25 CENTS

ARITHMETIC
SIMPLIFIED

WHOLE NUMBERS—COMMON FRACTIONS
DECIMALS—PROPORTION—USE OF FORMULAS

BY ERIK OBERG

DECIMAL EQUIVALENTS OF FRACTIONS OF AN INCH

1/64	0.015 625	11/32.	0.343 75	43/64	0.671 875
1/32.	0.031 25	23/64	0.359 375	11/16..	0.687 5
3/64	0.046 875	3/8...	0.375	45/64	0.703 125
1/16..	0.062 5	25/64	0.390 625	23/32.	0.718 75
5/64	0.078 125	13/32.	0.406 25	47/64	0.734 375
3/32.	0.093 75	27/64	0.421 875	3/4....	0.750
7/64	0.109 375	7/16..	0.437 5	49/64	0.765 625
1/8...	0.125	29/64	0.453 125	25/32.	0.781 25
9/64	0.140 625	15/32.	0.468 75	51/64	0.796 875
5/32.	0.156 25	31/64	0.484 375	13/16..	0.812 5
11/64	0.171 875	1/2....	0.500	53/64	0.828 125
3/16..	0.187 5	33/64	0.515 625	27/32.	0.843 75
13/64	0.203 125	17/32.	0.531 25	55/64	0.859 375
7/32.	0.218 75	35/64	0.546 875	7/8...	0.875
15/64	0.234 375	9/16..	0.562 5	57/64	0.890 625
1/4....	0.250	37/64	0.578 125	29/32.	0.906 25
17/64	0.265 625	19/32.	0.593 75	59/64	0.921 875
9/32.	0.281 25	39/64	0.609 375	15/16..	0.937 5
19/64	0.296 875	5/8...	0.625	61/64	0.953 125
5/16..	0.312 5	41/64	0.640 625	31/32.	0.968 75
21/64	0.328 125	21/32.	0.656 25	63/64	0.984 375

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NUMBER 137

ARITHMETIC SIMPLIFIED

By ERIK OBERG

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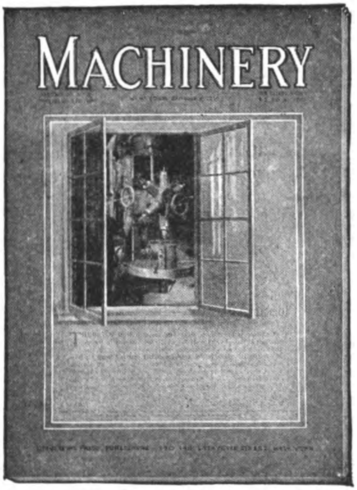


CHAPTER I

WHOLE NUMBERS.

Arithmetic is the art of figuring or calculating with numbers. The fundamental processes of arithmetic are addition, subtraction, multiplication, and division.

A number is a unit or a collection of units. The word unit means *one* or a *single object* or *thing.* The figure 1 is the unit of numbers; one inch is a unit of English length measurement; one pound is a unit of weight, etc.

The Reading of Numbers

The different places for the figures in a number are named as shown in the tabulated arrangement below.

Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens of Units.	Units.
3	7	6,	5	0	3,	2	7	6,	1	1	2

This number would be read three-hundred seventy-six billion, five-hundred three million, two-hundred seventy-six thousand, one-hundred twelve; in the same way 305,636,312 would be read three-hundred five million, six-hundred thirty-six thousand, three hundred twelve.

By referring to the tabulated arrangement above, where the names for the different places in the scale of numbers are given, it will be seen that each place to the left represents a value ten times greater than the next place to the right; thus 10 units make 1 ten, 10 tens make 1 hundred, 10 hundreds make 1 thousand, etc.

When writing a number with figures, it is usual to write the figures in groups of three each, beginning to point off from the right-hand side and placing a comma between each group, as, for instance:

52,752,126,612.

It will be noticed that the last group to the left contains only two figures. Of course, this group will contain one, two, or three figures, according to how many figures there are left in the number when groups of three have been pointed off from the right.

Addition of Whole Numbers

Addition is the process of finding the sum of two or more numbers; thus, when we add 3 and 6, we get 9 as the sum of these two numbers, and we have performed a simple example of addition. In order to make it unnecessary to say in every case that one number is to be added to another, the sign + (plus) is placed between the numbers. The sum of certain given numbers equals a certain other number, and the sign = (equals) is used to express this; thus

$$6 + 3 = 9 \text{ (six plus three equals nine).}$$

When two or more numbers are to be added, instead of placing them in a single line as above, they are usually placed directly under each other in such a manner that the units come under the units, the tens under the tens, etc., thus giving a column for units, another column for tens, a third for hundreds, and so forth. When whole numbers are written this way, all the figures in the right-hand column form an unbroken or complete line. If, for instance, we are to add the numbers 166, 5283, and 27, we would write them in this manner:

```
             11
            166
          5,283
             27
          -----
Sum ........5,476
```

The sum of these numbers is 5,476, and this sum is arrived at as follows: When the numbers have been placed as just shown, begin at the top of the right-hand column of figures and add all the figures in that column; thus 6 + 3 + 7 = 16. Place the figure 6, which is the right-hand figure in the result or sum, directly under the column of figures added, and place the figure 1, which is the left-hand figure in the sum 16, over the next column of figures and add it to the figures in that column; thus, 1 + 6 + 8 + 2 = 17. The figure 7 in this sum is placed directly under the column now added and the figure 1 is "carried over" to the top of the third column. Now this figure 1 is added to the figures in the third column; thus, 1 + 1 + 2 = 4. In the last column we have only one figure which is then simply brought down into the sum to the place directly beneath the place it occupies above.

Another example with more figures is given below.

```
Figures carried over—12112
                    316206
                     53612
                      9487
                     86679
                    ------
Sum ................465984
```

Rule for Addition of Whole Numbers

Place the numbers to be added under each other so that the units form a continuous column at the right-hand side; then, beginning with the right-hand column, add the figures in each column separately and write the sum of the column, provided it consists of only one figure, directly under the column added; should, however, the sum of any one column consist of more than one figure, then place the right-hand figure of the sum under the column added, and carry over the left-hand figure or figures to the top of the next column to be added, and add the carried over figure to this column.

To Prove that the Sum is Correct

The following example illustrates a method of proving that the sum obtained by adding certain numbers is correct.

Add the figures horizontally in each line, and continue to add the figures in the sum obtained, if it consists of more than one figure, until a single figure is obtained. In the example below it will be seen that the

The numbers to be added.	First sum of the figures.	Second sum of the figures.
356	14	5
705	12	3
26	8	8
1087		16

figures in the first number or line added, $3 + 5 + 6 = 14$; and then the figures in 14 are added, $1 + 4 = 5$; this figure is now placed in a column to the right; in the same way, $7 + 5 = 12$, and $1 + 2 = 3$, which is placed in the same column; for the third number we have $2 + 6 = 8$; place this in the same column and add the figures in this column, $5 + 3 + 8 = 16$; now the figures 1 and 6 in 16 added give a final sum 7. If the sum 1,087 in the addition is correct, the figures in this sum when added until the result is a single figure should also equal 7, or should be the same as the sum obtained by adding the figures horizontally in all the numbers originally added. In this case we see that $1 + 0 + 8 + 7 = 16$ and $1 + 6 = 7$; consequently the sum obtained is correct.

Examples for Practice

Find the following sums:

Example (1). 2,008 + 1,406 + 707 + 310 + 60.

Answer: 4,491.

Example (2). 25,651 + 7,725 + 5,867 + 32,403.

Answer: 71,646.

Example (3). 7,861 + 869 + 4,496 + 78.

Answer: 13,304.

Example (4). 235 + 4,571 + 53 + 687 + 10.

Answer: 5,556.

Example (5). 9,960 + 16,775 + 8,742.

Answer: 35,477.

Example (6). 2,020 + 1,709 + 100 + 1,653.

Answer: 5,482.

Subtraction of Whole Numbers

Subtraction is the process of finding the difference between two numbers, or in other words, how much one given number is greater than another number. The greater of the two numbers is called *minuend*, the smaller *subtrahend*, and the number obtained when the subtrahend has been taken from the minuend is called the *difference* or *remainder*. The process of subtraction is indicated by the sign — (minus) placed between the two given numbers, thus:

$$9 - 3 = 6 \text{ (nine minus three equals six).}$$

This means that when 3 is taken or subtracted from 9, 6 remains. In this case 9 is the minuend, 3 is the subtrahend, and 6 the difference or remainder. When one number is to be subtracted from another, instead of placing them in a single line with the minus sign between them as shown above, the minuend is usually placed above and the subtrahend below in such a manner that the right-hand figure in the subtrahend is exactly under the right-hand figure in the minuend; thus 6,835 — 3,614 will be written

minuend	6835
subtrahend	3614
difference or remainder..................	3221

The remainder is found by beginning at the right-hand side and subtracting or taking the value of each figure in the subtrahend from the figure above it in the minuend, and writing down the result in the remainder directly under the figures subtracted. Thus, 4 from 5 is 1; 1 from 3 is 2; 6 from 8 is 2; and 3 from 6 is 3. The whole remainder then is 3,221, as shown.

If the example given were 6,854 — 721 it would be written

minuend	6854
subtrahend	721
difference or remainder..................	6133

In this case the numbers in the subtrahend are subtracted from the numbers in the minuend in the same way as before. It will be noted, however, that there is no figure in the fourth or thousands place in the subtrahend to subtract from the figure 6 in the minuend; in that case the figure 6 is simply brought directly down to the remainder as there is nothing to subtract from it.

Suppose an example 853,745 — 538,263 was given.

	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
minuend	8	5	3,	7	4	5
subtrahend	5	3	8,	2	6	3
difference or remainder.....	3	1	5,	4	8	2

We commence by subtracting 3 from 5, leaving 2. In the second or tens column we find, however, that the figure 6 cannot be subtracted from 4, because 6 is a larger number than 4. We must then borrow from the nearest column to the left, or in this case from the hundreds column. One hundred in that column makes 10 tens in the column where 6 is to be subtracted from 4. Thus if we borrow 1 from 7 in the third or hundreds column we will have 10 in the second or tens column, and this added to the 4 already given in the tens column gives us 14. Now we subtract 6 from 14 giving the remainder 8. In the third column we have borrowed 1 from 7 and consequently have only 6 left, and then taking 2 from 6 leaves us 4. In the fourth or thousands place we cannot subtract 8 from 3 but must borrow one from the tens of thousands place, which gives us 10 in the thousands place, to which we add the 3 already given; then 8 from 13 equals 5. In the tens of thousands place 1 has already been borrowed from 5 and there is only 4 left in the minuend; then 3 from 4 leaves 1; finally 5 from 8 leaves 3. The whole remainder then is 315,482.

Suppose in another example that it is required to subtract 2,987 from 8,132.

minuend	8132
subtrahend	2987
difference or remainder	5145

In this case it will be necessary for us to borrow 1 ten from the second column in order to be able to subtract the units in the subtrahend from the units in the minuend. Borrowing 1 ten from the 3 tens given, gives us 10 units which added to the 2 units already in the right-hand or unit column gives us 12 units in all, and $12 - 7 = 5$. In the second column we have now only 2 tens left, and must therefore borrow from the third or hundreds column; this gives us 12 tens in all in the second column, and 8 from 12 is 4. In the third column we have now 0 left in the minuend and must borrow 1 from the fourth or thousands column which gives us 10 in the third or hundreds column. We then get $10 - 9 = 1$. Finally in the fourth column to the left there is now only 7 left in the minuend, we having already borrowed 1, and we have then $7 - 2 = 5$.

We will now suppose that we have an example, 1,000 — 356.

minuend	1000
subtrahend	356
difference or remainder	644

In this case it will be seen that we cannot take 6 from 0 and therefore we must borrow. As the tens and hundreds are also 0, we must borrow from the thousands. Borrowing 1 from the thousands gives us 10 in the third or hundreds column; then we borrow 1 from this hundreds column giving us ten in the second or tens column; finally we borrow 1 from this column which gives us 10 in the first or units column, leaving 9 in the second and third columns of the minuend. The example will be figured as follows: 6 from 10 leaves 4; 5 from 9 leaves 4; 3 from 9 leaves 6. As we borrowed 1 from the fourth or thousands column, there is nothing left in the minuend to bring down, and the difference or remainder is simply 644.

Rule for Subtraction

Write the smaller number under the greater so that units come under units, tens under tens, etc. Begin at the right-hand side and take each figure in the lower number from the figure above it, placing the remainder directly below the figures subtracted. If any figure in the subtrahend is greater than the figure above in the minuend, borrow 10 from the next figure to the left in the minuend, and add to the figure in the minuend from which we are to subtract. Borrowing 10 from the next figure to the left makes this figure one less than given, because the value of each place to the left is ten times greater than the value of the next place to the right.

To Prove that the Remainder Obtained is Correct

There are two ways for proving that the remainder obtained when subtracting one number from another is correct. One method is to add the remainder to the subtrahend, in which case the result should equal the minuend. This is the simplest, safest and best method. Another method is similar to the one already explained in addition. Suppose that an example was given as follows:

minuend	356	14	5
subtrahend	145	10	1
difference or remainder	211		4

The figures in the minuend, subtrahend, and remainder are added horizontally until a single figure is obtained; the difference between the sum of the figures in the minuend and the subtrahend should equal the sum of the figures in the remainder. In the example above $3 + 5 + 6 = 14$, and $1 + 4 = 5$; this is the sum of the figures in the minuend; $1 + 4 + 5 = 10$ and $1 + 0 = 1$; this is the sum of the figures in the subtrahend. The difference between 5 and 1 equals 4. If the subtraction is carried out correctly, the sum of the figures of the remainder added horizontally should also equal 4. It will be seen that by adding the figures $2 + 1 + 1$ we obtain 4, and consequently the result obtained in our example is correct. If the sum of the figures of the minuend is smaller than, or equal to, the sum of the figures in the subtrahend, add 9 to the sum of the figures in the minuend before subtracting the other sum from it.

Examples for Practice

Example (1). 6,856 — 1,432. Answer: 5,424.
Example (2). 5,673 — 2,665. Answer: 3,008.
Example (3). 205,083 — 195,736. Answer: 9,347.
Example (4). 61,287 — 50,089. Answer: 11,198.
Example (5). 61,037 — 8,908. Answer: 52,129.
Example (6). 2,002,001 — 1,909,199. Answer: 92,802.

Multiplication of Whole Numbers

Multiplication is the process of taking a given number a certain number of times. The sign for multiplication is ×, which is read *multiplied by* or *times*. If we take 3 four times we will get 12 as a result and we would write it

$$4 \times 3 = 12 \text{ (four times three equals twelve).}$$

The first number (4) given in the example above is called the *multiplicand*, the second number (3), by which the first is multiplied, the

MULTIPLICATION TABLE

0 × 1 = 0	0 × 2 = 0	0 × 3 = 0
1 × 1 = 1	1 × 2 = 2	1 × 3 = 3
2 × 1 = 2	2 × 2 = 4	2 × 3 = 6
3 × 1 = 3	3 × 2 = 6	3 × 3 = 9
4 × 1 = 4	4 × 2 = 8	4 × 3 = 12
5 × 1 = 5	5 × 2 = 10	5 × 3 = 15
6 × 1 = 6	6 × 2 = 12	6 × 3 = 18
7 × 1 = 7	7 × 2 = 14	7 × 3 = 21
8 × 1 = 8	8 × 2 = 16	8 × 3 = 24
9 × 1 = 9	9 × 2 = 18	9 × 3 = 27
10 × 1 = 10	10 × 2 = 20	10 × 3 = 30
0 × 4 = 0	0 × 5 = 0	0 × 6 = 0
1 × 4 = 4	1 × 5 = 5	1 × 6 = 6
2 × 4 = 8	2 × 5 = 10	2 × 6 = 12
3 × 4 = 12	3 × 5 = 15	3 × 6 = 18
4 × 4 = 16	4 × 5 = 20	4 × 6 = 24
5 × 4 = 20	5 × 5 = 25	5 × 6 = 30
6 × 4 = 24	6 × 5 = 30	6 × 6 = 36
7 × 4 = 28	7 × 5 = 35	7 × 6 = 42
8 × 4 = 32	8 × 5 = 40	8 × 6 = 48
9 × 4 = 36	9 × 5 = 45	9 × 6 = 54
10 × 4 = 40	10 × 5 = 50	10 × 6 = 60
0 × 7 = 0	0 × 8 = 0	0 × 9 = 0
1 × 7 = 7	1 × 8 = 8	1 × 9 = 9
2 × 7 = 14	2 × 8 = 16	2 × 9 = 18
3 × 7 = 21	3 × 8 = 24	3 × 9 = 27
4 × 7 = 28	4 × 8 = 32	4 × 9 = 36
5 × 7 = 35	5 × 8 = 40	5 × 9 = 45
6 × 7 = 42	6 × 8 = 48	6 × 9 = 54
7 × 7 = 49	7 × 8 = 56	7 × 9 = 63
8 × 7 = 56	8 × 8 = 64	8 × 9 = 72
9 × 7 = 63	9 × 8 = 72	9 × 9 = 81
10 × 7 = 70	10 × 8 = 80	10 × 9 = 90

10 × 10 = 100

multiplier, and the result (12), the *product*. Both the multiplicand and the multiplier are commonly called *factors*. The product of any one single figure with any other single figure is given in the accompanying multiplication table which should be committed to memory before advancing any further. When the multiplication table has been thoroughly mastered, proceed with the following examples.

When figuring the examples, write the multiplier under the multiplicand so that units come under units, tens under tens, hundreds under hundreds, etc.

Assume that 231 is to be multiplied by 3. This would be written

multiplicand	231
multiplier	3
product	693

To obtain the product multiply each figure in the multiplicand by the figure in the multiplier, beginning with the right-hand figure, and write the result of each multiplication directly beneath the figure multiplied; thus in the example above we have first $3 \times 1 = 3$, then $3 \times 3 = 9$, and then $3 \times 2 = 6$, giving a total result of 693.

Suppose in another example 361 is to be multiplied by 6.

multiplicand	361
multiplier	6
product	2166

We have first $6 \times 1 = 6$. Then $6 \times 6 = 36$; in this case only the right-hand figure 6 is put down directly under the figure in the multiplicand that has been multiplied, and the left-hand figure 3 is carried over and added to the result of the next multiplication; thus: $6 \times 3 = 18$, and this, with the 3 carried over from the last multiplication added, gives 21, which is then written down, making the total result 2166.

When the multiplier consists of two or more figures the method is as follows. Assume that 3617 is to be multiplied by 2034.

multiplicand	3617	17	8
multiplier	2034		9
separate products	14468		72
	10851		
	0000		
	7234		
product	7356978		

We cannot multiply 3617 by 2034 in one operation, and therefore multiply the figures in 3617 by each figure in 2034 in turn, commencing at the right-hand side. Multiplying 3617 by 4 gives us 14468, which is written down in the usual way. We then multiply 3617 by 3 giving us 10851 as a product. This is written below the product 14468 already obtained, but in such a manner that the right-hand figure in 10851 comes directly under figure 3 in the multiplier by which we have just multiplied. We now multiply 3617 by 0 (naught) which gives us naughts in all the places, because 0 times any number is 0. Finally we multiply 3617 by 2 giving us 7234, which is written as the previous products so that the right-hand figure comes directly under the figure 2 in the multiplier by which we have just multiplied. When the products of the multiplicand with all the figures of the multiplier have thus been obtained, a line is drawn under them and they are added. As there is nothing to add to the right-hand figure 8 it is simply brought down. Then we have $6 + 1 = 7$; $4 + 5 + 0 = 9$; $4 + 8 + 0$

+ 4 = 16, giving us 1 to carry over and add to the next column, where we then have 1 + 1 + 0 + 0 + 3 = 5; 1 + 0 + 2 = 3; the figure 7 is simply brought down, giving a total product of 7,356,978. It will be seen that the third line of the separate products which contains only naughts, has no influence on the final product, and, therefore, when a 0 occurs in the multiplier it can simply be disregarded, remembering, of course, that when multiplying the multiplicand by the next figure in the multiplier, the right-hand figure in the product obtained must be placed directly under that figure in the multiplier by which we have just multiplied.

Rule for Multiplication

Write the multiplier under the multiplicand so that units come under units, tens under tens, hundreds under hundreds, etc.; and, beginning at the right, multiply the multiplicand by each successive figure in the multiplier and place the right-hand figure of the separate products obtained directly below the figure in the multiplier just multiplied by. Then add these separate products to obtain the total product.

To Prove that the Product is Correct

To prove that the product is correct add the figures in the multiplicand and multiplier horizontally until a sum of one figure is obtained for each, in the same way as has been explained in proving addition. Thus in the last example, the figures in the multiplicand are added; 3 + 6 + 1 + 7 = 17; and 1 + 7 = 8; then the figures in the multiplier are added; 2 + 0 + 3 + 4 = 9. Then multiply the sum of the figures in the multiplicand by the sum of the figures of the multiplier, thus: 8 × 9 = 72, and 7 + 2 = 9. The sum of the figures of the total product added until a single figure is obtained should equal 9 if the multiplication has been figured correctly. That this is the case we see because 7 + 3 + 5 + 6 + 9 + 7 + 8 = 45 and 4 + 5 = 9.

Examples for Practice

Example (1). 836 × 84.	Answer:	70,224.
Example (2). 2,845 × 236.	Answer:	671,420.
Example (3). 4,327 × 420.	Answer:	1,817,340.
Example (4). 5,327 × 419.	Answer:	2,232,013.
Example (5). 1,296 × 1,296.	Answer:	1,679,616.
Example (6). 1,844,164 × 1,358.	Answer:	2,504,374,712.
Example (7). 298 × 298 × 298.	Answer:	26,463,592.

Squares, Cubes and Powers of Numbers

The square of a number is simply that number multiplied by itself. Thus the square of 2 is 2 × 2 = 4, and the square of 3 is 3 × 3 = 9; similarly the square of 177 is 177 × 177 = 31,329. The cube of a number is the product obtained if the number itself is repeated as a factor three times; thus: 2 × 2 × 2 = 8 is the cube of 2; in the same way 4 × 4 × 4 = 64 is the cube of 4.

Instead of writing 2×2 for the square of 2, it is often written 2^2, which is read *two square*, and means that 2 is multiplied by 2. In the same way 5^2 means 5×5 and is read *five square*. The cube of 2 can be written 2^3, which means $2 \times 2 \times 2$, and is read *two cube*. In the same way $12^3 = 12 \times 12 \times 12 = 1{,}728$. The expression 12^3 is read *twelve cube*. If we write 18^4, that would mean $18 \times 18 \times 18 \times 18 = 104{,}976$. The expression 18^4 is read *the fourth power of eighteen*. In the same way $21^5 = 21 \times 21 \times 21 \times 21 \times 21 = 4{,}084{,}101$, and is read *the fifth power of twenty-one*. The small figure at the upper right-hand corner of these expressions indicating the "power" (the 3 in 12^3, the 4 in 18^4, the 5 in 21^5) is called *exponent*.

Division of Whole Numbers

Division is the process of finding how many times one number is contained in another given number. The sign for division is ÷ and is read *divided by*. If we want to know how many times we can take 3 out of 12, or in other words, if we want to divide 12 by 3, we write it

$$12 \div 3 = 4 \text{ (twelve divided by three equals four).}$$

Another method for indicating division is to draw a line between the number divided and the number by which we divide, and place the former number over the line and the latter under it, thus

$$\frac{12}{3} = 4 \text{ (twelve divided by three equals four).}$$

The number divided (12) is called the *dividend*, and the number by which we divide (3) is called the *divisor*. The number (4) which shows how many times the divisor is contained in the dividend is called *quotient*. When carrying out the division, the dividend, divisor, and quotient are written relatively to one another in the places indicated below, and lines drawn between them as shown.

```
dividend | divisor
         |--------
         | quotient
```

Suppose it is required to divide 973 by 7. Here 973 is the dividend and 7 the divisor, and we write the example

```
dividend 973 | 7    divisor
         7   |----
         —   | 139  quotient
         27
         21
         —
          63
          63
          —
          00
```

The quotient 139 is obtained as follows: As 7 is contained in the first figure of the dividend once, 1 is the first figure in the quotient. Now multiply the divisor 7 by the figure 1 in the quotient, giving 7 as a product; place this product 7 under the figure 9 in the dividend; then subtract 7 from 9, obtaining 2 as a remainder; bring down the next figure in the dividend, which is 7, and place it to the right of the remainder 2, obtaining 27, which is now to be divided by 7 to obtain the second figure in the quotient; 7 is contained in 27 three times, therefore 3 is the second figure in the quotient; the divisor is multiplied by this figure and the product 21 is placed under 27 and subtracted from it, giving 6 as a remainder. Now bring down and annex to this figure 6, the next figure in the dividend, which is 3, giving us 63 as the number to be divided by 7 in order to obtain the third figure in the quotient; 7 is contained in 63 nine times, and $9 \times 7 = 63$; this product is placed under 63 in the division. As 63 from 63 leaves 0, and as there are no more figures to be brought down from the dividend, the example has been finished, and 139 is the quotient obtained when 973 is divided by 7.

Suppose an example $256 \div 8$ is given.

```
dividend 256 | 8    divisor
          24 |----
          —  | 32   quotient
          16
          16
          —
          00
```

If the divisor is not contained in the first figure of the dividend, two or more figures in the dividend must be used for the first division. In the example above, it will be seen that 8 is not contained in 2, but that it is contained in 25; consequently, we commence by saying 8 is contained in 25 three times, and the figure 3 is placed in the quotient as shown. Then $3 \times 8 = 24$ and this subtracted from 25 gives 1 as a remainder.

The figure 6 is now brought down from the dividend giving us 16 as the number to be divided by 8 to obtain the next figure in the quotient; 8 is contained in 16 two times and 2 is therefore the next figure in the quotient. Then, $2 \times 8 = 16$, leaving no remainder when subtracted from 16. As there are no other figures to bring down from the dividend, the quotient is simply 32.

If the product obtained by multiplying the divisor by that figure of the quotient last obtained should be greater than the number just divided by the divisor, the product cannot be subtracted from this number; this indicates that the figure in the quotient is incorrect and too great. If the remainder obtained when subtracting the product from the dividend above it is larger than the divisor, the figure just obtained in the quotient is incorrect and too small. Thus in the example above, if we should say that 8 is contained in 25 four times, when we multiply 4×8 we would get a product 32. As this is greater than 25 from which it is to be subtracted it indicates that 8 is not contained in 25 as many as four times. On the other hand, suppose we say that 8 is contained in 25 two times; $2 \times 8 = 16$, and 16 subtracted from 25 leaves 9, which is a larger number than the divisor 8; this indicates that 8 is contained in 25 more than two times.

Example: Find how many times 25 is contained in 7575.

dividend	7575	25	divisor
	75	308	quotient
	75		
	75		

If the divisor consists of two or more figures the process is exactly the same as explained above. If the divisor consists of two figures, see how many times it is contained in the two first figures in the dividend; if it consists of three figures find how many times it is contained in the three first figures in the dividend, if of four figures, in the four first figures, etc. Should it be found that the divisor is not contained in an equal number of figures in the dividend, use one more figure of the dividend than the number of figures in the divisor, for the first division. In the example above, 25 is contained three times in the first two figures of the dividend; 3 is therefore the first figure in our quotient. Multiplying the divisor by the figure 3 in the quotient gives us 75, which is placed under 75 in the dividend. Subtracting 75 from 75 gives us no remainder. Then bring down the next figure in the dividend, which is 7. It will be seen that 25 is not contained in 7 and therefore the next figure in the quotient is 0. Now bring down another figure from the dividend and place it directly to the right of the 7 already brought down, and use this number as a dividend for obtaining the next figure in the quotient; 25 is contained in 75 three times, and 3 is therefore the third figure in the quotient; $3 \times 25 = 75$, which leaves no remainder when subtracted from 75. As there are no more figures to bring down from the dividend the quotient obtained when dividing 7575 by 25 is 303.

Example: Divide 5375 by 82.

dividend	5375	82	divisor
	492	$65\frac{45}{82}$	quotient
	455		
	410		
	45		

There are two figures in the divisor, but 82 is not contained in the two first figures in the dividend and therefore we must find how many times it is contained in the three first figures or in 537. We find that 82 is contained six times in 537; therefore we multiply the divisor 82 by 6, obtaining the product 492, which is placed under 537 in the dividend. Subtracting 492 from 537 we obtain 45 as a remainder. Bring down figure 5 from the dividend and place it directly to the right of 45; find how many times 82 is contained in 455. We find that 82 is contained five times in 455 and 5 is therefore our next figure in the quotient. Multiply the divisor by 5, obtaining 410 which is subtracted from 455, leaving the remainder 45. As there are no more figures to bring down from the dividend we have obtained the whole number of times that the divisor is contained in the dividend; but the remainder 45 shows that the divisor is not contained in the dividend an even number of times. In order to indicate how much of a remainder results in our last division, this remainder is written after the whole number (65) of the quotient, with the divisor underneath, as shown in the example.

Rule for Division

Write the divisor to the right of the dividend with a vertical line between the two, and draw a line under the divisor. Then find how many times the divisor is contained in the smallest number of the left-hand figures of the dividend that will contain it, and write the result under the divisor as the first figure of the quotient. Multiply the divisor by this figure and write the product under that part of the dividend which has been used for the first division; then subtract the product from the figures in the dividend over it. Bring down the next figure of the dividend, and annex it to the remainder obtained by the subtraction, using the number thus obtained as a new dividend to be divided by the divisor. In this manner the second figure in the quotient is obtained and the same process of multiplication, subtraction, bringing down the next figure in the dividend, etc., as was followed for the first figure of the quotient is repeated until all the figures in the dividend have been brought down. If the divisor is not contained in the number obtained when one figure has been brought down from the dividend, write a cipher as the next figure in the quotient, and bring down the following figure from the dividend. If there be a remainder after all the figures in the dividend have been brought down and the last division carried out, place the remainder after the quotient with the divisor underneath it, and draw a line between the two.

To Prove that the Quotient Obtained is Correct

There are two methods of proving that the quotient is correct. One is to multiply the quotient by the divisor. The product, if the quotient is correct, will equal the dividend. The other method is similar to the methods used for proving addition, subtraction and multiplication. Add all the figures in the quotient until a sum of a single figure results; do the same with the figures in the divisor. Then multiply the figure obtained from the quotient by the figure obtained from the divisor, and if the product consists of more than one figure, add the figures in the product until the sum is a single figure. If the quotient is correctly obtained, this figure will equal the sum of the figures in the dividend when added until a single figure is obtained.

Example: $131{,}872 \div 317 = 416.$

Adding the figures in the divisor:

$$3 + 1 + 7 = 11;\ 1 + 1 = 2.$$

Adding the figures in the quotient:

$$4 + 1 + 6 = 11;\ 1 + 1 = 2.$$

Then, $2 \times 2 = 4$.
Adding the figures in the dividend:

$$1 + 3 + 1 + 8 + 7 + 2 = 22; \; 2 + 2 = 4.$$

To prove in a case when there is a remainder after all the figures in the dividend have been brought down and the last division carried out, add the sum of the figures in the remainder until a sum of a single figure results. Add this figure to the product of the sums of the divisor and quotient. Then this sum will equal the sum of the figures in the dividend.

Example: $321 \div 18 = 17$, with a remainder of 15.
Then: $1 + 8 = 9$;
$1 + 7 = 8$;
$8 \times 9 = 72$; $7 + 2 = 9$.
Sum of figures in remainder: $1 + 5 = 6$.
$9 + 6 = 15$; $1 + 5 = 6$.
Sum of figures in dividend:
$3 + 2 + 1 = 6$.

Examples for Practice

Example (1).	$12,167 \div 23$.	Answer: 529.
Example (2).	$24,389 \div 841$.	Answer: 29.
Example (3).	$970,299 \div 9,801$.	Answer: 99.
Example (4).	$5,293 \div 82$.	Answer: $64\frac{45}{82}$.
Example (5).	$559,136 \div 101$.	Answer: 5,536.
Example (6).	$970,225 \div 985$.	Answer: 985

Factoring

An *odd number* is a number which is not exactly divisible by 2, as, for instance, 1, 3, 5, 7, 19, 33, etc.

An *even number* is a number which is exactly divisible by 2, as, for instance, 2, 4, 6, 8, 20, 34, etc.

A *prime number* is one which *is not* exactly divisible by any number except itself and one; thus 1, 2, 3, 5, 7, 11, 19, 23, are prime numbers. The factors of a given number are those numbers which when multiplied together will give that number; for instance, 2 and 3 are factors of 6, since $2 \times 3 = 6$; and 3 and 11 are factors of 33, since $3 \times 11 = 33$.

A factor which is a prime number is called a *prime factor*.

Any number which is not a prime number can be divided into factors and is called a *composite* number. Thus 30 is a composite number, since $2 \times 3 \times 5 = 30$ (two times three times five equals thirty).

To divide a composite number into its prime factors, divide it by the smallest prime number, except one, which will exactly divide it. Then divide the quotient thus obtained by the smallest prime number which will exactly divide it, and continue this process until the last quotient is a prime number. To divide 30 into prime factors, therefore, we divide it by 2, which is the smallest prime number which will exactly divide it. This gives us 2 as our first prime factor and leaves us a quotient of 15. By trial, the smallest prime number which will divide 15 is found to be 3, and 3 is therefore our next prime factor, leaving us 5 as a quotient. As this last quotient 5 is a prime number it is the last prime factor of 30.

Example: Find the prime factors of 48.

In working out a problem of this kind it facilitates the work to write down the calculations carried out as shown below:

dividends and quotients	factors
48	2
24	2
12	2
6	2
3	3

As 48 is an even number, it is exactly divisible by 2, and 2 is therefore our first prime factor, and 24 our quotient and also our next dividend. As 24 is also an even number our next factor will be 2, leaving us 12 for the quotient and the third dividend. As 12 is evenly divisible by 2 our next factor is again 2, leaving us 6 for a quotient. Our next factor will also be 2, leaving us 3 for a quotient. As 3 is a prime number it is also one of the prime factors of 48, and is simply brought over into the line of factors. The prime factors of 48 consequently are $2 \times 2 \times 2 \times 2 \times 3$.

Example: Find the prime factors of 210.

dividends and quotients	factors
210	2
105	3
35	5
7	7

As 210 is an even number, it is exactly divisible by 2, and 2 is therefore our first prime factor, leaving 105 as the quotient and the next dividend. The smallest prime number which will exactly divide 105 is 3; 3 is therefore our next prime factor, giving 35 as the quotient and the next dividend; 35 divided by 5 gives us 7, which is itself a prime number, for the quotient, and our prime factors are consequently

$$2 \times 3 \times 5 \times 7 = 210.$$

Examples for Practice

Find the prime factors of:

36. ....Answer: 2 × 2 × 3 × 3.
49. ....Answer: 7 × 7.
625. ....Answer: 5 × 5 × 5 × 5.
1728. ....Answer: 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3.
240. ....Answer: 2 × 2 × 2 × 2 × 3 × 5.
420. ....Answer: 2 × 2 × 3 × 5 × 7.
770. ....Answer: 2 × 5 × 7 × 11.

Least Common Multiple

A *multiple* of any given number is any number which is exactly divisible by the given number; thus 28 is a multiple of 4, and also of 14.

A *common multiple* of two or more numbers is any number which is exactly divisible by each of the given numbers; thus 28 is a common multiple of 2, 4, 7, and 14.

TABLE OF PRIME NUMBERS FROM 1 TO 1000

1	59	139	233	337	439	557	653	769	883
2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	

The *least common multiple* of two or more numbers is the least or smallest number that is exactly divisible by each of the given numbers; thus 48 is the least common multiple of 8, 12, and 16.

A convenient way of carrying out the calculations for finding the least common multiple of a series of numbers is shown below.

Example: Find the least common multiple of 24, 20, and 15.

24, 20, 15	2
12, 10, 15	2
6, 5, 15	2
3, 5, 15	3
1, 5, 5	5
1, 1, 1	

First place the numbers for which the least common multiple is to be found in a line with commas between them, as shown, and draw a vertical line to the right of these numbers and a horizontal line under the numbers. Then divide by the *smallest prime number* that will exactly divide any of the numbers, and place the quotients obtained under the line; the numbers which are not exactly divisible, are simply brought down. Repeat the process with the second row of numbers and place the prime number used as a divisor to the right of the vertical line. When the given numbers have been divided by prime numbers until all the quotients are 1, the column of figures to the right of the vertical line gives the prime factors which should be multiplied together to obtain the least common multiple.

In the example above we see that in the first line of the given numbers, two of them are exactly divisible by 2. We therefore place 2 at the right-hand side as shown, and, dividing 24 and 20 by 2, we place the quotients 12 and 10 under the line. As 15 is not exactly divisible by 2 this number is simply brought down. In the line of numbers thus obtained we see that there are again two numbers divisible by 2, and repeating the process of division, and bringing down number 15, we get a new line of numbers 6, 5, 15. One of the numbers, 6, in this line is again divisible by 2, and therefore we place the third 2 to the right of the vertical line, and dividing by this number get 3 as the first number in the fourth line. In this case both 5 and 15 are brought down, because they are not exactly divisible by the divisor 2. Now we see that in the line of numbers 3, 5, 15, the smallest prime number that will exactly divide any of these is 3; placing 3 to the right of the vertical line and dividing, we get 1 as the quotient under 3 in the following horizontal line; 5 is brought down because it is not exactly divisible by 3, and 15 divided by 3 gives us the quotient 5. Then the next line is divided by 5, the 1 already obtained being brought down, and 1 also being the quotient of the division of 5 by 5. When we find, as now, that all the quotients are 1, the figures to the right of the vertical line are the factors of our least common multiple, and in this case, therefore, the least common multiple equals

$$2 \times 2 \times 2 \times 3 \times 5 = 120.$$

Example: Find the least common multiple of 6, 14, 15, and 33.

Write down the numbers in the same way as in the previous problem and carry out the example as indicated below.

6, 14, 15, 33	2
3, 7, 15, 33	3
1, 7, 5, 11	5
1, 7, 1, 11	7
1, 1, 1, 11	11
1, 1, 1, 1	

Always remember to divide each line of figures by the smallest prime number which will exactly divide any one of the numbers given, and simply bring down to the next line those numbers which are not exactly divisible by the divisor. Continue this until all the quotients equal 1.

Examples for Practice

Find the least common multiple of:

2, 3, 4, and 5.	Answer: 60.
3, 7, 8, and 10.	Answer: 840.
6, 9, and 11.	Answer: 198.
4, 5, 6, and 7.	Answer: 420.
12, 15, 25, and 35.	Answer: 2100.
13, 14, and 16.	Answer: 1456.
18, 20, and 30.	Answer: 180.
2, 3, 4, and 40.	Answer: 120.

CHAPTER II

FRACTIONS

A *fraction* is a part of a whole number or object. Thus $\frac{1}{2}$ (one-half) inch and $\frac{3}{16}$ (three-sixteenths) inch are fractions of 1 inch.

If we divide an inch into 16 equal parts, each of these parts is called $\frac{1}{16}$ (one-sixteenth). If we have 16 of these parts it is clear that we have the whole inch, and that $\frac{16}{16}$ inch equals 1 inch. In the same way $\frac{8}{8}$ equals 1, and $\frac{32}{32}$ equals 1. In any fraction, as, for instance, $\frac{5}{32}$, the number below the line gives the total number of parts into which the object has been divided, and the number above the line states how many of these parts have been taken. Thus $\frac{5}{32}$ of an inch means that the inch has been divided into 32 parts, and that we have taken 5 of these parts. The number above the line is called the *numerator* and the number below the line is called the *denominator*.

Proper and Improper Fractions and Mixed Numbers

As there are 12 inches in a foot, 1 inch can be obtained by dividing the foot in 12 equal parts; hence 1 inch equals $\frac{1}{12}$ of a foot; 5 inches, therefore, must equal $\frac{5}{12}$ of a foot, and 7 inches equals $\frac{7}{12}$ of a foot. In the same way 17 inches equals $\frac{17}{12}$ of a foot. Now, since 17 inches equals one whole foot and 5 inches, and 5 inches equals $\frac{5}{12}$ of a foot, 17 inches must equal $1 + \frac{5}{12}$ of a foot (or as it is commonly written $1\frac{5}{12}$), and therefore

$$\frac{17}{12} = 1\frac{5}{12}.$$

A fraction written in the form $\frac{17}{12}$ is called an *improper* fraction, because it contains more than one whole number, and is therefore not a *part* of the whole. An improper fraction is recognized by the fact that the numerator is larger than the denominator.

A fraction of the form $\frac{5}{32}$ is called a *proper* fraction, because it is a *part* of a whole number, and is recognized by its numerator being smaller than its denominator.

A number of the form $1\frac{5}{12}$ which contains both a whole number and a fraction is called a *mixed* number.

Following are three lines of examples. All the fractions in the first line are proper fractions, all the fractions in the second line are improper fractions, and those in the third line are mixed numbers:

proper fractions	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{64}$	$\frac{1}{3}$	$\frac{17}{27}$
improper fractions	$\frac{3}{2}$	$\frac{5}{4}$	$\frac{15}{4}$	$\frac{65}{64}$	$\frac{7}{3}$	$\frac{30}{27}$
mixed numbers	$1\frac{1}{2}$	$1\frac{1}{4}$	$3\frac{3}{4}$	$1\frac{1}{64}$	$2\frac{1}{3}$	$1\frac{3}{27}$

It will be seen that each of the mixed numbers is exactly equal to the improper fraction written directly above it; for instance, $\frac{3}{2} = 1\frac{1}{2}$; $\frac{5}{4} = 1\frac{1}{4}$; etc.

Changing an Improper Fraction to a Mixed Number

To change an improper fraction to a mixed number, we simply take out of the improper fraction as many whole numbers as it contains; for instance, $\frac{4}{3}$ inch contains one whole ($\frac{3}{3}$) inch, and when $\frac{3}{3}$ inch has been taken out of $\frac{4}{3}$ inch, $\frac{1}{3}$ inch remains; therefore, $\frac{4}{3} = 1\frac{1}{3}$.

Suppose we want to reduce $\frac{7}{3}$ to a mixed number. There are $\frac{3}{3}$ to one whole, and therefore we can take $\frac{3}{3}$ out of $\frac{7}{3}$ twice, still having $\frac{1}{3}$ left. Therefore $\frac{7}{3} = 2\frac{1}{3}$.

When actually carrying out the operation we follow the following rule: When changing an improper fraction to a mixed number, divide the numerator by the denominator; the quotient gives the whole number, and if there be a remainder, this remainder is the numerator of the fraction in the mixed number, the denominator being the same as the denominator of the improper fraction.

Example: Reduce $\frac{33}{15}$ to a mixed number. Dividing 33 by 15 gives us 2 as the quotient, and 3 as the remainder. The quotient 2 is the whole number and 3 is the numerator of the fraction, 15 remaining the denominator; consequently,

$$\frac{33}{15} = 2\frac{3}{15}.$$

Changing Whole or Mixed Numbers to Improper Fractions

Any whole or mixed number can be changed to an improper fraction. Suppose that we want to reduce 3 inches to 8ths. There are $\frac{8}{8}$ in one inch, and consequently there are three times that number, or $\frac{24}{8}$ in 3 inches. *A whole number can be changed to an improper fraction by multiplying the number by the denominator of the improper fraction desired, and placing the product as the numerator of the fraction.* As an example, reduce 5 to 32nds; 32 is the denominator of the improper fraction wanted, and $5 \times 32 = 160$ is the numerator, therefore $5 = \frac{160}{32}$.

When a mixed number is reduced to an improper fraction the whole number is first changed by itself and the fraction is added to the result.

Reduce $2\frac{1}{8}$ to an improper fraction; 2 reduced to an improper fraction is $\frac{16}{8}$, and $\frac{1}{8}$ added to this gives us $\frac{17}{8}$ as the result.

Reduce $5\frac{3}{7}$ to an improper fraction; 5 reduced to an improper fraction is $\frac{35}{7}$, and $\frac{3}{7}$ added gives us $\frac{38}{7}$.

Changing the Form of a Fraction without Changing its Value

The value of a fraction is not changed by multiplying the numerator and denominator by the same number. We know that $\frac{1}{2}$ inch equals $\frac{2}{4}$ inch or $\frac{4}{8}$ inch or $\frac{8}{16}$ inch. We obtain these different fractions simply by multiplying the numerator and denominator of the first fraction by the same number, for instance, $\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$ or $\frac{1 \times 4}{2 \times 4} = \frac{4}{8}$. In the same way, if we multiply the numerator and denominator of the fraction $\frac{15}{27}$ by 3 we get a new fraction $\frac{45}{81}$, but the value of the fraction is not changed. The denominators 27 and 81 express the number of parts into which the whole has been divided. The whole has therefore been divided into more parts in the latter case, but each part must necessarily be smaller because if we divide a pound of shot into 81 parts, each part must, of course, be much smaller than if the pound were divided into only 27 parts. We therefore need more of these small parts to make the same quantity than we needed of the larger parts, and 45 of the small parts give us just as much shot as 15 of the larger parts.

In the same way as multiplying the numerator and denominator of a fraction by the same number does not change its value, so the dividing of the numerator and denominator by the same number does not change the value. If the fraction $\frac{8}{16}$ is given, we can divide the numerator and denominator of this fraction by 8, in which case we obtain $\frac{1}{2}$. That this is so we can clearly see, because if we divide an inch in 16 parts, and take 8 of those parts, we have the same dimension as if we take $\frac{1}{2}$ inch directly.

Reducing a Fraction to its Lowest Terms

The numerator and denominator of the fraction are called the *terms* of the fraction, and when we divide numerator and denominator of a fraction by the same numbers until there is no one number which will exactly divide both, we have reduced the fraction to its lowest terms.

Example: Reduce $\frac{48}{64}$ to its lowest terms.

$$\frac{48}{64} = \frac{24}{32} = \frac{12}{16} = \frac{6}{8} = \frac{3}{4}.$$

In this example we divide numerator and denominator by 2, four times in succession, until we obtain the fraction $\frac{3}{4}$ in which the numerator and denominator cannot be exactly divided by the same number.

Example: Reduce $\frac{15}{45}$ to its lowest terms.

$$\frac{15}{45} = \frac{5}{15} = \frac{1}{3}.$$

In this case the numerator and denominator can both be divided by 3, giving us $\frac{5}{15}$ as the result; in this fraction the terms can again be divided by the same number, this time by 5, giving $\frac{1}{3}$ as the result.

It is of great value when reducing a fraction to its lowest terms to be able to see at once by what number both numerator and denominator are divisible. All even numbers are divisible by 2. To find whether a number is divisible by 3, add all the figures in the number, and if the sum of the figures is divisible by 3, the whole number can also be divided by 3. The number 294, for instance, is divisible by 3, because the sum of the figures in 294 is $2 + 9 + 4 = 15$, and 15 is divisible by 3. All numbers in which the last figure is 5 or 0 are divisible by 5.

Examples for Practice

Change the improper fractions below to mixed numbers:

Example (1). $\frac{31}{4}$. Answer: $7\frac{3}{4}$

Example (2). $\frac{42}{5}$. Answer: $8\frac{2}{5}$

Example (3). $\frac{37}{4}$. Answer: $9\frac{1}{4}$

Reduce these fractions to their lowest terms:

Example (4). $\frac{210}{420}$. Answer: $\frac{1}{2}$
Example (5). $\frac{112}{336}$. Answer: $\frac{1}{3}$
Example (6). $\frac{55}{245}$. Answer: $\frac{11}{49}$
Example (7). $\frac{91}{98}$. Answer: $\frac{13}{14}$

Addition of Fractions

When we add, the objects added must be of the same kind. We cannot directly add $\frac{1}{2}$ foot and 4 inches without first changing $\frac{1}{2}$ foot to inches; but if we do this, and say 6 inches instead of $\frac{1}{2}$ foot, then we can add 6 inches to 4 inches, obtaining 10 inches. In the same way in fractions, we can only add those fractions directly which are of the same kind; that is, those that have the same denominator. We can add $\frac{1}{4}$ to $\frac{2}{4}$, obtaining $\frac{3}{4}$. In the same way $\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$. If the fractions have the same denominator, their sum is obtained by simply adding the numerators and placing the common denominator of the added fractions as the denominator of the sum of the numerators.

Examples:

$$\frac{1}{16} + \frac{3}{16} + \frac{7}{16} = \frac{11}{16}$$

$$\frac{5}{26} + \frac{3}{26} + \frac{13}{26} + \frac{1}{26} = \frac{22}{26} = \frac{11}{13}$$

In the last example the sum equals $\frac{22}{26}$, but this fraction reduced to its lowest terms equals $\frac{11}{13}$.

When it is required to add fractions which have not the same denominator, as for instance $\frac{3}{4}$ inch + $\frac{3}{32}$ inch, one or all of the fractions to be added must be changed so that all the fractions have the same denominator. When that has been done we can add them by adding the numerators, as above. In the example where we added $\frac{1}{2}$ foot and 4 inches, we first changed $\frac{1}{2}$ foot to inches. In the example $\frac{3}{4} + \frac{3}{32}$ we proceed in the same way, changing $\frac{3}{4}$ to 32ds, before we can add; $\frac{3}{4} = \frac{24}{32}$, which is obtained by multiplying the denominator 4 by a number which will give 32 for the product, and then multiplying the numerator by the same number. ($4 \times 8 = 32$ and $3 \times 8 = 24$.) Now we can add $\frac{24}{32} + \frac{3}{32} = \frac{27}{32}$.

The first rule for addition of fractions is, therefore, to change the fractions so that all the denominators are alike, or to a common denominator, and preferably to the *least* common denominator. In the example above, 32 is the common denominator.

To Find the Least Common Denominator

It is not always as obvious as in the example above, especially when there are several fractions given, which number is the least common denominator, but it can be found by the rule that the least common denominator equals the least common multiple of all the given denominators.

Example: Find the least common denominator of $\frac{3}{4}$, $\frac{1}{6}$, $\frac{1}{15}$.

The least common denominator of these fractions equals the least common multiple of the denominators 4, 6, and 15. This is found as follows:

4, 6, 15	2
2, 3, 15	2
1, 3, 15	3
1, 1, 5	5
1, 1, 1	

The least common denominator then is $2 \times 2 \times 3 \times 5 = 60$. This means that the three fractions given above must all be changed to 60ths.

When the least common denominator has been found, the terms of the different fractions to be added are changed to this denominator, by dividing the least common denominator by the denominator of each fraction in turn, and then multiplying the terms of the fraction by the quotient obtained. In the example above $\frac{3}{4}$ is changed to 60ths by dividing 60 by 4, obtaining the quotient 15, and then multiplying the numerator and denominator of $\frac{3}{4}$ by 15 giving us $\frac{45}{60}$ as a result. In the same way $60 \div 6 = 10$, and this multiplied by the terms of $\frac{1}{6}$ gives $\frac{10}{60}$. In the same way $\frac{1}{15} = \frac{4}{60}$. When we have thus obtained $\frac{45}{60}$, $\frac{10}{60}$, and $\frac{4}{60}$, we can add them. The adding is done by adding the numerators, giving us the sum $\frac{59}{60}$.

Examples of Addition of Fractions

Example: Add $\frac{1}{4}$ inch, $\frac{15}{16}$ inch, and $\frac{5}{32}$ inch. This would be written:

$$\frac{1}{4} + \frac{15}{16} + \frac{5}{32} =$$
$$\frac{8}{32} + \frac{30}{32} + \frac{5}{32} = \frac{43}{32} = 1\frac{11}{32}$$

Find the least common denominator as explained above. This will be found to be 32; to change $\frac{1}{4}$ to 32ds according to the rule given above, divide 32 by 4 obtaining 8 as a quotient. Now multiply the numerator and denominator by 8, obtaining $\frac{8}{32}$. In the same way $\frac{15}{16}$ is changed to $\frac{30}{32}$, and $\frac{5}{32}$ remains in the form given. When the fractions have the same denominator they can be added, and the sum obtained is $\frac{43}{32}$. This sum is an improper fraction and should be changed to a mixed number. Our sum then is $1\frac{11}{32}$.

Example: Find the sum of $1\frac{1}{3} + 2\frac{3}{5} + 5\frac{1}{8}$.

$$1\frac{40}{120} + 2\frac{72}{120} + 5\frac{15}{120} = 8\frac{127}{120} = 9\frac{7}{120}$$

When mixed numbers are to be added, the whole numbers are added separately, and the fractions are added by themselves in the usual way. In the example above first find the least common denominator of the three fractions, which is 120, and then change the fractions so that all have this common denominator as shown in the second line. Now add the whole numbers and the fractions separately obtaining $8\frac{127}{120}$. The fraction $\frac{127}{120}$ is an improper fraction and should be changed to a mixed number; $\frac{127}{120} = 1\frac{7}{120}$; the mixed number $1\frac{7}{120}$ is added to the 8 already obtained, giving us $9\frac{7}{120}$ as the final sum.

Rule for Addition of Fractions

Find the least common denominator of the given fractions, and then change the fractions so that all have this least common denominator. Add the numerators, and then write the sum over the common denominator. When mixed and whole numbers are to be added, add the whole numbers and fractions separately. If the sum of the fractions is an improper fraction, change it to a mixed number and add it to the sum of the whole numbers already obtained.

Examples for Practice

Example (1). $\frac{5}{6} + \frac{7}{12} + \frac{3}{20}$ Answer: $1\frac{17}{30}$

Example (2). $3\frac{4}{15} + 2\frac{2}{7} + 4\frac{2}{3}$ Answer: $10\frac{23}{105}$

Example (3). $6\frac{7}{16} + 4\frac{1}{5} + 1\frac{3}{40}$ Answer: $11\frac{57}{80}$

Example (4). $3\frac{4}{45} + \frac{7}{27} + \frac{13}{72} + 3\frac{2}{3}$ Answer: $7\frac{211}{1080}$

Subtraction of Fractions

Subtraction of fractions is carried out by methods similar to those used for addition of fractions. If the fractions to be subtracted have not the same denominator, they must be changed so that they have a common denominator, preferably the *least* common denominator.

Subtract $\frac{5}{32}$ inch from $\frac{7}{8}$ inch.

$$\frac{7}{8} - \frac{5}{32} =$$
$$\frac{28}{32} - \frac{5}{32} = \frac{23}{32}$$

The least common denominator, found as previously explained, is 32; $\frac{7}{8}$ changed to 32ds equals $\frac{28}{32}$, and $\frac{5}{32}$ from $\frac{28}{32}$ equals $\frac{23}{32}$; $\frac{28}{32}$ is the minuend and $\frac{5}{32}$ is the subtrahend. The remainder $\frac{23}{32}$ is obtained simply by subtracting the numerator of the subtrahend from the numerator of the minuend and placing the common denominator as the denominator of the remainder.

Example: Find the difference between $6\frac{17}{32}$ and $4\frac{1}{8}$ inches.

$$6\frac{17}{32} - 4\frac{1}{8} =$$
$$6\frac{17}{32} - 4\frac{4}{32} = 2\frac{13}{32}$$

When one mixed number is to be subtracted from another mixed number, the whole numbers and the fractions are subtracted separately. In the example above, the fractions are first changed to a common denominator as shown in the second line, then the whole number 4 in the subtrahend is subtracted from the whole number 6 in the minuend, leaving 2 in the remainder; then $\frac{4}{32}$ is subtracted from $\frac{17}{32}$, leaving $\frac{13}{32}$; this added to the whole number already obtained gives the total remainder $2\frac{13}{32}$.

Example: What is the difference between 3 and $\frac{7}{12}$?

$$3 - \frac{7}{12} =$$
$$2\frac{12}{12} - \frac{7}{12} = 2\frac{5}{12}.$$

We cannot subtract $\frac{7}{12}$ directly from 3 and we therefore take 1 whole out of 3 and change it to a fraction having the same denominator as the subtrahend; 1 whole equals $\frac{12}{12}$, so that instead of 3 we can write the minuend $2\frac{12}{12}$. Now we can subtract $\frac{7}{12}$ from this giving us $2\frac{5}{12}$.

Example: Subtract $13\frac{11}{16}$ from 27.

$$27 - 13\frac{11}{16} =$$
$$26\frac{16}{16} - 13\frac{11}{16} = 13\frac{5}{16}$$

As there is no fraction in the minuend from which we can subtract the fraction in the subtrahend, we must take 1 whole from the 27 given and change it to 16ths; 1 whole is $\frac{16}{16}$, and when this is taken out of 27 and changed to 16ths, we have, of course, $26\frac{16}{16}$. Now we can subtract the whole numbers from the whole numbers and the fractions from the fractions, obtaining as a remainder $13\frac{5}{16}$.

Example: Subtract $6\frac{11}{14}$ from $8\frac{3}{5}$.

$$8\frac{3}{5} - 6\frac{11}{14} =$$
$$8\frac{42}{70} - 6\frac{55}{70} =$$
$$7\frac{112}{70} - 6\frac{55}{70} = 1\frac{57}{70}$$

First change the given fractions to fractions having the same denominator. When this has been done it will be seen that we cannot subtract $\frac{55}{70}$ from $\frac{42}{70}$ because 55 is a greater number than 42. We must therefore take 1 whole from the 8 given in the minuend and change this to 70ths; 1 whole is $\frac{70}{70}$ and this added to the $\frac{42}{70}$ already in the minuend gives us a total of $\frac{112}{70}$, as shown in the next line. Taking 1 whole from 8, of course, leaves us only 7 in the minuend as shown. Now by subtracting the whole numbers and the fractions separately, we obtain the remainder $1\frac{57}{70}$.

Rule for Subtraction of Fractions

Change the fractions so that they have a common denominator; subtract the numerator in the subtrahend from the numerator in the minuend and place the remainder over the common denominator. When one mixed number is to be subtracted from another mixed number subtract the whole numbers and the fractions separately. When the fraction in the subtrahend is greater than the fraction in the minuend, take 1 from the whole number given in the minuend and change it to a fraction with the common denominator, and add it to the given fraction in the minuend. When the minuend is a whole number, take 1 from the minuend and change it to a fraction with the common demoninator, and proceed as before.

Examples for Practice

Example (1). $\frac{5}{7} - \frac{3}{5}$ — Answer: $\frac{4}{35}$
Example (2). $1\frac{3}{35} - \frac{16}{45}$ — Answer: $\frac{46}{63}$
Example (3). $\frac{6}{21} - \frac{3}{49}$ — Answer: $\frac{11}{49}$
Example (4). $3\frac{3}{10} - 2\frac{3}{14}$ — Answer: $1\frac{3}{35}$
Example (5). $16\frac{5}{24} - 4\frac{5}{12}$ — Answer: $11\frac{19}{24}$

Multiplication of Fractions

When multiplying fractions, it is not necessary to change the fractions so that they have a common denominator. The general rule for multiplying fractions is simply to multiply numerator by numerator, and denominator by denominator. The products thus obtained are the numerator and denominator, respectively, of the product of the fractions.

Example: Multiply $\frac{3}{4}$ by $\frac{2}{3}$.

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$$

In the example above, we multiply numerator by numerator, $3 \times 2 = 6$, and denominator by denominator, $4 \times 3 = 12$, thus obtaining the product $\frac{6}{12}$. This reduced to its lowest terms equals $\frac{1}{2}$.

Example: What is the product of 5 times $\frac{13}{16}$?

$$5 \times \frac{13}{16} = \frac{65}{16} = 4\frac{1}{16}.$$

When a fraction is multiplied by a whole number, multiply the numerator by the whole number, and place the product over the denominator of the given fraction. In the example above, $5 \times 13 = 65$, and this is placed over the given denominator 16, giving us $\frac{65}{16}$, which reduced to a mixed number equals $4\frac{1}{16}$.

Example: Find the product of $3\frac{1}{3}$ times $1\frac{1}{5}$.

$$3\tfrac{1}{3} \times 1\tfrac{1}{5} =$$
$$\frac{10}{3} \times \frac{6}{5} = \frac{60}{15} = 4$$

When two mixed numbers, or a fraction and one mixed number, are to be multiplied, the mixed numbers must first be changed to improper fractions, and then the numerator is multiplied by the numerator, and the denominator by the denominator, as before. In the example above, $3\frac{1}{3}$ changed to an improper fraction equals $\frac{10}{3}$, and $1\frac{1}{5}$ equals $\frac{6}{5}$. Multiplying these fractions, we obtain $\frac{60}{15}$, which reduced to its lowest terms equals $\frac{4}{1}$, or 4. It should be understood that 1 may be considered as the denominator of any whole number; it has been explained that an expression written in the form $\frac{4}{1}$ indicates a division, and 4 divided by 1 equals 4.

Example: Find the product of $1\frac{1}{4} \times \frac{3}{4} \times 1\frac{5}{8}$.

$$1\tfrac{1}{4} \times \tfrac{3}{4} \times 1\tfrac{5}{8} =$$
$$\frac{5}{4} \times \frac{3}{4} \times \frac{13}{8} = \frac{195}{128} = 1\frac{67}{128}$$

If more than two fractions are to be multiplied, all the numerators are multiplied to obtain the numerator of the product, and all the denominators multiplied to obtain the denominator of the product. First, of course, change all mixed numbers to improper fractions, and consider all whole numbers without a fraction as if they had 1 for denominator. In the example above, when the mixed numbers have been changed to improper fractions, multiply $5 \times 3 = 15$, and then multiply this product 15 by 13, obtaining 195 as the total product of the numerators. In the same way, $4 \times 4 = 16$, and $16 \times 8 = 128$, gives the total product of the denominators. The improper fraction $\frac{195}{128}$ changed to a mixed number equals $1\frac{67}{128}$.

Cancellation

Cancellation is the process of taking out equal factors in both numerators and denominators of fractions to be multiplied, and is used for simplifying the work of multiplication of fractions. Any factor in any one of the numerators can be taken out if the same factor is taken out in any one of the denominators. Note that for each one factor in any one of the numerators, only one factor in one of the denominators must be canceled.

Example: Multiply $\frac{3}{4} \times \frac{4}{5} \times \frac{1}{6} \times \frac{10}{18}$.

1st line $\qquad \frac{\cancel{3}}{4} \times \frac{4}{5} \times \frac{1}{\cancel{6}} \times \frac{10}{18} =$

2nd line $\qquad \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{5} \times \frac{1}{2} \times \frac{10}{18} =$

3rd line $\qquad \frac{1}{1} \times \frac{1}{\cancel{5}} \times \frac{1}{2} \times \frac{\cancel{10}}{18} =$

4th line $\qquad \frac{1}{1} \times \frac{1}{1} \times \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{18} =$

5th line $\qquad \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{18} = \frac{1}{18}$

In this example we can take out the factor 3 in the numerator in the fraction $\frac{3}{4}$ if we at the same time take out a factor 3 in the denominator of any one of the fractions. We can take out this factor in the denominator of $\frac{1}{6} = \frac{1}{2 \times 3}$ leaving us $\frac{1}{2}$ as shown in the second line. We now can take out 4 in denominator and numerator of the fractions $\frac{1}{4}$ and $\frac{4}{5}$, leaving us $\frac{1}{1}$ and $\frac{1}{5}$, as shown in the third line. Next we can take out 5 in $\frac{1}{5}$ and $\frac{10}{18}$, leaving us $\frac{1}{1}$ and $\frac{2}{18}$ in the fourth line. Finally, we take out or cancel 2 in $\frac{1}{2}$ and $\frac{2}{18}$. The whole product is then $\frac{1}{18}$.

It is not necessary when canceling to write out the numbers to be multiplied after each cancellation, as has been done above for sake of clearness. Ordinarily, a line is drawn through the canceled figure and the factor remaining after cancellation written over the original figure in the numerator, and under it in the denominator. The example above, for instance, could be carried out as below:

$$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{5}}} \times \frac{1}{\underset{\underset{1}{\cancel{2}}}{\cancel{6}}} \times \frac{\overset{\overset{1}{\cancel{2}}}{\cancel{10}}}{18} = \frac{1}{18}$$

Example: Multiply $\frac{4}{15} \times \frac{3}{12} \times \frac{6}{11} \times \frac{33}{40}$.

This can be carried out as shown below:

$$\frac{\overset{1}{\cancel{4}}}{\underset{5}{\cancel{15}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{12}}} \times \frac{\overset{\overset{1}{\cancel{2}}}{\cancel{6}}}{\underset{1}{\cancel{11}}} \times \frac{\overset{3}{\cancel{33}}}{\underset{20}{\cancel{40}}} = \frac{3}{100}$$

Rule for Multiplication of Fractions

If proper fractions are to be multiplied, multiply numerators by numerators, and denominators by denominators. If mixed numbers are to be multiplied, change them first to improper fractions, and then multiply numerators by numerators, and denominators by denominators. If there are any common factors in the numerators or denominators of the different fractions to be multiplied, cancel these factors before multiplying.

Examples for Practice

Example (1). $\frac{5}{8} \times \frac{3}{4}$. Answer: $\frac{15}{32}$

Example (2). $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$. Answer: $\frac{1}{24}$

Example (3). $8\frac{3}{7} \times 7\frac{1}{8}$. Answer: $60\frac{3}{56}$

Example (4). $6\frac{3}{10} \times 3\frac{1}{2}$. Answer: $22\frac{1}{20}$

Division of Fractions

When dividing fractions, it is not necessary to change the fractions so that they have a common denominator. When dividing fractions, simply *invert* (turn upside down) the divisor, and then multiply the dividend by the inverted divisor, which is done by multiplying numerators by numerators, and denominators by denominators. When inverting the divisor, the numerator and denominator change place. Example:

$\frac{5}{8}$ inverted, takes the form $\frac{8}{5}$; $\frac{13}{20}$ inverted takes the form $\frac{20}{13}$.

Example of division:

$$\overset{\text{dividend}}{\frac{5}{8}} \div \overset{\text{divisor}}{\frac{5}{16}} = \overset{\text{dividend}}{\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{8}}}} \times \overset{\text{inverted divisor}}{\frac{\overset{2}{\cancel{16}}}{\underset{1}{\cancel{5}}}} = \frac{2}{1} = 2.$$

The divisor $\frac{5}{16}$ in the example above, is inverted, and the dividend $\frac{5}{8}$ is multiplied by the inverted divisor $\frac{16}{5}$. Common factors are canceled as shown.

Example: Divide $6\frac{1}{8}$ by $\frac{7}{8}$.

$$6\frac{1}{8} \div \frac{7}{8} = \frac{49}{8} \times \frac{8}{7} = \frac{7}{1} = 7.$$

When mixed or whole numbers are to be divided, they must first be changed to improper fractions, as in the example above.

Example: Divide $7\frac{3}{4}$ by 5.

$$7\frac{3}{4} \div 5 = \frac{31}{4} \times \frac{1}{5} = \frac{31}{20} = 1\frac{11}{20}.$$

If the divisor is a whole number, it must be considered as a fraction with the denominator 1; in the example above, therefore, 5 is considered as $\frac{5}{1}$, and this inverted takes the form $\frac{1}{5}$.

Rule for Division of Fractions

When dividing fractions, first change all whole or mixed numbers to improper fractions, invert the divisor and multiply numerator by numerator, and denominator by denominator. Cancel common factors before multiplying, same as in multiplication.

Examples

A line between two numbers indicates that the number above the line is to be divided by the number under the line. Therefore,

$$\frac{\frac{5}{8}}{\frac{3}{4}} = \tfrac{5}{8} \div \tfrac{3}{4} = \tfrac{5}{8} \times \tfrac{4}{3} = \tfrac{5}{6}.$$

Sometimes, in examples of the type shown above, additions or subtractions or multiplications may have to be carried out before the final division. For instance,

$$\frac{3 + 1\frac{1}{4}}{5 - 1\frac{1}{2}} = \frac{4\frac{1}{4}}{3\frac{1}{2}} = 4\tfrac{1}{4} \div 3\tfrac{1}{2} = \tfrac{17}{4} \div \tfrac{7}{2} = \tfrac{17}{4} \times \tfrac{2}{7} = \tfrac{17}{14} = 1\tfrac{3}{14}.$$

In examples of this kind, always simplify the expressions over and under the line to their simplest form, that is, until they consist of one number. Then divide as usual.

Example:

$$\frac{4\frac{1}{3} \times \frac{2}{5} + 3}{\frac{1}{2} \times \frac{3}{8}} = \frac{\frac{13}{3} \times \frac{2}{5} + 3}{\frac{1}{2} \times \frac{3}{8}} = \frac{\frac{26}{15} + \frac{45}{15}}{\frac{3}{16}} = \frac{\frac{71}{15}}{\frac{3}{16}} = \tfrac{71}{15} \div \tfrac{3}{16} =$$

$$\tfrac{71}{15} \times \tfrac{16}{3} = \tfrac{1136}{45} = 25\tfrac{11}{45}.$$

Example:

$$\frac{\frac{\frac{5}{8}}{\frac{3}{5}}}{\frac{\frac{7}{8}}{1\frac{1}{2}}} = \frac{\frac{5}{8} \div \frac{3}{5}}{\frac{7}{8} \div 1\frac{1}{2}} = \frac{\frac{5}{8} \times \frac{5}{3}}{\frac{7}{8} \times \frac{2}{3}} = \frac{\frac{25}{24}}{\frac{14}{24}} = \tfrac{25}{24} \div \tfrac{14}{24} = \tfrac{25}{24} \times \tfrac{24}{14} = \tfrac{25}{14} = 1\tfrac{11}{14}.$$

Examples for Practice

Example (1). $3\frac{11}{16} \div 2\frac{1}{4}$. Answer: $1\frac{23}{36}$

Example (2). $1\frac{9}{10} \div \frac{7}{10}$. Answer: $2\frac{5}{7}$

Example (3). $40\frac{3}{8} \div \frac{3}{8}$. Answer: $107\frac{2}{3}$

Example (4). $\frac{1}{2} \times \frac{1}{3} \div \frac{1}{4}$. Answer: $\frac{2}{3}$

CHAPTER III

DECIMAL FRACTIONS

Decimal fractions are ordinary or common fractions, having ten or a multiple of ten for the denominator; this denominator is not written out in the same way as in common fractions. The fraction $\frac{1}{10}$ is written as a decimal fraction 0.1, but is read *one-tenth*, the same as the common fraction. In the same way, $\frac{3}{100}$ is written as a decimal fraction 0.03, and is read *three-hundredths*. The figure 0 written to the left of the period (.) in 0.03 gives the whole number (in this case 0). The period is called decimal point, and the figures to the right of the decimal point are called *decimals*. A number of fractions with 10 or a multiple of 10 for denominators are written below, and under each one is written the same fraction expressed in decimals.

$\frac{2}{10}$	$\frac{16}{100}$	$\frac{153}{1000}$	$\frac{6}{100}$	$\frac{27}{1000}$
0.2	0.16	0.153	0.06	0.027

By inspecting the common fractions and the decimal fractions above, it will be seen that the decimals give the numerator of the fraction, and that the number of decimals give the value of the denominator of the fraction. If there be but one decimal to the right of the decimal point, we have tenths, if there be two decimals, hundredths, if there be three decimals, thousandths. In $\frac{6}{100}$ it is necessary for us to place a 0 between the decimal point and 6 when we write the decimal 0.06 in order to get two decimals after the decimal point, and thus express that we have six-*hundredths*. If we had written 0.6, it would have meant six-tenths. In the same way, $\frac{27}{1000}$ is written 0.027. If we had written 0.27 it would be read twenty-seven hundredths. The first figure after the decimal point gives the tenths, the second the hundredths, and the third the thousandths. The tabulated arrangement below shows the names and values of the different positions of both whole numbers and decimals in relation to the decimal point.

Billions	Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-Thousandths	Hundred-Thousandths	Millionths	Ten-Millionths	Hundred-Millionths	Billionths
6,	5	6	3,	7	8	2,	1	0	3	.	6	7	5	2	3	4	8	6	3

The figures to the left of the decimal point are whole numbers. The number above is read: six billion, five hundred sixty-three million, seven hundred eighty-two thousand, one hundred and three, and six hundred seventy-five million, two hundred thirty-four thousand, eight hundred sixty-three billionths.

In any decimal fraction, the number of decimals to the right of the decimal point determines the value of the denominator of the fraction. There should be as many decimals to the right of the decimal point as there are ciphers in the denominator; as there is only one cipher in 10, there should be but one decimal for tenths; but as there are three ciphers in 1,000, we must have three decimals to the right of the decimal point when we require thousandths. One or more ciphers (0) must therefore be written between the decimal point and the figure which expresses the value of the numerator of the fraction, when the numerator does not contain the required number of decimal places. For example, $\frac{3}{1000}$ and $\frac{83}{1000}$ are written 0.003 and 0.083.

Adding a cipher to the right of the decimal does not change its value. The fractions 0.3 and 0.30 have the same value because $\frac{3}{10}$ and $\frac{30}{100}$ express the same value, and $\frac{3}{10}$ is simply $\frac{30}{100}$ reduced to its lowest terms.

Effect of Moving the Decimal Point

If, in a decimal fraction, we move the decimal point one place to the right, we increase the value of the decimal ten times. If, in 0.003 we move the decimal point one place to the right so that it reads 0.03, this is 10 times greater than 0.003. If we move the decimal point two places to the right we make the value 100 times greater; if we move it three places, 1,000 times greater, and so forth. If, instead of moving the decimal point one place to the right we move it one place to the left, the value of the decimal is *divided* by 10. If in 0.3 we move the decimal point one place to the left, we get 0.03. The value of this last decimal equals the value of the first decimal (0.3) divided by 10, or $0.03 = 0.3 \div 10$. In the same way, if we move the decimal point two places to the left, we divide the value of the decimal by 100; if we move it three places to the left, we divide the value by 1,000, and so forth.

1 2 7.3 5 6	1st line
1 2 7 3.5 6	2d line
1 2 7 3 5.6	3d line
1 2 7 3 5 6	4th line
1 2.7 3 5 6	5th line
1.2 7 3 5 6	6th line
0.1 2 7 3 5 6	7th line

In the numbers above, the decimal point has been moved first to the right one place at a time, and then to the left one place at a time. The number in the second line is 10 times greater than the number in the first line; the number in the third line is 100 times greater than the number in the first line; and the number in the fourth line 1,000 times greater. It will be seen that in the fourth line, the decimal point would come after all the figures, thus making a whole number with no decimals. In such a case, of course, no decimal point is shown. In the fifth line, the decimal point has been moved one place to the left in relation to the place for the decimal point in the first line. The number in the fifth line, therefore, is one-tenth of the number in the first line. The number in the sixth line is one-hundredth of the number in the first line, and the number in the seventh line equals the number in the first line divided by 1,000. In the seventh line the decimal point comes in front of all the figures, making all the given figures decimals, and it is customary to place a cipher to the left of the decimal point to indicate that there are no whole numbers.

Addition of Decimal Fractions

Decimal fractions are added in the same way as whole numbers. The numbers are written under one another in such a manner that all the decimal points come in one continuous vertical line. This, of course, also places all the whole unit figures in a column, but does not necessarily make the extreme right-hand figures of the decimals come in a line under each other. When the numbers to be added have been placed in the manner mentioned, the addition is carried out exactly as addition of whole numbers. The decimal point in the sum is placed exactly under the decimal point in the numbers to be added.

Example:

$$\begin{array}{r} 7.635 \\ 0.03 \\ 123. \\ 0.0406 \\ \hline 130.7056 \end{array}$$

Examples for Practice

Example (1). 5.6 + 4.9 + 1.7 + 2.12. Answer: 14.32

Example (2). 4.67 + 5.36 + 0.84 + 7.05. Answer: 17.92

Example (3). 2.6661 + 0.8735 + 0.6877 + 3.34132. Answer: 7.56862

Example (4). 2.008 + 1.4 + 0.706 + 0.3 + 0.077. Answer: 4.491

Subtraction of Decimals

Subtraction of decimals is an operation very similar to subtraction of whole numbers. See that the decimal point in the subtrahend is directly under the decimal point in the minuend, and subtract as if whole numbers were to be subtracted. The decimal point in the remainder is placed directly under the decimal point in the minuend and subtrahend.

Find the difference between 7.873 and 3.412.

minuend	7.873
subtrahend	3.412
remainder	4.461

Find the difference between 0.6367 and 0.35.

minuend	0.6367
subtrahend	0.35
remainder	0.2867

Subtract 23.265 from 24.5.

minuend	24.500
subtrahend	23.265
remainder	1.235

If the number of decimals in the minuend is smaller than the number of decimals in the subtrahend, add ciphers in the minuend above the figures in the subtrahend, and then subtract as whole numbers. In the example above 24.5 is written 24.500, because there are three decimals in the subtrahend. Placing ciphers after a decimal does not change its value.

Rule for Subtraction of Decimals

Place the subtrahend under the minuend so that the decimal points come directly under each other. Subtract the same as whole numbers, and place the decimal point in the remainder directly under the decimal points above. If the number of decimals in the subtrahend is greater than the number of decimals in the minuend, add ciphers in the minuend above the decimals in the subtrahend, and subtract as before.

Examples for Practice

One of the most common applications of decimals is found in calculations with money and with length dimensions expressed in decimal equivalents of an inch. The United States monetary (money) system is based on the decimal system. One cent equals 0.01 dollar, and one dime equals 0.1 or 0.10 dollar. One quarter equals 0.25 dollar, and one half-dollar equals 0.5 or 0.50 dollar. In the same way, 69 cents equals 0.69 dollars.

Example (1). How much remains when 63 cents are taken out of $10?

$$\begin{array}{r} 10.00 \\ 0.63 \\ \hline 9.37 \end{array}$$

Nine dollars and 37 cents remains.

Example (2). If we have 3 pennies, 4 nickels, one dime, and 3 dollars and take 77 cents from this amount, how much remains? The sum on hand equals

$$0.03 + 0.20 + 0.10 + 3 = 3.33.$$

Then $3.33 - 0.77 =$ the remainder, or

$$\begin{array}{r} 3.33 \\ 0.77 \\ \hline 2.56 \end{array}$$

Two dollars and 56 cents remains.

Example (3). If one gage measures 3.563 inches, by micrometer, and another measures 5.124 inches, how much is the latter larger in diameter than the former?

```
5.124
3.563
-----
1.561 inch.
```

Example (4). Of two end measuring rods, the one measures 12.0013 and the other 5.9938 inches in length, how much is the one longer than the other?

```
12.0013
 5.9938
-------
 6.0075 inches
```

Example (5). 64.037 — 5.9082. Answer: 58.1288

Example (6). 2 — 0.9998. Answer: 1.0002

Example (7). 71.287 — 40.089. Answer: 31.198

Multiplication of Decimal Fractions

When multiplying decimal fractions, the multiplier is placed under the multiplicand, in the same way as in multiplication of whole numbers. While carrying out the multiplication, no attention is paid to the decimal point; the numbers are simply placed in such a manner that the right-hand figure in the multiplicand comes directly over the right-hand figure of the multiplier. It makes no difference whether the decimal points should happen to come under each other or not. If 126.5623 is to be multiplied by 4.67, write the numbers thus:

```
  126.5623          and not thus 126.5623
      4.67                         4.67
----------                      --------
   8859361
  7593738
 5062492
----------
591.045941
```

The multiplication is carried out exactly as when whole numbers are to be multiplied. When the final product has been obtained, we must determine the position of the decimal point in the product. The number of decimals in the product equals the sum of the number of decimals in the multiplicand and the multiplier. If there are four decimals in the multiplicand, and two decimals in the multiplier, as in the example above, then there should be six decimals in the product, and we place the decimal point in the product so that there are six figures to the right of the decimal point.

If there are not enough figures in the product to point off the required number of decimals, prefix ciphers until the required number is obtained, then place the decimal point; and place a cipher in front of the decimal point to indicate that there is no whole number.

Examples

A.	B.	C.
0.0023	0.003135	0.174
3.6	4	0.0023
138	0.012540	522
69		348
0.00828		0.0004002

In the example *A* above, there are four decimals in the multiplicand and one in the multiplier; there should therefore be $4 + 1 = 5$ decimals in the product. To get five decimals we must prefix ciphers in front of 828, as shown. In example *B*, there are six decimals in the multiplicand but none in the multiplier; consequently there are only six decimals in the product. In example *C*, there are three decimals in the multiplicand, and four in the multiplier, and therefore $3 + 4 = 7$ decimals in the product.

It is not necessary to multiply by the ciphers on the left of the decimal, as these merely determine the number of decimal places.

Rule for Multiplication of Decimal Fractions

Place the multiplier under the multiplicand, disregarding the decimal point. Multiply as in whole numbers, and in the product, point off as many decimals as there are decimals in both multiplier and multiplicand. If there are not enough figures in the product to point off the required number of decimals, prefix ciphers, put in the decimal point, and place a cipher to the left of the decimal point to indicate that there is no whole number.

Examples for Practice

Example (1). The circumference of a circle equals its diameter multiplied by 3.1416. If a steel disk measures 5.4 inches in diameter, how many inches does it measure around the outside?

```
  3.1416
     5.4
 -------
  125664
 157080
 -------
16.96464 inches.
```

Example (2). Each of fourteen men in a shop receives $15.95 a week. How much money would be required to pay off these men during a year if they lost no time?

```
 15.95
    14
 -----
  6380
 1595
 -----
223.30
```

To pay off the men one week takes 223 dollars and 30 cents. There are 52 weeks in the year, and the money required in a year therefore equals 223.30 × 52.

```
   223.30
       52
 --------
    44660
   111650
 --------
 11611.60 dollars.
```

Example (3). 2.918 × 0.364. Answer: 1.062152.

Example (4). 56.98 × 7.92. Answer: 451.2816.

Division of Decimals

When dividing decimal fractions, the dividend, divisor and quotient are placed in the same manner as in division of whole numbers. If there is not an equal number of decimal places in the dividend and divisor, add ciphers to the one having the smallest number of decimal places, until there is an equal number, and then divide as whole numbers, disregarding the decimal point.

Example: Divide 3.25 by 0.0625.

```
dividend 3.2500 | 0.0625 divisor
           3125 | ------
         ------ | 52      quotient
           1250
           1250
         ------
```

In the example above there are two decimals in 3.25 and four in 0.0625. We therefore add ciphers to 3.25 until there are four decimals in that number, thus: 3.2500. Now we divide as if we had whole numbers, and pay no attention to the decimal point.

In the example given we had no remainder when the last figure from the dividend had been brought down. If there be a remainder at that time, the division would be continued as in the example below.

Example: Divide 23.1875 by 0.25.

```
 23.1875 | 0.2500
  22500  | ------
 ------- |  92.75
    6875
    5000
   -----
   18750
   17500
   -----
    12500
    12500
    -----
```

If there is a remainder when the last figure has been brought down from the dividend, place a decimal point after the figures already obtained in the quotient, annex a 0 to the remainder left from the last subtraction, and continue to divide as before. To each remainder obtained annex a 0. This 0 takes the place of the figure brought down from the dividend. The figures obtained in the quotient after the decimal point has been placed are, of course, decimals.

Example: Divide 5 by 40.

```
5  | 40
0  |------
-- | 0.125
50
40
----
100
 80
----
 20C
 200
----
```

In this example 40 is contained in five, 0 times; therefore 0 is the first figure in the quotient. Then finding that we have a remainder 5 and no figures in the dividend to bring down, we place the decimal point in the quotient. Then we place 0 after the remainder 5 and divide as with whole numbers.

When we do not obtain an even decimal as a quotient, as in the previous examples, but it appears as if the division would never terminate, then we determine how many decimals are required in the quotient, and the division is carried to one decimal place further. If the last figure in the quotient is 5 or greater than 5, then the next last figure is increased by 1. For instance, if 0.3676 inch is required only with three decimals, or in thousandths inch, we would write 0.368, increasing the next last figure 7 in 0.3676 by 1, because the last figure (6) is greater than 5.

Example: Divide 2.5 by 16, giving the product to three decimal places.

```
2.5 | 160
00  |-------
--  | 0.1562
250
160
----
 900
 800
----
 1000
  960
----
  400
```

The result, to three decimal places, is 0.156. In this case the 6 in 0.1562 is not increased by 1, because the last figure (2) is less than 5.

In the division above, it is not necessary to subtract 00 from 2.5 as has been done. A 0 can be simply affixed to 2.5 as shown below, and the division carried out as usual. The decimal point is placed in the quotient before the first 0 is affixed.

```
 2.50 | 160
 160  |------
------| 0.1562
  900
  800
  ----
  1000
   960
   ----
    400
```

Examples for Practice

Example (1).	6146.48 ÷ 8.	Answer: 768.31.
Example (2).	559.136 ÷ 202.	Answer: 2.768.
Example (3).	3.87 ÷ 387.	Answer: 0.01.
Example (4).	5.6394 ÷ 7.23.	Answer: 0.78.
Example (5).	275 ÷ 0.05.	Answer: 5500.

Changing Common Fractions to Decimal Fractions

It has been previously stated that instead of using the sign ÷ for division, a division is indicated by writing the dividend and divisor in form of a fraction. Thus $\frac{3}{4}$ indicates a division; 3 is the dividend and 4 is the divisor. In the same way any fraction may be considered to indicate a division; the numerator is the dividend, and the denominator is the divisor. To change a common fraction to a decimal fraction, therefore, simply carry out the division.

Change $\frac{3}{4}$ to decimals.

```
 30 | 4
 28 |----
 ---| 0.75
 20
 20
 --
 00
```

How many thousandths of an inch are there in ⅞ inch?

```
 70 | 8
 64 |-----
----| 0.875
 60
 56
 ---
  40
  40
  --
  00
```

To change a common fraction to a decimal fraction, divide the numerator by the denominator, obtaining the quotient in the form of a decimal fraction.

Changing Decimal Fractions to Common Fractions

To change a decimal fraction to a common fraction, write the figures of the decimal (omitting the decimal point and any ciphers in front of the figures) as the numerator of a fraction, and 1 with as many ciphers annexed as there are decimals in the decimal fraction, as the denominator. Then reduce this fraction to its lowest terms.

Example: Change 0.175 to a common fraction.

$$\frac{175}{1000}=\frac{35}{200}=\frac{7}{40}$$

The numerator of the common fraction is 175, or the same as the figures in the decimal. The denominator is 1000, or 1 with three ciphers annexed; the number of ciphers is equal to the number of decimals in 0.175. Reduce the fraction thus obtained to its lowest terms as shown, by dividing both numerator and denominator by 5 and then again by 5. In the example above, the numerator and denominator can also be divided directly by 25, giving the final result with only one division. ($175 \div 25 = 7$ and $1000 \div 25 = 40$.)

Example: Change 0.0036 to a common fraction.

$$\frac{36}{10000}=\frac{9}{2500}$$

The numerator of the common fraction is 36, the ciphers in front of these figures being omitted. The denominator is 10000, or 1 with four ciphers annexed; the number of ciphers is equal to the number of decimals in 0.0036. The fraction thus obtained is reduced to its lowest terms by dividing numerator and denominator by 4.

Example: A dimension 0.6875 inch is given. Change this decimal to a common fraction of an inch.

$$\frac{6875}{10000}=\frac{275}{400}=\frac{11}{16}\text{ inch.}$$

Changing decimals to common fractions is, of course, simply a matter of writing the common fraction exactly as the decimal fraction is read. Thus 0.03 is read three-hundredths, and the common fraction is therefore $\frac{3}{100}$. In the same way 0.0027 (twenty-seven ten-thousandths) $=\frac{27}{10000}$.

Instead of writing fractions in the form $\frac{3}{8}$, it is quite common to write them in the form ⅜, and in printed matter 3/8.

CHAPTER IV

PROPORTION

When two quantities bear such a relation to each other that as one is increased the other becomes greater, or, as one is decreased, the other becomes less at the same rate, they are said to be in *direct proportion*. The circumference of round bar stock is *directly proportional* to the diameter of the bar. If the diameter increases, the circumference will increase, and if the diameter is made less, the circumference will be less.

If the relation between two quantities is such that as the one increases the other becomes smaller, and as the one decreases the other becomes greater in the same rate, they are in *inverse proportion*. The greater the number of men, for instance, that are employed on completing a certain number of machines, the shorter will be the time required for finishing the job, so that the time required is *inversely proportional* to the number of men.

When the relation between two quantities is such that the increase or decrease of one affects the other by a combination of two or more direct or inverse proportions, they are said to be in *compound proportion*. If one man can turn 50 bevel gear blanks in a day of 10 hours, then 5 men can turn 225 blanks in a day of 9 hours. The number of blanks turned by one man in 10 hours is in compound proportion to the number turned by 5 men in 9 hours, because the proportion is a combination of the proportion between the number at work and the proportion of the time they are working.

In calculations a proportion is usually written as below:

$$5 : 6 :: 10 : 12$$

which is read: five is to six as ten is to twelve.

In every proportion of four terms the product of the two extreme or outside terms equals the product of the two mean or intermediate terms; thus in the proportion $5 : 6 :: 10 : 12$, the product 5×12 equals the product 6×10.

In a proportion, the sign (:) can be substituted by the division sign (÷), and the sign (::) by the equal sign (=), so that the proportion above may be written $5 \div 6 = 10 \div 12$ or $5/6 = 10/12$. The fraction on either side of the equal sign reduced to its lowest terms is called the *ratio* of the proportion. In the example above, the fraction 5/6 is already reduced to its lowest terms, so that 5/6 is the ratio.

Examples of Direct Proportion

Example (1).—If it takes 18 days to assemble 4 lathes, how long would it require to assemble 14 lathes?

The time required to assemble 14 lathes is directly proportional to the time required for 4 lathes. If it takes 18 days to assemble 4 lathes, it takes $\frac{18}{4} = 4\frac{1}{2}$ days for one lathe, and $14 \times 4\frac{1}{2} = 63$ days to assemble 14 lathes; or, written as one calculation:

$$\frac{18}{4} \times 14 = 63.$$

This problem could also be solved as follows: Let the number of days to be found be x. Then write out the proportion as below:

$$4 : 18 :: 14 : x$$

(lathes : days :: lathes : days)

which is read 4 is to 18 as 14 is to x. This means that if 4 lathes are assembled in 18 days, then 14 lathes are assembled in x days, and the problem now is to find the value of x.

It has been stated that the product of the extreme terms in a proportion equals the product of the intermediate terms, therefore,

$$4 \times x = 18 \times 14.$$

If $4 \times x$, or $4x$, as it is commonly written with the multiplication sign left out, equals 18×14, then one $x = \frac{18 \times 14}{4} = 63$ which is the same answer as was previously obtained.

If the previous proportion is written in the form of fractions, as previously explained, we have:

$$\frac{4}{18} = \frac{14}{x}.$$

The numerators of the fractions on each side of the equal sign may be multiplied by the denominators of the fractions on the opposite side and the products will be equal; that is, $4 \times x = 18 \times 14$, which is the same as has already been obtained, and which gives $x = 63$.

[The last two methods explained seem more cumbersome than the first solution of the problem, on account of the explanations of the methods here required. In the following examples, however, where no explanations are given, the simplicity of the methods is more apparent.]

Example (2).—If 6 pounds of high-speed steel cost $3.66, how much will be charged for 17 pounds?

Assume the price of 17 pounds to be x dollars.

$6 : 3.66 :: 17 : x$ (6 is to 3.66 as 17 is to x).

Multiplying the extreme and mean terms we have $6x = 17 \times 3.66$, and $x = \frac{17 \times 3.66}{6} = 10.37$. Seventeen pounds of high-speed steel would thus cost $10.37.

Example (3).—Thirty-four linear feet of bar stock are required for the blanks for 100 clamping bolts. How many feet of stock would be required for 912 bolts?

$x =$ total length of stock required for 912 bolts.

$34 : 100 :: x : 912.$

$34 \times 912 = 100x.$

$\frac{34 \times 912}{100} = x. \quad x = 310$ feet, almost exactly.

It should be noted in the examples above that x occupies any place in the proportion according to the requirements of the problem, and care should be used to make the written proportion correctly express the given conditions of the problem. In every direct proportion it is necessary to have the corresponding quantities occupy the same relative place on each side of the proportion or equal sign. In Example (3) we have, for instance,

34 : 100 :: x : 912
feet : pieces :: feet : pieces

In Example (2) we have, in the same way,

6 : 3.66 :: 17 : x
pounds : price :: pounds : price

Example of Inverse Proportion

A factory employing 270 men completes a given number of typewriters weekly, the number of working hours being 60 per week. How many men would be required for the same production if the working hours were reduced to 54 per week?

In this example the time per week is in an inverse proportion to the number of men employed; the *shorter* the time the *more* men. The example can be solved by the method previously explained, x is the number of men working 54 hours. The inverse proportion is written:

270 : x :: 54 : 60
men, 10-hr. basis : men, 9-hr. basis :: time, 9-hr. basis : time, 10-hr. basis

Note that in an inverse proportion the corresponding quantities occupy inverse or opposite places in the proportion.

Carrying out the calculation we have:

$$270 \times 60 = 54x; \qquad x = \frac{270 \times 60}{54} = 300.$$

Compound Proportion

Example (1).—If a man capable of turning 65 studs in a day of 10 hours is paid 32.5 cents per hour, how much ought a man be paid who turns 72 studs in a 9-hour day if compensated in the same proportion?

When solving problems involving compound proportion, the following method of analysis tends to simplify the solution. Make up a table with four columns headed, "First Cause," "First Effect," "Second Cause," "Second Effect," and place under each the respective factors given in the problem. In the example above the table would be arranged as below:

First Cause.	First Effect.	Second Cause.	Second Effect.
1 man	65 studs	1 man	72 studs
10 hours		9 hours	
32.5 cents		x cents	

Consider as *causes* the number of men working, the length of time they work, and their capacity for work; the pay received or the amount of product turned out in a unit of time indicates the capacity for work. The effect is the total product given either in numbers, or by the dimensions of the work carried out. The unknown quantity is called x.

When the table is completed, take all the quantities in the first and fourth columns and place them as the numerator of a fraction with multiplication signs between them, and all the quantities in the second and third columns and place them as the denominator of a fraction with multiplication signs between them. Put this fraction equal to 1. Then cancel and reduce the fraction to its simplest form as below.

$$\frac{1 \times 10 \times 32.5 \times 72}{65 \times 1 \times 9 \times x} = 1$$

$$\frac{40}{x} = 1, \text{ or } x = 40 \text{ cents.}$$

Example (2).—If 10 men working 9 hours per day, 6 days per week, can dig a trench 5 feet wide, 7½ feet deep, 5,760 feet long in 3 weeks, how many men working 10 hours per day 7 days per week will be required to dig a trench 7 feet wide, 8 feet deep and 12,500 feet long in 5 weeks? If we tabulate the conditions of the problem we have:

First Cause.	First Effect.	Second Cause.	Second Effect.
10 men	5 feet	x men	7 feet
9 hours	7½ feet	10 hours	8 feet
6 days	5,760 feet	7 days	12,500 feet
3 weeks		5 weeks	

Now placing the quantities in the first and four columns as the numerator, and those in the second and third columns as the denominator of a fraction with multiplication signs between the various factors, and canceling, we have:

$$\frac{10 \times 9 \times 6 \times 3 \times 7 \times 8 \times 12{,}500}{5 \times 7\tfrac{1}{2} \times 5{,}760 \times x \times 10 \times 7 \times 5} = 1$$

$$\frac{15}{x} = 1, \text{ or } x = 15 \text{ men.}$$

Example (3).—Fifteen automatic screw machines of a certain make turn out *a total* of 270 pieces per hour. It is planned to double the total product per day by installing machines of more modern production capable of producing *each* 25 pieces per hour. At the same time the working hours per day are to be reduced from 10 to 9. How many machines of the new type will be required to double the daily output?

It will be noted in this problem that the capacity of the new machines is given in production of each machine per hour, while the capacity of the old machines is given as the production of the total number of machines per hour. It is necessary that we give the capacity of the old machines in the same form as the capacity of the new machines. As 15 machines produce 270 pieces per hour, each machine produces $270 \div 15 = 18$ pieces per hour. Note that the capacity of the respective machines, 18 and 25 pieces per hour, are "causes" of their total production.

Another of the given conditions is that the total daily output should be doubled. As 270 pieces are now produced per hour, and the working day is 10 hours, the total daily production is $270 \times 10 = 2{,}700$. Double this, or 5,400 pieces, is the required output per day of the new equipment. Having obtained these figures we can now tabulate the conditions.

First Cause.	First Effect.	Second Cause.	Second Effect.
15 machines	2,700 pieces	x machines	5,400 pieces
10 hours		9 hours	
18 pieces		25 pieces	

Following the same method as shown above we have:

$$\frac{15 \times 10 \times 18 \times 5{,}400}{x \times 9 \times 25 \times 2{,}700} = 1$$

$$\frac{24}{x} = 1, \text{ or } x = 24 \text{ machines.}$$

CHAPTER V

USE OF FORMULAS

In mechanical books and articles, signs and symbols are used in order to condense in small space the essentials of long and cumbersome rules. The symbols used are generally the letters in the alphabet, and the signs are simply the ordinary signs for arithmetical calculations. Knowledge of algebra is not necessary for the use of formulas in solving simple problems; the letters in the formulas simply stand in place of the figures which are applied for specific cases or problems, and the whole thing is nothing but plain arithmetic.

In spur gears, the outside diameter of the gear can be found by adding 2 to the number of teeth, and dividing the sum obtained by the diametral pitch of the gear. This rule can be expressed very simply by a formula. Assume that we write D for the outside diameter of the gear, N for the number of teeth, and P for the pitch. Then the formula would be

$$D = \frac{N + 2}{P}$$

This formula reads exactly as the rule given above. It says that the outside diameter of the gear (D) equals 2 added to the number of teeth (N), this sum divided by the pitch (P).

Example: If the number of teeth in a gear is 16 and the pitch 6, then simply put these figures in the place of N and P in the formula and find the outside diameter as in ordinary arithmetic.

$$D = \frac{16 + 2}{6} = \frac{18}{6} = 3.$$

D, or the outside diameter, then, is 3 inches.

Example: In another gear the number of teeth is 96 and the pitch 7; find the outside diameter of the gear.

$$D = \frac{96 + 2}{7} = \frac{98}{7} = 14 \text{ inches.}$$

From the examples given it will be seen that in formulas, each letter stands for a certain dimension or quantity. When using a formula for solving a problem, replace the letters in the formula by the figures given in a certain problem, and find the result as in a regular arithmetical calculation.

Example: Assume a formula to be given as follows:

$$\text{Horsepower} = \frac{P \times L \times A \times N}{33{,}000}$$

If $P = 200$, $L = 2$, $A = 350$, and $N = 110$, what is the value of the horsepower? Insert the values given for the different letters, and multiply and divide as indicated.

$$\text{Horsepower} = \frac{200 \times 2 \times 350 \times 110}{33,000} = 466.6.$$

In formulas, the sign for multiplication ($\times$) is often left out between letters, the values of which are to be multiplied. Thus AB means $A \times B$, and the formula

$$\frac{P \times L \times A \times N}{33,000} \text{ can also be written } \frac{PLAN}{33,000}$$

If $A = 3$ and $B = 5$, then $AB = A \times B = 3 \times 5 = 15$. If $A = 12$, $B = 2$, and $C = 3$, then $ABC = A \times B \times C = 12 \times 2 \times 3 = 72$.

Use of Parentheses and Brackets

A parenthesis ( ) or bracket [ ] in a formula means that the expression inside the parenthesis or bracket should be considered as one single symbol, or in other words, that the calculation inside the parenthesis should be carried out by itself, before other calculations are carried out.

Examples:

$6 \times (8 + 3) = 6 \times 11 = 66.$
$5 \times (16 - 14) + 12 = 5 \times 2 + 12 = 10 + 12 = 22.$

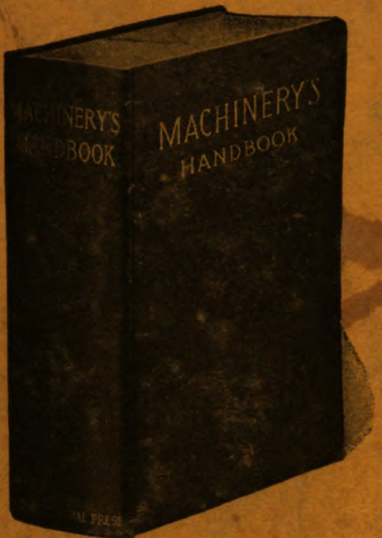
Order of Operations

When several numbers or expressions are connected by the signs $+$, $-$, $\times$ and $\div$, the operations are carried out in the order written, except that *all multiplications should be worked out before the other operations.* The reason for this is that numbers connected by a multiplication sign are only factors of the product thus indicated, which product should be considered by itself as one number. *Divisions should be carried out before additions and subtractions.*

Examples:

$5 \times 6 + 4 - 6 \times 4 = 30 + 4 - 24 = 34 - 24 = 10.$
$5 + 3 \times 2 = 5 + 6 = 11.$
$100 \div 2 \times 5 = 100 \div 10 = 10.$
but $5 \times (6 + 4) - 6 \times 4 = 5 \times 10 - 24 = 50 - 24 = 26.$
$(5 + 3) \times 2 = 8 \times 2 = 16.$
$(100 \div 2) \times 5 = 50 \times 5 = 250.$

The student is advised to refer to MACHINERY's Reference Book No. 52, "Advanced Shop Arithmetic," for a more complete treatise on the use of formulas in arithmetical calculations. This book also takes up a number of other subjects in arithmetic, and forms a continuation of the course begun in this book.



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