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NUMBER 89

THE THEORY OF SHRINKAGE AND FORCED FITS

With Tabulated Data and Examples from Practice

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SECOND EDITION

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INTRODUCTION

FORCED AND SHRINKAGE FITS

A shrinkage fit is a cylindrical or slightly conical joint between two machine members, as a crank-web and a shaft, in which the bore of the outer member or crank is smaller than the diameter of the inner member or shaft, so that the outer member must be expanded by heat before it can be set in place, while, in the subsequent cooling, it contracts and grips the inner member with a force which depends on the character of the metals, on the thickness of the outer member, and on the difference between the original diameter of the bore and that of the inner member. This difference is called the *allowance for shrinkage.* A forced fit is based on the same principle and is virtually of the same character, except that the parts are forced together when cold by hydraulic or other pressure.

These fits have a wide range of application, extending from small machine parts to built-up crank-shafts for heavy engines and the massive forgings for high-powered guns. As a rule, the forced fit is restricted to parts of small or moderate size, while shrinkage joints have no such limitations, being applicable especially where a maximum "grip" is desired, or, as in ordnance, where accurate results as to the intensity of the stresses produced in the parts thus united, are required. With both types, skillful machining and care in assembling are essential; but the shrinkage joint is compact, has the fewest possible parts, is secure against slip to the extent for which it was designed, and is tight against fluid pressure.

The fundamental principle governing the construction of the joint is the same with both types: the bore of the outer hub or other member is smaller, and the diameter of the pin or shaft larger, than the diameter of the finished fit. Hence, the inner member is compressed, the outer expanded, and the elasticity of the metals produces a radial pressure at the contact-surfaces of the fit, which pressure gives the fit its resistance to slip. The same principle is applied in the rolled joints used in expanding the ends of boiler tubes in place, although, in this case, the process is reversed, the hollow inner member or tube being stretched by rolling so that, if free, it would be greater in diameter than the hole in the tube-sheet or header.

As the integrity of the fit thus depends on the elasticity of the metals of the members, and as the formulas which follow are based on this elasticity and on the actions which occur during expansion and compression, it may be well to review these actions briefly and to give the sense in which the various terms relating to them are used in this treatise.

CHAPTER I

PRELIMINARY CONSIDERATIONS

Stress—Deformation—Lateral Contraction

An external force applied to a body acts, partially or wholly, to change the shape of the latter. A *stress* is the force acting within the body to oppose this change of shape. The *unit stress* is the stress on a unit of area of the cross-section. Thus, if the upper end of a steel rod, one inch square, be fixed, and a weight of 10,000 pounds be suspended from the lower end, the unit-stress on the metal will be 10,000 pounds; if the sectional area of the rod be two square inches and the weight remain the same, the unit-stress will be $10{,}000 \div 2 = 5{,}000$ pounds. Stresses may be either tensile (those that tend to elongate the body), compressive (those that will shorten it, as in a column), or shearing (which act to cut across the body, as in punching a rivet hole). Both tensile and compressive stresses may act at the same time, in the same line, on the same body, in which case the resultant stress will be the difference between the two, and in kind like the greater. Tensile stresses are usually considered as positive, and compressive stresses as negative, the resultant stress being their algebraic sum.

An external force not only puts the material under stress, but also causes some, usually slight, change in its shape. This change is called a *deformation*, and this deformation may be, under tension, an elongation; under compression, a shortening; or, under shearing, a detrusion or thrusting aside of the metal. The unit-deformation is the change in shape of a unit of the original length of the body. Thus, if a rod, 50 feet (600 inches) long, be stretched one inch by an applied load, the unit-deformation will be 1/600 of an inch.

A stress, tensile or compressive, has not only full effect in its line of action, but also produces compression in a direction at right angles to that line. This action is called *lateral contraction*. Thus, referring to Fig. 1, if the short length between the planes *ab* and *cd* of a rectangular bar be subjected to the unit tensile stress T at right angles to the ends *ab* and *cd*, the stress in planes parallel to the line of action of T will be equal to T; but the stretching of the metal in the direction of this line causes a contraction in the directions which are perpendicular to it. This contraction is equivalent to that which would be caused by a unit compressive stress P_1, acting on the sides *bc* and *ad*, and by a similar stress P_2 acting on the sides *ac* and *bd*. The magnitude of these induced compressive stresses depends on the metal. For wrought iron and steel, P_1 and P_2 are each taken usually as equal to $1/3\ T$; for cast iron, the ordinary values are about $1/4\ T$. This fraction, 1/3 or 1/4, is called the *factor of lateral contraction*, which factor will be designated by ϕ in the following. "Poisson's ratio," which is a constant used to determine the lateral effect of direct stress, refers to the same action.

If the unit-stress T, Fig. 1, had been compressive instead of tensile, there would still have been compression on planes parallel to its line of action, but that compression would then act outward from, instead of inward toward, the axis of the body. The lateral effect would be to elongate, not to contract. So far as is known, the factor of lateral contraction has the same value in compression as in tension. Thus, in Fig. 1, assume that P_1 and P_2 are direct compressive stresses and that there is no direct tensile stress like T. Then P_1 and P_2 will each

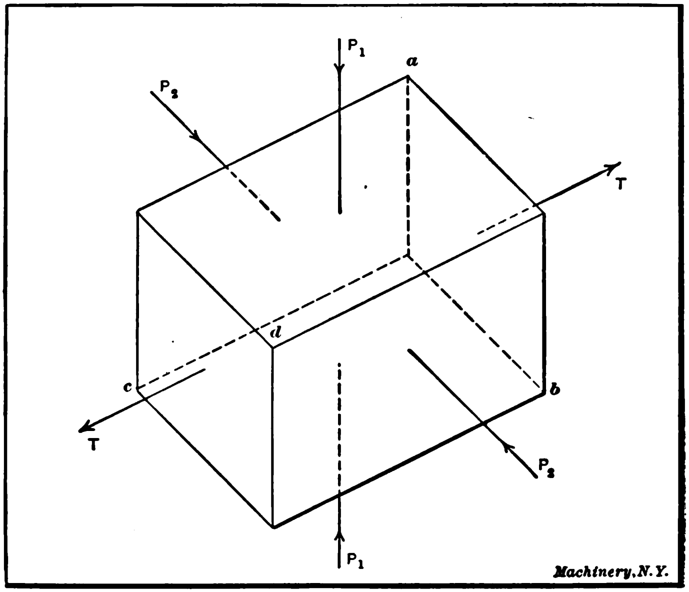

Fig. 1. Lateral Contraction Induced by Direct Tensional Stresses

develop lateral and equivalent tensile stresses, so that the actual unit stresses will be:

In the direction of T, $\phi\ (P_1 + P_2)$.

In the direction of P_1, $\phi\ P_2 - P_1$.

In the direction of P_2, $\phi\ P_1 - P_2$.

A stress thus developed by lateral action is identical in effect with a direct stress of its direction and magnitude. The direct stress, which does not consider lateral contraction, if the latter exist, is known as the *apparent stress*, while the *true stress* is the algebraic sum of the apparent stress and the stresses in its direction due to lateral action. It should be borne in mind that the true stress is the actual stress to which the body is subjected and by which the deformation is caused. Merriman says in "Mechanics of Materials," edition of 1899, page 291: "The true resistance of a body depends upon the actual deformations produced, and these are measured by the true internal stresses."

When there are several direct stresses acting on a body, the use of a general equation in which all stresses are assumed to be tensile, will prevent error in ascertaining the true stress in any given direction. Thus, let there be three direct or apparent tensile stresses, t_1, t_2 and t_3, applied to the three sets of parallel sides of the body in Fig. 2, and let T_1, T_2, and T_3 be the corresponding true stresses. Then:

$$T_1 = t_1 - \phi\, t_2 - \phi\, t_3$$

which is the general equation for this stress. If t_2 had been a compressive stress, the equation would be:

$$T_1 = t_1 - \phi\,(-t_2) - \phi\, t_3 = t_1 + \phi\,(t_2 - t_3)$$

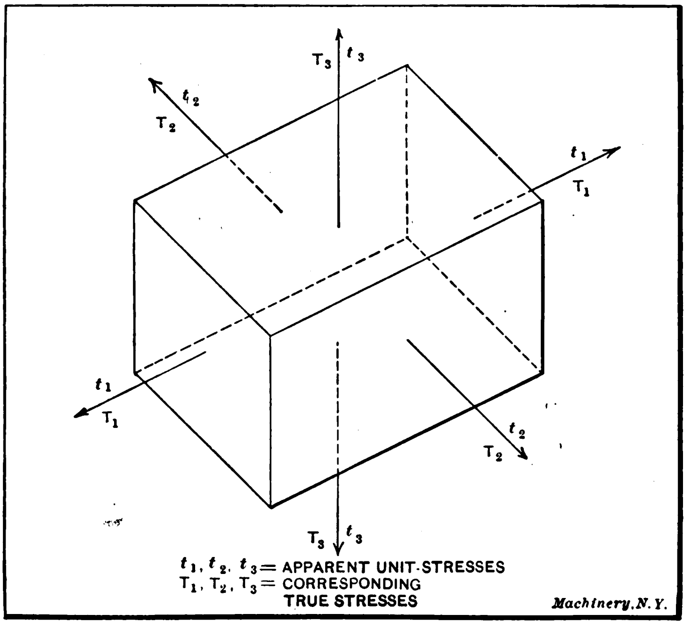

Fig. 2. True and Apparent Stresses

In this way, by writing the general equation for each stress on the assumption that all are tensile, and then changing the signs of those which are compressive, the true stresses are readily found.

Elastic Limit—Modulus of Elasticity

The *elastic limit* is that unit-stress at which the elasticity of the metal begins to disappear, that is, the stress at which it will not wholly regain its original form after the removal of the stress, and, hence, at which some "permanent set" makes its appearance. Theoretically, this limit occurs at a definite point, but experimentally it cannot be sharply marked, and is taken as the stress at which the "set" becomes fully distinguishable. Within the elastic limit, the deformation is approximately proportional to the stress producing it; beyond that limit, this ratio is no longer constant. General values of the elastic limit are: Cast iron, in tension, 6000, and in compression, 20,000 pounds per square inch; wrought-iron and steel, in either tension or compression, 25,000 and 50,000 pounds per square inch, respectively. These values, however, differ considerably for different kinds of steel, and also depend upon its treatment.

The modulus or coefficient of elasticity, E, is the ratio of a unit-stress to the unit-deformation which that stress produces. Thus, if S is the stress and s the deformation, $E = S \div s$. E is a constant for each similarly treated metal until the stress reaches the elastic limit. General values of E, for either tension or compression, are: Cast iron, 15,000,000; wrought-iron, 25,000,000; steel, 30,000,000.

Shrinkage Stresses—Approximate Method (Tires)

When the thickness of the outer member of a shrinkage fit is relatively small as compared with the diameter of the inner member, as is the case with a locomotive wheel-center and tire, the compression of the inner member is negligible in practice and the radial pressure on the fitted surfaces may be considered as expended wholly in producing stresses in the outer member. In a tire thus shrunk on, there are two stresses, one radial and compressive, and the other the circumferential or "hoop" stress which acts tangentially on a diametral plane to burst the tire. This tangential or hoop stress is the only one requiring consideration.

Let R_0 = original internal radius of tire,

R = radius of wheel-center,

t = mean unit tensile hoop stress in tire when expanded,

e_t = unit-deformation (elongation) due to t,

$E = \dfrac{t}{e_t}$ = modulus of elasticity,

p = unit radial pressure on fitted surfaces of wheel-center and tire,

b = width of tire, axially,

T = thickness of expanded tire, radially,

f = coefficient of friction at fitted surfaces.

The deformation or elongation per unit of length of the tire may be taken as equal to the increase in length of the latter by expansion, divided by the original internal length. Since the length of the circumference is directly proportional to that of its radius, we have:

$$e_t = \frac{R - R_0}{R_0}$$

$$t = Ee_t = E \times \frac{R - R_0}{R_0}$$

The expanded tire is virtually in the condition of a cylinder subjected at all points internally to the outward pressure p. The force tending to rupture such a cylinder on a diametral plane is equal to the projected area of the cylinder, multiplied by the internal pressure, or:

$$2R \times b \times p$$

and the resistance opposed by the tire to rupture is equal to the product of its sectional area by the average hoop stress, or:

$$2b \times T \times t$$

Equating the force and resistance, and substituting the value of t, we have:

$$p = \frac{Tt}{R} = \frac{ET\,(R - R_0)}{R\,R_0}$$

Multiplying the area of the fitted surface by the radial pressure and the coefficient of friction, the total resistance to slip is:

$$2\,\pi R \times b \times f \times p = 2\,\pi\,b\,f\,ET\left(\frac{R - R_0}{R_0}\right)$$

As an example, assume that a steel tire, 5½ inches wide and 3½ inches thick, is shrunk on a wheel-center 66 inches in diameter. Let the allowance for shrinkage be about 0.001 inch per inch of diameter, or 0.070 inch, total. Then $R = 33$ inches, $R_0 = 33 - \frac{0.070}{2} = 32.965$ inches, and, taking E as 30,000,000, the average tensile stress in the tire is 31,900 pounds per square inch, which is well within an elastic limit of 50,000 pounds. This value of t gives $p = 3380$ pounds per square inch, and, taking $f = 0.2$, the total resistance to slip is approximately 385 tons of 2000 pounds each.

This method is approximate for several reasons:

1. As we have assumed no compression in the wheel-center, the value e_t, as given in the first equation, is really the unit-deformation at the inner surface of the tire, where that deformation is a maximum, so that the value found for t is, as an average stress, too high, as is that of p also; thus, the compression of the wheel-center, if considered, would slightly reduce the average tensile stress.

2. The lateral contraction, due to the radial stress in the tire, is neglected, and this action would increase the tensile stress, as found above.

3. The tensile stress is assumed to be uniform over the cross-section of the tire, while it is really a maximum (see Fig. 5) at the fitted surface. As the thickness of the tire is relatively small as compared with its diameter, the aggregate error will not be material, if the shrinkage-allowance is moderate as in this case.

CHAPTER II

DERIVATION AND APPLICATION OF LAME'S FORMULAS

When the outer member of a shrinkage fit is relatively thick, as a wheel-hub or a crank-web, the approximate method given in the previous section will not serve, and recourse must be had to the formulas deduced for the investigation of the stresses in thick cylinders subjected to radial pressure—this pressure being internal for the outer member of the fit and external for the inner member. As in the tire, there are two "apparent" stresses in such a cylinder, the tangential or "hoop" stress, and the radial stress. The latter is always compressive; the former, in a shrinkage fit, is tensile in the outer member and compressive in the inner, while, in a gun, built up of superposed cylinders, it may be either tensile or compressive, as the location of the cylinder and the magnitude of the powder pressure determine. In any event, the tangential and radial stresses are interdependent; they affect each other by lateral contraction; and, through the latter action, they produce in the outer member a longitudinal compressive stress, parallel to the axis of the fit.

Various formulas have been proposed for the determination of the stresses in thick cylinders. Those founded on the principles established by Lamé have found general acceptance, since they avoid the assumptions on which others are based. Their close approach to accuracy is shown by their use in the design of high-powered guns, in which the stresses at the instant of explosion are very near the elastic limit of the metal. Lamé's fundamental formula may be deduced in several ways; the method* given below is due to Professor P. R. Alger, U. S. Navy, of the Bureau of Ordnance.

Fig. 3 represents a thick, hollow cylinder subjected to internal and external fluid pressure; the cylinder is assumed to be free at the ends, in order to prevent direct longitudinal stress.

Let $P_0 =$ internal unit pressure,
$P_1 =$ external unit pressure,
$R_0 =$ internal radius of cylinder,
$R_1 =$ external radius of cylinder,
$r =$ radius of any point within cylinder walls,
$t =$ "apparent" tensile tangential or "hoop" unit stress at radius r,
$p =$ "apparent" radial compressive unit-stress at radius r,
$l =$ "true" longitudinal unit-stress at radius r, due to lateral contraction,

* Cathcart, "Machine Design: Fastenings," New York, 1903, page 30.

T_0 = "true" tangential stress at inner surface of radius R_0,
T_1 = "true" tangential stress at outer surface of radius R_1,
e_t = unit-deformation due to "true" tangential stress at radius r,
e_p = unit-deformation due to "true" radial stress at radius r,
e_l = unit-deformation due to "true" longitudinal stress at radius r,
ϕ = factor of lateral contraction = 1/3 for steel,
E = modulus of elasticity = 30,000,000 for steel.

In this deduction, it is assumed:

a. That there is no direct longitudinal stress in any layer of the cylinder walls.

b. That a transverse section of the cylinder when not under pressure, remains a plane normal to the axis of the cylinder when the latter is

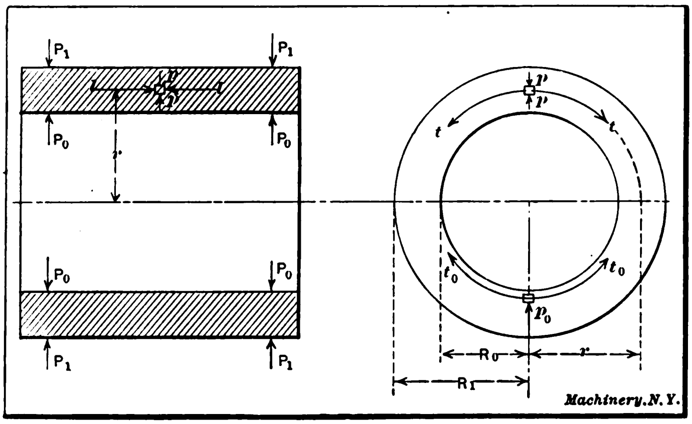

Fig. 3. Thick Hollow Cylinder Subjected to Internal and External Fluid Pressure

under stress, *i.e.*, that the longitudinal stress due to lateral contraction is uniform over the whole cross-section.

c. That the total or "true" stress in any direction is the measure of the tendency to yield in that direction.

d. That the factor of lateral contraction is equal to 1/3.

The true stresses in the indefinitely thin cylinder of radius r are:

$$\text{tangential unit-stress} = t - (-\phi\, p) = t + \frac{p}{3}$$

$$\text{radial unit-stress} = -p - \phi\, t = -\left(p + \frac{t}{3}\right)$$

$$\text{longitudinal unit-stress} = -\phi\, t + \phi\, p = -\left(\frac{t}{3} - \frac{p}{3}\right)$$

By the definition of the modulus of elasticity, the corresponding unit-deformations are:

$$\left.\begin{array}{l} e_t = (t + p/3) \div E \\ e_p = -(p + t/3) \div E \\ e_l = -(t/3 - p/3) \div E \end{array}\right\} \qquad (1)$$

Since, by hypothesis, e_l is constant:

$$t - p = \text{constant} = k$$

But,

$$\int_{R_0}^{R_1} t\,d\,r = P_0 R_0 - P_1 R_1$$

and, assuming $t = f'(r)$, this gives:

$$f(r)\Big]_{R_0}^{R_1} = P_0 R_0 - P_1 R_1$$

whence $f(r) = -pr$; and so $t = f'(r) = -p - \dfrac{r\,d\,p}{d\,r}$.

Thus, we have

$$t - p = k, \text{ and } t + p = -\frac{rdp}{dr}$$

whence $2p + k = -\dfrac{rdp}{dr}$ the integration of which gives:

$$2p + k = \frac{k_1^2}{r^2}$$

where k_1 is a constant of integration. Combining with $t - p = k$, we have $t + p = \dfrac{k_1^2}{r^2}$.

The equations which express the relation between "hoop" or tangential tension and radial stress at all points within the cylinder walls are then:

$$t - p = k = T_0 - P_0 = T_1 - P_1$$

$$(t + p)\,r^2 = k_1^2 = (T_0 + P_0)\,R_0^2 = (T_1 + P_1)\,R_1^2$$

Eliminating T_1 between the last parts of these equations, we have:

$$T_0 = \frac{P_0\,(R_1^2 + R_0^2)}{R_1^2 - R_0^2} - \frac{2R_1^2 P_1}{R_1^2 - R_0^2}$$

and substituting this in the first parts of the same equations, we have, after combining:

$$t = \frac{P_0R_0^2 - P_1R_1^2}{R_1^2 - R_0^2} + \frac{R_0^2\,R_1^2(P_0 - P_1)}{R_1^2 - R_0^2} \times \frac{1}{r^2} \qquad (2)$$

$$p = -\frac{P_0R_0^2 - P_1R_1^2}{R_1^2 - R_0^2} + \frac{R_0^2\,R_1^2\,(P_0 - P_1)}{R_1^2 - R_0^2} \times \frac{1}{r^2} \qquad (3)$$

which are Lamé's fundamental formulas for the "apparent" stresses in a thick cylinder subjected, internally and externally, to fluid pressure. In deriving these formulas, p has been taken as a compressive stress. If it had been assumed to be tensile, the signs in Equation (3) would have been reversed. With this change, however, it will be found that, in the shrinkage fit, this equation will give negative values, showing that p is a compressive stress. To obtain the "true" or actual stresses, the values of t and p from (2) and (3) are modified in the succeeding equations for the effect of lateral contraction, according to the methods of Clavarino.

Application of Lame's Formulas to Compound Cylinders

The shrinkage fit is applied to a compound cylinder, *i. e.*, to two cylinders, one superposed on the other. The inner cylinder may be solid, as in the ordinary shaft or hollow, as shafts and large crank-pins of steel are often made. Fig. 4 represents such a compound cylinder, the conditions being the same as in Fig. 3, except that the radial pressure P_1 is, in Fig. 4, produced by the shrinkage of the outer cylinder of ex-

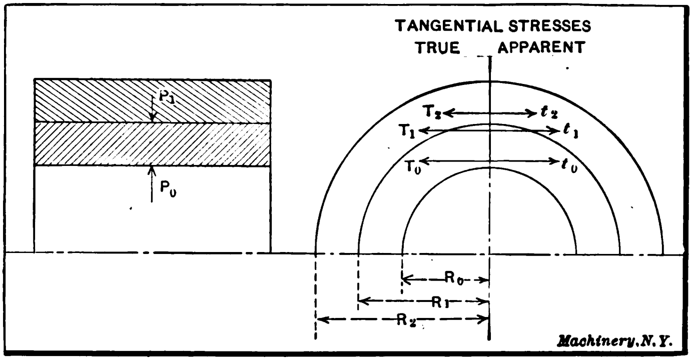

Fig. 4. Compound Cylinder consisting of an Outer Cylinder shrunk onto an Inner

ternal radius R_2. There is no external pressure on this cylinder, except that of the atmosphere, which is negligible. In the shrinkage fit, the metals of the inner and outer members may not be the same, and the tangential stresses in the two cylinders at the contact surface also differ.

Let E = modulus of elasticity, outer cylinder,

E_1 = modulus of elasticity, inner cylinder,

ϕ = factor of lateral contraction, outer cylinder,

ϕ_1 = factor of lateral contraction, inner cylinder,

t_0 = apparent tangential unit-stress, inner surface of inner cylinder,

t_1 = apparent tangential unit-stress, outer surface of inner cylinder.

T_0 and T_1 = corresponding true tangential stresses,

p_0 and p_1 = corresponding apparent radial stresses,

t_2 = apparent tangential unit-stress, inner surface of outer cylinder,

T_2 = corresponding true tangential stress,

p_2 = corresponding apparent radial stress.

It should be observed that, in deriving Equation (2), t was assumed to be a tensile stress. Therefore, in the deductions by substitution which follow, if the formula gives a negative value, the stress t_0 or t_1, which represents t for these conditions, is compressive. Similarly in Equation (3) p is by hypothesis always a compressive stress, and the formula gives, in the substitutions, simply its numerical value, as p_1, p_2, etc., for various conditions, and these values, when used in the equations for the true stresses, should have the minus sign.

Outer Cylinder

In a shrinkage fit, the only important stress in this cylinder is the true tangential stress at the inner surface, where that stress is a maxi-

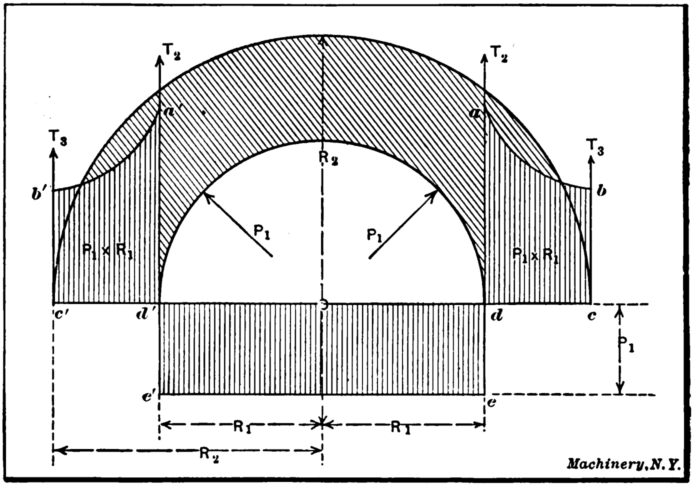

Fig. 5. Graphical Representation of Stresses produced by Shrinkage Fits

mum. (See Fig. 5). Since, for equilibrium, the pressure P_1 from the outer cylinder must be opposed by an equal and opposite pressure from the inner cylinder, the former cylinder is virtually under the same conditions as the latter, except that it is not subjected to external pressure. Hence, Equations (2) and (3) may be applied to the outer cylinder, by changing R_0 to R_1, R_1 to R_2, P_0 to P_1, and P_1 to zero. Making these substitutions and with $r = R_1$, we then have the apparent unit-stresses in the outer cylinder at the inner surface:

$$t_2 = \frac{P_1(R_2^2 + R_1^2)}{R_2^2 - R_1^2} \qquad (4)$$

$$p_2 = P_1 \qquad (5)$$

Considering lateral contraction, the corresponding true tangential tensile unit-stress is:

$$T_2 = t_2 - (-\phi\, p_2) = t_2 + \phi\; p_2$$

$$T_2 = P_1 \left(\frac{R_2^2 + R_1^2}{R_2^2 - R_1^2} + \phi \right) \qquad (6)$$

Inner Cylinder, Hollow

This cylinder corresponds to a hollow shaft forming the inner member of a shrinkage fit. The stresses to be found are the true tangential stress at the outer surface, which is required to determine the allowances, and the similar stress at the inner surface, since the tangential stress in such a cylinder is compressive and reaches its maximum at the bore (See Fig. 6). Equations (2) and (3) are applicable, if P_0 be made equal to zero, since there is only the atmospheric pressure on the bore of the shaft.

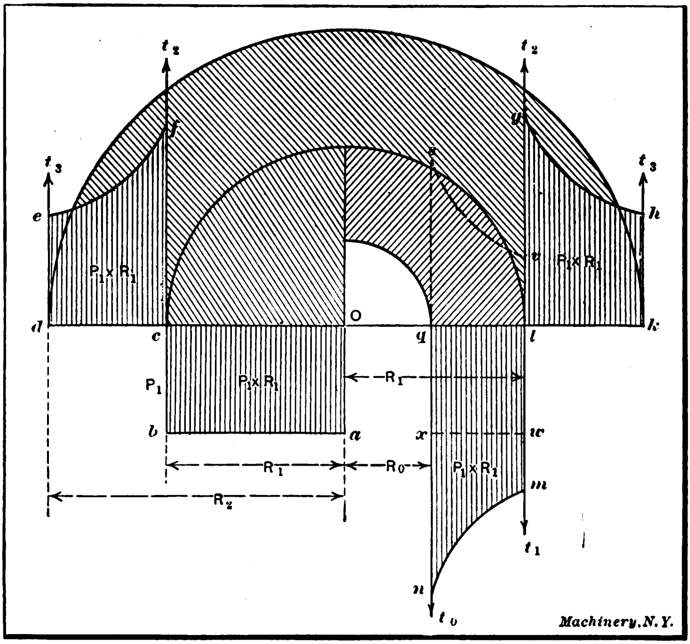

Fig. 6. Graphical Representation of Stresses produced by Shrinkage Fits

Making $r = R_1$, and $P_0 =$ zero, we have the apparent unit-stresses in the inner cylinder at the outer surface:

$$t_1 = -\frac{P_1\,(R_1^2 + R_0^2)}{R_1^2 - R_0^2} \tag{7}$$

$$p_1 = P_1 \tag{8}$$

The corresponding true tangential compressive stress is:

$$T_1 = t_1 - (-\phi_1\,p_1) = t_1 + \phi_1\,p_1$$

$$T_1 = -P_1\left(\frac{R_1^2 + R_0^2}{R_1^2 - R_0^2} - \phi_1\right) \tag{9}$$

For the inner surface, $r = R_0$, and $P_0 =$ zero in Equations (2) and (3). The apparent stresses, therefore, are:

$$t_0 = -P_1 \times \frac{2 R_1^2}{R_1^2 - R_0^2} \tag{10}$$

$$p_0 = 0 \tag{11}$$

Since $p_0 = 0$, the true tangential compressive stress is:

$$T_0 = t_0 = -\frac{2 P_1 R_1^2}{R_1^2 - R_0^2} \tag{12}$$

which is evidently greater, numerically, than T_1.

Inner Cylinder, Solid

If the inner cylinder be solid, the conditions will correspond with those of a solid shaft forming the inner member of the fit. Equations (2) and (3) will apply, if R_0 and P_0 be made equal to zero. The only stress of importance is the tangential stress at the outer surface, which is required in determining the allowances.

Making these substitutions, the apparent stresses at the outer surface are:

$$t_1 = -P_1 \tag{13}$$

$$p_1 = P_1 \tag{14}$$

The true tangential compressive stress is, therefore:

$$T_1 = t_1 - (-\phi_1 p_1) = t_1 + \phi_1 p_1$$

$$T_1 = -P_1 (1 - \phi_1) \tag{15}$$

The values given in Equations (13), (14) and (15) are valid for any point between the outer surface and the center of a solid shaft, since, if in Equations (2) and (3), R_0 and P_0 be made equal to zero, the second term of the right-hand member of each equation vanishes, no matter what value may be given to r, the radius of the point considered. In general, therefore, in a solid shaft subjected to a uniform external radial pressure, the true radial and tangential compressive stresses are equal at all points, and the intensity of each is uniform throughout.

CHAPTER III

FORMULAS FOR STRESSES IN THE HUB

As shown in Fig. 5, the tangential tensile stress in the hub reaches its maximum at the inner surface and decreases rapidly from that surface outward. The true stress at the bore is therefore of primary importance, since the metal is under its greatest stress there. This stress must not exceed the elastic limit, and is one of the factors which determine the "grip" of the fit. In Equation (6), the radii are those of the expanded hub, and the use of these dimensions would make computation complex. No material error will be caused by the substitution for them of the corresponding nominal radii, *i. e.*, those of the hub before expansion, and thus disregarding the allowances which are but a few thousandths of an inch.

Let D_1 = nominal internal diameter of hub,

D_2 = nominal external diameter of hub,

$$a = \frac{R_2^2 + R_1^2}{R_2^2 - R_1^2} = \frac{D_2^2 + D_1^2}{D_2^2 - D_1^2};$$

$\phi = 1/3$ for steel and 1/4 for cast-iron.

Substituting in Equation (6):

$$T_2 = P_1 (a + \phi) \qquad (16)$$

$$T_2 = P_1 \times \frac{4D_2^2 + 2D_1^2}{3(D_2^2 - D_1^2)} \text{ for steel,} \qquad (17)$$

$$T_2 = P_1 \times \frac{5D_2^2 + 3D_1^2}{4(D_2^2 - D_1^2)} \text{ for cast-iron.} \qquad (18)$$

Resistance of Hub to Bursting Load

The relation between the bursting load on the hub, due to the radial pressure on the fit, and the true tangential stress which resists it, is shown graphically in Fig. 5. If a cylinder be subjected to the unit internal radial pressure P_1, the force tending to burst it on a diametral plane is equal, for a section of unit length, to the product of this pressure by the diameter, or $P_1 \times 2R_1$, which is the area of the load-diagram *dee'd'*. This bursting load is resisted by, and equal to, the sum of the true tangential stresses in the cylinder-walls, which sum is represented by the two equal stress-diagrams, *abcd* and *a'b'c'd'*. Hence:

Load-area *dee'd'* = 2 × stress-area *abcd*.

The stress-area is laid out by plotting as ordinates on the diameter the values of the true tangential stress, $t + \phi p$, as found by the methods on page 13, and giving r various values from R_1 to R_2. The average tensile unit-stress in the cylinder-wall, or in the hub in this case, is equal to the area of the load-diagram, divided by the thickness of the hub *i. e.*, $\frac{P_1 R_1}{R_2 - R_1}$.

Fig. 5 shows that it is impossible for the shrinkage-load on the hub to burst that member, so long as the true hoop stress T_2 at the bore does not exceed the ultimate tensile stress of the metal. Again, dividing Equation (5) by (4), we have from the apparent stresses:

$$\frac{p_2}{t_2} = \frac{P_1}{t_2} = \frac{R_2^2 - R_1^2}{R_2^2 + R_1^2}, \tag{19}$$

which equation proves that the radial pressure P_1 at the fit can never be equal to the apparent hoop stress t_2 in the hub at the bore, even if t_2 be the ultimate tensile strength and R_2 be increased indefinitely. This is again shown by the fact that the equation may be transformed into

$$\frac{R_2}{R_1} = \sqrt{\frac{t_2 + P_1}{t_2 - P_1}},$$

from which it appears that if $P_1 = t_2$, R_2 becomes infinite, *i. e.*, no thickness whatever will prevent rupture. This condition fixes the useful limit of thickness of a cylinder, not reinforced by one or more enclosing cylinders so shrunk on as to put the innermost cylinder under exterior compression. No unsupported cylinder can be made thick enough to withstand an internal pressure per square inch which is as great as, or greater than, the ultimate tensile strength of the metal.

Rankine gives in "Applied Mechanics," London, 1869, page 293:

$$\frac{R_2}{R_1} = \sqrt{\frac{T + P_1}{T - P_1 + 2\,P_2}}$$

in which T is the ultimate tensile strength of the metal of the cylinder. From this equation it follows that if the internal pressure P_1 is equal to or greater than the sum $T + 2\,P_2$, of the ultimate strength and twice the external pressure, no thickness, however great, will enable the cylinder to resist the pressure.

With regard to the possible intensity of shrinkage-stresses, it should be borne in mind that shrinkage fits are usually made on the working parts of machines, and hence that the stresses due to shrinkage may be increased by others developed by the external forces applied to the member when the machine is in operation. In such cases, the total stress which will exist at any time should be considered in determining the shrinkage-allowances.

Effect of Thickness of Hub on Resistance to Slip

The principle governing the effect of the thickness of hub on the resistance to slip may be seen most readily from the formulas for the apparent stresses. Thus, Equation (19) shows that if the radius of the fit and the tangential stress at the bore of the hub are constant, the effect of variation in the external radius is simply to change the intensity of the radial pressure P_1 at the fit—a greater hub-thickness increasing the "grip," and a smaller decreasing it. Thus, if $R_2 = 2\,R_1$, $P_1 = 0.6\,t_2$; if $R_2 = 3\,R_1$, $P_1 = 0.8\,t_2$, etc.

From Equations (17) and (18), we have:

$$P_1 = T_2 \times \frac{3\,(D_2^2 - D_1^2)}{4\,D_2^2 + 2\,D_1^2} \text{ for steel,} \qquad (20)$$

TABLE I

Values of Ratio A, as computed from Equation (22).

Ratio of Nominal Diameters of Hub, $\frac{D_2}{D_1}$	Ratio $A = \frac{P_1}{T_2}$ Steel ($\phi = \frac{1}{3}$)	Ratio $A = \frac{P_1}{T_2}$ Cast Iron ($\phi = \frac{1}{4}$)	Ratio of Nominal Diameters of Hub, $\frac{D_2}{D_1}$	Ratio $A = \frac{P_1}{T_2}$ Steel ($\phi = \frac{1}{3}$)	Ratio $A = \frac{P_1}{T_2}$ Cast Iron ($\phi = \frac{1}{4}$)
1.5	0.341	0.351	2.8	0.615	0.648
1.6	0.382	0.395	3.0	0.632	0.666
1.8	0.449	0.466	3.2	0.645	0.682
2.0	0.500	0.522	3.4	0.657	0.695
2.2	0.539	0.565	3.6	0.666	0.706
2.4	0.570	0.599	3.8	0.675	0.715
2.6	0.595	0.626	4.0	0.682	0.723

$$P_1 = T_2 \times \frac{4\,(D_2^2 - D_1^2)}{5\,D_2^2 + 3\,D_1^2} \text{ for cast-iron,} \qquad (21)$$

which give the values of the radial pressure at the fit in terms of the true tangential stress at the bore of the hub.

From Equation (16):

$$\frac{P_1}{T_2} = \frac{1}{a + \phi} = \frac{D_2^2 - D_1^2}{D_2^2\,(1 + \phi) + D_1^2\,(1 - \phi)} = A \qquad (22)$$

a ratio which is of service in computing the allowances. Table I gives values of A for various diametral ratios. If the true tangential stress T_2 is known or assumed for any of the diametral ratios tabulated, the intensity of P_1, and hence the resistance of the fit to slip may be found by multiplying T_2 by the corresponding value of A.

CHAPTER IV

FORMULAS FOR STRESSES IN THE SHAFT

The radial and tangential stresses in the inner member are, as shown previously, both compressive. To both, the same principle applies: each is a measure of the deformation in its direction only at the point where the given intensity of stress exists. If, for example, the radial stress varies from the circumference to the center, its intensity at any given point will not measure the deformation of the entire radius of the member, but only the amount of deformation at the point considered. The only stress which will cover both cases—solid and hollow shafts—and give the reduction in the external diameter of the member, is, therefore, the true tangential stress at the outer surface, since the circumference of that surface and its diameter must decrease together. As with the hub, the nominal diameters may be substituted for the corresponding dimensions of the compressed shaft.

Let D_0 = nominal internal diameter of hollow shaft,

D_1 = nominal external diameter of hollow or solid shaft,

$$h = \frac{R_1^2 + R_0^2}{R_1^2 - R_0^2} = \frac{D_1^2 + D_0^2}{D_1^2 - D_0^2},$$

$$\frac{T_1}{P_1} = B$$

$\phi_1 = 1/3$ for steel and $1/4$ for cast iron.

Solid Inner Members

Equation (15) gives the true tangential stress at the outer surface. From that equation:

$$T_1 = -\,2/3\,P_1 \text{ for steel} \tag{23}$$

$$T_1 = -\,3/4\,P_1 \text{ for cast iron.} \tag{24}$$

Since T_1 is a compressive stress:

$$\frac{T_1}{P_1} = 1 - \phi_1 = B \text{ for solid inner members} \tag{25}$$

This ratio is of service in computing the allowances. In a solid shaft, both the radial and tangential stresses are, as mentioned before, uniform in intensity from the outer surface to the center, and are equal at all points.

Hollow Inner Members

Equation (9) gives the true tangential stress at the outer surface. From that equation:

$$T_1 = -\,P_1\,(h - \phi_1) \tag{26}$$

$$T_1 = -\,P_1 \times \frac{2\,D_1^2 + 4\,D_0^2}{3\,(D_1^2 - D_0^2)} \text{ for steel,} \tag{27}$$

and, since T_1 is compressive:

$$\frac{T_1}{P_1} = h - \phi_1 = B \text{ for hollow inner members.} \qquad (28)$$

Equation (12) gives the true tangential stress T_0 at the inner surface. From (12) and (27):

$$\frac{T_0}{T_1} = \frac{3\,D_1^2}{D_1^2 + 2\,D_0^2} \text{ for steel.} \qquad (29)$$

This expression shows the marked increase in the tangential stress from the outer surface to the bore.

The values of B for hollow steel shafts of various diametral ratios are given in Table II.

Work Done in Compressing Solid and Hollow Shafts

The compressibilities of solid and hollow shafts differ, the solid shaft being the stiffer. In a solid shaft under radial pressure, the radial and

TABLE II

Values of Ratio B for hollow steel shafts of external and internal diameters, D_1 and D_0, respectively.

$\frac{D_1}{D_0}$	$B = \frac{T_1}{P_1}$	$\frac{D_1}{D_0}$	$B = \frac{T_1}{P_1}$
2.0	1.333	3.0	0.917
2.5	1.048	3.5	0.844

For solid inner members Equation (25), $B = 2/3$ for steel and 3/4 for cast iron.

tangential stresses are equal at all points, as mentioned, and their intensity is uniform throughout. This can be proved from Equations (2) and (3) by making R_0 and P_0 equal to zero. The second term of the right-hand members of both equations will then disappear, and for any value of r from zero to R_1, $t = -P_1$ and $p = P_1$, p being a compressive stress by hypothesis. These relations are shown graphically in Fig. 6, where $Oa = cb = P_1 = t = p$. The diagram $Oabc$, therefore, represents the total apparent tangential stress in one-half of a solid shaft. Since this total stress is produced by the total stress in the left side of the hub, whose tangential value is represented by the diagram $cdef$, the two stress-areas are equal, or $Oabc = cdef = P_1 \times R_1$.

Now, consider the hollow shaft on the right-hand side (Fig. 6), whose original diameter was sufficiently greater than that of the solid shaft to make the radius R_1 of the fit and the radial pressure P_1 on the latter the same as before, with the same hub and hub stresses, so that $ghkl = cdef$. From Equation (7) it will be seen that the apparent tangential stress at the outer surface is $P_1\,h$, and is hence greater than that of a solid shaft [Equation (13)], since h is always more than unity. Equations (2) and (3), with suitable substitutions, show that the tangential stress increases rapidly toward the bore, where its magnitude is given by Equation (10). The area representing the total tangential stress is *lmnq*, Fig. 6, and, as before, $lmnq = ghkl = cdef = P_1 \times R_1$. The radial stress is no longer uniform as in a solid shaft, but is equal to P_1 at the outer surface, and decreases to zero at the bore [see Equations (8) and (11)].

It will be seen, then, that if two shafts—one solid, the other hollow—when subjected to the same external radial pressure P_1, are compressed to the same radius R_1, the tangential stresses in the hollow shaft will be considerably greater than those in the solid shaft. The reason for this increased effect of P_1 on the tangential stress is that the hollow shaft lacks the support of the solid and compressed cylinder of radius R_0 which has been removed at the bore. In the solid shaft, at the layer of radius R_0, there is an outward radial pressure equal to P_1, while, in the hollow shaft, at this radius, the radial pressure is zero. These relations can be shown by making $P_0 = P_1$ in Equation (2), when the second term of the right-hand member will disappear, and, at all radii between R_0 and R_1, the tangential stress will be equal to P_1, as in a solid shaft. In this assumed case, the outward radial pressure P_1 at the bore produces the total apparent tangential tensile stress in the hollow shaft shown by the area *qsvl*, and, if this be deducted from the area *lmnq*, the remainder will be the area *lwxq*. corresponding with that for a solid shaft between the radii R_1 and R_0. The deductions, as above, apply also to the true tangential stresses, which are the same in kind as the apparent stresses, although differing in intensity.

Effect of Lateral Contraction

It has been shown that in the outer member of a shrinkage fit, lateral contraction increases the apparent radial and tangential stresses, each by an amount equal to one-third for steel, so that the true stresses are that much greater, and that in the inner member there is the same proportionate, but reverse, effect, which acts to reduce the intensity of the direct stresses. This action also develops secondary longitudinal stresses in both members, which, however, are negligible in a shrinkage fit. Thus, in the outer member, the tangential tensile stress t produces a longitudinal compressive stress whose intensity is ϕt, and the radial compressive stress p causes a longitudinal tensile stress equal to ϕp. The resultant longitudinal compressive stress at any point of radius r is then (see Fig. 3):

$$l = -\phi t + \phi p = -\phi (t - p)$$

As an extreme example, take a steel hub shrunk on a solid steel shaft, the external diameter of the hub being 1.5 times that of the shaft. Let the shrinkage allowances be such as to produce a true tangential tensile stress of 30,000 pounds per square inch at the bore of the hub. From Table 1 we find that the unit radial pressure on the fit is 10,230 pounds. Applying the formulas previously given:

	Apparent Stress	True Stress
Hub at Bore:		
Tangential tensile stress	26,598	30,000
Radial compressive stress	10,230	19,096
Shaft at Outer Surface:		
Tangential compressive stress	10,230	6,820
Radial compressive stress	10,230	6,820

The stresses given in the table above were calculated as follows:

The true tangential unit stress T_2 at the bore of the hub is 30,000 pounds, the ratio of the hub diameter is $\frac{R_2}{R_1} = 1.5$; from this ratio, $R_2^2 = 2.25\, R_1^2$. From Table I, when $R_2 \div R_1 = 1.5$, with both members of steel, ratio $A = 0.341$. Hence

$$\frac{P_1}{T_2} = \frac{P_1}{30{,}000} = 0.341$$

$P_1 = 30{,}000 \times 0.341 = 10{,}230$ pounds = unit radial pressure.

Hub at bore.—The apparent tangential tensile stress is:

$$t_2 = \frac{P_1\,(R_2^2 + R_1^2)}{R_2^2 - R_1^2} \tag{4}$$

Substituting the values of P_1 and R_1:

$$t_2 = 10{,}230 \times \frac{3.25}{1.25} = 26{,}598 \text{ pounds.}$$

The apparent radial compressive stress is:

$$p_2 = P_1 = 10{,}230 \text{ pounds.} \tag{5}$$

The factor of lateral contraction ϕ, for steel, is $\frac{1}{3} = 0.333$. The true tangential stress is:

$$T_2 = P_1\left(\frac{R_2^2 + R_1^2}{R_2^2 - R_1^2} + \phi\right) \tag{6}$$

$$= P_1\left(\frac{3.25}{1.25} + 0.333\right) = 30{,}000 \text{ pounds.}$$

The true radial stress is:

$$P_2 = P_1\left[1 + \frac{\phi\,(R_2^2 + R_1^2)}{R_2^2 - R_1^2}\right]$$

$$= 10{,}230\left(1 + 0.333 \times \frac{3.25}{1.25}\right) = 19{,}096 \text{ pounds.}$$

Shaft at outer surface.—The shaft is solid. The apparent tangential (compressive) stress at the outer surface is:

$$t_1 = P_1 = 10{,}230 \text{ pounds.} \tag{13}$$

The apparent radial (compressive) stress is:

$$p_1 = P_1 = 10{,}230 \text{ pounds.} \tag{14}$$

The true tangential stress is:

$$T_1 = P_1 \ (1 - \phi) \ = 10{,}230 \ (1 - 0.333) = 6{,}820 \text{ pounds.} \quad (15)$$

The true radial stress is:

$$P'_1 = P_1 \ (1 - \phi) = 6{,}820 \text{ pounds.}$$

It will be seen that the use of the apparent, in place of the true, stresses introduces errors which, with regard to the hub, may be serious even in less extreme cases than the above.

Resistance to Slip

The resistance of the fit to slip is theoretically equal to the product of the area of the contact-surface times the unit radial pressure on that surface times the coefficient of friction.

Let D_1 = nominal diameter of fit,
L = length of fit,
P_1 = unit radial pressure on fitted surfaces,
f = coefficient of friction,
Q = total resistance to slip.

$$\text{Then } Q = \pi D_1 \times L \times P_1 \times f \quad (30)$$

Since slip begins with the parts at rest, the coefficient of friction for rest applies in computing the initial resistance. There is considerable variation in the values given for this coefficient. Reuleaux and Weisbach use 0.2. Rennie, in experiments on metals usually unlubricated, found the following values for f:

Wrought-iron on cast iron	0.28 to 0.37
Steel on cast iron	0.3 to 0.36

In Professor Wilmore's experiments, the average value of this coefficient was 0.102. These tests were made with a series of cast-iron disks, 4 inches in diameter and 1 inch thick, which were either forced or shrunk on steel spindles about 1 inch in diameter, the fit being about 1 inch long. Five sets of these spindles were used, the diameter of the first set being 1.001 inch and the allowances for each subsequent set increasing by 0.0005 inch. The spindles were pulled from the disks in the "tension" tests of the fit and twisted in the holes in measuring the resistance to slip in torsion. The shrinkage fits were found to be 1.5 times, and the forced fits 1.3 times, stronger in torsion than in tension. This result was to be expected, if the resistance measured was not that to initial slip only, since, in torsion, the grip is undiminished during progressive slipping, while, in tension, the area under pressure decreased steadily as the spindles left the disks.

Let P = force acting to twist a solid shaft,
p = lever arm of P,
J = polar moment of inertia of shaft,
c = distance of most remote fiber of shaft from axis of latter,
S_s = shearing stress at distance c = maximum unit shearing stress,
D_1 = diameter of shaft.

Then:

$$P \times p = S_s \times \frac{J}{c} = S_s \frac{\pi D_1^3}{16}$$

and from equation (30):

$$\frac{QD_1}{2} = \pi D_1 L P_1 f \times \frac{D_1}{2}$$

Taking P_1 and S_s as constant, and equating, we have $L = KD$, in which K is a constant. Therefore with a constant radial pressure, the length of the hub should vary as the diameter of the shaft, in order to make the grip of the fit proportional to the torsional strength of a solid shaft. For both practical and theoretical reasons, it is impossible to make the grip equal to this strength. Hence, with diameters of 2 inches and upwards, keys should be fitted in addition.

CHAPTER V

SHRINKAGE ALLOWANCES

The *total* allowance for shrinkage is the difference between the external diameter of the inner member (shaft) and the internal diameter of the outer (hub), before shrinkage. The *unit* shrinkage-allowance is the allowance per inch of nominal diameter, in either case, as above; and also, in either case, the *unit*-deformation of a given circumference or diameter is the difference between its lengths before and after shrinkage, divided by its original length. The principle which is applied in the derivation of formulas for shrinkage-allowances, is that the unit-deformation at any point is the quotient of the unit-stress at that point, divided by the modulus of elasticity. In a shrinkage fit, the unit-deformations considered are those at the fit, and the unit-stresses to which these deformations correspond are manifestly the "true" or actual stresses, and not those which have been termed "apparent" in this discussion, since, as has been shown, the effect of lateral contraction is important.

The length of a given circumference varies directly as that of its diameter. Hence the unit-deformation will be the same for both, and this deformation when due to the true tangential stress in the hub at the bore, will be the unit-deformation of the internal diameter of the hub. Similarly, for both solid and hollow shafts, the unit-deformations of the external diameters are those of the circumferences of their outer surfaces, produced by the true tangential stresses there, since that circumference and the external diameter decrease together. For the unit-deformation of the external diameter of the inner member, that due to the true radial stress at the outer surface will serve only for a solid shaft, since in it, as shown in Fig. 6, the tangential and radial stresses are equal to each other at all points from the circumference to the center, while, in the hollow shaft, the intensity of the radial stress varies from P_1 at the outer surface to zero at the bore, and hence the deformation due to this stress at any given point is that corresponding only with the infinitely small element of radius in which that stress exists, and not with the average unit-deformation of the whole radius.

The algebraic methods employed below are those of Reuleaux*, the true stresses being substituted, since his formulas do not consider lateral contraction, and apply only to solid shafts, as the radial stress in the inner member is used in their deduction. As before, let

P_1 = radial pressure on fitted surfaces,

T_1 = true tangential compressive stress at outer surface, inner member,

* "The Constructor," Suplee's translation, Philadelphia, Pa., 1895, page 17.

T_2 = true tangential tensile stress at inner surface, outer member,
R_1 = radius of fit,
R = actual internal radius of outer member before expansion,
R' = actual external radius of inner member before compression,

S = unit shrinkage-allowance $= \dfrac{R' - R}{R}$,

E and ϕ = modulus of elasticity and factor of contraction, outer member,
E_1 and ϕ_1 = modulus of elasticity and factor of contraction, inner member.

$$A = \frac{P_1}{T_2};\quad B = \frac{T_1}{P_1};\quad C = A \times B = T_1 \div T_2.$$

TABLE III

Values of Ratio C for solid steel shafts of nominal diameter D_1, and hubs of steel or cast-iron of nominal external and internal diameters D_2 and D_1, respectively.

Ratio of Diameters $\frac{D_2}{D_1}$	$C = A \times B = \frac{T_1}{T_2}$ Steel Hub	Cast-iron Hub	Ratio of Diameters $\frac{D_2}{D_1}$	$C = A \times B = \frac{T_1}{T_2}$ Steel Hub	Cast-iron Hub
1.5	0.227	0.234	2.8	0.410	0.432
1.6	0.255	0.263	3.0	0.421	0.444
1.8	0.299	0.311	3.2	0.430	0.455
2.0	0.333	0.348	3.4	0.438	0.463
2.2	0.359	0.377	3.6	0.444	0.471
2.4	0.380	0.399	3.8	0.450	0.477
2.6	0.397	0.417	4.0	0.455	0.482

By the definition of the modulus of elasticity, we have, at the radius R_1 of the fit, for:

$$\text{outer member, } \frac{R_1 - R}{R} = \frac{T_2}{E}$$

$$\text{inner member, } \frac{R' - R_1}{R'} = \frac{T_1}{E_1}$$

Adding, we have:

$$R' - R = \frac{RT_2}{E} + \frac{R'T_1}{E_1} \tag{31}$$

Dividing by R:

$$S = \frac{R' - R}{R} = \frac{T_2}{E} + \frac{R'}{R} \times \frac{T_1}{E_1} \tag{32}$$

From (31):

$$\frac{R'}{R} = \frac{E_1}{E} \times \frac{E + T_2}{E_1 - T_1}$$

Substituting this value in (32):

$$S = \frac{\dfrac{T_2}{E} + \dfrac{T_1}{E_1}}{1 - \dfrac{T_1}{E_1}}.$$

TABLE IV

Values of Ratio C for hollow steel shafts of external and internal diameters D_1 and D_0, respectively, and steel hubs of nominal external diameter D_2.

$\frac{D_2}{D_1}$	$\frac{D_1}{D_0}$	C	$\frac{D_2}{D_1}$	$\frac{D_1}{D_0}$	C
1.5	2.0	0.455	2.8	2.0	0.820
	2.5	0.357		2.5	0.645
	3.0	0.313		3.0	0.564
	3.5	0.288		3.5	0.519
1.6	2.0	0.509	3.0	2.0	0.842
	2.5	0.400		2.5	0.662
	3.0	0.350		3.0	0.580
	3.5	0.322		3.5	0.533
1.8	2.0	0.599	3.2	2.0	0.860
	2.5	0.471		2.5	0.676
	3.0	0.412		3.0	0.591
	3.5	0.379		3.5	0.544
2.0	2.0	0.667	3.4	2.0	0.876
	2.5	0.524		2.5	0.689
	3.0	0.459		3.0	0.602
	3.5	0.422		3.5	0.555
2.2	2.0	0.718	3.6	2.0	0.888
	2.5	0.565		2.5	0.698
	3.0	0.494		3.0	0.611
	3.5	0.455		3.5	0.562
2.4	2.0	0.760	3.8	2.0	0.900
	2.5	0.597		2.5	0.707
	3.0	0.523		3.0	0.619
	3.5	0.481		3.5	0.570
2.6	2.0	0.793	4.0	2.0	0.909
	2.5	0.624		2.5	0.715
	3.0	0.546		3.0	0.625
	3.5	0.502		3.5	0.576

The second term of the denominator is so small as to be negligible. Hence:

$$S = \frac{T_2}{E} + \frac{T_1}{E_1} \tag{33}$$

This equation is not in a practical form, since for a given value of S, there are two unknown quantities.

From Equation (22), $A = \frac{P_1}{T_2}$; Equations (25) and (28) give the value of $B = \frac{T_1}{P_1}$. Let $A \times B = C = \frac{T_1}{T_2}$. Then $T_2 = \frac{T_1}{C}$ and $T_1 = CT_2$. Substituting in (33):

TABLE V

Values of Ratio *C* for hollow steel shafts and cast-iron hubs. Notation as in Table IV.

$\frac{D^2}{D^1}$	$\frac{D^1}{D_0}$	C	$\frac{D_2}{D_1}$	$\frac{D_1}{D_0}$	C
1.5	2.0	0.468	2.8	2.0	0.864
	2.5	0.368		2.5	0.679
	3.0	0.322		3.0	0.594
	3.5	0.296		3.5	0.547
1.6	2.0	0.527	3.0	2.0	0.888
	2.5	0.414		2.5	0.698
	3.0	0.362		3.0	0.611
	3.5	0.333		3.5	0.562
1.8	2.0	0.621	3.2	2.0	0.909
	2.5	0.488		2.5	0.715
	3.0	0.427		3.0	0.625
	3.5	0.393		3.5	0.576
2.0	2.0	0.696	3.4	2.0	0.926
	2.5	0.547		2.5	0.728
	3.0	0.479		3.0	0.637
	3.5	0.441		3.5	0.587
2.2	2 0	0.753	3.6	2.0	0.941
	2.5	0.592		2.5	0.740
	3.0	0.518		3.0	0.647
	3.5	0.477		3.5	0.596
2.4	2.0	0.798	3.8	2.0	0.953
	2.5	0.628		2.5	0.749
	3.0	0.549		3.0	0.656
	3.5	0.506		3.5	0.603
2.6	2.0	0.834	4.0	2.0	0.964
	2.5	0.656		2.5	0.758
	3.0	0.574		3.0	0.663
	3.5	0.528		3.5	0.610

$$S = \frac{T_2}{E} + \frac{CT_2}{E_1} \tag{34}$$

$$S = \frac{T_1}{E_1} + \frac{T_1}{CE} \tag{35}$$

Multiplying (22) by (25), and also by (28), we have,

$$\text{for a solid inner member, } C = \frac{1 - \phi_1}{a + \phi} \tag{36}$$

for a hollow inner member, $$C = \frac{h - \phi_1}{a + \phi} \qquad (37)$$

The values of C for various diametral ratios are given, for solid steel shafts with steel or cast-iron hubs in Table III; and, similarly, for hollow steel shafts, in Tables IV and V.

Taking the modulus of elasticity for steel as 30,000,000, and for cast iron as 15,000,000, equations (34) and (35) become, for a cast-iron hub and a steel shaft:

$$S = \frac{T_2\,(2 + C)}{30{,}000{,}000} \qquad (38)$$

$$S = \frac{T_1(2 + C)}{C \times 30{,}000{,}000} \qquad (39)$$

and, for both hub and shaft of steel:

$$S = \frac{T_2\,(1 + C)}{30{,}000{,}000} \qquad (40)$$

$$S = \frac{T_1\,(1 + C)}{C \times 30{,}000{,}000} \qquad (41)$$

CHAPTER VI

CALCULATING SHRINKAGE FITS

In designing shrinkage fits, there are but two main principles to remember. First, the stress in the hub at the bore, which is the most important consideration, depends chiefly on the shrinkage-allowances. If the latter be too large, the elastic limit will be exceeded and permanent set will occur; or, in extreme cases, the ultimate strength of the metal will be passed and the hub will burst. Second, the intensity of the grip of the fit, and hence the resistance of the latter to slip, depends mainly on the thickness of the hub. The greater this thickness, the stronger the grip; and *vice versa*. Formulas (34) and (35) and Tables I and III serve all general purposes in practice. Information in detail can be obtained as follows:

a. For a given allowance per inch of diameter, the true tensile stress T_2 in the hub at the bore can be found from Equations (34), (38), or (40). These equations hold only up to the elastic limit. It will be seen that by increasing or decreasing the allowances, any stress up to this limit can be produced at the bore, and this stress will be the maximum tensile stress in the hub.

b. When T_2 is assumed at any desired value below the elastic limit, the corresponding unit-allowances can be found by substituting in Equation (34).

c. Equations (6) and (22) and Table I show the relation between the true tensile stress in the hub at the bore and the radial pressure on the fit. There are several factors which govern the intensity of this radial pressure: the magnitude of the allowances, the compressibility of the inner member, and the expansibility of the outer. The two latter depend on the metals; the last is affected by the thickness of the hub.

d. When T_2 is known, the value of P_1 can be obtained from Table I or equation (22).

e. The true tangential compressive stress T_1 at the outer surface of the inner member is usually of minor importance in design; its intensity can be found from (35). The true radial compressive stress at the surface is equal to the radial pressure P_1, minus the product of ϕ_1 by the value of t_1, as given by (7) and (13).

f. At the bore of a hollow shaft, the radial pressure is zero. Equation (12) gives the true tangential compressive stress.

g. The intensity of the apparent stresses is, in general, of academic interest only. To ascertain their magnitude, the true stresses are first found from (34) and (35); Equations (25) or (28) will then give the value of the radial pressure P_1, and, by substituting this in the equations on pages 13 to 15, the apparent stresses can be determined.

Examples

Example 1.—A steel crank-web, 15 inches least outside diameter, is to be shrunk on a 10-inch solid steel shaft. Required the allowance per inch of shaft-diameter to produce a maximum tensile stress in the crank of 25,000 pounds per square inch, assuming the stresses in the crank to be equivalent to those in a ring of the diameter given.

$\frac{D_2}{D_1} = \frac{15}{10} = 1.5$; $T_2 = 25{,}000$. From Table III, $C = 0.227$. Substituting in Equation (40), we find $S = 0.001$ inch.

Example 2.—Let the shaft in Example 1 have a 5-inch axial hole bored through it, other conditions being the same. Find the unit-allowance.

$\frac{D_2}{D_1} = 1.5$, as before; $\frac{D_1}{D_0} = \frac{10}{5} = 2$; $T_2 = 25{,}000$. From Table IV we find $C = 0.455$.

Substituting in Equation (40), we find $S = 0.0012$ inch, the increase in the allowance being due to the fact that the hollow shaft is the more compressible of the two.

Example 3.—Let the crank-web in Example 1 be of cast-iron and the maximum tensile stress in the hub be 4000 pounds per square inch. Find the unit-allowance.

$\frac{D_2}{D_1} = 1.5$; $T_2 = 4000$. From Table III, we find $C = 0.234$. Substituting in (38) $S = 0.0003$ inch, which, owing to the lower tensile strength of cast iron, is about one-third of the shrinkage-allowance in Example 1, although the stress is two-thirds of the elastic limit. For a forced fit, good practice gives (see Table VI) a unit-allowance of 0.0013 inch, or one-third greater than that of Example 1. The stresses which such an allowance would produce are, however, uncertain, as will be further discussed in the following chapter.

Example 4.—What is the radial pressure P_1 in the above examples?

For Examples 1 and 2, we find from Table I that $\frac{P_1}{T_2} = 0.341$. Hence,

$$P_1 = 25{,}000 \times 0.341 = 8525 \text{ pounds per square inch.}$$

For Example 3, we find from Table I that $\frac{P_1}{T_2} = 0.351$. In this case $T_2 = 4000$, hence,

$$P_1 = 4000 \times 0.351 = 1404 \text{ pounds per square inch.}$$

Example 5.—What is the resistance to slip per inch of length of hub in Example 3?

In Equation (30), $D_1 = 10$, $L = 1$, and from Example 4 we have $P_1 = 1404$; f may be taken as 0.2. Then $Q = 8817$ pounds, which is the total resistance of a ring of the hub, one inch in length.

Example 6.—Let the crank in Example 3 be 20 inches least diameter, the other dimensions and the tensile stress remaining the same. Find the increase in the radial pressure P_1, and hence that in the resistance to slip.

In this case $\frac{D_2}{D_1} = 2$, Table I gives the ratio A, for this condition, equal to 0.522, which is 49 per cent greater than the ratio $A = 0.351$ for $\frac{D_2}{D_1} = 1.5$. This percentage is the increase in radial pressure, and, hence, that in the resistance to slip.

Example 7.—What is the true tangential stress (compressive) at the bore of the shaft in Example 2?

The radial pressure P_1 is, from Example 4, 8525 pounds. Substituting this value, and also $R_1 = 5$, and $R_0 = 2.5$ in Equation (12), the true stress $T_0 = 22{,}733$ pounds per square inch.

Example 8.—What is the intensity of the apparent tangential stresses in the crank and shaft, Example 1?

The radial pressure P_1 is, from Example 4, 8525 pounds. Substituting this value, and also $R_2 = 7.5$, and $R_1 = 5$ in Equation (4), the apparent tensile stress t_2 at the bore of the hub is 22,165 pounds per square inch. The similar compressive stress t_1 at the outer surface of the shaft is, from Equation (13), equal to P_1.

Shrinkage Temperatures

The temperature to which the outer member in a shrinkage fit should be heated for clearance in assembling the parts, depends on the total expansion required and on the coefficient a of linear expansion of the metal, *i. e.*, the increase in length of any section of the metal in any direction for an increase in temperature of 1 degree F. The total expansion in diameter which is required, consists of the total allowance for shrinkage and an added amount for clearance.

The value of the coefficient a is, for nickel-steel, 0.000007; for steel in general, 0.0000065; for cast iron, 0.0000062. As an example, take an outer member of steel to be expanded 0.005 inch per inch of internal diameter, 0.001 being the shrinkage allowance and the remainder for clearance. Then:

$$a \times t^{\circ} = 0.005$$

$$t = \frac{0.005}{0.0000065} = 769 \text{ degrees F.}$$

The value t is the number of degrees F. which the temperature of the member must be raised.

CHAPTER VII

PRACTICAL CONSIDERATIONS

Cylindrical and Tapered Fits

The form of the shrinkage fit is usually truly cylindrical and of one diameter throughout; but both forced and shrinkage fits are, for some classes of work, either tapered or double-cylindrical, *i.e.*, with part of the fit of one diameter and part of another. The advantages of the tapered form in forced fits are: The possibility of abrasion of the fitted surfaces is reduced; less work is required to drive the inner member home; the drawings may be marked "Fit pin — inches from end of hole," which is the most trustworthy way of measuring the allowances; and the parts are more readily separated, if a renewal of the fit is desired. On the other hand, the difficulty of securing with accuracy the same form for both fitted surfaces, is somewhat greater; and the tapered fit is less reliable, since, if slip begins, the entire fit is virtually free with but little movement. These advantages and disadvantages apply also, but in less degree, to the double-cylinder form.

The practice of a prominent shipbuilding company, for both forced and shrinkage fits in either iron or steel, is: With large fits, both the inner and outer members have a taper of 1/16 inch to the foot; the allowances are 0.001 inch per inch of diameter with 0.001 inch added to the total. If the conditions are such that it is more convenient to ream the hole with standard parallel reamers, the inner member is tapered one half-thousandth inch (0.0005) per inch of length, unless the fit is so long that this taper would reduce the allowance at the small end to less than one-half that at the other extremity of the fit.

Differences between Forced and Shrinkage Fits

Lamé's formulas, as given in Equations (2) and (3) and as changed in the subsequent equations for lateral contraction according to the principles established by Clavarino, are the basis of the ordnance formulas employed by the United States Army and Navy. For economy in weight, the stresses in the metal of a gun, at the instant of explosion, approach closely to the elastic limit. It is evident, then, that the use of these formulas for such work makes their accuracy, for shrinkage fits in gun-steel, unquestionable. So far as is known, their fundamental principles are general, and they can be employed with equal accuracy for similar fits in cast iron. It has been customary to assume that they could be applied also for the determination of the stresses in the metals of forced fits. This assumption is, in the author's opinion, unwarranted, so far, at least, as cast iron outer members with large forcing allowances are concerned. There seems to be considerable evidence in support of this contention.

The basic principle of shrinkage and forced fits is the same—the radial pressure on the contact-surfaces produced by the expansion of the outer member and the compression of the inner; but there is a radical difference between the methods by which this principle is applied in the two cases. In the shrinkage fit, the outer member, owing to its expansion, slips freely into place, giving, in cooling, clean, smooth, and accurately fitted surfaces. In forced fits, on the contrary, there may be, in forcing, more or less abrasion, and, further, if the allowances be large, there may be an axial flow of the metal of the hub in advance of the entering shaft. It should be noted that, in forcing allowances, we are dealing with a layer of metal whose thickness is, in general, but 0.001 inch per inch of diameter, so that the total volume of the metal thus displaced would be very small, while its removal, with that lost by abrasion, would reduce materially the amount of the effective allowances, and, in consequence, the stresses and "grip" of the fit. Taking the elastic limit in tension of cast iron as 6000 to 7000 pounds and that of steel as 50,000 pounds, and considering the corresponding values of E, the former will endure, without permanent set, less than one-fourth the deformation of the latter, yet the forcing allowances of the two metals are often made the same, and, further, with the same metals and dimensions, some builders make the allowances for forcing considerably greater than those for shrinkage fits. In such cases, there must be either permanent set in the cast-iron hub, or the effective allowances must be materially lessened by abrasion, displacement, or both.

In Professor Wilmore's tests, the average resistance of the shrinkage fit to slip was, for an axial pull, 3.66 times greater than that of the forced fit, and, in rotation or torsion, 3.2 times greater. In each comparative test, the dimensions and allowances were the same for both. These results imply either permanent set or considerable abrasion or displacement of the metal of the forced fit. While these experiments were made on a small scale, they agree with the general estimate of the comparative strength of forced fits.

Table VI represents the practice of one of the largest builders of engines and other machinery in the United States, in forcing cast-iron cranks and wheel-hubs on steel shafts. The allowance for a crank is greater than that for a wheel-hub, and, with both, the allowance per inch of diameter decreases with increasing diameter. Take the unit-allowance for a 12-inch wheel-hub which is 0.001 inch. Assume the ratio of the external diameter of the hub to that of the shaft (solid) as 1.8, which gives a hub-thickness of 4.8 inches. If in Equation (38), $S = 0.001$, and, from Table III, $C = 0.311$, then the true tensile stress T_2 at the bore of the hub is about 13,000 pounds, or twice the elastic limit of cast iron. Again, we have here indications of permanent set, excessive abrasion, or very considerable displacement of the metal, so that the effective allowances cannot be those initially given.

Finally, the following formulas given by Mr. Stanley H. Moore may be cited. In these formulas, d denotes the total allowance, and D is the diameter of the shaft, in inches.

$$\text{Shrinkage fit} \qquad d = \frac{\frac{17}{16} D + 0.5}{1000}$$

$$\text{Forced fit} \qquad d = \frac{2\,D + 0.5}{1000}$$

These formulas show again a much greater allowance for forcing than for shrinkage.

Forced fits may be made by levers, screw-jacks, or hydraulic pressure, the latter being the most common. In the drive-fit, the pin is

TABLE VI. ALLOWANCES FOR FORCED FITS

Steel Shaft and Pin to Cast-iron Cranks. Average pressure required = 12.5 tons (of 2000 pounds) per inch of diameter.		Steel Shaft to Cast-iron Wheel-hubs. Average pressure required = 10 tons (of 2000 pounds) per inch of diameter.	
Diameter of Shaft, Inches	Allowance per Inch of Diameter	Diameter of Shaft, Inches	Allowance per Inch of Diameter
4	0.0030	12	0.0010
5	0.0024	13	0.0009
6	0.0020	15	0.0008
7	0.0017	17	0.0007
8	0.0015	18	0.0006
9	0.00135	19	0.00055
10	0.0013	22	0.0004
11	0.0012	23	0.00035
12	0.0010	24	0.0003
13	0.0010	26	0.00025
14	0.0010	27	0.0002
15	0.0010	....	......
16	0.0009	....	......
18	0.0008	....	......
20	0.00075	....	......

sent home by sledges; the allowances are usually about half that of a forced fit. With these various methods and the many purposes for which forced fits can be used, it is natural that the custom as to the amount of the allowances should differ, as it does, very widely, so that the practice cited here is not universal. The purpose of this discussion has been simply to point out that shrinkage formulas will not give with accuracy the stresses in a cast-iron hub, when the allowances are very large, or in any forced fit with undue allowances. Such a fit differs essentially from the shrinkage joint for which the formulas were constructed.

Cotterill says in his "Applied Mechanics," London, 1895, page 412: "When the limit of elasticity is overpassed, the formula (Lamé's) fails, and the distribution of stress becomes different. If the pressure be imagined gradually to increase until the innermost layer of the cylinder begins to stretch beyond the limit, more of the pressure is transmitted into the interior of the cylinder, so that the stress becomes partially equalized. If the pressure increases still further, the tension of the innermost layer is little altered, and, in soft materials, longitudinal flow of the metal commences under the direct action of the fluid pressure. The internal diameter of the cylinder then increases perceptibly and permanently. This is well known to happen in the cylinders employed in the manufacture of lead piping, which are exposed to the severe pressure necessary to produce flow in the lead. The cylinder is not weakened but strengthened, having adapted itself to sustain the pressure. Cast-iron hydraulic press cylinders are often worked at the great pressure of 3 tons per square inch, a fact which may perhaps be explained by a similar equilization."

Forcing Pressure

When the fit is cylindrical, the forcing pressure varies as the rate of advance of the inner member, reaching a maximum in continuous forcing when the pin or shaft is at the inner end of the hole. At this point, the pressure is theoretically equal to Q, the resistance to slip, as given in Equation (30), the coefficient of friction f being probably between 0.12 and 0.2, although it may vary widely. Tables VI and VIII give values of the forcing pressure, as found in practice. The assumption above, that the maximum forcing pressure is equal to the resistance to slip, is true only if that pressure is expended wholly in overcoming the obstruction to motion produced by the resistance of the outer member to expansion and of the inner to compression. If there is abrasion of the surfaces, or axial displacement of the metal in advance of the entering member, the assumption is not fully justified.

Applications in Practice

Railway Work. In railway work, steel tires are shrunk on the cast-iron wheel-centers of driving wheels. The fit is cylindrical; a common, although not universal, shrinkage-allowance is 0.001 inch per inch of diameter of the finished wheel. Forced fits are used for securing wheels to axles and crank-pins to driving wheels. In wheel-fits, the joint is cylindrical; the pressure is usually 9 to 10 tons per inch of diameter of fit. In removing a wheel after long service, the total pressure may reach 150 tons.

Stationary Engines. Shrinkage and forced fits—the latter more frequently—are used for crank-pins, cranks, wheel-hubs, and minor parts. With different builders, the amount of the unit-allowance has a wide range, owing to differences in the thickness of hubs, the forcing pressure employed, etc. General practice seems to favor a smaller allowance for shrinkage than for forcing, and, with increasing diameter, a decreasing unit-allowance. The latter is usually greater for cast iron than for steel. Table VII, which gives the data for typical fits from different builders, shows the variation in practice. In Table VIII* will be found complete data for forced fits from 2 to 9 inches in diameter.

* MACHINERY, May, 1897.

Marine Engines. In marine work, built-up crank-shafts are assembled and the casings of propeller shafts are secured by shrinkage fits. Forced fits have been employed for crank-shafts and are frequently used for smaller parts. In building up a steel shaft, the allowance is usually 0.001 inch per inch of diameter; the cranks and crank-pins are keyed, in addition to the shrinking. The crank-webs are heated by gas in a sheet-iron furnace until the expansion is sufficient for a free fit; they are then removed, the pin is pushed home and keyed, and the webs and pin are cooled with water. The webs are then set with the bores for the shaft vertical, and one is heated as before until sufficiently expanded, when the section of the shaft

TABLE VII. EXAMPLES OF TYPICAL FITS, FROM PRACTICE

Diameter of Pin or Shaft, Inches	Total Allowance, Inches		Metals
	Shrinkage	Forcing	
1.8798	....	0.0031	Shaft, steel. Hub, cast iron
4.2505	....	0.0103	Shaft, steel. Hub, cast iron
8.9	....	0.0152	Shaft, steel. Hub, cast iron
4 to 5	0.0045	0.0090	Cast iron crank
7.5 to 9	0.0027	0.0055	Cast iron crank
16 to 18	0.0015	0.0030	Cast iron crank
4	....	0.0120	Crank, cast iron. Shaft, steel
8	....	0.0120	Crank, cast iron. Shaft, steel
16	....	0.0144	Crank, cast iron. Shaft, steel
1 to 2	0.0090	0.0010	
4 to 6	0.0156	....	
5 to 7	....	0.0050	
9 to 12	0.0313	....	
10 to 12	....	0.0100	
5	....	0.0050	Shaft, steel. Crank, cast steel
5	....	0.0100	Shaft, steel. Crank, cast iron
11	0.0070		Cast iron counter-balance
13	0.0060		plates on steel crank-disks

is lowered into place and keyed; the same method is followed with the other section of the shaft.

Shaft casings are of bronze, usually from 5/8 inch to 1 inch thick at various sections of the shaft. In one case there were two such sections of casing, each 8 feet long and 20½ inches internal diameter. The shrinkage-allowance, total, was 0.013 inch, or 0.000634 inch per inch of diameter. Each section was set vertical and heated internally by gas. When expanded, it was slipped in place on the shaft, and the inner end was held firmly and cooled with water until it gripped the shaft.

Gun Construction. When a charge is exploded in the powder-chamber, the principal stress to which a gun is subjected is that due to the radial pressure of the gases which tends to burst it on an axial plane. This stress produces tangential (circumferential) tension in the tube, jacket, and hoops, and, in addition, there is a direct longitudinal stress in the layer of the tube in which the breech-plug houses. There also exists at all times, except during explosion, a radial compressive stress on the inner cylinders of the system, due to the shrinkage pressures of those outside of them. At the breech, there may be three or four of these superposed cylinders—the tube, the jacket, and one or two sets of concentric hoops. The radial pressure of the gases would produce in the tube, if the latter were unsupported, a circumferential tensile stress which would exceed the elastic limit of the metal. To

TABLE VIII. DATA FOR FORCED FITS, FROM PRACTICE

Mean Diameter of Pin, Inches	Length of Fit, Inches	Mean Diameter of Hole, Inches	Total Allowance	Allowance per Inch of Diameter	Area of Fitted Surface, Square Inches	Volume within Fitted Surface, Cubic Inches	Pressure to Enter Pin, Tons	Pressure at Mid-Position, Tons	Maximum Pressure, Tons
1.8798	6.125	1.8767	0.0031	00.0170	36.0	16.7	2	10	20
1.8819	6.125	1.8770	0.0042	0.00220	36.0	16.7	2	15	23
1.8774	4.375	1.8764	0.0010	0.00052	24.4	13.7	0.5	1	1
2.7455	4.500	2.7387	0.0068	0.00247	38.7	26.5	3	12	25
2.7465	4.500	2.7437	0.0028	0.00100	38.7	26.5	5	12	23
3.2610	5.000	3.2542	0.0068	0.00210	51.0	41.5	5	20	45
3.2625	5.000	3.2555	0.0070	0.00200	51.0	41.5	5	15	30
3.2670	5.000	3.2610	0.0060	0.00180	51.0	41.5	5	15	20
4.2505	6.000	4.2402	0.0103	0.00240	79.8	85.1	5	22	44
4.2388	6.625	4.2478	0.0091	0.00210	78.1	93.4	12	30	60
4.2303	6.500	4.2224	0.0079	0.00190	95.8	91.0	10	60	125
5.9343	4.062	5.9216	0.0127	0.00220	75.7	112.2	6	16	25
5.9381	4.000	5.9252	0.0129	0.00220	74.4	110.4	3	18	35
5.9294	4.125	5.9194	0.0100	0.00170	76.7	113.8	5	15	25
6.8829	5.125	6.8697	0.0132	0.00200	110.7	190.1	8	20	42
6.8890	5.000	6.8785	0.0105	0.00150	108.0	185.9	5	22	45
6.8692	4.875	6.8550	0.0142	0.00210	104.8	180.4	5	35	65
7.8884	5.500	7.8730	0.0154	0.00200	135.9	267.3	5	32	64
7.8715	6.500	7.8575	0.0140	0.00180	160.5	315.9	5	25	50
7.8620	5.625	7.8460	0.0160	0.00200	138.2	272.8	8	40	80
8.9240	6.125	8.9050	0.0190	0.00210	170.8	378.9	20	45	68
8.9000	6.750	8.8848	0.0152	0.00170	188.4	419.9	5	47	96
8.8780	6.500	8.8669	0.0112	0.00130	180.7	401.0	10	45	92

counteract this, the jacket and hoops are shrunk on, each of these cylinders putting the one which it encases under compression, and the aggregate of these radial pressures being transmitted to the tube. The actual tensile stress in the latter, during the burning of the powder, is then the difference between the tensile stress developed by the gases and the compressive stress due to the jacket and hoops—a remainder which is less than, but usually fairly close to, the elastic limit of the metal.

For maximum economy of material, the relations of the thicknesses and shrinkage-allowances should be such that the stresses at all points in the walls of the built-up gun will be, during explosion, not only approximately equal but also the greatest permissible, with due regard to the elastic limit and the factor of safety. The outer layers of the metal are, therefore, in a state of initial tension, the inner under initial compression, and during explosion all are in tension. The various thicknesses and allowances for the cylinders of any given gun can be computed by an extension of the methods shown by Formulas (2) and (3), and those in (1) for the corresponding unit-deformations due to the true stresses. The principles involved are, therefore, those which have been treated herein for shrinkage fits, with the added requirement that the superposed cylinders, during explosion and the subsequent release from pressure, must expand and contract together, so that each cylinder must have a definite shrinkage-allowance with regard to all the others of the system.

The 16-inch Army rifle, now at Sandy Hook, was designed for a powder-pressure of 38,000 pounds per square inch, a muzzle-velocity of 2500 feet per second, a muzzle-energy of 88,000 foot-tons, a penetration at the muzzle of 42.3 inches in steel, and a range of 21 miles. The weight of the gun is 126 tons and its total length is 49 feet 2.9 inches. At the breech, the gun is built up of a tube, a jacket, and two sets of hoops, the thicknesses being 5.3, 7.2, 3.7, and 4.3 inches, respectively. The tube and jacket are of nickel-steel, not fluid-compressed; the hoops are of fluid-compressed steel containing no nickel. The elastic limits in tension of the two metals were about 52,000 and 57,000 pounds, respectively, the hoop-metal being thus the harder and stronger. The forgings, after being rough-turned and bored, were tempered in oil and annealed. In expanding the jacket or a hoop, it was set vertically in a cylindrical furnace of fire-brick, and was then encased in a muffle of ½-inch boiler steel. The combustion-chamber between the muffle and the furnace-wall was 11 inches wide. The fuel was oil sprayed with steam through 20 burner openings, the flame striking the muffle at a tangent, so as to give a spiral movement to the gases. The circulation of the air between the muffle and the hoop kept the temperature of the latter uniform at all points. The heating of the jacket required 30 hours, and its bore was calipered three times during that period to determine the expansion.

In shrinking on the jacket, the tube was first set vertical, muzzle-end down, in a shrinkage-pit adjacent to the furnace; the lower end was secured in a cast-iron chuck anchored in the concrete foundations of the pit. Water-connections were made for cooling the interior of the tube and the exterior of the jacket when seated. The latter, when removed from the furnace, was measured, centered, and lowered into place. Water was then applied at the muzzle-end; the cooling continued for nine hours, the number of encircling "water-rings" or pipes varying from four, as a maximum, to two at the close of the operation. The shrinkage of the hoops near the muzzle was effected similarly; the remainder were assembled with the gun in a horizontal position in the lathe, each hoop during shrinkage being under the axial pressure of two 30-ton hydraulic jacks.

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The Industrial Press, Publishers of MACHINERY,

49-55 Lafayette Street, New York City, U. S. A.