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No. 23

A Dollar's Worth of Condensed Information on

Theory of Crane Design

SECOND EDITION—REVISED AND ENLARGED

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CONTENTS

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CHAPTER I

THE DESIGN OF JIB CRANES

Among the various types of jib cranes employed for different services in the industrial field, the simple underbraced type is most common, and has been selected for analysis in this chapter. In the investigation, the method of design, and all the possible stresses to which this type of crane may be subjected, are considered. The treatise may appear somewhat lengthy for such a simple machine, and although some of the stresses discussed are frequently disregarded in actual practice because of the employment of large factors of safety, yet all stresses should be investigated and provision made for them, especially in cranes of abnormal capacities or proportions, or both, which are frequently met with in practice.

As has often been said, sound judgment is a requisite of a successful designer. No precise rules can ever be formulated to cover all cases as they arise in practice, and the judgment of the designer is called upon repeatedly to decide the correct proceeding where there is no precedent.

The following discussion is of a typical crane, and is treated from a theoretical as well as a commercial standpoint, such as would be followed in the engineering office of a manufacturing company.

The type considered consists essentially of a structure in which GF, a mast, rests on a foundation (see Fig. 1), and is supported at the top by a suitable connection. AE is a member secured to the mast, and supported at D by a strut DC, which is bolted or riveted to a gusset plate on the member and mast, or connected to these members either with angles or castings as in Fig. 4. Let us first investigate the stresses produced in these members composing the frame by the external forces acting on the crane. The member AE, commonly called the jib, is subjected to stresses produced by the loads concentrated at the wheels of the trolley, and the weight of the members themselves, which stresses we will proceed to find. The trolley carrying the load is supported by four wheels traveling the length of the jib and producing the loads p, p, placed at a distance d from each other. The constant distance d is known as the wheel base. These wheel loads p, p are equal to the sum of the net load to be lifted, P, plus the weight of the trolley, ropes and bottom block, divided by the number of wheels supporting trolley, usually four.

The jib is considered as a beam supported at the joints A and D, having a cantilever end DE, and subjected to axial tensile, eccentric tensile, eccentric compressive, and flexural stresses. The length of the cantilever end from D to center line of load, when the load is at extreme outer end of the jib, is frequently made about one-fourth the distance between supports A and D, since, in general, the maximum bending moment produced by loads p, p when at the end of the cantilever, and that produced when the load is midway between A and D are about equal. But, more accurately, this ratio should be proportioned so as to obtain equal maximum fiber stresses in both cantilever and span, and thus a jib having a constant cross-section, such as a rolled beam or channel, can be economically employed. When loads p, p are acting between D and E, the maximum reaction R at D, when the trolley is at the extreme end of the cantilever, is the sum of the products of each of the wheel loads multiplied by the ratio of the long levers AE and AE_1 to the short lever AD. Expressing AE, AE_1, and

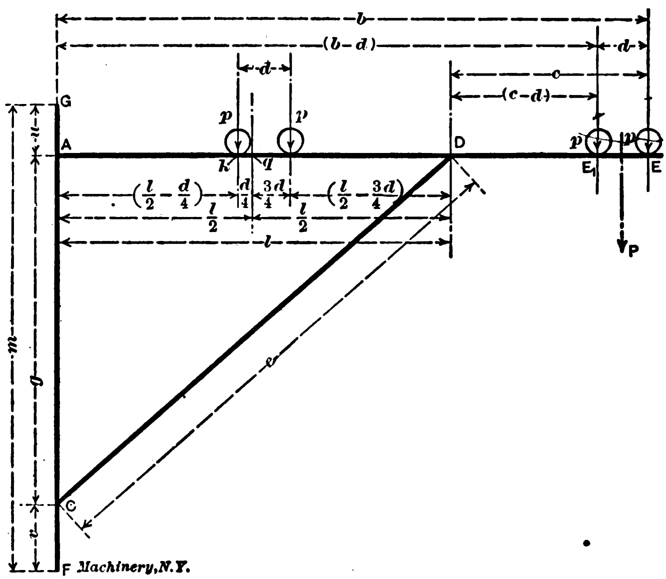


Fig. 1. Diagram of Type of Jib Crane selected for Analysis of Stresses

AD in terms of the dimension letters, we have (see Fig. 1) b, $(b-d)$ and l, respectively. Then taking moments about A, the fulcrum of the lever, we have

$$R = p \times \frac{b}{l} + p \times \frac{(b-d)}{l} \qquad (1)$$

This reaction R produces a direct tensile stress between the points A and D of the jib, and a compressive stress in strut CD.

Let side AC of the triangle ADC in Fig. 1 represent the magnitude of this reaction R; then side AD represents the value of the tensile stress, or

$$\text{Stress in } AD = \frac{\text{side } AD}{\text{side } AC} \times R,$$

and, employing dimension letters l and g, we obtain

$$\text{Stress in } AD = \frac{l}{g} \times R.$$

Substituting the value of R of formula (1) for R, we have

$$\text{Stress in } AD = \frac{pb + p(b-d)}{l} \times \frac{l}{g} = \frac{pb + p(b-d)}{g} \quad (2)$$

Before the section of the jib can be determined, it is required to find the maximum flexural stresses due to the live and dead load bending moments, and combine them with the axial or direct tensile stresses acting on span AD, when the absolute maximum bending moment occurs, that is when the wheel loads p, p are so placed that the center of the span is midway between the center of gravity of these loads and one of the trolley wheels. They must also be combined with the stresses produced by the eccentric pull of the ropes holding the load.

The direct tensile stress in the jib to be so combined is then not the maximum one just found by formula (2), but that due to the reaction R_1 when the trolley is at the position in the span producing the greatest bending moment, and the value of that reaction R_1 at D is found by taking the moments about support A, or,

$$R_1 = \frac{p\left(l + \frac{d}{2}\right)}{l}$$

Value of R_2 at A is found by taking moments about support D,

$$R_2 = \frac{p\left(l - \frac{d}{2}\right)}{l}$$

To obtain the maximum live load bending moment we take moments about point k under one of the wheels (as shown in Fig. 1); then we have

$$\text{Maximum bending moment} = R_2 \times \left(\frac{l}{2} - \frac{d}{4}\right)$$

But as $R_2 = \frac{p\left(l - \frac{d}{2}\right)}{l}$, if we substitute this value of R_2 in the last equation, we find the greatest live load bending moment from

$$\text{Live load bending moment} = \frac{p}{2l}\left(l - \frac{d}{2}\right)^2 \quad (3)$$

$$\text{Dead load bending moment} = \frac{wl}{8} \quad \ldots\ldots\ldots \quad (4)$$

$$\text{Approximate total bending moment} = \frac{p}{2l}\left(l - \frac{d}{2}\right)^2 + \frac{wl}{8} \qquad (5)$$

where d = wheel base,

w = weight of jib between supports A and D, which weight must be assumed,

$l = AD$, or span.

In regard to formula (5) it may be said that the customary approximate method of adding the maximum live load bending moment to the maximum dead load bending moment is incorrect, except in cases where the maximum live load bending moment occurs at the center of the span. The correct method for this case is to add to the maximum live load bending moment its increment of the dead load moment at that point, and not the maximum value which takes place at the center of the span. The usual method is sufficiently correct for practical purposes, however, as it is on the safe side.

The unit-stress f, due to bending, in pounds per square inch is found from

$$f = \frac{\dfrac{p}{2l}\left(l - \dfrac{d}{2}\right)^2 + \dfrac{wl}{8}}{Z} \qquad (6)$$

The unit-stress due to jib reaction R_1 is found from

$$f_1 = \frac{R_1 \times \dfrac{l}{g}}{a} = \frac{p\left(l + \dfrac{d}{2}\right)}{ag} \qquad (7)$$

Unit-stress f_2, due to tension of rope, is found from

$$f_2 = \frac{T}{a} + \frac{Tz}{Z} \qquad (8)$$

where T = tension in rope in pounds,

R_1 = value of reaction at D when greatest live load bending moment occurs,

z = eccentricity or distance between center line of rope and center line of member, in inches,

Z = section modulus of section,

a = area of section of member in square inches.

The maximum compressive stress in top flange of jib section = $f - f_1 + f_2$, or

$$\frac{\dfrac{p}{2l}\left(l - \dfrac{d}{2}\right)^2 + \dfrac{wl}{8}}{Z} - \frac{p\left(l + \dfrac{d}{2}\right)}{ag} + \frac{T}{a} + \frac{Tz}{Z}$$

or combining

$$f - f_1 + f_2 = \frac{\left[\frac{p}{2l}\left(l - \frac{d}{2}\right)^2 + \frac{wl}{8}\right] + Tz}{Z} + \frac{T - p\frac{\left(l + \frac{d}{2}\right)}{g}}{a} \quad (9)$$

The maximum tensile stress in the bottom flange of jib when such flange is opposite to the line of action of the rope (see Fig. 3) $= f + f_1 - f_2$ (f_2 in this case being modified to give tensile stress in bottom flange, due to eccentricity of rope loading), or

$$\frac{\frac{p}{2l}\left(l - \frac{d}{2}\right)^2 + \frac{wl}{8}}{Z} + \frac{p\left(l + \frac{d}{2}\right)}{ag} - \frac{T}{a} + \frac{Tz}{Z}$$

or combining,

$$f + f_1 - f_2 = \frac{\left[\frac{p}{2l}\left(l - \frac{d}{2}\right)^2 + \frac{wl}{8}\right] + Tz}{Z} + \frac{\frac{p\left(l + \frac{d}{2}\right)}{g} - T}{a} \quad (10)$$

These results should not exceed the specified fiber stress for the structure. Before selecting a structural shape to resist these maximum stresses just found, the stresses on the cantilever end should be considered as follows:

f = flexural stress due to bending,
f_1 = tensile stress due to jib reaction,
f_2 = compressive stress due to tension or pull of ropes.

Live and dead load maximum bending moment

$$= (p \times c) + [p \times (c - d)] + \left(w \times \frac{c}{2}\right) \quad (11)$$

and f, or stress due to bending on cantilever

$$\text{Unit stress } f = \frac{pc + p(c - d) + \frac{wc}{2}}{Z} \quad (12)$$

This maximum flexural stress takes place at D, and immediately to the left of D, there exists at the same time the direct tensile stress due to the maximum reaction R, when the trolley is at the extreme end of the cantilever producing this bending stress, found in formula (1), which also must be combined with the stress due to the pull of the rope. Therefore the unit-stress at point $D = f + f_1 - f_2$

$$\text{Unit-stress } f_1 = \frac{Rl}{ag} = \frac{pb + p(b - d)}{ag} \quad (13)$$

Stress due to pull of ropes $= f_2$

$$\text{Unit-stress } f_2 = \frac{T}{a} + \frac{Tz}{Z} \quad (8)$$

Therefore the maximum fiber stress =

$$f + f_1 - f_2 = \frac{pc + p\,(c - d) + \dfrac{wc}{2}}{Z} + \frac{pb + p\,(b - d)}{ag} - \left(\frac{T}{a} + \frac{Tz}{Z}\right)$$

or combining,

$$f + f_1 - f_2 = \frac{pc + p\,(c - d) + \dfrac{wc}{2} - Tz}{Z} + \frac{\dfrac{pb + p\,(b - d)}{g} - T}{a} \qquad (14)$$

where p = wheel load as before,

w = weight of section of jib from D to its extremity (see Fig. 1).

The compressive stress in strut CD is $R \times \dfrac{\text{side } CD}{\text{side } AC}$, of the triangle ADC, or $R \times \dfrac{e}{g}$

And since R is maximum when the trolley is at the extreme end of the cantilever, or

$$R = \frac{pb + p(b - d)}{l} \qquad (1)$$

then the maximum compressive stress in strut =

$$\frac{pb + p(b - d)}{l} \times \frac{e}{g} \qquad (15)$$

$$\text{Unit-stress in strut} = \frac{[pb + p(b - d)]e}{agl}, \text{ (see Fig. 1).} \qquad (16)$$

where a equals area of cross-section of strut.

The allowable unit-stress per square inch of section of this member is found by the usual Gordon formulas:

$$\text{for structural steel, } f = 17{,}100 - 57\frac{l}{r} \qquad (17)$$

$$\text{for yellow pine, } f = 1{,}200 - 18\frac{l}{t} \qquad (18)$$

However, a satisfactory reducing formula of the Rankine type, extensively used by bridge companies, and specified by some railroad companies, is recommended. It is as follows:

$$\text{for structural steel, } f = \frac{15{,}000}{1 + \dfrac{l^2}{13{,}500\, r^2}} \qquad (19)$$

$$\text{for yellow pine, } f = \frac{1{,}200}{1 + \dfrac{l^2}{250\, t^2}} \qquad (20)$$

where l = length of strut in inches,
t = thickness of timber in inches,
r = least radius of gyration.

The stress in the strut due to its own weight is neglected as being very small in most practical cases.

Ordinarily the ratio $\frac{l}{r}$ should not exceed 130; however, this ratio is frequently increased if the fiber-stress is well under the one specified, and as long as its departure from straightness will not subject the strut to an appreciable bending moment.

The stresses that may exist in the mast are as follows: (See Fig. 1.)

(1) Axial compression due to reaction R_2 and weight of structure.

(2) Eccentric stress due to R_2 when trolley is at extreme position on jib next to mast for cranes where jib connects to the face of the mast, and not at the center line of gravity of its section.

(3) Eccentric flexural stress due to tension in ropes.

(4) Flexural stress due to direct tension in jib AE, and to the horizonal component of direct compression in the strut DC.

(5) Eccentric flexural stress due to weight of drum and other hoisting machinery. This last stress is usually disregarded, however, except where the jib and hoisting machinery are of abnormally large proportions.

$$\text{Unit-stress } f'_1 = \frac{R_2}{a},$$

but

$$R_2 = \frac{pl + p(l - d)}{l}$$

therefore

$$f'_1 = \frac{\frac{pl + p(l-d)}{l}}{a} = \frac{pl + p(l-d)}{al} \qquad (21)$$

$$\text{Unit-stress } f'_2 = \frac{R_2}{a} + \frac{R_2 z_1}{Z} \text{ (see Fig. 3)},$$

but

$$R_2 = \frac{pl + p(l - d)}{l}$$

therefore

$$f'_2 = \frac{pl + p(l-d)}{l}\left(\frac{1}{a} + \frac{z_1}{Z}\right) \qquad (22)$$

$$\text{Unit-stress } f'_3 = \frac{T}{a} + \frac{Tz_2}{Z}\cos\theta \qquad (23)$$

$$\text{Tension in jib} = H = \frac{pb + p(b-d)}{g} \text{ (see Fig. 2)} \qquad (2)$$

The horizontal component of stress in strut is equal to the tension

H. The mast is then considered as a beam supported by reactions *H* and *r*. (See Fig. 2.)

$$r = \frac{p \times b + p(b - d) + w_1 \times j}{m} \qquad (24)$$

where w_1 = weight of structural frame,
j = distance from center of mast to center of gravity of frame,
m = distance between centers of bearings.

The quantity $w_1 \times j$ may be omitted when the frame is not very large. The maximum bending moment in the mast is then $r \times u$ or

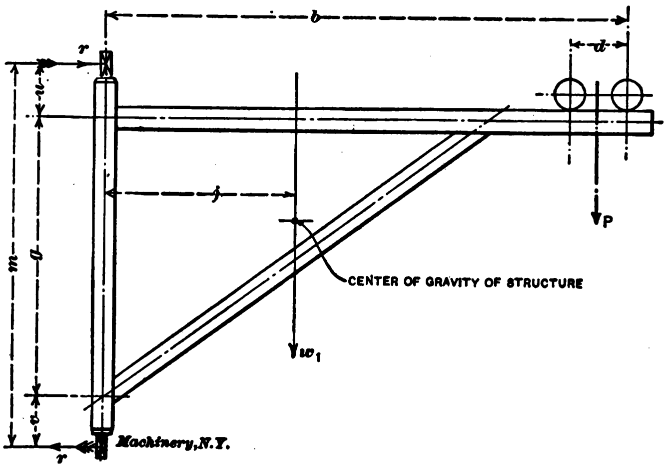


Fig. 2. Outline of Crane for which the Design is Calculated

$r \times v$, whichever is greatest. Distances *GA* and *CF*, Fig. 1, should be as small as consistent with the design to obtain economy.

$$\text{Unit-stress } f_4' \text{ at cantilever } GA = \frac{ru}{Z}$$

$$\text{Unit-stress } f_5' \text{ at cantilever } CF = \frac{rv}{Z} \qquad (25)$$

The axial compressive stress in the mast due to the whole weight of the structure, should be added to the flexural compressive stress f_4' due to bending when the trolley is at the extreme end of the jib, since that part of the mast immediately beneath *C* is subjected to both at the same time under these conditions.

The stress f_4' is not added to the stress f_2' as found by formula (22), because they do not take place at the same time, the maximum bending taking place when the trolley is at the end of the jib, and the maximum eccentric compressive stress when the trolley is close to the mast.

It is sometimes required, when long jib members are necessary, to brace the two shapes composing the jib at some intermediate point in order to reduce the ratio $\frac{l}{r}$, and, at the same time, lessen the tendency of the jib members to spread. This is done by securing structural shapes bent clear over the jib trolley. (See Fig. 4.) The ratio $\frac{l}{r}$ should not exceed that above specified.

The pintles at G and F should be made large enough to resist the bending moment on them, and also designed for a safe bearing pressure per square inch of their projected area. This pressure is the quantity r in formula (24).

The jib end connection is subjected to flexural stresses due to the tension of the rope or ropes, which should be taken into consideration. The connection is treated as a beam, and the pull of the rope or ropes

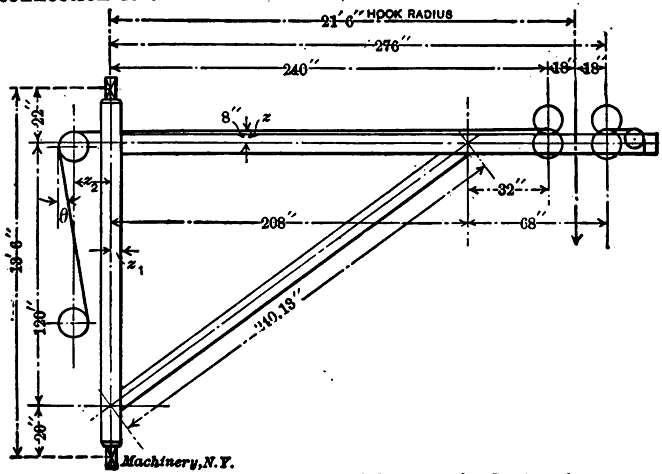


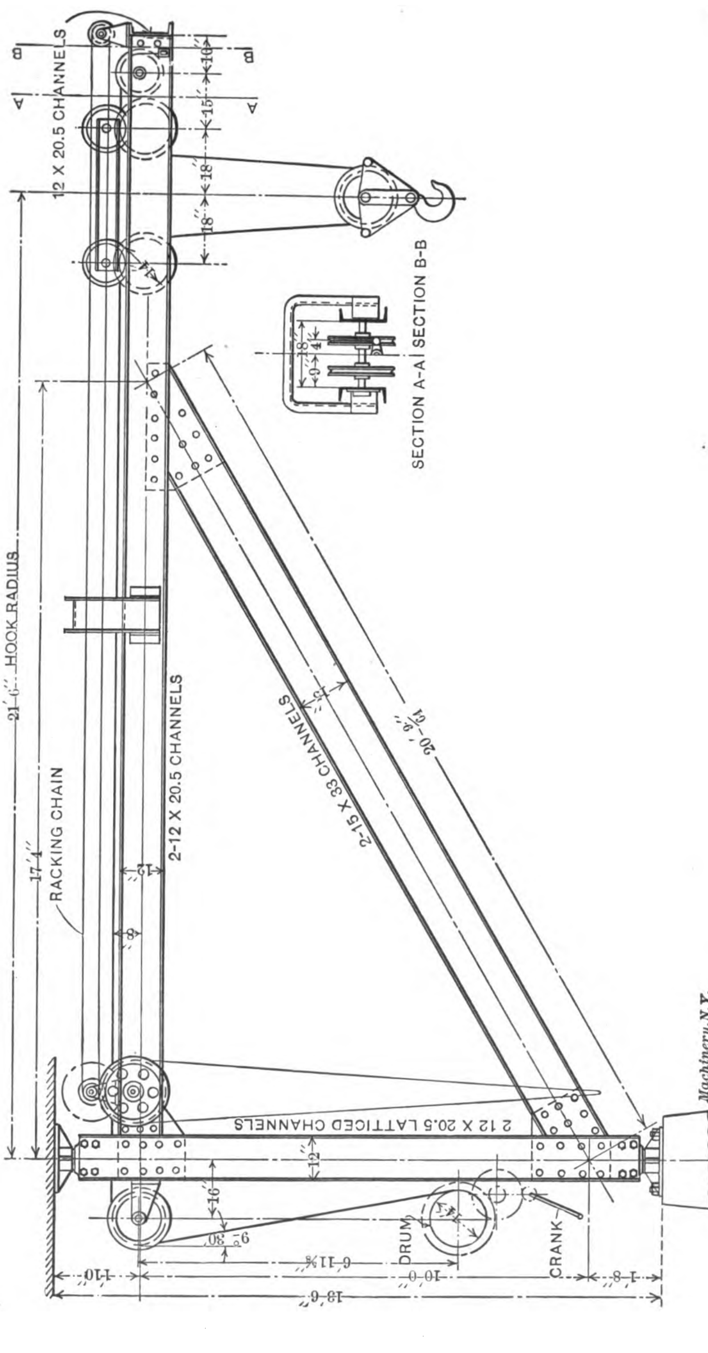
Fig. 3. General Dimensions of Crane to be Designed

as concentrated loads in the middle or at equal distances from the middle, according to the kind of connection employed, the beam in question being supported at both ends.

Example

Required to design a jib crane of the underbraced type to lift a load of 10,000 pounds at a radius of 21 feet 6 inches; distance between underside of roof truss or top support and floor 13 feet 6 inches; jib to be constructed of two structural steel frames composed of standard size channels and connected together (see Fig. 4); trolley mounted on four wheels running on top flanges of jib member. Maximum fibre-stress 13,000 pounds per square inch, which is allowable for hand-power machines. For a load of 10,000 pounds we will use four parts of 7/16-inch—6 strands of 19 wires—plow steel hoisting rope, having a breaking strength of 17,700 pounds, and will give a factor of safety of

$\frac{4 \times 17,700}{10,000} = 7.08$, which must also take care of the bending stresses



Machinery,N.Y.

Fig. 4. Crane Calculated to Lift a Load of 10,000 Pounds at a Radius of 21 Feet 6 Inches

in the ropes. This size of rope will require sheaves of 14 inches in diameter, and will allow a wheel base of 36 inches. Two ends of these two lengths of rope will wind on the drum, and the other two ends will be supported at the outer end of the jib by an equalizing beam.

Load to be lifted..................................	10,000 pounds
Approximate weight of trolley, ropes and block....	500 pounds
Total	10,500 pounds

which will make the wheel loads $\frac{10,500}{4} = 2,625$ pounds each.

Distance between mast and joint D, Fig. 3, $= 208$ inches.

Distance between jib and joint $C = 120$ inches.

Distance between mast and extreme position of outermost wheels of trolley = effective radius + half the wheel base = 21 feet 6 inches + 1 foot 6 inches = 23 feet = 276 inches.

Let us first assume the trolley at that position in the span AD producing the greatest bending moment (see Fig. 1):

Maximum live load bending moment

$$= \frac{2625}{2 \times 208}\left(208 - \frac{36}{2}\right)^2 = 227,793 \text{ inch-pounds.} \qquad (3)$$

By looking at the table of properties of steel channels in any steel company's handbook, we find that a 12-inch channel weighing 20.5 pounds per foot, with an area of 6.03 square inches, has a section modulus about the axis perpendicular to the web of 21.4, and this value divided into the live load bending moment will give a stress of 10,644 pounds per square inch, which leaves us a margin for the other stresses to be yet considered. Therefore, we will temporarily select the above shape for the purpose of finding the bending-moment due to the uniform weight of the member itself.

$$\text{Weight of channel between } A \text{ and } D = \frac{208}{12} \times 20.5 = 355 \text{ pounds.}$$

$$\text{Dead load bending moment} = \frac{355 \times 208}{8} = 9230 \text{ inch-pounds.} \qquad (4)$$

Approximate total bending moment

$$= 227,793 + 9,230 = 237,023 \text{ inch-pounds.} \qquad (5)$$

Unit-stress due to bending

$$= \frac{\frac{p}{2l}\left(l - \frac{d}{2}\right)^2 + \frac{wl}{8}}{Z} = \frac{237,023}{21.4} = 11,076 \text{ pounds per sq inch.} \qquad (6)$$

Unit-stress due to reaction R_1

$$= \frac{2625 \times \left(208 + \frac{36}{2}\right)}{120 \times 6.03} = 817 \text{ pounds per square inch.} \qquad (7)$$

Tension in ropes $\frac{10000}{4} = 2500$ pounds.

Unit-stress due to tension in rope

$$= \frac{2500}{6.03} + \frac{2500 \times 8}{21.4} = 1348 \text{ pounds per square inch.} \qquad (8)$$

Total stress on top flange $= f - f_1 + f_2 = 11{,}076 - 817 + 1{,}348 =$ 11,607 pounds per square inch (9), which stress is under the one specified; the shape tentatively selected may therefore be used for this member of the crane.

Weight of cantilever end of jib $= \frac{93}{12} \times 20.5 = 159$ pounds.

Unit-stress due to bending

$$= \frac{2625 \times 68 + 2625 \times 32 + 159 \times \frac{93}{2}}{21.4} = 12613 \text{ pounds per sq. inch.} \qquad (12)$$

Unit-stress due to reaction R

$$= \frac{2625 \times 276 + 2625 \times (276 - 36)}{120 \times 6.03} = 1872 \text{ pounds per sq. inch.} \qquad (13)$$

Unit-stress due to pull in rope

$$= \frac{2500}{6.03} + \frac{2500 \times 8}{21.4} = 1348 \text{ pounds per square inch.} \qquad (8)$$

Total unit-stress on top flange of cantilever $=$

$12{,}613 + 1{,}872 - 1{,}348 = 13{,}137$ pounds per square inch, (14)

which is 137 pounds per square inch more than the specified stress. In practice, this will not be considered of sufficient importance to change the design.

Total length of jib member $=$ 25 feet 1 inch, or 301 inches.

Least radius of gyration of 12×20.5 pounds channel $= 0.81$.

The ratio $\frac{\text{length}}{\text{least radius or gyration}} = \frac{301}{0.81} = 371$, consequently the channels of the two frames should be braced at least at a point midway between the end connection and the mast. (See Fig. 4.)

Length of strut $DC = \sqrt{120^2 + 208^2} = 240.13$ inches. Selecting a 15×33-pound channel having a cross-sectional area of 9.9 square inches, and least radius of gyration of 0.91, for strut, we have the compressive unit-stress

$$= \frac{[2625 \times 276 + 2625 (276 - 36)] \times 240.13}{208 \times 120 \times 9.9} = 1316 \text{ pounds per sq. inch.} \qquad (16)$$

$$\text{Allowable stress} = \frac{15000}{1 + \frac{240.13^2}{18500 \times 0.91^2}} = 2440 \text{ pounds per sq. inch.} \qquad (19)$$

The ratio of the length of the strut to its least radius of gyration is $\frac{240.13}{0.91} = 264$, which is excessive; the maximum unit-stress, however, is very low, only 1,316 pounds per square inch, or hardly more than half of that allowed by the formula (19). As there is not a channel rolled by any mill with a greater "least radius of gyration" than the one we have employed, we may stiffen the strut laterally by riveting an angle to its web in the inside or back of channel. Unless the ratio 130 must be adhered to, the channel should be left as it is as long as the member shows no great deflection under load.

Let us now investigate the stresses existing in the mast, which we assume is composed of two 12 x 20.5-pound channels. The distance from center of mast to nearest wheel when the trolley is at the extreme position next to mast = 11 inches.

$$\text{Then } R_2 = \frac{2{,}625\ (208 - 11) + 2{,}625\ (208 - 11 - 36)}{208} = 4{,}518 \text{ pounds.}$$

As the two vertical shapes composing the mast are latticed together, we will take the two equal reactions R_2 (one which acts on one channel and the other on the opposite one) to be resisted by the two shapes combined, therefore the least radius of gyration of the mast as built is then that perpendicular to the web of the channels, whose value is 4.61.

Then the allowable compressive stress

$$= \frac{15000}{1 + \frac{162^2}{13500 \times 4.61^2}} = 13{,}761 \text{ pounds per square inch.} \qquad (19)$$

Unit stress f'_1

$$= \frac{2625\ (208 - 11) + 2625\ (208 - 11 - 36)}{6.03 \times 208} = 749 \text{ pounds per sq. inch.} \qquad (21)$$

Stresses f'_2 do not take place in this gusset-connected frame.

Unit-stress f'_3 due to tension of rope

$$= \frac{2500}{6.03} + \frac{2500 \times 16}{21.4} \times \cos 9 \text{ deg. } 30 \text{ min.} \qquad (23)$$

$$= 2257 \text{ pounds per square inch. (See Fig. 4.)}$$

Horizontal reaction at top and bottom of mast when load is at extreme outside end of jib =

$$r = \frac{2{,}625 \times 276 + 2{,}625\ (276 - 36)}{162} = 8{,}361 \text{ pounds.} \qquad (24)$$

Unit-stress f'_4 due to bending moment at top of mast =

$$\frac{8{,}361 \times 22}{21.4} = 8{,}548 \text{ pounds per square inch.} \qquad (25)$$

Maximum unit-stress immediately beneath point A of mast $= f'_4 + f'_3 = 8{,}548 + 2{,}257 = 10{,}805$ pounds per square inch.

For the end connection of the jib at *E* we select a 12-inch × 20.5 pound channel for the sake of symmetry, and proceed to investigate the bending stress to which it is subjected, due to the pull of the ropes. The distance between the jib members is 18 inches; the pull on the ropes 2,500 pounds. The section modulus of the channel in consideration about an axis parallel to the web is 1.75. Two ropes, both four inches from the center of connecting channel are used (see Fig. 4).

Maximum bending moment on channel = 2,500 × (9 — 4) = 12,500 inch-pounds.

$$\text{Unit-stress} = \frac{12{,}500}{1.75} = 7{,}143 \text{ pounds per square inch.}$$

Horizontal reaction on pintles

$$r = 2 \times \frac{2{,}625 \times 276 + 2{,}625\,(276 - 36)}{162} = 16{,}722 \text{ pounds.} \qquad (24)$$

Assuming the pintles to be 6 inches long, and taking moments about a lever arm from the center of the bearing to the support (= 3 inches), we have, bending moment = 16,722 × 3 = 50,166 inch-pounds. Unit-stress on pintle should not exceed 9,000 pounds per square inch for machine steel.

Section modulus of a circular section $= \dfrac{\pi d^3}{32} = 0.098d^3$, where d = diameter of section.

$$\text{Diameter of pintle} = d = \sqrt[3]{\frac{50{,}166}{0.098 \times 9{,}000}} = 3.84 \text{ inches.}$$

The bearing pressure on pintles should not exceed 1,000 pounds per square inch of projected area. Therefore $\dfrac{16{,}722}{1{,}000} = 16.72$ square inches are required. We will make the pintles 3⅞ inches in diameter × 6 inches in length, which will give a bearing pressure of $\dfrac{16{,}722}{3.875 \times 6} = 719$ pounds per square inch.

CHAPTER II

EXAMPLES OF JIB CRANE CALCULATIONS

The following examples will prove helpful as suggestive of the ordinary procedure in jib crane calculations. Two problems are presented for solution, the first of which may be stated as follows.

Problem 1

The column of the crane, designed as shown in Fig. 5, is of cast iron, has all the appearance of being sound, and is supposed to have ¾ inch thickness of metal. The dimensions are as per sketch. The compression member consists of two 7-inch channels, weighing 11¼ pounds per foot, arranged back to back with a 3-inch space between

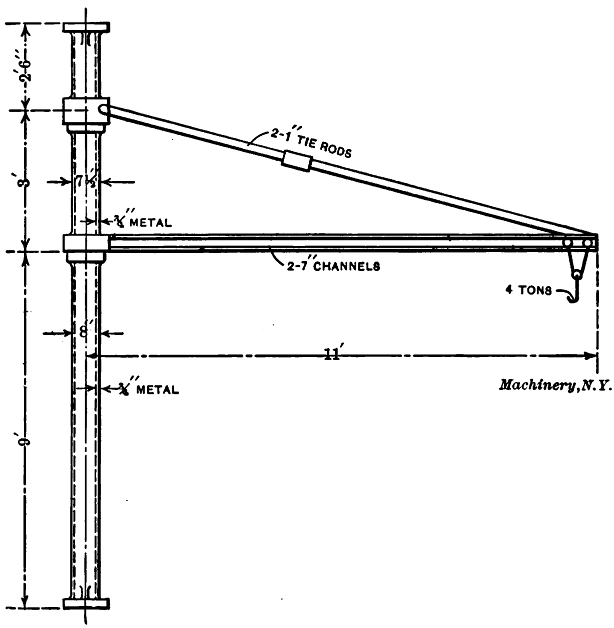


Fig. 5. General Construction of Jib Crane in Problem 1

them for the trolley to operate in. These are fastened together at each end, and the outer ends are supported by two 1-inch rods. The question raised is whether it will be safe to suspend 4 tons from the end of the 11-foot jib.

Calling a ton 2,000 pounds, the force conditions, reduced to simplest terms, will be as shown in Fig. 6. A compound beam with compressive stress, as indicated in the lower part of Fig. 6, would evidently be an equivalent case. Considering the column as a compound beam, the moment diagram will be as shown in Fig. 7. From this it will be seen that the maximum bending moment on the column equals 54,630 foot-pounds, exerted in the axis of the jib, or 5½ feet from the upper end of the column.

To find the maximum fiber unit stress for the case of a beam subject to flexure by transverse loads and also to compression in the

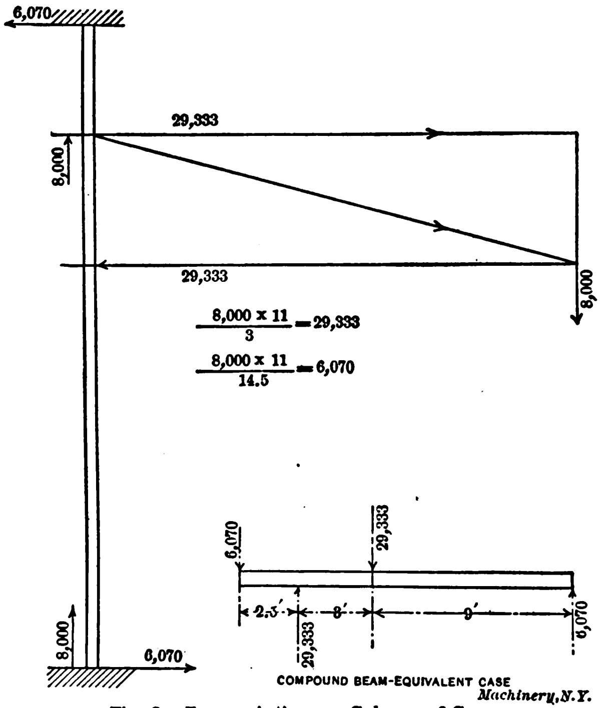


Fig. 6. Forces Acting on Column of Crane

direction of its length, we find in Merriman's "Mechanics of Materials," 10th edition, page 256, a formula which can be reduced to the form

$$S_1 = \frac{M\,c}{I - \dfrac{n\,P\,l^2}{m\,E}}$$

where S_1 = maximum fiber unit stress,
M = maximum bending moment in inch-pounds,
c = distance from the neutral axis to the remotest fiber,
I = moment of inertia of the cross section,
P = longitudinal compressive force = 8,000 pounds,
E = coefficient of elasticity = 15,000,000 for cast iron,
n and m = numbers depending on design and kind of loading,
l = length of span of the beam, in inches.

In the above, M, the maximum bending moment, $=$ 54,630 foot-pounds $=$ 54,630 $\times$ 12 inch-pounds, and $c = 3\frac{3}{4}$ inches.

I, for hollow column, $= 0.0491\ (d^4 - d_1^4)$, where d and d_1 are the external and internal diameters, $7\frac{1}{2}$ and 6 inches, and hence $I = 91.5$. The approximate value of $\frac{n}{m}$ is 1-12. The span l, in the equivalent case of the compound beam, is the distance between supports, or in our case, 144 inches. Hence we have

$$S_1 = \frac{54{,}630 \times 12 \times 3\frac{3}{4}}{91.5 - \frac{1}{12} \times \frac{8{,}000 \times 144^2}{15{,}000{,}000}}$$

$=$ 27,150 pounds per square inch.

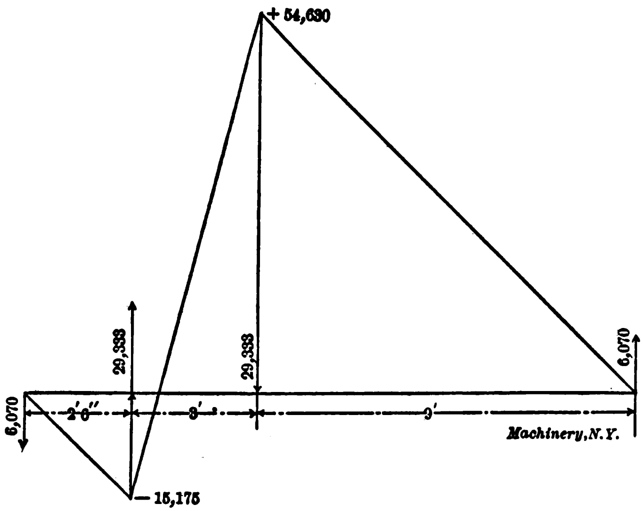


Fig. 7. Moment Diagram for Crane Column

To this should be added $\frac{P}{A}$, where A is the cross sectional area of the column, for the maximum compressive unit stress. $A = 15.9$ inches, making $\frac{P}{A} = \frac{8{,}000}{15.9} = 503$, and hence $S = S_1 + \frac{P}{A} = 27{,}650$ pounds per square inch, for compression. For tension, $S = S_1 - \frac{P}{A} = 26{,}650$ pounds per square inch.

Since the average ultimate strength of cast iron in tension is 20,000 pounds per square inch, it follows that the column will probably fail when a load of 8,000 pounds is lifted at the end of the jib.

The above method applies also to the discussion of the channels which are under combined flexure and compression. The slenderness-ratio of this column is too large for good engineering practice, and entirely insufficient for a load of 4 tons.

Problem 2

Our second problem we will present as follows: How should the stresses and sizes of the members for the crane shown in Fig. 8 be figured? The load is 5 tons. Members are to be built up of two channel irons, back to back.

The calculation of the size of the channels is largely one of trial and error, and we will simply give calculations showing the maximum stresses in the members we have selected as suitable for use in the case in question, after having tried various sizes. As shown in Fig. 9, it seems best to use 15-inch 33-pound channels for the yard arm, and 12-inch 20½-pound channels for the mast and brace. The channels forming the mast should be latticed. The calculations given below do not consider any of the minor factors which enter into the problem, such as the weight of the beams themselves, the weight of the trolley, and the pull of the ropes. These factors would appear to be amply taken care of in the margin of strength given by the channels selected. The designer, however, should always make sure of this.

The following table gives the properties of the shapes we will consider in our calculations:

Depth of channel in inches	15	12	10
Weight per foot in pounds	33.0	20.5	15.0
A = area of section in square inches	9.90	6.03	4.46
r = least radius of gyration	0.912	0.805	0.718
Z = section modulus, axis perpendicular to web	41.7	21.4	13.4

In addition to the reference letters given in the table above and in Fig. 9, the following will be used:

M = bending moment,
S_b = maximum fiber stress due to bending,
S_t = maximum fiber stress due to tension,
S = maximum fiber stress.

First find the maximum fiber stress due to bending at D in the yard-arm, when the load is at the extreme outer position E in Fig. 9.

$$M = W c = 5{,}000 \times 60 = 300{,}000 \text{ inch-pounds.}$$

$$S_b = \frac{M}{Z} = \frac{300{,}000}{41.7} = 7{,}200 \text{ pounds per square inch.} \qquad (26)$$

Note that W is only half the total load, since each member of the structure is composed of two channels, one on each side. The bending moment at D when the load is at B is found thus:

$$M = \frac{Wl}{4} = \frac{5{,}000 \times 96}{4} = 120{,}000 \text{ inch-pounds.} \qquad (27)$$

This being much smaller than in the previous case, it will give less than half the fiber stress. Unless there is some good reason for the design of framework adopted, it would be well to make *ED* about one-fourth of the length of *DH*. If this is done, the bending moment will be the same whether the load is at *E* or *B*, and, will in either case, be less than the maximum moment we have just found, so that a smaller section could be used.

The vertical reaction at *D* is found thus:

$$R_1 = W \times \frac{a}{l} = 5{,}000 \times \frac{13}{8} = 8{,}125 \text{ pounds.} \qquad (28)$$

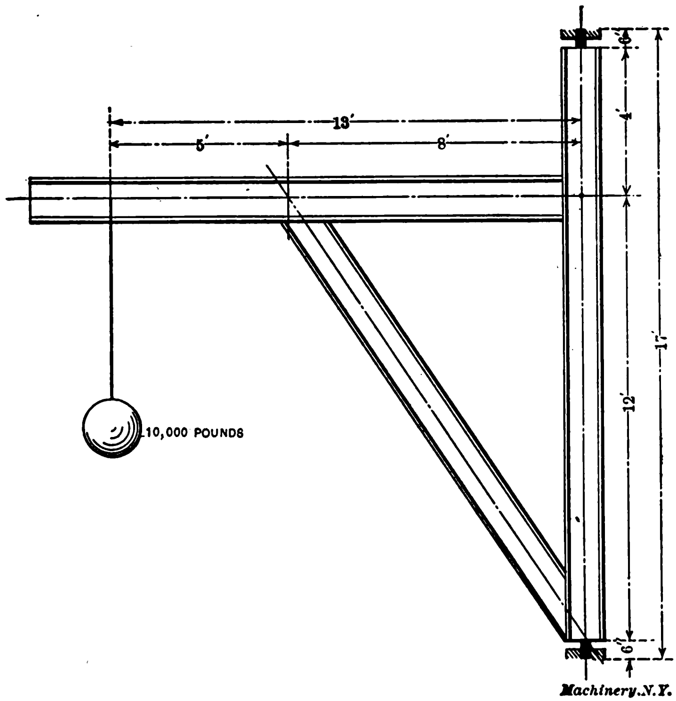


Fig. 8. General Design of Crane in Problem 2

This produces a tensile stress in *DH* which may be found by the parallelogram of forces shown in Fig. 9, or by the following calculation:

$$R_3 = R_1 \times \frac{l}{g} = 8{,}125 \times \frac{8}{12} = 5{,}420 \text{ pounds.} \qquad (29)$$

The stress per square inch in *DH* due to this force is:

$$S_t = \frac{R_3}{A} = \frac{5{,}420}{9.9} = 550 \text{ pounds per square inch} \qquad (30)$$

Adding this to the stress found in (26), we have the total stress in *DH*:

$$S = S_t + S_b = 550 + 7{,}200 = 7{,}750 \text{ pounds per sq. in.} \qquad (31)$$

which is the maximum fiber stress in the yard-arm, occurring just to the right of point D. This is well within the limit of safety, which may be taken as about 13,000 pounds per square inch.

The allowable fiber stress in the brace may be calculated from the following formula based on Rankine's formula for columns:

$$S = \frac{15,000}{1 + \dfrac{e^2}{13,500 \times r^2}} = \frac{15,000}{1 + \dfrac{173^2}{13,500 \times 0.805^2}} = 3395 \text{ pounds per sq. inch.} \qquad (32)$$

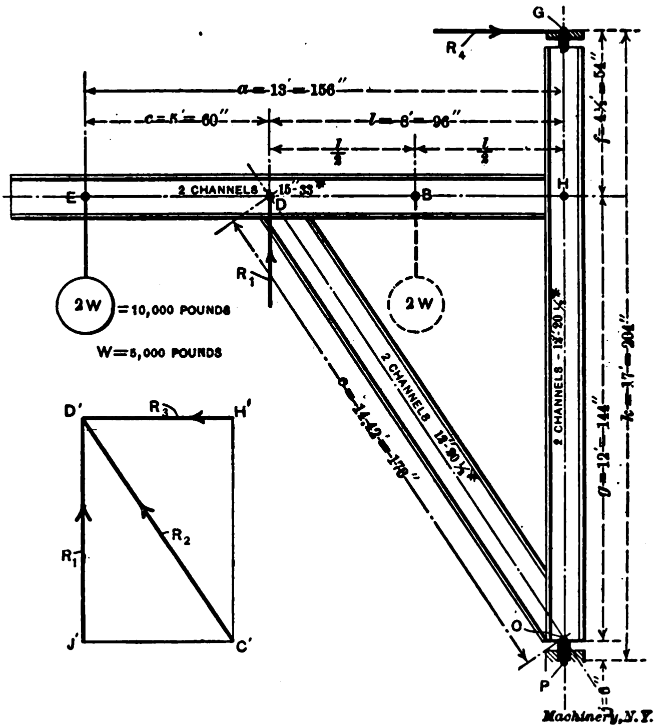


Fig. 9. Calculating the Stresses in the Channels in Crane Fig. 8

The reaction producing compression in CD is found by the force diagram in Fig. 9, or by the following calculation:

$$R_2 = R_1 \times \frac{e}{g} = 8,125 \times \frac{173}{144} = 9,760 \text{ pounds} \qquad (33)$$

The compressive stress per square inch in the brace is, then,

$$S = \frac{R_2}{A} = \frac{9,760}{6.03} = 1,620 \text{ pounds per square inch} \qquad (34)$$

which is, as may be seen, not quite one-half the allowable amount. The ratio of the length to the radius of gyration ($e \div r$) in this strut is so great, being about 215, that it is wise to keep the unit compressive stress down to a very low point.

The mast is most liable to fail by bending at *H* when the load is at *E*. To find the bending moment at *H*, we must first find the horizontal reaction at *G*:

$$R_4 = W \times \frac{a}{k} = 5{,}000 \times \frac{13}{17} = 3{,}825 \text{ pounds} \qquad (35)$$

The bending moment at *H* is then:

$$M = R_4 \times f = 3{,}825 \times 54 = 206{,}550 \text{ inch-pounds} \qquad (36)$$

and the maximum fiber stress due to bending at this point is

$$S_b = \frac{M}{Z} = \frac{206{,}550}{21.4} = 9{,}650 \text{ pounds per square inch} \qquad (37)$$

which is well within the limit of safety.

If the next size smaller standard channels had been used for these members, the results would have been as follows: A 12-inch 20½-pound channel for the yard-arm gives a maximum unit stress at *D* of about 14,000 pounds, which is too much. The unit compressive stress in the brace, if made of 10-inch 15-pound channels, would be about 2,190 pounds. Rankine's formula for this would allow 2,830 pounds, but there is not enough margin of safety with the high ratio of *e* to *r*, which is here about 240. The maximum stress in the mast at *H* would be 15,400 pounds per square inch. It will thus be seen that the sizes we have selected are the commercial sizes best suited for the case in hand.

CHAPTER III

CALCULATIONS FOR THE SHAFT, GEARS, AND BEARINGS OF CRANE MOTORS

To illustrate definitely the use of the table and diagrams in the method of calculation explained in the present chapter the following example will be taken:

Given: A crane motor with 4 poles, 15 H. P., 750 R. P. M. at normal load:

Diameter of armature..................	= 9½ inches
Air gap	= 3/32 inch
Area of pole face.....................	= 29 square inches
Density in air gap, given in lines of force per square inch...............	= 55.000
Total weight of rotating parts........	= 150 pounds

From a general layout drawing of the motor we have the dimensions given as in Fig. 10.

Motors for hoisting purposes are usually series wound, and thus run at different speeds under different loads. Therefore, if the motor

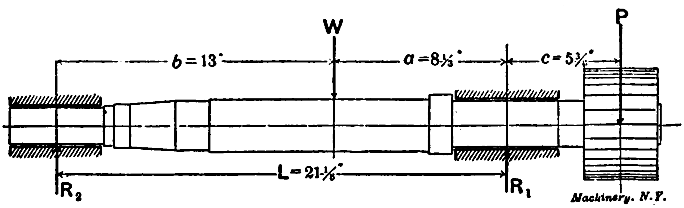


Fig. 10. Dimensions and General Arrangement of Shaft

is to be run with a certain overload, we have to take into consideration the corresponding speed and density in air gap, which can be obtained from the speed and excitation curves of the motor. In this example we suppose an overload of 25 per cent, and have accordingly: 18.75 H. P., 700 R. P. M., and a density in air gap equal to 58,800 lines of force per square inch.

Calculating the Gear

For figuring the gear we suppose that the diametral pitch equals 5 and the number of teeth equals 18. This gives us a pitch diameter $= \frac{18}{5} = 3.6$ inches. Thus, the pitch line speed at 700 R. P. M. $=$

$$\frac{\pi \times 3.6 \times 700}{12} = 660 \text{ feet per minute.}$$

VALUES OF C_1 FOR 15° INVOLUTE TEETH OF ONE-INCH FACE, WHICH PRODUCE FIBER STRESS OF 1000 POUNDS PER SQUARE INCH

Number of Teeth in Gear.	Circular Pitch.																
	3.14	2.52	2.10	1.80	1.57	1.40	1.25	1.14	1.05	0.98	0.89	0.84	0.79	0.70	0.63	0.57	0.52
	Diametral Pitch.																
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4½	5	5½	6
12	210	168	140	120	105	93	84	76	70	65	60	56	52	47	42	38	35
13	220	178	147	126	110	98	88	80	73	68	63	59	55	49	44	40	37
14	226	180	151	129	113	100	90	82	75	69	65	60	56	50	45	41	38
15	236	189	157	135	118	105	94	86	79	73	67	63	59	52	47	43	39
16	242	194	161	138	121	107	97	88	81	74	69	64	60	54	48	44	40
17	251	200	167	143	125	112	100	91	84	77	72	67	63	56	50	46	42
18	261	208	174	149	130	116	104	95	87	80	75	70	65	58	52	47	43
19	273	218	182	156	136	121	109	100	91	84	78	73	68	61	54	50	45
20	283	227	189	162	142	126	113	103	94	87	81	75	71	63	57	51	47
21	289	232	193	165	144	128	116	105	96	89	83	77	72	64	58	52	48
23	295	236	197	169	147	131	118	107	98	91	84	79	74	65	59	53	49
25	305	245	203	174	152	136	122	111	102	94	87	81	76	68	61	55	51
27	314	250	210	180	157	140	126	114	104	97	90	84	78	70	63	57	52
30	320	256	213	183	160	142	128	116	106	99	91	85	80	71	64	58	53
34	327	262	218	188	164	146	131	119	109	101	94	88	82	73	66	59	54
38	336	263	224	192	168	148	134	122	112	103	96	90	84	75	67	61	56
43	346	277	230	198	173	154	138	126	115	106	99	92	86	77	69	63	57
50	352	282	235	200	176	156	140	128	117	108	100	94	88	78	70	64	58
60	358	286	238	204	179	159	143	130	119	110	102	95	89	80	71	65	59
75	364	292	243	208	182	162	146	132	121	112	104	97	91	81	73	66	61
100	371	297	247	212	185	164	148	135	124	114	106	99	93	82	74	67	62
150	377	302	251	215	188	167	151	137	126	116	108	100	94	84	75	68	63
300	384	308	256	219	192	171	152	140	128	118	110	102	96	85	77	70	64
Rack	390	312	260	223	195	173	156	142	130	120	112	104	97	87	78	71	65

It is not advisable to use a pitch-line speed exceeding 1,000 feet per minute, on account of noisy running.

Before figuring the width of gear we have to determine the pressure P on the teeth. This is given by the following formula

$$P = \frac{\text{H. P.} \times 33{,}000}{\text{Pitch line speed}}$$

where P is expressed in pounds and pitch line speed in feet per minute Thus for 18.75 H. P. and a pitch line speed of 660 feet per minute

$$P = \frac{18.75 \times 33{,}000}{660} = 940 \text{ pounds, approximately.}$$

The width of gear is given by the formula:

$$w = \frac{P}{f \times C_1 \times C_2} \text{ where}$$

w = width of tooth of gear in inches,

P = pressure on tooth in pounds,

f = permissible fiber stress in thousands of pounds per square inch,

C_1 = coefficient depending on diametral pitch and number of teeth in gear. Its values can be obtained from table given on page 25.

C_2 = coefficient depending on pitch line speed. Its values can be obtained from curve in Fig. 11. If we suppose a gear of steel, we may use a fiber stress of 8,500 pounds per square inch. We therefore get as the width of tooth:

$$w = \frac{940}{8.5 \times 52 \times 0.5} = 4\tfrac{1}{4} \text{ inches, approximately.}$$

Forces Acting on Shaft

Besides the weight of the rotating part and the pressure on the gear, we must, when figuring the shaft and bearings, take into consideration the unbalanced magnetic pull caused by a displacement of the armature of the motor in relation to the poles. If B is the density given in lines of force per square inch at air gap, A is the area of pole face in square inches, and k is a constant which

for 4-pole machines = 2,
6-pole machines = 4.7,
8-pole machines = 7,

then the magnetic pull per pole $= \dfrac{B^2 \times A}{k \times 72{,}134{,}000}$ pounds. This formula gives us in our example a magnetic pull per pole $= \dfrac{58{,}800^2 \times 29}{2 \times 72{,}134{,}000}$ = 700 pounds, approximately. The pull per inch of the circumference of pole bore $= \dfrac{4 \times \text{pull per pole}}{\pi \times \text{pole bore}} = \dfrac{4 \times 700}{\pi \times 9.69} = 92$ pounds, approximately.

If we now suppose a displacement of armature of 25 per cent of the normal air gap, the ratio between air gap and displacement = 4. In the diagram, Fig. 12, reading on the vertical side the pull per inch of the circumference = 92, and on the horizontal side the ratio between air gap and displacement = 4, the line 55 passing through the intersection point of 92 and 4 indicates that the half value of the maximum magnetic pull per inch of circumference of pole bore is 55.

$\frac{M_{max}}{2} = 55$ pounds. Thus $M_{max} = 55 \times 2 = 110$ pounds. In order

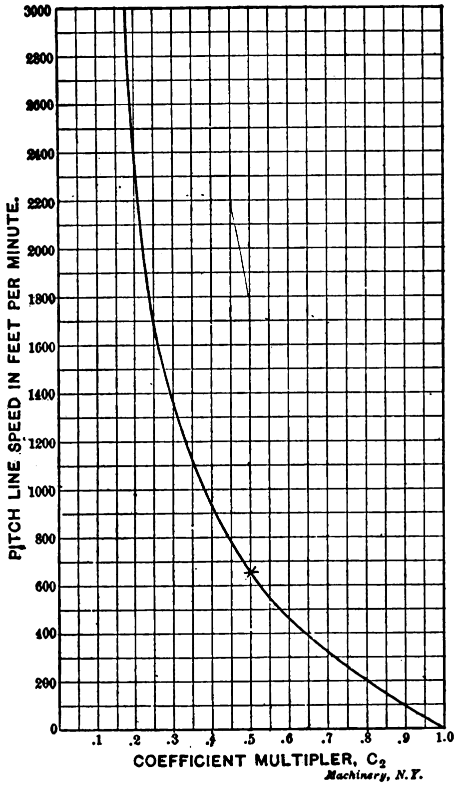


Fig 11

to give the values of $\frac{M_{max}}{2}$ as exactly as possible for a wide range of values of magnetic pull and displacement ratios, the diagram in Fig. 12 contains two sets of lines, one in the lower right corner on a small scale, and one in the upper left corner on a larger scale.

$$\text{The total magnetic pull on armature} = \frac{4 \times \text{radius of armature} \times M_{max}}{\pi}$$

In our example the radius of armature is 4.75 inches, and the M_{max}

is 110 pounds. The total magnetic pull therefore $= \frac{4 \times 4.75 \times 110}{\pi} =$ 665 pounds.

When the unbalanced magnetic pull is acting in the same direction as the weight of the rotating part, the shaft is subjected to its worst strain. Therefore, by adding these two forces we get the resulting force

$$W = 665 + 150 = 815 \text{ pounds.}$$

In general, the location of this force W on the shaft will practically lie at the center line of the armature.

The forces R_1 and R_2 acting on the bearings, as shown in Fig. 10, will be found from the following equations:

$$R_1 = \frac{W \times b + P \times (c + L)}{L} = 1{,}680 \text{ pounds.}$$

$$R_2 = W + P - R_1 = 75 \text{ pounds.}$$

Diameter of Shaft

The diameter of the shaft between the bearings (see Fig. 10) must be calculated for the maximum bending moment occurring. The bending moment at W is:

$$M_b = R_2 \times b = 975 \text{ inch-pounds.}$$

The bending moment at R is:

$$M_b = P \times c = 5{,}050 \text{ inch-pounds.}$$

This is, consequently, the maximum bending moment, and the shaft should be calculated accordingly.

The twisting moment for 18,75 H. P. and 700 R. P. M. is

$$M_t = \frac{18.75}{700} \times 63{,}024 = 1{,}690.$$

The combined moment M_c of M_b and M_t is:

If M_b is greater than M_t

$$M_c = 0.975 \times M_b + 0.25 \times M_t;$$

or, if M_b is less than M_t

$$M_c = 0.6 \times M_b + 0.6 \times M_t.$$

In our example, where $M_b > M_t$,

$$M_c = 0.975 \times 5{,}050 + 0.25 \times 1{,}690 = 5{,}340, \text{ approximately.}$$

Now, if f = fiber stress in shaft per square inch, and D = diameter of shaft at W in inches, the moment of resistance

$$M_r = 0.1 \times f \times D^3$$

Therefore if we put $M_r = M_c$,

$$D = \sqrt[3]{\frac{M_c \times 10}{f}}$$

If we suppose a fiber stress $f = 8{,}500$ pounds per square inch, we get

$$D = 1\tfrac{7}{8} \text{ inches, approximately.}$$

This is also the minimum required diameter of the shaft in the bearing at R_1. Of course, ordinarily both bearings are made the same. It is evidently of advantage to have the diameters of the journals somewhat larger than calculated, as strength alone is not the only consideration; the lubrication of the bearing, a low unit pressure per square inch of projected area attained without excessive length of journal, etc., are also important questions to consider. The diameter of the journals would therefore, in this case, be made, say 2¼ inches.*

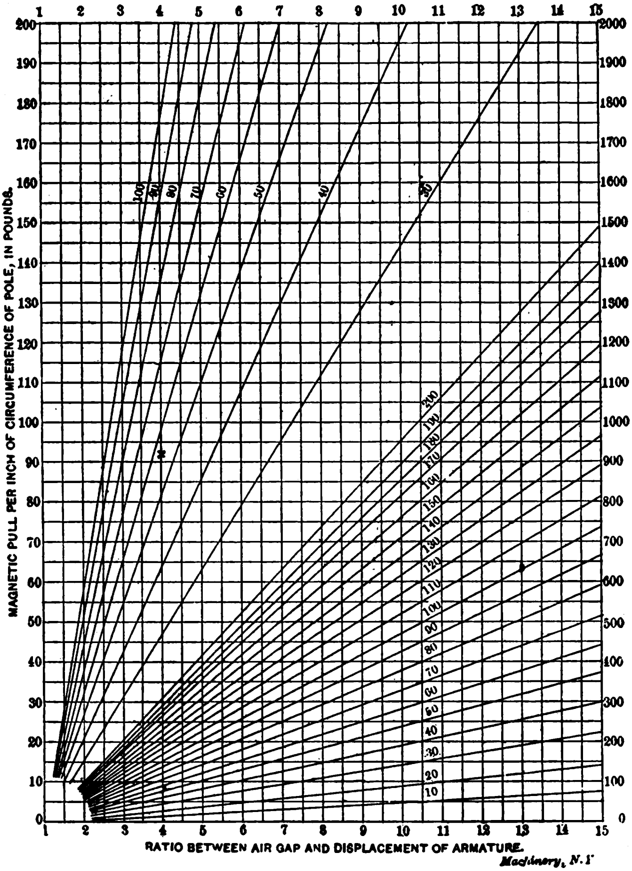


Fig. 12

Now, when the diameters at the journals are made 2¼ inches, evidently the remainder of the shaft at W would not be made as small as calculated, or 1⅞ inch in diameter. The mechanical design requires that this latter diameter be made larger than the journals, say 2½ inches, which diameter we will then use for calculating the deflection, as indicated later.

* For a more thorough discussion of the subject of journals and bearings, see MACHINERY'S Reference Series No. 11, Bearings, page 3, The Design of Bearings.

Calculation of Journals

In the bearings, it is not advisable to exceed a pressure per square inch of projected area of 150 pounds, nor should the product of this pressure by the peripheral velocity in feet per minute in the journal be greater than 55,000, when grease lubrication is used. For oil lubrication this product can be somewhat higher. If in our example we assume a pressure of 130 pounds per square inch, with the diameter of shaft at $R_1 = 2\frac{1}{4}$ inches, we obtain

$$\text{length of journal} = \frac{1{,}680}{130 \times 2.25} = 5\frac{3}{4} \text{ inches.}$$

At 700 R. P. M. and diameter of shaft $= 2\frac{1}{4}$ inches, and a pressure of 130 pounds per square inch in journal, the product of pressure by

$$\text{velocity will be} = \frac{130 \times \pi \times 2.25 \times 700}{12} = 53{,}500. \text{ approximately.}$$

Maximum Deflection of Shaft

For calculating the maximum deflection S of the shaft we have the following formula:

$$S = \frac{W \times a \times b \times (2L - a)}{9 \times E \times I \times L} \times \sqrt{\frac{a \times (2L - a)}{3}}$$

where S is in inches and

$W =$ the resulting force acting on the shaft in pounds,

$L =$ distance between centers of journals in inches,

$a =$ shortest distance between center line of one bearing and the acting point of force W,

$b = L - a$ in inches,

$E =$ modulus of elasticity,

$= 29{,}000{,}000$ for steel,

$= 27{,}000{,}000$ for wrought iron,

$I =$ moment of inertia of shaft $= 0.0491 \times D^4$, where D is the diameter of shaft in inches.

In this example we get the maximum deflection,

$S = 0.0027$ inches, approximately.

Most of the formulas given above are empirical, and give only approximate results, but they are exact enough for practical use.

CHAPTER IV

FORCE REQUIRED TO MOVE CRANE TROLLEYS

In designing crane trolleys and similar constructions the force required to move them is not always calculated to a nicety, and the design then based upon the figures. This may be the conception of the man fresh from college, but it more frequently happens that past experience of a case similar to the one in hand is relied upon entirely.

This is both a safe and quick method, when conditions make it possible, provided good judgment is exercised in allowing for differences between the past construction and the proposed new one. The designer is, however, often confronted by a problem in which he has no past experience to draw upon and for which he has no applicable data at hand, or the design may be of a type similar to that of past experience, but so different as to sizes that he is compelled to calculate from elementary principles. Two troublesome questions then arise: First, what theoretical conditions should be taken into account and what ones may be safely neglected? and second, what values should be assigned to the various constants and assumed factors entering into the calculations? The practicability of his designs will depend almost entirely upon the manner in which the above questions are answered.

Taking up the subject of crane trolleys, of the type in which the load is suspended by ropes passing over sheaves in the trolley and hanging block, as illustrated in Figs. 13, 14, and 15, the above questions may be considered as mutually dependent upon each other, and might be answered as follows:

Take into account journal friction of the trolley wheels, trolley sheaves and hanging block sheaves; also the separate weights of load to be carried, hanging block, and trolley.

Neglect friction of ropes in grooves of sheaves, power necessary to bend ropes over sheaves, and the rolling friction of the trolley wheels on the track, allowing these to be taken care of by the assumed coefficient of journal friction.

Neglect inertia, also, for the usual speeds of crane trolleys, since the difference between the coefficient of rest and of motion is sufficient to produce the necessary acceleration.

In choosing the coefficient of friction, consider the general conditions of lubrication as being poor, and consider that it is the coefficient of rest which is required. Assume this coefficient to be the same for all journals. A fair value is 0.1. Having settled these preliminary considerations, general formulas may be developed.

CASE 1. (See Fig. 13.) The conditions are: Two parts of rope supporting the load, one sheave in hanging block, and two sheaves in trolley.

Let W_l = weight of load to be carried,
W_b = weight of hanging block,
W_t = weight of trolley,
P_f = pull on trolley to overcome friction,
S_b = diameter of sheave in block,
J_b = diameter of journal in block,
S_t = diameter of sheave in trolley,
J_t = diameter of journal in trolley sheaves,
D = diameter of trolley wheels,
A = diameter of trolley axle journals,
C = coefficient of friction,

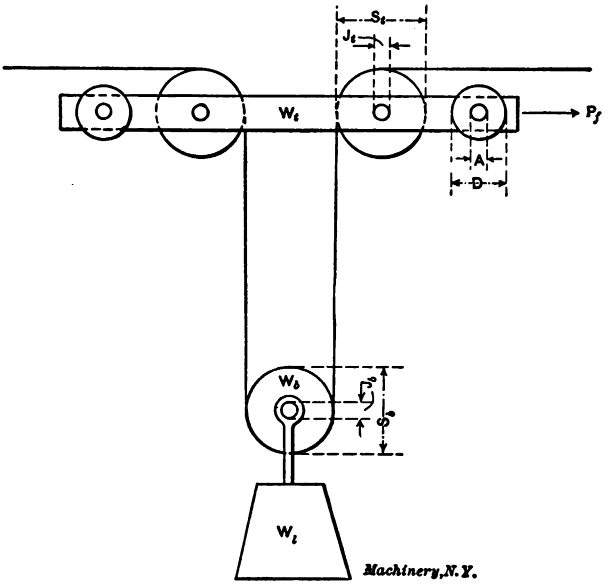


Fig. 13. Trolley with Sheave Suspended by Two Parts of Rope

F_b = friction of hanging block sheave,
F_{ts} = friction of trolley sheaves,
F_{tw} = friction of trolley wheels.

For Case I,

$$F_b = (W_l + W_b)\, C \frac{J_b}{S_b} \qquad (38)$$

The load being supported by two ropes, the load in each is ½ (W_l + W_b) and the arc of contact of the rope on the trolley sheaves being 90 degrees (a), the resultant pressure on the journals of each of these sheaves is ½ $(W_l + W_b)\, 2 \cos \frac{a}{2}$ = ½ $(W_l + W_b)\, 2 \cos 45$ degrees.

For the two sheaves the resultant pressure amounts to

$$1.4\,(W_l + W_b).$$

From the above we get:

$$F_{ts} = 1.4\,(W_l + W_b)\,C\,\frac{J_t}{S_t} \qquad (39)$$

For the friction of the axle bearings of the trolley wheels, the weight of the load, hanging block, and trolley must be considered, thus:

$$F_{tw} = (W_l + W_b + W_t)\,C\,\frac{A}{D} \qquad (40)$$

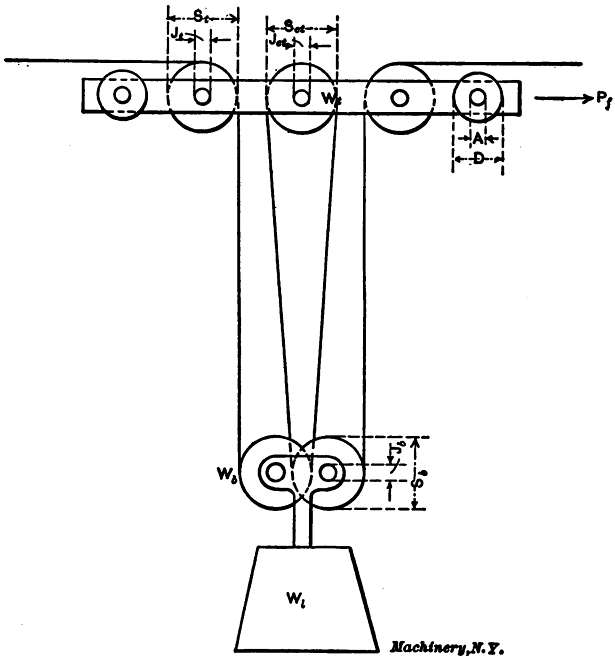


Fig. 14. Trolley with Sheave Suspended by Four Parts of Rope

We have, then, for the total friction

$P_f = F_b + F_{ts} + F_{tw}$, or

$$P_f = C\left[(W_l + W_b)\left(\frac{J_b}{S_b} + 1.4\,\frac{J_t}{S_t}\right) + (W_l + W_b + W_t)\,\frac{A}{D}\right] \qquad (41)$$

Case II. (See Fig. 14.) The conditions are: Four parts of rope supporting the load, two sheaves in the hanging block, and three sheaves in the trolley.

Let the notation be as for Case I with the addition of:

S_{ct} = diameter of sheave at center of trolley,

J_{ct} = diameter of journal for this sheave.

Then, F_b = same as for Case I (equation 38).

F_{tw} = same as for Case I (equation 40).

The friction of the two end sheaves in the trolley is one-half of that in Case I, or

$$0.7\ (W_l + W_b)\ C\frac{J_t}{S_t}.$$

The friction of the central sheave is

$$0.5\ (W_l + W_b)\ C\frac{J_{ct}}{S_{ct}}.$$

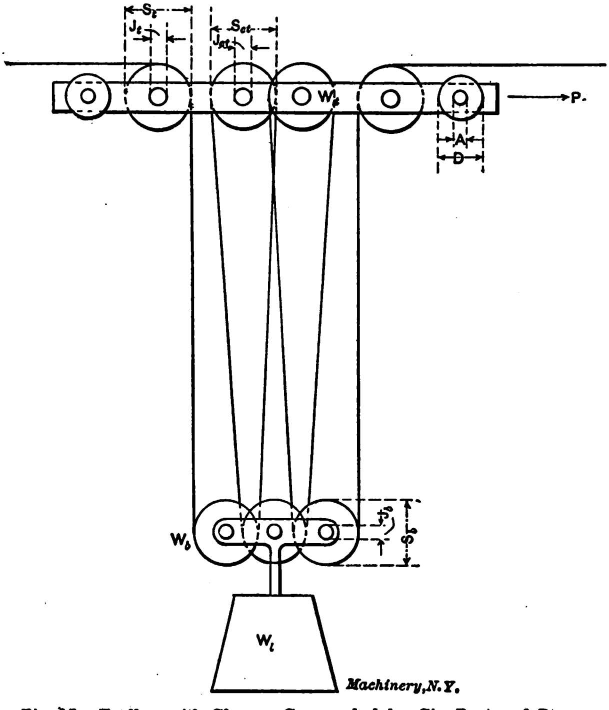


Fig. 15. Trolley with Sheave Suspended by Six Parts of Rope

The total friction of the trolley sheaves is then

$$F_{ts} = (W_l + W_b)\ C\left(0.7\frac{J_t}{S_t} + 0.5\frac{J_{ct}}{S_{ct}}\right) \tag{42}$$

The total frictional resistance of the trolley is:

$$P_t = F_b + F_{ts} + F_{tw},\ \text{or}$$

$$P_t = C\left[(W_l + W_b)\left(\frac{J_b}{S_b} + 0.7\frac{J_t}{S_t} + 0.5\frac{J_{ct}}{S_{ct}}\right) + (W_l + W_b + W_t)\frac{A}{D}\right] \tag{43}$$

When $\frac{J_{ct}}{S_{ct}} = \frac{J_t}{S_t}$, as it often will be, equation (42) becomes:

$$F_{ts} = 1.2\,(W_l + W_b)\,C\,\frac{J_t}{S_t} \tag{44}$$

Under these conditions equation (43) reduces to

$$P_f = C\left[(W_l + W_b)\left(\frac{J_b}{S_b} + 1.2\,\frac{J_t}{S_t}\right) + (W_l + W_b + W_t)\,\frac{A}{D}\right] \tag{45}$$

CASE III. (See Fig. 15.) The conditions are: Six parts of rope supporting the load, three sheaves in hanging block and four sheaves in trolley.

Notation the same as for Cases I and II.

F_b = same as Cases I and II (equation 38).

F_{tw} = same as Cases I and II (equation 40).

The load in each rope is 1/6 $(W_l + W_b)$.

In Case I the load in each rope was 1/2 $(W_l + W_b)$

The frictional resistance of the two end sheaves is therefore 1/6 ÷ 1/2 = 1/3 as much for this case as for Case I, and is equal to 0.47 $(W_l + W_b)\,C\,\frac{J_t}{S_t}$. The friction of the two central sheaves is 2/3 $(W_l + W_b)\,C\,\frac{J_{ct}}{S_{ct}}$. The total friction of the trolley sheaves is then

$$F_{ts} = (W_l + W_b)\,C\left(0.47\,\frac{J_t}{S_t} + 0.67\,\frac{J_{ct}}{S_{ct}}\right) \tag{46}$$

The total friction of the trolley is $P_f = F_b + F_{ts} + F_{tw}$, or

$$P_f = C\left[(W_l + W_b)\left(\frac{J_b}{S_b} + 0.47\,\frac{J_t}{S_t} + 0.67\,\frac{J_{ct}}{S_{ct}}\right) + (W_l + W_b + W_t)\,\frac{A}{D}\right] \tag{47}$$

When $\frac{J_{ct}}{S_{ct}} = \frac{J_t}{S_t}$, as would usually be the case, equation (46) reduces to

$$F_{ts} = 1.14\,(W_l + W_b)\,C\,\frac{J_t}{S_t} \tag{48}$$

Under this condition equation (47) becomes

$$P_f = C\left[(W_l + W_b)\left(\frac{J_b}{S_b} + 1.14\,\frac{J_t}{S_t}\right) + (W_l + W_b + W_t)\,\frac{A}{D}\right] \tag{49}$$

If we assume that the ratio of journal diameter to sheave diameter is the same for all sheaves and also the same for the trolley wheels and their axle journals, *i. e.*, that $\frac{J_b}{S_b} = \frac{J_t}{S_t} = \frac{J_{ct}}{S_{ct}} = \frac{A}{D}$, or that this condition is approximately true, and let R = this ratio, the foregoing formulas for the value of P_f may be reduced to the following form:

$$\text{For case I, } P_f = C\,R\,[3.4\,(W_l + W_b) + W_t] \qquad (50)$$

$$\text{For case II, } P_f = C\,R\,[3.2\,(W_l + W_b) + W_t] \qquad (51)$$

$$\text{For case III, } P_f = C\,R\,[3.1\,(W_l + W_b) + W_t] \qquad (52)$$

It is seen from the above that the friction is nearly the same for the three cases, provided the value of R be the same. Equation (51) being the intermediate condition may then be considered as representative of all.

APPENDIX

CALCULATION OF PILLAR CRANES

The maximum stresses in the different parts of a pillar crane are due to the maximum load lifted (the live load) and the weight (dead load) of the crane parts themselves. Fig. 16 shows a conventional design of a hand pillar crane, and assuming, for an example, the maximum load $Q = 5$ tons, the height $H = 12\frac{1}{2}$ feet, and the radius $A =$ 13 feet, the stresses in the different parts of the crane are calculated as shown in the following.

Stresses in the Boom

Fig. 16 shows plainly that the stresses in the boom and tie-bars are not due to the live load only, but that the weight of the eccentric parts of the crane (*i. e.*, boom, tie-bars, hoist, sheave wheels, crane hook and hoisting rope) and the pull of the hoisting rope must also be considered. As it is not possible to determine the dead load accurately before the crane is calculated and designed, it must be assumed. A practical method is to assume the weight of the above mentioned eccentric parts of the crane as half of the maximum live load, and its center of gravity at a distance equal to one-fourth of the radius of the crane from the center line of the pillar. These assumptions expressed in formulas read:

$$Q_1 = \frac{Q}{2}; \text{ or } Q_1 = \frac{10{,}000}{2} = 5{,}000 \text{ pounds.} \qquad (1)$$

$$a = \frac{A}{4}; \text{ or } a = \frac{13}{4} = 3\tfrac{1}{4} \text{ feet.} \qquad (2)$$

in which Q_1 = the weight of the eccentric parts of the crane, and a = the distance of the center of gravity of Q_1 from the center of the crane. If the actual figures, determined after the crane is calculated, differ considerably from these assumptions, corrections have to be made.

The next step is to determine the height h of the pillar, a practical rule being to make h about 0.6 of the radius of the crane:

$$h = 0.6\,A; \text{ or } h = 0.6 \times 13 = 8 \text{ feet, approximately.} \qquad (3)$$

The frame diagram shown in Fig. 17 can now be drawn. According to the law of equilibrium the sum of moments of the external forces must be equal to the sum of moments of the internal forces about the same center. The moment M_i of the internal force in the boom is the product of its compressive stress C and its lever arm e (5¾ feet) about center K:

$$M_i = Ce \tag{4}$$

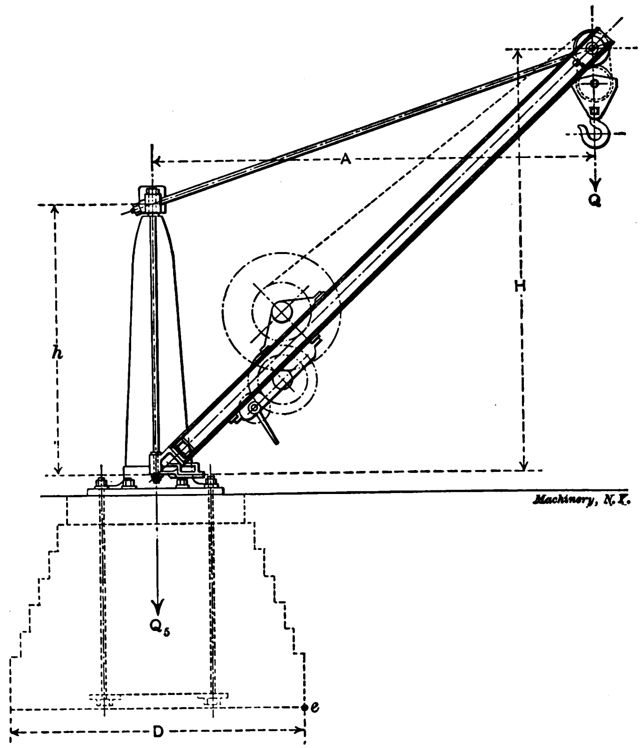


Fig. 16. General Lay-out of Pillar Crane

The moments of the external forces about center K are:

$$\text{The moment of } Q = M = Q\,A \tag{5}$$

$$\text{The moment of } Q_1 = M_1 = Q_1 a \tag{6}$$

$$\text{The moment of } Q_2 = M_2 = Q_2 b \tag{7}$$

Dimension b is found to be 4 feet by scaling.

Substituting the values:

$M = 10{,}000 \times 13 = 130{,}000$ foot-pounds,

$M_1 = 5{,}000 \times 3\frac{1}{4} = 16{,}250$ foot-pounds,

$M_2 = 5{,}000 \times 4 = 20{,}000$ foot-pounds.

The sum of above moments is:

$$M_s = M + M_1 + M_2 = 130{,}000 + 16{,}250 + 20{,}000 = 166{,}250 \text{ foot-pounds.} \tag{8}$$

As explained before

$$M_1 = M_s \qquad (9)$$

or

$C\,e = M_s$ and transposing

$$C = \frac{M_s}{e} = \text{the compressive stress in the boom.} \qquad (10)$$

Substituting the values in above formula

$$C = \frac{166{,}250}{5\frac{3}{4}} = 28{,}910 \text{ pounds.}$$

The unsupported length of the boom scales about 17 feet, and as in this case it is made up of two channels the load on each channel is $\frac{C}{2}$ or 14,455 pounds. The inclination of the two channels towards each other need not be taken into consideration as the increase of load is very small. Consulting any handbook of information relating to structural steel we find that a 6-inch × 8-pound channel has a sectional area of 2.38 square inches and a radius of gyration with respect to an axis perpendicular to its web of 2.34 inches. Only this radius of gyration need be considered as the flanges of the two channels are latticed together. For the ratio of the length L of the boom in feet to the radius of gyration r in inches

$$\frac{L}{r} = \frac{17}{2.34} = 7.2,$$

which is not excessive. The elastic limit for soft steel may be taken at 30,000 pounds per square inch. Dividing the load on one channel by its sectional area the actual unit stress will be

$$\frac{14{,}455}{2.38} = 6{,}075 \text{ pounds,}$$

which shows that two 6-inch × 8-pound channels are quite sufficient to stand the load.

Stresses in the Tie-bars

To find the tensile stress in the tie-bars the same method as just explained will be used. Taking K_1 as a center of moments and referring again to Fig. 17, the moments of the external forces are:

$$\text{Moment of } Q = M = QA, \qquad (5)$$

$$\text{Moment of } Q_1 = M_1 = Q_1 a, \qquad (6)$$

$$\text{Moment of } Q_2 = M_2 = -\,Q_2 f. \qquad (11)$$

Dimension f is found by scaling to be 2½ feet.

Substituting the values in these formulas:

$M = 10{,}000 \times 13 = 130{,}000$ foot-pounds,

$M_1 = 5{,}000 \times 3\frac{1}{4} = 16{,}250$ foot-pounds.

$M_2 = -\,5{,}000 \times 2\frac{1}{2} = -\,12{,}500$ foot-pounds,

and the sum M_s of these moments is

$$M_s = 130{,}000 + 16{,}250 - 12{,}500 = 133{,}750 \text{ foot-pounds.} \qquad (12)$$

The moment M_i of the internal stress T in the tie-bar is

$$M_i = Td \qquad (13)$$

Dimension d scales 7½ feet.

Since M_i must be equal to M_s

$$Td = M_s \qquad (14)$$

and transposing:

$$T = \frac{M_s}{d} \qquad (15)$$

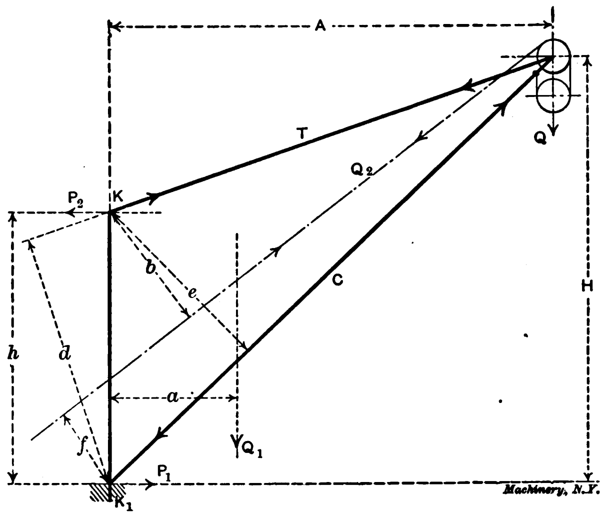


Fig. 17. Diagrammatical View of Pillar Crane

Substituting the values in formula (15) the tensile stress in the tie-bars is: $T = \frac{133{,}750}{7\frac{1}{2}} = 17{,}830$ pounds.

As there are two tie-bars the load on one is $\frac{17{,}830}{2} = 8{,}915$ pounds. Using a safe fiber stress of 10,000 pounds per square inch, the area of one bar $= \frac{8{,}915}{10{,}000} = 0.892$ square inch with the corresponding diameter of 1 1/16 inch.

Stresses in Pillar

The stresses in the pillar are due to the bending moments of the loads Q and Q_1 and to the direct vertical loads Q and Q_1. The bending moments of Q and Q_1 were found by formulas (5) and (6) to be 130,000 foot-pounds and 16,250 foot-pounds, respectively. The sum of these moments is

$$M_s = M + M_1 = 130{,}000 + 16{,}250 = 146{,}250 \text{ foot-pounds} \qquad (16)$$

or 1,755,000 inch-pounds. This bending moment in inch-pounds must be equal to the product of the section modulus of the pillar cross-section, times the safe unit fiber stress. Considering the sectional area of a hollow cylinder for the cast iron pillar, the section modulus is

$$S = \frac{\pi (D^4 - d^4)}{32D} \qquad (17)$$

in which D = the outside diameter of the pillar, and

d = the inside diameter of the pillar.

Using a safe fiber stress $s = 3{,}000$ pounds per square inch, the above mentioned equation reads:

$$M_s = \frac{\pi (D^4 - d^4)}{32D} \times s \qquad (18)$$

Assuming an outside diameter of 24 inches, the inside diameter d is found by transposing the formula (18):

$$d = \sqrt[4]{D^4 - \frac{32\, M_s\, D}{s\, \pi}} \qquad (19)$$

and substituting the values:

$$d = \sqrt[4]{24^4 - \frac{32 \times 1{,}755{,}000 \times 24}{3{,}000 \times 3.14}} = 20\tfrac{7}{8} \text{ inches.}$$

As mentioned before, not only the bending moment, but also the direct vertical loads Q and Q_1 must be considered. As the bending moment produces a tensile stress on one side of the column and a compressive stress on the other side, the additional vertical loads Q and Q_1 naturally increase the compressive and reduce the tensile unit stress somewhat.

The unit stress in the pillar caused by the vertical loads is

$$s_1 = \frac{Q + Q_1}{\text{Area}} = \frac{15{,}000}{110} = 137 \text{ pounds per square inch.} \qquad (20)$$

110 square inches is the sectional area of the pillar at the dangerous section.

The sectional area of the pillar was calculated for a bending stress of 3,000 pounds. Adding the unit stress for the bending moment and the unit stress for the vertical loads, the actual compressive unit stress is found to be:

$$3{,}000 + 137 = 3{,}137 \text{ pounds per square inch.} \qquad (21)$$

Substracting the unit-stress produced by the vertical loads from the unit bending stress the actual tensile stress in the pillar results:

$$3{,}000 - 137 = 2{,}863 \text{ pounds per square inch.} \qquad (22)$$

Vertical Tie-rods

The vertical tie-rods, connecting the crosshead with the lower end of the boom, receive the vertical component of the tensile stress T in the ties, and the vertical component of the compressive stress C in the boom, or what is the same, the added load Q and Q_1.

$$Q_3 = Q + Q_1 = 15{,}000 \text{ pounds.} \tag{23}$$

in which Q_3 = the stress in the two vertical tie-bars.

Each of the two tie-rods receives half of this load or 7,500 pounds. Using a safe unit fiber stress of 10,000 pounds, the area of one tie-rod is $\frac{7{,}500}{10{,}000} = 0.75$ square inch, with a corresponding diameter of one inch.

Pintle

The reaction P_2 on the pintle (see Fig. 18) is caused by the loads Q and Q_1, whose moments about K must equal the moment of P_2 about the same center:

$$QA + Q_1a = P_2h = P_1h \tag{24}$$

and this formula transposed and the values substituted

$$P_1 = P_2 = \frac{10{,}000 \times 13 + 5{,}000 \times 3\frac{1}{4}}{8} = 18{,}280 \text{ pounds.} \tag{25}$$

The reaction P_2 produces a bending moment on the pintle, and referring to Fig. 18, this bending moment

$$M_b = \frac{P_2L}{2} \tag{26}$$

in which L = the length of the pintle. Assuming the length $L = 1\frac{1}{2} D_1$, the formula (26) reads:

$$M_b = \frac{P_2 \times 1\frac{1}{2} D_1}{2}.$$

This moment has to be equal to the product of the section modulus of the sectional area of the pintle times the safe unit stress. The section modulus for a circular section being $\frac{\pi}{32} D_1^3$, and assuming the safe unit stress $s = 8{,}000$ pounds the equation reads:

$$\frac{P_2 \times 1\frac{1}{2} D_1}{2} = \frac{\pi}{32} D_1^3 s \tag{27}$$

and transposing

$$D_1 = \sqrt{\frac{P_2 \times 1\frac{1}{2} \times 32}{2 \pi s}} \text{ or} \tag{28}$$

$$D_1 = \sqrt{\frac{18{,}280 \times 1\frac{1}{2} \times 32}{2 \times 3.14 \times 8{,}000}} = 4\frac{1}{4} \text{ inches approx.}$$

$$\text{and } L = 1\frac{1}{2} \times 4\frac{1}{4} = 6\frac{3}{8} \text{ inches.}$$

Besides the bending moment produced by the reaction P_2 the direct vertical loads Q and Q_1 also produce stress, and this stress per square inch is found by dividing the sum of the vertical loads by the sectional area of the pintle in square inches:

$$s_1 = \frac{10{,}000 + 5{,}000}{14.19} = 1{,}060 \text{ pounds per square inch.} \qquad (29)$$

The maximum unit stress on the pintle is then:

$$8{,}000 + 1{,}060 = 9{,}060 \text{ pounds per square inch.} \qquad (30)$$

Foundation Bolts

Considering an axis $A\ A$ in Fig. 19, which shows a plan of the base of the pillar, the sum of the moments of the overturning loads Q and Q_1 about this axis must equal the sum of the resisting moments. The latter are due to the stress in the foundation bolts and to the weight of the pillar. This weight can easily be calculated as the cross-section of the pillar is already known, and in this case is found to be

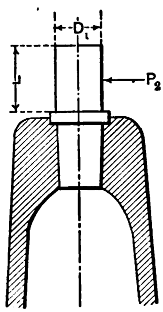


Fig. 18. Pintle of Pillar Crane

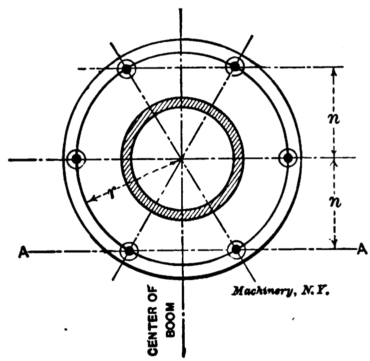


Fig. 19. Lay-out of Arrangement of Flange Bolts

$Q_4 = 5{,}000$ pounds. The moments of the overturning loads with respect to axis A-A are:

$$M_4 = Q\ (A - n) \qquad (31)$$

or

$$M_4 = 10{,}000\ (13 - 1\tfrac{1}{2}) = 115{,}000 \text{ foot-pounds,}$$

$$M_5 = Q_1\ (a - n) \qquad (32)$$

$$M_5 = 5{,}000\ (3\tfrac{1}{4} - 1\tfrac{1}{2}) = 8{,}750 \text{ foot-pounds,}$$

in which n = distance of the foundation bolts from the center of the crane. In this case n is found by scaling to equal $1\tfrac{1}{2}$ foot.

The sum of the overturning moments =

$$M_4 + M_5 = 123{,}750 \text{ foot-pounds.}$$

The resisting moments of the foundation bolts are:

$$M_6 = 2P_3n + (2P_3 \times 2n) \qquad (33)$$

in which P_3 equals the stress in one foundation bolt. The resisting moment of the weight Q_4 of the pillar is:

$$M_7 = Q_4n = 7{,}500 \text{ foot-pounds.} \qquad (34)$$

The sum of the resisting moments is therefore equal to

$$M_4 + M_5 = 2P_3n + (2P_3 \times 2n) + Q_4n \qquad (35)$$

and transposing

$$P_3 = \frac{(M_4 + M_5) - M_7}{6n} \qquad (36)$$

Substituting the value *s*, the stress on one foundation bolt

$$P_3 = \frac{123{,}750 - 7{,}500}{6 \times 1\frac{1}{2}} = 12{,}910 \text{ pounds.}$$

Using a safe unit stress of 12,000 pounds, the area of one bolt is $\frac{12{,}910}{12{,}000} = 1.08$ square inch with a corresponding diameter of 1¼ inch.*

Foundation

Referring to Fig. 16, we find the moments which tend to overturn the crane with its foundation about an axis passing through *e* to be: Sum of overturning moments =

$$Q\left(A - \frac{D}{2}\right) + Q_1\left(a - \frac{D}{2}\right) \qquad (37)$$

This sum of the overturning moments is resisted by the moment of the combined weights Q_5 of the foundation and the pillar:

$$\text{Sum of resisting moments} = Q_5 \frac{D}{2} \qquad (38)$$

The equation of moments therefore reads:

$$Q\left(A - \frac{D}{2}\right) + Q_1\left(a - \frac{D}{2}\right) = Q_5 \frac{D}{2} \qquad (39)$$

and transposing

$$Q_5 = \frac{Q\left(A - \frac{D}{2}\right) + Q_1\left(a - \frac{D}{2}\right)}{\frac{D}{2}} \qquad (40)$$

Assuming the diameter *D* of the foundation to be 9 feet and substituting the values:

$$Q_5 = \frac{10{,}000\,(13 - 4\frac{1}{2}) + 5{,}000\,(3\frac{1}{4} - 4\frac{1}{2})}{4\frac{1}{2}} = 17{,}500 \text{ pounds.}$$

Deducting from this combined weight of foundation and pillar the amount for the latter, we get the theoretical weight of the foundation:

$$17{,}500 - 5{,}000 = 12{,}500 \text{ pounds.}$$

---

* The calculation of the foundation bolts as here given is correct only on the assumption that the base flange of the crane and the bolts are made of inelastic materials. For a more fundamental treatment of the subject of foundation bolts, see MACHINERY, December, 1906, engineering edition: Flange Bolts, or MACHINERY'S Reference Series No. 22, Chapter III. For an article on the Working Strength of Bolts, which should also be considered in this connection, see MACHINERY, November, 1906, engineering edition, or MACHINERY'S Reference Series No. 22, Chapter II.

Using a factor of safety of 3, the weight of the actual foundation must be:

$$12{,}500 \times 3 = 37{,}500 \text{ pounds.}$$

Having calculated the different parts of the crane as described it is good practice to test the pillar for its rigidity, as the amount of deflection must not be too great. The load on the unsupported end of the pillar was found by formula (25) to be $P_2 = 18{,}280$ pounds. The deflection N in inches is:

$$N = \frac{P_2 h^3}{3\,EI} \qquad (41)$$

in which

h = the height of the pillar in inches = 96 inches,
E = the modulus of elasticity = 12,000,000 for cast iron,
I = the moment of inertia $= \frac{\pi}{64}(D^4 - d^4) = 7{,}257$.

Substituting these values we find the deflection

$$N = \frac{18{,}280 \times 96^3}{3 \times 12{,}000{,}000 \times 7{,}257} = 0.062 \text{ inch,}$$

or about 1/16 of an inch, which is not excessive.