
3	.61586	25	.84717	42	.63057	24	.87252	42	3
4	.61611	24	.84759	42	.63081	24	.87294	42	4
5	9.61636	25	9.84801	42	9.63105	24	9.87336	42	5
6	.61661	24	.84843	42	.63129	24	.87379	42	6
7	.61685	25	.84886	42	.63154	24	.87421	42	7
8	.61710	24	.84928	42	.63178	24	.87463	42	8
9	.61735	25	.84970	42	.63202	24	.87506	42	9
10	9.61760	24	9.85012	42	9.63226	24	9.87548	42	10
11	.61784	24	.85054	42	.63250	24	.87590	42	11
12	.61809	25	.85097	42	.63274	24	.87633	42	12
13	.61834	24	.85139	42	.63209	24	.87675	42	13
14	.61858	25	.85181	42	.63323	24	.87717	42	14
15	9.61883	24	9.85223	42	9.63347	24	9.87760	42	15
16	.61908	24	.85265	42	.63371	24	.87802	42	16
17	.61932	24	.85308	42	.63395	24	.87844	42	17
18	.61957	25	.85350	42	.63419	24	.87887	42	18
19	.61982	24	.85392	42	.63443	24	.87929	42	19
20	9.62006	24	9.85434	42	9.63468	24	9.87971	42	20
21	.62031	24	.85476	42	.63492	24	.88014	42	21
22	.62055	25	.85519	42	.63516	24	.88056	42	22
23	.62080	24	.85561	42	.63540	24	.88099	42	23
24	.62105	24	.85603	42	.63564	24	.88141	42	24
25	9.62129	24	9.85645	42	9.63588	24	9.88183	42	25
26	.62154	24	.85688	42	.63612	24	.88226	42	26
27	.62178	24	.85730	42	.63636	24	.88268	42	27
28	.62203	24	.85772	42	.63660	24	.88310	42	28
29	.62227	24	.85814	42	.63684	24	.88353	42	29
30	9.62252	24	9.85857	42	9.63708	24	9.88395	42	30
31	.62276	24	.85899	42	.63732	24	.88438	42	31
32	.62301	24	.85941	42	.63756	24	.88480	42	32
33	.62325	24	.85983	42	.63780	24	.88522	42	33
34	.62350	24	.86026	42	.63804	24	.88565	42	34
35	9.62374	24	9.86068	42	9.63828	24	9.88607	42	35
36	.62399	24	.86110	42	.63852	24	.88650	42	36
37	.62423	24	.86152	42	.63876	24	.88692	42	37
38	.62448	24	.86195	42	.63900	24	.88734	42	38
39	.62472	24	.86237	42	.63924	24	.88777	42	39
40	9.62497	24	9.86279	42	9.63948	24	9.88819	42	40
41	.62521	24	.86321	42	.63972	24	.88862	42	41
42	.62546	24	.86364	42	.63996	23	.88904	42	42
43	.62570	24	.86406	42	.64019	24	.88947	42	43
44	.62594	24	.86448	42	.64043	24	.88989	42	44
45	9.62619	24	9.86490	42	9.64067	24	9.89031	42	45
46	.62643	24	.86533	42	.64091	24	.89074	42	46
47	.62668	24	.86575	42	.64115	23	.89116	42	47
48	.62692	24	.86617	42	.64139	24	.89159	42	48
49	.62716	24	.86659	42	.64163	24	.89201	42	49
50	9.62741	24	9.86702	42	9.64187	23	9.89244	42	50
51	.62765	24	.86744	42	.64210	24	.89286	42	51
52	.62789	24	.86786	42	.64234	24	.89329	42	52
53	.62814	24	.86829	42	.64258	23	.89371	42	53
54	.62838	24	.86871	42	.64282	24	.89414	42	54
55	9.62862	24	9.86913	42	9.64306	24	9.89456	42	55
56	.62887	24	.86956	42	.64330	23	.89499	42	56
57	.62911	24	.86998	42	.64353	24	.89541	42	57
58	.62935	24	.87040	42	.64377	23	.89583	42	58
59	.62960	24	.87082	42	.64401	24	.89626	42	59
60	9.62984		9.87125		9.64425		9.89668		60
'	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	'

P. P.

	42	42
6	4.2	4.2
7	4.9	4.9
8	5.6	5.6
9	6.4	6.3
10	7.1	7.0
20	14.1	14.0
30	21.2	21.0
40	28.3	28.0
50	35.4	35.0

	25	24
6	2.5	2.4
7	2.9	2.8
8	3.3	3.2
9	3.7	3.7
10	4.1	4.1
20	8.3	8.1
30	12.5	12.2
40	16.6	16.3
50	20.8	20.4

	24	23
6	2.4	2.3
7	2.8	2.7
8	3.2	3.1
9	3.6	3.5
10	4.0	3.9
20	8.0	7.8
30	12.0	11.7
40	16.0	15.6
50	20.0	19.6

P. P.

	56°				57°				
′	Lg. Vers.	*D*	Log. Exs.	*D*	Lg. Vers.	*D*	Log. Exs.	*D*	′
0	9.64425	$2\bar{3}$	9.8966$\bar{8}$	$4\bar{2}$	9.6583$\bar{5}$	$2\bar{3}$	9.9222$\underline{4}$	43	0
1	.6444$\bar{8}$	24	.8971$\underline{1}$	$4\bar{2}$	.65859	$2\bar{3}$	.9226$\bar{7}$	$4\bar{2}$	1
2	.64472	$2\bar{3}$	.8975$\bar{3}$	$4\bar{2}$	.6588$\bar{2}$	$2\bar{3}$	.92310	43	2
3	.64496	24	.89796	$4\bar{2}$	.6590$\underline{5}$	23	.92353	$4\bar{2}$	3
4	.64520	$2\bar{3}$	.8983$\bar{8}$	$4\bar{2}$	.6592$\bar{8}$	$2\bar{3}$	.9239$\bar{5}$	43	4
5	9.6454$\bar{3}$	24	9.8988$\underline{1}$	$4\bar{2}$	9.65952	$2\bar{3}$	9.9243$\bar{8}$	$4\bar{2}$	5
6	.6456$\bar{7}$	$2\bar{3}$	.8992$\bar{3}$	$4\bar{2}$	.6597$\underline{5}$	$2\bar{3}$	.92481	43	6
7	.6459$\underline{1}$	$2\bar{3}$	.89966	$4\bar{2}$	.6599$\bar{8}$	23	.9252$\underline{4}$	$4\bar{2}$	7
8	.6461$\underline{4}$	24	.90008	$4\bar{2}$	.6602$\underline{1}$	23	.9256$\bar{6}$	43	8
9	.6463$\bar{8}$	$2\bar{3}$	.90051	43	.6604$\underline{4}$	$2\bar{3}$	.9260$\bar{9}$	$4\bar{2}$	9
10	9.64662	$2\bar{3}$	9.9009$\underline{4}$	$4\bar{2}$	9.66068	23	9.92652	43	10
11	.6468$\bar{5}$	24	.9013$\bar{6}$	$4\bar{2}$	.66091	$2\bar{3}$	.9269$\bar{5}$	$4\bar{2}$	11
12	.6470$\bar{9}$	$2\bar{3}$	.90179	$4\bar{2}$	.66114	$2\bar{3}$	.9273$\bar{7}$	43	12
13	.6473$\bar{3}$	$2\bar{3}$	.9022$\underline{1}$	$4\bar{2}$	.6613$\underline{7}$	23	.9278$\bar{0}$	$4\bar{2}$	13
14	.6475$\bar{6}$	24	.90264	$4\bar{2}$	.6616$\bar{0}$	$2\bar{3}$	.92823	43	14
15	9.6478$\bar{0}$	$2\bar{3}$	9.9030$\bar{6}$	$4\bar{2}$	9.6618$\bar{3}$	$2\bar{3}$	9.92866	43	15
16	.6480$\underline{4}$	$2\bar{3}$	.90349	$4\bar{2}$	.66207	23	.92909	$4\bar{2}$	16
17	.6482$\bar{7}$	$2\bar{3}$	.9039$\underline{1}$	$4\bar{2}$	.66230	23	.9295$\underline{1}$	43	17
18	.64851	24	.9043$\underline{4}$	$4\bar{2}$	.66253	23	.9299$\underline{4}$	$4\bar{2}$	18
19	.64875	$2\bar{3}$	.9047$\bar{6}$	$4\bar{2}$	.66276	$2\bar{3}$	.93037	43	19
20	9.6489$\bar{8}$	$2\bar{3}$	9.90519	$4\bar{2}$	9.6629$\bar{9}$	23	9.93080	43	20
21	.64922	$2\bar{3}$	.9056$\underline{1}$	43	.6632$\bar{2}$	23	.93123	$4\bar{2}$	21
22	.6494$\bar{5}$	$2\bar{3}$	.9060$\underline{4}$	$4\bar{2}$	.6634$\bar{5}$	23	.9316$\bar{5}$	43	22
23	.64969	$2\bar{3}$	.90647	$4\bar{2}$	.6636$\bar{8}$	23	.9320$\bar{8}$	43	23
24	.6499$\bar{2}$	24	.9068$\bar{9}$	$4\bar{2}$	.6639$\underline{1}$	$2\bar{3}$	.9325$\underline{1}$	$4\bar{2}$	24
25	9.6501$\bar{6}$	$2\bar{3}$	9.9073$\underline{2}$	$4\bar{2}$	9.66415	23	9.93294	43	25
26	.65040	$2\bar{3}$	.9077$\underline{4}$	$4\bar{2}$	.66438	23	.93337	43	26
27	.6506$\bar{3}$	$2\bar{3}$	.90817	43	.66461	23	.93380	$4\bar{2}$	27
28	.6508$\underline{7}$	$2\bar{3}$	.90860	$4\bar{2}$	.66484	23	.9342$\underline{2}$	43	28
29	.6511$\bar{0}$	$2\bar{3}$	.9090$\bar{2}$	$4\bar{2}$	.66507	23	.9346$\bar{5}$	43	29
30	9.65134	$2\bar{3}$	9.90945	$4\bar{2}$	9.66530	23	9.9350$\bar{8}$	$4\bar{2}$	30
31	.6515$\bar{7}$	$2\bar{3}$	.9098$\bar{7}$	$4\bar{2}$	.66553	23	.93551	43	31
32	.6518$\underline{1}$	$2\bar{3}$	.91030	43	.66576	23	.93594	43	32
33	.6520$\underline{4}$	$2\bar{3}$	.91073	$4\bar{2}$	.66599	23	.93637	43	33
34	.65228	$2\bar{3}$	.9111$\bar{5}$	$4\bar{2}$	.66622	23	.93680	$4\bar{2}$	34
35	9.6525$\underline{1}$	$2\bar{3}$	9.91158	$4\bar{2}$	9.66645	23	9.9372$\underline{2}$	43	35
36	.6527$\underline{5}$	$2\bar{3}$	.9120$\bar{0}$	$4\bar{2}$	.66668	23	.9376$\bar{5}$	43	36
37	.6529$\bar{8}$	23	.91243	43	.66691	23	.9380$\bar{8}$	43	37
38	.6532$\underline{1}$	23	.91286	$4\bar{2}$	.66714	23	.9385$\underline{1}$	$4\bar{2}$	38
39	.65345	$2\bar{3}$	.9132$\bar{8}$	$4\bar{2}$	.66737	23	.93894	43	39
40	9.6536$\bar{8}$	$2\bar{3}$	9.91371	43	9.66760	$2\bar{3}$	9.93937	43	40
41	.6539$\underline{2}$	$2\bar{3}$	.9141$\underline{4}$	$4\bar{2}$	.6678$\bar{3}$	$2\bar{3}$	.93980	43	41
42	.6541$\bar{5}$	$2\bar{3}$	.9145$\bar{6}$	$4\bar{2}$	.6680$\underline{5}$	23	.94023	43	42
43	.65439	$2\bar{3}$	.91499	$4\bar{2}$	.6682$\bar{8}$	23	.94066	43	43
44	.6546$\bar{2}$	23	.9154$\underline{1}$	43	.6685$\underline{1}$	23	.94109	$4\bar{2}$	44
45	9.6548$\bar{5}$	$2\bar{3}$	9.9158$\underline{4}$	$4\bar{2}$	9.6687$\underline{4}$	23	9.9415$\underline{1}$	43	45
46	.6550$\underline{9}$	$2\bar{3}$	.9162$\underline{7}$	$4\bar{2}$	.6689$\bar{7}$	$2\bar{3}$	.9419$\underline{4}$	43	46
47	.6553$\bar{2}$	$2\bar{3}$	.9166$\underline{9}$	43	.6692$\bar{0}$	$2\bar{2}$	.9423$\bar{7}$	43	47
48	.65556	23	.9171$\bar{2}$	$4\bar{2}$	.66943	23	.9428$\bar{0}$	43	48
49	.65579	$2\bar{3}$	.91755	$4\bar{2}$	.66966	23	.9432$\bar{3}$	43	49
50	9.6560$\bar{2}$	$2\bar{3}$	9.9179$\bar{7}$	43	9.66989	$2\bar{3}$	9.9436$\bar{6}$	43	50
51	.65626	23	.9184$\bar{0}$	$4\bar{2}$	.67012	$2\bar{2}$	.9440$\bar{9}$	43	51
52	.6564$\underline{9}$	$2\bar{3}$	.91883	43	.6703$\underline{4}$	23	.9445$\underline{2}$	43	52
53	.6567$\bar{2}$	$2\bar{3}$	.91926	$4\bar{2}$	.6705$\bar{7}$	23	.9449$\bar{5}$	43	53
54	.65696	$2\bar{3}$	.9196$\bar{8}$	$4\bar{2}$	.6708$\bar{0}$	$2\bar{2}$	.9453$\bar{8}$	43	54
55	9.6571$\underline{9}$	$2\bar{3}$	9.92011	43	9.67103	23	9.9458$\underline{1}$	43	55
56	.6574$\bar{2}$	$2\bar{3}$	.92054	$4\bar{2}$	.67126	$2\bar{3}$	.9462$\underline{4}$	43	56
57	.6576$\bar{5}$	$2\bar{3}$	.9209$\bar{6}$	43	.67149	$2\bar{2}$	.9466$\bar{7}$	43	57
58	.65789	$2\bar{3}$	.92139	$4\bar{2}$	.6717$\underline{1}$	23	.9471$\bar{0}$	43	58
59	.6581$\bar{2}$	23	.92182	$4\bar{2}$	.67194	$2\bar{2}$	.9475$\bar{3}$	43	59
60	9.6583$\bar{5}$		9.9222$\underline{4}$		9.67217		9.9479$\bar{6}$		60
′	Lg. Vers.	*D*	Log. Exs.	*D*	Lg. Vers.	*D*	Log. Exs.	*D*	′

P. P.

	43	$4\bar{2}$
6	4.3	4.$\bar{2}$
7	5.0	4.$\bar{9}$
8	5.$\bar{7}$	5.$\bar{6}$
9	6.$\underline{4}$	6.4
10	7.$\underline{1}$	7.$\underline{1}$
20	14.$\bar{3}$	14.$\underline{1}$
30	21.5	21.$\bar{2}$
40	28.$\bar{6}$	28.$\bar{3}$
50	35.$\bar{8}$	35.4

	24	$2\bar{3}$
6	2.4	2.$\bar{3}$
7	2.8	2.$\bar{7}$
8	3.2	3.$\bar{1}$
9	3.6	3.5
10	4.0	3.9
20	8.0	7.$\bar{8}$
30	12.0	11.$\bar{7}$
40	16.0	15.$\bar{6}$
50	20.0	19.6

	23	$2\bar{2}$
6	2.3	2.$\bar{2}$
7	2.7	2.6
8	3.$\bar{0}$	3.0
9	3.$\underline{4}$	3.4
10	3.$\bar{8}$	3.$\bar{7}$
20	7.$\bar{6}$	7.5
30	11.5	11.$\bar{2}$
40	15.$\bar{3}$	15.0
50	19.$\bar{1}$	18.7

P. P.

′	58° Lg. Vers.	D	58° Log. Exs.	D	59° Lg. Vers.	D	59° Log. Exs.	D	′
0	9.67217	23	9.94796	43	9.68571	22	9.97387	43	0
1	.67240	23	.94839	43	.68593	22	.97430	43	1
2	.67263	22	.94882	43	.68615	22	.97473	43	2
3	.67285	23	.94925	43	.68637	22	.97517	43	3
4	.67308	22	.94968	43	.68660	22	.97560	43	4
5	9.67331	23	9.95011	43	9.68682	22	9.97603	43	5
6	.67354	22	.95054	43	.68704	22	.97647	43	6
7	.67376	23	.95097	43	.68727	22	.97690	43	7
8	.67399	22	.95140	43	.68749	22	.97734	43	8
9	.67422	23	.95183	43	.68771	22	.97777	43	9
10	9.67445	22	9.95226	43	9.68793	22	9.97820	43	10
11	.67467	22	.95269	43	.68816	22	.97864	43	11
12	.67490	23	.95313	43	.68838	22	.97907	43	12
13	.67513	22	.95356	43	.68860	22	.97951	43	13
14	.67535	22	.95399	43	.68882	22	.97994	43	14
15	9.67558	23	9.95442	43	9.68905	22	9.98038	43	15
16	.67581	22	.95485	43	.68927	22	.98081	43	16
17	.67603	22	.95528	43	.68949	22	.98125	43	17
18	.67626	23	.95571	43	.68971	22	.98168	43	18
19	.67649	22	.95614	43	.68993	22	.98211	43	19
20	9.67671	22	9.95657	43	9.69016	22	9.98255	43	20
21	.67694	23	.95700	43	.69038	22	.98298	43	21
22	.67717	22	.95744	43	.69060	22	.98342	43	22
23	.67739	22	.95787	43	.69082	22	.98385	43	23
24	.67762	22	.95830	43	.69104	22	.98429	43	24
25	9.67784	22	9.95873	43	9.69126	22	9.98472	43	25
26	.67807	23	.95916	43	.69149	22	.98516	43	26
27	.67830	22	.95959	43	.69171	22	.98559	44	27
28	.67852	22	.96002	43	.69193	22	.98603	43	28
29	.67875	22	.96046	43	.69215	22	.98647	43	29
30	9.67897	22	9.96089	43	9.69237	22	9.98690	43	30
31	.67920	22	.96132	43	.69259	22	.98734	43	31
32	.67942	22	.96175	43	.69281	22	.98777	43	32
33	.67965	22	.96218	43	.69303	22	.98821	43	33
34	.67987	22	.96261	43	.69325	22	.98864	43	34
35	9.68010	22	9.96305	43	9.69347	22	9.98908	44	35
36	.68032	22	.96348	43	.69369	22	.98952	43	36
37	.68055	22	.96391	43	.69392	22	.98995	43	37
38	.68077	22	.96434	43	.69414	22	.99039	43	38
39	.68100	22	.96478	43	.69436	22	.99082	43	39
40	9.68122	22	9.96521	43	9.69458	22	9.99126	44	40
41	.68145	22	.96564	43	.69480	22	.99170	43	41
42	.68167	22	.96607	43	.69502	22	.99213	43	42
43	.68190	22	.96650	43	.69524	22	.99257	43	43
44	.68212	22	.96694	43	.69546	22	.99300	44	44
45	9.68235	22	9.96737	43	9.69568	22	9.99344	43	45
46	.68257	22	.96780	43	.69590	22	.99388	43	46
47	.68280	22	.96824	43	.69612	22	.99431	44	47
48	.68302	22	.96867	43	.69634	22	.99475	43	48
49	.68324	22	.96910	43	.69656	22	.99519	43	49
50	9.68347	22	9.96953	43	9.69678	22	9.99562	44	50
51	.68369	22	.96997	43	.69700	21	.99606	43	51
52	.68392	22	.97040	43	.69721	22	.99650	44	52
53	.68414	22	.97083	43	.69743	22	.99694	43	53
54	.68436	22	.97127	43	.69765	22	.99737	44	54
55	9.68459	22	9.97170	43	9.69787	22	9.99781	43	55
56	.68481	22	.97213	43	.69809	22	.99825	43	56
57	.68503	22	.97257	43	.69831	21	.99868	44	57
58	.68526	22	.97300	43	.69853	22	.99912	43	58
59	.68548	22	.97343	43	.69875	22	9.99956	44	59
60	9.68571		9.97387		9.69897		10.00000		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	44	43
6	4.4	4.3
7	5.1	5.1
8	5.8	5.8
9	6.6	6.5
10	7.3	7.2
20	14.6	14.5
30	22.0	21.7
40	29.3	29.0
50	36.6	36.2

	43
6	4.3
7	5.0
8	5.7
9	6.4
10	7.1
20	14.3
30	21.5
40	28.6
50	35.8

	23	22
6	2.3	2.2
7	2.7	2.6
8	3.0	3.0
9	3.4	3.4
10	3.8	3.7
20	7.6	7.5
30	11.5	11.2
40	15.3	15.0
50	19.1	18.7

	22	21
6	2.2	2.1
7	2.5	2.5
8	2.9	2.8
9	3.3	3.2
10	3.6	3.6
20	7.3	7.1
30	11.0	10.7
40	14.6	14.3
50	18.3	17.9

P. P.

′	60° Lg. Vers.	D	60° Log. Exs.	D	61° Lg. Vers.	D	61° Log. Exs.	D	′
0	9.69897	22	10.00000	44	9.71197	21	10.02639	44	0
1	.69919	21	.00044	43	.71218	21	.02684	44	1
2	.69940	22	.00087	44	.71239	21	.02728	44	2
3	.69962	22	.00131	43	.71261	21	.02772	44	3
4	.69984	22	.00175	44	.71282	21	.02816	44	4
5	9.70006	21	10.00219	43	9.71304	21	10.02861	44	5
6	.70028	22	.00262	44	.71325	21	.02905	44	6
7	.70050	22	.00306	44	.71346	21	.02949	44	7
8	.70072	21	.00350	43	.71368	21	.02994	44	8
9	.70093	22	.00394	44	.71389	21	.03038	44	9
10	9.70115	21	10.00438	44	9.71411	21	10.03082	44	10
11	.70137	22	.00482	43	.71432	21	.03127	44	11
12	.70159	22	.00525	44	.71453	21	.03171	44	12
13	.70181	21	.00569	44	.71475	21	.03215	44	13
14	.70202	22	.00613	44	.71496	21	.03260	44	14
15	9.70224	21	10.00657	43	9.71517	21	10.03304	44	15
16	.70246	22	.00701	44	.71539	21	.03348	44	16
17	.70268	21	.00745	44	.71560	21	.03393	44	17
18	.70289	22	.00789	44	.71581	21	.03437	44	18
19	.70311	21	.00833	43	.71603	21	.03481	44	19
20	9.70333	22	10.00876	44	9.71624	21	10.03526	44	20
21	.70355	21	.00920	44	.71645	21	.03570	44	21
22	.70376	22	.00964	44	.71667	21	.03615	44	22
23	.70398	21	.01008	44	.71688	21	.03659	44	23
24	.70420	21	.01052	44	.71709	21	.03704	44	24
25	9.70441	22	10.01096	44	9.71730	21	10.03748	44	25
26	.70463	21	.01140	44	.71752	21	.03793	44	26
27	.70485	22	.01184	44	.71773	21	.03837	44	27
28	.70507	21	.01228	44	.71794	21	.03881	44	28
29	.70528	21	.01272	44	.71815	21	.03926	44	29
30	9.70550	22	10.01316	44	9.71837	21	10.03970	44	30
31	.70572	21	.01360	44	.71858	21	.04015	44	31
32	.70593	21	.01404	44	.71879	21	.04059	44	32
33	.70615	21	.01448	44	.71900	21	.04104	45	33
34	.70636	22	.01492	44	.71922	21	.04149	44	34
35	9.70658	21	10.01536	44	9.71943	21	10.04193	44	35
36	.70680	21	.01580	44	.71964	21	.04238	44	36
37	.70701	21	.01624	44	.71985	21	.04282	44	37
38	.70723	22	.01668	44	.72006	21	.04327	44	38
39	.70745	21	.01712	44	.72028	21	.04371	44	39
40	9.70766	21	10.01756	44	9.72049	21	10.04416	45	40
41	.70788	21	.01800	44	.72070	21	.04461	44	41
42	.70809	21	.01844	44	.72091	21	.04505	44	42
43	.70831	21	.01889	44	.72112	21	.04550	44	43
44	.70852	22	.01933	44	.72133	21	.04594	45	44
45	9.70874	21	10.01977	44	9.72154	21	10.04639	44	45
46	.70896	21	.02021	44	.72176	21	.04684	44	46
47	.70917	21	.02065	44	.72197	21	.04728	44	47
48	.70939	21	.02109	44	.72218	21	.04773	45	48
49	.70960	21	.02153	44	.72239	21	.04818	44	49
50	9.70982	21	10.02197	44	9.72260	21	10.04862	45	50
51	.71003	21	.02242	44	.72281	21	.04907	44	51
52	.71025	21	.02286	44	.72302	21	.04952	44	52
53	.71046	21	.02330	44	.72323	21	.04996	45	53
54	.71068	21	.02374	44	.72344	21	.05041	44	54
55	9.71089	21	10.02418	44	9.72365	21	10.05086	45	55
56	.71111	21	.02463	44	.72386	21	.05131	44	56
57	.71132	21	.02507	44	.72408	21	.05175	45	57
58	.71154	21	.02551	44	.72429	21	.05220	44	58
59	.71175	21	.02595	44	.72450	21	.05265	45	59
60	9.71197		10.02639		9.72471		10.05310		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	45	44
6	4.5	4.4
7	5.2	5.2
8	6.0	5.9
9	6.7	6.7
10	7.5	7.4
20	15.0	14.8
30	22.5	22.2
40	30.0	29.6
50	37.5	37.1

	44	43
6	4.4	4.3
7	5.1	5.1
8	5.8	5.8
9	6.6	6.5
10	7.3	7.2
20	14.6	14.5
30	22.0	21.7
40	29.3	29.0
50	36.6	36.2

	22	21
6	2.2	2.1
7	2.5	2.5
8	2.9	2.8
9	3.3	3.2
10	3.6	3.6
20	7.3	7.1
30	11.0	10.7
40	14.6	14.3
50	18.3	17.9

	21
6	2.1
7	2.4
8	2.8
9	3.1
10	3.5
20	7.0
30	10.5
40	14.0
50	17.5

P. P.

′	62° Lg. Vers.	D	62° Log. Exs.	D	63° Lg. Vers.	D	63° Log. Exs.	D	′
0	9.72471	21	10.05310	44	9.73720	20	10.08015	45	0
1	.72492	21	.05354	45	.73740	20	.08061	45	1
2	.72513	21	.05399	45	.73761	21	.08106	45	2
3	.72534	21	.05444	44	.73782	20	.08151	45	3
4	.72555	21	.05489	45	.73802	20	.08197	45	4
5	9.72576	21	10.05534	45	9.73823	20	10.08242	45	5
6	.72597	21	.05579	44	.73843	20	.08288	45	6
7	.72618	21	.05623	45	.73864	20	.08333	45	7
8	.72639	21	.05668	45	.73884	21	.08379	45	8
9	.72660	21	.05713	45	.73905	20	.08424	45	9
10	9.72681	20	10.05758	44	9.73926	20	10.08470	45	10
11	.72701	21	.05803	45	.73946	20	.08515	45	11
12	.72722	21	.05848	45	.73967	20	.08561	45	12
13	.72743	21	.05893	45	.73987	20	.08606	45	13
14	.72764	21	.05938	45	.74008	20	.08652	45	14
15	9.72785	21	10.05983	45	9.74028	20	10.08697	46	15
16	.72806	21	.06028	44	.74049	20	.08743	45	16
17	.72827	20	.06072	45	.74069	20	.08789	45	17
18	.72848	21	.06117	45	.74090	20	.08834	45	18
19	.72869	21	.06162	45	.74110	20	.08880	46	19
20	9.72890	21	10.06207	45	9.74131	20	10.08926	45	20
21	.72911	20	.06252	45	.74151	20	.08971	45	21
22	.72931	21	.06297	45	.74172	20	.09017	45	22
23	.72952	21	.06342	45	.74192	20	.09062	46	23
24	.72973	21	.06387	45	.74213	20	.09108	45	24
25	9.72994	20	10.06432	45	9.74233	20	10.09154	46	25
26	.73015	21	.06477	45	.74254	20	.09200	45	26
27	.73036	21	.06522	45	.74274	20	.09245	45	27
28	.73057	20	.06568	45	.74294	20	.09291	46	28
29	.73077	21	.06613	45	.74315	20	.09337	45	29
30	9.73098	21	10.06658	45	9.74335	20	10.09382	46	30
31	.73119	20	.06703	45	.74356	20	.09428	46	31
32	.73140	21	.06748	45	.74376	20	.09474	45	32
33	.73161	20	.06793	45	.74396	20	.09520	46	33
34	.73181	21	.06838	45	.74417	20	.09566	45	34
35	9.73202	21	10.06883	45	9.74437	20	10.09611	46	35
36	.73223	20	.06928	45	.74458	20	.09657	46	36
37	.73244	21	.06974	45	.74478	20	.09703	45	37
38	.73265	20	.07019	45	.74498	20	.09749	46	38
39	.73285	21	.07064	45	.74519	20	.09795	46	39
40	9.73306	20	10.07109	45	9.74539	20	10.09841	45	40
41	.73327	21	.07154	45	.74559	20	.09886	46	41
42	.73348	20	.07200	45	.74580	20	.09932	46	42
43	.73368	21	.07245	45	.74600	20	.09978	46	43
44	.73389	20	.07290	45	.74620	20	.10024	46	44
45	9.73410	20	10.07335	45	9.74641	20	10.10070	46	45
46	.73430	21	.07380	45	.74661	20	.10116	46	46
47	.73451	20	.07426	45	.74681	20	.10162	45	47
48	.73472	21	.07471	45	.74702	20	.10208	46	48
49	.73493	20	.07516	45	.74722	20	.10254	46	49
50	9.73513	20	10.07562	45	9.74742	20	10.10300	46	50
51	.73534	21	.07607	45	.74762	20	.10346	46	51
52	.73555	20	.07652	45	.74783	20	.10392	46	52
53	.73575	20	.07697	45	.74803	20	.10438	46	53
54	.73596	21	.07743	45	.74823	20	.10484	46	54
55	9.73617	20	10.07788	45	9.74844	20	10.10530	46	55
56	.73637	20	.07834	45	.74864	20	.10576	46	56
57	.73658	21	.07879	45	.74884	20	.10622	46	57
58	.73679	20	.07924	45	.74904	20	.10668	46	58
59	.73699	20	.07970	45	.74924	20	.10714	46	59
60	9.73720		10.08015		9.74945		10.10760		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	46	46
6	4.6	4.6
7	5.4	5.3
8	6.2	6.1
9	7.0	6.9
10	7.7	7.6
20	15.5	15.3
30	23.2	23.0
40	31.0	30.6
50	38.7	38.3

	45	45
6	4.5	4.5
7	5.3	5.2
8	6.0	6.0
9	6.8	6.7
10	7.6	7.5
20	15.1	15.0
30	22.7	22.5
40	30.3	30.0
50	37.9	37.5

	44
6	4.4
7	5.2
8	5.9
9	6.7
10	7.4
20	14.8
30	22.2
40	29.6
50	37.1

	21	20
6	2.1	2.0
7	2.4	2.4
8	2.8	2.7
9	3.1	3.1
10	3.5	3.4
20	7.0	6.8
30	10.5	10.2
40	14.0	13.6
50	17.5	17.1

	20
6	2.0
7	2.3
8	2.6
9	3.0
10	3.3
20	6.6
30	10.0
40	13.3
50	16.6

P. P.

	64°				65°				
′	Lg. Vers.	*D*	Log. Exs.	*D*	Lg. Vers.	*D*	Log. Exs.	*D*	′
0	9.74945	20	10.10760	46	9.76146	19	10.13551	47	**0**
1	.74965	20	.10807	46	.76166	20	.13598	47	1
2	.74985	20	.10853	46	.76186	20	.13645	47	2
3	.75005	20	.10899	46	.76206	19	.13692	47	3
4	.75026	20	.10945	46	.76225	20	.13739	47	4
5	9.75046	20	10.10991	46	9.76245	19	10.13786	47	5
6	.75066	20	.11037	46	.76265	20	.13833	47	6
7	.75086	20	.11084	46	.76285	19	.13880	47	7
8	.75106	20	.11130	46	.76304	20	.13927	47	8
9	.75126	20	.11176	46	.76324	20	.13974	47	9
10	9.75147	20	10.11222	46	9.76344	19	10.14021	47	**10**
11	.75167	20	.11269	46	.76364	20	.14068	47	11
12	.75187	20	.11315	46	.76384	19	.14115	47	12
13	.75207	20	.11361	46	.76403	20	.14162	47	13
14	.75227	20	.11407	46	.76423	19	.14210	47	14
15	9.75247	20	10.11454	46	9.76443	20	10.14257	47	15
16	.75267	20	.11500	46	.76463	19	.14304	47	16
17	.75287	20	.11546	46	.76482	19	.14351	47	17
18	.75308	20	.11593	46	.76502	20	.14398	47	18
19	.75328	20	.11639	46	.76522	19	.14445	47	19
20	9.75348	20	10.11685	46	9.76541	20	10.14493	47	**20**
21	.75368	20	.11732	46	.76561	19	.14540	47	21
22	.75388	20	.11778	46	.76581	19	.14587	47	22
23	.75408	20	.11825	46	.76600	20	.14634	47	23
24	.75428	20	.11871	46	.76620	19	.14682	47	24
25	9.75448	20	10.11917	46	9.76640	19	10.14729	47	25
26	.75468	20	.11964	46	.76659	20	.14776	47	26
27	.75488	20	.12010	46	.76679	19	.14823	47	27
28	.75508	20	.12057	46	.76699	19	.14871	47	28
29	.75528	20	.12103	46	.76718	20	.14918	47	29
30	9.75548	20	10.12150	46	9.76738	19	10.14965	47	**30**
31	.75568	20	.12196	46	.76758	19	.15013	47	31
32	.75588	20	.12243	46	.76777	19	.15060	47	32
33	.75608	20	.12289	46	.76797	20	.15108	47	33
34	.75628	20	.12336	47	.76817	19	.15155	47	34
35	9.75648	20	10.12383	46	9.76836	19	10.15202	47	35
36	.75668	20	.12429	46	.76856	19	.15250	47	36
37	.75688	20	.12476	46	.76875	20	.15297	47	37
38	.75708	20	.12522	46	.76895	19	.15345	47	38
39	.75728	20	.12569	47	.76915	19	.15392	47	39
40	9.75748	20	10.12616	46	9.76934	19	10.15440	47	**40**
41	.75768	20	.12662	46	.76954	19	.15487	47	41
42	.75788	20	.12709	47	.76973	19	.15535	47	42
43	.75808	19	.12756	46	.76993	19	.15582	47	43
44	.75828	20	.12802	46	.77012	20	.15630	48	44
45	9.75848	20	10.12849	47	9.77032	19	10.15678	47	45
46	.75868	20	.12896	46	.77052	19	.15725	47	46
47	.75888	20	.12942	47	.77071	19	.15773	47	47
48	.75908	20	.12989	46	.77091	19	.15820	48	48
49	.75928	19	.13036	47	.77110	19	.15868	47	49
50	9.75947	20	10.13083	47	9.77130	19	10.15916	47	**50**
51	.75967	20	.13130	46	.77149	19	.15963	48	51
52	.75987	20	.13176	47	.77169	19	.16011	47	52
53	.76007	19	.13223	46	.77188	19	.16059	47	53
54	.76027	20	.13270	47	.77208	19	.16106	48	54
55	9.76047	20	10.13317	47	9.77227	19	10.16154	47	55
56	.76067	20	.13364	47	.77247	19	.16202	48	56
57	.76087	19	.13411	46	.77266	19	.16250	48	57
58	.76106	20	.13457	47	.77286	19	.16298	47	58
59	.76126	20	.13504	47	.77305	19	.16345	48	59
60	9.76146		10.13551		9.77325		10.16393		**60**
′	Lg. Vers.	*D*	Log. Exs.	*D*	Lg. Vers.	*D*	Log. Exs.	*D*	′

P. P.

	48	47
6	4.8	4.7
7	5.6	5.5
8	6.4	6.3
9	7.2	7.1
10	8.0	7.9
20	16.0	15.8
30	24.0	23.7
40	32.0	31.6
50	40.0	39.6

	47	46
6	4.7	4.6
7	5.5	5.4
8	6.2	6.2
9	7.0	7.0
10	7.8	7.7
20	15.6	15.5
30	23.5	23.2
40	31.3	31.0
50	39.1	38.7

	46
6	4.6
7	5.3
8	6.1
9	6.9
10	7.6
20	15.3
30	23.0
40	30.6
50	38.3

	20	20
6	2.0	2.0
7	2.4	2.3
8	2.7	2.6
9	3.1	3.0
10	3.4	3.3
20	6.8	6.6
30	10.2	10.0
40	13.6	13.3
50	17.1	16.6

	19
6	1.9
7	2.3
8	2.6
9	2.9
10	3.2
20	6.5
30	9.7
40	13.0
50	16.2

P. P.

	66°				67°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.77325	19	10.16393	48	9.78481	19	10.19293	49	0
1	.77344	19	.16441	47	.78500	19	.19342	49	1
2	.77363	19	.16489	48	.78519	19	.19391	48	2
3	.77383	19	.16537	48	.78538	19	.19439	49	3
4	.77402	19	.16585	48	.78557	19	.19488	49	4
5	9.77422	19	10.16633	47	9.78576	19	10.19537	49	5
6	.77441	19	.16680	48	.78595	19	.19586	49	6
7	.77461	19	.16728	48	.78614	19	.19635	49	7
8	.77480	19	.16776	48	.78633	19	.19684	49	8
9	.77499	19	.16824	48	.78652	19	.19733	49	9
10	9.77519	19	10.16872	48	9.78671	19	10.19782	49	10
11	.77538	19	.16920	48	.78690	19	.19831	49	11
12	.77557	19	.16968	48	.78709	19	.19880	49	12
13	.77577	19	.17016	48	.78728	19	.19929	49	13
14	.77596	19	.17064	48	.78747	19	.19979	49	14
15	9.77616	19	10.17112	48	9.78766	19	10.20028	49	15
16	.77635	19	.17160	48	.78785	19	.20077	49	16
17	.77654	19	.17209	48	.78804	19	.20126	49	17
18	.77674	19	.17257	48	.78823	19	.20175	49	18
19	.77693	19	.17305	48	.78842	19	.20224	49	19
20	9.77712	19	10.17353	48	9.78861	19	10.20273	49	20
21	.77732	19	.17401	48	.78880	19	.20323	49	21
22	.77751	19	.17449	48	.78899	19	.20372	49	22
23	.77770	19	.17498	48	.78918	18	.20421	49	23
24	.77790	19	.17546	48	.78937	19	.20470	49	24
25	9.77809	19	10.17594	48	9.78956	19	10.20520	49	25
26	.77828	19	.17642	48	.78975	19	.20569	49	26
27	.77847	19	.17690	48	.78994	19	.20618	49	27
28	.77867	19	.17739	48	.79013	19	.20668	49	28
29	.77886	19	.17787	48	.79032	19	.20717	49	29
30	9.77905	19	10.17835	48	9.79051	18	10.20767	49	30
31	.77925	19	.17884	48	.79069	19	.20816	49	31
32	.77944	19	.17932	48	.79088	19	.20865	49	32
33	.77963	19	.17980	48	.79107	19	.20915	49	33
34	.77982	19	.18029	48	.79126	19	.20964	49	34
35	9.78002	19	10.18077	48	9.79145	18	10.21014	49	35
36	.78021	19	.18126	48	.79164	19	.21063	49	36
37	.78040	19	.18174	48	.79183	19	.21113	49	37
38	.78059	19	.18222	48	.79202	18	.21162	50	38
39	.78078	19	.18271	48	.79220	19	.21212	49	39
40	9.78098	19	10.18319	48	9.79239	19	10.21262	49	40
41	.78117	19	.18368	48	.79258	18	.21311	49	41
42	.78136	19	.18416	49	.79277	19	.21361	49	42
43	.78155	19	.18465	48	.79296	19	.21410	50	43
44	.78174	19	.18514	48	.79315	18	.21460	49	44
45	9.78194	19	10.18562	48	9.79333	19	10.21510	50	45
46	.78213	19	.18611	48	.79352	19	.21560	49	46
47	.78232	19	.18659	48	.79371	18	.21609	50	47
48	.78251	19	.18708	49	.79390	19	.21659	49	48
49	.78270	19	.18757	48	.79409	18	.21709	50	49
50	9.78289	19	10.18805	48	9.79427	19	10.21759	49	50
51	.78309	19	.18854	49	.79446	19	.21808	50	51
52	.78328	19	.18903	48	.79465	18	.21858	50	52
53	.78347	19	.18951	49	.79484	19	.21908	49	53
54	.78366	19	.19000	48	.79503	18	.21958	50	54
55	9.78385	19	10.19049	49	9.79521	19	10.22008	50	55
56	.78404	19	.19098	48	.79540	18	.22058	50	56
57	.78423	19	.19146	49	.79559	19	.22108	50	57
58	.78442	19	.19195	49	.79578	18	.22158	50	58
59	.78462	19	.19244	48	.79596	19	.22208	50	59
60	9.78481		10.19293		9.79615		10.22258		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	50	49
6	5.0	4.9
7	5.8	5.8
8	6.6	6.6
9	7.5	7.4
10	8.3	8.2
20	16.6	16.5
30	25.0	24.7
40	33.3	33.0
50	41.6	41.2

	49	48
6	4.9	4.8
7	5.7	5.6
8	6.5	6.4
9	7.3	7.3
10	8.1	8.1
20	16.3	16.1
30	24.5	24.2
40	32.6	32.3
50	40.8	40.4

	48	47
6	4.8	4.7
7	5.6	5.5
8	6.4	6.3
9	7.2	7.1
10	8.0	7.9
20	16.0	15.8
30	24.0	23.7
40	32.0	31.6
50	40.0	39.6

	19	19
6	1.9	1.9
7	2.3	2.2
8	2.6	2.5
9	2.9	2.8
10	3.2	3.1
20	6.5	6.3
30	9.7	9.5
40	13.0	12.6
50	16.2	15.8

	18
6	1.8
7	2.1
8	2.4
9	2.8
10	3.1
20	6.1
30	9.2
40	12.3
50	15.4

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

′	68° Lg. Vers.	D	68° Log. Exs.	D	69° Lg. Vers.	D	69° Log. Exs.	D	′
0	9.79615	18	10.22258	50	9.80728	18	10.25295	51	0
1	.79634	19	.22308	50	.80747	18	.25347	51	1
2	.79653	18	.22358	50	.80765	18	.25398	51	2
3	.79671	18	.22408	50	.80783	18	.25449	51	3
4	.79690	19	.22458	50	.80802	18	.25501	51	4
5	9.79709	18	10.22508	50	9.80820	18	10.25552	51	5
6	.79727	19	.22558	50	.80839	18	.25604	51	6
7	.79746	18	.22608	50	.80857	18	.25655	51	7
8	.79765	18	.22658	50	.80875	18	.25707	51	8
9	.79783	19	.22708	50	.80894	18	.25758	51	9
10	9.79802	18	10.22759	50	9.80912	18	10.25810	51	10
11	.79821	18	.22809	50	.80930	18	.25861	51	11
12	.79839	19	.22859	50	.80949	18	.25913	51	12
13	.79858	18	.22909	50	.80967	18	.25964	51	13
14	.79877	18	.22960	50	.80985	18	.26016	51	14
15	9.79895	18	10.23010	50	9.81003	18	10.26067	52	15
16	.79914	19	.23060	50	.81022	18	.26119	51	16
17	.79933	18	.23110	50	.81040	18	.26171	51	17
18	.79951	18	.23161	50	.81058	18	.26222	52	18
19	.79970	18	.23211	50	.81077	18	.26274	51	19
20	9.79988	19	10.23262	50	9.81095	18	10.26326	52	20
21	.80007	18	.23312	50	.81113	18	.26378	51	21
22	.80026	18	.23362	50	.81131	18	.26429	52	22
23	.80044	18	.23413	50	.81150	18	.26481	52	23
24	.80063	18	.23463	50	.81168	18	.26533	52	24
25	9.80081	19	10.23514	50	9.81186	18	10.26585	51	25
26	.80100	18	.23564	50	.81204	18	.26637	52	26
27	.80119	18	.23615	50	.81223	18	.26689	52	27
28	.80137	18	.23666	50	.81241	18	.26741	52	28
29	.80156	18	.23716	50	.81259	18	.26793	52	29
30	9.80174	18	10.23767	50	9.81277	18	10.26845	52	30
31	.80193	18	.23817	51	.81295	18	.26897	52	31
32	.80211	18	.23868	50	.81314	18	.26949	52	32
33	.80230	18	.23919	50	.81332	18	.27001	52	33
34	.80248	19	.23969	51	.81350	18	.27053	52	34
35	9.80267	18	10.24020	50	9.81368	18	10.27105	52	35
36	.80286	18	.24071	51	.81386	18	.27157	52	36
37	.80304	18	.24122	50	.81405	18	.27209	52	37
38	.80323	18	.24172	51	.81423	18	.27261	52	38
39	.80341	18	.24223	51	.81441	18	.27314	52	39
40	9.80360	18	10.24274	50	9.81459	18	10.27366	52	40
41	.80378	18	.24325	51	.81477	18	.27418	52	41
42	.80397	18	.24376	51	.81495	18	.27470	52	42
43	.80415	18	.24427	51	.81513	18	.27523	52	43
44	.80434	18	.24478	51	.81532	18	.27575	52	44
45	9.80452	18	10.24529	51	9.81550	18	10.22627	52	45
46	.80470	18	.24580	51	.81568	18	.27680	52	46
47	.80489	18	.24631	51	.81586	18	.27732	52	47
48	.80507	18	.24682	51	.81604	18	.27785	52	48
49	.80526	18	.24733	51	.81622	18	.27837	52	49
50	9.80544	18	10.24784	51	9.81640	18	10.27890	52	50
51	.80563	18	.24835	51	.81658	18	.27942	52	51
52	.80587	18	.24886	51	.81676	18	.27995	52	52
53	.80600	18	.24937	51	.81695	18	.28047	52	53
54	.80618	18	.24988	51	.81713	18	.28100	52	54
55	9.80636	18	10.25039	51	9.81731	18	10.28152	53	55
56	.80655	18	.25090	51	.81749	18	.28205	52	56
57	.80673	18	.25142	51	.81767	18	.28258	52	57
58	.80692	18	.25193	51	.81785	18	.28310	53	58
59	.80710	18	.25244	51	.81803	18	.28363	52	59
60	9.80728		10.25295		9.81821		10.28416		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	53	52
6	5.3	5.2
7	6.2	6.1
8	7.0	7.0
9	7.9	7.9
10	8.8	8.7
20	17.6	17.5
30	26.5	26.2
40	35.3	35.0
50	44.1	43.7

	52	51
6	5.2	5.1
7	6.0	6.0
8	6.9	6.8
9	7.8	7.7
10	8.6	8.6
20	17.3	17.1
30	26.0	25.7
40	34.6	34.3
50	43.3	42.9

	51	50
6	5.1	5.0
7	5.9	5.9
8	6.8	6.7
9	7.6	7.6
10	8.5	8.4
20	17.0	16.8
30	25.5	25.2
40	34.0	33.6
50	42.5	42.1

	50
6	5.0
7	5.8
8	6.6
9	7.5
10	8.3
20	16.6
30	25.0
40	33.3
50	41.6

	19	18
6	1.9	1.8
7	2.2	2.1
8	2.5	2.4
9	2.8	2.8
10	3.1	3.1
20	6.3	6.1
30	9.5	9.2
40	12.6	12.3
50	15.8	15.4

	18
6	1.8
7	2.1
8	2.4
9	2.7
10	3.0
20	6.0
30	9.0
40	12.0
50	15.0

P. P.

′	70° Lg. Vers.	D	70° Log. Exs.	D	71° Lg. Vers.	D	71° Log. Exs.	D	′
0	9.81821		10.28416		9.82894		10.31629		0
1	.81839	18	.28469	53	.82911	17	.31684	54	1
2	.81857	18	.28521	52	.82929	17	.31738	54	2
3	.81875	18	.28574	53	.82947	18	.31793	54	3
4	.81893	18	.28627	53	.82964	17	.31847	54	4
5	9.81911	18	10.28680	52	9.82982	18	10.31902	54	5
6	.81929	18	.28733	53	.83000	17	.31956	54	6
7	.81947	18	.28786	53	.83017	17	.32011	54	7
8	.81965	18	.28839	53	.83035	18	.32066	55	8
9	.81983	18	.28892	53	.83053	17	.32120	54	9
10	9.82001	18	10.28945	53	9.83070	17	10.32175	54	10
11	.82019	18	.28998	53	.83088	18	.32230	55	11
12	.82037	18	.29051	53	.83106	17	.32284	54	12
13	.82055	18	.29104	53	.83123	17	.32339	55	13
14	.82073	18	.29157	53	.83141	17	.32394	54	14
15	9.82091	18	10.29210	53	9.83159	18	10.32449	55	15
16	.82109	17	.29263	53	.83176	17	.32504	55	16
17	.82127	18	.29316	53	.83194	17	.32558	54	17
18	.82145	18	.29370	53	.83211	17	.32613	55	18
19	.82163	18	.29423	53	.83229	18	.32668	55	19
20	9.82181	18	10.29476	53	9.83247	17	10.32723	55	20
21	.82199	18	.29529	53	.83264	17	.32778	55	21
22	.82217	18	.29583	53	.83282	17	.32833	55	22
23	.82235	18	.29636	53	.83299	17	.32888	55	23
24	.82252	17	.29689	53	.83317	18	.32944	55	24
25	9.82270	18	10.29743	53	9.83335	17	10.32999	55	25
26	.82288	18	.29796	53	.83352	17	.33054	55	26
27	.82306	18	.29850	53	.83370	17	.33109	55	27
28	.82324	18	.29903	53	.83387	17	.33164	55	28
29	.82342	17	.29957	53	.83405	17	.33220	55	29
30	9.82360	18	10.30010	53	9.83422	17	10.33275	55	30
31	.82378	18	.30064	53	.83440	17	.33330	55	31
32	.82396	18	.30117	53	.83458	18	.33385	55	32
33	.82413	17	.30171	54	.83475	17	.33441	55	33
34	.82431	18	.30225	53	.83493	17	.33496	55	34
35	9.82449	18	10.30278	53	9.83510	17	10.33552	55	35
36	.82467	17	.30332	54	.83528	17	.33607	55	36
37	.82485	18	.30386	53	.83545	17	.33663	55	37
38	.82503	18	.30440	54	.83563	17	.33718	55	38
39	.82520	17	.30493	53	.83580	17	.33774	55	39
40	9.82538	18	10.30547	54	9.83598	17	10.33829	55	40
41	.82556	18	.30601	53	.83615	17	.33885	56	41
42	.82574	17	.30655	54	.83633	17	.33941	55	42
43	.82592	18	.30709	54	.83650	17	.33996	55	43
44	.82609	17	.30763	54	.83668	17	.34052	56	44
45	9.82627	18	10.30817	54	9.83685	17	10.34108	55	45
46	.82645	18	.30871	54	.83703	17	.34164	56	46
47	.82663	17	.30925	54	.83720	17	.34220	56	47
48	.82681	18	.30979	54	.83737	17	.34275	55	48
49	.82698	17	.31033	54	.83755	17	.34331	56	49
50	9.82716	18	10.31087	54	9.83772	17	10.34387	56	50
51	.82734	17	.31141	54	.83790	17	.34443	56	51
52	.82752	18	.31195	54	.83807	17	.34499	56	52
53	.82769	17	.31249	54	.83825	17	.34555	56	53
54	.82787	18	.31303	54	.83842	17	.34611	56	54
55	9.82805	17	10.31358	54	9.83859	17	10.34667	56	55
56	.82823	18	.31412	54	.83877	17	.34723	56	56
57	.82840	17	.31466	54	.83894	17	.34780	56	57
58	.82858	18	.31521	54	.83912	17	.34836	56	58
59	.82876	17	.31575	54	.83929	17	.34892	56	59
60	9.82894	18	10.31629	54	9.83946	17	10.34948	56	60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	$5\bar{6}$	56	$5\bar{5}$	55	$5\bar{4}$	54	$5\bar{3}$	53	$5\bar{2}$
6	5.6	5.6	5.5	5.5	5.4	5.4	5.3	5.3	5.2
7	6.6	6.5	6.5	6.4	6.3	6.3	6.2	6.2	6.1
8	7.5	7.4	7.4	7.3	7.2	7.2	7.1	7.0	7.0
9	8.5	8.4	8.3	8.2	8.2	8.1	8.0	7.9	7.9
10	9.4	9.3	9.2	9.1	9.1	9.0	8.9	8.8	8.7
20	18.8	18.6	18.5	18.3	18.1	18.0	17.8	17.6	17.5
30	28.2	28.0	27.7	27.5	27.2	27.0	26.7	26.5	26.2
40	37.6	37.3	37.0	36.6	36.3	36.0	35.6	35.3	35.0
50	47.1	46.6	46.2	45.8	45.4	45.0	44.6	44.1	43.7

	18	$1\bar{7}$	17
6	1.8	1.7	1.7
7	2.1	2.0	2.0
8	2.4	2.3	2.2
9	2.7	2.6	2.5
10	3.0	2.9	2.8
20	6.0	5.8	5.6
30	9.0	8.7	8.5
40	12.0	11.6	11.3
50	15.0	14.6	14.1

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

′	72° Lg. Vers.	D	72° Log. Exs.	D	73° Lg. Vers.	D	73° Log. Exs.	D	′
0	9.83946	17	10.34948	56	9.84980	17	10.38387	58	0
1	.83964	17	.35005	56	.84997	17	.38445	58	1
2	.83981	17	.35061	56	.85014	17	.38504	58	2
3	.83999	17	.35117	56	.85031	17	.38562	58	3
4	.84016	17	.35174	56	.85049	17	.38621	58	4
5	9.84033	17	10.35230	56	9.85066	17	10.38679	58	5
6	.84051	17	.35286	56	.85083	17	.38738	58	6
7	.84068	17	.35343	56	.85100	17	.38796	59	7
8	.84085	17	.35399	57	.85117	17	.38855	58	8
9	.84103	17	.35456	56	.85134	17	.38914	59	9
10	9.84120	17	10.35513	56	9.85151	17	10.38973	58	10
11	.84137	17	.35569	56	.85168	17	.39031	59	11
12	.84155	17	.35626	57	.85185	17	.39090	59	12
13	.84172	17	.35683	56	.85202	17	.39149	58	13
14	.84189	17	.35739	57	.85219	17	.39208	59	14
15	9.84207	17	10.35796	56	9.85236	17	10.39267	59	15
16	.84224	17	.35853	57	.85253	17	.39326	59	16
17	.84241	17	.35910	57	.85270	17	.39385	59	17
18	.84259	17	.35967	56	.85287	17	.39444	59	18
19	.84276	17	.36023	57	.85304	17	.39503	59	19
20	9.84293	17	10.36080	57	9.85321	17	10.39562	59	20
21	.84310	17	.36137	57	.85338	17	.39621	59	21
22	.84328	17	.36194	57	.85355	17	.39681	59	22
23	.84345	17	.36251	57	.85372	17	.39740	59	23
24	.84362	17	.36308	57	.85389	16	.39799	59	24
25	9.84380	17	10.36366	57	9.85405	17	10.39859	59	25
26	.84397	17	.36423	57	.85422	17	.39918	59	26
27	.84414	17	.36480	57	.85439	17	.39977	59	27
28	.84431	17	.36537	57	.85456	17	.40037	59	28
29	.84449	17	.36594	57	.85473	17	.40096	59	29
30	9.84466	17	10.36652	57	9.85490	17	10.40156	60	30
31	.84483	17	.36709	57	.85507	16	.40216	59	31
32	.84500	17	.36766	57	.85524	17	.40275	59	32
33	.84517	17	.36824	57	.85541	17	.40335	60	33
34	.84535	17	.36881	57	.85558	17	.40395	59	34
35	9.84552	17	10.36938	57	9.85575	17	10.40454	60	35
36	.84569	17	.36996	58	.85592	16	.40514	59	36
37	.84586	17	.37054	57	.85608	17	.40574	60	37
38	.84603	17	.37111	57	.85625	17	.40634	60	38
39	.84620	17	.37169	57	.85642	17	.40694	60	39
40	9.84638	17	10.37226	57	9.85659	16	10.40754	60	40
41	.84655	17	.37284	58	.85676	17	.40814	60	41
42	.84672	17	.37342	57	.85693	17	.40874	60	42
43	.84689	17	.37399	58	.85710	16	.40934	60	43
44	.84706	17	.37457	58	.85726	17	.40994	60	44
45	9.84724	17	10.37515	57	9.85743	17	10.41054	60	45
46	.84741	17	.37573	58	.85760	16	.41114	60	46
47	.84758	17	.37631	58	.85777	17	.41174	60	47
48	.84775	17	.37689	58	.85794	17	.41235	60	48
49	.84792	17	.37747	58	.85811	16	.41295	60	49
50	9.84809	17	10.37805	58	9.85827	17	10.41355	60	50
51	.84826	17	.37863	58	.85844	17	.41416	60	51
52	.84844	17	.37921	58	.85861	16	.41476	60	52
53	.84861	17	.37979	58	.85878	17	.41537	60	53
54	.84878	17	.38037	58	.85895	16	.41597	60	54
55	9.84895	17	10.38095	58	9.85911	17	10.41658	61	55
56	.84912	17	.38153	58	.85928	17	.41719	60	56
57	.84929	17	.38212	58	.85945	16	.41779	60	57
58	.84946	17	.38270	58	.85962	17	.41840	61	58
59	.84963	17	.38328	58	.85979	16	.41901	61	59
60	9.84980		10.38387		9.85995		10.41962		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	61	60
6	6.1	6.0
7	7.1	7.0
8	8.1	8.0
9	9.1	9.1
10	10.1	10.1
20	20.3	20.1
30	30.5	30.2
40	40.6	40.3
50	50.8	50.4

	60	59
6	6.0	5.9
7	7.0	6.9
8	8.0	7.9
9	9.0	8.9
10	10.0	9.9
20	20.0	19.8
30	30.0	29.7
40	40.0	39.6
50	50.0	49.6

	59	58
6	5.9	5.8
7	6.9	6.8
8	7.8	7.8
9	8.8	8.8
10	9.8	9.7
20	19.6	19.5
30	29.5	29.2
40	39.3	39.0
50	49.1	48.7

	58	57
6	5.8	5.7
7	6.7	6.7
8	7.7	7.6
9	8.7	8.6
10	9.6	9.6
20	19.3	19.1
30	29.0	28.7
40	38.6	38.3
50	48.3	47.9

	57	56
6	5.7	5.6
7	6.6	6.6
9	7.6	7.5
9	8.5	8.5
10	9.5	9.4
20	19.0	18.8
30	28.5	28.2
40	38.0	37.6
50	47.5	47.1

	17	17	16
6	1.7	1.7	1.6
7	2.0	2.0	1.9
8	2.3	2.2	2.2
9	2.6	2.5	2.5
10	2.9	2.8	2.7
20	5.8	5.6	5.5
30	8.7	8.5	8.2
40	11.6	11.3	11.0
50	14.6	14.1	13.7

P. P.

′	74° Lg. Vers.	D	74° Log. Exs.	D	75° Lg. Vers.	D	75° Log. Exs.	D	′
0	9.85995	17	10.41962	60	9.86992	16	10.45693	63	0
1	.86012	16	.42022	61	.87009	16	.45756	63	1
2	.86029	17	.42083	61	.87025	16	.45820	64	2
3	.86046	16	.42144	61	.87042	16	.45884	63	3
4	.86062	17	.42205	61	.87058	16	.45947	64	4
5	9.86079	16	10.42266	61	9.87074	16	10.46011	64	5
6	.86096	17	.42327	61	.87091	16	.46075	64	6
7	.86113	16	.42388	61	.87107	16	.46139	64	7
8	.86129	17	.42450	61	.87124	16	.46203	64	8
9	.86146	16	.42511	61	.87140	16	.46267	64	9
10	9.86163	16	10.42572	61	9.87157	16	10.46331	64	10
11	.86179	17	.42633	61	.87173	16	.46395	64	11
12	.86196	16	.42695	61	.87189	16	.46460	64	12
13	.86213	17	.42756	61	.87206	16	.46524	64	13
14	.86230	16	.42817	61	.87222	16	.46588	64	14
15	9.86246	16	10.42879	61	9.87239	16	10.46652	64	15
16	.86263	17	.42940	61	.87255	16	.46717	64	16
17	.86280	16	.43002	61	.87271	16	.46781	64	17
18	.86296	16	.43063	62	.87288	16	.46846	64	18
19	.86313	17	.43125	61	.87304	16	.46910	64	19
20	9.86330	16	10.43187	62	9.87320	16	10.46975	65	20
21	.86346	16	.43249	61	.87337	16	.47040	64	21
22	.86363	17	.43310	62	.87353	16	.47104	65	22
23	.86380	16	.43372	61	.87370	16	.47169	64	23
24	.86396	16	.43434	62	.87386	16	.47234	65	24
25	9.86413	17	10.43496	62	9.87402	16	10.47299	65	25
26	.86430	16	.43558	62	.87419	16	.47364	65	26
27	.86446	16	.43620	62	.87435	16	.47429	65	27
28	.86463	16	.43682	62	.87451	16	.47494	65	28
29	.86479	17	.43744	62	.87468	16	.47559	65	29
30	9.86496	16	10.43806	62	9.87484	16	10.47624	65	30
31	.86513	16	.43868	62	.87500	16	.47689	65	31
32	.86529	16	.43931	62	.87516	16	.47754	65	32
33	.86546	16	.43993	62	.87533	16	.47820	65	33
34	.86562	17	.44055	62	.87549	16	.47885	65	34
35	9.86579	16	10.44118	62	9.87565	16	10.47950	65	35
36	.86596	16	.44180	62	.87582	16	.48016	65	36
37	.86612	16	.44242	62	.87598	16	.48081	65	37
38	.86629	16	.44305	63	.87614	16	.48147	66	38
39	.86645	16	.44368	62	.87631	16	.48213	65	39
40	9.86662	16	10.44430	62	9.87647	16	10.48278	65	40
41	.86678	17	.44493	63	.87653	16	.48344	66	41
42	.86695	16	.44556	62	.87679	16	.48410	66	42
43	.86712	16	.44618	63	.87696	16	.48476	66	43
44	.86728	16	.44681	62	.87712	16	.48542	65	44
45	9.86745	16	10.44744	63	9.87728	16	10.48607	66	45
46	.86761	16	.44807	63	.87744	16	.48674	66	46
47	.86778	16	.44870	63	.87761	16	.48740	66	47
48	.86794	16	.44933	63	.87777	16	.48806	66	48
49	.86811	16	.44996	63	.87793	16	.48872	66	49
50	9.86827	16	10.45059	63	9.87809	16	10.48938	66	50
51	.86844	16	.45122	63	.87825	16	.49004	66	51
52	.86860	16	.45185	63	.87842	16	.49071	66	52
53	.86877	16	.45248	63	.87858	16	.49137	66	53
54	86893	16	.45312	63	.87874	16	.49204	66	54
55	9.86910	16	10.45375	63	9.87890	16	10.49270	66	55
56	.86926	16	.45439	63	.87906	16	.49337	66	56
57	.86943	16	.45502	63	.87923	16	.49403	67	57
58	.86959	16	.45565	63	.87939	16	.49470	66	58
59	.86976	16	.45629	64	.87955	16	.49537	67	59
60	9.86992		10.45693		9.87971		10.49604		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	67	$\overline{66}$	66
6	6.7	6.6	6.6
7	7.8	7.7	7.7
8	8.9	8.8	8.8
9	10.0	10.0	9.9
10	11.1	11.1	11.0
20	22.3	22.1	22.0
30	33.5	33.2	33.0
40	44.6	44.3	44.0
50	55.8	55.4	55.0

	$\overline{65}$	65	$\overline{64}$
6	6.5	6.5	6.4
7	7.6	7.6	7.5
8	8.7	8.6	8.6
9	9.8	9.7	9.7
10	10.9	10.8	10.7
20	21.8	21.6	21.5
30	32.7	32.5	32.2
40	43.6	43.3	43.0
50	54.6	54.1	53.7

	64	$\overline{63}$	63
6	6.4	6.3	6.3
7	7.4	7.4	7.3
8	8.5	8.4	8.4
9	9.6	9.5	9.4
10	10.6	10.6	10.5
20	21.3	21.1	21.0
30	32.0	31.7	31.5
40	42.6	42.3	42 0
50	53.3	52.9	52.5

	$\overline{62}$	62	$\overline{61}$
6	6.2	6.2	6.1
7	7.3	7.2	7.2
8	8.3	8.2	8.2
9	9.4	9.3	9.2
10	10.4	10.3	10.2
20	20.8	20.6	20.5
30	31.2	31.0	30.7
40	41.6	41.3	41.0
50	52.1	51.6	51.2

	61	$\overline{60}$
6	6.1	6.0
7	7.1	7.0
8	8.1	8.0
9	9.1	9.1
10	10.1	10.1
20	20.3	20.1
30	30.5	30.2
40	40.6	40.3
50	50.8	50.4

	17	$\overline{16}$	16
6	1.7	1.6	1.6
7	2.0	1.9	1.8
8	2.2	2.2	2.1
9	2.5	2.5	2.4
10	2.8	2.7	2.6
20	5.6	5.5	5.3
30	8.5	8.2	8.0
40	11.3	11.0	10.6
50	14.1	13.7	13.3

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

′	76° Lg. Vers.	D	76° Log. Exs.	D	77° Lg. Vers.	D	77° Log. Exs.	D	′
0	9.87971	16	10.49604	66	9.88933	16	10.53724	70	0
1	.87987	16	.49670	67	.88949	15	.53794	71	1
2	.88003	16	.49737	67	.88964	16	.53865	70	2
3	.88020	16	.49804	67	.88980	16	.53936	71	3
4	.88036	16	.49871	67	.88996	16	.54007	71	4
5	9.88052	16	10.49939	67	9.89012	15	10.54078	71	5
6	.88068	16	.50006	67	.89028	16	.54149	71	6
7	.88084	16	.50073	67	.89044	16	.54220	71	7
8	.88100	16	.50140	67	.89060	15	.54291	71	8
9	.88116	16	.50208	67	.89075	16	.54362	71	9
10	9.88133	16	10.50275	67	9.89091	16	10.54433	71	10
11	.88149	16	.50342	67	.89107	15	.54505	71	11
12	.88165	16	.50410	67	.89123	16	.54576	71	12
13	.88181	16	.50477	68	.89139	16	.54647	72	13
14	.88197	16	.50545	67	.89155	15	.54719	71	14
15	9.88213	16	10.50613	68	9.89170	16	10.54791	71	15
16	.88229	16	.50681	67	.89186	15	.54862	72	16
17	.88245	16	.50748	68	.89202	16	.54934	71	17
18	.88261	16	.50816	68	.89218	16	.55006	72	18
19	.88277	16	.50884	68	.89234	15	.55078	72	19
20	9.88294	16	10.50952	68	9.89249	16	10.55150	72	20
21	.88310	16	.51020	68	.89265	15	.55222	72	21
22	.88326	16	.51088	68	.89281	16	.55294	72	22
23	.88342	16	.51157	68	.89297	15	.55366	72	23
24	.88358	16	.51225	68	.89312	16	.55438	72	24
25	9.88374	16	10.51293	68	9.89328	15	10.55511	72	25
26	.88390	16	.51361	68	.89344	16	.55583	72	26
27	.88406	16	.51430	68	.89360	16	.55655	73	27
28	.88422	16	.51498	68	.89376	15	.55728	72	28
29	.88438	16	.51567	69	.89391	15	.55801	72	29
30	9.88454	16	10.51636	68	9.89407	16	10.55873	73	30
31	.88470	16	.51704	68	.89423	15	.55946	72	31
32	.88486	16	.51773	69	.89438	16	.56019	73	32
33	.88502	16	.51842	69	.89454	15	.56092	73	33
34	.88518	16	.51911	69	.89470	16	.56165	73	34
35	9.88534	16	10.51980	69	9.89486	15	10.56238	73	35
36	.88550	16	.52049	69	.89501	16	.56311	73	36
37	.88566	16	.52118	69	.89517	15	.56384	73	37
38	.88582	16	.52187	69	.89533	15	.56457	73	38
39	.88598	16	.52256	69	.89548	16	.56531	73	39
40	9.88614	16	10.52325	69	9.89564	15	10.56604	73	40
41	.88630	16	.52394	69	.89580	16	.56678	73	41
42	.88646	15	.52464	69	.89596	15	.56751	74	42
43	.88662	16	.52533	69	.89611	15	.56825	73	43
44	.88678	16	.52603	69	.89627	16	.56899	74	44
45	9.88694	16	10.52672	70	9.89643	15	10.56973	74	45
46	.88710	16	.52742	69	.89658	15	.57047	73	46
47	.88726	16	.52812	69	.89674	16	.57120	74	47
48	.88742	16	.52881	70	.89690	15	.57195	74	48
49	.88758	16	.52951	70	.89705	15	.57269	74	49
50	9.88774	16	10.53021	70	9.89721	16	10.57343	74	50
51	.88790	15	.53091	70	.89737	15	.57417	74	51
52	.88805	16	.53161	70	.89752	15	.57491	74	52
53	.88821	16	.53231	70	.89768	15	.57566	74	53
54	.88837	16	.53301	70	.89783	16	.57640	75	34
55	9.88853	16	10.53372	70	9.89799	15	10.57715	74	55
56	.88869	15	.53442	70	.89815	15	.57790	74	56
57	.88885	16	.53512	70	.89830	15	.57864	75	57
58	.88901	16	.53583	70	.89846	16	.57939	75	58
59	.88917	16	.53653	70	.89862	15	.58014	75	59
60	9.88933		10 53724		9.89877		10.58089		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	75	74	73
6	7.5	7.4	7.3
7	8.7	8.6	8.5
8	10.0	9.8	9.7
9	11.2	11.1	10.9
10	12.5	12.3	12.1
20	25.0	24.6	24.3
30	37.5	37.0	36.5
40	50.0	49.3	48.6
50	62.5	61.6	60.8

	72	71	70
6	7.2	7.1	7.0
7	8.4	8.3	8.2
8	9.6	9.4	9.4
9	10.8	10.6	10.6
10	12.0	11.8	11.7
20	24.0	23.6	23.3
30	36.0	35.5	35.2
40	48.0	47.3	47.0
50	60.0	59.1	58.7

	69	68	67
6	6.9	6.8	6.7
7	8.0	7.9	7.8
8	9.2	9.0	8.9
9	10.3	10.2	10.0
10	11.5	11.3	11.1
20	23.0	22.6	22.3
30	34.5	34.0	33.5
40	46.0	45.3	44.6
50	57.5	56.6	55.8

	66	0
6	6.6	0.0
7	7.7	0.0
8	8.8	0.0
9	9.9	0.1
10	11.0	0.1
20	22.0	0.1
30	33.0	0.2
40	44.0	0.3
50	55.0	0.4

	16	16	15
6	1.6	1.6	1.5
7	1.9	1.8	1.8
8	2.2	2.1	2.0
9	2.5	2.4	2.3
10	2.7	2.6	2.6
20	5.5	5.3	5.1
30	8.2	8.0	7.7
40	11.0	10.6	10.3
50	13.7	13.3	12.9

P. P.

′	78° Lg. Vers.	D	78° Log. Exs.	D	79° Lg. Vers.	D	79° Log. Exs.	D	′
0	9.89877	15	10.58089	75	9.90805	15	10.62745	80	0
1	.89893	15	.58164	75	.90820	15	.62825	80	1
2	.89908	15	.58239	75	.90835	15	.62906	80	2
3	.89924	15	.58315	75	.90851	15	.62986	81	3
4	.89939	16	.58390	75	.90866	15	.63067	80	4
5	9.89955	15	10.58465	75	9.90881	15	10.63148	81	5
6	.89971	15	.58541	75	.90897	15	.63229	81	6
7	.89986	15	.58616	76	.90912	15	.63310	81	7
8	.90002	15	.58692	75	.90927	15	.63391	81	8
9	.90017	15	.58768	76	.90943	15	.63472	81	9
10	9.90033	15	10.58844	76	9.90958	15	10.63553	81	10
11	.90048	15	.58920	75	.90973	15	.63634	81	11
12	.90064	16	.58995	75	.90988	15	.63716	81	12
13	.90080	15	.59072	76	.91004	15	.63797	81	13
14	.90095	15	.59148	76	.91019	15	.63879	82	14
15	9.90111	15	10.59224	76	9.91034	15	10.63961	82	15
16	.90126	15	.59300	76	.91049	15	.64043	82	16
17	.90142	15	.59377	76	.91065	15	.64125	82	17
18	.90157	15	.59453	76	.91080	15	.64207	82	18
19	.90173	15	.59530	76	.91095	15	.64289	82	19
20	9.90188	15	10.59606	77	9.91110	15	10.64371	82	20
21	.90204	15	.59683	76	.91126	15	.64453	82	21
22	.90219	15	.59760	77	.91141	15	.64536	82	22
23	.90235	15	.59837	77	.91156	15	.64618	83	23
24	.90250	15	.59914	77	.91171	15	.64701	82	24
25	9.90266	15	10.59991	77	9.91187	15	10.64784	83	25
26	.90281	15	.60068	77	.91202	15	.64867	83	26
27	.90297	15	.60145	77	.91217	15	.64950	83	27
28	.90312	15	.60223	77	.91232	15	.65033	83	28
29	.90328	15	.60300	77	.91247	15	.65116	83	29
30	9.90343	15	10.60378	77	9.91263	15	10.65199	83	30
31	.90359	15	.60455	77	.91278	15	.65283	83	31
32	.90374	15	.60533	78	.91293	15	.65366	83	32
33	.90389	15	.60611	77	.91308	15	.65450	84	33
34	.90405	15	.60688	78	.91323	15	.65534	83	34
35	9.90420	15	10.60766	78	9.91338	15	10.65617	84	35
36	.90436	15	.60844	78	.91354	15	.65701	84	36
37	.90451	15	.60923	78	.91369	15	.65785	84	37
38	.90467	15	.61001	78	.91384	15	.65870	84	38
39	.90482	15	.61079	78	.91399	15	.65954	84	39
40	9.90497	15	10.61158	78	9.91414	15	10.66038	84	40
41	.90513	15	.61236	78	.91429	15	.66123	84	41
42	.90528	15	.61315	78	.91445	15	.66207	84	42
43	.90544	15	.61393	79	.91460	15	.66292	85	43
44	.90559	15	.61472	78	.91475	15	.66377	85	44
45	9.90574	15	10.61551	79	9.91490	15	10.66462	85	45
46	.90590	15	.61630	79	.91505	15	.66547	85	46
47	.90605	15	.61709	79	.91520	15	.66632	85	47
48	.90621	15	.61788	79	.91535	15	.66717	85	48
49	.90636	15	.61867	79	.91550	15	.66803	85	49
50	9.90651	15	10.61947	79	9.91565	15	10.66888	85	50
51	.90667	15	.62026	79	.91581	15	.66974	85	51
52	.90682	15	.62105	80	.91596	15	.67059	86	52
53	.90697	15	.62185	79	.91611	15	.67145	86	53
54	.90713	15	.62265	80	.91626	15	.67231	86	54
55	9.90728	15	10.62345	79	9.91641	15	10.67317	86	55
56	.90744	15	.62424	80	.91656	15	.67403	86	56
57	.90759	15	.62504	80	.91671	15	.67490	86	57
58	.90774	15	.62585	80	.91686	15	.67576	86	58
59	.90790	15	.62665	80	.91701	15	.67663	86	59
60	9.90805		10.62745		9.91716		10.67749		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	86	85	84
6	8.6	8.5	8.4
7	10.0	9.9	9.8
8	11.4	11.3	11.2
9	12.9	12.7	12.6
10	14.3	14.1	14.0
20	28.6	28.3	28.0
30	43.0	42.5	42.0
40	57.3	56.6	56.0
50	71.6	70.8	70.0

	83	82	81
6	8.3	8.2	8.1
7	9.7	9.5	9.4
8	11.0	10.9	10.8
9	12.4	12.3	12.1
10	13.8	13.6	13.5
20	27.6	27.3	27.0
30	41.5	41.0	40.5
40	55.3	54.6	54.0
50	69.1	68.3	67.5

	80	79	78
6	8.0	7.9	7.8
7	9.3	9.2	9.1
8	10.6	10.5	10.4
9	12.0	11.8	11.7
10	13.3	13.1	13.0
20	26.6	26.3	26.0
30	40.0	39.5	39.0
40	53.3	52.6	52.0
50	66.6	65.8	65.0

	77	76	75
6	7.7	7.6	7.5
7	9.0	8.8	8.7
8	10.2	10.1	10.0
9	11.5	11.4	11.2
10	12.8	12.6	12.5
20	25.6	25.3	25.0
30	38.5	38.0	37.5
40	51.3	50.6	50.0
50	64.1	63.3	62.5

	$\bar{0}$
6	0.0
7	0.0
8	0.0
9	0.1
10	0.1
20	0.1
30	0.2
40	0.3
50	0.4

	16	$1\bar{5}$	15
6	1.6	1.5	1.5
7	1.8	1.8	1.7
8	2.1	2.0	2.0
9	2.4	2.3	2.2
10	2.6	2.6	2.5
20	5.3	5.1	5.0
30	8.0	7.7	7.5
40	10.6	10.3	10.0
50	13.3	12.9	12.5

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

′	80° Lg. Vers.	D	80° Log. Exs.	D	81° Lg. Vers.	D	81° Log. Exs.	D	′
0	9.91716	15	10.67749	86	9.92612	14	10.73178	95	0
1	.91731	15	.67836	87	.92626	15	.73273	94	1
2	.91746	15	.67923	87	.92641	14	.73368	95	2
3	.91761	15	.68010	87	.92656	15	.73463	95	3
4	.91776	15	.68097	87	.92671	15	.73558	95	4
5	9.91791	15	10.68184	87	9.92686	14	10.73653	95	5
6	.91807	15	.68272	87	.92700	15	.73748	95	6
7	.91822	15	.68359	87	.92715	14	.73844	96	7
8	.91837	15	.68447	87	.92730	15	.73940	95	8
9	.91852	15	.68534	88	.92745	14	.74035	96	9
10	9.91867	15	10.68622	88	9.92759	15	10.74131	96	10
11	.91882	15	.68710	88	.92774	14	.74227	96	11
12	.91897	15	.68798	88	.92789	15	.74324	96	12
13	.91912	15	.68886	88	.92804	14	.74420	96	13
14	.91927	15	.68975	88	.92818	15	.74517	96	14
15	9.91942	15	10.69063	88	9.92833	14	10.74613	97	15
16	.91957	15	.69152	88	.92848	14	.74710	97	16
17	.91972	15	.69240	89	.92862	15	.74807	97	17
18	.91987	15	.69329	89	.92877	14	.74905	97	18
19	.92002	14	.69418	89	.92892	15	.75002	97	19
20	9.92016	15	10.69507	89	9.92907	14	10.75099	98	20
21	.92031	15	.69596	89	.92921	14	.75197	97	21
22	.92046	15	.69686	89	.92936	15	.75295	98	22
23	.92061	15	.69775	89	.92951	14	.75393	98	23
24	.92076	15	.69865	90	.92965	15	.75491	98	24
25	9.92091	15	10.69955	89	9.92980	14	10.75589	98	25
26	.92106	15	.70044	90	.92995	14	.75688	98	26
27	.92121	15	.70134	90	.93009	15	.75786	99	27
28	.92136	14	.70224	90	.93024	14	.75885	99	28
29	.92151	15	.70315	90	.93039	14	.75984	99	29
30	9.92166	15	10.70405	90	9.93053	15	10.76083	99	30
31	.92181	15	.70495	91	.93068	14	.76182	99	31
32	.92196	15	.70586	90	.93083	14	.76282	100	32
33	.92211	15	.70677	91	.93097	15	.76382	99	33
34	.92226	14	.70768	91	.93112	14	.76481	100	34
35	9.92240	15	10.70859	91	9.93127	14	10.76581	100	35
36	.92255	15	.70950	91	.93141	14	.76681	100	36
37	.92270	15	.71041	91	.93156	15	.76782	100	37
38	.92285	15	.71133	91	.93171	14	.76882	100	38
39	.92300	14	.71224	91	.93185	14	.76983	100	39
40	9.92315	15	10.71316	92	9.93200	14	10.77083	101	40
41	.92330	15	.71408	92	.93214	15	.77184	101	41
42	.92345	15	.71500	92	.93229	14	.77286	101	42
43	.92360	14	.71592	92	.93244	14	.77387	101	43
44	.92374	15	.71684	92	.93258	14	.77488	101	44
45	9.92389	15	10.71776	92	9.93273	14	10.77590	102	45
46	.92404	14	.71869	92	.93287	15	.77692	102	46
47	.92419	15	.71961	93	.93302	14	.77794	102	47
48	.92434	15	.72054	92	.93317	14	.77896	102	48
49	.92449	14	.72147	93	.93331	14	.77998	102	49
50	9.92463	15	10.72240	93	9.93346	14	10.78101	102	50
51	.92478	15	.72333	93	.93360	14	.78203	103	51
52	.92493	14	.72427	93	.93375	14	.78306	103	52
53	.92508	15	.72520	93	.93389	15	.78409	103	53
54	.92523	15	.72614	93	.93404	14	.78513	103	54
55	9.92538	14	10.72707	94	9.93419	14	10.78616	104	55
56	.92552	15	.72801	94	.93433	14	.78720	103	56
57	.92567	15	.72895	94	.93448	14	.78823	104	57
58	.92582	14	.72990	94	.93462	14	.78927	104	58
59	.92597	15	.73084	94	.93477	14	.79031	104	59
60	9.92612		10.73178		9.93491		10.79136		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	90	80
6	9.0	8.0
7	10.5	9.3
8	12.0	10.6
9	13.5	12.0
10	15.0	13.3
20	30.0	26.6
30	45.0	40.0
40	60.0	53.3
50	75.0	66.6

	9	8
6	0.9	0.8
7	1.0	0.9
8	1.2	1.0
9	1.3	1.2
10	1.5	1.3
20	3.0	2.6
30	4.5	4.0
40	6.0	5.3
50	7.5	6.6

	7	6
6	0.7	0.6
7	0.8	0.7
8	0.0	0.8
9	1.0	0.9
10	1.1	1.0
20	2.3	2.0
30	3.5	3.0
40	4.6	4.0
50	5.8	5.0

	5	4
6	0.5	0.4
7	0.6	0.4
8	0.6	0.5
9	0.7	0.6
10	0.8	0.6
20	1.6	1.3
30	2.5	2.0
40	3.3	2.6
50	4.1	3.3

	15	15
6	1.5	1.5
7	1.8	1.7
8	2.0	2.0
9	2.3	2.2
10	2.6	2.5
20	5.1	5.0
30	7.7	7.5
40	10.3	10.0
50	12.9	12.5

	14
6	1.4
7	1.7
8	1.9
9	2.2
10	2.4
20	4.8
30	7.2
40	9.6
50	12.1

P. P.

′	82° Lg. Vers.	D	82° Log. Exs.	D	83° Lg. Vers.	D	83° Log. Exs.	D	′
0	9.93491	14	10.79136	104	9.94356	14	10.85766	117	0
1	.93506	14	.79240	105	.94370	14	.85884	117	1
2	.93520	14	.79345	104	.94384	14	.86001	117	2
3	.93535	14	.79450	105	.94398	14	.86119	118	3
4	.93549	14	.79555	105	.94413	14	.86237	118	4
5	9.93564	14	10.79660	105	9.94427	14	10.86355	118	5
6	.93578	14	.79766	105	.94441	14	.86474	118	6
7	.93593	14	.79871	106	.94456	14	.86592	119	7
8	.93607	14	.79977	106	.94470	14	.86711	119	8
9	.93622	14	.80083	106	.94484	14	.86831	119	9
10	9.93636	14	10.80189	106	9.94498	14	10.86950	120	10
11	.93651	14	.80296	106	.94512	14	.87070	120	11
12	.93665	14	.80402	107	.94527	14	.87190	120	12
13	.93680	14	.80509	107	.94541	14	.87310	120	13
14	.93694	14	.80616	107	.94555	14	.87431	121	14
15	9.93709	14	10.80723	107	9.94569	14	10.87552	121	15
16	.93723	14	.80831	107	.94584	14	.87673	121	16
17	.93738	14	.80938	108	.94598	14	.87794	121	17
18	.93752	14	.81046	108	.94612	14	.87916	122	18
19	.93767	14	.81154	108	.94626	14	.88038	122	19
20	9.93781	14	10.81262	108	9.94640	14	10.88160	122	20
21	.93796	14	.81371	108	.94655	14	.88282	122	21
22	.93810	14	.81479	109	.94669	14	.88405	123	22
23	.93824	14	.81588	109	.94683	14	.88528	123	23
24	.93839	14	.81697	109	.94697	14	.88651	124	24
25	9.93853	14	10.81806	109	9.94711	14	10.88775	123	25
26	.93868	14	.81916	109	.94726	14	.88898	124	26
27	.93882	14	.82025	110	.94740	14	.89022	124	27
28	.93897	14	.82135	110	.94754	14	.89147	124	28
29	.93911	14	.82245	110	.94768	14	.89271	125	29
30	9.93925	14	10.82356	110	9.94782	14	10.89396	125	30
31	.93940	14	.82466	110	.94796	14	.89521	125	31
32	.93954	14	.82577	111	.94810	14	.89647	126	32
33	.93969	14	.82688	111	.94825	14	.89773	126	33
34	.93983	14	.82799	111	.94839	14	.89899	126	34
35	9.93997	14	10.82910	111	9.94853	14	10.90025	126	35
36	.94012	14	.83022	111	.94867	14	.90152	127	36
37	.94026	14	.83133	112	.94881	14	.90279	127	37
38	.94041	14	.83245	112	.94895	14	.90406	127	38
39	.94055	14	.83358	112	.94909	14	.90533	128	39
40	9.94069	14	10.83470	112	9.94923	14	10.90661	127	40
41	.94084	14	.83583	112	.94938	14	.90789	128	41
42	.94098	14	.83695	113	.94952	14	.90917	129	42
43	.94112	14	.83809	113	.94966	14	.91046	129	43
44	.94127	14	.83922	113	.94980	14	.91175	129	44
45	9.94141	14	10.84035	114	9.94994	14	10.91304	130	45
46	.94155	14	.84149	114	.95008	14	.91434	129	46
47	.94170	14	.84263	114	.95022	14	.91564	130	47
48	.94184	14	.84377	114	.95036	14	.91694	130	48
49	.94198	14	.84492	115	.95050	14	.91825	131	49
50	9.94213	14	10.84607	114	9.95064	14	10.91956	131	50
51	.94227	14	.84721	115	.95078	14	.92087	131	51
52	.94241	14	.84837	115	.95093	14	.92218	131	52
53	.94256	14	.84952	116	.95107	14	.92350	132	53
54	.94270	14	.85068	115	.95121	14	.92482	132	54
55	9.94284	14	10.85183	116	9.95135	14	10.92614	133	55
56	.94299	14	.85299	116	.95149	14	.92747	133	56
57	.94313	14	.85416	116	.95163	14	.92880	133	57
58	.94327	14	.85532	117	.95177	14	.93014	133	58
59	.94341	14	.85649	117	.95191	14	.93147	134	59
60	9.94356		10.85766		9.95205		10.93281		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	130	120
6	13.0	12.0
7	15.1	14.0
8	17.3	16.0
9	19.5	18.0
10	21.6	20.0
20	43.3	40.0
30	65.0	60.0
40	86.6	80.0
50	108.3	100.0

	110	100
6	11.0	10.0
7	12.8	11.6
8	14.6	13.3
9	16.5	15.0
10	18.3	16.6
20	36.6	33.3
30	55.0	50.0
40	73.3	66.6
50	91.6	83.3

	3	2
6	0.3	0.2
7	0.3	0.2
8	0.4	0.2
9	0.4	0.3
10	0.5	0.3
20	1.0	0.6
30	1.5	1.0
40	2.0	1.3
50	2.5	1.6

	1	0
6	0.1	0.0
7	0.1	0.0
8	0.1	0.0
9	0.1	0.1
10	0.1	0.1
20	0.3	0.1
30	0.5	0.2
40	0.6	0.3
50	0.8	0.4

	14	14
6	1.4	1.4
7	1.7	1.6
8	1.9	1.8
9	2.2	2.1
10	2.4	2.3
20	4.8	4.6
30	7.2	7.0
40	9.6	9.3
50	12.1	11.6

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

′	84° Lg. Vers.	D	84° Log. Exs.	D	85° Lg. Vers.	D	85° Log. Exs.	D	′
0	9.95205	14	10.93281	134	9.96039	14	11.02010	158	0
1	.95219	14	.93416	135	.96053	13	.02168	159	1
2	.95233	14	.93551	135	.96067	14	.02327	159	2
3	.95247	14	.93686	135	.96081	14	.02487	159	3
4	.95261	14	.93821	135	.96095	13	.02646	160	4
5	9.95275	14	10.93957	136	9.96108	14	11.02807	161	5
6	.95289	14	.94093	136	.96122	13	.02968	161	6
7	.95303	14	.94229	137	.96136	14	.03129	161	7
8	.95317	14	.94366	137	.96150	13	.03291	162	8
9	.95331	14	.94503	137	.96163	14	.03453	163	9
10	9.95345	14	10.94641	137	9.96177	13	11.03616	163	10
11	.95359	13	.94778	138	.96191	14	.03780	164	11
12	.95373	14	.94917	138	.96205	13	.03944	164	12
13	.95387	14	.95055	139	.96218	14	.04108	165	13
14	.95401	14	.95194	139	.96232	13	.04273	165	14
15	9.95415	14	10.95333	139	9.96246	13	11.04438	166	15
16	.95429	14	.95473	140	.96259	14	.04604	167	16
17	.95443	14	.95613	140	.96273	13	.04771	167	17
18	.95457	14	.95753	140	.96287	14	.04938	167	18
19	.95471	14	.95894	141	.96301	13	.05106	168	19
20	9.95485	14	10.96035	141	9.96314	14	11.05274	169	20
21	.95499	14	.96176	142	.96328	13	.05443	169	21
22	.95513	14	.96318	142	.96342	13	.05612	169	22
23	.95527	13	.96461	142	.96355	14	.05782	170	23
24	.95540	14	.96603	143	.96369	13	.05952	171	24
25	9.95554	14	10.96746	143	9.96383	14	11.06123	171	25
26	.95568	14	.96889	144	.96397	13	.06295	172	26
27	.95582	14	.97033	144	.96410	13	.06467	173	27
28	.95596	14	.97177	144	.96424	14	.06640	173	28
29	.95610	14	.97322	145	.96438	13	.06813	174	29
30	9.95624	13	10.97467	145	9.96451	13	11.06987	174	30
31	.95638	14	.97612	145	.96465	14	.07161	175	31
32	.95652	14	.97758	146	.96479	13	.07336	176	32
33	.95666	14	.97904	146	.96492	13	.07512	176	33
34	.95680	13	.98050	147	.96506	13	.07688	177	34
35	9.95693	14	10.98197	147	9.96519	14	11.07865	177	35
36	.95707	14	.98345	147	.96533	13	.08043	178	36
37	.95721	14	.98492	148	.96547	13	.08221	179	37
38	.95735	14	.98640	149	.96560	14	.08400	179	38
39	.95749	13	.98789	149	.96574	13	.08579	180	39
40	9.95763	14	10.98938	149	9.96588	13	11.08759	180	40
41	.95777	14	.99087	150	.96601	13	.08940	181	41
42	.95791	13	.99237	150	.96615	14	.09121	182	42
43	.95804	14	.99387	151	.96629	13	.09303	182	43
44	.95818	14	.99538	151	.96642	13	.09486	183	44
45	9.95832	14	10.99689	151	9.96656	13	11.09669	184	45
46	.95846	13	.99841	152	.96669	13	.09853	185	46
47	.95860	14	10.99993	152	.96683	14	.10038	185	47
48	.95874	14	11.00145	153	.96697	13	.10223	186	48
49	.95888	13	.00298	153	.96710	13	.10409	186	49
50	9.95901	14	11.00451	154	9.96724	13	11.10595	187	50
51	.95915	13	.00605	154	.96737	13	.10783	188	51
52	.95929	14	.00759	155	.96751	13	.10971	189	52
53	.95943	14	.00914	155	.96764	14	.11160	189	53
54	.95957	13	.01069	155	.96778	13	.11349	190	54
55	9.95970	14	11.01225	156	9.96792	13	11.11539	191	55
56	.95984	14	.01381	156	.96805	13	.11730	191	56
57	.95998	13	.01537	157	.96819	13	.11922	192	57
58	.96012	14	.01694	157	.96832	13	.12114	193	58
59	.96026	13	.01852	158	.96846	13	.12307	193	59
60	9.96039		11.02010		9.96859		11.12501		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	190	180
6	19.0	18.0
7	22.1	21.0
8	25.3	24.0
9	28.5	27.0
10	31.6	30.0
20	63.3	60.0
30	95.0	90.0
40	126.6	120.0
50	158.3	150.0

	170	160
6	17.0	16.0
7	19.8	18.6
8	22.6	21.3
9	25.5	24.0
10	28.3	26.6
20	56.6	53.3
30	85.0	80.0
40	113.3	106.6
50	141.6	133.3

	150	140
6	15.0	14.0
7	17.5	16.3
8	20.0	18.6
9	22.5	21.0
10	25.0	23.3
20	50.0	46.6
30	75.0	70.0
40	100.0	93.3
50	125.0	116.6

	130	9	8
6	13.0	0.9	0.8
7	15.1	1.0	0.9
8	17.3	1.2	1.0
9	19.5	1.3	1.2
10	21.6	1.5	1.3
20	43.3	3.0	2.6
30	65.0	4.5	4.0
40	86.6	6.0	5.3
50	108.3	7.5	6.6

	7	6	5
6	0.7	0.6	0.5
7	0.8	0.7	0.6
8	0.9	0.8	0.6
9	1.0	0.9	0.7
10	1.1	1.0	0.8
20	2.3	2.0	1.6
30	3.5	3.0	2.5
40	4.6	4.0	3.3
50	5.8	5.0	4.1

	$\overline{14}$	14	13
6	1.4	1.4	1.3
7	1.7	1.6	1.6
8	1.9	1.8	1.8
9	2.2	2.1	2.0
10	2.4	2.3	2.2
20	4.8	4.6	4.5
30	7.2	7.0	6.7
40	9.6	9.3	9.0
50	12.1	11.6	11.2

P. P.

	86°				87°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.96859	13	11.12501	195	9.97665	13	11.25785	255	0
1	.96837	14	.12696	195	.97679	13	.26040	256	1
2	.96887	13	.12891	196	.97692	13	.26297	257	2
3	.96900	13	.13087	196	.97705	13	.26554	259	3
4	.96914	13	.13284	198	.97718	13	.26814	260	4
5	9.96927	13	11.13482	198	9.97732	13	11.27074	262	5
6	.96941	13	.13680	199	.97745	13	.27336	263	6
7	.96954	13	.13879	200	.97758	13	.27599	265	7
8	.96968	13	.14079	201	.97772	13	.27864	266	8
9	.96981	13	.14280	201	.97785	13	.28131	267	9
10	9.96995	13	11.14482	202	9.97798	13	11.28398	269	10
11	.97008	13	.14684	203	.97811	13	.28668	270	11
12	.97022	13	.14887	204	.97825	13	.28938	272	12
13	.97035	13	.15092	205	.97838	13	.29211	274	13
14	.97049	13	.15297	205	.97851	13	.29485	275	14
15	9.97062	13	11.15502	206	9.97864	13	11.29760	277	15
16	.97076	13	.15709	208	.97878	13	.30037	278	16
17	.97089	13	.15917	208	.97891	13	.30316	280	17
18	.97103	13	.16125	209	.97904	13	.30596	282	18
19	.97116	13	.16334	210	.97917	13	.30878	283	19
20	9.97130	13	11.16544	211	9.97931	13	11.31162	285	20
21	.97143	13	.16755	212	.97944	13	.31447	287	21
22	.97157	13	.16967	213	.97957	13	.31734	288	22
23	.97170	13	.17180	214	.97970	13	.32023	290	23
24	.97183	13	.17394	214	.97984	13	.32313	292	24
25	9.97197	13	11.17609	215	9.97997	13	11.32606	294	25
26	.97210	13	.17824	216	.98010	13	.32900	296	26
27	.97224	13	.18041	218	.98023	13	.33196	298	27
28	.97237	13	.18259	218	.98036	13	.33494	299	28
29	.97251	13	.18477	219	.98050	13	.33793	301	29
30	9.97264	13	11.18697	220	9.98063	13	11.34095	303	30
31	.97277	13	.18917	221	.98076	13	.34398	305	31
32	.97291	13	.19138	222	.98089	13	.34704	307	32
33	.97304	13	.19361	223	.98102	13	.35011	309	33
34	.97318	13	.19584	224	.98116	13	.35321	311	34
35	9.97331	13	11.19809	225	9.98129	13	11.35632	313	35
36	.97345	13	.20034	227	.98142	13	.35946	315	36
37	.97358	13	.20261	227	.98155	13	.36261	318	37
38	.97371	33	.20489	228	.98168	13	.36579	320	38
39	.97385	13	.20717	230	.98181	13	.36899	322	39
40	9.97398	13	11.20947	230	9.98195	13	11.37221	324	40
41	.97412	13	.21178	232	.98208	13	.37546	326	41
42	.97425	13	.21410	233	.98221	13	.37872	328	42
43	.97438	13	.21643	234	.98234	13	.38201	331	43
44	.97452	13	.21877	235	.98247	13	.38532	333	44
45	9.97465	13	11.22112	236	9.98260	13	11.38866	335	45
46	.97478	13	.22349	237	.98273	13	.39201	338	46
47	.97492	13	.22586	239	.98287	13	.39540	340	47
48	.97505	13	.22825	239	.98300	13	.39880	343	48
49	.97519	13	.23065	241	.98313	13	.40224	345	49
50	9.97532	13	11.23306	242	9.98326	13	11.40569	348	50
51	.97545	13	.23548	243	.98339	13	.40918	351	51
52	.97559	13	.23792	245	.98352	13	.41269	353	52
53	.97572	13	.24037	246	.98365	13	.41622	356	53
54	.97585	13	.24283	247	.98378	13	.41979	359	54
55	9.97599	13	11.24530	248	9.98392	13	11.42338	361	55
56	.97612	13	.24778	250	.98405	13	.42699	364	56
57	.97625	13	.25028	251	.98418	13	.43064	367	57
58	.97639	13	.25279	252	.98431	13	.43431	370	58
59	.97652	13	.25531	254	.98444	13	.43802	373	59
60	9.97665		11.25785		9.98457		11.44175		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	250	240
6	25.0	24.0
7	29.1	28.0
8	33.3	32.0
9	37.5	36.0
10	41.6	40.0
20	83.3	80.0
30	125.0	120.0
40	166.6	160.0
50	208.3	200.0

	230	220
6	23.0	22.0
7	26.8	25.6
8	30.6	29.3
9	34.5	33.0
10	38.3	36.6
20	76.6	73.3
30	115.0	110.0
40	153.3	146.6
50	191.6	183.3

	210	200
6	21.0	20.0
7	24.5	23.3
8	28.0	26.6
9	31.5	30.0
10	35.0	33.3
20	70.0	66.6
30	105.0	100.0
40	140.0	133.3
50	175.0	166.6

	190	4	3
6	19.0	0.4	0.3
7	22.1	0.4	0.3
8	25.3	0.5	0.4
9	28.5	0.6	0.4
10	31.6	0.6	0.5
20	63.3	1.3	1.0
30	95.0	2.0	1.5
40	126.6	2.6	2.0
50	158.3	3.3	2.5

	2	1	0
6	0.2	0.1	0.0
7	0.2	0.1	0.0
8	0.2	0.1	0.0
9	0.3	0.1	0.1
10	0.3	0.1	0.1
20	0.6	0.3	0.1
30	1.0	0.5	0.2
40	1.3	0.6	0.3
50	1.6	0.8	0.4

	14	13	13
6	1.4	1.3	1.3
7	1.6	1.6	1.5
8	1.8	1.8	1.7
9	2.1	2.0	1.9
10	2.3	2.2	2.1
20	4.6	4.5	4.3
30	7.0	6.7	6.5
40	9.3	9.0	8.6
50	11.6	11.2	10.8

P. P.

	88°				89°				
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′
0	9.98457	13	11.44175	376	9.99235	12	11.75050	742	0
1	.98470	13	.44551	379	.99248	13	.75792	755	1
2	.98483	13	.44931	382	.99261	13	.76547	768	2
3	.98496	13	.45313	386	.99274	13	.77316	781	3
4	.98509	13	.45699	389	.99287	12	.78097	795	4
5	9.98522	13	11.46088	392	9.99299	13	11.78892	809	5
6	.98535	13	.46480	395	.99312	13	.79702	825	6
7	.98548	13	.46876	399	.99325	12	.80527	840	7
8	.98562	13	.47275	402	.99338	13	.81367	856	8
9	.98575	13	.47677	406	.99351	12	.82223	872	9
10	9.98588	13	11.48083	409	9.99363	13	11.83095	890	10
11	.98601	13	.48493	413	.99376	13	.83986	908	11
12	.98614	13	.48906	417	.99389	12	.84894	927	12
13	.98627	13	.49323	420	.99402	13	.85821	947	13
14	.98640	13	.49743	425	.99415	13	.86768	967	14
15	9.98653	13	11.50168	428	9.99428	12	11.87735	989	15
16	.98666	13	.50597	432	.99440	13	.88724	1009	16
17	.98679	13	.51029	436	.99453	12	.89735	1034	17
18	.98692	13	.51466	440	.99466	13	.90769	1059	18
19	.98705	13	.51906	445	.99479	12	.91829	1085	19
20	9.98718	13	11.52351	449	9.99491	13	11.92914	1112	20
21	.98731	13	.52801	454	.99504	13	.94026	1140	21
22	.98744	13	.53255	458	.99517	12	.95167	1171	22
23	.98757	13	.53713	463	.99530	13	.96338	1203	23
24	.98770	13	.54176	467	.99543	12	.97541	1236	24
25	9.98783	13	11.54643	472	9.99555	13	11.98777	1271	25
26	.98796	13	.55116	477	.99568	12	12.00048	1309	26
27	.98809	13	.55593	482	.99581	13	.01358	1349	27
28	.98822	13	.56076	487	.99594	12	.02707	1391	28
29	.98835	13	.56563	492	.99606	13	.04098	1436	29
30	9.98848	13	11.57056	498	9.99619	12	12.05535	1485	30
31	.98861	13	.57554	504	.99632	13	.07020	1537	31
32	.98874	13	.58058	509	.99645	12	.08557	1592	32
33	.98887	13	.58567	515	.99657	13	.10149	1652	33
34	.98900	13	.59082	520	.99670	12	.11801	1716	34
35	9.98913	12	11.59602	527	9.99683	12	12.13517	1785	35
36	.98925	13	.60129	533	.99695	13	.15302	1861	36
37	.98938	13	.60662	539	.99708	12	.17163	1943	37
38	.98951	13	.61202	545	.99721	13	.19106	2033	38
39	.98964	13	.61747	552	.99734	12	.21139	2131	39
40	9.98977	13	11.62300	559	9.99746	13	12.23271	2240	40
41	.98990	13	.62859	566	.99759	12	.25511	2361	41
42	.99003	13	.63425	573	.99772	12	.27872	2495	42
43	.99016	12	.63998	581	.99784	13	.30367	2645	43
44	.99029	13	.64579	588	.99797	12	.33013	2815	44
45	9.99042	13	11.65167	595	9.99810	13	12.35828	3009	45
46	.99055	13	.65762	604	.99823	12	.38837	3231	46
47	.99068	13	.66366	611	.99835	12	.42068	3489	47
48	.99081	12	.66978	620	.99848	13	.45557	3791	48
49	.99093	13	.67598	628	.99861	12	.49349	4152	49
50	9.99106	13	11.68227	638	9.99873	12	12.53501	4588	50
51	.99119	13	.68865	646	.99886	13	.58089	5127	51
52	.99132	13	.69511	656	.99899	12	.63217	5812	52
53	.99145	12	.70168	666	.99911	12	.69029	6707	53
54	.99158	13	.70834	675	.99924	13	.75736	7931	54
55	9.99171	13	11.71509	686	9.99937	12	12.83667	9704	55
56	.99184	13	.72196	696	.99949	12	.93371	12506	56
57	.99197	12	.72892	707	.99962	12	13.05877	17621	57
58	.99209	13	.73600	719	.99974	13	.23499	30116	58
59	.99222	13	.74319	730	.99987	12	.53615		59
60	9.99235		11.75050		10.00000		Infinity		60
′	Lg. Vers.	D	Log. Exs.	D	Lg. Vers.	D	Log. Exs.	D	′

P. P.

	13	13
6	1.3	1.3
7	1.6	1.5
8	1.8	1.7
9	2.0	1.9
10	2.2	2.1
20	4.5	4.3
30	6.7	6.5
40	9.0	8.6
50	11.2	10.8

	12
6	1.2
7	1.4
8	1.6
9	1.9
10	2.1
20	4.1
30	6.2
40	8.3
50	10.4

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

0°–10°

° ′	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.	
0 0	0.0000	29	0.0000	29	∞		1.0000	0	0 90
10	0.0029	2$\underline{9}$	0.0029	2$\underline{9}$	343.77$\bar{3}$		1.0000	$\underline{0}$	50
20	0.005$\underline{8}$	2$\bar{9}$	0.005$\underline{8}$	2$\bar{9}$	171.88$\bar{5}$		1.000$\underline{0}$	$\bar{0}$	40
30	0.008$\bar{7}$	29	0.008$\bar{7}$	29	114.58$\bar{8}$		0.999$\bar{9}$	0	30
40	0.011$\bar{6}$	29	0.011$\bar{6}$	29	85.9398		0.999$\bar{9}$	$\bar{0}$	20
50	0.014$\bar{5}$	29	0.014$\bar{5}$	29	68.7501		0.9999	$\bar{0}$	10
1 0	0.017$\bar{4}$	29	0.017$\bar{4}$	29	57.289$\bar{9}$	8.186$\bar{0}$	0.999$\bar{8}$	$\bar{0}$	0 89
10	0.020$\bar{3}$	2$\underline{9}$	0.020$\bar{3}$	2$\bar{9}$	49.1039	6.139$\underline{8}$	0.9998	$\bar{0}$	50
20	0.023$\bar{2}$	2$\bar{9}$	0.0233	29	42.964$\underline{1}$	4.775$\bar{6}$	0.999$\underline{7}$	$\underline{1}$	40
30	0.0262	29	0.0262	29	38.188$\underline{4}$	3.820$\underline{7}$	0.999$\bar{6}$	$\bar{0}$	30
40	0.0291	29	0.0291	29	34.367$\bar{7}$	3.126$\bar{1}$	0.9996	1	20
50	0.0320	29	0.0320	29	31.2416	2.605$\bar{3}$	0.9995	1	10
2 0	0.0349	29	0.0349	2$\bar{9}$	28.636$\bar{2}$	2.204$\bar{6}$	0.9994	1	0 88
10	0.0378	29	0.037$\bar{8}$	29	26.431$\underline{6}$	1.889$\bar{8}$	0.999$\bar{3}$	$\bar{1}$	50
20	0.0407	2$\underline{9}$	0.040$\underline{7}$	2$\underline{9}$	24.541$\underline{7}$	1.638$\underline{0}$	0.999$\underline{1}$	$\underline{1}$	40
30	0.043$\underline{6}$	2$\bar{9}$	0.043$\bar{6}$	2$\bar{9}$	22.903$\bar{7}$	1.433$\bar{3}$	0.999$\bar{0}$	$\bar{1}$	30
40	0.046$\bar{5}$	29	0.0466	29	21.470$\underline{4}$	1.264$\bar{8}$	0.9989	$\underline{1}$	20
50	0.049$\bar{4}$	29	0.0495	29	20.205$\bar{5}$	1.1244	0.9988	$\bar{1}$	10
3 0	0.052$\bar{3}$	29	0.0524	2$\bar{9}$	19.081$\bar{1}$	1.006$\bar{1}$	0.998$\bar{6}$	2	0 87
10	0.055$\bar{2}$	29	0.055$\bar{3}$	29	18.075$\underline{0}$	905$\bar{6}$	0.998$\bar{4}$	$\bar{1}$	50
20	0.058$\bar{1}$	29	0.058$\underline{2}$	2$\underline{9}$	17.169$\underline{3}$	819$\underline{5}$	0.998$\underline{3}$	$\underline{1}$	40
30	0.061$\bar{0}$	29	0.061$\bar{1}$	2$\bar{9}$	16.349$\bar{8}$	745$\bar{0}$	0.998$\bar{1}$	2	30
40	0.063$\bar{9}$	29	0.0641	2$\underline{9}$	15.6048	680$\underline{4}$	0.997$\bar{9}$	2	20
50	0.066$\bar{8}$	29	0.0670	2$\bar{9}$	14.9244	623$\bar{7}$	0.997$\bar{7}$	2	10
4 0	0.069$\bar{7}$	29	0.069$\bar{9}$	29	14.300$\bar{6}$	5739	0.997$\bar{5}$	2	0 86
10	0.072$\bar{6}$	29	0.072$\bar{8}$	2$\bar{9}$	13.726$\bar{7}$	529$\bar{8}$	0.997$\bar{3}$	$\underline{2}$	50
20	0.075$\bar{5}$	29	0.0758	2$\underline{9}$	13.1969	4907	0.997$\bar{1}$	$\bar{2}$	40
30	0.078$\bar{4}$	29	0.078$\underline{7}$	2$\bar{9}$	12.7062	455$\underline{7}$	0.9969	$\underline{2}$	30
40	0.081$\bar{3}$	29	0.081$\bar{6}$	2$\underline{9}$	12.250$\underline{5}$	424$\bar{3}$	0.996$\underline{7}$	$\bar{2}$	20
50	0.084$\bar{2}$	29	0.084$\bar{5}$	2$\bar{9}$	11.826$\bar{1}$	3961	0.996$\bar{4}$	$\bar{2}$	10
5 0	0.087$\bar{1}$	29	0.0875	29	11.430$\bar{0}$	3706	0.9962	$\bar{2}$	0 85
10	0.090$\bar{0}$	29	0.090$\underline{4}$	2$\bar{9}$	11.059$\bar{4}$	347$\bar{5}$	0.995$\bar{9}$	$\underline{3}$	50
20	0.092$\bar{9}$	29	0.093$\bar{3}$	2$\underline{9}$	10.7119	326$\underline{5}$	0.995$\bar{6}$	$\bar{2}$	40
30	0.095$\bar{8}$	29	0.096$\underline{3}$	2$\bar{9}$	10.385$\underline{4}$	307$\bar{3}$	0.9954	3	30
40	0.098$\underline{7}$	29	0.099$\bar{2}$	2$\underline{9}$	10.078$\underline{0}$	2899	0.9951	3	20
50	0.101$\bar{6}$	29	0.102$\bar{1}$	2$\bar{9}$	9.788$\bar{1}$	2738	0.9948	3	10
6 0	0.104$\bar{5}$	2$\bar{8}$	0.1051	2$\bar{9}$	9.514$\bar{3}$	259$\bar{0}$	0.9945	3	0 84
10	0.1074	29	0.108$\bar{0}$	2$\bar{9}$	9.255$\underline{3}$	245$\bar{4}$	0.9942	$\underline{3}$	50
20	0.1103	29	0.111$\underline{0}$	2$\underline{9}$	9.009$\bar{8}$	232$\underline{9}$	0.993$\underline{9}$	$\bar{3}$	40
30	0.1132	29	0.113$\bar{9}$	2$\bar{9}$	8.776$\underline{9}$	221$\bar{3}$	0.993$\bar{5}$	$\underline{3}$	30
40	0.1161	29	0.116$\underline{9}$	2$\underline{9}$	8.555$\bar{5}$	2106	0.993$\bar{2}$	$\bar{3}$	20
50	0.1190	2$\bar{8}$	0.119$\bar{8}$	2$\bar{9}$	8.344$\bar{9}$	2006	0.9929	$\bar{3}$	10
7 0	0.121$\bar{8}$	29	0.1228	2$\bar{9}$	8.144$\bar{3}$	191$\bar{3}$	0.992$\bar{5}$	$\bar{3}$	0 83
10	0.124$\bar{7}$	29	0.125$\bar{7}$	2$\underline{9}$	7.953$\underline{0}$	182$\bar{6}$	0.9922	$\underline{4}$	50
20	0.127$\bar{6}$	2$\underline{9}$	0.128$\underline{7}$	2$\bar{9}$	7.770$\bar{3}$	174$\underline{6}$	0.991$\underline{8}$	$\bar{3}$	40
30	0.130$\bar{5}$	2$\bar{8}$	0.131$\bar{6}$	29	7.595$\bar{7}$	167$\bar{0}$	0.991$\bar{4}$	4	30
40	0.1334	2$\underline{9}$	0.1346	3$\underline{0}$	7.428$\underline{7}$	159$\bar{9}$	0.991$\bar{0}$	4	20
50	0.1363	2$\bar{8}$	0.1376	2$\bar{9}$	7.268$\bar{7}$	1534	0.990$\bar{6}$	4	10
8 0	0.139$\bar{1}$	29	0.140$\bar{5}$	2$\bar{9}$	7.115$\bar{3}$	1471	0.990$\bar{2}$	4	0 82
10	0.142$\bar{0}$	2$\underline{9}$	0.1435	3$\underline{0}$	6.968$\bar{2}$	1413	0.989$\underline{8}$	$\underline{4}$	50
20	0.144$\bar{9}$	2$\bar{8}$	0.146$\underline{5}$	2$\bar{9}$	6.826$\underline{9}$	1358	0.989$\bar{4}$	$\bar{4}$	40
30	0.1478	2$\underline{9}$	0.149$\bar{4}$	3$\underline{0}$	6.691$\bar{1}$	1306	0.9890	$\underline{4}$	30
40	0.150$\underline{7}$	2$\bar{8}$	0.152$\bar{4}$	2$\bar{9}$	6.560$\bar{5}$	1257	0.988$\underline{6}$	$\bar{4}$	20
50	0.153$\bar{5}$	29	0.1554	30	6.434$\bar{8}$	1211	0.988$\bar{1}$	4	10
9 0	0.156$\bar{4}$	2$\bar{8}$	0.1584	2$\bar{9}$	6.313$\bar{7}$	1167	0.9877	$\bar{4}$	0 81
10	0.1593	2$\underline{9}$	0.161$\bar{3}$	30	6.197$\bar{0}$	1126	0.987$\underline{2}$	$\underline{5}$	50
20	0.162$\underline{2}$	2$\bar{8}$	0.164$\bar{3}$	30	6.084$\bar{4}$	108$\underline{7}$	0.986$\bar{7}$	4	40
30	0.165$\bar{0}$	2$\bar{8}$	0.167$\bar{3}$	30	5.975$\bar{7}$	104$\underline{9}$	0.9863	5	30
40	0.1679	29	0.170$\bar{3}$	30	5.870$\underline{8}$	101$\bar{4}$	0.9858	5	20
50	0.1708	2$\bar{8}$	0.173$\bar{3}$	30	5.769$\bar{3}$	98$\bar{0}$	0.9853	5	10
10 0	0.173$\bar{6}$		0.176$\bar{3}$		5.6713		0.9848		0 80
	Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	′ °

P. P.

	30	2$\bar{9}$	29
1	3.0	2.$\bar{9}$	2.9
2	6.0	5.$\underline{9}$	5.8
3	9.0	8.$\bar{8}$	8.7
4	12.0	11.$\underline{8}$	11.6
5	15.0	14.$\bar{7}$	14.5
6	18.0	17.$\underline{7}$	17.4
7	21.0	20.$\bar{6}$	20.3
8	24.0	23.$\underline{6}$	23.2
9	27.0	26.$\bar{5}$	26.1

	2$\bar{8}$	5	$\bar{4}$	4
1	2.$\bar{8}$	0.5	0.$\bar{4}$	0.4
2	5.$\underline{7}$	1.0	0.$\underline{9}$	0.8
3	8.$\bar{5}$	1.5	1.$\bar{3}$	1.2
4	11.$\underline{4}$	2.0	1.$\underline{8}$	1.6
5	14.$\bar{2}$	2.5	2.$\bar{2}$	2.0
6	17.$\underline{1}$	3.0	2.$\underline{7}$	2.4
7	19.$\bar{9}$	3.5	3.$\bar{1}$	2.8
8	22.$\underline{8}$	4.0	3.$\underline{6}$	3.2
9	25.$\bar{6}$	4.5	4.$\bar{0}$	3.6

	$\bar{3}$	3	$\bar{2}$	2
1	0.$\bar{3}$	0.3	0.$\bar{2}$	0.2
2	0.$\underline{7}$	0.6	0.$\underline{5}$	0.4
3	1.$\bar{0}$	0.9	0.$\bar{7}$	0.6
4	1.$\underline{4}$	1.2	1.$\underline{0}$	0.8
5	1.$\bar{7}$	1.5	1.$\bar{2}$	1.0
6	2.$\underline{1}$	1.8	1.$\underline{5}$	1.2
7	2.$\bar{4}$	2.1	1.$\bar{7}$	1.4
8	2.$\underline{8}$	2.4	2.$\underline{0}$	1.6
9	3.$\bar{1}$	2.7	2.$\bar{2}$	1.8

	$\bar{1}$	1	$\bar{0}$
1	0.$\bar{1}$	0.1	0.$\bar{0}$
2	0.$\underline{3}$	0.2	0.$\underline{1}$
3	0.$\bar{4}$	0.3	0.$\bar{1}$
4	0.$\underline{6}$	0.4	0.$\underline{2}$
5	0.$\bar{7}$	0.5	0.$\bar{2}$
6	0.$\underline{9}$	0.6	0.$\underline{3}$
7	1.$\bar{0}$	0.7	0.$\bar{3}$
8	1.$\underline{2}$	0.8	0.$\underline{4}$
9	1.$\bar{3}$	0.9	0.$\bar{4}$

P. P.

80°–90°

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

10°–20°

°	′	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.		
10	0	0·1736	28	0·1763	30	5·6713	949	0·9848	5	0	80
	10	0·1765	28	0·1793	30	5·5764	919	0·9843	5	50	
	20	0·1793	29	0·1823	30	5·4845	890	0 9838	5	40	
	30	0·1822	28	0·1853	30	5·3955	862	0·9832	5	30	
	40	0·1851	28	0·1883	30	5·3093	836	0·9827	5	20	
	50	0·1879	28	0·1913	30	5·2256	811	0·9822	5	10	
11	0	0·1908	28	0·1944	30	5·1445	787	0·9816	6	0	79
	10	0·1936	28	0·1974	30	5·0658	764	0·9810	5	50	
	20	0·1965	28	0·2004	30	4·9894	742	0·9805	6	40	
	30	0·1993	28	0·2034	30	4·9151	721	0·9799	5	30	
	40	0·2022	28	0·2065	30	4·8430	701	0·9793	6	20	
	50	0·2050	28	0·2095	30	4·7728	682	0·9787	6	10	
12	0	0·2079	28	0·2125	30	4·7046	664	0·9781	6	0	78
	10	0·2107	28	0·2156	30	4·6382	646	0·9775	6	50	
	20	0·2136	28	0·2186	30	4·5736	629	0·9769	6	40	
	30	0·2164	28	0·2217	30	4·5107	613	0·9763	6	30	
	40	0·2193	28	0·2247	30	4·4494	597	0·9756	6	20	
	50	0·2221	28	0·2278	30	4·3897	582	0·9750	6	10	
13	0	0·2249	28	0·2308	31	4·3315	568	0·9743	6	0	77
	10	0·2278	28	0·2339	30	4·2747	553	0·9737	6	50	
	20	0·2306	28	0·2370	31	4·2193	540	0·9730	7	40	
	30	0·2334	28	0·2401	30	4·1653	527	0·9723	6	30	
	40	0·2362	28	0·2431	31	4·1125	515	0·9717	7	20	
	50	0·2391	28	0·2462	31	4·0610	502	0·9710	7	10	
14	0	0·2419	28	0·2493	30	4·0108	491	0·9703	7	0	76
	10	0·2447	28	0·2524	31	3·9616	480	0·9696	7	50	
	20	0·2475	28	0·2555	31	3·9136	469	0·9688	7	40	
	30	0·2504	28	0·2586	31	3·8667	458	0·9681	7	30	
	40	0·2532	28	0·2617	31	3·8208	449	0·9674	7	20	
	50	0·2560	28	0·2648	31	3·7759	439	0·9666	7	10	
15	0	0·2588	28	0·2679	31	3·7320	429	0·9659	8	0	75
	10	0·2616	28	0·2710	31	3·6891	420	0·9651	7	50	
	20	0·2644	28	0·2742	31	3·6470	411	0·9644	7	40	
	30	0·2672	28	0·2773	31	3·6059	403	0·9636	8	30	
	40	0·2700	28	0·2804	31	3·5655	394	0·9628	8	20	
	50	0·2728	28	0·2836	31	3·5261	387	0·9620	8	10	
16	0	0·2756	28	0·2867	31	3·4874	379	0·9612	8	0	74
	10	0·2784	28	0·2899	31	3·4495	371	0·9604	8	50	
	20	0·2812	27	0·2930	31	3·4123	364	0·9596	8	40	
	30	0·2840	28	0·2962	32	3·3759	357	0·9588	8	30	
	40	0·2868	28	0·2994	31	3·3402	350	0·9580	8	20	
	50	0·2896	27	0·3025	32	3·3052	343	0·9571	8	10	
17	0	0·2923	28	0·3057	31	3·2708	337	0·9563	8	0	73
	10	0·2951	28	0·3089	32	3·2371	331	0·9554	8	50	
	20	0·2979	27	0·3121	32	3·2040	324	0·9546	9	40	
	30	0·3007	28	0·3153	32	3·1716	319	0·9537	8	30	
	40	0·3035	27	0·3185	32	3·1397	313	0·9528	9	20	
	50	0·3062	27	0·3217	32	3·1084	307	0·9519	9	10	
18	0	0·3090	28	0·3249	32	3·0777	302	0·9510	9	0	72
	10	0·3118	27	0·3281	32	3·0475	296	0·9501	9	50	
	20	0·3145	27	0·3313	32	3·0178	291	0·9492	9	40	
	30	0·3173	27	0·3346	32	2·9887	286	0·9483	9	30	
	40	0·3200	27	0·3378	32	2·9600	281	0·9474	9	20	
	50	0·3228	27	0·3411	32	2·9319	277	0·9464	9	10	
19	0	0·3255	27	0·3443	32	2·9042	272	0·9455	9	0	71
	10	0·3283	27	0·3476	32	2·8770	267	0·9445	9	50	
	20	0·3310	27	0·3508	32	2·8502	263	0·9436	9	40	
	30	0·3338	27	0·3541	33	2·8239	259	0·9426	10	30	
	40	0·3365	27	0·3574	33	2·7980	254	0·9416	9	20	
	50	0·3393	27	0·3607	32	2·7725	250	0·9407	10	10	
20	0	0·3420		0·3639		2·7475		0·9397		0	70
		Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	′	°

P. P.

	33	32	31
1	3·3	3·2	3·1
2	6·6	6·4	6·2
3	9·9	9·6	9·3
4	13·2	12·8	12·4
5	16·5	16·0	15·5
6	19·8	19·2	18·6
7	23·1	22·4	21·7
8	26·4	25·6	24·8
9	29·7	28·8	27·9

	30	30	29
1	3·0	3·0	2·9
2	6·1	6·0	5·8
3	9·1	9·0	8·7
7	12·2	12·0	11·6
5	15·2	15·0	14·5
6	18·3	18·0	17·4
7	21·3	21·0	20·3
8	24·4	24·0	23·2
9	27·4	27·0	26·1

	28	28	27
1	2·8	2·8	2·7
2	5·7	5·6	5·4
3	8·5	8·4	8·1
4	11·4	11·2	10·8
5	14·2	14·0	13·5
6	17·1	16·8	16·2
7	19·9	19·6	18·9
8	22·8	22·4	21·6
9	25·6	25·2	24·3

	10	9	8
1	1·0	0·9	0·8
2	2·0	1·8	1·6
3	3·0	2·7	2·4
4	4·0	3·6	3·2
5	5·0	4·5	4·0
6	6·0	5·4	4·8
7	7·0	6·3	5·6
8	8·0	7·2	6·4
9	9·0	8·1	7·2

	7	7	6	5
1	0·7	0·7	0·6	0·5
2	1·5	1·4	1·2	1·0
3	2·2	2·1	1·8	1·5
4	3·0	2·8	2·4	2·0
5	3·7	3·5	3·0	2·5
6	4·5	4·2	3·6	3·0
7	5·2	4·9	4·2	3·5
8	6·0	5·6	4·8	4·0
9	6·7	6·3	5·4	4·5

P. P.

° ′	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.	
20 0	**0·3420**	27	**0·3639**	33	**2·7475**	247	**0·9397**	10	**0 70**
10	0·3447	27	0·3672	33	2·7228	242	0·9387	10	50
20	0·3475	27	0·3705	33	2·6985	239	0·9377	10	40
30	0·3502	27	0·3739	33	2·6746	235	0·9366	10	30
40	0·3529	27	0·3772	33	2·6511	232	0·9356	10	20
50	0·3556	27	0·3805	33	2·6279	228	0·9346	10	10
21 0	**0·3583**	27	**0·3838**	33	**2·6051**	225	**0·9336**	10	**0 69**
10	0·3611	27	0·3872	33	2·5826	221	0·9325	10	50
20	0·3638	27	0·3905	33	2·5604	218	0·9315	11	40
30	0·3665	27	0·3939	33	2·5386	215	0·9304	10	30
40	0·3692	27	0·3972	34	2·5171	212	0·9293	11	20
50	0·3719	27	0·4006	34	2·4959	208	0·9282	10	10
22 0	**0·3746**	27	**0·4040**	33	**2·4751**	206	**0·9272**	11	**0 68**
10	0·3773	27	0·4074	34	2·4545	203	0·9261	11	50
20	0·3800	27	0·4108	34	2·4342	200	0·9250	11	40
30	0·3827	26	0·4142	34	2·4142	197	0·9239	11	30
40	0·3853	27	0·4176	34	2·3945	194	0·9227	11	20
50	0·3880	27	0·4210	34	2·3750	192	0·9216	11	10
23 0	**0·3907**	26	**0·4244**	34	**2·3558**	189	**0·9205**	11	**0 67**
10	0·3934	27	0·4279	34	2·3369	187	0·9193	11	50
20	0·3961	26	0·4313	34	2·3182	184	0·9182	11	40
30	0·3987	26	0·4348	35	2·2998	182	0·9170	11	30
40	0·4014	27	0·4383	34	2·2816	179	0·9159	12	20
50	0·4041	26	0·4417	35	2·2637	177	0·9147	11	10
24 0	**0·4067**	26	**0·4452**	34	**2·2460**	175	**0·9135**	12	**0 66**
10	0·4094	26	0·4487	35	2·2285	172	0·9123	12	50
20	0·4120	26	0·4522	35	2·2113	170	0·9111	12	40
30	0·4147	26	0·4557	35	2·1943	168	0·9099	12	30
40	0·4173	26	0·4592	35	2·1775	166	0·9087	12	20
50	0·4200	26	0·4627	35	2·1609	164	0·9075	12	10
25 0	**0·4226**	26	**0·4663**	35	**2·1445**	162	**0·9063**	12	**0 65**
10	0·4252	26	0·4698	35	2·1283	159	0·9050	12	50
20	0·4279	26	0·4734	36	2·1123	158	0·9038	12	40
30	0·4305	26	0·4770	35	2·0965	156	0·9026	12	30
40	0·4331	26	0·4805	36	2·0809	154	0·9013	13	20
50	0·4357	26	0·4841	36	2·0655	152	0·9000	12	10
26 0	**0·4383**	26	**0·4877**	36	**2·0503**	150	**0·8988**	13	**0 64**
10	0·4410	26	0·4913	36	2·0352	148	0·8975	12	50
20	0·4436	26	0·4949	36	2·0204	147	0·8962	13	40
30	0·4462	26	0·4986	36	2·0057	145	0·8949	13	30
40	0·4488	26	0·5022	36	1·9911	143	0·8936	13	20
50	0·4514	26	0·5058	37	1·9768	142	0·8923	13	10
27 0	**0·4540**	26	**0·5095**	36	**1·9626**	140	**0·8910**	13	**0 63**
10	0·4566	25	0·5132	37	1·9486	139	0·8897	13	50
20	0·4591	26	0·5169	36	1·9347	137	0·8883	13	40
30	0·4617	26	0·5205	37	1·9210	136	0·8870	13	30
40	0·4643	25	0·5242	37	1·9074	134	0·8856	13	20
50	0·4669	25	0·5280	37	1·8940	132	0·8843	13	10
28 0	**0·4694**	26	**0·5317**	37	**1·8807**	131	**0·8829**	13	**0 62**
10	0·4720	25	0·5354	37	1·8676	130	0·8816	14	50
20	0·4746	25	0·5392	37	1·8546	128	0·8802	14	40
30	0·4771	25	0·5429	38	1·8417	127	0·8788	13	30
40	0·4797	25	0·5467	37	1·8290	125	0·8774	14	20
50	0·4822	25	0·5505	38	1·8165	124	0·8760	14	10
29 0	**0·4848**	25	**0·5543**	38	**1·8040**	123	**0·8746**	14	**0 61**
10	0·4873	25	0·5581	38	1·7917	122	0·8732	14	50
20	0·4899	25	0·5619	38	1·7795	120	0·8718	14	40
30	0·4924	25	0·5657	38	1·7675	119	0·8703	14	30
40	0·4949	25	0·5696	39	1·7555	118	0·8689	14	20
50	0·4975	25	0·5735	38	1·7437	117	0·8675	14	10
30 0	**0·5000**		**0·5773**		**1·7320**		**0·8660**		**0 60**
	Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	′ °

P. P.

	39	38	37	36
1	3·9	3·8	3·7	3·6
2	7·8	7·6	7·4	7·2
3	11·7	11·4	11·1	10·8
4	15·6	15·2	14·8	14·4
5	19·5	19·0	18·5	18·0
6	23·4	22·8	22·2	21·6
7	27·3	26·6	25·9	25·2
8	31·2	30·4	29·6	28·8
9	35·1	34·2	33·3	32·4

	35	35	34	33
1	3·5	3·5	3·4	3·3
2	7·1	7·0	6·8	6·6
3	10·6	10·5	10·2	9·9
4	14·2	14·0	13·6	13·2
5	17·7	17·5	17·0	16·5
6	21·3	21·0	20·4	19·8
7	24·8	24·5	23·8	23·1
8	28·4	28·0	27·2	26·4
9	31·9	31·5	30·6	29·7

	27	27	26	25
1	2·7	2·7	2·6	2·5
2	5·5	5·4	5·2	5·0
3	8·2	8·1	7·8	7·5
4	11·0	10·8	10·4	10·0
5	13·7	13·5	13·0	12·5
6	16·5	16·2	15·6	15·0
7	19·2	18·9	18·2	17·5
8	22·0	21·6	20·8	20·0
9	24·7	24·3	23·4	22·5

	14	14	13	12
1	1·4	1·4	1·3	1·2
2	2·9	2·8	2·6	2·4
3	4·3	4·2	3·9	3·6
4	5·8	5·6	5·2	4·8
5	7·2	7·0	6·5	6·0
6	8·7	8·4	7·8	7·2
7	10·1	9·8	9·1	8·4
8	11·6	11·2	10·4	9·6
9	13·0	12·6	11·7	10·8

	11	11	10
1	1·1	1·1	1·0
2	2·3	2·2	2·0
3	3·4	3·3	3·0
4	4·6	4·4	4·0
5	5·7	5·5	5·0
6	6·9	6·6	6·0
7	8·0	7·7	7·0
8	9·2	8·8	8·0
9	10·3	9·9	9·0

P. P.

30°–40°

° ′	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.	
30 0	**0·5000**	25	**0·5773**	39	**1·7320**	116	**0·8660**	15	**0 60**
10	0·5025	25	0·5812	39	1·7204	114	0·8645	14	50
20	0·5050	25	0·5851	39	1·7090	113	0·8631	14	40
30	0·5075	25	0·5890	39	1·6976	112	0·8616	15	30
40	0·5100	25	0·5929	39	1·6864	111	0·8601	15	20
50	0·5125	25	0·5969	39	1·6753	110	0·8586	15	10
31 0	**0·5150**	25	**0·6008**	40	**1·6643**	109	**0·8571**	15	**0 59**
10	0·5175	24	0·6048	39	1·6533	108	0·8556	15	50
20	0·5200	25	0·6088	40	1·6425	107	0·8541	15	40
30	0·5225	25	0·6128	40	1·6318	106	0·8526	15	30
40	0·5250	24	0·6168	40	1·6212	105	0·8511	15	20
50	0·5274	24	0·6208	40	1·6107	104	0·8496	15	10
32 0	**0·5299**	25	**0·6248**	40	**1·6003**	103	**0·8480**	15	**0 58**
10	0·5324	24	0·6289	41	1·5900	102	0·8465	15	50
20	0·5348	24	0·6330	40	1·5798	101	0·8449	15	40
30	0·5373	24	0·6370	41	1·5697	100	0·8434	16	30
40	0·5397	24	0·6411	41	1·5596	99	0·8418	15	20
50	0·5422	24	0·6453	41	1·5497	98	0·8402	16	10
33 0	**0·5446**	24	**0·6494**	41	**1·5398**	97	**0·8386**	15	**0 57**
10	0·5471	24	0·6535	41	1·5301	96	0·8371	16	50
20	0·5495	24	0·6577	42	1·5204	96	0·8355	16	40
30	0·5519	24	0·6619	42	1·5108	95	0·8339	16	30
40	0·5543	24	0·6661	42	1·5013	94	0·8323	16	20
50	0·5568	24	0·6703	42	1·4919	93	0·8306	16	10
34 0	**0·5592**	24	**0·6745**	42	**1·4825**	92	**0·8290**	16	**0 56**
10	0·5616	24	0·6787	42	1·4733	92	0·8274	16	50
20	0·5640	24	0·6830	43	1·4641	91	0·8257	16	40
30	0·5664	24	0·6873	42	1·4550	90	0·8241	16	30
40	0·5688	24	0·6915	43	1·4460	89	0·8225	17	20
50	0·5712	24	0·6959	43	1·4370	89	0·8208	16	10
35 0	**0·5736**	23	**0·7002**	43	**1·4281**	88	**0·8191**	16	**0 55**
10	0·5759	24	0·7045	43	1·4193	87	0·8175	17	50
20	0·5783	23	0·7089	44	1·4106	86	0·8158	17	40
30	0·5807	23	0·7133	44	1·4019	86	0·8141	17	30
40	0·5830	24	0·7177	44	1·3933	85	0·8124	17	20
50	0·5854	23	0·7221	44	1·3848	84	0·8107	17	10
36 0	**0·5878**	23	**0·7265**	44	**1·3764**	84	**0·8090**	17	**0 54**
10	0·5901	23	0·7310	44	1·3680	83	0·8073	17	50
20	0·5925	23	0·7354	45	1·3597	83	0·8056	17	40
30	0·5948	23	0·7399	45	1·3514	81	0·8038	17	30
40	0·5971	23	0·7444	45	1·3432	81	0·8021	17	20
50	0·5995	23	0·7490	45	1·3351	80	0·8004	17	10
37 0	**0·6018**	23	**0·7535**	45	**1·3270**	80	**0·7986**	17	**0 53**
10	0·6041	23	0·7581	46	1·3190	79	0·7969	18	50
20	0·6064	23	0·7627	46	1·3111	78	0·7951	17	40
30	0·6087	23	0·7673	46	1·3032	78	0·7933	17	30
40	0·6110	23	0·7719	46	1·2954	77	0·7916	18	20
50	0·6133	23	0·7766	47	1·2876	77	0·7898	18	10
38 0	**0·6156**	23	**0·7813**	47	**1·2799**	76	**0·7880**	18	**0 52**
10	0·6179	23	0·7860	47	1·2723	76	0·7862	18	50
20	0·6202	22	0·7907	47	1·2647	75	0·7844	18	40
30	0·6225	23	0·7954	47	1·2571	74	0·7826	18	30
40	0·6248	22	0·8002	48	1·2497	74	0·7808	18	20
50	0·6270	22	0·8050	48	1·2422	73	0·7789	18	10
39 0	**0·6293**	23	**0·8098**	48	**1·2349**	73	**0·7771**	18	**0 51**
10	0·6316	22	0·8146	48	1·2276	73	0·7753	18	50
20	0·6338	22	0·8194	49	1·2203	72	0·7734	18	40
30	0·6361	22	0·8243	49	1·2131	71	0·7716	18	30
40	0·6383	22	0·8292	49	1·2059	71	0·7697	18	20
50	0·6405	22	0·8341	49	1·1988	70	0·7679	18	10
40 0	**0·6428**		**0·8391**		**1·1917**		**0·7660**		**0 50**
	Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	′ °

P. P.

	49	49	48	47	46
1	4·9	4·9	4·8	4·7	4·6
2	9·9	9·8	9·6	9·4	9·2
3	14·8	14·7	14·4	14·1	13·8
4	19·8	19·6	19·2	18·8	18·4
5	24·7	24·5	24·0	23·5	23·0
6	29·7	29·4	28·8	28·2	27·6
7	34·6	34·3	33·6	32·9	32·2
8	39·6	39·2	38·4	37·6	36·8
9	44·5	44·1	43·2	42·3	41·4

	45	45	44	43	42
1	4·5	4·5	4·4	4·3	4·2
2	9·1	9·0	8·8	8·6	8·4
3	13·6	13·5	13·2	12·9	12·6
4	18·2	18·0	17·6	17·2	16·8
5	22·7	22·5	22·0	21·5	21·0
6	27·3	27·0	26·4	25·8	25·2
7	31·8	31·5	30·8	30·1	29·4
8	36·4	36·0	35·2	34·4	33·6
9	40·9	40·5	39·6	38·7	37·8

	41	41	40	39
1	4·1	4·1	4·0	3·9
2	8·3	8·2	8·0	7·8
3	12·4	12·3	12·0	11·7
4	16·6	16·4	16·0	15·6
5	20·7	20·5	20·0	19·5
6	24·9	24·6	24·0	23·4
7	29·0	28·7	28·0	27·3
8	33·2	32·8	32·0	31·2
9	37·3	36·9	36·0	35·1

	25	25	24	23
1	2·5	2·5	2·4	2·3
2	5·1	5·0	4·8	4·6
3	7·6	7·5	7·2	6·9
4	10·2	10·0	9·6	9·2
5	12·7	12·5	12·0	11·5
6	15·3	15·0	14·4	13·8
7	17·8	17·5	16·8	16·1
8	20·4	20·0	19·2	18·4
9	22·9	22·5	21·6	20·7

	22	22	18	18
1	2·2	2·2	1·8	1·8
2	4·5	4·4	3·7	3·6
3	6·7	6·6	5·5	5·4
4	9·0	8·8	7·4	7·2
5	11·2	11·0	9·2	9·0
6	13·5	13·2	11·1	10·8
7	15·7	15·4	12·9	12·6
8	18·0	17·6	14·8	14·4
9	20·2	19·8	16·6	16·2

	17	17	16	15	14
1	1·7	1·7	1·6	1·5	1·4
2	3·5	3·4	3·2	3·0	2·9
3	5·2	5·1	4·8	4·5	4·3
4	7·0	6·8	6·4	6·0	5·8
5	8·7	8·5	8·0	7·5	7·2
6	10·5	10·2	9·6	9·0	8·7
7	12·2	11·9	11·2	10·5	10·1
8	14·0	13·6	12·8	12·0	11·6
9	15·7	15·3	14·4	13·5	13·0

P. P.

50°–60°

TABLE IX.—NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS.

40°–45°

° ′	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.	
40 0	**0·6428**	22	**0·8391**	4$\bar{9}$	**1·191$\bar{7}$**	70	**0·766$\bar{0}$**	19	**0 50**
10	0·6450	2$\bar{2}$	0·844$\bar{0}$	50	1·184$\bar{7}$	70	0·764$\bar{1}$	1$\bar{8}$	50
20	0·647$\bar{2}$	22	0·849$\bar{0}$	5$\bar{0}$	1·177$\bar{7}$	69	0·7623	19	40
30	0·649$\bar{4}$	22	0·8541	50	1·170$\bar{8}$	6$\bar{8}$	0·7604	19	30
40	0·651$\bar{6}$	22	0·8591	51	1·1640	6$\bar{8}$	0·7585	19	20
50	0·653$\bar{8}$	22	0·8642	51	1·157$\bar{1}$	68	0·7566	19	10
41 0	**0·656$\bar{0}$**	22	**0·8693**	51	**1·150$\bar{3}$**	67	**0·7547**	19	**0 49**
10	0·658$\bar{2}$	22	0·8744	5$\bar{1}$	1·143$\bar{6}$	67	0·7528	19	50
20	0·660$\bar{4}$	2$\bar{1}$	0·879$\bar{5}$	52	1·136$\bar{9}$	6$\bar{6}$	0·7509	1$\bar{9}$	40
30	0·6626	22	0·884$\bar{7}$	5$\bar{1}$	1·1303	66	0·748$\bar{9}$	1$\bar{9}$	30
40	0·6648	2$\bar{1}$	0·8899	5$\bar{2}$	1·1237	6$\bar{5}$	0·747$\bar{0}$	1$\bar{9}$	20
50	0·666$\bar{9}$	22	0·895$\bar{1}$	5$\bar{2}$	1·117$\bar{1}$	6$\bar{5}$	0·7451	1$\bar{9}$	10
42 0	**0·669$\bar{1}$**	2$\bar{1}$	**0·9004**	53	**1·1106**	6$\bar{4}$	**0·743$\bar{1}$**	1$\bar{9}$	**0 48**
10	0·6713	2$\bar{1}$	0·9057	53	1·104$\bar{1}$	6$\bar{4}$	0·7412	1$\bar{9}$	50
20	0·673$\bar{4}$	2$\bar{1}$	0·9110	5$\bar{3}$	1·0977	64	0·739$\bar{2}$	1$\bar{9}$	40
30	0·6756	2$\bar{1}$	0·916$\bar{3}$	5$\bar{3}$	1·0913	6$\bar{3}$	0·7373	2$\bar{0}$	30
40	0·677$\bar{7}$	2$\bar{1}$	0·9217	54	1·084$\bar{9}$	63	0·7353	1$\bar{9}$	20
50	0·679$\bar{8}$	2$\bar{1}$	0·9271	54	1·078$\bar{6}$	63	0·733$\bar{3}$	20	10
43 0	**0·6820**	21	**0·9325**	5$\bar{4}$	**1·072$\bar{3}$**	62	**0·731$\bar{3}$**	20	**0 47**
10	0·6841	2$\bar{1}$	0·937$\bar{9}$	55	1·066$\bar{1}$	62	0·729$\bar{3}$	20	50
20	0·686$\bar{2}$	21	0·943$\bar{4}$	55	1·059$\bar{9}$	6$\bar{1}$	0·727$\bar{3}$	20	40
30	0·688$\bar{3}$	21	0·948$\bar{9}$	5$\bar{5}$	1·0538	6$\bar{1}$	0·725$\bar{3}$	20	30
40	0·690$\bar{4}$	21	0·9545	56	1·047$\bar{6}$	6$\bar{0}$	0·723$\bar{3}$	20	20
50	0·692$\bar{5}$	21	0·9601	56	1·0416	6$\bar{0}$	0·721$\bar{3}$	20	10
44 0	**0 694$\bar{6}$**	21	**0·9657**	5$\bar{6}$	**1·035$\bar{5}$**	6$\bar{0}$	**0·719$\bar{3}$**	2$\bar{0}$	**0 46**
10	0·696$\bar{7}$	21	0·971$\bar{3}$	5$\bar{6}$	1·0295	5$\bar{9}$	0·7173	20	50
20	0·698$\bar{8}$	2$\bar{0}$	0·9770	57	1·023$\bar{5}$	5$\bar{9}$	0·7153	2$\bar{0}$	40
30	0·7009	21	0·9827	5$\bar{7}$	1 0176	5$\bar{9}$	0·713$\bar{2}$	2$\bar{0}$	30
40	0·7030	2$\bar{0}$	0·988$\bar{4}$	5$\bar{7}$	1·0117	5$\bar{8}$	0·7112	2$\bar{0}$	20
50	0·705$\bar{0}$	2$\bar{0}$	0·9942	58	1·005$\bar{8}$	5$\bar{8}$	0·709$\bar{1}$	2$\bar{0}$	10
45 0	**0·7071**		**1·0000**		**1·0000**		**0·7071**		**0 45**
	Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	′ °

P. P.

	70	2$\bar{2}$	22	2$\bar{1}$
1	7·0	2·$\bar{2}$	2·2	2·$\bar{1}$
2	14·0	4·5	4·4	4·3
3	21·0	6·$\bar{7}$	6·6	6·$\bar{4}$
4	28·0	9·0	8·8	8·6
5	35·0	11·$\bar{2}$	11·0	10·$\bar{7}$
6	42·0	13·5	13·2	12·9
7	49·0	15·$\bar{7}$	15·4	15·$\bar{0}$
8	56·0	18·0	17·6	17·2
9	63·0	20·$\bar{2}$	19·8	19·$\bar{3}$

	69	21	2$\bar{0}$	20
1	6·9	2·1	2·$\bar{0}$	2·0
2	13·8	4·2	4·1	4·0
3	20·7	6·3	6·$\bar{1}$	6·0
4	27·6	8·4	8·2	8·0
5	34·5	10·5	10·$\bar{2}$	10·0
6	41·4	12·6	12·3	12·0
7	48·3	14·7	14·$\bar{3}$	14·0
8	55·2	16·8	16·4	16·0
9	62·1	18·9	18·$\bar{4}$	18·0

	6$\bar{8}$	1$\bar{9}$	19	1$\bar{8}$
1	6·$\bar{8}$	1·$\bar{9}$	1·9	1·$\bar{8}$
2	13·7	3·9	3·8	3·7
3	20·$\bar{5}$	5·$\bar{8}$	5·7	5·$\bar{5}$
4	27·4	7·8	7·6	7·4
5	34·$\bar{2}$	9·$\bar{7}$	9·5	9·$\bar{2}$
6	41·1	11·7	11·4	11·1
7	47·$\bar{9}$	13·$\bar{6}$	13·3	12·$\bar{9}$
8	54·8	15·6	15·2	14·8
9	61·$\bar{6}$	17·$\bar{5}$	17·1	16·$\bar{6}$

P. P.

	68	67	66	6$\bar{5}$	6$\bar{4}$	64	63	62	6$\bar{1}$	6$\bar{0}$	5$\bar{9}$	59	5$\bar{8}$	58	5$\bar{7}$
1	6·8	6·7	6·6	6·$\bar{5}$	6·$\bar{4}$	6·4	6·3	6·2	6·$\bar{1}$	6·$\bar{0}$	5·$\bar{9}$	5·9	5·$\bar{8}$	5·8	5·$\bar{7}$
2	13·6	13·4	13·2	13·1	12·9	12·8	12·6	12·4	12·3	12·1	11·9	11·8	11·7	11·6	11·5
3	20·4	20·1	19·8	19·$\bar{6}$	19·$\bar{3}$	19·2	18·9	18·6	18·$\bar{4}$	18·$\bar{1}$	17·$\bar{8}$	17·7	17·$\bar{5}$	17·4	17·$\bar{2}$
4	27·2	26·8	26·4	26·2	25·8	25·6	25·2	24·8	24·6	24·2	23·8	23·6	23·4	23·2	23·0
5	34·0	33·5	33·0	32·$\bar{7}$	32·$\bar{2}$	32·0	31·5	31·0	30·$\bar{7}$	30·$\bar{2}$	29·$\bar{7}$	29·5	29·$\bar{2}$	29·0	28·$\bar{7}$
6	40·8	40·2	39·6	39·3	38·7	38·4	37·8	37·2	36·9	36·3	35·7	35·4	35·1	34·8	34·5
7	47·6	46·9	46·2	45·$\bar{8}$	45·$\bar{1}$	44·8	44·1	43·4	43·$\bar{0}$	42·$\bar{3}$	41·$\bar{6}$	41·3	40·$\bar{9}$	40·6	40·2
8	54·4	53·6	52·8	52·4	51·6	51·2	50·4	49·6	49·2	48·4	47·6	47·2	46·8	46·4	46·0
9	61·2	60·3	59·4	58·$\bar{9}$	58·$\bar{0}$	57·6	56·7	55·8	55·$\bar{3}$	54·$\bar{4}$	53·$\bar{5}$	53·1	52·$\bar{6}$	52·2	51·$\bar{7}$

	57	5$\bar{6}$	56	55	5$\bar{4}$	54	5$\bar{3}$	53	5$\bar{2}$	52	5$\bar{1}$	51	5$\bar{0}$	50	4$\bar{9}$
1	5·7	5·$\bar{6}$	5·6	5·5	5·$\bar{4}$	5·4	5·$\bar{3}$	5·3	5·$\bar{2}$	5·2	5·$\bar{1}$	5·1	5·$\bar{0}$	5·0	4·$\bar{9}$
2	11·4	11·3	11·2	11·0	10·9	10·8	10·7	10·6	10·5	10·4	10·3	10·2	10·1	10·0	9·9
3	17·1	16·$\bar{9}$	16·8	16·5	16·$\bar{3}$	16·2	16·$\bar{0}$	15·9	15·$\bar{7}$	15·6	15·$\bar{4}$	15·3	15·$\bar{1}$	15·0	14·$\bar{8}$
4	22·8	22·6	22·4	22·0	21·8	21·6	21·4	21·2	21·0	20·8	20·6	20·4	20·2	20·0	19·8
5	28·5	28·$\bar{2}$	28·0	27·5	27·$\bar{2}$	27·0	26·$\bar{7}$	26·5	26·$\bar{2}$	26·0	25·$\bar{7}$	25·5	25·$\bar{2}$	25·0	24·$\bar{7}$
6	34·2	33·9	33·6	33·0	32·7	32·4	32·1	31·8	31·5	31·2	30·9	30·6	30·3	30·0	29·7
7	39·9	39·$\bar{5}$	39·2	38·5	38·$\bar{1}$	37·8	37·$\bar{4}$	37·1	36·$\bar{7}$	36·4	36·$\bar{0}$	35·7	35·$\bar{3}$	35·0	34·$\bar{6}$
8	45·6	45·2	44·8	44·0	43·6	43·2	42·8	42·4	42·0	41·6	41·2	40·8	40·4	40·0	39·6
9	51·3	50·$\bar{8}$	50·4	49·5	49·$\bar{0}$	48·6	48·$\bar{1}$	47·7	47·$\bar{2}$	46·8	46·$\bar{3}$	45·9	45·$\bar{4}$	45·0	44·$\bar{5}$

0°–10 | 10°–20°

°	′	Vers.	d.	Exsec.	d.	°	′	Vers.	d.	Exsec.	d.
0	0	.00000	0	.00000	0	10	0	.01519	51	.01542	52
	10	.00000	1	.00000	1		10	.01570	52	.01595	53
	20	.00001	2	.00001	2		20	.01622	52	.01648	54
	30	.00004	3	.00004	3		30	.01674	53	.01703	55
	40	.00007	3	.00007	3		40	.01728	54	.01758	56
	50	.00010	4	.00010	4		50	.01782	55	.01814	57
1	0	.00015	5	.00015	5	11	0	.01837	55	.01871	58
	10	.00020	6	.00020	6		10	.01893	57	.01929	59
	20	.00027	7	.00027	7		20	.01950	57	.01988	60
	30	.00034	8	.00034	8		30	.02007	58	.02048	61
	40	.00042	8	.00042	8		40	.02066	59	.02109	62
	50	.00051	10	.00051	10		50	.02125	60	.02171	62
2	0	.00061	10	.00061	10	12	0	.02185	61	.02234	63
	10	.00071	11	.00071	11		10	.02246	62	.02297	65
	20	.00083	12	.00083	12		20	.02308	62	.02362	65
	30	.00095	13	.00095	13		30	.02370	63	.02428	66
	40	.00108	13	.00108	14		40	.02434	64	.02494	67
	50	.00122	15	.00122	14		50	.02498	65	.02562	68
3	0	.00137	15	.00137	16	13	0	.02563	66	.02630	69
	10	.00152	16	.00153	16		10	.02629	66	.02700	70
	20	.00169	17	.00169	17		20	.02695	67	.02770	71
	30	.00186	18	.00187	18		30	.02763	68	.02841	72
	40	.00204	19	.00205	19		40	.02831	69	.02914	73
	50	.00223	20	.00224	20		50	.02900	70	.02987	74
4	0	.00243	21	.00244	21	14	0	.02970	70	.03061	75
	10	.00264	21	.00265	21		10	.03041	72	.03136	76
	20	.00286	22	.00286	22		20	.03113	72	.03213	77
	30	.00308	23	.00309	23		30	.03185	72	.03290	78
	40	.00331	24	.00332	24		40	.03258	74	.03368	79
	50	.00355	25	.00357	25		50	.03332	75	.03447	80
5	0	.00380	26	.00382	26	15	0	.03407	75	.03527	81
	10	.00406	26	.00408	27		10	.03483	76	.03609	82
	20	.00433	27	.00435	27		20	.03559	77	.03691	83
	30	.00460	28	.00462	28		30	.03637	78	.03774	84
	40	.00488	29	.00491	29		40	.03715	79	.03858	85
	50	.00518	30	.00520	30		50	.03794	80	.03943	86
6	0	.00548	30	.00551	31	16	0	.03874	80	.04030	87
	10	.00578	32	.00582	32		10	.03954	81	.04117	88
	20	.00610	32	.00614	33		20	.04036	82	.04205	89
	30	.00643	33	.00647	34		30	.04118	83	.04295	90
	40	.00676	33	.00681	34		40	.04201	84	.04385	91
	50	.00710	35	.00715	35		50	.04285	84	.04476	92
7	0	.00745	36	.00751	36	17	0	.04369	85	.04569	93
	10	.00781	36	.00787	37		10	.04455	86	.04662	95
	20	.00818	37	.00824	38		20	.04541	87	.04757	95
	30	.00855	38	.00863	39		30	.04628	87	.04853	96
	40	.00894	39	.00902	40		40	.04716	89	.04949	98
	50	.00933	40	.00942	41		50	.04805	89	.05047	98
8	0	.00973	41	.00983	41	18	0	.04894	90	.05146	100
	10	.01014	42	.01024	42		10	.04984	91	.05246	101
	20	.01056	42	.01067	43		20	.05075	92	.05347	102
	30	.01098	43	.01110	44		30	.05167	93	.05449	103
	40	.01142	44	.01155	45		40	.05260	93	.05552	104
	50	.01186	45	.01200	46		50	.05354	94	.05656	105
9	0	.01231	46	.01246	47	19	0	.05448	95	.05762	106
	10	.01277	47	.01293	48		10	.05543	96	.05868	107
	20	.01324	47	.01341	49		20	.05639	97	.05976	109
	30	.01371	48	.01390	50		30	.05736	97	.06085	109
	40	.01420	49	.01440	50		40	.05833	98	.06194	111
	50	.01469	50	.01491	51		50	.05931	99	.06305	112
10	0	.01519		.01542		20	0	.06030		.06418	
°	′	Vers.	d.	Exsec.	d.	°	′	Vers.	d.	Exsec.	d.

P. P.

	110	100	90	80	70
1	11	10	9	8	7
2	22	20	18	16	14
3	33	30	27	24	21
4	44	40	36	32	28
5	55	50	45	40	35
6	66	60	54	48	42
7	77	70	63	56	49
8	88	80	72	64	56
9	99	90	81	72	63

	60	50	40	30	20
1	6	5	4	3	2
2	12	10	8	6	4
3	18	15	12	9	6
4	24	20	16	12	8
5	30	25	20	15	10
6	36	30	24	18	12
7	42	35	28	21	14
8	48	40	32	24	16
9	54	45	36	27	18

	10	$\bar{9}$	9	$\bar{8}$	8
1	1	0.9	0.9	0.8	0.8
2	2	1.9	1.8	1.7	1.6
3	3	2.8	2.7	2.5	2.4
4	4	3.8	3.6	3.4	3.2
5	5	4.7	4.5	4.2	4.0
6	6	5.7	5.4	5.1	4.8
7	7	6.6	6.3	5.9	5.6
8	8	7.6	7.2	6.8	6.4
9	9	8.5	8.1	7.6	7.2

	$\bar{7}$	7	$\bar{6}$	6	$\bar{5}$
1	0.7	0.7	0.6	0.6	0.5
2	1.5	1.4	1.3	1.2	1.1
3	2.2	2.1	1.9	1.8	1.6
4	3.0	2.8	2.6	2.4	2.2
5	3.7	3.5	3.2	3.0	2.7
6	4.5	4.2	3.9	3.6	3.3
7	5.2	4.9	4.5	4.2	3.8
8	6.0	5.6	5.2	4.8	4.4
9	6.7	6.3	5.8	5.4	4.9

	5	$\bar{4}$	4	$\bar{3}$	3
1	0.5	0.4	0.4	0.3	0.3
2	1.0	0.9	0.8	0.7	0.6
3	1.5	1.3	1.2	1.0	0.9
4	2.0	1.8	1.6	1.4	1.2
5	2.5	2.2	2.0	1.7	1.5
6	3.0	2.7	2.4	2.1	1.8
7	3.5	3.1	2.8	2.4	2.1
8	4.0	3.6	3.2	2.8	2.4
9	4.5	4.0	3.6	3.1	2.7

	$\bar{2}$	2	$\bar{1}$	1	$\bar{0}$
1	0.2	0.2	0.1	0.1	0.0
2	0.5	0.4	0.3	0.2	0.1
3	0.7	0.6	0.4	0.3	0.1
4	1.0	0.8	0.6	0.4	0.2
5	1.2	1.0	0.7	0.5	0.2
6	1.5	1.2	0.9	0.6	0.3
7	1.7	1.4	1.0	0.7	0.3
8	2.0	1.6	1.2	0.8	0.4
9	2.2	1.8	1.3	0.9	0.4

P. P.

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

20°–30° | 30°–40°

° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.
20 0	**.0603**	10	**.0642**	11	**30 0**	**.1339**	15	**.1547**	19
10	.0613	10	.0653	11	10	.1354	14	.1566	19
20	.0623	10	.0664	11	20	.1369	14	.1586	20
30	.0633	10	.0676	12	30	.1383	15	.1606	20
40	.0643	10	.0688	11	40	.1398	15	.1626	20
50	.0654	10	.0699	12	50	.1413	15	.1646	20
21 0	**.0664**	10	**.0711**	12	**31 0**	**.1428**	15	**.1666**	20
10	.0674	10	.0723	12	10	.1443	15	.1687	20
20	.0685	11	.0735	12	20	.1458	15	.1707	21
30	.0696	10	.0748	12	30	.1473	15	.1728	21
40	.0706	11	.0760	12	40	.1489	15	.1749	21
50	.0717	10	.0772	13	50	.1504	15	.1770	21
22 0	**.0728**	11	**.0785**	12	**32 0**	**.1519**	15	**.1792**	21
10	.0739	11	.0798	13	10	.1535	15	.1813	21
20	.0750	11	.0811	13	20	.1550	15	.1835	22
30	.0761	11	.0824	13	30	.1566	16	.1857	22
40	.0772	11	.0837	13	40	.1582	15	.1879	22
50	.0783	11	.0850	13	50	.1597	16	.1901	22
23 0	**.0795**	11	**.0863**	13	**33 0**	**.1613**	15	**.1923**	23
10	.0806	11	.0877	13	10	.1629	16	.1946	22
20	.0818	11	.0890	14	20	.1645	16	.1969	23
30	.0829	11	.0904	14	30	.1661	16	.1992	23
40	.0841	12	.0918	14	40	.1677	16	.2015	23
50	.0853	11	.0932	14	50	.1693	16	.2038	23
24 0	**.0864**	12	**.0946**	14	**34 0**	**.1709**	16	**.2062**	24
10	.0876	12	.0960	14	10	.1726	16	.2086	24
20	.0888	12	.0975	14	20	.1742	16	.2110	24
30	.0900	12	.0989	14	30	.1758	16	.2134	24
40	.0912	12	.1004	15	40	.1775	17	.2158	24
50	.0924	12	.1019	15	50	.1792	16	.2183	24
25 0	**.0937**	12	**.1034**	15	**35 0**	**.1808**	16	**.2207**	25
10	.0949	12	.1049	15	10	.1825	17	.2232	25
20	.0961	12	.1064	15	20	.1842	17	.2258	25
30	.0974	12	.1079	15	30	.1859	17	.2283	25
40	.0986	13	.1094	16	40	.1876	17	.2309	25
50	.0999	12	.1110	15	50	.1893	17	.2334	26
26 0	**.1012**	13	**.1126**	16	**36 0**	**.1910**	17	**.2360**	26
10	.1025	12	.1142	16	10	.1927	17	.2387	26
20	.1037	13	.1158	16	20	.1944	17	.2413	26
30	.1050	13	.1174	16	30	.1961	17	.2440	27
40	.1063	13	.1190	16	40	.1979	17	.2467	27
50	.1077	13	.1206	17	50	.1996	17	.2494	27
27 0	**.1090**	13	**.1223**	16	**37 0**	**.2013**	17	**.2521**	27
10	.1103	13	.1240	17	10	.2031	18	.2549	27
20	.1116	13	.1257	17	20	.2049	17	.2576	28
30	.1130	13	.1274	17	30	.2066	17	.2604	28
40	.1143	13	.1291	17	40	.2084	18	.2633	28
50	.1157	13	.1308	17	50	.2102	18	.2661	28
28 0	**.1170**	13	**.1325**	18	**38 0**	**.2120**	18	**.2690**	29
10	.1184	14	.1343	17	10	.2138	18	.2719	29
20	.1198	14	.1361	18	20	.2156	18	.2748	29
30	.1212	13	.1379	18	30	.2174	18	.2778	29
40	.1225	14	.1397	18	40	.2192	18	.2807	30
50	.1239	14	.1415	18	50	.2210	18	.2837	30
29 0	**.1254**	14	**.1433**	18	**39 0**	**.2228**	18	**.2867**	30
10	.1268	14	.1452	18	10	.2247	18	.2898	30
20	.1282	14	.1470	19	20	.2265	18	.2928	31
30	.1296	14	.1489	19	30	.2284	18	.2959	31
40	.1311	14	.1508	19	40	.2302	18	.2991	31
50	.1325	14	.1527	19	50	.2321	18	.3022	31
30 0	**.1339**		**.1547**		**40 0**	**.2339**		**.3054**	
° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.

P. P.

	31	30	29	28
1	3.1	3.0	2.9	2.8
2	6.2	6.0	5.8	5.6
3	9.3	9.0	8.7	8.4
4	12.4	12.0	11.6	11.2
5	15.5	15.0	14.5	14.0
6	18.6	18.0	17.4	16.8
7	21.7	21.0	20.3	19.6
8	24.8	24.0	23.2	22.4
9	27.9	27.0	26.1	25.2

	27	26	25	24
1	2.7	2.6	2.5	2.4
2	5.4	5.2	5.0	4.8
3	8.1	7.8	7.5	7.2
4	10.8	10.4	10.0	9.6
5	13.5	13.0	12.5	12.0
6	16.2	15.6	15.0	14.4
7	18.9	18.2	17.5	16.8
8	21.6	20.8	20.0	19.2
9	24.3	23.4	22.5	21.6

	23	22	21	20
1	2.3	2.2	2.1	2.0
2	4.6	4.4	4.2	4.0
3	6.9	6.6	6.3	6.0
4	9.2	8.8	8.4	8.0
5	11.5	11.0	10.5	10.0
6	13.8	13.2	12.6	12.0
7	16.1	15.4	14.7	14.0
8	18.4	17.6	16.8	16.0
9	20.7	19.8	18.9	18.0

	19	18	17	16
1	1.9	1.8	1.7	1.6
2	3.8	3.6	3.4	3.2
3	5.7	5.4	5.1	4.8
4	7.6	7.2	6.8	6.4
5	9.5	9.0	8.5	8.0
6	11.4	10.8	10.2	9.6
7	13.3	12.6	11.9	11.2
8	15.2	14.4	13.6	12.8
9	17.1	16.2	15.3	14.4

	15	14	13	12
1	1.5	1.4	1.3	1.2
2	3.0	2.8	2.6	2.4
3	4.5	4.2	3.9	3.6
4	6.0	5.6	5.2	4.8
5	7.5	7.0	6.5	6.0
6	9.0	8.4	7.8	7.2
7	10.5	9.8	9.1	8.4
8	12.0	11.2	10.4	9.6
9	13.5	12.6	11.7	10.8

	11	10	0
1	1.1	1.0	0.0
2	2.2	2.0	0.1
3	3.3	3.0	0.1
4	4.4	4.0	0.2
5	5.5	5.0	0.2
6	6.6	6.0	0.3
7	7.7	7.0	0.3
8	8.8	8.0	0.4
9	9.9	9.0	0.4

P. P.

40°–50° | 50°–60°

° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.
40 0	**.2339**	19	**.3054**	32	**50 0**	**.3572**	22	**.5557**	53
10	.2358	18	.3086	32	10	.3594	22	.5611	54
20	.2377	19	.3118	32	20	.3617	22	.5666	54
30	.2396	19	.3151	32	30	.3639	22	.5721	55
40	.2415	19	.3183	33	40	.3661	22	.5777	56
50	.2434	19	.3217	33	50	.3684	23	.5833	56
41 0	**.2453**	19	**.3250**	34	**51 0**	**.3707**	22	**.5890**	57
10	.2472	19	.3284	33	10	.3729	22	.5947	58
20	.2491	19	.3317	34	20	.3752	23	.6005	58
30	.2510	19	.3352	34	30	.3775	22	.6064	59
40	.2529	19	.3386	34	40	.3797	23	.6123	59
50	.2549	19	.3421	35	50	.3820	23	.6182	60
42 0	**.2568**	19	**.3456**	35	**52 0**	**.3843**	23	**.6242**	61
10	.2588	19	.3491	36	10	.3866	23	.6303	61
20	.2607	19	.3527	36	20	.3889	23	.6365	62
30	.2627	20	.3563	36	30	.3912	23	.6427	62
40	.2647	19	.3599	37	40	.3935	23	.6489	63
50	.2666	20	.3636	37	50	.3958	23	.6552	64
43 0	**.2686**	20	**.3673**	37	**53 0**	**.3982**	23	**.6616**	64
10	.2706	20	.3710	37	10	.4005	23	.6681	65
20	.2726	20	.3748	38	20	.4028	23	.6746	65
30	.2746	20	.3786	38	30	.4052	23	.6811	66
40	.2766	20	.3824	39	40	.4075	23	.6878	67
50	.2786	20	.3863	38	50	.4098	23	.6945	68
44 0	**.2806**	20	**.3901**	39	**54 0**	**.4122**	23	**.7013**	68
10	.2827	20	.3941	39	10	.4145	24	.7081	69
20	.2847	20	.3980	40	20	.4169	23	.7150	70
30	.2867	20	.4020	40	30	.4193	23	.7220	70
40	.2888	20	.4060	40	40	.4216	24	.7291	71
50	.2908	20	.4101	41	50	.4240	23	.7362	72
45 0	**.2929**	20	**.4142**	41	**55 0**	**.4264**	24	**.7434**	73
10	.2949	20	.4183	41	10	.4288	24	.7507	73
20	.2970	21	.4225	42	20	.4312	24	.7581	74
30	.2991	20	.4267	42	30	.4336	24	.7655	75
40	.3011	21	.4309	43	40	.4360	24	.7730	75
50	.3032	21	.4352	43	50	.4384	24	.7806	77
46 0	**.3053**	21	**.4395**	43	**56 0**	**.4408**	24	**.7883**	77
10	.3074	21	.4439	44	10	.4432	24	.7960	78
20	.3095	21	.4483	44	20	.4456	24	.8039	79
30	.3116	21	.4527	44	30	.4480	24	.8118	80
40	.3137	21	.4572	45	40	.4505	24	.8198	81
50	.3159	21	.4617	45	50	.4529	24	.8279	82
47 0	**.3180**	21	**.4663**	45	**57 0**	**.4553**	24	**.8361**	82
10	.3201	21	.4708	46	10	.4578	24	.8443	83
20	.3222	21	.4755	47	20	.4602	24	.8527	84
30	.3244	21	.4802	47	30	.4627	24	.8611	85
40	.3265	21	.4849	47	40	.4651	24	.8697	86
50	.3287	21	.4896	48	50	.4676	25	.8783	87
48 0	**.3308**	22	**.4945**	48	**58 0**	**.4701**	24	**.8871**	88
10	.3330	21	.4993	48	10	.4725	24	.8959	89
20	.3352	22	.5042	49	20	.4750	25	.9048	90
30	.3374	21	.5091	50	30	.4775	25	.9139	91
40	.3395	22	.5141	50	40	.4800	24	.9230	92
50	.3417	22	.5192	50	50	.4824	25	.9322	93
49 0	**.3439**	22	**.5242**	51	**59 0**	**.4849**	25	**.9416**	94
10	.3461	22	.5294	51	10	.4874	25	.9510	95
20	.3483	22	.5345	52	20	.4899	25	.9606	97
30	.3505	22	.5397	53	30	.4924	25	.9703	98
40	.3527	22	.5450	53	40	.4949	25	.9801	99
50	.3550	22	.5503	53	50	.4975	25	.9900	100
50 0	**.3572**		**.5557**		**60 0**	**.5000**		**1.0000**	
° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.

P. P.

	9	8	7	6	5	4
1	0.9	0.8	0.7	0.6	0.5	0.4
2	1.8	1.6	1.4	1.2	1.0	0.8
3	2.7	2.4	2.1	1.8	1.5	1.2
4	3.6	3.2	2.8	2.4	2.0	1.6
5	4.5	4.0	3.5	3.0	2.5	2.0
6	5.4	4.8	4.2	3.6	3.0	2.4
7	6.3	5.6	4.9	4.2	3.5	2.8
8	7.2	6.4	5.6	4.8	4.0	3.2
9	8.1	7.2	6.3	5.4	4.5	3.6

	3	2	1	$\bar{9}$	$\bar{8}$	$\bar{7}$
1	0.3	0.2	0.1	0.9	0.8	0.7
2	0.6	0.4	0.2	1.9	1.7	1.5
3	0.9	0.6	0.3	2.8	2.5	2.2
4	1.2	0.8	0.4	3.8	3.4	3.0
5	1.5	1.0	0.5	4.7	4.2	3.7
6	1.8	1.2	0.6	5.7	5.1	4.5
7	2.1	1.4	0.7	6.6	5.9	5.2
8	2.4	1.6	0.8	7.6	6.8	6.0
9	2.7	1.8	0.9	8.5	7.6	6.7

	$\bar{6}$	$\bar{5}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{1}$
1	0.6	0.5	0.4	0.3	0.2	0.1
2	1.3	1.1	0.9	0.7	0.5	0.3
3	1.9	1.6	1.3	1.0	0.7	0.4
4	2.6	2.2	1.8	1.4	1.0	0.6
5	3.2	2.7	2.2	1.7	1.2	0.7
6	3.9	3.3	2.7	2.1	1.5	0.9
7	4.5	3.8	3.1	2.4	1.7	1.0
8	5.2	4.4	3.6	2.8	2.0	1.2
9	5.8	4.9	4.0	3.1	2.2	1.3

	$\overline{25}$	25	$\overline{24}$	24	$\overline{23}$
1	2.5	2.5	2.4	2.4	2.3
2	5.1	5.0	4.9	4.8	4.7
3	7.6	7.5	7.3	7.2	7.0
4	10.2	10.0	9.8	9.6	9.4
5	12.7	12.5	12.2	12.0	11.7
6	15.3	15.0	14.7	14.4	14.1
7	17.8	17.5	17.1	16.8	16.4
8	20.4	20.0	19.6	19.2	18.8
9	22.9	22.5	22.0	21.6	21.1

	23	$\overline{22}$	22	$\overline{21}$	21
1	2.3	2.2	2.2	2.1	2.1
2	4.6	4.5	4.4	4.3	4.2
3	6.9	6.7	6.6	6.4	6.3
4	9.2	9.0	8.8	8.6	8.4
5	11.5	11.2	11.0	10.7	10.5
6	13.8	13.5	13.2	12.9	12.6
7	16.1	15.7	15.4	15.0	14.7
8	18.4	18.0	17.6	17.2	16.8
9	20.7	20.2	19.8	19.3	18.9

	$\overline{20}$	20	$\overline{19}$	19	$\overline{18}$
1	2.0	2.0	1.9	1.9	1.8
2	4.1	4.0	3.9	3.8	3.7
3	6.1	6.0	5.8	5.7	5.5
4	8.2	8.0	7.8	7.6	7.4
5	10.2	10.0	9.7	9.5	9.2
6	12.3	12.0	11.7	11.4	11.1
7	14.3	14.0	13.6	13.3	12.9
8	16.4	16.0	15.6	15.2	14.8
9	18.4	18.0	17.5	17.1	16.6

P. P.

60°–70° | 70°–80°

° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.
60 0	**.5000**	25	**1.0000**	101	**70 0**	**.6580**	27	**1.9238**	235
10	.5025	25	1.0101	102	10	.6607	27	1.9473	240
20	.5050	25	1.0204	103	20	.6634	27	1.9713	244
30	.5076	25	1.0307	105	30	.6662	27	1.9957	248
40	.5101	25	1.0413	106	40	.6689	27	2.0205	253
50	.5126	25	1.0519	107	50	.6717	27	2.0458	257
61 0	**.5152**	25	**1.0626**	109	**71 0**	**.6744**	27	**2.0715**	262
10	.5177	25	1.0735	110	10	.6772	27	2.0977	266
20	.5203	25	1.0846	111	20	.6799	27	2.1244	271
30	.5228	25	1.0957	113	30	.6827	27	2.1515	276
40	.5254	25	1.1070	114	40	.6854	27	2.1792	281
50	.5279	26	1.1184	116	50	.6882	28	2.2073	287
62 0	**.5305**	25	**1.1300**	117	**72 0**	**.6910**	27	**2.2360**	292
10	.5331	25	1.1418	118	10	.6937	27	2.2653	298
20	.5356	26	1.1536	120	20	.6965	28	2.2951	304
30	.5382	26	1.1657	121	30	.6993	27	2.3255	310
40	.5408	25	1.1778	123	40	.7020	28	2.3565	316
50	.5434	26	1.1902	125	50	.7048	28	2.3881	322
63 0	**.5460**	26	**1.2027**	126	**73 0**	**.7076**	27	**2.4203**	328
10	.5486	26	1.2153	128	10	.7104	28	2.4531	335
20	.5512	26	1.2281	130	20	.7132	28	2.4867	342
30	.5538	26	1.2411	131	30	.7160	27	2.5209	349
40	.5564	26	1.2543	133	40	.7187	28	2.5558	356
50	.5590	26	1.2676	135	50	.7215	28	2.5915	364
64 0	**.5616**	26	**1.2811**	137	**74 0**	**.7243**	28	**2.6279**	372
10	.5642	26	1.2948	139	10	.7271	28	2.6651	380
20	.5668	26	1.3087	140	20	.7299	28	2.7031	388
30	.5695	26	1.3228	143	30	.7327	28	2.7420	396
40	.5721	26	1.3371	144	40	.7355	28	2.7816	406
50	.5747	26	1.3515	146	50	.7383	28	2.8222	414
65 0	**.5774**	26	**1.3662**	148	**75 0**	**.7412**	28	**2.8637**	424
10	.5800	26	1.3810	151	10	.7440	28	2.9061	434
20	.5826	26	1.3961	152	20	.7468	28	2.9495	444
30	.5853	26	1.4114	155	30	.7496	28	2.9939	454
40	.5879	26	1.4269	157	40	.7524	28	3.0394	465
50	.5906	26	1.4426	159	50	.7552	28	3.0859	476
66 0	**.5932**	26	**1.4586**	161	**76 0**	**.7581**	28	**3.1335**	488
10	.5959	27	1.4747	164	10	.7609	28	3.1824	500
20	.5986	26	1.4912	166	20	.7637	28	3.2324	512
30	.6012	26	1.5078	169	30	.7665	28	3.2836	525
40	.6039	27	1.5247	171	40	.7694	28	3.3362	539
50	.6066	26	1.5419	174	50	.7722	28	3.3901	553
67 0	**.6092**	27	**1.5593**	177	**77 0**	**.7750**	28	**3.4454**	567
10	.6119	27	1.5770	179	10	.7779	28	3.5021	582
20	.6146	26	1.5949	182	20	.7807	28	3.5604	598
30	.6173	27	1.6131	184	30	.7835	28	3.6202	614
40	.6200	27	1.6316	188	40	.7864	28	3.6816	631
50	.6227	27	1.6504	190	50	.7892	28	3.7448	649
68 0	**.6254**	27	**1.6694**	194	**78 0**	**.7921**	28	**3.8097**	667
10	.6281	27	1.6888	196	10	.7949	28	3.8765	686
20	.6308	27	1.7085	200	20	.7978	28	3.9451	707
30	.6335	27	1.7285	203	30	.8006	28	4.0158	728
40	.6362	27	1.7488	206	40	.8035	28	4.0886	749
50	.6389	27	1.7694	210	50	.8063	28	4.1636	772
69 0	**.6416**	27	**1.7904**	213	**79 0**	**.8092**	28	**4.2408**	796
10	.6443	27	1.8117	216	10	.8120	28	4.3205	821
20	.6470	27	1.8334	220	20	.8149	28	4.4026	847
30	.6498	27	1.8554	224	30	.8177	29	4.4874	875
40	.6525	27	1.8778	228	40	.8206	28	4.5749	904
50	.6552	27	1.9006	231	50	.8235	28	4.6653	934
70 0	**.6580**		**1.9238**		**80 0**	**.8263**		**4.7587**	
° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.

P. P.

	9	8	7	6	5
1	0.9	0.8	0.7	0.6	0.5
2	1.8	1.6	1.4	1.2	1.0
3	2.7	2.4	2.1	1.8	1.5
4	3.6	3.2	2.8	2.4	2.0
5	4.5	4.0	3.5	3.0	2.5
6	5.4	4.8	4.2	3.6	3.0
7	6.3	5.6	4.9	4.2	3.5
8	7.2	6.4	5.6	4.8	4 0
9	8.1	7.2	6.3	5.4	4.5

	4	3	2	1	$\bar{9}$
1	0.4	0.3	0.2	0.1	0.9
2	0.8	0.6	0.4	0.2	1.9
3	1.2	0.9	0.6	0.3	2.8
4	1.6	1.2	0.8	0.4	3.8
5	2.0	1.5	1.0	0.5	4.7
6	2.4	1.8	1.2	0.6	5.7
7	2.8	2.1	1.4	0.7	6.6
8	3.2	2.4	1.6	0.8	7.6
9	3.6	2.7	1.8	0.9	8.5

	$\bar{8}$	$\bar{7}$	$\bar{6}$	$\bar{5}$	$\bar{4}$
1	0.8	0.7	0.6	0.5	0.4
2	1.7	1.5	1.3	1.1	0.9
3	2.5	2.2	1.9	1.6	1.3
4	3.4	3.0	2.6	2.2	1.8
5	4.2	3.7	3.2	2.7	2.2
6	5.1	4.5	3.9	3.3	2.7
7	5.9	5.2	4.5	3.8	3.1
8	6.8	6.0	5.2	4.4	3.6
9	7.6	6.7	5.8	4.9	4.0

	$\bar{3}$	$\bar{2}$	$\bar{1}$
1	0.3	0.2	0.1
2	0.7	0.5	0.3
3	1.0	0.7	0.4
4	1.4	1.0	0.6
5	1.7	1.2	0.7
6	2.1	1.5	0.9
7	2.4	1.7	1.0
8	2.8	2.0	1.2
9	3.1	2.2	1.3

	29	$2\bar{8}$	28	$2\bar{7}$
1	2.9	2.8	2.8	2.7
2	5.8	5.7	5.6	5.5
3	8.7	8.5	8.4	8.2
4	11.6	11.4	11.2	11.0
5	14.5	14.2	14.0	13.7
6	17.4	17.1	16.8	16.5
7	20.3	19.9	19.6	19.2
8	23.2	22.8	22.4	22.0
9	26.1	25.6	25.2	24.7

	27	$2\bar{6}$	26	$2\bar{5}$
1	2.7	2.6	2.6	2.5
2	5.4	5.3	5.2	5.1
3	8.1	7.9	7.8	7.6
4	10.8	10.6	10.4	10.2
5	13.5	13.2	13.0	12.7
6	16.2	15.9	15.6	15.3
7	18.9	18.5	18.2	17.8
8	21.6	21.2	20.8	20.4
9	24.3	23.8	23.4	22.9

P. P.

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

80°–85° | 85°–90°

° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.
80 0	**.8263**	28	**4.7587**	966	**85 0**	**.9128**	29	**10.4737**	.3946
10	.8292	29	4.8554	999	10	.9157	29	10.8683	.4229
20	.8321	28	4.9553	1035	20	.9186	29	11.2912	.4542
30	.8349	28	5.0588	1072	30	.9215	29	11.7455	.4892
40	.8378	29	5.1660	1111	40	.9244	29	12.2347	.5284
50	.8407	28	5.2772	1152	50	.9273	29	12.7631	.5725
81 0	**.8435**	29	**5.3924**	1196	**86 0**	**.9302**	29	**13.3356**	.6223
10	.8464	28	5.5121	1242	10	.9331	29	13.9579	.6789
20	.8493	29	5.6363	1291	20	.9360	29	14.6368	.7436
30	.8522	28	5.7654	1343	30	.9389	29	15.3804	.8180
40	.8550	29	5.8998	1398	40	.9418	29	16.1984	.9041
50	.8579	29	6.0396	1456	50	.9447	29	17.1026	1.0047
82 0	**.8608**	28	**6.1853**	1519	**87 0**	**.9476**	29	**18.1073**	1.1230
10	.8637	29	6.3372	1585	10	.9505	29	19.2303	1.2634
20	.8666	28	6.4957	1656	20	.9534	29	20.4937	1.4319
30	.8694	29	6.6613	1731	30	.9564	29	21.9256	1.6365
40	.8723	29	6.8344	1812	40	.9593	29	23.5621	1.8884
50	.8752	29	7.0156	1898	50	.9622	29	25.4505	2.2032
83 0	**.8781**	28	**7.2055**	1991	**88 0**	**.9651**	29	**27.6537**	2.6039
10	.8810	29	7.4046	2091	10	.9680	29	30.2576	3.1247
20	.8839	29	7.6138	2198	20	.9709	29	33.3823	3.8192
30	.8868	29	7.8336	2315	30	.9738	29	37.2015	4.7741
40	.8897	29	8.0651	2440	40	.9767	29	41.9757	6.1383
50	.8926	28	8.3091	2576	50	.9796	29	48.1140	8.1846
84 0	**.8954**	29	**8.5667**	2723	**89 0**	**.9825**	29	**56.2987**	
10	.8983	29	8.8391	2884	10	.9854	29	67.7573	
20	.9012	29	9.1275	3059	20	.9883	29	84.9456	
30	.9041	29	9.4334	3250	30	.9912	29	113.5930	
40	.9070	29	9.7585	3460	40	.9942	29	170.8883	
50	.9099	29	10.1045	3691	50	.9971	29	342.7752	
85 0	**.9128**		**10.4737**		**90 0**	**1.0000**		∞	
° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.

P. P.

	29	29
1	2.9	2.9
2	5.9	5.8
3	8.8	8.7
4	11.8	11.6
5	14.7	14.5
6	17.7	17.4
7	20.6	20.3
8	23.6	23.2
9	26.5	26.1

	28
1	2.8
2	5.7
3	8.5
4	11.4
5	14.2
6	17.1
7	19.9
8	22.8
9	25.6

TABLE XI.—REDUCTION OF BAROMETER READING TO 32° F.

Temp. ° Fahr.	Inches.										
	26·0	26·5	27·0	27·5	28·0	28·5	29·0	29·5	30·0	30·5	31·0
45	−·039	−·039	−·040	−·041	−·042	−·042	−·043	−·044	−·045	−·045	−·046
46	·041	·042	·043	·043	·044	·045	·046	·046	·047	·048	·049
47	·043	·044	·045	·046	·047	·048	·048	·049	·050	·051	·052
48	·046	·047	·047	·048	·049	·050	·051	·052	·053	·053	·054
49	·048	·049	·050	·051	·052	·052	·054	·054	·055	·056	·057
50	·050	·051	·052	·053	·054	·055	·056	·057	·058	·059	·060
51	·053	·054	·055	·056	·057	·058	·059	·060	·061	·062	·063
52	·055	·056	·057	·058	·059	·060	·061	·062	·064	·065	·066
53	·057	·058	·060	·061	·062	·063	·064	·065	·066	·067	·068
54	·060	·061	·062	·063	·064	·065	·067	·068	·069	·070	·071
55	·062	·063	·064	·065	·066	·068	·069	·070	·071	·073	·074
56	·064	·065	·067	·068	·069	·070	·072	·073	·074	·075	·077
57	·067	·068	·069	·070	·072	·073	·075	·076	·077	·078	·080
58	·069	·070	·071	·073	·074	·076	·077	·078	·080	·081	·082
59	·072	·073	·074	·075	·077	·078	·080	·081	·083	·084	·085
60	·074	·076	·077	·078	·079	·081	·082	·084	·085	·086	·088
61	·076	·077	·079	·080	·082	·083	·085	·086	·088	·089	·091
62	·079	·080	·082	·083	·085	·086	·088	·089	·091	·092	·094
63	·081	·082	·084	·085	·087	·088	·090	·091	·093	·095	·096
64	·083	·085	·086	·088	·090	·091	·093	·094	·096	·097	·099
65	·086	·087	·089	·090	·092	·093	·095	·097	·099	·100	·102
66	·088	·089	·091	·093	·095	·096	·098	·099	·101	·103	·105
67	·090	·092	·094	·095	·097	·099	·101	·102	·104	·106	·108
68	·093	·094	·096	·098	·100	·101	·103	·105	·107	·108	·110
69	·095	·097	·099	·100	·102	·104	·106	·107	·110	·111	·113
70	·097	·099	·101	·103	·105	·106	·109	·110	·112	·114	·116
71	·100	·101	·103	·105	·107	·109	·111	·113	·115	·117	·119
72	·102	·104	·106	·108	·110	·112	·114	·116	·118	·120	·122
73	·104	·106	·108	·110	·112	·114	·116	·118	·120	·122	·124
74	·107	·109	·111	·113	·115	·117	·119	·121	·123	·125	·127
75	·109	·111	·113	·115	·117	·119	·122	·124	·126	·128	·130
76	·111	·113	·116	·118	·120	·122	·124	·126	·128	·130	·133
77	·114	·116	·118	·120	·122	·124	·127	·129	·131	·133	·136
78	·116	·118	·120	·122	·125	·127	·129	·131	·134	·136	·138
79	·118	·120	·123	·125	·127	·129	·132	·134	·137	·139	·141
80	·121	·123	·125	·127	·130	·132	·135	·137	·139	·141	·144
81	·123	·125	·128	·130	·132	·134	·137	·139	·142	·144	·147
82	·125	·128	·130	·132	·135	·137	·140	·142	·145	·147	·149
83	·128	·130	·133	·135	·138	·140	·142	·145	·147	·149	·152
84	·130	·132	·135	·138	·140	·142	·145	·147	·150	·152	·155
85	·132	·134	·137	·140	·143	·145	·148	·150	·153	·155	·158
86	·135	·137	·140	·142	·145	·148	·150	·153	·155	·158	·161
87	·137	·139	·142	·144	·148	·150	·153	·155	·158	·161	·163
88	·139	·142	·145	·147	·150	·152	·155	·158	·161	·163	·166
89	·142	·144	·147	·150	·153	·155	·158	·161	·164	·166	·169
90	·144	·147	·150	·153	·155	·158	·161	·164	·166	·169	·172
91	−·146	−·149	−·152	−·155	−·158	−·160	−·163	−·166	−·169	−·172	−·175

TABLE XII.—BAROMETRIC ELEVATIONS.*

B	A	Diff. for .01.	B	A	Diff. for .01.	B	A	Diff. for .01.
Inches.	Feet.	Feet.	Inches.	Feet.	Feet.	Inches.	Feet.	Feet.
20.0	11,047	−13.6	23.7	6,423	−11.5	27.4	2,470	−9.9
20.1	10,911	13.5	23.8	6,308	11.4	27.5	2,371	9.9
20.2	10,776	13.4	23.9	6,194	11.4	27.6	2,272	9.9
20.3	10,642	13.4	24.0	6,080	11.3	27.7	2,173	9.8
20.4	10,508	13.3	24.1	5,967	11.3	27.8	2,075	9.8
20.5	10,375	13.3	24.2	5,854	11.3	27.9	1,977	9.7
20.6	10,242	13.2	24.3	5,741	11.2	28.0	1,880	9.7
20.7	10,110	13.1	24.4	5,629	11.1	28.1	1,783	9.7
20.8	9,979	13.1	24.5	5,518	11.1	28.2	1 686	9.7
20.9	9,848	13.0	24.6	5.407	11.1	28.3	1,589	9.6
21.0	9,718	12.9	24.7	5,296	11.0	28.4	1,493	9.6
21.1	9,589	12.9	24.8	5,186	10.9	28.5	1,397	9.5
21.2	9,460	12.8	24.9	5,077	10.9	28.6	1,302	9.5
21.3	9,332	12.8	25.0	4,968	10.9	28.7	1.207	9.5
21.4	9.204	12.7	25.1	4,859	10.8	28.8	1,112	9.4
21.5	9,077	12.6	25.2	4,751	10.8	28.9	1,018	9.4
21.6	8,951	12.6	25.3	4,643	10.8	29.0	924	9.4
21.7	8,825	12.5	25.4	4,535	10.7	29.1	830	9.4
21.8	8,700	12.5	25.5	4,428	10.7	29.2	736	9.3
21.9	8,575	12.4	25.6	4,321	10.6	29.3	643	9.3
22.0	8,451	12.4	25.7	4,215	10.6	29.4	550	9.2
22.1	8,327	12.3	25.8	4,109	10.5	29.5	458	9.2
22.2	8,204	12.2	25.9	4,004	10.5	29.6	366	9.2
22.3	8,082	12.2	26.0	3.899	10.4	29.7	274	9.2
22.4	7,960	12.2	26.1	3,794	10.4	29.8	182	9.1
22.5	7,838	12.1	26.2	3 690	10.4	29.9	91	9.1
22.6	7,717	12.0	26.3	3,586	10.3	30.0	0	9.1
22.7	7,597	12.0	26.4	3,483	10.3	30.1	−91	9.0
22.8	7,477	11.9	26.5	3,380	10.3	30.2	181	9.0
22.9	7.358	11.9	26.6	3,277	10.2	30.3	271	9.0
23.0	7,239	11.8	26.7	3,175	10.2	30.4	361	9.0
23.1	7,121	11.7	26.8	3,073	10.1	30.5	451	8.9
23.2	7,004	11.7	26.9	2.972	10.1	30.6	540	8.9
23.3	6,887	11.7	27.0	2.871	10.1	30.7	629	8.8
23.4	6,770	11.6	27.1	2 770	10.0	30.8	717	8.8
23.5	6,554	11.6	27.2	2,670	10.0	30.9	805	−8.8
23.6	6,538	−11.5	27.3	2,570	−10.0	31.0	−893	
23.7	6,423		27.4	2.470				

* Compiled from Report of U. S. C. & G. Survey for 1881, App. 10, Table XI.

TABLE XIII.—COEFFICIENTS FOR CORRECTIONS FOR TEMPERATURE AND HUMIDITY.*

$t+t'$	C	Diff. for 1°.	$t+t'$	C	Diff. for 1°.	$t+t'$	C	Diff. for 1°.
0°	−.1024	10.9	60°	−.0380	10.7	120°	+.0262	10.6
10	.0915	10.9	70	.0273	10.7	130	.0368	10.4
20	.0806	10.8	80	.0166	10.8	140	.0472	10.3
30	.0698	10.6	90	−.0058	10.7	150	.0575	10.2
40	.0592	10.6	100	+.0049	10.7	160	.0677	10.2
50	.0486	10.6	110	.0156	10.6	170	.0779	10.0
60	−.0380		120	+.0262		180	+.0879	

* Compiled from Report of U. S. C. & G. Survey for 1881, App. 10, Tables I, IV.

1 $\sin a = \dfrac{1}{\operatorname{cosec} a} = \dfrac{\tan a}{\sqrt{1+\tan^2 a}} = \sqrt{\dfrac{1-\cos 2a}{2}} = \dfrac{1}{\sqrt{1+\cot^2 a}}$

$= \cos a \tan a = \sqrt{1-\cos^2 a} = 2 \sin \tfrac{1}{2}a \cos \tfrac{1}{2}a$

$= \dfrac{1+\cos a}{\cot \frac{1}{2}a} = \dfrac{2 \tan \frac{1}{2}a}{1+\tan^2 \frac{1}{2}a} = \operatorname{vers} a \cot \tfrac{1}{2}a.$

2 $\cos a = \dfrac{1}{\sec a} = \dfrac{\cot a}{\sqrt{1+\cot^2 a}} = \dfrac{1}{\sqrt{1+\tan^2 a}}$

$= 1 - \operatorname{vers} a = \sin a \cot a = \sqrt{1-\sin^2 a} = 2 \cos^2 \tfrac{1}{2}a - 1$

$= \sin a \cot \tfrac{1}{2}a - 1 = \cos^2 \tfrac{1}{2}a - \sin^2 \tfrac{1}{2}a = 1 - 2 \sin^2 \tfrac{1}{2}a.$

3 $\tan a = \dfrac{1}{\cot a} = \dfrac{\sin a}{\cos a} = \dfrac{\sec a}{\operatorname{cosec} a} = \dfrac{1}{\sqrt{\operatorname{cosec}^2 a - 1}}$

$= \operatorname{vers} 2a \operatorname{cosec} 2a = \cot a - 2 \cot 2a = \sin a \sec a$

$= \dfrac{\sin 2a}{1+\cos 2a} = \operatorname{exsec} a \cot \tfrac{1}{2}a = \operatorname{exsec} 2a \cot 2a.$

4 $\cot a = \dfrac{1}{\tan a} = \dfrac{\cos a}{\sin a} = \dfrac{\sin 2a}{1-\cos 2a} = \dfrac{1+\cos 2a}{\sin 2a}$

$= \sqrt{\operatorname{cosec}^2 a - 1} = \cot \tfrac{1}{2}a - \operatorname{cosec} a.$

5 $\operatorname{vers} a = 1 - \cos a = \sin a \tan \tfrac{1}{2}a = 2 \sin^2 \tfrac{1}{2}a = \cos a \operatorname{exsec} a.$

6 $\operatorname{exsec} a = \sec a - 1 = \tan a \tan \tfrac{1}{2}a = \operatorname{vers} a \sec a.$

7 $\sin \tfrac{1}{2}a = \sqrt{\dfrac{\operatorname{vers} a}{2}} = \dfrac{\sin a}{2 \cos \frac{1}{2}a} = \dfrac{\operatorname{vers} a \cos \frac{1}{2}a}{\sin a}.$

8 $\cos \tfrac{1}{2}a = \sqrt{\dfrac{1+\cos a}{2}} = \dfrac{\sin a}{2 \sin \frac{1}{2}a} = \dfrac{\sin a \sin \frac{1}{2}a}{\operatorname{vers} a}.$

9 $\tan \tfrac{1}{2}a = \operatorname{vers} a \operatorname{cosec} a = \operatorname{cosec} a - \cot a = \dfrac{\tan a}{1+\sec a}.$

10 $\cot \tfrac{1}{2}a = \dfrac{1+\cos a}{\sin a} = \operatorname{cosec} a + \cot a = \dfrac{\tan a}{\operatorname{exsec} a} = \dfrac{1}{\operatorname{cosec} a - \cot a}.$

11 $\operatorname{vers} \tfrac{1}{2}a = 1 - \sqrt{\tfrac{1}{2}(1+\cos a)}.$

12 $\operatorname{exsec} \tfrac{1}{2}a = \dfrac{1}{\sqrt{\frac{1}{2}(1+\cos a)}} - 1.$

13 $\sin 2a \quad = 2\sin a \cos a = \dfrac{2\tan a}{1+\tan^2 a}.$

14 $\cos 2a \quad = \cos^2 a - \sin^2 a = 1 - 2\sin^2 a = 2\cos^2 a - 1$

$$= \frac{1-\tan^2 a}{1+\tan^2 a}.$$

15 $\tan 2a \quad = \dfrac{2\tan a}{1-\tan^2 a}.$

16 $\cot 2a \quad = \tfrac{1}{2}\cot a - \tfrac{1}{2}\tan a = \dfrac{\cot^2 a - 1}{2\cot a} = \dfrac{1-\tan^2 a}{2\tan a}.$

17 $\text{vers } 2a \quad = 2\sin^2 a = 1 - \cos 2a = 2\sin a \cos a \tan a.$

18 $\text{exsec } 2a = \dfrac{\tan 2a}{\cot a} = \dfrac{2\tan^2 a}{1-\tan^2 a} = \dfrac{2\sin^2 a}{1-2\sin^2 a}.$

19 $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b.$

20 $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b.$

21 $\sin a + \sin b = 2\sin\tfrac{1}{2}(a+b)\cos\tfrac{1}{2}(a-b).$

22 $\sin a - \sin b = 2\sin\tfrac{1}{2}(a-b)\cos\tfrac{1}{2}(a+b).$

23 $\cos a + \cos b = 2\cos\tfrac{1}{2}(a+b)\cos\tfrac{1}{2}(a-b).$

24 $\cos a - \cos b = -2\sin\tfrac{1}{2}(a+b)\sin\tfrac{1}{2}(a-b).$

Call the sides of any triangle A, B, C, and the opposite angles a, b, and c. Call $s = \tfrac{1}{2}(A+B+C)$.

25 $\tan\tfrac{1}{2}(a-b) = \dfrac{A-B}{A+B}\tan\tfrac{1}{2}(a+b) = \dfrac{A-B}{A+B}\cot\tfrac{1}{2}c.$

26 $C = (A+B)\dfrac{\cos\frac{1}{2}(a+b)}{\cos\frac{1}{2}(a-b)} = (A-B)\dfrac{\sin\frac{1}{2}(a+b)}{\sin\frac{1}{2}(a-b)}.$

27 $\sin\tfrac{1}{2}a = \sqrt{\dfrac{(s-B)(s-C)}{BC}}.$

28 $\cos\tfrac{1}{2}a = \sqrt{\dfrac{s(s-A)}{BC}}.$

29 $\text{vers } a = \dfrac{2(s-B)(s-C)}{BC}.$

30 $\text{Area} = \sqrt{s(s-A)(s-B)(s-C)} = A^2\dfrac{\sin b \sin c}{2\sin a}.$

		Logarithm.
Circumference of a circle (radius $= r$) $= 2\pi r$.		
Area of a circle $= \pi r^2$.		
Area of sector (length of arc $= l$) $= \frac{1}{2}lr$.		
" " " (angle of arc $= a°$) $= \frac{a}{360}\pi r^2$.		
Area of segment (chord $= c$, mid. ord. $= m$) $= \frac{2}{3}cm$ (approx.).		
Area of a circle to radius 1, Circumference of a circle to diameter 1, Surface of a sphere to diameter 1 $= \pi =$	3.1415927	0.497 1499
Volume of a sphere to radius 1 $= 4\pi \div 3$ =	4.1887902	0.622 0886
Arc equal to radius expressed in degrees =	57.2957795	1.758 1226
Arc equal to radius expressed in minutes =	3437.7467708	3.536 2739
Arc equal to radius expressed in seconds =	206264.8062471	5.314 4251
Length of arc of 1°, radius unity	0.1745329	8.241 8774
Sine of one second = 0.0000048481		4.685 5749
Cubic inches in United States standard gallon = 231		2.363 6120
Weight of one cubic foot of water at maximum density (therm. 39°.8 F., barom. 30″)	62.379	1.795 0384
Weight of one cubic foot of water at maximum density (therm. 62° F.)	62.321	1.794 6349
Acceleration due to gravity at latitude of New York in feet per square second	32.15945	1.507 3086
Feet in one metre	3.280869	0.515 9889
Metres in one foot	0.304797	9.484 0111

INDEX.

Numbers refer to sections except where specifically marked pages (p.).