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No. 5—FIRST PRINCIPLES OF THEORETICAL MECHANICS

BY LESTER G. FRENCH

49-55 Lafayette Street, New York City.

FIRST PRINCIPLES OF MECHANICS.

Mechanics is that branch of science which treats of the action of force, and of its effects. A *force* is commonly defined as any cause tending to produce or modify motion. Its action is always equivalent to a push or pull, such as is exerted when we use our muscles, and until we have made some progress in the study of the subject, it will be simpler to consider force in this sense, simply, without regard to its effects. For the present, therefore, a force may be defined as any cause producing a push or a pull. There are many familiar examples of force, as muscular effort, gravity, the expansive force of steam, the elasticity of a spring, the attraction of a magnet, etc.

The unit by which force is usually measured is the standard pound, avoirdupois; that is, the common pound. A force of 100 pounds is one capable of sustaining a weight of 100 pounds. It will appear hereafter that the weight of the pound varies with the locality, so that this unit is not an absolute one. The variation is so slight, however, that it is of no consequence, except in very accurate physical investigations.

Matter.

The material of which anything is composed is called *matter*. The term is a collective one, and is used when no particular substance is referred to. Lead, iron, water, air, or any other substance is spoken of in a general way as "matter."

Matter exists in three states: the solid, the liquid, and the gaseous. A *solid*, of which wood and iron are examples, is characterized by a tendency to resist any attempt to change either its shape or size. A *liquid* readily changes its shape, but its volume or size remains constant under the same temperature conditions. A pint of water will fill a pint vessel of any shape, but it cannot be forced into a vessel holding less than a pint.* A *gas* has neither definite shape nor definite volume. It will accommodate itself in any shape, like a liquid, can be compressed easily, and will also expand into a larger space. Air, oxygen, nitrogen and hydrogen are examples of gases.

Since force can act upon all three forms of matter, the subject of mechanics is divided into the mechanics of solids, the mechanics of liquids or hydraulics, and the mechanics of gases, or pneumatics. For the present, only the mechanics of solids will be considered.

A *body* is a definite portion of matter, as a pound of lead, an iron bar, a quart of water, or a cubic foot of air. It is believed that all bodies are made up of extremely small portions of matter, called *mole-*

* Liquids are very slightly compressible. Water will diminish about 0.00005 in volume under a pressure of 15 pounds per square inch.

cules, which are separated from one another by distances that are great compared with their size. These molecules are so minute that it is impossible to detect them, even with the most powerful microscope; but there are many facts determined by experiment, that make their existence seem very probable. If the speculations of scientists are correct, at least 500,000 molecules could be placed in a row between the measuring surfaces of a micrometer caliper, when it is set to read 0.001 inch. A molecule is the smallest portion of matter that can exist and still retain the properties of the substance of which it is a part.

It is believed, further, that every molecule contains two or more indivisible portions of matter, called *atoms*. Thus a molecule of water is composed of two atoms of hydrogen gas and one atom of oxygen gas. A molecule can be separated into its atoms by chemical action only, and then the separation is only momentary, for the atoms at once combine to form other molecules, usually of a different nature. The atom is purely a chemical unit; we are not concerned with it in mechanics.

Molecular Forces.

Two opposing forces reside in the molecules—an attractive force that binds the molecules together, and a repellent force, that tends to push them apart. The three states of matter, solid, liquid, and gaseous, depend upon the relation of these forces. If the attractive force predominates, the body is solid; if the repellent, it is gaseous; if the two are nearly balanced, it is liquid.

The repellent force is probably one manifestation of the phenomenon which we call heat. Thus, when a bar of steel is heated, the attractive force is gradually overcome by the repellent force, as is seen in the expansion and finally in the melting of the bar. So, also, if we heat a piece of ice, the ice is turned to water, and at last, when the repellent force becomes very strong, the water is turned into steam.

The attractive force is capable of acting not only between molecules of the same kind and in the same body; but between the surfaces of different bodies which are in contact, as well. In the former case it is called *cohesion*, and, in the latter, *adhesion*. It is cohesion that resists any attempt to pull apart a body, like a string or a wire, and adhesion that holds together bodies that stick to one another, as in the case of two pieces of wood, when united by glue, or of drops of rain on a window-pane, pencil or ink marks on a piece of paper, etc. The effect of adhesion is usually more noticeable between solids and liquids than elsewhere. Neither force will act, except at insensible distances. To join two pieces of iron, for example, welding must be resorted to, in which process the hammering brings the molecules in the two parts near enough together for the cohesive force to take effect. Adhesion and cohesion are of the same nature, the difference between them being one of name or definition rather than of kind. Two absolutely smooth surfaces, if such were possible, would adhere to one another perfectly, since their contact would be perfect, and it might then as properly be said that the adjoining particles were held together by cohesion as by adhesion.

Work and Power.

The terms force, work and power are of frequent occurrence in mechanices, and are oftentimes misused. As a definition of force has just been given, it will be advantageous to now take up the subjects of work and power, so that the meanings of the three may be compared and thus firmly impressed upon the memory.

Work.

Work is said to be performed when a force produces motion in opposition to a resistance. Force has one element only, namely, the push or pull exerted. Work is the result of the two elements, force and motion. When no motion results from the action of a force, no work is done. A jack-screw supporting a weight does no work, except when the screw is turned so as to raise the weight. Likewise, no mechanical work results when a man pushes against a heavy body which he is unable to move, however much it may seem like work to him in the common acceptance of the term. Should he push with equal force against a smaller body, however, and move it, work would be performed.

Measurement of Work.

(*a*) In order to calculate the work done, the magnitude of the force applied is measured in pounds and the distance moved in feet. The product of these quantities, obtained by multiplying them together, is the work in *foot-pounds*. Or, briefly stated,

$$Work = force \times distance. \qquad (1)$$

The foot-pound is called the *unit of work*, and may be defined as the work done by a force of one pound acting through a distance of one foot.

(*b*) In the estimation of work it is sometimes more convenient to multiply the resistance overcome by the distance, than to multiply the force applied by the distance, in which case

$$Work = resistance \times distance. \qquad (2)$$

It is clear that the resistance and the force applied must always be equal, so that it makes no numerical difference which method is used. For example, if a man raises a weight of 10 pounds through a certain height, he performs work. The resistance of the weight is equal to 10 pounds, and the force that he exerts is just sufficient to raise it, or equal to 10 pounds, also.

(*c*) The simplest example of work is that just cited, of a weight raised against the force of gravity. When solving such examples, care must be taken always to multiply the weight by the *vertical* height through which it moves. Thus, in Fig. 1, suppose the ball B to be rolled from the bottom to the top of the inclined plane. If W represent the weight of the ball and h the height that it is raised, the work done upon the ball would be $W \times h$. It is true that the ball has moved through the distance l, but the force required to roll the ball through this distance, and which acts in the direction of the arrow, is less than the weight W, and hence, if W were multiplied by l, the result would be too great. If it were known, however, what force, acting in the direction of the arrow, was required to roll the ball, then this force, multiplied by l, would give the work.

Power.

From what has been said upon work, it is plain that a force, however small, can perform any required amount of work, provided time enough be allowed. A toy engine, for example, might do 1,000,000 foot-pounds of work in a few hours, while an engine of moderate proportions would accomplish as much during a few strokes of the piston. Foot-pounds of work, merely, with time left out of account, would

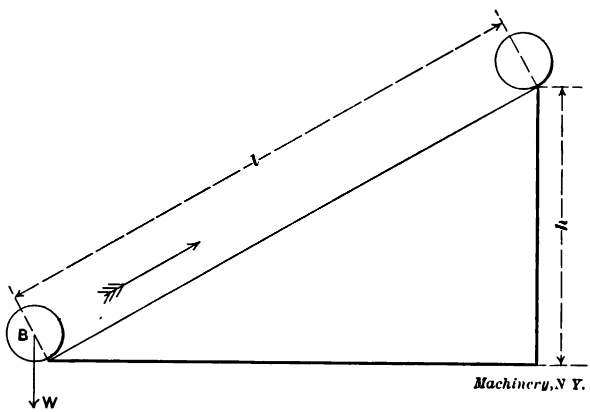


Fig. 1.

form no basis by which the capacities of the two engines could be compared. Hence, to compare the work done, either by or upon some agent, the time required must be considered.

The term *power* is employed to indicate the quantity of work done in a given time. "One million foot-pounds" is an expression indicating work; 1,000,000 foot-pounds of work performed in a day, or an hour or minute indicates power. Work has the two elements, force and the distance through which the force acts; power has three elements: force, distance and time.

The unit of power adopted for engineering work is the *horse-power* (abbreviated H. P.). One horse-power is equal to 33,000 foot-pounds per minute, or it may be said to equal 33,000 pounds raised one foot high in a minute.* Hence, to find the horse-power when work is done, divide the number of foot-pounds of work done in one minute by 33,000.

Lest it lead to confusion when met with, it should here be stated

* The horse-power unit was introduced by James Watt, the great improver of the steam engine, for the purpose of designating the power developed by his engines. He had ascertained by experiments that an average cart horse could develop 22,000 foot-pounds of work per minute, and being anxious to give good value to the purchasers of his engines he added 50 per cent to this amount, thus obtaining (22,000 + 11,000) the 33,000 foot-pounds per minute unit by which the power of steam and other engines has ever since been estimated.—*Jamieson's Applied Mechanics.*

that the term power is frequently used by writers on mechanics in the sense of force. In the so-called "mechanical powers," such as the lever, wheel and axle, wedge, screw, etc., it is quite usual to speak of the applied force as the power. Thus, the bar or lever shown in Fig. 2 is pivoted at O and at the end bears the weight W. At the other end a force, such as the pressure of the hand, acts downward in the direction of the arrow, and thus supports or raises the weight W. This pressure, which is the applied force, is what is called the power. Such use of the word, when force is what is meant, is ambiguous and can easily be avoided.

Friction.

Friction is the surface resistance which opposes the motion of one body upon another. It must be regarded as a force, although it is not always natural to think of it as such, for the reason, perhaps, that its action in resisting motion is of a negative character. The force of

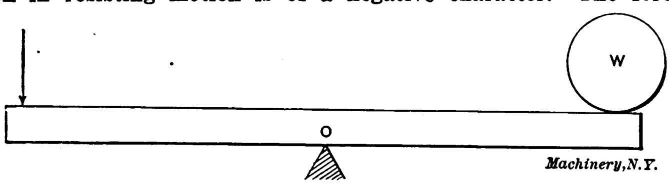


Fig. 2.

friction always acts in a direction parallel to the surfaces in contact. Thus, in Fig. 3, in pulling the block B along the surface, as shown, the frictional resistance is exerted in an opposite direction and parallel to the surfaces, as indicated by the arrow F.

Friction should not be confounded with adhesion, which not only resists the motion of one body upon another, but tends to hold the two together so that they cannot be separated. Adhesion is independent of the pressure between the bodies, while friction increases with the pressure. Moreover, the smoother the rubbing surfaces the less the friction; two perfectly smooth surfaces, if such were possible, would be frictionless, while, as has been previously stated, an adhesion between them would be very great. Lubricants increase the adhesion and diminish the friction. When the pressure between two bodies is small, the adhesion forms a considerable part of the resistance, and as the pressure increases, it becomes proportionately less, since adhesion does not increase with the pressure. At ordinary pressures the effect of adhesion can generally be neglected, and the whole resistance considered as the friction.

Kinds of Friction.

(*a*) A distinction is usually made between *friction of rest* and *friction of motion*, the former being the frictional resistance to be overcome in starting a body into motion, and the latter the resistance that continually accompanies the motion. Friction of rest is generally greater than friction of motion, other conditions being equal.

(*b*) When friction is mentioned, *sliding friction* is understood, *i.e.*, such as that between an engine crosshead and its guides, or between a journal and its bearing. It is due to the roughness of the surfaces in contact. Whenever wheels are employed, or rollers or balls placed between the surfaces, the resistance is called *rolling friction*, the nature of which is somewhat different; it is then due to the fact that the rolling body makes a greater or less depression in the surface of the other, so that it has continually to rise out of a hollow, as it were.

(c) Frictional resistance also occurs between the molecules of liquids and gases, or between them and any solid body with which they may be in contact, as in the case of air when blown through a

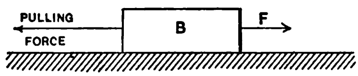


Fig. 3.

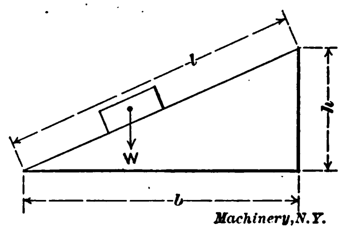


Fig. 4.

pipe, or a ship when sailing. This kind of resistance is called *fluid friction*. Its action is very different from that of the friction of solid bodies, and it is different in its nature.

Laws of Friction.

Certain conclusions have been drawn from early experiments upon friction, which are known as the laws of friction. They are only approximately true, however, and apply only within certain limits. Outside of those limits they have been proved by later experiments to vary, in some cases very widely. They are:

(1) Friction is proportional to the normal pressure between the surfaces.

(2) It is independent of the areas, or sizes, of the rubbing surfaces.

(3) It is independent of the velocity of motion, though friction of rest is greater than friction of motion.

In law 1, by "normal pressure" is meant the pressure in a direction at right angles to the surface. If an object rests upon a horizontal plane, like the top of a table, the normal pressure is equal to its weight. If it rests upon an inclined plane, as in Fig. 4, the normal pressure (at right angles to the inclined plane) is found by dividing the horizontal distance b by the length l of the plane, and multiplying the result by the weight W of the object, or

$$\text{Pressure} = \frac{b}{l} \times W \qquad (3)$$

Law 1, therefore, means that for any increase or diminution of the perpendicular pressure, the friction varies in the same ratio; thus, if the pressure is doubled or tripled, the friction becomes twice or three times as great. Law 3 varies most widely at high velocities, which tend to diminish the friction. In order that these laws shall hold, the velocity of motion of the sliding pieces must be comparatively slow, the surfaces must have little or no lubrication, and the normal pressure must be great enough so that the effect of adhesion will be inappreciable, but not so great as to cause the surfaces to "seize."

It is not intended to treat of fluid friction here, but it will be convenient to have the laws for comparison with those just given. The three most important laws are as follows:

(1) Fluid friction is independent of the pressure.

(2) It is proportional to the area of the rubbing surfaces.

(3) It is proportional to the square of the velocity at moderate and high speeds, and to the velocity, nearly, at low speed.

The friction of lubricated surfaces departs widely from any set of laws. Where the lubricant is very freely supplied, the friction depends upon the nature of the lubricant more than upon the material of the surfaces. As the surfaces become dry, the friction becomes like that of solid bodies; and when they are flooded with oil, it is more nearly like fluid friction. The friction of lubricated bearings, therefore, has become a subject of entirely independent investigations, and cannot be treated in a general way like the dry friction of solid bodies.

Coefficient of Friction.

If it should require a force of 10 pounds to pull a wooden block weighing 20 pounds along the surface of a board, the frictional resistance would be ½ or 0.5 of the normal pressure. Again, if a weight of 40 pounds were added to the block, making a total weight of 60 pounds, we know from law 1 that the resistance would be three times as great, or 30 pounds, which is still 0.5 of the pressure; and so, for any weight within the limit of law 1 the ratio of the friction to the pressure would remain this constant number 0.5. Knowing this, if it were desired to obtain the friction for any given weight of block, it would only be necessary to multiply the weight by 0.5, and if we had different numbers for different materials and various conditions, it would be very easy to calculate the friction for any particular case.

Any constant number like that above, which depends for its value upon the substance or conditions in question, is called a *coefficient*, and in the present case the *coefficient of friction*, which may be defined as that fraction of the normal pressure which is required to overcome the friction between two surfaces. *It is found by dividing the force of friction by the normal pressure.* Or expressed as a formula,

Letting f = the coefficient of friction,
F = the force of friction,
and P = the normal pressure,

$$f = \frac{F}{P} \qquad (4)$$

The following coefficients of friction may be taken as average values where more complete tables are not at hand. Under varying conditions a wide variation from these values may be found, and where coefficients are to be used, they should be obtained, if possible, from experiments suited to the particular case.

Wood on wood, dry	0.4 to 0.6
Metals on metals, dry	0.15 to 0.2
Metals on metals, lubricated	0.03 to 0.08
Metals on wood, dry	0.5 to 0.6
Leather on metals, dry	0.3

If a body is placed on a plane surface, and the latter inclined until the body is just at the point of sliding down, the angle made by the plane with the horizontal at that instant is called the *angle of friction*, or the angle of repose. It can be shown that when the plane is at this point, its height divided by the base ($h \div b$ in Fig. 4) is equal to the coefficient of friction. This fact affords one means of finding the coefficient of friction of materials by experiment. Written as a formula, we have, f being the coefficient of friction,

$$f = \frac{h}{b} \qquad (5)$$

Gravity.

The attractive force that exists between the earth and all bodies at or near its surface is called *gravity*. Weight is due to gravity. A body has weight because it is pulled downward by the force of gravity, and the amount that it weighs is a measure of this pull. A piece of iron, for example, weighs one pound when it is of such a size and density that it is drawn to the earth by a force equal to that which attracts a standard pound weight.

As has been previously mentioned, the weight of a body (that is, the force by which it is attracted to the earth), varies slightly with the locality.

(*a*) Weight varies with the altitude. A body weighs the most at the surface of the earth, as the attraction is there the strongest. *Below the surface its weight decreases in the same ratio that its distance from the center of the earth decreases.* Thus, calling the radius of the earth 4,000 miles, the relative weight of a body at the surface and at one mile below the surface would be as 4,000 : 3,999; or at the latter point its weight would have diminished 1/4,000 part. *Above the surface, the weight decreases in the same ratio that the square of the distance from the center increases.* That is to say, if a body be carried from the surface to the top of a mountain one mile high, the relative weights in the two positions would be as $4{,}001^2 : 4{,}000^2$, or as 16,008,001 : 16,000,000. Its weight would therefore diminish about 8,000 parts in 16,000,000, or 1/2,000 part.

(*b*) Weight varies with the latitude, or distance north and south of the equator. In passing from the equator to either pole, the attraction of gravity increases by 1/568 of its original amount. This is due to the want of sphericity of the earth, the polar diameter being 26 miles shorter than the diameter at the equator. At the poles, however, a body would actually weigh more than this, or about 1/193 more than at the equator. The difference, 1/289, is due to the rotation of the earth on its axis, the effect of which is to produce a force directly opposite to that of gravity, (centrifugal force), which is greatest at the equator and diminishes in moving from it, until at the poles it becomes nothing.

How Gravity Acts.

Under the influence of gravity, all bodies tend to move in a direction toward the earth's center, or to "fall," as we say, our idea of "down" being always in a direction towards this point. Gravity, therefore, acts in the direction of lines converging or meeting at the center of the earth, a point so far distant, compared with the dimensions of any bodies that are likely to be considered, that these lines of action are always assumed to be parallel. The question naturally arises, at what point in a body does gravity act? The answer is, at every point. All bodies are composed of particles, each of which has weight, and consequently is attracted by gravity. A body, therefore, is really drawn downward by a large number of forces of gravity—as many as there are molecules in the body.

It is always assumed, however, that gravity acts as a *single force* at a point called the *center of gravity*. In Fig. 5 let the dots *p*, *p*, etc., represent particles of the body *B*, under the influence of forces of

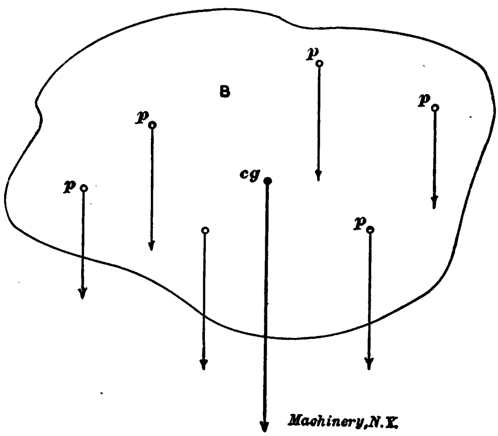


Fig. 5.

gravity, acting in parallel lines as shown by the direction of the arrows. Now, into whatever position this body be placed, there is always one invariable point through which the resultant of the attracting forces always passes. This point is called the center of gravity. It is a point, as *cg*, in Fig. 5, at which, if a single force of gravity were to act, in place of all the other forces, and equal in intensity to their sum, the effect upon the body would be the same as before. Again, since the intensity of the gravity force at each particle may be taken to represent its weight and the sum of these forces the weight of the body, we may consider the center of gravity as a point at which the weight of a body is concentrated.

Center of Gravity.

We have in the previous paragraph given an explanation of the meaning of the term center of gravity. We will now consider some of the principles involved in finding this point, together with a few of their applications. A body suspended at its center of gravity will balance in whatever position it may be placed. For this reason, the center of gravity is sometimes defined as that point about which a body will balance, in any position. Any *homogeneous* body will balance about its center of magnitude; that is, about its central point. Hence, in the case of *regular* geometrical figures, the center of gravity is readily determined, as the center of magnitude can usually be found by geometrical construction.

Center of Gravity of Geometrical Figures.

The center of gravity of a line is at its middle point; of a circle, at its center; of a rectangle, at the intersection of two lines joining the opposite corners; of a sphere or ball, at its center; of a prism and cylinder, at the middle point of a line joining the centers of gravity of the two ends. To illustrate the last two cases, the center of gravity of a bar of any homogeneous material, four feet long, two inches

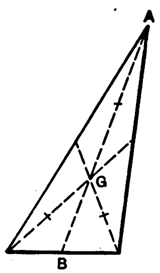


Fig. 6.

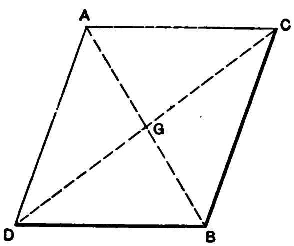
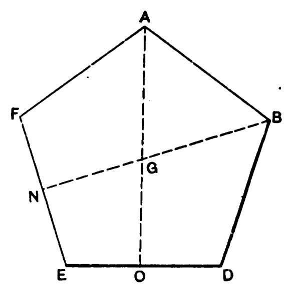


Fig. 7.



Machinery, N.Y.

Fig. 8.

wide and one inch thick, lies at a point two feet from one end, one inch from the edge and one-half inch from one side; and of a round bar of the same length, at a point on its axis two feet from one end.

The center of gravity of a triangle lies at the intersection of two lines drawn from the vertices (points) of any two angles to the middle of the opposite sides (Fig. 6). This point may also be found by drawing one of the lines, as *A B*, and laying off two-thirds of its length from the vertex. Thus, the center of gravity *G* in the figure is at a distance *A G* from *A*, equal to two-thirds of the length of the line *A B*, and the same proportion holds with the lines drawn from the other two vertices.

The center of gravity of a parallelogram is at the intersection of its diagonals, as *A B* and *C D* in Fig. 7. A parallelogram is a figure having four sides, the opposite ones being equal and parallel.

The center of gravity of a cone or of a pyramid is on a line drawn from the vertex to the center of gravity of the base, and at a distance from the vertex equal to three-fourths of the length of the line.

A help in finding the center of gravity of a plane figure is the fact that, if it has an axis of symmetry, the center of gravity will lie at some point upon this axis, and if it has two such axes, the center of gravity will lie at their point of intersection.

A plane figure is here understood to be a flat, material body, that is very thin compared with its extent or area, such as figures cut out of paper or sheet metal. Strictly speaking, a plane figure has extent, but no thickness.

An axis of symmetry is a line so drawn across a figure that it divides the latter into two parts, one of which would exactly coinside with the other, if the figure were folded over along this line. Thus, if the regular pentagon in Fig. 8 were folded about the line *A O*,

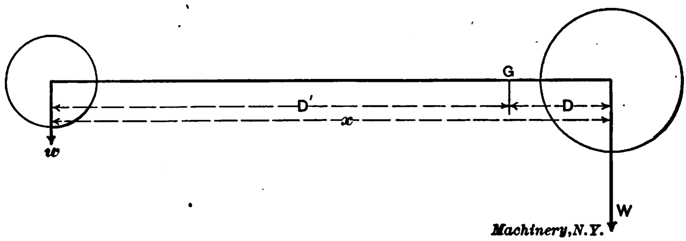


Fig. 9.

the parts *A B D* and *A F E* would exactly coincide; and if it were folded about *B N*, parts *B A F* and *B D E* would coincide. Hence, *A O* and *B N* are axes of symmetry, and the center of gravity of the figure lies at their intersection, or at *G*.

Center of Gravity of Two or More Bodies.

In Fig. 9 let the point *G* be the position of the center of gravity of the two bodies w and W. It must be so situated that they will balance about it, if rigidly connected. The turning effect exerted by each body about the point *G* is as though the weight of each were concentrated at its own center of gravity, and acted downward at that point, as indicated by the arrows. Moreover, as will appear when the subjects of moments and levers have been studied, if w and W are to balance, the ratio of the distances D' and D must be such that, calling w and W the weights of the two bodies, the proportion $w : D = W : D'$ will exist. Thus, if $w = 50$ pounds, W, 250 pounds, and D', 25 inches, then $50 : D = 250 : 25$, and $D = \frac{25 \times 50}{250} = 5$ inches.

The center of gravity lies upon a line connecting the center of gravity of each weight, and its distance D' from the smaller weight is expressed by the formula

$$D' = \frac{W x}{W + w} \qquad (6)$$

where $x =$ the distance between the centers of gravity of the weights

$W =$ the weight of the larger body,

and $w =$ the weight of the smaller body.

Stated as a rule, *to find the distance D', multiply the larger weight* *by the distance between the centers of gravity of the two weights, and divide by the sum of the weights.*

Center of Gravity by Trial.

If a body be suspended from a point, or otherwise supported so that it is free to vibrate and find its "own center," its center of gravity will place itself in the lowest possible position. If a piece of sheet metal be freely suspended from a nail, for example, the center of gravity will lie in a vertical direction from beneath the point of support. This fact may be taken advantage of in order to find the center of gravity of a flat plate by trial. Suspend it from some point, as in Fig. 10, and from the same point hang the plumb-bob *B'*. When both have come to rest, hold the string against the plate, and, using it as a guide, draw a line *A B* across the plate. As the center of gravity falls vertically below the point of support, it must lie at some point in this

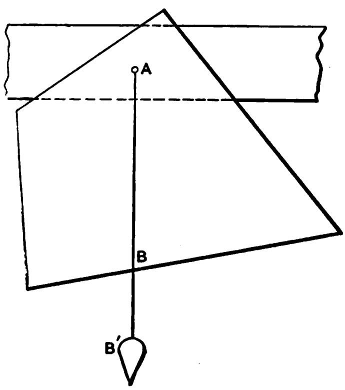


Fig. 10.

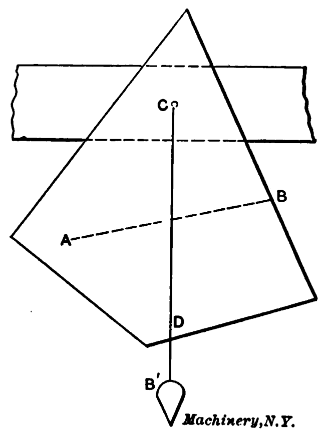


Fig. 11.

line. Next, suspend the plate from some other convenient point (Fig. 11), and repeat the operation, drawing the line *C D*. The center of gravity must lie in this line, also, and hence its location is at the intersection of lines *A B* and *C D*, since this is the only point common to them both. Furthermore, from however many points the plate might be suspended, the plumb-line would pass through this point of intersection. Two suspensions determine the point, however, and are all that are required.

Applications of Principles.

(*a*) A body is said to be in equilibrium when it balances, or has no tendency to overturn. When acted upon solely by the force of gravity, the only conditions necessary for the equilibrium of a body is that a vertical line through the center of gravity should pass through the point or surface which supports it. Thus, in Fig. 11, the plate is in equilibrium as drawn, and theoretically it would also be in equilibrium if it were turned half-way around, so that the center of gravity came directly above the point of support. In the former case, however, the equilibrium is said to be stable, while in the latter it is unstable. *Stable equilibrium exists where, on moving the body, the center of gravity ascends; and unstable equilibrium when it descends.* By swinging the plate of Fig. 11 about its point of support, the center of gravity would rise, and with the position of the plate reversed, if it were moved either way, the center of gravity would fall.

The case of bodies resting on a horizontal base is illustrated in Fig. 12. A leaning body, a chimney, for example, would remain in equilibrium so long as a vertical through its center of gravity passed within the base, as is the case here with the center of gravity at *G*. Moreover, the equilibrium would be stable, because the chimney, in overturning, would act as though pivoted at *O*, which is at the right of *G*, and therefore the center of gravity would have to ascend, slightly, along arc *G A*. Should the center of gravity be located at *G′*, the

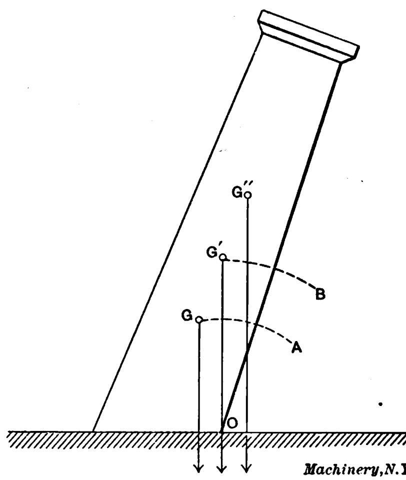


Fig. 12.

equilibrium would be unstable, because, at the moment of overturning, *G′* would begin to *descend* along the arc *G′B*. With the center of gravity at *G″*, the vertical falls without the base, and the chimney would overturn.

Equilibrium is said to be *neutral* when, upon moving a body, its center of gravity neither ascends nor descends. Examples: A flat plate suspended at its center of gravity; a cylinder, cone or sphere rolling upon a horizontal surface.

(*b*) A useful application is found in one of the theorems of Pappus, which is that the volume of any solid which can be generated by the revolution of the surface about an axis, is equal to the area of the surface by the circumference described by its center of gravity.

Moments.

The tendency of a force acting upon a body is, in general, to produce either a motion of translation (that is, to cause every part of the body to move in a straight line) or to produce a motion of rotation. A *moment*, in mechanics, is the measure of the turning effect of a force which tends to produce rotation. For example, suppose a force to act upon a body which is supported by a pivot. Unless the line of action of the force happens to pass through the pivot, the body will tend to rotate. Its tendency to rotate, moreover, will depend upon two things: (1) upon the magnitude of the force acting, and (2) upon the distance of the force from the pivot, *measuring along a line at right angles to the line of action of the force*. These two factors taken together always determine the turning effect, and their product is called the *moment* of the force.

To illustrate further, suppose the wrench shown in Fig. 13 to be in position No. 1, and that a person grasps it at point *F* and pulls in the

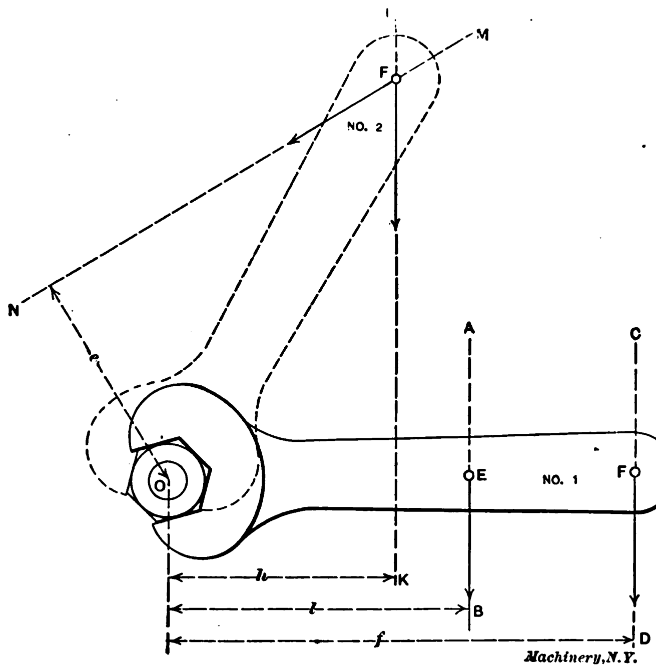


Fig. 13.

direction of the arrow along the line *C D*, first with a force of 25 pounds, and then with a force of 50 pounds. The bolt *O* acts as a pivot, and the tendency to turn the wrench and nut about it is twice as great in the latter as in the former case, because the first factor, namely, the magnitude of the force, has been increased twofold. Again, grasping the wrench at *E* and pulling along the line *A B*, its effectiveness would be lessened, for the reason that the second factor, or the distance, *l*, measured from the point *O* and at right angles to the line *A B*, is less than the distance *f* measured at right angles to line *C D*.

Finally, suppose the wrench to be in position No. 2, and to be grasped at the end at F, and to be pulled with a force of 50 pounds in the direction of line $I K$, parallel to lines $A B$ and $C D$. Here the wrench is held at the same point and pulled with the same force as at first, but we know from experience that, so far as turning the nut is concerned, the wrench will be far less effective when in position No. 1. The explanation is found in the fact that the effective distance of the force from O is the distance h, measured at right angles to the line $I K$, along which the force is supposed to act, and that this distance is less than either l or f.

From this illustration we see that the moment of a force is numerically equal to the product of the magnitude of the force and the perpendicular distance from the axis, or pivot, to the line of action of the force. To find the moment of a force, therefore, (1) determine the location of the axis about which the body is supposed to turn; (2) draw an indefinite line representing the line of action of the force; (3) multiply the force by the perpendicular distance from the axis to the line.

This perpendicular distance, as h, l, or f in Fig. 13, is called the lever arm of the moment, and the axis or pivot the center of rotation. If the force is taken in pounds and the lever arm in inches, the result will be in inch-pounds, while if the foot were used as the unit of length, the result would be in foot-pounds. The term foot-pounds, however, has here a very different meaning from that which has been given to it before. In this case it is the unit of rotative effect, and in the other the unit of work, or the work done in raising one pound one foot high. The two should not be confused.

In Fig. 13, if the pull along $C D$ should be 50 pounds and the distance f, 15 inches; the moment of the force would be $15 \times 50 = 750$ inch-pounds, or

$$\frac{15 \times 50}{12} = 62.5$$

foot-pounds. If the wrench in position No. 2 should be pulled in the direction of the arrow along the line $M N$, the moment would be the product of the force and the lever arm e. When a force tends to produce right-hand rotation, or rotation in the direction in which the hands of a watch move, its moment is said to be *positive*, and *negative* when the rotation tends in the opposite direction.

The Reaction of the Pivot.

If a block of wood be set on end on a smooth sheet of ice, as in Fig. 14, and a horizontal force be steadily applied at its upper end, it will simply slide along the surface; but let the wooden block be placed upon a rougher surface, and the result will be that it will overturn or rotate about the point O, which acts as a pivot. In both cases the frictional resistance F on the lower end of the block is a force acting in a direction opposite to P. On the ice, the force F is smaller than the force P, but on the rougher surface it becomes exactly equal to it; for, if F should be smaller than P, instead of equal to it, the block would not overturn, but would move to the left as it did when resting upon the ice. Similarly, whenever rotary motion of any body occurs, there must be at least two equal and opposite forces, not in the same straight line. This principle is universal.

In Fig. 13, for example, the bolt must re-act with a force equal and opposite to that applied to the handle of the wrench. There is a reaction at the shaft and bearing of a gear wheel or pulley, which is equal and opposite to the force applied by the driving gear or belt.

The Principle of Moments.

When two or more forces act upon a rigid body and tend to turn it about an axis, then, for equilibrium to exist, the sum of the moments of the forces which tend to turn the body in one direction must be equal to the sum of the moments of those which tend to turn it in the opposite direction about the same axis.

In Fig. 15, a lever 30 inches long is pivoted at the fulcrum *O*. At the right, and 10 inches from *O* is a weight, *B*, of 12 pounds, tending to turn the bar in a right-hand direction about its fulcrum *O*. At the left end, 12 inches from *O*, the weight *A* of 4 pounds tends to turn

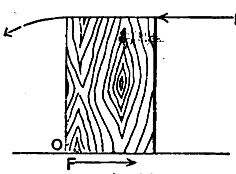


Fig. 14.

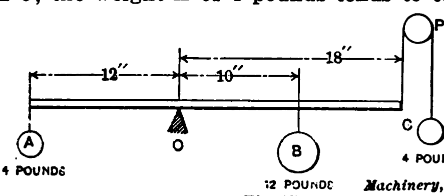


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Fig. 15.

the bar in a left-hand direction, while weight *C*, at the other end, 18 inches from *O*, has a like effect, through the use of the string and pulley *P*. Taking moments about *O*, which is the center of rotation, we have:

$$\text{Moment of } B = 10 \times 12 = 120 \text{ inch-pounds.}$$

Opposed to this are the moments of *A* and *C*:

$$\text{Moment of } A = 4 \times 12 = 48 \text{ inch-pounds.}$$
$$\text{Moment of } B = 4 \times 18 = 72 \text{ inch-pounds.}$$

$$\text{Sum of negative moments} = 120 \text{ inch-pounds.}$$

Hence, the opposing moments are equal, and, if we suppose, for simplicity, that the lever is weightless, it will balance or be in equilibrium. Should weight *A* be increased, the negative moments would be greater and the lever would turn to the left, while if *B* should be increased, or its distance from *O* be made greater, the lever would turn to the right. In the following treatment on the lever some additional examples will be taken up.

Another application of the principle of moments is given in Fig. 16. A beam of uniform cross-section, weighing 200 pounds, rests upon two supports, *R* and *R′*, which are 12 feet apart. The weight of the beam is considered to be concentrated at its center of gravity *G*, at a distance of 6 feet from each support. A weight of 50 pounds is placed upon the beam at a distance of 9 feet from the right-hand support, *R′*. Required, the portion of the total weight borne by each support.

Before proceeding, it should be explained that the two supports react or push upward, with a force equal to the downward pressure of the beam. To make this clear, suppose two men to take hold of the beam, one at each end, and that the supports be withdrawn. Then, in order to hold the beam in position, the two men must together lift or pull upward an amount equal to the weight of the beam and its load, or 250 pounds. Placing the supports in position again, and resting the beam upon them, does not change the conditions. The supports must react upwards just as the men had to pull up. The weight of the beam acts downward, and the supports react by an equal amount. This is an extension of the principle of the reaction of the pivot mentioned above.

Now, to solve the problem, assume the beam to be pivoted at one support, say at *R'*. The forces or weights of 50 pounds and 200 pounds tend to rotate the beam in a left-hand direction about this point, while

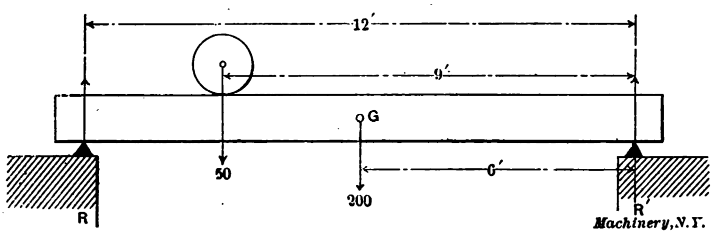

Fig. 16.

the reaction of *R* in an upward direction tends to give it a right-hand rotation. As the beam is balanced and has no tendency to rotate, it is in equilibrium, and the opposing moments of these forces must balance. Hence, taking moments,

$9 \times 50 = 450$ foot-pounds.
$6 \times 200 = 1{,}200$ foot-pounds.

Sum of negative moments $= 1{,}650$ foot-pounds.

Letting *R* represent the reaction of support,

Moment of $R = R \times 12$ foot-pounds.

By the principle of moments, $R \times 12 = 1{,}650$. That is, if *R*, the quantity which we wish to obtain, be multiplied by 12, the result will be 1,650. Hence, to obtain *R*, divide 1,650 by 12, whence $R = 137.5$ pounds, which is also the weight of that end of the beam. As the total load is 250 pounds, the weight of the other end must be $250 - 137.5 = 112.5$ pounds.

The Lever.

Under the subject of moments, it was shown that, for a lever to be in equilibrium—that is, for it to balance—the sum of the moments tending to turn it in one direction about its fulcrum, must balance or equal the sum of those which tend to turn it in the opposite direction. This simple principle enables us to solve examples where it is desired to find the length of one of the lever arms, or one of the forces or resistances acting upon the lever, the operations being somewhat similar to those used in finding the reaction of the supports of the beam shown in Fig. 16.

A very common, but at the same time a useful, illustration is found in the lever safety-valve. In Fig. 17, let *S* be the inside diameter of the valve seat; *G*, the center of gravity of the lever; and *W* the weight used to hold down the lever and keep the valve closed. The pivot or fulcrum *O* is the point about which moments are to be taken, and when the valve is just at the point of blowing off, the opposing moments which keep the lever in equilibrium are (1) the pressure against the valve multiplied by the distance *A*, tending to turn it in

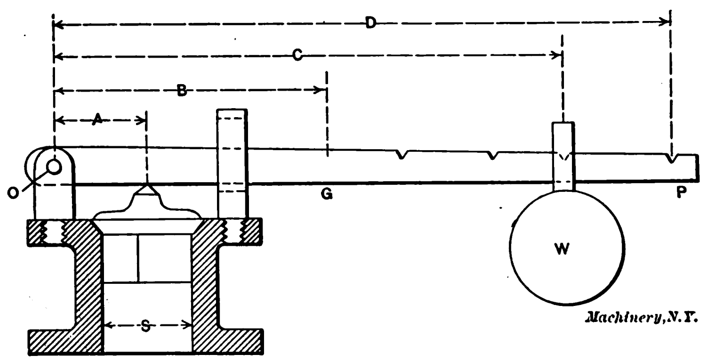

Fig.17.

a left-hand direction, and (2) the weight *W* multiplied by *C*, plus the weight of the lever multiplied by *B*, tending to turn it in a right-hand direction. The weight of the valve itself is comparatively small and may be neglected.

The Principle of Work.

There is another principle of more importance than the principle of moments, even in the study of machine elements. It is called the principle of work, and to make it clear, we will analyze the process of the operation of a machine.

1. A force such as the pull of a driving belt, or the pressure of steam, is applied in a given direction at one or more points. The product of the force, and the distance through which it moves, measure the work that is put into the machine.

2. The applied force is transmitted to the point where the operation is to be performed. During the transmission the force is modified in direction and amount, partly by the arrangement of the mechanism and partly by the resisting force of friction, which it must overcome.

3. At the point where the operation is performed the modified force overcomes a resistance in any required direction, such, for example, as the resistance of metal to a cutting tool. The product of the resistance, and the distance through which it is overcome, measures the work done by the machine.

The principle of work states that, neglecting frictional or other losses, the applied force, multiplied by the distance through which it moves, equals the resistance overcome, multiplied by the distance through which it is overcome. That is, a force acting through a given distance, can be made to overcome a greater force acting as a resistance through a less distance; but no possible arrangement can be made to overcome a greater force through the same distance.

The principle of work may also be stated as follows:

Work put in = lost work + work done by machine.

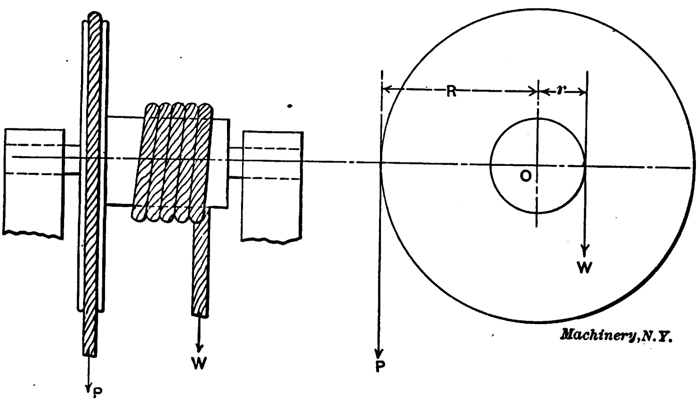

Fig. 18. Fig. 19.

This principle holds absolutely in every case. It applies equally to a simple lever, the most complex mechanism, or to a so-called "perpetual motion" machine. No machine can be made to perform work unless a somewhat greater amount—enough to make up for the losses—be applied by some external agent. As in the "perpetual motion" machine no such outside force is supposed to be applied, this problem is absolutely impossible, and against all the laws of mechanics.

The Wheel and Axle.

This mechanism, Fig. 18, is simply an arrangement for continuing the action of the lever as long as required. So long as a sufficient pull is applied to the rope, which fits into the grooved wheel, to overcome the resistance of the load attached to the rope that passes over the drum, the weight will be raised.

(*a*) First we will apply the principle of moments. In Fig. 19, let the larger circle represent the circumference of a wheel of radius R, to the periphery of which a force P is applied. Let the smaller circle represent the circumference of the drum of radius r, to the periphery of which is applied a resistance W. P and W correspond to the pull on the rope and the resistance of the weight indicated in Fig. 18.

The moment of the force P about the center O, which corresponds to the fulcrum of a lever, is P multiplied by the perpendicular distance R, it being a principle of geometry that a radius is perpendicular to a line drawn tangent to a circle, at the point of tangency. Also the opposing moment of W is $W \times r$. Hence, by the principle of moments,

$$P \times R = W \times r.$$

(*b*) Now, for comparison, we will apply the principle of work. Assuming this principle to be true, the pull P multiplied by the distance passed through by the rope should equal the resistance W multiplied by the distance that the load is raised. In one revolution the driving rope passes through a distance equal to the circumference of the wheel, which is equal to $2 \times 3.1416 \times R = 6.2832 \times R$, and the hoisting rope passes through a distance equal to $2 \times 3.1416 \times r$. Hence, by the principle of work,

$$6.2832 \times P \times R = 6.2832 \times W \times r.$$

This statement simply shows that $P \times R$ multiplied by 6.2832 equals $W \times r$ multiplied by the same number, and it is evident therefore, that the equality will not be altered by canceling the 6.2832 and writing

$$P \times R = W \times r.$$

But this is the same statement that was obtained above by applying the principle of moments. Hence, we see that the principle of moments and the principle of work harmonize.

It is to be observed that in the wheel and axle mechanism the drum may be of any size and that the wheel may be replaced by a crank, since the path described by the crank handle or crank pin is the circumference of a circle of a radius equal to the length of the crank.

Wheel-work.

A series of two or more axles geared together by toothed wheels, or by pulleys connected by belts, is called a *train*. A wheel which imparts motion is called a *driver*, and one which receives the motion a *driven* wheel. It can easily be shown that the basis of operation of a train of wheels is a continuation of the principle of the wheel and axle. In the latter the wheel is in reality a driven wheel and the axle or drum a driver, and hence we have that the product of the applied force and the radius of the driven equals the product of the resistance and the radius of the driver. To extend the rule to the wheel train, we have that the continued product of the applied force and the radii of the driven wheels equals the continued product of the resistance and the radii of the drivers. In calculations, the diameters, or the number of teeth in the wheels may be used instead of the radii, as stated above.

The Pulley.

The pulley, as a machine element, consists, in its simplest form, of a grooved wheel or sheave turning within a frame, called a block, by means of a cord or rope which passes over it. Combinations of these blocks are used in order to gain a mechanical advantage in raising weights.

In Fig. 20 is a fixed and movable pulley. The fixed pulley A, and also one end of the rope, is attached to the beam overhead, while pulley B may be raised or lowered through the action of the rope. The distance through which B and hence the weight W move is equal to one-half the movement of the free end of the rope. The applied force P, therefore, acts through twice the distance passed through by the weight, and will raise an object whose weight is equal to $2P$, neglecting, of course, all frictional losses. As the rope passes freely over the pulleys, the stress is the same at every point and is equal to the pull P. Assuming P to be 100 pounds, the pull exerted in either direction by the rope at sections a, b and c would therefore be 100 pounds, and hence the forces supporting W would be $100 + 100 = 200$ pounds, the pull upon eye-bolt C would be 100 pounds, and the forces acting at D, $100 + 100 = 200$ pounds.

In Fig. 21 is represented a combination of a double and a triple block. The pulleys of each turn freely upon the same pin as an axis, and for convenience in illustration are drawn with different diameters, this method serving well to show the principles of operation. In Fig. 22 are the same blocks, but with their positions reversed, the triple block being the movable one and the double block being fixed, while the end of the rope is here made fast to the upper or fixed block

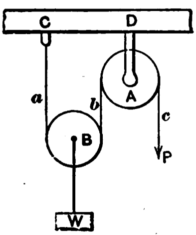

Fig. 20.

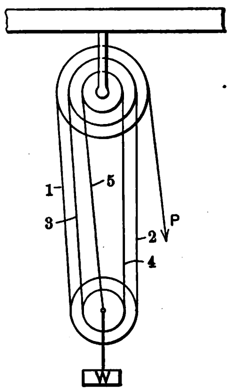

Fig. 21.

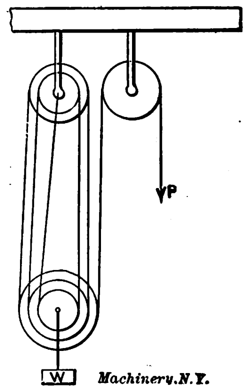

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Fig. 22.

instead of to the movable one, as in Fig. 21. In either case, by the principle of work, the applied force P, times the distance through which it moves, must equal the weight W, times the height that it is raised. Suppose W and the movable block to be raised bodily one foot without pulling at P. In Fig. 21 there would then be one foot of slack in each of the parts of the rope numbered from 1 to 5, or five feet in all, and to take up this the free end of the rope would have to be pulled down five feet, which is five times the distance moved through by the weight W. Hence, in lifting the weight a given distance, the force P moves through five times this distance; and applying the principle of work, $P \times 5 = W \times 1$, or an applied force of one pound will be sufficient to lift a weight of five pounds. By similar reasoning it will appear that, as arranged in Fig. 22, an applied force of one pound will lift a weight of six pounds, there being six parts of the rope in which slack can be taken up instead of five, as before. Whatever the arrangement or number of the pulleys, the weight that can be raised can be calculated by observing the relative distances passed through by the two forces P and W. It should be noticed however, that the resistance that can be overcome is always equal to the applied force multiplied by the number of the parts of the rope that engage with the movable block, which is a convenient rule to use. Thus, if there were seven parts springing from the movable block, a force of 100 pounds would overcome a resistance of $100 \times 7 = 700$ pounds, neglecting frictional losses.

This rule may also be arrived at by considering that the force P produces a uniform stress equal to P throughout the whole length of the rope, as was mentioned in connection with Fig. 20. In Fig. 21, for example, the tension in each of the numbered parts is equal to P, and the total upward force supporting the weight is equal to $5 \times P$.

In the foregoing it is assumed that the supporting ropes all hang vertically. In practice, they usually do, very nearly. In case they should not, however, the problem is more complicated. We shall deal with this problem later.

The Screw.

By this time the universal character of the principle of work must be apparent, even to one who but imperfectly understood its importance before. The law that work received equals work delivered, is everywhere true, if we disregard the losses of transmission. In the case of the screw, the initial force moves through the circumference of a circle, the point of application usually being at the end of a crank or bar, at the surface of a pulley, or applied in some similar manner. A screw may be defined as a cylinder around which threads are wound in successive coils or helices, equally spaced. The lead of a single-threaded screw is the distance between like points on successive threads measured on a line parallel to the axis of the screw. The amount that a screw advances in one turn is equal to the lead, and in fractional turns it is equal to the same fraction of the lead. Thus, if a screw is given one-fourth turn it advances one-fourth of the lead, and the ratio is the same as though the screw were supposed to make one complete turn and to advance a distance equal to the full lead. Hence, we have for the screw that the applied force multiplied by the circumference of the circle described by the force equals the resistance multiplied by the lead.

Machine Efficiency.

Thus far in problems of work we have neglected entirely the effect of frictional losses, which in many cases require a greater expenditure of power than that necessary for the operations actually performed by the machine.

The efficiency of a machine is the ratio of the work got out of a machine to the work put in, and is obtained by dividing the former quantity by the latter. If 1,000 foot-pounds of work were done by a machine in a given time, and 1,000 foot-pounds of work were put in in the same time, then the efficiency would be equal to $1{,}000/1{,}000 = 1$, or 100 per cent; but if only 250 foot-pounds were done by the machine, the rest being absorbed by friction, the efficiency would be 250/1,000 = 0.25, or 25 per cent. The efficiency of a machine can never be greater than 1.

Graphical Representation of Forces.

A force possesses three prominent characteristics which, when known, determine it. They are: its direction, place of application, and magnitude. The direction of a force is the direction in which it tends to move the body upon which it acts. If not influenced by any other forces, this will always be along a straight line. The place of application of a force is generally, though not always, taken at a point, as at the center of gravity. The magnitude of a force is measured in pounds.

Previously we have represented forces which have been supposed to act at a given point, or in certain directions, by means of straight lines and arrowheads, this being a natural and convenient way to do. It can be shown, moreover, that this method serves to represent very accurately the three characteristics mentioned above. The straight line indicates the line of action of the force, the arrowhead the direction in which the force is supposed to act along the line, and the length of the line and magnitude of the force, a suitable scale being adopted. Thus, if a scale of 1/16 of an inch to ten pounds were used, a line 2½ inches long would represent a force of 400 pounds. The point of application may occur at any point on the line, but it is generally convenient to assume it to be at one end.

To illustrate, in Fig. 23, a force is supposed to act along the line *A B* in a direction from left to right. The length *A B* may be made to show the magnitude of the force. If *A* is the point of application, the force is exerted as a pull, and if *B* should be assumed to be the point at which it acts, it would indicate that the force was exerted as a push. The single force which will produce the same effect upon a body as two or more forces acting together upon it is called their *resultant*. The separate forces themselves, which can be so combined, are called the *components*. The process of finding the resultant of two or more forces is called the composition of forces, and of finding two or more components of a given force, the resolution of forces.

Parallelogram of Forces.

In Fig. 24, let *A* and *B* be two pulleys which are pivoted to a board, and around which a cord is passed, having weights *P* and *Q* at the ends. Near the center of the cord a third weight, *R*, is suspended as shown. We will assume that the three weights are so proportioned that they will come to rest in the positions shown, and thus the point *O* will be acted upon by three forces in equilibrium, whose lines of action lie in the directions taken by the three parts of the cord. It is obvious, moreover, and this point should be carefully noted, that under these conditions the force acting along *O C* must be exactly equal and opposite to the resultant of the forces acting along *O A* and *O B*. Now measure along *O B* the part *O b* containing as many inches as there are pounds in the weight Q, and along $O\,A$ the part $O\,a$ containing as many inches as there are pounds in the weight P. With a pencil, draw the lines $O\,a$ and $O\,b$ upon the supporting board and complete the parallelogram $O\,a\,r\,b$. Then $O\,a$ and $O\,b$ will represent the magnitude and direction of the forces acting along $O\,A$ and $O\,B$, and upon examination it will be found that if the diagonal $O\,r$ be drawn, it will extend in the same line as the cord $O\,C$ and will contain as many inches as there are pounds in R. Therefore, $O\,r$, being opposite to $O\,C$, represents in magnitude and direction the resultant of forces $O\,a$ and $O\,b$.

The foregoing is an experimental proof of the principle of the parallelogram of forces, which is as follows:

If two forces applied at a point are represented in magnitude and direction by the adjacent sides of a parallelogram ($A\,B$ and $A\,C$ in Fig. 25), their resultant will be represented in magnitude and direction by the diagonal ($A\,R$) lying between those sides.

As an illustration of the use of the parallelogram of forces, let it be required to find the force acting through the connecting-rod of a steam engine due to the steam pressure upon the piston. In Fig. 26

Fig. 23.

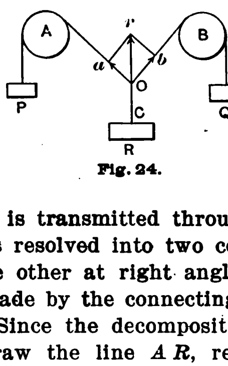
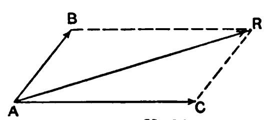


Fig. 24.



Machinery, N.Y.

Fig. 25.

the steam pressure is transmitted through the piston-rod $P\,A$, and at the cross-head A is resolved into two components, one along the connecting-rod and the other at right angles to the piston-rod. This is due to the angle made by the connecting-rod which creates a pressure upon the guides. Since the decomposition of the force occurs at A, from this point draw the line $A\,R$, representing in magnitude and direction the force of the steam pressure against the piston. Draw an indefinite line $A\,E$ at right angles to the piston-rod, and from R draw $R\,B$ and $R\,C$ parallel to $A\,E$ and $A\,D$, respectively. Then the points of intersection, B and C, will determine the lengths of the component $A\,B$ acting along the connecting-rod, and of the component $A\,C$ perpendicular to the guides.

Motion.

Motion is a progressive change of position. We can judge of the motion of a body only by comparison with the position of some other body, which latter does not have the same motion. Motion, then, is a relative term. A railroad train running at 10 miles an hour has this speed in relation to the earth, but in relation to another train moving at the same rate on a parallel track, and in the opposite direction, its motion is at the rate of 20 miles an hour. A brakeman running from the forward to the rear end of a freight train at the rate of 5 miles an hour, might be moving with either a greater or less velocity than this when compared with the ground, depending upon the motion of the train; and if it should happen that the train was moving forward at the rate of 5 miles an hour, the man would appear stationary to an observer standing beside the track.

To put a body into motion, or to alter its motion, requires the expenditure of force, as is a matter of common observance, and a little consideration will show that the tendency of force is always to produce motion, or to modify it. In case the body acted upon is perfectly free to move, however, as is nearly the condition, for example, of a heavy ball suspended from the ceiling by a long wire, the effect will always be to actually produce motion however slight the force. In that branch of mechanics called dynamics, which treats of the motion of bodies, we generally have to deal only with cases of this kind. Should

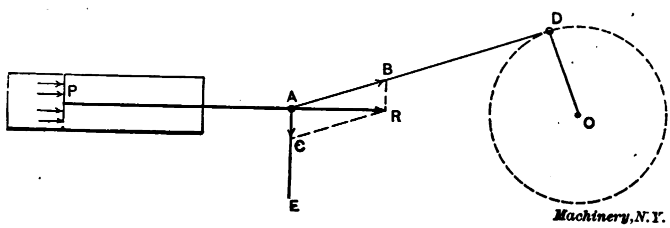


Fig. 26

it be necessary, however, to take frictional resistances into account, we deduct that part of the applied force which is used in overcoming friction, and assume that the remainder of the force acts as though such resistance did not exist.

Velocity is the rate of motion. When speaking above of the train moving 10 miles an hour, or of the brakeman running 5 miles an hour, the velocity of the train or brakeman was meant. Uniform velocity takes place when equal spaces are passed over in equal times, and variable velocity when the spaces are unequal. In physical problems, velocity is generally expressed in feet per second, and in engineering work in feet per minute. Other units are also used, as when we speak of the velocity of a railroad train as being a certain number of miles per hour.

The velocity of a body is equal to the distance passed through, *uniformly* divided by the time. In problems in dynamics it is customary to speak of distance as space, and in conformity with this we will represent it by the letter S.

Let $S =$ the space, or distance; $V =$ the velocity; and $t =$ the time. Then

$$V = \frac{S}{t}. \qquad (7)$$

Formula (7) may be re-written so as to find the values of S and t, thus:

$$S = Vt \text{ and } t = \frac{S}{V}.$$

Acceleration is the rate at which velocity changes when it is variable, that is, acceleration is the change in the velocity of a body during a very short interval of time, as a second. Thus, suppose a body to have a velocity one second of 100 feet per second, and the next second of 110 feet per second. The acceleration is then 10 feet per second *in a second.* If it should require two seconds for this increase of velocity to occur, the acceleration would be $10 \div 2 = 5$ feet per second in a second, and if it should occur during an interval of one-fourth of a second, it would be $10 \div \frac{1}{4} = 40$ feet per second in a second. When motion is decreasing instead of increasing, it is called *retarded* motion.

An important application of accelerated motion is found in the case of bodies falling under the influence of gravity; this will be taken up later. A body falling freely from rest to the earth acquires during the first second a velocity of about 32 feet per second; at the end of the second second a velocity of about $32 + 32 = 64$ feet per second; at the end of the third second a velocity of $64 + 32 = 96$ feet per second, and so on. It is thus a case of uniformly accelerated motion. This acceleration, due to the gravity of 32 feet per second in a second (32.2, more exactly, for the vicinity of London, and 32.16 for the vicinity of New York) enters so much into calculations that it is customary to always represent it by the same letter—the letter g.

Mass.

The mass of a body is the quantity of matter that it contains. We are accustomed to think of the weight of a body as a measure of its mass. When one speaks of a ton of coal, the word ton conveys at once an idea of the quantity of coal that is referred to. We know, however, that weight varies with the locality, decreasing as we go above the sea level, and increasing in passing either north or south from the equator. This fact was briefly explained in the first part of this treatise. The variation is slight, and in any case could not be detected with the ordinary balance scales, but it nevertheless exists. If a load of coal should weigh 2,000 pounds at the sea level on a pair of platform scales, and should then be drawn to the top of a mountain a mile high and similarly weighed, the scales would again balance at 2,000 pounds, because any variation in the attraction of gravity between the two places would affect the counterpoise of the scales in the same ratio that it affected the body weighed. But if the coal were weighed in a large spring balance, it would be found to weigh only about 1,999 pounds on the mountain top; yet it is perfectly plain that the quantity of matter in the coal would not be altered in any way by the journey. We thus see how easy it is, and also how erroneous, to form the idea that weight is a correct measure for quantity of matter or mass.

To obtain a numerical expression for mass, divide the weight of a body as determined by a spring balance g, by the acceleration due to gravity at that point; or for practical purposes, the weight as determined by a pair of good scales by 32.16. Expressed as a formula:

$$mass = \frac{weight}{g}. \qquad (8)$$

This expression fulfills the condition required; namely, it gives a constant value, wherever the locality. Weight varies directly as the force of gravity, and so does the value of g. Hence, if the weight and g are both determined at the same place, their ratio will be constant for all places. Thus the mass of a 100-pound weight $\frac{100}{32.16} = 3.11$ pounds. On the surface of the sun, where the force of gravity is 28 times as great as here, the same object would weigh 2,800 pounds, but its mass would be $\frac{28 \times 100}{28 \times 32.16} = 3.11$ pounds, as before. It will be observed that both mass and weight are taken in pounds. This double use of the word pound is customary, though somewhat ambiguous. Mass is an important factor in the study of motion.

Newton's Laws of Motion.

The first clear statement of the fundamental relations existing between force and motion was made in the 17th century by Sir Isaac Newton, the English mathematician and physicist. It was put in the form of three laws, which are given as originally stated by Newton:

I. Every body continues in its state of rest, or uniform motion in a straight line, except in so far as it may be compelled by force to change that state.

II. Change of motion is proportional to the force applied and takes place in the direction in which that force acts.

III. To every action there is always an equal reaction; or, the mutual action of two bodies are always equal and oppositely directed.

Law. I. The first law is known as the law of inertia, and it is, in fact, a statement of the principle of inertia. Inertia is a general property of matter, that is, a peculiar quality possessed by all bodies, just as elasticity, hardness, ductility, brittleness, etc., are properties common to different substances. By virtue of this property, called inertia, all bodies are compelled to remain at rest, when placed at rest, or in motion when placed in motion, until acted upon by some force. The term inertia means simply the inability of matter to change its state with regard to motion or rest.

The fact, as stated in the first law of motion, that any object at rest cannot of itself acquire motion, is a matter of every-day observation. Whenever a body passes from a state of rest to one of motion, a cause can always be assigned for the change, such as a blow or a push or pull. The truth of this statement on the second part of the law, however, is not so easily grasped. It is asserted that a body once in motion will continue in motion, following the path of a straight line, unless acted upon from without, and it is implied that it is as natural for a body to continue indefinitely in motion as it is for it to remain at rest. Looking about, however, it will be seen that whenever the motion of a body is altered, or changes from a rectilinear path, it is because of outside interference. A ball, for example, when thrown from the hand, moves in a curved path and finally comes to rest because of the attraction of gravity and the resistance of the air. If the ball be rolled along the rough ground, its loss of motion is accounted for by the friction, for we observe that the smoother the ground the further the ball will roll. Again, if we can conceive it possible that the ball could be hurled out into space away from these resistances, it is reasonable to suppose that it would go on forever.

The effect of inertia is also exhibited whenever we attempt to put a body suddenly into motion or to stop one already in motion. The quick start of a railway train throws everybody against the back of his seat, as we say, and in a similar manner the passengers are thrown forward when the brakes are quickly applied.

Law II. The term "motion" as here used by Newton embraces all the elements that go to make up the motion of a body, and hence introduces both mass and velocity, or what is called *momentum.* The momentum of a body is measured by the product of the mass M of the body by the velocity V, or

$$\text{momentum} = M\,V = \frac{W}{g}\,V. \qquad (9)$$

It is sometimes defined as the quantity of motion in a body. It is not a force, but rather the measure of the effect of a force in a given time, since to produce velocity in a mass requires time.

The second part of this law states that the motion takes place in the direction in which the force acts. From this follows the principle of the *independence of motions*, that when two or more forces act upon a body at the same time, each produces exactly the same effect as though it acted alone, whether the body be originally at rest or in motion. Thus, if a person threw a ball due north from the roof of a house, while the wind is blowing from the west, the effect of the throw in the northerly direction will be exactly the same as it would if the air were quiet, while the distance that the ball is carried to the east will be equal to the distance that it would travel in the same time if it were under the influence of the wind alone, disregarding, of course, any unequal frictional resistances of the air. Moreover, as the ball leaves the hand, it will gradually drop to the earth under the influence of gravity, and it will take precisely as long for it to reach the ground as it would if it had been simply dropped from the edge of the roof. That is to say, the effect of the force of gravity is exactly the same as though it acted alone; each motion goes on independently, although the position of the ball at any time depends upon the action of all the forces acting.

Law III. We have seen, under the subject of moments, how the supports of a beam react with a force equal to the downward pressure of the beam. There are many other evident illustrations of this law. A ton weight hanging on a crane hook exerts a downward pull of 2,000 pounds, and the reaction of the hook and chain is also 2,000 pounds. When a horse pulls a cart there is the reaction of the load. In jumping from a boat the reaction shoves the boat away from the shore. A man cannot "lift himself by his boot straps," because the downward push, or reaction, is equal to the upward pull.

Falling Bodies.

Under the influence of gravity alone, all bodies fall to the earth with the same velocity. The fact that heavy bodies actually fall more rapidly than those of less weight or density, as would be observed in the dropping of a stone and a leaf, is due solely to the greater retarding effect of the air upon the latter. Weight does not affect the time of fall. Weight is the measure of the attractive force of gravity, and if one body weighs twice as much as another, the attraction of gravity upon it is two times as great as upon the lighter body; but as this force must accelerate twice as great a mass in the former body as in the latter, the velocity of each must be alike. An apparatus used to prove this consists of a long glass tube with closed ends, arranged so that the air can be exhausted. When this has been done, it is found that objects of varying sizes and weights will fall from one end of the tube to the other with equal rapidity.

It has been stated before that in the vicinity of New York the acceleration due to gravity is 32.16 feet per second in a second. That is, the constant increase of velocity given by gravity during each second is 32.16 feet per second. For convenience we will call it 32 feet per second. Supposing a body to be dropped from such a height, therefore, that it falls during an interval of five seconds, its velocity at the end of each succeeding second will be as follows:

		Feet per second.
Velocity at end of 1st second =		32
Velocity at end of 2d second =	32 + 32 =	64
Velocity at end of 3d second =	64 + 32 =	96
Velocity at end of 4th second =	96 + 32 =	128
Velocity at end of 5th second =	128 + 32 =	160

It will be seen that the results 32, 64, 96, etc., may be obtained by multiplying the number of seconds by 32, the value of gravity. Hence, for finding the velocity at the end of any second, we have

$$v = g\,t. \qquad (10)$$

In this and succeeding formulas for falling bodies we will let

v = velocity of feet per second.
t = time in seconds.
g = acceleration due to gravity.
h = height in feet.

During the first second of fall the velocity at the start is 0 and at the close 32 feet per second. The *mean* velocity is 16 feet per second. Hence, the space traversed during this second is $16 \times 1 = 16$ feet. A body, therefore, falls 16 feet during the first second of motion.

In like manner, the space passed through during the second second is equal to the mean velocity during that second, multiplied by the time. The mean velocity is equal to the sum of the velocities at the beginning and end, divided by the two. Hence, by the aid of the table above, we may make out another table showing the distance passed through in each second. Since the time is one second, or unity, the multiplication by this factor may be omitted.

		Feet.
During 1st second, space =		16
During 2d second, space =	$\frac{32+64}{2}$	= 48
During 3d second, space =	$\frac{64+96}{2}$	= 80
During 4th second, space =	$\frac{96+128}{2}$	= 112
During 5th second, space =	$\frac{128+160}{2}$	= 144

It will be observed that $48 = 3 \times 16$, or three times the space passed through in the first second. Also, $80 = 5 \times 16$; $112 = 7 \times 16$; and $144 = 9 \times 16$. From this we conclude that the spaces traversed during each succeeding second are proportional to the odd numbers 1, 3, 5, 7, 9, 11, etc., which is a useful fact to remember.

We have seen that a body falls 16 feet the first second, 48 feet the second, 80 feet the third, and so on. In two seconds, therefore, it falls $16 + 48 = 64$ feet; in three seconds, $16 + 48 + 80 = 144$ feet, and so on. But $64 = 16 \times 4$, or 16×2^2, and $144 = 16 \times 9$, or 16×3^2, the 2 and 3 in each case being the number of seconds required for a body to fall 64 to 144 feet, respectively. And, in general, the space that a body will fall in a given time is equal to 16 multiplied by the square of the number of seconds. Hence,

At the end of 2d space $= 16 + 48 = 64 = 16 \times 2^2$.

At the end of 3d space $= 16 + 48 + 80 = 144 = 16 \times 3^2$.

At the end of 4th space $= 16 + 48 + 80 + 112 = 256 = 16 \times 4^2$.

At the end of 5th space $= 16 + 48 + 80 + 112 + 144 = 400 = 16 \times 5^2$.

The factor 16 that has been used is one-half of 32, the acceleration due to gravity, or $\frac{1}{2} g$. Hence, to find the total space for any time, multiply the square of that time in seconds by $\frac{1}{2} g$. Therefore,

$$h = \tfrac{1}{2} g t^2. \qquad (11)$$

Formulas 10 and 11 are the fundamental formulas for falling bodies. By combining them algebraically, we may obtain as an expression for velocity:

$$v = \sqrt{2 g h} \qquad (12)$$

From 10 and 12 may also be derived

$$t = \frac{v}{g} = \frac{\sqrt{2 g h}}{g} = \sqrt{\frac{2 h}{g}} \qquad (13)$$

These formulas apply to retarded motion which takes place when a body is thrown into the air, as well as to the accelerated motion produced by the action of gravity upon a falling body. Thus, when a body is thrown upward it is gradually retarded by the same amount that it is accelerated upon its return, and when it reaches the earth again, it has the same velocity that it had when it left the hand.

The Pendulum.

In its simplest practical form, the pendulum consists of a ball of lead or other heavy material suspended by a fine cord or wire. For convenience, this may be called a *simple pendulum*, and any pendulum in which the weight is not so concentrated, is a *compound pendulum*. Strictly, however, a true simple pendulum is merely an ideal conception—it is a particle of matter suspended by a weightless cord, and capable of vibrating without friction, while any pendulum that can be actually constructed is a compound pendulum.

The length of a pendulum is the distance from the point of suspension to a point lying below the center of gravity, called the center of oscillation. One vibration of a pendulum consists of one complete beat one way. When it swings back and forth once, two vibrations take place.

Law I. When the arc swung through is small, the vibrations occur in equal times, irrespective of the distance passed through. Moreover, the arc may vary widely in length without materially affecting the time of vibration. Thus, a pendulum of such a length that it will vibrate once in one second, when its arc of action is 5 degrees, would require only 1/200 of a second longer to vibrate through an arc of 30 degrees.

Law II. The times of vibration of different pendulums are proportional to the square root of their lengths. Thus, the times of vibrations of pendulums 1, 9 and 25 inches long would be proportional to the numbers 1, 3 and 5. It would take the second pendulum three times as long to vibrate as the first, and the third five times as long. A pendulum which vibrates once in four seconds must be four times as long as one which vibrates in two seconds, because the times of vibrations are as 2 : 1, and these must be proportional to the square roots of the lengths, or as $\sqrt{4} : \sqrt{1}$.

Law III. Time of vibration varies with the attraction of gravity, but is independent of the mass. This has been proved by swinging pendulums of different lengths in various localities and pendulums of the same length, but of different materials, at the same place.

Center of Oscillation.

The center of oscillation of a pendulum is that point which vibrates in the same time that it would if disconnected from all remaining particles. From Law II it is clear that the upper part of a pendulum tends to vibrate faster than the lower part, and so hasten its motion, while the lower part tends to vibrate slower and thus retard the motion of the whole. Between these two limits is the center of oscillation, which has the average velocity of all the particles of the pendulum, and which is neither quickened nor retarded by them. It vibrates in the same time that it would if it were a particle swinging by a weightless cord, as in the simple pendulum.

It may make it clearer to state that the center of oscillation and center of percussion of a body are at the same point. Hold an iron bar in the hand and strike an anvil a sharp blow with the end of the bar;

it will sting the hand. Strike the anvil again with that part of the bar which is near the hand, and the effect of the blow will again be felt. Now, at some point between these two a blow may be delivered and no jerk or sting will be experienced. That point is the center of percussion, which, as just mentioned, is the same as the center of oscillation. In the case of a bar of uniform cross-section, and suspended at one end, the center of oscillation lies at a distance of two-thirds of the length of the rod from the point of suspension.

The Compound Pendulum.

In order to apply the three laws to a compound pendulum, it is necessary to determine its length, which, according to the definition previously given, is the distance from its point of suspension to its center of oscillation. This done, it may be considered as a simple pendulum having the same length, for any simple pendulum of a given length will vibrate in the same time that a compound pendulum of the same length will vibrate.

It is important, therefore, to be able to locate the center of oscillation. This may be done by trial. The point of suspension and center of oscillation of a pendulum are mutually controvertible. If, therefore, a pendulum be inverted and another point of suspension found about which it will vibrate in the same time as before, this point will be the position of the first center of oscillation, and its distance from the first point of suspension can be measured.

Time of Vibration.

The time of vibration of a pendulum is found by the formula

$$t = 3.1416\sqrt{\frac{l}{g}} \tag{14}$$

where $t =$ time in seconds.
$l =$ length in feet.
$g =$ acceleration due to gravity.

In the vicinity of New York, for $t = 1$, $l = 39.1$ inches, or the length of the seconds pendulum is 39.1 inches.

Energy.

An agent is said to possess *energy* when it has the capacity of doing work—that is, of overcoming a resistance through a distance. In general, energy is something that is given to a body by doing work upon it, as when a weight is raised or is given a rapid motion, or when a spring is compressed; the energy, in turn, is given out when the body itself performs work. Energy is therefore sometimes defined as stored work. It is expressed in foot-pounds, the same unit that is used to express work.

Energy is either *potential* or *kinetic.*

(*a*) Potential energy is the power of doing work possessed by a body in virtue of its position or condition. If a body be so situated that it is acted upon by a force which will produce motion in it upon the removal of some restraining force, it is said to have potential energy.

Thus, a ball suspended by a string has the power of doing work, because, when the cord is cut, the ball will fall and will be capable of overcoming a resistance through a distance, the amount of the work depending upon the weight of the ball and the extent of the fall. A compressed spring and a head of water also have the capacity of doing work and are stored with potential energy.

The potential energy in any case is equal to the product of the force tending to produce motion, and the distance through which the body is able to move. If the suspended ball should weigh 10 pounds and hang 25 feet from the ground, it would possess 250 foot-pounds of energy. The force acting is here equal to the weight, or 10 pounds, and to raise the ball to its suspended position would require an expenditure of $10 \times 25 = 250$ foot-pounds of work, and when it falls it can give out just this amount of energy, which has been stored within it.

(*b*) Kinetic energy is the power of doing work possessed by a body in virtue of its motion. A moving railroad train, a fly-wheel, a current of air driving a wind-mill, a falling body, all possess kinetic energy. The kinetic energy of a body is obtained by multiplying one-half its mass by the square of its velocity in feet per second. Or,

$$E = \tfrac{1}{2} Mv^2 \qquad (15)$$

where E = energy in foot-pounds, M = mass, and v = velocity in feet per second. The value of mass, we have already seen, is obtained by dividing the weight of a body by 32.16, the acceleration due to gravity, or

$$M = \frac{W}{32.16}.$$

Hence we may write $\frac{W}{32.16}$ in formula (15), giving

$$E = \tfrac{1}{2} \times \frac{Wv^2}{32.16} = \frac{Wv^2}{64.32} = \frac{Wv^2}{2g}. \qquad (16)$$

It will be shown, shortly, how this formula is obtained.

Conservation of Energy.

Energy exists in various forms, such as mechanical, molecular, and chemical. It is stored in all kinds of fuel, and is made apparent by chemical reactions, by muscular effort, and by many other means. There is the potential energy of the electrical charge and the kinetic energy of the electrical current. Heat is a form of energy. In the present instance, we are concerned with these different kinds, other than mechanical, only in that the universal and important law of the conservation of energy embraces them all. This law states, first, that energy may be transformed directly or indirectly from any one form to any other form; and second, that, however energy may be transformed or dissipated, the total amount of energy must forever remain the same. Energy can neither be created nor destroyed. It simply exists, and the various processes by which it is utilized are simply means for transforming it from one form to another. The steam engine changes heat energy into mechanical energy, and the percussion of a bullet against a rock converts mechanical into heat energy and melts the bullet. A body just at the point of falling from an elevation has a store of potential energy. As it falls it loses potential energy, but its velocity increases and its potential energy is gradually changed into kinetic energy. This will be illustrated by an example.

Suppose a body weighing 100 pounds, a cannon ball, for example, to be so situated that it has no store of potential energy, and that it is shot vertically upwards with a velocity of 1,500 feet per second. From formula (16) we find its kinetic energy at the start to be

$$E = \frac{100 \times (1{,}500)^2}{64.32} = 3{,}498{,}100 \text{ foot-pounds.}$$

This results from the potential, chemical energy of the gunpowder, part of which has gone to produce heat and sound. As the ball rises, it does work against gravity, and also overcomes the frictional resistance of the air, the latter generating heat. When the ball is two miles high, its potential energy is equal to $100 \times 2 \times 5{,}280 = 1{,}056{,}000$ foot-pounds, and neglecting the frictional loss, its remaining kinetic energy is $3{,}498{,}100 - 1{,}056{,}000 = 2{,}442{,}100$ foot pounds. At the highest point reached the kinetic energy is entirely spent and the ball has its greatest store of potential energy. Could this be gathered together with the energy required for producing the heat and sound, it would exactly equal the amount of energy originally produced by the powder. As the ball drops to the earth again, its potential is changed back to kinetic energy, and when it reaches the ground it has the same velocity, and hence the same amount of kinetic energy as when it left the gun, excepting the loss through friction.

We are now in a position to understand the derivation of formulas 15 and 16.

The potential energy of a body of weight W and at a height h is equal to $W h$, or

$$E = W h \qquad (17)$$

But, from the law of the conservation of energy, the kinetic energy of the body in falling from the height h has the same value. Hence, formula (16) may be used for kinetic energy, provided an expression for velocity can be introduced into it. From formula (12) may be obtained the expression

$$h = \frac{v^2}{2g}$$

and writing this for h in (17), we get

$$E = W \times \frac{v^2}{2g} = \frac{Wv^2}{2g},$$

which is the same as before.

In examples involving the transformation of energy and its conversion into work, it should be remembered that work is done only when a resistance is overcome. A freely falling body is stored with energy, but is does no work until it meets with a resistance. The law of the energy stored in bodies is one of the most important ones in applied mechanics, particularly in hydraulics.

Rotating Bodies.

When a body revolves about an axis, the particles at different distances from the center have different velocities, and hence different amounts of kinetic energy. For any such body, however, there is a mean radius of rotation, which is of such a length that if the whole mass of the body could be concentrated at the circumference of a circle having this radius, and rotated at the same speed as before, the same amount of kinetic energy would be developed. This mean radius is called the *radius of gyration.* For a solid, cylindrical body, like a disk or an emery-wheel, the radius of gyration is equal to the radius of the disk divided by $\sqrt{2}$. For a fly-wheel rim, it is sufficiently accurate to assume it to be the distance from the center to a point halfway between the outer and inner edges of the rim.

The object of the fly-wheel is to store up energy when the machine to which it is attached accelerates, or speeds up, and to give out energy when the motion is retarded. This acceleration or retardation may be due either to a fluctuation of the load or to a change in the applied energy.

Force of a Blow.

It will be remembered that the principle of work, as applied to machines, teaches that, neglecting frictional or other losses, the work put into a machine equals the work done by the machine. This is merely a special case of the principle of the conservation of energy, and it can be used to find the force of the blow delivered by a hammer or a falling body. The work put in by the energy of a hammer at the instant of striking equals the work done in compressing or penetrating the material operated upon, and is equal to the resistance offered by the material, multiplied by the amount of this penetration.

It is clear that the resistance offered to the blow at any instant is equal to the force of the blow at that instant, and hence the work done equals the force of the blow multiplied by the amount of the penetration. It appears from this, moreover, that the force of a blow varies with the degree of penetration. Thus, suppose the energy of the first blow of a pile driver to be 10,000 foot-pounds, and that the pile sinks into the ground a distance of two feet. Before the ram can be brought to rest it must do 10,000 foot-pounds of work, and hence the average force acting must be 5,000 pounds; for 5,000 (the force acting) times 2 (the distance through which it acts) equals 10,000 (the available foot-pounds of energy). At the second stroke, suppose the ram to deliver 10,000 foot-pounds of energy and the pile to sink one foot. Again the work done must equal the force times the distance, or in this case $10{,}000 \times 1$; that is, the force of the blow is twice as great as before.

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