MACHINERY'S REFERENCE SERIES

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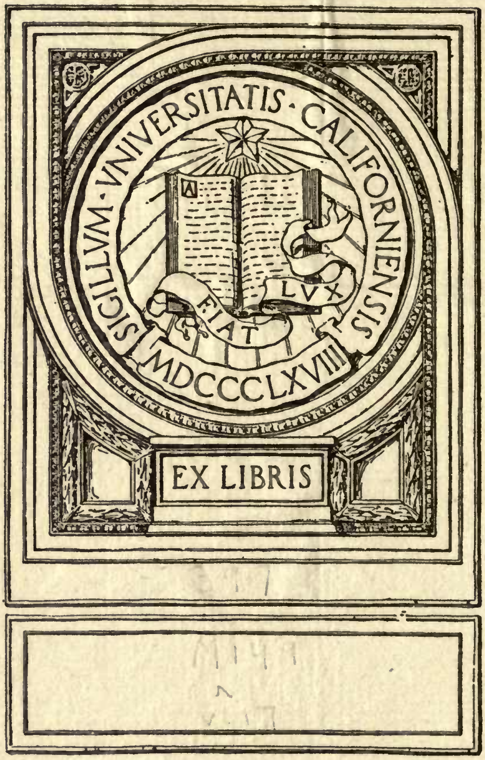
No. 17

STRENGTH OF CYLINDERS

SECOND EDITION—REVISED AND ENLARGED

CONTENTS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49-55 Lafayette Street, New York City

The subject treated in this number of MACHINERY'S Reference Series is one on which a considerable amount of information has been published by various writers in MACHINERY. The fundamental formulas on which the discussion is based, however, are the same, and it is, therefore, evident that the several authors to some extent deal with the subject in a similar manner. In the following chapters, the treatment of each writer has, however, been given in full, irrespective of the fact that, due to this, some formulas and statements are repeated.

CHAPTER I

HYDRAULIC CYLINDERS—GENERAL PRINCIPLES

Under the general heading of Hydraulic Cylinders may be classified all such cylindrical shells in which a uniformly distributed pressure from within acts directly against the inner circular walls of the shell. The transmitting medium of the pressure is considered as a perfect fluid, and the pressure is equal in all directions. The pressure is produced by the compression of the fluid against the ends and walls of the cylinder. The compression of the fluid forces its particles in a closer molecular contact, and as a natural sequence, a reduction in volume takes place. If the transmitting medium is water, however, the reduction of its volume under pressure is so small that it is not necessary to take into account the reduction in volume of this medium when compressed in hydraulic cylinders. The intensity of the stress on the ends and cylindrical walls of the hydraulic cylinder is, as a rule, stated in number of pounds pressure per square inch.

When calculating the strength of cast iron cylinders, having thick walls, two important points should be taken in consideration. The first one is the irregularity and lack of uniformity in the cooling off of thick cast iron cylinders when cast. This makes the texture of the material in the cylinders uneven, and makes them comparatively weaker than cylinders having thinner walls. Iron for thick cylinders should be of the best grade, and should be remelted three to four times to insure a tensile strength of about from 25,000 to 30,000 pounds per square inch. Irregularities arising from air bubbles, sand holes, and imperfect ramming of the sand in the mold, is another serious objection. To avoid these difficulties as much as possible, no cylinder for hydraulic work of any kind should be cast horizontally, but vertically. The second question to be considered is that beyond certain limits any increase in the thickness of the cylinder walls does not increase the strength of the cylinder, because at a certain point the stress will stretch the inner layers of the cylinder beyond the elastic limit, and any strain in excess of the pressure necessary to produce this result, will be followed by a molecular separation of the material, and a subsequent bursting of the cylinder. For extraordinarily high pressures, additional strength may be given to the cylinder by forcing annular rings of a stronger material on the outside in such a manner that these compress the inner walls of the cylinder when in a neutral condition. These rings may either be heated and shrunk in place, or may be forced on by hydraulic pressure. The reason why these annular rings will strengthen the cylinder to a great extent is that the stress within the cylinder has first to overcome the compression caused in the cylinder by the rings, before there will be any tensional stresses in the walls of the cylinder.

In hydraulic calculations the tensile strength of cast iron may be taken at 18,000 pounds per square inch, and that of steel at 70,000 pounds. It seems unnecessary to state that the word diameter always, when considering hydraulic cylinders, means inside diameter, if not otherwise specified. The lowest factor of safety, generally, in hydraulic calculations is 4, and a higher factor of safety, say from 6 upwards, is most common.

In regard to the steel used for hydraulic cylinders, it may be said that for thin shells only open-hearth basic steel drawn tubes should be used, because thin steel castings are not reliable. It has been stated at times that good charcoal iron welded tubes are equal, if not superior, to steel tubes. There is nothing, however, more erroneous than such an opinion, because a charcoal iron tube, no matter how well made, will not stand the severe test to which open-hearth steel tubes may be subjected. The material of steel tubes is perfectly homogeneous, and the product is as a rule perfectly uniform.

In formulas for determining the thickness of the walls necessary in hydraulic cylinders, many authorities include certain elements which are intended to provide for the consequences of welding, casting, cooling and abnormal strains, etc. This, however, is a form of formula which is not the most advisable to use. It is always best to use a formula given in its simplest form, clearly stating all the elements used in the derivation, and then for the designer to determine the factor of safety required, based on his experience with different materials and constructions.

The simplest rule that can be written for ascertaining the thickness of the walls of pipes, tubes and thin hollow cylinders to resist internal pressure, is the following:

$$t = \frac{PR}{S} \qquad (1)$$

In this formula,

t = thickness of material in the walls of the tube.
R = internal radius of the tube,
P = internal pressure in pounds per square inch,
S = tensile strain in pounds per square inch to which the material is subjected by the pressure P.

This form of rule has the sanction of Bernoulli, Unwin, Rankine, Clandel, Weisbach, and Clarke, and with modifications for special uses by Reuleaux, Brix, Barlow, Lamé, Grashof, Trautwine, and Clarke, but as the rule of each leads to so nearly the same result, for general purposes, what is given above may be accepted as the foundation rule which must not be departed from in any case, to which, however, certain elements may be added in order to cover particular cases, a few of which will be named.

This formula is used for ascertaining the thickness of boiler shells, for resisting internal pressure, by all the boiler inspection companies, but into it is inserted the *factor of safety* and the comparative strength of the riveted joints to the solid plates, which are of course limiting elements.

The formula for the thickness of the shell is:

$$t = \frac{PRF}{S(A \text{ or } B)}$$

In which

$F =$ the factor of safety, usually taken as 5,

$A =$ the strength of the punched plates,

$B =$ the strength of the driven rivets; the least of these to be taken, because the safe strength of any structure cannot be above the strength of its weakest part.

It should be noticed that the element S may be inserted in formula (1) as the ultimate tensile strength of the material, when we wish to find out the pressure P that will burst the tube, or S may represent 1/4, 1/5, or 1/10 of the ultimate strength, in place of F, according to the degree of safety required. For weldless tubes the elements A and B must be omitted.

A rather extraordinary case of rupture of a thick hydraulic cylinder may be mentioned, as it will prove instructive. The cylinder walls were 8 inches thick, the ram was 15 inches diameter, the internal diameter of the cylinder was 16 inches, and the pressure about 6,000 pounds per square inch. By transposing our formula to find S, we have:

$$S = \frac{PR}{t} = \frac{6{,}000 \times 8}{8} = 6{,}000,$$

that is, the tensile strain to which the metal in the cylinder was subjected was 6,000 pounds per square inch, which proved sufficient to completely rupture this cylinder; it went to pieces with a loud report. This casting was made of the strongest iron, it was melted in an air furnace, such as used for rolling-mill rolls, and was most carefully made in every particular. This cylinder was replaced by a new one of same dimensions, but of softer quality, having less tensile strength, but more elasticity.

In this case it may be assumed that the material must have suffered from the effects of internal strains, the result of unequal cooling. It is well known that the metal at the center of a mass of cast iron is weaker than the metal lying near its surface. With 8 inches of thickness, and metal of hard, "tight" quality, there is ample room for the unequal strains. But it must, in particular, be remembered that the formulas for thin cylinders do not hold good for cylinders with heavy walls, and subjected to high pressure, as we shall see in the complete treatment of thick hollow cylinders given in the next chapter. We shall in the following chapters also meet with the treatment of both the practical and analytical questions involved, as presented by various writers in past issues of MACHINERY.

CHAPTER II

FORMULAS FOR STRENGTH OF THICK, HOLLOW CYLINDERS

Rules for thick-walled hollow cylinders for sustaining internal pressure are many and various. For thin cylinders, the rules given by all authorities are the same in every particular, because they regard the materials of the cylinders as having uniform texture, and every part of the same as being under equal tension, which means that the net areas of their sections may be taken as the measure of their strength. This measure will not apply to thick cylinders, as will appear later on, and for which some reasons will be given, experience proving that increased thickness does not add proportional increased strength.

The rules which provide for this anomalous condition of the material, due to its position, are based upon the general formula for the strength of hollow cylinders, required additional thickness being given, but the rules formulated are found to vary greatly by the dictum of different authorities. Some rules show great ingenuity of method. For some it is difficult to see how algebraic expression, merely, can give strength, which resides only in the material, and strictly considered, tests can apply only to the samples of material used. There is no one rule applicable to all cases, and for imperfect material there is no rule at all. In what follows will be found the work and results of many well-known writers. These are recorded for easy comparison, and worked-out examples given. Many rules are only half truths, and therefore, do not apply, and many lead to no tangible results in the line of our inquiry; they are, therefore, "better missed than found." Our work here is chiefly to deal with such practical questions of the matter as are necessary to consider, and also to furnish the data of actual performance, which point out the shortest road to successful application.

First, let us consider the cross-section of a thick, hollow cylinder, which we will suppose is divided into concentric rings, fitting closely together, each and all of these rings having a certain and equal amount of elasticity per unit of length or circumference. Then, supposing a certain uniform pressure to be exerted all around the interior boundary of the inner ring, it will readily appear that each successive circular ring—counting from the inner one to the outer one—offers less and less resistance to the internal straining force. This is manifest, for a resistance which any solid body offers to the force by which it is strained is proportional to the extension which it undergoes, divided by its length.

Under this condition of things the inner rings will be subjected to strains beyond their elastic limit, before the outer rings take their share of the pressure; in consequence of this, the inner rings may suffer rupture before the outer rings come to a full bearing, since their capability of extension, because of their greater length, is much greater than the inner rings. In other words, the elastic limit of the outer rings is not reached until after the inner rings have failed, whence, in an instant, all fail, and a sudden burst is the result. The explanation exhibits but the elements of this complex problem, for the study of the effect of internal pressure upon thick-walled, hollow cylinders. is not a simple matter.

Let us now make and present a judicious selection of published rules, with formulas, and remarks thereon.

In Gregory's *Mathematics for Practical Men*, Royal Military Academy, Woolwich, 1825, and in reprint of same, 1833, page 289, is quoted Mr Barlow's rule·

$$t = \frac{P r}{S - P}$$

in which $t =$ thickness in inches,

$P =$ hydraulic pressure in pounds per square inch,

$r =$ internal radius, and

$S =$ cohesive strength of the material in pounds per square inch.

He then gives two worked-out examples by way of explanation, thus. "Let it be desired to find the thickness of metal in each of two cylinders having 12-inch bore, to just sustain an internal pressure of 1½ tons per circular inch for one of them, and 3 tons per circular inch for the other, the ultimate cohesion of cast iron being 18,000 pounds per square inch.

"Now, 1½ tons per circular inch = 4,278 pounds per square inch, and 3 tons per circular inch = 8,556 pounds per square inch, the ton being 2,240 pounds. Whence by the rule we have,

$$\frac{4{,}278 \times 6}{18{,}000 - 4{,}278} = 1.87 \text{ inch};$$

$$\frac{8{,}556 \times 6}{18{,}000 - 8{,}556} = 5.44 \text{ inches}.$$

"Whereas, on the usual principle of computation (using the rule for thin cylinders), the latter thickness would be exactly double the former; extensive experiments are necessary to tell which method deserves the preference."

Turnbull, in 1831, quotes Barlow's rule from Gregory. To obtain a result, let us introduce the figures of an actual case, say, 8 inches radius of interior, 6,000 pounds per square inch hydraulic pressure, and 18,000 pounds per square inch ultimate tensile strength of the cast iron used for the cylinder; then, inserting these figures in this formula, we will have

$$t = \frac{6{,}000 \times 8}{18{,}000 - 6{,}000} = 4 \text{ inches}.$$

Referring again to Mr. Barlow's original paper on "The resisting power of the cylinder and rules for computing the thickness of metal for presses of various powers and dimensions," published in *Transactions of the Institution of Civil Engineers*, Vol. I, London, 1836, and passing over his "investigation of the nature of the resistance opposed by any given thickness of metal in the cylinder or ring," we give his conclusion and application in his own words and formula:

"Let r be the radius of the proposed cylinder; p the pressure per square inch on the fluid; and x the required thickness; let, also, c represent the cohesive strength of a square inch of the metal. Then, the whole strain due to the interior pressure will be expressed by $p\,x$, and that the greatest resistance to which the cylinder can be safely opposed is,

$$c \times \frac{r\,x}{r+x}$$

hence when the strain and resistance are in equilibrium, we shall have,

$$r\,p = \frac{r\,x}{r+x} \times c, \text{ or } p\,r + p\,x = c\,x.$$

$$\text{Whence } x = \frac{p\,r}{c-p} = \text{the thickness sought.}$$

"Hence, the following rule in words, for computing the thickness of metal in all cases, *viz.*, multiply the pressure per square inch by the radius of the cylinder, and divide the product by the difference between the cohesive strength of a square inch of the metal and the pressure per square inch, and the quotient will be the thickness required."

Applying this rule to our case, we will have:

$$x = \frac{6{,}000 \times 8}{18{,}000 - 6{,}000} = \frac{48{,}000}{12{,}000} = 4 \text{ inches.}$$

Mr. Barlow says: "We may, without sensible error, call the cohesive power of cast iron 18,000 pounds per square inch. It will, of course, be understood that the thickness found by this rule is the least that will bear the required pressure, and that, in common practice, presses ought not to be warranted to bear above *one-third* the pressure given, unless it should appear that the estimated cohesive power of cast iron is too little; if this actually exceeds 18,000 pounds, a corresponding reduction may be made in the computed thickness."

In the beginning of his article, Mr. Barlow says: "I am not aware that any of our writers on mechanics have investigated the nature and amount of the circumferential strain which is exerted in a hydraulic cylinder by a given pressure on the fluid within." So we have in this article, presumably, the first investigation and rule upon this subject. Mr. Barlow further says: "It would appear at first sight, that, having found the strain (at the two sides imposed by the pressure of the fluid within), it would only be necessary to ascertain the thickness of metal necessary to resist this strain when applied directly to its length; this, however, is by no means the case, for if we imagine, as we must do, that the iron, in consequence of the internal pressure, suffers a certain degree of extension, we shall find that the external circumference participates much less in this extension than the interior, and as the resistance is proportional to the extension divided by the length, it follows that the external circumference and every successive layer, from the interior to the exterior surface, offers a less and less resistance to the interior strain."

The above statements prove that Mr. Barlow recognized the existence of variable strains upon the mass of the cylinder's walls during pressure, and states clearly that they decrease from the inner to the outer surface. Mr. Barlow states distinctly the importance of knowing, definitely, the cohesive strength of the metal, and that whatever it is, it must so take its place in the formula. He does not state, however, nor caution the maker of press cylinders against the danger of the weakening effect due to the unequal shrinkage in the walls of the cylinder, while the same are cooling after being cast. That such action does take place is a matter of common observation, and yet there seems to be great difference of result, as proven by test, about this phenomenon of cooling. We may quote such high authority as Mr. Hodgkinson, who says: "Comparing the tensile strength of bars of cast iron 1, 2, and 3 inches square, I found that the relative strengths were approximately as 100, 80, and 77." Capt. James gives 100, 66, and 60 for similar bars, and that ¾-inch square bars, cut from 2- and 3-inch bars, possessed only half the strength of 1-inch square cast bars. The cause of this is attributed to the greater strength of the "skin" portions of the castings, and to the more spongy and therefore weaker texture of the interior, which increases with the thickness.

In opposition to these statements we may add, here, that test pieces have been taken from the walls of certain 5-inch thick cylinders of American cast iron which exhibited in every part an equality of tensile strength, thus showing the uniformity of texture throughout the mass, not only by observation, but by experiment. It may also be added that the texture of the material in the 8-inch cylinder walls—mentioned in the previous chapter—which failed, was, to all appearances, sound and solid, through and through, at the ruptured sections. Another feature of cast iron must be observed; there is little or no indication of an ascertainable and measurable elastic limit. *Ordinance Notes* say: "Cast iron rarely shows a well defined limit of elasticity. The elastic limit to extension is 15,000 pounds per square inch."

We know too well by experience, and we therefore quote the words of Mr. H. T. Bovey, in his work on the "Strength of Materials," 1893, that "cast iron is, perhaps the most doubtful of all materials, and therefore the greatest care should be observed in its employment. It possesses little tenacity, or elasticity; is very hard and brittle, and may fail suddenly under shock, or under an extreme variation of temperature. Unequal cooling may pre-dispose the metal to rupture, and its strength may be still further diminished by the presence of air-holes. Cast iron and similar materials receive a sensible set, even under a small load, and the set increases with the load." We certainly know that all experience proves the need of intelligence and care in the proportioning and making of cast iron hydraulic press cylinders.

As to the choice of material, not including the steels, Rankine gives the ultimate tensile stress of cast iron 13,400 to 29,000 pounds per square inch, which assures undoubted evidence of the possibilities of this metal. High tensile strength, however, must not be the ruling element in the choice of metal for such castings as are liable to be affected by shrinkage strains. The case in practice to which reference is made, is one that came under the care and construction of the writer in the year 1874. Several presses were ordered to have 15-inch diameter of rams, with cylinders to sustain 6,000 pounds hydraulic pressure per square inch. The interior diameter for ram clearance was made 16 inches, and the walls were 8 inches thick, *i. e.*, same as the internal radius. The first one was cast from an air furnace of hard, close texture cast iron, such as rolling-mill rolls are made of; this one burst at the first trial. Another was ordered, of same dimensions, to take its place, but to be made of soft and tough iron. This one stood the test, is in use to-day, and is frequently put under a hydraulic pressure of 4 tons, or 8,000 pounds, per square inch.

Referring now to some of the published rules, the following notation is made uniform for the first five formulas:

Let P = internal pressure in pounds per square inch,
S = tensile stress in pounds per square inch to which the material is subjected by the pressure P,
D = internal diameter of the cylinder in inches,
t = thickness of metal in inches,
e = base of Napierian system of logarithms = 2.71828.

The ultimate tensile strength of cast iron is taken at 18,000 pounds per square inch; the internal hydraulic pressure at 6,000 pounds per square inch, and the internal diameter 16 inches. The worked-out result is given with each formula.

Bernoulli, Unwin, Rankine, Claudel, Weisbach, Van Buren, Haswell, Lanza, and Clark, give this first formula for ascertaining the thickness of *thin* cylinders, without joint:

$$t = \frac{D}{2}\,\frac{P}{S} \text{ or } t = \frac{Pr}{S} = 2\ 2/3 \text{ inches.} \qquad (1)$$

in which r = radius, or half of D.

Reuleaux gives for thick cylinders:

$$t = \frac{D}{2}\,\frac{P}{S}\left(1 + \frac{P}{2S}\right) \doteq 3.1 \text{ inches.} \qquad (2)$$

Trautwine repeats the same, but adds a factor of safety k, which we will assume to be 3, thus:

$$t = \frac{D}{2}\,\frac{Pk}{S}\left(1 + \frac{Pk}{2S}\right) = 12 \text{ inches.} \qquad (3)$$

Omitting k, we will get $t = 3.1$ same as by Reuleaux's rule.

Brix and Clark give:

$$t = \frac{D}{2}\left(e^{\frac{r}{s}} - 1\right) = 3.16 \text{ inches,} \qquad (4)$$

e being the base of the natural logarithms.

Grashof gives:

$$t = \frac{D}{2}\left[-1 + \sqrt{\frac{3\,S + 2\,P}{3\,S - 4\,P}}\right] = 3.84 \text{ inches.} \qquad (5)$$

Prof. Merriman's formula for the thickness of *thin* cylinders to resist internal pressure, may be derived from his general formula. thus:

$$P\,D = 2\,t\,S, \text{ in which,}$$

P = pressure per square inch of the liquid within the cylinder in pounds.

D = internal diameter of the cylinder in inches.

t = thickness of the walls of the cylinder in inches.

S = working tensional stress of the material in pounds.

By transposition we get:

$$2\,t = \frac{P\,D}{S}.$$

By substituting the radius for the diameter, which simplifies the formula, we get:

$$t = \frac{P\,r}{S},$$

and then by solving the problem of our data, although this formula is not intended for application to thick cylinders, yet we work it out for the sake of comparison with others. We have, then:

$$t = \frac{6{,}000 \times 8}{18{,}000} = 2\ 2/3 \text{ inches,}$$

which is the same result as given by the first formula. This rule is in harmony also, with the formula used by all boiler inspection companies for the thickness of boiler plates. Prof. Merriman further says: "For very thick cylinders this formula is only approximative."

Mr. J. D. Van Buren, Jr.'s formulas "are developed in reference to the *ultimate* strength of the material in order to leave the choice of a *factor of safety* to the judgment of the designer." This is the best way to give rules; if, then, the elastic limit of the material be ascertained, we will know just how far to go in putting stress upon materials, and be safe. But when a rule is given, without a hint as to the degree of its safety, we really know nothing about it.

Mr. Van Buren assumes 18,000 pounds per square inch as the ultimate and 2,500 pounds as the safe strength of good cast iron, thus allowing a liberal factor of safety—between 7 and 8. His formula is the same as the first one, as noted above, and is for thin-walled cylinders only, such as steam and water pipes.

Molesworth, in his "Pocket-Book of Engineering Formulas," gives a rule for the thickness of metal in hydraulic cylinders in this form:

$$t = \frac{\frac{1}{2} D P}{x - P},$$

in which t = thickness of metal in inches,
D = internal diameter of cylinder in inches,
P = pressure of the water in tons per square inch,
x = a constant for different metals, valued at 7 for cast iron; 14 for gun metal; 20 for wrought iron.

For our case we will have $t = \frac{\frac{1}{2} \times 16 \times 3}{7 - 3} = 6$ inches.

Nothing is said about conditions or whether this rule is for the safe, or the bursting strength. This rule is the same as Barlow's, except the constant 7 is taken instead of 18,000 pounds for the ultimate tensile strength.

Hurst's "Hand Book" gives the same rule in simpler form, thus:

$$t = \frac{PR}{7 - P} = \frac{24}{4} = 6 \text{ inches.}$$

D. K. Clark, for the sake of argument, divides the cross section of a cylinder into a number of concentric rings of equal thickness, and then supposes that each one bears less strain according to its distance from the center. He then plots a curve over the points, above a base line, representing the stress carried by each ring, and finds it to be hyperbolic, from which he formulates the following rules:

$$P = S \times \text{hyp. log. } R$$

$$S = \frac{P}{\text{hyp. log. } R}$$

$$\frac{P}{S} = \text{hyp. log. } R,$$

in which P = the internal pressure in tons or pounds per square inch,
S = the maximum tensile stress within elastic limit in tons or pounds per square inch,
R = the ratio of the diameters, that is, the outside diameter of the cylinder divided by the diameter of the bore.

These rules apply as readily as any. Let us take our previous case. Then we have from the tables: the hyp. log. of 32/16 = 0.693; S = 18,000 pounds per square inch = 9 tons; then $P = 9 \times 0.693 = 6.237$ = the tons pressure per square inch = 12,474 pounds, which will produce rupture, if the iron will fail at 18,000 pounds per square inch.

Lamé's treatment of this subject is classic; we will give only his formula for the thickness of the walls of thick cylinders, which is:

$$t = r\left[\left(\frac{S+P}{S-P}\right)^{\frac{1}{2}} - 1\right]$$

in which $t =$ thickness in inches,

$S =$ the tension in pounds per square inch,

$P =$ the hydraulic pressure in pounds per square inch,

$r =$ the internal radius of the cylinder in inches.

Inserting our data in this formula, we have:

$$t = 8\left[\left(\frac{18,000 + 6,000}{18,000 - 6,000}\right)^{\frac{1}{2}} - 1\right] = 3.312 \text{ inches.}$$

Lamé is quoted by Rankine, Reuleaux, Lineham, and Burr.

It is important to observe, as noted by Reuleaux, that "the internal pressure P, should in no case exceed the permissible stress S of the material. That is, if we make P equal to, or greater than S, the cylinder will burst, however great the thickness of the cylinder be made."

Rankine gives a rule of different form, but it leads to the same result as Lame's. It is in this form:

$$\frac{R}{r} = \sqrt{\frac{f + P}{f - P}}$$

in which $R =$ external radius of the cylinder,

$r =$ internal radius of the cylinder,

$f =$ tenacity of the material,

$P =$ bursting pressure.

Introducing our dimensions, we have:

$$\frac{R}{8} = \sqrt{\frac{18,000 + 6,000}{18,000 - 6,000}} = 1.414$$

whence $R = 11.312$. Now $R - r =$ the thickness of the cylinder's walls, and therefore $11.312 - 8 = 3.312 = t$.

We omit the fluid pressure from without, which is the atmosphere in this case, and is therefore unimportant. Rankine claims for his formula the same "important consequence" as noted by Reuleaux in a preceding paragraph, but how this can be made to appear, will require a journey through a long line of formulas, not within the scope of the present treatise. The way to a clear understanding of Rankine's deductions can only be pointed out by the intelligent hand of the advanced mathematician.

The following table gives the resulting thickness that will just sustain the pressure, as obtained from the several rules quoted. The data are: 16 inches internal diameter of cylinder; 6,000 pounds hydraulic pressure per square inch; and 18,000 pounds ultimate tensile strength of the cast iron used in the cylinder.

	Thickness in inches.
Rule for thin cylinders	2.666
Rule by Reuleaux	3.100
Rule by Brix	3.160
Rule by Lamé and Rankine	3.312
Rule by Grashof	3.840
Rule by Barlow	4.000
Rule by Molesworth and Hurst	6.000

The last one of this list, probably, includes the thickness for safety, although the author does not say it; the extremes excluded we have a range from 3.1 to 4 inches; the factor of safety must be decided by the designer for the material and purpose. Much depends upon a through and through strength and the toughness of the cylinder casting. If the metal be open-grained from the internal surface into the mass, the liquid within, under pressure, will penetrate its pores; thus acting upon a larger radius, it defies the rule and destroys the cylinder. The weight of evidence here given plainly shows that the formulas which do not take into account the variation of tension in the walls of a thick cylinder, under hydraulic pressure, are not reliable.

To help the reader understand some of the difficulties in the way of solving this problem, his attention is called to the fact that when a straight bar of metal is subjected to a direct tension, every part of the same may be considered as being under equal strain and as contributing its share of resistance to prevent separation. On the other hand, as stated by D. K. Clark: "the resistance offered by the sides of a cylinder to internal pressure is not uniformly exerted throughout the thickness of the sides. On the contrary, the resistance is a maximum at the inner surface of the cylinder, and when the stress on the inner surface does not exceed the limit of its elasticity, the tensile stress diminishes uniformly through the thickness of the sides, and is a minimum at the outer surface. For cast iron, the bursting strength is measured by the total resistance opposed to breakage, when the internal surface is strained to the ultimate limit of its tensile strength." But now we meet the difficulty of knowing the thickness of the inner skin and the degree of sponginess of the interior, if there be any; should this be excessive (offering little, or no resistance), we could not count on more than one-third the section for resistance to the internal pressure, and this condition of metal may account for many failures of press cylinders, especially when calculations are based upon the supposition of uniform density.

Of the formulas which have the sanction of science, it may be said that, while they are correct—considered as algebraic and arithmetic forms—yet, when employed for results, care must be taken that all the elements entering the formulas exactly conform to the facts; remembering always, that it is the material which gives the strength, and that the character of the stress to which it is subjected, must be known and provided for.

There is also danger due to casting the bottom in one with the cylinder, but this may be averted by forming the end spherical and of even thickness with the walls of the cylinder. Even when the bottoms are made convex to a larger radius, they are not safe; cases of failure have occurred at the joining of such curves. Reuleaux informs us that "the method used by Hummel, of Berlin, is to make the cylinder as a ring, and the bottom as a separate plate. Lorenz, of Carlsruhe, makes the bottom separately and screws it in." Leakage may be prevented by cupped leathers, same as those applied at the ram or plunger end.

When a certain load is to be lifted and sustained by the ram, the tension on the metal may be decreased by an increase of the diameter of the ram. "The Constructor" says: "In the case of Hummel's hydraulic press, in the table below, if we make the ram 26 inches diameter, the pressure *P* will be reduced to 4,087 pounds, and *S* to 7,900 pounds, which is quite practicable. The cross-section of the cylinders will be as 1 to 0.79, which effects a saving of about 20 per cent in the material."

DIMENSIONS OF A FEW NOTABLE PRESSES WITH PRESSURES, AND STRESS ON MATERIAL, FROM REULEAUX'S CONSTRUCTOR

Names and Localities.	Diam. Ram.	Bore.	Thickness.	Load on Ram, pounds.	Pressure per sq. in. pounds.	Stress on metal per sq. in. pounds.
Conway Bridge.....	18	20	$8\frac{1}{4}$	1,456,000	5,900	10,500
Brittania Bridge....	18	20	$8\frac{1}{4}$	1,456,000	4,191	7,460
" "	20	22	10	1,031,520	8,400	14,500
" "	27.56	28.33	$6\frac{3}{10}$	2,640,000	4,425	12,134
Hummel, of Berlin..	23	24	$8\frac{1}{2}$	2,200,000	5,174	10,000

Referring, again, to the admirable work, "The Constructor," by Reuleaux, translated by H. H. Suplee, we quote: "Brix calculates the stresses at different points on the radius upon the supposition that the internal diameter is not altered by the pressure. Barlow admits such an alteration by pressure that the area of the annular section is not reduced. Lamé makes neither of these assumptions, but calculates very closely the changes in the various stresses which are caused by the internal pressure at each point, and in this way has obtained the most reliable data as to the real behavior of the particles of the material.

"All these theories admit that the inner portion of the wall is strained the most, and hence it is for the inner wall that *S* should be chosen. The formulas of Lamé, as well as those of Barlow, show that beyond certain limits an increase in the thickness is not attended with any increase of strength. With a given resisting power *S*, this limit will be reached when $P = S$. Lack of homogeneity in the material may cause the danger pressure to be reached far within these limits—the material breaking without previously stretching.

"Various methods have been devised for strengthening thick cylinders, by giving the various layers different tensions. Of these methods the principal is that of hooping. The chief result of this construction is to produce a compression in the inner layer. The pressure must then first overcome this compression, and restore the normal condition, before it can produce any extension of the fibers, and as a result a much higher degree of resistance is secured than when the metal is left in its normal condition."

The calculations of the resistance of hooped cylinders offer many difficulties, and we can best refer the readers to "The Constructor," page 16, if he wishes to pursue his inquiry further. Proofs are given that the mere hooping of a cylinder with a ring of the same material as the inner tube, adds very materially to its strength. If, however, the ring is forced on in any manner so as to produce an initial strain upon the tube, a still greater advantage will be the result. Encircling the hoops by additional hoops has been proved to add still further advantage. The study of these principles and methods, as applied to heavy guns, will be found fruitful of results in this line of inquiry. It is, therefore, important to use material capable of withstanding a high stress, and to take great care in construction and in the disposition of the material.

We have here very clear proof drawn from highest authority, that the material and the conditions to be observed in its preparation and use, must be well considered. Of these essentials, most of the rules are silent. The presentation of formulas giving widely different results, without a word of comment or sign of assistance that will enable the reader to decide upon the one that applies to his case, is very far from being a reliable "short cut" to correct knowledge. The principles must be ever kept in sight, yet the way to results must be, like the straight line, the shortest distance between two points—between the known and the unknown. It is all very well to say: "You must use judgment with rules," but the rules themselves do not furnish this. Now, judgment is the outgrowth of the larger mind, fertilized by experience, and we are aiming here to render assistance to those who want to know, by presenting the results of experience.

There is no need of discussing the rule given for thin cylinders, as the stress produced by the hydraulic pressure acting as hoop-tension, may be taken as practically uniform throughout the walls of the cylinder. It is for thick ones only, that a "hard and fast" rule cannot be made to apply.

It should be said here that great care must be taken in the adoption and use of rules for proportioning parts of machines which are to carry heavy strains. The object of the writer in this painstaking effort to collect and compare existing rules, is to show what rules we have and what they mean. The ancient advice, "Prove all things," is not more important in the lines laid down by its author, than it is in engineering.

The present chapter has dealt largely with theoretical considerations, and a thorough investigation of the rules and formulas at our disposal. In the following chapters we shall give more attention to the application of the formulas to actual design as presented by practical designers of this class of machinery.

CHAPTER III

DESIGN OF THICK CYLINDERS

A phase of design on which there are but few available data is that of thick cylinders for pressures above one thousand pounds per square inch. Comparatively few hydraulic press cylinders work at a less pressure than this, and the designing must be done very carefully both regarding strength and distribution of the metal.

Lame's formula for thick cylinders, referred to in Chapter II, is, in its usual form,

$$t = r\left(\sqrt{\frac{S+P}{S-P}} - 1\right) \qquad (6)$$

sometimes inconvenient for handling. The following forms of the same formula, obtained by substitution, are preferable for the use of the designer.

$$S = P\frac{R^2 + r^2}{R^2 - r^2} \qquad (7)$$

$$R = r\sqrt{\frac{S+P}{S-P}} \qquad (8)$$

$$r = R\sqrt{\frac{S-P}{S+P}} \qquad (9)$$

$$P = S\frac{R^2 - r^2}{R^2 + r^2} \qquad (10)$$

in which:

$S =$ maximum allowable fiber stress per square inch,
$R =$ outer radius of cylinder, in inches,
$r =$ inner radius of cylinder, in inches,
$P =$ working pressure of liquid within cylinder,
$t = R - r =$ thickness of cylinder, in inches.

Form (8) of this equation may be transposed to read

$$\frac{R}{r} = \sqrt{\frac{S+P}{S-P}}$$

which reads "the ratio of the outer radius to the inner radius is equal to the square root of the quotient of the difference of the allowable working stress and the working pressure into the sum of the same." By allowing these last-named quantities to vary over a considerable range, the writer has prepared a table of ratios of outer to inner radii, from which one may, without calculation, determine the thickness of a cylinder wall. Careful study of this form of the equation reveals that as the pressure P approaches the allowable stress S, the ratio $\frac{R}{r}$ increases very rapidly; it becomes infinite when the pressure equalizes the allowable stress, and becomes an imaginary quantity when the pressure is greater than the allowable stress. In practice, this means that for each metal there is a limiting pressure, beyond which it is impossible to design a safe cylinder, and a metal of higher tensile strength must be employed. Further, for every material there is a pressure point for each diameter of cylinder beyond which it is economical to resort to a better grade of material. The allowable stress

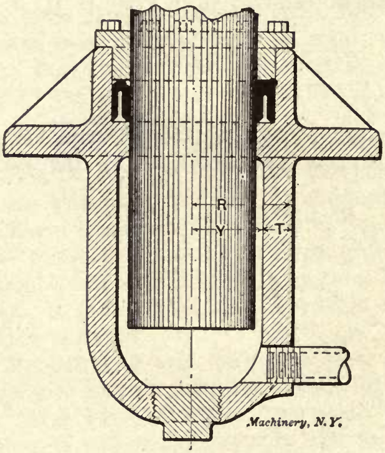

Fig. 1.

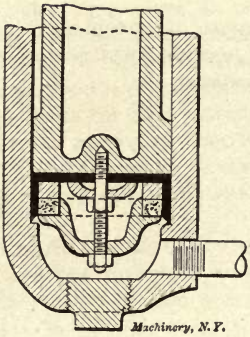

Fig. 2.

is a figure dependent on the elastic limit of the material. In hydraulic cylinders we are usually safe in working the material up to fifty per cent of the elastic limit.

In designing a cylinder to give a certain tonnage it is well to bear in mind the following points:

1. With a fixed pressure, the tonnage increases as the square of the diameter.

2. When the pressure exceeds 2,500 pounds per square inch, packings become leaky, valves do not hold, and pipe fittings give trouble; for these reasons it is advisable to keep the pressure below this point, but as this necessitates a larger cylinder, cost often is prohibitive.

Suppose a cylinder is required to give 95 to 100 tons pressure:

An 11-inch cylinder working at 2,000 pounds gives 95 tons, a 10-inch cylinder working at 2,500 pounds gives 98 tons, and a 9-inch cylinder working at 3,000 pounds gives 95 tons. For calculation let us take the 10-inch cylinder working at 2,500 pounds, and let our material be

RATIO OF OUTSIDE RADIUS TO INSIDE RADIUS, THICK CYLINDERS

Allowable Stress of Metal per square inch Section.	Working Pressure in Cylinder, Pounds per Square inch.												
	1000	1500	2000	2500	3000	3500	4000	4500	5000	5500	6000	6500	7000
2000	1.732	....	....	....	....	....	....	....	....	....	....	....	....
2500	1.527	2 000	....	....	....	....	....	....	....	....	....	....	....
3000	1.414	1.732	2.236	....	....	....	....	....	....	....	....	....	....
3500	1.341	1.581	1.915	2.449	....	....	....	....	....	....	....	....	....
4000	1.291	1.483	1.732	2.081	2.645	....	....	....	....	....	....	....	....
4500	1.253	1.414	1.612	1.871	2.236	2.828	....	....	....	....	....	....	....
5000	1.224	1.362	1.527	1.732	2.000	2.380	3.000	....	....	....	....	....	....
5500	1.201	1.322	1.464	1.633	1.844	2.121	2.516	3.162	....	....	....	....	....
6000	1.183	1.291	1.414	1.558	1.732	1.949	2.236	2.645	3.316	....	....	....	....
6500	....	1.264	1.374	1.500	1.647	1.825	2.049	2.345	2.768	3.464	....	....	....
7000	....	1.243	1.341	1.453	1.581	1.732	1.914	2.144	2.449	2.886	3.605	....	....
7500	....	1.224	1.314	1.414	1.527	1.658	1.813	2.000	2.236	2.549	3.000	3.741	....
8000	....	1.209	1.291	1.381	1.483	1.599	1.732	1.889	2.081	2.323	2.645	3.109	3.872
8500	....	1.194	1.271	1.354	1.446	1.548	1.666	1.802	1.963	2.160	2.408	2.738	3.214
9000	....	1.183	1.253	1.330	1.414	1.507	1.612	1.732	1.871	2.035	2.236	2.440	2.828
9500	....	....	1.235	1.306	1.386	1 472	1.566	1.673	1.795	1.936	2.104	2.309	2.569
10000	....	....	1.224	1.291	1.362	1.441	1.527	1.623	1.732	1.856	2.000	2.171	2.380
10500	....	....	1.212	1.274	1.341	1.414	1.493	1.581	1.678	1.789	1.915	2.061	2.236
11000	....	....	1.201	1.260	1.322	1.390	1.464	1.544	1.633	1.732	1.844	1.972	2.121
11500	....	....	1.193	1 247	1.306	1.369	1.437	1.511	1.593	1.683	1.784	1.897	2.027
12000	....	....	1.183	1.235	1.291	1.359	1.414	1.483	1.558	1.640	1.732	1.834	1.949
12500	....	....	....	1.224	1.277	1.333	1.393	1.457	1.527	1.603	1.687	1.779	1.878
13000	....	....	....	1.215	1.264	1.318	1.374	1.434	1.500	1.570	1.647	1.732	1.825
13500	....	....	....	1.206	1.253	1.303	1 357	1.414	1.475	1 541	1.612	1.690	1.775
14000	....	....	....	1.197	1.243	1.291	1.341	1.395	1.453	1.514	1.581	1.653	1.732
14500	....	....	....	1.189	1.233	1.279	1.327	1.378	1.432	1.490	1.553	1.620	1.693
15000	....	....	....	1.183	1.224	1.268	1.314	1.362	1.414	1.469	1.527	1.590	1.658
15500	....	....	....	1.177	1.216	1.258	1.304	1.348	1.397	1.449	1.504	1.563	1.627
16000	....	....	....	1.170	1.209	1.249	1.291	1.335	1.381	1.431	1.483	1.538	1.599

cast iron whose allowable stress is 6,000 pounds per square inch. By substituting in formula (6)

$$t = 5\left(\sqrt{\frac{6000+2500}{6000-2500}} - 1\right)$$

$t =$ thickness of cylinder wall, 2.79 inches.

Reference to the table of ratios under column of 2,500 pounds pressure and on the line of 6,000 pounds allowable stress, gives the ratio 1.558.

It is well to leave more metal in the bottom of a hydraulic cylinder than the design would seem to require, for the reason that a hole of some size must be cored in the bottom to permit the entrance of a boring bar when finishing the cylinder, and when this hole is

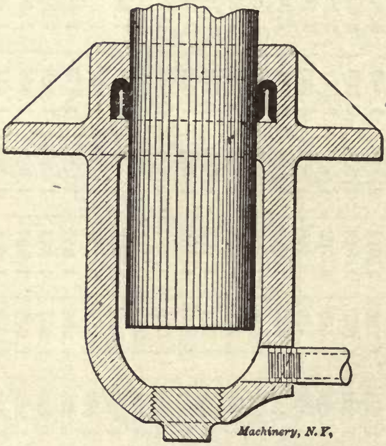

Fig. 3.

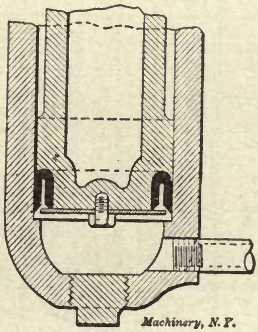

Fig. 4.

subsequently tapped and plugged, it will be found a fertile source of trouble.

Flanged cylinders, Figs. 1, 2, 3, and 4, are the type usually employed in hydraulic press work, and in addition to withstanding bursting pressure, they must withstand the beam load on the flanges. The frequent point of failure is at the junction between the flange and the cylinder. This section is usually further endangered, as the internal stresses set up by the cooling of the casting are severe, and the metal usually "draws" away because of the more rapid cooling of the flange. For this reason, care should be taken to avoid having thin portions leading abruptly from thick portions.

Patterns should be parted just above the flange, and all cylinders should be cast with the open end up so that the dirt in the iron will accumulate at the top of the casting where it can do little harm. On short cylinders, the sprues should come off from the flange and upper edge of the cylinder. On long cylinders it is necessary to have sprues further down, and it happens not infrequently that the spongy spots where the sprues have been removed have to be plugged. Porous castings may be treated in several ways: A strong sal-ammoniac solution is a very common treatment, as is also common salt. Starch or wood pulp left under pressure will sometimes prove effective.

The common forms of hydraulic packings are: U packing with a removable follower, Fig. 1; cup packing on the end of the ram, Fig. 2; U packing in a chamber in the neck of the cylinder, Fig. 3; and U packing on the end of the ram, Fig. 4. The U packing with the removable follower seems to be the most mechanical, and gives very excellent results under any pressures. There is much contention among the competing press builders regarding the best style of packing, but the writer's observation has been, that with good workmanship and a good packing, there is little choice as to efficiency, the main point being accessibility for repacking.

CHAPTER IV

CHARTS AND DIAGRAMS FOR THE DESIGN OF THICK CYLINDERS

In the present chapter three charts for the design of thick cylinders are given, which will be found helpful to designers. The formulas on which these charts are founded have already been given in Chapter II, but the present chapter contains all the matter necessary to make it a complete whole by itself, even at the risk of some repetition of statements.

The thickness of wall in cylinders for high pressure must be determined partly from experience. In all cases a large factor of safety must be employed to allow for imperfections in the metal, strains due to outside causes, etc. In the determination of the factor of safety, the designer must be guided by current successful practice.

The most generally accepted formula for the thickness of wall is that of Lamé. It is as follows:

$$t = \frac{D}{2}\left(\sqrt{\frac{S+P}{S-P}} - 1\right) \qquad (11)$$

in which

t = thickness of wall in inches.
D = inside diameter of cylinder in inches.
P = working pressure in pounds per square inch.
S = stress in cylinder wall in pounds per square inch.

(For the derivation of Lamé's formula, see Thurston's "Iron and Steel," page 452; Rankine's "Applied Mechanics," page 290; or Burr's "The Elasticity and Resistance of the Materials of Engineering," pages 19 and 895.)

If S is taken as the ultimate tensile strength of the material, the thickness found will be that which would just be ruptured by the given working pressure. To find the actual thickness, the ultimate tensile strength must be divided by the factor of safety.

The accompanying set of curves, Figs. 5 and 6, has been devised to save the time and mathematical work involved in the solution of prob-

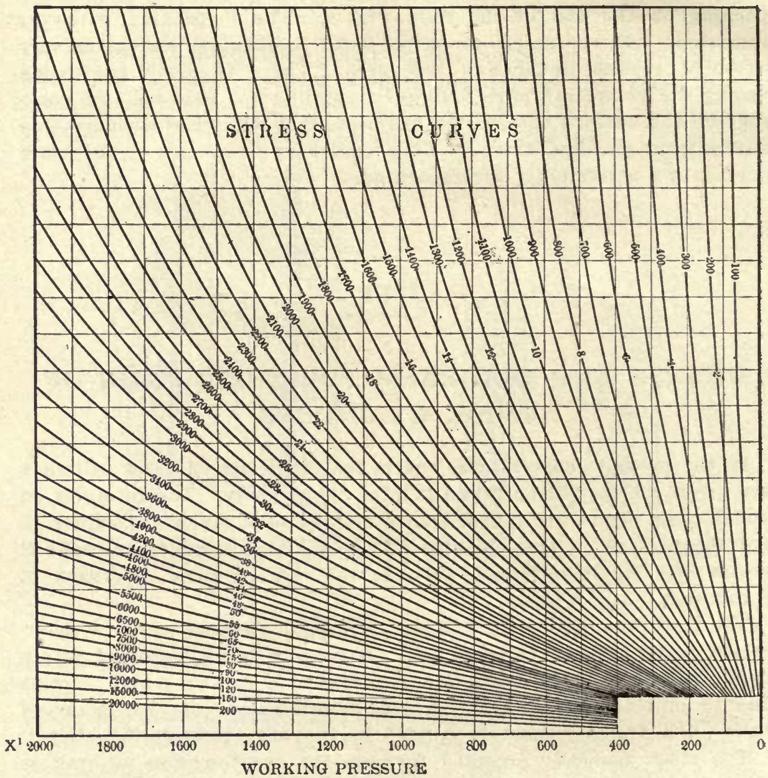

Fig. 5. Stress Curves for Designing Thick Cylinders

lems by Lamé's formula. By means of these curves any one of the four quantities may readily be determined when the other three are known.

To prepare the curves the formula was put in the form

$$\frac{R}{r} = \sqrt{\frac{S+P}{S-P}} \qquad (12)$$

in which

$R =$ the external radius of cylinder.

$r =$ the internal radius of cylinder.

(The formula is transposed to the form in (12) by substituting r for $D/2$ and $R - r$ for t, and then dividing by r.)

It will be seen that the value of the ratio R/r is independent of the diameter. For convenience, denote the value of this ratio by K. The formula then becomes

$$K = \sqrt{\frac{S+P}{S-P}} \tag{13}$$

The values of the inside diameter D are plotted in Fig. 6, and the values of the working pressure P are plotted in Fig. 5, as indicated.

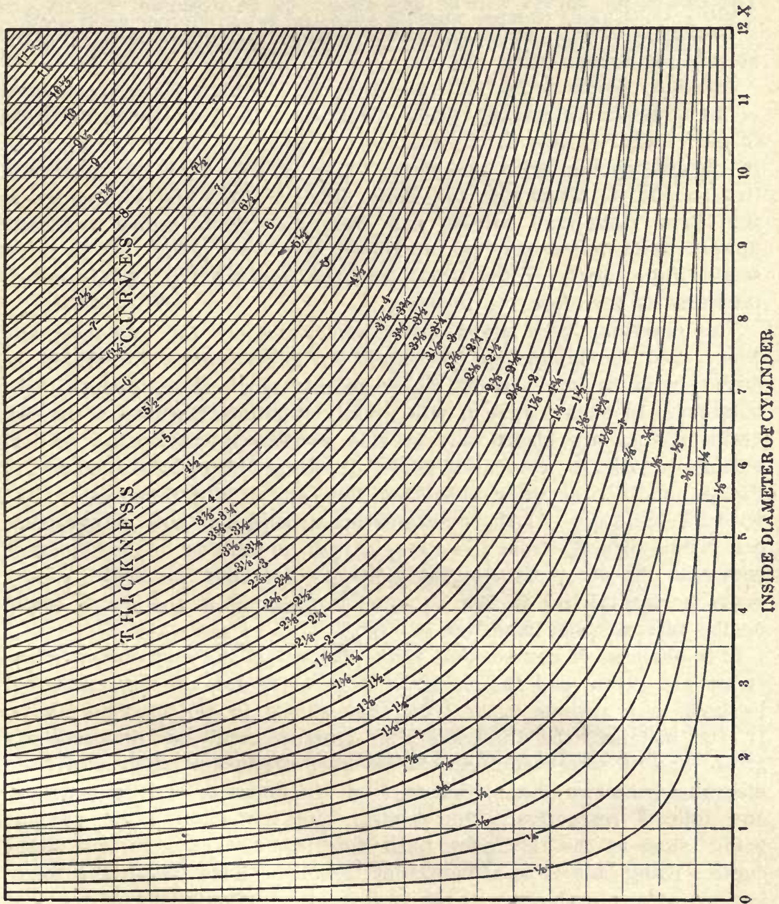

Fig. 6. Thickness Curves for Designing Thick Cylinders

The values of the ratio R/r are laid off along the right-hand vertical outline in Fig. 5, and along the lower horizontal outline in Fig. 6, commencing with one at the zero mark. The values 100, 200, etc., are then substituted for S in Formula (13), and the resulting points located on the diagram in Fig. 5. The curves resulting from connecting the points plotted, represent the stress in the walls of the cylinder.

Since $t = R - r$ and $R = Kr$, we have

$$t = Kr - r$$
$$= r\ (K - 1)$$

and therefore $2t = d\ (K - 1)$.

This equation is of the form $xy = C$. The values ⅛, ¼, etc., are then substituted for t, and the resulting curves are drawn in diagram Fig. 6. These curves represent the thickness of the cylinder wall. The use of the curves will be best shown by an example.

Example.—$D = 5$ inches; $S = 1{,}600$ pounds per square inch; $P = 700$ pounds per square inch; find t.

Solution.—From point 700 in diagram Fig. 5, follow vertically upward to the stress curve marked 1,600. Then move horizontally to the right to the vertical outline of the diagram. Locate the corresponding point to the intersection with the vertical line in Fig. 5 on the horizontal line in Fig. 6, measuring the same distance from the zero point at the lower right-hand corner, and from this point follow vertically upward until intersecting the horizontal line from point 5. The intersection falls nearly on the thickness curve marked 1½. The required thickness is therefore 1½ inches.

The diagrams here given are intended merely for guidance, and when used for laying out hydraulic cylinders should preferably be drawn in much larger scale, which enables much closer results to be obtained. In such a case it will greatly facilitate the use of the diagrams if they are placed side by side so that the right-hand vertical outline in Fig. 5 coincides with the lower horizonttal outline in Fig. 6; or, in other words, so that line X^10 in Fig. 5 comes in line with $0X$ in Fig. 6. If the diagrams are arranged in this manner, one can follow directly from the stress curves in the one to the thickness curves in the other, without the difficulty of finding the points on the lower horizontal line in Fig. 6, which correspond to the points found on the vertical right-hand line in Fig. 5.

It is obvious, of course, that if the thickness, the diameter, and the stress are given, and the pressure is to be found, the diagrams will be used in a reverse order from that shown in the example above. If, for instance, the thickness, the pressure, and the diameter are given, and it were required to find the stress, one enters into the diagrams from two places; that is, from the diameter and the pressure, and follows respectively the vertical and horizontal lines, account being taken of the thickness, until they intersect on a certain stress curve. From this it is evident that these diagrams permit the working out of any problem without mathematical calculations, if three of the four quantities, diameter, stress, pressure, and thickness, are given, and the fourth is to be found. Any one of the quantities may be unknown and located on the diagram.

Another writer presents the chart in Fig. 7 with the following explanation:

"Suppose it is desired to have a chart which will give the thickness of cylinders for various pressures, sizes and materials.

"For thin cylinders, not having a seam or joint, the formula is

$$t=\frac{p\,d}{2\,s},$$

where t = thickness in inches,
p = pressure in pounds per square inch,
d = internal diameter of cylinder, and
s = allowable working stress.

"For thick cylinders (as we have already seen in Chapter II), Burr, in his "Elasticity and Resistances of Materials," page 36, gives

$$t = r\left[\left(\frac{h+p}{h-p}\right)^{\frac{1}{2}} - 1\right]$$

in which r = interior radius,
h = maximum allowable hoop tension at the interior of the cylinder,
t = thickness in inches, and
p = pressure in pounds per square inch.

"Rankine gives

$$R = r\sqrt{\frac{s+p}{s-p}}$$

in which R = exterior radius,
r = interior radius,
s = allowable working stress, and
p = pressure in pounds per square inch.

"Using the same notation, Lamé gives

$$t = r\left(\sqrt{\frac{s+p}{s-p}} - 1\right)$$

and Merriman gives

$$t=\frac{r\,p}{s-p}.$$

"Changing the notation where required, we find that for thick cylinders, Rankine's, Lamé's, and Burr's formulas solve out to the same form:

$$t = r\left(\sqrt{\frac{s+p}{s-p}} - 1\right), \text{ or}$$

$$\frac{s}{p} = \frac{\left(\frac{t}{r}+1\right)^2+1}{\left(\frac{t}{r}+1\right)^2-1}$$

Merriman's formula solves out

$$\frac{s}{p} = \frac{r}{t} + 1.$$

For thin cylinders we have

$$\frac{s}{p} = \frac{r}{t}.$$

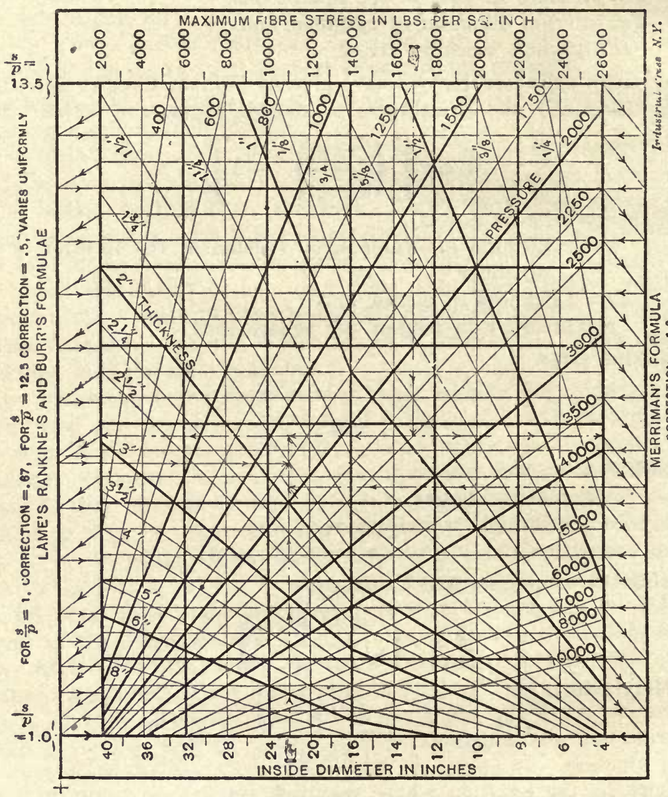

Fig. 7. Diagram for Designing Thick Cylinders

"Inserting values of $\frac{t}{r}$ in the Burr, Rankine and Lamé formulas, the corresponding values of $\frac{s}{p}$ may be calculated. Lay out the chart for thin cylinders, choosing for the first factor at the top of the chart, s, and for the second factor $\frac{1}{p}$; their product, $\frac{s}{p}$, is represented by a uniform scale on the sides. (See chart Fig. 7). The minimum reading has been chosen as 1, and the maximum reading as 13.5; each of the small spaces is 0.5. On the bottom lay off values of r, which have been chosen from 2 inches to 20 inches and doubled to read diameter instead of radius. This radius is considered a first factor commencing at the bottom of the chart, the second factor being $\frac{1}{t}$; their product is $\frac{r}{t}$, which is equal to $\frac{s}{p}$ and is represented by the same scale at the side.

"Since tracing from stress to pressure and thence to the side of chart strikes the same value as tracing from the diameter to thickness and thence to the side, they must intersect at the required thickness. The example illustrated by the broken lines is, stress = 17,000, pressure = 2,500, diameter = 22 inches; the intersection shows the thickness to be 1⅝ inch, according to formula for thin cylinders. In Merriman's formula $\frac{s}{p} = \frac{r}{t} + 1$, hence a correction of 2 spaces of 0.5 each is made at the right side, and in the above example the intersection shows the thickness to be 1⅞ inch. The corrections on the left side for Burr's, Rankine's and Lamé's formulas are laid off from calculated values previously referred to. The example indicated shows the thickness according to this formula to be 1¾ inch, which is probably the most reliable as being based upon a more nearly perfect theory. It may be noticed that in calculating the corrections for Lamé, Rankine and Burr's formulas the values inserted were $\frac{t}{r}$, while the values of the scale on the left of the chart are $\frac{r}{t}$. This is rectified by the use of a table of reciprocals when plotting these corrections."

The various equations for thick formulas have, in the previous treatment been repeated several times, but this has been done in order to permit each writer to present in full his way of analyzing the problem.

CHAPTER V

THICK CYLINDERS

The calculation of the thickness of cylinders for a given pressure has been so much discussed, and so many formulas have been deduced, some theoretical and others empirical, that there seems to be little to add. Yet this subject is so little understood that every experienced engineer relies on his own experience, and in most cases uses no formula at all, except a kind of proportional one, that is usually all right for limited pressures and sizes of cylinders. Some formulas, although published in reputable engineering hand-books, are absolutely worthless. Others, again, are good for high pressures but valueless for low pressures, and *vice versa.*

Commonly Used Formulas

In low pressure work the general practice is to make the thickness of the metal = diameter × unit pressure ÷ twice the allowable working stress of the material, and add to this a variable quantity to allow for unsound castings and possible unknown stresses, or

$$t = \frac{D P}{2 S} + a \qquad (14)$$

where t = thickness in inches,
D = diameter in inches,
P = pressure in pounds per square inch,
S = allowable tensile stress in pounds per square inch,
a = variable quantity.

The quantity a varies with the size of the cylinder and the pressure, and with the conditions under which the cylinder is operated.

For high pressures Lamé's formula is usually used and gives reliable results. This formula, transformed for practical application, is:

$$t = r\left[\sqrt{\frac{S+P}{S-P}} - 1\right] \qquad (15)$$

where t = thickness in inches,
S = allowable tensile stress in pounds per square inch,
P = working pressure in pounds per square inch,
r = internal radius.

This formula is arrived at theoretically and expresses the exact relations between the tensile stress and the working pressure of an elastic material, with the exception that it does not take the lateral contraction of the material under stress into consideration; this can be omitted for practical purposes, since the variation of the quality of the material, unsound castings, and conditions of service, more than counterbalance the gain by considering the lateral contraction.

For those that care to note the difference between Lamé's formula and the one considering lateral contraction, the latter, for cast iron and steel, using the same notation as before, is given below.

$$\text{For cast iron } t = r\left[\sqrt{\frac{4\,S + P}{4\,S - 4\,P}} - 1\right] \qquad (16)$$

$$\text{For steel } t = r\left[\sqrt{\frac{3\,S + P}{3\,S - 4\,P}} - 1\right] \qquad (17)$$

Whereas Lamé's formula is the same for any material, the latter formula varies with the material, since the lateral contraction varies. This contraction is about 1/4 for cast iron and 1/3 for steel.

For pressures ordinarily used in hydraulic work Formulas (16) and (17) give a thinner cylinder than Formula (15); but for very high pressures, such as occur in guns and sometimes in intensifiers, Formulas (16) and (17) give thicker cylinders than Formula (15). Unless one is positive of a high-grade material and sound castings, cast iron should not be used on pressures over 2,000 pounds per square inch.

Formulas (15), (16) and (17) are deduced on the supposition that the inner laminae of a cylinder rupture first, and the moment rupture occurs, the stress on the material is increased, due to the diameter being increased by the starting rupture, and the rupture continues to the outer lamina, or, commonly speaking, the cylinder is "burst." Accordingly, the formulas give such a thickness that the pressure on the inner lamina does not exceed the allowable tensile stress, provided the assumed working pressure is not exceeded. The pressure on each succeeding lamina varies as the square of its radius.

Since these formulas are deduced from the above assumptions, there must be some limited working pressure for each assumed allowable tensile stress, which, if exceeded, will produce a stress on the inner lamina exceeding this allowable tensile stress, even if the cylinder were made infinitely thick. We will now inspect Lamé's formula to find this limited working pressure. By making $P = S$, we have

$$t = r\left[\sqrt{\frac{S + S}{S - S}} - 1\right] \text{ or } t = \infty.$$

Therefore, S is, theoretically, the limit of working pressure; of course, practically it is much lower than this for economical reasons. The writer takes the thickness equal to the radius as a practical limit; if greater thickness is required a higher value for S is used, and consequently a lower factor of safety, or a better grade of material is employed.

In Formula (17) make $P = \frac{3}{4}\,S$, then

$$t = r\left[\sqrt{\frac{3\,S + 0.75\,S}{3\,S - 3\,S}} - 1\right] \text{ or } t = \infty,$$

In this formula the limit of working pressure is $\frac{3}{4}S$, showing that the thickness increases much more rapidly as the pressure increases than in Formula (15). In Formula (16), again, $t = \infty$ for $P = S$.

Another formula frequently used is

$$\frac{t}{r}=\frac{P}{S}\left(1+\frac{P}{S}\right) \tag{18}$$

This is an empirical formula giving results agreeing very closely with those obtained by Formula (15) for limited pressures; it does not give the true relation between S and P, and it is simply a modification of Formula (14), with the quantity a replaced by the factor $\left(1+\frac{P}{S}\right)$ which factor does not vary correctly with increased pressures and stresses. Make $P=S$ and we get

$$\frac{t}{r}=\frac{S}{S}\left(1+\frac{S}{S}\right)=2$$

or the thickness is equal to $2r$; even if we make $P=2S$ we get a thickness apparently sufficient for the pressure; but to find what the actual tensile stress produced will be under such a pressure, we are compelled to resort to Formula (15). Formula (18) is theoretically and practically wrong.

Conditions Governing the Thickness of Cylinders

Having investigated various formulas used for calculating the thickness of cylinders and given a fair average practice, we will now go into the conditions that govern the thickness of cylinders.

1. Two castings taken from the same cast vary widely as to chemical and physical qualities and soundness, depending on what part of the cast each is taken from, conditions of mold, etc. Castings from different casts vary still more.

2. There is a limited thickness below which casting is impossible; this varies with the kind and quality of metal and the skill of the men.

3. Castings handled by unskilled crane men receive very severe shocks and knocks, often producing stresses far in excess of the stress produced in service.

4. In hydraulic systems, the cylinders are subjected to shocks, the magnitude of which depends largely on the design of the system, the service for which the cylinder is used, and the construction and method of operation of the valves.

As far as the variation of the chemical and physical properties are concerned, that is taken care of by the factor of safety. The soundness of the casting is taken care of by allowing an additional amount of metal; this varies with the kind of material, being more for cast iron than for brass, for instance. The amount to be added increases in a certain ratio as the diameter increases, and decreases in a certain ratio as the pressure increases. The increasing pressure requires more body to the metal; therefore, the casting is sounder and less metal need be added. In fact, for pressures above a certain limit this addition of metal can be omitted altogether.

The amount of the addition depends on the quality of the metal and the allowable tensile stress, and should be proportioned accordingly by the designer. With a good quality of metal, the castings can be made thinner, and yet be sound. With a higher allowable tensile stress, the castings are thinner for a given pressure than with a lower, and consequently more metal must be added to make a sound and reliable casting. The limit of thickness below which casting is impossible varies with the quality of metal used, and should be decided by the designer's experience and judgment.

From the conditions enumerated, the writer has deduced a formula, conforming with theory and practice, which can be used for any working pressure high or low and any allowable tensile stress. For the primary thickness for the pressure, Formula (15) is used. Then add two quantities, one increasing as the diameter increases, and one decreasing as the diameter increases (but not in the same ratio as the first quantity), and both decreasing as the pressure per square inch increases. Following is the formula:

$$t = r\left[\sqrt{\frac{S+P}{S-P}} - 1\right] + \frac{S-P}{S}(0.452 - 0.0061\,D) + \left(\frac{S-P}{S+P}\right)^{5} 0.023\,D \qquad (19)$$

The notation is the same as previously given.

Now let us inspect this formula: Make $P = S$ and we get $t = \infty + 0 + 0$, which is theoretically correct. Now let us make $P = 0$ and we get $t = 0 + (0.452 - 0.0061\,D) + 0.023\,D$, which is the minimum thickness. For a two-inch cylinder this would be $t = 0.4398 + 0.046 = 0.4858$ inch, and for a thirty-inch cylinder, $t = 0.269 + 0.69 = 0.959$ inch, or for a sixty-inch cylinder, $t = 0.086 + 1.38 = 1.466$ inch. These thicknesses are within the limits of possibility of casting, and the formula is, therefore, correct from a practical standpoint.

Diagram for Calculating Thick Cylinders

From the diagram, Fig. 8, the thickness of cylinders can be taken directly for any working pressure up to 5,600 pounds per square inch, and for the commonly used fiber stresses.

The line AB is the base line on which the fiber stress curves are constructed. A 32-inch dameter cylinder was the maximum considered in plotting the curves, but the diagram can be made to read up to 40 inches diameter by extending the diagonals, reference from the fiber stress curves always being made to the base line AB. By letting the diagonals encroach on the fiber stress chart, the limit will be the full extent of the chart; the 5,600 line or the maximum diameter of cylinder would thus be 96 inches diameter. The formula is developed for a maximum diameter of cylinder of 74 inches, above which the second turn of the right-hand member becomes negative.

The location of the fiber stress curves with respect to each other is proportional to the respective fiber stress values measured along the ordinates. For if $S = 7{,}000$ pounds per square inch is required, divide a number of intervening ordinates between the 6,000- and 8,000-pound curves in half, and draw a smooth curve through the points thus

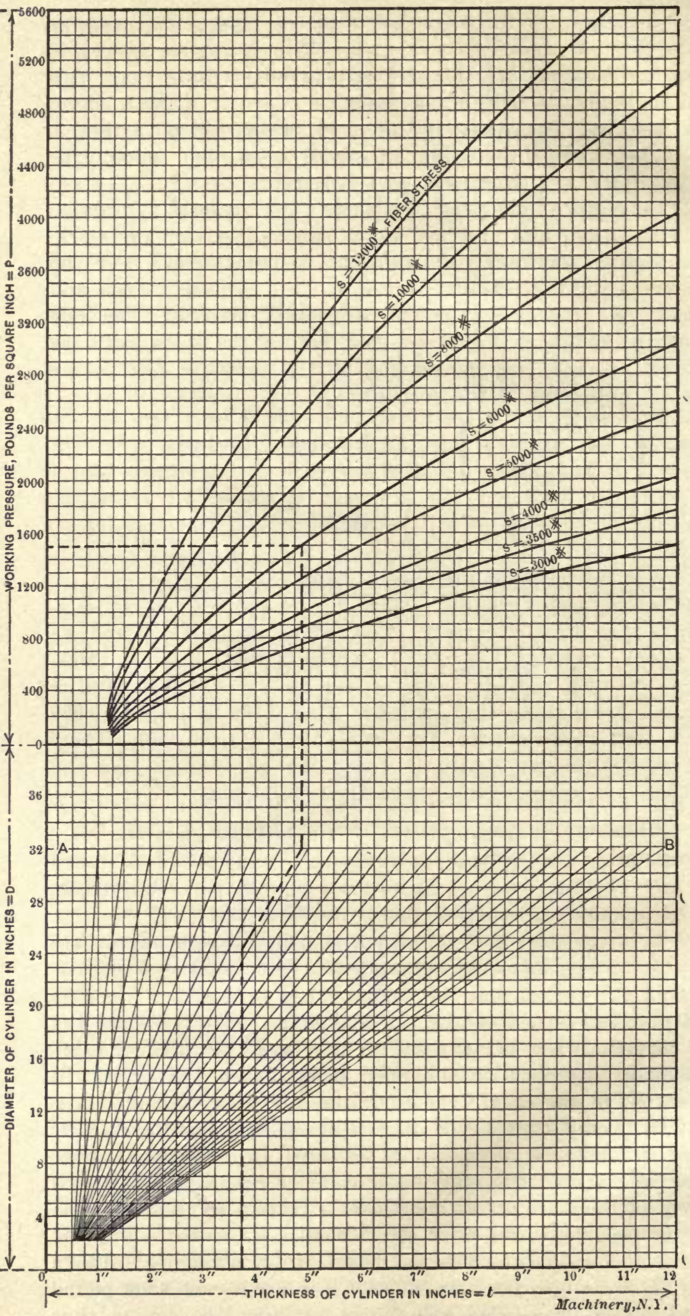

Fig. 8. Diagram for Calculating Thick Cylinders

Formula on which diagram is based:

$$t = r\left[\sqrt{\frac{S+P}{S-P}}-1\right]+\frac{S-P}{S}(0.452-0.0061\,D)+\left(\frac{S-P}{S+P}\right)^5 0.023\,D$$

in which t = thickness of cylinder in inches.
S = allowable fiber stress in pounds per square inch.
P = working pressure in pounds per square inch.
r = internal radius of cylinder in inches.
D = internal diameter of cylinder in inches.

Example of use of diagram: Required, thickness of cylinder, $D = 24$ inches, $P = 1{,}500$ pounds, $S = 6{,}000$ pounds.

Follow horizontal line from $P = 1{,}500$ to 6,000-pound curve; then follow vertical line down to base-line AB; then diagonal line until opposite 24-inch diameter; then vertical line to bottom scale, where the thickness ($= 3\frac{3}{4}$ inches)) is read off.

located. If $S = 6{,}500$ pounds is required, the points are located one-quarter of the length of the intervening ordinates above the 6,000-pound curve. Therefore, any number of curves can be plotted with little trouble.

For intermittent stresses, such as for cylinders for steam and hydraulic work, $S = 3{,}000$ pounds for cast iron, $S = 5{,}000$ pounds for ordinary brass, and $S = 10{,}000$ pounds for steel castings is ordinarily used by the writer.

For steady or gradually applied stresses, such as pipe line fittings, cast pipes, pneumatic cylinders, etc., the stresses should be: for cast iron, $S = 3{,}500$ to 4,000 pounds, for brass, $S = 6{,}000$ to 7,000 pounds, and for steel castings, $S = 12{,}000$ pounds per square inch.

If the cylinder is turned on the outside and bored, the thickness given in the chart is too high for working pressures up to 500 pounds, and the thickness can be decreased by the following amounts with safety. Let T be the thickness required and let t be the thickness taken from the diagram, then

$$T = t - \frac{500 - P}{500}(0.31 + 0.0146\,D),$$

in which $D =$ diameter of cylinder. It will be seen that for 500 pounds $T = t$.

For pressures of 2,000 pounds and over, cast iron should not be used, especially if subjected, additionally, to bending and tensile stresses due to external forces, as the factor of safety becomes too low, and the thickness prohibitive; even when an extra good quality of cast iron is used, such as gun iron, 2,000 pounds is about the safe limit, because it is not possible, in most cases, to determine the maximum pressure due to shocks, etc. Even if the pressure due to shocks comes within a reasonable limit, the cast iron will not last long under repeated shocks.

In low pressure cylinders, the thickness of metal is much greater than the working pressure requires, but must be such to obtain a good sound casting, and the actual pressure that could be put on such a cylinder without exceeding the allowable tensile stress of the material can be found by the following formula:

$$p = S\frac{R^2 - r^2}{R^2 + r^2} \qquad (20)$$

Where $R =$ the outer radius, the remainder of the notation being the same as before.

To find the tensile stress that a given pressure produces, simply transpose the above formula and solve for S; thus,

$$S = p\frac{R^2 + r^2}{R^2 - r^2} \qquad (21)$$

The thickness obtained by the Formula (19) is the true thickness of the cylinder rough or finished. If the plunger works by displacement, as it generally does in hydraulic work, or with non-compressible fluids, t is the rough thickness. If the cylinder is finished, t is the finished thickness. If the cylinder is to be rebored, t must be figured for the rebored cylinder, and the amount allowed for reboring must be added on the inside, even if a steel tube is to be forced into the rebored cylinder to obtain the original diameter. If the cylinder is subjected to shocks, this must be allowed for. In hydraulic work the shocks can usually be calculated approximately; not necessarily what the effect of the shocks actually will be, but the maximum effect under working conditions. In well designed piping systems the effect of shocks in a high pressure system is, contrary to general opinions, less than in a low pressure system for the same work.

Calculate the thickness for the static pressure, and investigate this thickness for tensile stress produced by the possible maximum shock

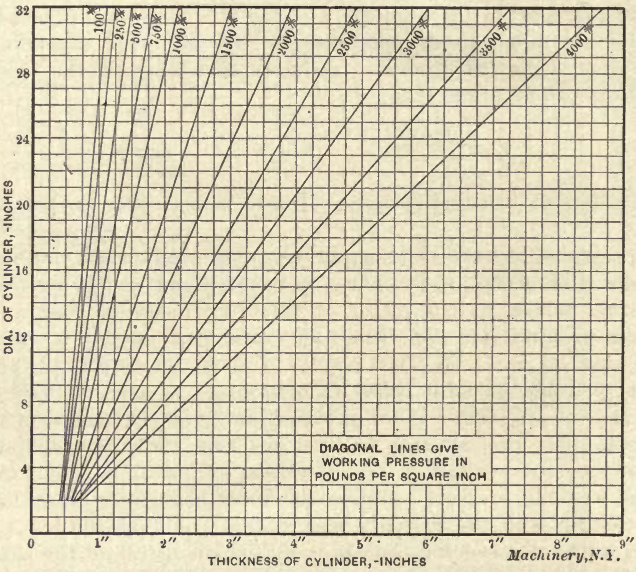

Fig. 9. Diagram for Cylinders with 10,000 Pounds per Square Inch Fiber Stress

under working conditions; if the stress comes within reasonable limits the cylinder is satisfactory. For cast iron the maximum tensile stress due to shock should not exceed 4,000 pounds when often repeated, or 4,500 pounds when rarely repeated. For brass 6,000 to 7,000 pounds, and for steel, 15,000 to 17,000 pounds are average values.

In case of hydraulic test pumps, especially as used for testing pipes, where the pipe is first filled with low pressure water before the test pressure is applied, no matter how suddenly the pressure is applied, the stress in the material cannot rise above that due to double the working pressure, since the water is not in motion, or inappreciably so. But in cylinders operating plungers, a maximum stress many times greater than the initial static pressure may result owing to the inertia of the moving water suddenly brought to rest. If the cylinder also acts as a support, the thickness need not be increased, even if the compressive stress is nearly equal to the allowable tensile stress, a case found in hydraulic accumulators, where the plunger remains stationary, and the cylinder carries the balancing weight and resists internal bursting pressure at the same time. Yet, mathematically, the square root of the sum of the squares of the compressive stress and the tensile stress due to the weight and working pressure should not exceed the allowable tensile stress of the material. If the cylinder supports a weight producing a tensile stress, additional metal must be provided to resist this stress, exclusive of that which resists internal bursting pressures. This additional metal may be in the form of ribs, provided the thickness of the ribs is equal to the thickness of the cylinder, so as to prevent stresses due to unequal cooling or contraction. If the cylinder is subjected to bending, an additional amount of metal must be provided, the moment of inertia of which, about an axis through the center of the cylinder, is sufficient to resist the bending stress.

In addition to the diagram, Fig. 8, a diagram for 10,000 pounds fiber stress only is given in Fig. 9, showing the plotting of the curves and how the thickness increases with the working pressure. It also shows plainly that the Formula (19) deduced is a straight line equation, and gives the reader a better idea of the ratio of increase in thickness than the general diagram, Fig. 8.

CHAPTER VI

BURSTING STRENGTH OF CAST IRON CYLINDERS

Some years ago the writer reported to the American Society of Mechanical Engineers some experiments on the bursting strength of cast iron cylinders. In the same report was developed a formula for the thickness of such cylinders which assumed the following form:

$$t = \frac{p\,d}{4\,S} + \sqrt{\frac{cpd^2}{S} + \frac{p^2d^2}{16\,S^2}}$$

where

t = thickness of shell in inches,
d = internal diameter in inches,
p = internal pressure in pounds per square inch,
S = tensile strength of metal in pounds per square inch.

The first term of the square root is in the nature of an allowance for bending or distortion of the shell from some lack of uniformity

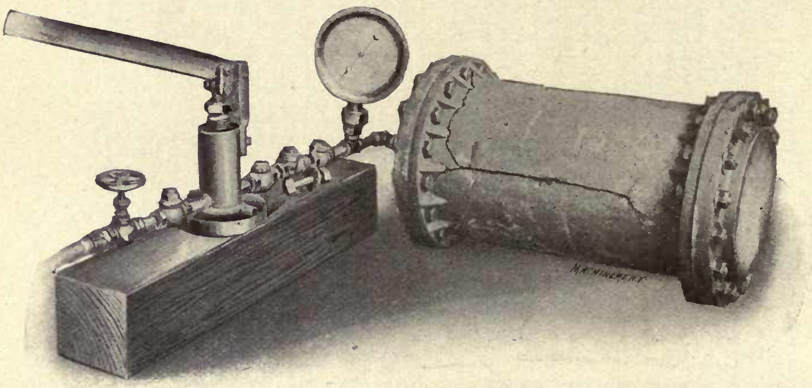

Fig. 10

in thickness or in strength, the constant, c, being determined by experiment.

If $c = 0$, the equation reduces to

$$t = \frac{p\,d}{2\,S}$$

the usual formula for thin shells.

An examination of several engine cylinders of different makes has shown values of c varying from 0.03 to 0.10 with an average value of 0.06. Experiments on nine different cylinders varying in diameter from 6 to 12 inches gave fairly uniform values for c with an average of

$$c = 0.05.$$

The metal of the cylinders was an unusually close-grained, tough cast iron, having a tensile strength of 24,000 pounds per square inch. The tensile stress on the shell as calculated by the formula

$$S = \frac{p\,d}{2\,t}$$

averaged about one-third of this, showing the inapplicability of such a formula to cast iron shells.

In 1903-04 Messrs. A. H. Austin and R. A. Brown made another series of experiments of this character in the laboratories of the Case School

Fig. 11

of Applied Science, Cleveland, Ohio, which throws additional light on the problem.

The experimenters used the apparatus in Fig. 10, which shows the pump, the water pipe and check valves, the pressure gage, and the cylinder in position for a test. Four cylinders were tested to rupture with water pressure, each cylinder having a length of 20 inches, an internal diameter of 10⅛ inches and a thickness of shell of ¾ inch.

The flanges were of the same thickness as the shell and were reinforced by sixteen triangular brackets, as may be seen in Fig. 10. The covers were held to the flanges by sixteen soft steel bolts, ⅝ inch in diameter, having a tensile strength of 80,000 pounds per square inch. The heads of the bolts were cut off on one side so as to bring the bolt holes close to the shell and avoid as much as possible the bending moment on the flanges. Gaskets of straw board soaked in linseed oil and inserted in shallow counterbores were used to prevent leakage. The inside surface of the shell was coated with paraffin for the same reason. Former experiments had shown that water under high pressure would find its way through very minute orifices.

The cylinder heads first used were $1\frac{1}{8}$ inch thick and reinforced by 8 radial ribs on the outside. These proved unsatisfactory, the first head breaking at 650 pounds per square inch and the second at 850 pounds. The ribs, being on the outside, were put in tension by the

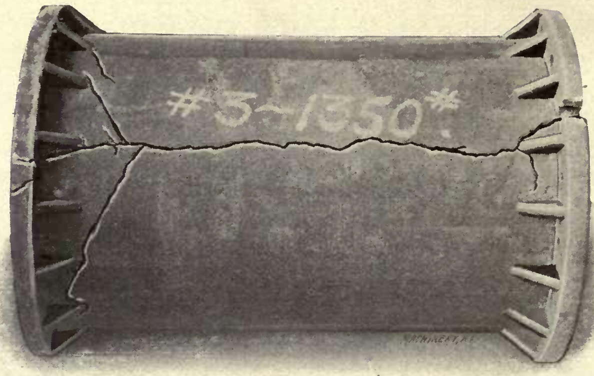

Fig. 12

Fig. 13

buckling of the head and had no value. As it was impracticable to put ribs on the inside, the head was thickened to $2\frac{1}{8}$ inches at the center, as shown in Fig. 10, when no more trouble was experienced. The four accompanying illustrations show the four cylinders after rupture and the pressure per square inch at the instant of breaking.

It will be noticed that the fractures are all longitudinal, there being but little of the tearing of the shell under the flange, which has been a marked feature of other experiments. It is evident that the brackets have served the purpose for which they were made in a way that no mere thickening of the flange could do. The metal used for these cylinders was a soft, gray cast iron having a rather low tensile strength. In fact, pieces cut from the shell of the cylinders and broken in the testing machine had an average value for S of only 14,000 pounds. As tension specimens of cast iron usually show less than the real strength on account of bending, the actual strength of the iron may be slightly more than this. The average cross-breaking strength of samples from the shell was only 30,000, which is also low.

The following table shows in detail the dimensions and test pressures of the various cylinders. As before stated, the cylinders were all of the same diameter and length, 10⅛ by 20 inches.

BURSTING PRESSURE OF CAST IRON CYLINDERS

No.	Thickness.			Bursting Pressure.	Value of c.	$S=\frac{pd}{2t}$
	Max.	Min.	Ave.			
1	.775	.757	.766	1350	.0213	9040
2	.783	.697	.740	1400	.0152	10200
3	.740	.703	.721	1350	.0126	9735
4	.770	.670	.720	1200	.0177	9080

Average value of $c \doteq .0167$.

The average value of c is shown by this table to be only one-third of that for cylinders with unsupported flanges. The values for S in the last column give the stress as calculated by the formula for thin shells, and show that the stress due to bending cannot be neglected even with the reinforced flanges. This may be more clearly shown by solving for S in the formula given at the beginning of the chapter.

$$S = \frac{p\,d}{2\,t} + \frac{cpd^2}{t^2}$$

where the first term of the second member gives the stress due to direct tension, and the second term, the stress due to bending. Assuming the average value $S = 14{,}000$ as determined by the testing machine, we have for the four cylinders:

No.	S	$\frac{pd}{2t}$	$\frac{cpd^2}{t^2}$
1	14,000	9,040	4,960
2	14,000	10,200	3,800
3	14,000	9,735	4,265
4	14,000	9,080	4,920

The "accidental" stress, as it may be called, is seen to be about one-third of the whole. The fractures in all of the cylinders were longitudinal, beginning at some weak spot near the center and extending either way, usually branching to two or more bolt holes at the flanges.

In this connection it may be of interest to notice a test recently made in the laboratories of the Case School of Applied Science of a gasoline engine cylinder for a Peerless motor car. This cylinder broke around a circumference just above the lower flange when subjected to a hydraulic pressure of 1,600 pounds per square inch. The cylinder had an internal diameter of 4.25 inches and a shell thickness of 5-16 inch. The flange was 9-16 inch thick. The fracture showed a clean, close-grained iron. Assuming a tensile strength of 18,000 pounds per square inch and substituting values, we have $c = 0.024$.

The conclusions to be derived from these experiments are:

First, that when the cylinder flanges are unsupported, the initial fracture will be circumferential and close to the flange at a pressure very much less than that determined by the formula:

$$p = \frac{2\,t\,S}{d}.$$

Second, that when the flanges are sufficiently braced to insure longitudinal fracture, a considerable allowance must be made for bending and other accidental stresses.