PRACTICAL ESSAY

ON

THE STRENGTH OF CAST IRON

AND OTHER METALS.

BY THOMAS TREDGOLD.

THE FIFTH EDITION, WITH NOTES, BY

EATON HODGKINSON, F.R.S.

TO WHICH ARE ADDED

EXPERIMENTAL RESEARCHES

ON THE

STRENGTH AND OTHER PROPERTIES OF CAST IRON.

BY THE EDITOR.

WITH ILLUSTRATIVE PLATES AND DIAGRAMS.

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STRENGTH OF CAST IRON

AND OTHER METALS:

CONTAINING

PRACTICAL RULES, TABLES, AND EXAMPLES, FOUNDED ON A SERIES OF EXPERIMENTS;

WITH AN EXTENSIVE

TABLE OF THE PROPERTIES OF MATERIALS.

BY THE LATE

THOMAS TREDGOLD, C.E.

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ADVERTISEMENT.

A New Edition of the Work of the late Mr. TREDGOLD, edited—with further Experimental Researches on the Strength of Cast Iron and other Metals—by EATON HODGKINSON, F.R.S., now being much called for, and holding the highest reputation as a Standard Work of reference, for the use of the Scientific and Practical Builder, is now presented together as a fifth edition of Tredgold, and a second edition of Mr. Eaton Hodgkinson's "Experimental Researches."

It is intended by Mr. Hodgkinson to publish in a collected volume, all his valuable experiments scattered in the publications of several Scientific Works, viz.: "Transactions of the Royal Society of London," "The House of Commons' Report on Iron," and "The Memoirs of the Manchester Society."

The Second portion of this Work may be had separately for the convenience of those who desire to possess, alone, Mr. Hodgkinson's "Researches."

J. W.

November, 1860.

CONTENTS OF PART I.

SECTION I.

SECTION II.

SECTION III.

SECTION IV.

SECTION V.

SECTION VI.

SECTION VII.

SECTION VIII.

SECTION IX.

SECTION X.

SECTION XI.

All the quantities, proportions, &c., are stated in English weights and measures, the pound being the avoirdupois pound, except the contrary be stated.

SECTION I.

MR. TREDGOLD'S ORIGINAL INTRODUCTION.

ART. 1. In consequence of the security which cast iron gives, when it is properly employed, for supporting considerable weights, pressures, or moving forces, it has lately been very much used; and is likely to wholly supersede the use of timber for many important purposes. Indeed, so considerable are the improvements which have arisen out of its use, that the period of its general introduction has been very justly considered as forming a new era in the history of machines.* "All other improvements," it has been remarked, "have been limited; confined to particular machines; but this, having increased the strength and durability of every machine, has improved the whole."†

Cast iron is a valuable material, because it gives safety against fire; it is not liable to sudden decay, nor soon destroyed by wear and tear, and it can be easily moulded into the form of greatest strength, or that which is best adapted for our intended purpose.

The fatal consequences that might result from the use of timber for supporting heavy buildings, either in case of fire or of decay, have often been foreseen; but in a few instances

* Essays on Mill Work, &c., by Robertson Buchanan, Essay II. p. 254, 2nd edit.; or 3rd edit. by G. Rennie, Esq., p. 177.

† Mr. Dunlop's Account of some Experiments on Cast Iron. Dr. Thomson's Annals of Philosophy, vol. xiii. p. 200.

36

it has happened that where iron has been used for greater security against fire, the structure has failed from want of strength. Such failures have not occurred from any defect in the material itself; for it too often happens that such works are conducted by persons of little experience, and less scientific knowledge. Men of little experience too frequently imagine that a large piece of iron is almost of infinite strength; and they often have a like indistinct notion of pressure. They design to please the eye, without regard to fitness, strength, or durability; instead of ornamenting a support, they make the support itself the ornament, and sacrifice everything to lightness of effect. The dimensions of the most important parts of structures are too often fixed by guess or chance; and the person who calculates the value of materials to the fraction of a penny, seldom if ever attempts to estimate their power, or the stress to which they will be exposed.

The manner in which the resistance of materials has been treated by most of our common mechanical writers, has also, in some degree, misled such practical men as were desirous of proceeding upon surer ground; and has given occasion for the sarcastic remark, "that the stability of a building is inversely proportional to the science of the builder."*

When it is considered that it is absolutely necessary that the parts of a building or a machine should preserve a certain form or position, as well as that they should bear a certain stress, it will become obvious that something more than the mere resistance to fracture should be calculated. In cases where the parts are short and bulky, it may do very well to employ the rules for resistance to fracture, and make the parts strong enough to sustain four times the load, but such cases rarely occur; and where long pieces are loaded to one-fourth of their strength, we may expect much flexure, vibration, and instability.

If a material of any kind be loaded with more than a

* Ency. Method. Dict. Architecture, art. Equilibre.

certain quantity, it loses the power of recovering its natural form, when the load is removed; the arrangement of its particles undergoes a permanent alteration; and if it supports the same load during a considerable time, the deflexion will increase, and the more in proportion as the load is above the elastic force of the material.*

On this part of the resistance of materials I have made many experiments, both with metals of various kinds, and with timber: I find that while the elastic force or power of restoration remains perfect, the extension is always directly proportional to the extending force, and that the deflexion does not increase after the load has been on for a second or two; but when the strain exceeds the elastic force, the extension or deflexion becomes irregular, and increases with time. I was led into this important inquiry by considering the proportions for cannon, and the common method of proving them. It appears from my experiments, that firing a certain number of times with the same quantity of powder would burst a cannon when the strain is above the elastic force of the material, though the effect of the first charge might not be sensible. The same remarks apply to the methods of proving

* This important fact appears to have been first noticed by Coulomb, while making his experiments on torsion. (Some account has been given of Coulomb's experiments by Dr. Young, Nat. Phil. vol. ii. p. 383, and also by Dr. Brewster in his additions to Ferguson's Lectures, vol. ii. p. 234, third edit.) But, in a great number of substances, we seem to have an instinctive knowledge of this property of matter: a bent wire retains its curvature; and it may be broken by repeated flexure, with much less force than would break it at once: indeed, when we attempt to break any flexible body, it is usually by bending and unbending it several times, and its strength is only beyond the effort applied to break it when we have not power to give it a permanent set at each bending. A permanent alteration is a partial fracture, and hence it is the proper limit of strength. Dr. Young, with his usual profound discrimination, pointed out the importance of this limit in applying the discoveries in science to the useful arts.

While the previous edition was in preparation, a copy was received of the "Essai Théorique et Expérimental sur la Résistance du Fer Forgé," of M. Duleau, which is founded on similar views of the strength of wrought iron. M. Duleau has ascertained, with an apparatus much more imperfect than mine, the fact that iron cannot be considered a perfectly elastic body when the strain exceeds a certain force. I shall, in the course of this edition, compare the results of his experiments with those I have made, wherever the conditions are similar.

the strength of steam engine boilers and pipes, by hydraulic pressure: if the strain in proving exceeds that which produces permanent alteration, an irreparable injury is done by the trial.

In the moving parts of machines the strain should obviously be under the elastic force of the material, and in the second Table will be found the flexure and load a piece of a given size will bear without destroying the elastic force.

I think every one, who carefully examines the subject, will feel satisfied that the measure of the resistance of a material to flexure is the only proper measure of its resistance, when it is to be applied where perfect form or unalterable position is desirable; and the measure of its resistance to permanent alteration, when it is used where flexure is not injurious nor objectionable.

In order to supply practical men with a convenient and ready means of assigning the dimensions of cast iron beams, columns, &c., to support known pressures, or moving forces, I have drawn up this volume. I am persuaded that its usefulness will find it a place among the common works of reference, which are more or less necessary to every architect, engineer, and builder. To bring it within as small a compass as possible, I have arranged the Tables so as to include as many distinct applications as the nature of the subjects seemed capable of admitting.

SOME PARTICULARS TO BE OBSERVED IN USING THE TABLES.

2. The weight of the beam itself is always to be estimated, and added to the load to be supported; or (because this method renders it necessary to estimate the weight before the bulk be determined) find the dimensions of the piece that would support the load by one of the Tables, and increase the breadth in the same proportion as the weight of the piece increases the load. If the weight of the piece, for example,

be an eighth part of the load, then to the breadth, found by the Table, add an eighth part of that breadth; and so of any other proportion. It is not an absolutely correct method, but it is simple and correct enough for use.

3. The Tables and Rules are calculated for soft gray cast iron. Metal of this kind yields easily to the file when the external crust is removed, and is slightly malleable in a cold state. Dr. C. Hutton has justly given the preference to such iron, because it is "less liable to fracture by a blow, or shock, than the hard metal."*

White cast iron is less subject to be destroyed by rusting than the gray kind; and it is also less soluble in acids; therefore it may be usefully employed where hardness is necessary, and where its brittleness is not a defect; but it should not be chosen for purposes where strength is necessary. When it is cast smooth, it makes excellent bearings for gudgeons or pivots to run upon, and is very durable, having very little friction.

White cast iron, in a recent fracture, has a white and radiated appearance, indicating a crystalline structure. It is very brittle and hard.

Gray cast iron has a granulated fracture, of a gray colour, with some metallic lustre; it is much softer and tougher than the white cast iron.

But between these kinds there are varieties of cast iron, having various shades of these qualities; those should be esteemed the best which approach nearest to the gray cast iron.

Gray cast iron is used for artillery, and is sometimes called gun-metal.

The best and most certain test of the quality of a piece of cast iron, is to try any of its edges with a hammer; if the blow of a hammer make a slight impression, denoting some degree of malleability, the iron is of a good quality, provided

* Tracts, vol. i. p. 141.

it be uniform: if fragments fly off, and no sensible indentation be made, the iron will be hard and brittle.*

The utmost care should be employed to render the iron in each casting of an uniform quality, because in iron of different qualities the shrinkage is different, which causes an unequal tension among the parts of the metal, impairs its strength, and renders it liable to sudden and unexpected failures. When the texture is not uniform, the surface of the casting is usually uneven where it ought to have been even. This unevenness, or the irregular swells and hollows on the surface of a casting, is caused by the unequal shrinkage of the iron of different qualities. A founder of much observation and experience in his business pointed out to me this test of an imperfect casting.

Now, when iron of a particular quality is obtained by mixture of different kinds, it will be difficult to blend them so thoroughly as to render the product perfectly uniform; hence we easily perceive one reason of iron being improved by annealing, for in passing slowly to the solid state, the parts are more at liberty to adjust themselves, so as to equalise, if not neutralise, the tension produced by shrinking. But, it is clear that an annealing heat applied after the metal has once acquired its solid state, must be sufficiently intense to reduce the cohesive power in a very considerable degree, otherwise it will not be sensibly beneficial.† These remarks apply to glass, and to various metals as well as to cast iron.

It has been remarked that "iron varies in strength, and not only from different furnaces, but also from the same furnace and the same melting; but this seems to be owing to some imperfection in the casting, and in general iron is much

* For more information upon this subject, see Mr. Fairbairn's Experiments upon the Transverse Strength, &c., of Bars of Cast Iron, from various parts of the United Kingdom. (Manchester Memoirs, vol. vi. new series).—EDITOR.

† Dr. Brewster has shown that the mechanical condition of unannealed glass is not capable of being altered by the heat of boiling water. Edin. Phil. Journal, vol. ii. p. 399.

more uniform than wood."* I am glad to find my own experience supported by the opinion of a writer so well known to practical men as Mr. Banks. But the very great strain which large masses of well mixed cast iron will bear, when applied to resist the greatest stresses in mill and engine work, is now extremely well known in this country. Its value was foreseen by our celebrated Smeaton at an early period of his practice. Upwards of fifty years ago he combated the prejudices against it in the following language: "If the length of time of the use of these (cast iron) utensils is not thought sufficient, I must add, that in the year 1755, for the first time, I applied them as totally new subjects, and the cry then was, that if the strongest timbers are not able for any great length of time to resist the action of the powers, what must happen from the brittleness of cast iron? It is sufficient to say, that not only those very pieces of cast iron are still at work, but that the good effect has in the North of England, where first applied, drawn them into common use, and I never heard of one failing."† These remarks were written in 1782, and the good opinion of Smeaton has been fully justified by the experience of succeeding engineers; the grand and varied works of Wilson, Rennie, Boulton and Watt, Telford, &c., &c., abundantly confirm it.‡

* Banks on the Power of Machines, p. 73. See also p. 94 of the same work.

† Reports, vol. i. pp. 410, 411.

‡ One of the boldest attempts with a new material was the application of cast iron to bridges: the idea appears to have originated, in the year 1773, with the late Thomas Farnolls Pritchard, then of Eyton Turrett, Shropshire, architect, who, in communication with the late Mr. John Wilkinson, of Brosely and Castlehead, ironmaster, suggested the practicability of constructing wide iron arches, capable of admitting the passage of the water in a river, such as the Severn, which is much subject to floods. This suggestion Mr. Wilkinson considered with great attention, and at length carried into execution between Madely and Brosely, by erecting the celebrated iron bridge at Colebrook Dale, which was the first construction of that kind in England, and probably in the world. This bridge was executed by a Mr. Daniel Onions, with some variations from Mr. Pritchard's plan, under the auspices and at the expense of Mr. Darby and Mr. Reynolds, of the iron works of Colebrook Dale. Mr. Pritchard died in October, 1777. He made several ingenious designs, to

Yet I must not omit to remark, that cast iron when it fails gives no warning of its approaching fracture, which is its chief defect when employed to sustain weights or moving forces; therefore care should be taken to give it sufficient strength. And it will be obvious from the preceding remarks, how much its strength depends upon the skill and experience of the founder.

4. The parts of each casting should be kept as nearly of the same bulk as possible, in order that they may all cool at the same rate.

Great care should be taken to prevent air bubbles in castings; and the more time there can be allowed for cooling the better, because the iron will be tougher than when rapidly cooled; slow cooling answers the same purpose as annealing.

In making patterns for cast iron, an allowance of about one-eighth of an inch per foot must be made for the contraction of the metal in cooling. Also the patterns that require it should be slightly bevelled to allow of their being drawn out of the sand without injuring the impression; about one-sixteenth of an inch in six inches is sufficient for this purpose.

In notes at the foot of each Table, the mode of applying these Tables to other materials is shown, which will be useful in exhibiting the comparative strength of different bodies when applied to the same purpose, as well as in giving the proportions of these materials for supporting a given load.

show how stone or brick arches might be constructed with cast iron centres, so that the centre should always form a permanent part of the arch. These designs were in the possession of the late Mr. John White, of Devonshire Place, one of his grandsons, to whom we are indebted for the preceding particulars of this note.

TABLE I.—ART. 5. *A Table of the Depths of Square Beams or Bars of Cast Iron, of different lengths to sustain weights* of from one cwt. to 500 tons, when supported at the ends, and loaded in the middle; the deflexion not to exceed $\frac{1}{40}$ of an inch for each foot in length.*†

Lengths in ft.		4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	
Weight in tons.	Weight in lbs.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Weig t.
$\frac{1}{20}$	112	1·2	1·4	1·7	1·9	2·0	2·2	2·4	2·5	2·6	2·7	2·9	3·0	3·1	3·2	3·3	3·4	3·5	3·6	3·7	1 cwt.
$\frac{1}{10}$	224	1·4	1·7	2·0	2·2	2·4	2·6	2·8	3·0	3·1	3·3	3·4	3·6	3·7	3·8	3·9	4·1	4·2	4·3	4·4	2 —
$\frac{3}{20}$	336	1·6	1·9	2·2	2·4	2·7	2·9	3·1	3·3	3·4	3·6	3·8	3·9	4·1	4·2	4·3	4·5	4·6	4·7	4·8	3 —
$\frac{1}{5}$	448	1·7	2·0	2·4	2·6	2·9	3·1	3·3	3·5	3·7	3·9	4·0	4·2	4·3	4·5	4·7	4·8	4·9	5·0	5·2	4 —
$\frac{1}{4}$	560	1·8	2·2	2·5	2·8	3·0	3·3	3·5	3·7	3·9	4·1	4·3	4·4	4·6	4·8	4·9	5·1	5·2	5·4	5·5	5 —
$\frac{3}{10}$	672	1·8	2·2	2·6	2·9	3·2	3·4	3·7	3·9	4·1	4·3	4·5	4·6	4·8	5·0	5·1	5·3	5·4	5·6	5·8	6 —
$\frac{7}{20}$	784	1·9	2·3	2·7	3·0	3·3	3·6	3·8	4·1	4·2	4·4	4·6	4·8	5·0	5·2	5·4	5·5	5·7	5·9	6·0	7 —
$\frac{2}{5}$	896	2·0	2·4	2·8	3·1	3·4	3·7	3·9	4·2	4·4	4·6	4·8	5·0	5·2	5·4	5·6	5·7	5·9	6·0	6·2	8 —
$\frac{9}{20}$	1008	2·0	2·5	2·9	3·2	3·5	3·8	4·0	4·3	4·5	4·7	4·9	5·1	5·3	5·5	5·7	5·9	6·0	6·2	6·4	9 —
$\frac{1}{2}$	1120	2·1	2·6	3·0	3·3	3·6	3·9	4·2	4·4	4·7	4·9	5·2	5·3	5·4	5·7	5·9	6·0	6·2	6·4	6·5	10 —
$\frac{11}{20}$	1232	2·1	2·6	3·0	3·4	3·7	4·0	4·3	4·5	4·8	5·0	5·3	5·4	5·6	5·8	6·0	6·2	6·4	6·5	6·7	11 —
$\frac{3}{5}$	1344	2·2	2·7	3·1	3·5	3·8	4·1	4·4	4·7	4·9	5·1	5·3	5·5	5·7	5·9	6·1	6·3	6·5	6·7	6·8	12 —
$\frac{13}{20}$	1456	2·2	2·7	3·1	3·5	3·8	4·2	4·4	4·7	4·9	5·2	5·4	5·6	5·9	6·0	6·2	6·5	6·6	6·8	7·0	13 —
$\frac{7}{10}$	1568	2·3	2·8	3·2	3·6	3·9	4·2	4·5	4·8	5·0	5·3	5·5	5·7	6·0	6·1	6·4	6·6	6·7	6·9	7·1	14 —
$\frac{3}{4}$	1680	2·3	2·8	3·2	3·6	4·0	4·3	4·6	4·9	5·2	5·4	5·6	5·8	6·1	6·2	6·5	6·7	6·8	7·0	7·2	15 —
$\frac{4}{5}$	1792	2·4	2·9	3·3	3·7	4·0	4·4	4·7	5·0	5·2	5·5	5·7	5·9	6·2	6·4	6·6	6·8	6·9	7·2	7·4	16 —
$\frac{17}{20}$	1904	2·4	2·9	3·4	3·8	4·1	4·4	4·7	5·0	5·3	5·5	5·8	6·0	6·2	6·5	6·7	6·9	7·1	7·3	7·5	17 —
$\frac{9}{10}$	2016	2·4	3·0	3·4	3·8	4·2	4·5	4·8	5·1	5·4	5·6	5·9	6·1	6·4	6·6	6·8	7·0	7·2	7·4	7·6	18 —
$\frac{19}{20}$	2128	2·5	3·0	3·5	3·9	4·2	4·6	4·9	5·2	5·4	5·7	6·0	6·2	6·5	6·7	6·9	7·1	7·3	7·5	7·7	19 —
1	2240	2·5	3·0	3·5	3·9	4·3	4·6	4·9	5·2	5·5	5·8	6·0	6·3	6·5	6·8	7·0	7·2	7·4	7·5	7·8	1 ton.
$1\frac{1}{4}$	2800	2·6	3·2	3·7	4·1	4·5	4·9	5·2	5·5	5·8	6·1	6·4	6·6	6·9	7·2	7·4	7·6	7·8	8·0	8·2	$1\frac{1}{4}$—
Defl. in inches		·1	·15	·2	·25	·3	·35	·4	·45	·5	·55	·6	·65	·7	·75	·8	·85	·9	·95	1·0	Defl. in.

* The weight of the load to be supported must include the weight of the beam. To find the weight of a beam, multiply the area of the section in inches by the length in feet and by 3·2, which will give the weight in lbs.

† The Table was calculated by the rule in art. 257.

TABLE I.—*Of the Stiffness of Beams (continued).*

Lengths in ft.		4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	
Weight in tons.	Weight in lbs.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Weight in tons.
1½	3,360	2·8	3·4	3·9	4·3	4·7	5·1	5·5	5·8	6·1	6·4	6·7	7·0	7·2	7·5	7·7	8·0	8·2	8·4	8·6	1½
1¾	3,920	2·9	3·5	4·0	4·5	4·9	5·3	5·7	6·0	6·3	6·7	6·9	7·2	7·5	7·7	8·0	8·2	8·5	8·7	8·9	1¾
2	4,480	2·9	3·5	4·1	4·7	5·1	5·5	5·9	6·2	6 5	6 8	7·2	7·6	7·7	8·0	8·3	8·5	8·7	9·0	9·2	2
2½	5,600	3·1	3·8	4·4	4·9	5·5	5·8	6·2	6·6	6·9	7·3	7·6	7·9	8·2	8·5	8·8	9·0	9·3	9·6	9·8	2½
3	6,720	3·3	4·0	4·6	5·1	5·7	6·1	6·5	6·9	7·3	7·6	7·9	8·3	8·6	8·9	9·2	9·4	9·7	10·0	10·1	3
3½	7,840	3·4	4·1	4·8	5·3	5·8	6·3	6·7	7·1	7·5	7·9	8·2	8·6	8·9	9·2	9·5	9·8	10·1	10·4	10·6	3½
4	8,960	3·5	4·3	4·9	5·5	6·0	6·5	7·0	7·4	7·8	8·2	8·5	8·9	9·2	9·5	9·8	10·1	10·4	10·7	11·0	4
4½	10,080		4·4	5·1	5·7	6 2	6·7	7·2	7·6	8 0	8·4	8·8	9·1	9·5	9·8	10·1	10·4	10·8	11·0	11·4	4½
5	11,200		4·5	5·2	5·8	6·4	6·9	7·4	7·8	8·2	8·6	9·0	9·4	9·7	10·1	10·4	10·7	11·0	11·2	11·6	5
6	13,440			5·5	6·1	6·7	7 2	7·7	8·2	8·6	9·0	9·4	9·8	10·2	10·5	10·9	11·2	11·5	11·9	12·1	6
7	15,680			5·7	6·3	6·9	7·5	8·0	8·5	8·9	9·4	9·8	10·2	10·6	11·0	11·3	11·7	12 0	12·3	12·7	7
8	17,920			5·9	6·6	7·2	7·8	8·3	8·8	9·3	9·7	10·1	10·6	10·9	11 3	11·7	12·0	12·4	12·8	13·1	8
9	20,160			6·0	6·8	7·4	8·0	8·5	9·0	9·5	10·0	10·4	10·9	11·3	11·7	12·0	12·4	12·8	13·1	13·5	9
10	22,400				6·9	7·6	8·2	8·8	9·3	9·8	10·3	10·7	11·2	11·6	12·0	12·4	12·8	13·1	13·5	13·8	10
11	24,640				7·1	7·8	8·4	9·0	9·5	10·0	10·5	11·0	11·5	11·9	12·3	12·7	13·1	13·5	13·8	14·2	11
12	26,880				7·2	7·9	8·6	9·2	9·7	10·2	10 8	11·2	11·7	12·1	12·5	13·0	13·4	13·7	14·1	14·5	12
13	29,120				7·4	8·1	8·8	9·4	9·9	10·4	11·0	11·5	11·9	12·4	12·8	13·2	13·6	14·0	14·4	14·7	13
14	31,360				7·5	8·3	8·9	9·5	10·1	10 6	11·1	11·7	12·1	12·6	13·0	13·4	13·8	14·2	14·6	15·0	14
15	33,600				7·7	8·4	9·1	9 7	10·3	10·8	11·4	11·9	12·3	12·8	13·2	13·7	14·1	14·5	14·9	15·3	15
16	35,840				7·8	8·5	9·2	9·8	10·4	11·0	11·5	12·0	12·5	13·0	13·5	13·9	14·3	14·7	15·1	15·5	16
17	38,080				7·9	8·7	9·4	10·0	10·6	11·2	11·7	12·2	12·7	13·2	13·7	14·1	14·5	14·9	15·4	15·8	17
18	40,320				8·0	8·8	9·5	10·1	10·8	11·3	11·9	12·4	12·9	13·4	13·9	14·3	14·7	15·1	15·6	16·0	18
19	42,560				8·1	8·9	9·6	10·3	10 9	11·5	12·0	12·6	13·1	13·6	14·1	14·5	15·0	15·4	15·8	16·2	19
Defl. in inches.		·1	·15	·2	·25	·3	·35	·4	·45	·5	·55	·6	·65	·7	·75	·8	·85	·9	·95	1·0	Deflex.

If the depth of a cast iron bar be multiplied by 0·937, the product will be the depth of a square bar of wrought iron of equal stiffness.
If the depth of a cast iron beam be multiplied by 1·83, the product will be the depth of a square beam of oak of equal stiffness.

TABLE I.—*Of the Stiffness of Beams (continued).*

Lengths in feet.		4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	
Weight in tons.	Weight in lbs.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Dept inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Weight in tons.
20	44,800					9·0	9·7	10·4	11·0	11·6	12·2	12·7	13·2	13·8	14·2	14·7	15·1	15·6	16·0	16·4	20
22	49,280					9·2	10·0	10·7	11·3	11·9	12·5	13·0	13·6	14·1	14·6	15·1	15·5	15·9	16·4	16·8	22
24	53,760					9·4	10·2	10·9	11·5	12·2	12·8	13·4	13·9	14·4	14·9	15·4	15·9	16·3	16·8	17·2	24
26	58,240					9·6	10·4	11·1	11·8	12·4	13·0	13·6	14·2	14·7	15·2	15·7	16·2	16·7	17·1	17·6	26
28	62,720					9·8	10·6	11·4	12·0	12·7	13·3	13·9	14·4	15·0	15·5	16·0	16·5	17·0	17·4	17·9	28
30	67,200						10·8	11·5	12·2	12·9	13·5	14·1	14·7	15·2	15·7	16·3	16·8	17·3	17·7	18·2	30
32	71,680						11·0	11·7	12·4	13·1	13·7	14·3	14·9	15·5	16·0	16·5	17·0	17·5	18·0	18·5	32
34	76,160						11·1	11·9	12·6	13·3	13·9	14·5	15·1	15·7	16·2	16·8	17·3	17·8	18·3	18·8	34
36	80,640						11·3	12·0	12·8	13·4	14·1	14·7	15·3	15·9	16·5	17·0	17·5	18·0	18·5	19·0	36
38	85,120						11·4	12·2	13·0	13·6	14·3	14·9	15·5	16·1	16·7	17·2	17·8	18·3	18·8	19·3	38
40	89,600							12·4	13·1	13·8	14·5	15·1	15·7	16·4	16·9	17·5	18·0	18·5	19·1	19·5	40
42	94,080							12·5	13·3	14·0	14·7	15·3	15·9	16·5	17·1	17·7	18·2	18·7	19·3	19·8	42
44	98,560							12·7	13·5	14·2	14·9	15·5	16·1	16·8	17·4	17·9	18·5	19·0	19·5	20·0	44
46	103,040							12·8	13·6	14·3	15·0	15·7	16·3	17·0	17·6	18·1	18·7	19·2	19·8	20·3	46
48	107,520							13·0	13·7	14·5	15·2	15·9	16·5	17·1	17·7	18·3	18·8	19·4	20·0	20·5	48
50	112,000								13·9	14·6	15·3	16·0	16·6	17·3	17·9	18·5	19·0	19·6	20·1	20·7	50
52	116,480								14·0	14·7	15·5	16·2	16·8	17·5	18·1	18·7	19·2	19·8	20·3	21·0	52
54	120,960								14·1	14·9	15·6	16·3	17·0	17·6	18·2	18·8	19·4	19·9	20·5	21·1	54
56	125,440								14·3	15·0	15·8	16·5	17·1	17·8	18·4	19·0	19·6	20·1	20·7	21·3	56
58	129,920								14·4	15·1	15·9	16·6	17·3	17·9	18·5	19·2	19·7	20·3	20·9	21·4	58
60	134,400								14·5	15·3	16·0	16·7	17·4	18·1	18·7	19·3	19·9	20·5	21·1	21·6	60
65	145,600								14·8	15·6	16·4	17·1	17·8	18·5	19·1	19·8	20·4	20·9	21·5	22·1	65
70	156,800								15·1	15·9	16·7	17·4	18·2	18·8	19·5	20·1	20·8	21·3	22·0	22·5	70
Defl. in inches.		·1	·15	·2	·25	·3	·35	·4	·45	·5	·55	·60	·65	·70	·75	·80	·85	·90	·95	1·0	Defl.

The depth of a yellow fir beam of equal stiffness may be found by multiplying the depth of the cast iron one by 1·71.

TABLE I.—*Of the Stiffness of Beams (continued).*

Lengths in feet.		4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	
Weight in tons.	Weight in lbs.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Depth inches.	Weight in tons.
75	168,000										17·0	17·7	18·5	19·2	19·8	20·5	21·3	21·7	22·3	22·9	75
80	179,200										17·2	18·0	18·7	19·4	20·1	20·7	21·4	22·0	22·6	23·2	80
85	190,400										17·5	18·3	19·0	19·7	20·4	21·0	21·7	22·4	23·0	23·6	85
90	201,600										17·8	18·6	19·3	20·0	20·7	21·4	22·1	22·7	23·3	23·9	90
95	212,800										18·0	18·8	19·5	20·3	21·0	21·7	22·4	23·0	23·6	24·3	95
100	224,000											19·0	19·8	20·6	21·3	22·0	22·6	23·3	23·9	24·5	100
110	246,400												20·3	21·0	21·8	22·5	23·2	23·3	24·5	25·1	110
120	268,800												20·8	21·5	22·3	23·0	23·7	24·4	25·0	25·7	120
130	291,200													22·0	22·7	23·5	24·2	24·9	25·5	26·3	130
140	313,600														23·2	23·9	24·6	25·4	26·0	26·7	140
150	336,000														23·6	24·3	25·0	25·8	26·5	27·2	150
160	358,400														23·9	24·7	25·5	26·2	26·9	27·6	160
170	380,800															25·1	25·9	26·6	27·4	28·1	170
180	403,200															25·5	26·3	27·0	27·8	28·5	180
190	425,600															25·9	26·7	27·4	28·2	28·9	190
200	448,000															26·2	27·0	27·7	28·5	29·2	200
250	560,000															27·6	28·5	29·4	30·1	31·0	250
300	672,000															28·9	29·9	30·7	31·5	32·0	300
350	784,000															30·0	31·0	32·0	33·0	33·7	350
400	896,000															31·1	32·0	32·9	33·9	34·7	400
450	1,008,000															32·0	33·1	34·0	34·8	35·8	450
500	1,120,000																33·8	34·8	35·7	36·7	500
Defl. in inches.		·10	·15	·20	·25	·30	·35	·40	·45	·50	·55	·60	·65	·70	·75	·80	·85	·90	·95	1·0	Defl.

TABLE II.—ART. 6. *A Table showing the weight or pressure a beam of cast iron, one inch in breadth, will sustain without destroying its elastic force, when it is supported at the ends, and loaded in the middle of its length, and also the deflexion in the middle, which that weight will produce. This Table was calculated by the Equation*, art. 143.*

Length.	1 foot.		2 feet.		3 feet.		4 feet.		5 feet.		6 feet.		7 feet.		8 feet.		9 feet.		10 feet.		Length.
Depths.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Depths.
1 in.	850	·02	425	·08	283	·18	212	·32	170	·5	142	·72	121	·98	106	1·28	95	1·62	85	2·0	1 in.
1½ —	1,912	·014	956	·053	637	·12	477	·21	383	·33	320	·48	273	·65	239	·85	214	1·08	192	1·34	1½ —
2 —			1,700	·04	1,132	·09	848	·16	680	·25	568	·36	484	·49	425	·64	380	·81	340	1·0	2 —
2½ —			2,656	·032	1,769	·072	1,325	·128	1,062	·2	887	·29	756	·39	662	·51	594	·65	531	·8	2½ —
3 —					2,547	·06	1,908	·11	1,530	·167	1,273	·24	1,089	·33	954	·426	855	·54	765	·66	3 —
3½ —					3,467	·052	2,597	·092	2,082	·143	1,739	·205	1,482	·28	1,298	·365	1,164	·46	1,041	·57	3½ —
4 —							3,392	·08	2,720	·125	2,272	·18	1,936	·245	1,700	·32	1,520	·405	1,360	·5	4 —
4½ —							4,293	·071	3,442	·111	2,875	·16	2,450	·217	2,146	·284	1,924	·36	1,721	·443	4½ —
5 —									4,250	·1	3,560	·144	3,050	·196	2,650	·256	2,375	·32	2,125	·4	5 —
6 —									6,120	·083	5,112	·12	4,356	·163	3,816	·213	3,420	·27	3,060	·33	6 —
7 —											6,958	·103	5,929	·14	5,194	·183	4,655	·23	4,165	·29	7 —
8 —											9,088	·09	7,744	·123	6,784	·16	6,080	·203	5,440	·25	8 —
9 —													9,801	·109	8,586	·142	7,695	·18	6,885	·22	9 —
10 —													12,100	·098	10,600	·128	9,500	·162	8,500	·2	10 —
11 —															12,826	·117	11,495	·15	10,285	·182	11 —
12 —															15,264	·107	13,680	·135	12,240	·17	12 —
13 —																	16,100	·125	14,400	·154	13 —
14 —																	18,600	·115	16,700	·143	14 —
Fixed at one end.	¼ foot.		½ foot.		¾ foot.		1 foot.		1¼ foot.		1½ foot.		1¾ foot.		2 feet.		2¼ feet.		2½ feet.		

REMARK.—The load shown by this Table is the greatest a beam should ever sustain, and therefore, in calculating this load, ample allowance must be made for accidents, and the weight of the beam itself must be included. The weight of the beam may be easily calculated by the rule in the note to Table I.

* A piece increased to n times the depth has its strength increased $3(n-1) + \frac{1}{n}$ times.

TABLE II.—*Of the Strength of Beams (continued).*

Lngth.	12 feet.		14 feet.		16 feet.		18 feet.		20 feet.		22 feet.		24 feet.		26 feet.		28 feet.		30 feet.		Lngth.
Depths.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Depths.
2 in.	283	1·44	243	1·96	212	2·56	189	3·24	170	4·0	154	4·34	142	5·76	131	6·76	121	7·84	118	9·0	2 in.
3 —	637	·96	546	1·31	478	1·71	425	2·16	382	2·67	347	3·23	318	3·84	294	4·51	273	5·23	255	6·0	3 —
4 —	1,133	·72	971	·98	849	1·28	755	1·62	680	2·08	618	2·42	566	2·88	523	3·38	485	3·92	453	4·5	4 —
5 —	1,771	·58	1,518	·78	1,328	1·02	1,180	1·29	1,062	1·6	966	1·93	885	2·30	817	2·70	759	3·14	708	3·6	5 —
6 —	2,548	·48	2,184	·65	1,912	·85	1,699	1·08	1,530	1·34	1,390	1·61	1,274	1·92	1,176	2·25	1,092	2·61	1,019	3·0	6 —
7 —	3,471	·41	2,975	·58	2,603	·73	2,314	·93	2,082	1·14	1,893	1·38	1,735	1·65	1,602	1·93	1,487	2·24	1,388	2·57	7 —
8 —	4,532	·36	3,884	·49	3,396	·64	3,020	·81	2,720	1·00	2,472	1·21	2,264	1·44	2,092	1·69	1,940	1·96	1,812	2·25	8 —
9 —	5,733	·32	4,914	·44	4,302	·57	3,825	·72	3,438	·89	3,123	1·07	2,862	1·28	2,646	1·50	2,457	1·74	2,295	2·0	9 —
10 —	7,083	·288	6,071	·392	5,312	·512	4,722	·648	4,250	·8	3,863	·968	3,541	1·152	3,269	1·352	3,035	1·568	2,833	1·8	10 —
11 —	8,570	·26	7,346	·36	6,428	·47	5,714	·59	5,142	·73	4,675	·88	4,285	1·05	3,955	1·23	3,673	1·425	3,428	1·64	11 —
12 —	10,192	·24	8,736	·33	7,648	·43	6,796	·54	6,120	·67	5,560	·81	5,096	·96	4,704	1·13	4,368	1·31	4,076	1·5	12 —
13 —	11,971	·22	10,260	·307	8,978	·39	7,980	·49	7,182	·61	6,529	·74	5,985	·886	5,525	1·04	5,130	1·21	4,788	1·33	13 —
14 —	13,883	·21	11,900	·28	10,412	·36	9,255	·46	8,330	·57	7,573	·69	6,941	·824	6,408	·965	5,950	1·12	5,553	1·28	14 —
15 —	15,937	·19	13,660	·26	11,952	·34	10,624	·43	9,562	·533	8,692	·645	7,967	·75	7,355	·9	6,829	1·03	6,374	1·2	15 —
16 —	18,128	·18	15,536	·245	13,584	·32	12,080	·403	10,880	·5	9,888	·63	9,056	·72	8,368	·84	7,760	·98	7,248	1·13	16 —
17 —	20,500	·17	17,500	·23	15,353	·3	13,647	·38	12,282	·47	11,166	·567	10,235	·673	9,447	·79	8,773	·92	8,183	1·06	17 —
18 —	22,932	·16	19,656	·217	17,208	·284	15,700	·36	13,752	·442	12,492	·54	11,448	·64	10,584	·75	9,828	·87	9,180	1·0	18 —
19 —	25,404	·152	21,800	·207	19,053	·27	16,935	·34	15,242	·42	13,857	·51	12,702	·607	11,725	·71	10,887	·825	10,161	·95	19 —
20 —	28,332	·144	24,284	·195	21,248	·256	18,888	·324	17,000	·4	15,452	·484	14,164	·576	13,076	·676	12,140	·784	11,332	·9	20 —
Fixed at one end.	3 feet.		3½ feet.		4 feet.		4½ feet.		5 feet.		5½ feet.		6 feet.		6½ feet.		7 feet.		7½ feet.		

TABLE II.—*Of the Strength of Beams (continued).*

Length.	12 feet.		14 feet.		16 feet.		18 feet.		20 feet.		22 feet.		24 feet.		26 feet.		28 feet.		30 feet.		Length.
Depths.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Weight in lbs.	Defl. in inches.	Depths.
21 in.	31,230	·138	26,770	·186	23,428	·245	20,825	·31	18,742	·382	17,036	·45	15,618	·55	14,417	·645	13,387	·75	12,495	·86	21 in.
22 —	34,500	·131	29,300	·178	25,712	·235	22,855	·295	20,570	·365	18,700	·44	17,141	·525	15,823	·615	14,693	·71	13,713	·815	22 —
23 —	37,600	·127	32,000	·17	28,103	·225	24,980	·282	22,482	·35	20,439	·42	18,735	·5	17,286	·59	16,059	·68	14,988	·78	23 —
24 —	40,768	·12	34,944	·163	30,592	·216	27,184	·27	24,480	·335	22,240	·402	20,384	·48	18,816	·565	17,492	·665	16,304	·75	24 —
25 —			37,700	·156	33,203	·21	29,514	·26	26,562	·32	24,148	·387	22,135	·46	20,432	·54	18,973	·625	17,708	·72	25 —
26 —			40,900	·15	35,912	·197	31,922	·25	28,730	·307	26,118	·375	23,941	·443	22,100	·52	20,521	·607	19,153	·695	26 —
27 —			44,000	·143	38,728	·19	34,425	·24	30,982	·297	28,166	·36	25,819	·427	23,832	·5	22,130	·58	20,655	·667	27 —
28 —			47,300	·14	41,650	·183	37,022	·23	33,320	·286	30,290	·347	27,766	·41	25,630	·48	23,800	·56	22,213	·645	28 —
29 —					44,678	·176	39,714	·223	35,742	·275	32,493	·333	29,785	·395	27,494	·462	25,530	·54	23,828	·62	29 —
30 —					47,808	·170	42,498	·216	38,250	·266	34,767	·322	31,869	·384	29,421	·450	27,315	·522	25,497	·60	30 —
31 —					51,053	·164	45,380	·207	40,842	·257	37,148	·31	34,085	·37	31,417	·435	29,173	·505	27,228	·58	31 —
32 —					54,400	·16	48,371	·202	43,520	·25	39,563	·302	36,266	·36	33,477	·42	31,086	·49	29,013	·56	32 —
33 —							51,425	·196	46,282	·242	42,075	·293	38,568	·35	35,602	·41	33,058	·47	30,855	·545	33 —
34 —							54,586	·19	49,130	·235	44,663	·283	40,941	·336	37,792	·395	35,093	·46	32,753	·53	34 —
35 —							57,847	·185	52,062	·228	47,329	·276	43,385	·329	40,048	·386	37,187	·448	34,708	·514	35 —
36 —							61,200	·18	55,080	·222	50,073	·269	45,900	·32	42,369	·375	39,343	·435	36,720	·5	36 —
Fixed at one end.	3 feet.		3½ feet.		4 feet.		4½ feet.		5 feet.		5½ feet.		6 feet.		6½ feet.		7 feet.		7½ feet.		

If the weight a cast iron bar will support be multiplied by 1·12, the product will be the weight a WROUGHT IRON bar of the same size will support. And the flexure of the wrought iron bar will be found by multiplying the flexure of the cast iron one by 0·86.

The strength of good OAK is one-fourth of the strength of cast iron; therefore an oak beam will bear one-fourth of the load of a cast iron one of the same size. And the flexure of the oak beam will be found by multiplying the flexure of the cast iron one by 2·8.

To find the weight a beam of YELLOW FIR will bear, multiply the weight a cast iron one of the same size will bear by 0·3. And to find its flexure, multiply the flexure of a cast iron one by 2·6.

In the same manner the Table may be applied to find the strength of any other material of which the proportional strength in respect to cast iron is known. See the alphabetical Table at the end of this work.

TABLE III.*—ART. 7. *A Table to show the weight or pressure a cylindrical pillar or column of cast iron will sustain, with safety, in hundred weights.*

Length or height.	2 ft.	4 ft.	6 ft.	8 ft.	10 ft.	12 ft.	14 ft.	16 ft.	18 ft.	20 ft.	22 ft.	24 ft.	
Diameter.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Diameter.
1 in.	18	12	8	5	3	2	2	1	1	1			1 in.
1½—	44	36	28	19	16	12	9	7	6	5	4	3	1½—
2 —	82	72	60	49	40	32	26	22	18	15	13	11	2 —
2½—	129	119	105	91	77	65	55	47	40	34	29	25	2½—
3 —	188	178	163	145	128	111	97	84	73	64	56	49	3 —
3½—	257	247	232	214	191	172	156	135	119	106	94	83	3½—
4 —	337	326	310	288	266	242	220	198	178	160	144	130	4 —
4½—	429	418	400	379	354	327	301	275	251	229	208	189	4½—
5 —	530	522	501	479	452	427	394	365	337	310	285	262	5 —
6 —	616	607	592	573	550	525	497	469	440	413	386	360	6 —
7 —	1040	1032	1013	989	959	924	887	848	808	765	725	686	7 —
8 —	1344	1333	1315	1289	1259	1224	1185	1142	1097	1052	1005	959	8 —
9 —	1727	1716	1697	1672	1640	1603	1561	1515	1467	1416	1364	1311	9 —
10 —	2133	2119	2100	2077	2045	2007	1964	1916	1865	1811	1755	1697	10 —
11 —	2580	2570	2550	2520	2490	2450	2410	2358	2305	2248	2189	2127	11 —
12 —	3074	3050	3040	3020	2970	2930	2900	2830	2780	2730	2670	2600	12 —

This Table was calculated by Equation xviii. art. 290. It is one of those cases where a Table is most useful even to the quickest calculator, because the weight to be supported and the length being given, a quadratic equation must be solved to find the diameter: here it is found by inspection. This Table does not admit of accurate application to other materials, on account of the form of the equation. It will be nearly correct for wrought iron, but is not applicable to timber.

* This Table has no solid basis. The very ingenious reasoning, from which the formula is deduced by which the Table was calculated, depends upon assumptions which Mr. Tredgold was induced to adopt through want of experimental data. See Mr. Barlow's Report on the Strength of Materials, 2nd vol. of the British Association. An abstract of an experimental research, to supply this deficiency, is given in the "Additions."—EDITOR.

SECTION II.

EXPLANATION OF THE TABLES, WITH EXAMPLES OF THEIR USE.

EXPLANATION OF THE FIRST TABLE.

8. The first Table (page 9, art. 5), shows by inspection, the dimensions of square beams to sustain weights or pressures of from one hundred weight to 500 tons; so as not to be bent or deflected in the middle, more than one-fortieth of an inch for each foot in length.

The length is the distance between the supports, as A B, fig. 1, Plate I., and the stress, whether it be from weight or pressure, is supposed to act at the middle of the length, as at C in the figure. The breadth and depth are supposed to be the same in every part of the length, and equal to one another.

The horizontal row of figures at the top of the Table contains the lengths in feet.

The columns, at the outsides, contain the weights in cwts. and tons, and the second column, on the left-hand side, contains the weights in pounds avoirdupois.

The horizontal row of figures at the bottom shows the deflexion for each length. The other columns show the depths in inches.

EXPLANATION OF THE SECOND TABLE.

9. The second Table (page 13, art. 6), is intended to show the greatest weight a beam of cast iron will bear in the

middle of its length, when it is loaded with as much as it will bear, so as to recover its natural form when the load is removed. If a beam be loaded beyond that point, the equilibrium of its parts is destroyed, and it takes a permanent set. Also, in a beam so loaded beyond its strength, the deflexion becomes irregular, increasing very rapidly in proportion to the load.

The horizontal row of figures along the top of the Table contains the lengths in feet, that is, the distances between the points of support; and the horizontal row at bottom, the length of a beam supported or fixed at one end only, which with the same load would have the same deflexion.

The columns on the outsides contain the depths in inches.

The other columns contain the weights in pounds avoirdupois, and the deflexions they would produce in inches and decimal parts, when the beams will be only just capable of restoring themselves.

The breadth of each beam is one inch, therefore the Table shows the utmost weight a beam of one inch in breadth should have to bear; and a piece five inches in breadth will bear five times as much, and so of any other breadth.

EXPLANATION OF THE THIRD TABLE.*

10. The third Table (page 16, art. 7), shows by inspection the weight or pressure a cylindrical pillar or column of cast iron will bear with safety. The pressure is expressed in cwts., and is computed on the supposition that the pillar is under the most unfavourable circumstances for resisting the stress, which happens, when, from settlements, imperfect fitting, or other causes, the direction of the stress is in the surface of the pillar, as shown in fig. 31, Plate IV.

The horizontal row of figures along the top of the Table contains the lengths or heights of the pillars in feet.

* See note to that Table.—EDITOR.

The outside vertical columns of the Table contain the diameters of the pillars in inches.

The other vertical columns of the Table show the weight in cwts. which a cast iron pillar, of the height at the top of the column, and of the diameter in the side columns, will support with safety. Consequently, of the height, the diameter, and the weight to be supported, any two being given, the other will be found by inspection.

EXAMPLES AND USE OF THE TABLES.

11. *Example* 1. To find the depth of a square bar of cast iron, twenty feet in length, that would support ten tons, the deflexion not exceeding half an inch.

Find the column in Table I. which has the length twenty feet at the top, and in that column, and opposite to ten tons in either of the side columns, will be found the proper depth for the bar, which is 9·8 inches.

If the depth 9·8 be multiplied by 1·71, it will give the depth of a square beam of fir that would support the same load with the same deflexion. Thus, $1·71 \times 9·8 = 16·76$ inches nearly, the depth of the fir beam.

If the depth of an oak beam be required, multiply by 1·83; thus $1·83 \times 9·8 = 17·93$ inches, the depth of an oak beam.

12. *Example* 2. Required the weight a cast iron beam would support without impairing its elastic force, the length, breadth, and depth being given?

Let the length be twenty feet, and the breadth the same as the depth, ten inches. In the second Table, under the length twenty feet, and opposite the depth ten inches, we find the weight 4250 ℔s. for the load a beam one inch in breadth would bear; and this multiplied by 10, gives 42,500 ℔s., or nearly nineteen tons; and the deflexion would be 0·8 inches, but the weight of the beam itself would be nearly three tons, and its effect the same as if half the three tons were applied in the middle; consequently the greatest load that the beam should be liable to sustain should not exceed seventeen tons and a half.

An oak beam of the same size would support only one-fourth of 42,500 lbs. or 10,625 lbs.; and its deflexion in the middle would be 0·8 multiplied by 2·8 = 2·24 inches.

A fir beam of the same size would support three-tenths of 42,500 lbs. = 12,750 lbs.; and its deflexion in the middle would be $0·8 \times 2·6 = 2·08$ inches.

A wrought iron bar of the same size would support 1·12 times the weight of the cast iron one, that is, $42,500 \times 1·12 = 47,600$ lbs.; and its deflexion in the middle would be 0·8 multiplied by 0·86 = 0·688 inches. But the reader will remember that wrought iron possesses this great stiffness only in consequence of the operations of forging or rolling, and these operations have very little effect where the thickness is considerable.

13. There are cases where a greater degree of flexure may be allowed, and there are others where it ought to be less; but I consider that to which the first Table is calculated as nearly the mean, and it is easy to make any variation in this respect.

Example 3. Let it be required to find the depth of a square cast iron bar to support ten tons without more deflexion than one-tenth of an inch, the length being twenty feet.

By examining the deflexion for twenty feet at the foot of the column in Table I. it will be found five times one-tenth of an inch; hence take the depth opposite five times the weight or fifty tons, which is 14·6 inches, the depth required.

14. *Example* 4. Find the depth of a square bar of cast iron to support ten tons, the deflexion not to exceed one inch, the length being twenty feet.

This degree of deflexion is double that at the foot of the column headed 20 feet in Table I.; therefore look opposite half the weight, or five tons, and the depth will be found to be 8·2 inches.

I have taken the same length and weight in each of these examples for the purpose of showing how much the depth must be increased to give stiffness.

15. When a bar or beam is employed to support a load in the middle, or at any other point of the length, a great saving of the material is made by making the bar thin and deep,* provided it be not made so thin as to break sideways.

The depth of a beam is sometimes limited by circumstances, and as no proportion could be given that would suit for every purpose, it is left entirely to the judgment of the person who may use the Table. But there is a limit to the depth, which, if it be exceeded, renders the use of cast iron for bearing purposes very objectionable and dangerous where the load is likely to acquire some degree of momentum from any cause; for if the depth be increased, it renders a beam rigid or nearly inflexible, and then a comparatively small impulsive force will break it. A very rigid beam resembles a hard body; it will bear an immense pressure, but the stroke of a small hammer will fracture it.

In order to mark the point where the depth has arrived at that proportion of the length which makes it become dangerously rigid, I have stopped the column of depths at that point, and should it be required to sustain a greater weight, the breadth must be increased instead of the depth.

16. *Example* 5. Find the depth of a rectangular bar of cast iron to support a weight of 10 tons in the middle of its length, the deflexion not to exceed one-fortieth of an inch per foot in length, and the length 20 feet; also let the depth be six times the breadth.

* The term depth is always employed for the dimension in the direction of the pressure.

Under the length 20 feet in Table I. and opposite six times the weight, will be found the depth, which in this case is 15·3 inches, and the breadth will be one-sixth of this depth, or 2·6 inches.

If a fir beam be proposed to support the same weight with the same quantity of deflexion, multiply the depth 15·3 inches by 1·71, which gives 26·2 inches for the depth of the fir beam, and its breadth will be

$$\frac{26{\cdot}2}{6} = 4{\cdot}37 \text{ inches nearly.}$$

The depth of an oak beam for the same purpose may also be found by using the multiplier given for oak at the foot of the Table.

In the same manner, if the depth had been fixed to be four times the breadth, look opposite four times the weight for the depth, and make the breadth one-fourth of the depth, and so of any other proportion.

17. *Example* 6. If the breadth and length of a beam be given, and it be required to find the depth such that the beam may sustain a given weight without impairing its elastic force; then, in the second Table, the depth and deflexion may be found thus: Divide the given weight by the breadth; the quotient will be the weight a beam of one inch in breadth would sustain, which being found in the column of weights under the given length, the depth required will be opposite to it, and also the deflexion.

Let the given breadth be three inches, the weight to be supported 10 tons or 22,400 ℔s., and the length 20 feet. Then

$$\frac{22400}{3} = 7466;$$

and the weight nearest to 7466 ℔s., in the column for 20 feet lengths in the second Table is 8330, and the depth 14 inches, and the deflexion would be 0·57 inches.

Example 7. The second Table may be usefully applied to proportion the parts of a very simple weighing machine for weighing very heavy weights. For the flexure of a beam being directly proportional to the load upon it, while its elastic force is perfect, this flexure may be made the measure of the weight upon the beam. And a multiplying index may be easily made to increase the extent of the divisions so as to render them distinct enough for any useful purpose.

Suppose that 4 tons (8960 lbs.) is the greatest load to be weighed, and that the distance between the supports is 16 feet; and make the breadth of the bar 7 inches. Then,

$$\frac{8960}{7} = 1280$$

and the nearest load above this under the length 16 feet in Table II. is 1328 lbs. and the corresponding depth 5 inches, which may be the depth of the bar. The flexure will be 1·02 inches, but if the beam be formed as fig. 4, Plate I., the flexure will be greater, being nearly 1·7. (The calculation may be made by art. 232.)

By making the index move over 5 inches when the deflexion is one inch, each cwt. will cause the index to move over one-tenth of an inch; but the scale should be graduated by the actual application of ton weights.

Two such beams and an index would form a simple weigh-bridge, which would be very little expense; a correct enough measure of weight for any practical use, not likely to get out of order, and would require no attention in weighing except examining the index. And this index might be enclosed, if necessary, so as to be inaccessible to the keeper of the weigh-bridge.

18. *Example* 8. To find the diameter for a mill shaft which is to be a solid cylinder of cast iron, that will bear a given pressure, the flexure in the middle not to exceed one-fortieth of an inch for each foot in length.

Let us suppose the distance of the supported points of a shaft to be 20 feet, and the pressure to be equal to 10 tons.

Then multiply the pressure* by the constant multiplier 1·7, that is,

$$10 \times 1{\cdot}7 = 17,$$

and in this case, opposite 17 tons in the first Table, and under 20 feet, we find 11·2 inches for the diameter of the cylinder or shaft.

But a mill shaft should have less flexure than one-fortieth of an inch for each foot in length; about half that degree of flexure will be as much as should be allowed to take place. Therefore opposite double the weight, or twice 17 tons, will be found the diameter to give the shaft that degree of stiffness, that is, 13·3 inches.

If it be for a water wheel, for example, the stress should include every force acting on the shaft; that is, the weight of the wheels on the shaft, and twice the weight of water in the buckets of the water wheel; and though it will exceed the actual stress as much as the difference between the weight of the water and its force to impel the wheel, the difference is too small to render it necessary to adopt a more accurate mode of computation.

Example 9. Large shafts are often made hollow in order to acquire a greater degree of stiffness with a less weight of metal, not only to lessen the first expense, but also to lessen the pressure, and consequently friction on the gudgeons. If the thickness of the metal be made one-fifth of the external diameter, the stiffness of the hollow tube will be half that of a square beam, of which the side is equal to the exterior diameter of the tube. Therefore in Table I., opposite double the stress on the shaft, will be found the diameter in inches under the given length.

For instance, let the shaft be 25 feet long, and the stress upon it when collected in the middle 18 tons; under 26 feet in Table I. and opposite 2 × 18, or 36 tons, will be found

* See art. 258, or Elementary Principles of Carpentry, Sect. II. art. 96; or edition by Mr. Barlow, 4to. 1840.

15·3 inches, the diameter of the shaft, provided it may bend 0·65 in., or a little more than half an inch at every revolution. If it should bend only half this, then look opposite twice 36 tons; the nearest in the Table is 75 tons, and the diameter is $18\frac{1}{2}$ inches. The thickness of metal will be one-fifth, or nearly $3\frac{3}{4}$ inches.

19. *Example* 10. When the diameter of a solid cylinder is given, and the length, to find the greatest load it will sustain without injury to its elasticity, and the deflexion that weight will cause.

Suppose the diameter to be 11 inches, and the length 20 feet, then in the second Table, opposite the depth 11 inches, and under the length 20 feet, will be found 5142 lbs. Let this be multiplied by the diameter 11 inches, and divided by the constant number 1·7; the result will be the weight required in pounds.

In this case it is 33,271 lbs.,

$$5142 \times 11 \div 1{\cdot}7 = 33{,}271.$$

The deflexion opposite 11 inches and under 20 feet is ·73 in.

Any different degree of deflexion may be allowed for in the same manner as shown in the third and fourth examples.

APPLICATION TO CASES WHERE THE LOAD IS TO BE UNIFORMLY DISTRIBUTED OVER THE LENGTH OF THE BEAM.

20. Whether a load be uniformly distributed over the length from A to B, fig. 2, Plate I., or it be collected at several equidistant points, as at 1, 2, 3, 4, 5, 6, and 7, in the same figure, the same rule may be used, as it causes no difference that need be regarded in practice. But the effect of this load in producing flexure differs from its effect in producing permanent alteration.

It is proved by writers on the resistance of solids, that the whole of a load upon a beam, when it is uniformly distributed over it, will produce the same degree of deflexion as five-eighths of the load applied in the middle * (see experiment, art. 54, 61, and 62). Consequently, take five-eighths of the whole load upon the beam, and with this reduced weight proceed as in the foregoing examples.

21. *Example* 11. Let it be required to find the dimensions of a cast iron bar to support 10 tons uniformly distributed over its length, the depth of the bar to be four times its breadth, and the deflexion to be not more than one-eightieth part of an inch for each foot in length, or one-fourth of an inch, the length being 20 feet.

Here the five-eighths of 10 tons is 6 tons and a quarter, and as the depth is to be four times the breadth, multiplying six and a quarter by four gives 25 tons; but the deflexion is to be only half that given in the Table; therefore the 25 must be doubled, which gives 50 for the number of tons opposite which the depth is to be found. The depth opposite 50 tons, and under 20 feet, is 14·6 inches, and the breadth is

$$\frac{14{\cdot}6}{4} \text{ or } 3{\cdot}65 \text{ inches};$$

that is, a bar 14·6 inches deep, and 3·65 inches in breadth, will bear a load of ten tons uniformly distributed over it when the length of bearing is 20 feet, and the deflexion in the middle a quarter of an inch.

Example 12. Let it be proposed to find the proper dimensions for an open girder of cast iron, for supporting the floor of a room, the girder being formed as described in fig. 11, Plate II. (See art. 41).

Suppose the distance between the walls to be 25 feet, and the distance between girder and girder to be 10 feet, then there will be

$$10 \times 25 = 250$$

* Dr. Young's Lectures on Nat. Phil. vol. ii. art. 325, 329. Mr. Barlow's Treatise on the Strength of Timber, Cast Iron, &c., art. 55. 1837.

superficial feet of floor supported by each girder; and the load on each foot being 160lbs., (see Alphabetical Table, art. Floor),

$$160 \times 250 = 40,000 \text{ lbs.}$$

is the whole load distributed over the girder. But five-eighths of 40,000 is 25,000lbs., and multiplying* 25,000 by 6·3, we have 157,000lbs.; the nearest number in Table I. is 156,800lbs., and the mean between the depths for 24 and 26 feet is 17·8 inches, which is the depth for the girder; the breadth should be one-fifth of the depth, or

$$\frac{17\cdot 8}{5} = 3\cdot 56 \text{ inches,}$$

and the section at A B, and C D, square.

If the girder were actually loaded to the extent we have calculated upon, the depression in the middle would be about one-third more than is stated at the foot of the Table, in consequence of the girder being diminished towards the ends; but the greatest variable load in practice is seldom more than half that we have assumed, and it is the flexure from the variable load which is most injurious to ceilings, &c.

Again, let the length of bearing be 20 feet, and the distance of the girders 8 feet, and the weight 160lbs. upon a superficial foot of the floor, then

$$20 \times 8 \times 160 = 25,600 \text{ lbs.}$$

the whole load distributed over the girder. And five-eighths of this load multiplied by 6·3 is

$$\frac{5 \times 6\cdot 3 \times 25,600}{8} = 100,800 \text{ lbs.}$$

The nearest number in Table I. is 103,040, and the depth

* It is shown in a note to art. 200, that where the breadth and depth of the section of the beam at A B, or C D, fig. 11, is one-fifth of the entire depth of the beam in the middle, the strength is to that of a square beam as 1 : 6·3, and the stiffness is in the same proportion.

corresponding to a 20 feet bearing is 14·3 inches, the depth of the girder required: and

$$\frac{14{\cdot}3}{5} = 2{\cdot}86 \text{ inches},$$

the breadth.

The examples here given of girders show the dimensions of some that were executed several years ago.

Example 13. The same calculations apply to the form of girder shown in fig. 24, Plate III. When the extreme breadth at the upper or lower side is one-fifth of the depth, divide this breadth into ten equal parts, and make the thickness in the middle of the depth four of these parts; the depth of the projections should be three-fourths of the breadth.* With these proportions, the depth at the middle of a girder for a 25 feet bearing should be 17·8 inches, and the extreme breadth 3·56 inches, as in the preceding example.

And for a 20 feet bearing $14\frac{3}{10}$ inches deep, and 2·86 inches in breadth. I have seen some of less dimensions employed in several instances, but it is to be hoped such examples are not very common. A review of my mode of calculation will show that no more excess of strength is allowed than ought to be in such a material.

When there is not any length and weight in the Table exactly the same as those which are given, take the nearest; the dimensions thus obtained will always be sufficiently near for practice.

22. In applying the second Table, the effect of a load uniformly distributed over the length is to be considered equal to that of half the load collected at the middle point, (art. 139). Therefore considering this half load the weight to be supported, proceed as in the other examples of the use of the second Table.

* See note to art. 186, for the reason of this rule.

EXAMPLES OF THE USE OF THE THIRD TABLE.

23. *Example* 14. Let it be required to support the floor of a warehouse by iron pillars, where the greatest load on any pillar will be 70 tons, the height of the pillars being 14 feet.

Seventy tons is equal to 1400 cwt.; and in the column having 14 feet at the head, in the third Table, 1561 cwt. is the nearest weight; and the diameter opposite this weight in the side column is 9 inches, the diameter required.

If it be wished to approach nearer to the proportion, take the mean between the weight above and that below 1400; that is, the mean between 1561 and 1185, which is 1373, or nearly 1400; hence it appears that a little more than $8\frac{1}{2}$ inches would be a sufficient diameter, but it is seldom necessary to calculate so near.

Example 15. If it be desired to fix on the diameter for story posts of cast iron to support the front of a house; such a one for example as is commonly erected in London where the ground story is to be occupied with shops; in such a case, each foot in length of frontage may be estimated at 25 cwt. for each floor, and 12 cwt. for the roof: hence in a house with three stories over the shops, the extreme load will be

$$\overline{3 \times 25} + 12 = 87 \text{ cwt.}$$

on each foot of frontage. Now if the posts be 7 feet apart, and 12 feet high, we have $7 \times 87 = 609$ cwt. the load upon one post; and hence we find by the Table, that a pillar $6\frac{1}{2}$ inches in diameter would be sufficient; the load 525 cwt., which corresponds to a diameter of 6 inches, being too small.

If there be only two stories above the pillars, and the height of a pillar be 10 feet, the distance from pillar to pillar 7 feet; then,

$$\overline{(2 \times 25) + 12} \times 7 = 434 \text{ cwt,}$$

the whole load for one pillar: and it appears by the Table, that a pillar 5 inches in diameter would sustain 452 cwt.; consequently 5 inches will be a proper diameter for the pillars.

When pillars are placed at irregular distances, that which carries the greatest load should be calculated for, and if it happen that such a pillar stands 10 feet from the next support on one side, and 6 feet from the next support on the other side, add these distances together, and take the mean for the distance apart; thus,

$$\frac{10 + 6}{2} = \frac{16}{2} = 8,$$

the mean distance of the supports.

The strain upon a pillar cannot be exactly in the direction of the axis when the pillars are placed at unequal distances to support an uniform load; and since this unequal distribution of supports is extremely common in story posts, the propriety of adopting the mode of calculation I have followed is evident.

The diameter of a story post is sometimes made so small in respect to its height and the load upon it, that a very slight lateral stroke would break it: while we hope that no serious accident may occur through such hardihood, we cannot but dread the consequences of trusting to these inadequate supports.

SECTION III.

OF THE FORMS OF GREATEST STRENGTH FOR BEAMS.

24. In the Introduction, I have stated that one of the most valuable properties of cast iron consists in our being able to mould it into the strongest form for our intended purpose; and in order to apply this property with the most advantage, it will be useful to consider the means of applying our theoretical knowledge on this subject to practice.

There are two means of increasing the strength of a beam; the one consists in disposing the parts of the cross section in the most advantageous form; the other, in diminishing the beam towards the parts that are least strained, so that the strain may be equal in every part of the length.

OF FORMS OF EQUAL STRENGTH FOR BEAMS TO RESIST CROSS STRAINS.

25. Before I point out the forms of equal strength corresponding to different modes of applying the load or straining force, let us consider the conditions that are essential in a practical point of view. In the first place, supported parts must have sufficient magnitude to insure stability; for it is much more important that every connection or joining should be firm, and that the bearing parts should be secure against crushing or indentation, than it is that a small portion of material should be saved. When mathematicians investigate a form of equal strength, the manner of connecting it or

supporting it is not considered. Girard has shown that whatever line generates a solid of equal resistance, the solid always terminates in a simple point, or in an arris which is either perpendicular or parallel to the direction of the straining force.* Therefore the forms given by this mode of investigation do not answer in practice unless they be properly modified.

26. It may be easily proved, that in a rectangular section, when a weight is supported by a beam, the area of the section at the point of greatest strain should be to the area at the place of least strain, as six times the length is to the depth at the point of greatest strain;† and this is the least proportion that ought to be given. Now when the length and depth are equal, the area at the point of least strain should be one-sixth of the area at the point of greatest strain, instead of being a simple point or an arris.

27. If a beam be supported at the ends, and the load applied at some one point between the supports, and always acting in the same direction, the best plan appears to be to keep the extended side perfectly straight, and to make the breadth the same throughout the length; then the mathematical form of the compressed side is that formed by drawing two semiparabolas A C D and B C D, fig. 3, C being the point where the force acts.‡ Now the curve terminating at A, it is necessary in applying it to use, to add some such

* Traité Analytique de la Résistance des Solides, art. 129.

† For it is shown (art. 110) that

$$\frac{f\,b\,d^2}{6\,l} = W,$$

but the force to resist detrusion being as the area simply; therefore we must have $f\,b'\,d' = W$ at the weakest point. Consequently

$$\frac{b\,d^2}{6\,l} = b'\,d';$$

or

$$6\,l : d :: b\,d : b'\,d';$$

where l is the length, $b\,d$ the area at the point of greatest strain, and $b'\,d'$ the area at the point of least strain.

‡ Greg. Mechanics, i. art. 180. It was first shown by Galileo.

parts as are indicated by the dotted lines at the extremities. The same form is proper for a beam supported in the middle, as the beam of a balance.

28. Irregular additions of this kind, however, render it difficult to estimate the effect of the straining force; therefore, some simple straight-lined figure to include the parabolic form is to be preferred: this may be easily effected as proposed by Dr. Young,* by making the lines bounding the compressed side tangents to the parabolas, as in fig. 4. If A E be equal to half C D, then E C is a tangent to the point C of an inscribed parabola A C, having its vertex at A.

By forming a beam in this manner, one-fourth of the material is saved; but the flexure will be somewhat more than one-third greater, therefore there is a loss of stiffness in using this form.

29. If the beam be strained sometimes from one side and sometimes from the other, both sides should be of the same figure, as in fig. 5. In the beam of a double acting steam engine, the strain is of this kind. A E and B F should be equal, and each equal to half C D as before.

30. It is sometimes desirable to preserve the same depth throughout; and in this case, the section through the length of the beam made perpendicular to the direction of the straining force should be a trapezium, described in the manner shown in the 6th figure,† the force acting perpendicularly at C, the points of support being at A and B. A figure of this kind would obviously be without stability, but modified as shown by fig. 7, the end being formed as shown at B,′ any degree of stability may be given, and with a less quantity of material than when the depth is diminished as in the parabolic form. Also, the deflexion is less, which gives this form a considerable advantage for bearing purposes. In a beam supported in the middle, the same form may be used when the weights act at the ends, as in a balance.

* Nat. Philos. vol. i. p. 767.

† Gregory's Mechanics, i. art. 179.

31. When a beam or bar is regularly diminished towards the points that are least strained, so that all the sections are similar figures, whether it be supported at the ends and loaded in the middle, or supported in the middle and loaded at the ends, the outline should be a cubic parabola; * and if the section of the beam be a circle at the point of greatest strain, the form of the beam should be that generated by the revolution of the cubic parabola round its axis, the vertex being at the point of least strain.

But in practice, a frustum of a cone or a pyramid will generally answer better, the diameter of the point of greatest strain being to that at the point of smallest strain as 3 : 2.†

The same figure is proper for a beam fixed at one end, and the force acting at the other; consequently, it is a proper figure for a mast to carry a single sail.

32. If a weight be uniformly distributed over the length of a beam supported at both ends, and the breadth be the same throughout, the line bounding the compressed side should be a semi-ellipse when the lower side is straight,‡ as shown in fig. 8.

Instead of an ellipse, I usually make the compressed side a portion of a circle, of which the radius is equal to the square of half the length divided by the depth of the beam. The dotted line in fig. 8 shows this form.

The same form of equal strength should be employed when the beam is intended to resist the pressure of a load rolling over it; hence the beams of a bridge, the rails of a waggon-way, and the like, should be of this figure.

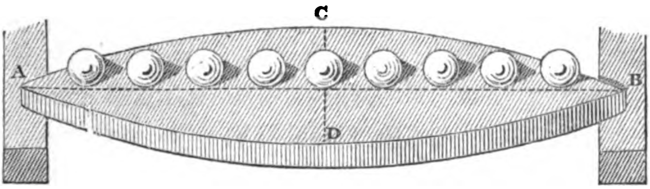
33. If a beam has to bear a weight uniformly distributed over its length, and its depth be everywhere the same, the beam being supported at both ends, then the outline of the breadth should be two parabolas A C B, A D B, set base to

* Gregory's Mechanics, i. art. 181, or Emerson's Mechanics, prop. lxxiii. cor. 1.

† Such a cone or pyramid will include the figure of equal strength, the subtangent of the curve being three times its abscissa.

‡ Gregory's Mechanics, i. art. 182, or Emerson's Mechanics, prop. lxxiii. cor. 3.

base, their vertices C and D being in the middle of the length, as shown in the next perspective sketch.* In practical cases, the arcs A C B, A D B, may be portions of circles.



When the ends are modified as in fig. 7, Plate I., this will be the most advantageous form for a beam for supporting a load uniformly distributed over its length, as lintels, bressummers, joists, and the like.

34. When a beam is fixed at one end only, and has to support a weight uniformly distributed over its length; if the breadth of the beam be every where the same, the form of equal strength is a triangle A C B,† fig. 21, Plate III.

35. If a beam be fixed at one end only, and the weight be uniformly diffused over the length, the section being everywhere circular, then the form of equal strength would be that generated by the revolution of a semi-cubic parabola round its axis.‡

It will be sufficient in practice to employ the frustum of a cone of which the diameter at the unsupported end is one-third of the diameter at the fixed end.§

* Young's Nat. Phil. i. p. 767.

† Emerson's Mechanics, prop. lxxiii. cor. 2.

‡ Ibid.

§ For the equation of the curve is,

$$a\,x = y^{\frac{3}{2}};$$

hence,

$$\tfrac{3}{2}\,x = \text{the subtangent} = 1{\cdot}5\,x;$$

and the length of the cone that would include the form of greatest strength is 1·5 times the length of the beam.

SECTION IV.

OF THE STRONGEST FORM OF SECTION.

36. When a rectangular beam is supported at the ends, and loaded in any manner between the supports, it may be observed that the side against which the force acts is always compressed, and that the opposite side is always extended; while at the middle of the depth there is a part which is neither extended nor compressed; or, in other words, it is not strained at all.

Any one who chooses to make experiments may satisfy himself that this is a correct statement of the fact, in any material whatever; whether it be hard and brittle as cast iron, zinc, or glass; or tough and ductile as wrought iron and soft steel; or flexible as wood and caoutchouc; or soft and ductile as lead and tin. In very flexible bodies it may be observed by drawing fine parallel lines across the side of the bar before the force is applied; when the piece is strained the lines become inclined, retaining their original distance apart only at the neutral axis. In almost all substances, the fracture shows distinctly that a part has been extended, and the rest compressed; and in some substances, as wood, lead, tin, wrought iron, &c., the place of the axis of motion may be observed in the fracture. It was first noticed in experiment, and applied to correct Galileo's theory by Marriotte.*

* Treatise on the Motion of Water, &c., translated by Desaguliers, p. 243, 8vo. London, 1718.

Coulomb * and Dr. Young have made it the basis of their investigations, the latter showing the important fact that an oblique force changes the position of this axis; † as has been investigated more in detail in this Essay. Lately the place of the neutral axis in horizontal beams has been more closely examined by Barlow in a numerous course of experiments; ‡ and Duleau has ingeniously shown its place by experiment on wrought iron.§ The same thing is exhibited in a refined and beautiful contrivance of Dr. Brewster's, which he calls a teinometer, and employs to measure the effect of force on elastic bodies.‖

The strains decrease from each side towards the middle, and in the middle they are insensible. I will call the part at the middle of the depth the neutral axis, or *axis of motion.* See Sect. VII. art. 107.

37. In the case of equilibrium, between the straining force and the resistance of a beam, it is a necessary condition that the resistance on one side of the axis of motion should be exactly equal to the resistance on the other side; or, that the force of compression should be equal to the force of extension. Now, in practice, a body should never be strained beyond its power of restoring itself; and as it is known from experience, that while their elastic force remains perfect, bodies resist the same degree of extension or of compression with equal forces, it is obvious that, in the section of a beam of the greatest strength, the form on each side of the axis of motion should be the same; because whatever is the strongest form for one side of the axis must be equally so for the other. Hence, the axis of motion in beams of the greatest strength will always be at the middle of the depth.¶

* Mémoires de l'Académie des Sciences. Paris, 1773.

† Lectures on Natural Philosophy, vol. ii. p. 47, 4to. London, 1805.

‡ Essay on the Strength of Timber, &c., 8vo. London, 1838.

§ Essai Théorique et Expérimental sur la Résistance du Fer Forgé, p. 26, 4to. Paris, 1820.

‖ Additions to Ferguson's Lectures, vol. ii. p. 232, 8vo. Edinburgh, 1823.

¶ These remarks apply only to bodies subjected to very moderate strains, par-

38. And, as it is shown by writers on the resistance of solids, that the power of any part in the same section is directly as the square of its distance from the axis of motion, (art. 108,) when the strain upon it is the same, it is obviously an advantage to dispose the parts of the section at the greatest possible distance from the axis of motion, provided that the middle parts be kept sufficiently strong to prevent the straining force from crushing the extreme parts together, and that the breadth be made sufficient to give stability.

39. It must also be observed that when the parts are not of equal thickness, the metal cools unequally, and therefore is partially strained by irregular contraction; it is sometimes even fractured by such irregular cooling: for this reason, the parts of a beam should be nearly of the same size. A good founder may generally reduce the danger of irregular cooling, but it is always best to avoid it altogether.

40. The form of section which I usually adopt in order to fulfil these conditions is represented in fig. 9.* A M is the axis of motion; the parts on each side of the axis of motion are the same; the metal is nearly of equal thickness, and the parts necessary to give strength and stability are disposed at the greatest distance from the axis of motion.

A section of this form is adapted for many purposes; such, for example, as the beam of a steam engine, as in fig. 26; or for supporting arches, as in fig. 10, for girders, bearing beams, and the like.

41. When it is necessary to leave some part of the middle of the beam quite open, or when the depth is considerable, I have recourse to another method, which has, in such cases, a decided advantage in point of economy. It consists in

ticularly in cast iron; since that metal requires, on the average, nearly seven times as much force to crush it as to tear it asunder, and the breaking strength of beams depends upon these forces.—EDITOR.

* This is not the form of greatest strength to resist fracture; and the beam proposed in the next article (fig. 11, Plate II.) breaks irregularly, and is remarkably weak. See Additions.—EDITOR.

making the compressed side of the beam, or that against which the force acts, a series of arches, and the other side a straight tie. (See fig. 11, Plate II.) If the tie be not straight, there is a great loss of strength, and a greater loss of stiffness.

In this figure, the thickness is supposed to be everywhere the same, and the narrowest part of the curved side of the same width as the straight side; or, so that the area of the section at A B may be the same as the area of the section at C D.

The sketch in the figure is for the case in which the load is uniformly distributed over the length, and then the upper side should be the proper curve of equilibrium for an uniform load. This curve is a common parabola, but a circular arc will always be sufficiently near when the rise is so small. The upper part of the beam forms an arch, of which the continued tie forms the abutments, and the smaller arches are merely to connect the two parts and give stability to the whole.

The connexion thus formed is necessary for supporting the tie; and in consequence of this connexion the effect of the straining force will be similar to that on a solid beam. Several girders and beams for floors have been formed on this principle; and a simple method of proportioning them will be found in the second Section.

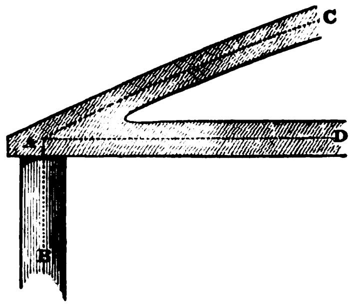
All the parts should be kept as nearly as convenient of the same bulk, to prevent irregular contraction.

42. If the load be distributed in any other manner, the curve should be the proper curve of equilibrium for that load.*

For if it be not the proper curve, partial strains will be produced in the beam, which will impair its strength. The curve of equilibrium should pass everywhere at the middle of the depth of the curved part of the beam, and should meet the axis of the straight tie in the centres of the supports

* The method of finding the curve of equilibrium is shown in my "Elementary Principles of Carpentry," Sect. I. art. 47-61.

upon which the beam rests. Thus A C being the curve of equilibrium, A D the axis or centre line of the tie; A B should be the centre of the support on which the beam rests.



43. If the load be applied at one point, the upper side should be formed of two straight lines, meeting in the point where the load is to rest, as at A in fig. 12.

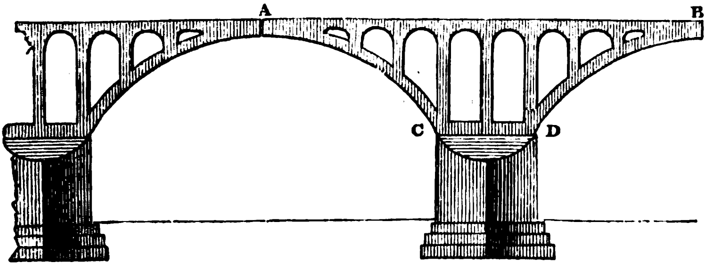
The openings should be disposed as may best answer the purpose for which the beam is intended, but they may generally be from 2 to 3 feet each. When such beams, as fig. 11, are used as girders, the openings receive the binding joists instead of mortises.

44. When a beam is to bear a load at one end, the other being fixed; or when a beam is loaded at both ends and the support is in the middle; then the tie should be the upper part of the beam: it should obviously be straight for the reasons already assigned; and the other parts should be straight also, except the small degree of curvature which would cause the weight of the part to be balanced by the forces concerned. Indeed, the arrangement for this strain should be the same as fig. 12 inverted, the support being at A, and the load at B and D.

45. But when the load is uniformly distributed over the length, the lower side A C, in the annexed figure, should be curved; the proper curve for an uniform load being a common parabola with its vertex at A. By a combination of

such beams, a bridge might be formed which would have no lateral pressure on its piers or abutments. C D being one of the piers, the distance between the points C and D may very easily be so arranged, that a given force at A or B would not disturb the equilibrium of the frame.

A bridge of this kind would not be affected by contraction and expansion; because no connexion would be necessary at



the junction of the beams at A, but such as would allow of the motion of contraction or expansion without injury.

In a design for a large bridge on this principle, which I made some years ago, it was contrived to put together in parts, without the assistance of centering; the open work of the spandrils being composed of vertical and diagonal supports and braces.

OF THE STRONGEST FORM OF SECTION FOR REVOLVING SHAFTS.

46. When a beam revolves, while the straining force continues to act in the same direction upon it, that form is obviously the best which is of the same strength to resist a stress at any point of the perimeter of its section, and the circle is the only form of section which has this property.*

* This conclusion has been objected to by Navier (Application de la Mécanique, note to art. 494) in the following terms:—

"The most convenient figure for axes of rotation is made a subject of inquiry in the Practical Essays on Mill-work by Buchanan, with notes by T. Tredgold, and re-

If a shaft be of any other form than cylindrical, the flexure will be different in different parts of the revolution, and therefore the motion will be unsteady, and particularly in new work. In a square shaft (and such shafts are chiefly employed), the resistance to pressure at one point is to the resistance to the same pressure at another point in the perimeter as ten is to seven nearly (art. 112). In feathered shafts, that is, shafts of which the section is similar to fig. 13, the resistance is more regular, but not perfectly so.*

For the same reasons, the sections of the masts of vessels should be circular.

47. As the circle is the best form for the section of a shaft, a hollow cylinder will be the strongest and stiffest form for a shaft; and the same form is also best calculated for resisting a twisting strain to which all shafts are more or less exposed.

The idea of making hollow tubes for resisting forces that often change their direction, has been undoubtedly borrowed from nature; but in art we cannot pursue the principle to so much advantage, because it is difficult to make a perfect casting of a thin tube; and in shafts, &c., of small diameter, it is much greater economy to make them solid.

It is usual to make hollow tubes of uniform diameter with

edited by George Rennie, F.R.S., and in the Practical Essay on the Strength of Cast Iron. Mr. Tredgold appears to think that the circle is the only figure which gives to the axes the property of offering in every direction the same resistance to flexure. This error proceeds from his considering the resistance to flexure as being measured by the expression which measures the resistance to rupture. We have already remarked that a square section gave the same resistance to flexure in the direction of the sides and of the diagonals. But moreover, this section gives an equal resistance in every direction; and the same is the case with regard to a great number of figures, which may be formed by combining in a symmetrical manner the circle and the square. It thence results that if the axes strengthened by salient sides, which the English call feathered shafts, do not answer as well as square axes, or full cylindrical ones, this arises probably from their not being as well disposed to resist torsion, and not from the inequalities of flexure of these axes."—EDITOR.

* In heavy astronomical instruments, and in all machines where steady and accurate movements are necessary, every attention should be paid to the effect of flexure. Irregularity may be diminished by excess of strength, but it cannot be wholly removed. The reader who wishes to pursue this subject, as far as regards astronomical instruments, may consult the Philosophical Magazine, vol. lx. p. 338, and vol. lxi. p. 10.

gudgeons cast separate, to fix at the ends. The manner of calculating the stiffness of hollow tubes for shafts is shown in art. 259 and 260, and an easy popular mode at art. 18. When they are applied to other purposes, consult art. 178, and those following it in the same proposition.

SECTION V.

AN ACCOUNT OF SOME EXPERIMENTS ON THE RESISTANCE OF CAST IRON.

48. There have been very few experiments made on the resistance of cast iron, in which the degree of flexure produced by a given weight has been measured; but the few that have come to my knowledge, and that are sufficiently described to admit of comparison, I purpose to compare with the rules I made use of in calculating the Tables in this work; and to add several new experiments.

MR. BANKS'S EXPERIMENTS.*

49. Mr. Banks made some experiments on cast iron, and noticed the deflexion, but only at the time of fracture. These experiments were made at a foundry at Wakefield. The iron was cast from the air-furnace; the bars one inch square, and the props exactly a yard distant. One yard in length weighed exactly 9 lbs., excepting one, which was about half an ounce less, and another a very little more. They all bent about an inch before they broke.

1st bar broke with	963 lbs.	Mean 971⅔ lbs.
2d bar broke with	958 "	
3d bar broke with	994 "	
4th bar, made from the cupola, broke with	864 "	

* From a treatise "On the Power of Machines," by John Banks. Kendall, 1803, p. 96.

50. Now the rule according to which the first Table was calculated is expressed by the equation

$$\cdot 001\ W\ L^2 = B\ D^3,$$

in which the weight in pounds is denoted by W, the length in feet by L, the breadth in inches by B, the depth in inches by D, and the number ·001 is a constant multiplier, which I shall sometimes denote by *a*.

The rule determines the dimensions for a deflexion of as many fortieths of an inch as there are feet in length, or $\frac{L}{40}$; and if *d* be the deflexion in inches determined by experiment, we have

$$d : W : : \frac{L}{40} : \frac{W\ L}{40\ d},$$

which being substituted for the weight in the equation above it, becomes

$$\frac{\cdot 001\ W\ L^3}{40\ d} = B\ D^3,$$

$$\text{or, } \cdot 001 = a = \frac{40\ B\ D^3\ d}{W\ L^3}.$$

The equation, in this form, may be called a formula of comparison, as when the value of *a* determined by it is the same I have used, or nearly the same, it will be evident that the Table is calculated from proper data.

51. Taking the mean of the first three of Mr. Banks's experiments, we have

$$\frac{40\ B\ D^3\ d}{W\ L^3} = \frac{40}{971 \times 27} = \cdot 00152 = a.$$

And in the bar from the cupola, or fourth experiment,

$$\frac{40\ B\ D^3\ d}{W\ L^3} = \frac{40}{864 \times 27} = \cdot 0017 = a.$$

The experiments of Mr. Banks indicate therefore that he had employed iron of a more flexible quality, but they are not sufficiently accurate for establishing the elements of a practical rule, because the deflexion was not correctly observed, nor observed at a proper stage of the experiment. For when a bar is strained nearly to the point of fracture, the deflexion becomes extremely irregular, and increases more rapidly than in the simple proportion of the weight, (see art. 56, 63, 65, and 67,) and consequently must give a much higher value to *a* than the true one, as we find to be the case with these experiments.

M. RONDELET'S EXPERIMENTS.*

52. M. Rondelet has described some experiments on different kinds of cast iron in his work on building, which were made upon specimens of 1·066 inches square, supported at the ends, and loaded in the middle of the length.

M. Rondelet's First Experiments. Distance between the Supports 3·83 feet.

Weight in lbs.	134	201	268	335	Remarks, &c.
Kind of iron.	Defl. inch.	Defl. inch.	Defl. inch.	Defl. inch.	
1. Gray cast iron	·089	·2	·357	·49	Broke with 482 lbs.
2. Do. do.	·156	·318	·38	·49	Broke with 482 lbs.
				2) ·98	·49 mean of deflexions, with 335 lbs.
3. Soft cast iron	·134	·313	·466	·62	Broke with 700 lbs.
4. Do. do.	·0223	·067	·134	·2	Broke with 1140 lbs.
5. Do. do.	·089	·156	·245	·38	Broke with 375 lbs.
6. Do. do.	·089	·178	·29	·445	Broke with 605 lbs.
				4)1 ·645	·411 mean of deflexions, with 335 lbs.

* Extracted from his Traité Théorique et Pratique de l'Art de Bâtir, tome iv.

M. Rondelet's Second Experiments. Distance between the Supports 1·915 *feet.*

Weight in lbs.	322	483	644	805	Remarks, &c.
Kind of iron.	Defl. inch.	Defl. inch.	Defl. inch.	Defl. inch.	
1. Gray cast iron	·067	·089			Broke with 580 lbs.
2. Do. do.	·0445	·089	·112	·134	Broke with 1063 lbs.
					Mean of deflexions, with 483 lbs. is ·089 inch.
3. Soft cast iron	·0445	·089	·134	·153	Broke with 1770 lbs.
4. Do. do.	·0445	·067	·134		Broke with 1360 lbs.
		2) ·156			Mean of deflexions with 483 lbs. is ·078 inch.
		·078			

In order to compare these results with the formula used in calculating the Tables, I have taken the mean deflexions corresponding to the load of 335 lbs. in the long pieces, and to 483 lbs. in the short ones; and in the gray cast iron.

For the long lengths . . . . $a =$ ·00134
For the short lengths . . . . $a =$ ·00135

In the soft cast iron,

For the long lengths . . . . $a =$ ·00112
For the short lengths . . . . $a =$ ·00118

These values of a were calculated by the formula of comparison given in art. 50, and the latter ones nearly agree with that employed to calculate the Table.

MR. EBBELS'S EXPERIMENT.

53. According to a trial communicated to me by Mr. R. Ebbels, a bar of cast iron, 1 inch square, and supported at the ends, the distance of the supports being 3 feet, the deflexion in the middle was $\frac{3}{16}$ths of an inch, with a weight of 308 lbs. suspended from the middle. The iron was of a hard

kind, not yielding very easily to the file; it was cast at a Welsh foundry.

In this trial we have

$$\frac{40\,B\,D^3\,d}{L^3\,W} = \frac{40 \times 3}{27 \times 308 \times 16} = \cdot000902 = a.$$

Consequently, iron of this kind is about $\frac{1}{10}$th stronger than that which the Table is calculated from, or rather it would bend $\frac{1}{10}$th part less under the same strain.

Experiment 1.

54. A joist of cast iron of the form described in fig. 9, Plate I., was submitted to the following trials. It was supported at the ends only; the distance between the supports 19 feet, and placed on its edge. The deflexion from its own weight was $\frac{3}{40}$ths of an inch.

When it was laid flatwise, the deflexion from its own weight was 3·5 inches, the distance of the supports remaining 19 feet.

The whole depth *a d*, fig. 9, was 9 inches, the breadth, *a b*, was 2 inches; the depth of the middle part, *e f*, was $7\frac{1}{2}$ inches; and the breadth of the middle part $\frac{3}{4}$ths of an inch.

55. It may be easily shown that to derive the value of *a*, from the experiment on the edge, we may use an equation of this form, (see art. 192 and 215,)

$$a = \frac{40\,B\,D^3\,d\,(1 - p^3\,q)}{\frac{5}{8}\,W\,L^3} = \frac{64\,B\,D^3\,d\,(1 - p^3\,q)}{W\,L^3};$$

in which D is the whole depth, and p D the depth of the middle part, and B the whole breadth, and q B the breadth after deducting that of the middle part.

In our experiment $D = 9$ inches, and $p\ D = 7{\cdot}5$, or $p = {\cdot}833$. Also, $B = 2$ inches, and deducting $\frac{3}{4}$ths, the breadth of the middle, we have $q\ B = 1{\cdot}25$, or $q = {\cdot}625$. And the weight of the part of the joist between the supports being 540 lbs., we find $a = {\cdot}00124$.

The equation for finding the value of a, in the experiment, with the joist flatwise, is

$$\frac{64\,B\,D^3\,d\,(1+p^3\,q)}{W\,L^3}=a=\cdot00092.$$

Where

$$D=2\text{ inches},\ B=9-7\cdot5=1\cdot5,\ p=\frac{\cdot75}{2},\ \text{and } q=\frac{7\cdot5}{1\cdot5}.$$

I consider the value of a derived from the experiment with the joist flatwise as nearest the truth, because the deflexion was so considerable, that a small error in measuring it would not sensibly affect the result, while there must be some uncertainty in ascertaining so small a deflexion as $\frac{3}{40}$ths of an inch in 19 feet; and a very small error in this measure would cause the difference between the results. I have, however, given it, as I determined it at the time, and the manner of calculation may be useful in other cases. If the mean be taken between the results, it is

$$\frac{\cdot00124+\cdot00092}{2}=\cdot00108.$$

In the experiment flatwise, we obtain a constant multiplier extremely near to that determined from a bar of the same iron an inch square, and 34 inches long (art. 57), and it differs only about $\frac{1}{12}$th part from the one employed for calculating the Table, page 12, art. 5.

Experiment 2.

56. I now purpose describing the direct experiments I have made for obtaining the constant multipliers used in this work; I call experiments direct when known weights are applied as the straining force, without the intervention of mechanical powers, without loss of effect from friction, or a risk of error in estimating the quantity of force, when the yielding of the supports cannot affect the measure of the deflexion, and when the deflexion can be accurately measured.

The iron I used was soft gray cast iron; it yielded easily to the file, and extended a little under the hammer, before it became brittle and short.*

The first experiment was made with a bar of an inch square, cast by Messrs. Dowson, London, with the supports 34 inches apart; the weights were placed in a scale suspended from the middle of the length; the load was increased by 10 lbs. at a time, and the deflexion measured each time, the quantity of deflexion being multiplied by means of a lever index. The whole time of making the experiment was nearly four hours; the thermometer varying from 65 to 66 degrees. Only half the number of observations is inserted here.

Weight in lbs.	Defl. in inch.	Remarks.	Weight in lbs.	Defl. in inch.	Remarks.
20†	·02		240	·13	
40	·03		260	·14	
60	·04		280	·15	
80	·05		300	·16	unloaded, and it returned to its natural state.
100	·06		320	·17	
120	·07		340	·18	
140	·08		360	·19	
160	·09		380	·20	
180	·10	unloaded, and it returned to its natural state.	400	·21	
200	·11		410	·22	deflexion became irregular; and when the load was removed, it had taken a permanent set, with a curvature of ·015 inch.
220	·12				

From this experiment we find that the deflexion of cast iron is exactly proportional to the load, till the strain arrives at a certain magnitude, and it then becomes irregular; and at or near the same strain a permanent alteration takes place in the structure of the iron, and a part of its elastic force is lost. The same thing occurs in experiments on other metals: I have tried wrought iron, tin, zinc, lead, and alloys of tin and lead, with a view to measure their elastic forces, and the strains that produce permanent alteration.

* A considerable degree of malleability is a good quality in cast iron for bearing purposes, because it lessens the risk of sudden failure. The iron was a mixture of Butterly iron, two parts, with one part of old iron.

† The weight of the scale, 8 lbs., ought to have been added.

57. According to this experiment,

$$\frac{40\,\mathrm{B\,D^3}\,d}{\mathrm{W\,L^3}} = \frac{40 \times \cdot 21}{400 \times 22\cdot 7} = \cdot 000925 = a.$$

Experiment 3.

58. The next experiments were made with an uniform bar of iron, cast by Messrs. Dowson, 3 inches by 1 and $1\frac{1}{2}$ inches, and 6·5 feet between the supports. When this bar was placed on its edge, and loaded in the middle with

150 lbs. the deflexion in the middle was	1	fortieth of an inch.
290 lbs.	2	do.
360 lbs.	$2\frac{1}{4}$	do.
440 lbs.	3	do.

The same deflexions were observed in removing the load, and it perfectly regained its natural state. Whence we have,

$$\frac{40\,\mathrm{B\,D^3}\,d}{\mathrm{W\,L^3}} = \frac{1\cdot 5 \times 27 \times 3}{440 \times 274\cdot 625} = \cdot 00105 \text{ nearly} = a.$$

Experiment 4.

59. The same piece, with the supports at the same distance, placed flatwise, and loaded in the middle with

180 lbs. the deflexion in the middle was	5	fortieths of an inch.
360 lbs.	10	do.

The bar restored itself perfectly when the weights were removed, and the trial was repeated with the same results; the load, of 360lbs., remained upon it ten hours without impairing its elastic force, or increasing the deflexion in the slightest degree.

60. From this and the preceding experiment, the ratio of the breadth and depth to the quantity of deflexion may be compared when the weight is the same. According to the theory of the resistance to flexure (art. 256),

$$d : \frac{1}{\mathrm{B\,D^3}};$$

and for the weight of 360℔s. we have

$$\frac{1}{1\cdot5 \times 3^3} : \frac{1}{3 \times 1\cdot5^3} :: 2\tfrac{1}{4} : \frac{9 \times 2\cdot5}{22\cdot5} = 10,$$

as it was found to be by experiment.

To find the constant multiplier from the last experiment, we have

$$\frac{40\,\mathrm{B D}^3 d}{\mathrm{W L}^3} = \frac{3 \times 3\cdot375 \times 10}{360 \times 274\cdot625} = \cdot00102 = a.$$

This value of a does not exactly agree with the one calculated from the first experiment on the same piece; but it is as near as can be expected in a case of this kind; and in a practical point of view it is as near an approach to accuracy as the nature of the subject requires.

Experiment 5.

61. I was desirous of trying the effect of an uniformly distributed load, and my weights, which are cubical pieces of cast iron, all of the same size, and each weighing 10℔s., are very well adapted for the purpose.

The same piece that was used for the last experiment was laid flatwise upon supports, the supports being 6 feet 6 inches apart, and 18 weights (in all 180℔s.) were laid along the upper side, just so as to be clear of one another, in the manner shown in fig. 2, Plate I. The deflexion produced by these weights was $\frac{3}{40}$ths of an inch.

A second tier of weights being added, making the whole weight upon the bar 360℔s., the deflexion was $\frac{6}{40}$ths of an inch.

62. Hence it appears, that when the weight is uniformly distributed over the length, the deflexion is directly as the weight.

And comparing this with the preceding experiment, it appears, that the deflexion from the weight uniformly distributed over the length, is to the deflexion from the same weight applied in the middle of the length, as 6 is to 10.

The proportion obtained by theoretical investigation is as 5 is to 8; but as $6:10::5:8\frac{1}{3}$. This small difference arises undoubtedly from error in measuring the deflexions in the experiments.

To compare the value of the constant multiplier by this experiment, the equation

$$\frac{40\,\mathrm{B}\mathrm{D}^3 d}{\frac{5}{8}\mathrm{W}\,\mathrm{L}^3} = a$$

must be used, whence we find $a = \cdot 00098$.

Experiment 6.

63. This experiment was made upon a piece of iron cast by Messrs. Bramah, of Pimlico, London. It crumbled sooner under the hammer than that used in the preceding experiments, and did not yield quite so readily to the file; it was regular and fine-grained.

The piece was uniform, and $\frac{9}{10}$ths of an inch square; the supports were 3 feet apart, and the weight was applied in the middle of the distance between the supports.

Weight in lbs.	Defl. in inch.	Remarks.	Weight in lbs.	Defl. in inch.	Remarks.
20	·02		220	·225	
40	·04		240	·245	
60	·06	When unloaded it returned to	260	·27	When this load
80	·08	its original form : loaded again,	280	·293	had been on 20
100	·10	the deflexion was the same, and	300	·318	minutes, it be-
120	·12	it remained loaded 12 hours with-	320	·34	came ·32 inch.
140	·14	out sensible increase, when on	340	·365	
160	·162	being unloaded it was found to	360	·392	
180	·183	have acquired a permanent set of	380	·42	
200	·21	·02 in. The index was set to	400	·445	
		nothing, and the weights pro-	420	·475	
		duced the same deflexions as at	440	·5	
		first; and it was further loaded	460	·532	which became in
		as described.	480	·57	an hour ·58.

When the weights were removed, the piece retained a permanent deflexion of ·075 inch; but it was several hours before it returned to that curvature. I did not break the specimen, because I had not weight enough by me for that purpose, neither would it have given a fair measure of the strength of the iron after the trials I have described; but I hope the effect of these trials will make the reader sensible of the necessity of limiting the strain within the range of the elastic force of the material.

According to this experiment,

$$\frac{40\,B D^3 d}{W L^3} = \frac{40 \times \cdot 9^4 \times \cdot 21}{200 \times 27} = \cdot 00102 = a.$$

COMPARISON OF THE PRECEDING EXPERIMENTS.

64. If the mean value of the constant a be taken for the experiments from art. 53 to 63, it is 0·0010446. The number used in calculating the first Table (art. 5, p. 9) was 0·001, a sufficiently near approximation, with the advantage of much simplicity.

Experiments 7, 8, *and* 9.

65. The next trials were made with specimens formed as shown in fig. 4, Plate I., with the deepest part C D exactly in the middle of the length, and the depth, at C D, 0·975 inch; the depth E A and B F were each half that at C D. The distance of the supported points A B was 3 feet, and the breadth of the bars 0·75 inch. The load was suspended from the point C in the middle of the length, and the deflexion was measured at the same point: the load was increased by 10lbs. at a time.

Weight acting on the bar.	1st Specimen. Deflex. produced.	2nd Specimen. Deflex. produced.	3rd Specimen. Deflex. produced.
lbs.	in.	in.	in.
40	·052	·065	·052
80	·104	·13	·105
120	·16	·19	·16
160	·215	·25	·21
180	·245	·28	·24
200	·272	·32	·265
500	·84		
540	Broke.		

On the first specimen the load of 180lbs. remained twelve hours; the deflexion did not sensibly increase, and it returned to its natural form when unloaded; it was again loaded to 200lbs., which remained upon it two hours; it was then unloaded again, and was found to have taken a permanent set with a deflexion of ·005 inch. The specimen was then loaded again, and the deflexions observed at every 20lbs.: the deflexion produced by the addition of 20lbs. was at first ·026, became ·03, ·04, and towards the end of the experiment ·05. When the load had been increased to 360lbs., in every succeeding addition of 10lbs. I observed that the deflexion increased by starts of as much as $\frac{1}{100}$th of an inch each, which appeared to be caused by the ends sliding on the supports, at the moment the weight was added; the bar emitted a slight crackling noise, like that produced by bending a piece of tin. There was a small defect in the bar at the place where it broke, which was 4 inches distant from the middle.

When the second specimen was unloaded, immediately, from a weight of 200lbs. it barely returned to its natural form; but a load of 180lbs. produced a permanent deflexion of ·005 when it remained upon it fourteen hours.

The load of 200lbs. remained twenty-one hours upon the third specimen, and when it was unloaded the index returned to zero; therefore this strain was less than would produce a permanent set. The set was nearly ·01 when the load was increased to 210lbs., and remained upon it ten hours. It was a smoother and better casting than the other specimens.

There did not appear to be any sensible difference in the quality of the iron in these specimens, except that the second specimen was more brittle under the hammer than the other two. They were all fine grained, and yielded easily to the file. They were cast by Messrs. Bramah.

66. I was proceeding with a trial of a piece of the same kind of iron, formed as described in fig. 4, Plate I., when it broke suddenly, at about a foot from the end, at an air

bubble. The bubble was not apparent on the surface, and yet so near it, that a slight stroke of a hammer would have broken into it. Founders should be very careful to avoid defects of this kind; and beams to sustain great weights should always be proved to a deflexion within their range of elasticity before they are used.

Experiments 10, 11, *and* 12.

67. These trials were made on three pieces of uniform breadth and depth, with the supports 3 feet apart, the load being applied in the middle of the length. The depth ·9 inch, and the breadth the same.

Weight acting on the bar.	1st Specimen. Deflex. produced.	2nd Specimen. Deflex. produced.	3rd Specimen. Deflex. produced.
lbs.	in.	in.	in.
40	·041	·042	·041
80	·082	·09	·08
120	·124	·136	·12
160	·165	·18	·16
180	·185	·202	·18
200	·206		·20

The load of 200 lbs. remained twelve hours on the first specimen, and when it was unloaded the quantity of permanent deflexion was barely sensible; and it was loaded and unloaded again with the same result.

The load of 180 lbs. remained three hours on the second specimen; it had not increased the deflexion, but when the load was removed, it was found that the bar had acquired a permanent set of nearly $\frac{1}{100}$th of an inch.

In the third specimen the bar returned perfectly to its natural form when the load was removed, after being upon it three hours.

Of these specimens the third was the most brittle under the hammer, and the hardest to the file; there was not a sensible difference between the other two; both were soft iron. These specimens were cast by Messrs. Bramah.

68. The chief object in view in the experiments No. 2, 6, 7, 8, 9, 10, and 11, was to determine the strain a square inch of cast iron would bear without permanent alteration, and the extension corresponding to that strain. Calling f this strain in pounds, the experiment 2 gives $f = 15{,}300$ lbs. $= 6{\cdot}8303$ tons, as calculated in art. 143; and the others being calculated by the same formula, in experiments 6, 10, and 12, $f = 14{,}814$; in experiments 7, 8, and 9, $f = 15{,}160$; and in experiment 11, $f = 13{,}333$ lbs. The greatest difference amounts to about $\frac{1}{8}$th of the highest value of f; but, in the experiment 2, the load was taken off after remaining only about ten minutes on the bar; in the others it remained for several hours. The former I consider most strictly applicable to practice; and yet it was desirable to show that a force acting a considerable time will produce a permanent set, when the same force could not produce it in a few minutes.

69. In art. 212, it is calculated that the extension produced by the strain of 15,300 lbs. in experiment 2, was $\frac{1}{1204}$ of the length;* and by the same mode of calculation the extension in experiment 6 is found to be $\frac{1}{1143}$, in experiment 10, $\frac{1}{1165}$, in experiment 11, $\frac{1}{1170}$, and in experiment 12, $\frac{1}{1200}$. Also, by the equation, art. 127, the extension in experiment 7 is found to be $\frac{1}{1332}$, in experiment 8, $\frac{1}{1132}$, and in experiment 9, $\frac{1}{1367}$.

The difference between the extension in the 8th and 9th experiments is the most considerable; and the mean between these is $\frac{1}{1239}$, which differs very little from $\frac{1}{1204}$, the number used in the rules.

70. A Table of the chief experiments that have been made

* The extension in experiment 2 has been re-calculated, and found to be the same as here stated, by Professor Leslie, whose mode of calculation is different. See Leslie's Elements of Natural Philosophy, vol. i. p. 240. Edinburgh, 1823.

on the absolute strength of cast-iron bars to resist a cross strain, the bars supported at the end, and loaded in the middle.

No.	Description.	Length between the supports in feet.		Dimensions at the strained point in inches.		Weight in lbs. that broke it.	Calculated weight that would destroy elastic force in lbs.	Ratio of the calculated weight to the breaking weight.
		ft.	in.	brdth.	dpth.			
1	Uniform bar ...	3	0	1	1	756	283	1 : 2·7
2	Ditto	3	0	1	1	735	283	1 : 2·6
3	Ditto	2	6	1	1	1008	340	1 : 2·96
4	Ditto	3	0	1	1	963	283	1 : 3·4
5	Ditto	3	0	1	1	958	283	1 : 3·38
6	Ditto	3	0	1	1	994	283	1 : 3·5
7	Ditto cast from the cupola.	3	0	1	1	864	283	1 : 3·05
8	Parabolic bar cast from the cupola.	3	0	1	1	874	283	1 : 3·08
9	Uniform bar ...	3	0	1	1	897	283	1 : 3·17
10	Ditto	2	8	1	1	1086	318·75	1 : 3·4
11	Ditto	1	4	1	1	2320	637·5	1 : 3·6
12	Ditto	2	8	2	½	2185	637·5	1 : 3·42
13	Ditto	1	4	2	½	4508	1275	1 : 3·53
14	Ditto	2	8	3	⅓	3588	956·25	1 : 3·63
15	Ditto	1	4	3	⅓	6854	1912·5	1 : 3·58
16	Ditto	2	8	4	¼	3979	1275	1 : 3·12
17	Semi-ellipse ...	2	8	4	¼	4000	1275	1 : 3·14
18	Parabolic ...	2	8	4	¼	3860	1275	1 : 3·03
19	Uniform strain in the direction of diagonal	2	8	$\sqrt{2}$	$\sqrt{2}$	851	224·5	1 : 3·79

The two columns on the right hand side are added to show the relation between the load which permanently destroys a part of the elastic force, and that which breaks the piece. It will be seen that the load which would produce permanent alteration, according to the formula as derived from my experiments, is about ⅓rd of that which actually broke the specimens; in the worst kind tried, it is $\frac{1}{2·6}$ of the breaking weight.

In the preceding Table, the experiments 1, 2, and 3, were made by Mr. Reynolds. No. 1 was twice repeated with the same result. No. 2 is a mean of three experiments.* Hence

* Banks on the Power of Machines, p. 30.

the mean ratio will be about 1 : 2·7. The experiments No. 4, 5, 6, 7, and 8, were made by Mr. Banks;* the mean ratio being 1 : 3·3. The rest were made by Mr. George Rennie, and all of the bars of his experiments were cast from the cupola; † the mean ratio being 1 : 3·4.

71. Allowing that the preceding experiments are sufficient to fix with considerable certainty the utmost strain that ought to exist in any structure of cast iron, still there is abundant scope for new experimental research; and that which perhaps may be considered of most importance is the effect produced by combining iron of different qualities.

Through the kindness of Mr. Francis Bramah, I am enabled to begin this inquiry. He has furnished me twelve specimens, of six different kinds of iron; that is, two specimens of each kind. Of these kinds three were run from pig iron from different iron works; one kind was run from old iron, usually termed scrap iron; another kind a mixture of old iron and pig iron in equal parts, and the sixth kind pig iron with an alloy of $\frac{1}{16}$th of copper.

Before I begin to describe the experiments, it will be proper to inform the reader what method I pursued in making them. I knew, from previous trials, that the force which produces a permanent set cannot be determined with that precision which is necessary in comparing iron of different kinds; we can merely observe when it is, and when it is not sensible; and it is most likely that it becomes so by gradations which we cannot trace. It was desirable to ascertain whether a load equivalent to 15,300 lbs. upon a square inch would produce a set or not; and a load of 162 lbs., on the middle of a bar of the size of the specimens, causes that degree of strain: hence, in specimens of the same size, the flexure by this load gives the comparative power of the

* Banks on the Power of Machines, p. 90.

† Philosophical Transactions for 1818, Part I., or Philosophical Magazine, vol. liii. p. 173.

different kinds, particularly when compared with the quantity of set produced by this or some additional load. But in specimens of different sizes, the comparison is most easily made by calculating the modulus of resilience, or resistance to impulsion, which gives the toughness or relative power of the material to resist a blow. Yet, even then it should be tried what strain will produce permanent alteration, or that which causes fracture, otherwise the comparative goodness of the iron will not be known: I have tried both in each of the varieties of iron.

For all purposes where strength is required, that iron is to be esteemed the best which will bear the greatest degree of flexure without set, and the greatest load. The worst and most brittle pieces of iron have the greatest degree of stiffness; consequently the highest modulus of elasticity; for even the most flexible kind of iron is sufficiently stiff.*

In the iron which was taken as a good medium to calculate from (see experiment, art. 56), we found

The force that it would bear without permanent alteration	15,300 lbs.
The extension in parts of the length extended . . .	$\frac{1}{1204}$
The modulus of elasticity for a base 1 inch square . .	18,400,000 lbs.
The modulus of resilience	12·7

These numbers being compared with the results of the experiments now to be described will afford the means of judging both of the qualities of the iron experimented upon, and of the fairness of the mean data I have employed.

OLD PARK IRON.

72. Two specimens run from this kind of pig iron, each

* I have here followed the principles of comparing materials which were first given in my "Elementary Principles of Carpentry," art. 368—373. The toughness is measured by the same data as in that work, only here a general number of comparison is used instead of making one material a standard of comparison. The term *modulus of resilience*, I have ventured to apply to the number which represents the power of a material to resist an impulsive force; and when I say that one material is tougher than another, it is in consequence of finding this modulus higher for that which is described as the toughest: see arts. 299 to 304, further on.

3 feet in length, and smooth, clean, and regular castings, were first put in trial. The section of the bars rectangular; depth 0·65 inch; breadth 1·3 inches; the supports 2·9 feet apart; and the load suspended from the middle.

Weight applied.	Effect on 1st bar.	Effect on 2nd bar.
lbs.	in.	in.
60	bent 0·1	bent 0·1
120	„ 0·2	„ 0·203
162	„ 0·265	„ 0·275
182	„ 0·305 small set.	„ 0·31 { set barely perceptible.
190	„ 0·32 set ·005	„ 0·33 set ·005

The iron was slightly malleable in a cold state; yielded easily to the file. The fracture dark gray with a little metallic lustre; fine grained and compact.

We may consider 162 lbs. as the greatest load it would bear without impairing its elastic force; and 0·27 is the mean between the flexures produced by this weight; therefore, calculating on these data, we have

The strain it would bear on a square inch without permanent alteration	15,390 lbs.
Extension in length by this strain	$\frac{1}{1153}$
Modulus of elasticity for a base of an inch square . .	17,744,000 lbs.
Modulus of resilience	13·4
Specific gravity	7·092

The absolute strength to resist fracture was tried by fixing the bar at one end, the load being applied by fixing a scale at the other end, and adding weights till the bar broke. The second bar tried in this manner broke with 184 lbs., the leverage 2 feet; fracture close to the fixed end, metal sound and perfect at the place of the fracture.*

Hence, calculating by equation, art. 110, the absolute cohesion of a square inch is 48,200 lbs.,† or 3·15 times

* These are circumstances which must have place, otherwise the experiment does not give a fair measure of the strength.

† This erroneous conclusion as to the great strength of cast iron, into which

15,300 lbs., the strain which has been found incapable of causing permanent set.

Hence I infer, that this iron is superior in toughness, and less stiff than the mean quality.

ADELPHI IRON.

73. The specimens of this iron were clean good castings of the same dimensions as those of Old Park iron. That is, depth 0·65 inch; breadth 1·3 inches; distance between the supports 2·9 feet.

Weight applied.	Effect on 1st bar.	Effect on 2nd bar.
lbs.	in.	in.
60	bent 0·1	bent 0·1
120	„ 0·2	„ 0·205
162	„ 0·26 no set.	„ 0·27 no set.
182	„ 0·3 set ·0075	„ 0·305 set ·005

Comparing this with the preceding experiments on Old Park iron, it is stiffer, and sooner acquires a permanent set. It is also somewhat harder to the file, and more brittle under the hammer. The colour of the fracture was a lighter gray, with less metallic lustre.

Its elasticity is not affected by the load of 162 lbs.; therefore

It will bear upon a square inch without permanent alteration	15,390 lbs.
And the mean of the two experiments gives the extension .	$\frac{1}{1174}$
Modulus of elasticity for a base of 1 square inch . . .	18,067,000 lbs.
Modulus of resilience	13·1
Specific gravity	7·07

The second bar, fixed at one end with a leverage of 2 feet, broke with 173 lbs.; the fracture close to the fixed end, and the place of fracture sound and perfect.

Navier, as well as Tredgold, has fallen, arises principally from a supposition that the neutral line remains stationary during the flexure of the body. See "Additions," or Notes to Arts. 68 and 143.—EDITOR.

According to this experiment, the absolute cohesion is 45,300 lbs. for a square inch, or 2·96 times 15,300 lbs.

A comparison of these trials shows that the difference between Adelphi and Old Park iron is not much, but that the Old Park is superior, particularly in absolute strength; for it required 184 lbs. to break the one, and only 173 lbs. to break the other.

ALFRETON IRON.

74. There was not a sensible difference between the size of these bars and the others. The depth 0·65 inch; breadth 1·3 inches; distance between the supports 2·9 feet.

Weight applied.	Effect on 1st bar.	Effect on 2nd bar.
lbs.	in.	in.
60	bent 0·1	bent 0·1
120	" 0·2	" 0·195
162	" 0·27 no set.	" 0·28 no set.
183	" 0·31 small set.	" 0·325 small set.

This iron differs very little from Old Park, a little more flexible, but very little. It seemed, if anything, somewhat harder to the file, but of a less degree of malleability; for instead of extending, it crumbled under the hammer. Fracture scarcely differing from that of Adelphi iron.

These bars bore 162 lbs. without set, and the mean deflexion was ·275. Hence,

The iron would bear upon a square inch without permanent alteration	15,390 lbs.
Extension in length by this strain	$\frac{1}{1131}$
Modulus of elasticity for a base of 1 inch square	17,406,000 lbs.
Modulus of resilience	13·6
Specific gravity	7·04

The second bar, fixed at one end, broke with 153 lbs., the leverage being two feet, the fracture close to the fixed end, and the metal sound and perfect at the place of fracture.

The absolute cohesion, according to this trial, is 40,000 lbs. for a square inch, or 2·63 times the force of 15,300 lbs.

This is a soft species of iron, and may answer extremely well alone, for castings where strength is not required; but it is the weakest iron I have tried, and would most likely be much improved by mixture.

SCRAP IRON.

75. These bars were run from old iron. They were uneven on the surface, indicating that irregularity of shrinkage which has been noticed in the Introduction (page 8). The depth of the bars 0·65 inch; the breadth 1·3 inches; the distance between the supports 2·9 feet.

Weight applied.	Effect on 1st bar.	Effect on 2nd bar.
lbs.	in.	in.
60	bent 0·09	bent 0·09
120	„ 0·18	„ 0·18
162	„ 0·25 no set.	„ 0·255 no set.
180	„ 0·28 no set.	„ 0·285 no set.
190	„ 0·3 small set.	„ 0·3 } set not certain.
210	„ 0·34 set ·005	„ 0·34 set ·004

This iron was very hard to the file, and very brittle, fragments flying off when hammered on the edge, instead of indenting as the preceding specimens.

The fracture dead or dull light gray; no metallic lustre; not very uniform; fine grained.

These bars showed no sign of permanent set with a load of 180 lbs.; but, to whatever cause this greater range of elastic power may be owing, it certainly would be unsafe to calculate upon it in practice. I shall therefore consider the load of 162 lbs., and a flexure of 0·25 inch, the data to calculate from; accordingly, the

Force of a square inch without permanent alteration . .	15,390 lbs.
Extension in length by this strain	$\frac{1}{1243}$
Modulus of elasticity for a base of 1 square inch . . .	19,180,000 lbs.
Modulus of resilience	12·4
Specific gravity	7·219

Fixed at one end, with a leverage of 2 feet, the second bar broke with 168 lbs., with one fracture at the fixed end; but the bar flew into several pieces.

This gives the absolute cohesion of a square inch 44,000 lbs., or nearly 2·9 times that strain which I consider to be the greatest cast iron should have to sustain.

From these elements we may conclude, that a casting of scrap iron will be $\frac{1}{12}$ stiffer than one from Old Park iron; that it has $\frac{1}{12}$ less power to resist a body in motion, and that it is less strong in the ratio of 168 to 184.

MIXTURE OF OLD PARK AND GOOD OLD IRON IN EQUAL PARTS.

76. The castings run from this mixture were even and clean; such as indicate a perfect union of the materials. The depth 0·65 inch; the breadth 1·3 inches; and the distance between the supports 2·9 feet.

Weight applied.	Effect on 1st bar.	Effect on 2nd bar.
lbs.	in.	in.
72	bent 0·1	bent 0·1
140	" 0·2	" 0·2
162	" 0·24 no set.	" 0·245 no set.
182	" 0·27 no set.	" 0·28 no set.
202	" 0·3 small set.	" 0·31 small set.
220		" 0·34 set ·005
300		" 0·475 set ·03

The iron was rather hard to the file; it indented with the hammer, but was rather short and crumbling.

Fracture a lighter gray, and more dull than Old Park iron; very compact, even, and fine grained.

The bars did not set with a load of 182 lbs., therefore the load of 162 lbs. is sufficiently within the limit; the flexure with that load is ·245, consequently we may state its properties thus:

Force on a square inch that does not produce permanent alteration	15,390 lbs.
Extension under this strain	$\frac{1}{1268}$
Modulus of elasticity for a base of 1 inch square . . .	19,514,000 lbs.
Modulus of resilience	12·1
Specific gravity	7·104

When the second bar was fixed at one end, it broke with 174 lbs., acting with a leverage of 2 feet; fracture close to the fixed end. Therefore the absolute cohesion of a square inch is 45,600 lbs., or very nearly three times the strain of 15,300 lbs.

In this mixture there is clearly too great a proportion of old iron; it is rather inferior to the quality of our mean specimen (art. 56). About one of old iron to two of the Old Park pig iron would be a better proportion. It is worthy of remark, that the absolute strength is nearly the mean of the two kinds which form the mixture, and so is the specific gravity.

ALLOY OF PIG IRON SIXTEEN PARTS, COPPER ONE PART.

77. It has been said that iron is much improved by a small proportion of copper; it was desirable, therefore, to ascertain its effect, and the advantage, if any, of employing it. The breadth of the specimens 1·25 inches; the depth ·675 inch; the distance between the supports 2·9 feet. The load which ought not to produce permanent alteration, about 167 lbs.

Weight applied.	Effect on 1st bar.	Effect on 2nd bar.
lbs.	in.	in.
60	bent 0·1	bent 0·1
122	„ 0·2	„ 0·2
167	„ 0·275 no set.	„ 0·265 no set.
180	„ 0·3 no set.	„ 0·29 no set.
203	„ 0·34 set ·003	„ 0·325 set ·002
300	„ 0·5	

These bars yielded freely to the file, but were short and crumbling under the hammer. I expected to have found them more ductile. The fracture dark gray, fine grained, and more compact than Old Park iron; with less metallic lustre.

The load of 167 lbs. did not produce any degree of set, the mean flexure by this load 0·27; and assuming this to be as great a load as it should bear in practice, we have,

Force on a square inch that does not produce permanent alteration	15,300 lbs.
Extension under this strain	$\frac{1}{1106}$
Modulus of elasticity for a base of 1 inch square . . .	16,921,000 lbs.
Modulus of resilience	13·8
Specific gravity	7·13

To try the absolute strength, the second bar was fixed at one end, and the scale suspended from the other end; weights were then added till the bar broke: the fracture took place close to the fixed end, and it required 194 lbs. to break the bar.

According to this experiment, the cohesive force of a square inch is 52,000 lbs., or 3·4 times the strain that will not give permanent alteration.

It appears that copper increases both the strength and extensibility of iron.

EXPERIMENTS ON THE RESISTANCE TO TENSION.

78. According to an experiment made by Muschenbroëk, a parallelopipedon, of which the side was ·17 of a Rhinland inch, broke with 1930 lbs.; * and since the Rhinland foot is 1·03 English feet, and the pound contains 7038 grains, this experiment gives 63,286 lbs. for the weight that would tear asunder a square inch, when reduced to English weights and measures.

79. An experiment made by Capt. S. Brown is thus described: "A bar of cast iron, Welsh pig, 1¼ inch square, 3 feet 6 inches long, required a strain of 11 tons 7 cwt.

* Muschenbroëk's Introd. ad Phil. Nat. vol. i. p. 417. 1762.

F 2

(25,424 lbs.) to tear it asunder: broke exactly transverse, without being reduced in any part; quite cold when broken; particles fine, dark bluish gray colour."*

Capt. Brown's machine for trying such experiments being constructed on the principle of a weigh-bridge, Mr. Barlow is of opinion it may show less than its real force; it also may be remarked, that to obtain the real force of cohesion, the resultant of the straining force should coincide exactly with the axis of the piece, for so small a deviation in this respect as $\frac{1}{6}$th of the breadth would reduce the strength one half.

From this experiment it appears that 16,265 lbs. will tear asunder a square inch of cast iron.

80. In some experiments made by Mr. G. Rennie, it is obvious, from the description of the apparatus, that the strain on the section of fracture would not be equal; and, therefore, that the straining force would be less than the cohesion of the section. The specimens were 6 inches long, and $\frac{1}{4}$th of an inch square at the section of fracture. A bar cast horizontally required a force of 1166 lbs. to tear it asunder. A bar cast vertically required a force of 1218 lbs. to tear it asunder.†

	Per square inch.
In the horizontal casting the force was equal to	18,656 lbs.
And in vertical casting	19,488 ,,

EXPERIMENTS ON THE RESISTANCE TO COMPRESSION IN SHORT LENGTHS.

81. The power of cast iron to resist compression was formerly much over-rated. Mr. Wilson estimated the power necessary to crush a cubic inch of cast iron at 1000 tons = 2,240,000 lbs.; and in describing an experiment by Mr. William Reynolds, of Ketley, in Shropshire, a cube of $\frac{1}{4}$th of an inch of cast iron, of the quality called gun-metal, was said

* Essay on the Strength of Timber, &c., by Mr. Barlow.

† Philosophical Transactions for 1818, Part I., or Philosophical Magazine, vol. liii. p. 167.

to require 448,000 lbs. to crush it.* But Mr. Telford, for whom the experiments were made, was so kind as to communicate the correct results of the experiments made by Mr. Reynolds; and it appears that

	Per square inch.
A cube of ¼th of an inch of soft gray metal was crushed by 80 cwt.	= 143,360 lbs.
Ditto of the kind of cast iron called gun-metal was crushed by 200 cwt.	= 350,400 lbs.

82. Such was the state of our knowledge on this important subject, when Mr. G. Rennie communicated a valuable series of experiments to the Royal Society, which were published in the first part of their Transactions for 1818.

Mr. Rennie's Experiments on cubes from the middle of a large block; specific gravity 7·033:

	in.		lbs.		Force per sq. in. in lbs.
Side of cube	⅛	was crushed by	1,454, highest result	=	93,056
Ditto	⅛	ditto	1,416, lowest ditto	=	74,624
Ditto	¼	ditto	10,561, highest ditto	=	168,976
Ditto	¼	ditto	9,020, lowest ditto	=	144,320

On cubes from horizontal castings, specific gravity 7·113

	in.		lbs.		lbs. per. sq. in.
Side of cube	¼	was crushed by	10,720, highest result	=	171,520
Ditto	¼	ditto	8,699, lowest ditto	=	139,184

On cubes from vertical castings, specific gravity† 7·074

	in.		lbs.		lbs. per. sq. in.
Side of cube	¼	was crushed by	12,665, highest result	=	202,640
Ditto	¼	ditto	9,844, lowest ditto	=	157,540

On pieces of different lengths.

					in.		lbs.		lbs. per sq. in.
Area	⅛	×	⅛	length	¾	was crushed by	1,743	=	111,552
Ditto	⅛	×	⅛	„	1	do.	1,439	=	92,096
Ditto	¼	×	¼	„	½	do.	9,374	=	149,984
Ditto	¼	×	¼	„	1	do.	6,321	=	101,136

These experiments were on too small a scale to allow of

* Edin. Encyclo. art. Bridge, p. 544; or Nicholson's Journal, vol. xxxv. p. 4. 1813.

† It is singular that the specific gravity of the vertical castings should be less than that of the horizontal ones.

that precision in adjustment which theory shows to be essential in such experiments; therefore there still remains much to be done by future experimentalists. It does not appear, within the limits of these experiments, that an increase of length had any sensible effect on the result.

I have selected the highest and lowest results, and such of the single trials that were made under the greatest difference of length; in all Mr. Rennie made thirty-nine trials on the resistance of cast iron to compression.*

EXPERIMENTS ON THE RESISTANCE TO COMPRESSION OF PIECES OF CONSIDERABLE LENGTH.

83. The only experiments of this kind that I know of were made by Mr. Reynolds, and are described as follows in Mr. Banks's work on the 'Power of Machines,' p. 89.

"Experiments on the strength of cast iron, tried at Ketley, in March, 1795. The different bars were all cast at one time out of the same air furnace, and the iron was very soft, so as to cut or file easily.

"Exp. 1. Two bars of iron, 1 inch square, and exactly 3 feet long, were placed upon a horizontal bar, so as to meet in a cap at the top, from which was suspended a scale; these bars made each an angle of 45° with the base plate, and of consequence formed an angle of 90° at the top: from this cap was suspended a weight of 7 tons (15,680 lbs.), which was left for sixteen hours, when the bars were a little bent, and but very little.

"Exp. 2. Two more bars of the same length and thickness were placed in a similar manner, making an angle of 22½° with the base plate; these bore 4 tons (8960 lbs.) upon the scale: a little more broke one of them which was observed to be a little crooked when first put up."

* Philosophical Transactions for 1818, Part I., or Philosophical Magazine, vol. liii. pp. 164, 165.

84. By the principles of statics,*

2 sin. 45° : Rad. : : 15,680 lbs. : 11,087 lbs.

equal the pressure in the direction of either bar in the first experiment. And,

2 sin. 22½° : Rad. : : 8960 lbs. : 11,709 lbs.

the pressure in the direction of either bar in the second experiment.

If we consider the direction of the force to have been exactly in the axis in these trials, then, according to the equation, art. 288, the greatest strain in the direction of one of these bars should not have exceeded 5840 lbs.; but if the direction of the pressure was at the distance of half the depth from the axis, which it is very probable it would be, the greatest strain in actual construction should not have exceeded 2720 lbs. See art. 287.

EXPERIMENTS ON THE RESISTANCE TO TWISTING.

85. Table of the principal experiments of the strength of cast iron to resist a twisting strain.

No.	Description.	Leverage.	Length.	Side or diameter in inches.	Weight in lbs. that broke the piece.	Calculated resistance without destroying the elastic force.	Ratio of the calculated resistance to the breaking weight
1	Bar placed vertically, fast at one end and twisted by a wheel at the other ...	ft. in. 1 0	not given	1 × 1	631	150	1 : 4·2
2	Cylinder fixed at one end, twisted by a lever at the other	14 2	inches. 2¾	inches. 2	250	73·7	1 : 3·39
3	Ditto	14 2	3¼	2¼	384	111	1 : 3·46
4	Ditto	14 2	3	2½	408	140	1 : 2·9
5	Ditto	14 2	3	2¾	700	184	1 : 3·8
6	Ditto	14 2	4	3¼	1170	309	1 : 3·78
7	Ditto	14 2	5	3½	1240	402	1 : 3·08
8	Ditto	14 2	5	3¾	1662	481	1 : 3·45
9	Ditto	14 2	5	4	1938	580	1 : 3·34
10	Ditto	14 2	6	4½	2158	713	1 : 3·02

* Gregory's Mechanics, vol. i. art. 43.

The experiment No. 1 was made by Mr. Banks.* The others were made by Mr. Dunlop, of Glasgow. Nos. 4 and 7 were faulty specimens.† Some experiments on a very small scale were made by Mr. George Rennie, but they are not inserted here, because they were not sufficiently described to admit of comparison.‡

I am indebted to Messrs. Bramah for a description of some new and interesting experiments on torsion, which they had made in order to ascertain what degree of confidence they might place in the theoretical and experimental deductions of writers on this subject. They were also desirous of knowing the effect of a small portion of copper on the quality of cast iron.

I have given a tabular form to the results of these experiments, in order that they may be more easily compared; and I have added two columns to the Table, to show how these experiments agree with the rules of this work.

The bars were firmly fixed at one end, in a horizontal position, and to the other end the straining force was applied, acting with a leverage of 3 feet. To prevent the effect of lateral stress, the bar rested loosely upon a support at the end to which the straining force was applied.

* "Power of Machines."

† Dr. Thomson's Annals of Philosophy, vol. xiii. p. 200-203.

‡ Philosophical Magazine, vol. liii. p. 168.

Number of experiment.	Description of iron.	Length of bar.	Side of bar.	Weight applied acting with three feet leverage.	Effect of weight or angle of torsion.	Calculated angle of torsion.	Ratio of the force which would not produce set to the breaking weight.
		feet.	inches.	lbs.	degrees.	degrees.	
1	Square bar of an alloy of 16 parts iron to one of copper	1	$1\frac{1}{16}$	166	$7\frac{1}{2}$	4·25	
				215	broke	...	1 : 3·6
2	Square bar of the same kind as No. 1	2	$1\frac{1}{16}$	111	$6\frac{1}{2}$	5·7	
				213	17	10·9	
				213	broke	...	1 : 3·5
3	Square bar a mixture of equal parts of Adelphi, Alfreton, and old iron...	1	$1\frac{1}{16}$	217	14	5·6	
				330	broke	...	1 : 5·5
4	Bar same kind as No. 3	1	$1\frac{1}{16}$	166	$7\frac{1}{2}$	4·25	
				310	broke	...	1 : 5·16
5	Bar same kind as No. 3	2	$1\frac{1}{16}$	164	$12\frac{1}{2}$	8·4	
				213	18	10·9	
				280	28	14·3	
					Broke by slipping one of the weights.		
6	Square bar of cast iron	1	1	237	broke	...	1 : 4·72
7	Square bar same kind as No. 6...	2	1	218	broke	...	1 : 4·35

The comparison between our rule (Equation iv. art. 265), and the force that broke these specimens, which is given in the last column, is very satisfactory, and very nearly agrees with former experiments on this strain as shown in the first Table of this article.

The observed angle of torsion is very irregular, and in all these experiments it greatly exceeds the angle calculated by Equation xiv. art. 272. But it will be remarked, that the angle was measured after the strain was far beyond that degree where it is known that flexure increases more rapidly than the load; and no allowance was made for the compression at the fixed points. M. Duleau, in his experiments on wrought iron, (see Sect. VI. art. 94 of this Essay,) allowed for the latter source of error by taking, as the measure of torsion, the angle through which the bar returned when the weight was taken off; * and the formula applied to his

* Essai sur la Résistance du Fer Forgé, p. 49.

experiment gives an error in excess,—here it is in defect: I shall, therefore, not endeavour to make the rules agree with either set of experiments, because I know that the flexure will be too great in Messrs. Bramah's experiment; and assuming this to be so, the rules will be nearly true; whereas, if M. Duleau's turn out to be most correct, it will only cause shafts to be made a small degree stronger than necessary.

EXPERIMENTS ON THE EFFECT OF IMPULSIVE FORCE.

86. The height from which a weight might fall upon a piece of cast iron without destroying its elastic force was calculated by Equation v. art. 306, for the specimens of ·9 inch square, used in the preceding experiments (art. 67). Repeated trials with that height of fall were made without producing a sensible effect. I then let the weight fall from double the calculated height, and every repetition of the blow added about $\frac{1}{100}$th of an inch to the curvature of the bar. I could not measure the effect of each trial very correctly, but a few trials rendered the bar so much curved as to be easily seen. I hope, at some future time, to be able to resume these experiments with an apparatus for measuring correctly the degree of permanent set.* See art. 313—350, where practical rules will be found.

TO DISTINGUISH THE PROPERTIES OF CAST IRON BY THE FRACTURE.

87. I shall close this section with a few remarks on the aspect of cast iron recently fractured, with a view to distinguish its properties.

There are two characters by which some judgment may be formed; these are the colour and the lustre of the fractured surface.

* This hope of the Author had not been realised when Practical Science was unfortunately deprived by his death, of one of its most able supporters.—EDITOR.

The colour of cast iron is various shades of gray; sometimes approaching to dull white, sometimes dark iron gray with specks of black gray.

The lustre of cast iron differs in kind and in degree. It is sometimes metallic, for example, like minute particles of fresh cut lead distributed over the fracture; and its degree, in this case, depends on the number and size of the bright parts. But in some kinds, the lustre seems to be given by facets of crystals disposed in rays. I will call this lustre, crystalline.

In very tough iron the colour of the fracture is uniform dark iron gray, the texture fibrous, with an abundance of metallic lustre. If the colour be the same, but with less lustre, the iron will be soft but more crumbling, and break with less force. If the surface be without lustre, and the colour dark and mottled, the iron will be found the weakest of the soft kinds of iron.

Again, if the colour be of a lighter gray with abundance of metallic lustre, the iron will be hard and tenacious; such iron is always very stiff. But if there be little metallic lustre with a light colour, the iron will be hard and brittle; it is very much so when the fracture is dull white; but in the extreme degrees of hardness, the surface of the fracture is grayish white and radiated with a crystalline lustre.

There may be some exceptions to these maxims, but I hope they will nevertheless be of great use to those engaged in a business which is every day becoming more important.

SECTION VI.

EXPERIMENTS ON MALLEABLE IRON AND OTHER METALS.

EXPERIMENTS ON THE RESISTANCE OF MALLEABLE IRON TO FLEXURE.

88. There have been a greater number of experiments made on malleable iron than on any other metal; but those on the lateral strength are chiefly by foreign experimentalists. From those of Duleau I shall select a few for the purpose of comparison; but in the first place I propose to describe some of my own trials.

The following experiments were made on bars of English and of Swedish iron; the bars were supported at the ends, and the weight applied in the middle between the supports; the length of each bar was exactly 6 feet, and the distance between the supports $66\frac{1}{2}$ inches.

English Iron.

Kind of bar and dimensions.	Weight of 6 feet in length.	Deflexion with			Weight of modulus of elasticity for a base of 1 sq. inch.
		58 lbs.	114 lbs.	170 lbs.	
	lbs.	inch.	inch.	inch.	lbs.
Bar $1\frac{1}{4}$ inch square . .	33	·0625	·1	·1875	27,240,000
Bar $1\frac{1}{8}$ " . .	25	·125	·25	·375	20,830,000
Bar 1 " . .	20	·15	·32	·5	24,990,000
Round bar $1\frac{1}{4}$ inch diamtr.	24	·125	·25	·375	23,154,000
Round bar 1 " .	17	·25	·5	·8	26,500,000
Mean weight of modulus 24,542,800 lbs.					

Swedish Iron.

Kind of bar and dimensions.	Weight of 6 feet in length.	Deflexion with 58 lbs.	Deflexion with 114 lbs.	Deflexion with 170 lbs.	Weight of modulus of elasticity for a base of 1 sq. inch.
	lbs.	inch.	inch.	inch.	lbs.
Bar 1·2 inch square . .	32	·0625	·125	·19	32,000,000
Bar 1⅛ " . .	27	·08	·161	·25	31,245,000
Bar 1 " . .	33	·125	·25	·375	33,328,000
Mean weight of modulus 32,191,000 lbs.					

The bars of Swedish iron varied in dimensions considerably; the dimension in the first column was taken at the point of greatest strain in each bar. The apparently superior stiffness of the Swedish iron is partly to be attributed to this cause; but it is in a greater degree owing to the mode of manufacture which gives more density as well as elastic force to the iron. If the English iron had been formed under the hammer, in the same manner, it would have been perhaps equally dense and strong, and as fit for the nicer purposes of smiths' work as the Swedish. All these specimens were tried in the same state as the bars are sent from the iron works; the trials were made in July, 1814.

89. The objects of my next experiments on malleable iron were, to determine the force that would produce permanent alteration; the effect of heating iron so as to give it uniform density; and the effect of temperature on its cohesive power. For this purpose, Mr. Barrow, of East Street, selected for me a bar of what he esteemed good iron, bearing the mark Penydarra. A piece 38 inches long, weighing 10·4lbs., was cut off this bar: its section did not sensibly differ from 1 inch square. With the supports 3 feet apart, and the weight applied in the middle, the following results were obtained.

Weight.	Deflexion in the middle with the bar as obtained from the iron works.	Deflexion in the middle after the bar had been uniformly heated and slowly cooled.
lbs.	inch.	inch.
126	·05	·059
252	·10	·117
310	·12	·145
330	·13	·154

In both states it bore the weight of 330 lbs. without sensible effect, though it was let down upon it, and relieved several times; but in either state an addition of 20 lbs. rendered the set perceptible; in the softened bar it appeared to be sensible when only 10 lbs. had been added.

Hence, by art. 110 we have the force that could be resisted; without permanent alteration 17,820 lbs. per square inch: by art. 121 the extension in the softened state, is $\frac{1}{1400}$ of its length; and by art. 105 the modulus of elasticity is 24,920,000 lbs. for a base of an inch square. The modulus before being softened is 29,500,000 lbs.

90. To try the effect of heat in decreasing the cohesion of malleable iron, I heated it to 212° of Fahrenheit, having previously got the machine ready, so that a weight of 300 lbs. could be instantly let down upon the bar as soon as it was put in, and the index adjusted to one of the divisions of the scale. These operations having been effected in a close and warm room, with as little loss of heat as possible, the window was thrown open, and the effect of cooling observed. The deflexion decreased as the bar cooled, but it was allowed to remain nearly two hours, in order to be perfectly cooled down to the temperature of the room, or 60°. Each division on the scale of the index is $\frac{1}{100}$th of an inch, and as nearly as I could determine, with the assistance of a magnifier, the deflexion had decreased three-fourths of one of the divisions; and it returned through fourteen divisions when the load was removed; therefore we may conclude, that by an elevation of temperature equal to 212—60=152 degrees, iron loses about a 20th part of its cohesive force, or a 3040th part for each degree.

M. DULEAU'S EXPERIMENTS.*

91. The most part of the experiments of M. Duleau were

* Taken from his Essai Théorique et Expérimental sur la Résistance du Fer Forgé. 4to. Paris, 1820.

made with malleable iron of Perigord; some of the specimens were hammered to make them regular, others were put in trial in the state they are sent from the iron works; the former are distinguished from the latter by an *h* added to the number of the experiment; and these numbers are the same as in M. Duleau's work. The experiments are divided into two classes: in the first the elasticity was observed to be impaired by the action of the load; in the second it was not. The specimens were supported at the ends, and the load suspended from the middle of the length. The dimensions of the pieces are in the original measures, as well as the weights: but the deductions are in our own measures and weights. All the experiments I have selected are on Perigord iron.

Number of experiments.		Distance between the supports.	Breadth.	Depth.	Depression in the middle.	Weight producing it.	Extension in parts of length.
		Millim.	Millim.	Millim.	Millim.	Kilogramm.	
1st class	15	2000·	45·	12·	54·	45	·000972
	17^{h}	2000·	40·	11·5	52·5	25	·000906
	36^{h}	3000·	60·	20·	33·	50	·000441
2nd class	21^{h}	2000·	11·5	40·	15·03	90	·000902
	22	3000·	77·	14·	72·	50	·000672
	29^{h}	3000·	15·	25·	70·	50	·001167

The last column shows the extension of an unit of length by the strain as calculated by M. Duleau; my formula, art. 121, gives the same results. The extension that malleable iron will bear without permanent alteration is $\frac{1}{1400} = \cdot 000714$, according to my experiment; but in M. Duleau's experiment, No. 36^{h}, the extension of ·000441 produced a permanent set, while in No. 29^{h} the extension was ·001167 without producing a set: this is a considerable irregularity, but such as may be expected in experiments on such long heavy specimens of small depth. In all such experiments, the effect of the weight of the piece should be observed. It is also essential that the points of support should be perfectly solid and firm, or that the flexure should be measured from a point, of which the position is invariable in respect to the points of support.

The mean weight of the modulus of elasticity, as determined by the above experiments, is, 28,000,000 lbs. for a base of 1 inch square. Experiment No. 22 gives the highest, being 31,864,000 lbs.; and No. 17 the lowest, being 22,974,000 lbs.; therefore it appears that the elastic force of Perigord iron is not greatly different from English iron.

M. Duleau concludes that a bar of malleable iron may be safely strained till the extension at the point of greatest strain is equal to $\frac{1}{3333}$ of its original length without losing its elasticity; and that the load upon a square inch which produces this extension is 8540 lbs. In many of his own experiments the extension was three times this without permanent loss of elasticity.

It has been my object to fix the limit which will produce permanent alteration of elasticity in a good material; to say, that beyond this strain you must not go, but approach it as nearly as your own judgment shall direct, when you are certain that you have assigned the greatest possible load it will be exposed to. Where a great strain is to be sustained, a good material is most suitable and most economical; to a defective material no rules whatever will apply; for who can measure the effect of a flaw in malleable iron, an air bubble in cast iron, a vent in a stone, or of knots and rottenness in timber? But the presence of most of these defects can be ascertained by inspection of the material itself; and since the greatest strain is at the surface of a beam or bar, the defects which impair the strength in the greatest degree are always most apparent.

Experiments on the flexure of malleable iron have also been made by Rondelet,* Aubry, and Navier,† which accord with the theoretical principles developed in this Essay.

* Traité de l'Art de Bâtir, tome iv. p. 509 and 514. 4to. 1814.

† Gauthey's Construction des Ponts, tome ii. p. 151. 4to. 1813.

EXPERIMENTS ON THE RESISTANCE TO TENSION.

92. The experiments on the absolute resistance of malleable iron to tension are very numerous: in many experiments it has been found above 80,000lbs., per square inch, and in very few under 50,000lbs., indeed in none where the iron was not defective. About 60,000lbs. seems to be the average force of good iron: and according to this estimate, the force that would produce permanent alteration is to that which would pull a bar asunder as 17,800 : 60,000, or nearly as 1 : 3·37. Hence we see, that on whatever principle it was that Emerson* concluded a material should not be put to bear more than a third or a fourth of the weight that would break it, the maxim is agreeable with the laws of resistance.

Experiments on the absolute strength of malleable iron have been made by Muschenbroëk,† Buffon,‡ Emerson,§ Perronet,‖ Soufflot,¶ Sickingen,** Rondelet,†† Telford,‡‡ Brown,§§ and Rennie.‖‖ Those by Messrs. Telford and Brown were made on the largest scale; and are minutely described in Professor Barlow's Essay, to which I must refer the reader.

EXPERIMENTS ON THE RESISTANCE TO COMPRESSION.

93. Very few experiments have been made on this species of resistance, and from some circumstances in such experiments requiring attention which the authors of them do not appear to have been aware of, we can make no use of

* Mechanics, 4to edit. p. 116. 1758.
† Introd. ad Phil. Nat. i. p. 426. 4to. 1762.
‡ Gauthey's Construction des Ponts, ii. pp. 153, 154.
§ Mechanics, p. 116. 4to edit. 1758.
‖ Gauthey's Construction des Ponts, ii. pp. 153, 154.
¶ Rondelet's L'Art de Bâtir, iv. pp. 499, 500. 4to. 1814.
** Annales de Chimie, xxv. p. 9.
†† Rondelet's L'Art de Bâtir, iv. pp. 499, 500. 4to. 1814.
‡‡ Barlow's Essay on Strength of Timber, &c. §§ Ibid.
‖‖ Philosophical Magazine, liii. p. 167. 1819.

them in illustrating our theoretical principles, unless it be to show that when we consider the direction of the force to nearly coincide with one of the surfaces of the bar, we shall always be calculating on safe data; and from the nature of practical cases in general, we can scarcely think of employing a less excess of force than is given by this rule.

On pieces of considerable length experiments have been made by Navier, Rondelet, and Duleau; and the force necessary to crush short specimens has also been ascertained by Rondelet.

Rondelet employed cubical specimens, the sides of the cubes varying from 6 to $10\frac{1}{2}$ and 12 lines; and cylinders of 6, 8, and 12 lines in diameter, the height being the same as the diameter in each cylinder. The mean resistance of the cubes was equivalent to 512 livres on a square line; the mean resistance of the cylinders 515 livres per square line: 512 livres on a square line is 70,000lbs. on a square inch in our weights and measures. The force necessary to crush the specimens was in proportion to the area; when the area was increased four times, this ratio did not differ from the result of the experiment so much as a fiftieth part.*

He observed in experiments on bars of different lengths, that when the height exceeded three times the diameter, the iron yielded by bending in the manner of a long column. Rondelet's experiments on longer specimens are not sufficiently detailed.†

Navier's experiments were made on long bars, and show the force that broke them; whether the flexure was sudden or gradual is not stated.‡

* There does not appear to be an abrupt change in the crushing of wrought iron to enable an experimenter to draw any very definite conclusions of this kind. According to my observations, wrought iron becomes slightly flattened or shortened with from 9 to 10 tons per square inch; with double that weight it is permanently reduced in length about $\frac{1}{32}$, and with three times that weight about $\frac{1}{13}$th of its length. (Philosophical Transactions, Part II., 1840, p. 422.)—EDITOR.

† Traité de l'Art de Bâtir, iv. pp. 521, 522.

‡ Gauthey's Construction des Ponts, ii. p. 152.

A bar of any material, in which the stress is very accurately adjusted in the direction of the axis, will bear a considerable load without apparent flexure, but the load is in unstable equilibrium, so much so indeed, that in a bar where the least dimension of the section is small in respect to the length, the slightest lateral force would cause the bar to bend suddenly and break under the load. In such a case it is not so much owing to the magnitude of the force that fracture is produced, as the momentum it acquires before the bar attains that degree of flexure which is necessary to oppose it. The reader will find this view of the subject to be agreeable to experience, particularly in flexible materials; in fact, I do not think any one can be aware of the danger of over-loading a column who has never observed an experiment of this kind.

M. Duleau found that a bar of malleable iron 11·8 feet long, and 1·21 inches square (31 millimètres), doubled under a load of 4400 lbs. (2000 kilogrammes). Another specimen about 11·8 feet long, the breadth 2·38 inches, and the depth 0·8 inch, doubled under a load of 2640 lbs.: this piece did not become sensibly bent before it doubled.* In the last experiment, our rule (Equa. xv. art. 288,) gives 876 lbs. as the greatest load the bar ought to sustain in practice; which is about one-third of the weight that doubled the piece; a similar result obtains in other cases.

EXPERIMENTS ON THE RESISTANCE TO TORSION.

94. Mr. Rennie made some experiments on the resistance of malleable iron to torsion. The weight acted with a lever of 2 feet, and the specimens were ¼th of an inch square; the strain was applied close to the fixed end:—

	lbs.	oz.
English iron, wrought, was wrenched asunder by	10	2
Swedish iron, wrought, ...	9	8 †

* Essai Théorique et Expérimental sur la Résistance du Fer Forgé, p. 26-37.
† Philosophical Magazine, vol. liii. p. 168.

If we could suppose the pieces so fitted that the distance between the centres of action, of the force and the fixing apparatus, was equal to the diameter of the specimen; then our formula gives 1·315 lbs. as the force that such a bar would resist without permanent change: this is only about ⅛th of the force that produced fracture. A like irregularity occurs in his experiments on the torsion of cast iron, which may very likely be in consequence of the strain not being applied exactly as I have supposed it to be.

The experiments on the resistance of malleable iron to torsion made by M. Duleau were all directed to determining its stiffness. The bars were fixed at one end in a horizontal position, and the force was applied to a wheel or large pulley fixed on the other end. In order to prevent lateral strain, the end to which the wheel was fixed reposed freely upon a support. It was found that the bars yielded a little at the fixed points; the permanent alteration produced by this yielding was allowed for by deducting the angle of set from the angle observed.*

Nature of the specimens.	Length of the part twisted.	Sid of diameter.	Angle of torsion with a wt. of 10 kilogrammes (22 lbs.) with leverage of 320 millimètres (1·22 feet).	Angle of torsion as calculated.
	Millimètres.	Millimètres.	degrees.	degrees.
Round iron, English, marked DOWLAIS, as from the iron works; hot short.	2400 (7·9 ft.)	19·83 (·78 in.)	4	10·4
Round iron, Perigord, as from the iron works.	2890 (9·5 ft.)	23·03 (·91 in.)	8	7
Square iron, English marked *C* 2, hot short.	4120 (13·5 ft.)	20 × 20 (·79 in.)	6½	10
Square iron, Perigord, as from the iron works.	2520 (8·3 ft.)	20·35 × 20·35 (·8 in.)	3·08	5·8
Flat iron, English.	2910 (9·6 ft.)	34 × 8·56 (1·32 × ·337 in.)	11·4	13·9

* Essai sur la Résistance du Fer Forgé, p. 50-53.

The last column shows the angle calculated by the formula (Equa. xiii. and xiv. art. 272). There is a considerable error in excess according to these experiments; see art. 85.

EXPERIMENTS ON VARIOUS METALS.

EXPERIMENTS ON STEEL.

95. The modulus of elasticity of steel was first determined by Dr. Young from the vibration of a tuning-fork; the height of the modulus found by this method was 8,530,000 feet; * hence the weight of the modulus for a base of an inch square will be 29,000,000 lbs.

M. Duleau has made some experiments on the flexure of steel bars when loaded in the middle and supported at the ends; in all he has described twelve experiments; † from these I will take four at random.

Description of specimens.	Distance between the supports.	Breadth.	Depth.	Depression with 10 kilogrammes.	Weight of modulus of elasticity in lbs. for a base an inch square.
	Millim.	Millim.	Millim.	Millim.	English lbs.
English cast steel, marked HUNTSMAN, perfectly regular, untempered, but brittle.	930	13·3	5·9	32·05	34,000,000
German steel (of cementation), marked FORTSMAN, and 3 deer heads, used for razors, dimensions irregular.	680	14·5	7·8	8	20,263,000
Same kind of steel.	1845	28·5	21·9	2·6	29,000,000
Ditto.	1350	52	26·6	0·5	17,880,000
				Mean for German steel 22,381,000 lbs.	

EXPERIMENTS ON GUN-METAL.

96. A cast bar of the alloy of copper and tin, commonly called gun-metal, of the specific gravity 8·152, was filed true and regular; its depth was 0·5 inch, and its breadth 0·7

* Lectures on Natural Philosophy, vol. ii. p. 86.

† Essai sur la Résistance du Fer Forgé, p. 38.

inch; it was supported at the ends, the distance between the supports being 12 inches; and the scale was suspended from the middle.

19 lbs. bent the bar	0·01 inch.	
38 . . .	0·02 „	
56 . . .	0·03 „	
78 . . .	0·04 „	
100 . . .	0·05 „	This load was raised from the bar several times, but permanent set was not sensible.
120 . . .	0·06 „	Every time the bar was relieved of this load, a set of about ·005 was observed.
200 . . .	0·17 „	
230 . . .	0·34 „	
320		slipped through between the supports, bent nearly 3 inches, but not broken.

We may therefore consider 100 lbs. as the utmost that the bar would support without permanent alteration, which is equivalent to a strain of 10,285 lbs. upon a square inch; and an extension of $\frac{1}{960}$th part of its length (see art. 110 and 121). Absolute cohesion greater than 34,000 lbs. for a square inch.

Calculating from this experiment, we find the weight of the modulus of elasticity for a base 1 inch square, 9,873,000 lbs.; and the specific gravity of gun-metal is 8·152; therefore the height of the modulus in feet is 2,790,000 feet.

The deflexion increases much more rapidly than in proportion to the weight, as soon as the strain exceeds the elastic force; a weight of 200 lbs. more than trebled the deflexion produced by 100, instead of only doubling it.

EXPERIMENTS ON BRASS.

97. Dr. Young made some experiments on brass, from which he calculated the height of the modulus of elasticity of brass plate to be 4,940,000 feet, or 18,000,000 lbs. for its weight to a base of 1 square inch. For wire of inferior brass he found the height to be 4,700,000 feet.*

As cast brass had not been submitted to experiment, I

* Natural Philosophy, vol. ii. p. 86.

procured a cast bar of good brass, and made the following experiment:

The bar was filed true and regular; its depth was 0·45 inch, and breadth 0·7 inch. The distance between the supports 12 inches, and the scale suspended from the middle.

12 lbs. bent the bar	0·01 inch.	
23 . . .	0.02 „	
38 . . .	0·03 „	The bar was relieved several times, but it took no perceptible set.
52 . . .	0·04 „	
65 . . .	0·05 „	relieved, the set was ·01.
110 . . .	0·18 „	
163		slipped between the supports, bent more than 2 inches, but not broken.

Hence 52 lbs. seems to be about the limit which could not be much exceeded without permanent change of structure. It is equivalent to a strain of 6700 lbs. upon a square inch, and the corresponding extension is $\frac{1}{1333}$ of its length, (see art. 110 and 121). Absolute cohesion greater than 21,000 lbs. per square inch. The modulus of elasticity according to this experiment is 8,930,000 lbs. for a base of an inch square. The specific gravity of the brass is 8·37, whence we have 2,460,000 feet for the height of the modulus.

SECTION VII.

OF THE STRENGTH AND DEFLEXION OF CAST IRON WHEN IT RESISTS PRESSURE OR WEIGHT.

98. The doctrine of the Strength of Materials, as given in this Work, rests upon three first principles, and these are abundantly proved by experience.

The First is, that the strength of a bar or rod to resist a given strain, when drawn in the direction of its length, is directly proportional to the area of its cross section; while its elastic power remains perfect, and the direction of the force coincides with the axis.

99. The Second is, that the extension of a bar or rod by a force acting in the direction of its length, is directly proportional to the straining force, when the area of the section is the same; while the strain does not exceed the elastic power.*

100. The Third is, that while the force is within the elastic power of the material, bodies resist extension and compression with equal forces.

101. It is further supposed that every part of the same piece of the material is of the same quality, and that there are no defects in it. If there be any material defect in a piece of

* This limit should be carefully attended to, for as soon as the strain exceeds the elastic power, the ductility of the material becomes sensible. The degrees of ductility are extremely variable in different bodies, and even in different states of the same body. A fluid possesses this property in the greatest degree, for every change in the relative position of its parts is permanent.

cast iron, it may often be discovered, either by inspection, or by the sound the piece omits when struck; except it be air bubbles, which cannot be known by these means.

The manner of examining the quality of a piece of cast iron has been given in the Introduction, p. 5; and such as will bear the test of hammering with the same apparent degree of malleability, will be found sufficiently near of the same strength and extensibility for any practical deductions to be correct.

The truth of these premises being admitted, every rule that is herein grounded upon them may be considered as firmly established as the properties of geometrical figures.

102. A free weight or mass of matter is always to be considered to act in the direction of a vertical line passing through its centre of gravity; and its whole effect as if collected at the point where this vertical line intersects the beam or the pillar, which is to support it. But if the weight or mass of matter be partially sustained, independently of the beam or pillar, in any manner, then the direction and intensity of the force must be found that would sustain the mass in equilibrium,* and this will be the direction and intensity of the pressure on the beam or pillar.

103. Let f denote the weight in pounds which would be borne by a rod of iron, or other matter, of an inch square, when the strain is as great as it will bear without destroying a part of its elastic force.† Also, let W be any other weight to be supported, and b = the breadth and t = the thickness of the piece to support it, in inches. Then, by our first principles, art. 98, we have

$$f : W :: 1 : b\,t$$

$$\text{or, } \frac{W}{f} = b\,t \qquad \text{(i.)}$$

* The method of finding this force and its direction is explained in my Elementary Principles of Carpentry, art. 24-29.

† "A permanent alteration of form," Dr. Young has remarked, "limits the strength of materials with regard to practical purposes, almost as much as fracture, since in general the force which is capable of producing this effect is sufficient, with a small addition, to increase it till fracture takes place." Natural Philosophy, vol. i. p. 141.

That is, the area should be directly as weight to be supported, and inversely as the force which would impair the elastic power of the material.

104. If ϵ be the quantity a bar of iron, or other matter, an inch square and a foot in length, would be extended by the force f; and l be any other length in feet; then

$$1 : l :: \epsilon : \Delta, \text{ or } l\epsilon = \Delta =$$

the extension in the length l. (ii.)

For, when the force is the same, the extension is obviously proportional to the length.

And, since by our principle, art. 99, the extension is as the force; we have $f : W :: \epsilon :$ extension produced by the weight $W = \frac{W\epsilon}{f}$, and we obtain from Equation ii. $\frac{Wl\epsilon}{f} = \Delta$. (iii.)

In which Δ is the extension that would be produced in the length l, by the weight W.

105. Where a comparison of elastic forces is to be made, it is sometimes convenient to have a single measure which is called the modulus of elasticity.* It is found by this analogy: as the length of a substance is to the diminution of its length, so is the modulus of elasticity to the force producing that diminution. Or, denoting the weight of the modulus in lbs. for a base of an inch square by m,

$$\epsilon : f :: 1 : m = \frac{f}{\epsilon}. \quad \text{(iv.)}$$

And if p be the weight of a bar of the substance 1 foot in length, and 1 inch square; then if M be the height of the modulus of elasticity in feet,†

$$\frac{f}{p\epsilon} = M. \quad \text{(v.)}$$

* The term was first used by Dr. Young. Lectures on Nat. Phil. vol. ii. art. 319.

† By this and the preceding equation, were calculated the height and weight of the modulus of elasticity of the different bodies in the Alphabetical Table.

106. Let the rectangular beam A A′, fig. 14, be supported upon a fulcrum D, in equilibrio, and for the present considering the beam to be acted upon by no other forces than the weights W W′; which are supposed to have produced their full effect in deflecting the beam, and the vertical section at B D to be divided into equal, and very thin filaments, as shown in fig. 15.

Consider B, fig. 14, to be the situation of one of the small filaments in the upper part of the beam, and *a a′* a tangent to the curvature of the filament B, at the point B. Now, it is clearly a necessary consequence of equilibrium that the forces tending to separate the filament at B should be equal, and in the direction of the tangent *a a′*; and the strain is obviously a tensile one.

But since F A is the direction of the weight, we have, by the principles of statics,

B *a* : A *a* :: S (= the resistance of the filament B) : $\frac{A\,a\,.\,S}{B\,a}$ = its effect in sustaining the weight W.

These forces, we know both from reasoning and experience, will compress the lower part of the beam; and let D be a compressed filament, of the same area as the filament B, and in the same position, and at the same distance from the under surface as the filament B is in respect to the upper surface. Also, let *e e′* be a tangent to the filament at D, and parallel to *a a′*; and representing one of the equal and opposite strains on the filament D by *e* D; we have, *e* D : *e* A :: S′ (the resistance to compression of D) : $\frac{e\,A\,.\,S'}{e\,D}$ = the effect of the filament D in sustaining the weight W.

The effect of both the filaments, B D, in supporting the weight will therefore be,

$$\frac{A\,a\,.\,S}{B\,a} + \frac{e\,A\,.\,S'}{e\,D},$$

or since B $a = e$ D, and as portions of the same matter of equal area resist extension or compression with equal forces (art. 100) $S = S'$; therefore, $\frac{S}{B\,a} \times (A\,a + e\,A) =$ the effect of the filaments D and B.* But

$$A\,a + e\,A = B\,D,$$

the vertical distance between the filaments.† Consequently $\frac{S \,.\, BD}{B\,a} =$ this effect in supporting the weight W. (vi.)

107. As one side of the beam suffers extension, and the other side compression, there will be a filament at some point of the depth, which will neither be extended nor compressed; the situation of this filament may be called the neutral axis, or axis of motion.

The extension or compression of a filament will obviously be as its distance from the neutral axis; and when the neutral axis divides the section into two equal and similar parts, its place will be at the middle of the depth.

And since the effect of two equal filaments is as the distance between them, the effect of either will be as its distance from the neutral axis; for the filaments being equal, and the strain on them equal, the axis will be at the middle of the distance between them; and the effect of both being measured by the whole depth, that of one of them will be

* But when the strain exceeds the elastic force of a body, the resistance to compression exceeds the resistance to tension; consequently, the effect of the filaments must be

$$\frac{A\,a \,.\, S + e\,A \,.\, S'}{B\,a}$$

Now the difference between S and S′ will be constantly increasing till fracture takes place, the area of the compressed part being constantly increasing, and that of the extended part diminishing. The variation will depend on the ductility of the material, but it cannot be ascertained, unless some very careful experiments were made for the purpose; and fortunately it is an inquiry not required in the practical application of theory.

† When the flexure becomes considerable, the curve is flattened in consequence of the forces compressing the beam, and $A\,a + e\,A$ will exceed the vertical distance between the filaments; and the point of greatest strain will be found to change to the place where the line A B intersects the filament. This change of the point of greatest strain is very apparent in experiment.

measured by half the depth. Therefore the effect of a filament is

$$\frac{S.BD}{2(Ba)} = \frac{S.Bd}{Ba} \qquad \text{(vii.)}$$

108. When a beam is sustained in any position,* not greatly differing from a horizontal one, by a fulcrum, as in fig. 14, the power of a fibre or filament to support a weight at A or A′ is directly as its force, its area, and the square of its distance from the neutral axis; and inversely as its distance, F B, of the straining force from the point of support.

For the strain being as the extension, and the extension of any filament being directly as the distance of that filament from the axis of motion, therefore, the force of a filament is as its distance from the axis of motion. But it has been shown (art. 103,) that the force is also as the area; and the power in sustaining a weight has been shown (art. 107,) to be directly as the vertical distance from the neutral axis, and inversely as the length B a, that is, as $\frac{Bd}{Ba}$; and, since the triangles F B a, B D f, are similar,

$$\frac{Bd}{Ba} = \frac{fd}{FB}, \text{ therefore}$$

$$\frac{(fd)^2 \times \text{the force of filament} \times \text{by its area}}{FB} = \text{the weight it will sustain.} \qquad \text{(viii.)}$$

109. Let d be the depth, divided into filaments, each equal to x the mth part of $\frac{d}{2}$; also put F B $= l$, the breadth of the beam $= b$, and f the weight that a fibre of a given magnitude would bear when drawn in the direction of its length, without destroying its elastic force.

Now, if we calculate the mean strain upon each filament

* It does not sensibly differ from the correct law of resistance till the beam be so much inclined as to slide on its support; but the general investigation will be found in art. 276.

by Equation viii. art. 108, we obtain the following progression, and its sum is the weight the beam will support.*

$$\frac{4 f b x^3}{l d} \times (1 + 2^3 + 3^2 \ldots\ldots \overline{m - 1}^2 + \frac{m^2}{2}) = \text{W}. \qquad \text{(ix.)}$$

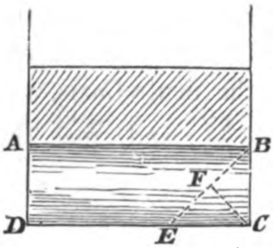
110. If the beam be rectangular, the value of

$$\text{W} = \frac{f b d^2}{6 l}. \qquad \text{(x.)}$$

Therefore the lateral strength of a rectangular beam is directly as its breadth, and the square of its depth, and inversely as its length.

And when the beam is square, its lateral strength is as the cube of its side.

111. If a plate be fixed along one of its sides A B, and the load be applied at the angle C; then if the distance A B be greater than B C, the plate will break in the direction of some line E B. To find this line, put F C = l, the leverage, and E B = b, the breadth; also t = the tangent of the angle E B C. Then, by similar triangles,



$$\sqrt{1 + t^2} :: \text{B C} : l = \frac{\text{B C} \times t}{\sqrt{1 + t^2}};$$

and

$$1 : \sqrt{1 + t^2} :: \text{B C} : b = \text{B C} \times \sqrt{1 + t^2};$$

therefore

$$\frac{f b d^2}{6 l} = \frac{f d^2 (1 + t^2)}{6 t}.$$

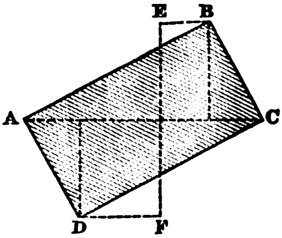
But this equation is a minimum when $t = 1$; that is, when the angle E B C is 45 degrees; consequently

* The first term of the progression is equivalent to the quantity called a fluxion, and is usually written thus, $\frac{4 f b x^2 \dot{x}}{d}$. The same remark applies to the other progressions.

$$\frac{f\,d^2\,(1 + t^2)}{6\,t} = \frac{f\,d^3}{3} = W. \qquad \text{(xi.)}$$

112. If the beam be rectangular, and the strain be in a perpendicular direction to one of its diagonals A C, making that diagonal $= b$, and the depth E F $= a$, the progression becomes (because the breadth is successively

$$\overline{a - 2\,x}, \overline{a - 4\,x}, \text{\&c.})$$



$$\frac{4\,b\,f\,x^3}{l\,a} \times \left\{ a\,(1 + 2^2 + \ldots\ldots \frac{m^2}{2}) - 2\,x\,(1 + 2^3 + \ldots \frac{m^3}{2}). \right\}$$

$$\text{or, } W = \frac{f\,b\,a^2}{24\,l}. \qquad \text{(xii.)}$$

If the beam be square, the direction of the straining force coincides with the vertical diagonal, and in that case

$$\frac{f\,a^3}{24\,l} = W. \qquad \text{(xiii.)}$$

But the diagonal of a square beam is equal to its side multiplied by $\sqrt{2}$; hence, if d be the side, we have

$$\frac{f\,d^3}{6\,\sqrt{2}\,l} = W. \qquad \text{(xiv.)}$$

Consequently the strength of a square beam, when the force is parallel to its side, is to the strength of the same beam when the force is in the direction of its diagonal as

$$1 : \frac{1}{\sqrt{2}};$$

or as 10 is to 7 nearly.

113. If the beam be a cylinder, and r the radius, then b is successively

$$2\sqrt{r^2 - x^2},\ 2\sqrt{r^2 - (2x)^2},\ \&c.,$$

And $\frac{4fx^3}{rl} \times \left\{ \sqrt{r^2 - x^2} + 2^2\sqrt{r^2 - (2x)^2} + \&c. \right\} = W$

$$\text{or, } W = \frac{\cdot 7854\, f r^3}{l}. \qquad \text{(xv.)}$$

If d be the diameter, then

$$W = \frac{\cdot 7854\, f d^3}{8\, l} \qquad \text{(xvi.)}$$

The lateral strength of a cylinder is directly as the cube of its diameter, and inversely as the length.

The strength of a square beam is to that of an inscribed cylinder as

$8 : 6 \times \cdot 7854$, that is, as $1 : \cdot 589$, or as $1\cdot 7 : 1$.

114. If the section of the beam be an ellipse, when the strain is in the direction of the conjugate axis, we have by the same process

$$W = \frac{\cdot 7854\, f t c^2}{l}, \qquad \text{(xvii.)}$$

where t is the semi-transverse, and c the semi-conjugate axis.

115. If the beam be a hollow cylinder or tube, and r be the exterior radius, $n r$ being that of the hollow part, then by the same process * we find

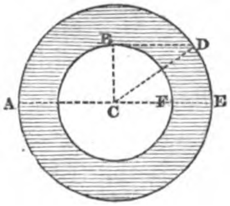
$$W = \frac{\cdot 7854\, f r^3 (1 - n^4)}{l}. \qquad \text{(xviii.)}$$

The radius of a solid cylinder that will contain the same quantity of matter as the tube, is easily found by geometrical construction in this manner: make B D perpendicular to B C; then C D being the radius of the tube, and B C that of

* Dr. Young gives a rule which is essentially the same, of which I was not aware when my "Principles of Carpentry" was written. See Natural Philosophy, vol. ii. art. 339, B. scholium. In a recent work on the Elements of Natural Philosophy, by Professor Leslie, vol. i. p. 242, the learned author has neglected to consider the effect of extension in his investigation of this equation.

the hollow part, B D will be the radius of a solid cylinder which will contain the same quantity of matter as the tube. Because

$$BD^2 = CD^2 - BC^2.$$



By comparing Equation xv. and xviii. we find that when a solid cylinder is expanded into a tube, retaining the same quantity of matter, the strength of the solid cylinder being 1, that of the tube will be $\frac{1 - n^4}{(1 - n^2)^{\frac{3}{2}}}$. When the thickness F E is $\frac{1}{5}$th of the diameter A E, the strength will be increased in the proportion of 1·7 to 1. And when F E is $\frac{3}{20}$ths of the diameter, the strength will be doubled by expanding the matter into a tube. But a greater excess of strength cannot be safely obtained than the latter, because the tube would not be capable of retaining its circular form with a less thickness of matter. From $\frac{1}{6}$th to $\frac{1}{10}$th seems to be the most common ratio in natural bodies, such as the stems of plants, &c.

116. If a beam be of the form shown in Plate I., fig. 9 (see art. 38, 39, and 40), and d be the extreme depth, and b the extreme breadth; $q\,b$ = the difference between the breadth in the middle and the extreme breadth, and $p\,d$ the depth of the narrow part in the middle; then by the process employed by calculating Equation x. we find

$$W = \frac{f\,b\,d^2}{6\,l} \times (1 - q\,p^3.) \qquad \text{(xix.)}$$

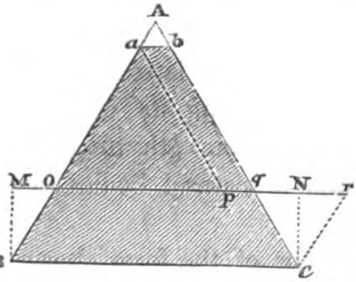
117. If the middle part of the beam be entirely left out, with the exception of cross parts to prevent the upper and

lower sides coming together, as in figs. 11 and 12, Plate II. (see art. 41), and d be the whole depth, $p\,d$ the depth of the part left out in the middle, and b the breadth, then

$$W = \frac{f\,b\,d^2}{6\,l}(1 - p^3.) \qquad \text{(xx.)}$$

118. Hitherto we have only considered those forms where the neutral axis divides the section into identical figures; but there are some interesting cases* where this does not happen, such, for example, as the triangular section.

Taking the case of a triangular section with a part removed at the vertex, we shall have a general case which will include that of the entire triangle. Let d be the depth of the complete triangle, and $m\,d$ the depth of the part cut off at the vertex; and $n\,d$ the depth of the neutral axis, M N, from the upper side $a\,b$ of the beam; then the distance of the neutral axis from the base will be $(1-n-m)\,d$. If both sides of the neutral axis were the same as the upper one, the strength



would be equal to that of a parallelogram $a\,b\,p\,q$, added to a triangle $a\,o\,p$; hence from Equation x. and xii. we have

$$\frac{f}{6\,l}(4\,m\,b\,n^2\,d^2 + b\,n^3\,d^2) = \frac{f\,b\,n^2\,d^2}{6\,l}(4\,m + n) = W.$$

But to find the place of the neutral axis, we must compare the strength of the lower side with the upper one; and the

* They are interesting, because the early theorists fell into some serious errors respecting them, and consequently have led practical engineers into erroneous opinions.

strength of the lower side is equal to that of a rectangle M N B C minus a triangle $q\,r$ C, or

$$\frac{f}{6\,l}\left\{4\,b\,d^2\,(1-m-n)^2 - b\,d^2\,(1-m-n)^3\right\} = \text{W}.$$

And consequently,

$$n^2\,(4\,m+n) = 4\,(1-m-n)^2 - (1-m-n)^3.$$

Whence,

$$n = \frac{5-2\,m-3\,m^2}{2\,(1-m)} - \sqrt{\left(\frac{5-2\,m-3\,m^2}{2\,(1-m)}\right)^2 - \frac{3-5\,m+m^2+m^3}{1-m}}.$$

When $m = 0{\cdot}1$, then $n = {\cdot}592$, and

$$\frac{{\cdot}348\,f\,b\,d^2}{6\,l} = \text{W}. \qquad \text{(xxi.)}$$

But if $m = o$, or the triangle A B C be entire, then $n = {\cdot}697$ nearly.* And

$$\frac{{\cdot}339\,f\,b\,d^2}{6\,l} = \text{W, or } \frac{{\cdot}0565\,f\,b\,d^2}{l} = \text{W.}\dagger \qquad \text{(xxii.)}$$

If $m = \frac{1}{9}$ we have $n = {\cdot}58166$, and

$$\frac{{\cdot}347\,f\,b\,d^2}{6\,l} = \text{W}. \qquad \text{(xxiii.)}$$

Where $m = 0{\cdot}1$, the strength is about the greatest possible, a triangular prism being about $\frac{1}{87}$th part stronger when the angle is taken off to $\frac{1}{10}$th of its depth, as shown by the shaded part of the figure. Emerson first announced this seeming paradox,‡ but it is easily shown that his solution only applies to the imaginary case where the neutral axis is an incompressible arris, at the base of the section.

A triangular prism is equally strong, whether the base or the vertex of the section be compressed: § and by comparing

* Duleau obtains a result equivalent to $n = {\cdot}57$ in this case, but the result only is given. Essai Théorique, &c., p. 77.

† This rule was first published in the Philosophical Magazine, vol. xlvii. p. 22. 1816.

‡ Mechanics, Sect. VIII., p. 114. 5th edit. 1800.

§ Duleau has proved the truth of this by his experiments on the flexure of triangular bars. Essai sur la Résistance, &c., p. 26.

Equations x. and xxii. it appears that its strength is to that of a circumscribed rectangular prism as 339 : 1000, or nearly as 1 : 3. But let it be remembered that this ratio only applies to strains which do not produce permanent alteration in the materials, and where the arris is not injured by the action of the straining force: if the strain be increased so as to produce fracture, the triangle will be found still weaker than in this proportion, when the arris is extended, and somewhat stronger when the arris is compressed; in the former case, from the imperfection of castings, where there is much surface in proportion to the quantity of matter, as in all acute arrises; in the latter case, from the saddle or other thing used to support the weight reducing the quantity of actual leverage.

It may be useful to remark, that a triangle contains half the quantity of matter that there is in the circumscribing rectangle, but its strength is only one-third; hence it is not economical to adopt triangular sections, and a like remark applies to the T formed sections so commonly used.

119. If the whole depth of a T formed section be d, its greatest breadth b, and its least breadth $(1-q)\,b$. Then, supposing the depth of the neutral axis M N from the narrow edge A E to be $\frac{1}{n}d$, the strength of the bar will be

$$\frac{4\,f\,b\,d^2\,(1-q)}{6\,l\,n^2}=\text{W}.$$

For $\frac{4\,d^2}{n^2}$ would be the square of the whole depth were both sides of the neutral axis the same; and the strength would be equal to a rectangle of that depth with the breadth $(1-q)\,b$.

But the strength of the other side of the axis by Equation xix. is

$$\frac{4\,f\,b\,d^2\,(1-q\,p^3)\,(n-1)^2}{6\,l\,n^2};$$

hence we have the equation $(n-1)^2\ (1-q\,p^3)=1-q$ to determine the place of the neutral axis; or

$$n = 1 + \sqrt{\frac{1-q}{1-q\,p^3}};$$

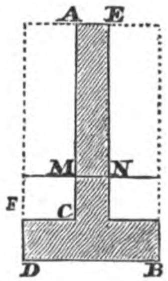
consequently,

$$\frac{4\,f\,b\,d^2\,(1-q)}{6\,l\left(1+\sqrt{\frac{1-q}{1-p^3\,q}}\right)^2} = \text{W}. \qquad \text{(xxiv.)}$$

This formula is complicated, but it affords some curious results. If we make $p=o$, we have the strength of a bar with its neutral axis at C, and the depth A C is

$$d\left(\frac{1}{1+\sqrt{1-q}}\right);$$

where d is the whole depth.



And if A E $=\frac{1}{4}$ D B, then A C $=\frac{2}{3}\,d$, when the neutral axis is at C.

The neutral axis may be at any point that may be chosen between the point C and half the depth, by varying the values of q or p for that purpose.

If we make

$$q = \cdot 75,\ \text{and}\ p = \cdot 5,\ \text{then A M} = \frac{d}{1\cdot 55},\ \text{and D F} = \text{C M};$$

also

$$\text{A E} = \tfrac{1}{4}\,\text{D B}.$$

The strength is

$$\frac{f\,b\,d^2}{6 \times 2\cdot 4\,l} = \text{W}. \qquad \text{xxv}$$

The figure is drawn in these proportions, b is the whole breadth D B, and d the whole depth. Its strength is to that of the circumscribing rectangle shown by the dotted lines as $\frac{1}{2\cdot 4}$: 1 or as 5 : 12.

These equations show the relation between the strength of beams, and the weight to be supported in some of the most useful cases when the load is applied as in fig. 14; but previous to considering how these equations will be effected by varying the mode of supporting the beam, it will be desirable to give some rules for estimating the deflexion of beams.

120. The deflexion of a beam supported as in fig. 16, Plate II., is caused by the extension of the fibres of the upper side, and the compression of those on the under side; the neutral line A B A′ retains the same length.

If we conceive the length of a beam to be divided into a great number of equal parts, and that the extension, at the upper side of the beam, of one of these parts is represented by *a b*, then the deflexion produced by this extension will be represented by *d e*, and the angles *a c b*, *d c e*, being equal, we shall have *b c* : *d c* : : *a b* : *d e*; the smallness of the angles rendering the deviation from strict similarity insensible.

Now, however small we may consider the parts to be, into which the length is divided, still the strain will vary in different parts of it, and consequently the deflexion; but if we consider the deflexion produced by the extension of any part, to be that which is due to an arithmetical mean between the greatest and least strains in that part, we shall then be extremely near the truth.

We have seen that the strain is as the weight and leverage directly, and as the breadth and square of the depth inversely (see art. 108). Our investigation will be more general by considering the weight, breadth, and depth variable, by taking *l*, *b*, and *d* for the length, breadth, and depth of the middle or supported point, and W for the whole weight, and *x*, *y*, and *w* for the depth, breadth, and weight on any other point. Then, the deflexion from the strain at any point *c* is as

$$\frac{W\,l\,x}{2\,b\,d^2} : \frac{w \times (d\,c)^2}{y\,x^2} : : \epsilon : \frac{2\,b\,d^2\,\epsilon\,w\,(d\,c)^2}{W\,l\,y\,x^3}.$$

And, if z be the length of one of the parts into which we suppose the whole length divided, then the deflexion from the mean force on the length of z situate at c will be

$$\frac{2 b d^2 \epsilon w z}{W l y x^3} \times \left(\frac{d c^2 + \overline{d c + z}^2}{2}\right)$$

Since the whole deflexion D A is the sum of the deflexions of the parts, we have

$$\frac{2 b d^2 \epsilon w z^3}{W l y x^3} \times \left(1^2 + 2^2 + \&c.\ \overline{m - 1}^2 + \frac{m^2}{2}\right) = \text{D A.} \qquad \text{(i.)}$$

121. *Case* 1. When a beam is rectangular, the depth and breadth uniform, and the load applied at one end. Then,

$$b = y,\ d = x, \text{ and } W = w.$$

Therefore the progression becomes

$$\frac{2 \epsilon z^3}{l d} \times (1^2 + 2^2, \&c.),$$

of which the sum is

$$\frac{2 \epsilon l^2}{3 d} = \text{the deflexion D A.*} \qquad \text{(ii.)}$$

122. *Case* 2. When the section of the beam is rectangular, and the load acts at one end, the depth being uniform, but the breadth varying as the length.

In this case the progression is

$$\frac{2 \epsilon z^2}{d} \times (1 + 2, \&c.) = \frac{\epsilon l^2}{d} = \text{the deflexion D A.} \qquad \text{(iii.)}$$

This is the beam of uniform strength, described in art. 30, fig. 6, and the deflexion is $\frac{1}{3}$rd more than that of a beam of equal breadth throughout its length. The deflexion of the beam of equal strength described in art. 33 is the same; the neutral axis becomes a circle in both these beams.†

* The same relation is otherwise determined in Dr. Young's Natural Philosophy, vol. ii. art. 325.

† M. Girard arrives at the erroneous conclusion, that all the solids of equal resistance curve into circular arcs, (Traité Analytique, p. 82,) in consequence of neglecting the effect of the depth of the solid on the radius of curvature.

In this case it is easily shown by other reasoning that the curve of the neutral axis is a portion of a circle, and it is well known that in an arc of very small curvature (one of such as are formed by the deflexions of beams in practical cases), the versed sine is sensibly proportioned to the square of the sine. This will enable the reader to form an estimate of the accuracy of the method I here follow. I am perfectly satisfied that it is correct enough for use in the construction of machines or buildings, and that it is a useless refinement to embarrass the subject with intricate rules; but this explanation may be necessary to some nice theorists, who aim rather at imaginary perfection than useful application.

123. *Case* 3. When the section of the beam is rectangular, the load acting at the end, the breadth uniform, and the depth varying as the square root of the length; which is the parabolic beam of equal strength. (See art. 27, fig. 3, Plate I.)

In this case the progression is

$$\frac{2\,\epsilon\,l^{\frac{1}{2}}\,z^{\frac{3}{2}}}{d} \times \left(1^{\frac{1}{2}} + 2^{\frac{1}{2}} + \&c.\right) \text{ or,}$$

$$\frac{4\,\epsilon\,l^2}{3\,d} = \text{the deflexion D A.} \qquad \text{(iv.)}$$

The deflexion is double that of a uniform beam, while the quantity of matter is only lessened $\frac{1}{3}$rd.

124. *Case* 4. When the section of the beam decreases from the supported point to the end where the load acts, so that the sections are similar figures, then the curve bounding the sides of the beam will be a cubical parabola; that is, the depth will be every where proportional to the cube root of the length.

In this case the progression is

$$\frac{2\,\epsilon\,l^{\frac{1}{3}}\,z^{\frac{5}{3}}}{d} \times (1^{\frac{2}{3}} + 2^{\frac{2}{3}} + \&c.) = \frac{6\,\epsilon\,l^2}{5\,d} = \text{the deflexion D A.} \qquad \text{(v.)}$$

The deflexion is to that of a uniform beam as 1·8 : 1.

125. *Case* 5. When a beam is of the same breadth throughout, and the vertical section is an ellipse (see fig. 8, art. 32), the deflexion from a weight at the vertex may be exhibited in a progression as below:

$$\frac{l^3 \epsilon z^3}{d} \times \left\{ \frac{2}{(2\,l\,z - z^2)^{\frac{3}{2}}} + \frac{8}{(4\,l\,z + 4\,z^2)^{\frac{3}{2}}} + \&c. + \frac{m^2}{(2lmz - m^2z^2)^{\frac{3}{2}}} \right\}$$

$$= \frac{l^3 \epsilon z^{\frac{3}{2}}}{d} \times \left(\frac{2}{(2\,l - z)^{\frac{3}{2}}} + \&c. \right) = \text{D A.}$$

By actually summing this progression when $m = 10$, we have

$$\frac{\cdot 857\ l^3 \epsilon}{d} = \text{the deflexion D A.} \tag{vi.}$$

126. *Case* 6. If a rectangular beam of uniform breadth and depth be so loaded that the strain be upon any point c, then

$$l^2 : d\,c \times (2\,l - d\,c) :: \text{W} : w = \frac{d\,c \times \text{W} \times (2\,l - d\,c)}{l^2}.$$

This value of w being substituted in Equation i., we have

$$\frac{\epsilon z^3}{d\,l^3} \left\{ 2\,l\,(1^2 + 2^2 + \&c.) - z\,(1^3 + 2^3, \&c.) \right\} = \frac{\epsilon\,l^3}{d} \times$$

$$(\tfrac{4}{3} - \tfrac{1}{2}) = \frac{5\,\epsilon\,l^3}{6\,d} = \text{the deflexion D A.*} \tag{vii.}$$

This is the deflexion of a beam uniformly loaded when it is supported at the ends, l being half the length.

127. *Case* 7. If the section of a beam be rectangular, and the breadth uniform, but a portion of the depth varying as the length, and the rest of it uniform: then the depth at any point c will be

$$\frac{d}{l} \overline{(1 - n\,l + n\,z)} = x;$$

the depth at the point where the weight acts being the $\overline{1 - n}$ th part of the depth at the point of support.

* This relation is otherwise determined by Dr. Young, Nat. Philos. vol. i. art. 329.

This value of x being substituted in Equation i., art. 120, it becomes

$$\frac{2\,l^2\,\epsilon\,z^3}{d}\times\left\{\frac{1}{(1-n\,l+n\,z)^3}+\frac{2^2}{(1-n\,l+2\,n\,z)^3}+\&c.\right\}=\text{D A}.$$

And the general expression for the sum of this progression is

$$\frac{2\,l^2\,\epsilon}{d}\left\{\frac{2\,(1-n)}{n^2}+\frac{3\,(1-n)^2}{2\,n^3}-\frac{3}{2\,n^3}+\frac{1}{n^3}\text{ hy. log. }\frac{1}{1-n}\right\}=\text{D A}.$$

When $n = {\cdot}5$, as in the beams figs. 4 and 5, Plate I., we have

$$\frac{1{\cdot}09\,l^2\,\epsilon}{d}=\text{D A the deflexion.} \qquad \text{(viii.)}$$

Hence the deflexion of a uniform beam being denoted by 1, this beam will be deflected 1·635 by the same force; the middle sections being the same.

If the beam be diminished at the end to two-thirds of the depth at the middle, then

$$n=\frac{1}{3};$$

and

$$\frac{0{\cdot}895\,l^2\,\epsilon}{d}=\text{D A the deflexion.} \qquad \text{(ix.)}$$

128. *Case* 8. If a rectangular beam be supported in the middle, and uniformly loaded over its length, then

$$l:(d\,c)::\text{W}:w=\frac{\text{W}\,(d\,c)}{l}.$$

Hence, when the beam is of uniform breadth and depth, we have, by substituting this value of w in Equation i., art. 120,

$$\frac{2\,\epsilon\,z^4}{l^2}\times(1+2^3+3^3+\&c.)=\frac{l^2\,\epsilon}{2\,d}=\text{D A the deflexion.} \qquad \text{(x.)}$$

In this case the deflexion is $\frac{3}{4}$ths of the deflexion of the same beam having the whole weight collected at the extremities.

129. *Case* 9. If a beam be generated by the revolution of a semi-cubical parabola round its axis, which is the figure

of equal strength for a beam supported in the middle when the weight is uniformly diffused over its length, then

$$l^2 : d^3 :: (d\,c)^2 : x^3 = \frac{d^3\,(d\,c)^2}{l^2};$$

$$\text{and } y = x; \text{ also } w = \frac{(d\,c)\,W}{l}.$$

These quantities, substituted in Equation i., art. 120, give

$$\frac{2\,\epsilon\,l^{\frac{2}{3}}\,z^{\frac{4}{3}}}{d} \times \left\{1 + 2^{\frac{1}{3}} + 3^{\frac{1}{3}} + \&c.\right\} = \frac{3\,\epsilon\,l^2}{2\,d} = \text{D A the deflexion.} \qquad \text{(xi.)}$$

Here the deflexion is $\frac{9}{4}$ths of that of a uniform beam with the load at the extremities.

130. *Case* 10. If a beam to support a uniformly distributed load be of equable breadth, but the depth varying directly as the distance from the extremity, as in fig. 21, Plate III., then

$$l : d :: (d\,c) : x = \frac{d\,(d\,c)}{l};$$

$$b \text{ is constant, and } w = \frac{(d\,c)\,W}{l};$$

therefore, by Equation i., art. 120,

$$\frac{2\,\epsilon\,l^2}{d} = \text{D A the deflexion.} \qquad \text{(xii.)}$$

If the beam had been uniform, and the loads at the extremities, the deflexion would have been only $\frac{1}{3}$rd of the deflexion in this case.

The cases I have considered are perhaps sufficient for the ordinary purposes of business; the next object is to show how these calculations are affected by changing the position and manner of supporting the beam, or the nature of the straining force; and to compare them with experiments, and draw them into practical rules. For this purpose the most clear and the most useful plan seems to be that of taking known practical cases for illustration.

BEAMS SUPPORTED IN THE MIDDLE, AND STRAINED AT THE ENDS, AS IN THE BEAM OF A STEAM-ENGINE.

131. The distance F B, fig. 14, of the direction of the straining force from the centre of motion being constantly the same, the strain will be the same in any position of the beam (art. 108). Also, the deflexion from its natural form will be the same in every position, because the strain is the same; and the length does not vary with the position.

Now the force acting upon the beam of a steam-engine being impulsive, the practical rules for its strength will be found in the eleventh section; the formula calculated in this section being used to establish those rules.

BEAMS FIXED AT ONE END; AS CANTILEVERS, CRANKS, &c.

132. The strain upon a beam supported upon a fulcrum, as in fig. 14, is obviously the same as when one of the ends is fixed in a wall, or other like manner; for fixing the end merely supplies the place of the weight otherwise required to balance the straining force. But though the strain upon the beam be the same, the deflexion of the point where the strain is applied will vary according to the mode of fixing the end; because the deflexion of the strained point will be that produced by the curvature of both the parts A B and B A′.

133. Let the dotted lines in fig. 17, Plate III., represent the natural position of a beam fixed at one end in a wall: when this beam is strained by a load at A, the compression at C will always be enough to allow the beam to curve between A′ and B, and the strain at the point A′ will obviously be the same as if a weight were suspended there that would balance the weight at A. Let A B A′ be the curvature of the beam by the load W, and *a a′* a tangent to the point B. Then A′ *a′* is proportional to the deflexion produced by the strain at A′, and

$$A'B : BD :: A'a' : Da = \frac{BD \times A'a'}{A'B} =$$

the deflexion from the curving of the part A′ B ; therefore

$$\frac{BD \times A'a'}{A'B} + Aa = \text{the whole deflexion } DA.$$

Now, since the deflexion is as the square of the length (see Equation i.-xii., art. 120-130), we have

$$(BA)^2 : (BA')^2 :: Aa : A'a' = \frac{Aa \times (BA')^2}{(BA)^2}.$$

Therefore,

$$Aa \times \left(1 + \frac{BD \times BA'}{(BA)^2}\right) = DA. \qquad \text{(i.)}$$

If the angle D B A be represented by c, then

$$BD = BA \times \cos. c;$$

and putting

$$r = \frac{BA'}{BA},$$

we have

$$Aa \times (1 + r. \cos. c) = DA. \qquad \text{(ii.)}$$

But since the deflexion is always very small, in practical cases, we may always consider cos. $c = 1$, or equal to the radius, and then we have

$$Aa \times (1 + r) = DA. \qquad \text{(iii.)}$$

134. In this equation r is the ratio of the length out of the wall to the length within the wall; that is,

$$BA : BA' :: 1 : r.$$

If the beam be either supported in the middle on a fulcrum, or fixed so that the length of the fixed part be equal to that of the projecting part, then

$$r = 1, \text{ and } 2\,(Aa) = DA. \qquad \text{(iv.)}$$

135. If the fixed part be of greater bulk than the projecting part, or it be so fixed that the extension of the fixed part would be very small, then the effect of such extension may be

neglected, and the deflection D A and A a will be the same; particularly in the cranks of machinery, as in fig. 18, because by employing this value of D A in calculating the resistance to impulsion, we err on the safe side. See art. 327.

BEAMS SUPPORTED AT BOTH ENDS, AS BEAMS FOR SUPPORTING WEIGHTS, &c.

136. When the same beam is supported at the ends, as in fig. 19, instead of being loaded at the ends, and supported in the middle, as in fig. 14, and the inclination and sum of the load be the same in both positions, the strains will be the same.

In either position of the beam we have

$$W \times FB = W' \times F'B,$$

or as

$$W : W' :: F'B : FB;$$

and therefore,

$$W + W' : W :: FF' : F'B.^{*}$$

Consequently,

$$W \times FB = \frac{\overline{W+W'} \times F'B \times FB}{FF'}.$$

If the beam be a rectangle, and the whole length $FF' = l$, and W the whole weight, then by art. 110, Equation x.

$$\frac{f b d^2}{6} = \frac{W \times FB \times F'B}{l}. \qquad \text{(v.)}$$

137. And the strain is as the rectangle of the segments into which the point B divides the beam; and therefore the greatest when the point B is in the middle, as has been otherwise shown by writers on mechanics.†

If the weight be applied in the middle, then

$$\frac{\overline{W+W'} \times F'B \times FB}{FF'} = \frac{\overline{W+W'} \times FF'}{4}.$$

* Euclid's Elements, Prop. XVIII. Book V.

† Gregory's Mechanics, vol. i. art. 178, cor. 2.

In a rectangular beam, the whole length being l, and W the whole weight, then

$$\frac{f b d^2}{6} = \frac{l W}{4}, \text{ or } \frac{2 f b d^2}{3} = l W. \qquad \text{(vi.)}$$

138. When a weight is distributed over the length of a beam A B, fig. 20, in any manner, the strain at any point C may be found. For let G be the centre of gravity of that part of the load upon A C, and g that of the load upon B C. Then by the property of the lever,

$\frac{w \times AG}{AC}$ = the stress at C from the weight w of the load upon A C.

Also,

$\frac{w' \times gB}{CB}$ = the stress at C from the weight w' of the load upon C B.

Therefore the whole stress is

$$\frac{w \times CB \times AG + w' \times AC \times gB}{AC \times CB}.$$

And by Equation v. art. 136, the strain will be

$$\frac{w \times CB \times AG + w' \times AC \times gB}{AB} \qquad \text{(vii.)}$$

139. *Case* 1. When the weight is uniformly distributed over the length, then

$$AG = \tfrac{1}{2} AC; \; gB = \tfrac{1}{2} CB, \text{ and } w + w' = W,$$

the whole weight upon the beam; these values being substituted in Equation vii. it becomes

$$\frac{W \times AC \times CB}{2 AB} = \text{strain at C.} \qquad \text{(viii.)}$$

The strain is greatest at the middle of the length, for then A C × C B is a maximum, and it is evidently the same as if half the weight were collected there; for in that case A C being equal C B, and either of these equal to half A B, we have in the case of a rectangular beam

$$\frac{f\,b\,d^2}{6} = \frac{l\,W}{8}, \text{ or } \frac{4\,f\,b\,d^2}{3} = l\,W. \qquad \text{(ix.)}$$

140. *Case* 2. When the load increases from A to B in proportion to the distance from A; then

$$A\,G = \tfrac{2}{3}\,A\,C, \text{ and } g\,B = \tfrac{1}{3}\,C\,B \times \frac{3\,A\,B - 2\,C\,B}{2\,A\,B - C\,B}.$$

Now since

$$w + w' = \text{the whole weight},$$

and

$$w = \frac{\tfrac{1}{2}\,A\,C^2 \times W}{A\,B},$$

also

$$w' = \tfrac{1}{2}\,C\,B \times W\,\frac{2\,A\,B - C\,B}{A\,B},$$

if these values be inserted in Equation vii. we have

$$\frac{W\,.\,A\,C}{6\,A\,B}\,(A\,B^2 - A\,C^2) = \text{the strain at C.} \qquad \text{(x.)}$$

By the principles of maxima and minima of quantities, we readily find that the strain is the greatest at the distance of $\sqrt{\tfrac{1}{3} A\,B}$ from A. And the strain will be nearly

$$\frac{A\,B^2\,.\,W}{15{\cdot}59} \text{ at the point of greatest strain} = \frac{W\,.\,A\,B}{7{\cdot}75} \text{ when W is the whole weight.} \qquad \text{(xi.)}$$

This distribution of pressure applies to the pressure of a fluid against a vertical sheet of iron; as in lock-gates, reservoirs, sluices, cisterns, piles for wharfs, &c.

141. *Case* 3. When the load increases as the square of the distance from A, we find by a similar process that the strain at any point C is

$$= \frac{W\,.\,A\,C}{12\,A\,B^2} \times (A\,B^3 - A\,C^3). \qquad \text{(xii.)}$$

The point of maximum strain in this case is at the distance of $\left(\frac{1}{4}\right)^{\frac{1}{3}}$ A B from A.

PRACTICAL RULES AND EXAMPLES.

RESISTANCE TO CROSS STRAINS.

142. *Prop.* I. To determine a rule for the breadth and depth of a beam, to support a given weight or pressure, when the distance between the supported or strained points is given; when the breadth and depth are both uniformly the same throughout the length, and the strain does not exceed the elastic force of cast iron.

143. *Case* 1. When a beam is supported at the ends, and loaded in the middle, as in fig. 19. From Equation vi. art. 137, taking W for the weight, we have

$$W l = \frac{2 f b d^2}{3}, \text{ where } l = F F'; \text{ fig. 19,}$$

and the value of f is the only part required from experiment; and

$$\frac{3 l W}{2 b d^2} = f.$$

Now, in the experiment described in art. 56, Sect. V., the bar returned to its natural state when the load was 300 lbs., and I was perfectly satisfied that it would bear more than that weight without destroying its elastic force. Therefore, from this experiment,

$$\frac{3 \times 34 \times 300}{2} = f = 15300 \text{ lbs.}^*$$

That is, cast iron of the quality described in art. 56, will bear 15,300 lbs. upon a square inch, when drawn in the direction

* Mr. Tredgold finds here that cast iron will bear a direct tensile force of 15,300 lbs. per square inch without injury to its elasticity, and concludes (arts. 70—76) that its utmost tensile force is nearly three times as great as this, or upwards of 20 tons. But it will be shown in the "Additions," art. 3, that a less weight per inch than 15,300 lbs. was sufficient to tear asunder bars of several sorts of cast iron; and the mean strength of that metal, from experiments on irons obtained from various parts of the United Kingdom, did not exceed 16,505 lbs. per square inch. Mr. Tredgold was mistaken in supposing the bar to have borne 300 lbs. without injury to its elasticity, as will be seen under the head "Transverse Strength" in the Additions.—EDITOR.

of its length, without producing permanent alteration in its structure. If this value of f be employed, our equation becomes

$$\frac{3\,l\,W}{2 \times 15300} = b\,d^2;$$

or, as it is convenient to take l in feet,

$$\frac{3 \times 12 \times l \times W}{2 \times 15300} = \frac{l\,W}{850} = b\,d^2.$$

144. *Rule* 1. To find the breadth of a uniform cast iron beam, to bear a given weight in the middle.

Multiply the length of bearing in feet by the weight to be supported in pounds; and divide the product by 850 times the square of the depth in inches; the quotient will be the breadth in inches required.*

145. *Rule* 2. To find the depth of a uniform cast iron beam, to bear a given weight in the middle.

Multiply the length of bearing in feet by the weight to be supported in pounds, and divide this product by 850 times the breadth in inches; and the square root of the quotient will be the depth in inches.

When no particular breadth or depth is determined by the nature of the situation for which the beam is intended, it will be found sometimes convenient to assign some proportion; as, for example, let the breadth be the nth part of the depth, n representing any number at will. Then the rule becomes—

146. *Rule* 3. Multiply n times the length in feet by the weight in pounds; divide this product by 850, and the cube root of the quotient will be the depth required: and the breadth will be the nth part of the depth.

It may be remarked here, that the rules are the same for inclined as for horizontal beams, when the horizontal distance, F F′ fig. 19, is taken for the length of bearing.

* If the bar is to be of wrought iron, divide by 952 instead of 850.
If the beam be of oak, divide by 212 instead of 850.
If it be of yellow fir, divide by 255 instead of 850.

147. *Example* 1. In a situation where the flexure of a beam is not a material defect, I wish to support a load which cannot exceed 33,600 lbs. (or 15 tons) in the middle of a cast iron beam, the distance of the supports being 20 feet; and making the breadth a fourth part of the depth.

In this case

$$n = 4 \text{ and } \frac{4 \times 20 \times 33600}{850} = 3162{\cdot}35.$$

The cube root of 3162·35 is nearly 14·68 inches, the depth required; the breadth is

$$\frac{14{\cdot}68}{4} = 3{\cdot}87 \text{ inches.}$$

In practice therefore I would use whole numbers, and make the beam 15 inches deep, and 4 inches in breadth.

148. *Case* 2. When a beam is supported at the ends, but the load is not in the middle between the supports. In this case

$$\frac{W . FB \times F'B}{l} = \frac{f b d^2}{6},$$

(Equation, v. art. 136,) consequently

$$\frac{4 FB \times F'B \times W}{850\, l} = b\, d^2.$$

149. *Rule.* Multiply the distance F B in feet (see fig. 19) by the distance F′ B in feet, and 4 times this product, divided by the whole length F F′ in feet, will give the effective leverage of the load, which being used instead of the length in any of the rules to Case 1, Prob. I., the breadth and depth may be found by them.

150. *Example.* Taking the same example as the last, except that, instead of placing the 15 tons in the middle, it is to be applied at 5 feet from one end; therefore we have F B = 5 feet, and consequently

$$F'B = 15 \text{ feet; and } \frac{5 \times 15 \times 4}{20} = 15$$

the number to be employed instead of the whole length in Rule 3. That is,

$$\frac{4 \times 15 \times 33600}{850} = 2372 \text{ nearly};$$

and the cube root of 2372 is nearly 13·34 inches, the depth for the beam, and

$$\frac{13 \cdot 34}{4} = 3 \cdot 33 \text{ inches}$$

for the breadth, or nearly $13\frac{1}{2}$ inches by $3\frac{1}{2}$ inches.

In the former case it was 15 inches by 4 inches.

151. *Case* 3. When the load is uniformly distributed over the length of a beam, which is supported at both ends.

In this case

$$\frac{W\,l}{8} = \frac{f\,b\,d^2}{6};$$

(see Equation ix. art. 139,) hence

$$\frac{l\,W}{2 \times 850} = b\,d^2.$$

The same rules apply as in Case 1, art. 144, 145, and 146, by making the divisor twice 850, or 1700.

152. *Example.* In a situation where I cannot make use of an arch for want of abutments, it is necessary to leave an opening 15 feet wide, in an 18-inch brick wall; required the depth of two cast iron beams to support the wall over the opening; each beam to be 2 inches thick, and the height of the wall intended to rest upon the beam being 30 feet?

The wall contains

$$30 \times 15 \times 1\frac{1}{2} = 675 \text{ cubic feet};$$

and as a cubic foot of brick-work weighs about 100 ℔s., the weight of the wall will be about 67,500 ℔s.; and half this weight, or 33,750 ℔s. will be the load upon one of the beams. Since the breadth is supposed to be given, the depth will be found by Rule 2, art. 145, if 1700 be used as the constant divisor; thus

$$\frac{15 \times 33750}{1700 \times 2} = 149 \text{ nearly.}$$

The square root of 149 is $12\frac{1}{4}$ nearly; therefore each beam should be $12\frac{1}{4}$ inches deep, and 2 inches in thickness. This operation gives the actual strength necessary to support the wall; but I have usually taken double the calculated weight in practice, to allow for accidents.

In this manner the strength proper for bressummers, lintels, and the like, may be determined. But if there be openings in the wall so placed that a pier rests upon the middle of the length of the beam, then the strength must be found by the rule, art. 145. A rule for a more economical form is given in art. 193.

153. *Case* 4. When a beam is fixed at one end, and the load applied at the other; also when a beam is supported upon a centre of motion. By Equation x. art. 110,

$$W l = \frac{f b d^2}{6};$$

and taking l in feet, and f = 15,300 lbs., we obtain

$$\frac{W l}{212 \cdot 5} = b d^2,$$

but the divisor 212 will be always sufficiently near for practice.

154. *Rule* 1. In a beam fixed at one end, take B D for the length, fig. 17, Plate III., or if the beam be supported in the middle, as in fig. 14, Plate II., take B F or B F′ for the length, observing to use the weight which is to act on that end in the calculation. Then calculate the strength by the rules to Case 1, art. 144, 145, and 146, using

$$\frac{850}{4} = 212$$

instead of 850 as a divisor.

Example. By this rule the proportions for the arms of a balance may be determined. Let the length of the arm, from the centre of suspension to the centre of motion, be $1\frac{1}{2}$ feet; and the extreme weight to be weighed 3 cwt., or 336 lbs., and let the thickness be $\frac{1}{10}$th part of the depth. Then by the rule

$$\frac{10 \times 1\cdot5 \times 336}{212} = 24 \text{ nearly.}$$

The cube root of 24 is 2·88 inches, the depth of the beam at the centre; and the breadth will be 0·288 inch.

For wrought iron the divisor is, in this case, 238, and taking the same example,

$$\frac{10 \times 1\cdot5 \times 336}{238} = 21\cdot2.$$

The cube root of 21·2 is 2·77 inches, the depth required; and the breadth is 0·277 inch.

155. *Rule* 2. If the weight be uniformly distributed over the length of the beam, employ 425 as a divisor, instead of 850 in the rules to Case 1, art. 144, 145, 146.

156. *Example.* Required the depth for the cantilevers of a balcony to project 4 feet, and to be placed 5 feet apart, the weight of the stone part being 1000 lbs., the breadth of each cantilever 2 inches, and the greatest possible load that can be collected upon 5 feet in length of the balcony 2200 lbs.?

Here the weight is

$$1000 + 2200 = 3200 \text{ lbs.};$$

and by Rule 2, Case 4,

$$\frac{3200 \times 4}{2 \times 425} = 15\cdot1 \text{ nearly}$$

and the square root of 15·1 is 3·80 nearly, the depth required.

157. *Remark.* The depth thus determined should be the depth at the wall, as A B, fig. 21, Plate III.; and if the breadth be the same throughout the length, the cantilever

will be equally strong in every part, if the under side be bounded by the straight line B C;* therefore, whatever ornamental form may be given to it, it should not be reduced in any part to a less depth than is shown by that line.

158. The strength of the teeth of wheels depends on this case. But since in consequence of irregular action, or any substance getting between the teeth, the whole stress may be thrown upon one corner of a tooth; and it has been shown in art. 111, that the resistance is much less in that case, for then the strength of a tooth of the thickness d would only be

$$\frac{f d^2}{3} = W,$$

if it were everywhere of equal thickness; and to make allowance for the diminution of thickness, we ought to make $\frac{f d^2}{5} = W$. We have also to make an allowance for wear,† which will be ample enough at the rate of $\frac{1}{3}$rd of the thickness; therefore,

$$\frac{f d^2 (1 - \frac{1}{3})^2}{5} = W; \text{ or } \frac{f d^2}{10 \cdot 25} = W.$$

In cast iron $f = 15{,}300$; whence we have, with sufficient accuracy,

$$\frac{W}{1500} = d^2, \text{ or } \left(\frac{W}{1500}\right)^{\frac{1}{2}} = d.$$

Rule. Divide the stress at the pitch circle in lbs. by 1500, and the square root of the product is the thickness of the teeth in inches.

* Emerson's Mechanics, 4to. edit. prop. lxxiii. cor. 2. It was first demonstrated by Galileo, the earliest writer on the resistance of solids. Opere del Galileo, Discorsi, &c., p. 104, tome ii. Bonon, 1655.

† The allowance for wear should be for a velocity of 3 feet per second; and in proportion to the velocity, that is,

$$\text{as } 3 : \tfrac{1}{4} t :: v : \frac{t v}{12}.$$

Hence $\frac{1}{4} t - \frac{1}{12} t v = \frac{1}{4} (1 - \frac{1}{3} v) t$, or $\frac{1}{12} t (3 - v)$, should be deducted from the thickness in the Table, for velocities differing from 3 feet per second.

Example 1. Let the greatest power acting at the pitch circle of a wheel be 6000 lbs. Then

$$\frac{6000}{1500} = 4;$$

and the square root of 4 is 2 inches, the thickness required.

The breadth of teeth should be proportioned to the stress upon them, and this stress should not exceed 400 lbs. for each inch in breadth, when the pitch * is $2\frac{1}{2}$ inches, because the surface of contact is always small, and teeth work irregularly when much worn.

The length of teeth ought not to exceed their thickness, but the strength is not affected by the greater or less length of the teeth.†

A Table of the Thickness, Breadth, and Pitch of Teeth for Wheel Work.

Stress in lbs. at the pitch circle.	Thickness of teeth.	Breadth of teeth.	Pitch ‡ in inches.
lbs.	inches.	inches.	inches.
400	0·52	1	1·1
800	0·73	2	1·5
1200	0·90	3	1·9
1600	1·03	4	2·2
2000	1·15	5	2·4
2400	1·26	6	2·7
2800	1·36	7	2·9
3200	1·43	8	3·0
3600	1·56	9	3·3
4000	1·64	10	3·4
4400	1·70	11	3·6
4800	1·78	12	3·7
5200	1·86	13	3·9
5600	1·93	14	4·0
6000	2·00	15	4·2

159. As good proportions for the teeth of wheels are of much importance in the construction of machinery, I shall

* The surface of contact is nearly in the direct ratio of the pitch, and therefore the breadth for a $2\frac{1}{2}$-inch pitch being given, the breadth for any other teeth will be directly as the stress, and inversely as the pitch.

† On the length and form of teeth for wheels, the reader may consult the Additions to Buchanan's Essays on Mill-work, vol. i. p. 39, edited by Mr. Rennie, 1842; or the Paper on the Teeth of Wheels, by Professor Willis, given at p. 139 of that work, and in the second volume of the Institution of Civil Engineers.

‡ The pitch is the distance from middle to middle of the teeth, and is here made 2·1 times the thickness of the teeth.

illustrate the mode of applying this Table by examples of different kinds.

Case 1. It is a common mode to compute the stress on the teeth of a machine by the power of the first mover, expressed in horses' power, and the velocity of the pitch circle in feet per second. Now, though I have given the stress in pounds in the Table, I have still kept this popular measure in view; and assuming a horse's power to be 200 lbs. with a velocity of 3 feet per second, which we ought to do in calculating the strength of machines,—

Then, the breadth in inches will be equal to the horses' power, to which the teeth are equal, when the velocity of the pitch circle is 1½ feet per second; twice the breadth will be the horses' power when the velocity is 3 feet per second; three times the breadth will be the horses' power when the velocity is 4½ feet per second; four times the breadth will be the horses' power when the velocity is 6 feet per second; five times the breadth will be the horses' power when the velocity is 7½ feet per second; and generally n times the horses' power when the velocity of the pitch circle is n times 1½ feet per second.

Example. Let a steam engine of 10 horses' power be applied to move a machine, and it is required to find the strength for the teeth of a wheel in it, which will move at the rate of 3 feet per second at the pitch circle.

Here then the horses' power should be double the breadth; consequently the breadth will be 5 inches, and according to the Table, the thickness of the teeth 1·15 inches, and pitch 2·4 inches.

And the same strength of teeth will do for any wheel where the horses' power of the first mover, divided by the velocity in feet per second, produces the same quotient. In this example it is 10 divided by 3; and the same strength of teeth will do for 20 divided by 6; 30 divided by 9; and so on. This will be of some advantage in the arrangement of collections of patterns.

Case. 2. When a machine is to be moved by horses, the horses' power should be estimated higher, on account of the jerks and irregular action of horses. We shall not estimate above the strain which often takes place in horse machines, if we rate the horse power at 400 ℔s. with a velocity of 3 feet per second, and make the strength of the teeth accordingly.

But the breadth of the teeth should be made in the same proportion as in the preceding case.

Example. When the horse power is taken at 400 ℔s. with a velocity of 3 feet per second, the stress on the teeth is given for this case in the Table. Thus, in a machine to be moved by four horses, the stress on all the wheels of which the pitch circles move at the rate of 3 feet per second, will be 1600 ℔s., and the pitch should be 2·2 inches, and thickness of teeth 1·03 inches; the breadth half the breadth in the Table, or 2 inches.

Then, for any other velocity, as suppose 6 feet per second, it will be, as

$$6 : 3 : : 1600 : 800.$$

That is, the stress on the teeth from a first mover of four horses is 800 ℔s. when the velocity is 6 feet per second; and the thickness of teeth by the Table is 0·73 inch, and pitch 1·5 inches.

Case 3. It remains now to show the general rule which includes the preceding cases, and appears to me to be a more direct and simple mode of proceeding.

If P be the power of the first mover in pounds, and V the velocity of that power in feet per second, the stress on the teeth of a wheel of which the velocity of the pitch circle is v, will be

$$\frac{P\,V}{v} = W, \text{ the stress on the teeth.}$$

But we cannot always know the velocity of the pitch circle, because it is not in general possible to vary the number of teeth after the pitch is determined, so as to give it the velocity we have assigned to it before the pitch was known.

The calculation may therefore be made with advantage in this manner: Let N be the number of revolutions the axis is to make per minute, on which the wheel is to be placed: and r the radius the wheel should have if the pitch were two inches, then

$$v = \frac{2{\cdot}1\, d\, N\, r}{19{\cdot}09 \times 24} = \frac{d\, N\, r}{218{\cdot}16}.$$

Consequently,

$$\frac{P\,V}{v} = \frac{218{\cdot}16\, P\, V}{d\, N\, r} = W.$$

Hence

$$\frac{218{\cdot}16\, P\, V}{1500\, N\, r} = d^3;\ \text{or}\ \left(\frac{{\cdot}14544\, P\, V}{N\, r}\right)^{\frac{1}{3}} = d.$$

The equation affords this rule.

Rule. Multiply 0·146 times the power of the first mover in pounds by its velocity in feet per second, and divide the product by the number of revolutions the wheel is proposed to make per minute, and by the radius the wheel should have in inches if its pitch were two inches; the cube root of the quotient will be the thickness of the teeth in inches.

Example 1. Suppose the effective force acting at the circumference of a water-wheel to be 300 ℔s. and its velocity 10 feet per second,* it is proposed to find the thickness for the teeth of a wheel which is to make twelve revolutions per minute, and have thirty teeth.

Here,

$$0{\cdot}146 \times 300 \times 10 = 438.$$

And since the radius of a wheel with thirty teeth and a pitch of 2 inches is 9·567 inches;† we have

$$\frac{438}{12 \times 9{\cdot}567} = 3{\cdot}815.$$

* The manner of estimating the effective force, and determining the best velocity for water-wheels, is shown in the Additions to Buchanan's Essays on Mill-work, vol. ii. p. 512-526, second edition; or p. 326-333, in the edit. by Mr. Rennie, 1842. On the subject of water-wheels the reader may consult Mr. Rennie's Preface, p. 22, for a notice of the labours of Poncelet, Morin, &c., and the valuable experiments of the Franklin Institute.

† This is easily ascertained by Donkin's Table of the radii of wheels. See Buchanan's Essays, vol. i. p. 206, second edition; or p. 114, Rennie's edition.

The cube root of 3·815 is very nearly 1·563, the thickness of the teeth required in inches.

Example 2. Let the effective force of the piston of a steam engine be 6875 lbs. and its velocity $3\frac{1}{2}$ feet per second; it is required to determine the strength for the teeth of a wheel to be driven by this engine, which is to have 152 teeth, and make 17 revolutions per minute.

In this case the radius for 152 teeth with a 2-inch pitch is 48·387 inches; therefore,

$$\frac{0 \cdot 146 \times 6875 \times 3 \cdot 5}{17 \cdot 5 \times 48 \cdot 387} = 4 \cdot 15.$$

And the cube root of 4·15 is very nearly 1·6 inches, the thickness of the teeth required.

By referring to the Table it will be found that teeth of this thickness should have a breadth of about 9 inches.

These rules will be found to give proportions extremely near to those adopted by Boulton and Watt, of Soho; Rothwell, Hick, and Rothwell, of Bolton, in Lancashire; and other esteemed manufacturers; which is one of the most gratifying proofs of the confidence that may be placed in the principles of calculation I have followed. The difference is chiefly in the greater breadths I have assigned for the greater strains, and which being a consequence of the principle adopted for proportioning these breadths, I cannot agree to change till it can be shown that the principle is erroneous.

160. *Case* 5. When the pressure upon a beam increases as the distance from one of its points of support. Since the point of greatest strain is at $\sqrt{\frac{1}{3}}\, l$ from the point A, where the strain begins at (see fig. 20), we have by art. 140 and 110,

$$\frac{W\,l}{7 \cdot 75} = \frac{f\,b\,d^2}{6},$$

or when l is in feet, and

$$f = 15300 \text{ lbs.}; \frac{W\,l}{1647} = b\,d^2;$$

a result which differs so little from Case 3, that the same rule may serve for both cases.

161. *Prop.* II. To determine a rule for the diagonal of a uniform square beam to support a given strain in the direction of that diagonal; when the strain does not exceed the elastic force of cast iron.

162. *Case* 1. When a beam is supported at the ends and loaded in the middle,

$$\frac{W\,l}{4} = \frac{f\,a^3}{24},$$

art. 137 and 112; or when l is in feet, and

$$f = 15300 \text{ lbs. } \left(\frac{W\,l}{212}\right)^{\frac{1}{3}} = a.$$

163. *Rule.* Multiply the length in feet by the weight in pounds, and divide the product by 212; the cube root of the quotient is the diagonal of the beam in inches.

164. *Case* 2. When a beam is supported at the ends, and the strain is not in the middle of the length,

$$\frac{W \times F\,B \times F'\,B}{l} = \frac{f\,a^3}{24},$$

art. 112 and 136; or when f = 15,300 lbs. and the length and distances F B, F′ B from the ends are in feet,

$$\left(\frac{W \times F\,B \times F'\,B}{53\,l}\right)^{\frac{1}{3}} = a.$$

165. *Rule.* Multiply the weight in pounds by the distance F B in feet, and multiply this product by the distance from the other end, or F′ B in feet (see fig. 19). Divide the last product by 53 times the length, and the cube root of the quotient will be the diagonal of the beam in inches.

I limit the rules to these cases only, because a beam is seldom placed in the position described in this proposition. Examples are omitted for the same reason.

166. *Prop.* III. To determine a rule to find the diameter

of a solid cylinder, to support a given strain, when the strain does not exceed the elastic force of cast iron.

If the diameter be not uniform, the diameter determined by the rule will be that at the point of greatest strain, and the diameter at any other point should never be less than corresponds to the form of equal strength.

167. *Case* 1. When a solid cylinder is supported at the ends, and the weight acts at the middle of the length,

$$\frac{W\,l}{4} = \frac{\cdot 7854\, f\, d^3}{8},$$

art. 113 and 137; or when l is in feet,

$f = 15300$ lbs. and $d =$ the diameter in inches.

we have

$$\left(\frac{W\,l}{500}\right)^{\frac{1}{3}} = d.$$

168. *Rule.* Multiply the weight in pounds by the length in feet; divide this product by 500, and the cube root of the quotient will be the diameter in inches.*

The figure of equal strength for a solid, of which the cross section is everywhere circular, is that generated by two cubic parabolas, set base to base,† the bases being equal, and joining at the section where the strain is the greatest.

169. *Example.* Required the diameter of a horizontal shaft of cast iron to sustain a pressure of 2000 lbs. in the middle of its length; the length being 20 feet? In this case we have

$$\frac{2000 \times 20}{500} = 80;$$

and the cube root of 80 is 4·31 inches nearly, which is the diameter required.

This is supposed to be a case where the flexure is of no importance, otherwise the diameter must be determined by the rules for flexure.

* For wrought iron divide by 560 instead of 500. For oak divide by 125 instead of 500.

† Emerson's Mechanics, 4to. edit., prop. lxxiii., cor. 4.

170. *Case* 2. When a cylinder is supported at the ends, but the strain is not in the middle of the length. By art. 113 and 136,

$$\frac{W \times FB \times F'B}{l} = \frac{\cdot 7854 f d^3}{8};$$

or when the lengths are in feet, d is the diameter in inches, and $f = 15{,}300$, the equation becomes

$$\left(\frac{4\,W \times FB \times F'B}{500\,l}\right)^{\frac{1}{3}} = d.$$

171. *Rule.* Multiply the rectangle of the segments, into which the strained point divides the beam, in feet, by 4 times the weight in pounds; when this product is divided by 500 times the length in feet, the cube root of the quotient will be the diameter of the cylinder in inches.

The figure of equal strength is the same as in Case 1, art. 168.

172. *Example.* Required the diameter of a shaft of cast iron to resist a pressure of 4000 lbs. at 3 feet from the end, the whole length of the shaft being 14 feet? In this example

$$\frac{3 \times 11 \times 4 \times 4000}{500 \times 14} = 75{\cdot}43.$$

The cube root of 75·43 is nearly 4·23 inches, the diameter required.

173. *Case* 3. When a load is uniformly distributed over the length of a solid cylinder supported at the ends only. By art. 113 and 139,

$$\frac{W\,l}{8} = \cdot 7854 f d^3;$$

therefore, when l is in feet, d the diameter in inches, and $f = 15{,}300$, we have

$$\left(\frac{W\,l}{1000}\right)^{\frac{1}{3}} = d = \tfrac{1}{10}(W\,l)^{\frac{1}{3}}.$$

174. *Rule.* Multiply the length in feet by the weight in

pounds, and $\frac{1}{10}$th of the cube root of the product will be the diameter in inches.*

The figure of equal strength for a uniform load, the section being everywhere circular,† is that generated by the revolution of a curve of which the equation is

$$a(l\,x - x^2)^{\frac{1}{3}} = y.$$

175. *Example.* A load of 6 tons (or 13,440 lbs.) is to be uniformly distributed over the length of a solid cylinder of cast iron, of which the length is 12 feet; required its diameter, so that the load shall not exceed its elastic force?

In this case

$$12 \times 13440 = 161280;$$

and the cube root of 161,280 is 54·44, and $\frac{1}{10}$th of this is 5·444 inches, the diameter required.

176. *Case* 4. When a cylinder is fixed at one end, and the load applied at the other; also, when a cylinder is supported on a centre of motion. By art. 113,

$$W\,l = \cdot 7854\, f\, r^3;$$

therefore, when d is the diameter, l is in feet, and $f =$ 15,300 lbs., we have

$$\left(\frac{8\,W\,l}{1000}\right)^{\frac{1}{3}} = d, \text{ or } \tfrac{1}{5}(W\,l)^{\frac{1}{3}} = d.$$

The figure of equal strength is the same as in Case 1, art. 168.

177. *Rule.* Multiply the leverage the weight acts with, in feet, by the weight in pounds; the fifth part of the cube root of this product will be the diameter required in inches.

The most important application of this case is to determine the proportions for gudgeons and axles; and this application will be best illustrated by an example.

The greatest stress upon a gudgeon or axle takes place when, from any accident, that stress is thrown upon the

* For wrought iron divide by 10·38. † Emerson's Mechanics, prop. lxxiii., cor. 3.

extreme point of its bearing. But besides the greatest possible stress we have to provide for wear; perhaps $\frac{1}{5}$th of the diameter may be allowed for this purpose.

Now taking the length l for the length from the shoulder to the extreme point of bearing in inches, we have

$$\tfrac{1}{5}\left(\tfrac{1}{15}\, l\, W\right)^{\frac{1}{3}} = d\left(1 - \tfrac{1}{5}\right);\ \text{or}\ \tfrac{1}{9}\left(l\, W\right)^{\frac{1}{3}} = d.$$

Whence we have this practical rule: Multiply the stress in pounds by the length in inches, and the cube root of the product divided by 9 is the diameter of the gudgeon in inches.*

Example. Let the stress on the gudgeon be 10 tons, or 22,400 ℔s., and its length 7 inches. Then

$$7 \times 22400 = 156800;$$

and the cube root of this number is 54 nearly; and

$$\frac{54}{9} = 6 \text{ inches}.$$

the diameter required.

But the stress of a gudgeon on its bearings ought to be limited, otherwise they will wear away very quickly: let us suppose this stress to be confined to a portion of the circumference, which is equal to $\frac{3}{4}$ths of the diameter of the gudgeon; and that the pressure is limited to 1500 ℔s. upon a square inch, which is about as great a pressure as we ought to put on the rubbing surfaces when one of them is of gun-metal. In this case we shall have

$$l = \frac{4\, W}{3 \times 15000\, d};\ \text{or}\ l = \frac{W}{1125\, d};$$

and to allow for a small portion of freedom we make

$$l = \frac{W}{1000\, d}.$$

* For wrought iron divide by 9·34. For wheel carriages less than 3-inch axles the length may be 5 times the diameter; then for wrought iron,

$$\frac{\sqrt{W}}{13} = d.$$

Above 3 inches it may be 4 times the diameter.

If this value of l be introduced in the preceding equation we have

$$\frac{1}{90}\left(\frac{W^2}{d}\right)^{\frac{1}{3}} = d; \text{ or } W = 854\, d^2; \text{ and } l = \cdot 854\, d.$$

According to these principles the following Table has been calculated, and I hope it will be useful.

Table of the Proportions of Gudgeons and Axles for different degrees of Stress.

Diameter of gudgeons.	Length of gudgeons.	Stress they may sustain.
inches.	inches.	℔s.
½	·43	213
¾	·64	480
1	·85	854
1½	1·25	1,921
2	1·7	3,416
3	2·5	7,686
4	3·4	13,664
5	4·3	21,350
6	5·1	30,744
7	5·9	41,846
8	6·8	54,656
9	7·7	69,174
10	8·5	85,400

Gudgeons exposed to the action of gritty matter may be made larger in diameter about ⅛th part.

178. *Prop.* IV. To determine a rule for the exterior diameter of a uniform tube or hollow cylinder * to resist a given force where the strain does not exceed the elastic force of cast iron.

179. *Case* 1. When a tube is supported at the ends, and the load acts at the middle of the length. By art. 115 and 137,

$$\frac{W\,l}{4} = \cdot 7854\, f\, d^3\,(1 - N^4);$$

hence, when d is the diameter in inches, l the length in feet, and $f = 15{,}300$ ℔s., we have

$$\left(\frac{W\,l}{500\,(1 - N^4)}\right)^{\frac{1}{3}} = d.$$

180. *General Rule.* Fix on some proportion between the diameters; so that the exterior diameter is to the interior

* A considerable accession of strength and stiffness is gained by making shafts hollow, which has been illustrated in art. 115; but it is difficult to get them cast sound, therefore shafts of this kind require to be carefully proved.

diameter as 1 is to N; the number N will always be a decimal, and ought not to exceed 0·8.*

Then multiply the length in feet by the weight to be supported in pounds. Also, multiply 500 by the difference between 1 and the fourth power of N, and divide the product of the length and the weight by the last product, and the cube root of the quotient will be the diameter in inches.

The interior diameter will be the number N multiplied by the exterior diameter, and half the difference of the diameters will be the thickness of metal.

If the proportion between the exterior and interior diameter be fixed, so that the thickness of metal may be always $\frac{1}{5}$th of the exterior diameter of the tube; then N = ·6; and the rule is

$$\left(\frac{W\,l}{435}\right)^{\frac{1}{3}} = d.$$

And there being no difference between this equation and that for a solid cylinder, except the constant divisor, we have this rule:

Particular Rule. When the thickness of metal is to be $\frac{1}{5}$th of the diameter of the tube, let the diameter be calculated by the rule for a solid cylinder, art. 168, except that 435 is to be used as a divisor instead of 500.

181. *Example.* Let the weight of a water-wheel, including the weight of the water in the buckets, be 44,800 lbs., and the whole length of the shaft 8 feet; from which deducting 5 feet,† the width of the wheel, leaves 3 feet for the length of bearing; required the diameter of a hollow shaft for it?

* In a large shaft there should be a tolerable bulk of metal to secure a perfect casting. Mr. Buchanan, in his "Essay on the Shafts of Mills," vol. i., p. 305, second edition (or page 202-3 in the edition of 1841), describes a hollow shaft of which the exterior diameter was 16 inches, and the interim one 12 inches, therefore

$$16 : 12 :: 1 : N = \frac{12}{16} = ·75.$$

This shaft was for an over-shot water-wheel of 16 feet in diameter.

† The wheel being so framed that the part of the length of the shaft it occupies may be considered perfectly strong.

Making N = ·7, its fourth power is ·2401; and

$$1 - \cdot 240 = \cdot 76.$$

Therefore, by the general rule we have

$$\frac{3 \times 44800}{500 \times \cdot 76} = 354 \text{ in the nearest whole numbers;}$$

and the cube root of 354 is 7 inches, the exterior diameter; and

$$7 \times \cdot 7 = 4 \cdot 9 \text{ inches, the interior diameter.}$$

By the particular rule the computation is easier, for it is

$$\frac{3 \times 44800}{435} = 309;$$

and the cube root of 309 is 6·76 inches, the exterior diameter; and the thickness of metal $\frac{1}{5}$th of this, or $1\frac{3}{8}$ inches nearly.

The particular rule will be found to give a good proportion for the thickness of metal for considerable strains; but in lighter work, where stiffness is the chief object, recourse should be had to the general rule.

182. *Case* 2. When a tube is supported at the ends, but the strain is not in the middle of the length. When the necessary substitutions are made, we have, by art. 115 and 136

$$\left(\frac{4\ W \times FB \times F'B}{500\ l \times (1 - N^4)}\right)^{\frac{1}{3}} = d.$$

183. *Rule.* Multiply the rectangle of the segments into which the strained point divides the beam, in feet, by four times the weight in pounds; call this the first product.

Multiply 500 times the length, in feet, by the difference between 1 and the fourth power of N (N being the interior diameter when the exterior diameter is unity); call this the second product.

Divide the first product by the second, and the cube root of the quotient will be the exterior diameter of the tube in inches.

Or, making the thickness of metal $\frac{1}{5}$th of the diameter, calculate by the rule art. 171, using 435 instead of 500 as a divisor.

184. *Example.* Let the weight of a wheel and other pressure upon a shaft be equal to 36,000 lbs., the distance of the point of stress from the bearing at one end being 3 feet, and the distance from the other bearing 1·5 feet; N being ·8; required the exterior and interior diameter of the shaft?

The fourth power of ·8 is ·409, and

$$1 - \cdot 409 = \cdot 591.$$

Therefore by the rule

$$\frac{3 \times 1\cdot 5 \times 4 \times 36000}{500 \times 4\cdot 5 \times \cdot 591} = 485;$$

and the cube root of 485 is 7·86 inches, the exterior diameter, and

$$7\cdot 86 \times \cdot 8 = 6\cdot 3 \text{ inches, the interior diameter.}$$

Cases 3 and 4 are not likely to occur in the practical application of tubes, but they may be supplied by Cases 3 and 4 for solid cylinders, by dividing the diameter of the solid cylinder by the cube root of the difference between 1 and the fourth power of N; or when the thickness of metal is to be $\frac{1}{5}$th of the diameter, divide by 435 instead of 500.

185. *Prop.* v. To determine a rule for finding the depth of a beam of the form of section shown in fig. 9, Plate I., to resist a given force when the strain does not exceed the elastic force of cast iron.

186. *Case* 1. When the beam is supported at the ends, and the load acts in the middle of the length. By art. 116 and 137,

$$\frac{W\,l}{4} = \frac{f\,b\,d^2}{6}(1 - q\,p^3);$$

or making $l =$ the length in feet, and $f = 15{,}300$ lbs.,

$$\frac{W l}{850} = b\, d^2 (1 - q\, p^3).*$$

187. *Rule.* Assume a breadth $a\, b$, fig. 9, that will answer the purpose the beam is intended for; and let this breadth, multiplied by some decimal q, be equal to the sum of the projecting parts, or, which is the same thing, equal to the difference between the breadth of the middle part and the whole breadth.

Also, let p be some decimal which multiplied by the whole depth will give the depth of the middle or thinner part $e\, f$ in the figure.

Multiply the length in feet by the weight in pounds, and divide this product by 850 times the breadth multiplied into the difference between unity and the cube of p multiplied by q; the square root of the quotient will be the depth in inches.

The figure of equal strength for this case is formed by two common parabolas put base to base, as shown by the dotted lines in fig. 22; for $l : d^2$ a property of the parabola, the other being constant quantities. Fig. 22 shows how it may be modified to answer in practice. When a figure of equal strength is used, the depth determined by the rule is that at the point of greatest strain, as C D in the figure.

188. *Example* 1. Required the depth of a beam of cast iron of the form of section shown in fig. 9, Plate I., to bear a

* If we make $p = \cdot 7$, and $q = \cdot 6$; then,

$$850\,(1 - q\, p^3) = 675;$$

and the rule is

$$\frac{W l}{675} = b\, d^2;$$

and the breadth of the middle part $= \cdot 4\, b$, and the depth of the middle part $\cdot 7\, d$.

When the parts are in these proportions, the strength is to that of the circumscribed rectangular section as 1 : 1·26.

If, with the same proportions, we make the breadth $a\, b$ always one-fifth of the depth $b\, d$, fig. 9, the strength will be to that of a square beam of the same depth as 1 : 6·3; and the stiffness will be in the same proportion.

load of 33,600 lbs. in the middle of the length, the length being 20 feet, and the breadth, *a b*, 3 inches?

Take ·625 for the decimal *q*, and ·7 for the decimal *p*, which are proportions that will be found to answer very well in practice.*

Then

$$\frac{20 \times 33600}{850 \times 3 \times (1 - \cdot 625 \times \cdot 7^3)} = \frac{20 \times 33600}{3 \times 667} = 335{\cdot}4 \text{ nearly};$$

and the square root of 335·4 is 18·4 inches, the depth required.

The depth *b d* being 18·4, the depth *e f* will be

$$18{\cdot}4 \times \cdot 7 = 12{\cdot}88 \text{ inches; also,}$$
$$3 \times \cdot 625 = 1{\cdot}875, \text{ and}$$
$$3 - 1{\cdot}875 = 1{\cdot}125 \text{ inches,}$$

the breadth of the middle part of the section.

Comparing this with the example, art. 147, it will be found that the same weight requires only about ⅔rds of the quantity of iron to support it, when the beam is formed in this manner.

Example 2. The same rule applies to determining the size of the rails for an iron railway, where economy with strength and durability is of much importance. As the weight has to move over the length of the rail, the figure of equal strength is that shown in Plate III., fig. 24, only it should be placed with the straight side upwards.

Suppose the weight of a coal waggon to be about 4 tons, 8960 lbs.; when the rails are shorter than twice the distance between the wheels, the utmost strain on a rail cannot exceed half this weight, or 4480 lbs., which will be allowing half the strength nearly for accidents. The usual length of

* Since

$$850\ (1 - \cdot 625 \times \cdot 7^3) = 677 \text{ nearly};$$

whenever the same proportions are used, the divisor 677 may be employed instead of repeating the calculation.

one rail is 3 feet,* and supposing the breadth to be 2 inches, then, by the manner of calculation shown in the note to art. 186,

$$\frac{\mathrm{W}\,l}{675 \times b} = \frac{4480 \times 3}{675 \times 2} = d^2 = 9{\cdot}96;$$

and the square root of

$$9{\cdot}96 = 3{\cdot}16 \text{ inches},$$

the depth in the middle of the length.

* It is worthy of consideration whether this be the most economical length, or not, for rails. This may be done as follows :

The weight of a bar of iron, an inch square and 700 feet long, is 1 ton; therefore, for a length of 700 feet, the area of the bar in inches multiplied by the price of a ton of iron will be the amount of 700 feet of rail. Make $\frac{700}{x}$ the length of a single rail; then, supposing the rail all of the same thickness,

$$\sqrt{\frac{\mathrm{W} \times 700}{850 \times b\,x}} = d,$$

the depth, and when it is reduced at the ends,

$$\cdot 7\, b \sqrt{\frac{\mathrm{W} \times 700}{850 \times b\,x}} = \text{the area}:$$

and calling A the price of a ton of iron; and B the price of fixing, and materials for one block; then the price of 700 feet will be

$$\cdot 7\, \mathrm{A}\, b \sqrt{\frac{\mathrm{W} \times 700}{850\, b\, x}} + x\,\mathrm{B} = \frac{\cdot 64\, \mathrm{A} \sqrt{\mathrm{W}\, b}}{\sqrt{x}} + \mathrm{B}\,x.$$

Hence by the rules of maxima and minima it appears that the price will be the least when the number of supports for 700 feet is

$$= \left(\frac{\cdot 32\, \mathrm{A} \sqrt{\mathrm{W}\, b}}{\mathrm{B}}\right)^{\frac{2}{3}};$$

wherein W is half the weight of a waggon and its load in lbs.

The same equation will apply to the new railway invented by Mr. Palmer, when W is made the whole weight of the waggon in lbs.

An example will illustrate the application : Let A the price of a ton of iron be £8; B the price of one support £0·5; the weight of a waggon 8960 lbs.; and the breadth of the rail 3 inches. Then

$$\left(\frac{\cdot 32 \times 8 \times \sqrt{8960 \times 3}}{\cdot 5}\right)^{\frac{2}{3}} = 89;$$

that is, there should be 89 supports in 700 feet, in order that the expense may be the least possible at these prices, and for these proportions; which makes the distance of the supports nearly 8 feet. But it should be understood that these prices are only what I have inserted for illustration; they are not from actual estimate.

Also,

$$3{\cdot}16 \times {\cdot}7 = 2{\cdot}212 \text{ inches},$$

the depth of the thin part in the section at the middle of the length, and

$$2 \times {\cdot}4 = 0{\cdot}8 \text{ inch},$$

the thickness of the middle part of the section.

The depth of a rail, all of the same thickness, would be 2·83 inches in the middle, calculated by Rule 2, art. 145.

Example 3. In Palmer's railway a single rail carries the waggon; * and let its weight be 8960 lbs., and the length of the rail 8 feet, its breadth 3 inches. By the rule

$$\frac{W\,l}{675\,b} = \frac{8960 \times 8}{675 \times 3} = 35{\cdot}4.$$

The square root of 35·4 is very nearly 6 inches, the depth required; and the depth of the middle part

$$6 \times {\cdot}7 = 4{\cdot}2 \text{ inches}.$$

The breadth 3 inches, and breadth of middle part

$$3 \times {\cdot}4 = 1{\cdot}2 \text{ inches}.$$

These are the dimensions for the middle of the length; but the under edge should be the figure of equal strength, Plate III., fig. 24, with the straight side upwards.

189. *Case* 2. When the beam is supported at the ends, but the load not applied in the middle between the supports. When l is the length in feet, and $f = 15{,}300$ lbs.,

$$\frac{4\,\text{F B} \times \text{F}'\,\text{B} \times \text{W}}{850\,l\,(1 - p^3\,q)} = b\,d^2$$

by art. 116 and 136.

190. *Rule.* Multiply the rectangle of the segments into which the strained point divides the beam, in feet, by 4, and divide this product by the length in feet; use this quotient instead of the length of the beam, and proceed by the last rule.

* Description of a Railway on a New Principle, by H. R. Palmer, 8vo. London, 1823.

191. *Example.* Let the load to be supported be 33,600 lbs. at 5 feet from one end, the whole length being 20 feet. Also, let the breadth of the widest part *a b*, fig. 9, be 4 inches.

Here F B = 5 feet, therefore F′ B = 15 feet, and

$$\frac{4 \times 5 \times 15}{20} = 15:$$

the multiplier to be used instead of the whole length in the rule.

Let $p = \cdot 7$, and $q = \cdot 625$; then

$$\frac{15 \times 33600}{850 \times 4 \times (1 - \cdot 625 \times \cdot 7^3)} = \frac{15 \times 33600}{4 \times 677} = 189 \text{ nearly,}$$

of which the square root is 13·5 inches, the depth required.

The depth *e f* will be

$$\cdot 7 \times 13 \cdot 5 = 9 \cdot 45 \text{ inches,}$$

and the breadth of the middle part of the section will be

$$4 - \overline{4 \times \cdot 625} = 4 - 2 \cdot 5 = 1 \cdot 5 \text{ inches.}$$

192. *Case* 3. When the load is uniformly distributed over the length of a beam. In this case

$$\frac{W\,l}{1700\,(1 - q\,p^3)} = b\,d^2$$

by art. 116 and 139.

193. *Rule.* Use half the weight instead of the whole weight upon the beam, and proceed by the rule to Case 1, art. 187.

The form of equal strength for this case, when the breadth is uniform, is an ellipse, but in practical cases it will require to be altered to the form shown in fig. 24.

194. I propose to give as an example of this rule, its application to the construction of fire-proof buildings; but it also applies to rafters, girders, bressummers, and all cases where the load is uniformly distributed over the length.

A fire-proof floor is usually formed by placing parallel beams of cast iron across the area in the shortest direction, and arching between the beams as shown by fig. 10, Plate I., with brick or other suitable material. Or they may be done by flat plates of iron resting on the ledges, with one or two courses of bricks paved upon the iron plates; and when the distance of the joists is considerable, the iron plates may be strengthened by ribs on the upper side as the floor plates of iron bridges are made.

When arches are employed, floors of this kind are least expensive when the arches are of considerable span; but then it is necessary to provide against the lateral thrust of the arches by tie bars. Also, since the arches ought to be flat, we can only extend them to a limited span, otherwise they would be too weak to answer the purpose. For instance, when an arch is to rise only $\frac{1}{10}$th of the span, and to be half a brick (or $4\frac{1}{2}$ inches) thick,* the greatest span that can be given to the arch with safety in a floor for ordinary purposes is 5 feet. If the arch rise only $\frac{1}{12}$th of the span, the span must be limited to 4 feet; and if it rise only $\frac{1}{17}$th of the span, it must be limited to 3 feet.

Again, for arches of one brick (or 9 inches), to bear the same load, and the rise $\frac{1}{10}$th of the span, the greatest span that can be given with safety is 8 feet; † when the rise is $\frac{1}{12}$th of the span, 7 feet; and when the rise is only $\frac{1}{17}$th of the span, the greatest span should not exceed 5 feet.

These limits were calculated from the ordinary strength of brick, and on the supposition that the load upon the floor will never be greater than 170 lbs. upon a superficial foot, in addition to the weight of the floor itself. If the load be greater, the span must be less, or the rise greater.‡

For half-brick arches the breadth of the beam *c d*, fig. 9, should be about 2 inches; and for 9-inch arches, from $2\frac{1}{2}$ to 3 inches.

* Rad. of curv. 6·75 feet.

† Rad. of curv. 15·6 feet.

‡ See also Elementary Principles of Carpentry, art. 249 and 270; edition by Mr. Barlow.

Example. It is proposed to form a fire-proof room, but from its situation it cannot be vaulted in the ordinary way on account of the strong abutments required for common vaulting, and also common vaulting is objectionable, because so much space is lost in a low room. The shortest direction across the room is 12 feet, and if iron beams of 3 inches breadth be laid across at 5 feet apart, and arched between with 9-inch brick arches, it is required to find the depth for the beams? See fig. 10, Plate I.

The quantity of brickwork resting upon 1 foot in length of joist will be

$$5 \times {\cdot}75 = 3{\cdot}75 \text{ cubic feet};$$

and the weight of a cubic foot being nearly 100 ℔s., the weight of the brickwork will be 375 ℔s.

But since the space above is to be used, and the greatest probable extraneous weight that will be in the room will arise from its being filled with people, we may take that weight at 120 ℔s. per superficial foot, and we have

$$5 \times 120 = 600 \text{ ℔s.}$$

for the weight on 1 foot in length. And supposing the paving and iron to be 350 ℔s. for each foot in length, the whole load on a foot in length will be

$$375 + 600 + 350 = 1325 \text{ ℔s. or}$$
$$12 \times 1325 = 15900 \text{ ℔s.}$$

the whole weight upon one joist. And as half this weight multiplied by the length, and divided by the breadth and constant number,* is equal to the square of the depth, we have

$$\frac{7950 \times 12}{675 \times 3} = 47{\cdot}11,$$

of which the square root is nearly 7 inches, the depth required. And

$$7 \times {\cdot}7 = 4{\cdot}9 \text{ inches}$$

* See note to Rule, art. 186.

the depth of the middle part, and

$$3 \times \cdot 4 = 1 \cdot 2$$

the breadth of the middle part.

By fixing the breadth, you avoid the risk of calculating for a thinner beam than is sufficient to support firmly the abutting course of bricks.

By means of this example we may easily form a small Table of the depth of beams for fire-proof floors, which will be often useful: in so doing, I shall not regard the difference between the weight of a 9-inch and a 4½-inch floor; because the lighter floor will be more liable to accidents from percussion, and therefore should have excess of strength.

Table of Cast Iron Joists for Fire-proof Floors, when the extraneous load is not greater than 120 lbs. *on a superficial foot (see* FLOORS, *Alphabetical Table.)*

Length of joists in feet.	Half-brick arches, breadth of beams 2 inches.			Nine-inch arches, breadth of beams 3 inches.		
	3 feet span.	4 feet span.	5 feet span.	6 feet span.	7 feet span.	8 feet span.
feet.	Depth in inches.	Depth in inches.	Depth in inches.	Depth in inches.	Depth in inches.	Depth in inches.
8	4¾	5¼	5¾	5¼	5¾	6
10	5½	6½	7	6½	7¼	7½
12	6¾	7¾	8½	7¾	8½	9
14	7¾	9	10	9¼	10	10½
16	9	10¼	11¼	10½	11½	12
18	10	11¾	12¾	11¾	13	13½
20	11¼	13	14	13	14¼	15
22	12¼	14¼	15½	14¼	15¾	16½
24	13¼	15½	17	15½	17	18

For half-brick arches the breadth *a b*, fig. 9, Plate I., is to be 2 inches, and the thickness of the middle part $\frac{8}{10}$ths of an inch; the depth *e f* being $\frac{7}{10}$ths of the whole depth; and the whole depth is given in inches in the Table for each length and span.

For 9-inch arches the breadth *a b*, fig. 9, is to be 3 inches, and the breadth of the middle part 1 inch and $\frac{2}{10}$ths. The depth $\frac{7}{10}$ths of the whole depth, as in the 4½-inch arches.

If the floor be for a room of greater span than about 16 feet, let the beams be put 8 feet apart; and put the beams for

8 feet bearing across at right angles to the other, in the manner of binding joists, and arch between the shorter beams. By casting the shorter beams with flanches at the ends, they can be bolted to the other, and a complete firm floor be made. This method has also the advantage of rendering it extremely easy to fix either a wooden floor or a ceiling.

The construction of these floors renders a place secure from fire without loss of space, and with very little extra expense; it may be of infinite use in the preservation of deeds, libraries, and indeed every other species of property. In a public museum, devoted to the collection and preservation of the scattered fragments of literature and art, it is extremely desirable that they should be guarded against fire; otherwise they may be involved in one common ruin, more dreadful to contemplate than their widest dispersion.

195. *Case* 4. When a beam is fixed at one end, and the load applied at the other. Also, when a beam is supported upon a centre of motion. By art. 116,

$$W\,l = \frac{f\,b\,d^2}{6} \times (1 - p^3\,q),$$

or when l is in feet, and $f = 15{,}300$ lbs.,

$$\frac{W\,l}{212\,(1 - q\,p^3)} = b\,d^2.$$

196. *Rule* 1. Calculate by the rule to Case 1, art. 187, using 212 instead of 850 for a divisor.

Or when the breadth of the middle is made $\frac{4}{10}$ths of the extreme breadth, and the depth $e\,f$ in fig. 9 is $\frac{7}{10}$ths of the whole depth; then, calculate by the rules art. 144, 145, or 146, using 168 instead of 850 as a divisor.

The figure of equal strength is a parabola; see figs. 25 and 26.

197. *Rule* 2. If the weight be uniformly distributed over the length, take the whole load upon the beam for the weight,

and calculate by the rule to Case 1, art. 187, except using 425 instead of 850 as a divisor.

198. *Prop.* VI. To determine a rule for finding the depth of a beam when part of the middle is left open, as in figs. 11, 12, and 27, so that it will resist a given force; the strain not exceeding the elastic force of the material.

199. When the depth is more than 12 or 14 inches, angular parts in the middle become necessary, as in fig. 27; the disposition of the middle part may in a great measure be regulated by fancy, provided it allows of sufficient diagonal and cross ties to bind the upper and lower parts together. The middle parts should be made of the same size as the other, in order that they may not be rendered useless by irregular contraction.

If the beams be required so long as not to be made in a single casting, and it is not a good plan to cast in very long lengths, then they may be joined in the middle, as in fig. 27. The connexion is made at the lower side only; at the upper side let the parts abut against one another, with only some contrivance to steady them while they become fixed in their places and loaded. Fig. 28 is a plan of the under side, showing how the connexion may be made.

200. *Case* 1. When the beam is supported at the ends and the load acts at the middle of the length. By art. 117 and 137,

$$\frac{W\,l}{4}=\frac{f\,b\,d^2}{6}(1-p^3),$$

or making l = the length in feet, and f = 15,300 lbs.,

$$\frac{W\,l}{850\,(1-p^3)}=b\,d^2.$$

Now, in general, we may make $p=\cdot7$, and then,

$$\frac{W\,l}{558}=b\,d^2;$$

or, in practice,*

$$\frac{W\,l}{560} = b\,d^2.$$

$$\text{If } b = \frac{1}{9}\,d, \text{ then } \frac{W\,l}{62} = d^3.$$

201. *Rule.* Multiply the length in feet by the weight to be supported in pounds; and divide this product by 560 times the breadth in inches; the square root of the quotient will be the depth required in inches. Consult art. 41 and 43 respecting the form of beams of this kind. The depth between the upper and lower part of the beam will be ·7 *d* inch, where *d* is the depth found by the rule.

202. *Example.* A beam for a 30-feet bearing is intended to sustain a load of 6 tons (13,440 lbs.) in the middle, the breadth to be 4 inches; required the depth?

By the rule

$$\frac{30 \times 13440}{4 \times 560} = 180;$$

the square root of 180 is nearly 13·5 inches, the whole depth.

The depth between the upper and lower part is

$$\cdot 7 \times 13\cdot 5 = 9\cdot 45 \text{ inches.}$$

If the depth be given, suppose 16 inches, and the breadth be required, then

$$\frac{30 \times 13440}{16 \times 16 \times 560} = 2\cdot 82, \text{ the breadth in inches;}$$

when the depth is 16 inches, and the depth between the upper and lower parts is

$$\cdot 7 \times 16 = 11\cdot 2 \text{ inches.}$$

* If we make $p = 0\cdot 6$, then

$$\frac{W\,l}{135} = d^3; \text{ and } b = 0\cdot 2\,d,$$

and the depth of the section at A B or C D, fig. 11, Plate II., will be the same as the breadth of the beam. And as the equation for a square beam of the same depth is

$$\frac{W\,l}{850} = d^3,$$

the strength of this beam will be to that of the square beam, of the same depth, as 1 : 6·3.

203. *Case* 2. When a beam is supported at the ends, but the load is not applied at the middle.

When l is the length in feet, $p = \cdot 7$, and $f = 15{,}300$ lbs.,

$$\frac{4\,BC \times CD \times W}{558\,l} = b\,d^2,$$

(see fig. 12, Plate II.;) or

$$\frac{BC \times CD \times W}{139\,l} = b\,d^2.$$

204. *Rule.* Multiply the rectangle of the segments into which the strained point divides the beam, in feet, by the weight in pounds, and divide this product by 139 times* the length in feet multiplied by the breadth in inches; the square root of the quotient will be the depth required in inches.

The depth between the upper and lower side will be ·7 × by the whole depth. Consult art. 41 and 43 respecting the form, &c. of beams of this kind.

205. *Example.* Let C B, fig. 12, be 10 feet, and D C, 6 feet: and therefore B D the length, 16 feet; and the weight to be supported at A, 20,000 lbs., the breadth of the beam being 2 inches; required the depth?

By the rule

$$\frac{10 \times 6 \times 20000}{139 \times 16 \times 2} = 270;$$

and the square root of 270 is 16½ inches nearly.

Also,

$$\cdot 7 \times 16\cdot 5 = 11\cdot 55 \text{ inches}$$

= the depth from a to b in fig. 12.

206. *Case* 3. When a load is distributed uniformly over the length of a beam. When the length is in feet, $p = \cdot 7$, and $f = 15{,}300$ lbs.,

$$\frac{W\,l}{1116} = b\,d^2,$$

by art. 117 and 139.

* In practice it will be sufficiently accurate to use 140 for a divisor.

207. *Rule.* Multiply the whole weight in pounds by the length in feet; divide this product by 1116 times the breadth in inches, and the square root of the quotient will be the depth in inches.

Multiply this depth by ·7, which will give the depth between the upper and lower parts. Respecting the form of the beam, see art. 41.

208. *Example.* It is required to support a wall, 20 feet in height, and 18 inches in thickness, over an opening 26 feet wide, by means of two beams of cast iron, each three inches in thickness; required the depth?

Suppose a cubic foot of brick-work to weigh 100 lbs.; then

$$20 \times 1{\cdot}5 \times 26 \times 100 = 78000 \text{ lbs.}$$

the weight of the wall.

Therefore by the rule

$$\frac{78000 \times 26}{1116 \times 6} = 303 \text{ nearly;}$$

and the square root of 303 is 17½ inches, the depth required.

The depth between the upper and lower parts is

$$\cdot 7 \times 17{\cdot}5 = 12{\cdot}25 \text{ inches.}$$

209. *Case* 4. When a beam is fixed at one end, and the load applied at the other. Also, when the load acts at one end of a beam supported on a centre of motion. By art. 117 we have, when the length is in feet, $p = \cdot 7$, and $f = 15{,}300$ lbs.,

$$\frac{W\,l}{139} = b\,d^2.$$

210. *Rule.* Calculate by the rule to Case 1, art. 201, using 140 instead of 560 for a divisor.

If the weight be uniformly distributed over the length of a beam fixed at one end, divide the weight by 2, and proceed as above directed.

DEFLEXION FROM CROSS STRAINS.

211. *Prop.* VII. To determine a rule for finding the deflexion of a cast iron beam, when the section is rectangular, and uniform throughout the length; the strain being 15,300 lbs. upon a square inch.

The same rules will apply to solid and hollow cylinders, to beams formed as figs. 9, 11, 12, and 26, when they are uniform throughout their length, and the depth used as a divisor is the greatest depth.

212. *Case* 1. When a beam is supported at the ends, and loaded in the middle, as in fig. 1.

By art. 121,

$$\frac{2\,\epsilon\, l^2}{3\,d} = \text{the deflexion},$$

when l=half the length; therefore,

$$\frac{3\,d \times \mathrm{D\,A}}{2\,l^2} = \epsilon =$$

the greatest extension of an inch in length while the elastic force remains perfect. According to the experiment described in art. 56, the elastic force was perfect when the bar was loaded with 300 lbs.; hence we have

$$\frac{3\,d \times \mathrm{D\,A}}{2\,l^2} = \frac{3 \times 1 \times \cdot 16}{2 \times 17^2} = \frac{1}{1204} = \cdot 00083 \text{ inch}$$

$=\epsilon$ the extension of an inch in length, by a force equal to 15,300 lbs. upon a square inch; or generally, cast iron is extended $\frac{1}{1204}$ part of its length by a force equal to 15,300 lbs. upon a square inch.

If this value of ϵ be substituted in the equation, and l be made the whole length in feet, we have

$$\frac{2 \times \cdot 00083 \times 12^2 \times l^2}{3 \times 4 \times d} = \mathrm{D\,A}, \text{ or}$$

$$\frac{\cdot 01992\, l^2}{d} = \mathrm{D\,A};$$

hence it appears that the equation

$$\frac{\cdot 02\, l^2}{d} = \mathrm{D\,A}$$

may be used without sensible error.

Consequently, the deflexion of an uniform rectangular beam supported at the ends may be determined by the following rule:

213. *Rule.* Multiply the square of the length in feet by ·02; and this product divided by the depth in inches is equal to the deflexion in inches.

214. *Example.* Required the deflexion in the middle of a beam 20 feet long, and 15 inches deep, when strained to the extent of its elastic force?

By the rule

$$\frac{\cdot 02 \times 20^2}{15} = \cdot 533 \text{ inch;}$$

therefore a beam loaded as in example (art. 147), will bend more than half an inch in the middle. If it be wished to reduce it to a quarter of an inch, double the breadth.

The deflexion of an uniform beam may also be found by Table II. art. 6.

215. *Case* 2. When a uniform rectangular beam is supported at the ends, and the load is equally distributed over the length. It has been shown in art. 139, Equation viii., that in this case the strain at any point is as the rectangle of the segments into which that point divides the beam; and the deflexion for that case is calculated by art. 126, Equation vii. And by comparing Equation ii. and vii.

$$\frac{2}{3} : \frac{5}{6} :: \frac{\cdot 02\, l^2}{d} : \frac{\cdot 025\, l^2}{d}.$$

Therefore the deflexion D A in the middle of a beam uniformly loaded is $= \frac{\cdot 025\, l^2}{d}$.

216. *Rule.* Multiply the square of the length in feet by ·025; and the quotient, from dividing this product by the depth in inches, will be the deflexion in the middle in inches.

217. *Example.* Let it be required to determine the deflexion that may be expected to take place in the example to Case 3, Prop. I. art. 152, where the length is 15 feet and the depth $12\frac{1}{4}$ inches?

By the rule

$$\frac{15 \times 15 \times \cdot 025}{12 \cdot 25} = \cdot 46 \text{ inch},$$

the deflexion required.

218. This mode of calculation may often remove groundless alarm, as well as inform us when a structure is dangerous; for if a beam be loaded so as to bend more than is determined by the rule which applies to it, the structure may be justly deemed insecure. We also, by this mode of calculation, have an easy method of trying the goodness of a beam: for let it be loaded with any part, as for example $\frac{1}{4}$th of the weight it should bear, then the deflexion ought to be $\frac{1}{4}$th of the calculated deflexion. When a beam is tried by loading it with more than the weight it is intended to bear, it may be so strained as to break with the lesser weight, besides the difficulty and danger in trying such an experiment.

219. *Case* 3. When a beam is fixed upon a centre of motion, and the force applied at the other end, the flexure of the fixed part being insensible. The cranks of engines are in this case.

The flexure will be the same as in Case 1, art. 212, but the length of the beam being only half the length in that case, we have

$$\frac{\cdot 08\, l^2}{d} = \text{D A the deflexion.}$$

220. *Case* 4. If a uniform rectangular beam be fixed at one end, and the force be applied at the other, the deflexion of the end where the force is applied will be

$$\frac{\cdot 08\, l^2}{d} \times (1 + r).$$

For the deflexion from the extension of the projecting part of the beam is $\frac{\cdot 08\, l^2}{d}$, where l is the length of that part in feet;

and if r be equal the $\frac{\text{length of fixed part}}{l}$, then, by Equation, iii. art. 133,

$$\frac{\cdot 08\ l^2}{d} \times (1 + r) = \text{the deflexion.}$$

221. *Rule.* Divide the length of the fixed part of the beam by the length of the part which yields to the force, and add 1 to the quotient; then multiply the square of the length in feet by the quotient so increased, and also by ·08; this product divided by the depth in inches will give the deflexion in inches.

222. *Example.* Conceive a beam, A B, fig. 26, to be uniform, and to be the beam of a pumping engine, the end B working the pumps, and the end A where the power acts 10 feet from the centre of motion, the end B 7 feet from the centre of motion, and the strain at B equal to the elastic force of the beam; through how much space will the point A move before the beam transmits the whole power to B, the depth of the beam being 12 inches?

In this case,

$$\frac{7}{10} = \cdot 7, \text{ and } 1 + \cdot 7 = 1\cdot 7;$$

therefore,

$$\frac{1\cdot 7 \times 10 \times 10 \times \cdot 08}{12} = 1\cdot 33 \text{ inches.}$$

223. *Prop.* VIII. To determine a rule for finding the deflexion of a cast iron beam, of uniform breadth, when the outline of the depth is a parabola, the strain being equal to 15,300 lbs. per square inch.

The same rules will apply to beams of the form of section shown in figs. 9 and 11, when the breadth is uniform.

224. *Case* 1. When a beam is supported at the ends, and the load is applied in the middle.

The deflexion for this case is calculated in art. 123, Equation iv.; and comparing it with the deflexion of a uniform beam we have

$$\frac{2}{3} : \frac{4}{3} :: \frac{\cdot 02\ l^2}{d} : \frac{\cdot 04\ l^2}{d} = \text{the deflexion.}$$

225. *Rule.* Multiply the square of the whole length of the beam in feet by ·04; divide the product by the middle depth in inches, and the quotient will be the deflexion in inches.

226. *Example.* Let the depth of a beam be 18·4 inches, and its length 20 feet, which is on the supposition that the beam, of which the depth is found by example to Case 1, Prop. v. art. 188, is parabolic. By the rule

$$\frac{20 \times 20 \times \cdot 04}{18 \cdot 4} = \cdot 87 \text{ inch},$$

the deflexion required.

If the beam were of uniform depth, the deflexion would be only half this quantity, or ·435.

227. *Case* 2. If a parabolic beam of uniform breadth be fixed at one end, and the force be applied at the other, the deflexion of the end where the force is applied will be

$$\frac{\cdot 16\, l^2}{d}(1 + r),$$

where l is the length of the part the force acts on in feet, and r = the quotient arising from dividing the length of the fixed part by the length l.

228. *Rule.* Divide the length in feet of the fixed part of the beam by the length in feet of the part which yields to the force, and add 1 to the quotient. Then multiply the square of the length in feet by the quotient so increased, and also by ·16; divide this product by the middle depth in inches, and the quotient will be the deflexion in inches.

229. *Example.* Let A B, fig. 26, be the beam of a steam engine, the moving force acting at A, and the resistance at B, C being the centre of motion; when A C = 12 feet, and C B = 10, and the depth in the middle 30 inches; it is required to determine the space the point A bends through before the full action is exerted on B, the strain being equal to the elastic force of the material?

In this case the length of the part C B, which may be considered as fixed, is 10 feet, and

$$\frac{10}{12} = \cdot 833, \text{ and } 1 + \cdot 833 = 1\cdot 833;$$

therefore,

$$\frac{12 \times 12 \times 1\cdot 833 \times \cdot 16}{30} = \frac{12 \times 22 \times \cdot 16}{30} = 1\cdot 408 \text{ inches,}$$

the deflexion of the point A.

Few people are aware of the extent of flexure in the parts of engines, and particularly when they are executed in a material which has been considered as nearly inflexible. In a well contrived machine, the importance of making the parts capable of transmitting motion and power with precision and regularity must be so obvious, that it appears almost incredible how much the laws of resistance have been neglected.

230. *Prop.* IX. To determine a rule for finding the deflexion of a cast iron beam of uniform breadth, when the depth at the end is only half the depth at the middle, the strain being equal to 15,300 ℔s. on a square inch.

231. *Case* 1. When a beam is supported at the ends, and the load is applied in the middle. By art. 127, Equation viii.,

$$\frac{1\cdot 09\, \epsilon\, l^2}{d} = \text{D A the deflexion};$$

when this is compared with Equation ii. art. 121, we have

$$\frac{2}{3} : 1\cdot 09 : : \frac{\cdot 02\, l^2}{d} : \frac{\cdot 0327\, l^2}{d} = \text{D A the deflexion.}$$

232. *Rule.* Multiply the square of the length in feet by ·0327, and the product divided by the depth in the middle in inches will give the deflexion in inches.

233. *Case* 2. When a beam is fixed at one end, and the force is applied at the other. In this case

$$\frac{\cdot 13\, l^2}{d}(1 + r) = \text{the deflexion.}$$

234. *Rule.* Calculate the deflexion by the rule, art. 228, except changing the multiplier to ·13 instead of ·16.

235. *Prop.* x. To determine a rule for finding the deflexion of a beam, generated by the revolution of a cubic parabola, the strain being equal to 15,300 ℔s. on a square inch.

The same rules will apply to any cases where the sections are similar figures, and the cube of the depth every where proportional to the leverage the force acts with.

236. *Case* 1. When a beam is supported at the ends, and the load is applied in the middle.

By art. 124, Equation v.,

$$\frac{6\,\epsilon\, l^2}{5\,d} = \text{D A the deflexion};$$

and comparing this with Equation ii. we have

$$\frac{2}{3} : \frac{6}{5} :: \frac{\cdot 02\, l^2}{d} : \frac{\cdot 036\, l^2}{d} = \text{the deflexion.}$$

237. *Rule.* Substitute ·036 in the place of ·04 in the rule to Prop. VIII. art. 225, and then calculate the deflexion by that rule.

238. *Case* 2. When a beam is fixed at one end, and the force acts at the other.

In this case

$$\frac{\cdot 144\, l^2}{d} = \text{the deflexion.}$$

239. *Rule.* In the rule to Prop. VIII. art. 228, use ·144 instead of ·16 as a multiplier, and calculate the deflexion by that rule, so altered.

240. *Prop* XI. To determine a rule for finding the deflexion of a cast iron beam, of uniform breadth, the depth being bounded by an ellipse; the strain being equal to 15,300 ℔s on a square inch.

If the Equations ii. and vi. be compared, it will be found that

$$\frac{2}{3} : \cdot 857 :: \frac{\cdot 02\, l^2}{d} : \frac{\cdot 0257\, l^2}{d} = \text{the deflexion.}$$

241. *Rule.* The deflexion may be calculated by the rule to Prop. VIII. art. 225, if the multiplier ·0257 be employed instead of ·04.

242. *Prop.* XII. To determine a rule for the deflexion of a beam of uniform depth, when the breadth is bounded by a triangle, the strain upon a square inch being 15,300 lbs.

From Equations ii. and iii. art. 121 and 122, we have

$$\frac{2}{3} : 1 :: \frac{\cdot 02\, l^2}{d} : \frac{\cdot 03\, l^2}{d} = \text{the deflexion.}$$

243. *Case* 1. When a beam is supported at the ends, and loaded in the middle.

244. *Rule.* Calculate by the rule to Prop. VIII. art. 225, using ·03 instead of ·04 as a multiplier.

245. *Case* 2. When a beam is supported at one end, and fixed at the other.

In this case

$$\frac{\cdot 12\, l^2}{d} = \text{the deflexion.}$$

246. *Rule.* Calculate the deflexion by the rule to Prop. VIII. art. 228, using ·12 as a multiplier instead of ·16.

247. The rules derived from the twelve preceding propositions are applicable to any kind of material. For example, let it be required to adapt any one of the rules for oak: in the Alphabetical Table at the end of this Essay, art. OAK, it appears that oak is 0·25 as strong as cast iron; therefore, in a rule for strength, multiply the constant number by 0·25. Thus in the rules to Prop. I. Case 1,

$$850 \times 0{\cdot}25 = 212{\cdot}5,$$

the number to be used in these rules when the material is oak.

Again, oak is 2·8 times as extensible as cast iron; consequently the deflexion being found for cast iron, 2·8 times that deflexion will be the deflexion of oak, when it is strained to the extent of its elastic power.

SECTION VIII.

OF LATERAL STIFFNESS.

248. *Definitions.* The *stiffness* of a body is its resistance at a given deflexion. And the *lateral stiffness* is the stiffness to resist cross pressure.

249. *Prop.* XIII. *To determine the stiffness of a uniform bar or beam, of which the section is a rectangle, when fixed at one end, to resist a weight at the other; or supported in the middle on a centre to support a stress at each end.*

When a beam is strained to the extent of its elastic force, we have the weight it will bear, or

$$W = \frac{f\,b\,d^2}{6\,l},$$

(by art. 110,) and the deflexion under that strain will be

$$\frac{2\,\epsilon\,l^2}{3\,d} \times (1 + r),$$

(by art. 121 and 133, Equation iii.) Then, since the deflexion is proportional to the strain, if a be the given deflexion, and w the weight which produces it, we have

$$(1+r)\frac{2\,\epsilon\,l^2}{3\,d} : a :: \frac{f\,b\,d^2}{6\,l} : w = \frac{f\,b\,d^3\,a}{4\,\epsilon\,l^3\,(1+r)};$$

and because $\frac{f}{\epsilon} = m$, (art. 105,) we have

$$\frac{4\,w\,l^3\,(1+r)}{a\,m} = b\,d^3. \qquad \text{(i.)}$$

If the length be in feet, then

$$\frac{6912\,w\,L^3\,(1+r)}{a\,m} = b\,d^3. \qquad \text{(ii.)}$$

Now in cast iron $m = 18,400,000$ lbs.; therefore

$$\frac{w\,L^3\,(1+r)}{2662\,a} = b\,d^3. \qquad \text{(iii.)}$$

Where L=the length in feet, *a* the deflexion in inches, *b* and *d* the breadth and depth in inches, and *w* the weight in pounds; and *r*=the length of the fixed part divided by L. When $r=1$, the lengths are equal, and $(1+r)=2$.

250. If the fixed part be of considerable bulk in respect to the other, we may neglect its effect on the deflexion, and in that case

$$\frac{w\,L^3}{2662\,a} = b\,d^3. \qquad \text{(iv.)}$$

If in any of the preceding equations the breadth be diminished while the depth is uniform, the flexure will be increased; and when the outline of the breadths becomes a triangle, this increase is half the deflexion of a beam of uniform breath; or the deflexions with the same strain are, as 2 : 3 (art. 122).

If the breadth be everywhere the same, but the beam be made the parabolic one of equal strength, then the deflexion will be twice as great as that of a beam of uniform depth (art. 123), and the general Equation iii. becomes

$$\frac{w\,L^3\,(1+r)}{1331\,a} = b\,d^3. \qquad \text{(v.)}$$

If the breadth be everywhere the same, but the outlines of the depth be straight lines, and the depth at either of the extremities half the depth at the point of greatest strain, then the deflexion is to that of a beam of uniform depth as 1·635 : 1 (art. 127); and Equation iii. becomes

$$\frac{w\,L^3\,(1+r)}{1628\,a} = b\,d^3. \qquad \text{(vi.)}$$

I shall illustrate this proposition by examples of its application to beams of pumping engines, cranks, and wheels.

BEAMS FOR PUMPING ENGINES.

251. *Example.* Let it be required to determine the breadth and depth of a beam for a pumping engine, its whole length being 24 feet, and the parts on each side of the centre of motion equal; and the straining force 30,000lbs., the deflexion not to exceed 0·25 inch.

First, on the supposition that the beam is to be uniform, then, by Equation iii., art. 249,

$$\frac{w\,L^3\,(1+r)}{2662\,a} = \frac{30000 \times 12^3 \times (1+1)}{2662 \times \cdot 25} = b\,d^3 = 155790.$$

If the breadth be made 5 inches, the depth should be 31·5 inches; for

$$31{\cdot}5^3 \times 5 = 156279,$$

which very little exceeds 155,790.

But if the depth at the middle be double the depth at either end, use 1628 as a divisor instead of 2662; and calculating by Equation vi. we find $b\,d^3 = 254{,}742$, and if the breadth be 5 inches, the depth should be 37 inches.

CRANKS.

252. *Example.* If the force acting upon a crank be 6000lbs., and its length be 3 feet, to determine its breadth and depth so that the deflexion may not exceed $\frac{1}{10}$th of an inch.

To this case, Equation iv., art. 250, applies, and

$$\frac{W\,L^3}{2662\,a} = \frac{6000 \times 3^3}{2662 \times \cdot 1} = 653 = b\,d^3.$$

If the breadth be made 3 inches, the depth should be 6 inches, for the cube of $6 \times 3 = 648$.

When the depth at the end where the force acts is half the depth at the axis, use 1628 instead of 2662 for a divisor.

WHEELS.

253. For wheels, if N be the number of arms, or radii, our equation should be

$$\frac{W L^3}{2662 N a} = b d^3.$$

254. *Example* 1. Let the greatest force acting at the circumference of a spur-weel be 1600lbs., the radius of the wheel 6 feet, and number of arms 8; and let the deflexion not exceed $\frac{1}{10}$ of an inch.

Then by the Equation, art. 253,

$$\frac{W L^3}{2662 N a} = \frac{1600 \times 6^3}{2662 \times 8 \times \cdot 1} = b d^3 = 163.$$

If we make the breadth 2·5 inches, then

$$\frac{163}{2 \cdot 5} = 65 \cdot 2 = d^3;$$

and the cube root of 65·2=4·03 inches, nearly, for the depth or dimension of each arm, in the direction of the force.

When the depth at the rim is intended to be half that at the axis, use 1628 as a divisor instead of 2662 for a divisor.

If a wheel be strained till the arms break, the fractures take place close to the axis; there is a sensible strain at the part of the arm near the rim, but it is so small in respect to that at the axis, that its effect is neglected in our rule.

Example 2. When the stress on the teeth is 1090lbs. Suppose the wheel to be 4 feet radius, with 6 arms; and that a flexure of $\frac{2}{10}$ths of an inch will not sensibly affect the action of the wheel-work. Also, let the arms be diminished in depth so as to be only half the depth at the rim of the wheel; the breadth being fixed at 2 inches.

By the Equation, art. 253, we have

$$\frac{W L^3}{1628\, b\, a\, N} = \frac{1090 \times 64}{2 \times 1628 \times 6 \times \cdot 2} = 18 \text{ nearly} = d^3.$$

But the cube root of 18 is 2·62 inches; consequently next the axis the arms should be 2 by 2·62, and at the rim 2 by 1·31, in order that the play in applying the power may not exceed $\frac{2}{10}$ths of an inch. This rule gives the quantity of iron, with the rectangular section, but let it be disposed in the form of greatest strength consistent with that required for casting.

Again, let the pinion to be moved by the preceding wheel have a radius of 0·75 foot, with four arms, and the breadth of the arm 2 inches; the angular motion produced by the flexure being the same as above; that is, if

$$4 : \cdot 2 :: \cdot 75 : \cdot 0375 = a.$$

Then,

$$\frac{W L^3}{1628\, b\, a\, N} = \frac{4\, W L^2}{1628\, b\, N \times \cdot 2}; \text{ or,}$$

$$\frac{4 \times 1090 \times \cdot 56}{1628 \times 2 \times 4 \times \cdot 2} = \cdot 94 = d^3.$$

The cube root of ·94 is ·98 nearly, for the thickness of the arm at the axis.

255. I think we may in most cases allow a flexure of $\frac{2}{10}$ths of an inch for a wheel of 4 feet radius for the effect of the arms, and other $\frac{2}{10}$ths for the flexure of the shafts. In consequence, therefore, of such an arrangement, the strength of the arms will be expressed by a more simple equation; as well as the strength of the shafts, to be treated in the section on Torsion.

When the flexure is 0·2 for a radius of 4 feet, it is very nearly a quarter of a degree; and with this degree of flexure, the arms of equal breadth, and the depth at the rim half the depth at the axis, we have

$$\frac{W L^2}{81\, N} = b\, d^3 \qquad \text{(vii.)}$$

Hence we have this practical rule. Multiply the stress at the pitch line in lbs. by the square of the radius in feet; and divide the product by 81 times the breadth multiplied by the number of arms; and the cube root of the quotient will be the depth of the arm at the axis, and half this depth will be the depth at the rim.

If the thickness of the rim be made equal to the thickness of the teeth, and the breadth be proportioned by the Table, art. 158, then the number of arms should be $1\frac{1}{2}$ times the radius of the wheel in feet, divided by the square of the thickness of the teeth in inches, taken in the nearest whole numbers: it is usual to make an even number of arms; but there does not appear to be any reason for adhering to this practice. Wheels are often broken in the rim by wedging them on to the shaft; but the practice of fixing the wheels on by wedges has now given way to a much superior one, which consists in boring the eye truly cylindrical, and the shaft being turned to fit the eye, the wheel is retained in its place by a slip of iron, fitted into corresponding grooves in the shaft and in the eye of the wheel.

256. *Prop.* XIV. *To determine the stiffness of a uniform bar, or beam, supported at the ends, to resist a cross strain in the middle.*

If a beam be rectangular and uniform; then, making a the greatest deflexion that it ought to assume, we have by Equation ii., art. 121, and art. 137,

$$\frac{2\epsilon l^2}{12 d} : a :: \frac{2 f b d^2}{3 l} : w = \frac{4 f b d^3 a}{\epsilon l^3}.$$

And as

$$m = \frac{f}{\epsilon}, \text{(art. 105;)}\ w = \frac{4 m b d^3 a}{l^3}.^{*} \qquad \text{(viii.)}$$

* By some error of computation, Professor Leslie makes this equation

$$\frac{8 m b d^3 a}{5 l^3} = w,$$

(Elements of Nat. Phil. vol. i. p. 237;) and, consequently, draws an erroneous measure of the modulus of elasticity from the experiments in art. 53 and 56. The equation I have arrived at is the same as Dr. Young had previously determined, (see his Nat. Philos. vol. ii. art. 326,) $2e$ in his equation being $= l$ in mine, and $2f = w$.

When L = the length in feet, then,

$$w = \frac{m\,b\,d^3\,a}{432\,L^3}. \qquad \text{(ix.)}$$

This equation answers for any material of which the weight of the modulus of elasticity is known; and this will be found in the Alphabetical Table at the end of this Work, for almost every kind of material in use. Its application to cast iron will be sufficient for an example.

The weight of the modulus for cast iron is 18,400,000lbs., and, dividing this number by 432, we have for cast iron

$$w = \frac{42600\,b\,d^3\,a}{L^3}. \qquad \text{(x.)}$$

257. If $a = \frac{L}{40}$ of an inch, or the deflexion be as many fortieths of an inch as there are feet in the length of the beam; then the equation will be

$$w = \frac{1065\,b\,d^3}{L^2};$$

which was made

$$\cdot 001\,w\,L^2 = b\,d^3$$

to calculate the Table, art. 5. (xi.)

When the deflexion is only as many 100ths of an inch as the beam is feet in length, a deflexion which should not be greatly exceeded in shafts, on account of the irregular wear on their gudgeons and bearings when the flexure is greater, then

$$\frac{426\,b\,d^3}{L^2} = w. \qquad \text{(xii.)}$$

If the load be uniformly distributed over the length of a uniform rectangular beam; then from art. 126 and 139, the dimensions being all in inches, we have

$$\frac{5\,\epsilon\,l^2}{24\,d} : a :: \frac{4\,f\,b\,d^2}{3\,l} : w = \frac{32\,f\,b\,d^3\,a}{5\,\epsilon\,l^3}.$$

And since

$$m = \frac{f}{\epsilon};\ w = \frac{32\,m\,b\,d^3\,a}{5\,l^3}. \qquad \text{(xiii.)}$$

Comparing this equation with Equation viii., it appears that a weight uniformly distributed will produce the same depression in the middle as $\frac{5}{8}$ths of that weight applied in the middle, as has been otherwise shown by Dr. Young.* and Messrs. Barlow,† Dupin,‡ and Duleau.§

When w is the weight of the beam itself; then p being the weight of a bar of the same matter 12 inches long, and 1 square,

$$w = \frac{l\,b\,d\,p}{12};$$

and the deflexion of a beam by its own weight is

$$a = \frac{5\,\epsilon\,p\,l^4}{12 \times 32\,d^2 f} = \frac{5\,l^4}{384\,\text{M}\,d^2}. \qquad \text{(xiv.)}$$

Where M is the height of the modulus of elasticity ‖ in feet (Equation v., art. 105).

258. In a uniform solid cylinder, the strength is to that of a square beam as 1 : 1·7 nearly (art. 113); therefore, by Equation x., art. 256, we have

$$\frac{w\,\text{L}^3}{25000\,a} = d^4. \qquad \text{(xv.)}$$

Where L is the length between the supports in feet, d the diameter in inches, and a the deflexion in inches produced by the weight w in lbs.

If the load be uniformly diffused over the length, and s be the load on 1 foot in length in lbs.; then $w = \text{L}\,s$, and the effect will be the same as if $\frac{5}{8}$ths of this load were applied in the middle (art. 257); consequently

$$\text{L}\left(\frac{s}{40000\,a}\right)^{\frac{1}{4}} = d. \qquad \text{(xvi.)}$$

Therefore, if the load on a foot in length be the same, the

* Nat. Phil. vol. ii. art. 325 and 329.

† Treatise on the Strength of Timber, &c., art. 55. ‡ Idem, p. 97.

§ Essai Théorique et Expérimental, art. 2 et 5.

‖ In theory this seems to furnish the most simple mode of obtaining the modulus, but it is not so accurate in practice, because it is difficult to ascertain the exact degree of flexure due to the weight.

diameter should be increased in direct proportion to the length, so that the flexure may be the same.

If in Equation xv. we make the flexure proportional to the length, and so that it may be $\frac{1}{100}$th of an inch for each foot in length.

Then,

$$\frac{w\,L^2}{250} = d^4; \text{ or}$$

$$\sqrt{L\left(\frac{w}{250}\right)^{\frac{1}{4}}} = d. \qquad \text{(xvii.)}$$

This equation will apply to uniform solid cylindrical shafts.

259. In a hollow shaft or cylinder, it will be only necessary to fix on what aliquot part of the diameter the thickness of metal should be, if its diameter were 1. Then, the difference between twice the thickness of metal and 1, will be the aliquot parts to be left hollow; and calling these parts n, it will be

$$\frac{d}{(1-n^4)^{\frac{1}{4}}} =$$

the diameter of a hollow shaft of the same stiffness as the solid one of the diameter d. (See Equation xviii., art. 115.) And the weight a solid shaft will sustain multiplied by $(1-n^4)$ will be the weight a hollow one of the same diameter will sustain.

Examples. If the thickness of metal be fixed at $\frac{1}{5}$th of the diameter, then

$$1 - \frac{2}{5} = \frac{3}{5} = n = \cdot 6, \text{ and}$$

$$(1 - \cdot 6^4)^{\frac{1}{4}} = \cdot 966 = \frac{1}{1\cdot 0352}.$$

And if the diameter of a solid cylinder be found by Equation xv., xvi., or xvii., as the nature of the subject may require, and the diameter so found be multiplied by 1·0352, it will give the diameter of a hollow tube that will be of the same stiffness, the hollow part being $\frac{3}{5}$ths of the whole diameter.

In the same manner, if the thickness of metal be $\frac{1}{6}$th of the diameter, multiply by 1·056.

And if the thickness of metal be $\frac{3}{20}$ths of the diameter, multiply by 1·07.

The weight a hollow cylinder will sustain when the thickness of metal is exactly $\frac{1}{5}$th of the diameter, is 0·87 the weight a solid cylinder of the same external diameter would sustain with the same pressure; for $(1-n^4) = {\cdot}87$. And its stiffness is to that of a square prism of the same depth as 1 is to 2, nearly.

260. *Example.* Required the diameter of a solid cylindrical shaft, 21 feet in length, that would not be deflected more than half an inch by a weight of 31 cwt., or 3472lbs., applied in the middle.

By Equation xv., art. 258,

$$\frac{w\,L^3}{25000\,a} = \frac{3472 \times 21^3}{25000 \times {\cdot}5} = d^4 = 2572, \text{ or } d = 7{\cdot}12 \text{ inches.}$$

the diameter required.

261. *Example.* Required the diameter of a hollow shaft, 21 feet in length, the interior diameter $\frac{7}{10}$ths of the exterior one, that would not be deflected more than half an inch by a load of 3472lbs. applied in the middle of the length?

Find the diameter of the solid cylinder, as in the preceding example, and multiply it by 1·07 (see art. 259). That is,

$$7{\cdot}12 \times 1{\cdot}07 = 7{\cdot}62 \text{ inches,}$$

the diameter required; the thickness of metal will be $\frac{3}{20}$ths of the diameter.

SECTION IX.

RESISTANCE TO TORSION.

262. *Definition.* The resistance which a shaft or axis offers to a force applied to twist it round is called the resistance to *Torsion.*

263. If a rectangular plate be supported at the corners A and B, fig. 29, Plate IV., and a weight be suspended from each of the other corners C D, then the strains produced by loading it in this manner will be similar to the twisting strain which occurs in shafts, and the like. In a cast iron plate the fractures would take place in the directions A B and C D at the same time; but, before the fracture, the one of the strains would serve as a fulcrum for the other; and the resistance to the forces at C and D would be sensibly the same as if the plate were supported upon a continued fulcrum in the direction A B.

Hence the strain may be considered a cross strain of the same nature as has been investigated in art. 108, and d D or c C the leverage the force at D or C acts with, the breadth of the strained section being A B.

To find the breadth of the section of fracture, and the leverage in terms of the length and breadth of the plate, we have A B, the breadth, and by similar triangles,

$$\frac{\mathrm{A\,D} \times \mathrm{B\,D}}{\mathrm{A\,B}} = \mathrm{D}\,d \text{ the leverage.}$$

These values of the leverage and breadth being substituted in the Equation, art. 110, it becomes

$$W=\frac{f\,b\,d^2}{6\,l}=\frac{f\,d^2\times A\,B\times A\,B}{6\times A\,D\times B\,D};$$

or because

$$A\,B^2=B\,D^2+A\,D^2,$$

we have

$$W=\frac{f\,d^2}{6}\times\frac{B\,D^2+A\,D^2}{A\,D\times B\,D}.$$

264. But when a force acts upon a shaft, it is usually at the circumference of a wheel upon that shaft, and if R be the radius of the wheel, then

$$\frac{2\,R\,W}{B\,D}=$$

the force collected at the surface of the shaft; and therefore, substituting this in the place of W, in the Equation above, we have

$$\frac{2\,R\,W}{B\,D}=\frac{f\,d^2}{6}\times\frac{B\,D^2+A\,D^2}{A\,D\times B\,D};$$

$$\text{or, } W=\frac{f\,d^2}{12\,R}\times\frac{B\,D^2+A\,D^2}{A\,D}.$$

If the length A D be l feet, and the leverage R be in feet; then for cast iron f = 15,300 lbs., and we have

$$\frac{8{\cdot}85\,d^2\,(b^2+144\,l^2)}{R\,l}=W. \qquad \text{(i.)}$$

But this equation has a *minimum* value when $l=\frac{b}{12}$; therefore the resistance will be the same whatever the length may be, provided the length be not less than the breadth. Consequently, whenever the length exceeds the breadth, we have

$$\frac{212{\cdot}4\,d^2\,b}{R}=W.^{*} \qquad \text{(ii.)}$$

* In malleable iron the equation will be

$$\frac{238\,d^2\,b}{R}=W;\ \text{for } 212{\cdot}4\times1{\cdot}12=238.$$

But when b is to d in a less ratio than $\sqrt{2} : 1$ the shaft will not bear so great a strain, and it will bear least when its section is exactly square.

265. When a shaft is square, and its length l in feet, its side d in inches, and the leverage R in feet, then, from Equation, art. 112, we obtain

$$W = \frac{f d^3}{3456 R l} \times (2 d^2 + 144 l^2).$$

And when $f = 15{,}300$ lbs.,

$$W = \frac{8 \cdot 85 d^2}{R l} \times (d^2 + 72 l^2). \qquad \text{(iii.)}$$

In a square shaft also the resistance has a minimum value; that is, when $\sqrt{72}\, l = d$; hence, whenever the length is greater than the diagonal of the section, the strength will be

$$\frac{150 d^3}{R} = W.^* \qquad \text{(iv.)}$$

Where R is the radius of the wheel in feet to which the power W in pounds is to be applied, and d is the side of the shaft or axis in inches.

266. In a cylindrical shaft the section of fracture is an ellipse, and when l and R are in feet, and $f = 15{,}300$, d being the diameter of the shaft in inches, we have by art. 114,

$$W = \frac{5 \cdot 2 d^2}{R l} \times (d^2 + 144 l^2). \qquad \text{(v.)}$$

267. Here again it may be shown, by the principles of maxima and minima, that there is a particular line of fracture where the resistance to torsion is a minimum; in a cylindrical body this happens when $12\, l = d$; that is, when the length is equal to the diameter.

Consequently, in all cases where the length exceeds the

* In malleable iron shafts the equation will be

$$\frac{168 d^3}{R} = W.$$

diameter, the Equation in art. 266 should be applied in the form

$$\frac{124{\cdot}8\ d^3}{R} = W.^* \qquad \text{(vi.)}$$

As the equation reduces to this form by substituting $\frac{d}{12}$ for l.

268. In the same manner it may be shown that in a tube or hollow cylinder of which the length is greater than the diameter, the resistance to torsion is expressed by the equation

$$\frac{124{\cdot}8\ d^3 (1-n^4)}{R} = W. \qquad \text{(vii.)}$$

Where d is the exterior diameter in inches, and $n\,d$ the interior diameter.

It will be a good proportion in practice to make $n = 0{\cdot}6$; then the rule becomes

$$\frac{108\ d^3}{R} = W. \qquad \text{(viii.)}$$

Where d is the exterior diameter in inches, and the thickness of metal is exactly $\frac{1}{5}$th of the diameter; R, as before, being the radius of the wheel in feet, to the circumference of which the power W in lbs. is applied.

269. *Example.* Let it be required to find the diameter of a shaft for a water-wheel, the radius of the water-wheel 9 feet, and the greatest force that it will be exposed to at the circumference, 2000 lbs.

If the shaft is to be a solid cylinder, then the diameter will be found by Equation vi. art. 267; that is,

$$\frac{W\ R}{124{\cdot}8} = \frac{2000 \times 9}{124{\cdot}8} = 144{\cdot}2 = d^3.$$

And the cube root of 144·2 is $5\frac{1}{4}$ inches, the diameter required.

If the shaft is to be a hollow cylinder, Equation viii. will apply, where

* For malleable iron make $\frac{140\ d^3}{R} = W.$

$$\frac{W R}{108} = \frac{2000 \times 9}{108} = 166{\cdot}7 = d^3.$$

And the cube root of 166·7 is $5\frac{1}{2}$ inches the diameter, when the thickness of metal is $\frac{1}{5}$th of this diameter.

270. But the lateral stress on a shaft will always be greater than the twisting force, when the length of the shaft exceeds $\frac{1}{4}$th of the radius of the wheel; yet the preceding equations will often be of use in calculating the strength of journals,* and these calculations should be made by Equation vi. in the same manner as in the example in the preceding article; only as an allowance for wear the diameter should be $\frac{1}{6}$th greater than is given by the rules.

271. The preceding investigation has been confined to the strength to resist twisting, but in shafts of great length in respect to their diameters, the effect of flexure is considerable.

Let ϵ be the extension the material will bear without injury when the length is unity. This extension must obviously limit the movement of torsion, or the angle of torsion. But, since the line of greatest strain, in a bar of greater length than its diameter, is always in the direction of the diagonal of a square; if a square were drawn on the surface of the bar in its natural state, it would become a rhombus by the action of the straining force, and the quantity of angular motion would be nearly $\sqrt{2}$ times the extension of the diagonal; or twice the extension of the length of the bar. For if a line were wound round the bar at an angle of 45° with the axis, its length would be $l\sqrt{2}$; l being the length of the bar in feet. Therefore, $l \epsilon \sqrt{2} =$ the extension, and $2 l \epsilon$ the arc described in feet, or $24 l \epsilon =$ the arc in inches. But if a be the number of degrees in an arc, and $\frac{d}{2}$ its radius; ·0174533 being the length of an arc of one degree when its radius is unity; we have

* A journal is different from a gudgeon only in being exposed to a considerable twisting strain.

$$24\,l\,\epsilon = \frac{a\,d}{2} \times \cdot 0174533\text{; or}$$

$$\frac{2750\,l\,\epsilon}{d} = a. \qquad \text{(ix.)}$$

That is, the angle of torsion a is as the length and extensibility of the body directly, and inversely as the diameter.

If the value of ϵ be taken for cast iron, that is, $= \frac{1}{1204}$, we have

$$\frac{2 \cdot 284\,l}{d} = a.^{*} \qquad \text{(x.)}$$

Here l is the length of the shaft or other body in feet; d its diameter in inches, and a the angle of torsion in degrees of a circle.

Example. Thus, let the vertical shaft of a mill be 30 feet in length, and the diameter 10 inches; then, when it is strained to the extent of its elastic force,

$$\frac{2 \cdot 284 \times 30}{10} = 6\tfrac{3}{4} \text{ degrees nearly.}$$

In certain cases this degree of twisting may be of considerable advantage in preventing the shocks incidental to machines moved by wind, horses, or other irregular powers; but in other cases it will be a disadvantage, because the motion will neither be so accurate, nor so certain to produce the desired effect.

272. Since the angle of torsion is as the extension, it will be as the straining force; and to estimate the stiffness of a body to resist torsion, we have this analogy when the body is a hollow cylinder; from Equation vii. and ix. of this section,

$$\frac{2750\,l\,\epsilon}{d} : a :: \frac{124 \cdot 8\,d^3\,(1-n^4)}{R} : W = \frac{124 \cdot 8\,d^4\,a\,(1-n^4)}{2750\,R\,l\,\epsilon}.$$

Or more generally,

* In malleable iron, $\epsilon = \frac{1}{1400}$; therefore, $\frac{1 \cdot 965 l}{d} = a.$

$$\frac{f\,d^4\,a\,(1-n^4)}{336600\,R\,l\,\epsilon}=W.$$

And if m be the weight of the modulus of elasticity (art. 105,)

$$\frac{m\,d^4\,a\,(1-n^4)}{336600\,l\,R}=W. \qquad \text{(xi.)}$$

When $n=o$ the equation applies to a solid cylinder.

When a shaft is rectangular, the analogy from Equation ii. and ix. becomes

$$\frac{2750\,l\,\epsilon}{b}:a::\frac{212{\cdot}4\,d^2\,b}{R}:W=\frac{212{\cdot}4\,d^2\,b^2\,a}{2750\,R\,l\,\epsilon};\text{ or}$$

$$\frac{m\,d^2\,b^2\,a}{198900\,R\,l}=W. \qquad \text{(xii.)}$$

We have now to show the application of these equations, and to form practical rules from them. The value of m for cast iron is, 18,400,000 lbs.; consequently Equation xi. applied to cast iron is

$$\frac{55\,d^4\,a\,(1-n^4)}{l\,R}=W.^* \qquad \text{(xiii.)}$$

And equation xii. gives

$$\frac{92{\cdot}5\,d^2\,b^2\,a}{l\,R}=W.^\dagger \qquad \text{(xiv.)}$$

PRACTICAL RULES AND EXAMPLES FOR THE STIFFNESS OF CYLINDRICAL SHAFTS TO RESIST TORSION.

273. In practical cases there will be known the length of the shaft, the power, and the leverage the power acts with; and there must be fixed, by the person who applies the rule, the number of degrees of torsion that will not affect the action of the machine; this being settled, the diameter of the shaft will be determined by the rule.

* In malleable iron,

$$\frac{74\,d^4\,a\,(1-n^4)}{R}=W.$$

† In malleable iron,

$$\frac{124\,d^2\,b^2\,a}{l\,R}=W.$$

Rule 1. To determine the diameter of a solid cylinder to resist torsion, with a given flexure.

Multiply the power in pounds by the length of the shaft in feet, and by the leverage in feet. Divide this product by 55 times the number of degrees in the angle of torsion, which is considered best for the action of the machine; and the fourth root of the quotient will be the diameter of the shaft.

Example. Let it be required to find the diameter for a series of lying shafts 30 feet in length to transmit a power equal to 4000 lbs. acting at the circumference of a wheel of 2 feet radius, so that the twist of the shafts on the application of the power may not exceed one degree?

Here the whole length must be taken as if it were one shaft, and by the rule,

$$\frac{4000 \times 30 \times 2}{55 \times 1} = 4364,$$

and by a Table of powers,* the fourth root is found to be 8·13 inches, the diameter required.

If the machinery be required to act with much precision, this will be as much flexure as can be allowed; but in ordinary cases two degrees might be admitted, and then a little less than 7 inches would be the diameter.

Where there is much wheel-work, the flexures should be less; indeed it does not appear to be desirable to exceed a quarter of a degree for the shafts or axes.

274. *Rule* 2. To determine the diameter of a hollow cylinder to resist torsion, when the thickness of metal is $\frac{1}{5}$th of the diameter, and the flexure given.

Multiply the power in pounds by the length of the shaft in feet, and by the leverage the power acts with in feet. Divide the product by 48 times the angle of flexure in degrees; the fourth root of the quotient will be the diameter required in inches.

* See Barlow's Mathematical Tables, Table III.

Example. Let the diameter of a hollow shaft be determined, so that it may be sufficient to withstand a force of 800 lbs. acting at the circumference of a wheel of 4 feet radius with a flexure of one degree; the thickness of metal to be $\frac{1}{5}$th of the diameter, and the length 10 feet.

In this case

$$\frac{800 \times 10 \times 4}{48 \times 1} = 666{\cdot}6;$$

and the fourth root of 666·6 is 5·1 inches nearly; which is the diameter required.

PRACTICAL RULE AND EXAMPLE FOR THE STIFFNESS OF SQUARE SHAFTS TO RESIST TORSION.

275. Rules for square shafts are applications of Equation xiv.; and the same things are known as in the case of cylindrical shafts.

Rule. To determine the side of a square shaft to resist torsion with a given flexure.

Multiply the power in pounds by the leverage it acts with in feet, and also by the length of the shaft in feet. Divide this product by 92·5 times the angle of flexure in degrees, and the square root of the quotient will be the area of the shaft in inches.

Example. Suppose the length of a shaft is to be 12 feet, and it is to be driven by a power of 700 lbs. acting on a pinion, on the shaft, of 1 foot radius to the pitch line, and that a flexure of 1 degree will not affect the machinery.

By the rule,

$$\frac{700 \times 1 \times 12}{92{\cdot}5 \times 1} = 90{\cdot}8.$$

The square root of 90·8 is 9·53, the area of the section in inches; and the square root of 9·53 is 3·1 inches nearly, for the side of the shaft.

The reader may find further information on the torsion of wires, and the laws of the oscillation of the torsion balance, in Dr. Young's Lectures on Nat. Philos. vol. i. pp. 140, 141; Dr. Brewster's Edinburgh Encyclopædia, art. Mechanics, p. 544 to 549; Dr. Brewster's edition of Ferguson's Lectures, vol. ii. p. 234; or Professor Leslie's Elements of Nat. Philos. vol. i. p. 243.

SECTION X.

OF THE STRENGTH OF COLUMNS, PILLARS, OR OTHER SUPPORTS COMPRESSED, OR EXTENDED, IN THE DIRECTION OF THEIR LENGTH.

276. If the length of a column be considerable with respect to its diameter, under a certain force it will bend; but when it becomes too short to bend, its strength is only limited by the force which would crush it. Considering, however, that it is imprudent to load even a short column beyond its elastic force, an inquiry respecting the phenomena of crushing would lead to nothing useful.

Let A A′ be a column, fig. 30, supported at A′, and supporting a load at A; and let this load have produced its full effect in straining the column. Let E be the neutral axis, B and D the centres of resistance, and A F the direction of the straining force. Draw d D parallel to A F, then, by the principles of statics, we have

$$d\mathrm{D} : \mathrm{DA} :: \mathrm{W}\ (\text{the weight}) : \frac{\mathrm{W.DA}}{d\mathrm{D}} =$$

the compressive force in the direction A D. Also,

$$\mathrm{DA} : \mathrm{AF} :: \frac{\mathrm{W.DA}}{d\mathrm{D}} : \frac{\mathrm{W.AF}}{d\mathrm{D}} =$$

the vertical pressure at D.

But, by similar triangles,

$$\mathrm{BD} : \mathrm{BF} :: d\mathrm{D} : \mathrm{AF} = \frac{\mathrm{BF}.d\mathrm{D}}{\mathrm{BD}},$$

therefore

$$\frac{W.\,A\,F}{d\,D} = \frac{W.\,B\,F}{B\,D}. \qquad \text{(i.)}$$

277. In a similar manner it may be proved that the strain at B is expressed by

$$\frac{W.\,\overline{B\,F - B\,D}}{B\,D}. \qquad \text{(ii.)}$$

Where it is obvious that when B D = B F this strain is nothing; that is, when the direction of the straining force passes through the point D, or the neutral axis coincides with the surface of the block. It also may be observed, that when B F exceeds B D, this strain is expressed by a positive quantity, indicating extension; but when B F is less than B D it is negative, indicating that it is a resistance to compression. If B F $= \frac{1}{2}$ B D, then both points are equally compressed.

The force has been supposed to be perpendicular to the plane of section for which the strains have been calculated, but this is not essential to the investigation; it is only the most usual strain on columns and ties. For instead of the force acting in the direction A F, let the force act in the direction A G; and let the angle F A G be denoted by C. Then, the investigation being resumed, we shall find Equation i. will become

$$\frac{(B\,F + A\,F.\,\sin.\,C)\,W.\,\cos.\,C}{B\,D} = \text{stress at D}. \qquad \text{(iii.)}$$

And,

$$\frac{(B\,F + A\,F.\,\sin.\,C - B\,D)\,W.\,\cos.\,C}{B\,D} = \text{stress at B}. \qquad \text{(iv.)}$$

But when the force acts in an oblique direction, a further stay is required to prevent the pillar overturning; and wherever this stay is placed there the greatest strain would be in a straight pillar. If it be stayed at D F, then the stay placed there becomes a point of support; and the action of the forces on the beam are similar to those already considered in fig. 14, Plate II. But it was remarked in a note to art. 108, that the mode of calculation there given is not correct when the beam is not nearly horizontal; the difference is owing to a change of the position of the neutral axis caused by the oblique direction of the force. The position of that axis, and the strength of the section, we will now proceed to calculate, and to develope the changes produced by altering the direction of the straining force.

278. It may be shown that the resistance of the section, on either side of the neutral axis, is equal to the force of a square inch multiplied by the area of that section and by the distance of the centre of gravity from the neutral axis, and divided by the distance of the compressed surface from the neutral axis, when B or D is the centre of percussion of the section.*

279. Let x be the distance of the neutral axis from the middle of the depth; $y = \mathrm{E\,G}$ the distance of the direction A G of the straining force from the middle of the depth; $d =$ the depth, $b =$ the breadth, and f the resistance of a square inch; then the area of the compressed part of the section will be $(\frac{1}{2} d + x)\, b$, and the extended part of the section $(\frac{1}{2} d - x)\, b$. Therefore, if $n\,(\frac{1}{2} d + x)$ and $n\,(\frac{1}{2} d - x)$ be the distances of the centres of percussion from the neutral axis, and $m\,(\frac{1}{2} d + x)$ and $m\,(\frac{1}{2} d - x)$ the distances of the centres of gravity, we shall have

$$\frac{\mathrm{W.\,B\,G\,cos.\,C}}{\mathrm{B\,D}} \times \frac{m\,f\,b\,(\frac{1}{2} d - x)^2}{(\frac{1}{2} d + x)} = \frac{\mathrm{W.\,(B\,G - B\,D)\,cos.\,C}}{\mathrm{B\,D}} \times$$

$$\frac{m\,f\,b\,(\frac{1}{2} d + x)^2}{(\frac{1}{2} d + x)}, \text{ or}$$

$$\mathrm{B\,G} \times (\tfrac{1}{2} d - x)^2 = (\mathrm{B\,G} - \mathrm{B\,D}) \times (\tfrac{1}{2} d + x)^2;$$

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* Emerson's Mechanics, 4to. edit. Prop. LXXVII.

and when the proper substitutions are made, this equation reduces to

$$x^2 (2 - 3n) + 2yx - \tfrac{1}{4} n d^2 = 0.$$

280. In a rectangular section $n = \frac{2}{3}$, and consequently we find by the preceding equation

$$x = \frac{d^2}{12y}.*$$

Also, since in this case $m = \frac{1}{2}$, we have an equilibrium between the compressing force and the resistance to compression, when

$$\frac{\text{W. B G. cos. C}}{\text{B D}} = \frac{f b}{2} (\tfrac{1}{2} d + x);$$

and substituting for B G, B D, and x their proper values, this equation becomes

$$W = \frac{f b d^2}{(d + 6y) \text{ cos. C}}. \qquad \text{(v.)}$$

But if B F be denoted by a; and $\frac{l}{2}$ = A F, whence F G will be =

$$\frac{\text{A F. sin. C}}{\text{cos. C.}} = \frac{l \text{ sin. C}}{2 \text{ cos. C}};$$

and therefore,

$$y = a + \frac{l \text{ sin. C}}{2 \text{ cos. C}}.$$

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* It is shown that

$$y = a + \frac{l \text{ sin. C}}{2 \text{ cos. C}} = a + \tfrac{1}{2} l \text{ tan. C};$$

hence the distance of the neutral axis from the axis of the column is

$$x = \frac{d^2}{12 (a + \frac{1}{2} l \text{ tan. C})};$$

and these axes must coincide when C is an angle of 90°, that is, when the direction of the force is perpendicular to the axis of the column; but not in any other case.

For when C = 90°, the tan. C is unlimited; and consequently the fraction which represents x is incomparably small, or the axes coincide.

Consequently,

$$\frac{f\,b\,d^2}{(d + 6\,y)\ \cos.\ C} = \frac{f\,b\,d^2}{d\ \cos.\ C + 6\,a.\ \cos.\ C. + 3\,l\ \sin.\ C} = W. \qquad \text{(vi.)}$$

This equation will enable us to trace the particular conditions of this important problem.

In the first place, if the points E and A be in a line perpendicular to B G, then $a = o$, and the equation is

$$\frac{f\,b\,d^2}{d.\ \cos.\ C + 3\,l\ \sin.\ C} = W. \qquad \text{(vii.)}$$

Secondly, if the force act in a direction parallel to B G, then C = 90 degrees; and sin. C = 1, and cos. C = *o*, and Equation vi. becomes

$$\frac{f\,b\,d^2}{3} = W. \qquad \text{(viii.)}$$

We have in this case the same equation as in art. 110, for in this instance l is double the length taken in that equation.

Thirdly, if the force act in a direction perpendicular to B G, then cos. C = 1, and sin. C = *o*, and consequently Equation vi. becomes

$$\frac{f\,b\,d^2}{d + 6\,a} = W. \qquad \text{(ix.)}$$

Fourthly, when $a = o$, or the direction of the force coincides with the axis E, then

$$f\,b\,d = W. \qquad \text{(x.)}$$

And, fifthly, if a = half the depth of the block, then

$$\frac{f\,b\,d}{4} = W. \qquad \text{(xi.)}$$

The Equations ix., x., and xi., apply to short columns, or blocks, of which the length is not more than ten or twelve times the least dimension of the section; and from them are derived the following practical rules:

TO FIND THE AREA OF A SHORT RECTANGULAR COLUMN OR BLOCK TO RESIST A GIVEN PRESSURE.

281. *Rule.* When the force is to be applied exactly in the axis or centre of the section of the block, divide the pressure or the weight in pounds by 15,300, and the quotient will be the area of the section of the block in inches. But since this requires a degree of precision in adjusting the direction of the force which it is altogether impossible to arrive at in practice, and when a force presses a block of which *a á* is the axis, fig. 31, Plate IV., it is always probable that the direction A A′ of the force may act upon one edge only of the end of the block, and therefore be at a distance of half the least thickness from the axis; which will reduce the resistance of the block to $\frac{1}{4}$th, and consequently the area should always be made four times as great as is determined by this rule.

When the distance of the direction of the force from the axis is determined by the nature of the construction, the following is a general rule.

282. *Rule.* To the thickness (or least dimension of the section) in inches, add six times the distance of the direction of the force from the axis in inches, and let this sum be multiplied by the weight or pressure in pounds; divide the quotient by 15,300 times the square of the least thickness in inches, and the quotient will be the breadth of the block in inches.

This rule is the Equation ix., art. 280, in words at length, and it applies to resistance to tension as well as to resistance to compression.

283. The writer of the article 'Bridge,' in the Supplement to the Encycl. Brit., has shown that when the force acts in the direction of the diagonal of the block, as is shown in fig. 32, the strain will be twice as great as when

the same force acts in the direction of the axis.* Now the reader will be satisfied, that, in consequence of settlements, or other causes, a column is always liable to be strained in this manner; and therefore will carefully avoid enlarging the ends of his columns, under the notion of gaining stability, for the effect of the straining force will be still more increased by such enlargement in the event of a change of direction from settlement, as in fig. 33. In my 'Treatise on Carpentry,' I have recommended circular abutting joints to lessen the effect of a partial change in the position of the strained pieces,† an idea which appears to have occurred, in the first instance, to Serlio.‡

284. A general solution of the equation expressing the stress and strain, when the column is cylindrical, is complicated, but in one particular case the result is extremely simple; that is, when the neutral axis is in one of the surfaces of the column. If d be the diameter of the column, then $\cdot 7854\, d^2 =$ the area, and $\frac{1}{2} d =$ the distance of the centre of gravity, and therefore

$$\frac{W . B G . \cos . C}{B D} = \frac{\cdot 7854\, d^2 f}{2}.$$

But when the neutral axis is in the surface of the cylinder,

$$B G = B D, \text{ or } W = \frac{\cdot 7854\, d^2 f}{2 \cos . C}.$$

In this case the distance of the direction of the force from the axis of the column will be $\frac{1}{8}$th of the diameter, the centre of percussion being $\frac{5}{8} d$ distant from the neutral axis.

285. Hence it appears, that when the distance of the direction of the force from the axis is $\frac{1}{8} d$, the strength of a cylinder is to that of a circumscribed square prism, as seven

* Napier's Supp. to Encycl. Brit., art. 'Bridge,' Prop. I. p. 499.
† Tredgold's Elementary Prin. Carpentry, Sect. IX. p. 164.
‡ Serlio's Architecture, Lib. I. p. 13. Paris, 1545.

times the area of the cylinder, to eight times the area of the prism; or nearly as 5·5 : 8, or as 1 : 1·46 nearly.

When the neutral axes are at or near the axes of the pieces, the ratio of the strength of the cylinder to that of the prism becomes

$$\frac{3 \times \cdot 7854}{4} : 1, \text{ or as } 1 : 1\cdot 7,$$

as has been shown by Dr. T. Young;* consequently in a column, when both the resistances to compression and extension are brought into action, the ratio varies between 1 : 1·46 and 1 : 1·7; the mean being nearly 1 : 1·6.

OF THE STRENGTH OF LONG PILLARS AND COLUMNS.

286. If a support be compressed in the direction of its length, and the deflexion be sufficient to sensibly increase the distance of the direction of the force from the axis, in the middle of the length of the support, it is evident that the strain will be increased; and since the curvature in practical cases will be very small, we may suppose it to be an arc of a circle. In a circle the square of the length of the chord, in a small segment, is sensibly equal to the radius × 8 times the versed sine; or $\frac{l^2}{8\,\delta}$ = radius. The deflexion will be greatest when the neutral axis coincides with the axis, and taking this extreme case, we shall have this analogy;—as the alteration of the length of the concave side is to the original length, so is the ½ depth to the radius of curvature; or,

$$\epsilon : 1 :: \frac{d}{2} : \text{radius} = \frac{d}{2\,\epsilon}.$$

Therefore

$$\frac{l^2}{8\,\delta} = \frac{d}{2\,\epsilon}; \text{ and } \delta = \frac{l^2\,\epsilon}{4\,d} = \text{the deflexion in the middle.}$$

* Dr. Young's Lectures on Nat. Philos. vol. ii. art. 389, B.

287. Let the distance of the direction of the force from the axis, when first applied, be denoted by a, as in a preceding article (art. 280); then, in consequence of the flexure, it will be equal to

$$a + \frac{l^2 \epsilon}{4 d};$$

consequently by Equation ix., we have

$$\frac{f b d^2}{d + 6 a + \frac{6 l^2 \epsilon}{4 d}} = \text{W}. \qquad \text{(xii.)}$$

In cast iron $f = 15{,}300$lbs. and $\epsilon = \frac{1}{1204}$ (art. 143 and 212); therefore, if l be the length in feet, b, d, and a in inches, we obtain the following practical formula, for the strength of a rectangular prism, viz.

$$\frac{15300 \, b \, d^2}{d + 6 a + \frac{\cdot 18 \, l^2}{d}} = \frac{15300 \, b \, d^3}{d^2 + 6 \, d \, a + \cdot 18 \, l^2} = \text{W}.^* \qquad \text{(xiii.)}$$

288. If $a = o$, or the direction of the force coincides with the axis, then the rule becomes

$$\frac{15300 \, b \, d^3}{d^2 + \cdot 18 \, l^2} = \text{W}. \qquad \text{(xiv.)}$$

It would, however, be improper in practice to calculate upon the nice adjustment of the direction of the pressure in the direction of the axis, which is supposed in the preceding equation; indeed, there are very few instances where its direction may not in all probability be at the distance of half the depth from the axis, and in that case $a = \frac{1}{2} d$, and

* For malleable iron,

$$\frac{17800 \, b \, d^3}{d^2 + 6 \, d \, a + \cdot 16 \, l^2} = \text{W}.$$

For oak,

$$\frac{3960 \, b \, d^3}{d^2 + 6 \, d \, a + \cdot 5 \, l^2} = \text{W}.$$

$$\frac{15300\, b\, d^3}{4\, d^2 + \cdot 18\, l^2} = \text{W.*} \qquad \text{(xv.)}$$

289. As an approximate rule for the strength of a cylinder to resist compression in the direction of its length, we have

$$\frac{15300\, d^4}{1 \cdot 6\, (d^2 + \cdot 18\, l^2)} = \frac{9562\, d^4}{d^2 + \cdot 18\, l^2} = \text{W.} \qquad \text{(xvi.)}$$

290. And if the direction of the force be *a* inches distant from the axis, the rule is

$$\frac{9562\, d^4}{d^2 + \cdot 6\, d\, a + \cdot 18\, l^2} = \text{W.} \qquad \text{(xvii.)}$$

If the force act in the direction of one of the surfaces of the column, then $a = \frac{1}{2}\, d$, and

$$\frac{9562\, d^4}{4\, d^2 + \cdot 18\, l^2} = \text{W.†} \qquad \text{(xviii.)}$$

By this rule the Table of columns (Table III. p. 26,) was calculated, only the weight is there given in cwts.

In all the rules from Equation xiii. to xviii. *l* is the length, A A′, fig. 31, Plate IV., in feet, *d* either the diameter or the least side in inches, *b* the greater side in inches, and W the weight to be supported in ℔s.

291. *Example* 1. Required the weight that could be supported, with safety, by a cylindrical column, the length being

* In malleable iron,

$$\frac{17800\, b\, d^3}{4\, d^2 + \cdot 16\, l^2} = \text{W.}$$

In oak,

$$\frac{3960\, b\, d^3}{d^2 + \cdot 5\, l^2} = \text{W.}$$

† In malleable iron,

$$\frac{11125\, d^4}{4\, d^2 + \cdot 16\, l^2} = \text{W.}$$

In oak,

$$\frac{2470\, d^4}{d^2 + \cdot 5\, l^2} = \text{W.}$$

11 feet, and the diameter 5 inches, and supposing it probable that the force may act in the direction A A′, fig. 31, at the distance of half the diameter from the axis?

In this example Equation xviii., art. 290, should be used; and therefore

$$\frac{9562\, d^4}{4\, d^2 + \cdot 18\, l^2} = \frac{9562 \times 5^4}{4 \times 5^2 + \cdot 18 \times 11^2} = W = 49{,}080 \text{lbs.}$$

or a little above 22 tons.

In this manner may be calculated the strength of story-posts for supporting buildings. When they are for houses, ample allowance should be made for the weight of crowded rooms, and when for warehouses the greatest possible weight of goods should be estimated.

292. *Example* 2. It is proposed to determine the compression a curved rib will sustain in the direction of its chord; the greatest distance of the axis of the rib from the chord line being 6 inches, the size of the rib 3 inches square, and the length of the chord line 5 feet.

By Equation xiii., art. 287,

$$W = \frac{15300\, b\, d^2}{d^2 + 6\, d\, a + \cdot 18\, l^2} = \frac{15300 \times 3^4}{3^2 + 6 \times 3 \times 6 + \cdot 18 \times 5^2} = 30{,}600 \text{lbs.}$$

Example 3. The piston-rod of a double-acting steam engine is another interesting case to which these equations will apply; and the reader will excuse my having recourse to algebraic notation in order to make the rule general.

Let D be the diameter of the steam cylinder in inches, and p the greatest pressure of the steam on a circular inch of the piston in lbs. Then $W = D^2 p$.

But it has been shown in a note to art. 290, that in malleable iron

$$W = \frac{11125\, d^4}{4\, d^2 + \cdot 16\, l^2}.$$

Therefore,

$$D^2 p = \frac{11125\, d^4}{4\, d^2 + \cdot 16\, l^2}; \text{ or}$$

$$D = 53\, d^2 \sqrt{\frac{1}{p.\,(d^2 + \cdot 04\, l^2)}}.$$

Now in an extreme case we can never have the length in feet greater than about three times the diameter in inches; substitute this value of l, and we have

$$\frac{D\sqrt{1 \cdot 5 p}}{53} = d.$$

If the pressure be 8lbs. on the circular inch, that is, a little more than 10lbs. on the square inch, it gives $\frac{D}{15} = d$. That is, the piston-rod should never be less than $\frac{1}{15}$th of the diameter of the cylinder in a double-acting steam engine. In practice it is usual to make them $\frac{1}{10}$th, which does not appear to be too great an excess of strength to allow for wear.

OF THE STRENGTH OF BARS AND RODS TO RESIST TENSION.

293. When the effect of flexure is considered in bars to resist tension, it makes an important difference. Instead of the strength being diminished by flexure, it either has no effect, or has a directly contrary effect. Hence in all works executed in metals the tensile force of the materials should be employed in preference to any other, except the bulk be considerable in respect to the length. In wood we cannot employ a tensile force to much advantage, because it is difficult to form connexions at the extremities of sufficient firmness, but in metals this creates no difficulty.

If a bar or rod be short, its resistance may be computed by the rules, art. 281 and 282.

But when it is long, and the bar is either curved, or the force is not in the direction of the axis, then the effect of flexure may be considered.

The Equation ix., art. 280, will be applicable to all cases where the direction of the force is parallel to the extremities of the bar, that is,

$$\frac{f b d^2}{d + 6 a} = W. \qquad \text{(xviii.)}$$

The flexure is found to be $= \frac{l^2 \epsilon}{4 d} = \frac{\cdot 03\, l^2}{d}$, when l is the length in feet, and $\epsilon = \frac{1}{1204}$. But this flexure is to be deducted from the distance from the axis. Hence

$$\frac{15300\, b\, d^3}{d^2 + 6\, a\, d - \cdot 18\, l^2} = W.^* \qquad \text{(xix.)}$$

When the direction of the force is at the distance of half the least side from the axis, then $a = \frac{1}{2} d$, and

$$\frac{15300\, b\, d^3}{4\, d^2 - \cdot 18\, l^2} = W. \qquad \text{(xx.)}$$

And when the direction of the force coincides with the axis,

$$15300\, b\, d = W. \qquad \text{(xxi.)}$$

When the bar is a cylinder, its strength is to that of a square bar as 1 : 1·6 nearly (art. 285); hence,

$$\frac{9562\, d^4}{d^2 + 6\, a\, d - \cdot 18\, l^2} = W. \qquad \text{(xxii.)}$$

Or, when the force is in one of the surfaces of the rod,

$$\frac{9562\, d^4}{4\, d^2 - \cdot 18\, l^2} = W. \qquad \text{(xxiii.)}$$

It was desirable to show what constituted the advantage of a tensile strain, but I do not intend to adopt these equations

* In malleable iron,

$$\frac{17800\, b\, d^3}{d^2 + 6\, a\, d - \cdot 16\, l^2} = W.$$

In oak,

$$\frac{3960\, b\, d^3}{d^2 + 6\, a\, d - \cdot 5\, l^2} = W.$$

in practical rules, because they are not so simple and easily applied as the rules already given in art. 281 and 282, which will only err a small quantity in excess, when proper care has been taken to take the greatest possible deviation of the straining force from the axis of the piece.

Example 1. Required the weight that may be suspended by a bar of cast iron of 4 inches by 8 inches; under the supposition that the direction of the strain will be in one of the wide surfaces of the bar? Equation xviii. of this art. applies to this case, wherein a is equal 2 inches, or half the least dimension of the bar, that being the distance the direction of the force is supposed to be from the axis; and therefore

$$\frac{f\,b\,d^2}{d + 6\,a} = \frac{15300 \times 8 \times 4^2}{4 + 12} = 122400 \text{ lbs.}$$

the weight required. See the rule in words at art. 282.

When it is considered that a very small degree of inaccuracy in fitting the connexion may throw the strain all on one side of the bar, the prudence of following this mode of calculation will be apparent.

Example 2. It is proposed to determine the area that should be given to the bars of a suspension bridge, if made of cast iron, for a span of 370 feet; the points of suspension being 30 feet above the lowest point of the curve; and the greatest load, including the weight of the bridge itself, 500 tons.

The load being nearly uniformly distributed, the curve assumed by the chains will not sensibly differ from a parabola;* and half the weight will be to the tension at the lower point of the curve, as the rise is to $\frac{1}{4}$th of the span; that is,

$$30 : \frac{370}{4} :: \frac{500}{2} : \frac{500 \times 370}{8 \times 30} = 771 \text{ tons.}$$

* See Elementary Principles of Carpentry, art. 57.

This is equal to 1,727,040 lbs., and by the rule art. 281, we have

$$\frac{1727040}{15000} = 115 \text{ square inches}$$

for the area of the bars, supposing the stress to be directly in the axis of each; and if we double this area it will provide for a deviation equal to $\frac{1}{6}$th of the diameter of each bar. This will be a sufficient excess of force, considering the great chance of its ever being covered with people, which is the load I have estimated. Hence the sum of the areas of the chains at the lowest point should be 230 square inches. The area at any other point of the curve should be to the area at the lowest point, as the secant of the angle a tangent to the curve makes with a horizontal line, is to the radius. In the present case, the sum of the areas at the point of suspension should be 242 square inches. Cast iron would be greatly superior to wrought iron for chain bridges; it would be more durable, less expensive to obtain the same strength, and when made sufficiently strong, its weight would prevent excessive vibration by small forces. Most of the wrought iron bridges appear to be very slight and temporary structures when examined by the rules I have given, which appear to be founded on unquestionable principles.

Example 3. To determine the area of a piston-rod for a single-acting engine, the force on the piston being equivalent to 11 lbs. on a square inch, and allowing for the possibility of the direction of the force being at half the diameter of the rod from its axis. In this case 11 times the square of the diameter of the piston in inches is equal to the stress, and if D be the diameter of the steam piston, and d that of the piston-rod, we have for wrought iron,

$$3{\cdot}1416 \times 11\,D^2 = \frac{3{\cdot}1416\,d^2 \times 17800}{4};$$

or very nearly,

$$\frac{D}{20} = d.$$

That is, the diameter of the piston-rod should be $\frac{1}{20}$th of the diameter of the steam cylinder, when nothing is allowed for wear; or making the allowance which appears to be requisite, the diameter should be $\frac{1}{15}$th of the diameter of the steam cylinder.

SECTION XI.

OF THE STRENGTH OF CAST IRON TO RESIST AN IMPULSIVE FORCE.*

294. The moving force of a body, or of a part of a machine, ought to be balanced by the elastic force of the parts which propagate the motion; for if the effect of the moving force be greater than the elastic force of the parts, some of them will ultimately break; besides, a part of the power of the machine will be lost at each stroke.

And since increasing the mass of matter to be moved increases the friction in a machine, it is an advantage to employ no more material in its moving parts than is absolutely necessary for strength; but, in other parts exposed to pressive forces only, it is desirable that the materials should always be capable of resisting the strains, with as small a degree of flexure as is convenient, because steadiness is, in the fixed parts of machines, a most desirable property.

A beam resists a moving force, as a spring, by yielding and opposing the force as it yields, till it finally overbalances it;† and hence it is, that a brittle or very stiff body breaks, because it does not yield sufficiently for destroying the force.

As the resistance of a beam under different degrees of

* Dr. Young has given the term *resilience* to this species of resistance; and the reader will find some interesting remarks on the importance of studying it, in his Lectures on Nat. Phil. vol. i. p. 143.

† For a machine to produce the greatest effect, the time of bending the beam should be as small as possible.

flexure can be calculated, the effect of that resistance in the destruction of motion may be estimated by the principles of dynamics: such inquiries are usually managed by the method of fluxions; but not being satisfied with the manner of establishing the principles of that method, though I have no doubt of the correctness of results obtained by it, I shall briefly deduce the rules of this section by another mode of calculation.

295. If the intensity of a force be variable, so that the action upon the body moved at any point be directly as some power, n, of the distance from a point B, fig. 23, towards which it moves. Then, if the intensity of the force at A be equal P, the intensity at any point C will be $\frac{(\mathrm{C\,B})^n . \mathrm{P}}{(\mathrm{A\,B})^n}$. For, by the definition,

$$(\mathrm{A\,B})^n : (\mathrm{C\,B})^n :: \mathrm{P} : \frac{(\mathrm{C\,B})^n . \mathrm{P}}{(\mathrm{A\,B})^n}.$$

Put S to denote the space A B; and conceive this space S to be divided into m equal parts, denoting any one of these parts by x; and in consequence of the smallness of these parts, if we take the mean between the intensity at the beginning, and that at the end of each part, and consider each of these means a uniform intensity for the space it was calculated for, then these uniform intensities may be represented by the following progression:

$$\frac{\mathrm{P}}{2\,\mathrm{S}^n} \times \left(\overline{0 + x^n} + \overline{x^n + 2^n x^n} + \overline{2^n x^n + 3^n x^n} + \ldots \overline{\overline{m-1}^n x^n + m^n x^n} \right)$$

or,

$$\frac{\mathrm{P}\,x^n}{\mathrm{S}^n} \left\{ 1^n + 2^n + 3^n + \ldots \overline{m-1}^n + \frac{m^n}{2} \right\}.$$

296. It is shown by writers on dynamics, that when the intensity of a force is uniform, the square of the quantity of force accumulated or destroyed is directly as the intensity

multiplied by the quantity of matter moved, and by the space moved through.* Therefore making W = the quantity of matter, and g a constant quantity to reduce the proportion to an equation, we find the square of the forces accumulated or destroyed, in the space S, may be exhibited by the progression

$$\frac{g\,P\,W\,x^n + 1}{S^n}\left\{1^n + 2^n + \ldots \overline{m-1}^n + \frac{m^n}{2}\right\}.$$

And from the principles of the method of progressions,† the accurate value of the square of the force accumulated or destroyed in the space S is

$$\frac{g\,P\,W\,S}{n+1}.$$

297. When $n = o$, or the intensity is uniform, the square of the accumulated force is $= g\,P\,W\,S$.

298. The force of gravity near the earth's surface is nearly uniform, and in this case we know from experiments on falling bodies that $g = 64\frac{1}{3}$, and $P = W$ the weight of the body; therefore, $64\frac{1}{3}\,W^2\,S$ = the square of the accumulated force, and $64\frac{1}{3}$ may be substituted for g.

Hence the moving force of a falling body is $W\sqrt{64\frac{1}{3}\,S}$.

299. If $n = 1$, we have

$$\frac{g\,P\,W\,S}{n+1} = \frac{64\frac{1}{3}\,P\,W\,S}{2} = 32\tfrac{1}{6}\,P\,W\,S;$$

and as, in the resistance of beams, the intensity at any deflexion is directly as the deflexion, the quantity $32\frac{1}{6}$ P W S represents the square of the force destroyed in producing a deflexion equal to S. That is, when a beam is supported at both ends, and S = the deflexion in the middle, in decimal parts of a foot, then $\sqrt{32\frac{1}{6}\,P\,W\,S}$ = the force that would be

* Dr. C. Hutton's Course of Math. vol. ii. p. 136. 5th edition.

† See Philosophical Magazine, vol. lvii. p. 201.

destroyed in producing the flexure S; where P is the weight that would produce the deflexion S.*

Having considered the effect of the resisting force of the material in destroying an impulsive force, we must now consider the circumstances which take place in the different cases occurring in practice.

300. If the blow be made by a falling body in the direction of gravity, and the weight of the falling body be w, and its velocity at the time of impact be v, then by the laws of collision, in the case of equilibrium,

$$v\,w = \sqrt{32\tfrac{1}{6}\,P\,S\,(W + w)}. \qquad \text{(i.)}$$

In which equation the small acceleration that would be produced by the action of gravity on the mass $W + w$, during the flexure of the beam, is neglected.

301. If the blow were made horizontally by a body of the weight w, moving with a velocity v, then the equation is correct; and even in the first case it is accurate enough for practical purposes.

302. If the blow were made by a weight w falling from a given height h, we have, by the laws of gravity (art. 298),

$$w\,v = w\,\sqrt{64\tfrac{1}{3}\,h};$$

therefore,

$$w\,\sqrt{64\tfrac{1}{3}\,h} = \sqrt{32\tfrac{1}{6}\,P\,S\,(W + w)}, \text{ or}$$

$$2\,w^2\,h = P\,S\,(W + w). \qquad \text{(ii.)}$$

303. When the strain is occasioned by a force of an intensity F, and velocity v, such for example as would be occasioned by the sudden derangement of a machine in motion with the velocity v, and force F, then

* The effect of elastic gases in producing or destroying motion is expressed by the same equation, when the change of bulk is not so rapid as to cause cold in the one case, or to develope heat in the other. The developement of heat by the sudden compression of air materially affects the velocity of sound, and was first applied by Laplace to correct the discrepancy between theory and experiments; a subject which has been further illustrated by the researches of Poisson, in an article "Sur la Vitesse du Son," Annales de Chimie, tome xxiii. p. 5.

$$F\,v = \sqrt{32\tfrac{1}{6}\,P\,S\,W}\,;\ \text{or},$$

$$F^2\,v^2 = 32\tfrac{1}{6}\,P\,S\,W. \qquad \text{(iii.)}$$

The last equation is applicable to the beams of steam engines, and in general to reciprocating movements in machines, such as the connecting rods, cranks, &c.

If a body be previously in motion in the direction of the impulsive force, then the force F v should be the difference between the forces of the impelling and impelled bodies.

304. A general number of comparison to exhibit the power of a body to resist impulse, and which might be termed the *modulus of resilience*, would be extremely convenient in calculations of this kind; and when we omit the effect of a difference of density, which it is usual to do, we have an easy method of forming such a number.* For in any case, if f be the force which produces permanent alteration, and ϵ the corresponding extension,

$$P\,S : f\,\epsilon.$$

And, since in bars of different materials placed in the same circumstances the resistance to impulse may be considered proportional to the height a body must fall to produce a permanent change in the structure of the matter; and as that height is proportional to P S, and consequently to $f\epsilon$, when the effect of density is neglected; we may take $f\,\epsilon$ the measure of the power of a body to resist impulsion, that is, the modulus of resilience; and representing this modulus by R,

$$f\,\epsilon = R. \qquad \text{(iv.)}$$

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* The number might include the effect of density, if we were to measure the resistance to impulse by the height a body should fall to produce permanent change by its own weight; for we easily derive from Equation ii. art. 302,

$$h : \frac{f\,\epsilon}{s},$$

when s is the specific gravity. This expression might be termed the *specific resilience* of a body, and if it were denoted by Σ, we should have

$$\frac{f\,\epsilon}{s} = \Sigma.$$

In cast iron,

$$\Sigma = 1{\cdot}762.$$

In cast iron,

$$f = 15300\text{; and } \epsilon = \frac{1}{1204};$$

$$\text{therefore } R = 12{\cdot}7.$$

305. These equations flow from the principle that while the elasticity is perfect, the deflexion or extension is as the force producing it, but it also varies according to the manner in which the material is strained. In some cases, of frequent occurrence, the application is shown in the examples.

But it will be useful, before we proceed any further, to inquire what velocity cast iron will bear, without permanent alteration, in order that we may be aware whether such velocity will ever take place in the parts of machines; for if any part of a machine be connected with others that will yield to the force, and the material be capable of transmitting the motion with greater velocity than the machine moves with, it need be formed only for resistance to power or pressure.

306. It has been shown that $\sqrt{32\frac{1}{6}\,P\,W\,S}$ is equal to the greatest force an elastic body can generate or destroy (art. 299); if it were exposed to a greater force, its arrangement would be permanently altered. Now, if V be the greatest velocity the body is capable of transmitting, if communicated to its mass, we have

$$\sqrt{32\tfrac{1}{6}\,P\,W\,S} = V\,W\text{, or}$$

$$\sqrt{\frac{32\frac{1}{6}\,P\,S}{W}} = V. \qquad \text{(v.)}$$

307. It has also been shown, that in cast iron the cohesive force f = 15,300 ℔s. (art. 143), and the extension,

$$\epsilon = \frac{1}{1204}\text{ (art. 212);}$$

and since $S = l\epsilon$ (by art. 104), and $P = bdf$ (art. 103), and $l\,b\,dp = W$, where p = 3·2 ℔s., the weight of a bar of iron 12 inches long and 1 inch square; therefore, when a bar is strained in the direction of its length,

$$\sqrt{\frac{32\frac{1}{6}\,PS}{W}} = \sqrt{\frac{32\frac{1}{6} \times b\,d\,f \times l\,\epsilon}{l\,b\,d\,p}} = \sqrt{\frac{32\frac{1}{6} \times 15300}{3{\cdot}2 \times 1204}} = V =$$

11·3 feet per second.

308. If a uniform bar be supported at the ends, we have

$$P = \frac{850\,b\,d^2}{l} \text{ (art. 143) and } S = \frac{{\cdot}02\,l^2}{12\,d} \text{ ft. (art. 212);}$$

also,

$$W = \frac{l\,b\,d\,p}{2},$$

for the mass of the beam would acquire only the same momentum as half of it collected in the middle. Consequently,

$$\sqrt{\frac{32\frac{1}{6}\,PS}{W}} = \sqrt{\frac{32\frac{1}{6} \times 850 \times {\cdot}02 \times 2}{12 \times 3{\cdot}2}} = V =$$

5·3366 feet per second, nearly.

I have shown by a comparison of many experiments in art. 70, that about 3·3 times the force that produces permanent alteration will break a beam; therefore, assuming the deflexion to continue proportional to the force till fracture takes place, we have

$$\sqrt{\frac{32\frac{1}{6} \times 3{\cdot}3\,P \times 3{\cdot}3\,S}{W}} = V; \text{ or}$$

$$3{\cdot}3\sqrt{\frac{32\frac{1}{6}\,PS}{W}} = V.$$

Therefore, a velocity of

$$3{\cdot}3 \times 5{\cdot}3366 = 17{\cdot}6 \text{ feet per second,}$$

would break a beam; or a beam would break by falling a height of about 5 feet.

309. Hence it is clear, that cast iron is capable of sustaining only a very small degree of velocity; and a correct

knowledge of this limit is certainly of the first importance in the application of this material in machinery. When a cast iron bar is exposed to an impulsive force in the direction of its length, the utmost velocity its mass should acquire must never exceed 11 feet per second; and when the force acts in a direction perpendicular to the length, it should never be capable of communicating to the mass of the bar a greater velocity than about 5 feet per second; and if it exceed 18 feet per second, the bar will break.

If the connecting rod of a steam engine were to move with a greater velocity than 5 feet per second, the swag of its own weight would produce permanent flexure.

If a ship with hollow cast iron masts should strike a rock when it moved with a velocity of 12 miles per hour, the masts would break; and even with less velocity, for here we neglect the effect of the wind on the masts.

310. To illustrate the use of the above investigation, or rather, to prevent any one from disappointment, in applying these rules for the resistance to impulsion, it may be useful to consider how they should be applied to the parts of machines. In a machine the motion is communicated from the impelled to the working point by a certain number of parts, and among these parts one at least should be capable of resisting the whole energy of the moving power. If there be many parts to transmit the power, then two or more of them should be capable of resisting the energy of the moving power, and they should be distributed so as to divide the line of communication into nearly equal parts. If the intermediate parts be made sufficient to resist the dead power of the machine, that is, the power without velocity, they will always be strong enough to convey the velocity, if it be less than is stated in the preceding article, to other parts, that will either forward it to the working point, or resist it entirely during a momentary derangement of the action of the machine. To make all the parts strong enough for this purpose would

often cause a machine to be clumsy, and unfit for any practical use.

311. Let the constant numbers for the strength and deflexion in feet be $f \delta$. Then,

$$P = \frac{f b d^2}{l}, \text{ and } S = \frac{\delta l^2}{d}.$$

Also, let the weight of the beam itself be n times the weight of the falling body. These values being substituted in Equation i. art. 300, we have

$$v w = \sqrt{32\tfrac{1}{6} P S \overline{W + w}} = \sqrt{32\tfrac{1}{6} l b d f \delta w (n + 1)}; \text{ or,}$$

$$\frac{v^2 w}{32\tfrac{1}{6} l f \delta (n+1)} = b d. \qquad \text{(vi.)}$$

312. If the like substitutions be made in Equation iii. art. 303, we obtain

$$F^2 V^2 = (32\tfrac{1}{6} P S W) = 32\tfrac{1}{6} l b d f \delta W;$$

and if $l b d p$ be the weight of the mass of the beam the force acts upon, then

$$\frac{F V}{l \sqrt{32\tfrac{1}{6} f \delta p}} = b d. \qquad \text{(vii.)}$$

PRACTICAL RULES AND EXAMPLES.

313. *Prop.* I. To determine a rule for finding the dimensions of a beam to resist the force of a body in motion.

It is evident by Equation vi. art. 311, that the error which would arise from neglecting to allow for the effect of the weight of the beam itself, would always be on the safe side in calculating the dimensions of a beam to resist an impulsive force; and since, by such neglect, the rule is reduced to a very simple form, instead of a very complicated one, I shall apply the equation under the form

$$\frac{v^2 w}{32\tfrac{1}{6} l f \delta} = b d.$$

314. *Case* 1. When the beam is uniform and supported at the ends. In this case $f = 850$ (see art. 143), and δ in feet $= \frac{\cdot 02}{12}$ (by art. 212) hence,

$$32\tfrac{1}{6} f \delta = 45\cdot 5; \text{ or}$$

$$\frac{v^2 w}{45\cdot 5} = b\, d.$$

315. *Rule.* Multiply the weight of the falling body in pounds by the square of its velocity in feet per second; divide this product by 45·5 times the length in feet, and the quotient will be the area in inches.

The depth should be at least sufficient to render the beam capable of supporting its own weight, added to the weight of the falling body, which may be readily found by Table II. art. 6.

316. If the height of the fall be given instead of the velocity of the falling body, then instead of multiplying by the square of the velocity, multiply by sixty-four times the height of the fall.

317. *Example* 1. To determine the area of a cast iron beam that would sustain, without injury, the shock of a weight of 170 lbs. falling upon its middle with a velocity of 8 feet per second, the distance between the supports being 26 feet. By the rule

$$\frac{170 \times 8^2}{45\cdot 5 \times 26} = 9\cdot 2 \text{ inches, the area required.}$$

Hence, if we make the depth 6 inches, the breadth will be 1·53 inches, and the beam would sustain a pressure of 1800 lbs. (see Table II.) to produce the same effect as the fall of 170 lbs. It may also be observed, that half the weight of the beam is 400 lbs., making 570 lbs. for the pressure the beam would have to sustain after the velocity was destroyed, which is not quite $\frac{1}{3}$rd of the weight the beam would bear.

318. *Example* 2. If a bridge of 30 feet span were formed

on beams of cast iron, of what area should the section of these beams be, so that any one of them might be sufficient to resist the impulsive force of a waggon wheel falling over a stone 3 inches high, the load upon that wheel being 3360lbs.?

The height of the fall being ·25 foot, the square of the velocity acquired by the fall will be 64 × ·25 = 16; therefore,

$$\frac{3360 \times 16}{45 \cdot 5 \times 30} = 39 \cdot 338 \text{ inches},$$

the area required.

This area is nearly 40 inches; suppose it 40, then 40 × 15 × 3·2 = 1920 lbs. = half the weight of the beam (that is, the area in inches multiplied by half the length in feet, multiplied by 3·2 lbs., the weight of a piece of cast iron, 1 foot in length, and 1 inch square); consequently 1920 + 3360 = 5280 lbs., the whole effective pressure on the beam, after the velocity is destroyed. If we were to make the beam 20 inches deep, and 2 inches in thickness, it may be found by Table II. that the deflexion would be ·9 of an inch, and it would require a pressure of 45,328 lbs. to produce the same effect as the fall of the wheel, above eight times the pressure of the load and weight.

319. *Case* 2. When a beam is supported at the ends, the breadth uniform, and the outline of the depths an ellipse.

This case applies to bridges or beams to withstand an impulsive force at any point of the length. By art. 144, f = 850, and by art. 139,

$$\delta \text{ in feet} = \frac{\cdot 0257}{12};$$

therefore the equation

$$\frac{v^2 w}{32\frac{1}{4} f \delta l} = b\, d,$$

in art. 313, becomes

$$\frac{v^2 w}{58 \cdot 5\, l} = b\, d.$$

320. *Rule.* Calculate by the rule, art. 315, with 58·5 as a divisor instead of 45·5.

321. *Case* 3. When the breadth and depth of a beam are uniform, and the section is as fig. 9, Plate I., and the beam supported at the ends.

In this case $f = 850\ (1 - q p^3)$ by art. 186, and

$$\delta = \frac{\cdot 02}{12} \text{ foot}$$

by art. 212; hence the equation (art. 313),

$$\frac{v^2 w}{32\frac{1}{6} f \delta l} = \frac{v^2 w}{45{\cdot}5\ (1 - p^3 q)\ l} = b\ d;$$

consequently the power of a beam to resist an impulsive force, when the quantity of material is the same, is considerably increased by giving this form to the section.

322. *Case* 4. If a beam of the form of section shown in fig. 9, be the elliptical form of equal strength (see fig. 24, Plate III.), then

$$\frac{v^2 w}{58{\cdot}5\ l\ (1 - p^3 q)} = b\ d,$$

when the beam is supported at both ends, and the impulsive force acts at any point of the length.

323. *Case* 5. In an open beam, as fig. 11, Plate II., we may consider the beam as bounded by a semi-ellipse, when the breadth is uniform, and in this case

$$\frac{v^2 w}{58{\cdot}5\ l\ (1 - p^3)} = b\ d.$$

324. *Example.* To determine the area of the section of an open girder that would sustain the shock of 300 ℔s. falling from a height of 1 foot, the length between the supports being 26 feet, and the depth of the open part $\frac{7}{10}$ of the whole depth.

In this example

$$\frac{v^2 w}{58{\cdot}5\ l\ (1 - p^3)} = \frac{64 \times 300}{58{\cdot}5 \times 26\ (1 - \cdot 343)} =$$

20 inches nearly. This is perhaps as great an impulsive force as it is probable a girder for a room will be likely to be exposed to; and since this area of section would not be sufficient for the greatest pressure, it appears unnecessary to calculate the effect of moving force in the construction of girders.

325. *Prop.* II. To determine a rule for finding the dimensions of a uniform beam to resist a moving force.

This proposition applies to the parts of machines; and as there are few people engaged in the construction of powerful machines that are not competent to apply an equation, I shall in this part give the rules in the form of equations only.

326. *Case* 1. When a uniform beam is supported at the ends, and the moving force acts at the middle of the length.

By art. 143, $f = 850$, and by art. 212,

$$\delta = \frac{\cdot 02}{12} = \cdot 00166 \text{ foot};$$

and since 3·2 lbs. = the weight of 1 foot in length, and 1 inch square, we shall have

$$p = \frac{3\cdot 2}{2} = 1\cdot 6;$$

therefore,

$$\frac{\text{F V}}{l\sqrt{33\frac{1}{3} f \delta p}} \text{ (art. 312)} = \frac{\text{F V}}{l\sqrt{32\frac{1}{6} \times 850 \times \cdot 001\dot{6} \times 1\cdot 6}} = \frac{\text{F V}}{8\cdot 6\, l} = b\, d.$$

327. *Rule.* When F is the force in pounds, V its velocity in feet per second, l the whole length in feet between the supports, b the breadth, and d the depth in inches, then

$$\frac{\text{F V}}{8\cdot 6\, l} = b\, d.$$

328. *Case* 2. When a uniform beam rests upon a centre of motion, and the moving force acts at one end, and is opposed by a greater resistance at the other end.

By art. 153,

$f = 212$, and by art. 220, $\delta = \cdot 08\ (1 + r)$, and $p = \frac{3\cdot 2}{2}$;

hence,

$$\frac{F\,V}{l\sqrt{32\frac{1}{6}f\delta p}} = \frac{F\,V}{8{\cdot}6\,l\sqrt{1+r}} = b\,d.$$

329. *Rule.* Make F = the force in pounds, V its velocity in feet per second, l = the length in feet between the centre of motion and the point where the force acts, and l' = the length in feet between the centre of motion and the point of resistance; b and d being the breadth and depth in inches; then

$$\frac{l'}{l} = r, \text{ and } \frac{F\,V}{8{\cdot}6\,l\sqrt{1+r}} = b\,d.$$

330. If $l = l'$ we have

$$\frac{F\,V}{8{\cdot}6\,l\sqrt{2}} = \frac{F\,V}{12{\cdot}2\,l} = b\,d.$$

331. *Example.* To determine the area of the section of the beam for a steam engine, when it is to be of uniform depth; the length 24 feet, the centre of motion in the middle of the length; the pressure upon the piston 5000 lbs., and its greatest velocity 4 feet per second.

By art. 330,

$$\frac{F\,V}{12{\cdot}2\,l} = b\,d = \frac{5000 \times 4}{12{\cdot}2 \times 12} = 137 \text{ inches nearly.}$$

If this beam were made 30 inches deep, the deflexion by such a strain would be about $\frac{8}{10}$ths of an inch, and the breadth would be

$$\frac{137}{30} = 4{\cdot}57 \text{ inches,}$$

and such a beam would bear a weight of about 12 times the pressure on the piston, without destroying its elastic force.

332. *Prop.* III. To determine a rule for finding the area of the middle section of a parabolic beam to resist a moving force when the breadth is uniform.

The motion communicated to the arm of a lever is the same as if its whole weight were collected at its centre of gravity; and as the length of the arm is to the distance of its centre of gravity, so is the mass to the effect of that mass collected at the extremity. Therefore, when the distance of the centre of gravity is some part of the length, the effect of the mass of the arm will be the same part of the whole of its weight when acting at the extremity.

333. *Case* 1. When a parabolic beam is supported at both ends, and the moving force acts at the middle of the length.

By art. 143, $f = 850$, and by art. 224,

$$\delta = \frac{\cdot 04}{12} = \cdot 0033 \text{ foot.}$$

Also, because the area of a parabolic beam is $\frac{2}{3}$ of one uniformly deep,* and the distance of the centre of gravity from the centre of motion is $\frac{3}{5}$ of the length;† we have

$$p = 3 \cdot 2 \times \frac{2}{3} \times \frac{3}{5} = \frac{6 \cdot 4}{5} = 1 \cdot 28.$$

Consequently, the equation (art. 312),

$$\frac{\text{F V}}{l\sqrt{32\frac{1}{6} f \delta p}} = \frac{\text{F V}}{l\sqrt{32\frac{1}{6} \times 850 \times \cdot 0033 \times 1 \cdot 28}} = \frac{\text{F V}}{l \times 10 \cdot 8} = b\,d.$$

334. In beams supported at both ends, and of the same breadth, the power of a parabolic beam to resist a moving force, is to that of a uniform beam, as 10 is to 8 nearly; and the parabolic beam requires very little more than $\frac{2}{3}$rds of the quantity of material.

335. *Rule.* When F is the force in pounds, V its velocity in feet per second, l the whole length between the supports, and b and d the breadth and depth in inches; then

$$\frac{\text{F V}}{10 \cdot 8\, l} = b\,d.$$

336. *Example.* Let the force of a steam engine be applied to the middle of its beam, so as to cause it to move an axis

* Dr. Hutton's Course, vol. ii. p. 126. † Idem, vol. ii. p. 327.

by means of two cranks, placed so as to be impelled by the ends of the beam. Let the greatest pressure on the piston be 3000 lbs., its greatest velocity 3 feet per second, and the whole length 12 feet.

By the rule (art. 335),

$$\frac{F\,V}{10{\cdot}8\,l} = \frac{3000 \times 3}{10{\cdot}8 \times 12} = b\,d = 70 \text{ inches.}$$

337. *Case* 2. When a parabolic beam rests upon a centre of motion, and a moving force acts at one end, and is opposed by a greater resistance at the other.

By art. 153, $f = 212$, and by art. 227,

$$\delta = \frac{{\cdot}16\,(1 + r)}{12};$$

also,

$$p = 32 \times \frac{2}{3} \times \frac{2}{5} = \frac{12{\cdot}8}{15} = {\cdot}8533\dot{3};$$

hence,

$$\frac{F\,V}{l\sqrt{32\tfrac{1}{6} f\,\delta\,p}} = \frac{F\,V}{l\sqrt{32\tfrac{1}{6} \times 212 \times {\cdot}85\dot{3} \times \dfrac{{\cdot}16\,(1 + r)}{12}}} = \frac{F\,V}{8{\cdot}82\,l\sqrt{1 + r}} = b\,d.$$

338. *Rule.* Make F the force in pounds, and V its velocity in feet per second, l = the length in feet, from the centre of motion to the point where the force acts, and l' the length from the centre of motion to the resisted point; also, make b and d the breadth and depth in inches; then

$$\frac{l'}{l} = r, \text{ and } \frac{F\,V}{8{\cdot}82\,l\sqrt{1+r}} = b\,d.$$

339. If $l = l'$, that is, when the centre of motion is in the middle of the beam,

$$\frac{F\,V}{12{\cdot}5\,l} = b\,d.$$

340. In a steam engine the weight of the connecting apparatus, the power applied to the air-pump, and the weight

of the catch-pins, should be allowed for; and when the engine moves machinery, the beam should not be less than is determined by this rule. The depth of the beam is usually the same as the diameter of the steam piston.

341. *Example.* If the pressure on the piston of a steam engine be 15,000 lbs. the whole length of the beam 24 feet, and its velocity 3 feet per second, required the area of the beam?

In this case,

$$\frac{\text{F V}}{12{\cdot}5\,l} = \frac{15000 \times 3}{12{\cdot}5 \times 12} = b\,d = 300 \text{ inches.}$$

If the beam be made 48 inches deep, it should be $6\frac{1}{4}$ inches in breadth; and the best method of forming such a beam is to make it in two parts, each $3\frac{1}{8}$ in breadth, placed at 12 or 14 inches apart, and well connected together. This arrangement causes an engine to work with more steadiness, and the parts are less troublesome to move and fix in their places than a single mass would be.

342. *Prop.* IV. To determine a rule for finding the area of the middle section of a beam of uniform breadth, the depth at the end being half the depth in the middle, and the middle of the depth open, to resist a moving force.

Let the parts be so arranged that the centre of gravity may be considered to be at the middle of the length of the arm of the beam, which will be very nearly true in practice, and will render the computation somewhat easier.

343. *Case* 1. When an open beam is supported at the ends, and the force is applied in the middle of the length.

By art. 200, $f = 850\ (1 - p'^3)$, and by art. 231,

$$\delta = \frac{{\cdot}0327}{12} = {\cdot}002725;$$

also, we have

$$p = \frac{3{\cdot}2 \times (1 - p')}{2} = 1{\cdot}6\ (1 - p');$$

and the Equation (art. 312),

$$\frac{\text{F V}}{l\sqrt{32\tfrac{1}{6} f \delta p}} = \frac{\text{F V}}{l\sqrt{32\tfrac{1}{6} \times 850\,(1-p'^3) \times \cdot002726 \times 1\cdot6\,(1-p')}} =$$

$$\frac{\text{F V}}{10\cdot92\, l\sqrt{(1-p'^3) \times (1-p')}} = b\, d.$$

344. *Rule.* Make F the force in pounds, V its velocity in feet per second, l the whole length between the supports in feet, p' that number which would be produced by dividing the depth of the part left out in the middle, by the whole depth; (if this ratio were not fixed, the solution could not be effected;) and b and d the breadth and depth in the middle in inches; then

$$\frac{\text{F V}}{10\cdot92\, l\sqrt{(1-p'^3) \times (1-p')}} = b\, d.$$

345. If p' be made $= \cdot7$, which is a convenient proportion, then

$$\frac{\text{F V}}{4\cdot85\, l} = b\, d.$$

346. *Case* 2. When an open beam is supported on a centre of motion, and the moving force acts at one end, and the resistance at the other.

By the same method as above we find

$$\frac{\text{F V}}{10\cdot92\, l\sqrt{(1-p^3) \times (1-p) \times (1+r)}} = b\, d.$$

347. *Rule.* Make F = the force in pounds, V its velocity in feet per second, l the length from the point where the force acts to the centre of motion in feet and l' the length from the centre of motion to the point of resistance, b and d the breadth and depth in inches in the middle of the beam, and p the number arising from the division of the depth of the part left out in the middle by the whole depth; then, $\frac{l'}{l} = r$, and

$$\frac{\text{F V}}{10{\cdot}92\, l\sqrt{(1-p^3)\times(1-p)\times(1+r)}}=b\,d.$$

348. If $p = {\cdot}7$, the equation reduces to

$$\frac{\text{F V}}{4{\cdot}85\, l.\sqrt{1+r}}=b\,d.$$

349. Also, when the centre of motion is in the middle of the beam, and $p = {\cdot}7$, we have

$$\frac{\text{F V}}{6{\cdot}86\, l}=b\,d.$$

350. *Example.* As an example to the equation in the last article, let us suppose the pressure on the piston of a steam engine to be 15,000 lbs., its velocity 3 feet per second, and the whole length of the beam 24 feet, which is the same as the example (art. 341). In this case

$$\frac{\text{F V}}{6{\cdot}86\, l}=\frac{15000\times 3}{6{\cdot}86\times 12}=771 \text{ inches}=b\,d.$$

And, if the depth be made 48 inches, then

$$\frac{771}{48}=16{\cdot}06$$

inches the breadth, which is to be the same throughout the length. The bulk of the metal in the upper and lower part of the beam will be found by multiplying the depth by ·7; that is, ·7 × 48 = 33·6; which, deducted from 48, leaves 14·4 inches, or 7·2 inches for each side.

Fig. 34, Plate IV., shows a sketch for a beam of this kind, drawn according to these proportions.

351. Any of the rules of this, or of the preceding Section, may be applied to other materials by substituting the proper values of the cohesive force, extensibility, and density; these are given for several kinds in the following Table.

TABLE OF DATA, &c.

USEFUL IN VARIOUS CALCULATIONS;

ARRANGED ALPHABETICALLY.

The data correspond to the mean temperature and pressure of the atmosphere, dry materials; and the temperature is measured by Fahrenheit's scale.

AIR. Specific gravity 0·0012; weight of a cubic foot 0·0753 lb., or 527 grains (SHUCKBURGH); 13·3 cubic feet or 17 cylindric feet of air weigh 1 lb.; it expands $\frac{1}{480}$ or ·00208 of its bulk at 32° by the addition of one degree of heat (DULONG and PETIT).

ASH. Specific gravity 0·76; weight of a cubic foot 47·5 lbs.; weight of a bar 1 foot long and 1 inch square 0·33 lb.; will bear without permanent alteration, a strain of 3540 lbs. upon a square inch, and an extension of $\frac{1}{464}$ of its length; weight of modulus of elasticity for a base of an inch square 1,640,000 lbs.; height of modulus of elasticity 4,970,000 feet; modulus of resilience 7·6; specific resilience 10. (Calculated from BARLOW'S Experiments.)

Compared with cast iron as unity, its strength is 0·23; its extensibility 2·6; and its stiffness 0·089.

ATMOSPHERE. Mean pressure of, at London, 28·89 inches of mercury = 14·18 lbs. upon a square inch. (ROYAL SOCIETY.) The pressure of the atmosphere is usually estimated at 30 inches of mercury, which is very nearly 14¾ lbs. upon a square inch, and equivalent to a column of water 34 feet high.

BEECH. Specific gravity 0·696; weight of a cubic foot 45·3 lbs.; weight of a bar 1 foot long and 1 inch square 0·315 lb.; will bear without permanent alteration on a square inch 2360 lbs., and an extension of $\frac{1}{570}$ of its length; weight of modulus of elasticity, for a base of an inch square 1,345,000 lbs.; height of modulus of elasticity 4,600,000 feet; modulus of resilience 4·14; specific resilience 6. (Calculated from BARLOW'S Experiments.)

Compared with cast iron as unity, its strength is 0·15; its extensibility 2·1; and its stiffness 0·073.

Brass, *cast*. Specific gravity 8·37; weight of a cubic foot 523 lbs.; weight of a bar 1 foot long and 1 inch square 3·63 lbs.; expands $\frac{1}{93800}$ of its length by one degree of heat (Troughton); melts at 1869° (Daniell); cohesive force of a square inch 18,000 lbs. (Rennie); will bear on a square inch without permanent alteration 6700 lbs., and an extension in length of $\frac{1}{1333}$; weight of modulus of elasticity for a base of an inch square 8,930,000 lbs.; height of modulus of elasticity 2,460,000 feet; modulus of resilience 5; specific resilience 0·6 (Tredgold).

Compared with cast iron as unity, its strength is 0·435; its extensibility 0·9; and its stiffness 0·49.

Brick. Specific gravity 1·841; weight of a cubic foot 115 lbs.; absorbs $\frac{1}{15}$ of its weight of water; cohesive force of a square inch 275 lbs. (Tredgold); is crushed by a force of 562 lbs. on a square inch (Rennie).

Brick-work. Weight of a cubic foot of newly built, 117 lbs.; weight of a rod of new brick-work 16 tons.

Bridges. When a bridge is covered with people, it is about equivalent to a load of 120 lbs. on a superficial foot; and this may be esteemed the greatest possible extraneous load that can be collected on a bridge; while one incapable of supporting this load cannot be deemed safe.

Bronze. See Gun-metal.

Cast Iron. Specific gravity 7·207; weight of a cubic foot 450 lbs.; a bar 1 foot long and 1 inch square weighs 3·2 lbs. nearly; it expands $\frac{1}{162000}$ of its length by one degree of heat (Roy); greatest change of length in the shade in this climate $\frac{1}{1723}$; greatest change of length exposed to the sun's rays $\frac{1}{1270}$; melts at 3479° (Daniell), and shrinks in cooling from $\frac{1}{98}$ to $\frac{1}{85}$ of its length (Muschet); is crushed by a force of 93,000 lbs. upon a square inch (Rennie); will bear without permanent alteration 15,300 lbs.* upon a square inch, and an extension of $\frac{1}{1204}$ of its length; weight of modulus of elasticity for a base of an inch square 18,400,000 lbs.; height of modulus of elasticity 5,750,000 feet; modulus of resilience 12·7: specific resilience 1·76 (Tredgold).

Chalk. Specific gravity 2·315; weight of a cubic foot 144·7 lbs.; is crushed by a force of 500 lbs. on a square inch. (Rennie.)

Clay. Specific gravity 2·0; weight of a cubic foot 125 lbs.

* See note to art. 143.—Editor.

COAL. *Newcastle.* Specific gravity 1·269; weight of a cubic foot 79·31 lbs. A London chaldron of 36 bushels weighs about 28 cwt., whence a bushel is 87 lbs. (but is usually rated at 84 lbs.) A Newcastle chaldron, 53 cwt. (SMEATON.)

COPPER. Specific gravity 8·75 (HATCHETT); weight of a cubic foot 549 lbs.; weight of a bar 1 foot long and 1 inch square 3·81 lbs.; expands in length by one degree of heat $\frac{1}{105900}$ (SMEATON); melts at 2548° (DANIELL); cohesive force of a square inch, when hammered, 33,000 lbs. (RENNIE).

EARTH, *common.* Specific gravity 1·52 to 2·00; weight of a cubic foot from 95 to 125 lbs.

ELM. Specific gravity 0·544; weight of a cubic foot 34 lbs.; weight of a bar 1 foot long and 1 inch square 0·236 lbs.; will bear on a square inch without permanent alteration 3240 lbs., and an extension in length of $\frac{1}{414}$; weight of modulus of elasticity for a base of an inch square 1,340,000 lbs.; height of modulus of elasticity 5,680,000 feet; modulus of resilience 7·87; specific resilience 14·4. (Calculated from BARLOW's Experiments.)

Compared with cast iron as unity, its strength is 0·21; its extensibility 2·9; and its stiffness 0·073.

FIR, *red or yellow.* Specific gravity 0·557; weight of a cubic foot 34·8 lbs.; weight of a bar 1 foot long and 1 inch square 0·242 lb.; will bear on a square inch without permanent alteration 4290 lbs., = 2 tons nearly, and an extension in length of $\frac{1}{470}$; weight of modulus of elasticity for a base of an inch square 2,016,000 lbs.; height of modulus of elasticity 8,330,000 feet; modulus of resilience 9·13; specific resilience 16·4. (TREDGOLD.)

Compared with cast iron as unity, its strength is 0·3; its extensibility 2·6; and its stiffness 0·1154, $= \frac{1}{8{\cdot}66}$.

FIR, *white.* Specific gravity 0·47; weight of a cubic foot 29·3 lbs.; weight of a bar 1 foot long and 1 inch square 0·204 lb.; will bear on a square inch without permanent alteration 3630 lbs., and an extension in length of $\frac{1}{504}$; weight of modulus of elasticity for a base of an inch square 1,830,000 lbs.; height of modulus of elasticity 8,970,000 feet; modulus of resilience 7·2; specific resilience 15·3. (TREDGOLD.)

Compared with cast iron as unity, its strength is 0·23; its extensibility 2·4: and its stiffness 0·1.

FLOORS. The weight of a superficial foot of a floor is about 40 lbs. when there is a ceiling, counter-floor, and iron girders. When a floor is covered with people, the load upon a superficial foot may be calculated at 120 lbs. Therefore 120 + 40 = 160 lbs. on a superficial foot is the least stress that ought to be taken in estimating the strength for the parts of a floor of a room.

FORCE. See Gravity, Horses, &c.

GRANITE, *Aberdeen*. Specific gravity 2·625; weight of a cubic foot 164 lbs.; is crushed by a force of 10,910 lbs. upon a square inch. (RENNIE.)

GRAVEL. Weight of a cubic foot about 120lbs.

GRAVITY, generates a velocity $32\frac{1}{6}$ feet in a second, in a body falling from rest; space described in the first second $16\frac{1}{12}$ feet.

GUN-METAL, *cast* (copper 8 parts, tin 1). Specific gravity 8·153; weight of a cubic foot $509\frac{1}{2}$ lbs.; weight of a bar 1 foot long and 1 inch square 3·54 lbs. (TREDGOLD); expands in length by 1° of heat $\frac{1}{99090}$ (SMEATON): will bear on a square inch without permanent alteration 10,000 lbs., and an extension in length of $\frac{1}{960}$; weight of modulus of elasticity for a base of an inch square 9,873,000 lbs.: height of modulus of elasticity 2,790,000 feet; modulus of resilience, and specific resilience, not determined. (TREDGOLD.)

Compared with cast iron as unity, its strength is 0·65; its extensibility 1·25; and its stiffness 0·535.

HORSE, of average power, produces the greatest effect in drawing a load when exerting a force of $187\frac{1}{2}$ lbs. with a velocity of $2\frac{1}{2}$ feet per second, working 8 hours in a day.* (TREDGOLD.) A good horse can exert a force of 480 lbs. for a short time. (DESAGULIERS.) In calculating the strength for horse machinery, the horse's power should be considered 400 lbs.

IRON, *cast*. See Cast Iron.

IRON, *malleable*. Specific gravity 7·6 (MUSCHENBROEK); weight of a cubic foot 475 lbs.; weight of a bar 1 foot long and 1 inch square 3·3 lbs.; ditto, when hammered, 3·4 lbs.; expands in length by 1° of heat $\frac{1}{143000}$ (SMEATON); good English iron will bear on a square inch without permanent alteration 17,800 lbs.,† = 8 tons nearly, and an extension in length of $\frac{1}{1400}$; cohesive force diminished $\frac{1}{3000}$ by an elevation 1° of temperature; weight of modulus of elasticity for a base of an inch square 24,920,000 lbs.; height of modulus of elasticity 7,550,000 feet; modulus of resilience, and specific resilience, not determined (TREDGOLD).

Compared with cast iron as unity, its strength is 1·12; its extensibility 0·86: and its stiffness 1·3.

LARCH. Specific gravity ·560; weight of a cubic foot 35 lbs.; weight of

* This is equivalent to raising 3 cubic feet of water $2\frac{1}{2}$ feet per second, or $7\frac{1}{2}$ cubic feet 1 foot per second. See Buchanan's Essays, 3rd edition, by Mr. Rennie, page 88.

† Equivalent to a height of 5000 feet of the same matter.

a bar 1 foot long and 1 inch square 0·243 lb.; will bear on a square inch without permanent alteration 2065 lbs., and an extension in length of $\frac{1}{520}$; weight of modulus of elasticity for a base of an inch square 10,074,000 lbs.; height of modulus of elasticity 4,415,000 feet; modulus of resilience 4; specific resilience 7·1. (Calculated from BARLOW's experiments.)

Compared with cast iron as unity, its strength is 0·136; its extensibility 2·3; and its stiffness 0·058.*

LEAD, *cast.* Specific gravity 11·353 (BRISSON); weight of a cubic foot 709·5 lbs.; weight of a bar 1 foot long and 1 inch square 4·94 lbs.; expands in length by 1° of heat $\frac{1}{62800}$ (SMEATON); melts at 612° (CRICHTON); will bear on a square inch without permanent alteration 1500 lbs., and an extension in length of $\frac{1}{480}$; weight of modulus of elasticity for a base of an inch square 720,000 lbs.; height of modulus of elasticity 146,000 feet; modulus of resilience 3·12; specific resilience 0·27 (TREDGOLD).

Compared with cast iron as unity, its strength is 0·096; its extensibility 2·5; and its stiffness 0·0385.

MAHOGANY, *Honduras.* Specific gravity 0·56; weight of a cubic foot 35 lbs.; weight of a bar 1 foot long and 1 inch square 0·243 lb.; will bear on a square inch without permanent alteration 3800 lbs., and an extension in length of $\frac{1}{420}$; weight of modulus of elasticity for a base of an inch square 1,596,000 lbs.; height of modulus of elasticity 6,570,000 feet; modulus of resilience 9·047; specific resilience 16·1. (TREDGOLD.)

Compared with cast iron as unity, its strength is 0·24; its extensibility 2·9; and its stiffness 0·487.

MAN. A man of average power produces the greatest effect when exerting a force of 31¼ lbs. with a velocity of 2 feet per second, for 10 hours in a day.† (TREDGOLD.) A strong man will raise and carry from 250 to 300 lbs. (DESAGULIERS.)

MARBLE, *white.* Specific gravity 2·706; weight of a cubic foot 169 lbs.; weight of a bar 1 foot long and 1 inch square 1·17 lbs.; cohesive force of a square inch 1811 lbs.; extensibility $\frac{1}{1394}$ of its length; weight of modulus of elasticity for a base of an inch square 2,520,000 lbs.; height of modulus of elasticity 2,150,000 feet;

* The mean of my trials on specimens of very different qualities places the strength and stiffness of Larch much higher on the scale of comparison; but I had not observed the point where it loses elastic power.

† This is equivalent to half a cubic foot of water raised 2 feet per second; or 1 cubic foot of water 1 foot per second. See Buchanan's Essays, p. 92, edited by Mr. Rennie.

modulus of resilience at the point of fracture 1·3; specific resilience at the point of fracture 0·48 (TREDGOLD); is crushed by a force of 6060 lbs. upon a square inch (RENNIE).

MERCURY. Specific gravity 13·568 (BRISSON); weight of a cubic inch 0·4948 lb.; expands in bulk by 1° of heat $\frac{1}{9990}$ (DULONG and PETIT); weight of modulus of elasticity for a base of an inch square 4,417,000 lbs.; height of modulus of elasticity 750,000 feet (Dr. YOUNG from CANTON'S Experiments).

OAK, *good English.* Specific gravity 0·83; weight of a cubic foot 52 lbs.; weight of a bar 1 foot long and 1 inch square 0·36 lb.; will bear upon a square inch without permanent alteration 3960 lbs.; and an extension in length of $\frac{1}{430}$; weight of modulus of elasticity for a base of an inch square 1,700,000 lbs.; height of modulus of elasticity 4,730,000 feet; modulus of resilience 9·2; specific resilience 11. (TREDGOLD.)

Compared with cast iron as unity, its strength is 0·25; its extensibility 2·8; and its stiffness 0·093.

PENDULUM. Length of pendulum to vibrate seconds in the latitude of London 39·1372 inches (KATER); ditto to vibrate half seconds 9·7843 inches.

PINE, *American, yellow.* Specific gravity 0·46; weight of a cubic foot 26¾ lbs.; weight of a bar 1 foot long and 1 inch square 0·186 lb.; will bear on a square inch without permanent alteration 3900 lbs., and an extension in length of $\frac{1}{414}$; weight of modulus of elasticity for a base of an inch square 1,600,000 lbs.; height of modulus of elasticity 8,700,000 feet; modulus of resilience 9·4: specific resilience 20. (TREDGOLD.)

Compared with cast iron as unity, its strength is 0·25; its extensibility 2·9; and its stiffness 0·087.

PORPHYRY, *red.* Specific gravity 2·871: weight of a cubic foot 179 lbs.; is crushed by a force of 35,568 lbs. upon a square inch. (GAUTHEY.)

ROPE, *hempen.* Weight of a common rope 1 foot long and 1 inch in circumference from 0·04 to 0·046 lb.; and a rope of this size should not be exposed to a strain greater than 200 lbs.; but in compounded ropes, such as cables, the greatest strain should not exceed 120 lbs.;*

* The square of the circumference in inches multiplied by 200 will give the number of pounds a rope may be loaded with, and multiply by 120 instead of 200 for cables. Common ropes will bear a greater load with safety after they have been some time in use, in consequence of the tension of the fibres becoming equalized by repeated stretchings and partial untwisting. It has been imagined that the improved strength was gained by their being laid up in store; but if they can there be preserved from deterioration, it is as much as can be expected.

and the weight of a cable 1 foot in length and 1 inch in circumference does not exceed 0·027 lb.

ROOFS. Weight of a square foot of Welsh rag slating 11¼ lbs.; weight of a square foot of plain tiling 16¼ lbs.; greatest force of the wind upon a superficial foot of roofing may be estimated at 40 lbs.

SLATE, *Welsh.* Specific gravity 2·752 (KIRWAN); weight of a cubic foot 172 lbs.; weight of a bar 1 foot long and 1 inch square 1·19 lbs.; cohesive force of a square inch 11,500 lbs.; extension before fracture $\frac{1}{1370}$; weight of modulus of elasticity for a base of an inch square 15,800,000 lbs.; height of modulus of elasticity 13,240,000 feet; modulus of resilience 8·4; specific resilience 2. (TREDGOLD.)

SLATE, *Westmoreland.* Cohesive force of a square inch 7870 lbs.; extension in length before fracture $\frac{1}{1640}$; weight of modulus of elasticity for a base of an inch square 12,900,000 lbs. (TREDGOLD.)

SLATE, *Scotch.* Cohesive force of a square inch 9600 lbs.; extension in length before fracture $\frac{1}{1645}$; weight of modulus of elasticity for a base of an inch square 15,790,000 lbs. (TREDGOLD.)

STEAM. Specific gravity at 212° is to that of air at the mean temperature as 0·472 is to 1 (THOMSON); weight of a cubic foot 249 grains; modulus of elasticity for a base of an inch square 14¾ lbs.; when not in contact with water, expands $\frac{1}{480}$ of its bulk by 1° of heat. (GAY LUSSAC.)

STEEL. Specific gravity 7·84; weight of a cubic foot 490 lbs.; a bar 1 foot long and 1 inch square weighs 3·4 lbs.; it expands in length by 1° of heat $\frac{1}{157200}$ (ROY); tempered steel will bear without permanent alteration 45,000 lbs.; cohesive force of a square inch 130,000 lbs. (RENNIE); cohesive force diminished $\frac{1}{5000}$ by elevating the temperature 1°; modulus of elasticity for a base of an inch square 29,000,000 lbs.; height of modulus of elasticity 8,530,000 feet (Dr. YOUNG).

STONE, *Portland.* Specific gravity 2·113; weight of a cubic foot 132 lbs.; weight of a prism 1 inch square and 1 foot long 0·92 lb.; absorbs $\frac{1}{16}$ of its weight of water (R. TREDGOLD); is crushed by a force of 3729 lbs. upon a square inch (RENNIE); cohesive force of a square inch 857 lbs.; extends before fracture $\frac{1}{1789}$ of its length; modulus of elasticity for a base of an inch square 1,533,000 lbs.; height of modulus of elasticity 1,672,000 feet;* modulus of resilience at the

* In the stones, the modulus here given is calculated from the flexure at the time of fracture; when it is taken for the first degrees of flexure, it is a little greater. The experiments are described in the Philosophical Magazine, vol. lvi. p. 290.

point of fracture 0·5; specific resilience at the point of fracture 0·23 (TREDGOLD).

STONE, *Bath.* Specific gravity 1·975; weight of a cubic foot 123·4 lbs.; absorbs $\frac{1}{13}$ of its weight of water (R. TREDGOLD); cohesive force of a square inch 478 lbs. (TREDGOLD).

STONE, *Craigleith.* Specific gravity 2·362; weight of a cubic foot 147·6 lbs.; absorbs $\frac{1}{63}$ of its weight of water; cohesive force of a square inch 772 lbs. (TREDGOLD); is crushed by a force of 5490 lbs. upon a square inch (RENNIE).

STONE, *Dundee.* Specific gravity 2·621; weight of a cubic foot 163·8 lbs.; absorbs $\frac{1}{511}$ part of its weight of water; cohesive force of a square inch 2661 lbs. (TREDGOLD); is crushed by a force of 6630 lbs. upon a square inch (RENNIE).

STONE-WORK. Weight of a cubic foot of rubble-work about 140 lbs.; of hewn stone 160 lbs.

TIN, *cast.* Specific gravity 7·291 (BRISSON); weight of a cubic foot 455·7 lbs.; weight of a bar 1 foot long and 1 inch square 3·165 lbs.; expands in length by 1° of heat $\frac{1}{72510}$ (SMEATON); melts at 442° (CRICHTON); will bear upon a square inch without permanent alteration 2880 lbs., and an extension in length of $\frac{1}{1600}$; modulus of elasticity for a base of an inch square 4,608,000 lbs.; height of modulus of elasticity 1,453,000 feet; modulus of resilience 1·8; specific resilience 0·247 (TREDGOLD).

Compared with cast iron as unity, its strength is 0·182; its extensibility 0·75; and its stiffness 0·25.

WATER, *river.* Specific gravity 1·000; weight of a cubic foot 62·5 lbs.; weight of a cubic inch 252·525 grains; weight of a prism 1 foot long and 1 inch square 0·434 lb.; weight of an ale gallon of water 10·2 lbs.; expands in bulk by 1° of heat $\frac{1}{3858}$ (DALTON);* expands in freezing $\frac{1}{17}$ of its bulk (WILLIAMS); and the expanding force of freezing water is about 35,000 lbs. upon a square inch, according to Muschenbroëk's valuation; modulus of elasticity for a base of an inch square 326,000 lbs.; height of modulus of elasticity 750,000 feet, or 22,100 atmospheres of 30 inches of mercury (Dr. YOUNG, from CANTON'S Experiments).

WATER, *sea.* Specific gravity 1·0271; weight of a cubic foot 64·2 lbs.

* Water has a state of maximum density at or near 40°, which is considered an exception to the general law of expansion by heat: it is extremely improbable that there is anything more than an apparent exception, most likely arising from water at low temperatures absorbing a considerable quantity of air, which has the effect of expanding it; and consequently of causing the apparent anomaly.

WATER is 828 times the density of air of the temperature 60°, and barometer 30.

WHALE-BONE. Specific gravity 1·3; weight of a cubic foot 81 lbs.; will bear a strain of 5600 lbs. upon a square inch without permanent alteration, and an extension in length of $\frac{1}{146}$; modulus of elasticity for a base of an inch square 820,000 lbs.; height of modulus of elasticity 1,458,000 feet; modulus of resilience 38·3; specific resilience 29. (TREDGOLD.)

WIND. Greatest observed velocity 159 feet per second (ROCHON); force of wind with that velocity about 57¾ lbs. on the square foot.*

ZINC, *cast.* Specific gravity 7·028 (WATSON); weight of a cubic foot 439¼ lbs.; weight of a bar 1 inch square and 1 foot long 3·05 lbs.; expands in length by 1° of heat $\frac{1}{61200}$ (SMEATON); melts at 648° (DANIELL); will bear on a square inch without permanent alteration 5700 lbs. = 0·365 cast iron, and an extension in length of $\frac{1}{2400} = \frac{1}{2}$ that of cast iron (TREDGOLD); † modulus of elasticity for a base of an inch square 13,680,000 lbs.; height of modulus of elasticity 4,480,000 feet; modulus of resilience 2·4; specific resilience 0·34. (TREDGOLD.)

Compared with cast iron as unity, its strength is 0·365; its extensibility 0·5; and its stiffness 0·76.

* Table of the force of winds, formed from the Tables of Mr. Rouse and Dr. Lind, and compared with the observations of Colonel Beaufoy.

Velocity in miles per hour.	A wind may be denominated when it does not exceed the velocity opposite to it.	Velocity per second.	Force on a square foot.
		feet.	lbs.
6·8	A gentle pleasant wind	10	0·229
13·6	A brisk gale	20	0·915
19·5	A very brisk gale	30	2·059
34·1	A high wind	50	5·718
47·7	A very high wind	70	11·207
54·5	A storm or tempest	80	14·638
68·2	A great storm	100	22·872
81·8	A hurricane	120	32·926
102·3	A violent hurricane, that tears up trees, overturns buildings, &c.	150	51·426

* Accurate observations on the variation and mean intensity of the force of winds would be very desirable both to the mechanician and meteorologist.

† The fracture of zinc is very beautiful; it is radiated, and preserves its lustre a long time.

NOTE ON THE ACTION OF CERTAIN SUBSTANCES ON CAST IRON.

In some circumstances cast iron will decompose, and be converted into a soft substance resembling plumbago. A few instances of this kind I add here, as they will be interesting to persons who employ iron for various purposes.

Dr. Henry observed that when cast iron was left in contact with muriate of lime, or muriate of magnesia, most of the iron was removed, the specific gravity of the mass was reduced to 2·155, and what remained consisted chiefly of plumbago, and the impurities usually found in cast iron.*

A similar change was produced in some cast iron cylinders used to apply the weaver's dressing to cloth : this dressing is a kind of paste, made of wheat or barley-meal. The corrosion of the cylinders took place repeatedly, and was so rapid that it was found necessary to use wooden ones. Dr. Thomson ascribes the change to the acid formed by the paste turning sour.†

Another instance of greater importance has been recorded by Mr. Brande. A portion of a cast iron gun had undergone a like change from being long immersed in sea-water. To the depth of an inch it was converted into a substance having all the external characters of plumbago ; it was easily cut, greasy to the feel, and made a black streak upon paper.‡ The component parts of this substance were in the ratio of

Oxide of iron . . .	81
Plumbago	16
	97

Mr. Brande could not detect any silica in it ; and remarks, that anchors and other articles of wrought iron, when similarly exposed, are only superficially oxidized, and exhibit no other peculiar appearance.

Near the town of Newhaven, in America, a cannon ball was discovered, which it was ascertained had lain undisturbed about forty-two years in ground kept constantly moist by sea-water : the diameter of the ball was 3·87 inches, and with a common saw a section was easily made through a coat of plumbaginous matter, which at the place of incision was half an inch thick ; but its thickness varied in different places. The plumbago cut with the same ease, gave the same streak to paper, and had in every respect the properties of common black lead.

A cannon ball had undergone a similar change, which was taken from the wreck of a vessel that appeared to have been many years under water ; the ball was covered by oysters firmly adhering to it, and its external part was converted into plumbago.

* Dr. Thomson's Annals of Philosophy, vol. v. p. 66.
† Idem, vol. x. p. 302.
‡ Quarterly Journal of Science, vol. xii. p. 407.

But an old cannon found covered with oysters did not, on the removal of its coating, show any signs of such conversion.*

The reader who wishes to pursue this interesting subject may consult an article "On the Mechanical Structure of Iron developed by Solution," &c., by Mr. Daniell,† who has made several experiments with a view to determine the nature of the substance resembling plumbago, which is found on the surface of iron after it has been exposed to the action of an acid.

Mr. Daniell found that the structure of iron, as developed by solution, was very different in different kinds; and that it required three times as long to saturate a given portion of acid when it acted on white cast iron, as when it acted on the gray kind.

* Phillips's Annals of Philosophy, vol. iv. p. 77. 1822.

† Quarterly Journal of Science, vol. ii. p. 278. Much additional information on the effect of water upon iron will be obtained from the Report of Mr. Mallet "upon the Action of Sea and River Water, whether clear or foul, and at various temperatures, upon Cast and Wrought Iron." British Association, vols. vii. and viii.—Editor.

EXPLANATION OF THE PLATES.

PLATE I.

FIG. 1. A bar supported at the ends, and loaded in the middle of the length. See art. 8.

FIG. 2. A beam with the load uniformly distributed over the length, as the experiment, art. 61, was tried. See art. 20 and 61.

FIG. 3. The form for a beam of uniform strength to resist the action of a load at C. ACD and BCD are semi-parabolas, A and B being the vertices. The dotted lines show the additions to this form to render it of practical use. See art. 27, 123, and 223-229.

FIG. 4. A form for a beam which is as nearly of uniform resistance as practical conditions will admit of: it is bounded by straight lines, and the depths at the ends are each half the depth in the middle. See art. 17 (Ex. 7), 28, 65, 127, and 230-234.

FIG. 5. A variation of the last form for the case where the force sometimes acts upwards and sometimes downwards. See art. 29, 127, and 230-234.

FIG. 6. A figure of uniform strength for a beam, when the depth is uniform. See art. 30, 122, and 242-246.

FIG. 7. A modification of fig. 6, which is the most economical form of equal strength for resistance to pressure. B′ is the form of the end. See art. 30.

FIG. 8. The form of equal resistance for a load rolling along the upper side, as in a railway; or for a load uniformly distributed over the length. ACB is half an ellipse. The dotted lines show the addition required in practice. See art. 32, 125, 240, and 241.

FIG. 9. The strongest form of section for a beam to resist a cross strain. AM is the line called the *neutral axis*. See art. 40, 54, 116, 185-197, and 321.

FIG. 10 shows an application of the section, fig. 9, to form a fire-proof floor, the projection serving the double purpose of giving additional strength, and forming a support for the arches. See art. 40 and 194.

PLATE II.

FIG. 11. This is the figure of a very economical beam for supporting a load diffused over its length: it is adapted for girders, beams to support walls, and the like. An easy rule for proportioning girders is given in art. 50. When this form is used for a girder, the openings answer for inserting cross joists. AB and CD show the sections at these places. See art. 21 (Ex. 12), 41, 43, 117, 198-210, and 323.

FIG. 12. This figure shows a beam on the same principle as the preceding figure, except that the load is supposed to act only at one point A. See art. 43, 44, 117, and 198-210.

FIG. 13. The section of a shaft, commonly called a feathered shaft. See art. 46.

FIG. 14. A figure to illustrate the action of forces upon a beam, and to explain the mode of calculation. See art. 106, 108, 131, and 154.

FIG. 15. A section of the beam in fig. 14, at BD. This section is supposed to be divided into thin laminæ. See art. 106.

FIG. 16. A figure to illustrate the method of calculating the deflexion of beams. In these figures (figs. 14 and 16) I have regarded distinctness of the parts referred to more than the true relation of the parts to one another. See art. 120.

PLATE III.

FIG. 17 is to illustrate the circumstances which take place in the deflexion of beams fixed at one end. See art. 133 and 154.

FIG. 18. To explain the mode of calculating the strength of cranks. See art. 135.

FIG. 19. A figure to explain the manner of estimating the strength and deflexion of a beam supported at the ends. See art. 136, 143, 146, 149, and 165.

FIG. 20. A figure to show how to calculate the strain upon a beam when a load is distributed in any regular manner over it. The load being uniform, *a d* is the line which represents its upper surface ; when the load increases as the distance from the end A, *c d* is the line bounding it ; and when the load increases as the square of the distance from A, *b d* is the line bounding it. The second case is the same as the pressure of a fluid against a vertical sheet fixed at the ends. See art. 138-141, and 160.

FIG. 21. ACB is the form of equal strength for a uniform load : it is in this figure applied to the cantilever of a balcony, and whatever ornamental form may be given to the under side, it should not be cut within the line B C. See art. 34, 130, and 157.

FIG. 22. When the section is of the form C′ D′, and the breadth uniform, the figure of equal strength for a load in the middle is formed by two semi-parabolas (as in fig. 3), shown by the dotted lines ; and it may be formed for practical application, as shown in the figure. See art. 187, 223-227.

FIG. 23 is a figure to illustrate the nature of variable forces. See art. 295.

FIG. 24. If the section of a beam be C′ D′, and the breadth uniform, the form of equal strength for a uniform load, as in girders for floors, is a semi-ellipse, shown by the dotted lines ; and also when the load rolls or slides over it ; and it may be formed for practical application, as the figure ; and an easy rule for girders of this kind is given in page 42. See art. 21 (Ex. 13), 188, 193, 240, 241, and 322.

PLATE IV.

FIG. 25 represents a beam fixed at one end ; *a′ b′* is its section ; the load acting at the end C, the figure of equal resistance is a semi-parabola. See art. 196 and 223-229.

FIG. 26. Form suitable for the beam of a steam-engine to the form of section *a′ b′*. See art. 40, 196, 222, and 223-229.

FIG. 27. A sketch for a beam to bear a considerable load distributed uniformly over its length, when the span is so much as to render it necessary to cast it in two pieces. The connexion may be made by a plate of wrought iron on each side at C, with indents to fit the corresponding parts of the beam. Wrought iron should be preferred for the connecting plates, because, being more ductile, it is more safe. See the next figure and art. 198-210.

FIG. 28 shows the under side of the beam in the preceding figure. The plates are held together by bolts; but it is intended that the strength should depend on the incidents, the bolts being only to hold them together. No connexion is wanted at the upper side of the beam, except a bolt *c d*, or like contrivance, to steady it. See art. 199.

FIG. 29. A figure to explain the nature of the resistance to twisting or torsion. See art. 263.

FIG. 30. A figure to illustrate the action of the straining force on columns, posts, and the like. See art. 276.

FIG. 31. To explain the effect of settlement or other derangement of the straining force. See art. 10 and 281.

FIG. 32. Another case of settlement or derangement of the straining force on a column considered. See art. 283.

FIG. 33. To show why columns should not be enlarged at the top or bottom. See art. 283.

FIG. 34. A sketch for an open beam recommended for an engine beam. See art. 350. In small beams the middle part may be left wholly open, except at the centre. Capt. Kater has used this form for the beam of a delicate balance.

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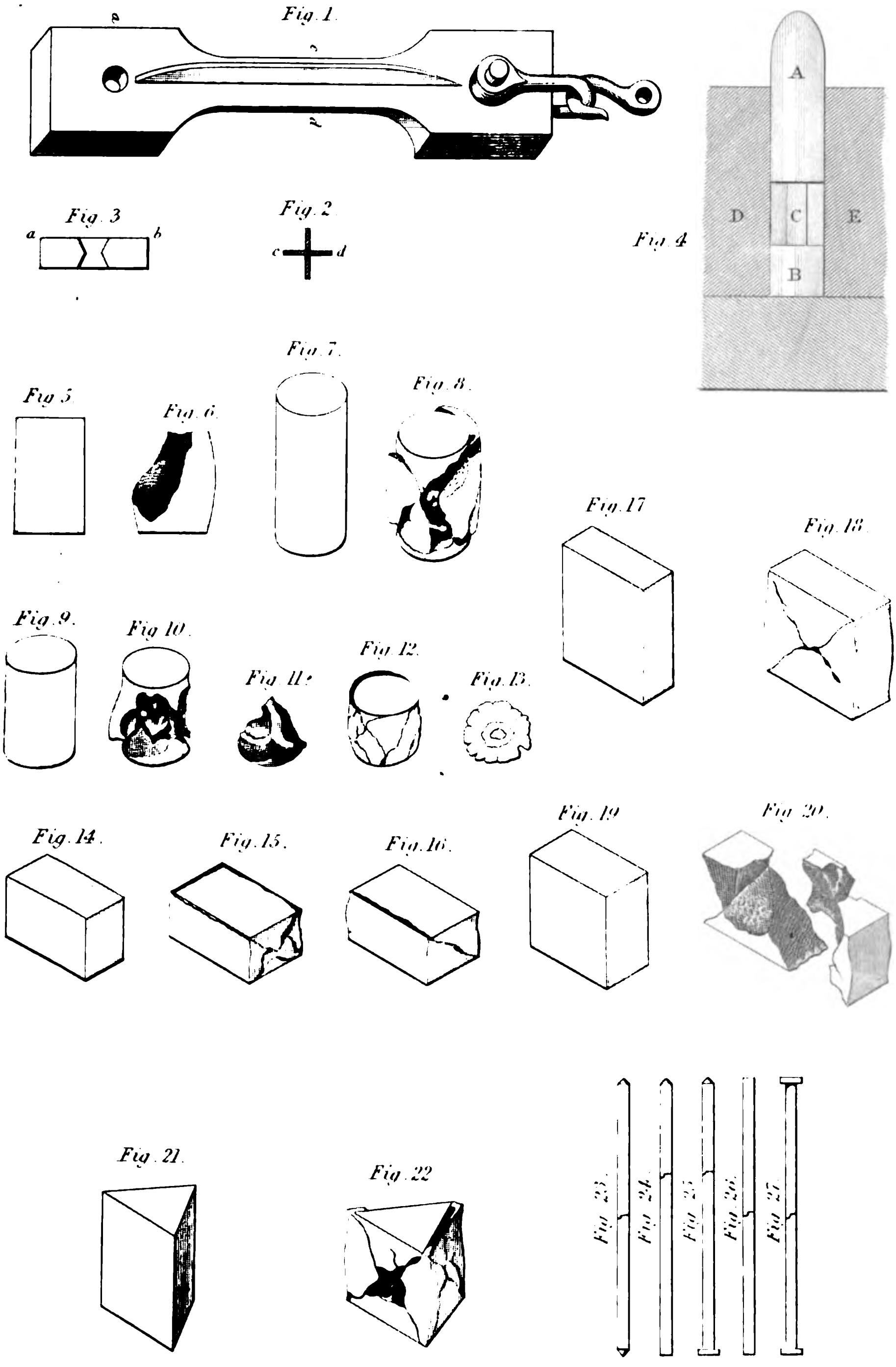

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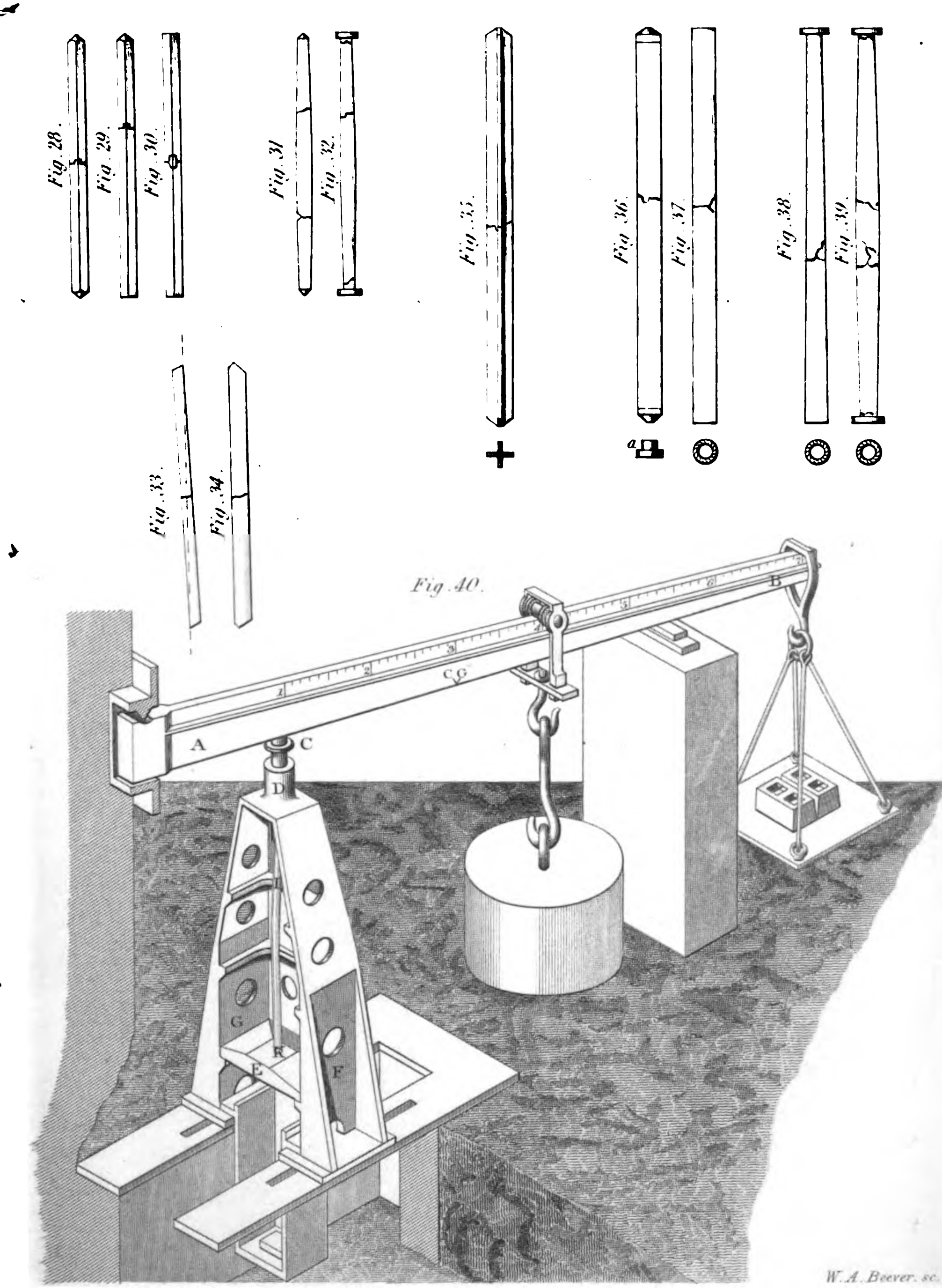

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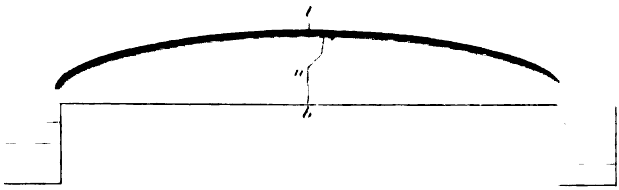
Fig. 41.



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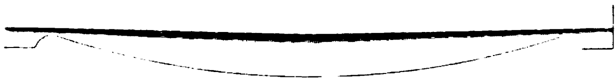

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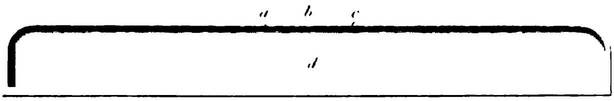

Fig. 44.

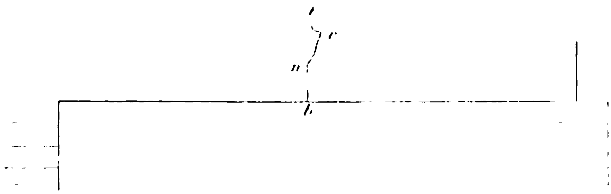

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Plate IV

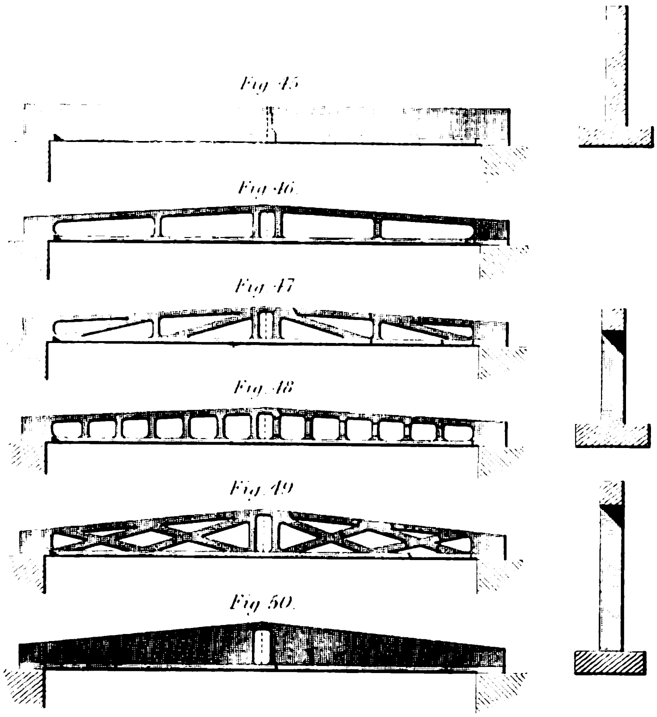

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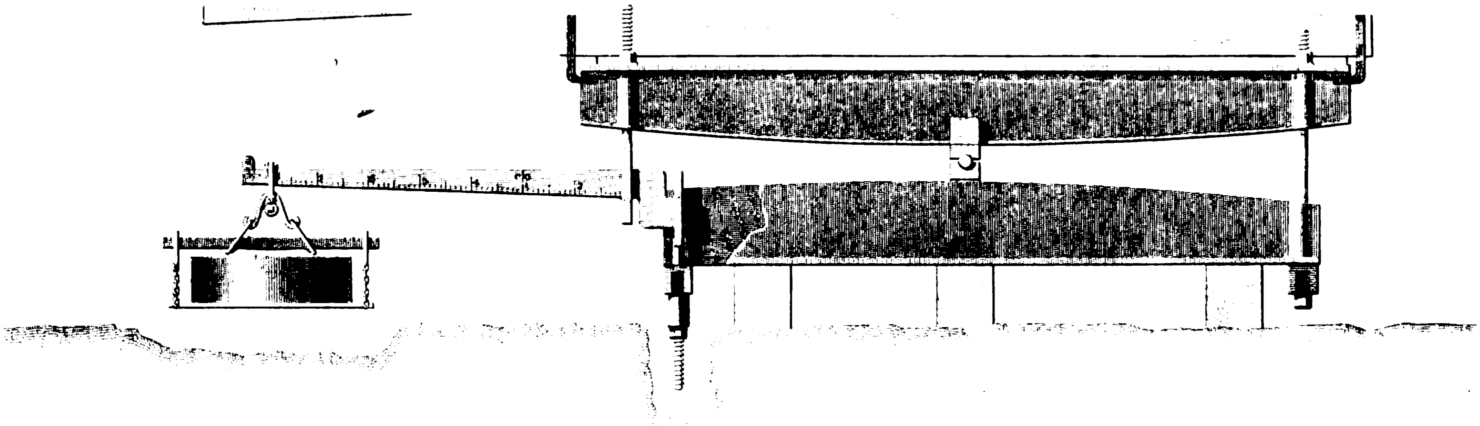
Fig. 51



Scale of Feet

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EXPERIMENTAL RESEARCHES

ON THE

STRENGTH AND OTHER PROPERTIES

OF

CAST IRON.

FORMING A SECOND VOLUME TO TREDGOLD'S

PRACTICAL ESSAY ON THE STRENGTH OF CAST IRON AND OTHER METALS.

BY EATON HODGKINSON, F.R.S.

EXPERIMENTAL RESEARCHES

ON THE

STRENGTH AND OTHER PROPERTIES

OF

CAST IRON:

WITH THE DEVELOPMENT OF NEW PRINCIPLES;

CALCULATIONS DEDUCED FROM THEM;

AND

INQUIRIES APPLICABLE TO RIGID AND TENACIOUS BODIES GENERALLY.

BY EATON HODGKINSON, F.R.S.

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CONTENTS OF PART II.

PART II.

EXPERIMENTAL RESEARCHES

ON THE

STRENGTH AND OTHER PROPERTIES OF CAST IRON;

WITH

THE DEVELOPMENT OF NEW PRINCIPLES; CALCULATIONS DEDUCED FROM THEM; AND INQUIRIES APPLICABLE TO RIGID AND TENACIOUS BODIES GENERALLY.

By EATON HODGKINSON, F.R.S.

EXPERIMENTAL RESEARCHES, ETC.

1. IN the preceding Work the very ingenious Author has confined his reasonings chiefly to the effects produced upon bodies by forces, which were small comparatively to those necessary to produce fracture. In this Second, or Additional Part, I shall generally give the ultimate strength of the bodies experimented upon, and endeavour to show the laws, or illustrate the phenomena, attendant upon fracture. The conclusions in this Part will be drawn from experiments mostly made since Mr. Tredgold wrote his Work upon Cast Iron, at which time there was confessedly a want of experimental information upon the subject in this country; and we were but slightly acquainted with what had been done upon the Continent. Having for many years devoted a portion of my time to experimental research on the strength of materials, in which the expense was borne by my liberal friend William Fairbairn, Esq., whose extensive mechanical establishment, at Manchester, enabled him to offer me every facility for the purpose, I have obtained a large mass of facts on most of the subjects connected with the strength of materials. Mr. Fairbairn has likewise published the results of a great number of well-conducted experiments upon the transverse strength of bars of cast iron. An abstract, therefore, of the leading experiments made at Mr. Fairbairn's Works, and of those given by Navier, Rennie, Bramah, and others, with theoretical considerations, is all that can be attempted in this Additional

Part; pointing out, as I proceed, whatever has a bearing upon the conclusions of Tredgold in the body of the Work.

TENSILE STRENGTH OF CAST IRON.

2. To determine the direct tensile strength of cast iron, I had models made of the form in Plate I. fig. 1.

The castings from these models were very strong at the ends, in order that they might be perfectly rigid there, and had their transverse section, for about a foot in the middle, of the form in fig. 2. This part, which was weaker than the ends, was intended to be torn asunder by a force acting perpendicularly through its centre. The ends of the castings had eyes made though them, with a part more prominent than the rest in the middle of the casting, where the eye passed through; fig. 3 represents a section of the eye. The intention of this was that bolts passing through the eyes, and having shackles attached to them, by which to tear the casting asunder, would rest upon this prominent part in the middle, and therefore upon a point passing in a direct line through the axis of the casting. Several of the castings were torn asunder by the machine for testing iron cables, belonging to the Corporation of Liverpool; the results from these are marked with an asterisk. Others were made in the same manner, but of smaller transverse area; these were broken by means of Mr. Fairbairn's lever (Plate II. fig. 40), which was adapted so as to be well suited for the purpose.

The form of casting here used was chosen to obviate the objections made by Mr. Tredgold (art 79 and 80) and others against the conclusions of former experimenters. The results are as follow:

3. *Results of Experiments on the Tensile Strength of Cast Iron.*

Description of iron.	Area of section in inches.	Breaking weight in lbs.	Strength per sq. in. of section.	Mean in lbs. per square inch.
				tons. cwt.
Carron iron, No. 2, hot blast .	4·031	56,000	13,892*	13,505=6 0½
Do. do. do. .	1·7236	22,395	12,993	
Do. do. do. .	1·7037	23,219	13,629	
Carron iron, No. 2, cold blast .	1·7091	28,667	16,772	16,683=7 9
Do. do. do. .	1·6331	27,099	16,594	
Carron iron, No. 3, hot blast .	1·7023	28,667	16,840	17,755=7 18½
Do. do. do. .	1·6613	31,019	18,671	
Carron iron, No. 3, cold blast .	1·6232	22,699	13,984	14,200=6 7
Do. do. do. .	1·6677	24,043	14,417	
Devon (Scotland) iron, No. 3, hot blast	4·269	93,520	21,907*	21,907=9 15½
Buffery iron, No. 1, hot blast .	3·835	51,520	13,434*	13,434=6 0
Buffery iron, No. 1, cold blast .	4·104	71,680	17,466*	17,466=7 16
Coed-Talon (North Wales) iron, No. 2, hot blast . .	1·586	25,818	16,279	16,676=7 9
Do. do. do. .	1·645	28,086	17,074	
Coed-Talon (North Wales) iron, No. 2, cold blast . .	1·535	30,102	19,610	18,855=8 8
Do. do. do. .	1·568	28,380	18,100	
Low Moor iron (Yorkshire), No. 3, from 5 experiments	1·540	22,385	14,535	14,535=6 10
Mixture of iron,—4 experiments further on (art. 7.) .	...	...	...	16,542=7 7¾
Mean from the whole				16,505=7 7⅓

4. The preceding Table, excepting the two last lines, is extracted from my Report on the strength and other properties of cast iron obtained by hot and cold blast, in vol. vi. of the British Association.

5. In the second volume published by the Association, there are given the results of a few experiments, which I made to determine the tensile strength of cast iron of the following mixture: Blaina No. 2 (Welsh), Blaina No. 3, and W. S. S., No. 3 (Shropshire), each in equal portions.*

6. In these experiments the intention was to determine, first, the direct tensile strength of a rectangular mass, when drawn through its axis, and next the strength of such a mass, when the force was in the direction of its side. The castings for the experiments on the central strain were of the form previously described; and in the others the force was exactly

* This mixture of iron is the same as I had employed in some experiments on the strength and best forms of cast iron beams (Memoirs of the Literary and Philosophical Society of Manchester, vol. v., Second Series), of which an account will be given further on.

along the side. The experiments were made by means of the Liverpool testing machine.

7. *Force up the middle.*

Experiments.	Area of section in parts of an inch.	Breaking weight in tons.	Strength per square inch.
1	3·012	22·5	7·47 } tons.
2	2·97	21·0	7·07 } mean 7·65 = 17136 lbs.
3	3·031	25·5	8·41 }
4	2·95	19·5	6·59 different quality of iron.

8. *Force along the side.*

Experiments.	Area of section in parts of an inch.	Breaking weight in tons.	Strength per square inch.
5	4·83	11·5	2·38 } mean 2·62 tons.
6	4·815	13·75	2·855 }

9. Whence we see that the strength of a rectangular piece of cast iron, drawn along the side, is about one-third, or a little more, of its strength to resist a central strain, since $3 \times 2·62 = 7·86$ is somewhat greater than 7·65. Mr. Tredgold computed that, if the elasticity remained perfect, the strength in these two cases would be as 4 to 1 (art. 281).

10. The following Table, calculated from one given by Navier (Application de la Mécanique), contains the results of several experiments made in 1815 by Minard and Desormes upon the direct tensile strength of cylindrical pieces of cast iron, of which the specific gravity was 7·074. The results from the experiments which they have given on defective specimens are here rejected.

No. of Experiments.	Temperature.		Area of transverse section.	Weight producing rupture.		
				Total.	Per square millimètre.	Per English inch square.
	Degrees of centigrade thermometer.	Degrees of Fahrenheit.	Square millimètres.	Kilogrammes.	Kilogrammes.	Tons. English.
1	— 6	21·2	330	3392	10·2788	6·5277
2	— 5	23	346	3542	10·2370	6·5011
3	— 5	23	363	3092	8·5179	5·4094
4	—15	5	363	3720	10·2479	6·5080
5	+ 60	140	353	4020	11·3881	7·2321
6	+ 3	37·4	147	1920	13·0612	8·2946
7	+ 5	41	165	1920	11·6364	7·3898
8	+ 5	41	165	2140	12·9691	8·2362
9	+ 5	41	165	2360	14·3030	9·0833
13	+ 5	41	346	3670	10·6069	6·7360

11. Since a square millimètre is ·001550059 of an English square inch, and a kilogramme = 2·205 of a pound avoirdupois, multiplying the kilogramme per square millimètre, in the last column of the Table above, by 2·205, and dividing the product by ·001550059, and by 2240, the number of pounds in a ton, gives the number of tons per square inch which the iron required to tear it asunder. Or if we multiply the numbers in the last column but one by ·63506, we obtain the same result; and thus the last column was formed.

12. We find from these experiments that the strength of the weakest specimen was 5·09 tons per square inch, that of the strongest 9·08 tons, and the mean strength from all the specimens was 7·19 tons.

13. In my own experiments given above, in which every care was taken both to form the castings in such a manner as to obviate theoretical objections, and to obtain accurate results, the strengths varied from 6 tons to 9¾ tons per square inch, the mean from twenty-five experiments being 16,505lbs. or 7·37 tons nearly. These experiments were upon iron obtained from various parts of England, Scotland, and Wales; and in no case, except one, was it found to bear more than 8½ tons per square inch. With these facts before the reader he will, I conceive, be unable to see how the mean tensile strength of cast iron can properly be assumed at more than 7 or 7½ tons per square inch; but some of our best writers have, by calculating the tensile strength from experiments on the transverse, arrived at the conclusion that the strength of cast iron is 10, or even 20 tons, or more. Mr. Barlow conceives it to be upwards of 10 tons (Treatise on Strength of Timber, Cast Iron, &c., p. 222), and Mr. Tredgold makes it at least 20 (art. 72 to 76). The reasoning of Mr. Tredgold, by which he arrives at this erroneous conclusion, with others resulting from it, will be examined at length under the head "Transverse Strength." Navier, too, (Application de la

Mécanique, article 4,) calculates the tensile strength of cast iron from principles somewhat similar to those assumed by Tredgold, and finds it much too high.

STRENGTH OF CAST IRON AND OTHER MATERIALS TO RESIST COMPRESSION.

14. On this subject there was acknowledged to exist a greater want of experimental research than on any other connected with the strength of materials. Feeling this to be the case, I have done all that I could, without too lavish a use of the means intrusted to me, to supply the deficiency.

15. The matter will be classed under two heads. 1st. The resistance of bodies which are so short, compared with their lateral dimensions, as to be crushed with little or no flexure. 2nd. The resistance of pillars long enough to break by flexure.

RESISTANCE OF SHORT MASSES TO A CRUSHING FORCE.

16. On this subject I shall give, as before, an abstract from my Report on the strength and other properties of hot and cold blast iron, in the sixth volume of the British Association, referring for more information to the Work itself. Great diversity exists in the conclusions of former experimenters upon the matter. Rondelet found (Traité de l'Art de bâtir) that cubes of malleable iron, and prisms of various kinds of stone, were crushed with forces which were directly as the area, whilst from Mr. Rennie's experiments, both upon cast iron and wood, it would appear that the resistance increases, particularly in the latter, in a much higher ratio than the area (Mr. Barlow's Treatise on the Strength of Materials, art. 112). With respect to M. Rondelet's conclusions, that cubes of malleable iron resisted crushing with forces proportional to their areas, and that to such a degree,

that when in his experiments the area was increased four times, this ratio did not differ from the result so much as a fiftieth part, I am strongly persuaded that *wrought* iron does not admit of such precision of judging when crushing commences, as to enable any conclusion to be easily drawn with respect to its proportionate resistance to crushing. A prism of that metal becomes slightly flattened and enlarged in diameter with about 9 or 10 tons per square inch, and this effect increases as the weight is increased; but there is no abrupt change in the metal by disunion of the parts, as in cast iron, wood, &c.

17. With respect to the experiments of Mr. Rennie, the lever used in performing them would not during its descent act uniformly upon all parts of the specimen; and therefore the results would be liable to objection. I endeavoured therefore, by repeating, with considerable variations, in the Report above named, the ingenious experiments of Mr. Rennie, to arrive at some definite conclusions upon the subject.

18. In order to effect this, it was thought best to crush the object between two flat surfaces, taking care that these were kept perfectly parallel, and that the ends of the prism to be crushed were turned parallel, and at right angles to their axes; so that when the specimen was placed between the crushing surfaces its ends might be completely bedded upon them. For this purpose a hole, $1\frac{1}{4}$ inch diameter, was drilled through a block of cast iron, about 5 or 6 inches square, and two steel bolts were made which just fitted this hole, but passed easily through it; the shortest of these bolts was about $1\frac{1}{4}$ inch long, and the other about 5 inches; the ends of these bolts were hardened, having previously been turned quite flat and perpendicular to their axes, except one end of the larger bolt, which was rounded. The specimen was crushed between the flat ends of these bolts, which were kept parallel by the block of iron in which they were inserted. See fig. 4, where A and B represent the bolts, with the prism

C between them, and D E the block of iron. During the experiment the block and bolt B rested upon a flat surface of iron, and the rounded end of the bolt A was pressed upon by the lever. There was another hole drilled through the block at right angles to that previously described; this was done in order that the specimen might be examined during the experiment, and previous to it, to see that it was properly bedded.

19. The specimens were crushed by means of the lever represented in Plate II. fig. 40, the bolt A (Plate I. fig. 4) being placed under it in the manner the pillar is there described to be. In the experiments the lever was kept as nearly horizontal as possible.

The results of the first experiments I made are given in the following Table:

20. *Tabulated results of experiments made to ascertain the weights necessary to crush given cylinders, &c., of cast iron, of the quality No. 2, from the Carron Iron Works. The specimens in the first three columns of results were from cylinders cast for the purpose, and turned to the right size; the ½ inch from those of about ¾ inch, &c.*

Height of specimens.	Cylinder ¼ inch dia., area of base ·0491. Crushing weight.	Cylinder ⅜ inch dia., area of base ·1104. Crushing weight.	Cylinder ½ inch dia., area of base ·1963. Crushing weight.	Cylinder ·64 inch diameter, area of base ·3217. Crushing weight.	Right prisms, base an equilateral triangle, circumscribing an ½ inch cylinder, its sides being ·866 in., area ·3247. Crushing weight.	Right prisms, bases squares, ½ inch the side, cut out of an inch square bar, area ·250. Crushing weight.	Right prisms, base a rectangle 1·00 × ·251 inch, cut out of a bar 1¼ inch square, area ·251 inch. Crushing weight.
	lbs.	lbs.	lbs.	lbs.			
⅛ inch.	{ 8737 / 8145 }	18,882	30,461				
¼ do.	6818	16,474	26,983				
⅜ do.	6563	13,736	26,412				
½ do.	6301	13,638	24,210		35,548	25,721	27,187
⅝ do.	6309	14,156		38,671			
¾ do.	5980	15,059	23,465	35,888	33,448	24,191	
⅞ do.				35,888			
1 do.	5798	14,877	22,867		31,348	23,950	25,151
1¼ do.		14,190	24,177				
1½ do.		14,143	23,453				
2 do.		13,800	21,828				

21. By comparing the results in each vertical column, we see that the shorter specimens generally bear more than the larger ones of the same diameter, or dimensions of base. In the shortest specimens fracture takes place by the middle becoming flattened and increased in breadth, so as to burst

the surrounding parts, and cause them to be crumbled and broken in pieces. This is usually the case when the lateral dimensions of the prism are large compared with the height. When they are equal to, or less than the height, fracture is caused by the body becoming divided diagonally in one or more directions. In this case the prism, in cast iron at least, either does not bend before fracture, or bends very slightly; and therefore the fracture takes place by the two ends of the specimen forming cones or pyramids, which split the sides and throw them out; or, as is more generally the case in cylindrical specimens, by a wedge sliding off, starting at one of the ends, and having the whole end for its base; this wedge being at an angle which is constant in the same material, though different in different materials. In cast iron the angle is such that the height of the wedge is somewhat less than $\frac{3}{2}$ of the diameter. The forms of fracture in these cases may be seen from Plate I., in which fig. 5 represents a cylinder before fracture, and fig. 6 the same cylinder afterwards; a part in the form of a wedge having separated from one side of it, and the remainder being shortened and bulged out in the middle, which is very obvious in experiments on soft cast iron. Figs. 7 and 8 represent another cylinder before and after fracture: in fig. 8 the sides are separated by the action of two cones, having the ends of the cylinder for the bases, and the vertices with sharp edges or points formed near to the centre of the cylinder, but inclined a little from the axis, so that they may slip past each other, and divide the mass without injury to the cones. Figs. 9 and 10 show the same thing as the two preceding figures; and fig. 11 is a representation of one of the cones, the vertex being sharp, as above mentioned. Fig. 12 is another cylinder, of soft cast iron, showing the directions of the fractures, but not separated. Fig. 13 was a cylinder too short to be crushed in the ordinary way; but the centre shows the rudiments of a cone, throwing out the surrounding parts. Fig. 14 represents a rectangular prism; and figs. 15 and 16 the modes of its fracture. Figs. 17 and 18 represent a whole and fractured prism; and the same is the case with respect to figs. 19 and 20. In fig. 20 the sharp-pointed pyramid, with the lower side of the specimen for its base, is very clearly shown; it has cut up the prism, separating the sides, and left a number of sharp-edged parts, all of which have slid off in the angle of least resistance. Figs. 21 and 22 exhibit the appearance of a triangular prism, before and after fracture; two pyramids, formed as usual, with their bases at the ends, and the vertices towards the centre, have thrown out the angular parts. The parts, so separated, have in prisms of every form a general resemblance, and the form of the pyramidal wedge has considerable interest, as it is that of least resistance in cast iron, and furnishes hints as to the best forms of cutters for that metal. Further investigations upon the subject of this article, of a theoretical nature, will be given on a future occasion.

22. The mode of fracture is the same, and the strength of the specimen very little diminished, by any increase of its height, whilst its lateral dimensions are the same, provided the height be greater than the diameter, when the body is a cylinder, but not greater, in cast iron, than four or five times the diameter, or least lateral dimension in specimens not cylindrical. If the length be greater than this, the body bends with the pressure, and though it may break by sliding off as before, the strength is much decreased. In cases where the length is much greater than as above, the body breaks across, as if bent by a transverse pressure.

23. The preceding Table was formed by taking the means from results on the resistance to crushing of specimens of equal size in Carron Iron, No. 2, one Table being on hot blast iron, and the other cold; the mean from the results being given here in one Table, in preference to the two Tables at large, as in the volume of the Association; since most of the results thus obtained are means from several ex-

periments; and there was very little difference in the strength of the hot and cold blast iron of this description to resist crushing.

24. Comparing the results in the different vertical columns of the Table, it appears that the strength of the specimens was nearly as the area of their transverse section. Thus the cylinders ½ inch in diameter bore nearly four times as much as those of ¼ inch. The falling off from this proportion in the strength of some of the larger specimens, I attribute to those having been cut out of larger (and consequently softer) masses of the iron than the small specimens.

25. To obtain further evidence on the matter I crushed in the same manner twelve right cylinders of Teak-wood, ½ inch, 1 inch, and 2 inches diameter, four of each, the latter eight out of the same piece of wood; the height in each case was double the diameter. The pressure was in the direction of the fibres. The strengths were as below.

Cylinders ½ inch diameter.		Cylinders 1 inch diameter.		Cylinders 2 inches diameter.	
2335	mean 2439 lbs.	10507	mean 10171 lbs.	38909	mean 40304 lbs.
2543		9499		39721	
2543		10507		41294	
2335		10171		41294	

26. The mean crushing weights above are nearly as 25, 100, and 400, which is the ratio of the areas of the sections of the cylinders. The strengths are therefore as the areas, though these vary as 4 and 16 to 1.

27. In this and every other kind of timber, like as in iron and crystalline bodies generally, crushing takes place by wedges sliding off in angles with their base, which may be considered constant in the same material: hence the strength to resist crushing will be as the area of fracture, and consequently as the direct transverse area; since the area of fracture would, in the same material, always be equal to the direct transverse area, multiplied by a constant quantity.

28. In estimating the resistance, per square inch, of the iron above to a crushing force, I shall mostly confine myself

to such specimens as vary in height from about the length of the wedge, which would slide off to twice its length,—say, such as have their height from $1\frac{1}{2}$ to 3 times the diameter,—thus avoiding the results from prisms which, through their shortness, could not break by a wedge sliding off at its proper angle, and therefore must offer an increased resistance; and those whose extra length would enable them to bend before sliding off, and thus have their strength reduced. I shall here take the results from my experiments upon the hot and cold blast iron separately, that the difference may be seen; the iron being of the same materials in the two cases, and affected only by the heat of the blast used in its preparation.

The results are in the following Tables.

29. *Resistance of Hot Blast Iron* (*Carron, No.* 2) *to a crushing force.*

Dimensions of base of specimen.	Number of experiments derived from.	Mean crushing weight.	Mean crushing weight per square inch.	General mean per square inch.
		lbs.	lbs.	
Right cylinder, diameter $\frac{1}{4}$ inch	3	6,426	130,909	From the cylinders 121,685 lbs. = 54 tons $6\frac{1}{4}$ cwts.
„ $\frac{3}{8}$ „	4	14,542	131,665	
„ $\frac{1}{2}$ „	5	22,110	112,605	
„ $\frac{16}{25}$ = ·64	1	35,888	111,560	
Right prism, area ·50 × ·50 in.	3	25,104	100,416	From the prisms 100,739 lbs. = 44 tons $19\frac{1}{4}$ cwts.
Ditto area 1·00 × ·26	2	26,276	101,062	

Mean per sq. in. from the whole of the experiments, 114,703 lbs. = 51 tons 4 cwts.

30. *Resistance of Cold Blast Iron* (*Carron, No.* 2) *to a crushing force.*

Dimensions of base of specimen.	Number of experiments derived from.	Mean crushing weight.	Mean crushing weight per square inch.	General mean per square inch.
		lbs.	lbs.	
Right cylinder, diameter $\frac{1}{4}$ inch	2	6,088	124,023	From the cylinders 118,211 lbs. = 52 tons $15\frac{1}{2}$ cwts.
„ $\frac{3}{8}$ „	4	14,190	128,478	
„ $\frac{1}{2}$ „	7	24,290	123,708	
„ ·45 „ and ·75 high	2	15,369	96,634	
Equilateral triangle, side ·866 inch	2	32,398	99,769	From the prisms 101,964 lbs. = 45 tons $10\frac{1}{2}$ cwts.
Square, area ·50 × ·50 inch	2	24,538	98,152	
Rectangle, area 1·00 × ·243	3	26,237	107,971	

Mean per sq. in. from the whole of the experiments, 111,248 lbs. = 49 tons $13\frac{1}{4}$ cwts.

31. Comparing the results in these two Tables, it will be seen, as has been mentioned before, that the Carron Iron, No. 2, offers but little difference of resistance to a crushing force, whether the iron be prepared with a hot or a cold blast. The falling off in the resistance per square inch in the latter class of experiments, in each Table, compared with the former, has been attempted to be accounted for by the iron out of which the larger specimens were cut being softer and weaker than the thin cylinders out of which the smaller specimens were obtained.

32. The resistances of other species of cast iron to a crushing force, obtained in the same manner as above, are as in the following Table.

Description of Iron.	Form of specimen.	Number of experiments derived from.	Mean strength per square inch.
			lbs. tons. cwts.
Devon (Scotch) iron, No. 3, hot blast . .	Cylinder.	4	145,435 = 64 $18\frac{1}{8}$
Buffery (near Birmingham) iron, No. 1, hot blast . . .	Do.	4	86,397 = 38 $11\frac{1}{4}$
Do., cold blast . .	Do.	4	93,385 = 41 $13\frac{1}{2}$
Coed-Talon (Welsh) iron, No. 2, hot blast	Do.	4	82,734 = 36 $18\frac{1}{2}$
Do., cold blast . .	Do.	4	81,770 = 36 10
Carron (Scotch) iron, No. 2, hot blast . .	Cylinders and prisms.	18	114,703 = 51 4
Do., cold blast . .	Do.	22	111,248 = 49 $13\frac{1}{4}$
Carron iron, No. 3, hot blast . . .	Prisms.	3	133,440 = 59 $11\frac{1}{4}$
Do., cold blast . .	Do.	4	115,442 = 51 $10\frac{3}{4}$
Low Moor (Yorkshire) iron, No. 3, cold blast . . .	Cylinder. Rectangle.	3 2	115,911 103,692 } mean. 109,801 = 49 tons $0\frac{1}{3}$ cwts.
Mixture of iron, same as in my experiments on the strength of beams (see note to art. 5).	Cylinders ·508 and ·6 inch. diameter, 3 of each. Rectangles cut out of a beam.	6 3	100,049 121,767 } 110,908 = 49 $10\frac{1}{4}$

33. The ratios of the forces necessary to crush and tear equal masses of cast iron may now be obtained; the experiments of which I have given the results, in this and the

preceding article, will supply those ratios which are in the following Table.

Description of Metal.	Crushing force per square inch in lbs.	Tensile force per square inch in lbs.	Ratio.
Devon iron, No. 3, hot blast	145,435	21,907	6·638 : 1
Buffery iron, No. 1, hot blast	86,397	13,434	6·431 : 1
Do. No. 1, cold blast	93,385	17,466	5·346 : 1
Coed-Talon iron, No. 2, hot blast	82,734	16,676	4·961 : 1
Do. No. 2, cold blast	81,770	18,855	4·337 : 1
Carron iron, No. 2, hot blast	114,703	13,505	8·493 : 1
Do. No. 2, cold blast	111,248	16,683	6·668 : 1
Do. No. 3, hot blast	133,440	17,755	7·515 : 1
Do. No. 3, cold blast	115,442	14,200	8·129 : 1
Low Moor iron, No. 3, cold blast	109,801	14,535	7·554 : 1
Mixture of iron used in my experiments on beams (art. 5-7)	110,908	17,136	6·472 : 1
			Mean 6·595 : 1

34. From this Table it appears that cast iron requires from 4·337 to 8·493 times as much force to crush it as to tear it asunder, the mean being 6·595. I conceive, however, this mean to be too low; it would have been between 7 and 8 if the prisms which were crushed had been cut out of the same masses as those which were torn asunder; but they were in many instances out of larger castings, which, being softer, offered less resistance.

STRENGTH OF LONG PILLARS.

35. I shall here consider such pillars as are too long to break by sliding off, as in the last article. This class will include all such as are usually made of iron or timber, the lengths of the shortest of which are generally many times their lateral dimensions; and it has been remarked before, that in some cases right cylinders of cast iron, whose length did not exceed 5 times the diameter, became bent under a direct force of compression, so as to break nearly straight across in the manner of longer columns.

The acknowledged want of practical information upon this subject,* and its great importance, made me anxious to undertake an extensive series of experiments upon it, such as would confirm or show the error of existing theories, and give such information as would be of real service to the engineer and architect, whilst they tended to unfold the laws that regulate the strength of pillars. This wish was, as on other occasions, cheerfully responded to by my friend William Fairbairn, Esq., at whose expense the extensive series of experiments, of which the following is an abstract † was made.

36. In the original Paper the experiments are contained in thirteen Tables,‡ as below.

		No. of Experiments.
	CAST IRON.	
Table I.	Solid uniform cylindrical pillars, with rounded ends (fig. 23)	55
II.	Do., with flat ends (fig. 26)	65
III.	Solid uniform square pillars, with rounded ends (fig. 28.)	7
IV.	Solid uniform cylindrical pillars, with discs (fig. 27)	12
V.	Do., with ends rounded (fig. 23), rounded and flat (fig. 24), and both ends flat (fig. 26)	23
VI.	Solid cylindrical pillars, enlarged middle, rounded ends (fig. 31)	7
VII.	Do., do., discs on ends (fig. 32)	4
VIII.	Hollow uniform cylinders, rounded ends (fig. 36)	19
IX.	Do., with flat ends (fig. 37)	11
X.	Short hollow uniform cylinders, with flat ends (fig. 37)	13
XI.	Pillars, hollow and solid, of various forms, and different modes of fixing (figs. 33, 34, 35, 38, 39)	10
	WROUGHT IRON AND STEEL.	
XII.	Solid uniform cylindrical pillars of these metals (figs. 23, 24, 25, 26, 27)	30
	WOOD.	
XIII.	Pillars of oak and red deal, square and other rectangular forms (figs. 28, 29, 30, &c.)	21
	Number of experiments	277

* Dr. Robison (Mechanical Philosophy, vol. i.) and Mr. Barlow (Report to the British Association) have strongly expressed their opinion of our want of information upon it; and the British Association have mentioned such experiments as among the desiderata of Practical Science.

† See Experimental Researches on the Strength of Pillars of Cast Iron and other Materials.—Phil. Trans. of the Royal Society, Part II. 1840.

‡ Abstracts of Tables I., II., VIII., IX., X., are given in the present work.

37. The drawing, Plate II. fig. 40, will show how the experiments were made. The pillars were placed vertically, resting upon a flat smooth plate of hardened steel, laid upon a cast iron shelf, E, made very strong, and lying horizontally. The pressure was communicated to the upper end of the pillar by means of the lever, A B, acting upon a bolt, C, of hardened steel, 2½ inches diameter, and about a foot long, kept vertical by being made to pass through a hole bored in a deep mass, D, of cast iron, the hole being so turned as just to let the bolt slide easily through without lateral play. The top of the bolt was hemispherical, that the pressure from the lever might act through the axis of it; and the bottom was turned flat to rest upon the pillar, I K. The bottom of this bolt, and the shelf on which the pillar stood, were necessarily kept parallel to each other; for the mass through which the bolt passed, and that on which the shelf rested, were parts of the same large case, D F G, of iron, cast in one piece, and so formed as to admit shelves at various heights for breaking pillars of different lengths. The case had three of its four sides closed; circular apertures were, however, made through them, that the experimenter might observe the pillar without danger.

38. With a view to develope the laws connecting the strength of cast iron pillars with their dimensions, they were broken of various lengths, from 5 feet to 1 inch, and their diameters varied from ½ an inch to 2 inches in solid pillars; and in hollow ones the length was increased to 7 feet 6 inches, and the diameter to 3½ inches. My first object was to supply the deficiencies of Euler's theory of the strength of pillars (Académie de Berlin, 1757,) if it should appear capable of being rendered practically useful; and if not, to endeavour to adapt the experiments so as to lead to useful conclusions. As the results of the experiments were intended to be compared together, it was desirable that all the pillars of cast iron should be from one species of metal; and the description

chosen was a Yorkshire iron, the Low Moor, No. 3, which is a good iron, not very hard, and differs not widely from that called No. 2. The pillars were mostly made cylindrical, as that seemed a more convenient form in experiments of this kind than the square; for square pillars do not bend or break in a direction parallel to their sides, but to their diagonals, nearly. The experiments in the first Table were made on solid uniform pillars rounded at the ends to render them capable of turning easily there, and that the force might be through the axis; and in the second Table the pillars were uniform and cylindrical, as before, but had their ends flat and at right angles to the axis; so that the whole end of the pillar might be pressed upon, instead of the axis only, as in the last case; thus rendering the pillar incapable of moving at the ends. In the fifth Table (art. 36) uniform pillars, with one end rounded and one flat, were used; and to prove the constant connexion (in any particular material) between the results of these three kinds of pillars, all of equal dimensions, enters largely into the other Tables, which are occupied likewise in inquiries into the strength of hollow cylindrical pillars, and others of different forms.

39. RESULTS.

1st. In all long pillars of the same dimensions, the resistance to fracture by flexure is about three times greater when the ends of the pillar are flat and firmly bedded, than when they are rounded and capable of turning.*

2nd. The strength of a pillar, with one end round and the other flat, is the arithmetical mean between that of a pillar of the same dimensions with both ends rounded, and with both ends flat. Thus, of three cylindrical pillars, all of the same length and diameter, the first having its ends rounded, the

* This will be seen by comparing the results from the longer pillars of equal size in Tables I. and II., page 241, of which abstracts are given further on.

second with one end rounded and one flat, and the third with both ends flat, the strengths are as 1, 2, 3, nearly.*

3rd. A long uniform pillar, with its ends firmly fixed, whether by discs or otherwise, has the same power to resist breaking as a pillar of the same diameter, and half the length, with the ends rounded or turned so that the force would pass through the axis.

The preceding properties were found to exist in long pillars of steel, wrought iron, and wood.

4th. The experiments in Tables VI. and VII. (art. 36) show that some additional strength is given to a pillar by enlarging its diameter in the middle part; this increase does not, however, appear to be more than $\frac{1}{7}$th or $\frac{1}{8}$th of the breaking weight.

5th. The index of the power of the diameter, to which the strength of long pillars of cast iron, with rounded ends, is proportional, is 3·76, nearly; and 3·55 in those with flat ends; as appeared from means between the results of a great number of experiments; † or the strength of both may be taken as following the 3·6 power of the diameter, nearly.

6th. In cast iron pillars of the same thickness the strength is inversely proportional to the 1·7 power of the length, nearly.‡

40. Thus the strength of a solid pillar of that material, with rounded ends, the diameter of the pillar being d and the length l, is, as

$$\frac{d^{3\cdot6}}{l^{1\cdot7}}.$$

* This was proved by Table V., page 241, and the next conclusion was obtained by a like comparison of results.

† The pillars with rounded ends, compared together, varied in diameter (or side of square) from $\frac{1}{2}$ inch to 2 inches. There were seventeen of them, and the largest exponent of the diameter, to which the strength was in any case proportional, is 3·928, and the smallest 3·425. The pillars with flat ends, and those with discs, compared as above, were eleven, and their exponents varied from 3·922 to 3·412.

‡ The exponent 1·7 for the strength, according to the inverse power of the length, is from experiments upon nineteen pillars, varying in length from 60·5 to 3·78 inches; the highest and lowest exponents being 1·914 and 1·424.

41. The breaking weights of solid cylindrical cast iron pillars, as appeared from the experiments, are nearly as below.

In solid pillars with their ends rounded and moveable as above, fig. 23,

$$\text{Strength in tons} = 14{\cdot}9 \times \frac{d^{3.6}}{l^{1.7}}.$$

In solid pillars with their ends flat, and incapable of motion, fig. 26,

$$\text{Strength in tons} = 44{\cdot}16 \times \frac{d^{3.6}}{l^{1.7}};$$

where l is in feet, and d in inches.

In hollow pillars nearly the same laws were found to obtain; thus, if D and d be the external and internal diameters of a cast iron pillar, whose length is l, the strength of a hollow pillar of which the ends were moveable, fig. 36, (as in the connecting rod of a steam engine,) would be expressed by the formula below.

$$\text{Strength in tons} = 13 \times \frac{D^{3.6} - d^{3.6}}{l^{1.7}}.$$

In hollow pillars, whose ends are flat, and firmly fixed (fig. 37, &c.) by discs or otherwise, I found from the results of numerous experiments as before,

$$\text{Strength in tons} = 44{\cdot}3 \times \frac{D^{3.6} - d^{3.6}}{l^{1.7}}.$$

42. The co-efficients given above in the formulæ for the strength of solid pillars were obtained as below:—The 14·9 is the mean result from 18 pillars in Table I., art. 36, varying in length from 121 times the diameter down to 15 times. The 44·16, for pillars with flat ends, is similarly obtained from 11 pillars in Table II., varying in length from 78 to 25 times the diameter. Flat-ended pillars, shorter than 25 or 30 times the diameter, require a modification of the above rule for their strength; as I found them to be

crushed as well as bent by the pressure, and therefore to have their strength decreased: the mode I used in this case will be seen in arts. 45 to 51, further on. The co-efficients for the strength of hollow pillars were obtained in the same manner as those for solid ones; the 13 is the mean obtained from 19 experiments in Table VIII. upon hollow pillars, with rounded caps upon the ends, to make them moveable there like solid ones with rounded ends; this number is too low, as some of the first hollow pillars were bad castings. The 44·3, for those with flat ends, was obtained from 11 pillars in Table IX.; taking pillars only whose length was more than 25 times the external diameter, as in solid ones.

The Tables from which these results were derived are given, slightly abridged, further on. The formulæ for hollow pillars were obtained by adapting the results of Euler's theory to those of experiment, and were found to answer well when so altered. According to that theory the strength varies as $\frac{D^4-d^4}{l^2}$ (Poisson, Mécanique, vol. i., 2nd edition, art. 315).

43. As the above expressions for the strength of pillars contain fractional powers of the diameters and lengths, these may be taken from the Tables below; the first Table comprising the involved values in inches of most of the diameters in common use; and the second those of the lengths in feet.

TABLE I.—*Powers of Diameters.*

$1·0^{3·6}$ = 1	$4·25^{3·6}$ = 182·89	$6·8^{3·6}$ = 993·19
$1·25^{3·6}$ = 2·2329	$4·3^{3·6}$ = 190·76	$6·9^{3·6}$ = 1046·8
$1·5^{3·6}$ = 4·3045	$4·4^{3·6}$ = 207·22	$7·0^{3·6}$ = 1102·4
$1·75^{3·6}$ = 7·4978	$4·5^{3·6}$ = 224·68	$7·1^{3·6}$ = 1160·2
$2·0^{3·6}$ = 12·125	$4·6^{3·6}$ = 243·18	$7·2^{3·6}$ = 1220·1
$2·1^{3·6}$ = 14·454	$4·7^{3·6}$ = 262·76	$7·25^{3·6}$ = 1250·9
$2·2^{3·6}$ = 17·089	$4·75^{3·6}$ = 272·96	$7·3^{3·6}$ = 1282·2
$2·25^{3·6}$ = 18·529	$4·8^{3·6}$ = 283·44	$7·4^{3·6}$ = 1346·6
$2·3^{3·6}$ = 20·055	$4·9^{3·6}$ = 305·28	$7·5^{3·6}$ = 1413·3
$2·4^{3·6}$ = 23·3755	$5·0^{3·6}$ = 328·82	$7·6^{3·6}$ = 1482·3
$2·5^{3·6}$ = 27·076	$5·1^{3·6}$ = 352·58	$7·7^{3·6}$ = 1553·7
$2·6^{3·6}$ = 31·182	$5·2^{3·6}$ = 378·10	$7·75^{3·6}$ = 1590·3
$2·7^{3·6}$ = 35·720	$5·25^{3·6}$ = 391·36	$7·8^{3·6}$ = 1627·6
$2·75^{3·6}$ = 38·159	$5·3^{3·6}$ = 404·94	$7·9^{3·6}$ = 1704·0
$2·8^{3·6}$ = 40·716	$5·4^{3·6}$ = 433·13	$8·0^{3·6}$ = 1782·9
$2·9^{3·6}$ = 46·199	$5·5^{3·6}$ = 462·71	$8·25^{3·6}$ = 1991·7
$3·0^{3·6}$ = 52·196	$5·6^{3·6}$ = 493·72	$8·5^{3·6}$ = 2217·7
$3·1^{3·6}$ = 58·736	$5·7^{3·6}$ = 526·20	$8·75^{3·6}$ = 2461·7
$3·2^{3·6}$ = 65·848	$5·75^{3·6}$ = 543·01	$9·0^{3·6}$ = 2724·4
$3·25^{3·6}$ = 69·628	$5·8^{3·6}$ = 560·20	$9·25^{3·6}$ = 3006·85
$3·3^{3·6}$ = 73·561	$5·9^{3·6}$ = 595·75	$9·5^{3·6}$ = 3309·8
$3·4^{3·6}$ = 81·908	$6·0^{3·6}$ = 632·91	$9·75^{3·6}$ = 3634·3
$3·5^{3·6}$ = 90·917	$6·1^{3·6}$ = 671·72	$10·0^{3·6}$ = 3981·07
$3·6^{3·6}$ = 100·62	$6·2^{3·6}$ = 712·22	$10·25^{3·6}$ = 4351·2
$3·7^{3·6}$ = 111·05	$6·25^{3·6}$ = 733·11	$10·5^{3·6}$ = 4745·5
$3·75^{3·6}$ = 116·55	$6·3^{3·6}$ = 754·44	$10·75^{3·6}$ = 5165·0
$3·8^{3·6}$ = 122·24	$6·4^{3·6}$ = 798·45	$11·0^{3·6}$ = 5610·7
$3·9^{3·6}$ = 134·23	$6·5^{3·6}$ = 844·28	$11·25^{3·6}$ = 6083·4
$4·0^{3·6}$ = 147·03	$6·6^{3·6}$ = 891·99	$11·5^{3·6}$ = 6584·3
$4·1^{3·6}$ = 160·70	$6·7^{3·6}$ = 941·61	$11·75^{3·6}$ = 7114·4
$4·2^{3·6}$ = 175·26	$6·75^{3·6}$ = 967·15	$12·0^{3·6}$ = 7674·5

TABLE II.—*Powers of Lengths.*

$1^{1·7}$ = 1	$9^{1·7}$ = 41·900	$17^{1·7}$ = 123·53
$2^{1·7}$ = 3·2490	$10^{1·7}$ = 50·119	$18^{1·7}$ = 136·13
$3^{1·7}$ = 6·4730	$11^{1·7}$ = 58·934	$19^{1·7}$ = 149·24
$4^{1·7}$ = 10·556	$12^{1·7}$ = 68·329	$20^{1·7}$ = 162·84
$5^{1·7}$ = 15·426	$13^{1·7}$ = 78·289	$21^{1·7}$ = 176·92
$6^{1·7}$ = 21·031	$14^{1·7}$ = 88·801	$22^{1·7}$ = 191·48
$7^{1·7}$ = 27·332	$15^{1·7}$ = 99·851	$23^{1·7}$ = 206·51
$8^{1·7}$ = 34·297	$16^{1·7}$ = 111·43	$24^{1·7}$ = 222·00

44. As an example, suppose it were required to find the strength of a hollow cylindrical cast iron pillar, 14 feet long, 6·2 inches external diameter, and 4·1 inches internal; the pillar being flat, and well supported, at the ends.

From the Tables we obtain $14^{1·7} = 88·801$, $6·2^{3·6} = 712·22$, and $4·1^{3·6} = 160·70$. Whence strength $= 44·3 \times \frac{D^{3·6} - d^{3·6}}{l^{1·7}} = 44·3 \times \frac{712·22 - 160·70}{88·801} = 275·1$ tons.

STRENGTH OF SHORT FLEXIBLE PILLARS.

45. The formulæ above apply to all pillars whose length is not less than about 30 times the external diameter; for pillars shorter than this, it will be necessary to modify the formulæ by other considerations, since in these shorter pillars the breaking weight is a considerable proportion of that necessary to crush the pillar.

46. Thus, considering the pillar as having two functions, one to support the weight, and the other to resist flexure, it follows that when the material is incompressible (supposing such to exist), or when the pressure necessary to break the pillar is very small, on account of the greatness of its length compared with its lateral dimensions, then the strength of the whole transverse section of the pillar will be employed in resisting flexure; when the breaking pressure is half of what would be required to crush the material, one half only of the strength may be considered as available for resistance to flexure, whilst the other half is employed to resist crushing; and when, through the shortness of the pillar, the breaking weight is so great as to be nearly equal to the crushing force, we may consider that no part of the strength of the pillar is applied to resist flexure.

47. This reasoning is supported by the results from a number of short pillars of various lengths, from 26 times the diameter down to twice the diameter. These pillars were reduced to about half the strength, as calculated by the preceding formulæ, when the length was so small that the breaking weight was half of that which would crush the pillar; and the results from short pillars of other lengths were in accordance with the preceding reasoning. (See Researches on the strength of pillars, &c., art. 39, Phil. Trans. 1840.)

48. We may therefore separate these effects by taking, in imagination, from the pillar (by reducing its breadth, since the streng h is as the breadth,) as much as would support the

pressure, and considering the remainder as resisting flexure to the degrees indicated by the previous rules.

49. Suppose, then, c to be the force which would crush the pillar without flexure; d the utmost pressure the pillar, as flexible, would bear to break it without being weakened by crushing (as was shown to take place with a certain pressure dependent on the material); b the breaking weight, as calculated by the preceding formulæ for long pillars; y the real breaking weight.

Then supposing a portion of the pillar, equal to what would just be crushed by the pressure d, to be taken away, we have $c-d$= the crushing strength of the remaining part, and $y-d$ the weight actually laid upon it. Whence $\frac{y-d}{c-d}$= the part of this remaining portion of the pillar which has to resist crushing,

$$\therefore 1-\frac{y-d}{c-d}=\frac{c-y}{c-d}=$$

the part to sustain flexure.

50. But the strength of the pillar, if rectangular, may be supposed to be reduced, by reducing either its breadth, or the computed strength of the whole, to the degree indicated by the fraction last obtained. In circular pillars this mode is not strictly applicable; but we obtain a near approximation to the breaking weight y, by reducing the calculated value of b in that proportion.

Whence $b\times\frac{c-y}{c-d}=y$, the strength of a short flexible pillar, b being that of a long one, $\therefore b\,c-b\,y=c\,y-d\,y$,

$$\text{and } y=\frac{b\,c}{b+c-d}.$$

51. It was shown (Experimental Researches, art. 5-7) that cast iron pillars with flat ends uniformly bore about three times as much as those of the same dimensions with rounded ends; and this was found by experiment to apply to all pillars from 121 times the diameter down to 30 times.

52. In flat-ended cast iron pillars, shorter than this, there was observed to be a falling off in the strength; and the same was found to be the case in pillars of other materials, on which many experiments were made, to ascertain whether the results previously mentioned, as obtained from the cast iron pillars, were general. The cause of the shorter pillars falling off in strength, as mentioned above, was rendered very probable by the experiments upon wrought iron; for in that metal a pressure of from 10 to 12 tons per square inch produced a permanent change in, and reduced the length of short cylinders, subjected to it, (art. 60 of Paper above;) and about the same pressure per square inch of section, when required to break by flexure a wrought iron pillar with flat ends, produced a similar falling off in strength to that which was experienced when a weight per square inch, not widely different from this, was required to break a cast iron pillar with flat ends. The fact of cast iron pillars sustaining a marked diminution of their breaking strength by a weight nearly the same as that which produced incipient crushing in wrought iron, and a falling off in the strength of wrought iron pillars, rendered it extremely probable that the same cause (incipient crushing or derangement of the parts) produced the same change in both these species of iron.

53. The pressure which produced the change mentioned above in the breaking of cast iron pillars was about $\frac{1}{4}$th of that which crushed the material, as given from the experiments upon the metal there used. I shall therefore assume here, as I did there, that one-fourth of the crushing-weight is as great a pressure as these cast iron pillars could be loaded with, without their ultimate strength being decreased by incipient crushing; and it was there shown that the length of such a pillar, if solid and with flat ends, would be about 30 times its diameter.

54. We shall have, therefore, $d=\frac{1}{4}$, in the preceding formula

$$y = \frac{b\,c}{b + c - d};$$

whence in cast iron of the kind used, (Low Moor, No. 3,)

$$y = \frac{b\,c}{b + \frac{3\,c}{4}}.$$

55. To find the force necessary to crush a square inch of the iron mentioned above, in order that the value of c, which is that which would crush the whole pillar if inflexible, might be computed, I made (art. 55 of the Paper before referred to) experiments upon it, both upon cylinders and rectangles; and the mean strength from five of the results gave, per square inch, 109,801 ℔s.=49·018 tons.

56. The value of y above is compounded of two quantities, b the strength as obtained from one of the formulæ for long flexible pillars (art. 41 of the present Work), and c the crushing force.

57. The following Table, which gives the dimensions and breaking weights of eleven short solid pillars with flat ends, together with the calculated values of b, c, and y, will show what degree of approximation the calculated strength bears to the real.

Here b the strength in ℔s. (arts 39 and 41),

$$= 98922 \times \frac{d^{3.55}}{l^{1.7}}.$$

58. *Short solid pillars flat at the ends, fig.* 26.

Diameter of pillar.	Length of pillar.	Value of b.	Value of c.	Breaking weight.	Calculated breaking weight from the formula $y = \frac{bc}{b + \frac{3c}{4}}$
inches.	ft. inches.	℔s.	℔s.	℔s.	℔s.
·50	1·008=12·1	8327	21559	7195	7328
·50	·840=10·083	11353	21559	8931	8872
·50	·630= 7·5625	18515	21559	11255	11508
·50	·315= 3·7812	60155	21559	17468	16992
·777	1·681=20·166	16713	52064	15581	15604
·775	1·260=15·125	27005	51797	21059	21241
·785	1·008=12·1	41300	53142	24287	27043
·768	·840=10·083	52096	50865	25923	29363
·777	·630= 7·5625	88547	52064	32007	36130
1·022	1·681=20·1666	44218	90074	31804	35631
1·000	1·260=15·125	66746	86238	40250	43797

59. The next Table (abridged from Table X., art. 36) contains the dimensions of thirteen short hollow cylinders of the same iron, together with their real and calculated breaking weights, for comparison as before. Here, as before, the ends are flat, and the lengths less than 30 times the external diameter.

60. *Hollow uniform cylindrical pillars of Low Moor Iron, No. 3.*

Number of experiments.	Length of pillar.	External Diameter.	Internal Diameter.	Weight of pillar.		Breaking Weight.	Value of *b*.	Value of *c*.	Calculated breaking weight from formula. $y = \frac{bc}{b + \frac{3c}{4}}$	Remarks.
	foot.	inch.	inch.	lbs.	oz.	lbs.	lbs.	lbs.	lbs.	
1	2·5208	1·26	·767	6	2	33679	38807·6	86178·5	32331	Not perfectly sound.
2	2·5208	1·26	·781	6	1	32867	38274	84310	31790	Core not quite in middle. Thickness of metal on opposite sides, 3 : 4.
3	2·1666	1·25	·768	5	2	35302	48461·7	83882	36501	Air bubbles in casting.
4	2·1666	1·17	·752	4	7	31195	36887	69283·2	23764	Core in centre, T : C : : 43 : 74.
5	1·9166	1·16	·7705	3	9	30383	42633	64844·7	30291	Do., T : C : : 11 : 18.
6	1·6805	1·21	·77	3	9	41751	64599·2	75130·6	40128	Core in centre.
7	1·6805	1·14	·805	2	11	27135	46408	56193	29449	
8	1·4166	1·15	·91	1	11	25511	50927	42636	26191	
9	1·3333	1·15	·92	1	9½	25105	54730	41053	26273	
10	1·2604	1·16	·932	1	8	26729	61304·1	41133·8	27364	Core in centre, T : C : : 52 : 64.
11	1·2604	1·08	·77	1	12	27135	61570·2	49457	30863	
12	1·1667	1·15	·792	2	0	37285	91909	59953	40257	
13	·7333	1·13	·91	0	13½	34037	133000	38704	31750	

Some of these pillars broke into many pieces; several of them were bored inside, and turned on the outside. They were, with the two exceptions named, very good castings. By the ratio T : C is to be understood the depth of the part extended to that compressed in the section of fracture. By the value of b is to be understood the breaking weight, as calculated from the formula

$$b = 99318 \frac{D^{3.55} - d^{3.55}}{l^{1.7}},$$

for the strength of long hollow pillars in lbs., which is given, somewhat abridged, in art. 41.

It will be observed that the calculated strengths agree moderately well with the real ones in both of the preceding Tables, showing that the resistance to crushing is an element of the strength of *short* flexible pillars at least.

COMPARATIVE STRENGTH OF LONG SIMILAR PILLARS.

61. It has been stated (art. 39, results 5 and 6) that the strength of solid pillars with rounded ends varied as $\frac{d^{3.76}}{l^{1.7}}$, and that of those with flat ends as $\frac{d^{3.55}}{l^{1.7}}$. This was when the former pillars were not shorter than about 15, nor the latter than about 30 times the diameter.

62. In the research for the above numbers, I was led to conclude that, if the material had been incompressible, the 3·76 and 3·55 would each have become 4, and the 1·7 have been 2 (see arts. 24 and 33 of the Paper above referred to). In that case the strength would have varied as $\frac{d^4}{l^2}$, which is the ratio of the strength of pillars according to the theory of Euler; which theory was intended to apply to the power of pillars to resist incipient flexure, whilst my inquiry was as to the breaking strength. In similar pillars the length is to the diameter in a constant ratio: calling then the length $n\ d$,

where n is a constant quantity, we have, in these different cases, the strength as

$$\frac{d^{3\cdot76}}{n^{1\cdot7} \times d^{1\cdot7}}, \qquad \frac{d^{3\cdot55}}{n^{1\cdot7} \times d^{1\cdot7}}, \qquad \frac{d^4}{n^2 d^2}.$$

Dividing, these become

$$\frac{d^{2\cdot06}}{n^{1\cdot7}}, \qquad \frac{d^{1\cdot85}}{n^{1\cdot7}}, \qquad \frac{d^2}{n^2}.$$

63. In the first of these cases the strength varies as a power of the diameter somewhat higher than the square; in the second somewhat lower; and in the third, as the square. We may therefore conclude, that in similar pillars the strength is nearly as the square of the diameter, or of any other linear dimension; and as the area of the section is as the square of the diameter, the strength is nearly as the area of the transverse section.

64. In deducing the conclusion in the last article, Euler remarks that if of two similar pillars of the same material, one be double the linear dimensions of the other, the larger will but bear four times as much as the smaller, though its weight is eight times as great. Berlin Memoirs, 1757.

65. The following Table, containing the results from such of my experiments on solid uniform cylindrical pillars as were from models *similar* in form, will show how far the above conclusions agree with the results of experiments.

	Diameters of pillars compared.	Length of pillars compared.	Breaking weight of pillars.	Powers of the dimensions to which the breaking weights are proportional.	
	inch.	inches.	lbs.		
Pillars with rounded ends.	·497 ·99	7·5625 15·125	5262 19752	1·908	Mean from the powers, 1·865.
	·76 1·52	15·125 30·25	9223 32531	1·819	
	·99 1·97	30·25 60·5	6105 25403	2·057	
Pillars with flat ends.	·51 1·56	20·166 60·5	3830 28962	1·841	
	·50 ·997	30·25 60·5	1662 6238	1·9081	
	·51 1·02	15·125 30·25	6764 21844	1·6913	
	·50 1·022	10·083 20·166	8931 31804	1·8323	

66. In the preceding Table, the pillars being from similar models were assumed to be similar, notwithstanding slight deviations in the measures. It appears that the power of the lineal dimensions, according to which their strengths vary, is somewhat lower than the second.

67. If long pillars be so formed as to resist being crushed by the breaking weight, as has been mentioned before, they will be similar.

We have seen (art. 52-3) that when pillars require a force to break them by flexure, which exceeds a certain portion of the force which would crush them, if they were not flexible, the pillar sustains a considerable diminution in its power of resistance to flexure in consequence of a partial crushing, or crippling of the material. Suppose $c\,d^2$ = the crushing force of the pillar (d being the diameter), or that pressure which would cause rupture in it, if it were too short to break by flexure; and $n\,c\,d^2$ that part of this pressure which is the utmost it would, as flexible, sustain without apparent crippling or crushing. Then, since the strength in lbs. to resist fracture by flexure in pillars, with both ends rounded and both flat, was $33379 \frac{d^{3.76}}{l^{1.7}}$, and $98922 \frac{d^{3.55}}{l^{1.7}}$, respectively, as appeared from my experiments, l being in feet and d in inches, we have these quantities each equal to $n c d^2$, in the cases where short pillars, which break by flexure, are bearing, at the time of fracture, the greatest weights they can sustain without any apparent crushing. Whence, in pillars with rounded ends,

$$33379 \frac{d^{3.76}}{l^{1.7}} = n c d^2; \therefore l = \left(\frac{33379}{n c}\right)^{\frac{1}{1.7}} \times d^{\frac{1.76}{1.7}};$$

in pillars with flat ends,

$$98922 \frac{d^{3.55}}{l^{1.7}} = n c d^2; \therefore l = \left(\frac{98922}{n c}\right)^{\frac{1}{.7}} \times d^{\frac{1.55}{1.7}}.$$

68. In the former of these cases, l varies somewhat faster than as the first power of the diameter, and in the second somewhat slower; the two showing that, in the case of pillars equally loaded to resist crushing by the weight, the length to the diameter will be nearly in a constant ratio, or the pillars must be similar.

ON THE STRENGTH OF PILLARS OF VARIOUS FORMS, AND DIFFERENT MODES OF FIXING.

69. In hollow pillars of greater diameter at one end than the other, or in the middle than at the ends, as in Table XI. (art. 36), it was not found that any additional strength was obtained over that of uniform cylindrical pillars; on the other hand, the strength of these seemed to be the greater; with respect to this, however, the conclusions were not very decisive. The result from the comparison is in agreement with what may be deduced from Euler's theoretical values of

the strengths of uniform cylindrical solid pillars, and of those in the shape of a truncated cone (Berlin Memoirs, 1757); his formulæ for these being

$$P = \frac{a^2 D^4}{A^2 d^4} \cdot p, \text{ and } P' = \frac{a^2 D'^2 E^2}{A^2 d^4} \cdot p.$$

These values are to one another, *cæteris paribus*, as D^4 to $D'^2 E^2$; where D is the diameter of the uniform pillar and D' E the diameters of the two ends of that in the form of a truncated cone. But if we compute the diameter of a uniform cylindrical pillar of the same length and solid content as one with unequal diameters, we shall find the uniform pillar stronger than the other, and the more so according as the inequality of the diameters of the latter is greater.

70. The strength of a pillar in the form of a connecting rod of a steam engine was found to be very small; indeed, less than half the strength that the same metal would have given if cast in the form of a uniform hollow cylinder. The ratio of the strength, according to my experiments, was 17578 to 39645.

71. A pillar irregularly fixed, so that the pressure would be in the direction of the diagonal, is reduced to one-third of its strength, the case being nearly similar to that of a pillar with rounded ends, the strength of which has been shown to be only $\frac{1}{3}$rd or that of a pillar with flat ends.*

72. Uniform pillars fixed at one end and moveable at the other, as in those flat at one end and rounded at the other, break at $\frac{1}{3}$rd of the length (nearly) from the moveable end; therefore, to economise the metal, they should be rendered stronger there than in other parts.

73. Of rectangular pillars of timber it was proved experi-

* Tredgold, art. 283 of his Work on Cast Iron, and in his Treatise on Carpentry, following the idea of Serlio in his Architecture, recommends circular abutting joints, to lessen the effect of irregularity in the strains upon columns, from settlements and other causes; but this, we see, is voluntarily throwing away two-thirds of the full strength of the material to prevent what may often be avoided.

mentally that the pillar of greatest strength, where the length and quantity of material is the same, is a square.

COMPARATIVE STRENGTHS OF LONG PILLARS OF CAST IRON, WROUGHT IRON, STEEL, AND TIMBER.

74. It results from the experiments upon pillars of the same dimensions, but different materials, that if we call the strength of cast iron 1000, we shall have for wrought iron 1745, cast steel 2518, Dantzic oak 108·8, red deal 78·5. The numbers, all but the last, were obtained from the pillars with rounded ends, and the computations made by the rules used for cast iron.

POWER OF PILLARS TO SUSTAIN LONG CONTINUED PRESSURE.

75. In all the experiments of which an account has been given, the pillars were broken without any regard to time, and an experiment seldom lasted longer than from one to three hours. To determine, therefore, the effect of a load lying constantly upon a pillar, Mr. Fairbairn had at my suggestion four pillars cast of the same iron as before, and all of the same length and diameter; the length of each was 6 feet, and the diameter 1 inch, and they were rounded at the ends. The first was loaded with 4 cwt., the second with 7 cwt., the third with 10 cwt., and the fourth with 13 cwt.; this last was loaded with $\frac{97}{100}$ of what had previously broken a pillar of the same dimensions, when the weight was carefully laid on without loss of time. The pillar loaded with 13 cwt. bore the weight between five and six months and then broke; that loaded with 10 cwt. is increasing slightly in flexure; the others, though a little bent, do not alter. They have now borne the loads three years. The deflexion of the first pillar is ·01 inch, that of the second ·025, and of

the third ·409. The deflexion of this last pillar, when first taken, was ·230; and after each succeeding year it was ·380, ·380, and ·409, as at present.

EULER'S THEORY OF THE STRENGTH OF PILLARS.

76. It appeared from the researches of this great analyst, that a pillar of any given dimensions and description of material required a certain weight to bend it, even in the slightest degree; and with less than this weight it would not be bent at all (Acad. de Berlin, 1757). Lagrange, in an elaborate essay in the same work, arrives at the same conclusion. The theory as deduced from this conclusion is very beautiful, and Poisson's exposition of it, in his 'Mécanique,' 2nd edition, vol. i., will well repay the labour of a perusal.

77. I have many times sought, experimentally, with great care for the weight producing incipient flexure, according to the theory of Euler, but have hitherto been unsuccessful. So far as I can see, flexure commences with weights far below those with which pillars are usually loaded in practice. It seems to be produced by weights much smaller than are sufficient to render it capable of being measured. I am therefore doubtful whether such a fixed point will ever be obtained, if indeed it exist. With respect to the conclusions of some writers, that flexure does not take place with less than about half the breaking weight; this, I conceive, could only mean large and palpable flexure; and it is not improbable that the writers were in some degree deceived from their having generally used specimens thicker, compared with their length, than have been usually employed in the present effort.

Some results of the theory of Euler, as given by Poisson (Mécanique, vol. i. 2nd edit.), have been of great service in the course of the inquiry.

78. I will now give the leading results, abridged from four of the Tables of experiments on cast iron pillars, enumerated in art. 36, p. 241 ; and the reader who wishes for further information upon them, or upon those of wrought iron or timber, is respectfully referred to the original paper in the Philosophical Transactions of the Royal Society, Part II. 1840.

RESULTS OF EXPERIMENTS ON THE RESISTANCE OF SOLID UNIFORM CYLINDERS OF CAST IRON TO A FORCE OF COMPRESSION.

TABLE I.—*Low Moor Iron, No. 3, cast in dry sand. Ends of specimen turned* (fig. 23), *so that the force would pass through the axis.*

Length.	Diameter.	Mean Diameter.	Deflexion of middle of pillar.	Corresponding weight	Breaking weight.	Mean from breaking weights.	Ratio T : C.	Remarks.
Inches.	Inch.	Inch.	Inch.	lbs.	lbs.	lbs.		
60·5	·50	......	·07	53				These pillars, and those of the same length below, were made by mistake ½ an inch longer than was intended, and therefore all the future ones were made of the same length, or exact subdivisions of it.
		.50	·49	113	136			
60·5	·50	......	·04	97		143		
			·23	136	150			
60·5	·77	·77	......	......	780	780		
60·5	.77				780			
60·5	·99		·05	515				
			·14	991				
		·99	·19	1183				
			·52	1615	1663	1902		
60·5	·99	......	......	......	2141			
60·5	1·28	1·29	·25	5069	5293			
60·5	1·30		·07	5673		5707		
			·10?	5897	6121			
60·5	1·29		·03	2141				The two following pillars were cast in green sand.
		1·295	·17	4549				
			·34	4997	5149	......	......	
60·5	1·30		·00?	2141				Weight of pillar 19 lbs. 8 oz.
			bent.	2757		5465		
			·07	5145	5781	......	......	19 lbs. 14 oz.

60·5	1·53	1·52	·14	10525	10861	10861	126 : 26	
60·5	1·51				10861			
60·5	1·53				10121	......	......	28 lbs. 5 oz. } cast in green sand.
		1·535				10650		
60·5	1·54				11179	......	......	28 lbs. 9 oz. } cast in green sand.
60·5	1·76		·01 ?	8483				
		1·765	·09	11035				
			·30	14225	14701	15560		
60·5	1·77	......	......	......	16419			
60·5	1·76		bent.	2808				
			·02	8617				
		1·78	·12	13721				
			·22	15233	16493			Weight of pillar 38 lbs.
60·5	1·80		bent.	3355		17564		
			·09	14201				
			·48	18355	18635	......	145 : 36	
60·5	1·94	......	......	......	22811	22811		44 lbs. = weight of pillar.
60·5	1·97		bent.	3355				
			·07	12970				
			·28	22127				
		1·96	·50 }	24311				
			·52 }					
			·60	24857	25403			47 lbs. ,, ,,
60·5	1·96		·03	12287		21291		
			·19	19943				
			·48	22787	23179		153 : 43 or 150 : 42	46 lbs. 8 oz. ,, ,,
30·25	·50		bent.	248				
			·15	472	526			
30·25	·50	·50	·02	304		539		
			·09	472	535			
30·25	·50		......	......	556			

TABLE I.—*Continued.*

Length.	Diameter.	Mean Diameter.	Deflexion of middle of pillar.	Corresponding weight.	Breaking weight.	Mean from breaking weights.	Ratio T : C.	Remarks.
Inches.	Inch.	Inch.	Inch.	lbs.	lbs.	lbs.		
30·25	·77	......	·02	1717				
		·77	·10	2390	2726	2726		
30·25	·99 }		·04	2745				
		·99	·13	4985	6105 }		76 : 23	
30·25	·99 }		·02	3641		6105		
			·07	4985	6105 }		78 : 21	This bent apparently in different directions.
30·25	1·29 }		·01	12287				
		1·29	·07	16115	17515 }		96 : 34	
30·25	1·29 }		·08	12287		17235		
			·21	16115	16955 }		105 : 24 ?	Small flaw in tensile part.
30·25	1·52 }	......	·04	22619				
			·12	30739	32419 }		99 : 51	
30·25	1·53 }	1·52	......	......	34638 }	32531	101 : 52	
30·25	1·51 }	......	·07	22619	30536 }		110 : 43	
20·1666	1·00 }	1·01	.. ...	......	15737 }	15737	63 : 67	4 lbs. 3 oz. = weight of pillar.
20·1666	1·02 }		......	......	15737 }		65 : 37	4 „ 5 „ „ „
20·1666	·785 }	·767	......	......	7255 }	6602		2 lbs. 8¼ oz. = weight of pillar.
20·1666	·75 }		......	......	5950 }			2 „ 6 „ „ „
15·125	·50	......	·05	1689				
			·31	1997	1997		39 : 11	
15·125	·50	......	·20	1801				
			·34	1857	1857		39 : 11	
15·125	·50	......	·08	1353				
		·50	·20	1801	1857	1904		

15·125	·77	......	......	......	10138		48 : 29	
15·125	·76	......	......	......	9746		53 : 23	
15·125	·75	·76	......	......	7786	9223	49 : 26	
15·125	·99	......	......	......	20163		57 : 42	
15·125	·99	......	......	......	19239		60 : 39	
15·125	·99	·99	......	......	19855	19752	60 : 39	
10·083	·76		......	......	16683			The first and second failed by the ends becoming split by a conical wedge which formed at them.
10·083	·76	·76	......	......	16683	17506		
10·083	·77		......	......	19152			
7·5625	·51	......	......	......	6188		31 : 20	
7·5625	·49	......	......	......	4578		32 : 17	
7·5625	·49	·497	......	......	5019	5262	34 : 15	
7·5625	·77	·77	......	......	23893		35 : 42	These pillars were split at both ends.
7·5625	·77		......	......	22003	22948	41 : 36	
3·7812	·50	......	......	......	15233	15107		In these two experiments the area compressed seemed greater than the extended area. The ends were split by the pressure.
3·7812	·50	·50	......	......	14981			

By the letters T C are to be understood the versed sines, or depths of the neutral line, on the surfaces submitted to tension or compression; and the ratio T : C is that of those versed sines or depths, as nearly as the curve of the neutral line could be represented by a straight line. In almost every case the pillars broke nearly in the middle, both in this Table and the following one.

TABLE II.—*Low Moor Iron, No. 3, cast in green sand. Ends of cylinders turned flat, and parallel to each other, and the pressure caused by the approach of two parallel surfaces, between which the cylinder was placed, its ends perfectly coinciding with them,* fig. 26.

Length.	Diameter.	Mean Diameter.	Weight.	Deflexion of middle of pillar.	Corresponding weight.	Breaking weight.	Mean from breaking weight.	Ratio T : C.	Remarks.
Inches.	Inch.	Inch.	lbs. oz.	Inch.	lbs.	lbs.	lbs.		
60·5	·51	·51	3 4⅙	......	......	483	487		These two had disks 2 inches diameter upon the ends: all the rest had the ends turned flat, and were without disks.
60·5	·51		3 5¼			491			
60·5	·77	·77		·07	1162				
				·22	2036				
60·5	·77		7 5½	·30	2260	2316			
				·06	1588		2456		
				·10	2036				
				·17	2484	2596			
60·5	·99			·05	4123				
				·30	6475	6811			
60·5	1·01	·997		·05	4123				
				·14	5467	5971	6238		
60·5	·99	......	11 8	·04	4123				
				·10	5467	5932			
60·5	1·30	......	......	·10	11235				
				·24	14763				
				·47	16331	16527			
60·5	1·29	1·29	19 11	·15	11217		16064		
				·45	15137	15333			
60·5	1·28	......	20 0	......	... ..	16331			

60·5	1·55		28	10	·13	21857				
					·21	25105				
					·62	27135	27135			
60·5	1·57		29	7	·10	21857				
		1·56			·14	25105				
					·26	29977		28962		
					·48	31398				
							31398		120 : 37	A wedge broke out, and showed the neutral line.
60·5	1·55		28	5	......	......	28353			
30·25	·50	·50	1	10½	·05	1990				
					·13	1606				
							1662	1662		
30·25	·78				·08	4357				
					·22	7717	8589			
30 25	·78				·04	2905				
		·77			·06	4697		8811		
					·10	8281				
					·19	9177	9625			
30·25	·76				......	......	8420		61 : 15	A crack showed the neutral line.
30·25	1·01				......	......	19132			
30·25	1·00				·05	13833				
		1·01			·19	17865	18369			
30·25	1·02				·05	17795				
					·16	21715	21844	20310	64 : 39	Cracked at neutral line.
30·25	1·00				·05	14841				
					·07	17865				
					·13	20889	21897		65 : 35	

TABLE II.—*Continued.*

Length.	Diameter.	Mean diameter.	Weight.	Mean weight.	Breaking weight.	Mean breaking weight.	Ratio T : C.	Remarks.
Inches. 20·1666 20·1666	Inch. ·51 ·51	Inch. ·51	lbs. oz. 1 $1\frac{1}{2}$ 1 $1\frac{1}{4}$	lbs. oz. 1 $1\frac{3}{8}$	lbs. 3830 3830	lbs. 3830		
20·1666 20·1666 20·1666	·78 ·78 ·77	·777	2 9 2 $8\frac{1}{2}$ 2 $7\frac{1}{2}$	2 $8\frac{1}{3}$	16701 15357 14685	15581	45 : 33	The first broke in two pieces near to the middle, and near to one end; and a piece was split off the other end, at the neutral line. The other two broke nearly in the same manner.
20·1666 20·1666	1·03 1·015	1·022	4 6 4 5	4 $5\frac{1}{2}$	32007 31601	31804	56 : 46 54 : 49	Broke in two places near the middle; both ends cracked at neutral line.
15·125 15·125 15·125	·51 ·51 ·51	·51	14 $13\frac{3}{4}$ $13\frac{3}{4}$	$13\frac{10}{12}$	6512 7016 6764	6764	32 : 19	
15·125 15·125 15·125 15·125	·79 ·77 ·76 ·78	·775	$29\frac{3}{4}$ 29 $28\frac{1}{2}$ $29\frac{1}{2}$	$29\frac{3}{16}$	21179 22363 19003 21691	21509		
15·125 15·125	1·00 1·00	1·00	3 4 3 4	3 4	39112 41388	40250		
12·1 12·1 12·1	·50 ·50 ·50	·50	$10\frac{1}{2}$ 10 10	$10\frac{1}{6}$	7279 6943 7363	7195	3 : 2 3 : 2 3 : 2	

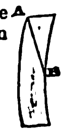
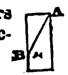
12·1 12·1 12·1 12·1	·78 ·79 ·79 ·78	·785	1 7½ 1 8 1 8¼ 1 7½	1 7¾	25355 24043 23875 23875	24287	39 : 39 1 : 1 1 : 1	
10·0833 10·0833 10·0833	·50 ·50 ·50	·50	8½ 8½ 8½	8½	8287 8623 9883	8931	13 : 12 13 : 12 13 : 13	
10 0833 10·0833 10·0833 10·0833	·78 ·77 ·76 ·76	·768			27491 25531 25531 25139	25923	1 : 1	These generally broke in several pieces; but always in the middle by bending. There was, however, usually a wedge formed about the centre, which tended to split the pillar there.
7·5625 7·5625 7·5625	·50 ·50 ·50	·50	6⅓ 6⅓ 6⅓	6⅓	11479 11143 11143	11255	12 : 13 12 : 13 12 : 13	
7·5625 7·5625 7·5625	·78 ·78 ·77	·777	14½ 14½ 14¼	[illegible] $\frac{5}{12}$	33225 31601 31195	32007	4 : 5	There was a good deal of doubt respecting the neutral line; but somewhat more than one-half was compressed.
3·7812 3·7812 3·7812	·50 ·50 ·50	·50			17795 17935 16675	17468	20 : 30	These broke in the middle by bending, as before; but they generally showed a short ridge or wedge in the centre, as mentioned above.
2 2	·52 ·52	·52			23035 22699	22867		The first bent, and slid off in A B. The other bent and cracked half across in the middle. 
1 1 1	·52 ·52 ·52	·52			23963 24747 25139	24616		Broke by a wedge, about three-quarters of an inch high, sliding off in the direction A B. 

TABLE III.—*Hollow cylindrical pillars, rounded at the ends* (Pl. II. fig. 36).—*Results of experiments on the strength of hollow uniform cylinders of cast iron (the Low Moor, No. 3), the ends having hemispherical caps* (a) *on them, that the compressing force might act through the axis of the pillar, and its ends move freely. Length of cylinder, including caps on the ends, 7 feet 6¾ inches. The pillars were in most cases, except otherwise mentioned, cast in dry sand, both in this Table and the following one. The weights of the cylinders, as set down, are for the whole length, 7 feet 6¾ inches.*

Number of experiment.	Description of Pillar.	Deflexion.	Weight producing the deflexion.	Breaking weight, or that with which the pillar sunk.	Value of x from formula $x = \frac{W}{D^{3\cdot76} - d^{3\cdot76}}$ where W = the breaking weight, and D, d the external and internal diameters.	Remarks.
		inch.	lbs.	lbs.	lbs.	
1	Hollow uniform cylinder. External diameter 1·78 in. Internal do. 1·21 Weight of cylinder 31 lbs.	·03 ·32 ·49	2237 4829 5333	5585	834·37	With the weight 5585 lbs. the pillar sunk down, but was not allowed to bend so as to break. Its elasticity was very little injured, and it was experimented upon in another way, without showing any defect of strength, as was the case with other pillars treated in the same manner.
2	External diameter 1·74 in. Internal do. 1·187 Weight of cylinder 30¼ lbs.	·02 ·13 ·48	2141 4325 5585	5711	933·13	This pillar sunk with the weight 5711, but it was bent no farther than necessary, and was preserved for another experiment, as before.
3	External diameter 2·01 in. Internal do. 1·415 Weight of cylinder 36½ lbs.	·04 ·31 ·75	2237 6845 8105	8357	826·20	The thickness of the metal at the place of fracture varied on the opposite sides as 19 to 42.
4	External diameter 2·33 in. Internal do. 1·70 Weight of cylinder 46½ lbs.	·24 ·37 ·72	11169 12737 14697	15089	903·28	Thickness of metal on opposite sides at place of fracture as 1 to 4 nearly. The thin side was that which was compressed, and the same was the case in most of the other pillars.
5	External diameter 2·23 in. Internal do. 1·54 Weight of cylinder 47 lbs.	·01 ·22 ·69	2237 8357 12137	12389	808·21	

6	External diameter 2·24 in. Internal do. 1·735 Weight of cylinder 34¾ lbs.	·015 ·34 ·38	2141 12445 12669	13341	1041·7	
7	External diameter 2·24 in. Internal do. 1·58	·02 ·21	4325 13521	13913	917·62	This pillar was reduced to half its thickness near to the ends, and to three-fourths half-way between the middle and each end, but it did not fail in the reduced parts: it sunk by flexure.
8	External diameter 2·49 in. Internal do. 1·89 Weight of cylinder 48¾ lbs.	·01 ? ·40 ·52	4123 18623 19239	19855	996·24	This pillar was reduced in the same manner as the last, and sunk by flexure, as before.
9	External diameter 2·47 in. Internal do. 1·98 Weight of cylinder 41 lbs.	·01 ? ·23 ·62	3211 17391 18667	19003	1123·5	The thickness of metal at place of fracture varied as 7 to 9.
10	External diameter 2·46 in. Internal do. 1·855 Weight of cylinder 49 lbs.	bent. ·04 ·49 ·65	2141 9103 18083 18615	19147	989·95	The metal in this varied in thickness at the place of fracture as 3 to 4.
11	External diameter 2·73 in. Internal do. 2·17 Weight of cylinder 48 lbs.	bent. ·03 ·70	3603 12105 22787	23963	949·48	
12	External diameter 2·74 in. Internal do. 2·155 Weight of cylinder 51¼ lbs.	·02 ·14 1·10	3603 21219 27491	27883	1059·5	Variation of thickness of metal at place of fracture 2 to 3 nearly.
13	External diameter 3·01 in. Internal do. 2·48 Weight of cylinder 50¼ lbs.	·07 ·23 ·75	16115 21219 25923	26707	819·46	The thickness of metal at the place of fracture varied in this as 9 to 15.
14	External diameter 3·36 in. Internal do. 2·823 Weight of cylinder 59⅙ lbs.	·09 ·32 1·10	16115 30627 40335	40973	895·02	Variation of thickness of metal at the place of fracture 19 to 34.

TABLE III.—*continued.*

Number of experiment.	Description of Pillar.	Deflexion.	Weight producing the deflexion.	Breaking weight, or that with which the pillar sunk.	Value of x from formula $x = \frac{W}{D^{3\cdot76} - d^{3\cdot76}}$ where W = the breaking weight, and D, d the external and internal diameters.	Remarks.
		inch.	lbs.	lbs.	lbs.	
15	Hollow uniform cylinder. External diameter 3·36 in. Internal do. 2·63 Weight of cylinder 77¾ lbs.	bent. ·09 ·30 ·90 1·07	3355 16115 33824 48511 49494	50477	880·11	Variation of metal in thickness at point of fracture 5 to 7.
16	Solid uniform pillar rounded at the ends . . . Diameter 2·24 in. Weight 93 lbs.	bent. ·02 ·25	2141 7541 20273	21231	1026·6	
17	Pillars of shorter lengths than those above. External diameter 1·78 in. Internal do. 1·21 Length 4 feet 9 inches.			13693	927·86	
18	External diameter 2·31 in. Internal do. 1·67 Length 4 feet 9 inches.			36382	1005·3	Thickness of metal on opposite sides 10 to 21.
19	External diameter 1·85 in. Internal do. 1·36 Length 2 feet 7 inches. Weight of 2 feet 5 inches = 8 lbs. 15½ oz.	·35	32587	33763	784·9	The metal in this varied in thickness at point of fracture as 11 to 15. The great weight necessary to break this very short pillar probably caused incipient crushing, and thus reduced the value of x.

The value of x, obtained in the sixth column of the preceding Table, is the strength of a solid pillar, 1 inch diameter, and of the same length as those above. For, when the length is constant, the strength W varies as $D^{3\cdot76}d-^{3\cdot76}$, (arts. 39 & 41); and therefore, to find x, the strength of a solid pillar, 1 inch diameter and 7 feet 6¾ inches long, we have $D^{3\cdot76}-d^{3\cdot76} : 1^{3\cdot76} : : W : x=\frac{W}{D^{3\cdot76}-d^{3\cdot76}}$. The mean from the values of x found in the Table above is 932·76 lbs., and if this be multiplied by $l^{1\cdot7}$, where $l=7\cdot5625$ feet (=7 feet 6¾ inches), we obtain the strength in pounds of a solid pillar, 1 foot long and 1 inch diameter, rounded at the ends; and this, divided by 2240, to reduce it into tons, is the co-efficient 12·979, called 13, in the formula for the strength, $13 \times \frac{D^{3\cdot6}-d^{3\cdot6}}{l^{1\cdot7}}$, (art. 41), where the index 3·6 is put as a mean between the 3·76 of pillars with rounded ends, and the 3·55 of those with flat ends. The remarks here made will apply to the values of x in the next Table.

TABLE IV.—*Hollow Cylindrical Pillars, flat at the ends* (fig. 37). *Results of Experiments on the Strength of Hollow Uniform Cylinders of Cast Iron (Low Moor, No. 3), the ends being turned flat and perpendicular to the sides, and the pressure communicated by the approach of parallel surfaces, against which the ends of the pillars were firmly bedded. Length of each pillar 7 feet 6¾ inches, except otherwise mentioned.*

Number of experiment.	Description of Pillar.	Deflexion.	Weight producing the deflexion.	Breaking weight, or that with which the pillar sunk.	Value of x from formula $x = \frac{W}{D^{3\cdot55} - d^{3\cdot55}}$ where W = the breaking weight, D, d the external and internal diameters.	Ratio of the thicknesses of the ring of metal on opposite sides at place of fracture.	Remarks.
		inch.	lbs.	lbs.	lbs.		
1	Hollow uniform cylinder, same as in Experiment 1 of the preceding Table. External diameter, 1·78 Internal do. 1·21 Length of cylinder, 7 ft. 4¾ in. Weight (length 7ft. 6¾ in.) 31 lbs.	·02 ·03 ·00 ·05 ·20 ·50 ·66	2813 3821 unloaded 4829 12001 16705 17489	17840	2973·7	1 : 5	When this cylinder was broken, it was found that the thinner side was that which was compressed. The weight of this cylinder and all those below, whether their lengths are 7 feet 6¾ inches, or 7 feet 4¾ inches, are given for the greater length.
2	Cylinder same as No. 2 in preceding Table. External diameter, 1·74 Internal do. 1·187 Length of cylinder, 7 ft. 4¾ in. Weight of cylinder considered uniform as before reduction, 30¼ lbs.	·12 ·32 ·48 direction changed ·54	11217 15137 15921 16313	16705	3031·5	7 : 11	This cylinder was reduced to half its thickness near to the ends, and to three-fourths of its thickness half-way between the middle and the ends, but it did not break at the reduced parts.
3	External diameter 1·76 Internal do. 1·18	bent ·03 ·09 ·36 ·54	2141 2749 6677 15233 16241	16745	2968·7	1 : 3	This cylinder was reduced in the same manner as the last, or somewhat more, and the fracture took place at the reduced part, half-way between the middle and one end; the reduced part, as appeared by measure, was a little less than three-fourths of the whole before reduction.

4	External diameter 1·75 Internal do. 1·11 Weight of cylinder 32 lbs.	·01 ·19 ·55	2237 16477 20509	20957	3586·9	1 : 2	This pillar was a good sound casting, and was not reduced in its thickness in the manner of the last two.
5	External diameter 2·04 Internal do. 1·46 Length 7 ft. 4¾ in. Weight 35¼ lbs.	·04 ·08 ·37 ·52	3589 14703 29977 31601	32413	3573·8	1 : 1	This column was not reduced in thickness, as in the second and third experiments.
6	External diameter 2·01 Internal do. 1·368 Length of cylinder 7 ft. 6¾ in. Weight 37¾ lbs.	·03 ·08 ·00 ·38 ·53	3589 18667 unloaded 28353 29977	30789	3290·3	7 : 10	Slight bubble in place of fracture. The ends of the cylinder were not reduced.
7	External diameter 2·01 Internal do. 1·415 Length 7 ft. 4¾ in. Weight 36¼ lbs.	bent ·10 ·01 ·14 ·25	4251 21857 unloaded 25917 27541	28353	3214·8	5 : 11	This cylinder was the same as that in Experiment 3 of the last Table. It was rendered quite straight, and its ends were firmly bedded; it was reduced, as in Experiment 2, and it broke in the middle, and at three of the reduced places.
8	External diameter 1·99 Internal do. 1·31 Length 7 ft. 5·8 in. Weight before reduction, 39 lbs.	bent ·20 ·55 ·90	1456 15605 24205 26731	27067	2988·3	6 : 7	This cylinder was reduced in the manner of the preceding ones. It broke at a small flaw near the middle.
9	External diameter 2·23 Internal do. 1·54 Length 7 ft. 4¾ in. Weight before reduction 47 lbs.			40569	3099·0	4 : 9	This cylinder was the same as that in No. 5, last Table; it was now reduced in thickness in the same manner, and to the same degree, as in the preceding ones. It broke in the middle, and at one of the reduced places near to the middle.

TABLE IV.—*Continued.*

Number of experiment.	Description of Pillar.	Deflexion.	Weight producing the deflexion.	Breaking weight, or that with which the pillar sunk.	Value of x from formula $x = \frac{W}{D^{3 \cdot 55} - d^{3 \cdot 55}}$ where W = the breaking weight, D, d the external and internal diameters.	Ratio of the thicknesses of the ring of metal on opposite sides at place of fracture.	Remarks.
		inch.	lbs.	lbs.	lbs.		
10	Uniform solid cylinder, cast in green (moist) sand Diameter 1·76 Length 7 ft. 6¾ in. Weight 56 lbs.	bent bent ·35 ·65	4135 10855 21219 22787	23179	3115·5		With 23179 lbs. it became bent more than an inch, and slipped out of its place; it was afterwards rendered straight, and replaced; and it would have broken with a less weight.
11	Uniform solid cylinder, cast in dry sand Diameter 1·72 Length 7 ft. 6¾ in. Weight 53 lbs. 8 oz.	·20 ·28 ·44 ·65	16115 18355 20595 21715	21995	3207·7		With the last weight, 21995 lbs., the pillar slipped from its fixings, as the preceding one had done, and when replaced it was broken with a less weight than it had borne before. The weight, in both this case and the last, was so near to what the breaking weight must have been if fracture had been effected as usual, that I have not hesitated to put down the results as those of fracture.

TRANSVERSE STRENGTH.

79. The transverse strain is that to which cast iron and some other materials are most frequently subjected, and therefore experiments have been oftener made that way than any other. Still, as regards cast iron, whose uses are multiplying every day, the knowledge of the practical man has hitherto been far from equalling his wants; and accordingly various efforts have lately been made, and doubtless will continue to be made, to obtain extra information upon so important a subject.

I will give an account of some of these, the objects of which are as below.

1st. To ascertain what alteration takes place in bars of cast iron subjected to long-continued strains.

2nd. To determine the effects of changes in the temperature of bars upon their strength.

3rd. To inquire into the elasticity and strength of cast iron bars, under ordinary circumstances, the time when the former becomes impaired, and the erroneous conclusions that have been deduced from it.

4th. To find the best forms of beams, and the strength of beams of particular forms.

80. LONG-CONTINUED PRESSURE UPON BARS OR BEAMS.

To ascertain how far cast iron beams might be trusted with loads permanently laid upon them, Mr. Fairbairn made the following experiments.—(Report on the Strength of Cast Iron, obtained from the Hot and Cold Blasts, vol. vi. of the British Association).

He took bars both of cold and hot blast iron (Coed Talon, No. 2), each 5 feet long, and cast from a model 1 inch square; and having loaded them in the middle with different weights, with their ends supported on props 4 feet 6 inches asunder, they were left in this position to determine how long they would sustain the loads without breaking. They bore the weights, with one exception, upwards of five years, with small increase of deflexion, though some of them were loaded nearly to the breaking point. Since that time, however, less care has been taken to protect them from accident, and three others have been found broken. They are carefully examined and have their deflexions taken occasionally, which are set down in the following Table, which contains the exact dimensions of the bars, with the load upon each. These experiments were undertaken by Mr. Fairbairn at my suggestion, as I was led to conceive, from experiments I had made in a different way upon malleable iron, that time would have little effect in destroying the power of beams to bear a dead weight.

81. *Experiments of W. Fairbairn, Esq., on the Strength of bars to resist long-continued pressure.*

Date of observation.	Temperature of the air at time of observation.	Experiment 1. Cold blast iron. Depth of bar 1·050. Breadth of bar 1·030.	Experiment 2. Hot blast iron. Depth of bar 1·050. Breadth of bar 1·010.	Experiment 3. Cold blast iron. Depth of bar 1·030. Breadth of bar 1·020.	Experiment 4. Hot blast iron. Depth of bar 1·040. Breadth of bar 1·020.	Experiment 5. Cold blast iron. Depth of bar 1·030. Breadth of bar 1·020.	Experiment 6. Hot blast iron. Depth of bar 1·050. Breadth of bar 1·000.	Experiment 7. Cold blast iron. Depth of bar 1·000. Breadth of bar 1·010.	Experiment 8. Cold blast iron. Depth of bar 1·020. Breadth of bar 1·030.	Experiment 9. Hot blast iron. Depth of bar 1·040. Breadth of bar 1·010.
	Fahrenheit.	Deflexions with a permanent load of 280 lbs. laid upon each.		Deflexions with a permanent load of 336 lbs. laid upon each.		Deflexions with a permanent load of 392 lbs. laid upon each.		Deflexions with a permanent load of 448 lbs. laid upon each.		
1837.										
March 6				1·267		1·684	1·715	1·964	1·410	This broke with 392 lbs.; other hot blast bars were tried, but they were successively broken with 448 lbs.
„ 9	49°	·916	1·043	1·270	1·454	1·694	1·758	2·005	1·413	
„ 11		·930	1·064	1·270	1·461	1·694	1·760	2·005	1·413	
„ 17								2·010	1·413	
April 15	47°	·930	1·078	1·271	1·475	1·716	1·767	2·014	1·422	
May 31	62°	·932	1·082	1·274	1·481	1·725	1·775	Broke after bearing the weight 37 days.	1·424	
Aug. 22	70°	·937	1·086	1·288	1·504	1·737	1·783		1·438	
Nov. 18	45°	·942	1·083	1·286	1·499	1·724	1·773		1·431	
1838.										
Jan. 8	38°	·941	1·086	1·288	1·502	1·722	1·773		1·430	
March 12	51°	·945	1·091	1·298	1·505	1·801	1·784		1·439	
June 23	78°	·963	1·107	1·316	1·538	1·824	1·803		1·457	
1839.										
Feb. 7	54°	·950	1·093	1·293	1·524	1·815	1·784		1·433	
July 5	72°	·959	1·104	1·305	1·533	1·824	1·798		1·446	
Nov. 7	50°	·955	1·102	1·303	1·531	1·824	1·796		1·445	
Dec. 9	39°	·956	1·102	1·303	1·531	1·823	1·796		1·445	
1840.										
Feb. 14	50°	·955	1·104	1·305	1·531	1·824	1·797		1·446	
April 27	63°	·954	1·116	1·309	1·519	1·818	1·802		1·445	
June 6	61°	·951	1·112	1·303	1·520	1·825	1·798		1·445	
Aug. 3	74°	·953	1·115	1·305	1·523	1·826	1·801		1·447	
Sept. 14	55°	1·047*	1·115	1·305	1·613*	1·826	1·802		1·447	
1841.										
Nov. 22	50°	1·045	1·115	1·306	1·620	1·829	1·804		1·449	
1842.										
April 19	58°			1·308	1·620	1·828	1·812		1·449	

* After August 3, 1840, a body seems to have fallen upon the bars of the 1st and 4th Experiment, and this may have increased their deflexions.

82. Looking at the results of these experiments, and the note upon the first and fourth, it appears that the deflexion in each of the beams increased considerably for the first twelve or fifteen months; after which time there has been, usually, a smaller increase in their deflexions, though from four to five years have elapsed. The beam in Experiment 8, which was loaded nearest to its breaking weight, and which would have been broken by a few additional pounds laid on at first, had not, perhaps, up to the time of its fracture, a greater deflexion than it had three or four years before; and the change in deflexion in Experiment 1, where the load is less than $\frac{2}{3}$rds of the breaking weight, seems to have been almost as great as in any other; rendering it not improbable that the deflexion will, in each beam, go on increasing till it become a certain quantity, beyond which, as in that of experiment 8, it will increase no longer, but remain stationary. The unfortunate fracture of this last beam, probably through accident, has left this conclusion in doubt.

83. These important experiments show that cast iron may be trusted with permanent loads far greater than has previously been expected; it having been generally admitted that about $\frac{1}{3}$rd of the breaking weight was as far as it was safe to load a beam with in practice. It was conceived that a load greater than this would break the beam or other body in time, since the elasticity was thought to be injured with about this weight; and it was accounted unsafe to load a body beyond its elastic force (see arts. 70, 71, Part I. of this volume).

EFFECTS OF TEMPERATURE UPON THE STRENGTH OF CAST IRON.

84. Mr. Fairbairn gave, in the report previously referred to, the results of several experiments to determine how far the strength of cast iron bearers is influenced by such changes of

temperature as they are occasionally subjected to. He had a number of bars cast, of Coed Talon Iron, Nos. 2 and 3, part of them being of iron made with a heated blast, and part with cold. These bars were from models 1 inch square and 2 feet 6 inches long. They were laid on supports 2 feet 3 inches asunder, and broken by weights hung at the middle. The experiments were made in winter. Some bars were broken in the open air; some when immersed in frozen water or covered with snow; some in melted lead; and others when heated red hot. The results are in the following Table, the first column of which shows the temperature under which the experiments were made; the second and third columns give the breaking weights of the bars in pounds, when reduced by calculation to exactly 1 inch square; and the fourth column gives the ratio of the strengths of the cold and hot blast irons in the two preceding columns.

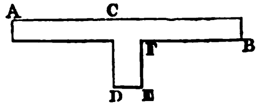
Temperature, Fahrenheit.	Coed Talon, cold blast.		Coed Talon, hot blast.		Ratio of the strengths of the two irons.
	No. 2 Iron. lbs.		No. 2 Iron. lbs.		
16°			800·3		
26°	851·0		823·1		1000 : 967·2
		mean		mean	
32°	940·7 958·5	949·6	933·4 906·0	919·7	1000 : 977·6
190°	743·1		823·6		1000 : 1108·3
Red in the dark	723·1		829·7		
Perceptibly red in daylight	663·3				
	No. 3 Iron.	mean	No. 3 Iron.		
212°	905·0 943·6	924·3	818·4		1000 : 885·4
				mean	
600°	909·3 1157·0	1033·1	834·1 917·5	875·8	1000 : 847·7

85. It would appear from these experiments, though the results are somewhat anomalous, that the strength of cast iron is not reduced when its temperature is raised to 600°, which is nearly that of melting lead; and it does not differ very widely whatever the temperature may be, provided the bar be not heated so as to be red hot.

ON THE STRENGTH OF CAST IRON BARS OR BEAMS UNDER ORDINARY CIRCUMSTANCES,—THE TIME WHEN THE ELASTICITY BECOMES IMPAIRED,—AND THE ERRONEOUS CONCLUSIONS THAT HAVE BEEN DERIVED FROM A MISTAKE AS TO THAT TIME.

86. It is, as has been before observed, to ascertain the resistance of materials to a transverse strain, that the efforts of experimenters have chiefly been directed: one reason for this seems to be the great facility with which bodies can be broken this way comparatively with others, which require large weights, or complex machinery, and often considerable attention to theoretical requirements.

The inquiry into the strengths of hot and cold blast cast iron, which I undertook, in conjunction with Mr. Fairbairn, for the British Association, and whose results were inserted in the sixth volume of their Reports, induced me to make a number of experiments on the transverse strength of bars. In these experiments, most of which were made before Mr. Fairbairn's avocations enabled him to attend to the matter, I had a number of bars cast from models 5 feet long and 1 inch square; and the castings were supported on props 4 feet 6 inches asunder, and broken by weights suspended from the middle. These bars were made long and slender, in order that small variations in the elasticity might be rendered obvious. My earliest experiments upon the Carron Iron, and particularly those upon the Buffery, convinced me that the elasticity of bars is injured much earlier than is generally supposed; and that instead of it remaining perfect till one-third or upwards of the breaking weight was laid on, as is generally admitted by writers (Tredgold, art. 70, &c.), it was evident that $\frac{1}{6}$th or less produced in some cases a considerable set, or defect of elasticity; and judging from its slow increase afterwards, I was persuaded that it had not come on by any sudden change, but had existed, though in a less degree, from a very early period. I mentioned the fact and my convictions some time afterwards to Mr. Fairbairn, and, after examining the matter with more attention than before, expressed a desire to have bars cast of a greater length than the preceding ones, to render the defect more obvious. I had, therefore, two bars of Carron Iron, No. 2, hot blast, cast from the same model, each 7 feet long; they were uniform throughout, and the form of the section of each was as in the figure. They were laid on supports, 6 feet 6 inches asunder, and broken by weights suspended from the middle;



the former with the rib downward, in which experiment the flexure would be, almost wholly, owing to the extension of that rib; and the latter with the rib upward, in which the flexure would be owing to the compression of the rib. In both of these bars, the dimensions of the parallelogram A B was the same, $= 5 \times \cdot 30 = 1 \cdot 50$ square inches, the thickness being ·30 inch. In the former of these cases, $C D = 1 \cdot 55$, and D E (which may represent the uniform thickness of the rib D F) $= \cdot 36$ inch. In the latter casting $C D = 1 \cdot 56$, and $D E = \cdot 365$ inch. The results are as follow.

FIRST BAR. Broken with the vertical rib downwards. T			SECOND BAR. Broken with the vertical rib upwards. ⊥		
Weight in lbs.	Deflexion in inches.	Deflexion (load removed).	Weight in lbs.	Deflexion in inches.	Deflexion (load removed).
7	·015	visible	7	... ...	not visible
14	·032	·001?	14	·025	visible
21	·046	·002	21	·045	·002
28	·064	·004	28	·065	·003
56	·130	·005	56	·134	·005
112	·273	·020	112	·270	·015
168	·444	·035	224	·580	·058
224	·618	·058	336	·895	·101
280	·813	·093	448	1·224	·155
336	1·030	·130	560	1·585	·235
364	broke	... ...	672	1·985	·330
			784	2·410	·490
			896	... ...	·722
			1008	4·140	1·040
			1064	... ...	... ...
			1120	broke	... ...

First bar: ∴ Ultimate deflexion = 1·138

Second bar: During the fracture a wedge 2·92 inches long, and 1·05 deep, broke out, of the form

∴ Ultimate deflexion = 5·00.

In the first of these experiments, it will be seen that the elasticity was sensibly injured with 7 lbs., and in the latter with 14 lbs.; the breaking weights being 364 lbs. and 1120 lbs. In the former of these cases a set was visible with $\frac{1}{52}$nd, and in the other with $\frac{1}{80}$th of the breaking weight, showing that there is no weight, however small, that will not injure the elasticity. The ratio of the breaking weights in these two experiments was as 1 : 3·07; showing that a bar of this form was more than three times as strong one way up as the opposite way.

87. The mode I used to observe when the elastic force became injured was as follows. When a bar was laid upon the supports for experiment, a "straight edge" was placed over it, the ends of which rested upon the bar directly over the points of support. These ends were slides which enabled the straight edge to be raised or lowered at pleasure. In this manner it was easy to bring it down to touch in the slightest

degree a piece of wood tied upon the middle of the bar. A candle was then placed at the side of the bar, opposite to where the observer stood, by the light of which, distances extremely minute could be observed.

88. The results from these experiments will enable us to see the mode by which Tredgold deduced the erroneous conclusion, as to the high tensile strength of cast iron, adverted to in the note to art. 143, page 113; and which has produced an effect on many of the numerical conclusions throughout his work.

The experiment (No. 2, art. 56), from which, and others, he infers that a cast iron bar, 1 inch square and 34 inches between the supports, will bear 300 lbs. on the middle, without injury to the elasticity, has not, I conceive, been examined in its progress with adequate care to form the basis of important conclusions. It is evident, from my experiments given above, that the elastic force was injured with much less weight than that which formed the set he first noticed in the beam.

Tredgold calculated the direct tensile strength of the cast iron in these experiments, from the results of the transverse strength. He assumed in his calculations, that the position of the neutral line remains fixed, and in the middle of a rectangular or circular section, during the whole experiment; and the resistance of a particle, at equal distances on each side of the neutral line, to be the same.

From these principles he calculated that the most extended surface of the bar was supporting a tension of 15,300 lbs. per square inch, without the elasticity being injured. This number which is greater than was required to tear asunder the specimen in many of the preceding experiments (art. 3, Part II.), Tredgold adopts, throughout the work, as the direct tensile strength of cast iron, when not strained beyond the elastic force.

From these principles he calculates (art. 212 of his work) that the greatest extension which cast iron will bear without injury to its elastic force is $\frac{1}{1204}$th part of its length. In art. 70, he calculates the weights which would be required to destroy the elasticity of a number of cast iron bars, the breaking weights of which are severally given. He compares these weights, and concludes that a body requires about three times as much to break it as to destroy its elastic force. Hence he would conclude that the absolute strength, per square inch, of this iron, is $15{,}300 \times 3 = 45{,}900$ lbs. nearly, or more than 20 tons. Tredgold computes, in several cases, (arts. 72 to 76), the ultimate tensile strength of cast iron bars from the weights which broke them transversely. He found the strength to vary from 40,000 to 48,200 lbs. per square inch, the mean being 44,620 lbs., or nearly three times 15,300, as mentioned above.

My own experiments, which were made by tearing asunder 25 castings, prepared with great care, from cast iron obtained from various parts of England, Scotland, and Wales, gave as a mean 16,505 lbs. = 7·37 tons per square inch; and in no case, except one, was the strength found to be more than 8½ tons per square inch (art. 3, page 229).

89. According to the principles assumed by Tredgold (art. 37 and 100), the position of the neutral line must remain unchanged during flexure by different weights; but experiment shows that the neutral line shifts as the weights are increased; and at the time of fracture it is frequently near to the concave side of the beam. This seldom can be discovered in fractures of beams by simple transverse pressure, but it may sometimes be discovered in those that have been broken by a blow, and then it will perhaps seem to have been at $\frac{1}{5}$th or $\frac{1}{6}$th of the depth of a rectangular beam, as I have occasionally observed, the smaller part being compressed.

90. From the experiments on the strength of cast iron

pillars (Tables I. and II., art. 78) we get additional evidence upon this matter; for in these the neutral line was frequently well defined; and in the longest pillars, those which required the least weight to break them, the ratio T : C, which is that of the depths of the parts submitted to tension and compression, was, in different cases, 126 : 26, 145 : 36, 153 : 43, and 120 : 37. In two rectangular pillars, each $60\frac{1}{2}$ inches long, and $1\frac{1}{2}$ inches square, the area of the compressed part, was less than $\frac{1}{7}$th of the whole section. In the pillars mentioned above the weight necessary to break them was very small, compared with that which would have been required to crush them without flexure; and there was scarcely a pillar broken in which the part compressed exceeded the part extended, though some had sustained very great pressures.

In the longest pillars mentioned above we may, I conceive, consider the position of the neutral line as not widely different from what it would have been if the pillar had been broken as a beam, transversely; the only difference in the two cases being, that the compressed part, compared with the extended part, would be greater in the pillar, through the weight laid upon it, than in the beam. And we have seen that, in the pillars alluded to, the depth of the compressed part varied from $\frac{1}{3}$rd to less than $\frac{1}{6}$th of that of the extended part; and in a beam the depth of the compressed part would be still smaller. In the experiments on tension and compression (arts. 33-34, Part II.), it has been shown that cast iron resists crushing with about seven times as much force as it does tearing asunder; and some experiments not yet published have inclined me to believe that the neutral line in a rectangular beam, at the time of fracture, divides the section in the proportion of six or seven to one, or one not widely different from this; but I draw the conclusion with considerable diffidence.

91. The subject here treated of will be adverted to in a

future article, after the experiments which I am intending to give upon the transverse strength of bars have, with those on the tensile strength (art. 3, page 229), furnished data for the purpose.

EXPERIMENTS TO DETERMINE THE TRANSVERSE STRENGTH OF UNIFORM BARS OF CAST IRON.

92. I will now give the results of a very extensive series of experiments upon rectangular bars, all cast from the same model, and including irons from the principal Works in the United Kingdom.

The great accumulation of specimens of iron, which were obtained, but could not be used in our inquiry, before mentioned, respecting the strength of hot and cold blast iron, afforded a good opportunity to acquire the relative values of many of the leading irons. Mr. Fairbairn, therefore, undertook the matter, and had castings made from the whole; and having increased them by subsequent additions of iron, the variety of results from the whole, both of Mr. Fairbairn's and my own experiments, has become great, especially when those are added to them which we derived from the hot blast inquiry, and these will be found abridged in the following pages. The experiments of Mr. Fairbairn were very carefully made, some of them by myself, as those on the anthracite iron, &c., and all with the same attention to accuracy. The results are published at length in the Manchester Memoirs, vol. vi., second series; but are here given in an abridged form, only one series of results being set down for each kind of iron: every result in each series being a mean between values derived from the same weights in different experiments. The bars were cast from a model, 5 feet long and 1 inch square, and they were during the experiment, laid upon supports 4 feet 6 inches asunder, and bent by weights suspended from the middle. After each load had been laid on, which was done with care and with small additions of weight, the deflexion was obtained by means of a long scale in the form of a wedge, graduated along its side, so that very minute distances could be measured. The beam was then unloaded in order that the defect of elasticity, or set, might be obtained. These experiments and those made by my friend for the British Association, were conducted in the same manner as others which I had made some time before on the Carron, Buffery, and other irons, mentioned in art. 86; and the remark which I had made of the very early defect of elasticity of cast iron, as shown by my experiments, received a further confirmation from these; as the quantity of set was, from that time, always carefully observed. The results of all the bars were afterwards reduced by calculation in the same manner as those in my Report (British Association, vol. vi.), in order to preserve uniformity in the whole.

93. *Experiments of W. Fairbairn, Esq., on the strength of uniform rectangular bars of cast iron.*

English Irons.

No. 1. Apedale iron, No. 2, hot blast, Newcastle, Staffordshire.			No. 2. Adelphi iron, No. 2, cold blast, Derbyshire.			No. 3. Butterley iron, Derbyshire.			No. 4. Eagle Foundry iron, No. 2, hot blast, Staffordshire.			No. 5. Level iron, No. 1, hot blast, Staffordshire.			No. 6. Level iron, No. 2, hot blast, Staffordshire.		
Means from 2 experiments.			Means from 2 experiments.			Means from 2 experiments.			Means from 2 experiments.			Means from 2 experiments.			Means from 3 experiments.		
Depth of bar 1·017 in. Breadth ,, 1·009 Wt. of 1 of the bars 15 lbs. 3 oz.			Depth of bar 1·030 in. Breadth ,, ·993 Weight ,, 15 lbs. 8 oz.			Depth of bar ·995 in. Breadth ,, ·988 Weight ,, 14 lbs. 12½ oz.			Depth of bar 1·024 in. Breadth ,, 1·035 Weight ,, 15 lbs. 13¼ oz.			Depth of bar 1·011 in. Breadth ,, 1·010 Weight ,, 15 lbs. 6½ oz.			Depth of bar 1·033 in. Breadth ,, 1·011 Weight ,, 15 lbs. 13 oz.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
112	·277	·009	30	·065	+	28	·068		14	·032	+	56	·137	+	56	·129	·002
182	·487	·023	56	·135	·002	56	·135	·002	56	·136	·003	112	·273	·012	112	·246	·011
238	·673	·045	112	·294	·014	126	·335	·015	112	·279	·013	168	·429	·025	168	·404	·022
294	·878	·076	168	·470	·034	182	·509	·038	168	·441	·030	224	·598	·044	224	·559	·038
350	1·107	·110	224	·658	·060	238	·702	·064	224	·616	·051	280	·776	·070	280	·726	·061
406	1·356	·162	280	·865	·093	294	·912	·099	280	·803	·078	336	·965	·097	336	·905	·088
441	1·536		336	1·080	·138	350	1·142	·147	336	1·006	·113	392	1·171	·133	392	1·095	·121
455	broke		392	1·335	·201	406	1·402	·203	392	1·225	·159	448	1·392	·185	420	1·196	
			434	1·542		448	1·633		420	1·347		476	broke		453	broke	
			448	1·615		462	1·717		448	broke							
			465	broke		479	broke										
	ult. defl. =1·583			ult. defl. =1·705			ult. defl. =1·823			ult. defl. =1·462			ult. defl. =1·499			ult. defl. =1·314	

English Iron

No. 7. Low Moor iron, No. 2, cold blast, Yorkshire.			No. 8. Milton iron, No. 1, hot blast, Yorkshire.			No. 9. Milton iron, No. 3, hot blast, Yorkshire.		
Means from 2 experiments.			Means from 2 experiments.			Means from 2 experiments.		
Depth of bar ·999 in. Breadth „ 1·009 Weight „ 14 lbs. 13 oz.			Depth of bar 1·061 in. Breadth „ 1·042 Weight „ 16 lbs. 8½ oz.			Depth of bar 1·023 in. Breadth „ 1·009 Weight „ 15 lbs. 14 oz.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
56	·145	·006	42	·103	+	42	·093	+
112	·301	·013	112	·296	·008	56	·127	+
182	·524	·046	182	·508	·036	126	·294	·011
238	·724	·074	238	·697	·060	182	·442	·028
294	·946	·111	294	·907	·092	238	·599	·045
350	1·195	·163	350	1·143	·137	294	·772	·068
406	1·480	·237	406	1·406		350	·955	·097
455	1·784		413	broke		406	1·156	·137
465	broke					441	1·294	
						451	broke	
	ult. defl. =1·853			ult. defl. =1·437			ult. defl. =1·338	

No. 10. Elsicar iron, No. 2, cold blast.			No. 11. Oldberry iron, No. 2, cold blast.			No. 12. Old Park iron, No. 2, cold blast.		
Means from 2 experiments.			Means from 2 experiments.			Means from 2 experiments.		
Depth of bar 1·025 in. Breadth „ 1·007 Weight „ 15 lbs. 8 oz.			Depth of bar 1·050 in. Breadth „ 1·007 Weight „ 16 lbs. 0½ oz.			Depth of bar 1·034 in. Breadth „ 1·008 Weight „ 16 lbs. 3 oz.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
56	·152	·007	30	·064	+	28	·066	
126	·370	·026	56	·124	·003	56	·137	
182	·570	·060	112	·264	·012	112	·271	·015
238	·797	·092	168	·421	·031	168	·427	·029
294	1·064	·151	224	·588	·054	224	·597	·052
350	1·362	·217	280	·768	·083	280	·780	·080
406	1·710	·318	336	·968	·122	336	·980	·115
448	1·201	·417	392	1·185	·175	392	1·195	·159
472	broke		448	1·434	·253	434	1·370	
			490	1·652		462	1·510	
			504	broke		476	broke	
	ult. defl. =2·168			ult. defl. =1·724			ult. defl. =1·568	

English Irons.

No. 13. Horace St. Paul's, Wind-mill End iron, No. 2, cold blast, Staffordshire.			No. 14. Ley's Works iron, No. 1, hot blast.		
Means from 2 experiments.			Means from 3 experiments.		
Depth of bar 1·040 in. Breadth ,, 1·017 Weight ,, 16 lbs.			Depth of bar 1·008 in. Breadth ,, 1·012 Weight ,, 15 lbs. 4 oz.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
56	·112	+	28	·086	
112	·233	·005	56	·172	·007
175	·382	·018	112	·369	·031
231	·525	·035	168	·597	·065
287	·674	·054	224	·850	·107
343	·947	·078	280	1·136	·156
399	1·117	·108	336	1·447	·227
455	1·267	·147	373	1·686	·295
511	1·491	·215	404	broke	
532	broke				
	ult. defl. =1·519			ult. defl. =1·876	

No. 15. Lane End iron, No. 2.			No. 16. Carroll iron, No. 2, cold blast.		
Means from 3 experiments.			Means from 2 experiments.		
Depth of bar 1·005 in. Breadth ,, 1·018 Weight ,, 15 lbs. 7½ oz.			Depth of bar 1·037 in. Breadth ,, 1·012 Weight ,, 16lbs. 5 oz.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·070		56	·112	+
56	·140		112	·229	·008
112	·271	·010	126	·260	·010
168	·445	·025	182	.391	·017
224	·610	·040	238	·526	·034
280	·780	·057	294	·671	·052
336	·969	·080	350	·825	·073
392	1·165	·107	406	·939	·104
439	1·339		448	1·120	·125
457	broke		469	broke	
	ult. defl. =1·407			ult. defl. =1·187	

No. 17. Bierly iron, No. 2, Bradford, Yorkshire.			No. 18. W. S. S. iron, No. 2, Staffordshire.		
Means from 3 experiments.			Means from 3 experiments.		
Depth of bar 1·024 in. Breadth ,, 1·036.			Depth of bar 1·018 in. Breadth ,, 1·007		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·061		28	·069	
56	·123	+	56	·139	·002
112	·258	·011	112	·282	·012
168	·384	·021	168	·445	·026
224	·533	·039	224	·608	·042
280	·692	·060	280	·789	·062
336	·853	·086	336	·987	·089
392	1·037	·120	383	1·157	·114
420	1·136		411	1·264	
439	broke		427	broke	
	ult. defl. =1·194			ult. defl. =1·322	

English Irons.

No. 19. Coltham, B. F., iron, No. 1, hot blast, Staffordshire.			No. 20. Corbyn's Hall iron, No. 2, near Dudley, Staffordshire.			No. 21. Wall-Brook iron, No. 3, Dudley, Worcestershire.			No. 22. Oldberry iron, No. 3, hot blast, (patent iron) Shropshire.			No. 23. Elsicar iron, No. 1, cold blast.		
Means from 3 experiments.			Means from 3 experiments.			Means from 3 experiments.			Means from 3 experiments.			Means from 2 experiments.		
Depth of bar 1·017 in. Breadth ,, 1·009			Depth of bar 1·027 in. Breadth " 1·022			Depth of bar 1·025 in. Breadth ,, 1·033			Depth of bar 1·003 in. Breadth ,, 1·001			Depth of bar 1·037 in. Breadth ,, 1·027 Weight ,, 15 lbs. 10 oz.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·066		28	·069		28	·062		28	·051		56	·140	
56	·133	+	56	·139	+	56	·125	·003	56	·100	+	112	·275	·020
112	·268	·014	112	·288	·015	112	·252	·013	112	·192	+	168	·427	·038
168	·428	·028	168	·466	·036	168	·401	·028	168	·286	·006	224	·591	·054
224	·586	·046	224	·652	·062	224	·556	·049	224	·389	·010	280	·761	·075
280	·756	·068	280	·845	·088	280	·720	·071	280	·492	·014	336	·948	·102
336	·939	·097	336	1·059	·122	336	·902	·103	336	·596	·019	392	1·148	·135
392	1·137	·132	392	1·296	·171	392	1·088	·138	392	·701	·026	448	1·355	·176
439	1·319	·180	448	1·566	·234	451	1·303		448	·804	·031	476	broke	
457	1·393		457	1·613		474	broke		504	·918	·039			
485	broke		464	broke					523	·958	·043			
									546	broke				
	ult. defl. =1·505			ult. defl. =1·643			ult. defl. =1·394			ult. defl. =1·004			ult. defl. =1·460	

Scotch Irons.

No. 1. Carron iron, No. 3, cold blast. Means from 3 experiments. Depth of bar 1·004 in. Breadth ,, 1·005

Wt. in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·069	+
56	·136	·005
112	·267	·011
168	·418	.024
224	·576	·043
280	·741	·066
336	·921	·094
392	1·114	·129
429	1·251	
450	broke	
	ult. defl. = 1·321	

No. 2. Carron iron, No. 3, hot blast. Means from 3 experiments. Depth of bar ·997 in. Breadth ,, 1·006

Wt. in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·068	
56	·129	+
112	·249	·006
168	·380	·011
224	·520	·021
280	·663	·035
336	·809	·052
392	·972	·075
448	1·140	.103
485	1·259	
520	broke	
	ult. defl. = 1·369	

No. 3. Muirkirk iron, No. 1, cold blast. Means from 2 experiments. Depth of bar 1·032 in. Breadth ,, 1·016

Wt. in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·071	
56	·141	+
112	·282	·011
168	·446	·028
224	·624	·051
280	·814	·081
336	1·028	·121
392	1·258	·172
448	1·535	·255
479	broke	
	ult. defl. = 1·682	

No. 4. Muirkirk iron, No. 1, hot blast. Means from 2 experiments. Depth of bar 1·020 in. Breadth ,, 1·022

Wt. in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·074	
56	·152	+
112	·306	·012
168	·476	·025
224	·655	·043
280	·844	·070
336	1·059	·109
392	1·309	·171
420	1·440	
444	broke	
	ult. defl. = 1·640	

No. 5. Gartsherrie iron, No. 3, hot blast. Means from 3 experiments. Depth of bar 1·020 in. Breadth ,, 1·025 Weight 15 lbs. 8 oz.

Wt. in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·069	
56	·145	+
112	·292	·018
168	·460	·034
224	·639	·056
280	·822	·081
336	1·030	·114
392	1·254	·155
448	1·494	·209
455	broke	
	ult. defl. = 1·527	

No. 6. Dundyvan iron, No. 3, cold blast. Means from 3 experiments. Depth of bar 1·010 in. Breadth ,, 1·018 Weight 15 lbs. 12 oz.

Wt. in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·067	
56	·135	+
112	·255	·008
168	·420	·021
224	·579	·037
280	·746	·059
336	·932	·089
392	1·132	·126
448	1·358	·181
474	broke	
	ult. defl. = 1·456	

No. 7. Monkland iron, No. 2, hot blast. Means from 2 experiments. Depth of bar 1·014 in. Breadth ,, ·998 Weight 15 lbs. 4 oz.

Wt. in lbs.	Deflexion in inches.	Deflexion, load removed.
112	·345	·022
126	·390	·027
182	·598	·051
238	·827	·083
294	1·075	·118
350	1·358	·174
406	1·693	·273
413	broke	
	ult. defl. = 1·738	

Welsh Irons.

No. 1. Blaina iron, No. 3, cold blast, Monmouthshire.			No. 2. Plaskynaston iron, No. 2, hot blast.			No. 3. Pant iron, No. 2.			No. 4. Beaufort iron, No. 2, hot blast.			No. 5. Beaufort iron, No. 3, hot blast.			No. 6. Maesteg iron, No. uncertain (marked white), Glamorganshire.		
Means from 3 experiments.			Means from 3 experiments.			Means from 3 experiments.			Means from 3 experiments.			Means from 2 experiments.			Means from 2 experiments.		
Depth of bar 1·039 in. Breadth „ 1·018			Depth of bar 1·009 in. Breadth „ 1·006			Depth of bar 1·034 in. Breadth „ 1·027			Depth of bar 1·037 in. Breadth „ 1·033 Weight „ 16 lbs.			Depth of bar 1·081 in. Breadth „ 1·014 Weight „ 16lbs. 4½oz.			Depth of Bar 1·041 in. Breadth „ 1·027 Weight „ 16 lbs.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·069		28	·078		28	·063		28	·056		28	·051		28	·062	
56	·135	·006	56	·154	·007	56	·122	·005	56	·116		56	·089	+	56	·131	+
112	·270	·018	112	·320	·021	112	·254	·014	112	·235	·008	126	·231	·004	112	·272	·013
168	·431	·035	168	·511	·040	168	·396	·025	168	·361	·017	182	·346	·012	168	·429	·030
224	·607	·059	224	·706	·064	224	·545	·042	224	·497	·030	238	·467	·024	224	·612	·056
230	·793	·093	280	·915	·092	280	·701	·059	280	·642	·049	294	·592	·037	280	·815	·090
336	1·008	·141	336	1·139	·126	336	·861	·076	336	·797	·073	350	·726	·056	336	1·024	·140
392	1·232	·197	373	1·298	·153	392	1·035	·100	392	·963	·103	406	·872	·078	392	1·262	·191
448	1·535	·290	387	broke		429	1·154	·117	448	1·147	·136	462	1·029	·106	448	1·535	·274
476	broke					448	broke		495	1·258	·183	518	1·200	·142	504	1·880	·415
									513	1·350	·207	574	1·391	·197	504	repeated broke	
									532	broke		588	1·442	·213			
												599	broke				
	ult. defl. = 1·661			ult. defl. = 1·354			ult. defl. = 1·210			ult. defl. = 1·459			ult. defl. = 1·479			ult. defl. = 1.880	

Welsh Irons.

No. 7 Maesteg iron, No. uncertain, (marked red) Glamorganshire.			No. 8. Pontypool iron, No. 2.			No. 9. Varteg-hill iron, No. 2, hot blast, South Wales.			No. 10. Pentwyn iron, No. 2.			No. 11. Bute iron, No. 1, cold blast.			No. 12. Brimbo iron, No. 2, cold blast.		
Means from 2 experiments.			Means from 2 experiments.			Means from 2 experiments.			Means from 2 experiments.			Means from 3 experiments.			Means from 3 experiments.		
Depth of bar 1·026 in. Breadth ,, 1·012 Weight ,, 15lbs. 8 oz.			Depth of bar 1·042 in. Breadth ,, 1·020 Weight ,, 15 lbs. 12 oz.			Depth of bar 1·017 in. Breadth ,, 1·008 Weight ,, 15 lbs. 11 oz.			Depth of bar 1·053 in. Breadth ,, 1·024 Wt. of 1 of the bars 16 lbs.			Depth of bar 1·027 in. Breadth " 1·024 Weight ,, 15 lbs. 11 oz.			Depth of bar 1·021 in. Breadth ,, 1·023 Weight ,, 15 lbs. 15 oz.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·070		28	·063		112	·276	·011	28	.056		28	·063		28	·065	
56	·143	+	56	·133	·005	126	·311	·015	56	·116	+	56	·128	+	56	·134	·002
112	·288	·010	112	·291	·020	182	·476	·032	112	·242	·008	112	·262	·010	112	·272	·014
175	·499	·044	168	·457	·038	238	·648	·057	168	·372	·018	168	·416	·022	168	·437	·033
231	·696	·075	224	·641	·065	294	·840	·085	224	·515	·030	224	·578	·042	224	·615	·058
287	·902	·117	280	·838	·097	350	1·045	·123	280	·674	·048	280	·748	·067	280	·805	·088
343	1·170	·186	336	1·045	·142	392	1·220	·169	336	·844	·073	336	·931	·098	336	1·009	·124
399	1·462	·279	392	1·286	·204	420	1·355	·205	392	1·023	·101	392	1·131	·133	392	1·227	·172
455	1·756	·380	448	1·571	·296	434	broke		448	1·221	·139	448	1·357	·185	448	1·477	·237
469	broke		462	1·651					490	1·385		504	1·613	·253	476	1·619	
			486	broke					497	broke		513	1·660		497	broke	
												534	broke				
	ult. defl. =1·840			ult. defl. =1·781			ult. defl. =1·422			ult. defl. =1·410			ult. defl. =1·718			ult. defl. =1·713	

Welsh Irons.

No. 13. Coed-Talon iron, No. 2, cold blast.		
Means from 3 experiments.		
Depth of bar 1·048 in. Breadth ,, 1·020 Weight ,, 16 lbs.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·063	
56	·121	+
121	·283	·015
159	·387	·025
196	·496	·036
252	·660	·061
308	·840	·083
364	1·029	·117
420	1·242	·169
439	1·322	·190
457	broke	
	ult. defl. =1·403	

No. 14. Coed-Talon iron, No. 2, hot blast.		
Means from 2 experiments.		
Depth of bar 1·064 in. Breadth ,, 1·005 Weight ,, 15 lbs. 14 oz.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·068	
56	·130	·005
126	·327	·027
182	·505	·054
238	·699	·087
294	·910	·122
350	1·151	·177
406	1·427	·255
448	1·667	·332
455	1·709	
465	broke	
	ult. defl. =1·773	

No. 15. Coed-Talon iron, No. 3, cold blast.		
Means from 2 experiments.		
Depth of bar 1·015 in. Breadth ,, 1·011		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·063	
56	·124	+
112	·244	·011
168	·378	·020
224	·516	·033
280	·659	·048
336	·806	·066
392	·966	·089
448	1·137	·116
504	1·319	·156
546	1·473	
560	broke	
	ult. defl. =1·521	

No. 16. Coed-Talon iron, No. 3, hot blast.		
Means from 2 experiments.		
Depth of bar 1·006 in. Breadth ,, 1·003		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·074	+
56	·146	·004
112	·293	·011
168	·454	·024
224	·616	·039
280	·786	·057
336	·967	·077
392	1·156	·101
448	1·360	·136
476	1·469	
504	broke	
	ult. defl. =1·567	

No. 17. Ponkey iron, No. 3, cold blast.		
Means from 3 experiments.		
Depth of bar 1·014 in. Breadth ,, 1·022 Weight ,, 16 lbs.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·056	
56	·120	+
112	·241	·010
168	·373	·020
224	·515	·031
280	·657	·048
336	·807	·068
392	·972	·094
448	1·152	·126
504	1·348	·171
560	1·572	·233
569	1·613	
595	broke	
	ult. defl. =1·712	

No. 18. Frood iron, No. 2, cold blast.		
Means from 3 experiments.		
Depth of bar 1·011 in. Breadth ,, 1·015 Weight ,, 15 lbs. 6½ oz.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·073	
56	·144	+
112	·298	·016
168	·470	·032
224	·657	·056
280	·857	·089
336	1·088	·131
392	1·341	·188
448	1·639	·273
457	1·692	
478	broke	
	ult. defl. =1·798	

Welsh Anthracite Irons.

No. 1. Yniscedwyn Anthracite iron, No. 1, hot blast.			No. 2. Yniscedwyn Anthracite iron, No. 2, hot blast.			No. 3. Yniscedwyn Anthracite iron, No. 3. hot blast.		
Means from 3 experiments.			Means from 3 experiments.			Means from 3 experiments.		
Depth of bar 1·022 in. Breadth ,, 1·013 Weight ,, 15 lbs. 7½ oz.			Depth of bar 1·024 in. Breadth ,, 1·021 Weight ,, 15 lbs. 12 oz.			Depth of bar 1·017 in. Breadth ,, 1·010 Weight ,, 15 lbs. 9 oz.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·072	+	28	·064		28	·063	+
56	·147	·009	56	·132	+	56	·126	·004
112	·307	·018	112	·263	·013	112	·257	·010
224	·650	·064	224	·558	·039	224	·544	·029
336	1·054	·129	336	·876	·073	336	·854	·063
448	1·545	·250	448	1·241	·139	448	1·197	·111
460	1·601		485	1·366		504	1·385	·146
477	broke		504	1·441		538	broke	
			518	broke				
	ult. defl. =1·693			ult. defl. =1·497			ult. defl. =1·499	

No. 4. Ystalyfera Anthracite iron, No. 1, hot blast.			No. 5. Ystalyfera Anthracite iron, No. 2, hot blast.			No. 6. Ystalyfera Anthracite iron, No. 3, hot blast.		
Means from 3 experiments.			Means from 3 experiments.			Means from 3 experiments.		
Depth of bar 1·032 in. Breadth ,, 1·050 Weight ,, 15lbs. 13 oz.			Depth of bar 1·027 in. Breadth ,, 1·038 Weight ,, 15 lbs. 10 oz.			Depth of bar 1·020 in. Breadth ,, 1·041 Weight ,, 15 lbs. 11 oz.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
28	·074	+	28	·064	·002	28	·066	+
56	·153	·009	56	·134	·006	56	·138	·005
112	·330	·032	112	·281	·025	112	·297	·015
224	·739	·099	224	·623	·068	224	·658	·049
336	1·258	·210	336	1·021	·147	336	1·065	·107
411	1·694		448	1·500	·263	448	1·550	·216
448	1·914	·458	476	1·641		495	broke	
457	2·009		495	broke				
483	broke							
	ult. defl. =2·181			ult. defl. =1·742			ult. defl. =1·792	

94. *Mean results of experiments made, by the Author, on the transverse strength and elasticity of uniform bars of cast iron, of different forms of section. All the bars, except the last, No. 16, being cast 5 feet long, and laid on supports 4 feet 6 inches asunder, and having the weights suspended from the middle. The results are abridged from the experiments in the Author's Report on the Strength, &c., of Hot and Cold Blast Iron* (Brit. Association, vol. vi.), *and from other experiments recently made.*

Rectangular Bars of English Iron.									Rectangular Bars.			Rectangular Bars of Scotch Iron.		
No. 1. Buffery iron, No. 1, hot blast, near Birmingham.			No. 2. Buffery iron, No. 1, cold blast, near Birmingham.			No. 3. Low Moor iron, No. 3, cold blast, Yorkshire.			No. 4. Mixture of iron used in beams of Liverpool and Leeds Junction Railway, at Salford.			No. 5. Carron iron, No. 2, hot blast, made with coke.		
Means from 3 experiments.			Means from 3 experiments.			Means from 2 experiments. *Phil. Trans.*, Part II., 1840.			From 1 experiment.			Means from 3 experiments.		
Depth of bar 1·002 in. Breadth ,, 1·017 Weight ,, 15 lbs. 8 oz.			Depth of bar 1·009 in. Breadth ,, ·980 Weight ,, 15 lbs. 8 oz.			Depth of bar 1·003 in. Breadth ,, 1·000			Depth of bar 1·030 in. Breadth ,, 1·025 Weight ,, 15 lbs. 13 oz.			Depth of bar 1·004 in. Breadth ,, 1·004 Weight ,, 15 lbs. 5 oz.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
112	·316	·014	112	·278	·008	28	·065	·001 ?	28	·055	+	16	·037	visible
224	·687	·046	224	·603	·044	56	·143	·005	56	·130	increased	23	·052	,,
336	1·150	·116	336	·984	·096	112	·314	·025	112	·290	·025	30	·069	·001 ?
392	1·420	·183	392	1·207	·142	224	·701		224	·63	·07	56	·131	·002
411	1·520	·211	448	1·45	·199	336	1·186	·17	336	1·55	·14	112	·269	·008
446	broke		476	broke		392	1·481	·26	392	1·83	·23	224	·582	·036
						448	1·826	·38				336	·933	·086
						467	broke					448	1·348	·174
												469	broke	
	ult. defl. =1·647			ult. defl. =1·54			ult. defl. =1·943						ult. defl. =1·430	

Rectangular Bars of Scotch Iron.

No. 6. Carron iron, No. 2, cold blast, made with coke.			No. 7. Devon iron, No. 3, hot blast, made with coal.			No. 8. Devon iron, No. 3, cold blast, made with coke.			No. 9. Carron iron, No 2. bars cast 1¼ inch square, and reduced in middle to 1 inch, nearly.			No. 10. Carron iron, No. 2.			No. 11. Carron iron, No. 2.		
Means from 3 experiments.			Means from 2 experiments.			Means from 2 experiments.			Means from 4 expts., 2 on hot and 2 on cold blast iron.			Means from 2 expts., 1 on hot and 1 on cold blast iron.			Means from 3 expts., 2 on hot and 1 on cold blast iron.		
Depth of bar 1·029 in. Breadth ,, 1·014 Weight ,, 15lbs. 12oz.			Depth of bar 1·005 in. Breadth ,, 1·005			Depth of bar 1·00 in. Breadth ,, 1·00			Depth of bar in middle— 1·017 in. Breadth ,, 1·018			Depth of bar 3·000 in. Breadth ,, 1·025 Weight 46½ lbs. nearly.			Depth of bar 4·977 in. Breadth ,, 1·023 Weight ,, 77 lbs. 11 oz.		
Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion not taken.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.	Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
16	·033	visible ?	112	·192		112	·192		112		·006	1082	·088	·002	4936	·107	·01 nearly
30	·062	,,	168	·295	·002	168	·292		140		·010	1343	·109	·004	5867	·130	·01
56	·117	·003	224	·405	·008	224	·395		168		·022	1605	·134	·006	6798	·153	·015
112	·230	·006	280	·505		280	·495		196		·030	1866	·160	·008	7730	·182	·02
168	·358	·012	336	·615	·020	336	·590	·004	224		·043	2126	·188	·011	8662	·208	·02
224	·490	·022	392	·730		392	·690		336		·118	2388	·216	·014	9593	·227	·03
280	·621	·034	448	·845	·045	420	·740		392		·163	2649	·246	·018	10298	broke	
336	·762	·053	504	·980		448	broke		448		·220	2910	·276	·024			
392	·913	·075	539	1·070					476	broke		3172	·309	·030			
418	1·073	·105	546	broke								3433	·342	·037			
511	broke											3694	·378	·048			
												3890	broke				
	ult. defl. = 1·278			ult. defl. = 1·08			ult. defl. = ·790						ult. defl. = ·406			ult. defl. = ·265	

Bars of Scotch Iron, Carron, No. 2.

No. 12. Bar, section an isosceles triangle, base 1·43 inches, and each side 2·00 inches, broken with the vertex downwards.

Means from 3 experiments, two on hot and one on cold blast iron.

Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
112	·07	
168	·106	
224	·145	·005
336	·227	·010
448	·317	·020
560	·420	·040
672	·530	
766	broke	
	ult. defl. = ·62	

No. 13. Bar same as last, and filed to the exact size, but having 1-10th of the depth taken off the vertex. It was broken with the vertex downwards.

Means from 2 experiments, one on hot and the other on cold blast iron.

Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
112	·072	
168	·110	·002
224	·150	·003
336	·235	·006
448	·330	·014
560	·430	·030
658	·520	
702	broke	
	ult. defl. = ·58	

No. 14. Uniform bar, whose form of section is below, broken with the vertical rib downwards.

Means from 2 experiments, one on hot and the other on cold blast iron.

Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
112	·195	
168	·305	·013
196	·365	
224	·425	·025
252	·495	·040
273	broke	
	ult. defl. = ·54	

No. 15. Bar same as last, broken with the vertical rib upwards.

Means from 2 experiments, one on hot and the other on cold blast iron.

Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
112	·205	
224	·40	·02
280	·50	·025
336	·60	
392	·715	·065
448	·83	
560	1·105	·165
672	1·43	
784	1·84	
896	2·425	
994	3·050	
1015	broke	
	ult. defl. = 3·19	

Fracture attended by the separation of a wedge of the form

No. 16. Larger bar, from same model as those in page 284, broken with the vertical rib upwards.

Means from 2 experiments, distance between supports 6 feet 6 inches. A fluid iron, but its name unknown.

Weight in lbs.	Deflexion in inches.	Deflexion, load removed.
56	·112	
112	·238	·014
224	·498	·033
336	·776	·072
448	1·050	·114
560	1·401	·181
672	1·720	·261
784	2·122	·397
896	2·542	·592
1008	3·130	·872
1120	3·892	1·275
1176	4·290	
1232	broke	
	ult. defl. = 4·757	

Fracture caused by tension without the separation of a wedge.

REMARKS ON THE EXPERIMENTS IN THE LAST ARTICLE.

95. The objects sought for in every experiment were to obtain —the deflexion of the beam with given weights, generally increasing by equal increments,—and the set, or defect of elasticity, as exhibited by the deviation of the beam from its original form, after the weight was taken off; for reasons before mentioned (art. 86). As the beams were generally broken, the deflexion at the time of fracture could not often be obtained by direct admeasurement; it was therefore usually calculated, for each separate experiment, from the breaking weight, and the last weight, with its observed deflexion previous to fracture.

Many of the experiments were on the Carron Iron, No. 2, both hot and cold blast; and as these two denominations of iron differ in transverse strength only as 99 to 100, from the results of a great number of experiments, we may consider them as of the same strength. Some of the experiments were made to test admitted conclusions with respect to the strength of materials, and their objects will now be described.

The experiments Nos. 5 and 6 were on bars somewhat greater than 1 inch square; and the bars Nos. 10 and 11 were nearly of the same breadth as those, but had their depths 3 and 5 inches respectively. Hence, supposing each bar to be 1 inch broad, and the depths 1, 3, 5 inches respectively, the strengths should be as 1, 9, 25, the square of the depths. The mean strength from the bars Nos. 5 and 6 was 490 lbs., but as these were larger than 1 inch square, I will state the reduced results from a number of experiments. The strength per square inch from five cold blast bars of this iron was 467 lbs., and from five hot blast bars it was 459 lbs.; mean from the whole 463 lbs. Multiplying this mean by 9 and 25, gives for the strength of the bars, 3 and 5 inches deep, 4167 and 11,575 lbs. respectively. The 3 and 5-inch bars, in Nos. 10 and 11, are rather more than 1 inch broad, and their mean strengths are 3890 and 10,298 lbs. And, if the breadths were reduced to 1 inch, the strengths would be 3795, 10,067 lbs. respectively. But, as these numbers are less than 4167, 11,575, as above, it appears that the strength of rectangular bars, of the same length and breadth, increases in a ratio somewhat lower than as the square of the depth.

It having been often asserted that, if the external part or crust of a cast iron bar be taken away, the strength of the internal part will be much less than that of a bar of the same dimensions, retaining its outer crust; the result of the experiments in No. 9, compared with those of Nos. 5 and 6, will show that the falling off in strength, if any, is not great.

It was asserted by Emerson, in his 'Mechanics,' 4to, page 114, that if a beam be made in the form of a triangular prism, and $\frac{1}{9}$th of the height be taken from the vertex, parallel to the base, the remaining part will be stronger than the whole beam: this result was obtained on a supposition that materials are incompressible. Tredgold, in art. 118 of this volume, computes the same, on the supposition of equal extensions and compressions from equal forces, and finds that if $\frac{1}{10}$th of the depth of such a beam be taken away from the vertex, the strength will be about the greatest. The experiments Nos. 12 and 13 were intended to show how far this was true; and it appears that the frustrum, with $\frac{1}{10}$th of the depth taken away, instead of being stronger than the whole triangle, was weaker than it in the proportion of 702 to 766. The object of the experiments Nos. 14 and 15 was principally to show that beams of cast iron, of the same dimensions, might be made to bear, when turned one way upwards, several times the weight which they would bear when turned the opposite way up. In this case the strengths were as 1015 to 273, or as 4 to 1 nearly. This was first shown in the Author's Paper on the 'Strength and best Form of Iron Beams' (Manchester Memoirs, vol. v.), and of which an abstract will be given in this volume. The experiments No. 16 have in part the same object as those in art. 86, before described.

Abstract of Results obtained from the whole of the Experiments, both of Mr. Fairbairn and the Author, on the Transverse Strength, and other properties, of cast iron bars, from the principal Iron Works in the United Kingdom.

Number of iron in the scale of strength.	Names of Irons.	Number of experiments on each.	Specific gravity.	Modulus of elasticity in lbs. per square inch, or stiffness.	Breaking weight in lbs. of bars 4 ft. 6 in. between supports.	Breaking weight in lbs. of bars 2 ft. 3 in. reduced to 4 ft. 6 in. between supports.	Mean breaking weight in lbs. (S.)	Ultimate deflexion of 4 ft. 6 in. bars, in parts of an inch.	Power of the 4 ft. 6 in. bars to resist impact.	Colour.	Quality.
1	Ponkey, No. 3, cold blast	4	7·122	17211000	567	590	578	1·747	992	Whitish gray	hard
2	Devon, No. 3, hot blast*	2	7·251	22473650	537	—	537	1·09	589	White	hard
3	Oldberry, No. 3, hot blast	5	7·300	22733400	543	517	530	1·005	549	White	hard
4	Carron, No. 3, hot blast	2	7·056	17873100	520	534	527	1·365	711	Whitish gray	hard
5	Coed-Talon, No. 3, hot blast	4	6·970	14707900	496	530	518	1·577	782	Dullish gray	hard
6	Beaufort, No. 3, hot blast	5	7·069	16802000	505	529	517	1·599	807	Dullish gray	hard
7	Butterley	4	7·038	15379500	489	515	502	1·815	889	Dark gray	soft
8	Bute, No. 1, cold blast	4	7·066	15163000	495	487	491	1·764	872	Bluish gray	soft
9	Wind Mill End, No. 2, cold blast	4	7·071	16490000	483	495	489	1·581	765	Dark gray	hard
10	Old Park, No. 2, cold blast	5	7·049	14607000	441	529	485	1·621	718	Gray	soft
11	Carron, No. 2, cold blast*	3	7·066	17270500	476	—	476	1·313	630	Dull gray	rather hard
12	Beaufort, No. 2, hot blast	4	7·108	16301000	478	470	474	1·512	729	Dull gray	hard
13	Low Moor, No. 2, cold blast	4	7·055	14509500	462	483	472	1·852	855	Dark gray	soft
14	Low Moor, No. 3, cold blast*	2	7·052	13918740	467	—	467	1·944	908	Dark gray	rather harder
15	Buffery, No. 1, cold blast*	3	7·079	15381200	463	—	463	1·550	721	Gray	rather hard
16	Carron, No. 2, hot blast*	3	7·046	16085000	463	—	463	1·337	619	Grayish blue	rather hard
17	Brimbo, No. 2, cold blast	5	7·017	14911666	466	453	459	1·748	815	Light gray	rather hard
18	Apedale, No. 2, hot blast	3	7·017	14852000	457	455	456	1·730	791	Light gray	stiff
19	Oldberry, No. 2, cold blast	4	7·059	14307500	453	457	455	1·811	822	Dark gray	rather soft
20	Pentwyn, No. 2	4	7·038	15193000	438	473	455	1·484	650	Bluish gray	hard
21	Maesteg, No. 2	5	7·038	13959500	453	455	454	1·957	886	Dark gray	rather soft
22	Muirkirk, No. 1, cold blast	4	7·113	14003550	443	465	454	1·734	770	Bright gray	fluid
23	Adelphi, No. 2, cold blast	5	7·080	13815500	441	457	449	1·759	777	Light gray	soft
24	Blaina, No. 3, cold blast	5	7·159	14281466	433	464	448	1·726	747	Bright gray	hard
25	Devon, No. 3, cold blast*	2	7·295	22907700	448	—	448	·790	354	Light gray	hard
26	Gartsherrie, No. 3, hot blast	5	7·017	13894000	427	467	447	1·557	998	Light gray	soft
27	Frood, No. 2, cold blast	5	7·031	14112666	460	434	447	1·825	841	Light gray	open
28	Lane End, No. 2	3	7·028	15787666	444	—	444	1·414	629	Dark gray	soft
29	Carron, No. 3, cold blast*	5	7·094	16246966	444	448	446	1·336	594	Gray	soft

30	Dundyvan, No. 3, cold blast .	4	7·087	16534000	456	430	443		1·469	674		Dull gray .	. rather soft
31	Maesteg (marked red) . . .	5	7·038	13971500	440	444	442		1·887	830		Bluish gray .	. . . fluid
32	Corbyn's Hall, No. 2 . . .	5	7·007	13845866	430	454	442		1·687	727		Gray . .	. . . . soft
33	Pontypool, No. 2	5	7·080	13136500	439	441	440		1·857	816		Dull blue .	. rather soft
34	Wallbrook, No. 3	5	6·979	15394766	432	449	440		1·443	625		Light gray .	rather hard
35	Milton, No. 3, hot blast . . .	4	7·051	15852500	427	449	438		1·368	585		Gray . . .	rather hard
36	Buffery, No. 1, hot blast* . .	3	6·998	13730500	436	—	436		1·640	721		Dull gray .	. . . . soft
37	Level, No. 1, hot blast . . .	5	7·080	15452500	461	403	432		1·516	699		Light gray .	. . . soft
38	Pant, No. 2	5	6·975	15280900	408	455	431		1·251	511		Light gray .	rather hard
39	Level, No. 2, hot blast . . .	6	7·031	15241000	419	439	429		1·358	570		Dull gray .	. . . . soft
40	W. S. S., No. 2	5	7·041	14953333	413	446	429		1·339	554		Light gray .	. . . . soft
41	Eagle Foundry, No. 2, hot blast	4	7·038	14211000	408	446	427		1·512	618		Bluish gray .	. . . soft
42	Elsicar, No. 2, cold blast . .	4	6·928	12586500	446	408	427		2·224	992		Gray . . .	. . . . soft
43	Varteg, No. 2, hot blast . .	4	7·007	15012000	422	430	426		1·450	621		Gray . . .	. . . . hard
44	Coltham, No. 1, hot blast . .	5	7·128	15510066	464	385	424		1·532	716		Whitish gray	. rather soft
45	Carroll, No. 2, cold blast . .	4	7·069	17036000	430	408	419		1·231	530		Gray . . .	. . . . hard
46	Muirkirk, No. 1, hot blast . .	4	6·953	13294400	418	420	419		1·570	656		Bluish gray .	. . . soft
47	Bierley, No. 2	5	7·185	16156133	404	432	418		1·222	494		Dark gray .	. . . soft
48	Coed-Talon, No. 2, hot blast .	4	6·969	14322500	409	424	416		1·882	772		Bright gray .	. . . soft
49	Coed-Talon, No. 2, cold blast .	5	6·955	14304000	408	418	413		1·470	600		Gray . . .	. rather soft
50	Monkland, No. 2, hot blast .	3	6·916	12259500	402	404	403		1·762	709		Bluish gray .	. . . soft
51	Ley's Works, No. 1, hot blast .	3	6·957	11539333	392	—	392		1·890	742		Bluish gray .	. . . soft
52	Milton, No. 1, hot blast . .	4	6·976	11974500	353	386	369		1·525	538		Gray . . .	soft and fluid
53	Plaskynaston, No. 2, hot blast	5	6·916	13341633	378	337	357		1·366	517		Light gray .	. rather soft
	ANTHRACITE IRONS.												
54	Ynisceedwyn Anthracite, No. 1, hot blast . . .	6	7·078	13741400	453	464	458		1·730	785		Grayish blue	. . . soft
55	„ 2, „	5	7·095	15334000	485	532	508		1·529	709		Grayish blue	. . harder
56	„ 3, „	5	7·168	16194327	515	525	520		1·525	785		Whitish gray .	rather harder
57	Ystalyfera Anthracite, First sample, No. 1, hot blast	6	6·992	11555635	435	423	429	411 mean	2·252	973	771 mean	Bluish gray	
	Second do. „ „ . .	4	7·098	14044420	392·3		392·3		1·445	569			
58	First do. No. 2, „ . .	6	7·053	13973270	453	454	454	467	1·788	810	769	Dark gray .	. rather soft
	Second do. „ „ . .	4	7·258	15686750	480·7		480·7		1·505	728			
59	First do. No. 3, „ . .	6	7·133	13436806	457	475	466	484	1·825	837	751	Whitish gray	
	Second do. „ „ . .	4	7·352	18391425	502		502		1·324	665			

* The irons marked with an asterisk are from the Author's Experiments.

96. In the preceding Table, the result from each bar is reduced to exactly 1 inch square; and the transverse strength which may be taken as a criterion of the value of each iron, is obtained from a mean between the reduced results of the original experiments upon it;—first on bars 4 feet 6 inches between the supports, and next on those of half the length, or 2 feet 3 inches between the supports. All the other results are deduced from the 4 ft. 6 inch bars. In all cases the weights were laid on the middle of the bar.

97. Since the experiments above were given to the public, some others, upon bars of the same dimensions, and having their results reduced in the same manner as these, have been published by Mr. David Mushet. Other experiments on the Ystalyfera iron have been given to the public by Mr. Evans: those above are results obtained from experiments upon two samples of each kind, sent to Mr. Fairbairn from the proprietors.

Mr. Fairbairn has likewise recently sent to the Institution of Civil Engineers the results of experiments made for him by the Author, upon bars of the same size as those in the preceding pages, and on four other kinds of cast iron, viz.: iron obtained from Turkish ores; iron from the island of Elba; and two kinds of Ulverston (English) iron.

98. *Explanation and Uses of the preceding Table.*

1st. *Explanation.*—The column representing the number of experiments refers to those from which the strength of the beams was obtained.

The specific gravity was obtained, generally, from a mean of about half a dozen experiments, on small specimens, weighed in and out of water.

The modulus of elasticity was usually obtained from the deflexion caused by 112 lbs. on the 4 feet 6 inch bars, calculated from the value of m in the formula, $m=\frac{wl^3}{4bd^3a}$, (Part I. art. 256). The numbers representing the power to resist impact were obtained from the product of the breaking weight of the bars, by their ultimate deflexion; as it appeared from the experiments in the Author's Paper 'On Impact upon Beams,' (British Association of Science, fifth Report,) that the conclusions of Tredgold (art. 304), with respect to a modulus of resilience, applicable so long as the elasticity was uninjured, might be extended to the breaking point in cast iron.

2nd. *Uses of the Table.*—These are numerous, but two only of the most common will be mentioned. If b and d be the breadth and depth of a rectangular beam in inches, l the distance between the supports in feet, w the breaking weight in lbs., w' any other weight, d' its deflexion, and m the modulus of elasticity in lbs., for a square inch: putting 4·5 for the distance 4 feet 6 inches, above, we have

$$w = \frac{4 \cdot 5 \times b d^2 s}{l}.$$ The value of s being taken from the Table above.

$$w' = \frac{m\, b\, d^3\, d'}{432\, l^3}.$$ (Part I. art. 256) the value of the modulus m being obtained from the Table.

DEFECT OF ELASTICITY.

99. In all the preceding experiments on rectangular bars, the defect of elasticity, measured by the deflexion remaining in the bar after the load had been removed, was observed, for reasons previously given (arts. 86, 92); and to show the law which regulates this defect, its value, with equal additions of weight, will be collected from the mean results upon each iron, and placed under the corresponding weights in the following Table.

100. *Defect of elasticity, or set, as obtained from the mean deflexion of bars cast from models 1 inch square, laid on supports 4·5 ft. asunder; using only those irons upon which experiments had been made, as to the set, upon all the weights set down.*

FIRST SERIES.	56	112	168	224	280	336	392	448
No. 1 Irons.								
Elsicar, cold blast		·020	·038	·054	·075	·102	·135	·176
Muirkirk, cold blast		·011	·028	·051	·081	·121	·172	·255
Bute, cold blast		·010	·022	·042	·067	·098	·133	·185
No. 2 Irons.								
Corbyn's Hall		.015	·036	·062	·088	·122	·171	·234
Beaufort, hot blast		.008	·017	·030	·049	·073	·103	·136
Pentwyn		.008	·018	·030	·048	·073	·101	·139
Frood, cold blast		.016	·032	·056	·089	·131	·188	·273
No. 3 Irons.								
Carron, hot blast		·006	·011	·021	·035	·052	·075	·103
Gartsherrie, hot blast		·018	·034	·056	·081	·114	·155	·209
Dundyvan, cold blast		·008	·021	·037	·059	·089	·126	·181
Coed-Talon, do.		·011	·020	·033	·048	·066	·089	·116
Ponkey, do.		·010	·020	·031	·048	·068	·094	·126
Maesteg, number unknown		·013	·030	·056	·090	·140	·191	·274
No. 2 Irons.								
Oldberry, cold blast	·003	·012	·031	·054	·083	·122	·175	·253
Pontypool	·005	·020	·038	·065	·097	·142	·204	·296
Brimbo	·002	·014	·033	·058	·088	·124	·172	·237
Carron, cold blast	·003	·006	·012	·022	·034	·053	·075	·105
No. 3 Irons.								
Blaina, cold blast	·006	·018	·035	·059	·093	·141	·197	·290
Coed-Talon, hot blast	·004	·011	·024	·039	·057	·077	·101	·136
Means from sets from nineteen kinds of iron		·0124	·026	·045	·069	·100	·140	·196
Sets computed from the formula $x = \frac{W^2}{342}$, where x is the set, and w the weight in ½ cwts.		·0117	·026	·047	·073	·105	·143	·187
SECOND SERIES.								
No. 2 Irons.								
Adelphi, cold blast	·002	·014	·034	·060	·093	·138	·201	
Eagle Foundry, hot blast	·003	·013	·030	·051	·078	·113	·159	
Level, hot blast	·002	·011	·022	·038	·061	·088	·121	
Pant	·005	·014	·025	·042	·059	·076	·100	
No. 3 Irons.								
Wallbrook	·003	·013	·028	·049	·071	·103	·138	
Carron, cold blast	·005	·011	·024	·043	·066	·094	·129	
Means from the six kinds of iron	·0037	·0127	·027	·047	·071	·102	·141	
Means from the last six in former part of Table	·0038	·0128	·029	·051	·080	·115	·157	
Means from the twelve kinds of iron	·0037	·0127	·028	·049	·076	·109	·149	
Sets computed, as before, from formula $x = \frac{W^2}{328}$, x and w being as above	·0030	·0122	·027	·049	·073	·110	·149	

101. Comparing the mean sets, or defects of elasticity, in each series of the preceding Table, with the computed ones, it

appears that the defects vary nearly as the square of the weights; the set being the abscissa and the weight the ordinate of a parabola.

Hence there is no force, however small, that will not injure the elasticity of cast iron.

102. When bars of a ⊥ form of section are bent, so as to make the flexure to depend upon the extension or compression of the vertical rib (as in arts. 86, and 94, experiments 14 to 16), the set is nearly as the square of the extension or compression; these being measured by the deflexions.

103. In all the preceding experiments, the weight laid upon the beam acted in a vertical direction; and the weight of the beam, independently of the other weight, had a small tendency to deflect the beam; the deflexions given in the Table being measured as commencing from that position which the beam had taken in consequence of its own weight. This, therefore, introduced an error which, though very small, on account of the great strength of cast iron compared with its weight, ought, if possible, to be avoided; especially where the object was,—not only to prove that defects of elasticity were produced by weights which were not hitherto supposed capable of injuring the elasticity,—but also to seek for the law which regulated these defects. Other objections to these results might be urged, as for instance: when a beam is laid upon two supports, and bent by a weight in the middle or elsewhere; the friction between the ends of the beam and the supports will have a slight influence upon the deflexion, a matter which has been submitted to calculation by Professor Moseley* in his able work on engineering. To meet the objections above, I had an apparatus constructed with four friction wheels, two to support each end of the beam; one wheel acting horizontally and the other vertically. The horizontal wheels were intended to destroy the friction arising from the weight of the beam, and the vertical ones that from

* "Mechanical Principles of Engineering and Architecture," art. 389.

the weight applied; this weight, in its descent, being made to act horizontally upon the beam, by means of a cord passing over a pulley. The results obtained in this way confirm the truth of the former ones; and by being freed from small errors, are much more consistent among themselves than they would otherwise have been.

104. A bar of the ⊥ form of section, bent so as to compress the vertical rib, with weights varying from 112 to 1344 lbs., gave, from a mean of two experiments very carefully made, the set, as the 1·88 power of the deflexion, measuring the compression of the rib. In these experiments, each weight was allowed to remain on the beam five minutes, and the set was taken twice, at intervals of one and five minutes after unloading; it having been found that a greater length of time produced but little change in the quantity of the set.

105. Experiments made to extend the vertical rib, the bar, during flexure, being turned the opposite side upwards, gave the set as a power of the deflexion, or extension, somewhat higher than as above.

106. Supposing the set to arise wholly from the extension or compression of the rib, which is very probable, it will therefore be nearly as the square of the extension or compression, as above observed. If, therefore, x represent the quantity of extension or compression, which a body has sustained, and $a\,x$ the force producing that extension or compression, on the supposition that the body was perfectly elastic; then, the real force f, necessary to produce the extension or compression x, will be smaller, than on the supposition of perfect elasticity, by a quantity $b\,x^2$; and we shall have $f = a\,x - b\,x^2$.

107. The law of defective elasticity, as here given, and its application to other materials, as stone, timber, &c., was discovered by the author in July, 1843, and laid before the British Association of Science, at its meeting in Cork.

OF THE SECTION OF GREATEST STRENGTH IN CAST IRON BEAMS.

108. The very extensive and increasing use made of cast iron beams renders it excedingly desirable that they should be cast in the form best suited for insuring strength; and that, if possible, formulæ should be obtained by which the strength can be estimated. Without these the engineer and founder must be in constant uncertainty; and either endanger the stability of erections, costing many thousands of pounds, and perhaps supporting hundreds of human beings, or incur the risk of employing an unnecessary quantity of metal, which, besides its expense, does injury by its own weight.

109. The earliest use of this most valuable material for beams has been but of recent date: so far as I can learn it was first used by Boulton and Watt, who in 1800 employed beams, whose section was of the form ⊥, in building the cotton mill for Messrs. Philips and Lee in Salford. These, the earliest cast iron beams, differed from the ⊥ formed beams of the present day, in having the lower portion of the vertical part thicker than the modern ones. Both have had the same object in their construction, that of supporting arches of brick-work for the floors of fire-proof buildings; and as they were well suited for that purpose, and of a convenient form for casting, besides being very strong, particularly the modern ones, comparatively with rectangular beams of that metal, their use has been very general, and they are still employed; though they have now been supplanted in most of the large erections of Manchester and its neighbourhood, and many other parts of the kingdom, by another form derived from experiments of which I gave an account in the fifth volume of the 'Memoirs of the Literary and Philosophical Society of Manchester' (second series) published in 1831. I propose giving here extracts from the leading results and reasonings in that Paper.

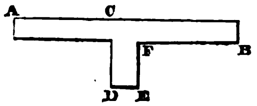
110. In the application of a material like cast iron to purposes to which it had not been before applied, it could not

be expected that the form best suited for resistance to strain, any more than the quantity necessary to support that strain, could be at once attained. The ⊥ form of cast iron beam mentioned above, was, however, by no means a bad one; it had undergone modifications and improvements by different parties, and had had various experiments made upon it, some of which, made by my friend Mr. Fairbairn, on a large scale, I gave in the Paper mentioned above. This form, however, Mr. Tredgold saw, was not the best, and gave, in his article on the 'Strongest Form of Section' (Part I. Section IV.), a representation of what he considered the best (Plate I. fig. 9), a beam with two equal ribs or flanges, one at the top and the other at the bottom. Mr. Tredgold proposed this form, assuming that, whilst the elasticity of a body is perfect, it resists the same degree of extension or compression with equal forces; and therefore he concluded that a beam, to bear the most, should have equal ribs at top and bottom, as it ought not to be strained so as to injure its elasticity.*

111. Having myself given a Paper on the 'Transverse Strength of Materials,' in the fourth volume of the 'Memoirs of the Literary and Philosophical Society of Manchester,' published in 1824, containing the mathematical development of some principles to which I attached importance, besides some experiments to ascertain the position of the neutral line in bent pieces of timber, I felt persuaded that the form proposed by Mr. Tredgold was not the best to resist fracture in cast iron. It was evident that that metal resisted fracture by compression with much greater force than it did by tension, though the ratio was then unknown: and I was convinced that the transverse strength of a bar depended in some manner upon both of these forces; the situation of the neutral line being changed before fracture in consequence of their inequality.

* Mr. Tredgold did not suspect that the elasticity was injured by forces however small, see art. 86, &c.

112. To obtain further information on this subject, I adopted, about the year 1828, a mode of analysing, separately the forces of extension and compression, in a bent body; results of which have been given in arts. 86 and 94, from experiments, since made for other purposes. I had bars cast from a model, 5 feet long, whose section was of the form,



It was in all parts $\frac{1}{4}$ of an inch thick, and uniform in breadth and depth, the thickness being as small as the castings could be run to make them uniform and sound; the breadth A B of the flange was 4 inches, and the depth F E of the rib running along its middle was 1·1 inch.

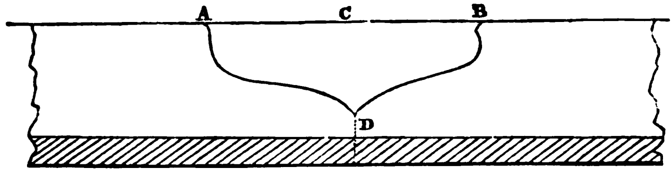
113. When the castings so formed had their ends placed horizontally upon supports, and weights were suspended from the middle, the flexure would depend almost entirely upon the contraction or extension of the rib F E. When the rib was upwards, the deflexion would arise from the contraction of that rib, and, when downwards, from its extension.

114. To ascertain the resistance to fracture in these two cases, I took two castings, apparently precisely alike, and placing the ends of each of them upon two props 4 feet 3 inches asunder, broke them by weights in the middle, one with the rib upwards, and the other with it downwards, as in the figure.

That with the rib downwards bore $2\frac{1}{4}$ cwt., and broke with $2\frac{1}{2}$ cwt. The other casting bore $8\frac{3}{4}$ cwt., and broke with 9 cwt. Deflexion of the latter in middle with 4 cwt. = ·6 inch, with $8\frac{1}{4}$ cwt. = 1·8 inch.

115. The strength of the castings was, therefore, nearly as $2\frac{1}{2}$ to 9, or as 10 to 36, accordingly as they were broken one or the other way upwards.

116. When the second broke, a piece flew out, whole, from the compressed side of the casting, of the following form, A, D, B, where A B = 4 inches, and C D = ·98 inch; the point D at the bottom being in or near to the neutral line of the bar, a side view of which is represented in the figure.



The side A B of the wedge-like piece broken out, was, as will be seen, in the direction of the length of the casting, and the weights were laid on at C. Hence, as the depth of the casting was found to be 1·35 inch, C D $= \frac{\cdot 98}{1\cdot 35} = \frac{10}{14}$ of the depth, nearly. In the experiment on the second bar (art. 86), made since that time, C D $= \frac{1\cdot 05}{1\cdot 56} = \frac{10}{15}$ nearly.

117. These experiments are interesting; they show the effect of the position of the casting on the strength; give the situation of the neutral line; and may, from the peculiar form of the wedge, which, as represented here, is more perfect than usual, throw some additional light on the nature of the strain.

118. Those who with Tredgold (Part I. art. 37, &c.) suppose the strength to be bounded by the elasticity, and that the same force would destroy the elastic power, whether it was applied to extend or compress the body, must have conceived these castings, and indeed those of every other form, to be equally strong, whichsoever way upwards they were turned;—a conclusion which we see would lead to very erroneous results, if applied to measure the ultimate strength of cast iron.

Other experiments were made at that time upon bars of the same form as the preceding, to ascertain the deflexions with given forces when the rib C D was subjected alternately to tension and compression; and it was shown as might be expected, that the extensions and compressions, measured by the deflexions, were nearly equal from the same forces, though the extension was usually somewhat greater than the compression, the difference increasing with the weight, through the whole range to fracture. This always took place by the rib being torn asunder, the compression necessary to produce fracture being several times as great as the extension required to do it, as may be inferred from art. 33.

119. The object of the preceding experiments being to prepare, in some degree, the way to an inquiry into the best forms of beams of cast iron; we will now reconsider the strain to which they are subjected, with a view to their adaptation to bear a given load with the least quantity of metal.

120. Suppose a beam supported at its ends, and bent by a weight laid at any intermediate point upon it: since all materials are both extensible and compressible, it is evident that the whole of the lower fibres are in some degree of extension, less or more, and the whole of the upper fibres are in a compressed state; there being some point, intermediate between both, where extension ends and compression begins. If then we suppose all the forces of extension and compression, in the section of the beam where the deflecting force is applied, to be separately collected into two points, one over the other, the beam will offer the greatest resistance, the quantity of metal being the same, when these points are as far asunder as possible, since the leverage is then the greatest.

121. When the depth of the beam is limited, this object would, perhaps, be best attained by putting two strong ribs, one at the top and the other at the bottom, the intermediate part between the ribs being a thin sheet of metal, to keep the ribs always at the same distance, as well as to serve another purpose which will be mentioned further on.

122. As to the comparative strength of the ribs, in beams of different materials, that depends on the nature of the body, and can only be derived from experiment. Thus, suppose the same force were required to injure the elasticity to a certain extent, or to cause rupture, whether it acted by extension or compression, then the strengths of the ribs should be equal; and this would be the case whatever the thickness of the part between the ribs might be, providing it was constant. But, supposing the thickness of the part

between the ribs was so small that its resistance might be neglected, and the metal to be of such a nature that a force F was needed to injure the elasticity to a certain degree by stretching it, and another force G to do the same by compressing it, it is evident that the size of the ribs should be as G to F, or inversely as their resisting power, that they may be equally affected by the strain. Or if, the resistance of the part between the ribs being neglected, it took equal weights F′ and G′ to break the material by tension and compression, the beam should have ribs as G′ to F′ to bear the most without fracture.

123. This last matter must be considered with some modifications: it would not, perhaps, be proper to make the size of the ribs just in the ratio of the ultimate tensile to the crushing forces, as the top rib would be so slender that it would be in danger of being broken by accidents; and the part between the ribs, though thin, has some influence on the strength.

124. The thickness, too, of the middle part between the ribs is not a matter of choice: independent of the difficulty of casting, and the care necessary to prevent irregular cooling, and contraction, in beams whose parts differ much in thickness, the middle part cannot be rendered thin at pleasure, but must have a certain thickness, though in long beams the breaking weight is small, and a very small strength in the middle part is all that is necessary.

125. The neutral line being the boundary between two opposing forces, those of tension and compression, it seems probable that bending the beam would produce a tendency to separation at that place. Moreover, the tensile and compressive forces are, strictly speaking, not parallel; they are deflected from their parallelism by the action of the weight, which not only bends the beam, but tends to cut it across in the direction of the section of fracture; and this last tendency is resisted by all the particles in the section. This compounded force will then tend to separate the compressed part

of the beam, in the form of a wedge, and this tendency must be resisted by the strength of the part between the ribs or flanges. We have had several instances of fracture this way (arts. 86, 94), and there will occur several others in the course of the following experiments, as in art. 135, Experiment 12, &c.

126. We see then that there are three probable ways in which a beam may be broken: 1st, by tension, or tearing asunder the extended part; 2nd, by the separation of a wedge, as above; and 3rd, by compression, or the crushing of the compressed part. I have not, however, obtained a fracture, by this last mode, in cast iron broken transversely.

EXPERIMENTS TO ASCERTAIN THE BEST FORM OF CAST IRON BEAMS, AND THE STRENGTH OF SUCH BEAMS.

127. In the commencement of these experiments the form I first adopted was one in which the arc, bounding the top of the beam, was a semi-ellipse, with the bottom rib a straight line; but the sizes of the ribs at top and bottom were in various proportions. The ribs in the model were first made equal, as in the beam of strongest form according to the opinion of Mr. Tredgold (Section IV., art. 37); and when a casting had been taken from it, a small portion was taken from the top rib, and attached to the edge of the bottom one, so as to make the ribs as one to two; and when another casting had been obtained, a portion more was taken from the top, and attached to the bottom, as before, and a casting got from it, the ribs being then as one to four. In these alterations the only change was in the ratio of the ribs, the depth and every other dimension in the model remaining the same.

128. Finding that all these beams had been broken by the bottom rib being torn asunder, and that the strength by each change was increased, I had the bottom rib successively enlarged, the size of the top rib remaining the same. The bottom rib still giving way first, I had the top rib increased,

feeling that it might be too small for the thickness of the middle part between the ribs. The bottom rib was again increased, so that the ratio of the strengths of the bottom and top ribs was greater than before; still the beam broke by the bottom rib failing first, as before. As the strength continued to be increased more than the area of the section, though the depth of the beam and the distance between the supports remained the same, I pursued, in the future experiments, the same course, increasing by small degrees the size of the ribs, particularly that of the bottom one, till such time as that rib became so large that its strength was as great as that of the top one; or a little greater, since the fracture took place by a wedge separating from the top part of the beam. I here discontinued the experiments of this class, conceiving that the beams last arrived at, were in form of section nearly the strongest for cast iron.

129. In most of the experiments the beams were intended to have been broken by a weight at their middle; and, therefore, the form of the arcs, bounding the top of the beams, was, in this inquiry, of little importance: in making them elliptical, they were too strong near to the ends for a load uniformly laid over them; the proper form is something between the ellipse and the parabola. It is shown, by most of the writers on the strength of materials, that if the beam be of equal thickness throughout its depth, the curve should be an ellipse to enable it to support, with equal strength in every part, a uniform load; and if there be nothing but the rims, or the intermediate parts be taken away, the curve of equilibrium, for a weight uniformly laid over it, is a parabola. When, therefore, the middle part is not wholly taken away, the curve is between the ellipse and the parabola, and approaches more nearly to the latter, as the middle part is thinner.

130. The instrument used in the experiments was a lever (Plate II. fig. 40) about 15 feet long, placed horizontally, one end of which turned on a pivot in a wall, and the weights

were hung near to the other; the beams being placed between them and the wall, at 2 or 3 feet distance from it.

131. All the beams in the first Table (Table I. following) were exactly 5$\frac{1}{8}$ inches deep in the middle, and 5 feet long, and were supported on props just 4 feet 6 inches asunder. The lever was placed at the middle of the beam, and rested on a saddle, which was supported equally by the top of the beam and the bottom rib, and terminated in an arris at its top, where the lever was applied. The deflexions were taken in inches and decimal parts, at or near the middle of the beam, as mentioned afterwards. The weights given are the whole pressure, both from the lever and the weights laid on, when reduced to the point of application on the beam. The dimensions of section in each experiment were obtained from a careful admeasurement of the beam itself, at the place of fracture, which was always very near (usually within half an inch of) the middle of the beam; the depth of the section being supposed to be that of the middle of the beam, or 5$\frac{1}{8}$ inches.

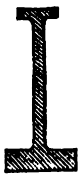
132. As the experiments were made at different times, and there might be some variation in the iron, though it was intended always to be the same, a beam of the same length and depth as the others, but of the usual ⊥ form, always from the same model, was cast with each set of castings for the sake of comparison. The results of the experiments upon these beams are given in the third Table.

133. The first six beams, in the first Table, were cast *horizontal*, that is, each beam lay flat on its side in the sand; all the rest were cast *erect*, that is, each beam lay in the sand in the same posture as when it was afterwards loaded, except that the casting was turned upside down, when in the sand.

134. In all the experiments the area of the section was obtained with the greatest care; it includes, besides the parts of which the dimensions are given, the area of the small angular portions at the junction of the top and bottom ribs with the vertical part between them.

TABLE

135. *Tabulated Results of experiments to ascertain the best form of cast iron beams, all the depth in the middle, where the load was applied, =5½ inches. All dimensions in the*

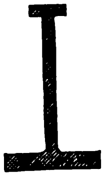
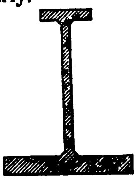
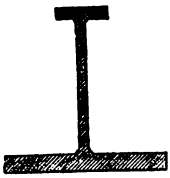
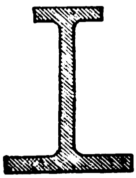
Form of section of beam in middle.	Area of top rib in middle of beam.	Area of bottom rib in middle of beam.	Thickness of vertical part between the ribs.	Area of section of beam at place of fracture.	Weight of beam.	Deflexions in parts of an inch.	Corresponding weights in lbs.
1st Experiment. Beam (Plate III. fig. 41) with equal ribs at top and bottom. 	1·75 × ·42 =·735	1·77 × ·39 =·690	·29	2·82	36¼ lbs.		
2nd Experiment. Beam with area of section of top and bottom rib as 1 to 2. 	1·74 × ·26 =·45	1·73 × ·55 =·98	·30	2·87	39 lbs.		
3rd Experiment. Beam with area of section of top and bottom rib as 1 to 4. 	1 07 × ·30 = 32	2·1 × ·57 =1·2	·32	3·02	40 lbs.		
4th Experiment. Beam from the same model as the last, but cast the opposite way up.	1·05 × ·32 =0·34	2 15 × ·56 =1·20	·33	3·08	39¼ lbs.		

I.

beams being made 5 feet long and laid on supports 4 feet 6 inches asunder, and having the Table are in inches, and the weights in pounds, except otherwise mentioned.

Breaking weight in lbs.	Strength per square inch of section in lbs.	Strength per sq. in. of section of beam of usual form (Tab. III.) cast with these for comparison.	Gain in strength by comparison with beam of usual form.	Form of fracture.	Remarks.
6678 lbs. = 59 cwt. 70 lbs.	$\frac{6678}{2 \cdot 82}$ = 2368	2584	$-\frac{1}{12}$	This is represented by the line *b n r t*, (Plate III. fig. 41,) where *t r* = ·6, and *b n* = 2·5, the figure being a side view of the beam. The distances *t r*, *b n* are measured vertically.	
7368 lbs. = 65 cwt. 88 lbs.	$\frac{7368}{2 \cdot 87}$ = 2567	2584	$-\frac{1}{152}$	Nearly same as in Exp. 1; here *t r* (Plate III. fig. 41,) = ·55 inch. Here, and in all other cases, *t r* is measured vertically, as before.	
8270 lbs. = 73 cwt. 94 lbs.	$\frac{8270}{3 \cdot 02}$ = 2787	2584	$\frac{1}{17}$ nearly.	Nearly as in Experiment 1; and *t r* = ·6 (fig. 41).	
8263 lbs. = 73 cwt. 89 lbs.	$\frac{8263}{3 \cdot 08}$ = 2683	2792	$-\frac{1}{26}$ nearly.	Nearly as figure to Exp. 1, but here *b n* = 2·5, and *t r* ·55 (fig. 41).	

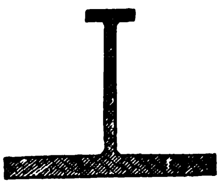
TABLE I.—

Form of section of beam in middle.	Area of top rib in middle of beam.	Area of bottom rib in middle of beam.	Thickness of vertical part between the ribs.	Area of section of beam at place of fracture.	Weight of beam.	Deflexions in parts of an inch.	Corresponding weights in lbs.
5th Experiment. Beam with area of section of ribs as 1 to 4½ nearly. 	1·05 × ·34 = 0·357	3·08 × ·51 = 1·570	·305	3·37	44¾ lbs.		
6th Experiment. Ratio of ribs 1 to 4 nearly. 	1·6 × ·315 = 0·5	4·16 × ·53 = 2·2	·38	4·50	57 lbs.	·4 ·45 ·52	11186 12698 13706
7th Experiment. Beam differing from last, having a broader bottom flange. Ratio of ribs 1 to 5½ nearly. 	1·56 × ·315 = 0·49	5·17 × ·56 = 2·89	·34	5	67½ lbs.	·24 ·36 ·40 ·42 ·45 ·48 ·49 ·53	8288 12698 13706 14210 15218 15722 16226 16730
8th Experiment. 	2·3 × ·315 = ·72	4·06 × ·57 = 2·314	·33	4·628			

(*Continued.*)

Breaking weight in lbs.	Strength per square inch of section in lbs.	Strength per sq. in. of section of beam of usual form (Tab. III.) cast with these for comparison.	Gain in strength by comparison with beam of usual form.	Form of fracture.	Remarks.
10727 lbs. = 95 cwt. 87 lbs.	$\frac{10727}{3 \cdot 37} = 3183$	2792	$\frac{1}{7}$ nearly	Here *t r* (fig. 41) = ·6 inch.	Broke by tension, small flaw in bottom rib, at place of fracture.
14462 lbs. = 129 cwt. 14 lbs.	$\frac{14462}{4 \cdot 5} = 3214$	2693	$\frac{1}{5}$ nearly	Here *b n* (fig. 41) = 2·5 inches.	Broke by tension 1 inch from the middle.
16730 lbs. = 149 cwt. 42 lbs.	$\frac{16730}{5} = 3346$	2693	$\frac{1}{4}$ nearly		After having borne the last-named weight some minutes, it broke by tension very near the middle.
15024 lbs. = 134 cwt. 16 lbs.	$\frac{15024}{4 \cdot 628} = 3246$				Broke by tension very nearly in the middle. This beam and those in all the experiments, except the last, were of the form (Pl. III., figs. 42 and 43), being uniform in height, and having a large bottom rib tapering towards the ends.

TABLE I.—

Form of section of beam in middle.	Area of top rib in middle of beam.	Area of bottom rib in middle of beam.	Thickness of vertical part between the ribs.	Area of section of beam at place of fracture.	Weight of beam.	Deflexions in parts of an inch.	Corresponding weights in lbs.
9th Experiment. From the same model as that used in Experiment 8, except that the bottom rib is increased in breadth.	2·35 × ·29 = ·68	5·43 × ·537 = 2·916	·35	5·292		·12 ·15 ·18 ·20 ·22 ·25 ·26 ·29 ·31 ·33 ·53	6218 7598 8288 9309 10330 11338 12346 13354 14371 15393 16401
10th Experiment. Beam from the same model, but with further increase of bottom rib.		6·8 × ·502 = 3·413			64¼ lbs.	·16 ·18 ·19 ·21 ·22 ·24 ·26 ·28	6218 7598 8288 9309 10331 11339 12341 13351
11th Experiment. Beam from same model as in last experiment.	2·3 × ·28 = ·64	6·61 × ·54 = 3·57	·34	5·86	68½ lbs.	·26 ·29 ·30 ·33 ·35 ·36 ·43	12087 12777 repeated 14345 15913 16697 18265
12th Experiment. 	2·33 × ·31 = ·72	6·67 × ·66 = 4·4	·266	6·4	71 lbs.		

(*Continued.*)

Breaking weight in lbs.	Strength per square inch of section in lbs.	Strength per sq. in. of section of beam of usual form (Tab. III.) cast with these for comparison.	Gain in strength by comparison with beam of usual form.	Form of fracture.	Remarks.
16905 lbs. = 150 cwt. 105 lbs.	$\frac{16905}{5\cdot292}$ =3194				Broke by tension.
14836 lbs. nearly = 128 cwt.					This broke by tension, and ought to have borne considerably more than the last beam; but its iron must have been of a less tenacious kind than the others; as is evident by comparing their deflexions, this beam having bent little more than half what the preceding one did before it broke.
19441 lbs. =173½ cwt.	$\frac{19441}{5\cdot86}$ =3317				This beam broke very nearly in the middle, by tension, as before.
26084 lbs. = 11 tons 13 cwt.	$\frac{26084}{6\cdot4}$ =4075	2885	upwards of $\frac{2}{5}$	A wedge separated from its upper side, as shown in the fig. below, which is a side view of the beam, where *a d c* is the wedge, *a c* = 5·1 inches, *b d* = 3·9 inches, angle *a d c* at vertex = 82°.	

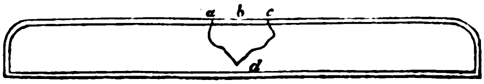
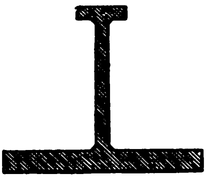


TABLE I.–

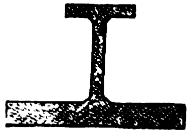
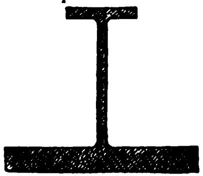
Form of section of beam in middle.	Area of top rib in middle of beam.	Area of bottom rib in middle of beam.	Thickness of vertical part between the ribs.	Area of section of beam at place of fracture.	Weight of beam.	Deflexions in parts of an inch.	Corresponding weights in lbs.
13th Experiment. From the same model as in the last Experiment.	2·3 × ·28 = ·64	6·63 × ·65 = 4·31	·335	6·5	74¾ lbs.	·22 ·24 ·25 ·26 ·30 ·34 ·36 ·38 ·43 ·47 ·48 ·50	9328 11397 12777 14345 15913 17481 18265 19049 20617 22185 22969 "
14th Experiment. Elliptical beam, differing from that in Experiment 7, in having a larger bottom rib; ratio of ribs 6¾ to 1. 	1·54 × ·32 = ·493	6·50 × ·51 = 3·315	·34	5·41	70¾ lbs.	·26 ·27 ·28 ·30 ·31 ·34 ·35 ·42 ·43 ·46 ·50 ·54	9327 10707 11397 12087 12777 14345 15913 16697 17481 19849 19833 20617

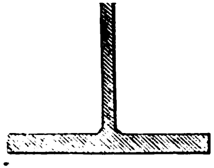
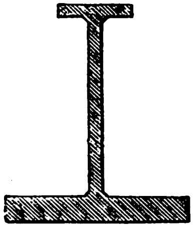
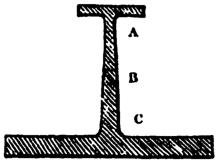
(*Continued.*)

Breaking weight in lbs.	Strength per square inch of section in lbs.	Strength per sq. in. of section of beam of usual form (Tab. III.) cast with these for comparison.	Gain in strength by comparison with beam of usual form.	Form of fracture.	Remarks.
23249 lbs. = 10 tons 8 cwt. nearly	$\frac{23249}{6 \cdot 5}$ = 3576	2885	$\frac{1}{5}$ nearly.		Broke in the middle by tension.
21009 lbs. = 9 tons 8 cwt. nearly	$\frac{21009}{5 \cdot 41}$ = 3883	2885	upwards of $\frac{1}{4}$	Nearly as in figure to first experiment, $b\,n$ = 1·8 inch.	

TABLE II.

136. *Results of experiments on beams whose forms (Plate III. figs. 42, 43) differ but little from the best of those in Table I.*

Form of section of beam in middle.	Distance between supports.	Depth of beam.	Area of top rib in middle of beam.	Area of bottom rib in middle.	Thickness of vertical part between the ribs.	Area of section at place of fracture.	Weight of beam.	Deflexions.	Corresponding weights.	Breaking weight.	Remarks.
Experiment 1. 	7 ft.	4·1 in.	2·25 × ·33 = ·74	6·00 × ·74 = 4·44	·40 in.	6·54	114 lbs.	·25	2764		This beam was cast 7 feet 6 in. long; and the "weight of beam" set down was in all cases that of the whole length.
								·26	2994		
								·27	3224		
								·29	3569		
								·32	3914		
								·40	5180		
								·44	5525		
								·45	6042		
								·55	6971		
								·62	7727		
								·70	8483		
								·75	8637		
								·80	10017		
								·95	11397		
								1·08	12815	13548 lbs.	
Experiment 2. 	7 ft.	5·2 in.	2·25 × ·35 = ·79	6·00 × ·77 = 4·62	·34 in.	6·94	128 lbs.	·35	7947		Length of beam as before.
								·43	8637		
								·51	9327		
								·53	10017		
								·56	10707		
								·58	11397	15129 lbs.	
								·63	12087	= 6 tons 15 cwt. 9 lbs.	

								·36 ·42 ·49 ·56 ·58	10017 11397 13543 14999 15129	15129 lbs. =6 tons 15 cwt. 9 lbs.	
Experiment 4. 	7 ft.	6·93 in.	2·25 × ·34 =·765	6·05 × ·75 =4·537	·38 in.	7·67	146 lbs.	·20 ·24 ·27 ·35 ·40 ·45 ·52 ·60 ·65	9327 10707 12087 14271 15913 17481 19049 20617 21401	22185 lbs. =9 tons 18 cwt.	
Experiment 5. Beam from the same model as the last.	7 ft.	6·98 in.	2·25 × ·32 =·72	5·95 × ·73 =4·343	·37 in.	7·40	141 lbs.	·25 ·36 ·44 ·47 ·50 ·58	9327 11397 14345 15913 17481 19049		
Experiment 6. 	4 ft. 6 in.	$5\frac{1}{8}$ in.	2·15 × ·27 =·58	6·74 × ·71 =4·785	At A ·25 B ·37 C ·53	7·20	81 lbs.	·19 ·23 ·25 ·29 ·32 ·40 ·45 ·50 ·52 ·55	11056 12436 13816 15196 16576 19600 21616 23128 23632 24640	25144 lbs. =11 tons $4\frac{1}{4}$ cwt.	It is doubtful whether this beam broke by tension or compression, a crack showing a wedge which *broke out afterwards*, and of which *a b c d* is the form in figure below. *a c* = 4·2 inches, *b d* = 1·7 inches, or $\frac{1}{3}$rd of the depth of the beam nearly. The strength per square inch of section of this beam was 3492 lbs., and that of the beam of usual form in Table III., cast with it, was 2706 lbs.; hence gain in strength from form of section equal one-fifth nearly.

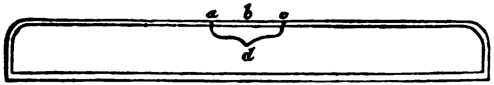
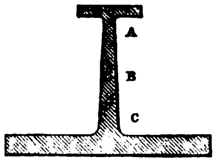


TABLE II.—(*Continued.*)

Form of section of beam in middle.	Distance between supports.	Depth of beam.	Area of top rib in middle of beam.	Area of bottom rib in middle.	Thickness of vertical parts between the ribs.	Area of section at place of fracture.	Weight of beam.	Deflexions.	Corresponding weights.	Breaking Weight.	Remarks.
Experiment 7.  	4 ft. 6 in.	5·1 in.	2·15 × ·24 = ·52	7·60 × ·72 = 5·472	At A ·27 B ·44 C ·48	7·90	88 lbs.	·17 ·21 ·25 ·27 ·28 ·32 ·35 ·37 ·42 ·52 ·55 ·58	10017 11397 12777 14345 15913 17481 18592 20608 22624 24640 26152 27664	28168 lbs. = 12 tons 11½ cwt.	This beam bore 3565 lbs. per square inch of section, and the beam of usual form 2796 lbs.; hence saving in metal from form of section = ·215.
Experiment 8. Beam of double length, and slightly varied section.	9 ft.	5⅛ in.	2·2 × ·36 = ·79	7·0 × ·69 = 4·83	A ·27 B ·33 C ·60		170½ lbs.	1·00 1·12 1·27 1·45	8296 8986 9676 10366	11056 lbs. = 4 tons 18¾ cwt.	Broke 9 inches from the middle, where there were two small defects whose area was about ⅛th of an inch in the bottom rib. The experiment was therefore imperfect.
Experiment 9. Beam slightly differing from the last.	9 ft.	5⅛ in.	2·25 × ·3 = ·67	7·7 × ·76 = 5·85	A ·36 B ·42 C ·50		192 lbs.	·90 ·96 1·05 1·30 1·52 1·84 2·04	8296 8986 9676 11056 12436 13816 14506	15196 lbs. = 6 tons 15¾ cwt.	It broke in the middle, throwing out a wedge as in Experiment 6. Here *a c*, the length of the wedge, = 6·9 inches; *b d*, its depth, = 2·25 inches.

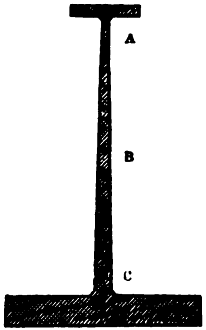
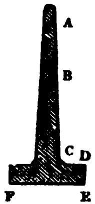
Experiment 10. 	9 ft.	10¼ in.	2·1 × ·27 = ·57	6·14 × ·77 = 4·72	At A ·26 B ·25 C ·35		227 lbs.	·23	11056		This beam broke as in Experiments 6 and 9; the dimensions of the wedge broken out were, *a c*, its length, = 13· inches *b d*, its depth, = 5·8 „
								·26	13816		
								·34	16576		
								·40	19600		
								·46	21616		
								·51	23632		
								·55	25648		
								·61	26656		
								·65	27664	28672 lbs.	
								·68	28168	= 12 tons 16 cwt.	
Experiment 11. Beam with top and bottom ribs somewhat larger than the last.	9 ft.	10¼ in.	2·2 × ·33 = ·73	7·6 × ·75 = 5·70	A ·15 B ·38 C ·35		244 lbs.	·22	12436		This broke as before, length of the wedge 18 inches, depth 6·15 inches. The form of the wedge was not so regular as before.
								·24	13816		
								·32	18592		
								·35	20608		
								·40	22624		
								·47	24640		
								·50	26656		
								·55	28672		
								·64	30184		
								·70	31192	32200 lbs.	
								·76	31696	= 14 tons 7½ cwt.	

TABLE III.

137. *Results of Experiments upon beams of the usual form (Plate III. fig. 44, and section below), one of which was cast with each casting of those in Table I., Art. 135, for comparison with them. These beams were from the model of Messrs. Fairbairn and Lillie, in* 1830; *they were cast* 5 *feet long, and* 5½ *inches deep in the middle, and were laid on supports* 4 *feet* 6 *inches asunder as in the beams of Table I.*

	Thickness at A.	Thickness at B.	Thickness at C.	Thickness D E.	Breadth F E.	Area of section.	Weight of beam in lbs.	Deflexions.	Corresponding weights.	Breaking weight.	Strength per square inch of section in middle.
Experiment 1. 	·32	·44	·47	·52	2·27	3·2	40½	·25 ·37	5758 7138	8270	$\frac{8270}{3\cdot2}=$ 2584 lbs.
Experiment 2.	·30	·37	·425	·53	2·28	2·98	38	·37 ·50 ·62	6679 9495 9279	9503	$\frac{9503}{2\cdot98}=$ 3188 lbs.
Experiment 3.	·29	·425	·46	·53	2·3	3·16	40½			8823	$\frac{8823}{3\cdot16}=$ 2792 lbs.
Experiment 4.	·29	·425	·53	·565	2·34	3·32	41	·4 ·43 ·47	7598 8494 8942	8942	$\frac{8942}{3\cdot32}=$ 2693 lbs.
Experiment 5.						3·08 nearly.	39½	·28 ·33	6218 7138	7598	$\frac{7598}{3\cdot08}=$ 2466 lbs.

Experiment 6.	·30	·42	·45	·51	2·28	3·17	40			9146 nearly.	$\frac{9146}{3·17}$ = 2885 lbs.
Experiment 7.	·27	·40	·44	·46	2·27	2·921	36½	·21	4148		$\frac{8792}{2·921}$ = 3009 lbs.
								·23	4493		
								·24	4838		
								·26	5183		
								·27	5528		
								·29	5873		
								·31	6218		
								·33	6563		The beam in Experiment 1, cast on its side, like this, gave 2584 lbs. per square inch; hence the mean from the two is 2796 lbs.
								·35	6908		
								·37	7253		
								·38	7598		
								·40	7943		
								·42	8288		
								·44	8540	8792 = 3 tons 18½ cwt.	
Experiment 8.	·27	·355	·43	2·26	·47	2·837	37	·09	2078		$\frac{9044}{2·837}$ = 3188 lbs.
								·18	4148		
								·20	4493		
								·23	4838		
								·25	5183		
								·27	5528		
								·29	5873		
								·32	6218		
								·35	6563		
								·38	6908		
								·39	7253		
								·42	7943		
								·44	8288		
								·45	8540		
								·46	8792	9044 = 4 tons ¾ cwt.	Mean from the whole, 2851 lbs.

Some of these beams, particularly the first two, twisted in a serpentine manner before fracture.

REMARKS UPON THE EXPERIMENTS, IN TABLE I., TO OBTAIN THE BEST FORM OF CAST IRON BEAMS OF WHICH THE LENGTH AND DEPTH WERE INVARIABLE.

138. It has been mentioned before, that these experiments were begun with that form of section of beam which Tredgold (Part I., art. 40) was induced to consider as the strongest, the top and bottom flanges in it being equal. This form was, however, found to be $\frac{1}{12}$th weaker to resist fracture than that in common use (the ⊥ form); though it would, perhaps, be nearly the strongest in wrought iron. The top flange, in the model before used, was next reduced, and the part taken off it was added to the bottom one. This alteration gave an increase of strength, and the beam, in Experiment 3, was somewhat stronger than that of the usual form cast with it for comparison. It did not now seem advisable to decrease further the top flange; and as every beam had been found to break by tension, or through the weakness of the bottom part, I thought it best to keep increasing the bottom flange, by small degrees, till such time as the beam broke by the rupture of some other part. Proceeding thus, the beam, in Experiment 5, had its bottom and top ribs, or flanges, as $4\frac{1}{2}$ to 1; and the result from that form of section was a gain in strength of about $\frac{1}{7}$th. Before increasing the bottom rib any further, I added a little to the top one, as the vertical part of the beam, or that between the ribs, would be perhaps strong enough for much larger ribs. In Experiments 6, 7, and 14, the top rib and vertical part of the model were the same, the only difference being in the increasing breadth of the bottom rib. From these, the gain in strength, above what was borne by beams of the usual form, was respectively $\frac{1}{5}$th, $\frac{1}{4}$th, and between $\frac{1}{4}$th and $\frac{1}{3}$rd.

In Experiments 8, 9, 10, 11, 12, 13, the top rib of the model was the same; but it was somewhat larger than in

Experiments 6, 7, and 14, and the bottom rib was the only part intended to be varied. In the 12th Experiment of the Table (being the 19th made) the section of the bottom rib at the place of fracture, the middle, was more than double the rest of the section there; and the ratio of the top and bottom ribs was as 1 to 6. In all the beams before this, fracture had taken place through the weakness of the bottom rib. In this it took place by the separation of a wedge from the top rib and the vertical part of the beam, which part happened to be thinner than usual: the gain in strength, arising from the form of the section in this beam, was upwards of $\frac{2}{5}$ths of what the beam of usual form bore; and the saving from the general form of the beam was nearly $\frac{3}{10}$ths of the metal. This experiment was repeated in Experiment 13, but the beam, though cast from the same model, had its vertical part thicker than in the former; and its strength, per square inch of section, was less. Fracture took place in it by the rupture of the bottom rib, as in all the preceding experiments except the last.

The form of section in Experiment 12 is somewhat better than that in Experiment 14; it is the best which was arrived at for the beam to bear an ultimate strain; and it is, doubtless, nearly the strongest which can be attained: for the vertical part between the flanges is as thin as it can, probably, be cast; and the remainder of the metal is disposed in the flanges, and consequently as far asunder as possible, the section of the flanges being in the ratio of 6 to 1, or nearly in that of the mean crushing and tensile strength of cast iron (Part II., art. 33).

The strengths, per square inch, borne by the beams in Experiments 12, 13, and 14, were 4075, 3576, 3883 lbs. respectively, the mean being 3845 lbs. And the strength of the beam of the common form (Table III.), cast with them for comparison, was 2885 lbs. per square inch. The difference, or strength gained from a mean among the results, was therefore 960 lbs., or upwards of $\frac{1}{4}$th of that mean.

Some of the beams, it will be noticed, had their bottom rib considerably thicker than the vertical part between the ribs; the line of junction being tapered from the thick to the thinner part. This tapering was more gradual, and higher up the beam, than is represented in the forms of section. The castings obtained were very good, as might be inferred from the strength of them; but as additional care is requisite to obtain good castings, when the parts differ much in thickness, we should bear in mind that it is not absolutely necessary, but convenient, to make one part thicker than another. The same strength would have been obtained by making the bottom rib broader and thinner than that in the beams tried, leaving the quantity of metal the same.

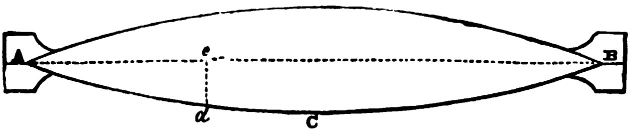
FORM OF BEAM ALTERED.

139. After Experiment 7, it was evident that the bottom flange in the future beams, if made of equal size throughout, as heretofore, would become very heavy; I had, therefore, the form of the beams, near to the ends, changed, leaving the section in the middle, as before, and making the height of the beam equal throughout, instead of being elliptical, as in the previous experiments. The bottom flanges were both made to taper towards the ends in the form of a double parabola whose vertex was in the middle of the beam; and the ratio of the sections of the flanges were, throughout the length of the beam, the same as that in the middle. (See Plate III. figs. 42 and 43, the former being a plan, and the latter an elevation of the beam.)

140. From the great quantity of matter in the flanges, particularly the bottom one, in the subsequent experiments, and the small thickness of the part between the flanges, I was convinced that nearly all the tensile force would be exerted by the bottom flange, whilst the rest of the beam would serve for little more than a fulcrum; the centre of resistance to

compression, or of that fulcrum, being very near to the top; it being perhaps at the point *r*, in the experiments in the Table (Table I.).

Suppose then D to be the vertical distance from the centre of compression, at any part of the beam, to the centre of tension in the bottom flange; and if T be the direct tensile strength of the bottom flange at that part, T multiplied by some function of D, (or T D nearly,) will represent the strength of the beam there. But D throughout the same beam will be a constant quantity, or nearly so; the strength of the beam at any part, will, therefore, be nearly in proportion to that of its bottom flange at that part; and as the strain will be less towards the ends than elsewhere, the bottom flange will be reduced there likewise. Suppose the bottom flange to be formed of two equal parabolas, the vertex of one of them, A C B, being at C, in the figure annexed; then, by



the nature of the curve, any ordinate de is as $Ae \times Be$; the strength of the bottom flange, therefore, and consequently that of the beam at that place, will be as this rectangle. It is shown too, in Part I., and by writers on the strength of materials generally, that the rectangle $Ae \times Be$ is the proportion of strength which a beam ought to have, at any distance Ae from A, to bear equally the same weight in every part, or a weight laid uniformly over it. The conclusions above were verified by several experiments, in which beams were broken by weights applied at half the distance between the middle and the ends.

141. From the experiments in Table I., in which the length and depth of the beam were always the same, it would appear, that when the size of the top flange and the thickness of the

vertical part remain unaltered, the latter being small, the strength of the beam is nearly in proportion to the size of the bottom flange; double the size of that flange giving nearly, but not quite, double strength.

REMARKS ON TABLE II.

142. The first four beams in this Table were all cast 7 feet 6 inches long, and they were supported by props 7 feet asunder. They were all from the same model, which varied only in the breadth of its vertical part between the flanges, the depth of the beam being all that was intended to vary. The depths were nearly 4, 5, 6, and 7 inches, but accurate admeasurements are given with the sections. The vertical part in these beams was rendered too strong comparatively with the size of the bottom flange, in order that they might all break by tension, or in the same manner, to furnish the means of judging correctly of their relative strength.

143. From these experiments it appears that the ultimate strength, in sections like the preceding, is, *cæteris paribus*, nearly as the depth, but somewhat lower than in that ratio.

SIMPLE RULE FOR THE STRENGTH OF BEAMS APPROACHING TO THE BEST FORM IN THE PRECEDING EXPERIMENTS. (TABLES I. AND II.)

144. It appears, from art. 141, that when the length, depth, and top flange, in different cast iron beams, with very large flanges, are the same, and the thickness of the vertical part between the flanges is small and invariable, the strength is nearly in proportion to the size of the bottom flange. It appears, too, from the last article, that in beams which vary only in depth, every other dimension being the same, the strength is nearly as the depth.

145. Hence in different beams whose length is the same, the strength must be nearly as their depth multiplied by the area of a middle section of their bottom flange; and when the length is different, the strength will be as this product divided by the length.

$$\therefore W = \frac{c\,a\,d}{l},$$

where W = the breaking weight in the middle of the beam, a = the area of a section of the bottom flange in the middle, d = the depth of the beam there, l = the distance between the supports, and c = a quantity nearly constant in the best forms of beams, and which will be supplied from the results of the experiments in Tables I. and II.

146. We will seek, by means of this approximate formula, for the value of c considered as constant, obtaining it from each of the experiments; and, for that purpose, confining ourselves to those forms in which the section of the bottom flange in its middle is more than half the whole section of the beam, take the mean from among them all for c.

$$\text{Since } W = \frac{c\,a\,d}{l}, \therefore c = \frac{l\,W}{a\,d}$$

Taking the dimensions in inches, and the breaking weight in cwts., and separating the results of the beams which were cast *erect* from those cast *on their side*, we shall have

In beams cast erect.

Experiment		Value of c.
7	Table I.	545
9	,,	545
11	,,	512
12	,,	558
13	,,	507
14	,,	596
1	Table II.	558
2	,,	472
4	,,	529
		Mean 536 cwt.

In beams cast on their side.

Experiment		Value of c.
6	Table II.	494
7	,,	484
9	,,	489
10	,,	571
11	,,	531
		Mean 514 cwt.

Since 536 or 514 is the mean value of c to give the breaking weight in cwts., according as the beam has been cast erect or on its sides, one-twentieth of these numbers, or 26·8 and 25·7, will be the value of c to give the breaking weight in tons. Neglecting the decimals, and taking 26 for the mean value of c, we have

$$W=\frac{c\,a\,d}{l}=\frac{26\,a\,d}{l}$$

for the strength in tons where the dimensions d and l are in inches.

But if l, the distance between the supports, be taken in feet, the value of c will be $\frac{26}{12}$ = 2·166, and the strength in tons will be

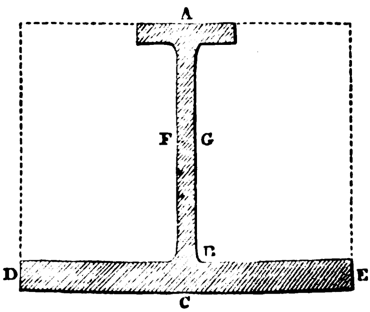
$$W=\frac{2{\cdot}166\,a\,d}{l}.$$

This rule is formed on the supposition, that the strength of the flanges is so great that the resistance of the middle part between them is small in comparison, and may be neglected.

Another approximate rule, for the strength of the beams in Tables I. and II., and which includes the effect of the vertical part between the flanges, may be deduced as below.

147. Since cast iron resists rupture by compression with about $6\frac{1}{2}$ times the power that it does by extension, (art. 33,) we may consider it as comparatively incompressible, and suppose that the operation of the top flange of the beam, when bent, is only to form a fulcrum upon which to break the bottom flange and the part between the flanges.

Let then A D E represent the section of the beam in the middle, D E being its bottom flange and A the top one, round which it turns.



Let W = breaking weight.

l = distance between supports.

d = A C = the whole depth.

d' = A B = the depth to the bottom flange.

b = D E = the breadth of the bottom flange.

b' = F G = the thickness of the vertical part.

f = the tensile strength of the metal per unit of section.

n = a constant quantity.

To seek for the strength of the beam we may estimate, first, the resistance of a rectangular solid whose depth is A C and breadth D E, as shown by the dotted lines, and subtract from that the resistance which would be offered by that part which the beam wanted to make it such a uniform solid as the above.

$\therefore \frac{f b d^2}{n}$ = moment of resistance of the particles in the rectangular solid A D E,

and $\frac{f'(b-b')d'^2}{n}$ = moment of resistance of the part necessary to make the beam a solid rectangle, where f' is the strain of the particles at the distance AB.

When the beam is supposed to be incompressible, as in the present case, n is equal to 3, and when it is equally extensible and compressible, n is equal to 6 (Tredgold, art. 110).

$$\text{But } f' : f :: d' : d, \therefore f' = \frac{f d'}{d}.$$

Substituting this value for f' in the latter moment of resistance, and subtracting the result from the former moment, gives the moment of resistance of the solid equal to

$$\frac{f b d^2}{n} - \frac{f d'}{d} \times \frac{(b-b')d'^2}{n}.$$

But, from the property of the lever, this moment is equal to $\frac{1}{2} l \times \frac{W}{2} = \frac{l W}{4}$, or to half the weight acting with a leverage of half the length.

Whence $\frac{l\text{W}}{4} = \frac{f\,b\,d^2}{n} - \frac{f\,d'}{d.} \times \frac{(b-b')\,d'^2}{n}$

$$= \frac{f}{n\,d}\left\{b\,d^3 - (b - b')\,d'^3\right\}$$

$$\therefore \text{W} = \frac{4\,f}{n\,l\,d}\left\{b\,d^3 - (b - b')\,d'^3\right\}.$$

In the same iron, f and n are constants; putting $c = \frac{4f}{n}$, we shall have

$$\text{W} = \frac{c}{d\,l}\left\{b\,d^3 - (b - b')\,d'^3\right\}.$$

This formula gives

$$c = \frac{d\,l\,\text{W}}{b\,d^3 - (b - b')\,d'^3}.$$

The numerical value of c, calculated by this formula, from each of the experiments in Tables I. and II., taking the breaking weight in lbs., the length in feet, and the other dimensions in inches, is as below.

	Value of c.
Table I.	1897·98
	1735·94
	1666·31
	1638·92
	1744·10
	1725·90
	1631·74
	1785·96
	1570·80
	1835·14
	1671·01
	1797·87

	Value of c.
Table II.	1932·78
	1566·36
	1593·82
	1607·46
	1604·63
	1623·98
	1627·43
	1547·49

Mean value of c from the whole twenty beams, 1690·28.

This value of c is in lbs., and dividing it by 2240, gives ·7544 for its value in tons.

The iron in my experiments on beams was of a strong kind, made with a cold blast; and many of the beams were cast erect in the sand, which gives them a little additional strength. We may, therefore, expect that the value of c, just obtained, will be somewhat too great for the generality of hot-blast castings; and for large beams, the iron of which is usually softer than that of small ones. We will, therefore, collect here its values, to obtain the strength in tons, computed from the results of other experiments on a large scale given further on. Taking them in the order in which they are inserted, we have as follows:

From Messrs. Marshall's beams (art. 153), cast from the cupola, we obtain	$c = \cdot 625$	Mean ·671
" "	$c = \cdot 710$	
From Mr. Gooch's beam (art. 154), we obtain	$c = \cdot 679$	
Messrs. Marshall's beam (art. 153), cast from the air furnace, gives	$c = \cdot 795$	

Mr. Cubitt's beams (art. 165), taking the results from the sound ones only, and the value of b' from a mean where that dimension varies, give as below.

From Experiment 1	$c = \cdot 6523$	Mean ·7086
" " 2	$c = \cdot 6467$	
" " 3	$c = \cdot 7411$	
" " 4	$c = \cdot 7276$	
" " 6	$c = \cdot 7473$	
" " 9	$c = \cdot 6703$	
" " 10	$c = \cdot 7746$	

Taking a mean value of c, as obtained from the whole of the beams cast from the cupola, thirty in number, we should have it considerably more than ·7; the means from twenty experiments being ·7544; from three experiments ·671; and from seven experiments ·7086. The lowest of these means differs but little from $\frac{2}{3}$; and adopting this as a safe approximate value for c, from which to compute the strength of beams generally, we have in the preceding formula for W its value as below.

$$W = \frac{2}{3\,d\,l}\left\{ b\,d^3 - (b - b')\,d'^3 \right\},$$

where W is in tons, l in feet, and the rest are in inches.

148. The preceding formula for the strength of a beam depends on the two following suppositions: 1st, that all the particles, except those of the top part or flange of a bent beam, are in a state of tension; 2nd, that the resistance of each particle is as its distance from the top of the beam. Neither of these suppositions can be regarded otherwise than as an approximation. We know that the former, which is almost tantamount to the exploded assumption of Galileo, that materials are incompressible, is not strictly true of any

bodies whatever; and the second supposition is subject to the double inaccuracy of the leverage of the particles being estimated as from the top of the beams, and therefore rather too great; and of the force of the fibres being as their extension, whilst, in reality, it is in a less ratio than that, as shown in preceding articles (99 to 107).

If, as is expected, the formula should be allowed to give results agreeing moderately well with those of experiment at the time of fracture, it will appear evident that the 2nd assumption above is favourable to that of incompressibility, in estimating the transverse strength of cast iron.

149. To obtain further evidence on this subject, we will seek, by means of the experiments in this work, for the value of n in the formula, $w=\frac{fbd^2}{nl}$, for the strength of a rectangular bar, fixed at one end and loaded at the other, b being the breadth, d the depth, l the length of leverage, w the weight at the end, and the rest as in art. 147.

150. Selecting from the experiments, in articles 3 and 96, the mean tensile and transverse strengths of all the irons in which both these properties were obtained, we have as in the following Table.

Description of Iron.	Tensile strength per square inch. (f)	Transverse strength of bar 1 inch square and 54 inches between supports.	Transverse strength of bar 1 inch square fixed at one end and loaded at the other, the weight acting with a leverage l of 27 inches. (w)	Value of n from formula $n=\frac{fbd^2}{lw}$.
	lbs.	lbs.	lbs.	
Carron iron, No. 2, cold blast ...	16,683	476	238	$n=2·59$
Carron iron, No. 2, hot blast ...	13,505	463	231½	$n=2·16$
Carron iron, No. 3, cold blast ...	14,200	446	223	$n=2·36$
Carron iron, No. 3, hot blast ...	17,755	527	263½	$n=2·50$
Devon iron, No. 3, hot blast......	21,907	537	268½	$n=3·02$
Buffery iron, No. 1, cold blast...	17,466	463	231½	$n=2·79$
Buffery iron, No. 1, hot blast ...	13,434	436	218	$n=2·28$
Coed-Talon iron, No. 2, cold blast	18,855	413	206½	$n=3·38$
Coed-Talon iron, No. 2, hot blast	16,676	416	208	$n=2·96$
Low Moor iron, No. 3, cold blast	14,535	467	233½	$n=2·30$
				Mean value of $n=2·63$

151. The transverse strength of a rectangular body being directly as the product of the breadth, the square of the depth and the strength of its fibres, and inversely as the length, the value of w, in the formula $w = \frac{f b d^2}{n l}$, will depend upon the value of n; and this last quantity will, as we have seen, depend on the comparative resistance of the fibres to extension and compression. Thus if, according to the general assumption, the extensions and compressions of the particles are equal from equal forces, and as the forces, the neutral line will be in the middle of the body, and the value of n equal to 6 (Part I., art. 110). If, according to Galileo, the body were incompressible, and the forces of the fibres were as their extension, we should have $n = 3$; and if, on the supposition of incompressibility, the forces of the fibres were the same for all degrees of extension, we should have $n = 2$. (See my Paper on the Strength of Materials, 'Memoirs of the Literary and Philosophical Society of Manchester,' vol. iv. 2nd series, p. 243.)

The value of n, in the preceding Table, obtained from numerous experiments upon ten kinds of cast iron, varies from 2·16 to 3·38, the mean being 2·63. This result shows that the assumption of the incompressibility of cast iron may be admitted so long as we assume that the forces are directly as the extension of the fibres; and it might be admitted still, if we were to make the more improbable assumption, that the forces are the same for all degrees of extension; for the value of n in the former case would be 3, and in the latter 2, and the mean result is 2·63, somewhat nearer to the former than the latter.

The mean value of n obtained from the fracture of different kinds of stone, in numerous experiments not yet published, is not widely different from 3. The value of n, being assumed by Tredgold as 6, has, when applied at the time of fracture, caused the errors pointed out in notes to arts. 68, 143, &c., of Part I.

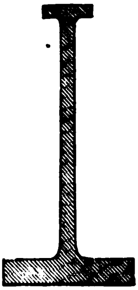
152. To obviate the anomalies above, and to obtain results consistent with experiment in the fracture of beams of cast iron—taking the neutral line in its proper position—we shall assume the forces of extension and compression to be of the form $f = a\,x - \phi\,(x)$, where f is the force, x the extension or compression, a a constant quantity, and $\phi\,(x)$ a function representing the diminution of the force f in consequence of the defect of elasticity. If $\phi\,(x)$ be assumed as equal to $b\,x^n$, as in art. 106, we shall have $n = 2$ nearly, if the experiments are made on the transverse flexure of bars; but it is more desirable that the value of n should be obtained from the direct longitudinal variations of the body experimented upon. This subject will be resumed in a future article.

EXPERIMENTS ON LARGE BEAMS.

153. I have been favoured, through Mr. J. O. March, of Leeds, with the results of three experiments upon beams cast for the mill of Messrs. Marshall and Co., of that town, in 1838. They were from drawings supplied by Mr. Fairbairn, of Manchester; and were of a moderately good form of section, according to my experiments, though the bottom flange was rather too small. The beams were broken to ascertain the ultimate strengths, as well as to test the difference of strength between those cast from the cupola and the air furnace. The experiments, Mr. March states, were very carefully made, under the inspection of Messrs. Marshall and Co.; and the beams were cast from Bierley pig iron. The form, dimensions, and results are as follow:

Dimensions of the beams.

12	inches	deep at one end, and
10½	„	deep at the other.
2½	„	= breadth of rib on the upper edge at the ends.
4	„	= breadth of flange at the ends.



Dimensions of section in middle, in inches.

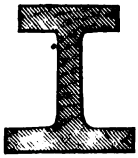
Area of top rib 3·00 × ·75 = 2·25.
Area of bottom rib 8·25 × 1·25 = 10·31,
or 11 square inches, the increase from the brackets, at the junction of the bottom rib with the vertical part, being included.
Thickness of vertical part, ⅝ = ·625 inch.
Depth of beam, in middle, 17 inches.

Beams proved at Messrs. Marshall and Co.'s, Leeds; the distance between the supports 18 feet. Deflexions in 50ths of an inch.

Tons.	2	4	6	8	10	12	14	16	18	20	22	24	25	26	27	28	
Deflected.	5	9	13	20	25	30	36	40	47	54	58						1st cupola casting. Broke with 22 tons.
Permanent Deflexion.				4	5	6	7	8	8	10							
Deflected.		7		12		26		36		47	54	61					2nd cupola casting. Broke with 25 tons.
Permanent Deflexion.				4		6		6		8	10	12½					
Deflected.		7		16		32		35		47	53	60	65	67	72		Air furnace casting. Broke with 28 tons.
Permanent Deflexion.				1		2		4		7	8	24	12	15	17½		

154. In an experiment made upon a beam by Mr. Gooch, whilst he was superintending the formation of the Leeds and Manchester Railway, the particulars are as follow:

Dimensions of section in middle, in inches.



Top rib . . 6 × 1½ = 9.
Bottom rib . 8 × 1½ = 12.
Thickness of middle part 1½.
Depth of beam . . . . 9.

Distance between supports, 11 feet 8 inches; weight of casting, 11¼ cwt.

Weights laid on, in tons.	Deflexions, in parts of an inch.
3½	·10
4½	·15
5½	·175
6½	·22
7½	·25
8½	·30
10	·33
17	1·10
20	Broke 8½ inches from centre.

After bearing 17 tons, the beam was unloaded, and the elasticity seemed to be very little or not at all injured.

The mixture of metal was

1 ton Colebrook Vale, cold blast.
1½ „ Staffordshire, hot blast.
1½ „ Scotch, „

Mr. Gooch observes, in his letter giving an account of the experiment, "The cross section and some of the other dimensions are not of the most favourable or economical form, but circumstances required the adoption of them in the case of this girder."

155. The two following beams were cast for a viaduct forming the junction of the Liverpool and Manchester with the Leeds Railway, passing through Salford. They are not of forms best adapted for resisting fracture, but their great size will give additional interest to experiments upon them.

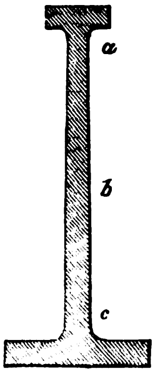
156. As several beams were cast from the same models, I was requested, by the Messrs. Ormerod, of Manchester, the founders, to superintend an experiment upon a beam from each; but the strength was so great that the experiment could not well be made in the usual way, that of applying a weight in the middle. I had, therefore, the beam inverted during the experiment, its bottom flange being turned upwards. The middle was supported laterally by stays, and it rested upon a cross bar, and other apparatus, on which it turned as on an axis; this cross bar being made to rest on two other beams of nearly equal magnitude to that intended to be tried. One end of these beams, and the corresponding end of the beam to be bent, were connected by means of a strong bolt; and the other end of the latter beam was connected, as described below, with the opposite ends of the two supporting beams. The object was to break the top beam in the middle by a force applied at one end, whilst the other end was fixed; and to effect the required pressure, a powerful lever, forged

for the purpose, was applied to the moveable end; it being evident that the pressure at the end would only be half of the effect produced in the middle. (See Plate V.)

157. To enable the deflexions to be observed, a straight edge of the same length as the beam was used, and made to rest upon it, touching it only at the ends. The quantities, which the deflexions varied in consequence, of different weights, were measured by inserting a long wedge-like body, graduated on the side, between the straight edge and the top of the beam. The observed distances, when the beam was bearing a given load, were subtracted from the observed distance when there was no load upon it, for the deflexion.

The experiments were made with a very complete apparatus and every attention to accuracy.

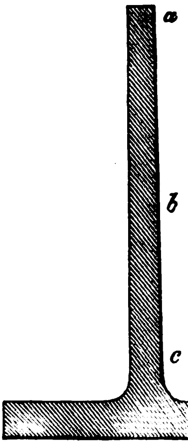
158. The first beam had small ribs, or flanges, at top and bottom, and a strong vertical plate between them, as in the annexed section. The dimensions and results from the beam are as follow:



Section of top rib . . 5·1 × 2·33 inches.
" bottom rib . 12·1 × 2·07 "
Thickness at *a*, 2·06 } mean 2·08.
" *b*, 2·12
" *c*, 2·07
Depth of beam in middle 30·5 inches.
Distance between supports 27 feet 5 inches.
Whole length of beam 28 feet 9 inches.

Weight applied at the end, in tons.	Weight applied in the middle, in tons.	Deflexion in middle, in parts of inch.
9·2	18·4	·31
13·4	26·8	·47
17·6	35·2	·61
21·8	43·6	·73
26·0	52·0	·87
30·2	60·4	1·02
34·4	68·8	1·16
38·3	76·6	1·29

With this, 76·6 tons in middle, it broke apparently in consequence of an accidental shake.



159. The second beam was much heavier and stronger than the former; its section is as in the annexed figure, and its dimensions and results are as follow:

Mean thickness of bottom flange	3·12	in.
Breadth of "	23·9	"
Whole depth of beam . . .	36·1	"
Thickness of vertical rib at *a* .	3·14	"
" " *b* .	3·36	"
" " *c* .	3·38	"

Distance between supports 23 feet 1 inch.
Whole length of beam 24 " 6 "
Weight of beam, 6 tons 1 cwt. 1 qr.

Weight applied at end of beam, in tons.	Weight applied at middle of beam, in tons.	Deflexion in middle of beam, in inches.
13·4	26·8	·10
17·6	35·2	·14
21·8	43·6	·17
26·0	52·0	·20
30·2	60·4	·23
34·4	68·8	·27
38·6	77·2	·31
42·8	85·6	·35
47·0	94·0	·38
51·2	102·4	·42
55·4	110·8	·46
59·6	119·2	·51
63·8	127·6	·55
68·0	136·0	·59
72·2	144·4	·64
76·4	152·8	·68

With this load, 153 tons in the middle nearly, the experiment was discontinued, as the apparatus was overstrained.

MR. FRANCIS BRAMAH'S EXPERIMENTS ON BEAMS.

160. In the second volume of the Institution of Civil Engineers, there is a paper containing experiments made in the year 1834, by Mr. A. H. Renton, for Mr. Bramah. They were upon beams of which the section is in the form ⊥, some of them being solid throughout their length, and others having apertures in them. The following is an abstract of

such of the experiments as were pursued to the time of breaking the beam.

161. Beams, the section of which is of the ⊥ form, are, as we have seen, much weaker than those of another which has been arrived at (art. 135); but they are not without interest, as they are easily cast, and have considerable strength; except those with open work in them, which form the subject of Mr. Bramah's second Table, given hereafter; his first Table containing the results of experiments on solid beams. The former beams, though recommended by Tredgold (Part I., art. 41), are very weak; as will be seen by comparing the breaking weights of the beams, 3 inches deep, in the two Tables, and taking into consideration the weights of the beams. The great weakness of beams with apertures in them was shown, too, in my experiments on beams, 'Memoirs of the Literary and Philosophical Society of Manchester,' second series, vol. v., 1831.

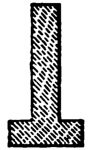


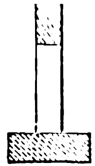
TABLE I.

162. *Mr. Bramah's Experiments upon solid cast iron beams, the middle section of which was as in the annexed figure; the width of the flange throughout the beam being 1·5 inch, the thickness of the beam in every part ·5 inch, the depth of the section in the middle 3 inches, and the distance between the supports 3 feet 1 inch.*

Weight laid on scale with a leverage of 12.	Real weight laid on beam.	Beam 3 inches deep, and uniform in depth throughout (Plate IV., fig. 45), broke with flange downwards. ⊥ Weight 22 lbs. 4 oz.	Beam same as the last, broke with the flange upwards. T Weight 21 lbs. 2 oz.	Beam, with middle section as before, but reduced at the ends to half the depth of the middle, broke with flange downwards. ⊥ Weight 17 lbs.	Beam, middle section as before, reduced at ends to ⅔rds of depth in middle, broke with flange downwards. ⊥ Weight 17 lbs. 12 oz.	Beam same as last, but reversed, broke with flange upwards. T Weight 17 lbs. 8 oz.
lbs.	lbs.					
56	672	·027	·025	·037	·042	·036
112	1344	·0535	·049	·072	·078	·073
168	2016	·079	·075	·108	·113	·113
224	2688	·102	·105	·145	·144	·151
280	3360	·126	·132	·185	·181	·199
336	4032	·148	·166	·219	·220	
364	4368	·172	·183	·237	·240	
392	4704	·186		·257	·262	
420	5040	·205			·284	
448	5376				·306	
504	6048				·352	
532	6384				·393	
		144 × 12 = 1728 lbs.* injured the elasticity; 434 × 12 = 5208 lbs. broke the beam.	160 × 12 = 1920 lbs. injured the elasticity; 378 × 12 = 4536 lbs. broke the beam.	160 × 12 = 1920 lbs. injured the elasticity; 511 × 12 = 6132 lbs. broke the beam.	158 × 12 = 1896 lbs. injured the elasticity; set with that weight ·004; 546 × 12 = 6552 lbs. broke the beam.	156 × 12 = 1872 lbs. injured the elasticity; 294 × 12 = 2528 lbs. broke the beam. There was an air bubble in the casting.

* This must not be considered as the first weight which injured the elasticity, but that which rendered the defect very obvious. It has been shown (arts. 80 and 101) that the elasticity would be injured by any weight, however small. The same remark will apply to the other beams.

TABLE II.



163. *Beams differing in section from the former only in having a portion taken away from the middle; the sections being of the general form in the margin, and the elevations as in the Plates referred to below.*

Distance of supports = 3 feet 1 inch.

Depth of beam in middle, except otherwise mentioned, 3 inches.

Weight laid on scale with leverage of 12.	Real weight on beam.	Beam of form as in Plate IV. fig. 46, weight 15 lbs. 8 oz.; broke with the flange downwards.	Beam same as last, broke with flange upwards; weight 16 lbs. 8 oz.	Beam of form as in Plate IV. fig. 47, weight 17 lbs. 6 oz.; broke with flange downwards.	Beam same as last, weight 18 lbs. 0 oz.; broke with flange upwards.	Beam of form as in Plate IV. fig. 48, weight 15 lbs. 7 oz.; broke with flange downwards.	Beam same as last, broke with flange upwards.	Beam same as in Plate IV. fig. 49, depth in middle 4 in., weight 17 lbs. 2 oz.; broke with flange downwards.	Beam same as last, broke with flange upwards.	Beam same as in Plate IV. fig. 50, part taken out of centre, same area of section in middle as last, weight 21 lbs. 8 oz.; broke with flange downwards.	Beam same as last, broke with flange upwards.
lbs.	lbs.	Deflexion. inch.	Deflexion. inch.	Deflexion. inch.	Deflexion. inch.	Deflexion. inch.	Deflexion. inch.	Deflexion. inch.	Deflexion. inch.	Deflexion. inch.	Deflexion. inch.
28	336	·050	·048	·017	·017	·023	·026	·012	·010	·009	·009
56	672	·099	·100	·037	·035	·048	·053	·027	·024	·019	·019
84	1008	·152	·155	·055	·052	·070	·081	·039	·037	·029	·030
98	1176	·182	·190	......	......	......	......	......	......	......	......
112	1344	......	......	·075*	·070*	·091	·109	·051	·050	·037	·031
140	1680	......	......	·092	·092	·113	·135	·062	·061	·044	·042
168	2016	......	......	·113	·115	·139	·164	·075*	·075*	·051	·052
180	2160	......	......	......	·122	......	·176	......	......	......	......
196	2352	......	......	·134	......	·165	......	·088	·086	·059	......
224	2688	......	......	·156	......	·190	......	·101	·099	·069	......
252	3024	......	......	·179	......	·218	......	·111	·111	·079	......
280	3360	......	......	·202	......	·246	......	·121	......	·090	......
308	3696	......	......	·224	......	......	......	·133	......	.099	......
336	4032	......	......	·245	......	......	......	·147	......	·110	......
392	4707	......	......	·292	......	......	......	·172	......	·133	......
418	5376	......	......	......	......	......	......	·200	......	·159	......
504	6048	......	......	......	......	......	......	·228	......	·186	......
532	6384	......	......	......	......	......	......	·242	......	·200	......
560	6720	......	......	......	......	......	......	·258	......	......	......
588	7056	......	......	......	......	......	......	·271	......	......	......
		Broke by 100 × 12 = 1200 lbs.	Broke by 98 × 12 = 1176 lbs.	Broke by 400 × 12 = 4800 lbs.	Broke by 184 × 12 = 2208 lbs.	Broke by 284 × 12 = 3408 lbs.	Broke by 188 × 12 = 2256 lbs.	Broke by 592 × 12 = 7104 lbs.	Broke by 276 × 12 = 3312 lbs.	Broke by 540 × 12 = 6480 lbs.	Broke by 196 × 12 = 2352 lbs.

* The asterisks show the deflexions with which the elasticity was observed to be injured.—See note to last Table.

164. Mr. Bramah's experiments were made to support certain principles adopted by Tredgold, some of which have been controverted in this work. Mr. Bramah states that it was "a principal feature in these experiments, and essential to the accuracy of the results, to note that point where the elastic power becomes impaired, and the specimens take a permanent set," &c. But I have shown (arts. 86 and 101) that there is no elastic limit in cast iron; and if there were, the depth, 3 inches, of the beams on which Mr. Bramah's experiments were made, was much too great, with their small distance between the supports, 3·083 feet, to enable him to discover when the defect of elasticity first took place. In neglecting his reasonings, as very disputable, and not suited to this place—where the strength considered is the ultimate, and Bramah's (following Tredgold) the incipient, of the material—I shall merely observe, that the results with respect to fracture, seem to be in accordance with those from my own experiments (arts. 112-115). Mr. Bramah draws no conclusions from the breaking weights, but they show the great difference in the ultimate strength of a beam, according as it is turned one way up, or the opposite; and will serve as a beacon to warn the public of the danger of using beams with apertures in them.

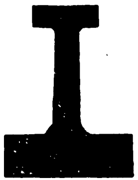
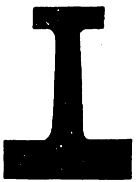
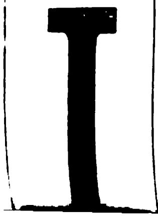
MR. CUBITT'S EXPERIMENTS ON BEAMS.

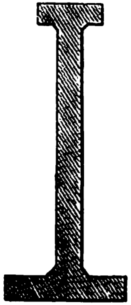
165. Sir Henry De la Beche and Thomas Cubitt, Esq., having in 1844 been appointed by Government to inquire into the circumstances respecting the fall of a cotton mill at Oldham, in Lancashire, and part of a prison at Northleach, in Gloucestershire; these gentlemen accompanied their Report with the results of the experiments made at Manchester, and given more at length in this 2nd Part; adding in particular those with respect to the strength and best forms of iron beams and pillars.

166. Mr. Cubitt likewise, feeling "impressed with the importance of further researches on the forms of cast iron beams, whether for the purpose of confirming or of extending the views hitherto taken," (Report, page 7,) caused eight experiments to be made on a moderately large scale. These experiments were published with the Report, and an abstract of them is in the following Table.

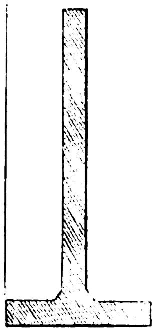
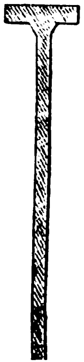
167. Tabular results of experiments on cast iron beams—each cast 16 feet long—laid during the experiment, on supports 15 feet asunder, and intended to be of the same weight. The first six beams were uniform throughout, the seventh and eighth tapered towards the ends.

TABLE TO MR. CUBITT'S EXPERIMENTS.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	Number.	Weight.	Size of bottom flange.	Size of top flange.	Thickness between flanges.	Total depth.	Area of section.	Distance of bearings.	Pressure applied to middle.	Deflexion.		Set.		Comparative stiffness.	Comparative power to resist impact.	Comparative strength reduced to equal area.	Comparative strength reduced to equal weight.
	1	cwt. qrs. lbs. 6 2 14	5·02 by 1·59	2·58 by ·86	·86 at top, 1·22 at bottom.	7·15	15·39	ft. in. 15 0	tons. 1 3 5 6 7	·265 ·73 1·285 1·54 1·80	 Broke	·11 ·075 ·15 ·185		100	100	100	100
	2	6 3 1	5·1 by 1·59	2·6 by ·86	·92 at top, 1·28 at bottom.	7·17	15·84	15 0	1 3 5 7 7⅛	·265 ·72 1·215 1 76 1·79	 Broke	·015 ·058 ·121 ·243		104·6	101·2	98·9	99·7
	3	6 3 4	5 05 by 1·04	2·58 by ·87	·93	10 75	16·02	15 0	1 3 5 7 9 11 11½	·083 ·28 ·455 ·645 ·86 1·06 1·11	 Broke	·02 ·04 ·058 ·08 ·105 ·14		265·8	101·3	157·8	160·4

					by 1·05	by ·88						3 5 7 9 11 12	·275 ·47 ·65 ·845 1·04 1·134	Broke	·014 ·031 ·0545 ·085 ·124					
	5	6	0	24	5·03 by ·88	2·58 by ·65	·73	12·75	14·64	15	0	1 3 5 7 9 10 10¼	·05 ·18 ·305 ·465 ·60 ·685 ·702	Broke.	Set. ·137 ·0267 ·0525 ·0737 ·0852 Bottom flange unsound.		420·8	57·0	153·9	156·1
	6	7	2	22	5·13 by 1·09	2·64 by ·94	·95	12·8	18·60	15	0	1 3 5 7 9 11 13 15 15¾	·062 ·16 ·265 ·395 ·51 ·645 ·77 ·9 ·945	Broke	·004 ·012 ·025 ·04 ·055 ·07 ·09 ·13		469·7	118·1	186·2	193·7

TABLE—*Continued.*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	Number.	Weight.	Size of bottom flange.	Size of top flange.	Thickness between flanges.	Total depth.	Area of section.	Distance of bearings.	Pressure applied to middle.	Deflexion.		Set.		Comparative stiffness.	Comparative power to resist impact.	Comparative strength reduced to equal area.	Comparative strength reduced to equal weight.
	7	cwt. qrs. lbs. 6 2 18	6·5 by 1·	None	·875 at top, ·95 at bottom.	14·	18·51	ft. in. 15 0	tons. 2 6 10 12 $12\frac{7}{8}$	·15 ·41 ·94 ·97	 Broke.	·017 ·055 ·08 ·11 Bottom flange unsound.		360·7	95·2	147·	177·7
	8	5 3 24	5·9 by ·84	2·72 by ·83	·68	17·25	18·11	15 0	2 6 10 14 16	·075 ·26 ·46 ·66 ·76	 Broke.	 ·075 Bottom flange unsound.	The set was not taken in this experiment, but it was discernible at ·1 ton.	631·2	96·5	194·2	253·9

	9	...	5·05 by 1·59	2·6 by ·9	·87 at top, 1·26 at bottom.	7·15	15·63	7	6	2 6 10 14 15$\frac{5}{8}$	·06 ·185 ·31 ·47 ·52	Broke							
	10	...	5·06 by 1·02	2·6 by ·88	·92	10·75	15·89	7	6	4 8 12 16 20 22 23$\frac{1}{3}$	·05 ·105 ·155 ·21 ·261 ·29 ·318	Broke, nearly but not quite sound.							
	11	...	...	...	...	...	...	7	6	4 12 20 28 31	·025 ·085 ·145 ·22 ·244	This piece (part of No. 6) was not broken, therefore no area is given.							

168. Mr. Cubitt states that his object in making these experiments "was to show the great difference of strength of cast iron that can be got by taking certain forms." (Appendix to Report, page 39.) This object he has realized in a certain manner, as will be seen from the last two columns in the preceding Table, in which the beams, though of equal weight and section, are successively made to increase rapidly in strength above the top ones, which are the weakest.

169. Now as these weakest beams are not very different in form of section from, though somewhat weaker than, those which I have arrived at from a long course of inductive experiments, and considered as nearly the strongest in cast iron (arts. 135 and 136); and as the circumstance may be considered as bearing deeply upon the character of my published results with respect to beams, it will be incumbent on me to analyze the effort of Her Majesty's Commissioner with more freedom than I would otherwise willingly have done.

170. Speaking plainly, then, it appears to me that Mr. Cubitt, by increasing the strength through increasing the depth, the area being the same, has shown nothing more than would have been predicted from the slightest knowledge of theory; and that several of his beams, instead of showing greater strength, exhibit only weakness and inferiority of form.

171. To give a simple illustration of this statement, we will suppose a number of rectangular beams to be formed, of the same length and area of section, but of different breadths and depths. Then the strength of each being as $b\,d^2$, varies as d, since $b\,d$, the area of the section, is constant.

172. We will select from Mr. Cubitt's Table the results of the different experiments, and attach to them a column containing the results which would have been derived from rectangular beams, of the same length and area, and varying in depth as Mr. Cubitt's did. It must, however, be understood that the rectangular section, which is comparatively a

weak one, is introduced only for illustration, as its strength for different depths is easily computed; and it may be presumed that the strength of other sections, not so easily calculated, would increase, by augmenting the depth, in some such ratio as that does.

Number of Experiment.	Total depth of beam in middle.	Comparative strength reduced to equal areas.	Comparative strength of rectangular beam of the same depth as Mr. Cubitt's, and of constant area and length.	Remarks.
1	7·15	100	100	The strength of the first rectangular beam is assumed as 100, same as the first of Mr. Cubitt's.
2	7·17	98·9	100·27	
3	10·75	157·8	150·3	
4	10·75	152·4	150·3	
5	12·75	153·9*	178·3	* Bottom flange unsound.
6	12·8	186·2	179·0	
7	14·0	147*	195·8	* The bottom flange of both beams seems to have been slightly defective.
8	17·25	194·2*	241·2	

173. If we compare the results in the third and fourth columns, showing the comparative strengths of Mr. Cubitt's beams and of rectangular ones of the same depth, we shall see that the increased strength of Mr. Cubitt's beams, in the lower part of the Table, above the strength of that at the top, with which they are compared, is derived wholly from the *depth;* and as his latter beams give generally much lower results than are obtained from the rectangular section, we may infer that they are of inferior forms to that with which they are compared. The defect in the bottom flange of three out of four of them (an uncommon occurrence in properly cast beams) renders it probable that, if sound, they would have borne a little more than they did; but affords no probability that their increase of strength would have been equal to that of the rectangular section; which no doubt would

have been the case, or nearly so, if the form in the first experiment had been used.

174. A further confirmation of the conclusion here arrived at is derived from the fact, that the strength of beams of the best form was found from my experiments to be, *cæteris paribus*, nearly as the depth; and the material in the section was but little increased with a large addition to the depth.

175. From the experiments and reasoning above, Mr. Cubitt has drawn the conclusion of "*our knowledge of the best forms and arrangements of cast iron beams not being based upon principles the correctness of which cannot be questioned,* (Report, page 10,) *and they are offered in confirmation or extension.*"

176. It would appear that Mr. Cubitt had mistaken the object of my experiments "on the strength and best forms of cast iron beams." It was virtually to seek for the form into which a given quantity of iron could be cast, so as to bear the greatest weight, the length and the depth of the beam being constant. Mr. Cubitt's experiments seem to have been intended to show that a given quantity of iron cast into beams—all of the same length—the section being of various forms (as the ⊥ section which I had represented as comparatively weak), would be made to bear more than others which I had represented as approaching to the strongest; this being effected merely by increasing the depth.

177. A little more attention to theoretical considerations might equally well have shown that increasing the depth—a privilege I did not allow myself when seeking for the best form of beam—had a great influence on the strength; and this might perhaps have prevented Mr. Cubitt offering to the public, under Her Majesty's sanction, additional examples, on a large scale, of weak beams.

178. In the tabular extract of Mr. Cubitt's experiments I have given the sets or defects of elasticity as obtained by

that gentleman; but the length of his beams was not a sufficient number of times their depth for the results of the early sets, however carefully taken, to be any thing but an approximation.

179. As Mr. Cubitt's experimental results with respect to the strength of beams seem to be in accordance with my own, and might generally be computed from them, whatever opinion he may have formed to the contrary, I see no reason to doubt that the best form of beam is obtained from the reasonings and experiments previously given (arts. 108 to 144); and according to which many thousands of tons have been cast. I am preparing to repeat the leading experiments in my former effort on beams, on a very large scale, and to extend them considerably, through the liberality of an Iron Company.

COMPARATIVE STRENGTH OF HOT AND COLD BLAST IRON.

180. Having, in conjunction with Mr. Fairbairn, been requested, by the British Association for the Advancement of Science, to ascertain by experiment the comparative strengths of irons made by a heated and a cold blast, I will give here the results from my 'Report on the Tensile, Crushing, and Transverse Strengths of several kinds of Iron,' (Brit. Assoc. vol. vi. 1838,) attaching to them, in conclusion, the results from Mr. Fairbairn's experiments, which were on the latter kind of strain.

181. As the modes in which the different kinds of experiment were made, and many of the results obtained, are given in the earlier pages of this work, it will not be necessary here to enter into detail upon that subject. I shall, therefore, content myself with stating, that the experiments were made with great care; and in devising the apparatus, the utmost attention was paid to theoretical requirements.

182. Taking only the means from all the experiments, in the report above mentioned, and attaching to each result a

number, in a parenthesis, indicative of the number of experiments from which it has been derived, we have as follows:

Carron Iron, No. 2 (*Scotch*).

	Cold blast.	Hot blast.	Ratio representing cold blast by 1000.	
Tensile strength in lbs. per square inch	16683 (2)	13505 (3)	1000 : 809	
Compressive (crushing) strength in lbs. per square inch; from specimens cut out of castings previously torn asunder	106375 (3)	108540 (2)	1000 : 1020	Mean 997
Crushing strength obtained from prisms of various forms	100631 (9)	100738 (5)	1000 : 1001	
Do. from cylinders	125403 (13)	121685 (13)	1000 : 970	
Transverse strength from all the experiments	...... (11)	...... (13)	1000 : 991	
Computed power to resist impact	...... (9)	...... (9)	1000 : 1005	
Transverse strength of bars, 1 inch square, and 4 feet 6 inches between the supports, in lbs.	476 (3)	463 (3)	1000 : 973	
Ultimate deflexion of do. in inches	1·313 (3)	1·337 (3)	1000 : 1018	
Modulus of elasticity in lbs. per square inch (Part I. art. 256)	17270500 (2)	16085000 (2)	1000 : 931	
Specific gravity	7066	7046 (9)	1000 : 997	

Devon Iron, No. 3 (*Scotch*).

	Cold blast.	Hot blast.	Ratio representing cold blast by 1000.
Tensile strength per square inch	.....	21907 (1)	......
Compressive strength do.	......	145435 (4)	.....
Transverse do. from the experiments generally	...... (5)	...... (5)	1000 : 1417
Power to resist impact	...... (4)	...... (4)	1000 : 2786
Transverse strength of bars, 1 in. square, and 4 feet 6 inches between the supports	448 (2)	537 (2)	1000 : 1199
Ultimate deflexion, do.	·79 (2)	1·09 (2)	1000 : 1380
Modulus of elasticity, do.	22907700 (2)	22473650 (2)	1000 : 981
Specific gravity	7295 (4)	7229 (2)	1000 : 991

Buffery Iron, No. 1 (*English*).

	Cold blast.	Hot blast.	Ratio representing cold blast by 1000.
Tensile strength per square inch	17466 (1)	13434 (1)	1000 : 769
Compressive strength do.	93366 (4)	86397 (4)	1000 : 925
Transverse strength	...... (5)	...... (5)	1000 : 931
Power to resist impact	...... (2)	...... (2)	1000 : 962
Transverse strength of bars, 1 inch square, and 4 feet 6 inches between the supports	463 (3)	436 (3)	1000 : 942
Ultimate deflexion, do.	1·55 (3)	1·64 (3)	1000 : 1058
Modulus of elasticity, do.	15381200 (2)	13730500 (2)	1000 : 893
Specific gravity	7079	6998	1000 : 989

Coed-Talon Iron, No. 2 (*Welsh*).

	Cold blast.	Hot blast.	Ratio representing cold blast by 1000.
Tensile strength per square inch .	18855 (2)	16676 (2)	1000 : 884
Compressive strength do. . .	81770 (4)	82739 (4)	1000 : 1012
Specific gravity	6955 (4)	6968 (3)	1000 : 1002

Carron Iron, No. 3 (*Scotch*).

	Cold blast.	Hot blast.	Ratio representing cold blast by 1000.
Tensile strength per square inch .	14200 (2)	17755 (2)	1000 : 1250
Compressive strength do. . .	115442 (4)	133440 (3)	1000 : 1156
Specific gravity	7135 (1)	7056 (1)	1000 : 989

183. Abstract of the transverse strengths, and powers to bear impact, as obtained from the experiments on the three irons first mentioned in the preceding Table. The bars, of whatever form, were usually cast 5 feet long, and laid upon supports 4 feet 6 inches asunder. Those of 1 inch square being the bars from which the comparative powers to bear impact were computed, had their deflexions, from different weights, very carefully observed up to the time of fracture; and as the measured dimensions of the bar usually differed a small quantity from those of the model, the results, both as to strength and deflexion, were reduced by computation to what they would have been if the bar had been exactly 1 inch square. A reduction of this nature was made in the results of all the experiments, except otherwise mentioned. The comparative power to bear impact was obtained by multiplying the breaking weight of a bar, 1 inch square, by its ultimate deflexion, the length being always the same; a mode which is admissible, as appears from my experiments on the power of beams to bear impact (British Association, 5th Report).

Carron Iron, No. 2 (Scotch).

Form and dimensions of section of casting.		Strength of irons.			Power to bear impact.		
		Cold blast iron.	Hot blast iron.	Ratio of strengths. The strength of cold blast iron being represented as 1000.	Cold blast iron.	Hot blast iron.	Ratio. The power of cold blast iron being represented as 1000.
Rectangular bar, 1 inch square	Results reduced.	492	469	1000 : 953·2	686	677·2	1000 : 987·1
Ditto ditto		509	456	1000 : 895·8	711	649·3	1000 : 913·2
" "		429	465	1000 : 1083·9	493	532·0	1000 : 1079·1
" "		449	475	1000 : 1057·9	1481	1598·7	1000 : 1079·4
" "		457	429	1000 : 938·7	2601	2744·2	1000 : 1055·0
Bar, 3 inches deep and 1 inch broad		3750	3843	1000 : 1024·8	141	154	1000 : 1092·2
" 5 inches deep and 1 inch broad		10362	{ 10957 / 9149 } 10053	1000 : 970·1	3391	3087	1000 : 910·3
					530	452	1000 : 852 8
					359	458·6	1000 : 1277·4
			Mean	1000 : 989·1		Mean	1000 : 1005·1
Bar whose section is ⊤	Results not reduced.	266	280	1000 : 1052·6			
Bar from the same model, but reversed		1050	980	1000 : 933·3			
Bar, section an isosceles triangle ▽		815	{ 672 / 817 } 742	1000 : 910·4			
Bar, a frustrum of ditto ▽		677	728	1000 : 1075·0			
			Mean	1000 : 992·8			
			General Mean	1000 : 990·9			

Devon, No. 3 Iron (Scotch).

Form and dimensions of section of casting.	Strength of irons.			Power to bear impact.		
	Cold blast iron.	Hot blast iron.	Ratio of strengths. The strength of cold blast iron being represented as 1000.	Cold blast iron.	Hot blast iron.	Ratio. The power of cold blast iron being represented as 1000.
Bar, 1 inch square	448	504	1000 : 1125·0	353·9	589·2	1000 : 1664·8
„ 1 „	448	570	1000 : 1272·3	489·5	1761·7	1000 : 3598·9
„ 1½ inch deep by 1 inch broad	890	1456	1000 : 1635·9	921·8	2747	1000 : 2980·0
				1702·3	4935	1000 : 2899·0
„ 3 „ by 1 „	3389	5183	1000 : 1529·3			
„ 5 „ by 1 „	10133	15422	1000 : 1521·9		Mean	1000 : 2785·6
		Mean	1000 : 1416·9			

Buffery, No. 1 Iron (English, near Birmingham).

Form and dimensions of section of casting.	Strength of irons.			Power to bear impact.		
	Cold blast iron.	Hot blast iron.	Ratio of strengths. The strength of cold blast iron being represented as 1000.	Cold blast iron.	Hot blast iron.	Ratio. The power of cold blast iron being represented as 1000.
Bar, 1 inch square	491	464	1000 : 945·0	721·19	721·5	1000 : 1000·4
„ 1 „	437	437	1000 : 1000·0	2341·6	2163·2	1000 : 923·8
„ 1 „	462	409	1000 : 885·7			
„ 2 „	3057	2975	1000 : 973·1		Mean	1000 : 962·1
„ 2 „	3424	2903	1000 : 850·1			
		Mean	1000 : 930·7			

GENERAL SUMMARY OF TRANSVERSE STRENGTHS, AND COMPUTED POWERS TO RESIST IMPACT.

184. Selecting, from the irons above, the results of the experiments on the transverse strength, and power to resist impact, of the different bars broken, and adding to them the results of Mr. Fairbairn's experiments (Brit. Assoc. vol. vi.), we have as below.

Distance between supports 4 feet 6 inches.

Description of iron.	Strength of cold blast.	Strength of hot blast.	Ratio of strength, cold blast = 1000.	Ratio of powers to sustain impact, representing that of cold blast by 1000.
Carron, No. 2.	lbs.	lbs.		
Results from bars 1 inch square	476 (3)	463 (3)	1000 : 973	
„ from all the experiments	(11)	(13)	1000 : 991	1000 : 100[illegible]
Devon, No. 3.				
Results from bars 1 inch square	448 (2)	537 (2)	1000 : 1199	
„ from all the experiments	(5)	(5)	1000 : 1417	1000 : 278[illegible]
Buffery, No. 1.				
Results from bars 1 inch square	463 (3)	436 (3)	1000 : 942	
„ from all the experiments	(5)	(5)	1000 : 931	1000 : 96[illegible]
Muirkirk, No. 1.				
Results from bars 1 in. sq. (Mr. Fairbairn's experiments.)	454·2 (4)	418·9 (4)	1000 : 922	1000 : 82[illegible]
Coed-Talon, No. 2, ditto	412·6 (5)	416·8 (4)	1000 : 1010	1000 : 1234
Coed-Talon, No. 3, ditto	553·2 (4)	513·1 (4)	1000 : 927	1000 : 92[illegible]
Carron, No. 3, ditto	445·7 (5)	525·7 (5)	1000 : 1179	1000 : 1201
Elsicar, cold, and Milton, hot, No. 1, ditto	451·5 (4)	369·4 (4)	1000 : 818	1000 : 875

185. These Tables contain the results of a very large number of experiments, made with great care upon English, Welsh, and Scotch iron, mostly supplied from the makers. They show that the irons marked No. 1, which are softer and richer than those of Nos. 2 and 3, are injured by the heated blast; since the hot blast irons of this description are less capable than the cold blast ones to resist fracture, whether the forces are tensile, compressive, transverse, or impulsive.

186. The irons marked No. 2, being harder than those of No. 1, have much less difference in their strength than the latter. In the Carron iron, No. 2, on which a great many experiments were made, the transverse strength, of the hot

and cold blast specimens, was as 99 : 100, and their power of bearing impact equal. The Coed-Talon iron of this No. gave the transverse strength, of hot and cold blast, as 101 : 100, and their power to bear impact as 123 : 100. In both the Carron and the Coed-Talon irons, the hot blast castings were of equal strength to the cold blast ones, to resist crushing; but, in both, the strength of the hot blast was less than that of the cold blast, to resist tension, in a ratio of 8 or 9 to 10.

187. The No. 3 irons seem, both in aspect and strength, to be generally benefited by the heated blast. In the Carron iron, No. 3, the hot blast was superior to the cold, in the power of resisting tension, compression, transverse strain, and impact, in a ratio approaching, in each case, that of 12 to 10. The Coed-Talon iron of this No. had, however, its hot blast kind weaker than the cold, to resist transverse strain and impact, in the ratio of about 93 to 100.

The iron, No. 3, from the Devon works in Scotland, was weak and irregular in the cold blast castings; but the hot blast iron from the same works was among the strongest I have tried. In this the ratio of the powers, of hot and cold blast iron, to bear pressure, was as 14 : 10; and to bear impact, as 28 : 10 nearly.

188. In these experiments, the hot blast irons usually differed from the cold blast, only so far as a different mode of manufacture—the introduction of a heated blast with coal, instead of a cold blast with coke—would produce. The difficulty we experienced in obtaining from the makers irons of both kinds made from the same materials, rendered it necessary to make the experiments on a smaller number of irons than would otherwise have been tried; but, from the evidence adduced, we may perhaps conclude that the introduction of a heated blast, into the manufacture of cast iron, has injured the softer irons, whilst it has frequently mollified and improved those of a harder nature; and considering the small deterioration which the irons of the quality No. 2 have

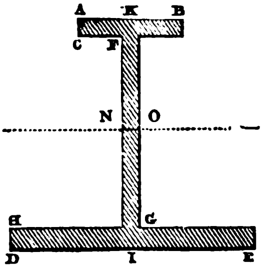
sustained, and the apparent benefit to those of No. 3, together with the great saving effected by the heated blast, there seems good reason for the process becoming as general as it has done. It is, however, to be feared that the facilities which the heated blast gives, of adulterating cast iron by mixture, have introduced into use a species of metal very inferior to that used in this comparison, or that from which the formulæ and leading results of this work have been obtained.

THEORETICAL INQUIRIES WITH REGARD TO THE STRENGTH OF BEAMS.

189. In the course of our remarks on the transverse strength of cast iron, as deduced from experiment, it was shown that the formulæ given by Tredgold, in Part I. of this work, were usually inapplicable to the computation of the strength of that metal to resist fracture. That very ingenious writer—following in the track of Dr. Young, and himself followed by numerous others—considered bodies, when not overstrained, to be perfectly elastic; and to resist extension and compression with equal energy. But theories deduced from these suppositions, however elegant, and nearly correct for small displacements of the fibres or particles, give the breaking strength of cast iron, in some cases, not half what it has been shown to bear by experiment (arts. 150 and 151). A square bar, instead of having its neutral line in the centre—one-half being extended and the other compressed, according to the suppositions above—requires to be considered as totally incompressible, the neutral line being close to the side, or even beyond it,—a matter practically impossible. This defect in the received theories has been shown to arise from the neglect by writers of an element which appears always to be conjoined with elasticity, diminishing its power. This element—ductility, producing defective elasticity—will be

attempted to be introduced into the following investigation but as the formulæ are generally complex, and require additional experiments to supply their constant co-efficients, the reader may perhaps take for practical use the approximate ones previously given in this Second Part.

190. To find the position of the neutral line and the strength of a cast iron beam, supported at the ends, and loaded in the middle; the form of a section of the beam in the middle being that of the figure A B D E, where B C, H E, represent sections of the top and bottom ribs, F G that of the vertical one connecting them, and N O passes through the neutral line.



Let W = weight necessary to break the beam,
l = distance between the supports,
a, a' = N I, N K, respectively,
c, c' = D H, A C do.
b, b' = D E, A B do.
β = the thickness of the vertical rib,
f, f' = tensile and compressive forces of the metal, in a unity of section, as exerted at a distance a on opposite sides of the neutral line,
$\phi(x)$, $\phi'(x')$ = quantities respectively proportional to the forces of extension and compression of a particle, at a distance x from the neutral line,
n, n' = constant quantities dependent on the destruction of the elasticity of the material, by tensile and compressive forces.

191. The bottom rib will be in a state of tension, and the top one in a state of compression; and the parts of the section generally will be extended or compressed according to their distance from the line N O.

1st. To find the position of the neutral line.

192. Since $f . \frac{\phi(x)}{\phi(a)}$ = the force of the extended fibres or particles in a unity of section, at a distance x from the neutral line; therefore, multiplying this quantity by $b\,dx$, or by $\beta\,dx$, we have the force of the particles in an area of the section of which the breadth is b or β, and depth dx. Now the forces of tension, or those exerted by the particles below the line

N O, may be expressed in two functions, the first representing the forces of the particles in the section N G, and the other those in the section H E.

$$\therefore f.\frac{\beta}{\phi(a)}\int_0^{a-c}\phi(x)\,dx + f.\frac{b}{\phi(a)}\cdot\int_{a-c}^{a}\phi(x)\,dx = \text{S} = \text{sum of the forces of tension} \quad (1).$$

Proceeding in the same manner with respect to the compressed particles, we have

$$f'.\frac{\beta}{\phi'(a)}\cdot\int_0^{a'-c'}\phi'(x')\,dx' + f'.\frac{b'}{\phi'(a)}\cdot\int_{a'-c'}^{a'}\phi'(x')\,dx' = \text{S}' = \text{sum of the forces of compression} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

The weight acts in a direction parallel to the section of fracture, and therefore the sum of the forces of extension and compression, being the only horizontal forces, are equal to each other.

$$\therefore \text{S} = \text{S}' \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

193. It appears from my recent experiments that no rigid body is perfectly elastic; and it has been shown (art. 99 to 106), in extracts from communications which I made to the York and late Cambridge meetings of the British Association, that, in the flexure of bars of cast iron and stone, the defect of elasticity was nearly as the square of the weight applied, or of the deflexion, though the defect from the smaller deflexions seemed to increase in a somewhat lower ratio. Other experiments on the defect of elasticity, as exhibited in the flexure of bars of cast iron, wrought iron, steel, timber, and stone—and on the defect of elasticity in the longitudinal variations of bodies—will appear in a future volume of the British Association. As the latter experiments, those on longitudinal variation, are not at present completed, I shall assume $\phi(x) = x - \frac{x^v}{n\,a}$, and $\phi'(x') = x' - \frac{x'^{v'}}{n'a}$; where v, v', n, n' are supposed to be constant, and v, v' to represent the powers of x, x', to which the defects of elasticity of extension and compression are proportional.

Substituting in equations (1) and (2), the values of $\phi(x)$ and $\phi'(x')$, as above, we have

$$f.\frac{n\beta}{na-a^{v-1}}\cdot\int_0^{a-c}\left(x-\frac{x^v}{na}\right)dx+f.\frac{nb}{na-a^{v-1}}\cdot\int_{a-c}^{a}\left(x-\frac{x^v}{na}\right)dx=S \quad . \ . \ . \ . \ (4).$$

$$f'.\frac{n'\beta}{n'a-a^{v'-1}}\cdot\int_0^{a'-c'}\left(x'-\frac{x'^{v'}}{n'a}\right)dx'+f'.\frac{n'b'}{n'a-a^{v'-1}}\cdot\int_{a'-c'}^{a'}\left(x'-\frac{x'^{v'}}{n'a}\right)dx'=S' \quad . \ . \ (5).$$

Performing the integrations of equations (4) and (5) gives

$$f.\frac{n\beta}{na-a^{v-1}}\cdot\int_0^{a-c}\left(x-\frac{x^v}{na}\right)dx=\frac{fn\beta}{na-a^{v-1}}\left(\frac{(a-c)^2}{2}-\frac{(a-c)^{v+1}}{(v+1)na}\right),$$

$$f.\frac{nb}{na-a^{v-1}}\cdot\int_{a-c}^{a}\left(x-\frac{x^v}{na}\right)dx=\frac{fnb}{na-a^{v-1}}\left\{\left(\frac{a^2}{2}-\frac{a^{v+1}}{(v+1)na}\right)-\left(\frac{(a-c)^2}{2}-\frac{(a-c)^{v+1}}{(v+1)na}\right)\right\}$$

$$f.\frac{n'\beta}{n'a-a^{v'-1}}\cdot\int_0^{a'-c'}\left(x'-\frac{x'^{v'}}{n'a}\right)dx'=\frac{fn'\beta}{n'a-a^{v'-1}}\left(\frac{(a'-c')^2}{2}-\frac{(a'-c')^{v'+1}}{(v'+1)n'a}\right),$$

$$f'.\frac{n'b'}{n'a-a^{v'-1}}\cdot\int_{a'-c'}^{a'}\left(x'-\frac{x'^{v'}}{n'a}\right)dx'=\frac{f'n'b'}{n'a-a^{v'-1}}\left\{\left(\frac{a'^2}{2}\quad\frac{a'^{v'+1}}{(v'+1)n'a}\right)-\left(\frac{(a'-c')^2}{2}-\frac{(a'-c')^{v'+1}}{(v'+1)n'a}\right)\right\}$$

Collecting together the integrals of equation (4), we have

$$S=\frac{fn}{na-a^{v-1}}\left\{\frac{ba^2}{2}-\frac{ba^{v+1}}{(v+1)na}-\frac{b(a-c)^2}{2}+\frac{\beta(a-c)^2}{2}+\frac{b(a-c)^{v+1}}{(v+1)na}-\frac{\beta(a-c)^{v+1}}{(v+1)na}\right\}$$

$$=\frac{fn}{na-a^{v-1}}\left\{ba^2\left(\frac{1}{2}-\frac{a^{v-1}}{(v+1)na}\right)-(b-\beta)\left(\frac{(a-c)^2}{2}-\frac{(a-c)^{v+1}}{(v+1)na}\right)\right\}.$$

In like manner,

$$S'=\frac{f'n'}{n'a-a^{v'-1}}\left\{b'a'^2\left(\frac{1}{2}-\frac{a'^{v'-1}}{(v'+1)n'a}\right)-(b'-\beta)\left(\frac{(a'-c')^2}{2}-\frac{(a'-c')^{v'+1}}{(v'+1)n'a}\right)\right\}.$$

Equating the values of S and S′ gives, for the equation of the neutral line,

$$\frac{f}{n-a^{v-2}}\left\{ba^2\left(\frac{n}{2}-\frac{a^{v-1}}{(v+1)a}\right)-(b-\beta)\left(\frac{n(a-c)^2}{2}-\frac{(a-c)^{v+1}}{(v+1)a}\right)\right\}=\frac{f'}{n'-a^{v'-2}}$$
$$\left\{b'a'^2\left(\frac{n'}{2}-\frac{a'^{v'-1}}{(v'+1)a}\right)-(b'-\beta)\left(\frac{n'(a'-c')^2}{2}-\frac{(a'-c')^{v'+1}}{(v'+1)a}\right)\right\} \quad . \ . \ (6),$$

where $a'=D-a$, D being the depth of the beam.

Cor. 1st. If $v=2$, and $v'=2$, as would appear to be nearly the case from the experiments on the transverse flexure of cast iron bars, mentioned above, the equation of the neutral line would be

$$\frac{f}{n-1}\left\{ba^2(3n-2)-(b-\beta)\left(3n(a-c)^2-\frac{2(a-c)^3}{a}\right)\right\}=\frac{f'}{n'-1}$$
$$\left\{b'a'^2\left(3n'-\frac{2a'}{a}\right)-(b'-\beta)\left(3n'(a'-c')^2-\frac{2(a'-c')^3}{a}\right)\right\} \quad . \quad . \quad (7).$$

If the beam is of the ⊥ form, having no top rib, then $c'=0$, $b'=0$, and equation (7) becomes

$$\frac{f}{n-1}\left\{ba^2(3n-2)-(b-\beta)\left(3n(a-c)^2-\frac{2(a-c)^3}{a}\right)\right\}=\frac{f'\beta a'^2}{n'-1}\left(3n'-2\frac{a'}{a}\right) (8).$$

If $b=b'=\beta$, then the section of the beam is rectangular, as a joist; and the equation of the neutral line (7) becomes

$$\frac{fa^2}{n-1}(3n-2)=\frac{f'a'^2}{n'-1}\left(3n'-2\frac{a'}{a}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

Cor. 2nd. If $v=1$, and $v'=1$, or the defect of elasticity be as the extension and compression, then equation (6) will become

$$\frac{f}{2\left(n-\frac{1}{a}\right)}\left\{ba^2\left(n-\frac{1}{a}\right)-(b-\beta)(a-c)^2\left(n-\frac{1}{a}\right)\right\}=\frac{f'}{2\left(n'-\frac{1}{a}\right)}$$
$$\left\{b'a'^2\left(n'-\frac{1}{a}\right)-(b'-\beta)(a'-c')^2\left(n'-\frac{1}{a}\right)\right\}\therefore\frac{f}{2}\left\{ba^2-(b-\beta)\right.$$
$$\left.(a-c)^2\right\}=\frac{f'}{2}\left\{b'a'^2-(b'-\beta)(a'-c')^2\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10).$$

But this equation becomes

$$f\left\{ba\times\frac{2}{a}-(b-\beta)(a-c)\times\frac{a-c}{2}\right\}=f'\left\{b'a'\times\frac{a'}{2}-(b'-\beta)(a'-c')\times\frac{a'-c'}{2}\right\},$$

where ba = area of the whole section of tension considered as a rectangular surface of which the breadth is b and depth a; $\frac{a}{2}$= distance of centre of gravity of that section from the neutral line; $(b-\beta)(a-c)$ = area of part necessary to complete the rectangular surface above; $\frac{a-c}{2}$ = distance of centre of gravity of this defective part. In like manner $b'a'$= area of whole section of compression considered as a rectangular surface of breadth b' and depth a'; $\frac{a'}{2}$ = distance of its centre of gravity; $(b'-\beta)(a'-c')$ = area of part wanting; and $\frac{a'-c'}{2}$ = the distance of its centre of gravity.

Whence it appears that if $f=f'$, the neutral line will be in the centre of gravity of the section.

Cor. 3rd. If the elasticity be considered as perfect, then n, n' will be infinitely great, and taking as usual $f=f'$, equation (6) will become

$$ba^2-(b-\beta)(a-c)^2=b'a'^2-(b'-\beta)(a'-c')^2 \quad . \quad . \quad . \quad . \quad (11).$$

The curious circumstance of this equation being in agreement with equation (10), is rendered obvious by other reasoning. For, in Cor. 2, we have

$$\phi(x)=x-\frac{x}{na}=\left(1-\frac{1}{na}\right)x, \text{ and } \phi'(x')=x'-\frac{x'}{n'a}=\left(1-\frac{1}{n'a}\right)x',$$

the forces being as the extensions and compressions.

If, on the supposition of perfect elasticity, $b'=0$, and $c'=0$, the beam being of the ⊥ form of section, having no top rib, the last equation will become

$$ba^2-(b-\beta)(a-c)^2=\beta a'^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12).$$

If the beam be rectangular and perfectly elastic, then $b=b'=\beta$, and equation (11) becomes

$$ba^2=b'a'^2$$
$$\therefore a^2=a'^2,$$

or the neutral line is in the middle.

2nd. To find the strength of the beam, the values of a, a', and consequently the position of the neutral line, having been previously determined, from one of the preceding equations, or by other means.

194. Since, by equation (4) of the preceding article, the sum of the forces of extension is

$$\frac{fn\beta}{na-a^{v-1}}\cdot\int_0^{a-c}\left(x-\frac{x^v}{na}\right)dx+\frac{fnb}{na-a^{v-1}}\cdot\int_{a-c}^{a}\left(x-\frac{x^v}{na}\right)dx.$$

And as the moment of each of these forces, with respect to the neutral line, is equal to the product of the force by its distance x from that line, we have for the sum of the moments of the forces of tension,

$$\frac{fn\beta}{na-a^{v-1}}\cdot\int_0^{a-c}\left(x^2-\frac{x^{v+1}}{na}\right)dx+\frac{fnb}{na-a^{v-1}}\cdot\int_{a-c}^{a}\left(x^2-\frac{x^{v+1}}{na}\right)dx=S_{,} \quad . \quad . \quad (13).$$

$$\therefore \frac{fn\beta}{na-a^{v-1}}\cdot\int_0^{a-c}\left(x^2-\frac{x^{v+1}}{na}\right)dx=\frac{fn\beta}{na-a^{v-1}}\cdot\left\{\frac{(a-c)^3}{3}-\frac{(a-c)^{v+2}}{(v+2)\,na}\right\},$$

and

$$\frac{fnb}{na-a^{v-1}}\cdot\int_{a-c}^{a}\left(x^2-\frac{x^{v+1}}{na}\right)dx=\frac{fnb}{na-a^{v-1}}\cdot\left\{\frac{a^3}{3}-\frac{a^{v+2}}{(v+2)\,na}-\frac{(a-c)^3}{3}+\frac{(a-c)^{v+2}}{(v+2)\,na}\right\}.$$

Whence we have, for the sum of the moments of the forces of the extended particles,

$$S_{,}=\frac{f}{3a(n-a^{v-2})}\cdot\left\{ba^3\left(n-\frac{3a^{v-2}}{v+2}\right)-(b-\beta)\left(n(a-c)^3-\frac{3(a-c)^{v+2}}{(v+2)a}\right)\right\} (14).$$

In like manner we obtain, for the sum of the moments of the forces of the compressed particles,

$$S_{,,}=\frac{f'}{3a(n'-a^{v'-2})}\cdot\left\{b'a'^3\left(n'-\frac{3a'^{v'-1}}{(v'+2)a}\right)-(b'-\beta)\left(n'(a'-c')^3-\frac{3(a'-c')^{v'+2}}{(v'+2)a}\right)\right\} (15).$$

But $S_{,}+S_{,,}$ the sum of the moments of the forces of extension and compression, must be equal to the product of half the weight laid on the middle of the beam by half the distance between the supports; for we may consider the beam as fixed firmly in the middle, and loaded at one end with half the weight laid on the middle.

$$\therefore S_{,}+S_{,,}=\tfrac{1}{2}W\times\tfrac{1}{2}l=\frac{Wl}{4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16).$$

Cor. 1. If $c'=0$, and $b'=0$, the section of the beam being of the ⊥ form, having no top rib, we have from equation (15),

$$S_{,,}=\frac{f'\beta}{3a(n'-a^{v'-2})}\left(n'a'^3-\frac{3a'^{v'+2}}{(v'+2)a}\right),$$

and $S_{,}$ as in equation (14), for substitution in equation (16).

Cor. 2. If c, c', b, b' are each $=0$, or the beam is rectangular, as a joist, we have from equation (14)

$$S_{,} = \frac{f\beta a^2}{3(n - a^{v-2})}\left(n - \frac{3a^{v-2}}{v+2}\right),$$

from equation (15)

$$S_{,,} = \frac{f'\beta a'^3}{3a(n' - a^{v'-2})}\left(n' - \frac{3a'^{v'-1}}{(v'+2)a}\right),$$

where

$$\frac{Wl}{4} = S_{,} + S_{,,} \quad \ldots\ldots\ldots\ldots \quad (17).$$

Cor. 3. If v, v' are each $= 2$, as would appear from the experiments (art. 106), then the values of $S_{,}$, $S_{,,}$, for the strength of a rectangular section in the last corollary, give

$$\frac{Wl}{4} = \frac{f\beta a^2}{12(n-1)}(4n-3) + \frac{f'\beta a'^3}{12(n'-1)a}\left(4n' - \frac{3a'}{a}\right) \quad \ldots \quad (18),$$

where $\frac{a'}{a}$ is the ratio of the depth of the compressed section to that of the extended one.

On the supposition of this corollary, that $v = v' = 2$, the general formulæ in equations (14) (15) give

$$S_{,} = \frac{f}{12a(n-1)}\left\{ba^3(4n-3) - (b-\beta)\left(4n(a-c)^3 - \frac{3(a-c)^4}{a}\right)\right\},$$

$$S_{,,} = \frac{f'}{12a(n'-1)}\left\{b'a'^3\left(4n' - \frac{3a'}{a}\right) - (b'-\beta)\left(4n'(a'-c')^3 - \frac{3(a'-c')^4}{a}\right)\right\}.$$

Cor. 4. If $v = v' = 1$, or the defect of elasticity is as the extension and compression, equations (14) and (15) give

$$S_{,} = \frac{f}{3a}\left\{ba^3 - (b-\beta)(a-c)^3\right\}, \text{ and}$$

$$S_{,,} = \frac{f'}{3a}\left\{b'a'^3 - (b'-\beta)(a'-c')^3\right\}.$$

Cor. 5. If the beam be supposed to be perfectly elastic, then n and n' are both infinite, and we have from equations (14) and (15)

$$S_{,} = \frac{f}{3a}\left\{ba^3 - (b-\beta)(a-c)^3\right\},$$

$$S_{,,} = \frac{f'}{3a}\left\{b'a'^3 - (b'-\beta)(a'-c')^3\right\},$$

agreeing with the results of the last corollary, as previously remarked with respect to equations (10) and (11).

But as the body is elastic we will assume as usual $f = f'$,

$$\therefore S_{,} + S_{,,} = \frac{f}{3a}\left\{ba^3 + b'a'^3 - (b-\beta)(a-c)^3 - (b'-\beta)(a'-c')^3\right\} = \frac{Wl}{4} \quad . \quad (19).$$

If $b = b'$, and $c = c'$, or the top and bottom ribs are equal, then, the neutral line being in the centre, $a = a'$; and the last equation gives

$$\frac{W\,l}{4} = \frac{2f}{3\,a}\left\{b\,a^3 - (b - \beta)\,(a - c)^3\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20),$$

a result in agreement with equation XIX., art. 116 of Tredgold.

If in this case $\beta = 0$, or the part between the ribs was so thin that it might be neglected,

$$\frac{W\,l}{4} = \frac{2fb}{3\,a}\left\{a^3 - (a - c)^3\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21),$$

agreeing with art. 117 of Tredgold.

If $\beta = b$, or the beam is rectangular, then equation (20) becomes

$$\frac{W\,l}{4} = \frac{2f}{3\,a} \times b\,a^3 = \frac{2\,f\,b\,a^2}{3} = \frac{f\,b\,D^2}{6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22),$$

where D is the whole depth $= 2\,a$, a result usually arrived at by a much simpler process (Tredgold, art. 110).

For other investigations on the subject of the neutral line, and the strength of beams variously fixed—the elasticity being supposed perfect — see Professor Moseley's 'Principles of Engineering and Architecture.'

RESISTANCE TO TORSION.

195. If a prismatic body, fixed firmly at one end, have a weight applied to twist it by means of a lever acting at the other, perpendicular to the length of the body, to find the resistance to twisting and to fracture.

196. The problem here proposed has been made the subject of an ingenious article by Tredgold in the 1st Part of this work; but as it has been subjected to more recent and profound theoretical investigation by Cauchy and others on the Continent, the formulæ given by Navier ('Application de Mécanique'), including those of Cauchy, will be inserted here, referring for their demonstrations to the work itself, or to M. Cauchy's 'Exercices de Mathématiques,' 4e année. Experimental results by Bevan, Rennie, Savart, and the Author, will likewise be given or noticed.

197. Let l = the length of the prism from the fixed end to the point of application of the lever used to twist it.
r = the radius of the prism, if round.
b, d = its breadth and thickness, if rectangular.
P = the weight acting by means of the lever to twist it.
R = the length of the lever.
θ = the angle of torsion, at the point of application, considered as very small.
G = a constant for each species of body, representing the specific resistance to flexure by torsion.
T = a constant weight expressing the resistance to torsion, with regard to a unit of surface, at the time of fracture.

We have then as in the following Table.

Form of section of prism.	Resistance to angular flexure by a force of torsion.	Resistance to fracture by a force of torsion.
Round . .	$G = PR.\frac{2l}{\pi r^4 \theta}$.	$T = PR.\frac{2}{\pi r^3}$.
Square . . .	$G = PR.\frac{6l}{d^4 \theta}$.	$T = PR.\frac{6}{\sqrt{2}.d^3}$.
Rectangular .	$G = PR.\frac{3(b^2+d^2)l}{b^3 d^3 \theta}$.	$T = PR.\frac{3\sqrt{b^2+d^2}}{b^2.d^2}$.

The formulæ for G are in art. 160, and those for T in art. 168, of the 'Application de Mécanique,' 1[re] partie; those for the rectangular prism being from Cauchy.

198. If E be the force necessary to elongate or shorten a prism, the transverse section of which is a superficial unity, as one square inch, by a quantity equal to the length of the prism, the elasticity being supposed perfect, and the force applied in the direction of the length; and if F be the force necessary to break such a prism; then E will be the modulus of elasticity of the material, and F the modulus of its resistance to fracture; and the values of G and T above will be connected with those of E and F by the following relations.

$$G = \frac{2E}{5},\ T = \frac{4F}{5}.$$

See 'Application de Mécanique,' notes to articles 159 and 167.

The connexion is, however, but little in accordance with the results of experiment, with respect to the values of T and F, as might be expected from the elasticity being much injured previous to the time of fracture.

199. The angle θ being measured by the arc due to a radius equal unity, if the angle were expressed in degrees, and represented by Δ, we should have $\theta = \Delta \cdot \frac{\pi}{180} = \frac{\Delta}{57 \cdot 29578}$, where $57 \cdot 29578$, or $\frac{180}{\pi}$, is the number of degrees in the arc whose length is equal to radius.

200. To obtain the values of G and T in the preceding Table, with respect to any particular material, as cast iron, we must refer to experiment, and will next insert some results which were kindly sent by the author, Mr. Geo. Rennie, in 1842, as part of a more general inquiry; noticing afterwards other experiments previously mentioned, besides some of an earlier date, quoted by Tredgold.

201. Experiments, by Geo. Rennie, Esq., F.R.S., on the strength of three bars of cast iron to resist fracture by torsion. The bars were planed exactly one inch square, and were firmly fixed, at one end, in a horizontal position, and broken by weights acting at the other by means of an arched lever, 3 feet in length, perpendicular to the bar, and exactly balanced by a counter weight. The bars were cast from the cupola, the first vertically, the other two horizontally. The former was broken with 191 ℔s., and the two latter with 231 ℔s. each.

202. To determine from the preceding experiments the value of T, the modulus of resistance to fracture by torsion, in the formula, $T = PR. \frac{6}{\sqrt{2}.d^3}$, for a square prism,—taking the weights in pounds, and the dimensions in inches,—we have $R = 36$, $d = 1$, and $T = P \times \frac{36 \times 6}{\sqrt{2}} = 152 \cdot 735 \times P$.

In the horizontal casting P = 191, ∴ T = 29172·4	}	Mean
In the vertical castings P = 231, ∴ T = 35281·8	}	32227 lbs.

If R were taken in feet and the rest in inches, and pounds, as before, the value of T would be 2685·6.

203. The experiments of Messrs. Bramah on the torsion of square bars of cast iron (Tredgold, art. 85) give for T, taking the dimensions in inches,

No. 3	T = 42020·4	Mean 37747 lbs.
„ 4	T = 39474·0	
„ 6	T = 36198·0	
„ 7	T = 33296·4	

204. Mr. Dunlop's experiments on the torsion of cylinders, varying, in diameter, from 2 to $4\frac{1}{2}$ inches, and in length from $2\frac{3}{4}$ to 6 inches, the leverage being 14 feet 2 inches (Tredgold, art. 85), give, from the formula $T = PR.\ \frac{2}{\pi r^3}$, as follows.

In Experiment No. 2,	T = 27056·4	Mean 27534 lbs.
„ 3,	T = 29187·6	
„ 5,	T = 29142·0	
„ 6,	T = 29509·2	
„ 8,	T = 27286·8	
„ 9,	T = 26217·6	
„ 10,	T = 24338·4	

Taking a mean from the three mean results last obtained gives, T = 32503 lbs.; the dimensions being in inches.

205. Putting this value for T, in the formula (art. 197), and transposing, we obtain the following value of PR.

In a cylinder	$PR = 51055\ .\ r^3$.
In a square prism	$PR = 7661\ .\ d^3$.
In a rectangular prism . .	$PR = 10834\ .\ \frac{b^2 d^2}{\sqrt{b^2 + d^2}}$.

206. If R be taken in feet, as was supposed by Tredgold, art. 265, we shall have, for a square prism, $P = \frac{638 d^3}{R}$.

Hence his co-efficient, 150, being less than $\frac{1}{4}$th of that of fracture, may be regarded as perfectly safe for practical application.

207. Mr. Benjamin Bevan gave, in the 'Philosophical Transactions' for 1829, a memoir containing numerous experiments "on the modulus of torsion."

They were principally on timber, but contained a small Table of the modulus of torsion of metals. Mr. Bevan defined this modulus by the value of T in the following equation for a square prism, twisted as before described:

$$\frac{R^2 l P}{d^4 T} = \delta,$$

where δ is the deflexion, considered as very small, and the rest of the notation as before; the weights and the dimensions being in pounds and inches.

We have from above $\frac{R l P}{d^4 T} = \frac{\delta}{R}$;

and as $\frac{\delta}{R}$ is the deflexion at a unity of distance, and very small, it may be taken for the arc.

$$\therefore \frac{RlP}{d^4 T} = \theta, \text{ where } T = \frac{PRl}{d^4 \theta}.$$

But from the Table (art. 197), $G = PR . \frac{6l}{d^4 \theta} = \frac{6PRl}{d^4 \theta}$, $\therefore$ $G = 6\,T$.

208. Mr. Bevan finds the modulus T in cast iron, whose specific gravity is 7·163, to be as below.

940000
963000 } Mean 951600 ℔s.
952000

The moduli of wrought iron and steel were nearly equal to each other; and a mean from the results of eight experiments on iron and three on steel, gave, for T, 1779090 ℔s.

209. Mr. Bevan found the modulus T to be $\frac{1}{16}$th of the modulus of elasticity in metallic substances.

But it was shown above that $G = 6T$, $\therefore G = \frac{6}{16}$ of the modulus of elasticity; which differs from $\frac{2}{5}$, as computed by Cauchy (art. 198), only as 16 to 15.

210. Multiplying Mr. Bevan's mean values of T by six we obtain,

In cast iron . . . . . . $G = 951600 \times 6 = 5709600$ ℔s.
In wrought iron and steel . . $G = 1779090 \times 6 = 10674540$ ℔s.

211. Substitute in the formulæ (art. 197) the value of G, as obtained from the above experiments on cast iron, and transposing, we have as below.

For a cylinder . . . . . $PR = 8968620 \cdot \frac{r^4 \theta}{l}$.

For a square prism . . . . . $PR = 951600 \cdot \frac{d^4 \theta}{l}$.

For a rectangular prism . . $\Gamma R = 1903200 \cdot \frac{b^3 d^3 \theta}{(b^2 + d^2) l}$.

212. The experiments of Mr. Bevan seem to have been very carefully made, extra precaution being used both to prevent friction and to obtain correct measures; but a source of error may have arisen from computing, by a less accurate formula than Cauchy's, the results from those rectangular prisms which differed considerably from squares.

213. Some experiments of my own, upon the torsion of cylinders of wrought iron and steel, made some years since, at the request of Mr. Babbage, but not yet published, showed that the angle of torsion was very nearly as the weight, as had previously been shown by Savart (Annales de Chimie et de Physique, Aug. 1829).

214. The object of M. Savart's able Memoir was to compare the theory of torsion, as given by Poisson and Cauchy, with the results of experiment; and though his experiments were made on brass, copper, steel, glass, oak, &c.—and included none on cast iron—it may not be amiss to give here the general laws which he deduces from them. They are as below.

1st. Whatever be the form of the transverse section of the rods (subjected to torsion), the arcs of torsion are directly proportional to the moment of the force and to the length.

2nd. When the sections of the rods are similar, whether circular, triangular, square, or rectangular, much elongated, the arcs of torsion are in the inverse ratio of the fourth power of the linear dimensions of the section.

3rd. When the sections are rectangles, and the rods possess an uniform elasticity in every direction, the arcs of torsion are in the inverse ratio of the product of the cubes of the transverse dimensions, divided by the sum of their squares; from whence it follows that, if the breadth is very great compared with the thickness, the arcs of torsion will be sensibly in the inverse ratio of the breadth and of the cube of the thickness.

These laws, M. Savart observes, are in exact agreement with those of Cauchy, both for cylindrical and rectangular sections; showing that his formulæ (art. 197) are constructed on principles which may be applied with safety. When the elasticity is not uniform, the laws are somewhat modified.

215. In concluding this notice of experiments on the strength and other properties of cast iron, which may, perhaps, on a future occasion be extended in various ways, I would refer the reader desirous of information on the effects of expansion and contraction, upon structures, by heat, to a Memoir 'on the Expansion of Arches' through the changes of ordinary temperature, by Mr. George Rennie. This Memoir contains experiments on the rise of the arches in the Southwark Bridge, which is of cast iron, having three rows of arches in length, containing in the whole about 5560 tons of iron. Mr. Rennie made experiments upon the rise of the arches in each row; from which it appears that the rise of an arch, whose span is 246 feet and versed sine 23 feet 1 inch, is about $\frac{1}{40}$th of an inch for each degree of Fahrenheit, making $1\frac{1}{4}$ inch for a difference of 50°. Mr. Rennie gives a Table of experiments of his own upon the expansion of iron and stone, with others from M. Destigny; and concludes that there is no more danger to the stability of iron bridges, from the effects of expansion and contraction, than to those of stone; for when the abutments are firmly fixed, the arches have no alternative but to rise or fall.

The effects of percussion and vibration upon bodies, particularly cast iron, have been much further inquired into since the time of Tredgold; and upon these subjects I beg to refer the reader to a Memoir of my own on the effects of "Impact upon Beams," in the 5th Report of the British Association, 1835. The object of this Memoir was to compare theory with experiment, deducing practical conclusions.

THE END.

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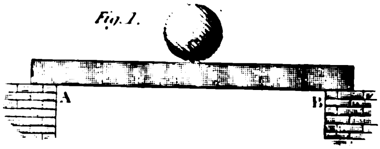
Fig. 1.



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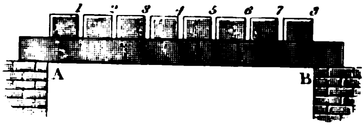


Fig. 3.

Fig. 4.

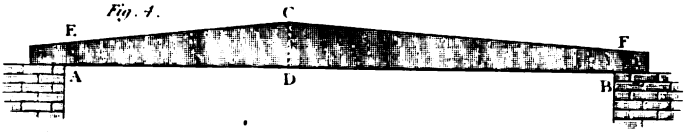
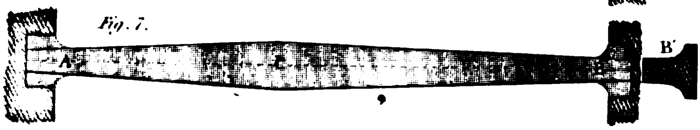


Fig. 5.

Fig. 6.

Fig. 7.



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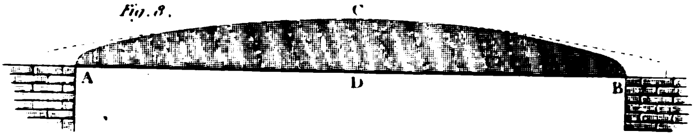

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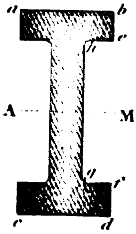

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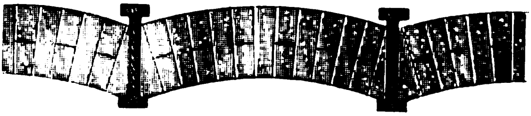

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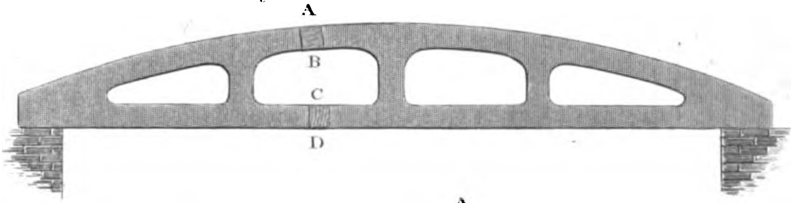
PLATE 11

Fig 11.



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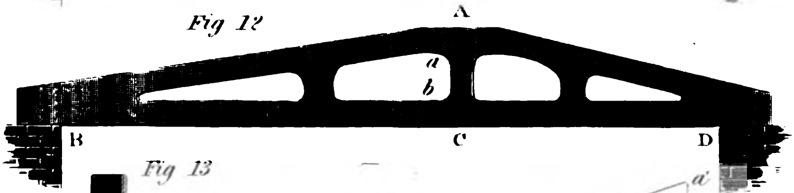

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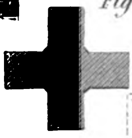

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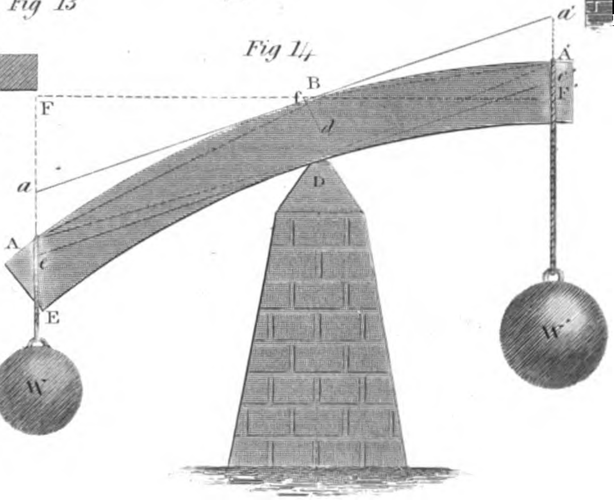

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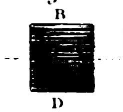

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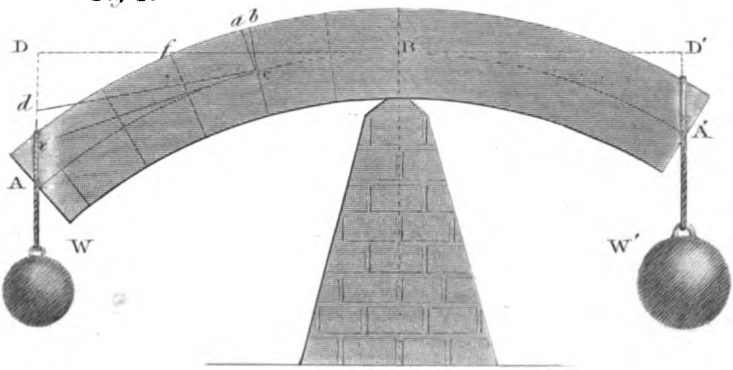

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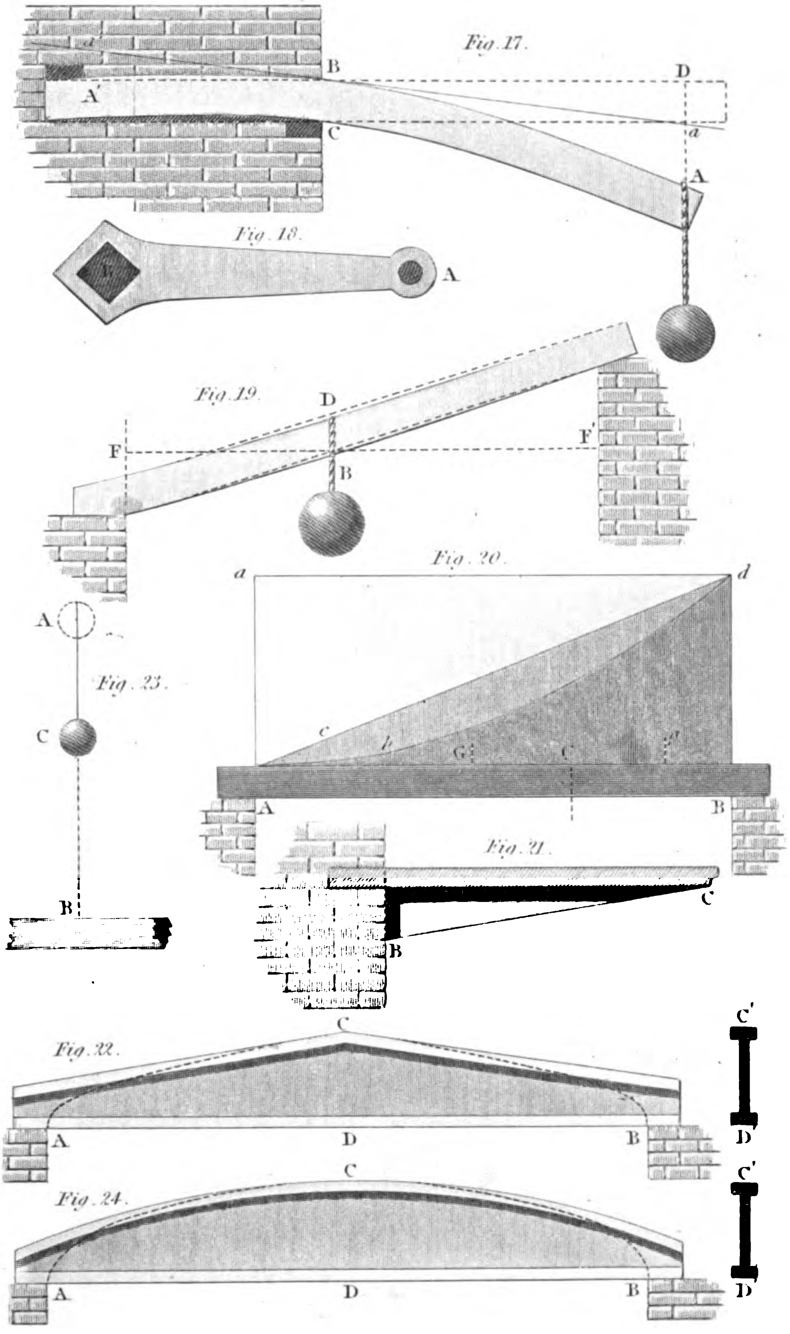

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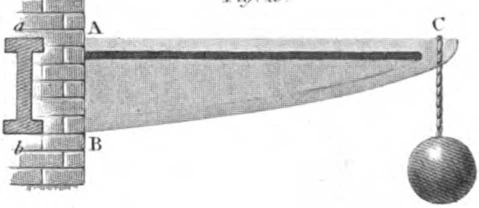

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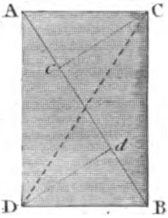

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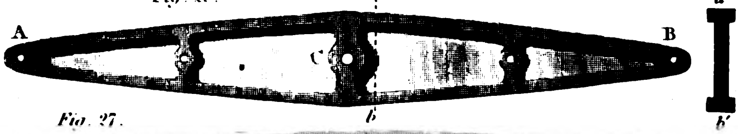

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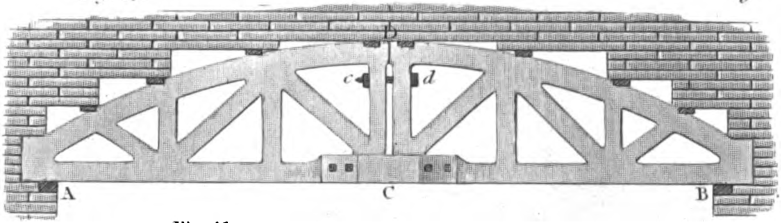
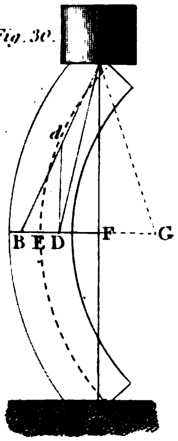


Fig. 28.

Fig. 30.



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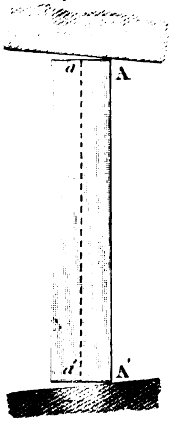

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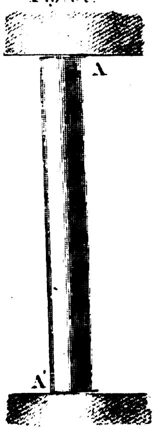

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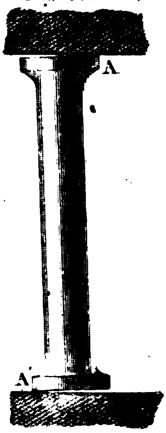

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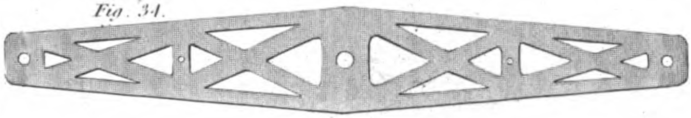

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