MACHINERY'S REFERENCE SERIES

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No. 12—MATHEMATICS OF MACHINE DESIGN

WITH SPECIAL REFERENCE TO SHAFTING AND EFFICIENCY OF HOISTING MACHINERY

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CHAPTER I.

MACHINERY SHAFTING.

At first thought it seems as if a chapter upon this subject should be prefaced with an apology. Undoubtedly shafting has been the subject of as much discussion as any thing mechanical, and the rules laid down for mill shafting are so well founded as to be available under all circumstances, and so widely known as to require no discussion. When it comes to machinery shafts, however, we have the ordinary rules for twisting and bending movements, and for the two combined, but we often find complicated combinations of loading needing investigation, and we find little or nothing in the text books about the present practice followed in fitting up machinery shafting.

In discussing the subject let us first take up briefly the three general principles governing shafting; simple twisting moments, simple bending moments, and combined twisting and bending moments.

When there is no bending moment the shaft may be designed for a simple twisting moment, and we have:

$$T = \frac{\pi}{16} d^3 f; \; d = \sqrt[3]{\frac{T}{0.196 f}} \qquad (1)$$

in which T = the twisting moment in inch-pounds,
d = the diameter of the shaft,
f = the fiber stress in pounds per square inch.

Table I gives the value of $\frac{\pi}{16} d^3$ for different diameters of shaft, and

$\frac{T}{f}$ = constant in table.

Example: A shaft is to sustain a twisting moment of 120,000 inch-pounds, the fiber stress being 16,000 pounds per square inch. Required, the diameter of the shaft.

$$\frac{120,000}{16,000} = 7.5.$$

The nearest constant in the table *above* 7.5 is 7.55, the diameter corresponding to which is 3⅜ inches, which is the diameter of the required shaft.

A form of shafting frequently found in machinery is the stationary shaft, upon which certain heavy parts revolve, the shaft remaining stationary in the bearings, while the revolving pieces are usually brass bushed. Such shafts are often called pins, and being required to transmit no twisting moment, may be proportioned for a simple

Table I. Values of $\frac{T}{f} = \frac{\pi}{16} d^3$

d	0	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	d
1	.196	.237	.279	.331	.383	.446	.510	.586	.663	.752	.842	.945	1.05	1.18	1.29	1.43	1
2	1.57	1.72	1.88	2.05	2.23	2.44	2.63	2.85	3.07	3.31	3.55	3.81	4.08	4.37	4.66	4.98	2
3	5.30	5.63	5.99	6.36	6.74	7.14	7.55	7.98	8.42	8.88	9.35	9.80	10.35	10.88	11.42	11.99	3
4	12.57	13.17	13.78	14.42	15.07	15.70	16.44	17.66	17.89	18.66	19.43	20.23	21.04	21.89	22.75	23.64	4
5	24.54	25.48	26.43	27.42	28.41	29.45	30.49	31.58	32.67	34.76	34.95	36.14	37.33	38.57	39.82	41.11	5
6	42.41	43.76	45.12	46.53	47.94	49.40	50.87	52.39	53.92	55.50	57.09	58.74	60.39	62.09	63.80	65.57	6
7	67.35	69.18	71.02	72.92	74.82	76.79	78.76	80.79	82.83	84.94	87.05	89.22	91.40	93.64	95.89	98.19	7

Table II. Values of $\frac{d^3}{10}$

d	0	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	d
1	0.1	.119	.142	.167	.195	.222	.259	.297	.337	.381	.429	.480	.536	.595	.659	.745	1
2	0.8	.874	.959	1.04	1.14	1.23	1.34	1.45	1.56	1.68	1.80	1.94	2.08	2.22	2.37	2.57	2
3	2.7	2.86	3.05	3.24	3.43	3.63	3 84	4.06	4.28	4.52	4.76	5 01	5.27	5.54	5.12	6.17	3
4	6.4	6.69	7.02	7.34	7.67	8.02	8.37	8.73	9.11	9 49	9.89	11.29	10.72	11.14	11.58	12.15	4
5	12.5	12.95	13.46	13.95	14.47	14.98	15.53	16.07	16.63	17.20	17 79	18.39	19.01	19.63	20.27	21.09	5
6	21.6	22.25	22.97	23.68	24.41	25.14	25.91	26.67	27.46	28.25	29 07	29.90	30.75	31.61	32.49	33.61	6
7	34.3	35.19	36.17	37.12	38.1	39.09	40.11	41.13	42.18	43.24	44.33	45.42	46.55	47.67	48.84	50.30	7

Table III. Values of h for Various Values of $\frac{R^2}{r+a}$ and n.

$\frac{R^2}{r+a}$	Values of n.					$\frac{R^2}{r+a}$	Values of n.				
	1	2	3	4	5		1	2	3	4	5
20	0.000001	0.000004	0.000009	0.000016	0.000025	100	0.000005	0.00002	0.000045	0.000082	0.000125
30	0.0000015	0.000006	0.0000135	0.000024	0.0000375	110	0.0000055	0.000022	0.0000495	0.00009	0.0001375
40	0.000002	0.000008	0.000018	0.000032	0.00005	120	0.000006	0.000024	0.000054	0.000097	0.00015
50	0.0000025	0.00001	0.0000225	0.00004	0.0000625	130	0.0000065	0.000026	0.0000585	0.000105	0.0001625
60	0.000003	0.000012	0.000027	0.000048	0.000075	140	0.0000070	0.000028	0.000063	0.000113	0.000175
70	0.0000035	0.000014	0.0000315	0.000056	0.0000875	150	0.0000075	0.00003	0.0000675	0.000121	0.0001875
80	0.000004	0.000016	0.000036	0.000064	0.0001	160	0.0000080	0.000032	0.000072	0.000129	0.000200
90	0.0000045	0.000018	0.0000405	0.000073	0.000112	170	0.0000085	0.000034	0.0000765	0.000137	0.0002125

bending moment. Equating the bending moment to the moment of resistance for a round section we have,

$$M = \frac{\pi}{32} d^3 f = 0.098\, d^3 f; \qquad \frac{M}{f} = \frac{d^3}{10} \text{ very nearly.}$$

Table II gives values of $\frac{d^3}{10}$ for various diameters of shafts.

Example: A pin is to take a bending moment of 65,000 inch-pounds at 16,000 pounds per square inch fiber stress. Required, diameter of the pin.

$$\frac{65{,}000}{16{,}000} = 4.06.$$

The nearest constant to 4.06 in the table is 4.06, the corresponding diameter being 3 7/16 inches.

Table IV. Values of n for Various Values of k.

k	n	k	n	k	n
.1	1.03	1.1	1.37	2.1	1.64
.2	1.06	1.2	1.40	2.2	1.66
.3	1.10	1.3	1.43	2.3	1.68
.4	1.13	1.4	1.46	2.4	1.71
.5	1.17	1.5	1.48	2.5	1.73
.6	1.20	1.6	1.51	2.6	1.75
.7	1.24	1.7	1.54	2.7	1.77
.8	1 27	1.8	1.56	2.8	1.79
.9	1.30	1.9	1.59	2.9	1.81
1.0	1 34	2.0	1.61	3.0	1.83

When a revolving shaft through which power is transmitted, carries gears, drums or other devices, it is subject to both bending and twisting moments, and a calculation by either of the above tables ignoring the other will result in a weak shaft. We may, however, substitute for the simple twisting moment T a greater twisting moment T_e, which will be the equivalent of the combined twisting moment T, and bending moment M.

This equivalent twisting moment is

$$T_e = M + \sqrt{M^2 + T^2} \text{ *}$$

The diameter of shaft suitable for the combined moments T and M, may be found by substituting T_e for T in equation (1). This formula may be more conveniently used in the following form: Letting the ratio

$$\frac{M}{T} = k, \text{ and } D = nd,$$

where D = the diameter of shaft for combined moments T and M, d = the diameter of shaft for simple twisting moment T, and

$$n = \sqrt[3]{k + \sqrt{k^2 + 1}} = \sqrt[3]{1.83\, k + 0.83}, \text{ approximately.}$$

* This formula used by the author gives a considerably larger value to the combined twisting and bending moments than the formulas commonly used. It gives, in effect, a larger factor of safety.

Table IV gives values of n for various values of k.

Example: Suppose the shafts of the two previous examples are one, that is a shaft subjected to a twisting moment of 120,000 inch-pounds, and a bending moment of 65,000 inch-pounds. Required, the diameter of the shaft, with the fiber stress at 16,000 pounds per square inch as before.

$$k = \frac{65,000}{120,000} = 0.54$$

The nearest corresponding value of n is 1.17.

The diameter d for a twisting moment of 120,000 inch-pounds found in the first example is 3⅜ inches.

$D = 3.375 \times 1.17 = 3.94$ inches, say a 4-inch shaft.

A comparison of the three examples will show the importance of considering both the bending and twisting moments upon any shaft that is subject to both actions.

Gears, drums and other detail parts are so distributed upon shafting

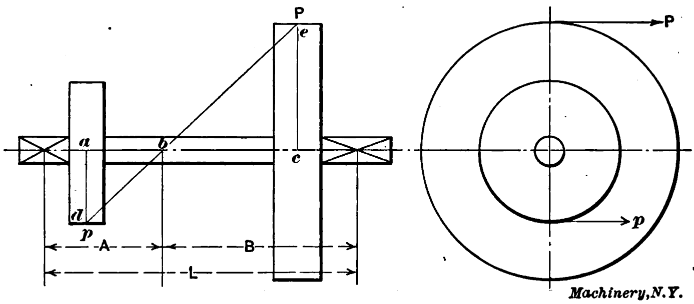


Fig. 1.

as to cause combined strains, and the maximum fiber stress resulting therefrom must be determined. A simple case is that of two gears between bearings, Fig. 1, the large and small gear respectively carrying loads P and p, the loads acting in the same direction. Connect P and p by a line cutting the axis at b.

Since the shaft must be in equilibrium, we have $p \times da = P \times ec$, and

$$\frac{p}{P} = \frac{e\,c}{d\,a}$$

By the law of similar triangles,

$$\frac{e\,c}{d\,a} = \frac{e\,b}{b\,d}$$

and substituting we have,

$$\frac{p}{P} = \frac{e\,b}{b\,d}$$

and, consequently, $p \times bd = P \times eb$.

The reaction at b is then $p + P = W$,

and the bending moment is $M = \frac{W\,A\,B}{L}$

The torsional moment is,

$$T = p \times ad.$$

$$k = \frac{M}{T} = \frac{W A B}{L \times p \times a d} = \frac{(p + P) A B}{L \times p \times a d}$$

Another case of combined strains is shown in Figs. 2 and 3, where Fig. 2 shows an elevation, and Fig. 3, diagrammatical end views. In

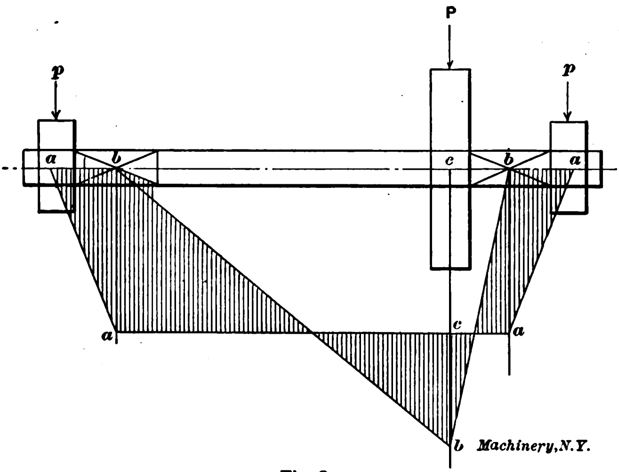


Fig. 2.

this example a gear under load *P* drives two pinions each under load *p*. First consider case A where all loads are in the same direction. Draw the bending moment diagram *b-b-b* for load *P*, also the diagram

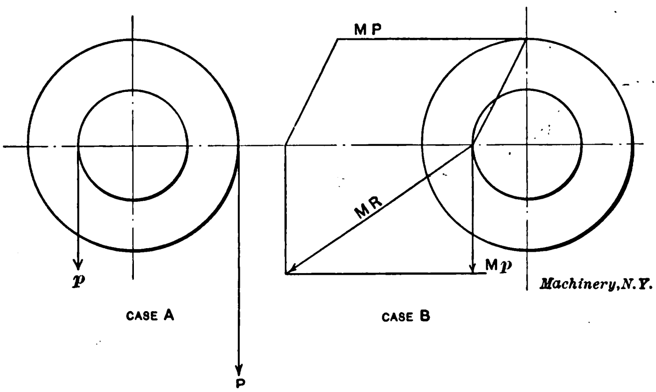


Fig. 3.

a-a-a-a for loads *p*. It is obvious, since the loads are on opposite sides of the bearing, that the bending moments will oppose each other, and the shaded portion of the diagram represents the algebraic sum of the two moment diagrams. Any ordinate of the shaded portion is the bending moment of the corresponding point of the shaft.

In case B the loads are not in the same direction. To find the moment at any given point on the shaft it is necessary to lay off separately, in the direction of each force, the bending moment ordinate for that force, as $MP = eb$, and $Mp = ba$, and take the resultant MR as the bending moment at the given point. This method may be extended to all forms of loading, as illustrated in the following examples:

Example: A shaft, Fig. 4, carrying a gear loaded to 2,140 pounds tooth load, drives two pinions, each under 1,925 pounds tooth load. Required, the size of the shaft suitable for the given dimensions, the fiber stress being 9,000 pounds per square inch.

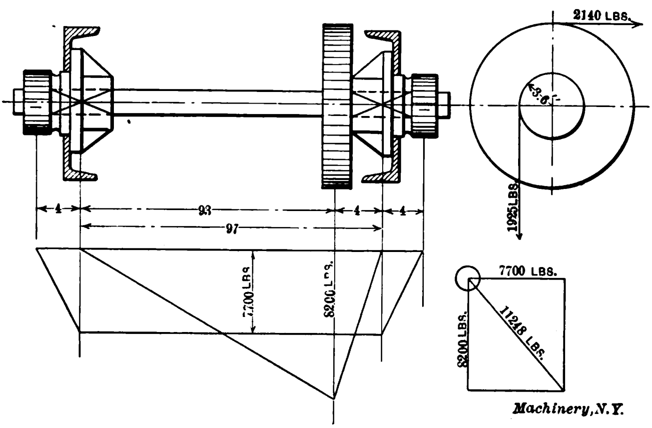


Fig. 4.

$$T = 1{,}925 \times 3.6 = 6{,}930 \text{ inch-pounds,}$$

$$Mp = 1{,}925 \times 4 = 7{,}700 \text{ inch-pounds,}$$

$$MP = \frac{2{,}140 \times 93 \times 4}{97} = 8{,}200 \text{ inch-pounds,}$$

$$MR = 11{,}248 \text{ inch-pounds.}$$

$$k = \frac{MR}{T} = \frac{11{,}248}{6{,}930} = 1.6, \text{ the corresponding value of } n \text{ being } 1.51.$$

$$\frac{T}{f} = \frac{6{,}930}{9{,}000} = 0.77$$

From Table I, of values for twisting moments, we find the nearest constant above 0.77 is 0.842 for a 1⅝-inch shaft. Then $1.625 \times 1.51 = 2.45$ inches for the required diameter of the shaft.

In all such cases as the above example, in which a shaft is supported upon channels or other unsymmetrical supports, the bending moment must be calculated to the center of gravity of the supporting member.

Example: In Fig. 5 we have a drum shaft of given dimensions and following conditions: Drum loose on shaft and bushed; weight of drum 500 pounds; two ropes leading from drum each under 3,750 pounds load; weight of gear, 250 pounds; tooth load on drum gear horizontal; rope loads vertical; fiber stress to be 10,000 pounds per square inch. Resolving all loads to the heavier loaded journal we have

$$\frac{6{,}226 \times 24}{30} = 4{,}980 \text{ pounds horizontal load,}$$

$$\frac{3{,}750 \times 17}{30} = 2{,}125 \text{ pounds, rope } c,$$

$$\frac{3{,}750 \times 4}{30} = 500 \text{ pounds, rope } d,$$

$$\frac{250 \times 24}{30} = 200 \text{ pounds, weight of gear,}$$

$$\frac{500}{2} = 250 \text{ pounds, weight of drum,}$$

$$\text{Total} = 3{,}075 \text{ pounds vertical load.}$$

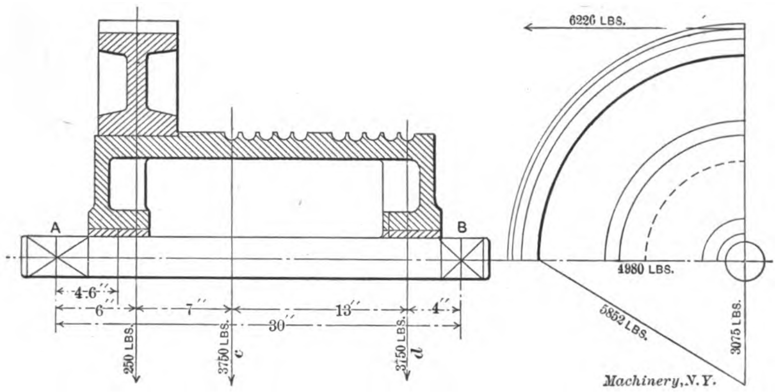


Fig. 5.

The resultant of the horizontal and vertical loads on *A* is 5,852 pounds. Then $M = 5{,}852 \times 4.6 = 26{,}919$ inch-pounds.

$$\frac{M}{f} = \frac{26{,}919}{10{,}000} = 2.69$$

In Table II, of values from simple bending moments, the nearest constant is 2.7 for a 3-inch shaft.

Fig. 6 shows a common arrangement of drum and gear, in which *a* is the center of gravity of the rope loads *P*. Where there are two ropes on the drum, the position of *a* is constant, while for one rope the position of *a* varies along the drum, and for the latter case several solutions should be made with varying positions of *a*. The load is supported upon three points, the journals *A* and *B* and the gear teeth at *C*. The load *P* puts a downward load upon each journal *A* and *B*, and is divided proportionally between them. In the figure *a* is central, so the loads upon A and B are equal, and are $P/2$; the upward load p at C is divided proportionally between A and B; thus we have:

$$\left.\begin{aligned}\text{Load at } A &= \frac{C\,m}{L}\\ \text{Load at } B &= \frac{C\,n}{L}\end{aligned}\right\}\text{ upward loads.}$$

$$\text{Loads at } A \text{ and } B = \frac{P}{2}, \text{ downward loads.}$$

The algebraic sum of the two loads upon either journal gives the amount and direction of the resultant load on the journal. Draw BC, and produce Ca to cut AB at d. We thus represent the load P as

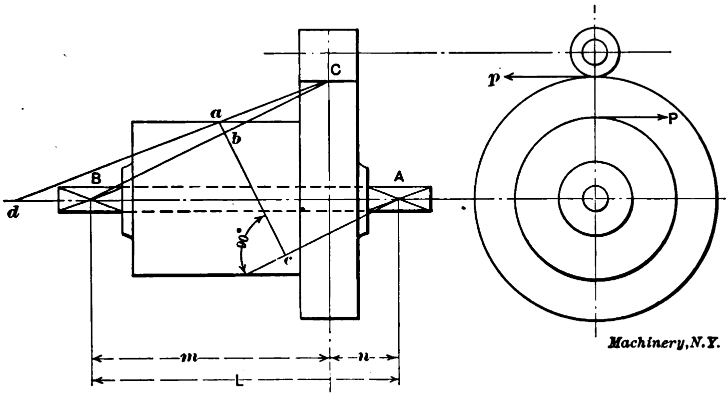


Fig. 6.

eccentrically supported upon a beam BC, at an arm ab, and prevented from rotating about BC as an axis by the reaction of journal A acting at an arm bc. We thus have

$$\text{Load at } A = \frac{P \times a\,b}{b\,c} = W, \text{ upward load.}$$

$$\left.\begin{aligned}\text{Load at } C &= \frac{(P + W)\,B\,b}{B\,C}\\ \text{Load at } B &= \frac{(P + W)\,C\,b}{B\,C}\end{aligned}\right\}\text{ downward loads.}$$

The condition of loading at journal A is seen from the position of point d, which, lying beyond B as in the figure, indicates an upward load at A, lying on B indicates no load at A, and lying between A and B indicates a downward load at A, the weight of the drum and gear being neglected.

Fig. 7 shows the same arrangement with the pinion on the opposite side from a, and this case is analogous to that shown in Fig. 1.

A shaft requiring special investigation in certain classes of machines, as cranes, turntables and other revolving machines, is that effecting the slewing or turning in a horizontal plane. Fig. 8 represents the diagram of a common slewing mechanism for a crane, in which

A = the center pin, column or mast,

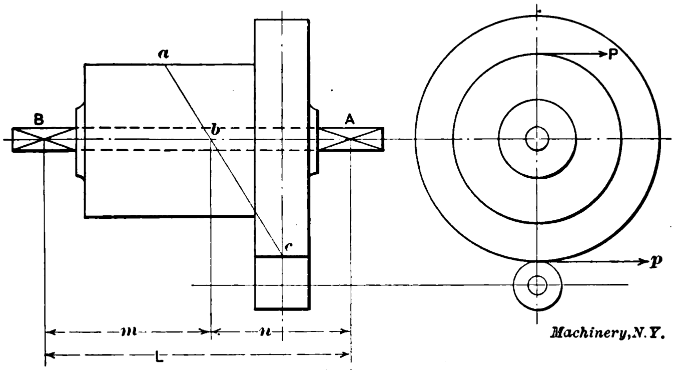


Fig. 7.

B = a large circular rack, concentric with A,

C = a pinion mounted upon a vertical shaft, and meshing with the rack B.

W = the load in pounds,

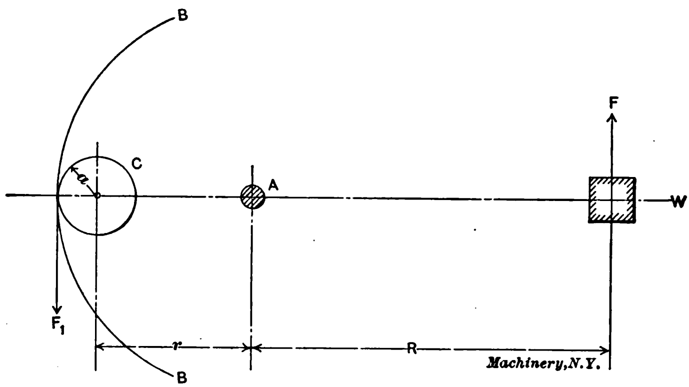


Fig. 8.

R = the radius of the boom in feet,

r = the radius of the circle described by the slewing shaft,

a = the radius of the pinion C,

n = the number of revolutions per minute of the crane,

V = the velocity of the load W in feet per second $= \frac{2 \pi R n}{60}$

$V^2 = 0.01\ R^2 n^2$,
S = space in feet in which full velocity is to be obtained,
F = force in pounds acquired by the load W,
F_1 = force in pounds on the slewing shaft,
g = acceleration = 32.2,

$$\text{Energy} = \frac{W V^2}{2 g} = F S$$

Then,

$$F = \frac{W V^2}{2 g S} = \frac{W V^2}{64.4 S} \qquad (2)$$

Assuming S = one-quarter of a turn, $= \frac{2 \pi R}{4} = 1.57\ R$, and substituting the values of S and V^2 in (2) we have,

$$F = 0.0001\ R\ W\ n^2$$

Now

$$F_1 = \frac{F R}{r + a} = \frac{0.0001\ R^2\ W\ n^2}{r + a}.$$

The twisting moment on the slewing shaft then is

$$T = F_1\ a = \frac{0.0001\ R^2\ W\ n^2\ a}{r + a}. \qquad (3)$$

Substituting in (3) the value $\frac{\pi}{16} d^3 f$ for T, and assuming $f = 10.000$ pounds per square inch, we have

$$\frac{\pi}{16} d^3 f = \frac{0.0001\ R^2\ W\ n^2\ a}{r + a}$$

$$d^3 = \frac{0.00000005\ R^2\ W\ n^2\ a}{r + a}$$

$$d = \sqrt[3]{0.00000005 \frac{R^2\ n^2}{r + a} W a}$$

If we assume $0.00000005 \frac{R^2\ n^2}{r + a} = h$, we may write the last formula

$$d = \sqrt[3]{h\ W\ a} \qquad (4)$$

Table III gives values of h for various values of the ratio $\frac{R^2}{r + a}$ and n, assuming $f = 10,000$ pounds per square inch. For $f = 12,000$, multiply the values by 0.833, and for $f = 16,000$, multiply by 0.625.

The pinion C may be either overhung, or mounted between bearings, as shown respectively in Fig. 9 and Fig. 10. The values of k are as follows:

Overhung Shaft, Fig. 9.

The bending moment

$$M = F_1 L = \frac{0.0001\, R^2 W n^2 L}{r + a}$$

and the twisting moment

$$T = \frac{0.0001\, R^2 W n^2 a}{r + a}$$

and

$$k = \frac{M}{T} = \frac{L}{a}$$

Shaft between Bearings, Fig. 10.

The bending moment

$$M = \frac{F_1 L}{2} = \frac{0.0001\, R^2 W n^2 L}{2\,(r + a)}$$

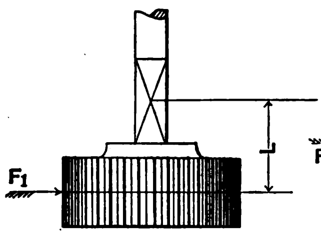


Fig. 9.

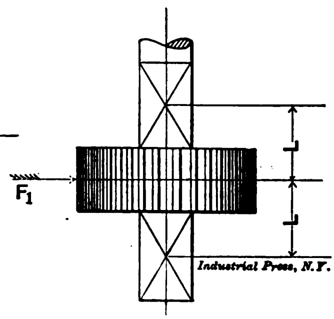


Fig. 10.

and the twisting moment

$$T = \frac{0.0001\, R^2 W n^2 a}{r + a}$$

and

$$k = \frac{M}{T} = \frac{L}{2a}$$

In taking the values of W and R, not only the load and the radius of the boom must be considered, but also the weight and radius of such heavy parts of the machinery as may revolve with the crane, in each case resolving the turning moment of such parts about the center pin or mast, to the radius R.

Example: Fig. 11 represents a steam crane, the letters corresponding to those of Fig. 8, while the dimensions and weights given are those of a particular crane having a capacity of ten tons at a radius of sixteen feet, the pinion C meshing into an internal spur rack in the foundation and being driven by bevel gears as shown. Required, the diameter of the vertical shaft, f being 12,000 pounds per square inch.

The load at 16 feet radius = ...20,000 pounds.
The block at 16 feet radius = ... 435 "
The jib weighs 3,000 pounds, its center of gravity being at a radius of 11½ feet, resolved to a radius of 16 feet $= \dfrac{3{,}000 \times 11.5}{16} =$ .... 2,156 "
The boiler and extension weigh 11,855 pounds at a center of gravity radius of 8½ feet, resolved to a radius of 16 feet $= \dfrac{11{,}855 \times 8.5}{16} =$ .... 6,297 "
The machinery and side frames weigh 21,800 pounds, at a radius of 2 feet, resolved to a radius of 16 feet $= \dfrac{21{,}800 \times 2}{16} =$ .... 2,725 "

Total load assumed at a radius of 16 feet = ...31,613 pounds.

$R = 16$ feet, $r = 2$ feet, and $a = 9$ inches; consequently,

$$\frac{R^2}{r + a} = \frac{16^2}{2 + \frac{3}{4}} = 93$$

Opposite 90 and under $n = 3$ in Table III, 3 being the required revolutions per minute of the crane in question, the value of h is 0.0000405, and for 93 the value of h would consequently be about 0.000042, and since the stress per square inch is 12,000 pounds, we have

$$h = 0.000042 \times 0.833 = 0.000035.$$

From (4) we have

$$d = \sqrt[3]{0.000035 \times 31{,}613 \times 9} = 2.15 \text{ inches diameter.}$$

for twisting only.

The pinion being mounted between bearings, and $L = 6$ inches, we have

$$k = \frac{6}{18} = 0.33$$

Corresponding to this value of k we find the nearest value of $n = 1.10$ in Table IV. Then

$d = 2.15 \times 1.1 = 2\frac{3}{8}$ inches, approximately = diameter of shaft required.

When calculating the size of shafting, the first thing of importance to determine is the length of the journal, and once established, all bending moments should be taken to the center of the journal. Given the diameter, the length of the journal depends upon three conditions: Bearing area, character of lubricant, and ability to carry off heat. Grease has become a most widely used lubricant for heavy machinery, possessing as it does sufficient body to enable the designer to use higher bearing values than with other lubricants without squeezing the lubricant from the bearing. It has been found in practice that to limit the surface speed to 350 feet per minute, and the bearing value to 80 pounds per square inch of projected area, produces, with grease as a lubricant, a very satisfactory bearing, well within the limits of good performance as regards heating.

Let d = the diameter of the shaft,

l = the length of the journal,

a = the projected area, $= d \times l$,

W = total pounds pressure on journal,

R.P.M. = revolutions per minute of the shaft.

Then the pressure per square inch is,

$$\frac{W}{a} = \frac{W}{d \times l} = 80 \text{ pounds}; \qquad d = \frac{W}{80\,l};$$

The surface speed, in feet, is,

$$\frac{\pi\, d \times \text{R.P.M.}}{12} = 350; \qquad d = \frac{12 \times 350}{\pi \times \text{R.P.M.}} = \frac{1337}{\text{R.P.M.}}$$

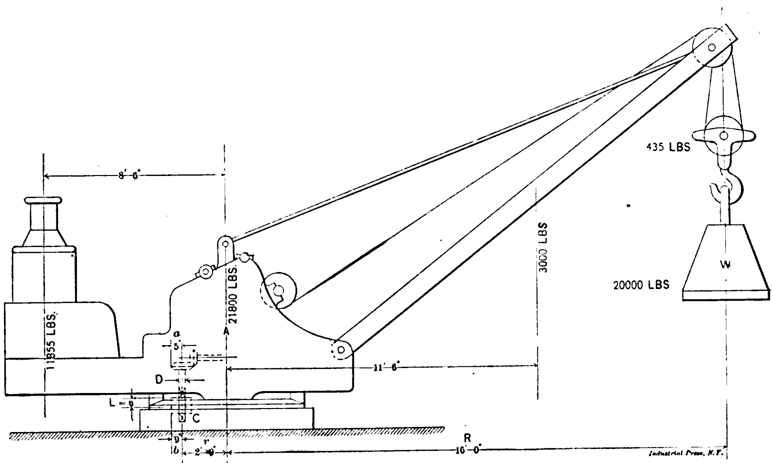


Fig. 11.

Equating these two values of d we have,

$$\frac{W}{80\,l} = \frac{1337}{\text{R.P.M.}}; \qquad l = \frac{W \times \text{R.P.M.}}{106{,}960};$$

and rounding off the constant to a more convenient figure, we have

$$l = \frac{W \times \text{R.P.M.}}{100{,}000}$$

Cases will arise, especially with heavily loaded slow running shafts, in which this rule gives a bearing altogether too short for practice, sometimes not allowing room for the stud bosses on the cap, and also having too high a bearing value, which should be kept below 1,000 pounds per square inch. For shafts running about 80 R.P.M. or faster, the above rule gives excellent bearings, while for slower running shafts an investigation of the bearing value as well as the above rule at once determines the limiting length of the journal.

Chart Fig. 14 will be found of great convenience in determining the dimensions of journals. The example shown by the heavy line in the upper portion of the chart, shows that a journal under 700 pounds total pressure, and running 1,250 R.P.M. should be about 8 inches long. This at once determines the length of the journal and leaves the diameter

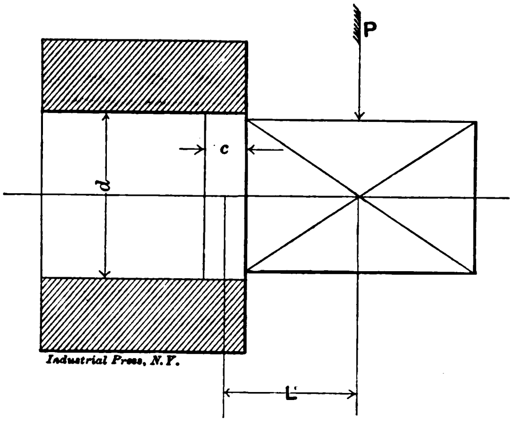


Fig. 12.

to be determined by calculating the bending moments to the center of the journal. The example given in the lower part of the chart shows that a journal 4 inches in diameter and 8 inches long under 5,250 pounds total pressure, will have a bearing value of 163 pounds per square inch of projected area.

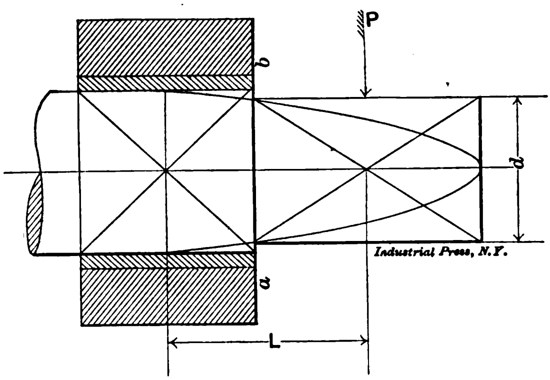


Fig. 13.

A condition frequently met with is that of a shaft forced by pressure into one member, and revolving in another member, as in Fig. 12. The arm L is determined by laying off a distance c such that

$$\frac{P}{c \times d} \overset{=}{<} \text{the safe crushing strength of the metal composing the supporting member.}$$

and taking L as the distance from the load P to the center of the strip c. It is excellent practice to provide slight shoulders wherever practicable, against which to key the gears, thus locating gears definitely for the assembling workman. Frequently a heavily loaded pinion demands a smaller shaft than the foregoing rules require, in order to leave sufficient metal over the key. This may be accomplished by making the diameter d at the section $a-b$, Fig. 13, coincident with the diameter of a paraboloid drawn as shown.

A most convenient solution of this problem is offered in Chart Fig. 15, which also solves at a glance all problems regarding shaft diam-

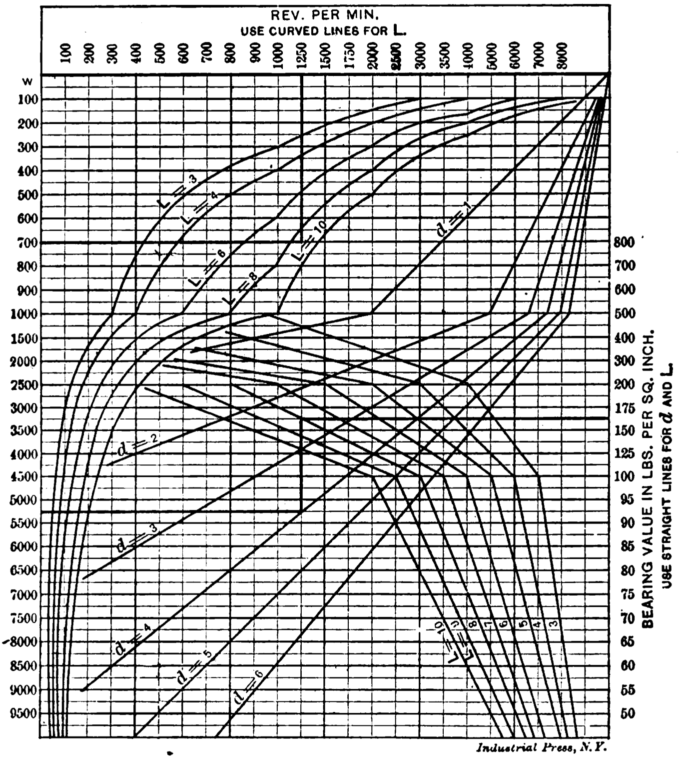


Fig. 14.

eters to withstand any combination of moments. The shaft shown at the top is worked out in the chart by following the dotted lines. The shaft is subject to a bending moment of 247,500 inch-pounds and a twisting moment of 165,000 inch-pounds, hence $k=1.5$, and $f=9{,}000$. Enter the chart at the left at 165,000 inch-pounds and follow dotted line to the $f = 9{,}000$ line, thence up to the $k = 1.5$ line, thence over to the right, and read diameter of required shaft on the scale, 6¾ inches. It is now required to find the smallest permissible diameter for the end of the shaft according to the dimensions given. Follow the line

marked 15 in the extreme left-hand column to first intersection with the line for 6¾ inches from the scale at the bottom of the chart, thence diagonally to the line marked 5½ at the extreme left, thence up, and read the required diameter on the scale at the top, 4⅞ inches.

For shafts subjected to simple bending or simple twisting moments

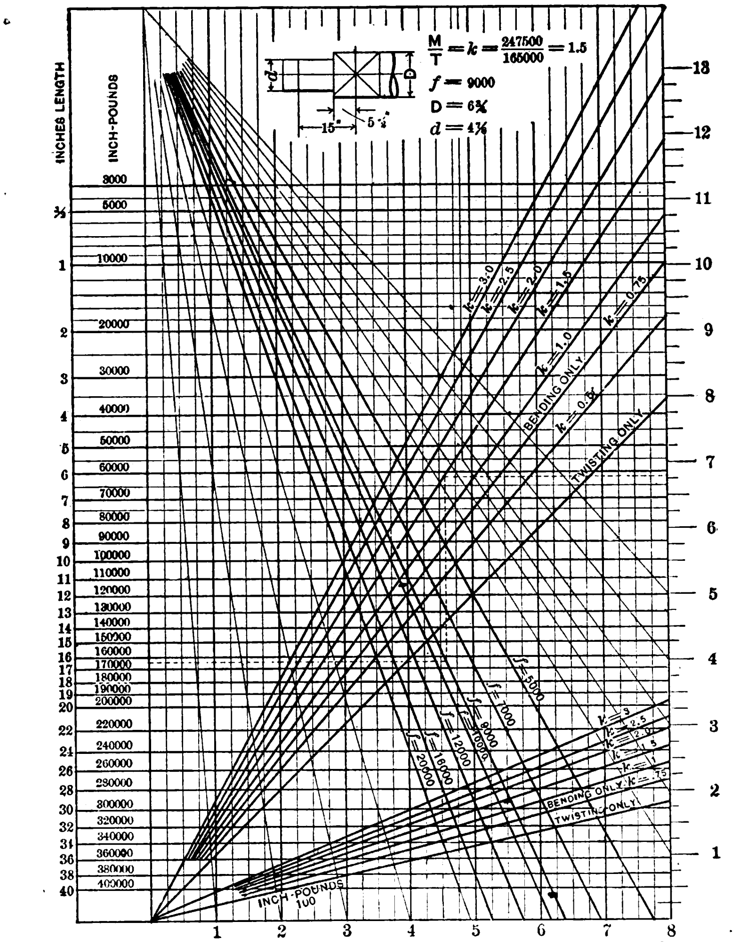


Fig. 15?

use the lines so marked instead of the k lines. The set **of lines**

marked $\frac{\text{inch-pounds}}{100}$ are to be used for smaller shafts by **dividing all**

readings in the column of inch-pounds by 100. This makes the **range of** the chart from 30 to 400,000 inch-pounds.

CHAPTER II.

EFFICIENCY OF MECHANISM.

WITH SPECIAL REFERENCE TO HOISTING MACHINERY.

When undertaking the development of any machine, the designer is promptly brought to face the question of the probable efficiency of the mechanism he wishes to employ. If the machine belongs to a class with which the designer has been long familiar, he may be able to judge closely from past experience, as to what efficiency to assume in his present calculations. If the designer cannot bring to his aid such past experience, he may be told by the chief engineer to assume a particular value for the efficiency. Failing in both past experience and the availability of the chief engineer, the designer may attempt a wild guess, more or less remote from actual conditions, possibly seeking information from a handbook, where he may find something like the following, from D. A. Low's Pocket-book for Engineers:

Mechanical Efficiency of Machines.

$P =$ force acting at the driving point,
$W =$ force acting at the working point,

$$r = \text{velocity ratio of the machine} = \frac{\text{velocity of working point}}{\text{velocity of driving point}}$$

$p =$ value of P when $W = 0$,
$e =$ a coefficient,
$E =$ mechanical efficiency of the machine,
When friction is neglected $P \div W = r$.
When friction is taken into account $P = (1 + e)\ Wr + p$.
For a particular machine the preceding equation reduces to $P = mW + k$, where m and k are constants determined from experiments with the machine. Finally,

$$E = \frac{Wr}{(1 + e)\ Wr + p} = \frac{Wr}{m\ W + k}$$

This is exceedingly disappointing, as the inconvenience of experimenting with a particular machine yet unbuilt, with a view of determining constants to be used in calculations during its design, is apparent. Consequently the aforesaid wild guess is too often used as a basis from which to calculate the probable performance of the machine.

The determination of the efficiency of any elementary portion of a machine by analysis is, however, a comparatively simple matter, and by dividing the proposed machine into several such elementary portions, and determining by analysis the efficiency of each element, the approximate efficiency of the whole machine may be determined. The following analysis of some simple portions of machinery may easily be extended by the application of the same reasoning to other cases, and the tables may form a guide for an intelligent guess which will come nearer the truth than a wild guess.

Efficiency Defined.

The force exerted to run any kind of machine is used in the performance of two functions: To perform the intended useful work for which the machine is designed; and to overcome the frictional resistances in the several parts of the machine. If the machine could be considered as running with absolutely no frictional resistance between its moving parts, we should have the product of force into space moved equal to the product of load into space moved; or

$$P_1 s = L h; \quad P_1 = \frac{L h}{s} \qquad (5)$$

in which P_1 = the theoretical force, which acting through a space s, will move a load L a certain distance h, under the assumption that there are no frictional resistances in the machine.

The force exerted through the space s must, however, overcome the frictional resistances within the machine, as well as the resistance of the load L through the distance h. Let W = the sum of all the frictional resistances within the machine, and w = the sum of all the distances through which the several frictional resistances are overcome. Then Ww = the work done in overcoming the frictional resistances of the several parts of the machine. The actual force, acting through a space s, besides being required to move the load a distance h, must in addition be sufficient to overcome the frictional resistances within the machine itself; so the actual effort required to move the load is;

$$P s = L h + W w, \text{ or } P = \frac{L h + W w}{s} \qquad (6)$$

Thus from (5) and (6) it is seen that the actual force P must be greater than the theoretical force P_1.

The ratio of the theoretical to the actual force is termed the efficiency of the machine; thus

$$e = \frac{P_1}{P}$$

As has been seen, P is always greater than P_1, and it follows that the efficiency of any machine being always less than unity, represents the percentage of the force exerted which is actually employed in moving the load. The use of this ratio expressing the efficiency is of the greatest value in practical problems relating to the force required to run any given machine; because, in practically all cases, the theoretical force

$$P_1 = \frac{L h}{s}$$

in which the three factors L, h, and s, are known, may be more or less easily determined, and a knowledge of the efficiency e of the particular machine under consideration then enables the designer to determine at once the force required for the particular case, as

$$e = \frac{P_1}{P}, \text{ and } P = \frac{P_1}{e}.$$

The value of e for any machine is easily computed when the efficiencies of the several moving parts are known. Let e_1, e_2, e_3 ——— e_n be the efficiencies of the several moving parts of the machine; then the efficiency of the whole machine is,

$$e = e_1 \times e_2 \times e_3 \times \cdot \cdot \cdot \; e_n \qquad (7)$$

Since the moving parts of most machinery may be reduced to a few classes or heads, a knowledge of the average values of the efficiency of each class will, in most cases, enable the designer to arrive at results sufficiently accurate for practical purposes.

In the preceding discussion it has been assumed that the force P acts in a direction to move the load forward, and that the frictional resistances act against the force P, in the same direction as the load L. The relation of power to load and frictional resistances is well illustrated in the case of a crane; and such a machine will hereafter be used in the discussion, it being understood that what is said applies as well to any class of machinery.

Efficiency of Backward Motion.

When a crane is at rest with a load L suspended, the force P is being exerted to maintain the load in suspension, and prevent it running down. In this case the frictional resistances within the machine are acting in the same direction as P, and usually the work Ww done in overcoming them is the same for the backward as for the forward motion; while L becomes the actuating force, and P acts as a retarding force to prevent acceleration when lowering the load.

Thus, when the load is being lowered, we have

$$Lh = Ps + Ww, \text{ or } P = \frac{Lh - Ww}{s} \qquad (8)$$

while as before we have

$$P_1 = \frac{Lh}{s} \qquad (9)$$

which clearly indicates that when lowering the load, the force P, which must act to prevent acceleration, is less than the theoretical force P_1. By the efficiency of a machine for the backward motion is understood the ratio of the actual force required to prevent acceleration when lowering the load, to the theoretical force required to effect the same result could the frictional resistances within the machine be neglected.

Thus for backward motion,

$$= \frac{P}{P_1}$$

Substituting in this equation the values of P and P_1 in (8) and (9) we have,

$$e_b = \frac{\frac{Lh - Ww}{s}}{\frac{Lh}{s}} = \frac{Lh - Ww}{Lh}$$

from which we see that when $Ww = Lh$, $e_b = 0$, and the internal forces of frictional resistance and L are balanced without the application of P; also when Ww is greater than Lh, e_b has a negative value, and an additional force P acting in the same direction as L, must be applied at the point of application of the power in order to lower the load.

A negative efficiency on the backward motion may therefore be taken as an indication that the load will remain suspended upon the removal of the motive power. This is a feature especially to be desired in all cranes as a safety device for those operating them. It is, however, often obtained by the sacrifice of high efficiency on the forward movement. For the forward motion we had

$$e = \frac{P_1}{P}$$

and substituting in this equation the values of P_1 and P from (5) and (6) we have,

$$e = \frac{\frac{Lh}{s}}{\frac{Lh + Ww}{s}} = \frac{Lh}{Lh + Ww}$$

which, under the assumption that the work done in overcoming the frictional resistances within the machine is the same for both forward and backward motion, and assuming the limiting case $Ww = Lh$, becomes

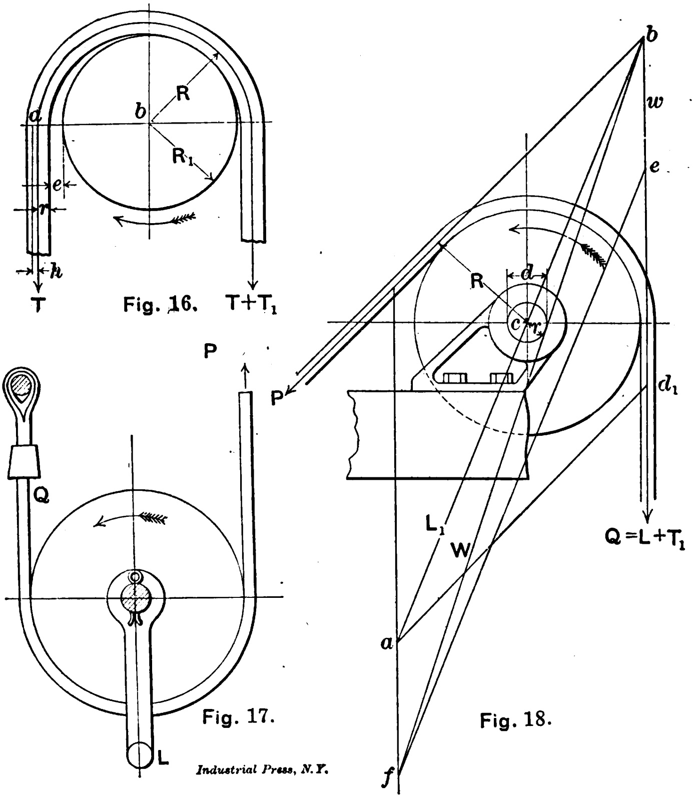
$$e = \frac{Lh}{Lh + Lh} = \frac{1}{2}$$

Thus the efficiency for the forward motion of all elementary self-locking machines which automatically sustain the load, never exceeds 50 per cent, while for cases where e_b is negative the efficiency is less than 50 per cent.

This statement is true for all elements in machine design intended to be used as power transmission elements for the forward movement, while being expected, from the nature of the design, to resist all backward impulses due to the load when the power is removed. It is possible, however, by the introduction of devices which, being idle during the forward movement, are called into action by the slightest backward movement of the parts to which they are attached, and which, being so put into action, present additional frictional resistances acting in the direction of the force P, to design a machine of any given efficiency for the forward movement, which will automatically sustain the load when the power is removed.

Rigidity of Ropes.

When considering the efficiency of the different classes of mechanism combined to form a hoisting machine, it will be seen that the resistance of ropes to bending around sheaves and drums enters largely into the equations for the efficiency of these parts. Any rope offers resistance, by reason of its rigidity, when wound onto a sheave or drum,



Figs. 16 to 18.

while by reason of its elasticity, little or no resistance is offered when it unwinds and passes off the sheave or drum.

In Fig. 16 let T = the tension in the *on* side of the rope about to be wound around a sheave, and $T + T_1$ = the tension in the *off* side of the rope; then T_1 = the force required to bend the rope around the sheave while under the tension T. Let R_1 = the radius of the sheave, and d = the diameter of the rope, while r = the radius of the rope $\frac{d}{2}$.

Then let $R_1 + r = R$.

The lever arm of the rope axis on the *off* side, is,

$$R_1 + r = R.$$

Considering the *on* side of the rope, the fibers on the outside are stretched, while those on the inside are compressed, and the resultant of these two forces with the force T will lie to the outside of the rope axis a distance denoted by h.

Then the lever arm of the *on* side is

$$ab = R_1 + e + r + h = R + e + h.$$

The distance e is given by DuBois as

$$e = \frac{k}{T} \text{ for hemp rope, and } e = \frac{k\,R}{T} \text{ for wire ropes,}$$

where k is a constant to be determined by experiment.

The condition for equilibrium is then for wire ropes

$$T\left(R + \frac{k\,R}{T} + h\right) = (T + T_1)R, \text{ or } T_1 = k + \frac{T\,h}{R}.$$

Experiment gives this formula the form,

$$T_1 = 1.08 + \frac{0.09\,T}{R} \text{ for wire ropes,}$$

$$T_1 = \frac{100 + 0.22\,T}{R} \text{ for tarred hemp rope,}$$

$$T_1 = \frac{4 + 0.065\,T}{R} \text{ for untarred hemp rope,}$$

where T and T_1 are expressed in pounds, and R in inches. (DuBois.)

The efficiency of the rope, neglecting the journal friction of the sheave, is

$$e = \frac{T}{T + T_1}.$$

Example: A one-inch wire rope under 20,000 pounds tension is wound over a 15-inch sheave. Neglecting the journal friction of the sheave, what force $(T + T_1)$ will be required to raise the load of 20,000 pounds?

Here $T = 20,000$.

$R_1 = 7.5$.

$R = 8$.

then, $T_1 = 1.08 + \dfrac{0.09 \times 20,000}{8} = 226.08$ pounds,

and $T + T_1 = 20,226.08$ pounds.

The efficiency in this case is

$$e = \frac{T}{T + T_1} = \frac{20,000}{20,226.08} = 0.989.$$

Table V gives the efficiency of plough steel wire ropes when strained to their full working capacity, and wound over sheaves or upon drums of the smallest diameter that should ever be used with each size of rope. It will be observed that the diameters given in this table are much smaller than those recommended by the rope manufacturers as the minimum to be used with each size of rope. The diameters given here are those in constant use by many of the foremost crane builders, it being found impracticable to use the large sheaves and drums recommended in the space at the disposal of the designers.

TABLE V. EFFICIENCY OF WIRE ROPES.

Diam. of Rope.	Min. Diam. of Drum or Sheave under Rope.	Efficiency $e = \frac{T}{T + T_1}$
½	10	0.982
⅝	12	0.985
¾	14	0.987
⅞	16	0.989
1	18	0.990
1⅛	20	0.991
1¼	22	0.992
		Average, 0.988

The Fixed Pulley.

Let Fig. 18 represent a fixed pulley or rope sheave, over which a rope is passed, by means of which a force P is to lift a load L. The spaces s and h through which P and L move, respectively, are equal ($s = h$), hence, neglecting all friction and lost power, we have the theoretical force,

$$P_1 = L \qquad (10)$$

The wasteful resistances to be overcome are: 1st, the stiffness of the rope requiring the additional force T_1, which may be added to the load, making the total force acting in the *on* side of the rope

$$Q = L + T_1, \text{ and}$$

2nd, the journal friction due to the resultant pressure L_1 of P and Q, and the weight w of the sheave.

Produce P and Q to meet at b. Lay off to any convenient scale $bd_1 = Q$, and draw d_1a parallel to Pb. Then, similarly, lay off P on Pb. Then $ab = L_1$, and when measured to scale gives the resultant pressure on the journal due to P and Q. Lay off on bd_1 to the same scale as before, $be = w$, the weight of the sheave. Draw ef parallel to ab, and draw bf. Then $bf = W$, and when measured to scale gives the total pressure W on the journal, due to the resultant of the forces P and Q and the weight w of the sheave.

We now have three forces acting, P, Q, and W, of which Q and W are acting in the same direction, opposed to P, and as the distances through which these forces move are proportional to the lever arm in each case, we have the condition of equilibrium, letting the coefficient of journal friction $= \phi$

$$PR = QR + Wr\phi$$

$$P = \frac{QR + Wr\phi}{R} \qquad (11)$$

From (10) and (11) we have the efficiency

$$e = \frac{P_1}{P} = \frac{L R}{Q R + W r \phi}$$

In making calculations, we may at first neglect the stiffness of the rope, in which case $Q = L$, and we have the efficiency with the rope neglected,

$$e_1 = \frac{L R}{L R + W r \phi}$$

Let e_2 = the efficiency of the rope from Table V. Then we have the efficiency, including the rope,

$$e = e_1 \times e_2.$$

The maximum value of L_1 is reached when P and Q are parallel, and is then $P + Q = L_1$; the weight w of the sheave may be neglected as having little influence upon the efficiency; the rigidity of the rope may be neglected at first and brought into the solution afterwards, as shown above; then $P + Q = 2L$. Then under these assumptions, *viz.*, $W = L_1 = P + Q = 2L$, and $Q = L$, we have by substitution in (11)

$$P = \frac{L R + 2L r \phi}{R} \qquad (12)$$

Thus from (10) and (12) we get the minimum efficiency of a fixed sheave, neglecting the weight of the sheave, and letting the efficiency of the rope $= e_2$ as above

$$e = \frac{e_2 P_1}{P} = \frac{e_2 L}{\frac{L R + 2L r \phi}{R}} = \frac{e_2 R}{R + d \phi}.$$

Table VI gives the minimum efficiency of the smallest diameter of sheave allowable with each size of rope, assuming in each case the load L on the rope to be the full working strength of the rope, the arc of contact to be 180 degrees, the coefficient of journal friction 0.08, the diameter of journal pin 4 inches, and values of e_2 taken from Table V.

TABLE VI. EFFICIENCY OF THE FIXED SHEAVE.

Diam. of rope.	Diam. Sheave.	e_2 for Rope, Table V.	e for Sheave.	Coeff. of Resistance, k.
½	10	0.982	0.925	1.081
⅝	12	0.985	0.936	1.068
¾	14	0.987	0.945	1.058
⅞	16	0.989	0.952	1.050
1	18	0.990	0.957	1.045
1⅛	20	0.991	0.961	1.040
1¼	22	0.992	0.965	1.036
	Average	0.988	0.948	1.055

From (12) we have, including the efficiency of the rope e_2,

$$P = \left(\frac{R + d \phi}{e_2 R}\right) L, \text{ and letting } \frac{R + d \phi}{e_2 R} = k, \text{ we have}$$

$$P = k\,L \qquad (13)$$

in which k is the coefficient of resistance of the sheave and rope combined. From (10) and (13) we have,

$$e = \frac{P_1}{P} = \frac{L}{k\,L} = \frac{1}{k}\text{, and } k = \frac{1}{e}$$

In the fifth column of Table VI, the values of k are calculated under the same conditions as are those of e, so that knowing either the power applied, P, or the load to be lifted, L, the other may be easily calculated with sufficient accuracy by the use of the above tabular values in the two equations

$$P = kL \text{ and } L = eP.$$

For the backward motion when the load is descending, we have

$$L = kP\text{, and } P = \frac{L}{k}.$$

The distance through which L acts is equal to the distance through which P acts; hence letting s equal this distance, we have the work performed at the point of application of each force, P and L, as Ps, and

$$Ps = \frac{Ls}{k},$$

and the efficiency

$$e = \frac{Ps}{Ls} = \frac{1}{k}.$$

Thus the efficiency of a fixed sheave is the same for the backward as for the forward motion.

Movable Pulley or Sheave.

In the case of movable pulleys or sheaves, Fig. 17, as in pulley blocks, the ropes are always parallel, or nearly so, and letting Q = the tension produced in the *on* side of the rope by the load L, we have

$$P = kQ$$

and the condition of equilibrium is

$$L = P + Q = Q + kQ = Q\,(1 + k).$$

To raise the load L a distance s, each end of the rope must be shortened by a distance equal to s, and as the end Q is fixed, this is accomplished by the end P moving upwards a distance equal to $2s$.

The total work performed is then $P \times 2s$, or $2kQs$,
and the useful work performed is Ls, or $Qs\,(1 + k)$,
while the efficiency is

$$e = \frac{Qs\,(1 + k)}{2kQs} = \frac{1 + k}{2k}$$

The efficiency of a fixed sheave was shown to be $e = 1/k$, and as k is always greater than unity, we see that the efficiency of a movable pulley is greater than that of a fixed pulley.

For the reverse motion, with the load descending, we shall have the tension in the ends of the rope reversed, and $Q = kP$, while as before $L = P + Q = P + kP = P\,(1 + k)$.

The work performed is $Ls = Ps\ (1 + k)$, while the useful work performed is $2Ps$, and the efficiency is

$$e = \frac{2\,Ps}{Ps\,(1 + k)} = \frac{2}{1 + k}$$

for the backward movement.

Table VII gives the minimum forward efficiency under the same conditions as for Table VI.

TABLE VII. EFFICIENCY OF THE MOVABLE PULLEY.

Diam. of Rope.	Diam. Sheave.	Coef. Resistance k.	Efficiency $e = \frac{1+k}{2k}$
½	10	1.081	0.962
⅝	12	1.068	0.968
¾	14	1.058	0.972
⅞	16	1.050	0.976
1	18	1.045	0.978
1⅛	20	1.040	0.980
1¼	22	1.036	0.982
	Average	1.055	0.974

The Tackle.

Any combination of fixed and movable pulleys or sheaves whereby power is multiplied, enabling large resistances to be overcome, is called tackle. The most usual form of tackle is that shown in Fig. 21, in which *A* represents the fixed sheaves mounted in some portion of the machine, and *B* represents the movable sheaves in the block to which the load is attached. The sheaves are usually of one diameter, and mounted upon one pin, those in the figure being made of varying diameters to enable the winding of the ropes to be clearly shown. By means of the tables given for the fixed and movable pulleys, we may obtain the efficiency of any arrangement of tackle. Inasmuch as the tackle shown represents a large majority of those in use, it is well to investigate the efficiency of such tackle as a unit.

The efficiency is in inverse proportion to the number of sheaves in the tackle, which is determined by the number of runs of rope to be used, which in turn is determined (friction being neglected) by the relation

$$\frac{L}{n} = t, \text{ or } n = \frac{L}{t}$$

in which L = the load, n = the number of ropes. t = the tension in each rope, L and t being the known factors determining n.

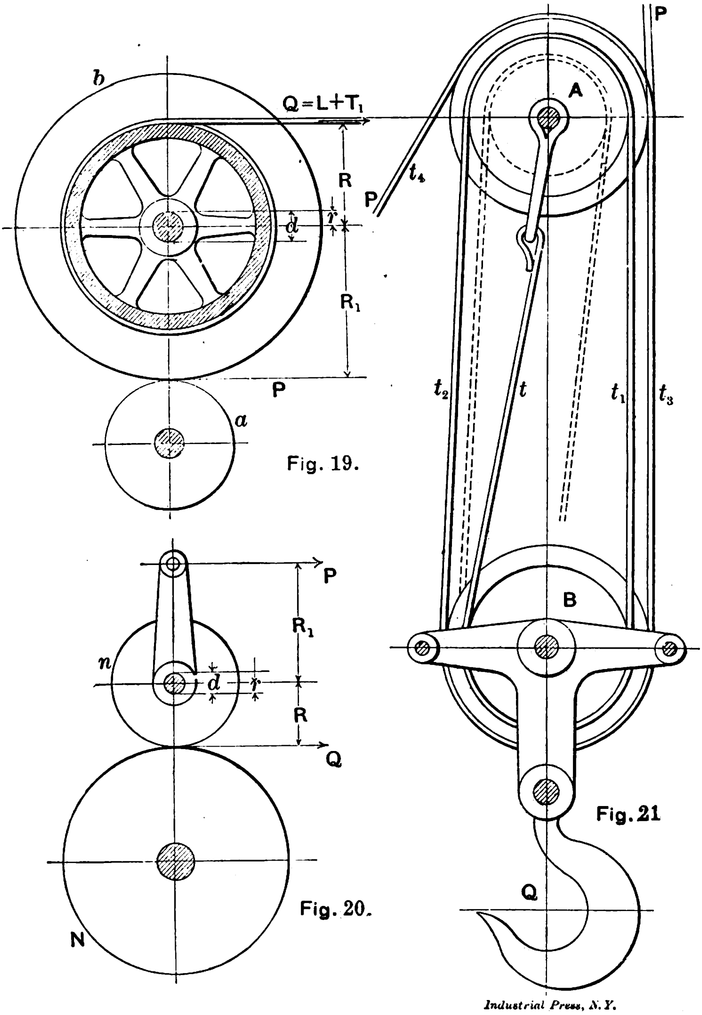
Thus neglecting friction and all hurtful resistances, we have

$$P_1 = \frac{L}{n}$$

Taking all hurtful resistances into account, it will be seen that the tensions in the several runs of rope are not equal. Thus if t = the tension in the first rope, $t_1 = kt$ = the tension in the second rope, $t_2 = kt_1 = k^2t$ = the tension in the third rope, and $t_{(n-1)} = k^{(n-1)}\,t$ = the tension in the nth rope. The power end of the rope is not included in the n runs of rope as it has no direct lifting power, the n

runs including only those directly connected to the movable block.

In Fig. 21, t_3 = the tension in the last run of rope, and as shown above, $t_4 = kt_3 = k^4 t$ = the tension in the power end of the rope.



Figs. 19 to 21.

In general, then, for n runs of rope, the tension in the power end $= k^n t$, or

$$P = k^n t \qquad (14)$$

The hurtful resistances may be considered as added to the load, which then becomes equal to the sum of the tensions in the several ropes connected to the movable block, and we have

$$Q = t + k\,t + k^2\,t + k^3\,t \;\cdot\; + k^{(n-1)}\,t$$

$$Q = t\,(1 + k + k^2 + k^3 + k^4 \;\cdot\; + k^{(n-1)} = \frac{t\,(k^n - 1)}{k - 1} \qquad (15)$$

Denoting the distance through which P moves in a unit of time by s, the distance through which Q moves in the same time is s/n, and the efficiency of the tackle is

$$e = \frac{\frac{s}{n}Q}{P\,s} = \frac{Q}{n\,P}$$

Substituting the values of P and Q from (14) and (15) we have

$$e^* = \frac{\frac{t\,(k^n - 1)}{k - 1}}{n\,k^n\,t} = \frac{k^n - 1}{n\,k^n\,(k - 1)}$$

TABLE VIII. EFFICIENCY OF TACKLE.

Diam of Rope.	Min. Diam. of Sheave.	Value of k.	Efficiency $= e = \frac{k^n - 1}{n\,k^n\,(k-1)}$ — Number of Runs of Rope: 2	3	4	5	6	Average.
½	10	1.081	.888	.856	.828	.793	.767	.825
⅝	12	1 068	.909	.879	.853	.826	.800	.853
¾	14	1.058	915	.893	.869	.843	.823	.868
⅞	16	1.050	.927	.907	.880	.861	.845	.884
1	18	1.045	.935	.915	.897	.875	.860	.896
1⅛	20	1.040	.941	.925	.898	.880	.871	.903
1¼	22	1 036	.948	.933	.909	.897	.883	.912
	Average		.923	.915	.875	853	.835	

Table VIII gives the efficiency of tackle under the same conditions as were assumed for single fixed and movable pulleys.

Winding Drums.

Let Fig. 19 represent a winding drum operated by two gears, of which the pitch lines are a and b, by means of which a load L is to be moved by the application of a force P at the pitch line of the larger gear b. The distances through which P and L move are proportional to the radii of the gear and drum, so that

$$P_1\,R_1 = L\,R \text{ and } P_1 = \frac{L\,R}{R_1} \qquad (16)$$

* The formula here given by the author does not give the strictly *theoretical* efficiency. To obtain this only the original weight Q at the hook of the tackle should be considered. This would give $e = \frac{Q}{n\,k^n\,t}$, Q denoting the original load only.

The wasteful resistances to be overcome are: 1st, the stiffness of the rope requiring an additional force T_1, which may be added to the load L, making the total force acting in the rope

$$Q = L + T_1, \text{ and}$$

2d, the journal friction due to the resultant pressure of P and Q, and to the weight W of the drum when wound full of rope or chain.

The stiffness of the rope may be neglected at first to be brought into the solution later, which makes $Q = L$, and assuming P and Q to be parallel, the maximum value of their resultant is $P + Q = 2L$, when the condition of equilibrium, (ϕ being the coefficient of journal friction) becomes

$$P R_1 = L R + 2 L r \phi + W r \phi = L R + r \phi (2 L + W)$$

$$P = \frac{L R + r \phi (2 L + W)}{R_1} \qquad (17)$$

Letting the efficiency of the rope $= e_2$, and neglecting the weight W of the drum and rope, we get from (16) and (17) the efficiency

$$e = \frac{P_1}{P} e_2 = \frac{\frac{L R}{R_1}}{\frac{L R + 2 r \phi L}{R_1}} e_2$$

$$e = \frac{e_2 R}{R + 2 r \phi} = \frac{e_2 R}{R + d \phi}$$

where d equals the diameter of drum shaft.

Table IX gives the efficiency of drums for various sizes of ropes strained to their full capacity, the drum shafts being of such assumed diameter as to cover extreme practice, and the coefficient of journal friction being taken as 0.08.

TABLE IX. EFFICIENCY OF WINDING DRUMS.

Diam. of Rope.	Min. Diam. of Drum.	Value of e_2	Assumed Diam. of Shaft	$e = \frac{e_2 R}{R + d \phi}$	Coef. of Resistance $k = \frac{1}{e}$
½	10	0.982	3	0.939	1.064
⅝	12	0.985	3½	0.943	1.060
¾	14	0.987	4	0.945	1.058
⅞	16	0.989	4½	0.947	1.056
1	18	0.990	5	0.950	1.053
1⅛	20	0.991	5½	0.951	1.051
1¼	22	0.992	6	0.953	1.049
			Average,	0.949	1.053

In the case of heavy drums wound with large wire rope or heavy chain, it is sometimes desirable to take into account the weight W, in which case the formula should be used.

Gearing.

In Fig. 20, let $P =$ the power applied at a radius R_1, to drive a gear of pitch radius R, having n teeth, and meshing with a gear having

N teeth, at the pitch line of which there is a resistance to rotation, $= Q$. Neglecting all wasteful resistances, we have the theoretical force

$$P_1 = \frac{Q\,R}{R_1} \qquad (18)$$

The wasteful resistances to be overcome are the friction between the teeth of the two gears, and the friction of the journal A.

The friction between the teeth is given by Weisbach as

$$Z = \pi\,\phi\,Q\left(\frac{1}{n} + \frac{1}{N}\right)$$

which, assuming the direction of Q to be normal to the common center line of the two gears, and $\phi = 0.11$, becomes, approximately,

$$Z = \frac{Q}{3}\left(\frac{1}{n} + \frac{1}{N}\right) = \frac{Q}{3\,n} + \frac{Q}{3\,N}$$

which acts through a lever arm R.

In most cases occurring in practice, the forces P and Q are either arranged on opposite sides of a journal bearing, or one force at each end of a shaft next to a journal bearing. It is thus a close approximation to actual conditions to take the weight upon the journals as $W = 2Q$. Then taking into consideration the wasteful resistances, we have the condition of equilibrium,

$$P\,R_1 = Q\,R + \left(\frac{Q}{3\,n} + \frac{Q}{3\,N}\right)R + 2\,Q\,\phi\,r$$

$$P = \frac{Q\left[R\left(1 + \dfrac{1}{3\,n} + \dfrac{1}{3\,N}\right) + 2\,\phi\,r\right]}{R_1} \qquad (19)$$

The efficiency of the gear and shaft is

$$e = \frac{P_1}{P} = \frac{\dfrac{Q\,R}{R_1}}{\dfrac{Q\left[R\left(1 + \dfrac{1}{3\,n} + \dfrac{1}{3\,N}\right) + 2\,\phi\,r\right]}{R_1}}$$

$$= \frac{R}{R\left(1 + \dfrac{1}{3\,n} + \dfrac{1}{3\,N}\right) + \phi\,d}$$

where $d =$ the diameter of shaft.

As the expression $\left(1 + \dfrac{1}{3\,n} + \dfrac{1}{3\,N}\right)$ increases in value as the number of teeth diminishes, it follows that the number of teeth in the large gear remaining constant, the efficiency of the pinion and shaft is greater as the number of teeth n is greater. A smaller pinion than one of thirteen teeth being little used, $n = 13$ has been taken as the basis of Table X. The pitch of the pinion being taken as 1¾ inches, gives the pitch diameter for a pinion of thirteen teeth as 7¼ inches nearly. The diameter of the pinion shaft is taken as 3 inches, and the table gives the efficiency of the pinion and shaft when running in mesh with gears of various numbers of teeth, giving various velocity ratios.

Efficiency of a Complete Machine.

From the consideration of the efficiency of the several mechanisms which, combined, form a machine, we may by the general formula already given,

$$e = e_1 \times e_2 \times e_3 \times e_4 \ldots\ldots\ldots\ldots\ldots\ldots e_n$$

pass to the determination of the efficiency of the complete machine.

Fig. 22 represents the several elementary parts of a crane. The power is applied at the crank upon the crankshaft *A*, and a pinion *a* upon

TABLE X. EFFICIENCY OF GEARS AND SHAFTS.

$$e = \frac{R}{R\left(1 + \frac{1}{3n} + \frac{1}{3N}\right) + d\,\phi}$$

No. of Teeth in Pinion.	Pitch in inches.	Pitch Diam. of Pinion Approx.	Diam. of Pinion Shaft.	Number of Teeth in Gear and Velocity Ratio.														
				T	R	T	R	T	R	T	R	T	R	T	R	T	R	
13	1¾	7¼	3	13	1:1	20	1:1½	26	1:2	39	1:3	65	1:5	91	1:7	117	1:9	Average
				.922		.929		.933		.937		.940		.941		.942		.934
Coef. resistance $k = \frac{1}{e} =$				1.084		1.076		1.072		1.067		1.064		1.062		1.061		1.070

this shaft meshes into a gear *b* upon the intermediate shaft *B*. A pinion b_1 upon this shaft meshes with a gear *c* upon the drum shaft *C*. Thus the power is transmitted to the drum *E* upon which the rope which passes to the tackle *D* is wound, and thus the load *Q* is raised. The speed reduction of the two gears is shown in the figure as 1 : 4 in each case, and the tackle is shown as having four runs of rope. This completes the elementary crane. There are many devices such as friction pawls, ratchet wheels, band brakes, etc., which are applied to cranes for various purposes, but these do not as a rule affect the efficiency of the machine.

In the following consideration of the efficiency of the complete machine the average values given in the preceding tables will be used as follows:

e_1 = the efficiency of pinion *a* and shaft *A* (Table No. X) .....= 0.934
e_2 = the efficiency of pinion b_1 and shaft *B* (Table No. X) ..... = 0.934
e_3 = the efficiency of drum *E* and shaft *C* (Table No. IX) ....= 0.949

e_4 = the efficiency of tackle D, 4 ropes (Table No. VIII) ..... = 0.875
e = the efficiency of the complete machine = $e_1 \times e_2 \times e_3 \times e_4$, or
$e = 0.934 \times 0.934 \times 0.949 \times 0.875 = 0.723$.

While on account of the small force available for operation, the hand crane is usually double geared, the steam crane, being operated by much greater force, is often single-geared. Thus, should a steam crane be applied to the elementary crane of Fig. 22, the shaft A, pinion a, and gear b would be omitted, and shaft B would become the engine

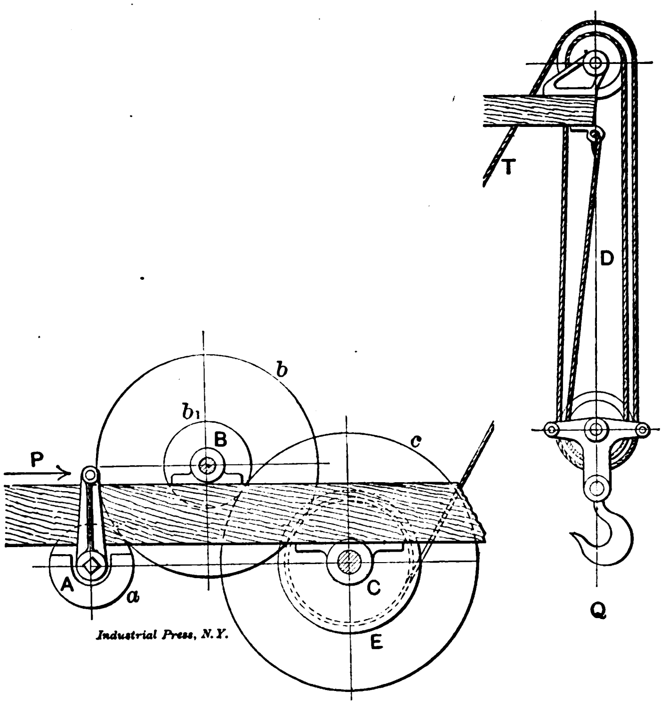


Fig. 22.

shaft. The mechanical efficiency of small, simple slide valve engines is given by several authorities as 85 per cent to 90 per cent. Assuming the smaller of these two values, we have

e_1 = the efficiency of the engine 0.850
e_2 = the efficiency of the pinion and gear 0.934
e_3 = the efficiency of the drum and shaft 0.949
e_4 = the efficiency of the tackle 0.875
$e = 0.850 \times 0.934 \times 0.949 \times 0.875 =$ 0.656

Taking the above value obtained for the hand crane as a basis, we have the coefficient of resistance for the complete crane, $k = 1/e = 1/0.72 = 1.38$.

Example: One man can exert a force of about 30 pounds upon a crank handle. Four men are working at a crank 16 inches long, the ratio of the gears $a - b$ and $b - c$ is 1 to 4 in each case, and the diameter of the drum is 24 inches; the force or pull in the rope wound around the drum is

$$T = \frac{120 \times 16 \times 4 \times 4}{12} = 2{,}560 \text{ pounds.}$$

Fig. 22 shows the crane as having four runs of rope, which gives the load

$$Q = 2{,}560 \times 4 = 10{,}240 \text{ pounds.}$$

The actual load L that can be raised by four men working this crane would be, assuming the efficiency as 72 per cent,

$$L = 10{,}240 \times 0.72 = 7{,}372 \text{ pounds, or about } 3\tfrac{1}{2} \text{ tons.}$$

Conversely: A load of 3½ tons is to be raised by such a crane. We have the force or pull in the rope

$$T = \frac{7{,}000}{4} = 1{,}750 \text{ pounds.}$$

Then the power P required is

$$P = \frac{1{,}750 \times 12}{4 \times 4 \times 16} = 82 \text{ pounds, nearly.}$$

The coefficient of resistance is 1.38, and we have the actual force required on the crank F as

$$P = 82 \times 1.38 = 113 \text{ pounds, nearly,}$$

which would be fair work for four men.