RAILROAD CONSTRUCTION.

THEORY AND PRACTICE.

A TEXT-BOOK FOR THE USE OF STUDENTS IN COLLEGES AND TECHNICAL SCHOOLS.

BY

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etc.

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PREFACE.

THE preparation of this book was begun several years ago, when much of the subject-matter treated was not to be found in print, or was scattered through many books and pamphlets, and was hence unavailable for student use. Portions of the book have already been printed by the mimeograph process or have been used as lecture-notes, and hence have been subjected to the refining process of classroom use.

The author would call special attention to the following features:

a. Transition curves; the multiform-compound-curve method is used, which has been followed by many railroads in this country; the particular curves here developed have the great advantage of being exceedingly simple, and although the method is not theoretically exact, it is demonstrable that the differences are so small that they may safely be neglected.

b. A system of earthwork computations by means of a slide-rule (which accompanies the volume) which enables one to compute readily the volume of the most complicated earthwork forms with an accuracy only limited by the precision of the cross-sectioning.

c. The "mass curve" in earthwork; the theory and use of this very valuable process.

d. Tables I, II, III, and IV have been computed *ab novo.* Tables I and II were checked (after computation) with other tables, which are generally considered as standard, and all discrepancies were further examined. They are believed to be perfect.

e. Tables V, VI, VII, and IX have been borrowed, by permission, from "Ludlow's Mathematical Tables." It is believed that five-place tables give as accurate results as actual field practice requires. Tables VIII and X have been compiled to conform with Ludlow's system.

The author wishes to acknowledge his indebtedness to Mr. Chas. A. Sims, civil engineer and railroad contractor, for reading and revising the portions relating to the cost of earthwork.

Since the book is written primarily for students of railroad engineering in technical institutions, the author has assumed the usual previous preparation in algebra, geometry, and trigonometry.

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RAILROAD CONSTRUCTION.

CHAPTER I.

RAILROAD SURVEYS.

The proper conduct of railroad surveys presupposes an adequate knowledge of almost the whole subject of railroad engineering, and particularly of some of the complicated questions of Railroad Economics, which are not generally studied except at the latter part of a course in railroad engineering, if at all. This chapter will therefore be chiefly devoted to methods of instrumental work, and the problem of choosing a general route will be considered only as it is influenced by the topography or by the application of those elementary principles of Railroad Economics which are self-evident or which may be accepted by the student until he has had an opportunity of studying those principles in detail.

RECONNOISSANCE SURVEYS.

1. Character of a reconnoissance survey. A reconnoissance survey is a very hasty examination of a belt of country to determine which of all possible or suggested routes is the most promising and best worthy of a more detailed survey. It is essentially very rough and rapid. It aims to discover those salient features which instantly stamp one route as distinctly superior to another and so narrow the choice to routes which are so nearly equal in value that a more detailed survey is necessary to decide between them.

2. Selection of a general route. The general question of running a railroad between two towns is usually a financial rather than an engineering question. Financial considerations usually determine that a road *must* pass through certain more or less important towns between its termini. When a railroad runs through a thickly settled and very flat country, where, from a topographical standpoint, the road may be run by any desired route, the "right-of-way agent" sometimes has a greater influence in locating the road than the engineer. But such modifications of alignment, on account of business considerations, are foreign to the engineer's side of the subject, and it will be hereafter assumed that topography alone determines the location of the line. The consideration of those larger questions combining finance and engineering (such as passing by a town on account of the necessary introduction of heavy grades in order to reach it) is likewise ignored.

3. Valley route. This is perhaps the simplest problem. If the two towns to be connected lie in the same valley, it is frequently only necessary to run a line which shall have a nearly uniform grade. The reconnoissance problem consists largely in determining the difference of elevation of the two termini of this division and the approximate horizontal distance so that the proper grade may be chosen. If there is a large river running through the valley, the road will probably remain on one side or the other throughout the whole distance, and both banks should be examined by the reconnoissance party to determine which is preferable. If the river may be easily bridged, both banks may be alternately used, especially when better alignment is thereby secured. A river valley has usually a steeper slope in the upper part than in the lower part. A uniform grade throughout the valley will therefore require that the road climbs up the side slopes in the lower part of the valley. In case the "ruling grade" * for the whole road is as great as or greater

* The *ruling grade* may here be loosely defined as the maximum grade which is permissible. This definition is not strictly true, as may be seen later when studying Railroad Economics, but it may here serve the purpose.

than the steepest natural valley slope, more freedom may be used in adopting that alignment which has the least cost—regardless of grade. The natural slope of large rivers is almost invariably so low that grade has no influence in determining the choice of location. When bridging is necessary, the river banks should be examined for suitable locations for abutments and piers. If the soil is soft and treacherous much difficulty may be experienced and the choice of route may be largely determined by the difficulty of bridging the river except at certain favorable places.

4. Cross-country route. A cross-country route always has one or more summits to be crossed. The problem becomes more complex on account of the greater number of possible solutions and the difficulty of properly weighing the advantages and disadvantages of each. The general aim should be to choose the lowest summits and the highest stream crossings, provided that by so doing the grades between these determining points shall be as low as possible and shall not be greater than the ruling grade of the road. Nearly all railroads combine cross-country and valley routes to some extent. Usually the steepest natural slopes are to be found on the cross-country routes, and also the greatest difficulty in securing a low through grade. An approximate determination of the ruling grade is usually made during the reconnoissance. If the ruling grade has been previously decided on by other considerations, the leading feature of the reconnoissance survey will be the determination of a general route along which it will be possible to survey a line whose maximum grade shall not exceed the ruling grade.

5. Mountain route. The streams of a mountainous region frequently have a slope exceeding the desired ruling grade. In such cases there is no possibility of securing the desired grade by following the streams. The penetration of such a region may only be accomplished by "development"—accompanied perhaps by tunneling. "Development" consists in deliberately increasing the length of the road between two extremes of elevation so that the rate of grade shall be as low as desired.

The usual method of accomplishing this is to take advantage of some convenient formation of the ground to introduce some lateral deviation. The methods may be somewhat classified as follows:

(a) Running the line up a convenient lateral valley, turning a sharp curve and working back up the opposite slope. As shown in Fig. 1, the considerable rise between *A* and *B* was

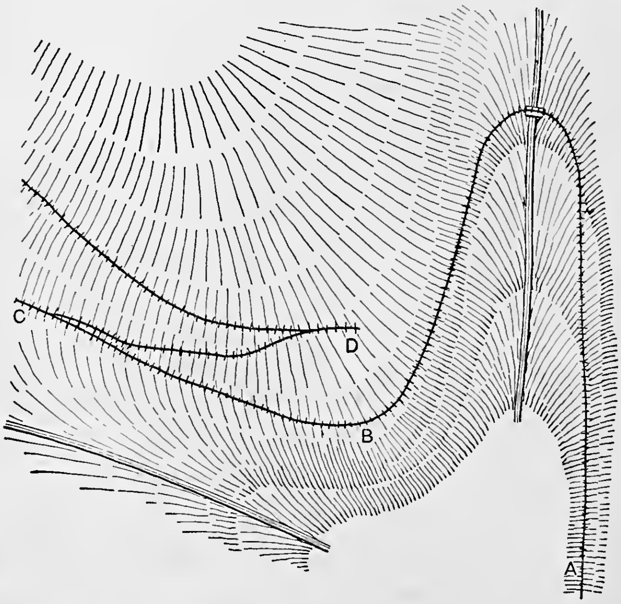


Fig. 1.

surmounted by starting off in a very different direction from the general direction of the road; then, when about one-half of the desired rise had been obtained, the line crossed the valley and continued the climb along the opposite slope. (b) *Switch-back.* On the steep side-hill *BCD* (Fig. 1) a very considerable gain in elevation was accomplished by the switchback *CD*. The gain in elevation from *B* to *D* is very great. On the other hand, the speed must always be slow; there are two complete stoppages of the train for each run; all trains must run backward from *C* to *D*. (c) *Bridge spiral.* When a valley is so narrow at some point that a bridge or viaduct of reasonable length can span the valley at a considerable elevation above the bottom of the valley, a bridge spiral may be desirable. In

Fig. 2 the line ascends the stream valley past *A*, crosses the stream at *B*, works back to the narrow place at *C*, and there crosses itself, having gained perhaps 100 feet in elevation. (**d**) *Tunnel spiral.* This is the reverse of the previous plan.

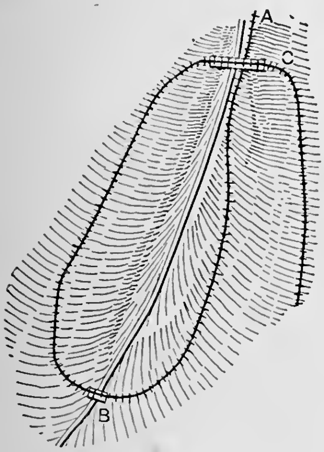


Fig. 2.

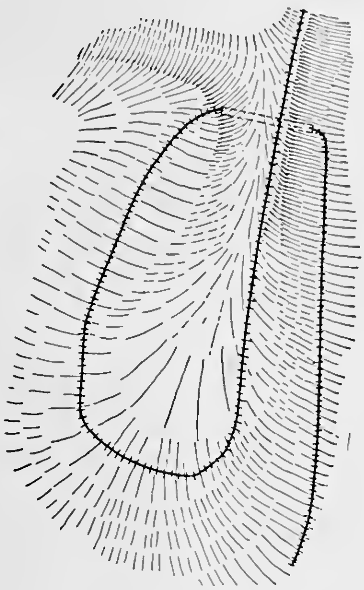


Fig. 3

It implies a thin steep ridge, so thin at some place that a tunnel through it will not be excessively long. Switchbacks and spirals are sometimes necessary in mountainous countries, but they should not be considered as normal types of construction. A region must be very difficult if these devices cannot be avoided.

Rack railways and cable roads, although types of mountain railroad construction, will not be here considered.

6. Existing maps. The maps of the U. S. Geological Survey are exceedingly valuable as far as they have been completed. So far as topographical considerations are concerned, they almost dispense with the necessity for the reconnoissance and "first preliminary" surveys. Some of the State Survey maps will give practically the same information. County and township maps can often be used for considerable information as to the relative *horizontal* position of governing points, and even some approximate data regarding elevations may be obtained by a study of the streams. Of course such information will not dispense with surveys, but will assist in so planning them as to obtain the best information with the least work. When the relative horizontal positions of points are reliably indicated on a map, the reconnoissance may be reduced to the determination of the relative elevations of the governing points of the route.

7. Determination of relative elevations. A recent description of European methods includes spirit-leveling in the reconnoissance work. This may be due to the fact that, as indicated above, previous topographical surveys have rendered unnecessary the "exploratory" survey which is required in a new country, and that their reconnoissance really corresponds more nearly to our preliminary.

The perfection to which barometrical methods have been brought has rendered it possible to determine differences of elevation with sufficient accuracy for reconnoissance purposes by the combined use of a mercurial and an aneroid barometer. The mercurial barometer should be kept at "headquarters," and readings should be taken on it at such frequent intervals that any fluctuation is noted, and throughout the period that observations with the aneroid are taken in the field. At each observation there should also be recorded the time, the reading of the attached thermometer, and the temperature of the external air. For uniformity, the mercurial readings should then be "reduced to 32° F." Before starting out, a reading of the aneroid should be taken at headquarters coincident with a reading of the mercurial. The difference is one value of the correction to the aneroid. As soon as the aneroid is brought back another comparison of readings should be made. Even though there has been considerable rise or fall of pressure in the interval, the *difference* in readings (the correction) should be substantially the same provided the aneroid is a good instrument. The best aneroids read directly to $\frac{1}{100}$ of an inch of mercury and may be estimated to $\frac{1}{1000}$ of an inch—which corresponds to about 0.9 foot difference of elevation. In the field there should be read, at each point whose elevation is desired, the aneroid, the time, and the temperature. These readings, corrected by the mean value of the correction between the aneroid and the mercurial, should then be combined with the reading of the mercurial (interpolated if necessary) for the times of the aneroid observations and the difference of elevation obtained. [See the author's "Problems in the Use and Adjustment of Engineering Instruments," Prob. 22.] Important points should be observed more than once if possible. Such duplicate observations will be found to give surprisingly concordant results even when a general fluctuation of atmospheric pressure so modifies the tabulated readings that an agreement is not at first apparent. Variations of pressure produced by high winds, thunder-storms, etc., will generally vitiate possible accuracy by this method. By "headquarters" is meant any place whose elevation above any given datum is known and where the mercurial may be placed and observed while observations within a range of several miles are made with the aneroid. If necessary the elevation of a new headquarters may be determined by the above method, but there should be if possible several independent observations whose accordance will give a fair idea of their accuracy.

The above method should be neither slighted nor used for more than it is worth. When properly used, the errors are compensating rather than cumulative. When used, for example, to determine that a pass B is 260 feet higher than a determined bridge crossing at A which is six miles distant, and that another pass C is 310 feet higher than A and is ten miles distant, the figures, even with all necessary allowances for inaccuracy, will give an engineer a good idea as to the choice of route especially as affected by ruling grade. There is no comparison between the time and labor involved in obtaining the above information by barometric and by spirit-leveling methods, and *for reconnoissance purposes* the added accuracy of the spirit-leveling method is hardly worth its cost.

8. Horizontal measurements, bearings, etc. When there is no map which may be depended on, or when only a skeleton map is obtainable, a rapid survey, sufficiently accurate for the purpose, may be made by using a pocket compass for bearings and a telemeter, odometer, or pedometer for distances. The telemeter [stadia] is more accurate, but it requires a definite clear sight from station to station, which may be difficult through a wooded country. The odometer, which records the revolutions of a wheel of known circumference, may be used even in rough and wooded country, and the results may be depended on to a small percentage. The pedometer (or pace-measurer) depends for its accuracy on the actual movement of the mechanism for each pace and on the uniformity of the pacing. Its results are necessarily rough and approximate, but it may be used to fill in some intermediate points in a large skeleton map. A hand-level is also useful in determining the relative elevation of various topographical features which may have some bearing on the proper location of the road.

9. Importance of a good reconnoissance. The foregoing instruments and methods should be considered only as aids in exercising an educated common sense, without which a proper location cannot be made. The reconnoissance survey should command the best talent and the greatest experience available. If the general route is properly chosen, a comparatively low order of engineering skill can fill in a location which will prove a paying railroad property; but if the general route is so chosen that the ruling grades are high and the business obtained is small and subject to competition, no amount of perfection in detailed alignment or roadbed construction can make the road a profitable investment.

PRELIMINARY SURVEYS.

10. Character of survey. A preliminary railroad survey is properly a topographical survey of a belt of country which has been selected during the reconnoissance and within which it is estimated that the located line will lie. The width of this belt will depend on the character of the country. When a railroad is to follow a river having very steep banks the choice of location is sometimes limited at places to a very few feet of width and the belt to be surveyed may be correspondingly narrowed. In very flat country the desired width may be only limited by the ability to survey points with sufficient accuracy at a considerable distance from what may be called the "backbone line" of the survey.

11. Cross-section method. This is the only feasible method in a wooded country, and is employed by many for all kinds of country. The *backbone* line is surveyed either by observing magnetic bearings with a compass or by carrying forward

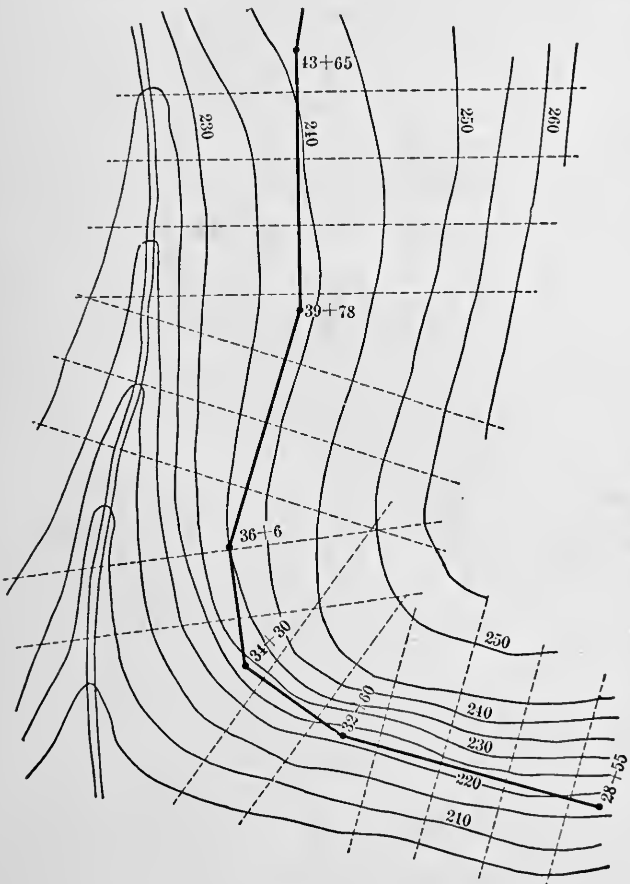


FIG. 4.

absolute azimuths with a transit. The compass method has the disadvantages of limited accuracy and the possibility of considerable local error owing to local attraction. On the other hand there are the advantages of greater simplicity, no necessity for a back rodman, and the fact that the errors are purely local and not cumulative, and may be so limited, with care, that they will cause no vital error in the subsequent location survey. The transit method is essentially more accurate, but is liable to be more laborious and troublesome. If a large tree is encountered, either it must be cut down or a troublesome operation of offsetting must be used. If the compass is employed under these circumstances, it need only be set up on the far side of the tree and the former bearing produced. An error in reading a transit azimuth will be carried on throughout the survey. An error of only five minutes of arc will cause an offset of nearly eight feet in a mile. Large azimuth errors may, however, be avoided by immediately checking each new azimuth with a needle reading. It is advisable to obtain true azimuth at the beginning of the survey by an observation on the sun or Polaris, and to check the azimuths every few miles by azimuth observations. Distances along the backbone line should be measured with a chain or steel tape and stakes set every 100 feet. When a course ends at a substation, as is usually the case, the remaining portion of the 100 feet should be measured along the next course. The level party should immediately obtain the elevations (to the nearest tenth of a foot) of all stations, and also of the lowest points of all streams crossed and even of dry gullies which would require culverts.

12. Cross-sectioning. It is usually desirable to obtain contours at five-foot intervals. This may readily be done by the use of a Locke level (which should be held on top of a simple five-foot stick), a tape, and a rod ten feet in length graduated to feet and tenths. The method of use may perhaps be best explained by an example. Let Fig. 5 represent a section *perpendicular* to the survey line—such a section as would be made by the dotted lines in Fig. 4. *C* represents the station point. Its elevation as determined by the level is, say, 158.3 above datum. When the Locke level on its five-foot rod is placed at

C, the level has an elevation of 163.3. Therefore when a point is found (as at *a*) where the level will read 3.3 on the rod, that

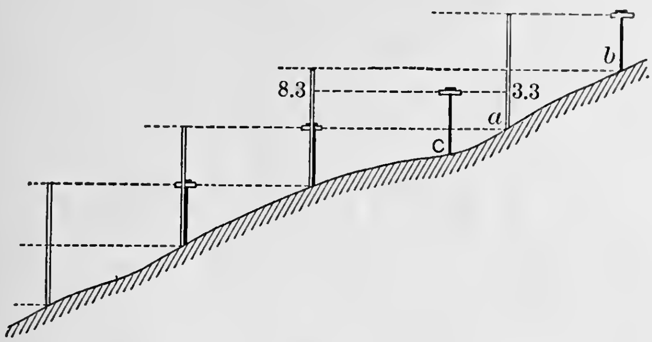


Fig. 5.

point has an elevation of 160.0 and its distance from the center gives the position of the 160-foot contour. Leaving the long rod at that point (*a*), carry the level to some point (*b*) such that the level will sight at the *top* of the rod. *b* is then on the 165-

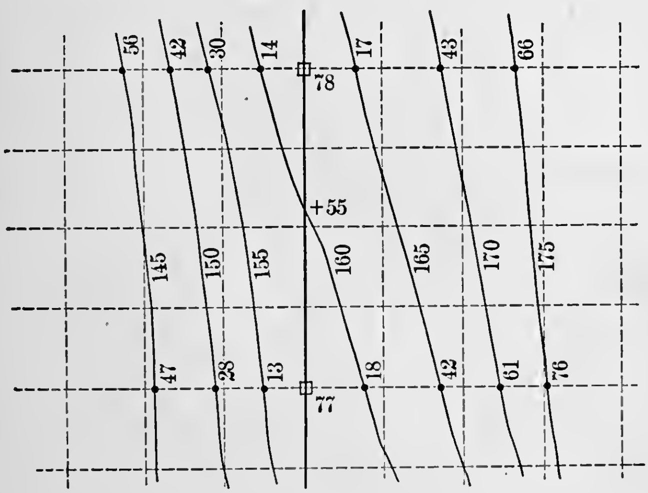


Fig. 6.

foot contour, and the *horizontal* distance *ab* added to the horizontal distance *ac* gives the position of that contour from the center. The contours on the lower side are found similarly. The first rod reading will be 8.3, giving the 155-foot contour. Plot the results in a note-book which is ruled in quarter-inch squares, using a scale of 100 feet per inch in both directions.

Plot the work UP the page; then when looking ahead along the line, the work is properly oriented. When a contour crosses the survey line, the place of crossing may be similarly determined. If the ground flattens out so that five-foot contours are very far apart, the absolute elevations of points at even fifty-foot distances from the center should be determined. The method is exceedingly rapid. Whatever error or inaccuracy occurs is confined in its effect to the one station where it occurs. The work being thus plotted in the field, unusually irregular topography may be plotted with greater certainty and no great error can occur without detection. It would even be possible by this method to detect a gross error that might have been made by the level party.

13. Stadia method. This method is best adapted to fairly open country where a "shot" to any desired point may be taken without clearing. The *backbone* survey line is the same as in the previous method except that each course is limited to the practicable length of a stadia sight. The distance between stations should be checked by foresight and backsight—also the vertical angle. Azimuths should be checked by the needle. Considering the vital importance of leveling on a railroad survey it might be considered desirable to run a line of levels over the stadia stations in order that the leveling may be as precise as possible; but when it is considered that a preliminary survey is a somewhat hasty survey of a route that *may* be abandoned, and that the errors of leveling by the stadia method (which are compensating) may be so minimized that no proposed route would be abandoned on account of such small error, and that the effect of such an error may be easily neutralized by a slight change in the location, it may be seen that excessive care in the leveling of the preliminary survey is hardly justifiable.

Since the students taking this work are assumed to be familiar with the methods of stadia topographical surveys, this part of the subject will not be further elaborated.

14. "First" and "second" preliminary surveys. Some engineers advocate two preliminary surveys. When this is done, the first is a very rapid survey, made perhaps with a compass, and is only a better grade of reconnoissance. Its aim is to rapidly develop the facts which will decide for or against any proposed route, so that if a route is found to be unfavorable another more or less modified route may be adopted without having wasted considerable time in the survey of useless details. By this time the student should have grasped the fundamental idea that both the reconnoissance and preliminary surveys are not surveys of *lines* but of *areas*; that their aim is to survey only those topographical features which would have a determining influence on any railroad line which might be constructed through that particular territory, and that the value of a locating engineer is largely measured by his ability to recognize those determining influences with the least amount of work from his surveying corps. Frequently too little time is spent on the comparative study of preliminary lines. A line will be hastily decided on after very little study; it will then be surveyed with minute detail and estimates carefully worked up, and the claims of any other suggested route will then be handicapped, if not disregarded, owing to an unwillingness to discredit and throw away a large amount of detailed surveying. The cost of two or three extra preliminary surveys (*at critical points* and not over the whole line) is utterly insignificant compared with the probable improvement in the "operating value" of a line located after such a comparative study of preliminary lines.

LOCATION SURVEYS.

15. "Paper location." When the preliminary survey has been plotted to a scale of 200 feet per inch and the contours drawn in, a study may be made for the location survey. Disregarding for the present the effect on location of transition curves, the alignment may be said to consist of straight lines (or "tangents") and circular curves. The "paper location" therefore consists in plotting on the preliminary map a succession of straight lines which are tangent to the circular curves connecting them. The determining points should first be considered. Such points are the termini of the road, the lowest practicable point over a summit, a river-crossing, etc. So far as is possible, having due regard to other considerations, the road should be a "surface" road, i.e., the cut and fill should be made as small as possible. The maximum permissible grade must also have been determined and duly considered. The method of location differs radically according as the lines joining the determining points have a very low grade or have a grade that approaches the maximum permissible. With very low natural grades it is only necessary to strike a proper balance between the requirements for easy alignment and the avoidance of excessive earthwork. When the grade between two determined points approaches the maximum, a study of the location may be begun by finding a strictly surface line which will connect those points with a line at the given grade. For example, suppose the required grade is 1.6% and that the contours are drawn at 5-foot intervals. It will require 312 feet of 1.6% grade to rise 5 feet. Set a pair of dividers at 312 feet and step off this interval on successive contours. This line will in general be very irregular, but in an easy country it may lie fairly close to the proper location line, and even in difficult country such a surface line will assist greatly in selecting a suitable location. When the larger part of the line will evidently consist of tangents, the tangents should be first located and should then be connected by suitable curves. When the curves predominate, as they generally will in mountainous country, and particularly when the line is purposely lengthened in order to reduce the grade, the curves should be plotted first and the tangents may then be drawn connecting them. Considering the ease with which such lines may be drawn on the preliminary map, it is frequently advisable, after making such a paper location, to begin all over, draw a new line over some specially difficult section and compare results. Profiles of such lines may be readily drawn by noting their intersection with each contour crossed. Drawing on each profile the required grade line will furnish an approximate idea of the *comparative* amount of earthwork required. After deciding on the paper location, the length of each tangent, the central angle (see § 21), and the radius of each curve should be measured as accurately as possible. Since a slight error made in such measurements, taken from a map with a scale of 200 feet per inch, would by accumulation cause serious discrepancies between the plotted location and the location as afterward surveyed in the field, frequent tie lines and angles should be determined between the plotted location line and the preliminary line, and the location should be altered, as may prove necessary, by changing the length of a tangent or changing the central angle or radius of a curve, so that the agreement of the check-points will be sufficiently close. The errors of an inaccurate preliminary survey may thus be easily neutralized (see § 33). When the preliminary line has been properly run, its "backbone" line will lie very near the location line and will probably cross it at frequent intervals, thus rendering it easy to obtain short and numerous tie lines.

16. Surveying methods. A transit should be used for alignment, and only precise work is allowable. The transit stations should be centered with tacks and should be tied to witness-stakes, which should be located outside of the range of the earthwork, so that they will neither be dug up nor covered up. All original property lines lying within the limits of the right of way should be surveyed with reference to the location line, so that the right-of-way agent may have a proper basis for settlement. When the property lines do not extend far outside of the required right of way they are frequently surveyed completely.

The leveler usually reads the target to the nearest thousandth of a foot on turning-points and bench-marks, but reads to the nearest tenth of a foot for the elevation of the ground at stations. Considering that $\frac{1}{1000}$ of a foot has an angular value of only 7 seconds at a distance of 300 feet, and that one division of a level-bubble is usually about 30 seconds, it may be seen that it is a useless refinement to read to thousandths unless corresponding care is taken in the use of the level. The leveler should also locate his bench-marks outside of the range of earthwork. A knob of rock protruding from the ground affords an excellent mark. A large nail, driven in the roots of a tree, which is not to be disturbed, is also a good mark. These marks should be clearly described in the note-book. The leveler should obtain the elevation of the ground at all station-points; also at all sudden breaks in the profile line, determining also the distance of these breaks from the previous even station. This will include the position and elevation of all streams, and even dry gullies, which are crossed.

Measurements should preferably be made with a steel tape, care being taken on steep ground to insure horizontal measurements. Stakes are set each 100 feet, and also at the beginning and end of all curves. Transit-points (sometimes called "plugs" or "hubs") should be driven flush with the ground, and a "witness-stake," having the "number" of the station, should be set three feet to the right. For example, the witness-stake might have on one side "137 + 69.92," and on the other side "P C 4° R," which would signify that the transit hub is 69.92 feet beyond station 137, or 13769.92 feet from the beginning of the line, and also that it is the "point of curve" of a "4°-curve" which turns to the *right*.

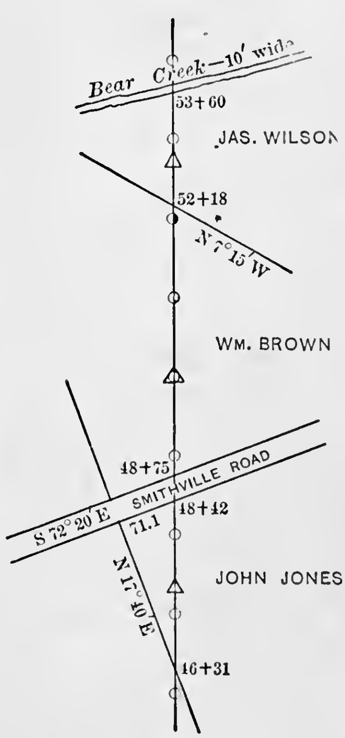
Alignment. The alignment is evidently a part of the location survey, but, on account of the magnitude and importance of the subject, it will be treated in a separate chapter.

17. Form of Notes. Although the Form of Notes cannot be thoroughly understood until after curves are studied, it is here introduced as being the most convenient place. The right-hand page should have a sketch showing all roads, streams, and property lines crossed with the bearings of those lines. This should be drawn to a scale of 100 feet per inch—the quarter-inch squares which are usually ruled in note-books giving convenient 25-foot spaces. This sketch will always be more or less distorted on curves, since the center line is always shown as *straight* regardless of curves. The station points ("Sta." in first column, left-hand page) should be placed opposite to their sketched positions, which means that even stations will be recorded on every *fourth* line. This allows three intermediate lines for substations, which is ordinarily more than sufficient. The notes should read UP the page, so that the sketch will be properly oriented when looking ahead along the line. The other columns on the left-hand page will be self-explanatory when the subject of curves is understood. If the "calculated bearings" are based on azimuthal observations, their agreement (or *constant* difference) with the needle readings will form a valuable check on the curve calculations and the instrumental work.

FORM OF NOTES.

[Left-hand page.] [Right-hand page.]

Sta.	Alignment	Vernier	Tang. Defl.	Calc. Bearing.	Needle.
54					
53					
⊙ + 72.2	P.T.	9° 11′	18° 22′	N 54° 48′ E	N 62° 15′ E
52		7 57			
51	[3° 24′ curve to right for 18° 22′; tang. dist., 272.5]	6 15			
⊙ 50		4 33			
49		2 51			
48		1 09			
⊙ + 32	P.C.	0°			
47					
46				N 36° 26′ E	N 44° 0′ E



CHAPTER II.

ALIGNMENT.

In this chapter the alignment of the *center line* only of a pair of rails is considered. When a railroad is crossing a summit in the grade line, although the horizontal projection of the alignment may be straight, the vertical projection will consist of two sloping lines joined by a curve. When a curve is on a grade, the center line is really a spiral, a curve of double curvature, although its horizontal projection is a circle. The center line therefore consists of straight lines and curves of single and double curvature. The simplest method of treating them is to consider their horizontal and vertical projections separately. In treating simple, compound, and transition curves, only the horizontal projections of those curves will be considered.

SIMPLE CURVES.

18. Designation of curves. A curve may be designated either by its radius or by the angle subtended by a chord of unit length. Such an angle is known as the "degree of curve" and is indicated by D. Since the curves that are practically used have very long radii, it is generally impracticable to make any use of the actual center, and the curve is located without reference to it. If AB in Fig. 7 represents a unit chord

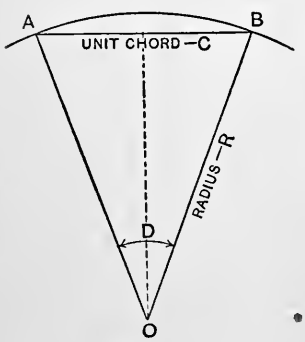


Fig. 7.

(C) of a curve of radius R, then by the above definition the angle AOB equals D. Then $AO \sin \frac{1}{2}D = \frac{1}{2}AB = \frac{1}{2}C$.

$$\therefore\ R = \frac{\frac{1}{2}C}{\sin \frac{1}{2}D} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or, by inversion,

$$\sin \frac{1}{2}D = \frac{C}{2R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The unit chord is variously taken throughout the world as 100 feet, 66 feet, and 20 meters. In the United States 100 feet is invariably used as the unit chord length, and throughout this work it will be so considered. Table I has been computed on this basis. It gives the radius, with its logarithm, of all curves from a 0° 01′ curve up to a 10° curve, varying by single minutes. The sharper curves, which are seldom used, are given with larger intervals.

An approximate value of R may be readily found from the following simple rule, which should be memorized:

$$R = \frac{5730}{D}.$$

Although such values are not mathematically correct, since R does not strictly vary inversely as D, yet the resulting value is within a tenth of one per cent for all commonly used values of R, and is sufficiently close for many purposes, as will be shown later.

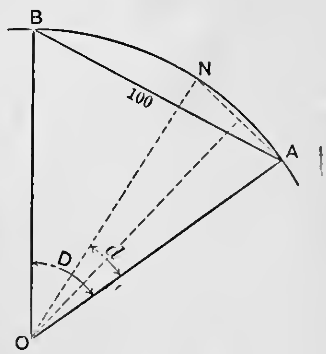


FIG. 8.

19. Length of a sub-chord. Since it is impracticable to measure along a curved arc, curves are always measured by laying off 100-foot chord lengths. This means that the actual arc is always a little longer than the chord. It also means that a *subchord* (a chord shorter than the unit length) will be a little longer than the ratio of the angles subtended would call for. The truth of this may be seen without calculation by noting that two equal subchords, each subtending the angle $\frac{1}{2}D$, will evidently be slightly longer than 50 feet each. If c be the length of a subchord subtending the angle d, then, as in Eq. (2),

$$\sin \tfrac{1}{2}d = \frac{c}{2R},$$

or, by inversion,

$$c = 2R \sin \tfrac{1}{2}d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The *nominal length* of a subchord $= 100\frac{d}{D}$. For example, a *nominal* subchord of 40 feet will subtend an angle of $\frac{40}{100}$ of $D°$; its *true length* will be slightly more than 40 feet, and may be computed by Eq. 3. The *difference* between the nominal and true lengths is maximum when the subchord is about 57 feet long, but with the low degrees of curvature ordinarily used the difference may be neglected. With a 10° curve and a nominal chord length of 60 feet, the true length is 60.049 feet. Very sharp curves should be laid off with 50-foot or even 25-foot chords (nominal length). In such cases especially the true lengths of these subchords should be computed and used instead of the nominal lengths.

20. Length of a curve. The length of a curve is always indicated by the quotient of $100\Delta \div D$. If the quotient of $\Delta \div D$ is a whole number, the length as thus indicated is the true length—*measured in 100-foot chord lengths.* If it is an odd number or if the curve begins and ends with a subchord (even though $\Delta \div D$ is a whole number), theoretical accuracy requires that the *true* subchord lengths shall be used, although the difference may prove insignificant. The length of the arc (or the mean length of the two rails) is therefore always in excess of the length as given above. Ordinarily the amount of this excess is of no practical importance. It simply adds an insignificant amount to the length of rail required.

Example. Required the nominal and true lengths of a 3° 45′ curve having a central angle of 17° 25′. First reduce the degrees and minutes to decimals of a degree. $(100 \times 17° 25') \div 3° 45' = 1741.667 \div 3.75 = 464.444$. The curve has four 100-foot chords and a nominal chord of 64.444. The true chord should be 64.451. The actual arc is

$$17°.4167 \times \frac{\pi}{180°} \times R = 464.527.$$

The excess is therefore $464.527 - 464.451 = 0.076$ foot.

21. Elements of a curve. Considering the line as running from A toward B, the beginning of the curve, at A, is called the *point of curve* (PC). The other end of the *curve*, at B, is called the *point of tangency* (PT). The intersection of the tangents is called the *vertex* (V). The angle made by the tangents at V, which equals the angle made by the radii to the extremities of the curve, is called the *central angle* (Δ). AV and BV, the two equal tangents from the vertex to the PC and PT, are called the *tangent distances* (T). The chord AB is called the *long chord* (LC). The intercept HG from the middle of the long chord to the middle of the arc is called the *middle ordinate* (M). That part of the secant GV from the middle of the arc to the vertex is called the *external distance* (E). From the figure it is very easy to derive the following frequently used relations:

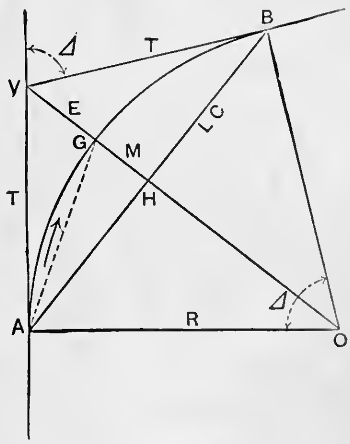


FIG. 9.

$$T = R \tan \tfrac{1}{2}\Delta \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$
$$LC = 2R \sin \tfrac{1}{2}\Delta \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$
$$M = R \operatorname{vers} \tfrac{1}{2}\Delta \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$
$$E = R \operatorname{exsec} \tfrac{1}{2}\Delta \quad . \quad . \quad . \quad . \quad . \quad (7)$$

22. Relation between T, E, and Δ. Join A and G in Fig. 9. The angle $VAG = \frac{1}{4}\Delta$, since it is measured by one half of the arc AG between the secant and tangent. $AGO = 90° - \frac{1}{4}\Delta$.

$$AV : VG :: \sin AGV : \sin VAG;$$

$$\sin AGV = \sin AGO = \cos \tfrac{1}{4}\Delta;$$

$$T : E :: \cos \tfrac{1}{4}\Delta : \sin \tfrac{1}{4}\Delta;$$

$$T = E \cot \tfrac{1}{4}\Delta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The same relation may be obtained by dividing Eq. 4 by Eq. 7, since $\tan \alpha \div \text{exsec } \alpha = \cot \frac{1}{2}\alpha$.

23. Elements of a 1° curve. From Eqs. 1 to 8 it is seen that the elements of a curve vary directly as R. It is also seen to be very nearly true that R varies inversely as D. If the elements of a 1° curve for various central angles are calculated and tabulated, the elements of a curve of $D°$ curvature may be approximately found by dividing by D the corresponding elements of a 1° curve having the same central angle. For small central angles and low degrees of curvature the errors involved by the approximation are insignificant, and even for larger angles the errors are so small that *for many purposes* they may be disregarded.

In Table II is given the value of the tangent distances, external distances, and long chords for a 1° curve for various central angles. The student should familiarize himself with the degree of approximation involved by solving a large number of cases under various conditions by the exact and approximate methods, in order that he may know when the approximate method is sufficiently exact for the intended purpose. The approximate method also gives a ready check on the exact method.

24. Exercises. (*a*) What is the tangent distance of a 4° 20′ curve having a central angle of 18° 24′ ?

(*b*) Given a 3° 30′ curve and a central angle of 16° 20′, how far will the curve pass from the vertex ? [Use Eq. 7.]

(*c*) An 18° curve is to be laid off using 25-foot (nominal) chord lengths. What is the true length of the subchords ?

(*d*) Given two tangents making a central angle of 15° 24′. It is desired to connect these tangents by a curve which shall pass 16.2 feet from their intersection. How far down the tangent will the curve begin and what will be its radius? (Use Eq. 8 and then use Eq. 4 inverted.)

25. Curve location by deflections. The angle between a secant and a tangent (or between two secants intersecting on an arc) is measured by one half of the intercepted arc. Beginning at the *PC* (*A* in Fig. 10), if the first chord is to be a full chord we may deflect an angle *VAa* ($= \frac{1}{2}D$), and the point *a*, which is 100 feet from *A*, is a point on the curve. For the next station, *b*, deflect an *additional* angle *bAa* ($= \frac{1}{2}D$) and, with one end of the tape at *a*, swing the other end until the 100-foot point is on the line *Ab*. The point *b* is then on the curve. If the final chord *cB* is a subchord, its *additional deflection* ($\frac{1}{2}\alpha$) is something less than $\frac{1}{2}D$. The last *deflection* (*BAV*) is of course $\frac{1}{2}\Delta$. It is particularly important, when a curve begins or ends with a subchord and the deflections are odd quantities, that the last additional deflection should be carefully computed and added to the previous deflection, to check the mathematical work by the agreement of this last computed deflection with $\frac{1}{2}\Delta$.

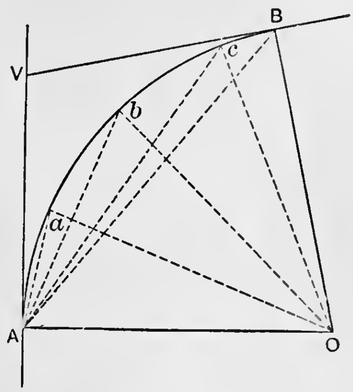


FIG. 10.

Example. Given a 3° 24′ curve having a central angle of 18° 22′ and beginning at sta. 47 + 32, to compute the deflections. The nominal length of curve is 18° 22′ ÷ 3° 24′ = 18.367 ÷ 3.40 = 5.402 stations or 540.2 feet. The curve therefore ends at sta. 52 + 72.2. The deflection for sta. 48 is $\frac{68}{100} \times \frac{1}{2}(3° \; 24')$ $= 0.68 \times 1°.7 = 1°.156 = 1° \; 09'$ nearly. For each additional 100 feet it is 1° 42′ additional. The final additional deflection for the final subchord of 72.2 feet is

$$\frac{72.2}{100} \times \tfrac{1}{2}(3° \; 24') = 1°.2274 = 1° \; 14' \text{ nearly.}$$

The deflections are

P. C . . . . Sta.	47 + 32	0°
	48 . . . . 0°	+ 1° 09′ = 1° 09′
	49 . . . . 1° 09′	+ 1° 42′ = 2° 51′
	50 . . . . 2° 51′	+ 1° 42′ = 4° 33′
	51 . . . . 4° 33′	+ 1° 42′ = 6° 15′
	52 . . . . 6° 15′	+ 1° 42′ = 7° 57′
P. T	52 + 72.2 . . . . 7° 57′	+ 1° 14′ = 9° 11′

As a check $9° \ 11' = \frac{1}{2}(18° \ 22') = \frac{1}{2}\Delta$. (See the Form of Notes in § 17.)

26. Instrumental work. It is generally impracticable to locate more than 500 to 600 feet of a curve from one station. Obstructions will sometimes require that the transit be moved up every 200 or 300 feet. There are two methods of setting off the angles when the transit has been moved up from the *PC.*

(a) The transit may be sighted at the previous transit station with a reading on the plates equal to the deflection angle from that station to the station occupied, but with the angle set off on the *other* side of 0°, so that when the telescope is turned to 0° it will sight along the tangent at the station occupied. Plunging the telescope, the forward stations may be set off by deflecting the proper deflections from the tangent at the station occupied. This is a very common method and, when the degree of curvature is an even number of degrees and when the transit is only set at even stations, there is but little objection to it. But the degree of curvature is sometimes an odd quantity, and the exigencies of difficult location frequently require that substations be occupied as transit stations. Method (*a*) will then require the recalculation of all deflections for each new station occupied. The mathematical work is largely increased and the probability of error is very greatly increased and not so easily detected. Method (*b*) is just as simple as method (*a*) even for the most simple cases, and for the more difficult cases just referred to the superiority is very great.

(b) Calculate the deflection for each station and substation throughout the curve as though the whole curve were to be located from the *PC.* The computations may thus be completed and *checked* (as above) before beginning the instrumental work. If it unexpectedly becomes necessary to introduce a substation at any point, its deflection from the *PC* may be readily interpolated. The stations actually set from the *PC* are located as usual. **RULE. When the transit is set on any forward station, backsight to ANY previous station with the plates set at the deflection angle for the station sighted at. Plunge the telescope and sight at any forward station with the deflection angle originally computed for that station.** When the plates read the deflection angle for the station occupied, the telescope is sighting along the tangent at that station—which is the method of getting the forward tangent when occupying the *PT.* Even though the station occupied is an unexpected substation, when the instrument is properly oriented at that station, the angle reading for *any* station, forward or back, is that originally computed for it from the *PC.* In difficult work, where there are obstructions, a valuable check on the accuracy may be found by sighting backward at *any* visible station and noting whether its deflection agrees with that originally computed. As a numerical illustration, assume a 4° curve, with 28° curvature, with stations 0, 2, 4, and 7 occupied. After setting stations 1 and 2, set up the transit at sta. 2 and backsight to sta. 0 with the deflection for sta. 0, which is 0°. The reading on sta. 1 is 2°; when the reading is 4° the telescope is tangent to the curve, and when sighting at 3 and 4 the deflections will be 6° and 8°. Occupy 4; sight to 2 with a reading of 4°. When the reading is 8° the telescope is tangent to the curve and, by plunging the telescope, 5, 6, and 7 may be located with the originally com-

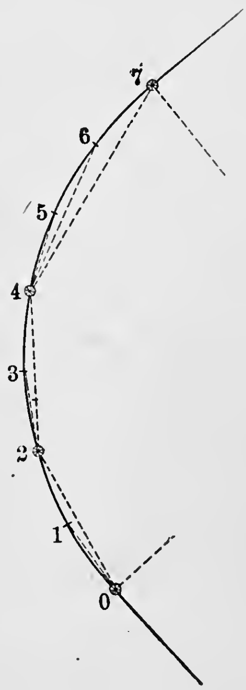


FIG. 11.

puted deflections of 10°, 12°, and 14°. When occupying 7 a backsight may be taken to any visible station with the plates reading the deflection for that station; then when the plates read 14° the telescope will point along the forward tangent.

The location of curves by deflection angles is the normal method. A few other methods, to be described, should be considered as exceptional.

27. Curve location by two transits. A curve might be located more or less on a swamp where accurate chaining would be exceedingly difficult if not impossible. The long chord AB may be determined by triangulation or otherwise, and the elements of

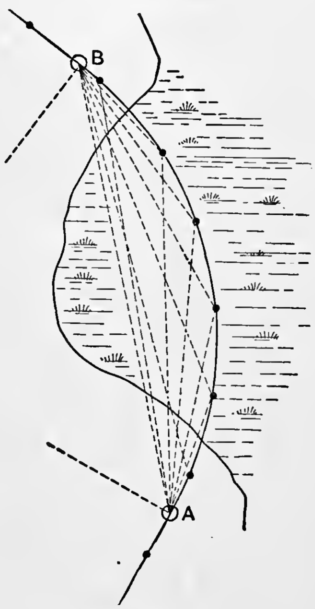


Fig. 12.

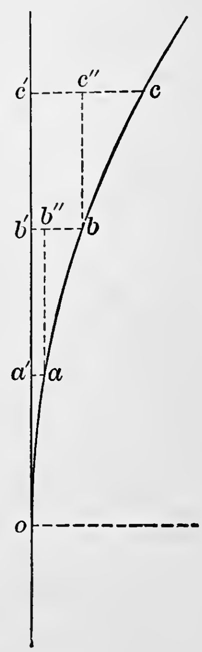


Fig. 13.

the curve computed, including (possibly) subchords at each end. The deflection from A and B to each point may be computed. A rodman may then be sent (by whatever means) to locate long stakes at points determined by the simultaneous sightings of the two transits.

28. Curve location by tangential offsets. When a curve is very flat and no transit is at hand the following method may be used: Produce the back tangent as far forward as necessary. Compute the ordinates Oa', Ob', Oc', etc., and the abscissæ $a'a$, $b'b$, $c'c$, etc. If Oa is a full station (100 feet), then

$$\left.\begin{aligned} Oa' &= Oa' &&= 100\cos\tfrac{1}{2}D, \text{ also} = R\sin D;\\ Ob' &= Oa' + a'b' &&= 100\cos\tfrac{1}{2}D + 100\cos\tfrac{3}{2}D,\\ &&&\qquad \text{also} = R\sin 2D;\\ Oc' &= Oa' + a'b' + b'c' &&= 100(\cos\tfrac{1}{2}D + \cos\tfrac{3}{2}D + \cos\tfrac{5}{2}D),\\ &&&\qquad \text{also} = R\sin 3D; \end{aligned}\right\} \quad (9)$$

etc.

$$\left.\begin{aligned} a'a &= &&100\sin\tfrac{1}{2}D, \text{ also} = R\operatorname{vers} D;\\ b'b &= a'a + b''b &&= 100\sin\tfrac{1}{2}D + 100\sin\tfrac{3}{2}D,\\ &&&\qquad \text{also} = R\operatorname{vers} 2D;\\ c'c &= b'b + c''c &&= 100(\sin\tfrac{1}{2}D + \sin\tfrac{3}{2}D + \sin\tfrac{5}{2}D),\\ &&&\qquad \text{also} = R\operatorname{vers} 3D; \end{aligned}\right\} \quad (10)$$

etc.

The functions $\frac{1}{2}D$, $\frac{3}{2}D$, etc., may be more conveniently used *without* logarithms, by adding the several *natural* trigonometrical functions and pointing off two decimal places. It may also be noted that ob' (for example) is one half of the long chord for four stations; also that $b'b$ is the middle ordinate for four stations. If the engineer is provided with tables giving the long chords and middle ordinates for various degrees of curvature, these quantities may be taken (perhaps by interpolation) from such tables.

If the curve begins or ends at a substation, the angles and terms will be correspondingly altered. The modifications may be readily deduced on the same principles as above, and should be worked out as an exercise by the student.

29. Curve location by middle ordinates. Take first the simpler case when the curve begins at an even station. If we consider (in Fig. 14) the curve produced back to z, the chord $za = 2 \times 100\cos\frac{1}{2}D$, $A'a = 100\cos\frac{1}{2}D$, and $A'A = am = zn = 100\sin\frac{1}{2}D$. Set off AA' perpendicular to the tangent and $A'a$ parallel to the tangent. $AA' = aa' = bb' = cc'$, etc. $= 100\sin\frac{1}{2}D$. Set off aa' perpendicular to $a'A$. Produce Aa'

until $a'b = A'a$, thus determining b. Succeeding points of the curve may thus be determined indefinitely.

Suppose the curve begins with a subchord. As before $ra = Am' = c' \cos \frac{1}{2}d'$, and $rA = am' = c' \sin \frac{1}{2}d'$. Also $sz = An' = c'' \cos \frac{1}{2}d''$, and $sA = zn' = c'' \sin \frac{1}{2}d''$, in which

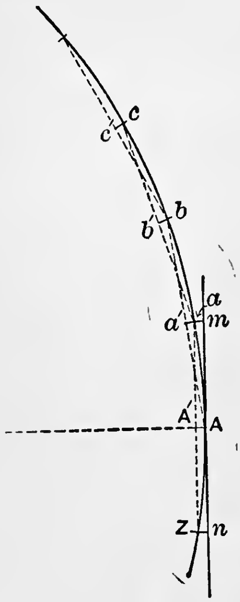


FIG. 14.

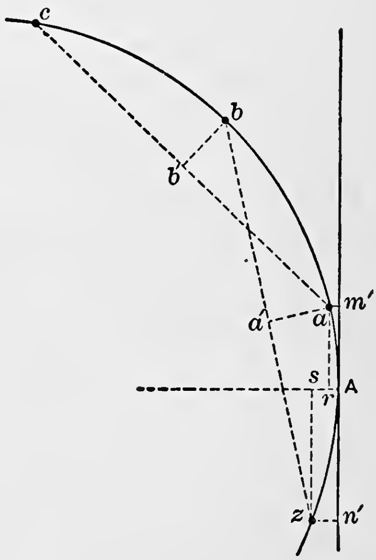


FIG. 15.

$(d' + d'') = D$. The points z and a being determined on the ground, aa' may be computed and set off as before and the curve continued in full stations. A subchord at the end of the curve may be located by a similar process.

30. Curve location by offsets from the long chord. (Fig. 16.) Consider at once the general case in which the curve commences with a subchord (curvature, d'), contains with one or more full chords (curvature of each, D), and ends with a subchord with curvature d''. The numerical work consists in computing first AB, then the various abscissæ and ordinates. $AB = 2R \sin \frac{1}{2}\Delta$.

$$\left.\begin{aligned}
Aa' &= Aa' &&= c' \cos \tfrac{1}{2}(\Delta - d'); \\
Ab' &= Aa' + a'b' &&= c' \cos \tfrac{1}{2}(\Delta - d') + 100 \cos \tfrac{1}{2}(\Delta - 2d' - D); \\
Ac' &= Aa' + a'b' + b'c' &&= c' \cos \tfrac{1}{2}(\Delta - d') + 100 \cos \tfrac{1}{2}(\Delta - 2d' - D) \\
& && \qquad + 100 \cos \tfrac{1}{2}(\Delta - 2d'' - D); \\
\text{also} & \\
&= AB - Bc' &&= 2R \sin \tfrac{1}{2}\Delta - c'' \cos \tfrac{1}{2}(\Delta - d'').
\end{aligned}\right\} \quad (11)$$

$$\left.\begin{aligned} a'a &= a'a &&= c' \sin \tfrac{1}{2}(\Delta - d'); \\ b'b &= a'a + mb &&= c' \sin \tfrac{1}{2}(\Delta - d') + 100 \sin \tfrac{1}{2}(\Delta - 2d' - D); \\ c'c &= b'b - nb &&= c' \sin \tfrac{1}{2}(\Delta - d') + 100 \sin \tfrac{1}{2}(\Delta - 2d' - D) \\ & && \qquad - 100 \sin \tfrac{1}{2}(\Delta - 2d'' - D); \\ \text{also} & &&= c'' \sin \tfrac{1}{2}(\Delta - d''). \end{aligned}\right\} \quad (12)$$

The above formulæ are considerably simplified when the curve begins and ends at even stations. When the curve is very long a regular law becomes very apparent in the formation of all terms between the first and last. There are too few terms in the above equations to show the law.

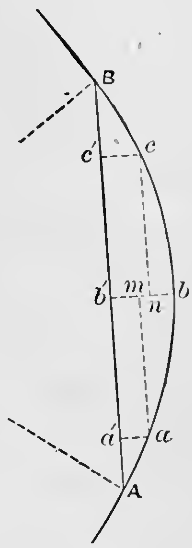


FIG. 16.

31. Use and value of the above methods. The chief value of the above methods lies in the possibility of doing the work without a transit. The same principles are sometimes employed, even when a transit is used, when obstacles prevent the use of the normal method (see § 32, *c*). If the terminal tangents have already been accurately determined, these methods are useful to locate points of the curve when rigid accuracy is not essential. Track foremen frequently use such methods to lay out unimportant sidings, especially when the engineer and his transit are not at hand. Location by tangential offsets (or by offsets from the long chord) is to be preferred when the curve is flat (i.e., has a small central angle Δ) and there is no obstruction along the tangent, or long chord. Location by middle ordinates may be employed regardless of the length of the curve, and in cases when both the tangents and the long chord are obstructed. The above methods are but samples of a large number of similar methods which have been devised. The choice of the particular method to be adopted must be determined by the local conditions.

32. Obstacles to location. In this section will be given only a few of the principles involved in this class of problems, with illustrations. The engineer must decide in each case, which is the best method to use, and it is frequently advisable to devise a special solution for some particular case.

a. **When the vertex is inaccessible.** As shown in § 26, it is not absolutely essential that the vertex of a curve should be located on the ground. But it is very evident that the angle between the terminal tangents is determined with far less probable error if it is measured by a single measurement at the vertex rather than as the result of numerous angle measurements along the curve, involving several positions of the transit and comparatively short sights. Sometimes the location of the tangents is already determined on the ground (as by *bn* and *am*, Fig. 17), and it is required to join the tangents by a curve of given radius. *Method.* Measure *ab* and the angles *Vba* and *baV*. Δ is the sum of these angles. The distances *bV* and *aV* are computable from the above data. Given Δ and *R*, the tan-

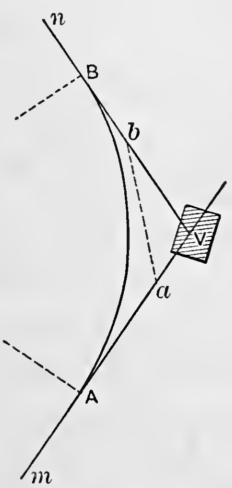


Fig. 17.

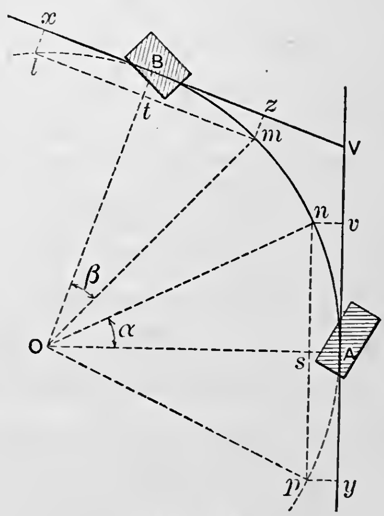


Fig. 18.

gent distances are computable, and then *Bb* and *aA* are found by subtracting *bV* and *aV* from the tangent distances. The curve may then be run from *A*, and the work may be checked by noting whether the curve as run ends at *B*—previously located from *b*.

b. **When the point of curve (or point of tangency) is inaccessible.** At some distance (*As*, Fig. 18) an unobstructed line *pn* may be run parallel with AV. $nv = py = As = R$ vers α.

$$\therefore \quad \text{vers } \alpha = As \div R. \quad ns = ps = R \sin \alpha.$$

At y, which is at a distance ps back from the *computed* position of A, make an offset sA to p. Run pn parallel to the tangent. A tangent to the curve at n makes an angle of α with np. From n the curve is run in as usual.

If the point of tangency is obstructed, a similar process, somewhat reversed, may be used. β is that portion of Δ still to be laid off when m is reached. $tm = tl = R \sin \beta$. $mz = tB = lx = R$ vers β.

c. When the central part of the curve is obstructed. α is the central angle between two points of the curve between which a chord may be run. α may equal *any* angle, but it is preferable that α should be a multiple of D, the degree of curve, and that the points m and n should be on even stations. $mn = 2R \sin \frac{1}{2}\alpha$. A point s may be located by an offset ks from the chord mn by a similar method to that outlined in § 30.

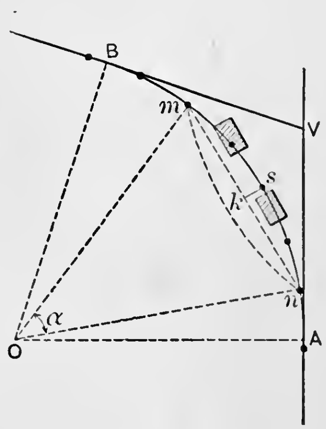


Fig. 19.

The device of introducing the dotted curve mn having the same radius of curvature as the other, although neither necessary nor advisable in the case shown in Fig. 19, is sometimes the best method of surveying around an obstacle. The offset from any point on the dotted curve to the corresponding point on the true curve is twice the "ordinate to the long chord," as computed in § 30.

33. Modifications of location. The following methods may be used in allowing for the discrepancies between the "paper location" based on a more or less rough preliminary survey and the more accurate instrumental location. (See § 15.) They are also frequently used in locating new parallel tracks and modifying old tracks.

a. To move the forward tangent parallel to itself a distance x, the point of curve (A) remaining fixed. (Fig. 20.)

$$V'h = B'r = x'.$$

$$VV' = \frac{V'h}{\sin hVV'} = \frac{x'}{\sin \Delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$AV' = AV + VV'.$$

The triangle BmB' is isosceles and $Bm = B'm$.

$$R' - R = O'O = mB = \frac{B'r}{\text{vers } B'mB} = \frac{x'}{\text{vers } \Delta}$$

$$\therefore \quad R' = R + \frac{x'}{\text{vers } \Delta} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The solution is very similar in case the tangent is moved inward to $V''B''$. Note that this method necessarily changes the

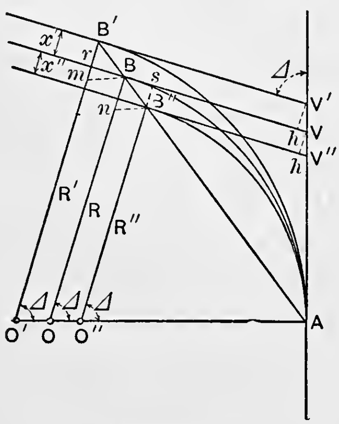


FIG. 20.

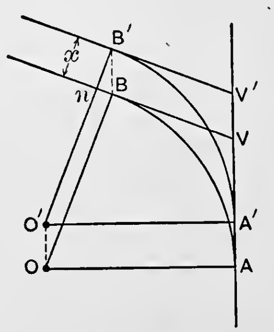


FIG. 21.

radius. If the radius is not to be changed, the point of curve must be altered as follows:

b. To move the forward tangent parallel to itself a distance x, the radius being unchanged. (Fig. 21.) In this case the whole curve is moved bodily a distance $OO' = AA' = VV' = BB'$, and moved parallel to the first tangent AV.

$$BB' = \frac{B'n}{\sin nBB'} = \frac{x}{\sin \Delta} = AA'. \quad . \quad . \quad (15)$$

c. To change the direction of the forward tangent at the point of tangency. (Fig. 22.) This problem involves a change (α) in the central angle and also requires a new radius. An error in the determination of the central angle furnishes an occasion for its use.

R, Δ, α, AV, and BV are known. $\Delta' = \Delta - \alpha$.

$$Bs = R \text{ vers } \Delta. \qquad Bs = R' \text{ vers } \Delta'.$$

$$\therefore \quad R' = R\frac{\text{vers } \Delta}{\text{vers } (\Delta - \alpha)}. \quad . \quad . \quad . \quad . \quad (16)$$

$$As = R \sin \Delta. \qquad A's = R' \sin \Delta'.$$

$$\therefore \quad AA' = A's - As = R' \sin \Delta' - R \sin \Delta. \quad . \quad (17)$$

The above solutions are given to illustrate a large class of problems which are constantly arising. All of the ordinary

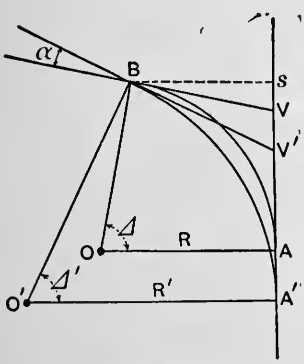


Fig. 22.

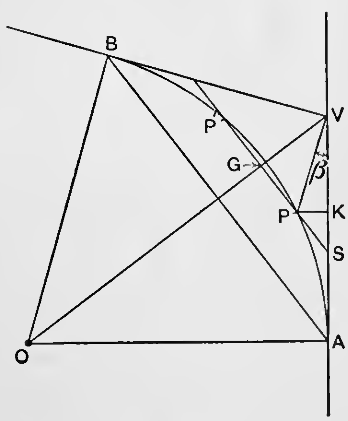


Fig. 23.

problems can be solved by the application of elementary geometry and trigonometry.

34. Limitations in location. It may be required to run a curve that shall join two given tangents and also pass through a given point. The point (P, Fig. 23) is assumed to be determined by its distance (VP) from the vertex and by the angle $AVP = \beta$.

It is required to determine the radius (R) and the tangent distance (AV). Δ is known.

$$PVG = \tfrac{1}{2}(180^\circ - \Delta) - \beta = 90^\circ - (\tfrac{1}{2}\Delta + \beta).$$

$$PP' = 2\,VP \sin PVG \;\; = 2\,VP \cos(\tfrac{1}{2}\Delta + \beta).$$

$$PSV = \tfrac{1}{2}\Delta. \qquad \therefore \quad SP = VP\frac{\sin\beta}{\sin\frac{1}{2}\Delta}.$$

$$AS = \sqrt{SP \times SP'} = \sqrt{SP(SP + PP')}.$$

$$= \sqrt{VP\frac{\sin\beta}{\sin\frac{1}{2}\Delta}\left[VP\frac{\sin\beta}{\sin\frac{1}{2}\Delta} + 2\,VP\cos(\tfrac{1}{2}\Delta + \beta)\right]}$$

$$= VP\sqrt{\frac{\sin^2\beta}{\sin^2\frac{1}{2}\Delta} + \frac{2\sin\beta\cos(\frac{1}{2}\Delta + \beta)}{\sin\frac{1}{2}\Delta}}.$$

$$SV = VP\frac{\sin(\frac{1}{2}\Delta + \beta)}{\sin\frac{1}{2}\Delta}.$$

$$AV = AS + SV$$

$$= \frac{VP}{\sin\frac{1}{2}\Delta}\left[\sin(\tfrac{1}{2}\Delta + \beta) + \sqrt{\sin^2\beta + 2\sin\beta\sin\tfrac{1}{2}\Delta\cos(\tfrac{1}{2}\Delta + \beta)}\right]. \qquad (18)$$

$$R = AV\cot\tfrac{1}{2}\Delta.$$

In the special case in which P is on the median line OV, $\beta = 90^\circ - \frac{1}{2}\Delta$, and $(\frac{1}{2}\Delta + \beta) = 90^\circ$. Eq. (18) then reduces to

$$AV = \frac{VP}{\sin\frac{1}{2}\Delta}(1 + \cos\tfrac{1}{2}\Delta) = VP\cot\tfrac{1}{4}\Delta,$$

as might have been immediately derived from Eq. (8).

In case the point P is given by the offset PK and by the distance VK, the triangle PKV may be readily solved, giving the distance VP and the angle β, and the remainder of the solution will be as above.

35. Determination of the curvature of existing track. (a) *Using a transit.* Set up the transit at any point in the center of the track. Measure in each direction 100 feet to points also in the center of the track. Sight on one point with the plates at 0°. Plunge the telescope and sight at the other point. The angle between the chords equals the degree of curvature.

(b) *Using a tape and string.* Stretch a string (say 50 feet long) between two points on the inside of the head of the outer rail. Measure the ordinate (x) between the *middle* of the string and the head of the rail. Then

$$R = \frac{\text{chord}^2}{8x} \text{ (very nearly)}. \quad . \quad . \quad . \quad . \quad (19)$$

For, in Fig. 24, since the triangles AOE and ADC are similar, $AO : AE :: AD : DC$ or $R = \frac{1}{2}\overline{AD}^2 \div x$. When, as is usual, the arc is very short compared with the radius, $AD = \frac{1}{2}AB$, very nearly. Making this substitution we have Eq. (19). With a chord of 50 feet and a 10° curve, the resulting difference in x is .0025 of an inch—far within the possible accuracy of such a method. The above method gives the radius of the inner head of the outer rail. It should be diminished by $\frac{1}{2}g$ for the radius of the center of the track. With easy curvature, however, this will not affect the result by more than one or two tenths of one per cent.

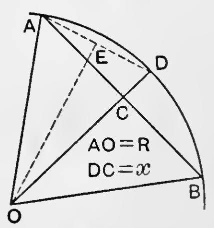


FIG. 24.

The inversion of this formula gives the required middle ordinate for a rail on a given curve. For example, the middle ordinate of a 30-foot rail, bent for a 6° curve, is

$$x = 900 \div (8 \times 955) = .118 \text{ foot} = 1.4 \text{ inches}.$$

Another much used rule is to require the foreman to have a string, knotted at the centre, of such length that the middle ordinate, measured in inches, equals the degree of curve. To find that length, substitute (in eq. (19)) $5730 \div D$ for R and $D \div 12$ for x. Solving for *chord*, we obtain *chord* = 61.8 feet. The rule is not theoretically exact, but, considering the uncertain stretching of the string, the error is insignificant. In fact, the distance usually given is 62 feet, which is close enough for all purposes for which such a method should be used.

36. Problems. A systematic method of setting down the solution of a problem simplifies the work. Logarithms should always be used, and *all* the work should be so set down that a revision of the work to find a supposed error may be readily done. The value of such systematic work will become more apparent as the problems become more complicated. The two solutions given below will illustrate such work.

a. Given a 3° curve beginning at Sta. 27 + 60 and running to Sta. 32 + 45. Compute the ordinates and offsets used in locating the curve by tangential offsets.

b. With the same data as above, compute the distances to locate the curve by offsets from the long chord.

c. Assume that in Fig. 17 ab is measured as 217.6 feet, the angle $abV = 17° 42'$, and the angle $baV = 21° 14'$. Join the tangents by a 4° 30′ curve. Determine bB and aA.

d. Assume that in a case similar to Fig. 18 it was noted that a distance (As) equal to 12 feet would clear the building. Assume that $\Delta = 38° 20'$ and that $D = 4° 40'$. Required the value of α and the position of n. *Solution:*

$\text{vers}\ \alpha = As \div R$	$As = 12$	$\log = 1.07918$
	R (for 4° 40′ curve)	$\log = 3.08923$
	$\alpha = 8° 01'$	$\log \text{vers}\ \alpha = 7.98994$
$ns = R \sin \alpha$		$\log \sin \alpha = 9.14445$
		$\log R = 3.08923$
	$ns = 171.27$	$\log = 2.23369$

e. Assume that the forward tangent of a 3° 20′ curve having a central angle of 16° 50′ must be moved 3.62 feet *inward*, without altering the *P.C.* Required the change in radius.

f. Given two tangents making an angle of 36° 18′. It is required to pass a curve through a point 93.2 feet from the vertex, the line from the vertex to the point making an angle of 42° 21′ with the tangent. Required the radius and tangent distance. *Solution:* Applying eq. (18), we have

2	log = 0.30103
$\beta = 42° 21'$	log sin = 9.82844
$\frac{1}{2}\Delta = 18° 09'$	log sin = 9.4934$\hat{6}$
$(\frac{1}{2}\Delta + \beta) = 60° 30'$	log cos = 9.69234
.20667	9.3152$\hat{7}$
log $\sin^2 \beta$ = 9.65688 45382	
2| 9.81987 66049	
9.9099$\hat{3}$ 81271	
nat sin 60° 30′ = .870$\hat{3}$	
1.683$\hat{6}$	log = 0.22610
VP = 93.2	log = 1.9694$\hat{1}$
	2.1955$\hat{1}$
	log sin $\frac{1}{2}\Delta$ = 9.4934$\hat{6}$
tang. dist. AV = 503.56	log = 2.70205
	log cot $\frac{1}{2}\Delta$ = 10.48437
R = 1536.1	3.18642
D = 3° 44′	

COMPOUND CURVES.

37. Nature and use. Compound curves are formed by a succession of two or more simple curves of different curvature. The curves must have a common tangent at the point of compound curvature (*P.C.C.*). In mountainous regions there is frequently a necessity for compound curves having several changes of curvature. Such curves may be located separately as a succession of simple curves, but a combination of two simple curves has special properties which are worth investigating and utilizing. In the following demonstrations R_2 always represents the *longer* radius and R_1 the *shorter*, no matter which succeeds the other. T_1 is the tangent adjacent to the curve of shorter radius (R_1), and is invariably the shorter tangent. Δ_1 is the central angle of the curve of radius R_1, but it may be greater or less than Δ_2.

38. Mutual relations of the parts of a compound curve having two branches. In Fig. 25, AC and CB are the two branches of

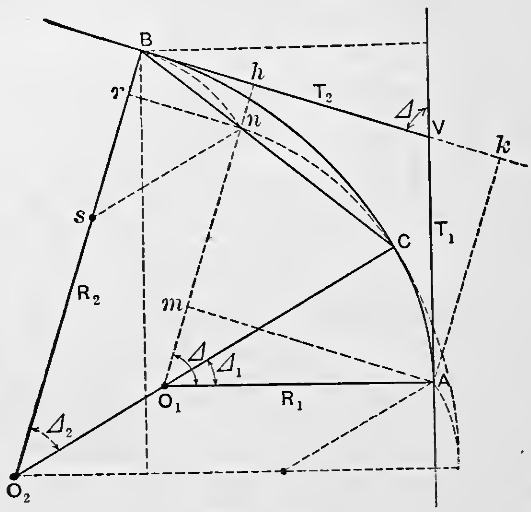


FIG 25.

the compound curve having radii of R_1 and R_2 and central angles of Δ_1 and Δ_2. Produce the arc AC to n so that $Ao_1n = \Delta$. The chord Cn produced *must* intersect B. The line ns, parallel to CO_2, will intersect BO_2 so that $Bs = sn = O_2O_1 = R_2 - R_1$. Draw Am perpendicular to O_1n. It will be parallel to hk.

$$Br = sn \text{ vers } Bsn \qquad = (R_2 - R_1) \text{ vers } \Delta_2;$$

$$mn = AO_1 \text{ vers } AO_1n \quad = R_1 \text{ vers } \Delta;$$

$$Ak = AV \sin AVk \qquad = T_1 \sin \Delta;$$

$$Ak = hm = mn + nh = mn + Br.$$

$$\therefore\ T_1 \sin \Delta = R_1 \text{ vers } \Delta + (R_2 - R_1) \text{ vers } \Delta_2. \quad . \quad (20)$$

Similarly it may be shown that

$$T_2 \sin \Delta = R_2 \operatorname{vers} \Delta - (R_2 - R_1) \operatorname{vers} \Delta_1. \quad . \quad (21)$$

The mutual relations of the elements of compound curves may be solved by these two equations. For example, assume the tangents as fixed (Δ therefore known) and that a curve of given radius R_1 shall start from a given point at a distance T_1 from the vertex, and that the curve shall continue through a given angle Δ_1. Required the other parts of the curve. From Eq. (20) we have

$$R_2 - R_1 = \frac{T_1 \sin \Delta - R_1 \operatorname{vers} \Delta}{\operatorname{vers} \Delta_2}.$$

$$\therefore R_2 = R_1 + \frac{T_1 \sin \Delta - R_1 \operatorname{vers} \Delta}{\operatorname{vers} (\Delta - \Delta_1)}. \quad . \quad . \quad (22)$$

T_2 may then be obtained from Eq. (21).

As another problem, given the location of the two tangents, with the two tangent distances (thereby locating the PC and PT), and the central angle of each curve; required the two radii. Solving Eq. (20) for R_1, we have

$$R_1 = \frac{T_1 \sin \Delta - R_2 \operatorname{vers} \Delta_2}{\operatorname{vers} \Delta - \operatorname{vers} \Delta_2}.$$

Similarly from Eq. (21) we may derive

$$R_1 = \frac{T_2 \sin \Delta - R_2 (\operatorname{vers} \Delta - \operatorname{vers} \Delta_1)}{\operatorname{vers} \Delta_1}.$$

Equating these, reducing, and solving for R_2, we have

$$R_2 = \frac{T_1 \sin \Delta \operatorname{vers} \Delta_1 - T_2 \sin \Delta (\operatorname{vers} \Delta - \operatorname{vers} \Delta_2)}{\operatorname{vers} \Delta_2 \operatorname{vers} \Delta_1 - (\operatorname{vers} \Delta - \operatorname{vers} \Delta_1)(\operatorname{vers} \Delta - \operatorname{vers} \Delta_2)}. \quad (23)$$

Although the various elements may be chosen as above with considerable freedom, there are limitations. For example, in Eq. (22), since R_2 is always greater than R_1, the term to be added to R_1 must be essentially positive—i.e., $T_1 \sin \Delta$ must be greater than R_1 vers Δ. This means that $T_1 > R_1 \frac{\text{vers } \Delta}{\sin \Delta}$, or that $T_1 > R_1 \tan \frac{1}{2}\Delta$, or that T_1 is greater than the corresponding tangent on a simple curve. Similarly it may be shown that T_2 is *less* than $R_2 \tan \frac{1}{2}\Delta$ or *less* than the corresponding tangent on a simple curve. Nevertheless T_2 is always greater than T_1. In the limiting case when $R_2 = R_1$, $T_2 = T_1$ and $\Delta_2 = \Delta_1$.

39. Modifications of location. Some of these modifications may be solved by the methods used for simple curves. For example:

a. It is desired to move the tangent VB, Fig. 26, parallel to itself to $V'B'$. Run a new curve from the $P.C.C.$ which shall reach the new tangent at B', where the chord of the old curve

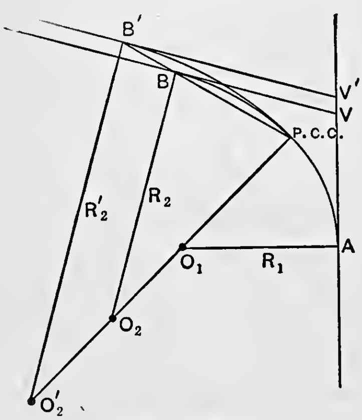


FIG. 26.

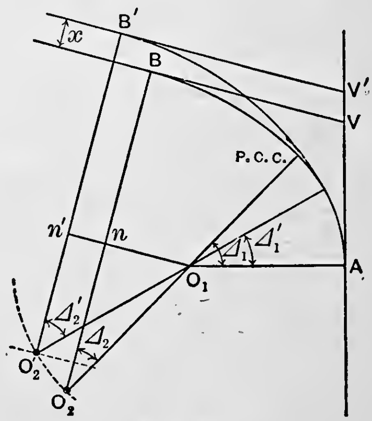


FIG. 27.

intersects the new tangent. The solution is almost identical with that in § 33, *a*.

b. Assume that it is desired to change the forward tangent (as above) but to retain the same radius. In Fig. 27

$$(R_2 - R_1) \cos \Delta_2 = O_2 n;$$

$$(R_2 - R_1) \cos \Delta_2' = O_2' n'.$$

$$x = O_2 n - O_2' n' = (R_2 - R_1)(\cos \Delta_2 - \cos \Delta_2').$$

$$\cos \Delta_2' = \cos \Delta_2 - \frac{x}{R_2 - R_1}. \quad . \quad . \quad . \quad (24)$$

The *P.C.C.* is moved *backward* along the sharper curve an angular distance of $\Delta_2' - \Delta_2 = \Delta_1 - \Delta_1'$.

In case the tangent is moved inward rather than outward, the solution will apply by transposing Δ_2 and Δ_2'. Then we will have

$$\cos \Delta_2' = \cos \Delta_2 + \frac{x}{R_2 - R_1}. \quad . \quad . \quad . \quad (25)$$

The *P.C.C.* is then moved *forward.*

c. Assume the same case as (**b**) except that the larger radius comes first and that the tangent adjacent to the smaller radius is moved. In Fig. 28

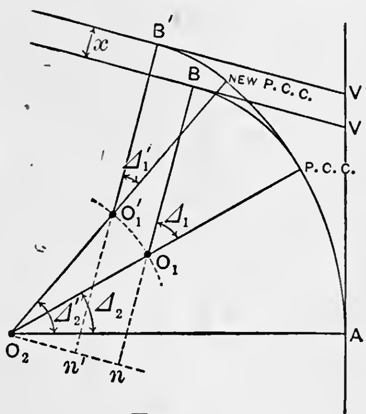


Fig. 28.

$$(R_2 - R_1) \cos \Delta_1 = O_1 n;$$

$$(R_2 - R_1) \cos \Delta_1' = O_1' n'.$$

$$x = O_1' n' - O_1 n = (R_2 - R_1)(\cos \Delta_1' - \cos \Delta_1).$$

$$\cos \Delta_1' = \cos \Delta_1 + \frac{x}{R_2 - R_1}. \quad . \quad . \quad . \quad (26)$$

The *P.C.C.* is moved *forward* along the easier curve an angular distance of $\Delta_1' - \Delta_1 = \Delta_2 - \Delta_2'$.

In case the tangent is moved *inward*, transpose as before and we have

$$\cos \Delta_1' = \cos \Delta_1 - \frac{x}{R_2 - R_1}. \quad . \quad . \quad . \quad (27)$$

The *P.C.C.* is moved *backward.*

d. Assume that the radius of one curve is to be altered without changing either tangent. Assume conditions as in Fig. 29. For the diagrammatic solution assume that R_2 is to be increased by O_2S. Then, since R_2' must pass through O_1 and extend beyond O_1 a distance O_1S, the locus of the new center must lie on the arc drawn about O_1 as center and with OS as radius. The locus of O_2' is also given by a line $O_2'p$ parallel to BV and at a distance of R_2' (equal to $S \ldots P.C.C.$) from it. The new center is therefore at the intersection O_2'. An arc with radius R_2' will therefore be tangent at B' and tangent to the old curve *produced* at NEW $P.C.C.$ Draw O_1n' perpendicular to O_2B. With O_2 as center draw the arc O_1m, and with O_2' as center draw the arc O_1m'. $mB = m'B' = R_1$. $\therefore mn = m'n' = (R_2' - R_1) \text{ vers } \Delta_2' = (R_2 - R_1) \text{ vers } \Delta_2$.

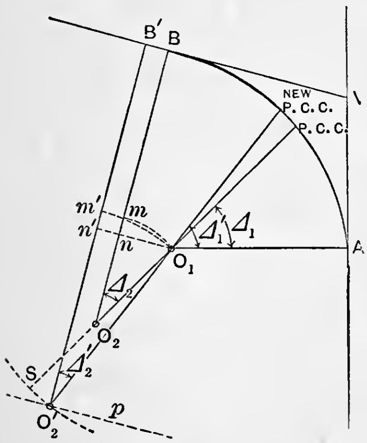


FIG. 29.

$$\therefore \text{vers } \Delta_2' = \frac{(R_2 - R_1)}{(R_2' - R_1)} \text{ vers } \Delta_2. \quad . \quad . \quad . \quad (28)$$

$$O_1n = (R_2 - R_1) \sin \Delta_2;$$

$$O_1n' = (R_2' - R_1) \sin \Delta_2'.$$

$$BB' = O_1n' - O_1n = (R_2' - R_1) \sin \Delta_2' - (R_2 - R_1) \sin \Delta_2. \quad (29)$$

This problem may be further modified by assuming that the radius of the curve is decreased rather than increased, or that the smaller radius follows the larger. The solution is similar and is suggested as a profitable exercise.

It might also be assumed that, instead of making a given change in the radius R_2, a given change BB' is to be made. Δ_2' and R_2' are required. Eliminate R_2' from Eqs. 28 and 29 and solve the resulting equation for Δ_2'. Then determine R_2' by a suitable inversion of either Eq. 28 or 29.

As in §§ 32 and 33, the above problems are but a few, although perhaps the most common, of the problems the engineer may meet with in compound curves. All of the ordinary problems may be solved by these and similar methods.

40. Problems. *a.* Assume that the two tangents of a compound curve are to be 348 feet and 624 feet, and that $\Delta_1 = 22° 16'$ and $\Delta_2 = 28° 20'$. Required the radii.

[*Ans.* $R_1 = 326.92$; $R_2 = 1574.85$.]

b. A line crosses a valley by a compound curve which is first a 6° curve for 46° 30′ and then a 9° 30′ curve for 84° 16′. It is afterward decided that the last tangent should be 6 feet farther up the hill. What are the required changes? [*Note.* The second tangent is evidently moved *outward.* The solution corresponds to that in the first part of § 39, *c.* The *P.C.C.* is moved forward 16.39 feet. If it is desired to know how far the *P.T.* is moved in the direction of the tangent (i.e., the *projection* of *BB′*, Fig. 28, on *V′B′*), it may be found by observing that it is equal to $nn' = (R_2 - R_1)(\sin \Delta_1 - \sin \Delta_1')$. In this case it equals 0.65 foot, which is very small because Δ_1 is nearly 90°. The value of Δ_2 (46° 30′) is not used, since the solution is independent of the value of Δ_2. The student should learn to recognize which quantities are mutually related and therefore essential to a solution, and which are independent and non-essential.]

TRANSITION CURVES.

41. Superelevation of the outer rail on curves. When a mass is moved in a circular path it requires a centripetal force to keep it moving in that path. By the principles of mechanics we know that this force equals $Gv^2 \div gR$, in which G is the weight, v the velocity in feet per second, g the acceleration of gravity in feet per second in a second, and R the radius of curvature. If the two rails of a curved track were laid on a level (transversely), this centripetal force could only be furnished by the pressure of the wheel-flanges against the rails. As this is very objectionable, the outer rail is elevated so that the reaction of the rails against the wheels shall contain a horizontal component equal to the required centripetal force. In Fig. 30, if ob represents the reaction, oc will represent the weight G, and ao will represent the required centripetal force. From similar triangles we may write $sn : sm :: ao : oc$. Call $g = 32.17$. Call $R = 5730 \div D$, which is sufficiently accurate

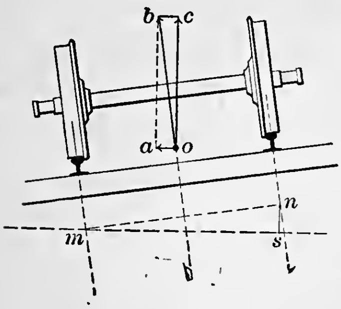


Fig. 30.

for this purpose (see § 19). Call $v = 5280\,V \div 3600$, in which V is the velocity in miles per hour. mn is the distance between rail centers, which, for an 80-lb. rail and standard gauge, is 4.916 feet. sm is slightly less than this. As an average value we may call it 4.900, which is its exact value when the superelevation is $4\frac{3}{4}$ inches. Calling $sn = e$, we have

$$e = sm\frac{ao}{oc} = 4.9\frac{Gv^2}{gR}\frac{1}{G} = \frac{4.9 \times 5280^2\,V^2D}{32.17 \times 3600^2 \times 5730}.$$

$$e = .0000572\,V^2D. \qquad \ldots\ldots\ldots \qquad (30)$$

It should be noticed that, according to this formula, the required superelevation varies as the *square* of the velocity, which means that a change of velocity of only 10% would call for a change of superelevation of 21%. Since the velocities of trains over any road are extremely variable, it is impossible to adopt any superelevation which will fit all velocities even approximately. The above fact also shows why any over-refinement in the calculations is useless and why the above approximations, which are really small, are amply justifiable. For example, the above formula contains the approximation that $R = 5730 \div D$. In the extreme case of a 10° curve the error involved would be about 1%. A change of about $\frac{1}{2}$ of 1% in the velocity, or say from 40 to 40.2 miles per hour, would mean as much. The error in e due to the assumed constant value of sm is never more than a very small fraction of 1%. The rail-laying is not done closer than this. The following tabular form is based on Eq. 30:

SUPERELEVATION OF THE OUTER RAIL (IN FEET) FOR VARIOUS VELOCITIES AND DEGREES OF CURVATURE.

Velocity in Miles per Hour.	Degree of Curve.									
	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
30	.05	.10	.15	.20	.26	.31	.36	.41	.46	.51
40	.09	.18	.27	.37	.46	.55	.64	.73	.82	
50	.14	.29	.43	.57	.71	.86				
60	.20	.41	.62	.82						

42. Practical rules for superelevation. A much used rule for superelevation is to "elevate one half an inch for each degree of curvature." The rule is rational in that e in Eq. 30 varies directly as D. The above rule therefore agrees with Eq. 30 when V is about 27 miles per hour. However applicable the rule may have been in the days of low velocities, the elevation thus computed is too small now.

Another (and better) rule is to "elevate for the speed of the fastest trains." This rule is further justified by the fact that a four-wheeled truck, having two parallel axles, will always tend to run to the outer rail and will require considerable flange pressure to guide it along the curve. The effect of an excess of superelevation on the slower trains will only be to relieve this flange pressure somewhat. This rule is coupled with the limitation that the elevation should never exceed a limit of six inches—sometimes eight inches. This limitation implies that locomotive engineers must reduce the speed of fast trains around sharp curves until the speed does not exceed that for which the actual superelevation used is suitable. The heavy line in the tabular form (§ 41) shows the six-inch limitation.

Some roads furnish their track foremen with a list of the superelevations to be used on each curve in their sections. This method has the advantage that each location may be separately studied, and the proper velocity, as affected by local conditions (*e.g.*, proximity to a stopping-place for all trains), may be determined and applied.

Another method is to allow the foremen to determine the superelevation for each curve by a simple measurement taken at the curve. The rule is developed as follows: By an inversion of Eq. 19 we have

$$x = chord^2 \div 8R \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Putting x equal to e in Eq. 30 and solving for "*chord*," we have

$$chord^2 = .0000572\,V^2D8R$$
$$= 2.621\,V^2.$$
$$chord = 1.62\,V. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

To apply the rule, assume that 50 miles per hour is fixed as the velocity from which the superelevation is to be computed. Then $1.62\,V = 1.62 \times 50 = 81$ feet, which is the distance given to the trackmen. Stretch a tape (or even a string) with a length of 81 feet between two points on the inside head of the outer rail or the outer head of the inner rail. The ordinate at the middle point then equals the superelevation. The values of this chord length for varying velocities are given in the accompanying tabular form.

Velocity in miles per hour......	20	25	30	35	40	45	50	55	60
Chord length in feet	32.4	40.5	48.6	56.7	64.8	72.9	81.0	89.1	97.2

43. Transition from level to inclined track. On curves the track is inclined transversely; on tangents it is level. The transition from one condition to the other must be made gradually. If there is no transition curve, there must be either inclined track on the tangent or insufficiently inclined track on the curve or both. Sometimes the full superelevation is continued through the total length of the curve and the "run-off" (having a length of 100 to 200 feet) is located entirely on the tangents at each end. In other practice it is located partly on the tangent and partly on the curve. Whatever the method, the superelevation is correct at only one point of the run-off. At all other points it is too great or too small. This (and other causes) produces objectionable lurches and resistances when entering and leaving curves. The object of transition curves is to obviate these resistances.

44. Fundamental principle of transition curves. If a curve has variable curvature, beginning at the tangent with a curve of infinite radius, and the curvature gradually sharpens until it equals the curvature of the required simple curve and there becomes tangent to it, the superelevation of such a transition curve may begin at zero at the tangent, gradually increase to the required superelevation for the simple curve, and yet have at every point the superelevation required by the curvature at that point. Since in Eq. (30) e is directly proportional to D, the required curve must be one in which the degree of curve increases directly as the distance along the curve. The mathematical development of such a curve is quite complicated. It has, however, been developed, and tables have been computed for its use, by Prof. C. L. Crandall. The following method has the advantage of great simplicity, while its agreement with the true transition curve is as close as need be, as will be shown.

45. Multiform compound curves. If the transition curve commences with a very flat curve and at regular even chord lengths compounds into a curve of sharper curvature until the desired curvature is reached, the increase in curvature at each chord point being uniform, it is plain that such a curve is a close approximation to the true spiral, especially since the rails as laid will *gradually* change their curvature rather than maintain a uniform curvature throughout each chord length and then abruptly change the curvature at the chord points. Such a curve, *as actually laid*, will be a much closer approximation to the true curve than the multiform compound curve by which it is set out. There will actually be a *gradual* increase in curvature which increases directly as the length of the curve.

46. Required length of spiral. The required length of spiral evidently depends on the amount of superelevation to be gained, and also depends somewhat on the speed. If the spiral is laid off in 25-foot chord lengths, with the first chord subtending a 1° curve, the second a 2° curve, etc., the fifth chord will subtend a 5° curve, and the increase from this last chord to a 6° curve is the same as the uniform increase of curvature between the chords. The same spiral extended would run on to a 12° curve in $(12 - 1)25 = 275$ feet. The last chord of a spiral should have a smaller degree of curvature than the simple curve to which it is joined. If the curves are very sharp, such as are used in street work and even in suburban trolley work, an increase in degree of curvature of 1° per 25 feet will not be sufficiently rapid, as such a rate would require too long curves. 2°, 10°, or even 20° increase per 25 feet may be necessary, but then the chords should be reduced to 5 feet. Such a rapid rate of increase is justified by the necessary reduction in speed. On the other hand, very high speed will make a lower rate of increase desirable, and therefore a spiral whose degree of curvature increases only 0° 30′ per 25 feet may be used. Such a spiral would require a length of 375 feet to run on to an 8° curve, which is inconveniently long, but it might be used to run on to a 4° curve, where its length would be only 175 feet. Three spirals have been developed in Table IV, each with chords of 25 feet, the rate of increase in the degree of curvature being 0° 30′, 1° and 2° per chord. One of these will be suitable for any curvature found on ordinary steam-railroads.

47. To find the ordinates of a 1°-per-25-feet spiral. Since the first chord subtends a 1° curve, its central angle is 0° 15′ and the angle aQV (Fig. 31) is 7′ 30″. The tangent at a makes an angle of 15′ with VQ. The angle between the chord ba and the tangent at a is $\frac{1}{2}(30') = 15'$, and the angle $bab'' = \frac{1}{2}(30') + 15' = 30'$. Similarly the angle $cbc'' = \frac{1}{2}(45') + 30' + 15' = 67'\ 30' = 1°\ 07'\ 30''$, and the angle dcd'' is $2°\ 0'$. The ordinate $aa' = 25 \sin 7'\ 30''$, and $Qa' = 25 \cos 7'\ 30''$. $Qb' = Qa' + a'b' = Qa' + ab'' = 25\ (\cos 7'\ 30'' + \cos 30')$. $bb' = b'b'' + bb'' = 25\ (\sin 7'\ 30'' + \sin 30')$. Similarly the ordinates of c, d, etc., may be obtained.

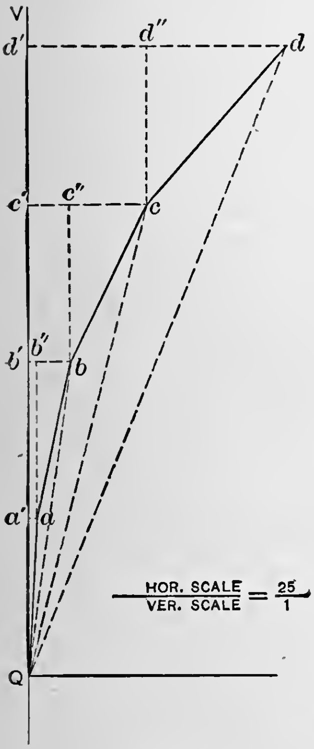


FIG. 31.

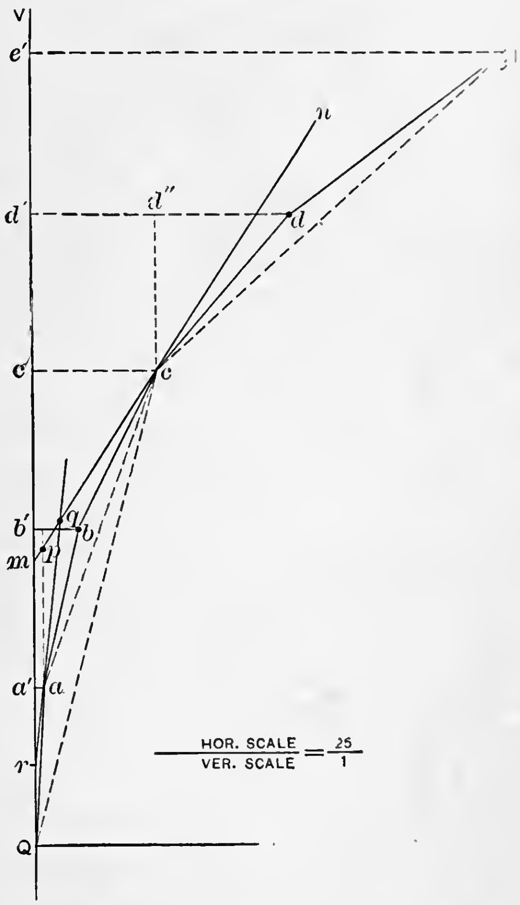


FIG. 32.

48. To find the deflections from any point of the spiral. $aQV = 7'\ 30''$. Tan $bQV = bb' \div Qb'$; tan $cQV = cc' \div Qc'$; etc. Thus we are enabled to find the deflection angles from the tangent at Q to any point of the spiral.

The tangent to the curve at c (Fig. 32) makes an angle of $1°\ 30'$ with QV, or $cmV = 1°\ 30'$. $Qcm = cmV - cQm$. The value of cQm is known from previous work. The deflection from c to Q then becomes known.

$acm = cmV - cap = cmV - caq - qap$. caq is the deflection angle to c from the tangent at a and will have been previously computed numerically. $qap = 15'$. acm therefore becomes known.

$$bcm = \tfrac{1}{2} \text{ of } 45' = 22'\ 30'';$$

$$dcn = \tfrac{1}{2} \text{ of } 60' = 30'.$$

$ecn = ecd'' - ncd''$, $ncd'' = cmV$, $\tan ecd'' = (ee' - d''d') \div c'e'$, all of which are known from the previous work.

By this method the deflections from the tangent at any

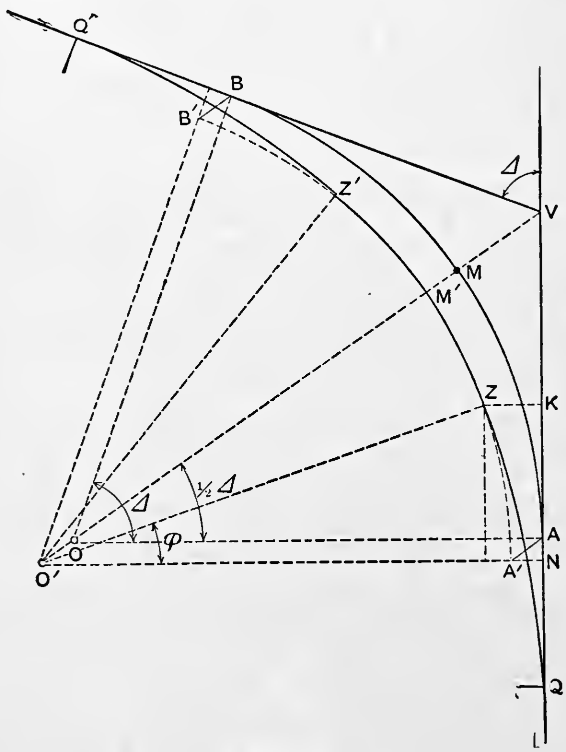


Fig. 33.

point of the curve to any other point are determinable. These values are compiled in Table IV. The corresponding values of these angles when the increase in the degree of curvature per chord length is 30′, and when it is 2°, are also given in Table IV.

49. Connection of spiral with circular curve and with tangent. See Fig. 33.* Let AV and BV be the tangents to be connected by a $D°$ curve, having a suitable spiral at each end. If no spirals were to be used, the problem would be solved as in simple curves giving the curve AMB. Introducing the spiral has the effect of throwing the curve away from the vertex a distance MM' and reducing the central angle of the $D°$ curve by 2ϕ. Continuing the curve beyond Z and Z' to A' and B', we will have $AA' = BB' = MM'$. $ZK =$ the x ordinate and is therefore known. Call $MM' = m$. $A'N = x - R \text{ vers } \phi$. Then

$$m = MM' = AA' = \frac{A'N}{\cos \frac{1}{2}\Delta} = \frac{x - R \text{ vers } \phi}{\cos \frac{1}{2}\Delta}. \quad . \quad . \quad (33)$$

$$NA = AA' \sin \tfrac{1}{2}\Delta = (x - R \text{ vers } \phi) \tan \tfrac{1}{2}\Delta.$$

$$\begin{aligned} VQ &= QK - KN + NA + AV \\ &= y - R \sin \phi + (x - R \text{ vers } \phi) \tan \tfrac{1}{2}\Delta + R \tan \tfrac{1}{2}\Delta \\ &= y - R \sin \phi + x \tan \tfrac{1}{2}\Delta + R \cos \phi \tan \tfrac{1}{2}\Delta. \quad . \quad (34) \end{aligned}$$

When $A'N$ has already been computed, it may be more convenient to write

$$VQ = y + R (\tan \tfrac{1}{2}\Delta - \sin \phi) + A'N \tan \tfrac{1}{2}\Delta. \qquad (35)$$

$$\begin{aligned} VM' &= VM + MM' \\ &= R \text{ exsec } \tfrac{1}{2}\Delta + \frac{x}{\cos \frac{1}{2}\Delta} - \frac{R \text{ vers } \phi}{\cos \frac{1}{2}\Delta}. \quad . \quad . \quad (36) \end{aligned}$$

$$\begin{aligned} AQ &= VQ - AV \\ &= y - R \sin \phi + (x - R \text{ vers } \phi) \tan \tfrac{1}{2}\Delta. \qquad (37) \end{aligned}$$

Example. To join two tangents making an angle of 34° 20′ by a 5° 40′ curve and suitable spirals. Use 1°-per-25-feet

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* The student should at once appreciate the fact of the necessary distortion of the figure. The distance MM' in Fig. 33 is perhaps 100 times its real proportional value.

spirals with five chords. Then $\phi = 3° 45'$, $x = 2.999$, $\frac{1}{2}\Delta = 17° 10'$, and $y = 124.942$.

(Eq. 33)			R	3.00497
			vers ϕ	7.33063
		2.166		0.33560
		$x =$ 2.999		
		$A'N =$ 0.833		9.92064
			cos $\frac{1}{2}\Delta$	9.98021
	$m = MM' = AA' =$	0.872		9.94043
(Eq. 36)			R	3.00497
			exsec $\frac{1}{2}\Delta$	8.66863
		$VM =$ 47.164		1.67366
		$m =$ 0.872		
		$VM' =$ 48.036		
(Eq. 35)	$y =$ 124.942	nat. tan $\frac{1}{2}\Delta$ = .30891		
		nat. sin ϕ = .06540		
		.24351		9.38651
			R	3.00497
	246.314			2.39148
		[See above]	$A'N$	9.92064
			tan $\frac{1}{2}\Delta$	9.48984
	0.257		AN	9.41048
	$VQ =$ 371.513			
(Eq. 37)			R	3.00497
			tan $\frac{1}{2}\Delta$	9.48984
	312.471		AV	2.49481
	$AQ =$ 59.042			

50. Field-work. When the spiral is designed during the original location, the tangent distance VQ should be computed and the point Q located. It is hardly necessary to locate all of the points of the spiral until the track is to be laid. The extremities should be located, and as there will usually be one and perhaps two full station points on the spiral, these should also be located. Z may be located by setting off $QK = y$ and $KZ = x$, or else by the tabular deflection for Z from Q and the distance ZQ, which is the long chord. Setting up the instrument at Z and sighting back at Q with the proper deflection, the tangent at Z may be found and the circular curve located as usual, its central angle being $\Delta - 2\phi$. A similar operation will locate Q' from Z'.

To locate points on the spiral. Set up at Q, with the plates reading 0° when the telescope sights along VQ. Set off from Q the deflections given in Table IV for the instrument at Q, using a chord length of 25 feet, the process being like the method for simple curves except that the deflections are irregular. If a full station-point occurs within the spiral, interpolate between the deflections for the adjacent spiral-points. For example, a spiral begins at Sta. 56 + 15. Sta. 57 comes 10 feet beyond the third spiral point. The deflection for the third point is 35′ 0″; for the fourth it is 56′ 15″. $\frac{10}{25}$ of the difference (21′ 15″) is 8′ 30″; the deflection for Sta. 57 is therefore 43′ 30″. This method is not theoretically accurate, but the error is small. Arriving at z, the forward alignment may be obtained by sighting back at Q (or at any other point) with the given deflection for that point from the station occupied. Then when the plates read 0° the telescope will be tangent to the spiral and to the succeeding curve. All rear points should be checked from z. If it is necessary to occupy an intermediate station, use the deflections given for that station, orienting as just explained for z, checking the back points and locating all forward points up to z if possible.

After the center curve has been located and z' is reached, the other spiral must be located but *in reverse order*, *i.e.*, the sharp curvature of the spiral is at z' and the curvature decreases toward Q'.

51. To replace a simple curve by a curve with spirals. This *may* be done by the method of § 49, but it involves shifting the whole track a distance m, which in the given example equals 0.87 foot. Besides this the track is appreciably shortened, which would require rail-cutting. But the track may be kept at practically the same length and the lateral deviation from the old track may be made very small by slightly sharpening the curvature of the old track, moving the new curve so that it is wholly or partially *outside* of the old curve, the remainder of it with the spirals being *inside* of the old curve. It is found by experience that a decrease in radius of from 1% to 5% will answer

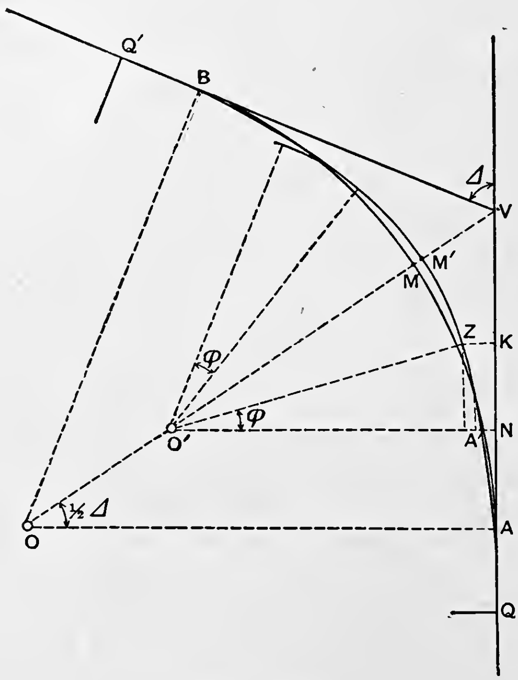

FIG. 34.

the purpose. The larger the central angle the less the change. The solution is as indicated in Fig. 34.

$$O'N = R' \cos \phi + x.$$

$$O'V = O'N \sec \tfrac{1}{2}\Delta$$
$$= R' \cos \phi \sec \tfrac{1}{2}\Delta + x \sec \tfrac{1}{2}\Delta.$$

$$m = MM' = MV - M'V$$
$$= R \operatorname{exsec} \tfrac{1}{2}\Delta - (O'V - R')$$
$$= R \operatorname{exsec} \tfrac{1}{2}\Delta - R' \cos \phi \sec \tfrac{1}{2}\Delta - x \sec \tfrac{1}{2}\Delta + R'. \qquad (38)$$

$$AQ = QK - KN + NV - VA$$
$$= y - R' \sin \phi + (R' \cos \phi + x) \tan \tfrac{1}{2}\Delta - R \tan \tfrac{1}{2}\Delta$$
$$= y - R' \sin \phi + R' \cos \phi \tan \tfrac{1}{2}\Delta - (R - x) \tan \tfrac{1}{2}\Delta. \qquad (39)$$

The length of the old curve from Q to $Q' = 2AQ + 100\frac{\Delta}{D}$.

The length of the new curve from Q to $Q' = 2L + 100\frac{\Delta - 2\phi}{D'}$,

in which L is the length of each spiral.

Example. Suppose the old curve is a 7° 30′ curve with a central angle of 38° 40′. As a trial, compute the relative length of a new 8° curve with spirals of seven chords. $\phi = 7° 0'$; $\frac{1}{2}\Delta = 19° 20'$; R (for the 7° 30′ curve) $= 764.489$; R' (for the 8° curve) $= 716.779$; $x = 7.628$.

[Eq. 38]			R	2.88337
			exsec $\frac{1}{2}\Delta$	8.77642
	45.687			1.65979
	$R' =$ 716.779			
	762.466		R'	2.85538
			cos ϕ	9.99675
			sec $\frac{1}{2}\Delta$	0.02521
		753.953		2.87734
			x	0.88241
			sec $\frac{1}{2}\Delta$	0.02521
		8.084		0.90762
	762.037	762.037		
	$m =$ 0.429			
[Eq. 39]	$y =$ 174.722		R'	2.85538
			sin ϕ	9.08589
		87.353		1.94128
			R'	2.85538
			cos ϕ	9.99675
			tan $\frac{1}{2}\Delta$	9.54512
	249.606			2.39725
			$R =$ 764.489	
			$x =$ 7.628	
			756.861	2.87901
			tan $\frac{1}{2}\Delta$	9.54512
		265.543		2.42413
	424.328	352.896		
	352.896			
	$AQ =$ 71.432			

The length of the old curve from Q to Q' is

$$100\frac{\Delta}{D} = 100\frac{38.667}{7.5} = \qquad 515.556$$

$$2AQ = 2 \times 71.432 = \qquad \underline{142.864}$$

$$658.420$$

$$\text{New curve: } 100\frac{\Delta - 2\phi}{D'} = 100\frac{38.667 - 14.000}{8.0} = 308.333$$

$$2L = 2 \times 175 = \underline{350.000}$$

$$658.333 \qquad \underline{658.333}$$

$$\text{Difference in length} = \quad 0.087$$

Considering that this difference may be divided among 22 joints (using 30-foot rails) no rail-cutting would be necessary. If the difference is too large, a slight variation in the value of the new radius R' will reduce the difference as much as necessary. A truer comparison of the lengths would be found by comparing the lengths of the arcs.

52. Application of transition curves to compound curves. Since compound curves are only employed when the location is limited by local conditions, the elements of the compound curve should be determined (as in §§ 38 and 39) regardless of the transition curves, depending on the fact that the lateral shifting of the curve when transition curves are introduced is very small. If the limitations are very close, an estimated allowance may be made for them.

Methods have been devised for inserting transition curves between the branches of a compound curve, but the device is complicated and usually needless, since when the train is once on a curve the wheels press against the outer rail steadily and a change in curvature will not produce a serious jar even though the superelevation is temporarily a little more or less than it should be.

If the easier curve of the compound curve is less than 3° or 4°, there may be no need for a transition curve off from that branch. This problem then has two cases according as transition curves are used at both ends or at one end only.

a. *With transition curves at both ends.* Adopting the method of § 49, calling $\Delta_1 = \frac{1}{2}\Delta$, we may compute $m_1 = MM_1'$. Similarly, calling $\Delta_2 = \frac{1}{2}\Delta$, we may compute $m_2 = MM_2'$. But

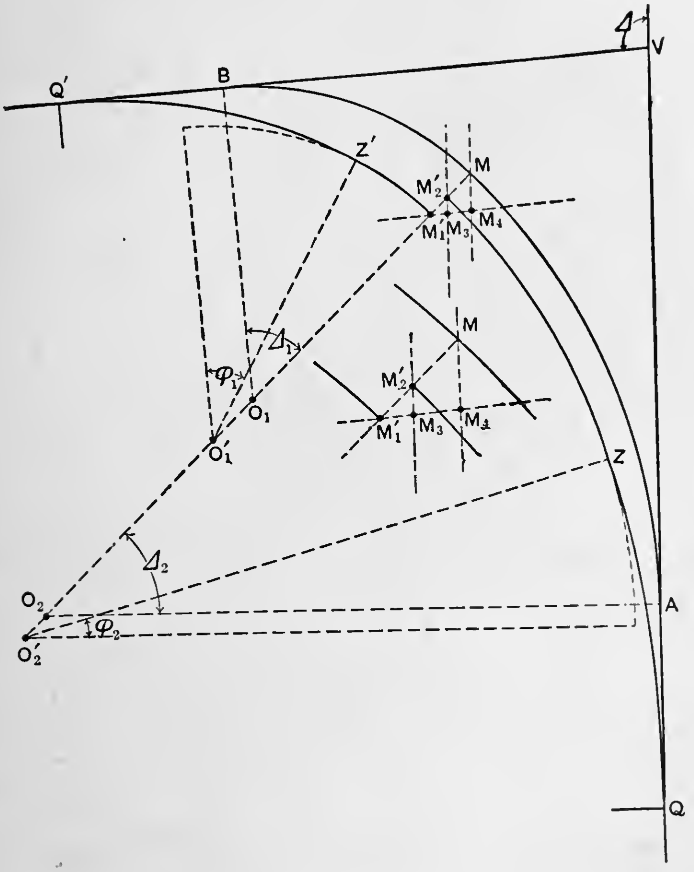

FIG. 35.

M_1' and M_2' must be made to coincide. This may be done by moving the curve $Z'M_1'$ and its transition curve parallel to $Q'V$ a distance $M_1'M_3$, and the other curve parallel to QV a distance $M_2'M_3$. In the triangle $M_1'M_3M_2'$, the angle at $M_1' = 90° - \Delta_1$, the angle at $M_2' = 90° - \Delta_2$, and the angle at $M_3 = \Delta$.

$$\left.\begin{aligned} \text{Then}\quad M_1'M_3 &= M_1'M_2'\frac{\sin(90° - \Delta_2)}{\sin \Delta} = (m_1 - m_2)\frac{\cos \Delta_2}{\sin \Delta}. \\ \text{Similarly}\quad M_2'M_3 &= M_1'M_2'\frac{\sin(90° - \Delta_1)}{\sin \Delta} = (m_1 - m_2)\frac{\cos \Delta_1}{\sin \Delta}. \end{aligned}\right\} \quad (40)$$

b. *With a transition curve on the sharper curve only.* Compute $m_1 = MM_1'$ as before; then move the curve Z_1M_1' parallel to $Q'V$ a distance of

$$M_1'M_4 = m_1 \frac{\cos \Delta_2}{\sin \Delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

The simple curve MA is moved parallel to VA a distance of

$$MM_4 = m_1 \frac{\cos \Delta_1}{\sin \Delta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

If Δ_1 and Δ_2 are both small, $M_1'M_4$ and MM_4 may be more than m_1, but the lateral deviation of the new curve from the old will always be less than m_1.

53. To replace a compound curve by a curve with spirals. The solution is somewhat analogous to that of § 51. Compute m_1 for the sharper branch of the curve, placing $\Delta_1 = \frac{1}{2}\Delta$ in Eq. 38. Since m_1 and m_2 for the two branches of the curve must be identical, a value for R_2' must be found which will satisfy the determined value of $m_2 = m_1$. Solving Eq. 38 for R', we obtain

$$R' = \frac{R \text{ vers } \frac{1}{2}\Delta - m \cos \frac{1}{2}\Delta - x}{\cos \phi - \cos \frac{1}{2}\Delta}. \quad . \quad . \quad . \quad . \quad (43)$$

Substituting in this equation the known value of m_1 ($= m_2$) and calling $R' = R_2'$, $R = R_2$, and $\Delta_2 = \frac{1}{2}\Delta$, solve for R_2'. Obtain the value of AQ for each branch of the curve separately by Eq. 39, and compare the lengths of the old and new lines.

Example. Assume a compound curve with $D_1 = 8°$; $D_2 = 4°$; $\Delta_1 = 36°$ and $\Delta_2 = 32°$. Use 1°-per-25-feet spirals; $\phi_1 = 7°\ 0'$; $\phi_2 = 1°\ 30'$. Assume that the sharper curve is sharpened from 8° 0′ to 8° 12′.

[Eq. 38]			R_1	2.85538
			exsec 36°	9.37303
	169.209			2.22842
$R_1' =$	699.326			
			R_1'	2.84468
	868.535		$\cos \phi_1$	9.99675
			$\sec \Delta_1$	0.09204
		857.970		2.93347
			x_1	0.88241
			$\sec \Delta_1$	0.09204
		9.429		0.97445
	867.399	867.399		
$m_1 =$	1.136			
[Eq. 43]			R_2	3.15615
			vers 32°	9.18170
	217.700			2.33785
			$m_1 = 1.136$	0.05538
			cos 32°	9.92842
		0.963		9.98380
		$x_2 =$ 0.763		
	1.726	1.726		
	215.974			2.33440
			nat. $\cos \phi$ = .99966	
			nat. $\cos \Delta_2$ = .84805	
			.15161	9.18073
$R_2' =$	1424.54	[4° 1′ 22″]		3.15367
[Eq. 39]			R_1'	2.84468
$y_1 =$	174.722		$\sin \phi_1$	9.08589
		85.226		1.93057
			R_1'	2.84468
			$\cos \phi_1$	9.99675
			$\tan \frac{1}{2}\Delta [\Delta_1 = 36°]$	9.86126
	504.302			2.70269
			$R_1 =$ 716.779	
			$x_1 =$ 7.628	
			709.151	2.85074
			$\tan \frac{1}{2}\Delta$	9.86126
		515.235		2.71206
	679.024	600.461		
	600.461			
$AQ_1 =$	78.563			

[Eq. 39]		R_2'	3.15367
$y_2 =$ 74.994		$\sin \phi_2$	8.41792
	37 290		1.57159
		R_2'	3.15367
		$\cos \phi_2$	9.99985
		$\tan \frac{1}{2}\Delta (\Delta_2 = 32°)$	9.79579
889.843			2.94931
		$R_2 =$ 1432.69	
		$x_2 =$ 0.76	
		1431.93	3.15592
		$\tan \frac{1}{2}\Delta$	9.79579
	894.770		2.95171
964.837	932.060		
932.060			
$AQ_2 =$ 32.777			

For the length of the old track we have:

$$100 \frac{\Delta_1}{D_1} = 100 \frac{36°}{8°} = 450.$$

$$100 \frac{\Delta_2}{D_2} = 100 \frac{32°}{4°} = 800.$$

$$AQ_1 = 78.563$$

$$AQ_2 = 32.777$$

$$1361.340$$

For the length of the new track we have:

$$100 \frac{\Delta_1 - \phi_1}{D_1'} = 100 \frac{29°}{8°.20} = 353.659$$

$$100 \frac{\Delta_2 - \phi_2}{D_2'} = 100 \frac{30°.5}{4°.023} = 758.140$$

Spiral on 8° 12′ curve	175.000
" " 4° 01′ 22″ "	75.
Length of new track	= 1361.799
" " old "	= 1361.340
Excess in length of new track =	0.459 feet.

Since the new track is slightly longer than the old, it shows that the new track runs too far *outside* the old track at the *P.C.C.* On the other hand the offset m is only 1.136. The maximum amount by which the new track comes *inside* of the old track at two points, presumably not far from Z' and Z, is very difficult to determine exactly. Since it is desirable that the maximum offsets (inside and outside) should be made as nearly equal as possible, this feature should not be sacrificed to an effort to make the two lines of precisely equal length so that the rails need not be cut. Therefore, if it is found that the offsets inside the old track are nearly equal to m (1.136), the above figures should stand. Otherwise m may be diminished (and the above excess in length of track diminished) by *increasing* R_1' very slightly and making the necessary consequent changes.

VERTICAL CURVES.

54. Necessity for their use. Whenever there is a change in the rate of grade, it is necessary to eliminate the angle that would be formed at the point of change and to connect the two grades by a curve. This is especially necessary at a sag between two grades, since the shock caused by abruptly forcing an upward motion to a rapidly moving heavy train is very severe both to the track and to the rolling stock.

55. Required length. Theoretically the length should depend on the change in the rate of grade, the greater change requiring a longer curve. The importance of this was greater in the days when link couplers were in universal use and the "slack" in a long train was very great. Under such circumstances, when a train was moving down a heavy grade the cars would crowd ahead against the engine. Reaching the sag, the engine would begin to pull out, rapidly taking out the slack. Six inches of slack on each car would amount to several feet on a long train, and the resulting jerk on the couplers, especially those near the rear of the train, has frequently resulted in broken couplers or even derailments. A vertical curve will practically eliminate this danger if the curve is made long enough, but the rapidly increasing adoption of close spring couplers and air-brakes, even for freight trains, is obviating the necessity for such very long curves. Two hundred feet may be considered sufficiently long for all ordinary changes of grade. Four hundred feet would probably suffice for the greatest change ever found in practice.

56. Form of curve. In Fig. 36 assume that A and C, equi-

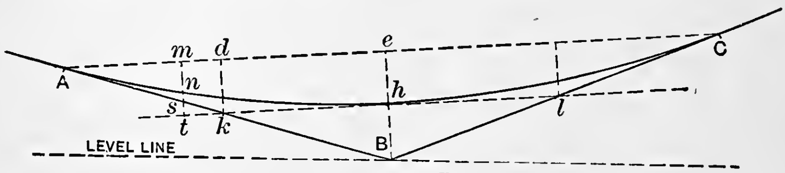

FIG. 36.

distant from B, are the extremities of the vertical curve. Bisect AC at e; draw Be and bisect it at h. Bisect AB and BC at k and l. The line kl will pass through h. A parabola may be drawn with its vertex at h which will be tangent to AB and BC at A and B. It may readily be shown from the properties of a parabola that if an ordinate be drawn at *any* point (as at n) we will have

$$sn : eh \text{ (or } hB) :: \overline{Am}^2 : \overline{Ae}^2,$$

or

$$sn = eh\frac{Am^2}{Ae^2}. \qquad (44)$$

Since the elevation of any point along AB or BC is readily determinable, the elevation of any point on the curve may be computed by adding the correction sn.

57. Numerical example. Assume that B is located at Sta. 16 + 20; that the curve is to be 200 feet long; that the grade of AB is $-$ 0.8%, and of BC + 1.2%; also that the elevation of B above the datum plane is 162.6. Then the elevation of the various points is as follows: A, 163.4; C, 163.8; e,

$\frac{1}{2}(163.4+163.8)=163.6$; h, $\frac{1}{2}(163.6+162.6)=163.1$. Then $eh=0.5$. The elevations of the points on the curve are:

Sta. 15 + 20,		(A)	163.4
" 16	,	$163.4 - (.80 \times 0.8) + (.80^2 \times 0.5) =$	163.08
" 17	,	$162.6 + (.80 \times 1.2) + (.20^2 \times 0.5) =$	163.58
" 17 + 20,		(C)	163.8

A theoretical inaccuracy in the above method lies in the fact that eh and all parallel lines are not truly vertical. In the above case the variation from the vertical is 0° 07′, while the effect of this variation on the elevations in this case (as in the most extreme cases) is absolutely inappreciable. The grades in the figure are necessarily very greatly exaggerated, which increases the apparent inaccuracy.

CHAPTER III.

EARTHWORK.

FORM OF EXCAVATIONS AND EMBANKMENTS.

58. Usual form of cross-section in cut or fill. The normal form of cross-section in cut is as shown in Fig. 37, in which *e . . . g* represents the natural surface of the ground, no matter how irregular; *ab* represents the position and width of the re-

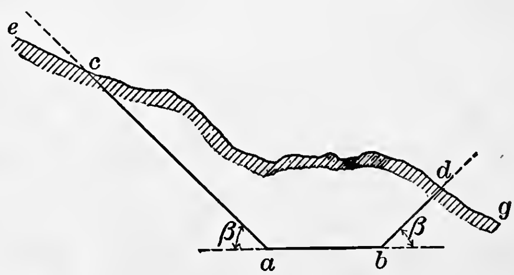

FIG. 37.

quired roadbed; *ac* and *bd* represent the "side slopes" which begin at *a* and *b* and which intersect the natural surface at such

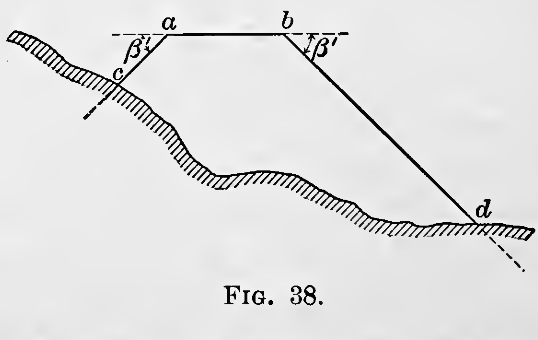

FIG. 38.

points (*c* and *d*) as will be determined by the required slope angle (β).

The normal section in fill is as shown in Fig. 38. The points *c* and *d* are likewise determined by the intersection of the required side slopes with the natural surface. In case the required roadbed (*ab* in Fig. 39) intersects the natural surface, both cut

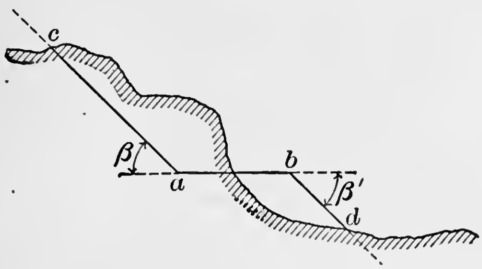

Fig. 39.

and fill are required, and the points *c* and *d* are determined as before. Note that β and β' are not necessarily equal. Their proper values will be discussed later.

59. Terminal pyramids and wedges. Fig. 40 illustrates the general form of cross-sections when there is a transition from cut to fill. *a . . . g* represents the grade line of the road which passes from cut to fill at *d*. *sdt* represents the surface profile. A cross-section taken at the point where either side of the roadbed *first* cuts the surface (the point *m* in this case) will usually be triangular if the ground is regular. A similar cross-section should be taken at *o*, where the other side of the roadbed cuts the surface. In general the earthwork of cut and fill terminates in two pyramids. In Fig. 40 the pyramid vertices are at *n* and *k*, and the bases are *lhm* and *opq*. The roadbed is generally wider in cut than in fill, and therefore the section *lhm* and the altitude *ln* are generally greater than the section *opq* and the altitude *pk*. When the line of intersection of the roadbed and natural surface (*nodkm*) becomes perpendicular to the axis of the roadbed (*ag*) the pyramids become wedges whose bases are the nearest convenient cross-sections.

60. Slopes. a. Cuttings. The required slopes for cuttings vary from perpendicular cuts, which may be used in hard rock which will not disintegrate by exposure, to a slope of perhaps

4 horizontal to 1 vertical in a soft material like quicksand or in a clayey soil which flows easily when saturated. For earthy materials a slope of 1 : 1 is the maximum allowable, and even this should only be used for firm material not easily affected by

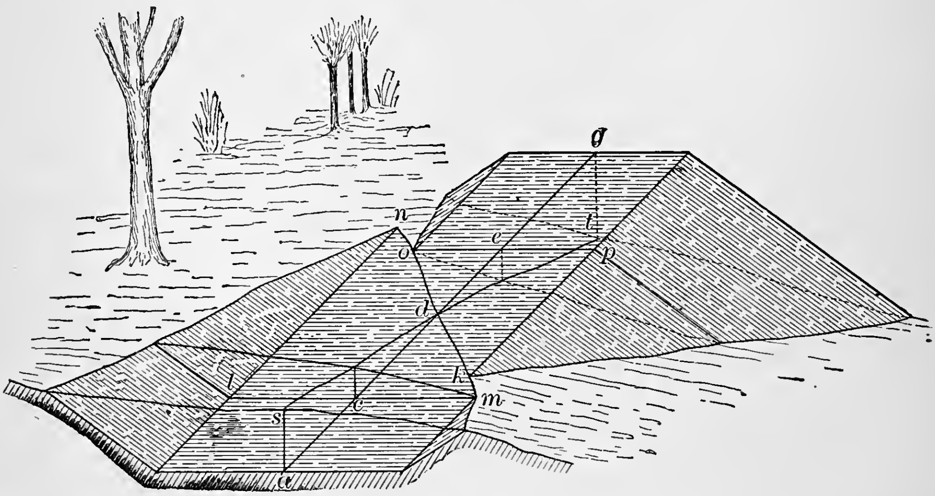

FIG. 40.

saturation. A slope of $1\frac{1}{2}$ horizontal to 1 vertical is a safer slope for average earthwork. It is a frequent blunder that slopes in cuts are made too steep, and it results in excessive work in clearing out from the ditches the material that slides down, at a much higher cost per yard than it would have cost to take it out at first, to say nothing of the danger of accidents from possible landslides.

b. Embankments. The slopes of an embankment vary from 1 : 1 to 1.5 : 1. A rock fill will stand at 1 : 1, and if some care is taken to form the larger pieces on the outside into a rough dry wall, a much steeper slope can be allowed. This method is sometimes a necessity in steep side-hill work. Earthwork embankments generally require a slope of $1\frac{1}{2}$ to 1. If made steeper at first, it generally results in the edges giving way, requiring repairs until the ultimate slope is nearly or quite $1\frac{1}{2}$: 1. The difficulty of incorporating the added material with the old embankment and preventing its sliding off frequently makes these repairs disproportionately costly.

61. Compound sections. When the cut consists partly of earth and partly of rock, a compound cross-section must be

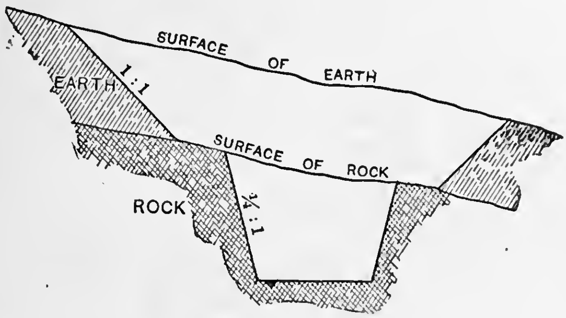

Fig. 41.

made. If borings have been made so that the contour of the rock surface is accurately known, then the true cross-section may be determined. The rock and earth should be calculated separately, and this will require an accurate knowledge of where the rock "runs out"—a difficult matter when it must be determined by boring. During construction the center part of the earth cut would be taken out first and the cut widened until a sufficient width of rock surface had been exposed so that the rock cut would have its proper width and side slopes. Then the earth slopes could be cut down at the proper angle. A "berm" of about three feet is usually left on the edges of the rock cut as a margin of safety against a possible sliding of the earth slopes. After the work is done, the amount of excavation that has been made is readily computable, but accurate preliminary estimates are difficult. The area of the cross-section of earth in the figure must be determined by a method similar to that developed for borrow-pits (see § 89).

62. Width of roadbed. Owing to the large and often disproportionate addition to volume of cut or fill caused by the addition of even one foot to the width of roadbed, there is a natural tendency to reduce the width until embankments become unsafe and cuts are too narrow for proper drainage. The cost of maintenance of roadbed is so largely dependent on the drainage of the roadbed that there is true economy in making an ample allowance for it. The practice of some of the leading railroads of the country in this respect is given in the following table, in which are also given some data belonging more properly to the subject of superstructure.

WIDTH OF ROADBED FOR SINGLE AND DOUBLE TRACK—SLOPE RATIOS—DISTANCES BETWEEN TRACK CENTERS.

Road.	Single Track.		Double Track.		Slope Ratios.		Dist. between Track Centers.
	Cut.	Fill.	Cut.	Fill.	Cut.	Fill.	
A., T. & Santa Fé....	28′ earth	20	....	....	1 : 1	1.5 : 1	
	22′ rock	. ..	... .	......	¼ : 1		
Chi., Burl. & Quincy	14 + (2 × 5) *	16	28 + (2 × 5)	30	1.5 : 1	1.5 : 1	14′
Chi., Mil. & St. Paul.	18 + (2 × 6)	20 to 24	31 + (2 × 6)	33 to 37	1.5 : 1	1.5 : 1	13′
C., C., C. & St. Louis	20 + (2 × 4)	20	33 + (2 × 4)	33	1.5 : 1	1.5 : 1	13′
Illinois Central......	32.5	18	....	....	1.5 : 1	1.5 : 1	
Erie	20′ 8½″	20′ 8½″	33′ 8½″	33′ 8½″	1.5 : 1	1.5 : 1	13′
Lehigh Valley.......	14 + (2 × 3.5)	16	27 + (2 × 3.5)	30	1 : 1	1.5 : 1	13′
L. S. & Michigan So.	... .	....	33 + (2 × 7.25)	32	1.5 : 1	1.5 : 1	13′
Louisville & Nashv..	13 + (2 × 4.5)	16	....	....	1 : 1	1.5 : 1	
Michigan Central....	....	....	33 + (2 × 2.5)	33	1.5 : 1	1.5 : 1	13′
N. Y. N. H. & H....	....	.. ..	30	30	1.5 : 1	1.5 : 1	12′
Norfolk & Western...	21′ 2″ earth	17′ 2″	34′ 2″ earth	30′ 2″	1 5 : 1	1.5 : 1	13′
	16′ rock	....	29′ rock	......	¼ : 1 ½ : 1	....	13′
Pennsylvania......	19′ 2″ light traffic	19′ 2″					
	27′ 2″ heavy "	19′ 2″	31′ 4″+(2 × 4)	31′ 4″	1.5 : 1	1.5 : 1	12′ 2″
Union Pacific...	14 + (2 × 3.5)	16	....	....	1 : 1	1.5 : 1	

* (2 × 5) signifies two ditches each 5 feet wide: the following cases should be interpreted similarly.

It may be noted from the above table that the average width for an *earthwork* cut, single track, is about 24.7 feet, with a minimum of 19 feet 2 inches. The widths of fills, single track, average over 18 feet, with numerous minimums of 16 feet. The widths for double track may be found by adding the distance between track centers, which is usually 13 feet.

63. Form of subgrade. The stability of the roadbed depends largely on preventing the ballast and subsoil from becoming saturated with water. The ballast must be porous so that it will not retain water, and the subsoil must be so constructed that it will readily drain off the rain-water that soaks through the ballast. This is accomplished by giving the subsoil a curved form, convex upward, or a surface made up of two or three planes, the two outer planes having a slope of about 1 : 24 (sometimes more and sometimes less, depending on the soil) and the middle plane, if three are used, being level. When a circular form is used, a crowning of 6 inches in a total width of 17 or 18 feet is generally used. Occasionally the subgrade is made level, especially in rock-cuts, but if the subsoil is previously compressed by rolling, as required on the N. Y. C. & H. R. R. R., or if the subsoil is drained by tile drains laid underneath the ditches, the necessity for slopes is not so great. Rock cuts are generally required to be excavated to one foot below subgrade and then filled up again to subgrade with the same material, if it is suitable.

64. Ditches. "The stability of the track depends upon the strength and permanence of the roadbed and structures upon which it rests; whatever will protect them from damage or prevent premature decay should be carefully observed. The worst enemy is WATER, and the further it can be kept away from the track, or the sooner it can be diverted from it, the better the track will be protected. Cold is damaging only by reason of the water which it freezes; therefore the first and most important provision for good track is drainage." (Rules of the Road Department, Illinois Central R. R.)

The form of ditch generally prescribed has a flat bottom 12″ to 24″ wide and with sides having a minimum slope, except in rock-work, of 1 : 1, more generally 1.5 : 1 and sometimes 2 : 1. Sometimes the ditches are made V-shaped, which is objectionable unless the slopes are low. The best form is evidently that which will cause the greatest flow for a given slope, and this will evidently be the form in which the ratio of area to wetted perimeter is the largest. The semicircle fulfills this condition better than any other form, but the nearly vertical sides would be difficult to maintain. (See Fig. 42.) A ditch, with a flat bottom and such slopes as the soil requires, which approximates to the circular form will therefore be the best.

FIG. 42.

When the flow will probably be large and at times rapid it will be advisable to pave the ditches with stone, especially if the soil is easily washed away. Six-inch tile drains, placed 2′ under the ditches, are prescribed on some roads. (See Fig. 43.) No better method could be devised to insure a dry subsoil. The ditches through cuts should be led off at the end of the cut so that the adjacent embankment will not be injured.

Wherever there is danger that the drainage from the land above a cut will drain down into the cut, a ditch should be made near the edge of the cut to intercept this drainage, and this ditch should be continued, and paved if necessary, to a point where the outflow will be harmless. Neglect of these simple and inexpensive precautions frequently causes the soil to be loosened on the shoulders of the slopes during the progress of a heavy rain, and results in a landslide which will cost more to repair than the ditches which would have prevented it for all time.

Ditches should be formed along the bases of embankments; they facilitate the drainage of water from the embankment, and may prevent a costly slip and disintegration of the embankment.

65. Effect of sodding the slopes, etc. Engineers are unanimously in favor of rounding off the shoulders and toes of embankments and slopes, sodding the slopes, paving the ditches, and providing tile drains for subsurface drainage, all to be put in during original construction. (See Fig. 43.) Some of the highest grade specifications call for the removal of the top layer of vegetable soil from cuts and from under proposed fills to some convenient place, from which it may be afterwards spread on the slopes, thus facilitating the formation of sod from grass-seed. But while engineers favor these measures and their economic value may be readily demonstrated, it is generally impossible to obtain the authorization of such specifications from railroad directors and promoters. The addition to the original cost of the roadbed is considerable, but is by no means as great as the capitalized value of the extra cost of maintenance resulting from the usual practice. Fig. 43 is a copy of

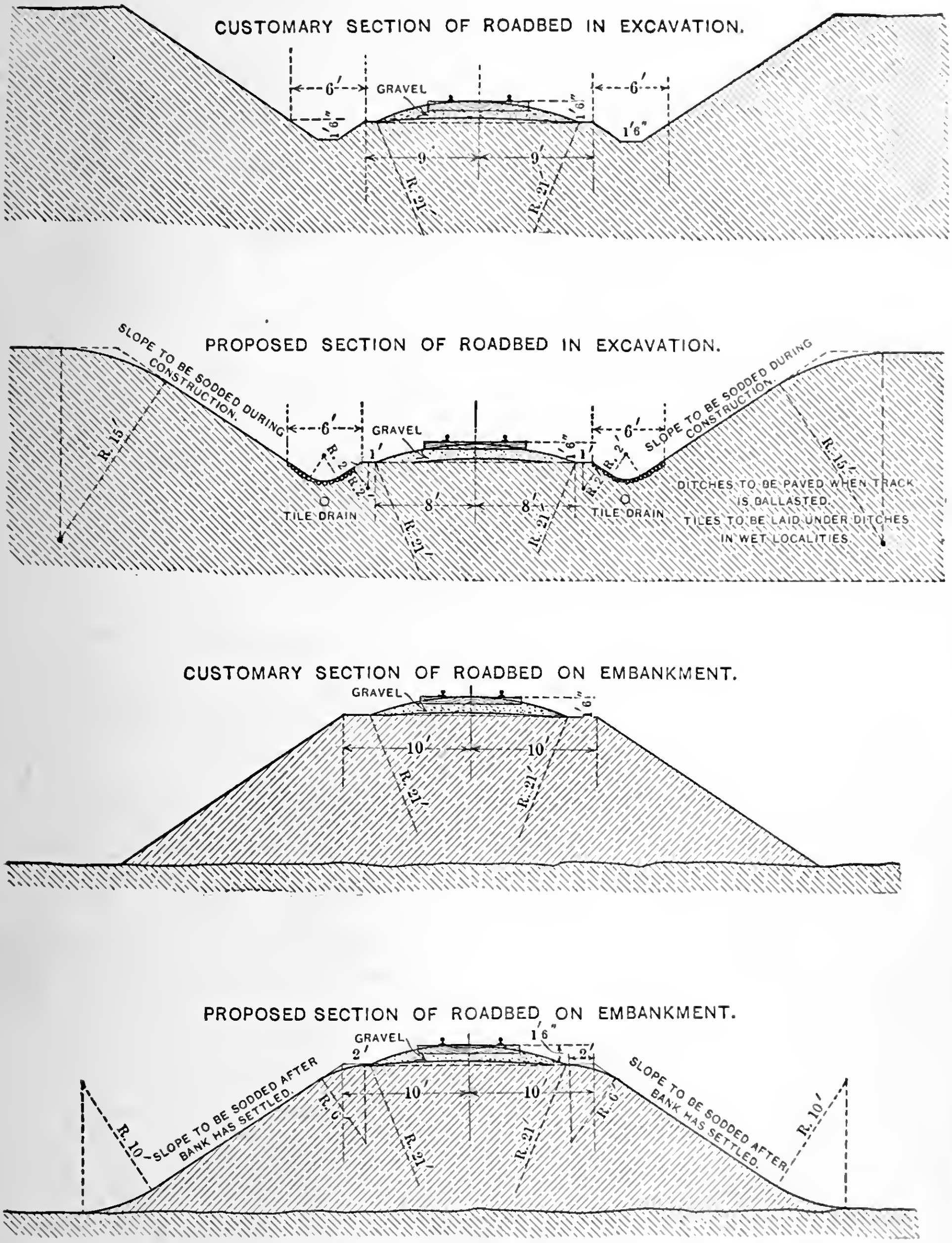

FIG. 43.—"WHITTEMORE ON RAILWAY EXCAVATION AND EMBANKMENTS," Trans. Am. Soc. C. E., Sept. 1894

designs * presented at a convention of the American Society of Civil Engineers by Mr. D. J. Whittemore, Past President of the Society and Chief Engineer of the Chi., Mil. & St. Paul R.R. The "customary sections" represent what is, with some variations of detail, the practice of many railroads. The "proposed sections" elicited unanimous approval. They should be adopted when not prohibited by financial considerations.

EARTHWORK SURVEYS.

66. Relation of actual volume to the numerical result. It should be realized at the outset that the accuracy of the result of computations of the volume of any given mass of earthwork has but little relation to the accuracy of the mere numerical work. The process of obtaining the volume consists of two distinct parts. In the first place it is assumed that the volume of the earthwork may be represented by a more or less complicated geometrical form, and then, secondly, the volume of such a geometrical form is computed. A desire for simplicity (or a frank willingness to accept approximate results) will often cause the cross-section men to assume that the volume may be represented by a very simple geometrical form which is really only a very rough approximation to the true volume. In such a case, it is only a waste of time to compute the volume with minute numerical accuracy. One of the first lessons to be learned is that economy of time and effort requires that the accuracy of the numerical work should be kept proportional to the accuracy of the cross-sectioning work, and also that the accuracy of both should be proportional to the use to be made of the results. The subject is discussed further in § 94.

67. Prismoids. To compute the volume of earthwork, it is necessary to assume that it has some geometric form whose volume is readily determinable. The general method is to consider

* Trans. Am. Soc. Civil Eng., Sept. 1894.

the volume as consisting of a series of *prismoids*, which are solids having parallel plane ends and bounded by surfaces which may be formed by lines moving continuously along the edges of the bases. These surfaces may also be considered as the surfaces generated by lines moving along the edges joining the corresponding points of the bases, these edges being the directrices, and the lines being always parallel to either base, which is a plane director. The surfaces thus developed may or may not be planes. The volume of such a prismoid is readily determinable (as explained in § 70 *et seq.*), while its definition is so very general that it may be applied to very rough ground. The "two plane ends" are sections perpendicular to the axis of the road. The roadbed and side slopes (also plane) form three of the side surfaces. The only approximation lies in the degree of accuracy with which the plane (or warped) surfaces coincide with the actual surface of the ground between these two sections. This accuracy will depend (*a*) on the number of points which are taken in each cross-section and the accuracy with which the lines joining these points coincide with the actual cross-sections; (*b*) on the skill shown in selecting places for the cross-sections so that the warped surfaces shall coincide as nearly as possible with the surface of the ground. In fairly smooth country, cross-sections every 100 feet, placed at the even stations, are sufficiently accurate, and such a method simplifies the computations greatly; but in rough country cross-sections must be interpolated as the surface demands. As will be explained later, carelessness or lack of judgment in cross-sectioning will introduce errors of such magnitude that all refinements in the computations are utterly wasted.

68. Cross-sectioning. The process of cross-sectioning consists in determining at any place the intersection by a vertical plane of the prism of earth lying between the roadbed, the side slopes, and the natural surface. The intersection with the roadbed and side slopes gives three straight lines. The intersection with the natural surface is in general an irregular line. On smooth regular ground or when approximate results are accept-able this line is assumed to be straight. According to the irregularity of the ground and the accuracy desired more and more "intermediate points" are taken.

The distance (d in Fig. 44) of the roadbed below (or above) the natural surface at the center is known or determined from

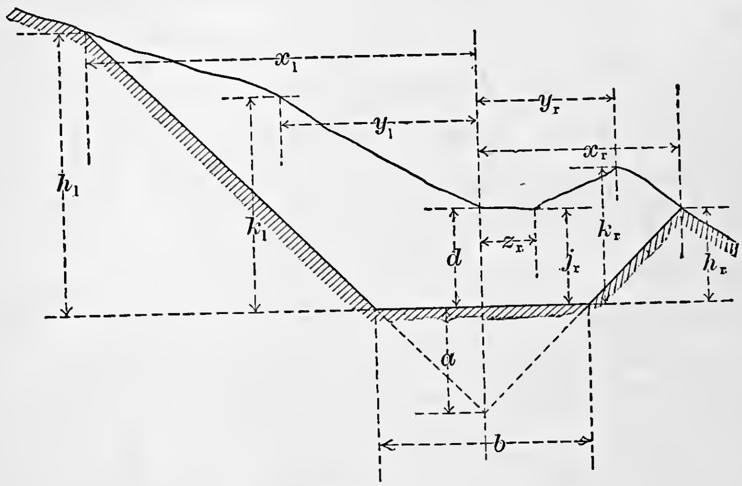

FIG. 44.

the profile or by the computed establishment of the grade line. The distances out from the center of all "breaks" are determined with a tape. To determine the elevations for a cut, set up a level at any convenient point so that the line of sight is higher than any point of the cross-section, and take a rod reading on the center point. This rod reading added to d gives the height of the instrument (H. I.) above the roadbed. Subtracting from H. I. the rod reading at any "break" gives the height of that point above the roadbed (h_l, k_l, h_r, etc.). This is true for all cases in excavation. For fill, the rod reading at center minus d equals the H. I., which may be positive or negative. When negative, add to the "H. I." the rod readings of the intermediate points to get their depths below "grade"; when positive, subtract the "H. I." from the rod readings.

The heights or depths of these intermediate points above or below grade need only be taken to the nearest tenth of a foot, and the distances out from the center will frequently be sufficiently exact when taken to the nearest foot. The roughness of the surface of farming land or woodland generally renders useless any attempt to compute the volume with any greater accuracy than these figures would imply unless the form of the ridges and hollows is especially well defined. The position of the slope-stake points is considered in the next section. Additional discussion regarding cross-sectioning is found in § 82.

69. Position of slope-stakes. The slope-stakes are set at the intersection of the required side slopes with the natural surface, which depends on the center cut or fill (d). The distance of

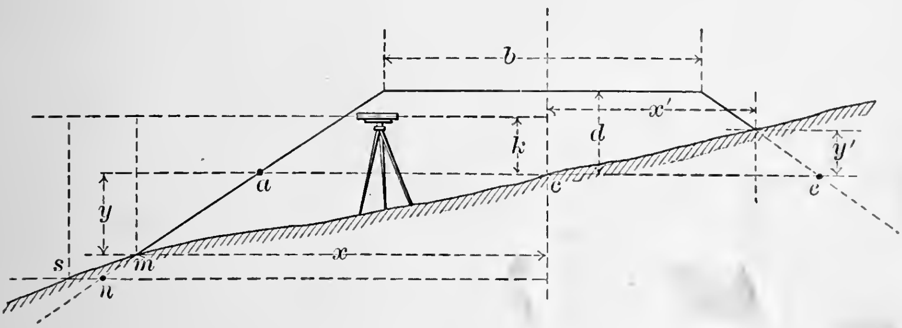

FIG. 45.

the slope-stake from the center for the lower side is $x = \frac{1}{2}b + s(d + y)$; for the up-hill side it is $x' = \frac{1}{2}b + s(d - y')$. s is the "slope ratio" for the side slopes, the ratio of horizontal to vertical. In the above equation both x and y are unknown. Therefore some position must be found by trial which will satisfy the equation. As a preliminary, the value of x for the point $a = \frac{1}{2}b + sd$, which is the value of x for *level* cross-sections. In the case of fills on sloping ground the value of x on the *down-hill* side is *greater* than this; on the *up-hill* side it is *less*. The difference in distance is s times the difference of elevation. Take a numerical case corresponding with Fig. 45. The rod reading on c is 2.9; $d = 4.2$; therefore the telescope is $4.2 - 2.9 = 1.3$ *below* grade. $s = 1.5 : 1$, $b = 16$. Hence for the point a (or for level ground) $x = \frac{1}{2} \times 16 + 1.5 \times 4.2 = 14.3$. At a distance out of 14.3 the ground is seen to be about 3 feet lower, which will not only require $1.5 \times 3 = 4.5$ more, but enough additional distance so that the added distance shall be 1.5 times the additional drop. As a first trial the rod may be held at 24 feet out and a reading of, say, 8.3 is obtained. 8.3 + 1.3 = 9.6, the depth of the point below grade. The point on the slope line (n) which has this depth below grade is at a distance from the center $x = 8 + 1.5 \times 9.6 = 22.4$. The point on the surface (s) having that depth is 24 feet out. Therefore the true point (m) is nearer the center. A second trial at 20.5 feet out gives a rod reading of, say, 7.1 or a depth of 8.4 below grade. This corresponds to a distance out of 20.6. Since the natural soil (especially in farming lands or woods) is generally so rough that a difference of elevation of a tenth or so may be readily found by slightly varying the location of the rod (even though the distance from the center is the same), it is useless to attempt too much refinement, and so in a case like the above the combination of 8.4 below grade and 20.6 out from center may be taken to indicate the proper position of the slope-stake. This is usually indicated in the form of a fraction, the distance out being the denominator and the height above (or below) grade being the numerator; the fact of *cut* or *fill* may be indicated by C or F. Ordinarily a second trial will be sufficient to determine with sufficient accuracy the true position of the slope-stake. Experienced men will frequently estimate the required distance out to within a few tenths at the first trial. The left-hand pages of the note-book should have the station number, surface elevation, grade elevation, center cut or fill, and rate of grade. The right-hand pages should be divided in the center and show the distances out and heights above grade of all points, as is illustrated in § 84. The notes should read UP the page, so that when looking ahead along the line the figures are in their proper relative position. The "fractions" farthest from the center line represent the slope-stake points.

COMPUTATION OF VOLUME.

70. Prismoidal formula. Let Fig. 46 represent a triangular prismoid. The two triangles forming the ends lie in *parallel* planes, but since the angles of one triangle are not equal to the corresponding angles of the other triangle, at least two of the surfaces must be *warped*. If a section, parallel to the bases, is

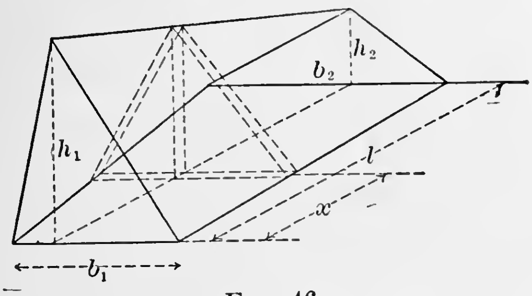

FIG. 46.

made at any point at a distance x from one end, the area of the section will evidently be

$$A_x = \tfrac{1}{2}b_xh_x = \tfrac{1}{2}\left[b_1 + (b_2 - b_1)\frac{x}{l}\right]\left[h_1 + (h_2 - h_1)\frac{x}{l}\right].$$

The volume of a section of infinitesimal length will be A_xdx, and the total volume of the prismoid will be *

$$\int_0^l A_xdx = \tfrac{1}{2}\int_0^l\left[b_1 + (b_2 - b_1)\frac{x}{l}\right]\left[h_1 + (h_2 - h_1)\frac{x}{l}\right]dx$$

$$= \tfrac{1}{2}\Big[b_1h_1x + (b_2 - b_1)h_1\frac{x^2}{2l} + b_1(h_2 - h_1)\frac{x^2}{2l}$$

$$+ (b_2 - b_1)(h_2 - h_1)\frac{x^3}{3l^2}\Big]_0^l$$

$$= \tfrac{1}{2}\left\{b_1h_1l + [(b_2-b_1)h_1 + b_1(h_2-h_1)]\frac{l}{2} + (b_2-b_1)(h_2-h_1)\frac{l}{3}\right\},$$

* Students unfamiliar with the Integral Calculus may take for granted the fundamental formulæ that $\int dx = x$, that $\int xdx = \frac{1}{2}x^2$, and that $\int x^2dx = \frac{1}{3}x^3$; also that in integrating between the limits of l and 0 (zero), the value of the integral may be found by simply substituting l for x after integration.

$$\int_0^l A_x dx = \frac{l}{2}[\tfrac{1}{3}b_1h_1 + \tfrac{1}{6}b_1h_2 + \tfrac{1}{6}b_2h_1 + \tfrac{1}{3}b_2h_2]$$

$$= \frac{l}{6}[\tfrac{1}{2}b_1h_1 + \tfrac{1}{2}b_1(h_1 + h_2) + \tfrac{1}{2}b_2(h_1 + h_2) + \tfrac{1}{2}b_2h_2]$$

$$= \frac{l}{6}\left[\tfrac{1}{2}b_1h_1 + 4\left(\frac{1}{2} \cdot \frac{b_1 + b_2}{2} \cdot \frac{h_1 + h_2}{2}\right) + \tfrac{1}{2}b_2h_2\right]$$

$$= \frac{l}{6}[A_1 + 4A_m + A_2], \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

in which A_1, A_2, and A_m are the areas respectively of the two bases and of the middle section. Note that A_m is *not* the *mean* of $A_1$and A_2, although it does not necessarily differ very greatly from it.

The above proof is absolutely independent of the values, absolute or relative, of b_1, b_2, h_1 or h_2. For example, h_2 may be zero and the second base reduces to a line and the prismoid becomes wedge-shaped; or b_3 and h_2 may both vanish, the second base becoming a point and the prismoid reduces to a pyramid Since every prismoid (as defined in § 67) may be reduced to a combination of triangular prismoids, wedges, and pyramids, and since the formula is true for any one of them individually, it is true for all collectively; therefore it may be stated that*

The volume of a prismoid equals one sixth of the perpendicular distance between the bases multiplied by the sum of the areas of the two bases plus four times the area of the middle section.

While it is always possible to compute the volume of any prismoid by the above method, it becomes an extremely complicated and tedious operation to compute the true value of the middle section if the end sections are complicated in form. It

* The student should note that the derivation of equation (45) does not complete the proof, but that the statements in the following paragraph are logically necessary for a general proof.

therefore becomes a simpler operation to compute volumes by approximate formulæ and apply, if necessary, a correction. The most common methods are as follows:

71. Averaging end areas. The volume of the triangular prismoid (Fig. 46), computed by averaging end areas, is $\frac{l}{2}[\frac{1}{2}b_1h_1 + \frac{1}{2}b_2h_2]$. Subtracting this from the true volume (as given in the equation above, Eq. (45)), we obtain the correction

$$\frac{l}{12}[(b_1 - b_2)(h_2 - h_1)]. \quad . \quad . \quad . \quad . \quad (46)$$

This shows that if either the h's or b's are equal, the correction vanishes; it also shows that if the bases are roughly similar and b varies roughly with h (which *usually* occurs, as will be seen later), the correction will be *negative*, which means that the method of averaging end areas *usually* gives *too large* results.

72. Middle areas. Sometimes the middle area is computed and the volume is assumed to be equal to the length times the middle area. This will equal $\frac{l}{2} \times \frac{b_1 + b_2}{2} \times \frac{h_1 + h_2}{2}$. Subtracting this from the true volume, we obtain the correction

$$\frac{l}{24}(b_1 - b_2)(h_1 - h_2). \quad . \quad . \quad . \quad . \quad . \quad (47)$$

As before, the form of the correction shows that if either the h's or b's are equal, the correction vanishes; also under the *usual* conditions, as before, the correction is *positive* and only one-half as large as by averaging end areas. Ordinarily the labor involved in the above method is no less than that of applying the exact prismoidal formula.

73. Two-level ground. When *approximate* computations of earthwork are sufficiently exact the field-work may be materially reduced by observing simply the center cut (or fill) and the natural slope α, measured with a clinometer. The area of such a section (see Fig. 48) equals

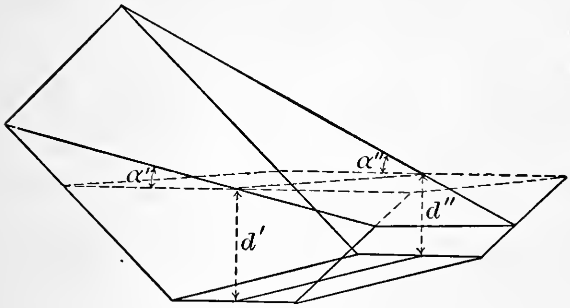

FIG. 47.

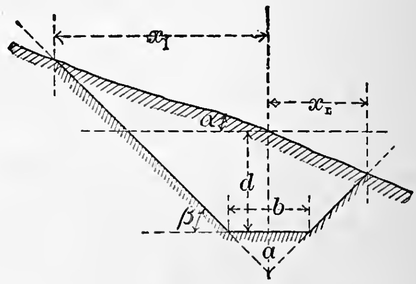

FIG. 48.

$$\tfrac{1}{2}(a+d)(x_l+x_r)-\frac{ab}{2}.$$

But

$$x_l \tan\beta = a + d + x_l \tan\alpha,$$

from which $$x_l = \frac{a+d}{\tan\beta - \tan\alpha}.$$

Similarly, $$x_r = \frac{a+d}{\tan\beta + \tan\alpha}.$$

Substituting,

$$\text{Area} = (a+d)^2 \frac{\tan\beta}{\tan^2\beta - \tan^2\alpha} - \frac{ab}{2}. \quad . \quad . \quad (48)$$

The values α, $\tan\beta$, $\tan^2\beta$ are constant for all sections, so that it requires but little work to find the area of any section. As this method of cross-sectioning implies considerable approximation, it is generally a useless refinement to attempt to compute the volume with any greater accuracy than that obtained by averaging end areas. It may be noted that it may be easily proved that the correction to be applied is of the same form as that found in § 71 and equals

$$\frac{l}{12}[(x_l'+x_r')-(x_l''+x_r'')][(d''+a)-(d'+a)],$$

which reduces to

$$\text{Correction} = \frac{l}{6}\left\{\left[(a+d')\frac{\tan\beta}{\tan^2\beta-\tan^2\alpha'} - (a+d'')\frac{\tan\beta}{\tan^2\beta-\tan^2\alpha''}\right][d''-d']\right\}. \quad (49)$$

When $d'' = d'$ the correction vanishes. This shows that when the center heights are equal there is no correction—regardless of the slope. If the slope is uniform throughout, the form of the correction is simplified and is invariably *negative.* Under the usual conditions the correction is *negative,* i.e., the method *generally* gives *too large* results.

74. Level sections. When the country is very level or when only approximate preliminary results are required, it is sometimes assumed that the cross-sections are level. The method of level sections is capable of easy and rapid computation. The area may be written as

$$(a+d)^2 s - \frac{ab}{2}. \quad \cdots\cdots \quad (50)$$

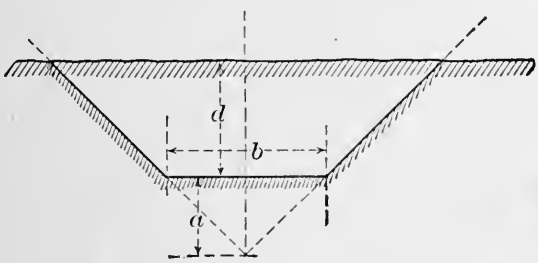

FIG. 49.

This also follows from Eq. (48) when $\alpha = 0$ and $\tan\beta = \frac{1}{s}$. s here represents the "slope ratio," *i.e.*, the ratio of the horizontal projection of the slope to the vertical. A table is very readily formed giving the area in square feet of a section of given depth and for any given width of roadbed and ratio of side-slopes. The area may also be readily determined (as illustrated in the following example) without the use of such a table; a table of squares will facilitate the work. Assuming the cross-sections at equal distances $(=l)$ apart, the total approximate volume for any distance will be

$$\frac{l}{2}[A_0 + 2(A_1 + A_2 + \ldots A_{n-1}) + A_n]. \quad . \quad . \quad (51)$$

The prismoidal correction may be directly derived from Eq. (46) as $\frac{l}{12}[2(a+d')s - 2(a+d'')s][(a+d'') - (a+d')]$, which reduces to

$$-\frac{ls}{6}(d' - d'')^2 \quad \text{or} \quad -\frac{l}{12}\frac{b}{a}(d' - d'')^2. \quad . \quad (52)$$

This may also be derived from Eq. (49), since $\alpha = 0$, $\tan \alpha = 0$, and $\tan \beta = 2a \div b$. This correction is *always* negative, showing that the method of averaging end areas, when the sections are level, always gives too large results. The prismoidal correction for any one prismoid is therefore a constant times the *square* of a difference. The squares are always positive whether the differences are positive or negative. The correction therefore becomes

$$-\frac{l}{12}\frac{b}{a}\Sigma(d' \sim d'')^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

75. Numerical example: level sections. Given the following center heights for the same number of consecutive stations 100 feet apart; width of roadbed 18 feet; slope $1\frac{1}{2}$ to 1.

The products in the fifth column may be obtained very readily and with sufficient accuracy by the use of the slide-rule described in § 79. The products should be considered as $(a+d)(a+d) \div \frac{1}{s}$. In this problem $s = 1\frac{1}{2}, \frac{1}{s} = .6667$. To apply the rule to the first case above, place 6667 on scale B over 89 on scale A, then opposite 89 on scale B will be found

118.8 on scale *A*. The position of the decimal point will be evident from an approximate mental solution of the problem.

Sta.	Center Height.	$a + d$	$(a + d)^2$	$(a + d)^2 s$	Areas.	$d' \sim d''$	$(d' \sim d'')^2$
17	2.9	8.9	79.21	118.81	118.81		
18	4.7	10.7	114.49	171.74	× 2 = 343.48	1.8	3.24
19	6.8	12.8	163.84	245.76	491.52	2.1	4.41
20	11.7	17.7	313.29	469.93	939.86	4.9	24.01
21	4.2	10.2	104.04	156.06	312.12	7.5	56.25
22	1.6	7.6	57.76	86.64	86.64	2.6	6.76
					2292.43		94.67

$$\frac{ab}{2} = \frac{6 \times 18}{2} = 54 \qquad 10 \times 54 = \underline{540} \quad 1752.43$$

$$\frac{1752.43 \times 100}{2 \times 27} = 3245 \text{ cub. yards} \quad = \text{approx. vol.}$$

$$\text{Corr.} = -\frac{100 \times 18}{12 \times 6 \times 27} \times 94.67 \quad = -91 \text{ cub. yds.}$$

$$3245 - 91 = 3154 \text{ cub. yds.} \quad = \text{exact volume.}$$

The above demonstration of the correction to be applied to the approximate volume, found by averaging end areas, is introduced mainly to give an idea of the amount of that correction. Absolutely level sections are practically unknown, and the error involved in assuming any given sections as truly level will ordinarily be greater than the computed correction. If greater accuracy is required, more points should be obtained in the cross-sectioning, which will generally show that the sections are not truly level.

76. Equivalent sections. When sections are very irregular the following method may be used, especially if great accuracy is not required. The sections are plotted to scale and then a uniform slope line is obtained by stretching a thread so that the undulations are averaged and an *equivalent section* is obtained. The *center depth* (d) and the *slope angle* (α) of this line *can* be obtained from the drawing, but it is more convenient to measure the distances (x_l and x_r) from the center. The area may then be obtained independent of the center depth as follows: Let s = the slope ratio of the side slopes $= \cot \beta = \frac{b}{2a}$. (See Fig. 48.) Then the

$$\text{Area} = \frac{1}{2}\left(\frac{x_l + x_r}{s}\right)(x_l + x_r) - \frac{x_r}{s}\frac{x_r}{2} - \frac{x_l}{s}\frac{x_l}{2} - \frac{ab}{2}$$

$$= \frac{x_l x_r}{s} - \frac{ab}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

The true volume, according to the prismoidal formula, of a length of the road measured in this way will be

$$\frac{l}{6}\left[\frac{x_l'x_r'}{s} - \frac{ab}{2} + 4\left(\frac{x_l' + x_l''}{2}\frac{x_r' + x_r''}{2}\frac{1}{s} - \frac{ab}{2}\right) + \frac{x_l''x_r''}{s} - \frac{ab}{2}\right].$$

If computed by averaging end areas, the approximate volume will be

$$\frac{l}{2}\left[\frac{x_l'x_r'}{s} - \frac{ab}{2} + \frac{x_l''x_r''}{s} - \frac{ab}{2}\right].$$

Subtracting this result from the true volume, we obtain as the correction

$$\text{Correction} = \frac{l}{6s}(x_l'' - x_l')(x_r' - x_r''). \quad . \quad . \quad . \quad (55)$$

This shows that if the side distances to either the right or left are equal at adjacent stations the correction is *zero*, and also that if the difference is small the correction is also small and very probably within the limit of accuracy obtainable by that method of cross-sectioning. In fact, as has already been shown in the latter part of § 75, it will usually be a useless refinement to compute the prismoidal correction when the method of cross-sectioning is as rough and approximate as this method generally is.

77. Equivalent level sections. These sloping "two-level" sections are sometimes transformed into "level sections of equal area," and the volume computed by the method of level sections (§ 74). But the true volume of a prismoid with sloping ends does not agree with that of a prismoid with equivalent bases and level ends except under special conditions, and when this method is used a correction must be applied if accuracy is desired, although, as intimated before, the assumption that the sections have uniform slopes will frequently introduce greater inaccuracies than that of this method of computation. The following demonstration is therefore given to show the scope and limitations of the errors involved in this much used method.

In Fig. 50, let d_1 be the center height which gives an

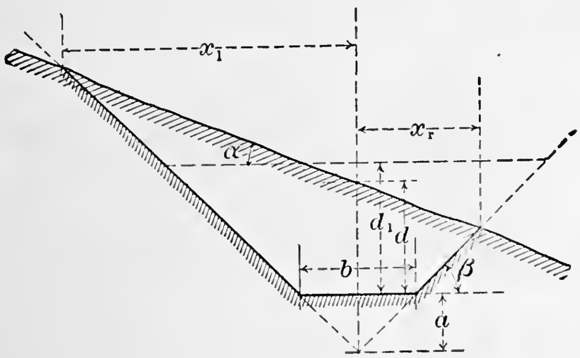

FIG. 50.

equivalent level section. The area will equal $(a + d_1)^2 s - \frac{ab}{2}$, which must equal the area given in § 76, $\frac{x_l x_r}{s} - \frac{ab}{2}$. $s = \frac{b}{2a}$.

$$\therefore\ (a + d_1)^2 s = \frac{x_l x_r}{s},$$

$$\text{or} \qquad a + d_1 = \frac{\sqrt{x_l x_r}}{s}. \quad . \quad . \quad . \quad . \quad (56)$$

To obtain d_1 directly from notes, given in terms of d and α, we may substitute the values of x_l and x_r given in § 73, which gives

$$a + d_1 = (a + d)\frac{\tan\beta}{\sqrt{\tan^2\beta - \tan^2\alpha}} = \frac{a + d}{\sqrt{1 - s^2\tan^2\alpha}}. \qquad (57)$$

The *true* volume of the equivalent section may be represented by

$$\frac{ls}{6}\left[(a + d_1')^2 + 4\left(\frac{a + d_1'}{2} + \frac{a + d_1''}{2}\right)^2 + (a + d_1'')^2\right].$$

From this there should be subtracted the volume of the "grade prism" under the roadbed to obtain the volume of the cut that would be actually excavated, but in the following comparison, as well as in other similar comparisons elsewhere made, the volume of the grade prism invariably cancels out, and so for the sake of simplicity it will be disregarded. This expression for volume may be transposed to

$$\frac{ls}{6}\left[\frac{x_l'x_r'}{s^2} + 4\left(\frac{\sqrt{x_l'x_r'}}{2s} + \frac{\sqrt{x_l''x_r''}}{2s}\right)^2 + \frac{x_l''x_r''}{s^2}\right].$$

The true volume of the prismoid with sloping ends is (see § 76)

$$\frac{l}{6}\left[\frac{x_l'x_r'}{s} + 4\left(\left(\frac{x_l' + x_l''}{2}\right)\left(\frac{x_r' + x_r''}{2}\right)\frac{1}{s}\right) + \frac{x_l''x_r''}{s}\right].$$

The difference of the two volumes

$$= \frac{l}{6s}(x_l'x_r' + x_l''x_r' + x_l'x_r'' + x_l''x_r'' - x_l'x_r' - 2\sqrt{x_l'x_r'x_l''x_r''} - x_l''x_r'')$$

$$= \frac{l}{6s}(\sqrt{x_l'x_r''} - \sqrt{x_l''x_r'})^2. \qquad (58)$$

This shows that "equivalent level sections" do *not* in general give the true volume, there being an exception when

$x_l'x_r'' = x_l''x_r'$. This condition is fulfilled when the slope is uniform, i.e., when $\alpha' = \alpha''$. When this is nearly so the error is evidently not large. On the other hand, if the slopes are inclined in opposite directions the error may be very considerable, particularly if the angles of slope are also large.

78. Three-level sections. The next method of cross-sectioning in the order of complexity, and therefore in the order of

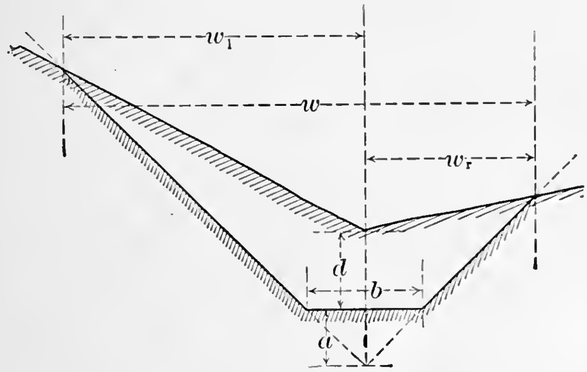

FIG. 51.

accuracy, is the method of three-level sections. The area of the section is $\frac{1}{2}(a + d)(w_r + w_l) - \frac{ab}{2}$, which may be written $\frac{1}{2}(a + d)w - \frac{ab}{2}$, in which $w = w_r + w_l$. If the volume is computed by averaging end areas, it will equal

$$\frac{l}{4}[(a + d')w' - ab + (a + d'')w'' - ab]. \quad . \quad . \quad (59)$$

If we divide by 27 to reduce to cubic yards, we have, when $l = 100$,

$$\text{Vol}_{(\prime \ldots \prime\prime)} = \tfrac{25}{27}(a + d')w' - \tfrac{25}{27}ab + \tfrac{25}{27}(a + d'')w'' - \tfrac{25}{27}ab.$$

For the next section

$$\text{Vol}_{(\prime\prime \ldots \prime\prime\prime)} = \tfrac{25}{27}(a + d'')w'' - \tfrac{25}{27}ab + \tfrac{25}{27}(a + d''')w''' - \tfrac{25}{27}ab.$$

For a partial station length compute as usual and multiply result by $\frac{\text{length in feet}}{100}$. The prismoidal correction may be obtained by applying Eq. (46) to each side in turn. For the left side we have

$$\frac{l}{12}[(a + d') - (a + d'')](w_l'' - w_l'), \quad \text{which equals}$$

$$\frac{l}{12}(d' - d'')(w_l'' - w_l').$$

For the right side we have, similarly,

$$\frac{l}{12}(d' - d'')(w_r'' - w_r').$$

The total correction therefore equals

$$\frac{l}{12}(d' - d'')[(w_l'' + w_r'') - (w_l' + w_r')]$$

$$= \frac{l}{12}(d' - d'')(w'' - w').$$

Reduced to cubic yards, and with $l = 100$,

$$\text{Pris. Corr.} = \tfrac{25}{81}(d' - d'')(w'' - w'). \quad . \quad . \quad (60)$$

When this result is compared with that given in Eq. (55) there is an apparent inconsistency. If two-level ground is considered as but a special case of three-level ground, it would seem as if the same laws should apply. If, in Eq. (55), $x_r' = x_r''$, and x_l'' is different from x_l', the equation reduces to zero; but in this case d' would also be different from d''; and since $x_l' + x_r'$ would $= w'$, and $x_l'' + x_r'' = w''$ in Eq. (60), $w'' - w'$ would not equal zero and the correction would be some finite quantity and not zero. The explanation lies in the difference in the form and volume of the prismoids, according to the method of the formation of the warped surfaces. If the surface is supposed to be generated by the locus of a line moving parallel to the ends as plane directors and along two *straight* lines lying *in* the side-slopes, then $x_l^{\text{mid.}}$ will equal $\frac{1}{2}(x_l' + x_l'')$, and $x_r^{\text{mid.}}$ will equal $\frac{1}{2}(x_r' + x_r'')$, but the profile of the center line will *not* be straight and $d^{\text{mid.}}$ will *not* equal $\frac{1}{2}(d' + d'')$. On the other hand, if the surfaces be generated by *two* lines moving parallel to the ends as plane directors and along a *straight* center line and straight side lines lying in the slopes, a warped surface will be generated each side of the center line, which will have uniform slopes on each side of the center at the two ends and *nowhere else.* This shows that when the upper surface of earthwork is warped (as it generally is), two-level ground should not be considered as a special case of three-level ground. This discussion, however, is only valuable to explain an apparent inconsistency and error. The method of two-level ground should only be used when such refinements as are here discussed are of no importance as affecting the accuracy.

The following example is given to illustrate the method of three-level sections.

Station.	Center.	Left.	Right.	$a+d$	w	Yards.		$d'-d''$	$w''-w'$	Pris. Corr.	$x_l \sim x_r$	$\frac{V(x_l \sim x_r)}{3R}$	Curv. Corr.*
17	2.6F	$\frac{10.6F}{22.9}$	$\frac{0.8F}{8.2}$	7.3	31.1	210					14.7	+1	
18	8.1F	$\frac{15.8F}{30.7}$	$\frac{3.4F}{12.1}$	12.8	42.8	507	595	−5.5	+11.7	−20	18.6	+3	+4
+40	10.7F	$\frac{20.2F}{37.3}$	$\frac{4.8F}{14.2}$	15.4	51.5	734	448	−2.6	+ 8.7	− 3	23.1	+6	+4
19	6.4F	$\frac{14.0F}{28.0}$	$\frac{2.1F}{10.1}$	11.1	38.1	392	602	+4.3	−13.4	−11	17.9	+2	+5
20	3.7F	$\frac{5.8F}{15.7}$	$\frac{0.2F}{7.3}$	8.4	23.0	179	449	+2.7	−15.1	−13	8.4	+1	+3

Roadbed, 14′ wide in fill. Approx. Vol. = 2094 −47 +16
Slope 1½ to 1. Pris. corr. = 47

$a = \frac{b}{2s} = \frac{14}{3} = 4.7;$ True Vol. = 2047 (disregarding curv. corr).*

$\frac{25}{27}ab = 61.$

* For the Derivation of the curvation correction, see § 93.

In the first column of yards

$$210 = \tfrac{25}{27}(a + d)w = \tfrac{25}{27} \times 7.3 \times 31.1;$$

507, 734, etc., are found similarly;

$$595 = 210 - 61 + 507 - 61;$$

$$448 = \tfrac{40}{100}(507 - 61 + 734 - 61);$$

$$602 = \tfrac{60}{100}(734 - 61 + 392 - 61);$$

$$449 = 392 - 61 + 179 - 61.$$

For the prismoidal correction,

$$-20 = \tfrac{25}{81}(d' - d'')(w'' - w') = \tfrac{25}{81}(2.6 - 8.1)(42.8 - 31.1)$$

$$= \tfrac{25}{81}(-5.5)(+11.7).$$

For the next line, $-3 = \tfrac{40}{100}[\tfrac{25}{81}(-2.8)(+8.7)]$, and similarly for the rest. The "*F*" in the columns of center heights, as well as in the columns of "right" and "left," are inserted to indicate *fill* for all those pooints. Cut would be indicated by "*C*."

79. Computation of products. The quantities $\frac{25}{27}(a + d)w$ and $\frac{25}{27}ab$ represent in each case the product of two variable terms and a constant. These products are sometimes obtained from tables which are calculated for all ordinary ranges of the variable terms as arguments. A similar table computed for $\frac{25}{81}(d' - d'')(w'' - w')$ will assist similarly in computing the prismoidal correction. Prof. Charles L. Crandall, of Cornell University, is believed to be the first to prepare such a set of tables, which were first published in 1886 in "Tables for the Computation of Railway and Other Earthwork." Another easy method of obtaining these products is by the use of a slide-rule. A slide-rule has been designed by the author to accompany this volume. It is designed particularly for this special work, although it may be utilized for many other purposes for which slide-rules are valuable. To illustrate its use, suppose $(a+d) = 28.2$, and $w = 62.4$; then

$$\frac{25}{27}(a+d)w = \frac{28.2 \times 62.4}{1.08}.$$

Set 108 (which, being a constant of frequent use, is specially marked) on the sliding scale (B) opposite 282 on the other scale (A), and then opposite 624 on scale B will be found 1629 on scale A, the 162 being read directly and the 9 read by estimation. Although strict rules may be followed for pointing off the final result, it only requires a very simple mental calculation to know that the result must be 1629 rather than 162.9 or 16290. For products less than 1000 cubic yards the result may be read directly from the scale; for products between 1000 and 5000 the result may be read directly to the nearest 10 yards, and the tenths of a division estimated. Between 5000 and 10,000 yards the result may be read directly to the nearest 20 yards, and the fraction estimated; but prisms of such volume will never be found as simple triangular prisms—at least, an assumption that any mass of ground was as regular as this would probably involve more error than would occur from faulty estimation of fractional parts. Facilities for reading as high as 10,000 cubic yards would not have been put on the scale except for the necessity of finding such products as $\frac{25}{27}(9.1 \times 9.5)$, for example. This product would be read off from the same part of the rule as $\frac{25}{27}(91 \times 95)$. In the first case the product (80.0) could be read directly to the nearest .2 of a cubic yard, which is unnecessarily accurate. In the other case, the product (8004) could only be obtained by estimating $\frac{4}{20}$ of a division.

The computation for the prismoidal correction may be made similarly except that the divisor is 3.24 instead of 1.08. For example, $\frac{25}{81}(5.5 \times 11.7) = \frac{5.5 \times 11.7}{3.24}$. Set the 324 on scale *B* (also specially marked like 108) opposite 55 on scale *A*, and proceed as before.

80. Five-level sections. Sometimes the elevations over each edge of the roadbed are observed when cross-sectioning. These are distinctively termed "five-level sections." If the center, the slope-stakes, and *one* intermediate point on each side (*not* necessarily over the edge of the roadbed) are observed, it is termed an "irregular section." The field-work of cross-sectioning five-level sections is no less than for irregular sections with one intermediate point; the computations, although capable of peculiar treatment on account of the location of the intermediate point, are no easier, and in some respects more laborious; the cross-sections obtained will not in general represent the actual cross-sections as truly as when there is perfect freedom in locating the intermediate point; as it is generally inadvisable or unnecessary to employ five-level sections throughout the length of a road, the change from one method to another adds a possible element of inaccuracy and loses the advantage of uniformity of method, particularly in the notes and *form* of computations. On these accounts the method will not be further developed, except to note that this case, as well as any other, may be solved by dividing the whole prismoid into triangular prismoids, computing the volume by averaging end areas, and computing the prismoidal correction by adding the computed corrections for each elementary triangular prismoid.

81. Irregular sections. In cross-sectioning irregular sections, the distance from the center and the elevation above "grade" of every "break" in the cross-section must be observed. The area of the irregular section may be obtained by computing the area of the trapezoids (*five*, in Fig. 44) and subtracting the two external triangles. For Fig. 44 the area would be

$$\frac{h_l+k_l}{2}(x_l-y_l)+\frac{k_l+d}{2}y_l+\frac{d+j_r}{2}z_r+\frac{j_r+k_r}{2}(y_r-z_r)$$
$$+\frac{k_r+h_r}{2}(x_r-y_r)-\frac{h_l}{2}\left(x_l-\frac{b}{2}\right)-\frac{h_r}{2}\left(x_r-\frac{b}{2}\right).$$

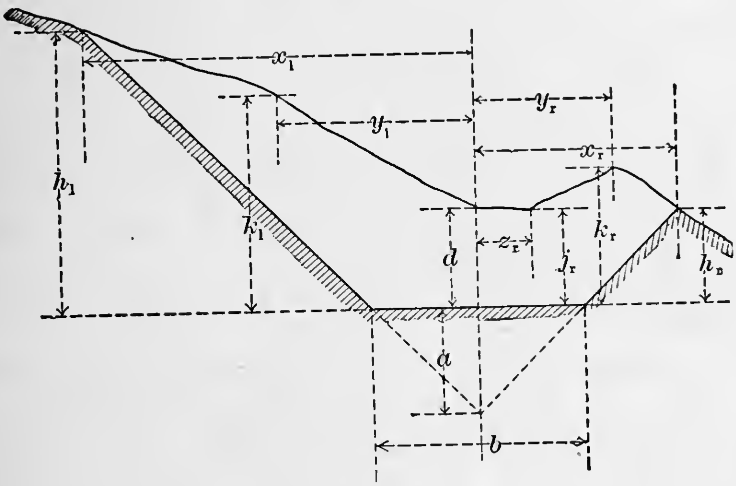

Fig. 44.

Expanding this and collecting terms, of which many will cancel, we obtain

$$\text{Area}=\frac{1}{2}\Big[x_lk_l+y_l(d-h_l)+x_rk_r+y_r(j_r-h_r)$$
$$+z_r(d-k_r)+\frac{b}{2}(h_l+h_r)\Big]. \quad . \quad . \quad (61)$$

An examination of this formula will show a perfect regularity in its formation which will enable one to write out a similar formula for any section, no matter how irregular or how many points there are, without any of the preliminary work. The formula may be expressed in words as follows:

Area equals one-half the sum of products obtained as follows:

the distance to each slope-stake times the height above grade of the point next inside the slope-stake;

the distance to each intermediate point in turn times the height of the point just inside minus the height of the point just outside;

finally, one-half the width of the roadbed times the sum of the slope-stake heights.

If one of the sides is perfectly regular from center to slope-stake, it is easy to show that the rule holds literally good. The "point next inside the slope-stake" in this case is the center; the intermediate terms for that side vanish. The *last term* must always be used. The rule holds good for three-level sections, in which case there are three terms, which may be reduced to two. Since these two terms are both variable quantities for each cross-section, the special method, given in § 78, in which one term $\left(\frac{ab}{2}\right)$ is a constant for all sections, is preferable. In the general method, each intermediate "break" adds another term.

82. Volume of an irregular prismoid. If there is a break at one cross-section which is not represented at the next, the ridge (or hollow) implied by that break is supposed to "vanish" at the next section. In fact, the volume will not be correctly

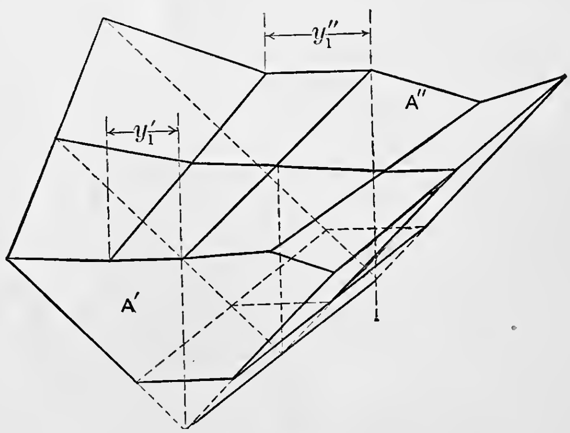

Fig. 52.

represented unless a cross-section is taken at the point where the ridge or hollow "vanishes" or "runs out." To obtain the true prismoidal correction it is necessary to observe on the ground the place where a break in an adjacent section, which is not represented in the section being taken, runs out. For example, in Fig. 52, the break on the left of section A'', at a distance of y_l'' from the center, is observed to run out in section A' at a distance of y_l' from the center. The volume of the prismoid, computed by the prismoidal formula as in § 70, will involve the midsection, to obtain the dimension of which will require a laborious computation. A simpler process is to compute the volume by averaging end areas as in § 81 and apply a prismoidal correction. To do this write out an expression for each end area similar to that given in Eq. 61. The sum of these areas times $\frac{l}{2}$ gives the approximate volume. As before, for partial station lengths, multiply the result by $\frac{\text{length in feet}}{100}$. There will be no constant subtractive term, $\frac{25}{27}ab$, as in § 78. The *true* prismoidal correction may be computed, as in § 83, or the following approximate method may be used: Consider the irregular section to be three-level ground *for the purpose of computing the correction* only. This has the advantage of less labor in computation than the use of the true prismoidal correction, and although the error involved may be considerable in individual sections, the error is as likely to be positive as negative, and in the long run the error will not be large and generally will be much less than would result by the neglect of any prismoidal correction.

83. True prismoidal correction for irregular prismoids. As intimated in § 82, each cross-section should be assumed to have the same number of sides as the adjacent cross-section when computing the prismoidal correction. This being done, it permits the division of the whole prismoid into elementary triangular prismoids, the dimensions of the bases of which being given in each case by a vertical distance above grade line and by the horizontal distance between two adjacent breaks. The summation of the prismoidal corrections for each of the elementary triangular prismoids will give the true prismoidal correction. Assuming for an example the cross-section of Fig. 44, with a cross-section of the same number of sides, and with dimensions similarly indicated, for the other end, the prismoidal correction becomes (see Eq. 46)

$$\frac{l}{12}\Big[(h_l' - h_l'')[(x_l'' - y_l'') - (x_l' - y_l')] + (k_l' - k_l'')[(x_l'' - y_l'') - (x_l' - y_l')]$$
$$+ (k_l' - k_l'')(y_l'' - y_l') + (d' - d'')(y_l'' - y_l') + (d' - d'')(z_r'' - z_r')$$
$$+ (j_r' - j_r'')(z_r'' - z_r') + (j_r' - j_r'')[(y_r'' - z_r'') - (y_r' - z_r')]$$
$$+ (k_r' - k_r'')[(y_r'' - z_r'') - (y_r' - z_r')]$$
$$+ (k_r' - k_r'')[(x_r'' - y_r'') - (x_r' - y_r')] + (h_r' - h_r'')[(x_r'' - y_r'') - (x_r' - y_r')]$$
$$- (h_l' - h_l'')\left[\left(x_l'' - \frac{b}{2}\right) - \left(x_l' - \frac{b}{2}\right)\right] - (h_r' - h_r'')\left[\left(x_r'' - \frac{b}{2}\right) - \left(x_r' - \frac{b}{2}\right)\right]\Big].$$

Expanding this and collecting terms, of which many will cancel, we obtain

$$\text{Pris. Corr.} = \frac{l}{12}\Big[(x_l'' - x_l')(k_l' - k_l'') + (y_l'' - y_l')[(d' - h_l') - (d'' - h_l'')]$$
$$+ (x_r'' - x_r')(k_r' - k_r'') + (y_r'' - y_r')[(j_r' - h_r') - (j_r'' - h_r'')]$$
$$+ (z_r'' - z_r')[(d' - k_r') - (d'' - k_r'')]\Big]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

By comparing this equation with Eq. 61 a remarkable coincidence in the law of formation may be seen, which enables this formula to be written by mere inspection and to be applied numerically with a minimum of labor from the computations for end areas, as will be shown (§ 84) by a numerical example. For each term in Eq. 61, as, for example, $y_r(j_r - h_r)$, there is a correction term in Eq. 62 of the form

$$(y_r'' - y_r')[(j_r' - h_r') - (j_r'' - h_r'')].$$

Each one of these terms (y_r'', y_r', $(j_r' - h_r')$, and $(j_r'' - h_r'')$) has been previously used in finding the end areas and has its place in the computation sheet. The summation of the products of these differences times a constant gives the total true prismoidal correction in cubic yards for the whole prismoid considered.

The *constant* is the same as that computed in § 78, i.e., $\frac{25}{81}$.

84. Numerical example; irregular sections; volume, with true prismoidal correction.

Sta.	Center { cut or fill.	Left.			Right.	
19	0.6*c*	$\frac{3.6c}{14.4}$	$\left(\frac{2.3c}{8.2}\right)$	$\left(\frac{1.8c}{6.0}\right)$	$\frac{0.1c}{4.2}$	$\frac{0.4c}{9.6}$
18	2.3*c*	$\frac{4.2c}{15.3}$	$\frac{6.8c}{8.4}$	$\frac{3.2c}{5.2}$	$\left(\frac{1.9c}{3.6}\right)$	$\frac{1.2c}{10.8}$
17	7.6*c*	$\frac{8.2c}{21.3}$	$\frac{10.2c}{17.4}$	$\frac{8.0c}{6.1}$	$\left(\frac{5.8c}{8.0}\right)$	$\frac{4.2c}{15.3}$
+ 42	10.2*c*	$\frac{12.2c}{27.3}$	$\left(\frac{12.3c}{22.0}\right)$	$\frac{12.6c}{8.2}$	$\frac{6.2c}{7.5}$	$\frac{8.4c}{21.6}$
16	6.8*c*	$\frac{8.9c}{22.4}$		$\frac{7.6c}{12.0}$	$\frac{3.2c}{4.1}$	$\frac{2.6c}{12.9}$

Roadbed 18 feet wide in cut; slope $1\frac{1}{2}$ to 1.

The figures in the bracket $\left(\frac{12.3c}{22.0}\right)$ mean that it was noted in the field that the break, indicated at Sta. 17 as being 17.4 to the left, ran out at Sta. 16 + 42 at 22.0 to the left. By interpolation between 8.2 and 27.3 the height of this point is *computed* as 12.3. The quantities in the other brackets are obtained similarly. These quantities are only used when the computation of the true prismoidal correction is desired. They are not needed in computing the volume by averaging end areas, nor are they used at all if the prismoidal correction is to be obtained by assuming (*for this purpose*) the ground to be *three-level* ground.

In the tabular form on page 98 the figures within the braces (⏟) are NOT used in computing the volume, but are only used to obtain the *differences* of widths or heights with which to compute the *true* prismoidal correction. It may be noted, as a check, that the volume, computed from these figures in the braces, is the same as that computed from the other figures.

VOLUME OF IRREGULAR PRISMOID, WITH TRUE PRISMOIDAL CORRECTION.

Sta.	Width.	Height.	Yards.		True pris. corr. $w'' - w'$	True pris. corr. $h' - h''$	True pris. corr. Yards.
	L[22.4	7.6	158				
	[12.0	− 2.1	− 23				
16	12.9]R	3.2	40				
	4.1]R	4.2	16				
	9.0	11.5	96				
	L[27.3	12.6	319		+ 4.9	− 5.0	− 7
	[8.2	− 2.0	− 15		− 3.8	− 0.1	0
	L{27.3	12.3					
+ 42	22.0	0.4					
	8.2	− 2.1					
	21.6]R	6.2	124		+ 8.7	− 3.0	− 8
	7.5]R	1.8	13		+ 3.4	+ 2.4	+ 3
	9.0	20.6	172	378			(− 5)
	L[21.3	10.2	201		− 6.0	+ 2.1	− 4
	17.4	− 0.2	− 3		− 4.6	+ 0.6	− 1
	6.1	− 2.6	− 14		− 2.1	+ 0.5	0
17	15.3}R	5.8			− 6.3	+ 0.4	− 1
	8.0}R	3.4			+ 0.5	− 1.6	0
	15.3]R	7 6	107				
	9.0	12.4	103	584			(− 3)
	L[15.3	6.8	95		− 6.0	+ 3.4	− 6
	8.4	− 1.0	− 7		− 9.0	+ 0.8	− 2
	5.2	− 4.5	− 22		− 0.9	+ 1.9	− 1
18	10.8]R	2.3	23		− 4.5	+ 5.3	− 7
	10.8}R	1.9					
	3.6}R	1.1					
	9.0	5.4	45	528			(− 16)
	L[14.4	0.6	8				
	L{14.4	2.3			− 0.9	+ 4.5	− 1
	8.2	− 1.8			− 0.2	+ 0.8	0
19	6.0	− 1.7			+ 0.8	− 2.8	− 1
	9.6]R	0.1	1		− 1.2	+ 1.8	− 1
	4.2]R	0.2	1		+ 0.6	+ 0.9	0
	9.0	4.0	33	177			(− 3)

Approx. vol. = 1667   − 27
True pris. corr. = − 27

True volume = 1640 cubic yards

The figures within each brace (or bracket) constitute a group which must be used in connection with a group which has the same number of points, on the same side of the center, in the *next* cross-section, previous or succeeding. In the column of

"Yards" under "True pris. corr.," we have, for example, $(-5) = \frac{42}{100}(-7 + 0 - 8 + 3)$.

85. Volume of irregular prismoid, with approximate prismoidal correction. If the prismoidal correction is obtained approximately, by the method outlined in § 82, the process will be as shown in the tabular form. Not only is the numerical work considerably less than the exact method, but the discrepancy in cubic yards is almost insignificant.

Sta.	Width.	Height.	Yards.		Cen. Height.	Total width.	$d'-d''$	$w''-w'$	Approx. pris. corr.
	22.4	7.6	158		+ 6.8	35.3			
	12.0	– 2.1	– 23						
16	12.9	3.2	40						
	4.1	4.2	16						
	9.0	11.5	96						
	27.3	12.6	319		+ 10.2	48.9	–3.4	+ 13.6	– 14
	8.2	– 2.0	– 15						
+ 42	21.6	6.2	124						
	7.5	1.8	13						
	9.0	20.6	172	378					(– 6)
	21.3	10.2	201		+ 7.6	36.6	+2.6	– 12.3	– 10
	17.4	– 0.2	– 3						
17	6.1	– 2.6	– 14						
	15 3	7.6	107						
	9.0	12.4	103	584					(– 6)
	15.3	6.8	95		+ 2.3	26.1	+5.3	– 10.5	– 17
	8.4	– 1.0	– 7						
18	5.2	– 4.5	– 22						
	10.8	2.3	23						
	9.0	5.4	45	528					(– 17)
	14.4	0.6	8		+ 0.6	24.0	+1.7	– 2.1	– 1
19	9.6	0.1	1						
	4.2	0.2	1						
	9.0	4.0	33	177					(– 1)

Approx. volume = 1667 — 30
Approx. pris. corr. = – 30

Corrected volume = 1637 cubic yards

86. Illustration of value of approximate rules. The accompanying tabulation shows that when the volume of an irregular prismoid is computed by averaging end areas and is corrected by considering the ground as three-level ground (*for the pur-*

poses of the correction only), the error for the different sections is sometimes positive and sometimes negative, and in this case

Sections.	True volume.	Approx. vol. by averaging end areas.	Difference or *true* pris. corr.	Approx. pris. corr. on basis of three-level ground.	Error.	Approx. vol., computed from center and side heights *only*.	Error.
16........16 + 42	373	378	− 5	− 6	− 1	396	+ 23
16 + 42...17	581	584	− 3	− 6	− 3	577	− 4
17........18	512	528	− 16	− 17	− 1	463	− 49
18 19	174	177	− 3	− 1	+ 2	147	− 27
	1640	1667	− 27	− 30	− 3	1583	− 57

amounts to only 3 yards in 1640—less than $\frac{1}{5}$ of 1%. If the prismoidal correction had been neglected, the error would have been 27 yards—nearly 2%. The approximate results are here *too large* for each section—as is usually the case. If points between the center and slope stakes are omitted and the volume computed as if the ground were *three-level* ground, the error is quite large in individual sections, but the errors are both positive and negative and therefore compensating.

87. Cross-sectioning irregular sections. The prismoids considered have *straight* lines joining corresponding points in the two cross-sections. The center line must be straight between two cross-sections. If a ridge or valley is found lying diagonally across the roadbed, a cross-section *must* be interpolated at the lowest (or highest) point of the profile. Therefore a "break" at any section cannot be said to run out at the other section on the *opposite* side of the center. It must run out on the *same* side of the center or possibly *at* the center. Very frequently complicated cross-sectioning may be avoided by computing the volume, by some special method, of a mound or hollow when the ground is comparatively regular except for the irregularity referred to.

88. Side-hill work. When the natural slope cuts the roadbed there is a necessity for both cut and fill at the same cross-section. When this occurs the cross-sections of both cut and fill are often so nearly triangular that they may be considered as such without great error, and the volumes may be computed separately as triangular prismoids without adopting the more elaborate form of computation so necessary for complicated irregular sections. When the ground is too irregular for this the best plan is to follow the uniform system. In computing the cut, as in Fig. 53,

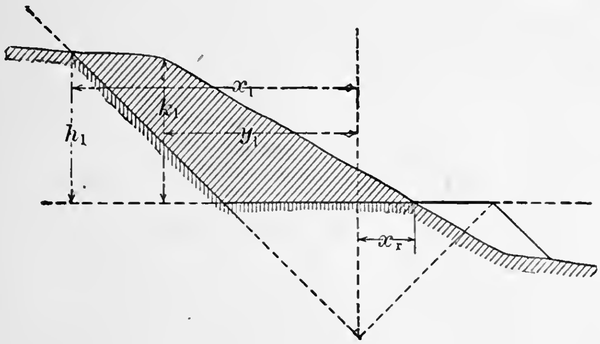

FIG. 53.

the left side would be as usual; there would be a small center cut and an ordinate of zero at a short distance to the right of the center. Then, *ignoring the fill*, and applying Eq. 61 strictly, we have two terms for the left side, one for the right, and the term involving $\frac{1}{2}b$, which will be $\frac{1}{2}bh_l$ in this case, since $h_r = 0$, and the equation becomes

$$\text{Area} = \tfrac{1}{2}[x_l k_l + y_l(d - h_l) + x_r d + \tfrac{1}{2}bh_l].$$

The area for fill may also be computed by a strict application

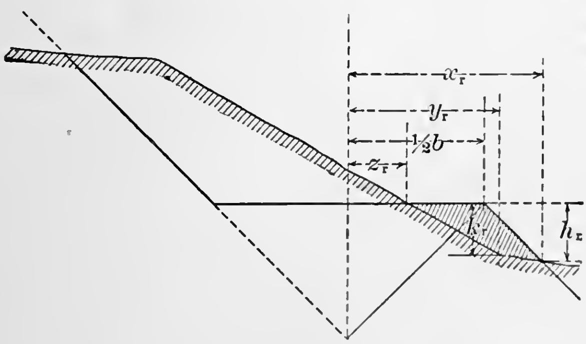

FIG. 54.

of Eq. 61, but for Fig. 54 all distances for the left side are zero and the elevation for the first point out is zero. d also must be considered as zero. Following the rule, § 81, literally, the equation becomes

$$\text{Area}_{(\text{Fill})} = \tfrac{1}{2}[x_r k_r + y_r(o - h_r) + z_r(o - k_r) + \tfrac{1}{2}b(o + h_r)],$$

which reduces to

$$\tfrac{1}{2}[x_r k_r - y_r h_r - z_r k_r + \tfrac{1}{2}b h_r].$$

(*Note* that x_r, h_r, etc., have different significations and values in this and in the preceding paragraphs.) The "terminal pyramids" illustrated in Fig. 40 are instances of side-hill work for very short distances. Since side-hill work always implies *both* cut and fill at the same cross-section, whenever either the cut or fill disappears and the earthwork becomes wholly cut or wholly fill, that point marks the end of the "side-hill work," and a cross-section should be taken at this point.

89. Borrow-pits. The cross-sections of borrow-pits will vary not only on account of the undulations of the surface of the

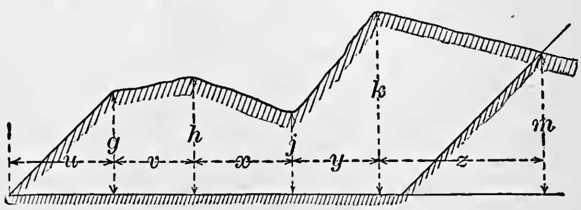

FIG. 55.

ground, but also on the sides, according to whether they are made by widening a convenient cut (as illustrated in Fig. 55) or simply by digging a pit. The sides should always be properly sloped and the cutting made cleanly, so as to avoid unsightly roughness. If the slope ratio on the right-hand side (Fig. 55) is s, the area of the triangle is $\tfrac{1}{2}sm^2$. The area of the section is $\tfrac{1}{2}[ug+(g+h)v+(h+j)x+(j+k)y+(k+m)z-sm^2]$. If all the horizontal measurements were referred to one side as an origin, a formula similar to Eq. 61 could readily be developed, but little or no advantage would be gained on account of any simplicity of computation. Since the *exact* volume of the earth borrowed is frequently necessary, the prismoidal correction should be computed; and since such a section as Fig. 55 does not even approximate to a three-level section, the method suggested in § 82 cannot be employed. It will then be necessary to employ the exact method, § 83, by dividing the volume into triangular prismoids and taking the summation of their corrections, found according to the general method of § 71.

90. Correction for curvature. The volume of a solid, generated by revolving a plane area about an axis lying in the plane but outside of the area, equals the product of the given area times the length of the path of the center of gravity of the area. If the centers of gravity of all cross-sections lie in the center of the road, where the length of the road is measured, there is absolutely no necessary correction for curvature. If all the cross-sections in any given length were exactly the same and therefore had the same eccentricity, the correction for curvature would be very readily computed according to the above principle. But when both the areas and the eccentricities vary from point to point, as is generally the case, a theoretically exact solution is quite complex, both in its derivation and application. Suppose, for simplicity, a curved section of the road, of uniform cross-sections and with the center of gravity of every cross-section at the same distance e from the center line of the road. The length of the path of the center of gravity will be to the length of the center line as $R \pm e : R$. Therefore we have *True vol.* : *nominal vol.* : : $R \pm e : R$.

$\therefore$ *True vol.* $= lA\dfrac{R \pm e}{R}$ for a volume of uniform area and eccentricity. For any other area and eccentricity we have, similarly, *True vol.'* $= lA'\dfrac{R \pm e'}{R}$. This shows that the effect of curvature is the same as increasing (or diminishing) the area by a quantity depending on the area and eccentricity, the increased (or diminished) area being found by multiplying the actual area by the ratio $\dfrac{R \pm e}{R}$. This being independent of the value of l, it is true for infinitesimal lengths. If the eccentricity is assumed to vary uniformly between two sections, the *equivalent area* of a cross-section located midway between the two end cross-sections would be $A_m \dfrac{\left(R \pm \dfrac{e' + e''}{2}\right)}{R}$. Therefore the volume of a solid which, when straight, would be $\frac{l}{6}(A' + 4A_m + A'')$, would then become

$$True\ vol. = \frac{l}{6R}\left[A'(R \pm e') + 4A_m\left(R \pm \frac{e' + e''}{2}\right) + A''(R \pm e'')\right].$$

Subtracting the nominal volume (the true volume when the prismoid is straight), the

$$Correction = \pm \frac{l}{6R}\Big[(A' + 2A_m)e' + (2A_m + A'')e''\Big]. \quad (63)$$

Another demonstration of the same result is given by Prof. C. L. Crandall in his "Tables for the Computation of Railway and other Earthwork," in which is obtained by calculus methods the summation of elementary volumes having variable areas with variable eccentricities. The exact application of Eq. (63) requires that A_m be known, which requires laborious computations, but no error worth considering is involved if the equation is written approximately

$$Curv.\ corr. = \frac{l}{2R}(A'e' + A''e''), \quad . \quad . \quad . \quad (64)$$

which is the equation generally used. The approximation consists in assuming that the difference between A' and A_m equals the difference between A_m and A'' but with opposite sign. The error due to the approximation is always utterly insignificant.

91. Eccentricity of the center of gravity. The determination of the true positions of the centers of gravity of a long series of irregular cross-sections would be a very laborious operation, but fortunately it is generally sufficiently accurate to consider the cross-sections as three-level ground, or, for side-hill work, to be triangular, *for the purpose of this correction.* The

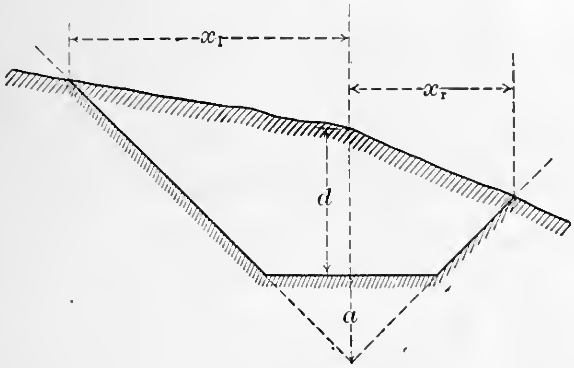

FIG. 56.

eccentricity of the cross-section of Fig. 56 (including the grade triangle) may be written

$$e = \frac{\frac{(a+d)x_l}{2}\frac{x_l}{3} - \frac{(a+d)x_r}{2}\frac{x_r}{3}}{\frac{(a+d)x_l}{2} + \frac{(a+d)x_r}{2}} = \frac{1}{3}\frac{x_l^2 - x_r^2}{x_l + x_r} = \frac{1}{3}(x_l - x_r). \quad (65)$$

The side toward x_r being considered positive in the above demonstration, if $x_r > x_l$, e would be negative, i.e., the center of gravity would be on the left side. Therefore, for three-level ground, the correction for curvature (see Eq. 64) may be written

$$Correction = \frac{l}{6R}[A'(x_l' - x_r') + A''(x_l'' - x_r'')].$$

Since the approximate volume of the prismoid is

$$\frac{l}{2}(A + A') = \frac{l}{2}A' + \frac{l}{2}A'' = V' + V'',$$

in which V' and V'' represent the number of cubic yards corresponding to the area at each station, we may write

$$Corr.\ in\ cub.\ yds. = \frac{1}{3R}[V'(x_l' - x_r') + V''(x_l'' - x_r'')]. \quad (66)$$

It should be noted that the value of e, derived in Eq. 65, is the eccentricity of the whole area including the triangle under the roadbed. The eccentricity of the true area is greater than this and equals

$$e \times \frac{\text{true area} + \frac{1}{2}ab}{\text{true area}} = e_1.$$

The required quantity ($A'e'$ of Eq. 64) equals *true area* $\times\ e_1$, which equals (*true area* $+ \frac{1}{2}ab) \times e$. Since the value of e is very simple, while the value of e_1 would, in general, be a complex quantity, it is easier to use the simple value of Eq. 65 and add $\frac{1}{2}ab$ to the area. Therefore, in the case of three-level ground the subtractive term $\frac{2}{2}\frac{5}{7}ab$ (§ 78) should *not* be subtracted in computing this correction. For irregular ground, when computed by the method given in §§ 81 and 82, which does not involve the grade triangle, a term $\frac{2}{2}\frac{5}{7}ab$ must be *added* at every station when computing the quantities V' and V'' for Eq. 66.

It should be noted that the factor $1 \div 3R$, which is constant for the length of the curve, may be computed with all necessary accuracy and without resorting to tables by remembering that

$$R = \frac{5730}{\text{degree of curve}}.$$

Since it is useless to attempt the computation of railroad earthwork closer than the nearest cubic yard, it will frequently be possible to write out all curvature corrections by a simple mental process upon a mere inspection of the computation sheet. Eq. 66 shows that the correction for each station is of the form $\frac{V(x_l - x_r)}{3R}$. $3R$ is generally a large quantity—for a 6° curve it is 2865. $(x_l - x_r)$ is generally small. It may frequently be seen by inspection that the product $V(x_l - x_r)$ is roughly twice or three times $3R$, or perhaps less than half of $3R$, so that the corrective term for that station may be written 2, 3, or 0 cubic yards, the fraction being disregarded. For much larger absolute amounts the correction must be computed with a correspondingly closer percentage of accuracy.

The algebraic sign of the curvature correction is best determined by noting that the center of gravity of the cross-section is on the right or left side of the center according as x_r is greater or less than x_l, and that the correction is *positive* if the center of gravity is on the *outside* of the curve, and *negative* if on the *inside*.

It is frequently found that x_l is uniformly greater (or uniformly less) than x_r throughout the length of the curve. Then the curvature correction for each station is uniformly positive or negative. But in irregular ground the center of gravity is apt to be irregularly on the outside or on the inside of the curve, and the curvature correction will be correspondingly positive or negative. If the curve is to the *right*, the correction will be positive or negative according as $(x_l - x_r)$ is positive or negative; if the curve is to the *left*, the correction will be positive or negative according as $(x_r - x_l)$ is positive or negative. Therefore when computing curves to the *right* use the form $(x_l - x_r)$ in Eqs. 66 and 68; when computing curves to the *left* use the form $(x_r - x_l)$ in these equations; the algebraic sign of the correction will then be strictly in accordance with the results thus obtained.

92. Center of gravity of side-hill sections. In computing the

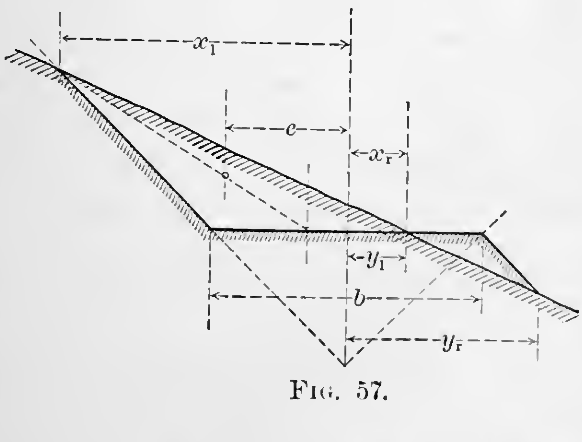

Fig. 57.

correction for side-hill work the cross section would be treated as triangular unless the error involved would evidently be too great to be disregarded. The center of gravity of the triangle lies on the line joining the vertex with the middle of the base and at $\frac{1}{3}$ of the length of this line from the base. It is therefore equal to the distance from the center to the foot of this line plus $\frac{1}{3}$ of its horizontal projection. Therefore

$$e = \left[\frac{b}{2} - \frac{1}{2}\left(\frac{b}{2} + x_r\right)\right] + \frac{1}{3}\left[x_l - \left(\frac{b}{2} - \frac{1}{2}\left(\frac{b}{2} + x_r\right)\right)\right]$$

$$= \frac{b}{4} - \frac{x_r}{2} + \frac{x_l}{3} - \frac{b}{12} + \frac{x_r}{6}$$

$$= \frac{b}{6} + \frac{x_l}{3} - \frac{x_r}{3}$$

$$= \frac{1}{3}\left[\frac{b}{2} + (x_l - x_r)\right]. \quad \ldots \ldots \quad (67)$$

By the same process as that used in § 91 the correction equation may be written

$$\text{Corr. in cub. yds.} = \frac{1}{3R}\left[V'\left(\frac{b}{2} + (x_l' - x_r')\right) + V''\left(\frac{b}{2} + (x_l'' - x_r'')\right)\right]. \quad (68)$$

It should be noted that since the grade triangle is not used in this computation the volume of the grade prism is *not* involved in computing the quantities V' and V''.

The eccentricities of cross-sections in side-hill work are *never* zero, and are frequently quite large. The total volume is generally quite small. It follows that the correction for curvature is generally a vastly larger proportion of the total volume than in ordinary three-level or irregular sections.

If the triangle is wholly to one side of the center, Eq. 67 can still be used. For example, to compute the eccentricity of the triangle of fill, Fig. 57, denote the two distances to the slope-stakes by y_r and $-y_l$ (note the minus sign). Applying Eq. 67 literally (noting that $\frac{b}{2}$ must here be considered as negative in order to make the notation consistent) we obtain

$$e = \frac{1}{3}\left[-\frac{b}{2} + (-y_l - y_r)\right].$$

which reduces to

$$e = -\frac{1}{3}\left[\frac{b}{2} + y_l + y_r\right]. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (69)$$

As the algebraic signs tend to create confusion in these formulæ, it is more simple to remember that for a triangle lying on *both* sides of the center e is always numerically equal to $\frac{1}{3}\left[\frac{b}{2} + (x_l \sim x_r)\right]$, and for a triangle entirely on one side, e is numerically equal to $\frac{1}{3}\left[\frac{b}{2}\right.$ + the numerical *sum* of the two distances out]. The algebraic sign of e is readily determinable as in § 91.

93. Example of curvature correction. Assume that the fill in § 78 occurred on a 6° curve to the *right*. $\frac{1}{3R} = \frac{1}{2865}$. The quantities 210, 507, etc., represent the quantities V', V'', etc., since they include in each case the 61 cubic yards due to the grade prism. Then

$$\frac{V(x_l \sim x_r)}{3R} = \frac{210(22.9 - 8.2)}{2865} = \frac{3101.7}{2865} = +1.$$

The sign is plus since the center of gravity of the cross-section is on the left side of the center and the road curves to the right, thus making the true volume larger. For Sta. 18 the correction, computed similarly, is + 3, and the correction for the whole section is 1 + 3 = 4. For Sta. 18 + 40 the correction is computed as 6 yards. Therefore, for the 40 feet, the correction is $\frac{40}{100}(3 + 6) = 3.6$, which is called 4. Computing the others similarly we obtain a total correction of + 16 cubic yards.

94. Accuracy of earthwork computations. The preceding methods give the *precise volume* (except where approximations are distinctly admitted) of the prismoids which are *supposed* to represent the volume of the earthwork. To appreciate the accuracy necessary in cross-sectioning to obtain a given accuracy in volume, consider that a fifteen-foot length of the cross-section, which is assumed to be straight, really sags 0.1 foot, so that the cross-section is in error by a triangle 15 feet wide and 0.1 foot high. This sag 0.1 foot high would hardly be detected by the eye, but in a length of 100 feet in each direction it would make an error of volume of 1.4 cubic yards in *each* of the two prismoids, assuming that the sections at the other ends were perfect. If the cross-sections at both ends of a prismoid were in error by this same amount, the volume of that prismoid would be in error by 2.8 cubic yards if the errors of area were both plus or both minus. If one were plus and one minus, the errors would neutralize each other, and it is the compensating character of these errors which permits any confidence in the results as obtained by the usual methods of cross-sectioning. It demonstrates the utter futility of attempting any closer accuracy than the nearest cubic yard. It will thus be seen that if an error really exists at *any* cross-section it involves the prismoids on *both* sides of the section, even though all the other cross-sections are perfect. As a further illustration, suppose that cross-sections were taken by the method of slope angle and center depth (§ 73), and that a cross-section, assumed as uniform, sags 0.4 foot in a width of 20 feet. Assume an equal error (of same sign) at the other end of a 100-foot section. The error of volume for that one prismoid is 38 cubic yards.

The computations further assume that the warped surface, passing through the end sections, coincides with the surface of the ground. Suppose that the cross-sectioning had been done with mathematical perfection; and, to assume a simple case, suppose a sag of 0.5 foot between the sections, which causes an error equal to the volume of a pyramid having a base of 20 feet (in each cross-section) times 100 feet (between the cross-sections) and a height of 0.5 foot. The volume of this pyramid is $\frac{1}{3}(20 \times 100) \times 0.5 = 333$ cub. ft. $= 12$ cub. yds. And yet this sag or hump of 6 inches would generally be utterly unnoticed, or at least disregarded.

When the ground is very rough and broken it is sometimes practically impossible, even with frequent cross-sections, to locate warped surfaces which will closely coincide with all the sudden irregularities of the ground. In such cases the computations are necessarily more or less approximate and dependence must be placed on the compensating character of the errors.

95. Approximate computations from profiles. As a means of comparing the relative amounts of earthwork on two or more proposed routes which have been surveyed by preliminary surveys, it will usually be sufficiently accurate to compare the *areas* of cutting (assuming that the cut and fill are approximately balanced) as shown by the several profiles. The errors involved may be large in individual cases and for certain small sections, but fortunately the errors (in comparing two lines) will be largely compensated. The errors are much larger on side-hill work than when the cross-sections are comparatively level. The errors become large when the depth of cut or fill is very great. If the lines compared have the same general character as to the slope of the cross-sections, the proportion of side-hill work, and the average depth of cut or fill, the error involved in considering their relative volumes of cutting to be as the relative areas of cutting on the profiles (obtained perhaps by a planimeter) will probably be small. If the volume in each case is computed by assuming the sections as *level*, with a depth equal to the center cut, the error involved will depend only on the amount of side-hill work and the degree of the slope. If these features are about the same on the two lines compared, the error involved is still less.

FORMATION OF EMBANKMENTS.

96. Shrinkage of earthwork. The evidence on this subject as to the amount of shrinkage is very conflicting, a fact which is probably due to the following causes:

1. The various kinds of earthy material act very differently as respects shrinkage. There has been but little uniformity in the *classification of earths* in the tests and experiments that have been made.

2. Very much depends on the *method* of forming an embankment (as will be shown later). Different reports have been based on different methods—often without mention of the method.

3. An embankment requires considerable *time* to shrink to its final volume, and therefore much depends on the time elapsed between construction and the measurement of what is supposed to be the settled volume.

P. J. Flynn quotes some experiments (*Eng. News*, May 1, 1886) made in India in which pits were dug, having volumes of 400 to 600 cubic feet. The material, when piled into an embankment, measured largely in excess of the original measurement—as is the universal experience. The pits were refilled with the same material. As the rains, very heavy in India, settled the material in the pits, more was added to keep the pits full. Even after the rainy season was over, there was in every case material in excess. This would seem to indicate a permanent *expansion*, although it is possible that the observations were not continued for a sufficient time to determine the final settled volume.

On the contrary, notes made by Mr. Elwood Morris many years ago on the behavior of embankments of several thousand cubic yards, formed in layers by carts and scrapers, one winter intervening between commencement and completion, showed in each case a permanent *contraction* averaging about 10%.

All authorities agree that rockwork *expands* permanently when formed into an embankment, but the percentages of expansion given by different authorities differ even more than with earth—varying from 8 to 90%. Of course this very large range in the coefficient is due to differences in the character of the rock. The softer the rock and the closer its similarity to earth, the less will be its expansion. On account of the conflicting statements made, and particularly on account of the influence of methods of work, but little confidence can be felt in any given coefficient, especially when given to a fraction of a per cent, but the consensus of American practice seems to average about as follows:

Permanent contraction of earth..........	about 10%
" expansion of rock............	40 to 60%

These values for rock should be materially reduced, according to judgment, when the rock is soft and liable to disintegrate. The hardest rocks, loosely piled, may occasionally give even higher results. The following is given by several authors as the permanent contraction of several grades of earth:

Gravel or sand........................	about 8%
Clay...............................	" 10%
Loam...............................	" 12%
Loose vegetable surface soil............	" 15%

It may be noticed from the above table that the harder and cleaner the material the less is the contraction. Perfectly clean gravel or sand would not probably change volume appreciably. The above coefficients of shrinkage and expansion may be used to form the following convenient table.

Material.	To make 1000 cubic yards of embankment will require	1000 cubic yards measured in excavation will make
Gravel or sand.......... ...	1087 cubic yards	920 cubic yards
Clay......................	1111 " "	900 " "
Loam......................	1136 " "	880 " "
Loose vegetable soil........	1176 " "	850 " "
Rock, large pieces..........	714 " "	1400 " "
" small "	625 " "	1600 " "
	measured in excavation	of embankment.

97. Allowance for shrinkage. On account of the initial expansion and subsequent contraction of earth, it becomes necessary to form embankments higher than their required ultimate form in order to allow for the subsequent shrinkage. As the shrinkage appears to be all vertical (practically), the embankment must be formed as shown in Fig. 58. The effect of shrinkage should not be confounded with that of slipping of the sides, which is especially apt to occur if the embankment is subjected to heavy rains very soon after being formed, and also when the embankments are originally steep. It is often difficult

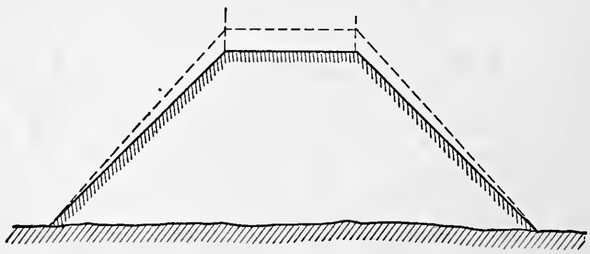

FIG. 58.

to form an embankment at a slope of 1 : 1 which will not slip more or less before it hardens.

Very high embankments shrink a greater percentage than lower ones. Various rules giving the relation between shrinkage and height have been suggested, but they vary as badly as the suggested coefficients of contraction, probably for the same causes. As the fact is unquestionable, however, the extra height of the embankment must be varied somewhat as in Fig. 59, which represents a longitudinal section of an embankment.

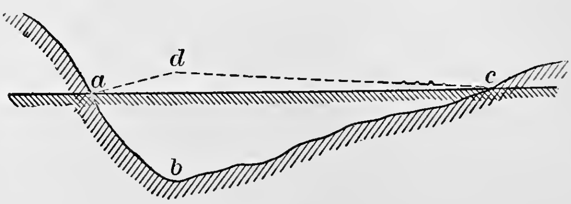

FIG. 59.

As considerable time generally elapses between the completion of the embankment and the actual running of trains, the grade *ad* will generally be nearly flattened down to its ultimate form before traffic commences, but such grades are occasionally objectionable if added to what is already a ruling grade. With some kinds of soil the time required for complete settlement may be as much as two or three years, but, even in such cases, it is probable that one-half of the settlement will take place during the first six months. The engineer should therefore require the contractor to make all fills about 8 to 15% (according to the material) higher than the profiles call for, in order that subsequent shrinkage may not reduce it to less than the required volume.

98. Methods of forming embankments. When the method is not otherwise objectionable, a high embankment can be formed very cheaply (assuming that carts or wheelbarrows are used) by dumping over the end and building to the full height (or even higher, to allow for shrinkage) as the embankment proceeds. This allows more time for shrinkage, saves nearly all the cost of spreading (see Item 4, § 111), and reduces the cost of roadways (Item 5). Of course this method is especially applicable when the material comes from a place as high as or higher than grade, so that no up-hill hauling is required.

Another method is to spread it in layers two or three feet thick (see Fig. 60), which are made concave upwards to avoid

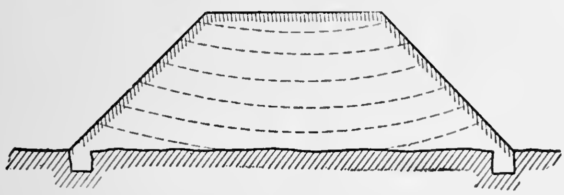

FIG. 60.

possible sliding on each other. Spreading in layers has the advantage of partially ramming each layer, so that the subsequent shrinkage is very small. Sometimes small trenches are dug along the lines of the toes of the embankment. This will frequently prevent the sliding of a large mass of the embankment, which will then require extensive and costly repairs, to say nothing of possible accidents if the sliding occurs after the road is in operation. Incidentally these trenches will be of value in draining the subsoil. When circumstances require an embankment on a hillside, it is advisable to cut out "steps" to prevent a possible sliding of the whole embankment. Merely ploughing the side-hill will often be a cheaper and sufficiently effective method.

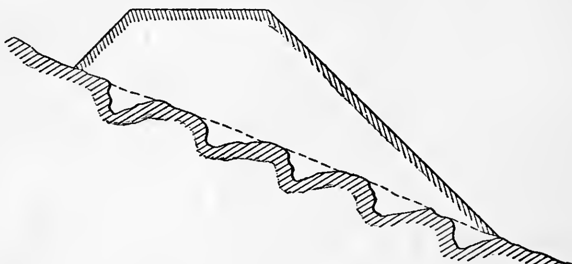

FIG. 61.

Occasionally the formation of a very high and long embankment may be most easily and cheaply accomplished by building a trestle to grade and opening the road. Earth can then be procured where most convenient, perhaps several miles away, loaded on cars with a steam-shovel, hauled by the trainload, and dumped from the cars with a patent unloader. On such a large scale, the cost per yard would be very much less than by ordinary methods—enough less sometimes to more than pay for the temporary trestle, besides allowing the road to be opened for traffic very much earlier, which is often a matter of prime financial importance. It may also obviate the necessity for extensive borrow-pits in the immediate neighborhood of the heavy fill and also utilize material which would otherwise be wasted.

COMPUTATION OF HAUL.

99. Nature of subject. As will be shown later when analyzing the cost of earthwork, the most variable item of cost is that depending on the distance hauled. As it is manifestly impracticable to calculate the exact distance to which every individual cartload of earth has been moved, it becomes necessary to devise a means which will give at least an equivalent of the haulage of all the earth moved. Evidently the *average* haul for any mass of earth moved is equal to the distance from the center of gravity of the excavation to the center of gravity of the embankment formed by the excavated material. As a rough approximation the center of gravity of a cut (or fill) may sometimes be considered to coincide with the center of gravity of that part of the profile representing it, but the error is frequently very large. The center of gravity may be determined by various methods, but the method of the "mass diagram" accomplishes the same ultimate purpose (the determination of the haul) with all-sufficient accuracy and also furnishes other valuable information.

100. Mass diagram. In Fig. 62 let $A'B' \ldots G'$ represent a profile and grade line drawn to the usual scales. Assume A'

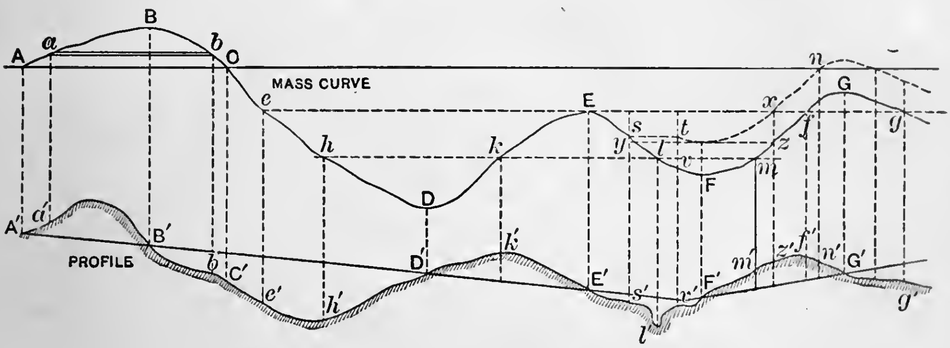

Fig. 62.—Mass Diagram.

to be a point past which no earthwork will be hauled. Above every station point in the profile draw an ordinate which will represent the algebraic sum of the cubic yards of cut and fill (calling cut + and fill −) from the point A' to the point considered. In doing this shrinkage must be allowed for by considering how much embankment would actually be made by so many cubic yards of excavation of such material. For example, it will be found that 1000 cubic yards of sand or gravel, measured in place (see § 97), will make about 920 cubic yards of embankment; therefore all cuttings in sand or gravel should be discounted in about this proportion. Excavations in rock should be increased in the proper ratio. In short, all excavations should be valued according to the amount of *settled* embankment that could be made from them. The computations may be made systematically as shown in the tabular form. Place in the first column a list of the stations; in the second column, the number of cubic yards of cut or fill between each station and the preceding station; in the third and fourth columns, the kind of material and the proper shrinkage factor; in the fifth column, a repetition of the quantities in cubic yards, except that the excavations are diminished (or increased, in the case of rock) to the number of cubic yards of settled embankment which may be made from them. In the sixth column, place the *algebraic sum* of the quantities in the fifth column (calling cuts + and fills −) from the starting-point to the station considered. These algebraic sums at each station will be the ordinates, drawn to some scale, of the mass curve. The scale to be used will depend somewhat on whether the work is heavy or light, but for ordinary cases a scale of 5000 cubic yards per inch may be used. Drawing these ordinates to scale, a curve *A*, *B*, . . . *G* may be obtained by joining the extremities of the ordinates.

Sta.	Yards { cut + fill −	Material.	Shrinkage factor.	Yards, reduced for shrinkage.	Ordinate in mass curve.
46 + 70	....	....	....	....	0
47	+ 195	Clayey soil	−10 per cent	+ 175	+ 175
48	+1792	" "	−10 "	+1613	+1788
+ 60	+ 614	" "	−10 "	+ 553	+2341
49	− 143	....	....	− 143	+2198
50	− 906	....	....	− 906	+1292
51	−1985	....	....	−1985	− 693
52	−1721	....	....	−1721	−2414
+ 30	− 112	....	....	− 112	−2526
53	+ 177	Hard rock	+60 per cent	+ 283	−2243
+ 70	+ 180	" "	+60 "	+ 289	−1954
54	− 52	....	....	− 52	−2006
+ 42	− 71	....	....	− 71	−2077
55	+ 276	Clayey soil	−10 per cent	+ 249	−1828
56	+1242	" "	−10 "	+1118	− 710
57	+1302	" "	−10 "	+1172	+ 462

101. Properties of the mass curve.

1. The curve will be rising while over cuts and falling while over fills.

2. A tangent to the curve will be horizontal (as at *B*, *D*, *E*, *F*, and *G*) when passing from cut to fill or from fill to cut.

3. When the curve is *below* the "zero line" it shows that material must be drawn *backward* (to the left); and *vice versa*, when the curve is *above* the zero line it shows that material must be drawn *forward* (to the right).

4. When the curve crosses the zero line (as at A and C) it shows (in this instance) that the cut between A' and B' will just provide the material required for the fill between B' and C', and that no material should be hauled past C', or, in general, past any intersection of the mass curve and the zero line.

5. If any horizontal line be drawn (as ab), it indicates that the cut and fill between a' and b' will just balance.

6. When the center of gravity of a given volume of material is to be moved a given distance, it makes no difference (at least theoretically) how far each individual load may be hauled or how any individual load may be disposed of. The summation of the products of each load times the distance hauled will be a constant, whatever the method, and will equal the total volume times the movement of the center of gravity. The *average haul*, which is the movement of the center of gravity, will therefore equal the summation of these products divided by the total volume. If we draw two horizontal parallel lines at an infinitesimal distance dx apart, as at ab, the small increment of cut dx at a' will fill the corresponding increment of fill at b', and this material must be hauled the distance ab. Therefore the product of ab and dx, which is the product of distance times volume, is represented by the area of the infinitesimal rectangle at ab, and the total area ABC represents the summation of volume times distance for all the earth movement between A' and C'. This summation of products divided by the total volume gives the average haul.

7. The horizontal line, tangent at E and cutting the curve at e, f, and g, shows that the cut and fill between e' and E' will just balance, and that a *possible* method of hauling (whether desirable or not) would be to "borrow" earth for the fill between C' and e', use the material between D' and E' for the

fill between e' and D', and similarly balance cut and fill between E' and f' and also between f' and g'.

8. Similarly the horizontal line *hklm* may be drawn cutting the curve, which will show another *possible* method of hauling. According to this plan, the fill between C' and h' would be made by borrowing; the cut and fill between h' and k' would balance; also that between k' and l' and between l' and m'. Since the area *ehDkE* represents the measure of haul for the earth between e' and E', and the other areas measure the corresponding hauls similarly, it is evident that the sum of the areas *ehDkE* and *ElFmf*, which is the measure of haul of all the material between e' and f', is largely in excess of the sum of the areas *hDk*, *kEl*, and *lFm*, plus the somewhat uncertain measures of haul due to borrowing material for $e'h'$ and wasting the material between m' and f'. Therefore to make the measure of haul a minimum a line should be drawn which will make the sum of the areas between it and the mass curve a minimum. Of course this is not necessarily the cheapest plan, as it implies more or less borrowing and wasting of material, which *may* cost more than the amount saved in haul. The comparison of the two methods is quite simple, however. Since the amount of fill between e' and h' is represented by the *difference* of the ordinates at e and h, and similarly for m' and f', it follows that the amount to be borrowed between e' and h' will exactly equal the amount wasted between m' and f'. By the first of the above methods the haul is excessive, but is definitely known from the mass diagram, and all of the material is utilized; by the second method the haul is reduced to about one-half, but there is a known quantity in cubic yards wasted at one place and the same quantity borrowed at another. The length of haul necessary for the borrowed material would need to be ascertained; also the haul necessary to waste the other material at a place where it would be unobjectionable. Frequently this is best done by widening an embankment beyond its necessary width. The computation of the relative cost of the above methods will be discussed later (§ 116).

9. Suppose that it were deemed best, after drawing the mass curve, to introduce a trestle between s' and v', thus saving an amount in fill equal to tv. If such had been the original design, the mass curve would have been a straight horizontal line between s and t and would continue as a curve which would be at all points a distance tv above the curve $vFmzfGg$. If the line Ef is to be used as a zero line, its intersection with the new curve at x will show that the material between E' and z' will just balance if the trestle is used, and that the amount of haul will be measured by the area between the line Ex and the broken line $Estx$. The same computed result may be obtained without drawing the auxiliary curve txn . . . by drawing the horizontal line zy at a distance xz ($= tv$) below Ex. The amount of the haul can then be obtained by adding the triangular area between Es and the horizontal line Ex, the rectangle between st and Ex, and the irregular area between vFz and y . . . z (which last is evidently equal to the area between tx and E . . . x). The disposal of the material at the right of z' would then be governed by the indications of the profile and mass diagram which would be found at the right of g'. In fact it is difficult to decide with the best of judgment as to the proper disposal of material without having a mass diagram extending to a considerable distance each side of that part of the road under immediate consideration.

102. Area of the mass curve. The area may be computed most readily by means of a planimeter, which is capable with reasonable care of measuring such areas with as great accuracy as is necessary for this work. If no such instrument is obtainable, the area may be obtained by an application of "Simpson's rule." The ordinates will usually be spaced 100 feet apart. Select an *even* number of such spaces, leaving, if necessary, one or more triangles or trapezoids at the ends for separate and independent computation. Let y_0 . . . y_n be the ordinates, i.e., the number of cubic yards at each station of the mass curve, or the figures of "column six" referred to in § 100. Let the uniform distance between ordinates ($= 100$ feet) be called 1, i.e., one *station.* Then the units of the resulting area will be cubic yards hauled one station. Then the

$$\text{Area} = \tfrac{1}{3}[y_0 + 4(y_1 + y_3 + \ldots y_{(n-1)}) + 2(y_2 + y_4 + \ldots y_{(n-2)}) + y_n]. \quad (70)$$

When an ordinate occurs at a substation, the best plan is to ignore it at first and calculate the area as above. Then, if the difference involved is too great to be neglected, calculate the area of the triangle having the extremity of the ordinate at the substation as an apex, and the extremities of the ordinates at the adjacent stations as the ends of the base. This may be done by finding the ordinate at the substation that would be a proportional between the ordinates at the adjacent full stations. Subtract this from the real ordinate (or *vice versa*) and multiply the difference by $\frac{1}{2} \times 1$. An inspection will often show that the correction thus obtained would be too small to be worthy of consideration. If there is more than one substation between two full stations, the corrective area will consist of two triangles and one or more trapezoids which may be similarly computed, if necessary.

When the zero line (Fig. 62) is shifted to *eE*, the drop from *AC* (produced) to *E* is known in the same units, cubic yards. This constant may be subtracted from the numbers ("column 4," § 100) representing the ordinates, and will thus give, without any scaling from the diagram, the exact value of the modified ordinates.

103. Value of the mass diagram. The great value of the mass diagram lies in the readiness with which different plans for the disposal of material may be examined and compared. When the mass curve is once drawn, it will generally require only a shifting of the horizontal line to show the disposal of the material by any proposed method. The mass diagram also shows the extreme length of haul that will be required by any proposed method of disposal of material. This brings into consideration the "limit of profitable haul," which will be fully discussed in § 116. For the present it may be said that with each method of carrying material there is some limit beyond which the expense of hauling will exceed the loss resulting from borrowing and wasting. With wheelbarrows and scrapers the limit of profitable haul is comparatively short, with carts and tram-cars it is much longer, while with locomotives and cars it may be several miles. If, in Fig. 62, *eE* or *Ef* exceeds the limit of profitable haul, it shows at once that some such line as *hklm* should be drawn and the material disposed of accordingly.

104. Changing the grade line. The formation of the mass curve and the resulting plans as to the disposal of material are based on the mutual relations of the grade line and the surface profile and the amounts of cut and fill which are thereby implied. If the grade line is altered, every cross-section is altered, the amount of cut and fill is altered, and the mass curve is also changed. At the farther limit of the actual change of the grade line the revised mass curve will have (in general) a different ordinate from the previous ordinate at that point. From that point on, the revised mass curve will be parallel to its former position, and the revised curve may be treated similarly to the case previously mentioned in which a trestle was introduced. Since it involves tedious calculations to determine accurately how much the volume of earthwork is altered by a change in grade line, especially through irregular country, the effect on the mass curve of a change in the grade line cannot therefore be readily determined except in an approximate way. Raising the grade line will evidently increase the fills and diminish the cuts, and *vice versa.* Therefore if the mass curve indicated, for example, either an excessively long haul or the necessity for borrowing material (implying a fill) and wasting material farther on (implying a cut), it would be *possible* to diminish the fill (and hence the amount of material to be borrowed) by lowering the grade line near that place, and diminish the cut (and hence the amount of material to be wasted) by raising the grade line at or near the place farther on. Whether the advantage thus gained would compensate for the possibly injurious effect of these changes on the grade line would require patient investigation. But the method outlined shows how the mass curve might be used to indicate a possible change in grade line which might be demonstrated to be profitable.

105. Limit of free haul. It is sometimes specified in contracts for earthwork that *all* material shall be entitled to free haul up to some specified limit, say 500 or 1000 feet, and that all material drawn farther than that shall be entitled to an allowance on the *excess* of distance. It is manifestly impracticable to measure the excess for each load, as much so as to measure the actual haul of each load. The mass diagram also solves this problem very readily. Let Fig. 63 represent a pro-

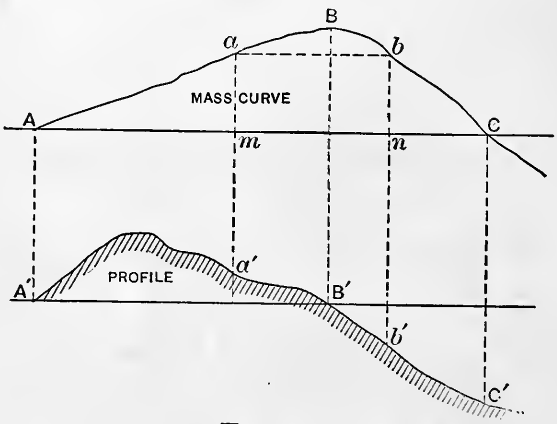

FIG. 63.

file and mass diagram of about 2000 feet of road, and suppose that 800 feet is taken as the limit of free haul. Find two points, a and b, in the mass curve *which are on the same horizontal line* and which are 800 feet apart. Project these points down to a' and b'. Then the cut and fill between a' and b' will just balance, and the cut between A' and a' will be needed for the fill between b' and C'. In the mass curve, the area between the horizontal line ab and the curve aBb represents the haulage of the material between a' and b', which is all free. The rectangle $abmn$ represents the haulage of the material in the cut $A'a'$ across the 800 feet from a' to b'. This is also free. The sum of the two areas Aam and bnC represents the haulage entitled to an allowance, since it is the summation of the products of cubic yards times the *excess* of distance hauled.

If the amount of cut and fill was symmetrical about the point B', the mass curve would be a symmetrical curve about the vertical line through B, and the two limiting lines of free haul would be placed symmetrically about B and B'. In general there is no such symmetry, and frequently the difference is considerable. The area $aBbnm$ will be materially changed according as the two vertical lines am and bn, always 800 feet apart, are shifted to the right or left. It is easy to show that the area $aBbnm$ is a *maximum* when ab is horizontal. The minimum value would be obtained either when m reached A or n reached C, depending on the exact form of the curve. Since the position for the minimum value is manifestly unfair, the best definite value obtainable is the maximum, which must be obtained as above described. Since $aBbnm$ is made maximum, the remainder of the area, which is the allowance for overhaul, becomes a minimum. The areas Aam and bCn may be obtained as in § 102. If the whole area $AaBbCA$ has been previously computed, it may be more convenient to compute the area $aBbnm$ and subtract it from the total area.

Since the intersections of the mass curve and the "zero line" mark limits past which no material is drawn, it follows that there will be no allowance for overhaul except where the distance between consecutive intersections of the zero line and mass curve exceeds the limit of free haul.

Frequently all allowances for overhaul are disregarded; the profiles, estimates of quantities, and the required disposal of material are shown to bidding contractors, and they must then make their own allowances and bid accordingly. This method has the advantage of avoiding possible disputes as to the amount of the overhaul allowance, and is popular with railroad companies on this account. On the other hand the facility with which different plans for the disposal of material may be studied and compared by the mass-curve method facilitates the adoption of the most economical plan, and the elimination of uncertainty will frequently lead to a safe reduction of the bid, and so the method is valuable to both the railroad company and the contractor.

ELEMENTS OF THE COST OF EARTHWORK.

(The following analysis of the cost of earthwork follows the general method given in the well-known papers published by Ellwood Morris, C.E., in the Journal of the Franklin Institute in September and October, 1841. Numerous corroborative data have been obtained from various other sources, and also figures on methods not then in vogue.)

106. General divisions of the subject. The variations in the cost of earthwork are caused by the greatly varying conditions under which the work is done, chief among which is character of material, method of carriage, and length of haul. Any general system of computation must therefore differentiate the total cost into such elementary items that all differences due to variations in conditions may be allowed for. The variations due to character of material will be allowed for by an estimate on loose light sandy soil, and also an estimate on the heaviest soils, such as stiff clay and hard-pan. These represent the extremes (excluding rock, which will be treated separately), and the cost of intermediate grades must be estimated by interpolating between the extreme values. The general divisions of the subject will be: *

1. Loosening.
2. Loading.
3. Hauling.
4. Spreading.
5. Keeping roadways in order.
6. Repairs, wear, depreciation, and interest on cost of plant.
7. Superintendence and incidentals.
8. Contractor's profit.

By making the estimates on the basis of $1 per day for the cost of common labor, it is a simple matter to revise the estimates according to the local price of labor by multiplying the final estimate of cost by the price of labor in dollars per day.

* Trautwine.

107. Item 1. Loosening. (a) Ploughs. Very light sandy soils can frequently be shovelled without any previous loosening, but it is generally economical, even with very light material, to use a plough. Morris quotes, as the results of experiments, that a three-horse plough would loosen from 250 to 800 cubic yards of earth per day, which at a valuation of $5 per day would make the cost per yard vary from 2 cents to 0.6 cent. Trautwine estimates the cost on the basis of two men handling a two-horse plough at a total cost of $3.87 per day, being $1 each for the men, 75 c. for each horse, and an allowance of 37 c. for the plough, harness, etc. From 200 to 600 cubic yards is estimated as a fair day's work, which makes a cost of 1.9 c. to 0.65 c. per yard, which is substantially the same estimate as above. Extremely heavy soils have sometimes been loosened by means of special ploughs operated by traction-engines.

(b) Picks. When picks are used for loosening the earth, as is frequently necessary and as is often done when ploughing would perhaps be really cheaper, an estimate * for a fair day's work is from 14 to 60 cubic yards, the 14 yards being the estimate for stiff clay or cemented gravel, and the 60 yards the estimate for the lightest soil that would require loosening. At $1 per day this means about 7 c. to 1.7 c. per cubic yard, which is about three times the cost of ploughing. Five feet of the face is estimated † as the least width along the face of a bank that should be allowed to enable each laborer to work with freedom and hence economically.

(c) Blasting. Although some of the softer shaly rocks may be loosened with a pick for about 15 to 20 c. per yard, yet rock in general, frozen earth, and sometimes even compact clay is most economically loosened by blasting. The subject of blasting will be taken up later, §§ 117–123.

(d) Steam-shovels. The items of loosening and loading merge together with this method, which will therefore be treated in the next section.

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* Trautwine. † Hurst.

108. Item 2. Loading. (a) Hand-shovelling. Much depends on proper management, so that the shovellers need not wait unduly either for material or carts. With the best of management considerable time is thus lost, and yet the intervals of rest need not be considered as entirely lost, as it enables the men to work, while actually loading, at a rate which it would be physically impossible for them to maintain for ten hours. Seven shovellers are sometimes allowed for each cart; otherwise there should be five, two on each side and one in the rear. Economy requires that the number of loads per cart per day should be made as large as possible, and it is therefore wise to employ as many shovellers as can work without mutual interference and without wasting time in waiting for material or carts. The figures obtainable for the cost of this item are unsatisfactory on account of their large disagreements. The following are quoted as the number of cubic yards that can be loaded into a cart by an average laborer in a working day of ten hours, the lower estimate referring to heavy soils, and the higher to light sandy soils: 10 to 14 cubic yards (Morris), 12 to 17 cubic yards (Haskoll), 18 to 22 cubic yards (Hurst), 17 to 24 cubic yards (Trautwine), 16 to 48 cubic yards (Ancelin). As these estimates are generally claimed to be based on actual experience, the discrepancies are probably due to differences of management. If the average of 15 to 25 cubic yards be accepted, it means, on the basis of $1 per day, 6.7 c. to 4 c. per cubic yard. These estimates apply only to earth. *Rockwork* costs more, not only because it is harder to handle, but because a cubic yard of solid rock, measured in place, occupies about 1.8 cubic yards when broken up, while a cubic yard of earth will occupy about 1.2 cubic yards. Rockwork will therefore require about 50% more loads to haul a given volume, *measured in place*, than will the same nominal volume of earthwork. The above authorities give estimates for loading rock varying from 6.9 c. to 10 c. per cubic yard. The above estimates apply only to the loading of carts or cars with shovels or by hand (loading masses of rock). The cost of loading wheelbarrows and the cost of scraper work will be treated under the item of hauling.

(b) Steam-shovels.* Whenever the magnitude of the work will warrant it there is great economy in the use of steam-shovels. These have a "bucket" or "dipper" on the end of a long beam, the bucket having a capacity varying from $\frac{1}{2}$ to $2\frac{1}{2}$ cubic yards. Steam-shovels handle all kinds of material from the softest earth to shale rock, earthy material containing large boulders, tree-stumps, etc. The capacity of the larger sizes is about 3000 cubic yards in 10 hours. They perform all the work of loosening and loading. Their economical working requires that the material shall be hauled away as fast as it can be loaded, which usually means that cars on a track, hauled by horses or mules, or still better by a locomotive, shall be used. The expenses for a steam-shovel, costing about $5000, will average about $1000 per month. Of this the engineer will get $100; the fireman $50; the cranesman $90; repairs perhaps $250 to $300; coal, from 15 to 25 tons, cost very variable on account of expensive hauling; water, a very uncertain amount, sometimes costing $100 per month; about five laborers and a foreman, the laborers getting $1.25 per day and the foreman $2.50 per day, which will amount to $227.50 per month. This gang of laborers is employed in shifting the shovel when necessary, taking up and relaying tracks for the cars, shifting loaded and unloaded cars, etc. In shovelling through a deep cut, the shovel is operated so as to undermine the upper parts of the cut, which then fall down within reach of the shovel, thus increasing the amount of material handled for each new position of the shovel. If the material is too tough to fall down by its own weight, it is sometimes found economical to employ a gang of men to loosen it or even blast it rather than shift the shovel so frequently. Non-condensing engines of 50 horse-power use so much water that the cost of water-supply becomes a serious

* For a thorough treatment of the capabilities, cost, and management of steam-shovels the reader is referred to "Steam-shovels and Steam-shovel Work," by E. A. Hermann. D. Van Nostrand Co., New York.

matter if water is not readily obtainable. The lack of water facilities will often justify the construction of a pipe line from some distant source and the installation of a steam-pump. Hence the seemingly large estimate of $100 per month for water-supply, although under favorable circumstances the cost may almost vanish. The larger steam-shovels will consume nearly a ton of coal per day of 10 hours. The expense of hauling this coal from the nearest railroad or canal to the location of the cut is often a very serious item of expense and may easily double the cost per ton. Some steam-shovels have been constructed to be operated by electricity obtained from a plant perhaps several miles away. Such a method is especially advantageous when fuel and water are difficult to obtain.

109. Item 3. Hauling. The cost of hauling depends on the number of round trips per day that can be made by each vehicle employed. As the cost of each vehicle is practically the same whether it makes many trips or few, it becomes important that the number of trips should be made a maximum, and to that end there should be as little delay as possible in loading and unloading. Therefore devices for facilitating the passage of the vehicles have a real money value.

(a) Carts. The average speed of a horse hauling a two-wheeled cart has been found to be 200 feet per minute, a little slower when hauling the load and a little faster when returning empty. This figure has been repeatedly verified. It means an allowance of one minute for each 100 feet (or "station") of "lead—the lead being the distance the earth is hauled." The time lost in loading, dumping, waiting to load, etc., has been found to average 4 minutes per load. Representing the number of stations (100 feet) of lead by s, the number of loads handled in 10 hours (600 minutes) would be $600 \div (s + 4)$. The number of loads per cubic yard, measured in the bank, is differentiated by Morris into three classes, viz.:

3 loads per cubic yard in descending hauling;
$3\frac{1}{2}$ " " " " " level hauling; and
4 " " " " " ascending hauling.

Attempts have been made to estimate the effect of the grade of the roadway by a theoretical consideration of its rate, and of the comparative strength of a horse on a level and on various grades. While such computations are always practicable on a railway (even on a temporary construction track), the traction on a temporary earth roadway is always very large and so very variable that any refinements are useless. On railroad earthwork the hauling is generally nearly level or it is *descending*—forming embankments on low ground with material from cuts in high ground. The only common exception occurs when an embankment is formed from borrow-pits on low ground. One method of allowing for ascending grade is to add to the horizontal distance 14 times the difference of elevation for work with carts and 24 times the difference of elevation for work with wheelbarrows, and use that as the lead. For example, using carts, if the lead is 300 feet and there is a difference of elevation of 20 feet, the lead would be considered equivalent to $300 + (14 \times 20) = 580$ feet on a level.

Trautwine assumes the average load for all classes of work to be $\frac{1}{3}$ cubic yard, which figure is justified by large experience. Using one figure for all classes of work simplifies the calculations and gives the number of cubic yards carried per day of 10 hours equal to $\frac{600}{3(s + 4)}$. Dividing the cost of a cart per day by the number of cubic yards carried gives the cost of hauling per yard. In computing the cost of a cart per day, Trautwine refers to the practice of having one driver manage four carts, thus making a charge of 25 c. per day for each cart for the driver. 75 c. is allowed for the horse, which is supposed to be the total cost, including that for Sundays and rainy days. 25 c. more is allowed for the cart, harness, repairs, etc., thus making a total cost of $1.25 per day. Some contractors employ a greater number of drivers and expect each to assist in loading. There is found to be no saving in total cost per yard, while the chances of loafing are perhaps greater. Morris instances five actual cases in which the cost of the cart (reduced to the basis of

$1 per day for labor) varied from $1.37 to $1.48. The items of these costs were not given.

Since the time required for loading loose rock is greater than for earthwork, less loads will be hauled per day. The time allowance for loading, etc., is estimated by Trautwine as 6 minutes instead of 4 as for earth. Considering the great expansion of rock when broken up (see § 97), one cubic yard of solid rock, measured in place, would furnish the equivalent of five loads of earthwork of $\frac{1}{3}$ cubic yard. Therefore, on the basis of five loads per cubic yard, the number of cubic yards handled per day per cart would be $\frac{600}{5(s+6)}$.

$$\text{Cost per yard in cents} = \frac{125 \times 5(s+6)}{600}. \quad . \quad (71)$$

(b) Wagons. For longer leads (i.e., from $\frac{1}{3}$ to $\frac{2}{3}$ of a mile) wagons drawn by two horses have been found most economical. The wagons have bottoms of loose thick narrow boards and are unloaded very easily and quickly by lifting the individual boards and breaking up the continuity of the bottom, thus depositing the load directly underneath the wagon. The capacity is about one cubic yard. The cost may be estimated on the same principles as that for carts.

(c) Wheelbarrows. According to Trautwine, the speed of moving wheelbarrows may be considered the same as for carts, 200 feet per minute; the time spent in loading and dumping is $1\frac{1}{4}$ minutes, and in addition about $\frac{1}{10}$ of the time is wasted in short rests, adjusting the wheeling plants, etc. On the basis of $1 per day for labor, an allowance of 5 c. for the barrow, and 14 loads per cubic yard, the cost of hauling per cubic yard (computed on the same principles as above) will be

$$\frac{105 \times 14(s+1.25)}{600 \times 0.9}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

For rockwork the number of loads per cubic yard is estimated as 24, and the time spent in loading, etc., estimated at 1.6 minutes instead of 1.25 minutes, which makes the estimate

$$\text{Cost per cubic yard} = \frac{105 \times 24(s + 1.6)}{600 \times 0.9}. \quad . \quad (73)$$

(d) Scrapers.* Scrapers, or scoops, are especially useful in canal work, and also for railroad work when a low embankment is to be formed from borrow-pits at the sides, when the distance does not exceed 100 feet, nor the vertical height 15 feet. The slope should not exceed 1.5 to 1. Under these conditions scraper work is cheaper than any other method. Scooping may be done all in one direction, in which case two half-turns are made for each load moved; or it may be done in both directions (from both sides on to a bank, or, in canal work, from the center to each bank), in which case one load is hauled to each half-turn. The capacity of the scoops (the "drag" variety) is $\frac{1}{10}$ cubic yard; the time lost in loading, unloading, and all other ways per load (except in turning) will average $\frac{2}{3}$ minute; the time lost in each half-turn (semi-circle) is $\frac{1}{3}$ minute; the speed of the horses may be estimated as 70 feet of *lead* per minute, the lead being here considered as the *sum* of the vertical and horizontal distances, and the estimate including the time of going and returning. If a represents the sum of the horizontal and vertical distances, the number of cubic yards handled per day of 10 hours by "side-scooping" will be

$$0.1\left(\frac{600}{\frac{a}{70} + 1\frac{1}{3}}\right), \text{ which equals } \frac{4200}{a + 93\frac{1}{3}}.$$

For "double-scooping" the formula becomes

$$0.1\left(\frac{600}{\frac{a}{70} + 1}\right), \text{ which equals } \frac{4200}{a + 70}.$$

* Condensed from Journ. Franklin Inst., Oct. 1841, by Morris.

Dividing the cost of a scraper per day (estimated at $2.75) by the number of yards handled per day gives the average cost per yard.

Except in very loose sandy soil it is best to plough the earth first, which will cost *about* 1 c. per yard. (See § 107.) Drag-scrapers are now made chiefly of steel, and their capacity is more nearly 0.15 cubic yard. Wheeled scrapers, having a capacity of about 0.5 cubic yard, are frequently used with even greater economy and for greater distances, as they are cheaper than carts up to 250 or 300 feet of lead. Both drag- and wheel-scrapers are best operated in gangs of perhaps 10, using extra or "snap" teams to help load, and a few extra men to help in loading and unloading. The average cost of one scraper per day may thus be easily calculated and the average number of cubic yards handled per day computed as above, from which the cost per yard may be estimated.

(e) **Cars and horses.** The items of cost by this method are (*a*) charge for horses employed, (*b*) charge for men employed strictly in hauling, (*c*) charge for shifting rails when necessary, (*d*) repairs, depreciation, and interest on cost of cars and track. Part of this cost should strictly be classified under items 5 and 6, mentioned in § 106, but it is perhaps more convenient to estimate them as follows.

The traction of a car on rails is so very small and constant that grade resistance constitutes a very large part of the total resistance if the grade is 1% or more. For all ordinary grades it is sufficiently accurate to say that the grade resistance is to the gross weight as the rise is to the distance. If the distance is supposed to be measured along the slope, the proportion is strictly true; i.e., on a 1% grade the grade resistance is 1 lb. per 100 of weight or 20 lbs. per ton. If the resistance on a level at the usual velocity is $\frac{1}{120}$, a grade of 1 : 120 (0.83%) will exactly double it. If the material is hauled *down* a grade of 1 : 120, the cars will run by gravity after being started. The work of hauling will then consist practically of hauling the empty cars up the grade. The grade resistance depends only on the rate of grade and the weight, but the tractive resistance will be *greater per ton of weight* for the unloaded than for the loaded cars. The tractive power of a horse is less on a grade than on a level, not only because the horse raises his own weight in addition to the load, but is anatomically less capable of pulling on a grade than on a level. In general it will be possible to plan the work so that loaded cars need not be hauled *up* a grade, unless an embankment is to be formed from a low borrow-pit, in which case another method would probably be advisable. These computations are chiefly utilized in designing the method of work—the proportion of horses to cars. An example may be quoted from English practice (Hurst), in which the cars had a capacity of $3\frac{1}{3}$ cubic yards, weighing 30 cwt. empty. Two horses took five "wagons" $\frac{3}{4}$ of a mile on a level railroad and made 15 journeys per day of 10 hours, i.e., they handled 250 yards per day. In addition to those on the "straight road," another horse was employed to make up the train of loaded wagons. With a short lead the straight-road horses were employed for this purpose. In the above example the number of men required to handle these cars, shift the tracks, etc., is not given, and so the exact cost of the above work cannot be analyzed. It may be noticed that the two horses travelled $22\frac{1}{2}$ miles per day, drawing in one direction a load, including the weight of the cars, of about 57,300 lbs. or 28.65 net tons. Allowing $\frac{1}{120}$ as the necessary tractive force, it would require a pull of 477.5 lbs., or 239 lbs. for each horse. With a velocity of 220 feet per minute this would amount to $1\frac{1}{2}$ horse-power per horse, exerted for only a short time, however, and allowing considerable time for rest and for drawing only the empty cars. The cars generally used in this country have a capacity of $1\frac{1}{2}$ cubic yards and cost about \$65 apiece. Besides the shovellers and dumping-gang, several men and a foreman will be required to keep the track in order and to make the constant shifts that are necessary. Two trains are generally used, one of which is loaded while the other is run to the dump. Some passing-place is necessary, but this is generally provided by having a switch at the cut and running the trains on each track alternately. This insures a train of cars always at the cut to keep the shovellers employed. The cost of hauling per cubic yard can only be computed when the number of laborers, cars, and horses employed are known, and these will depend on the lead, on the character of the excavation, on the grade, if any, etc., and must be so proportioned that the shovellers need not wait for cars to fill, nor the dumping-gang for material to handle, nor the horses and drivers for cars to haul. Much skill is necessary to keep a large force in smooth running order.

(f) **Cars and locomotives.** 30-lb. rails are the lightest that should be used for this work, and 35- or 40-lb. rails are better. One or two narrow-gauge locomotives (depending on the length of haul), costing about $2500 each, will be necessary to handle two trains of about 15 cars each, the cars having a capacity of about 2 cubic yards and costing about $100 each. Some cars can be obtained as low as $70. A force of about five men and a foreman will be required to shift the tracks. The track-shifters, except the foreman, may be common laborers. The dumping-gang will require about seven men. Even when the material is all taken *down* grade the grades may be too steep for the safe hauling of loaded cars down the grade, or for hauling empty cars up the grade. Under such circumstances temporary trestles are necessary to reduce the grade. When these are used, the uprights and bracing are left in the embankment—only the stringers being removed. This is largely a necessity, but is partially compensated by the fact that the trestle forms a core to the embankment which prevents lateral shifting during settlement. The average speed of the trains may be taken as 10 miles per hour or 5 miles of lead per hour. The time lost in loading and unloading is estimated (Trautwine) as 9 minutes or .15 of an hour. The number of trips per day of 10 hours will equal $\frac{10}{\frac{1}{5}(\text{miles of lead}) + .15}$ or $\frac{50}{(\text{miles of lead}) + .75}$. Of course this quotient *must* be a whole number. Knowing the number of trains and their capacity, the total number of cubic yards handled is known, which, divided into the total daily cost of the trains, will give the cost of hauling per yard. The daily cost of a train will include

(*a*) Wages of engineer, who frequently fires his own engine;

(*b*) Fuel, about $\frac{1}{4}$ to 1 ton of bituminous coal, depending on work done;

(*c*) Water, a very variable item, frequently costing \$3 to \$5 per day;

(*d*) Repairs, variable, frequently at rate of 50 to 60% per year;

(*e*) Interest on cost and depreciation, 16 to 40%.

To these must be added, to obtain the total cost of the haul,

(*f*) Wages of the gang employed in shifting track.

110. Choice of method of haul dependent on distance. In light side-hill work in which material need not be moved more than 12 or 15 feet, i.e., moved *laterally* across the road-bed, the earth may be moved most cheaply by mere shovelling. Beyond 12 feet scrapers are more economical. At about 100 feet drag-scrapers and wheelbarrows are equally economical. Between 100 and 200 feet wheelbarrows are generally cheaper than either carts or drag-scrapers, but wheeled scrapers are always cheaper than wheelbarrows. Beyond 500 feet two-wheeled carts become the most economical up to about 1700 feet; then four-wheeled wagons become more economical up to 3500 feet. Beyond this cars on rails, drawn by horses or by locomotives, become cheaper. The economy of cars on rails becomes evident for distances as small as 300 feet provided the volume of the excavation will justify the outlay. Locomotives will always be cheaper than horses and mules providing the work to be done is of sufficient magnitude to justify the purchase of the necessary plant and risk the loss in selling the plant ultimately as second-hand equipment, or keeping the plant on hand and idle for an indefinite period waiting for other work. Horses will not be economical for distances much over a mile. For greater distances locomotives are more economical, but the question of "limit of profitable haul" (§ 116) must be closely studied, as the circumstances are certainly not common when it is advisable to haul material much over a mile.

111. Item 4. SPREADING. The cost of spreading varies with the method employed in dumping the load. When the earth is tipped over the edge of an embankment there is little if any necessary work. Trautwine allows about $\frac{1}{4}$ c. per cubic yard for keeping the dumping-places clear and in order. This would represent the wages of one man at $1 per day attending to the unloading of 1200 two-wheeled carts each carrying $\frac{1}{3}$ cubic yard. 1200 carts in 10 hours would mean an average of two per minute, which implies more rapid and efficient work than may be depended on. The allowance is probably too small. When the material is dumped in layers some levelling is required, for which Trautwine allows 50 to 100 cubic yards as a fair day's work, costing from 1 to 2 cents per cubic yard. The cost of spreading will not ordinarily exceed this and is frequently nothing—all depending on the method of unloading. It should be noted that Mr. Morris's examples and computations (Jour. Franklin Inst., Sept. 1841) disregard altogether any special charge for this item.

112. Item 5. KEEPING ROADWAYS IN ORDER. This feature is important as a measure of true economy, whatever the system of transportation, but it is often neglected. A petty saving in such matters will cost many times as much in increased labor in hauling and loss of time. With some methods of haul the cost is best combined with that of other items.

(a) **Wheelbarrows.** Wheelbarrows should generally be run on planks laid on the ground. The adjusting and shifting of these planks is done by the wheelers, and the time for it is allowed for in the 10% allowance for "short rests, adjusting the wheeling plank, etc." The actual cost of the planks must be added, but it would evidently be a very small addition per cubic yard in a large contract. When the wheelbarrows are run on planks placed on "horses" or on trestles the cost is very appreciable; but the method is frequently used with great economy. The variations in the requirements render any general estimate of such cost impracticable.

(b) Carts and wagons. The cost of keeping roadways in order for carts and wagons is sometimes estimated merely as so much per cubic yard, but it is evidently a function of the *lead.* The work consists in draining off puddles, filling up ruts, picking up loose stones that may have fallen off the loads, and in general doing everything that will reduce the traction as much as possible. Temporary inclines, built to avoid excessive grade at some one point, are often measures of true economy. Trautwine suggests $\frac{1}{10}$ c. per cubic yard per 100 feet of lead for earthwork and $\frac{2}{10}$ c. for rockwork, as an estimate for this item when carts are used.

(c) Cars. When cars are used a shifting-gang, consisting of a foreman and several men (say five), are constantly employed in shifting the track so that the material may be loaded and unloaded where it is desired. The average cost of this item may be estimated by dividing the total daily cost of this gang by the number of cubic yards handled in one day.

113. Item 6. Repairs, Wear, Depreciation, and Interest on Cost of Plant. The amount of this item evidently depends upon the character of the soil—the harder the soil the worse the wear and depreciation. The *interest on cost* depends on the current borrowing value of money. The estimate for this item has already been included in the allowances for horses, carts, ploughs, harness, wheelbarrows, steam-shovels, etc. Trautwine estimates $\frac{1}{4}$ c. per cubic yard for picks and shovels. Depreciation is generally a large percentage of the cost of earth-working tools, the life of all being limited to a few years, and of many tools to a few months.

114. Item 7. Superintendence and Incidentals. The incidentals include water-carriers, trimming cuts to grade, digging the side ditches, trimming up the sides of borrow-pits to prevent their becoming unsightly, etc. These last operations yield but little earth and cost far more than the price paid per cubic yard. Morris allows 1 c. per cubic yard for this item; Trautwine allows $1\frac{3}{4}$ to 2 c. for it; while others combine items 6 and 7 and call them 5% of the total cost, which method has the merit of making the cost of items 6 and 7 a function of the character of soil and length of lead.

115. Item 8. Contractor's Profit. This is usually estimated at from 6 to 15%, according to the sharpness of the competition and the possible uncertainty as to true cost owing to unfavorable circumstances. The contractor's real profit may vary considerably from this. He often pays clerks, boards and lodges the laborers in shanties built for the purpose, or keeps a supply-store, and has various other items both of profit and expense. His profit is largely dependent on skill in so handling the men that all can work effectively without interference or delays in waiting for others. An unusual season of bad weather will often affect the cost very seriously. It is a common occurrence to find that two contractors may be working on the same kind of material and under precisely similar conditions and at the same price, and yet one may be making money and the other losing it—all on account of difference of management.

116. Limit of profitable haul. As intimated in §§ 103 and 110, there is with every method of haul a limit of distance beyond which the expense for excessive hauling will exceed the loss resulting from borrowing and wasting. This distance is somewhat dependent on local conditions, thus requiring an independent solution for each particular case, but the general principles involved will be about as follows: Assume that it has been determined, as in Fig. 62, that the cut and fill will exactly balance between two points, as between e and x, assuming that, as indicated in § 101 (9), a trestle has been introduced between s and t, thus altering the mass curve to $Estxn$ . . . Since there is a balance between A' and C', the material for the fill between C' and e' must be obtained either by "borrowing" in the immediate neighborhood or by transportation from the excavation between z' and n'. If cut and fill have been approximately balanced in the selection of grade line, as is ordinarily done, borrowing material for the fill $C'e'$ implies a wastage of material at the cut $z'n'$. To compare the two methods, we may place against the plan of borrowing and wasting, (a) cost, if any, of extra right of way that may be needed from which to obtain earth for the fill $C'e'$; (b) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the borrow-pit and of the fill, and the other expenses incidental to borrowing M cubic yards for the fill $C'e'$; (c) cost of loosening, loading, hauling a distance equal to that between the centers of gravity of the cut $z'n'$ and of the spoil-bank, and the other expenses incidental to wasting M cubic yards at the cut $z'n'$; (d) cost, if any, of land needed for the spoil-bank. The cost of the other plan will be the cost of loosening, loading, hauling (the hauling being represented by the trapezoidal figure $Cexn$), and the other expenses incidental to making the fill $C'e'$ with the material from the cut $z'n'$, the amount of material being M cubic yards, which is represented in the figure by the vertical ordinate from e to the line Cn. The difference between these costs will be the cost, if any, of land for borrow-pit and spoil-bank *plus* the cost of loosening, loading, etc. (except hauling and roadways) of M cubic yards, *minus* the difference in cost of the excessive haul from Ce to xn and the comparatively short hauls from borrow-pit and to spoil-bank.

As an illustration, taking some of the estimates previously given for operating with average material, the cost of all items, except hauling and roadways, would be about as follows: loosening, with plough, 1.2 c., loading 5.0 c., spreading 1.5 c., wear, depreciation, etc., .25 c., superintendence, etc., 1.5 c.; total 8.95 c. Suppose that the haul for both borrowing and wasting averages 100 feet or 1 station. Then the cost of haul per yard, using carts, would be (§ 109, a) $[125 \times 3(1 + 4)] \div 600 = 3.125$ c. The cost of roadways would be about 0.1 c. per yard, making a total of 3.225 c. per cubic yard. Assume $M = 10000$ cubic yards and the area $Cexn = 180000$ yards-stations or the equivalent of 10000 yards hauled 1800 feet. This haul would cost $[125 \times 3(18 + 4)] \div 600 = 13.75$ c. per cubic yard. The cost of roadways will be $18 \times .1$ or 1.8 c., making a total of 15.55 c. for hauling and roadways. The difference of cost of hauling and roadways will be 15.55 − (2 × 3.225) = 9.10 c. per yard or $910 for the 10000 yards. Offsetting this is the cost of loosening, etc., 10000 yards, at 8.95 c., costing $895. These figures may be better compared as follows:

LONG HAUL.	Loosening, etc.,	10000 yards,		@ 8.95 c.		$ 895.
	Hauling, "	10000 "		@ 15.55 c.		1555.
						$2450.
BORROWING AND WASTING.	Loosening, etc.,	10000 yards	(borrowed),	@ 8.95	c.	$895.
	" "	10000 "	(wasted),	@ 8.95	c.	895.
	Hauling, etc.,	10000 "	(borrowed),	@ 3.225	c.	322.50
	" "	10000 "	(wasted),	@ 3.225	c.	322.50
						$2435.00

These costs are practically balanced, but no allowance has been made for right of way. If any considerable amount had to be paid for that, it would decide this particular case in favor of the long haul. This shows that *under these conditions* 1800 feet is *about* the limit of profitable haul, the land costing nothing extra.

BLASTING.

117. Explosives. The effect of blasting is due to the extremely rapid expansion of a gas which is developed by the decomposition of a very small amount of solid matter. Blasting compounds may be divided into two general classes, (*a*) slow-burning and (*b*) detonating. Gunpowder is a type of the slow-burning compounds. These are generally ignited by heat; the ignition proceeds from grain to grain; the heat and pressure produced are comparatively low. Nitro-glycerine is a type of the detonating compounds. They are exploded by a shock which *instantaneously* explodes the whole mass. The heat and pressure developed are far in excess of that produced by the explosion of powder. Nitro-glycerine is so easily exploded that it is very dangerous to handle. It was discovered that if the nitro-glycerine was absorbed by a spongy material like infusorial earth, it was much less liable to explode, while its power when actually exploded was practically equal to that of the amount of pure nitro-glycerine contained in the dynamite, which is the name given to the mixture of nitro-glycerine and infusorial earth. Nitro-glycerine is expensive; many other explosive chemical compounds which properly belong to the slow-burning class are comparatively cheap. It has been conclusively demonstrated that a mixture of nitro-glycerine and some of the cheaper chemicals has a greater explosive force than the sum of the strengths of the component parts when exploded separately. Whatever the reason, the fact seems established. The reason is possibly that the explosion of the nitro-glycerine is sufficiently powerful to produce a *detonation* of the other chemicals, which is impossible to produce by ordinary means, and that this explosion caused by detonation is more powerful than an ordinary explosion. The majority of the explosive compounds and "powders" on the market are of this character—a mixture of 20 to 60 per cent. of nitro-glycerine with variable proportions of one or more of a great variety of explosive chemicals.

The choice of the explosive depends on the character of the rock. A hard brittle rock is most effectively blasted by a detonating compound. The rapidity with which the full force of the explosive is developed has a shattering effect on a brittle substance. On the contrary, some of the softer tougher rocks and indurated clays are but little affected by dynamite. The result is but little more than an enlargement of the blast-hole. Quarrying must generally be done with blasting-powder, as the quicker explosives are too shattering. Although the results obtained by various experimenters are very variable, it may be said that pure nitro-glycerine is eight times as powerful as black powder, dynamite (75% nitro-glycerine) six times, and gun-cotton four to six times as powerful. For open work where time is not particularly valuable, black powder is by far the cheapest, but in tunnel-headings, whose progress determines the progress of the whole work, dynamite is so much more effective and so expedites the work that its use becomes economical.

118. Drilling. Although many very complicated forms of drill-bars have been devised, the best form (with slight modifications to suit circumstances) is as shown in Fig. 64, (*a*) and (*b*).

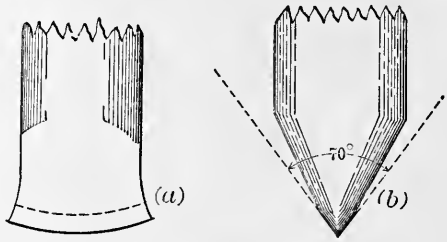
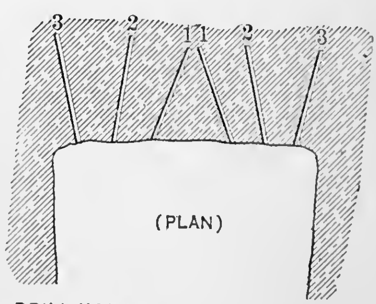


FIG. 64.

The width should flare at the bottom (*a*) about 15 to 30%. For hard rock the curve of the edge should be somewhat flatter and for soft rock somewhat more curved than shown, Fig. 64, (*a*). Sometimes the angle of the two faces is varied from that given, Fig. 64, (*b*), and occasionally the edge is purposely blunted so as to give a crushing rather than a cutting effect. The drills will require sharpening for each 6 to 18 inches depth of hole, and will require a new edge to be worked every 2 to 4 days. For drilling vertical holes the *churn-drill* is the most economical. The drill-bar is of iron, about 6 to 8 feet long, $1\frac{1}{4}''$ in diameter, weighs about 25 to 30 lbs., and is shod with a piece of steel welded on. The bar is lifted a few inches between each blow, turned partially around, and allowed to fall, the impact doing the work. From 5 to 15 feet of holes, depending on the character of the rock, is a fair day's work—10 hours. In very soft rocks even more than this may be done. This method is inapplicable for inclined holes or even for vertical holes in confined places, such as tunnel-headings. For such places the only practical *hand* method is to use hammers. This may be done by light drills and light hammers (one-man work), or by heavier drills held by one man and struck by one or two men with heavy hammers. The conclusion of an exhaustive investigation as to the relative economy of light or heavy hammers is that the light-hammer method is more economical for the softer rocks, the heavy-hammer method is more economical for the harder rocks, but that the light-hammer method is always more expeditious and hence to be preferred when time is important.

The subject of machine rock-drills is too vast to be treated here. The method is only practicable when the amount of work to be done is large, and especially when time is valuable. The machines are generally operated by compressed air for tunnel-work, thus doing the additional service of supplying fresh air to the tunnel-headings where it is most needed. The cost per foot of hole drilled is quite variable, but is usually somewhat less than that of hand-drilling—sometimes but a small fraction of it.

119. Position and direction of drill-holes. As the cost of drilling holes is the largest single item in the total cost of blasting, it is necessary that skill and judgment should be used in so locating the holes that the blasts will be most effective. The greatest effect of a blast will evidently be in the direction of the "line of least resistance." In a strictly homogeneous material this will be the shortest line from the center of the explosive to the surface. The variations in homogeneity on account of laminations and seams require that each case shall be judged according to experience. In open-pit blasting it is generally easy to obtain two and sometimes three exposed faces to the rock, making it a simple matter to drill holes so that a blast will do effective work. When a solid face of rock must be broken into, as in a tunnel-heading, the work is necessarily ineffectual and expensive. A conical or wedge-shaped mass will first be blown out by simultaneous blasts in the holes marked 1, Fig. 65; blasts in the holes marked 2 and 3 will then complete the cross-section of the heading. A great saving in cost may often be secured by skilfully taking advantage of seams, breaks, and irregularities. When the work is economically done there is but little noise or throwing of rock,



DRILL HOLES IN TUNNEL HEADING
FIG. 65.

a covering of old timbers and branches of trees generally sufficing to confine the smaller pieces which would otherwise fly up.

120. Amount of explosive. The amount of explosive required varies as the cube of the line of least resistance. The best results are obtained when the line of least resistance is ¾ of the depth of the hole; also when the powder fills about ⅓ of the hole. For average rock the amount of powder required is as follows:

Line of least resistance................	2 ft.	4 ft.	6 ft.	8 ft.
Weight of powder......................	¼ lb.	2 lbs.	6¾ lbs.	16 lbs.

Strict compliance with all of the above conditions would require that the diameter of the hole should vary for every case. While this is impracticable, there should evidently be some variation in the size of the hole, depending on the work to be done. For example, a 1″ hole, drilled 2′ 8″ deep, with its line of least resistance 2′, and loaded with ¼ lb. of powder, would be filled to a depth of 9½″, which is nearly ⅓ of the depth. A 3″ hole, drilled 8′ deep, with its line of least resistance 6′, and loaded with 6¾ lbs. of powder, would be filled to a depth of over 28″, which is also nearly ⅓ of the depth. One pound of blasting-powder will occupy about 28 cubic inches. Quarrying necessitates the use of numerous and sometimes repeated light charges of powder, as a heavy blast or a powerful explosive like dynamite is apt to shatter the rock. This requires more powder to the cubic yard than blasting for mere excavation, which may usually be done by the use of ¼ to ⅓ lb. of powder per cubic yard of easy open blasting. On account of the great resistance offered by rock when blasted in headings in tunnels, the powder used per cubic yard will run up to 2, 4, and even 6 lbs. per cubic yard. As before stated, nitro-glycerine is about eight times (and dynamite about six times) as powerful as the same *weight* of powder.

121. Tamping. Blasting-powder and the slow-burning explosives require thorough tamping. Clay is probably the best, but sand and fine powdered rock are also used. Wooden plugs, inverted expansive cones, etc., are periodically reinvented by enthusiastic inventors, only to be discarded for the simpler methods. Owing to the extreme rapidity of the development of the force of a nitro-glycerine or dynamite explosion, tamping is not so essential with these explosives, although it unquestionably adds to their effectiveness. Blasting under water has been effectively accomplished by merely pouring nitro-glycerine into the drilled holes through a tube and then exploding the charge without any tamping except that furnished by the superincumbent water. It has been found that air-spaces about a charge make a material reduction in the effectiveness of the explosion. It is therefore necessary to carefully ram the explosive into a solid mass. Of course the liquid nitro-glycerine needs no ramming, but dynamite should be rammed with a *wooden* rammer. Iron should be carefully avoided in ramming gunpowder. A copper bar is generally used.

122. Exploding the charge. Black powder is generally exploded by means of a fuse which is essentially a cord in which there is a thin vein of gunpowder, the cord being protected by tar, extra linings of hemp, cotton, or even gutta-percha. The fuse is inserted into the middle of the charge, and the tamping carefully packed around it so that it will not be injured. To produce the detonation required to explode nitro-glycerine and dynamite, there must be an initial explosion of some easily ignited explosive. This is generally accomplished by means of caps containing fulminating-powder which are exploded by electricity. The electricity (in one class of caps) heats a very fine platinum wire to redness, thereby igniting the sensitive powder, or (in another class) a spark is caused to jump through the powder between the ends of two wires suitably separated. Dynamite can also be exploded by using a small cartridge of gunpowder which is itself exploded by an ordinary fuse.

123. Cost. Trautwine estimates the cost of blasting (for mere excavation) as averaging 45 cents per cubic yard, falling as low as 30 cents for easy but *brittle* rock, and running up to

60 cents and even $1 when the cutting is shallow, the rock especially tough, and the strata unfavorably placed. Soft tough rock frequently requires more powder than harder brittle rock.

124. Classification of excavated material. The classification of excavated material is a fruitful source of dispute between contractors and railroad companies, owing mainly to the fact that the variation between the softest earth and the hardest rock is so gradual that it is very difficult to describe distinctions between different classifications which are unmistakable and indisputable. The classification frequently used is (*a*) earth, (*b*) loose rock, and (*c*) solid rock. As blasting is frequently used to loosen "loose rock" and even "earth" (if it is frozen), the fact that blasting is employed cannot be used as a criterion, especially as this would (if allowed) lead to unnecessary blasting for the sake of classifying material as rock.

Earth. This includes clay, sand, gravel, loam, decomposed rock and slate, boulders or loose stones not greater than 1 cubic foot (3 cubic feet, P. R. R.), and sometimes even "hard-pan." In general it will signify material which *can* be loosened by a plough with two horses, or with which one picker can keep one shoveller busy.

Loose rock. This includes boulders and loose stones of more than one cubic foot and less than one cubic yard; stratified rock, not more than six inches thick, separated by a stratum of clay; also all material (not classified as earth) which may be loosened by pick or bar and which "*can* be quarried without blasting, although blasting may occasionally be resorted to."

Solid rock includes all rock found in masses of over one cubic yard which cannot be removed except by blasting.

It is generally specified that the engineer of the railroad company shall be the judge of the classification of the material, but frequently an appeal is taken from his decisions to the courts.

125. Specifications for earthwork. The following specifications, issued by the Norfolk and Western R. R., represent the average requirements. It should be remembered that very strict specifications invariably increase the cost of the work, and frequently add to the cost more than is gained by improved quality of work.

1. The grading will be estimated and paid for by the cubic yard, and will include clearing and grubbing, and all open excavations, channels, and embankments required for the formation of the roadbed, and for turnouts and sidings; cutting all ditches or drains about or contiguous to the road; digging the foundation-pits of all culverts, bridges, or walls; reconstructing turnpikes or common roads in cases where they are destroyed or interfered with; changing the course or channel of streams; and all other excavations or embankments connected with or incident to the construction of said Railroad.

2. All grading, except where otherwise specified, whether for cuts or fills, will be measured in the excavations and will be classified under the following heads, viz.: Solid Rock, Loose Rock, Hard-pan, and Earth.

Solid Rock shall include all rock occurring in masses which, in the judgment of the said Engineer Maintenance of Way, may be best removed by blasting.

Loose Rock shall include all kinds of shale, soapstone, and other rock which, in the judgment of the said Engineer Maintenance of Way, can be removed by pick and bar, and is soft and loose enough to be removed without blasting, although blasting may be occasionally resorted to; also, detached stone of less than one (1) cubic yard and more than one (1) cubic foot.

Hard-pan shall consist of tough indurated clay or cemented gravel, which requires blasting or other equally expensive means for its removal, or which cannot be ploughed with less than four horses and a railroad plough, or which requires two pickers to a shoveller, the said Engineer Maintenance of Way to be the judge of these conditions.

Earth shall include all material of an earthy nature, of whatever name or character, not unquestionably loose rock or hard-pan as above defined.

Powder. The use of powder in cuts will not be considered as a reason for any other classification than earth, unless the material in the cut is clearly other than earth under the above specifications.

3. Earth, gravel, and other materials taken from the excavations, except when otherwise directed by the said Engineer Maintenance of Way or his assistant, shall be deposited in the adjacent embankment; the cost of removing and depositing which, when the distance necessary to be hauled is not more than sixteen hundred (1600) feet, shall be included in the price paid for the excavation.

4. EXTRA HAUL will be estimated and paid for as follows: whenever material from excavations is necessarily hauled a greater distance than sixteen hundred (1600) feet, there shall be paid in addition to the price of excavation the price of extra haul per 100 feet, or part thereof, after the first 1600 feet; the necessary haul to be determined in each case by the said Engineer Maintenance of Way or his assistant, from the profile and cross-sections, and the estimates to be in accordance therewith.

5. All embankments shall be made in layers of such thickness and carried on in such manner as the said Engineer Maintenance of Way or his assistant may prescribe, the stone and heavy materials being placed in slopes and top. And in completing the fills to the proper grade such additional heights and fulness of slope shall be given them, to provide for their settlement, as the said Engineer Maintenance of Way, or his assistant, may direct. Embankments about masonry shall be built at such times and in such manner and of such materials as the said Engineer Maintenance of Way or his assistant may direct.

6. In procuring materials for embankments from without the line of the road, and in wasting materials from cuttings, the place and manner of doing it shall in each case be indicated by the Engineer Maintenance of Way or his assistant; and care must be taken to injure or disfigure the land as little as possible. Borrow-pits and spoil-banks must be left by the Contractor in regular and sightly shape.

7. The lands of the said Railroad Company shall be cleared

to the extent required by the said Engineer Maintenance of Way, or his assistant, of all trees, brushes, logs, and other perishable materials, which shall be destroyed by burning or deposited in heaps as the said Engineer Maintenance of Way, or his assistant, may direct. Large trees must be cut not more than two and one-half ($2\frac{1}{2}$) feet from the ground, and under embankments less than four (4) feet high they shall be cut close to the ground. All small trees and bushes shall be cut close to the ground.

8. Clearing shall be estimated and paid for by the acre or fraction of an acre.

9. All stumps, roots, logs, and other obstructions shall be grubbed out, and removed from all places where embankments occur less than two (2) feet in height; also, from all places where excavations occur and from such other places as the said Engineer Maintenance of Way or his assistant may direct.

10. Grubbing shall be estimated and paid for by the acre or fraction of an acre.

11. Contractors, when directed by the said Engineer Maintenance of Way or his assistant in charge of the work, will deposit on the side of the road, or at such convenient points as may be designated, any stone, rock, or other materials that they may excavate; and all materials excavated and deposited as above, together with all timber removed from the line of the road, will be considered the property of the Railroad Company, and the Contractors upon the respective sections will be responsible for its safe-keeping until removed by said Railroad Company, or until their work is finished.

12. Contractors will be accountable for the maintenance of safe and convenient places wherever public or private roads are in any way interfered with by them during the progress of the work. They will also be responsible for fences thrown down, and for gates and bars left open, and for all damages occasioned thereby.

13. Temporary bridges and trestles, erected to facilitate the progress of the work, in case of delays at masonry structures

from any cause, or for other reasons, will be at the expense of the Contractor.

14. The line of road or the gradients may be changed in any manner, and at any time, if the said Engineer Maintenance of Way or his assistant shall consider such a change necessary or expedient; but no claim for an increase in prices of excavation or embankment on the part of the Contractor will be allowed or considered unless made in writing before the work on that part of the section where the alteration has been made shall have been commenced. The said Engineer Maintenance of Way or his assistant may also, on the conditions last recited, increase or diminish the length of any section for the purpose of more nearly equalizing or balancing the excavations and embankments, or for any other reason.

15. The roadbed will be graded as directed by the said Engineer Maintenance of Way or his assistant, and in conformity with such breadths, depths, and slopes of cutting and filling as he may prescribe from time to time, and no part of the work will be finally accepted until it is properly completed and dressed off at the required grade.

CHAPTER IV.

TRESTLES.

126. Extent of use. Trestles constitute from 1 to 3% of the length of the average railroad. It was estimated in 1889 that there was then about 2400 miles of single-track railway trestle in the United States, divided among 150,000 structures and estimated to cost about $75,000,000. The annual charge for maintenance, estimated at $\frac{1}{8}$ of the cost, therefore amounted to about $9,500,000 and necessitated the annual use of perhaps 300,000,000 ft. B.M. of timber. The corresponding figures at the present time must be somewhat in excess of this. The magnitude of this use, which is causing the rapid disappearance of forests, has resulted in endeavors to limit the use of timber for this purpose. Trestles may be considered as justifiable under the following conditions:

a. Permanent trestles.

1. Those of *extreme* height—then called viaducts and frequently constructed of iron or steel, as the Kinzua viaduct, 302 ft. high.

2. Those across waterways—*e.g.*, that across Lake Pontchartrain, near New Orleans, 22 miles long.

3. Those across swamps of soft deep mud, or across a river-bottom, liable to occasional overflow.

b. Temporary trestles.

1. To open the road for traffic as quickly as possible—often a reason of great financial importance.

2. To quickly replace a more elaborate structure, destroyed

by accident, on a road already in operation, so that the interruption to traffic shall be a minimum.

3. To form an earth embankment with earth brought from a distant point by the train-load, when such a measure would cost less than to borrow earth in the immediate neighborhood.

4. To bridge an opening temporarily and thus allow time to learn the regimen of a stream in order to better proportion the size of the waterway and also to facilitate bringing *suitable* stone for masonry from a distance. In a new country there is always the double danger of either building a culvert too small, requiring expensive reconstruction, perhaps after a disastrous washout, or else wasting money by constructing the culvert unnecessarily large. Much masonry has been built of a very poor quality of stone because it could be conveniently obtained and because good stone was unobtainable except at a prohibitive cost for transportation. Opening the road for traffic by the use of temporary trestles obviates both of these difficulties.

127. Trestles vs. embankments. Low embankments are very much cheaper than low trestles both in first cost and maintenance. Very high embankments are very expensive to construct, but cost comparatively little to maintain. A trestle of equal height may cost much less to construct, but will be expensive to maintain—perhaps $\frac{1}{8}$ of its cost per year. To determine the height beyond which it will be cheaper to maintain a trestle rather than build an embankment, it will be necessary to allow for the cost of maintenance. The height will also depend on the relative cost of timber, labor, and earthwork. At the present average values, it will be found that for less heights than 25 feet the *first cost* of an embankment will *generally* be less than that of a trestle; this implies that a permanent trestle should never be constructed with a height less than 25 feet except for the reasons given in § 126. The height at which a permanent trestle is certainly cheaper than earthwork is more uncertain. A high grade line joining two hills will invariably imply at least a culvert if an embankment is used. If the culvert is built of masonry, the cost of the embankment will be so increased that the height at which a trestle becomes economical will be materially reduced. The cost of an embankment increases much more rapidly than the height—with very high embankments more nearly as the square of the height—while the cost of trestles does not increase as rapidly as the height. Although local circumstances may modify the application of any set rules, it is probably seldom that it will be cheaper to build an embankment 40 or 50 feet high than to permanently maintain a wooden trestle of that height. A steel viaduct would probably be the best solution of such a case. These are frequently used for permanent structures, especially when very high. The cost of maintenance is much less than that of wood, which makes the use of iron or steel preferable for permanent trestles unless wood is abnormally cheap. Neither the cost nor the construction of iron or steel trestles will be considered in this chapter.

128. Two principal types. There are two principal types of wooden trestles—pile trestles and framed trestles. The great objection to pile trestles is the rapid rotting of the portion of the pile which is underground, and the difficulty of renewal. The maximum height of pile trestles is about 30 feet, and even this height is seldom reached. Framed trestles have been constructed to a height of considerably over 100 feet. They are frequently built in such a manner that any injured piece may be readily taken out and renewed without interfering with traffic. Trestles consist of two parts—the supports called "bents," and the stringers and floor system. As the stringers and floor system are the same for both pile and framed trestles, the "bents" are all that need be considered separately.

PILE TRESTLES.

129. Pile bents. A pile bent consists generally of four piles driven into the ground deep enough to afford not only sufficient vertical resistance but also lateral resistance. On top of these piles is placed a horizontal "cap." The caps are fastened to the tops of the piles by methods illustrated in Fig. 66. The method of fastening shown in each case should not be considered as applicable only to the particular type of pile bent used to illustrate it. Fig. 66 (*a* and *d*) illustrates a mortise-joint with a hard-

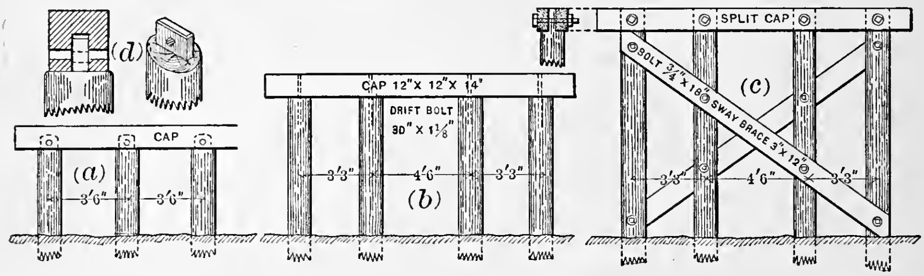


FIG. 66.

wood pin about $1\frac{1}{4}''$ in diameter. The hole for the pin should be bored separately through the cap and the mortise, and the hole through the cap should be at a slightly higher level than that through the mortise, so that the cap will be drawn down tight when the pin is driven. Occasionally an iron dowel (an iron pin about $1\frac{1}{2}''$ in diameter and about 6″ long) is inserted partly in the cap and partly in the pile. The use of drift-bolts, shown in Fig. 66 (*b*), is cheaper in first cost, but renders repairs and renewals very troublesome and expensive. "Split caps," shown in Fig. 66 (*c*), are formed by bolting two half-size strips on each side of a tenon on top of the pile. Repairs are very easily and cheaply made without interference with the traffic and without injuring other pieces of the bent. The smaller pieces are more easily obtainable in a sound condition; the decay of one does not affect the other, and the first cost is but little if any greater than the method of using a single piece. For further discussion, see § 136.

For very light traffic and for a height of about 5 feet three vertical piles will suffice, as shown in Fig. 66 (*a*). Up to a height of 8 or 10 feet four piles may be used without sway-bracing, as in Fig. 66 (*b*), if the piles have a good bearing. For heights greater than 10 feet sway-bracing is generally necessary. The outside piles are frequently driven with a batter varying from 1 : 12 to 1 : 4.

Piles are made, if possible, from timber obtained in the vicinity of the work. Durability is the great requisite rather than strength, for almost any timber is strong enough (except as noted below) and will be suitable if it will resist rapid decay. The following list is quoted as being in the order of preference on account of durability:

1. Red cedar	5. White pine	9. White oak	12. Black oak
2. Red cypress	6. Redwood	10. Post-oak	13. Hemlock
3. Pitch-pine	7. Elm	11. Red oak	14. Tamarac
4. Yellow pine	8. Spruce		

Red-cedar piles are said to have an average life of 27 years with a possible maximum of 50 years, but the timber is rather weak, and if exposed in a river to flowing ice or driftwood is apt to be injured. Under these circumstances oak is preferable, although its life may be only 13 to 18 years.

130. Methods of driving piles. The following are the principal methods of driving piles:

a. A hammer weighing 2000 to 3000 lbs. or more, sliding in guides, is drawn up by horse-power or a portable engine, and allowed to fall *freely*.

b. The same as above except that the hammer does not fall freely, but drags the rope and revolving drum as it falls and is thus quite materially retarded. The mechanism is a little more simple, but is less effective, and is sometimes made deliberately deceptive by a contractor by retarding the blow, in order to apparently indicate the requisite resistance on the part of the pile.

The above methods have the advantage that the mechanism is cheap and can be transported into a new country with comparative ease, but the work done is somewhat ineffective and costly compared with some of the more elaborate methods given below.

c. Gunpowder pile-drivers, which automatically explode a cartridge every time the hammer falls. The explosion not only forces the pile down, but throws up the hammer for the next blow. For a given height of fall the effect is therefore doubled. It has been shown by experience, however, that when it is attempted to use such a pile-driver rapidly the mechanism becomes so heated that the cartridges explode prematurely, and the method has therefore been abandoned.

d. Steam pile-drivers, in which the hammer is operated directly by steam. The hammer falls freely a height of about 40 inches and is raised again by steam. The effectiveness is largely due to the rapidity of the blows, which does not allow time between the blows for the ground to settle around the pile and increase the resistance, which does happen when the blows are infrequent. "The hammer-cylinder weighs 5500 lbs., and with 60 to 75 lbs. of steam gives 75 to 80 blows per minute. With 41 blows a large unpointed pile was driven 35 feet into a hard clay bottom in half a minute." Such a driver would cost about $800.

The above four methods are those usual for dry earth. In very soft wet or sandy soils, where an unlimited supply of water is available, the *water-jet* is sometimes employed. A pipe is fastened along the side of the pile and extends to the pile-point. If water is forced through the pipe, it loosens the sand around the point and, rising along the sides, decreases the side resistance so that the pile sinks by its own weight, aided perhaps by extra weights loaded on. This loading may be accomplished by connecting the top of the pile and the pile-driver by a block and tackle so that a portion of the weight of the pile-driver is continually thrown on the pile.

Excessive driving frequently fractures the pile below the surface and thereby greatly weakens its bearing power. To prevent excessive "brooming" of the top of the pile, owing to the action of the hammer, the top should be protected by an iron ring fitted to the top of the pile. The "brooming" not only renders the driving ineffective and hence uneconomical, but vitiates the value of any test of the bearing power of the pile by noting the sinking due to a given weight falling a given distance. If the pile is so soft that brooming is unavoidable, the top should be adzed off

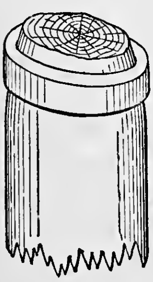


Fig. 67.

frequently, and especially should it be done just before the final blows which are to test its bearing-power.

In a new country judgment and experience will be required to decide intelligently whether to employ a simple drop-hammer machine, operated by horse-power and easily transported but uneconomical in operation, or a more complicated machine working cheaply and effectively after being transported at greater expense.

131. Pile-driving formulæ. If $R =$ the resistance of a pile, and s the set of the pile during the last blow, w the weight of the pile-hammer, and h the fall during the last blow, then we may state the approximate relation that $Rs = wh$, or $R = \frac{wh}{s}$. This is the basic principle of all rational formulæ, but the maximum weight which a pile will sustain after it has been driven some time is by no means equal to the resistance of the pile during the last blow. There are also many other modifying elements which have been variously allowed for in the many proposed formulæ. The formulæ range from the extreme of empirical simplicity to very complicated attempts to allow properly for all modifying causes. As the simplest rule, specifications sometimes require that the piles shall be driven until the pile will not sink more than 5 inches under five consecutive blows of a 2000 lb., hammer falling 25 feet. The "*Engineering News* formula"* gives the safe load as $\frac{2wh}{s+1}$, in which $w =$ weight of hammer, $h =$ fall in *feet*, $s =$ set of pile in *inches* under the last blow. This formula is derived from the above basic formula by calling the safe load $\frac{1}{6}$ of the final resistance, and by adding (arbitrarily) 1 to the final set (s) as a compensation for the extra resistance caused by the settling of earth around the pile between each blow. This formula is used only for ordinary hammer-driving. When the piles are driven by a steam pile-driver the formula becomes

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* *Engineering News*, Nov. 17, 1892.

safe load $= \frac{2wh}{s + 0.1}$. For the "gunpowder pile-driver," since the explosion of the cartridge drives the pile in with the same force with which it throws the hammer upward, the effect is *twice* that of the fall of the hammer, and the formula becomes safe load $= \frac{4wh}{s + 0.1}$. In these last two formulæ the constant in the denominator is changed from $s + 1$ to $s + 0.1$. The constant (1.0 or 0.1) is supposed to allow, as before stated, for the effect of the extra resistance caused by the earth settling around the pile between each blow. The more rapid the blows the less the opportunity to settle and the less the proper value of the constant.

The above formulæ have been given on account of their simplicity and their practical agreement with experience. Many other formulæ have been proposed, the majority of which are more complicated and attempt to take into account the weight of the pile, resistance of the guides, etc. While these elements, as well as many others, have their influence, their effect is so overshadowed by the indeterminable effect of other elements—as, for example, the effect of the settlement of earth around the pile between blows—that it is useless to attempt to employ anything but a purely empirical formula.

132. Pile-points and pile-shoes. Piles are generally sharpened to a blunt point. If the pile is liable to strike boulders, sunken logs, or other obstructions which are liable to turn the point, it is necessary to protect the point by some form of shoe. Several forms in cast iron have been used, also a wrought-iron shoe, having four "straps" radiating from the apex, the straps being nailed on to the pile, as shown in Fig. 68 (*b*). The cast-iron form shown in Fig. 68 (*a*) has a base cast around a drift-bolt. The recess on the top of the base receives the bottom of the pile and prevents a tendency to split the bottom of the pile or to force the shoe off laterally.

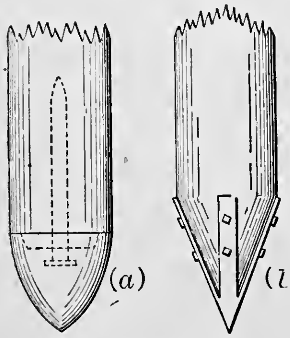


Fig. 68.

133. Details of design. No theoretical calculations of the strength of pile bents need be attempted on account of the extreme complication of the theoretical strains, the uncertainty as to the real strength of the timber used, the variability of that strength with time, and the insignificance of the economy that would be possible even if exact sizes could be computed. The piles are generally required to be not less than 10″ or 12″ in diameter at the large end. The P. R. R. requires that they shall be "not less than 14 and 7 inches in diameter at butt and small end respectively, exclusive of bark, which must be removed." The removal of the bark is generally required in good work. Soft *durable* woods, such as are mentioned in § 129, are best for the piles, but the caps are generally made of oak or yellow pine. The caps are generally 14 feet long (for single track) with a cross-section 12″ × 12″ or 12″ × 14″. "Split caps" would consist of two pieces 6″ × 12″. The sway-braces, never used for less heights than 6′, are made of 3″ × 12″ timber, and are spiked on with $\frac{3}{8}$″ spikes 8″ long. The floor system will be the same as that described later for framed trestles.

134. Cost of pile trestles. The cost, per linear foot, of piling depends on the method of driving, the scarcity of suitable timber, the price of labor, the length of the piles, and the amount of shifting of the pile-driver required. The cost of soft-wood piles varies from 8 to 15 c. per lineal foot, and the cost of oak piles varies from 10 to 30 c. per foot according to the length, the longer piles costing more per foot. The cost of driving will average about $2.50 per pile, or 7.5 to 10 c. per lineal foot. Since the cost of shifting the pile-driver is quite an item in the total cost, the cost of driving a long pile would be *less* per foot than for a short pile, but on the other hand the cost of the pile is *greater* per foot, which tends to make the total cost per foot constant. Specifications generally say that the piling will be paid for per lineal foot of piling *left in the work*. The wastage of the tops of piles sawed off is always something, and is frequently very large. Sometimes a small amount per foot of piling sawed off is allowed the contractor as compensation for his loss. This reduces the contractor's risk and possibly reduces his bid by an equal or greater amount than the extra amount actually paid him.

FRAMED TRESTLES.

135. Typical Design. A typical design for a framed trestle bent is given in Fig. 69. This represents, with slight variations of detail, the plan according to which a large part of the framed

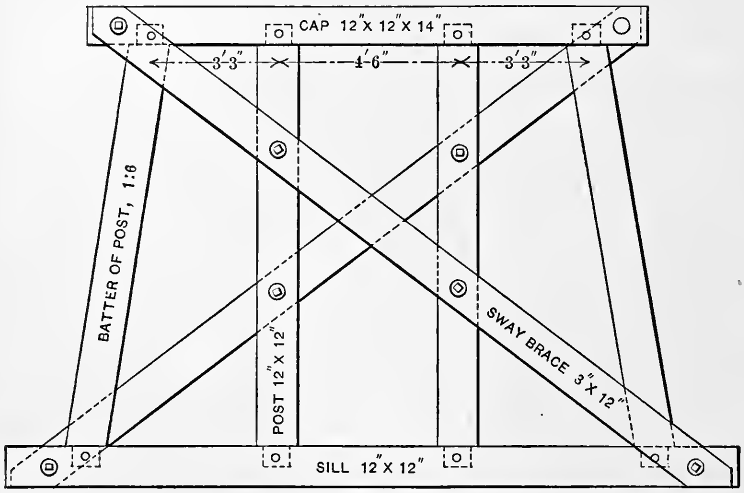


Fig. 69.

trestle bents of the country have been built—i.e., of those less than 20 or 30 feet in height, not requiring multiple-story construction.

136. Joints. (a) **The mortise-and-tenon joint** is illustrated in Fig. 69 and also in Fig. 66 (*a*). The tenon should be about 3″ thick, 8″ wide, and $5\frac{1}{2}$″ long. The mortise should be cut a little deeper than the tenon. "Drip-holes" from the mortise to the outside will assist in draining off water that may accumulate in the joint and thus prevent the rapid decay that would otherwise ensue. These joints are very troublesome if a single post decays and requires renewal. It is generally required that the mortise and tenon should be thoroughly daubed with paint before putting them together. This will tend to

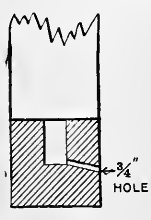


Fig. 70.

make the joint water-tight and prevent decay from the accumulation and retention of water in the joint.

(b) The plaster joint. This joint is made by bolting and spiking a 3″ × 12″ plank on both sides of the joint. The cap and sill should be notched to receive the posts. Repairs are greatly facilitated by the use of these joints. This method has been used by the Delaware and Hudson Canal Co. [R. R.].

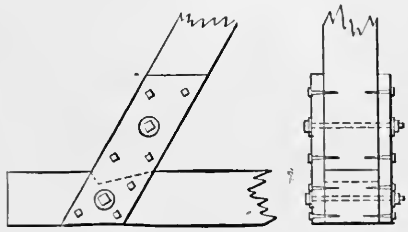


FIG. 71.

(c) Iron plates. An iron plate of the form shown in Fig. 72 (*b*) is bent and used as shown in Fig. 72 (*a*). Bolts passing through the bolt-holes shown secure the plates to the timbers and make a strong joint which may be readily loosened for repairs. By slight modifications in the design the method may be used for inclined posts and complicated joints.

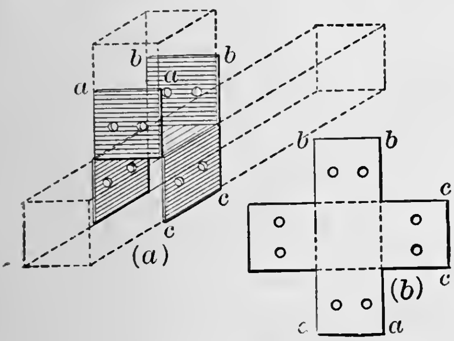


FIG. 72.

(d) Split caps and sills. These are described in § 129. Their advantages apply with even greater force to framed trestles.

(e) Dowels and drift-bolts. These joints facilitate cheap and rapid construction, but renewals and repairs are very difficult, it being almost impossible to extract a drift-bolt which has been driven its full length without splitting open the pieces containing it. Notwithstanding this objection they are extensively used, especially for temporary work which is not expected to be used long enough to need repairs.

137. Multiple-story construction. Single-story framed trestle bents are used for heights up to 18 or 20 feet and exceptionally up to 30 feet. For greater heights some such construction as is illustrated in a skeleton design in Fig. 73 is used. By using split sills between each story and separate vertical and batter posts in each story, any piece may readily be removed and renewed if necessary. The height of these stories varies, in different designs, from 15 to 25 and even 30 feet. In some designs the structure of each story is independent of the stories above and below. This greatly facilitates both the original construction and subsequent repairs. In other designs the verticals and batter-posts are made continuous through two consecutive stories. The structure is somewhat stiffer, but is much more difficult to repair.

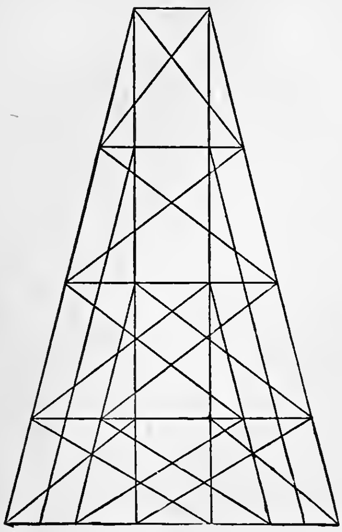


FIG. 73.

Since the bents of any trestle are usually of variable height and those heights are not always an even multiple of the uniform height desired for the stories, it becomes necessary to make the upper stories of uniform height and let

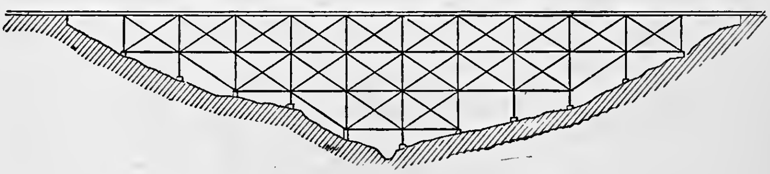


FIG. 74.

the odd amount go to the lowest story, as shown in Figs. 73 and 74.

138. Span. The shorter the span the greater the number of trestle bents; the longer the span the greater the required strength of the stringers supporting the floor. Economy demands the adoption of a span that shall make the sum of these requirements a minimum. The higher the trestle the greater the cost of each bent, and the greater the span that would be justifiable. Nearly all trestles have bents of variable height, but the advantage of employing uniform standard sizes is so great that many roads use the same span and sizes of timber not only for the panels of any given trestle, but also for all trestles regardless of height. The spans generally used vary from 10 to 16 feet. The Norfolk and Western R. R. uses a span of 12′ 6″ for all single-story trestles, and a span of 25′ for all multiple-story trestles. The stringers are the same in both cases, but when the span is 25 feet, knee-braces are run

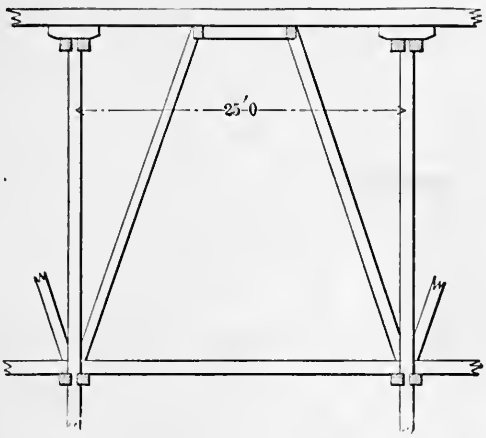


FIG. 75.

from the sill of the first story below to near the middle of each set of stringers. These knee-braces are connected at the top by a "straining-beam" on which the stringers rest, thus supporting the stringer in the center and virtually reducing the span about one-half.

139. Foundations. (a) **Piles.** Piles are frequently used as a foundation, as in Fig. 76, particularly in soft ground, and also for temporary structures. These foundations are cheap, quickly constructed, and are particularly valuable when it is financially necessary to open the road for traffic as soon as possible and with the least expenditure of money; but there is the disadvantage

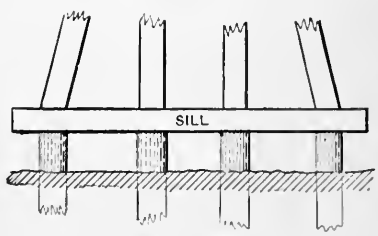


FIG. 76.

of inevitable decay within a few years unless the piles are chemically treated, as will be discussed later. Chemical treatment, however, increases the cost so that such a foundation would often cost more than a foundation of stone. A pile should be driven under each post as shown in Fig. 76.

(b) Mud-sills. Fig. 77 illustrates the use of mud-sills as built by the Louisville and Nashville R. R. Eight blocks 12″ × 12″ × 6′ are used under each bent. When the ground is very soft, two additional timbers (12″ × 12″ × length of bent-sill), as shown by the dotted lines, are placed underneath. The number required evidently depends on the nature of the ground.

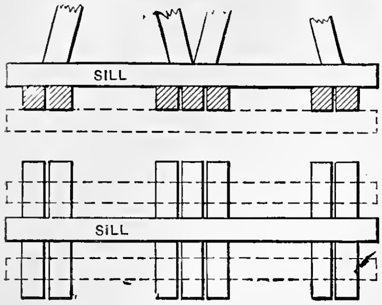


FIG. 77.

(c) Stone foundations. Stone foundations are the best and the most expensive. For very high trestles the Norfolk and Western R.R. employs foundations as shown in Fig. 78, the

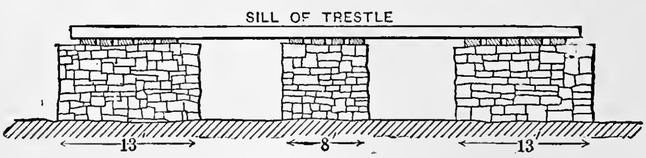


FIG. 78.

walls being 4 feet thick. When the height of the trestle is 72 feet or less (the plans requiring for 72′ in height a foundation-wall 39′ 6′ long) the foundation is made continuous. The sill of the trestle should rest on several short lengths of 3″ × 12″ plank, laid transverse to the sill on top of the wall.

140. Longitudinal bracing. This is required to give the structure longitudinal stiffness and also to reduce the columnar length of the posts. This bracing generally consists of horizontal "waling-strips" and diagonal braces. Sometimes the braces are placed wholly on the outside posts unless the trestle is very high. For single-story trestles the P. R. R. employs the "laced" system, i.e., a line of posts joining the cap of one bent with the sill of the next, and the sill of that bent with the cap of the next. Some plans employ braces forming an X in alternate panels. Connecting these braces in the center more than doubles their columnar strength. Diagonal braces, when bolted to posts, should be fastened to them as near the ends of the posts as possible. The sizes employed vary largely, depending on the clear length and on whether they are expected to act by tension or compression. 3″ × 12″ planks are often used when the design would require tensile strength only, and 8″ × 8″ posts are often used when compression may be expected.

141. Lateral bracing. Several of the more recent designs of trestles employ diagonal lateral bracing between the caps of adjacent bents. It adds greatly to the stiffness of the trestle and better maintains its alignment. 6″ × 6″ posts, forming an × and connected at the center, will answer the purpose.

142. Abutments. When suitable stone for masonry is at hand and a suitable subsoil for a foundation is obtainable without too much excavation, a masonry abutment will be the best. Such an abutment would probably be used when masonry footings for trestle bents were employed (§ 139, *c*).

Another method is to construct a "crib" of 10″ × 12″ timber, laid horizontally, drift-bolted together, securely braced and embedded into the ground. Except for temporary construction such a method is generally objectionable on account of rapid decay.

Another method, used most commonly for pile trestles, and for framed trestles having pile foundations (§ 139, *a*), is to use a pile bent at such a place that the natural surface on the *up-hill* side is not far below the cap, and the thrust of the material, filled in to bring the surface to grade, is insignificant. 3″ × 12″ planks are placed behind the piles, cap, and stringers to retain the filled material.

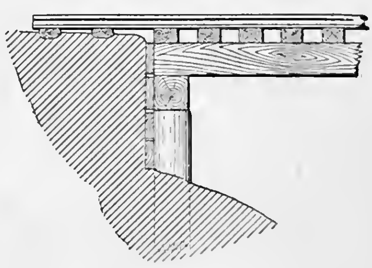


Fig. 79.

FLOOR SYSTEMS.

143. Stringers. The general practice is to use two, three, and even four stringers under each rail. Sometimes a stringer is placed under each guard-rail. Generally the stringers are made of two panel lengths and laid so that the joints alternate. A few roads use stringers of only one panel length, but this practice is strongly condemned by many engineers. The stringers should be separated to allow a circulation of air around them and prevent the decay which would occur if they were placed close together. This is sometimes done by means of 2″ planks, 6′ to 8′ long, which are placed over each trestle bent. Several bolts, passing through all the stringers forming a group and through the separators, bind them all into one solid construction. Cast-iron "spools" or washers, varying from 4″ to $\frac{3}{4}$″ in length (or thickness), are sometimes strung on each bolt so as to separate the stringers. Sometimes washers are used between the separating planks and the stringers, the object of the separating planks then being to bind the stringers, especially abutting stringers, and increase their stiffness.

The most common size for stringers is 8″ × 16″. The Pennsylvania Railroad varies the width, depth, and number of stringers under each rail according to the clear span. It may be noticed that, assuming a uniform load per running foot, both

Clear span.	No. of pieces under each rail.	Width.	Depth.
10 feet	2	8 inches	15 inches
12 "	2	8 "	16 "
14 "	2	10 "	17 "
16 "	3	8 "	17 "

the pressure per square inch at the ends of the stringers (the caps having a width of 12″) and also the stress due to transverse strain are kept *approximately* constant for the variable gross load on these varying spans.

144. Corbels. A corbel (in trestle-work) is a stick of timber (perhaps two placed side by side), about 3′ to 6′ long, placed underneath and along the stringers and resting on the cap. There are strong prejudices for and against their use, and a corresponding diversity in practice. They are bolted to the stringers and thus stiffen the joint. They certainly reduce the objectionable crushing of the fibers at each end of the stringer, but if the corbel is no wider than the stringers, as is generally the case, the area of pressure between the corbels and the cap is

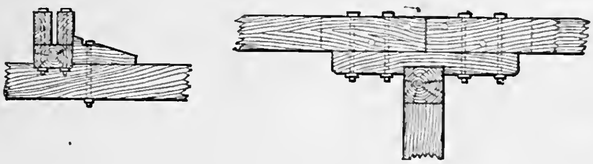


FIG. 80.

no greater and the pressure per square inch on the cap is no less than the pressure on the cap if no corbels were used. If the corbels and cap are made of hard wood, as is recommended by some, the danger of crushing is lessened, but the extra cost and the frequent scarcity of hard wood, and also the extra cost and labor of using, corbels, may often neutralize the advantages obtained by their use.

145. Guard-rails. These are frequently made of 5″ × 8″ stuff, notched 1″ for each tie. The sizes vary up to 8″ × 8″, and the depth of notch from $\frac{3}{4}$″ to $1\frac{1}{2}$″. They are generally bolted to every third or fourth tie. It is frequently specified that they shall be made of oak, white pine, or yellow pine. The joints are made over a tie, by halving each piece, as illustrated in Fig. 81. The joints on opposite sides of the trestle should be

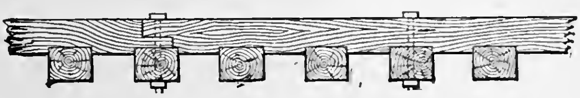


FIG. 81.

"staggered." Some roads fasten every tie to the guard-rail, using a bolt, a spike, or a lag-screw.

Guard-rails were originally used with the idea of preventing the wheels of a derailed truck from running off the ends of the ties. But it has been found that an outer guard-rail alone (without an inner guard-rail) becomes an actual element of danger, since it has frequently happened that a derailed wheel has caught on the outer guard-rail, thus causing the truck to slew around and so produce a dangerous accident. The true function of the *outside* guard-rail is thus changed to that of a tie-spacer, which keeps the ties from spreading when a derailment occurs. The inside guard-rail generally consists of an ordinary steel rail spiked about 10 inches inside of the running rail. These inner guard-rails should be bent inward to a point in the center of the track about 50 feet from the end of the bridge or trestle. If the inner guard-rails are placed with a clear space of 10 inches inside the running rail, the outer guard-rails should be *at least* 6′ 10″ apart. They are generally much farther apart than this.

146. Ties on trestles. If a car is derailed on a bridge or trestle, the heavily loaded wheels are apt to force their way between the ties by displacing them unless the ties are closely spaced and fastened. The clear space between ties is generally equal to or less than their width. Occasionally it is a little more than their width. 6″ × 8″ ties, spaced 14″ to 16″ from center to center, are most frequently used. The length varies from 9′ to 12′ for single track. They are generally notched ½″ deep on the under side where they rest on the stringers. Oak ties are generally required even when cheaper ties are used on the other sections of the road. Usually every third or fourth tie is bolted to the stringers. When stringers are placed underneath the guard-rails, bolts are run from the top of the guard-rail to the under side of the stringer. The guard-rails thus hold down the whole system of ties, and no direct fastening of the ties to the stringers is needed.

147. Superelevation of the outer rail on curves. The location of curves on trestles should be avoided if possible, especially when the trestle is high. Serious additional strains are introduced especially when the curvature is sharp or the speed high. Since such curves are sometimes practically unavoidable, it is necessary to design the trestle accordingly. If a train is stopped on a curved trestle, the action of the train on the trestle is evidently vertical. If the train is moving with a considerable velocity, the resultant of the weight and the centrifugal action is a force somewhat inclined from the vertical. Both of these conditions may be expected to exist at times. If the *axis* of the system of posts is vertical (as illustrated in methods *a*, *b*, *c*, *d*, and *e*), any lateral force, such as would be produced by a moving train, will tend to rack the trestle bent. If the stringers are set vertically, a centrifugal force likewise tends to tip them sidewise. If the axis of the system of posts (or of the stringers) is inclined so as to coincide with the pressure of the train on the trestle when the train is moving at its normal velocity, there is no tendency to rack the trestle when the train is moving at that velocity, but there will be a tendency to rack the trestle or twist the stringers when the train is stationary. Since a moving train is usually the normal condition of affairs, as well as the condition which produces the maximum stress, an inclined axis is evidently preferable from a theoretical standpoint; but whatever design is adopted, the trestle should evidently be sufficiently cross-braced for either a moving or a stationary load, and any proposed design must be studied as to the effect of *both* of these conditions. Some of the various methods of securing the requisite superelevation may be described as follows:

(a) **Framing the outer posts longer than the inner posts, so that the cap is inclined at the proper angle; axis of posts vertical.** (Fig. 82.) The method requires more work in framing the trestle, but simplifies subsequent track-laying and maintenance, unless it should be found that the superelevation adopted is unsuitable, in which case it could be corrected by one of the other methods given below. The stringers tend to twist when the train is stationary.

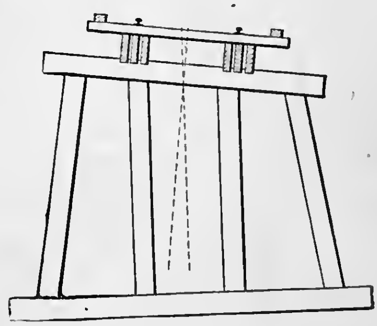


FIG. 82

(b) **Notching the cap so that the stringers are at a different elevation.** (Fig. 83.) This weakens the cap and requires that all ties shall be notched to a bevelled surface to fit the stringers, which also weakens the ties. A centrifugal force will tend to twist the stringers and rack the trestle.

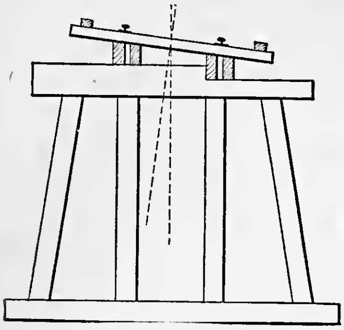


Fig 83.

(c) **Placing wedges underneath the ties at each stringer.** These wedges are fastened with two bolts. Two or more wedges will be required for each tie. The additional number of pieces required for a long curve will be immense, and the work of inspection and keeping the nuts tight will greatly increase the cost of maintenance.

(d) **Placing a wedge under the outer rail at each tie.** This requires but one extra piece per tie. There is no need of a wedge under the inner tie in order to make the rail normal to the tread. The resulting inward inclination is substantially that produced by some forms of rail-chairs or tie-plates. The spikes (a little longer than usual) are driven through the wedge into the tie. Sometimes "lag-screws" are used instead of spikes. If experience proves that the superelevation is too much or too little, it may be changed by this method with less work than by any other.

(e) **Corbels of different heights.** When corbels are used (see § 144) the required inclination of the floor system may be obtained by varying the depth of the corbels.

(f) **Tipping the whole trestle.** This is done by placing the trestle on an inclined foundation. If very much inclined, the trestle bent must be secured against the possibility of slipping sidewise, for the slope would be considerable with a sharp curve, and the vibration of a moving train would reduce the coefficient of friction to a comparatively small quantity.

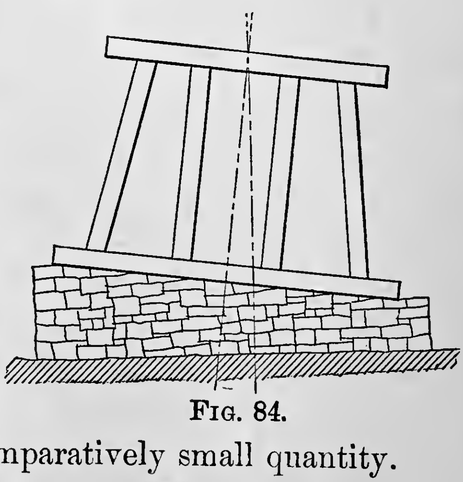


Fig. 84.

(g) **Framing the outer posts longer.** This case is identical with case (*a*) except that the axis of the system of posts is inclined, as in case (*f*), but the sill is horizontal.

The above-described plans will suggest a great variety of methods which are possible and which differ from the above only in minor details.

148. Protection from fire. Trestles are peculiarly subject to fire, from passing locomotives, which may not only destroy the trestle, but perhaps cause a terrible disaster. This danger is sometimes reduced by placing a strip of galvanized iron along the top of each set of stringers and also along the tops of the caps. Still greater protection was given on a long trestle on the Louisville and Nashville R. R. by making a solid flooring of timber, covered with a layer of ballast on which the ties and rails were laid as usual.

Barrels of water should be provided and kept near all trestles, and on very long trestles barrels of water should be placed every two or three hundred feet along its length. A place for the barrels may be provided by using a few ties which have an extra length of about four feet, thus forming a small platform, which should be surrounded by a railing. The track-walkers should be held accountable for the maintenance of a supply of water in these barrels, renewals being frequently necessary on account of evaporation. Such platforms should also be provided as REFUGE-BAYS for track-walkers and trackmen working on the trestle. On very long trestles such a platform is sometimes provided with sufficient capacity for a hand-car.

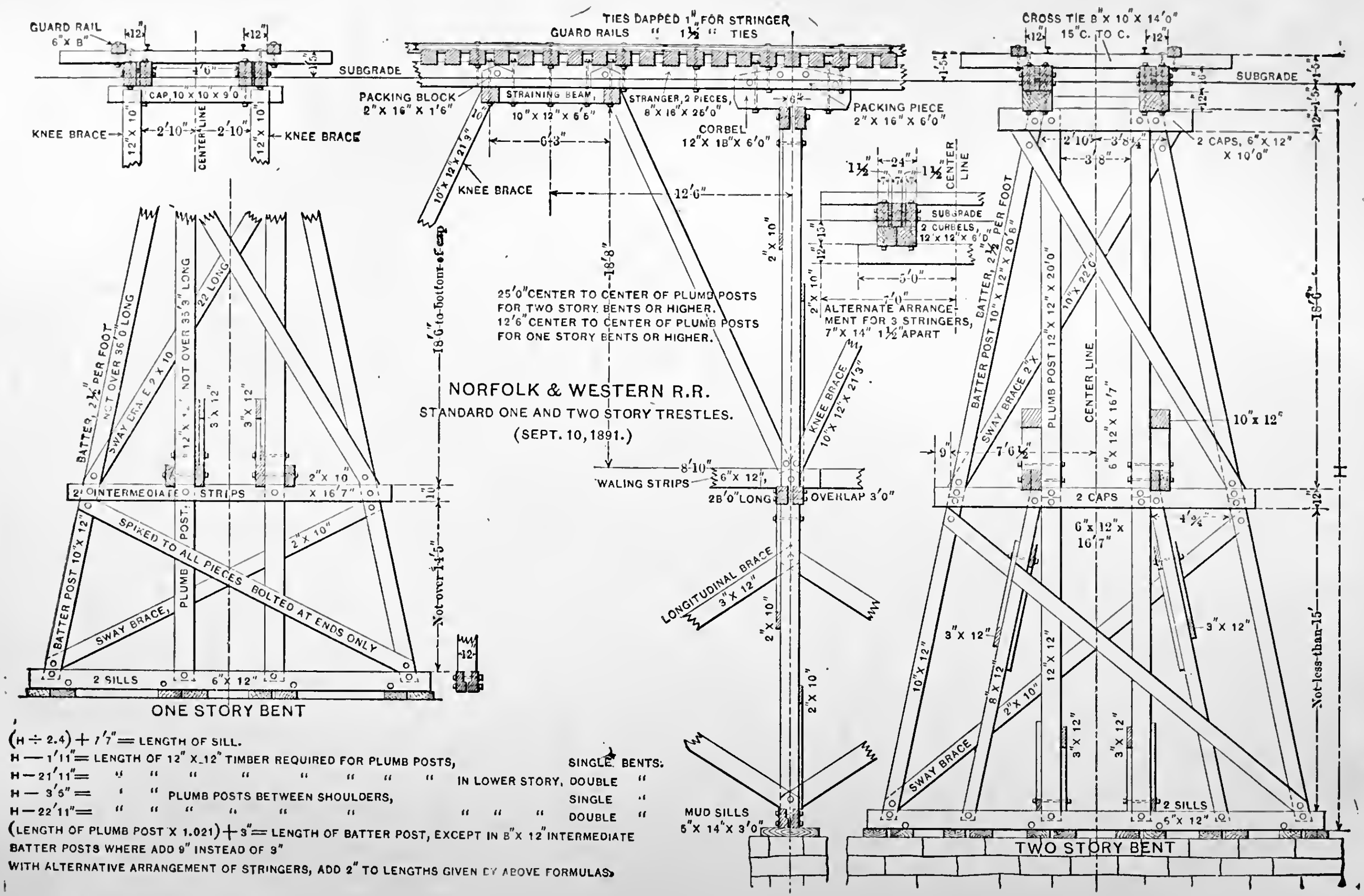
149. Timber. Any strong durable timber may be used when the choice is limited, but oak, pine, or cypress are preferred when obtainable. When all of these are readily obtainable, the various parts of the trestle will be constructed of different kinds of wood—the stringers of long-leaf pine, the posts and braces of pine or red cypress, and the caps, sills, and corbels (if used) of white oak. The use of oak (or a similar hard wood) for caps, sills, and corbels is desirable because of its greater strength in resisting crushing across the grain, which is the critical test for these parts. There is no physiological basis to the objection, sometimes made, that different species of timber, in contact with each other, will rot quicker than if only one kind of timber is used. When a very extensive trestle is to be built at a place where suitable growing timber is at hand but there is no convenient sawmill, it will pay to transport a portable sawmill and engine and cut up the timber as desired.

150. Cost of framed timber trestles. The cost varies widely on account of the great variation in the cost of timber. When a railroad is first penetrating a new and undeveloped region, the cost of timber is frequently small, and when it is obtainable from the company's right-of-way the only expense is felling and sawing. The work per M., B. M., is small, considering that a single stick 12″ × 12″ × 25′ contains 300 feet, B. M., and that sometimes a few hours' work, worth less than $1, will finish all the work required on it. Smaller pieces will of course require more work per foot, B. M. Long-leaf pine can be purchased from the mills at from $8 to $12 per M. feet, B. M., according to the dimensions. To this must be added the freight and labor of erection. The cartage from the nearest railroad to the trestle may often be a considerable item. Wrought iron will cost about 3 c. per pound and cast iron 2 c., although the prices are often lower than these. The amount of iron used depends on the detailed design, but, as an average, will amount to $1.50 to $2 per 1000 feet, B. M., of timber. A large part of the trestling of the country has been built at a contract price of about $30 per 1000 feet, B. M., erected. While the cost will frequently rise to $40 and even $50 when timber is scarce, it will drop to $13 (cost quoted) when timber is cheap.

DESIGN OF WOODEN TRESTLES.

151. Common practice. A great deal of trestling has been constructed without any rational design except that custom and experience have shown that certain sizes and designs are *probably* safe. This method has resulted occasionally in failures but more frequently in a very large waste of timber. Many railroads

PLATE I.



(*To face page* 174)

employ a uniform size for all posts, caps, and sills, and a uniform size for stringers, all regardless of the height or span of the trestle. For repair work there are practical reasons favoring this. "To attempt to run a large lot of sizes would be more wasteful in the end than to maintain a few stock sizes only. Lumber can be bought more cheaply by giving a general order for 'the run of the mill for the season,' or 'a cargo lot,' specifying approximate percentages of standard stringer size, of 12 × 12-inch stuff, 10 × 10-inch stuff, etc., and a liberal proportion of 3- or 4-inch plank, all lengths thrown in. The 12 × 12-inch stuff, etc., is ordered all lengths, from a certain specified length up. In case of a wreck, washout, burn-out, or sudden call for a trestle to be completed in a stated time, it is much more economical and practical to order a certain number of carloads of 'trestle stuff' to the ground and there to select piece after piece as fast as needed, dependent only upon the length of stick required. When there is time to make the necessary surveys of the ground and calculations of strength, and to wait for a special bill of timber to be cut and delivered, the use of different sizes for posts in a structure would be warranted to a certain extent." * For new construction, when there is generally sufficient time to design and order the proper sizes, such wastefulness is less excusable, and under any conditions it is both safer and more economical to prepare *standard designs* which can be made applicable to varying conditions and which will at the same time utilize as much of the strength of the timber as can be depended on. In the following sections will be given the elements of the preparation of such standard designs, which will utilize uniform sizes with as little waste of timber as possible. It is *not* to be understood that special designs should be made for each individual trestle.

152. Required elements of strength. The *stringers* of trestles are subject to transverse strains, to crushing across the grain at the ends, and to shearing along the neutral axis. The

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* From "Economical Designing of Timber Trestle Bridges."

strength of the timber must therefore be computed for all these kinds of stress. *Caps* and *sills* will fail, if at all, by crushing across the grain; although subject to other forms of stress, these could hardly cause failure in the sizes usually employed. There is an apparent exception to this: if piles are improperly driven and an uneven settlement subsequently occurs, it may have the effect of transferring practically all of the weight to two or three piles, while the *cap* is subjected to a severe transverse strain which may cause its failure. Since such action is caused generally by avoidable errors of construction it may be considered as abnormal, and since such a failure will generally occur by a *gradual* settlement, all danger may be avoided by reasonable care in inspection. *Posts* must be tested for their columnar strength. These parts form the bulk of the trestle and are the parts which can be definitely designed from known stresses. The stresses in the bracing are more indefinite, depending on indeterminate forces, since the inclined posts take up an unknown proportion of the lateral stresses, and the design of the bracing may be left to what experience has shown to be safe, without involving any large waste of timber.

153. Strength of timber. Until recently tests of the strength of timber have generally been made by testing small, selected, well-seasoned sticks of "clear stuff," free from knots or imperfections. Such tests would give results so much higher than the vaguely known strength of large unseasoned "commercial" timber that very large factors of safety were recommended—factors so large as to detract from any confidence in the whole theoretical design. Recently the U. S. Government has been making a thoroughly scientific test of the strength of full-size timber under various conditions as to seasoning, etc. The work has been so extensive and thorough as to render possible the economical designing of timber structures.

One important result of the investigation is the determination of the great influence of the moisture in the timber and the law of its effect on the strength. It has been also shown that timber soaked with water has substantially the same strength as green timber, even though the timber had once been thoroughly seasoned. Since trestles are exposed to the weather they should be designed on the basis of using green timber. It has been shown that the strength of green timber is very regularly about 55 to 60% of the strength of timber in which the moisture is 12% of the dry weight, 12% being the proportion of moisture usually found in timber that is protected from the weather but not heated, as, e.g., the timber in a barn. Since the moduli of rupture have all been reduced to this standard of moisture (12%), if we take *one-eighth* of the rupture values, it still allows a factor of safety of about five, even on green timber.

Moduli of rupture for various timbers. [12% moisture.]

(Condensed from U. S. Forestry Circular, No. 15.)

No.	Species.	Weight per cubic foot.	Cross-bending. Ultimate Strength.	Cross-bending. Modulus of Elasticity.	Crushing end wise.	Crushing across grain.	Shearing along grain.
1	Long-leaf pine.....	38	12 600	2 070 000	8000	1180	700
2	Cuban "	39	13 600	2 370 000	8700	1220	700
3	Short-leaf "	32	10 100	1 680 000	6500	960	700
4	Loblolly "	33	11 300	2 050 000	7400	1150	700
5	White "	24	7 900	1 390 000	5400	700	400
6	Red "	31	9 100	1 620 000	6700	1000	500
7	Spruce "	39	10 000	1 640 000	7300	1200	800
8	Bald cypress.......	29	7 900	1 290 000	6000	800	500
9	White cedar.......	23	6 300	910 000	5200	700	400
10	Douglas spruce....	32	7 900	1 680 000	5700	800	500
11	White oak........	50	13 100	2 090 000	8500	2200	1000
12	Overcup "	46	11 300	1 620 000	7300	1900	1000
13	Post "	50	12 300	2 030 000	7100	3000	1100
14	Cow "	46	11 500	1 610 000	7400	1900	900
15	Red "	45	11 400	1 970 000	7200	2300	1100
16	Texan "	46	13 100	1 860 000	8100	2000	900
19	Willow "	45	10 400	1 750 000	7200	1600	900
20	Spanish "	46	12 000	1 930 000	7700	1800	900
21	Shagbark hickory..	51	16 000	2 390 000	9500	2700	1100
27	Pignut " ..	56	18 700	2 730 000	10900	3200	1200
28	White elm........	34	10 300	1 540 000	6500	1200	800
29	Cedar "	46	13 500	1 700 000	8000	2100	1300
30	White ash........	39	10 800	1 640 000	7200	1900	1100

AVERAGE SAFE ALLOWABLE WORKING UNIT STRESSES, IN POUNDS, PER SQUARE INCH. RECOMMENDED BY THE COMMITTEE ON "STRENGTH OF BRIDGE AND TRESTLE TIMBERS." (ASSOCIATION OF RAILWAY SUPERINTENDENTS OF BRIDGES AND BUILDINGS: FIFTH ANNUAL CONVENTION, NEW ORLEANS, OCTOBER, 1895.)

Kind of timber.	Tension.		Compression.			Transverse.		Shearing.	
			With grain.						
	With grain.	Across grain.	End bearing.	Column under 15 diameters.	Across grain.	Extreme fibre stress.	Modulus of elasticity.	With grain.	Across grain.
Factor of safety....	Ten.	Ten.	Five.	Five.	Four.	Six.	Two.	Four.	Four.
White oak	1000	200	1400	900	500	1000	550 000	200	1000
White pine	700	50	1100	700	200	700	500 000	100	500
Southern, long-leaf, or Georgia yellow pine	1200	60	1600	1000	350	1200	850 000	150	1250
Douglas, Oregon, and Washington fir or pine: Yellow fir..	1200	...	1600	1200	300	1100	700 000	150	....
Douglas, Oregon, and Washington fir or pine: Red fir.....	1000	...	....	....	...	800	......	...	....
Northern or short-leaf yellow pine	900	50	1200	800	250	1000	600 000	100	1000
Red pine	900	50	1200	800	200	800	600 000	...	....
Norway pine	800	...	1200	800	200	700	600 000	...	....
Canadian (Ottawa) white pine	1000	...	....	1000	...	....	....	100	....
Canadian (Ontario) red pine	1000	...	....	1000	...	800	700 000	100	....
Spruce and Eastern fir	800	50	1200	800	200	700	600 000	100	750
Hemlock	600	...	....	800	150	600	450 000	100	600
Cypress	600	...	1200	800	200	800	450 000	...	....
Cedar	800	...	1200	800	200	800	350 000	...	400
Chestnut	900	...	....	1000	250	800	500 000	150	400
California redwood	700	...	....	800	200	750	350 000	100	....
California spruce	....	...	....	800	...	800	600 000	...	....

On page 177 there are quoted the values taken from the U. S. Government reports on the strength of timber, the tests probably being the most thorough and reliable that were ever made.

On page 178 are given the "average safe allowable working unit stresses in pounds per square inch," as recommended by the committee on "Strength of Bridge and Trestle Timbers," the work being done under the auspices of the Association of Railway Superintendents of Bridges and Buildings. The report was presented at their fifth annual convention, held in New Orleans, in October, 1895.

154. Loading. As shown in § 138, the span of trestles is always small, is generally 14 feet, and is never greater than 18′ except when supported by knee-braces. The greatest load that will ever come on any one span will be the concentrated loading of the drivers of a consolidation locomotive. With spans of 14 feet or less it is impossible for even the four pairs of drivers to be on the same span at once. The weight of the rails, ties, and guard-rails should be added to obtain the total load on the stringers, and the weight of these, plus the weight of the stringers, should be added to obtain the pressure on the caps or corbels. This dead load is almost insignificant compared with the live load and may be included with it. The weight of rails, ties, etc., may be estimated at 200 pounds per foot. To obtain the weight on the caps the weight of the stringers must be added, which depends on the design and on the weight per cubic foot of the wood employed. But as the weight of the stringers is comparatively small, a considerable percentage of variation in weight will have but an insignificant effect on the result. Disregarding all refinements as to actual dimensions, the ordinary maximum loading for standard gauge railroads may be taken as that due to four pairs of driving-axles, spaced 5′ 0″ apart and giving a pressure of 25,000 pounds per axle. This should be increased to 40,000 pounds per axle (same spacing) for the heaviest traffic. On the basis of 25,000 pounds per axle the following results have been computed:

STRESSES ON VARIOUS SPANS DUE TO MOVING LOADS OF 25,000 POUNDS, SPACED 5′ 0″ APART.

Span in feet.	Max. mom.—ft. lbs.	Max. shear.	Max load on one cap.
10	65 000	38 500	52 100
12	103 600	45 000	62 700
14	142 400	49 600	74 200
16	181 400	54 725	85 700
18	220 600	60 100	97 900

Although the dead load does not vary in proportion to the live load, yet, considering the very small influence of the dead load, there will be no appreciable error in assuming the corresponding values, for a load of 40,000 lbs. per axle, to be $\frac{40}{25}$ of those given in the above tabulation.

155. Factors of safety.—The most valuable result of the government tests is the knowledge that under given moisture conditions the strength of various species of sound timber is not the variable uncertain quantity it was once supposed to be, but that its strength can be relied on to a comparatively close percentage. This confidence in values permits the employment of lower factors of safety than have heretofore been permissible. Stresses, which when excessive would result in immediate destruction, such as cross-breaking and columnar stresses, should be allowed a higher factor of safety—say 6 or 8 for green timber. Other stresses, such as crushing across the grain and shearing along the neutral axis, which will be apparent to inspection before it is dangerous, may be allowed lower factors—say 3 to 5.

156. Design of stringers.—The strength of rectangular beams of equal width varies as the square of the depth; therefore deep beams are the strongest. On the other hand, when any cross-sectional dimension of timber much exceeds 12″ the cost is much higher per M., B.M., and it is correspondingly difficult to obtain thoroughly sound sticks, free from wind-shakes, etc. Wind-shakes especially affect the shearing strength. Also, if the required transverse strength is obtained by using high narrow stringers, the area of pressure between the stringers and the cap may become so small as to induce crushing across the grain. This is a very common defect in trestle design. As already indicated in § 138, the span should vary roughly with the average height of the trestle, the longer spans being employed when the trestle bents are very high, although it is usual to employ the same span throughout any one trestle.

To illustrate, if we select a span of 14 feet, the load on one cap will be 74,200 lbs. If the stringers and cap are made of long-leaf yellow pine, which require the closely determined value of 1180 lbs. per square inch to produce a crushing amounting to 3% of the height on timber with 12% moisture, we may use 200 lbs. per square inch as a safe pressure even for green timber; this will require 371 square inches of surface. If the cap is 12″ wide, this will require a width of 31 inches, or say 2 stringers under each rail, each 8 inches wide. For rectangular beams

$$\text{Moment} = \tfrac{1}{6}R'bh^2.$$

Using for R' the safe value 1575 lbs. per square inch, we have

$$142400 \times 12 = \tfrac{1}{6} \times 1575 \times 32 \times h^2,$$

from which $h = 15''.9$. If desired, the width may be increased to 9″ and the depth correspondingly reduced, which will give similarly $h = 14''.8$, or say 15″. This shows that two beams, 9″ × 15″, under each rail will stand the transverse bending and have more than enough area for crushing.

The shear per square inch will equal

$$\frac{3}{2}\,\frac{\text{total shear}}{\text{cross section}} = \frac{3}{2}\,\frac{49600}{4 \times 9 \times 15} = 138 \text{ lbs. per sq. inch},$$

which is a safe value, although it should preferably be less. Hence the above combination of dimensions will answer.

The deflection should be computed to see if it exceeds the somewhat arbitrary standard of $\frac{1}{200}$ of the span. The deflection for *uniform loading* is

$$\Delta = \frac{5 Wl^3}{32bh^3E},$$

in which l = length in inches;
W = total load, assumed as uniform;
E = modulus of elasticity, given as 2,070,000 lbs.

per sq. in. for long-leaf pine, 12% dry, and assumed to be 1,200,000 for green timber. Then

$$\Delta = \frac{5 \times 72800 \times 168^3}{32 \times 36 \times 15^3 \times 1200000} = 0''.37$$

$$\tfrac{1}{200} \times 168'' = 0''.84,$$

so that the calculated deflection is well within the limit. Of course the loading is not strictly uniform, but even with a liberal allowance the deflection is still safe.

For the heaviest practice (40000 lbs. per axle) these stringer dimensions must be correspondingly increased.

157. Design of posts. Four posts are generally used for single-track work. The inner posts are usually braced by the cross-braces, so that their columnar strength is largely increased; but as they are apt to get more than their share of work, the advantage is compensated and they should be treated as unsupported columns for the total distance between cap and sill in simple bents, or for the height of stories in multiple-story construction. The caps and sills are assumed to have a width of 12″. It facilitates the application of bracing to have the columns of the same width and vary the other dimension as required.

Unfortunately the experimental work of the U. S. Government on timber testing has not yet progressed far enough to establish unquestionably a general relation between the strength of long columns and the crushing strength of short blocks. The following formula has been suggested, but it cannot be considered as established:

$$f = F \times \frac{700 + 15c}{700 + 15c + c^2}, \quad \text{in which}$$

f = allowable working stress per sq. in. for long columns;
F = " " " " " " " short blocks;
$c = \frac{l}{d}$;
l = length of column in inches;
d = least cross-sectional dimension in inches.

Enough work has been done to give great reliability to the two following formulæ for white pine and yellow pine, quoted from Johnson's "Materials of Construction," p. 684:

$$\text{Working load per sq. in.} = p = 1000 - \frac{1}{4}\left(\frac{l}{h}\right)^2, \text{ long-leaf pine;}$$

$$\text{" " " " "} = p = 600 - \frac{1}{8}\left(\frac{l}{h}\right)^2, \text{ white pine;}$$

in which l = length of column in inches, and
h = least cross-sectional dimension in inches.

The frequent practice is to use 12″ × 12″ posts for all trestles. If we substitute in the above formula $l = 20' = 240''$ and $h = 12''$, we have $p = 1000 - \frac{1}{4}(\frac{240}{12})^2 = 900$ lbs.

900 × 144 = 129600 lbs., the *working load* for *each* post. This is more than the total load on one trestle bent and illustrates the usual great waste of timber. Making the post 8″ × 12″ and calculating similarly, we have $p = 775$, and the working load per column is 775 × 96 = 74400 lbs. As considerable must be allowed for "weathering," which destroys the strength of the outer layers of the wood, and also for the dynamic effect of the live load, 8″ × 12″ may not be too great,

but it is certainly a safe dimension. 12″ × 6″ would possibly prove amply safe in practice. One method of allowing for weathering is to disregard the outer half-inch on all sides of the post, i.e., to calculate the strength of a post one inch smaller in each dimension than the post actually employed. On this basis an 8″ × 12″ × 20′ post, computed as a 7″ × 11′ post, would have a *safe* columnar strength of 706 lbs. per square inch. With an area of 77 square inches, this gives a working load of 54362 lbs. for *each post*, or 217448 lbs. for the four posts. Considering that 74200 lbs. is the maximum load on one cap (14 feet span), the great excess of strength is apparent.

158. Design of caps and sills. The stresses in caps and sills are very indefinite, except as to crushing across the grain. As the stringers are placed almost directly over the inner posts, and as the sills are supported just under the posts, the transverse stresses are almost insignificant. In the above case four posts have an area of 4 × 12″ × 8″ = 384 sq. in. The total load, 74200 lbs., will then give a pressure of 193 pounds per square inch, which is within the allowable limit. This one feature might require the use of 8″ × 12″ posts rather than 6″ × 12″ posts, for the smaller posts, although probably strong enough as posts, would produce an objectionably high pressure.

159. Bracing. Although some idea of the stresses in the bracing could be found from certain assumptions as to wind-pressure, etc., yet it would probably not be found wise to decrease, for the sake of economy, the dimensions which practice has shown to be sufficient for the work. The economy that would be possible would be too insignificant to justify any risk. Therefore the usual dimensions, given in §§ 139 and 140, should be employed.

CHAPTER V.

TUNNELS.

SURVEYING.

160. Surface surveys. As tunnels are always dug from each end and frequently from one or more intermediate shafts, it is necessary that an accurate surface survey should be made between the two ends. As the natural surface in a locality where a tunnel is necessary is almost invariably very steep and rough, it requires the employment of unusually refined methods of work to avoid inaccuracies. It is usual to run a line on the surface that will be at every point vertically over the center line of the tunnel. Tunnels are generally made straight unless curves are absolutely necessary, as curves add greatly to the cost. Fig. 85 represents roughly a longitudinal section of the

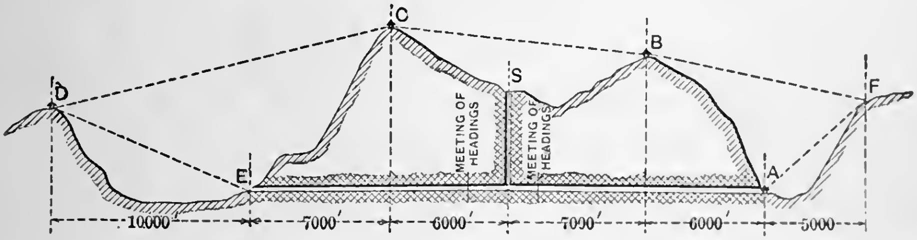


FIG. 85.—SKETCH OF SECTION OF THE HOOSAC TUNNEL.

Hoosac Tunnel. Permanent stations were located at *A*, *B*, *C*, *D*, *E*, and *F*, and stone houses were built at *A*, *B*, *C*, and *D*. These were located with ordinary field transits at first, and then all the points were placed as nearly as possible in one vertical plane by repeated trials and minute corrections, using a very large specially constructed transit. The stations *D* and *F* were necessary because *E* and *A* were invisible from *C* and *B*.

The alignment at A and E having been determined with great accuracy, the true alignment was easily carried into the tunnel.

The relative elevations of A and E were determined with great accuracy. Steep slopes render necessary many settings of the level per unit of horizontal distance and require that the work be unusually accurate to obtain even fair accuracy per unit of distance. The levels are usually re-run many times until the probable error is a very small quantity.

The exact horizontal distance between the two ends of the tunnel must also be known, especially if the tunnel is on a grade. The usual steep slopes and rough topography likewise render accurate horizontal measurements very difficult. Frequently when the slope is steep the measurement is best obtained by measuring along the slope and allowing for grade. This may be very accurately done by employing two tripods (level or transit tripods serve the purpose very well), setting them up slightly less than one tape-length apart and measuring between horizontal needles set in wooden blocks inserted in the top of each tripod. The elevation of each needle is also observed. The true horizontal distance between two successive positions of the needles then equals the square root of the difference of the squares of the inclined distance and the difference of elevation. Such measurements will probably be more accurate than those made by attempting to hold the tape horizontal and plumbing down with plumb-bobs, because (1) it is practically difficult to hold both ends of the tape truly horizontal; (2) on steep slopes it is impossible to hold the downhill end of a 100-foot tape (or even a 25-foot length) on a level with the other end, and the great increase in the number of applications of the unit of measurement very greatly increases the probable error of the whole measurement; (3) the vibrations of a plumb-bob introduce a large probability of error in transferring the measurement from the elevated end of the tape to the ground, and the increased number of such applications of the unit of measurement still further increases the probable error.

161. Surveying down a shaft. If a shaft is sunk, as at S, Fig. 85, and it is desired to dig out the tunnel in both directions from the foot of the shaft so as to meet the headings from the outside, it is necessary to know, when at the bottom of the shaft, the elevation, alignment, and horizontal distance from each end of the tunnel.

The *elevation* is generally carried down a shaft by means of a steel tape. This method involves the least number of applications of the unit of measurement and greatly increases the accuracy of the final result.

The *horizontal distance from each end* may be easily transferred down the shaft by means of a plumb-bob, using some of the precautions described in the next paragraph.

To transfer the *alignment* from the surface to the bottom of a shaft requires the highest skill because the shaft is always small, and to produce a line perhaps several thousand feet long in a direction given by two points 6 or 8 feet apart requires that the two points must be determined with extreme accuracy. The eminently successful method adopted in the Hoosac Tunnel will be briefly described: Two beams were securely fastened across the top of the shaft (1030 feet deep), the beams being placed transversely to the direction of the tunnel and as far apart as possible and yet allow plumb-lines, hung from the intersection of each beam with the tunnel center line, to swing freely at the bottom of the shaft. These intersections of the beams with the center line were determined by averaging the results of a large number of careful observations for alignment. Two fine parallel wires, spaced about $\frac{1}{16}''$ apart, were then stretched between the beams so that the center line of the tunnel bisected at all points the space between the wires. Plumb-bobs, weighing 15 pounds, were suspended by fine wires beside each cross-beam, the wires passing between the two parallel alignment wires and bisecting the space. The plumb-bobs were allowed to swing in pails of water at the bottom. Drafts of air up the shaft required the construction of boxes surrounding the wires. Even these precautions did not suffice to absolutely prevent vibration of the wire at the bottom through a very small arc. The mean point of these vibrations in each case was then located on a rigid cross-beam suitably placed at the bottom of the shaft and at about the level of the roof of the tunnel. Short plumb-lines were then suspended from these points whenever desired; a transit was set (by trial) so that its line of collimation passed through both plumb lines and the line at the bottom could thus be prolonged.

162. Underground surveys. Survey marks are frequently placed on the timbering, but they are apt to prove unreliable on account of the shifting of the timbering due to settlement of the surrounding material. They should never be placed at the bottom of the tunnel on account of the danger of being disturbed or covered up. Frequently holes are drilled in the roof and filled with wooden plugs in which a hook is screwed exactly on line. Although this is probably the safest method, even these plugs are not always undisturbed, as the material, unless very hard, will often settle slightly as the excavation proceeds. When a tunnel is perfectly straight and not too long, alignment-points may be given as frequently as desired from permanent stations located *outside* the tunnel where they are not liable to disturbance. This has been accomplished by running the alignment through the upper part of the cross-section, at one side of the center, where it is out of the way of the piles of masonry material, débris, etc., which are so apt to choke up the lower part of the cross-section. The position of this line relative to the cross-section being fixed, the alignment of any required point of the cross-section is readily found by means of a light frame or template with a fixed target located where this line would intersect the frame when properly placed. A level-bubble on the frame will assist in setting the frame in its proper position.

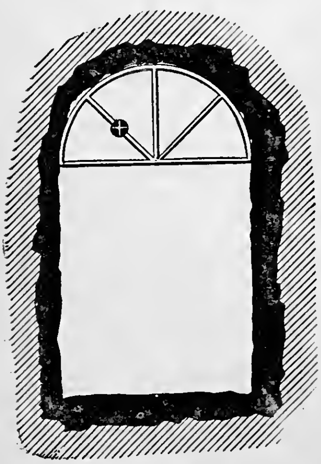


Fig. 86.

In all tunnel surveying the cross-wires must be illuminated by a lantern, and the object sighted at must also be illuminated. A powerful dark-lantern with the opening covered with *ground glass* has been found useful. This may be used to illuminate a plumb-bob string or a very fine rod, or to place behind a brass plate having a narrow slit in it, the axis of the slit and plate being coincident with the plumb-bob string by which it is hung.

On account of the interference to the surveying caused by the work of construction and also by the smoke and dust in the air resulting from the blasting, it is generally necessary to make the surveys at times when construction is temporarily suspended.

163. Accuracy of tunnel surveying. Apart from the very natural desire to do surveying which shall check well, there is an important financial side to accurate tunnel surveying. If the survey lines do not meet as desired when the headings come together, it may be found necessary, if the error is of appreciable size, to introduce a slight curve, perhaps even a reversed curve, into the alignment, and it is even conceivable that the tunnel section would need to be enlarged somewhat to allow for these curves. The cost of these changes and the perpetual annoyance due to an enforced and undesirable alteration of the original design will justify a considerable increase in the expenses of the survey. Considering that the cost of surveys is usually but a small fraction of the total cost of the work, an increase of 10 or even 20% in the cost of the surveys will mean an insignificant addition to the total cost and frequently, if not generally, it will result in a saving of many times the increased cost. The accuracy actually attained in two noted American tunnels is given as follows: The Musconetcong tunnel is about 5000 feet long, bored through a mountain 400 feet high. The error of alignment at the meeting of the headings was 0'.04, error of levels 0'.015, error of distance 0'.52. The Hoosac tunnel is over 25,000 feet long. The heading from the east end met the heading from the central shaft at a point 11274 feet from the east end and 1563 feet from the shaft. The error in alignment was $\frac{5}{16}$ of an inch, that of levels "a few hundredths," error of distance "trifling." The alignment, corrected at the shaft, was carried on through and met the heading from the west end at a point 10138 feet from the west end and 2056 feet from the shaft. Here the error of alignment was $\frac{9}{16}''$ and that of levels 0.134 ft.

DESIGN.

164. Cross-sections. Nearly all tunnels have cross-sections peculiar to themselves—all varying at least in the details. The *general* form of a great many tunnels is that of a rectangle surmounted by a semi-circle or semi-ellipse. In very soft material an inverted arch is necessary along the bottom. In such cases the sides will generally be arched instead of vertical. The sides are frequently battered. With very long tunnels, several forms of cross-section will often be used in the same tunnel, owing to differences in the material encountered. In solid rock, which will not disintegrate upon exposure, no lining is required, and the cross-section will be the irregular section left by the blasting, the only requirement being that no rock shall be left within the required cross-sectional figure. Farther on, in the same tunnel, when passing through some very soft treacherous material, it may be necessary to put in a full arch lining—top, sides, and bottom—which will be nearly circular in cross-section. For an illustration of this see Figs. 87 and 88.

The width of tunnels varies as greatly as the designs. Single-track tunnels generally have a width of 15 to 16 feet. Occasionally they have been built 14 feet wide, and even less, and also up to 18 feet, especially when on curves. 24 to 26 feet is the most common width for double track. Many double-track tunnels are only 22 feet wide, and some are 28 feet wide. The heights are generally 19 feet for single track and 20 to 22 feet for double track. The variations from these figures are considerable. The lower limits depend on the cross-section of the rolling stock, with an indefinite allowance for clearance and ventilation. Cross-sections which coincide too closely with what is

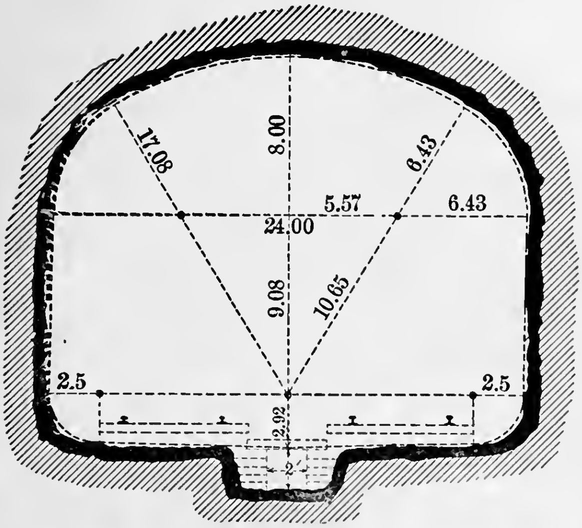


FIG. 87.—HOOSAC TUNNEL. SECTION THROUGH SOLID ROCK.

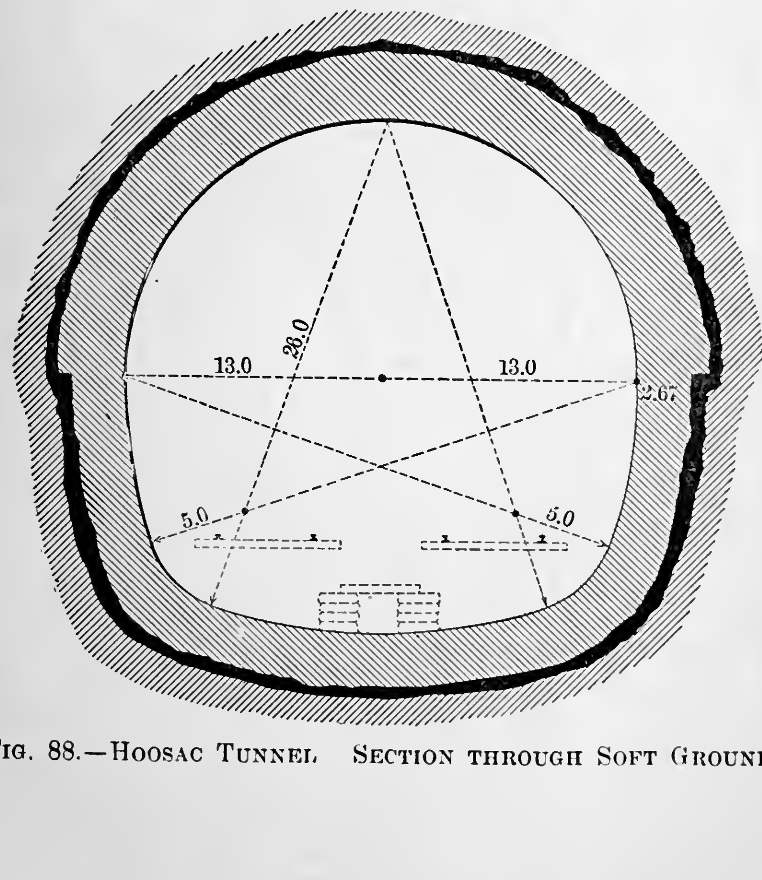


FIG. 88.—HOOSAC TUNNEL SECTION THROUGH SOFT GROUND.

absolutely required for clearance are objectionable, because any slight settlement of the lining which would otherwise be harmless would then become troublesome and even dangerous. Figs. 87, 88, and 89 * show some typical cross-sections.

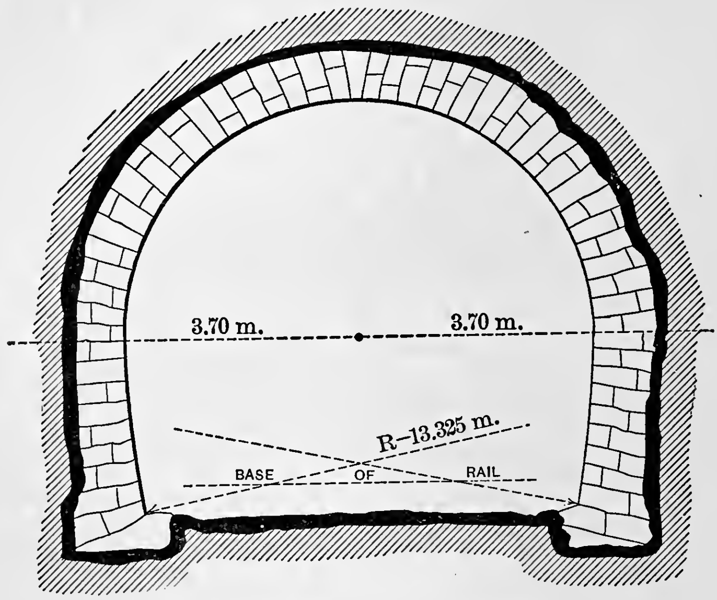
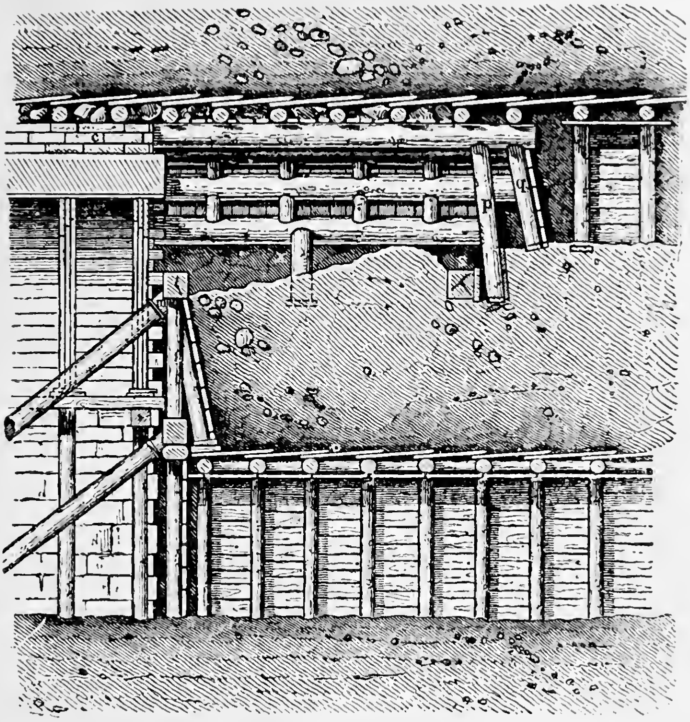


Fig. 89.—St. Cloud Tunnel.

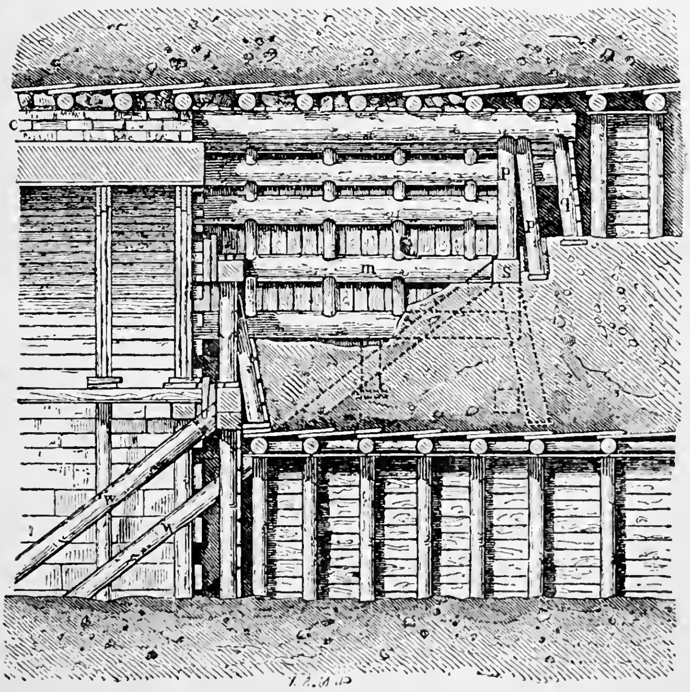
165. Grade. A grade of at least 0.2% is needed for drainage. If the tunnel is at the summit of two grades, the tunnel grade should be practically level, with an allowance for drainage, the actual summit being perhaps in the center so as to drain both ways. When the tunnel forms part of a long ascending grade, it is advisable to reduce the grade through the tunnel unless the tunnel is very short. The additional atmospheric resistance and the decreased adhesion of the driver wheels on the damp rails in a tunnel will cause an engine to work very hard and still more rapidly vitiate the atmosphere until the accumulation of poisonous gases becomes a source of actual danger to the engineer and fireman of the locomotive and of extreme discomfort to the passengers. If the nominal ruling grade of the road were maintained through a tunnel, the maximum resistance would be

* Drinker's "Tunneling."

PLATE II.



TUNNEL-TIMBERING—ENGLISH SYSTEM (*a*).



TUNNEL-TIMBERING—ENGLISH SYSTEM (*b*).

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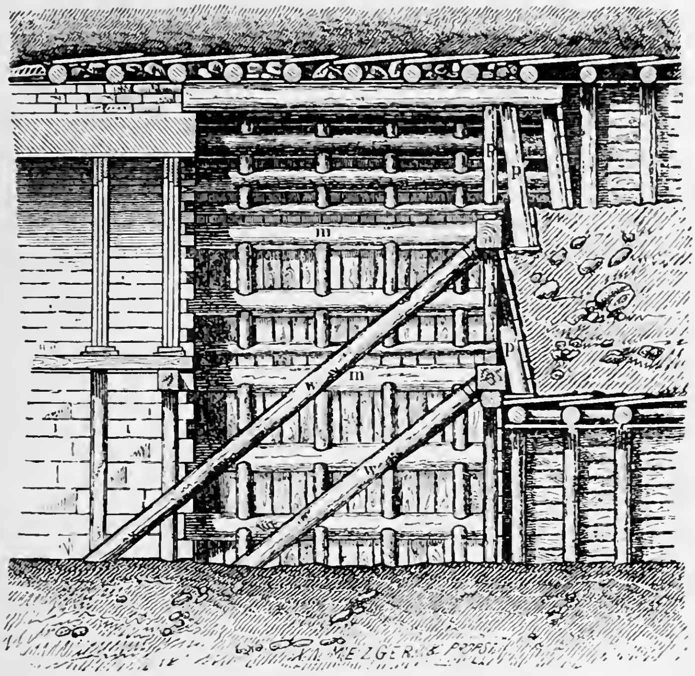
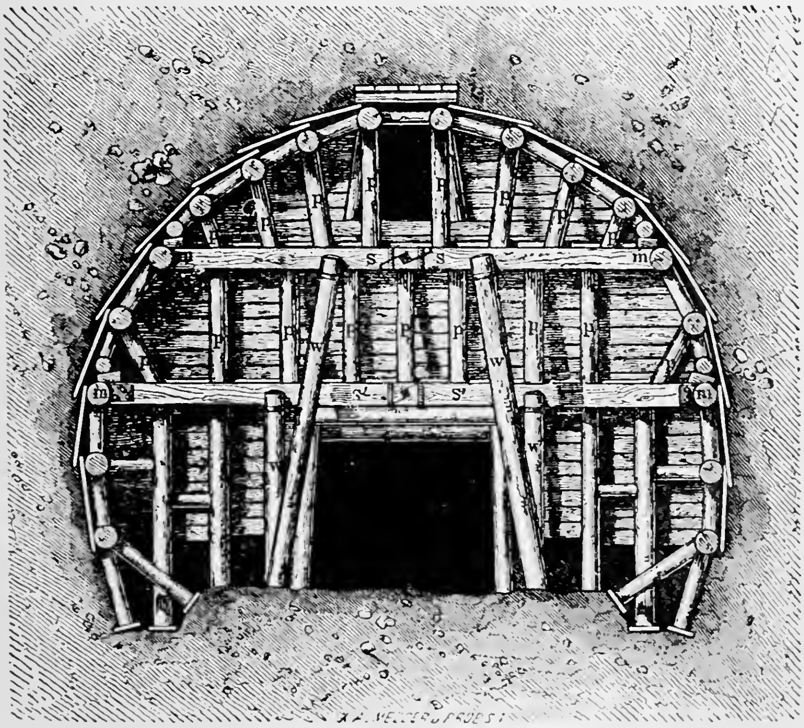
PLATE III.



TUNNEL-TIMBERING—ENGLISH SYSTEM (*c*).



TUNNEL-TIMBERING—ENGLISH SYSTEM (*d*).

(*To face page* 192.)

found in the tunnel. This would probably cause trains to stall there, which would be objectionable and perhaps dangerous.

166. Lining. It is a characteristic of many kinds of rock and of all earthy material that, although they may be self-sustaining when first exposed to the atmosphere, they rapidly disintegrate and require that the top and perhaps the sides and even the bottom shall be lined to prevent caving in. In this country, when timber is cheap, it is occasionally framed as an arch and used as the *permanent* lining, but masonry is always to be preferred. Frequently the cross-section is made extra large so that a masonry lining may subsequently be placed inside the wooden lining and thus postpone a large expense until the road is better able to pay for the work. In very soft unstable material, like quicksand, an arch of cut stone voussoirs may be necessary to withstand the pressure. A good quality of brick is occasionally used for lining, as they are easily handled and make good masonry if the pressure is not excessive. Only the best of cement mortar should be used, economy in this feature being the worst of folly. Of course the excavation must include the outside line of the lining. Any excavation which is made outside of this line (by the fall of earth or loose rock or by excessive blasting) must be refilled with stone well packed in. Occasionally it is necessary to fill these spaces with concrete. Of course it is not necessary that the lining be uniform throughout the tunnel.

167. Shafts. Shafts are variously made with square, rectangular, elliptical, and circular cross-sections. The rectangular cross-section, with the longer axis parallel with the tunnel, is most usually employed. Generally the shaft is directly over the center of the tunnel, but that always implies a complicated connection between the linings of the tunnel and shaft, provided such linings are necessary. It is easier to sink a shaft near to one side of the tunnel and make an opening through the nearly vertical side of the tunnel. Such a method was employed in the Church Hill Tunnel, illustrated in Fig. 90.* Fig. 91 † shows

* Drinker's "Tunneling."

† Ržiha, "Lehrbuch der Gesammten Tunnelbaukunst."

a cross-section for a large main shaft. Many shafts have been built with the idea of being left open permanently for ventilation and have therefore been elaborately lined with masonry.

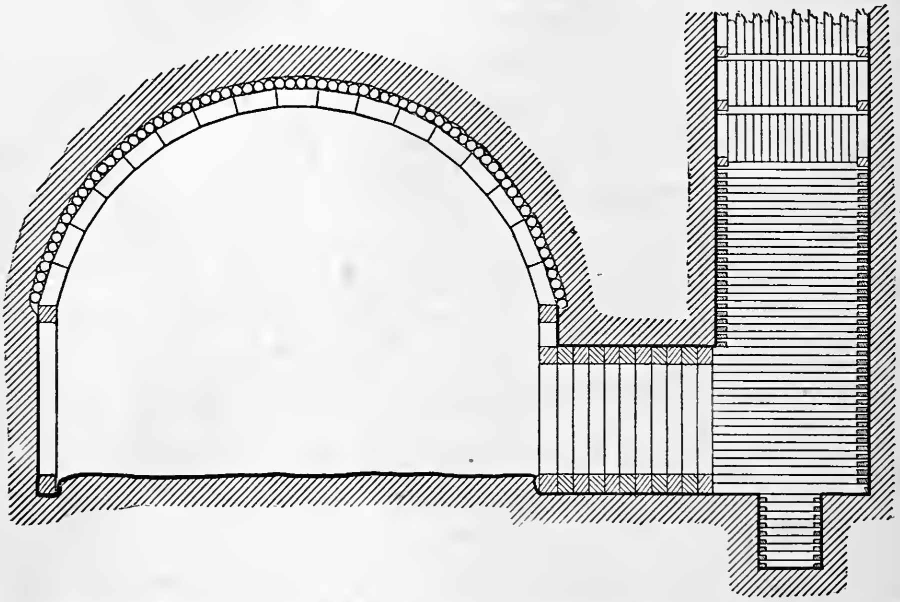


Fig. 90.—Connection with Shaft, Church Hill Tunnel.

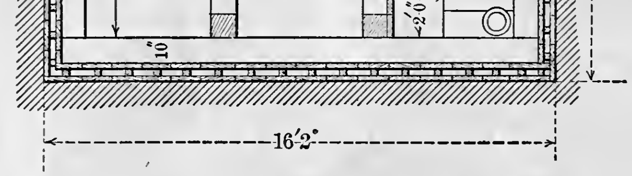
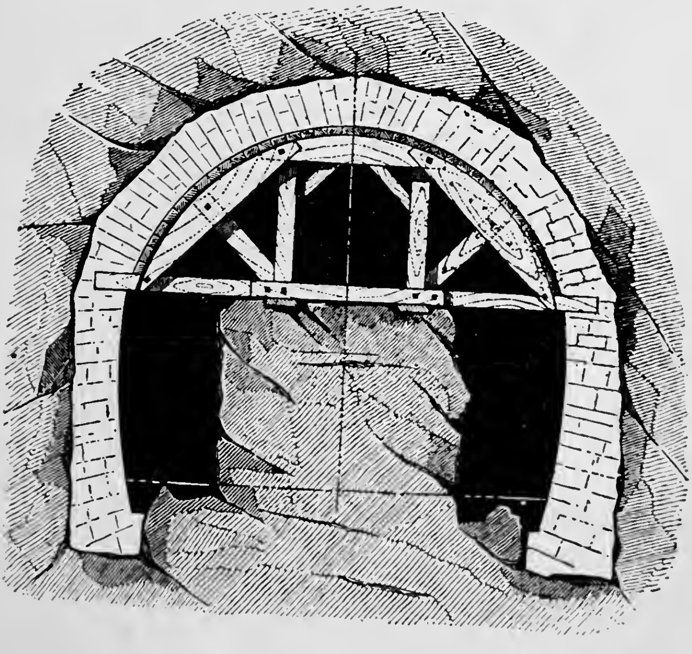


Fig. 91.—Cross-section, Large Main Shaft.

The general consensus of opinion now appears to be that shafts are worse than useless for ventilation; that the quick passage of a train through the tunnel is the most effective ventilator; and that shafts only tend to produce cross-currents and are ineffective to clear the air. In consequence, many of these elaborately lined shafts have been permanently closed, and the more recent

PLATE IV.

TUNNEL-TIMBERING—FRENCH SYSTEM (*a*).



TUNNEL-TIMBERING—FRENCH SYSTEM (*b*).

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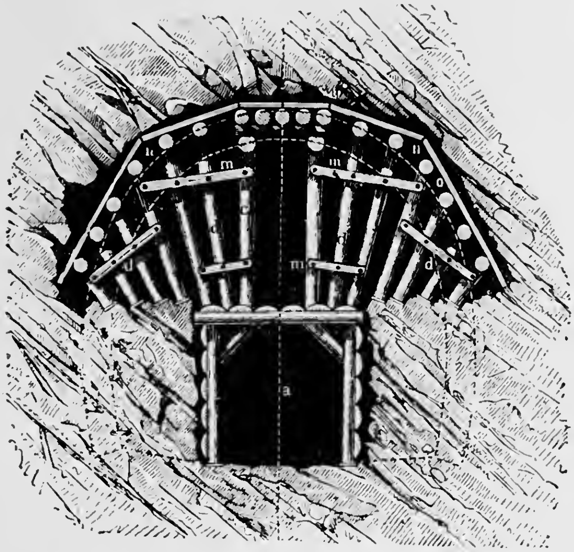
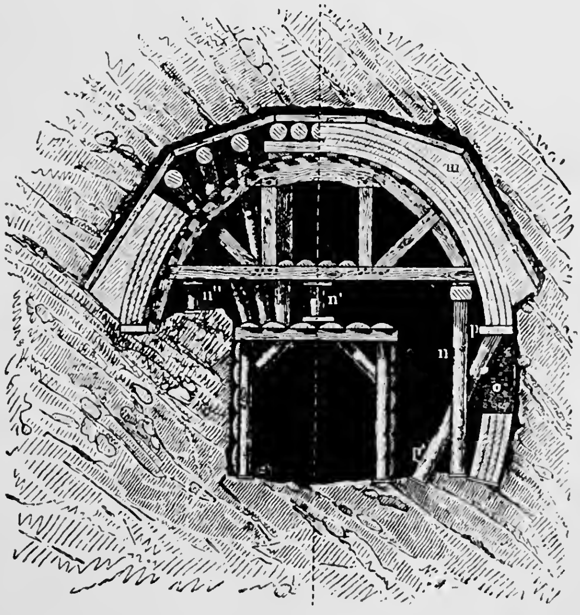
PLATE V.



TUNNEL TIMBERING—BELGIAN SYSTEM (*a*).



TUNNEL-TIMBERING—BELGIAN SYSTEM (*b*).

(*To face page* 194.)

practice is to close up a shaft as soon as the tunnel is completed. Shafts always form drainage-wells for the material they pass through, and sometimes to such an extent that it is a serious matter to dispose of the water that collects at the bottom, requiring the construction of large and expensive drains.

168. Drains. A tunnel will almost invariably strike veins of water which will promptly begin to drain into the tunnel and not only cause considerable trouble and expense during construction, but necessitate the provision of permanent drains for its perpetual disposal. These drains must frequently be so large as to appreciably increase the required cross-section of the tunnel. Generally a small open gutter on each side will suffice for this purpose, but in double-track tunnels a large covered drain is often built between the tracks. It is sometimes necessary to thoroughly grout the outside of the lining so that water will not force its way through the masonry and perhaps injure it, but may freely drain down the sides and pass through openings in the side walls near their base into the gutters.

CONSTRUCTION.

169. Headings. The methods of all tunnel excavation depend on the general principle that all earthy material, except the softest of liquid mud and quicksand, will be self-sustaining over a greater or less area and for a greater or less time after excavation is made, and the work consists in excavating some material and immediately propping up the exposed surface by timbering and poling-boards. The excavation of the cross-section begins with cutting out a "heading," which is a small horizontal drift whose breast is constantly kept 15 feet or more in advance of the full cross-sectional excavation. In solid self-sustaining rock, which will not decompose upon exposure to air, it becomes simply a matter of excavating the rock with the least possible expenditure of time and energy. In soft ground the heading must be heavily timbered, and as the heading is gradually enlarged the timbering must be gradually extended and perhaps replaced, according to some regular system, so that when the full cross-section has been excavated it is supported by such timbering as is intended for it. The heading is sometimes made on the center line near the top; with other plans, on the center line near the bottom; and sometimes two simultaneous headings are run in the two lower corners. Headings near the bottom serve the purpose of draining the material above it and facilitating the excavation. The simplest case of heading timbering is that shown in Fig. 92, in which cross-timbers are placed at intervals just under the roof, set in notches cut in the side walls and supporting poling-boards which sustain whatever pressure may come on them. Cross-timbers near the bottom support a flooring on which vehicles for transporting material may be run and under which the drainage may freely escape. As the necessity for timbering becomes greater, side timbers and even bottom timbers must be added, these timbers supporting poling-boards, and even the breast of the heading must be protected by boards suitably braced, as shown in Fig. 93. The

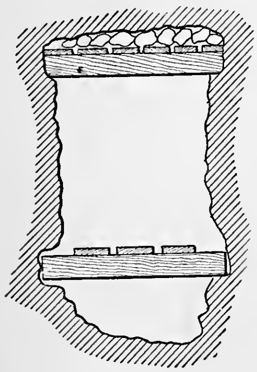


Fig. 92.

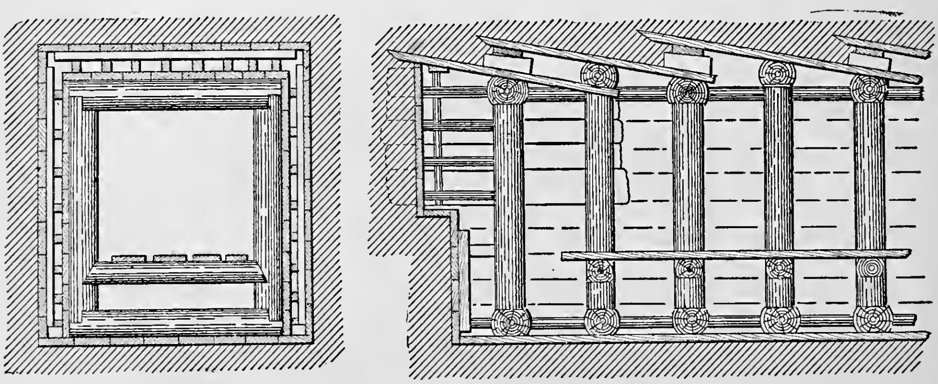
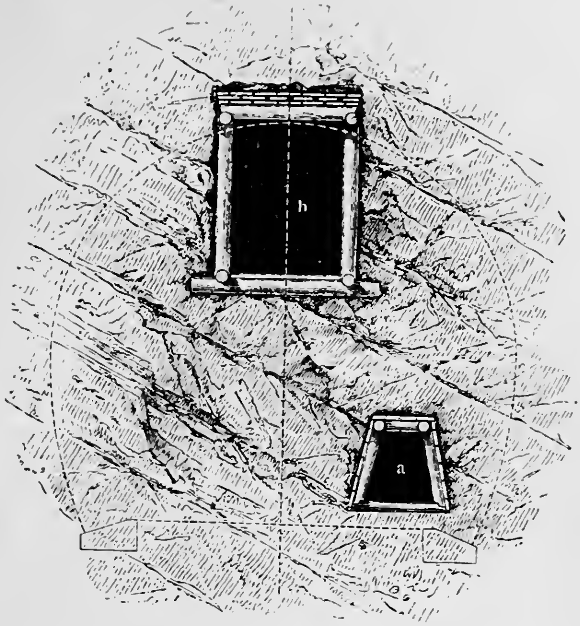


Fig 93.—Timbering for Tunnel Heading.

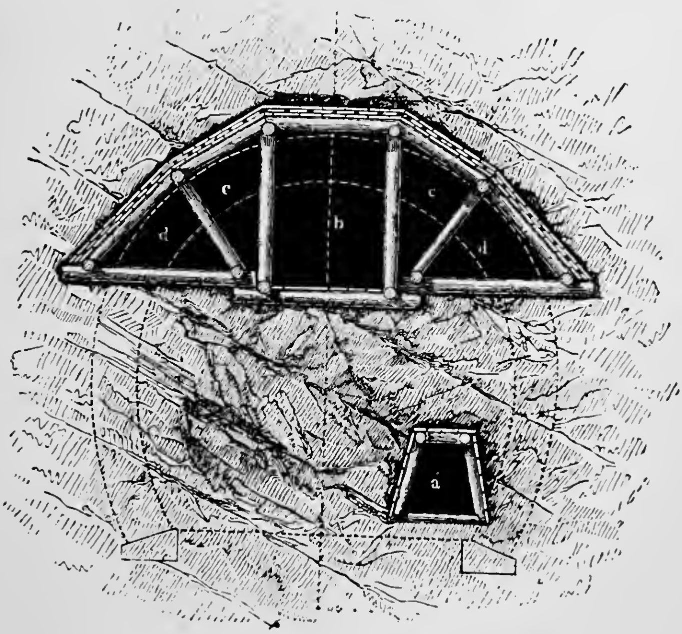
supporting timbers are framed into collars in such a manner that added pressure only increases their rigidity.

170. Enlargement. Enlargement is accomplished by removing the poling-boards, one at a time, excavating a greater or less

PLATE VI.



TUNNEL TIMBERING—GERMAN SYSTEM (*a*).



TUNNEL-TIMBERING—GERMAN SYSTEM (*b*).

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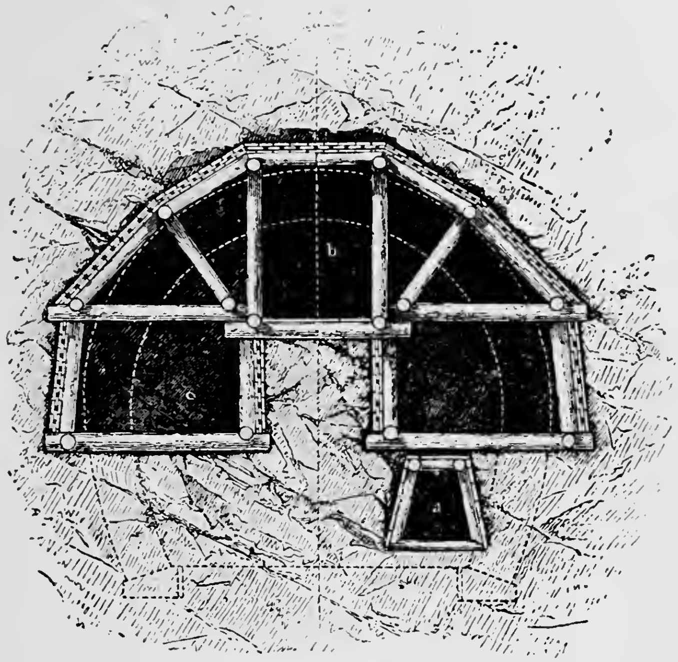
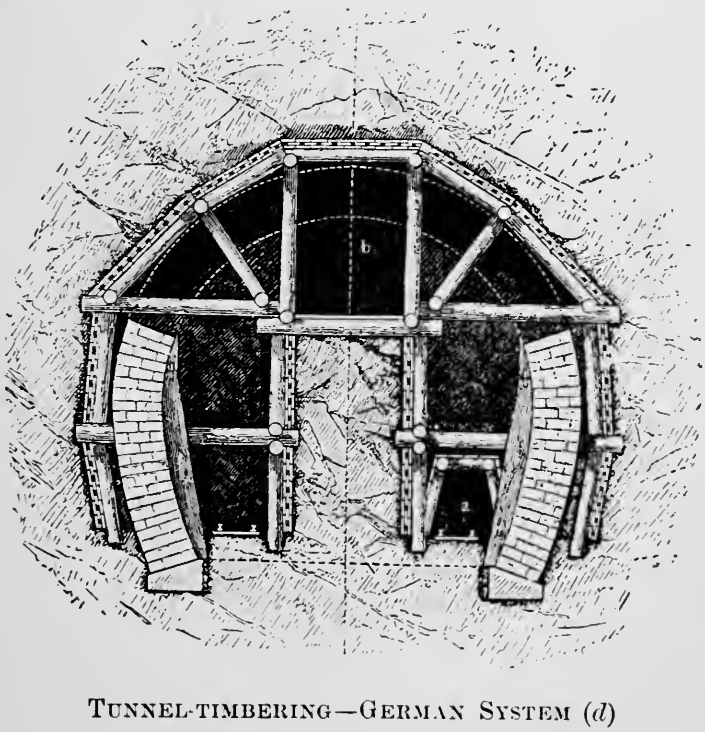
PLATE VII.



TUNNEL-TIMBERING—GERMAN SYSTEM (*c*).



TUNNEL-TIMBERING—GERMAN SYSTEM (*d*)

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amount of material, and immediately supporting the exposed material with poling-boards suitably braced. (See Figs. 93 and 94.) This work being systematically done, space is thereby

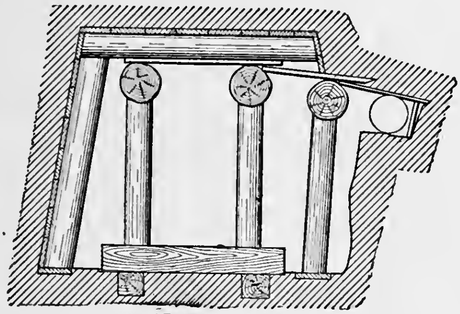


Fig. 94.

obtained in which the framing for the full cross-section may be gradually introduced. The framing is constructed with a cross-section so large that the masonry lining may be constructed within it.

171. Distinctive features of various methods of construction. There are six general systems, known as the English, German, Belgian, French, Austrian, and American. They are so named from the origin of the methods, although their use is not confined to the countries named. Fig. 95 shows by numbers (1 to 5) the order of the excavation within the cross-sections. The English, Austrian, and American systems are alike in excavating the entire cross-section before beginning the construction of the masonry lining. The German method leaves a solid core (5) until practically the whole of the lining is complete. This has the disadvantage of extremely cramped quarters for work, poor ventilation, etc. The Belgian and French methods agree in excavating the upper part of the section, building the arch at once, and supporting it temporarily until the side walls are built. The Belgian method then takes out the core (3), removes very short sections of the sides (4), immediately underpinning the arch with short sections of the side walls and thus gradually constructing the whole side wall. The French method digs out the sides (3), supporting the arch temporarily with timbers and then replacing the timbers with masonry; the core (4) is taken out last. The French method has the same disadvantage as the German—working in a cramped space. The Belgian and French systems have the disadvantage that the arch, supported temporarily on timber, is very apt to be strained and cracked by the slight settlement that so frequently occurs in soft material. The English, Austrian, and American methods differ mainly in the

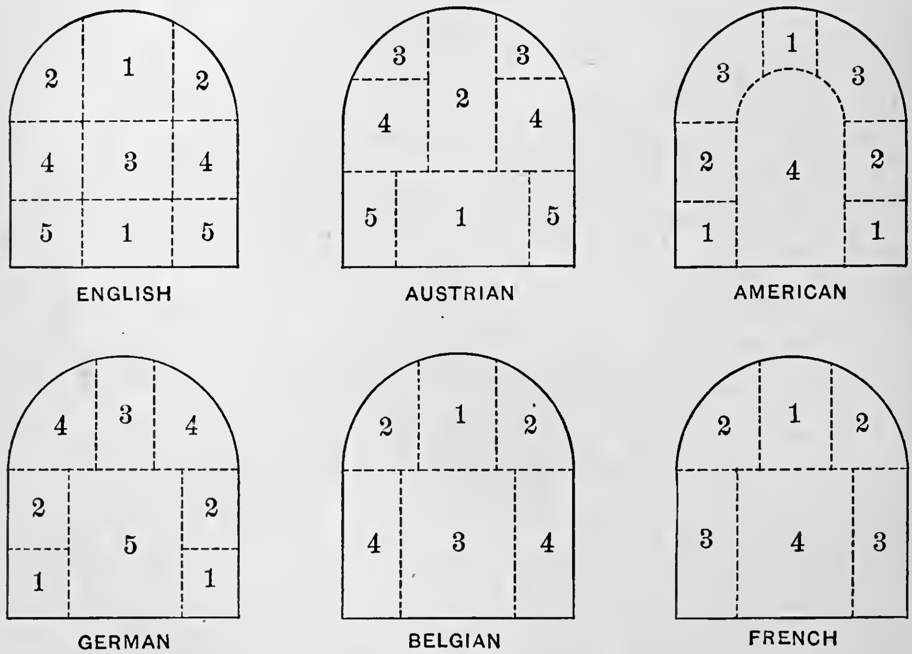
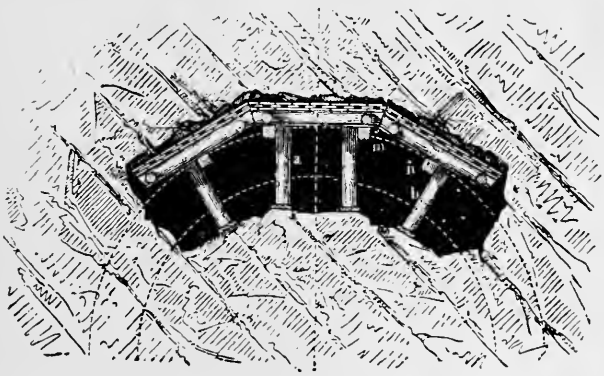


FIG. 95.—ORDER OF WORKING BY THE VARIOUS SYSTEMS.

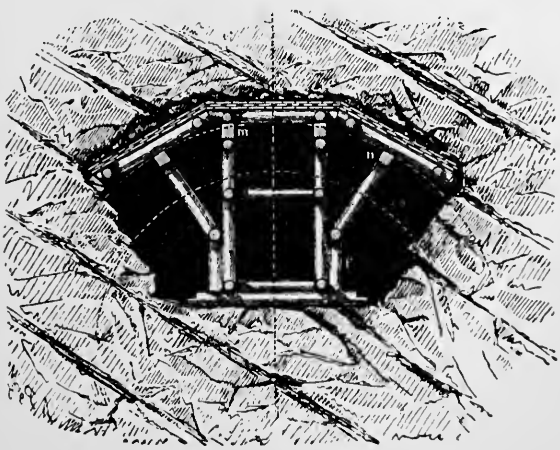
design of the timbering. The English support the roof by lines of very heavy *longitudinal* timbers which are supported at comparatively wide intervals by a heavy framework occupying the whole cross-section. The Austrian system uses such frequent cross-frames of timber-work that poling-boards will suffice to support the material between the frames. The American system agrees with the Austrian in using frequent cross-frames supporting poling-boards, but differs from it in that the "cross-frames" consist simply of arches of 3 to 15 wooden voussoirs, the voussoirs being blocks of 12″ × 12″ timber about 2 to 8 feet long and cut with joints normal to the arch. These arches are put together on a centering which is removed as soon as the arch

PLATE VIII.

TUNNEL-TIMBERING—AUSTRIAN SYSTEM (*a*).



TUNNEL-TIMBERING—AUSTRIAN SYSTEM (*b*).



TUNNEL-TIMBERING—AUSTRIAN SYSTEM (*c*).

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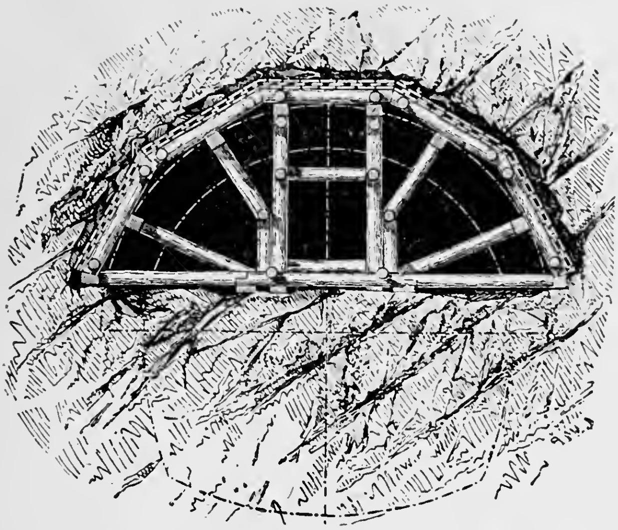
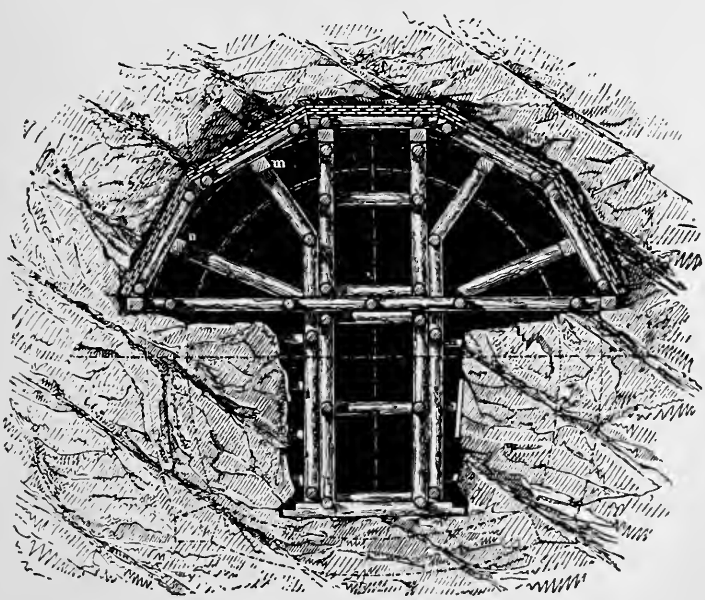
PLATE IX.



TUNNEL-TIMBERING—AUSTRIAN SYSTEM (*d*).



TUNNEL-TIMBERING—AUSTRIAN SYSTEM (*e*).

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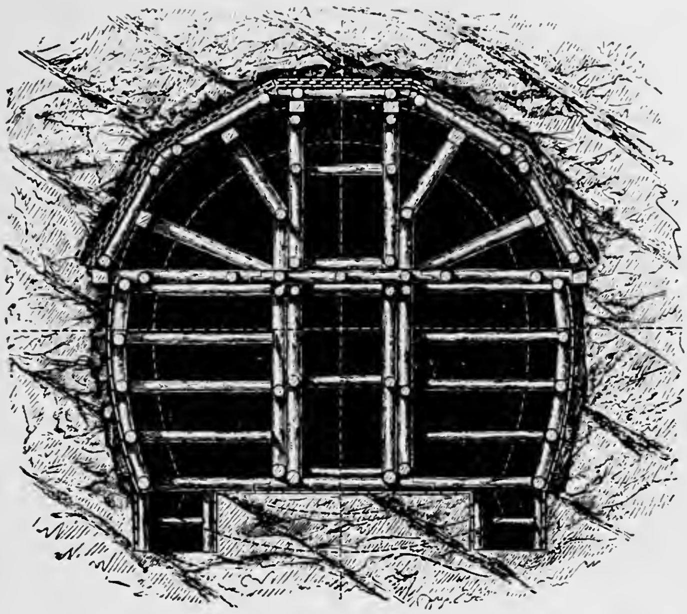
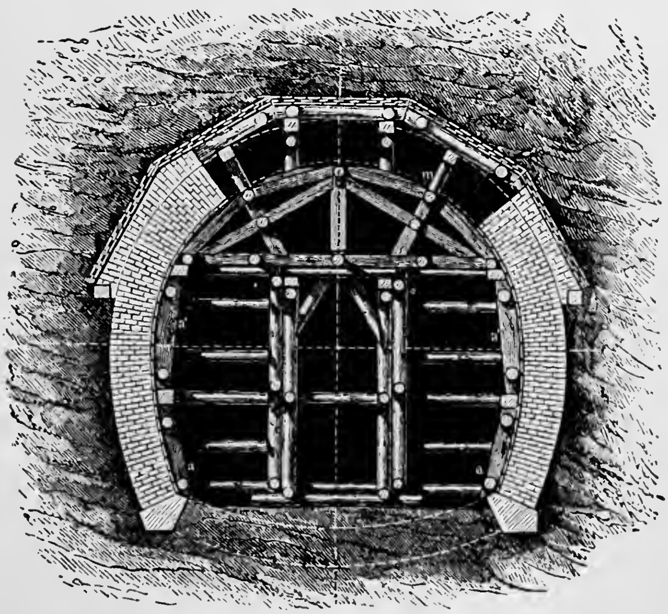


TUNNEL-TIMBERING—AUSTRIAN SYSTEM (*f*).



TUNNEL-TIMBERING—AUSTRIAN SYSTEM (*g*).

(***To*** *face page* 198)

is keyed up and thus immediately opens up the full cross-section, so that the center core (4) may be immediately dug out and the masonry constructed in a large open space. The American system has been used successfully in very soft ground, but its advantages are greater in loose rock, when it is much cheaper than the other methods which employ more timber. Fig. 90 illustrates the use of the American system. The figure shows the wooden arch in place. The masonry arch may be placed when convenient, since it is *possible* to lay the track and commence traffic as soon as the wooden arch is in place. Plates II to XIV illustrate the methods of excavating and timbering by these various systems.

172. Ventilation during construction. Tunnels of any great length must be artificially ventilated during construction. If the excavated material is rock so that blasting is necessary, the need for ventilation becomes still more imperative. The invention of compressed-air drills simultaneously solved two difficulties. It introduced a motive power which is unobjectionable in its application (as gas would be), and it also furnished at the same time a supply of just what is needed—pure air. If no blasting is done (and sometimes even when there is blasting), air must be supplied by direct pumping. The cooling effect of the sudden expansion of compressed air only reduces the otherwise objectionably high temperature sometimes found in tunnels. Since pure air is being continually pumped in, the foul air is thereby forced out.

173. Excavation for the portals. Under normal conditions there is always a greater or less amount of open cut preceding and following a tunnel. Since all tunnel methods depend (to some slight degree at least) on the capacity of the exposed material to act as an arch, there is implied a considerable thickness of material above the tunnel. This thickness is reduced to nearly zero over the tunnel portals and therefore requires special treatment, particularly when the material is very soft. Fig. 96 *

* Ržiha, "Lehrbuch der Gersammten Tunnelbaukunst."

illustrates one method of breaking into the ground at a portal. The loose stones are piled on the framing to give stability to the framing by their weight and also to retain the earth on the slope above. Another method is to sink a temporary shaft to the tunnel near the portal; immediately enlarge to the full size and build the masonry lining; then work back to the portal.

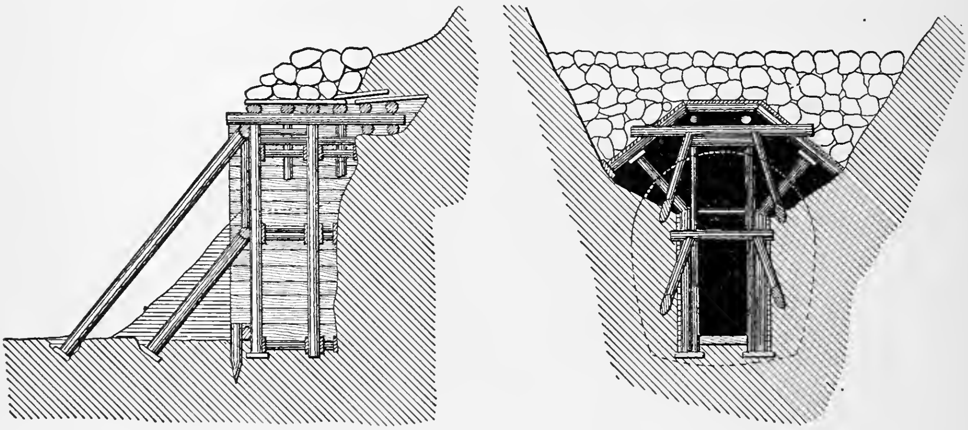


FIG. 96.—TIMBERING FOR TUNNEL PORTAL.

This method is more costly, but is preferable in very treacherous ground, it being less liable to cause landslides of the surface material.

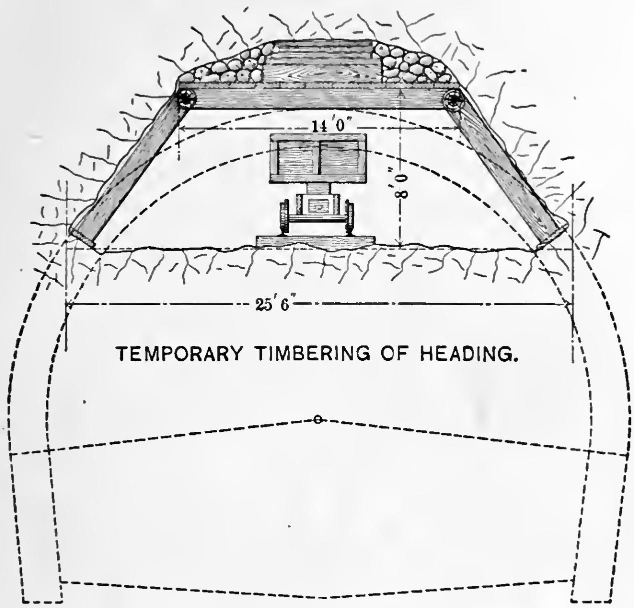
174. Tunnels vs. open cuts. In cases in which an open cut rather than a tunnel is a possibility the ultimate consideration is generally that of first cost combined with other financial considerations and annual maintenance charges directly or indirectly connected with it. Even when an open cut may be constructed at the same cost as a tunnel (or perhaps a little cheaper) the tunnel may be preferable under the following conditions:

1. When the soil indicates that the open cut would be liable to landslides.

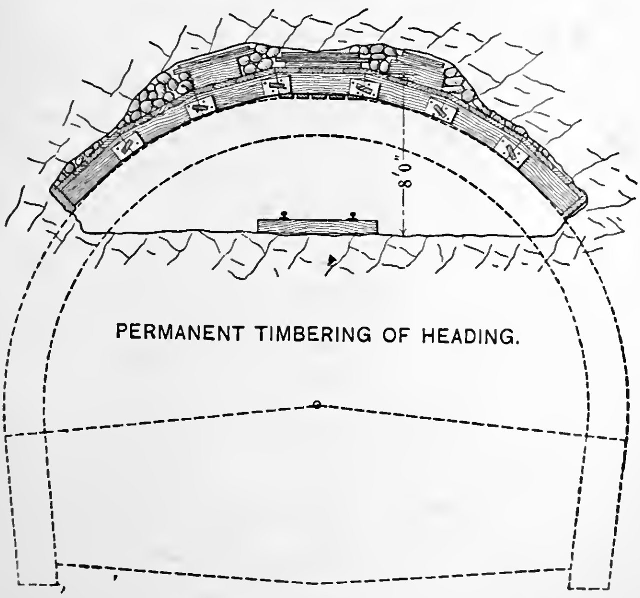
2. When the open cut would be subject to excessive snow-drifts or avalanches.

3. When land is especially costly or it is desired to run under existing costly or valuable buildings or monuments. When running through cities, tunnels are sometimes constructed as open cuts and then arched over.

PLATE XI.



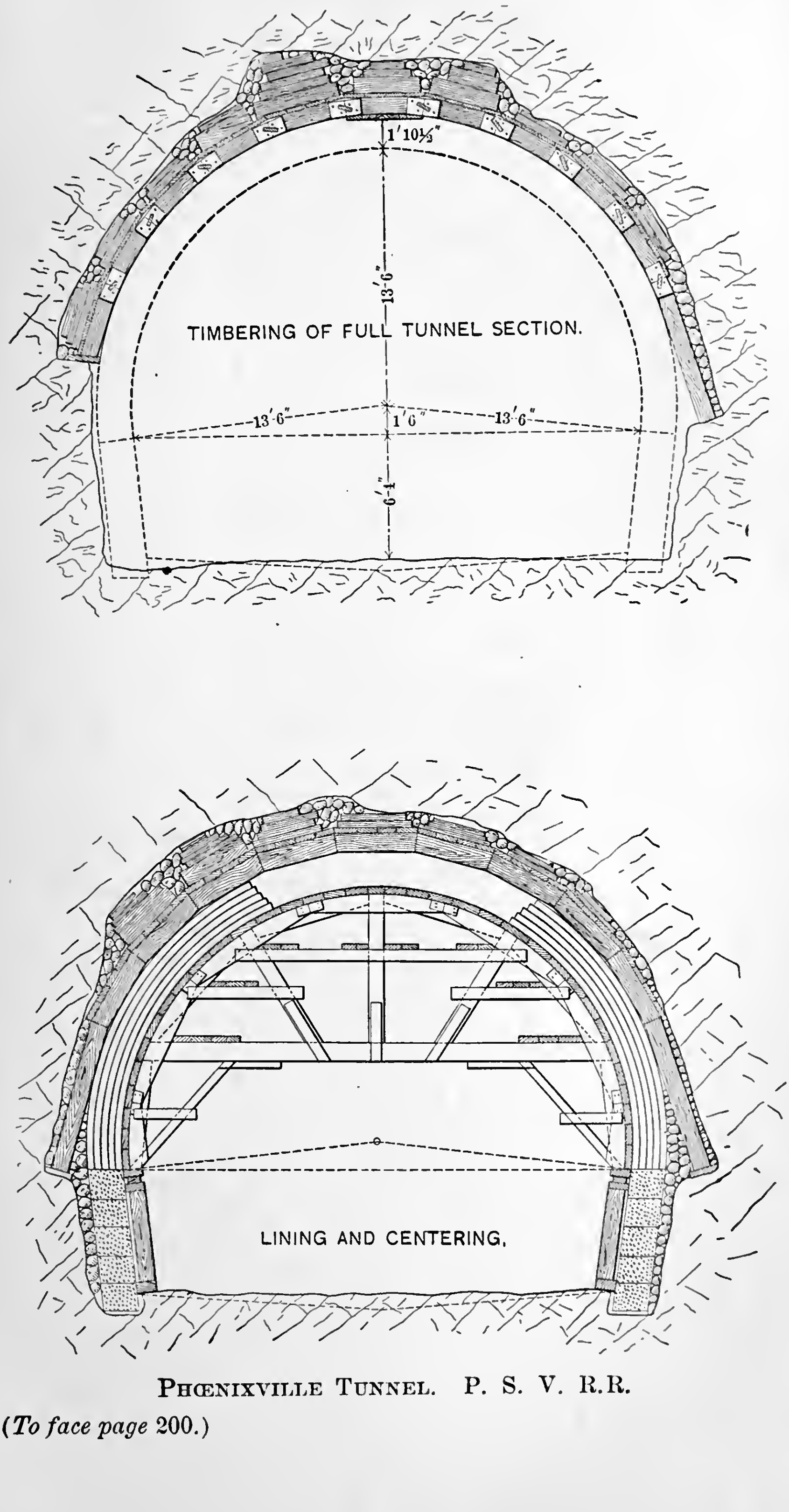
TEMPORARY TIMBERING OF HEADING.



PERMANENT TIMBERING OF HEADING.

PHŒNIXVILLE TUNNEL. P. S. V. R.R.

(*To face page* 200.)



PHŒNIXVILLE TUNNEL. P. S. V. R.R.

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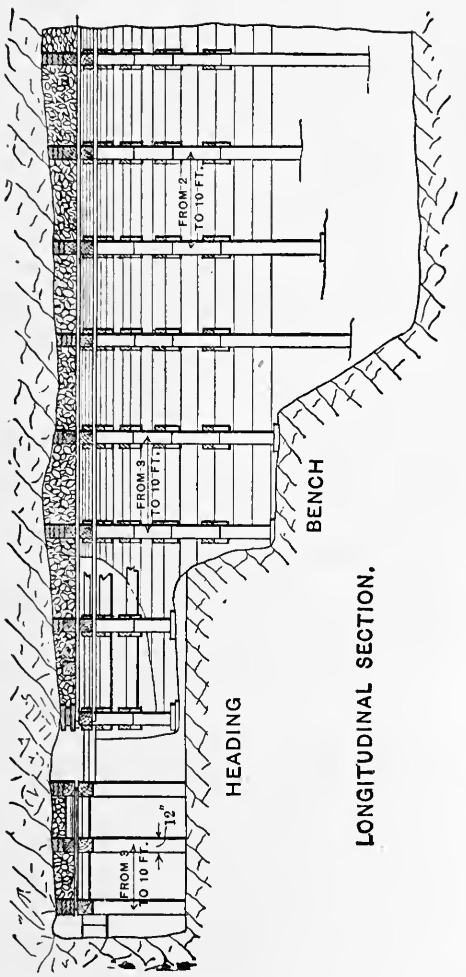
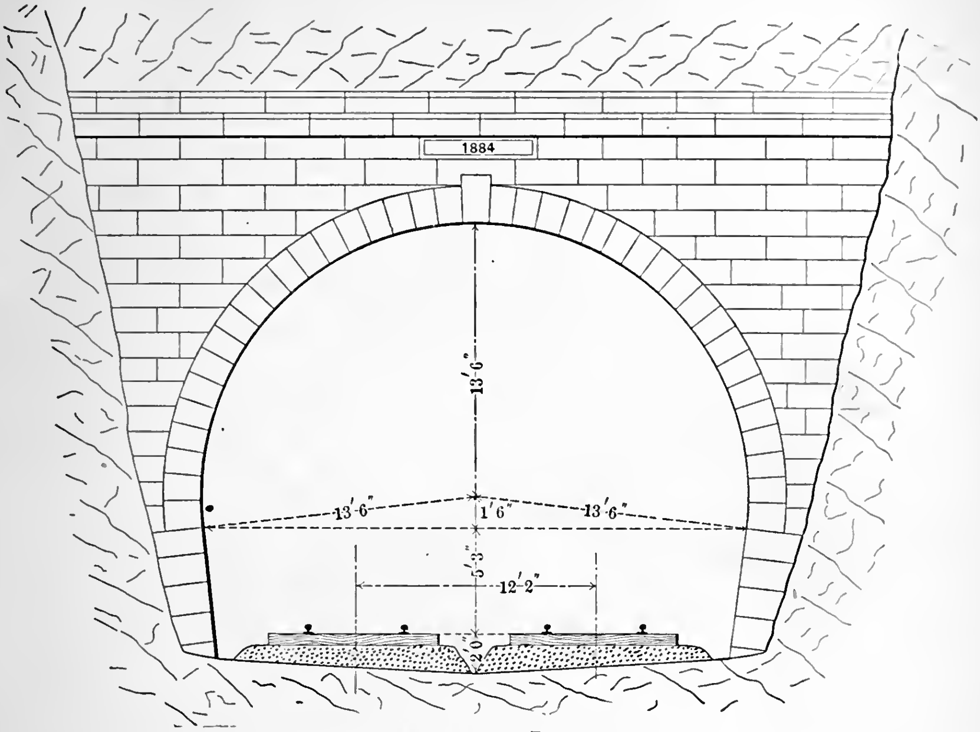


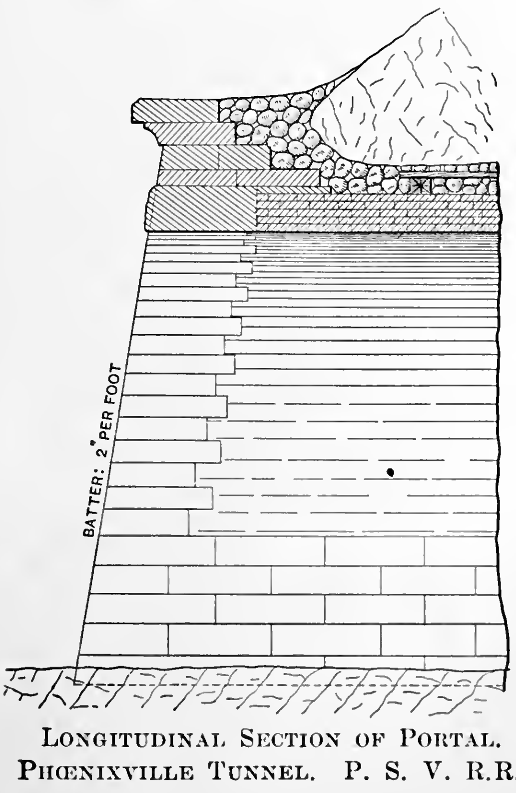
PHŒNIXVILLE TUNNEL. P. S. V. R.R.

(*To face page* 200.)

PLATE XIV.



ELEVATION OF PORTAL.



LONGITUDINAL SECTION OF PORTAL.
PHŒNIXVILLE TUNNEL. P. S. V. R.R.

(*To face page* 200.)

These cases apply to tunnels *vs.* open cuts when the alignment is fixed by other considerations than the mere topography. The broader question of excavating tunnels to avoid excessive grades or to save distance or curvature, and similar problems, are hardly susceptible of general analysis except as questions of railway economics and must be treated individually.

175. Cost of tunneling. The cost of any construction which involves such uncertainties as tunneling is very variable. It depends on the material encountered, the amount and kind of timbering required, on the size of the cross-section, on the price of labor, and especially on the *reconstruction* that *may* be necessary on account of mishaps.

Headings generally cost $4 to $5 per cubic yard for excavation, while the remainder of the cross-section in the same tunnel may cost about half as much. The average cost of a large number of tunnels in this country may be seen from the following table: *

Material.	Cost per cubic yard.				Cost per lineal foot.	
	Excavation.		Masonry.			
	Single.	Double.	Single.	Double.	Single.	Double.
Hard rock.....	$5.89	$5.45	$12.00	$ 8.25	$ 69.76	$142.82
Loose rock....	3.12	3.48	9.07	10.41	80.61	119.26
Soft ground...	3.62	4.64	15.00	10.50	135.31	174.42

A considerable variation from these figures may be found in individual cases, due sometimes to unusual skill (or the lack of it) in prosecuting the work, but the figures will generally be sufficiently accurate for preliminary estimates or for the comparison of two proposed routes.

* Figures derived from Drinker's "Tunneling."

CHAPTER VI.

CULVERTS AND MINOR BRIDGES.

176. Definition and object. Although a variable percentage of the rain falling on any section of country soaks into the ground and does not immediately reappear, yet a very large percentage flows over the surface, always seeking and following the lowest channels. The roadbed of a railroad is constantly intersecting these channels, which frequently are normally dry. In order to prevent injury to railroad embankments by the impounding of such rainfall, it is necessary to construct waterways through the embankment through which such rainflow may freely pass. Such waterways, called culverts, are also applicable for the bridging of very small although perennial streams, and therefore in this work the term culvert will be applied to all water-channels passing through a railroad embankment which are not of sufficient magnitude to require a special structural design, such as is necessary for a large masonry arch or a truss bridge.

177. Elements of the design. A well-designed culvert must afford such free passage to the water that it will not "back up" over the adjoining land nor cause any injury to the embankment or culvert. The ability of the culvert to discharge freely all the water that comes to it evidently depends chiefly on the area of the waterway, but also on the form, length, slope, and materials of construction of the culvert and the nature of the approach and outfall. When the embankment is very low and the amount of water to be discharged very great, it sometimes becomes necessary to allow the water to discharge "under a head," i.e., with the surface of the water above the top of the culvert. Safety then requires a much stronger construction than would otherwise be necessary to avoid injury to the culvert or embankment by washing. The necessity for such construction should be avoided if possible.

AREA OF THE WATERWAY.

178. Elements involved. The determination of the required area of the waterway involves such a multiplicity of indeterminate elements that any close determination of its value from purely theoretical considerations is a practical impossibility. The principal elements involved are:

a. Rainfall. The real test of the culvert is its capacity to discharge without injury the flow resulting from the extraordinary rainfalls and "cloud bursts" that may occur once in many years. Therefore, while a knowledge of the average annual rainfall is of very little value, a record of the maximum rainfall during heavy storms for a long term of years may give a relative idea of the maximum demand on the culvert.

b. Area of watershed. This signifies the total area of country draining into the channel considered. When the drainage area is very small it is sometimes included within the area surveyed by the preliminary survey. When larger it is frequently possible to obtain its area from other maps with a percentage of accuracy sufficient for the purpose. Sometimes a special survey for the purpose is considered justifiable.

c. Character of soil and vegetation. This has a large influence on the rapidity with which the rainflow from a given area will reach the culvert. If the soil is hard and impermeable and the vegetation scant, a heavy rain will run off suddenly, taxing the capacity of the culvert for a short time, while a spongy soil and dense vegetation will retard the flow, making it more nearly uniform and the maximum flow at any one time much less.

d. Shape and slope of watershed. If the watershed is very long and narrow (other things being equal), the water from the remoter parts will require so much longer time to reach the culvert that the flow will be comparatively uniform, especially when the slope of the whole watershed is very low. When the slope of the remoter portions is quite steep it may result in the nearly simultaneous arrival of a storm-flow from all parts of the watershed, thus taxing the capacity of the culvert.

e. **Effect of design of culvert.** The principles of hydraulics show that the slope of the culvert, its length, the form of the cross-section, the nature of the surface, and the form of the approach and discharge all have a considerable influence on the area of cross-section required to discharge a given volume of water in a given time, but unfortunately the combined hydraulic effect of these various details is still a very uncertain quantity.

179. Methods of computation of area. There are three possible methods of computation.

(a) **Theoretical.** As shown above it is a practical impossibility to estimate correctly the combined effect of the great multiplicity of elements which influence the final result. The nearest approach to it is to estimate by the use of empirical formulæ the amount of water which will be presented at the upper end of the culvert in a given time and then to compute, from the principles of hydraulics, the rate of flow through a culvert of given construction, but (as shown in § 178, *e*) such methods are still very unreliable, owing to lack of experimental knowledge. This method has *apparently* greater scientific accuracy than other methods, but a little study will show that the elements of uncertainty are as great and the final result no more reliable. The method is most reliable for streams of uniform flow, but it is under these conditions that method (*c*) is most useful. The theoretical method will not therefore be considered further.

(b) **Empirical.** As illustrated in § 180, some formulæ make the area of waterway a function of the drainage area, the formula being affected by a coefficient the value of which is estimated between limits according to the judgment. Assuming that the formulæ are sound, their use only narrows the limits of error, the final determination depending on experience and judgment.

(c) **From observation.** This method, considered by far the best for permanent work, consists in observing the high-water marks on contracted channel-openings which are on the same stream and as near as possible to the proposed culvert. If the country is new and there are no such openings, the wisest plan is to bridge the opening by a temporary structure in wood which has an ample waterway (see § 126, *b*, 4) and carefully observe all high-water marks on that opening during the 6 to 10 years which is ordinarily the minimum life of such a structure. As shown later, such observations may be utilized for a close computation of the required waterway. Method (*b*) may be utilized for an approximate calculation for the required area for the temporary structure, using a value which is intentionally excessive, so that a permanent structure of sufficient capacity may subsequently be constructed *within* the temporary structure.

180. Empirical formulæ. Two of the best known empirical formulæ for area of the waterway are the following:

(a) **Myer's formula:**

$$\text{Area of waterway in square feet} = C \times \sqrt{\text{drainage area in acres}},$$

where C is a coefficient varying from 1 for flat country to 4 for mountainous country and rocky ground. As an illustration, if the drainage area is 100 acres, the waterway area should be from 10 to 40 square feet, according to the value of the coefficient chosen. It should be noted that this formula does not regard the great variations in rainfall in various parts of the world nor the design of the culvert, and also that the final result depends largely on the choice of the coefficient.

(b) **Talbot's formula:**

$$\text{Area of waterway in square feet} = C \times \sqrt[4]{(\text{drainage area in acres})^3}.$$

"For steep and rocky ground C varies from $\frac{2}{3}$ to 1. For rolling agricultural country subject to floods at times of melting snow, and with the length of the valley three or four times its width, C is about $\frac{1}{3}$; and if the stream is longer in proportion to the area, decrease C. In districts not affected by accumulated snow, and where the length of the valley is several times the width, $\frac{1}{5}$ or $\frac{1}{6}$, or even less, may be used. C should be increased for steep side slopes, especially if the upper part of the valley has a much greater fall than the channel at the culvert." * As an illustration, if the drainage area is 100 acres the area of waterway should be $C \times 31.6$. The area should then vary from 5 to 31 square feet, according to the character of the country. Like the previous estimate, the result depends on the choice of a coefficient and disregards local variations in rainfall, except as they may be arbitrarily allowed for in choosing the coefficient.

181. Value of empirical formulæ. The fact that these formulæ, as well as many others of similar nature that have been suggested, depend so largely upon the choice of the coefficient shows that they are valuable "more as a guide to the judgment than as a working rule," as Prof. Talbot explicitly declares in commenting on his own formula. In short, they are chiefly valuable in indicating a probable maximum and minimum between which the true result probably lies.

182. Results based on Observation. As already indicated in § 179, observation of the stream in question gives the most reliable results. If the country is new and no records of the flow of the stream during heavy storms has been taken, even the life of a temporary wooden structure may not be long enough to include one of the unusually severe storms which must be allowed for, but there will usually be some high-water mark which will indicate how much opening will be required. The following quotation illustrates this: "A tidal estuary may generally be safely narrowed considerably from the extreme water lines if stone revetments are used to protect the bank from wash. Above the true estuary, where the stream cuts through the marsh, we generally find nearly vertical banks, and we are safe if the faces of abutments are placed even with the banks. In level sections of the country, where the current is sluggish, it is usually safe to encroach somewhat on the

* Prof. A. N. Talbot, "Selected Papers of the Civil Engineers' Club of the Univ. of Illinois."

general width of the stream, but in rapid streams among the hills the width that the stream has cut for itself through the soil should not be lessened, and in ravines carrying mountain torrents the openings must be left very much larger than the ordinary appearance of the banks of the stream would seem to make necessary." *

As an illustration of an observation of a storm-flow through a temporary trestle, the following is quoted: "Having the flood height and velocity, it is an easy matter to determine the volume of water to be taken care of. I have one ten-bent pile trestle 135 feet long and 24 feet high over a spring branch that ordinarily runs about six cubic inches per second. Last summer during one of our heavy rainstorms (four inches in less than three hours) I visited this place and found by float observations the surface velocity at the highest stage to be 1.9 feet per second. I made a high-water mark, and after the flood-water receded found the width of stream to be 12 feet and an average depth of 2¾ feet. This, with a surface velocity of 1.9 feet per second, would give approximately a discharge of 50 cubic feet, or 375 gallons, per second. Having this information it is easy to determine size of opening required." †

183. Degree of accuracy required. The advantages resulting from the use of standard designs for culverts (as well as other structures) have led to the adoption of a comparatively small number of designs. The practical use made of a computation of required waterway area is to determine which one of several standard designs will most nearly fulfill the requirements. For example, if a 24-inch iron pipe, having an area of 3.14 square feet, is considered to be a little small, the next size (30-inch) would be adopted; but a 30-inch pipe has an area of 4.92 square feet, which is 56% larger. A similar result, except that the percentage of difference might not be quite so marked,

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* J. P. Snow, Boston & Maine Railway. From Report to Association of Railway Superintendents of Bridges and Buildings. 1897.

† A. J. Kelley, Kansas City Belt Railway. From Report to Association of Railway Superintendents of Bridges and Buildings. 1897.

will be found by comparing the areas of consecutive standard designs for stone box culverts.

The advisability of designing a culvert to withstand any storm-flow that may *ever* occur is considered doubtful. Several years ago a record-breaking storm in New England carried away a very large number of bridges, etc., hitherto supposed to be safe. It was not afterward considered that the design of those bridges was faulty, because the extra cost of constructing bridges capable of withstanding such a flood, added to interest for a long period of years, would be enormously greater than the cost of repairing the damages of such a storm once or twice in a century. Of course the element of danger has some weight, but not enough to justify a great additional expenditure, for common prudence would prompt unusual precautions during or immediately after such an extraordinary storm.

PIPE CULVERTS.

184. Advantages. Pipe culverts, made of cast iron or earthenware, are very durable, readily constructed, moderately cheap, will pass a larger volume of water in proportion to the area than many other designs on account of the smoothness of the surface, and (when using iron pipe) may be used very close to the track when a low opening of large capacity is required. Another advantage lies in the ease with which they may be inserted through a somewhat larger opening that has been temporarily lined with wood, without disturbing the roadbed or track.

185. Construction. Permanency requires that the foundation shall be firm and secure against being washed out. To accomplish this, the soil of the trench should be hollowed out to fit the lower half of the pipe, making suitable recesses for the bells. In very soft treacherous soil a foundation-block of concrete is sometimes placed under each joint, or even throughout the whole length. When pipes are laid through a slightly larger timber culvert great care should be taken that the pipes are properly supported, so that there will be no settling nor development of unusual strains when the timber finally decays and gives way. To prevent the washing away of material around the pipe the ends should be protected by a bulkhead. This is best constructed of masonry (see Fig. 97), although wood is sometimes used for cheap and minor constructions. The joints should be calked, especially when the culvert is liable to run full or when the outflow is impeded and the culvert is liable to be partly or wholly filled during freezing weather. The cost of a calking of clay or even hydraulic cement is insignificant compared with the value of the additional safety afforded. When the grade of the pipe is perfectly uniform, a very low rate of grade will suffice to drain a pipe culvert, but since some unevenness of grade is inevitable through uneven settlement or imperfect construction, a grade of 1 in 20 should preferably be required, although much less is often used. The length of a pipe culvert is approximately determined as follows:

Length = 2*s* (*depth of embankment to top of pipe*) + (*width of roadbed*),

in which *s* is the slope ratio (horizontal to vertical) of the banks. In practice an even number of lengths will be used which will most nearly agree with this formula.

186. Iron-pipe culverts. Simple cast-iron pipes are used in sizes from 12″ to 48″ diameter. These are usually made in lengths of 12 feet with a few lengths of 6 feet, so that any required length may be more nearly obtained. The lightest pipes made are sufficiently strong for the purpose, and even those which would be rejected because of incapacity to withstand pressure may be utilized for this work. In Fig. 97 are shown the standard plans used on the C. C. C. & St. L. Ry., which may be considered as typical plans.

Pipes formed of cast-iron segments have been used up to 12 feet diameter. The shell is then made comparatively thin, but is stiffened by ribs and flanges on the outside. The segments break joints and are bolted together through the flanges. The joints are made tight by the use of a tarred rope, together with neat cement.

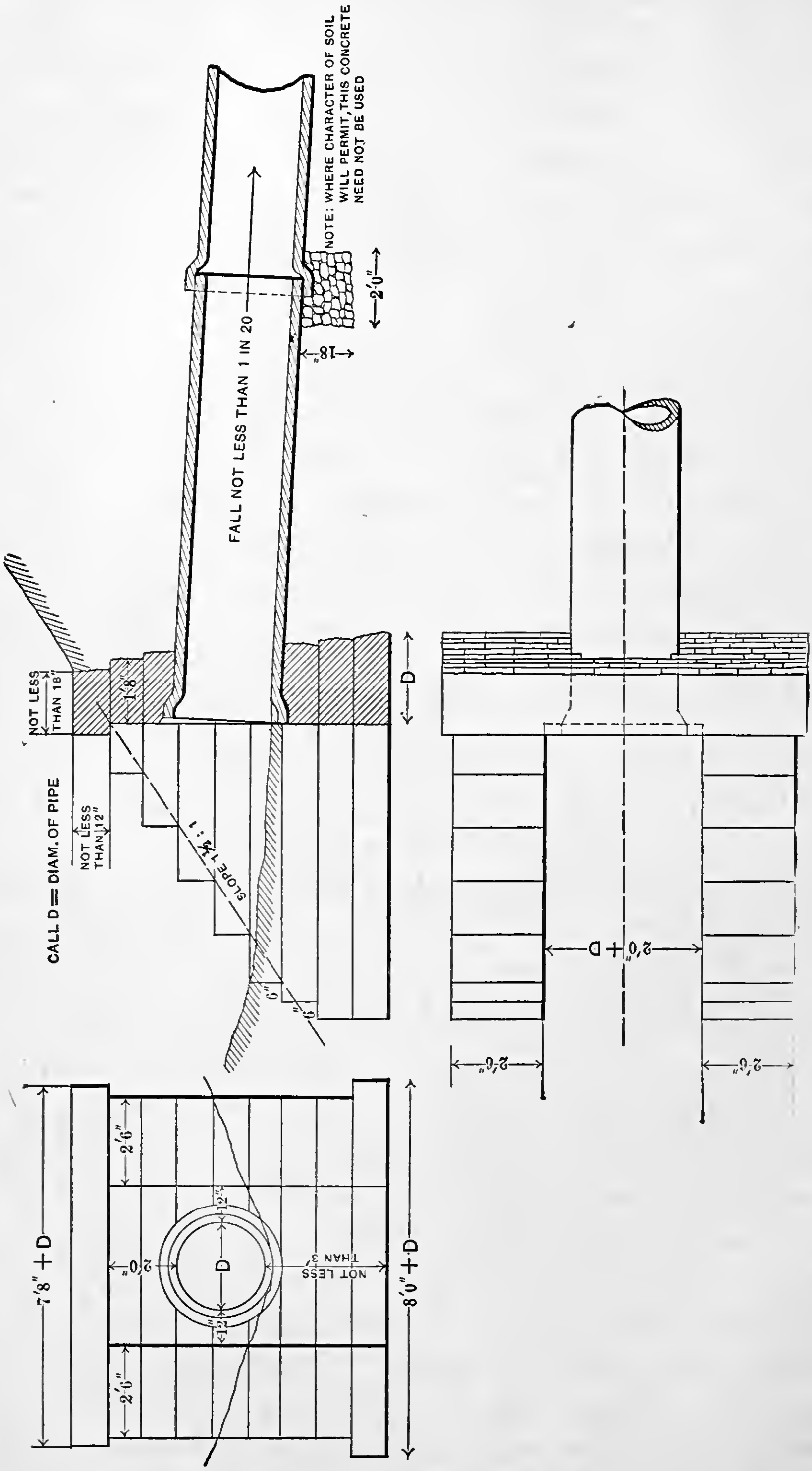


FIG. 97.—STANDARD CAST-IRON PIPE CULVERT. C. C. C. & ST. L. RY. (May 1893.)

187. Tile-pipe culverts. The pipes used for this purpose vary from 12″ to 24″ in diameter. When a larger capacity is required two or more pipes may be laid side by side, but in such a case another design might be preferable. It is frequently specified that "double-strength" or "extra-heavy" pipe shall be used, evidently with the idea that the stresses on a culvert-pipe are greater than on a sewer-pipe. But it has been conclusively demonstrated that, no matter how deep the embankment, the pressure cannot exceed a somewhat uncertain maximum, also that the greatest danger consists in placing the pipe so near the ties that shocks may be directly transferred to the pipe without the cushioning effect of the earth and ballast. When the pipes are well bedded in *clear* earth and there is a sufficient depth of earth over them to avoid direct impact (at least three feet) the ordinary sewer-pipe will be sufficiently strong. "Double-strength" pipe is frequently less perfectly burned, and

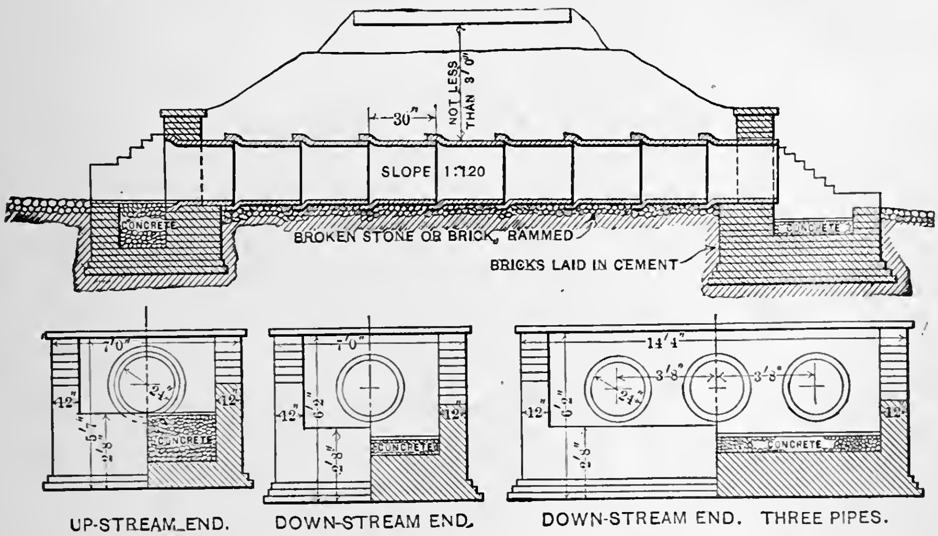


FIG. 98.—STANDARD VITRIFIED-PIPE CULVERT. PLANT SYSTEM. (1891.)

the supposed extra strength is not therefore obtained. In Fig. 98 are shown the standard plans for vitrified-pipe culverts as used on the "Plant system." Tile pipe is much cheaper than iron pipe, but is made in much shorter lengths and requires much more work in laying and especially to obtain a uniform grade.

BOX CULVERTS.

188. Wooden box culverts. This form serves the purpose of a cheap temporary construction which allows the use of a ballasted roadbed. As in all temporary constructions, the area should be made considerably larger than the calculated area (§§ 179–182), not only for safety but also in order that, if the smaller area is demonstrated to be sufficiently large, the permanent construction (probably pipe) may be placed inside without disturbing the embankment. All designs agree in using heavy timbers (12″ × 12″, 10″ × 12″, or 8″ × 12″) for the side walls, cross-timbers for the roof, every fifth or sixth timber being notched down so as to take up the thrust of the side walls, and planks for the flooring. Fig. 99 shows some of the standard designs as used by the C., M. & St. P. Ry.

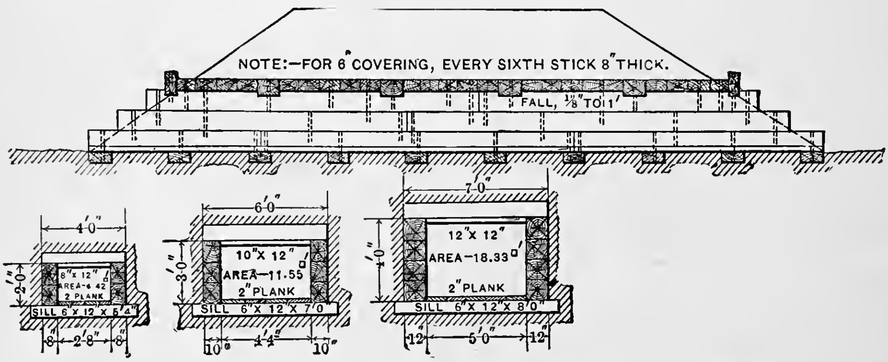


FIG. 99.—STANDARD TIMBER BOX CULVERT. C., M. & ST. P. RY. (Feb. 1889.)

189. Stone box culverts. In localities where a *good* quality of stone is cheap, stone box culverts are the cheapest form of permanent construction for culverts of medium capacity, but their use is decreasing owing to the frequent difficulty in obtaining really suitable stone within a reasonable distance of the culvert. The clear span of the cover-stones varies from 2 to 4 feet. The required thickness of the cover-stones is sometimes calculated by the theory of transverse strains on the basis of certain assumptions of loading—as a function of the height of the embankment and the unit strength of the stone used. Such a method is simply another illustration of a class of calculations which look very precise and beautiful, but which are worse than useless (because misleading) on account of the hopeless uncertainty as to the true value of certain quantities which must be used in the computations. In the first place the true value of the unit tensile strength of stone is such an uncertain and variable quantity that calculations based on any assumed value for it are of small reliability. In the second place the weight of the prism of earth lying directly above the stone, plus an allowance for live load, is by no means a measure of the load on the stone nor of the forces that tend to fracture it. All earthwork will tend to form an arch above any cavity and thus relieve an uncertain and probably variable proportion of the pressure that might otherwise exist. The higher the embankment the *less* the *proportionate* loading, until at some uncertain height an increase in height will not increase the load on the cover-stones. The effect of frost is likewise large, but uncertain and not computable. The usual practice is therefore to make the thickness such as experience has shown to be safe with a good quality of stone, i.e., about 10 or 12 inches for 2 feet span and up to 16 or 18 inches for 4 feet span. The side walls should be carried down deep enough to prevent their being undermined by scour or heaved by frost. The use of cement mortar is also an important feature of first-class work, especially when there is a rapid scouring current or a liability that the culvert will run under a head. In Fig. 100 are shown standard plans for single and double stone box culverts as used on the Norfolk and Western R.R.

190. Old-rail culverts. It sometimes happens (although very rarely) that it is necessary to bring the grade line within 3 or 4 feet of the bottom of a stream and yet allow an area of 10 or 12 square feet. A single large pipe of sufficient area could not be used in this case. The use of several smaller pipes side by side would be both expensive and inefficient. For similar reasons neither wooden nor stone box culverts could be used. In such cases, as well as in many others where the head-room is not so limited, the plan illustrated in Fig. 101 is a very satisfactory solution of the problem. The old rails, having a length of 8 or

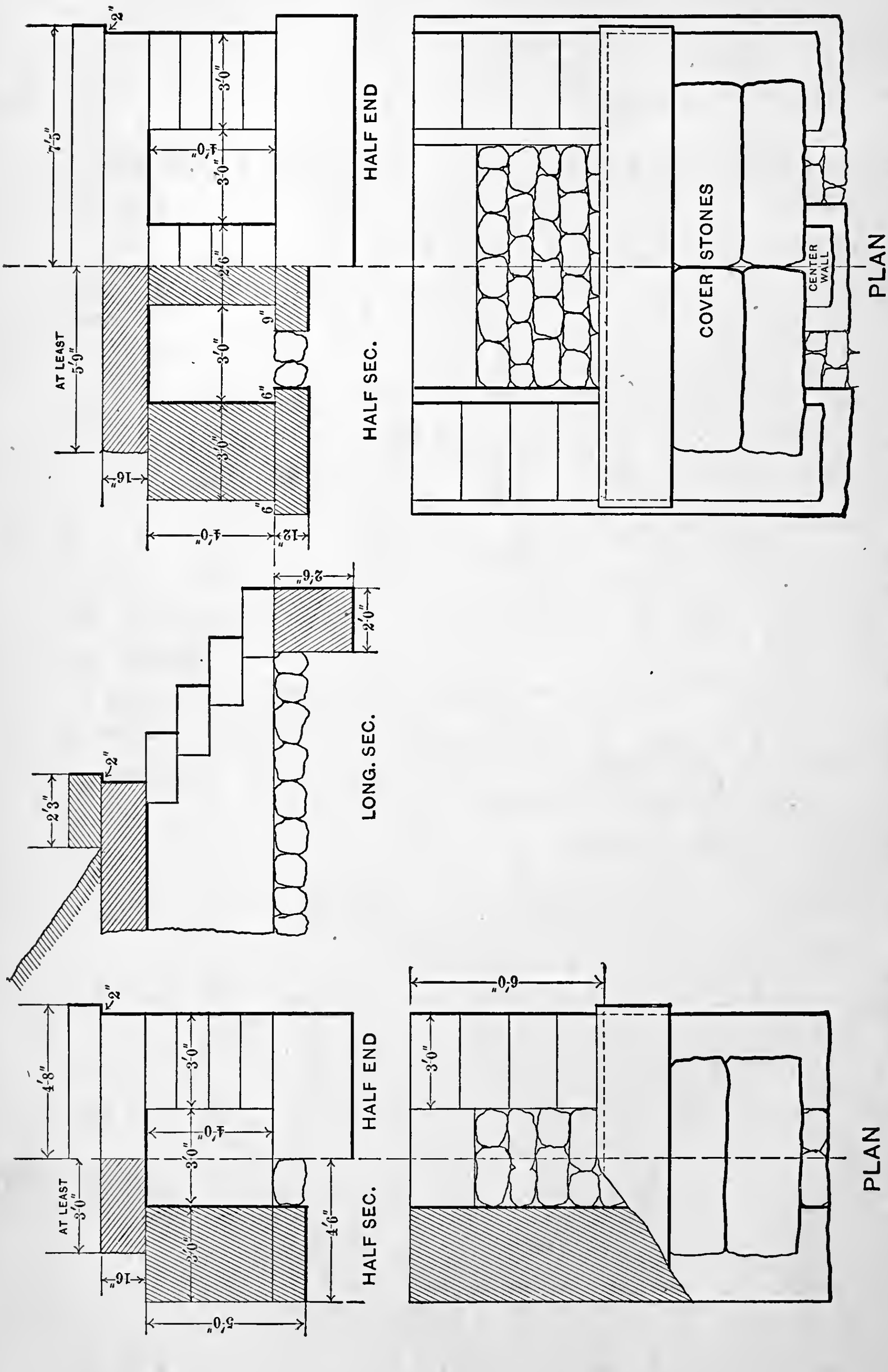


Fig. 100.—Standard Single and Double Stone Culverts (3'×4'). N. & W. R.R. (1890.)

9 feet, are laid close together across a 6-foot opening. Sometimes the rails are held together by long bolts passing through the webs of the rails. In the plan shown the rails are confined

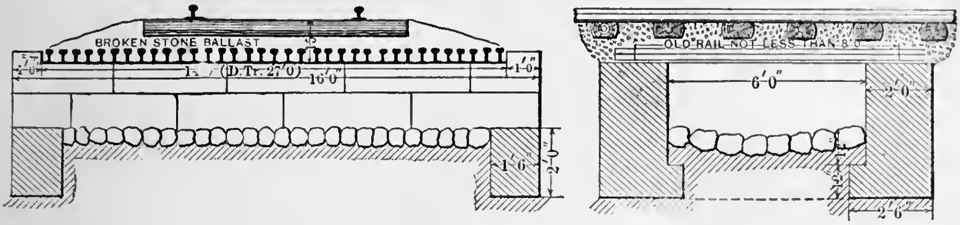


FIG. 101.—STANDARD OLD-RAIL CULVERT. N. & W. R.R. (1895.)

by low end walls on each abutment. This plan requires only 15 inches between the base of the rail and the top of the culvert channel. It also gives a continuous ballasted roadbed.

ARCH CULVERTS.

191. Influence of design on flow. The variations in the design of arch culverts have a very marked influence on the cost and efficiency. To combine the least cost with the greatest efficiency, due weight should be given to the following elements: (*a*) the amount of masonry, (*b*) the simplicity of the constructive work, (*c*) the design of the wing walls, (*d*) the design of the junction of the wing walls with the barrel

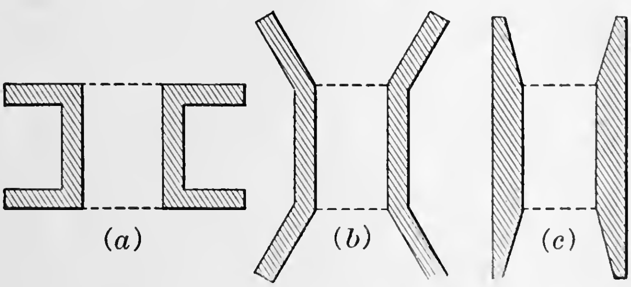


FIG. 102.—TYPES OF CULVERTS.

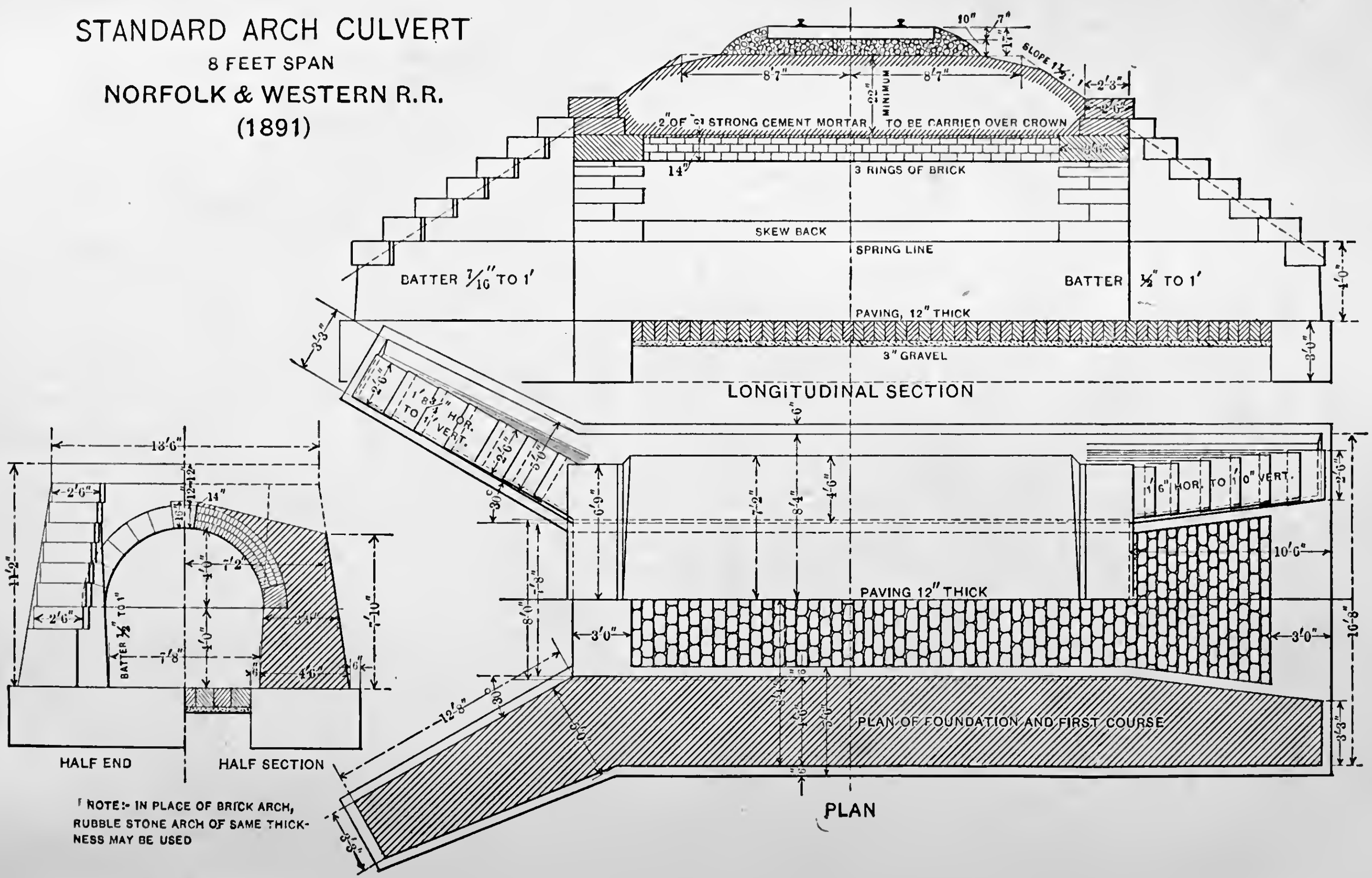
and faces of the arch, and (*e*) the safety and permanency of the construction. These elements are more or less antagonistic to each other, and the defects of most designs are due to a lack of proper proportion in the design of these opposing interests. The simplest construction (satisfying elements *b* and *e*) is the straight barrel arch between two parallel vertical head walls, as sketched in Fig. 102, *a*. From a hydraulic standpoint the design is poor, as the water eddies around the corners, causing a great resistance which decreases the flow. Fig. 102, *b*, shows a much better design in many respects, but much depends on the details of the design as indicated in elements (*b*) and (*d*). As a general thing a good hydraulic design requires complicated and expensive masonry construction, i.e., elements (*b*) and (*d*) are opposed. Design 102, *c*, is sometimes inapplicable because the water is liable to work in behind the masonry during floods and perhaps cause scour. This design uses less masonry than (*a*) or (*b*).

192. Example of arch culvert design. In Plate XV is shown the design for an 8-foot arch culvert according to the standard of the Norfolk and Western R.R. Note that the plan uses the flaring wing walls (Fig. 102, *b*) on the up-stream side (thus protecting the abutments from scour) and straight wing walls (similar to Fig. 102, *c*) on the down-stream end. This economizes masonry and also simplifies the constructive work. Note also the simplicity of the junction of the wing walls with the barrel of the arch, there being no re-entrant angles below the springing line of the arch. The design here shown is but one of a set of designs for arches varying in span from 6′ to 30′.

MINOR OPENINGS.

193. Cattle-guards. (a) Pit guards. Cattle-guards will be considered under the head of minor openings, since the old-fashioned plan of pit guards, which are even now defended and preferred by some railroad men, requires a break in the continuity of the roadbed. A pit about three feet deep, five feet long, and as wide as the width of the roadbed, is walled up with stone (sometimes with wood), and the rails are supported on heavy timbers laid longitudinally with the rails. The break in the continuity of the roadbed produces a disturbance in the elastic wave running through the rails, the effect of which is noticeable at high velocities. The greatest objection, however, lies in the

PLATE XV.



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dangerous consequences of a derailment or a failure of the timbers owing to unobserved decay or destruction by fire—caused perhaps by sparks and cinders from passing locomotives. The very insignificance of the structure often leads to careless in-

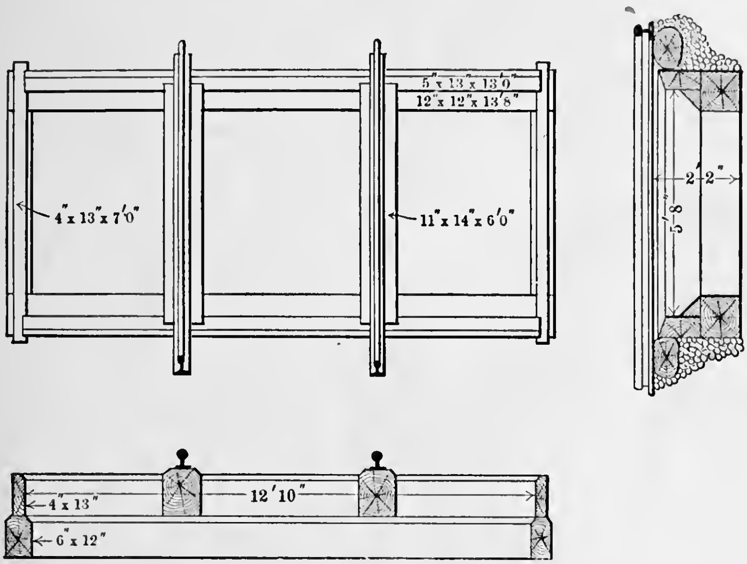


FIG. 103.—PIT CATTLE-GUARDS. P. R.R.

spection. But if a single pair of wheels gets off the rails and drops into the pit, a costly wreck is inevitable. The (once) standard design for such a structure on the Pennsylvania R.R. is shown in Fig. 103.

(b) Surface cattle-guards. These are fastened on top of the ties; the continuity of the roadbed is absolutely unbroken and thus is avoided much of the danger of a bad wreck owing to a possible derailment. The device consists essentially of overlaying the ties (both inside and outside the rails) with a surface on which cattle will not walk. The multitudinous designs for such a surface are variously effective in this respect. An objection, which is often urged indiscriminately against all such designs, is the liability that a brake-chain which may happen to be dragging *may* catch in the rough bars which are used. The bars are sometimes "home-made," of wood, as shown in Fig. 104. Iron or steel bars are made as shown in Fig. 105. The general construction is the same as for the wooden bars. The metal bars have far greater durability, and it is claimed that they are more effective in discouraging cattle from attempting to cross.

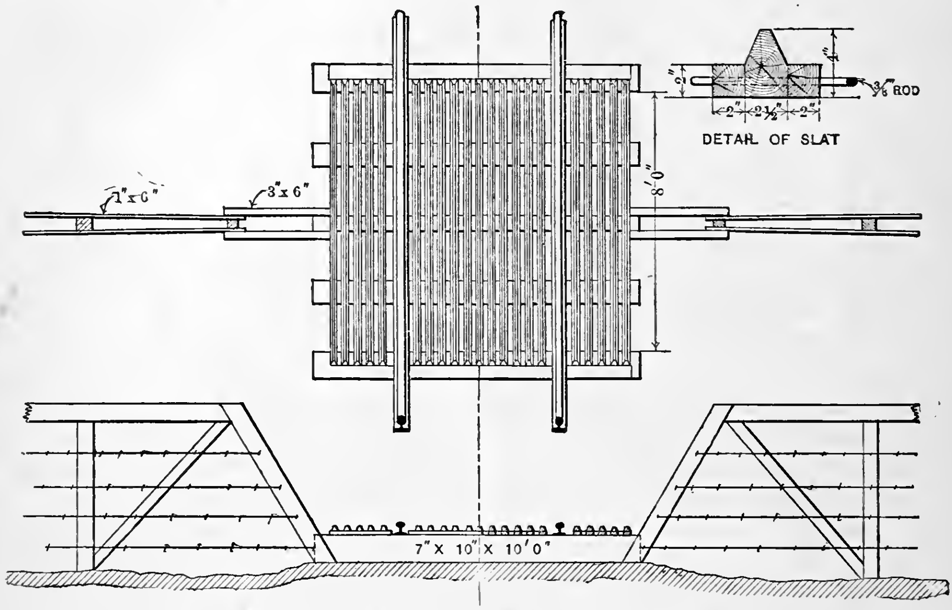


FIG. 104.—CATTLE-GUARD WITH WOODEN SLATS.

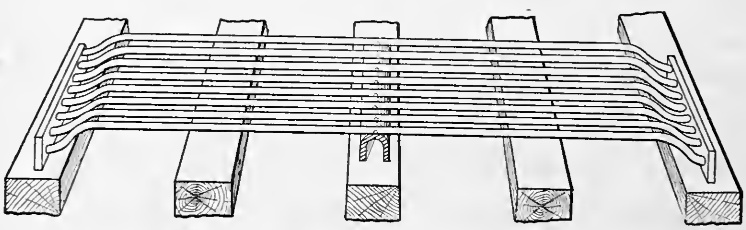
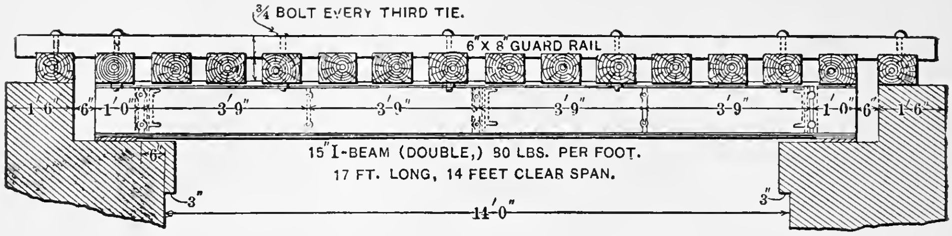
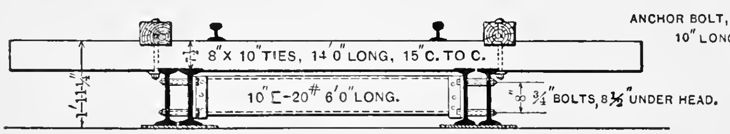
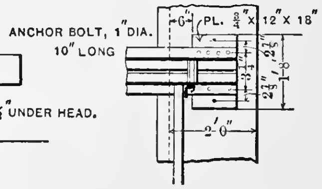


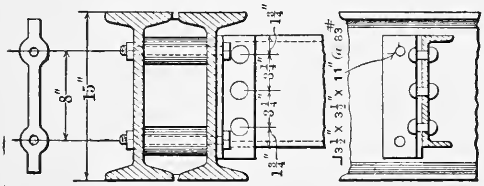
FIG. 105.—MERRILL-STEVENS STEEL CATTLE-GUARD.

194. Cattle-passes. Frequently when a railroad crosses a farm on an embankment, cutting the farm into two parts, the railroad company is obliged to agree to make a passageway through the embankment sufficient for the passage of cattle and perhaps even farm-wagons. If the embankment is high enough so that a stone arch is practicable, the initial cost is the only great objection to such a construction; but if an open wooden structure is necessary, all the objections against the old-fashioned cattle-guards apply with equal force here. The avoidance of a grade crossing which would otherwise be necessary is one of the





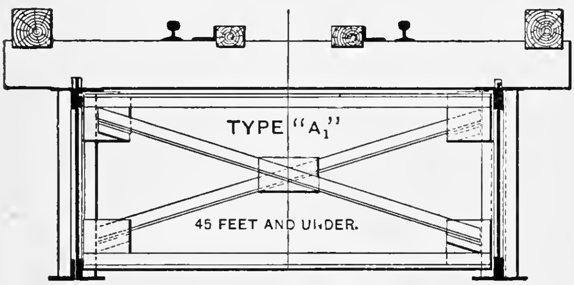


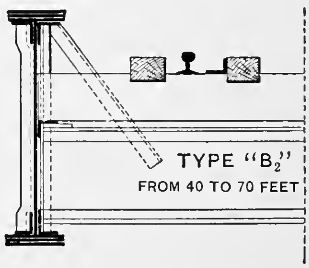


STANDARD I-BRIDGES—14-FT. SPAN.

NORFOLK AND WESTERN R.R.

(1891.)

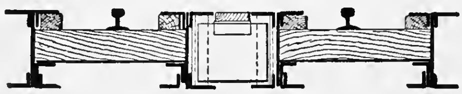




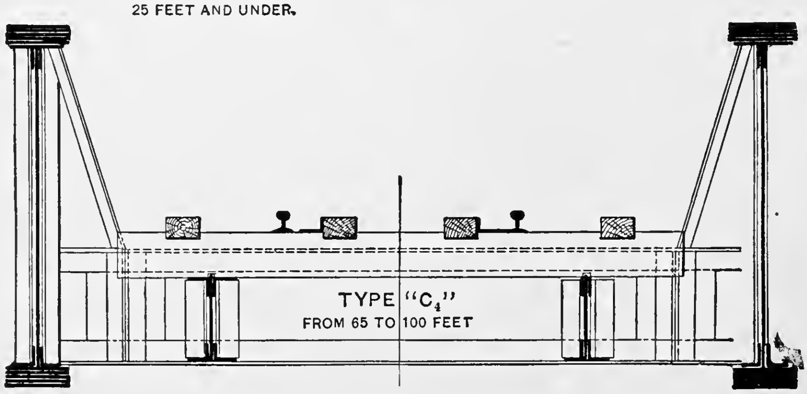
TYPES OF PLATE GIRDER BRIDGES.

C. M. & St.P. RY.

(Dec. 1895.)



TYPE "E" GIRDER
25 FEET AND UNDER.



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great compensations for the expense of the construction and maintenance of these structures. The construction is sometimes made by placing two pile trestle bents about 6 to 8 feet apart, supporting the rails by stringers in the usual way, the special feature of this construction being that the embankments are filled in behind the trestle bents, and the thrust of the embankments is mutually taken up through the stringers, which are notched at the ends or otherwise constructed so that they may take up such a thrust. The designs for old-rail culverts and arch culverts are also utilized for cattle-passes when suitable and convenient, as well as the designs illustrated in the following section.

195. Standard stringer and I-beam bridges. The advantages of standard designs apply even to the covering of short spans with wooden stringers or with I beams—especially since the methods do not require much vertical space between the rails and the upper side of the clear opening, a feature which is often of prime importance. These designs are chiefly used for culverts or cattle-passes and for crossing *over* highways—providing such a narrow opening would be tolerated. The plans all imply stone abutments, or at least abutments of sufficient stability to withstand all thrust of the embankments. Some of the designs are illustrated in Plate XVI. The preparation of these standard designs should be attacked by the same general methods as already illustrated in § 156. When computing the required transverse strength, due allowance should be made for lateral bracing, which should be amply provided for. Note particularly the methods of bracing illustrated in Plate XVI. The designs calling for iron (or steel) stringers may be classed as permanent constructions, which are cheap, safe, easily inspected and maintained and therefore a desirable method of construction.

CHAPTER VII.

BALLAST.

196. Purpose and requirements. "The object of the ballast is to transfer the applied load over a large surface; to hold the timber work in place horizontally; to carry off the rain-water from the superstructure and to prevent freezing up in winter; to afford means of keeping the ties truly up to the grade line; and to give elasticity to the roadbed." This extremely condensed statement is a description of an ideally perfect ballast. The value of any given kind of ballast is proportional to the extent to which it fulfills these requirements. The ideally perfect ballast is not necessarily the most economical ballast for all roads. Light traffic generally justifies something cheaper, but a very common error is to use a very cheap ballast when a small additional expenditure would procure a much better ballast which would be much more economical in the long run.

197. Materials. The materials most commonly employed are gravel and broken stone. Burnt clay, cinders, shells, and small coal are occasionally used as ballast when they are especially cheap and convenient or when better kinds are especially expensive. Although it is hardly correct to speak of the natural soil as ballast, yet many miles of cheap railways are "ballasted" with the natural soil, which is then called "mud ballast."

Mud ballast. When the natural soil is gravelly so that rain will drain through it quickly, it will make a fair roadbed for light traffic, but for heavy traffic, and for the greater part of the length of most roads, the natural soil is a very poor material for ballast; for, no matter how suitable the soil might be along limited sections of the road, it would practically never happen that the soil would be uniformly good throughout the whole length of the road. Considering that a heavy rain will in one day spoil the results of weeks of patient "surfacing" with mud ballast, it is seldom economical to use "mud" if there is a gravel-bed or other source of ballast anywhere on the line of the road.

Cinders. The advantages consist in the excellent facilities for drainage, ease of handling, and cheapness—after the road is in operation. One disadvantage is excessive dust in dry weather. Cinders are considered preferable to gravel in yards.

Slag. When slag is readily obtainable it furnishes an excellent ballast, free from dust and perfect in drainage qualities. Some kinds of slag are objectionable on account of their deleterious chemical effect on the ties and spikes—especially on metallic ties.

Shells, small coal, etc. These comparatively inferior kinds of ballast are used for light traffic when they are especially cheap and convenient. They are extremely dusty in dry weather, break up into very fine dust, and are but little better than mud.

Gravel. This is the most common form of ballast which may be called good ballast. In 1885, the Roadmasters Association of America voted in favor of gravel ballast as against rock ballast. Although not so stated, this action was perhaps due to a conviction of its real economy for the *average* railroad of this country, which may be called a "light traffic" road. Gravel should preferably be screened over a screen having a $\frac{1}{2}''$ mesh, so as to screen out all dirt and the finest stones. Generally a railroad will be able to find at some point along its line a "gravel-pit" affording a suitable supply. This may be dug out with a steam-shovel, screened if necessary, and sent out over the line by the train-load at a comparatively small cost.

Rock or broken stone. Rock ballast is generally specified to be such as will pass through a $1\frac{1}{2}''$ (or $2''$) ring. Although preferably broken by hand, machine-broken stone is much cheaper. It is most easily handled with forks. This also has the effect of screening out the dirt and fine chips which would interfere with effectual drainage. Rock ballast is more expensive in first cost, and also more troublesome to handle, than any other kind, but under heavy traffic will keep in surface better and will require less work for maintenance after the ties have become thoroughly bedded. For roads with very light traffic, running few trains, at comparatively low velocities, the advantages of rock ballast over other kinds are not so pronounced. For such roads rock ballast is an expensive luxury. The amount of traffic which will justify the use of rock ballast will depend on the cost of obtaining ballast of the various kinds.

198. Cross-sections. A depth of 12″ under the tie is generally required on the best roads, but for light traffic this is sometimes reduced to 6″ and even less. The width is generally 1 to 2 feet less than the width of the roadbed proper—excluding ditches. If the ballast has an average width of 10 feet (12 feet at bottom and 8 feet at top) and an average depth of 15 inches (including that placed between the ties), it will require 2444 cubic yards per mile of track. The P. R.R. estimates 2500 cubic yards of gravel and 2800 cubic yards of stone ballast per mile of single track. On account of the requirements of drainage the best form of cross-section depends on the kind of ballast used.

Mud ballast. Since the great objection to mud ballast lies in its liability to become soft by soaking up the rain that falls, it becomes necessary that it should be drained as quickly and readily as its nature will permit. Fig. 106 shows a typical

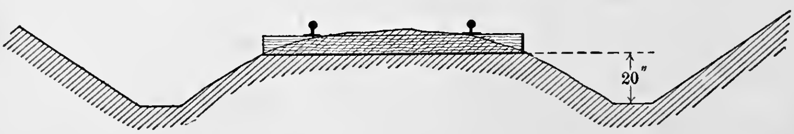


FIG. 106.—"MUD" BALLAST.

cross-section for mud ballast. It should be crowned 2″ above the top of the tie at the center, thence sloped so as to leave a slight clearance under the rail between the ties, thence sloping down to the bottom of the tie at each end and continuing to slope down to the ditch (in cut), which should be 18″ or 20″ below the bottom of the tie.

Gravel, cinders, slag, etc. The subgrade is crowned 6″ or 8″ in the center, as shown in Fig. 107. The ballast is crowned

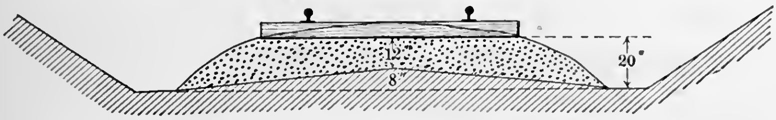


Fig. 107.—Gravel Ballast.

to the top of the tie in the center, but is sloped down to the bottom of the tie at each end. This is necessary (and more especially so with mud ballast) to prevent a possible accumulation and settlement of water at the ends of the tie, which would readily soak into the end fibers and produce decay.

Broken stone. Stone ballast is shouldered out beyond the ends of the ties so as to afford greater lateral binding. The space between the ties is filled up level with the tops. The

Fig. 108.—Broken Stone Ballast.

perfect drainage of stone ballast permits this to be done without any danger of causing decay of the ties by the accumulation and retention of water.

199. Methods of laying ballast. The cheapest method of laying ballast on new roads is to lay ties and rails directly on the prepared subgrade and run a construction train over the track to distribute the ballast. Then the track is lifted up until sufficient ballast is worked under the ties and the track is properly surfaced. This method, although cheap, is apt to injure the rails by causing bends and kinks, due to the passage of loaded construction trains when the ties are very unevenly and roughly supported, and the method is therefore condemned and prohibited in some specifications. The best method is to draw in carts (or on a contractor's temporary track) the ballast that is required under the level of the *bottom* of the ties. Spread this ballast carefully to the required surface. Then lay the ties and rails, which will then have a very fair surface and uniform support. A construction train can then be run on the rails and distribute sufficient additional ballast to pack around and between the ties and make the required cross-section.

The necessity for constructing some lines at an absolute minimum of cost and of opening them for traffic as soon as possible has often led to the policy of starting traffic when there is little or no ballast—perhaps nothing more than a mere tamping of the natural soil under the ties. When this is done ballast may subsequently be drawn where required by the train-load on flat cars and unloaded at a minimum of cost by means of a "plough." The plough has the same width as the cars and is guided either by a ridge along the center of each car or by short posts set up at the sides of the cars. It is drawn from one end of the train to the other by means of a cable. The cable is sometimes operated by means of a small hoisting-engine carried on a car at one end of the train. Sometimes the locomotive is detached temporarily from the train and is run ahead with the cable attached to it.

200. Cost. The cost of ballast *in the track* is quite a variable item for different roads, since it depends (*a*) on the first cost of the material as it comes to the road, (*b*) on the distance from the source of supply to the place where it is used, and (*c*) on the method of handling. The first cost of cinder or slag is frequently insignificant. A gravel-pit may cost nothing except the price of a little additional land beyond the usual limits of the right of way. Broken stone will usually cost $1 or more per cubic yard. If suitable stone is obtainable on the company's land, the cost of blasting and breaking should be somewhat less than this. The cost of loading the ballast on to trains will be small (per cubic yard) if it is handled with steam-shovels—as in the case of gravel taken from a gravel-pit. Hand-shovelling will cost more. The cost of hauling will depend on the distance hauled, and also, to a considerable extent, on the limitations on the operation of the train due to the necessity of keeping out of the way of regular trains. There is often a needless waste in this way. The "mud train" is considered a pariah and entitled to no rights whatever, regardless of the large daily cost of such a train and of the necessary gang of men. The cost of broken stone ballast *in the track* is estimated at $1.25 per cubic yard. The cost of gravel ballast is estimated at 60 c. per cubic yard in the track. The cost of placing and tamping gravel ballast is estimated at 20 c. to 24 c. per cubic yard, for cinders 12 c. to 15 c. per cubic yard. The cost of loading gravel on cars, using a steam-shovel, is estimated at 6 c. to 10 c. per cubic yard.*

* Report Roadmasters Association, 1885.

CHAPTER VIII.

TIES,

AND OTHER FORMS OF RAIL SUPPORT.

201. Various methods of supporting rails. It is necessary that the rails shall be sufficiently supported and braced, so that the gauge shall be kept constant and that the rails shall not be subjected to excessive transverse stress. It is also preferable that the rail support shall be neither rigid (as if on solid rock) nor too yielding, but shall have a *uniform* elasticity throughout. These requirements are more or less fulfilled by the following methods.

(a) **Longitudinals.** Supporting the rails throughout their entire length. This method is very seldom used in this country except occasionally on bridges and in terminals when the longitudinals are supported on cross-ties. In § 224 will be described a system of rails, used to some extent in Europe, having such broad bases that they are self-supporting on the ballast and are only connected by tie-rods to maintain the gauge.

(b) **Cast-iron "bowls" or "pots."** These are castings resembling large inverted bowls or pots, having suitable chairs on top for holding and supporting the rails, and tied together with tie-rods. They will be described more fully later (§ 223).

(b) **Cross-ties of metal or wood.** These will be discussed in the following sections.

202. Economics of ties. The true cost of ties depends on the relative total cost of maintenance for long periods of time. The first cost of the ties delivered to the road is but one item in the economics of the question. Cheap ties require frequent renewals, which cost for the *labor* of each renewal practically the same whether the tie is of oak or hemlock. Cheap ties make a poor roadbed which will require more track labor to keep even in tolerable condition. The roadbed will require to be disturbed so frequently on account of renewals that the ties never get an opportunity to get settled and to form a smooth roadbed for any length of time. Irregularity in width, thickness, or length of ties is especially detrimental in causing the ballast to act and wear unevenly. The life of ties has thus a more or less direct influence on the life of the rails, on the wear of rolling stock, and on the speed of trains. These last items are not so readily reducible to dollars and cents, but when it can be shown that the total cost, for a long period of time, of several renewals of cheap ties, with all the extra track labor involved, is as great as or greater than that of a few renewals of durable ties, then there is no question as to the real economy. In the following discussions of the merits of untreated ties (either cheap or costly), chemically treated ties, or metal ties, the true question is therefore of the ultimate cost of maintaining any particular kind of ties for an indefinite period, the cost including the first cost of the ties, the labor of placing them and maintaining them to surface, and the somewhat uncertain (but not therefore non-existent) effect of frequent renewals on repairs of rolling stock, on possible speed, etc.

WOODEN TIES.

203. Choice of wood. This naturally depends, for any particular section of country, on the supply of wood which is most readily available. The woods most commonly used, especially in this country, are oak and pine, oak being the most durable and generally the most expensive. Redwood is used very extensively in California and proves to be extremely durable, so far as decay is concerned, but it is very soft and is much injured by "rail-cutting." This defect is being partly remedied by the use of tie-plates, as will be explained later. Cedar, chestnut, hemlock, and tamarack are frequently used in this country. In tropical countries very durable ties are frequently obtained from the hard woods peculiar to those countries. According to a recent bulletin of the U. S. Department of Agriculture the proportions of the various kinds used in the United States are about as follows:

Oak	60%	Chestnut	5%	Cypress	2%
Pine	20	Hemlock and Tamarack	3	Various	1
Cedar	6	Redwood	3	Total	100%

204. Durability. The durability of ties depends on the climate; the drainage of the ballast; the volume, weight, and speed of the traffic; the curvature, if any; the use of tie-plates; the time of year of cutting the timber; the age of the timber and the degree of its seasoning before placing in the track; the nature of the soil in which the timber was grown; and, chiefly, on the species of wood employed. The variability in these items will account for the discrepancies in the reports on the life of various woods used for ties.

White oak is credited with a life of 5 to 12 years, depending principally on the traffic. Is is both hard and durable, the hardness enabling it to withstand the cutting tendency of the rail-flanges, and the durability enabling it to resist decay. *Pine* and *redwood* resist decay very well, but are so soft that they are badly cut by the rail-flanges and do not hold the spikes very well, necessitating frequent respiking. Since the spikes must be driven within certain very limited areas on the face of each tie, it does not require many spike-holes to "spike-kill" the tie. On sharp curves, especially with heavy traffic, the wheel-flange pressure produces a side pressure on the rail tending to overturn it, which tendency is resisted by the spike, aided sometimes by rail-braces. Whenever the pressure becomes too great the spike will yield somewhat and will be slightly withdrawn. The resistance is then somewhat less and the spike is soon so loose that it must be redriven in a new hole. If this occurs very often, the tie may need to be replaced long before any decay has set in. When the traffic is very light, the wood very durable, and the climate favorable ties have been known to last 25 years.

205. Dimensions. The usual dimensions for the best roads (standard gauge) are 8′ to 8′ 6″ long, 6″ to 7″ thick, and 8″ to 10″ wide on top and bottom (if they are hewed) or 8″ to 9″ wide if they are sawed. For cheap roads and light traffic the length is shortened sometimes to 7′ and the cross-section also reduced. On the other hand a very few roads use ties 9′ long.

Two objections are urged against sawed ties: first, that the grain is torn by the saw, leaving a woolly surface which induces decay; and secondly, that, since timber is not perfectly straight-grained, some of the fibers are cut obliquely, exposing their ends, which are thus liable to decay. The use of a "planer-saw" obviates the first difficulty. Chemical treatment of ties obviates both of these difficulties. Sawed ties are more convenient to handle, are a necessity on bridges and trestles, and it is even claimed, although against commonly received opinion, that actual trial has demonstrated that they are more durable than hewed ties.

206. Spacing. The spacing is usually 14 to 16 ties to a 30-foot rail. This number is sometimes reduced to 12 and even 10, and on the other hand occasionally increased to 18 or 20 by employing narrower ties. There is no economy in reducing the number of ties very much, since for any required stiffness of track it is more economical to increase the number of supports than to increase the weight of the rail. The decreasing cost of rails and the increasing cost of ties have materially changed the relation between number of ties and weight of rail to produce a given stiffness at minimum cost, but many roads have found it economical to employ a large number of ties rather than increase the weight of the rail. On the other hand there is a practical limit to the number that may be employed, on account of the necessary space between the ties that is required for proper tamping. This width is ordinarily about twice the width of the tie. At this rate, with light ties 6″ wide and with 12″ clear space, there would be 20 ties per 30-foot rail, or 3520 per mile. The smaller ties can generally be bought much cheaper (proportionately) than the larger sizes, and hence the economy.

Track instructions to foremen generally require that the spacing of ties shall *not* be uniform along the length of any rail. Since the joint is generally the weakest part of the rail structure, the joint requires more support than the center of the rail. Therefore the ties are placed with but 8″ or 10″ clear space between them at the joints, this applying to 3 or 4 ties at each joint; the remaining ties, required for each rail length, are equally spaced along the remaining distance.

207. Specifications. The specifications for ties are apt to include the items of size, kind of wood, and method of construction, besides other minor directions about time of cutting, seasoning, delivery, quality of timber, etc.

(a) **Size.** The particular size or sizes required will be somewhat as indicated in § 205.

(b) **Kind of wood.** When the kind or kinds of wood are specified, the most suitable kinds that are available in that section of country are usually required.

(c) **Method of construction.** It is generally specified that the ties shall be hewed on two sides; that the two faces thus made shall be parallel planes and that the bark shall be removed. It is sometimes required that the ends shall be sawed off square; that the timber shall be cut in the winter (when the sap is down); and that the ties shall be seasoned for six months. These last specifications are not required or lived up to as much as their importance deserves. It is sometimes required that the ties shall be delivered on the right of way, neatly piled in rows, the alternate rows at right angles, piled if possible on ground not lower than the rails and at least seven feet away from them, the lower row of ties resting on two ties which are themselves supported so as to be clear of the ground.

(d) **Quality of timber.** The usual specifications for sound timber are required, except that they are not so rigid as for a better class of timber work. The ties must be sound, reasonably straight-grained, and not very crooked—one test being that a line joining the center of one end with the center of the middle shall not pass outside of the other end. Splits or shakes, especially if severe, should cause rejection.

Specifications sometimes require that the ties shall be cut from single trees, making what is known as "pole ties" and definitely condemning those which are cut or split from larger trunks, giving two "slab ties" or four "quarter ties" for each cross-section, as is illustrated in Fig. 109. Even if pole ties are better, their exclusive use means the rapid destruction of forests of young trees.

FIG. 109.—METHODS OF CUTTING TIES.

208. Regulations for laying and renewing ties. The regulations issued by railroad companies to their track foremen will generally include the following, in addition to directions regarding dimensions, spacing, and specifications given in §§ 204–207. When hewn ties of somewhat variable size are used, as is frequently the case, the largest and best are to be selected for use as joint ties. If the upper surface of a tie is found to be warped (contrary to the usual specifications) so that one or both rails do not get a full bearing across the whole width of the tie, it must be adzed to a true surface along its whole length and not merely notched for a rail-seat. When respiking is necessary and spikes have been pulled out, the holes should be immediately plugged with "wooden spikes," which are supplied to the foremen for that express purpose, so as to fill up the holes and prevent the decay which would otherwise take place when the hole becomes filled with rain-water. Ties should always be laid at right angles to the rails and never obliquely. Minute regulations to prevent premature rejection and renewal of ties are frequently made. It is generally required that the requisitions for renewals shall be made by the actual count of the individual ties to be renewed instead of by any wholesale estimates. It is unwise to have ties of widely variable size, hardness, or durability adjacent to each other in the track, for the uniform elasticity, so necessary for smooth riding, will be unobtainable under those circumstances.

209. Cost of ties. When railroads can obtain ties cut by farmers from woodlands in the immediate neighborhood, the price will frequently be as low as 20 c. for the smaller sizes, running up to 50 c. for the larger sizes and better qualities, especially when the timber is not very plentiful. Sometimes if a railroad cannot procure suitable ties from its immediate neighborhood, it will find that adjacent railroads control all adjacent sources of supply for their own use and that ties can only be procured from a considerable distance, with a considerable added cost for transportation. First-class oak ties cost about 75 to 80 c. and frequently much more. Hemlock ties can generally be obtained for 35 c. or less.

PRESERVATIVE PROCESSES FOR WOODEN TIES.

210. General principle. Wood has a fibrous cellular structure, the cells being filled with sap or air. The woody fiber is but little subject to decay unless the sap undergoes fermentation. Preservative processes generally aim at removing as much of the water and sap as possible and filling up the pores of the wood with an antiseptic compound. The most common methods (except one) all agree in this general process and only differ in the method employed to get rid of the sap and in the antiseptic chemical with which the fibers are filled. One valuable feature of these processes lies in the fact that the softer cheaper woods (such as hemlock and pine) are more readily treated than are the harder woods and yet will produce practically as good a tie as a treated hard-wood tie and a very much better tie than an untreated hard-wood tie. The various processes will be briefly described, taking up first the process which is fundamentally different from the others, viz., vulcanizing.

211. Vulcanizing. The process consists in heating the timber to a temperature of 300° to 500° F. in a cylinder, the air being under a pressure of 100 to 175 lbs. per square inch. By this process the albumen in the sap is coagulated, the water evaporated, and the pores are partially closed by the coagulation of the albumen. It is claimed that the heat sterilizes the wood and produces chemical changes in the wood which give it an antiseptic character. It has been very extensively used on the elevated lines of New York City, and it is claimed to give perfect satisfaction. The treatment has cost that road 25 c. per tie.

212. Creosoting. This process consists in impregnating the wood with *wood-creosote* or with *dead oil of coal-tar.* *Wood-creosote* is one of the products of the destructive distillation of wood—usually long-leaf pine. *Dead oil of coal-tar* is a product of the distillation of coal-tar at a temperature between 480° and 760° F. It would require about 35 to 50 pounds of creosote to completely fill the pores of a cubic foot of wood. But it would be impossible to force such an amount into the wood, nor is it necessary or desirable. About 10 pounds per cubic foot, or about 35 pounds per tie, is all that is necessary. For piling placed in salt water about 18 to 20 pounds per cubic foot is used, and the timber is then perfectly protected against the ravages of the *teredo navalis.* To do the work, long cylinders, which may be opened at the ends, are necessary. Usually the timbers are run in and out on iron carriages running on rails fastened to braces on the inside of the cylinder. When the load has been run in, the ends of the cylinder are fastened on. The water and air in the pores of the wood are first drawn out by subjecting the wood alternately to steam-pressure and to the action of a vacuum-pump. This is continued for several hours. Then, after one of the vacuum periods, the cylinder is filled with creosote oil at a temperature of about 170° F. The pumps are kept at work until the pressure is about 80 to 100 pounds per square inch, and is maintained at this pressure from one to two hours according to the size of the timber. The oil is then withdrawn, the cylinders opened, the train pulled out and another load made up in 40 to 60 minutes. The average time required for treating a load is about 18 or 20 hours, the absorption about 10 or 11 pounds of oil per cubic foot, and the cost (1894) from $12.50 to $14.50 per thousand feet B. M.

213. Burnettizing (chloride-of-zinc process). This process is very similar to the creosoting process except that the chemical is chloride of zinc, and that the chemical is not heated before use. The preliminary treatment of the wood to alternate vacuum and pressure is not continued for quite so long a period as in the creosoting process. Care must be taken, in using this process, that the ties are of as uniform quality as possible, for seasoned ties will absorb much more zinc chloride than unseasoned (in the same time), and the product will lack uniformity unless the seasoning is uniform. The A., T. & S. Fé R.R. has works of its own at which ties are treated by this process at a cost of about 25 c. per tie. The Southern Pacific R.R. also has works for burnettizing ties at a cost of 9.5 to 12 c. per tie. The zinc-chloride solution used in these works contains only 1.7% of zinc chloride instead of over 3% as used in the Santa Fé works, which perhaps accounts partially for the great difference in cost per tie. One great objection to burnettized ties is the fact that the chemical is somewhat easily washed out, when the wood again becomes subject to decay. Another objection, which is more forcible with respect to timber subject to great stresses, as in trestles, than to ties, is the fact that when the solution of zinc chloride is made strong (over 3%) the timber is made very brittle and its strength is reduced. The reduction in strength has been shown by tests to amount to $\frac{1}{4}$ to $\frac{1}{10}$ of the ultimate strength, and that the elastic limit has been reduced by about $\frac{1}{7}$.

214. Kyanizing (bichloride-of-mercury or corrosive-sublimate process). This is a process of "steeping." It requires a much longer time than the previously described processes, but does not require such an expensive plant. Wooden tanks of sufficient size for the timber are all that is necessary. The corrosive sublimate is first made into a concentrated solution of one part of chemical to six parts of *hot* water. When used in the tanks this solution is weakened to 1 part in 100 or 150. The wood will absorb about 5 to 6.5 pounds of the bichloride per 100 cubic feet, or about one pound for each 4 to 6 ties. The timber is allowed to soak in the tanks for several days, the general rule being about one day for each inch of least thickness and one day over—which means seven days for six-inch ties, or thirteen (to fifteen) days for 12″ timber (least dimension). The process is somewhat objectionable on account of the chemical being such a virulent poison, workmen sometimes being sickened by the fumes arising from the tanks. On the Baden railway (Germany) kyanized ties last 20 to 30 years. On this railway the wood is always air-dried for two weeks after impregnation and before being used, which is thought to have an important effect on its durability. The solubility of the chemical and the liability of the chemical washing out and leaving the wood unprotected is an element of weakness in the method.

215. Wellhouse (or zinc-tannin) process. The last two methods described (as well as some others employing similar chemicals) are open to the objection that since the wood is impregnated with an aqueous solution, it is liable to be washed out very rapidly if the wood is placed under water, and will even disappear, although more slowly, under the action of moisture and rain. Several processes have been proposed or patented to prevent this. Many of them belong to one class, of which the Wellhouse process is a sample. By these processes the timber is successively subjected to the action of two chemicals, each individually soluble in water, and hence readily impregnating the timber, but the chemicals when brought in contact form insoluble compounds which cannot be washed out of the wood-cells. By the Wellhouse process, the wood is first impregnated with a solution of chloride of zinc and glue, and is then subjected to a bath of tannin under pressure. The glue and tannin combine to form an insoluble leathery compound in the cells, which will prevent the zinc chloride from being washed out. It is being used by the A., T. & S. Fé R.R., their works being located at Las Vegas, New Mexico, and also by the Union Pacific R.R. at their works at Laramie, Wyo. In 1897 Mr. J. M. Meade, a resident engineer on the A., T. & S. Fé, exhibited to the Roadmasters Association of America a piece of a tie treated by this process which had been taken from the tracks after nearly 13 years' service. The tie was selected at random, was taken out for the sole purpose of having a specimen, and was still in sound condition and capable of serving many years longer. The cost of the treatment was then quoted as 13 c. per tie. It was claimed that the treatment trebled the life of the tie besides adding to its spike-holding power.

216. Cost of treating. The cost of treating ties by the various methods has been estimated as follows*—assuming that the plant was of sufficient capacity to do the work economically: creosoting, 25 c. per tie; vulcanizing, 25 c. per tie; burnettizing (chloride of zinc), 8.25 c. per tie; kyanizing (steeping in corrosive sublimate), 14.6 c. per tie; Wellhouse process (chloride of zinc and tannin), 11.25 c. per tie. These estimates are only for the net cost at the works and do not include the cost of hauling the ties to and from the works, which may mean 5 to 10 c. per tie. Some of these processes have been installed on cars which are transported over the road and operated where most convenient.

217. Economics of treated ties. The fact that treated ties are not universally adopted is due to the argument that the added life of the tie is not worth the extra cost. If ties can be bought for 25 c., and cost 25 c. for treatment, and the treatment only doubles their life, there is apparently but little gained except the work of placing the extra tie in the track, which is more or less offset by the interest on 25 c. for the life of the untreated tie, and the larger initial outlay makes a stronger impression on the mind than the computed ultimate economy. But when ties cost 75 c. and treatment costs only 25 c., or perhaps less, then the economy is more apparent and unquestionable. But this analysis may be made more closely. As shown in § 202, the disturbance of the roadbed on account of frequent renewals of untreated ties is a disadvantage which would justify an appreciable expenditure to avoid, although it is

* Bull. No. 9, U. S. Dept. of Agric., Div. of Forestry. App. No. 1, by Henry Flad.

very difficult to closely estimate its true value. The annual cost of a system of ties may be considered as the sum of (*a*) the interest on the first cost, (*b*) the annual sinking fund that would buy a new tie at the end of its life, and (*c*) the average annual cost of maintenance for the life of the tie, which includes the cost of laying and the considerable amount of subsequent tamping that must be done until the tie is fairly settled in the roadbed, beside the regular trackwork on the tie, which is practically constant. This last item is difficult to compute, but it is easy to see that, since the cost of laying the tie and the subsequent tamping to obtain proper settlement is the same for all ties (of similar form), the *average* annual charge on the longer-lived tie would be much less. In the following comparison item (*c*) is disregarded, simply remembering that the advantage is with the longer-lived tie.

	Untreated tie.	Treated tie.
Original cost	40 cents	65 cents
Life (assumed at)	7 years	14 years
Item (*a*)—interest on first cost @ 4%	1.6 cents	2.6 cents
" (*b*)—sinking fund @ 4%	5.1 "	3.6 "
" (*c*)—(considered here as offsetted)	——	——
Average annual cost (except item (*c*))	6.7 cents	6.2 cents

On this basis treated ties will cost 0.5 cent *less* per annum *besides* the advantage of item (*c*) and the still more indefinite advantages resulting from smoother running of trains, less wear and tear on rolling stock, etc., due to less disturbance of the roadbed.

In Europe, where wood is expensive, untreated ties are seldom used, as the treatment is always considered to be worth more than it costs. The rapid destruction of the forests of timber in this country is having the effect of increasing the price, so that it will not be long before treated ties (or metal ties) will be economical for a large majority of the railroads of the country.

METAL TIES.

218. Extent of use. In 1894 * there were nearly 35000 miles of "metal track" in various parts of the world. Of this total, there were 3645 miles of "longitudinals" (see § 224), found exclusively in Europe, nearly all of it being in Germany. There were over 12000 miles of "bowls and plates" (see § 223), found almost entirely in British India and in the Argentine Republic. The remainder, over 18000 miles, was laid with metal cross-ties of various designs. There were over 8000 miles of metal cross-ties in Germany alone, about 1500 miles in the rest of Europe, over 6000 miles in British India, nearly 1000 miles in the rest of Asia, and about 1500 miles more in various other parts of the world. Several railroads in this country have tried various designs of these ties, but their use has never passed the experimental stage. These 35000 miles represent about 9% of the total railroad mileage of the world—nearly 400000 miles. They represent about 17.6% of the total railroad mileage, exclusive of the United States and Canada, where they are not used at all, except experimentally. In the four years from 1890 to 1894 the use of metal track increased from less than 25000 miles to nearly 35000 miles. This increase was practically equal to the total increase in railroad mileage during that time, exclusive of the increase in the United States and Canada. This indicates a large growth in the percentage of metal track to total mileage, and therefore an increased appreciation of the advantages to be derived from their use.

219. Durability. The durability of metal track is still far from being a settled question, due largely to the fact that the best form for such track is not yet determined, and that a large part of the apparent failures in metal track have been evidently due to defective design. Those in favor of them estimate the life as from 30 to 50 years. The opponents place it as not more than 20 years, or perhaps as long as the best of wooden ties.

* Bulletin No. 9, U. S. Dept. of Agriculture, Div. of Forestry.

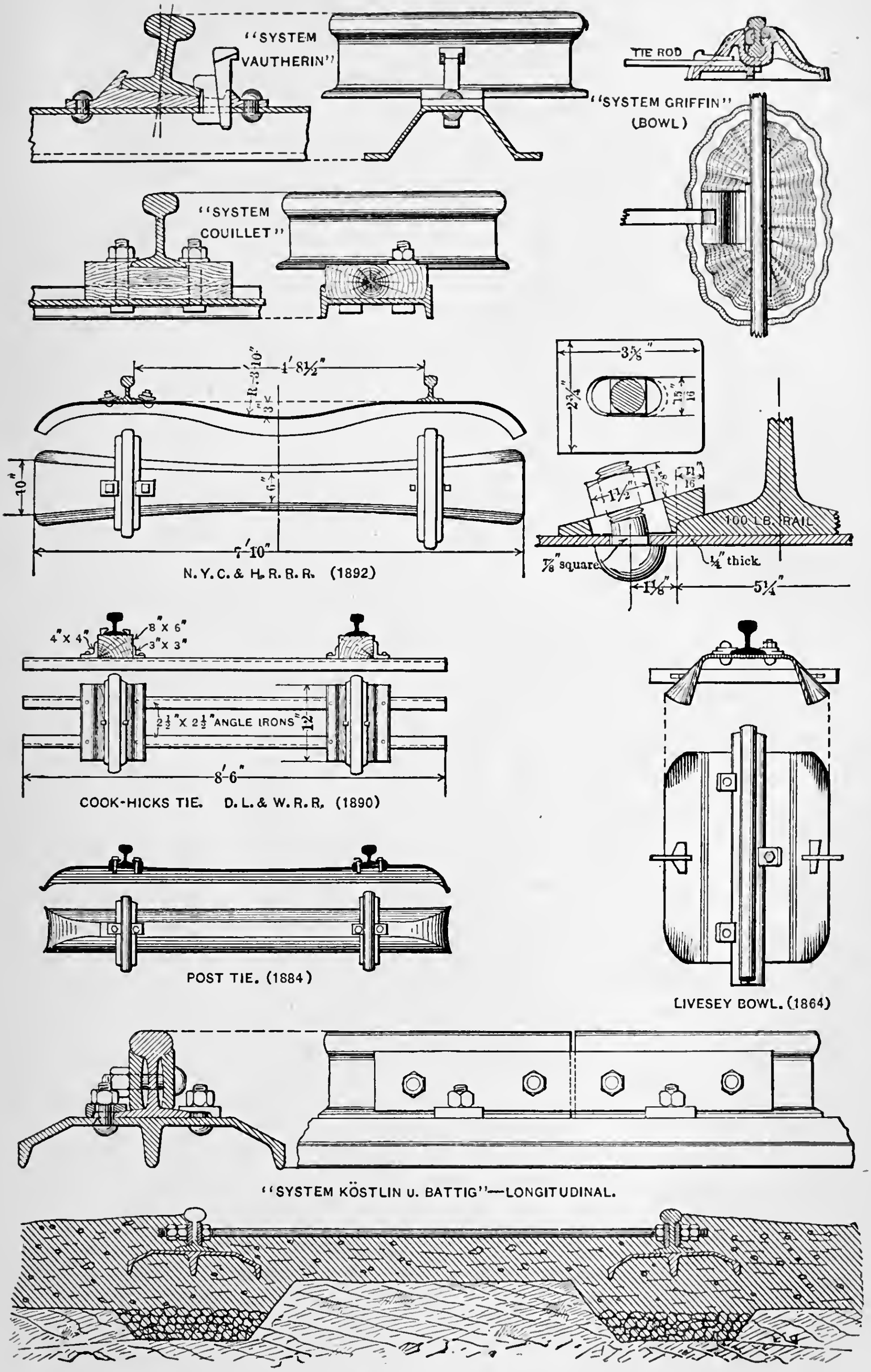
Unlike the wooden tie, however, which deteriorates as much with time as with usage, the life of a metal tie is more largely a function of the traffic. The life of a well-designed metal tie has been estimated at 150000 to 200000 trains; for 20 trains per day, or say 6000 per year, this would mean from 25 to 33 years. 20 trains per day on a *single* track is a much larger number than will be found on the majority of railroads. Metal ties are found to be subject to rust, especially when in damp localities, such as tunnels; but on the other hand it is in such confined localities, where renewals are troublesome, that it is especially desirable to employ the best and longest-lived ties. Paint, tar, etc., have been tried as a protection against rust, but the efficacy of such protection is as yet uncertain, the conditions preventing any renewal of the protection—such as may be done by repainting a bridge, for example. Failures in metal cross-ties have been largely due to cracks which begin at a corner of one of the *square* holes which are generally *punched* through the tie, the holes being made for the bolts by which the rails are fastened to the tie. The holes are generally *punched* because it is cheaper. Reaming the holes after punching is thought to be a safeguard against this frequent cause of failure. Another method is to round the corners of the square punch with a radius of about $\frac{1}{8}''$. If a crack has already started, the spread of the crack may be prevented by drilling a small hole at the end of it.

220. Form and dimensions of metal cross-ties. Since stability in the ballast is an essential quality for a tie, this must be accomplished either by turning down the end of the tie or by having some form of lug extending downward from one or more points of the tie. The ties are sometimes depressed in the center (see Plate XVII, N. Y. C. & H. R. R.R. tie) to allow for a thick covering of ballast on top in order to increase its stability in the ballast. This form requires that the ties should be sufficiently well tamped to prevent a tendency to bend out straight, thus widening the gauge. Many designs of ties are objectionable because they cannot be placed in the track without disturbing adjacent ties. The failure of many metal cross-ties, otherwise of good design, may be ascribed to too light weight. Those weighing much less than 100 pounds have proved too light. From 100 to 130 pounds weight is being used satisfactorily on German railroads. The general outside dimensions are about the same as for wooden ties, except as to thickness. The metal is generally from $\frac{1}{4}''$ to $\frac{3}{8}''$ thick. They are, of course, only made of wrought iron or steel, cast iron being used only for "bowls" or "plates" (see § 213). The details of construction of some of the most commonly used ties may be seen by a study of Plate XVII.

221. Fastenings. The devices for fastening the rails to the ties should be such that the gauge may be widened if desired on curves, also that the gauge can be made true regardless of slight inaccuracies in the manufacture of the ties, and also that shims may be placed under the rail if necessary during cold weather when the tie is frozen into the ballast and cannot be easily disturbed. Some methods of fastening require that the base of the rail be placed against a lug which is riveted to the tie or which forms a part of it. This has the advantage of reducing the number of pieces, but is apt to have one or more of the disadvantages named above. Metal keys or wooden wedges are sometimes used, but the majority of designs employ some form of bolted clamp. The form adopted for the experimental ties used by the N. Y. C. & H. R. R.R. (see Plate XVII) is especially ingenious in the method used to vary the gauge or allow for inaccuracies of manufacture. Plate XVII shows some of the methods of fastening adopted on the principal types of ties.

222. Cost. The cost of metal cross-ties in Germany averages about 1.6 c. per pound or about $1.60 for a 100-lb. tie. The ties manufactured for the N. Y. C. & H. R. R.R. in 1892 weighed about 100 lbs. and cost $2.50 per tie, but if they had been made in larger quantities and with the present price of steel the cost would possibly have been much lower. The item of freight from the place of manufacture to the place where used is no inconsiderable item of cost with some roads. Metal cross-ties have been used by some street railroads in this country.

PLATE XVII.



METAL TIES.

(*To face page* 240)

Those used on the Terre Haute Street Railway weigh 60 pounds and cost about 66 c. for the tie, or 74 c. per tie with the fastenings.

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223. Bowls or plates. As mentioned before, over 12000 miles of railway, chiefly in British India and in the Argentine Republic, are laid with this form of track. It consists essentially of large cast-iron inverted "bowls" laid at intervals under each rail and opposite each other, the opposite bowls being tied together with tie-rods. A suitable chair is riveted or bolted on to the top of each bowl so as to properly hold the rail. Being made of cast iron, they are not so subject to corrosion as steel or wrought iron. They have the advantage that when old and worn out their scrap value is from 60 to 80% of their initial cost, while the scrap value of a steel or wrought-iron tie is practically nothing. Failure generally occurs from breakage, the failures from this cause in India being about 0.4 per cent per annum. They weigh about 250 lbs. apiece and are therefore quite expensive in first cost and transportation charges. There are miles of them in India which have already lasted 25 years and are still in a serviceable condition. Some illustrations of this form of tie are shown in Plate XVII.

224. Longitudinals.* This form, the use of which is confined almost exclusively to Germany, is being gradually replaced on many lines by metal cross-ties. The system generally consists of a compound rail of several parts, the upper bearing rail being very light and supported throughout its length by other rails, which are suitably tied together with tie-rods so as to maintain the proper gauge, and which have a sufficiently broad

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* Although the discussion of longitudinals might be considered to belong more properly to the subject of RAILS, yet the essential idea of all designs must necessarily be the *support* of a rail-head on which the rolling stock may run, and therefore this form, unused in this country, will be briefly described here.

base to be properly supported in the ballast. One great objection to this method of construction is the difficulty of obtaining proper drainage especially on grades, the drainage having a tendency to follow along the lines of the rails. The construction is much more complicated on sharp curves and at frogs and switches. Another fundamentally different form of longitudinal is the Haarman compound "self-bearing" rail, having a base 12″ wide and a height of 8″, the alternate sections breaking joints so as to form a practically continuous rail.

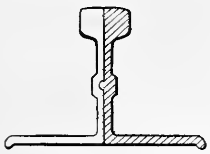

FIG. 110.

Some of the other forms of longitudinals are illustrated in Plate XVII.

For a very complete discussion of the subject of metal ties, see the "Report on the Substitution of Metal for Wood in Railroad Ties" by E. E. Russell Tratman, it being Bulletin No. 4, Forestry Division of the U. S. Dept. of Agriculture.

CHAPTER IX.

RAILS.

225. Early forms. The first rails ever laid were wooden stringers which were used on very short tram-roads around coal-mines. As the necessity for a more durable rail increased, owing chiefly to the invention of the locomotive as a motive power, there were invented successively the cast-iron "fish-belly" rail and various forms of wrought-iron strap rails which finally developed into the T rail used in this country and the double-headed rail, supported by chairs, used so extensively in England. The cast-iron rails were cast in lengths of about 3 feet and were supported in iron chairs which were sometimes set upon stone piers. A great deal of the first railroad track of this country was laid with longitudinal stringers of wood placed upon cross-ties, the inner edge of the stringers being

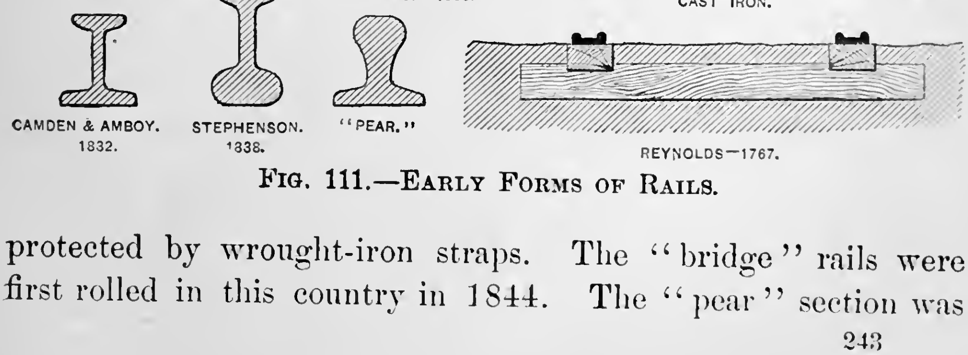

FIG. 111.—EARLY FORMS OF RAILS.

protected by wrought-iron straps. The "bridge" rails were first rolled in this country in 1844. The "pear" section was an approach to the present form, but was very defective on account of the difficulty of designing a good form of joint. The "Stevens" section was designed in 1830 by Col. Robert L. Stevens, Chief Engineer of the Camden and Amboy Railroad; although quite defective in its proportions, according to the present knowledge of the requirements, it is essentially the present form. In 1836, Charles Vignoles invented essentially the same form in England; this form is therefore known throughout England and Europe as the Vignoles rail.

226. Present standard forms. The larger part of modern railroad track is laid with rails which are either "T" rails or the double-headed or "bull-headed" rails which are carried in chairs. The double-headed rail was designed with a symmetrical form with the idea that after one head had been worn out by traffic the rail could be reversed, and that its life would be practically doubled. Experience has shown that the wear of the rail in the chairs is very great; so much so that when one head has been worn out by traffic the whole rail is generally useless. If the rail is turned over, the worn places, caused by the chairs, make a rough track and the rail appears to be more brittle and subject to fracture, possibly due to the crystallization that may have occurred during the previous usage and to the reversal of stresses in the fibers. Whatever the explanation, experience has demonstrated the *fact*. The "bull-headed" rail has the lower head only large enough to properly hold the wooden keys with which the rail is secured to the chairs (see Fig. 112) and furnish the necessary strength. The use of these rails requires the use of two cast-iron chairs for each tie. It is claimed that such track is better for heavy and fast traffic, but it is more expensive to build and maintain. It is the standard form of track in England and some parts of Europe.

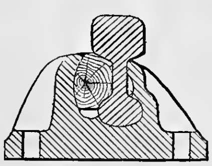

FIG. 112.—BULL-HEADED RAIL AND CHAIR.

Until a few years ago there was a very great multiplicity in the designs of "T" rails as used in this country, nearly every prominent railroad having its own special design, which perhaps differed from that of some other road by only a very minute and insignificant detail, but which nevertheless would require a complete new set of rolls for rolling. This certainly must have had a very appreciable effect on the cost of rails. In 1893, the American Society of Civil Engineers, after a very exhaustive investigation of the subject, extending over several years, having obtained the opinions of the best experts of the country, adopted a series of sections which have been very extensively adopted by the railroads of this country. Instead of having the rail sections for various weights to be geometrically similar figures, certain dimensions are made constant, regardless of the weight. It was decided that the metal should be distributed through the section in the proportions of—head 42%, web 21%, and flange 37%. The top of the head should have a radius of 12″; the top corner radius of head should be $\frac{5}{16}$″; the

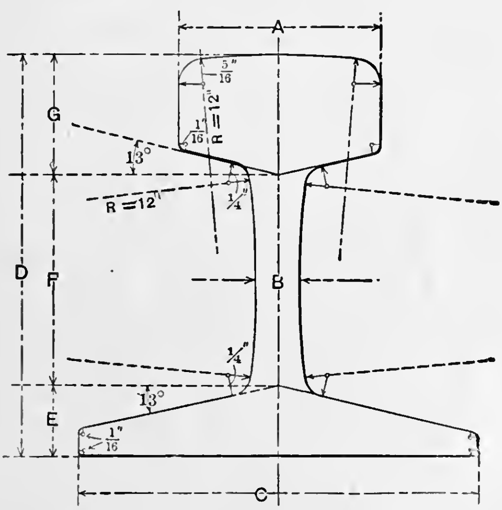

FIG. 113.—AM. SOC. C. E. STANDARD RAIL SECTION.

lower corner radius of head should be $\frac{1}{16}$″; the corners of the flanges, $\frac{1}{16}$″ radius; side radius of web, 12″; top and bottom radii of web corners, $\frac{1}{4}$″; and angles with the horizontal of the under side of the head and the top of the flange, 13°. The sides of the head are vertical.

The height of the rail (D) and the width of the base (C) are always made equal to each other.

	Weight per Yard.												
	40	45	50	55	60	65	70	75	80	85	90	95	100
A	$1\frac{7}{8}''$	$2''$	$2\frac{1}{8}''$	$2\frac{1}{4}''$	$2\frac{3}{8}''$	$2\frac{13}{32}''$	$2\frac{7}{16}''$	$2\frac{15}{32}''$	$2\frac{1}{2}''$	$2\frac{9}{16}''$	$2\frac{5}{8}''$	$2\frac{11}{16}''$	$2\frac{3}{4}''$
B	$\frac{25}{64}$	$\frac{27}{64}$	$\frac{7}{16}$	$\frac{15}{32}$	$\frac{31}{64}$	$\frac{1}{2}$	$\frac{33}{64}$	$\frac{17}{32}$	$\frac{35}{64}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$
C & D	$3\frac{1}{2}$	$3\frac{11}{16}$	$3\frac{7}{8}$	$4\frac{1}{16}$	$4\frac{1}{4}$	$4\frac{7}{16}$	$4\frac{5}{8}$	$4\frac{13}{16}$	5	$5\frac{3}{16}$	$5\frac{3}{8}$	$5\frac{9}{16}$	$5\frac{3}{4}$
E	$\frac{5}{8}$	$\frac{21}{32}$	$\frac{11}{16}$	$\frac{23}{32}$	$\frac{49}{64}$	$\frac{25}{32}$	$\frac{13}{16}$	$\frac{27}{32}$	$\frac{7}{8}$	$\frac{57}{64}$	$\frac{59}{64}$	$\frac{15}{16}$	$\frac{31}{32}$
F	$1\frac{55}{64}$	$1\frac{31}{32}$	$2\frac{1}{16}$	$2\frac{11}{64}$	$2\frac{17}{64}$	$2\frac{3}{8}$	$2\frac{15}{32}$	$2\frac{35}{64}$	$2\frac{5}{8}$	$2\frac{3}{4}$	$2\frac{55}{64}$	$2\frac{63}{64}$	$3\frac{5}{64}$
G	$1\frac{1}{64}$	$1\frac{1}{16}$	$1\frac{1}{8}$	$1\frac{11}{64}$	$1\frac{7}{32}$	$1\frac{9}{32}$	$1\frac{11}{32}$	$1\frac{27}{64}$	$1\frac{1}{2}$	$1\frac{25}{64}$	$1\frac{19}{32}$	$1\frac{41}{64}$	$1\frac{45}{64}$

The chief features of disagreement among railroad men relate to the radius of the upper corner of the head and the slope of the side of the head. The radius ($\frac{5}{16}''$) adopted for the upper corner (constant for all weights) is a little more than is advocated by those in favor of "sharp corners" who often use a radius of $\frac{1}{4}''$. On the other hand it is much less than is advocated by those who consider that it should be nearly equal to (or even greater than) the larger radius universally adopted for the corner of the wheel-flange. The discussion turns on the relative rapidity of rail wear and the wear of the wheel-flanges as affected by the relation of the form of the wheel-tread to that of the rail. It is argued that sharp rail corners wear the wheel-flanges so as to produce sharp flanges, which are liable to cause derailment at switches and also to require that the tires of engine-drivers must be more frequently turned down to their true form. On the other hand it is generally believed that rail wear is much less rapid while the area of contact between the rail and wheel-flange is small, and that when the rail has worn down, as it invariably does, to nearly the same form as the wheel-flange, the rail wears away very quickly.

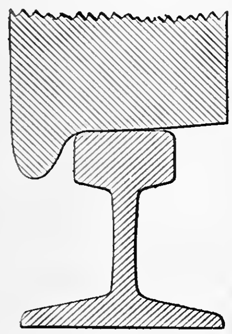

FIG. 114. — RELATION OF RAIL TO WHEEL-TREAD.

227. Weight for various kinds of traffic. The heaviest rails in regular use weigh 100 lbs. per yard, and even these are only used on some of the heaviest traffic sections of such roads as the N. Y. Central, the Pennsylvania, the N. Y., N. H. & H., and a few others. Probably the larger part of the mileage of the country is laid with 60- to 75-lb. rails—considering the fact that "the larger part of the mileage" consists of comparatively light-traffic roads and may exclude all the heavy trunk lines. Very light-traffic roads are sometimes laid with 56-lb. rails. Roads with fairly heavy traffic generally use 75- to 85-lb. rails, especially when grades are heavy and there is much and sharp curvature. The tendency on all roads is toward an increase in the weight, rendered necessary on account of the increase in the weight and capacity of rolling stock, and due also to the fact that the price of rails has been so reduced that it is both better and cheaper to obtain a more solid and durable track by increasing the weight of the rail rather than by attempting to support a weak rail by an excessive number of ties or by excessive track labor in tamping. It should be remembered that in buying rails the mere weight is, in one sense, of no importance. The important thing to consider is the STRENGTH and the STIFFNESS. If we assume that all weights of rails have *similar* cross-sections (which is nearly although not exactly true), then, since for beams of similar cross-sections the *strength* varies as the *cube* of the homologous dimensions and the *stiffness* as the *fourth power*, while the area (and therefore the weight per unit of length) only varies as the *square*, it follows that the stiffness varies as the square of the weight, and the strength as the $\frac{3}{2}$ power of the weight. Since for ordinary variations of weight the price per ton is the same, adding (say) 10% to the weight (and cost) adds 21% to the stiffness and over 15% to the strength. As another illustration, using an 80-lb. rail instead of a 75-lb. rail adds only $6\frac{2}{3}$% to the cost, but adds about 14% to the stiffness and nearly 11% to the strength. This shows why heavier rails are more economical and are being adopted even when they are not absolutely needed on account of heavier rolling stock. The stiffness, strength, and consequent durability are increased in a much greater ratio than the cost.

228. Effect of stiffness on traction. A very important but generally unconsidered feature of a stiff rail is its effect on tractive force. An extreme illustration of this principle is seen when a vehicle is drawn over a soft sandy road. The constant compression of the sand in front of the wheel has virtually the same effect on traction as drawing the wheel up a grade whose steepness depends on the radius of the wheel and the depth of the rut. On the other hand, if a wheel, made of perfectly elastic material, is rolled over a surface which, while supported with absolute rigidity, is also perfectly elastic, there would be a forward component, caused by the expanding of the compressed metal just behind the center of contact, which would just balance the backward component. If the rail was supported throughout its length by an absolutely rigid support, the high elasticity of the wheel-tires and rails would reduce this form of resistance to an insignificant quantity, but the ballast and even the ties are comparatively inelastic. When a weak rail yields, the ballast is more or less compressed or displaced, and even though the elasticity of the rail brings it back to nearly its former place, the work done in compressing an inelastic material is wholly lost. The effect of this on the fuel account is certainly very considerable and yet is frequently entirely overlooked. It is practically impossible to compute the saving in tractive power, and therefore in cost of fuel, resulting from a given increase in the weight and stiffness of the rail, since the yielding of the rail is so dependent on the spacing of the ties, the tamping, etc. But it is not difficult to perceive in a general way that such an economy is possible and that it should not be neglected in considering the value of stiffness in rails.

229. Length of rails. The standard length of rails with most railroads is 30 feet. In recent years many roads have been trying 45-foot and even 60-foot rails. The argument in favor of longer rails is chiefly that of the reduction in track-joints, which are costly to construct and to maintain and are a fruitful source of accidents. Mr. Morrison of the Lehigh Valley R.R.* declares that, as a result of extensive experience with 45-foot rails

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* Report, Roadmasters Association, 1895.

on that road, he finds that they are much less expensive to handle, and that, being so long, they can be laid around sharp curves without being curved in a machine, as is necessary with the shorter rails. The great objection to longer rails lies in the difficulty in allowing for the expansion, which will require, in the coldest weather, an opening at the joint of nearly $\frac{3}{4}''$ for a 60-foot rail. The Pennsylvania R.R. and the Norfolk and Western R.R. each have a considerable mileage laid with 60-foot rails.

230. Expansion of rails. Steel expands at the rate of .0000065 of its length per degree Fahrenheit. The extreme range of temperature to which any rail will be subjected will be about 160°, or say from $-$ 20° F. to $+$ 140° F. With the above coefficient and a rail length of 60 feet the expansion would be 0.0624 foot, or about $\frac{3}{4}$ inch. But it is doubtful whether there would ever be such a range of motion even if there were such a range of temperature. Mr. A. Torrey, chief engineer of the Mich. Cent. R.R., experimented with a section over 500 feet long, which, although not a single rail, was made "continuous" by rigid splicing, and he found that there was no appreciable additional contraction of the rail at any temperature below $+$ 20 F. The reason is not clear, but the *fact* is undeniable.

The heavy girder rails, used by the street railroads of the country, are bonded together with perfectly tight rigid joints which do not permit expansion. If the rails are laid at a temperature of 60° F. and the temperature sinks to 0°, the rails have a *tendency* to contract .00039 of their length. If this tendency is resisted by the friction of the pavement in which the rails are buried, it only results in a tension amounting to .00039 of the modulus of elasticity, or say 10920 pounds per square inch, assuming 28 000 000 as the modulus of elasticity. This stress is not dangerous and may be permitted. If the temperature rises to 120° F., a tendency to expansion and buckling will take place, which will be resisted as before by the pavement, and a compression of 10920 pounds per square inch will be induced, which will likewise be harmless. The range of temperature of rails which are buried in pavement is much less than when they are entirely above the ground and will probably never reach the above extremes. Rails supported on ties which are only held in place by ballast must be allowed to expand and contract almost freely, as the ballast cannot be depended on to resist the distortion induced by any considerable range of temperature, especially on curves.

231. Rules for allowing for temperature. Track regulations generally require that the track foremen shall use iron (*not* wooden) shims for placing between the ends of the rails while splicing them. The thickness of these shims should vary with the temperature. Some roads use such approximate rules as the following: "The proper thickness for coldest weather is $\frac{5}{16}$ of an inch; during spring and fall use $\frac{1}{8}$ of an inch, and in the very hottest weather $\frac{1}{16}$ of an inch should be allowed." This is on the basis of a 30-foot rail. When a more accurate adjustment than this is desired, it may be done by assuming some very high temperature (120° to 150° F.) as a maximum, when the joints should be *tight;* then compute in tabular form the spacing for each temperature, varying by 20°, allowing 0″.0468 (almost exactly $\frac{3}{64}$″) for each 20° change. Such a tabular form would be about as follows (rail length 30 feet):

Temperature.	150°	130°	110°	90°	70°	50°	30°	10°	− 10°	− 30°
Rail opening...	0	$\frac{3}{64}$″	$\frac{3}{32}$″	$\frac{9}{64}$″	$\frac{3}{16}$″	$\frac{15}{64}$″	$\frac{9}{32}$″	$\frac{21}{64}$″	$\frac{3}{8}$″	$\frac{27}{64}$″

One practical difficulty in the way of great refinement in this work is the determination of the real temperature of the rail when it is laid. A rail lying in the hot sun has a very much higher temperature than the air. The temperature of the rail cannot be obtained even by exposing a thermometer directly to the sun, although such a result might be the best that is easily obtainable. On a cloudy or rainy day the rail has practically the same temperature as the air; therefore on such days there need be no such trouble.

232. Chemical composition. About 98 to 99.5% of the composition of steel rails is iron, but the value of the rail, as a rail, is almost wholly dependent upon the large number of other chemical elements which are, or may be, present in very small amounts. The manager of a steel-rail mill once declared that their aim was to produce rails having in them—

Carbon	0.32 to 0.40%
Silicon	0.04 to 0.06%
Phosphorus	0.09 to 0.105%
Manganese	1.00 to 1.50%

The analysis of 32 specimens of rails on the Chic., Mil. & St. Paul R.R. showed variations as follows:

Carbon	0.211 to 0.52%
Silicon	0.013 to 0.256%
Phosphorus	0.055 to 0.181%
Manganese	0.35 to 1.63%

These quantities have the same general relative proportions as the rail-mill standard given above, the differences lying mainly in the broadening of the limits. Increasing the percentage of carbon by even a few hundredths of one per cent makes the rail harder, but likewise more brittle. If a track is well ballasted and not subject to heaving by frost, a harder and more brittle rail may be used without excessive danger of breakage, and such a rail will wear much longer than a softer tougher rail, although the softer tougher rail may be the better rail for a road having a less perfect roadbed.

A small but objectionable percentage of sulphur is sometimes found in rails, and very delicate analysis will often show the presence, in very minute quantities, of several other chemical elements. The use of a very small quantity of nickel or aluminum has often been suggested as a means of producing a more durable rail. The added cost and the uncertainty of the amount of advantage to be gained has hitherto prevented the practical use or manufacture of such rails.

233. Testing. Chemical and mechanical testing are both necessary for a thorough determination of the value of a rail. The chemical testing has for its main object the determination of those minute quantities of chemical elements which have such a marked influence on the rail for good or bad. The mechanical testing consists of the usual tests for elastic limit, ultimate strength, and elongation at rupture, determined from pieces cut out of the rail, besides a "drop test." The drop test consists in dropping a weight of 2000 lbs. from a height of 16 to 20 feet on to the center of a rail which is supported on abutments placed three or four feet apart. The number of blows required to produce rupture or to produce a permanent set of specified magnitude gives a measure of the strength and toughness of the rail.

234. Rail wear on tangents. When the wheel loads on a rail are abnormally heavy, and particularly when the rail has but little carbon and is unusually soft, the concentrated pressure on the rail is frequently greater than the elastic limit, and the metal "flows" so that the head, although greatly abraded, will spread somewhat outside of its original lines, as shown in Fig. 115. The rail wear that occurs on tangents is almost exclusively on top. Statistics show that the rate of rail wear on tangents decreases as the rails are more worn. Tests of a large number of rails on tangents have shown a rail wear averaging nearly one pound per yard per 10 000 000 tons of traffic. There is about 33 pounds of metal in one yard of the head of an 80-lb. rail. As an extreme value this may be worn down one-half, thus giving a tonnage of 165 000 000 tons for the life of the rail. Other estimates bring the tonnage down to 125 000 000 tons. Since the locomotive is considered to be responsible for one half (and possibly more) of the damage done to the rail, it is found that the rate of wear on roads with shorter trains is more rapid in proportion to the tonnage, and it

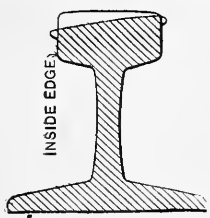

FIG. 115.

is therefore thought that the life of a rail should be expressed in terms of the number of trains. This has been estimated at 300 000 to 500 000 trains.

235. Rail wear on curves. On curves the maximum rail wear occurs on the inner side of the head of the outer rail, giving a worn form somewhat as shown in Fig. 116. The dotted line shows the nature and progress of the rail wear on the inner rail of a curve. Since the pressure on the outer rail is somewhat lateral rather than vertical, the "flow" does not take place to the same extent, if at all, on the outside, and whatever flow would take place on the inside is immediately worn off by the wheel-flange. Unlike the wear on tangents, the wear on curves is at a greater rate as the rail becomes more worn.

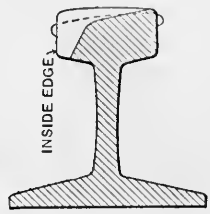

FIG. 116.

The inside rail on curves wears chiefly on top, the same as on a tangent, except that the wear is much greater owing to the longitudinal slipping of the wheels on the rail, and the lateral slipping that must occur when a rigid four-wheeled truck is guided around a curve. The outside rail is subjected to a greater or less proportion of the longitudinal slipping, likewise to the lateral slipping, and, worst of all, to the grinding action of the flange of the wheel, which grinds off the side of the head.

The results of some very elaborate tests, made by Mr. A. M. Wellington, on the Atlantic and Great Western R.R., on the wear of rails, seem to show that the rail wear on curves may be expressed by the formula: "Total wear of rails on a d degree curve in pounds per yard per 10 000 000 tons duty $= 1 + 0.03d^2$." "It is not pretended that this formula is strictly correct even in theory, but several theoretical considerations indicate that it may be nearly so." According to this formula the average rail wear on a 6° curve will be about twice the rail wear on a tangent. While this is approximately true, the various causes modifying the rate of rail wear (length of trains, age and quality of rails, etc.) will result in numerous and large variations from the above formula, which should only be taken as indicating an approximate law.

236. Cost of rails. In 1873 the cost of steel rails was about $120 per ton, and the cost of iron rails about $70 per ton. Although the steel rails were at once recognized as superior to iron rails on account of more uniform wear, they were an expensive luxury. The manufacture of steel rails by the Bessemer process created a revolution in prices, and they have steadily dropped in price until, during the last few years, steel rails have been manufactured and sold for $22 per ton. At such prices there is no longer any demand for iron rails, since the cost of manufacturing them is substantially the same as that of steel rails, while their durability is unquestionably inferior to that of steel rails.

CHAPTER X.

RAIL-FASTENINGS.

RAIL-JOINTS.

237. Theoretical requirements for a perfect joint. A perfect rail-joint is one that has the *same strength* and *stiffness*—no more and no less—as the rails which it joins, and which will not interfere with the regular and uniform spacing of ties. It should also be reasonably cheap both in first cost and in cost of maintenance. Since the action of heavy loads on an elastic rail is to cause a wave of translation in front of each wheel, any change in the stiffness or elasticity of the rail structure will cause more or less of a shock, which must be taken up and resisted by the joint. The greater the change in stiffness the greater the shock, and the greater the destructive action of the shock. The perfect rail-joint must keep both rail ends truly in line both laterally and vertically, so that the flange or tread of the wheel need not jump or change its direction of motion suddenly in passing from one rail to the other. A consideration of all the above requirements will show that only a perfect welding of rail-ends would produce a joint of uniform strength and stiffness which would give a uniform elastic wave ahead of each wheel. As welding is impracticable for ordinary railroad work (see § 230), some other contrivance is necessary which will approach this ideal as closely as may be.

238. Efficiency of the ordinary angle-bar. Throughout the middle portion of a rail the rail acts as a continuous girder. If we consider for simplicity that the ties are unyielding, the deflection of such a continuous girder between the ties will be but one-fourth of the deflection that would be found if the rail were cut half-way between the ties and an equal concentrated load were divided equally between the two unconnected ends. The maximum stress for the continuous girder would be but one-half of that in the cantilevers. Joining these ends with rail-joints will give the ordinary "suspended" joint. In order to maintain uniform strength and stiffness the angle-bars must supply the deficiency. These theoretical relations are modified to an unknown extent by the unknown and variable yielding of the ties. From some experiments made by the Association of Engineers of Maintenance of Way of the P. R.R.* the following deductions were made:

1. The capacity of a "suspended" joint is greater than that of a "supported" joint—whether supported on one or three ties. (See § 240.)

2. That (with the particular patterns tested) the angle-bars alone can carry only 53 to 56% of a concentrated load placed on a joint.

3. That the capacity of the whole joint (angle-bars and rail) is only 52.4% of the strength of the unbroken rail.

4. That the ineffectiveness of the angle-bar is due chiefly to a deficiency in compressive resistance.

Although it has been universally recognized that the angle-bar is not a perfect form of joint, its simplicity, cheapness, and reliability have caused its almost universal adoption. Within a very few years other forms (to be described later) have been adopted on trial sections and have been more and more extended, until their present use is very large. The present time (1900) is evidently a transition period, and it is quite probable that within a very few years the now common angle-plate will be as unknown in standard practice as the old-fashioned "fish-plate" is at the present time.

239. Effect of rail gap at joints. It has been found that the jar at a joint is due almost entirely to the *deflection* of the joint

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* Roadmasters Association of America—Reports for 1897.

and scarcely at all to the small gap required for expansion. This gap causes a drop equal to the versed sine of the arc having a chord equal to the gap and a radius equal to the radius of the wheel. Taking the extreme case (for a 30-foot rail) of a $\frac{3}{8}''$ gap and a 33″ freight-car wheel, the drop is about $\frac{1}{1000}''$. In order to test how much the jarring at a joint is due to a gap between the rails, the experiment was tried of cutting shallow notches in the top of an otherwise solid rail and running a locomotive and an inspection car over them. The resulting jarring was practically imperceptible and not comparable to the jar produced at joints. Notwithstanding this fact, many plans have been tried for avoiding this gap. The most of these plans consist essentially of some form of compound rail, the sections breaking joints. (Of course the design of the compound rail has also several other objects in view.) In Fig. 117 are shown a

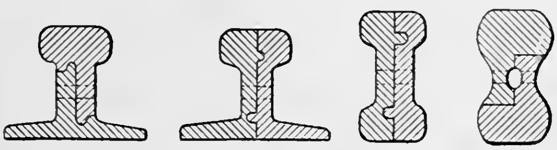


FIG. 117.—COMPOUND RAIL SECTIONS.

few of the very many designs which have been proposed. These designs have invariably been abandoned after trial. Another plan, which has been extensively tried on the Lehigh Valley R.R., is the use of mitered joints. The advantages gained by their use are as yet doubtful, while the added expense is unquestionable. The "Roadmasters Association of America" in 1895 adopted a resolution recommending mitered joints for double track, but their use does not seem to be growing.

240. "Supported," "suspended," and "bridge" joints. In a supported joint the ends of the rails are on a tie. If the angle-plates are short, the joint is entirely supported on one tie; if very long, it may be possible to place three ties under one angle-bar and thus the joint is virtually supported on three ties rather than one. In a suspended joint the ends of the rails are midway between two ties and the joint is supported by the two. There have always been advocates of both methods, but suspended joints are more generally used than supported joints. The opponents of three-tie joints claim that either the middle tie will be too strongly tamped, thus making it a supported joint, or that, if the middle tie is weakest, the joint becomes a very long (and therefore weak) suspended joint between the outer joint-ties, or that possibly one of the outer joint-ties gives way, thus breaking the angle-plate at the joint. Another objection which is urged is that unless the bars are very long (say 44 inches, as used on the Mich. Cent. R.R.) the ties are too close for proper tamping. The best answer to these objections is the successful use of these joints on several heavy-traffic roads.

"Bridge"-joints are similar to suspended joints in that the joint is supported on two ties, but there is the important difference that the bridge-joint supports the rail from *underneath* and there is no transverse stress in the rail, whereas the supported joint requires the combined transverse strength of both angle-bars and rail. A serious objection to bridge-joints lies in the fact of their considerable thickness between the rail base and the tie. When joints are placed "staggered" rather than "opposite" (as is now the invariable standard practice), the ties supporting a bridge-joint must either be notched down, thus weakening the tie and promoting decay at the cut, or else the tie must be laid on a slope and the joint and the opposite rail do not get a fair bearing.

241. Failures of rail-joints. It has been observed on double-track roads that the maximum rail wear occurs a few inches beyond the rail gap at the joint in the direction of the traffic. On single-track roads the maximum rail wear is found a few inches *each* side of the joint rather than at the extreme ends of the rail, thus showing that the rail end deflects down under the wheel until (with fast trains especially) the wheel actually jumps the space and strikes the rail a few inches beyond the joint, the impact producing excessive wear. This action, which is called the "drop," is apt to cause the first tie beyond the joint to become depressed, and unless this tie is carefully watched and maintained at its proper level, the stresses in the angle-bar may actually become reversed and the bar may break at the top. The angle-bars of a suspended joint are normally in compression at the top. The mere reversal of the stresses would cause the bars

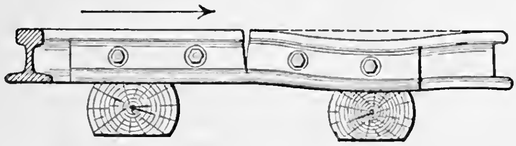


FIG. 118.—EFFECT OF "WHEEL DROP" (EXAGGERATED).

to give way with a less stress than if the stress were always the same in kind. A supported joint, and especially a three-tie joint (see § 240), is apt to be broken in the same manner.

242. Standard angle-bars.—An angle-bar must be so made as to closely fit the rails. The great multiplicity in the designs of rails (referred to in Chapter IX) results in nearly as great variety in the detailed dimensions of the angle-bars. The sections here illustrated must be considered only as types of the variable forms necessary for each different shape of rail. The absolutely essential features required for a fit are (1) the angles

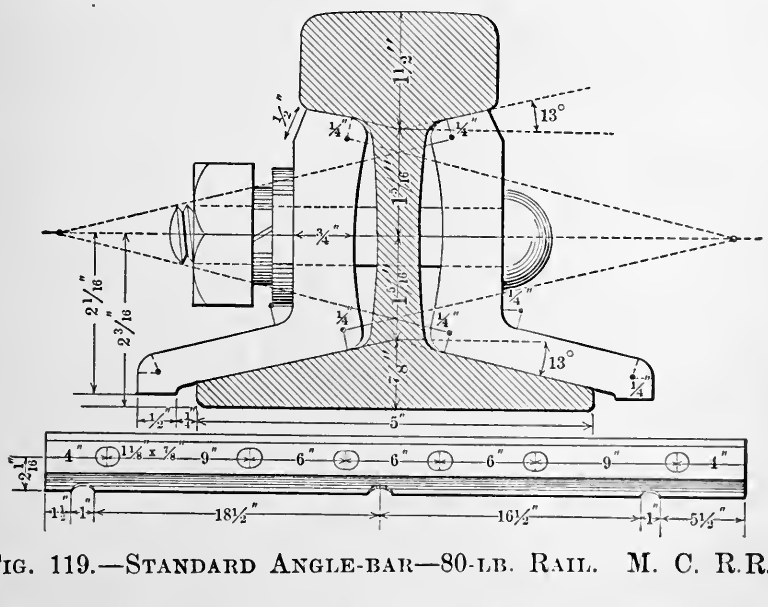


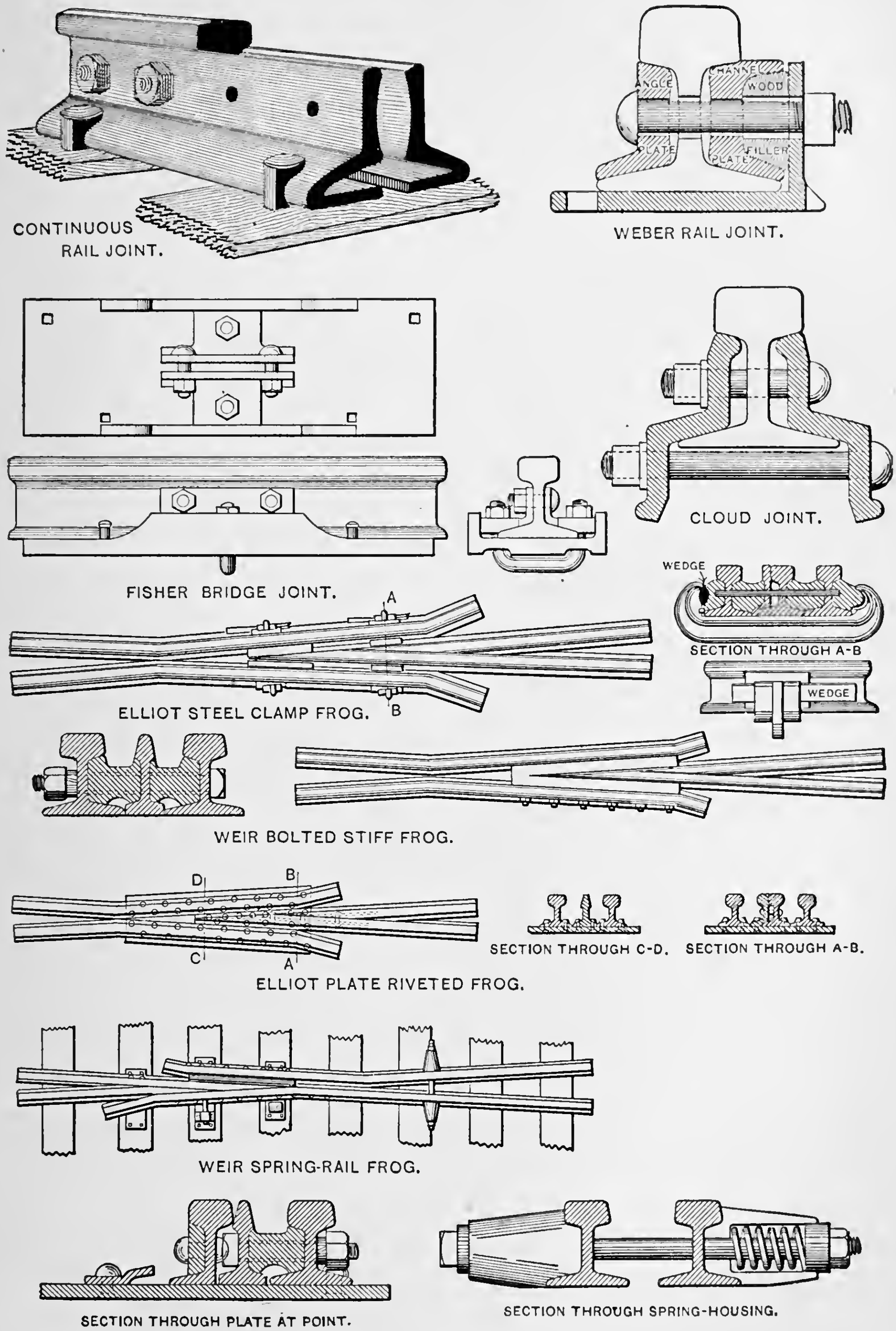
FIG. 119.—STANDARD ANGLE-BAR—80-LB. RAIL. M. C. R.R.

of the upper and lower surfaces of the bar where they fit against the rail, and (2) the height of the bar. The bolt-holes in the bar and rail must also correspond. The holes in the angle-plates are elongated or made oval, so that the track-bolts, which are made of corresponding shape immediately under the head, will not be turned by jarring or vibration. The holes in the rails are made of larger diameter (by about $\frac{1}{4}''$) than the bolts, so as to allow the rail to expand with temperature.

243. Later designs of rail-joints. In Plate XVIII are shown various designs which are competing for adoption. The most prominent of these (judging from the discussion in the convention of the Roadmasters Association of America in 1897) are the "Continuous" and the "Weber." Each of them has been very extensively adopted, and where used are universally preferred to angle-plates. Nearly all the later designs embody more or less directly the principle of the bridge-joint, i.e., support the rail from underneath. An experience of several years will be required to demonstrate which form of joint best satisfies the somewhat opposed requirements of minimum cost (both initial and for maintenance) and minimum wear of rails and rolling stock.

TIE-PLATES.

244. Advantages. (*a*) As already indicated in § 204, the life of a soft-wood tie is very much reduced by "rail-cutting" and "spike-killing," such ties frequently requiring renewal long before any serious decay has set in. It has been practically demonstrated that the "rail-cutting" is not due to the mere pressure of the rail on the tie, even with a maximum load on the rail, but is due to the impact resulting from vibration and to the longitudinal working of the rail. It has been proved that this rail-cutting is practically prevented by the use of tie-plates. (*b*) On curves there is a tendency to overturn the outer rail due to the lateral pressure on the side of the head. This produces a concentrated pressure of the outer edge of the base on the tie which produces rail-cutting and also draws the inner spikes. Formerly the only method of guarding



Rail Joints and Frogs.

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against this was by the use of "rail-braces," one pattern of which is shown in Fig. 120. But it has been found that tie-

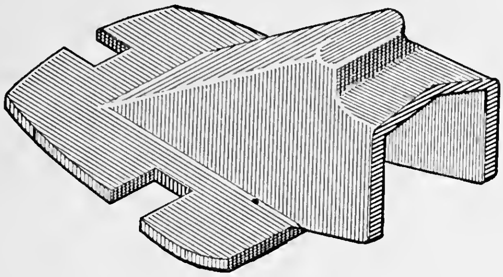

FIG. 120.

plates serve the purpose even better, and rail-braces have been abandoned where tie-plates are used. (*c*) Driving spikes through holes in the plate enables the spikes on *each* side of the rail to mutually support each other, no matter in which (lateral) direction the rail may tend to move, and this probably accounts in large measure for the added stability obtained by the use of tie-plates. (*d*) The wear in spikes, called "necking," caused by the vertical vibration of the rail against them, is very greatly reduced. (*e*) The cost is very small compared with the value of the added life of the tie, the large reduction in the work of track maintenance, and the smoother running on the better track which is obtained. It has been estimated that by the use of tie-plates the life of hard-wood ties is increased from one to three years, and the life of soft-wood ties is increased from three to six years. From the very nature of the case, the value of tie-plates is greater when they are used to protect soft ties.

245. Elements of the design. The earliest forms of tie-plates were flat on the bottom, but it was soon found that they would work loose, allow sand and dirt to get between the rail and the plate and also between the plate and the tie, which would cause excessive wear. Such plates are also apt to produce an objectionable rattle. Another fault of the earlier designs was the use of plates so thin that they would buckle. The latest designs have flanges or "teeth" formed on the lower surface which penetrate the tie about $\frac{3}{4}''$ to $1\frac{3}{8}''$. Opinion is still divided on the question of whether these teeth should run with the grain or across the grain. If the flanges run with the grain, they generally extend the whole length of the tie-plate—as in the Wolhaupter design. If the grain is to be cut crosswise, several teeth about 1″ wide will be used—as in the Goldie design.

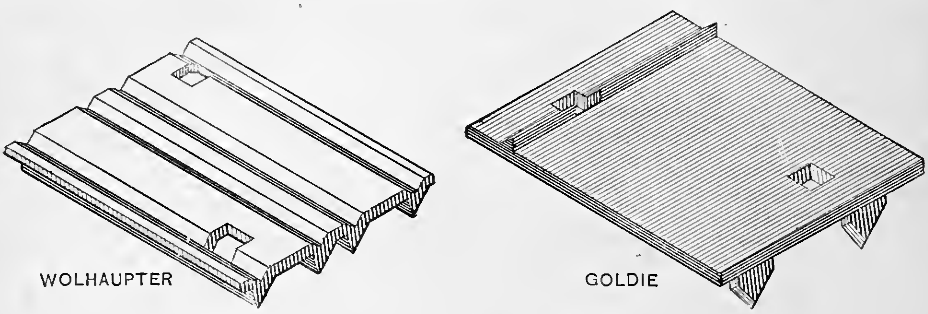

FIG. 121.—TIE-PLATES.

It is a very important feature that the spike-holes should be so punched that the spikes will fit closely to the base of the rail. Otherwise a lateral motion of the rail will be permitted which will defeat one of the main objects of the use of the plate.

Another unsettled detail is the use of "shoulders" on the upper surface. On the one hand it is claimed that the use of shoulders relieves the spikes of side pressure from the rail and prevents "necking." On the other hand it is claimed that if the plain plate is once properly set with new spikes (at least with spikes not already necked) the spikes will not neck appreciably, and that, as the shouldered plates cost more, the additional expenditure is unnecessary.

The above designs should be studied with reference to the manner in which they fulfill the requirements which have been already stated. As in the case of rail-joints, the best forms of tie-plates are of comparatively recent design, and experience with them is still insufficient to determine beyond all question which designs are the best.

246. Methods of setting. A very important detail in the process of setting the tie-plates on the ties is that the flanges or teeth should penetrate the tie as far as desired when the plates are first put in position. It requires considerable force to press the teeth into a tie. In a few cases trackmen have depended on the easy process of waiting for passing trains to force the teeth down. Until the teeth are down the spikes cannot be driven home, and this apparently cheap and easy process results in loose spikes and rails. If the trackmen neglect even temporarily to tighten these spikes, it will become impossible to make them tight ultimately. The plates are generally pounded into place with a 10- to 16-pound sledge-hammer. A very good method was adopted once during the construction of a bridge when a pile-driver was at hand. The bridge-ties were placed under the pile-hammer. The plates, accurately set to gauge, were then forced in by a blow from the 3000-lb. hammer falling 2 or 3 feet.

SPIKES.

247. Requirements. The rails must be held to the ties by a fastening which will not only give sufficient resistance, but which will retain its capacity for resistance. It must also be cheap and easily applied. The ordinary track-spike fulfills the last requirements, but has comparatively small resisting power, compared with screws or bolts. Worse than all, the tendency to vertical vibration in the rail produces a series of upward pulls on the spike that soon loosens it. When motion has once begun the capacity for resistance is greatly reduced, and but little more vibration is required to pull the spike out so much that redriving is necessary. Driving the spike to place again in the same hole is of small value except as a very temporary expedient, as its holding power is then very small. Redriving the spikes in new holes very soon "spike-kills" the tie. Many plans have been devised to increase the holding power of spikes, such as making them jagged, twisting the spike, swelling the spike at about the center of its length, etc. But it has been easily demonstrated that the fibers of the wood are generally so crushed and torn by driving such spikes that their holding power is less than that of the plain spike.

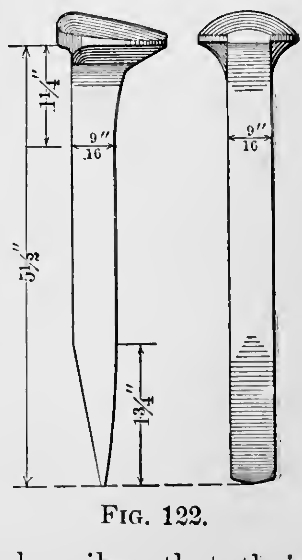

FIG. 122.

The ordinary spike (see Fig. 122) is made with a square cross-section which is uniform through the middle of its length, the lower $1\frac{3}{4}''$ tapering down to a chisel edge, the upper part swelling out to the head. The Goldie spike (see Fig. 123) aims to improve this form by reducing to a minimum the destruction of the fibers. To this end, the sides are made smooth, the edges are clean-cut, and the point, instead of being chisel-shaped, is ground down to a pyramidal form. Such fiber-cutting as occurs is thus accomplished without much crushing, and the fibers are thus pressed away from the spike and slightly downward. Any tendency to draw the spike will therefore cause the fibers to press still harder on the spike and thus increase the resistance.

FIG. 123.

248. Driving. The holding power of a spike depends largely on how it is driven. If the blows are eccentric and irregular in direction, the hole will be somewhat enlarged and the holding power largely decreased. The spikes on each side of the rail in any one tie should not be directly opposite, but should be staggered. Placing them directly opposite will tend to split the tie, or at least decrease the holding power of the spikes. The direction of staggering should be reversed in the two pairs of spikes in any one tie (see Fig. 124). This will tend to prevent any twisting of the tie in the ballast, which would otherwise loosen the rail from the tie.

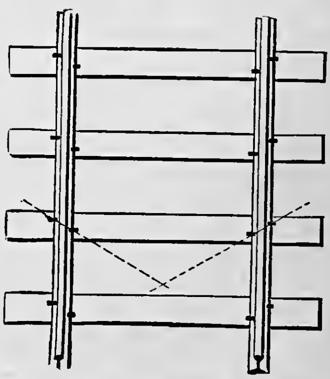

FIG. 124. SPIKE-DRIVING.

249. Screws and bolts. The use of these abroad is very extensive, but their use in this country has not passed the experimental stage. The screws are "wood"-screws (see Fig. 125), having large square heads, which are screwed down with a track-wrench. Holes, having the same diameter as the *base* of the screw-threads, should first be bored into the tie, at exactly the right position and at the proper angle with the vertical.

A light wooden frame is sometimes used to guide the auger at the proper angle. Sometimes the large head of the screw bears directly against the base of the rail, as with the ordinary spike. Other designs employ a plate, made to fit the rail on one side, bearing on the tie on the other side, and through which the screw passes. These screws cost much more than spikes and require more work to put in place, but their holding power is much greater and the work of track maintenance is very much less. Screw-bolts, passing entirely through the tie, having the head at the bottom of the tie and the nut on the upper side, are also used abroad. These are quite difficult to replace, requiring that the ballast be dug out beneath the tie, but on the other hand the occasions for replacing such a bolt are comparatively rare, as their durability is very great. The

FIG. 125.

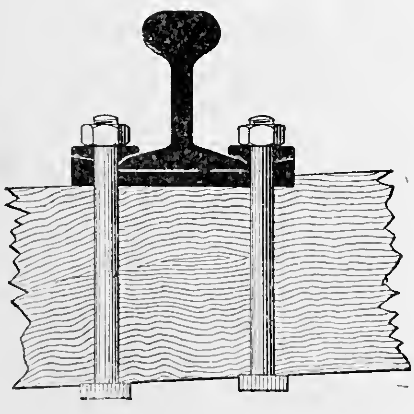

FIG. 126.

use of screws or bolts increases the life of the tie by the avoidance of "spike-killing." It is capable of demonstration that the reduced cost of maintenance and the resulting improvement in track would much more than repay the added cost of screws and bolts, but it seems impossible to induce railroad directors to authorize a large and immediate additional expenditure to make an annual saving whose value, although unquestionably considerable, cannot be exactly computed.

250. "Wooden spikes." Among the regulations for track-laying given in § 208, mention was made of wooden "spikes," or plugs, which are used to fill up the holes when spikes are withdrawn. The value of the policy of filling up these holes is unquestionable, since the expense is insignificant compared with the loss due to the quick and certain decay of the tie if these holes are allowed to fill with water and remain so. But the method of making these plugs is variable. On some roads they are "hand-made" by the trackmen out of otherwise useless scraps of lumber, the work being done at odd moments. This policy, while apparently cheap, is not necessarily so, for the hand-made plugs are irregular in size and therefore more or less inefficient. It is also quite probable that if a track gang is required to make their own plugs, they may spend time on these very cheap articles which could be more profitably employed otherwise. Since the holes made by the spikes are larger at the top than they are near the bottom, the plugs should *not* be of uniform cross section but should be slightly wedge-shaped. The "Goldie tie-plug" (see Fig. 127) has been designed to fill these requirements. Being machine-made, they are uniform in size; they are of a shape which will best fit the hole; they can be furnished of any desired wood, and at a cost which makes it a wasteful economy to attempt to cut them by hand.

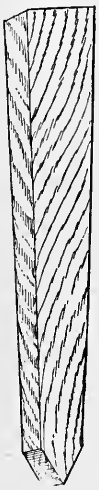

FIG. 127.

TRACK-BOLTS AND NUT-LOCKS.

251. Essential requirements. The track-bolts must have sufficient strength and must be screwed up tight enough to hold the angle-plates against the rail with sufficient force to develop the full transverse strength of the angle-bars. On the other hand the bolts should not be screwed so tight that slipping may not take place when the rail expands or contracts with temperature. It would be impossible to screw the bolts tight enough to prevent slipping during the contraction due to a considerable fall of temperature on a straight track, but when the track is curved, or when expansion takes place, it is conceivable that the resistance of the ties in the ballast to lateral motion may be less than the resistance at the joint. A test to determine this resistance was made by Mr. A. Torrey, chief engineer of the Mich. Cent. R.R., using 80-lb. rails and ordinary angle-bars, the bolts being screwed up as usual. It required a force of about 31000 to 35000 lbs. to start the joint, which would be equivalent to the stress induced by a change of temperature of about 22°. But if the central angle of any given curve is small, a comparatively small lateral component will be sufficient to resist a compression of even 35000 lbs. in the rails. Therefore there will ordinarily be no trouble about having the joints screwed too tight. The vibration caused by the passage of a train reduces the resistance to slipping. This vibration also facilitates an objectionable feature, viz., loosening of the nuts of the track-bolts. The bolt is readily prevented from turning by giving it a form which is *not* circular immediately under the head and making corresponding holes in the angle-plate. Square holes would answer the purpose, except that the square corners in the holes in the angle-plates would increase the danger of fracture of the plates. Therefore the holes (and also the bolts, under the head) are made of an oval form, or perhaps a square form with rounded corners, avoiding angles in the outline.

The nut-locks should be simple and cheap, should have a life at least as long as the bolt, should be effective, and should not lose their effectiveness with age. Many of the designs that have been tried have been failures in one or more of these particulars, as will be described in detail below.

252. Design of track-bolts. In Fig. 128 is shown a common design of track-bolt. In its general form this represents the bolt used on nearly all roads, being used not only with the common angle-plates, but also with many of the improved designs of rail-joints. The variations are chiefly a general increase in size to correspond with the increased weight of rails, besides variations in detail dimensions which are frequently unimportant. The diameter is usually $\frac{3}{4}''$ to $\frac{7}{8}''$; $1''$ bolts are sometimes used for the heaviest sections of rails. As to length, the bolts should not extend more than $\frac{1}{2}''$ outside of the nut when it is screwed up. If it extends farther than this, it is liable to be broken off by a possible derailment at that point. The lengths used vary from $3\frac{1}{4}''$, which may be used with 60 lbs. rails, to $5''$, which is required with 100-lb. rails. The length required depends somewhat on the type of nut-lock used.

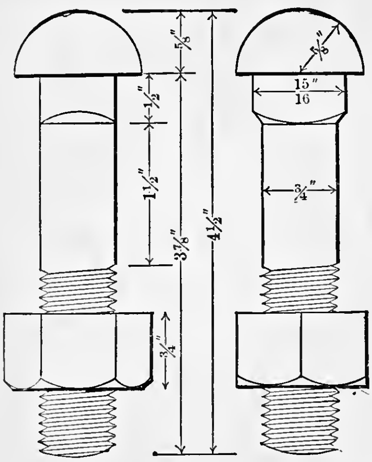

FIG. 128.—TRACK-BOLT.

253. Design of nut-locks. The designs for nut-locks may be divided into three classes: (*a*) those depending entirely on an elastic washer which absorbs the vibration which might otherwise induce turning; (*b*) those which jam the threads of the bolt and nut so that, when screwed up, the frictional resistance is too great to be overcome by vibration; (*c*) the "positive" nut-locks—those which mechanically hold the nut from turning. Some of the designs combine these principles to some extent. The "vulcanized fiber" nut-lock is an example of the first class. It consists essentially of a rubber washer which is protected by an iron ring. When first placed this lock is effective, but the rubber soon hardens and loses its elasticity and it is then ineffective and worthless. Another illustration of class (*a*) is the use of wooden blocks, generally of $1''$ to $2''$ oak, which extend the entire length of the angle-bar, a single piece forming the washer for the four or six bolts of a joint. This form is cheap, but the wood soon shrinks, loses its elasticity, or decays so that it soon becomes worthless, and it requires constant adjustment to keep it in even tolerable condition. The "Verona" nut-lock is another illustration of class (*a*) which also combines some of the positive elements of class (*c*). It is made of tempered steel and, as shown in Fig. 129, is warped and has sharp edges or points. The warped form furnishes the element of elastic pressure when the nut is screwed up. The steel being harder than the iron of the angle-bar or of the nut, it bites into them, owing to the great pressure that must exist

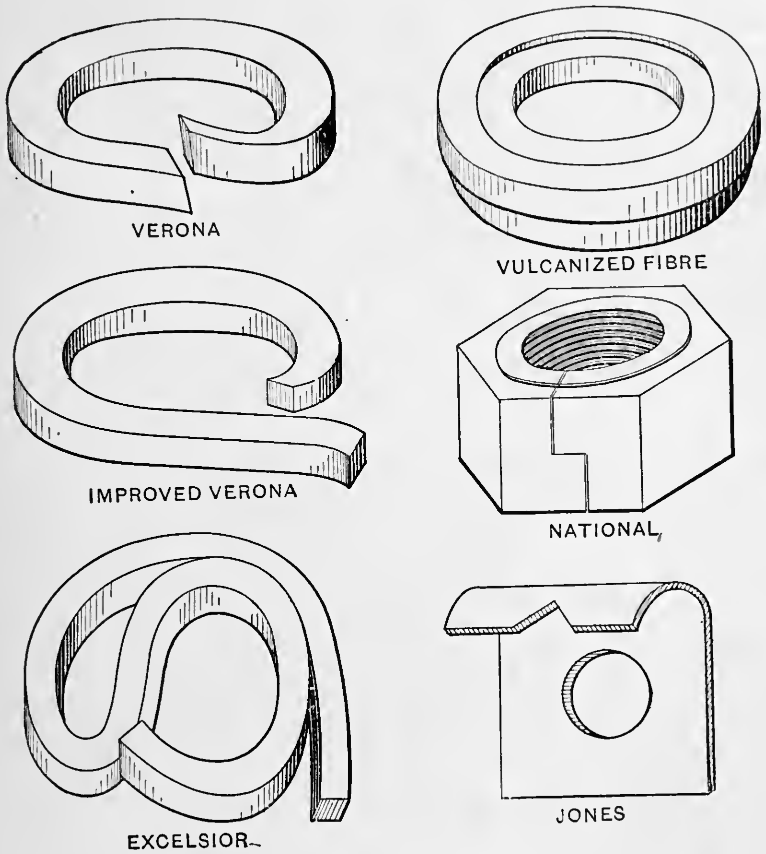

Fig. 129.—Types of Nut-locks.

when the washer is squeezed nearly flat, and thus prevents any *backward* movement, although forward movement (or tightening the bolt) is not interfered with. The "National" nut-lock is a type of the second class (*b*), in which, like the "Harvey" nut-lock, the nut and lock are combined in one piece. With six-bolt angle-bars and 30-foot rails, this means a saving of 2112 pieces on each mile of single track. The "National" nuts are open on one side. The hole is drilled and the thread is cut slightly smaller than the bolt, so that when the nut is screwed up it is forced slightly open and therefore presses on the threads of the bolt with such force that vibration cannot jar it loose. Unlike the "National" nut, the "Harvey" nut is solid, but the form of the thread is progressively varied so that the thread pinches the thread of the bolt and the frictional resistance to turning is too great to be affected by vibration.

The "Jones" nut-lock, belonging to class (*c*), is a type of a nut-lock that does not depend on elasticity or jamming of screw-threads. It is made of a thin flexible plate, the square part of which is so large that it will not turn after being placed on the bolt. After the nut is screwed up, the thin plate is bent over so that the re-entrant angle of the plate engages the corner of the nut and thus mechanically prevents any turning. The metal is supposed to be sufficiently tough to endure without fracture as many bendings of the plate as will ever be desired. Nut-locks of class (*c*) are not in common use.

CHAPTER XI.

SWITCHES AND CROSSINGS.

SWITCH CONSTRUCTION.

254. Essential elements of a switch. Flanges of some sort are a necessity to prevent car-wheels from running off from the rails on which they may be moving. But the flanges, although a necessity, are also a source of complication in that they require some special mechanism which will, when desired, guide the wheels out from the controlling influence of the main-line rails. This must either be done by raising the wheels high enough so that the flanges may pass *over* the rails, or by breaking the continuity of the rails in such a way that channels or "flange spaces" are formed *through* the rails. An ordinary stub switch breaks the continuity of the main-line rails in three places, two of them at the switch-block and one at the frog. The Wharton switch avoids two of these breaks by so placing inclined planes that the wheels, rolling on their flanges, will surmount these inclines until they are a little higher than the rails. Then the wheels on the side toward which the switch runs are guided over and across the main rail on that side. This rise being accomplished in a short distance, it becomes impracticable to operate these switches except at slow speeds, as any sudden change in the path of the center of gravity of a car causes very destructive jars both to the switch and to the rolling stock. The other general method makes a break in one main rail (or both) at the switch-block. In both methods the wheels are led to one side by means of the "lead rails," and finally one line of wheels passes *through* the main rail on that side by means of a "frog." There are some designs by which even this break in the main rail is avoided, the wheels being led *over* the main rail by means of a short *movable* rail which is on occasion placed across the main rail, but such designs have not come into general use.

255. Frogs. Frogs are provided with two channel-ways or "flange spaces" through which the flanges of the wheels move. Each channel cuts out a parallelogram from the tread area. Since the wheel-tread is always wider than the rail, the wing rails will support the wheel not only across the space cut out by

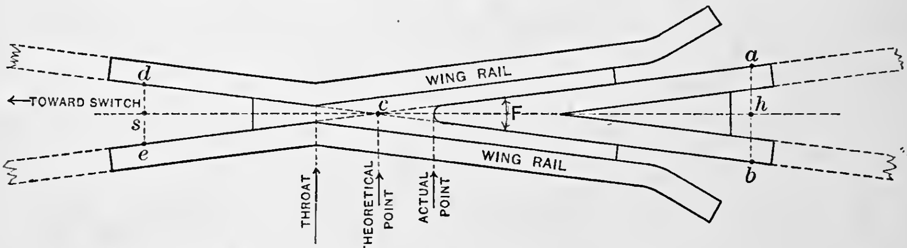

FIG. 130.—DIAGRAMMATIC DESIGN OF FROG.

the channel, but also until the tread has passed the point of the frog and can obtain a broad area of contact on the tongue of the frog. This is the theoretical idea, but it is very imperfectly realized. The wing rails are sometimes subjected to excessive wear owing to "hollow treads" on the wheels—owing also to the frog being so flexible that the point "ducks" when the wheel approaches it. On the other hand the sharp point of the frog will sometimes cause destructive wear on the tread of the wheel. Therefore the tongue of the frog is not carried out to the sharp theoretical point, but is purposely somewhat blunted. But the break which these channels make in the continuity of the tread area becomes extremely objectionable at high speeds, being mutually destructive to the rolling stock and to the frog. The jarring has been materially reduced by the device of "spring frogs"—to be described later. Frogs were originally made of cast iron—then of cast iron with wearing parts of cast steel, which were fitted into suitable notches in the cast iron. This form proved extremely heavy and devoid of that elasticity of track which is necessary for the safety of rolling stock and track at high speeds. The present universal practice is to build the frog up of pieces of rails which are cut or bent as required. These pieces of rails (at least four) are sometimes assembled by riveting them to a flat plate, but this method is now but little used, except for very light work. The usual practice is now chiefly divided between "bolted" and "keyed" frogs. In each case the space between the rails, except a sufficient flange-way, is filled with a cast-iron filler and the whole assemblage of parts is suitably bolted or clamped together, as is illustrated in Plate XVIII. The operation of a spring-rail frog is evident from the figure. Since a siding is usually operated at slow speed, while the main track may be operated at fast speed, a spring-rail frog will be so set that the tread is continuous for the main track and broken for the siding. This also means that the spring rail will only be moved by trains moving at a (presumably) slow speed on to the siding. For the fast trains on the main line such a frog is substantially a "fixed" frog and has a tread which is practically continuous.

256. To find the frog number. The frog number (n) equals the ratio of the distance of any point on the tongue of the frog from the theoretical point of the frog divided by the width of the tongue at that point, i.e. $= hc \div ab$ (Fig. 130). This value may be directly measured by applying any convenient unit of measure (even a knife, a short pencil, etc.) to some point of the tongue where the width just equals the unit of measure, and then noting how many times the unit of measure is contained in the distance from that place to the theoretical point. But since c, the theoretical point, is not so readily determinable with exactitude, it being the imaginary intersection of the gauge lines, it may be more accurate to measure de, ab, and hs; then n, the frog number, $= hs \div (ab + de)$. If the frog angle be called F, then

$$n = hc \div ab = hs \div (ab + de) = \tfrac{1}{2} \cot \tfrac{1}{2}F;$$

i.e.. $$\cot \tfrac{1}{2}F = 2n.$$

257. Stub switches. The use of these, although once nearly universal, has been practically abandoned as turnouts from *main track* except for the poorest and cheapest roads. In some States, their use on main track is prohibited by law. They have the sole merit of cheapness with adaptability to the circumstances of very light traffic operated at slow speed when a considerable element of danger may be tolerated for the sake of economy. The rails from *A* to *B* (see Fig. 131*) are not fastened

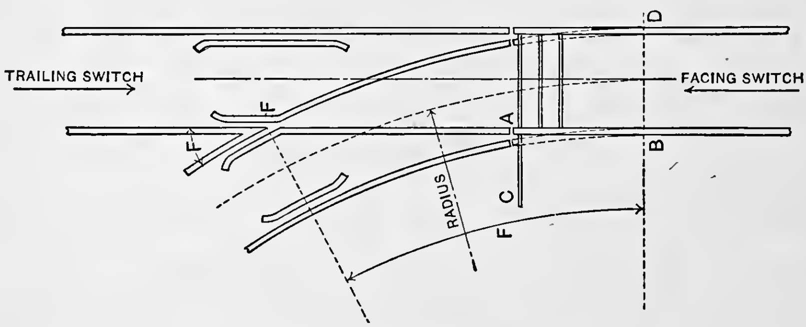

Fig. 131.—Stub Switch.

to the ties; they are fastened to each other by tie-rods which keep them at the proper gauge; at and back of *B* they are securely spiked to the ties, and at *A* they are kept in place by the connecting bar (*C*) fastened to the switch-stand. One great objection to the switch is that, in its usual form, when operated as a trailing switch, a derailment is inevitable if the switch is misplaced. The very least damage resulting from such a derailment must include the bending or breaking of the tie-rods of the switch-rail. Several devices have been invented to obviate this objection, some of which succeed very well mechanically, although their added cost precludes any economy in the total cost of the switch. Another objection to the switch is the looseness of construction which makes the switches objectionable at high speeds. The gap of the rails at the head-block is always considerable, and is sometimes as much as two inches. A

* The student should at once appreciate that in Fig. 131, as well as in nearly all the remaining figures in this chapter, it becomes necessary to use excessively large frog angles, short radii, and a very wide gauge in order to illustrate the desired principles with figures which are sufficiently small for the page. In fact, the proportions used in the figures are such that serious mechanical difficulties would be encountered if they were used. These difficulties are here ignored because they can be neglected in the proportions used in practice.

driving-wheel with a load of 12000 to 20000 pounds, jumping this gap with any considerable velocity, will do immense damage to the farther rail end, besides producing such a stress in the construction that a breakage is rendered quite likely, and such a breakage might have very serious consequences.

258. Point switches. The essential principle of a point switch is illustrated in Fig. 132. As is shown, one main rail and also one of the switch-rails is unbroken and immovable.

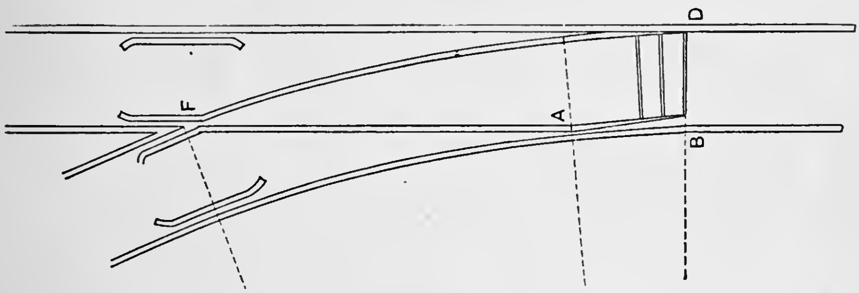

FIG. 132.—POINT SWITCH.

The other main rail (from *A* to *F*) and the corresponding portion of the other lead rail are substantially the same as in a stub switch. A portion of the main rail (*AB*) and an equal length of the opposite lead rail (usually 15 to 24 feet long) are fastened together by tie-rods. The end at *A* is jointed as usual and the other end is pointed, both sides being trimmed down so that the feather edge at *B* includes the web of the rail. In order to retain in it as much strength as possible, the point-rail is raised so that it rests on the base of the stock-rail, one side of the base of the point-rail being entirely cut away. As may be seen in Fig. 133, although the influence of the point of the rail in moving the wheel-flange away from the stock-rail is really zero at that point, yet the rail has all the strength of the web and about one-half that of the base—a very fair angle-iron.

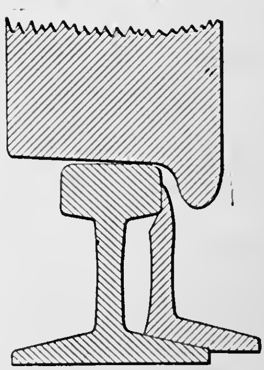

FIG. 133.

The planing runs back in *straight* lines, until at about six or seven feet back from the point the full width of the head is obtained. The full width of the base will only be obtained at about 13 feet from the point. An 80-lb. rail is 5 inches

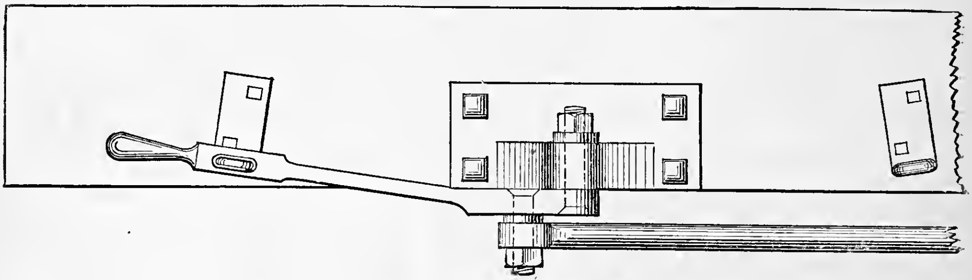

FIG. 134.—GROUND LEVER FOR THROWING A SWITCH.

wide at the base. Allowing $\frac{3}{4}''$ more for a spike between the rails, this gives $5\frac{3}{4}''$ as the minimum width between rail centers at the joint. The minimum angle of the switch-point (using a 15-foot point rail) is therefore the angle whose tangent is $\frac{5.75}{15 \times 12} = .03914$, which is the tangent of $1° \ 50'$. Switch-rails are sometimes used with a length of 24 feet, which reduces the angle of the switch-point to $1° \ 09'$.

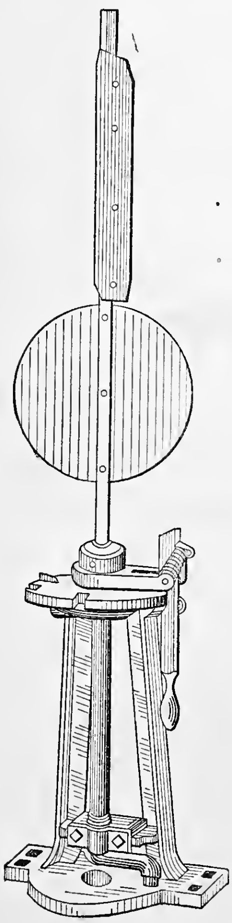

FIG. 135.

259. Switch-stands. The simplest and cheapest form is the "ground lever," which has no target. The radius of the circle described by the connecting-rod pin is precisely one-half the throw. From the nature of the motion the device is practically self-locking in either position, padlocks being only used to prevent malicious tampering. The numerous designs of upright stands are always combined with targets, one design of which is illustrated in Fig. 135. When the road is equipped with interlocking signals, the switch-throw mechanism forms a part of the design.

260. Tie-rods. These are fastened to the webs of the rails by means of lugs which are bolted on, there being usually a hinge-joint between the rod and the lug. Four such tie-rods are generally necessary. The first rod is sometimes made without hinges, which gives additional stiffness to the comparatively weak rail-points. The old fashioned tie-rod, having jaws fitting the base of the rail, was almost universally used in the days of stub switches. One great inconvenience in their use lies in the fact that they must be slipped on, one by one, over the *free* ends of the switch-rails. Sometimes the lugs are fastened to the rail-webs by rivets instead of bolts.

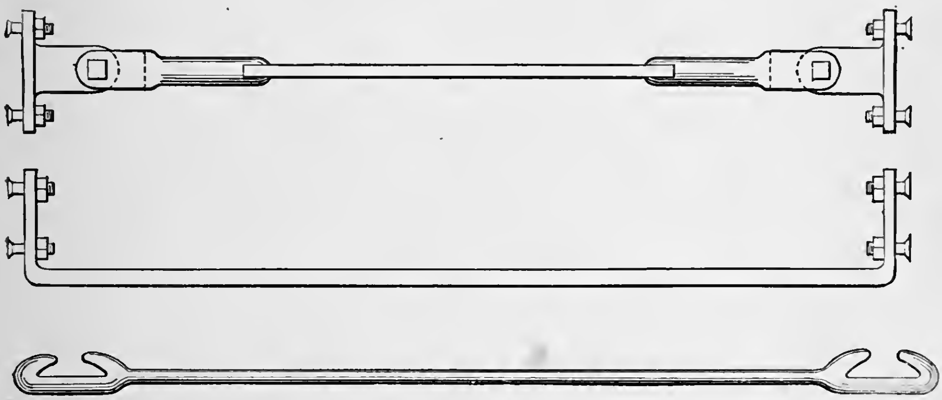

FIG. 136.—FORMS OF TIE-RODS.

261. Guard-rails. As shown in Figs. 131 and 132, guard-rails are used on both the main and switch tracks opposite the frog-point. Their function is not only to prevent the possibility of the wheel-flanges passing on the wrong side of the frog-point, but also to save the side of the frog-tongue from excessive wear. The necessity for their use may be realized by noting the very apparent wear usually found on the side of the head of the guard-rail. The flange-way space between the heads of the guard-rail and wheel-rail therefore becomes a definite quantity and should equal about two inches. Since this is less than the space between the heads of ordinary (say 80-pound) rails when placed base to base, to say nothing of the $\frac{3}{4}''$ necessary for spikes, it becomes necessary to cut the flange of the guard-rail. The length of the rail is made from 10 to 15 feet, the ends being bent as shown in Fig. 132, so as to prevent the possibility of the end of the rail being struck by a wheel-flange.

MATHEMATICAL DESIGN OF SWITCHES.

In all of the following demonstrations regarding switches, turnouts, and crossovers, the lines are assumed to represent the *gauge-lines*—i.e., the lines of the *inside* of the head of the rails.

262. Design with circular lead-rails. The simplest method is to consider that the lead-rails curve out from the main track-rails by arcs of circles which are tangent to the main rails and which extend to the frog-point F. The simple curve from D to F is of such radius that $(r + \frac{1}{2}g)$ vers $F = g$, in which $F =$ the frog angle, $g =$ gauge, $L =$ the "lead" (BF), and $r =$ the radius of the *center* of the switch-rails.

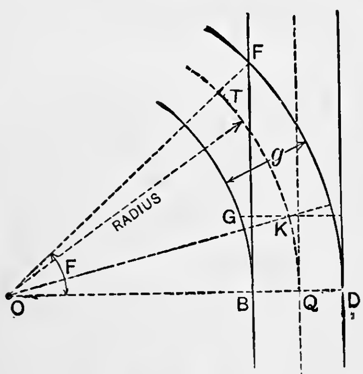

FIG. 137.

$$\therefore \quad r + \tfrac{1}{2}g = \frac{g}{\text{vers } F} \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

Also $\quad BF \div BD = \cot \frac{1}{2}F; \quad BD = g; \quad BF = L.$

$$\therefore \quad L = g \cot \tfrac{1}{2}F \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

Also
$$L = (r + \tfrac{1}{2}g) \sin F; \quad . \quad . \quad . \quad . \quad . \quad (76)$$

$$QT = 2r \sin \tfrac{1}{2}F. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

These formulæ involve the angle F. As shown in Table III, the angles (F) are always odd quantities, and their trigonometric functions are somewhat troublesome to obtain closely with ordinary tables. The formulæ may be simplified by substituting the frog-number n, from the relation that $n = \frac{1}{2} \cot \frac{1}{2}F$. Since

$$r - \tfrac{1}{2}g = L \cot F \text{ and } r + \tfrac{1}{2}g = L \operatorname{cosec} F,$$

then $r = \frac{1}{2}L\,(\cot F + \operatorname{cosec} F)$

$= \frac{1}{2}g \cot \frac{1}{2}F\,(\cot F + \operatorname{cosec} F)$

$= \frac{1}{2}g \cot^2 \frac{1}{2}F$, since $(\cot \alpha + \operatorname{cosec} \alpha) = \cot \frac{1}{2}\alpha$

$$= 2gn^2. \qquad (78)$$

Also $$L = 2gn, \qquad (79)$$

from which $$r = n \times L. \qquad (80)$$

These extremely simple relations may obviate altogether the necessity for tables, since they involve only the frog-number and the gauge. On account of the great simplicity of these rules, they are frequently used as they are, regardless of the fact that the curve is never a uniform simple curve from switch-block to frog. In the first place there is a considerable length of the gauge-line within the frog, which is straight unless it is purposely curved to the proper curve while being manufactured, which is seldom if ever done—except for the very large-angled frogs used for street-railway work, etc. It is also doubtful whether the switch-rails (BA, Fig. 131) are bent to the computed curve when the rails are set for the switch. The switch-rails of point switches are *straight*, thus introducing a stretch of straight track which is about one-fifth of the total length of the lead-rails. The effect of these modifications on the length and radius of the lead-rails will be developed and discussed in the next four sections.

The throw (t) of a stub switch depends on the weight of the rail, or rather on the width of its base. The throw must be at least $\frac{3}{4}''$ more than that width. The head-block should therefore be placed at such a distance from the heel of the switch (B) that the versed sine of the arc equals the throw. These points *must* be opposite on the two rails, but the points on the two rails where these relations are exactly true will not be opposite. Therefore, instead of considering either of the two radii $(r + \frac{1}{2}g)$ and $(r - \frac{1}{2}g)$, the mean radius r is used. Then (see Fig. 137)

$$\text{vers } KOQ = t \div r,$$

and the length of the switch-rails is

$$QK = r \sin KOQ. \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

These relations develop another disadvantage in the use of a stub switch. The required value of BG, using a No. 10 frog and 80-pound rail, is 30.1 feet—slightly more than a full rail length. It would be unsafe to leave so much of the track unspiked from the ties. Whether this is obviated by spiking down a portion of the switch-rails (virtually shortening the lead) or by moving the switch-block nearer the heel of the switch (shortening the switch-rails), but still maintaining the required throw, the theoretical accuracy of the curve is hopelessly lost.

263. Effect of straight frog-rails. A portion of the ends of the rails of a frog are free and *may* be bent to conform to the switch-rail curve, but there is a considerable portion which is fitted to the cast-iron filler, and this portion is always straight. Call the length of this straight portion back from the frog-point f ($= FH$, Fig. 138). Then we have

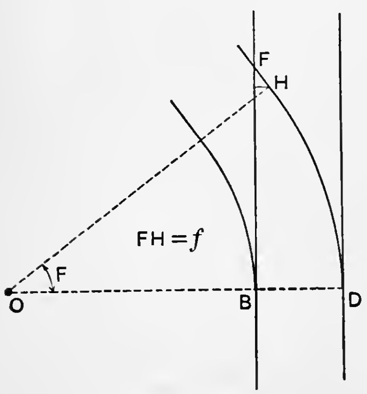

FIG. 138.

$$r + \tfrac{1}{2}g = (g - f \sin F) \div \text{vers } F$$

$$= \frac{g}{\text{vers } F} - f \cot \tfrac{1}{2}F$$

$$= \frac{g}{\text{vers } F} - 2fn. \quad . \quad . \quad (82)$$

$$BF = L = (g - f \sin F) \cot \tfrac{1}{2}F + f \cos F$$

$$= 2gn - f \sin F \cot \tfrac{1}{2}F + f \cos F$$

$$= 2gn - f(1 + \cos F) + f \cos F$$

$$= 2gn - f. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

Since $r - \tfrac{1}{2}g = (L - f \sec F) \cot F$, and

$$r + \tfrac{1}{2}g = (L - f \cos F) \operatorname{cosec} F,$$

$$r = \tfrac{1}{2}L\,(\cot F + \operatorname{cosec} F) - \tfrac{1}{2}f \sec F \cot F - \tfrac{1}{2}f \cos F \operatorname{cosec} F$$

$$= Ln - \tfrac{1}{2}f\left(\frac{1 + \cos F}{\sin F}\right).$$

$$r = Ln - \tfrac{1}{2}f \cot \tfrac{1}{2}F$$

$$= Ln - fn. \quad \text{Then from (83)}$$

$$r = 2gn^2 - 2fn. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

264. Effect of straight point-rails. The "point switches," now so generally used, have *straight* switch-rails. This requires an *angle* in the alignment rather than turning off by a tangential curve. The angle is, however, very small (between 1° and 2°), and the disadvantages of this angle are small compared with the very great advantages of the device.

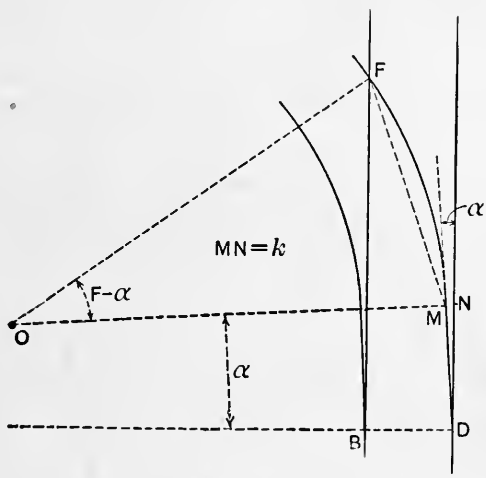

FIG. 139.

$$FM = \frac{g - k}{\sin \frac{1}{2}(F + \alpha)};$$

$$r + \tfrac{1}{2}g = \frac{FM}{2 \sin \frac{1}{2}(F - \alpha)}$$

$$= \frac{g - k}{2 \sin \frac{1}{2}(F + \alpha) \sin \frac{1}{2}(F - \alpha)}$$

$$= \frac{g - k}{\cos \alpha - \cos F}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

$$BF = L = FM \cos \tfrac{1}{2}(F + \alpha) + DN$$
$$= (g - k) \cot \tfrac{1}{2}(F + \alpha) + DN. \quad . \quad (86)$$

265. Combined effect of straight frog-rails and straight point-rails. It becomes necessary in this case to find a curve which shall be tangent to both the point-rail and the frog-rail. The curve therefore begins at M, its tangent making an angle of α (usually 1° 50′) with the main rail, and runs to H. The central

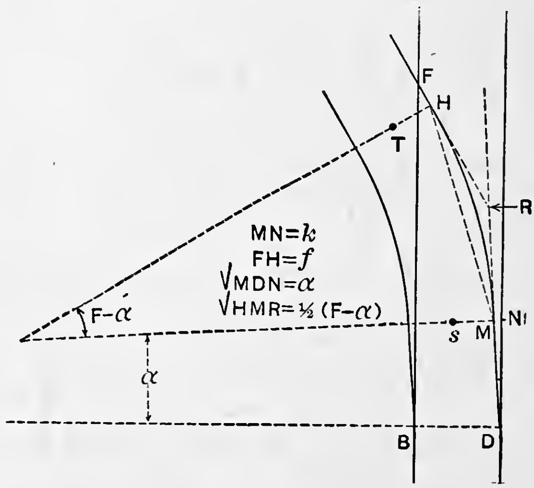

Fig. 140.

angle of the curve is therefore $(F - \alpha)$. The angle of the chord HM with the main rails is therefore

$$\tfrac{1}{2}(F - \alpha) + \alpha = \tfrac{1}{2}(F + \alpha);$$

$$HM = \frac{g - f \sin F - k}{\sin \frac{1}{2}(F + \alpha)};$$

$$r + \tfrac{1}{2}g = \frac{HM}{2 \sin \frac{1}{2}(F - \alpha)}$$

$$= \frac{g - f \sin F - k}{2 \sin \frac{1}{2}(F + \alpha) \sin \frac{1}{2}(F - \alpha)}$$

$$= \frac{g - f \sin F - k}{\cos \alpha - \cos F}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

$$ST = 2r \sin \tfrac{1}{2}(F - \alpha). \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

$$BF = L = HM \cos \tfrac{1}{2}(F + \alpha) + f \cos F + DN$$
$$= (g - f \sin F - k) \cot \tfrac{1}{2}(F + \alpha) + f \cos F + DN. \quad (89)$$

It may be more simple, if $(r + \frac{1}{2}g)$ has already been computed, to write

$$L = 2(r + \tfrac{1}{2}g) \sin \tfrac{1}{2}(F - \alpha) \cos \tfrac{1}{2}(F + \alpha) + f \cos F + DN$$
$$= (r + \tfrac{1}{2}g)(\sin F - \sin \alpha) + f \cos F + DN. \quad . \quad . \quad (90)$$

266. Comparison of the above methods. Computing values for r and L by the various methods, on the uniform basis of a No. 9 frog, standard gauge 4′ $8\frac{1}{2}''$, $f = 3'.37$, $k = 5\frac{3}{4}'' = 0'.479$, $DN = 15'\ 0''$, and $\alpha = 1°\ 50'$, we may tabulate the comparative results:

	§ 262. Simple circle Curved frog r. Curved switch-r.	§ 263. Straight frog-r. Curved switch-r.	§ 264. Curved frog-r. Straight switch-r.	§ 265. Straight frog-r. Straight switch r.
r	762.75	702.00	747.48	681.16
Deg. of curve	7° 31′	8° 10′	7° 40′	8° 25′
L	84.75	81.37	74.00	72.13

This shows that the effect of using straight frog-rails and straight switch-rails is to sharpen the curve and shorten the lead in each case separately, and that the combined effect is still greater. The effect of the straight switch-rails is especially marked in reducing the length of lead, and therefore Eq. 78 to 80, although having the advantage of extreme simplicity, cannot be used for point-switches without material error. The effect of the straight frog-rail is less, and since it can be materially reduced by bending the free end of the frog-rails, the influence of this feature is frequently ignored, the frog-rails are assumed to be curved and Eq. 85 and 86 are used. (See § 276 for a further discussion of this point.)

267. Dimensions for a turnout from the OUTER side of a curved track. In this demonstration the switch-rails will be considered as uniformly circular from the switch-points to the frog-point.

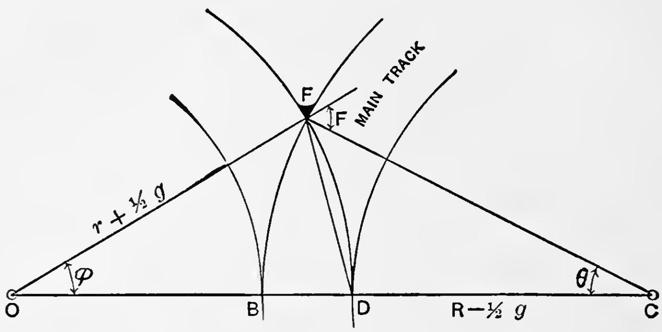

FIG. 141.

In the triangle FCD (Fig. 141) we have

$$(FC+CD):(FC-CD)::\tan\tfrac{1}{2}(FDC+DFC):\tan\tfrac{1}{2}(FDC-DFC);$$

but $$\tfrac{1}{2}(FDC+DFC)=90^\circ-\tfrac{1}{2}\theta$$

and $$\tfrac{1}{2}(FDC-DFC)=\tfrac{1}{2}F.$$

Also $$FC+CD=2R \quad\text{and}\quad FC-CD=g;$$

$$\therefore\ 2R:g::\cot\tfrac{1}{2}\theta:\tan\tfrac{1}{2}F$$
$$::\cot\tfrac{1}{2}F:\tan\tfrac{1}{2}\theta;$$

$$\therefore\ \tan\tfrac{1}{2}\theta=\frac{gn}{R}. \qquad (91)$$

Also $OF:FC::\sin\theta:\sin\phi;$ but $\phi=(F-\theta);$

then $$r+\tfrac{1}{2}g=(R+\tfrac{1}{2}g)\frac{\sin\theta}{\sin(F-\theta)}. \qquad (92)$$

$$BF=L=2(R+\tfrac{1}{2}g)\sin\tfrac{1}{2}\theta. \qquad (93)$$

If the curvature of the main track is very sharp or the frog angle unusually small, F may be less than θ; in which case the center O will be on the same side of the main track as C. Eq. 92 will become (by calling $r=-r$ and changing the signs)

$$(r-\tfrac{1}{2}g)=(R+\tfrac{1}{2}g)\frac{\sin\theta}{\sin(\theta-F)}. \qquad (94)$$

If we call d the degree of curve corresponding to the radius r, and D the degree of curve corresponding to the radius R, also d' the degree of curve of a turnout from a straight track (the frog angle F being the same), it may be shown that $d = d' - D$ (very nearly). To illustrate we will take three cases, a number 6 frog (very blunt), a number 9 frog (very commonly used), and a number 12 frog (unusually sharp). Suppose $D = 4° \ 0'$; also $D = 10° \ 0'$; $g = 4' \ 8\frac{1}{2}'' = 4'.708$.

Frog number.	$D = 4°$.				"L" for *straight* track.
	d	$d' - D$	Error.	L	
6	12° 54′ 20″	12° 57′ 52″	0° 03′ 32″	56.57	56.50
9	3 30 27	3 31 04	0 0 37	84.85	84.75
12	0 13 33	0 13 36	0 0 03	112.72	113.00

Frog number.	$D = 10°$				"L" for *straight* track.
	d	$d' - D$	Error.	L	
6	6° 53′ 24″	6° 57′ 52″	0° 04′ 28″	56.66	56.50
9	2 27 54	2 28 56	0 01 02	84.86	84.75
12	5 44 26	5 46 24	0 01 58	112.91	113.00

A brief study of the above tabular form will show that the error involved in the use of the approximate rule for ordinary curves (4° or less) and for the usual frogs (about No. 9) is really insignificant, and that, even for sharper curves (10° or more), or for very blunt frogs, the error would never cause damage, considering the lower probable speed. In the most unfavorable case noted above the change in radius is about 1%. On account of the closeness of the approximation the method is frequently used. The remarkable agreement of the computed values of L with the corresponding values for a straight main track (the lead rails circular throughout) shows that the error is insignificant in using the more easily computed values.

268. Dimensions for a turnout from the INNER side of a curved

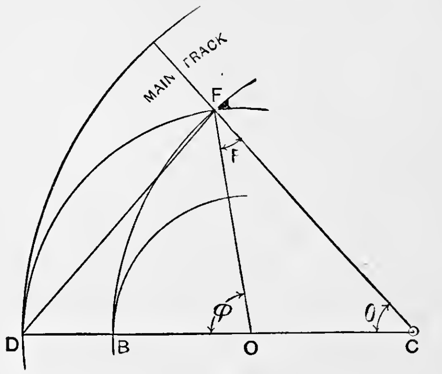

FIG. 142.

track. (Lead rails circular throughout.) From Fig. 142 we have

$$DC+FC : DC-FC :: \tan \tfrac{1}{2}(DFC+FDC) : \tan \tfrac{1}{2}(DFC-FDC);$$

but $$\tfrac{1}{2}(DFC + FDC) = 90° - \tfrac{1}{2}\theta$$

and $$\tfrac{1}{2}(DFC - FDC) = \tfrac{1}{2}F;$$

$$\therefore\ 2R : g :: \cot \tfrac{1}{2}\theta : \tan \tfrac{1}{2}F$$
$$:: \cot \tfrac{1}{2}F : \tan \tfrac{1}{2}\theta;$$

$$\therefore\ \tan \tfrac{1}{2}\theta = \frac{gn}{R}. \qquad (95)$$

$$OF : FC :: \sin \theta : \sin (F + \theta).$$

$$(r + \tfrac{1}{2}g) = (R - \tfrac{1}{2}g)\frac{\sin \theta}{\sin (F + \theta)}. \qquad (96)$$

$$L = BF = 2(R - \tfrac{1}{2}g) \sin \tfrac{1}{2}\theta. \qquad (97)$$

As in § 267, it may be readily shown that the degree of the turnout (d) is *nearly* the *sum* of the degree of the main track (D) and the degree (d') of a turnout from a straight track when the frog angle is the same. The discrepancy in this case is somewhat greater than in the other, especially when the curvature of the main track is sharp. If the frog angle is also large, the curvature of the turnout is excessively sharp. If the frog angle is very small, the liability to derailment is great. Turnouts to the inside of a curved track should therefore be avoided, unless the curvature of the main track is small.

269. Double turnout from a straight track. In Fig. 143 the frogs F_l and F_r are generally made equal. Then, if there are

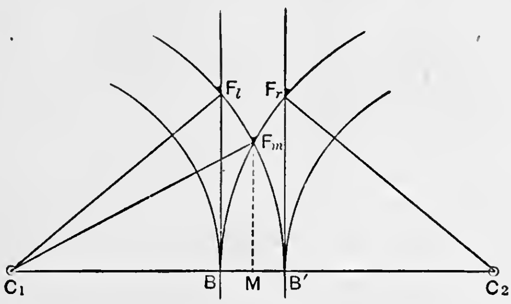

FIG. 143.

uniform curves from B' to F_l and from B to F_r, the required value of F_m is obtained from

$$\text{vers } \tfrac{1}{2}F_m = \frac{g}{2(r + \frac{1}{2}g)}, \quad . \quad . \quad . \quad . \quad . \quad (98)$$

r being found from Eq. 78, in which n is the frog number of F_l or F_r.

$$MF_m = r \tan \tfrac{1}{2}F_m;$$

but since $n_m = \frac{1}{2} \cot \frac{1}{2}F_m$,

$$MF_m = \frac{r}{2n_m}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (99)$$

Since $\text{vers } F_l = \dfrac{g}{(r + \frac{1}{2}g)}$,

$$\text{vers } \tfrac{1}{2}F_m = \tfrac{1}{2} \text{ vers } F_l, \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

Also, since $(C_1F_m)^2 = (MF_m)^2 + (C_1M)^2$, we have

$$(r + \tfrac{1}{2}g)^2 = \left(\frac{r}{2n_m}\right)^2 + r^2;$$

$$r^2 + rg + \tfrac{1}{4}g^2 = \frac{r^2}{4n_m^{\ 2}} + r^2.$$

Simplifying and substituting $r = 2gn^2$, we have

$$2g^2n^2 + \tfrac{1}{4}g^2 = \frac{4g^2n^4}{4n_m^{\ 2}};$$

$$n_m^{\ 2} = \frac{n^4}{2n^2 + \frac{1}{4}}.$$

Dropping the $\frac{1}{4}$, which is always insignificant in comparison with $2n^2$, we have

$$n_m = \frac{n}{\sqrt{2}} = n \times .707 \text{ (approx.)}. \quad . \quad . \quad (101)$$

Frogs are usually made with angles corresponding to integral values of n, or sometimes in "half" sizes, e.g. 6, $6\frac{1}{2}$, 7, $7\frac{1}{2}$, etc. If No. $8\frac{1}{2}$ frogs are used for F_l and F_r, the exact frog number for F_m is 6.01. This is so nearly 6 that a No. 6 frog may be used without sensible inaccuracy. Numbers 7 and 10 are a less perfect combination. If sharp frogs must be used, $8\frac{1}{2}$ and 12 form a very good combination.

If it becomes necessary to use other frogs because the right combination is unobtainable, it may be done by compounding the curve at the middle frog. F_l and F_r should be greater than $\frac{1}{2}F_m$. If equal to $\frac{1}{2}F_m$, the rails would be straight from the middle frog to the outer frogs. In Fig. 144, $\theta_1 = F_l - \frac{1}{2}F_m$. Drawing the *chord* $\overline{F_lF_m}$,

$$\overline{KF_lF_m} = F_l - \tfrac{1}{2}\theta_1 = F_l - \tfrac{1}{2}F_l + \tfrac{1}{4}F_m = \tfrac{1}{2}(F_l + \tfrac{1}{2}F_m);$$

$$\overline{F_lF_m} = \frac{\overline{KF_m}}{\sin \overline{KF_lF_m}} = \frac{g}{2 \sin \frac{1}{2}(F_l + \frac{1}{2}F_m)}; \quad . \quad . \quad . \quad (102)$$

$$\overline{KF_l} = \overline{KF_m} \cot \overline{KF_lF_m} = \tfrac{1}{2}g \cot \tfrac{1}{2}(F_l + \tfrac{1}{2}F_m); \quad (103)$$

$$(r_1 + \tfrac{1}{2}g) = \frac{\overline{F_lF_m}}{2 \sin \frac{1}{2}\theta} = \frac{g}{4 \sin \frac{1}{2}(F_l + \frac{1}{2}F_m) \sin \frac{1}{2}(F_l - \frac{1}{2}F_m)}$$

$$= \frac{\frac{1}{2}g}{\cos \frac{1}{2}F_m - \cos F_l}. \quad . \quad . \quad . \quad . \quad (104)$$

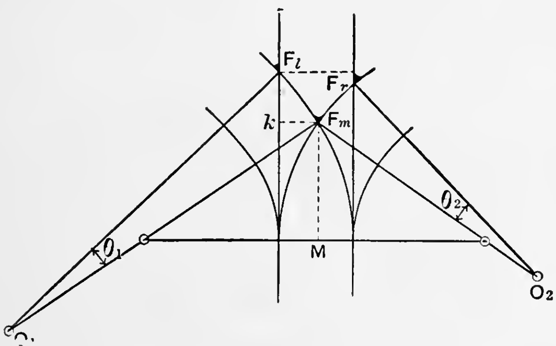

FIG. 144.

If three frogs, all different, *must* be used, the largest may be selected as F_m; the radius of the lead rails may be found by an inversion of Eq. 98; F_m may be located in the center of the tracks by Eq. 99; then each of the smaller frogs may be located by separate applications of Eq. 102 or 103, the radius being determined by Eq. 104.

270. Two turnouts on the same side. In Fig. 145, let O_1 bisect O_2D. Then $(r_1 + \frac{1}{2}g) = \frac{1}{2}(r_2 + \frac{1}{2}g)$; also, $O_1O_2 = O_1F_l$ and $F_r = F_l$.

$$\text{vers } F_m = \frac{g}{r_1 + \frac{1}{2}g} = \frac{2g}{r_2 + \frac{1}{2}g}; \quad . \quad . \quad . \quad (105)$$

$$BF_m = (r_1 + \tfrac{1}{2}g) \sin F_m. \quad . \quad . \quad . \quad . \quad (106)$$

It may readily be shown that the relative values of F_r, F_l, and F_m are almost identical with those given in § 269; as may be apparent when it is considered that the middle switch may be regarded simply as a curved main track, and that, as

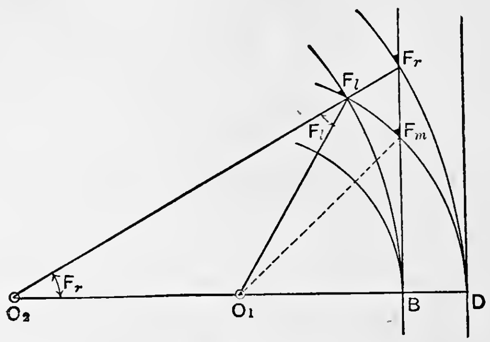

FIG. 145.

developed in § 267, the dimensions of turnouts are nearly the same whether the main track is straight or slightly curved.

271. Connecting curve from a straight track. The "connecting curve" is the track lying between the frog and the side track where it becomes parallel to the main track (*FS* in Fig. 146 or 147). Call d the distance between track centers. The angle $FO_1R = F$ (see Fig. 146). Call r' the radius of the connecting curve. Then

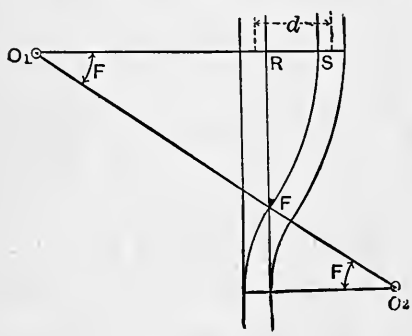

FIG. 146.

$$(r' - \tfrac{1}{2}g) = \frac{d - g}{\text{vers } F}; \quad . \quad . \quad . \quad . \quad . \quad (107)$$

$$FR = (r' - \tfrac{1}{2}g) \sin F. \quad . \quad . \quad (108)$$

If it is considered that the distance FR consumes too much track room, it may be shortened by the method indicated in Fig. 151.

272. Connecting curve from a curved track to the OUTSIDE. When the main track is curved, the required quantities are the radius r of the connecting curve from F to S, Fig. 147, and its length or central angle. In the triangle CSF

$$CS + CF : CS - CF :: \tan \tfrac{1}{2}(CFS + CSF) : \tan \tfrac{1}{2}(CFS - CSF);$$

but $\frac{1}{2}(CFS + CSF) = 90 - \frac{1}{2}\psi$; and, since the triangle O_1SF is isosceles, $\frac{1}{2}(CFS - CSF) = \frac{1}{2}F$;

$$\therefore\ 2R + d : d - g :: \cot \tfrac{1}{2}\psi : \tan \tfrac{1}{2}F$$

$$:: \cot \tfrac{1}{2}F : \tan \tfrac{1}{2}\psi;$$

$$\therefore\ \tan \tfrac{1}{2}\psi = \frac{2n(d - g)}{2R + d}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (109)$$

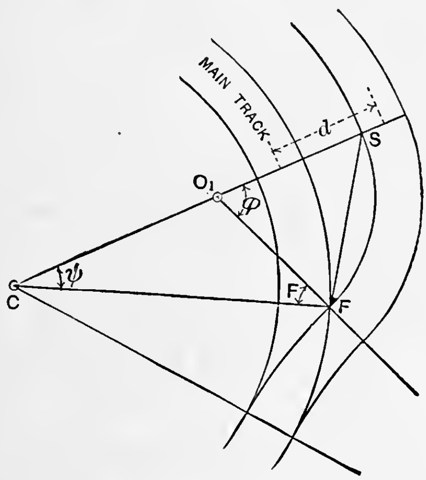

FIG. 147.

From the triangle CO_1F we may derive

$$r - \tfrac{1}{2}g : R + \tfrac{1}{2}g :: \sin \psi : \sin (F + \psi);$$

$$r - \tfrac{1}{2}g = (R + \tfrac{1}{2}g) \frac{\sin \psi}{\sin (F + \psi)} \quad . \quad . \quad . \quad . \quad (110)$$

Also $$FS = 2(r - \tfrac{1}{2}g) \sin \tfrac{1}{2}(F + \psi). \quad . \quad . \quad . \quad (111)$$

273. Connecting curve from a curved track to the INSIDE. As above, it may readily be deduced from the triangle CFS (see Fig. 148) that

$$(2R - d) : (d - g) :: \cot \tfrac{1}{2}\psi : \tan \tfrac{1}{2}F,$$

and finally that

$$\tan \tfrac{1}{2}\psi = \frac{2n(d - g)}{2R - d}. \quad . \quad . \quad . \quad . \quad . \quad (112)$$

Similarly we may derive (as in Eq. 110)

$$(r - \tfrac{1}{2}g) = (R - \tfrac{1}{2}g)\frac{\sin \psi}{\sin (F - \psi)}. \quad . \quad . \quad (113)$$

Also $$FS = 2(r - \tfrac{1}{2}g) \sin \tfrac{1}{2}(F - \psi). \quad . \quad . \quad (114)$$

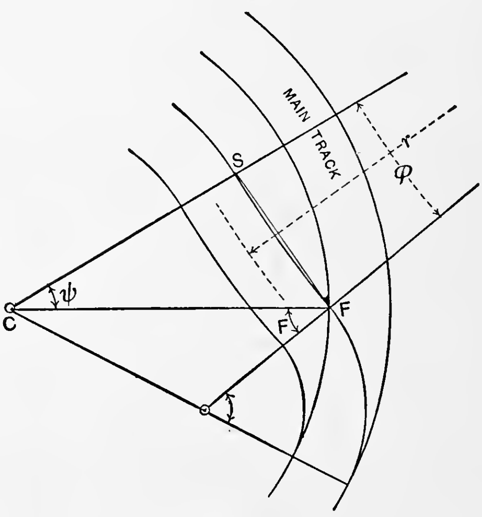

Fig. 148.

Two other cases are possible. (*a*) r may increase until it becomes infinite (see Fig. 149), then $F = \psi$. In such a case we may write, by substituting in Eq. 112,

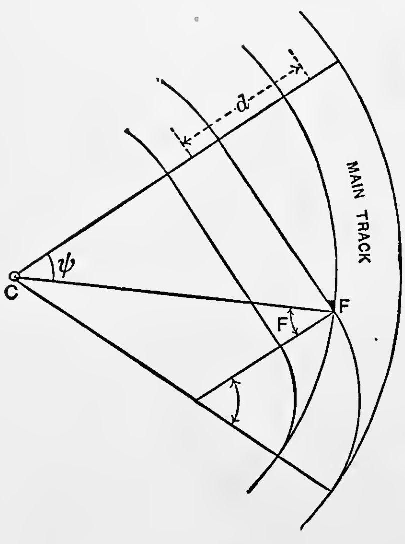

Fig. 149.

$$2R - d = 4n^2(d - g). \quad . \quad . \quad (115)$$

This equation shows the value of R, which renders this case possible with the given values of n, d, and g. (*b*) ψ may be greater than F. As before (see Fig. 150)

$$2R - d : d - g :: \cot \tfrac{1}{2}\psi : \tan \tfrac{1}{2}F;$$

$$\tan \tfrac{1}{2}\psi = \frac{2n(d - g)}{2R - d},$$

the same as Eq. 112, but

$$r + \tfrac{1}{2}g = (R - \tfrac{1}{2}g)\frac{\sin\psi}{\sin(\psi - F)}. \quad . \quad . \quad (116)$$

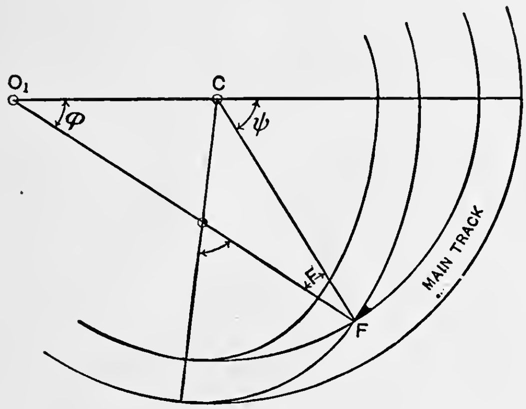

Fig. 150.

274. Crossover between two parallel straight tracks. (See Fig. 151.) The turnouts are as usual. The crossover track may be straight, as shown by the full lines, or it may be a reversed curve, as shown by the dotted lines. The reversed curve shortens the total length of track required, but is somewhat objectionable. The first method requires that both frogs must be equal. The second method permits unequal frogs, although equal frogs are preferable. The length of straight crossover track is F_1T.

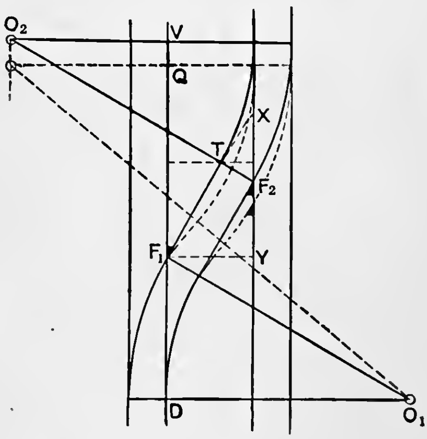

Fig. 151.

$$F_1T \sin F_1 + g \cos F_1 = d - g;$$

$$F_1T = \frac{d-g}{\sin F_1} - g \cot F_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (117)$$

The total distance along the track may be derived as follows:

$$DV = 2DF_1 + F_2Y = 2DF_1 + XY - XF_2;$$

$$XY = (d - g) \cot F_1; \qquad XF_2 = g \div \sin F_2;$$

$$\therefore \quad DV = 2DF_1 + (d - g) \cot F_1 - \frac{g}{\sin F_2}. \quad . \quad . \quad (118)$$

If a reversed curve with equal frogs is used, we have

$$\text{vers } \theta = \frac{d}{2r}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (119)$$

also

$$DQ = 2r \sin \theta. \quad . \quad . \quad . \quad . \quad . \quad (120)$$

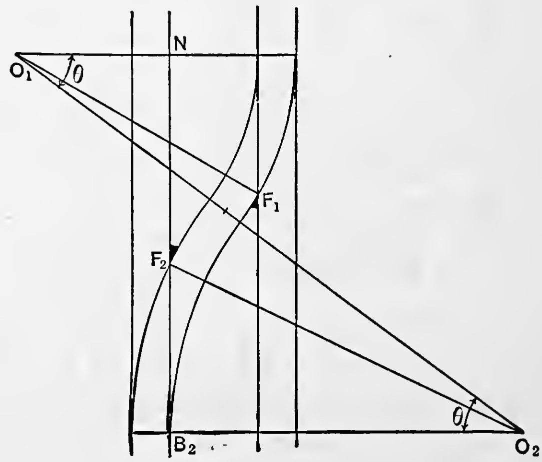

FIG. 152.

If the frogs are unequal, we will have (see Fig. 152)

$$r_2 \text{ vers } \theta + r_1 \text{ vers } \theta = d;$$

$$\therefore \quad \text{vers } \theta = \frac{d}{r_1 + r_2}; \quad . \quad . \quad . \quad . \quad . \quad (121)$$

also the distance along the track

$$B_2N = (r_1 + r_2) \sin \theta. \quad . \quad . \quad . \quad . \quad . \quad (122)$$

275. Crossover between two parallel curved tracks. (a) Using a straight connecting curve. This solution has limitations. If one frog (F_1) is chosen, F_2 becomes determined, being a function of F_1. If F_1 is less than some limit, depending on the width

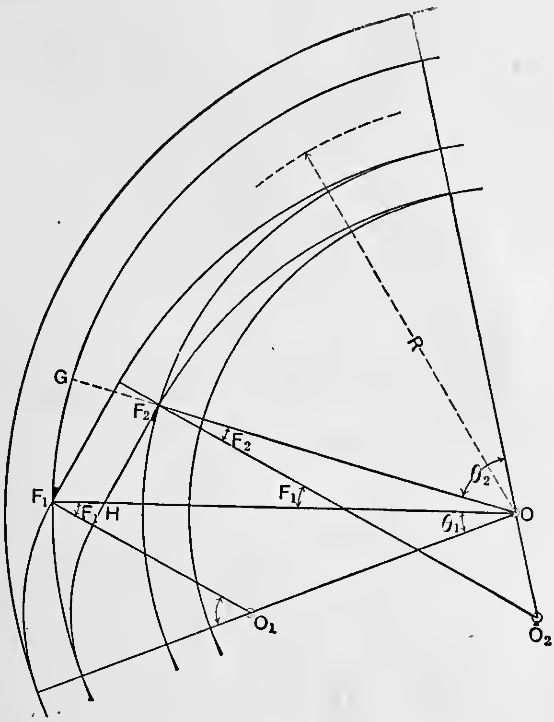

FIG. 153.

(d) between the parallel tracks, this solution becomes impossible. In Fig. 153 assume F_1 as known. Then $F_1H = g \sec F_1$. In the triangle HOF_2 we have

$$\sin HF_2O : \sin F_2HO :: HO : F_2O;$$

$$\sin F_2HO = \cos F_1; \quad HF_2O = 90° + F_2;$$

$$\therefore \quad \sin HF_2O = \cos F_2.$$

$$HO = R + \tfrac{1}{2}d - \tfrac{1}{2}g - g \sec F_1; \quad F_2O = R - \tfrac{1}{2}d + \tfrac{1}{2}g;$$

$$\therefore \quad \cos F_2 = \cos F_1 \frac{R + \frac{1}{2}d - \frac{1}{2}g - g \sec F_1}{R - \frac{1}{2}d + \frac{1}{2}g}. \quad . \quad . \quad . \quad (123)$$

Knowing F_2, θ_2 is determinable from Eq. 91. Fig. 153 shows the case where θ_2 is greater than F_2. Fig. 154 shows the case where it is less. The demonstration of Eq. 123 is applicable to

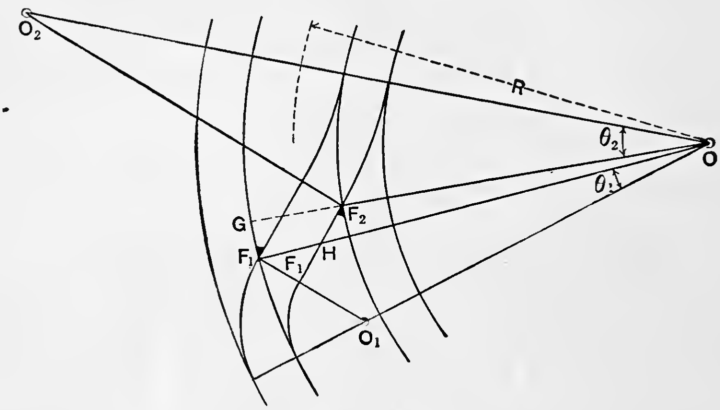

Fig. 154.

both figures. The relative position of the frogs F_1 and F_2 may be determined as follows, the solution being applicable to both Figs. 153 and 154:

$$HOF_2 = 180° - (90° - F_1) - (90° + F_2) = F_1 - F_2.$$

Then

$$GF_1 = 2(R + \tfrac{1}{2}d - \tfrac{1}{2}g) \sin \tfrac{1}{2}(F_1 - F_2). \quad . \quad . \quad . \quad (124)$$

Since F_2 comes out *any* angle, its value will not be in general that of an even frog number, and it will therefore need to be made to order.

(b) Continuing the switch-rail curves until they meet as a reversed curve. In this case F_1 and F_2 may be chosen at pleasure (within limitations), and they will of course be of regular sizes and equal or unequal as desired. F_1 and F_2 being known, θ_1 and θ_2 are computed by Eq. 95 and 91. In the triangle OO_1O_2 (see Fig. 155)

$$\text{vers } \psi = \frac{2(S - OO_2)(S - OO_1)}{OO_2 - OO_1},$$

in which $$S = \tfrac{1}{2}(OO_1 + OO_2 + O_1O_2);$$

but
$$OO_1 = R + \tfrac{1}{2}d - r_1,$$

$$OO_2 = R - \tfrac{1}{2}d + r_2,$$

$$O_1O_2 = r_1 + r_2;$$

$$\therefore \quad S = \tfrac{1}{2}(2R + 2r_2) = R + r_2;$$

$$S - OO_2 = R + r_2 - R + \tfrac{1}{2}d - r_2 = \tfrac{1}{2}d;$$

$$S - OO_1 = R + r_2 - R - \tfrac{1}{2}d + r_1 = r_1 + r_2 - \tfrac{1}{2}d;$$

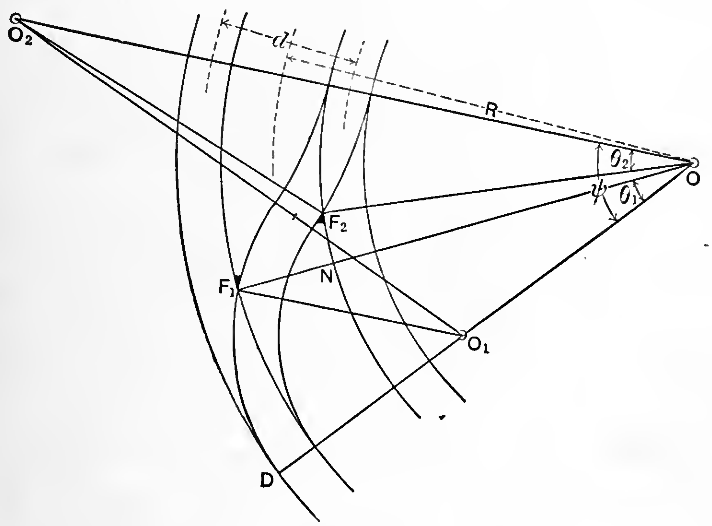

FIG. 155.

$$\text{vers}\,\psi = \frac{d(r_1 + r_2 - \tfrac{1}{2}d)}{(R - \tfrac{1}{2}d + r_2)(R + \tfrac{1}{2}d - r_1)}; \quad . \quad . \quad . \quad (125)$$

$$\sin OO_2O_1 = \sin\psi\frac{OO_1}{O_1O_2} = \sin\psi\frac{R + \tfrac{1}{2}d - r_1}{r_1 + r_2}; \quad . \quad . \quad (126)$$

$$O_2O_1D = \psi + O_1O_2O; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (127)$$

$$NF_2 = 2(R - \tfrac{1}{2}d + \tfrac{1}{2}g)\sin\tfrac{1}{2}(\psi - \theta_1 - \theta_2). \quad . \quad . \quad (128)$$

Although the above method introduces a reversed curve, yet it uses up less track than the first method and permits the use of ordinary frogs rather than those having some special angle which must be made to order.

276. Practical rules for switch-laying. A consideration of the previous sections will show that the formulæ are comparatively simple when the lead rails are assumed as circular; that they become complicated, even for turnouts from a straight main track, when the effect of straight frog and point rails is allowed for, and that they become hopelessly complicated when allowing for this effect on turnouts from a curved main track. It is also shown (§ 267) that the length of the lead is practically

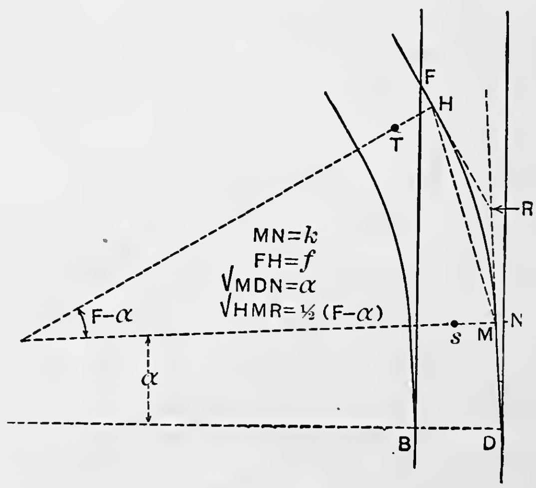

Fig. 140.

the same whether the main track is straight or is curved with such curves as are commonly used, and that the degree of curve of the lead rails from a curved main track may be found with close approximation by mere addition or subtraction. From this it may be assumed that, if the length of lead (L) and the radius of the lead rails (r) are computed from Eq. 87 and 90 for various frog angles, the same leads may be used for curved main track; also, that the degree of curve of the lead rails may be found by addition or subtraction, as indicated in § 267, and that the approximations involved will not be of practical detriment.

In accordance with this plan Table III has been computed from Eq. 87, 88, and 90. The *leads* there given may be used for all main tracks straight or curved. The table gives the degree of curve of the lead rails for *straight* main track; for a turnout to the *inside*, *add* the degree of curve of the main track; for a turnout to the *outside*, *subtract* it.

If the position of the switch-block is definitely determined, then the rails must be cut accordingly; but when some freedom is allowable (which never need exceed 15 feet and may require but a few inches), one rail-cutting may be avoided. Mark on the rails at B, F, and D; measure off the length of the switch-rails DN; offset $\frac{1}{2}g + k$ from N for the point S. The point H may be located (temporarily) by measuring along the rail a distance FH $(= f)$ and then swinging out a distance of $f \div n$ (n being the frog number). $HT = \frac{1}{2}g$ and is measured at right angles to FH. Points for track centers between S and T may be laid off by a transit or by the use of a string and tape. Substituting in Eq. 31 the value of R and of *chord* $(= ST)$, we may compute x $(= db)$. Locate the middle point d and the quarter points a'' and c''. Then $a''a$ and $c''c$ each equal three-fourths of db. Theoretically this gives a parabola rather than a circle, but the difference for all practical cases is too small for measurement.

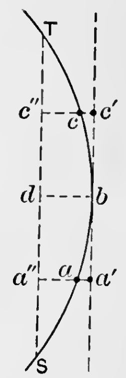

FIG. 156.

Example. Given a main track on a 4° curve; a turnout to the outside, using a number 9 frog; gauge $4' \, 8\frac{1}{2}''$; $f = 3'.37$; $k = 5\frac{3}{4}''$; $DN = 15' \, 0''$ and $\alpha = 1° \, 50'$. Then for a *straight* track r would equal 681.16 [$d = 8° \, 25'$]. For this curved track d will be nearly $(8° \, 25' - 4°) = 4° \, 25'$, or r will be 1297.6. L for the *straight* track would be 72.20; but since the lead is slightly increased (see § 267) when the turnout is on the outside of a curve, L may here be called 72.5. $FH = f = 3'.37$; $f \div n = 3.37 \div 9 = 0'.375 = 4''.5$. H, T, and S may be located as described above. ST may be measured on the ground, or it may be computed from Eq. 88, giving the value of 53.80 feet for straight track. Since it is slightly more for a turnout to the outside of a curve, it may be called 54.0. Then $x = db = \frac{(54.0)^2}{8 \times 1297.6} = 0.281$ feet, and aa'' and $cc'' = 0.21$ foot.

CROSSINGS.

277. Two straight tracks. When two straight tracks cross each other, four frogs are necessary, the angles of two of them being supplementary to the angles of the other. Since such crossings are sometimes operated at high speeds, they should be

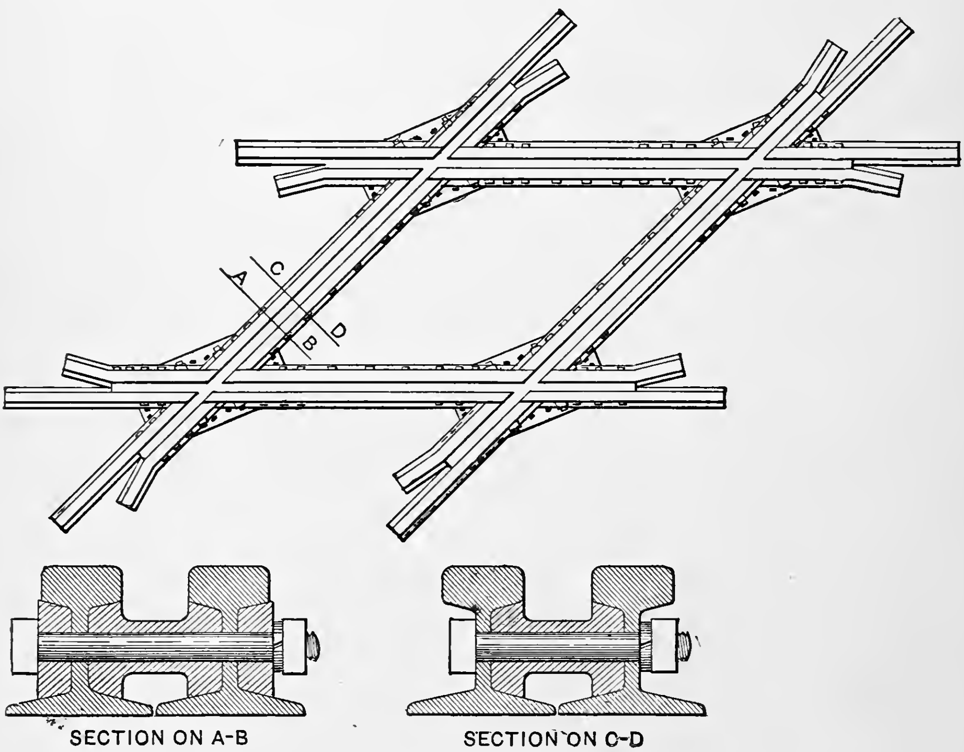

FIG. 157.—CROSSING.

very strongly constructed, and the angles should preferably be 90° or as near that as possible. The frogs will not in general be "stock" frogs of an even number, especially if the angles are large, but must be made to order with the required angles as measured. In Fig. 157 are shown the details of such a crossing. Note the fillers, bolts, and guard-rails.

278. One straight and one curved track. Structurally the crossing is about the same as above, but the frog angles are all unequal. In Fig. 158, R is known, and the angle M, made by the center lines of the tracks at their point of intersection, is also known.

$$M = NCM. \quad NC = R \cos M.$$

$$\left.\begin{aligned}
(R-\tfrac{1}{2}g)\cos F_1 &= NC + \tfrac{1}{2}g; \\
\therefore \quad \cos F_1 &= \frac{R\cos M + \tfrac{1}{2}g}{R-\tfrac{1}{2}g}. \\
\text{Similarly} \quad \cos F_2 &= \frac{R\cos M + \tfrac{1}{2}g}{R+\tfrac{1}{2}g}, \\
\cos F_3 &= \frac{R\cos M - \tfrac{1}{2}g}{R+\tfrac{1}{2}g}, \\
\cos F_4 &= \frac{R\cos M - \tfrac{1}{2}g}{R-\tfrac{1}{2}g}.
\end{aligned}\right\} \quad (129)$$

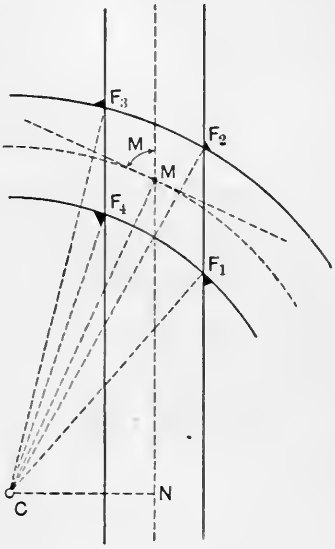

Fig. 158.

279. Two curved tracks. The four frogs are unequal, and the angle of each must be computed. The radii R_1 and R_2 are

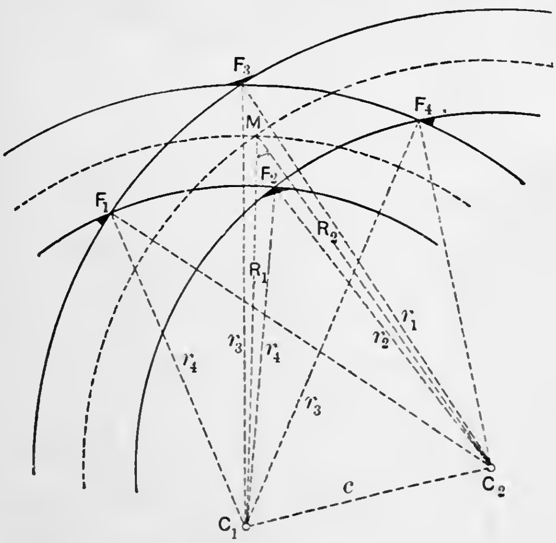

Fig. 159.

known; also the angle M. r_1, r_2, r_3, and r_4 are therefore known by adding or subtracting $\frac{1}{2}g$, but the lines are so indicated for brevity. Call the angle $MC_1C_2 = C_1$, the angle $MC_2C_1 = C_2$, and the line $C_1C_2 = c$. Then

$$\tfrac{1}{2}(C_1 + C_2) = 90° - \tfrac{1}{2} M$$

and

$$\tan \tfrac{1}{2}(C_1 - C_2) = \cot \tfrac{1}{2}M \frac{R_2 - R_1}{R_2 + R_1}.$$

C_1 and C_2 then become known and

$$c = C_1C_2 = R_2 \frac{\sin M}{\sin C_1}.$$

In the triangle $F_1C_1C_2$, call $\tfrac{1}{2}(c + r_1 + r_4) = s_1$; then

$$\text{vers } F_1 = \frac{2(s_1 - r_1)(s_1 - r_4)}{r_1 r_4}.$$

Similarly

$$\left.\begin{aligned} \text{vers } F_2 &= \frac{2(s_2 - r_2)(s_2 - r_4)}{r_2 r_4}, \\ \text{vers } F_3 &= \frac{2(s_3 - r_1)(s_3 - r_3)}{r_1 r_3}, \\ \text{vers } F_4 &= \frac{2(s_4 - r_2)(s_4 - r_3)}{r_2 r_3}. \end{aligned}\right\} \quad . \quad . \quad (130)$$

In the above equations

$$s_2 = \tfrac{1}{2}(c + r_2 + r_4),$$
$$s_3 = \tfrac{1}{2}(c + r_1 + r_3),$$
$$s_4 = \tfrac{1}{2}(c + r_2 + r_3).$$

APPENDIX.

THE ADJUSTMENTS OF INSTRUMENTS.

The accuracy of instrumental work may be vitiated by any one of a large number of inaccuracies in the geometrical relations of the parts of the instruments. Some of these relations are so apt to be altered by ordinary usage of the instrument that the makers have provided adjusting-screws so that the inaccuracies may be readily corrected. There are other possible defects, which, however, will seldom be found to exist, provided the instrument was properly made and has never been subjected to treatment sufficiently rough to distort it. Such defects, when found, can only be corrected by a competent instrument maker or repairer.

A warning is necessary to those who would test the accuracy of instruments, and especially to those whose experience in such work is small. Lack of skill in handling an instrument will often indicate an apparent error of adjustment when the real error is very different or perhaps non-existent. It is always a safe plan when testing an adjustment to note the amount of the apparent error; then, beginning anew, make another independent determination of the amount of the error. When two or more *perfectly independent* determinations of such an error are made it will generally be found that they differ by an appreciable amount. The *differences* may be due in variable measure to careless inaccurate manipulation and to instrumental *defects* which are wholly independent of the particular test being made. Such careful determinations of the amounts of the errors are generally advisable in view of the next paragraph.

Do not disturb the adjusting-screws any more than necessary. Although metals are apparently rigid, they are really elastic and yielding. If some parts of a complicated mechanism, which is held together largely by friction, are subjected to greater internal stresses than other parts of the mechanism, the jarring resulting from handling will frequently cause a slight readjustment in the parts which will tend to more nearly equalize the internal stresses. Such action frequently occurs with the adjusting mechanism of instruments. One screw may be strained more than others. The friction of parts may prevent the opposing screw from *immediately* taking up an equal stress. Perhaps the adjustment appears perfect under these conditions. Jarring diminishes the friction between the parts, and the unequal stresses tend to equalize. A motion takes place which, although microscopically minute, is sufficient to indicate an error of adjustment. A readjustment, made by unskillful hands, may not make the final adjustment any more perfect. The frequent shifting of adjusting-screws wears them badly, and when the screws are worn it is still more difficult to keep them from moving enough to vitiate the adjustments. It is therefore preferable in many cases to refrain from disturbing the adjusting-screws, especially as the accuracy of the work done is not *necessarily* affected by errors of adjustment, as may be illustrated:

(*a*) Certain operations are *absolutely* unaffected by certain errors of adjustment.

(*b*) Certain operations are so slightly affected by certain *small* errors of adjustment that their effect may properly be neglected.

(*c*) Certain errors of adjustment may be readily allowed for and neutralized so that no error results from the use of the unadjusted instrument. Illustrations of all these cases will be given under their proper heads.

ADJUSTMENTS OF THE TRANSIT.

1. *To have the plate-bubbles in the center of the tubes when the axis is vertical.* Clamp the upper plate and, with the lower

clamp loose, swing the instrument so that the plate-bubbles are parallel to the lines of opposite leveling-screws. Level up until both bubbles are central. Swing the instrument 180°. If the bubbles again settle at the center, the adjustment is perfect. If either bubble does not settle in the center, move the leveling-screws until the bubble is *half-way* back to the center. Then, before touching the adjusting-screws, note carefully the position of the bubbles and observe whether the bubbles always settle at the *same* place in the tube, no matter to what position the instrument may be rotated. When the instrument is so leveled, the axis is truly vertical and the discrepancies between this constant position of the bubbles and the centers of the tubes measure the errors of adjustment. By means of the adjusting-screws bring each bubble to the center of the tube. If this is done so skillfully that the true level of the instrument is not disturbed, the bubbles should settle in the center for all positions of the instrument. Under unskillful hands, two or more such trials may be necessary.

When the plates are not horizontal, the measured angle is greater than the true horizontal angle by the difference between the measured angle and its projection on a horizontal plane. When this angle of inclination is small, the difference is insignificant. Therefore when the plate-bubbles are *very nearly* in adjustment, the error of measurement of horizontal angles may be far within the lowest unit of measurement used. A *small* error of adjustment of the plate-bubble *perpendicular* to the telescope will affect the horizontal angles by only a small proportion of the error, which will be perhaps imperceptible. Vertical angles will be affected by the same insignificant amount. A *small* error of adjustment of the plate-bubble *parallel* to the telescope will affect horizontal angles very slightly, but will affect vertical angles by the full amount of the error.

All error due to unadjusted plate-bubbles may be avoided by noting in what positions in the tubes the bubbles will remain fixed for all positions of azimuth and then keeping the bubbles adjusted to these positions, for the axis is then truly vertical. It will often save time to work in this way temporarily rather than to stop to make the adjustments. This should especially be done when accurate vertical angles are required.

When the bubbles are truly adjusted, they should remain stationary, regardless of whether the telescope is revolved with the upper plate loose and the lower plate clamped or whether the whole instrument is revolved, the plates being clamped together. If there is any appreciable difference, it shows that the two vertical axes or "centers" of the plates are not concentric. This may be due to cheap and faulty construction or to the excessive wear that may be sometimes observed in an old instrument originally well made. In either case it can only be corrected by a maker.

2. *To make the revolving axis of the telescope perpendicular to the vertical axis of the instrument.* This is best tested by using a long plumb-line, so placed that the telescope must be pointed upward at an angle of about 45° to sight at the top of the plumb-line and downward about the same amount, if possible, to sight at the lower end. The vertical axis of the transit must be made truly vertical. Sight at the upper part of the line, clamping the horizontal plates. Swing the telescope down and see if the cross-wire again bisects the cord. If so, the adjustment is *probably* perfect (a conceivable exception will be noted later); if not, raise or lower one end of the axis by means of the adjusting-screws, placed at the top of one of the standards, until the cross-wire will bisect the cord both at top and bottom. The plumb-bob may be steadied, if necessary, by hanging it in a pail of water. As many telescopes cannot be focused on an object nearer than 6 or 8 feet from the telescope, this method requires a long plumb-line swung from a high point, which may be inconvenient.

Another method is to set up the instrument about 10 feet from a high wall. After leveling, sight at some convenient mark high up on the wall. Swing the telescope down and make a mark (when working alone some convenient natural mark may generally be found) low down on the wall. Plunge the telescope and revolve the instrument about its vertical axis and again sight at the upper mark. Swing down to the lower mark. If the wire again bisects it, the adjustment is perfect. If not, fix a point *half-way* between the two positions of the lower mark. The plane of this point, the upper point, and the center of the instrument is truly vertical. Adjust the axis to these upper and lower points as when using the plumb-line.

3. *To make the line of collimation perpendicular to the revolving axis of the telescope.* With the instrument level and

the telescope nearly horizontal point at some well-defined point at a distance of 200 feet or more. Plunge the telescope and establish a point in the opposite direction. Turn the whole instrument about the vertical axis until it again points at the first mark. Again plunge to "direct position" (i.e., with the level-tube *under* the telescope). If the vertical cross-wire again points at the second mark, the adjustment is perfect. If not, the error is *one-fourth* of the distance between the two positions of the second mark. Loosen the capstan-screw on one side of the telescope and tighten it on the other side until the vertical wire is set at the one-fourth mark. Turn the whole instrument by means of the tangent screw until the vertical wire is *midway* between the two positions of the second mark. Plunge the telescope. If the adjusting has been skillfully done, the cross-wire should come exactly to the first mark. As an "erecting eyepiece" reinverts an image already inverted, the ring carrying the cross-wires must be moved in the *same* direction as the *apparent* error in order to correct that error.

The necessity for the third adjustment lies principally in the practice of producing a line by plunging the telescope, but when this is required to be done with great accuracy it is always better to obtain the forward point by reversion (as described above for making the test) and take the *mean* of the two forward points. Horizontal and vertical angles are practically unaffected by *small* errors of this adjustment, unless, in the case of horizontal angles, the vertical angles to the points observed are very different.

Unnecessary motion of the adjusting-screws may sometimes be avoided by carefully establishing the forward point on line by repeated reversions of the instrument, and thus determining by repeated trials the exact amount of the error. *Differences* in the amount of error determined would be evidence of inaccuracy in manipulating the instrument, and would show that an adjustment based on the first trial would *probably* prove unsatisfactory.

The 2d and 3d adjustments are mutually dependent. If either adjustment is badly out, the other adjustment cannot be made except as follows:

(*a*) The second adjustment can be made regardless of the third when the lines to the high point and the low point make *equal* angles with the horizontal.

(*b*) The third adjustment can be made regardless of the second when the front and rear points are *on a level* with the instrument.

When both of these requirements are *nearly* fulfilled, and especially when the error of either adjustment is small, no trouble will be found in perfecting either adjustment on account of a small error in the other adjustment.

If the test for the second adjustment is made by means of the plumb-line and the vertical cross-wire intersects the line at all points as the telescope is raised or lowered, it not only demonstrates at once the accuracy of that adjustment, but also shows that the third adjustment is either perfect or has so small an error that it does not affect the second.

4. *To have the bubble of the telescope-level in the center of the tube when the line of collimation is horizontal.* The line of collimation should coincide with the optical axis of the telescope. If the object-glass and eyepiece have been properly centered, the previous adjustment will have brought the vertical cross-wire to the center of the field of view. The horizontal cross-wire should also be brought to the center of the field of view, and the bubble should be adjusted to it.

a. Peg method. Set up the transit at one end of a nearly level stretch of about 300 feet. Clamp the telescope with its bubble in the center. Drive a stake vertically under the eyepiece of the transit, and another about 300 feet away. Observe the height of the center of the eyepiece (the telescope being level) above the stake (calling it a); observe the reading of the rod when held on the other stake (calling it b); take the instrument to the other stake and set it up so that the eyepiece is vertically over the stake, observing the height, c; take a reading on the first stake, calling it d. If this adjustment is perfect, then

$$a - d = b - c,$$

or $$(a - d) - (b - c) = 0.$$

Call $$(a - d) - (b - c) = 2m.$$

When m is positive, the line points downward;
" m " negative, " " " upward.

To adjust: if the line points *up*, sight the horizontal cross-wire (by moving the vertical tangent screw) at a point which is m lower, then adjust the bubble so that it is in the center.

By taking several independent values for a, b, c, and d, a mean value for m is obtained, which is more reliable and which may save much unnecessary working of the adjusting-screws.

b. Using an auxiliary level. When a carefully adjusted level is at hand, this adjustment may sometimes be more easily made by setting up the transit and level, so that their lines of collimation are as nearly as possible at the same height. If a point may be found which is half a mile or more away and which is on the horizontal cross-wire of the level, the horizontal cross-wire of the transit may be pointed directly at it, and the bubble adjusted accordingly. Any slight difference in the heights of the lines of collimation of the transit and level (say $\frac{1}{4}''$) may almost be disregarded at a distance of $\frac{1}{2}$ mile or more, or, if the difference of level would have an appreciable effect, even this may be practically eliminated by making an estimated allowance when sighting at the distant point. Or, if a distant point is not available, a level-rod with target may be used at a distance of (say) 300 feet, making allowance for the carefully determined difference of elevation of the two lines of collimation.

5. *Zero of vertical circle.* When the line of collimation is truly horizontal and the vertical axis is truly vertical, the reading of the vertical circle should be 0°. If the arc is adjustable, it should be brought to 0°. If it is not adjustable, the *index error* should be observed, so that it may be applied to all readings of vertical angles.

ADJUSTMENTS OF THE WYE LEVEL.

1. *To make the line of collimation coincide with the center of the rings.* Point the intersection of the cross-wires at some well-defined point which is at a considerable distance. The instrument need not be level, which allows much greater liberty in choosing a convenient point. The vertical axis should be

clamped, and the clips over the wyes should be loosened and raised. Rotate the telescope in the wyes. The intersection of the cross-wires should be continually on the point. If it is not, it requires adjustment. Rotate the telescope 180° and adjust *one-half* of the error by means of the capstan-headed screws that move the cross-wire ring. It should be remembered that, with an erecting telescope, on account of the inversion of the image, the ring should be moved in the direction of the *apparent* error. Adjust the other half of the error with the leveling-screws. Then rotate the telescope 90° from its usual position, sight accurately at the point, and then rotate 180° from that position and adjust any error as before. It may require several trials, but it is necessary to adjust the ring until the intersection of the cross-wires will remain on the point for any position of rotation.

If such a test is made on a very distant point and again on a point only 10 or 15 feet from the instrument, the adjustment may be found correct for one point and incorrect for the other. This indicates that the object-slide is improperly centered. Usually this defect can only be corrected by an instrument-maker. If the difference is very small it may be ignored, but the adjustment should then be made on a point which is at about the mean distance for usual practice—say 150 feet.

If the whole image appears to shift as the telescope is rotated, it indicates that the eyepiece is improperly adjusted. This defect is likewise usually corrected only by the maker. It does not interfere with instrumental accuracy, but it usually causes the intersection of the cross-wires to be eccentric with the field of view.

2. *To make the axis of the level tube parallel to the line of collimation.* Raise the clips as far as possible. Swing the level so that it is parallel to a pair of opposite leveling-screws and clamp it. Bring the bubble to the middle of the tube by means of the leveling-screws. Take the telescope out of the wyes and replace it end for end, using *extreme care* that the wyes are not jarred by the action. If the bubble does not come to the center, correct *one-half* of the error by the vertical adjusting-screws at one end of the bubble. Correct the other half by the leveling-screws. Test the work by again changing the telescope end for end in the wyes.

Care should be taken while making this adjustment to see that the level-tube is vertically under the telescope. With the bubble in the center of the tube, rotate the telescope in the wyes for a considerable angle each side of the vertical. If the first half of the adjustment has been made and the bubble moves, it shows that the axis of the wyes and the axis of the level-tube are not in the same vertical plane although both have been made horizontal. By moving one end of the level-tube *sidewise* by means of the horizontal screws at one end of the tube, the two axes may be brought into the same plane. As this adjustment is liable to disturb the other, both should be alternately tested until both requirements are complied with.

By these methods the axis of the bubble is made parallel to the axis of the wyes; and as this has been made parallel to the lines of collimation by means of the previous adjustment, the axis of the bubble is therefore parallel to the line of collimation.

3. *To make the line of collimation perpendicular to the vertical axis.* Level up so that the instrument is approximately level over both sets of leveling-screws. Then, after leveling carefully over one pair of screws, revolve the telescope 180°. If it is not level, adjust half of the error by means of the capstan-headed screw under one of the wyes, and the other half by the leveling-screws. Reverse again as a test.

When the first two adjustments have been accurately made, good leveling may always be done by bringing the bubble to the center by means of the leveling-screws, at every sight if necessary, even if the third adjustment is not made. Of course this third adjustment should be made as a matter of convenience, so that the line of collimation may be always level no matter in what direction it may be pointed, but it is not *necessary* to stop work to make this adjustment every time it is found to be defective.

ADJUSTMENTS OF THE DUMPY LEVEL.

1. *To make the axis of the level-tube perpendicular to the vertical axis.* Level up so that the instrument is approximately level over both sets of leveling-screws. Then, after leveling carefully over one pair of screws, revolve the telescope 180°. If

it is not level, adjust *one-half* of the error by means of the adjusting-screws at one end of the bubble, and the other half by means of the leveling-screws. Reverse again as a test.

2. *To make the line of collimation perpendicular to the vertical axis.* The method of adjustment is identical with that for the transit (No. 4, p. 308) except that the cross-wire must be adjusted to agree with the level-bubble rather than *vice versa*, as is the case with the corresponding adjustment of the transit; i.e., with the level-bubble in the center, raise or lower the horizontal cross-wire until it points at the mark known to be on a level with the center of the instrument.

If the instrument has been well made and has not been distorted by rough usage, the cross-wires will intersect at the center of the field of view when adjusted as described. If they do not, it indicates an error which ordinarily can only be corrected by an instrument-maker. The error may be due to any one of several causes, which are

(*a*) faulty centering of object-slide;

(*b*) faulty centering of eyepiece;

(*c*) distortion of instrument so that the geometric axis of the telescope is not perpendicular to the vertical axis. If the error is only just perceptible, it will not probably cause any error in the work.

EXPLANATORY NOTE ON THE USE OF THE TABLES.

The logarithms here given are "five-place," but the last figure sometimes has a special mark over it (e.g., $\hat{6}$) which indicates that one-half a unit in the last place should be *added*. For example:

the value	includes all values between
.69586	.6958575000 + and .6958624999 . . .
$.6958\hat{6}$	.6958625000 + and .6958674999 . . .

The maximum error in any one value therefore does not exceed one-quarter of a fifth-place unit.

When adding or subtracting such logarithms allow a half-unit for such a sign. For example:

.69586	.69586	$.6958\hat{6}$
.10841	$.1084\hat{1}$	$.1084\hat{1}$
$.1294\hat{7}$	$.1294\hat{7}$	$.1294\hat{7}$
$.9337\hat{4}$	.93375	$.9337\hat{5}$

All other logarithmic operations are performed as usual and are supposed to be understood by the student.

Deg.	0°		1°		2°		3°		Deg.
Min.	Radius.	Log *R*	Radius.	Log *R*	Radius.	Log *R*	Radius.	Log *R*	Min.
0	∞	∞	5729.6	3.75813	2864.9	3.45711	1910.1	3.28105	0
1	343775	5.53627	5635.7	.75095	2841.3	.45351	1899.5	.27864	1
2	171887	5.23524	5544.8	.74389	2818.0	.44993	1889.1	.27625	2
3	114592	5.05915	5456.8	.73694	2795.1	.44639	1878.8	.27387	3
4	85944	4.93421	5371.6	.73010	2772.5	.44287	1868.6	.27151	4
5	68755	4.83730	5288.9	.72336	2750.4	.43939	1858.5	.26915	5
6	57296	4.75812	5208.8	3.71673	2728.5	3.43593	1848.5	3.26681	6
7	49111	.69117	5131.0	.71020	2707.0	.43249	1838.6	.26448	7
8	42972	.63318	5055.6	.70377	2685.9	.42909	1828.8	.26217	8
9	38197	.58203	4982.3	.69743	2665.1	.42571	1819.1	.25986	9
10	34377	.53627	4911.2	.69118	2644.6	.42235	1809.6	.25757	10
11	31252	4.49488	4842.0	3.68502	2624.4	3.41903	1800.1	3.25529	11
12	28648	.45709	4774.7	.67895	2604.5	.41572	1790.7	.25303	12
13	26444	.42233	4709.3	.67296	2584.9	.41245	1781.5	.25077	13
14	24555	.39014	4645.7	.66705	2565.6	.40919	1772.3	.24853	14
15	22918	.36018	4583.8	.66122	2546.6	.40597	1763.2	.24629	15
16	21486	4.33215	4523.4	3.65547	2527.9	3.40276	1754.2	3.24407	16
17	20222	.30582	4464.7	.64979	2509.5	.39958	1745.3	.24186	17
18	19099	.28100	4407.5	.64419	2491.3	.39642	1736.5	.23967	18
19	18093	.25752	4351.7	.63865	2473.4	.39329	1727.8	.23748	19
20	17189	.23524	4297.3	.63319	2455.7	.39017	1719.1	.23530	20
21	16370	4.21405	4244.2	3.62780	2438.3	3.38708	1710.6	3.23314	21
22	15626	.19385	4192.5	.62247	2421.1	.38401	1702.1	.23098	22
23	14947	.17454	4142.0	.61720	2404.2	.38097	1693.7	.22884	23
24	14324	.15606	4092.7	.61200	2387.5	.37794	1685.4	.22670	24
25	13751	.13833	4044.5	.60686	2371.0	.37494	1677.2	.22458	25
26	13222	4.12130	3997.5	3.60178	2354.8	3.37195	1669.1	3.22247	26
27	12732	.10491	3951.5	.59676	2338.8	.36899	1661.0	.22037	27
28	12278	.08911	3906.6	.59180	2323.0	.36604	1653.0	.21827	28
29	11854	.07387	3862.7	.58689	2307.4	.36312	1645.1	.21619	29
30	11459	.05915	3819.8	.58204	2292.0	.36021	1637.3	.21412	30
31	11090	4.04491	3777.9	3.57724	2276.8	3.35733	1629.5	3.21206	31
32	10743	.03112	3736.8	.57250	2261.9	.35446	1621.8	.21000	32
33	10417	.01776	3696.6	.56780	2247.1	.35162	1614.2	.20796	33
34	10111	4.00479	3657.3	.56316	2232.5	.34879	1606.7	.20593	34
35	9822.2	3.99221	3618.8	.55856	2218.1	.34598	1599.2	.20390	35
36	9549.3	3.97997	3581.1	3.55401	2203.9	3.34318	1591.8	3.20189	36
37	9291.3	.96807	3544.2	.54951	2189.8	.34041	1584.5	.19988	37
38	9046.7	.95649	3508.0	.54506	2176.0	.33765	1577.2	.19789	38
39	8814.8	.94521	3472.6	.54065	2162.3	.33491	1570.0	.19590	39
40	8594.4	.93421	3437.9	.53629	2148.8	.33219	1562.9	.19392	40
41	8384.8	3.92349	3403.8	3.53197	2135.4	3.32949	1555.8	3.19195	41
42	8185.2	.91302	3370.5	.52769	2122.3	.32680	1548.8	.18999	42
43	7994.8	.90281	3337.7	.52345	2109.2	.32412	1541.9	.18804	43
44	7813.1	.89282	3305.7	.51925	2096.4	.32147	1535.0	.18610	44
45	7639.5	.88306	3274.2	.51510	2083.7	.31883	1528.2	.18417	45
46	7473.4	3.87352	3243.3	3.51098	2071.1	3.31621	1521.4	3.18224	46
47	7314.4	.86418	3213.0	.50691	2058.7	.31360	1514.7	.18032	47
48	7162.0	.85503	3183.2	.50287	2046.5	.31101	1508.1	.17842	48
49	7015.9	.84608	3154.0	.49886	2034.4	.30843	1501.5	.17652	49
50	6875.6	.83731	3125.4	.49490	2022.4	.30587	1495.0	.17462	50
51	6740.7	3.82871	3097.2	3.49097	2010.6	3.30332	1488.5	3.17274	51
52	6611.1	.82027	3069.6	.48707	1998.9	.30079	1482.1	.17087	52
53	6486.4	.81200	3042.4	.48321	1987.3	.29827	1475.7	.16900	53
54	6366.3	.80388	3015.7	.47939	1975.9	.29577	1469.4	.16714	54
55	6250.5	.79591	2989.5	.47559	1964.6	.29328	1463.2	.16529	55
56	6138.9	3.78809	2963.7	3.47183	1953.5	3.29081	1457.0	3.16344	56
57	6031.2	.78040	2938.4	.46811	1942.4	.28835	1450.8	.16161	57
58	5927.2	.77285	2913.5	.46441	1931.5	.28590	1444.7	.15978	58
59	5826.8	.76542	2889.0	.46075	1920.7	.28347	1438.7	.15796	59
60	5729.6	.75813	2864.9	.45711	1910.1	.28105	1432.7	.15615	60

Deg.	4°		5°		6°		7°		Deg.
Min.	Radius.	Log *R*	Radius.	Log *R*	Radius.	Log *R*	Radius.	Log *R*	Min.
0	1432.7	3.15615	1146.3	3.05929	955.37	2.98017	819.02	2.91329	0
1	1426.7	.15434̂	1142.5	.05784̂	952.72	.97896	817.08	.91226̂	1
2	1420.8	.15255	1138.7	.05640̂	950.09	.97776	815.14	.91123̂	2
3	1415.0	.15076	1134.9	.05497	947.48	.97657	813.22	.91021	3
4	1409.2	.14897̂	1131.2	.05354	944.88	.97537̂	811.30	.90918̂	4
5	1403.5	.14720	1127.5	.05211̂	942.29	.97418̂	809.40	.90816	5
6	1397.8	3.14543	1123.8	3.05069̂	939.72	2.97300	807.50	2.90714	6
7	1392.1	.14367	1120.2	.04928	937.16	.97181̂	805.61	.90612̂	7
8	1386.5	.14191̂	1116.5	.04787	934.62	.97063̂	803.73	.90511	8
9	1380.9	.14017	1112.9	.04646	932.09	.96945̂	801.86	.90410	9
10	1375.4	.13843	1109.3	.04506	929.57	.96828	800.00	.90309	10
11	1369.9	3.13669̂	1105.8	3.04366	927.07	2.96711	798.14	2.90208	11
12	1364.5	.13497	1102.2	.04227	924.58	.96594̂	796.30	.90107̂	12
13	1359.1	.13325	1098.7	.04088	922.10	.96478	794.46	.90007̂	13
14	1353.8	.13154	1095.2	.03949̂	919.64	.96361̂	792.63	.89907̂	14
15	1348.4	.12983̂	1091.7	.03811̂	917.19	.96246	790.81	.89807̂	15
16	1343.2	3.12813̂	1088.3	3.03674	914.75	2.96130̂	789.00	2.89708	16
17	1338.0	.12644	1084.8	.03537	912.33	.96015	787.20	.89608̂	17
18	1332.8	.12475̂	1081.4	.03400	909.92	.95900	785.41	.89509̂	18
19	1327.6	.12307̂	1078.1	.03264	907.52	.95785̂	783.62	.89410̂	19
20	1322.5	.12140̂	1074.7	.03128	905.13	.95671	781.84	.89312	20
21	1317.5	3.11974	1071.3	3.02992̂	902.76	2.95557	780.07	2.89213̂	21
22	1312.4	.11808	1068.0	.02857̂	900.40	.95443̂	778.31	.89115	22
23	1307.4	.11642̂	1064.7	.02723	898.05	.95330	776.55	.89017	23
24	1302.5	.11477̂	1061.4	.02589	895.71	.95217	774.81	.88919̂	24
25	1297.6	.11313̂	1058.2	.02455	893.39	.95104	773.07	.88821̂	25
26	1292.7	3.11150	1054.9	3.02322	891.08	2.94991̂	771.34	2.88724̂	26
27	1287.9	.10987	1051.7	.02189	888.78	.94879	769.61	.88627	27
28	1283.1	.10825	1048.5	.02056̂	886.49	.94767̂	767.90	.88530̂	28
29	1278.3	.10663	1045.3	.01924̂	884.21	.94655̂	766.19	.88433̂	29
30	1273.6	.10502	1042.1	.01792̂	881.95	.94544	764.49	.88337	30
31	1268.9	3.10341̂	1039.0	3.01661̂	879.69	2.94433	762.80	2.88241	31
32	1264.2	.10182	1035.9	.01530̂	877.45	.94322̂	761.11	.88145	32
33	1259.6	.10022̂	1032.8	.01400	875.22	.94212	759.43	.88049	33
34	1255.0	.09864	1029.7	.01270	873.00	.94101̂	757.76	.87953̂	34
35	1250.4	.09705̂	1026.6	.01140	870.80	.93991̂	756.10	.87858	35
36	1245.9	3.09548	1023.5	3.01010̂	868.60	2.93882	754.44	2.87762̂	36
37	1241.4	.09391	1020.5	.00882	866.41	.93772̂	752.80	.87668	37
38	1236.9	.09234̂	1017.5	.00753	864.24	.93663̂	751.16	.87573	38
39	1232.5	.09079	1014.5	.00625	862.07	.93554̂	749.52	.87478̂	39
40	1228.1	.08923̂	1011.5	.00497	859.92	.93446	747.89	.87384	40
41	1223.7	3.08769	1008.6	3.00370	857.78	2.93337̂	746.27	2.87290	41
42	1219.4	.08614̂	1005.6	.00242̂	855.65	.93229̂	744.66	.87196	42
43	1215.1	.08461	1002.7	3.00116	853.53	.93122	743.06	.87102	43
44	1210.8	.08308	999.76	2.99989̂	851.42	.93014	741.46	.87008̂	44
45	1206.6	.08155̂	996.87	.99863̂	849.32	.92907	739.86	.86915	45
46	1202.4	3.08003̂	993.99	2.99738	847.23	2.92800	738.28	2.86822	46
47	1198.2	.07852	991.13	.99613	845.15	.92693	736.70	.86729	47
48	1194.0	.07701	988.28	.99488	843.08	.92587	735.13	.86636̂	48
49	1189.9	.07550̂	985.45	.99363̂	841.02	.92480̂	733.56	.86544	49
50	1185.8	.07400̂	982.64	.99239̂	838.97	.92374̂	732.01	.86451̂	50
51	1181.7	3.07251	979.84	2.99115̂	836.93	2.92269	730.45	2.86359̂	51
52	1177.7	.07102	977.06	.98992	834.90	.92163̂	728.91	.86267̂	52
53	1173.6	.06954	974.29	.98869	832.89	.92058̂	727.37	.86175̂	53
54	1169.7	.06806	971.54	.98746̂	830.88	.91953̂	725.84	.86084	54
55	1165.7	.06658̂	968.81	.98624	828.88	.91849	724.31	.85992̂	55
56	1161.8	3.06511̂	966.09	2.98501̂	826.89	2.91744̂	722.79	2.85901̂	56
57	1157.9	.06365̂	963.39	.98380	824.91	.91640̂	721.28	.85810̂	57
58	1154.0	.06219̂	960.70	.98258̂	822.93	.91536̂	719.77	.85719̂	58
59	1150.1	.06074	958.03	.98137̂	820.97	.91433	718.27	.85629	59
60	1146.3	.05929	955.37	.98017	819.02	.91329̂	716.78	.85538̂	60

Deg.	8°		9°		10°		11°		Deg.
Min.	Radius.	Log R	Radius.	Log R	Radius.	Log R	Radius.	Log R	Min.
0	716.78	2.85538	637.27	2.80432	573.69	2.75867	521.67	2.71739	0
1	715.29	.85448	636.10	.80352	572.73	.75795	520.88	.71674	1
2	713.81	.85358	634.93	.80272	571.78	.75723	520.10	.71608	2
3	712.34	.85268	633.76	.80192	570.84	.75651	519.32	.71543	3
4	710.87	.85178	632.60	.80113	569.90	.75579	518.54	.71478	4
5	709.40	.85089	631.44	.80033	568.96	.75508	517.76	.71413	5
6	707.95	2.85000	630.29	2.79954	568.02	2.75436	516.99	2.71348	6
7	706.49	.84911	629.14	.79874	567.09	.75365	516.21	.71283	7
8	705.05	.84822	627.99	.79795	566.16	.75293	515.44	.71218	8
9	703.61	.84733	626.85	.79716	565.23	.75222	514.68	.71153	9
10	702.17	.84644	625.71	.79637	564.31	.75151	513.91	.71088	10
11	700.75	2.84556	624.58	2.79558	563.38	2.75080	513.15	2.71024	11
12	699.33	.84468	623.45	.79480	562.47	.75009	512.38	.70959	12
13	697.91	.84380	622.32	.79401	561.55	.74939	511.63	.70895	13
14	696.50	.84292	621.20	.79323	560.64	.74868	510.87	.70831	14
15	695.09	.84204	620.09	.79245	559.73	.74798	510.11	.70767	15
16	693.70	2.84117	618.97	2.79167	558.82	2.74727	509.36	2.70702	16
17	692.30	.84029	617.87	.79089	557.92	.74657	508.61	.70638	17
18	690.91	.83942	616.76	.79011	557.02	.74587	507.86	.70575	18
19	689.53	.83855	615.66	.78934	556.12	.74517	507.12	.70511	19
20	688.16	.83768	614.56	.78856	555.23	.74447	506.38	.70447	20
21	686.78	2.83682	613.47	2.78779	554.34	2.74377	505.64	2.70383	21
22	685.42	.83595	612.38	.78702	553.45	.74307	504.90	.70320	22
23	684.06	.83509	611.30	.78625	552.56	.74238	504.16	.70257	23
24	682.70	.83423	610.21	.78548	551.68	.74168	503.42	.70193	24
25	681.35	.83337	609.14	.78471	550.80	.74099	502.69	.70130	25
26	680.01	2.83251	608.06	2.78395	549.92	2.74030	501.96	2.70067	26
27	678.67	.83166	606.99	.78318	549.05	.73961	501.23	.70004	27
28	677.34	.83080	605.93	.78242	548.17	.73892	500.51	.69941	28
29	676.01	.82995	604.86	.78165	547.30	.73823	499.78	.69878	29
30	674.69	.82910	603.80	.78089	546.44	.73754	499.06	.69815	30
31	673.37	2.82825	602.75	2.78013	545.57	2.73685	498.34	2.69752	31
32	672.06	.82740	601.70	.77938	544.71	.73617	497.62	.69690	32
33	670.75	.82656	600.65	.77862	543.86	.73548	496.91	.69627	33
34	669.45	.82571	599.61	.77786	543.00	.73480	496.19	.69565	34
35	668.15	.82487	598.57	.77711	542.15	.73412	495.48	.69503	35
36	666.86	2.82403	597.53	2.77636	541.30	2.73343	494.77	2.69440	36
37	665.57	.82319	596.50	.77561	540.45	.73275	494.07	.69378	37
38	664.29	.82235	595.47	.77486	539.61	.73207	493.36	.69316	38
39	663.01	.82152	594.44	.77411	538.76	.73140	492.66	.69254	39
40	661.74	.82068	593.42	.77336	537.92	.73072	491.96	.69192	40
41	660.47	2.81985	592.40	2.77261	537.09	2.73004	491.26	2.69131	41
42	659.21	.81902	591.38	.77187	536.25	.72937	490.56	.69069	42
43	657.95	.81819	590.37	.77112	535.42	.72869	489.86	.69007	43
44	656.69	.81736	589.36	.77038	534.59	.72802	489.17	.68946	44
45	655.45	.81653	588.36	.76964	533.77	.72735	488.48	.68884	45
46	654.20	2.81571	587.36	2.76890	532.94	2.72668	487.79	2.68823	46
47	652.96	.81489	586.36	.76816	532.12	.72601	487.10	.68762	47
48	651.73	.81406	585.36	.76742	531.30	.72534	486.42	.68701	48
49	650.50	.81324	584.37	.76669	530.49	.72467	485.73	.68640	49
50	649.27	.81243	583.38	.76595	529.67	.72401	485.05	.68579	50
51	648.05	2.81161	582.40	2.76522	528.86	2.72334	484.37	2.68518	51
52	646.84	.81079	581.42	.76449	528.05	.72267	483.69	.68457	52
53	645.63	.80998	580.44	.76376	527.25	.72201	483.02	.68396	53
54	644.42	.80917	579.47	.76303	526.44	.72135	482.34	.68335	54
55	643.22	.80836	578.49	.76230	525.64	.72069	481.67	.68275	55
56	642.02	2.80755	577.53	2.76157	524.84	2.72003	481.00	2.68214	56
57	640.83	.80674	576.56	.76084	524.05	.71937	480.33	.68154	57
58	639.64	.80593	575.60	.76012	523.25	.71871	479.67	.68094	58
59	638.45	.80513	574.64	.75939	522.46	.71805	479.00	.68033	59
60	637.27	.80432	573.69	.75867	521.67	.71739	478.34	.67973	60

Deg.	Radius.	Log *R*	Deg.	Radius.	Log *R*	Deg.	Radius.	Log *R*	Deg.	Radius.	Log *R*
12°	478.34	2.67973	14°	410.28	2.61307	16°	359.26	2.55541	21°	274.37	2.43833
2	477.02	.67853	2	409.31	.61205	5	357.42	.55317	10	272.23	.43494
4	475.71	.67734	4	408.34	.61102	10	355.59	.55094	20	270.13	.43157
6	474.40	.67614	6	407.38	.61000	15	353.77	.54872	30	268.06	.42823
8	473.10	.67495	8	406.42	.60898	20	351.98	.54652	40	266.02	.42492
10	471.81	2.67376	10	405.47	2.60796	25	350.21	.54432	50	264.02	.42163
12	470.53	.67258	12	404.53	.60694	30	348.45	2.54214	22°	262.04	2.41837
14	469.25	.67140	14	403.58	.60593	35	346.71	.53997	10	260.10	.41513
16	467.98	.67022	16	402.65	.60492	40	344.99	.53780	20	258.18	.41192
18	466.72	.66905	18	401.71	.60391	45	343.29	.53565	30	256.29	.40873
20	465.46	2.66788	20	400.78	2.60291	50	341.60	.53351	40	254.43	.40557
22	464.21	.66671	22	399.86	.60190	55	339.93	.53138	50	252.60	.40243
24	462.97	.66555	24	398.94	.60090	17°	338.27	2.52927	23°	250.79	2.39931
26	461.73	.66439	26	398.02	.59990	5	336.64	.52716	10	249.01	.39622
28	460.50	.66323	28	397.11	.59891	10	335.01	.52506	20	247.26	.39315
30	459.28	2.66207	30	396.20	2.59791	15	333.41	.52297	30	245.53	.39010
32	458.06	.66092	32	395.30	.59692	20	331.82	.52090	40	243.82	.38707
34	456.85	.65977	34	394.40	.59593	25	330.24	.51883	50	242.14	.38407
36	455.65	.65863	36	393.50	.59494	30	328.68	2.51677	24°	240.49	2.38109
38	454.45	.65748	38	392.61	.59396	35	327.13	.51472	10	238.85	.37813
40	453.26	2.65634	40	391.72	2.59298	40	325.60	.51269	20	237.24	.37519
42	452.07	.65521	42	390.84	.59199	45	324.09	.51066	30	235.65	.37227
44	450.89	.65407	44	389.96	.59102	50	322.59	.50864	40	234.08	.36937
46	449.72	.65294	46	389.08	.59004	55	321.10	.50663	50	232.54	.36649
48	448.56	.65181	48	388.21	.58907	18°	319.62	2.50464	25°	231.01	2.36363
50	447.40	2.65069	50	387.34	2.58809	5	318.16	.50265	30	226.55	.35517
52	446.24	.64957	52	386.48	.58713	10	316.71	.50067	26°	222.27	.34688
54	445.09	.64845	54	385.62	.58616	15	315.28	.49869	30	218.15	.33875
56	443.95	.64733	56	384.77	.58519	20	313.86	.49673	27°	214.18	2.33078
58	442.81	.64622	58	383.91	.58423	25	312.45	.49478	30	210.36	.32296
13°	441.68	2.64511	15°	383.06	2.58327	30	311.06	2.49284	28°	206.68	.31529
2	440.56	.64400	2	382.22	.58231	35	309.67	.49090	30	203.13	.30776
4	439.44	.64290	4	381.38	.58135	40	308.30	.48898	29°	199.70	2.30037
6	438.33	.64180	6	380.54	.58040	45	306.95	.48706	30	196.38	.29310
8	437.22	.64070	8	379.71	.57945	50	305.60	.48515	30°	193.19	.28597
10	436.12	2.63960	10	378.88	2.57850	55	304.27	.48325	30	190.09	.27896
12	435.02	.63851	12	378.05	.57755	19°	302.94	2.48136	31°	187.10	2.27207
14	433.93	.63742	14	377.23	.57661	5	301.63	.47948	32	181.40	.25863
16	432.84	.63633	16	376.41	.57566	10	300.33	.47760	33	176.05	.24563
18	431.76	.63524	18	375.60	.57472	15	299.04	.47573	34	171.02	.23303
20	430.69	2.63416	20	374.79	2.57378	20	297.77	.47388	35	166.28	.22083
22	429.62	.63308	22	373.98	.57284	25	296.50	.47203	36	161.80	2.20899
24	428.56	.63201	24	373.17	.57191	30	295.25	2.47018	37	157.58	.19749
26	427.50	.63093	26	372.37	.57097	35	294.00	.46835	38	153.58	.18633
28	426.44	.62986	28	371.57	.57004	40	292.77	.46652	39	149.79	.17547
30	425.40	2.62879	30	370.78	2.56911	45	291.55	.46471	40	146.19	.16492
32	424.35	.62773	32	369.99	.56819	50	290.33	.46289	41	142.77	2.15464
34	423.32	.62666	34	369.20	.56726	55	289.13	.46109	42	139.52	.14464
36	422.28	.62560	36	368.42	.56634	20°	287.94	2.45930	43	136.43	.13489
38	421.26	.62454	38	367.64	.56542	5	286.76	.45751	44	133.47	.12539
40	420.23	2.62349	40	366.86	2.56450	10	285.58	.45573	45	130.66	.11613
42	419.22	.62243	42	366.09	.56358	15	284.42	.45396	46	127.97	2.10709
44	418.20	.62138	44	365.31	.56266	20	283.27	.45219	47	125.39	.09827
46	417.19	.62034	46	364.55	.56175	25	282.12	.45044	48	122.93	.08965
48	416.19	.61929	48	363.78	.56084	30	280.99	2.44869	49	120.57	.08124
50	415.19	2.61825	50	363.02	2.55993	35	279.86	.44694	50	118.31	.07302
52	414.20	.61721	52	362.26	.55902	40	278.75	.44521	52	114.06	2.05713
54	413.21	.61617	54	361.51	.55812	45	277.64	.44348	54	110.13	.04192
56	412.23	.61514	56	360.76	.55721	50	276.54	.44176	56	106.50	.02736
58	411.25	.61410	58	360.01	.55631	55	275.45	.44004	58	103.13	.01340
14°	410.28	2.61307	16°	359.26	2.55541	21°	274.37	2.43833	60	100.00	2.00000

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS FOR A 1° CURVE.

Δ	Tangent *T.*	Ext. Dist. *E.*	Long Ch'd *LC.*	Δ	Tangent *T.*	Ext. Dist. *E.*	Long Ch'd *LC.*	Δ	Tangent *T.*	Ext. Dist. *E.*	Long Ch'd *LC.*
1°	50.00	0.218	100.00	**11°**	551.70	26.500	1098.3	**21°**	1061.9	97.58	2088.3
10′	58.34	0.297	116.67	10	560.11	27.313	1114.9	10	1070.6	99.15	2104.7
20	66.67	0.388	133.33	20	568.53	28.137	1131.5	20	1079.2	100.75	2121.1
30	75.01	0.491	150.00	30	576.95	28.974	1148.1	30	1087.8	102.35	2137.4
40	83.34	0.606	166.66	40	585.36	29.824	1164.7	40	1096.4	103.97	2153.8
50	91.68	0.733	183.33	50	593.79	30.686	1181.2	50	1105.1	105.60	2170.2
2°	100.01	0.873	199.99	**12°**	602.21	31.561	1197.8	**22°**	1113.7	107.24	2186.5
10	108.35	1.024	216.66	10	610.64	32.447	1214.4	10	1122.4	108.90	2202.9
20	116.68	1.188	233.32	20	619.07	33.347	1231.0	20	1131.0	110.57	2219.2
30	125.02	1.364	249.98	30	627.50	34.259	1247.5	30	1139.7	112.25	2235.6
40	133.36	1.552	266.65	40	635.93	35.183	1264.1	40	1148.4	113.95	2251.9
50	141.70	1.752	283.31	50	644.37	36.120	1280.7	50	1157.0	115.66	2268.3
3°	150.04	1.964	299.97	**13°**	652.81	37.069	1297.2	**23°**	1165.7	117.38	2284.6
10	158.38	2.188	316.63	10	661.25	38.031	1313.8	10	1174.4	119.12	2301.0
20	166.72	2.425	333.29	20	669.70	39.006	1330.3	20	1183.1	120.87	2317.3
30	175.06	2.674	349.95	30	678.15	39.993	1346.9	30	1191.8	122.63	2333.6
40	183.40	2.934	366.61	40	686.60	40.992	1363.4	40	1200.5	124.41	2349.9
50	191.74	3.207	383.27	50	695.06	42.004	1380.0	50	1209.2	126.20	2366.2
4°	200.08	3.492	399.92	**14°**	703.51	43.029	1396.5	**24°**	1217.9	128.00	2382.5
10	208.43	3.790	416.58	10	711.97	44.066	1413.1	10	1226.6	129.82	2398.8
20	216.77	4.099	433.24	20	720.44	45.116	1429.6	20	1235.3	131.65	2415.1
30	225.12	4.421	449.89	30	728.90	46.178	1446.2	30	1244.0	133.50	2431.4
40	233.47	4.755	466.54	40	737.37	47.253	1462.7	40	1252.8	135.36	2447.7
50	241.81	5.100	483.20	50	745.85	48.341	1479.2	50	1261.5	137.23	2464.0
5°	250.16	5.459	499.85	**15°**	754.32	49.441	1495.7	**25°**	1270.2	139.11	2480.2
10	258.51	5.829	516.50	10	762.80	50.554	1512.3	10	1279.0	141.01	2496.5
20	266.86	6.211	533.15	20	771.29	51.679	1528.8	20	1287.7	142.93	2512.8
30	275.21	6.606	549.80	30	779.77	52.818	1545.3	30	1296.5	144.85	2529.0
40	283.57	7.013	566.44	40	788.26	53.969	1561.8	40	1305.3	146.79	2545.3
50	291.92	7.432	583.09	50	796.75	55.132	1578.3	50	1314.0	148.75	2561.5
6°	300.28	7.863	599.73	**16°**	805.25	56.309	1594.8	**26°**	1322.8	150.71	2577.8
10	308.64	8.307	616.38	10	813.75	57.498	1611.3	10	1331.6	152.69	2594.0
20	316.99	8.762	633.02	20	822.25	58.699	1627.8	20	1340.4	154.69	2610.3
30	325.35	9.230	649.66	30	830.76	59.914	1644.3	30	1349.2	156.70	2626.5
40	333.71	9.710	666.30	40	839.27	61.141	1660.8	40	1358.0	158.72	2642.7
50	342.08	10.202	682.94	50	847.78	62.381	1677.3	50	1366.8	160.76	2658.9
7°	350.44	10.707	699.57	**17°**	856.30	63.634	1693.8	**27°**	1375.6	162.81	2675.1
10	358.81	11.224	716.21	10	864.82	64.900	1710.3	10	1384.4	164.87	2691.3
20	367.17	11.753	732.84	20	873.35	66.178	1726.8	20	1393.2	166.95	2707.5
30	375.54	12.294	749.47	30	881.88	67.470	1743.2	30	1402.0	169.04	2723.7
40	383.91	12.847	766.10	40	890.41	68.774	1759.7	40	1410.9	171.15	2739.9
50	392.28	13.413	782.73	50	898.95	70.091	1776.2	50	1419.7	173.27	2756.1
8°	400.66	13.991	799.36	**18°**	907.49	71.421	1792.6	**28°**	1428.6	175.41	2772.3
10	409.03	14.582	815.99	10	916.03	72.764	1809.1	10	1437.4	177.55	2788.4
20	417.41	15.184	832.61	20	924.58	74.119	1825.5	20	1446.3	179.72	2804.6
30	425.79	15.799	849.23	30	933.13	75.488	1842.0	30	1455.1	181.89	2820.7
40	434.17	16.426	865.85	40	941.69	76.869	1858.4	40	1464.0	184.08	2836.9
50	442.55	17.066	882.47	50	950.25	78.264	1874.9	50	1472.9	186.29	2853.0
9°	450.93	17.717	899.09	**19°**	958.81	79.671	1891.3	**29°**	1481.8	188.51	2869.2
10	459.32	18.381	915.70	10	967.38	81.092	1907.8	10	1490.7	190.74	2885.3
20	467.71	19.058	932.31	20	975.96	82.525	1924.2	20	1499.6	192.99	2901.4
30	476.10	19.746	948.92	30	984.53	83.972	1940.6	30	1508.5	195.25	2917.6
40	484.49	20.447	965.53	40	993.12	85.431	1957.1	40	1517.4	197.53	2933.7
50	492.88	21.161	982 14	50	1001.70	86.904	1973.5	50	1526.3	199.82	2949.8
10°	501.28	21.886	998.74	**20°**	1010.29	88.389	1989.9	**30°**	1535.3	202.12	2965.9
10	509.68	22.624	1015.35	10	1018.89	89.888	2006.3	10	1544.2	204.44	2982.0
20	518.08	23.375	1031.95	20	1027.49	91.399	2022.7	20	1553.1	206.77	2998.1
30	526.48	24.138	1048.54	30	1036.09	92.924	2039.1	30	1562.1	209.12	3014.2
40	534.89	24.913	1065.14	40	1044.70	94.462	2055.5	40	1571.0	211.48	3030.2
50	543.29	25.700	1081.73	50	1053.31	96.013	2071.9	50	1580.0	213.86	3046.3
11°	551.70	26.500	1098.33	**21°**	1061.93	97.577	2088.3	**31°**	1589.0	216.25	3062.4

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS FOR A 1° CURVE.

Δ	Tangent *T.*	Ext. Dist. *E.*	Long Ch'd *LC.*	Δ	Tangent *T.*	Ext. Dist. *E.*	Long Ch'd *LC.*	Δ	Tangent *T.*	Ext. Dist. *E.*	Long Ch'd *LC.*
31°	1589.0	216.25	3062.4	**41°**	2142.2	387.38	4013.1	**51°**	2732.9	618.39	4933.4
10'	1598.0	218.66	3078.4	10	2151.7	390.71	4028.7	10	2743.1	622.81	4948.4
20	1606.9	221.08	3094.5	20	2161.2	394.06	4044.3	20	2753.4	627.24	4963.4
30	1615.9	223.51	3110.5	30	2170.8	397.43	4059.9	30	2763.7	631.69	4978.4
40	1624.9	225.96	3126.6	40	2180.3	400.82	4075.5	40	2773.9	636.16	4993.4
50	1633.9	228.42	3142.6	50	2189.9	404.22	4091.1	50	2784.2	640.66	5008.4
32°	1643.0	230.90	3158.6	**42°**	2199.4	407.64	4106.6	**52°**	2794.5	645.17	5023.4
10	1652.0	233.39	3174.6	10	2209.0	411.07	4122.2	10	2804.9	649.70	5038.4
20	1661.0	235.90	3190.6	20	2218.6	414.52	4137.7	20	2815.2	654.25	5053.4
30	1670.0	238.43	3206.6	30	2228.1	417.99	4153.3	30	2825.6	658.83	5068.3
40	1679.1	240.96	3222.6	40	2237.7	421.48	4168.8	40	2835.9	663.42	5083.3
50	1688.1	243.52	3238.6	50	2247.3	424.98	4184.3	50	2846.3	668.03	5098.2
33°	1697.2	246.08	3254.6	**43°**	2257.0	428.50	4199.8	**53°**	2856.7	672.66	5113.1
10	1706.3	248.66	3270.6	10	2266.6	432.04	4215.3	10	2867.1	677.32	5128.0
20	1715.3	251.26	3286.6	20	2276.2	435.59	4230.8	20	2877.5	681.99	5142.9
30	1724.4	253.87	3302.5	30	2285.9	439.16	4246.3	30	2888.0	686.68	5157.8
40	1733.5	256.50	3318.5	40	2295.6	442.75	4261.8	40	2898.4	691.40	5172.7
50	1742.6	259.14	3334.4	50	2305.2	446.35	4277.3	50	2908.9	696.13	5187.6
34°	1751.7	261.80	3350.4	**44°**	2314.9	449.98	4292.7	**54°**	2919.4	700.89	5202.4
10	1760.8	264.47	3366.3	10	2324.6	453.62	4308.2	10	2929.9	705.66	5217.3
20	1770.0	267.16	3382.2	20	2334.3	457.27	4323.6	20	2940.4	710.46	5232.1
30	1779.1	269.86	3398.2	30	2344.1	460.95	4339.0	30	2951.0	715.28	5246.9
40	1788.2	272.58	3414.1	40	2353.8	464.64	4354.5	40	2961.5	720.11	5261.7
50	1797.4	275.31	3430.0	50	2363.5	468.35	4369.9	50	2972.1	724.97	5276.5
35°	1806.6	278.05	3445.9	**45°**	2373.3	472.08	4385.3	**55°**	2982.7	729.85	5291.3
10	1815.7	280.82	3461.8	10	2383.1	475.82	4400.7	10	2993.3	734.76	5306.1
20	1824.9	283.60	3477.7	20	2392.8	479.59	4416.1	20	3003.9	739.68	5320.9
30	1834.1	286.39	3493.5	30	2402.6	483.37	4431.4	30	3014.5	744.62	5335.6
40	1843.3	289.20	3509.4	40	2412.4	487.16	4446.8	40	3025.2	749.59	5350.4
50	1852.5	292.02	3525.3	50	2422.3	490.98	4462.2	50	3035.8	754.57	5365.1
36°	1861.7	294.86	3541.1	**46°**	2432.1	494.82	4477.5	**56°**	3046.5	759.58	5379.8
10	1870.9	297.72	3557.0	10	2441.9	498.67	4492.8	10	3057.2	764.61	5394.5
20	1880.1	300.59	3572.8	20	2451.8	502.54	4508.2	20	3067.9	769.66	5409.2
30	1889.4	303.47	3588.6	30	2461.7	506.42	4523.5	30	3078.7	774.73	5423.9
40	1898.6	306.37	3604.5	40	2471.5	510.33	4538.8	40	3089.4	779.83	5438.6
50	1907.9	309.29	3620.3	50	2481.4	514.25	4554.1	50	3100.2	784.94	5453.3
37°	1917.1	312.22	3636.1	**47°**	2491.3	518.20	4569.4	**57°**	3110.9	790.08	5467.9
10	1926.4	315.17	3651.9	10	2501.2	522.16	4584.7	10	3121.7	795.24	5482.5
20	1935.7	318.13	3667.7	20	2511.2	526.13	4599.9	20	3132.6	800.42	5497.2
30	1945.0	321.11	3683.5	30	2521.1	530.13	4615.2	30	3143.4	805.62	5511.8
40	1954.3	324.11	3699.3	40	2531.1	534.15	4630.4	40	3154.2	810.85	5526.4
50	1963.6	327.12	3715.0	50	2541.0	538.18	4645.7	50	3165.1	816.10	5541.0
38°	1972.9	330.15	3730.8	**48°**	2551.0	542.23	4660.9	**58°**	3176.0	821.37	5555.6
10	1982.2	333.19	3746.5	10	2561.0	546.30	4676.1	10	3186.9	826.66	5570.2
20	1991.5	336.25	3762.3	20	2571.0	550.39	4691.3	20	3197.8	831.98	5584.7
30	2000.9	339.32	3778.0	30	2581.0	554.50	4706.5	30	3208.8	837.31	5599.3
40	2010.2	342.41	3793.8	40	2591.1	558.63	4721.7	40	3219.7	842.67	5613.8
50	2019.6	345.52	3809.5	50	2601.1	562.77	4736.9	50	3230.7	848.06	5628.3
39°	2029.0	348.64	3825.2	**49°**	2611.2	566.94	4752.1	**59°**	3241.7	853.46	5642.8
10	2038.4	351.78	3840.9	10	2621.2	571.12	4767.3	10	3252.7	858.89	5657.3
20	2047.8	354.94	3856.6	20	2631.3	575.32	4782.4	20	3263.7	864.34	5671.8
30	2057.2	358.11	3872.3	30	2641.4	579.54	4797.5	30	3274.8	869.82	5686.3
40	2066.6	361.29	3888.0	40	2651.5	583.78	4812.7	40	3285.8	875.32	5700.8
50	2076.0	364.50	3903.6	50	2661.6	588.04	4827.8	50	3296.9	880.84	5715.2
40°	2085.4	367.72	3919.3	**50°**	2671.8	592.32	4842.9	**60°**	3308.0	886.38	5729.7
10	2094.9	370.95	3935.0	10	2681.9	596.62	4858.0	10	3319.1	891.95	5744.1
20	2104.3	374.20	3950.6	20	2692.1	600.93	4873.1	20	3330.3	897.54	5758.5
30	2113.8	377.47	3966.3	30	2702.3	605.27	4888.2	30	3341.4	903.15	5772.9
40	2123.3	380.76	3981.9	40	2712.5	609.62	4903.2	40	3352.6	908.79	5787.3
50	2132.7	384.06	3997.5	50	2722.7	614.00	4918.3	50	3363.8	914.45	5801.7
41°	2142.2	387.38	4013.1	**51°**	2732.9	618.39	4933.4	**61°**	3375.0	920.14	5816.0

TABLE II.—TANGENTS, EXTERNAL DISTANCES, AND LONG CHORDS FOR A 1° CURVE.

Δ	Tangent T.	Ext. Dist. E.	LongCh'd LC.	Δ	Tangent T.	Ext. Dist. E.	LongCh'd LC.	Δ	Tangent T.	Ext. Dist. E.	LongCh'd LC.
61°	3375.0	920.14	5816.0	**71°**	4086.9	1308.2	6654.4	**81°**	4893.6	1805.3	7442.2
10′	3386.3	925.85	5830.4	10	4099.5	1315.5	6668.0	10	4908.0	1814.7	7454.9
20	3397.5	931.58	5844.7	20	4112.1	1322.9	6681.6	20	4922.5	1824.1	7467.5
30	3408.8	937.34	5859.1	30	4124.8	1330.3	6695.1	30	4937.0	1833.6	7480.2
40	3420.1	943.12	5873.4	40	4137.4	1337.7	6708.6	40	4951.5	1843.1	7492.8
50	3431.4	948.92	5887.7	50	4150.1	1345.1	6722.1	50	4966.1	1852.6	7505.4
62°	3442.7	954.75	5902.0	**72°**	4162.8	1352.6	6735.6	**82°**	4980.7	1862.2	7518.0
10	3454.1	960.60	5916.3	10	4175.6	1360.1	6749.1	10	4995.4	1871.8	7530.5
20	3465.4	966.48	5930.5	20	4188.4	1367.6	6762.5	20	5010.0	1881.5	7543.1
30	3476.8	972.39	5944.8	30	4201.2	1375.2	6776.0	30	5024.8	1891.2	7555.6
40	3488.2	978.31	5959.0	40	4214.0	1382.8	6789.4	40	5039.5	1900.9	7568.2
50	3499.7	984.27	5973.3	50	4226.8	1390.4	6802.8	50	5054.3	1910.7	7580.7
63°	3511.1	990.24	5987.5	**73°**	4239.7	1398.0	6816.3	**83°**	5069.2	1920.5	7593.2
10	3522.6	996.24	6001.7	10	4252.6	1405.7	6829.6	10	5084.0	1930.4	7605.6
20	3534.1	1002.3	6015.9	20	4265.6	1413.5	6843.0	20	5099.0	1940.3	7618.1
30	3545.6	1008.3	6030.0	30	4278.5	1421.2	6856.4	30	5113.9	1950.3	7630.5
40	3557.2	1014.4	6044.2	40	4291.5	1429.0	6869.7	40	5128.9	1960.2	7643.0
50	3568.7	1020.5	6058.4	50	4304.6	1436.8	6883.1	50	5143.9	1970.3	7655.4
64°	3580.3	1026.6	6072.5	**74°**	4317.6	1444.6	6896.4	**84°**	5159.0	1980.4	7667.8
10	3591.9	1032.8	6086.6	10	4330.7	1452.5	6909.7	10	5174.1	1990.5	7680.1
20	3603.5	1039.0	6100.7	20	4343.8	1460.4	6923.0	20	5189.3	2000.6	7692.5
30	3615.1	1045.2	6114.8	30	4356.9	1468.4	6936.2	30	5204.4	2010.8	7704.9
40	3626.8	1051.4	6128.9	40	4370.1	1476.4	6949.5	40	5219.7	2021.1	7717.2
50	3638.5	1057.7	6143.0	50	4383.3	1484.4	6962.8	50	5234.9	2031.4	7729.5
65°	3650.2	1063.9	6157.1	**75°**	4396.5	1492.4	6976.0	**85°**	5250.3	2041.7	7741.8
10	3661.9	1070.2	6171.1	10	4409.8	1500.5	6989.2	10	5265.6	2052.1	7754.1
20	3673.7	1076.6	6185.2	20	4423.1	1508.6	7002.4	20	5281.0	2062.5	7766.3
30	3685.4	1082.9	6199.2	30	4436.4	1516.7	7015.6	30	5296.4	2073.0	7778.6
40	3697.2	1089.3	6213.2	40	4449.7	1524.9	7028.8	40	5311.9	2083.5	7790.8
50	3709.0	1095.7	6227.2	50	4463.1	1533.1	7041.9	50	5327.4	2094.1	7803.0
66°	3720.9	1102.2	6241.2	**76°**	4476.5	1541.4	7055.0	**86°**	5343.0	2104.7	7815.2
10	3732.7	1108.6	6255.2	10	4489.9	1549.7	7068.2	10	5358.6	2115.3	7827.4
20	3744.6	1115.1	6269.1	20	4503.4	1558.0	7081.3	20	5374.2	2126.0	7839.6
30	3756.5	1121.7	6283.1	30	4516.9	1566.3	7094.4	30	5389.9	2136.7	7851.7
40	3768.5	1128.2	6297.0	40	4530.4	1574.7	7107.5	40	5405.6	2147.5	7863.8
50	3780.4	1134.8	6310.9	50	4544.0	1583.1	7120.5	50	5421.4	2158.4	7876.0
67°	3792.4	1141.4	6324.8	**77°**	4557.6	1591.6	7133.6	**87°**	5437.2	2169.2	7888.1
10	3804.4	1148.0	6338.7	10	4571.2	1600.1	7146.6	10	5453.1	2180.2	7900.1
20	3816.4	1154.7	6352.6	20	4584.8	1608.6	7159.6	20	5469.0	2191.1	7912.2
30	3828.4	1161.3	6366.4	30	4598.5	1617.1	7172.6	30	5484.9	2202.2	7924.3
40	3840.5	1168.1	6380.3	40	4612.2	1625.7	7185.6	40	5500.9	2213.2	7936.3
50	3852.6	1174.8	6394.1	50	4626.0	1634.4	7198.6	50	5517.0	2224.3	7948.3
68°	3864.7	1181.6	6408.0	**78°**	4639.8	1643.0	7211.6	**88°**	5533.1	2235.5	7960.3
10	3876.8	1188.4	6421.8	10	4653.6	1651.7	7224.5	10	5549.2	2246.7	7972.3
20	3889.0	1195.2	6435.6	20	4667.4	1660.5	7237.4	20	5565.4	2258.0	7984.2
30	3901.2	1202.0	6449.4	30	4681.3	1669.2	7250.4	30	5581.6	2269.3	7996.2
40	3913.4	1208.9	6463.1	40	4695.2	1678.1	7263.3	40	5597.8	2280.6	8008.1
50	3925.6	1215.8	6476.9	50	4709.2	1686.9	7276.1	50	5614.2	2292.0	8020.0
69°	3937.9	1222.7	6490.6	**79°**	4723.2	1695.8	7289.0	**89°**	5630.5	2303.5	8031.9
10	3950.2	1229.7	6504.4	10	4737.2	1704.7	7301.9	10	5646.9	2315.0	8043.8
20	3962.5	1236.7	6518.1	20	4751.2	1713.7	7314.7	20	5663.4	2326.6	8055.7
30	3974.8	1243.7	6531.8	30	4765.3	1722.7	7327.5	30	5679.9	2338.2	8067.5
40	3987.2	1250.8	6545.5	40	4779.4	1731.7	7340.3	40	5696.4	2349.8	8079.3
50	3999.5	1257.9	6559.1	50	4793.6	1740.8	7353.1	50	5713.0	2361.5	8091.2
70°	4011.9	1265.0	6572.8	**80°**	4808.7	1749.9	7365.9	**90°**	5729.7	2373.3	8103.0
10	4024.4	1272.1	6586.4	10	4822.0	1759.0	7378.7	10	5746.3	2385.1	8114.7
20	4036.8	1279.3	6600.1	20	4836.2	1768.2	7391.4	20	5763.1	2397.0	8126.5
30	4049.3	1286.5	6613.7	30	4850.5	1777.4	7404.1	30	5779.9	2408.9	8138.2
40	4061.8	1293.7	6627.3	40	4864.8	1786.7	7416.8	40	5796.7	2420.9	8150.0
50	4074.4	1300.9	6640.9	50	4879.2	1796.0	7429.5	50	5813.6	2432.9	8161.7
71°	4086.9	1308.2	6654.4	**81°**	4893.6	1805.3	7442.2	**91°**	5830.5	2444.9	8173.4

LEAD-RAILS CIRCULAR THROUGHOUT; GAUGE 4′ 8½″. See § 262.

Frog Number (n).	Frog Angle (F)	Lead (L) (Eq. 79).	Chord (QT) (Eq. 77).	Radius of Lead Rails (r, Eq. 78).	Log r.	Degree of Curve (d).	Frog Number (n).
4	14° 15′ 00″	37.67	37.38	150.67	2.17801	38° 46′	4
4.5	12 40 59	42.37	42.12	190.69	.28032	30 24	4.5
5	11 25 16	47.08	46.85	235.42	.37183	24 32	5
5.5	10 23 20	51.79	51.58	284.85	.45462	20 13	5.5
6	9 31 38	56.50	56.30	339.00	.53020	16 58	6
6.5	8 47 51	61.21	61.03	397.85	.59972	14 26	6.5
7	8 10 16	65.92	65.75	461.42	.66409	12 26	7
7.5	7 37 41	70.62	70.47	529.69	.72402	10 50	7.5
8	7 09 10	75.33	75.19	602.67	.78007	9 31	8
8.5	6 43 59	80.04	79.90	680.36	.83273	8 26	8.5
9	6 21 35	84.75	84.62	762.75	.88238	7 31	9
9.5	6 01 32	89.46	89.33	849.85	.92934	6 45	9.5
10	5 43 29	94.17	94.05	941.67	2.97389	6 05	10
10.5	5 27 09	98.87	98.76	1038.19	3.01627	5 32	10.5
11	5 12 18	103.58	103.47	1139.42	.05668	5 02	11
11.5	4 58 45	108.29	108.19	1245.36	.09529	4 36	11.5
12	4 46 19	113.00	112.90	1356.00	3.13226	4 14	12

TURNOUTS WITH STRAIGHT POINT-RAILS AND STRAIGHT FROG-RAILS; GAUGE 4′ 8½″. See § 265.

Frog Number (n).	Switch Point Angle (a).	Length of Switch Point (DN).	Length of Straight Frog-rail (f).	Lead (L) (Eq. 90).	Chord (ST) (Eq. 88).	Radius of Lead-rails (r, Eq. 87).	Log r.	Degree of Curve (d).	Frog Number (n).
4	3° 40′	7.5	1.50	32.20	23.09	125.21	2.09764	47° 05′	4
4.5	3 40	7.5	1.69	34.29	25.03	159.25	.20208	36 36	4.5
5	2 45	10.0	1.87	41.85	29.88	197.65	.29589	29 22	5
5.5	2 45	10.0	2.06	44.16	32.03	240.44	.38106	24 00	5.5
6	1 50	15.0	2.25	56.00	38.66	288.09	.45953	19 59	6
6.5	1 50	15.0	2.44	58.84	41.34	340.19	.53172	16 54	6.5
7	1 50	15.0	2.62	61.65	43.98	397.65	.59950	14 27	7
7.5	1 50	15.0	2.81	64.36	46.50	460.00	.66276	12 29	7.5
8	1 50	15.0	3.00	67.04	48.99	527.91	.72256	10 52	8
8.5	1 50	15.0	3.19	69.60	51.38	600.94	.77883	9 33	8.5
9	1 50	15.0	3.37	72.20	53.80	681.16	.83325	8 25	9
9.5	1 50	15.0	3.56	74.70	56.11	767.11	.88486	7 28	9.5
10	1 50	15.0	3.75	77.04	58.28	858.14	.93356	6 41	10
10.5	1 50	15.0	3.94	79.51	60.57	959.00	2.98182	5 59	10.5
11	1 50	15.0	4.12	81.82	62.69	1065.52	3.02756	5 23	11
11.5	1 50	15.0	4.31	84.09	64.78	1180.16	3.07194	4 51	11.5
12	1 50	15.0	4.50	86.16	66.67	1299.93	3.11392	4 24	12

TRIGONOMETRICAL FUNCTIONS OF THE FROG ANGLES (F).

Frog Number (n).	Frog Angle (F).	Nat. sin F.	Nat. cos F.	Log sin F.	Log cos F.	Log cot F.	Log vers F.	Frog Number (n).
4	14° 15′ 00″	.24615	.96923	9.39120	9.98642	10.59522	8.48811	4
4.5	12 40 49	.21951	.97561	.34145	.98927	.64782	.38721	4.5
5	11 25 16	.19802	.98020	.29670	.99131	.69461	.29670	5
5.5	10 23 20	.18033	.98360	.25606	.99282	.73675	.21467	5.5
6	9 31 38	.16552	.98621	.21884	.99397	.77513	.13966	6
6.5	8 47 51	.15294	.98823	.18453	.99486	.81033	.07058	6.5
7	8 10 16	.14213	.98985	.15268	.99557	.84288	8.00655	7
7.5	7 37 41	.13274	.99115	.12301	.99614	.87313	7.94691	7.5
8	7 09 10	.12452	.99222	.09522	.99660	.90138	.89110	8
8.5	6 43 59	.11724	.99310	.06909	.99699	.92796	.83864	8.5
9	6 21 35	.11077	.99385	.04442	.99732	.95289	.78915	9
9.5	6 01 32	.10497	.99448	9.02107	.99759	.97652	.74232	9.5
10	5 43 29	.09975	.99501	8.99891	.99783	10.99892	.69788	10
10.5	5 27 09	.09502	.99548	.97781	.99803	11.02021	.65560	10.5
11	5 12 18	.09072	.99588	.95770	.99820	.04050	.61528	11
11.5	4 58 45	.08679	.99623	.93848	.99836	.05987	.57676	11.5
12	4 46 19	.08319	.99653	8.92007	9.98849	11.07842	7.53986	12

Point.	Total Central Angle φ	Nat. sin φ	Nat. cos φ	Log sin φ	Log cos φ	Log vers φ	x	Log x	y
1	0° 07′ 30″	.0022	one	7.33878	10.00000	4.37654	0.027	8.43568	25.000
2	0 22 30	.0065	one	7.81596	9.99999	5.33078	0.136	9.13465	50.000
3	0 45 00	.0131	.9999	8.11692	9.99996	5.93284	0.382	9.58181	74.999
4	1 15 00	.0218	.9997	8.33875	9.99989	6.37653	0.818	9.91286	99.995
5	1 52 30	.0327	.9994	8.51480	9.99976	6.72869	1.500	0.17602	124.985
6	2 37 30	.0458	.9989	8.66085	9.99954	7.02091	2.481	0.39467	149.966
7	3 30 00	.0616	.9981	8.78567	9.99919	7.27072	3.817	0.58171	174.930
8	4 30 00	.0784	.9969	8.89464	9.99866	7.48892	5.561	0.74514	199.870
9	5 37 30	.0980	.9952	8.99130	9.99796	7.68262	7.792	0.89164	224.772
10	6 52 30	.1197	.9928	9.07816	9.99686	7.85675	10.489	1.02061	249.623

0° 30′-per-25-feet spiral.

—sighting at—	Deflections from the tangent at the point occupied when the instrument is at—										
	Q	1	2	3	4	5	6	7	8	9	10
Q	0° 0′ 0″	0° 3′ 45″	0° 13′ 07″	0° 27′ 30″	0° 46′ 52″	1° 11′ 15″	1° 40′ 37″	2° 15′ 00″	2° 54′ 22″	3° 38′ 45″	4° 28′ 10″
1	0 3 45	0 0 00	0 07 30	0 20 37	0 38 45	1 01 52	1 30 00	2 03 07	2 41 15	3 24 22	4 12 30
2	0 9 22	0 7 30	0 00 00	0 11 15	0 28 07	0 50 00	1 16 52	1 48 45	2 25 37	3 07 30	3 54 22
3	0 17 30	0 16 52	0 11 15	0 00 00	0 15 00	0 35 37	1 01 15	1 31 52	2 07 30	2 48 07	3 33 45
4	0 28 07	0 28 45	0 24 22	0 15 00	0 00 00	0 18 45	0 43 07	1 12 30	1 46 52	2 26 15	3 10 37
5	0 41 15	0 43 07	0 40 00	0 31 52	0 18 45	0 00 00	0 22 30	0 50 37	1 23 45	2 01 52	2 45 00
6	0 56 52	1 00 00	0 58 07	0 51 15	0 39 22	0 22 30	0 00 00	0 26 15	0 58 07	1 35 00	2 16 52
7	1 15 00	1 19 22	1 18 45	1 13 07	1 02 30	0 46 52	0 26 15	0 00 00	0 30 00	1 05 37	1 46 15
8	1 35 37	1 41 15	1 41 52	1 37 30	1 28 07	1 13 45	0 54 22	0 30 00	0 00 00	0 33 45	1 13 07
9	1 58 45	2 05 37	2 07 30	2 04 22	1 56 15	1 43 07	1 25 00	1 01 52	0 33 45	0 00 00	0 37 30
10	2 24 20	2 32 30	2 35 37	2 33 45	2 26 52	2 15 00	1 58 07	1 36 15	1 09 22	0 37 30	0 00 00

Point.	Total Central Angle φ	Nat. sin φ	Nat. cos φ	Log sin φ	Log cos φ	Log vers φ	x	Log x	y
1	0° 15′	.0043	one	7.63981	9.99999	4.97860	.055	8.73672	25.000
2	0 45	.0131	.9999	8.11692	9.99996	5.93284	.273	9.43616	49.999
3	1 30	.0262	.9996	8.41792	9.99985	6.53488	.763	9.88252	74.994
4	2 30	.0436	.9990	8.63968	9.99958	6.97853	1.636	0.21378	99.979
5	3 45	.0654	.9978	8.81560	9.99907	7.33063	2.999	0.47697	124.942
6	5 15	.0915	.9958	8.96143	9.99817	7.62274	4.960	0.69548	149.865
7	7 00	.1218	.9925	9.08589	9.99675	7.87238	7.628	0.88241	174.722
8	9 00	.1564	.9877	9.19433	9.99462	8.09031	11.107	1.04559	199.479
9	11 15	.1951	.9808	9.29023	9.99157	8.28363	15.502	1.19039	224.090
10	13 45	.2377	.9713	9.37600	9.98737	8.45724	20.913	1.32041	248.497

1°-per-25-feet spiral.

— sighting at —	Deflections from the tangent at the point occupied when the instrument is at--										
	Q	1	2	3	4	5	6	7	8	9	10
Q	0° 0′ 0″	0° 07′ 30″	0° 26′ 15″	0° 55′ 0″	1° 33′ 45″	2° 22′ 30″	3° 21′ 15″	4° 30′ 00″	5° 48′ 47″	7° 17′ 34″	8° 56′ 22″
1	0 07 30	0 00 00	0 15 00	0 41 15	1 17 30	2 03 45	3 00 00	4 06 15	5 22 30	6 48 47	8 25 05
2	0 18 45	0 15 00	0 00 00	0 22 30	0 56 15	1 40 00	2 33 45	3 37 30	4 51 15	6 15 02	7 48 49
3	0 35 00	0 33 45	0 22 30	0 00 00	0 30 00	1 11 15	2 02 30	3 03 45	4 15 00	5 36 15	7 07 32
4	0 56 15	0 57 30	0 48 45	0 30 00	0 00 00	0 37 30	1 26 15	2 25 00	3 33 45	4 52 30	6 21 17
5	1 22 30	1 26 15	1 20 00	1 03 45	0 37 30	0 00 00	0 45 00	1 41 15	2 47 30	4 03 45	5 30 00
6	1 53 45	2 00 00	1 56 15	1 42 30	1 18 45	0 45 00	0 00 00	0 52 30	1 56 15	3 10 00	4 33 45
7	2 30 00	2 38 45	2 37 30	2 26 15	2 05 00	1 33 45	0 52 30	0 00 00	1 00 00	2 11 15	3 32 30
8	3 11 13	3 22 30	3 23 45	3 15 00	2 56 15	2 27 30	1 48 45	1 00 00	0 00 00	1 07 30	2 26 15
9	3 57 26	4 11 13	4 14 58	4 08 45	3 52 30	3 26 15	2 50 00	2 03 45	1 07 30	0 00 00	1 15 00
10	4 48 38	5 04 55	5 11 11	5 07 28	4 53 43	4 30 00	3 56 15	3 12 30	2 18 45	1 15 00	0 00 00

Point.	Total Central Angle Φ	Nat. sin Φ	Nat. cos Φ	Log sin Φ	Log cos Φ	Log vers Φ	x	Log x	y
1	0° 30′	.0087	.9999	7.94084	9.99998	5.58066	0.109	9.03774	25.000
2	1 30	.0262	.9996	8.41792	9.99985	6.53488	0.545	9.73676	49.996
3	3 00	.0523	.9986	8.71880	9.99940	7.13687	1.527	0.18388	74.977
4	5 00	.0871	.9962	8.94029	9.99834	7.58039	3.271	0.51468	99.916
5	7 30	.1305	.9914	9.11570	9.99627	7.93222	5.992	0.77762	124.767
6	10 30	.1822	.9832	9.26063	9.99266	8.22389	9.903	0.99579	149.459
7	14 00	.2419	.9703	9.38367	9.98690	8.47282	15.208	1.18207	173.890
8	18 00	.3090	.9510	9.48998	9.97820	8.68969	22.099	1.34437	197.922
9	22 30	.3827	.9239	9.58284	9.96561	8.88150	30.752	1.48787	221.376
10	27 30	.4617	.8870	9.66440	9.94793	9.05303	41.317	1.61613	244.034

2°-per-25-feet spiral.

sighting at	Deflections from the tangent at the point occupied when the instrument is at—										
	Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Q	0° 00′ 00″	0° 15′ 00″	0° 52′ 30″	1° 50′ 00″	3° 07′ 30″	4° 45′ 00″	6° 42′ 30″	9° 00′ 05″	11° 37′ 45″	14° 35′ 30″	17° 53′ 25″
1	0 15 00	0 00 00	0 30 00	1 22 30	2 35 00	4 07 30	6 00 01	8 12 34	10 45 10	13 37 52	16 50 42
2	0 37 30	0 30 00	0 00 00	0 45 00	1 52 30	3 20 00	5 07 30	7 15 02	9 42 36	12 30 15	15 38 00
3	1 10 00	1 07 30	0 45 00	0 00 00	1 00 00	2 22 30	4 05 00	6 07 30	8 30 04	11 12 39	14 15 20
4	1 52 30	1 55 00	1 37 30	1 00 00	0 00 00	1 15 00	2 52 30	4 50 00	7 07 30	9 45 05	12 42 42
5	2 45 00	2 52 30	2 40 00	2 07 30	1 15 00	0 00 00	1 30 00	3 22 30	5 35 00	8 07 30	11 00 06
6	3 47 28	3 59 59	3 52 30	3 25 00	2 37 30	1 30 00	0 00 00	1 45 00	3 52 30	6 20 00	9 07 30
7	4 59 53	5 17 26	5 14 58	4 52 30	4 10 00	3 07 30	1 45 00	0 00 00	2 00 00	4 22 30	7 05 00
8	6 22 15	6 44 50	6 47 24	6 29 56	5 52 30	4 55 00	3 37 30	2 00 00	0 00 00	2 15 00	4 52 30
9	7 54 30	8 22 08	8 29 45	8 17 21	7 44 55	6 52 30	5 40 00	4 07 30	2 15 00	0 00 00	2 30 00
10	9 36 35	10 09 17	10 22 00	10 14 40	9 47 18	8 59 54	7 52 30	6 25 00	4 37 30	2 30 00	0 00 00

N.	0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173̂	216̂	260	303	346	389
101	432	475	518	561	604	646̂	689̂	732	775	817̂
102	860	902̂	945	987̂	*030	*072̂	*114̂	*157	*199̂	*241̂
103	01 283̂	326	368	410	452	494	536	578	619̂	661̂
104	703̂	745	787	828̂	870	911̂	953	994̂	*036	*077̂
105	02 119	160̂	201̂	243	284	325	366̂	407̂	448̂	489̂
106	530̂	571̂	612̂	653̂	694	735	775̂	816̂	857	898
107	938̂	979	*019̂	*060	*100̂	*141	*181	*221̂	*262	*302
108	03 342̂	382̂	422̂	463	503	543	583	623	663	703
109	742̂	782̂	822̂	862	901̂	941̂	981	*020̂	*060	*100
110	04 139̂	178̂	218	257̂	297	336	375̂	415	454	493
111	532̂	571̂	610̂	649̂	688̂	727̂	766̂	805̂	844	883
112	922	960̂	999̂	*038	*076̂	*115̂	*154	*192̂	*231	*269̂
113	05 308	346̂	384̂	423	461̂	499̂	538	576	614	652̂
114	690̂	728̂	766̂	804̂	842̂	880̂	918̂	956̂	994	*032
115	06 070	107̂	145	183	220̂	258	296	333̂	371	408̂
116	446	483	520̂	558	595̂	632̂	670	707	744̂	781̂
117	818̂	855̂	893	930	967	*004	*040̂	*077̂	*114̂	*151̂
118	07 188	225	261̂	298̂	335	372	408̂	445	481̂	518
119	554̂	591	627̂	664	700̂	737	773	809̂	845̂	882
120	918	954̂	990̂	*026̂	*062̂	*098̂	*134̂	*170̂	*206̂	*242̂
121	08 278̂	314̂	350̂	386	422	457̂	493̂	529	564̂	600̂
122	636	671̂	707	742̂	778	813̂	849	884̂	920	955
123	990̂	*026	*061	*096̂	*131̂	*166̂	*202	*237	*272	*307
124	09 342	377	412	447	482	517	552	586̂	621̂	656
125	691	725̂	760̂	795	830	864̂	899	933̂	968	*002̂
126	10 037	071̂	106	140̂	174̂	209	243̂	277̂	312	346
127	380̂	414̂	448̂	483	517	551	585	619	653	687
128	721	755	789	822̂	856̂	890̂	924	958	991̂	*025̂
129	11 059	092̂	126̂	160	193̂	227	260̂	294	327̂	361
130	394̂	427̂	461	494̂	528	561	594̂	627̂	661	694
131	727	760̂	793̂	826̂	859̂	892̂	925̂	958̂	991̂	*024̂
132	12 057̂	090̂	123	156	189	221̂	254̂	287	320	352̂
133	385	418	450̂	483	515̂	548	580̂	613	645̂	678
134	710̂	743	775̂	807̂	840	872	904̂	937	969	*001
135	13 033̂	065̂	097̂	130	162	194	226	258	290	322
136	354	386	417̂	449̂	481̂	513̂	545	577	608̂	640̂
137	672	703̂	735̂	767	798̂	830̂	862	893̂	925	956̂
138	988	*019̂	*051	*082	*113̂	*145	*176̂	*207̂	*239	*270
139	14 301̂	332	364	395	426̂	457̂	488̂	519̂	550̂	582
140	613	644	675	706	736̂	767̂	798̂	829̂	860̂	891
141	922	952̂	983̂	*014	*045	*075̂	*106̂	*137	*167̂	*198
142	15 229	259̂	290	320̂	351	381̂	412	442̂	473	503
143	533̂	564	594̂	624̂	655	685	715̂	745̂	776	806
144	836	866̂	896̂	926̂	956̂	987	*017	*047	*077	*107
145	16 137	166̂	196̂	226̂	256̂	286̂	316	346	376	405̂
146	435̂	465	494̂	524̂	554	584	613̂	643	672̂	702
147	731̂	761̂	791	820̂	849̂	879	908̂	938	967̂	997
148	17 026	055̂	085	114	143̂	172̂	202	231	260̂	289̂
149	318̂	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	753̂	782̂	811̂	840	869
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.

	43̂	43	42	41
.1	4.3̂	4.3	4.2	4.1
.2	8.7	8.6	8.4	8.2
.3	13.0̂	12.9	12.6	12.3
.4	17.4	17.2	16.8	16.4
.5	21.7̂	21.5	21.0	20.5
.6	26.1	25.8	25.2	24.6
.7	30.4̂	30.1	29.4	28.7
.8	34.8	34.4	33.6	32.8
.9	39.1̂	38.7	37.8	36.9

	40̂	40	39	38
.1	4.0̂	4.0	3.9	3.8
.2	8.1	8.0	7.8	7.6
.3	12.1̂	12.0	11.7	11.4
.4	16.2	16.0	15.6	15.2
.5	20.2̂	20.0	19.5	19.0
.6	24.3	24.0	23.4	22.8
.7	28.3̂	28.0	27.3	26.6
.8	32.4	32.0	31.2	30.4
.9	36.4̂	36.0	35.1	34.2

	37̂	37	36	35
.1	3.7̂	3.7	3.6	3.5
.2	7.5	7.4	7.2	7.0
.3	11.2̂	11.1	10.8	10.5
.4	15.0	14.8	14.4	14.0
.5	18.7̂	18.5	18.0	17.5
.6	22.5	22.2	21.6	21.0
.7	26.2̂	25.9	25.2	24.5
.8	30.0	29.6	28.8	28.0
.9	33.7̂	33.3	32.4	31.5

	34̂	34	33	32
.1	3.4̂	3.4	3.3	3.2
.2	6.9	6.8	6.6	6.4
.3	10.3̂	10.2	9.9	9.6
.4	13.8	13.6	13.2	12.8
.5	17.2̂	17.0	16.5	16.0
.6	20.7	20.4	19.8	19.2
.7	24.1̂	23.8	23.1	22.4
.8	27.6	27.2	26.4	25.6
.9	31.0̂	30.6	29.7	28.8

	31̂	31	30	29
.1	3.1̂	3.1	3.0	2.9
.2	6.3	6.2	6.0	5.8
.3	9.4	9.3	9.0	8.7
.4	12.6	12.4	12.0	11.6
.5	15.7̂	15.5	15.0	14.5
.6	18.9	18.6	18.0	17.4
.7	22.0̂	21.7	21.0	20.3
.8	25.2	24.8	24.0	23.2
.9	28.3̂	27.9	27.0	26.1

P. P.

N.	0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	753	782	811	840	869
151	897	926	955	984	*012	*041	*070	*098	*127	*156
152	18 184	213	241	270	298	327	355	384	412	440
153	469	497	526	554	582	611	639	667	695	724
154	752	780	808	836	864	893	921	949	977	*005
155	19 033	061	089	117	145	173	201	229	256	284
156	312	340	368	396	423	451	479	507	534	562
157	590	617	645	673	700	728	755	783	810	838
158	865	893	920	948	975	*003	*030	*057	*085	*112
159	20 139	167	194	221	249	276	303	330	357	385
160	412	439	466	493	520	547	574	601	628	655
161	682	709	736	763	790	817	844	871	898	924
162	951	978	*005	*032	*058	*085	*112	*139	*165	*192
163	21 219	245	272	298	325	352	378	405	431	458
164	484	511	537	564	590	616	643	669	695	722
165	748	774	801	827	853	880	906	932	958	984
166	22 011	037	063	089	115	141	167	193	219	245
167	271	297	323	349	375	401	427	453	479	505
168	531	557	582	608	634	660	686	711	737	763
169	788	814	840	865	891	917	942	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
171	299	325	350	375	401	426	451	477	502	527
172	553	578	603	628	653	679	704	729	754	779
173	804	829	855	880	905	930	955	980	*005	*030
174	24 055	080	105	129	154	179	204	229	254	279
175	304	328	353	378	403	427	452	477	502	526
176	551	576	600	625	650	674	699	723	748	773
177	797	822	846	871	895	920	944	968	993	*017
178	25 042	066	091	115	139	164	188	212	237	261
179	285	309	334	358	382	406	430	455	479	503
180	527	551	575	599	623	647	672	696	720	744
181	768	792	816	840	863	887	911	935	959	983
182	26 007	031	055	078	102	126	150	174	197	221
183	245	269	292	316	340	363	387	411	434	458
184	482	505	529	552	576	599	623	646	670	693
185	717	740	764	787	811	834	858	881	904	928
186	951	974	998	*021	*044	*068	*091	*114	*137	*161
187	27 184	207	230	254	277	300	323	346	369	392
188	416	439	462	485	508	531	554	577	600	623
189	646	669	692	715	738	761	784	806	829	852
190	875	898	921	944	966	989	*012	*035	*058	*080
191	28 103	126	149	171	194	217	239	262	285	307
192	330	352	375	398	420	443	465	488	510	533
193	555	578	600	623	645	668	690	713	735	758
194	780	802	825	847	869	892	914	936	959	981
195	29 003	025	048	070	092	114	137	159	181	203
196	225	248	270	292	314	336	358	380	402	424
197	446	468	490	512	534	556	578	600	622	644
198	666	688	710	732	754	776	798	820	841	863
199	885	907	929	950	972	994	*016	*038	*059	*081
200	30 103	124	146	168	190	211	233	254	276	298
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	29	28	27
.1	2.9	2.8	2.7
.2	5.8	5.6	5.4
.3	8.7	8.4	8.1
.4	11.6	11.2	10.8
.5	14.5	14.0	13.5
.6	17.4	16.8	16.2
.7	20.3	19.6	18.9
.8	23.2	22.4	21.6
.9	26.1	25.2	24.3

	26̂	26
.1	2.6	2.6
.2	5.3	5.2
.3	7.9	7.8
.4	10.6	10.4
.5	13.2	13.0
.6	15.9	15.6
.7	18.5	18.2
.8	21.2	20.8
.9	23.8	23.4

	25̂	25	24
.1	2.5	2.5	2.4
.2	5.1	5.0	4.8
.3	7.6	7.5	7.2
.4	10.2	10.0	9.6
.5	12.7	12.5	12.0
.6	15.3	15.0	14.4
.7	17.8	17.5	16.8
.8	20.4	20.0	19.2
.9	22.9	22.5	21.6

	23̂	23
.1	2.3	2.3
.2	4.7	4.6
.3	7.0	6.9
.4	9.4	9.2
.5	11.7	11.5
.6	14.1	13.8
.7	16.4	16.1
.8	18.8	18.4
.9	21.1	20.7

	22̂	22	21̂
.1	2.2	2.2	2.1
.2	4.5	4.4	4.3
.3	6.7	6.6	6.4
.4	9.0	8.8	8.6
.5	11.2	11.0	10.7
.6	13.5	13.2	12.9
.7	15.7	15.4	15.0
.8	18.0	17.6	17.2
.9	20.2	19.8	19.3

P. P.

N.	0	1	2	3	4	5	6	7	8	9
200	30 103	124	146	168	190	211	233	254	276	298
201	319	341	363	384	406	427	449	470	492	513
202	535	556	578	599	621	642	664	685	707	728
203	749	771	792	813	835	856	878	899	920	941
204	963	984	*005	*027	*048	*069	*090	*112	*133	*154
205	31 175	196	217	239	260	281	302	323	344	365
206	386	408	429	450	471	492	513	534	555	576
207	597	618	639	660	681	702	722	743	764	785
208	806	827	848	869	890	910	931	952	973	994
209	32 014	035	056	077	097	118	139	160	180	201
210	222	242	263	284	304	325	346	366	387	407
211	428	449	469	490	510	531	551	572	592	613
212	633	654	674	695	715	736	756	776	797	817
213	838	858	878	899	919	940	960	980	*001	*021
214	33 041	061	082	102	122	142	163	183	203	223
215	244	264	284	304	324	344	365	385	405	425
216	445	465	485	505	525	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	825
218	845	865	885	905	925	945	965	985	*004	*024
219	34 044	064	084	104	123	143	163	183	203	222
220	242	262	281	301	321	341	360	380	400	419
221	439	459	478	498	518	537	557	576	596	615
222	635	655	674	694	713	733	752	772	791	811
223	830	850	869	889	908	928	947	966	986	*005
224	35 025	044	063	083	102	121	141	160	179	199
225	218	237	257	276	295	314	334	353	372	391
226	411	430	449	468	487	507	526	545	564	583
227	602	621	641	660	679	698	717	736	755	774
228	793	812	831	850	869	888	907	926	945	964
229	983	*002	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	191	210	229	248	267	286	305	323	342
231	361	380	399	417	436	455	474	492	511	530
232	549	567	586	605	623	642	661	679	698	717
233	735	754	773	791	810	828	847	866	884	903
234	921	940	958	977	996	*014	*033	*051	*070	*088
235	37 107	125	143	162	180	199	217	236	254	273
236	291	309	328	346	364	383	401	420	438	456
237	475	493	511	530	548	566	584	603	621	639
238	657	676	694	712	730	749	767	785	803	821
239	840	858	876	894	912	930	948	967	985	*003
240	38 021	039	057	075	093	111	129	147	165	183
241	201	219	237	255	273	291	309	327	345	363
242	381	399	417	435	453	471	489	507	525	543
243	560	578	596	614	632	650	667	685	703	721
244	739	757	774	792	810	828	845	863	881	899
245	916	934	952	970	987	*005	*023	*040	*058	*076
246	39 093	111	129	146	164	181	199	217	234	252
247	269	287	305	322	340	357	375	392	410	427
248	445	463	480	497	515	532	550	567	585	602
249	620	637	655	672	689	707	724	742	759	776
250	794	811	828	846	863	881	898	915	933	950
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.

	22	21	20̂	20	19̂	19	18̂	18	17̂	17
.1	2.2	2.1	2.0	2.0	1.9	1.9	1.8	1.8	1.7	1.7
.2	4.4	4.2	4.1	4.0	3.9	3.8	3.7	3.6	3.5	3.4
.3	6.6	6.3	6.1	6.0	5.8	5.7	5.5	5.4	5.2	5.1
.4	8.8	8.4	8.2	8.0	7.8	7.6	7.4	7.2	7.0	6.8
.5	11.0	10.5	10.2	10.0	9.7	9.5	9.2	9.0	8.7	8.5
.6	13.2	12.6	12.3	12.0	11.7	11.4	11.1	10.8	10.5	10.2
.7	15.4	14.7	14.3	14.0	13.6	13.3	12.9	12.6	12.2	11.9
.8	17.6	16.8	16.4	16.0	15.6	15.2	14.8	14.4	14.0	13.6
.9	19.8	18.9	18.4	18.0	17.5	17.1	16.6	16.2	15.7	15.3

N.	0	1	2	3	4	5	6	7	8	9
250	39 794	811	828	846	863	881	898	915	933	950
251	967	984	*002	*019	*036	*054	*071	*088	*105	*123
252	40 140	157	174	191	209	226	243	260	277	295
253	312	329	346	363	380	398	415	432	449	466
254	483	500	517	534	551	569	586	603	620	637
255	654	671	688	705	722	739	756	773	790	807
256	824	841	858	875	892	908	925	942	959	976
257	993	*010	*027	*044	*061	*077	*094	*111	*128	*145
258	41 162	179	195	212	229	246	263	279	296	313
259	330	346	363	380	397	413	430	447	464	480
260	497	514	530	547	564	581	597	614	631	647
261	664	680	697	714	730	747	764	780	797	813
262	830	846	863	880	896	913	929	946	962	979
263	995	*012	*028	*045	*061	*078	*094	*111	*127	*144
264	42 160	177	193	209	226	242	259	275	292	308
265	324	341	357	373	390	406	423	439	455	472
266	488	504	521	537	553	569	586	602	618	635
267	651	667	683	700	716	732	748	765	781	797
268	813	829	846	862	878	894	910	927	943	959
269	975	991	*007	*023	*040	*056	*072	*088	*104	*120
270	43 136	152	168	184	200	216	233	249	265	281
271	297	313	329	345	361	377	393	409	425	441
272	457	473	489	505	520	536	552	568	584	600
273	616	632	648	664	680	695	711	727	743	759
274	775	791	806	822	838	854	870	886	901	917
275	933	949	965	980	996	*012	*028	*043	*059	*075
276	44 091	106	122	138	154	169	185	201	216	232
277	248	263	279	295	310	326	342	357	373	389
278	404	420	435	451	467	482	498	513	529	545
279	560	576	591	607	622	638	653	669	685	700
280	716	731	747	762	778	793	809	824	839	855
281	870	886	901	917	932	948	963	978	994	*009
282	45 025	040	055	071	086	102	117	132	148	163
283	178	194	209	224	240	255	270	286	301	316
284	332	347	362	377	393	408	423	438	454	469
285	484	499	515	530	545	560	576	591	606	621
286	636	652	667	682	697	712	727	743	758	773
287	788	803	818	833	848	864	879	894	909	924
288	939	954	969	984	999	*014	*029	*044	*059	*075
289	46 090	105	120	135	150	165	180	195	210	225
290	240	255	269	284	299	314	329	344	359	374
291	389	404	419	434	449	464	479	493	508	523
292	538	553	568	583	597	612	627	642	657	672
293	687	701	716	731	746	761	775	790	805	820
294	834	849	864	879	894	908	923	938	952	967
295	982	997	*011	*026	*041	*055	*070	*085	*100	*114
296	47 129	144	158	173	188	202	217	232	246	261
297	275	290	305	319	334	348	363	378	392	407
298	421	436	451	465	480	494	509	523	538	552
299	567	581	596	610	625	639	654	668	683	697
300	712	726	741	755	770	784	799	813	828	842
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.

	17̂	17	16̂	16	15̂	15	14̂	14
.1	1.7̂	1.7	1.6̂	1.6	1.5̂	1.5	1.4̂	1.4
.2	3.5	3.4	3.3	3.2	3.1	3.0	2.9	2.8
.3	5.2̂	5.1	4.9̂	4.8	4.6̂	4.5	4.3̂	4.2
.4	7.0	6.8	6.6	6.4	6.2	6.0	5.8	5.6
.5	8.7̂	8.5	8.2̂	8.0	7.7̂	7.5	7.2̂	7.0
.6	10.5	10.2	9.9	9.6	9.3	9.0	8.7	8.4
.7	12.2̂	11.9	11.5̂	11.2	10.8̂	10.5	10.1̂	9.8
.8	14.0	13.6	13.2	12.8	12.4	12.0	11.6	11.2
.9	15.7̂	15.3	14.8̂	14.4	13.9̂	13.5	13.0̂	12.6

N.	0	1	2	3	4	5	6	7	8	9
300	47 712	726̂	741	755̂	770	784̂	799	813̂	828	842
301	856̂	871	885̂	900	914̂	928̂	943	957̂	972	986̂
302	48 000̂	015	029̂	044	058	072̂	087	101	115̂	130
303	144̂	158̂	173	187	201̂	216	230	244̂	259	273
304	287̂	301̂	316	330	344̂	358̂	373	387	401̂	415̂
305	430	444	458̂	472̂	487	501	515̂	529̂	543̂	558
306	572	586̂	600̂	614̂	629	643	657	671̂	685̂	699̂
307	714	728	742	756̂	770̂	784̂	798̂	812̂	827	841
308	855	869	883̂	897̂	911̂	925̂	939̂	953̂	967̂	982
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49 136	150	164	178	192	206	220	234	248	262
311	276	290	304	318	332	346	359̂	373̂	387̂	401̂
312	415̂	429̂	443̂	457	471	485	499	513	526̂	540̂
313	554̂	568̂	582	596	610	624	637̂	651̂	665̂	679
314	693	707	720̂	734̂	748̂	762	776	789̂	803̂	817̂
315	831	845	858̂	872̂	886	900	913̂	927̂	941	955
316	968̂	982̂	996	*010	*023̂	*037̂	*051	*065	*078̂	*092
317	50 106	119̂	133̂	147	160̂	174̂	188	201̂	215̂	229
318	242̂	256̂	270	283̂	297̂	311	324̂	338	352	365̂
319	379	392̂	406̂	420	433̂	447	460̂	474̂	488	501̂
320	515	528̂	542	555̂	569̂	583	596̂	610	623̂	637
321	650̂	664	677̂	691	704̂	718	731̂	745	758̂	772
322	785̂	799	812̂	826	839̂	853	866̂	880	893̂	907
323	920̂	933̂	947	960̂	974	987̂	*001	*014̂	*027̂	*041
324	51 054̂	068	081̂	094̂	108	121̂	135	148	161̂	175
325	188̂	201̂	215	228̂	242	255	268̂	282	295	308̂
326	322	335	348̂	361̂	375	388̂	401̂	415	428	441̂
327	455	468	481̂	494̂	508	521	534̂	547̂	561	574
328	587̂	600̂	614	627	640̂	653̂	667	680	693	706̂
329	719̂	733	746	759	772̂	785̂	798̂	812	825	838
330	851̂	864̂	877̂	891	904	917	930̂	943̂	956̂	969̂
331	983	996	*009	*022	*035̂	*048̂	*061̂	*074̂	*087̂	*100̂
332	52 114	127	140	153	166	179	192	205̂	218̂	231̂
333	244̂	257̂	270̂	283̂	296̂	309̂	322̂	335̂	348̂	361̂
334	374̂	387̂	400̂	413̂	426̂	439̂	452̂	465̂	478̂	491̂
335	504̂	517̂	530̂	543̂	556̂	569̂	582	595	608	621
336	634	647	660	672̂	685̂	698̂	711̂	724̂	737	750
337	763	776	789	801̂	814̂	827̂	840	853	866	879
338	891̂	904̂	917̂	930	943	956	968̂	981̂	994̂	*007
339	53 020	033	045̂	058̂	071	084	097	109̂	122̂	135
340	148	160̂	173̂	186	199	211̂	224̂	237	250	262̂
341	275̂	288	301	313̂	326̂	339	352	364̂	377	390
342	402̂	415̂	428	440̂	453̂	466	478̂	491̂	504	516̂
343	529̂	542	554̂	567̂	580	592̂	605̂	618	630̂	643
344	656	668̂	681	693̂	706̂	719	731̂	744	756̂	769̂
345	782	794̂	807	819̂	832	845	857̂	870	882̂	895
346	907̂	920	932̂	945	958	970̂	983	995̂	*008	*020̂
347	54 033	045̂	058	070̂	083	095̂	108	120̂	133	145̂
348	158	170̂	183	195̂	208	220̂	232̂	245	257̂	270
349	282̂	295	307̂	320	332̂	344̂	357	369̂	382	394̂
350	407	419	431̂	444	456̂	469	481	493̂	506	518̂
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	14̂	14
.1	1.4̂	1.4
.2	2.9	2.8
.3	4.3̂	4.2
.4	5.8	5.6
.5	7.2	7.0
.6	8.7	8.4
.7	10.1̂	9.8
.8	11.6	11.2
.9	13.0̂	12.6

	13̂	13
.1	1.3̂	1.3
.2	2.7	2.6
.3	4.0̂	3.9
.4	5.4	5.2
.5	6.7	6.5
.6	8.1	7.8
.7	9.4̂	9.1
.8	10.8	10.4
.9	12.1̂	11.7

	12̂	12
.1	1.2̂	1.2
.2	2.5	2.4
.3	3.7̂	3.6
.4	5.0	4.8
.5	6.2̂	6.0
.6	7.5	7.2
.7	8.7̂	8.4
.8	10.0	9.6
.9	11.2̂	10.8

P. P.

N.	0	1	2	3	4	5	6	7	8	9
350	54 407	419	431̂	444	456̂	469	481	493̂	506	518̂
351	530̂	543	555̂	568	580	592̂	605	617	629̂	642
352	654̂	666̂	679	691̂	703̂	716	728	740̂	753	765
353	777̂	790	802	814̂	826̂	839	851	863̂	876	888
354	900̂	912̂	925	937	949̂	961̂	974	986	998̂	*010̂
355	55 023	035	047̂	059̂	071̂	084	096	108̂	120̂	133
356	145	157	169̂	181̂	194	206	218	230̂	242̂	254̂
357	267	279	291	303̂	315̂	327̂	340	352	364	376
358	388̂	400̂	412̂	424̂	437	449	461	473	485	497̂
359	509̂	521̂	533̂	545̂	558	570	582	594	606	618
360	630̂	642̂	654̂	666̂	678̂	690̂	702̂	714̂	726̂	738̂
361	750̂	762̂	775	787	799	811	823	835	847	859
362	871	883	895	907	919	931	943	955	966̂	978̂
363	990̂	*002̂	*014̂	*026̂	*038̂	*050̂	*062̂	*074̂	*086̂	*098
364	56 110	122	134	146	158	170	181̂	193̂	205̂	217̂
365	229̂	241	253	265	277	288̂	300̂	312̂	324̂	336
366	348	360	372	383̂	395̂	407̂	419	431	443	455
367	466̂	478̂	490̂	502	514	525̂	537̂	549̂	561	573
368	585	596̂	608̂	620	632	643̂	655̂	667̂	679	691
369	702̂	714̂	726	738	749̂	761̂	773	785	796̂	808̂
370	820	832	843̂	855̂	867	879	890̂	902̂	914	925̂
371	937̂	949	961	972̂	984	996	*007̂	*019̂	*031	*042̂
372	57 054̂	066	077̂	089̂	101	112̂	124̂	136	147̂	159
373	171	182̂	194	206	217̂	229	240̂	252̂	264	275̂
374	287	299	310̂	322	333̂	345	357	368̂	380	391̂
375	403	414̂	426̂	438	449̂	461	472̂	484	495̂	507
376	519	530̂	542	553̂	565	576̂	588	599̂	611	622̂
377	634	645̂	657	668̂	680	691̂	703	714̂	726	737̂
378	749	760̂	772	783̂	795	806̂	818	829̂	841	852̂
379	864	875̂	887	898̂	909̂	921	932̂	944	955̂	967
380	978̂	990	*001	*012̂	*024	*035̂	*047	*058̂	*069̂	*081
381	58 092̂	104	115̂	126̂	138	149̂	161	172	183̂	195
382	206̂	217̂	229	240̂	252	263	274̂	286	297	308̂
383	320	331	342̂	354	365	376̂	388	399	410̂	422
384	433	444̂	455̂	467	478̂	489̂	501	512	523̂	535
385	546	557̂	568̂	580	591	602̂	613̂	625	636	647̂
386	658̂	670	681	692̂	703̂	715	726	737̂	748̂	760
387	771	782̂	793̂	804̂	816	827	838̂	849̂	861	872
388	883	894̂	905̂	916̂	928	939	950̂	961̂	972̂	984
389	995	*006	*017̂	*028̂	*039̂	*050̂	*062	*073	*084	*095̂
390	59 106̂	117̂	128̂	140	151	162	173	184̂	195̂	206̂
391	217̂	229	240	251	262	273	284̂	295̂	306̂	317̂
392	328̂	339̂	351	362	373	384	395	406	417	428
393	439̂	450̂	461̂	472̂	483̂	494̂	505̂	516̂	527̂	538̂
394	549̂	560̂	571̂	582̂	593̂	604̂	615̂	626̂	637̂	648̂
395	659̂	670̂	681̂	692̂	703̂	714̂	725̂	736̂	747̂	758̂
396	769̂	780̂	791̂	802̂	813̂	824̂	835̂	846	857	868
397	879	890	901	912	923	933̂	944̂	955̂	966̂	977̂
398	988̂	999	*010	*021	*032	*043	*053̂	*064̂	*075̂	*086̂
399	60 097̂	108	119	130	141	151̂	162̂	173̂	184̂	195
400	206	217	227̂	238̂	249̂	260̂	271	282	293	303̂
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

P. P.

	12̂	12	11̂	11	10̂
.1	1.2̂	1.2	1.1̂	1.1	1.0̂
.2	2.5	2.4	2.3	2.2	2.1
.3	3.7̂	3.6	3.4̂	3.3	3.1̂
.4	5.0	4.8	4.6	4.4	4.2
.5	6.2̂	6.0	5.7̂	5.5	5.2̂
.6	7.5	7.2	6.9	6.6	6.3
.7	8.7̂	8.4	8.0̂	7.7	7.3̂
.8	10.0	9.6	9.2	8.8	8.4
.9	11.2̂	10.8	10.3̂	9 9	9.4̂

P. P.

N.	0	1	2	3	4	5	6	7	8	9
400	60 206	217	227	238	249	260	271	282	293	303
401	314	325	336	347	357	368	379	390	401	412
402	422	433	444	455	466	476	487	498	509	519
403	530	541	552	563	573	584	595	606	616	627
404	638	649	659	670	681	692	702	713	724	735
405	745	756	767	777	788	799	810	820	831	842
406	852	863	874	884	895	906	916	927	938	949
407	959	970	981	991	*002	*013	*023	*034	*044	*055
408	61 066	076	087	098	108	119	130	140	151	161
409	172	183	193	204	215	225	236	246	257	268
410	278	289	299	310	320	331	342	352	363	373
411	384	394	405	416	426	437	447	458	468	479
412	489	500	511	521	532	542	553	563	574	584
413	595	605	616	626	637	647	658	668	679	689
414	700	710	721	731	742	752	763	773	784	794
415	805	815	825	836	846	857	867	878	888	899
416	909	920	930	940	951	961	972	982	993	*003
417	62 013	024	034	045	055	065	076	086	097	107
418	117	128	138	149	159	169	180	190	200	211
419	221	232	242	252	263	273	283	294	304	314
420	325	335	345	356	366	376	387	397	407	418
421	428	438	449	459	469	480	490	500	510	521
422	531	541	552	562	572	582	593	603	613	624
423	634	644	654	665	675	685	695	706	716	726
424	736	747	757	767	777	788	798	808	818	828
425	839	849	859	869	879	890	900	910	920	931
426	941	951	961	971	981	992	*002	*012	*022	*032
427	63 043	053	063	073	083	093	104	114	124	134
428	144	154	164	175	185	195	205	215	225	235
429	245	256	266	276	286	296	306	316	326	336
430	347	357	367	377	387	397	407	417	427	437
431	447	458	468	478	488	498	508	518	528	538
432	548	558	568	578	588	598	608	618	628	639
433	649	659	669	679	689	699	709	719	729	739
434	749	759	769	779	789	799	809	819	829	839
435	849	859	869	879	889	899	909	919	928	938
436	948	958	968	978	988	998	*008	*018	*028	*038
437	64 048	058	068	078	088	098	107	117	127	137
438	147	157	167	177	187	197	207	217	226	236
439	246	256	266	276	286	296	306	315	325	335
440	345	355	365	375	384	394	404	414	424	434
441	444	453	463	473	483	493	503	512	522	532
442	542	552	562	571	581	591	601	611	621	630
443	640	650	660	670	679	689	699	709	718	728
444	738	748	758	767	777	787	797	806	816	826
445	836	846	855	865	875	885	894	904	914	923
446	933	943	953	962	972	982	992	*001	*011	*021
447	65 031	040	050	060	069	079	089	098	108	118
448	128	137	147	157	166	176	186	195	205	215
449	224	234	244	253	263	273	282	292	302	311
450	321	331	340	350	360	369	379	389	398	408
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P. P.

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P. P.

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450	65 321̂	331	340̂	350	360	369̂	379	389	398̂	408
451	417̂	427̂	437	446̂	456	466	475̂	485	494̂	504
452	514	523̂	533	542̂	552̂	562	571̂	581	590̂	600
453	610	619̂	629	638̂	648	657̂	667̂	677	686̂	696
454	705̂	715	724̂	734̂	744	753̂	763	772̂	782	791̂
455	801	810̂	820	830	839̂	849	858̂	868	877̂	887
456	896̂	906	915̂	925	934̂	944	953̂	963	972̂	982
457	991̂	*001	*010̂	*020	*029̂	*039	*048̂	*058	*067̂	*077
458	66 086̂	096	105̂	115	124̂	134	143̂	153	162̂	172
459	181̂	190̂	200	209̂	219	228̂	238	247̂	257	266̂
460	276	285	294̂	304	313̂	323	332̂	342	351	360̂
461	370	379̂	389	398̂	408	417	426̂	436	445̂	455
462	464	473̂	483	492̂	502	511	520̂	530	539̂	548̂
463	558	567̂	577	586	595̂	605	614̂	623̂	633	642̂
464	652	661	670̂	680	689	698̂	708	717̂	726̂	736
465	745̂	754̂	764	773̂	782̂	792	801̂	810̂	820	829̂
466	838̂	848	857	866̂	876	885	894̂	904	913	922̂
467	931̂	941	950̂	959̂	969	978	987̂	996̂	*006	*015̂
468	67 024̂	034	043	052̂	061̂	071	080	089̂	099	108
469	117̂	126̂	136	145	154̂	163̂	173	182	191̂	200̂
470	210	219	228̂	237̂	246̂	256	265	274̂	283̂	293
471	302	311̂	320̂	329̂	339	348	357̂	366̂	376	385
472	394	403̂	412̂	422	431	440	449̂	458̂	467̂	477
473	486	495̂	504̂	513̂	523	532	541	550̂	559̂	568̂
474	578	587	596	605̂	614̂	623̂	633	642	651	660
475	669̂	678̂	687̂	697	706	715	724	733̂	742̂	751̂
476	760̂	770	779	788	797	806̂	815̂	824̂	833̂	842̂
477	852	861	870	879	888	897̂	906̂	915̂	924̂	933̂
478	943	952	961	970	979	988	997̂	*006̂	*015̂	*024̂
479	68 033̂	042̂	051̂	060̂	070	079	088	097	106	115
480	124	133	142	151̂	160̂	169̂	178̂	187̂	196̂	205̂
481	214̂	223̂	232̂	241̂	250̂	259̂	268̂	277̂	286̂	295̂
482	304̂	313̂	322̂	331̂	340̂	349̂	358̂	367̂	376̂	385̂
483	394̂	403̂	412̂	421̂	430̂	439̂	448̂	457̂	466̂	475̂
484	484̂	493̂	502̂	511̂	520̂	529̂	538̂	547̂	556̂	565
485	574	583	592	601	610	619	628	637	646	654̂
486	663̂	672̂	681̂	690̂	699̂	708̂	717	726	735	744
487	753	762	770̂	779̂	788̂	797̂	806̂	815̂	824	833
488	842	851	860	868̂	877̂	886̂	895̂	904	913	922
489	931	940	948̂	957̂	966̂	975̂	984	993	*002	*010̂
490	69 019̂	028̂	037̂	046	055	064	073	081̂	090̂	099̂
491	108	117	126	134̂	143̂	152̂	161	170	179	187̂
492	196̂	205̂	214	223	232	240̂	249̂	258̂	267	276
493	284̂	293̂	302̂	311	320	328̂	337̂	346̂	355	364
494	372̂	381̂	390̂	399	408	416̂	425̂	434	443	451̂
495	460̂	469̂	478	487	495̂	504̂	513	522	530̂	539̂
496	548	557	565̂	574̂	583	592	600̂	609̂	618	627
497	635̂	644̂	653	662	670̂	679̂	688	697	705̂	714
498	723	731̂	740̂	749	758	766̂	775	784	792̂	801̂
499	810	819	827̂	836	845	853̂	862	871	879̂	888̂
500	897	905̂	914̂	923	931̂	940̂	949	958	966̂	975
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P. P.

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P. P.

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500	69 897	905̂	914̂	923	931̂	940̂	949	958	966̂	975
501	984	992̂	*001	*010	*018̂	*027	*036	*044̂	*053	*061̂
502	70 070̂	079	087̂	096̂	105	113̂	122	131	139̂	148
503	157	165̂	174	182̂	191̂	200	208̂	217	226	234̂
504	243	251̂	260̂	269	277̂	286	294̂	303̂	312	320̂
505	329	337̂	346̂	355	363̂	372	380̂	389̂	398	406̂
506	415	423̂	432	441	449̂	458	466̂	475	483̂	492
507	501	509̂	518	526̂	535	543̂	552	560̂	569̂	578
508	586̂	595	603̂	612	620̂	629	637̂	646	654̂	663
509	672	680̂	689	697̂	706	714̂	723	731̂	740	748̂
510	757	765̂	774	782̂	791	799̂	808	816̂	825	833̂
511	842	850̂	859	867̂	876	884̂	893	901̂	910	918̂
512	927	935̂	944	952̂	961	969̂	978	986̂	995	*003̂
513	71 011̂	020	028̂	037	045̂	054	062̂	071	079̂	088
514	096̂	105	113	121̂	130	138̂	147	155̂	164	172̂
515	180̂	189	197̂	206	214̂	223	231̂	239̂	248	256̂
516	265	273̂	282	290	298̂	307	315̂	324	332	340̂
517	349	357̂	366	374	382	391	399̂	408	416	424̂
518	433	441̂	449̂	458	466̂	475	483̂	491̂	500	508̂
519	516̂	525	533̂	542	550	558̂	567	575̂	583̂	592
520	600̂	608̂	617	625̂	633̂	642	650̂	659	667	675̂
521	684	692	700̂	709	717	725̂	734	742	750̂	758̂
522	767	775̂	783̂	792	800̂	808̂	817	825̂	833̂	842
523	850	858̂	867	875	883̂	891̂	900	908̂	916̂	925
524	933	941̂	949̂	958	966̂	974̂	983	991	999̂	*007̂
525	72 016	024	032̂	040̂	049	057̂	065̂	074	082	090̂
526	098̂	107	115	123̂	131̂	140	148	156̂	164̂	173
527	181	189̂	197̂	206	214	222	230̂	238̂	247	255
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529	345̂	354	362	370	378̂	386̂	395	403	411	419̂
530	427̂	436	444	452	460̂	468̂	476̂	485	493	501̂
531	509̂	517̂	526	534	542	550̂	558̂	566̂	575	583
532	591	599̂	607̂	615̂	624	632	640	648̂	656̂	664̂
533	672̂	681	689	697	705̂	713̂	721̂	729̂	738	746
534	754	762̂	770̂	778̂	786̂	795	803	811	819	827̂
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536	916̂	924̂	932	941	949	957	965	973	981̂	989̂
537	997̂	*005̂	*013̂	*021̂	*030	*038	*046	*054	*062	*070
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539	159	167	175	183	191	199	207	215	223̂	231̂
540	239̂	247̂	255̂	263̂	271̂	279̂	287̂	295̂	303̂	311̂
541	319̂	328	336	344	352	360	368	376	384	392
542	400	408	416	424	432	440	448	456	464	472
543	480	488	496	504	512	520	528	536	544	552
544	560	568	576	584	592	600	608	615̂	623̂	631̂
545	639̂	647̂	655̂	663̂	671̂	679̂	687̂	695̂	703̂	711̂
546	719̂	727	735	743	751	759	767	775	783	791
547	798̂	806̂	814̂	822̂	830̂	838̂	846̂	854̂	862	870
548	878	886	894	902	909̂	917̂	925̂	933̂	941̂	949̂
549	957	965	973	981	989	997	*004̂	*012̂	*020̂	*028̂
550	74 036̂	044	052	060	068	075̂	083̂	091̂	099̂	107̂
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P. P.

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.9	8.1	7.6̂	7.2	6.7̂

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550	74 036	044	052	060	068	075	083	091	099	107	
551	115	123	131	139	146	154	162	170	178	186	
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553	272	280	288	296	304	312	319	327	335	343	
554	351	359	366	374	382	390	398	406	413	421	
555	429	437	445	453	460	468	476	484	492	499	
556	507	515	523	531	538	546	554	562	570	577	8
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558	663	671	679	687	694	702	710	718	725	733	.2 1.6
559	741	749	756	764	772	780	788	795	803	811	.3 2.4
560	819	826	834	842	850	857	865	873	881	888	.4 3.2
561	896	904	912	919	927	935	942	950	958	966	.5 4.0
562	973	981	989	997	*004	*012	*020	*027	*035	*043	.6 4.8
563	75 051	058	066	074	081	089	097	105	112	120	.7 5.6
564	128	135	143	151	158	166	174	182	189	197	.8 6.4
565	205	212	220	228	235	243	251	258	266	274	.9 7.2
566	281	289	297	304	312	320	327	335	343	350	
567	358	366	373	381	389	396	404	412	419	427	
568	435	442	450	458	465	473	480	488	496	503	
569	511	519	526	534	541	549	557	564	572	580	
570	587	595	602	610	618	625	633	641	648	656	
571	663	671	679	686	694	701	709	717	724	732	7
572	739	747	755	762	770	777	785	792	800	808	.1 0.7
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575	967	974	982	989	997	*004	*012	*019	*027	*034	.4 3.0
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578	193	200	208	215	223	230	238	245	253	260	.7 5.2
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581	417	425	432	440	447	455	462	470	477	485	
582	492	500	507	514	522	529	537	544	552	559	
583	567	574	582	589	596	604	611	619	626	634	
584	641	648	656	663	671	678	686	693	700	708	
585	715	723	730	738	745	752	760	767	775	782	
586	790	797	804	812	819	827	834	841	849	856	7
587	864	871	878	886	893	901	908	915	923	930	.1 0.7
588	937	945	952	960	967	974	982	989	997	*004	.2 1.4
589	77 011	019	026	033	041	048	055	063	070	078	.3 2.1
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591	158	166	173	181	188	195	203	210	217	225	.5 3.5
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593	305	313	320	327	335	342	349	356	364	371	.7 4.9
594	378	386	393	400	408	415	422	430	437	444	.8 5.6
595	451	459	466	473	481	488	495	503	510	517	.9 6.3
596	524	532	539	546	554	561	568	575	583	590	
597	597	604	612	619	626	634	641	648	655	663	
598	670	677	684	692	699	706	713	721	728	735	
599	742	750	757	764	771	779	786	793	800	808	
600	815	822	829	837	844	851	858	866	873	880	
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600	77 815	822	829	837	844	851	858	866	873	880	
601	887	894	902	909	916	923	931	938	945	952	
602	959	967	974	981	988	995	*003	*010	*017	*024	
603	78 031	039	046	053	060	067	075	082	089	096	
604	103	111	118	125	132	139	147	154	161	168	
605	175	182	190	197	204	211	218	226	233	240	
606	247	254	261	269	276	283	290	297	304	311	7
607	319	326	333	340	347	354	362	369	376	383	.1 0.7
608	390	397	404	412	419	426	433	440	447	454	.2 1.5
609	461	469	476	483	490	497	504	511	518	526	.3 2.2
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613	746	753	760	767	774	781	788	795	802	810	.7 5.2
614	817	824	831	838	845	852	859	866	873	880	.8 6.0
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616	958	965	972	979	986	993	*000	*007	*014	*021	
617	79 028	035	042	049	056	063	070	078	085	092	
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619	169	176	183	190	197	204	211	218	225	232	
620	239	246	253	260	267	274	281	288	295	302	
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622	379	386	393	400	407	414	421	428	435	442	.1 0.7
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626	657	664	671	678	685	692	699	706	713	720	.5 3.5
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633	140	147	154	161	168	174	181	188	195	202	
634	209	216	222	229	236	243	250	257	263	270	
635	277	284	291	298	304	311	318	325	332	339	
636	345	352	359	366	373	380	386	393	400	407	6
637	414	421	427	434	441	448	455	461	468	475	.1 0.6
638	482	489	495	502	509	516	523	529	536	543	.2 1.3
639	550	557	563	570	577	584	591	597	604	611	.3 1.9
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642	753	760	767	774	780	787	794	801	807	814	.6 3.9
643	821	828	834	841	848	855	862	868	875	882	.7 4.5
644	888	895	902	909	915	922	929	936	942	949	.8 5.2
645	956	962	969	976	983	989	996	*003	*010	*016	.9 5.8
646	81 023	030	036	043	050	057	063	070	077	083	
647	090	097	104	110	117	124	130	137	144	151	
648	157	164	171	177	184	191	197	204	211	218	
649	224	231	238	244	251	258	264	271	278	284	
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650	81 291̂	298	304̂	311̂	318	324̂	331̂	338	345	351̂
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652	425	431̂	438	444̂	451̂	458	464̂	471̂	478	484̂
653	491̂	498	504̂	511̂	518	524̂	531	538	544̂	551
654	558	564̂	571	577̂	584̂	591	597̂	604	611	617̂
655	624	631	637̂	644	650̂	657̂	664	670̂	677	684
656	690̂	697	703̂	710	717	723̂	730	736̂	743̂	750
657	756̂	763	770	776̂	783	789̂	796	803	809̂	816
658	822̂	829	836	842̂	849	855̂	862	869	875̂	882
659	888̂	895	901̂	908̂	915	921̂	928	934̂	941	948
660	954̂	961	967̂	974	980̂	987̂	994	*000̂	*007	*013̂
661	82 020	026̂	033̂	040	046̂	053	059̂	066	072̂	079
662	086	092̂	099	105̂	112	118̂	125	131̂	138	145
663	151̂	158	164̂	171	177̂	184	190̂	197	203̂	210̂
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671	672̂	678̂	685	691̂	698	704̂	711	717̂	724	730̂
672	737	743̂	750	756̂	763	769	775̂	782	788̂	795
673	801̂	808	814̂	821	827̂	834	840	846̂	853	859̂
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675	930̂	937	943	949̂	956	962̂	969	975̂	982	988
676	994̂	*001	*007̂	*014	*020̂	*027	*033	*039̂	*046	*052
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697	323̂	329̂	335̂	342	348	354̂	360̂	367	373	379̂
698	385̂	392	398	404	410̂	416̂	423	429	435̂	441̂
699	447̂	454	460	466̂	472̂	479	485	491	497̂	503̂
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703	695	701	708	714	720	726	732	739	745	751
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706	880	886	893	899	905	911	917	923	929	936
707	942	948	954	960	966	972	979	985	991	997
708	85 003	009	015	021	028	034	040	046	052	058
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712	248	254	260	266	272	278	284	290	297	303
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714	370	376	382	388	394	400	406	412	418	424
715	430	436	443	449	455	461	467	473	479	485
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718	612	618	624	630	636	642	648	655	661	667
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728	213	219	225	231	237	243	249	255	261	267
729	273	278	284	290	296	302	308	314	320	326
730	332	338	344	350	356	362	368	374	380	386
731	391	397	403	409	415	421	427	433	439	445
732	451	457	463	469	475	481	486	492	498	504
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734	569	575	581	587	593	599	605	611	617	623
735	628	634	640	646	652	658	664	670	676	682
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737	746	752	758	764	770	776	782	788	794	800
738	805	811	817	823	829	835	841	847	852	858
739	864	870	876	882	888	894	899	905	911	917
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741	982	987	993	999	*005	*011	*017	*023	*028	*034
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747	332	338	343	349	355	361	367	372	378	384
748	390	396	402	407	413	419	425	431	436	442
749	448	454	460	465	471	477	483	489	494	500
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759	88 024	030	035	041	047	053	058	064	070	075
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780	209	215	220	226	231	237	243	248	254	259
781	265	270	276	282	287	293	298	304	309	315
782	320	326	332	337	343	348	354	359	365	370
783	376	381	387	393	398	404	409	415	420	426
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789	707	713	718	724	729	735	740	746	751	757
790	762	768	773	779	784	790	795	801	806	812
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792	872	878	883	889	894	900	905	911	916	922
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794	982	987	993	998	*004	*009	*015	*020	*026	*031
795	90 036	042	047	053	058	064	069	075	080	086
796	091	097	102	107	113	118	124	129	135	140
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798	200	205	211	216	222	227	233	238	244	249
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808	741	746	752	757	762	768	773	778	784	789	
809	795	800	805	811	816	821	827	832	838	843	
810	848	854	859	864	870	875	880	886	891	896	
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814	062	068	073	078	084	089	094	100	105	110	.3 1.6
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816	169	174	179	185	190	195	201	206	211	217	.5 2.7
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822	487	492	497	503	508	513	519	524	529	534	
823	540	545	550	556	561	566	571	577	582	587	
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830	908	913	918	923	928	934	939	944	949	955	
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849	891	896	901	906	911	916	921	926	931	937	
850	942	947	952	957	962	967	972	977	982	988	
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851	993	998	*003	*008̂	*013̂	*018̂	*023̂	*028̂	*034	*039	
852	93 044	049	054	059	064̂	069̂	074̂	079̂	084̂	090	
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855	196̂	201̂	207	212	217	222	227	232	237	242̂	
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863	601	606	611	616	621	626	631̂	636̂	641̂	646̂	.7 3.8̂
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865	701̂	706̂	711̂	716̂	721̂	726̂	731̂	736̂	742	747	.9 4.9̂
866	752	757	762	767	772	777	782	787	792	797	
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868	852	857	862	867	872	877	882	887	892	897	
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871	94 002	007	012	017	022	026̂	031̂	036̂	041̂	046̂	5
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874	151	156	161	166	171	176	181	186	191	196	.3 1.5
875	201	206	210̂	215̂	220̂	225̂	230̂	235̂	240̂	245̂	.4 2.0
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883	596	601	606	611	615̂	620̂	625̂	630̂	635̂	640̂	
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888	841̂	846	851	856	861	865̂	870̂	875̂	880̂	885̂	.2 0.9̂
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892	95 036̂	041̂	046	051	056	061	065̂	070̂	075̂	080̂	.6 2.7
893	085	090	095	099̂	104̂	109̂	114̂	119	124	129	.7 3.1̂
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896	231	235̂	240̂	245̂	250	255	260	264̂	269̂	274̂	
897	279	284	289	294	298̂	303̂	308̂	313	318	323	
898	327̂	332̂	337̂	342	347	352	356̂	361̂	366̂	371	
899	376	381	385̂	390̂	395̂	400	405	410	414̂	419̂	
900	424̂	429	434	438̂	443̂	448̂	453	458	463	467̂	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**P. P.**

N.	0	1	2	3	4	5	6	7	8	9
900	95 424	429	434	438	443	448	453	458	463	467
901	472	477	482	487	492	496	501	506	511	516
902	520	525	530	535	540	544	549	554	559	564
903	569	573	578	583	588	593	597	602	607	612
904	617	621	626	631	636	641	645	650	655	660
905	665	669	674	679	684	689	693	698	703	708
906	713	717	722	727	732	737	741	746	751	756
907	760	765	770	775	780	784	789	794	799	804
908	808	813	818	823	827	832	837	842	847	851
909	856	861	866	870	875	880	885	890	894	899
910	904	909	913	918	923	928	933	937	942	947
911	952	956	961	966	971	975	980	985	990	994
912	999	*004	*009	*014	*018	*023	*028	*033	*037	*042
913	96 047	052	056	061	066	071	075	080	085	090
914	094	099	104	109	113	118	123	128	132	137
915	142	147	151	156	161	166	170	175	180	185
916	189	194	199	204	208	213	218	222	227	232
917	237	241	246	251	256	260	265	270	275	279
918	284	289	293	298	303	308	312	317	322	327
919	331	336	341	345	350	355	360	364	369	374
920	379	383	388	393	397	402	407	412	416	421
921	426	430	435	440	445	449	454	459	463	468
922	473	478	482	487	492	496	501	506	511	515
923	520	525	529	534	539	543	548	553	558	562
924	567	572	576	581	586	590	595	600	605	609
925	614	619	623	628	633	637	642	647	651	656
926	661	666	670	675	680	684	689	694	698	703
927	708	712	717	722	726	731	736	741	745	750
928	755	759	764	769	773	778	783	787	792	797
929	801	806	811	815	820	825	829	834	839	843
930	848	853	857	862	867	871	876	881	885	890
931	895	899	904	909	913	918	923	927	932	937
932	941	946	951	955	960	965	969	974	979	983
933	988	993	997	*002	*007	*011	*016	*020	*025	*030
934	97 034	039	044	048	053	058	062	067	072	076
935	081	086	090	095	099	104	109	113	118	123
936	127	132	137	141	146	151	155	160	164	169
937	174	178	183	188	192	197	202	206	211	215
938	220	225	229	234	239	243	248	252	257	262
939	266	271	276	280	285	289	294	299	303	308
940	313	317	322	326	331	336	340	345	349	354
941	359	363	368	373	377	382	386	391	396	400
942	405	409	414	419	423	428	432	437	442	446
943	451	456	460	465	469	474	479	483	488	492
944	497	502	506	511	515	520	525	529	534	538
945	543	548	552	557	561	566	570	575	580	584
946	589	593	598	603	607	612	616	621	626	630
947	635	639	644	649	653	658	662	667	671	676
948	681	685	690	694	699	703	708	713	717	722
949	726	731	736	740	745	749	754	758	763	768
950	772	777	781	786	790	795	800	804	809	813
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	5
.1	0.5
.2	1.0
.3	1.5
.4	2.0
.5	2.5
.6	3.0
.7	3.5
.8	4.0
.9	4.5

	4
.1	0.4
.2	0.9
.3	1.3
.4	1.8
.5	2.2
.6	2.7
.7	3.1
.8	3.6
.9	4.0

N.	0	1	2	3	4	5	6	7	8	9	P. P.
950	97 772̂	777	781̂	786	790̂	795	800	804̂	809	813̂	
951	818	822̂	827	831̂	836̂	841	845̂	850	854̂	859	
952	863̂	868̂	873	877̂	882	886̂	891	895̂	900	904̂	
953	909̂	914	918̂	923	927̂	932	936̂	941	945̂	950̂	
954	955	959̂	964	968̂	973	977̂	982	986̂	991	996	
955	98 000̂	005	009̂	014	018̂	023	027̂	032	036̂	041	5
956	046	050̂	055	059̂	064	068̂	073	077̂	082	086̂	.1 0.5
957	091	095̂	100̂	105	109̂	114	118̂	123	127̂	132	.2 1.0
958	136̂	141	145̂	150	154̂	159	163̂	168̂	173	177̂	.3 1.5
959	182	186̂	191	195̂	200	204̂	209	213̂	218	222̂	.4 2.0
960	227	231̂	236	240̂	245	249̂	254̂	259	263̂	268	.5 2.5
961	272̂	277	281̂	286	290̂	295	299̂	304	308̂	313	.6 3.0
962	317̂	322	326̂	331	335̂	340	344̂	349	353̂	358	.7 3.5
963	362̂	367	371̂	376	380̂	385	389̂	394	398̂	403	.8 4.0
964	407̂	412	416̂	421	425̂	430	434̂	439	443̂	448	.9 4.5
965	452̂	457	461̂	466	470̂	475	479̂	484	488̂	493	
966	497̂	502	506̂	511	515̂	520	524̂	529	533̂	538	
967	542̂	547	551̂	556	560̂	565	569̂	574	578̂	583	
968	587̂	592	596̂	601	605̂	610	614̂	619	623̂	628	
969	632̂	637	641̂	646	650̂	655	659̂	663̂	668	672̂	
970	677	681̂	686	690̂	695	699̂	704	708̂	713	717̂	4̂
971	722	726̂	731	735̂	740	744̂	749	753	757̂	762	.1 0.4̂
972	766̂	771	775̂	780	784̂	789	793̂	798	802̂	807	.2 0.9
973	811̂	815̂	820	824̂	829	833̂	838	842̂	847	851̂	.3 1.3̂
974	856	860̂	865	869̂	873̂	878	882̂	887	891̂	896	.4 1.8
975	900̂	905	909̂	914	918̂	922̂	927	931̂	936	940̂	.5 2.2̂
976	945	949̂	954	958̂	963	967	971̂	976	980̂	985	.6 2.7
977	989̂	994	998̂	*003	*007	*011̂	*016	*020̂	*025	*029̂	.7 3.1̂
978	99 034	038̂	043	047	051̂	056	060̂	065	069̂	074	.8 3.6
979	078̂	082̂	087	091̂	096	100̂	105	109̂	113̂	118	.9 4.0̂
980	122̂	127	131̂	136	140̂	145	149	153̂	158	162̂	
981	167	171̂	176	180	184̂	189	193̂	198	202̂	206̂	
982	211	215̂	220	224̂	229	233̂	237̂	242	246̂	251	
983	255̂	260	264	268̂	273	277̂	282	286̂	290̂	295	
984	299̂	304	308̂	312̂	317	321̂	326	330̂	335	339	
985	343̂	348	352̂	357	361̂	365̂	370	374̂	379	383̂	4
986	387̂	392	396̂	401	405̂	409̂	414	418̂	423	427̂	.1 0.4
987	431̂	436	440̂	445	449̂	453̂	458	462̂	467	471̂	.2 0.8
988	475̂	480	484̂	489	493̂	497̂	502	506̂	511	515	.3 1.2
989	519̂	524	528̂	533	537	541̂	546	550̂	554̂	559	.4 1.6
990	563̂	568	572̂	576̂	581	585̂	590	594	598̂	603	.5 2.0
991	607̂	611̂	616	620̂	625	629̂	633̂	638	642̂	647	.6 2.4
992	651	655̂	660	664̂	668̂	673	677̂	682	686	690̂	.7 2.8
993	695	699̂	703̂	708	712̂	717	721	725̂	730	734̂	.8 3.2
994	738̂	743	747̂	751̂	756	760̂	765	769	773̂	778	.9 3.6
995	782̂	786̂	791	795̂	800	804	808̂	813	817	821̂	
996	826	830̂	834̂	839	843̂	847̂	852	856̂	861	865	
997	869̂	874	878	882̂	887	891̂	895̂	900	904̂	908̂	
998	913	917̂	922	926	930̂	935	939	943̂	948	952	
999	956̂	961	965	969̂	974	978̂	982̂	987	991̂	995̂	
1000	00 000	004̂	008̂	013	017̂	021̂	026	030̂	034̂	039	
N.	0	1	2	3	4	5	6	7	8	9	P. P.

N.	0	1	2	3	4	5	6	7	8	9
1000	000 000	043̂	087	130̂	173̂	217	260̂	304	347̂	390̂
01	434	477̂	521	564	607̂	651	694̂	737̂	781	824̂
02	867̂	911	954̂	997̂	*041	*084̂	*127̂	*171	*214̂	*257̂
03	001 301	344	387̂	431	474	517̂	560̂	604	647	690̂
04	733̂	777	820	863̂	906̂	950	993	*036̂	*079̂	*123
05	002 166	209̂	252̂	295̂	339	382	425̂	468̂	511̂	555
06	598	641	684̂	727̂	770̂	814	857	900	943	986̂
07	003 029̂	072̂	115̂	159	202	245	288	331̂	374̂	417̂
08	460̂	503̂	546̂	590	633	676	719	762	805	848
09	891	934	977	*020̂	*063̂	*106̂	*149̂	*192̂	*235̂	*278̂
1010	004 321̂	364̂	407̂	450̂	493̂	536̂	579̂	622̂	665	708
11	751	794	837	880	923	966	*009	*051̂	*094̂	*137̂
12	005 180̂	223̂	266̂	309	352	395	438	481	523̂	566̂
13	609̂	652̂	695	738	781	824	866̂	909̂	952̂	995
14	006 038	081	123̂	166̂	209	252	295	337̂	380̂	423̂
15	466	509	551̂	594̂	637	680	722̂	765̂	808	851
16	893̂	936̂	979	*022	*064̂	*107̂	*150	*193	*235̂	*278
17	007 321	363̂	406̂	449	491̂	534̂	577	620	662̂	705
18	748	790̂	833	875̂	918̂	961	*003̂	*046̂	*089	131̂
19	008 174	217	259̂	302	344̂	387	430	472̂	515	557̂
1020	600	642̂	685̂	728	770̂	813	855̂	898	940̂	983
21	009 025̂	068̂	111	153̂	196	238̂	281	323̂	366	408̂
22	451	493̂	536	578̂	621	663̂	706	748̂	790̂	833
23	875̂	918	960̂	*003	*045̂	*088	*130̂	*172̂	*215	*257̂
24	010 300	342̂	385	427	469̂	512	554̂	596̂	639	681̂
25	724	766	808̂	851	893̂	935̂	978	*020̂	*062̂	*105
26	011 147̂	189̂	232	274̂	316̂	359	401̂	443̂	486	528
27	570̂	612̂	655	697̂	739̂	782	824	866̂	908̂	951
28	993	*035̂	*077̂	*120	*162	*204̂	*246̂	*288̂	*331	*373
29	012 415̂	457̂	500	542	584	626̂	668̂	710̂	753	795
1030	837	879̂	921̂	963̂	*006	*048	*000	*132̂	174̂	216̂
31	013 258̂	301	343	385	427	469	511̂	553̂	595̂	637̂
32	679̂	722	764	806	848	890	932	974	*016	*058̂
33	014 100̂	142̂	184̂	226̂	268̂	310̂	352̂	394̂	436̂	478̂
34	520̂	562̂	604̂	646̂	688̂	730̂	772̂	814̂	856̂	898̂
35	940̂	982̂	*024̂	*066	*108	*150	*192	*234	*276	*318
36	015 360	401̂	443̂	485̂	527̂	569̂	611	653	695	737
37	779	820̂	862̂	904̂	946	988	*030	*072	*113̂	155̂
38	016 197̂	239	281	323	364̂	406̂	448̂	490	532	573̂
39	615̂	657̂	699	741	782̂	824̂	866̂	908	950	991̂
1040	017 033̂	075	117	158̂	200̂	242	284	325̂	367̂	409
41	450̂	492̂	534	576	617̂	659̂	701	742̂	784̂	826
42	867̂	909̂	951	992̂	*034̂	*076	*117̂	*159̂	*201	*242̂
43	018 284̂	326	367̂	409	451	492̂	534	575̂	617̂	659
44	700̂	742	783̂	825̂	867	908̂	950	991̂	*033	*074̂
45	019 116̂	158	199̂	241	282̂	324	365̂	407	448̂	490
46	531̂	573	614̂	656	697̂	739	780̂	822	863̂	905
47	946̂	988	*029̂	*071	*112̂	*154	*195̂	*237	*278̂	*320
48	020 361̂	402̂	444	485̂	527	568̂	610	651̂	692̂	734
49	775̂	817	858̂	899̂	941	982̂	*024	*065	*106̂	*148
1050	021 189̂	230̂	272	313̂	354̂	396	437̂	478̂	520	561̂
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	43̂	43
.1	4.3̂	4.3
.2	8.7	8.6
.3	13.0̂	12.9
.4	17.4	17.2
.5	21.7̂	21.5
.6	26.1	25.8
.7	30.4̂	30.1
.8	34.8	34.4
.9	39.1̂	38.7

	42̂	42
.1	4.2̂	4.2
.2	8.5	8.4
.3	12.7̂	12.6
.4	17.0	16.8
.5	21.2̂	21.0
.6	25.5	25.2
.7	29.7̂	29.4
.8	34.0	33.6
.9	38.2̂	37.8

	41̂	41
.1	4.1̂	4.1
.2	8.3	8.2
.3	12.4̂	12.3
.4	16.6	16.4
.5	20.7̂	20.5
.6	24.9	24.6
.7	29.0̂	28.7
.8	33.2	32.8
.9	37.3̂	36.9

P. P.

N.	0	1	2	3	4	5	6	7	8	9
1050	021 189	230	272	313	354	396	437	478	520	561
51	602	644	685	726	768	809	850	892	933	974
52	022 015	057	098	139	181	222	263	304	346	387
53	428	469	511	552	593	634	676	717	758	799
54	840	882	923	964	*005	*046	*088	*129	*170	*211
55	023 252	293	335	376	417	458	499	540	581	623
56	664	705	746	787	828	869	910	951	993	*034
57	024 075	116	157	198	239	280	321	362	403	444
58	485	526	568	609	650	691	732	773	814	855
59	896	937	978	*019	*060	*101	*142	*183	*224	*265
1060	025 306	347	388	429	469	510	551	592	633	674
61	715	756	797	838	879	920	961	*002	*042	*083
62	026 124	165	206	247	288	329	370	410	451	492
63	533	574	615	656	696	737	778	819	860	901
64	941	982	*023	*064	*105	*145	*186	*227	*268	*309
65	027 349	390	431	472	512	553	594	635	675	716
66	757	798	838	879	920	961	*001	*042	*083	*123
67	028 164	205	246	286	327	368	408	449	490	530
68	571	612	652	693	734	774	815	856	896	937
69	977	*018	*059	*099	*140	*181	*221	*262	*302	*343
1070	029 384	424	465	505	546	586	627	668	708	749
71	789	830	870	911	951	992	*032	*073	*114	*154
72	030 195	235	276	316	357	397	438	478	519	559
73	599	640	680	721	761	802	842	883	923	964
74	031 004	044	085	125	166	206	247	287	327	368
75	408	449	489	529	570	610	651	691	731	772
76	812	852	893	933	973	*014	*054	*094	*135	*175
77	032 215	256	296	336	377	417	457	498	538	578
78	619	659	699	739	780	820	860	900	941	981
79	033 021	061	102	142	182	222	263	303	343	383
1080	424	464	504	544	584	625	665	705	745	785
81	825	866	906	946	986	*026	*066	*107	147	187
82	034 227	267	307	347	388	428	468	508	548	588
83	628	668	708	748	789	829	869	909	949	989
84	035 029	069	109	149	189	229	269	309	349	389
85	429	470	510	550	590	630	670	710	750	790
86	830	870	910	950	990	*029	*069	*109	*149	*189
87	036 229	269	309	349	389	429	469	509	549	589
88	629	669	708	748	788	828	868	908	948	988
89	037 028	068	107	147	187	227	267	307	347	386
1090	426	466	506	546	586	625	665	705	745	785
91	825	864	904	944	984	*023	*063	*103	143	183
92	038 222	262	302	342	381	421	461	501	540	580
93	620	660	699	739	779	819	858	898	938	977
94	039 017	057	096	136	176	216	255	295	335	374
95	414	454	493	533	572	612	652	691	731	771
96	810	850	890	929	969	*008	*048	*088	*127	*167
97	040 206	246	286	325	365	404	444	483	523	563
98	602	642	681	721	760	800	839	879	918	958
99	997	*037	*076	*116	*155	*195	*234	*274	*313	*353
1100	041 392	432	471	511	550	590	629	669	708	748
N.	0	1	2	3	4	5	6	7	8	9

P. P.

	41̂	41	40̂	40	39̂
.1	4.1	4.1	4.0	4.0	3.9
.2	8.3	8.2	8.1	8.0	7.9
.3	12.4	12.3	12.1	12.0	11.8
.4	16.6	16.4	16.2	16.0	15.8
.5	20.7	20.5	20.2	20.0	19.7
.6	24.9	24.6	24.3	24.0	23.7
.7	29.0	28.7	28.3	28.0	27.6
.8	33.2	32.8	32.4	32.0	31.6
.9	37.3	36.9	36.4	36.0	35.5

$\log \sin \phi = \log \phi'' + S.$ $\log \tan \phi = \log \phi'' + T.$

0°

$\log \phi'' = \log \sin \phi + S'.$ $\log \phi'' = \log \tan \phi + T'.$

″	′	S	T	Log. Sin.	S′	T′	Log. Tan.
0	**0**	4.685 57	57	— ∞	5.314 42	42	— ∞
60	1	57	57	6.46 372	42	42	6.46 372
120	2	57	57	.76 475	42	42	.76 475
180	3	57	57	.94 084	42	42	.94 084
240	4	57	57	7.06 578	42	42	7.06 578
300	5	4.685 57	57	7.16 269	5.314 42	42	7.16 269
360	6	57	57	.24 187	42	42	.24 188
420	7	57	57	.30 882	42	42	.30 882
480	8	57	57	.36 681	42	42	.36 681
540	9	57	57	.41 797	42	42	.41 797
600	**10**	4.685 57	57	7.46 372	5.314 42	42	7.46 372
660	11	57	57	.50 512	42	42	.50 512
720	12	57	57	.54 290	42	42	.54 291
780	13	57	57	.57 767	42	42	.57 767
840	14	57	57	.60 985	42	42	.60 985
900	15	4.685 57	58	7.63 981	5.314 42	42	7.63 982
960	16	57	58	.66 784	42	42	.66 785
1020	17	57	58	.69 417	42	42	.69 418
1080	18	57	58	.71 899	42	42	.71 900
1140	19	57	58	.74 248	42	42	.74 248
1200	**20**	4.685 57	58	7.76 475	5.314 43	42	7.76 476
1260	21	57	58	.78 594	43	42	.78 595
1320	22	57	58	.80 614	43	42	.80 615
1380	23	57	58	.82 545	43	42	.82 546
1440	24	57	58	.84 393	43	42	.84 394
1500	25	4.685 57	58	7.86 166	5.314 43	41	7.86 167
1560	26	57	58	.87 869	43	41	.87 871
1620	27	57	58	.89 508	43	41	.89 510
1680	28	57	58	.91 088	43	41	.91 089
1740	29	57	58	.92 612	43	41	.92 613
1800	**30**	4.685 57	58	7.94 084	5.314 43	41	7.94 086
1860	31	57	58	.95 508	43	41	.95 510
1920	32	57	58	.96 887	43	41	.96 889
1980	33	57	59	.98 223	43	41	.98 225
2040	34	57	59	.99 520	43	41	.99 522
2100	35	4.685 56	59	8.00 778	5.314 43	41	8.00 781
2160	36	56	59	.02 002	43	41	.02 004
2220	37	56	59	.03 192	43	41	.03 194
2280	38	56	59	.04 350	43	40	.04 352
2340	39	56	59	.05 478	43	40	.05 481
2400	**40**	4.685 56	59	8.06 577	5.314 43	40	8.06 580
2460	41	56	59	.07 650	43	40	.07 653
2520	42	56	59	.08 696	43	40	.08 699
2580	43	56	60	.09 718	43	40	.09 721
2640	44	56	60	.10 716	43	40	.10 720
2700	45	4.685 56	60	8.11 692	5.314 44	40	8.11 696
2760	46	56	60	.12 647	44	40	.12 651
2820	47	56	60	.13 581	44	40	.13 585
2880	48	56	60	.14 495	44	39	.14 499
2940	49	56	60	.15 390	44	39	.15 395
3000	**50**	4.685 56	60	8.16 268	5.314 44	39	8.16 272
3060	51	56	60	.17 128	44	39	.17 133
3120	52	56	61	.17 971	44	39	.17 976
3180	53	56	61	.18 798	44	39	.18 803
3240	54	55	61	.19 610	44	39	.19 615
3300	55	4.685 55	61	8.20 407	5.314 44	39	8.20 412
3360	56	55	61	.21 189	44	38	.21 195
3420	57	55	61	.21 958	44	38	.21 964
3480	58	55	61	.22 713	44	38	.22 719
3540	59	55	62	.23 455	44	38	.23 462

log sin $\phi = \log \phi'' + S$.
log tan $\phi = \log \phi'' + T$.

1°

log $\phi'' = $ log sin $\phi + S'$.
log $\phi'' = $ log tan $\phi + T'$.

″	′	S	T	Log. Sin.	S′	T′	Log. Tan.
3600	**0**	4.685 55̂	62	8.24 185̂	5.314 44̂	38	8.24 192
3660	1	55	62	.24 903̂	45	38	.24 910
3720	2	55	62	.25 609̂	45	38	.25 616̂
3780	3	55	62̂	.26 304	45	37̂	.26 311̂
3840	4	55	62̂	.26 988	45	37̂	.26 995̂
3900	5	4.685 55	62̂	8.27 661̂	5.314 45	37̂	8.27 669
3960	6	55	63	.28 324̂	45	37	.28 332̂
4020	7	54̂	63	.28 977̂	45̂	37	.28 985̂
4080	8	54̂	63	.29 620̂	45̂	37	.29 629
4140	9	54̂	63̂	.30 254̂	45̂	36̂	.30 263̂
4200	**10**	4.685 54̂	63̂	8.30 879̂	5.314 45̂	36̂	8.30 888̂
4260	11	54̂	63̂	.31 495̂	45̂	36̂	.31 504̂
4320	12	54̂	64	.32 102̂	45̂	36̂	.32 112
4380	13	54	64	.32 701̂	46	36	.32 711̂
4440	14	54	64	.33 292̂	46	36	.33 302̂
4500	15	4.685 54	64̂	8.33 875̂	5.314 46	35̂	8.33 885̂
4560	16	54	64̂	.34 450̂	46	35̂	.34 461
4620	17	54	65	.35 018	46	35	.35 029
4680	18	54	65	.35 578̂	46	35	.35 589̂
4740	19	53̂	65	.36 131̂	46̂	35	.36 143
4800	**20**	4.685 53̂	65̂	8.36 677̂	5.314 46̂	34̂	8.36 689̂
4860	21	53̂	65̂	.37 217	46̂	34̂	.37 229
4920	22	53̂	65̂	.37 750	46̂	34̂	.37 762
4980	23	53̂	66	.38 276	46̂	34	.38 289
5040	24	53	66	.38 796	47	34	.38 809
5100	25	4.685 53	66̂	8.39 310	5.314 47	33̂	8.39 323̂
5160	26	53	66̂	.39 818	47	33̂	.39 831̂
5220	27	53	67	.40 320	47	33	.40 334
5280	28	52̂	67	.40 816	47̂	33	.40 830̂
5340	29	52̂	67	.41 307	47̂	33	.41 321̂
5400	**30**	4.685 52̂	67̂	8.41 792	5.314 47̂	32̂	8.41 807
5460	31	52̂	67̂	.42 271̂	47̂	32̂	.42 287
5520	32	52̂	68	.42 746	47̂	32	.42 762
5580	33	52	68	.43 215̂	48	32	.43 231̂
5640	34	52	68̂	.43 680	48	31̂	.43 696
5700	35	4.685 52	68̂	8.44 139̂	5.314 48	31̂	8.44 156
5760	36	52	69	.44 594	48	31	.44 611
5820	37	51̂	69	.45 044	48̂	31	.45 061̂
5880	38	51̂	69̂	.45 489̂	48̂	30̂	.45 507
5940	39	51̂	69̂	.45 930	48̂	30̂	.45 948
6000	**40**	4.685 51̂	69̂	8.46 366̂	5.314 48̂	30̂	8.46 385
6060	41	51	70	.46 798̂	49	30	.46 817
6120	42	51	70	.47 226̂	49	30	.47 245̂
6180	43	51	70̂	.47 650	49	29̂	.47 669̂
6240	44	51	70̂	.48 069̂	49	29̂	.48 089
6300	45	4.685 50̂	71	8.48 485	5.314 49̂	29	8.48 505
6360	46	50̂	71̂	.48 896̂	49̂	28̂	.48 917
6420	47	50̂	71̂	.49 304	49̂	28̂	.49 325
6480	48	50̂	72	.49 708	49̂	28	.49 729̂
6540	49	50	72	.50 108	50	28	.50 130
6600	**50**	4.685 50	72̂	8.50 504̂	5.314 50	27̂	8.50 526̂
6660	51	50	72̂	.50 897̂	50	27̂	.50 920
6720	52	50	73	.51 286̂	50	27	.51 310
6780	53	49̂	73	.51 672̂	50̂	27	.51 696
6840	54	49̂	73̂	.52 055	50̂	26̂	.52 079
6900	55	4.685 49̂	73̂	8.52 434̂	5.314 50̂	26̂	8.52 458̂
6960	56	49	74	.52 810	51	26	.52 835
7020	57	49	74̂	.53 183	51	25̂	.53 208
7080	58	49	74̂	.53 552̂	51	25̂	.53 578
7140	59	49̂	75	.53 918̂	51	25	.53 944̂

$\log \sin \phi = \log \phi'' + S.$ $\log \tan \phi = \log \phi'' + T.$

2°

$\log \phi'' = \log \sin \phi + S'.$ $\log \phi'' = \log \tan \phi + T'.$

″	′	S	T	Log. Sin.	S′	T′	Log. Tan.
7200	**0**	4.685 48̂	75	8.54 282	5.314 51̂	25	8.54 308̂
7260	1	48̂	75̂	.54 642	51̂	24̂	.54 669
7320	2	48̂	75̂	.54 999̂	51̂	24̂	.55 027
7380	3	48	76	.55 354	52	24	.55 381̂
7440	4	48	76̂	.55 705̂	52	23̂	.55 733̂
7500	5	4.685 48	76̂	8.56 054	5.314 52	23̂	8.56 083
7560	6	48	77	.56 400	52	23	.56 429
7620	7	47̂	77̂	.56 743	52̂	22̂	.56 772̂
7680	8	47̂	77̂	.57 083̂	52̂	22̂	.57 113̂
7740	9	47̂	78	.57 421̂	52̂	22	.57 452
7800	**10**	4.685 47	78	8.57 756̂	5.314 53	22	8.57 787̂
7860	11	47	78̂	.58 089	53	21̂	.58 121
7920	12	47	79	.58 419̂	53	21	.58 451̂
7980	13	46̂	79	.58 747	53̂	21	.58 779̂
8040	14	46̂	79̂	.59 072	53̂	20̂	.59 105
8100	15	4.685 46̂	80	8.59 395	5.314 53̂	20	8.59 428̂
8160	16	46	80	.59 715	54	20	.59 749
8220	17	46	80̂	.60 033	54	19̂	.60 067̂
8280	18	46	81	.60 349	54	19	.60 384
8340	19	45̂	81	.60 662̂	54̂	19	.60 698
8400	**20**	4.685 45̂	81̂	8.60 973̂	5.314 54̂	18̂	8.61 009̂
8460	21	45̂	82	.61 282̂	54̂	18	.61 319
8520	22	45	82	.61 589	55	18	.61 626
8580	23	45	82̂	.61 893̂	55	17̂	.61 931̂
8640	24	45	83	.62 196	55	17	.62 234̂
8700	25	4.685 44̂	83̂	8.62 496̂	5.314 55̂	16̂	8.62 535
8760	26	44̂	83̂	.62 795	55̂	16̂	.62 834
8820	27	44̂	84	.63 091	55̂	16	.63 131
8880	28	44	84̂	.63 385̂	56	15̂	.63 425̂
8940	29	44	84̂	.63 677̂	56	15̂	.63 718̂
9000	**30**	4.685 43̂	85	8.63 968	5.314 56̂	15	8.64 009̂
9060	31	43̂	85̂	.64 256̂	56̂	14̂	.64 298
9120	32	43̂	86	.64 543	56̂	14	.64 585̂
9180	33	43	86	.64 827̂	57	14	.64 870̂
9240	34	43	86̂	.65 110	57	13̂	.65 153̂
9300	35	4.685 43	87	8.65 391	5.314 57	13	8.65 435
9360	36	42̂	87̂	.65 670	57̂	12̂	.65 715
9420	37	42̂	87̂	.65 947̂	57̂	12̂	.65 993
9480	38	42	88	.66 223	58	12	.66 269
9540	39	42	88̂	.66 497	58	11̂	.66 543̂
9600	**40**	4.685 42	89	8.66 769	5.314 58	11	8.66 816
9660	41	41̂	89̂	.67 039̂	58̂	10̂	.67 087
9720	42	41̂	89̂	.67 308	58̂	10̂	.67 356̂
9780	43	41	90	.67 575	59	10	.67 624
9840	44	41	90̂	.67 840̂	59	09̂	.67 890
9900	45	4.685 41	91	8.68 104̂	5.314 59	09	8.68 154̂
9960	46	40̂	91̂	.68 366̂	59̂	08̂	.68 417
10020	47	40̂	91̂	.68 627	59̂	08̂	.68 678̂
10080	48	40	92	.68 886̂	60	08	.68 938
10140	49	40	92̂	.69 144	60	07̂	.69 196̂
10200	**50**	4.685 40	93	8.69 400	5.314 60	07	8.69 453
10260	51	39̂	93̂	.69 654̂	60̂	06̂	.69 708
10320	52	39̂	93̂	.69 907̂	60̂	06̂	.69 961̂
10380	53	39	94	.70 159	61	06	.70 214
10440	54	39	94̂	.70 409	61	05̂	.70 464̂
10500	55	4.685 38̂	95	8.70 657̂	5.314 61̂	05	8.70 714
10560	56	38̂	95̂	.70 905	61̂	04̂	.70 962
10620	57	38̂	96	.71 150̂	61̂	04	.71 208̂
10680	58	38	96̂	.71 395	62	03̂	.71 453̂
10740	59	38	97	.71 638̂	62	03	.71 697

0°

′	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
0	— ∞		— ∞		+ ∞	0.00 000	60
1	6.46 372	30103	6.46 372	30103	3.53 627	0.00 000	59
2	6.76 475	17609	6.76 475	17609	3.23 524	0.00 000	58
3	6.94 084	12494	6.94 084	12494	3.05 915	0.00 000	57
4	7.06 578	9691	7.06 578	9691	2.93 421	0.00 000	56
5	7.16 269	7918	7.16 269	7918	2.83 730	0.00 000	55
6	7.24 187	6695	7.24 188	6694	2.75 812	0.00 000	54
7	7.30 882	5799	7.30 882	5799	2.69 117	0.00 000	53
8	7.36 681	5115	7.36 681	5115	2.63 318	0.00 000	52
9	7.41 797	4575	7.41 797	4575	2.58 203	0.00 000	51
10	7.46 372	4139	7.46 372	4139	2.53 627	0.00 000	50
11	7.50 512	3778	7.50 512	3779	2.49 488	0.00 000	49
12	7.54 290	3476	7.54 291	3476	2.45 709	9.99 999	48
13	7.57 767	3218	7.57 767	3218	2.42 233	9.99 999	47
14	7.60 985	2996	7.60 985	2996	2.39 014	9.99 999	46
15	7.63 981	2803	7.63 982	2803	2.36 018	9.99 999	45
16	7.66 784	2633	7.66 785	2633	2.33 215	9.99 999	44
17	7.69 417	2482	7.69 418	2482	2.30 582	9.99 999	43
18	7.71 899	2348	7.71 900	2348	2.28 099	9.99 999	42
19	7.74 248	2227	7.74 248	2227	2.25 751	9.99 999	41
20	7.76 475	2119	7.76 476	2119	2.23 524	9.99 999	40
21	7.78 594	2020	7.78 595	2020	2.21 405	9.99 999	39
22	7.80 614	1930	7.80 615	1930	2.19 384	9.99 999	38
23	7.82 545	1848	7.82 546	1848	2.17 454	9.99 999	37
24	7.84 393	1772	7.84 394	1773	2.15 605	9.99 999	36
25	7.86 166	1703	7.86 167	1703	2.13 832	9.99 999	35
26	7.87 869	1639	7.87 871	1639	2.12 129	9.99 999	34
27	7.89 508	1579	7.89 510	1579	2.10 490	9.99 998	33
28	7.91 088	1524	7.91 089	1524	2.08 910	9.99 998	32
29	7.92 612	1472	7.92 613	1472	2.07 386	9.99 998	31
30	7.94 084	1424	7.94 086	1424	2.05 914	9.99 998	30
31	7.95 508	1379	7.95 510	1379	2.04 490	9.99 998	29
32	7.96 887	1336	7.96 889	1336	2.03 111	9.99 998	28
33	7.98 223	1296	7.98 225	1296	2.01 774	9.99 998	27
34	7.99 520	1258	7.99 522	1259	2.00 478	9.99 998	26
35	8.00 778	1223	8.00 781	1223	1.99 219	9.99 997	25
36	8.02 002	1190	8.02 004	1190	1.97 995	9.99 997	24
37	8.03 192	1158	8.03 194	1158	1.96 805	9.99 997	23
38	8.04 350	1128	8.04 352	1128	1.95 647	9.99 997	22
39	8.05 478	1099	8.05 481	1099	1.94 519	9.99 997	21
40	8.06 577	1072	8.06 580	1072	1.93 419	9.99 997	20
41	8.07 650	1046	8.07 653	1046	1.92 347	9.99 997	19
42	8.08 696	1022	8.08 699	1022	1.91 300	9.99 997	18
43	8.09 718	998	8.09 721	999	1.90 278	9.99 996	17
44	8.10 716	976	8.10 720	976	1.89 279	9.99 996	16
45	8.11 692	954	8.11 696	954	1.88 303	9.99 996	15
46	8.12 647	934	8.12 651	934	1.87 349	9.99 996	14
47	8.13 581	914	8.13 585	914	1.86 415	9.99 996	13
48	8.14 495	895	8.14 499	895	1.85 500	9.99 996	12
49	8.15 390	877	8.15 395	877	1.84 605	9.99 995	11
50	8.16 268	860	8.16 272	860	1.83 727	9.99 995	10
51	8.17 128	843	8.17 133	843	1.82 867	9.99 995	9
52	8.17 971	827	8.17 976	827	1.82 023	9.99 995	8
53	8.18 798	811	8.18 803	812	1.81 196	9.99 995	7
54	8.19 610	797	8.19 615	797	1.80 384	9.99 994	6
55	8.20 407	782	8.20 412	783	1.79 587	9.99 994	5
56	8.21 189	768	8.21 195	768	1.78 804	9.99 994	4
57	8.21 958	755	8.21 964	755	1.78 036	9.99 994	3
58	8.22 713	742	8.22 719	742	1.77 280	9.99 994	2
59	8.23 455	730	8.23 462	730	1.76 538	9.99 993	1
60	8.24 185		8.24 192		1.75 808	9.99 993	0
	Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

89°

1°

′	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
0	8.24 185	718	8.24 192	718	1.75 808	9.99 993	**60**
1	8.24 903	706	8.24 910	706	1.75 090	9.99 993	59
2	8.25 609	694	8.25 616	695	1.74 383	9.99 993	58
3	8.26 304	684	8.26 311	684	1.73 688	9.99 992	57
4	8.26 988	673	8.26 995	673	1.73 004	9.99 992	56
5	8.27 661	663	8.27 669	663	1.72 331	9.99 992	55
6	8.28 324	653	8.28 332	653	1.71 667	9.99 992	54
7	8.28 977	643	8.28 985	643	1.71 014	9.99 992	53
8	8.29 620	634	8.29 629	634	1.70 371	9.99 991	52
9	8.30 254	625	8.30 263	625	1.69 736	9.99 991	51
10	8.30 879	616	8.30 888	616	1.69 111	9.99 991	**50**
11	8.31 495	607	8.31 504	607	1.68 495	9.99 990	49
12	8.32 102	599	8.32 112	599	1.67 888	9.99 990	48
13	8.32 701	591	8.32 711	591	1.67 288	9.99 990	47
14	8.33 292	583	8.33 302	583	1.66 697	9.99 990	46
15	8.33 875	575	8.33 885	575	1.66 114	9.99 989	45
16	8.34 450	567	8.34 461	568	1.65 539	9.99 989	44
17	8.35 018	560	8.35 029	560	1.64 971	9.99 989	43
18	8.35 578	553	8.35 589	553	1.64 410	9.99 989	42
19	8.36 131	546	8.36 143	546	1.63 857	9.99 988	41
20	8.36 677	539	8.36 689	539	1.63 310	9.99 988	**40**
21	8.37 217	533	8.37 229	533	1.62 771	9.99 988	39
22	8.37 750	526	8.37 762	527	1.62 238	9.99 987	38
23	8.38 276	520	8.38 289	520	1.61 711	9.99 987	37
24	8.38 796	514	8.38 809	514	1.61 191	9.99 987	36
25	8.39 310	508	8.39 323	508	1.60 676	9.99 986	35
26	8.39 818	502	8.39 831	502	1.60 168	9.99 986	34
27	8.40 320	496	8.40 334	496	1.59 666	9.99 986	33
28	8.40 816	491	8.40 830	491	1.59 169	9.99 986	32
29	8.41 307	485	8.41 321	485	1.58 678	9.99 985	31
30	8.41 792	479	8.41 807	480	1.58 193	9.99 985	**30**
31	8.42 271	474	8.42 287	475	1.57 713	9.99 985	29
32	8.42 746	469	8.42 762	469	1.57 238	9.99 984	28
33	8.43 215	464	8.43 231	464	1.56 768	9.99 984	27
34	8.43 680	459	8.43 696	460	1.56 304	9.99 984	26
35	8.44 139	454	8.44 156	455	1.55 844	9.99 983	25
36	8.44 594	450	8.44 611	450	1.55 389	9.99 983	24
37	8.45 044	445	8.45 061	445	1.54 938	9.99 982	23
38	8.45 489	440	8.45 507	441	1.54 493	9.99 982	22
39	8.45 930	436	8.45 948	437	1.54 052	9.99 982	21
40	8.46 366	432	8.46 385	432	1.53 615	9.99 981	**20**
41	8.46 798	428	8.46 817	428	1.53 183	9.99 981	19
42	8.47 226	423	8.47 245	424	1.52 754	9.99 981	18
43	8.47 650	419	8.47 669	419	1.52 330	9.99 980	17
44	8.48 069	415	8.48 089	416	1.51 911	9.99 980	16
45	8.48 485	411	8.48 505	412	1.51 495	9.99 979	15
46	8.48 896	407	8.48 917	408	1.51 083	9.99 979	14
47	8.49 304	404	8.49 325	404	1.50 675	9.99 979	13
48	8.49 708	400	8.49 729	400	1.50 270	9.99 978	12
49	8.50 108	396	8.50 130	396	1.49 870	9.99 978	11
50	8.50 504	393	8.50 526	393	1.49 473	9.99 978	**10**
51	8.50 897	389	8.50 920	390	1.49 080	9.99 977	9
52	8.51 286	386	8.51 310	386	1.48 690	9.99 977	8
53	8.51 672	382	8.51 696	383	1.48 304	9.99 976	7
54	8.52 055	379	8.52 079	379	1.47 921	9.99 976	6
55	8.52 434	375	8.52 458	376	1.47 541	9.99 975	5
56	8.52 810	373	8.52 835	373	1.47 165	9.99 975	4
57	8.53 183	369	8.53 208	370	1.46 792	9.99 975	3
58	8.53 552	366	8.53 578	366	1.46 422	9.99 974	2
59	8.53 918	363	8.53 944	364	1.46 055	9.99 974	1
60	8.54 282		8.54 308		1.45 691	9.99 973	**0**
	Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

88°

2°

′	Log. Sin.	D	Log. Tan.	Com. D.	Log. Cot.	Log. Cos.	
0	8.54 282	360	8.54 308	360	1.45 691	9.99 973	60
1	8.54 642	357	8.54 669	358	1.45 331	9.99 973	59
2	8.54 999	354	8.55 027	354	1.44 973	9.99 972	58
3	8.55 354	351	8.55 381	352	1.44 618	9.99 972	57
4	8.55 705	348	8.55 733	349	1.44 266	9.99 971	56
5	8.56 054	346	8.56 083	346	1.43 917	9.99 971	55
6	8.56 400	343	8.56 429	343	1.43 571	9.99 971	54
7	8.56 743	340	8.56 772	341	1.43 227	9.99 970	53
8	8.57 083	338	8.57 113	338	1.42 886	9.99 970	52
9	8.57 421	335	8.57 452	335	1.42 548	9.99 969	51
10	8.57 756	332	8.57 787	333	1.42 212	9.99 969	50
11	8.58 089	330	8.58 121	330	1.41 879	9.99 968	49
12	8.58 419	327	8.58 451	328	1.41 548	9.99 968	48
13	8.58 747	325	8.58 779	325	1.41 220	9.99 967	47
14	8.59 072	323	8.59 105	323	1.40 895	9.99 967	46
15	8.59 395	320	8.59 428	320	1.40 571	9.99 966	45
16	8.59 715	318	8.59 749	318	1.40 251	9.99 966	44
17	8.60 033	316	8.60 067	316	1.39 932	9.99 965	43
18	8.60 349	313	8.60 384	314	1.39 616	9.99 965	42
19	8.60 662	311	8.60 698	311	1.39 302	9.99 964	41
20	8.60 973	309	8.61 009	309	1.38 990	9.99 964	40
21	8.61 282	306	8.61 319	307	1.38 681	9.99 963	39
22	8.61 589	304	8.61 626	305	1.38 374	9.99 963	38
23	8.61 893	302	8.61 931	303	1.38 068	9.99 962	37
24	8.62 196	300	8.62 234	300	1.37 765	9.99 962	36
25	8.62 496	298	8.62 535	299	1.37 465	9.99 961	35
26	8.62 795	296	8.62 834	297	1.37 166	9.99 961	34
27	8.63 091	294	8.63 131	294	1.36 869	9.99 960	33
28	8.63 385	292	8.63 425	293	1.36 574	9.99 959	32
29	8.63 677	290	8.63 718	291	1.36 281	9.99 959	31
30	8.63 968	288	8.64 009	288	1.35 990	9.99 958	30
31	8.64 256	286	8.64 298	287	1.35 702	9.99 958	29
32	8.64 543	284	8.64 585	285	1.35 414	9.99 957	28
33	8.64 827	282	8.64 870	283	1.35 129	9.99 957	27
34	8.65 110	281	8.65 153	281	1.34 846	9.99 956	26
35	8.65 391	279	8.65 435	280	1.34 565	9.99 956	25
36	8.65 670	277	8.65 715	278	1.34 285	9.99 955	24
37	8.65 947	275	8.65 993	276	1.34 007	9.99 954	23
38	8.66 223	274	8.66 269	274	1.33 731	9.99 954	22
39	8.66 497	272	8.66 543	272	1.33 456	9.99 953	21
40	8.66 769	270	8.66 816	271	1.33 184	9.99 953	20
41	8.67 039	268	8.67 087	269	1.32 913	9.99 952	19
42	8.67 308	267	8.67 356	267	1.32 643	9.99 952	18
43	8.67 575	265	8.67 624	266	1.32 376	9.99 951	17
44	8.67 840	264	8.67 890	264	1.32 110	9.99 950	16
45	8.68 104	262	8.68 154	262	1.31 845	9.99 950	15
46	8.68 366	260	8.68 417	261	1.31 583	9.99 949	14
47	8.68 627	259	8.68 678	259	1.31 321	9.99 948	13
48	8.68 886	257	8.68 938	258	1.31 062	9.99 948	12
49	8.69 144	256	8.69 196	256	1.30 803	9.99 947	11
50	8.69 400	254	8.69 453	255	1.30 547	9.99 947	10
51	8.69 654	253	8.69 708	253	1.30 292	9.99 946	9
52	8.69 907	251	8.69 961	252	1.30 038	9.99 945	8
53	8.70 159	250	8.70 214	250	1.29 786	9.99 945	7
54	8.70 409	248	8.70 464	249	1.29 535	9.99 944	6
55	8.70 657	247	8.70 714	248	1.29 286	9.99 943	5
56	8.70 905	245	8.70 962	246	1.29 038	9.99 943	4
57	8.71 150	244	8.71 208	245	1.28 791	9.99 942	3
58	8.71 395	243	8.71 453	243	1.28 546	9.99 942	2
57	8.71 638	241	8.71 697	242	1.28 303	9.99 941	1
60	8.71 880		8.71 939		1.28 060	9.99 940	0
	Log. Cos.	D	Log. Cot.	Com. D.	Log. Tan.	Log. Sin.	′

87°

3°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	′
0	8.71 880	240	8.71 939	241	1.28 060	9.99 940	60
1	8.72 120	239	8.72 180	240	1.27 819	9.99 940	59
2	8.72 359	237	8.72 420	238	1.27 579	9.99 939	58
3	8.72 597	236	8.72 659	237	1.27 341	9.99 938	57
4	8.72 833	235	8.72 896	235	1.27 104	9.99 938	56
5	8.73 069	233	8.73 131	235	1.26 868	9.99 937	55
6	8.73 302	233	8.73 366	233	1.26 633	9.99 936	54
7	8.73 535	231	8.73 599	232	1.26 400	9.99 935	53
8	8.73 766	230	8.73 831	231	1.26 168	9.99 935	52
9	8.73 997	229	8.74 062	229	1.25 937	9.99 934	51
10	8.74 226	227	8.74 292	228	1.25 708	9.99 933	50
11	8.74 453	226	8.74 520	227	1.25 479	9.99 933	49
12	8.74 680	225	8.74 748	226	1.25 252	9.99 932	48
13	8.74 905	224	8.74 974	225	1.25 026	9.99 931	47
14	8.75 129	223	8.75 199	223	1.24 801	9.99 931	46
15	8.75 353	221	8.75 422	223	1.24 577	9.99 930	45
16	8.75 574	221	8.75 645	221	1.24 354	9.99 929	44
17	8.75 795	219	8.75 867	220	1.24 133	9.99 928	43
18	8.76 015	218	8.76 087	219	1.23 913	9.99 928	42
19	8.76 233	217	8.76 306	218	1.23 693	9.99 927	41
20	8.76 451	216	8.76 524	217	1.23 475	9.99 926	40
21	8.76 667	215	8.76 741	216	1.23 258	9.99 925	39
22	8.76 883	214	8.76 958	214	1.23 042	9.99 925	38
23	8.77 097	213	8.77 172	214	1.22 827	9.99 924	37
24	8.77 310	212	8.77 386	213	1.22 613	9.99 923	36
25	8.77 522	211	8.77 599	212	1.22 400	9.99 922	35
26	8.77 733	210	8.77 811	210	1.22 188	9.99 922	34
27	8.77 943	209	8.78 022	210	1.21 978	9.99 921	33
28	8.78 152	208	8.78 232	209	1.21 768	9.99 920	32
29	8.78 360	207	8.78 441	207	1.21 559	9.99 919	31
30	8.78 567	206	8.78 648	207	1.21 351	9.99 919	30
31	8.78 773	205	8.78 855	206	1.21 144	9.99 918	29
32	8.78 978	204	8.79 061	204	1.20 938	9.99 917	28
33	8.79 183	203	8.79 266	204	1.20 734	9.99 916	27
34	8.79 386	202	8.79 470	203	1.20 530	9.99 916	26
35	8.79 588	201	8.79 673	202	1.20 327	9.99 915	25
36	8.79 789	200	8.79 875	201	1.20 125	9.99 914	24
37	8.79 989	199	8.80 076	200	1.19 923	9.99 913	23
38	8.80 189	198	8.80 276	199	1.19 723	9.99 912	22
39	8.80 387	197	8.80 476	198	1.19 524	9.99 912	21
40	8.80 585	197	8.80 674	197	1.19 326	9.99 911	20
41	8.80 782	195	8.80 871	197	1.19 128	9.99 910	19
42	8.80 977	195	8.81 068	195	1.18 931	9.99 909	18
43	8.81 172	194	8.81 264	195	1.18 736	9.99 908	17
44	8.81 366	193	8.81 459	194	1.18 541	9.99 907	16
45	8.81 560	192	8.81 653	193	1.18 347	9.99 907	15
46	8.81 752	191	8.81 846	192	1.18 154	9.99 906	14
47	8.81 943	191	8.82 038	191	1.17 961	9.99 905	13
48	8.82 134	189	8.82 230	190	1.17 770	9.99 904	12
49	8.82 324	189	8.82 420	190	1.17 579	9.99 903	11
50	8.82 513	188	8.82 610	188	1.17 389	9.99 902	10
51	8.82 701	187	8.82 799	188	1.17 201	9.99 902	9
52	8.82 888	186	8.82 987	187	1.17 012	9.99 901	8
53	8.83 075	185	8.83 175	186	1.16 825	9.99 900	7
54	8.83 260	185	8.83 361	185	1.16 638	9.99 899	6
55	8.83 445	184	8.83 547	185	1.16 453	9.99 898	5
56	8.83 629	183	8.83 732	184	1.16 268	9.99 897	4
57	8.83 813	182	8.83 916	183	1.16 083	9.99 896	3
58	8.83 995	182	8.84 100	182	1.15 900	9.99 896	2
59	8.84 177	181	8.84 282	182	1.15 717	9.99 895	1
60	8.84 358		8.84 464		1.15 535	9.99 894	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	330	320	310	300
6	33.0	32.0	31.0	30.0
7	38.5	37.3	36.1	35.0
8	44.0	42.6	41.3	40.0
9	49.5	48.0	46.5	45.0
10	55.0	53.3	51.6	50.0
20	110.0	106.6	103.3	100.0
30	165.0	160.0	155.0	150.0
40	220.0	213.3	206.6	200.0
50	275.0	266.6	258.3	250.0

	290	280	270	260
6	29.0	28.0	27.0	26.0
7	33.8	32.6	31.5	30.3
8	38.6	37.3	36.0	34.6
9	43.5	42.0	40.5	39.0
10	48.3	46.6	45.0	43.3
20	96.6	93.3	90.0	86.6
30	145.0	140.0	135.0	130.0
40	193.3	186.6	180.0	173.3
50	241.6	233.3	225.0	216.6

	250	240	230	220
6	25.0	24.0	23.0	22.0
7	29.1	28.0	26.8	25.6
8	33.3	32.0	30.6	29.3
9	37.5	36.0	34.5	33.0
10	41.6	40.0	38.3	36.6
20	83.3	80.0	76.6	73.3
30	125.0	120.0	115.0	110.0
40	166.6	160.0	153.3	146.6
50	208.3	200.0	191.6	183.3

	210	200	190	180
6	21.0	20.0	19.0	18.0
7	24.5	23.3	22.1	21.0
8	28.0	26.6	25.3	24.0
9	31.5	30.0	28.5	27.0
10	35.0	33.3	31.6	30.0
20	70.0	66.6	63.3	60.0
30	105.0	100.0	95.0	90.0
40	140.0	133.3	126.6	120.0
50	175.0	166.6	158.3	150.0

	9̂	9	8	7	6	5
6	0.9	0.9	0.8	0.7	0.6	0.5
7	1.1	1.0	0.9	0.8	0.7	0.6
8	1.2	1.2	1.0	0.9	0.8	0.6
9	1.4	1.3	1.2	1.0	0.9	0.7
10	1.6	1.5	1.3	1.1	1.0	0.8
20	3.1	3.0	2.6	2.3	2.0	1.6
30	4.7	4.5	4.0	3.5	3.0	2.5
40	6.3	6.0	5.3	4.6	4.0	3.3
50	7.9	7.5	6.6	5.8	5.0	4.1

	4̂	4	3	2	1	0̂
6	0.4	0.4	0.3	0.2	0.1	0.0
7	0.5	0.4	0.3	0.2	0.1	0.0
8	0.6	0.5	0.4	0.2	0.1	0.0
9	0.7	0.6	0.4	0.3	0.1	0.1
10	0.7	0.6	0.5	0.3	0.1	0.1
20	1.5	1.3	1.0	0.6	0.3	0.1
30	2.2	2.0	1.5	1.0	0.5	0.2
40	3.0	2.6	2.0	1.3	0.6	0.3
50	3.7	3.3	2.5	1.6	0.8	0.4

P. P.

86°

4°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	
0	8.84 358	180	8.84 464	181	1.15 535	9.99 894	60
1	8.84 538	180	8.84 645	180	1.15 354	9.99 893	59
2	8.84 718	178	8.84 826	179	1.15 174	9.99 892	58
3	8.84 897	178	8.85 005	179	1.14 994	9.99 891	57
4	8.85 075	177	8.85 184	178	1.14 815	9.99 890	56
5	8.85 252	176	8.85 363	177	1.14 637	9.99 889	55
6	8.85 429	176	8.85 540	176	1.14 459	9.99 888	54
7	8.85 605	175	8.85 717	176	1.14 283	9.99 888	53
8	8.85 780	174	8.85 893	175	1.14 107	9.99 887	52
9	8.85 954	174	8.86 068	175	1.13 931	9.99 886	51
10	8.86 128	173	8.86 243	174	1.13 756	9.99 885	50
11	8.86 301	172	8.86 417	173	1.13 582	9.99 884	49
12	8.86 474	171	8.86 590	172	1.13 409	9.99 883	48
13	8.86 645	171	8.86 763	172	1.13 237	9.99 882	47
14	8.86 816	170	8.86 935	171	1.13 065	9.99 881	46
15	8.86 987	169	8.87 106	170	1.12 893	9.99 880	45
16	8.87 156	169	8.87 277	170	1.12 723	9.99 879	44
17	8.87 325	168	8.87 447	169	1.12 553	9.99 878	43
18	8.87 494	167	8.87 616	169	1.12 384	9.99 877	42
19	8.87 661	167	8.87 785	168	1.12 215	9.99 876	41
20	8.87 828	166	8.87 953	167	1.12 047	9.99 875	40
21	8.87 995	165	8.88 120	167	1.11 880	9.99 874	39
22	8.88 160	165	8.88 287	166	1.11 713	9.99 874	38
23	8.88 326	164	8.88 453	165	1.11 547	9.99 873	37
24	8.88 490	163	8.88 618	165	1.11 381	9.99 872	36
25	8.88 654	163	8.88 783	164	1.11 216	9.99 871	35
26	8.88 817	162	8.88 947	163	1.11 052	9.99 870	34
27	8.88 980	162	8.89 111	163	1.10 889	9.99 869	33
28	8.89 142	161	8.89 274	162	1.10 726	9.99 868	32
29	8.89 303	161	8.89 436	162	1.10 563	9.99 867	31
30	8.89 464	160	8.89 598	161	1.10 401	9.99 866	30
31	8.89 624	159	8.89 759	161	1.10 240	9.99 865	29
32	8.89 784	159	8.89 920	160	1.10 079	9.99 864	28
33	8.89 943	158	8.90 080	159	1.09 919	9.99 863	27
34	8.90 101	158	8.90 240	158	1.09 760	9.99 862	26
35	8.90 259	157	8.90 398	158	1.09 601	9.99 861	25
36	8.90 417	156	8.90 557	157	1.09 443	9.99 860	24
37	8.90 573	156	8.90 714	157	1.09 285	9.99 859	23
38	8.90 729	156	8.90 872	156	1.09 128	9.99 858	22
39	8.90 885	155	8.91 028	156	1.08 971	9.99 857	21
40	8.91 040	154	8.91 184	155	1.08 815	9.99 856	20
41	8.91 195	154	8.91 340	155	1.08 660	9.99 855	19
42	8.91 349	153	8.91 495	154	1.08 505	9.99 853	18
43	8.91 502	153	8.91 649	154	1.08 350	9.99 852	17
44	8.91 655	152	8.91 803	153	1.08 196	9.99 851	16
45	8.91 807	151	8.91 957	152	1.08 043	9.99 850	15
46	8.91 959	151	8.92 109	152	1.07 890	9.99 849	14
47	8.92 110	150	8.92 262	151	1.07 738	9.99 848	13
48	8.92 261	150	8.92 413	151	1.07 586	9.99 847	12
49	8.92 411	150	8.92 565	150	1.07 435	9.99 846	11
50	8.92 561	149	8.92 715	150	1.07 284	9.99 845	10
51	8.92 710	148	8.92 866	149	1.07 134	9.99 844	9
52	8.92 858	148	8.93 015	149	1.06 984	9.99 843	8
53	8.93 007	147	8.93 164	149	1.06 835	9.99 842	7
54	8.93 154	147	8.93 313	148	1.06 686	9.99 841	6
55	8.93 301	146	8.93 461	148	1.06 538	9.99 840	5
56	8.93 448	146	8.93 609	147	1.06 390	9.99 839	4
57	8.93 594	146	8 93 756	146	1.06 243	9.99 837	3
58	8.93 740	145	8.93 903	146	1.06 097	9.99 836	2
59	8.93 885	144	8.94 049	145	1.05 950	9.99 835	1
60	8.94 029		8.94 195		1.05 805	9.99 834	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	181	180	178	176
6	18.1	18.0	17.8	17.6
7	21.1	21.0	20.7	20.5
8	24.1	24.0	23.7	23.4
9	27.1	27.0	26.7	26.4
10	30.1	30.0	29.6	29.3
20	60.3	60.0	59.3	58.6
30	90.5	90.0	89.0	88.0
40	120.6	120.0	118.6	117.3
50	150.8	150.0	148.3	146.6

	174	172	170	168
6	17.4	17.2	17.0	16.8
7	20.3	20.0	19.8	19.6
8	23.2	22.9	22.6	22.4
9	26.1	25.8	25.5	25.2
10	29.0	28.6	28.3	28.0
20	58.0	57.3	56.6	56.0
30	87.0	86.0	85.0	84.0
40	116.0	114.6	113.3	112.0
50	145.0	143.3	141.6	140.0

	166	164	162	160
6	16.6	16.4	16.2	16.0
7	19.3	19.1	18.9	18.6
8	22.1	21.8	21.6	21.3
9	24.9	24.6	24.3	24.0
10	27.6	27.3	27.0	26.6
20	55.3	54.6	54.0	53.3
30	83.0	82.0	81.0	80.0
40	110.6	109.3	108.0	106.6
50	138.3	136.6	135.0	133.3

	158	156	154	152
6	15.8	15.6	15.4	15.2
7	18.4	18.2	17.9	17.7
8	21.0	20.8	20.5	20.2
9	23.7	23.4	23.1	22.8
10	26.3	26.0	25.6	25.3
20	52.6	52.0	51.3	50.6
30	79.0	78.0	77.0	76.0
40	105.3	104.0	102.6	101.3
50	131.6	130.0	128.3	126.6

	150	149	148	147
6	15.0	14.9	14.8	14.7
7	17.5	17.4	17.2	17.1
8	20.0	19.8	19.7	19.6
9	22.5	22.3	22.2	22.0
10	25.0	24.8	24.6	24.5
20	50.0	49.6	49.3	49.0
30	75.0	74.5	74.0	73.5
40	100.0	99.3	98.6	98.0
50	125.0	124.1	123.3	122.5

	146	145	1	1	0
6	14.6	14.5	0.1	0.1	0.0
7	17.0	16.9	0.2	0.1	0.0
8	19.4	19.3	0.2	0.1	0.0
9	21.9	21.7	0.2	0.1	0.1
10	24.3	24.1	0.2	0.1	0.1
20	48.6	48.3	0.5	0.3	0.1
30	73.0	72.5	0.7	0.5	0.2
40	97.3	96.6	1.0	0.6	0.3
50	121.6	120.8	1.2	0.8	0.4

P. P.

85°

5°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	′
0	8.94 029	144	8.94 195	145	1.05 805	9.99 834	60
1	8.94 174	143	8.94 340	144	1.05 659	9.99 833	59
2	8.94 317	143	8.94 485	144	1.05 515	9.99 832	58
3	8.94 460	143	8.94 629	144	1.05 370	9.99 831	57
4	8.94 603	142	8.94 773	143	1.05 226	9.99 830	56
5	8.94 745	142	8.94 917	142	1.05 083	9.99 829	55
6	8.94 887	141	8.95 059	142	1.04 940	9.99 827	54
7	8.95 028	141	8.95 202	142	1.04 798	9.99 826	53
8	8.95 169	140	8.95 344	141	1.04 656	9.99 825	52
9	8.95 310	140	8.95 485	141	1.04 514	9.99 824	51
10	8.95 450	139	8.95 626	141	1.04 373	9.99 823	50
11	8.95 589	139	8.95 767	140	1.04 232	9.99 822	49
12	8.95 728	138	8.95 907	140	1.04 092	9.99 821	48
13	8.95 867	138	8.96 047	139	1.03 952	9.99 819	47
14	8.96 005	138	8.96 186	139	1.03 813	9.99 818	46
15	8.96 143	137	8.96 325	138	1.03 674	9.99 817	45
16	8.96 280	137	8.96 464	138	1.03 536	9.99 816	44
17	8.96 417	136	8.96 602	137	1.03 398	9.99 815	43
18	8.96 553	136	8.96 739	137	1.03 260	9.99 814	42
19	8.96 689	135	8.96 876	137	1.03 123	9.99 813	41
20	8.96 825	135	8.97 013	136	1.02 986	9.99 811	40
21	8.96 960	134	8.97 149	136	1.02 850	9.99 810	39
22	8.97 094	134	8.97 285	135	1.02 714	9.99 809	38
23	8.97 229	134	8.97 421	135	1.02 579	9.99 808	37
24	8.97 363	133	8.97 556	134	1.02 444	9.99 807	36
25	8.97 496	133	8.97 690	134	1.02 309	9.99 805	35
26	8.97 629	132	8.97 825	133	1.02 175	9.99 804	34
27	8.97 762	132	8.97 958	133	1.02 041	9.99 803	33
28	8.97 894	132	8.98 092	133	1.01 908	9.99 802	32
29	8.98 026	131	8.98 225	132	1.01 775	9.99 801	31
30	8.98 157	131	8.98 357	132	1.01 642	9.99 799	30
31	8.98 288	130	8.98 490	131	1.01 510	9.99 798	29
32	8.98 419	130	8.98 621	131	1.01 378	9.99 797	28
33	8.98 549	130	8.98 753	131	1.01 247	9.99 796	27
34	8.98 679	129	8.98 884	131	1.01 116	9.99 794	26
35	8.98 808	129	8.99 015	130	1.00 985	9.99 793	25
36	8.98 937	128	8.99 145	130	1.00 855	9.99 792	24
37	8.99 066	128	8.99 275	129	1.00 725	9.99 791	23
38	8.99 194	127	8.99 404	129	1.00 595	9.99 789	22
39	8.99 322	127	8.99 533	129	1.00 466	9.99 788	21
40	8.99 449	127	8.99 662	128	1.00 337	9.99 787	20
41	8.99 577	126	8.99 791	128	1.00 209	9.99 786	19
42	8.99 703	126	8.99 919	127	1.00 081	9.99 784	18
43	8.99 830	126	9.00 046	127	0.99 953	9.99 783	17
44	8.99 956	125	9.00 174	126	0.99 826	9.99 782	16
45	9.00 081	125	9.00 300	126	0.99 699	9.99 781	15
46	9.00 207	125	9.00 427	126	0.99 573	9.99 779	14
47	9.00 332	124	9.00 553	125	0.99 446	9.99 778	13
48	9.00 456	124	9.00 679	125	0.99 321	9.99 777	12
49	9.00 580	124	9.00 804	125	0.99 195	9.99 776	11
50	9.00 704	123	9.00 930	124	0.99 070	9.99 774	10
51	9.00 828	123	9.01 054	124	0.98 945	9.99 773	9
52	9.00 951	122	9.01 179	124	0.98 821	9.99 772	8
53	9.01 073	122	9.01 303	124	0.98 697	9.99 770	7
54	9.01 196	122	9.01 427	123	0.98 573	9.99 769	6
55	9.01 318	122	9.01 550	123	0.98 450	9.99 768	5
56	9.01 440	121	9.01 673	123	0.98 327	9.99 766	4
57	9.01 561	121	9.01 796	122	0.98 204	9.99 765	3
58	9.01 682	120	9.01 918	122	0.98 081	9.99 764	2
59	9.01 803	120	9.02 040	121	0.97 959	9.99 763	1
60	9.01 923		9.02 162		0.97 838	9.99 761	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	145	144	143	142	141
6	14.5	14.4	14.3	14.2	14.1
7	16.9	16.8	16.7	16.5	16.4
8	19.3	19.2	19.0	18.9	18.8
9	21.7	21.6	21.4	21.3	21.1
10	24.1	24.0	23.8	23.6	23.5
20	48.3	48.0	47.6	47.3	47.0
30	72.5	72.0	71.5	71.0	70.5
40	96.6	96.0	95.3	94.6	94.0
50	120.8	120.0	119.1	118.3	117.5

	140	139	138	137	136
6	14.0	13.9	13.8	13.7	13.6
7	16.3	16.2	16.1	16.0	15.8
8	18.6	18.5	18.4	18.2	18.1
9	21.0	20.8	20.7	20.5	20.4
10	23.3	23.1	23.0	22.8	22.6
20	46.6	46.3	46.0	45.6	45.3
30	70.0	69.5	69.0	68.5	68.0
40	93.3	92.6	92.0	91.3	90.6
50	116.6	115.8	115.0	114.1	113.3

	135	134	133	132
6	13.5	13.4	13.3	13.2
7	15.7	15.6	15.5	15.4
8	18.0	17.8	17.7	17.6
9	20.2	20.1	19.9	19.8
10	22.5	22.3	22.1	22.0
20	45.0	44.6	44.3	44.0
30	67.5	67.0	66.5	66.0
40	90.0	89.3	88.6	88.0
50	112.5	111.6	110.8	110.0

	131	130	129	128
6	13.1	13.0	12.9	12.8
7	15.3	15.1	15.0	14.9
8	17.4	17.3	17.2	17.0
9	19.6	19.5	19.3	19.2
10	21.8	21.6	21.5	21.3
20	43.6	43.3	43.0	42.6
30	65.5	65.0	64.5	64.0
40	87.3	86.6	86.0	85.3
50	109.1	108.3	107.5	106.6

	127	126	125	124	123
6	12.7	12.6	12.5	12.4	12.3
7	14.8	14.7	14.6	14.4	14.3
8	16.9	16.8	16.6	16.5	16.4
9	19.0	18.9	18.7	18.5	18.4
10	21.1	21.0	20.8	20.6	20.5
20	42.3	42.0	41.6	41.3	41.0
30	63.5	63.0	62.5	62.0	61.5
40	84.6	84.0	83.3	82.6	82.0
50	105.8	105.0	104.1	103.3	102.5

	122	121	120	1	1	0
6	12.2	12.1	12.0	0.1	0.1	0.0
7	14.2	14.1	14.0	0.2	0.1	0.0
8	16.2	16.1	16.0	0.2	0.1	0.0
9	18.3	18.1	18.0	0.2	0.1	0.1
10	20.3	20.1	20.0	0.2	0.1	0.1
20	40.6	40.3	40.0	0.5	0.3	0.1
30	61.0	60.5	60.0	0.7	0.5	0.2
40	81.3	80.6	80.0	1.0	0.6	0.3
50	101.6	100.8	100.0	1.2	0.8	0.4

P. P.

84°

6°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	
0	9.01 923	120	9.02 162	121	0.97 838	9.99 761	60
1	9.02 043	119	9.02 283	121	0.97 716	9.99 760	59
2	9.02 163	119	9.02 404	120	0.97 595	9.99 759	58
3	9.02 282	119	9.02 525	120	0.97 475	9.99 757	57
4	9.02 401	119	9.02 645	120	0.97 354	9.99 756	56
5	9.02 520	118	9.02 765	119	0.97 234	9.99 754	55
6	9.02 638	118	9.02 885	119	0.97 115	9.99 753	54
7	9.02 756	118	9.03 004	119	0.96 995	9.99 752	53
8	9.02 874	117	9.03 123	119	0.96 876	9.99 750	52
9	9.02 992	117	9.03 242	118	0.96 757	9.99 749	51
10	9.03 109	116	9.03 361	118	0.96 639	9.99 748	50
11	9.03 225	116	9.03 479	118	0.96 521	9.99 746	49
12	9.03 342	116	9.03 597	117	0.96 403	9.99 745	48
13	9.03 458	116	9.03 714	117	0.96 285	9.99 744	47
14	9.03 574	115	9.03 831	117	0.96 168	9.99 742	46
15	9.03 689	115	9.03 948	116	0.96 051	9.99 741	45
16	9.03 805	114	9.04 065	116	0.95 935	9.99 739	44
17	9.03 919	114	9.04 181	116	0.95 818	9.99 738	43
18	9.04 034	114	9.04 297	115	0.95 702	9.99 737	42
19	9.04 148	114	9.04 413	115	0.95 587	9.99 735	41
20	9.04 262	113	9.04 528	115	0.95 471	9.99 734	40
21	9.04 376	113	9.04 643	114	0.95 356	9.99 732	39
22	9.04 489	113	9.04 758	114	0.95 242	9.99 731	38
23	9.04 602	113	9.04 872	114	0.95 127	9.99 730	37
24	9.04 715	112	9.04 987	114	0.95 013	9.99 728	36
25	9.04 828	112	9.05 101	113	0.94 899	9.99 727	35
26	9.04 940	112	9.05 214	113	0.94 785	9.99 725	34
27	9.05 052	111	9.05 327	113	0.94 672	9.99 724	33
28	9.05 163	111	9.05 440	113	0.94 559	9.99 723	32
29	9.05 275	111	9.05 553	112	0.94 446	9.99 721	31
30	9.05 386	110	9.05 666	112	0.94 334	9.99 720	30
31	9.05 496	110	9.05 778	112	0.94 222	9.99 718	29
32	9.05 607	110	9.05 890	111	0.94 110	9.99 717	28
33	9.05 717	110	9.06 001	111	0.93 998	9.99 715	27
34	9.05 827	109	9.06 113	111	0.93 887	9.99 714	26
35	9.05 936	109	9.06 224	111	0.93 776	9.99 712	25
36	9.06 046	109	9.06 335	110	0.93 665	9.99 711	24
37	9.06 155	109	9.06 445	110	0.93 554	9.99 710	23
38	9.06 264	108	9.06 555	110	0.93 444	9.99 708	22
39	9.06 372	108	9.06 665	109	0.93 334	9.99 707	21
40	9.06 480	108	9.06 775	109	0.93 225	9.99 705	20
41	9.06 588	107	9.06 884	109	0.93 115	9.99 704	19
42	9.06 696	107	9.06 994	108	0.93 006	9.99 702	18
43	9.06 803	107	9.07 102	109	0.92 897	9.99 701	17
44	9.06 910	107	9.07 211	108	0.92 788	9.99 699	16
45	9.07 017	106	9.07 319	108	0.92 680	9.99 698	15
46	9.07 124	106	9.07 428	107	0.92 572	9.99 696	14
47	9.07 230	106	9.07 535	107	0.92 464	9.99 695	13
48	9.07 336	106	9.07 643	107	0.92 357	9.99 693	12
49	9.07 442	105	9.07 750	107	0.92 249	9.99 692	11
50	9.07 548	105	9.07 857	107	0.92 142	9.99 690	10
51	9.07 653	105	9.07 964	106	0.92 035	9.99 689	9
52	9.07 758	104	9.08 071	106	0.91 929	9.99 687	8
53	9.07 863	104	9.08 177	106	0.91 822	9.99 686	7
54	9.07 967	104	9.08 283	105	0.91 716	9.99 684	6
55	9.08 072	104	9.08 389	105	0.91 611	9.99 683	5
56	9.08 176	103	9.08 494	105	0.91 505	9.99 681	4
57	9.08 279	103	9.08 600	105	0.91 400	9.99 679	3
58	9.08 383	103	9.08 705	105	0.91 295	9.99 678	2
59	9.08 486	103	9.08 810	104	0.91 190	9.99 676	1
60	9.08 589		9.08 914		0.91 085	9.99 675	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	121̂	121	120	119	118
6	12.1	12.1	12.0	11.9	11.8
7	14.2	14.1	14.0	13.9	13.7
8	16.2	16.1	16.0	15.8	15.7
9	18.2	18.1	18.0	17.8	17.7
10	20.2	20.1	20.0	19.8	19.6
20	40.5	40.3	40.0	39.6	39.3
30	60.7	60.5	60.0	59.5	59.0
40	81.0	80.6	80.0	79.3	78.6
50	101.2	100.8	100.0	99.1	98.3

	117̂	117	116	115
6	11.7	11.7	11.6	11.5
7	13.7	13.6	13.5	13.4
8	15.6	15.6	15.4	15.3
9	17.6	17.5	17.4	17.2
10	19.6	19.5	19.3	19.1
20	39.1	39.0	38.6	38.3
30	58.7	58.5	58.0	57.5
40	78.3	78.0	77.3	76.6
50	97.9	97.5	96.6	95.8

	114̂	114	113	112	111
6	11.4	11.4	11.3	11.2	11.1
7	13.3	13.3	13.2	13.0	12.9
8	15.2	15.2	15.0	14.9	14.8
9	17.2	17.1	16.9	16.8	16.6
10	19.1	19.0	18.8	18.6	18.5
20	38.1	38.0	37.6	37.3	37.0
30	57.2	57.0	56.5	56.0	55.5
40	76.3	76.0	75.3	74.6	74.0
50	95.4	95.0	94.1	93.3	92.5

	110̂	110	109	108
6	11.0	11.0	10.9	10.8
7	12.9	12.8	12.7	12.6
8	14.7	14.6	14.5	14.4
9	16.6	16.5	16.3	16.2
10	18.4	18.3	18.1	18.0
20	36.8	36.6	36.3	36.0
30	55.2	55.0	54.5	54.0
40	73.6	73.3	72.6	72.0
50	92.1	91.6	90.8	90.0

	107̂	107	106	105	104
6	10.7	10.7	10.6	10.5	10.4
7	12.5	12.5	12.3	12.2	12.1
8	14.3	14.2	14.1	14.0	13.8
9	16.1	16.0	15.9	15.7	15.6
10	17.9	17.8	17.6	17.5	17.3
20	35.8	35.6	35.3	35.0	34.6
30	53.7	53.5	53.0	52.5	52.0
40	71.6	71.3	70.6	70.0	69.3
50	89.6	89.1	88.3	87.5	86.6

	103̂	103	2	1̂	1
6	10.3	10.3	0.2	0.1	0.1
7	12.1	12.0	0.2	0.2	0.1
8	13.8	13.7	0.2	0.2	0.1
9	15.5	15.4	0.3	0.2	0.1
10	17.2	17.1	0.3	0.2	0.1
20	34.5	34.3	0.6	0.5	0.3
30	51.7	51.5	1.0	0.7	0.5
40	69.0	68.6	1.3	1.0	0.6
50	86.2	85.8	1.6	1.2	0.8

P. P.

83°

7°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	
0	9.08 589	102	9.08 914	104	0.91 085	9.99 675	60
1	9.08 692	102	9.09 018	104	0.90 981	9.99 673	59
2	9.08 794	102	9.09 123	103	0.90 877	9.99 672	58
3	9.08 897	102	9.09 226	103	0.90 773	9.99 670	57
4	9.08 999	102	9.09 330	103	0.90 670	9.99 669	56
5	9.09 101	101	9.09 433	103	0.90 566	9.99 667	55
6	9.09 202	101	9.09 536	103	0.90 463	9.99 665	54
7	9.09 303	101	9.09 639	102	0.90 360	9.99 664	53
8	9.09 404	101	9.09 742	102	0.90 258	9.99 662	52
9	9.09 505	100	9.09 844	102	0.90 155	9.99 661	51
10	9.09 606	100	9.09 947	101	0.90 053	9.99 659	50
11	9.09 706	100	9.10 048	102	0.89 951	9.99 658	49
12	9.09 806	100	9.10 150	101	0.89 849	9.99 656	48
13	9.09 906	99	9.10 252	101	0.89 748	9.99 654	47
14	9.10 006	99	9.10 353	101	0.89 647	9.99 653	46
15	9.10 105	99	9.10 454	101	0.89 546	9.99 651	45
16	9.10 205	98	9.10 555	100	0.89 445	9.99 650	44
17	9.10 303	99	9.10 655	100	0.89 344	9.99 648	43
18	9.10 402	98	9.10 756	100	0.89 244	9.99 646	42
19	9.10 501	98	9.10 856	100	0.89 144	9.99 645	41
20	9.10 599	98	9.10 956	99	0.89 044	9.99 643	40
21	9.10 697	97	9.11 055	99	0.88 944	9.99 641	39
22	9.10 795	97	9.11 155	99	0.88 845	9.99 640	38
23	9.10 892	97	9.11 254	99	0.88 745	9.99 638	37
24	9.10 990	97	9.11 353	98	0.88 646	9.99 637	36
25	9.11 087	96	9.11 452	98	0.88 548	9.99 635	35
26	9.11 184	97	9.11 550	98	0.88 449	9.99 633	34
27	9.11 281	96	9.11 649	98	0.88 351	9.99 632	33
28	9.11 377	96	9.11 747	98	0.88 253	9.99 630	32
29	9.11 473	96	9.11 845	98	0.88 155	9.99 628	31
30	9.11 570	95	9.11 943	97	0.88 057	9.99 627	30
31	9.11 665	96	9.12 040	97	0.87 959	9.99 625	29
32	9.11 761	95	9.12 137	97	0.87 862	9.99 623	28
33	9.11 856	95	9.12 235	96	0.87 765	9.99 622	27
34	9.11 952	95	9.12 331	97	0.87 668	9.99 620	26
35	9.12 047	94	9.12 428	96	0.87 571	9.99 618	25
36	9.12 141	94	9.12 525	96	0.87 475	9.99 617	24
37	9.12 236	94	9.12 621	96	0.87 379	9.99 615	23
38	9.12 330	94	9.12 717	96	0.87 283	9.99 613	22
39	9.12 425	93	9.12 813	95	0.87 187	9.99 611	21
40	9.12 518	94	9.12 908	95	0.87 091	9.99 610	20
41	9.12 612	93	9.13 004	95	0.86 996	9.99 608	19
42	9.12 706	93	9.13 099	95	0.86 900	9.99 606	18
43	9.12 799	93	9.13 194	95	0.86 805	9.99 605	17
44	9.12 892	93	9.13 289	94	0.86 710	9.99 603	16
45	9.12 985	92	9.13 384	94	0.86 616	9.99 601	15
46	9.13 078	92	9.13 478	94	0.86 521	9.99 600	14
47	9.13 170	92	9.13 572	94	0.86 427	9.99 598	13
48	9.13 263	92	9.13 666	94	0.86 333	9.99 596	12
49	9.13 355	92	9.13 760	93	0.86 239	9.99 594	11
50	9.13 447	91	9.13 854	93	0.86 146	9.99 593	10
51	9.13 538	92	9.13 947	93	0.86 052	9.99 591	9
52	9.13 630	91	9.14 041	93	0.85 959	9.99 589	8
53	9.13 721	91	9.14 134	93	0.85 866	9.99 587	7
54	9.13 813	90	9.14 227	92	0.85 773	9.99 586	6
55	9.13 903	91	9.14 319	92	0.85 680	9.99 584	5
56	9.13 994	90	9.14 412	92	0.85 588	9.99 582	4
57	9.14 085	90	9.14 504	92	0.85 495	9.99 580	3
58	9.14 175	90	9.14 596	92	0.85 403	9.99 579	2
59	9.14 265	90	9.14 688	92	0.85 311	9.99 577	1
60	9.14 355		9.14 780		0.85 219	9.99 575	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	104	103	102	101
6	10.4	10.3	10.2	10.1
7	12.1	12.0	11.9	11.8
8	13.8	13.7	13.6	13.4
9	15.6	15.4	15.3	15.1
10	17.3	17.1	17.0	16.8
20	34.6	34.3	34.0	33.6
30	52.0	51.5	51.0	50.5
40	69.3	68.6	68.0	67.3
50	86.6	85.8	85.0	84.1

	100	100	99	98
6	10.0	10.0	9.9	9.8
7	11.7	11.6	11.5	11.4
8	13.4	13.3	13.2	13.0
9	15.1	15.0	14.8	14.7
10	16.7	16.6	16.5	16.3
20	33.5	33.3	33.0	32.6
30	50.2	50.0	49.5	49.0
40	67.0	66.6	66.0	65.3
50	83.7	83.3	82.5	81.6

	97	97	96	95
6	9.7	9.7	9.6	9.5
7	11.4	11.3	11.2	11.1
8	13.0	12.9	12.8	12.6
9	14.6	14.5	14.4	14.2
10	16.2	16.1	16.0	15.8
20	32.5	32.3	32.0	31.6
30	48.7	48.5	48.0	47.5
40	65.0	64.6	64.0	63.3
50	81.2	80.8	80.0	79.1

	94	94	93	92
6	9.4	9.4	9.3	9.2
7	11.0	10.9	10.8	10.7
8	12.6	12.5	12.4	12.2
9	14.2	14.1	13.9	13.8
10	15.7	15.6	15.5	15.3
20	31.5	31.3	31.0	30.6
30	47.2	47.0	46.5	46.0
40	63.0	62.6	62.0	61.3
50	78.7	78.3	77.5	76.6

	91	91	90	2	1
6	9.1	9.1	9.0	0.2	0.1
7	10.7	10.6	10.5	0.2	0.2
8	12.2	12.1	12.0	0.3	0.2
9	13.7	13.6	13.5	0.3	0.2
10	15.2	15.1	15.0	0.3	0.2
20	30.5	30.3	30.0	0.6	0.5
30	45.7	45.5	45.0	1.0	0.7
40	61.0	60.6	60.0	1.3	1.0
50	76.2	75.8	75.0	1.6	1.2

P. P.

82°

8°

′	Log. Sin	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	
0	9.14 355	90	9.14 786	91	0.85 219	9.99 575	60
1	9.14 445	89	9.14 872	91	0.85 128	9.99 573	59
2	9.14 535	89	9.14 963	91	0.85 037	9.99 571	58
3	9.14 624	89	9.15 054	91	0.84 945	9.99 570	57
4	9.14 713	89	9.15 145	91	0.84 854	9.99 568	56
5	9.14 802	89	9.15 236	90	0.84 763	9.99 566	55
6	9.14 891	88	9.15 327	90	0.84 673	9.99 564	54
7	9.14 980	88	9.15 417	90	0.84 582	9.99 563	53
8	9.15 068	88	9.15 507	90	0.84 492	9.99 561	52
9	9.15 157	88	9.15 598	89	0.84 402	9.99 559	51
10	9.15 245	88	9.15 687	90	0.84 312	9.99 557	50
11	9.15 333	88	9.15 777	89	0.84 222	9.99 555	49
12	9.15 421	87	9.15 867	89	0.84 133	9.99 553	48
13	9.15 508	87	9.15 956	89	0.84 043	9.99 552	47
14	9.15 595	87	9.16 045	89	0.83 954	9.99 550	46
15	9.15 683	87	9.16 134	89	0.83 865	9.99 548	45
16	9.15 770	87	9.16 223	89	0.83 776	9.99 546	44
17	9.15 857	86	9.16 312	88	0.83 687	9.99 544	43
18	9.15 943	86	9.16 401	88	0.83 599	9.99 542	42
19	9.16 030	86	9.16 489	88	0.83 511	9.99 541	41
20	9.16 116	86	9 16 577	88	0.83 422	9.99 539	40
21	9.16 202	86	9.16 665	87	0.83 334	9.99 537	39
22	9.16 288	86	9.16 753	88	0.83 247	9.99 535	38
23	9.16 374	85	9.16 841	87	0.83 159	9.99 533	37
24	9.16 460	85	9.16 928	87	0.83 071	9.99 531	36
25	9.16 545	85	9.17 015	87	0.82 984	9.99 529	35
26	9.16 630	85	9.17 103	87	0.82 897	9.99 528	34
27	9.16 716	85	9.17 190	86	0.82 810	9.99 526	33
28	9.16 801	84	9.17 276	87	0.82 723	9.99 524	32
29	9.16 885	84	9.17 363	86	0.82 636	9.99 522	31
30	9.16 970	84	9.17 450	86	0.82 550	9.99 520	30
31	9.17 054	84	9.17 536	86	0.82 464	9.99 518	29
32	9.17 139	84	9.17 622	86	0.82 377	9.99 516	28
33	9.17 223	84	9.17 708	85	0.82 291	9.99 514	27
34	9.17 307	84	9 17 794	86	0.82 206	9.99 512	26
35	9.17 391	83	9 17 880	85	0.82 120	9.99 511	25
36	9 17 474	83	9.17 965	85	0.82 034	9.99 509	24
37	9.17 558	83	9.18 051	85	0.81 949	9.99 507	23
38	9.17 641	83	9.18 136	85	0.81 864	9.99 505	22
39	9.17 724	83	9.18 221	85	0.81 779	9.99 503	21
40	9.17 807	83	9.18 306	84	0.81 694	9.99 501	20
41	9.17 890	82	9.18 390	84	0.81 609	9.99 499	19
42	9.17 972	82	9.18 475	84	0.81 525	9.99 497	18
43	9.18 055	82	9.18 559	84	0.81 440	9.99 495	17
44	9.18 137	82	9.18 644	84	0.81 356	9.99 493	16
45	9.18 219	82	9.18 728	84	0.81 272	9.99 491	15
46	9.18 301	82	9.18 812	84	0.81 188	9.99 489	14
47	9.18 383	81	9.18 896	83	0.81 104	9.99 487	13
48	9.18 465	81	9.18 979	83	0.81 020	9.99 485	12
49	9.18 546	81	9.19 063	83	0.80 937	9.99 484	11
50	9.18 628	81	9.19 146	83	0.80 854	9.99 482	10
51	9.18 709	81	9.19 229	83	0.80 770	9.99 480	9
52	9.18 790	80	9.19 312	83	0.80 687	9.99 478	8
53	9.18 871	81	9.19 395	82	0.80 604	9.99 476	7
54	9.18 952	80	9.19 478	82	0.80 522	9.99 474	6
55	9.19 032	80	9.19 560	82	0.80 439	9.99 472	5
56	9.19 113	80	9.19 643	82	0.80 357	9.99 470	4
57	9.19 193	80	9.19 725	82	0.80 274	9.99 468	3
58	9 19 273	80	9.19 807	82	0.80 192	9.99 466	2
59	9.19 353	79	9.19 889	82	0.80 110	9.99 464	1
60	9.19 433		9.19 971		0.80 028	9.99 462	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	91	91	90	89
6	9.1	9.1	9.0	8.9
7	10.7	10.6	10.5	10.4
8	12.2	12.1	12.0	11.8
9	13.7	13.6	13.5	13.3
10	15.2	15.1	15.0	14.8
20	30.5	30.3	30.0	29.6
30	45.7	45.5	45.0	44.5
40	61.0	60.6	60.0	59.3
50	76.2	75.8	75.0	74.1

	88	88	87	86
6	8.8	8.8	8.7	8.6
7	10.3	10.2	10.1	10.0
8	11.8	11.7	11.6	11.4
9	13.3	13.2	13.0	12.9
10	14.7	14.6	14.5	14.3
20	29.5	29.3	29.0	28.6
30	44.2	44.0	43.5	43.0
40	59.0	58.6	58.0	57.3
50	73.7	73.3	72.5	71.6

	85	85	84	83
6	8.5	8.5	8.4	8.3
7	10.0	9.9	9.8	9.7
8	11.4	11.3	11.2	11.0
9	12.8	12.7	12.6	12.4
10	14.2	14.1	14.0	13.8
20	28.5	28.3	28.0	27.6
30	42.7	42.5	42.0	41.5
40	57.0	56.6	56.0	55.3
50	71.2	70.8	70.0	69.1

	82	82	81	80
6	8.2	8.2	8.1	8.0
7	9.6	9.5	9.4	9.3
8	11.0	10.9	10.8	10.6
9	12.4	12.3	12.1	12.0
10	13.7	13.6	13.5	13.3
20	27.5	27.3	27.0	26.6
30	41.2	41.0	40.5	40.0
40	55.0	54.6	54.0	53.3
50	68.7	68.3	67.5	66.6

	79	2	1
6	7.9	0.2	0.1
7	9.3	0.2	0.2
8	10.6	0.2	0.2
9	11.9	0.3	0.2
10	13.2	0.3	0.2
20	26.5	0.6	0.5
30	39.7	1.0	0.7
40	53.0	1.3	1.0
50	66.2	1.6	1.2

P. P.

81°

9°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	
0	9.19 433	80	9.19 971	81	0.80 028	9.99 462	60
1	9.19 513	79	9.20 053	81	0.79 947	9.99 460	59
2	9.19 592	79	9.20 134	81	0.79 865	9.99 458	58
3	9.19 672	79	9.20 216	81	0.79 784	9.99 456	57
4	9.19 751	79	9.20 297	81	0.79 703	9.99 454	56
5	9.19 830	79	9.20 378	81	0.79 622	9.99 452	55
6	9.19 909	79	9.20 459	81	0.79 541	9.99 450	54
7	9.19 988	78	9.20 540	80	0.79 460	9.99 448	53
8	9.20 066	78	9.20 620	81	0.79 379	9.99 446	52
9	9.20 145	78	9.20 701	80	0.79 298	9.99 444	51
10	9.20 223	78	9.20 781	80	0.79 218	9.99 442	50
11	9.20 301	78	9.20 862	80	0.79 138	9.99 440	49
12	9.20 379	78	9.20 942	80	0.79 058	9.99 437	48
13	9.20 457	78	9.21 022	80	0.78 978	9.99 435	47
14	9.20 535	77	9.21 102	79	0.78 898	9.99 433	46
15	9.20 613	77	9.21 181	79	0.78 818	9.99 431	45
16	9.20 690	77	9.21 261	79	0.78 739	9.99 429	44
17	9.20 768	77	9.21 340	79	0.78 659	9.99 427	43
18	9.20 845	77	9.21 420	79	0.78 580	9.99 425	42
19	9.20 922	77	9.21 499	79	0.78 501	9.99 423	41
20	9.20 999	77	9.21 578	79	0.78 422	9.99 421	40
21	9.21 076	76	9.21 657	78	0.78 343	9.99 419	39
22	9.21 152	76	9.21 735	78	0.78 264	9.99 417	38
23	9.21 229	76	9.21 814	78	0.78 186	9.99 415	37
24	9.21 305	76	9.21 892	78	0.78 107	9.99 413	36
25	9.21 382	76	9.21 971	78	0.78 029	9.99 411	35
26	9.21 458	76	9.22 049	78	0.77 951	9.99 408	34
27	9.21 534	75	9.22 127	78	0.77 873	9.99 406	33
28	9.21 609	76	9.22 205	78	0.77 795	9.99 404	32
29	9.21 685	75	9.22 283	77	0.77 717	9.99 402	31
30	9.21 761	75	9.22 360	77	0.77 639	9.99 400	30
31	9.21 836	75	9.22 438	77	0.77 562	9.99 398	29
32	9.21 911	75	9.22 515	77	0.77 484	9.99 396	28
33	9.21 987	75	9.22 593	77	0.77 407	9.99 394	27
34	9.22 062	74	9.22 670	77	0.77 330	9.99 392	26
35	9.22 136	75	9.22 747	77	0.77 253	9.99 389	25
36	9.22 211	74	9.22 824	76	0.77 176	9.99 387	24
37	9.22 286	74	9.22 900	77	0.77 099	9.99 385	23
38	9.22 360	74	9.22 977	76	0.77 022	9.99 383	22
39	9.22 435	74	9.23 054	76	0.76 946	9.99 381	21
40	9.22 509	74	9.23 130	76	0.76 870	9.99 379	20
41	9.22 583	74	9.23 206	76	0.76 793	9.99 377	19
42	9.22 657	73	9.23 282	76	0.76 717	9.99 374	18
43	9.22 731	74	9.23 358	76	0.76 641	9.99 372	17
44	9.22 805	73	9.23 434	76	0.76 565	9.99 370	16
45	9.22 878	73	9.23 510	75	0.76 489	9.99 368	15
46	9.22 952	73	9.23 586	75	0.76 414	9.99 366	14
47	9.23 025	73	9.23 661	75	0.76 338	9.99 364	13
48	9.23 098	73	9.23 737	75	0.76 263	9.99 361	12
49	9.23 171	73	9.23 812	75	0.76 188	9.99 359	11
50	9.23 244	72	9.23 887	75	0.76 113	9.99 357	10
51	9.23 317	73	9.23 962	75	0.76 038	9.99 355	9
52	9.23 390	72	9.24 037	75	0.75 963	9.99 353	8
53	9.23 462	72	9.24 112	74	0.75 888	9.99 350	7
54	9.23 535	72	9.24 186	74	0.75 813	9.99 348	6
55	9.23 607	72	9.24 261	74	0.75 739	9.99 346	5
56	9.23 679	72	9.24 335	74	0.75 664	9.99 344	4
57	9.23 751	72	9.24 409	74	0.75 590	9.99 342	3
58	9.23 823	72	9.24 484	74	0.75 516	9.99 339	2
59	9.23 895	71	9.24 558	74	0.75 442	9.99 337	1
60	9.23 967		9.24 632		0.75 368	9.99 335	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	81	81	80	79
6	8.1	8.1	8.0	7.9
7	9.5	9.4	9.3	9.2
8	10.8	10.8	10.6	10.5
9	12.2	12.1	12.0	11.8
10	13.6	13.5	13.3	13.1
20	27.1	27.0	26.6	26.3
30	40.7	40.5	40.0	39.5
40	54.3	54.0	53.3	52.6
50	67.9	67.5	66.6	65.8

	78	78	77
6	7.8	7.8	7.7
7	9.1	9.1	9.0
8	10.4	10.4	10.2
9	11.8	11.7	11.5
10	13.1	13.0	12.8
20	26.1	26.0	25.6
30	39.2	39.0	38.5
40	52.3	52.0	51.3
50	65.4	65.0	64.1

	76	76	75	74
6	7.6	7.6	7.5	7.4
7	8.9	8.8	8.7	8.6
8	10.2	10.1	10.0	9.8
9	11.5	11.4	11.2	11.1
10	12.7	12.6	12.5	12.3
20	25.5	25.3	25.0	24.6
30	38.2	38.0	37.5	37.0
40	51.0	50.6	50.0	49.3
50	63.7	63.3	62.5	61.6

	73	73	72
6	7.3	7.3	7.2
7	8.6	8.5	8.4
8	9.8	9.7	9.6
9	11.0	10.9	10.8
10	12.2	12.1	12.0
20	24.5	24.3	24.0
30	36.7	36.5	36.0
40	49.0	48.6	48.0
50	61.2	60.8	60.0

	71	71	2	2
6	7.1	7.1	0.2	0.2
7	8.3	8.3	0.3	0.2
8	9.5	9.4	0.3	0.2
9	10.7	10.6	0.4	0.3
10	11.9	11.8	0.4	0.3
20	23.8	23.6	0.8	0.6
30	35.7	35.5	1.2	1.0
40	47.6	47.3	1.6	1.3
50	59.6	59.1	2.1	1.6

P. P.

80°

10°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	
0	9.23 967	71	9.24 632	73	0.75 368	9.99 335	60
1	9.24 038	71	9.24 705	74	0.75 294	9.99 333	59
2	9.24 110	71	9.24 779	73	0.75 220	9.99 330	58
3	9.24 181	71	9.24 853	73	0.75 147	9.99 328	57
4	9.24 252	71	9.24 926	73	0.75 073	9.99 326	56
5	9.24 323	71	9.25 000	73	0.75 000	9.99 324	55
6	9.24 394	71	9.25 073	73	0.74 927	9.99 321	54
7	9.24 465	71	9.25 146	73	0.74 854	9.99 319	53
8	9.24 536	70	9.25 219	73	0.74 781	9.99 317	52
9	9.24 607	70	9.25 292	73	0.74 708	9.99 315	51
10	9.24 677	70	9.25 365	72	0.74 635	9.99 312	50
11	9.24 748	70	9.25 437	72	0.74 562	9.99 310	49
12	9.24 818	70	9.25 510	72	0.74 490	9.99 308	48
13	9.24 888	70	9.25 582	72	0.74 417	9.99 306	47
14	9.24 958	69	9.25 654	72	0.74 345	9.99 303	46
15	9.25 028	70	9.25 727	72	0.74 273	9.99 301	45
16	9.25 098	69	9.25 799	72	0.74 201	9.99 299	44
17	9.25 167	70	9.25 871	72	0.74 129	9.99 296	43
18	9.25 237	69	9.25 943	71	0.74 057	9.99 294	42
19	9.25 306	69	9.26 014	72	0.73 985	9.99 292	41
20	9.25 376	69	9.26 086	71	0.73 913	9.99 290	40
21	9.25 445	69	9.26 158	71	0.73 842	9.99 287	39
22	9.25 514	69	9.26 229	71	0.73 771	9.99 285	38
23	9.25 583	69	9.26 300	71	0.73 699	9.99 283	37
24	9.25 652	68	9.26 371	71	0.73 628	9.99 280	36
25	9.25 721	69	9.26 443	71	0.73 557	9.99 278	35
26	9.25 790	68	9.26 514	70	0.73 486	9.99 276	34
27	9.25 858	68	9.26 584	71	0.73 415	9.99 273	33
28	9.25 927	68	9.26 655	70	0.73 344	9.99 271	32
29	9.25 995	68	9.26 726	70	0.73 274	9.99 269	31
30	9.26 063	68	9.26 796	70	0.73 203	9.99 266	30
31	9.26 131	68	9.26 867	70	0.73 133	9.99 264	29
32	9.26 199	68	9.26 937	70	0.73 062	9.99 262	28
33	9.26 267	67	9.27 007	70	0.72 992	9.99 259	27
34	9.26 335	67	9.27 078	70	0.72 922	9.99 257	26
35	9.26 402	68	9.27 148	70	0.72 852	9.99 255	25
36	9.26 470	67	9.27 218	69	0.72 782	9.99 252	24
37	9.26 537	67	9.27 287	70	0.72 712	9.99 250	23
38	9.26 605	67	9.27 357	69	0.72 642	9.99 248	22
39	9.26 672	67	9.27 427	69	0.72 573	9.99 245	21
40	9.26 739	67	9.27 496	69	0.72 503	9.99 243	20
41	9.26 806	67	9.27 566	69	0.72 434	9.99 240	19
42	9.26 873	66	9.27 635	69	0.72 365	9.99 238	18
43	9.26 940	67	9.27 704	69	0.72 295	9.99 236	17
44	9.27 007	66	9.27 773	69	0.72 226	9.99 233	16
45	9.27 073	66	9.27 842	69	0.72 157	9.99 231	15
46	9.27 140	66	9.27 911	68	0.72 088	9.99 228	14
47	9.27 206	66	9.27 980	69	0.72 020	9.99 226	13
48	9.27 272	66	9.28 049	68	0.71 951	9.99 224	12
49	9.27 339	66	9.28 117	68	0.71 882	9.99 221	11
50	9.27 405	66	9.28 186	68	0.71 814	9.99 219	10
51	9.27 471	65	9.28 254	68	0.71 746	9.99 216	9
52	9.27 536	66	9.28 322	68	0.71 677	9.99 214	8
53	9.27 602	65	9.28 390	68	0.71 609	9.99 212	7
54	9.27 668	65	9.28 459	68	0.71 541	9.99 209	6
55	9.27 733	65	9.28 527	67	0.71 473	9.99 207	5
56	9.27 799	65	9.28 594	68	0.71 405	9.99 204	4
57	9.27 864	65	9.28 662	67	0.71 337	9.99 202	3
58	9.27 929	65	9.28 730	67	0.71 270	9.99 199	2
59	9.27 995	65	9.28 797	67	0.71 202	9.99 197	1
60	9.28 060		9.28 865		0.71 135	9.99 194	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	74	73	73
6	7.4	7.3	7.3
7	8.6	8.6	8.5
8	9.8	9.8	9.7
9	11.1	11.0	10.9
10	12.3	12.2	12.1
20	24.6	24.5	24.3
30	37.0	36.7	36.5
40	49.3	49.0	48.6
50	61.6	61.2	60.8

	72	72	71	71
6	7.2	7.2	7.1	7.1
7	8.4	8.4	8.3	8.3
8	9.6	9.6	9.5	9.4
9	10.9	10.8	10.7	10.6
10	12.1	12.0	11.9	11.8
20	24.1	24.0	23.8	23.6
30	36.2	36.0	35.7	35.5
40	48.3	48.0	47.6	47.3
50	60.4	60.0	59.6	59.1

	70	70	69	69
6	7.0	7.0	6.9	6.9
7	8.2	8.1	8.1	8.0
8	9.4	9.3	9.2	9.2
9	10.6	10.5	10.4	10.3
10	11.7	11.6	11.6	11.5
20	23.5	23.3	23.1	23.0
30	35.2	35.0	34.7	34.5
40	47.0	46.6	46.3	46.0
50	58.7	58.3	57.9	57.5

	68	68	67	67
6	6.8	6.8	6.7	6.7
7	8.0	7.9	7.9	7.8
8	9.1	9.0	9.0	8.9
9	10.3	10.2	10.1	10.0
10	11.4	11.3	11.2	11.1
20	22.8	22.6	22.5	22.3
30	34.2	34.0	33.7	33.5
40	45.6	45.3	45.0	44.6
50	57.1	56.6	56.2	55.8

	66	66	65	65
6	6.6	6.6	6.5	6.5
7	7.7	7.7	7.6	7.6
8	8.8	8.8	8.7	8.6
9	10.0	9.9	9.8	9.7
10	11.1	11.0	10.9	10.8
20	22.1	22.0	21.8	21.6
30	33.2	33.0	32.7	32.5
40	44.3	44.0	43.6	43.3
50	55.4	55.0	54.6	54.1

	2	2
6	0.2	0.2
7	0.3	0.2
8	0.3	0.2
9	0.4	0.3
10	0.4	0.3
20	0.8	0.6
30	1.2	1.0
40	1.6	1.3
50	2.1	1.6

P. P.

79°

11°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	
0	9.28 060	65	9.28 865	67	0.71 135	9.99 194	60
1	9.28 125	64	9.28 932	67	0.71 067	9.99 192	59
2	9.28 189	65	9.29 000	67	0.71 000	9.99 189	58
3	9.28 254	64	9.29 067	67	0.70 933	9.99 187	57
4	9.28 319	64	9.29 134	67	0.70 866	9.99 185	56
5	9.28 383	64	9.29 201	66	0.70 798	9.99 182	55
6	9.28 448	64	9.29 268	67	0.70 732	9.99 180	54
7	9.28 512	64	9.29 335	66	0.70 665	9.99 177	53
8	9.28 576	64	9.29 401	67	0.70 598	9.99 175	52
9	9.28 641	64	9.29 468	66	0.70 531	9.99 172	51
10	9.28 705	64	9.29 535	66	0.70 465	9.99 170	50
11	9.28 769	63	9.29 601	66	0.70 398	9.99 167	49
12	9.28 832	64	9.29 667	66	0.70 332	9.99 165	48
13	9.28 896	63	9.29 734	66	0.70 266	9.99 162	47
14	9.28 960	63	9.29 800	66	0.70 200	9.99 160	46
15	9.29 023	63	9.29 866	66	0.70 134	9.99 157	45
16	9.29 087	63	9.29 932	66	0.70 068	9.99 155	44
17	9.29 150	63	9.29 998	66	0.70 002	9.99 152	43
18	9.29 213	63	9.30 064	65	0.69 936	9.99 150	42
19	9.29 277	63	9.30 129	65	0.69 870	9.99 147	41
20	9.29 340	63	9.30 195	65	0.69 805	9.99 145	40
21	9.29 403	63	9.30 260	65	0.69 739	9.99 142	39
22	9.29 466	62	9.30 326	65	0.69 674	9.99 139	38
23	9.29 528	63	9.30 391	65	0.69 608	9.99 137	37
24	9.29 591	62	9.30 456	65	0.69 543	9.99 134	36
25	9.29 654	63	9.30 522	65	0.69 478	9.99 132	35
26	9.29 716	62	9.30 587	65	0.69 413	9.99 129	34
27	9.29 779	62	9.30 652	65	0.69 348	9.99 127	33
28	9.29 841	62	9.30 717	64	0.69 283	9.99 124	32
29	9.29 903	62	9.30 781	65	0.69 218	9.99 122	31
30	9.29 965	62	9.30 846	64	0.69 153	9.99 119	30
31	9.30 027	62	9.30 911	64	0.69 089	9.99 116	29
32	9.30 089	62	9.30 975	64	0.69 024	9.99 114	28
33	9.30 151	61	9.31 040	64	0.68 960	9.99 111	27
34	9.30 213	62	9.31 104	64	0.68 896	9.99 109	26
35	9.30 275	61	9.31 168	64	0.68 831	9.99 106	25
36	9.30 336	61	9.31 232	64	0.68 767	9.99 104	24
37	9.30 398	61	9.31 297	64	0.68 703	9.99 101	23
38	9.30 459	61	9.31 361	63	0.68 639	9.99 098	22
39	9.30 520	61	9.31 424	64	0.68 575	9.99 096	21
40	9.30 582	61	9.31 488	64	0.68 511	9.99 093	20
41	9.30 643	61	9.31 552	63	0.68 447	9.99 091	19
42	9.30 704	61	9.31 616	63	0.68 384	9.99 088	18
43	9.30 765	61	9.31 679	63	0.68 320	9.99 085	17
44	9.30 826	60	9.31 743	63	0.68 257	9.99 083	16
45	9.30 886	61	9.31 806	63	0.68 193	9.99 080	15
46	9.30 947	60	9.31 869	63	0.68 130	9.99 077	14
47	9.31 008	60	9.31 933	63	0.68 067	9.99 075	13
48	9.31 068	60	9.31 996	63	0.68 004	9.99 072	12
49	9.31 129	60	9.32 059	63	0.67 941	9.99 069	11
50	9.31 189	60	9.32 122	63	0.67 878	9.99 067	10
51	9.31 249	60	9.32 185	63	0.67 815	9.99 064	9
52	9.31 309	60	9.32 248	62	0.67 752	9.99 062	8
53	9.31 370	59	9.32 310	63	0.67 689	9.99 059	7
54	9.31 429	60	9.32 373	62	0.67 626	9.99 056	6
55	9.31 489	60	9.32 436	62	0.67 564	9.99 054	5
56	9.31 549	59	9.32 498	62	0.67 501	9.99 051	4
57	9.31 609	60	9.32 560	62	0.67 439	9.99 048	3
58	9.31 669	59	9.32 623	62	0.67 377	9.99 046	2
59	9.31 728	59	9.32 685	62	0.67 314	9.99 043	1
60	9.31 788		9.32 747		0.67 252	9.99 040	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	67	67
6	6.7	6.7
7	7.9	7.8
8	9.0	8.9
9	10.1	10.0
10	11.2	11.1
20	22.5	22.3
30	33.7	33.5
40	45.0	44.6
50	56.2	55.8

	66	66	65	65
6	6.6	6.6	6.5	6.5
7	7.7	7.7	7.6	7.6
8	8.8	8.8	8.7	8.6
9	10.0	9.9	9.8	9.7
10	11.1	11.0	10.9	10.8
20	22.1	22.0	21.8	21.6
30	33.2	33.0	32.7	32.5
40	44.3	44.0	43.6	43.3
50	55.4	55.0	54.6	54.1

	64	64	63	63
6	6.4	6.4	6.3	6.3
7	7.5	7.4	7.4	7.3
8	8.6	8.5	8.4	8.4
9	9.7	9.6	9.5	9.4
10	10.7	10.6	10.6	10.5
20	21.5	21.3	21.1	21.0
30	32.2	32.0	31.7	31.5
40	43.0	42.6	42.3	42.0
50	53.7	53.3	52.9	52.5

	62	62	61	61
6	6.2	6.2	6.1	6.1
7	7.3	7.2	7.2	7.1
8	8.3	8.2	8.2	8.1
9	9.4	9.3	9.2	9.1
10	10.4	10.3	10.2	10.1
20	20.8	20.6	20.5	20.3
30	31.2	31.0	30.7	30.5
40	41.6	41.3	41.0	40.6
50	52.1	51.6	51.2	50.8

	60	60	59
6	6.0	6.0	5.9
7	7.0	7.0	6.9
8	8.0	8.0	7.9
9	9.1	9.0	8.9
10	10.1	10.0	9.9
20	20.1	20.0	19.8
30	30.2	30.0	29.7
40	40.3	40.0	39.6
50	50.4	50.0	49.6

	3	2	2
6	0.3	0.2	0.2
7	0.3	0.3	0.2
8	0.4	0.3	0.2
9	0.4	0.4	0.3
10	0.5	0.4	0.3
20	1.0	0.8	0.6
30	1.5	1.2	1.0
40	2.0	1.6	1.3
50	2.5	2.1	1.6

P. P.

78°

12°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	
0	9.31 788	59	9.32 747	62	0.67 252	9.99 040	60
1	9.31 847	59	9.32 809	62	0.67 190	9.99 038	59
2	9.31 906	59	9.32 871	62	0.67 128	9.99 035	58
3	9.31 966	59	9.32 933	62	0.67 066	9.99 032	57
4	9.32 025	59	9.32 995	61	0.67 004	9.99 029	56
5	9.32 084	59	9.33 057	61	0.66 943	9.99 027	55
6	9.32 143	59	9.33 118	62	0.66 881	9.99 024	54
7	9.32 202	58	9.33 180	61	0.66 819	9.99 021	53
8	9.32 260	59	9.33 242	61	0.66 758	9.99 019	52
9	9.32 319	58	9.33 303	61	0.66 696	9.99 016	51
10	9.32 378	58	9.33 364	61	0.66 635	9.99 013	50
11	9.32 436	58	9.33 426	61	0.66 574	9.99 010	49
12	9.32 495	58	9.33 487	61	0.66 513	9.99 008	48
13	9.32 553	58	9.33 548	61	0.66 452	9.99 005	47
14	9.32 611	58	9.33 609	60	0.66 390	9.99 002	46
15	9.32 670	58	9.33 670	61	0.66 330	9.98 999	45
16	9.32 728	58	9.33 731	61	0.66 269	9.98 997	44
17	9.32 786	58	9.33 792	60	0.66 208	9.98 994	43
18	9.32 844	58	9.33 852	61	0.66 147	9.98 991	42
19	9.32 902	58	9.33 913	60	0.66 086	9.98 988	41
20	9.32 960	57	9.33 974	60	0.66 026	9.98 986	40
21	9.33 017	58	9.34 034	60	0.65 965	9.98 983	39
22	9.33 075	57	9.34 095	60	0.65 905	9.98 980	38
23	9.33 133	57	9.34 155	60	0.65 845	9.98 977	37
24	9.33 190	57	9.34 215	60	0.65 784	9.98 975	36
25	9.33 248	57	9.34 275	60	0.65 724	9.98 972	35
26	9.33 305	57	9.34 336	60	0.65 664	9.98 969	34
27	9.33 362	57	9.34 396	60	0.65 604	9.98 966	33
28	9.33 419	57	9.34 456	59	0.65 544	9.98 963	32
29	9.33 476	57	9.34 515	60	0.65 484	9.98 961	31
30	9.33 533	57	9.34 575	60	0.65 424	9.98 958	30
31	9.33 590	57	9.34 635	59	0.65 364	9.98 955	29
32	9.33 647	57	9.34 695	59	0.65 305	9.98 952	28
33	9.33 704	56	9.34 754	59	0.65 245	9.98 949	27
34	9.33 761	56	9.34 814	59	0.65 186	9.98 947	26
35	9.33 817	56	9.34 873	59	0.65 126	9.98 944	25
36	9.33 874	56	9.34 933	59	0.65 067	9.98 941	24
37	9.33 930	56	9.34 992	59	0.65 008	9.98 938	23
38	9.33 987	56	9.35 051	59	0.64 948	9.98 935	22
39	9.34 043	56	9.35 110	59	0.64 889	9.98 933	21
40	9.34 099	56	9.35 169	59	0.64 830	9.98 930	20
41	9.34 156	56	9.35 228	59	0.64 771	9.98 927	19
42	9.34 212	56	9.35 287	59	0.64 712	9.98 924	18
43	9.34 268	56	9.35 346	59	0.64 653	9.98 921	17
44	9.34 324	55	9.35 405	58	0.64 594	9.98 918	16
45	9.34 379	56	9.35 464	58	0.64 536	9.98 915	15
46	9.34 435	55	9.35 522	59	0.64 477	9.98 913	14
47	9.34 491	56	9.35 581	58	0.64 418	9.98 910	13
48	9.34 547	55	9.35 640	58	0.64 360	9.98 907	12
49	9.34 602	55	9.35 698	58	0.64 302	9.98 904	11
50	9.34 658	55	9.35 756	58	0.64 243	9.98 901	10
51	9.34 713	55	9.35 815	58	0.64 185	9.98 898	9
52	9.34 768	55	9.35 873	58	0.64 127	9.98 895	8
53	9.34 824	55	9.35 931	58	0.64 068	9.98 892	7
54	9.34 879	55	9.35 989	58	0.64 010	9.98 890	6
55	9.34 934	55	9.36 047	58	0.63 952	9.98 887	5
56	9.34 989	55	9.36 105	57	0.63 894	9.98 884	4
57	9.35 044	54	9.36 163	58	0.63 837	9.98 881	3
58	9.35 099	55	9.36 221	57	0.63 779	9.98 878	2
59	9.35 154	55	9.36 278	58	0.63 721	9.98 875	1
60	9.35 209		9.36 336		0.63 663	9.98 872	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	62	61	61
6	6.2	6.1	6.1
7	7.2	7.2	7.1
8	8.2	8.2	8.1
9	9.3	9.2	9.1
10	10.3	10.2	10.1
20	20.6	20.5	20.3
30	31.0	30.7	30.5
40	41.3	41.0	40.6
50	51.6	51.2	50.8

	60	60	59	59
6	6.0	6.0	5.9	5.9
7	7.0	7.0	6.9	6.9
8	8.0	8.0	7.9	7.8
9	9.1	9.0	8.9	8.8
10	10.1	10.0	9.9	9.8
20	20.1	20.0	19.8	19.6
30	30.2	30.0	29.7	29.5
40	40.3	40.0	39.6	39.3
50	50.4	50.0	49.6	49.1

	58	58	57	57
6	5.8	5.8	5.7	5.7
7	6.8	6.7	6.7	6.6
8	7.8	7.7	7.6	7.6
9	8.8	8.7	8.6	8.5
10	9.7	9.6	9.6	9.5
20	19.5	19.3	19.1	19.0
30	29.2	29.0	28.7	28.5
40	39.0	38.6	38.3	38.0
50	48.7	48.3	47.9	47.5

	56	56	55	55
6	5.6	5.6	5.5	5.5
7	6.6	6.5	6.5	6.4
8	7.5	7.4	7.4	7.3
9	8.5	8.4	8.3	8.2
10	9.4	9.3	9.2	9.1
20	18.8	18.6	18.5	18.3
30	28.2	28.0	27.7	27.5
40	37.6	37.3	37.0	36.6
50	47.1	46.6	46.2	45.8

	54	3	2
6	5.4	0.3	0.2
7	6.3	0.3	0.3
8	7.2	0.4	0.3
9	8.2	0.4	0.4
10	9.1	0.5	0.4
20	18.1	1.0	0.8
30	27.2	1.5	1.2
40	36.3	2.0	1.6
50	45.4	2.5	2.1

P. P.

77°

13°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	
0	9.35 209	54	9.36 336	57	0.63 663	9.98 872	60
1	9.35 263	54	9.36 394	57	0.63 606	9.98 869	59
2	9.35 318	54	9.36 451	57	0.63 548	9.98 866	58
3	9.35 372	54	9.36 509	57	0.63 491	9.98 863	57
4	9.35 427	54	9.36 566	57	0.63 433	9.98 860	56
5	9.35 481	54	9.36 623	57	0.63 376	9.98 858	55
6	9.35 536	54	9.36 681	57	0.63 319	9.98 855	54
7	9.35 590	54	9.36 738	57	0.63 262	9.98 852	53
8	9.35 644	54	9.36 795	57	0.63 204	9.98 849	52
9	9.35 698	54	9.36 852	57	0.63 147	9.98 846	51
10	9.35 752	54	9.36 909	57	0.63 090	9.98 843	50
11	9.35 806	54	9.36 966	56	0.63 033	9.98 840	49
12	9.35 860	53	9.37 023	57	0.62 977	9.98 837	48
13	9.35 914	54	9.37 080	56	0.62 920	9.98 834	47
14	9.35 968	53	9.37 136	57	0.62 863	9.98 831	46
15	9.36 021	53	9.37 193	56	0.62 806	9.98 828	45
16	9.36 075	53	9.37 250	56	0.62 750	9.98 825	44
17	9.36 128	53	9.37 306	56	0.62 693	9.98 822	43
18	9.36 182	53	9.37 363	56	0.62 637	9.98 819	42
19	9.36 235	53	9.37 419	56	0.62 580	9.98 816	41
20	9.36 289	53	9.37 475	56	0.62 524	9.98 813	40
21	9.36 342	53	9.37 532	56	0.62 468	9.98 810	39
22	9.36 395	53	9.37 588	56	0.62 412	9.98 807	38
23	9.36 448	53	9.37 644	56	0.62 356	9.98 804	37
24	9.36 501	53	9.37 700	56	0.62 299	9.98 801	36
25	9.36 554	53	9.37 756	55	0.62 243	9.98 798	35
26	9.36 607	53	9.37 812	56	0.62 188	9.98 795	34
27	9.36 660	52	9.37 868	56	0.62 132	9.98 792	33
28	9.36 713	53	9.37 924	55	0.62 076	9.98 789	32
29	9.36 766	52	9.37 979	56	0.62 020	9.98 786	31
30	9.36 818	52	9.38 035	55	0.61 964	9.98 783	30
31	9.36 871	52	9.38 091	55	0.61 909	9.98 780	29
32	9.36 923	52	9.38 146	55	0.61 853	9.98 777	28
33	9.36 976	52	9.38 202	55	0.61 798	9.98 774	27
34	9.37 028	52	9.38 257	55	0.61 742	9.98 771	26
35	9.37 081	52	9.38 313	55	0.61 687	9.98 768	25
36	9.37 133	52	9.38 368	55	0.61 632	9.98 765	24
37	9.37 185	52	9.38 423	55	0.61 576	9.98 762	23
38	9.37 237	52	9.38 478	55	0.61 521	9.98 759	22
39	9.37 289	52	9.38 533	55	0.61 466	9.98 755	21
40	9.37 341	52	9.38 589	55	0.61 411	9.98 752	20
41	9.37 393	51	9.38 644	54	0.61 356	9.98 749	19
42	9.37 445	52	9.38 698	55	0.61 301	9.98 746	18
43	9.37 497	51	9.38 753	55	0.61 246	9.98 743	17
44	9.37 548	52	9.38 808	54	0.61 191	9.98 740	16
45	9.37 600	51	9.38 863	55	0.61 137	9.98 737	15
46	9.37 652	51	9.38 918	54	0.61 082	9.98 734	14
47	9.37 703	51	9.38 972	54	0.61 027	9.98 731	13
48	9.37 755	51	9.39 027	54	0.60 973	9.98 728	12
49	9.37 806	51	9.39 081	54	0.60 918	9.98 725	11
50	9.37 857	51	9.39 136	54	0.60 864	9.98 721	10
51	9.37 909	51	9.39 190	54	0.60 809	9.98 718	9
52	9.37 960	51	9.39 244	54	0.60 755	9.98 715	8
53	9.38 011	51	9.39 299	54	0.60 701	9.98 712	7
54	9.38 062	51	9.39 353	54	0.60 647	9.98 709	6
55	9.38 113	51	9.39 407	54	0.60 592	9.98 706	5
56	9.38 164	50	9.39 461	54	0.60 538	9.98 703	4
57	9.38 215	51	9.39 515	54	0.60 484	9.98 700	3
58	9.38 266	51	9.39 569	54	0.60 430	9.98 696	2
59	9.38 317	50	9.39 623	53	0.60 376	9.98 693	1
60	9.38 367		9.39 677		0.60 323	9.98 690	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	′

P. P.

	57	57	56	56
6	5.7	5.7	5.6	5.6
7	6.7	6.6	6.6	6.5
8	7.6	7.6	7.5	7.4
9	8.6	8.5	8.5	8.4
10	9.6	9.5	9.4	9.3
20	19.1	19.0	18.8	18.6
30	28.7	28.5	28.2	28.0
40	38.3	38.0	37.6	37.3
50	47.9	47.5	47.1	46.6

	55	55	54	54
6	5.5	5.5	5.4	5.4
7	6.5	6.4	6.3	6.3
8	7.4	7.3	7.2	7.2
9	8.3	8.2	8.2	8.1
10	9.2	9.1	9.1	9.0
20	18.5	18.3	18.1	18.0
30	27.7	27.5	27.2	27.0
40	37.0	36.6	36.3	36.0
50	46.2	45.8	45.4	45.0

	53	53	52	52
6	5.3	5.3	5.2	5.2
7	6.2	6.2	6.1	6.0
8	7.1	7.0	7.0	6.9
9	8.0	7.9	7.9	7.8
10	8.9	8.8	8.7	8.6
20	17.8	17.6	17.5	17.3
30	26.7	26.5	26.2	26.0
40	35.6	35.3	35.0	34.6
50	44.6	44.1	43.7	43.3

	51	51	50
6	5.1	5.1	5.0
7	6.0	5.9	5.9
8	6.8	6.8	6.7
9	7.7	7.6	7.6
10	8.6	8.5	8.4
20	17.1	17.0	16.8
30	25.7	25.5	25.2
40	34.3	34.0	33.6
50	42.9	42.5	42.1

	3	3	2
6	0.3	0.3	0.2
7	0.4	0.3	0.3
8	0.4	0.4	0.3
9	0.5	0.4	0.4
10	0.6	0.5	0.4
20	1.1	1.0	0.8
30	1.7	1.5	1.2
40	2.3	2.0	1.6
50	2.9	2.5	2.1

P. P.

14°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.38 367̂	50̂	9.39 677	54	0.60 323	9.98 690̂	3	60
1	9.38 418	50̂	9.39 731	53̂	0.60 269	9.98 687̂	3̂	59
2	9.38 468̂	50̂	9.39 784̂	54	0.60 215̂	9.98 684	3	58
3	9.38 519	50̂	9.39 838̂	53̂	0.60 161̂	9.98 681	3	57
4	9.38 569̂	50̂	9.39 892	53̂	0.60 108	9.98 678	3̂	56
5	9.38 620	50̂	9.39 945̂	53̂	0.60 054̂	9.98 674̂	3	55
6	9.38 670̂	50	9.39 999	53̂	0.60 001	9.98 671̂	3	54
7	9.38 720̂	50̂	9.40 052̂	53̂	0.59 947̂	9.98 668̂	3̂	53
8	9.38 771	50	9.40 106	53	0.59 894	9.98 665	3	52
9	9.38 821	50	9.40 159	53̂	0.59 841	9.98 662	3̂	51
10	9.38 871	50	9.40 212̂	53	0.59 787̂	9.98 658̂	3	50
11	9.38 921	50	9.40 265̂	53	0.59 734̂	9.98 655̂	3	49
12	9.38 971	50	9.40 318̂	53̂	0.59 681̂	9.98 652̂	3̂	48
13	9.39 021	50	9.40 372	53	0.59 628	9.98 649	3	47
14	9.39 071	49̂	9.40 425	53	0.59 575	9.98 646	3̂	46
15	9.39 120̂	50	9.40 478	53	0.59 522	9.98 642̂	3	45
16	9.39 170̂	49̂	9.40 531	52̂	0.59 469	9.98 639̂	3	44
17	9.39 220	49̂	9.40 583̂	53	0.59 416̂	9.98 636̂	3̂	43
18	9.39 269̂	49̂	9.40 636̂	52̂	0.59 363̂	9.98 633	3	42
19	9.39 319	49̂	9.40 689	53	0.59 311	9.98 630	3̂	41
20	9.39 368̂	49̂	9.40 742	52̂	0.59 258	9.98 626̂	3	40
21	9.39 418	49̂	9.40 794̂	52̂	0.59 205̂	9.98 623̂	3̂	39
22	9.39 467̂	49	9.40 847	52̂	0.59 153	9.98 620	3	38
23	9.39 516̂	49̂	9.40 899̂	52̂	0.59 100̂	9.98 617	3̂	37
24	9.39 566	49	9.40 952	52̂	0.59 048	9.98 613̂	3	36
25	9.39 615	49	9.41 004̂	52̂	0.58 995̂	9.98 610̂	3̂	35
26	9.39 664	49	9.41 057	52	0.58 943	9.98 607	3	34
27	9.39 713	49	9.41 109	52̂	0.58 891	9.98 604	3̂	33
28	9.39 762	49	9.41 161̂	52	0.58 838̂	9.98 600̂	3	32
29	9.39 811	49	9.41 213̂	52̂	0.58 786̂	9.98 597̂	3̂	31
30	9.39 860	49	9.41 266	52	0.58 734	9.98 594	3	30
31	9.39 909	48̂	9.41 318	52	0.58 682	9.98 591	3̂	29
32	9.39 957̂	48̂	9.41 370	52	0.58 630	9.98 587̂	3	28
33	9.40 006	49	9.41 422	52	9.58 578	9.98 584̂	3̂	27
34	9.40 055	48̂	9.41 474	51̂	0.58 526	9.98 581	3	26
35	9.40 103̂	48̂	9.41 525̂	52	0.58 474̂	9.98 578	3̂	25
36	9.40 152	48̂	9.41 577̂	52	0.58 422̂	9.98 574̂	3̂	24
37	9.40 200̂	48̂	9.41 629̂	51̂	0.58 370̂	9.98 571	3	23
38	9.40 249	48	9.41 681	51̂	0.58 319	9.98 568	3̂	22
39	9.40 297	48̂	9.41 732̂	51̂	0.58 267̂	9.98 564̂	3	21
40	9.40 345̂	48̂	9.41 784	52	0.58 216	9.98 561̂	3̂	20
41	9.40 394	48	9.41 836	51̂	0.58 164	9.98 558	3̂	19
42	9.40 442	48	9.41 887̂	51	0.58 112̂	9.98 554̂	3	18
43	9.40 490	48	9.41 938̂	51̂	0.58 061̂	9.98 551̂	3̂	17
44	9.40 538	48	9.41 990	51̂	0.58 010	9.98 548	3̂	16
45	9.40 586	48	9.42 041̂	51	0.57 958̂	9.98 544̂	3	15
46	9.40 634	48	9.42 092̂	51̂	0.57 907̂	9.98 541̂	3̂	14
47	9.40 682	48	9.42 144	51	0.57 856	9.98 538	3̂	13
48	9.40 730	47̂	9.42 195	51̂	0.57 805	9.98 534̂	3	12
49	9.40 777̂	48	9.42 246̂	51	0.57 753̂	9.98 531̂	3̂	11
50	9.40 825̂	47̂	9.42 297̂	51	0.57 702̂	9.98 528	3̂	10
51	9.40 873	47̂	9.42 348̂	51	0.57 651̂	9.98 524̂	3	9
52	9.40 920̂	47̂	9.42 399̂	51	0.57 600̂	9.98 521̂	3̂	8
53	9.40 968	47̂	9.42 450̂	50̂	0.57 549̂	9.98 518	3̂	7
54	9.41 015̂	47̂	9.42 501	51	0.57 499	9.98 514̂	3	6
55	9.41 063	47̂	9.42 552	50̂	0.57 448	9.98 511̂	3̂	5
56	9.41 110̂	47̂	9.42 602̂	51	0.57 397̂	9.98 508	3̂	4
57	9.41 158	47	9.42 653̂	50̂	0.57 346̂	9.98 504̂	3̂	3
58	9.41 205	47̂	9.42 704	50̂	0.57 296	9.98 501	3	2
59	9.41 252̂	47	9.42 754̂	50̂	0 57 245̂	9.98 498	3̂	1
60	9.41 299̂		9.42 805		0.57 195	9.98 494̂		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	54	53̂	53
6	5.4	5.3̂	5.3
7	6.3	6.2̂	6.2
8	7.2	7.1̂	7.0̂
9	8.1	8.0	7.9̂
10	9.0	8.9	8.8̂
20	18.0	17.8̂	17.6̂
30	27.0	26.7̂	26.5
40	36.0	35.6̂	35.3̂
50	45.0	44.6	44.1̂

	52̂	52	51̂	51
6	5.2̂	5.2	5.1̂	5.1
7	6.1	6.0̂	6.0	5.9̂
8	7.0	6.9̂	6.8̂	6.8
9	7.9	7.8	7.7	7.6̂
10	8.7̂	8.6̂	8.6	8.5
20	17.5	17.3̂	17.1̂	17.0
30	26.2̂	26.0	25.7̂	25.5
40	35.0	34.6̂	34.3̂	34.0
50	43.7̂	43.3̂	42.9	42.5

	50̂	50	49̂	49
6	5.0̂	5.0	4.9̂	4.9
7	5.9	5.8̂	5.8	5.7
8	6.7̂	6.6̂	6.6	6.5̂
9	7.6	7.5	7.4	7.3̂
10	8.4	8.3̂	8.2̂	8.1̂
20	16.8̂	16.6̂	16.5	16.3̂
30	25.2̂	25.0	24.7̂	24.5
40	33.6̂	33.3̂	33.0	32.6̂
50	42.1	41.6̂	41.2̂	40.8̂

	48̂	48	47̂	47
6	4.8̂	4.8	4.7̂	4.7
7	5.6̂	5.6	5.5̂	5.5
8	6.4̂	6.4	6.3̂	6.2̂
9	7.3	7.2	7.1	7.0̂
10	8.1	8.0	7.9	7.8̂
20	16.1̂	16.0	15.8̂	15.6̂
30	24.2̂	24.0	23.7̂	23.5
40	32.3̂	32.0	31.6̂	31.3̂
50	40.4	40.0	39.6	39.1̂

	3̂	3
6	0.3̂	0.3
7	0.4	0.3̂
8	0.4̂	0.4
9	0.5̂	0.4̂
10	0.6	0.5
20	1.1̂	1.0
30	1.7̂	1.5
40	2.3̂	2.0
50	2.9	2.5

P. P.

75°

15°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.41 299	47	9.42 805	51	0.57 195	9.98 494	3	60
1	9.41 346	47	9.42 856	50	0.57 144	9.98 491	3	59
2	9.41 394	47	9.42 906	50	0.57 094	9.98 487	3	58
3	9.41 441	47	9.42 956	50	0.57 043	9.98 484	3	57
4	9.41 488	46	9.43 007	50	0.56 993	9.98 481	3	56
5	9.41 534	47	9.43 057	50	0.56 942	9.98 477	3	55
6	9.41 581	47	9.43 107	50	0.56 892	9.98 474	3	54
7	9.41 628	46	9.43 157	50	0.56 842	9.98 470	3	53
8	9.41 675	46	9.43 208	50	0.56 792	9.98 467	3	52
9	9.41 721	47	9.43 258	50	0.56 742	9.98 464	3	51
10	9.41 768	46	9.43 308	50	0.56 692	9.98 460	3	50
11	9.41 815	46	9.43 358	50	0.56 642	9.98 457	3	49
12	9.41 861	46	9.43 408	50	0.56 592	9.98 453	3	48
13	9.41 908	46	9.43 458	50	0.56 542	9.98 450	3	47
14	9.41 954	46	9.43 508	49	0.56 492	9.98 446	3	46
15	9.42 000	46	9.43 557	50	0.56 442	9.98 443	3	45
16	9.42 047	46	9.43 607	49	0.56 393	9.98 439	3	44
17	9.42 093	46	9.43 657	49	0.56 343	9.98 436	3	43
18	9.42 139	46	9.43 706	50	0.56 293	9.98 433	3	42
19	9.42 185	46	9.43 756	49	0.56 243	9.98 429	3	41
20	9.42 232	46	9.43 806	49	0.56 194	9.98 426	3	40
21	9.42 278	46	9.43 855	49	0.56 144	9.98 422	3	39
22	9.42 324	45	9.43 905	49	0.56 095	9.98 419	3	38
23	9.42 369	46	9.43 954	49	0.56 045	9.98 415	3	37
24	9.42 415	46	9.44 003	49	0.55 996	9.98 412	3	36
25	9.42 461	46	9.44 053	49	0.55 947	9.98 408	3	35
26	9.42 507	45	9.44 102	49	0.55 898	9.98 405	3	34
27	9.42 553	45	9.44 151	49	0.55 848	9.98 401	3	33
28	9.42 598	46	9.44 200	49	0.55 799	9.98 398	3	32
29	9.42 644	45	9.44 249	49	0.55 750	9.98 394	3	31
30	9.42 690	45	9.44 299	49	0.55 701	9.98 391	3	30
31	9.42 735	45	9.44 348	49	0.55 652	9.98 387	3	29
32	9.42 781	45	9.44 397	49	0.55 603	9.98 384	3	28
33	9.42 826	45	9.44 446	48	0.55 554	9.98 380	3	27
34	9.42 871	45	9.44 494	49	0.55 505	9.98 377	3	26
35	9.42 917	45	9.44 543	49	0.55 456	9.98 373	3	25
36	9.42 962	45	9.44 592	48	0.55 407	9.98 370	3	24
37	9.43 007	45	9.44 641	49	0.55 359	9.98 366	3	23
38	9.43 052	45	9.44 690	48	0.55 310	9.98 363	3	22
39	9.43 098	45	9.44 738	48	0.55 261	9.98 359	3	21
40	9.43 143	45	9.44 787	48	0.55 213	9.98 356	3	20
41	9.43 188	45	9.44 835	48	0.55 164	9.98 352	4	19
42	9.43 233	45	9.44 884	48	0.55 116	9.98 348	3	18
43	9.43 278	44	9.44 932	48	0.55 067	9.98 345	3	17
44	9.43 322	45	9.44 981	48	0.55 019	9.98 341	3	16
45	9.43 367	44	9.45 029	48	0.54 970	9.98 338	3	15
46	9.43 412	45	9.45 077	48	0.54 922	9.98 334	3	14
47	9.43 457	44	9.45 126	48	0.54 874	9.98 331	3	13
48	9.43 501	44	9.45 174	48	0.54 825	9.98 327	3	12
49	9.43 546	45	9.45 222	48	0.54 777	9.98 324	4	11
50	9.43 591	44	9.45 270	48	0.54 729	9.98 320	3	10
51	9.43 635	44	9.45 318	48	0.54 681	9.98 316	3	9
52	9.43 680	44	9.45 367	48	0.54 633	9.98 313	3	8
53	9.43 724	44	9.45 415	48	0.54 585	9.98 309	3	7
54	9.43 768	44	9.45 463	47	0.54 537	9.98 306	4	6
55	9.43 813	44	9.45 510	48	0.54 489	9.98 302	3	5
56	9.43 857	44	9.45 558	48	0.54 441	9.98 298	3	4
57	9.43 901	44	9.45 606	47	0.54 393	9.98 295	3	3
58	9.43 945	44	9.45 654	48	0.54 346	9.98 291	3	2
59	9.43 989	44	9.45 702	47	0.54 298	9.98 288	4	1
60	9.44 034		9.45 749		0.54 250	9.98 284		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	50̂	50
6	5.0	5.0
7	5.9	5.8
8	6.7	6.6
9	7.6	7.5
10	8.4	8.3
20	16.8	16.6
30	25.2	25.0
40	33.6	33.3
50	42.1	41.6

	49̂	49	48̂	48
6	4.9	4.9	4.8	4.8
7	5.8	5.7	5.6	5.6
8	6.6	6.5	6.4	6.4
9	7.4	7.3	7.3	7.2
10	8.2	8.1	8.1	8.0
20	16.5	16.3	16.1	16.0
30	24.7	24.5	24.2	24.0
40	33.0	32.6	32.3	32.0
50	41.2	40.8	40.4	40.0

	47̂	47	46̂	46
6	4.7	4.7	4.6	4.6
7	5.5	5.5	5.4	5.3
8	6.3	6.2	6.2	6.1
9	7.1	7.0	7.0	6.9
10	7.9	7.8	7.7	7.6
20	15.8	15.6	15.5	15.3
30	23.7	23.5	23.2	23.0
40	31.6	31.3	31.0	30.6
50	39.6	39.1	38.7	38.3

	45̂	45	44̂	44
6	4.5	4.5	4.4	4.4
7	5.3	5.2	5.2	5.1
8	6.0	6.0	5.9	5.8
9	6.8	6.7	6.7	6.6
10	7.6	7.5	7.4	7.3
20	15.1	15.0	14.8	14.6
30	22.7	22.5	22.2	22.0
40	30.3	30.0	29.6	29.3
50	37.9	37.5	37.1	36.6

	4	3̂	3
6	0.4	0.3	0.3
7	0.4	0.4	0.3
8	0.5	0.4	0.4
9	0.6	0.5	0.4
10	0.6	0.6	0.5
20	1.3	1.1	1.0
30	2.0	1.7	1.5
40	2.6	2.3	2.0
50	3.3	2.9	2.5

P. P.

74°

16°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.44 034	44	9.45 749̂	48	0.54 250̂	9.98 284	3̂	60
1	9.44 078	44	9.45 797̂	47̂	0.54 202̂	9.98 280̂	3̂	59
2	9.44 122	44	9.45 845	47̂	0.54 155	9.98 277	3̂	58
3	9.44 166	43̂	9.45 892̂	47̂	0.54 107̂	9.98 273̂	4	57
4	9.44 209̂	44	9.45 940	47̂	0.54 060	9.98 269̂	3̂	56
5	9.44 253̂	44	9.45 987̂	47̂	0.54 012̂	9.98 266	3̂	55
6	9.44 297̂	43̂	9.46 035	47̂	0.53 965	9.98 262̂	4	54
7	9.44 341	43̂	9.46 082̂	47	0.53 917̂	9.98 258̂	3̂	53
8	9.44 384̂	44	9.46 129̂	47̂	0.53 870̂	9.98 255	3̂	52
9	9.44 428̂	43̂	9.46 177	47	0.53 823	9.98 251̂	4	51
10	9.44 472	43̂	9.46 224	47̂	0.53 776	9.98 247̂	3̂	50
11	9.44 515̂	43̂	9.46 271̂	47	0.53 728̂	9.98 244	3̂	49
12	9.44 559	43̂	9.46 318̂	47̂	0.53 681̂	9.98 240̂	4	48
13	9.44 602̂	43̂	9.46 366	47	0.53 634	9.98 236̂	3̂	47
14	9.44 646	43̂	9.46 413	47	0.53 587	9.98 233	3̂	46
15	9.44 689̂	43	9.46 460	47	0.53 540	9.98 229̂	4	45
16	9.44 732̂	43̂	9.46 507	47	0.53 493	9.98 225̂	3̂	44
17	9.44 776	43	9.46 554	47	0.53 446	9.98 222	3̂	43
18	9.44 819	43̂	9.46 601	46̂	0.53 399	9.98 218̂	4	42
19	9.44 862̂	43	9.46 647̂	47	0.53 352̂	9.98 214̂	3̂	41
20	9.44 905̂	43	9.46 694̂	47	0.53 305̂	9.98 211	4	40
21	9.44 948̂	43	9.46 741̂	46̂	0.53 258̂	9.98 207	3̂	39
22	9.44 991̂	43	9.46 788	46̂	0.53 212	9.98 203̂	3̂	38
23	9.45 034̂	43	9.46 834̂	47	0.53 165̂	9.98 200	4	37
24	9.45 077̂	43	9.46 881̂	46̂	0.53 118̂	9.98 196	3̂	36
25	9.45 120̂	42̂	9.46 928	46̂	0.53 072	9.98 192̂	4	35
26	9.45 163	43	9.46 974̂	46̂	0.53 025̂	9.98 188̂	3̂	34
27	9.45 206	43	9.47 021	46̂	0.52 979	9.98 185	4	33
28	9.45 249	42̂	9.47 067̂	46̂	0.52 932̂	9.98 181	3̂	32
29	9.45 291̂	42̂	9.47 114	46̂	0.52 886	9.98 177̂	4	31
30	9.45 334	43	9.47 160̂	46̂	0.52 839̂	9.98 173̂	3̂	30
31	9.45 377	42̂	9.47 207	46	0.52 793	9.98 170	4	29
32	9.45 419̂	42̂	9.47 253	46̂	0.52 747	9.98 166	3̂	28
33	9.45 462	42̂	9.47 299̂	46	0.52 700̂	9.98 162̂	4	27
34	9.45 504̂	42̂	9.47 345̂	46̂	0.52 654̂	9.98 158̂	3̂	26
35	9.45 547	42̂	9.47 392	46	0.52 608	9.98 155	4	25
36	9.45 589̂	42	9.47 438	46	0.52 562	9.98 151	3̂	24
37	9.45 631̂	42̂	9.47 484	46̂	0.52 516	9.98 147̂	4	23
38	9.45 674	42	9.47 530̂	46	0.52 469̂	9.98 143̂	3̂	22
39	9.45 716	42̂	9.47 576̂	46	0.52 423̂	9.98 140	4	21
40	9.45 758̂	42	9.47 622̂	46	0.52 377̂	9.98 136	3̂	20
41	9.45 800̂	42	9.47 668̂	45̂	0.52 331̂	9.98 132̂	4	19
42	9.45 842̂	42̂	9.47 714	46	0.52 286	9.98 128̂	4	18
43	9.45 885	42	9.47 760	46	0.52 240	9.98 124̂	3̂	17
44	9.45 927	42	9.47 806	45̂	0.52 194	9.98 121	4	16
45	9.45 969	42	9.47 851̂	46	0.52 148̂	9.98 117	3̂	15
46	9.46 011	41̂	9.47 897̂	45̂	0.52 102̂	9.98 113̂	4	14
47	9.46 052̂	42	9.47 943	46	0.52 057	9.98 109̂	4	13
48	9.46 094̂	42	9.47 989	45̂	0.52 011	9.98 105̂	3̂	12
49	9.46 136̂	41̂	9.48 034̂	45̂	0.51 965̂	9.98 102	4	11
50	9.46 178	42	9.48 080	45̂	0.51 920	9.98 098	4	10
51	9.46 220	41̂	9.48 125̂	45̂	0.51 874̂	9.98 094	3̂	9
52	9.46 261̂	41̂	9.48 171	45̂	0.51 829	9.98 090̂	4	8
53	9.46 303	42	9.48 216̂	45̂	0.51 783̂	9.98 086̂	4	7
54	9.46 345	41̂	9.48 262	45̂	0.51 738	9.98 082̂	3̂	6
55	9.46 386̂	41̂	9.48 307̂	45̂	0.51 692̂	9.98 079	4	5
56	9.46 428	41̂	9.48 353	45	0.51 647	9.98 075	4	4
57	9.46 469̂	41̂	9.48 398	45̂	0.51 602	9.98 071	3̂	3
58	9.46 511	41	9.48 443̂	45	0.51 556̂	9.98 067̂	4	2
59	9.46 552	41̂	9.48 488̂	45̂	0.51 511̂	9.98 063̂	4	1
60	9.46 593̂		9.48 534		0.51 466	9.98 059̂		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	48	47̂	47
6	4.8	4.7̂	4.7
7	5.6	5.5̂	5.5
8	6.4	6.3̂	6.2̂
9	7.2	7.1	7.0̂
10	8.0	7.9	7.8̂
20	16.0	15.8̂	15.6̂
30	24.0	23.7̂	23.5
40	32.0	31.6̂	31.3̂
50	40.0	39.6	39.1̂

	46̂	46	45̂	45
6	4.6̂	4.6	4.5̂	4.5
7	5.4	5.3̂	5.3	5.2̂
8	6.2	6.1̂	6.0̂	6.0
9	7.0	6.9	6.8	6.7̂
10	7.7̂	7.6̂	7.6	7.5
20	15.5	15.3̂	15.1̂	15.0
30	23.2̂	23.0	22.7̂	22.5
40	31.0	30.6̂	30.3̂	30.0
50	38.7̂	38.3̂	37.9	37.5

	44	43̂	43
6	4.4	4.3̂	4.3
7	5.1̂	5.1	5.0
8	5.8̂	5.8	5.7̂
9	6.6	6.5	6.4̂
10	7.3̂	7.2̂	7.1̂
20	14.6̂	14.5	14.3̂
30	22.0	21.7̂	21.5
40	29.3̂	29.0	28.6̂
50	36.6̂	36.2̂	35.8̂

	42̂	42	41̂	41
6	4.2̂	4.2	4.1̂	4.1
7	4.9̂	4.9	4.8̂	4.8
8	5.6̂	5.6	5.5̂	5.4̂
9	6.4	6.3	6.2	6.1̂
10	7.1	7.0	6.9	6.8̂
20	14.1̂	14.0	13.8̂	13.6̂
30	21.2̂	21.0	20.7̂	20.5
40	28.3̂	28.0	27.6̂	27.3̂
50	35.4	35.0	34.6	34.1̂

	4	3̂
6	0.4	0.3̂
7	0.4̂	0.4
8	0.5̂	0.4̂
9	0.6	0.5
10	0.6̂	0.6
20	1.3̂	1.1̂
30	2.0	1.7̂
40	2.6̂	2.3̂
50	3.3̂	2.9

P. P.

73°

17°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.46 593	41	9.48 534	45	0.51 466	9.98 059	3	60
1	9.46 635	41	9.48 579	45	0.51 421	9.98 056	4	59
2	9.46 676	41	9.48 624	45	0.51 376	9.98 052	4	58
3	9.46 717	41	9.48 669	45	0.51 330	9.98 048	4	57
4	9.46 758	41	9.48 714	45	0.51 285	9.98 044	3	56
5	9.46 799	41	9.48 759	45	0.51 240	9.98 040	4	55
6	9.46 840	41	9.48 804	44	0.51 195	9.98 036	4	54
7	9.46 881	41	9.48 849	45	0.51 151	9.98 032	4	53
8	9.46 922	41	9.48 894	45	0.51 106	9.98 028	4	52
9	9.46 963	41	9.48 939	45	0.51 061	9.98 024	3	51
10	9.47 004	41	9.48 984	44	0.51 016	9.98 021	4	50
11	9.47 045	41	9.49 028	45	0.50 971	9.98 017	4	49
12	9.47 086	40	9.49 073	44	0.50 926	9.98 013	4	48
13	9.47 127	41	9.49 118	44	0.50 882	9.98 009	4	47
14	9.47 168	40	9.49 162	45	0.50 837	9.98 005	4	46
15	9.47 208	40	9.49 207	44	0.50 792	9.98 001	3	45
16	9.47 249	41	9.49 252	44	0.50 748	9.97 997	4	44
17	9.47 290	40	9.49 296	44	0.50 703	9.97 993	4	43
18	9.47 330	40	9.49 341	44	0.50 659	9.97 989	4	42
19	9.47 371	40	9.49 385	44	0.50 614	9.97 985	4	41
20	9.47 411	40	9.49 430	44	0.50 570	9.97 981	4	40
21	9.47 452	40	9.49 474	44	0.50 525	9.97 977	4	39
22	9.47 492	40	9.49 518	44	0.50 481	9.97 973	4	38
23	9.47 532	40	9.49 563	44	0.50 437	9.97 969	3	37
24	9.47 573	40	9.49 607	44	0.50 392	9.97 966	4	36
25	9.47 613	40	9.49 651	44	0.50 348	9.97 962	4	35
26	9.47 653	40	9.49 695	44	0.50 304	9.97 958	4	34
27	9.47 694	40	9.49 740	44	0.50 260	9.97 954	4	33
28	9.47 734	40	9.49 784	44	0.50 216	9.97 950	4	32
29	9.47 774	40	9.49 828	44	0.50 172	9.97 946	4	31
30	9.47 814	40	9.49 872	44	0.50 128	9.97 942	4	30
31	9.47 854	40	9.49 916	44	0.50 083	9.97 938	4	29
32	9.47 894	40	9.49 960	43	0.50 039	9.97 934	4	28
33	9.47 934	40	9.50 004	44	0.49 996	9.97 930	4	27
34	9.47 974	40	9.50 048	44	0.49 952	9.97 926	4	26
35	9.48 014	40	9.50 092	44	0.49 908	9.97 922	4	25
36	9.48 054	39	9.50 136	43	0.49 864	9.97 918	4	24
37	9.48 093	40	9.50 179	44	0.49 820	9.97 914	4	23
38	9.48 133	39	9.50 223	43	0.49 776	9.97 910	4	22
39	9.48 173	40	9.50 267	44	0.49 733	9.97 906	4	21
40	9.48 213	39	9.50 311	43	0.49 689	9.97 902	4	20
41	9.48 252	39	9.50 354	43	0.49 645	9.97 898	4	19
42	9.48 292	39	9.50 398	44	0.49 602	9.97 894	4	18
43	9.48 331	39	9.50 442	43	0.49 558	9.97 890	4	17
44	9.48 371	39	9.50 485	43	0.49 514	9.97 886	4	16
45	9.48 410	39	9.50 529	43	0.49 471	9.97 881	4	15
46	9.48 450	39	9.50 572	43	0.49 427	9.97 877	4	14
47	9.48 489	39	9.50 616	43	0.49 384	9.97 873	4	13
48	9.48 529	39	9.50 659	43	0.49 340	9.97 869	4	12
49	9.48 568	39	9.50 702	43	0.49 297	9.97 865	4	11
50	9.48 607	39	9.50 746	43	0.49 254	9.97 861	4	10
51	9.48 646	39	9.50 789	43	0.49 210	9.97 857	4	9
52	9,48 686	39	9.50 832	43	0.49 167	9.97 853	4	8
53	9.48 725	39	9.50 876	43	0.49 124	9.97 849	4	7
54	9.48 764	39	9.50 919	43	0.49 081	9.97 845	4	6
55	9.48 803	39	9.50 962	43	0.49 038	9.97 841	4	5
56	9.48 842	39	9.51 005	43	0.48 994	9.97 837	4	4
57	9.48 881	39	9.51 048	43	0.48 951	9.97 833	4	3
58	9.48 920	39	9.51 091	43	0.48 908	9.97 829	4	2
59	9.48 959	38	9.51 134	43	0.48 865	9.97 824	4	1
60	9.48 998		9.51 177		0.48 822	9.97 820		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	45̂	45	44̂	44
6	4.5	4.5	4.4	4.4
7	5.3	5.2	5.2	5.1
8	6.0	6.0	5.9	5.8
9	6.8	6.7	6.7	6.6
10	7.6	7.5	7.4	7.3
20	15.1	15.0	14.8	14.6
30	22.7	22.5	22.2	22.0
40	30.3	30.0	29.6	29.3
50	37.9	37.5	37.1	36.6

	43̂	43
6	4.3	4.3
7	5.1	5.0
8	5.8	5.7
9	6.5	6.4
10	7.2	7.1
20	14.5	14.3
30	21.7	21.5
40	29.0	28.6
50	36.2	35.8

	41̂	41	40̂	40
6	4.1	4.1	4.0	4.0
7	4.8	4.8	4.7	4.6
8	5.5	5.4	5.4	5.3
9	6.2	6.1	6.1	6.0
10	6.9	6.8	6.7	6.6
20	13.8	13.6	13.5	13.3
30	20.7	20.5	20.2	20.0
40	27.6	27.3	27.0	26.6
50	34.6	34.1	33.7	33.3

	39̂	39	38̂
6	3.9	3.9	3.8
7	4.6	4.5	4.5
8	5.2	5.2	5.1
9	5.9	5.8	5.8
10	6.6	6.5	6.4
20	13.1	13.0	12.8
30	19.7	19.5	19.2
40	26.3	26.0	25.6
50	32.9	32.5	32.1

	4̂	4	3̂
6	0.4	0.4	0.3
7	0.5	0.4	0.4
8	0.6	0.5	0.4
9	0.7	0.6	0.5
10	0.7	0.6	0.6
20	1.5	1.3	1.1
30	2.2	2.0	1.7
40	3.0	2.6	2.3
50	3.7	3.3	2.9

P. P.

72°

18°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.48 998	39	9.51 177	43	0.48 822	9.97 820	4	60
1	9.49 037	39	9.51 220	43	0.48 779	9.97 816	4	59
2	9.49 076	38	9.51 263	43	0.48 736	9.97 812	4	58
3	9.49 114	39	9.51 306	43	0.48 693	9.97 808	4	57
4	9.49 153	38	9.51 349	42	0.48 650	9.97 804	4	56
5	9.49 192	39	9.51 392	43	0.48 608	9.97 800	4	55
6	9.49 231	38	9.51 435	42	0.48 565	9.97 796	4	54
7	9.49 269	38	9.51 477	43	0.48 522	9.97 792	4	53
8	9.49 308	38	9.51 520	42	0.48 479	9.97 787	4	52
9	9.49 346	38	9.51 563	42	0.48 437	9.97 783	4	51
10	9.49 385	38	9.51 605	43	0.48 394	9.97 779	4	50
11	9.49 423	38	9.51 648	42	0.48 351	9.97 775	4	49
12	9.49 462	38	9.51 691	42	0.48 309	9.97 771	4	48
13	9.49 500	38	9.51 733	42	0.48 266	9.97 767	4	47
14	9.49 539	38	9.51 776	42	0.48 224	9.97 763	4	46
15	9.49 577	38	9.51 818	42	0.48 181	9.97 758	4	45
16	9.49 615	38	9.51 861	42	0.48 139	9.97 754	4	44
17	9.49 653	38	9.51 903	42	0.48 096	9.97 750	4	43
18	9.49 692	38	9.51 946	42	0.48 054	9.97 746	4	42
19	9.49 730	38	9.51 988	42	0.48 012	9.97 742	4	41
20	9.49 768	38	9.52 030	42	0.47 969	9.97 737	4	40
21	9.49 806	38	9.52 073	42	0.47 927	9.97 733	4	39
22	9.49 844	38	9.52 115	42	0.47 885	9.97 729	4	38
23	9.49 882	38	9.52 157	42	0.47 842	9.97 725	4	37
24	9.49 920	38	9.52 199	42	0.47 800	9.97 721	4	36
25	9.49 958	38	9.52 241	42	0.47 758	9.97 716	4	35
26	9.49 996	37	9.52 284	42	0.47 716	9.97 712	4	34
27	9.50 034	38	9.52 326	42	0.47 674	9.97 708	4	33
28	9.50 072	38	9.52 368	42	0.47 632	9.97 704	4	32
29	9.50 110	37	9.52 410	42	0.47 590	9.97 700	4	31
30	9.50 147	38	9.52 452	42	0.47 548	9.97 695	4	30
31	9.50 185	37	9.52 494	42	0.47 506	9.97 691	4	29
32	9.50 223	37	9.52 536	42	0.47 464	9.97 687	4	28
33	9.50 260	38	9.52 578	41	0.47 422	9.97 683	4	27
34	9.50 298	37	9.52 619	42	0.47 380	9.97 678	4	26
35	9.50 336	37	9.52 661	42	0.47 338	9.97 674	4	25
36	9.50 373	37	9.52 703	41	0.47 296	9.97 670	4	24
37	9.50 411	37	9.52 745	42	0.47 255	9.97 666	4	23
38	9.50 448	37	9.52 787	41	0.47 213	9.97 661	4	22
39	9.50 486	37	9.52 828	41	0.47 171	9.97 657	4	21
40	9.50 523	37	9.52 870	42	0.47 130	9.97 653	4	20
41	9.50 561	37	9.52 912	41	0.47 088	9.97 649	4	19
42	9.50 598	37	9.52 953	41	0.47 046	9.97 644	4	18
43	9.50 635	37	9.52 995	41	0.47 005	9.97 640	4	17
44	9.50 672	37	9.53 036	41	0.46 963	9.97 636	4	16
45	9.50 710	37	9.53 078	41	0.46 922	9.97 632	4	15
46	9.50 747	37	9.53 119	41	0.46 880	9.97 627	4	14
47	9.50 784	37	9.53 161	41	0.46 839	9.97 623	4	13
48	9.50 821	37	9.53 202	41	0.46 797	9.97 619	4	12
49	9.50 858	37	9.53 244	41	0.46 756	9.97 614	4	11
50	9.50 895	37	9.53 285	41	0.46 714	9.97 610	4	10
51	9.50 932	37	9.53 326	41	0.46 673	9.97 606	4	9
52	9.50 969	37	9.53 368	41	0.46 632	9.97 601	4	8
53	9.51 006	37	9.53 409	41	0.46 591	9.97 597	4	7
54	9.51 043	37	9.53 450	41	0.46 549	9.97 593	4	6
55	9.51 080	36	9.53 491	41	0.46 508	9.97 588	4	5
56	9.51 117	37	9.53 533	41	0.46 467	9.97 584	4	4
57	9.51 154	36	9.53 574	41	0.46 426	9.97 580	4	3
58	9.51 190	37	9.53 615	41	0.46 385	9.97 575	4	2
59	9.51 227	36	9.53 656	41	0.46 344	9.97 571	4	1
60	9.51 264		9.53 697		0.46 303	9.97 567		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	43	42	42
6	4.3	4.2	4.2
7	5.0	4.9	4.9
8	5.7	5.6	5.6
9	6.4	6.4	6.3
10	7.1	7.1	7.0
20	14.3	14.1	14.0
30	21.5	21.2	21.0
40	28.6	28.3	28.0
50	35.8	35.4	35.0

	41	41
6	4.1	4.1
7	4.8	4.8
8	5.5	5.4
9	6.2	6.1
10	6.9	6.8
20	13.8	13.6
30	20.7	20.5
40	27.6	27.3
50	34.6	34.1

	39	38	38
6	3.9	3.8	3.8
7	4.5	4.5	4.4
8	5.2	5.1	5.0
9	5.8	5.8	5.7
10	6.5	6.4	6.3
20	13.0	12.8	12.6
30	19.5	19.2	19.0
40	26.0	25.6	25.3
50	32.5	32.1	31.6

	37	37	36
6	3.7	3.7	3.6
7	4.4	4.3	4.2
8	5.0	4.9	4.8
9	5.6	5.5	5.5
10	6.2	6.1	6.1
20	12.5	12.3	12.1
30	18.7	18.5	18.2
40	25.0	24.6	24.3
50	31.2	30.8	30.4

	4	4
6	0.4	0.4
7	0.5	0.4
8	0.6	0.5
9	0.7	0.6
10	0.7	0.6
20	1.5	1.3
30	2.2	2.0
40	3.0	2.6
50	3.7	3.3

P. P.

71°

19°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.51 264	37	9.53 697	41	0.46 303	9.97 567	4	60
1	9.51 301	36	9.53 738	41	0.46 262	9.97 563	4	59
2	9.51 337	36	9.53 779	41	0.46 221	9.97 558	4	58
3	9.51 374	36	9.53 820	41	0.46 180	9.97 554	4	57
4	9.51 410	36	9.53 861	41	0.46 139	9.97 549	4	56
5	9.51 447	36	9.53 902	41	0.46 098	9.97 545	4	55
6	9.51 483	36	9.53 943	40	0.46 057	9.97 541	4	54
7	9.51 520	36	9.53 983	41	0.46 016	9.97 536	4	53
8	9.51 556	36	9.54 024	41	0.45 975	9.97 532	4	52
9	9.51 593	36	9.54 065	40	0.45 934	9.97 527	4	51
10	9.51 629	36	9.54 106	41	0.45 894	9.97 523	4	50
11	9.51 665	36	9.54 147	40	0.45 853	9.97 519	4	49
12	9.51 702	36	9.54 187	40	0.45 812	9.97 514	4	48
13	9.51 738	36	9.54 228	41	0.45 772	9.97 510	4	47
14	9.51 774	36	9.54 269	40	0.45 731	9.97 505	4	46
15	9.51 810	36	9.54 309	40	0.45 690	9.97 501	4	45
16	9.51 847	36	9.54 350	40	0.45 650	9.97 497	4	44
17	9.51 883	36	9.54 390	40	0.45 609	9.97 492	4	43
18	9.51 919	36	9.54 431	40	0.45 569	9.97 488	4	42
19	9.51 955	36	9.54 471	40	0.45 528	9.97 483	4	41
20	9.51 991	36	9.54 512	40	0.45 488	9.97 479	4	40
21	9.52 027	36	9.54 552	40	0.45 447	9.97 475	4	39
22	9.52 063	36	9.54 593	40	0.45 407	9.97 470	4	38
23	9.52 099	36	9.54 633	40	0.45 367	9.97 466	4	37
24	9.52 135	35	9.54 673	40	0.45 326	9.97 461	4	36
25	9.52 170	35	9.54 714	40	0.45 286	9.97 457	4	35
26	9.52 206	36	9.54 754	40	0.45 246	9.97 452	4	34
27	9.52 242	35	9.54 794	40	0.45 205	9.97 448	4	33
28	9.52 278	36	9.54 834	40	0.45 165	9.97 443	4	32
29	9.52 314	35	9.54 874	40	0.45 125	9.97 439	4	31
30	9.52 349	35	9.54 915	40	0.45 085	9.97 434	4	30
31	9.52 385	36	9.54 955	40	0.45 045	9.97 430	4	29
32	9.52 421	35	9.54 995	40	0.45 005	9.97 425	4	28
33	9.52 456	35	9.55 035	40	0.44 965	9.97 421	4	27
34	9.52 492	35	9.55 075	40	0.44 925	9.97 416	4	26
35	9.52 527	35	9.55 115	39	0.44 884	9.97 412	4	25
36	9.52 563	35	9.55 155	40	0.44 845	9.97 407	4	24
37	9.52 598	35	9.55 195	40	0.44 805	9.97 403	4	23
38	9.52 634	35	9.55 235	40	0.44 765	9.97 398	4	22
39	9.52 669	35	9.55 275	40	0.44 725	9.97 394	4	21
40	9.52 704	35	9.55 315	40	0.44 685	9.97 389	4	20
41	9.52 740	35	9.55 355	39	0.44 645	9.97 385	4	19
42	9.52 775	35	9.55 394	40	0.44 605	9.97 380	4	18
43	9.52 810	35	9.55 434	39	0.44 565	9.97 376	4	17
44	9.52 846	35	9.55 474	40	0.44 526	9.97 371	4	16
45	9.52 881	35	9.55 514	39	0.44 486	9.97 367	4	15
46	9.52 916	35	9.55 553	40	0.44 446	9.97 362	4	14
47	9.52 951	35	9.55 593	39	0.44 406	9.97 358	4	13
48	9.52 986	35	9.55 633	39	0.44 367	9.97 353	4	12
49	9.53 021	35	9.55 672	39	0.44 327	9.97 349	4	11
50	9.53 056	35	9.55 712	39	0.44 288	9.97 344	4	10
51	9.53 091	35	9.55 751	40	0.44 248	9.97 340	4	9
52	9.53 126	35	9.55 791	39	0.44 208	9.97 335	5	8
53	9.53 161	35	9.55 831	39	0.44 169	9.97 330	4	7
54	9.53 196	34	9.55 870	39	0.44 129	9.97 326	4	6
55	9.53 231	35	9.55 909	39	0.44 090	9.97 321	4	5
56	9.53 266	35	9.55 949	39	0.44 051	9.97 317	4	4
57	9.53 301	34	9.55 988	39	0.44 011	9.97 312	4	3
58	9.53 335	35	9.56 028	39	0.43 972	9.97 308	5	2
59	9.53 370	34	9.56 067	39	0.43 932	9.97 303	4	1
60	9.53 405		9.56 106		0.43 893	9.97 298		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	41	40	40
6	4.1	4.0	4.0
7	4.8	4.7	4.6
8	5.4	5.4	5.3
9	6.1	6.1	6.0
10	6.8	6.7	6.6
20	13.6	13.5	13.3
30	20.5	20.2	20.0
40	27.3	27.0	26.6
50	34.1	33.7	33.3

	39	39
6	3.9	3.9
7	4.6	4.5
8	5.2	5.2
9	5.9	5.8
10	6.6	6.5
20	13.1	13.0
30	19.7	19.5
40	26.3	26.0
50	32.9	32.5

	37	36	36
6	3.7	3.6	3.6
7	4.3	4.2	4.2
8	4.9	4.8	4.8
9	5.5	5.5	5.4
10	6.1	6.1	6.0
20	12.3	12.1	12.0
30	18.5	18.2	18.0
40	24.6	24.3	24.0
50	30.8	30.4	30.0

	35	35	34
6	3.5	3.5	3.4
7	4.1	4.1	4.0
8	4.7	4.6	4.6
9	5.3	5.2	5.2
10	5.9	5.8	5.7
20	11.8	11.6	11.5
30	17.7	17.5	17.2
40	23.6	23.3	23.0
50	29.6	29.1	28.7

	5	4	4
6	0.5	0.4	0.4
7	0.6	0.5	0.4
8	0.6	0.6	0.5
9	0.7	0.7	0.6
10	0.8	0.7	0.6
20	1.6	1.5	1.3
30	2.5	2.2	2.0
40	3.3	3.0	2.6
50	4.1	3.7	3.3

P. P.

70

20°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.53 405	35	9.56 106	39	0.43 893	9.97 298	4	60
1	9.53 440	34	9.56 146	39	0.43 854	9.97 294	4	59
2	9.53 474	34	9.56 185	39	0.43 815	9.97 289	4	58
3	9.53 509	35	9.56 224	39	0.43 775	9.97 285	5	57
4	9.53 544	34	9.56 263	39	0.43 736	9.97 280	4	56
5	9.53 578	34	9.56 303	39	0.43 697	9.97 275	4	55
6	9.53 613	34	9.56 342	39	0.43 658	9.97 271	4	54
7	9.53 647	34	9.56 381	39	0.43 619	9.97 266	5	53
8	9.53 682	34	9.56 420	39	0.43 580	9.97 261	4	52
9	9.53 716	34	9.56 459	39	0.43 540	9.97 257	4	51
10	9.53 750	34	9.56 498	39	0.43 501	9.97 252	4	50
11	9.53 785	34	9.56 537	39	0.43 462	9.97 248	5	49
12	9.53 819	34	9.56 576	39	0.43 423	9.97 243	4	48
13	9.53 854	34	9.56 615	38	0.43 384	9.97 238	4	47
14	9.53 888	34	9.56 654	39	0.43 346	9.97 234	5	46
15	9.53 922	34	9.56 693	39	0.43 307	9.97 229	4	45
16	9.53 956	34	9.56 732	39	0.43 268	9.97 224	4	44
17	9.53 990	34	9.56 771	39	0.43 229	9.97 220	5	43
18	9.54 025	34	9.56 810	38	0.43 190	9.97 215	4	42
19	9.54 059	34	9.56 848	39	0.43 151	9.97 210	4	41
20	9.54 093	34	9.56 887	38	0.43 112	9.97 206	5	40
21	9.54 127	34	9.56 926	39	0.43 074	9.97 201	4	39
22	9.54 161	34	9.56 965	38	0.43 035	9.97 196	5	38
23	9.54 195	34	9.57 003	38	0.42 996	9.97 191	4	37
24	9.54 229	33	9.57 042	39	0.42 958	9.97 187	4	36
25	9.54 263	34	9.57 081	38	0.42 919	9.97 182	5	35
26	9.54 297	34	9.57 119	38	0.42 880	9.97 177	4	34
27	9.54 331	34	9.57 158	38	0.42 842	9.97 173	5	33
28	9.54 365	33	9.57 196	38	0.42 803	9.97 168	4	32
29	9.54 398	34	9.57 235	39	0.42 765	9.97 163	4	31
30	9.54 432	34	9.57 274	38	0.42 726	9.97 159	5	30
31	9.54 466	33	9.57 312	38	0.42 687	9.97 154	4	29
32	9.54 500	34	9.57 350	38	0.42 649	9.97 149	5	28
33	9.54 534	33	9.57 389	38	0.42 611	9.97 144	4	27
34	9.54 567	33	9.57 427	38	0.42 572	9.97 140	5	26
35	9.54 601	33	9.57 466	38	0.42 534	9.97 135	4	25
36	9.54 634	34	9.57 504	38	0.42 495	9.97 130	5	24
37	9.54 668	33	9.57 542	38	0.42 457	9.97 125	4	23
38	9.54 702	33	9.57 581	38	0.42 419	9.97 121	5	22
39	9.54 735	33	9.57 619	38	0.42 380	9.97 116	4	21
40	9.54 769	33	9.57 657	38	0.42 342	9.97 111	5	20
41	9.54 802	33	9.57 696	38	0.42 304	9.97 106	4	19
42	9.54 836	33	9.57 734	38	0.42 266	9.97 102	5	18
43	9.54 869	33	9.57 772	38	0.42 227	9.97 097	5	17
44	9.54 902	33	9.57 810	38	0.42 189	9.97 092	4	16
45	9.54 936	33	9.57 848	38	0.42 151	9.97 087	5	15
46	9.54 969	33	9.57 886	38	0.42 113	9.97 082	4	14
47	9.55 002	33	9.57 925	38	0.42 075	9.97 078	5	13
48	9.55 036	33	9.57 963	38	0.42 037	9.97 073	4	12
49	9.55 069	33	9.58 001	38	0.41 999	9.97 068	5	11
50	9.55 102	33	9.58 039	38	0.41 961	9.97 063	5	10
51	9.55 135	33	9.58 077	38	0.41 923	9.97 058	4	9
52	9.55 168	33	9.58 115	38	0.41 885	9.97 054	5	8
53	9.55 202	33	9.58 153	37	0.41 847	9.97 049	5	7
54	9.55 235	33	9.58 190	38	0.41 809	9.97 044	4	6
55	9.55 268	33	9.58 228	38	0.41 771	9.97 039	5	5
56	9.55 301	33	9.58 266	38	0.41 733	9.97 034	5	4
57	9.55 334	33	9.58 304	37	0.41 695	9.97 029	4	3
58	9.55 367	33	9.58 342	38	0.41 658	9.97 025	5	2
59	9.55 400	33	9.58 380	37	0.41 620	9.97 020	5	1
60	9.55 433		9.58 417		0.41 582	9.97 015		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	39	39
6	3.9	3.9
7	4.6	4.5
8	5.2	5.2
9	5.9	5.8
10	6.6	6.5
20	13.1	13.0
30	19.7	19.5
40	26.3	26.0
50	32.9	32.5

	38	38	37
6	3.8	3.8	3.7
7	4.5	4.4	4.4
8	5.1	5.0	5.0
9	5.8	5.7	5.6
10	6.4	6.3	6.2
20	12.8	12.6	12.5
30	19.2	19.0	18.7
40	25.6	25.3	25.0
50	32.1	31.6	31.2

	35	34	34
6	3.5	3.4	3.4
7	4.1	4.0	3.9
8	4.6	4.6	4.5
9	5.2	5.2	5.1
10	5.8	5.7	5.6
20	11.6	11.5	11.3
30	17.5	17.2	17.0
40	23.3	23.0	22.6
50	29.1	28.7	28.3

	33	33
6	3.3	3.3
7	3.9	3.8
8	4.4	4.4
9	5.0	4.9
10	5.6	5.5
20	11.1	11.0
30	16.7	16.5
40	22.3	22.0
50	27.9	27.5

	5	4
6	0.5	0.4
7	0.6	0.5
8	0.6	0.6
9	0.7	0.7
10	0.8	0.7
20	1.6	1.5
30	2.5	2.2
40	3.3	3.0
50	4.1	3.7

P. P.

69°

21°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.55 433	33	9.58 417	38	0.41 582	9.97 015	4	60
1	9.55 466	32	9.58 455	37	0.41 544	9.97 010	5	59
2	9.55 498	33	9.58 493	38	0.41 507	9.97 005	5	58
3	9.55 531	33	9.58 531	37	0.41 469	9.97 000	5	57
4	9.55 564	32	9.58 568	37	0.41 431	9.96 995	4	56
5	9.55 597	33	9.58 606	38	0.41 394	9.96 991	5	55
6	9.55 630	32	9.58 644	37	0.41 356	9.96 986	5	54
7	9.55 662	33	9.58 681	37	0.41 318	9.96 981	5	53
8	9.55 695	32	9.58 719	37	0.41 281	9.96 976	4	52
9	9.55 728	32	9.58 756	37	0.41 243	9.96 971	5	51
10	9.55 760	32	9.58 794	37	0.41 206	9.96 966	5	50
11	9.55 793	33	9.58 831	37	0.41 168	9.96 961	5	49
12	9.55 826	32	9.58 869	37	0.41 131	9.96 956	4	48
13	9.55 858	32	9.58 906	37	0.41 093	9.96 952	5	47
14	9.55 891	32	9.58 944	37	0.41 056	9.96 947	5	46
15	9.55 923	32	9.58 981	37	0.41 018	9.96 942	5	45
16	9.55 956	32	9.59 019	37	0.40 981	9.96 937	5	44
17	9.55 988	32	9.59 056	37	0.40 944	9.96 932	5	43
18	9.56 020	32	9.59 093	37	0.40 906	9.96 927	4	42
19	9.56 053	32	9.59 131	37	0.40 869	9.96 922	5	41
20	9.56 085	32	9.59 168	37	0.40 832	9.96 917	5	40
21	9.56 118	32	9.59 205	37	0.40 794	9.96 912	5	39
22	9.56 150	32	9.59 242	37	0.40 757	9.96 907	5	38
23	9.56 182	32	9.59 280	37	0.40 720	9.96 902	5	37
24	9.56 214	32	9.59 317	37	0.40 683	9.96 897	5	36
25	9.56 247	32	9.59 354	37	0.40 646	9.96 892	5	35
26	9.56 279	32	9.59 391	37	0.40 608	9.96 887	5	34
27	9.56 311	32	9.59 428	37	0.40 571	9.96 882	5	33
28	9.56 343	32	9.59 465	37	0.40 534	9.96 877	4	32
29	9.56 375	32	9.59 502	37	0.40 497	9.96 873	5	31
30	9.56 407	32	9.59 540	37	0.40 460	9.96 868	5	30
31	9.56 439	32	9.59 577	37	0.40 423	9.96 863	5	29
32	9.56 471	32	9.59 614	37	0.40 386	9.96 858	5	28
33	9.56 503	32	9.59 651	37	0.40 349	9.96 853	5	27
34	9.56 535	32	9.59 688	36	0.40 312	9.96 848	5	26
35	9.56 567	32	9.59 724	37	0.40 275	9.96 843	5	25
36	9.56 599	32	9.59 761	37	0.40 238	9.96 838	5	24
37	9.56 631	31	9.59 798	37	0.40 201	9.96 833	5	23
38	9.56 663	32	9.59 835	36	0.40 164	9.96 828	5	22
39	9.56 695	32	9.59 872	37	0.40 128	9.96 823	5	21
40	9.56 727	31	9.59 909	37	0.40 091	9.96 818	5	20
41	9.56 758	32	9.59 946	36	0.40 054	9.96 813	5	19
42	9.56 790	31	9.59 982	37	0.40 017	9.96 808	5	18
43	9.56 822	32	9.60 019	36	0.39 980	9.96 802	5	17
44	9.56 854	31	9 60 056	37	0.39 944	9.96 797	5	16
45	9.56 885	31	9.60 093	36	0.39 907	9.96 792	5	15
46	9.56 917	32	9.60 129	37	0.39 870	9.96 787	5	14
47	9.56 949	31	9.60 166	36	0.39 833	9.96 782	5	13
48	9 56 980	31	9.60 203	36	0.39 797	9.96 777	5	12
49	9.57 012	31	9.60 239	36	0.39 760	9.96 772	5	11
50	9.57 043	31	9.60 276	36	0.39 724	9.96 767	5	10
51	9.57 075	31	9.60 312	37	0.39 687	9.96 762	5	9
52	9.57 106	31	9.60 349	36	0.39 650	9.96 757	5	8
53	9.57 138	31	9.60 386	36	0.39 614	9.96 752	5	7
54	9.57 169	31	9.60 422	36	0.39 577	9.96 747	5	6
55	9.57 201	31	9.60 459	36	0.39 541	9 96 742	5	5
56	9.57 232	31	9.60 495	36	0.39 504	9.96 737	5	4
57	9.57 263	31	9.60 531	36	0.39 468	9.96 732	5	3
58	9.57 295	31	9.60 568	36	0.39 432	9.96 727	5	2
59	9.57 326	31	9.60 604	36	0.39 395	9.96 721	5	1
60	9.57 357		9.60 641		0.39 359	9.96 716		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	38	37	37
6	3.8	3.7	3.7
7	4.4	4.4	4.3
8	5.0	5.0	4.9
9	5.7	5.6	5.5
10	6.3	6.2	6.1
20	12.6	12.5	12.3
30	19.0	18.7	18.5
40	25.3	25.0	24.6
50	31.6	31.2	30.8

	36	36
6	3.6	3.6
7	4.2	4.2
8	4.8	4.8
9	5.5	5.4
10	6.1	6.0
20	12.1	12.0
30	18.2	18.0
40	24.3	24.0
50	30.4	30.0

	33	32	32
6	3.3	3.2	3.2
7	3.8	3.8	3.7
8	4.4	4.3	4.2
9	4.9	4.9	4.8
10	5.5	5.4	5.3
20	11.0	10.8	10.6
30	16.5	16.2	16.0
40	22.0	21.6	21.3
50	27.5	27.1	26.6

	31	31
6	3.1	3.1
7	3.7	3.6
8	4.2	4.1
9	4.7	4.6
10	5.2	5.1
20	10.5	10.3
30	15.7	15.5
40	21.0	20.6
50	26.2	25.8

	5	5	4
6	0.5	0.5	0.4
7	0.6	0.6	0.5
8	0.7	0.6	0.6
9	0.8	0.7	0.7
10	0.9	0.8	0.7
20	1.8	1.6	1.5
30	2.7	2.5	2.2
40	3.6	3.3	3.0
50	4.6	4.1	3.7

P. P.

68°

22°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.57 357		9.60 641		0.39 359	9.96 716		60
1	9.57 389	31	9.60 677	36	0.39 322	9.96 711	5	59
2	9.57 420	31	9.60 713	36	0.39 286	9.96 706	5	58
3	9.57 451	31	9.60 750	36	0.39 250	9.96 701	5	57
4	9.57 482	31	9.60 786	36	0.39 213	9.96 696	5	56
5	9.57 513	31	9.60 822	36	0.39 177	9.96 691	5	55
6	9.57 544	31	9.60 859	36	0.39 141	9.96 686	5	54
7	9.57 576	31	9.60 895	36	0.39 105	9.96 681	5	53
8	9.57 607	31	9.60 931	36	0.39 069	9.96 675	5	52
9	9.57 638	31	9.60 967	36	0.39 032	9.96 670	5	51
10	9.57 669	31	9.61 003	36	0.38 996	9.96 665	5	50
11	9.57 700	31	9.61 039	36	0.38 960	9.96 660	5	49
12	9.57 731	31	9.61 076	36	0.38 924	9.96 655	5	48
13	9.57 762	31	9.61 112	36	0.38 888	9.96 650	5	47
14	9.57 792	30	9.61 148	36	0.38 852	9.96 644	5	46
15	9.57 823	31	9.61 184	36	0.38 816	9.96 639	5	45
16	9.57 854	31	9.61 220	36	0.38 780	9.96 634	5	44
17	9.57 885	31	9.61 256	36	0.38 744	9.96 629	5	43
18	9.57 916	30	9.61 292	36	0.38 708	9.96 624	5	42
19	9.57 947	31	9.61 328	36	0.38 672	9.96 619	5	41
20	9.57 977	30	9.61 364	36	0.38 636	9.96 613	5	40
21	9.58 008	31	9.61 400	36	0.38 600	9.96 608	5	39
22	9.58 039	30	9.61 436	36	0.38 564	9.96 603	5	38
23	9.58 070	31	9.61 472	36	0.38 528	9.96 598	5	37
24	9.58 100	30	9.61 507	35	0.38 492	9.96 593	5	36
25	9.58 131	30	9.61 543	36	0.38 456	9.96 587	5	35
26	9.58 162	31	9.61 579	36	0.38 420	9.96 582	5	34
27	9.58 192	30	9.61 615	35	0.38 385	9.96 577	5	33
28	9.58 223	30	9.61 651	36	0.38 349	9.96 572	5	32
29	9.58 253	30	9.61 686	35	0.38 313	9.96 567	5	31
30	9.58 284	30	9.61 722	36	0.38 277	9.96 561	5	30
31	9.58 314	30	9.61 758	35	0.38 242	9.96 556	5	29
32	9.58 345	30	9.61 794	36	0.38 206	9.96 551	5	28
33	9.58 375	30	9.61 829	35	0.38 170	9.96 546	5	27
34	9.58 406	30	9.61 865	35	0.38 135	9.96 540	5	26
35	9.58 436	30	9.61 901	36	0.38 099	9.96 535	5	25
36	9.58 466	30	9.61 936	35	0.38 063	9.96 530	5	24
37	9.58 497	30	9.61 972	35	0.38 028	9.96 525	5	23
38	9.58 527	30	9.62 007	35	0.37 992	9.96 519	5	22
39	9.58 557	30	9.62 043	35	0.37 957	9.96 514	5	21
40	9.58 587	30	9.62 078	35	0.37 921	9.96 509	5	20
41	9.58 618	30	9.62 114	35	0.37 886	9.96 503	5	19
42	9.58 648	30	9.62 149	35	0.37 850	9.96 498	5	18
43	9.58 678	30	9.62 185	35	0.37 815	9.96 493	5	17
44	9.58 708	30	9.62 220	35	0.37 779	9.96 488	5	16
45	9.58 738	30	9.62 256	35	0.37 744	9.96 482	5	15
46	9.58 769	30	9.62 291	35	0.37 708	9.96 477	5	14
47	9.58 799	30	9.62 327	35	0.37 673	9.96 472	5	13
48	9.58 829	30	9.62 362	35	0.37 637	9.96 466	5	12
49	9.58 859	30	9.62 397	35	0.37 602	9.96 461	5	11
50	9.58 889	30	9.62 433	35	0.37 567	9.96 456	5	10
51	9.58 919	30	9.62 468	35	0.37 531	9.96 450	5	9
52	9.58 949	30	9.62 503	35	0.37 496	9.96 445	5	8
53	9.58 979	30	9.62 539	35	0.37 461	9.96 440	5	7
54	9.59 009	30	9.62 574	35	0.37 426	9.96 434	5	6
55	9.59 038	29	9.62 609	35	0.37 390	9.96 429	5	5
56	9.59 068	30	9.62 644	35	0.37 355	9.96 424	5	4
57	9.59 098	30	9.62 679	35	0.37 320	9.96 418	5	3
58	9.59 128	29	9.62 715	35	0.37 285	9.96 413	5	2
59	9.59 158	30	9.62 750	35	0.37 250	9.96 408	5	1
60	9.59 188	30	9.62 785	35	0.37 215	9.96 402	5	0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	36̂	36
6	3.6	3.6
7	4.2	4.2
8	4.8	4.8
9	5.5	5.4
10	6.1	6.0
20	12.1	12.0
30	18.2	18.0
40	24.3	24.0
50	30.4	30.0

	35̂	35
6	3.5	3.5
7	4.1	4.1
8	4.7	4.6
9	5.3	5.2
10	5.9	5.8
20	11.8	11.6
30	17.7	17.5
40	23.6	23.3
50	29.6	29.1

	31̂	31
6	3.1	3.1
7	3.7	3.6
8	4.2	4.1
9	4.7	4.6
10	5.2	5.1
20	10.5	10.3
30	15.7	15.5
40	21.0	20.6
50	26.2	25.8

	30̂	30	29̂
6	3.0	3.0	2.9
7	3.5	3.5	3.4
8	4.0	4.0	3.9
9	4.6	4.5	4.4
10	5.1	5.0	4.9
20	10.1	10.0	9.8
30	15.2	15.0	14.7
40	20.3	20.0	19.6
50	25.4	25.0	24.6

	5̂	5
6	0.5	0.5
7	0.6	0.6
8	0.7	0.6
9	0.8	0.7
10	0.9	0.8
20	1.8	1.6
30	2.7	2.5
40	3.6	3.3
50	4.6	4.1

P. P.

67°

23°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.59 188	29	9.62 785	35	0.37 215	9.96 402	5	**60**
1	9.59 217	30	9.62 820	35	0.37 179	9.96 397	5	59
2	9.59 247	29	9.62 855	35	0.37 144	9.96 392	5	58
3	9.59 277	29	9.62 890	35	0.37 109	9.96 386	5	57
4	9.59 306	30	9.62 925	35	0.37 074	9.96 381	5	56
5	9.59 336	29	9.62 960	35	0.37 039	9.96 375	5	55
6	9.59 366	29	9.62 995	35	0.37 004	9.96 370	5	54
7	9.59 395	29	9.63 030	35	0.36 969	9.96 365	5	53
8	9.59 425	29	9.63 065	35	0.36 934	9.96 359	5	52
9	9.59 454	29	9.63 100	35	0.36 899	9.96 354	5	51
10	9.59 484	29	9.63 135	35	0.36 864	9.96 349	5	**50**
11	9.59 513	29	9.63 170	35	0.36 829	9.96 343	5	49
12	9.59 543	29	9.63 205	34	0.36 794	9.96 338	5	48
13	9.59 572	29	9.63 240	35	0.36 760	9.96 332	5	47
14	9.59 602	29	9.63 275	35	0.36 725	9.96 327	5	46
15	9.59 631	29	9.63 310	34	0.36 690	9.96 321	5	45
16	9.59 661	29	9.63 344	35	0.36 655	9.96 316	5	44
17	9.59 690	29	9.63 379	35	0.36 620	9.96 311	5	43
18	9.59 719	29	9.63 414	34	0.36 585	9.96 305	5	42
19	9.59 749	29	9.63 449	35	0.36 551	9.96 300	5	41
20	9.59 778	29	9.63 484	34	0.36 516	9.96 294	5	**40**
21	9.59 807	29	9.63 518	34	0.36 481	9.96 289	5	39
22	9.59 837	29	9.63 553	35	0.36 447	9.96 283	5	38
23	9.59 866	29	9.63 588	34	0.36 412	9.96 278	5	37
24	9.59 895	29	9.63 622	34	0.36 377	9.96 272	5	36
25	9.59 924	29	9.63 657	35	0.36 343	9.96 267	5	35
26	9.59 953	29	9.63 692	34	0.36 308	9.96 261	5	34
27	9.59 982	29	9.63 726	34	0.36 273	9.96 256	5	33
28	9.60 012	29	9.63 761	34	0.36 239	9.96 251	5	32
29	9.60 041	29	9.63 795	34	0.36 204	9.96 245	5	31
30	9.60 070	29	9.63 830	34	0.36 170	9.96 240	5	**30**
31	9.60 099	29	9.63 864	34	0.36 135	9.96 234	5	29
32	9.60 128	29	9.63 899	34	0.36 101	9.96 229	5	28
33	9.60 157	29	9.63 933	34	0.36 066	9.96 223	5	27
34	9.60 186	29	9.63 968	34	0.36 032	9.96 218	5	26
35	9.60 215	29	9.64 002	34	0.35 997	9.96 212	6	25
36	9.60 244	29	9.64 037	34	0.35 963	9.96 206	5	24
37	9.60 273	28	9.64 071	34	0.35 928	9.96 201	5	23
38	9.60 301	29	9.64 106	34	0.35 894	9.96 195	5	22
39	9.60 330	29	9.64 140	34	0.35 859	9.96 190	5	21
40	9.60 359	28	9.64 174	34	0.35 825	9.96 184	5	**20**
41	9.60 388	29	9.64 209	34	0.35 791	9.96 179	5	19
42	9.60 417	28	9.64 243	34	0.35 756	9.96 173	5	18
43	9.60 445	29	9.64 277	34	0.35 722	9.96 168	5	17
44	9.60 474	28	9.64 312	34	0.35 688	9.96 162	5	16
45	9.60 503	29	9.64 346	34	0.35 653	9.96 157	5	15
46	9.60 532	28	9.64 380	34	0.35 619	9.96 151	5	14
47	9.60 560	28	9.64 415	34	0.35 585	9.96 146	6	13
48	9.60 589	29	9.64 449	34	0.35 551	9.96 140	5	12
49	9.60 618	28	9.64 483	34	0.35 517	9.96 134	5	11
50	9.60 646	28	9.64 517	34	0.35 482	9.96 129	5	**10**
51	9.60 675	28	9.64 551	34	0.35 448	9.96 123	5	9
52	9.60 703	28	9.64 585	34	0.35 414	9.96 118	5	8
53	9.60 732	28	9.64 620	34	0.35 380	9.96 112	6	7
54	9.60 760	28	9.64 654	34	0.35 346	9.96 106	5	6
55	9.60 789	28	9.64 688	34	0.35 312	9.96 101	5	5
56	9.60 817	28	9.64 722	34	0.35 278	9.96 095	5	4
57	9.60 846	28	9.64 756	34	0.35 244	9.96 090	5	3
58	9.60 874	28	9.64 790	34	0.35 209	9.96 084	6	2
59	9.60 903	28	9.64 824	34	0.35 175	9.96 078	5	1
60	9.60 931		9.64 858		0.35 141	9.96 073		**0**
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	35	35
6	3.5	3.5
7	4.1	4.1
8	4.7	4.6
9	5.3	5.2
10	5.9	5.8
20	11.8	11.6
30	17.7	17.5
40	23.6	23.3
50	29.6	29.1

	34	34
6	3.4	3.4
7	4.0	3.9
8	4.6	4.5
9	5.2	5.1
10	5.7	5.6
20	11.5	11.3
30	17.2	17.0
40	23.0	22.6
50	28.7	28.3

	30
6	3.0
7	3.5
8	4.0
9	4.5
10	5.0
20	10.0
30	15.0
40	20.0
50	25.0

	29	29	28
6	2.9	2.9	2.8
7	3.4	3.4	3.3
8	3.9	3.8	3.8
9	4.4	4.3	4.3
10	4.9	4.8	4.7
20	9.8	9.6	9.5
30	14.7	14.5	14.2
40	19.6	19.3	19.0
50	24.6	24.1	23.7

	6	5	5
6	0.6	0.5	0.5
7	0.7	0.6	0.6
8	0.8	0.7	0.6
9	0.9	0.8	0.7
10	1.0	0.9	0.8
20	2.0	1.8	1.6
30	3.0	2.7	2.5
40	4.0	3.6	3.3
50	5.0	4.6	4.1

P. P.

66°

24°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.60 931	28	9.64 858	34	0.35 141	9.96 073	5	60
1	9.60 959	28	9.64 892	34	0.35 107	9.96 067	5	59
2	9.60 988	28	9.64 926	33	0.35 073	9.96 062	6	58
3	9.61 016	28	9.64 960	34	0.35 040	9.96 056	5	57
4	9.61 044	28	9.64 994	34	0.35 006	9.96 050	5	56
5	9.61 073	28	9.65 028	34	0.34 972	9.96 045	6	55
6	9.61 101	28	9.65 062	34	0.34 938	9.96 039	5	54
7	9.61 129	28	9.65 096	33	0.34 904	9.96 033	5	53
8	9.61 157	28	9.65 129	34	0.34 870	9.96 028	6	52
9	9.61 186	28	9.65 163	34	0.34 836	9.96 022	5	51
10	9.61 214	28	9.65 197	33	0.34 802	9.96 016	5	50
11	9.61 242	28	9.65 231	34	0.34 769	9.96 011	6	49
12	9.61 270	28	9.65 265	34	0.34 735	9.96 005	5	48
13	9.61 298	28	9.65 299	33	0.34 701	9.95 999	5	47
14	9.61 326	28	9.65 332	34	0.34 667	9.95 994	6	46
15	9.61 354	28	9.65 366	33	0.34 633	9.95 988	5	45
16	9.61 382	28	9.65 400	33	0.34 600	9.95 982	5	44
17	9.61 410	28	9.65 433	34	0.34 566	9.95 977	6	43
18	9.61 438	28	9.65 467	33	0.34 532	9.95 971	5	42
19	9.61 466	28	9.65 501	34	0.34 499	9.95 965	6	41
20	9.61 494	28	9.65 535	33	0.34 465	9.95 959	5	40
21	9.61 522	27	9.65 568	33	0.34 431	9.95 954	6	39
22	9.61 550	28	9.65 602	33	0.34 398	9.95 948	5	38
23	9.61 578	28	9.65 635	33	0.34 364	9.95 942	5	37
24	9.61 606	28	9.65 669	34	0.34 331	9.95 937	6	36
25	9.61 634	27	9.65 703	33	0.34 297	9.95 931	5	35
26	9.61 661	28	9.65 736	33	0.34 263	9.95 925	6	34
27	9.61 689	27	9.65 770	33	0.34 230	9.95 919	5	33
28	9.61 717	28	9.65 803	33	0.34 196	9.95 914	6	32
29	9.61 745	27	9.65 837	33	0.34 163	9.95 908	5	31
30	9.61 772	28	9.65 870	33	0.34 129	9.95 902	6	30
31	9.61 800	27	9.65 904	33	0.34 096	9.95 896	5	29
32	9.61 828	28	9.65 937	33	0.34 062	9.95 891	6	28
33	9.61 856	27	9.65 971	33	0.34 029	9.95 885	6	27
34	9.61 883	27	9.66 004	33	0.33 996	9.95 879	5	26
35	9.61 911	27	9.66 037	33	0.33 962	9.95 873	6	25
36	9.61 938	27	9.66 071	33	0.33 929	9.95 867	5	24
37	9.61 966	28	9.66 104	33	0.33 895	9.95 862	6	23
38	9.61 994	27	9.66 137	33	0.33 862	9.95 856	5	22
39	9.62 021	27	9.66 171	33	0.33 829	9.95 850	6	21
40	9.62 049	27	9.66 204	33	0.33 795	9.95 844	6	20
41	9.62 076	27	9.66 237	33	0.33 762	9.95 838	5	19
42	9.62 104	27	9.66 271	33	0.33 729	9.95 833	6	18
43	9.62 131	27	9.66 304	33	0.33 696	9.95 827	5	17
44	9.62 158	27	9.66 337	33	0.33 662	9.95 821	6	16
45	9.62 186	27	9.66 370	33	0.33 629	9.95 815	6	15
46	9.62 213	27	9.66 404	33	0.33 596	9.95 809	5	14
47	9.62 241	27	9.66 437	33	0.33 563	9.95 804	6	13
48	9.62 268	27	9.66 470	33	0.33 529	9.95 798	6	12
49	9.62 295	27	9.66 503	33	0.33 496	9.95 792	5	11
50	9.62 323	27	9.66 536	33	0.33 463	9.95 786	6	10
51	9.62 350	27	9.66 570	33	0.33 430	9.95 780	6	9
52	9.62 377	27	9.66 603	33	0.33 397	9.95 774	6	8
53	9.62 404	27	9.66 636	33	0.33 364	9.95 768	5	7
54	9.62 432	27	9.66 669	33	0.33 331	9.95 763	6	6
55	9.62 459	27	9.66 702	33	0.33 298	9.95 757	6	5
56	9.62 486	27	9.66 735	33	0.33 265	9.95 751	6	4
57	9.62 513	27	9.66 768	33	0.33 232	9.95 745	5	3
58	9.62 540	27	9.66 801	33	0.33 198	9.95 739	6	2
59	9.62 567	27	9.66 834	33	0.33 165	9.95 733	6	1
60	9.62 595		9.66 867		0.33 132	9.95 727		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	34	33	33
6	3.4	3.3	3.3
7	3.9	3.9	3.8
8	4.5	4.4	4.4
9	5.1	5.0	4.9
10	5.6	5.6	5.5
20	11.3	11.1	11.0
30	17.0	16.7	16.5
40	22.6	22.3	22.0
50	28.3	27.9	27.5

	28	28
6	2.8	2.8
7	3.3	3.2
8	3.8	3.7
9	4.3	4.2
10	4.7	4.6
20	9.5	9.3
30	14.2	14.0
40	19.0	18.6
50	23.7	23.3

	27	27
6	2.7	2.7
7	3.2	3.1
8	3.6	3.6
9	4.1	4.0
10	4.6	4.5
20	9.1	9.0
30	13.7	13.5
40	18.3	18.0
50	22.9	22.5

	6	5
6	0.6	0.5
7	0.7	0.6
8	0.8	0.7
9	0.9	0.8
10	1.0	0.9
20	2.0	1.8
30	3.0	2.7
40	4.0	3.6
50	5.0	4.6

P. P.

65°

25°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.62 595	27	9.66 867	32	0.33 132	9.95 727	6	60
1	9.62 622	27	9.66 900	33	0.33 100	9.95 721	5	59
2	9.62 649	27	9.66 933	33	0.33 067	9.95 716	6	58
3	9.62 676	27	9.66 966	33	0.33 034	9.95 710	6	57
4	9.62 703	27	9.66 999	33	0.33 001	9.95 704	6	56
5	9.62 730	27	9.67 032	33	0.32 968	9.95 698	6	55
6	9.62 757	27	9.67 065	32	0.32 935	9.95 692	6	54
7	9.62 784	27	9.67 097	33	0.32 902	9.95 686	5	53
8	9.62 811	27	9.67 130	33	0.32 869	9.95 680	6	52
9	9.62 838	26	9.67 163	33	0.32 836	9.95 674	6	51
10	9.62 864	27	9.67 196	32	0.32 803	9.95 668	6	50
11	9.62 891	27	9.67 229	33	0.32 771	9.95 662	6	49
12	9.62 918	27	9.67 262	32	0.32 738	9.95 656	6	48
13	9.62 945	26	9 67 294	33	0.32 705	9.95 650	6	47
14	9.62 972	27	9.67 327	32	0.32 672	9.95 644	6	46
15	9.62 999	26	9.67 360	33	0.32 640	9.95 638	6	45
16	9.63 025	27	9.67 393	32	0.32 607	9.95 632	5	44
17	9.63 052	26	9.67 425	33	0.32 574	9.95 627	6	43
18	9.63 079	27	9.67 458	32	0.32 541	9.95 621	6	42
19	9.63 106	26	9.67 491	32	0.32 509	9.95 615	6	41
20	9.63 132	27	9.67 523	33	0.32 476	9.95 609	6	40
21	9.63 159	26	9.67 556	32	0.32 443	9.95 603	6	39
22	9.63 186	26	9.67 589	32	0.32 411	9.95 597	6	38
23	9.63 212	26	9.67 621	33	0.32 378	9.95 591	6	37
24	9.63 239	27	9.67 654	32	0.32 345	9.95 585	6	36
25	9.63 266	26	9.67 687	32	0.32 313	9.95 579	6	35
26	9.63 292	26	9.67 719	32	0.32 280	9.95 573	6	34
27	9.63 319	26	9.67 752	32	0.32 248	9.95 567	6	33
28	9.63 345	26	9.67 784	32	0.32 215	9.95 561	6	32
29	9.63 372	26	9.67 817	32	0.32 183	9.95 555	6	31
30	9.63 398	26	9.67 849	32	0.32 150	9.95 549	6	30
31	9.63 425	26	9.67 882	32	0.32 118	9.95 543	6	29
32	9.63 451	26	9.67 914	32	0.32 085	9.95 537	6	28
33	9.63 478	26	9.67 947	32	0.32 053	9.95 530	6	27
34	9.63 504	26	9.67 979	32	0.32 020	9.95 524	6	26
35	9.63 530	26	9.68 012	32	0.31 988	9.95 518	6	25
36	9.63 557	26	9.68 044	32	0.31 955	9.95 512	6	24
37	9.63 583	26	9.68 077	32	0.31 923	9.95 506	6	23
38	9.63 609	26	9.68 109	32	0.31 891	9.95 500	6	22
39	9 63 636	26	9.68 141	32	0.31 858	9.95 494	6	21
40	9.63 662	26	9.68 174	32	0.31 826	9.95 488	6	20
41	9.63 688	26	9.68 206	32	0.31 793	9.95 482	6	19
42	9.63 715	26	9.68 238	32	0.31 761	9.95 476	6	18
43	9.63 741	26	9.68 271	32	0.31 729	9.95 470	6	17
44	9.63 767	26	9.68 303	32	0.31 696	9.95 464	6	16
45	9.63 793	26	9.68 335	32	0.31 664	9.95 458	6	15
46	9.63 819	26	9.68 368	32	0.31 632	9.95 452	6	14
47	9.63 846	26	9.68 400	32	0.31 600	9.95 445	6	13
48	9.63 872	26	9.68 432	32	0.31 567	9.95 439	6	12
49	9.63 898	26	9.68 464	32	0.31 535	9.95 433	6	11
50	9.63 924	26	9.68 497	32	0.31 503	9.95 427	6	10
51	9.63 950	26	9.68 529	32	0.31 471	9.95 421	6	9
52	9.63 976	26	9.68 561	32	0.31 439	9.95 415	6	8
53	9.64 002	26	9.68 593	32	0.31 406	9.95 409	6	7
54	9.64 028	26	9.68 625	32	0.31 374	9.95 403	6	6
55	9.64 054	26	9.68 657	32	0.31 342	9.95 397	6	5
56	9.64 080	26	9.68 690	32	0.31 310	9.95 390	6	4
57	9.64 106	26	9.68 722	32	0.31 278	9.95 384	6	3
58	9.64 132	26	9.68 754	32	0.31 246	9.95 378	6	2
59	9.64 158	25	9.68 786	32	0.31 214	9.95 372	6	1
60	9.64 184		9.68 818		0.31 182	9.95 366		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	33	32	32
6	3.3	3.2	3.2
7	3.8	3.8	3.7
8	4.4	4.3	4.2
9	4.9	4.9	4.8
10	5.5	5.4	5.3
20	11.0	10.8	10.6
30	16.5	16.2	16.0
40	22.0	21.6	21.3
50	27.5	27.1	26.6

	27
6	2.7
7	3.1
8	3.6
9	4.0
10	4.5
20	9.0
30	13.5
40	18.0
50	22.5

	26	26	25
6	2.6	2.6	2.5
7	3.1	3.0	3.0
8	3.5	3.4	3.4
9	4.0	3.9	3.8
10	4.4	4.3	4.2
20	8.8	8.6	8.5
30	13.2	13.0	12.7
40	17.6	17.3	17.0
50	22.1	21.6	21.2

	6	6	5
6	0.6	0.6	0.5
7	0.7	0.7	0.6
8	0.8	0.8	0.7
9	1.0	0.9	0.8
10	1.1	1.0	0.9
20	2.1	2.0	1.8
30	3.2	3.0	2.7
40	4.3	4.0	3.6
50	5.4	5.0	4.6

P. P.

64°

26°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.64 184	26	9.68 818	32	0.31 182	9.95 366	6	60
1	9.64 210	26	9.68 850	32	0.31 150	9.95 360	6	59
2	9.64 236	26	9.68 882	32	0.31 117	9.95 353	6	58
3	9.64 262	25	9.68 914	32	0.31 085	9.95 347	6	57
4	9.64 287	26	9.68 946	32	0.31 053	9.95 341	6	56
5	9.64 313	26	9.68 978	32	0.31 021	9.95 335	6	55
6	9.64 339	25	9.69 010	32	0.30 989	9.95 329	6	54
7	9.64 365	26	9.69 042	31	0.30 957	9.95 323	6	53
8	9.64 391	25	9.69 074	32	0.30 926	9.95 316	6	52
9	9.64 416	26	9.69 106	32	0.30 894	9.95 310	6	51
10	9.64 442	25	9.69 138	32	0.30 862	9.95 304	6	50
11	9.64 468	25	9.69 170	32	0.30 830	9.95 298	6	49
12	9.64 493	26	9.69 202	32	0.30 798	9.95 292	6	48
13	9.64 519	25	9.69 234	31	0.30 766	9.95 285	6	47
14	9.64 545	25	9.69 265	32	0.30 734	9.95 279	6	46
15	9.64 570	25	9.69 297	32	0.30 702	9.95 273	6	45
16	9.64 596	26	9.69 329	31	0.30 670	9.95 267	6	44
17	9.64 622	25	9.69 361	32	0.30 639	9.95 260	6	43
18	9.64 647	25	9.69 393	32	0.30 607	9.95 254	6	42
19	9.64 673	25	9.69 425	31	0.30 575	9.95 248	6	41
20	9.64 698	25	9.69 456	32	0.30 543	9.95 242	6	40
21	9.64 724	25	9.69 488	31	0.30 511	9.95 235	6	39
22	9.64 749	25	9.69 520	32	0.30 480	9.95 229	6	38
23	9.64 775	25	9.69 552	31	0.30 448	9.95 223	6	37
24	9.64 800	25	9.69 583	32	0.30 416	9.95 217	6	36
25	9.64 826	25	9.69 615	31	0.30 384	9.95 210	6	35
26	9.64 851	25	9.69 647	31	0.30 353	9.95 204	6	34
27	9.64 876	25	9.69 678	32	0.30 321	9.95 198	6	33
28	9.64 902	25	9.69 710	31	0.30 289	9.95 191	6	32
29	9.64 927	25	9.69 742	31	0.30 258	9.95 185	6	31
30	9.64 952	25	9.69 773	32	0.30 226	9.95 179	6	30
31	9.64 978	25	9.69 805	31	0.30 194	9.95 173	6	29
32	9.65 003	25	9.69 837	31	0.30 163	9.95 166	6	28
33	9.65 028	25	9.69 868	31	0.30 131	9.95 160	6	27
34	9.65 054	25	9.69 900	31	0.30 100	9.95 154	6	26
35	9.65 079	25	9.69 931	31	0.30 068	9.95 147	6	25
36	9.65 104	25	9.69 963	31	0.30 037	9.95 141	6	24
37	9.65 129	25	9.69 994	32	0.30 005	9.95 135	6	23
38	9.65 155	25	9.70 026	31	0.29 973	9.95 128	6	22
39	9.65 180	25	9.70 058	31	0.29 942	9.95 122	6	21
40	9.65 205	25	9.70 089	31	0.29 910	9.95 116	6	20
41	9.65 230	25	9.70 121	31	0.29 879	9.95 109	6	19
42	9.65 255	25	9.70 152	31	0.29 847	9.95 103	6	18
43	9.65 280	25	9.70 183	31	0.29 816	9.95 097	6	17
44	9.65 305	25	9.70 215	31	0.29 785	9.95 090	6	16
45	9.65 331	25	9.70 246	31	0.29 753	9.95 084	6	15
46	9.65 356	25	9.70 278	31	0.29 722	9.95 078	6	14
47	9.65 381	25	9.70 309	31	0.29 690	9.95 071	6	13
48	9.65 406	25	9.70 341	31	0.29 659	9.95 065	6	12
49	9.65 431	25	9.70 372	31	0.29 628	9.95 058	6	11
50	9.65 456	25	9 70 403	31	0.29 596	9.95 052	6	10
51	9.65 481	25	9.70 435	31	0.29 565	9.95 046	6	9
52	9.65 506	24	9.70 466	31	0.29 533	9.95 039	6	8
53	9.65 530	25	9.70 497	31	0.29 502	9.95 033	6	7
54	9.65 555	25	9.70 529	31	0.29 471	9.95 026	6	6
55	9.65 580	25	9.70 560	31	0.29 439	9.95 020	6	5
56	9.65 605	24	9.70 591	31	0.29 408	9.95 014	6	4
57	9.65 630	25	9.70 623	31	0.29 377	9.95 007	6	3
58	9.65 655	25	9.70 654	31	0.29 346	9.95 001	6	2
59	9.65 680	24	9.70 685	31	0.29 314	9.94 994	6	1
60	9.65 704		9.70 716		0.29 283	9.94 988		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	32̂	32
6	3.2	3.2
7	3.8	3.7
8	4.3	4.2
9	4.9	4.8
10	5.4	5.3
20	10.8	10.6
30	16.2	16.0
40	21.6	21.3
50	27.1	26.6

	31̂	31
6	3.1	3.1
7	3.7	3.6
8	4.2	4.1
9	4.7	4.6
10	5.2	5.1
20	10.5	10.3
30	15.7	15.5
40	21.0	20.6
50	26.2	25.8

	26	25̂	25
6	2.6	2.5	2.5
7	3.0	3.0	2.9
8	3.4	3.4	3.3
9	3.9	3.8	3.7
10	4.3	4.2	4.1
20	8.6	8.5	8.3
30	13.0	12.7	12.5
40	17.3	17.0	16.6
50	21.6	21.2	20.8

	24̂	6̂	6
6	2.4	0.6	0.6
7	2.8	0.7	0.7
8	3.2	0.8	0.8
9	3.7	1.0	0.9
10	4.1	1.1	1.0
20	8.1	2.1	2.0
30	12.2	3.2	3.0
40	16.3	4.3	4.0
50	20.4	5.4	5.0

P. P.

63°

27°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	′
0	9.65 704̂	25	9.70 716̂	31̂	0.29 283̂	9.94 988	6̂	60
1	9.65 729̂	24̂	9.70 748	31	0.29 252	9.94 981̂	6	59
2	9.65 754	25	9.70 779	31	0.29 221	9.94 975	6	58
3	9.65 779	24̂	9.70 810	31̂	0.29 190	9.94 969	6̂	57
4	9.65 803̂	25	9.70 841̂	31	0.29 158̂	9.94 962̂	6̂	56
5	9.65 828̂	24̂	9.70 872̂	31	0.29 127̂	9.94 956	6̂	55
6	9.65 853	25	9.70 903̂	31̂	0.29 096̂	9.94 949̂	6̂	54
7	9.65 878	24̂	9.70 935	31	0.29 065	9.94 943	6̂	53
8	9.65 902̂	24̂	9.70 966	31	0.29 034	9.94 936̂	6	52
9	9.65 927	24̂	9.70 997	31	0.29 003	9.94 930	6̂	51
10	9.65 951̂	25	9.71 028	31̂	0.28 972	9.94 923̂	6̂	50
11	9.65 976̂	24̂	9.71 059̂	31	0.28 940̂	9.94 917	6̂	49
12	9.66 001	24̂	9.71 090̂	31	0.28 909̂	9.94 910̂	6̂	48
13	9.66 025̂	24̂	9.71 121̂	31	0.28 878̂	9.94 904	6	47
14	9.66 050	24̂	9.71 152̂	31	0.28 847̂	9.94 897̂	6̂	46
15	9.66 074̂	24̂	9.71 183̂	31	0.28 816̂	9.94 891	6̂	45
16	9.66 099	24̂	9.71 214̂	31	0.28 785̂	9.94 884̂	6̂	44
17	9.66 123̂	24̂	9.71 245̂	31	0.28 754̂	9.94 878	6̂	43
18	9.66 148	24̂	9.71 276̂	31	0.28 723̂	9.94 871̂	6	42
19	9.66 172̂	24̂	9.71 307̂	31	0.28 692̂	9.94 865	6̂	41
20	9.66 197	24̂	9.71 338̂	31	0.28 661̂	9.94 858̂	6̂	40
21	9.66 221̂	24̂	9.71 369̂	31	0.28 630̂	9.94 852	6̂	39
22	9.66 246	24̂	9.71 400̂	31	0.28 599̂	9.94 845̂	6̂	38
23	9.66 270̂	24	9.71 431̂	31	0.28 568̂	9.94 839	6	37
24	9.66 294̂	24̂	9.71 462̂	31	0.28 537̂	9.94 832̂	7	36
25	9.66 319	24̂	9.71 493̂	30̂	0.28 506̂	9.94 825̂	6̂	35
26	9.66 343̂	24	9.71 524	31	0.28 476	9.94 819	6̂	34
27	9.66 367̂	24̂	9.71 555	31	0.28 445	9.94 812̂	6̂	33
28	9.66 392	24̂	9.71 586	31	0.28 414	9.94 806	6	32
29	9.66 416̂	24	9.71 617	30̂	0.28 383	9.94 799̂	6̂	31
30	9.66 440̂	24̂	9.71 647̂	31	0.28 352̂	9.94 793	6	30
31	9.66 465	24	9.71 678̂	31	0.28 321̂	9.94 786̂	7	29
32	9.66 489	24̂	9.71 709̂	30̂	0.28 290̂	9.94 779̂	6̂	28
33	9.66 513̂	24	9.71 740	31	0.28 260	9.94 773	6	27
34	9.66 537̂	24	9.71 771	30̂	0.28 229	9.94 766̂	6̂	26
35	9.66 561̂	24̂	9.71 801̂	31	0.28 198̂	9.94 760	6̂	25
36	9.66 586	24	9.71 832̂	31	0.28 167̂	9.94 753̂	7	24
37	9.66 610	24	9.71 863̂	30̂	0.28 136̂	9.94 746̂	6̂	23
38	9.66 634	24̂	9.71 894	31	0.28 106	9.94 740	6	22
39	9.66 658̂	24	9.71 925	30̂	0.28 075	9.94 733̂	6̂	21
40	9.66 682̂	24	9.71 955̂	30̂	0.28 044̂	9.94 727	6̂	20
41	9.66 706̂	24	9.71 986	31	0.28 014	9.94 720̂	7	19
42	9.66 730̂	24	9.72 017	30̂	0.27 983	9.94 713̂	6̂	18
43	9.66 754̂	24	9.72 047̂	31	0.27 952̂	9.94 707	6	17
44	9.66 778̂	24	9.72 078̂	30̂	0.27 921̂	9.94 700̂	7	16
45	9.66 802̂	24	9.72 109	30̂	0.27 891	9.94 693̂	6̂	15
46	9.66 826̂	24	9.72 139̂	30̂	0.27 860̂	9.94 687	6̂	14
47	9.66 850̂	24	9.72 170	31	0.27 830	9.94 680̂	6̂	13
48	9.66 874̂	24	9.72 201	30̂	0.27 799	9.94 674	7	12
49	9.66 898̂	24	9.72 231̂	30̂	0.27 768̂	9.94 667	6̂	11
50	9.66 922̂	24	9.72 262	30̂	0.27 738	9.94 660̂	6	10
51	9.66 946̂	24	9.72 292̂	30̂	0.27 707̂	9.94 654	7	9
52	9.66 970̂	23̂	9.72 323	31	0.27 677	9.94 647	6̂	8
53	9.66 994	24	9.72 354	30̂	0.27 646	9.94 640̂	7	7
54	9.67 018	24	9.72 384̂	30̂	0.27 615̂	9.94 633̂	6̂	6
55	9.67 042	24	9.72 415	30̂	0.27 585	9.94 627	6̂	5
56	9.67 066	23̂	9.72 445̂	30̂	0.27 554̂	9.94 620̂	7	4
57	9.67 089̂	24	9.72 476	30̂	0.27 524	9.94 613̂	6̂	3
58	9.67 113̂	23̂	9.72 506̂	30̂	0.27 493̂	9.94 607	7	2
59	9.67 137	24	9.72 537	30̂	0.27 463	9.94 600	6̂	1
60	9.67 161		9.72 567̂		0.27 432̂	9.94 593̂		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	31̂	31	30̂
6	3.1̂	3.1	3.0̂
7	3.7	3.6	3.5̂
8	4.2	4.1̂	4.0̂
9	4.7	4.6̂	4.6
10	5.2̂	5.1̂	5.1
20	10.5	10.3̂	10.1̂
30	15.7̂	15.5	15.2̂
40	21.0	20.6̂	20.3̂
50	26.2̂	25.8̂	25.4

	25
6	2.5
7	2.9
8	3.3̂
9	3.7̂
10	4.1̂
20	8.3̂
30	12.5
40	16.6̂
50	20.8̂

	24̂	24	23̂
6	2.4̂	2.4	2.3̂
7	2.8̂	2.8	2.7̂
8	3.2̂	3.2	3.1̂
9	3.7	3.6	3.5
10	4.1	4.0	3.9
20	8.1̂	8.0	7.8̂
30	12.2̂	12.0	11.7̂
40	16.3̂	16.0	15.6̂
50	20.4	20.0	19.6

	7	6̂	6
6	0.7	0.6̂	0.6
7	0.8	0.7̂	0.7
8	0.9̂	0.8̂	0.8
9	1.0̂	1.0	0.9
10	1.1̂	1.1	1.0
20	2.3̂	2.1̂	2.0
30	3.5	3.2̂	3.0
40	4.6̂	4.3̂	4.0
50	5.8̂	5.4	5.0

P. P.

62°

28°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.67 161	23	9.72 567	30	0.27 432	9.94 593	6	60
1	9.67 184	24	9.72 598	30	0.27 402	9.94 587	7	59
2	9.67 208	23	9.72 628	30	0.27 371	9.94 580	6	58
3	9.67 232	24	9.72 659	30	0.27 341	9.94 573	7	57
4	9.67 256	23	9.72 689	30	0.27 311	9.94 566	6	56
5	9.67 279	23	9.72 719	30	0.27 280	9.94 560	7	55
6	9.67 303	24	9.72 750	30	0.27 250	9.94 553	6	54
7	9.67 327	23	9.72 780	30	0.27 219	9.94 546	7	53
8	9.67 350	23	9.72 811	30	0.27 189	9.94 539	6	52
9	9.67 374	23	9.72 841	30	0.27 159	9.94 533	7	51
10	9.67 397	24	9.72 871	30	0.27 128	9.94 526	6	50
11	9.67 421	23	9.72 902	30	0.27 098	9.94 519	7	49
12	9.67 445	23	9.72 932	30	0.27 067	9.94 512	6	48
13	9.67 468	23	9.72 962	30	0.27 037	9.94 506	7	47
14	9.67 492	23	9.72 993	30	0.27 007	9.94 499	7	46
15	9.67 515	23	9.73 023	30	0.26 976	9.94 492	6	45
16	9.67 539	23	9.73 053	30	0.26 946	9.94 485	7	44
17	9.67 562	23	9.73 084	30	0.26 916	9.94 478	6	43
18	9.67 586	23	9.73 114	30	0.26 886	9.94 472	7	42
19	9.67 609	23	9.73 144	30	0.26 855	9.94 465	7	41
20	9.67 633	23	9.73 174	30	0.26 825	9.94 458	6	40
21	9.67 656	23	9.73 205	30	0.26 795	9.94 451	7	39
22	9.67 679	23	9.73 235	30	0.26 765	9.94 444	7	38
23	9.67 703	23	9.73 265	30	0.26 734	9.94 437	6	37
24	9.67 726	23	9.73 295	30	0.26 704	9.94 431	7	36
25	9.67 750	23	9.73 325	30	0.26 674	9.94 424	7	35
26	9.67 773	23	9.73 356	30	0.26 644	9.94 417	6	34
27	9.67 796	23	9.73 386	30	0.26 614	9.94 410	7	33
28	9.67 819	23	9.73 416	30	0.26 584	9.94 403	7	32
29	9.67 843	23	9.73 446	30	0.26 553	9.94 396	6	31
30	9.67 866	23	9.73 476	30	0.26 523	9.94 390	7	30
31	9.67 889	23	9.73 506	30	0.26 493	9.94 383	7	29
32	9.67 913	23	9.73 536	30	0.26 463	9.94 376	6	28
33	9.67 936	23	9.73 567	30	0.26 433	9.94 369	7	27
34	9.67 959	23	9.73 597	30	0.26 403	9.94 362	7	26
35	9.67 982	23	9.73 627	30	0.26 373	9.94 355	7	25
36	9.68 005	23	9.73 657	30	0.26 343	9.94 348	7	24
37	9.68 029	23	9.73 687	30	0.26 313	9.94 341	6	23
38	9.68 052	23	9.73 717	30	0.26 283	9.94 335	7	22
39	9.68 075	23	9.73 747	30	0.26 253	9.94 328	7	21
40	9.68 098	23	9.73 777	30	0.26 223	9.94 321	7	20
41	9.68 121	23	9.73 807	30	0.26 193	9.94 314	7	19
42	9.68 144	23	9.73 837	30	0.26 163	9.94 307	6	18
43	9.68 167	23	9.73 867	30	0.26 133	9.94 300	7	17
44	9.68 190	23	9.73 897	30	0.26 103	9.94 293	7	16
45	9.68 213	23	9.73 927	30	0.26 073	9.94 286	7	15
46	9.68 236	23	9.73 957	30	0.26 043	9.94 279	7	14
47	9.68 259	23	9.73 987	30	0.26 013	9.94 272	7	13
48	9.68 282	23	9.74 017	30	0.25 983	9.94 265	7	12
49	9.68 305	23	9.74 047	29	0.25 953	9.94 258	7	11
50	9.68 328	23	9.74 076	30	0.25 923	9.94 251	6	10
51	9.68 351	23	9.74 106	30	0.25 893	9.94 245	7	9
52	9.68 374	22	9.74 136	30	0.25 863	9.94 238	7	8
53	9.68 397	23	9.74 166	29	0.25 833	9.94 231	7	7
54	9.68 420	23	9.74 196	30	0.25 804	9.94 224	7	6
55	9.68 443	23	9.74 226	30	0.25 774	9.94 217	7	5
56	9.68 466	22	9.74 256	30	0.25 744	9.94 210	7	4
57	9.68 488	23	9.74 286	29	0.25 714	9.94 203	7	3
58	9.68 511	23	9.74 315	30	0.25 684	9.94 196	7	2
59	9.68 534	22	9.74 345	29	0.25 654	9.94 189	7	1
60	9.68 557		9.74 375		0.25 625	9.94 182		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	30̂	30	29̂
6	3.0	3.0	2.9
7	3.5	3.5	3.4
8	4.0	4.0	3.9
9	4.6	4.5	4.4
10	5.1	5.0	4.9
20	10.1	10.0	9.8
30	15.2	15.0	14.7
40	20.3	20.0	19.6
50	25.4	25.0	24.6

	24
6	2.4
7	2.8
8	3.2
9	3.6
10	4.0
20	8.0
30	12.0
40	16.0
50	20.0

	23̂	23	22̂
6	2.3	2.3	2.2
7	2.7	2.7	2.6
8	3.1	3.0	3.0
9	3.5	3.4	3.4
10	3.9	3.8	3.7
20	7.8	7.6	7.5
30	11.7	11.5	11.2
40	15.6	15.3	15.0
50	19.6	19.1	18.7

	7	6̂
6	0.7	0.6
7	0.8	0.7
8	0.9	0.8
9	1.0	1.0
10	1.1	1.1
20	2.3	2.1
30	3.5	3.2
40	4.6	4.3
50	5.8	5.4

P. P.

29°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.68 557	23	9.74 375	30	0.25 625	9.94 182	7	60
1	9.68 580	22	9.74 405	30	0.25 595	9.94 175	7	59
2	9.68 602	23	9.74 435	29	0.25 565	9.94 168	7	58
3	9.68 625	22	9.74 464	30	0.25 535	9.94 161	7	57
4	9.68 648	23	9.74 494	29	0.25 505	9.94 154	7	56
5	9.68 671	22	9.74 524	30	0.25 476	9.94 147	7	55
6	9.68 693	23	9.74 554	29	0.25 446	9.94 140	7	54
7	9.68 716	22	9.74 583	29	0.25 416	9.94 133	7	53
8	9.68 739	22	9.74 613	30	0.25 387	9.94 126	7	52
9	9.68 761	23	9.74 643	29	0.25 357	9.94 118	7	51
10	9.68 784	22	9.74 672	30	0.25 327	9.94 111	7	50
11	9.68 807	22	9.74 702	29	0.25 297	9.94 104	7	49
12	9.68 829	22	9.74 732	29	0.25 268	9.94 097	7	48
13	9.68 852	22	9.74 761	30	0.25 238	9.94 090	7	47
14	9.68 874	22	9.74 791	29	0.25 208	9.94 083	7	46
15	9.68 897	23	9.74 821	29	0.25 179	9.94 076	7	45
16	9.68 920	22	9.74 850	29	0.25 149	9.94 069	7	44
17	9.68 942	22	9.74 880	29	0.25 120	9.94 062	7	43
18	9.68 965	22	9.74 909	30	0.25 090	9.94 055	7	42
19	9.68 987	22	9.74 939	29	0.25 060	9.94 048	7	41
20	9.69 010	22	9.74 969	29	0.25 031	9.94 041	7	40
21	9.69 032	22	9.74 998	29	0.25 001	9.94 034	7	39
22	9.69 055	22	9.75 028	29	0.24 972	9.94 026	7	38
23	9.69 077	22	9.75 057	29	0.24 942	9.94 019	7	37
24	9.69 099	22	9.75 087	29	0.24 913	9.94 012	7	36
25	9.69 122	22	9.75 116	29	0.24 883	9.94 005	7	35
26	9.69 144	22	9.75 146	29	0.24 854	9.93 998	7	34
27	9.69 167	22	9.75 175	29	0.24 824	9.93 991	7	33
28	9.69 189	22	9.75 205	29	0.24 795	9.93 984	7	32
29	9.69 211	22	9.75 234	29	0.24 765	9.93 977	7	31
30	9.69 234	22	9.75 264	29	0.24 736	9.93 969	7	30
31	9.69 256	22	9.75 293	29	0.24 706	9.93 962	7	29
32	9.69 278	22	9.75 323	29	0.24 677	9.93 955	7	28
33	9.69 301	22	9.75 352	29	0.24 647	9.93 948	7	27
34	9.69 323	22	9.75 382	29	0.24 618	9.93 941	7	26
35	9.69 345	22	9.75 411	29	0.24 588	9.93 934	7	25
36	9.69 367	22	9.75 441	29	0.24 559	9.93 926	7	24
37	9.69 390	22	9.75 470	29	0.24 529	9.93 919	7	23
38	9.69 412	22	9.75 499	29	0.24 500	9.93 912	7	22
39	9.69 434	22	9.75 529	29	0.24 471	9.93 905	7	21
40	9.69 456	22	9.75 558	29	0.24 441	9.93 898	7	20
41	9.69 478	22	9.75 588	29	0.24 412	9.93 891	7	19
42	9.69 500	22	9.75 617	29	0.24 383	9.93 883	7	18
43	9.69 523	22	9.75 646	29	0.24 353	9.93 876	7	17
44	9.69 545	22	9.75 676	29	0.24 324	9.93 869	7	16
45	9.69 567	22	9.75 705	29	0.24 295	9.93 862	7	15
46	9.69 589	22	9.75 734	29	0.24 265	9.93 854	7	14
47	9.69 611	22	9.75 764	29	0.24 236	9.93 847	7	13
48	9.69 633	22	9.75 793	29	0.24 207	9.93 840	7	12
49	9.69 655	22	9.75 822	29	0.24 177	9.93 833	7	11
50	9.69 677	22	9.75 851	29	0.24 148	9.93 826	7	10
51	9.69 699	22	9.75 881	29	0.24 119	9.93 818	7	9
52	9.69 721	22	9.75 910	29	0.24 090	9.93 811	7	8
53	9.69 743	22	9.75 939	29	0.24 060	9.93 804	7	7
54	9.69 765	22	9.75 968	29	0.24 031	9.93 796	7	6
55	9.69 787	22	9.75 998	29	0.24 002	9.93 789	7	5
56	9.69 809	22	9.76 027	29	0.23 973	9.93 782	7	4
57	9.69 831	21	9.76 056	29	0.23 943	9.93 775	7	3
58	9.69 853	22	9.76 085	29	0.23 914	9.93 767	7	2
59	9.69 875	22	9.76 115	29	0.23 885	9.93 760	7	1
60	9.69 897		9.76 144		0.23 856	9.93 753		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	30	29̂	29
6	3.0	2.9	2.9
7	3.5	3.4	3.4
8	4.0	3.9	3.8
9	4.5	4.4	4.3
10	5.0	4.9	4.8
20	10.0	9.8	9.6
30	15.0	14.7	14.5
40	20.0	19.6	19.3
50	25.0	24.6	24.1

	23
6	2.3
7	2.7
8	3.0
9	3.4
10	3.8
20	7.6
30	11.5
40	15.3
50	19.1

	22̂	22	21̂
6	2.2	2.2	2.1
7	2.6	2.5	2.5
8	3.0	2.9	2.8
9	3.4	3.3	3.2
10	3.7	3.6	3.6
20	7.5	7.3	7.1
30	11.2	11.0	10.7
40	15.0	14.6	14.3
50	18.7	18.3	17.9

	7̂	7
6	0.7	0.7
7	0.9	0.8
8	1.0	0.9
9	1.1	1.0
10	1.2	1.1
20	2.5	2.3
30	3.7	3.5
40	5.0	4.6
50	6.2	5.8

P. P.

60°

30°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.69 897	22	9.76 144	29	0.23 856	9.93 753	7	60
1	9.69 919	21	9.76 173	29	0.23 827	9.93 746	7	59
2	9.69 940	22	9.76 202	29	0.23 797	9.93 738	7	58
3	9.69 962	22	9.76 231	29	0.23 768	9.93 731	7	57
4	9.69 984	21	9.76 260	29	0.23 739	9.93 724	7	56
5	9.70 006	22	9.76 289	29	0.23 710	9.93 716	7	55
6	9.70 028	22	9.76 319	29	0.23 681	9.93 709	7	54
7	9.70 050	21	9.76 348	29	0.23 652	9.93 702	7	53
8	9.70 071	22	9.76 377	29	0.23 623	9.93 694	7	52
9	9.70 093	21	9.76 406	29	0.23 594	9.93 687	7	51
10	9.70 115	22	9.76 435	29	0.23 565	9.93 680	7	50
11	9.70 137	21	9.76 464	29	0.23 535	9.93 672	7	49
12	9.70 158	21	9.76 493	29	0.23 506	9.93 665	7	48
13	9.70 180	22	9.76 522	29	0.23 477	9.93 658	7	47
14	9.70 202	21	9.76 551	29	0.23 448	9.93 650	7	46
15	9.70 223	21	9.76 580	29	0.23 419	9.93 643	7	45
16	9.70 245	22	9.76 609	29	0.23 390	9.93 635	7	44
17	9.70 267	21	9.76 638	29	0.23 361	9.93 628	7	43
18	9.70 288	21	9.76 667	29	0.23 332	9.93 621	7	42
19	9.70 310	21	9.76 696	29	0.23 303	9.93 613	7	41
20	9.70 331	22	9.76 725	29	0.23 274	9.93 606	7	40
21	9.70 353	21	9.76 754	29	0.23 245	9.93 599	7	39
22	9.70 375	21	9.76 783	29	0.23 216	9.93 591	7	38
23	9.70 396	21	9.76 812	29	0.23 187	9.93 584	7	37
24	9.70 418	21	9.76 841	29	0.23 158	9.93 576	7	36
25	9.70 439	21	9.76 870	28	0.23 129	9.93 569	7	35
26	9.70 461	21	9.76 899	29	0.23 101	9.93 562	7	34
27	9.70 482	21	9.76 928	29	0.23 072	9.93 554	7	33
28	9.70 504	21	9.76 957	29	0.23 043	9.93 547	7	32
29	9.70 525	21	9.76 986	29	0.23 014	9.93 539	7	31
30	9.70 547	21	9.77 015	28	0.22 985	9.93 532	7	30
31	9.70 568	21	9.77 043	29	0.22 956	9.93 524	7	29
32	9.70 590	21	9.77 072	29	0.22 927	9.93 517	7	28
33	9.70 611	21	9.77 101	29	0.22 898	9.93 509	7	27
34	9.70 632	21	9.77 130	28	0.22 869	9.93 502	7	26
35	9.70 654	21	9.77 159	29	0.22 841	9.93 495	7	25
36	9.70 675	21	9.77 188	29	0.22 812	9.93 487	7	24
37	9.70 696	21	9.77 217	28	0.22 783	9.93 480	7	23
38	9.70 718	21	9.77 245	29	0.22 754	9.93 472	7	22
39	9.70 739	21	9.77 274	29	0.22 725	9.93 465	7	21
40	9.70 760	21	9.77 303	28	0.22 696	9.93 457	7	20
41	9.70 782	21	9.77 332	29	0.22 668	9.93 450	7	19
42	9.70 803	21	9.77 361	28	0.22 639	9.93 442	7	18
43	9.70 824	21	9.77 389	29	0.22 610	9.93 435	7	17
44	9.70 846	21	9.77 418	28	0.22 581	9.93 427	7	16
45	9.70 867	21	9.77 447	29	0.22 553	9.93 420	7	15
46	9.70 888	21	9.77 476	28	0.22 524	9.93 412	7	14
47	9.70 909	21	9.77 504	29	0.22 495	9.93 405	7	13
48	9.70 930	21	9.77 533	28	0.22 466	9.93 397	7	12
49	9.70 952	21	9.77 562	29	0.22 438	9.93 390	8	11
50	9.70 973	21	9.77 591	28	0.22 409	9.93 382	7	10
51	9.70 994	21	9.77 619	28	0.22 380	9.93 374	7	9
52	9.71 015	21	9.77 648	29	0.22 352	9.93 367	7	8
53	9.71 036	21	9.77 677	28	0.22 323	9.93 359	7	7
54	9.71 057	21	9.77 705	28	0.22 294	9.93 352	7	6
55	9.71 078	21	9.77 734	29	0.22 266	9.93 344	7	5
56	9.71 099	21	9.77 763	28	0.22 237	9.93 337	7	4
57	9.71 121	21	9.77 791	28	0.22 208	9.93 329	8	3
58	9.71 142	21	9.77 820	29	0.22 180	9.93 321	7	2
59	9.71 163	21	9.77 849	28	0.22 151	9.93 314	7	1
60	9.71 184		9.77 877		0.22 122	9.93 306		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	29	29	28
6	2.9	2.9	2.8
7	3.4	3.4	3.3
8	3.9	3.8	3.8
9	4.4	4.3	4.3
10	4.9	4.8	4.7
20	9.8	9.6	9.5
30	14.7	14.5	14.2
40	19.6	19.3	19.0
50	24.6	24.1	23.7

	22	21	21
6	2.2	2.1	2.1
7	2.5	2.5	2.4
8	2.9	2.8	2.8
9	3.3	3.2	3.1
10	3.6	3.6	3.5
20	7.3	7.1	7.0
30	11.0	10.7	10.5
40	14.6	14.3	14.0
50	18.3	17.9	17.5

	8	7	7
6	0.8	0.7	0.7
7	0.9	0.9	0.8
8	1.0	1.0	0.9
9	1.2	1.1	1.0
10	1.3	1.2	1.1
20	2.6	2.5	2.3
30	4.0	3.7	3.5
40	5.3	5.0	4.6
50	6.6	6.2	5.8

P. P.

59°

31°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.71 184	21	9.77 877	28	0.22 122	9.93 306	7	60
1	9.71 205	21	9.77 906	28	0.22 094	9.93 299	7	59
2	9.71 226	21	9.77 934	28	0.22 065	9.93 291	7	58
3	9.71 247	21	9.77 963	29	0.22 037	9.93 284	8	57
4	9.71 268	21	9.77 992	28	0.22 008	9.93 276	7	56
5	9.71 289	21	9.78 020	28	0.21 979	9.93 268	7	55
6	9.71 310	21	9.78 049	28	0.21 951	9.93 261	7	54
7	9.71 331	20	9.78 077	28	0.21 922	9.93 253	8	53
8	9.71 351	21	9.78 106	28	0.21 894	9.93 245	7	52
9	9.71 372	21	9.78 134	28	0.21 865	9.93 238	7	51
10	9.71 393	21	9.78 163	28	0.21 837	9.93 230	7	50
11	9.71 414	20	9.78 191	28	0.21 808	9.93 223	8	49
12	9.71 435	21	9.78 220	28	0.21 780	9.93 215	7	48
13	9.71 456	21	9.78 248	28	0.21 751	9.93 207	7	47
14	9.71 477	21	9.78 277	28	0.21 723	9.93 200	8	46
15	9.71 498	20	9.78 305	28	0.21 694	9.93 192	7	45
16	9.71 518	21	9.78 334	28	0.21 666	9.93 184	7	44
17	9.71 539	20	9.78 362	28	0.21 637	9.93 177	8	43
18	9.71 560	21	9.78 391	28	0.21 609	9.93 169	7	42
19	9.71 581	20	9.78 419	28	0.21 580	9.93 161	8	41
20	9.71 601	21	9.78 448	28	0.21 552	9.93 153	7	40
21	9.71 622	20	9.78 476	28	0.21 523	9.93 146	7	39
22	9.71 643	21	9.78 505	28	0.21 495	9.93 138	8	38
23	9.71 664	20	9.78 533	28	0.21 467	9.93 130	7	37
24	9.71 684	21	9.78 561	28	0.21 438	9.93 123	8	36
25	9.71 705	20	9.78 590	28	0.21 410	9.93 115	7	35
26	9.71 726	20	9.78 618	28	0.21 381	9.93 107	7	34
27	9.71 746	21	9.78 647	28	0.21 353	9.93 100	8	33
28	9.71 767	20	9.78 675	28	0.21 325	9.93 092	7	32
29	9.71 788	20	9.78 703	28	0.21 296	9.93 084	8	31
30	9.71 808	20	9.78 732	28	0.21 268	9.93 076	7	30
31	9.71 829	20	9.78 760	28	0.21 239	9.93 069	8	29
32	9.71 849	21	9.78 788	28	0.21 211	9.93 061	7	28
33	9.71 870	20	9.78 817	28	0.21 183	9.93 053	8	27
34	9.71 891	20	9.78 845	28	0.21 154	9.93 045	7	26
35	9.71 911	20	9.78 873	28	0.21 126	9.93 038	8	25
36	9.71 932	20	9.78 902	28	0.21 098	9.93 030	7	24
37	9.71 952	20	9.78 930	28	0.21 070	9.93 022	8	23
38	9.71 973	20	9.78 958	28	0.21 041	9.93 014	8	22
39	9.71 993	20	9.78 987	28	0.21 013	9.93 006	7	21
40	9.72 014	20	9.79 015	28	0.20 985	9.92 999	8	20
41	9.72 034	20	9.79 043	28	0.20 956	9.92 991	7	19
42	9.72 055	20	9.79 071	28	0.20 928	9.92 983	8	18
43	9.72 075	20	9.79 100	28	0.20 900	9.92 975	8	17
44	9.72 096	20	9.79 128	28	0.20 872	9.92 967	7	16
45	9.72 116	20	9.79 156	28	0.20 843	9.92 960	8	15
46	9.72 136	20	9.79 184	28	0.20 815	9.92 952	8	14
47	9.72 157	20	9.79 213	28	0.20 787	9.92 944	7	13
48	9.72 177	20	9.79 241	28	0.20 759	9.92 936	8	12
49	9.72 198	20	9.79 269	28	0.20 731	9.92 928	8	11
50	9.72 218	20	9.79 297	28	0.20 702	9.92 920	7	10
51	9.72 238	20	9.79 325	28	0.20 674	9.92 913	8	9
52	9.72 259	20	9.79 354	28	0.20 646	9.92 905	8	8
53	9.72 279	20	9.79 382	28	0.20 618	9.92 897	7	7
54	9.72 299	20	9.79 410	28	0.20 590	9.92 889	8	6
55	9.72 319	20	9.79 438	28	0.20 561	9.92 881	8	5
56	9.72 340	20	9.79 466	28	0.20 533	9.92 873	8	4
57	9.72 360	20	9.79 494	28	0.20 505	9.92 865	7	3
58	9.72 380	20	9.79 522	28	0.20 477	9.92 858	8	2
59	9.72 400	20	9.79 551	28	0.20 449	9.92 850	8	1
60	9.72 421		9.79 579		0.20 421	9.92 842		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	29	28	28
6	2.9	2.8	2.8
7	3.4	3.3	3.2
8	3.8	3.8	3.7
9	4.3	4.3	4.2
10	4.8	4.7	4.6
20	9.6	9.5	9.3
30	14.5	14.2	14.0
40	19.3	19.0	18.6
50	24.1	23.7	23.3

	21	20	20
6	2.1	2.0	2.0
7	2.4	2.4	2.3
8	2.8	2.7	2.6
9	3.1	3.1	3.0
10	3.5	3.4	3.3
20	7.0	6.8	6.6
30	10.5	10.2	10.0
40	14.0	13.6	13.3
50	17.5	17.1	16.6

	8	7
6	0.8	0.7
7	0.9	0.9
8	1.0	1.0
9	1.2	1.1
10	1.3	1.2
20	2.6	2.5
30	4.0	3.7
40	5.3	5.0
50	6.6	6.2

P. P.

58°

32°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.72 421	20	9.79 579	28	0.20 421	9.92 842	8	60
1	9.72 441	20	9.79 607	28	0.20 393	9.92 834	7	59
2	9.72 461	20	9.79 635	28	0.20 365	9.92 826	8	58
3	9.72 481	20	9.79 663	28	0.20 337	9.92 818	8	57
4	9.72 501	20	9.79 691	28	0.20 308	9.92 810	8	56
5	9.72 522	20	9.79 719	28	0.20 280	9.92 802	8	55
6	9.72 542	20	9.79 747	28	0.20 252	9.92 794	8	54
7	9.72 562	20	9.79 775	28	0.20 224	9.92 786	8	53
8	9.72 582	20	9.79 803	28	0.20 196	9.92 778	7	52
9	9.72 602	20	9.79 831	28	0.20 168	9.92 771	8	51
10	9.72 622	20	9.79 859	28	0.20 140	9.92 763	8	50
11	9.72 642	20	9.79 887	28	0.20 112	9.92 755	8	49
12	9.72 662	20	9.79 915	28	0.20 084	9.92 747	8	48
13	9.72 682	20	9.79 943	28	0.20 056	9.92 739	8	47
14	9.72 702	20	9.79 971	28	0.20 028	9.92 731	8	46
15	9.72 723	20	9.79 999	28	0.20 000	9.92 723	8	45
16	9.72 743	20	9.80 027	28	0.19 972	9 92 715	8	44
17	9.72 763	20	9.80 055	28	0.19 944	9.92 707	8	43
18	9.72 783	19	9.80 083	28	0.19 916	9.92 699	8	42
19	9.72 802	20	9.80 111	28	0.19 888	9.92 691	8	41
20	9.72 822	20	9.80 139	28	0.19 860	9.92 683	8	40
21	9.72 842	20	9.80 167	28	0.19 832	9.92 675	8	39
22	9.72 862	20	9.80 195	28	0.19 804	9.92 667	8	38
23	9.72 882	20	9 80 223	28	0.19 776	9.92 659	8	37
24	9.72 902	20	9.80 251	27	0.19 748	9.92 651	8	36
25	9.72 922	19	9.80 279	28	0.19 721	9 92 643	8	35
26	9.72 942	20	9.80 307	28	0.19 693	9.92 635	8	34
27	9.72 962	20	9.80 335	28	0.19 665	9.92 627	8	33
28	9.72 982	20	9.80 363	28	0.19 637	9.92 619	8	32
29	9.73 002	19	9.80 391	27	0.19 609	9.92 611	8	31
30	9.73 021	20	9.80 418	28	0.19 581	9.92 603	8	30
31	9.73 041	20	9.80 446	28	0.19 553	9.92 595	8	29
32	9.73 061	19	9.80 474	28	0.19 525	9.92 587	8	28
33	9.73 081	20	9.80 502	27	0.19 497	9.92 579	8	27
34	9.73 101	19	9.80 530	28	0.19 470	9.92 570	8	26
35	9.73 120	20	9.80 558	28	0.19 442	9.92 562	8	25
36	9.73 140	19	9.80 586	27	0.19 414	9.92 554	8	24
37	9.73 160	20	9.80 613	28	0.19 386	9.92 546	8	23
38	9.73 180	19	9.80 641	28	0.19 358	9.92 538	8	22
39	9.73 199	20	9.80 669	27	0.19 330	9.92 530	8	21
40	9.73 219	19	9.80 697	28	0.19 303	9.92 522	8	20
41	9.73 239	19	9.80 725	27	0.19 275	9.92 514	8	19
42	9:73 258	20	9.80 752	28	0.19 247	9.92 506	8	18
43	9.73 278	19	9.80 780	28	0.19 219	9.92 498	8	17
44	9.73 298	19	9.80 808	27	0.19 191	9.92 489	8	16
45	9.73 317	20	9.80 836	28	0.19 164	9.92 481	8	15
46	9.73 337	19	9.80 864	27	0.19 136	9.92 473	8	14
47	9.73 357	19	9.80 891	28	0.19 108	9.92 465	8	13
48	9.73 376	19	9.80 919	27	0.19 080	9.92 457	8	12
49	9 73 396	19	9.80 947	28	0.19 053	9.92 449	8	11
50	9.73 415	20	9.80 975	27	0.19 025	9.92 441	8	10
51	9.73 435	19	9.81 002	27	0.18 997	9.92 433	8	9
52	9.73 455	19	9.81 030	28	0.18 970	9.92 424	8	8
53	9.73 474	19	9.81 058	27	0.18 942	9.92 416	8	7
54	9.73 494	19	9.81 085	28	0.18 914	9.92 408	8	6
55	9.73 513	19	9.81 113	27	0.18 886	9.92 400	8	5
56	9.73 533	19	9.81 141	27	0.18 859	9.92 392	8	4
57	9.73 552	19	9.81 168	28	0.18 831	9.92 383	8	3
58	9.73 572	19	9.81 196	27	0.18 803	9.92 375	8	2
59	9.73 591	19	9.81 224	27	0.18 776	9.92 367	8	1
60	9.73 611		9.81 251		0.18 748	9.92 359		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	28	28	27
6	2.8	2.8	2.7
7	3.3	3.2	3.2
8	3.8	3.7	3.6
9	4.3	4.2	4.1
10	4.7	4.6	4.6
20	9.5	9.3	9.1
30	14.2	14.0	13.7
40	19.0	18.6	18.3
50	23.7	23.3	22.9

	20	20	19
6	2.0	2.0	1.9
7	2.4	2.3	2.3
8	2.7	2.6	2.6
9	3.1	3.0	2.9
10	3.4	3.3	3.2
20	6.8	6.6	6.5
30	10.2	10.0	9.7
40	13.6	13.3	13.0
50	17.1	16.6	16.2

	8	8	7
6	0.8	0.8	0.7
7	1.0	0.9	0.9
8	1.1	1.0	1.0
9	1.3	1.2	1.1
10	1.4	1.3	1.2
20	2.8	2.6	2.5
30	4.2	4.0	3.7
40	5.6	5.3	5.0
50	7.1	6.6	6.2

P. P.

57°

33°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.73 611	19	9.81 251	28	0.18 748	9.92 359	8	60
1	9.73 630	19	9.81 279	27	0.18 720	9.92 351	8	59
2	9.73 650	19	9.81 307	27	0.18 693	9.92 342	8	58
3	9.73 669	19	9.81 334	28	0.18 665	9.92 334	8	57
4	9.73 688	19	9.81 362	27	0.18 637	9.92 326	8	56
5	9.73 708	19	9.81 390	27	0.18 610	9.92 318	8	55
6	9.73 727	19	9.81 417	27	0.18 582	9.92 310	8	54
7	9.73 746	19	9.81 445	28	0.18 555	9.92 301	8	53
8	9.73 766	19	9.81 473	27	0.18 527	9.92 293	8	52
9	9.73 785	19	9.81 500	27	0.18 499	9.92 285	8	51
10	9.73 805	19	9.81 528	27	0.18 472	9.92 277	8	50
11	9.73 824	19	9.81 555	27	0.18 444	9.92 268	8	49
12	9.73 843	19	9.81 583	27	0.18 417	9.92 260	8	48
13	9.73 862	19	9.81 610	27	0.18 389	9.92 252	8	47
14	9.73 882	19	9.81 638	28	0.18 362	9.92 244	8	46
15	9.73 901	19	9.81 666	27	0.18 334	9.92 235	8	45
16	9.73 920	19	9.81 693	27	0.18 306	9.92 227	8	44
17	9.73 940	19	9.81 721	27	0.18 279	9.92 219	8	43
18	9.73 959	19	9.81 748	27	0.18 251	9.92 210	8	42
19	9.73 978	19	9.81 776	27	0.18 224	9.92 202	8	41
20	9.73 997	19	9.81 803	27	0.18 196	9.92 194	8	40
21	9.74 016	19	9.81 831	27	0.18 169	9.92 185	8	39
22	9.74 036	19	9.81 858	27	0.18 141	9.92 177	8	38
23	9.74 055	19	9.81 886	27	0.18 114	9.92 169	8	37
24	9.74 074	19	9.81 913	27	0.18 086	9.92 160	8	36
25	9.74 093	19	9.81 941	27	0.18 059	9.92 152	8	35
26	9.74 112	19	9.81 968	27	0.18 031	9.92 144	8	34
27	9.74 131	19	9.81 996	27	0.18 004	9.92 135	8	33
28	9.74 151	19	9.82 023	27	0.17 976	9.92 127	8	32
29	9.74 170	19	9.82 051	27	0.17 949	9.92 119	8	31
30	9.74 189	19	9.82 078	27	0.17 921	9.92 110	8	30
31	9.74 208	19	9.82 105	27	0.17 894	9.92 102	8	29
32	9.74 227	19	9.82 133	27	0.17 867	9.92 094	8	28
33	9.74 246	19	9.82 160	27	0.17 839	9.92 085	8	27
34	9.74 265	19	9.82 188	27	0.17 812	9.92 077	8	26
35	9.74 284	19	9.82 215	27	0.17 784	9.92 069	8	25
36	9.74 303	19	9.82 243	27	0.17 757	9.92 060	8	24
37	9.74 322	19	9.82 270	27	0.17 729	9.92 052	8	23
38	9.74 341	18	9.82 297	27	0.17 702	9.92 043	8	22
39	9.74 360	19	9.82 325	27	0.17 675	9.92 035	8	21
40	9.74 379	19	9.82 352	27	0.17 647	9.92 027	8	20
41	9.74 398	19	9.82 380	27	0.17 620	9.92 018	8	19
42	9.74 417	19	9.82 407	27	0.17 593	9.92 010	8	18
43	9.74 436	19	9.82 434	27	0.17 565	9.92 001	8	17
44	9.74 455	19	9.82 462	27	0.17 538	9.91 993	8	16
45	9.74 474	19	9.82 489	27	0.17 510	9.91 984	8	15
46	9.74 493	18	9.82 516	27	0.17 483	9.91 976	8	14
47	9.74 511	19	9.82 544	27	0.17 456	9.91 967	8	13
48	9.74 530	19	9.82 571	27	0.17 428	9.91 959	8	12
49	9.74 549	19	9.82 598	27	0.17 401	9.91 951	8	11
50	9.74 568	18	9.82 626	27	0.17 374	9.91 942	8	10
51	9.74 587	19	9.82 653	27	0.17 347	9.91 934	8	9
52	9.74 606	19	9.82 680	27	0.17 319	9.91 925	8	8
53	9.74 625	18	9.82 708	27	0.17 292	9.91 917	8	7
54	9.74 643	19	9.82 735	27	0.17 265	9.91 908	8	6
55	9.74 662	18	9.82 762	27	0.17 237	9.91 900	8	5
56	9.74 681	19	9.82 789	27	0.17 210	9.91 891	8	4
57	9.74 700	18	9.82 817	27	0.17 183	9.91 883	8	3
58	9.74 718	19	9.82 844	27	0.17 156	9.91 874	8	2
59	9.74 737	18	9.82 871	27	0.17 128	9.91 866	8	1
60	9.74 756		9.82 898		0.17 101	9.91 857		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	28	27	27
6	2.8	2.7	2.7
7	3.2	3.2	3.1
8	3.7	3.6	3.6
9	4.2	4.1	4.0
10	4.6	4.6	4.5
20	9.3	9.1	9.0
30	14.0	13.7	13.5
40	18.6	18.3	18.0
50	23.3	22.9	22.5

	19	19	18
6	1.9	1.9	1.8
7	2.3	2.2	2.1
8	2.6	2.5	2.4
9	2.9	2.8	2.8
10	3.2	3.1	3.1
20	6.5	6.3	6.1
30	9.7	9.5	9.2
40	13.0	12.6	12.3
50	16.2	15.8	15.4

	8	8
6	0.8	0.8
7	1.0	0.9
8	1.1	1.0
9	1.3	1.2
10	1.4	1.3
20	2.8	2.6
30	4.2	4.0
40	5.6	5.3
50	7.1	6.6

P. P.

56°

34°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.74 756	19	9.82 898̂	27̂	0.17 101̂	9.91 857̂	8̂	60
1	9.74 775	18̂	9.82 926	27	0.17 074	9.91 849	8̂	59
2	9.74 793̂	19	9.82 953	27̂	0.17 047	9.91 840̂	8̂	58
3	9.74 812̂	18̂	9.82 980̂	27	0.17 019̂	9.91 832	8̂	57
4	9.74 831	18̂	9.83 007̂	27̂	0.16 992̂	9.91 823̂	9	56
5	9.74 849̂	19	9.83 035	27	0.16 965	9.91 814̂	8̂	55
6	9.74 868̂	18̂	9.83 062	27̂	0.16 938	9.91 806	8̂	54
7	9.74 887	18̂	9.83 089̂	27	0.16 910̂	9.91 797̂	8̂	53
8	9.74 905̂	19	9.83 116̂	27	0.16 883̂	9.91 789	8̂	52
9	9.74 924̂	18̂	9.83 143̂	27̂	0.16 856̂	9.91 780̂	8̂	51
10	9.74 943	18̂	9.83 171	27	0.16 829	9.91 772	8̂	50
11	9.74 961̂	18̂	9.83 198	27̂	0.16 802	9.91 763̂	8	49
12	9.74 980	18̂	9.83 225̂	27	0.16 774̂	9.91 755	9	48
13	9.74 998̂	18̂	9.83 252̂	27	0.16 747̂	9.91 746	8̂	47
14	9.75 017	19	9.83 279̂	27̂	0.16 720̂	9.91 737̂	8̂	46
15	9.75 036	18̂	9.83 307	27	0.16 693	9.91 729	8̂	45
16	9.75 054̂	18̂	9.83 334	27	0.16 666	9.91 720̂	8	44
17	9.75 073	18̂	9.83 361	27	0.16 639	9.91 712	9	43
18	9.75 091̂	18̂	9.83 388	27̂	0.16 612	9.91 703	8̂	42
19	9.75 110	18̂	9.83 415̂	27	0.16 584̂	9.91 694̂	8̂	41
20	9.75 128̂	18̂	9.83 442̂	27	0.16 557̂	9.91 686	8	40
21	9.75 147	18̂	9.83 469̂	27	0.16 530̂	9.91 677̂	9	39
22	9.75 165̂	18̂	9.83 496̂	27̂	0.16 503̂	9.91 668̂	8̂	38
23	9.75 184	18̂	9.83 524	27	0.16 476	9.91 660	8	37
24	9.75 202̂	18̂	9.83 551	27	0.16 449	9.91 651̂	9	36
25	9.75 221	18	9.83 578	27	0.16 422	9.91 642̂	8̂	35
26	9.75 239	18̂	9.83 605	27	0.16 395	9.91 634	8	34
27	9.75 257̂	18̂	9.83 632	27̂	0.16 368	9.91 625̂	9	33
28	9.75 276	18̂	9.83 659̂	27	0.16 340̂	9.91 616̂	8̂	32
29	9.75 294̂	18̂	9.83 686̂	27	0.16 313̂	9.91 608	8	31
30	9.75 313	18	9.83 713̂	27	0.16 286̂	9.91 599̂	9	30
31	9.75 331	18̂	9.83 740̂	27	0.16 259̂	9.91 590̂	8̂	29
32	9.75 349̂	18̂	9.83 767̂	27	0.16 232̂	9.91 582	8	28
33	9.75 368	18	9.83 794̂	27	0.16 205̂	9.91 573̂	9	27
34	9.75 386	18̂	9.83 821̂	27	0.16 178̂	9.91 564̂	8̂	26
35	9.75 404̂	18̂	9.83 848̂	27	0.16 151̂	9.91 556	9	25
36	9.75 423	18	9.83 875̂	27	0.16 124̂	9.91 547	8̂	24
37	9.75 441	18̂	9.83 902̂	27	0.16 097̂	9.91 538̂	9	23
38	9.75 459̂	18	9.83 929̂	27̂	0.16 070̂	9.91 529̂	8̂	22
39	9.75 478	18	9.83 957	27	0.16 043	9.91 521	8	21
40	9.75 496	18̂	9.83 984	27	0.16 016	9.91 512̂	9	20
41	9.75 514̂	18	9.84 011	27	0.15 989	9.91 503̂	8̂	19
42	9.75 532̂	18̂	9.84 038	27	0.15 962	9.91 495	9	18
43	9.75 551	18	9.84 065	26̂	0.15 935	9.91 486	8	17
44	9.75 569	18	9.84 091̂	27	0.15 908̂	9.91 477̂	9	16
45	9.75 587	18̂	9.84 118̂	27	0.15 881̂	9.91 468̂	8̂	15
46	9.75 605̂	18	9.84 145̂	27	0.15 854̂	9.91 460	9	14
47	9.75 623̂	18̂	9.84 172̂	27	0.15 827̂	9.91 451	9̂	13
48	9.75 642	18	9.84 199̂	27	0.15 800̂	9.91 442	8	12
49	9.75 660	18	9.84 226̂	27	0.15 773̂	9.91 433̂	9	11
50	9.75 678	18̂	9.84 253̂	27	0.15 746̂	9.91 424̂	8	10
51	9.75 696̂	18	9.84 280̂	27	0.15 719̂	9.91 416	9	9
52	9.75 714̂	18	9.84 307̂	27	0.15 692̂	9.91 407	9̂	8
53	9.75 732̂	18	9.84 334̂	26̂	0.15 665̂	9.91 398	8	7
54	9.75 750̂	18̂	9.84 361	27	0.15 639	9 91 389̂	9	6
55	9.75 769	18	9.84 388	27	0.15 612	9.91 380̂	8̂	5
56	9.75 787	18	9.84 415	27	0.15 585	9.91 372	9	4
57	9.75 805	18	9.84 442	27	0.15 558	9.91 363	9̂	3
58	9.75 823	18	9.84 469	27	0.15 531	9.91 354	8	2
59	9.75 841	18	9.84 496	26̂	0.15 504	9.91 345̂	9	1
60	9.75 859		9.84 522̂		0.15 477̂	9.91 336̂		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	27̂	27	26̂
6	2.7̂	2.7	2.6̂
7	3.2	3.1̂	3.1
8	3.6̂	3.6	3.5̂
9	4.1̂	4.0̂	4.0
10	4.6	4.5	4.4
20	9.1̂	9.0	8.8̂
30	13.7̂	13.5	13.2̂
40	18.3̂	18.0	17.6̂
50	22.9	22.5	22.1

	19	18̂	18
6	1.9	1.8̂	1.8
7	2.2	2.1̂	2.1
8	2.5̂	2.4̂	2.4
9	2.8̂	2.8	2.7
10	3.1̂	3.1	3.0
20	6.3̂	6.1̂	6.0
30	9.5	9.2̂	9.0
40	12.6̂	12.3̂	12.0
50	15.8̂	15.4	15.0

	9	8̂
6	0.9	0.8̂
7	1.0̂	1.0
8	1.2	1.1̂
9	1.3̂	1.3
10	1.5	1.4
20	3.0	2.8̂
30	4.5	4.2̂
40	6.0	5.6̂
50	7.5	7.1

P. P.

55°

35°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.75 859	18	9.84 522	27	0.15 477	9.91 336	9	60
1	9.75 877	18	9.84 549	27	0.15 450	9.91 327	9	59
2	9.75 895	18	9.84 576	27	0.15 423	9.91 318	8	58
3	9.75 913	18	9.84 603	26	0.15 396	9.91 310	9	57
4	9.75 931	18	9.84 630	27	0.15 370	9.91 301	9	56
5	9.75 949	18	9.84 657	27	0.15 343	9.91 292	8	55
6	9.75 967	18	9.84 684	27	0.15 316	9.91 283	9	54
7	9.75 985	18	9.84 711	26	0.15 289	9.91 274	9	53
8	9.76 003	18	9.84 737	27	0.15 262	9.91 265	9	52
9	9.76 021	18	9.84 764	27	0.15 235	9.91 256	9	51
10	9.76 039	18	9.84 791	26	0.15 208	9.91 247	8	50
11	9.76 057	18	9.84 818	27	0.15 182	9.91 239	9	49
12	9.76 075	17	9.84 845	26	0.15 155	9.91 230	9	48
13	9.76 092	18	9.84 871	27	0.15 128	9.91 221	9	47
14	9.76 110	18	9.84 898	27	0.15 101	9.91 212	9	46
15	9.76 128	18	9.84 925	26	0.15 074	9.91 203	9	45
16	9.76 146	17	9.84 952	27	0.15 048	9.91 194	8	44
17	9.76 164	18	9.84 979	26	0.15 021	9.91 185	9	43
18	9.76 182	18	9.85 005	27	0.14 994	9.91 176	9	42
19	9.76 200	17	9.85 032	27	0.14 967	9.91 167	9	41
20	9.76 217	18	9.85 059	26	0.14 940	9.91 158	9	40
21	9.76 235	18	9.85 086	27	0.14 914	9.91 149	9	39
22	9.76 253	17	9.85 113	26	0.14 887	9.91 140	9	38
23	9.76 271	18	9.85 139	27	0.14 860	9.91 131	9	37
24	9.76 289	17	9.85 166	26	0.14 833	9.91 122	9	36
25	9.76 306	18	9.85 193	27	0.14 807	9.91 113	9	35
26	9.76 324	17	9.85 220	26	0.14 780	9.91 104	9	34
27	9.76 342	18	9.85 246	27	0.14 753	9.91 095	9	33
28	9.76 360	17	9.85 273	26	0.14 726	9.91 086	9	32
29	9.76 377	18	9.85 300	27	0.14 700	9.91 077	9	31
30	9.76 395	17	9.85 327	26	0.14 673	9.91 068	9	30
31	9.76 413	18	9.85 353	26	0.14 646	9.91 059	9	29
32	9.76 431	17	9.85 380	27	0.14 620	9.91 050	9	28
33	9.76 448	17	9.85 407	26	0.14 593	9.91 041	9	27
34	9.76 466	18	9.85 433	27	0.14 566	9.91 032	9	26
35	9.76 484	17	9.85 460	26	0.14 539	9.91 023	9	25
36	9.76 501	17	9.85 487	26	0.14 513	9.91 014	9	24
37	9.76 519	17	9.85 513	27	0.14 486	9.91 005	9	23
38	9.76 536	18	9.85 540	26	0.14 459	9.90 996	9	22
39	9.76 554	17	9.85 567	27	0.14 433	9.90 987	9	21
40	9.76 572	17	9.85 594	26	0.14 406	9.90 978	9	20
41	9.76 589	17	9.85 620	26	0.14 379	9.90 969	9	19
42	9.76 607	17	9.85 647	26	0.14 353	9.90 960	9	18
43	9 76 624	18	9.85 673	27	0.14 326	9.90 951	9	17
44	9.76 642	17	9.85 700	26	0.14 299	9.90 942	9	16
45	9.76 660	17	9.85 727	26	0.14 273	9.90 933	9	15
46	9.76 677	17	9.85 753	27	0.14 246	9.90 923	9	14
47	9.76 695	17	9.85 780	26	0.14 219	9.90 914	9	13
48	9.76 712	17	9.85 807	26	0.14 193	9.90 905	9	12
49	9.76 730	17	9.85 833	26	0.14 166	9.90 896	9	11
50	9.76 747	17	9.85 860	27	0.14 140	9.90 887	9	10
51	9.76 765	17	9.85 887	26	0.14 113	9.90 878	9	9
52	9.76 782	17	9.85 913	26	0.14 086	9.90 869	9	8
53	9.76 800	17	9.85 940	26	0.14 060	9.90 860	9	7
54	9.76 817	17	9.85 966	26	0.14 033	9.90 850	9	6
55	9.76 835	17	9.85 993	27	0.14 007	9.90 841	9	5
56	9.76 852	17	9.86 020	26	0.13 980	9.90 832	9	4
57	9.76 869	17	9.86 046	26	0.13 953	9.90 823	9	3
58	9.76 887	17	9.86 073	26	0.13 927	9.90 814	9	2
59	9.76 904	17	9.86 099	26	0.13 900	9.90 805	9	1
60	9.76 922		9.86 126		0.13 874	9.90 796		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	27	26
6	2.7	2.6
7	3.1	3.1
8	3.6	3.5
9	4.0	4.0
10	4.5	4.4
20	9.0	8.8
30	13.5	13.2
40	18.0	17.6
50	22.5	22.1

	18	17	17
6	1.8	1.7	1.7
7	2.1	2.0	2.0
8	2.4	2.3	2.2
9	2.7	2.6	2.5
10	3.0	2.9	2.8
20	6.0	5.8	5.6
30	9.0	8.7	8.5
40	12.0	11.6	11.3
50	15.0	14.6	14.1

	9	9	8
6	0.9	0.9	0.8
7	1.1	1.0	1.0
8	1.2	1.2	1.1
9	1.4	1.3	1.3
10	1.6	1.5	1.4
20	3.1	3.0	2.8
30	4.7	4.5	4.2
40	6.3	6.0	5.6
50	7.9	7.5	7.1

P. P.

54°

36°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.76 922	17	9.86 126	26	0.13 874	9.90 796	9	60
1	9.76 939	17	9.86 152	26	0.13 847	9.90 786	9	59
2	9.76 956	17	9.86 179	27	0.13 821	9.90 777	9	58
3	9.76 974	17	9.86 206	26	0.13 794	9.90 768	9	57
4	9.76 991	17	9.86 232	26	0.13 767	9.90 759	9	56
5	9.77 008	17	9.86 259	26	0.13 741	9.90 750	9	55
6	9.77 026	17	9.86 285	26	0.13 714	9.90 740	9	54
7	9.77 043	17	9.86 312	26	0.13 688	9.90 731	9	53
8	9.77 060	17	9.86 338	26	0.13 661	9.90 722	9	52
9	9.77 078	17	9.86 365	26	0.13 635	9.90 713	9	51
10	9.77 095	17	9.86 391	26	0.13 608	9.90 703	9	50
11	9.77 112	17	9.86 418	26	0.13 582	9.90 694	9	49
12	9.77 130	17	9.86 444	26	0.13 555	9.90 685	9	48
13	9.77 147	17	9.86 471	26	0.13 529	9.90 676	9	47
14	9.77 164	17	9.86 497	26	0.13 502	9.90 666	9	46
15	9.77 181	17	9.86 524	26	0.13 476	9.90 657	9	45
16	9.77 198	17	9.86 550	26	0.13 449	9.90 648	9	44
17	9.77 216	17	9.86 577	26	0.13 423	9.90 639	9	43
18	9.77 233	17	9.86 603	26	0.13 396	9.90 629	9	42
19	9.77 250	17	9.86 630	26	0.13 370	9.90 620	9	41
20	9.77 267	17	9.86 656	26	0.13 343	9.90 611	9	40
21	9.77 284	17	9.86 683	26	0.13 317	9.90 602	9	39
22	9.77 302	17	9.86 709	26	0.13 290	9.90 592	9	38
23	9.77 319	17	9.86 736	26	0.13 264	9.90 583	9	37
24	9.77 336	17	9.86 762	26	0.13 237	9.90 574	9	36
25	9.77 353	17	9.86 788	26	0.13 211	9.90 564	9	35
26	9.77 370	17	9.86 815	26	0.13 185	9.90 555	9	34
27	9.77 387	17	9.86 841	26	0.13 158	9.90 546	9	33
28	9.77 404	17	9.86 868	26	0.13 132	9.90 536	9	32
29	9.77 421	17	9.86 894	26	0.13 105	9.90 527	9	31
30	9.77 439	17	9.86 921	26	0.13 079	9.90 518	9	30
31	9.77 456	17	9.86 947	26	0.13 052	9.90 508	9	29
32	9.77 473	17	9.86 973	26	0.13 026	9.90 499	9	28
33	9.77 490	17	9.87 000	26	0.13 000	9.90 490	9	27
34	9.77 507	17	9.87 026	26	0.12 973	9.90 480	9	26
35	9.77 524	17	9.87 053	26	0.12 947	9.90 471	9	25
36	9.77 541	17	9.87 079	26	0.12 920	9.90 461	9	24
37	9.77 558	17	9.87 105	26	0.12 894	9.90 452	9	23
38	9.77 575	17	9.87 132	26	0.12 868	9.90 443	9	22
39	9.77 592	17	9.87 158	26	0.12 841	9.90 433	9	21
40	9.77 609	17	9.87 185	26	0.12 815	9.90 424	9	20
41	9.77 626	17	9.87 211	26	0.12 789	9.90 414	9	19
42	9.77 643	17	9.87 237	26	0.12 762	9.90 405	9	18
43	9.77 660	17	9.87 264	26	0.12 736	9.90 396	9	17
44	9.77 677	16	9.87 290	26	0.12 709	9.90 386	9	16
45	9.77 693	17	9.87 316	26	0.12 683	9.90 377	9	15
46	9.77 710	17	9.87 343	26	0.12 657	9.90 367	9	14
47	9.77 727	17	9.87 369	26	0.12 630	9.90 358	9	13
48	9.77 744	17	9.87 395	26	0.12 604	9.90 348	9	12
49	9.77 761	16	9.87 422	26	0.12 578	9.90 339	9	11
50	9.77 778	17	9.87 448	26	0.12 551	9.90 330	9	10
51	9.77 795	17	9.87 474	26	0.12 525	9.90 320	9	9
52	9.77 812	16	9.87 501	26	0.12 499	9.90 311	9	8
53	9.77 828	17	9.87 527	26	0.12 472	9.90 301	9	7
54	9.77 845	17	9.87 553	26	0.12 446	9.90 292	9	6
55	9.77 862	16	9.87 580	26	0.12 420	9.90 282	9	5
56	9.77 879	17	9.87 606	26	0.12 393	9.90 273	9	4
57	9.77 896	17	9.87 632	26	0.12 367	9.90 263	9	3
58	9.77 913	16	9.87 659	26	0.12 341	9.90 254	9	2
59	9.77 929	17	9.87 685	26	0.12 315	9.90 244	9	1
60	9.77 946		9.87 711		0.12 288	9.90 235		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	27	26	26
6	2.7	2.6	2.6
7	3.1	3.1	3.0
8	3.6	3.5	3.4
9	4.0	4.0	3.9
10	4.5	4.4	4.3
20	9.0	8.8	8.6
30	13.5	13.2	13.0
40	18.0	17.6	17.3
50	22.5	22.1	21.6

	17	17	16
6	1.7	1.7	1.6
7	2.0	2.0	1.9
8	2.3	2.2	2.2
9	2.6	2.5	2.5
10	2.9	2.8	2.7
20	5.8	5.6	5.5
30	8.7	8.5	8.2
40	11.6	11.3	11.0
50	14.6	14.1	13.7

	9	9
6	0.9	0.9
7	1.1	1.0
8	1.2	1.2
9	1.4	1.3
10	1.6	1.5
20	3.1	3.0
30	4.7	4.5
40	6.3	6.0
50	7.9	7.5

P. P.

53°

37°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.77 946	16	9.87 711	26	0.12 288	9.90 235	9	60
1	9.77 963	17	9.87 737	26	0.12 262	9.90 225	9	59
2	9.77 980	16	9.87 764	26	0.12 236	9.90 216	9	58
3	9.77 996	17	9.87 790	26	0.12 209	9.90 206	10	57
4	9.78 013	16	9.87 816	26	0.12 183	9.90 196	9	56
5	9.78 030	16	9.87 843	26	0.12 157	9.90 187	9	55
6	9.78 046	17	9.87 869	26	0.12 131	9.90 177	9	54
7	9.78 063	16	9.87 895	26	0.12 104	9.90 168	9	53
8	9.78 080	17	9.87 921	26	0.12 078	9.90 158	9	52
9	9.78 097	16	9.87 948	26	0.12 052	9.90 149	9	51
10	9.78 113	16	9.87 974	26	0.12 026	9.90 139	9	50
11	9.78 130	17	9.88 000	26	0.11 999	9.90 130	10	49
12	9.78 147	16	9.88 026	26	0.11 973	9.90 120	9	48
13	9.78 163	16	9.88 053	26	0.11 947	9.90 110	9	47
14	9.78 180	16	9.88 079	26	0.11 921	9.90 101	9	46
15	9.78 196	16	9.88 105	26	0.11 895	9.90 091	9	45
16	9.78 213	17	9.88 131	26	0.11 868	9.90 082	10	44
17	9.78 230	16	9.88 157	26	0.11 842	9.90 072	9	43
18	9.78 246	16	9.88 184	26	0.11 816	9.90 062	9	42
19	9.78 263	16	9.88 210	26	0.11 790	9.90 053	9	41
20	9.78 279	16	9.88 236	26	0.11 763	9.90 043	10	40
21	9.78 296	16	9.88 262	26	0.11 737	9.90 033	9	39
22	9.78 312	16	9.88 288	26	0.11 711	9.90 024	9	38
23	9.78 329	17	9.88 315	26	0.11 685	9.90 014	10	37
24	9.78 346	16	9.88 341	26	0.11 659	9.90 004	9	36
25	9.78 362	16	9.88 367	26	0.11 633	9.89 995	9	35
26	9.78 379	16	9.88 393	26	0.11 606	9.89 985	10	34
27	9.78 395	16	9.88 419	26	0.11 580	9.89 975	9	33
28	9.78 412	16	9.88 445	26	0.11 554	9.89 966	9	32
29	9.78 428	16	9.88 472	26	0.11 528	9.89 956	10	31
30	9.78 444	16	9.88 498	26	0.11 502	9.89 946	9	30
31	9.78 461	16	9.88 524	26	0.11 476	9.89 937	9	29
32	9.78 477	16	9.88 550	26	0.11 449	9.89 927	10	28
33	9.78 494	16	9.88 576	26	0.11 423	9.89 917	9	27
34	9.78 510	16	9.88 602	26	0.11 397	9.89 908	10	26
35	9.78 527	16	9.88 629	26	0.11 371	9.89 898	9	25
36	9.78 543	16	9.88 655	26	0.11 345	9.89 888	10	24
37	9.78 559	16	9.88 681	26	0.11 319	9.89 878	9	23
38	9.78 576	16	9.88 707	26	0.11 293	9.89 869	10	22
39	9.78 592	16	9.88 733	26	0.11 266	9.89 859	9	21
40	9.78 609	16	9.88 759	26	0.11 240	9.89 849	10	20
41	9.78 625	16	9.88 785	26	0.11 214	9.89 839	9	19
42	9.78 641	16	9.88 811	26	0.11 188	9.89 830	10	18
43	9.78 658	16	9.88 838	26	0.11 162	9.89 820	9	17
44	9.78 674	16	9.88 864	26	0.11 136	9.89 810	10	16
45	9.78 690	16	9.88 890	26	0.11 110	9.89 800	9	15
46	9.78 707	16	9.88 916	26	0.11 084	9.89 791	10	14
47	9.78 723	16	9.88 942	26	0.11 058	9.89 781	10	13
48	9.78 739	16	9.88 968	26	0.11 032	9.89 771	9	12
49	9.78 755	16	9.88 994	26	0.11 005	9.89 761	10	11
50	9.78 772	16	9.89 020	26	0.10 979	9.89 751	9	10
51	9.78 788	16	9.89 046	26	0.10 953	9.89 742	10	9
52	9.78 804	16	9.89 072	26	0.10 927	9.89 732	10	8
53	9.78 821	16	9.89 098	26	0.10 901	9.89 722	9	7
54	9.78 837	16	9.89 124	26	0.10 875	9.89 712	10	6
55	9.78 853	16	9.89 150	26	0.10 849	9.89 702	10	5
56	9.78 869	16	9.89 177	26	0.10 823	9.89 692	9	4
57	9.78 885	16	9.89 203	26	0.10 797	9.89 683	10	3
58	9.78 902	16	9.89 229	26	0.10 771	9.89 673	10	2
59	9.78 918	16	9.89 255	26	0.10 745	9.89 663	10	1
60	9.78 934		9.89 281		0.10 719	9.89 653		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	26	26
6	2.6	2.6
7	3.1	3.0
8	3.5	3.4
9	4.0	3.9
10	4.4	4.3
20	8.8	8.6
30	13.2	13.0
40	17.6	17.3
50	22.1	21.6

	17	16	16
6	1.7	1.6	1.6
7	2.0	1.9	1.8
8	2.2	2.2	2.1
9	2.5	2.5	2.4
10	2.8	2.7	2.6
20	5.6	5.5	5.3
30	8.5	8.2	8.0
40	11.3	11.0	10.6
50	14.1	13.7	13.3

	10	9
6	1.0	0.9
7	1.1	1.1
8	1.3	1.2
9	1.5	1.4
10	1.6	1.6
20	3.3	3.1
30	5.0	4.7
40	6.6	6.3
50	8.3	7.9

P. P.

52°

38°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.78 934	16	9.89 281	26	0.10 719	9.89 653	9	60
1	9.78 950	16	9.89 307	26	0.10 693	9.89 643	10	59
2	9.78 966	16	9.89 333	26	0.10 667	9.89 633	10	58
3	9.78 982	16	9.89 359	26	0.10 641	9.89 623	10	57
4	9.78 999	16	9.89 385	26	0.10 615	9.89 613	9	56
5	9.79 015	16	9.89 411	26	0.10 589	9.89 604	10	55
6	9.79 031	16	9.89 437	26	0.10 563	9.89 594	10	54
7	9.79 047	16	9.89 463	26	0.10 537	9.89 584	10	53
8	9.79 063	16	9.89 489	26	0.10 511	9.89 574	10	52
9	9.79 079	16	9.89 515	26	0.10 485	9.89 564	10	51
10	9.79 095	16	9.89 541	26	0.10 459	9.89 554	9	50
11	9.79 111	16	9.89 567	26	0.10 433	9.89 544	10	49
12	9.79 127	16	9.89 593	26	0.10 407	9.89 534	10	48
13	9.79 143	16	9.89 619	26	0.10 381	9.89 524	10	47
14	9.79 159	16	9.89 645	26	0.10 355	9.89 514	10	46
15	9.79 175	16	9.89 671	26	0.10 329	9.89 504	10	45
16	9.79 191	16	9.89 697	26	0.10 303	9.89 494	10	44
17	9.79 207	16	9.89 723	26	0.10 277	9.89 484	10	43
18	9.79 223	16	9.89 749	26	0.10 251	9.89 474	10	42
19	9.79 239	16	9.89 775	26	0.10 225	9.89 464	10	41
20	9.79 255	16	9.89 801	26	0.10 199	9.89 454	10	40
21	9.79 271	16	9.89 827	26	0.10 173	9.89 444	10	39
22	9.79 287	16	9.89 853	26	0.10 147	9.89 434	10	38
23	9.79 303	16	9.89 879	26	0.10 121	9.89 424	10	37
24	9 79 319	16	9.89 905	26	0.10 095	9.89 414	10	36
25	9.79 335	16	9.89 931	26	0.10 069	9.89 404	10	35
26	9.79 351	16	9.89 957	25	0.10 043	9.89 394	10	34
27	9.79 367	15	9.89 982	26	0.10 017	9.89 384	10	33
28	9.79 383	16	9.90 008	26	0.09 991	9 89 374	10	32
29	9.79 399	16	9.90 034	26	0.09 965	9.89 364	10	31
30	9.79 415	16	9.90 060	26	0.09 939	9.89 354	10	30
31	9.79 431	15	9.90 086	26	0.09 913	9.89 344	10	29
32	9.79 446	16	9.90 112	26	0.09 887	9.89 334	10	28
33	9.79 462	16	9.90 138	25	0.09 861	9.89 324	10	27
34	9.79 478	15	9.90 164	26	0.09 836	9.89 314	10	26
35	9.79 494	16	9.90 190	26	0.09 810	9.89 304	10	25
36	9.79 510	16	9.90 216	26	0.09 784	9.89 294	10	24
37	9.79 526	15	9.90 242	26	0.09 758	9.89 284	10	23
38	9.79 541	16	9 90 268	26	0.09 732	9.89 274	10	22
39	9.79 557	16	9.90 294	25	0.09 706	9.89 264	10	21
40	9.79 573	15	9.90 319	26	0.09 680	9.89 253	10	20
41	9.79 589	16	9.90 345	26	0.09 654	9.89 243	10	19
42	9.79 605	15	9.90 371	26	0.09 628	9.89 233	10	18
43	9.79 620	16	9.90 397	25	0.09 602	9.89 223	10	17
44	9.79 636	15	9.90 423	26	0.09 577	9.89 213	10	16
45	9.79 652	16	9.90 449	26	0.09 551	9.89 203	10	15
46	9.79 668	15	9 90 475	26	0.09 525	9.89 193	10	14
47	9.79 683	16	9.90 501	25	0.09 499	9.89 182	10	13
48	9.79 699	15	9.90 526	26	0 09 473	9.89 172	10	12
49	9 79 715	15	9.90 552	26	0.09 447	9.89 162	10	11
50	9.79 730	16	9.90 578	26	0.09 421	9.89 152	10	10
51	9.79 746	15	9.90 604	25	0.09 395	9.89 142	10	9
52	9.79 762	15	9.90 630	26	0.09 370	9.89 132	10	8
53	9.79 777	16	9.90 656	26	0.09 344	9.89 121	10	7
54	9.79 793	15	9.90 682	25	0.09 318	9 89 111	10	6
55	9.79 809	15	9.90 707	26	0.09 292	9.89 101	10	5
56	9.79 824	16	9.90 733	26	0.09 266	9.89 091	10	4
57	9.79 840	15	9.90 759	26	0.09 240	9.89 081	10	3
58	9.79 856	15	9.90 785	25	0.09 214	9.89 070	10	2
59	9.79 871	15	9.90 811	26	0.09 189	9.89 060	10	1
60	9.79 887		9.90 837		0.09 163	9.89 050		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	26	25
6	2.6	2.5
7	3.0	3.0
8	3.4	3.4
9	3.9	3.8
10	4.3	4.2
20	8.6	8.5
30	13.0	12.7
40	17.3	17.0
50	21.6	21.2

	16	16	15
6	1.6	1.6	1.5
7	1.9	1.8	1.8
8	2.2	2.1	2.0
9	2.5	2.4	2.3
10	2.7	2.6	2.6
20	5.5	5.3	5.1
30	8.2	8.0	7.7
40	11.0	10.6	10.3
50	13.7	13.3	12.9

	10	10	9
6	1.0	1.0	0.9
7	1.2	1.1	1.1
8	1.4	1.3	1.2
9	1.6	1.5	1.4
10	1.7	1.6	1.6
20	3.5	3.3	3.1
30	5.2	5.0	4.7
40	7.0	6.6	6.3
50	8.7	8.3	7.9

P. P.

51°

39°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.79 887	16	9.90 837	26	0.09 163	9.89 050	10	60
1	9.79 903	15	9.90 863	25	0.09 137	9.89 040	10	59
2	9.79 918	15	9.90 888	26	0.09 111	9.89 030	10	58
3	9.79 934	15	9.90 914	25	0.09 085	9.89 019	10	57
4	9.79 949	15	9.90 940	26	0.09 060	9.89 009	10	56
5	9.79 965	15	9.90 966	26	0.09 034	9.88 999	10	55
6	9.79 980	15	9.90 992	25	0.09 008	9.88 989	10	54
7	9.79 996	15	9.91 017	26	0.08 982	9.88 978	10	53
8	9.80 011	15	9.91 043	26	0.08 956	9.88 968	10	52
9	9.80 027	15	9.91 069	25	0.08 930	9.88 958	10	51
10	9.80 042	15	9.91 095	26	0.08 905	9.88 947	10	50
11	9.80 058	15	9.91 121	25	0.08 879	9.88 937	10	49
12	9.80 073	15	9.91 146	26	0.08 853	9.88 927	10	48
13	9.80 089	15	9.91 172	25	0.08 827	9.88 917	10	47
14	9.80 104	15	9.91 198	26	0.08 802	9.88 906	10	46
15	9.80 120	15	9.91 224	26	0.08 776	9.88 896	10	45
16	9.80 135	15	9.91 250	25	0.08 750	9.88 886	10	44
17	9.80 151	15	9.91 275	26	0.08 724	9.88 875	10	43
18	9.80 166	15	9.91 301	25	0.08 698	9.88 865	10	42
19	9.80 182	15	9.91 327	26	0.08 673	9.88 855	10	41
20	9.80 197	15	9.91 353	25	0.08 647	9.88 844	10	40
21	9.80 213	15	9.91 378	26	0.08 621	9.88 834	10	39
22	9.80 228	15	9.91 404	25	0.08 595	9.88 823	10	38
23	9.80 243	15	9.91 430	26	0.08 570	9.88 813	10	37
24	9.80 259	15	9.91 456	25	0.08 544	9.88 803	10	36
25	9.80 274	15	9.91 481	26	0.08 518	9.88 792	10	35
26	9.80 289	15	9.91 507	25	0.08 492	9.88 782	10	34
27	9.80 305	15	9.91 533	26	0.08 467	9.88 772	10	33
28	9.80 320	15	9.91 559	25	0.08 441	9.88 761	10	32
29	9.80 335	15	9.91 584	26	0.08 415	9.88 751	10	31
30	9.80 351	15	9.91 610	25	0.08 389	9.88 740	10	30
31	9.80 366	15	9.91 636	26	0.08 364	9.88 730	10	29
32	9.80 381	15	9.91 662	25	0.08 338	9.88 720	10	28
33	9.80 397	15	9.91 687	26	0.08 312	9.88 709	10	27
34	9.80 412	15	9.91 713	25	0.08 286	9.88 699	10	26
35	9.80 427	15	9.91 739	26	0.08 261	9.88 688	10	25
36	9.80 443	15	9.91 765	25	0.08 235	9.88 678	10	24
37	9.80 458	15	9.91 790	26	0.08 209	9.88 667	10	23
38	9.80 473	15	9.91 816	25	0.08 183	9.88 657	10	22
39	9.80 488	15	9.91 842	25	0.08 158	9.88 646	10	21
40	9.80 504	15	9.91 867	26	0.08 132	9.88 636	10	20
41	9.80 519	15	9.91 893	25	0.08 106	9.88 625	10	19
42	9.80 534	15	9.91 919	26	0.08 081	9.88 615	10	18
43	9.80 549	15	9.91 945	25	0.08 055	9.88 604	10	17
44	9.80 564	15	9.91 970	25	0.08 029	9.88 594	10	16
45	9.80 580	15	9.91 996	26	0.08 004	9.88 583	10	15
46	9.80 595	15	9.92 022	25	0.07 978	9.88 573	10	14
47	9.80 610	15	9.92 047	26	0.07 952	9.88 562	10	13
48	9.80 625	15	9.92 073	25	0.07 926	9.88 552	10	12
49	9.80 640	15	9.92 099	25	0.07 901	9.88 541	10	11
50	9.80 655	15	9.92 124	26	0.07 875	9.88 531	10	10
51	9.80 671	15	9.92 150	25	0.07 849	9.88 520	10	9
52	9.80 686	15	9.92 176	25	0.07 824	9.88 510	10	8
53	9.80 701	15	9.92 201	26	0.07 798	9.88 499	10	7
54	9.80 716	15	9.92 227	25	0.07 772	9.88 489	10	6
55	9.80 731	15	9.92 253	25	0.07 747	9.88 478	11	5
56	9.80 746	15	9.92 278	26	0.07 721	9.88 467	10	4
57	9.80 761	15	9.92 304	25	0.07 695	9.88 457	10	3
58	9.80 776	15	9.92 330	25	0.07 670	9.88 446	10	2
59	9.80 791	15	9.92 355	26	0.07 644	9.88 436	10	1
60	9.80 806		9.92 381		0.07 618	9.88 425		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	26	25
6	2.6	2.5
7	3.0	3.0
8	3.4	3.4
9	3.9	3.8
10	4.3	4.2
20	8.6	8.5
30	13.0	12.7
40	17.3	17.0
50	21.6	21.2

	16	15	15
6	1.6	1.5	1.5
7	1.8	1.8	1.7
8	2.1	2.0	2.0
9	2.4	2.3	2.2
10	2.6	2.6	2.5
20	5.3	5.1	5.0
30	8.0	7.7	7.5
40	10.6	10.3	10.0
50	13.3	12.9	12.5

	11	10	10
6	1.1	1.0	1.0
7	1.3	1.2	1.1
8	1.4	1.4	1.3
9	1.6	1.6	1.5
10	1.8	1.7	1.6
20	3.6	3.5	3.3
30	5.5	5.2	5.0
40	7.3	7.0	6.6
50	9.1	8.7	8.3

P. P.

50°

40°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.80 806	15	9.92 381	25	0.07 618	9.88 425	10	**60**
1	9.80 822	15	9.92 407	25	0.07 593	9.88 415	11	59
2	9.80 837	15	9.92 432	26	0.07 567	9.88 404	10	58
3	9.80 852	15	9.92 458	25	0.07 541	9.88 393	10	57
4	9.80 867	15	9.92 484	25	0.07 516	9.88 383	10	56
5	9.80 882	15	9.92 509	25	0.07 490	9.88 372	11	55
6	9.80 897	15	9.92 535	26	0.07 465	9.88 361	10	54
7	9.80 912	15	9.92 561	25	0.07 439	9.88 351	10	53
8	9.80 927	15	9.92 586	25	0.07 413	9.88 340	11	52
9	9.80 942	15	9.92 612	26	0.07 388	9.88 329	10	51
10	9.80 957	15	9.92 638	25	0.07 362	9.88 319	10	**50**
11	9.80 972	15	9.92 663	25	0.07 336	9.88 308	11	49
12	9.80 987	14	9.92 689	25	0.07 311	9.88 297	10	48
13	9.81 001	15	9.92 714	26	0.07 285	9.88 287	10	47
14	9.81 016	15	9.92 740	25	0.07 259	9.88 276	11	46
15	9.81 031	15	9.92 766	25	0.07 234	9.88 265	10	45
16	9.81 046	15	9.92 791	25	0.07 208	9.88 255	10	44
17	9.81 061	15	9.92 817	25	0.07 183	9.88 244	11	43
18	9.81 076	14	9.92 842	26	0.07 157	9.88 233	10	42
19	9.81 091	15	9.92 868	25	0.07 131	9.88 223	11	41
20	9.81 106	15	9.92 894	25	0.07 106	9.88 212	10	**40**
21	9.81 121	15	9.92 919	25	0.07 080	9.88 201	11	39
22	9.81 136	14	9.92 945	26	0.07 055	9.88 190	10	38
23	9.81 150	15	9.92 971	25	0.07 029	9.88 180	11	37
24	9.81 165	15	9.92 996	25	0.07 003	9.88 169	10	36
25	9.81 180	14	9.93 022	25	0.06 978	9.88 158	11	35
26	9.81 195	15	9.93 047	25	0.06 952	9.88 147	10	34
27	9.81 210	15	9.93 073	25	0.06 927	9.88 137	11	33
28	9.81 225	14	9.93 098	26	0.06 901	9.88 126	10	32
29	9.81 239	15	9.93 124	25	0.06 875	9.88 115	11	31
30	9.81 254	14	9.93 150	25	0.06 850	9.88 104	10	**30**
31	9.81 269	15	9.93 175	25	0.06 824	9.88 094	11	29
32	9.81 284	15	9.93 201	25	0.06 799	9.88 083	11	28
33	9.81 299	14	9.93 226	25	0.06 773	9.88 072	10	27
34	9.81 313	15	9.93 252	26	0.06 748	9.88 061	11	26
35	9.81 328	14	9.93 278	25	0.06 722	9.88 050	11	25
36	9.81 343	15	9.93 303	25	0.06 696	9.88 039	10	24
37	9.81 358	14	9.93 329	25	0.06 671	9.88 029	11	23
38	9.81 372	14	9.93 354	25	0.06 645	9.88 018	11	22
39	9.81 387	15	9.93 380	25	0.06 620	9.88 007	10	21
40	9.81 402	14	9.93 405	25	0.06 594	9.87 996	11	**20**
41	9.81 416	15	9.93 431	25	0.06 569	9.87 985	11	19
42	9.81 431	14	9.93 456	25	0.06 543	9.87 974	11	18
43	9.81 446	14	9.93 482	26	0.06 518	9.87 963	10	17
44	9.81 460	15	9.93 508	25	0.06 492	9.87 953	11	16
45	9.81 475	14	9.93 533	25	0.06 466	9.87 942	11	15
46	9.81 490	14	9.93 559	25	0.06 441	9.87 931	11	14
47	9.81 504	15	9.93 584	25	0.06 415	9.87 920	10	13
48	9.81 519	14	9.93 610	25	0.06 390	9.87 909	11	12
49	9.81 534	14	9.93 635	25	0.06 364	9.87 898	11	11
50	9.81 548	14	9.93 661	25	0.06 339	9.87 887	11	**10**
51	9.81 563	15	9.93 686	25	0.06 313	9.87 876	11	9
52	9.81 578	14	9.93 712	25	0.06 288	9.87 865	11	8
53	9.81 592	14	9.93 737	25	0.06 262	9.87 854	10	7
54	9.81 607	14	9.93 763	25	0.06 237	9.87 844	11	6
55	9.81 621	14	9.93 788	25	0.06 211	9.87 833	11	5
56	9.81 636	14	9.93 814	26	0.06 186	9.87 822	11	4
57	9.81 650	14	9.93 840	25	0.06 160	9.87 811	11	3
58	9.81 665	15	9.93 865	25	0.06 134	9.87 800	11	2
59	9.81 680	14	9.93 891	25	0.06 109	9.87 789	11	1
60	9.81 694		9.93 916		0.06 083	9.87 778		**0**
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	26	25
6	2.6	2.5
7	3.0	3.0
8	3.4	3.4
9	3.9	3.8
10	4.3	4.2
20	8.6	8.5
30	13.0	12.7
40	17.3	17.0
50	21.6	21.2

	15	15	14
6	1.5	1.5	1.4
7	1.8	1.7	1.7
8	2.0	2.0	1.9
9	2.3	2.2	2.2
10	2.6	2.5	2.4
20	5.1	5.0	4.8
30	7.7	7.5	7.2
40	10.3	10.0	9.6
50	12.9	12.5	12.1

	11	10
6	1.1	1.0
7	1.3	1.2
8	1.4	1.4
9	1.6	1.6
10	1.8	1.7
20	3.6	3.5
30	5.5	5.2
40	7.3	7.0
50	9.1	8.7

P. P.

49°

41°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.81 694	14	9.93 916	25	0.06 083	9.87 778	11	60
1	9.81 709	14	9.93 942	25	0.06 058	9.87 767	11	59
2	9.81 723	14	9.93 967	25	0.06 032	9.87 756	11	58
3	9.81 738	14	9.93 993	25	0.06 007	9.87 745	11	57
4	9.81 752	14	9.94 018	25	0.05 981	9.87 734	11	56
5	9.81 767	14	9.94 044	25	0.05 956	9.87 723	11	55
6	9.81 781	14	9.94 069	25	0.05 930	9.87 712	11	54
7	9.81 796	14	9.94 095	25	0.05 905	9.87 701	11	53
8	9.81 810	14	9.94 120	25	0.05 879	9.87 690	11	52
9	9.81 824	14	9.94 146	25	0.05 854	9.87 679	11	51
10	9.81 839	14	9.94 171	25	0.05 828	9.87 668	11	50
11	9.81 853	14	9.94 197	25	0.05 803	9.87 657	11	49
12	9.81 868	14	9.94 222	25	0.05 777	9.87 645	11	48
13	9.81 882	14	9.94 248	25	0.05 752	9.87 634	11	47
14	9.81 897	14	9.94 273	25	0.05 726	9.87 623	11	46
15	9.81 911	14	9.94 299	25	0.05 701	9.87 612	11	45
16	9.81 925	14	9.94 324	25	0.05 675	9.87 601	11	44
17	9.81 940	14	9.94 350	25	0.05 650	9.87 590	11	43
18	9.81 954	14	9.94 375	25	0.05 625	9.87 579	11	42
19	9.81 969	14	9.94 400	25	0.05 599	9.87 568	11	41
20	9.81 983	14	9.94 426	25	0.05 574	9.87 557	11	40
21	9.81 997	14	9.94 451	25	0.05 548	9.87 546	11	39
22	9.82 012	14	9.94 477	25	0.05 523	9.87 535	11	38
23	9.82 026	14	9.94 502	25	0.05 497	9.87 523	11	37
24	9.82 040	14	9.94 528	25	0.05 472	9.87 512	11	36
25	9.82 055	14	9.94 553	25	0.05 446	9.87 501	11	35
26	9.82 069	14	9.94 579	25	0.05 421	9.87 490	11	34
27	9.82 083	14	9.94 604	25	0.05 395	9.87 479	11	33
28	9.82 098	14	9.94 630	25	0.05 370	9.87 468	11	32
29	9.82 112	14	9.94 655	25	0.05 344	9.87 457	11	31
30	9.82 126	14	9.94 681	25	0.05 319	9.87 445	11	30
31	9.82 140	14	9.94 706	25	0.05 293	9.87 434	11	29
32	9.82 155	14	9.94 732	25	0.05 268	9.87 423	11	28
33	9.82 169	14	9.94 757	25	0.05 243	9.87 412	11	27
34	9.82 183	14	9.94 782	25	0.05 217	9.87 401	11	26
35	9.82 197	14	9.94 808	25	0.05 192	9.87 389	11	25
36	9.82 212	14	9.94 833	25	0.05 166	9.87 378	11	24
37	9.82 226	14	9.94 859	25	0.05 141	9.87 367	11	23
38	9.82 240	14	9.94 884	25	0.05 115	9.87 356	11	22
39	9.82 254	14	9.94 910	25	0.05 090	9.87 345	11	21
40	9.82 269	14	9.94 935	25	0.05 064	9.87 333	11	20
41	9.82 283	14	9.94 961	25	0.05 039	9.87 322	11	19
42	9.82 297	14	9.94 986	25	0.05 014	9.87 311	11	18
43	9.82 311	14	9.95 011	25	0.04 988	9.87 300	11	17
44	9.82 325	14	9.95 037	25	0.04 963	9.87 288	11	16
45	9.82 339	14	9.95 062	25	0.04 937	9.87 277	11	15
46	9.82 354	14	9.95 088	25	0.04 912	9.87 266	11	14
47	9.82 368	14	9.95 113	25	0.04 886	9.87 254	11	13
48	9.82 382	14	9.95 139	25	0.04 861	9.87 243	11	12
49	9.82 396	14	9.95 164	25	0.04 836	9.87 232	11	11
50	9.82 410	14	9.95 189	25	0.04 810	9.87 221	11	10
51	9.82 424	14	9.95 215	25	0.04 785	9.87 209	11	9
52	9.82 438	14	9.95 240	25	0.04 759	9.87 198	11	8
53	9.82 452	14	9.95 266	25	0.04 734	9.87 187	11	7
54	9.82 467	14	9.95 291	25	0.04 708	9.87 175	11	6
55	9.82 481	14	9.95 316	25	0.04 683	9.87 164	11	5
56	9.82 495	14	9.95 342	25	0.04 658	9.87 153	11	4
57	9.82 509	14	9.95 367	25	0.04 632	9.87 141	11	3
58	9.82 523	14	9.95 393	25	0.04 607	9.87 130	11	2
59	9.82 537	14	9.95 418	25	0.04 581	9.87 118	11	1
60	9.82 551		9.95 443		0.04 556	9.87 107		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	25	25
6	2.5	2.5
7	3.0	2.9
8	3.4	3.3
9	3.8	3.7
10	4.2	4.1
20	8.5	8.3
30	12.7	12.5
40	17.0	16.6
50	21.2	20.8

	14	14
6	1.4	1.4
7	1.7	1.6
8	1.9	1.8
9	2.2	2.1
10	2.4	2.3
20	4.8	4.6
30	7.2	7.0
40	9.6	9.3
50	12.1	11.6

	11	11
6	1.1	1.1
7	1.3	1.3
8	1.5	1.4
9	1.7	1.6
10	1.9	1.8
20	3.8	3.6
30	5.7	5.5
40	7.6	7.3
50	9.6	9.1

P. P.

48°

42°

′	Log. Sin.	d	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.82 551	14	9.95 443	25	0.04 556	9.87 107	11	60
1	9.82 565	14	9.95 469	25	0.04 531	9.87 096	11	59
2	9.82 579	14	9.95 494	25	0.04 505	9.87 084	11	58
3	9.82 593	14	9.95 520	25	0.04 480	9.87 073	11	57
4	9.82 607	14	9.95 545	25	0.04 454	9.87 062	11	56
5	9.82 621	14	9.95 571	25	0.04 429	9.87 050	11	55
6	9.82 635	14	9.95 596	25	0.04 404	9.87 039	11	54
7	9.82 649	14	9.95 621	25	0.04 378	9.87 027	11	53
8	9.82 663	14	9.95 647	25	0.04 353	9.87 016	11	52
9	9.82 677	14	9.95 672	25	0.04 327	9.87 004	11	51
10	9.82 691	14	9.95 697	25	0.04 302	9.86 993	11	50
11	9.82 705	14	9.95 723	25	0.04 277	9.86 982	11	49
12	9.82 719	14	9.95 748	25	0.04 251	9.86 970	11	48
13	9.82 733	13	9.95 774	25	0.04 226	9.86 959	11	47
14	9.82 746	14	9.95 799	25	0.04 200	9.86 947	11	46
15	9.82 760	14	9.95 824	25	0.04 175	9.86 936	11	45
16	9.82 774	14	9.95 850	25	0.04 150	9.86 924	11	44
17	9.82 788	14	9.95 875	25	0.04 124	9.86 913	11	43
18	9.82 802	13	9.95 901	25	0.04 099	9.86 901	11	42
19	9.82 816	14	9.95 926	25	0.04 074	9.86 890	11	41
20	9.82 830	14	9.95 951	25	0.04 048	9.86 878	11	40
21	9.82 844	14	9.95 977	25	0.04 023	9.86 867	11	39
22	9.82 858	13	9.96 002	25	0.03 997	9.86 855	11	38
23	9.82 871	14	9.96 027	25	0.03 972	9.86 844	11	37
24	9.82 885	14	9.96 053	25	0.03 947	9.86 832	11	36
25	9.82 899	13	9.96 078	25	0.03 921	9.86 821	11	35
26	9.82 913	14	9.96 104	25	0.03 896	9.86 809	11	34
27	9.82 927	13	9.96 129	25	0.03 871	9.86 798	12	33
28	9.82 940	14	9.96 154	25	0.03 845	9.86 786	11	32
29	9.82 954	14	9.96 180	25	0.03 820	9.86 774	11	31
30	9.82 968	13	9.96 205	25	0.03 795	9.86 763	11	30
31	9.82 982	14	9.96 230	25	0.03 769	9.86 751	11	29
32	9.82 996	13	9.96 256	25	0.03 744	9.86 740	11	28
33	9.83 009	14	9.96 281	25	0.03 718	9.86 728	12	27
34	9.83 023	13	9.96 306	25	0.03 693	9.86 716	11	26
35	9.83 037	14	9.96 332	25	0.03 668	9.86 705	11	25
36	9.83 051	13	9.96 357	25	0.03 642	9.86 693	11	24
37	9.83 064	14	9.96 383	25	0.03 617	9.86 682	11	23
38	9.83 078	13	9.96 408	25	0.03 592	9.86 670	12	22
39	9.83 092	14	9.96 433	25	0.03 566	9.86 658	11	21
40	9.83 106	13	9.96 459	25	0.03 541	9.86 647	11	20
41	9.83 119	13	9.96 484	25	0.03 516	9.86 635	12	19
42	9.83 133	14	9.96 509	25	0.03 490	9.86 623	11	18
43	9.83 147	13	9.96 535	25	0.03 465	9.86 612	11	17
44	9.83 160	13	9.96 560	25	0.03 440	9.86 600	12	16
45	9.83 174	14	9.96 585	25	0.03 414	9.86 588	11	15
46	9.83 188	13	9.96 611	25	0.03 389	9.86 577	11	14
47	9.83 201	13	9.96 636	25	0.03 364	9.86 565	12	13
48	9.83 215	14	9.96 661	25	0.03 338	9.86 553	11	12
49	9.83 229	13	9.96 687	25	0.03 313	9.86 542	12	11
50	9.83 242	13	9.96 712	25	0.03 287	9.86 530	11	10
51	9.83 256	13	9.96 737	25	0.03 262	9.86 518	11	9
52	9.83 269	14	9.96 763	25	0.03 237	9.86 507	12	8
53	9.83 283	13	9.96 788	25	0.03 211	9.86 495	11	7
54	9.83 297	13	9.96 813	25	0.03 186	9.86 483	12	6
55	9.83 310	13	9.96 839	25	0.03 161	9.86 471	11	5
56	9.83 324	13	9.96 864	25	0.03 135	9.86 460	12	4
57	9.83 337	13	9.96 889	25	0.03 110	9.86 448	11	3
58	9.83 351	14	9.96 915	25	0.03 085	9.86 436	12	2
59	9.83 365	13	9.96 940	25	0.03 059	9.86 424	12	1
60	9.83 378		9.96 965		0.03 034	9.86 412		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	25	25
6	2.5	2.5
7	3.0	2.9
8	3.4	3.3
9	3.8	3.7
10	4.2	4.1
20	8.5	8.3
30	12.7	12.5
40	17.0	16.6
50	21.2	20.8

	14	13
6	1.4	1.3
7	1.6	1.6
8	1.8	1.8
9	2.1	2.0
10	2.3	2.2
20	4.6	4.5
30	7.0	6.7
40	9.3	9.0
50	11.6	11.2

	12	11	11
6	1.2	1.1	1.1
7	1.4	1.3	1.3
8	1.6	1.5	1.4
9	1.8	1.7	1.6
10	2.0	1.9	1.8
20	4.0	3.8	3.6
30	6.0	5.7	5.5
40	8.0	7.6	7.3
50	10.0	9.6	9.1

P. P.

47°

43°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.83 378	13	9.96 965	25	0.03 034	9.86 412	11	60
1	9.83 392	13	9.96 991	25	0.03 009	9.86 401	12	59
2	9.83 405	13	9.97 016	25	0.02 984	9.86 389	11	58
3	9.83 419	13	9.97 041	25	0.02 958	9.86 377	12	57
4	9.83 432	13	9.97 067	25	0.02 933	9.86 365	11	56
5	9.83 446	13	9.97 092	25	0.02 908	9.86 354	12	55
6	9.83 459	13	9.97 117	25	0.02 882	9.86 342	12	54
7	9.83 473	13	9.97 143	25	0.02 857	9.86 330	11	53
8	9.83 486	13	9.97 168	25	0.02 832	9.86 318	12	52
9	9.83 500	13	9.97 193	25	0.02 806	9.86 306	12	51
10	9.83 513	13	9.97 219	25	0.02 781	9.86 294	12	50
11	9.83 527	13	9.97 244	25	0.02 756	9.86 282	11	49
12	9.83 540	13	9.97 269	25	0.02 730	9.86 271	12	48
13	9.83 554	13	9.97 295	25	0.02 705	9.86 259	12	47
14	9.83 567	13	9.97 320	25	0.02 680	9.86 247	11	46
15	9.83 580	13	9.97 345	25	0.02 654	9.86 235	12	45
16	9.83 594	13	9.97 370	25	0.02 629	9.86 223	12	44
17	9.83 607	13	9.97 396	25	0.02 604	9.86 211	12	43
18	9.83 621	13	9.97 421	25	0.02 578	9.86 199	12	42
19	9.83 634	13	9.97 446	25	0.02 553	9.86 187	11	41
20	9.83 647	13	9.97 472	25	0.02 528	9.86 176	12	40
21	9.83 661	13	9.97 497	25	0.02 502	9.86 164	12	39
22	9.83 674	13	9.97 522	25	0.02 477	9.86 152	12	38
23	9.83 688	13	9.97 548	25	0.02 452	9.86 140	12	37
24	9.83 701	13	9.97 573	25	0.02 427	9.86 128	12	36
25	9.83 714	13	9.97 598	25	0.02 401	9.86 116	12	35
26	9.83 728	13	9.97 624	25	0.02 376	9.86 104	12	34
27	9.83 741	13	9.97 649	25	0.02 351	9.86 092	12	33
28	9.83 754	13	9.97 674	25	0.02 325	9.86 080	12	32
29	9.83 768	13	9.97 699	25	0.02 300	9.86 068	12	31
30	9.83 781	13	9.97 725	25	0.02 275	9.86 056	12	30
31	9.83 794	13	9.97 750	25	0.02 249	9.86 044	12	29
32	9.83 808	13	9.97 775	25	0.02 224	9.86 032	12	28
33	9.83 821	13	9.97 801	25	0.02 199	9.86 020	12	27
34	9.83 834	13	9.97 826	25	0.02 174	9.86 008	12	26
35	9.83 847	13	9.97 851	25	0.02 148	9.85 996	12	25
36	9.83 861	13	9.97 877	25	0.02 123	9.85 984	12	24
37	9.83 874	13	9.97 902	25	0.02 098	9.85 972	12	23
38	9.83 887	13	9.97 927	25	0.02 072	9.85 960	12	22
39	9.83 900	13	9.97 952	25	0.02 047	9.85 948	12	21
40	9.83 914	13	9.97 978	25	0.02 022	9.85 936	12	20
41	9.83 927	13	9.98 003	25	0.01 996	9.85 924	12	19
42	9.83 940	13	9.98 028	25	0.01 971	9.85 912	12	18
43	9.83 953	13	9.98 054	25	0.01 946	9.85 900	12	17
44	9.83 967	13	9.98 079	25	0.01 921	9.85 887	12	16
45	9.83 980	13	9.98 104	25	0.01 895	9.85 875	12	15
46	9.83 993	13	9.98 129	25	0.01 870	9.85 863	12	14
47	9.84 006	13	9.98 155	25	0.01 845	9.85 851	12	13
48	9.84 019	13	9.98 180	25	0.01 819	9.85 839	12	12
49	9.84 033	13	9.98 205	25	0.01 794	9.85 827	12	11
50	9.84 046	13	9.98 231	25	0.01 769	9.85 815	12	10
51	9.84 059	13	9.98 256	25	0.01 744	9.85 803	12	9
52	9.84 072	13	9.98 281	25	0.01 718	9.85 791	12	8
53	9.84 085	13	9.98 306	25	0.01 693	9.85 778	12	7
54	9.84 098	13	9.98 332	25	0.01 668	9.85 766	12	6
55	9.84 111	13	9.98 357	25	0.01 642	9.85 754	12	5
56	9.84 124	13	9.98 382	25	0.01 617	9.85 742	12	4
57	9.84 138	13	9.98 408	25	0.01 592	9.85 730	12	3
58	9.84 151	13	9.98 433	25	0.01 567	9.85 718	12	2
59	9.84 164	13	9.98 458	25	0.01 541	9.85 705	12	1
60	9.84 177		9.98 483		0.01 516	9.85 693		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	25	25
6	2.5	2.5
7	3.0	2.9
8	3.4	3.3
9	3.8	3.7
10	4.2	4.1
20	8.5	8.3
30	12.7	12.5
40	17.0	16.6
50	21.2	20.8

	13	13
6	1.3	1.3
7	1.6	1.5
8	1.8	1.7
9	2.0	1.9
10	2.2	2.1
20	4.5	4.3
30	6.7	6.5
40	9.0	8.6
50	11.2	10.8

	12	12	11
6	1.2	1.2	1.1
7	1.4	1.4	1.3
8	1.6	1.6	1.5
9	1.9	1.8	1.7
10	2.1	2.0	1.9
20	4.1	4.0	3.8
30	6.2	6.0	5.7
40	8.3	8.0	7.6
50	10.4	10.0	9.6

P. P.

46°

44°

′	Log. Sin.	d.	Log. Tan.	c. d.	Log. Cot.	Log. Cos.	d.	
0	9.84 177	13	9.98 483	25	0.01 516	9.85 693	12	60
1	9.84 190	13	9.98 509	25	0.01 491	9.85 681	12	59
2	9.84 203	13	9.98 534	25	0.01 465	9.85 669	12	58
3	9.84 216	13	9.98 559	25	0.01 440	9.85 657	12	57
4	9.84 229	13	9.98 585	25	0.01 415	9.85 644	12	56
5	9.84 242	13	9.98 610	25	0.01 390	9.85 632	12	55
6	9.84 255	13	9.98 635	25	0.01 364	9.85 620	12	54
7	9.84 268	13	9.98 660	25	0.01 339	9.85 608	12	53
8	9.84 281	13	9.98 686	25	0.01 314	9.85 595	12	52
9	9.84 294	13	9.98 711	25	0.01 289	9.85 583	12	51
10	9.84 307	13	9.98 736	25	0.01 263	9.85 571	12	50
11	9.84 320	13	9.98 762	25	0.01 238	9.85 559	12	49
12	9.84 333	13	9.98 787	25	0.01 213	9.85 546	12	48
13	9.84 346	13	9.98 812	25	0.01 187	9.85 534	12	47
14	9.84 359	13	9.98 837	25	0.01 162	9.85 522	12	46
15	9.84 372	13	9.98 863	25	0.01 137	9.85 509	12	45
16	9.84 385	13	9.98 888	25	0.01 112	9.85 497	12	44
17	9.84 398	13	9.98 913	25	0.01 086	9.85 485	12	43
18	9.84 411	13	9.98 938	25	0.01 061	9.85 472	12	42
19	9.84 424	12	9.98 964	25	0.01 036	9.85 460	12	41
20	9.84 437	13	9.98 989	25	0.01 010	9.85 448	12	40
21	9.84 450	13	9.99 014	25	0.00 985	9.85 435	12	39
22	9.84 463	13	9.99 040	25	0.00 960	9.85 423	12	38
23	9.84 476	13	9.99 065	25	0.00 935	9.85 411	12	37
24	9.84 489	13	9.99 090	25	0.00 909	9.85 398	12	36
25	9.84 502	12	9.99 115	25	0.00 884	9.85 386	12	35
26	9.84 514	13	9.99 141	25	0.00 859	9.85 374	12	34
27	9.84 527	13	9.99 166	25	0.00 834	9.85 361	12	33
28	9.84 540	13	9.99 191	25	0.00 808	9.85 349	12	32
29	9.84 553	12	9.99 216	25	0.00 783	9.85 336	12	31
30	9.84 566	13	9.99 242	25	0.00 758	9.85 324	12	30
31	9.84 579	13	9.99 267	25	0.00 733	9.85 312	12	29
32	9.84 592	12	9.99 292	25	0.00 707	9.85 299	12	28
33	9.84 604	13	9.99 318	25	0.00 682	9.85 287	12	27
34	9.84 617	13	9.99 343	25	0.00 657	9.85 274	12	26
35	9.84 630	12	9.99 368	25	0.00 631	9.85 262	12	25
36	9.84 643	13	9.99 393	25	0.00 606	9.85 249	12	24
37	9.84 656	13	9.99 419	25	0.00 581	9.85 237	12	23
38	9.84 669	12	9.99 444	25	0.00 556	9.85 224	12	22
39	9.84 681	13	9.99 469	25	0.00 530	9.85 212	12	21
40	9.84 694	12	9.99 494	25	0.00 505	9.85 199	12	20
41	9.84 707	13	9.99 520	25	0.00 480	9.85 187	12	19
42	9.84 720	12	9.99 545	25	0.00 455	9.85 174	12	18
43	9.84 732	13	9.99 570	25	0.00 429	9.85 162	12	17
44	9.84 745	12	9.99 595	25	0.00 404	9.85 149	12	16
45	9.84 758	13	9.99 621	25	0.00 379	9.85 137	12	15
46	9.84 771	12	9.99 646	25	0.00 353	9.85 124	12	14
47	9.84 783	13	9.99 671	25	0.00 328	9.85 112	12	13
48	9.84 796	12	9.99 697	25	0.00 303	9.85 099	12	12
49	9.84 809	13	9.99 722	25	0.00 278	9.85 087	12	11
50	9.84 822	12	9.99 747	25	0.00 252	9.85 074	12	10
51	9.84 834	12	9.99 772	25	0.00 227	9.85 062	12	9
52	9.84 847	13	9.99 798	25	0.00 202	9.85 049	12	8
53	9.84 860	12	9.99 823	25	0.00 177	9.85 037	13	7
54	9.84 872	12	9.99 848	25	0.00 151	9.85 024	12	6
55	9.84 885	13	9.99 873	25	0.00 126	9.85 011	12	5
56	9.84 898	12	9.99 899	25	0.00 101	9.84 999	12	4
57	9.84 910	12	9.99 924	25	0.00 076	9.84 986	12	3
58	9.84 923	13	9.99 949	25	0.00 050	9.84 974	13	2
59	9.84 936	12	9.99 974	25	0.00 025	9.84 961	12	1
60	9.84 948		0.00 000		0.00 000	9.84 948		0
	Log. Cos.	d.	Log. Cot.	c. d.	Log. Tan.	Log. Sin.	d.	′

P. P.

	25	25
6	2.5	2.5
7	3.0	2.9
8	3.4	3.3
9	3.8	3.7
10	4.2	4.1
20	8.5	8.3
30	12.7	12.5
40	17.0	16.6
50	21.2	20.8

	13	13
6	1.3	1.3
7	1.6	1.5
8	1.8	1.7
9	2.0	1.9
10	2.2	2.1
20	4.5	4.3
30	6.7	6.5
40	9.0	8.6
50	11.2	10.8

	12	12
6	1.2	1.2
7	1.4	1.4
8	1.6	1.6
9	1.9	1.8
10	2.1	2.0
20	4.1	4.0
30	6.2	6.0
40	8.3	8.0
50	10.4	10.0

P. P.

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TABLE VIII.

LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

	0°				1°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	—∞		—∞		6.18271	1435	6.18278	1436	0
1	2.62642	60206	2.62642	60206	.19707	1412	.19714	1412	1
2	3.22848	35218	3.22848	35218	.21119	1389	.21126	1390	2
3	3.58066	24987	3.58066	24987	.22509	1368	.22516	1368	3
4	3.83054	19382	3.83054	19382	.23877	1346	.23884	1347	4
5	4.02436	15836	4.02436	15836	6.25223	1326	6.25231	1326	5
6	.18272	13389	.18272	13389	.26549	1306	.26557	1306	6
7	.31662	11598	.31662	11598	.27856	1286	.27864	1287	7
8	.43260	10230	.43260	10230	.29142	1268	.29151	1268	8
9	.53490	9151	.53491	9151	.30410	1250	.30419	1250	9
10	4.62642	8278	4.62642	8279	6.31660	1232	6.31669	1232	**10**
11	.70920	7558	.70921	7557	.32892	1214	.32901	1215	11
12	.78478	6953	.78478	6952	.34107	1198	.34116	1198	12
13	.85431	6437	.85431	6437	.35305	1182	.35315	1182	13
14	.91868	5992	.91868	5993	.36487	1166	.36497	1166	14
15	4.97860	5605	4.97861	5605	6.37653	1150	6.37663	1151	15
16	5.03466	5266	5.03466	5266	.38803	1135	.38814	1135	16
17	.08732	4964	.08732	4964	.39938	1121	.39949	1121	17
18	.13696	4696	.13697	4696	.41059	1106	.41070	1106	18
19	.18393	4455	.18393	4456	.42165	1093	.42177	1093	19
20	5.22848	4238	5.22849	4238	6.43258	1078	6.43270	1079	**20**
21	.27086	4040	.27087	4040	.44337	1066	.44349	1066	21
22	.31126	3861	.31127	3861	.45403	1052	.45415	1053	22
23	.34987	3697	.34988	3697	.46455	1040	.46468	1040	23
24	.38684	3545	.38685	3545	.47496	1028	.47509	1028	24
25	5.42230	3406	5.42231	3407	6.48524	1015	6.48537	1016	25
26	.45636	3278	.45638	3278	.49539	1004	.49553	1004	26
27	.48915	3158	.48916	3159	.50544	992	.50557	993	27
28	.52073	3048	.52075	3048	.51536	981	.51550	982	28
29	.55121	2944	.55123	2945	.52518	970	.52532	970	29
30	5.58066	2848	5.58068	2848	6.53488	960	6.53503	960	**30**
31	.60914	2757	.60916	2758	.54448	949	.54463	950	31
32	.63672	2672	.63674	2672	.55397	939	.55413	939	32
33	.66344	2593	.66346	2593	.56336	929	.56352	929	33
34	.68937	2518	.68940	2517	.57265	919	.57281	919	34
35	5.71455	2447	5.71457	2447	6.58184	909	6.58201	909	35
36	.73902	2379	.73904	2380	.59093	900	.59110	900	36
37	.76282	2316	.76284	2316	.59993	891	.60011	891	37
38	.78598	2256	.78601	2256	.60884	882	.60902	882	38
39	.80854	2199	.80857	2199	.61766	872	.61784	873	39
40	5.83053	2145	5.83056	2145	6.62639	864	6.62657	864	**40**
41	.85198	2093	.85201	2093	.63503	855	.63522	856	41
42	.87291	2044	.87295	2043	.64359	847	.64378	848	42
43	.89335	1996	.89338	1997	.65206	839	.65226	839	43
44	.91332	1952	.91335	1952	.66045	831	.66065	831	44
45	5.93284	1909	5.93288	1909	6.66876	823	6.66897	823	45
46	.95193	1868	.95197	1868	.67700	815	.67720	816	46
47	.97061	1829	.97065	1829	.68515	808	.68536	808	47
48	5.98890	1790	5.98894	1791	.69323	800	.69345	800	48
49	6.00680	1755	6.00685	1755	.70124	793	.70145	794	49
50	6.02435	1720	6.02440	1720	6.70917	786	6.70939	786	**50**
51	.04155	1686	.04160	1687	.71703	779	.71725	779	51
52	.05842	1654	.05847	1654	.72482	772	.72505	772	52
53	.07496	1623	.07501	1623	.73254	765	.73277	765	53
54	.09120	1594	.09125	1594	.74019	758	.74043	759	54
55	6.10714	1565	6.10719	1565	6.74777	752	6.74802	752	55
56	.12279	1537	.12284	1537	.75529	745	.75554	746	56
57	.13816	1511	.13822	1511	.76275	739	.76300	739	57
58	.15327	1484	.15333	1485	.77014	733	.77040	733	58
59	.16811	1460	.16818	1460	.77747	726	.77773	727	59
60	6.18271		6.18278		6.78474		6.78506		**60**
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

	2°				3°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	6.78474	721	6.78506	721	7.13687	481	7.13746	481	0
1	.79195	714	.79221	715	.14168	478	.14228	479	1
2	.79909	709	.79937	709	.14646	475	.14707	476	2
3	.80618	703	.80646	703	.15122	473	.15183	474	3
4	.81322	697	.81350	698	.15595	470	.15657	471	4
5	6.82019	692	6.82048	692	7.16066	468	7.16129	469	5
6	.82711	686	.82740	687	.16534	466	.16598	466	6
7	.83398	681	.83427	682	.17000	463	.17064	464	7
8	.84079	676	.84109	676	.17463	460	.17528	461	8
9	.84755	670	.84785	671	.17923	458	.17989	459	9
10	6.85425	665	6.85457	666	7.18382	455	7.18448	456	10
11	.86091	660	.86123	660	.18837	453	.18905	454	11
12	.86751	655	.86783	656	.19291	451	.19359	452	12
13	.87407	650	.87439	651	.19742	448	.19811	449	13
14	.88057	646	.88090	646	.20191	446	.20260	447	14
15	6.88703	641	6.88737	641	7.20637	444	7.20707	445	15
16	.89344	636	.89378	636	.21081	442	.21152	442	16
17	.89980	631	.90015	632	.21523	440	.21595	440	17
18	.90612	627	.90647	628	.21963	437	.22035	438	18
19	.91239	622	.91275	623	.22400	435	.22473	436	19
20	6.91862	618	6.91898	618	7.22836	433	7.22909	434	20
21	.92480	613	.92516	614	.23269	431	.23343	431	21
22	.93093	609	.93131	610	.23700	429	.23775	429	22
23	.93703	605	.93741	605	.24129	426	.24204	427	23
24	.94308	601	.94346	601	.24555	424	.24632	425	24
25	6.94909	597	6.94948	597	7.24980	422	7.25057	423	25
26	.95506	592	.95545	593	.25402	420	.25480	421	26
27	.96099	589	.96139	589	.25823	418	.25902	419	27
28	.96688	584	.96728	585	.26241	416	.26321	417	28
29	.97272	581	.97313	581	.26658	414	.26738	415	29
30	6.97853	577	6.97895	577	7.27072	412	7.27153	413	30
31	.98430	573	.98472	574	.27485	410	.27567	411	31
32	.99004	569	.99046	570	.27895	409	.27978	409	32
33	6.99573	565	6.99616	566	.28304	406	.28387	407	33
34	7.00139	562	7.00182	563	.28711	405	.28795	405	34
35	7.00701	558	7.00745	559	7.29116	402	7.29200	404	35
36	.01259	555	.01304	555	.29518	401	.29604	402	36
37	.01814	551	.01860	552	.29919	399	.30006	400	37
38	.02366	548	.02412	548	.30319	397	.30406	398	38
39	.02914	544	.02960	545	.30716	395	.30804	396	39
40	7.03458	541	7.03505	541	7.31112	393	7.31201	394	40
41	.03999	537	.04047	538	.31505	392	.31595	393	41
42	.04537	534	.04585	535	.31897	390	.31988	391	42
43	.05071	531	.05120	531	.32288	388	.32379	389	43
44	.05603	527	.05652	528	.32676	386	.32768	388	44
45	7.06130	525	7.06180	525	7.33063	385	7.33156	385	45
46	.06655	521	.06706	522	.33448	383	.33542	384	46
47	.07177	518	.07228	519	.33831	382	.33926	382	47
48	.07695	515	.07747	516	.34213	380	.34309	380	48
49	.08211	512	.08263	513	.34593	378	.34689	379	49
50	7.08723	509	7.08776	509	7.34971	377	7.35069	377	50
51	.09232	506	.09286	507	.35348	375	.35446	376	51
52	.09739	503	.09793	503	.35723	373	.35822	374	52
53	.10242	500	.10297	501	.36097	371	.36196	373	53
54	.10743	497	.10798	498	.36468	370	.36569	371	54
55	7.11240	495	7.11297	495	7.36839	368	7.36940	369	55
56	.11735	492	.11792	493	.37207	367	.37310	368	56
57	.12227	489	.12285	490	.37574	366	.37678	366	57
58	.12716	486	.12775	487	.37940	364	.38044	365	58
59	.13203	484	.13262	484	.38304	362	.38409	363	59
60	7.13687		7.13746		7.38667		7.38773		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	

′	4° Log. Vers.	D	4° Log. Exsec.	D	5° Log. Vers.	D	5° Log. Exsec.	D	′
0	7.38667	361	7.38773	361	7.58039	289	7.58204	290	0
1	.39028	359	.39134	360	.58328	287	.58494	289	1
2	.39387	358	.39495	359	.58615	287	.58783	288	2
3	.39745	356	.39854	357	.58902	286	.59071	287	3
4	.40102	355	.40211	356	.59188	285	.59358	286	4
5	7.40457	353	7.40567	354	7.59473	284	7.59645	285	5
6	.40810	352	.40922	353	.59758	283	.59930	284	6
7	.41163	350	.41275	352	.60041	282	.60214	283	7
8	.41513	349	.41627	350	.60323	281	.60498	282	8
9	.41863	348	.41977	349	.60604	280	.60780	281	9
10	7.42211	346	7.42326	347	7.60885	279	7.61062	280	10
11	.42557	345	.42673	346	.61164	279	.61342	280	11
12	.42903	343	.43019	345	.61443	277	.61622	279	12
13	.43246	342	.43364	343	.61721	277	.61901	278	13
14	.43589	341	.43708	342	.61998	276	.62179	277	14
15	7.43930	339	7.44050	340	7.62274	275	7.62456	276	15
16	.44270	338	.44390	339	.62549	274	.62733	275	16
17	.44608	337	.44730	338	.62823	273	.63008	274	17
18	.44946	335	.45068	337	.63096	272	.63282	274	18
19	.45281	334	.45405	335	.63369	272	.63556	273	19
20	7.45616	333	7.45740	334	7.63641	270	7.63829	272	20
21	.45949	332	.46075	332	.63911	270	.64101	271	21
22	.46281	330	.46407	332	.64181	269	.64372	270	22
23	.46612	329	.46739	330	.64451	268	.64643	269	23
24	.46941	328	.47070	329	.64719	267	.64912	269	24
25	7.47270	327	7.47399	328	7.64986	266	7.65181	268	25
26	.47597	325	.47727	327	.65253	266	.65449	267	26
27	.47922	324	.48054	325	.65519	265	.65716	266	27
28	.48247	323	.48379	324	.65784	264	.65982	265	28
29	.48570	322	.48703	323	.66048	263	.66247	264	29
30	7.48892	321	7.49026	322	7.66311	263	7.66512	264	30
31	.49213	320	.49348	321	.66574	261	.66776	263	31
32	.49533	318	.49669	319	.66836	261	.67039	262	32
33	.49852	317	.49989	318	.67097	260	.67301	261	33
34	.50169	316	.50307	317	.67357	259	.67562	261	34
35	7.50485	315	7.50624	316	7.67617	258	7.67823	260	35
36	.50800	314	.50941	315	.67875	258	.68083	259	36
37	.51114	313	.51256	313	.68133	257	.68342	258	37
38	.51427	311	.51569	313	.68390	256	.68601	257	38
39	.51739	311	.51882	311	.68647	255	.68858	257	39
40	7.52050	309	7.52194	310	7.68902	255	7.69115	256	40
41	.52359	308	.52504	309	.69157	254	.69371	255	41
42	.52667	307	.52814	308	.69411	253	.69627	254	42
43	.52975	306	.53122	307	.69665	252	.69881	254	43
44	.53281	305	.53429	306	.69917	252	.70135	253	44
45	7.53586	304	7.53735	305	7.70169	251	7.70388	252	45
46	.53890	303	.54041	304	.70421	250	.70641	252	46
47	.54193	302	.54345	303	.70671	250	.70893	251	47
48	.54495	300	.54648	302	.70921	249	.71144	250	48
49	.54796	300	.54950	301	.71170	248	.71394	250	49
50	7.55096	299	7.55251	299	7.71418	247	7.71644	248	50
51	.55395	297	.55550	299	.71666	247	.71892	248	51
52	.55692	297	.55849	298	.71913	246	.72141	247	52
53	.55989	295	.56147	296	.72159	245	.72388	246	53
54	.56285	295	.56444	296	.72404	245	.72635	246	54
55	7.56580	293	7.56740	295	7.72649	244	7.72881	245	55
56	.56873	293	.57035	294	.72893	243	.73126	245	56
57	.57166	292	.57329	292	.73137	242	.73371	244	57
58	.57458	290	.57621	292	.73379	242	.73615	243	58
59	.57749	290	.57913	291	.73621	241	.73859	242	59
60	7.58039		7.58204		7.73863		7.74101		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	360	350	340
6	36.0	35.0	34.0
7	42.0	40.8	39.6
8	48.0	46.6	45.3
9	54.0	51.5	51.0
10	60.0	58.3	56.6
20	120.0	116.6	113.3
30	180.0	175.0	170.0
40	240.0	233.3	226.6
50	300.0	291.6	283.3

	330	320	310
6	33.0	32.0	31.0
7	38.5	37.3	36.1
8	44.0	42.6	41.3
9	49.5	48.0	46.5
10	55.0	53.3	51.6
20	110.0	106.6	103.3
30	165.0	160.0	155.0
40	220.0	213.3	206.6
50	275.0	266.6	258.3

	300	290	280
6	30.0	29.0	28.0
7	35.0	33.8	32.6
8	40.0	38.6	37.3
9	45.0	43.5	42.0
10	50.0	48.3	46.6
20	100.0	96.6	93.3
30	150.0	145.0	140.0
40	200.0	193.3	186.6
50	250.0	241.6	233.3

	270	260	250
6	27.0	26.0	25.0
7	31.5	30.3	29.1
8	36.0	34.6	33.3
9	40.5	39.0	37.5
10	45.0	43.3	41.6
20	90.0	86.6	83.3
30	135.0	130.0	125.0
40	180.0	173.3	166.6
50	225.0	216.6	208.3

	240	230	220
6	24.0	23.0	22.0
7	28.0	26.8	25.6
8	32.0	30.6	29.3
9	36.0	34.5	33.0
10	40.0	38.3	36.6
20	80.0	76.6	73.3
30	120.0	115.0	110.0
40	160.0	153.3	146.6
50	200.0	191.6	183.3

	210	200	190
6	21.0	20.0	19.0
7	24.5	23.3	22.1
8	28.0	26.6	25.3
9	31.5	30.0	28.5
10	35.0	33.3	31.6
20	70.0	66.6	63.3
30	105.0	100.0	95.0
40	140.0	133.3	126.6
50	175.0	166.6	158.3

P. P.

	6°				7°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	7.73863	241	7.74101	242	7.87238	206	7.87563	208	0
1	.74104	240	.74343	241	.87444	205	.87771	207	1
2	.74344	239	.74585	241	.87650	205	.87978	207	2
3	.74583	239	.74826	240	.87855	204	.88185	206	3
4	.74822	238	.75066	239	.88060	204	.88391	206	4
5	7.75060	237	7.75305	239	7.88264	204	7.88597	205	5
6	.75297	236	.75544	238	.88468	203	.88803	205	6
7	.75534	236	.75782	237	.88672	203	.89008	204	7
8	.75770	235	.76019	237	.88875	202	.89212	204	8
9	.76006	234	.76256	236	.89077	202	.89416	203	9
10	7.76240	234	7.76492	235	7.89279	201	7.89620	203	10
11	.76475	233	.76728	235	.89481	201	.89823	202	11
12	.76708	233	.76963	234	.89682	200	.90025	202	12
13	.76941	232	.77197	233	.89882	200	.90228	201	13
14	.77173	232	.77431	233	.90082	199	.90429	201	14
15	7.77405	231	7.77664	232	7.90282	199	7.90630	201	15
16	.77636	230	.77897	231	.90481	198	.90831	200	16
17	.77867	230	.78128	231	.90680	198	.91032	199	17
18	.78097	229	.78360	230	.90878	197	.91231	199	18
19	.78326	228	.78590	230	.91076	197	.91431	199	19
20	7.78554	228	7.78820	229	7.91273	197	7.91630	198	20
21	.78783	227	.79050	229	.91470	196	.91828	198	21
22	.79010	227	.79279	228	.91667	196	.92027	197	22
23	.79237	226	.79507	228	.91863	195	.92224	197	23
24	.79463	225	.79735	227	.92058	195	.92421	197	24
25	7.79689	225	7.79962	226	7.92253	195	7.92618	196	25
26	.79914	224	.80188	226	.92448	194	.92815	195	26
27	.80138	224	.80414	225	.92642	194	.93010	195	27
28	.80362	223	.80639	225	.92836	193	.93206	195	28
29	.80586	222	.80864	224	.93029	193	.93401	195	29
30	7.80808	222	7.81088	224	7.93222	192	7.93596	194	30
31	.81031	221	.81312	223	.93415	192	.93790	194	31
32	.81252	221	.81535	222	.93607	191	.93984	193	32
33	.81473	220	.81758	222	.93799	191	.94177	193	33
34	.81694	220	.81980	221	.93990	190	.94370	192	34
35	7.81914	219	7.82201	221	7.94181	190	7.94562	192	35
36	.82133	219	.82422	220	.94371	190	.94754	192	36
37	.82352	218	.82642	219	.94561	189	.94946	191	37
38	.82570	217	.82862	219	.94751	189	.95137	191	38
39	.82788	217	.83081	219	.94940	189	.95328	190	39
40	7.83005	217	7.83300	218	7.95129	188	7.95519	190	40
41	.83222	216	.83518	217	.95317	187	.95709	189	41
42	.83438	215	.83735	217	.95505	188	.95898	189	42
43	.83653	215	.83952	216	.95693	187	.96088	188	43
44	.83868	214	.84169	216	.95880	186	.96276	188	44
45	7.84083	214	7.84385	215	7.96066	186	7.96465	188	45
46	.84297	213	.84600	215	.96253	186	.96653	188	46
47	.84510	213	.84815	214	.96439	185	.96841	187	47
48	.84723	212	.85030	213	.96624	185	.97028	187	48
49	.84935	212	.85243	213	.96809	184	.97215	186	49
50	7.85147	211	7.85457	213	7.96994	184	7.97401	186	50
51	.85359	211	.85670	212	.97178	184	.97587	185	51
52	.85570	210	.85882	211	.97362	183	.97773	185	52
53	.85780	210	.86094	211	.97546	183	.97958	184	53
54	.85990	209	.86305	211	.97729	183	.98143	184	54
55	7.86199	209	7.86516	210	7.97912	182	7.98327	184	55
56	.86408	208	.86726	210	.98094	182	.98512	183	56
57	.86616	208	.86936	209	.98276	182	.98695	183	57
58	.86824	207	.87146	208	.98458	181	.98879	183	58
59	.87031	206	.87354	208	.98639	181	.99062	182	59
60	7.87238		7.87563		7.98820		7.99244		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	180	9̂	9
6	[illegible]	[illegible]	0.[illegible]
7	21.0	1.1	1.[illegible]
8	24.0	1.[illegible]	1.2
9	27.0	1.4	1.3
10	30.0	1.6	1.5
20	60.0	3.1	[illegible].0
30	90.0	4.7	4.[illegible]
40	120.0	[illegible]	[illegible].0
50	150.0	7.[illegible]	7.[illegible]

	8̂	8	7̂
6	0.8	0.8	0.7
7	1.0	0.[illegible]	0.9
8	1.1	1.[illegible]	1.[illegible]
9	1.3	1.2	1.1
10	1.4	1.3	1.2
20	2.8	2.[illegible]	[illegible].5
30	4.2	4.[illegible]	[illegible].7
40	5.6	5.3	5.[illegible]
50	7.1	6.[illegible]	[illegible].2

	7	6̂	6
6	0.7	0.6	0.6
7	0.8	0.7	0.7
8	0.[illegible]	0.8	0.8
9	1.0	1.0	0.9
10	1.1	1.1	1.0
20	2.3	2.1	2.0
30	3.5	3.2	3.0
40	4.6	4.3	4.0
50	5.8	5.4	5.0

	5̂	5	4̂
6	0.5	0.5	0.4
7	0.6	0.6	0.5
8	0.7	0.6	0.6
9	0.8	0.7	0.7
10	0.9	0.8	0.7
20	1.8	1.6	1.5
30	2.7	2.5	2.2
40	3.6	3.3	3.0
50	4.6	4.1	3.7

	4	3̂	3
6	0.4	0.3	0.3
7	0.4	0.4	0.3
8	0.5	0.4	0.4
9	0.6	0.5	0.4
10	0.6	0.6	0.5
20	1.3	1.1	1.0
30	2.0	1.7	1.5
40	2.6	2.3	2.0
50	3.3	2.9	2.5

	2̂	2	1̂
6	0.2	0.2	0.1
7	0.3	0.2	0.2
8	0.3	0.2	0.2
9	0.4	0.3	0.2
10	0.4	0.3	0.2
20	0.8	0.6	0.5
30	1.2	1.0	0.7
40	1.6	1.3	1.[illegible]
50	2.1	1.6	1.[illegible]

	1	0̂
6	0.1	0.0
7	0.1	0.0
8	0.1	0.0
9	0.1	0.1
10	0.1	0.1
20	0.3	0.1
30	0.5	0.2
40	0.6	0.3
50	0.8	0.4

P. P.

′	8° Log. Vers.	D	8° Log. Exsec.	D	9° Log. Vers.	D	9° Log. Exsec.	D	′
0	7.98820	180̂	7.99244̂	182̂	8.09031̂	160̂	8.09569̂	162̂	0
1	.99000̂	180	.99427	182	.09192	160	.09732	162	1
2	.99180̂	179̂	.99609	181̂	.09352	160	.09894	162	2
3	.99360	179̂	.99790̂	181	.09512	159̂	.10056	161̂	3
4	.99539̂	179	7.99971̂	180̂	.09671̂	159	.10217̂	161	4
5	7.99718̂	178̂	8.00152	180̂	8.09830̂	159	8.10378̂	161	5
6	7.99897	178̂	.00332̂	180	.09989̂	158̂	.10539̂	160̂	6
7	8.00075̂	177̂	.00512̂	180	.10148	158̂	.10700	160̂	7
8	.00253	178	.00692̂	179	.10306̂	158	.10860̂	160	8
9	.00431	177	.00871̂	179	.10464̂	157̂	.11020̂	160	9
10	8.00608	176̂	8.01050̂	178̂	8.10622	157̂	8.11180̂	159̂	10
11	.00784̂	176̂	.01229	178̂	.10779̂	157	.11340	159	11
12	.00961	176	.01407̂	178	.10936̂	157	.11499	159	12
13	.01137	176	.01585̂	177̂	.11093̂	156̂	.11658	158̂	13
14	.01313	175	.01763	177	.11250	156̂	.11816̂	158̂	14
15	8.01488	175	8.01940	177	8.11406̂	156	8.11975	158	15
16	.01663	175	.02117	176̂	.11562̂	155̂	.12133	158	16
17	.01838	174	.02293̂	176	.11718	155̂	.12291	157̂	17
18	.02012	174	.02469̂	175̂	.11873̂	155̂	.12448̂	157	18
19	.02186	173̂	.02645	175̂	.12029	155	.12605̂	157	19
20	8.02359̂	173̂	8.02820	175	8.12184	154̂	8.12762̂	157	20
21	.02533	173	.02995̂	175	.12338̂	154	.12919̂	156	21
22	.02706	172̂	.03170̂	174̂	.12492̂	154̂	.13075̂	156̂	22
23	.02878̂	172	.03345	174	.12647	153̂	.13232	155̂	23
24	.03050̂	172	.03519	173̂	.12800̂	153̂	.13387̂	156	24
25	8.03222	171̂	8.03692̂	173̂	8.12954	153̂	8.13543̂	155	25
26	.03394	171	.03866	173	.13107̂	153	.13698̂	155̂	26
27	.03565	171	.04039	173	.13260̂	152̂	.13854	154̂	27
28	.03736	170̂	.04212	172	.13413	152̂	.14008̂	154̂	28
29	.03906̂	170	.04384	172̂	.13565̂	152	.14163	154̂	29
30	8.04076̂	170	8.04556̂	171̂	8.13717̂	152	8.14317̂	154	30
31	.04246̂	169̂	.04728	171̂	.13869̂	151̂	.14471̂	153̂	31
32	.04416	169	.04899̂	171	.14021	151̂	.14625	153̂	32
33	.04585	169	.05070̂	170̂	.14172̂	151	.14778̂	153̂	33
34	.04754	168̂	.05241	170̂	.14323̂	151	.14932	153	34
35	8.04922̂	168	8.05411̂	170	8.14474̂	150̂	8.15085	152̂	35
36	.05090̂	168	.05581̂	170	.14625	150̂	.15237̂	152̂	36
37	.05258̂	167̂	.05751̂	169̂	.14775̂	150	.15390	152	37
38	.05426	167	.05921	169	.14925̂	149̂	.15542	152	38
39	.05593	167	.06090	169	.15075	150	.15694	152	39
40	8.05760	166̂	8.06259	168̂	8.15225	149	8.15846	151̂	40
41	.05926̂	166̂	.06427̂	168	.15374	149	.15997̂	151	41
42	.06093	166	.06595̂	168	.15523	149	.16148̂	151	42
43	.06259	165̂	.06763̂	167̂	.15672	148̂	.16299̂	150̂	43
44	.06424̂	165	.06931	167	.15820̂	148	.16450	150̂	44
45	8.06589̂	165	8.07098	167	8.15968̂	148	8.16600̂	150	45
46	.06754̂	165	.07265	166̂	.16116̂	148	.16750̂	150	46
47	.06919̂	164	.07431̂	166̂	.16264̂	147̂	.16900̂	149̂	47
48	.07083̂	164	.07598	166	.16412	147	.17050	149̂	48
49	.07247̂	164	.07764	165̂	.16559	147	.17199̂	149̂	49
50	8.07411̂	163̂	8.07929̂	165̂	8.16706	146̂	8.17349	148̂	50
51	.07575	163	.08095	165	.16852̂	146̂	.17497̂	149	51
52	.07738	162̂	.08260	164̂	.16999	146̂	.17646̂	148̂	52
53	.07900̂	162̂	.08424̂	164̂	.17145̂	146	.17795	148	53
54	.08063	162̂	.08589	164̂	.17291̂	145̂	.17943	148	54
55	8.08225̂	161̂	8.08753̂	163̂	8.17437	145̂	8.18091	147̂	55
56	.08387	162	.08917	164	.17582̂	145̂	.18238̂	147̂	56
57	.08549	161	.09081	163	.17728	145	.18386	147̂	57
58	.08710	161	.09244	163	.17873	144̂	.18533̂	147	58
59	.08871	160̂	.09407	162̂	.18017̂	144̂	.18680̂	146̂	59
60	8.09031		8.09569̂		8.18162		8.18827		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	180	170	160
6	18.0	17.0	16.0
7	21.0	19.8̂	18.6̂
8	24.0	22.6̂	21.3̂
9	27.0	25.5	24.0
10	30.0	28.3̂	26.6̂
20	60.0	56.6̂	53.3̂
30	90.0	85.0	80.0
40	120.0	113.3̂	106.6̂
50	150.0	141.6̂	133.3̂

	150	140
6	15.0	14.0
7	17.5	16.3̂
8	20.0	18.6̂
9	22.5	21.0
10	25.0	23.3̂
20	50.0	46.6̂
30	75.0	70.0
40	100.0	93.3̂
50	125.0	116.6̂

	9̂	9	8̂
6	0.9̂	0.9	0.8̂
7	1.1	1.0̂	1.0
8	1.2̂	1.2	1.1̂
9	1.4̂	1.3̂	1.3
10	1.6	1.5	1.4
20	3.1̂	3.0	2.8̂
30	4.7̂	4.5	4.2̂
40	6.3̂	6.0	5.6̂
50	7.9	7.5	7.1

	8	7̂	7
6	0.8	0.7̂	0.7
7	0.9̂	0.9	0.8
8	1.0̂	1.0	0.9̂
9	1.2	1.1	1.0̂
10	1.3̂	1.2̂	1.1̂
20	2.6̂	2.5	2.3̂
30	4.0	3.7̂	3.5
40	5.3̂	5.0	4.6̂
50	6.6̂	6.2̂	5.8

	6̂	6
6	0.6̂	0.6
7	0.7̂	0.7
8	0.8̂	0.8
9	1.0	0.9
10	1.1	1.0
20	2.1̂	2.0
30	3.2̂	3.0
40	4.3̂	4.0
50	5.4	5.0

	5̂	5
6	0.5̂	0.5
7	0.6̂	0.6
8	0.7	0.6̂
9	0.8	0.7̂
10	0.9	0.8̂
20	1.8̂	1.6
30	2.7̂	2.5
40	3.6̂	3.3̂
50	4.6	4.1̂

P. P.

'	10° Log. Vers.	D	10° Log. Exsec.	D	11° Log. Vers.	D	11° Log. Exsec.	D	'
0	8.18162	144̂	8.18827	146̂	8.26417̂	131	8.27223	133̂	0
1	.18306̂	144	.18973̂	146̂	.26548̂	131	.27356	133̂	1
2	.18450̂	144	.19120	146	.26679̂	131	.27490	133	2
3	.18594̂	143̂	.19266	145̂	.26810̂	130̂	.27623	133	3
4	.18738	143	.19411̂	145̂	.26941	130	.27756̂	133	4
5	8.18881	143̂	8.19557	145̂	8.27071	130	8.27889	132̂	5
6	.19024̂	142̂	.19702̂	145	.27201	130	.28021̂	132	6
7	.19167	142̂	.19847̂	145̂	.27331	130	.28153̂	132̂	7
8	.19309̂	142̂	.19992̂	144̂	.27461	129̂	.28286	132	8
9	.19452	142	.20137	144̂	.27590̂	129	.28418	132	9
10	8.19594	142	8.20281̂	144	8.27719̂	129̂	8.28550	131̂	10
11	.19736	142	.20425̂	144	.27849	128̂	.28681̂	131̂	11
12	.19878	141	.20569̂	144	.27977̂	129	.28813	131	12
13	.20019	141̂	.20713̂	143̂	.28106̂	128̂	.28944	131	13
14	.20160̂	141	.20857	143	.28235	128̂	.29075	131	14
15	8.20301̂	140̂	8.21000	143	8.28363̂	128	8.29206	130̂	15
16	.20442	140̂	.21143	143	.28491̂	128	.29336̂	130̂	16
17	.20582̂	140̂	.21286	142̂	.28619̂	128	.29467	130̂	17
18	.20723	140	.21428̂	142̂	.28747̂	127̂	.29597̂	130	18
19	.20863	140	.21571	142	.28875	127̂	.29727̂	130	19
20	8.21003	139̂	8.21713	142	8.29002̂	127	8.29857̂	130	20
21	.21142̂	139̂	.21855	141̂	.29129̂	127	.29987̂	129̂	21
22	.21282	139	.21996̂	141̂	.29256̂	127	.30117	129̂	22
23	.21421	139	.22138	141̂	.29383̂	126̂	.30246̂	129	23
24	.21560	138̂	.22279̂	141	.29510	126̂	.30375̂	129	24
25	8.21698	138̂	8.22420̂	140̂	8.29636̂	126̂	8.30504̂	129	25
26	.21837	138̂	.22561	140̂	.29763	126	.30633̂	128̂	26
27	.21975̂	138	.22701̂	140̂	.29889	126	.30762	128̂	27
28	.22113̂	137̂	.22842	140	.30015	125̂	.30890̂	128̂	28
29	.22251	138	.22982	140	.30140̂	125̂	.31019	128	29
30	8.22389	137	8.23122	140	8.30266	125̂	8.31147	128	30
31	.22526	137̂	.23262	139̂	.30391̂	125	.31275	127̂	31
32	.22663	136̂	.23401̂	139	.30516̂	125̂	.31402̂	127̂	32
33	.22800	137	.23540̂	139	.30642	124̂	.31530	127̂	33
34	.22937	136̂	.23679̂	139	.30766̂	124̂	.31657̂	127̂	34
35	8.23073̂	136	8.23818̂	138̂	8.30891	124̂	8.31785	127	35
36	.23209̂	136̂	.23957	138̂	.31015̂	124̂	.31912	127	36
37	.23346	135̂	.24095̂	138̂	.31140	124	.32039	126̂	37
38	.23481̂	136	.24234	138	.31264	124	.32165̂	126̂	38
39	.23617̂	135	.24372	137̂	.31388	123̂	.32292	126̂	39
40	8.23752	135̂	8.24509̂	138	8.31511̂	124	8.32418̂	126	40
41	.23888	135	.24647̂	137	.31635̂	123	.32544̂	126	41
42	.24023	135̂	.24784̂	137̂	.31758̂	123̂	.32670̂	126	42
43	.24158	134̂	.24922	137	.31882	123	.32796̂	125̂	43
44	.24292̂	134	.25059	136̂	.32005	123	.32922	125̂	44
45	8.24426	134̂	8.25195̂	136̂	8.32128	122̂	8.33047̂	125̂	45
46	.24561	134	.25332	136̂	.32250̂	122̂	.33173	125	46
47	.24695	133̂	.25468̂	136	.32373	122̂	.33298	125	47
48	.24828̂	133̂	.25604̂	136	.32495̂	122	.33423	124̂	48
49	.24962	133̂	.25740̂	136	.32617̂	122	.33547̂	125	49
50	8.25095̂	133	8.25876̂	135̂	8.32739̂	122	8.33672̂	124̂	50
51	.25228̂	133	.26012	135̂	.32861̂	121̂	.33797	124	51
52	.25361̂	132̂	.26147̂	135	.32983	121̂	.33921	124̂	52
53	.25494	133	.26282̂	135	.33104̂	121	.34045̂	123̂	53
54	.25627	132	.26417̂	134̂	.33225̂	121̂	.34169	124	54
55	8.25759	132	8.26552	134̂	8.33347	121	8.34293	124	55
56	.25891	132	.26686̂	134̂	.33468	120̂	.34417	123	56
57	.26023	132	.26821	134	.33588̂	120̂	.34540	123̂	57
58	.26155	131̂	.26955	134	.33709	120̂	.34663̂	123	58
59	.26286̂	131	.27089	134	.33829̂	120̂	.34786̂	123	59
60	8.26417̂		8.27223		8.33950		8.34909̂		60
'	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	'

P. P.

	130	120
6	13.0	12.0
7	15.1̂	14.0
8	17.3̂	16.0
9	19.5	18.0
10	21.6̂	20.0
20	43.3̂	40.0
30	65.0	60.0
40	86.6̂	80.0
50	108.3̂	100.0

	4̂	4	3̂
6	0.4̂	0.4	0.3̂
7	0.5	0.4̂	0.4
8	0.6	0.5̂	0.4̂
9	0.7	0.6	0.5
10	0.7̂	0.6̂	0.6
20	1.5	1.3̂	1.1̂
30	2.2̂	2.0	1.7̂
40	3.0	2.6̂	2.3̂
50	3.7̂	3.3̂	2.9

	3	2̂
6	0.3	0.2̂
7	0.3̂	0.3
8	0.4	0.3̂
9	0.4̂	0.4
10	0.5	0.4̂
20	1.0	0.8̂
30	1.5	1.2
40	2.0	1.6̂
50	2.5	2.1

	2	1̂
6	0.2	0.1̂
7	0.2̂	0.2
8	0.2̂	0.2
9	0.3	0.2̂
10	0.3̂	0.2̂
20	0.6̂	0.5
30	1.0	0.7̂
40	1.3̂	1.0
50	1.6̂	1.2̂

	1	0̂
6	0.1	0.0̂
7	0.1	0.0̂
8	0.1̂	0.0̂
9	0.1̂	0.1
10	0.1̂	0.1
20	0.3̂	0.1̂
30	0.5	0.2̂
40	0.6̂	0.3̂
50	0.8̂	0.4

P. P.

12° 13°

′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	8.33950	120	8.34909	123	8.40875	110	8.42002	113	0
1	.34070	120	.35032	122	.40985	110	.42116	113	1
2	.34190	119	.35155	122	.41096	110	.42229	113	2
3	.34309	120	.35277	122	.41206	110	.42343	113	3
4	.34429	119	.35399	122	.41317	110	.42456	113	4
5	8.34549	119	8.35522	122	8.41427	110	8.42569	113	5
6	.34668	119	.35644	121	.41537	110	.42682	113	6
7	.34787	119	.35765	122	.41647	109	.42795	113	7
8	.34906	119	.35887	121	.41757	110	.42908	112	8
9	.35025	118	.36009	121	.41867	109	.43021	112	9
10	8.35143	118	8.36130	121	8.41976	109	8.43133	112	10
11	.35262	118	.36251	121	.42086	109	.43246	112	11
12	.35380	118	.36372	120	.42195	109	.43358	112	12
13	.35498	118	.36493	121	.42304	109	.43470	112	13
14	.35616	117	.36614	120	.42413	109	.43582	112	14
15	8.35734	118	8.36734	120	8.42522	108	8.43694	111	15
16	.35852	117	.36855	120	.42630	109	.43805	111	16
17	.35969	117	.36975	120	.42739	108	.43917	111	17
18	.36086	117	.37095	120	.42847	108	.44028	111	18
19	.36204	117	.37215	120	.42956	108	.44139	111	19
20	8.36321	116	8.37335	119	8.43064	108	8.44251	111	20
21	.36437	117	.37454	119	.43172	108	.44362	111	21
22	.36554	116	.37574	119	.43280	108	.44473	110	22
23	.36671	116	.37693	119	.43388	107	.44583	110	23
24	.36787	116	.37812	119	.43495	107	.44694	110	24
25	8.36903	116	8.37931	118	8.43603	107	8.44804	110	25
26	.37019	116	.38050	119	.43710	107	.44915	110	26
27	.37135	115	.38169	118	.43817	107	.45025	110	27
28	.37251	115	.38287	118	.43924	107	.45135	109	28
29	.37366	115	.38406	118	.44031	106	.45245	110	29
30	8.37482	115	8.38524	118	8.44138	107	8.45355	110	30
31	.37597	115	.38642	118	.44245	106	.45465	109	31
32	.37712	115	.38760	118	.44351	106	.45574	109	32
33	.37827	115	.38878	117	.44458	106	.45684	109	33
34	.37942	114	.38995	117	.44564	106	.45793	109	34
35	8.38057	114	8.39113	117	8.44670	106	8.45902	109	35
36	.38171	114	.39230	117	.44776	105	.46011	109	36
37	.38286	114	.39347	117	.44882	106	.46120	108	37
38	.38400	114	.39464	117	.44988	105	.46229	109	38
39	.38514	114	.39581	116	.45093	105	.46338	108	39
40	8.38628	113	8.39698	116	8.45199	105	8.46446	108	40
41	.38741	114	.39814	116	.45304	105	.46555	108	41
42	.38855	113	.39931	116	.45409	105	.46663	108	42
43	.38969	113	.40047	116	.45514	105	.46771	108	43
44	.39082	113	.40163	116	.45619	105	.46879	108	44
45	8.39195	113	8.40279	116	8.45724	104	8.46987	107	45
46	.39308	113	.40395	115	.45829	105	.47095	108	46
47	.39421	113	.40511	115	.45934	104	.47203	107	47
48	.39534	112	.40626	115	.46038	104	.47310	107	48
49	.39646	112	.40742	115	.46142	104	.47417	107	49
50	8.39758	112	8.40857	115	8.46247	104	8.47525	107	50
51	.39871	112	.40972	115	.46351	104	.47632	107	51
52	.39983	112	.41087	115	.46455	103	.47739	107	52
53	.40095	112	.41202	114	.46558	104	.47846	106	53
54	.40207	111	.41317	114	.46662	103	.47953	107	54
55	8.40318	111	8.41431	114	8.46766	103	8.48060	106	55
56	.40430	111	.41546	114	.46869	103	.48166	106	56
57	.40541	111	.41660	114	.46972	103	.48273	106	57
58	.40652	111	.41774	114	.47076	103	.48379	106	58
59	.40764	111	.41888	114	.47179	103	.48485	106	59
60	8.40875		8.42002		8.47282		8.48591		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	120	119	118
6	12.0	11.9	11.8
7	14.0	13.9	13.7
8	16.0	15.8	15.7
9	18.0	17.8	17.7
10	20.0	19.8	19.6
20	40.0	39.6	39.3
30	60.0	59.5	59.0
40	80.0	79.3	78.6
50	100.0	99.1	98.3

	117	116	115
6	11.7	11.6	11.5
7	13.6	13.5	13.4
8	15.6	15.4	15.3
9	17.5	17.4	17.2
10	19.5	19.3	19.1
20	39.0	38.6	38.3
30	58.5	58.0	57.5
40	78.0	77.3	76.6
50	97.5	96.6	95.8

	114	113	112
6	11.4	11.3	11.2
7	13.3	13.2	13.0
8	15.2	15.0	14.9
9	17.1	16.9	16.8
10	19.0	18.8	18.6
20	38.0	37.6	37.3
30	57.0	56.5	56.0
40	76.0	75.3	74.6
50	95.0	94.1	93.3

	111	110	109
6	11.1	11.0	10.9
7	12.9	12.8	12.7
8	14.8	14.6	14.5
9	16.6	16.5	16.3
10	18.5	18.3	18.1
20	37.0	36.6	36.3
30	55.5	55.0	54.5
40	74.0	73.3	72.6
50	92.5	91.6	90.8

	108	107	106
6	10.8	10.7	10.6
7	12.6	12.5	12.3
8	14.4	14.2	14.1
9	16.2	16.0	15.9
10	18.0	17.8	17.6
20	36.0	35.6	35.3
30	54.0	53.5	53.0
40	72.0	71.3	70.6
50	90.0	89.1	88.3

	105	104	0
6	10.5	10.4	0.0
7	12.2	12.1	0.0
8	14.0	13.8	0.0
9	15.7	15.6	0.1
10	17.5	17.3	0.1
20	35.0	34.6	0.1
30	52.5	52.0	0.2
40	70.0	69.3	0.3
50	87.5	86.6	0.4

P. P.

	14°				15°				
'	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	'
0	8.47282	102	8.48591	106	8.53242	96	8.54748	99	0
1	.47384	103	.48697	106	.53338	95	.54847	99	1
2	.47487	102	.48803	105	.53434	96	.54946	99	2
3	.47590	102	.48909	105	.53530	95	.55045	99	3
4	.47692	102	.49014	105	.53625	95	.55144	99	4
5	8.47795	102	8.49120	105	8.53721	95	8.55243	99	5
6	.47897	102	.49225	105	.53816	95	.55342	98	6
7	.47999	102	.49331	105	.53911	95	.55441	98	7
8	.48101	102	.49436	105	.54007	95	.55539	98	8
9	.48203	101	.49541	105	.54102	95	.55638	98	9
10	8.48304	101	8.49646	104	8.54197	94	8.55736	98	10
11	.48406	101	.49750	105	.54291	95	.55834	98	11
12	.48507	101	.49855	104	.54386	94	.55933	98	12
13	.48609	101	.49960	104	.54481	94	.56031	98	13
14	.48710	101	.50064	104	.54575	94	.56129	97	14
15	8.48811	101	8.50168	104	8.54670	94	8.56226	98	15
16	.48912	101	.50273	104	.54764	94	.56324	97	16
17	.49013	100	.50377	104	.54858	94	.56422	97	17
18	.49114	101	.50481	104	.54952	94	.56519	97	18
19	.49215	100	.50585	103	.55046	94	.56617	97	19
20	8.49315	100	8.50688	104	8.55140	93	8.56714	97	20
21	.49415	100	.50792	103	.55234	94	.56812	97	21
22	.49516	100	.50896	103	.55328	93	.56909	97	22
23	.49616	100	.50999	103	.55421	93	.57006	97	23
24	.49716	100	.51102	103	.55515	93	.57103	97	24
25	8.49816	100	8.51205	103	8.55608	93	8.57200	96	25
26	.49916	99	.51309	103	.55701	93	.57296	97	26
27	.50015	100	.51412	102	.55795	93	.57393	96	27
28	.50115	99	.51514	103	.55888	93	.57490	96	28
29	.50215	99	.51617	102	.55981	93	.57586	96	29
30	8.50314	99	8.51720	102	8.56074	92	8.57682	96	30
31	.50413	99	.51822	102	.56166	92	.57779	96	31
32	.50512	99	.51925	102	.56259	93	.57875	96	32
33	.50611	99	.52027	102	.56352	92	.57971	96	33
34	.50710	98	.52129	102	.56444	92	.58067	95	34
35	8.50809	99	8.52231	102	8.56536	92	8.58163	96	35
36	.50908	98	.52333	102	.56629	92	.58259	95	36
37	.51006	98	.52435	101	.56721	92	.58354	96	37
38	.51105	98	.52537	101	.56813	92	.58450	95	38
39	.51203	98	.52638	101	.56905	92	.58546	95	39
40	8.51301	98	8.52740	101	8.56997	92	8.58641	95	40
41	.51399	98	.52841	101	.57089	91	.58736	95	41
42	.51497	98	.52943	101	.57180	91	.58832	95	42
43	.51595	97	.53044	101	.57272	91	.58927	95	43
44	.51693	98	.53145	101	.57363	91	.59022	95	44
45	8.51791	97	8.53246	101	8.57455	91	8.59117	94	45
46	.51888	97	.53347	101	.57546	91	.59211	95	46
47	.51986	97	.53448	100	.57637	91	.59306	94	47
48	.52083	97	.53548	100	.57728	91	.59401	94	48
49	.52180	97	.53649	100	.57819	91	.59495	94	49
50	8.52277	97	8.53749	100	8.57910	90	8.59590	94	50
51	.52374	97	.53850	100	.58001	91	.59684	94	51
52	.52471	96	.53950	100	.58092	90	.59779	94	52
53	.52568	97	.54050	100	.58182	90	.59873	94	53
54	.52665	96	.54150	100	.58273	90	.59967	94	54
55	8.52761	96	8.54250	100	8.58363	90	8.60061	94	55
56	.52858	96	.54350	99	.58453	90	.60155	94	56
57	.52954	96	.54449	100	.58544	90	.60249	93	57
58	.53050	96	.54549	99	.58634	90	.60342	94	58
59	.53146	96	.54649	99	.58724	90	.60436	93	59
60	8.53242		8.54748		8.58814		8.60530		60
'	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	'

P. P.

	103	102	101
6	10.3	10.2	10.1
7	12.0	[illegible]	11.8
8	13.7	13.6	13.4
9	15.4	15.3	15.1
10	17.1	[illegible]	[illegible]
20	34.3	[illegible]	[illegible]
30	51.5	[illegible]	[illegible]
40	[illegible]	[illegible]	67.3
50	85.8	85.0	[illegible]

	100	99	98
6	10.0	[illegible]	[illegible]
7	[illegible]	11.5	11.4
8	13.3	13.2	[illegible]
9	15.0	[illegible]	14.7
10	[illegible]	16.5	[illegible]
20	33.3	[illegible]	[illegible]
30	[illegible]	49.5	[illegible]
40	[illegible]	66.0	[illegible]
50	83.3	82.5	[illegible]

	97	96	95
6	9.7	[illegible]	[illegible]
7	11.3	11.2	11.1
8	[illegible]	12.8	[illegible]
9	[illegible]	14.4	14.2
10	16.1	16.0	[illegible]
20	32.3	32.0	[illegible]
30	48.5	48.0	47.5
40	64.6	64.0	63.3
50	80.8	80.0	[illegible]

	94	93	92
6	9.4	[illegible]	[illegible]
7	[illegible]	[illegible]	10.7
8	12.5	12.4	[illegible]
9	14.1	[illegible]	[illegible]
10	[illegible]	15.5	[illegible]
20	31.3	[illegible]	[illegible]
30	47.0	[illegible]	[illegible]
40	[illegible]	[illegible]	[illegible]
50	78.3	77.5	[illegible]

	91	90	0
6	9.1	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]
9	[illegible]	13.5	[illegible]
10	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]
30	45.5	45.0	[illegible]
40	[illegible]	60.0	[illegible]
50	[illegible]	[illegible]	[illegible]

P. P.

	16°				17°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	8.58814	90	8.60530	93	8.64043	84	8.65984	88	0
1	.58904	89	.60623	93	.64128	84	.66072	88	1
2	.58993	90	.60716	93	.64212	84	.66160	88	2
3	.59083	89	.60810	93	.64296	84	.66248	88	3
4	.59173	89	.60903	93	.64381	84	.66336	88	4
5	8.59262	89	8.60996	93	8.64465	84	8.66425	87	5
6	.59351	89	.61089	93	.64549	84	.66512	88	6
7	.59441	89	.61182	92	.64633	84	.66600	88	7
8	.59530	89	.61275	93	.64717	84	.66688	87	8
9	.59619	89	.61368	92	.64801	83	.66776	87	9
10	8.59708	89	8.61460	92	8.64884	83	8.66863	88	10
11	.59797	89	.61553	92	.64968	84	.66951	87	11
12	.59886	88	.61645	92	.65052	83	.67039	87	12
13	.59974	89	.61738	92	.65135	83	.67126	87	13
14	.60063	88	.61830	92	.65218	83	.67213	87	14
15	8.60152	88	8.61922	92	8.65302	83	8.67301	87	15
16	.60240	88	.62014	92	.65385	83	.67388	87	16
17	.60328	88	.62106	92	.65468	83	.67475	87	17
18	.60417	88	.62198	92	.65551	83	.67562	87	18
19	.60505	88	.62290	91	.65634	83	.67649	87	19
20	8.60593	88	8.62382	92	8.65717	83	8.67736	86	20
21	.60681	88	.62474	91	.65800	82	.67822	87	21
22	.60769	88	.62565	91	.65883	82	.67909	86	22
23	.60857	87	.62657	91	.65965	83	.67996	86	23
24	.60944	87	.62748	91	.66048	82	.68082	86	24
25	8.61032	87	8.62840	91	8.66131	82	8.68169	86	25
26	.61119	87	.62931	91	.66213	82	.68255	86	26
27	.61207	87	.63022	91	.66295	82	.68341	86	27
28	.61294	87	.63113	91	.66378	82	.68428	86	28
29	.61381	87	.63204	90	.66460	82	.68514	86	29
30	8.61469	87	8.63295	91	8.66542	82	8.68600	86	30
31	.61556	87	.63386	91	.66624	82	.68686	85	31
32	.61643	87	.63477	90	.66706	82	.68772	86	32
33	.61730	86	.63567	90	.66788	82	.68858	86	33
34	.61816	87	.63658	90	.66870	81	.68944	85	34
35	8.61903	86	8.63748	90	8.66951	81	8.69029	86	35
36	.61990	86	.63839	90	.67033	82	.69115	85	36
37	.62076	86	.63929	90	.67115	81	.69201	85	37
38	.62163	86	.64019	90	.67196	81	.69286	85	38
39	.62249	86	.64109	90	.67277	81	.69372	85	39
40	8.62336	86	8.64199	90	8.67359	81	8.69457	85	40
41	.62422	86	.64289	90	.67440	81	.69542	85	41
42	.62508	86	.64379	90	.67521	81	.69627	85	42
43	.62594	86	.64469	89	.67602	81	.69712	85	43
44	.62680	86	.64559	90	.67683	81	.69798	85	44
45	8.62766	86	8.64649	89	8.67764	81	8.69883	84	45
46	.62852	85	.64738	89	.67845	80	.69967	85	46
47	.62937	85	.64828	89	.67926	81	.70052	85	47
48	.63023	85	.64917	89	.68007	80	.70137	84	48
49	.63108	85	.65006	89	.68087	80	.70222	84	49
50	8.63194	85	8.65096	89	8.68168	80	8.70306	84	50
51	.63279	85	.65185	89	.68248	80	.70391	84	51
52	.63364	85	.65274	89	.68329	80	.70475	84	52
53	.63449	85	.65363	89	.68409	80	.70560	84	53
54	.63534	85	.65452	89	.68489	80	.70644	84	54
55	8.63619	85	8.65541	88	8.68569	80	8.70728	84	55
56	.63704	85	.65629	88	.68650	80	.70813	84	56
57	.63789	84	.65718	89	.68730	80	.70897	84	57
58	.63874	85	.65807	88	.68810	79	.70981	84	58
59	.63959	84	.65895	88	.68889	80	.71065	84	59
60	8.64043		8.65984		8.68969		8.71149		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	93	92	91
6	9.3	9.2	9.1
7	10.8	10.7	10.6
8	12.4	12.2	12.1
9	13.9	13.8	13.6
10	15.5	15.3	15.1
20	31.0	30.6	30.3
30	46.5	46.0	45.5
40	62.0	61.3	60.6
50	77.5	76.6	75.8

	90	89	88
6	9.0	8.9	8.8
7	10.5	10.4	10.2
8	12.0	11.8	11.7
9	13.5	13.3	13.2
10	15.0	14.8	14.6
20	30.0	29.6	29.3
30	45.0	44.5	44.0
40	60.0	59.3	58.6
50	75.0	74.1	73.3

	87	86	85
6	8.7	8.6	8.5
7	10.1	10.0	9.9
8	11.6	11.4	11.3
9	13.0	12.9	12.7
10	14.5	14.3	14.1
20	29.0	28.6	28.3
30	43.5	43.0	42.5
40	58.0	57.3	56.6
50	72.5	71.6	70.8

	84	83	82
6	8.4	8.3	8.2
7	9.8	9.7	9.5
8	11.2	11.0	10.9
9	12.6	12.4	12.3
10	14.0	13.8	13.6
20	28.0	27.6	27.3
30	42.0	41.5	41.0
40	56.0	55.3	54.6
50	70.0	69.1	68.3

	81	80	79
6	8.1	8.0	7.9
7	9.4	9.3	9.2
8	10.8	10.6	10.5
9	12.1	12.0	11.8
10	13.5	13.3	13.1
20	27 0	26.6	26.3
30	40.5	40.0	39.5
40	54.0	53.3	52 6
50	67.5	66.6	65.8

	0
6	0.0
7	0.0
8	0.0
9	0 1
10	0.1
20	0.1
30	0.2
40	0.3
50	0.4

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

′	18° Log. Vers.	D	18° Log. Exsec.	D	19° Log. Vers.	D	19° Log. Exsec.	D	′
0	8.68969	79	8.71149	83	8.73625	75	8.76058	79	0
1	.69049	80	.71232	84	.73700	75	.76137	80	1
2	.69129	79	.71316	83	.73775	75	.76217	79	2
3	.69208	79	.71400	84	.73851	75	.76297	79	3
4	.69288	79	.71484	83	.73926	75	.76376	80	4
5	8.69367	79	8.71567	83	8.74001	75	8.76456	79	5
6	.69446	79	.71651	83	.74076	75	.76536	79	6
7	.69526	79	.71734	83	.74151	75	.76615	79	7
8	.69605	79	.71817	83	.74226	75	.76694	79	8
9	.69684	79	.71901	83	.74301	75	.76774	79	9
10	8.69763	79	8.71984	83	8.74376	74	8.76853	79	10
11	.69842	79	.72067	83	.74451	75	.76932	79	11
12	.69921	78	.72150	83	.74526	74	.77011	79	12
13	.70000	79	.72233	83	.74600	74	.77090	79	13
14	.70079	78	.72316	83	.74675	74	.77169	79	14
15	8.70157	78	8.72399	82	8.74749	74	8.77248	79	15
16	.70236	78	.72481	83	.74824	74	.77327	78	16
17	.70314	78	.72564	82	.74898	74	.77406	79	17
18	.70393	78	.72647	82	.74973	74	.77485	78	18
19	.70471	78	.72729	82	.75047	74	.77563	78	19
20	8.70550	78	8.72812	82	8.75121	74	8.77642	78	20
21	.70628	78	.72894	82	.75195	74	.77720	79	21
22	.70706	78	.72977	82	.75269	74	.77799	78	22
23	.70784	78	.73059	82	.75343	74	.77877	78	23
24	.70862	78	.73141	82	.75417	74	.77956	78	24
25	8.70940	78	8.73223	82	8.75491	73	8.78034	78	25
26	.71018	77	.73306	82	.75565	73	.78112	78	26
27	.71096	78	.73388	82	.75639	73	.78191	78	27
28	.71174	77	.73470	81	.75712	73	.78269	78	28
29	.71251	77	.73551	82	.75786	74	.78347	78	29
30	8.71329	77	8.73633	82	8.75860	73	8.78425	78	30
31	.71406	77	.73715	81	.75933	74	.78503	78	31
32	.71484	77	.73797	81	.76006	73	.78581	78	32
33	.71561	77	.73878	81	.76080	73	.78659	77	33
34	.71639	77	.73960	81	.76153	73	.78736	78	34
35	8.71716	77	8.74041	81	8.76226	73	8.78814	77	35
36	.71793	77	.74123	81	.76300	73	.78892	77	36
37	.71870	77	.74204	81	.76373	73	.78969	77	37
38	.71947	77	.74286	81	.76446	73	.79047	77	38
39	.72024	77	.74367	81	.76519	73	.79124	77	39
40	8.72101	76	8.74448	81	8.76592	72	8.79202	77	40
41	.72178	77	.74529	81	.76664	73	.79279	77	41
42	.72255	76	.74610	81	.76737	72	.79357	77	42
43	.72331	77	.74691	80	.76810	73	.79434	77	43
44	.72408	76	.74772	81	.76883	72	.79511	77	44
45	8.72485	76	8.74853	81	8.76955	72	8.79588	77	45
46	.72561	76	.74934	80	.77028	72	.79665	77	46
47	.72637	76	.75014	80	.77100	72	.79742	77	47
48	.72714	76	.75095	80	.77173	72	.79819	77	48
49	.72790	76	.75175	80	.77245	72	.79896	76	49
50	8.72866	76	8.75256	80	8.77317	72	8.79973	77	50
51	.72942	76	.75336	80	.77390	72	.80050	76	51
52	.73018	76	.75417	80	.77462	72	.80126	77	52
53	.73094	76	.75497	80	.77534	72	.80203	76	53
54	.73170	76	.75577	80	.77606	72	.80280	76	54
55	8.73246	76	8.75658	80	8.77678	72	8.80356	76	55
56	.73322	75	.75738	80	.77750	72	.80433	76	56
57	.73398	75	.75818	80	.77822	71	.80509	76	57
58	.73473	76	.75898	80	.77893	72	.80586	76	58
59	.73549	75	.75978	80	.77965	71	.80662	76	59
60	8.73625		8.76058		8.78037		8.80738		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	84	83	82	81	80	79	78	77	76	75	74	73	72	71	0
6	8.4	8.3	8.2	8.1	8.0	7.9	7.8	7.7	7.6	7.5	7.4	7.3	7.2	7.1	0.0
7	9.8	9.7	9.5	9.4	9.3	9.2	9.1	9.0	8.8	8.7	8.6	8.5	8.4	8.3	0.0
8	11.2	11.0	10.9	10.8	10.6	10.5	10.4	10.2	10.1	10.0	9.8	9.7	9.6	9.4	0.0
9	12.6	12.4	12.3	12.1	12.0	11.8	11.7	11.5	11.4	11.2	11.1	10.9	10.8	10.6	0.1
10	14.0	13.8	13.6	13.5	13.3	13.1	13.0	12.8	12.6	12.5	12.3	12.1	12.0	11.8	0.1
20	28.0	27.6	27.3	27.0	26.6	26.3	26.0	25.6	25.3	25.0	24.6	24.3	24.0	23.6	0.1
30	42.0	41.5	41.0	40.5	40.0	39.5	39.0	38.5	38.0	37.5	37.0	36.5	36.0	35.5	0.2
40	56.0	55.3	54.6	54.0	53.3	52.6	52.0	51.3	50.6	50.0	49.3	48.6	48.0	47.3	0.3
50	70.0	69.1	68.3	67.5	66.6	65.3	65.0	64.1	63.3	62.5	61.6	60.8	60.0	59.1	0.4

P. P.

	20°				21°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	8.78037	71̂	8.80738̂	76	8.82229̂	68	8.85214̂	73	0
1	.78108̂	71̂	.80814̂	76̂	.82297̂	68̂	.85287̂	73	1
2	.78180	71̂	.80891	76	.82366	68	.85360̂	72̂	2
3	.78251̂	71̂	.80967	76	.82434	68	.85433	73	3
4	.78323	71̂	.81043	76	.82502	67̂	.85506	73	4
5	8.78394̂	71̂	8.81119	76	8.82569̂	68	8.85579	72̂	5
6	.78466	71	.81195	76	.82637̂	68	.85651̂	73	6
7	.78537	71̂	.81271	75̂	.82705̂	67̂	.85724̂	72̂	7
8	.78608̂	71	.81346̂	76	.82773	68	.85797	72̂	8
9	.78679̂	71	.81422̂	75̂	.82841	67̂	.85869̂	72̂	9
10	8.78750̂	71	8.81498	75̂	8.82908̂	67̂	8.85942	72̂	10
11	.78821̂	71	.81573̂	76	.82976	67̂	.86014̂	72̂	11
12	.78892̂	71	.81649̂	75̂	.83043̂	67̂	.86087	72̂	12
13	.78963̂	70̂	.81725	75̂	.83111	67̂	.86159̂	72	13
14	.79034	71	.81800̂	75̂	.83178̂	67̂	.86231̂	72̂	14
15	8.79105	70̂	8.81876	75	8.83246	67	8.86304	72	15
16	.79175̂	71	.81951	75̂	.83313	67̂	.86376	72̂	16
17	.79246̂	70̂	.82026̂	75̂	.83380̂	67	.86448̂	72	17
18	.79317	70̂	.82102	75	.83447̂	67̂	.86520̂	72	18
19	.79387̂	70̂	.82177	75	.83515	67	.86592̂	72	19
20	8.79458	70̂	8.82252	75	8.83582	67	8.86664̂	72	20
21	.79528̂	70	.82327	75̂	.83649	67	.86736̂	72	21
22	.79598̂	70̂	.82402̂	75	.83716	67	.86808̂	72	22
23	.79669	70	.82477̂	74̂	.83783	67	.86880̂	71̂	23
24	.79739	70̂	.82552	75	.83850	66̂	.86952	72	24
25	8.79809̂	70	8.82627	75	8.83916̂	67	8.87024	71̂	25
26	.79879̂	70	.82702	74̂	.83983̂	66̂	.87095̂	72	26
27	.79949̂	70	.82776̂	75	.84050	67	.87167̂	71̂	27
28	.80019̂	70	.82851̂	74̂	.84117	66̂	.87239	71̂	28
29	.80089̂	70	.82926	74̂	.84183̂	66̂	.87310̂	71̂	29
30	8.80159̂	70	8.83000̂	74̂	8.84250	66̂	8.87382	71̂	30
31	.80229̂	69̂	.83075	74̂	.84316̂	66̂	.87453̂	71̂	31
32	.80299	70	.83149̂	74̂	.84383	66̂	.87525	71̂	32
33	.80369	69̂	.83224	74̂	.84449̂	66	.87596̂	71̂	33
34	.80438̂	69̂	.83298̂	74̂	.84515̂	66̂	.87668	71	34
35	8.80508	69̂	8.83373	74	8.84582	66̂	8.87739	71̂	35
36	.80577̂	69̂	.83447	74̂	.84648̂	66	.87810̂	71	36
37	.80647	69̂	.83521̂	74	.84714̂	66	.87881̂	71̂	37
38	.80716̂	69̂	.83595̂	74̂	.84780̂	66	.87953	71	38
39	.80786	69	.83670	74	.84846̂	66	.88024	71	39
40	8.80855	69̂	8.83744	74	8.84912̂	66	8.88095	71	40
41	.80924̂	69	.83818	74	.84978̂	66	.88166	71	41
42	.80993̂	69̂	.83892	74	.85044̂	66	.88237	71	42
43	.81063	69	.83966	73̂	.85110̂	66	.88308	70̂	43
44	.81132	69	.84039̂	74	.85176̂	65	.88378̂	71	44
45	8.81201	69	8.84113̂	73̂	8.85242	66	8.88449̂	71	45
46	.81270	69	.84187	74	.85308	65̂	.88520̂	70̂	46
47	.81339	68̂	.84261	73̂	.85373̂	65̂	.88591	70̂	47
48	.81407̂	69	.84334̂	73̂	.85439	66	.88661̂	71	48
49	.81476̂	68̂	.84408	73̂	.85505	65̂	.88732̂	70̂	49
50	8.81545	69	8.84481̂	73̂	8.85570̂	65̂	8.88803	70̂	50
51	.81614	68̂	.84555	73̂	.85626	65	.88873̂	70̂	51
52	.81682̂	68̂	.84628̂	73̂	.85701	65̂	.88944	70̂	52
53	.81751	68̂	.84702	73̂	.85766̂	65̂	.89014̂	70̂	53
54	.81819̂	68̂	.84775̂	73	.85832	65	.89085	70	54
55	8.81888	68̂	8.84848̂	73̂	8.85897	65̂	8.89155	70̂	55
56	.81956̂	68̂	.84922	73	.85962̂	65	.89225̂	70	56
57	.82025	68	.84995	73̂	.86027̂	65	.89295̂	70̂	57
58	.82093	68̂	.85068̂	73	.86092̂	65̂	.89366	70	58
59	.82161̂	68	.85141̂	73	.86158	65	.89436	70	59
60	8.82229̂		8.85214̂		8.86223		8.89506		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	76	75	74
6	7.6	7.5	7.4
7	8.8̂	8.7̂	8.6̂
8	10.1̂	10.0	9.8̂
9	11.4	11.2̂	11.1
10	12.6̂	12.5	12.3̂
20	25.3̂	25.0	24.6̂
30	38.0	37.5	37.0
40	50.6̂	50.0	49.3̂
50	63.3̂	62.5	61.6̂

	73	72	71
6	7.3	7.2	7.1
7	8.5	8.4	8.3
8	9.7̂	9.6	9.4̂
9	10.9̂	10.8	10.6̂
10	12.1̂	12.0	11.8̂
20	24.3̂	24.0	23.6̂
30	36.5	36.0	35.5
40	48.6̂	48.0	47.3̂
50	60.8̂	60.0	59.1̂

	70	69	68
6	7.0	6.9	6.8
7	8.1̂	8.0̂	7.9̂
8	9.3̂	9.2	9.0̂
9	10.5	10.3̂	10.2
10	11.6̂	11.5	11.3̂
20	23.3̂	23.0	22.6̂
30	35.0	34.5	34.0
40	46.6̂	46.0	45.3̂
50	58.3̂	57.5	56.6̂

	67	66	65
6	6.7	6.6	6.5
7	7.8	7.7	7.6
8	8.9̂	8.8	8.6̂
9	10.0̂	9.9	9.7̂
10	11.1̂	11.0	10.8̂
20	22.3̂	22.0	21.6̂
30	33.5	33.0	32.5
40	44.6̂	44.0	43.3̂
50	55.8̂	55.0	54.1̂

	0̂
6	0.0̂
7	0.0̂
8	0.0̂
9	0 1
10	0.1
20	0.1̂
30	0.2̂
40	0.3̂
50	0.4

P. P.

	22°				23°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	8.86223	64	8.89506	70	8.90034	62	8.93631	67	0
1	.86287	65	.89576	70	.90096	62	.93699	67	1
2	.86352	65	.89646	70	.90158	62	.93766	67	2
3	.86417	65	.89716	69	.90220	62	.93833	67	3
4	.86482	64	.89786	70	.90282	62	.93901	67	4
5	8.86547	65	8.89856	70	8.90344	62	8.93968	67	5
6	.86612	64	.89926	69	.90406	61	.94035	67	6
7	.86676	64	.89995	70	.90467	62	.94102	67	7
8	.86741	64	.90065	69	.90529	61	.94170	67	8
9	.86805	64	.90135	70	.90591	61	.94237	67	9
10	8.86870	64	8.90205	69	8.90652	62	8.94304	67	10
11	.86934	64	.90274	69	.90714	61	.94371	67	11
12	.86999	64	.90344	69	.90776	61	.94438	67	12
13	.87063	64	.90413	69	.90837	61	.94505	67	13
14	.87127	64	.90483	69	.90899	61	.94572	66	14
15	8.87192	64	8.90552	69	8.90960	61	8.94638	67	15
16	.87256	64	.90622	69	.91021	61	.94705	66	16
17	.87320	64	.90691	69	.91083	61	.94772	67	17
18	.87384	64	.90760	69	.91144	61	.94839	66	18
19	.87448	64	.90830	69	.91205	61	.94905	66	19
20	8.87512	64	8.90899	69	8.91267	61	8.94972	67	20
21	.87576	64	.90968	69	.91328	61	.95039	66	21
22	.87640	64	.91037	69	.91389	61	.95105	66	22
23	.87704	63	.91106	69	.91450	61	.95172	66	23
24	.87768	64	.91175	69	.91511	61	.95238	66	24
25	8.87832	63	8.91244	69	8.91572	61	8.95305	66	25
26	.87895	64	.91313	68	.91633	61	.95371	66	26
27	.87959	63	.91382	69	.91694	61	.95437	66	27
28	.88023	63	.91451	69	.91755	60	.95504	66	28
29	.88086	63	.91520	68	.91815	61	.95570	66	29
30	8.88150	63	8.91588	69	8.91876	60	8.95636	66	30
31	.88213	63	.91657	68	.91937	60	.95703	66	31
32	.88277	63	.91726	68	.91997	61	.95769	66	32
33	.88340	63	.91794	68	.92058	60	.95835	66	33
34	.88404	63	.91863	69	.92119	60	.95901	66	34
35	8.88467	63	8.91932	68	8.92179	60	8.95967	66	35
36	.88530	63	.92000	68	.92240	60	.96033	66	36
37	.88593	63	.92068	68	.92300	60	.96099	66	37
38	.88656	63	.92137	68	.92361	60	.96165	66	38
39	.88720	63	.92205	68	.92421	60	.96231	66	39
40	8.88783	63	8.92274	68	8.92487	60	8.96297	65	40
41	.88846	63	.92342	68	.92542	60	.96362	66	41
42	.88909	62	.92410	68	.92602	60	.96428	65	42
43	.88971	63	.92478	68	.92662	60	.96494	66	43
44	.89034	63	.92546	68	.92722	60	.96560	65	44
45	8.89097	62	8.92615	68	8.92782	60	8.96625	65	45
46	.89160	63	.92683	68	.92842	60	.96691	66	46
47	.89223	62	.92751	68	.92902	60	.96757	65	47
48	.89285	62	.92819	68	.92962	60	.96822	65	48
49	.89348	63	.92887	68	.93022	60	.96888	65	49
50	8.89411	62	8.92955	67	8.93082	59	8.96953	65	50
51	.89473	62	.93022	68	.93142	60	.97018	65	51
52	.89536	62	.93090	67	.93202	59	.97084	65	52
53	.89598	62	.93158	68	.93261	60	.97149	65	53
54	.89660	62	.93226	67	.93321	59	.97214	65	54
55	8.89723	63	8.93293	68	8.93381	59	8.97280	65	55
56	.89785	62	.93361	67	.93440	60	.97345	65	56
57	.89847	62	.93429	67	.93500	59	.97410	65	57
58	.89910	62	.93496	67	.93560	59	.97475	65	58
59	.89972	62	.93564	67	.93619	59	.97540	65	59
60	8.90034		8.93631		8.93679		8.97606		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	70	69	68
6	7.0	6.9	6.8
7	8.1	8.0	7.9
8	9.3	9.2	9.0
9	10.5	10.3	10.2
10	11.6	11.5	11.3
20	23.3	23.0	22.6
30	35.0	34.5	34.0
40	46.6	46.0	45.3
50	58.3	57.5	56.6

	67	66	65
6	6.7	6.6	6.5
7	7.8	7.7	7.6
8	8.9	8.8	8.6
9	10.0	9.9	9.7
10	11.1	11.0	10.8
20	22.3	22.0	21.6
30	33.5	33.0	32.5
40	44.6	44.0	43.3
50	55.8	55.0	54.1

	64	63	62
6	6.4	6.3	6.2
7	7.4	7.3	7.2
8	8.5	8.4	8.2
9	9.6	9.4	9.3
10	10.6	10.5	10.3
20	21.3	21.0	20.6
30	32.0	31.5	31.0
40	42.6	42.0	41.3
50	53.3	52.5	51.6

	61	60	59
6	6.1	6.0	5.9
7	7.1	7.0	6.9
8	8.1	8.0	7.8
9	9.1	9.0	8.8
10	10.1	10.0	9.8
20	20.3	20.0	19.6
30	30.5	30.0	29.5
40	40.6	40.0	39.3
50	50.8	50.0	49.1

	0
6	0.0
7	0.0
8	0.0
9	0.1
10	0.1
20	0.1
30	0.2
40	0.3
50	0.4

P. P.

	24°				25°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	8.93679	59	8.97606	65	8.97170̂	57	9.01443	62̂	0
1	.93738	59̂	.97671	65	.97227̂	56̂	.01505̂	63	1
2	.93797̂	59̂	.97736	65	.97284	57	.01568̂	62̂	2
3	.93857	59	.97801	64̂	.97341	57	.01631	63	3
4	.93916	59̂	.97865̂	65	.97398	57	.01694	62̂	4
5	8.93975̂	59	8.97930̂	65	8.97455	56̂	9.01756̂	63	5
6	.94034̂	59̂	.97995̂	64̂	.97511̂	57	.01819̂	62̂	6
7	.94094	59	.98060	65	.97568̂	56̂	.01882	62̂	7
8	.94153	59	.98125	65	.97625	56̂	.01944̂	63	8
9	.94212	59	.98190	64̂	.97681̂	56̂	.02007̂	62̂	9
10	8.94271	59	8.98254̂	64̂	8.97738	57	9.02070	62̂	10
11	.94330	59	.98319	64̂	.97795	56̂	.02132̂	62̂	11
12	.94389	59	.98383̂	65	.97851̂	56̂	.02195	62	12
13	.94448	58̂	.98448̂	64̂	.97908	56̂	.02257	62̂	13
14	.94506̂	59	.98513	64̂	.97964̂	56	.02319̂	62̂	14
15	8.94565̂	59	8.98577̂	64̂	8.98020̂	56̂	9.02382	62̂	15
16	.94624̂	58̂	.98642	64̂	.98077	56̂	.02444̂	62	16
17	.94683	59	.98706̂	64	.98133̂	56̂	.02506̂	62̂	17
18	.94742	58̂	.98770̂	64̂	.98190	56	.02569	62	18
19	.94800̂	58̂	.98835	64̂	.98246	56̂	.02631	62̂	19
20	8.94859	58̂	8.98899	64	8.98302̂	56	9.02693̂	62	20
21	.94917̂	58̂	.98963̂	64̂	.98358̂	56	.02755̂	62	21
22	.94976	58̂	.99028	64	.98414̂	56	.02817̂	62̂	22
23	.95034̂	58̂	.99092	64̂	.98470̂	56̂	.02880	62	23
24	.95093	58̂	.99156̂	64	.98527	56	.02942	62	24
25	8.95151̂	58̂	8.99220̂	64	8.98583	56	9.03004	62	25
26	.95210	58	.99284̂	64	.98639	56	.03066	62	26
27	.95268	58̂	.99348̂	64	.98695	55̂	.03128	62	27
28	.95326̂	58	.99412̂	64	.98750̂	56	.03190	62	28
29	.95384̂	58̂	.99476̂	64	.98806̂	56	.03252	61̂	29
30	8.95443	58	8.99540̂	64	8.98862̂	56	9.03313̂	62	30
31	.95501	58	.99604̂	64	.98918̂	55̂	.03375̂	62	31
32	.95559	58̂	.99668̂	64	.98974	56	.03437̂	61̂	32
33	.95617̂	58	.99732̂	63̂	.99030	55̂	.03499	62	33
34	.95675̂	58	.99796	64	.99085̂	55̂	.03561	61̂	34
35	8.95733̂	58	8.99860	63̂	8.99141	56	9.03622̂	61̂	35
36	.95791̂	57̂	.99923̂	64	.99197	55̂	.03684	62	36
37	.95849	58	8.99987̂	63̂	.99252̂	55̂	.03746	61̂	37
38	.95907	58	9.00051	63̂	.99308	55̂	.03807̂	61̂	38
39	.95965	58	.00114̂	64	.99363̂	55̂	.03869	61̂	39
40	8.96023	57̂	9.00178̂	63̂	8.99419	55̂	9.03930̂	61̂	40
41	.96080̂	57̂	.00242	63̂	.99474̂	55	.03992	61̂	41
42	.96138	58	.00305̂	63̂	.99529̂	55̂	.04053̂	61̂	42
43	.96196	57̂	.00369	63̂	.99585	55̂	.04115	61̂	43
44	.96253̂	57̂	.00432̂	63	.99640̂	55	.04176̂	61̂	44
45	8.96311	57̂	9.00495̂	63̂	8.99695̂	55̂	9.04238	61	45
46	.96368̂	57̂	.00559	63̂	.99751	55	.04299	61̂	46
47	.96426	57̂	.00622̂	63̂	.99806	55̂	.04360̂	61̂	47
48	.96483̂	57̂	.00686	63	.99861̂	55	.04421̂	61̂	48
49	.96541	57̂	.00749	63̂	.99916̂	55	.04483	61	49
50	8.96598̂	57̂	9.00812̂	63	8.99971̂	55	9.04544	61̂	50
51	.96656	57	.00875̂	63	9.00026̂	55	.04605̂	61	51
52	.96713	57̂	.00938̂	63̂	.00081̂	55	.04666̂	61	52
53	.96770̂	57	.01002	63	.00136̂	55	.04727̂	61	53
54	.96827̂	57̂	.01065	63	.00191̂	55	.04788̂	61̂	54
55	8.96885	57	9.01128	63	9.00246̂	55	9.04850	61	55
56	.96942	57	.01191	63	.00301̂	55	.04911	61	56
57	.96999	57̂	.01254	63	.00356̂	54̂	.04972	61	57
58	.97056̂	57	.01317	63	.00411	55	.05033	60̂	58
59	.97113̂	57	.01380	63	.00466	54̂	.05093̂	61	59
60	8.97170̂		9.01443		9.00520̂		9.05154̂		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	65	64	63
6	6.5	6.4	6.3
7	7.6	7.4̂	7.3̂
8	8.6̂	8.5̂	8.4
9	9.7̂	9.6	9.4̂
10	10.8̂	10.6̂	10.5
20	21.6̂	21.3̂	21.0
30	32.5	32.0	31.5
40	43.3̂	42.6̂	42.0
50	54.1̂	53.3̂	52.5

	62	61	60
6	6.2	6.1	6.0
7	7.2̂	7.1	7.0
8	8.2̂	8.1̂	8.0
9	9.3	9.1̂	9.0
10	10.3̂	10.1̂	10.0
20	20.6̂	20.3̂	20.0
30	31.0	30.5	30.0
40	41.3̂	40.6̂	40.0
50	51.6̂	50.8̂	50.0

	59	58	57
6	5.9	5.8	5.7
7	6.9	6.7̂	6.6̂
8	7.8̂	7.7̂	7.6
9	8.8̂	8.7	8.5̂
10	9.8̂	9.6̂	9.5
20	19.6	19.3̂	19.0
30	29.5	29.0	28.5
40	39.3̂	38.6̂	38.0
50	49.1̂	48.3̂	47.5

	56	55	54
6	5.6	5.5	5.4
7	6.5̂	6.4	6.3
8	7.4̂	7.3̂	7.2
9	8.4	8.2̂	8.1
10	9.3̂	9.1̂	9.0
20	18.6̂	18.3̂	18.0
30	28.0	27.5	27.0
40	37.3̂	36.6̂	36.0
50	46.6̂	45.8̂	45.0

	0̂
6	0.0̂
7	0.0̂
8	0.0̂
9	0.1
10	0.1
20	0.1̂
30	0.2̂
40	0.3̂
50	0.4

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

′	26° Log. Vers.	D	26° Log. Exsec.	D	27° Log. Vers.	D	27° Log. Exsec.	D	′
0	9.00520	55	9.05154̂	61	9.03740	52̂	9.08752	59	0
1	.00575̂	54̂	.05215̂	61	.03792̂	52̂	.08811	59	1
2	.00630	54̂	.05276̂	60̂	.03845	53	.08870	59	2
3	.00684̂	54̂	.05337	61	.03898	52̂	.08929	59	3
4	.00739	55	.05398	60̂	.03950̂	52	.08988	59	4
5	9.00794	54̂	9.05458̂	61	9.04002̂	52̂	9.09047	59	5
6	.00848̂	54̂	.05519̂	60̂	.04055	52̂	.09106	58	6
7	.00903	54	.05580	60̂	.04107̂	52̂	.09164̂	59	7
8	.00957	54̂	.05640̂	60̂	.04160	52̂	.09223̂	59	8
9	.01011̂	54̂	.05701	61	.04212̂	52	.09282̂	58̂	9
10	9.01066	54̂	9.05762	60̂	9.04264̂	52̂	9.09341	59	10
11	.01120̂	54	.05822̂	60̂	.04317	52	.09400	58̂	11
12	.01174̂	54̂	.05883	60̂	.04369	52̂	.09458̂	59	12
13	.01229	54	.05943̂	60̂	.04421̂	52	.09517̂	58̂	13
14	.01283	54̂	.06004	60̂	.04473̂	52	.09576	58̂	14
15	9.01337	54	9.06064̂	60	9.04525̂	52	9.09634̂	58	15
16	.01391̂	54	.06124̂	60̂	.04577̂	52̂	.09693	59	16
17	.01445̂	54	.06185	60̂	.04630	52	.09752	58̂	17
18	.01499̂	54̂	.06245̂	60	.04682	52	.09810̂	58	18
19	.01554	54	.06305̂	60̂	.04734	52	.09869	58̂	19
20	9.01608	54	9.06366	60	9.04786	51̂	9.09927̂	58̂	20
21	.01662	53̂	.06426	60̂	.04837̂	52	.09986	58̂	21
22	.01715̂	54	.06486̂	60	.04889̂	52	.10044̂	58̂	22
23	.01769̂	54	.06546̂	60	.04941̂	52	.10102̂	58	23
24	.01823̂	54	.06606̂	60̂	.04993̂	51̂	.10161	58̂	24
25	9.01877	53̂	9.06667	60	9.05045	52	9.10219̂	58̂	25
26	.01931	54	.06727	60	.05097	51̂	.10278	58̂	26
27	.01985	53̂	.06787	60	.05148̂	52	.10336	58̂	27
28	.02038̂	54	.06847	60	.05200̂	52	.10394̂	58̂	28
29	.02092̂	53̂	.06907	60	.05252	51̂	.10452̂	58̂	29
30	9.02146	53̂	9.06967	60	9.05303̂	52	9.10511	58̂	30
31	.02199̂	54	.07027	60	.05355̂	51̂	.10569	58	31
32	.02253̂	53̂	.07087	59̂	.05407	51̂	.10627	58̂	32
33	.02307	53̂	.07146̂	60	.05458̂	51̂	.10685̂	58̂	33
34	.02360̂	53̂	.07206̂	60	.05510	51̂	.10743̂	58	34
35	9.02414	53̂	9.07266̂	59̂	9.05561̂	51̂	9.10801̂	58	35
36	.02467̂	53̂	.07326	60	.05613	51̂	.10859̂	58	36
37	.02521	53	.07386	59̂	.05664̂	51	.10917̂	58	37
38	.02574	53̂	.07445̂	60	.05715̂	51̂	.10975̂	58	38
39	.02627̂	53̂	.07505̂	59̂	.05767	51̂	.11033̂	58	39
40	9.02681	53	9.07565	59̂	9.05818̂	51	9.11091̂	58	40
41	.02734	53̂	.07624̂	59̂	.05869̂	51̂	.11149̂	57̂	41
42	.02787̂	53	.07684	59̂	.05921	51	.11207	58	42
43	.02840̂	53̂	.07743̂	60	.05972	51̂	.11265	58	43
44	.02894	53	.07803̂	59̂	.06023̂	51	.11323	57̂	44
45	9.02947	53	9.07863	59̂	9.06074̂	51	9.11380̂	58	45
46	.03000	53	.07922̂	59	.06125̂	51	.11438̂	58	46
47	.03053	53	.07981̂	59̂	.06176̂	51	.11496̂	57̂	47
48	.03106	53	.08041	59̂	.06227̂	51̂	.11554	57̂	48
49	.03159	53	.08100̂	59̂	.06279	51	.11611̂	58	49
50	9.03212	53	9.08160	59̂	9.06330	50̂	9.11669̂	57̂	50
51	.03265	53	.08219̂	59	.06380̂	51	.11727	57̂	51
52	.03318	53	.08278̂	59̂	.06431̂	51	.11784̂	57̂	52
53	.03371	52̂	.08338	59	.06482̂	51	.11842	57̂	53
54	.03423̂	53	.08397	59̂	.06533̂	51	.11899̂	58	54
55	9.03476̂	53	9.08456̂	59	9.06584̂	50̂	9.11957̂	57̂	55
56	.03529̂	52̂	.08515̂	59	.06635	51	.12015	57̂	56
57	.03582	52̂	.08574̂	59̂	.06686	50̂	.12072	57̂	57
58	.03634̂	53	.08634	59	.06736̂	51	.12129̂	57̂	58
59	.03687̂	52̂	.08693	59	.06787̂	50̂	.12187	57̂	59
60	9.03740		9.08752		9.06838		9.12244̂		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	61	60	59
6	6.1	6.0	5.9
7	7.1	7.0	6.9
8	8.1	8.0	7.8
9	9.1	9.0	8.8
10	10.1	10.0	9.8
20	20.3	20.0	19.6
30	30.5	30.0	29.5
40	40.6	40.0	39.3
50	50.8	50.0	49.1

	58	57
6	5.8	5.7
7	6.7	6.6
8	7.7	7.6
9	8.7	8.5
10	9.6	9.5
20	19.3	19.0
30	29.0	28.5
40	38.6	38.0
50	48.3	47.5

	55	54
6	5.5	5.4
7	6.4	6.3
8	7.3	7.2
9	8.2	8.1
10	9.1	9.0
20	18.3	18.0
30	27.5	27.0
40	36.6	36.0
50	45.8	45.0

	53	52
6	5.3	5.2
7	6.2	6.0
8	7.0	6.9
9	7.9	7.8
10	8.8	8.6
20	17.6	17.3
30	26.5	26.0
40	35.3	34.6
50	44.1	43.3

	51	0̂
6	5.1	0.0
7	5.9	0.0
8	6.8	0.0
9	7.6	0.1
10	8.5	0.1
20	17.0	0.1
30	25.5	0.2
40	34.0	0.3
50	42.5	0.4

P. P.

28° 29°

′	Log. Vers. (28°)	D	Log. Exsec. (28°)	D	Log. Vers. (29°)	D	Log. Exsec. (29°)	D	′
0	9.06838	50	9.12244	57	9.09823	49	9.15641	56	0
1	.06888	51	.12302	57	.09872	48	.15697	55	1
2	.06939	50	.12359	57	.09920	49	.15752	56	2
3	.06990	50	.12416	57	.09969	48	.15808	55	3
4	.07040	50	.12474	57	.10018	49	.15864	56	4
5	9.07091	50	9.12531	57	9.10067	48	9.15920	55	5
6	.07141	50	.12588	57	.10115	48	.15975	56	6
7	.07192	50	.12645	57	.10164	49	.16031	55	7
8	.07242	50	.12703	57	.10213	48	.16087	55	8
9	.07293	50	.12760	57	.10261	48	.16142	56	9
10	9.07343	50	9.12817	57	9.10310	48	9.16198	55	10
11	.97393	50	.12874	57	.10358	48	.16254	55	11
12	.07444	50	.12931	57	.10407	48	.16309	55	12
13	.07494	50	.12988	57	.10455	48	.16365	55	13
14	.07544	50	.13045	57	.10504	48	.16420	55	14
15	9.07594	50	9.13102	57	9.10552	48	9.16476	55	15
16	.07644	50	.13159	57	.10601	48	.16531	55	16
17	.07695	50	.13216	56	.10649	48	.16587	55	17
18	.07745	50	.13273	57	.10697	48	.16642	55	18
19	.07795	50	.13330	57	.10746	48	.16698	55	19
20	9.07845	50	9.13387	57	9.10794	48	9.16753	55	20
21	.07895	50	.13444	56	.10842	48	.16808	55	21
22	.07945	50	.13500	57	.10890	48	.16864	55	22
23	.07995	50	.13557	56	.10939	48	.16919	55	23
24	.08045	50	.13614	57	.10987	48	.16974	55	24
25	9.08095	50	9.13671	56	9.11035	48	9.17029	55	25
26	.08145	50	.13727	57	.11083	48	.17085	55	26
27	.08195	49	.13784	56	.11131	48	.17140	55	27
28	.08244	50	.13841	56	.11179	48	.17195	55	28
29	.08294	49	.13897	57	.11227	48	.17250	55	29
30	9.08344	50	9.13954	56	9.11275	48	9.17305	55	30
31	.08394	49	.14011	56	.11323	48	.17361	55	31
32	.08443	49	.14067	56	.11371	48	.17416	55	32
33	.08493	50	.14124	56	.11419	47	.17471	55	33
34	.08543	49	.14180	56	.11467	48	.17526	55	34
35	9.08592	49	9.14237	56	9.11515	48	9.17581	55	35
36	.08642	49	.14293	56	.11562	47	.17636	55	36
37	.08691	49	.14350	56	.11610	48	.17691	55	37
38	.08741	49	.14406	56	.11658	48	.17746	55	38
39	.08790	49	.14462	56	.11706	47	.17801	55	39
40	9.08840	49	9.14519	56	9.11754	48	9.17856	54	40
41	.08889	49	.14575	56	.11801	47	.17910	55	41
42	.08939	49	.14631	56	.11849	47	.17965	55	42
43	.08988	49	.14688	56	.11897	48	.18020	54	43
44	.09037	49	.14744	56	.11944	47	.18075	55	44
45	9.09087	49	9.14800	56	9.11992	47	9.18130	55	45
46	.09136	49	.14856	56	.12039	47	.18185	54	46
47	.09185	49	.14913	56	.12087	47	.18239	55	47
48	.09234	49	.14969	56	.12134	47	.18294	54	48
49	.09284	49	.15025	56	.12182	47	.18349	54	49
50	9.09333	49	9.15081	56	9.12229	47	9.18403	55	50
51	.09382	49	.15137	56	.12277	47	.18458	54	51
52	.09431	49	.15193	56	.12324	47	.18513	54	52
53	.09480	49	.15249	56	.12371	47	.18567	54	53
54	.09529	49	.15305	56	.12419	47	.18622	54	54
55	9.09578	49	9.15361	56	9.12466	47	9.18676	54	55
56	.09627	49	.15417	56	.12513	47	.18731	55	56
57	.09676	48	.15473	56	.12560	47	.18786	54	57
58	.09725	49	.15529	55	.12608	47	.18840	54	58
59	.09774	49	.15585	56	.12655	47	.18894	54	59
60	9.09823		9.15641		9.12702		9.18949		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	57̂	57	56̂
6	5.7	5.7	5.6
7	6.7	6.6	6.6
8	7.6	7.6	7.5
9	8.6	8.5	8.5
10	9.6	9.5	9.4
20	19.1	19.0	18.8
30	28.7	28.5	28.2
40	38.3	38.0	37.6
50	47.9	47.5	47.1

	56	55̂	55
6	5.6	5.5	5.5
7	6.5	6.5	6.4
8	7.4	7.4	7.3
9	8.4	8.3	8.2
10	9.3	9.2	9.1
20	18.6	18.5	18.3
30	28.0	27.7	27.5
40	37.3	37.0	36.6
50	46.6	46.2	45.8

	54̂	54
6	5.4	5.4
7	6.3	6.3
8	7.2	7.2
9	8.2	8.1
10	9.1	9.0
20	18.1	18.0
30	27.2	27.0
40	36.3	36.0
50	45.4	45.0

	51	50̂	50
6	5.1	5.0	5.0
7	5.9	5.9	5.8
8	6.8	6.7	6.6
9	7.6	7.6	7.5
10	8.5	8.4	8.3
20	17.0	16.8	16.6
30	25.5	25.2	25.0
40	34.0	33.6	33.3
50	42.5	42.1	41.6

	49̂	49	48̂
6	4.9	4.9	4.8
7	5.8	5.7	5.6
8	6.6	6.5	6.4
9	7.4	7.3	7.3
10	8.2	8.1	8.1
20	16.5	16.3	16.1
30	24.7	24.5	24.2
40	33.0	32.6	32.3
50	41.2	40.8	40.4

	48	47̂	47
6	4.8	4.7	4.7
7	5.6	5.5	5.5
8	6.4	6.3	6.2
9	7.2	7.1	7.0
10	8.0	7.9	7.8
20	16.0	15.8	15.6
30	24.0	23.7	23.5
40	32.0	31.6	31.3
50	40.0	39.6	39.1

P. P.

′	30° Log. Vers.	D	30° Log. Exsec.	D	31° Log. Vers.	D	31° Log. Exsec.	D	′
0	9.12702	47	9.18949	54	9.15483	45	9.22176	53	0
1	.12749	47	.19003	54	.15528	45	.22229	53	1
2	.12796	47	.19058	54	.15574	45	.22282	53	2
3	.12843	47	.19112	54	.15619	45	.22335	53	3
4	.12890	47	.19167	54	.15665	45	.22388	53	4
5	9.12937	47	9.19221	54	9.15710	45	9.22441	53	5
6	.12984	47	.19275	54	.15755	45	.22494	53	6
7	.13031	47	.19329	54	.15801	45	.22547	53	7
8	.13078	47	.19384	54	.15846	45	.22600	53	8
9	.13125	47	.19438	54	.15891	45	.22653	53	9
10	9.13172	46	9.19492	54	9.15937	45	9.22706	53	10
11	.13219	47	.19546	54	.15982	45	.22759	53	11
12	.13266	47	.19601	54	.16027	45	.22812	53	12
13	.13313	46	.19655	54	.16073	45	.22865	52	13
14	.13359	47	.19709	54	.16118	45	.22918	53	14
15	9.13406	46	9.19763	54	9.16163	45	9.22971	53	15
16	.13453	47	.19817	54	.16208	45	.23024	52	16
17	.13500	46	.19871	54	.16253	45	.23076	53	17
18	.13546	46	.19925	54	.16298	45	.23129	52	18
19	.13593	46	.19979	54	.16343	45	.23182	53	19
20	9.13639	47	9.20033	54	9.16388	45	9.23235	52	20
21	.13686	46	.20087	54	.16434	45	.23287	53	21
22	.13733	46	.20141	54	.16479	44	.23340	52	22
23	.13779	46	.20195	54	.16523	45	.23393	53	23
24	.13826	46	.20249	53	.16568	45	.23446	52	24
25	9.13872	46	9.20303	54	9.16613	45	9.23498	52	25
26	.13919	46	.20357	54	.16658	45	.23551	52	26
27	.13965	46	.20411	54	.16703	45	.23603	52	27
28	.14011	46	.20465	53	.16748	44	.23656	53	28
29	.14058	46	.20518	54	.16793	45	.23709	52	29
30	9.14104	46	9.20572	53	9.16838	44	9.23761	52	30
31	.14151	46	.20626	54	.16882	45	.23814	52	31
32	.14197	46	.20680	53	.16927	44	.23866	52	32
33	.14243	46	.20733	54	.16972	45	.23919	52	33
34	.14289	46	.20787	53	.17017	44	.23971	53	34
35	9.14336	46	9.20841	53	9.17061	44	9.24024	52	35
36	.14382	46	.20894	54	.17106	45	.24076	52	36
37	.14428	46	.20948	53	.17151	44	.24128	52	37
38	.14474	46	.21002	52	.17195	44	.24181	52	38
39	.14520	46	.21055	53	.17240	44	.24233	52	39
40	9.14566	46	9.21109	53	9.17284	44	9.24285	52	40
41	.14612	46	.21162	53	.17329	44	.24338	52	41
42	.14658	46	.21216	53	.17373	44	.24390	52	42
43	.14704	46	.21269	53	.17418	44	.24442	52	43
44	.14750	46	.21323	53	.17462	44	.24495	52	44
45	9.14796	46	9.21376	53	9.17507	44	9.24547	52	45
46	.14842	46	.21430	53	.17551	44	.24599	52	46
47	.14888	46	.21483	53	.17596	44	.24651	52	47
48	.14934	45	.21537	53	.17640	44	.24704	52	48
49	.14980	46	.21590	53	.17684	44	.24756	52	49
50	9.15026	45	9.21643	53	9.17729	44	9.24808	52	50
51	.15071	46	.21697	53	.17773	44	.24860	52	51
52	.15117	46	.21750	53	.17817	44	.24912	52	52
53	.15163	45	.21803	53	.17861	44	.24964	52	53
54	.15209	45	.21857	53	.17906	44	.25016	52	54
55	9.15254	46	9.21910	53	9.17950	44	9.25068	52	55
56	.15300	45	.21963	53	.17994	44	.25120	52	56
57	.15346	45	.22016	53	.18038	44	.25172	52	57
58	.15391	45	.22070	53	.18082	44	.25224	52	58
59	.15437	46	.22123	53	.18126	44	.25276	52	59
60	9.15483		9.22176		9.18170		9.25328		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	54	54	53
6	5.4	5.4	5.3
7	6.3	6.3	6.2
8	7.2	7.2	7.1
9	8.2	8.1	8.0
10	9.1	9.0	[illegible]
20	18.1	18.0	[illegible]
30	27.2	27.0	26.7
40	36.3	36.0	35.6
50	45.4	45.0	44.6

	53	52	52
6	5.3	5.2	5.2
7	6.2	6.1	6.1
8	7.0	7.0	6.9
9	7.9	7.9	7.8
10	8.8	8.7	8.6
20	17.6	17.5	17.3
30	26.5	26.2	26.0
40	35.3	35.0	34.6
50	44.1	43.7	43.3

	47	47	46
6	4.7	4.7	4.6
7	5.5	5.5	5.4
8	6.3	6.2	6.2
9	7.1	7.0	7.0
10	7.9	7.8	7.7
20	15.8	15.6	15.5
30	23.7	23.5	23.2
40	31.6	31.3	31.0
50	39.6	39.1	38.7

	46	45	45
6	4.6	4.5	4.5
7	5.3	5.3	5.2
8	6.1	6.0	6.0
9	6.9	6.8	6.7
10	7.6	7.6	7.5
20	15.3	15.1	15.0
30	23.0	22.7	22.5
40	30.6	30.3	30.0
50	38.3	37.9	37.5

	44	44
6	4.4	4.4
7	5.2	5.1
8	5.9	5.8
9	6.7	6.6
10	7.4	7.3
20	14.8	14.6
30	22.2	22.0
40	29.6	29.3
50	37.1	36.6

	32°				33°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.18170	44	9.25328	52	9.20771	42	9.28412	51	0
1	.18214	44	.25380	52	.20814	42	.28463	51	1
2	.18258	44	.25432	52	.20856	42	.28514	50	2
3	.18302	44	.25484	51	.20899	43	.28564	51	3
4	.18346	44	.25536	52	.20942	42	.28615	51	4
5	9.18390	44	9.25588	52	9.20984	42	9.28666	50	5
6	.18434	44	.25640	52	.21027	42	.28717	51	6
7	.18478	44	.25692	51	.21069	42	.28768	50	7
8	.18522	43	.25743	52	.21112	42	.28818	50	8
9	.18566	44	.25795	51	.21154	42	.28869	51	9
10	9.18610	44	9.25847	52	9.21196	42	9.28920	50	10
11	.18654	43	.25899	51	.21239	42	.28970	50	11
12	.18697	44	.25950	52	.21281	42	.29021	51	12
13	.18741	43	.26002	51	.21324	42	.29072	50	13
14	.18785	44	.26054	51	.21366	42	.29122	51	14
15	9.18829	43	9.26105	52	9.21408	42	9.29173	50	15
16	.18872	43	.26157	51	.21451	42	.29223	50	16
17	.18916	43	.26209	51	.21493	42	.29274	50	17
18	.18959	44	.26260	51	.21535	42	.29324	51	18
19	.19003	43	.26312	52	.21577	42	.29375	50	19
20	9.19047	43	9.26364	51	9.21620	42	9.29426	50	20
21	.19090	43	.26415	51	.21662	42	.29476	50	21
22	.19134	43	.26467	51	.21704	42	.29527	50	22
23	.19177	43	.26518	51	.21746	42	.29577	50	23
24	.19221	43	.26570	51	.21788	42	.29627	50	24
25	9.19264	43	9.26621	51	9.21830	42	9.29678	50	25
26	.19308	43	.26673	51	.21872	42	.29728	50	26
27	.19351	43	.26724	51	.21914	42	.29779	50	27
28	.19395	43	.26776	51	.21956	42	.29829	50	28
29	.19438	43	.26827	51	.21998	42	.29879	50	29
30	9.19481	43	9.26878	51	9.22040	42	9.29930	50	30
31	.19525	43	.26930	51	.22082	42	.29980	50	31
32	.19568	43	.26981	51	.22124	42	.30030	50	32
33	.19611	43	.27032	51	.22166	42	.30081	50	33
34	.19654	43	.27084	51	.22208	42	.30131	50	34
35	9.19698	43	9.27135	51	9.22250	41	9.30181	50	35
36	.19741	43	.27186	51	.22292	42	.30231	50	36
37	.19784	43	.27238	51	.22334	42	.30282	50	37
38	.19827	43	.27289	51	.22376	41	.30332	50	38
39	.19870	43	.27340	51	.22417	42	.30382	50	39
40	9.19914	43	9.27391	51	9.22459	41	9.30432	50	40
41	.19957	43	.27443	51	.22501	42	.30482	50	41
42	.20000	43	.27494	51	.22543	41	.30533	50	42
43	.20043	43	.27545	51	.22584	41	.30583	50	43
44	.20086	43	.27596	51	.22626	42	.30633	50	44
45	9.20129	43	9.27647	51	9.22668	41	9.30683	50	45
46	.20172	43	.27698	51	.22709	41	.30733	50	46
47	.20215	43	.27749	51	.22751	41	.30783	50	47
48	.20258	43	.27800	51	.22792	41	.30833	50	48
49	.20301	42	.27852	51	.22834	41	.30883	50	49
50	9.20343	43	9.27903	51	9.22876	41	9.30933	50	50
51	.20386	43	.27954	51	.22917	41	.30983	50	51
52	.20429	43	.28005	51	.22959	41	.31033	50	52
53	.20472	42	.28056	51	.23000	41	.31083	50	53
54	.20515	43	.28107	50	.23042	41	.31133	49	54
55	9.20558	42	9.28157	51	9.23083	41	9.31183	50	55
56	.20600	43	.28208	51	.23124	41	.31233	50	56
57	.20643	43	.28259	51	.23166	41	.31283	50	57
58	.20686	42	.28310	51	.23207	41	.31333	50	58
59	.20728	43	.28361	50	.23248	41	.31383	49	59
60	9.20771		9.28412		9.23290		9.31432		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	52	51	5
6	5.2	5.1	5
7	6.0	6.0	5
8	6.9	6.8	6
9	7.8	7.7	7
10	8.6	8.6	8
20	17.3	17.1	17
30	26.0	25.7	25
40	34.6	34.3	34
50	43.3	42.9	42

	50	50	4
6	5.0	5.0	4
7	5.9	5.8	5
8	6.7	6.6	6
9	7.6	7.5	7
10	8.4	8.3	8
20	16.8	16.6	16
30	25.2	25.0	24
40	33.6	33.3	33
50	42.1	41.6	41

	44	43	4
6	4.4	4.3	4
7	5.1	5.1	5
8	5.8	5.8	5
9	6.6	6.5	6
10	7.3	7.2	7
20	14.6	14.5	1
30	22.0	21.7	2
40	29.3	29.0	2
50	36.6	36.2	3

	42	42	4
6	4.2	4.2	
7	4.9	4.9	
8	5.6	5.6	
9	6.4	6.3	
10	7.1	7.0	
20	14.1	14.0	1
30	21.2	21.0	2
40	28.3	28.0	2
50	35.4	35.0	3

	41
6	4.1
7	4.8
8	5.4
9	6.1
10	6.8
20	13.6
30	20.5
40	27.3
50	34.1

′	34° Log. Vers.	*D*	34° Log. Exsec.	*D*	35° Log. Vers.	*D*	35° Log. Exsec.	*D*	′
0	9.23290	41̂	9.31432̂	50	9.25731̂	40	9.34395	49	**0**
1	.23331̂	41	.31482̂	50	.25771̂	40	.34444	48̂	1
2	.23372̂	41̂	.31532̂	49̂	.25811̂	40	.34492̂	49	2
3	.23414	41	.31582	50	.25851̂	40	.34541̂	49	3
4	.23455	41̂	.31632	49̂	.25891̂	40	.34590̂	49	4
5	9.23496̂	41	9.31681̂	50	9.25931̂	40	9.34639̂	48̂	5
6	.23537̂	41̂	.31731̂	49̂	.25971̂	40	.34688	49	6
7	.23579	41	.31781	50	.26011̂	39̂	.34737	48̂	7
8	.23620	41	.31831	49̂	.26051	40	.34785̂	49	8
9	.23661	41	.31880̂	50	.26091	40	.34834̂	49	9
10	9.23702	41̂	9.31930̂	49̂	9.26131	40	9.34883̂	48̂	**10**
11	.23743̂	41	.31980	49̂	.26171	39̂	.34932	48̂	11
12	.23784̂	41	.32029̂	49̂	.26210̂	40	.34980̂	49	12
13	.23825̂	41	.32079	50	.26250̂	40	.35029̂	48̂	13
14	.23866̂	41	.32129	49̂	.26290̂	39̂	.35078	49	14
15	9.23907̂	41	9.32178̂	49̂	9.26330	40	9.35127	48̂	15
16	.23948̂	41	.32228	49̂	.26370	39̂	.35175̂	49	16
17	.23989̂	41	.32277̂	49̂	.26409̂	40	.35224̂	48̂	17
18	.24030̂	41	.32327	50	.26449̂	39̂	.35273	48̂	18
19	.24071̂	40̂	.32377	49̂	.26489	39̂	.35321̂	48̂	19
20	9.24112	41	9.32426̂	49̂	9.26528̂	40	9.35370	49	**20**
21	.24153	41	.32476	49̂	.26568̂	39̂	.35419	48̂	21
22	.24194	41	.32525̂	49̂	.26608	39̂	.35467̂	48̂	22
23	.24235	40̂	.32575	49̂	.26647̂	39̂	.35516	48	23
24	.24275̂	41	.32624̂	49	.26687	39̂	.35564̂	48̂	24
25	9.24316̂	40̂	9.32673̂	49̂	9.26726̂	40	9.35613	48̂	25
26	.24357	41	.32723	49̂	.26766̂	39̂	.35661̂	48̂	26
27	.24398	40̂	.32772̂	49̂	.26806	39̂	.35710	48̂	27
28	.24438̂	41	.32822	49̂	.26845̂	39̂	.35758̂	48	28
29	.24479̂	40̂	.32871̂	49	.26885	39̂	.35807	48̂	29
30	9.24520	41	9.32920̂	49̂	9.26924̂	39̂	9.35855̂	48̂	**30**
31	.24561	40̂	.32970	49̂	.26964	39	.35904	48̂	31
32	.24601̂	40̂	.33019̂	49̂	.27003	39̂	.35952̂	48̂	32
33	.24642	40̂	.33069	49	.27042̂	39̂	.36001	48̂	33
34	.24682̂	41	.33118	49̂	.27082	39̂	.36049̂	48̂	34
35	9.24723̂	40̂	9.33167̂	49	9.27121̂	39̂	9.36098	48̂	35
36	.24764	40̂	.33216̂	49̂	.27161	39	.36146̂	48	36
37	.24804̂	40̂	.33266	49	.27200	39̂	.36194̂	48̂	37
38	.24845	40̂	.33315	49̂	.27239̂	39	.36243	48̂	38
39	.24885̂	40̂	.33364̂	49	.27278̂	39̂	.36291̂	48̂	39
40	9.24926	40̂	9.33413̂	49̂	9.27318	39	9.36340	48	**40**
41	.24966̂	40̂	.33463	49	.27357	39̂	.36388	48̂	41
42	.25007	40̂	.33512	49	.27396̂	39	.36436̂	48	42
43	.25047̂	40	.33561	49̂	.27435̂	39̂	.36484̂	48̂	43
44	.25087̂	40̂	.33610̂	49	.27475	39	.36533	48̂	44
45	9.25128	40̂	9.33659̂	49	9.27514	39	9.36581̂	48	45
46	.25168̂	40̂	.33708̂	49̂	.27553	39̂	.36629̂	48̂	46
47	.25209	40	.33758	49	.27592̂	39	.36678	48	47
48	.25249	40̂	.33807	49	.27631̂	39	.36726	48	48
49	.25289̂	40	.33856	49	.27670̂	39	.36774	48̂	49
50	9.25329̂	40̂	9.33905	49	9.27709̂	39̂	9.36822̂	48	**50**
51	.25370	40	.33954	49	.27749	39	.36870̂	48̂	51
52	.25410	40̂	.34003	49	.27788	39	.36919	48	52
53	.25450̂	40	.34052	49	.27827	39	.36967	48	53
54	.25490̂	40̂	.34101	49	.27866	39	.37015	48	54
55	9.25531	40	9.34150	49	9.27905	39	9.37063	48̂	55
56	.25571	40	.34199	49	.27944	38̂	.37111̂	48	56
57	.25611	40	.34248	49	.27982̂	39	.37159̂	48	57
58	.25651	40̂	.34297	49	.28021̂	39	.37207̂	48	58
59	.25691̂	40	.34346	49	.28060̂	39	.37255̂	48	59
60	9.25731̂		9.34395		9.28099̂		9.37303̂		**60**
′	Log. Vers.	*D*	Log. Exsec.	*D*	Log. Vers.	*D*	Log. Exsec.	*D*	′

P. P.

	50	49̂	49
6	5.0	4.9	4.9
7	5.8	5.8	5.7
8	6.6	6.6	6.5
9	7.5	7.4	7.3
10	8.3	8.2	8.1
20	16.6	16.5	16.3
30	25.0	24.7	24.5
40	33.3	33.0	32.6
50	41.6	41.2	40.8

	48̂	48
6	4.8̂	4.8
7	5.6	5.6
8	6.4	6.4
9	7.3	7.2
10	8.1	8.0
20	16.1̂	16.0
30	24.2̂	24.0
40	32.3̂	32.0
50	40.4	40.0

	41̂	41
6	4.1̂	4.1
7	4.8̂	4.8
8	5.5	5.4̂
9	6.2	6.1̂
10	6.9	6.8̂
20	13.8̂	13.6
30	20.7̂	20.5
40	27.6̂	27.3̂
50	34.6	34.1̂

	40̂	40
6	4.0̂	4.0
7	4.7	4.6̂
8	5.4	5.3
9	6.1	6.0
10	6.7̂	6.6̂
20	13.5	13.3
30	20.2̂	20.0
40	27.0	26.6̂
50	33.7̂	33.3

	39̂	39
6	3.9̂	3.9
7	4.6	4.5
8	5.2	5.2
9	5.9	5.8̂
10	6.6	6.5
20	13.1̂	13.0
30	19.7̂	19.5
40	26.3̂	26.0
50	32.9	32.5

	38̂
6	3.8̂
7	4.5
8	5.1
9	5.8
10	6.4
20	12.8̂
30	19.2
40	25.6
50	32.1

P. P.

	36°				37°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.28099	39	9.37303	48	9.30398	37	9.40163	47	0
1	.28138	38	.37352	48	.30436	38	.40210	47	1
2	.28177	39	.37400	48	.30474	37	.40258	47	2
3	.28216	39	.37448	48	.30511	37	.40305	47	3
4	.28255	38	.37496	48	.30549	38	.40352	47	4
5	9.28293	39	9.37544	48	9.30587	37	9.40399	47	5
6	.28332	38	.37592	48	.30624	37	.40447	47	6
7	.28371	39	.37640	47	.30662	38	.40494	47	7
8	.28410	38	.37687	48	.30700	37	.40541	47	8
9	.28448	39	.37735	48	.30737	37	.40588	47	9
10	9.28487	38	9.37783	48	9.30775	37	9.40635	47	10
11	.28526	38	.37831	48	.30812	37	.40682	47	11
12	.28564	39	.37879	48	.30850	37	.40730	47	12
13	.28603	38	.37927	47	.30887	37	.40777	47	13
14	.28642	38	.37975	48	.30925	37	.40824	47	14
15	9.28680	38	9.38023	48	9.30962	37	9.40871	47	15
16	.28719	38	.38071	48	.31000	37	.40918	47	16
17	.28757	38	.38119	47	.31037	37	.40965	47	17
18	.28796	39	.38166	48	.31075	37	.41012	47	18
19	.28835	38	.38214	47	.31112	37	.41059	47	19
20	9.28873	38	9.38262	48	9.31150	37	9.41106	47	20
21	.28912	38	.38310	47	.31187	37	.41153	47	21
22	.28950	38	.38357	48	.31224	37	.41200	47	22
23	.28988	38	.38405	47	.31262	37	.41247	47	23
24	.29027	38	.38453	48	.31299	37	.41294	47	24
25	9.29065	38	9.38501	47	9.31336	37	9.41341	47	25
26	.29104	38	.38548	48	.31374	37	.41388	47	26
27	.29142	38	.38596	47	.31411	37	.41435	47	27
28	.29180	38	.38644	48	.31448	37	.41482	47	28
29	.29219	38	.38692	47	.31485	37	.41529	46	29
30	9.29257	38	9.38739	47	9.31523	37	9.41576	47	30
31	.29295	38	.38787	47	.31560	37	.41623	47	31
32	.29334	38	.38834	48	.31597	37	.41670	47	32
33	.29372	38	.38882	47	.31634	37	.41717	46	33
34	.29410	38	.38930	47	.31671	37	.41763	47	34
35	9.29448	38	9.38977	47	9.31708	37	9.41810	47	35
36	.29487	38	.39025	47	.31746	37	.41857	46	36
37	.29525	38	.39072	48	.31783	37	.41904	47	37
38	.29563	38	.39120	47	.31820	37	.41951	47	38
39	.29601	38	.39168	47	.31857	37	.41998	46	39
40	9.29639	38	9.39215	48	9.31894	37	9.42044	47	40
41	.29677	38	.39263	47	.31931	37	.42091	46	41
42	.29715	38	.39310	47	.31968	37	.42138	47	42
43	.29754	38	.39358	47	.32005	37	.42185	46	43
44	.29792	38	.39405	47	.32042	37	.42231	47	44
45	9.29830	38	9.39453	47	9.32079	37	9.42278	46	45
46	.29868	38	.39500	47	.32116	37	.42325	47	46
47	.29906	38	.39548	47	.32153	37	.42372	46	47
48	.29944	38	.39595	47	.32190	37	.42418	47	48
49	.29982	38	.39642	47	.32227	36	.42465	46	49
50	9.30020	37	9.39690	47	9.32263	37	9.42512	46	50
51	.30057	38	.39737	47	.32300	37	.42558	47	51
52	.30095	38	.39785	47	.32337	36	.42605	46	52
53	.30133	38	.39832	47	.32374	37	.42652	46	53
54	.30171	38	.39879	47	.32411	36	.42698	46	54
55	9.30209	37	9.39927	47	9.32447	37	9.42745	47	55
56	.30247	38	.39974	47	.32484	36	.42792	46	56
57	.30285	37	.40021	47	.32521	37	.42838	46	57
58	.30322	38	.40069	47	.32558	36	.42885	46	58
59	.30360	38	.40116	47	.32594	37	.42931	46	59
60	9.30398		9.40163		9.32631		9.42978		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	48	48	47	47	46	39	38	38	37	37	36
6	4.8	4.8	4.7	4.7	4.6	3.9	3.8	3.8	3.7	3.7	3.6
7	5.6	5.6	5.5	5.5	5.4	4.5	4.5	4.4	4.4	4.3	4.2
8	6.4	6.4	6.3	6.2	6.2	5.2	5.1	5.0	5.0	4.9	4.8
9	7.3	7.2	7.1	7.0	7.0	5.8	5.8	5.7	5.6	5.5	5.5
10	8.1	8.0	7.9	7.8	7.7	6.5	6.4	6.3	6.2	6.1	6.1
20	16.1	16.0	15.8	15.6	15.5	13.0	12.8	12.6	12.5	12.3	12.1
30	24.2	24.0	23.7	23.5	23.2	19.5	19.2	19.0	18.7	18.5	18.2
40	32.3	32.0	31.6	31.3	31.0	26.0	25.6	25.3	25.0	24.6	24.3
50	40.4	40.0	39.6	39.1	38.7	32.5	32.1	31.6	31.2	30.8	30.4

P. P.

′	38° Log. Vers.	D	38° Log. Exsec.	D	39° Log. Vers.	D	39° Log. Exsec.	D	′
0	9.32631	36	9.42978	46	9.34802	35	9.45752	45	0
1	.32668	36	.43024	47	.34837	36	.45797	46	1
2	.32704	37	.43071	46	.34873	35	.45843	46	2
3	.32741	36	.43118	46	.34909	35	.45889	46	3
4	.32778	36	.43164	46	.34944	35	.45935	45	4
5	9.32814	36	9.43211	46	9.34980	36	9.45981	46	5
6	.32851	37	.43257	46	.35016	35	.46027	46	6
7	.32888	36	.43304	46	.35051	35	.46073	45	7
8	.32924	36	.43350	46	.35087	35	.46118	46	8
9	.32961	36	.43396	46	.35122	35	.46164	46	9
10	9.32997	36	9.43443	46	9.35158	35	9.46210	45	10
11	.33034	36	.43489	46	.35193	35	.46256	46	11
12	.33070	36	.43536	46	.35229	35	.46302	45	12
13	.33107	36	.43582	46	.35264	35	.46347	46	13
14	.33143	36	.43629	46	.35300	35	.46393	45	14
15	9.33180	36	9.43675	46	9.35335	35	9.46439	46	15
16	.33216	36	.43721	46	.35370	35	.46485	45	16
17	.33252	36	.43768	46	.35406	35	.46530	46	17
18	.33289	36	.43814	46	.35441	35	.46576	45	18
19	.33325	36	.43861	46	.35477	35	.46622	46	19
20	9.33361	36	9.43907	46	9.35512	35	9.46668	45	20
21	.33398	36	.43953	46	.35547	35	.46713	45	21
22	.33434	36	.43999	46	.35583	35	.46759	46	22
23	.33470	36	.44046	46	.35618	35	.46805	45	23
24	.33507	36	.44092	46	.35653	35	.46850	45	24
25	9.33543	36	9.44138	46	9.35689	35	9.46896	46	25
26	.33579	36	.44185	46	.35724	35	.46942	45	26
27	.33615	36	.44231	46	.35759	35	.46987	45	27
28	.33652	36	.44277	46	.35794	35	.47033	45	28
29	.33688	36	.44323	46	.35829	35	.47078	46	29
30	9.33724	36	9.44370	46	9.35865	35	9.47124	45	30
31	.33760	36	.44416	46	.35900	35	.47170	45	31
32	.33796	36	.44462	46	.35935	35	.47215	45	32
33	.33833	36	.44508	46	.35970	35	.47261	45	33
34	.33869	36	.44554	46	.36005	35	.47306	46	34
35	9.33905	36	9.44601	46	9.36040	35	9.47352	45	35
36	.33941	36	.44647	46	.36076	35	.47398	45	36
37	.33977	36	.44693	46	.36111	35	.47443	45	37
38	.34013	36	.44739	46	.36146	35	.47489	45	38
39	.34049	36	.44785	46	.36181	35	.47534	45	39
40	9.34085	36	9.44831	46	9.36216	35	9.47580	45	40
41	.34121	36	.44877	46	.36251	35	.47625	45	41
42	.34157	36	.44924	46	.36286	35	.47671	45	42
43	.34193	36	.44970	46	.36321	35	.47716	45	43
44	.34229	36	.45016	46	.36356	35	.47762	45	44
45	9.34265	36	9.45062	46	9.36391	35	9.47807	45	45
46	.34301	36	.45108	46	.36426	35	.47852	45	46
47	.34337	36	.45154	46	.36461	34	.47898	45	47
48	.34373	35	.45200	46	.36495	35	.47943	45	48
49	.34408	36	.45246	46	.36530	35	.47989	45	49
50	9.34444	36	9.45292	46	9.36565	35	9.48034	45	50
51	.34480	35	.45338	46	.36600	34	.48080	45	51
52	.34516	36	.45384	46	.36635	35	.48125	45	52
53	.34552	35	.45430	46	.36670	35	.48170	45	53
54	.34587	36	.45476	46	.36705	34	.48216	45	54
55	9.34623	35	9.45522	46	9.36739	35	9.48261	45	55
56	.34659	36	.45568	46	.36774	34	.48306	45	56
57	.34695	35	.45614	46	.36809	35	.48352	45	57
58	.34730	36	.45660	46	.36844	34	.48397	45	58
59	.34766	35	.45706	46	.36878	35	.48442	45	59
60	9.34802		9.45752		9.36913		9.48488		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	47	46	46	45	45	37	36	36	35	35	34
6	4.7	4.6	4.6	4.5	4.5	3.7	3.6	3.6	3.5	3.5	3.4
7	5.5	5.4	5.3	5.3	5.2	4.3	4.2	4.2	4.1	4.1	4.0
8	6.2	6.2	6.1	6.0	6.0	4.9	4.8	4.8	4.7	4.6	4.6
9	7.0	7.0	6.9	6.8	6.7	5.5	5.5	5.4	5.3	5.2	5.2
10	7.8	7.7	7.6	7.6	7.5	6.1	6.1	6.0	5.9	5.8	5.7
20	15.6	15.5	15.3	15.1	15.0	12.3	12.1	12.0	11.8	11.6	11.5
30	23.5	23.2	23.0	22.7	22.5	18.5	18.2	18.0	17.7	17.5	17.2
40	31.3	31.0	30.6	30.3	30.0	24.6	24.3	24.0	23.6	23.3	23.0
50	39.1	38.7	38.3	37.9	37.5	30.8	30.4	30.0	29.6	29.1	28.7

	40°				41°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.36913	34	9.48488	45	9.38968	34	9.51190	45	0
1	.36948	34	.48533	45	.39002	33	.51235	44	1
2	.36982	35	.48578	45	.39035	34	.51279	45	2
3	.37017	34	.48624	45	.39069	33	.51324	44	3
4	.37052	34	.48669	45	.39103	34	.51369	45	4
5	9.37086	35	9.48714	45	9.39137	33	9.51414	44	5
6	.37121	34	.48759	45	.39170	33	.51458	45	6
7	.37156	34	.48805	45	.39204	34	.51503	44	7
8	.37190	34	.48850	45	.39238	33	.51548	44	8
9	.37225	34	.48895	45	.39271	33	.51592	45	9
10	9.37259	34	9.48940	45	9.39305	34	9.51637	44	10
11	.37294	34	.48986	45	.39339	33	.51682	44	11
12	.37328	34	.49031	45	.39372	33	.51726	45	12
13	.37363	34	.49076	45	.39406	33	.51771	44	13
14	.37397	34	.49121	45	.39439	33	.51816	44	14
15	9.37432	34	9.49166	45	9.39473	34	9.51860	45	15
16	.37466	34	.49211	45	.39507	33	.51905	44	16
17	.37501	34	.49257	45	.39540	33	.51950	44	17
18	.37535	34	.49302	45	.39574	33	.51994	44	18
19	.37570	34	.49347	45	.39607	33	.52039	45	19
20	9.37604	34	9.49392	45	9.39641	33	9.52084	44	20
21	.37639	34	.49437	45	.39674	33	.52128	44	21
22	.37673	34	.49482	45	.39708	33	.52173	44	22
23	.37707	34	.49527	45	.39741	33	.52217	44	23
24	.37742	34	.49572	45	.39774	33	.52262	44	24
25	9.37776	34	9.49618	45	9.39808	33	9.52306	45	25
26	.37810	34	.49663	45	.39841	33	.52351	44	26
27	.37845	34	.49708	45	.39875	33	.52396	44	27
28	.37879	34	.49753	45	.39908	33	.52440	44	28
29	.37913	34	.49798	45	.39941	33	.52485	44	29
30	9.37947	34	9.49843	45	9.39975	33	9.52529	44	30
31	.37982	34	.49888	45	.40008	33	.52574	44	31
32	.38016	34	.49933	45	.40041	33	.52618	44	32
33	.38050	34	.49978	45	.40075	33	.52663	44	33
34	.38084	34	.50023	45	.40108	33	.52707	44	34
35	9.38118	34	9.50068	45	9.40141	33	9.52752	44	35
36	.38153	34	.50113	45	.40175	33	.52796	44	36
37	.38187	34	.50158	45	.40208	33	.52841	44	37
38	.38221	34	.50203	45	.40241	33	.52885	44	38
39	.38255	34	.50248	45	.40274	33	.52930	44	39
40	9.38289	34	9.50293	45	9.40307	33	9.52974	44	40
41	.38323	34	.50338	45	.40341	33	.53018	44	41
42	.38357	34	.50383	44	.40374	33	.53063	44	42
43	.38391	34	.50427	45	.40407	33	.53107	44	43
44	.38425	34	.50472	45	.40440	33	.53152	44	44
45	9.38459	34	9.50517	45	9.40473	33	9.53196	44	45
46	.38493	34	.50562	45	.40506	33	.53240	44	46
47	.38527	34	.50607	44	.40540	33	.53285	44	47
48	.38561	34	.50652	45	.40573	33	.53329	44	48
49	.38595	34	.50697	45	.40606	33	.53374	44	49
50	9.38629	34	9.50742	45	9.40639	33	9.53418	44	50
51	.38663	34	.50787	44	.40672	33	.53462	44	51
52	.38697	33	.50831	45	.40705	33	.53507	44	52
53	.38731	34	.50876	45	.40738	33	.53551	44	53
54	.38765	34	.50921	44	.40771	33	.53595	44	54
55	9.38799	34	9.50966	45	9.40804	33	9.53640	44	55
56	.38833	33	.51011	44	.40837	33	.53684	44	56
57	.38866	34	.51055	45	.40870	33	.53728	44	57
58	.38900	33	.51100	45	.40903	33	.53773	44	58
59	.38934	34	.51145	44	.40936	33	.53817	44	59
60	9.38968		9.51190		9.40969		9.53861		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	45	45
6	4.5	4.5
7	5.3	5.2
8	6.0	6.0
9	6.8	6.7
10	7.6	7.5
20	15.1	15.0
30	22.7	22.5
40	30.3	30.0
50	37.9	37.5

	44	44
6	4.4	4.4
7	5.2	5.1
8	5.9	5.8
9	6.7	6.6
10	7.4	7.3
20	14.8	14.6
30	22.2	22.0
40	29.6	29.3
50	37.1	36.6

	35	34
6	3.5	3.4
7	4.1	4.0
8	4.6	4.6
9	5.2	5.2
10	5.8	5.7
20	11.6	11.5
30	17.5	17.2
40	23.3	23.0
50	29.1	28.7

	34	33
6	3.4	3.3
7	3.9	3.9
8	4.5	4.4
9	5.1	5.0
10	5.6	5.6
20	11.3	11.1
30	17.0	16.7
40	22.6	22.3
50	28.3	27.9

	33
6	3.3
7	3.8
8	4.4
9	4.9
10	5.5
20	11.0
30	16.5
40	22.0
50	27.5

P. P.

	42°				43°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.40969	32̂	9.53861̂	44̂	9.42918	32	9.56505̂	43̂	**0**
1	.41001̂	33	.53906	44	.42950	32	.56549	44	1
2	.41034̂	33	.53950	44̂	.42982	32	.56593	44	2
3	.41067̂	33	.53994̂	44	.43014	32̂	.56637	43̂	3
4	.41100̂	32̂	.54038̂	44̂	.43046̂	32	.56680̂	44	4
5	9.41133	33	9.54083	44	9.43078̂	32	9.56724̂	44	5
6	.41166	33	.54127	44̂	.43110̂	31̂	.56768̂	43̂	6
7	.41199	32̂	.54171̂	44	.43142	32	.56812	44	7
8	.41231̂	33	.54215̂	44	.43174	32	.56856	43̂	8
9	.41264̂	32̂	.54259̂	44̂	.43206	32	.56899̂	44	9
10	9.41297	33	9.54304	44	9.43238	32	9.56943̂	44	**10**
11	.41330	32̂	.54348	44̂	.43270	32	.56987̂	43̂	11
12	.41362̂	33	.54392̂	44	.43302	32	.57031	44	12
13	.41395̂	32̂	.54436̂	44	.43334	31̂	.57075	43̂	13
14	.41428	33	.54480̂	44̂	.43365̂	32	.57118̂	44	14
15	9.41461	32̂	9.54525	44	9.43397̂	32	9.57162̂	43̂	15
16	.41493̂	33	.54569	44	.43429̂	32	.57206	44	16
17	.41526̂	32̂	.54613	44̂	.43461̂	31̂	.57250	43̂	17
18	.41559	32̂	.54657̂	44	.43493	32	.57293̂	44	18
19	.41591̂	32̂	.54701̂	44	.43525	32	.57337̂	43̂	19
20	9.41624	33	9.54745̂	44̂	9.43557	31̂	9.57381	43̂	**20**
21	.41657	32̂	.54790	44	.43588̂	32	.57424̂	44	21
22	.41689̂	32̂	.54834	44	.43620̂	31̂	.57468̂	43̂	22
23	.41722	32̂	.54878	44	.43652	32	.57512	44	23
24	.41754̂	32̂	.54922	44̂	.43684	31̂	.57556	43̂	24
25	9.41787	32̂	9.54966̂	44	9.43715̂	32	9.57599̂	43̂	25
26	.41819̂	32̂	.55010̂	44	.43747̂	31̂	.57643	44	26
27	.41852	33	.55054̂	44	.43779	31̂	.57687	43̂	27
28	.41885	32̂	.55098̂	44	.43810̂	32	.57730̂	43̂	28
29	.41917̂	32̂	.55142̂	44	.43842̂	31̂	.57774	44	29
30	9.41950	32	9.55186̂	44	9.43874	32	9.57818	43̂	**30**
31	.41982	32̂	.55230̂	44̂	.43906	31̂	.57861̂	43̂	31
32	.42014̂	32̂	.55275	44	.43937̂	31̂	.57905	44	32
33	.42047	32̂	.55319	44	.43969	31̂	.57949	43̂	33
34	.42079̂	32̂	.55363	44	.44000̂	32	.57992̂	43̂	34
35	9.42112	32̂	9.55407	44	9.44032̂	31̂	9.58036	43̂	35
36	.42144̂	32̂	.55451	44	.44064	31̂	.58079̂	44	36
37	.42177	32	.55495	44	.44095̂	31̂	.58123̂	43̂	37
38	.42209	32̂	.55539	44	.44127	31̂	.58167	43̂	38
39	.42241̂	32̂	.55583	44	.44158̂	31̂	.58210̂	43̂	39
40	9.42274	32̂	9.55627	44	9.44190	31̂	9.58254	43̂	**40**
41	.42306̂	32	.55671	44	.44221̂	31̂	.58297̂	44	41
42	.42338̂	32̂	.55715	44	.44253	31̂	.58341̂	43̂	42
43	.42371	32	.55759	44	.44284̂	31̂	.58385	43̂	43
44	.42403	32̂	.55803	44	.44316	31̂	.58428̂	43̂	44
45	9.42435̂	32	9.55847	43̂	9.44347̂	31̂	9.58472	43̂	45
46	.42467̂	32̂	.55890̂	44	.44379	31̂	.58515̂	43̂	46
47	.42500	32	.55934̂	44	.44410̂	31̂	.58559	43̂	47
48	.42532	32̂	.55978̂	44	.44442	31̂	.58602̂	43̂	48
49	.42564̂	32	.56022̂	44	.44473̂	31	.58646	43̂	49
50	9.42596̂	32̂	9.56066̂	44	9.44504̂	31̂	9.58689̂	43̂	**50**
51	.42629	32	.56110̂	44	.44536	31̂	.58733	43̂	51
52	.42661	32	.56154̂	43̂	.44567̂	31̂	.58776̂	44	52
53	.42693̂	32	.56198	44	.44599	31	.58820̂	43̂	53
54	.42725̂	32	.56242	44	.44630	31̂	.58864	43̂	54
55	9.42757̂	32	9.56286	44	9.44661̂	31̂	9.58907̂	43̂	55
56	.42789̂	32̂	.56330	44	.44693	31	.58951	43̂	56
57	.42822	32	.56374	43̂	.44724	31̂	.58994	43̂	57
58	.42854	32	.56417̂	44	.44755̂	31̂	.59037̂	43̂	58
59	.42886	32	.56461̂	43̂	.44787	31	.59081	43̂	59
60	9.42918		9.56505̂		9.44818		9.59124̂		**60**
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	44̂	44
6	4.4̂	4.4
7	5.2	5.1̂
8	5.9	5.8
9	6.7	6.6
10	7.4	7.3̂
20	14.8̂	14.6
30	22.2	22.0
40	29.6	20.3̂
50	37.1	30.6

	43̂	43
6	4.3̂	4.3
7	5.1	5.0
8	5.8	5.7̂
9	6.5	6.4̂
10	7.2	7.1̂
20	14.5	14.3̂
30	21.7̂	21.5
40	29.0	28.6̂
50	36.2	35.8

	33	32̂
6	3.3	3.2̂
7	3.8	3.8
8	4.4	4.3̂
9	4.9̂	4.9
10	5.5	5.4
20	11.0	10.8̂
30	16.5	16.2̂
40	22.0	21.6
50	27.5	27.1

	32	31̂
6	3.2	3.1̂
7	3.7̂	3.7
8	4.2̂	4.2
9	4.8	4.7
10	5.3̂	5.2
20	10.6̂	10.5
30	16.0	15.7̂
40	21.3̂	21.0
50	20.6̂	26.2

	31
6	3.1
7	3.6
8	4.1̂
9	4.6
10	5.1̂
20	10.3
30	15.5̂
40	20.6
50	25.8

P. P.

	44°				45°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.44818	31	9.59124	43	9.46671	30	9.61722	43	0
1	.44849	31	.59168	43	.46701	30	.61765	43	1
2	.44880	31	.59211	43	.46732	30	.61808	43	2
3	.44912	31	.59255	43	.46762	30	.61852	43	3
4	.44943	31	.59298	43	.46793	30	.61895	43	4
5	9.44974	31	9.59342	43	9.46823	30	9.61938	43	5
6	.45005	31	.59385	43	.46853	30	.61981	43	6
7	.45036	31	.59429	43	.46884	30	.62024	43	7
8	.45068	31	.59472	43	.46914	30	.62067	43	8
9	.45099	31	.59515	43	.46945	30	.62110	43	9
10	9.45130	31	9.59559	43	9.46975	30	9.62153	43	10
11	.45161	31	.59602	43	.47005	30	.62196	43	11
12	.45192	31	.59646	43	.47036	30	.62239	43	12
13	.45223	31	.59689	43	.47066	30	.62282	43	13
14	.45254	31	.59732	43	.47096	30	.62326	43	14
15	9.45285	31	9.59776	43	9.47127	30	9.62369	43	15
16	.45316	31	.59819	43	.47157	30	.62412	43	16
17	.45348	31	.59863	43	.47187	30	.62455	43	17
18	.45379	31	.59906	43	.47218	30	.62498	43	18
19	.45410	31	.59949	43	.47248	30	.62541	43	19
20	9.45441	31	9.59993	43	9.47278	30	9.62584	43	20
21	.45472	31	.60036	43	.47308	30	.62627	43	21
22	.45503	31	.60079	43	.47339	30	.62670	43	22
23	.45534	31	.60123	43	.47369	30	.62713	43	23
24	.45565	30	.60166	43	.47399	30	.62756	43	24
25	9.45595	31	9.60209	43	9.47429	30	9.62799	43	25
26	.45626	31	.60253	43	.47459	30	.62842	43	26
27	.45657	31	.60296	43	.47490	30	.62885	43	27
28	.45688	31	.60339	43	.47520	30	.62928	43	28
29	.45719	31	.60383	43	.47550	30	.62971	43	29
30	9.45750	30	9.60426	43	9.47580	30	9.63014	43	30
31	.45781	31	.60469	43	.47610	30	.63057	43	31
32	.45812	31	.60512	43	.47640	30	.63100	43	32
33	.45843	30	.60556	43	.47670	30	.63143	43	33
34	.45873	31	.60599	43	.47700	30	.63186	43	34
35	9.45904	31	9.60642	43	9.47731	30	9.63229	43	35
36	.45935	30	.60685	43	.47761	30	.63272	43	36
37	.45966	31	.60729	43	.47791	30	.63315	43	37
38	.45997	30	.60772	43	.47821	30	.63358	43	38
39	.46027	31	.60815	43	.47851	30	.63401	42	39
40	9.46058	30	9.60858	43	9.47881	30	9.63443	43	40
41	.46089	31	.60902	43	.47911	30	.63486	43	41
42	.46120	30	.60945	43	.47941	30	.63529	43	42
43	.46150	31	.60988	43	.47971	30	.63572	43	43
44	.46181	30	.61031	43	.48001	30	.63615	43	44
45	9.46212	30	9.61075	43	9.48031	30	9.63658	42	45
46	.46242	31	.61118	43	.48061	29	.63701	43	46
47	.46273	30	.61161	43	.48090	30	.63744	43	47
48	.46304	30	.61204	43	.48120	30	.63787	43	48
49	.46334	31	.61247	43	.48150	30	.63830	43	49
50	9.46365	30	9.61291	43	9.48180	30	9.63873	42	50
51	.46396	30	.61334	43	.48210	29	.63915	43	51
52	.46426	30	.61377	43	.48240	30	.63958	43	52
53	.46457	30	.61420	43	.48270	30	.64001	43	53
54	.46487	31	.61463	43	.48300	29	.64044	42	54
55	9.46518	30	9.61506	43	9.48329	30	9.64087	43	55
56	.46549	30	.61550	43	.48359	30	.64130	43	56
57	.46579	30	.61593	43	.48389	29	.64173	43	57
58	.46610	30	.61636	43	.48419	30	.64216	42	58
59	.46640	30	.61679	43	.48449	29	.64258	43	59
60	9.46671		9.61722		9.48478		9.64301		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	43	43
6	4.3	4.3
7	5.1	5.0
8	5.8	5.7
9	6.5	6.4
10	7.2	7.1
20	14.5	14.3
30	21.7	21.5
40	29.0	28.6
50	36.2	35.8

	42
6	4.2
7	4.9
8	5.6
9	6.4
10	7.1
20	14.1
30	21.2
40	28.3
50	35.4

	31	31
6	3.1	3.1
7	3.7	3.6
8	4.2	4.1
9	4.7	4.6
10	5.2	5.1
20	10.5	10.3
30	15.7	15.5
40	21.0	20.6
50	26.2	25.8

	30	30
6	3.0	3.0
7	3.5	3.5
8	4.0	4.0
9	4.6	4.5
10	5.1	5.0
20	10.1	10.0
30	15.2	15.0
40	20.3	20.0
50	25.4	25.0

	29
6	2.9
7	3.4
8	3.9
9	4.4
10	4.9
20	9.8
30	14.7
40	19.6
50	24.6

P. P.

′	46° Log. Vers.	D	46° Log. Exsec.	D	47° Log. Vers.	D	47° Log. Exsec.	D	′
0	9.48478	30	9.64301	43	9.50243	29	9.66864	42	0
1	.48508	29	.64344	42	.50272	29	.66907	43	1
2	.48538	30	.64387	43	.50301	29	.66950	42	2
3	.48568	29	.64430	43	.50330	29	.66992	42	3
4	.48597	30	.64473	42	.50359	29	.67035	42	4
5	9.48627	29	9.64515	43	9.50388	29	9.67077	42	5
6	.48657	29	.64558	43	.50417	29	.67120	42	6
7	.48686	30	.64601	42	.50446	29	.67162	43	7
8	.48716	29	.64644	43	.50475	29	.67205	42	8
9	.48746	29	.64687	42	.50504	29	.67248	42	9
10	9.48775	30	9.64729	43	9.50533	29	9.67290	42	10
11	.48805	29	.64772	43	.50562	29	.67333	42	11
12	.48835	29	.64815	42	.50591	28	.67375	42	12
13	.48864	29	.64858	43	.50619	29	.67418	42	13
14	.48894	29	.64901	42	.50648	29	.67460	42	14
15	9.48923	30	9.64943	43	9.50677	29	9.67503	43	15
16	.48953	29	.64986	42	.50706	28	.67546	42	16
17	.48983	29	.65029	43	.50735	29	.67588	42	17
18	.49012	29	.65072	42	.50764	29	.67631	42	18
19	.49042	29	.65114	43	.50793	28	.67673	42	19
20	9.49071	29	9.65157	42	9.50821	29	9.67716	42	20
21	.49101	29	.65200	43	.50850	29	.67758	42	21
22	.49130	29	.65243	42	.50879	28	.67801	42	22
23	.49160	29	.65285	43	.50908	29	.67843	42	23
24	.49189	29	.65328	42	.50937	28	.67886	42	24
25	9.49219	29	9.65371	43	9.50965	29	9.67928	42	25
26	.49248	29	.65414	42	.50994	28	.67971	42	26
27	.49278	29	.65456	43	.51023	29	.68013	42	27
28	.49307	29	.65499	42	.51052	28	.68056	42	28
29	.49336	29	.65542	43	.51080	29	.68098	42	29
30	9.49366	29	9.65585	42	9.51109	28	9.68141	42	30
31	.49395	29	.65627	43	.51138	29	.68183	42	31
32	.49425	29	.65670	42	.51167	28	.68226	42	32
33	.49454	29	.65713	42	.51195	28	.68268	42	33
34	.49483	29	.65755	43	.51224	29	.68311	42	34
35	9.49513	29	9.65798	42	9.51253	28	9.68353	42	35
36	.49542	29	.65841	43	.51281	28	.68396	42	36
37	.49571	29	.65884	42	.51310	28	.68438	42	37
38	.49601	29	.65926	42	.51338	29	.68481	42	38
39	.49630	29	.65969	43	.51367	28	.68523	42	39
40	9.49659	29	9.66012	42	9.51396	28	9.68566	42	40
41	.49689	29	.66054	42	.51424	28	.68608	42	41
42	.49718	29	.66097	43	.51453	28	.68651	42	42
43	.49747	29	.66140	42	.51481	28	.68693	42	43
44	.49776	29	.66182	42	.51510	29	.68735	42	44
45	9.49806	29	9.66225	43	9.51539	28	9.68778	42	45
46	.49835	29	.66268	42	.51567	28	.68820	42	46
47	.49864	29	.66310	42	.51596	28	.68863	42	47
48	.49893	29	.66353	43	.51624	28	.68905	42	48
49	.49922	29	.66396	42	.51653	28	.68948	42	49
50	9.49952	29	9.66438	42	9.51681	28	9.68990	42	50
51	.49981	29	.66481	42	.51710	28	.69033	42	51
52	.50010	29	.66523	43	.51738	28	.69075	42	52
53	.50039	29	.66566	42	.51767	28	.69117	42	53
54	.50068	29	.66609	42	.51795	28	.69160	42	54
55	9.50097	29	9.66651	42	9.51823	28	9.69202	42	55
56	.50126	29	.66694	43	.51852	28	.69245	42	56
57	.50155	29	.66737	42	.51880	28	.69287	42	57
58	.50185	29	.66779	42	.51909	28	.69330	42	58
59	.50214	29	.66822	42	.51937	28	.69372	42	59
60	9.50243		9.66864		9.51965		9.69414		60
	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	43	42
6	4.3	4.2
7	5.0	4.9
8	5.7	5.6
9	6.4	6.4
10	7.1	7.1
20	14.3	14.1
30	21.5	21.2
40	28.6	28.3
50	35.8	35.4

	42
6	4.2
7	4.9
8	5.6
9	6.3
10	7.0
20	14.0
30	21.0
40	28.0
50	35.0

	30	29
6	3.0	2.9
7	3.5	3.4
8	4.0	3.9
9	4.5	4.4
10	5.0	4.9
20	10.0	9.8
30	15.0	14.7
40	20.0	19.6
50	25.0	24.6

	29	28
6	2.9	2.8
7	3.4	3.3
8	3.8	3.8
9	4.3	4.3
10	4.8	4.7
20	9.6	9.5
30	14.5	14.2
40	19.3	19.0
50	24.1	23.7

	28
6	2.8
7	3.2
8	3.7
9	4.2
10	4.6
20	9.3
30	14.0
40	18.6
50	23.3

P. P.

	48°				49°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.51965̂	28̂	9.69414̂	42̂	9.53648̂	27̂	9.71954	42̂	0
1	.51994	28̂	.69457	42̂	.53676	28	.71996̂	42	1
2	.52022̂	28	.69499̂	42̂	.53704	27̂	.72038̂	42̂	2
3	.52050̂	28̂	.69542	42	.53731̂	27̂	.72081	42	3
4	.52079	28̂	.69584	42̂	.53759	28	.72123	42̂	4
5	9.52107̂	28	9.69626̂	42̂	9.53787	27̂	9.72165̂	42	5
6	.52135̂	28̂	.69669	42̂	.53814̂	27̂	.72207̂	42̂	6
7	.52164	28̂	.69711̂	42	.53842	28	.72250	42	7
8	.52192̂	28	.69753̂	42̂	.53870	27̂	.72292	42̂	8
9	.52220̂	28̂	.69796	42̂	.53897̂	27̂	.72334̂	42	9
10	9.52249	28	9.69838̂	42̂	9.53925	27̂	9.72376̂	42̂	10
11	.52277	28̂	.69881	42	.53952̂	28	.72419	42	11
12	.52305̂	28	.69923	42̂	.53980̂	27̂	.72461	42	12
13	.52333̂	28̂	.69965̂	42̂	.54008	27̂	.72503	42̂	13
14	.52362	28	.70008	42̂	.54035̂	27̂	.72545̂	42	14
15	9.52390	28	9.70050̂	42	9.54063	27̂	9.72587̂	42̂	15
16	.52418	28̂	.70092̂	42̂	.54090̂	27̂	.72630	42	16
17	.52446̂	28	.70135	42̂	.54118	27̂	.72672	42̂	17
18	.52474̂	28̂	.70177̂	42̂	.54145̂	27̂	.72714̂	42	18
19	.52503	28	.70220	42	.54173	27̂	.72756̂	42̂	19
20	9.52531	28	9.70262	42̂	9.54200̂	27̂	9.72799	42	20
21	.52559	28	.70304̂	42̂	.54228	27̂	.72841	42	21
22	.52587	28̂	.70347	42	.54255̂	27̂	.72883	42̂	22
23	.52615̂	28	.70389	42̂	.54283	27̂	.72925̂	42	23
24	.52643̂	28	.70431̂	42̂	.54310̂	27̂	.72967̂	42̂	24
25	9.52671̂	28	9.70474	42	9.54338	27̂	9.73010	42	25
26	.52699̂	28	.70516	42̂	.54365̂	27̂	.73052	42	26
27	.52727̂	28̂	.70558̂	42̂	.54393	27̂	.73094	42̂	27
28	.52756	28	.70601	42	.54420̂	27̂	.73136̂	42	28
29	.52784	28	.70643	42̂	.54448	27	.73178̂	42̂	29
30	9.52812	28	9.70685̂	42̂	9.54475	27̂	9.73221	42	30
31	.52840	28	.70728	42	.54502̂	27̂	.73263	42	31
32	.52868	28	.70770	42̂	.54530	27̂	.73305	42̂	32
33	.52896	28	.70812̂	42	.54557̂	27̂	.73347̂	42	33
34	.52924	28	.70854̂	42̂	.54585	27	.73389̂	42	34
35	9.52952	28	9.70897	42̂	9.54612	27̂	9.73431̂	42̂	35
36	.52980	28	.70939̂	42	.54639̂	27̂	.73474	42	36
37	.53008	28	.70981̂	42̂	.54667	27	.73516	42̂	37
38	.53036	28	.71024	42̂	.54694	27̂	.73558̂	42	38
39	.53064	28	.71066̂	42	.54721̂	27	.73600̂	42	39
40	9.53092	28	9.71108̂	42̂	9.54748̂	27̂	9.73642̂	42̂	40
41	.53120	27̂	.71151	42	.54776	27̂	.73685	42	41
42	.53147̂	28	.71193	42̂	.54803̂	27̂	.73727	42	42
43	.53175̂	28	.71235̂	42̂	.54830̂	27̂	.73769	42̂	43
44	.53203̂	28	.71278	42	.54858	27	.73811̂	42	44
45	9.53231̂	27̂	9.71320	42̂	9.54885	27̂	9.73853̂	42	45
46	.53259	28	.71362̂	42	.54912̂	27	.73895̂	42̂	46
47	.53287	28	.71404̂	42̂	.54939̂	27̂	.73938	42	47
48	.53315	28	.71447	42	.54967	27	.73980	42	48
49	.53343	27̂	.71489	42̂	.54994	27	.74022	42̂	49
50	9.53370̂	28	9.71531̂	42	9.55021	27̂	9.74064̂	42	50
51	.53398̂	28	.71573̂	42̂	.55048̂	27	.74106̂	42	51
52	.53426̂	27̂	.71616	42̂	.55075̂	27̂	.74148̂	42̂	52
53	.53454	28	.71658̂	42	.55103	27	.74191	42	53
54	.53482	27̂	.71700̂	42̂	.55130	27	.74233	42	54
55	9.53509̂	28	9.71743	42	9.55157	27	9.74275	42̂	55
56	.53537̂	27̂	.71785	42̂	.55184	27̂	.74317̂	42	56
57	.53565	28	.71827̂	42	.55211̂	27	.74359̂	42	57
58	.53593	27̂	.71869̂	42̂	.55238̂	27	.74401̂	42̂	58
59	.53620̂	28	.71912	42	.55265̂	27	.74444	42	59
60	9.53648̂		9.71954		9.55292̂		9.74486		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	42̂	42
6	4.2̂	4.2
7	4.9̂	4.9
8	5.6̂	5.6
9	6.4	6.3
10	7.1	7.0
20	14.1̂	14.0
30	21.2̂	21.0
40	28.3̂	28.0
50	35.4	35.0

	28̂	28
6	2.8̂	2.8
7	3.3	3.2̂
8	3.8	3.7̂
9	4.3	4.2
10	4.7	4.6̂
20	9.5	9.3̂
30	14.2	14.0
40	19.0	18.6̂
50	23.7̂	23.3̂

	27̂	27
6	2.7̂	2.7
7	3.2	3.1̂
8	3.6̂	3.6
9	4.1	4.0̂
10	4.6	4.5
20	9.1̂	9.0
30	13.7̂	13.5
40	18.3	18.0
50	22.9	22.5

P. P.

	50°				51°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.55292	27	9.74486	42	9.56900	26	9.77012	42	0
1	.55319	27	.74528	42	.56926	26	.77055	42	1
2	.55347	27	.74570	42	.56953	26	.77097	42	2
3	.55374	27	.74612	42	.56979	26	.77139	42	3
4	.55401	27	.74654	42	.57005	26	.77181	42	4
5	9.55428	27	9.74696	42	9.57032	26	9.77223	42	5
6	.55455	27	.74739	42	.57058	26	.77265	42	6
7	.55482	27	.74781	42	.57085	26	.77307	42	7
8	.55509	27	.74823	42	.57111	26	.77349	42	8
9	.55536	27	.74865	42	.57138	26	.77391	42	9
10	9.55563	27	9.74907	42	9.57164	26	9.77433	42	10
11	.55590	27	.74949	42	.57190	26	.77475	42	11
12	.55617	27	.74991	42	.57217	26	.77517	42	12
13	.55644	27	.75033	42	.57243	26	.77560	42	13
14	.55671	27	.75076	42	.57269	26	.77602	42	14
15	9.55698	27	9.75118	42	9.57296	26	9.77644	42	15
16	.55725	26	.75160	42	.57322	26	.77686	42	16
17	.55751	27	.75202	42	.57348	26	.77728	42	17
18	.55778	27	.75244	42	.57375	26	.77770	42	18
19	.55805	27	.75286	42	.57401	26	.77812	42	19
20	9.55832	27	9.75328	42	9.57427	26	9.77854	42	20
21	.55859	26	.75370	42	.57454	26	.77896	42	21
22	.55886	27	.75413	42	.57480	26	.77938	42	22
23	.55913	27	.75455	42	.57506	26	.77980	42	23
24	.55940	26	.75497	42	.57532	26	.78022	42	24
25	9.55966	27	9.75539	42	9.57559	26	9.78064	42	25
26	.55993	27	.75581	42	.57585	26	.78107	42	26
27	.56020	26	.75623	42	.57611	26	.78149	42	27
28	.56047	27	.75665	42	.57637	26	.78191	42	28
29	.56074	27	.75707	42	.57664	26	.78233	42	29
30	9.56101	26	9.75750	42	9.57690	26	9.78275	42	30
31	.56127	27	.75792	42	.57716	26	.78317	42	31
32	.56154	26	.75834	42	.57742	26	.78359	42	32
33	.56181	27	.75876	42	.57768	26	.78401	42	33
34	.56208	26	.75918	42	.57794	26	.78443	42	34
35	9.56234	27	9.75960	42	9.57821	26	9.78485	42	35
36	.56261	26	.76002	42	.57847	26	.78527	42	36
37	.56288	27	.76044	42	.57873	26	.78569	42	37
38	.56315	26	.76086	42	.57899	26	.78611	42	38
39	.56341	26	.76128	42	.57925	26	.78653	42	39
40	9.56368	27	9.76171	42	9.57951	26	9.78696	42	40
41	.56395	26	.76213	42	.57977	26	.78738	42	41
42	.56421	26	.76255	42	.58003	26	.78780	42	42
43	.56448	27	.76297	42	.58029	26	.78822	42	43
44	.56475	26	.76339	42	.58055	26	.78864	42	44
45	9.56501	26	9.76381	42	9.58082	26	9.78906	42	45
46	.56528	26	.76423	42	.58108	26	.78948	42	46
47	.56554	27	.76465	42	.58134	26	.78990	42	47
48	.56581	26	.76507	42	.58160	26	.79032	42	48
49	.56608	26	.76549	42	.58186	26	.79074	42	49
50	9.56634	26	9.76592	42	9.58212	26	9.79116	42	50
51	.56661	26	.76634	42	.58238	26	.79158	42	51
52	.56687	26	.76676	42	.58264	26	.79200	42	52
53	.56714	27	.76718	42	.58290	26	.79242	42	53
54	.56741	26	.76760	42	.58316	26	.79285	42	54
55	9.56767	26	9.76802	42	9.58342	25	9.79327	42	55
56	.56794	26	.76844	42	.58367	26	.79369	42	56
57	.56820	26	.76886	42	.58393	26	.79411	42	57
58	.56847	26	.76928	42	.58419	26	.79453	42	58
59	.56873	26	.76970	42	.58445	26	.79495	42	59
60	9.56900		9.77012		9.58471		9.79537		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	42	42
6	4.2	4.2
7	4.9	4.9
8	5.6	5.6
9	6.4	6.3
10	7.1	7.0
20	14.1	14.0
30	21.2	21.0
40	28.3	28.0
50	35.4	35.0

	27	27
6	2.7	2.7
7	3.2	3.1
8	3.6	3.6
9	4.1	4.0
10	4.6	4.5
20	9.1	9.0
30	13.7	13.5
40	18.3	18.0
50	22.9	22.5

	26	26
6	2.6	2.6
7	3.1	3.0
8	3.5	3.4
9	4.0	3.9
10	4.4	4.3
20	8.8	8.6
30	13.2	13.0
40	17.6	17.3
50	22.1	21.6

	25
6	2.5
7	3.0
8	3.4
9	3.8
10	4.2
20	8.5
30	12.7
40	17.0
50	21.2

P. P.

′	52° Log. Vers.	D	52° Log. Exsec.	D	53° Log. Vers.	D	53° Log. Exsec.	D	′
0	9.58471	26	9.79537	42	9.60008	25	9.82062	42	0
1	.58497	25	.79579	42	.60034	25	.82104	42	1
2	.58523	26	.79621	42	.60059	25	.82146	42	2
3	.58549	26	.79663	42	.60084	25	.82188	42	3
4	.58575	26	.79705	42	.60110	25	.82230	42	4
5	9.58601	25	9.79747	42	9.60135	25	9.82272	42	5
6	.58626	26	.79789	42	.60160	25	.82315	42	6
7	.58652	26	.79831	42	.60185	25	.82357	42	7
8	.58678	25	.79874	42	.60211	25	.82399	42	8
9	.58704	26	.79916	42	.60236	25	.82441	42	9
10	9.58730	25	9.79958	42	9.60261	25	9.82483	42	10
11	.58755	26	.80000	42	.60286	25	.82525	42	11
12	.58781	26	.80042	42	.60312	25	.82567	42	12
13	.58807	25	.80084	42	.60337	25	.82609	42	13
14	.58833	26	.80126	42	.60362	25	.82651	42	14
15	9.58859	25	9.80168	42	9.60387	25	9.82694	42	15
16	.58884	26	.80210	42	.60412	25	.82736	42	16
17	.58910	25	.80252	42	.60438	25	.82778	42	17
18	.58936	26	.80294	42	.60463	25	.82820	42	18
19	.58962	25	.80336	42	.60488	25	.82862	42	19
20	9.58987	25	9.80378	42	9.60513	25	9.82904	42	20
21	.59013	26	.80420	42	.60538	25	.82946	42	21
22	.59039	25	.80463	42	.60563	25	.82988	42	22
23	.59064	26	.80505	42	.60589	25	.83031	42	23
24	.59090	25	.80547	42	.60614	25	.83073	42	24
25	9.59116	25	9.80589	42	9.60639	25	9.83115	42	25
26	.59141	26	.80631	42	.60664	25	.83157	42	26
27	.59167	25	.80673	42	.60689	25	.83199	42	27
28	.59193	25	.80715	42	.60714	25	.83241	42	28
29	.59218	25	.80757	42	.60739	25	.83283	42	29
30	9.59244	26	9.80799	42	9.60764	25	9.83325	42	30
31	.59270	25	.80841	42	.60789	25	.83368	42	31
32	.59295	25	.80883	42	.60814	25	.83410	42	32
33	.59321	25	.80925	42	.60839	25	.83452	42	33
34	.59346	25	.80968	42	.60864	25	.83494	42	34
35	9.59372	25	9.81010	42	9.60889	25	9.83536	42	35
36	.59397	26	.81052	42	.60914	25	.83578	42	36
37	.59423	25	.81094	42	.60939	25	.83620	42	37
38	.59449	25	.81136	42	.60964	25	.83663	42	38
39	.59474	25	.81178	42	.60989	25	.83705	42	39
40	9.59500	25	9.81220	42	9.61014	25	9.83747	42	40
41	.59525	25	.81262	42	.61039	25	.83789	42	41
42	.59551	25	.81304	42	.61064	25	.83831	42	42
43	.59576	25	.81346	42	.61089	25	.83873	42	43
44	.59602	25	.81388	42	.61114	25	.83916	42	44
45	9.59627	25	9.81430	42	9.61139	25	9.83958	42	45
46	.59653	25	.81473	42	.61164	25	.84000	42	46
47	.59678	25	.81515	42	.61189	24	.84042	42	47
48	.59704	25	.81557	42	.61214	25	.84084	42	48
49	.59729	25	.81599	42	.61239	25	.84126	42	49
50	9.59754	25	9.81641	42	9.61264	25	9.84168	42	50
51	.59780	25	.81683	42	.61289	24	.84211	42	51
52	.59805	25	.81725	42	.61313	25	.84253	42	52
53	.59831	25	.81767	42	.61338	25	.84295	42	53
54	.59856	25	.81809	42	.61363	25	.84337	42	54
55	9.59881	25	9.81851	42	9.61388	24	9.84379	42	55
56	.59907	25	.81894	42	.61413	25	.84422	42	56
57	.59932	25	.81936	42	.61438	24	.84464	42	57
58	.59958	25	.81978	42	.61462	25	.84506	42	58
59	.59983	25	.82020	42	.61487	25	.84548	42	59
60	9.60008		9.82062		9.61512		9.84590		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	42	42
6	4.2	4.2
7	4.9	4.9
8	5.6	5.6
9	6.4	6.3
10	7.1	7.0
20	14.1	14.0
30	21.2	21.0
40	28.3	28.0
50	35.4	35.0

	26	25
6	2.6	2.5
7	3.0	3.0
8	3.4	3.4
9	3.9	3.8
10	4.3	4.2
20	8.6	8.5
30	13.0	12.7
40	17.3	17.0
50	21.6	21.2

	25	24
6	2.5	2.4
7	2.9	2.8
8	3.3	3.2
9	3.7	3.7
10	4.1	4.1
20	8.3	8.1
30	12.5	12.2
40	16.6	16.3
50	20.8	20.4

P. P.

	54°				55°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.61512̂	24̂	9.84596̂	42	9.62984	24̂	9.87125	42̂	0
1	.61537	25	.84632̂	42̂	.63008̂	24	.87167̂	42	1
2	.61562	24̂	.84675	42	.63032̂	24̂	.87209̂	42̂	2
3	.61586̂	25	.84717	42	.63057	24	.87252	42	3
4	.61611̂	24̂	.84759	42̂	.63081	24̂	.87294	42̂	4
5	9.61636	25	9.84801̂	42	9.63105̂	24	9.87336̂	42̂	5
6	.61661	24̂	.84843̂	42̂	.63129̂	24̂	.87379	42	6
7	.61685̂	25	.84886	42	.63154	24	.87421	42̂	7
8	.61710̂	24̂	.84928	42	.63178	24	.87463̂	42̂	8
9	.61735	25	.84970	42̂	.63202	24̂	.87506	42	9
10	9.61760	24̂	9.85012̂	42	9.63226̂	24	9.87548	42̂	10
11	.61784̂	24̂	.85054̂	42̂	.63250̂	24	.87590̂	42̂	11
12	.61809	25	.85097	42	.63274̂	24̂	.87633	42	12
13	.61834	24̂	.85139	42	.63299	24	.87675	42̂	13
14	.61858̂	25	.85181	42̂	.63323	24	.87717̂	42̂	14
15	9.61883̂	24̂	9.85223̂	42	9.63347	24̂	9.87760	42̂	15
16	.61908	24̂	.85265̂	42̂	.63371̂	24	.87802̂	42	16
17	.61932̂	24̂	.85308	42	.63395̂	24	.87844̂	42̂	17
18	.61957	25	.85350	42̂	.63419̂	24	.87887	42̂	18
19	.61982	24̂	.85392̂	42	.63443̂	24̂	.87929̂	42	19
20	9.62006̂	24̂	9.85434̂	42	9.63468	24	9.87971̂	42̂	20
21	.62031	24̂	.85476̂	42̂	.63492	24	.88014	42̂	21
22	.62055̂	25	.85519	42	.63516	24	.88056̂	42̂	22
23	.62080̂	24̂	.85561	42̂	.63540	24	.88099	42	23
24	.62105	24̂	.85603̂	42	.63564	24	.88141	42̂	24
25	9.62129̂	24̂	9.85645̂	42̂	9.63588	24	9.88183̂	42̂	25
26	.62154	24̂	.85688	42	.63612	24	.88226	42̂	26
27	.62178̂	24̂	.85730	42̂	.63636	24̂	.88268̂	42	27
28	.62203	24̂	.85772̂	42	.63660̂	24	.88310̂	42̂	28
29	.62227̂	24̂	.85814̂	42̂	.63684̂	24	.88353	42̂	29
30	9.62252	24̂	9.85857	42	9.63708̂	24	9.88395̂	42̂	30
31	.62276̂	24̂	.85899	42̂	.63732̂	24	.88438	42̂	31
32	.62301	24̂	.85941̂	42	.63756̂	24	.88480̂	42	32
33	.62325̂	24̂	.85983̂	42̂	.63780̂	24	.88522̂	42̂	33
34	.62350	24̂	.86026	42	.63804̂	24	.88565	42̂	34
35	9.62374̂	24̂	9.86068	42̂	9.63828̂	24̂	9.88607̂	42̂	35
36	.62399	24̂	.86110̂	42	.63852	24	.88650	42̂	36
37	.62423̂	24̂	.86152̂	42̂	.63876	24	.88692̂	42	37
38	.62448	24̂	.86195	42	.63900	24	.88734̂	42̂	38
39	.62472̂	24̂	.86237	42̂	.63924	24	.88777	42̂	39
40	9.62497	24̂	9.86279̂	42	9.63948	24	9.88819̂	42̂	40
41	.62521̂	24̂	.86321̂	42̂	.63972	24	.88862	42̂	41
42	.62546	24̂	.86364	42	.63996	23̂	.88904̂	42̂	42
43	.62570̂	24	.86406	42̂	.64019̂	24	.88947	42̂	43
44	.62594̂	24̂	.86448̂	42	.64043̂	24	.88989̂	42	44
45	9.62619	24̂	9.86490̂	42̂	9.64067̂	24	9.89031̂	42̂	45
46	.62643̂	24̂	.86533	42	.64091̂	24	.89074	42̂	46
47	.62668	24̂	.86575	42̂	.64115̂	23̂	.89116̂	42	47
48	.62692̂	24	.86617̂	42	.64139	24	.89159	42̂	48
49	.62716̂	24̂	.86659̂	42̂	.64163	24	.89201̂	42̂	49
50	9.62741	24̂	9.86702	42̂	9.64187	23̂	9.89244	42̂	50
51	.62765̂	24	.86744̂	42	.64210̂	24	.89286̂	42̂	51
52	.62789̂	24̂	.86786̂	42̂	.64234̂	24	.89329	42̂	52
53	.62814	24̂	.86829	42	.64258̂	23̂	.89371̂	42̂	53
54	.62838̂	24	.86871	42̂	.64282	24	.89414	42̂	54
55	9.62862̂	24̂	9.86913̂	42̂	9.64306	24	9.89456̂	42̂	55
56	.62887	24̂	.86956	42	.64330	23̂	.89499	42	56
57	.62911̂	24	.86998	42̂	.64353̂	24	.89541	42̂	57
58	.62935̂	24̂	.87040̂	42	.64377̂	23̂	.89583̂	42̂	58
59	.62960	24	.87082̂	42̂	.64401	24	.89626	42̂	59
60	9.62984		9.87125		9.64425		9.89668̂		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	42̂	42
6	4.2	4.2
7	4.9	4.9
8	5.6	5.6
9	6.4	6.3
10	7.1	7.0
20	14.1̂	14.0
30	21.2	21.0
40	28.3̂	28.0
50	35.4	35.0

	25	24̂
6	2.5	2.4̂
7	2.9	2.8̂
8	3.3̂	3.2
9	3.7	3.7
10	4.1̂	4.1̂
20	8.3̂	8.1̂
30	12.5	12.2̂
40	16.6̂	16.3̂
50	20.8̂	20.4

	24	23̂
6	2.4	2.3̂
7	2.8	2.7̂
8	3.2	3.1̂
9	3.6	3.5
10	4.0	3.9
20	8.0	7.8̂
30	12.0	11.7
40	16.0	15.6
50	20.0	19.6

P. P.

′	56° Log. Vers.	D	56° Log. Exsec.	D	57° Log. Vers.	D	57° Log. Exsec.	D	′
0	9.64425	23	9.89668	42	9.65835	23	9.92224	43	0
1	.64448	24	.89711	42	.65859	23	.92267	42	1
2	.64472	23	.89753	42	.65882	23	.92310	43	2
3	.64496	24	.89796	42	.65905	23	.92353	42	3
4	.64520	23	.89838	42	.65928	23	.92395	43	4
5	9.64543	24	9.89881	42	9.65952	23	9.92438	42	5
6	.64567	23	.89923	42	.65975	23	.92481	43	6
7	.64591	23	.89966	42	.65998	23	.92524	42	7
8	.64614	24	.90008	42	.66021	23	.92566	43	8
9	.64638	23	.90051	43	.66044	23	.92609	42	9
10	9.64662	23	9.90094	42	9.66068	23	9.92652	43	10
11	.64685	24	.90136	42	.66091	23	.92695	42	11
12	.64709	23	.90179	42	.66114	23	.92737	43	12
13	.64733	23	.90221	42	.66137	23	.92780	42	13
14	.64756	24	.90264	42	.66160	23	.92823	43	14
15	9.64780	23	9.90306	42	9.66183	23	9.92866	43	15
16	.64804	23	.90349	42	.66207	23	.92909	42	16
17	.64827	23	.90391	42	.66230	23	.92951	43	17
18	.64851	24	.90434	42	.66253	23	.92994	42	18
19	.64875	23	.90476	42	.66276	23	.93037	43	19
20	9.64898	23	9.90519	42	9.66299	23	9.93080	43	20
21	.64922	23	.90561	43	.66322	23	.93123	42	21
22	.64945	23	.90604	42	.66345	23	.93165	43	22
23	.64969	23	.90647	42	.66368	23	.93208	43	23
24	.64992	24	.90689	42	.66391	23	.93251	42	24
25	9.65016	23	9.90732	42	9.66415	23	9.93294	43	25
26	.65040	23	.90774	42	.66438	23	.93337	43	26
27	.65063	23	.90817	43	.66461	23	.93380	42	27
28	.65087	23	.90860	42	.66484	23	.93422	43	28
29	.65110	23	.90902	42	.66507	23	.93465	43	29
30	9.65134	23	9.90945	42	9.66530	23	9.93508	42	30
31	.65157	23	.90987	42	.66553	23	.93551	43	31
32	.65181	23	.91030	43	.66576	23	.93594	43	32
33	.65204	23	.91073	42	.66599	23	.93637	43	33
34	.65228	23	.91115	42	.66622	23	.93680	42	34
35	9.65251	23	9.91158	42	9.66645	23	9.93722	43	35
36	.65275	23	.91200	42	.66668	23	.93765	43	36
37	.65298	23	.91243	43	.66691	23	.93808	43	37
38	.65321	23	.91286	42	.66714	23	.93851	42	38
39	.65345	23	.91328	42	.66737	23	.93894	43	39
40	9.65368	23	9.91371	43	9.66760	23	9.93937	43	40
41	.65392	23	.91414	42	.66783	23	.93980	43	41
42	.65415	23	.91456	42	.66805	23	.94023	43	42
43	.65439	23	.91499	42	.66828	23	.94066	43	43
44	.65462	23	.91541	43	.66851	23	.94109	42	44
45	9.65485	23	9.91584	42	9.66874	23	9.94151	43	45
46	.65509	23	.91627	42	.66897	23	.94194	43	46
47	.65532	23	.91669	43	.66920	22	.94237	43	47
48	.65556	23	.91712	42	.66943	23	.94280	43	48
49	.65579	23	.91755	42	.66966	23	.94323	43	49
50	9.65602	23	9.91797	43	9.66989	23	9.94366	43	50
51	.65626	23	.91840	42	.67012	22	.94409	43	51
52	.65649	23	.91883	43	.67034	23	.94452	43	52
53	.65672	23	.91926	42	.67057	23	.94495	43	53
54	.65696	23	.91968	42	.67080	22	.94538	43	54
55	9.65719	23	9.92011	43	9.67103	23	9.94581	43	55
56	.65742	23	.92054	42	.67126	23	.94624	43	56
57	.65765	23	.92096	43	.67149	22	.94667	43	57
58	.65789	23	.92139	42	.67171	23	.94710	43	58
59	.65812	23	.92182	42	.67194	22	.94753	43	59
60	9.65835		9.92224		9.67217		9.94796		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	43	42
6	4.3	4.2
7	5.0	4.9
8	5.7	5.6
9	6.4	6.4
10	7.1	7.1
20	14.3	14.1
30	21.5	21.2
40	28.6	28.3
50	35.8	35.4

	24	23
6	2.4	2.3
7	2.8	2.7
8	3.2	3.1
9	3.6	3.5
10	4.0	3.9
20	8.0	7.8
30	12.0	11.7
40	16.0	15.6
50	20.0	19.6

	23	22
6	2.3	2.2
7	2.7	2.6
8	3.0	3.0
9	3.4	3.4
10	3.8	3.7
20	7.6	7.5
30	11.5	11.2
40	15.3	15.0
50	19.1	18.7

P. P.

′	58° Log. Vers.	D	58° Log. Exsec.	D	59° Log. Vers.	D	59° Log. Exsec.	D	′
0	9.67217	23	9.94796	43	9.68571	22	9.97387	43	0
1	.67240	23	.94839	43	.68593	22	.97430	43	1
2	.67263	22	.94882	43	.68615	22	.97473	43	2
3	.67285	23	.94925	43	.68637	22	.97517	43	3
4	.67308	22	.94968	43	.68660	22	.97560	43	4
5	9.67331	23	9.95011	43	9.68682	22	9.97603	43	5
6	.67354	22	.95054	43	.68704	22	.97647	43	6
7	.67376	23	.95097	43	.68727	22	.97690	43	7
8	.67399	22	.95140	43	.68749	22	.97734	43	8
9	.67422	23	.95183	43	.68771	22	.97777	43	9
10	9.67445	22	9.95226	43	9.68793	22	9.97820	43	10
11	.67467	22	.95269	43	.68816	22	.97864	43	11
12	.67490	23	.95313	43	.68838	22	.97907	43	12
13	.67513	22	.95356	43	.68860	22	.97951	43	13
14	.67535	22	.95399	43	.68882	22	.97994	43	14
15	9.67558	23	9.95442	43	9.68905	22	9.98038	43	15
16	.67581	22	.95485	43	.68927	22	.98081	43	16
17	.67603	22	.95528	43	.68949	22	.98125	43	17
18	.67626	23	.95571	43	.68971	22	.98168	43	18
19	.67649	22	.95614	43	.68993	22	.98211	43	19
20	9.67671	22	9.95657	43	9.69016	22	9.98255	43	20
21	.67694	23	.95700	43	.69038	22	.98298	43	21
22	.67717	22	.95744	43	.69060	22	.98342	43	22
23	.67739	22	.95787	43	.69082	22	.98385	43	23
24	.67762	22	.95830	43	.69104	22	.98429	43	24
25	9.67784	22	9.95873	43	9.69126	22	9.98472	43	25
26	.67807	23	.95916	43	.69149	22	.98516	43	26
27	.67830	22	.95959	43	.69171	22	.98559	44	27
28	.67852	22	.96002	43	.69193	22	.98603	43	28
29	.67875	22	.96046	43	.69215	22	.98647	43	29
30	9.67897	22	9.96089	43	9.69237	22	9.98690	43	30
31	.67920	22	.96132	43	.69259	22	.98734	43	31
32	.67942	22	.96175	43	.69281	22	.98777	43	32
33	.67965	22	.96218	43	.69303	22	.98821	43	33
34	.67987	22	.96261	43	.69325	22	.98864	43	34
35	9.68010	22	9.96305	43	9.69347	22	9.98908	44	35
36	.68032	22	.96348	43	.69369	22	.98952	43	36
37	.68055	22	.96391	43	.69392	22	.98995	43	37
38	.68077	22	.96434	43	.69414	22	.99039	43	38
39	.68100	22	.96478	43	.69436	22	.99082	43	39
40	9.68122	22	9.96521	43	9.69458	22	9.99126	44	40
41	.68145	22	.96564	43	.69480	22	.99170	43	41
42	.68167	22	.96607	43	.69502	22	.99213	43	42
43	.68190	22	.96650	43	.69524	22	.99257	43	43
44	.68212	22	.96694	43	.69546	22	.99300	44	44
45	9.68235	22	9.96737	43	9.69568	22	9.99344	43	45
46	.68257	22	.96780	43	.69590	22	.99388	43	46
47	.68280	22	.96824	43	.69612	22	.99431	44	47
48	.68302	22	.96867	43	.69634	22	.99475	43	48
49	.68324	22	.96910	43	.69656	22	.99519	43	49
50	9.68347	22	9.96953	43	9.69678	22	9.99562	44	50
51	.68369	22	.96997	43	.69700	21	.99606	43	51
52	.68392	22	.97040	43	.69721	22	.99650	44	52
53	.68414	22	.97083	43	.69743	22	.99694	43	53
54	.68436	22	.97127	43	.69765	22	.99737	44	54
55	9.68459	22	9.97170	43	9.69787	22	9.99781	43	55
56	.68481	22	.97213	43	.69809	22	.99825	43	56
57	.68503	22	.97257	43	.69831	21	.99868	44	57
58	.68526	22	.97300	43	.69853	22	.99912	43	58
59	.68548	22	.97343	43	.69875	22	9.99956	44	59
60	9.68571		9.97387		9.69897		10.00000		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	44	43
6	4.4	4.3
7	5.1	5.1
8	5.8	5.8
9	6.6	6.5
10	7.3	7.2
20	14.6	14.5
30	22.0	21.7
40	29.3	29.0
50	36.6	36.2

	43
6	4.3
7	5.0
8	5.7
9	6.4
10	7.1
20	14.3
30	21.5
40	28.6
50	35.8

	23	22
6	2.3	2.2
7	2.7	2.6
8	3.0	3.0
9	3.4	3.4
10	3.8	3.7
20	7.6	7.5
30	11.5	11.2
40	15.3	15.0
50	19.1	18.7

	22	21
6	2.2	2.1
7	2.5	2.5
8	2.9	2.8
9	3.3	3.2
10	3.6	3.6
20	7.3	7.1
30	11.0	10.7
40	14.6	14.3
50	18.3	17.9

	60°				61°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.69897	22	10.00000	44	9.71197	21	10.02639̂	44̂	**0**
1	.69919	21̂	.00044	43̂	.71218	21̂	.02684	44	1
2	.69940̂	22	.00087̂	44	.71239̂	21̂	.02728	44̂	2
3	.69962̂	22	.00131̂	43̂	.71261	21̂	.02772̂	44	3
4	.69984̂	22	.00175	44	.71282̂	21̂	.02816̂	44̂	4
5	9.70006̂	21̂	10.00219	43̂	9.71304	21̂	10.02861	44	5
6	.70028	22	.00262̂	44	.71325̂	21	.02905	44̂	6
7	.70050	22	.00306̂	44	.71346̂	21̂	.02949̂	44̂	7
8	.70072	21̂	.00350̂	43̂	.71368	21̂	.02994	44	8
9	.70093̂	22	.00394	44	.71389̂	21̂	.03038	44̂	9
10	9.70115̂	21̂	10.00438	44	9.71411	21̂	10.03082̂	44̂	**10**
11	.70137	22	.00482	43̂	.71432̂	21	.03127	44	11
12	.70159	22	.00525̂	44	.71453̂	21̂	.03171	44̂	12
13	.70181	21̂	.00569̂	44	.71475	21̂	.03215̂	44̂	13
14	.70202̂	22	.00613̂	44	.71496̂	21	.03260	44	14
15	9.70224̂	21̂	10.00657̂	43̂	9.71517̂	21̂	10.03304	44̂	15
16	.70246	22	.00701	44	.71539	21̂	.03348̂	44̂	16
17	.70268	21̂	.00745	44	.71560̂	21	.03393	44̂	17
18	.70289̂	22	.00789	44	.71581̂	21̂	.03437̂	44	18
19	.70311̂	21̂	.00833	43̂	.71603	21̂	.03481̂	44̂	19
20	9.70333	22	10.00876̂	44	9.71624̂	21	10.03526	44̂	**20**
21	.70355	21̂	.00920̂	44	.71645̂	21̂	.03570̂	44̂	21
22	.70376̂	22	.00964̂	44	.71667	21	.03615	44̂	22
23	.70398̂	21̂	.01008̂	44	.71688	21̂	.03659̂	44̂	23
24	.70420	21̂	.01052̂	44	.71709̂	21	.03704	44̂	24
25	9.70441̂	22	10.01096̂	44	9.71730̂	21̂	10.03748̂	44̂	25
26	.70463̂	21̂	.01140̂	44	.71752	21	.03793	44	26
27	.70485	22	.01184̂	44	.71773	21̂	.03837	44̂	27
28	.70507	21̂	.01228̂	44	.71794̂	21	.03881̂	44̂	28
29	.70528̂	21̂	.01272̂	44	.71815̂	21̂	.03926	44̂	29
30	9.70550	22	10.01316̂	44	9.71837	21	10.03970̂	44̂	**30**
31	.70572	21̂	.01360̂	44	.71858	21̂	.04015	44̂	31
32	.70593̂	21̂	.01404̂	44	.71879̂	21	.04059̂	44̂	32
33	.70615	21̂	.01448̂	44	.71900̂	21̂	.04104	45	33
34	.70636̂	22	.01492̂	44	.71922	21	.04149	44̂	34
35	9.70658̂	21̂	10.01536̂	44	9.71943	21̂	10.04193̂	44̂	35
36	.70680	21̂	.01580̂	44	.71964̂	21	.04238	44̂	36
37	.70701̂	21̂	.01624̂	44	.71985̂	21	.04282̂	44̂	37
38	.70723	22	.01668̂	44	.72006̂	21̂	.04327	44̂	38
39	.70745	21̂	.01712̂	44	.72028	21	.04371̂	44̂	39
40	9.70766̂	21̂	10.01756̂	44	9.72049	21	10.04416	45	**40**
41	.70788	21̂	.01800̂	44	.72070	21̂	.04461	44̂	41
42	.70809̂	21̂	.01844̂	44̂	.72091̂	21	.04505̂	44̂	42
43	.70831	21̂	.01889	44	.72112̂	21	.04550	44̂	43
44	.70852̂	22	.01933	44	.72133̂	21	.04594̂	45	44
45	9.70874̂	21̂	10.01977	44	9.72154̂	21̂	10.04639̂	44̂	45
46	.70896	21̂	.02021	44̂	.72176	21	.04684	44̂	46
47	.70917̂	21̂	.02065̂	44	.72197	21	.04728̂	44̂	47
48	.70939	21̂	.02109̂	44	.72218	21	.04773	45	48
49	.70960̂	21̂	.02153̂	44	.72239	21̂	.04818	44̂	49
50	9.70982	21̂	10.02197̂	44̂	9.72260̂	21	10.04862̂	45	**50**
51	.71003̂	21̂	.02242	44	.72281̂	21	.04907̂	44̂	51
52	.71025	21̂	.02286	44	.72302̂	21	.04952	44̂	52
53	.71046̂	21̂	.02330	44̂	.72323̂	21	.04996̂	45	53
54	.71068	21̂	.02374̂	44	.72344̂	21	.05041̂	44̂	54
55	9.71089̂	21̂	10.02418̂	44̂	9.72365̂	21	10.05086	45	55
56	.71111	21̂	.02463	44	.72386̂	21̂	.05131	44̂	56
57	.71132̂	21̂	.02507	44	.72408	21	.05175̂	45	57
58	.71154	21̂	.02551	44̂	.72429	21	.05220̂	44̂	58
59	.71175̂	21̂	.02595̂	44	.72450	21	.05265	45	59
60	9.71197		10.02639̂		9.72471		10.05310		**60**
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	45	44̂
6	4.5	4.4̂
7	5.2̂	5.2
8	6.0	5.9̂
9	6.7̂	6.7
10	7.5	7.4
20	15.0	14.8̂
30	22.5	22.2̂
40	30.0	29.6̂
50	37.5	37.1

	44	43̂
6	4.4	4.3̂
7	5.1̂	5.1
8	5.8̂	5.8
9	6.6	6.5
10	7.3̂	7.2̂
20	14.6̂	14.5̂
30	22.0	21.7̂
40	29.3̂	29.0
50	36.6̂	36.2̂

	22	21̂
6	2.2	2.1̂
7	2.5̂	2.5
8	2.9̂	2.8̂
9	3.3	3.2
10	3.6̂	3.6
20	7.3̂	7.1̂
30	11.0	10.7̂
40	14.6̂	14.3̂
50	18.3̂	17.9

	21
6	2.1
7	2.4̂
8	2.8
9	3.1̂
10	3.5
20	7.0
30	10.5
40	14.0
50	17.5

P. P.

′	62° Log. Vers.	D	62° Log. Exsec.	D	63° Log. Vers.	D	63° Log. Exsec.	D	′
0	9.72471	21	10.05310	44	9.73720	20	10.08015	45	0
1	.72492	21	.05354	45	.73740	20	.08061	45	1
2	.72513	21	.05399	45	.73761	21	.08106	45	2
3	.72534	21	.05444	44	.73782	20	.08151	45	3
4	.72555	21	.05489	45	.73802	20	.08197	45	4
5	9.72576	21	10.05534	45	9.73823	20	10.08242	45	5
6	.72597	21	.05579	44	.73843	20	.08288	45	6
7	.72618	21	.05623	45	.73864	20	.08333	45	7
8	.72639	21	.05668	45	.73884	21	.08379	45	8
9	.72660	21	.05713	45	.73905	20	.08424	45	9
10	9.72681	20	10.05758	44	9.73926	20	10.08470	45	10
11	.72701	21	.05803	45	.73946	20	.08515	45	11
12	.72722	21	.05848	45	.73967	20	.08561	45	12
13	.72743	21	.05893	45	.73987	20	.08606	45	13
14	.72764	21	.05938	45	.74008	20	.08652	45	14
15	9.72785	21	10.05983	45	9.74028	20	10.08697	46	15
16	.72806	21	.06028	44	.74049	20	.08743	45	16
17	.72827	20	.06072	45	.74069	20	.08789	45	17
18	.72848	21	.06117	45	.74090	20	.08834	45	18
19	.72869	21	.06162	45	.74110	20	.08880	46	19
20	9.72890	21	10.06207	45	9.74131	20	10.08926	45	20
21	.72911	20	.06252	45	.74151	20	.08971	45	21
22	.72931	21	.06297	45	.74172	20	.09017	45	22
23	.72952	21	.06342	45	.74192	20	.09062	46	23
24	.72973	21	.06387	45	.74213	20	.09108	45	24
25	9.72994	20	10.06432	45	9.74233	20	10.09154	46	25
26	.73015	21	.06477	45	.74254	20	.09200	45	26
27	.73036	21	.06522	45	.74274	20	.09245	45	27
28	.73057	20	.06568	45	.74294	20	.09291	46	28
29	.73077	21	.06613	45	.74315	20	.09337	45	29
30	9.73098	21	10.06658	45	9.74335	20	10.09382	46	30
31	.73119	20	.06703	45	.74356	20	.09428	46	31
32	.73140	21	.06748	45	.75376	20	.09474	45	32
33	.73161	20	.06793	45	.74396	20	.09520	46	33
34	.73181	21	.06838	45	.74417	20	.09566	45	34
35	9.73202	21	10.06883	45	9.74437	20	10.09611	46	35
36	.73223	20	.06928	45	.74458	20	.09657	46	36
37	.73244	21	.06974	45	.74478	20	.09703	45	37
38	.73265	20	.07019	45	.74498	20	.09749	46	38
39	.73285	21	.07064	45	.74519	20	.09795	46	39
40	9.73306	20	10.07109	45	9.74539	20	10.09841	45	40
41	.73327	21	.07154	45	.74559	20	.09886	46	41
42	.73348	20	.07200	45	.74580	20	.09932	46	42
43	.73368	21	.07245	45	.74600	20	.09978	46	43
44	.73389	20	.07290	45	.74620	20	.10024	46	44
45	9.73410	20	10.07335	45	9.74641	20	10.10070	46	45
46	.73430	21	.07380	45	.74661	20	.10116	46	46
47	.73451	20	.07426	45	.74681	20	.10162	46	47
48	.73472	21	.07471	45	.74702	20	.10208	45	48
49	.73493	20	.07516	45	.74722	20	.10254	46	49
50	9.73513	20	10.07562	45	9.74742	20	10.10300	46	50
51	.73534	21	.07607	45	.74762	20	.10346	46	51
52	.73555	20	.07652	45	.74783	20	.10392	46	52
53	.73575	20	.07697	45	.74803	20	.10438	46	53
54	.73596	21	.07743	45	.74823	20	.10484	46	54
55	9.73617	20	10.07788	45	9.74844	20	10.10530	46	55
56	.73637	20	.07834	45	.74864	20	.10576	46	56
57	.73658	21	.07879	45	.74884	20	.10622	46	57
58	.73679	20	.07924	45	.74904	20	.10668	46	58
59	.73699	20	.07970	45	.74924	20	.10714	46	59
60	9.73720		10.08015		9.74945		10.10760		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	46	46
6	4.6	4.6
7	5.4	5.3
8	6.2	6.1
9	7.0	6.9
10	7.7	7.6
20	15.5	15.3
30	23.2	23.0
40	31.0	30.6
50	38.7	38.3

	45	45
6	4.5	4.5
7	5.3	5.2
8	6.0	6.0
9	6.8	6.7
10	7.6	7.5
20	15.1	15.0
30	22.7	22.5
40	30.3	30.0
50	37.9	37.5

	44
6	4.4
7	5.2
8	5.9
9	6.7
10	7.4
20	14.8
30	22.2
40	29.6
50	37.1

	21	20
6	2.1	2.0
7	2.4	2.4
8	2.8	2.7
9	3.1	3.1
10	3.5	3.4
20	7.0	6.8
30	10.5	10.2
40	14.0	13.6
50	17.5	17.1

	20
6	2.0
7	2.3
8	2.6
9	3.0
10	3.3
20	6.6
30	10.0
40	13.3
50	16.6

′	64° Log. Vers.	D	64° Log. Exsec.	D	65° Log. Vers.	D	65° Log. Exsec.	D	′
0	9.74945	20	10.10760	46	9.76146	19	10.13551	47	0
1	.74965	20	.10807	46	.76166	20	.13598	47	1
2	.74985	20	.10853	46	.76186	20	.13645	47	2
3	.75005	20	.10899	46	.76206	19	.13692	47	3
4	.75026	20	.10945	46	.76225	20	.13739	47	4
5	9.75046	20	10.10991	46	9.76245	19	10.13786	47	5
6	.75066	20	.11037	46	.76265	20	.13833	47	6
7	.75086	20	.11084	46	.76285	19	.13880	47	7
8	.75106	20	.11130	46	.76304	20	.13927	47	8
9	.75126	20	.11176	46	.76324	20	.13974	47	9
10	9.75147	20	10.11222	46	9.76344	19	10.14021	47	10
11	.75167	20	.11269	46	.76364	20	.14068	47	11
12	.75187	20	.11315	46	.76384	19	.14115	47	12
13	.75207	20	.11361	46	.76403	20	.14162	47	13
14	.75227	20	.11407	46	.76423	19	.14210	47	14
15	9.75247	20	10.11454	46	9.76443	20	10.14257	47	15
16	.75267	20	.11500	46	.76463	19	.14304	47	16
17	.75287	20	.11546	46	.76482	19	.14351	47	17
18	.75308	20	.11593	46	.76502	20	.14398	47	18
19	.75328	20	.11639	46	.76522	19	.14445	47	19
20	9.75348	20	10.11685	46	9.76541	20	10.14493	47	20
21	.75368	20	.11732	46	.76561	19	.14540	47	21
22	.75388	20	.11778	46	.76581	19	.14587	47	22
23	.75408	20	.11825	46	.76600	20	.14634	47	23
24	.75428	20	.11871	46	.76620	19	.14682	47	24
25	9.75448	20	10.11917	46	9.76640	19	10.14729	47	25
26	.75468	20	.11964	46	.76659	20	.14776	47	26
27	.75488	20	.12010	46	.76679	19	.14823	47	27
28	.75508	20	.12057	46	.76699	19	.14871	47	28
29	.75528	20	.12103	46	.76718	20	.14918	47	29
30	9.75548	20	10.12150	46	9.76738	19	10.14965	47	30
31	.75568	20	.12196	46	.76758	19	.15013	47	31
32	.75588	20	.12243	46	.76777	19	.15060	47	32
33	.75608	20	.12289	46	.76797	20	.15108	47	33
34	.75628	20	.12336	47	.76817	19	.15155	47	34
35	9.75648	20	10.12383	46	9.76836	19	10.15202	47	35
36	.75668	20	.12429	46	.76856	19	.15250	47	36
37	.75688	20	.12476	46	.76875	20	.15297	47	37
38	.75708	20	.12522	46	.76895	19	.15345	47	38
39	.75728	20	.12569	47	.76915	19	.15392	47	39
40	9.75748	20	10.12616	46	9.76934	19	10.15440	47	40
41	.75768	20	.12662	46	.76954	19	.15487	47	41
42	.75788	20	.12709	47	.76973	19	.15535	47	42
43	.75808	19	.12756	46	.76993	19	.15582	47	43
44	.75828	20	.12802	46	.77012	20	.15630	48	44
45	9.75848	20	10.12849	47	9.77032	19	10.15678	47	45
46	.75868	20	.12896	46	.77052	19	.15725	47	46
47	.75888	20	.12942	47	.77071	19	.15773	47	47
48	.75908	20	.12989	46	.77091	19	.15820	48	48
49	.75928	19	.13036	47	.77110	19	.15868	47	49
50	9.75947	20	10.13083	47	9.77130	19	10.15916	47	50
51	.75967	20	.13130	46	.77149	19	.15963	48	51
52	.75987	20	.13176	47	.77169	19	.16011	47	52
53	.76007	19	.13223	46	.77188	19	.16059	47	53
54	.76027	20	.13270	47	.77208	19	.16106	48	54
55	9.76047	20	10.13317	47	9.77227	19	10.16154	47	55
56	.76067	20	.13364	47	.77247	19	.16202	48	56
57	.76087	19	.13411	46	.77266	19	.16250	48	57
58	.76106	20	.13457	47	.77286	19	.16298	47	58
59	.76126	20	.13504	47	.77305	19	.16345	48	59
60	9.76146		10.13551		9.77325		10.16393		60

P. P.

	48	47̂
6	4.8	4.7
7	5.6	5.5
8	6.4	6.3
9	7.2	7.1
10	8.0	7.9
20	16.0	15.8
30	24.0	23.7
40	32.0	31.6
50	40.0	39.6

	47	46̂
6	4.7	4.6
7	5.5	5.4
8	6.2	6.2
9	7.0	7.0
10	7.8	7.7
20	15.6	15.5
30	23.5	23.2
40	31.3	31.0
50	39.1	38.7

	46
6	4.6
7	5.3
8	6.1
9	6.9
10	7.6
20	15.3
30	23.0
40	30.6
50	38.3

	20̂	20
6	2.0	2.0
7	2.4	2.3
8	2.7	2.6
9	3.1	3.0
10	3.4	3.3
20	6.8	6.6
30	10.2	10.0
40	13.6	13.3
50	17.1	16.6

	19̂
6	1.9
7	2.3
8	2.6
9	2.9
10	3.2
20	6.5
30	9.7
40	13.0
50	16.2

	66°				67°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.77325	19	10.16393	48	9.78481	19	10.19293	49	0
1	.77344	19	.16441	47	.78500	19	.19342	49	1
2	.77363	19	.16489	48	.78519	19	.19391	48	2
3	.77383	19	.16537	48	.78538	19	.19439	49	3
4	.77402	19	.16585	48	.78557	19	.19488	49	4
5	9.77422	19	10.16633	47	9.78576	19	10.19537	49	5
6	.77441	19	.16680	48	.78595	19	.19586	49	6
7	.77461	19	.16728	48	.78614	19	.19635	49	7
8	.77480	19	.16776	48	.78633	19	.19684	49	8
9	.77499	19	.16824	48	.78652	19	.19733	49	9
10	9.77519	19	10.16872	48	9.78671	19	10.19782	49	10
11	.77538	19	.16920	48	.78690	19	.19831	49	11
12	.77557	19	.16968	48	.78709	19	.19880	49	12
13	.77577	19	.17016	48	.78728	19	.19929	49	13
14	.77596	19	.17064	48	.78747	19	.19979	49	14
15	9.77616	19	10.17112	48	9.78766	19	10.20028	49	15
16	.77635	19	.17160	48	.78785	19	.20077	49	16
17	.77654	19	.17209	48	.78804	19	.20126	49	17
18	.77674	19	.17257	48	.78823	19	.20175	49	18
19	.77693	19	.17305	48	.78842	19	.20224	49	19
20	9.77712	19	10.17353	48	9.78861	19	10.20273	49	20
21	.77732	19	.17401	48	.78880	19	.20323	49	21
22	.77751	19	.17449	48	.78899	19	.20372	49	22
23	.77770	19	.17498	48	.78918	18	.20421	49	23
24	.77790	19	.17546	48	.78937	19	.20470	49	24
25	9.77809	19	10.17594	48	9.78956	19	10.20520	49	25
26	.77828	19	.17642	48	.78975	19	.20569	49	26
27	.77847	19	.17690	48	.78994	19	.20618	49	27
28	.77867	19	.17739	48	.79013	19	.20668	49	28
29	.77886	19	.17787	48	.79032	19	.20717	49	29
30	9.77905	19	10.17835	48	9.79051	18	10.20767	49	30
31	.77925	19	.17884	48	.79069	19	.20816	49	31
32	.77944	19	.17932	48	.79088	19	.20865	49	32
33	.77963	19	.17980	48	.79107	19	.20915	49	33
34	.77982	19	.18029	48	.79126	19	.20964	49	34
35	9.78002	19	10.18077	48	9.79145	18	10.21014	49	35
36	.78021	19	.18126	48	.79164	19	.21063	49	36
37	.78040	19	.18174	48	.79183	19	.21113	49	37
38	.78059	19	.18222	48	.79202	18	.21162	50	38
39	.78078	19	.18271	48	.79220	19	.21212	49	39
40	9.78098	19	10.18319	48	9.79239	19	10.21262	49	40
41	.78117	19	.18368	48	.79258	18	.21311	49	41
42	.78136	19	.18416	49	.79277	19	.21361	49	42
43	.78155	19	.18465	48	.79296	19	.21410	50	43
44	.78174	19	.18514	48	.79315	18	.21460	49	44
45	9.78194	19	10.18562	48	9.79333	19	10.21510	50	45
46	.78213	19	.18611	48	.79352	19	.21560	49	46
47	.78232	19	.18659	48	.79371	18	.21609	50	47
48	.78251	19	.18708	49	.79390	19	.21659	49	48
49	.78270	19	.18757	48	.79409	18	.21709	50	49
50	9.78289	19	10.18805	48	9.79427	19	10.21759	49	50
51	.78309	19	.18854	49	.79446	19	.21808	50	51
52	.78328	19	.18903	48	.79465	18	.21858	50	52
53	.78347	19	.18951	49	.79484	19	.21908	49	53
54	.78366	19	.19000	48	.79503	18	.21958	50	54
55	9.78385	19	10.19049	49	9.79521	19	10.22008	50	55
56	.78404	19	.19098	48	.79540	18	.22058	50	56
57	.78423	19	.19146	49	.79559	19	.22108	50	57
58	.78442	19	.19195	49	.79578	18	.22158	50	58
59	.78462	19	.19244	48	.79596	19	.22208	50	59
60	9.78481		10.19293		9.79615		10.22258		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	50	49	49	48	48	47	19	19	18
6	5.0	4.9	4.9	4.8	4.8	4.7	1.9	1.9	1.8
7	5.8	5.8	5.7	5.6	5.6	5.5	2.3	2.2	2.1
8	6.6	6.6	6.5	6.4	6.4	6.3	2.6	2.5	2.4
9	7.5	7.4	7.3	7.3	7.2	7.1	2.9	2.8	2.8
10	8.3	8.2	8.1	8.1	8.0	7.9	3.2	3.1	3.1
20	16.6	16.5	16.3	16.1	16.0	15.8	6.5	6.3	6.1
30	25.0	24.7	24.5	24.2	24.0	23.7	9.7	9.5	9.2
40	33.3	33.0	32.6	32.3	32.0	31.6	13.0	12.6	12.3
50	41.6	41.2	40.8	40.4	40.0	39.6	16.2	15.8	15.4

	68°				69°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.79615	18	10.22258	50	9.80728	18	10.25295	51	0
1	.79634	19	.22308	50	.80747	18	.25347	51	1
2	.79653	18	.22358	50	.80765	18	.25398	51	2
3	.79671	18	.22408	50	.80783	18	.25449	51	3
4	.79690	19	.22458	50	.80802	18	.25501	51	4
5	9.79709	18	10.22508	50	9.80820	18	10.25552	51	5
6	.79727	19	.22558	50	.80839	18	.25604	51	6
7	.79746	18	.22608	50	.80857	18	.25655	51	7
8	.79765	18	.22658	50	.80875	18	.25707	51	8
9	.79783	19	.22708	50	.80894	18	.25758	51	9
10	9.79802	18	10.22759	50	9.80912	18	10.25810	51	10
11	.79821	18	.22809	50	.80930	18	.25861	51	11
12	.79839	19	.22859	50	.80949	18	.25913	51	12
13	.79858	18	.22909	50	.80967	18	.25964	51	13
14	.79877	18	.22960	50	.80985	18	.26016	51	14
15	9.79895	18	10.23010	50	9.81003	18	10.26067	52	15
16	.79914	19	.23060	50	.81022	18	.26119	51	16
17	.79933	18	.23110	50	.81040	18	.26171	51	17
18	.79951	18	.23161	50	.81058	18	.26222	52	18
19	.79970	18	.23211	50	.81077	18	.26274	51	19
20	9.79988	19	10.23262	50	9.81095	18	10.26326	52	20
21	.80007	18	.23312	50	.81113	18	.26378	51	21
22	.80026	18	.23362	50	.81131	18	.26429	52	22
23	.80044	18	.23413	50	.81150	18	.26481	52	23
24	.80063	18	.23463	50	.81168	18	.26533	52	24
25	9.80081	19	10.23514	50	9.81186	18	10.26585	51	25
26	.80100	18	.23564	50	.81204	18	.26637	52	26
27	.80119	18	.23615	50	.81223	18	.26689	52	27
28	.80137	18	.23666	50	.81241	18	.26741	52	28
29	.80156	18	.23716	50	.81259	18	.26793	52	29
30	9.80174	18	10.23767	50	9.81277	18	10.26845	52	30
31	.80193	18	.23817	51	.81295	18	.26897	52	31
32	.80211	18	.23868	50	.81314	18	.26949	52	32
33	.80230	18	.23919	50	.81332	18	.27001	52	33
34	.80248	19	.23969	51	.81350	18	.27053	52	34
35	9.80267	18	10.24020	50	9.81368	18	10.27105	52	35
36	.80286	18	.24071	51	.81386	18	.27157	52	36
37	.80304	18	.24122	50	.81405	18	.27209	52	37
38	.80323	18	.24172	51	.81423	18	.27261	52	38
39	.80341	18	.24223	51	.81441	18	.27314	52	39
40	9.80360	18	10.24274	50	9.81459	18	10.27366	52	40
41	.80378	18	.24325	51	.81477	18	.27418	52	41
42	.80397	18	.24376	51	.81495	18	.27470	52	42
43	.80415	18	.24427	51	.81513	18	.27523	52	43
44	.80434	18	.24478	51	.81532	18	.27575	52	44
45	9.80452	18	10.24529	51	9.81550	18	10.22627	52	45
46	.80470	18	.24580	51	.81568	18	.27680	52	46
47	.80489	18	.24631	51	.81586	18	.27732	52	47
48	.80507	18	.24682	51	.81604	18	.27785	52	48
49	.80526	18	.24733	51	.81622	18	.27837	52	49
50	9.80544	18	10.24784	51	9.81640	18	10.27890	52	50
51	.80563	18	.24835	51	.81658	18	.27942	52	51
52	.80581	18	.24886	51	.81676	18	.27995	52	52
53	.80600	18	.24937	51	.81695	18	.28047	52	53
54	.80618	18	.24988	51	.81713	18	.28100	52	54
55	9.80636	18	10.25039	51	9.81731	18	10.28152	53	55
56	.80655	18	.25090	51	.81749	18	.28205	52	56
57	.80673	18	.25142	51	.81767	18	.28258	52	57
58	.80692	18	.25193	51	.81785	18	.28310	53	58
59	.80710	18	.25244	51	.81803	18	.28363	52	59
60	9.80728		10.25295		9.81821		10.28416		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	53	52
6	5.3	5.2
7	6.2	6.1
8	7.0	7.0
9	7.9	7.9
10	8.8	8.7
20	17.6	17.5
30	26.5	26.2
40	35.3	35.0
50	44.1	43.7

	52	51
6	5.2	5.1
7	6.0	6.0
8	6.9	6.8
9	7.8	7.7
10	8.6	8.6
20	17.3	17.1
30	26.0	25.7
40	34.6	34.3
50	43.3	42.9

	51	50
6	5.1	5.0
7	5.9	5.9
8	6.8	6.7
9	7.6	7.6
10	8.5	8.4
20	17.0	16.8
30	25.5	25.2
40	34.0	33.6
50	42.5	42.1

	50
6	5.0
7	5.8
8	6.6
9	7.5
10	8.3
20	16.6
30	25.0
40	33.3
50	41.6

	19	18
6	1.9	1.8
7	2.2	2.1
8	2.5	2.4
9	2.8	2.8
10	3.1	3.1
20	6.3	6.1
30	9.5	9.2
40	12.6	12.3
50	15.8	15.4

	18
6	1.8
7	2.1
8	2.4
9	2.7
10	3.0
20	6.0
30	9.0
40	12.0
50	15.0

P. P.

′	70° Log. Vers.	D	70° Log. Exsec.	D	71° Log. Vers.	D	71° Log. Exsec.	D	′
0	9.81821	18	10.28416	53	9.82894	17	10.31629	54	0
1	.81839	18	.28469	52	.82911	17	.31684	54	1
2	.81857	18	.28521	53	.82929	18	.31738	54	2
3	.81875	18	.28574	53	.82947	17	.31793	54	3
4	.81893	18	.28627	52	.82964	18	.31847	54	4
5	9.81911	18	10.28680	53	9.82982	17	10.31902	54	5
6	.81929	18	.28733	53	.83000	17	.31956	54	6
7	.81947	18	.28786	53	.83017	18	.32011	55	7
8	.81965	18	.28839	53	.83035	17	.32066	54	8
9	.81983	18	.28892	53	.83053	17	.32120	54	9
10	9.82001	18	10.28945	53	9.83070	18	10.32175	55	10
11	.82019	18	.28998	53	.83088	17	.32230	54	11
12	.82037	18	.29051	53	.83106	17	.32284	55	12
13	.82055	18	.29104	53	.83123	17	.32339	54	13
14	.82073	18	.29157	53	.83141	18	.32394	55	14
15	9.82091	17	10.29210	53	9.83159	17	10.32449	55	15
16	.82109	18	.29263	53	.83176	17	.32504	54	16
17	.82127	18	.29316	53	.83194	17	.32558	55	17
18	.82145	18	.29370	53	.83211	18	.32613	55	18
19	.82163	18	.29423	53	.83229	17	.32668	55	19
20	9.82181	18	10.29476	53	9.83247	17	10.32723	55	20
21	.82199	18	.29529	53	.83264	17	.32778	55	21
22	.82217	18	.29583	53	.83282	17	.32833	55	22
23	.82235	17	.29636	53	.83299	18	.32888	55	23
24	.82252	18	.29689	53	.83317	17	.32944	55	24
25	9.82270	18	10.29743	53	9.83335	17	10.32999	55	25
26	.82288	18	.29796	53	.83352	17	.33054	55	26
27	.82306	18	.29850	53	.83370	17	.33109	55	27
28	.82324	17	.29903	53	.83387	17	.33164	55	28
29	.82342	18	.29957	53	.83405	17	.33220	55	29
30	9.82360	18	10.30010	53	9.83422	17	10.33275	55	30
31	.82378	18	.30064	53	.83440	18	.33330	55	31
32	.82396	17	.30117	54	.83458	17	.33385	55	32
33	.82413	18	.30171	53	.83475	17	.33441	55	33
34	.82431	18	.30225	53	.83493	17	.33496	55	34
35	9.82449	17	10.30278	54	9.83510	17	10.33552	55	35
36	.82467	18	.30332	53	.83528	17	.33607	55	36
37	.82485	18	.30386	54	.83545	17	.33663	55	37
38	.82503	17	.30440	53	.83563	17	.33718	55	38
39	.82520	18	.30493	54	.83580	17	.33774	55	39
40	9.82538	18	10.30547	53	9.83598	17	10.33829	56	40
41	.82556	17	.30601	54	.83615	17	.33885	55	41
42	.82574	18	.30655	54	.83633	17	.33941	55	42
43	.82592	17	.30709	54	.83650	17	.33996	56	43
44	.82609	18	.30763	54	.83668	17	.34052	55	44
45	9.82627	18	10.30817	54	9.83685	17	10.34108	56	45
46	.82645	17	.30871	54	.83703	17	.34164	56	46
47	.82663	18	.30925	54	.83720	17	.34220	55	47
48	.82681	17	.30979	54	.83737	17	.34275	56	48
49	.82698	18	.31033	54	.83755	17	.34331	56	49
50	9.82716	17	10.31087	54	9.83772	17	10.34387	56	50
51	.82734	18	.31141	54	.83790	17	.34443	56	51
52	.82752	17	.31195	54	.83807	17	.34499	56	52
53	.82769	18	.31249	54	.83825	17	.34555	56	53
54	.82787	17	.31303	54	.83842	17	.34611	56	54
55	9.82805	18	10.31358	54	9.83859	17	10.34667	56	55
56	.82823	17	.31412	54	.83877	17	.34723	56	56
57	.82840	18	.31466	54	.83894	17	.34780	56	57
58	.82858	17	.31521	54	.83912	17	.34836	56	58
59	.82876	18	.31575	54	.83929	17	.34892	56	59
60	9.82894		10.31629		9.83946		10.34948		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	56̂	56
6	5.6	5.6
7	6.6	6.5
8	7.5	7.4
9	8.5	8.4
10	9.4	9.3
20	18.8	18.7
30	28.2	28.0
40	37.6	37.3
50	47.1	46.6

	55̂	55
6	5.5	5.5
7	6.5	6.4
8	7.4	7.3
9	8.3	8.2
10	9.2	9.1
20	18.5	18.3
30	27.7	27.5
40	37.0	36.6
50	46.2	45.8

	54̂	54
6	5.4	5.4
7	6.3	6.3
8	7.2	7.2
9	8.2	8.1
10	9.1	9.0
20	18.1	18.0
30	27.2	27.0
40	36.3	36.0
50	45.4	45.0

	53̂	53
6	5.3	5.3
7	6.2	6.2
8	7.1	7.0
9	8.0	7.9
10	8.9	8.8
20	17.8	17.6
30	26.7	26.5
40	35.6	35.3
50	44.6	44.1

	52̂
6	5.2
7	6.1
8	7.0
9	7.9
10	8.7
20	17.5
30	26.2
40	35.0
50	43.7

	18	17̂	17
6	1.8	1.7	1.7
7	2.1	2.0	2.0
8	2.4	2.3	2.3
9	2.7	2.6	2.5
10	3.0	2.9	2.8
20	6.0	5.9	5.7
30	9.0	8.7	8.5
40	12.0	11.7	11.3
50	15.0	14.6	14.1

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

	72°				73°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.83946̂	17̂	10.34948̂	56	9.84980̂	17	10.38387	58̂	**0**
1	.83964	17̂	.35005	56	.84997̂	17	.38445̂	58̂	1
2	.83981̂	17̂	.35061	56̂	.85014̂	17	.38504	58̂	2
3	.83999	17	.35117̂	56̂	.85031̂	17̂	.38562̂	58	3
4	.84016	17̂	.35174	56̂	.85049	17	.38621	58̂	4
5	9.84033̂	17̂	10.35230̂	56	9.85066	17	10.38679̂	58̂	5
6	.84051	17̂	.35286̂	56̂	.85083	17	.38738	58	6
7	.84068̂	17	.35343	56̂	.85100	17	.38796̂	59	7
8	.84085̂	17̂	.35399̂	57	.85117	17	.38855̂	58̂	8
9	.84103	17̂	.35456̂	56̂	.85134	17	.38914	59	9
10	9.84120̂	17	10.35513	56̂	9.85151	17	10.38973	58	**10**
11	.84137̂	17̂	.35569̂	56	.85168	17	.39031̂	59	11
12	.84155	17̂	.35626	57	.85185	17	.39090̂	59	12
13	.84172̂	17	.35683	56	.85202	17	.39149̂	58̂	13
14	.84189̂	17̂	.35739̂	57	.85219	17	.39208	59	14
15	9.84207	17̂	10.35796̂	56̂	9.85236	17	10.39267	59	15
16	.84224̂	17	.35853	57	.85253	17	.39326	59	16
17	.84241̂	17̂	.35910	57	.85270	17	.39385	59̂	17
18	.84259	17	.35967	56̂	.85287	17	.39444̂	59	18
19	.84276	17̂	.36023̂	57	.85304	17	.39503̂	59	19
20	9.84293̂	17	10.36080̂	57	9.85321	17	10.39562̂	59	**20**
21	.84310̂	17̂	.36137̂	57	.85338	17	.39621̂	59̂	21
22	.84328	17̂	.36194̂	57	.85355	17	.39681	59	22
23	.84345̂	17	.36251̂	57	.85372	17	.39740	59̂	23
24	.84362̂	17̂	.36308̂	57̂	.85389	16̂	.39799̂	59̂	24
25	9.84380	17	10.36366	57	9.85405̂	17	10.39859	59	25
26	.84397	17̂	.36423	57	.85422̂	17	.39918	59̂	26
27	.84414̂	17	.36480	57̂	.85439̂	17	.39977̂	59̂	27
28	.84431̂	17̂	.36537̂	57	.85456̂	17	.40037	59̂	28
29	.84449	17	.36594̂	57̂	.85473̂	17	.40096̂	59̂	29
30	9.84466	17	10.36652	57	9.85490̂	17	10.40156	60	**30**
31	.84483	17̂	.36709	57̂	.85507̂	16̂	.40216	59̂	31
32	.84500̂	17	.36766̂	57̂	.85524	17	.40275̂	59̂	32
33	.84517̂	17̂	.36824	57̂	.85541	17	.40335	60	33
34	.84535	17	.36881̂	57	.85558	17	.40395	59̂	34
35	9.84552	17̂	10.36938̂	57̂	9.85575	17	10.40454̂	60	35
36	.84569̂	17	.36996	58	.85592	16̂	.40514̂	59̂	36
37	.84586̂	17	.37054	57̂	.85608̂	17	.40574	60	37
38	.84603̂	17	.37111̂	57̂	.85625̂	17	.40634	60	38
39	.84620̂	17̂	.37169	57̂	.85642̂	17	.40694	60	39
40	9.84638	17	10.37226̂	57̂	9.85659̂	16̂	10.40754	60	**40**
41	.84655	17̂	.37284	58	.85676	17	.40814	60	41
42	.84672̂	17	.37342	57	.85693	17	.40874	60	42
43	.84689̂	17	.37399̂	58	.85710	16̂	.40934	60	43
44	.84706̂	17̂	.37457̂	58	.85726̂	17	.40994	60̂	44
45	9.84724	17	10.37515̂	57̂	9.85743̂	17	10.41054̂	60	45
46	.84741	17	.37573	58	.85760̂	16̂	.41114̂	60	46
47	.84758	17̂	.37631	58	.85777	17	.41174̂	60̂	47
48	.84775̂	17	.37689	58	.85794	17	.41235	60̂	48
49	.84792̂	17	.37747	58	.85811	16̂	.41295̂	60	49
50	9.84809̂	17	10.37805	58	9.85827̂	17	10.41355̂	60̂	**50**
51	.84826̂	17̂	.37863	58	.85844̂	17	.41416	60	51
52	.84844	17	.37921	58	.85861̂	16̂	.41476̂	60̂	52
53	.84861	17	.37979	58̂	.85878	17	.41537	60̂	53
54	.84878	17	.38037̂	58	.85895	16̂	.41597̂	60̂	54
55	9.84895	17	10.38095̂	58	9.85911̂	17	10.41658	61	55
56	.84912	17̂	.38153̂	58̂	.85928̂	17	.41719	60̂	56
57	.84929̂	17	.38212	58̂	.85945̂	16̂	.41779̂	60̂	57
58	.84946̂	17	.38270̂	58	.85962	17	.41840	61	58
59	.84963̂	17	.38328̂	58̂	.85979	16̂	.41901	61	59
60	9.84980̂		10.38387		9.85995̂		10.41962		**60**
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	61	60̂	
6	6.1	6.0̂	
7	7.1	7.0̂	
8	8.1̂	8.0̂	
9	9.1̂	9.1	
10	10.1̂	10.1	
20	20.3̂	20.1̂	
30	30.5	30.2̂	
40	40.6̂	40.3̂	
50	50.8̂	50.4	
	60	**59̂**	
6	6.0	5.9̂	
7	7.0	6.9̂	
8	8.0	7.9̂	
9	9.0	8.9	
10	10.0	9.9	
20	20.0	19.8̂	
30	30.0	29.7̂	
40	40.0	39.6̂	
50	50.0	49.6	
	59	**58̂**	
6	5.9	5.8̂	
7	6.9	6.8	
8	7.8̂	7.8	
9	8.8̂	8.8	
10	9.8̂	9.7̂	
20	19.6̂	19.5	
30	29.5	29.2̂	
40	39.3̂	39.0	
50	49.1̂	48.7̂	
	58	**57̂**	
6	5.8	5.7	
7	6.7̂	6.7	
8	7.7̂	7.6̂	
9	8.7	8.6	
10	9.6̂	9.6	
20	19.3̂	19.1̂	
30	29.0	28.7̂	
40	38.6̂	38.3̂	
50	48.3̂	47.9	
	57	**56̂**	
6	5.7	5.6̂	
7	6.6̂	6.6	
8	7.6	7.5̂	
9	8.5̂	8.5	
10	9.5	9.4	
20	19.0	18.8̂	
30	28.5	28.2̂	
40	38.0	37.6̂	
50	47.5	47.1	
	17̂	**17**	**16̂**
6	1.7̂	1.7	1.6̂
7	2.0̂	2.0	1.9
8	2.3̂	2.2̂	2.2
9	2.6	2.5̂	2.5
10	2.9	2.8̂	2.7̂
20	5.8̂	5.6	5.5
30	8.7̂	8.5	8.2̂
40	11.6	11.3̂	11.0
50	14.6	14.1̂	13.7̂

	74°				75°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.85995	17	10.41962	60	9.86992	16	10.45693	63	0
1	.86012	16	.42022	61	.87009	16	.45756	63	1
2	.86029	17	.42083	61	.87025	16	.45820	64	2
3	.86046	16	.42144	61	.87042	16	.45884	63	3
4	.86062	17	.42205	61	.87058	16	.45947	64	4
5	9.86079	16	10.42266	61	9.87074	16	10.46011	64	5
6	.86096	17	.42327	61	.87091	16	.46075	64	6
7	.86113	16	.42388	61	.87107	16	.46139	64	7
8	.86129	17	.42450	61	.87124	16	.46203	64	8
9	.86146	16	.42511	61	.87140	16	.46267	64	9
10	9.86163	16	10.42572	61	9.87157	16	10.46331	64	10
11	.86179	17	.42633	61	.87173	16	.46395	64	11
12	.86196	16	.42695	61	.87189	16	.46460	64	12
13	.86213	17	.42756	61	.87206	16	.46524	64	13
14	.86230	16	.42817	61	.87222	16	.46588	64	14
15	9.86246	16	10.42879	61	9.87239	16	10.46652	64	15
16	.86263	17	.42940	61	.87255	16	.46717	64	16
17	.86280	16	.43002	61	.87271	16	.46781	64	17
18	.86296	16	.43063	62	.87288	16	.46846	64	18
19	.86313	17	.43125	61	.87304	16	.46910	64	19
20	9.86330	16	10.43187	62	9.87320	16	10.46975	65	20
21	.86346	16	.43249	61	.87337	16	.47040	64	21
22	.86363	17	.43310	62	.87353	16	.47104	65	22
23	.86380	16	.43372	61	.87370	16	.47169	64	23
24	.86396	16	.43434	62	.87386	16	.47234	65	24
25	9.86413	17	10.43496	62	9.87402	16	10.47299	65	25
26	.86430	16	.43558	62	.87419	16	.47364	65	26
27	.86446	16	.43620	62	.87435	16	.47429	65	27
28	.86463	16	.43682	62	.87451	16	.47494	65	28
29	.86479	17	.43744	62	.87468	16	.47559	65	29
30	9.86496	16	10.43806	62	9.87484	16	10.47624	65	30
31	.86513	16	.43868	62	.87500	16	.47689	65	31
32	.86529	16	.43931	62	.87516	16	.47754	65	32
33	.86546	16	.43993	62	.87533	16	.47820	65	33
34	.86562	17	.44055	62	.87549	16	.47885	65	34
35	9.86579	16	10.44118	62	9.87565	16	10.47950	65	35
36	.86596	16	.44180	62	.87582	16	.48016	65	36
37	.86612	16	.44242	62	.87598	16	.48081	65	37
38	.86629	16	.44305	63	.87614	16	.48147	66	38
39	.86645	16	.44368	62	.87631	16	.48213	65	39
40	9.86662	16	10.44430	62	9.87647	16	10.48278	65	40
41	.86678	17	.44493	63	.87653	16	.48344	66	41
42	.86695	16	.44556	62	.87679	16	.48410	66	42
43	.86712	16	.44618	63	.87696	16	.48476	66	43
44	.86728	16	.44681	62	.87712	16	.48542	65	44
45	9.86745	16	10.44744	63	9.87728	16	10.48607	66	45
46	.86761	16	.44807	63	.87744	16	.48674	66	46
47	.86778	16	.44870	63	.87761	16	.48740	66	47
48	.86794	16	.44933	63	.87777	16	.48806	66	48
49	.86811	16	.44996	63	.87793	16	.48872	66	49
50	9.86827	16	10.45059	63	9.87809	16	10.48938	66	50
51	.86844	16	.45122	63	.87825	16	.49004	66	51
52	.86860	16	.45185	63	.87842	16	.49071	66	52
53	.86877	16	.45248	63	.87858	16	.49137	66	53
54	.86893	16	.45312	63	.87874	16	.49204	66	54
55	9.86910	16	10.45375	63	9.87890	16	10.49270	66	55
56	.86926	16	.45439	63	.87906	16	.49337	66	56
57	.86943	16	.45502	63	.87923	16	.49403	67	57
58	.86959	16	.45565	63	.87939	16	.49470	66	58
59	.86976	16	.45629	64	.87955	16	.49537	67	59
60	9.86992		10.45693		9.87971		10.49604		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	67	66	66
6	6.7	6.6	6.6
7	7.8	7.7	7.7
8	8.9	8.8	8.8
9	10.0	10.0	9.9
10	11.1	11.1	11.0
20	22.3	22.1	22.0
30	33.5	33.2	33.0
40	44.6	44.3	44.0
50	55.8	55.4	55.0

	65	65	64
6	6.5	6.5	6.4
7	7.6	7.6	7.5
8	8.7	8.6	8.6
9	9.8	9.7	9.7
10	10.9	10.8	10.7
20	21.8	21.6	21.5
30	32.7	32.5	32.2
40	43.6	43.3	43.0
50	54.6	54.1	53.7

	64	63	63
6	6.4	6.3	6.3
7	7.4	7.4	7.3
8	8.5	8.4	8.4
9	9.6	9.5	9.4
10	10.6	10.6	10.5
20	21.3	21.1	21.0
30	32.0	31.7	31.5
40	42.6	42.3	42.0
50	53.3	52.9	52.5

	62	62	61
6	6.2	6.2	6.1
7	7.3	7.2	7.2
8	8.3	8.2	8.2
9	9.4	9.3	9.2
10	10.4	10.3	10.2
20	20.8	20.6	20.5
30	31.2	31.0	30.7
40	41.6	41.3	41.0
50	52.1	51.6	51.2

	61	60
6	6.1	6.0
7	7.1	7.0
8	8.1	8.0
9	9.1	9.1
10	10.1	10.1
20	20.3	20.1
30	30.5	30.2
40	40.6	40.3
50	50.8	50.4

	17	16	16
6	1.7	1.6	1.6
7	2.0	1.9	1.8
8	2.2	2.2	2.1
9	2.5	2.5	2.4
10	2.8	2.7	2.6
20	5.6	5.5	5.3
30	8.5	8.2	8.0
40	11.3	11.0	10.6
50	14.1	13.7	13.3

P. P.

	76°				77°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.87971	16	10.49604	66	9.88933	16	10.53724	70	0
1	.87987	16	.49670	67	.88949	15	.53794	71	1
2	.88003	16	.49737	67	.88964	16	.53865	70	2
3	.88020	16	.49804	67	.88980	16	.53936	71	3
4	.88036	16	.49871	67	.88996	16	.54007	71	4
5	9.88052	16	10.49939	67	9.89012	15	10.54078	71	5
6	.88068	16	.50006	67	.89028	16	.54149	71	6
7	.88084	16	.50073	67	.89044	16	.54220	71	7
8	.88100	16	.50140	67	.89060	15	.54291	71	8
9	.88116	16	.50208	67	.89075	16	.54362	71	9
10	9.88133	16	10.50275	67	9.89091	16	10.54433	71	10
11	.88149	16	.50342	67	.89107	15	.54505	71	11
12	.88165	16	.50410	67	.89123	16	.54576	71	12
13	.88181	16	.50477	68	.89139	16	.54647	72	13
14	.88197	16	.50545	67	.89155	15	.54719	71	14
15	9.88213	16	10.50613	68	9.89170	16	10.54791	71	15
16	.88229	16	.50681	67	.89186	15	.54862	72	16
17	.88245	16	.50748	68	.89202	16	.54934	71	17
18	.88261	16	.50816	68	.89218	16	.55006	72	18
19	.88277	16	.50884	68	.89234	15	.55078	72	19
20	9.88294	16	10.50952	68	9.89249	16	10.55150	72	20
21	.88310	16	.51020	68	.89265	15	.55222	72	21
22	.88326	16	.51088	68	.89281	16	.55294	72	22
23	.88342	16	.51157	68	.89297	15	.55366	72	23
24	.88358	16	.51225	68	.89312	16	.55438	72	24
25	9.88374	16	10.51293	68	9.89328	15	10.55511	72	25
26	.88390	16	.51361	68	.89344	16	.55583	72	26
27	.88406	16	.51430	68	.89360	16	.55655	73	27
28	.88422	16	.51498	68	.89376	15	.55728	72	28
29	.88438	16	.51567	69	.89391	15	.55801	72	29
30	9.88454	16	10.51636	68	9.89407	16	10.55873	73	30
31	.88470	16	.51704	68	.89423	15	.55946	72	31
32	.88486	16	.51773	69	.89438	16	.56019	73	32
33	.88502	16	.51842	69	.89454	15	.56092	73	33
34	.88518	16	.51911	69	.89470	16	.56165	73	34
35	9.88534	16	10.51980	69	9.89486	15	10.56238	73	35
36	.88550	16	.52049	69	.89501	16	.56311	73	36
37	.88566	16	.52118	69	.89517	15	.56384	73	37
38	.88582	16	.52187	69	.89533	15	.56457	73	38
39	.88598	16	.52256	69	.89548	16	.56531	73	39
40	9.88614	16	10.52325	69	9.89564	15	10.56604	73	40
41	.88630	16	.52394	69	.89580	16	.56678	73	41
42	.88646	15	.52464	69	.89596	15	.56751	74	42
43	.88662	16	.52533	69	.89611	15	.56825	73	43
44	.88678	16	.52603	69	.89627	16	.56899	74	44
45	9.88694	16	10.52672	70	9.89643	15	10.56973	74	45
46	.88710	16	.52742	69	.89658	15	.57047	73	46
47	.88726	16	.52812	69	.89674	16	.57120	74	47
48	.88742	16	.52881	70	.89690	15	.57195	74	48
49	.88758	16	.52951	70	.89705	15	.57269	74	49
50	9.88774	16	10.53021	70	9.89721	16	10.57343	74	50
51	.88790	15	.53091	70	.89737	15	.57417	74	51
52	.88805	16	.53161	70	.89752	15	.57491	74	52
53	.88821	16	.53231	70	.89768	15	.57566	74	53
54	.88837	16	.53301	70	.89783	16	.57640	75	54
55	9.88853	16	10.53372	70	9.89799	15	10.57715	74	55
56	.88869	15	.53442	70	.89815	15	.57790	74	56
57	.88885	16	.53512	70	.89830	15	.57864	75	57
58	.88901	16	.53583	70	.89846	16	.57939	75	58
59	.88917	16	.53653	70	.89862	15	.58014	75	59
60	9.88933		10.53724		9.89877		10.58089		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	75	74	73
6	7.5	7.4	7.3
7	8.7	8.6	8.5
8	10.0	9.8	9.7
9	11.2	11.1	10.9
10	12.5	12.3	12.1
20	25.0	24.6	24.3
30	37.5	37.0	36.5
40	50.0	49.3	48.6
50	62.5	61.6	60.8

	72	71	70
6	7.2	7.1	7.0
7	8.4	8.3	8.2
8	9.6	9.4	9.4
9	10.8	10.6	10.6
10	12.0	11.8	11.7
20	24.0	23.6	23.3
30	36.0	35.5	35.2
40	48.0	47.3	47.0
50	60.0	59.1	58.7

	69	68	67
6	6.9	6.8	6.7
7	8.0	7.9	7.8
8	9.2	9.0	8.9
9	10.3	10.2	10.0
10	11.5	11.3	11.1
20	23.0	22.6	22.3
30	34.5	34.0	33.5
40	46.0	45.3	44.6
50	57.5	56.6	55.8

	66	0
6	6.6	0.0
7	7.7	0.0
8	8.8	0.0
9	9.9	0.1
10	11.0	0.1
20	22.0	0.1
30	33.0	0.2
40	44.0	0.3
50	55.0	0.4

	16	16	15
6	1.6	1.6	1.5
7	1.9	1.8	1.8
8	2.2	2.1	2.0
9	2.5	2.4	2.3
10	2.7	2.6	2.6
20	5.5	5.3	5.1
30	8.2	8.0	7.7
40	11.0	10.6	10.3
50	13.7	13.3	12.9

P. P.

	78°				79°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.89877	15	10.58089	75	9.90805	15	10.62745	80	0
1	.89893	15	.58164	75	.90820	15	.62825	80	1
2	.89908	15	.58239	75	.90835	15	.62906	80	2
3	.89924	15	.58315	75	.90851	15	.62986	81	3
4	.89939	16	.58390	75	.90866	15	.63067	80	4
5	9.89955	15	10.58465	75	9.90881	15	10.63148	81	5
6	.89971	15	.58541	75	.90897	15	.63229	81	6
7	.89986	15	.58616	76	.90912	15	.63310	81	7
8	.90002	15	.58692	75	.90927	15	.63391	81	8
9	.90017	15	.58768	76	.90943	15	.63472	81	9
10	9.90033	15	10.58844	76	9.90958	15	10.63553	81	10
11	.90048	15	.58920	75	.90973	15	.63634	81	11
12	.90064	16	.58995	75	.90988	15	.63716	81	12
13	.90080	15	.59072	76	.91004	15	.63797	81	13
14	.90095	15	.59148	76	.91019	15	.63879	82	14
15	9.90111	15	10.59224	76	9.91034	15	10.63961	82	15
16	.90126	15	.59300	76	.91049	15	.64043	82	16
17	.90142	15	.59377	76	.91065	15	.64125	82	17
18	.90157	15	.59453	76	.91080	15	.64207	82	18
19	.90173	15	.59530	76	.91095	15	.64289	82	19
20	9.90188	15	10.59606	77	9.91110	15	10.64371	82	20
21	.90204	15	.59683	76	.91126	15	.64453	82	21
22	.90219	15	.59760	77	.91141	15	.64536	82	22
23	.90235	15	.59837	77	.91156	15	.64618	83	23
24	.90250	15	.59914	77	.91171	15	.64701	82	24
25	9.90266	15	10.59991	77	9.91187	15	10.64784	83	25
26	.90281	15	.60068	77	.91202	15	.64867	83	26
27	.90297	15	.60145	77	.91217	15	.64950	83	27
28	.90312	15	.60223	77	.91232	15	.65033	83	28
29	.90328	15	.60300	77	.91247	15	.65116	83	29
30	9.90343	15	10.60378	77	9.91263	15	10.65199	83	30
31	.90359	15	.60455	77	.91278	15	.65283	83	31
32	.90374	15	.60533	78	.91293	15	.65366	83	32
33	.90389	15	.60611	77	.91308	15	.65450	84	33
34	.90405	15	.60688	78	.91323	15	.65534	83	34
35	9.90420	15	10.60766	78	9.91338	15	10.65617	84	35
36	.90436	15	.60844	78	.91354	15	.65701	84	36
37	.90451	15	.60923	78	.91369	15	.65785	84	37
38	.90467	15	.61001	78	.91384	15	.65870	84	38
39	.90482	15	.61079	78	.91399	15	.65954	84	39
40	9.90497	15	10.61158	78	9.91414	15	10.66038	84	40
41	.90513	15	.61236	78	.91429	15	.66123	84	41
42	.90528	15	.61315	78	.91445	15	.66207	84	42
43	.90544	15	.61393	79	.91460	15	.66292	85	43
44	.90559	15	.61472	78	.91475	15	.66377	85	44
45	9.90574	15	10.61551	79	9.91490	15	10.66462	85	45
46	.90590	15	.61630	79	.91505	15	.66547	85	46
47	.90605	15	.61709	79	.91520	15	.66632	85	47
48	.90621	15	.61788	79	.91535	15	.66717	85	48
49	.90636	15	.61867	79	.91550	15	.66803	85	49
50	9.90651	15	10.61947	79	9.91565	15	10.66888	85	50
51	.90667	15	.62026	79	.91581	15	.66974	85	51
52	.90682	15	.62105	80	.91596	15	.67059	86	52
53	.90697	15	.62185	79	.91611	15	.67145	86	53
54	.90713	15	.62265	80	.91626	15	.67231	86	54
55	9.90728	15	10.62345	79	9.91641	15	10.67317	86	55
56	.90744	15	.62424	80	.91656	15	.67403	86	56
57	.90759	15	.62504	80	.91671	15	.67490	86	57
58	.90774	15	.62585	80	.91686	15	.67576	86	58
59	.90790	15	.62665	80	.91701	15	.67663	86	59
60	9.90805		10.62745		9.91716		10.67749		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	86	85	84
6	8.6	8.5	8.4
7	10.0	9.9	9.8
8	11.4	11.3	11.2
9	12.9	12.7	12.6
10	14.3	14.1	14.0
20	28.6	28.3	28.0
30	43.0	42.5	42.0
40	57.3	56.6	56.0
50	71.6	70.8	70.0

	83	82	81
6	8.3	8.2	8.1
7	9.7	9.5	9.4
8	11.0	10.9	10.8
9	12.4	12.3	12.1
10	13.8	13.6	13.5
20	27.6	27.3	27.0
30	41.5	41.0	40.5
40	55.3	54.6	54.0
50	69.1	68.3	67.5

	80	79	78
6	8.0	7.9	7.8
7	9.3	9.2	9.1
8	10.6	10.5	10.4
9	12.0	11.8	11.7
10	13.3	13.1	13.0
20	26.6	26.3	26.0
30	40.0	39.5	39.0
40	53.3	52.6	52.0
50	66.6	65.8	65.0

	77	76	75
6	7.7	7.6	7.5
7	9.0	8.8	8.7
8	10.2	10.1	10.0
9	11.5	11.4	11.2
10	12.8	12.6	12.5
20	25.6	25.3	25.0
30	38.5	38.0	37.5
40	51.3	50.6	50.0
50	64.1	63.3	62.5

	0̂
6	0.0
7	0.0
8	0.0
9	0.1
10	0.1
20	0.1
30	0.2
40	0.3
50	0.4

	16	15̂	15
6	1.6	1.5	1.5
7	1.8	1.8	1.7
8	2.1	2.0	2.0
9	2.4	2.3	2.2
10	2.6	2.6	2.5
20	5.3	5.1	5.0
30	8.0	7.7	7.5
40	10.6	10.3	10.0
50	13.3	12.9	12.5

P. P.

′	80° Log. Vers.	D	80° Log. Exsec.	D	81° Log. Vers.	D	81° Log. Exsec.	D	′
0	9.91716	15	10.67749	86	9.92612	14	10.73178	95	0
1	.91731	15	.67836	87	.92626	15	.73273	94	1
2	.91746	15	.67923	87	.92641	14	.73368	95	2
3	.91761	15	.68010	87	.92656	15	.73463	95	3
4	.91776	15	.68097	87	.92671	15	.73558	95	4
5	9.91791	15	10.68184	87	9.92686	14	10.73653	95	5
6	.91807	15	.68272	87	.92700	15	.73748	95	6
7	.91822	15	.68359	87	.92715	14	.73844	96	7
8	.91837	15	.68447	87	.92730	15	.73940	95	8
9	.91852	15	.68534	88	.92745	14	.74035	96	9
10	9.91867	15	10.68622	88	9.92759	15	10.74131	96	10
11	.91882	15	.68710	88	.92774	14	.74227	96	11
12	.91897	15	.68798	88	.92789	15	.74324	96	12
13	.91912	15	.68886	88	.92804	14	.74420	96	13
14	.91927	15	.68975	88	.92818	15	.74517	96	14
15	9.91942	15	10.69063	88	9.92833	14	10.74613	97	15
16	.91957	15	.69152	88	.92848	14	.74710	97	16
17	.91972	15	.69240	89	.92862	15	.74807	97	17
18	.91987	15	.69329	89	.92877	14	.74905	97	18
19	.92002	14	.69418	89	.92892	15	.75002	97	19
20	9.92016	15	10.69507	89	9.92907	14	10.75099	98	20
21	.92031	15	.69596	89	.92921	14	.75197	97	21
22	.92046	15	.69686	89	.92936	15	.75295	98	22
23	.92061	15	.69775	89	.92951	14	.75393	98	23
24	.92076	15	.69865	90	.92965	15	.75491	98	24
25	9.92091	15	10.69955	89	9.92980	14	10.75589	98	25
26	.92106	15	.70044	90	.92995	14	.75688	98	26
27	.92121	15	.70134	90	.93009	15	.75786	99	27
28	.92136	14	.70224	90	.93024	14	.75885	99	28
29	.92151	15	.70315	90	.93039	14	.75984	99	29
30	9.92166	15	10.70405	90	9.93053	15	10.76083	99	30
31	.92181	15	.70495	91	.93068	14	.76182	99	31
32	.92196	15	.70586	90	.93083	14	.76282	100	32
33	.92211	15	.70677	91	.93097	15	.76382	99	33
34	.92226	14	.70768	91	.93112	14	.76481	100	34
35	9.92240	15	10.70859	91	9.93127	14	10.76581	100	35
36	.92255	15	.70950	91	.93141	14	.76681	100	36
37	.92270	15	.71041	91	.93156	15	.76782	100	37
38	.92285	15	.71133	91	.93171	14	.76882	100	38
39	.92300	14	.71224	91	.93185	14	.76983	100	39
40	9.92315	15	10.71316	92	9.93200	14	10.77083	101	40
41	.92330	15	.71408	92	.93214	15	.77184	101	41
42	.92345	15	.71500	92	.93229	14	.77286	101	42
43	.92360	14	.71592	92	.93244	14	.77387	101	43
44	.92374	15	.71684	92	.93258	14	.77488	101	44
45	9.92389	15	10.71776	92	9.93273	14	10.77590	102	45
46	.92404	14	.71869	92	.93287	15	.77692	102	46
47	.92419	15	.71961	93	.93302	14	.77794	102	47
48	.92434	15	.72054	92	.93317	14	.77896	102	48
49	.92449	14	.72147	93	.93331	14	.77998	102	49
50	9.92463	15	10.72240	93	9.93346	14	10.78101	102	50
51	.92478	15	.72333	93	.93360	14	.78203	103	51
52	.92493	14	.72427	93	.93375	14	.78306	103	52
53	.92508	15	.72520	93	.93389	15	.78409	103	53
54	.92523	15	.72614	93	.93404	14	.78513	103	54
55	9.92538	14	10.72707	94	9.93419	14	10.78616	104	55
56	.92552	15	.72801	94	.93433	14	.78720	103	56
57	.92567	15	.72895	94	.93448	14	.78823	104	57
58	.92582	14	.72990	94	.93462	14	.78927	104	58
59	.92597	15	.73084	94	.93477	14	.79031	104	59
60	9.92612		10.73178		9.93491		10.79136		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	90	80	9	8	7	6	5	4	15	15	14
6	9.0	8.0	0.9	0.8	0.7	0.6	0.5	0.4	1.5	1.5	1.4
7	10.5	9.3	1.0	0.9	0.8	0.7	0.6	0.4	1.8	1.7	1.7
8	12.0	10.6	1.2	1.0	0.9	0.8	0.6	0.5	2.0	2.0	1.9
9	13.5	12.0	1.3	1.2	1.0	0.9	0.7	0.6	2.3	2.2	2.2
10	15.0	13.3	1.5	1.3	1.1	1.0	0.8	0.6	2.6	2.5	2.4
20	30.0	26.6	3.0	2.6	2.3	2.0	1.6	1.3	5.1	5.0	4.8
30	45.0	40.0	4.5	4.0	3.5	3.0	2.5	2.0	7.7	7.5	7.2
40	60.0	53.3	6.0	5.3	4.6	4.0	3.3	2.6	10.3	10.0	9.6
50	75.0	66.6	7.5	6.6	5.8	5.0	4.1	3.3	12.9	12.5	12.1

	82°				83°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.93491	14	10.79136	104	9.94356	14	10.85766	117	0
1	.93506	14	.79240	105	.94370	14	.85884	117	1
2	.93520	14	.79345	104	.94384	14	.86001	117	2
3	.93535	14	.79450	105	.94398	14	.86119	118	3
4	.93549	14	.79555	105	.94413	14	.86237	118	4
5	9.93564	14	10.79660	105	9.94427	14	10.86355	118	5
6	.93578	14	.79766	105	.94441	14	.86474	118	6
7	.93593	14	.79871	106	.94456	14	.86592	119	7
8	.93607	14	.79977	106	.94470	14	.86711	119	8
9	.93622	14	.80083	106	.94484	14	.86831	119	9
10	9.93636	14	10.80189	106	9.94498	14	10.86950	120	10
11	.93651	14	.80296	106	.94512	14	.87070	120	11
12	.93665	14	.80402	107	.94527	14	.87190	120	12
13	.93680	14	.80509	107	.94541	14	.87310	120	13
14	.93694	14	.80616	107	.94555	14	.87431	121	14
15	9.93709	14	10.80723	107	9.94569	14	10.87552	121	15
16	.93723	14	.80831	107	.94584	14	.87673	121	16
17	.93738	14	.80938	108	.94598	14	.87794	121	17
18	.93752	14	.81046	108	.94612	14	.87916	122	18
19	.93767	14	.81154	108	.94626	14	.88038	122	19
20	9.93781	14	10.81262	108	9.94640	14	10.88160	122	20
21	.93796	14	.81371	108	.94655	14	.88282	122	21
22	.93810	14	.81479	109	.94669	14	.88405	123	22
23	.93824	14	.81588	109	.94683	14	.88528	123	23
24	.93839	14	.81697	109	.94697	14	.88651	124	24
25	9.93853	14	10.81806	109	9.94711	14	10.88775	123	25
26	.93868	14	.81916	109	.94726	14	.88898	124	26
27	.93882	14	.82025	110	.94740	14	.89022	124	27
28	.93897	14	.82135	110	.94754	14	.89147	124	28
29	.93911	14	.82245	110	.94768	14	.89271	125	29
30	9.93925	14	10.82356	110	9.94782	14	10.89396	125	30
31	.93940	14	.82466	110	.94796	14	.89521	125	31
32	.93954	14	.82577	111	.94810	14	.89647	126	32
33	.93969	14	.82688	111	.94825	14	.89773	126	33
34	.93983	14	.82799	111	.94839	14	.89899	126	34
35	9.93997	14	10.82910	111	9.94853	14	10.90025	126	35
36	.94012	14	.83022	111	.94867	14	.90152	127	36
37	.94026	14	.83133	112	.94881	14	.90279	127	37
38	.94041	14	.83245	112	.94895	14	.90406	127	38
39	.94055	14	.83358	112	.94909	14	.90533	128	39
40	9.94069	14	10.83470	112	9.94923	14	10.90661	127	40
41	.94084	14	.83583	112	.94938	14	.90789	128	41
42	.94098	14	.83695	113	.94952	14	.90917	129	42
43	.94112	14	.83809	113	.94966	14	.91046	129	43
44	.94127	14	.83922	113	.94980	14	.91175	129	44
45	9.94141	14	10.84035	114	9.94994	14	10.91304	130	45
46	.94155	14	.84149	114	.95008	14	.91434	129	46
47	.94170	14	.84263	114	.95022	14	.91564	130	47
48	.94184	14	.84377	114	.95036	14	.91694	130	48
49	.94198	14	.84492	115	.95050	14	.91825	131	49
50	9.94213	14	10.84607	114	9.95064	14	10.91956	131	50
51	.94227	14	.84721	115	.95078	14	.92087	131	51
52	.94241	14	.84837	115	.95093	14	.92218	131	52
53	.94256	14	.84952	116	.95107	14	.92350	132	53
54	.94270	14	.85068	115	.95121	14	.92482	132	54
55	9.94284	14	10.85183	116	9.95135	14	10.92614	133	55
56	.94299	14	.85299	116	.95149	14	.92747	133	56
57	.94313	14	.85416	116	.95163	14	.92880	133	57
58	.94327	14	.85532	117	.95177	14	.93014	133	58
59	.94341	14	.85649	117	.95191	14	.93147	134	59
60	9.94356		10.85766		9.95205		10.93281		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	130	120
6	13.0	12.0
7	15.1	14.0
8	17.3	16.0
9	19.5	18.0
10	21.6	20.0
20	43.3	40.0
30	65.0	60.0
40	86.6	80.0
50	108.3	100.0

	110	100
6	11.0	10.0
7	12.8	11.6
8	14.6	13.3
9	16.5	15.0
10	18.3	16.6
20	36.6	33.3
30	55.0	50.0
40	73.3	66.6
50	91.6	83.3

	3	2
6	0.3	0.2
7	0.3	0.2
8	0.4	0.2
9	0.4	0.3
10	0.5	0.3
20	1.0	0.6
30	1.5	1.0
40	2.0	1.3
50	2.5	1.6

	1	0
6	0.1	0.0
7	0.1	0.0
8	0.1	0.0
9	0.1	0.1
10	0.1	0.1
20	0.3	0.1
30	0.5	0.2
40	0.6	0.3
50	0.8	0.4

	14	14
6	1.4	1.4
7	1.7	1.6
8	1.9	1.8
9	2.2	2.1
10	2.4	2.3
20	4.8	4.6
30	7.2	7.0
40	9.6	9.3
50	12.1	11.6

P. P.

′	84° Log. Vers.	D	84° Log. Exsec.	D	85° Log. Vers.	D	85° Log. Exsec.	D	′
0	9.95205	14	10.93281	134	9.96039	14	11.02010	158	0
1	.95219	14	.93416	135	.96053	13	.02168	159	1
2	.95233	14	.93551	135	.96067	14	.02327	159	2
3	.95247	14	.93686	135	.96081	14	.02487	159	3
4	.95261	14	.93821	135	.96095	13	.02646	160	4
5	9.95275	14	10.93957	136	9.96108	14	11.02807	161	5
6	.95289	14	.94093	136	.96122	13	.02968	161	6
7	.95303	14	.94229	137	.96136	14	.03129	161	7
8	.95317	14	.94366	137	.96150	13	.03291	162	8
9	.95331	14	.94503	137	.96163	14	.03453	163	9
10	9.95345	14	10.94641	137	9.96177	13	11.03616	163	10
11	.95359	13	.94778	138	.96191	14	.03780	164	11
12	.95373	14	.94917	138	.96205	13	.03944	164	12
13	.95387	14	.95055	139	.96218	14	.04108	165	13
14	.95401	14	.95194	139	.96232	13	.04273	165	14
15	9.95415	14	10.95333	139	9.96246	13	11.04438	166	15
16	.95429	14	.95473	140	.96259	14	.04604	167	16
17	.95443	14	.95613	140	.96273	13	.04771	167	17
18	.95457	14	.95753	140	.96287	14	.04938	167	18
19	.95471	14	.95894	141	.96301	13	.05106	168	19
20	9.95485	14	10.96035	141	9.96314	14	11.05274	169	20
21	.95499	14	.96176	142	.96328	13	.05443	169	21
22	.95513	14	.96318	142	.96342	13	.05612	169	22
23	.95527	13	.96461	142	.96355	14	.05782	170	23
24	.95540	14	.96603	143	.96369	13	.05952	171	24
25	9.95554	14	10.96746	143	9.96383	14	11.06123	171	25
26	.95568	14	.96889	144	.96397	13	.06295	172	26
27	.95582	14	.97033	144	.96410	13	.06467	173	27
28	.95596	14	.97177	144	.96424	14	.06640	173	28
29	.95610	14	.97322	145	.96438	13	.06813	174	29
30	9.95624	13	10.97467	145	9.96451	13	11.06987	174	30
31	.95638	14	.97612	145	.96465	14	.07161	175	31
32	.95652	14	.97758	146	.96479	13	.07336	176	32
33	.95666	14	.97904	146	.96492	13	.07512	176	33
34	.95680	13	.98050	147	.96506	13	.07688	177	34
35	9.95693	14	10.98197	147	9.96519	14	11.07865	177	35
36	.95707	14	.98345	147	.96533	13	.08043	178	36
37	.95721	14	.98492	148	.96547	13	.08221	179	37
38	.95735	14	.98640	149	.96560	14	.08400	179	38
39	.95749	13	.98789	149	.96574	13	.08579	180	39
40	9.95763	14	10.98938	149	9.96588	13	11.08759	180	40
41	.95777	14	.99087	150	.96601	13	.08940	181	41
42	.95791	13	.99237	150	.96615	14	.09121	182	42
43	.95804	14	.99387	151	.96629	13	.09303	182	43
44	.95818	14	.99538	151	.96642	13	.09486	183	44
45	9.95832	14	10.99689	151	9.96656	13	11.09669	184	45
46	.95846	13	.99841	152	.96669	13	.09853	185	46
47	.95860	14	10.99993	152	.96683	14	.10038	185	47
48	.95874	14	11.00145	153	.96697	13	.10223	186	48
49	.95888	13	.00298	153	.96710	13	.10409	186	49
50	9.95901	14	11.00451	154	9.96724	13	11.10595	187	50
51	.95915	13	.00605	154	.96737	13	.10783	188	51
52	.95929	14	.00759	155	.96751	13	.10971	189	52
53	.95943	14	.00914	155	.96764	14	.11160	189	53
54	.95957	13	.01069	155	.96778	13	.11349	190	54
55	9.95970	14	11.01225	156	9.96792	13	11.11539	191	55
56	.95984	14	.01381	156	.96805	13	.11730	191	56
57	.95998	13	.01537	157	.96819	13	.11922	192	57
58	.96012	14	.01694	157	.96832	13	.12114	193	58
59	.96026	13	.01852	158	.96846	13	.12307	193	59
60	9.96039		11.02010		9.96859		11.12501		60
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	190	180
6	19.0	18.0
7	22.1	21.0
8	25.3	24.0
9	28.5	27.0
10	31.6	30.0
20	63.3	60.0
30	95.0	90.0
40	126.6	120.0
50	158.3	150.0

	170	160
6	17.0	16.0
7	19.8	18.6
8	22.6	21.3
9	25.5	24.0
10	28.3	26.6
20	56.6	53.3
30	85.0	80.0
40	113.3	106.6
50	141.6	133.3

	150	140
6	15.0	14.0
7	17.5	16.3
8	20.0	18.6
9	22.5	21.0
10	25.0	23.3
20	50.0	46.6
30	75.0	70.0
40	100.0	93.3
50	125.0	116.6

	130	9	8
6	13.0	0.9	0.8
7	15.1	1.0	0.9
8	17.3	1.2	1.0
9	19.5	1.3	1.2
10	21.6	1.5	1.3
20	43.3	3.0	2.6
30	65.0	4.5	4.0
40	86.6	6.0	5.3
50	108.3	7.5	6.6

	7	6	5
6	0.7	0.6	0.5
7	0.8	0.7	0.6
8	0.9	0.8	0.6
9	1.0	0.9	0.7
10	1.1	1.0	0.8
20	2.3	2.0	1.6
30	3.5	3.0	2.5
40	4.6	4.0	3.3
50	5.8	5.0	4.1

	14	14	13
6	1.4	1.4	1.3
7	1.7	1.6	1.6
8	1.9	1.8	1.8
9	2.2	2.1	2.0
10	2.4	2.3	2.2
20	4.8	4.6	4.5
30	7.2	7.0	6.7
40	9.6	9.3	9.0
50	12.1	11.6	11.2

P. P.

TABLE VIII.—LOGARITHMIC VERSED SINES AND EXTERNAL SECANTS.

	86°				87°				
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′
0	9.96859̂	13̂	11.12501	195	9.97665̂	13̂	11.25785̂	255	**0**
1	.96873	14	.12696	195	.97679	13	.26040̂	256̂	1
2	.96887	13̂	.12891	196̂	.97692	13̂	.26297	257̂	2
3	.96900̂	13̂	.13087̂	196̂	.97705̂	13	.26554̂	259̂	3
4	.96914	13̂	.13284	198	.97718̂	13̂	.26814	260̂	4
5	9.96927̂	13̂	11.13482	198	9.97732	13	11.27074̂	262	5
6	.96941	13̂	.13680	199̂	.97745	13̂	.27336̂	263	6
7	.96954̂	13̂	.13879̂	200	.97758̂	13̂	.27599̂	265	7
8	.96968	13̂	.14079̂	201	.97772	13	.27864̂	266̂	8
9	.96981̂	13̂	.14280̂	201̂	.97785	13̂	.28131	267̂	9
10	9.96995	13̂	11.14482	202̂	9.97798̂	13	11.28398̂	269̂	**10**
11	.97008̂	13̂	.14684̂	203	.97811̂	13̂	.28668	270̂	11
12	.97022	13̂	.14887̂	204̂	.97825	13	.28938̂	272̂	12
13	.97035̂	13̂	.15092	205	.97838	13̂	.29211	274	13
14	.97049	13̂	.15297	205̂	.97851̂	13	.29485	275̂	14
15	9.97062̂	13̂	11.15502̂	206̂	9.97864̂	13̂	11.29760̂	277	15
16	.97076	13̂	.15709	208	.97878	13	.30037̂	278̂	16
17	.97089̂	13̂	.15917	208	.97891	13̂	.30316	279̂	17
18	.97103	13̂	.16125	209̂	.97904̂	13	.30596̂	282	18
19	.97116̂	13̂	.16334̂	210	.97917̂	13̂	.30878̂	283̂	19
20	9.97130	13̂	11.16544̂	211	9.97931	13	11.31162	285̂	**20**
21	.97143̂	13̂	.16755̂	212	.97944	13̂	.31447̂	287	21
22	.97157	13	.16967̂	213	.97957̂	13	.31734̂	288̂	22
23	.97170	13̂	.17180̂	214	.97970̂	13̂	.32023	290̂	23
24	.97183̂	13̂	.17394̂	214̂	.97984	13	.32313̂	292̂	24
25	9.97197	13̂	11.17609	215̂	9.97997	13̂	11.32606	294	25
26	.97210̂	13̂	.17824̂	216̂	.98010̂	13	.32900	296	26
27	.97224	13̂	.18041	218	.98023̂	13	.33196	298	27
28	.97237̂	13̂	.18259	218̂	.98036̂	13̂	.33494	299̂	28
29	.97251	13̂	.18477̂	219̂	.98050	13	.33793̂	301̂	29
30	9.97264̂	13̂	11.18697	220̂	9.98063	13̂	11.34095	303̂	**30**
31	.97277̂	13	.18917̂	221	.98076̂	13	.34398̂	305̂	31
32	.97291	13̂	.19138̂	222̂	.98089̂	13	.34704	307	32
33	.97304̂	13̂	.19361	223̂	.98102̂	13̂	.35011̂	309̂	33
34	.97318	13̂	.19584̂	224̂	.98116	13	.35321	311̂	34
35	9.97331̂	13̂	11.19809	225̂	9.98129	13	11.35632	313̂	35
36	.97345	13̂	.20034̂	227	.98142	13̂	.35946	315̂	36
37	.97358̂	13	.20261̂	227̂	.98155̂	13	.36261̂	318	37
38	.97371̂	13̂	.20489	228̂	.98168̂	13	.36579̂	320	38
39	.97385	13̂	.20717̂	230	.98181̂	13̂	.36899̂	322	39
40	9.97398̂	13̂	11.20947̂	230̂	9.98195	13	11.37221̂	324̂	**40**
41	.97412	13̂	.21178	232	.98208	13	.37546	326̂	41
42	.97425	13	.21410	233	.98221	13̂	.37872	328̂	42
43	.97438̂	13̂	.21643	234̂	.98234̂	13	.38201	331̂	43
44	.97452	13̂	.21877̂	235	.98247̂	13	.38532	333̂	44
45	9.97465̂	13̂	11.22112̂	236̂	9.98260̂	13	11.38866	335̂	45
46	.97478̂	13̂	.22349	237̂	.98273̂	13̂	.39201̂	338	46
47	.97492	13̂	.22586̂	239	.98287	13	.39540	340̂	47
48	.97505̂	13̂	.22825̂	239̂	.98300	13	.39880̂	343̂	48
49	.97519	13̂	.23065	241̂	.98313	13	.40224	345̂	49
50	9.97532	13	11.23306̂	242	9.98326	13̂	11.40569̂	348	**50**
51	.97545̂	13̂	.23548̂	243̂	.98339̂	13	.40918	351	51
52	.97559	13̂	.23792	245	.98352̂	13	.41269	353̂	52
53	.97572	13	.24037	246	.98365̂	13	.41622̂	356	53
54	.97585̂	13̂	.24283	247	.98378̂	13̂	.41979	359	54
55	9.97599	13̂	11.24530	248̂	9.98392	13	11.42338	361̂	55
56	.97612	13	.24778̂	250	.98405	13	.42699̂	364̂	56
57	.97625̂	13̂	.25028̂	251	.98418	13	.43064	367̂	57
58	.97639	13̂	.25279̂	252	.98431	13	.43431̂	370̂	58
59	.97652	13̂	.25531̂	254	.98444	13̂	.43802	373̂	59
60	9.97665̂		11.25785̂		9.98457̂		11.44175̂		**60**
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′

P. P.

	250	240
6	25.0	24.0
7	29.1̂	28.0
8	33.3̂	32.0
9	37.5	36.0
10	41.6̂	40.0
20	83.3̂	80.0
30	125.0	120.0
40	166.6̂	160.0
50	208.3̂	200.0

	230	220
6	23.0	22.0
7	26.8̂	25.6̂
8	30.6	29.3
9	34.5	33.0
10	38.3̂	36.6̂
20	76.6̂	73.3̂
30	115.0	110.0
40	153.3̂	146.6̂
50	191.6̂	183.3̂

	210	200
6	21.0	20.0
7	24.5	23.3̂
8	28.0	26.6̂
9	31.5	30.0
10	35.0	33.3̂
20	70.0	66.6̂
30	105.0	100.0
40	140.0	133.3̂
50	175.0	166.6̂

	190	4	3
6	19.0	0.4	0.3
7	22.1̂	0.4̂	0.3̂
8	25.3̂	0.5̂	0.4
9	28.5	0.6	0.4̂
10	31.6̂	0.6̂	0.5
20	63.3̂	1.3̂	1.0
30	95.0	2.0	1.5
40	126.6̂	2.6̂	2.0
50	158.3̂	3.3̂	2.5

	2	1	0̂
6	0.2	0.1	0.0̂
7	0.2̂	0.1	0.0̂
8	0.2̂	0.1̂	0.0̂
9	0.3	0.1̂	0.1
10	0.3̂	0.1̂	0.1
20	0.6̂	0.3̂	0.1̂
30	1.0	0.5	0.2
40	1.3̂	0.6̂	0.3̂
50	1.6̂	0.8̂	0.4

	14	13̂	13
6	1.4	1.3	1.3
7	1.6̂	[illegible]	[illegible]
8	1.8̂	[illegible]	1.7̂
9	2.1	[illegible]	1.9
10	2.3̂	[illegible]	2.1̂
20	4.6̂	4.5	4.3̂
30	7.0	6.7	[illegible]
40	9.3̂	[illegible]	[illegible]
50	11.6̂	11.2	[illegible]

P. P.

	88°				89°					P. P.
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	
0	9.98457̂	13	11.44175̂	376	9.99235̂	12̂	11.75050	742̂	0	
1	.98470̂	13	.44551̂	379̂	.99248	13	.75792̂	755	1	
2	.98483̂	13	.44931	382̂	.99261	13	.76547̂	768̂	2	
3	.98496̂	13	.45313̂	386	.99274	13	.77316	781̂	3	
4	.98509̂	13	.45699̂	389	.99287	12̂	.78097̂	795	4	
5	9.98522̂	13	11.46088̂	392	9.99299̂	13	11.78892̂	809̂	5	
6	.98535̂	13	.46480̂	395̂	.99312̂	13	.79702	825	6	
7	.98548̂	13̂	.46876	399	.99325̂	12̂	.80527	840	7	
8	.98562	13	.47275	402̂	.99338	13	.81367	856	8	
9	.98575	13	.47677̂	406	.99351	12̂	.82223	872̂	9	
10	9.98588	13	11.48083̂	409̂	9.99363̂	13	11.83095̂	890̂	10	
11	.98601	13	.48493	413	.99376̂	13	.83986	908	11	
12	.98614	13	.48906	417	.99389̂	12̂	.84894	927	12	
13	.98627	13	.49323	420̂	.99402	13	.85821	947	13	
14	.98640	13	.49743̂	425	.99415	13	.86768	967	14	
15	9.98653	13	11.50168̂	428̂	9.99428	12̂	11.87735	989	15	
16	.98666	13	.50597	432̂	.99440̂	13	.88724	1009	16	
17	.98679	13	.51029̂	436̂	.99453̂	12̂	.89735	1034̂	17	
18	.98692	13	.51466	440̂	.99466	13	.90769̂	1059̂	18	
19	.98705	13	.51906̂	445	.99479	12̂	.91829	1085	19	
20	9.98718	13	11.52351̂	449̂	9.99491̂	13	11.92914	1112̂	20	
21	.98731	13	.52801	454	.99504̂	13	.94026̂	1140̂	21	
22	.98744	13	.53255	458	.99517̂	12̂	.95167	1171	22	
23	.98757	13	.53713	463	.99530	13	.96338	1203	23	
24	.98770	13	.54176̂	467̂	.99543	12̂	.97541	1236	24	
25	9.98783	13	11.54643̂	472̂	9.99555̂	13	11.98777	1271̂	25	
26	.98796	13	.55116	477̂	.99568̂	12̂	12.00048	1309̂	26	
27	.98809	13	.55593̂	482̂	.99581	13	.01358	1349	27	
28	.98822	13	.56076̂	487̂	.99594	12̂	.02707	1391̂	28	
29	.98835	13	.56563̂	492̂	.99606̂	13	.04098̂	1436̂	29	
30	9.98848	13	11.57056	498	9.99619̂	12̂	12.05535	1485	30	
31	.98861	13	.57554	504	.99632	13	.07020	1537	31	
32	.98874	13	.58058	509	.99645	12̂	.08557	1592	32	
33	.98887	13	.58567	515	.99657̂	13	.10149	1652	33	
34	.98900	13	.59082	520̂	.99670̂	12̂	.11801	1716	34	
35	9.98913	12̂	11.59602̂	527	9.99683	12̂	12.13517	1785̂	35	
36	.98925̂	13	.60129̂	533	.99695̂	13	.15302̂	1861	36	
37	.98938̂	13	.60662̂	539̂	.99708̂	12̂	.17163̂	1943	37	
38	.98951̂	13	.61202	545̂	.99721	13	.19106̂	2033	38	
39	.98964̂	13	.61747̂	552̂	.99734	12̂	.21139̂	2131̂	39	
40	9.98977̂	13	11.62300	559	9.99746̂	13	12.23271	2240̂	40	
41	.98990̂	13	.62859	566	.99759̂	12̂	.25511̂	2361	41	
42	.99003̂	13	.63425	573	.99772	12̂	.27872̂	2495	42	
43	.99016̂	12̂	.63998	581	.99784̂	13	.30367̂	2645̂	43	
44	.99029	13	.64579	588	.99797̂	12̂	.33013	2815̂	44	
45	9.99042	13	11.65167	595̂	9.99810	13	12.35828̂	3009	45	
46	.99055	13	.65762̂	604	.99823	12̂	.38837̂	3231	46	
47	.99068	13	.66366̂	611̂	.99835̂	12̂	.42068̂	3489	47	
48	.99081	12̂	.66978	620̂	.99848	13	.45557̂	3791̂	48	
49	.99093̂	13	.67598̂	628̂	.99861	12̂	.49349	4152	49	
50	9.99106̂	13	11.68227	638	9.99873̂	12̂	12.53501	4588̂	50	
51	.99119̂	13	.68865	646̂	.99886	13	.58089̂	5127̂	51	
52	.99132̂	13	.69511̂	656̂	.99899	12̂	.63217	5812	52	
53	.99145̂	12̂	.70168	666	.99911̂	12̂	.69029̂	6707̂	53	
54	.99158	13	.70834	675̂	.99924	13	.75736̂	7931̂	54	
55	9.99171	13	11.71509̂	686̂	9.99937	12̂	12.83667	9704	55	
56	.99184	13	.72196	696̂	.99949̂	12̂	.93371	12506̂	56	
57	.99197	12̂	.72892̂	707̂	.99962	12̂	13.05877̂	17621̂	57	
58	.99209̂	13	.73600	719̂	.99974̂	13	.23499	30116	58	
59	.99222̂	13	.74319̂	730̂	.99987̂	12̂	.53615		59	
60	9.99235̂		11.75050		10.00000		Infinity		60	
′	Log. Vers.	D	Log. Exsec.	D	Log. Vers.	D	Log. Exsec.	D	′	P. P.

P. P.

	13̂	13
6	1.3̂	1.3
7	1.6	1.5
8	1.8	1.7̂
9	2.0	1.9̂
10	2.2̂	2.1̂
20	4.5	4.3̂
30	6.7	6.5
40	9.0	8.6̂
50	11.2̂	10.8

	12̂
6	1.2̂
7	1.4̂
8	1.6̂
9	1.9
10	2.1
20	4.1̂
30	6.2̂
40	8.3̂
50	10.4

0°–10°

° ′	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.	
0 0	**0.0000**	29	**0.0000**	29	**∞**		**1.0000**	0	**0 90**
10	0.0029	29	0.0029	29	343.773̂		1.0000	0	50
20	0.0058	29̂	0.0058	29̂	171.885̂		1.0000	0̂	40
30	0.0087̂	29	0.0087̂	29	114.588̂		0.9999̂	0	30
40	0.0116̂	29	0.0116̂	29	85.9398		0.9999̂	0̂	20
50	0.0145̂	29	0.0145̂	29	68.7501		0.9999	0̂	10
1 0	**0.0174̂**	29	**0.0174̂**	29	**57.2899̂**	8.1866̂	**0.9998̂**	0̂	**0 89**
10	0.0203̂	29	0.0203̂	29̂	49.1039	6.1398	0.9998	0̂	50
20	0.0232̂	29̂	0.0233	29	42.9641	4.7756̂	0.9997̂	1	40
30	0.0262	29	0.0262	29	38.1884̂	3.8217	0.9996̂	0̂	30
40	0.0291	29	0.0291	29	34.3677̂	3.1261̂	0.9996	1	20
50	0.0320	29	0.0320	29	31.2416	2.6053̂	0.9995	1	10
2 0	**0.0349**	29	**0.0349**	29̂	**28.6362̂**	2.2046̂	**0.9994**	1	**0 88**
10	0.0378	29	0.0378̂	29	26.4316	1.8898̂	0.9993	1̂	50
20	0.0407	29	0.0407̂	29	24.5417̂	1.6380	0.9991̂	1	40
30	0.0436	29̂	0.0436̂	29̂	22.9037̂	1.4333̂	0.9990̂	1̂	30
40	0.0465̂	29	0.0466	29	21.4704	1.2648̂	0.9989	1	20
50	0.0494̂	29	0.0495	29	20.2055̂	1.1244	0.9988	1̂	10
3 0	**0.0523̂**	29	**0.0524**	29̂	**19.0811̂**	1.0061̂	**0.9986̂**	2	**0 87**
10	0.0552̂	29	0.0553̂	29	18.0750	9056̂	0.9984̂	1̂	50
20	0.0581̂	29	0.0582̂	29	17.1693̂	8195	0.9983	1̂	40
30	0.0610̂	29	0.0611̂	29̂	16.3498̂	7450̂	0.9981̂	2	30
40	0.0639̂	29	0.0641	29	15.6048	6804	0.9979̂	2	20
50	0.0668̂	29	0.0670	29̂	14.9244	6237̂	0.9977̂	2	10
4 0	**0.0697̂**	29	**0.0699̂**	29	**14.3006̂**	5739	**0.9975̂**	2	**0 86**
10	0.0726̂	29	0.0728̂	29̂	13.7267̂	5298̂	0.9973̂	2	50
20	0.0755̂	29	0.0758	29	13.1969	4907	0.9971̂	2̂	40
30	0.0784̂	29	0.0787	29̂	12.7062	4557	0.9969	2	30
40	0.0813̂	29	0.0816̂	29	12.2505	4243̂	0.9967	2̂	20
50	0.0842̂	29	0.0845̂	29̂	11.8261̂	3961	0.9964̂	2̂	10
5 0	**0.0871̂**	29	**0.0875**	29	**11.4300̂**	3706	**0.9962**	2̂	**0 85**
10	0.0900̂	29	0.0904	29̂	11.0594̂	3475̂	0.9959̂	3	50
20	0.0929̂	29	0.0933̂	29̂	10.7119	3265	0.9956̂	2̂	40
30	0.0958̂	29	0.0963	29̂	10.3854	3073̂	0.9954	3	30
40	0.0987̂	29	0.0992̂	29	10.0780̂	2899	0.9951	3	20
50	0.1016̂	29	0.1021̂	29̂	9.7881̂	2738	0.9948	3	10
6 0	**0.1045̂**	28̂	**0.1051**	29̂	**9.5143̂**	2590̂	**0.9945**	3	**0 84**
10	0.1074	29	0.1080̂	29̂	9.2553	2454̂	0.9942	3	50
20	0.1103	29	0.1110	29̂	9.0098̂	2329̂	0.9939	3̂	40
30	0.1132	29	0.1139̂	29̂	8.7769	2213̂	0.9935̂	3	30
40	0.1161	29	0.1169	29̂	8.5555̂	2106	0.9932̂	3̂	20
50	0.1190	28̂	0.1198̂	29̂	8.3449̂	2006	0.9929	3̂	10
7 0	**0.1218̂**	29	**0.1228**	29̂	**8.1443̂**	1913̂	**0.9925̂**	3̂	**0 83**
10	0.1247̂	29	0.1257̂	29̂	7.9530	1826̂	0.9922	4	50
20	0.1276̂	29	0.1287	29̂	7.7703̂	1746	0.9918	3̂	40
30	0.1305̂	28̂	0.1316̂	29̂	7.5957̂	1670̂	0.9914̂	4	30
40	0.1334	29	0.1346	30	7.4287	1599̂	0.9910̂	4	20
50	0.1363	28̂	0.1376	29̂	7.2687̂	1534	0.9906̂	4	10
8 0	**0.1391̂**	29	**0.1405̂**	29̂	**7.1153̂**	1471	**0.9902̂**	4	**0 82**
10	0.1420̂	29	0.1435	30	6.9682̂	1413	0.9898̂	4	50
20	0.1449̂	28̂	0.1465	29̂	6.8269̂	1358	0.9894̂	4̂	40
30	0.1478	29	0.1494̂	30	6.6911̂	1306	0.9890	4	30
40	0.1507	28̂	0.1524̂	29̂	6.5605̂	1257	0.9886	4̂	20
50	0.1535̂	29	0.1554	30	6.4348̂	1211	0.9881̂	4̂	10
9 0	**0.1564̂**	28̂	**0.1584**	29̂	**6.3137̂**	1167	**0.9877**	4̂	**0 81**
10	0.1593	29	0.1613̂	30	6.1970̂	1126	0.9872̂	5	50
20	0.1622	28̂	0.1643̂	30	6.0844̂	1087	0.9867̂	4̂	40
30	0.1650̂	28̂	0.1673̂	30	5.9757̂	1049̂	0.9863	5	30
40	0.1679	29	0.1703̂	30	5.8708	1014̂	0.9858	5	20
50	0.1708	28̂	0.1733̂	30	5.7693̂	980̂	0.9853	5	10
10 0	**0.1736̂**		**0.1763̂**		**5.6713**		**0.9848**		**0 80**
	Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	′ °

P. P.

	30	29̂	29
1	3.0	2.9̂	2.9
2	6.0	5.9	5.8
3	9.0	8.8̂	8.7
4	12.0	11.8	11.6
5	15.0	14.7̂	14.5
6	18.0	17.7	17.4
7	21.0	20.6̂	20.3
8	24.0	23.6	23.2
9	27.0	26.5̂	26.1

	28̂	5	4̂	4
1	2.8̂	0.5	0.4̂	0.4
2	5.7	1.0	0.9	0.8
3	8.5̂	1.5	1.3̂	1.2
4	11.4	2.0	1.8	1.6
5	14.2̂	2.5	2.2̂	2.0
6	17.1	3.0	2.7	2.4
7	19.9̂	3.5	3.1̂	2.8
8	22.8	4.0	3.6	3.2
9	25.6̂	4.5	4.0̂	3.6

	3̂	3	2̂	2
1	0.3̂	0.3	0.2̂	0.2
2	0.7	0.6	0.5	0.4
3	1.0̂	0.9	0.7̂	0.6
4	1.4	1.2	1.0	0.8
5	1.7̂	1.5	1.2̂	1.0
6	2.1	1.8	1.5	1.2
7	2.4̂	2.1	1.7̂	1.4
8	2.8	2.4	2.0	1.6
9	3.1̂	2.7	2.2̂	1.8

	1̂	1	0̂
1	0.1̂	0.1	0.0̂
2	0.3	0.2	0.1
3	0.4̂	0.3	0.1̂
4	0.6	0.4	0.2
5	0.7̂	0.5	0.2̂
6	0.9	0.6	0.3
7	1.0̂	0.7	0.3̂
8	1.2	0.8	0.4
9	1.3̂	0.9	0.4̂

P. P.

80°–90°

10°–20°

° ′	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.	
10 0	**0.1736**		**0.1763**		**5.6713**		**0.9848**		**0 80**
10	0.1765	28	0.1793	30	5.5764	949	0.9843	5	50
20	0.1793	28	0.1823	30	5.4845	919	0.9838	5	40
30	0.1822	29	0.1853	30	5.3955	890	0.9832	5	30
40	0.1851	28	0.1883	30	5.3093	862	0.9827	5	20
50	0.1879	28	0.1913	30	5.2256	836	0.9822	5	10
11 0	**0.1908**	28	**0.1944**	30	**5.1445**	811	**0.9816**	5	**0 79**
10	0.1936	28	0.1974	30	5.0658	787	0.9810	6	50
20	0.1965	28	0.2004	30	4.9894	764	0.9805	5	40
30	0.1993	28	0.2034	30	4.9151	742	0.9799	6	30
40	0.2022	28	0.2065	30	4.8430	721	0.9793	5	20
50	0.2050	28	0.2095	30	4.7728	701	0.9787	6	10
12 0	**0.2079**	28	**0.2125**	30	**4.7046**	682	**0.9781**	6	**0 78**
10	0.2107	28	0.2156	30	4.6382	664	0.9775	6	50
20	0.2136	28	0.2186	30	4.5736	646	0.9769	6	40
30	0.2164	28	0.2217	30	4.5107	629	0.9763	6	30
40	0.2193	28	0.2247	30	4.4494	613	0.9756	6	20
50	0.2221	28	0.2278	30	4.3897	597	0.9750	6	10
13 0	**0.2249**	28	**0.2308**	30	**4.3315**	582	**0.9743**	6	**0 77**
10	0.2278	28	0.2339	31	4.2747	568	0.9737	6	50
20	0.2306	28	0.2370	30	4.2193	553	0.9730	6	40
30	0.2334	28	0.2401	31	4.1653	540	0.9723	7	30
40	0.2362	28	0.2431	30	4.1125	527	0.9717	6	20
50	0.2391	28	0.2462	31	4.0610	515	0.9710	7	10
14 0	**0.2419**	28	**0.2493**	31	**4.0108**	502	**0.9703**	7	**0 76**
10	0.2447	28	0.2524	30	3.9616	491	0.9696	7	50
20	0.2475	28	0.2555	31	3.9136	480	0.9688	7	40
30	0.2504	28	0.2586	31	3.8667	469	0.9681	7	30
40	0.2532	28	0.2617	31	3.8208	458	0.9674	7	20
50	0.2560	28	0.2648	31	3.7759	449	0.9666	7	10
15 0	**0.2588**	28	**0.2679**	31	**3.7320**	439	**0.9659**	7	**0 75**
10	0.2616	28	0.2710	31	3.6891	429	0.9651	8	50
20	0.2644	28	0.2742	31	3.6470	420	0.9644	7	40
30	0.2672	28	0.2773	31	3.6059	411	0.9636	7	30
40	0.2700	28	0.2804	31	3.5655	403	0.9628	8	20
50	0.2728	28	0.2836	31	3.5261	394	0.9620	8	10
16 0	**0.2756**	28	**0.2867**	31	**3.4874**	387	**0.9612**	8	**0 74**
10	0.2784	28	0.2899	31	3.4495	379	0.9604	8	50
20	0.2812	28	0.2930	31	3.4123	371	0.9596	8	40
30	0.2840	27	0.2962	31	3.3759	364	0.9588	8	30
40	0.2868	28	0.2994	32	3.3402	357	0.9580	8	20
50	0.2896	28	0.3025	31	3.3052	350	0.9571	8	10
17 0	**0.2923**	27	**0.3057**	32	**3.2708**	343	**0.9563**	8	**0 73**
10	0.2951	28	0.3089	31	3.2371	337	0.9554	8	50
20	0.2979	28	0.3121	32	3.2040	331	0.9546	8	40
30	0.3007	27	0.3153	32	3.1716	324	0.9537	9	30
40	0.3035	28	0.3185	32	3.1397	319	0.9528	8	20
50	0.3062	27	0.3217	32	3.1084	313	0.9519	9	10
18 0	**0.3090**	27	**0.3249**	32	**3.0777**	307	**0.9510**	9	**0 72**
10	0.3118	28	0.3281	32	3.0475	302	0.9501	9	50
20	0.3145	27	0.3313	32	3.0178	296	0.9492	9	40
30	0.3173	27	0.3346	32	2.9887	291	0.9483	9	30
40	0.3200	27	0.3378	32	2.9600	286	0.9474	9	20
50	0.3228	27	0.3411	32	2.9319	281	0.9464	9	10
19 0	**0.3255**	27	**0.3443**	32	**2.9042**	277	**0.9455**	9	**0 71**
10	0.3283	27	0.3476	32	2.8770	272	0.9445	9	50
20	0.3310	27	0.3508	32	2.8502	267	0.9436	9	40
30	0.3338	27	0.3541	32	2.8239	263	0.9426	9	30
40	0.3365	27	0.3574	33	2.7980	259	0.9416	10	20
50	0.3393	27	0.3607	33	2.7725	254	0.9407	9	10
20 0	**0.3420**	27	**0.3639**	32	**2.7475**	250	**0.9397**	10	**0 70**
	Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	′ °

P. P.

	33	32	31
1	3.3	3.2	3.1
2	6.6	6.4	6.2
3	9.9	9.6	9.3
4	13.2	12.8	12.4
5	16.5	16.0	15.5
6	19.8	19.2	18.6
7	23.1	22.4	21.7
8	26.4	25.6	24.8
9	29.7	28.8	27.9

	30̂	30	29
1	3.0	3.0	2.9
2	6.1	6.0	5.8
3	9.1	9.0	8.7
4	12.2	12.0	11.6
5	15.2	15.0	14.5
6	18.3	18.0	17.4
7	21.3	21.0	20.3
8	24.4	24.0	23.2
9	27.4	27.0	26.1

	28̂	28	27
1	2.8	2.8	2.7
2	5.7	5.6	5.4
3	8.5	8.4	8.1
4	11.4	11.2	10.8
5	14.2	14.0	13.5
6	17.1	16.8	16.2
7	19.9	19.6	18.9
8	22.8	22.4	21.6
9	25.6	25.2	24.3

	10	9	8
1	1.0	0.9	0.8
2	2.0	1.8	1.6
3	3.0	2.7	2.4
4	4.0	3.6	3.2
5	5.0	4.5	4.0
6	6.0	5.4	4.8
7	7.0	6.3	5.6
8	8.0	7.2	6.4
9	9.0	8.1	7.2

	7̂	7	6	5
1	0.7	0.7	0.6	0.5
2	1.5	1.4	1.2	1.0
3	2.2	2.1	1.8	1.5
4	3.0	2.8	2.4	2.0
5	3.7	3.5	3.0	2.5
6	4.5	4.2	3.6	3.0
7	5.2	4.9	4.2	3.5
8	6.0	5.6	4.8	4.0
9	6.7	6.3	5.4	4.5

P. P.

70°–80°

20°–30°

° ′	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.	
20 0	0.3420	27	0.3639	33	2.7475	247	0.9397	10	0 70
10	0.3447	27	0.3672	33	2.7228	242	0.9387	10	50
20	0.3475	27	0.3705	33	2.6985	239	0.9377	10	40
30	0.3502	27	0.3739	33	2.6746	235	0.9366	10	30
40	0.3529	27	0.3772	33	2.6511	232	0.9356	10	20
50	0.3556	27	0.3805	33	2.6279	228	0.9346	10	10
21 0	0.3583	27	0.3838	33	2.6051	225	0.9336	10	0 69
10	0.3611	27	0.3872	33	2.5826	221	0.9325	10	50
20	0.3638	27	0.3905	33	2.5604	218	0.9315	11	40
30	0.3665	27	0.3939	33	2.5386	215	0.9304	10	30
40	0.3692	27	0.3972	34	2.5171	212	0.9293	11	20
50	0.3719	27	0.4006	34	2.4959	208	0.9282	10	10
22 0	0.3746	27	0.4040	33	2.4751	206	0.9272	11	0 68
10	0.3773	27	0.4074	34	2.4545	203	0.9261	11	50
20	0.3800	27	0.4108	34	2.4342	200	0.9250	11	40
30	0.3827	26	0.4142	34	2.4142	197	0.9239	11	30
40	0.3853	27	0.4176	34	2.3945	194	0.9227	11	20
50	0.3880	27	0.4210	34	2.3750	192	0.9216	11	10
23 0	0.3907	26	0.4244	34	2.3558	189	0.9205	11	0 67
10	0.3934	27	0.4279	34	2.3369	187	0.9193	11	50
20	0.3961	26	0.4313	34	2.3182	184	0.9182	11	40
30	0.3987	26	0.4348	35	2.2998	182	0.9170	11	30
40	0.4014	27	0.4383	34	2.2816	179	0.9159	12	20
50	0.4041	26	0.4417	35	2.2637	177	0.9147	11	10
24 0	0.4067	26	0.4452	34	2.2460	175	0.9135	12	0 66
10	0.4094	26	0.4487	35	2.2285	172	0.9123	12	50
20	0.4120	26	0.4522	35	2.2113	170	0.9111	12	40
30	0.4147	26	0.4557	35	2.1943	168	0.9099	12	30
40	0.4173	26	0.4592	35	2.1775	166	0.9087	12	20
50	0.4200	26	0.4627	35	2.1609	164	0.9075	12	10
25 0	0.4226	26	0.4663	35	2.1445	162	0.9063	12	0 65
10	0.4252	26	0.4698	35	2.1283	159	0.9050	12	50
20	0.4279	26	0.4734	36	2.1123	158	0.9038	12	40
30	0.4305	26	0.4770	35	2.0965	156	0.9026	12	30
40	0.4331	26	0.4805	36	2.0809	154	0.9013	13	20
50	0.4357	26	0.4841	36	2.0655	152	0.9000	12	10
26 0	0.4383	26	0.4877	36	2.0503	150	0.8988	13	0 64
10	0.4410	26	0.4913	36	2.0352	148	0.8975	12	50
20	0.4436	26	0.4949	36	2.0204	147	0.8962	13	40
30	0.4462	26	0.4986	36	2.0057	145	0.8949	13	30
40	0.4488	26	0.5022	36	1.9911	143	0.8936	13	20
50	0.4514	26	0.5058	37	1.9768	142	0.8923	13	10
27 0	0.4540	26	0.5095	36	1.9626	140	0.8910	13	0 63
10	0.4566	25	0.5132	37	1.9486	139	0.8897	13	50
20	0.4591	26	0.5169	36	1.9347	137	0.8883	13	40
30	0.4617	26	0.5205	37	1.9210	136	0.8870	13	30
40	0.4643	25	0.5242	37	1.9074	134	0.8856	13	20
50	0.4669	25	0.5280	37	1.8940	132	0.8843	13	10
28 0	0.4694	26	0.5317	37	1.8807	131	0.8829	13	0 62
10	0.4720	25	0.5354	37	1.8676	130	0.8816	14	50
20	0.4746	25	0.5392	37	1.8546	128	0.8802	14	40
30	0.4771	25	0.5429	38	1.8417	127	0.8788	13	30
40	0.4797	25	0.5467	37	1.8290	125	0.8774	14	20
50	0.4822	25	0.5505	38	1.8165	124	0.8760	14	10
29 0	0.4848	25	0.5543	38	1.8040	123	0.8746	14	0 61
10	0.4873	25	0.5581	38	1.7917	122	0.8732	14	50
20	0.4899	25	0.5619	38	1.7795	120	0.8718	14	40
30	0.4924	25	0.5657	38	1.7675	119	0.8703	14	30
40	0.4949	25	0.5696	39	1.7555	118	0.8689	14	20
50	0.4975	25	0.5735	38	1.7437	117	0.8675	14	10
30 0	0.5000		0.5773		1.7320		0.8660		0 60
	Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	′ °

P. P.

	39	38	37	36
1	3.9	3.8	3.7	3.6
2	7.8	7.6	7.4	7.2
3	11.7	11.4	10.1	10.8
4	15.6	15.2	14.8	14.4
5	19.5	19.0	18.5	18.0
6	23.4	22.8	22.2	21.6
7	27.3	26.6	25.9	25.2
8	31.2	30.4	29.6	28.8
9	35.1	34.2	33.3	32.4

	35	35	34	33
1	3.5	3.5	3.4	3.3
2	7.1	7.0	6.8	6.6
3	10.6	10.5	10.2	9.9
4	14.2	14.0	13.6	13.2
5	17.7	17.5	17.0	16.5
6	21.3	21.0	20.4	19.8
7	24.8	24.5	23.8	23.1
8	28.4	28.0	27.2	26.4
9	31.9	31.5	30.6	29.7

	27	27	26	25
1	2.7	2.7	2.6	2.5
2	5.5	5.4	5.2	5.0
3	8.2	8.1	7.8	7.5
4	11.0	10.8	10.4	10.0
5	13.7	13.5	13.0	12.5
6	16.5	16.2	15.6	15.0
7	19.2	18.9	18.2	17.5
8	22.0	21.6	20.8	20.0
9	24.7	24.3	23.4	22.5

	14	14	13	12
1	1.4	1.4	1.3	1.2
2	2.9	2.8	2.6	2.4
3	4.3	4.2	3.9	3.6
4	5.8	5.6	5.2	4.8
5	7.2	7.0	6.5	6.0
6	8.7	8.4	7.8	7.2
7	10.1	9.8	9.1	8.4
8	11.6	11.2	10.4	9.6
9	13.0	12.6	11.7	10.8

	11	11	10
1	1.1	1.1	1.0
2	2.3	2.2	2.0
3	3.4	3.3	3.0
4	4.6	4.4	4.0
5	5.7	5.5	5.0
6	6.9	6.6	6.0
7	8.0	7.7	7.0
8	9.2	8.8	8.0
9	10.3	9.9	9.0

P. P.

60°–70°

30°-40°

° ′	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.	
30 0	**0.5000**		**0.5773**		**1.7320**		**0.8660**		**0 60**
10	0.5025	25	0.5812	39	1.7204	116	0.8645	15	50
20	0.5050	25	0.5851	39	1.7090	114	0.8631	14	40
30	0.5075	25	0.5890	39	1.6976	113	0.8616	14	30
40	0.5100	25	0.5929	39	1.6864	112	0.8601	15	20
50	0.5125	25	0.5969	39	1.6753	111	0.8586	15	10
31 0	**0.5150**	25	**0.6008**	39	**1.6643**	110	**0.8571**	15	**0 59**
10	0.5175	25	0.6048	40	1.6533	109	0.8556	15	50
20	0.5200	24	0.6088	39	1.6425	108	0.8541	15	40
30	0.5225	25	0.6128	40	1.6318	107	0.8526	15	30
40	0.5250	25	0.6168	40	1.6212	106	0.8511	15	20
50	0.5274	24	0.6208	40	1.6107	105	0.8496	15	10
32 0	**0.5299**	24	**0.6248**	40	**1.6003**	104	**0.8480**	15	**0 58**
10	0.5324	25	0.6289	40	1.5900	103	0.8465	15	50
20	0.5348	24	0.6330	41	1.5798	102	0.8449	15	40
30	0.5373	24	0.6370	40	1.5697	101	0.8434	15	30
40	0.5397	24	0.6411	41	1.5596	100	0.8418	16	20
50	0.5422	24	0.6453	41	1.5497	99	0.8402	15	10
33 0	**0.5446**	24	**0.6494**	41	**1.5398**	98	**0.8386**	16	**0 57**
10	0.5471	24	0.6535	41	1.5301	97	0.8371	15	50
20	0.5495	24	0.6577	41	1.5204	96	0.8355	16	40
30	0.5519	24	0.6619	42	1.5108	96	0.8339	16	30
40	0.5543	24	0.6661	42	1.5013	95	0.8323	16	20
50	0.5568	24	0.6703	42	1.4919	94	0.8306	16	10
34 0	**0.5592**	24	**0.6745**	42	**1.4825**	93	**0.8290**	16	**0 56**
10	0.5616	24	0.6787	42	1.4733	92	0.8274	16	50
20	0.5640	24	0.6830	42	1.4641	92	0.8257	16	40
30	0.5664	24	0.6873	43	1.4550	91	0.8241	16	30
40	0.5688	24	0.6915	42	1.4460	90	0.8225	16	20
50	0.5712	24	0.6959	43	1.4370	89	0.8208	17	10
35 0	**0.5736**	24	**0.7002**	43	**1.4281**	89	**0.8191**	16	**0 55**
10	0.5759	23	0.7045	43	1.4193	88	0.8175	16	50
20	0.5783	24	0.7089	43	1.4106	87	0.8158	17	40
30	0.5807	23	0.7133	44	1.4019	86	0.8141	17	30
40	0.5830	23	0.7177	44	1.3933	86	0.8124	17	20
50	0.5854	24	0.7221	44	1.3848	85	0.8107	17	10
36 0	**0.5878**	23	**0.7265**	44	**1.3764**	84	**0.8090**	17	**0 54**
10	0.5901	23	0.7310	44	1.3680	84	0.8073	17	50
20	0.5925	23	0.7354	44	1.3597	83	0.8056	17	40
30	0.5948	23	0.7399	45	1.3514	83	0.8038	17	30
40	0.5971	23	0.7444	45	1.3432	81	0.8021	17	20
50	0.5995	23	0.7490	45	1.3351	81	0.8004	17	10
37 0	**0.6018**	23	**0.7535**	45	**1.3270**	80	**0.7986**	17	**0 53**
10	0.6041	23	0.7581	45	1.3190	80	0.7969	17	50
20	0.6064	23	0.7627	46	1.3111	79	0.7951	18	40
30	0.6087	23	0.7673	46	1.3032	78	0.7933	17	30
40	0.6110	23	0.7719	46	1.2954	78	0.7916	17	20
50	0.6133	23	0.7766	46	1.2876	77	0.7898	18	10
38 0	**0.6156**	23	**0.7813**	47	**1.2799**	77	**0.7880**	18	**0 52**
10	0.6179	23	0.7860	47	1.2723	76	0.7862	18	50
20	0.6202	23	0.7907	47	1.2647	76	0.7844	18	40
30	0.6225	22	0.7954	47	1.2571	75	0.7826	18	30
40	0.6248	23	0.8002	47	1.2497	74	0.7808	18	20
50	0.6270	22	0.8050	48	1.2422	74	0.7789	18	10
39 0	**0.6293**	22	**0.8098**	48	**1.2349**	73	**0.7771**	18	**0 51**
10	0.6316	23	0.8146	48	1.2276	73	0.7753	18	50
20	0.6338	22	0.8194	48	1.2203	73	0.7734	18	40
30	0.6361	22	0.8243	49	1.2131	72	0.7716	18	30
40	0.6383	22	0.8292	49	1.2059	71	0.7697	18	20
50	0.6405	22	0.8341	49	1.1988	71	0.7679	18	10
40 0	**0.6428**	22	**0.8391**	49	**1.1917**	70	**0.7660**	18	**0 50**
	Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	′ °

P. P.

	49	49	48	47	46
1	4.9	4.9	4.8	4.7	4.6
2	9.9	9.8	9.6	9.4	9.2
3	14.8	14.7	14.4	14.1	13.8
4	19.8	19.6	19.2	18.8	18.4
5	24.7	24.5	24.0	23.5	23.0
6	29.7	29.4	28.8	28.2	27.6
7	34.6	34.3	33.6	32.9	32.2
8	39.6	39.2	38.4	37.6	36.8
9	44.5	44.1	43.2	42.3	41.4

	45	45	44	43	42
1	4.5	4.5	4.4	4.3	4.2
2	9.1	9.0	8.8	8.6	8.4
3	13.6	13.5	13.2	12.9	12.6
4	18.2	18.0	17.6	17.2	16.8
5	22.7	22.5	22.0	21.5	21.0
6	27.3	27.0	26.4	25.8	25.2
7	31.8	31.5	30.8	30.1	29.4
8	36.4	36.0	35.2	34.4	33.6
9	40.9	40.5	39.6	38.7	37.8

	41	41	40	39
1	4.1	4.1	4.0	3.9
2	8.3	8.2	8.0	7.8
3	12.4	12.3	12.0	11.7
4	16.6	16.4	16.0	15.6
5	20.7	20.5	20.0	19.5
6	24.9	24.6	24.0	23.4
7	29.0	28.7	28.0	27.3
8	33.2	32.8	32.0	31.2
9	37.3	36.9	36.0	35.1

	25	25	24	23
1	2.5	2.5	2.4	2.3
2	5.1	5.0	4.8	4.6
3	7.6	7.5	7.2	6.9
4	10.2	10.0	9.6	9.2
5	12.7	12.5	12.0	11.5
6	15.3	15.0	14.4	13.8
7	17.8	17.5	16.8	16.1
8	20.4	20.0	19.2	18.4
9	22.9	22.5	21.6	20.7

	22	22	18	18
1	2.2	2.2	1.8	1.8
2	4.5	4.4	3.7	3.6
3	6.7	6.6	5.5	5.4
4	9.0	8.8	7.4	7.2
5	11.2	11.0	9.2	9.0
6	13.5	13.2	11.1	10.8
7	15.7	15.4	12.9	12.6
8	18.0	17.6	14.8	14.4
9	20.2	19.8	16.6	16.2

	17	17	16	15	14
1	1.7	1.7	1.6	1.5	1.4
2	3.5	3.4	3.2	3.0	2.9
3	5.2	5.1	4.8	4.5	4.3
4	7.0	6.8	6.4	6.0	6.8
5	8.7	8.5	8.0	7.5	7.2
6	10.5	10.2	9.6	9.0	8.7
7	12.2	11.9	11.2	10.5	10.1
8	14.0	13.6	12.8	12.0	11.6
9	15.7	15.3	14.4	13.5	13.0

50°-60°

40°–45°

° ′	Sin.	d.	Tan.	d.	Cot.	d.	Cos.	d.	
40 0	0.6428	22	0.8391	49	1.1917	70	0.7660	19	0 50
10	0.6450	22	0.8440	50	1.1847	70	0.7641	18	50
20	0.6472	22	0.8490	50	1.1777	69	0.7623	19	40
30	0.6494	22	0.8541	50	1.1708	68	0.7604	19	30
40	0.6516	22	0.8591	51	1.1640	68	0.7585	19	20
50	0.6538	22	0.8642	51	1.1571	68	0.7566	19	10
41 0	0.6560	22	0.8693	51	1.1503	67	0.7547	19	0 49
10	0.6582	22	0.8744	51	1.1436	67	0.7528	19	50
20	0.6604	21	0.8795	52	1.1369	66	0.7509	19	40
30	0.6626	22	0.8847	51	1.1303	66	0.7489	19	30
40	0.6648	21	0.8899	52	1.1237	65	0.7470	19	20
50	0.6669	22	0.8951	52	1.1171	65	0.7451	19	10
42 0	0.6691	21	0.9004	53	1.1106	64	0.7431	19	0 48
10	0.6713	21	0.9057	53	1.1041	64	0.7412	19	50
20	0.6734	21	0.9110	53	1.0977	64	0.7392	19	40
30	0.6756	21	0.9163	53	1.0913	63	0.7373	20	30
40	0.6777	21	0.9217	54	1.0849	63	0.7353	19	20
50	0.6798	21	0.9271	54	1.0786	63	0.7333	20	10
43 0	0.6820	21	0.9325	54	1.0723	62	0.7313	20	0 47
10	0.6841	21	0.9379	55	1.0661	62	0.7293	20	50
20	0.6862	21	0.9434	55	1.0599	61	0.7273	20	40
30	0.6883	21	0.9489	55	1.0538	61	0.7253	20	30
40	0.6904	21	0.9545	56	1.0476	60	0.7233	20	20
50	0.6925	21	0.9601	56	1.0416	60	0.7213	20	10
44 0	0.6946	21	0.9657	56	1.0355	60	0.7193	20	0 46
10	0.6967	21	0.9713	56	1.0295	59	0.7173	20	50
20	0.6988	20	0.9770	57	1.0235	59	0.7153	20	40
30	0.7009	21	0.9827	57	1.0176	59	0.7132	20	30
40	0.7030	20	0.9884	57	1.0117	58	0.7112	20	20
50	0.7050	20	0.9942	58	1.0058	58	0.7091	20	10
45 0	0.7071		1.0000		1.0000		0.7071		0 45
	Cos.	d.	Cot.	d.	Tan.	d.	Sin.	d.	′ °

P. P.

	70	22	22	21	21
1	7.0	2.2	2.2	2.1	2.1
2	14.0	4.5	4.4	4.3	4.2
3	21.0	6.7	6.6	6.4	6.3
4	28.0	9.0	8.8	8.6	8.4
5	35.0	11.2	11.0	10.7	10.5
6	42.0	13.5	13.2	12.9	12.6
7	49.0	15.7	15.4	15.0	14.7
8	56.0	18.0	17.6	17.2	16.8
9	63.0	20.2	19.8	19.3	18.9

	69	20	20	19	19
1	6.9	2.0	2.0	1.9	1.9
2	13.8	4.1	4.0	3.9	3.8
3	20.7	6.1	6.0	5.8	5.7
4	27.6	8.2	8.0	7.8	7.6
5	34.5	10.2	10.0	9.7	9.5
6	41.4	12.3	12.0	11.7	11.4
7	48.3	14.3	14.0	13.6	13.3
8	55.2	16.4	16.0	15.6	15.2
9	62.1	18.4	18.0	17.5	17.1

	68	68	67	66	18
1	6 8	6.8	6.7	6.6	1.8
2	13.7	13.6	13.4	13.2	3.7
3	20.5	20.4	20.1	19.8	5.5
4	27.4	27.2	26.8	26.4	7.4
5	34.2	34.0	33.5	33.0	9.2
6	41.1	40.8	40.2	39.6	11.1
7	47.9	47.6	46.9	46.2	12.9
8	54.8	54.4	53.6	52.8	14.8
9	61.6	61.2	60.3	59.4	16.6

	65	64	64	63	62	61	60	59	59	58	58	57	57	56	56	55	54	54	53	53	52	52
1	6.5	6.4	6.4	6.3	6.2	6.1	6.0	5.9	5.9	5.8	5.8	5.7	5.7	5.6	5.6	5.5	5.4	5.4	5.3	5.3	5.2	5.2
2	13.1	12.9	12.8	12.6	12.4	12.3	12.1	11.9	11.8	11.7	11.6	11.5	11.4	11.3	11.2	11.0	10.9	10.8	10.7	10.6	10.5	10.4
3	19.6	19.3	19.2	18.9	18.6	18.4	18.1	17.8	17.7	17.5	17.4	17.2	17.1	16.9	16.8	16.5	16.3	16.2	16.0	15.9	15.7	15.6
4	26.2	25.8	25.6	25.2	24.8	24.6	24.2	23.8	23.6	23.4	23.2	23.0	22.8	22.6	22.4	22.0	21.8	21.6	21.4	21.2	21.0	20.8
5	32.7	32.2	32.0	31.5	31.0	30.7	30.2	29.7	29.5	29.2	29.0	28.7	28.5	28.2	28.0	27.5	27.2	27.0	26.7	26.5	26.2	26.0
6	39.3	38.7	38.4	37.8	37.2	36.9	36.3	35.7	35.4	35.1	34.8	34.5	34.2	33.9	33.6	33.0	32.7	32.4	32.1	31.8	31.5	31.2
7	45.8	45.1	44.8	44.1	43.4	43.0	42.3	41.6	41.3	40.9	40.6	40.2	39.9	39.5	39.2	38.5	38.1	37.8	37.4	37.1	36.7	36.4
8	52.4	51.6	51.2	50.4	49.6	49.2	48.4	47.6	47.2	46.8	46.4	46.0	45.6	45.2	44.8	44.0	43.6	43.2	42.8	42.4	42.0	41.6
9	58.9	58.0	57.6	56.7	55.8	55.3	54.4	53.5	53.1	52.6	52.2	51.7	51.3	50.8	50.4	49.5	49.0	48.6	48.1	47.7	47.2	46.8

	51	51	50	50	49
1	5.1	5.1	5.0	5.0	4.9
2	10.3	10.2	10.1	10.0	9.9
3	15.4	15.3	15.1	15.0	14.8
4	20.6	20.4	20.2	20.0	19.8
5	25.7	25.5	25.2	25.0	24.7
6	30.9	30.6	30.3	30.0	29.7
7	36.0	35.7	35.3	35.0	34.6
8	41.2	40.8	40.4	40.0	39.6
9	46.3	45.9	45.4	45.0	44.5

Table for passing from Sexagesimal to Circular Measure.

°	Circular Meas.	′	Circular Meas.	″	Circular Meas.
100	1.74 532 9	10	0.00 290 9	10	0.00 004 8
200	3.49 065 8	20	0.00 581 8	20	0.00 009 7
300	5.23 598 8	30	0.00 872 6	30	0.00 014 5
		40	0.01 163 5	40	0.00 019 4
40	0.69 813 1				
50	0.87 266 4	50	0.01 454 4	50	0.00 024 2
60	1.04 719 7				
		6	0.00 174 5	6	0.00 002 9
70	1.22 173 0	7	0.00 203 6	7	0.00 003 4
80	1.39 626 3	8	0.00 232 7	8	0.00 003 9
90	1.57 079 6	9	0.00 261 8	9	0.00 004 3

45°–50°

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

0°-10°   10°-20°

° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.
0 0	.00000		.00000		10 0	.01519		.01542̂	
10	.00000̂	0̂	.00000̂	0̂	10	.01570	51	.01595	52̂
20	.00001̂	1	.00001̂	1	20	.01622	52	.01648̂	53̂
30	.00004	2̂	.00004	2̂	30	.01674̂	52̂	.01703	54̂
40	.00007	3	.00007	3	40	.01728	53̂	.01758̂	55̂
50	.00010̂	3̂	.00010̂	3̂	50	.01782	54	.01814̂	56
1 0	.00015	4̂	.00015	4̂	11 0	.01837̂	55̂	.01871̂	57
10	.00020̂	5̂	.00020̂	5̂	10	.01893	55̂	.01929̂	58
20	.00027	6̂	.00027	6̂	20	.01950	57	.01988̂	59
30	.00034̂	7̂	.00034̂	7̂	30	.02007̂	57̂	.02048̂	60
40	.00042̂	8	.00042̂	8	40	.02066	58̂	.02109̂	61
50	.00051	8̂	.00051	8̂	50	.02125	59	.02171̂	62
2 0	.00061	10	.00061	10	12 0	.02185	60	.02234	62̂
10	.00071̂	10̂	.00071̂	10̂	10	.02246	61	.02297̂	63̂
20	.00083	11̂	.00083	11̂	20	.02308	62	.02362̂	65
30	.00095̂	12	.00095̂	12̂	30	.02370̂	62̂	.02428	65̂
40	.00108̂	13̂	.00108̂	13	40	.02434	63̂	.02494̂	66̂
50	.00122	13̂	.00122̂	14	50	.02498	64	.02562	67̂
3 0	.00137	15	.00137	14̂	13 0	.02563	65	.02630̂	68̂
10	.00152̂	15̂	.00153	16	10	.02629	66	.02700	69̂
20	.00169	16̂	.00169̂	16̂	20	.02695̂	66̂	.02770	70
30	.00186̂	17̂	.00187	17̂	30	.02763	67̂	.02841̂	71̂
40	.00204̂	18	.00205	18	40	.02831̂	68̂	.02914	72̂
50	.00223̂	19	.00224	19	50	.02900̂	69	.02987	73
4 0	.00243̂	20	.00244	20	14 0	.02970̂	70	.03061̂	74̂
10	.00264̂	21	.00265	21	10	.03041	70̂	.03136̂	75
20	.00286	21̂	.00286̂	21̂	20	.03113	72	.03213	76̂
30	.00308̂	22̂	.00309	22̂	30	.03185	72	.03290	77
40	.00331̂	23	.00332̂	23̂	40	.03258̂	72̂	.03368	78
50	.00355̂	24	.00357	24̂	50	.03332̂	74	.03447̂	79̂
5 0	.00380̂	25	.00382	25	15 0	.03407̂	75	.03527̂	80
10	.00406̂	26	.00408	26	10	.03483	75̂	.03609	81̂
20	.00433	26̂	.00435	27	20	.03559̂	76̂	.03691	82
30	.00460̂	27̂	.00462̂	27̂	30	.03637	77̂	.03774	83
40	.00488̂	28	.00491	28̂	40	.03715	78	.03858̂	84̂
50	.00518	29̂	.00526̂	29̂	50	.03794	79	.03943̂	85
6 0	.00548	30	.00551	30̂	16 0	.03874	80	.04030	86̂
10	.00578̂	30̂	.00582	31	10	.03954̂	80̂	.04117	87
20	.00610̂	32	.00614	32	20	.04036	81̂	.04205̂	88̂
30	.00643	32̂	.00647	33	30	.04118	82	.04295	89̂
40	.00676	33	.00681	34	40	.04201	83	.04385̂	90̂
50	.00710̂	33̂	.00715̂	34̂	50	.04285	84	.04476̂	91
7 0	.00745̂	35	.00751	35̂	17 0	.04369̂	84̂	.04569	92̂
10	.00781̂	36	.00787̂	36̂	10	.04455	85̂	.04662̂	93̂
20	.00818	36̂	.00824̂	37	20	.04541	86	.04757̂	95
30	.00855̂	37̂	.00863	38̂	30	.04628̂	87̂	.04853	95̂
40	.00894	38̂	.00902	39	40	.04716	87̂	.04949̂	96̂
50	.00933	39	.00942	40	50	.04805	89	.05047̂	98
8 0	.00973	40	.00983	41	18 0	.04894̂	89̂	.05146	98̂
10	.01014	41	.01024̂	41̂	10	.04984̂	90	.05246	100
20	.01056	42	.01067	42̂	20	.05076	91̂	.05347	101
30	.01098̂	42̂	.01110̂	43̂	30	.05167̂	91̂	.05449	102
40	.01142	43̂	.01155	44̂	40	.05260̂	93	.05552̂	103̂
50	.01186	44	.01200̂	45̂	50	.05354	93̂	.05656̂	104
9 0	.01231	45	.01246̂	46	19 0	.05448	94	.05762	105̂
10	.01277	46	.01293̂	47	10	.05543	95̂	.05868̂	106̂
20	.01324	47	.01341̂	48	20	.05639	95̂	.05976	107̂
30	.01371̂	47̂	.01390̂	49	30	.05736	97	.06085	109
40	.01420	48̂	.01440̂	50	40	.05833̂	97̂	.06194̂	109̂
50	.01469	49	.01491	50̂	50	.05931̂	98	.06305̂	111
10 0	.01519	50	.01542̂	51̂	20 0	.06030̂	99	.06418	112̂
° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.

P. P.

	110	100	90	80	70	60	50	40
1	11	10	9	8	7	6	5	4
2	22	20	18	16	14	12	10	8
3	33	30	27	24	21	18	15	12
4	44	40	36	32	28	24	20	16
5	55	50	45	40	35	30	25	20
6	66	60	54	48	42	36	30	24
7	77	70	63	56	49	42	35	28
8	88	80	72	64	56	48	40	32
9	99	90	81	72	63	54	45	36

	30	20	10	9̂	9	8̂	8	7̂
1	3	2	1	0.9̂	0.9	0.8̂	0.8	0.7̂
2	6	4	2	1.9	1.8	1.7	1.6	1.5
3	9	6	3	2.8̂	2.7	2.5̂	2.4	2.2̂
4	12	8	4	3.8	3.6	3.4	3.2	3.0
5	15	10	5	4.7̂	4.5	4.2̂	4.0	3.7̂
6	18	12	6	5.7	5.4	5.1	4.8	4.5
7	21	14	7	6.6̂	6.3	5.9̂	5.6	5.2̂
8	24	16	8	7.6	7.2	6.8	6.4	6.0
9	27	18	9	8.5̂	8.1	7.6̂	7.2	6.7̂

	7	6̂	6	5̂	5	4̂	4
1	0.7	0.6̂	0.6	0.5̂	0.5	0.4̂	0.4
2	1.4	1.3	1.2	1.1	1.0	0.9	0.8
3	2.1	1.9̂	1.8	1.6̂	1.5	1.3̂	1.2
4	2.8	2.6	2.4	2.2	2.0	1.8	1.6
5	3.5	3.2̂	3.0	2.7̂	2.5	2.2̂	2.0
6	4.2	3.9	3.6	3.3	3.0	2.7	2.4
7	4.9	4.5̂	4.2	3.8̂	3.5	3.1̂	2.8
8	5.6	5.2	4.8	4.4	4.0	3.6	3.2
9	6.3	5.8̂	5.4	4.9̂	4.5	4.0̂	3.6

	3̂	3	2̂	2	1̂	1	0̂
1	0.3̂	0.3	0.2̂	0.2	0.1̂	0.1	0.0̂
2	0.7	0.6	0.5	0.4	0.3	0.2	0.1
3	1.0̂	0.9	0.7̂	0.6	0.4̂	0.3	0.1̂
4	1.4	1.2	1.0	0.8	0.6	0.4	0.2
5	1.7̂	1.5	1.2̂	1.0	0.7̂	0.5	0.2̂
6	2.1	1.8	1.5	1.2	0.9	0.6	0.3
7	2.4̂	2.1	1.7̂	1.4	1.0̂	0.7	0.3̂
8	2.8	2.4	2.0	1.6	1.2	0.8	0.4
9	3.1̂	2.7	2.2̂	1.8	1.3̂	0.9	0.4̂

P. P.

20°-30° **30°-40°**

° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.
20 0	.0603	10	.0642	11	30 0	.1339	15	.1547	19
10	.0613	10	.0653	11	10	.1354	14	.1566	19
20	.0623	10	.0664	11	20	.1369	14	.1586	20
30	.0633	10	.0676	12	30	.1383	15	.1606	20
40	.0643	10	.0688	11	40	.1398	15	.1626	20
50	.0654	10	.0699	12	50	.1413	15	.1646	20
21 0	.0664	10	.0711	12	31 0	.1428	15	.1666	20
10	.0674	10	.0723	12	10	.1443	15	.1687	20
20	.0685	11	.0735	12	20	.1458	15	.1707	21
30	.0696	10	.0748	12	30	.1473	15	.1728	21
40	.0706	11	.0760	12	40	.1489	15	.1749	21
50	.0717	10	.0772	13	50	.1504	15	.1770	21
22 0	.0728	11	.0785	12	32 0	.1519	15	.1792	21
10	.0739	11	.0798	13	10	.1535	15	.1813	21
20	.0750	11	.0811	13	20	.1550	15	.1835	22
30	.0761	11	.0824	13	30	.1566	16	.1857	22
40	.0772	11	.0837	13	40	.1582	15	.1879	22
50	.0783	11	.0850	13	50	.1597	16	.1901	22
23 0	.0795	11	.0863	13	33 0	.1613	15	.1923	23
10	.0806	11	.0877	13	10	.1629	16	.1946	22
20	.0818	11	.0890	14	20	.1645	16	.1969	23
30	.0829	11	.0904	14	30	.1661	16	.1992	23
40	.0841	12	.0918	14	40	.1677	16	.2015	23
50	.0853	11	.0932	14	50	.1693	16	.2038	24
24 0	.0864	12	.0946	14	34 0	.1709	16	.2062	24
10	.0876	12	.0960	14	10	.1726	16	.2086	24
20	.0888	12	.0975	14	20	.1742	16	.2110	24
30	.0900	12	.0989	14	30	.1758	16	.2134	24
40	.0912	12	.1004	15	40	.1775	17	.2158	24
50	.0924	12	.1019	15	50	.1792	16	.2183	24
25 0	.0937	12	.1034	15	35 0	.1808	16	.2207	25
10	.0949	12	.1049	15	10	.1825	17	.2232	25
20	.0961	12	.1064	15	20	.1842	17	.2258	25
30	.0974	12	.1079	15	30	.1859	17	.2283	25
40	.0986	13	.1094	16	40	.1876	17	.2309	25
50	.0999	12	.1110	15	50	.1893	17	.2334	26
26 0	.1012	13	.1126	16	36 0	.1910	17	.2360	26
10	.1025	12	.1142	16	10	.1927	17	.2387	26
20	.1037	13	.1158	16	20	.1944	17	.2413	26
30	.1050	13	.1174	16	30	.1961	17	.2440	27
40	.1063	13	.1190	16	40	.1979	17	.2467	27
50	.1077	13	.1206	17	50	.1996	17	.2494	27
27 0	.1090	13	1223	16	37 0	.2013	17	.2521	27
10	.1103	13	.1240	17	10	.2031	18	.2549	27
20	.1116	13	.1257	17	20	.2049	17	.2576	28
30	.1130	13	.1274	17	30	.2066	17	.2604	28
40	.1143	13	.1291	17	40	.2084	18	.2633	28
50	.1157	13	.1308	17	50	.2102	18	.2661	28
28 0	.1170	13	.1325	18	38 0	2120	18	.2690	29
10	.1184	14	.1343	17	10	.2138	18	.2719	29
20	.1198	14	.1361	18	20	.2156	18	.2748	29
30	.1212	13	.1379	18	30	.2174	18	.2778	29
40	.1225	14	.1397	18	40	.2192	18	.2807	30
50	.1239	14	.1415	18	50	.2210	18	.2837	30
29 0	.1254	14	.1433	18	39 0	.2228	18	.2867	30
10	.1268	14	.1452	18	10	.2247	18	.2898	30
20	.1282	14	.1470	19	20	.2265	18	.2928	31
30	.1296	14	.1489	19	30	.2284	18	.2959	31
40	.1311	14	.1508	19	40	.2302	18	.2991	31
50	.1325	14	.1527	19	50	.2321	18	.3022	31
30 0	.1339		.1547		40 0	.2339		.3054	
° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.

P. P.

	31	30	29	28
1	3.1	3.0	2.9	2.8
2	6.2	6.0	5.8	5.6
3	9.3	9.0	8.7	8.4
4	12.4	12.0	11.6	11.2
5	15.5	15.0	14.5	14.0
6	18.6	18.0	17.4	16.8
7	21.7	21.0	20.3	19.6
8	24.8	24.0	23.2	22.4
9	27.9	27.0	26.1	25.2

	27	26	25	24
1	2.7	2.6	2.5	2.4
2	5.4	5.2	5.0	4.8
3	8.1	7.8	7.5	7.2
4	10.8	10.4	10.0	9.6
5	13.5	13.0	12.5	12.0
6	16.2	15.6	15.0	14.4
7	18.9	18.2	17.5	16.8
8	21.6	20.8	20.0	19.2
9	24.3	23.4	22.5	21.6

	23	22	21	20
1	2.3	2.2	2.1	2.0
2	4.6	4.4	4.2	4.0
3	6.9	6.6	6.3	6.0
4	9.2	8.8	8.4	8.0
5	11.5	11.0	10.5	10.0
6	13.8	13.2	12.6	12.0
7	16.1	15.4	14.7	14.0
8	18.4	17.6	16.8	16.0
9	20.7	19.8	18.9	18.0

	19	18	17	16
1	1.9	1.8	1.7	1.6
2	3.8	3.6	3.4	3.2
3	5.7	5.4	5.1	4.8
4	7.6	7.2	6.8	6.4
5	9.5	9.0	8.5	8.0
6	11.4	10.8	10.2	9.6
7	13.3	12.6	11.9	11.2
8	15.2	14.4	13.6	12.8
9	17.1	16.2	15.3	14.4

	15	14	13	12
1	1.5	1.4	1.3	1.2
2	3.0	2.8	2.6	2.4
3	4.5	4.2	3.9	3.6
4	6.0	5.6	5.2	4.8
5	7.5	7.0	6.5	6.0
6	9.0	8.4	7.8	7.2
7	10.5	9.8	9.1	8.4
8	12.0	11.2	10.4	9.6
9	13.5	12.6	11.7	10.8

	11	10	0
1	1.1	1.0	0.0
2	2.2	2.0	0.1
3	3.3	3.0	0.1
4	4.4	4.0	0.2
5	5.5	5.0	0.2
6	6.6	6.0	0.3
7	7.7	7.0	0.3
8	8.8	8.0	0.4
9	9.9	9.0	0.4

P. P.

40°-50° 50°-60°

° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.
40 0	.2339̂	19	.3054	32	50 0	.3572	22̂	.5557	53̂
10	.2358̂	18̂	.3086	32̂	10	.3594̂	22̂	.5611̂	54̂
20	.2377	19	.3118	32̂	20	.3617	22	.5666	54
30	.2396	19	.3151	32̂	30	.3639	22̂	.5721̂	55̂
40	.2415	19	.3183̂	33̂	40	.3661̂	22̂	.5777	56̂
50	.2434	19	.3217	33	50	.3684	23	.5833̂	56
41 0	.2453	19	.3250	34	51 0	.3707	22̂	.5890	57̂
10	.2472	19	.3284	33̂	10	.3729̂	22̂	.5947̂	58
20	.2491	19̂	.3317̂	34̂	20	.3752	23	.6005̂	58̂
30	.2510̂	19	.3352	34̂	30	.3775	22̂	.6064	59
40	.2529̂	19̂	.3386	34̂	40	.3797̂	23	.6123	59̂
50	.2549	19̂	.3421	35̂	50	.3820̂	23	.6182̂	60
42 0	.2568̂	19̂	.3456̂	35	52 0	.3843̂	23	.6242̂	61
10	.2588	19̂	.3491̂	36̂	10	.3866̂	23	.6303̂	61̂
20	.2607̂	19̂	.3527̂	36	20	.3889̂	23	.6365	62
30	.2627	20	.3563̂	36	30	.3912̂	23	.6427	62̂
40	.2647	19̂	.3599̂	37	40	.3935̂	23	.6489̂	63
50	.2666̂	20	.3636̂	37	50	.3958̂	23̂	.6552̂	64
43 0	.2686	20	.3673̂	37	53 0	.3982	23	.6616̂	64̂
10	.2706̂	20	.3710̂	37̂	10	.4005	23̂	.6681	65
20	.2726̂	20	.3748	38	20	.4028̂	23̂	.6746	65̂
30	.2746̂	20	.3786	38	30	.4052	23	.6811̂	66̂
40	.2766̂	20	.3824	39	40	.4075	23̂	.6878	67
50	.2786̂	20	.3863	38̂	50	.4098̂	23̂	.6945	68
44 0	.2806̂	20̂	.3901̂	39̂	54 0	.4122	23̂	.7013	68̂
10	.2827	20	.3941	39̂	10	.4145̂	24	.7081̂	69
20	.2847	20̂	.3980̂	40	20	.4169̂	23̂	.7150̂	70
30	.2867̂	20̂	.4020̂	40	30	.4193	23̂	.7220̂	70̂
40	.2888	20̂	.4060̂	40̂	40	.4216̂	24	.7291	71̂
50	.2908̂	20̂	.4101	41	50	.4240̂	23̂	.7362̂	72
45 0	.2929	20̂	.4142	41̂	55 0	.4264	24	.7434̂	73
10	.2949	20̂	.4183	41̂	10	.4288	24	.7507̂	73̂
20	.2970	21	.4225	42	20	.4312	24	.7581	74
30	.2991	20̂	.4267	42̂	30	.4336	24	.7655	75̂
40	.3011̂	21	.4309̂	43	40	.4360	24	.7730̂	75̂
50	.3032̂	21	.4352̂	43	50	.4384	24	.7806	77
46 0	.3053̂	21	.4395̂	43̂	56 0	.4408	24	.7883	77̂
10	.3074̂	21	.4439	44	10	.4432	24̂	.7960̂	78̂
20	.3095̂	21	.4483	44̂	20	.4456̂	24	.8039	79
30	.3116̂	21	.4527̂	44̂	30	.4480̂	24̂	.8118	80
40	.3137̂	21̂	.4572	45̂	40	.4505	24	.8198	81
50	.3159	21	.4617̂	45̂	50	.4529	24̂	.8279	82
47 0	.3180	21̂	.4663	45̂	57 0	.4553̂	24̂	.8361	82̂
10	.3201̂	21	.4708̂	46̂	10	.4578	24̂	.8443̂	83̂
20	.3222̂	21̂	.4755	47	20	.4602̂	24̂	.8527	84̂
30	.3244	21̂	.4802	47	30	.4627	24̂	.8611̂	85̂
40	.3265̂	21̂	.4849	47̂	40	.4651̂	24̂	.8697	86
50	.3287	21̂	.4896̂	48̂	50	.4676	25	.8783̂	88
48 0	.3308̂	22	.4945	48̂	58 0	.4701	24̂	.8871	88
10	.3330̂	21̂	.4993̂	48̂	10	.4725̂	24̂	.8959	89̂
20	.3352	22	.5042	49	20	.4750	25	.9048̂	90̂
30	.3374	21̂	.5091̂	50	30	.4775	25	.9139	91
40	.3395̂	22	.5141̂	50̂	40	.4800	24̂	.9230	92̂
50	.3417̂	22	.5192	50̂	50	.4824̂	25	.9322̂	93̂
49 0	.3439̂	22	.5242̂	51̂	59 0	.4849̂	25	.9416	94̂
10	.3461̂	22	.5294	51̂	10	.4874̂	25	.9510̂	95̂
20	.3483̂	22	.5345̂	52	20	.4899̂	25	.9606	97
30	.3505̂	22	.5397̂	53	30	.4924̂	25	.9703	98
40	.3527̂	22̂	.5450̂	53	40	.4949̂	25̂	.9801	99
50	.3550	22	.5503̂	53̂	50	.4975	25	.9900	100
50 0	.3572		.5557		60 0	.5000		1 0000	
° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.

P. P.

	9	8	7	6	5	4
1	0.9	0.8	0.7	0.6	0.5	0.4
2	1.8	1.6	1.4	1.2	1.0	0.8
3	2.7	2.4	2.1	1.8	1.5	1.2
4	3.6	3.2	2.8	2.4	2.0	1.6
5	4.5	4.0	3.5	3.0	2.5	2.0
6	5.4	4.8	4.2	3.6	3.0	2.4
7	6.3	5.6	4.9	4.2	3.5	2.8
8	7.2	6.4	5.6	4.8	4.0	3.2
9	8.1	7.2	6.3	5.4	4.5	3.6

	3	2	1	9̂	8̂	7̂
1	0.3	0.2	0.1	0.9̂	0.8̂	0.7̂
2	0.6	0.4	0.2	1.9̂	1.7	1.5
3	0.9	0.6	0.3	2.8̂	2.5̂	2.2̂
4	1.2	0.8	0.4	3.8	3.4	3.0
5	1.5	1.0	0.5	4.7̂	4.2̂	3.7̂
6	1.8	1.2	0.6	5.7	5.1	4.5
7	2.1	1.4	0.7	6.6̂	5.9̂	5.2̂
8	2.4	1.6	0.8	7.6	6.8̂	6.0
9	2.7	1.8	0.9	8.5	7.6̂	6.7̂

	6̂	5̂	4̂	3̂	2̂	1̂
1	0.6̂	0.5̂	0.4̂	0.3̂	0.2̂	0.1̂
2	1.3	1.1	0.9	0.7	0.5	0.3
3	1.9̂	1.6̂	1.3̂	1.0̂	0.7̂	0.4
4	2.6	2.2	1.8	1.4	1.0	0.6
5	3.2̂	2.7̂	2.2̂	1.7̂	1.2̂	0.7̂
6	3.9	3.3	2.7	2.1	1.5	0.9
7	4.5̂	3.8̂	3.1̂	2.4̂	1.7̂	1.0̂
8	5.2	4.4	3.6	2.8	2.0	1.2
9	5.8̂	4.9̂	4.0̂	3.1̂	2.2̂	1.3̂

	25̂	25	24̂	24	23̂	23
1	2.5̂	2.5	2.4̂	2.4	2.3̂	2.3
2	5.1	5.0	4.9	4.8	4.7	4.6
3	7.6	7.5	7.3̂	7.2	7.0̂	6.9
4	10.2	10.0	9.8	9.6	9.4	9.2
5	12.7̂	12.5	12.2̂	12.0	11.7̂	11.5
6	15.3	15.0	14.7	14.4	14.1	13.8
7	17.8̂	17.5	17.1̂	16.8	16.4̂	16.1
8	20.4	20.0	19.6	19.2	18.8	18.4
9	22.9̂	22.5	22.0̂	21.6	21.1̂	20.7

	22̂	22	21̂	21	20̂	20
1	2.2̂	2.2	2.1̂	2.1	2.0̂	2.0
2	4.5	4.4	4.3	4.2	4.1	4.0
3	6.7	6.6	6.4̂	6.3	6.1̂	6.0
4	9.0	8.8	8.6	8.4	8.2	8.0
5	11.2̂	11.0	10.7̂	10.5	10.2̂	10.0
6	13.5	13.2	12.9	12.6	12.3	12.0
7	15.7̂	15.4	15.0̂	14.7	14.3̂	14.0
8	18.0	17.6	17.2	16.8	16.4	16.0
9	20.2̂	19.8	19.3̂	18.9	18.4̂	18.0

	19̂	19	18̂
1	1.9̂	1.9	1.8̂
2	3.9	3.8	3.7
3	5.8̂	5.7	5.5̂
4	7.8	7.6	7.4
5	9.7̂	9.5	9.2̂
6	11.7	11.4	11.1
7	13.6̂	13.3	12.9̂
8	15.6	15.2	14.8
9	17.5̂	17.1	16.6̂

P. P.

TABLE X.—NATURAL VERSED SINES AND EXTERNAL SECANTS.

60°-70° 70°-80°

° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.
60 0	.5000	25	1.0000	101	**70 0**	.6580	27	1.9238	235
10	.5025	25	1.0101	102	10	.6607	27	1.9473	240
20	.5050	25	1.0204	103	20	.6634	27	1.9713	244
30	.5076	25	1.0307	105	30	.6662	27	1.9957	248
40	.5101	25	1.0413	106	40	.6689	27	2.0205	253
50	.5126	25	1.0519	107	50	.6717	27	2.0458	257
61 0	.5152	25	1.0626	109	**71 0**	.6744	27	2.0715	262
10	.5177	25	1.0735	110	10	.6772	27	2.0977	267
20	.5203	25	1.0846	111	20	.6799	27	2.1244	270
30	.5228	25	1.0957	113	30	.6827	27	2.1515	276
40	.5254	25	1.1070	114	40	.6854	27	2.1792	281
50	.5279	26	1.1184	116	50	.6882	28	2.2073	287
62 0	.5305	25	1.1300	117	**72 0**	.6910	27	2.2360	292
10	.5331	25	1.1418	118	10	.6937	27	2.2653	298
20	.5356	26	1.1536	120	20	.6965	28	2.2951	304
30	.5382	26	1.1657	121	30	.6993	27	2.3255	310
40	.5408	25	1.1778	123	40	.7020	28	2.3565	316
50	.5434	26	1.1902	125	50	.7048	28	2.3881	322
63 0	.5460	26	1.2027	126	**73 0**	.7076	27	2.4203	328
10	.5486	26	1.2153	128	10	.7104	28	2.4531	335
20	.5512	26	1.2281	130	20	.7132	28	2.4867	342
30	.5538	26	1.2411	131	30	.7160	27	2.5209	349
40	.5564	26	1.2543	133	40	.7187	28	2.5558	356
50	.5590	26	1.2676	135	50	.7215	28	2.5915	364
64 0	.5616	26	1.2811	137	**74 0**	.7243	28	2.6279	372
10	.5642	26	1.2948	139	10	.6271	28	2.6651	380
20	.5668	26	1.3087	140	20	.7299	28	2.7031	388
30	.5695	26	1.3228	143	30	.7327	28	2.7420	396
40	.5721	26	1.3371	144	40	.7355	28	2.7816	406
50	.5747	26	1.3515	146	50	.7383	28	2.8222	414
65 0	.5774	26	1.3662	148	**75 0**	.7412	28	2.8637	424
10	.5800	26	1.3810	151	10	.7440	28	2.9061	434
20	.5826	26	1.3961	152	20	.7468	28	2.9495	444
30	.5853	26	1.4114	155	30	.7496	28	2.9939	454
40	.5879	26	1.4269	157	40	.7524	28	3.0394	465
50	.5906	26	1.4426	159	50	.7552	28	3.0859	476
66 0	.5932	26	1.4586	161	**76 0**	.7581	28	3.1335	488
10	.5959	27	1.4747	164	10	.7609	28	3.1824	500
20	.5986	26	1.4912	166	20	.7637	28	3.2324	512
30	.6012	26	1.5078	169	30	.7665	28	3.2836	525
40	.6039	27	1.5247	171	40	.7694	28	3.3362	539
50	.6066	26	1.5419	174	50	.7722	28	3.3901	553
67 0	.6092	27	1.5593	177	**77 0**	.7750	28	3.4454	567
10	.6119	27	1.5770	179	10	.7779	28	3.5021	583
20	.6146	26	1.5949	182	20	.7807	28	3.5604	598
30	.6173	27	1.6131	185	30	.7835	28	3.6202	614
40	.6200	27	1.6316	188	40	.7864	28	3.6816	631
50	.6227	27	1.6504	190	50	.7892	28	3.7448	649
68 0	.6254	27	1.6694	194	**78 0**	.7921	28	3.8097	667
10	.6281	27	1.6888	196	10	.7949	28	3.8765	686
20	.6308	27	1.7085	200	20	.7978	28	3.9451	707
30	.6335	27	1.7285	203	30	.8006	28	4.0158	728
40	.6362	27	1.7488	206	40	.8035	28	4.0886	749
50	.6389	27	1.7694	210	50	.8063	28	4.1636	772
69 0	.6416	27	1.7904	213	**79 0**	.8092	28	4.2408	796
10	.6443	27	1.8117	216	10	.8120	28	4.3205	821
20	.6470	27	1.8334	220	20	.8149	28	4.4026	847
30	.6498	27	1.8554	224	30	.8177	29	4.4874	875
40	.6525	27	1.8778	227	40	.8206	28	4.5749	904
50	.6552	27	1.9006	232	50	.8235	28	4.6653	934
70 0	.6580		1.9238		**80 0**	.8263		4.7587	
° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.

P. P.

	9	8	7	6	5	4
1	0.9	0.8	0.7	0.6	0.5	0.4
2	1.8	1.6	1.4	1.2	1.0	0.8
3	2.7	2.4	2.1	1.8	1.5	1.2
4	3.6	3.2	2.8	2.4	2.0	1.6
5	4.5	4.0	3.5	3.0	2.5	2.0
6	5.4	4.8	4.2	3.6	3.0	2.4
7	6.3	5.6	4.9	4.2	3.5	2.8
8	7.2	6.4	5.6	4.8	4.0	3.2
9	8.1	7.2	6.3	5.4	4.5	3.6

	3	2	1	9	8	7
1	0.3	0.2	0.1	0.9	0.8	0.7
2	0.6	0.4	0.2	1.9	1.7	1.5
3	0.9	0.6	0.3	2.8	2.5	2.2
4	1.2	0.8	0.4	3.8	3.4	3.0
5	1.5	1.0	0.5	4.7	4.2	3.7
6	1.8	1.2	0.6	5.7	5.1	4.5
7	2.1	1.4	0.7	6.6	5.9	5.2
8	2.4	1.6	0.8	7.6	6.8	6.0
9	2.7	1.8	0.9	8.5	7.6	6.7

	6	5	4	3	2	1
1	0.6	0.5	0.4	0.3	0.2	0.1
2	1.3	1.1	0.9	0.7	0.5	0.3
3	1.9	1.6	1.3	1.0	0.7	0.4
4	2.6	2.2	1.8	1.4	1.0	0.6
5	3.2	2.7	2.2	1.7	1.2	0.7
6	3.9	3.3	2.7	2.1	1.5	0.9
7	4.5	3.8	3.1	2.4	1.7	1.0
8	5.2	4.4	3.6	2.8	2.0	1.2
9	5.8	4.9	4.0	3.1	2.2	1.3

	29	28	28	27
1	2.9	2.8	2.8	2.7
2	5.8	5.7	5.6	5.5
3	8.7	8.5	8.4	8.2
4	11.6	11.4	11.2	11.0
5	14.5	14.2	14.0	13.7
6	17.4	17.1	16.8	16.5
7	20.3	19.9	19.6	19.2
8	23.2	22.8	22.4	22.0
9	26.1	25.6	25.2	24.7

	27	26	26	25
1	2.7	2.6	2.6	2.5
2	5.4	5.3	5.2	5.1
3	8.1	7.9	7.8	7.6
4	10.8	10.6	10.4	10.2
5	13.5	13.2	13.0	12.7
6	16.2	15.9	15.6	15.3
7	18.9	18.5	18.2	17.8
8	21.6	21.2	20.8	20.4
9	24.3	23.8	23.4	22.9

80°-85° 85°-90°

° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.
80 0	.8263	28	4.7587	966	85 0	.9128	29	10.4737	.3946
10	.8292	29	4.8554	999	10	.9157	29	10.8683	.4229
20	.8321	28	4.9553	1035	20	.9186	29	11.2912	.4542
30	.8349	28	5.0588	1072	30	.9215	29	11.7455	.4892
40	.8378	29	5.1660	1111	40	.9244	29	12.2347	.5284
50	.8407	28	5.2772	1152	50	.9273	29	12.7631	.5725
81 0	.8435	29	5.3924	1196	86 0	.9302	29	13.3356	.6223
10	.8464	28	5.5121	1242	10	.9331	29	13.9579	.6789
20	.8493	29	5.6363	1291	20	.9360	29	14.6368	.7436
30	.8522	28	5.7654	1343	30	.9389	29	15.3804	.8180
40	.8550	29	5.8998	1398	40	.9418	29	16.1984	.9041
50	.8579	29	6.0396	1456	50	.9447	29	17.1026	1.0047
82 0	.8608	28	6.1853	1519	87 0	.9476	29	18.1073	1.1230
10	.8637	29	6.3372	1585	10	.9505	29	19.2303	1.2634
20	.8666	28	6.4957	1656	20	.9534	29	20.4937	1.4319
30	.8694	29	6.6613	1731	30	.9564	29	21.9256	1.6365
40	.8723	29	6.8344	1812	40	.9593	29	23.5621	1.8884
50	.8752	29	7.0156	1898	50	.9622	29	25.4505	2.2032
83 0	.8781	28	7.2055	1991	88 0	.9651	29	27.6537	2.6039
10	.8810	29	7.4046	2091	10	.9680	29	30.2576	3.1247
20	.8839	29	7.6138	2198	20	.9709	29	33.3823	3.8192
30	.8868	29	7.8336	2315	30	.9738	29	37.2015	4.7741
40	.8897	29	8.0651	2440	40	.9767	29	41.9757	6.1383
50	.8926	28	8.3091	2576	50	.9796	29	48.1140	8.1846
84 0	.8954	29	8.5667	2723	89 0	.9825	29	56.2987	
10	.8983	29	8.8391	2884	10	.9854	29	67.7573	
20	.9012	29	9.1275	3059	20	.9883	29	84.9456	
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85 0	.9128		10.4737		90 0	1.0000		∞	
° ′	Vers.	d.	Exsec.	d.	° ′	Vers.	d.	Exsec.	d.

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9	26.5	26.1	25.6

1	$\sin a$	$= \dfrac{1}{\operatorname{cosec} a} = \dfrac{\tan a}{\sqrt{1+\tan^2 a}} = \sqrt{\dfrac{1-\cos 2a}{2}} = \dfrac{1}{\sqrt{1+\cot^2 a}}$
		$= \cos a \tan a = \sqrt{1-\cos^2 a} = 2 \sin \tfrac{1}{2}a \cos \tfrac{1}{2}a$
		$= \dfrac{1+\cos a}{\cot \frac{1}{2}a} = \dfrac{2 \tan \frac{1}{2}a}{1+\tan^2 \frac{1}{2}a} = \operatorname{vers} a \cot \tfrac{1}{2}a.$
2	$\cos a$	$= \dfrac{1}{\sec a} = \dfrac{\cot a}{\sqrt{1+\cot^2 a}} = \dfrac{1}{\sqrt{1+\tan^2 a}}$
		$= 1 - \operatorname{vers} a = \sin a \cot a = \sqrt{1-\sin^2 a} = 2\cos^2 \tfrac{1}{2}a - 1$
		$= \sin a \cot \tfrac{1}{2}a - 1 = \cos^2 \tfrac{1}{2}a - \sin^2 \tfrac{1}{2}a = 1 - 2\sin^2 \tfrac{1}{2}a.$
3	$\tan a$	$= \dfrac{1}{\cot a} = \dfrac{\sin a}{\cos a} = \dfrac{\sec a}{\operatorname{cosec} a} = \dfrac{1}{\sqrt{\operatorname{cosec}^2 a - 1}}$
		$= \operatorname{vers} 2a \operatorname{cosec} 2a = \cot a - 2\cot 2a = \sin a \sec a$
		$= \dfrac{\sin 2a}{1+\cos 2a} = \operatorname{exsec} a \cot \tfrac{1}{2}a = \operatorname{exsec} 2a \cot 2a.$
4	$\cot a$	$= \dfrac{1}{\tan a} = \dfrac{\cos a}{\sin a} = \dfrac{\sin 2a}{1-\cos 2a} = \dfrac{1+\cos 2a}{\sin 2a}$
		$= \sqrt{\operatorname{cosec}^2 a - 1} = \cot \tfrac{1}{2}a - \operatorname{cosec} a.$
5	$\operatorname{vers} a$	$= 1 - \cos a = \sin a \tan \tfrac{1}{2}a = 2\sin^2 \tfrac{1}{2}a = \cos a \operatorname{exsec} a.$
6	$\operatorname{exsec} a$	$= \sec a - 1 = \tan a \tan \tfrac{1}{2}a = \operatorname{vers} a \sec a.$
7	$\sin \tfrac{1}{2}a$	$= \sqrt{\dfrac{\operatorname{vers} a}{2}} = \dfrac{\sin a}{2\cos \frac{1}{2}a} = \dfrac{\operatorname{vers} a \cos \frac{1}{2}a}{\sin a}.$
8	$\cos \tfrac{1}{2}a$	$= \sqrt{\dfrac{1+\cos a}{2}} = \dfrac{\sin a}{2\sin \frac{1}{2}a} = \dfrac{\sin a \sin \frac{1}{2}a}{\operatorname{vers} a}.$
9	$\tan \tfrac{1}{2}a$	$= \operatorname{vers} a \operatorname{cosec} a = \operatorname{cosec} a - \cot a = \dfrac{\tan a}{1+\sec a}.$
10	$\cot \tfrac{1}{2}a$	$= \dfrac{1+\cos a}{\sin a} = \operatorname{cosec} a + \cot a = \dfrac{\tan a}{\operatorname{exsec} a} = \dfrac{1}{\operatorname{cosec} a - \cot a}.$
11	$\operatorname{vers} \tfrac{1}{2}a$	$= 1 - \sqrt{\tfrac{1}{2}(1+\cos a)}.$
12	$\operatorname{exsec} \tfrac{1}{2}a$	$= \dfrac{1}{\sqrt{\frac{1}{2}(1+\cos a)}} - 1.$

13 $\sin 2a = 2 \sin a \cos a = \dfrac{2 \tan a}{1 + \tan^2 a}.$

14 $\cos 2a = \cos^2 a - \sin^2 a = 1 - 2 \sin^2 a = 2 \cos^2 a - 1$

$= \dfrac{1 - \tan^2 a}{1 + \tan^2 a}.$

15 $\tan 2a = \dfrac{2 \tan a}{1 - \tan^2 a}.$

16 $\cot 2a = \tfrac{1}{2} \cot a - \tfrac{1}{2} \tan a = \dfrac{\cot^2 a - 1}{2 \cot a} = \dfrac{1 - \tan^2 a}{2 \tan a}.$

17 $\operatorname{vers} 2a = 2 \sin^2 a = 1 - \cos 2a = 2 \sin a \cos a \tan a.$

18 $\operatorname{exsec} 2a = \dfrac{\tan 2a}{\cot a} = \dfrac{2 \tan^2 a}{1 - \tan^2 a} = \dfrac{2 \sin^2 a}{1 - 2 \sin^2 a}.$

19 $\sin (a \pm b) = \sin a \cos b \pm \cos a \sin b.$

20 $\cos (a \pm b) = \cos a \cos b \mp \sin a \sin b.$

21 $\sin a + \sin b = 2 \sin \tfrac{1}{2}(a + b) \cos \tfrac{1}{2}(a - b).$

22 $\sin a - \sin b = 2 \sin \tfrac{1}{2}(a - b) \cos \tfrac{1}{2}(a + b).$

23 $\cos a + \cos b = 2 \cos \tfrac{1}{2}(a + b) \cos \tfrac{1}{2}(a - b).$

24 $\cos a - \cos b = - 2 \sin \tfrac{1}{2}(a + b) \sin \tfrac{1}{2}(a - b).$

Call the sides of any triangle A, B, C, and the opposite angles a, b, and c. Call $s = \tfrac{1}{2}(A + B + C)$.

25 $\tan \tfrac{1}{2}(a - b) = \dfrac{A - B}{A + B} \tan \tfrac{1}{2}(a + b) = \dfrac{A - B}{A + B} \cot \tfrac{1}{2}c.$

26 $C = (A + B) \dfrac{\cos \tfrac{1}{2}(a + b)}{\cos \tfrac{1}{2}(a - b)} = (A - B) \dfrac{\sin \tfrac{1}{2}(a + b)}{\sin \tfrac{1}{2}(a - b)}.$

27 $\sin \tfrac{1}{2}a = \sqrt{\dfrac{(s - B)(s - C)}{BC}}.$

28 $\cos \tfrac{1}{2}a = \sqrt{\dfrac{s(s - A)}{BC}}.$

29 $\operatorname{vers} A = \dfrac{2(s - B)(s - C)}{BC}.$

30 $\text{Area} = \sqrt{s(s - A)(s - B)(s - C)} = A^2 \dfrac{\sin b \sin c}{2 \sin a}.$

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