DYNAMO ELECTRIC MACHINERY;

ITS CONSTRUCTION, DESIGN, AND OPERATION

DIRECT-CURRENT MACHINES

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PREFACE.

THE object aimed at in the preparation of the first edition of this book has been kept in view and has controlled the preparation of this eighth edition. This has been the production of a text-book for the use of students pursuing electrical or non-electrical engineering courses. The method of presentation is considered as especially adapted for classroom exercises, which consist of recitations, computations, and occasional lectures, and which are supplemented by laboratory exercises, the two being correlated with a view to training the mind of the student and adding somewhat to his knowledge. It will be found that in treatment the sequence is such that parts which it may seem undesirable to require from other than electrical engineering students may be omitted without introducing a discontinuity in the matter which remains.

With the exception of the first two chapters, the book has been entirely rewritten; nearly two hundred of its illustrations are new, most of them having been specially drawn to make clear methods of construction or characteristics of operation; and it has been considerably extended in scope. In the new matter will be found a set of problems at the end of each chapter, a presentation of the theory of commutation, means for the predetermination of the operating characteristics of direct-current generators and motors, a discussion on storage batteries from the engineering point of view, a treatment of the theory of balancers and of boosters, and a discussion of costs, prices, and operating expenses of machines and plants.

The chapters on the design of machines and on tests, which appeared in the former editions, have been omitted, as these subjects require for their adequate treatment more space than one would be warranted in giving them in a book of this character.

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DYNAMO ELECTRIC MACHINERY.

CHAPTER I.

ELECTRICAL LAWS AND FACTS.

1. **Mechanical Units.** — *Force* is that which tends to produce, alter, or destroy motion. The units of force are the dyne and the poundal. The *dyne* is that force which, acting on a one-gram mass for one second, will tend to produce a velocity of one centimeter per second. The *poundal* is that force which, acting on a mass of one pound for one second, will tend to produce a velocity of one foot per second. The weight of a pound mass is frequently taken as a unit of force, and is called, for brevity, a *pound*. A force of one pound is approximately equal to 32.2 poundals.

Work is the production of motion against resistance. The units of work are the foot-pound and the erg. The *foot-pound* is the work done in lifting a body weighing one pound one foot vertically. The *erg* is the work performed by a force of one dyne in moving a body one centimeter in the direction of the force. The *joule* is a larger unit much used, and is equal to 10^7 ergs.

Energy is the capacity to do work. It is expressed in the same units as work. The two classes of energy are *Kinetic* energy and *Potential* energy. A body possesses kinetic energy in virtue of its motion, while potential energy is due to the separation or the disarrangement of attracting particles or masses. A wound-up spring has potential energy because of the strained positions of the molecules, while a weight raised to a height has potential energy because of the separation of its mass from the attracting mass of the earth. The potential energy of a body is measured by the work required to put the body into its strained condition. The kinetic energy of a body is proportional to its mass and to the square of its velocity, or

$$\text{Kinetic Energy} = \frac{Wv^2}{2g},$$

since the mass of a body is equal to its weight divided by the acceleration due to gravity. Kinetic energy will be expressed in ergs when W is the weight in grams of the body whose velocity is v centimeters per second, and when g is 981 cm. per second per second. If W be the weight in pounds, v the velocity in feet per second, and g is 32.2 feet per second per second, then the kinetic energy is expressed in foot-pounds.

Power is the rate of performance of work. Its units are the horse-power and the watt. A *horse-power* is 33,000 foot-pounds per minute. A *watt* is one joule per second. One horse-power is equivalent to 746 watts. Representing the torque or twisting moment of a machine in pound-feet by T, and its angular velocity in radians per second by $\omega = 2\pi V/60$, where V is the number of revolutions per minute, then the horse-power of the machine is

$$\text{H.P.} = \frac{60\,\omega T}{33000} = \frac{2\pi VT}{33000}.$$

In a belt-driven machine the torque in the shaft is equal to the difference in tension of the two sides of the belt multiplied by the radius of the pulley, that is, $T = (F - F') r$ pound-feet.

2. **Electrical Units.** — Since distinction must continually be made between *absolute* or *c.g.s.* units and *practical* units, throughout this work capital letters will be used for quantities expressed in practical units, and lower-case letters for quantities expressed in absolute units.

The absolute unit of *current* is such that, when flowing through a conductor of one centimeter length, which is bent into an arc of one centimeter radius, it will exert a force of one dyne on a unit magnet pole (§ 10) placed at the center. The practical unit of current, the *ampere*, is one-tenth the magnitude of the absolute unit.

The absolute unit of *quantity* is that quantity of electricity which in one second passes any cross-section of a conductor in which the absolute unit of current is flowing. The practical unit of quantity is one-tenth of the absolute unit, and is called the *coulomb*. For large quantities the *ampere-hour* and the *mega-coulomb*, the latter being equal to a million coulombs, are units frequently used; and for small quantities the *micro-coulomb*, equal to one-millionth of a coulomb, is often used.

The absolute unit of *difference of potential* exists between two points when it requires the expenditure of one erg of work to move an absolute unit quantity of electricity from one point to the other. The practical unit of difference of potential, the *volt*, is 10^8 times as large as the absolute unit.

It is convenient and rational to make a distinction between electromotive force and difference of potential. Electromotive force is produced when a conductor cuts magnetic lines of force, or when the electrodes of a primary battery are immersed in a solution. But a difference of potential may exist due to the flow of an electric current. Between any two points of a conductor carrying a current there is that which would send a current through an auxiliary wire connecting these points, and it is called difference of potential. If the current in the original conductor be doubled, the difference of potential between the same two points will be doubled, showing that this difference of potential exists because of the current flowing in the original conductor. The word *pressure* is used either for difference of potential or for *E.M.F.* with obvious relevancy.

The absolute unit of *resistance* is offered by a body when it allows an absolute unit of current to flow along it between its two terminals, when these are maintained at unit (absolute) difference of potential. The practical unit of resistance, the *ohm*, is 10^9 times as large as the absolute unit. The *megohm*, equal to a million ohms, and the *microhm*, equal to one-millionth of an ohm, are units frequently used.

3. Ohm's Law. — The relation between the current, electromotive force, and resistance of a simple circuit is given by Ohm's law, in absolute units, as

$$i = \frac{e}{r}.$$

Since the current in amperes is $10i$, the *E.M.F.* in volts is $10^{-8}e$, and the resistance in ohms is $10^{-9}r$, Ohm's law may be expressed in practical units by the formula

$$I = \frac{E}{R},$$

where I is the number of amperes flowing in an undivided circuit, E the algebraic sum of all the electromotive forces in that circuit in volts, and R the sum of all the resistances in series in that circuit expressed in ohms.

The form of the equation $E = IR$, as applied to a portion of a circuit, is much used under the name of Ohm's law. In this case, however, E is not *E.M.F.*, but difference of potential, as explained in the last article.

If, in a house lighted by electricity, the service maintains a constant pressure of 100 volts at the mains where they enter from the street, and no lights be turned on, then at every lamp socket in the house there will be a pressure of 100 volts. If now a lamp be turned on, it will be working on less than 100 volts, because of the *drop* or *fall of potential.* If many lamps be turned on, a considerable drop may occur. The drop is caused by the resistance of the wires carrying the current from the place of constant potential to the place where it is used, and the volts lost have been consumed in doing useless work, i.e. heating the wires. That the drop is proportional to the current flowing is shown by a simple application of Ohm's law.

Let R be the resistance of the line, and E_d the volts drop caused thereby when a current I flows. Then

$$E_d = IR,$$

from which it is evident that the drop varies as the current when the resistance of the line is constant.

4. Resistance of Conductors.—The resistance R of a conductor is expressed by the formula

$$R = \frac{\rho l}{A},$$

where ρ is a constant called the *resistivity,* and depending upon the material and the temperature of the conductor, l is the length, and A the cross-section of the conductor. The reciprocal of the resistivity, $\frac{1}{\rho}$ is called the *conductivity* of a substance.

If, in the foregoing expression for R, the centimeter and square centimeter be the units of length and cross-section respectively, and the resistance is desired in ohms, then ρ must be the resistance between opposite faces of a centimeter cube of the given material, and this is called its *specific resistance*. Areas of conductors are frequently expressed in terms of a unit, *circular mil*, equal to the area of a circle $\frac{1}{1000}$ inch in diameter. If area be so expressed and if l be the length in feet of the conductor, then ρ must be the resistance of a portion thereof one foot long and one circular mil in cross-section, i.e. of one *mil-foot*, so that R may be in ohms. The resistivities of various metals at 0° Centigrade are given in the following table:

RESISTIVITIES.

MATERIAL	SPECIFIC RESISTANCE IN MICROHMS AT 0°C.	RESISTANCE PER MIL-FOOT, IN OHMS AT 0° C.
Copper (soft)	1.594	9.6
Aluminum (soft)	2.55	15.4
Iron (soft)	8.7	52.4
Platinum	8.98	54.1
Steel	13.0	78
German Silver (18%)	34	204
Manganin	43	259
German Silver (30%)	45	271
"Advance"	49	295
"Ia Ia"	50	301
"Climax"	85	512
"Superior"	86	518

As rectangular conductors are much used in armatures and upon switchboards, it frequently becomes necessary to express their cross-sections in circular mils. Since the cross-section of a circle having a diameter of $\frac{1}{1000}$ inch is 0.000000785 square inches, the equivalent cross-section of a conductor expressed in circular mils is equal to its cross-section in square inches divided by 0.000000785, and the cross-section in circular mils is equal to 1273236 times its cross-section in square inches.

The resistivity of a conductor depends upon its physical condition and upon its purity. Thus, for example, the resistivity of soft copper is two per cent lower than that of hard copper. The resistivity of an alloy is usually greater than that of any of its constituents, consequently the admixture of a small percentage of one metal with another usually implies a higher resistivity.

The American Institute of Electrical Engineers has adopted as its standard resistivity for soft copper one given by Matthiessen. A wire of standard soft copper, of uniform cross-section, of one meter length, and weighing one gram, should have a resistance of 0.141729 international ohms at 0° C. A commercial copper showing this resistivity is said to have 100 per cent conductivity. Copper is frequently obtained having a conductivity of 102 per cent. It is in these cases almost invariably electrolytic copper.

The resistance of conductors depends upon temperature. Rise of temperature causes an increase of resistance in all pure metals, and the rate of increase is approximately the same for each. Representing the increase of resistance per unit resistance at 0° C. and unit rise in temperature by α, called the *temperature coefficient*, and the resistance of a conductor at 0° C. by R_0, then its resistance at any temperature T may be expressed by

$$R_T = R_0 (1 + \alpha T).$$

While it is sufficient for engineering purposes to consider the temperature coefficient of any conductor constant, it should be remembered that this coefficient varies slightly at different temperatures.

The average value of the temperature coefficient of copper is 0.0042, that is, between any initial and final temperature copper increases its resistance by 0.42 per cent of its resistance at zero degrees Centigrade for each degree rise of temperature.

Many alloys have a very small temperature coefficient, and are thus desirable for resistances in measuring instruments. Acid and salt solutions, carbon, hard rubber, and glass have negative temperature coefficients.

5. **Divided Circuits.** — When portions of an electric circuit are connected in *series*, the total resistance of the circuit is equal to the sum of the resistances of the separate portions. To determine the equivalent resistance of a number of resistances connected in *parallel*, let I be the current flowing in the undivided part of the circuit shown

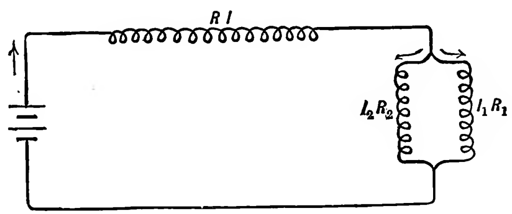


Fig. 1.

in Fig. 1, and let I_1 and I_2 be the currents flowing in the resistances R_1 and R_2 respectively. Then

$$I = I_1 + I_2;$$

and, since the potential difference, E, across each branch circuit is the same, by Ohm's law

$$I_1 = \frac{E}{R_1} \text{ and } I_2 = \frac{E}{R_2},$$

whence

$$I_1 : I_2 :: \frac{1}{R_1} : \frac{1}{R_2}.$$

The currents in the branches of a divided circuit are inversely as the resistances of the branches.

If R_e be a single resistance, that, when substituted for the shunted resistances R_1 and R_2 will leave I unchanged, then $I = \frac{E}{R_e}$; consequently

$$\frac{E}{R_e} = \frac{E}{R_1} + \frac{E}{R_2},$$

or

$$R_e = \frac{R_1 R_2}{R_1 + R_2} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}.$$

The resistance equivalent to a number of shunted resistances is equal to the reciprocal of the sum of the reciprocals of the separate resistances.

The distribution of current through the elements of a network of conductors, no matter how complex, may be determined by the aid of the following two laws due to Kirchhoff: — LAW I. — *The algebraic sum of the currents meeting at any point of a network is zero.* LAW II. — *In any mesh of a network the algebraic sum of the IR drops is equal to the algebraic sum of the electromotive forces.* For example, if E be the electromotive force of the battery indicated in Fig. 1, according to the first law

$$I - I_1 - I_2 = 0, \qquad (1)$$

and from the second law

$$E = IR + I_1R_1, \qquad (2)$$

and

$$E = IR + I_2R_2. \qquad (3)$$

From (2) and (3) $I_1R_1 = I_2R_2.$

Hence from (1)

$$I - I_1 - \frac{R_1I_1}{R_2} = 0,$$

or

$$I_1 = I\frac{R_2}{R_1 + R_2}.$$

6. Power of Electric Current. — If a difference of potential of e absolute units exist between two points, the transfer of an absolute unit quantity of electricity from one point to the other requires the expenditure of e ergs of work. Since the volt is equal to 10^8 absolute units of potential difference, and the coulomb is equal to 10^{-1} absolute units of quantity, it follows that the work performed in transferring one coulomb of electricity under a difference of potential of one volt is 10^7 ergs or one joule. A current of I amperes flowing for t seconds represents It coulombs of electricity, and if these be transferred under a potential difference of E volts, the work done in joules will be

$$W = EIt.$$

The rate of working, or the *power*, is

$$P = \frac{W}{t} = EI,$$

where P is expressed in *watts*, i.e. joules per second. Since, from Ohm's law, $E = IR$, by substitution

$$P = I^2R.$$

For commercial currents and voltages the watt is a needlessly small unit, hence the kilowatt (= 1000 watts) is generally used to express electrical power. It is represented by the abbreviation K.W. The horse-power is equal to 746 watts, or approximately three-fourths of a K.W.

7. Heat Developed by a Current. — When a current I is maintained in a circuit of resistance R, the work performed is converted into heat. The work thus done per second, or the power expended, will be I^2R watts. Since this production of heat is often of no service, this expenditure of power is generally called the I^2R *loss.*

This production of heat causes a rise of temperature in the conductor, and the temperature will continue to rise till the heat generated per second by the I^2R loss is exactly counterbalanced by the rate of dissipation of heat by conduction, convection, and radiation.

The inherent resistances of electrical machines involve the production of heat in their operation (as do also friction and reversal of magnetism), which causes a rise of temperature. As insulating materials can survive only moderately high temperatures, such machines must be designed to operate without becoming too hot. This is accomplished by decreasing the I^2R loss, by increasing the radiating surface, and by improving ventilation.

8. Insulating Materials. The desirable properties of materials which are to be used for insulating various electrical conductors from each other are: (*a*) a high insulation resistance and this resistance should remain high over a considerable range of temperature; (*b*) a dielectric strength sufficient to preclude any possibility of perforation by voltages liable to exist between the conductors which they separate, and this strength must also persist throughout all probable ranges of temperature; (*c*) such physical properties as will permit of mechanical manipulation; (*d*) nonalteration of chemical constitution when subjected to high temperatures and operating conditions.

As no one insulating material possesses all of these desirable properties, for any particular purpose that insulating material should be chosen which is best suited for the given conditions. In this choice, available space and cost of the insulation are also determining factors.

The dielectric strength of an insulating material is measured by the voltage which must be applied to it in order to cause its rupture. The dielectric strength depends upon the thickness of the dielectric, the form of the opposed conducting surfaces, and the manner in which the *E.M.F.* is applied, whether gradually, suddenly, or periodically varying. It has been stated that the dielectric strength approximately varies inversely as the cube root of the thickness, showing that a thin sheet is relatively stronger than a thick one of the same material. For example, the dielectric strength of mica when 1 mm. thick is 610 kilovolts per centimeter, but when 0.1 mm. thick it is 1150 kilovolts per centimeter.

Mica possesses the highest insulation resistance and the greatest dielectric strength of insulating materials. It does not absorb moisture and its chemical constitution is unaffected by high temperatures. It is not, however, mechanically strong.

Sheets of insulation made up from pieces of scrap mica cemented together by linseed oil or preparations of shellac, when carefully constructed with lapped joints, exhibit nearly as good insulating and dielectric properties as sheet mica. While not perfect mechanically, these sheets permit of bending better than pure mica.

For insulating purposes where considerable flexibility is essential, micanite paper and micanite cloth are well adapted. These materials consist of small pieces of mica in combination with paper or various kinds of fabric.

Preparations of fibrous materials with linseed oil, which, after being dried, have been thoroughly baked, are fairly good insulators. As water is generally present in their pores, their insulation resistance, upon heating, decreases until the temperature has reached 100° C., and then it increases. These preparations are mechanically flexible. Preparations of fibrous material with shellac are good insulators, but crack upon bending.

Vulcanized fibers are made by treating paper fiber chemically, and, when dried, they have a fairly high insulation resistance, but they readily absorb moisture, and, upon drying, are liable to warp and twist. They furthermore become brittle when heated.

Hard rubber is a good insulator and possesses the desirable mechanical qualifications, but it does not withstand moderately high temperatures, for at 70° C. it becomes soft and melts at 80° C. Its employment is limited, therefore, to apparatus to be used at comparatively low temperatures.

Asbestos, a fairly good insulator, is used principally because of its incombustibility. Vulcabeston, which is a preparation of asbestos and rubber, exhibits good insulating and mechanical qualities, and is especially fitted for higher temperatures. Asbestos and vulcabeston are much used in electric heating apparatus.

9. **Test of Dielectric Strength.** — In order to test the voltage necessary to break down a sample sheet of insulating material, the sample is placed between two flat metallic surfaces which are connected respectively with the two

terminals of a high-voltage transformer, whose voltage can be varied at will. The sample should project considerably beyond the edges of the metallic surfaces, so that no discharge can take place from one terminal to the other around the sample under test. The test voltage is applied and is gradually increased until the material punctures.

Practical average values of dielectric strengths over the thicknesses stated of various insulating materials are given in the following table:

MATERIAL	THICKNESS IN MM.	DIELECTRIC STRENGTH IN EFFECTIVE SINUSOIDAL KILOVOLTS PER CM.
Air	0.5	50.3
"	1.0	43.6
"	5.0	33.5
"	10.0	29.8
Asbestos	0.5–1.5	25–10
Cotton	0.1–0.3	110
Glass	2.0–6.0	250–170
Hard rubber	1.5–75.0	510–400
Mica	0.05–3.0	1300–500
Micanite	0.2–0.5	400
" paper	0.2–0.6	150
" cloth	0.2–0.6	80
Paper (paraffined)	0.05–0.2	360
Silk	0.02–0.2	200–150
Vulcabeston	1.0–2.5	80–20
Vulcanized fiber	0.8–2.0	80

For measuring the test voltage a spark gap having sharp needle-point terminals is connected in parallel with the sample under test. The distance between the needle-points is adjustable so as to limit the voltage which can be impressed upon the conductors on each side of the insulating material. In carrying out the test, the needle-points are adjusted at a certain minimum distance apart. The voltage impressed upon the terminals is raised until a spark passes between the points. The air gap is then increased in length, and the operation repeated until the sample breaks down, and the spark passes through it instead of across the air gap. The length of the air gap is measured and the break-down voltage may then be obtained from the curve of Fig. 2, which shows the effective sinusoidal voltages

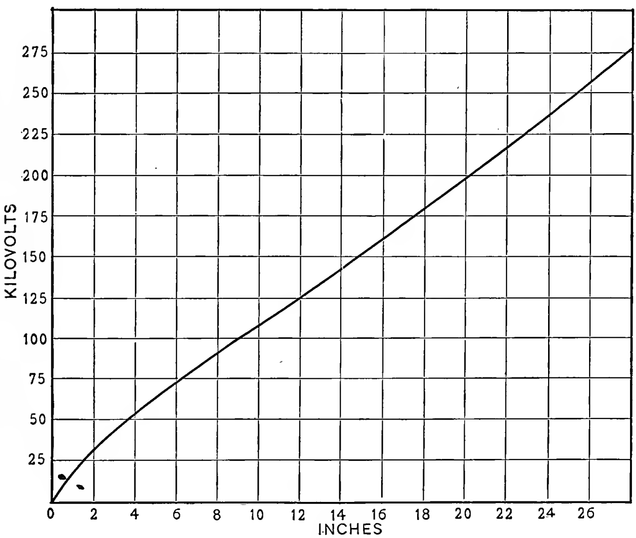


Fig. 2.

corresponding to the sparking distances in air between opposed sharp needle-points.

The test voltage to be applied in determining the suitability of insulation in commercial apparatus depends upon the kind and the size of the apparatus, its normal voltage, and the service for which it is designed. The following voltages for testing insulation of apparatus and cables by a continuous application for one minute are recommended by the American Institute of Electrical Engineers:

RATED TERMINAL VOLTAGE OF CIRCUIT	RATED OUTPUT	TESTING VOLTAGE
Not exceeding 400 volts	Under 10 K.W.	1000 volts
" " " "	10 K.W. and over	1500 "
400 volts–800 volts	Under 10 K.W.	1500 "
" – "	10 K.W. and over	2000 "
800 volts–1200 volts	Any	3500 "
1200 volts–2500 volts	Any	5000 "
2500 volts and over	Any	Double normal rated voltage

The test voltage should be applied successively between each electric circuit and surrounding conductors and also between adjacent electric circuits. High-voltage tests should be made at the temperatures assumed during normal operation.

PROBLEMS.

1. How much work is done by a pump in raising 2500 gallons of water from a mine 200 feet deep? If this is accomplished in 25 minutes, what is the power of the pump expressed in horse-power?

2. Calculate the kinetic energy of an electric locomotive weighing 95 tons when running at a velocity of 50 miles per hour.

3. The hot resistance of a 100-candle-power carbon incandescent lamp is 45 ohms. How much current does the lamp take when connected to 120-volt mains?

4. What must be the *E.M.F.* of a generator to supply a group of 50 lamps connected in parallel, each requiring $\frac{1}{2}$ ampere, the resistance of the generator being 0.1 ohm, so that each lamp of 220 ohms resistance shall receive its full current, assuming the line wires to have a resistance of 0.5 ohm?

5. Determine the resistance of one mile of copper wire 0.325 inch in diameter at zero degrees Centigrade.

6. Calculate the resistance at 15° C. of a mile of track rail weighing 70 pounds per yard, taking 120 ohms as its resistance per mil-foot at that temperature. Specific gravity of track rail = 7.8.

7. Find the resistance at 700° C. of a platinum wire two meters long and one millimeter in diameter; the temperature coefficient being 0.0036.

8. When four conductors of 4, 8, 10, and 16 ohms resistance respectively are joined in parallel to the terminals of a battery whose *E.M.F.* is 20 volts on open circuit and whose internal resistance is 3 ohms, how much current will flow in each conductor?

9. Resistances of 2, 3, 4, 6, 8, and 10 ohms respectively are connected as shown in Fig. 3, the number adjacent to each

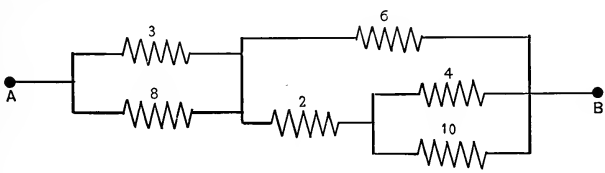


Fig. 3.

branch representing its resistance. What will be the potential difference across each resistance and the current therein when the terminals *A*, *B* are connected to 50-volt mains?

10. What power is expended in the 100-candle-power lamp of problem 3? Express the result in watts and in horse-power. When electrical energy costs 10 cents per kilowatt-hour, how much does it cost to operate the lamp for three hours?

CHAPTER II.

MAGNETIC LAWS AND FACTS.

10. **Strength of Magnet Pole.** — A unit magnet pole is one which will repel an equal like pole, when at a distance of one centimeter in vacuum or in air,* with a force of one dyne.

It follows from this definition that a pole m units strong will repel a like *unit* pole with a force of m dynes. The force exerted between two magnetic poles varies inversely as the square of the distance between them. Hence the force in dynes exerted between two magnetic poles of strengths m and m' when d centimeters apart in air is defined by the equation

$$F = \frac{mm'}{d^2}.$$

11. **Magnetic Field and Lines of Force.** — The space around a magnet where its action is felt is termed the field of that magnet. This field may conveniently be considered as permeated by *lines of force*. These lines represent by their direction the *direction* of the force exerted by the magnet, and by their closeness to each other show the *magnitude* of this force.

The directions of the lines of force in the vicinity of a magnet may be demonstrated by scattering iron filings over a glass plate laid on a magnet. Magnetic poles are

* In this chapter air is considered to have the same magnetic properties as a vacuum.

induced in the iron particles and the latter arrange themselves parallel to the lines of force, this arrangement being facilitated by gently tapping the glass plate.

12. Intensity of Magnetic Field. — A magnetic field in air is said to have unit strength or intensity at any point therein when a unit magnet pole placed at that point is acted upon by a force of one dyne, or when a magnet pole m units strong is acted upon by a force of m dynes. Therefore the strength of a magnetic field in air, represented by $\mathcal{H}$, may be expressed as

$$\mathcal{H} = \frac{F}{m}.$$

By convention one line of force per square centimeter is considered to represent a field of unit strength, the square centimeter being on a surface that is at all points perpendicular to the lines cutting it. Hence the strength or intensity of any field in air, $\mathcal{H}$, can be expressed by the number of lines of force per square centimeter.

Suppose a sphere of one centimeter radius to be circumscribed about a unit magnet pole. Another unit pole at any point on the surface of this sphere will be acted upon by a force of one dyne. Hence there exists unit field intensity at any point on this surface. But there are 4π square centimeters on this surface, and each square centimeter will be cut by one line of force. Therefore, *there emanate from a unit magnet pole* 4π *lines of force.* Similarly a magnet pole of strength m sends out $4\pi m$ lines of force. The total number of lines of force, or the total magnetic *flux*, represented by the symbol Φ, may therefore be expressed as $\Phi = 4\pi m$.

When a magnetic field has different intensities at various points in it, as is usually the case, it is called a *non-uniform* field; and when it has everywhere the same intensity and direction, it is said to be a *uniform* field.

13. Magnetic Potential. — The magnetic potential at any point is measured by the work that would be required to bring a unit magnet pole up to that point from an infinite distance.

The difference of magnetic potential between any two points is measured by the work in ergs required to carry a unit magnet pole from one to the other.

14. Permeability. — The maintenance of the same difference of magnetic potential between two points will result in more lines of force in iron than in air. Iron is then said to be more permeable than air, or to have a greater *permeability*. If, for a given gradient of magnetic potential, $\mathcal{H}$ lines of force per square centimeter be set up with air as a medium, and later at the same point $\mathcal{B}$ lines with some other substance as a medium, then the ratio of $\mathcal{B}$ to $\mathcal{H}$ expresses the permeability of that substance. This ratio is usually represented by the symbol μ, so that

$$\mu = \frac{\mathcal{B}}{\mathcal{H}}.$$

The permeability μ expresses, therefore, the relative magnetic conductivity of a substance compared with air.

The total flux in the second substance is sometimes considered to be made up of two parts, the first consisting of that which would be present with air as a medium and the second being that which is added by the second substance. The total number of lines of force per square centimeter, $\mathcal{B}$, produced in a substance of permeability μ is called the *flux density*, or the *induction per square centimeter.* For air, vacuum, and most substances $\mu = 1$. For iron, nickel, and cobalt μ has a higher value, reaching, in the case of iron, as high as 3000. Such substances are said to be *paramagnetic*, or simply *magnetic*. Bismuth, antimony, phosphorus, and a few other materials have a permeability very slightly less than unity; these being known as *diamagnetic* substances. A substance for which μ is zero would insulate magnetism; but no such substance is known.

One line of force, that is, a flux equal to $\frac{1}{4\pi}$ that from an isolated unit pole, is called a *maxwell*. A flux density of one line of force, or one maxwell, per square centimeter, is called a *gauss*. The total magnetic flux, Φ, in maxwells, which passes through an area of A square centimeters, in which the flux-density is $\mathfrak{B}$ gausses, is given by the equation

$$\Phi = \mathfrak{B}A.$$

15. **Electro-Magnetic Induction.** — In 1831 Faraday discovered that when a conductor was moved in a magnetic field, an electromotive force was set up in the conductor. This phenomenon is the foundation of all modern electrical engineering.

An absolute unit of *E.M.F.* is produced when a conductor cuts one line of force per second. If the conductor cuts two lines in the second, or one line in half a second, then two such units of electromotive force are produced. An *E.M.F.* of one volt is produced by the cutting of 10^8 lines of force per second.

If, in the short interval of time, dt seconds, $d\Phi$ lines be cut, then during that interval the value of the induced *E.M.F.* will be

$$e = -\frac{d\Phi}{dt} \text{ absolute units,}$$

or,
$$E = -\frac{1}{10^8}\frac{d\Phi}{dt} \text{ volts,}$$

the negative sign being used because the induced *E.M.F.* tends to send a current in such a direction as to demagnetize the field. When of no consequence the negative sign will hereafter be omitted.

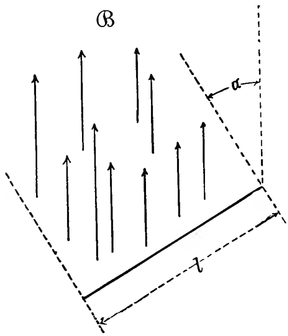


Fig. 4.

If a conductor, Fig. 4, l centimeters long moves in a direction perpendicular to itself with a uniform velocity of v centimeters per second across a uniform magnetic field having a flux density of $\mathcal{B}$ gausses, the plane of its path making an angle α with the direction of the lines of force, then the number of lines cut per second is $\mathcal{B}lv \sin \alpha$, and, since the rate of cutting is uniform, the *E.M.F.* at any instant is

$$e' = \mathcal{B}lv \sin \alpha.$$

If there be a non-uniformity in the rate of cutting lines, due either to an uneven field or to an irregular motion, then the average value of the induced *E.M.F.* associated with the cutting of Φ lines in the time, t seconds, will be

$$e_{av} = \frac{\Phi}{t} \text{ absolute units, or } E_{av} = \frac{\Phi}{10^8 t} \text{ volts.}$$

If a circular loop of wire revolve about its diameter as an axis in a non-uniform magnetic field with a constant angular velocity, or if it revolve in a uniform field with a variable velocity, its sides cut lines of force at various rates. The instantaneous *E.M.F.* in the whole loop will be as before,

$$e' = \frac{d\Phi}{dt},$$

where Φ is the number of lines that links with, or that passes through, the loop. If the loop be of n turns, then the pressure will be n times as great, or during the interval dt,

$$E = -\frac{n d\Phi}{10^8 dt} \text{ volts.}$$

16. Direction of Induced E.M.F. — The direction of flow of a current induced in a closed circuit by moving it in a magnetic field is best represented by drawing the conventional representation of the three dimensions of

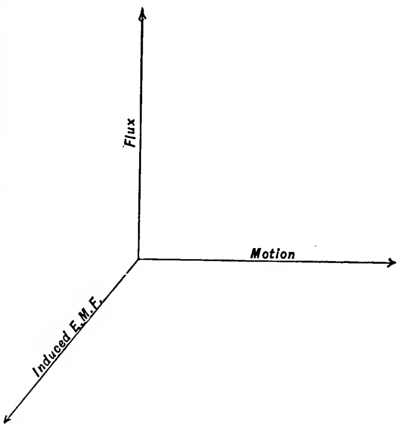


Fig. 5.

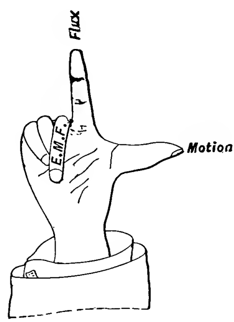


Fig. 6.

space. If the flux be directed upwards, and the motion of the conductor be to the right, then the *E.M.F.* will tend to send a current toward the reader. If any one of these conditions be changed it necessitates the change of one of the others, and conversely the change of any two leaves the third unaltered. About the same idea is represented in Fleming's Rule, which is as follows : —

Let the index finger of the right hand point in the direction of the flux, and the thumb in the direction of the motion. Bend the second finger at right angles with the thumb and index finger, and it will point in the direction of the *E.M.F.*

Another rule is: —

Stand facing a north magnetic pole. Pass a conductor downward. The current tends to flow to the left.

17. Inductance. — An electric current produces a magnetic field in the vicinity of the conductor carrying that current. The conductor may therefore be considered as encircled by lines of force. When the current is first started in such a conductor, these lines of force must be established. In establishing itself, each line is considered as having cut the conductor, or, what is equivalent thereto, been cut by the conductor. This cutting of lines of force results in the production of an electromotive force in the conductor, called the *E.M.F. of self-induction.* When the flow of current ceases, the surrounding lines of force collapse, cutting the conductor, thus also producing an electromotive force of self-induction. The *E.M.F.* of self-induction is always a *counter E.M.F.*, that is, its direction is such as to tend to prevent the change of current which causes it.

The magnitude of this *E.M.F.* is dependent upon the rapidity with which the field is established or destroyed, and upon a constant called the *self-inductance* or the *coefficient of self-induction* of the circuit. It is generally represented by the letter L, and is that coefficient by which the time rate of change of current in the circuit must be multiplied in order to give the *E.M.F.* induced in the circuit. Its absolute value is numerically represented by the number of lines of force linked with the circuit per absolute unit of current in that circuit. Its practical unit is 10^9 times as large as the absolute unit, and is called the *henry*. A circuit having an inductance of one henry will have a pressure of one volt induced in it by a uniform change of current of one ampere per second. Hence the *E.M.F.* of self-induction may be written

$$E_s = -L\frac{dI}{dt}.$$

But, from §15, the *E.M.F.* induced in a loop of wire moving in a magnetic field is

$$E = -\frac{nd\Phi}{dt}10^{-8}.$$

Equating these expressions, there results

$$L = n\frac{d\Phi}{dI}10^{-8}.$$

Two circuits may exercise a mutually inductive action upon each other, and an *E.M.F.* may be induced in one by a change of current in the other. This is called an *E.M.F.* of *mutual induction*. In magnitude it depends upon the shape and position of the two circuits, and upon the character of medium in which they are placed. It is also dependent upon a constant which is called the *mutual inductance* or *coefficient of mutual induction* of the two circuits. It is generally represented by the letter *M*. It is that coefficient by which the time rate of change of the current in one of the circuits is multiplied in order to give the *E.M.F.* induced in the other circuit. Its absolute value is numerically equal to the number of lines of force linked with one of the circuits per absolute unit of current in the other circuit. Its practical unit is the same as the practical unit of self-inductance, that is, the henry, and is 10^9 times as large as the absolute unit.

18. Growth of Current in an Inductive Circuit. — When a current is started in a circuit having resistance and inductance by impressing a constant *E.M.F.* upon its terminals, the self-induced pressure in that circuit tends to oppose the flow of the current and prevents it reaching its ultimate value immediately. At the instant of closing the circuit there is no current flowing, and let time be reckoned from this instant. At any subsequent instant, t seconds later, the impressed *E.M.F.* may be considered as the sum of two parts, E_1 and E_r. The first, E_1, is that part which is opposed to, and just neutralizes, the *E.M.F.* of self-induction, so that $E_1 = - E_s$; but

$$E_s = - L \frac{dI}{dt},$$

so

$$E_1 = L \frac{dI}{dt}.$$

The second part, E_r, is that which is necessary to send current through the resistance of the circuit, according to Ohm's law, so that

$$E_r = RI.$$

The impressed electromotive force, being the sum of E_1 and E_r, is therefore

$$E = RI + L\frac{dI}{dt};$$

whence

$$dt = \frac{L}{E - RI}\, dI = - \frac{L}{R} \cdot \frac{-RdI}{E - RI}.$$

Integrating from the initial conditions $t = 0$, $I = 0$, to any condition $t = t$, $I = I'$,

$$t = -\frac{L}{R}\left[\log_\epsilon(E - RI') - \log_\epsilon E\right].$$

Therefore

$$-\frac{Rt}{L} = \log_\epsilon\left(\frac{E - RI'}{E}\right),$$

from which the instantaneous current value is

$$I' = \frac{E}{R}\left(1 - \varepsilon^{-\frac{Rt}{L}}\right),$$

where ε is the base of the natural system of logarithms and numerically equal to 2.7183. This equation shows that the rise of current in an inductive circuit follows a logarithmic curve, and that, when t is of sufficient magnitude to render the second term negligible, the value of the current will be as given by Ohm's law, a condition which agrees with experimental observations.

A curve of the growth of current in a circuit having resistance and inductance is shown in Fig. 7, the values of I being calculated for the conditions noted.

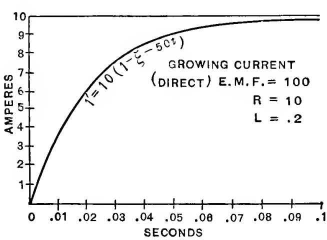


Fig. 7.

The natural logarithms used in preceding formulæ can be obtained by multiplying the common logarithm of the number, the mantissa and characteristic being included, by 2.3026.

19. Decay of Current in an Inductive Circuit.—If a current be flowing in a circuit having inductance and resistance, and the supply of $E.M.F.$ be discontinued, without, however, interrupting the continuity of the circuit, the flow of current will not cease instantly, but the *E.M.F.* of self-induction will keep it flowing for a time, with values decreasing according to a logarithmic law.

An expression for the value of this current at any time, t seconds, after withdrawing the impressed *E.M.F.*, may be obtained as in the foregoing section. The current at the instant of interruption of the impressed *E.M.F.* is due solely to the electromotive force of self-induction and may be represented by $\frac{E}{R}$. Therefore

$$E = RI + L\frac{dI}{dt} = 0,$$

whence

$$dt = -\frac{L}{R} \cdot \frac{dI}{I}.$$

By integrating from the initial conditions $t=0$, $I = \frac{E}{R}$, to any condition $t=t$, $I=I'$, the instantaneous value of the current is found to be

$$I' = \frac{E}{R}\varepsilon^{-\frac{Rt}{L}},$$

which is the term that had to be subtracted in the formula for the growth of current. This shows clearly that, while self-induction prevents the instantaneous attainment of the ultimate value of the current, there is eventually no loss of energy, since what is subtracted from the growing current is given back to the decaying current.

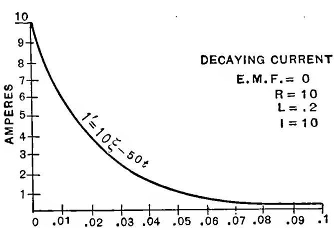


Fig. 8.

Fig. 8 is the curve of decay of current in the same circuit as was considered in Fig. 7. The ordinates of the one figure are seen to be complementary to those of the other.

20. Quantity of Electricity Traversing a Circuit Due to a Change of Flux Linked with it. — In many dynamo tests, and in many magnetic investigations, it is necessary to measure, generally by means of a ballistic galvanometer, the quantity of electricity traversing a circuit due to a change of flux linked with it. If the circuit have a resistance of r and in dt time the flux linked with n turns changes by $d\Phi$, then the instantaneous current

$$i = \frac{\frac{n d\Phi}{dt}}{r}.$$

But the quantity of electricity flowing during the time dt is $dq = idt$, hence

$$dq = \frac{n d\Phi}{r},$$

which is independent of time. So if the flux change from Φ_1 to Φ_2, then

$$q = \frac{\Phi_2 - \Phi_1}{r} n.$$

If the resistance of the circuit be expressed in ohms and the flux in maxwells, the quantity of electricity in micro-coulombs will be

$$Q = \frac{n}{100} \cdot \frac{\Phi_2 - \Phi_1}{R}.$$

21. Work Performed by a Conductor Carrying a Current and Moving in a Magnetic Field. — Let a conductor carrying a constant current i be moved in a direction perpendicular to itself and to the lines of force of a magnetic field. Suppose it to move for dt seconds, and in that time to cut $d\Phi$ lines of force. Then the induced *E.M.F.* e will be $-\frac{d\Phi}{dt}$. The quantity of electricity dq that has to traverse the circuit against this *E.M.F.* during the time dt will be idt. Since potential is a measure of work, the work required to carry dq units of electricity against a difference of potential e is edq ergs. Hence the work in ergs,

$$dw = edq = idt \times \frac{d\Phi}{dt} = id\Phi.$$

Therefore the current i, in cutting Φ lines of force, performs the work

$$w = i\Phi \text{ ergs.}$$

From this it is seen that the work done by a conductor carrying a current and cutting lines of force is independent of the time it takes to cut them.

In the above discussion, if the field be non-uniform or the motion be irregular, the value of e will not be the same for each instant of time. But since the result obtained is independent of time, it is immaterial how the lines are arranged, and how the rate of cutting varies.

22. **Force Exerted between a Field and a Conductor Carrying a Current.** — When a conductor moves in a field perpendicular to itself and to the lines of force, then, from the foregoing article, the work performed is

$$w = i\Phi \text{ ergs.}$$

If the conductor be l centimeters long, and traverses a distance of d centimeters through a uniform field having a flux density of $\mathfrak{B}$ gausses, then the total number of lines of force cut is

$$\Phi = ld\mathfrak{B},$$

and the

$$\text{Work} = ild\mathfrak{B} \text{ ergs.}$$

But

$$\text{Work} = \text{force} \times \text{distance} = Fd.$$

$$\therefore F = il\mathfrak{B} = \frac{Il\mathfrak{B}}{10} \text{ dynes.}$$

This force acts perpendicularly to the wire and to the lines of force.

23. Magnetomotive Force of a Circular Circuit Carrying a Current. — A thin circular conductor carrying a current forms a *magnetic shell.* The current produces magnetic lines of force each of which is closed upon itself and is linked with the conducting circuit. If a unit magnet pole be carried along any closed path linked with this conductor, coming back to point of starting, it will be subjected to magnetic forces that will vary in magnitude as its position is changed. These forces may also not have the same direction as the path of movement at any instant. But the sum of the products of the lengths of the path elements into the corresponding portions of the force, resolved along the path, will represent the difference of magnetic potential set up by the magnetic shell. This sum is the same as the product of the *average force* by the length of the path or the total work performed upon the unit pole.

It is immaterial whether the pole be carried from one side of the shell to the other, Fig. 9, or the shell be turned bottom side up around the pole. In the latter case it is clear that all the lines emanating from the pole will be cut once, and once only, by the conductor, wherefore 4π lines will have been cut.

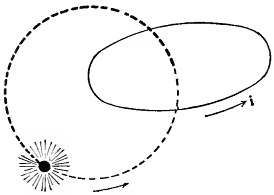


Fig. 9.

If current i flows in the conductor, then, by § 21, the work in ergs is

$$w = i\Phi = 4\pi i.$$

If there be n turns of the conductor, each line of force will be cut n times, and the work will be $4\pi ni$ ergs. Hence the magnetic difference of potential between the two sides of a thin magnetic shell is $4\pi ni$ or $\frac{4\pi nI}{10}$. Since $\frac{4\pi}{10} = 1.257$, and since the current and the number of turns are usually regarded as a single quantity, namely *ampere-turns*, it follows that the difference of magnetic potential is equal to 1.257 times the number of ampere-turns.

Difference of magnetic potential is a measure of the ability of the shell to set up lines of force, or, in other words, is a measure of its *magnetomotive force*. Magnetomotive force and difference of magnetic potential are similar, just as are electromotive force and difference of electrical potential. Therefore magnetomotive force may be expressed as

$$M.M.F. = 1.257\ nI.$$

The practical unit of $M.M.F.$ is the *gilbert*, so that a gilbert $= 1.257 \times$ ampere-turn.

24. The Toroid. — A uniform toroidal winding upon a ring-shaped iron core and carrying a current i produces the same flux density at all points on the axis of the toroid. Assuming the core to be removed and the portion of the flux which is not due to the iron to have the same distribution as the flux occasioned by iron, then, if there be n turns in the coil, and the length of the axis be l cm., the work necessary to be exerted upon a unit pole to carry it once along the axis would be $4\pi ni$ ergs, which is also equal to the product of field intensity $\mathcal{H}$ at the axis into its length l. Hence the magnetizing force, that is, the strength or intensity of field, in the toroid is

$$\mathcal{H} = \frac{4\pi nI}{10\,l}.$$

25. Magnetization Curves. — The permeability of air is constant for all magnetizing forces; but this is not true of iron and other substances having permeabilities noticeably greater than unity. The values of μ for these paramagnetic substances depend also upon the chemical composition and physical condition of the latter. Values of magnetizing force, $\mathcal{H}$, and the corresponding flux density, $\mathcal{B}$, in average commercial wrought iron, in cast iron, and in cast steel are given in the following table. The change of flux density with variation of magnetizing force is best shown by curves, called *magnetization*, or $\mathcal{B}$-$\mathcal{H}$, *curves*. In Figs. 10, 11, and 12 are given the magnetization curves respectively of wrought and sheet iron, cast iron, and cast steel, the curves being plotted from the tabulated values. From an inspection of these curves, it is seen that an increase in magnetizing force results at first in a marked increase of flux per unit area, but as the material becomes magnetized further the increment of flux density lessens and finally approaches a value proportional to the corresponding increment of magnetizing force. At this point the material is said to be *saturated*, that is, an increase of $M.M.F.$ shows no increase of flux density due to the presence of the given material.

WROUGHT AND SHEET IRON

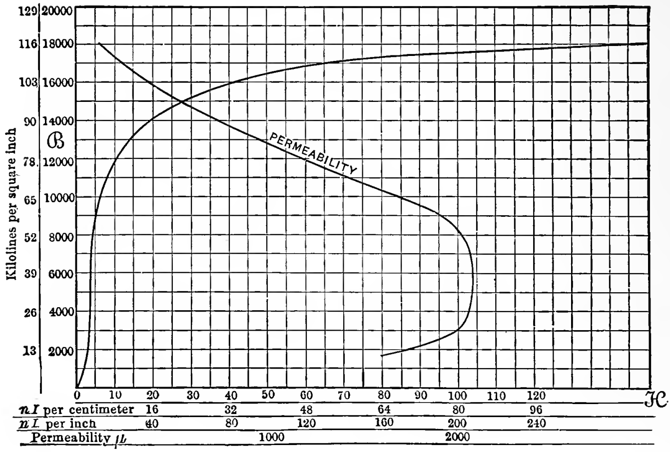


Fig. 10.

CAST IRON

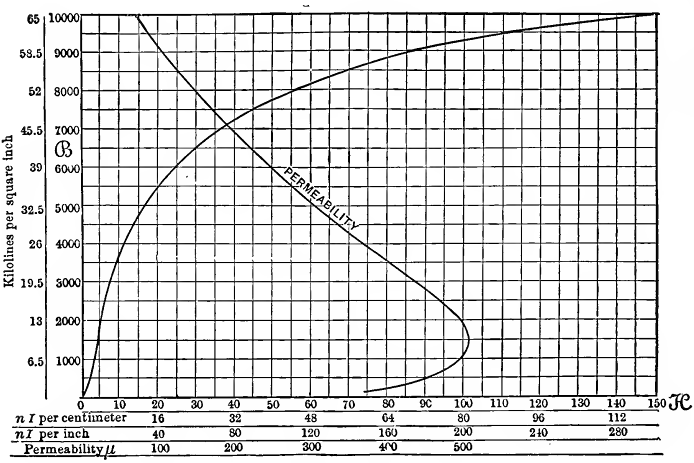


Fig. 11.

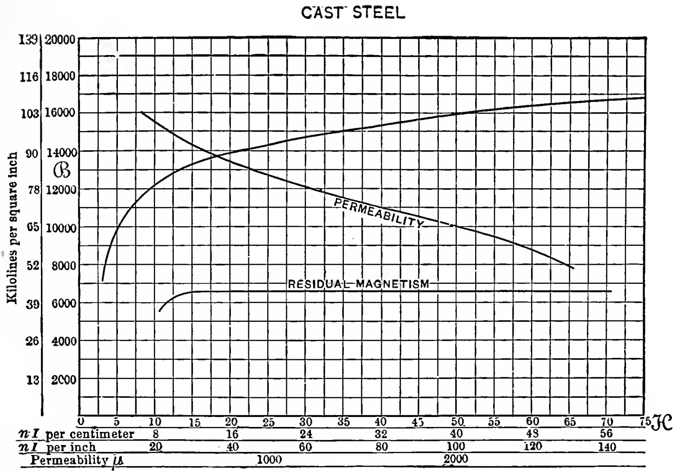


Fig. 12.

DATA FOR $\mathcal{B}$–$\mathcal{H}$ CURVES.

AVERAGE FIRST–QUALITY METAL.

$\mathcal{H}$	AMPERE-TURNS PER CENTIMETER LENGTH.	AMPERE-TURNS PER INCH LENGTH.	WROUGHT AND SHEET IRON.		CAST IRON.		CAST STEEL.	
			$\mathcal{B}$	KILO-LINES PER SQ. IN.	$\mathcal{B}$	KILO-LINES PER SQ. IN.	$\mathcal{B}$	KILO-LINES PER SQ. IN.
10	7.95	20.2	11800	74	3900	25.2	12000	77
20	15.90	40.4	14000	90	5500	35.5	13800	89
30	23.85	60.6	15200	98	6500	42.0	14600	94
40	31.80	80.8	15800	102	7100	45.7	15400	99
50	39.75	101.0	16400	106	7700	49.5	16000	103
60	47.70	121.2	16800	108	8200	53.0	16400	106
80	63.65	161.6	17200	111	8900	57.2	16700	108
100	79.50	202.0	17600	114	9300	60.0	17600	113
125	99.70	252.5	17800	115	9700	62.4	18200	117
150	119.25	303.0	18000	116	10100	65.8	18600	120

$\mathcal{H} = 1.257\ (nI \text{ per cm.}) = .495\ (nI \text{ per in.})$. $\mathcal{B} = .155\ (\Phi \text{ per sq. in.})$

Curves of permeability, plotted in terms of flux density, for wrought iron, cast iron, and cast steel are also shown in the accompanying figures.

In general all substances mixed with or alloyed with iron lower its permeability. In steel and cast iron the permeability seems to be in inverse proportion to the amount of carbon present. Carbon in the graphitic (not combined) form lowers the permeability less than carbon when combined. In cast iron and cast steel such substances as tend to give softness and greater homogeneity to the metal when present in limited amounts, say 2 per cent, increase the value of μ. Aluminum and silicon act in this way.

The physical condition of the metal also affects its permeability. Chilling in the mold, when casting, lowers it, as does tempering, or hardening the metal by working it. On the other hand, annealing increases the permeability.

A piece of iron or steel, subjected to a small magnetizing force, has its permeability increased by increasing the temperature until a *critical temperature* is reached, when it falls off rapidly to almost *unity*. For stronger magnetization the permeability does not rise so high at the critical temperature, and does not fall off so sharply after it. The value of this critical temperature lies between 650° C. and 900° C., depending on the test piece. The influence of temperature upon permeability is very small for the changes in temperature occurring in practical operation, and is therefore negligible.

26. Reluctance and Permeance. — In the flow of magnetic lines of force the reciprocal of the *permeability*, $\frac{1}{\mu}$, is called the *reluctivity*. The total *reluctance*, tending to oppose the passage of magnetic lines under the influence of a magnetic difference of potential, is directly as the length and the reluctivity of the medium and inversely as its cross-section. Hence the total magnetic resistance or

$$\text{reluctance} = \frac{\text{length}}{\text{cross-section}} \text{reluctivity}.$$

Reluctivity is usually represented by $\nu \left(= \frac{1}{\mu}\right)$. Hence for a medium of cross-section A square centimeters and length l centimeters, the reluctance

$$\mathcal{R} = \frac{l}{A} \nu.$$

The unit in which $\mathcal{R}$ is expressed is called the *oersted.*

Permeance is the reciprocal of the reluctance, hence the permeance

$$\mathcal{P} = \frac{A}{l\nu} = \frac{A}{l} \mu.$$

It must be remembered that ν and μ are not constant for some substances, but depend for their values upon the strength of the magnetizing force $\mathcal{H}$ which is acting upon the substances.

27. **Relation Between Magnetomotive Force, Magnetic Flux, and Reluctance.** — The flux produced in a magnetic circuit by a magnetomotive force may be expressed by an equation similar to that expressing the current flow in an electric circuit due to an impressed electromotive force, which is

$$\text{current} = \frac{\text{electromotive force}}{\text{resistance}}.$$

The corresponding equation for the magnetic circuit is

$$\text{magnetic flux} = \frac{\text{magnetomotive force}}{\text{reluctance}},$$

or symbolically

$$\Phi = \frac{M.M.F.}{\mathcal{R}} = \frac{4\pi n \dfrac{I}{10}}{\dfrac{l}{\mu A}}.$$

Since the unit of magnetic flux is one line of force or the maxwell, the unit of magnetomotive force is the gilbert, and the unit of reluctance is the oersted, this equation may be written

$$\text{maxwells} = \frac{\text{gilberts}}{\text{oersteds}}.$$

The application of this equation is not as simple as that of the corresponding equation of the electric circuit. Electric circuits, in general, exist in media of zero electric conductivity, and therefore permit of accurate calculations, since the leakage is inappreciable. Magnetic circuits, on the other hand, are situated in media which have permeabilities of at least unity and hence much leakage is present, and precise calculations require a consideration of all flux paths. In the designing of dynamo electric machinery, however, one or more paths of low reluctance are presented to the magnetizing force, and these are so shaped that the leakage paths offer a comparatively high reluctance.

28. Hysteresis. — If a piece of iron become magnetized, and the magnetizing force be then removed, the iron does not become completely demagnetized. A certain magnetizing force in the opposite direction must be applied to bring it back to its original condition. This phenomenon, where "changes of magnetism lag behind the changes of force," has been termed *hysteresis*. Because of hysteresis a $\mathcal{B}$–$\mathcal{H}$ curve taken with continuously *increasing* values of $\mathcal{H}$ to the maximum and then with continuously *decreasing* values of $\mathcal{H}$ to a negative maximum, and so on, will assume the shape shown in Fig. 13. The distance *OA* represents

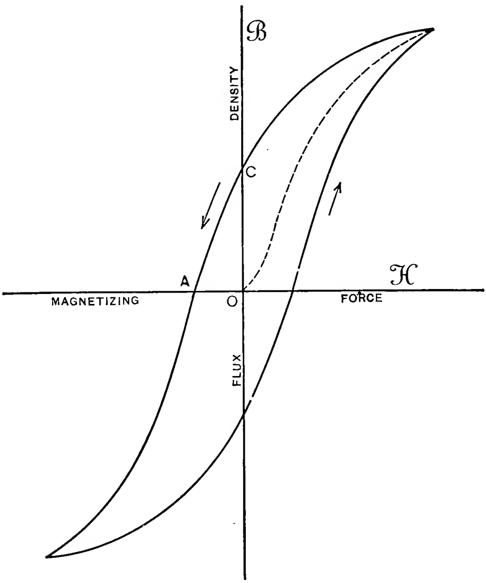


Fig. 13.

the *coercivity*, that is, the magnetizing force necessary to bring the iron from a magnetic to a neutral state. The distance *OC* represents the *retentivity*, that is, the value of the *residual* magnetic flux density in the iron after the magnetizing force has been removed.

The area inclosed by the curve represents the energy lost in carrying the iron through one cycle, i.e. from a maximum magnetization in one direction to a maximum in the opposite direction and back to the original condition. Suppose the magnetization to be due to a current I flowing in a solenoid of n turns. If, in a short interval of time dt, a change of $d\Phi$ be made in the flux which is linked with the solenoid, then this change will induce an *E.M.F.* in the solenoid, which, during the interval of time dt, will be equal to

$$E = \frac{n d\Phi}{10^8 dt} \text{ volts.}$$

During this time work must be performed to maintain this current I, and its magnitude is

$$EIdt = \frac{nId\Phi}{10^8},$$

for Idt represents the quantity of electricity which is transferred from one point to another, between which there exists a difference of potential E. Now $\Phi = A\mathcal{B}$ (§ 14) and hence $d\Phi = Ad\mathcal{B}$. Furthermore, $nI = \dfrac{10\,\mathcal{H}l}{4\pi}$ (§ 24). Hence the work during the time dt is

$$EIdt = \frac{Al}{10^7 4\pi} \mathcal{H} d\mathcal{B} \text{ joules.}$$

Supposing the magnetizing force to vary cyclically, taking t seconds to make one cycle, then the work per cycle is

$$EIt = \frac{Al}{10^7 4\pi} \int_{-\mathcal{B}_m}^{+\mathcal{B}_m} \mathcal{H} d\mathcal{B} \text{ joules.}$$

If the number of cycles completed in one second be f, then $f = \dfrac{1}{t}$, and the power in joules per second, that is, the power in watts, equals

$$EI = \frac{fAl}{10^7 4\pi} \int_{-\mathcal{B}_m}^{+\mathcal{B}_m} \mathcal{H} d\mathcal{B} = \frac{10^{-7}}{4\pi} fv \int_{-\mathcal{B}_m}^{+\mathcal{B}_m} \mathcal{H} d\mathcal{B},$$

where v is the volume of iron in cubic centimeters. The integral expression is evidently the area contained by the hysteresis loop.

The value of this integral is dependent upon $\mathcal{B}_m$, upon the retentivity of the kind of iron, and upon its coercivity. Steinmetz has shown that for all practical purposes the value of the integral for flux densities ranging from 2000 to 14000 gausses may be expressed by the empirical formula

$$\frac{1}{4\pi}\int_{-\mathcal{B}_m}^{+\mathcal{B}_m} \mathcal{H} d\mathcal{B} = \eta \mathcal{B}_m^{1.6},$$

where η is a constant depending upon the physical and chemical properties of the iron. Therefore the power lost in watts due to hysteresis may be written

$$P_h = 10^{-7}\, v f \eta \mathcal{B}_m^{1.6}.$$

Values of the constant η are given in the following table:—

HYSTERETIC CONSTANTS.

Best soft iron or steel sheets	0.001
Good soft iron sheets	0.002
Ordinary soft iron	0.003
Soft annealed cast steel	0.008
Cast steel	0.012
Cast iron	0.016
Hard cast steel	0.025

The hysteretic constant increases with continued heating, and this effect is called *ageing*. Annealing, while it increases the permeability, also increases the hysteretic constant as well as the ageing effect.

The magnitude of the hysteretic constant is largely dependent upon the mechanical structure of the iron. To attain the smallest value, the iron should not be of homogeneous structure, but should be more compact in directions perpendicular to the direction of the flux than in transverse directions.

29. **Eddy Currents.** — When a mass of iron is subjected to a pulsating flux, electromotive forces are set up in the iron which produce currents therein, called *eddy*, or *Foucault, currents.* The flow of these currents represents an expenditure of energy appearing as heat. In order to prevent excessive heating of such portions of dynamo electric machinery subject to rapid reversals or changes of magnetization, these portions are constructed of *laminated* iron, the laminæ being transverse to the direction of flow of the eddy currents, but longitudinal with the magnetic flux. Each lamina is more or less thoroughly insulated from its neighbors by the natural oxide on the surface or by Japan lacquer.

The loss of power due to eddy currents could be made inappreciable by the use of laminæ sufficiently thin; but a limitation exists due to the decrease in effective iron cross-section caused by the waste of space which is taken up by the insulation between adjacent laminæ. The thickness of the laminæ generally used for dynamo armatures is between 0.014 and 0.02 inch, and the space between laminæ is usually somewhat less than 0.002 inch.

A formula for the calculation of the power, in watts, lost in iron due to eddy currents, based upon the assumption that the laminæ are perfectly insulated from each other, is

$$P_e = kvf^2l^2\mathfrak{B}^2_m,$$

where k = a constant depending upon the resistivity of the iron, its value being about 1.6×10^{-11},

v = volume of iron in cubic centimeters,

l = thickness of one lamina in centimeters,

f = number of magnetic cycles per second,

and $\mathfrak{B}_m$ = maximum flux density (i.e. Φ_m per sq. cm.).

The armatures of dynamos are usually provided with projecting teeth, and therefore the flux density between an armature and its field poles is greatest opposite the teeth and is a minimum opposite the slots. As the armature rotates, this variation of flux produces to some extent eddy currents in the pole faces. To reduce the loss occasioned thereby, the pole faces are sometimes also constructed of laminated iron.

PROBLEMS.

1. Two cylindrical magnets, 1.8 cm. in diameter, are magnetized to an intensity of 500 units pole for each square cm. of cross-sectional area, and their north poles are placed 8 cm. apart. Compute the force of repulsion between the two north poles.

2. What is the total flux from each pole of the magnets specified in Prob. 1, considering the poles to be isolated and concentrated at points?

3. A conductor 24 inches long, moving parallel to itself and at right angles to a magnetic field having an intensity of 40000 maxwells per sq. in., traverses 5 ft. in 3.5 seconds. Determine the average *E.M.F.* in volts induced in the conductor during this interval.

4. In $\frac{1}{25}$ second the current strength in a circuit, having an inductance of 0.6 henry, falls from 30 to 15 amperes. What is the magnitude and direction of the average induced *E.M.F.* in the circuit due to this change of current?

5. What is the inductance of a circuit having 5 ohms resistance and in which the instantaneous value of the current 0.03 second after impressing 110 volts upon the circuit is 13.9 amperes?

6. What would be the current in a circuit, having 10 ohms resistance and 0.3 henry inductance, 0.02 second after suppressing the initial *E.M.F.* of 80 volts?

7. The force exerted on a wire 5 ft. long, which carries a current of 50 amperes, is 1800 dynes. What is the intensity of the magnetic field in which this wire is situated?

8. A brass toroid, having a mean diameter of 20 cm., is completely wound with 8 turns of wire per cm. of axial length. Determine the magnetic field intensity at the axis, when a current of 50 amperes flows through the winding. Calculate the total magnetomotive force which sets up this field.

9. A wrought-iron toroid, wound with 20 turns of wire per inch of axial length, has a current of 3.5 amperes flowing through the winding. Determine the flux density and permeability of the iron from the data given in § 25.

10. The flux density in a cylindrical cast-iron rod 5 cm. in diameter and 30 cm. long is 6000 gausses ($\mu = 250$). Compute the reluctance and permeance of the rod between the two faces.

11. Calculate the total number of ampere-turns necessary to produce a flux density of 6000 gausses in the toroid of Prob. 10.

12. A closed core, composed of the best steel sheets 0.035 cm. thick, has a volume of 5400 cu. cm., and is subjected to 100 magnetic reversals per second, i.e. $f = 50$. Calculate the hysteresis and eddy current losses when the maximum flux density in the core is 3500 gausses.

CHAPTER III.

ARMATURES.

30. **Dynamos.** — Dynamos may be defined as machines to convert mechanical energy into electrical energy, or electrical energy into mechanical energy, by utilizing the principle of electromagnetic induction. A dynamo is known as a *generator* when mechanical energy, supplied in the form of rotation, in all commercial machines, is converted into electrical energy, which may be delivered either as "*direct current*" or as "*alternating current.*" When the conversion of energy takes place in the reverse order, the dynamo is called a *motor.*

31. **Principle of Action of a Generator.** — If a loop of wire be revolved in a magnetic field about an axis perpendicular to the lines of force, as in Fig. 14, then each side (but not the ends) of the loop is a *conductor* moving across the lines of a magnetic field, and as such will have an *E.M.F.* induced in it. Since the motion of one conductor is up while that of the other is down, the directions of the induced *E.M.F.*'s in the two sides would be opposite to each other, but since they are on opposite sides of a loop, the pressure will be cumulative; i. e. instead of neutralizing each other, the two pressures will be added to each other. If now the two ends of the wire from which the loop is made be respectively connected with *slip rings*, and a circuit be completed through contacts sliding on them; a current will flow. When the loop, in its revolution, reaches a

position (as illustrated in Fig. 14) such that the conductor that was previously moving upward begins to move downward, then the direction of the induced *E.M.F.* will be changed in both sides of the loop, and the direction of the

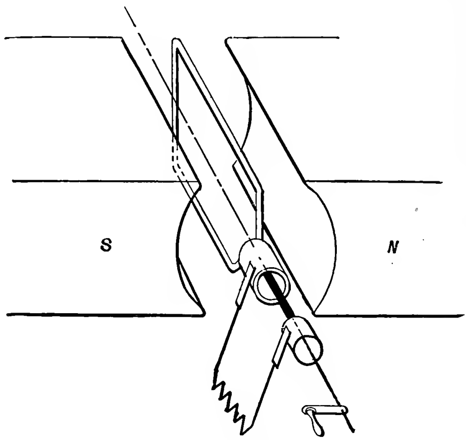


Fig. 14.

current through the circuit will be changed. For each complete revolution the current changes direction twice. It is an *alternating current*, and the supposed machine is an *alternating-current generator*, or simply an *alternator*.

32. **The Function of the Commutator.** — If, instead of connecting the two ends of a loop of wire revolving in a magnetic field to slip rings, they be attached one to each half of a split metal ring mounted on the same shaft, the two halves being insulated from each other, and brushes be provided, which are so placed that at the instant the induced *E.M.F.* in the loop changes in direction the brushes will slide across from one of the halves to the other, then the current, while reversed in the loop, will flow in the same direction in the external circuit. This arrangement, called

a *commutator*, is employed when it is desired to obtain a *rectified, continuous* or *direct* current. A dynamo so equipped is called a *direct-current generator*, or simply a *generator*.

If the loop were wound double, i.e. have four conductors, before the ends were attached to commutator segments, and if the speed and the strength of the magnetic field be maintained constant, twice the *E.M.F.* will be produced.

For a single loop, the commutator would consist of two cylindrical pieces or *segments*, as shown in Fig. 15. In this case there would be no *E.M.F.* produced at the instants when the brushes pass from one segment to the other, and hence the current would fall to zero twice during every revolution of the loop. If two loops, placed at right angles to each other, are rotated in a magnetic field, one or the other would always be cutting lines of force and at no time could the pressure be zero. To satisfactorily collect current from this arrangement requires four commutator segments and a system of connections similar to that shown in Fig. 16. In this case the *E.M.F.* would fluctuate, but not so badly as in the previous one. If the number of loops be increased and the number of commutator segments be correspondingly increased, the *E.M.F.* fluctuation of such an arrangement will become practically negligible.

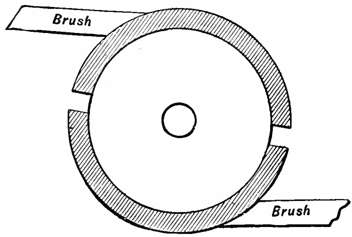


Fig. 15.

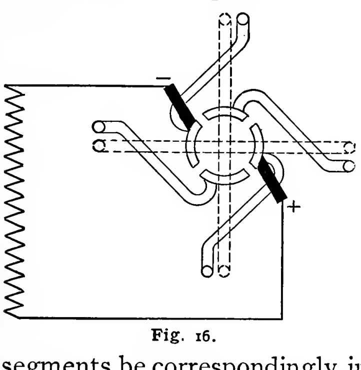


Fig. 16.

33. Electromotive Force Generated. — The magnitude of the electromotive force induced in a conductor of length l cm. moving parallel to itself with a velocity of v cm. per sec. across a uniform magnetic field having a flux density of $\mathfrak{B}$ gausses is

$$E = \mathfrak{B} l v \; 10^{-8} \sin \alpha \text{ volts,} \qquad \S\, 15$$

where α is the angle between the paths of the flux and of the conductor. If a single loop (two conductors) revolve about an axis that is in the plane of the loop and perpendicular to the flux with an angular velocity of V revolutions per minute, the linear velocity of the conductors will be

$$v = 2\,\pi r \frac{V}{60},$$

where r is the distance in cm. of each conductor from the axis. The *instantaneous E.M.F.* for a loop of s conductors is therefore

$$E' = 2\,\pi \mathfrak{B} l r s \cdot \frac{V}{60} 10^{-8} \sin \alpha \text{ volts.}$$

But $2\,rl\mathfrak{B}$ is the maximum flux passing through the loop, Φ_m; hence

$$E' = \pi \Phi_m s \frac{V}{60} 10^{-8} \sin \alpha \text{ volts.} \qquad (1)$$

The maximum value of the induced *E.M.F.* for the loop of s conductors is attained when $\alpha = 90°$, i.e. when the conductors move perpendicularly across the magnetic flux. This maximum value is

$$E_m = \pi \Phi_m s \frac{V}{60} 10^{-8} \text{ volts.} \qquad (2)$$

The instantaneous *E.M.F.* is therefore

$$E' = E_m \sin \alpha,$$

which is the equation of a *sine curve.* Thus the sine curve, Fig. 17, shows the instantaneous values of the induced electromotive force as the angle α varies from 0° to 360°.

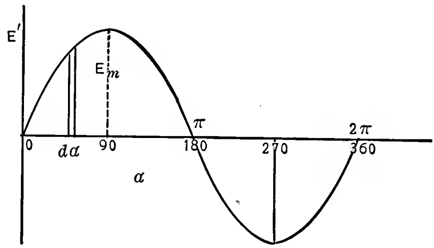


Fig. 17.

The average *E.M.F.* during a half revolution is obtained by dividing the area of one lobe by the base line. Thus

$$E_{av} = \frac{\int_0^\pi E_m \sin \alpha d\alpha}{\pi} = \frac{E_m}{\pi}\left[-\cos \alpha\right]_0^\pi = \frac{2}{\pi} E_m. \qquad (3)$$

Substituting the value of E_m, there results the average *E.M.F.* induced in the loop of s conductors

$$E_{av} = 2\,\Phi_m s \frac{V}{60} 10^{-8} \text{ volts.} \qquad (4)$$

If this loop be provided with a commutator as explained in the foregoing section, the direction of the voltage impressed on the external circuit remains the same, but the magnitude varies sinusoidally as shown in Fig. 18.

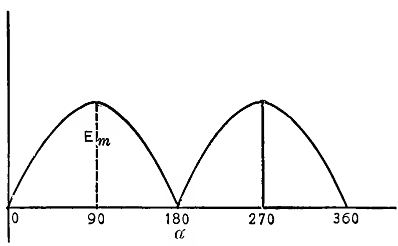


Fig. 18.

If a similar loop be placed 90° from the other, the magnitude of the induced

E. M. F. therein will be the same as that in the first, but corresponding instantaneous values in the two loops occur 90° apart, as shown in Fig. 19. The resulting electromotive force at any instant may be found by adding the *E.M.F.*'s induced in the two loops at that instant. Thus in Fig. 19 the dotted line shows the resulting pressure for two loops situated 90° apart.

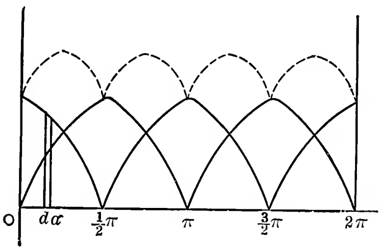


Fig. 19.

The instantaneous value of the electromotive force induced in the first loops is

$$E_1' = E_m \sin \alpha,$$

and that induced in the second is

$$E_2' = E_m \sin (90^\circ + \alpha);$$

therefore the resulting instantaneous value of voltage induced in the loops when connected in series is

$$E' = E_1' + E_2' = E_m [\sin \alpha + \cos \alpha].$$

In general, if there be m loops connected in series and displaced $\frac{\pi}{m}$ electrical degrees from each other, and rotating about a common axis in a uniform magnetic field, then the instantaneous pressure will be

$$E' = E_m \left[\sin \alpha + \sin \left(\frac{\pi}{m} + \alpha \right) + \ldots + \sin \left(\pi \frac{m-1}{m} + \alpha \right) \right], \quad (5)$$

and the average pressure will be

$$E_{av} = \frac{E_m}{\frac{\pi}{2m}} \int_0^{\frac{\pi}{2m}} \left[\sin \alpha + \sin \left(\frac{\pi}{m} + \alpha \right) + \cdots \right.$$

$$\left. + \sin \left(\pi \frac{m - 1}{m} + \alpha \right) \right] d\alpha ;$$

whence

$$E_{av} = \frac{2\, m E_m}{\pi} = 2\, m \Phi_m s \cdot \frac{V}{60} 10^{-8} \text{ volts.} \tag{6}$$

The total number of conductors S, connected in series and in circuit between two brushes, is equal to the product of the number of loops m and the number of conductors per loop, s; or

$$S = ms.$$

By substituting this value in (6), there is between brushes

$$E_{av} = 2\, \Phi_m S \frac{V}{60} 10^{-8} \text{ volts.} \tag{7}$$

The maximum and minimum values of instantaneous pressures are obtained respectively by substituting for α in (5) the quantities $\frac{\pi}{2\, m}$ and 0. The resulting expressions are

$$E_{max} = E_m \csc \frac{\pi}{2\, m}$$

and

$$E_{min} = E_m \cot \frac{\pi}{2\, m},$$

where E_m is the maximum $E.M.F.$ per loop. The percentage fluctuation of the *E.M.F.*, therefore, can be represented by

$$100 \times \frac{E_{\max} - E_{\min}}{E_{\text{av}}} \quad \text{or} \quad \frac{50\pi}{m}\left[\csc\frac{\pi}{2m} - \cot\frac{\pi}{2m}\right]. \qquad (8)$$

Thus, for twelve loops revolving in a bipolar field, the fluctuation of electromotive force is 1.7 per cent.

In order to render the foregoing formulæ applicable to a multipolar field, it is necessary to insert the symbol p, which denotes the number of pairs of field poles. The product of the terms $\frac{V}{60}$ and p represents the number of magnetic cycles some of the iron of the machine passes through in one second. It is termed *frequency*, as in alternating-current work, and is represented by f. Thus

$$f = p \cdot \frac{V}{60}.$$

Therefore the average electromotive force available between brushes is

$$E_{\text{av}} = 2\,\Phi_m S f\, 10^{-8} \text{ volts}, \qquad (9)$$

where Φ_m is the total flux per pole passing through the armature.

34. **The Armature.** — In a dynamo, the loops of wire in which *E.M.F.* is induced by movement in a magnetic field, together with the iron core that sustains them, with the necessary insulation, and with the parts connected immediately thereto, constitute the *armature* of a dynamo. An armature in which both sides of the loop of wire cut lines of force, as in the cases just described, is called a *Drum Armature*. A kind of armature less generally used is the

Ring Armature, illustrated diagrammatically in Fig. 20. Here the lines of force emanating from the N. pole flow through the iron core of the ring, and very few across the air space inside the ring. Hence, when wires are wound on the ring, and the whole is revolved about an axis perpendicular to the plane of the ring, only the wires on the outside face of the ring cut lines of force, those on the inside serving only to complete the electrical circuit.

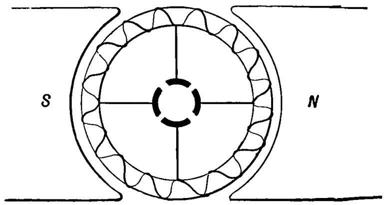


Fig. 20.

Drum armatures have all the conductors on the peripheral surface, and therefore have a greater percentage of active wire than ring armatures. Drum armature cores are very often constructed in the form of a ring, because of better ventilation and economy of iron; and such construction should not be confused with that of ring armatures.

A drum armature of large diameter and of short length in the axial direction may have more wire exposed on its ends than on its periphery. The pole pieces are sometimes placed at the ends, and the armature is then called a *Disk Armature.* This type is seldom used in this country.

35. **The Field Magnets.** — Almost all dynamos have their magnetic fields produced by electro-magnets, and these are called *field magnets.* In small machines the field magnets are usually bipolar, i.e. they have one North and one South pole, with the armature revolving between them. Bipolar machines are made in many forms, a few of which are

shown in Figs. 21, 22, and 23. The magnetizing coils, or *field coils,* may be placed on both legs of the magnet, on

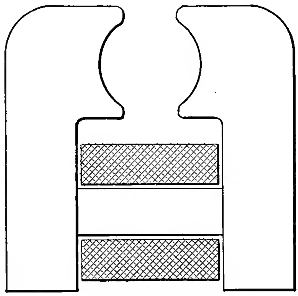


Fig. 21.

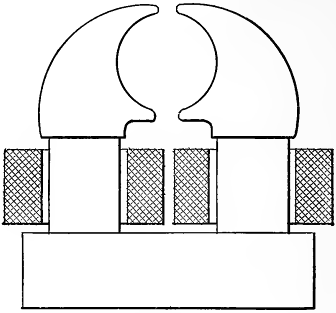


Fig. 22.

one leg, or on the *yoke* which connects the two. The best and most used bipolar arrangement is the enclosed type of

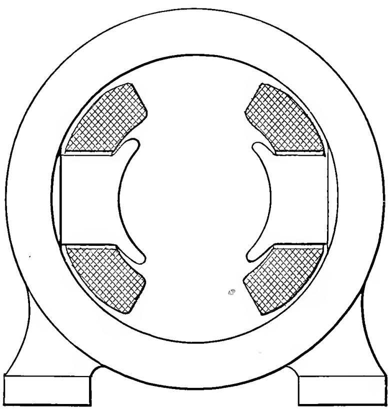


Fig. 23.

Fig. 23, for the coils are almost completely surrounded by iron, thereby securing protection from mechanical injury, as well as avoiding excessive magnetic leakage.

A typical form of multipolar field magnet structure is shown in Fig. 24, in which an even number of poles are so wound as to be alternately magnetized North and South.

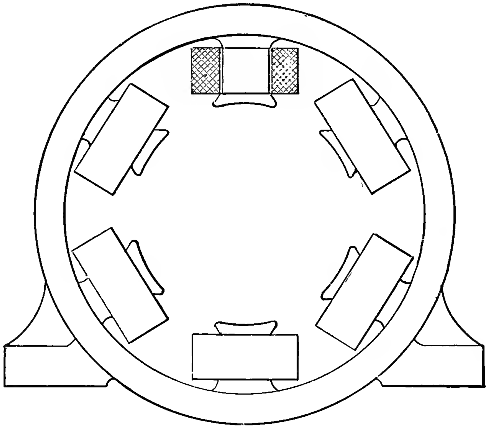


Fig. 24.

36. **Armature Windings.** — It is possible to connect the conductors of an armature to each other and to the commutator segments in a great many ways that will permit of satisfactory operation. The design of an operative scheme of winding is, to a great extent, a geometrical problem. Of the many possible and proposed schemes of winding, those which have been adopted and are still used in the construction of standard machines are characterized by economy of copper, by good time constants to secure satisfactory commutation, and by such coil shapes as permit of convenience in construction and assembling and of accessibility for making repairs.

Direct-current armature windings are divided into two classes, namely *open-coil* and *closed-coil* windings. The former are used almost exclusively on series constant-current machines, such as the Thomson-Houston arc-light generators, and will be discussed in a later chapter. With open-coil windings, only those conductors which are conductively connected between the commutator segments, which the brushes are momentarily resting on, are effective in supplying an electromotive force.

Closed-coil windings are much more generally used. If the wire of an ordinary, that is, *single*, closed-coil winding were removed from the armature and uncoiled, it would form a closed loop, and the points of connection with the commutator segments would be equidistant from each other. Some closed-coil windings are so constructed that the wire, if removed from the armature core and uncoiled, would form two or more closed endless loops. Such windings are termed *duplex*, *triplex*, or *quadruplex windings*, according to the number of endless loops, whether two, three, or four. Such *multiplex* windings are sometimes employed on machines of large current output; but their use is relatively infrequent, and therefore, unless explicitly stated to the contrary, *single*, or *simplex*, windings will hereinafter be understood.

It is convenient, in treating of armature windings, to call each of the portions which terminates at two commutator segments an *element* of the winding. An armature element may consist of one or more *armature coils*. Those parts of an armature element which lie on the periphery of the armature and in which *E.M.F.*'s are induced are called *inductors*. Thus, in the ring type of armature, only the portions of the wire on the outer surface of the ring constitute its inductors. A drum armature element, however, has two inductors lying axially on the core surface. In the following discussion of armature windings, when one inductor is mentioned it does not imply that only one wire is meant; further, an element said to be formed by two inductors may be a coil of many turns. Simplification of the winding diagrams is effected in this manner.

Two types of closed-coil windings for direct-current armatures are to be distinguished, namely *two-circuit* or *wave windings*, and *multiple-circuit* or *lap windings;* the former being used principally on machines of small output, and the lap winding on machines of intermediate and large output. In the wave winding there are always two circuits between the brushes, regardless of the number of field poles on the machine, each of the circuits carrying one-half of the total current. For this type of winding only two brushes are required, but it is usual in practice to provide as many brushes as there are poles, in order to avoid excessive sparking at the commutator. There is a slight, but generally neglected, *E.M.F.* induced in the portions of the winding included at any instant between the different sets of brushes of the same polarity. It is quite small, however, as compared with the *E.M.F.* induced between brushes of opposite polarity. In the lap winding there are as many circuits between the brushes as there are field poles, and each circuit carries $\frac{1}{2p}$ of the total current. For armatures of this type as many brushes as field poles are required.

Starting from a certain commutator segment and passing clockwise over p elements of a simplex wave winding, or one element of a simplex lap winding, one reaches the next adjacent segment either to the right or to the left. If it be to the right it is called a *progressive* winding; and if to the left, it is called a *retrogressive* winding. The use of the latter is somewhat more economical in copper.

A 4-pole two-circuit, or wave winding is shown diagrammatically in Fig. 25, in which the inductors are represented by the short radial lines and the *end connections* by the lines joining them. The brushes are placed inside the commutator for clearness. Fig. 26 shows the same winding more clearly in developed form, the seventeen commutator segments being lettered from *a* to *q*. Tracing the method of interconnecting

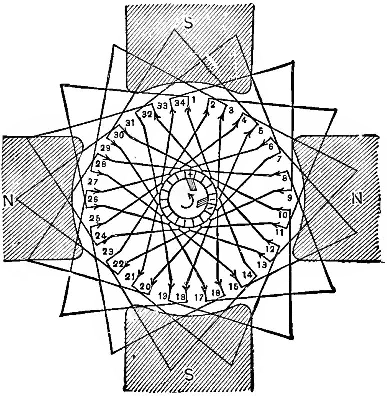


Fig. 25.

the 34 armature inductors, and starting from segment *a*,

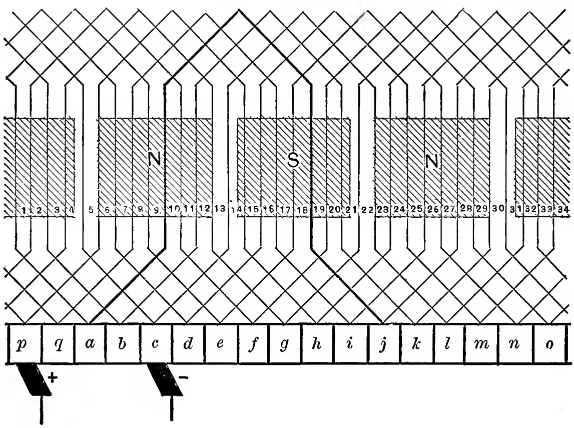


Fig. 26.

it is seen that connection is made to inductor No. 10, which is connected at the rear to inductor No. 19; and this at the front is joined to segment *j* and also to inductor No. 28. Proceeding in this way, the winding scheme may be tabulated as follows:

REAR END		FRONT END	
	to	to	to
10	19	j	28
28	3	b	12
12	21	k	30
30	5	c	14
14	23	l	32
32	7	d	16
16	25	m	34
34	9	e	18
18	27	n	2
2	11	f	20
20	29	o	4
4	13	g	22
22	31	p	6
6	15	h	24
24	33	q	8
8	17	i	26
26	1	a	10

Thus the winding forms one closed circuit, and it is evident that there are two paths from the positive to the negative brush. It is a progressive winding.

The number of inductors spanned by the end connections at each end is called the *winding pitch*, and is represented by λ. In Fig. 26 the inductors are numbered consecutively from 1 to 34 in passing around the armature, and inductor No. 10 joins inductor No. 19 at the rear; the rear-end winding pitch is therefore 9, or

$$\lambda_r = 19 - 10 = 9.$$

At the front, or commutator, end, inductor No. 19 joins inductor No. 28, so that the front-end winding pitch is also

$$\lambda_f = 9.$$

It occurs frequently, however, that the front and rear winding pitches are different, in which case the mean winding pitch is

$$\lambda = \frac{\lambda_r + \lambda_f}{2}.$$

There exists a definite relation between the total number of inductors on the surface of an operative armature, the number of pairs of poles, and the winding pitch. For the wave winding, the permissible total number of inductors is

$$C = 2p\lambda \pm 2.$$

Hence, for the four-pole machine of Fig. 26 the total number of inductors would be $2 \times 2 \times 9 \pm 2$; that is, $C = 38$ or 34. The latter number was here chosen.

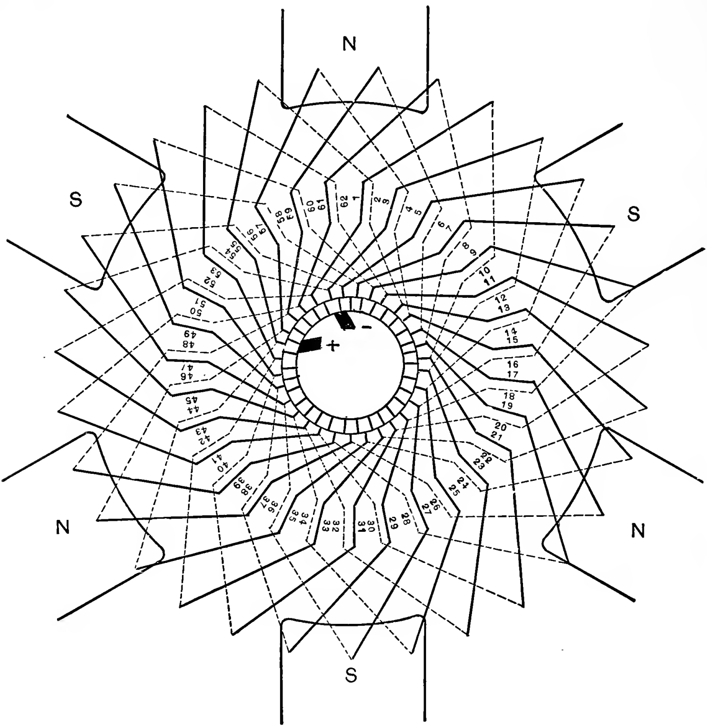


Fig. 27.

A six-pole wave winding for a drum armature having 31 slots is shown in Fig. 27. There are 62 inductors, the even and odd numbered ones representing the lower and upper inductors in the slots respectively. The winding pitches for this armature are

$$\lambda_r = 9,$$
$$\lambda_f = 11,$$
and
$$\lambda = 10.$$

A six-pole, retrogressive, lap or multiple-circuit, in this case six-circuit, winding is shown diagrammatically in Fig. 28, and it is seen that there are six paths in parallel from the positive to the negative brushes. There are 80 inductors, the winding pitches being

$$\lambda_r = 11$$
and
$$\lambda_f = 13.$$

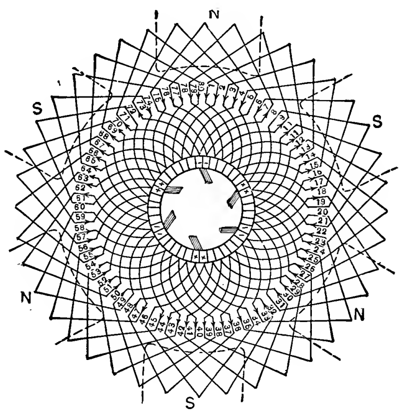


Fig. 28.

In the lap winding, the front and rear winding pitches cannot be equal and the difference between them must be some multiple of 2. If there be two inductors per slot, and the lower ones be even-numbered and the upper ones odd-numbered, as usual, then the front and rear winding pitches must be odd, and therefore the mean winding pitch, λ, is always an even number. The expression for the total number of inductors on a lap winding for p pairs of poles allows more latitude than is the case with wave windings, and is

$$C = 2\,p\lambda.$$

Herefrom the mean winding pitch is

$$\lambda = \frac{C}{2p},$$

but frequently values of λ are taken which are considerably smaller than the value obtained from this formula. In the present case, for example, $\lambda = 12$ instead of $\frac{80}{6}$. Such windings are called *short-chord windings.*

As previously stated, a winding element of a drum armature has two inductors, yet there may be any number of turns of wire in the element. This is often so in practice, where high terminal voltages are desired. Fig. 29 shows

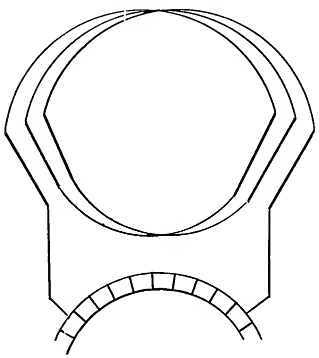


Fig. 29.

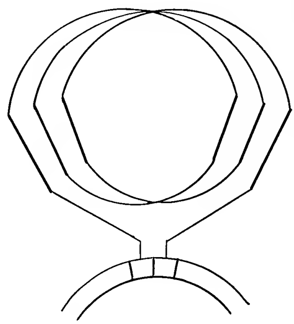


Fig. 30.

a three-turn element for a wave winding, and Fig. 30 a similar element for a lap winding. Winding pitches are sometimes, but not in this text, expressed by the number of spanned core-slots or subtended angles at the axis, the latter being expressed in radians or degrees either electrical or mechanical.

37. Multiplex Armature Windings. — In the simplex armature windings thus far considered, a winding would consist of one complete circuit. In multiplex windings, on the other hand, there may be two or more distinct circuits completely insulated from each other, and each of these might be provided with a separate commutator and set of brushes. The usual practice, however, is to provide only one commutator with the segments pertaining to one winding intermeshed with those belonging to the other windings. Thus no greater number of brushes is required than for a corresponding simplex winding, yet they must be considerably wider so that simultaneous commutation of all the circuits may be going on.

Starting from a certain commutator segment, and traversing p elements of a simplex wave winding, or any one element of a simplex lap winding, one reaches the next adjacent commutator segment. Proceeding in like manner with a *duplex* winding, one reaches the second following segment from the starting point; and similarly, with a *triplex* winding, one reaches the third following segment, etc., the intermediate segments being connected to the other windings. Such multiplex windings, which consist of a number of complete and independent simplex windings, are *multiply re-entrant*, that is, each individual circuit re-enters upon itself to form a closed circuit. Multiplex armature windings may be *singly*, *doubly*, *triply*, or in general, *multiply re-entrant.*

A singly re-entrant winding is one in which, by successive angular advances, the entire winding is traversed before returning to the starting point. A doubly re-entrant winding is one in which only half of the winding is traversed before reaching the initial segment. Similarly, in a triply re-entrant winding only one-third thereof is traversed. The number of separate circuits on an armature determines the *degree* of re-entrancy. The number of times it is necessary to pass around the armature in traversing a complete circuit must not be confounded with the degree of re-entrancy.

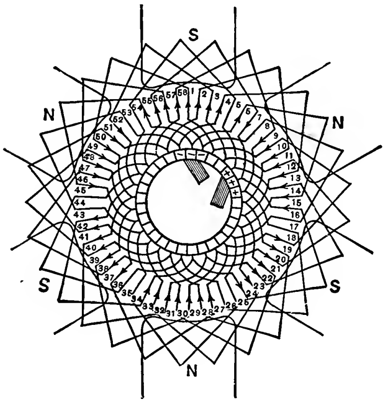


Fig. 31.

Fig. 31 depicts a six-pole, retrogressive, two-circuit, singly re-entrant duplex winding comprising 58 inductors. A four-pole, retrogressive, two-circuit, triply re-entrant triplex winding, having 66 inductors, is shown in Fig. 32. A duplex winding may be either singly or doubly re-entrant, a triplex winding may be either singly or triply re-entrant, and a quadruplex winding may be singly, doubly, or quadruply re-entrant. Multiplex windings beyond these are rarely used in practice.

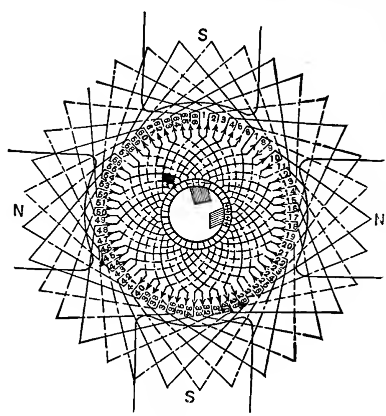


Fig. 32.

The general formula for multiplex wave windings is

$$C = 2\, p\lambda \pm 2\, y,$$

where C, p, and λ have the same significance as before, and where y is the multiplicity of the winding, whether duplex, triplex, and so on. For a given multiplex winding, the choice of the mean winding pitch, and therefore also the total number of inductors, depends upon the degree of re-entrancy desired. The highest common factor of λ and y expresses the degree of re-entrancy. Thus, for the duplex winding of Fig. 31,

$$y = 2$$

and

$$\lambda = 9;$$

therefore the winding must be singly re-entrant, since the highest common factor of 2 and 9 is 1. The total number of inductors for this winding might be

$$C = 2 \times 3 \times 9 \pm (2 \times 2)$$
$$= 50 \text{ or } 58.$$

The latter value was here chosen.

For the triplex winding of Fig. 32,

$$y = 3 \qquad \text{and} \qquad \lambda = 15;$$

hence the winding is triply re-entrant. The total number of inductors might also have been 54 instead of 66 as shown.

In multiplex lap windings, the degree of re-entrancy is equal to the highest common factor of half the number of inductors, $\frac{C}{2}$, and the multiplicity of the winding, y. The mean winding pitch is chosen as near as possible to

$$\lambda = \frac{C}{2p},$$

the same as for simplex lap windings.

38. Equalizing Connections. — The electromotive forces generated in different sections of an armature are not exactly equal, due to inaccurate centering of the armature and to the unavoidable differences of distance of the armature conductors from the field pole pieces. The *E.M.F.*

differences, although small, nevertheless set up local currents in the armature, and these may result in excessive heating of the conductors and sparking at the commutator. In wave windings, little difficulty is experienced in this respect, because the inductors are connected in series and are distributed under all the poles; but in lap windings large internal currents are produced in the armature which cause operative troubles. To minimize troubles from these currents, lap-wound armatures of large generators are supplied with *equalizing connections* of low resistance. These are connections between points of the winding which should be at the same potential, and they usually take the form of rings situated at the commutator end of the armature core.

39. E.M.F. Equation of Dynamos. — The average electromotive force between brushes, induced in the conductors of an armature revolving in a multipolar magnetic field, is

$$E_{av} = 2\ \Phi_m S f\ 10^{-8} \text{ volts,} \qquad §33$$

where Φ_m is the flux per pole in maxwells which cuts the conductors, S is the number of *conductors* in series between brushes, and f is the number of magnetic cycles through which the armature core passes in one second; or $f = p\dfrac{V}{60}$.

This equation is perfectly general, and applicable to any type of direct-current dynamo with any style of armature winding. To ensure its proper application, a consideration of the significance of the term S for various styles of winding is necessary.

In general, S is the total number of conductors on the armature divided by the number of current paths through the armature between brushes. Reference to §§ 36 and

37 shows that the number of paths between brushes for various styles of winding are as follows:

TYPE OF WINDING	WAVE	LAP
Simplex . . .	2	$2p$
Duplex . . .	4	$4p$
Triplex . . .	6	$6p$
Quadruplex .	8	$8p$

As a numerical example, compute the no-load voltage of an 8-pole generator having a simplex lap-wound armature with a total of 1920 conductors, when revolving at 300 rev. per min. The effective magnetic flux per pole is 7 megamaxwells.

There are 4 pairs of poles, whence the number of conductors per circuit is

$$S = \frac{1920}{2 \times 4} = 240.$$

The number of magnetic cycles passed through per second is

$$f = 4 \cdot \frac{300}{60} = 20.$$

Therefore the terminal *E.M.F.* of the generator (neglecting the resistance drop in windings) is

$$E_{av} = 2 \cdot \frac{7000000}{10^8} \cdot 240 \cdot 20 = 672 \text{ volts.}$$

40. **Core Construction.**—To reduce to a minimum the otherwise excessive eddy current loss (§ 29) in armature cores, these are constructed of thin discs of soft wrought iron or mild steel, which are more or less thoroughly insulated from each other by the natural oxide or by a coating of varnish on the discs. Sometimes, for special machines, shellac coatings on the discs, or thin paper sheets between

them, are applied. Laminating the core in this way does not completely prevent the flow of eddy currents, for small *E.M.F.*'s will still be induced in each lamina which produces them. The eddy current loss, being proportional to the square of the thickness of laminations, would be lowered by using very thin discs. A limit to the reduction of thickness is the increased loss due to the higher flux density necessary, owing to waste of space which is taken up by the insulation between laminations.

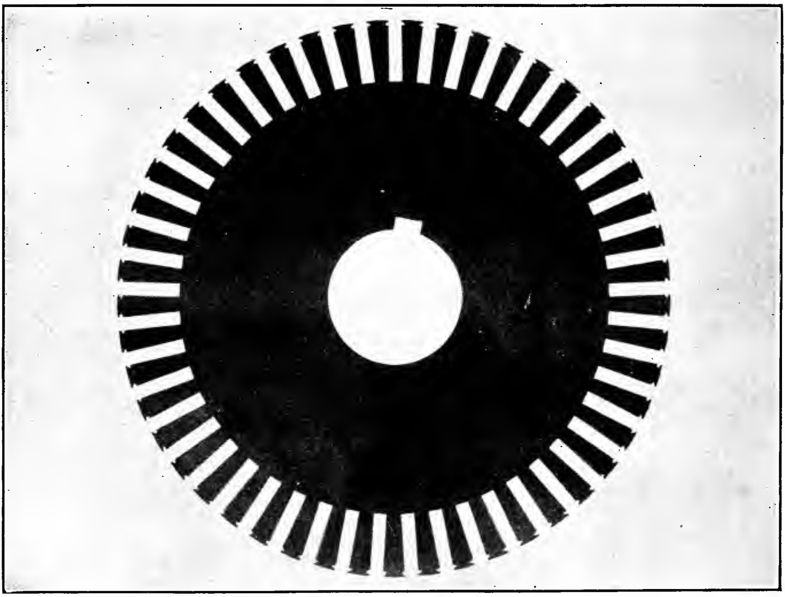


Fig. 33.

For the smaller machines, having armatures less than 16 inches in diameter, the discs are punched in one piece, and usually take the form shown in Fig. 33. Sometimes apertures are provided in the laminations about the axis, and these constitute air passages through the core, thus improving ventilation. These discs are mounted directly on the shaft and are keyed to prevent turning. They are held in position by flanges of cast steel or cast iron, which are pressed together by nuts on the shaft, as shown in Fig. 34, or by bolts passing through, but insulated from, the laminations.

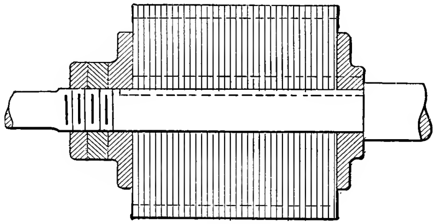


Fig. 34.

For the larger armatures, the discs consist of a number of segments which are assembled on a mechanical support

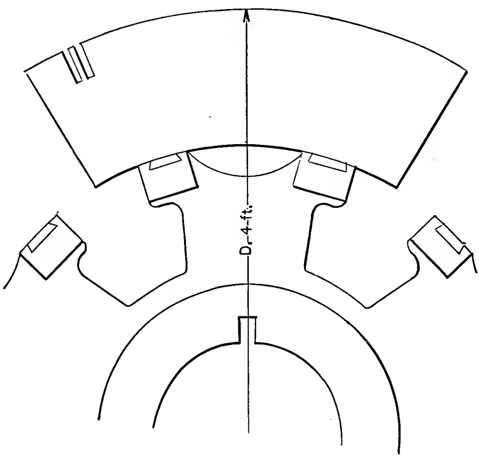


Fig. 35.

called a *spider*, being attached thereto by inwardly-projecting lugs on the segments. In Fig. 35 is shown in part the spider of a 300-K.W. generator with a segment dovetailed to one of the arms. The joints are staggered in the laminations of successive layers. Fig. 36 illustrates a section of the spider of the same machine with the end flanges holding the laminations in place. The flanges are shaped so as to form a support for the armature winding. The spider, which has an extension for supporting the commutator, is pressed onto the shaft and keyed.

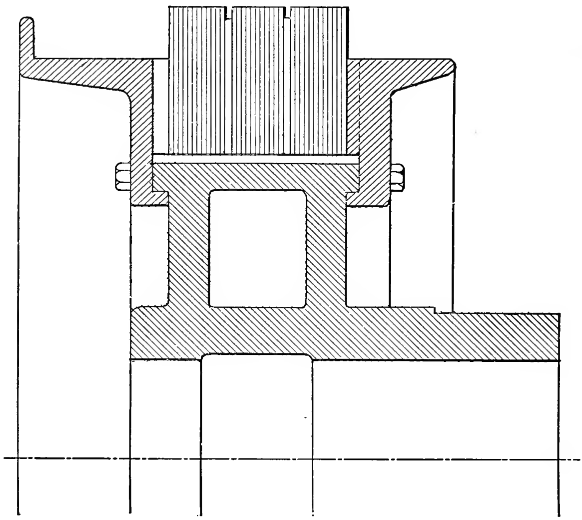


Fig. 36.

To obtain sufficient ventilation in the armature and thereby lessen the temperature rise incident to operation, it is usual to provide radial *ventilating ducts* in the core. The rotation of the armature causes air to pass in through the axial apertures, or spider openings, and out through the ventilating ducts as indicated by the arrows in Fig. 37. The ducts are formed by separating the laminations at intervals by the interposition of blocks of non-magnetic material called *spacing pieces.* A type of spacing piece is shown in Fig. 38, which consists of brass strips set edgewise into slots punched in stout core discs and riveted.

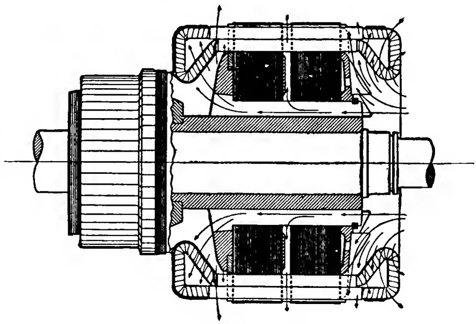


Fig. 37.

This type is commendable because it gives support to the armature teeth.

Good practice requires the provision of one ventilating duct for every 2 to 4 inches of axial core length, the width

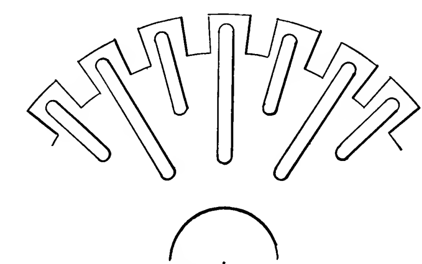


Fig. 38.

of the ducts varying from three-eighths to five-eighths of an inch.

Fig. 39 illustrates a Westinghouse armature core partly assembled on the spider. One of the segments of a disc and a spacing piece are shown leaning against that portion of the spider upon which the commutator is to be mounted.

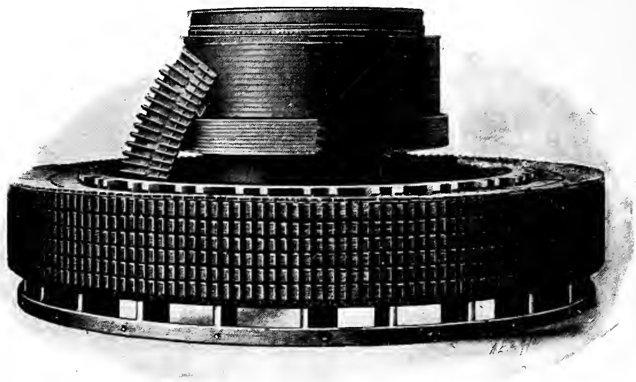


Fig. 39.

Two types of armature slots are in general use, the open slot and the partly closed slot. The former has the advantage that the armature conductors may be formed into coils, insulated, and readily inserted into the slots,

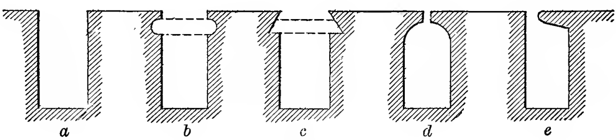


Fig. 40.

whereas the latter type simplifies the matter of securing the windings into place. The open slot is more often employed on direct-current machines. Fig. 40 shows some of the styles of open and partially closed slots; the recesses at the top of some teeth being provided for the insertion of fiber or wooden wedges, which serve to retain the conductors in the slots.

In many machines the windings are held in place by *binding wires* wound around the periphery of the armature. Grooves are provided therefor by having some of the discs of slightly smaller diameter. The wire used for this purpose is generally of hard-drawn brass or phosphor bronze, and, on railway motors, of steel. It is wound over insulating strips, forming a band of several turns, these being often soldered together.

41. **Armature Coils.** — The armature coils are of copper, in the form of either wire or strips, the former being usually employed on the smaller machines. For machines of large output, it is not advisable to use heavy bars as conductors, because of the eddy currents set up in them when one side of a coil is momentarily in a stronger field than the other. Thus, a number of smaller conductors, insulated from each other and connected in parallel at the commutator, avoid this condition.

In multipolar armatures the windings consist of a number of similar and interchangable *formed coils*, which are wound on separate collapsible forming blocks. The several conductors that constitute one coil are insulated individually and are fastened together and wrapped with a few layers of insulating tape. For this purpose cotton or linen tape, varnished cloth or paper, or micanite are generally used. The advantage of formed coils is their superior insulation and the facility with which damaged or burned-out coils can be removed without disturbing the other coils. Fig. 41 shows some Western Electric Company formed coils, those on the right being for lap-wound and those on the left for wave-wound armatures. In Fig. 42 is depicted an armature core ready for winding, with some formed coils in different stages of completion.

One-turn armature coils for large dynamos, having conductors of large sectional area, which do not permit of bending, are composed of two halves individually insulated and placed in their respective slots. A copper bridge is then bent over the bare ends at the rear, and soldered, thus completing the electrical continuity of the coil.

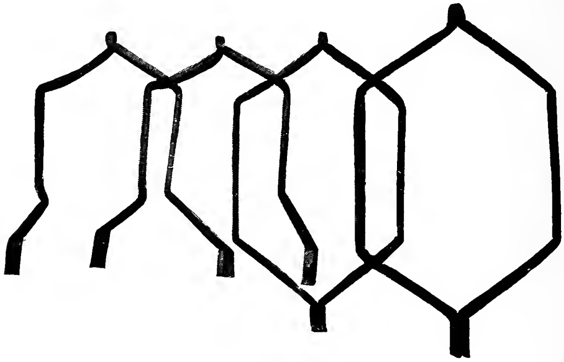


Fig. 41.

Before placing the armature coils in the slots, the latter are generally lined with insulating material, such as micanite, fiber, and a paper pulp known as presspahn. The thickness of slot insulation depends upon the terminal voltage of the machine, and should be capable of withstanding several times this voltage without puncturing. On the other hand, the thickness should not be so great as to materially lower the *space factor* of the slot, i.e. the ratio of the copper cross-section to the slot area. This factor depends upon a number of conditions, but common values for machines up to 75 K.W. at voltages from 100 to 600 may be interpolated from the curves of Fig. 43 given by Hobart.

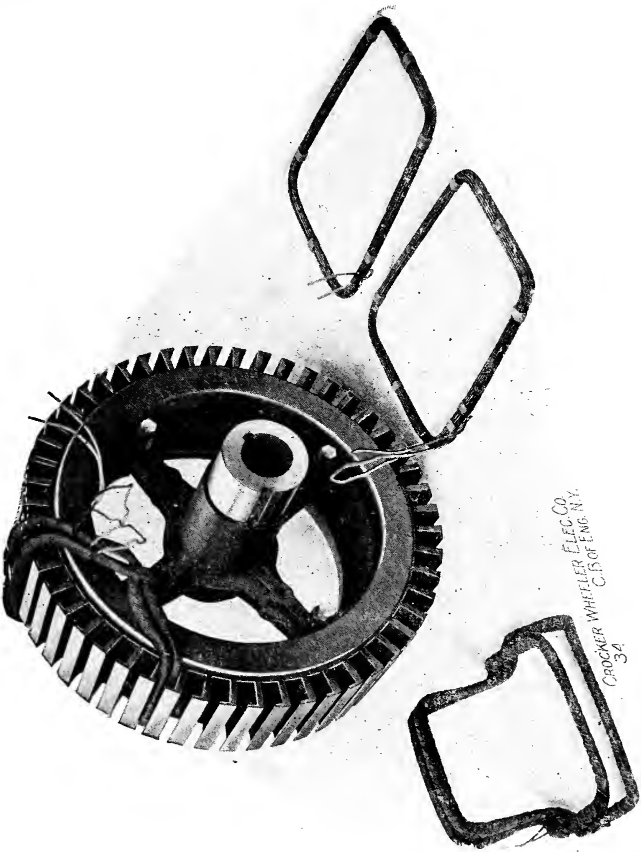


Fig. 42.

A partly-wound armature for a 200-K.W., 500-volt generator manufactured by the Allis-Chalmers Company is shown in Fig. 44. After all the coils are in place, the exposed portions of the windings, or the end-connections, must be firmly bound to withstand the centrifugal force.

42. **Commutators.** — The segments or bars of a commutator are always of drop-forged or hard-drawn copper. They must be properly tapered so that when all the segments are put together the whole will form a cylindrical structure. The insulation between segments is always of mica. Of the various grades of mica employed for insulating purposes, the amber-colored mica, which must be free

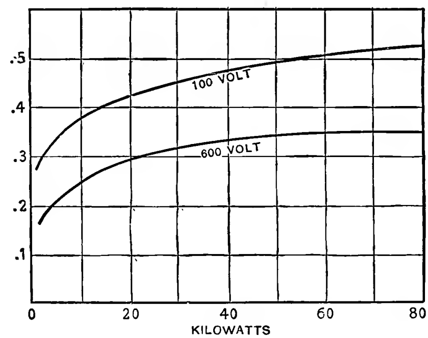


Fig. 43.

from iron, is to be preferred. Besides being a good insulator, amber mica has the additional advantage of wearing at the same rate as copper; thus after long use it leaves neither elevations nor depressions on the commutator surface. Not only must the individual segments be well insulated from each other, but especially good insulation must be provided between the segments and the spider upon which they are mounted and the clamping rings which hold them in position, because the potential differences at these places are the same as the terminal voltage of the machine. The usual thicknesses of mica required for commutator insulation, in inches, are:

VOLTAGE OF MACHINE	300 VOLTS OR LESS	300–1000 VOLTS
Between adjacent Segments . . .	0.02 to 0.04	0.04 to 0.06
Between Segments and Spider . .	0.06 to 0.12	0.10 to 0.15

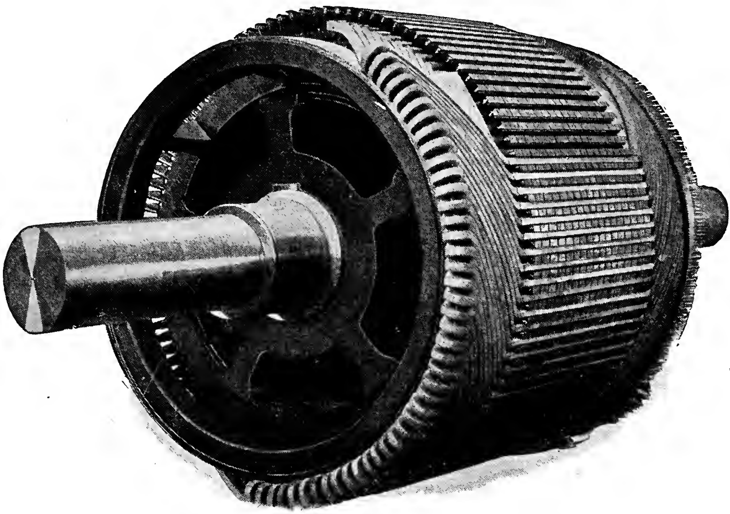


Fig. 44.

When the commutator segments and mica strips are assembled in place, a pair of temporary steel rings is placed around them, the inner one being split into a number of sections, as shown in Fig. 45. The screws are tightened so that the component parts of the commutator are firmly pressed together. A groove is then turned therein, and clamping rings of corresponding shape are fitted. These are then bolted on, and the steel rings removed, leaving a completed commutator, such as shown in section in Fig. 46. Considerable reliance is placed on the clamping rings, for these must prevent the possible dislocation of the segments due to expansion and contraction which accompany temperature change, or due to centrifugal force.

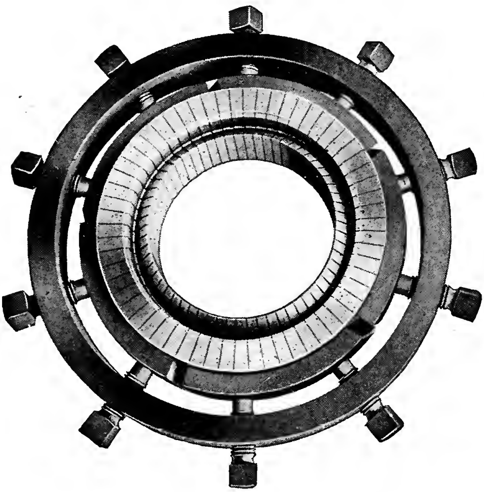


Fig. 45.

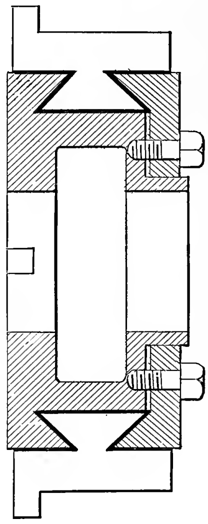


Fig. 46.

To secure successful operation a commutator must be designed with a sufficient number of bars, so that the difference of potential between two adjacent bars shall not exceed 10 volts. This would mean that a 100-volt bipolar machine should have at least 20 bars. The potential between the brushes or around *half* the commutator is 100 volts, hence half the commutator must have ten bars.

The number of commutator segments to be used depends upon thc style of winding and the voltage of the machine. According to Arnold, this number should never be less than 0.037 times the product of the total number of armature conductors and the square root of the current per armature circuit. The width of a commutator segment for good mechanical construction should not be less than $\frac{3}{16}$ inch at the periphery.

Commutators for turbo-generators or other high-speed dynamos usually have small diameters, so that the linear speed is not excessive; nevertheless speeds as high as 8000 ft. per min. are encountered in practice, which is two or three times as great as that of the usual low-speed commutators. The ordinary methods of commutator construction for these high speeds are inadequate because of the great centrifugal force tending to pull the commutator apart. To prevent this action, stout steel rings, well insulated from the segments, are shrunk on the outside of the commutator at several places. High-speed commutators have a great axial length in order to secure a large radiating surface.

Commutators should be designed with sufficient exposed area so as to radiate the heat which is communicated to them without too high a temperature elevation. The total commutator loss consists of two principal components, the loss due to resistance, and that due to friction.

The transition resistance between the brushes and commutator causes a drop in voltage at each point of contact. This drop depends upon the quality of the brush, and is practically independent of the linear speed of the commutator, the current density at brush contacts, and brush pressure. The drop for different grades of brushes varies between 0.6 and 1.4 volts for the contacts of one polarity. Double these values times the current output of the machine gives the loss represented by the transition resistances.

The pressure of the brushes on the commutator should be low, thus resulting in a small friction loss. Light pressure does not materially increase the voltage drop at the brushes. The magnitude of the friction loss in watts is equal to $\left(\frac{2 \times \pi \times 746}{33000}\right)$ times the product of the following quantities: the radius of the commutator in feet, the speed in revolutions per minute, the coefficient of friction between the brushes and the commutator (0.3 for carbon brushes and 0.25 for copper brushes), and the sum of the pressures of all the brushes upon the commutator. This latter should amount to 1.25 lbs. per square inch of rubbing surface. Carbon brushes permit a current density of 30 to 70 amperes per square inch of rubbing surface, and copper brushes about eight times as much.

There is an additional loss at the commutator due to the sparking at the brushes and to the currents in the short-circuited segments. These losses cannot be calculated closely, but may be estimated as equal to about six per cent of the regular commutator losses.

Knowing the total losses in the commutator the temperature rise may be estimated by means of empirical expressions, § 68.

The connections from the armature windings to the commutator segments are made by means of metal strips called *risers*, which are firmly clamped and soldered to the rear ends of the segments. In Fig. 47 is shown a completed Western Electric Company commutator with the risers attached.

After a commutator has been in use for a time, it becomes grooved and pitted, a condition which causes further sparking and wear, and the commutator must be turned down again to a true surface. The design of a commutator should allow sufficient material for repeated operations of this kind.

Fig. 47.

43. **Brushes and Brush Holders.** — Brushes are generally made of hard blocks of graphitic carbon. These brushes wear well mechanically and give the commutator a smooth surface, and further, the greater resistance of a carbon brush results in less sparking when it bridges two commutator bars than would the lower resistance of a copper brush. Carbon brushes are generally set at an angle, though some makers set them radially, especially in motors which must be reversed in direction, as in the case of railway and elevator motors.

On low-potential machines brushes of copper gauze are sometimes used, because there is less tendency to spark on low voltages, and because the resistance of carbon brushes would be too great a portion of the resistance of the entire circuit. Such brushes are also used on some turbo-generators.

The number of brush sets necessary depends upon the style of winding employed on the armature, § 36; but there may be, and usually are, several brushes per set. That is, instead of broad brushes, a number of smaller ones are used on all machines except those of little output. This scheme enables the removal of brushes one at a time for trimming purposes while the machine is in operation.

Individual brushes are supported in *brush holders,* as in Fig. 48, which shows a box-guide type of Westinghouse manufacture. Brush holders should provide adjustment as to position and tension of the brushes, and allow the latter to follow any irregularity in the commutator surface. The brush-holder springs should be arranged so that the brush pressure is maintained constant during the life of the brush. The spring should not form a part of the electric circuit; flexible copper conductors generally complete the connection from the brush to the stationary part of the holder. A General Elec-

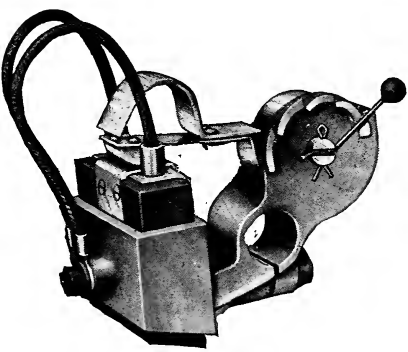


Fig. 48.

tric Company brush-holder arm is shown complete in Fig. 49.

The brush-holder arms are carried on rings, called *rockers*, which are mounted concentric with the commutator, either on a sleeve at the front bearing or on the field

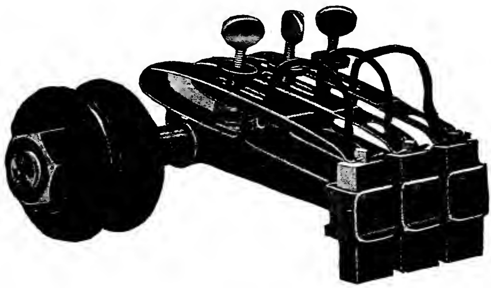


Fig. 49.

magnet frame. The rocker is capable of being moved around the commutator by means of a tangential screw and hand wheel. After the proper brush position has been found the rocker is securely clamped. All the brushes, both positive and negative, are usually mounted on the same rocker, so that their adjustment is simultaneously effected. Fig. 50 shows such a rocker with the brush gear.

44. **Shafts and Bearings.** — As shafts for armatures are often subjected to sudden large variations in load, it is usual to construct them somewhat larger than those of other machines of similar size. The diameter of the shaft depends upon the output and speed of the armature, and to obtain practical values the following empirical formula may be used:

$$D_s = k\sqrt[4]{\frac{\text{K.W. output}}{\text{rev. per min.}}},$$

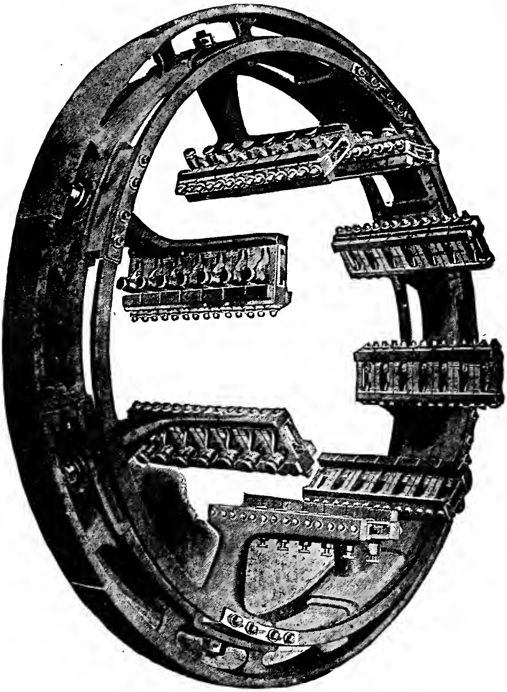


Fig. 50.

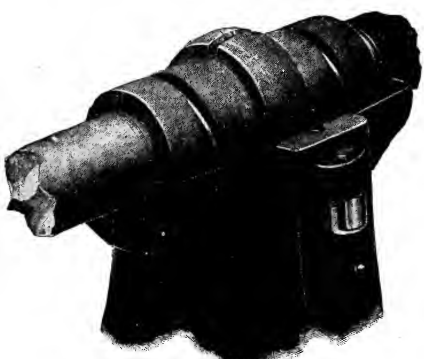


Fig. 51.

where D_s is the shaft diameter in inches, and k is a constant having the following values for different-sized machines using mild-steel shafts :

50 K.W. or less,	$k = 6.5$
50 K.W. to 500 K.W.,	$k = 8.4$
500 K.W. and over,	$k = 10.2$

This shaft diameter refers to that part under the core and commutator, the portions within the bearings being somewhat less.

Dynamo bearings should have ample bearing surface and be rigidly constructed. They are always made in two sections, thus permitting the removal of the armature. It is necessary that the bearings be exactly in line, and frequently a form of self-alignment bearing is used, as in Fig. 51. The shaft revolves in a cylindrical brass bearing having an outer spherical enlargement at the center which rests upon a corresponding bed of Babbitt metal.

Lubrication may be secured by the use of ordinary oil cups, but generally by the employment of self-oiling devices. One of these, Fig. 51, consists of two brass rings playing in semi-circumferential slots in the bearing, which permit the rings to hang loosely on the shaft. The bearing pedestals are hollow under the rings and serve as oil receptacles. As the shaft revolves, the rings also revolve at such a rate as to carry a steady stream of oil up into the slots, thereby lubricating the bearing.

Recently ball bearings have been applied to bearings of small dynamos, say up to 50 K.W., and have given complete satisfaction. One type of ball-bearing is shown in Fig. 52, where *o* is the outer ring and *i* the inner ring

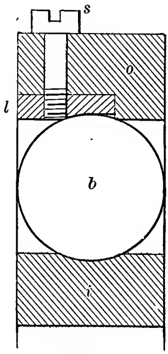


Fig. 52.

with the hardened polished steel balls, *b*, between them. A removable piece, *l*, somewhat longer than the diameter of a ball, permits of the insertion of the balls, and may be held in place by the screw *s*. The rings are of hardened steel and have polished running surfaces. The inner ring is rigidly fastened to the shaft.

PROBLEMS.

1. The instantaneous *E.M.F.* induced in a conductor of a revolving loop at the moment it cuts a certain magnetic flux at an angle of 60° is 2.5 volts. What electromotive force is induced in this conductor at the instant the angle between the paths of flux and conductor is 70 degrees?

2. The maximum *E.M.F.* induced in each of six similar loops placed 30 electrical degrees apart, revolving together about a common axis in a uniform magnetic field is 25 volts. Calculate the maximum and minimum values of the total voltage resulting from the connection of all the loops in series. Determine the percentage fluctuation.

3. How many magnetic cycles does the armature core of a 16-pole dynamo pass through per second, if the armature makes 375 revolutions per minute?

4. How many inductors may there be on a 12-pole simplex wave winding in which the rear winding pitch is 13 and the front winding pitch is 17?

5. There are 122 inductors on an eight-circuit short-chord simplex winding which is imbedded in 61 armature slots. Determine the maximum value of the mean winding pitch, the even-numbered conductors being located in the bottoms of the slots.

6. A six-pole multiplex wave winding has 60 inductors and a mean winding pitch of 9. What is the degree of multiplicity and of re-entrancy of the winding?

7. The armature of a 12-pole generator makes 250 revolutions per minute. It is simplex lap-wound, and has 180 slots with 4 conductors per slot. What is the average value of the induced *E.M.F.* at no load if the magnetic flux passing through the armature is 10 megamaxwells per pole?

8. A triplex wave-wound armature with 294 inductors, each of four conductors, is substituted for the armature of Prob. 7, all other conditions remaining the same. What is the no-load terminal voltage? State the degrees of re-entrancy obtainable in this armature winding.

9. Determine the total commutator losses of a 250 rev. per min. dynamo when delivering a current of 300 amperes. Two positive and two negative sets of carbon brushes are employed, the grade of the brushes being such as to result in a drop of 1.1 volts at the contacts of one polarity. A current density of 50 amperes per sq. in. of brush rubbing surface is to be allowed. The diameter of the commutator is 18 inches.

CHAPTER IV.

FIELD MAGNETS.

45. **Field-Magnet Frames.** — In the foregoing chapter was shown the dependence of the electromotive force induced in a dynamo armature upon the total magnetic flux cut by the conductors. This magnetic flux is produced by the current in the field coils of the machine. The path of the flux is called the *magnetic circuit*, and may be divided into three main portions, namely: the iron of the field magnets, the armature core, and the *air gap* between armature and field-magnet poles. The first of these, which constitutes the *field-magnet frame*, may, in most machines, be subdivided into three parts, viz.: (1) the *field cores*, upon which the field coils are situated; (2) the *yoke*, which connects the field cores at the outer ends; and (3) the *pole pieces* or *pole shoes*, which are the enlarged inner ends of the field cores.

The frames of direct-current generators may be cast in one piece either of cast iron or cast steel, but it is usual to construct the field cores separately from the yoke. The choice of material for the yoke, as also for the field cores, is governed by considerations of (*a*) weight, (*b*) first cost, and (*c*) economy and satisfactory regulation in operation. Cast iron has the advantage over cast steel in cheapness, but as it is magnetically inferior, more material is necessary to carry the required magnetic flux; further, if the field poles are also of cast iron, the expenditure for copper will be greater, because more turns will be required and each turn would be longer than if the better cast steel were used. In machines having different parts of the field frame of different materials, wrought iron, which is the best available magnetic substance, is often employed in the form of punchings for the cores and pole pieces.

For multipolar machines the yoke is generally circular in shape, of rectangular or elliptical section, and is divided either horizontally or vertically into two parts to facilitate the removal of the armature, the two halves normally being bolted together. It is mounted upon a cast-iron *bed plate*, to which are also fastened the pedestals which carry the armature bearings. The bipolar type of machine is now restricted to the high-speed smaller units, say 5 K.W. or less, because of the greater amount of necessary material occasioned by the longer magnetic circuit. Bipolar field-magnet frames are made in a great variety of shapes, some of which are shown in Figs. 21, 22, and 23; and are generally cast in one or more pieces which are bolted together after the field coils are in place.

Separate cores for multipolar frames are constructed of cast steel, laminated wrought iron, or laminated steel. Solid cores are usually of circular or of rectangular cross-section, and laminated cores are of the latter only. A pole piece, built up of soft-steel laminations riveted together between stout end plates, is shown in Fig. 53, which is representative of Westinghouse practice. Field cores may either be fastened to the yoke by bolts passing through the frame and screwed into the core or pole pieces, or be cast integral with the yoke. The latter method gives especially good magnetic joints, but the former allows the removal of any one of the cores with its field coil for purposes of repair.

The pole tips are generally somewhat larger than the cores, as shown in Fig. 53, an arrangement which serves

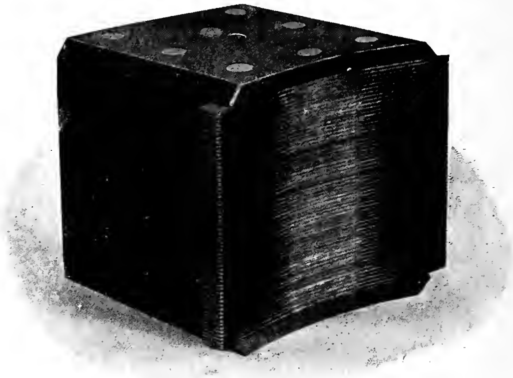


Fig. 53

the double purpose of producing a more uniform flux distribution in the air gap, and retaining the field coils in position. In this particular type, one corner of each punching is cut away and the laminations are stacked with the beveled corners alternately to one side and to the other, thus producing a pole with saturated pole tips, which is advantageous in yielding good commutation. Where solid poles of cast iron or cast steel are used with separate pole pieces, the latter are preferably made of laminated wrought iron, thereby reducing the detrimental effects of eddy currents induced in the pole faces by the flux variation occasioned by the slots in the revolving armature core.

In the usual designs of direct-current dynamos, the ratio of the peripheral length of the pole face to the *pole pitch*, or distance between adjacent pole centers, ranges from 0.60 to 0.75, and this ratio is known as the *polar span;* that is, the polar span is from 60% to 75% of the pole pitch.

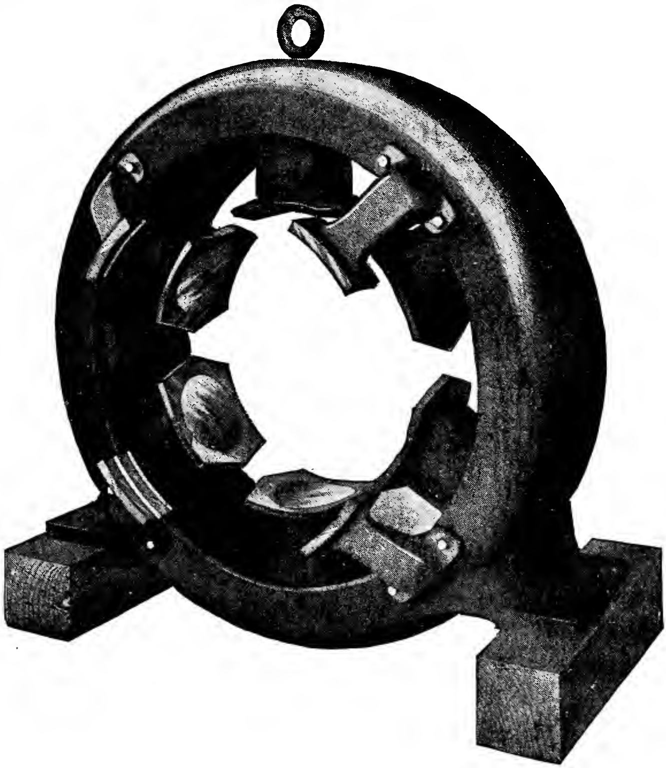
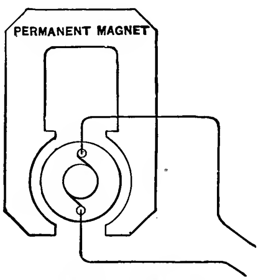


Fig. 54.

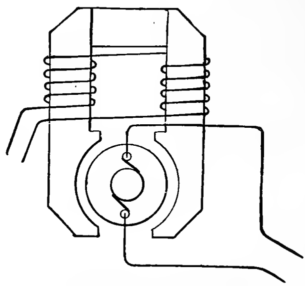
Fig. 54 shows the complete field-magnet frame of a Crocker-Wheeler generator. This frame is of cast iron, and the round poles are of cast steel provided with removable cast-iron shoes, which are clamped in place after the field coils have been assembled.

46. Methods of Field Excitation. — Dynamos are classified according to the five methods of exciting the fields of the machine. They are: — the *Magneto*, the *Separately Excited*, the *Shunt Wound*, the *Series Wound*, and the *Compound Wound*. The last three methods refer to *self-exciting* machines, that is, generators which supply their own field current.

The magneto generator, Fig. 55, is one in which the field is a permanent steel magnet, generally of horseshoe form. Because of the low flux densities in this type of



MAGNETO DYNAMO
Fig. 55.

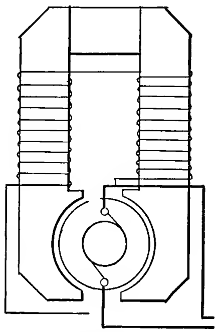


SEPARATELY EXCITED DYNAMO
Fig. 56.

machine necessitating more iron, its use is limited to small dynamos, such as those used in gasolene engine ignition work, or for telephone signaling.

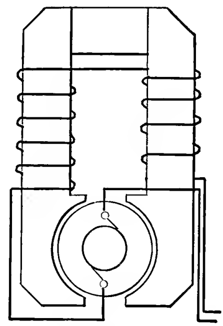
The separately excited dynamo, Fig. 56, has, as its name implies, its field coils traversed by a current other than that produced by the machine. It is produced by an auxiliary generator called an *exciter*. Alternating-current machines are nearly always of this type.

The shunt-wound machine, Fig. 57, has a large number of turns of fine wire wound on its field core, and the ends are connected to the terminals of the machine, thus being in *shunt* with the outside circuit. The ampere-turns requisite for excitation are obtained by passing a small current, say from 2 to 8 per cent of the total current output, through a large number of turns.



SHUNT WOUND
DYNAMO

Fig. 57.

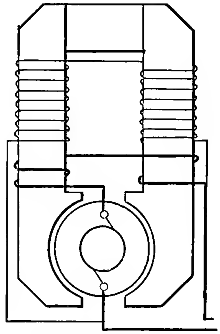


SERIES WOUND
DYNAMO

Fig. 58.

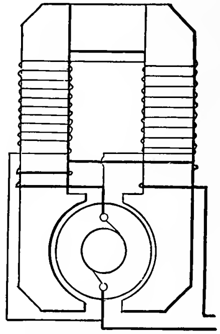
The series-wound generator, Fig. 58, has all the current that is produced by the armature passing through a few turns of large wire wound around the field cores. The exciting coils are then in *series* with the external circuit. The ampere-turns required for excitation are obtained by passing a large current through a small number of turns. Series generators are practically only used for series arc lighting and in the Thury system of direct-current power transmission at high voltages.

The compound-wound machine, Fig. 59, is one in which there are both shunt and series coils on the field magnets. This method of winding is used for purposes of regulation under varying loads, as will be explained later. Compound windings are of two classes, the *long shunt* and the *short* *shunt.* In the former, the current used in the shunt windings is also passed through the series windings along with the main current. In the latter, the current from the shunt coils passes directly back to the armature, avoiding



COMPOUND WOUND
DYNAMO LONG SHUNT

Fig. 59.



COMPOUND WOUND
DYNAMO SHORT SHUNT

Fig. 60.

the series turns. Figs. 59 and 60 clearly show the connections of the two types. The short-shunt compound winding is generally preferred.

Shunt-wound and compound-wound generators find their principal utilization in constant-potential systems of electrical distribution for lighting and power.

47. **Magnetic Leakage.** — Since air is not an insulator of magnetism, but is simply much less permeable than iron, it is evident that some of the lines of force generated by the field coils will not follow around the desired path through pole pieces and armature, but will take a path through the air and be of no utility in creating *E.M.F.* in the revolving armature. Fig. 61 roughly represents some of the paths such lines may take.

If Φ_t be the total flux set up by the field coils, and Φ_a be that portion of it which passes through the armature and is cut by the conductors, then the coefficient of magnetic leakage, or *dispersion coefficient*, is

$$\delta = \frac{\Phi_t}{\Phi_a},$$

and is always greater than unity.

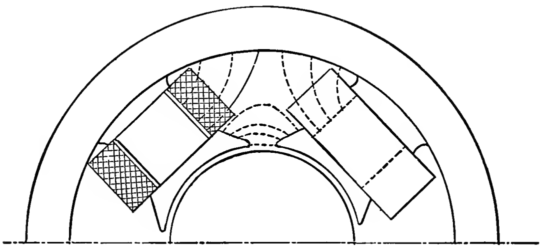


Fig. 61.

In practice, the value of δ ranges from 1.25 to 1.5 in bipolar dynamos, depending upon the design. For multipolar machines, the values of the dispersion coefficient vary from 1.3 in small dynamos of about 2 K.W. to 1.1 in large machines of about 500 K.W. output.

Increasing the field excitation of a generator results in more magnetic flux, not all of which is useful in developing a greater voltage. Further, the leakage flux increases faster than the useful flux; for as the flux density of the magnetic circuit becomes greater its permeability drops, whereas the permeability of air is constant and unity at all flux densities. The dispersion coefficient is therefore dependent upon the load on the machine.

The dispersion coefficients of small machines for definite excitation may be determined experimentally by the use of test coils in connection with a ballistic galvanometer.

48. Calculation of Exciting Ampere-Turns. — In order that a generator armature may produce the desired voltage when revolving at a definite speed, a certain amount of magnetic flux must be cut by the armature conductors. This flux is set up in the magnetic circuit of the machine by a current flowing through the field coils. The strength of current necessary and the number of turns of wire to be provided on the field coils depend upon the length and reluctance of the magnetic circuit and the dispersion coefficient. As the reluctances of the various portions of the magnetic circuit are different owing to differences of dimensions, flux density, or permeability, it is necessary to calculate the magnetomotive force, or ampere-turns, for each of the sections; their summation will then determine the required exciting ampere-turns, after correcting for magnetic leakage.

In designing the magnetic circuit of a dynamo every portion of it should have sufficient cross-section to carry the total flux at a reasonable flux density. It is assumed in such calculations that all the magnetic leakage occurs at the pole faces; hence the total flux which passes through the armature core and teeth from the air gap is the useful flux, Φ_a, and that which passes through the center of the field cores is the total flux produced, or $\Phi_f = \delta\Phi_a$. This assumption simplifies the calculation without affecting the degree of accuracy to any great extent.

In the following table are given the various parts of the magnetic circuits of ordinary direct-current multipolar machines, with the usual materials of which they are constructed, and the common values of flux density therein.

PORTION OF MAGNETIC CIRCUIT	MATERIAL	FLUX DENSITIES KILOMAXWELLS PER SQ. IN.
Armature Core	Soft Laminated Iron	70 to 110
Armature Teeth	Soft Laminated Iron	100 to 140
Air Gap	Air	40 to 60
Field Cores	Cast iron	30 to 50
	Cast steel	70 to 100
	Soft Laminated Iron	70 to 110
Field-magnet Yokes	Cast iron	27 to 45
	Cast steel	70 to 90

Having decided upon the dimensions and material of each portion of the magnetic circuit, the ampere-turns required to drive the flux through them is calculated from the expression

$$nI = \frac{10\,\mathcal{R}\Phi}{4\,\pi\,2.54} = 0.313\,\frac{l\mathcal{B}}{\mu}, \qquad \S\S\ 23\text{–}27$$

where for a given portion $\mathcal{R}$ is the reluctance in oersteds, l is the length in inches, $\mathcal{B}$ is the flux density in maxwells per square inch, Φ is the total flux in maxwells, and μ is the permeability of the material. In multipolar machines it is only necessary to consider one complete magnetic circuit, that is, for one pair of poles. The resulting ampere-turns are those necessary for two field coils.

As an example determine the ampere-turns per pole required to send a flux of 20 megamaxwells through the armature of the 350-K.W., 500-volt, 8-pole generator shown in part in Figs. 62 and 63, in which the dimensions are expressed in inches. The mean path of the flux is shown by the closed dotted line.

Armature Core. To allow for the insulation between the armature laminations it is usual, in practice, to con-

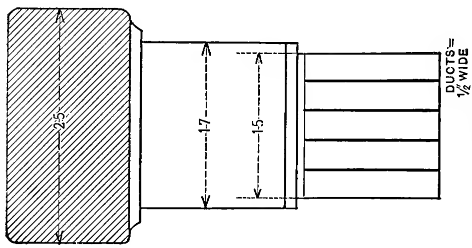

2·5
1·7
1·5
DUCTS = ½" WIDE

Fig. 63.

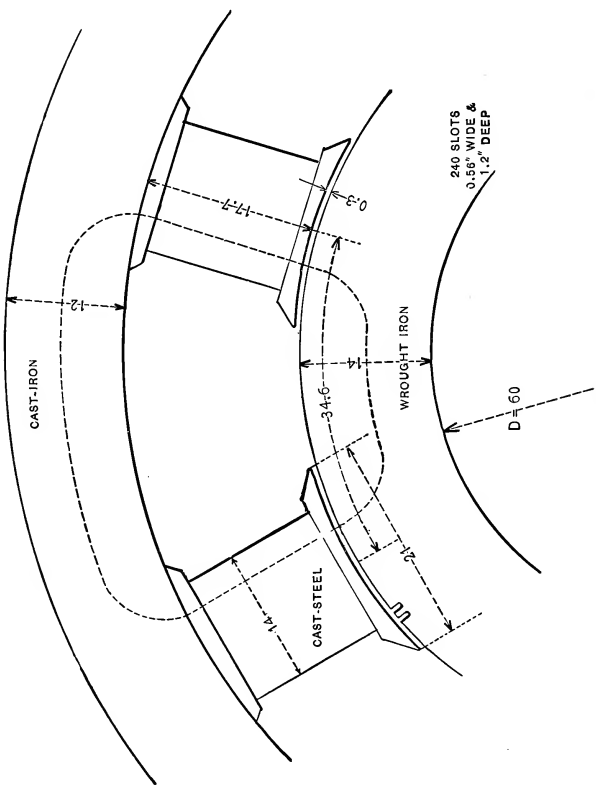

CAST-IRON
1·2
0·3
WROUGHT IRON
1·4
34·6
2·1
CAST-STEEL
1·4
D = 60
240 SLOTS
0.56" WIDE &
1.2" DEEP

Fig. 62.

sider that it occupies 10% of the gross length of iron. The presence of ventilating ducts further decreases the net iron length. The cross-sectional area of the armature core is

$$A_c = (14 - 1.2) \times 0.9\,(15 - 4 \times 0.5) = 150 \text{ sq. in.}$$

In multipolar machines of the usual construction there are two paths for the flux per pole through the armature core and field yoke; hence only one-half of the flux per pole passes through these parts. The flux density in the armature core is

$$\mathcal{B}_c = \frac{20{,}000{,}000}{150 \times 2} = 66{,}600 \text{ maxwells per sq. in.}$$

The permeability of the core at this flux density, as determined from Fig. 10, is $\mu_c = 1650$.

The mean length of path traversed by the flux is approximately 33 inches.

Therefore the ampere-turns required to overcome the armature core reluctance are

$$(nI)_c = \frac{0.313 \times 33 \times 66{,}600}{1650} = 420.$$

Armature Teeth. The accurate determination of the ampere-turns necessary to send the magnetic flux through the armature teeth is very complicated, but the following practical method, involving a few assumptions, leads to good results. The number of teeth under a pole piece is $21 \div \frac{88\,\pi}{240} = 18.2$. Owing to the fringing of the flux at the pole tips, not merely the teeth immediately under the pole face carry the flux from that pole, but one or more additional teeth may assist. Making an allowance of 10 per cent for this flux fringing, a value frequently taken, then the number of armature teeth receiving flux from one pole is $18.2 \times 1.1 = 20$.

As the teeth are wider at the periphery than at the bottom of the slots, their sectional area will be different at various distances from the axis of rotation. It is usual to employ the sectional area at one-third the tooth length from the narrow end. The width of the tooth at this place is

$$\frac{\pi\left[88-\left(\frac{2\times 2}{3}\times 1.2\right)\right]}{240} = 0.56 = 0.57 \text{ inch.}$$

The net cross-sectional area of the flux-carrying teeth per pole is

$$A_t = 20 \times 0.57 \times 0.9\ (15 - 4 \times 0.5) = 134 \text{ sq. in.}$$

Armature teeth are worked at high flux densities, and at these densities the permeability of iron is not very high. Consequently the permeance of the copper or air between the teeth cannot be neglected, and a correction must be made therefor. The terms *apparent* and *corrected* flux densities are to be distinguished in this connection. Thus, the apparent flux density in the teeth is

$$\mathfrak{B}_{ta} = \frac{20{,}000{,}000}{134} = 149{,}000 \text{ maxwells per sq. in.}$$

The corresponding corrected flux density may be obtained from the curves of Fig. 64 given by Hobart. The different curves refer to different ratios of tooth-width to slot-width. For the dynamo under consideration, in which the tooth-width and slot-width are practically equal, the corrected tooth flux density is

$$\mathfrak{B}_{tc} = 138{,}000 \text{ maxwells per sq. in.}$$

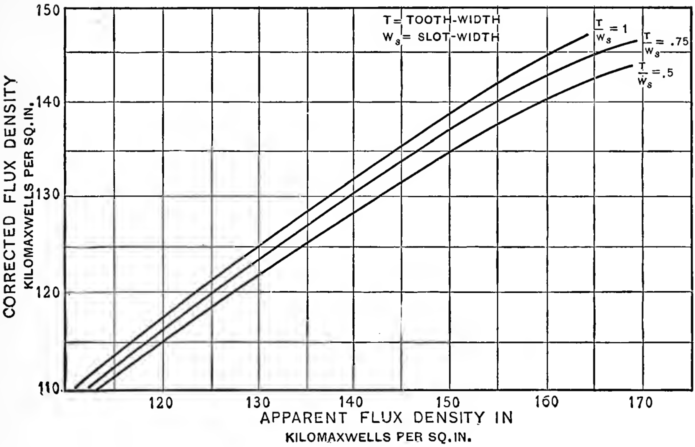

T = TOOTH-WIDTH
W_s = SLOT-WIDTH
$\frac{T}{W_s} = 1$
$\frac{T}{W_s} = .75$
$\frac{T}{W_s} = .5$
CORRECTED FLUX DENSITY
KILOMAXWELLS PER SQ.IN.
150
140
130
120
110
120
130
140
150
160
170
APPARENT FLUX DENSITY IN
KILOMAXWELLS PER SQ.IN.

Fig. 64.

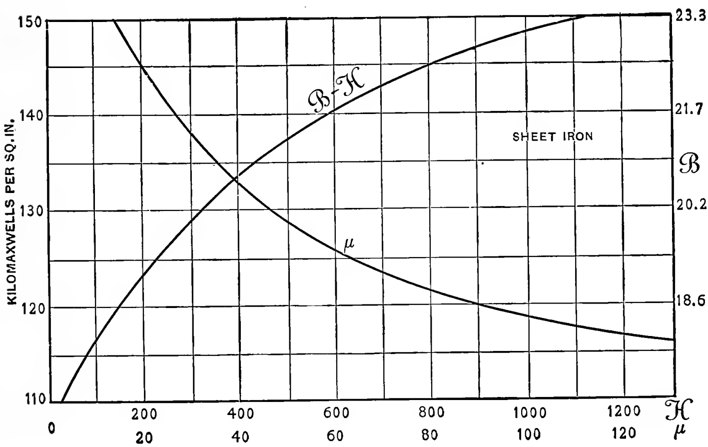

$\mathcal{B}$-$\mathcal{H}$
SHEET IRON
μ
KILOMAXWELLS PER SQ.IN.
150
140
130
120
110
23.3
21.7
$\mathcal{B}$
20.2
18.6
0
200
400
600
800
1000
1200
$\mathcal{H}$
20
40
60
80
100
120
μ

Fig. 65.

The permeability of the iron at this flux density, as determined from Fig. 65, is $\mu_t = 30$.

The length of two teeth is $l_t = 2 \times 1.2 = 2.4$.

Hence the ampere-turns required to overcome the reluctance of the armature teeth are

$$(nI)_t = \frac{0.313 \times 2.4 \times 138{,}000}{30} = 3450.$$

Air Gap. The usual practice in computing the flux density in the air gap is to divide the total flux per pole entering the armature by the area of the pole face, thus:

$$\mathcal{B}_a = \frac{20{,}000{,}000}{17 \times 21} = 56{,}000 \text{ maxwells per sq. in.}$$

The radial length of the air gap is 0.3 inch.

Since the permeability of air is unity regardless of flux density, the ampere-turns required to overcome the air-gap reluctance are

$$(nI)_a = 0.313 \times 0.3 \times 2 \times 56{,}000 = 10{,}500,$$

which is the principal component of the total number of ampere-turns to be provided on the machine.

Field Cores. Taking a value of 1.15 for the dispersion coefficient, the total flux traversing a field core will be $20{,}000{,}000 \times 1.15 = 23{,}000{,}000$ maxwells. The sectional area of the poles is

$$A_p = 17 \times 14 = 238 \text{ sq. in.}$$

Therefore the flux density in the poles is

$$\mathcal{B}_p = \frac{23{,}000{,}000}{238} = 96{,}600 \text{ maxwells per sq. in.}$$

The permeability of cast steel at this flux density, as determined from Fig. 12, is $\mu_p = 480$.

The length of two field cores, including pole pieces, is 35.4 inches.

Therefore the ampere-turns required to overcome the reluctance of the field poles are

$$(nI)_p = \frac{0.313 \times 35.4 \times 96{,}600}{480} = 2230.$$

Field Yoke. The cross-sectional area of the yoke is $12 \times 25 = 300$ sq. in., and the flux traversing it is $\frac{1}{2} \times 23{,}000{,}000 = 11{,}500{,}000$ maxwells. Therefore the flux density is

$$\mathcal{B}_y = \frac{11{,}500{,}000}{300} = 38{,}300 \text{ maxwells per sq. in.}$$

The permeability of the cast-iron yoke at this flux density, as determined from Fig. 11, is $\mu_y = 253$.

The mean length of path traversed by the flux is approximately 54 inches.

Hence the ampere-turns required to overcome the yoke reluctance are

$$(nI)_y = \frac{0.313 \times 54 \times 38{,}300}{253} = 2560.$$

Summary. The ampere-turns per pair of poles necessary for overcoming the reluctance of the entire magnetic circuit and the various parts of it are as follows: —

Armature Core,	420
Armature Teeth,	3,450
Air Gap,	10,500
Field Cores,	2,230
Field Yoke,	2,560
Total,	19,160 ampere-turns.

Therefore 9580 ampere-turns must be provided on each pole in order to set up the required flux. This is true only when the flux distribution through each section is uniform, which condition exists when no current flows through the armature, that is, at no load. Additional ampere-turns must be provided when the generator delivers energy so as to neutralize the effects of demagnetization and distortion, which will be discussed in Chapter V.

49. Field Coils. – In a shunt-wound machine, the ampere-turns necessary for excitation are obtained by a relatively small current flowing through many turns of wire, whereas in a series-wound machine the required ampere-turns are obtained by sending the entire current, or a definite part of it, through but a few turns of wire. In a compound-wound machine, the shunt winding supplies the ampere-turns required to produce the definite magnetic flux through the armature at no load, and the series winding supplies the additional ampere-turns necessary for full-load operation.

Knowing the necessary ampere-turns per field pole at no load, the size or cross-section of the wire to be used for the shunt-field winding may be calculated as follows:

Let I_{sh} = current in the shunt-field winding in amperes,
E = terminal voltage of machine at no load,
and E_{rh} = voltage allowance in regulating rheostat;

then the resistance of each shunt-field coil in ohms is

$$R_{sh} = \frac{E - E_{rh}}{I_{sh}\, 2p}, \qquad (1)$$

where p is the number of pairs of poles.

The temperature rise of the field coils under full-load operation should not exceed 50° C. above the usual engine-room temperature of 25° C. At a temperature of 75° C. the resistance between opposite faces of an inch cube of copper is $\frac{1.594}{2.54}(1 + 0.0042 \times 75) = 0.825$ microhms (§ 4.) Representing the number of turns on one shunt coil by n_{sh}, the mean length of a turn in inches by L_{sh}, and the section of the conductor in square inches by A_{sh}, then the resistance of each shunt-field coil in ohms is

$$R_{sh} = \frac{0.825\, n_{sh} L_{sh}}{10^6 A_{sh}}. \qquad (2)$$

Equating (1) and (2) it follows that the conductor cross-section of the shunt winding is

$$A_{sh} = \frac{0.825\, L_{sh} (nI)_{sh}\, 2p}{(E - E_{rh})\, 10^6} \text{ sq. in.} \qquad (3)$$

If circular conductors are to be employed the proper size may be determined by reference to a wire table.

As an example on the foregoing, calculate the section of the shunt-field conductor necessary to provide 9580 ampere-turns on each field pole of the 350-K.W., 8-pole, 500-volt generator of § 48.

Assume 15 % of the generator voltage to be taken up by the adjusting rheostat, and that the depth of the field winding is 3 inches. The mean length of a turn is then 74 inches. Therefore the conductor area of the shunt winding is

$$A_{sh} = \frac{0.825 \times 74 \times 9580 \times 8}{(500 - 75) 10^6} = 0.011 \text{ sq. in.}$$

The space occupied by the conductors depends upon the number of poles, voltage, and speed of the machine, and upon

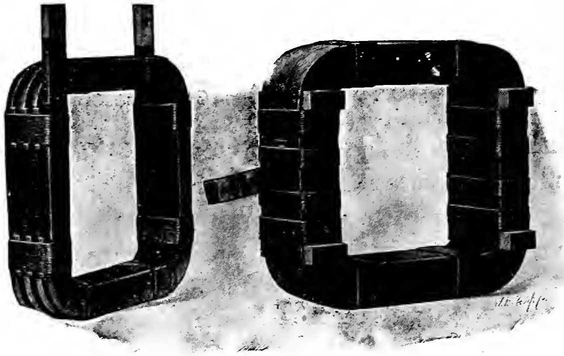


Fig. 66.

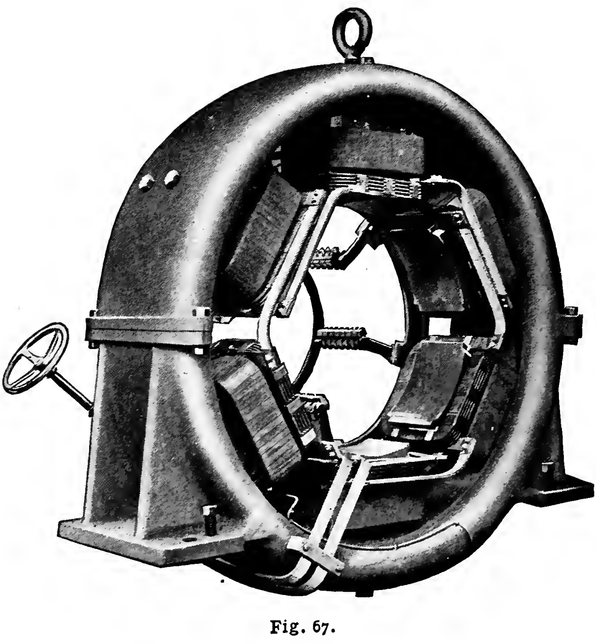


Fig. 67.

the shape of the conductor section. As a rule, from 40% to 70% of the total cross-sectional area of a field coil is occupied by copper, the remainder being taken up by insulation. The ratio of the total copper section to the gross section of the coil is called the *space factor* of the coil.

Having determined the space factor of a proposed shunt-field winding, the length of the coils may be computed, and the available space for the series winding, if any, may be obtained. Then the calculation of the number of turns and the size of conductor on the series winding follows, the method of procedure being similar to that for the shunt winding.

Shunt coils are usually made of cotton-insulated round or rectangular conductors, wound on metal frames or on removable molds and held in shape by paper and rope bands, the exterior of the coils being coated with moisture-proof insulating varnish. The coils of series machines and the series coils of compound-wound dynamos are very often of forged copper conductors insulated with tape, the individual turns being separated by spacing pieces. Typical Westinghouse series and shunt-field coils are shown in Fig. 66. The *current density* in shunt coils of the usual construction is about 1000 amperes per square inch, and in series coils it may be 20% greater because of the superior heat-radiating facilities.

A completed and assembled field winding of a 250-K.W., 250-volt, compound-wound generator is depicted in Fig. 67, in which are shown the connections of the series field coils at the end of the machine away from the commutator.

PROBLEMS.

1. The resistance of the field winding of a 15-K.W., 220-volt shunt-wound generator is 60 ohms. What percentage of the full-load power output is the power consumed in field excitation?

2. A series-wound generator supplies current to five arc lamps connected in series, each taking 9.6 amperes at 47 volts. The resistance of the field winding is 1.2 ohms, and the drop in the external circuit is 15 volts. Calculate the percentage of the total power expended in exciting the field magnets.

3. The flux density in the field-magnet cores, which are 6 inches in diameter, of a bipolar dynamo is 80 kilomaxwells per sq. in., and the dispersion coefficient of the machine as determined experimentally is 1.35. Calculate the magnitude of the flux passing through the armature.

4. Calculate the number of no-load ampere-turns to be provided on each field pole so that a flux of 6 megamaxwells per pole may be sent through the armature of a 100-K.W., 6-pole, 110-volt, compound-wound dynamo having the following constants and dimensions:

Armature outside diameter = 32 in.
Armature internal diameter = 20 in.
Armature gross length = 9 in.
Two ventilating ducts, each 0.4 in. wide.
125 armature slots, each 1 in. × 0.4 in.
Armature core of laminated iron.
Radial length of air gap = 0.2 in.
Dispersion coefficient = 1.20.
Diameter of field cores = 9.5 in.
Polar span = 70%.
Field structure of cast steel.
Internal diameter of yoke = 54 in.
Outside diameter of yoke = 62 in.
Yoke width = 10 in.

5. Allowing 20% of the voltage of the generator of Prob. 4 to be lost in the adjusting rheostat, determine the size of circular copper wire to be used in the shunt winding. The outside diameter of the field coils is 13 inches, and the net length of the winding is 9 inches.

6. Calculate the total resistance and number of turns of the shunt winding of the generator of the two foregoing problems, the space factor of the field coils being 0.60.

CHAPTER V.

ARMATURE REACTION. COMMUTATION.

50. Armature Reaction. — When an armature revolves in a magnetic field an electromotive force is developed in its conductors, and this is available at the brushes of the machine. If, therefore, the brushes be connected through an external circuit, a current will flow through it, and the current strength will depend upon the resistance of the circuit. This current, in flowing through the armature winding, will exert a magnetizing action on the core independently of the field magnets; thus there are two coexistent magnetic fields with directions at an angle to each other. As the resultant field differs in direction from that caused only by the current in the field coils, it is frequently said that the current in the armature conductors causes a distortion of the flux in the magnetic circuit, and this effect of the armature current is called the *cross-magnetizing effect.* Because of cross-magnetization, it is necessary, in order to secure good commutation, to shift the position of the brushes away from the geometrical median line between radii to two adjacent poles. A shifting of brushes from this *neutral plane* results in a weakening of the magnetic field, due to the armature current in some of the conductors, and this effect is called the *demagnetizing effect.* These two effects of the armature current, i.e., the cross-magnetizing and demagnetizing effects, when considered together, are called *armature reaction* or *armature interference.*

51. Cross-Magnetizing Effect of Armature Current. — Consider a drum armature revolving in a bipolar magnetic field, and let the brushes be placed midway between the poles, that is, in the neutral plane. When the field magnets are excited and the armature is running on open circuit, the magnetic flux which passes through the armature core may be represented by dotted lines in Fig. 68, and it is seen to be quite uniformly distributed. Upon interrupting the exciting circuit, and sending a current from some outside source through the generator armature, the resulting magnetic field may be depicted as in Fig. 69. This is called the *cross flux* because its axis is across that of the main field flux. In operation, however, both of these fields exist simultaneously, and the resultant flux through the armature is obtained by combining these two conditions as in Fig. 70. The flux distribution through the generator field poles and armature is thus non-uniform, and the distortion takes place in the direction of rotation, resulting in the crowding of lines of force in the trailing pole tips. This distortion of the magnetic flux occasions a loss in the operation of a

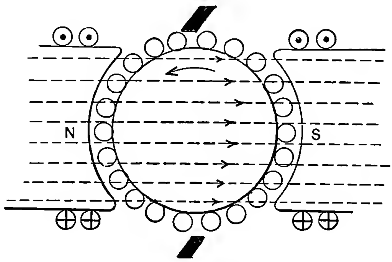


Fig. 68.

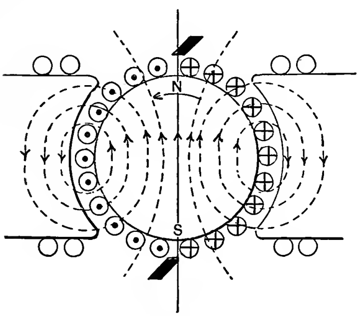


Fig. 69.

generator because it increases the reluctance in two ways, —(*a*) by saturating the iron at the pole pieces and thus

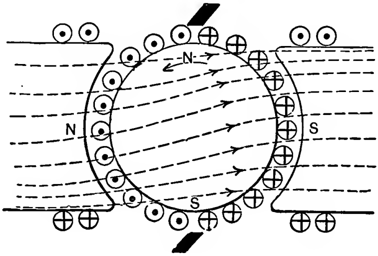


Fig. 70.

reducing the permeability, and (*b*) by lengthening the paths, both in air and in iron, that the lines of force follow.

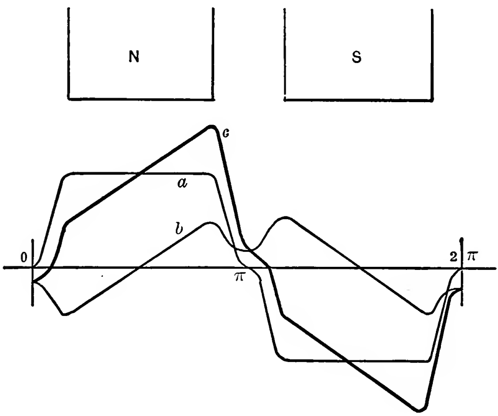


Fig. 71.

The composition of the two apparent magnetic fluxes in the air-gap of a bipolar dynamo is illustrated in Fig. 71, which shows rectified curves of magnetic distribution under the pole pieces and in the air-gap of the machine, ordinates of the curves representing flux density. Curve *a* represents the symmetrical flux distribution occasioned by the current in the field magnets, and curve *b* shows the flux distribution in the air-gap due to the armature current. Adding the ordinates of these curves yields the resultant flux distribution actually existing in the air-gap of dynamos under load when brushes are in neutral plane, as indicated in curve *c*.

52. **Demagnetizing Effect of Armature Current.** — The effect of the cross flux is to produce with the main flux a resultant magnetic field in the air-gap of a dynamo the direction of which is inclined to the direction of the field flux, as shown by the arrows in Fig. 70. Since the brushes should be at those points where they may short-circuit coils whose conductors begin to cut magnetic lines of force in a reversed direction, the position of the brushes should not be midway between the poles (or in the neutral plane), but at right angles to the direction of the resultant flux. Therefore the plane of the brushes through the armature axis, known as the *commutating plane,* should be shifted away from the neutral plane for a generator in the direction of rotation. This shifting of the commutating plane causes a further distortion of the flux, necessitating a continued shifting of the brushes until the commutating plane is displaced 90 electrical degrees from the final resultant flux. In the foregoing, the armature conductors are supposed to be connected to commutator segments lying on the same radius.

For generators, the commutating plane is in advance of the neutral plane, and for motors, behind the neutral plane. The angle between these planes is termed the angle of *brush lead* or *brush lag.*

In Fig. 72 is shown a generator armature revolving in a bipolar field, the brushes being given a forward displacement of θ degrees. The armature conductors may be divided into two separate groups by the two vertical lines *ab* passing

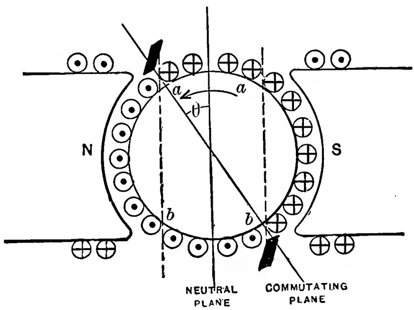


Fig. 72.

through the brush contacts, each group being considered as forming a number of complete turns. The current flowing in the turns which lie outside of these lines sets up the cross flux as shown in § 51, and the current flowing through the turns included between the lines *ab* produces a magnetomotive force opposing that of the field-magnet coils, and thus exerting a demagnetizing effect on the magnetic circuit. The product of the number of turns between *aa* and the current flowing in them is called the *demagnetizing* or *back ampere-turns*, since it produces a weakening of the magnetic field.

The current flowing in the armature conductors of a multipolar dynamo similarly exerts distorting and demagnetizing effects upon the magnetic field, and the armature turns may also be divided into two belts, namely *cross turns* and *back turns*.

53. **Compensation for Armature Reaction.** — To neutralize the effects of cross-magnetization and demagnetization produced by the current flowing in dynamo armatures, it is necessary to provide additional magnetomotive force by increasing the ampere-turns of the field-magnet coils. Compensation for demagnetization is easily calculated, since the number of back turns times the current flowing through them at any load, multiplied by the dispersion coefficient at that load, gives the additional number of ampere-turns necessary at that load for compensation. The additional field ampere-turns per pole necessary to overcome the demagnetizing effect of the armature current at full load may be represented by

$$(nI)_{dm} = \frac{2\,\theta}{180} \cdot \frac{S}{2} \cdot \frac{I}{2\,p}\,\delta,$$

where θ = brush lead in electrical degrees (angular distance between centers of like-named poles = 360 electrical degrees),

S = number of armature conductors in series between two brushes (§ 39),

I = full-load current of dynamo, leaving or entering at the brushes,

and δ = dispersion coefficient.

The additional field ampere-turns necessary to neutralize the distortional effect of the armature current are difficult to determine, but the following method of calculation, based upon experimental data, yields a good estimation. The ampere-turns of the armature producing the cross-magnetization, being due to the current in the cross turns only, are

$$D' = \left(1 - \frac{2\,\theta}{180}\right) \frac{S}{4\,p}\, I \quad \text{per pole.}$$

The ratios of this quantity D' to the number of ampere-turns per pole on the field magnets required for compensation of distortion, $(nI)_{cm}$, are the ordinates of Fig. 73; and the abscissæ represent the no-load ampere-turns per pole on the field-magnet coils. This ratio, for values of D' between 1000 (lower curve) and 10,000 (upper curve), lies between the two curves, its magnitude being determined by interpolation. Knowing the no-load field ampere-turns, and having calculated the value of D' from the above equation at full-load current, the number of ampere-turns per field spool, $(nI)_{cm}$, necessary to compensate for distortion due to

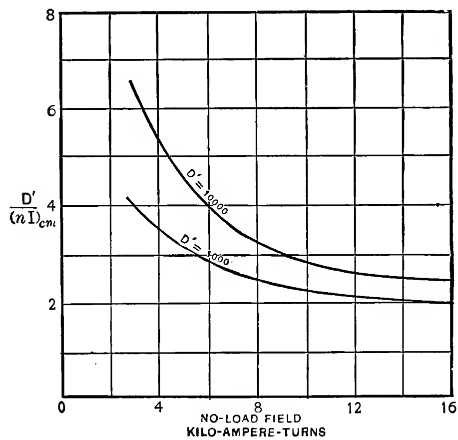


Fig. 73.

cross-magnetization at this load, may therefore be obtained from Fig. 73.

As a numerical example of the foregoing, calculate the field ampere-turns per pole necessary to compensate for armature reaction at full load in the 350-K.W., 500-volt generator of §48, when the angle of brush lead is 5 degrees.

The armature has an eight-circuit winding embedded in 240 slots, there being 6 conductors per slot. The number of armature conductors in series between positive and negative brushes is

$$S = \frac{240 \times 6}{2 \times 4} = 180.$$

The full-load current of the dynamo is

$$I = \frac{350 \times 1000}{500} = 700 \text{ amperes.}$$

Therefore the field ampere-turns necessary to neutralize demagnetization at full load are

$$(nI)_{dm} = \frac{2 \times 5}{180} \times \frac{180}{2 \times 8} \times 700 \times 1.15 = 500.$$

The ampere-turns of the armature producing cross-magnetization are

$$D' = \left(1 - \frac{2 \times 5}{180}\right) \times \frac{180}{4 \times 4} \times 700 = 7430.$$

The field ampere-turns per pole at no load $= 9580$ (§ 48). With this value as abscissa, the ordinate corresponding to $D' = 7430$ would be 2.8. Hence

$$\frac{D'}{(nI)_{cm}} = 2.8,$$

from which the ampere-turns per field pole required for overcoming effect of distortion at full load are $(nI)_{cm} = \frac{7430}{2.8} = 2650.$

The total ampere-turns, then, to be provided on each field pole are $9580 + 500 + 2650 = 12{,}730$. When a dynamo delivers current there is a resistance drop in the machine itself, so that the actual induced voltage must be somewhat larger than the rated terminal voltage. The additional ampere-turns required for producing the greater magnetic flux necessary to obtain the higher internal voltage have not been considered in the foregoing.

54. Devices for Reducing Armature Reaction. — The distortion of the magnetic field may be diminished by lengthening the air-gap and working the armature teeth at high flux densities, thereby increasing the reluctance of the path of the cross flux. This, however, also increases the reluctance of the main flux path and necessitates the provision of a greater number of ampere-turns on the field-magnet coils.

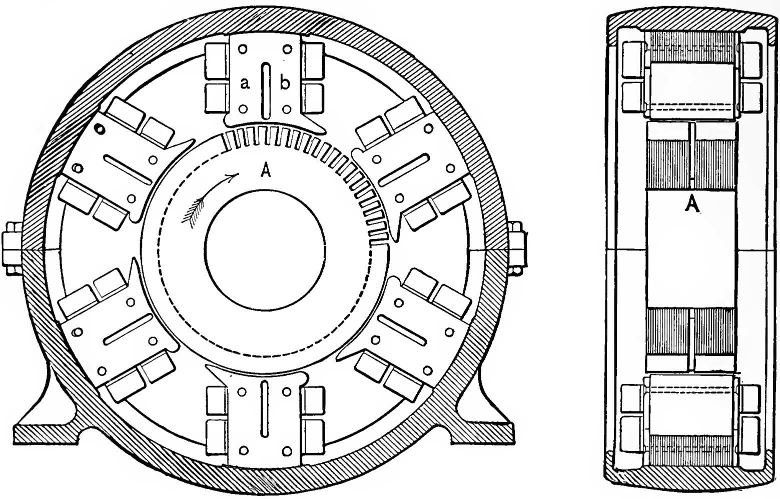


Fig. 74.

Magnetic field distortion may be reduced by slotting the field poles longitudinally. This considerably increases the reluctance of the path of the cross flux by introducing in it an air-gap, whereas the reluctance of the main flux path is very slightly altered. Fig. 74 shows a Lundell split-pole type of generator in section and illustrates the construction of the pole piece. The magnetic flux which enters the pole piece divides between the two paths *a* and *b*. Owing, however, to the greater span covered by the shoe belonging to the part marked *b*, the magnetic reluctance of that part is much smaller than that of the part marked *a*. As a result, the flux does not divide itself equally between the two paths. The part of the pole piece marked *b* under increasing excitation becomes saturated before the part marked *a*. At normal excitation, the flux density at *b* is above 16,000 lines per square centimeter, while the flux density in *a* is but about 10,000 lines per square centimeter. In other words, *b* is well saturated, while the magnetization of *a* is still below the knee of the magnetization curve. This saturation of half of the pole piece is effective in preventing a skewing of the field by the cross turns as the load on the machine increases. This is shown in the flux distribution curve of Fig. 75, wherein the arrows show the direction of rotation

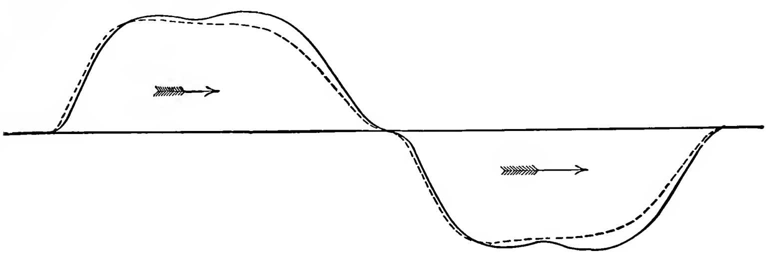


Fig. 75.

of the armature. The dotted line represents the distribution at no load, and the heavy line the distribution at full load. This small distorting effect of the cross turns permits the employment of a small air-gap without causing serious sparking.

Ryan compensates for the magnetizing effects of the armature winding by surrounding the armature with a stationary winding, which passes through perforations in the pole faces. These stationary windings carry the whole current of the machine, being connected in series with the external circuit. The current in these windings produces a magnetomotive force equal and opposite to that due to the armature current. The number of ampere-turns on the compensating winding is about one and one-quarter times the armature cross ampere-turns. This arrangement is not much used in direct-current practice because of constructive difficulties and increased cost.

Distortion of magnetic field is further decreased by properly shaping the pole pieces. The distribution of flux should be such that an armature coil at first enters a weak field and then gradually comes to the strongest part. If the lines of force are allowed to crowd into the trailing-pole tips, this gradual transition is impossible. If the horns are farther from the armature surface than the body of the pole face, then the air-gap, and consequently the reluctance at the horns, is increased, and the lines of force are distributed more symmetrically. The poles may have chamfered corners or be non-concentric with the armature, the radius of the latter being less than that of the pole faces.

The use of auxiliary poles between the main field poles of dynamos is also effective in reducing armature reaction. The coils on the auxiliary poles are connected in series with the armature, and the entire current, or a definite part of it, traverses the auxiliary winding. This arrangement yields perfect commutation at all loads for various speeds with a definite setting of the brushes. The field structure of a dynamo with auxiliary poles manufactured by the Electro-Dynamic Company is shown in Fig. 76.

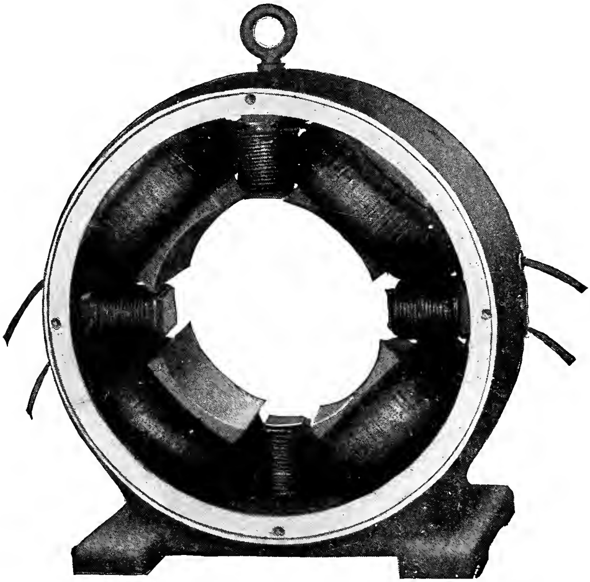


Fig. 76.

55. **Commutation.**—The electromotive force induced in the armature conductors of practically all direct-current generators is alternating, and in order to obtain a unidirectional *E.M.F.* at the terminals of the machine it is necessary to reverse the connections at the moment the induced electromotive force changes its direction. This process is called *commutation*, and is accomplished by means of a commutator, whose segments are connected to the armature windings, and brushes which collect the current from the commutator. During the process of commutation, the armature conductors connected to the commutator segments covered

by a brush are momentarily short-circuited. In this interval of time, the current flowing in a coil must be changed from a maximum value in one direction to zero, and from zero to a maximum value in the other direction.

The current in an armature coil, at the beginning of a short-circuit by a brush, is responsible for the existence of a definite amount of magnetic flux which is linked with the turns of the coil. This flux would disappear if the current were suppressed, or would be built up in a reversed direction upon the reversal of direction of the current. The product of this flux and the number of turns in the coil divided by the 10^8 times the current in amperes flowing through the coil at the beginning of short-circuit, gives a quantity which may be termed the *commutation self-inductance* of the coil. Representing this quantity by L, the number of turns of the coil by n, and the number of magnetic lines of force accompanying a current of I_1 amperes flowing in the coil by Φ, then the commutation self-inductance is

$$L = \frac{n\Phi}{I_1 10^8} \text{ henrys.}$$

The magnetic flux surrounding a coil carrying a current represents an amount of energy the magnitude of which is equal to $\frac{1}{2} L I_1^2$. This energy must be disposed of and an equal amount oppositely directed must exist before the coil is in proper condition to be transferred from one side of the armature to the other at the brush. The magnetic energy may be dissipated either by the introduction of resistance to reduce the current flow or by a counter-electromotive force, which may be obtained by so placing the brushes that the short-circuited coil will cut some of the flux from that pole corner of the field magnet toward which the coil is moving. That is, the commutating plane is shifted so that, during the time a coil is short-circuited by a brush, the coil will traverse a magnetic field of opposite direction having such intensity as to produce an *E.M.F.* in it sufficient to reverse the direction of the initial current. If, during the time of short-circuit, the intensity of the current after reversal be identical with its original intensity, then sparkless commutation will ensue with this particular brush setting. At any other position of the brushes, the magnitude of the induced *E.M.F.* will be such as to result in a larger or smaller current value in the coil after reversal than its initial value, consequently sparking will occur during the process of commutation at such positions of the brushes. The greater the current output of a machine the greater must be the induced *E.M.F.* to effect current reversal, and consequently the field in which the coil is situated during commutation must be more intense, and therefore the brush position should change with the load. The present practice, however, is to have a definite brush position which is the same at all loads upon the machine. Therefore the commutating plane should be shifted forward in generators until no sparking occurs in the final position at full-load and also at no-load operation.

There are thus two *E.M.F.*'s active in producing a current through the short-circuited armature coil: first, the electromotive force induced in the coil by cutting the magnetic lines of force set up by the currents in the field magnet coils, and second, the *E.M.F.* occasioned by the varying flux due to the changing current in the coil. The latter *E.M.F.* is independent of any action of the field-magnets, and is called an *electromotive force of self-induction* or *reactance voltage*, and it tends to prevent any increase or decrease in the strength of the current flowing. Sparkless commutation is dependent upon the magnitude of the resultant *E.M.F.* of these two opposing components. Were the resultant *E.M.F.* zero at every instant, perfect commutation might be obtained. But this ideal condition cannot be realized, since the component electromotive forces constitute different time functions. The induced *E.M.F.* depends upon the intensity of the field and speed of the armature, whereas the *E.M.F.* of self-induction depends upon the rate of current change in the coil. An approach to perfect commutation, therefore, is obtained by an adjustment of conditions whereby a number of instantaneous values of these opposing *E.M.F.*'s are equal during the time the coil is short-circuited by the brush or while it undergoes commutation.

The current reversal in the armature coils is accelerated by the use of high-resistance brushes because the initial current is more quickly reduced to zero. Consider a coil of low resistance to be short-circuited by a high-resistance carbon brush, as shown in Fig. 77, the brush having the same width as the commutator segments. Let i_1 and i_2 be the currents flowing in the taps to segments 1 and 2 respectively, and let I_1 be the current flowing through each armature path between brushes. At the instant when commutator segment 1 is completely under a brush, the currents from both sides of the armature unite and pass through the corresponding tap; then

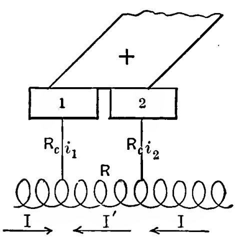


Fig. 77.

$$i_1 = 2\,I_1 \quad \text{and} \quad i_2 = 0.$$

A moment later, the brush will be in contact with both segments; then, if I' be the short-circuited current,

$$i_1 = I_1 + I' \quad \text{and} \quad i_2 = I_1 - I'.$$

In this position, the high transition resistance between the brush and commutator will be less at segment 1 than that at segment 2, because of the greater contact area of the former; therefore the voltage drop across the contact at segment 2 will be greater than that at the other segment. Because of this difference of voltage drops across the two segments, there is a tendency for a current to flow in a direction opposite to the direction of i_1. This condition results in quicker reversal of the current.

Continuing the cycle of commutation, a little later the brush contact area on both segments will be the same; then, if the short-circuit current be zero,

$$i_1 = I_1 = i_2.$$

The next moment the direction of current I' will be reversed, and

$$i_1 = I_1 - I' \quad \text{and} \quad i_2 = I_1 + I'.$$

Finally, when the brush rests only on segment 2,

$$i_1 = 0 \quad \text{and} \quad i_2 = 2\,I_1.$$

The current density in the brushes varies considerably at different parts of it, and is not proportional to the *E.M.F.* because of the exceedingly variable transition resistance between brushes and commutator. This increases very rapidly if there be even minute sparking under the brushes.

56. Time of Commutation.—The time interval during which the current changes from a maximum value in one direction to an equal value in the opposite direction is the time that elapses from the instant that one commutator segment reaches a brush to the instant the preceding segment emerges from the other side of the brush. This time is evidently dependent upon the speed of the commutator and upon the width of the brushes, and is the time required for the strip of insulation between two successive segments to pass under the brush. If w_b represent the breadth of the brush in inches, and m the thickness of mica between adjacent commutator segments in inches, then the time of the short-circuit is

$$t_c = \frac{(w_b - m)}{v} \text{ seconds,}$$

where v is the peripheral velocity of the commutator in inches per second. Representing the commutator diameter in inches by D_c, and the number of revolutions per minute of the armature by V, then

$$v = \frac{\pi D_c V}{60}.$$

Therefore

$$t_c = \frac{60\,(w_b - m)}{\pi D_c V}.$$

The reciprocal of this time gives the number of commutations per second, or what is termed the *frequency of commutation.* The frequencies found in practice lie between 200 and 800 per second. The breadth of a brush may be equal to the width of a commutator segment, but usually the brush is sufficiently broad to bridge over several segments. For a definite brush width, the frequency of commutation is much higher for multiplex windings than for simplex windings.

Let $+I_1$ be the current flowing in a coil just entering under a brush. After the time t_c, the current value in that coil for perfect commutation should be $-I_1$. What the instantaneous values of the current during this short interval of time t_c will be, or how they may vary in a particular machine under certain conditions, cannot be foretold, yet Fig. 78 shows a possible time variation of the current in a

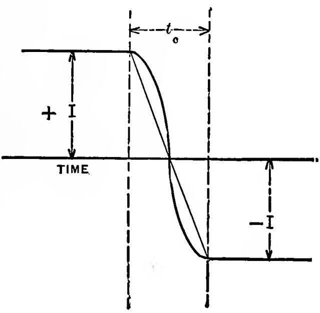


Fig. 78.

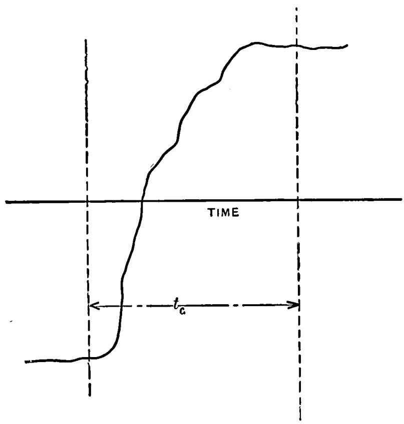


Fig. 79.

coil undergoing commutation. It represents a sinusoidal current change. Numerous curves, such as Fig. 79, have been obtained showing the actual current variation during the period of short-circuit based upon experimental observations, but such curves differ widely among themselves, and consequently no one of them may be taken to represent the general conditions occurring during commutation.

57. Calculation of Reactance Voltage. — The reactance voltage of a short-circuited armature coil is occasioned by the varying flux which surrounds that coil when the current in it is changing. Its value depends upon the time rate of current change, and its instantaneous value may be expressed as

$$E_s' = -L\frac{dI'}{dt},$$

where L is the commutation self-inductance of the coil and I' is the value of the current at the instant t seconds after the beginning of the short-circuit. It cannot be predetermined exactly how the successive values of I' are related to each other during this interval, and it becomes necessary, in order to estimate the reactance voltage, to assume that the values of the short-circuit current follow some simple law. It is usual to assume a simple harmonic variation, as shown between the dotted lines of Fig. 78, in which case the instantaneous value of the current may be expressed as

$$I' = I_1 \cos 2\pi \frac{1}{2t_c} t,$$

where I_1 is the current per armature path, and t_c is the time of short-circuit (there are $\frac{1}{2t_c}$ complete variations per second). By substitution

$$E_s' = -L\frac{d\left(I_1 \cos \frac{\pi t}{t_c}\right)}{dt},$$

whence

$$E_s' = LI_1\frac{\pi}{t_c} \sin \frac{\pi t}{t_c}.$$

The maximum value of E_s occurs when $\frac{\pi t}{t_c}$ is 90°; then the reactance voltage of the short-circuited coil is

$$E_s = LI_1\frac{\pi}{t_c}. \tag{1}$$

The time of commutation, t_c, was obtained in the foregoing article; thus for the determination of the reactance voltage there still remains the calculation of the self-inductance of the coil.

To calculate the magnitude of L for a short-circuited armature coil or element, consider each coil side to lie in a slot, a typical form of which is shown in Fig. 80. The flux,

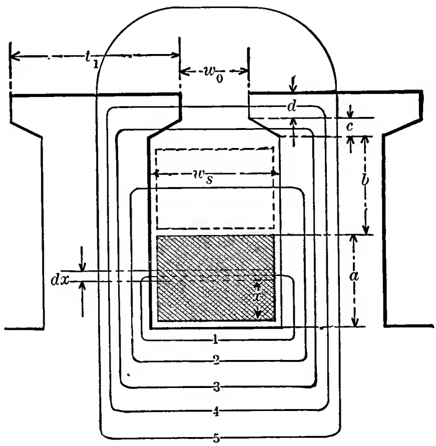


Fig. 80.

which is due to the current flowing through the n turns of this element, may take a number of paths across the slot, as shown by the numbered lines. Some lines of force also encircle the coil where it projects beyond the slots. The total inductance of a coil will be the sum of the inductances due to the flux through these various paths linking with the turns of the coil. All dimensions shown in the diagram will be expressed in inches.

Consider an element of the conductors in the slot dx

wide and at a distance x from the bottom of the slot. The magnetomotive force which produces the flux in this element, due to the current I_1 amperes flowing in $\frac{x}{a}$ of the n conductors of this coil side, is

$$M.M.F = 4\pi\left(\frac{x}{a}\,n\right)\frac{I_1}{10}\text{ gilberts.}$$

Since the permeability of the iron is very much greater than that of the air, the reluctance of the iron portion of the flux paths may be neglected. Then the reluctance of the elementary path through the coil itself is $d\mathcal{R}_1 = \dfrac{w_s}{2.54\, l_a dx}$ oersteds, where l_a is the gross axial length of the armature core in inches. Hence the flux through this small area in maxwells is

$$d\Phi_1 = \frac{4\pi\frac{x}{a}n\frac{I_1}{10}}{\frac{w_s}{2.54\, l_a dx}} = \frac{4\pi\, 2.54}{10}\cdot\frac{nI_1 l_a x dx}{a w_s}.$$

These lines of force are linked with $\frac{x}{a}n$ turns, and therefore the elementary inductance in henrys, being 10^8 times the number of linkages per ampere, is

$$dL_1 = 4\pi\, 2.54\frac{n^2 I_1 l_a}{a^2 w_s I_1 10^9}\, x^2 dx.$$

Integrating over the full width of the coil, the inductance due to the flux through path 1 in henrys is

$$L_1 = 31.9\frac{n^2 l_a}{a^2 w_s 10^9}\int_0^a x^2 dx = 10.63\frac{n^2 l_a a}{w_s 10^9}.$$

Above the upper surface of the conductors, the magnetomotive force is constant, and the lines of force through the upper portions of the slot are linked with all the conductors of the coil-side. The reluctance of path 2 is $\frac{w_s}{2.54\, l_a b}$ and the $M.M.F.$ sending the flux through it is $4\pi n \frac{I_1}{10}$; therefore the flux is

$$\Phi_2 = \frac{4\pi\, 2.54}{10} \cdot \frac{n I_1 l_a b}{w_s} \text{ maxwells.}$$

The inductance due to this flux is

$$L_2 = 31.9 \frac{n^2 l_a b}{w_s 10^9}.$$

Similarly the inductances due to the magnetic flux through paths 3 and 4 linking with the n turns of the coil are

$$L_3 = 63.8 \frac{n^2 l_a c}{(w_s + w_0) 10^9}$$

and

$$L_4 = 31.9 \frac{n^2 l_a d}{w_0\, 10^9}.$$

For two surfaces, w_0 inches apart, in the same plane, the paths of the magnetic lines of force may be taken as quadrants joined by straight lines of length w_0. Representing the width of a tooth at the air-gap by t_1, and considering only the flux outside the slot between two adjacent teeth, then the reluctance of the mean path 5 is

$$\mathcal{R}_5 = \frac{w_0 + \pi \frac{t_1}{2}}{2.54\, l_a t_1} \text{ oersteds.}$$

There is additional flux, also occasioned by the current in the coil under consideration, which passes through adjacent teeth up to the limit of the interpolar gap. Because of its lesser influence, it will here be neglected. It is approximately compensated for by the overestimation of the inductance of the coil as thus far calculated, due to neglecting the reduction of effective iron area occasioned by the presence of air ducts and insulation between laminations. The flux passing through path 5 is

$$\Phi_5 = \frac{4\pi\, 2.54}{10} \cdot \frac{n I_1 l_a t_1}{w_0 + \pi \frac{t_1}{2}};$$

hence the resulting inductance is

$$L_5 = 31.9 \frac{n^2 l_a t_1}{\left(w_0 + \pi \frac{t_1}{2}\right) 10^9}.$$

Therefore the total inductance of the embedded portion of one coil-side, being the sum of the terms L_1 to L_5, in henrys, is

$$L_e = 10.63\, n^2 l_a \left[\frac{a + 3b}{w_s} + \frac{6c}{w_s + w_0} + \frac{3d}{w_0} + \frac{3t_1}{w_0 + \frac{\pi t_1}{2}} \right] 10^{-9}. \quad (2)$$

To obtain the inductance of a complete winding element, i.e., two coil-sides, twice this value L_e must be taken, and to it must be added the inductance of the end-connections, or parts of the coil extending beyond the slots. It is usual to assume a magnetic flux of 2 maxwells per ampere-inch of conductor as linking with the exposed portion of a winding element. The length of the end connection at one end of the armature core in machines of the usual type may be taken as 1.5 times the pole pitch, or $1.5\lambda_p$ inches. The inductance of the "free" portion of an armature coil of n turns would then be

$$L_f = \frac{(2 I_1 n \times 3\lambda_p) n}{10^8 I_1} = 6\lambda_p n^2 10^{-8}. \quad (3)$$

Generally, two or more armature coils are simultaneously undergoing commutation, and since the coil-sides of two or more elements are usually in the same or adjacent slots, there is a mutual inductive action between them. In Fig. 81, one coil-side of the winding element, short-circuited by

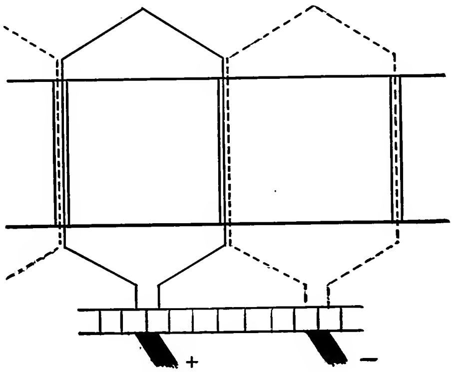


Fig. 81.

the positive brush, lies in the same slot with one coil-side of the element short-circuited by the negative brush. The mutual inductance with such an arrangement is very nearly equal to the inductance of the embedded portion of a coil. Thus little error will be introduced by taking double the value of L_e, as previously calculated, to include the effect of mutual inductance. Since the end connections of the two coils of Fig. 81 do not coincide but are widely separated, no mutual inductive action between these portions of the winding elements need be considered.

The total inductance of a short-circuited coil is therefore

$$L = 4\,L_e + L_f. \qquad (4)$$

The values of t_c, I_1, and L now being known, the reactance voltage of a coil undergoing commutation may be determined from equation (1).

In dynamos having lap-wound armatures the above value of L would be that for a coil between two adjacent commutator segments, or, as sometimes stated, the inductance per segment. In dynamos having wave-wound armatures with only two brushes, the foregoing value of coil inductance is that for one winding element. For one coil of this type of armature with p elements terminating at two successive commutator segments, the inductance would be p times as great. Consequently the employment of as many brushes as there are poles is desirable from the commutation viewpoint.

A quick method of estimating the inductance of the embedded portion of an armature element, due to Hobart, is based upon the assumption that a flux of 10 maxwells surrounds each inch of conductor length per ampere of current which flows through it. Then

$$L_e = \frac{(10\, I_1\, n l_n)\, n}{10^8\, I_1} = n^2 l_n\, 10^{-7} \text{ henrys}, \tag{5}$$

where l_n is the net axial length of the armature core. Combining with equation (3), the total inductance of a short-circuited coil in henrys is

$$L = n^2 (40\, l_n + 6\, \lambda_p)\, 10^{-8}. \tag{6}$$

Except for the large slow-speed dynamos, the reactance voltage should not exceed 2 volts per segment in order to obtain fair commutation. For these large machines the value of the reactance voltage may reach 5 volts, but the aim of the designer is to obtain a lower value.

58. Conditions for Good Commutation. — There are two *E.M.F.*'s active in producing a current through an armature coil undergoing commutation, namely, (1) the electromotive force of self-induction, which is equal to $-L\dfrac{dI'}{dt}$, and (2) the electromotive force induced in the coil due to its cutting lines of force, the value of which may be expressed as some function of the time, or $f(t)$. To complete the electromotive force equation of a short-circuited coil, the various resistance drops must be inserted. Let R, Fig. 77, be the resistance of the coil undergoing commutation, and R_c be the resistance of each connection tap to the commutator. Then the resistance drop across two adjacent segments (§ 55) is

$$I'R + i_1R_c + i_2R_c = I'R + R_c[I_1 + I' + I_1 - I'] = I'R + 2I_1R_c.$$

To include the voltage drop from the brushes to the commutator, let R_1 be the transition resistance of one set of brushes, having a breadth equal to the width of a commutator segment, when resting on only one segment. At the time t seconds after the beginning of the short-circuit, the transition resistance of segment 1, Fig. 77, is $\dfrac{t_c}{t_c - t}R_1$, and that of segment 2 is $\dfrac{t_c}{t}R_1$. The corresponding resistance drops are respectively

$$\frac{t_c}{t_c - t}R_1i_1 = t_cR_1\frac{I_1 + I'}{t_c - t}$$

and

$$\frac{t_c}{t}R_1i_2 = \frac{t_cR_1}{t}(I_1 - I').$$

The complete $E.M.F.$ equation of a coil undergoing sparkless commutation may then be expressed as

$$f(t) - L\frac{dI'}{dt} - I'R - 2I_1R_c - t_cR_1\left[\frac{I_1 + I'}{t_c - t} + \frac{I_1 - I'}{t}\right] = 0.$$

An analysis of this equation has shown that for sparkless commutation in the neutral plane $\frac{R_1}{L} t_c$ must be equal to or greater than unity. This implies that the transition resistance of brushes should be great, that the inductance of the short-circuited coil should be small, and that the time of commutation be comparatively large. As the width of the brushes is generally such as to short-circuit several coils simultaneously, the above general equation must be modified accordingly. However, $\frac{R_1}{L} t_c$ remains practically unaltered, since both R_1 and L are multiplied by the number of coils simultaneously short-circuited by one brush.

Because of the great transition resistance between copper commutator segments and carbon brushes, these are more generally used than copper brushes, although the latter find application in high-speed turbo-generators as well as in low-voltage generators or motors. For a fixed brush position at all loads there is a tendency to spark when the machine is subjected to wide variations of load, but the use of carbon brushes counteracts this tendency to a great extent because of their high transition resistance. With copper brushes, a fixed brush position for all loads can rarely be attained. The value of $\frac{R_1}{L} t_c$ usually exceeds 2 with carbon brushes, but may occasionally fall below $\frac{1}{2}$ for copper brushes.

As the inductance of a short-circuited coil depends upon the square of the number of turns, it is desirable to have few turns per coil, so that the reactance voltage may be low. Good practice limits the number of turns of a coil in moderate-sized machines to 2 or 3. An inspection of equation (2) of § 57 shows the desirability of having the axial length of armature core small so as to reduce L_e; this implies that the armature diameter be large for the same output. Large diameters permit of the employment of large commutators having many segments, consequently there will be fewer armature turns per segment than would be possible with smaller commutators having bars of equal width; this condition, as already stated, is conducive to a lower reactance voltage.

The time of commutation could be raised by increasing the width of the brush, but if the brush bridges over more than a few segments, the inductance of each short-circuited coil will be much greater because of its turns linking with lines of force produced by the current in other coils as well. A further limitation to increasing the time of short-circuit by the use of wide brushes is the lowering of the transition resistance R_1 due to the greater contact area of the brush.

There are a number of purely mechanical conditions upon which depend the quality of commutation. A rough commutator surface causes vibration of the brushes, which results in a widely varying transition resistance and subsequent sparking. A further source of sparking, more especially in high-speed machines, is loose commutator bars. Such sparking produces local blackening of the commutator surface. An open or discontinuous armature winding and a reversed coil occasion severe sparking. The limit of the capacity of a machine may be excessive sparking instead of excessive heating, and therefore the suppression of sparking by proper design of the machine is of utmost importance.

PROBLEMS.

1. Compute the field ampere-turns per pole necessary to compensate for demagnetization in a 15-K. W., 125-volt, 4-pole dynamo having a wave-wound armature, the winding being contained in 121 armature slots, with 4 conductors per slot. The commutator has 121 segments, and the commutating plane is shifted 3 segments ahead. Dispersion coefficient = 1.25.

2. Calculate the field ampere-turns per pole necessary to neutralize the distortional effect of the armature current of a 6-pole generator with a triplex lap-wound armature having a total of 1086 conductors. The rated current of the machine is 500 amperes. The angle of brush lead is 8 degrees. No-load field ampere-turns per pole = 7200.

3. For the dynamo of Prob. 4, Chap. IV, calculate the additional ampere-turns required to compensate for armature reaction. The armature has a simplex wave winding with two conductors per slot; the angle of brush lead is 5 degrees.

4. Determine the time of short-circuit of an armature coil undergoing commutation of a 12-pole, 350-K. W., 250-volt, 70-rev.-per-min. generator having a commutator 56 inches in diameter with 448 segments. The brushes exactly cover two segments with the intervening insulation, which is .03 inch thick.

5. The generator armature of the foregoing problem has a simplex lap winding, and the inductance of a short-circuited coil is 0.000030 henry. Compute the reactance voltage per segment.

6. Determine the reactance voltage per segment of a 12-pole 300-K. W., 500-volt generator making 200 rev. per min., the constants of which follow:

Diameter of armature	= 75 in.
Gross length of armature	= 18 in.
Diameter of commutator	= 50 in.

Number of commutator segments	$= 489.$
Thickness of insulation between segments	$= 0.04$ in.
Number of armature slots, as per Fig. 82	$= 489.$
Size of armature conductors	$= \frac{3}{8}'' \times \frac{1}{8}''.$

Armature is lap wound; two conductors per slot.

Brush width	$= \frac{5}{8}$ in.

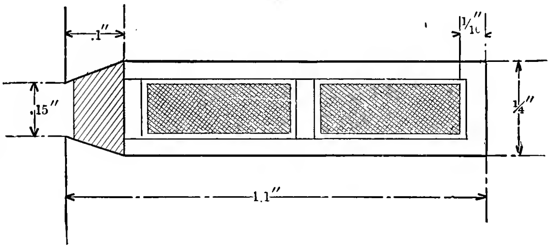


Fig. 82.

CHAPTER VI.

GENERATORS.

Efficiency of Operation.

59. **Capacity of a Dynamo.** — The capacity of a generator is measured by the power it can develop, that is, the capacity varies as the product of the terminal electromotive force and the current supplied to an outside circuit. The *E.M.F.* of a dynamo depends upon the speed, the number of conductors, and the number of magnetic lines of force passing through the armature, § 39. The allowable current output depends primarily upon the size of the armature conductors, so that these may carry the required current without excessive heating. The larger the conductors, the larger must be the armature core, other things remaining the same. Sometimes, however, commutation difficulties limit the output of a machine rather than temperature elevation.

The *E.M.F.* of a generator may be raised by increasing the speed, the number of conductors, or the magnetic flux through the armature. The speed of a machine is limited by considerations of mechanical strength and economy of material. It is frequently specified in that the generator is to be directly connected to a steam engine, turbine, or other prime mover, or, in the case of a motor, by direct-coupling to the machine it operates. The speed of small machines is greater than that of large ones, but the peripheral velocity for nearly all sizes lies between 25 and 100 feet per second for belt-driven machines, and between 25 and 50 feet per second for direct-connected machines. In turbo-generators the speeds may be as high as 250 feet per second. The speed limits of modern direct-current generators, in revolutions per minute, are given in the following table:

K.W.	GENERATOR SPEEDS.	
	DIRECT-CONNECTED.	BELT-DRIVEN.
5	400–800	650–2000
10	350–500	600–1800
20	250–400	550–1600
50	180–350	500–1200
100	120–300	450–900
200	100–250	400–600
500	70–120	300–400
1000	60–90	
1500	55–85	
2000	50–80	

The number of inductors on a given armature can be increased by decreasing the size of the wire. Sufficient cross-section must, however, be provided in the conductors to enable them to carry the maximum current of the machine without causing them to heat to such an extent as to endanger the insulation. Good practice calls for from 400 to 800 circular mils for armature conductor cross-section per ampere, the proper value depending upon the heat-dissipating facilities in the machine. The smaller values are suitable for intermittently operating machines, such as elevator or crane motors; whereas the larger values are for continuously running machines, such as central-station generators.

The field flux of a generator, for a path of constant reluctance, depends upon the magnetomotive force produced by the current in the field winding. Increasing the number of turns on the field coils or raising the current flowing in them would increase the magnetic flux passing through the armature. Decreasing the reluctance of the path of the flux yields a similar result; hence the desirability of small air-gaps, magnetic material of high permeability, and short flux paths of large cross-section.

Because of armature reaction, it is desirable to limit the current flowing in the armature conductors. This is accomplished by providing numerous paths between brushes for the armature current, so that only a small portion of the total current flows through each coil. This implies the provision of a suitable number of field poles, which is therefore a necessary condition for obtaining satisfactory operation as regards sparking. The usual limits as to the number of poles on commercial direct-current generators are given below:

K.W.	NUMBER OF POLES.
1–15	2–4
15–100	4–6
100–200	6–8
200–300	6–10
300–500	8–12
500–1000	10–16
1000–2000	12–24

60. Heating of Dynamos. — When a generator delivers current, there is a continuous production of heat in the armature and field magnets, which represents the conversion of some electrical energy into heat. This production of heat is occasioned by eddy current and hysteresis losses in the iron, copper losses in both armature and field windings, bearing friction and windage, pole-face losses, and commutator losses. The temperature of the machine, therefore, continually rises until a temperature is reached at which as much heat will escape per unit of time as is generated in an equal period. The dissipation of heat takes place by conduction, air convection, and radiation. The ultimate temperature of any part of an operating machine depends upon the emissivity and area of the radiating surface and its temperature elevation over the surrounding atmosphere. Hence it is necessary to design each part of the dynamo so that its temperature rise during continuous full-load operation shall not exceed a certain prescribed limit.

The ultimate constant temperature is usually acquired after from 6 to 18 hours of full-load operation, according to the size and construction of the machine. The temperature of the armature of a 300-K.W. generator operating under constant full load, in terms of time, is

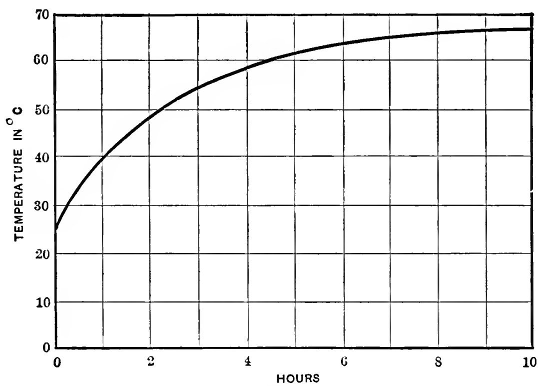


Fig. 83.

shown in Fig. 83. It is obviously possible to obtain a greater output for a short time without excessive rise of temperature; for example, a machine may yield 25 per cent overload capacity for two hours without undue temperature elevation. Fig. 84 shows the ultimate temperature of the armature of the same 300-K.W. generator under different loads, a constant temperature being attained at each load.

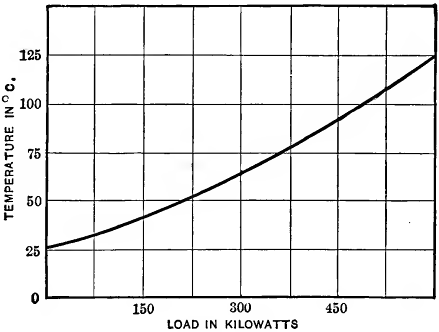


Fig. 84.

Two methods for obtaining the rise of temperature are in common use: *a*, by a thermometer; *b*, by increase of electrical resistance. The latter method is to be preferred, and should be used wherever practicable. In taking the temperature of a surface of the machine by the first method, the thermometer bulb should be laid flat against that surface and be covered by a pad of cotton sufficiently small to allow normal escape of heat from the surface. The use of the cotton pad prevents radiation of heat from the bulb of the thermometer. The determination of the temperature rise of a winding by the second method involves the measurement of its resistance at room temperature and at the ultimate temperature assumed under full-load operation. Knowing the temperature coefficient of resistance of the material at 0° C., the rise in temperature may be computed, § 4.

The temperature elevation of a part of a machine as determined thermometrically by applying a thermometer to the hottest accessible portion, may frequently be less than 70 % of the temperature rise as computed from the resistance measurements, this difference depending upon the construction of the part under test. If, however, a thermometer applied to a winding indicates a higher temperature elevation than that obtained from resistance measurements, then the thermometer indication should be accepted.

At rated load and under normal conditions of ventilation, the maximum temperature rise, referred to a standard room temperature of 25° C., should not exceed 50° C. for field coils and armature, as measured by resistance increase; 55° C. for commutator and brushes, and 40° C. for bearings and other parts of the machine, as thermometrically determined.

61. Output Coefficients. — The capacity or rating of a machine depends to a great extent upon its heating. In the armature, the conductor cross-section must be such that the full-load current may flow through the winding without producing an undue temperature rise. The employment of large conductors requires large armatures. Therefore an approximate estimate of the capacity of a machine can be obtained if the dimensions and speed of the armature be known.

An empirical expression, given by Kapp, for the rated output of a generator in kilowatts is

$$P = \xi D^2 l_a V,$$

where ξ = a factor called the *output coefficient*,
D = armature diameter in inches,
l_a = gross axial length of armature core in inches,
and V = rev. per min. of the armature.

The value of the output coefficient depends upon the armature diameter, and may be obtained from the curve of Fig. 85, which yields results typical of good practice. The foregoing equation is useful for rough preliminary design purposes, the decision as to final dimensions being subject to considerations of commutation and voltage regulation.

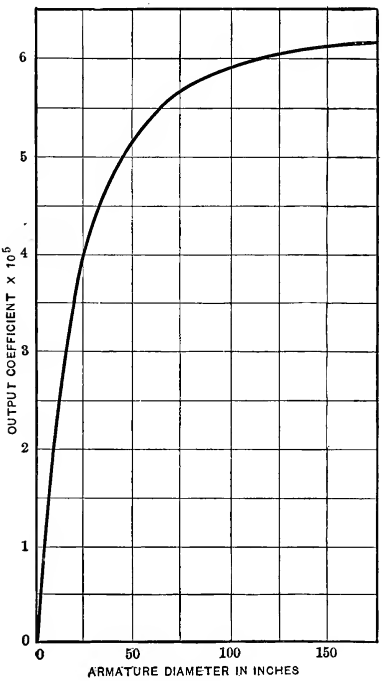


Fig. 85.

62. Losses in Armature Cores. — The loss of energy which attends the rotation of an armature in a magnetic field is due to hysteresis and eddy currents in the armature core. In order to reduce these losses, and thereby to prevent an excessive temperature rise, armature cores are composed of a series of thin disks or laminations, insulated more or less thoroughly from each other. The magnitudes of the hysteresis and eddy current losses, as shown in §§ 28 and 29, depend upon the size of the core, the magnetic flux density in its various parts, the thickness of the laminations, and the number of magnetic cycles passed through per second by the iron. Curves of hysteresis and eddy current losses in armature cores, in watts per cubic inch and per pound, expressed in terms of flux density, are given respectively in Figs. 86 and 87, for both 25 and 60 cycles

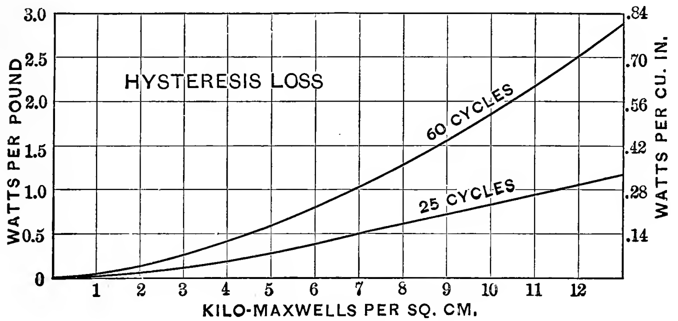


Fig. 86.

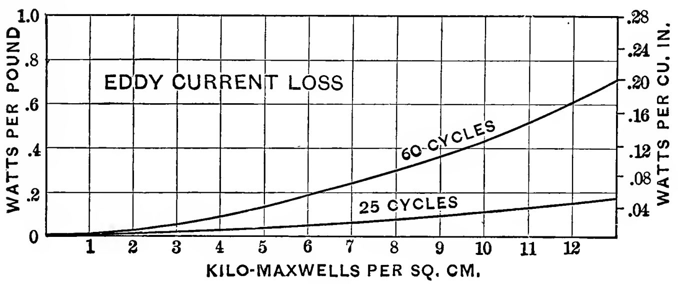


Fig. 87.

per second. These curves are plotted from values calculated by means of the formulæ

$$P_h = 8.3\ vf\eta\mathcal{B}_m^{1.6}\,10^{-8},$$

$$P_e = 4.07\ vf^2l^2\mathcal{B}_m^2\,10^{-17},$$

where v = volume of iron in cubic inches,

f = cycles per second,

l = thickness of laminations in mils (usually 14),

$\mathcal{B}_m$ = max. flux density in maxwells per sq. in.,

and η = hysteretic constant (taken as 0.0021).

63. Armature Copper Loss. — When a generator delivers current to an external circuit, this current, in flowing through the armature winding, occasions a loss of energy, which appears as heat. The magnitude of this loss at any load is equal to the product of the armature resistance from the positive to the negative brushes (excluding brush transition resistance) and the square of the total current of the machine at the definite load. If S be the number of conductors in series between brushes, that is, the total number of conductors on the armature divided by q (= number of current paths through armature from positive to negative brushes), if L_a be the length in inches of the embedded portion of one conductor plus the length on one end of the exposed or free portion, and if A_a be the cross-section of an armature conductor in square inches, then the total resistance of the armature in ohms is

$$R_a = \frac{0.825\, L_a S}{A_a q\, 10^6},$$

where the constant 0.825×10^{-6} is the resistance in ohms between opposite faces of an inch cube of copper at 75° C. Values of q for various types of armature windings are given in the table of § 39.

For preliminary design purposes, in order to save time, the value of L_a is frequently taken as the sum of the embedded length of a conductor, l_a, plus 1.5 times the pole pitch. That is, the quantity $1.5\, \lambda_p$ is an estimation of the free length of an armature conductor on one end of the core. Then

$$L_a = l_a + 1.5\, \lambda_p.$$

The armature copper loss at full load in watts is therefore

$$P_a = I^2 R_a,$$

where I is the full-load current of the dynamo leaving or entering at the brushes.

When large armature conductors pass through a non-uniform magnetic field, such as exists under the field-pole corners, eddy currents will be produced in them because of the greater *E.M.F.* induced in one side of the conductor. This loss is reduced in large machines by using several conductors connected in parallel instead of one large equivalent conductor.

64. Pole-Face Losses. — In dynamos having toothed armatures the reluctance of the air-gap between the armature and the field poles is less opposite a tooth than opposite a slot. Consequently more lines of force pass through portions of the pole face opposite armature teeth than through those portions opposite the slots in the armature core. As the armature rotates each point of the pole face is subjected to a pulsating flux, thus giving rise to eddy-current and hysteresis losses in the pole pieces. To minimize this effect pole pieces are often constructed of laminated iron or sheet steel.

The magnetomotive force of the eddy currents tends to equalize the flux density, and therefore the pulsation is confined to a very thin surface layer of the pole pieces. An expression for the pole-face loss of a dynamo in watts, given by Adams, is

$$P_p = 10^{-7}\,kk'\mathcal{B}^2 A_p \sqrt{\frac{v^3}{\mu\rho}},$$

where $\mathcal{B}$ is the average pole-face flux density in maxwells per square inch; A_p is the total pole-face area of the dynamo in square inches; v is the peripheral velocity of the armature in feet per second; μ is the permeability of the pole face; ρ is the electrical resistivity of the pole face in *c.g.s.* units (ρ equals about 1500); k is 1.6 times the square root of the tooth pitch in inches for solid pole pieces, and 4.1 times the thickness of the laminations in inches divided by the square root of the tooth pitch in inches for laminated pole shoes; k' is a constant depending upon the ratio of slot opening, w_0, to the radial length of the air-gap, Δ, and may be taken from Fig. 88. The foregoing expression has received experimental verification.

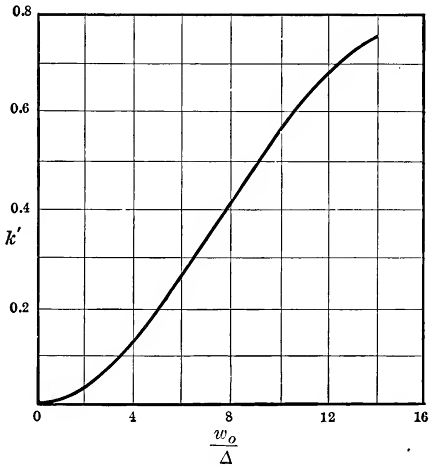


Fig. 88.

The pole-face loss of a dynamo is usually less than one per cent of the output of the machine. There is another source of energy loss in the pole face which is due to reluctance pulsation of the magnetic circuit of the machine occasioned by a variation in the number of teeth under a field pole. Large air-gaps and chamfered pole corners practically eliminate the flux pulsation. In the case of laminated pole pieces, there are some additional losses due to transverse bolts and screw heads which serve to hold such shoes on the field cores.

65. **Excitation Loss.** — The loss of power due to the current flowing through the field-magnet windings for producing the magnetic field of the machine is easily calculated as the product of the square of that current and the resistance of the winding. Thus, the excitation loss of a series-wound dynamo in watts is

$$P_f = I_{se}^2 R_{se}, \tag{1}$$

and that of a shunt-wound machine in watts is

$$P_f = I_{sh}^2 R_{sh} = \frac{E^2}{R_{sh}}; \tag{2}$$

where I_{se} and I_{sh} in amperes are the currents flowing in the series and shunt field coils respectively, R_{se} and R_{sh} are the resistances thereof in ohms at the steady running temperature (§ 49), and E is the terminal voltage of the generator. In a compound-wound machine the total excitation loss is the sum of the foregoing expressions, or

$$P_f = I_{se}^2 R_{se} + I_{sh}^2 R_{sh}. \tag{3}$$

Care must be exercised to apply this equation correctly for long-shunt and short-shunt compound-wound dynamos.

With separately excited field magnets the power loss in the resistance of the field-magnet coils alone should be considered, but with either shunt- or series-wound field coils the power loss in the accompanying regulating rheostat should also be included, since this apparatus is considered an essential part of the machine.

66. **Bearing Friction and Windage.** — As an armature revolves, some energy is wasted in bearing friction and windage, and this loss may be considered independent of

the load on the machine. Its value cannot be determined accurately, but may be estimated by means of the curve in Fig. 89, given by Hobart. The loss in watts due to

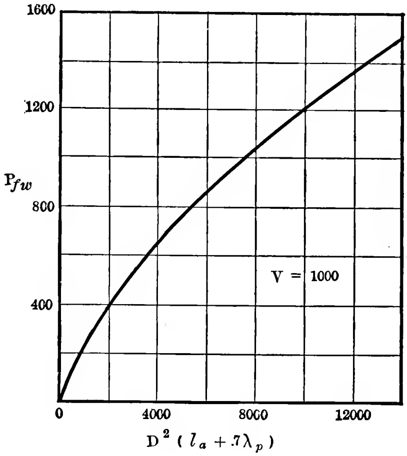


Fig. 89.

friction and windage, P_{fw}, is plotted against the product of the square of the armature diameter in inches into the axial length of the armature over the end connections of the winding. The latter factor may be considered as the gross length of the armature plus seven-tenths of the pole pitch, or $l_a + 0.7\ \lambda_p$ inches. The curve refers to a speed of 1000 rev. per min.; the friction and windage loss at any other speed is taken in direct proportion.

This loss is usually less than one-half per cent in machines of over 1000 K. W. output, and may be 2 to 3 per cent in machines of 20 K. W. or under.

67. Commutator Loss. — The transition resistance between the brushes and commutator causes a drop in voltage at each point of contact, the magnitude of which depends upon the quality of the brush, but is practically independent of commutator speed, brush current density, and brush pressure, § 42. This drop for both positive and negative brushes varies between 1.2 and 2.8 volts. Therefore the product of this drop times the current leaving or entering at the brushes gives the loss due to the brush transition resistance.

The pressure of the brushes on the commutator causes a friction loss. This quantity may be expressed as equal to

$$\frac{746\,\pi D_c V \mu' F}{33000 \times 12} \text{ watts,} \qquad \S\ 42$$

where D_c = commutator diameter in inches,
V = rev. per. min. of armature,
μ' = coefficient of friction (0.30 for carbon brushes and 0.25 for copper brushes),
F = sum of pressures of all brushes on commutator in pounds; generally 1.25 lbs. per sq. in. of rubbing surface.

To allow for the additional loss at the commutator due to sparking at the brushes and currents in the short-circuited segments, which cannot be determined accurately, six per cent is usually added to the regular commutator loss. Therefore the total commutator loss in watts may be expressed as

$$P_c = 1.06\,[(1.2 \text{ to } 2.8)\,I + 0.0059\,D_c V \mu' F].$$

68. Temperature Elevation. — The temperature elevation of any part of a dynamo is proportional to the watts expended in that part and inversely proportional to its radiating surface. The rise of temperature will be influenced considerably by the speed of the armature and by the effectiveness of the ventilating arrangements. The temperature rise is considered separately for armature, field coils, and for commutator.

The total losses in the armature comprise the eddy-current and hysteresis losses in the core and the copper loss in the winding. The true radiating surface of the armature is difficult of determination, since the end connections of the winding and the surfaces on both sides of the ventilating ducts assist in radiating some of the heat developed in the armature. It is more convenient to consider a surface to which the cooling effect may be regarded as approximately proportional, and such is the *external cylindrical surface* of the armature.

For well-ventilated armatures of modern dynamos, the temperature elevation in degrees Centigrade as thermometrically measured may be obtained from the following expression due to Arnold:

$$T_a = \frac{k_1(P_e + P_h + P_a)}{\pi D(l_a + 0.7\ \lambda_p)\ (1 + .03\ v)},$$

where $P_e + P_h$ = core losses in watts, § 62,

P_a = armature copper loss in watts, § 63,

D = armature diameter in inches,

$l_a + 0.7\ \lambda_p$ = axial length of armature over end connections in inches, § 66,

v = peripheral velocity of armature in feet per second,

and k_1 = a constant the value of which may be taken as 55.

The temperature rise of the field coils depends upon the depth of the winding, the heat emissivity of the bobbins upon which the wire is wound, and the effect of the fanning action of the revolving armature. For multipolar machines of modern design, the temperature rise in degrees Centigrade, as obtained from resistance measurements, may be calculated from the following expression also given by Arnold,

$$T_f = \frac{k_2 P_f}{A_f},$$

where P_f is the total excitation loss of the dynamo in watts, § 65, A_f is the area of the exposed surface of all the field coils in square inches, and k_2 is a constant the value of which may be taken as 90.

For the temperature rise of commutators, the same authority gives the following empirical equation:

$$T_c = \frac{k_3 P_c}{\pi D_c l_c (1 + .03\, v)},$$

where T_c = temperature elevation of the commutator in degrees Centigrade,

P_c = commutator loss in watts, § 67,

D_c = commutator diameter in inches,

l_c = length of commutator in inches,

v = peripheral velocity of commutator in feet per sec.,

and k_3 = a constant depending upon degree of ventilation; its value may be taken as 20.

69. Efficiency. — The *efficiency* of a machine is defined as the ratio of its net power output to its gross power input. It may also be defined as the ratio of the net power output to the sum of the net power output and the total losses. If P_o be the output in watts, and P_{in} be the input to a dynamo in watts, then the efficiency is

$$\varepsilon = \frac{P_o}{P_{in}} = \frac{P_o}{P_o + (P_h + P_e + P_a + P_p + P_f + P_{fw} + P_c)},$$

the various losses being determined as in §§ 62 to 67. The efficiency of a machine at full load should be determined at the ultimate temperature assumed under continuous operation at rated load, referred to the standard engine-room temperature of 25° C.

The electrical power delivered by, or supplied to, a dynamo should be measured at the terminals of the machine, and is given by the product of the terminal voltage and the ampere output. The mechanical power should be measured at the pulley, gearing, coupling, etc., thus excluding the losses in these devices, but including the bearing friction and windage. If, however, a generator be mounted directly upon the shaft of a prime mover so that it cannot be separated therefrom, the frictional losses in the bearings and in windage may be disregarded in determining the efficiency of the dynamo, owing to the difficulty in apportioning these losses between prime mover and generator.

Where a machine has auxiliary apparatus, such as an exciter, the power lost in the auxiliary apparatus should not be charged to the machine, but to the plant consisting of machine and auxiliary apparatus taken together. In such cases plant efficiency should be distinguished from machine efficiency.

The efficiency of a dynamo increases with the size, being low on small machines, and quite high on the larger ones. The efficiencies to be expected of modern direct-current compound- or shunt-wound generators of various sizes at full load are shown by the curve of Fig. 90.

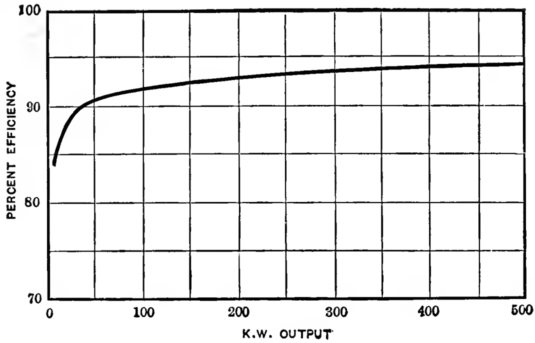


Fig. 90.

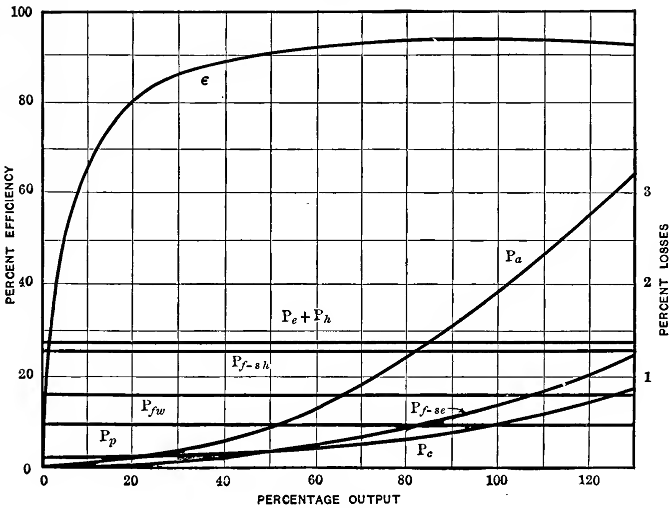


Fig. 91.

The efficiency of a compound- or shunt-wound dynamo is small at low outputs because the practically constant core losses, friction and windage loss, and shunt-field excitation loss are then large in comparison with the power output. Fig. 91 shows how the efficiency of a certain

200-K.W. compound-wound generator increases with the output. Curves are also given in this figure which show the variation of the different losses with the output of the generator. Since the distribution of the magnetic and electrical losses of a generator lies within the discretion of the designer, it is possible to so design a machine as to have its point of maximum efficiency at full load or at some other specified load. As a rule, however, the exact location of the maximum efficiency is hardly considered in designing a dynamo, since the efficiency near the maximum value is fairly constant over wide variations of load.

70. Coefficient of Conversion. — The *coefficient of conversion* of a generator is the ratio of the total electrical energy developed in the armature winding to the total mechanical energy supplied to the armature. This is sometimes called the *efficiency of conversion*, but to distinguish it from efficiency as defined in the foregoing article, it is better to use the term coefficient of conversion. This coefficient is always less than unity, and is expressed by

$$\beta = \frac{E_{in}I}{E_{in}I + P_e + P_h + P_{fw} + P_p},$$

where E_{in} is the actual voltage generated in the armature, i.e., internal *E.M.F.*, I is the current of the generator in amperes leaving or entering at the brushes, and P_e, P_h, P_{fw}, and P_p are respectively the eddy-current, hysteresis, friction and windage, and pole-face losses of the machine in watts.

71. Economic Coefficient. — Some of the electrical power developed in a generator armature is consumed in overcoming the resistance of the armature winding, some is wasted at the commutator, and some is expended in exciting the field magnets; the remainder being delivered as useful power to the external circuit, or load. The ratio of this useful electrical energy to the total electrical energy developed in the armature is known as the *economic coefficient*, and sometimes as the *electrical efficiency*. Hence, the economic coefficient may be expressed as

$$\eta = \frac{E_{in}I - (P_a + P_c + P_f)}{E_{in}I},$$

where P_a, P_c, and P_f are respectively the armature copper loss, commutator loss, and excitation loss of the generator in watts.

The efficiency, or, as it is sometimes called, *commercial efficiency*, of a generator is evidently the product of the conversion and economic coefficients, or

$$\epsilon = \beta\eta.$$

72. **Magnetos.** — Magnetos or magneto-generators are dynamos in which the magnetic flux is set up by permanent magnets. Since the flux density in this type of machine is necessarily low, for a given flux more iron must be used than in machines having their fields produced by electro-magnets. Therefore the application of magnetos is limited to purposes requiring a relatively small amount of energy, such as telephone signaling, automobile ignition work, and testing of electrical circuits.

Magnetos are generally alternating-current machines and provided with slip rings or contact studs instead of commutators. The armatures are usually of the Siemens type, wound with many turns of fine wire, and mounted so that they may be rapidly rotated between the poles of permanent horseshoe magnets. Fig. 92 shows a telephone magneto-generator manufactured by the Western Electric Company. To the armature shaft is affixed a pinion which meshes with a gear wheel turned by hand. Such machines are designed to ring a call-bell or telephone ringer through an external resistance as high as 50,000 ohms. The armature windings of magnetos have resistances between 300 and 600 ohms depending on the type of service for which they are designed. These generators are provided

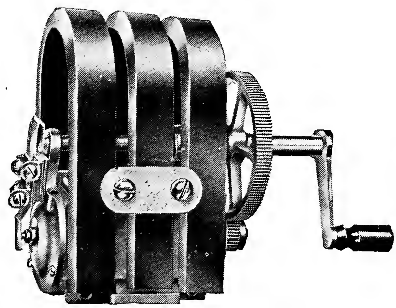


Fig. 92.

with "shunts," which afford a by-path of low resistance around the armature when not in use, or devices which normally hold the armature circuits open and close them when the generators are operated. The air gaps of such machines may be as low as 0.01 inch without introducing operative difficulties.

73. Constant-Potential and Constant-Current Supply.— There are in use two systems of electrical distribution: (*a*) at constant potential, (*b*) with constant current. In the former system, lamps, motors, or other types of electrical apparatus are connected in parallel with each other across the supply mains. To secure satisfactory operation, it is necessary to maintain a constant voltage between these mains, so that if some of the load be disconnected, or more load be added, the current flowing in the remaining lamps, motors, etc., will stay unchanged. This method of supplying, at any point of usage, current at a constant potential irrespective of the load which is there or at other points of the system, is very generally used in the distribution of electrical energy for incandescent electric lighting, for operation of constant-pressure motors, and for electric traction. The great sensitiveness of the light intensity of incandescent lamps to a change in voltage, the candle power varying perhaps as the fourth power of the voltage, requires that the voltage across electric-lighting supply mains vary less than 3 per cent of its rated value. In electric traction, where the load is exceedingly variable, particularly in trunk-line operation, constant-potential distribution can only be approximated. Frequently a drop of 25 per cent is allowed.

For lighting by arc lights where considerable energy is expended at the points of illumination, and where these points are separated from each other by considerable distances, it is sometimes economical and desirable to connect the lamps in series. For satisfactory operation the current in the circuit should be maintained constant, so that if more lamps be put in service, or some taken out, the voltage across the remaining lamps will be unchanged. A lamp connected to such a system may be cut out by short-circuiting it.

The advantage of constant-current distribution for town lighting is the economy of copper for the supply mains.

The line can be made of much smaller wire than in the case of a constant-pressure circuit, for on a constant-current circuit as the load increases the power or energy transmitted is increased by raising the potential, the current remaining unaltered; while in a constant-pressure circuit an increase of load is met by an increase of current, and the supply mains must be of sufficient size to safely carry the required maximum current. The size of wire necessary is dictated, not by the energy transmitted, but by the current flowing, hence a wire large enough to supply just one lamp of a constant-pressure circuit can supply all the lamps of a constant-current circuit.

Constant-Potential Generators.

74. Characteristic Curves of Shunt-Wound Generators. — The operation of any dynamo can best be studied by inspection of a curve which shows the relation existing between the current generated or supplied by the machine and the voltage under which it operates. Such curves are called *characteristic curves*, and they are generally plotted with current strengths as abscissæ and voltages as ordinates. The characteristic curve of a shunt-wound generator is shown as E in Fig. 93, and it is seen therefrom that the terminal voltage decreases slightly as the current output of the machine increases, the speed of the generator being maintained constant.

To obtain the characteristic curve of a shunt-wound generator experimentally the machine is run at normal speed, and readings are taken of terminal voltage and current output, the setting of the field rheostat being fixed during the test. The setting of this rheostat may be that giving rated voltage either at no load or at full load. Fig. 93 indicates the latter condition. In some small machines the voltage can be reduced to zero without causing excessive sparking or extreme temperature elevation, but as a rule the complete characteristic is obtained only when the field rheostat is adjusted for a voltage much below the rated voltage of the machine.

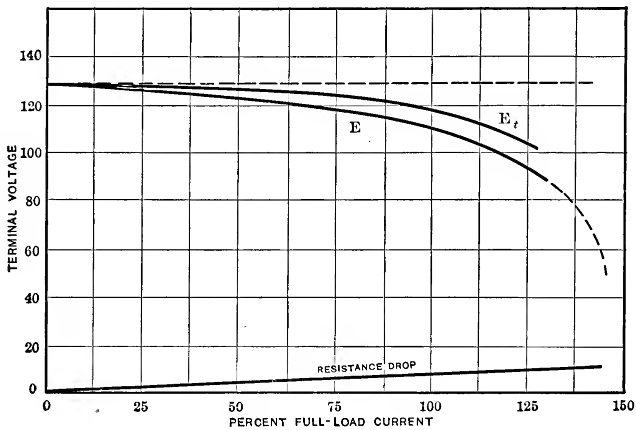


Fig. 93.

The characteristic curve of a strictly constant-potential generator would be a straight horizontal line, since this indicates that the voltage remains the same at all loads. The terminal voltage of a shunt-wound dynamo at constant field excitation and speed decreases slightly as the load increases, because of the armature resistance drop and armature reaction, § 50. The armature resistance drop, being the product of the armature current and resistance, is practically a linear function of the load (the change in resistance due to heating occasioned by increased current may be neglected), and may be plotted as a straight line, as in

Fig. 93. The total voltage generated is obtained by adding the armature resistance drop to the terminal voltage. Thus, the curve of total voltage, E_t, is plotted by adding the ordinates of E and those of the resistance-drop curve. The difference between E_t and the no-load terminal voltage of the machine shows the effect of armature reaction.

The drop in terminal voltage is at first due chiefly to the drop resulting from armature resistance. As the current increases, the effects of armature reaction and saturation of the magnetic circuit become evident. This soon becomes the predominating cause of voltage drop, and to such an extent that the curve turns back toward the origin. When the resistance in the external circuit is zero, of course no current flows through the field, and the few volts then produced are due to residual magnetism. Unless the field excitation is kept constant in determining the terminal voltage curve of a generator, it should be remembered that the difference between E_t and the no-load terminal voltage is also due to a decrease of the field current occasioned by the fall of potential at the terminals of the field winding.

The voltage of a shunt machine generally increases more rapidly than the speed. An increase of speed not only increases primarily the number of volts generated, but also increases the armature flux because of increased excitation. The condition of the magnetic circuit as regards saturation determines whether this secondary influence shall be great or small.

75. Voltage Regulation. — Shunt-wound generators are so designed that the lowering of terminal voltage from no load to full load shall be as small as is consistent with economy and practicability. Such machines are particularly adapted for constant-potential distribution. The maintenance of a perfectly constant terminal voltage is effected by the use of regulating field rheostats.

Suppose the field rheostat of a shunt-wound generator to be adjusted for obtaining the rated voltage of the machine at full load. Upon disconnecting the load and leaving the rheostat setting unaltered, the terminal voltage of the generator increases. This change of voltage from full load to no load at constant speed when expressed as a percentage of the rated full-load voltage, is termed the *voltage regulation* of the generator. Thus the regulation of the generator of the foregoing section at full load, as obtained from Fig. 93, is

$$\frac{128 - 110}{110} = 0.163 \text{ or } 16.3 \text{ per cent.}$$

The regulation of a separately excited generator should be determined at constant excitation. The regulation of a generator unit, consisting of a generator united with a prime mover, should be determined at constant conditions of the prime mover; i.e., constant steam pressure, head, etc. It would include the inherent speed variations of the prime mover. For this reason the regulation of a generator unit is to be distinguished from the regulation of either the prime mover or of the generator contained in it, when taken separately.

76. Hand Regulation. — To maintain a perfectly constant terminal voltage at increased load necessitates an increase in the total *E.M.F.* generated in the machine. An inspection of the formula for the electromotive force of a generator,

$$E = 2\,\Phi\, Sf\,10^{-8},$$

shows that the only quantity that it is practical to vary is the magnetic flux through the armature Φ_m. This can easily be accomplished by regulating the amount of resistance in a rheostat, which is in series with the field coils, and which therefore governs the amount of current in them, as in Fig. 94.

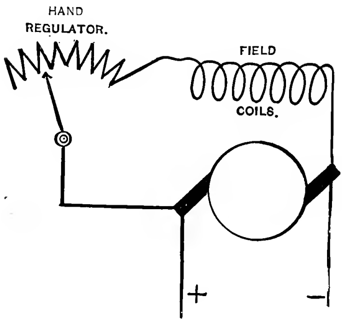


Fig. 94.

In distributing current for use among a number of consumers the current is carried to *feeding-points* which are near the locality they supply, but may be distant from the station. It is desirable to keep the pressure at these points at a constant value, irrespective of the varying loss of potential that is going on because of the resistance of the conductors leading to them. To achieve this end *feeders* are employed to carry the current to the feeding-points. Each feeder is accompanied by a pilot wire imbedded in the insulation. At the feeding-point the pilot wires are attached to the feeder terminals, and at the station end are attached to a voltmeter, so that the station attendant can regulate the pressure not at the machine terminals but at the distant distributing point.

77. Field Rheostats. — For varying the current in the shunt field coils of generators, it is usual to employ field rheostats which may be mounted on the station switchboard together with the usual indicating instruments, or on a separate frame. Such rheostats consist essentially of high-resistance wire or ribbon with numerous taps connected to a series of contact studs over which moves a contact arm. The resistance units may be in the form of cards, bars, bobbins, or grids, according to the capacity required.

A field rheostat, manufactured by the General Electric Company, is shown in Fig. 95. It is arranged to be

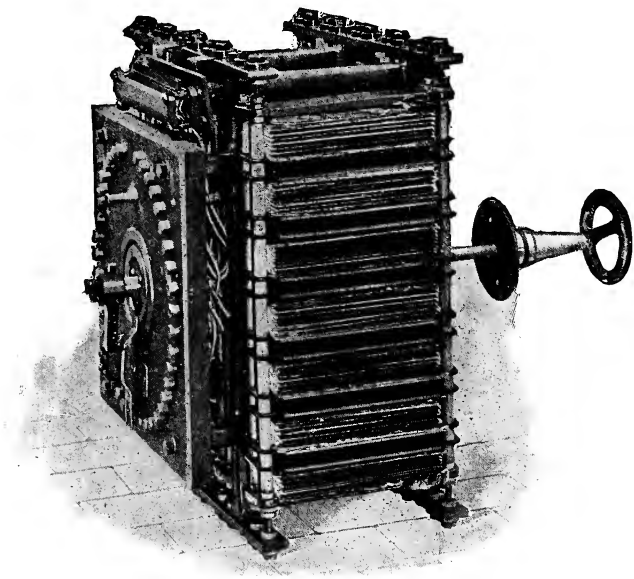


Fig. 95.

placed on the back of switchboards with the regulating handle projecting in front. The resistance units are of the card form, and are constructed by winding the resistance ribbon on tubes of asbestos which are subsequently pressed flat. These cards are then assembled, with interposed asbestos, in sufficient numbers to make up the required resistance of the rheostat. Iron plates, somewhat wider than the cards, are introduced at intervals, and thus increase the radiating surface. Numerous taps are taken from the resistance units to the various contact studs.

Fig. 96 illustrates a rear view of a Westinghouse rheostat with the bottom plate removed. The resistance unit

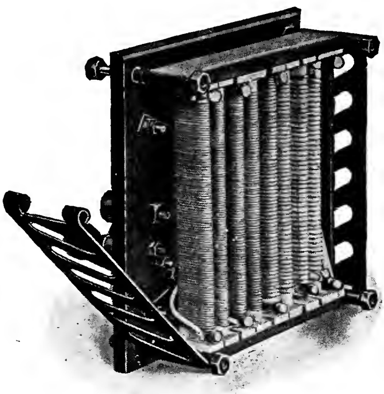


Fig. 96.

is of the bar type, so called because the resistance wire is wound on flat iron bars, but insulated therefrom by a layer of fireproof insulating material. By varying the size of the wire, resistances may be wound of from .03 to 400 ohms per linear inch of the bar, with a maximum capacity of 4 watts per square inch of surface on one side.

Field rheostats for very large generators consist of resistance units in the form of iron grids supported in frames, which are mounted directly on the floor at some convenient point near the switchboard. Fig. 97 shows such a rheostat made by the Westinghouse Electric and Manufacturing Company.

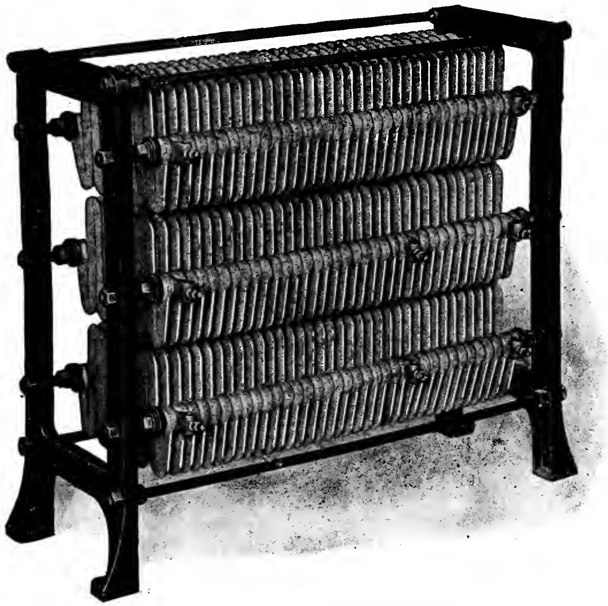


Fig. 97.

Another form of field rheostat, made by the Cutler-Hammer Manufacturing Company, is shown in Fig. 98. In this rheostat the heat generated is not radiated directly from the surface of the wire, but is conducted to a supporting plate, which then becomes the radiating surface. The resistance wires, contacts and lever are mounted on a base of insulating material, the whole being carried by an iron casing, which prevents the possibility of contact with the heat radiating portion of the rheostat. Owing to the increased radiating surface thus obtained, a shorter and smaller wire can be used for a given volt-ampere capacity than if the wire were merely exposed to the air. No consideration of the mechanical strength of the wire enters into the design of this resistance, since it is supported and protected by an insulating compound.

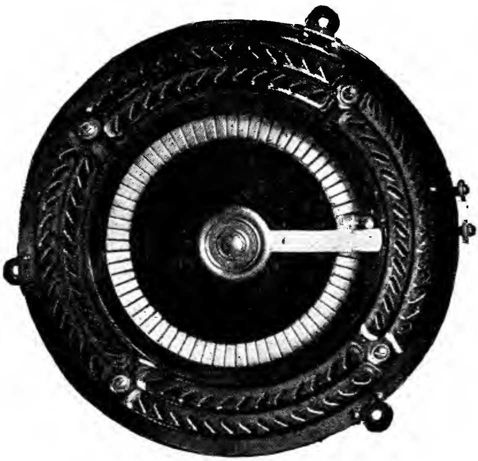


Fig. 98.

When large generators, such as are used in railroad work, have their field-circuits opened, the *E.M.F.* self-induced by the disappearance of the flux in the fields is liable to reach such a magnitude as to pierce the insulation of the field coils and destroy their usefulness. To obviate this, before the field circuit is broken, the field coils are connected (Fig. 99) through a high discharge resistance, and the current in them is allowed to decay slowly. It is thus unattended with any destructive potential differences. Arc lights have in several instances been used for this purpose instead of high resistances.

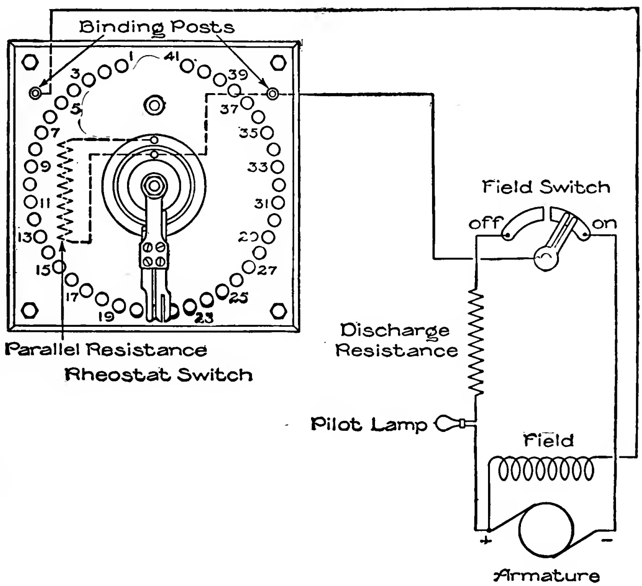


Fig. 99.

78. **Self-Regulation.** — By far the most elegant method of constant potential regulation is that in which the main current of the machine is utilized in maintaining constant the magnetic flux through the armature. This is accomplished by passing all or the greater part of the current flowing in the armature a few times around the field magnets, so that an increased load on the armature increases the magnetizing ampere-turns of the field coils. These *series turns*, when rightly proportioned, can be made

to compensate for a part, for all, or for even more than all of the drop. This device can be used in connection with any other form of excitation, as permanent magnets, separate excitation, or shunt excitation. In the last case, the dynamo is said to be *compound wound*, as described in § 46. If the machine is designed to maintain a constant pressure at some distant feeding-point, instead of at the machine terminals, the machine is said to be *over-compounded*, since the potential at the terminals will rise on increase of load. From 3 to 5 per cent over-compounding is frequent in machines used to supply lighting circuits, and 10 per cent over-compounding is usual in railway generators.

79. Characteristic Curves of Compound-wound Generators. — As a compound-wound machine is essentially a shunt-wound generator provided with a series field winding, the characteristic curve thereof would be the resultant of the *shunt* characteristic and the *series* characteristic. The form of these curves is shown in Fig. 100, the shunt characteristic being the same as that for shunt-wound dynamos. The voltage induced in the armature by the increase of magnetic flux due to the current in the series turns is proportional to the current. As the load increases this voltage will increase, and, if the series winding be properly proportioned, the increase of the voltage due to the current in the series turns may neutralize the decrease of the main voltage occasioned by armature resistance and reaction. The form of the curves of Fig. 100 indicates that such neutralization can occur at only one load. If the compensation be complete at full load, the machine is said to be *flat-compounded*. The *compound* characteristic for flat-compounding is shown by the broken line, and that for over-compounding is shown by the full line in the figure.

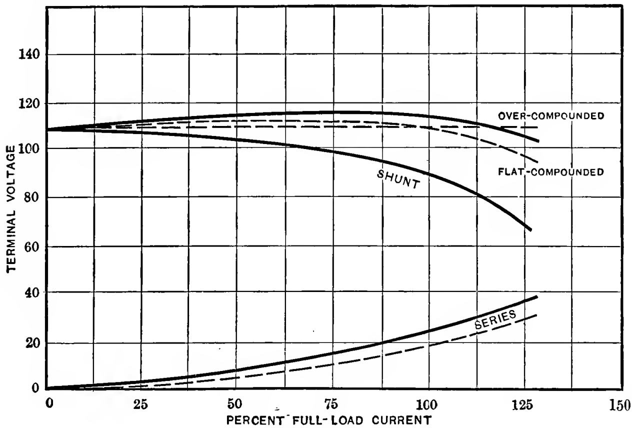


Fig. 100.

The degree of compounding may be changed by varying either the current flowing through the series winding or the number of turns on it. In practice it is usual to provide more series turns than required, and to place an adjustable resistance across the terminals of the series field coils. The full armature current therefore divides between this resistance and the series coils, and the amount flowing through the latter may be adjusted for the required compounding.

In over-compounded machines, the voltage regulation is the ratio of the maximum difference in voltage from a straight line connecting the no-load and full-load values of terminal voltage as function of the current, to the full-load terminal voltage.

80. Railway and Lighting Generators. — The tendency of modern engineering practice is to install lighting generators which are directly connected to the prime mover. Owing to the inherent speed of steam engines being smaller than that of generators, direct-connected armatures are designed to run at a lower speed than belt-driven ones. Economical construction demands that they be of the multipolar type. They require less floor space per kilowatt than the belt-driven machines; and this is a question of considerable importance in many installations. They have a higher efficiency of operation consequent upon the elimination of losses in belting and countershafting. They also permit of operation of isolated plants in residences and other places where the noise resulting from belt-driven machinery would not be tolerated.

In order that standard generators may be easily connected with engines of any make, and *vice versa*, committees from engineering societies have recommended the adoption of the following standard sizes, speeds, and armature shaft fits: —

Sizes in K.W. Capacity .	5	7.5	10	15	20	25	50
Speeds in Rev. per Min. .	450	425	400	375	350	325	290
Armature Fit in Inches .	3	3	3½	3½	4	4	5
Sizes in K. W. Capacity .	75	100	125	150	200	250	300
Speeds in Rev. per Min. .	275	250	235	220	200	190	180
Armature Fit in Inches .	6	7	7½	8	9	10	11

Fig. 101 shows a 1600-K. W., 16-pole, 100 rev. per min. General Electric Company direct-connected engine-driven railway generator. These generators are built in sizes from 100 K. W. to 2700 K. W., and are designed to yield the prevailing full-load railway voltages of 550, 575, or 600 volts. The field-magnet yoke is of cast iron, circular in shape and of oval or rectangular cross-section. The frame is divided, the upper half being fastened to the lower by concealed bolts. The poles are solid steel castings

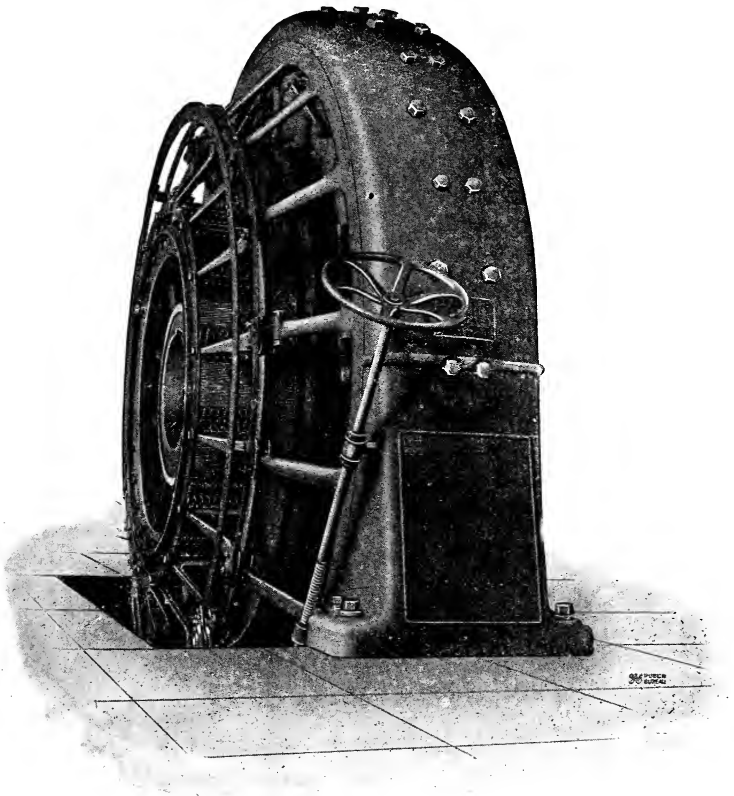


Fig. 101.

bolted to the frame, and may be removed laterally without taking out the armature. Commutating poles are provided in most sizes, to compensate for armature reaction, thus insuring good commutation at all loads. The armature spider is equipped with vanes which fan air through the ventilating passages formed through the laminations and windings and around the poles, thus improving ventilation. The brush-holder mechanism consists of a ring concentric with the axis of the armature and attached to the field frame. The simultaneous shifting of the brushes is accomplished by the turning of the hand wheel. These generators are rated on the basis that after a continuous full-load run of 24 hours the temperature elevation of no part of the machine will rise more than 35° above the engine-room temperature. A subsequent increase of 50 per cent full load for two hours will cause no more than 55° C. temperature elevation over the surrounding air.

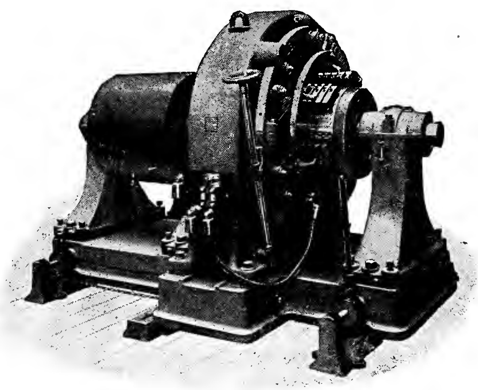


Fig. 102.

A belt-driven, 400-K.W., 375 rev. per min. generator manufactured by the Western Electric Company is shown in Fig. 102. The pole pieces are of laminated sheet steel, and are cast into the circular yoke, thus insuring good magnetic joints. In the larger machines the frames are divided vertically, a construction that permits of easy access to the armature without necessitating the use of heavy hoisting apparatus. Slide rails are provided upon which the machines may be moved by means of a screw in order to tighten the belt. Alignment is maintained by tongues in the base of the machine which fit into grooves in the slide rails.

The Allis-Chalmers Company manufactures generators of the belted "H" type in sizes of from 7.5 to 500 K.W. for voltages of 120, 240 and 500 volts, and engine-type generators from 12 to 1000 K.W. The field poles of these machines are made up of laminated steel stampings

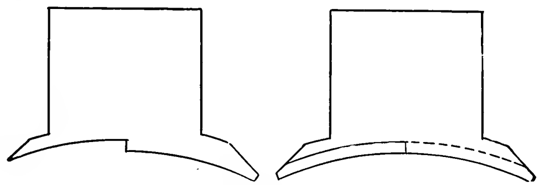


Fig. 103.

of the shape shown in Fig. 103. In assembling these punchings to form the poles, they are alternately reversed with respect to one side. Thus, the face of the pole for a short depth contains but one-half as much iron as the main body of the pole. This results, under normal excitation, in a saturated pole face. It has the same effect in preventing distortion of the field under the influence of armature reaction as saturation of the teeth of the armature core. The teeth can therefore be operated at a smaller magnetic flux density. The hysteresis losses in the teeth can accordingly be made smaller. The thinness of the stampings, and the ideally perfect lamination of the pole face, permit the use of a smaller ratio of tooth width to slot width, without the excessive eddy current loss in the pole face which would occur in other machines. The possibility of using narrow teeth results in a reduction of the inductances of the armature coils. This facilitates effective commutation.

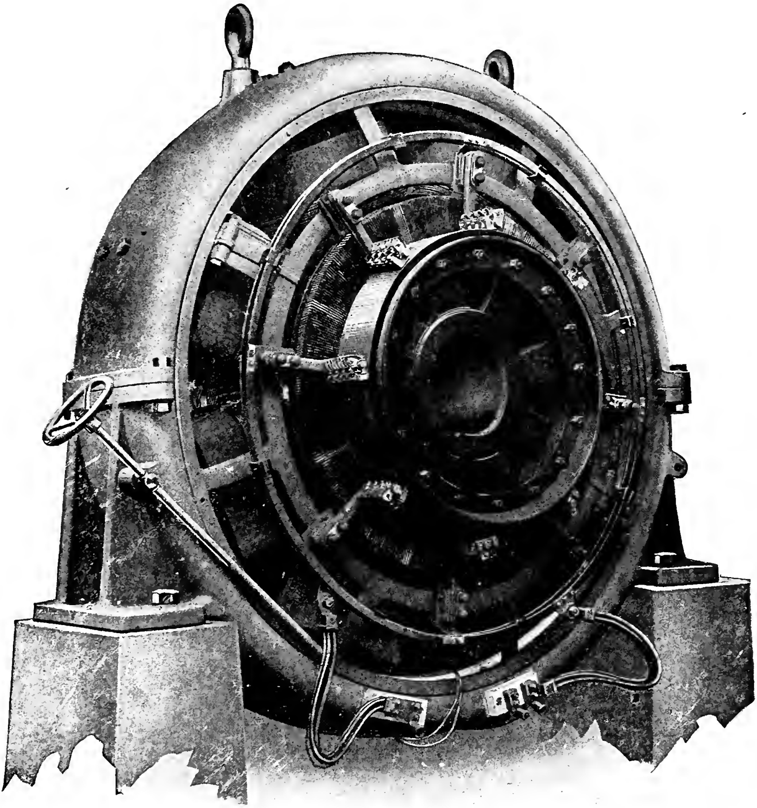


Fig. 104.

Fig. 104 shows a 350-K.W. engine-type generator made by the Westinghouse Electric and Manufacturing Company.

Some of the important features of this design are the use of conductor retaining wedges in the armature slots, the arrangement of the series field coil connections, removable pole pieces, and the arrangement of armature equalizer rings and of the brush-holder shifting device.

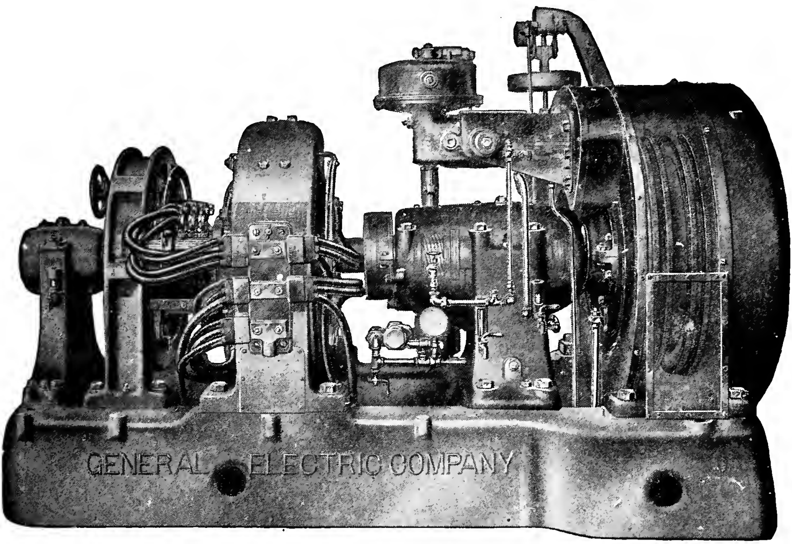


Fig. 105.

Fig. 105 depicts a General Electric Company generator direct coupled to a steam turbine, and mounted on a common bedplate. The cut shows the generating unit as semitransparent so as to reveal the interior parts.

The field-magnet frame of a 6-pole single-coil type of Lundell generator with its field coil in place is shown in Fig. 106. The frame is divided in a vertical plane which is perpendicular to the axis of the armature.

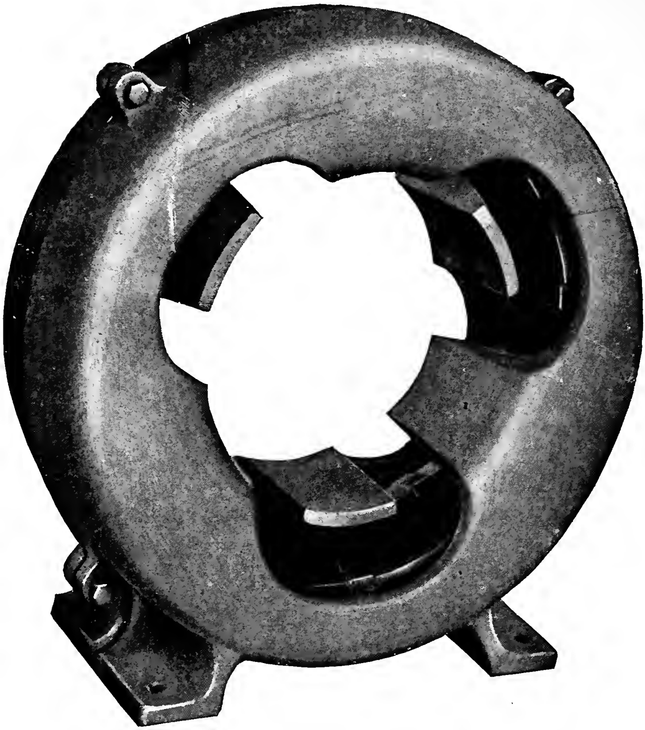


Fig. 106.

81. **Three-Wire Generators.** — The adoption of three-wire systems of electrical distribution, particularly for lighting, is due to the saving of copper in the line conductors. The standard voltage for incandescent lamps is about 110 volts. At this pressure large conductors must be employed on long lines in order to maintain a fairly constant voltage at the lamps as the load changes. By doubling the voltage across the mains and connecting the lamps thereto so that they are joined by pairs in series with each other, only one-fourth as much copper need be used, since the

same power is transmitted at half the current, and for the same permissible drop the conductors need be but half as large. But, in order that each lamp may be operated independently of the others, a balancing wire or *neutral wire* must be provided, and this is usually of the same size as the other conductors. Therefore the weight of conductors on a three-wire system will be $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$ as much as on a two-wire system.

The introduction of a neutral wire involves the generation of the total *E.M.F.* in two parts so that this neutral wire may constitute a common conductor for the two component voltages. To obtain this condition, two generators may be connected as in Fig. 107, but as this signifies additional expense, various other methods have been adopted to obtain three-wire distribution. These practical methods employ: (*a*) dynamotors, § 110, which have two armature windings upon the same core connected to two separate commutators, and connected in the same manner as two individual generators; (*b*) storage batteries, § 113, of sufficient number of cells connected between the two outside wires, the neutral wire connecting with the middle point of the battery; (*c*) balancers, § 111, which are two mechanically coupled dynamos connected across the outside wires, one of which, if the system be unbalanced, will run as a motor and drive the other as a generator which supplies energy to the more heavily loaded side; (*d*) three-brush dynamos, and (*e*) three-wire generators.

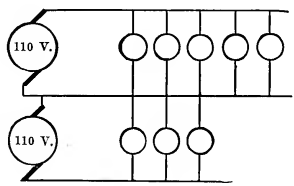


Fig. 107.

The total voltage of a generator could be divided into two parts by placing a brush midway between the positive and negative brushes; but, for satisfactory current collection, the coil short-circuited by this additional brush must lie in a weak magnetic field. Such an arrangement was developed by Dettmar, and is shown in Fig. 108. This illustrates a four-pole field-magnet frame wound as a bipolar machine with two adjacent north poles and two adjacent south poles. The yokes of such machines between oppositely named poles must be of sufficient cross-section to carry the total flux per pole at a reasonable flux density. The tendency of the armature current is to crowd the flux toward the leading poles, thus resulting in a somewhat greater voltage between the positive terminal and the neutral wire than between the latter and the negative terminal.

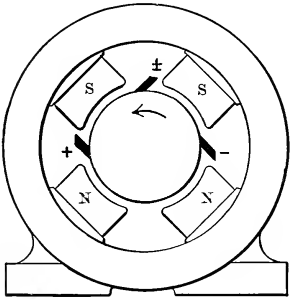


Fig. 108.

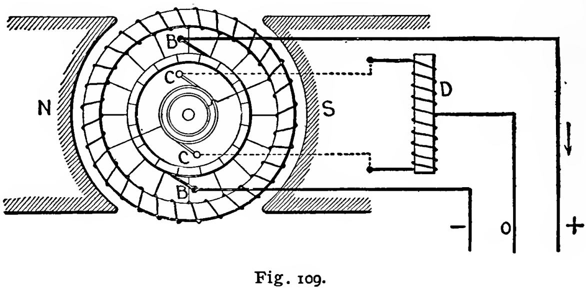


Fig. 109.

The three-wire generator designed by Dobrowolsky is well adapted for three-wire supply circuits. Points of the armature winding at distances from one another equal to twice the pole pitch are connected to one slip ring and the intermediate points are connected to another slip ring. Brushes bearing upon these collector rings connect with the ends of a coil wound on an iron core, called a *reactor*, as shown in Fig. 109. The middle point of the reactor, *D*, connects to the neutral wire of the system, the outside wires being

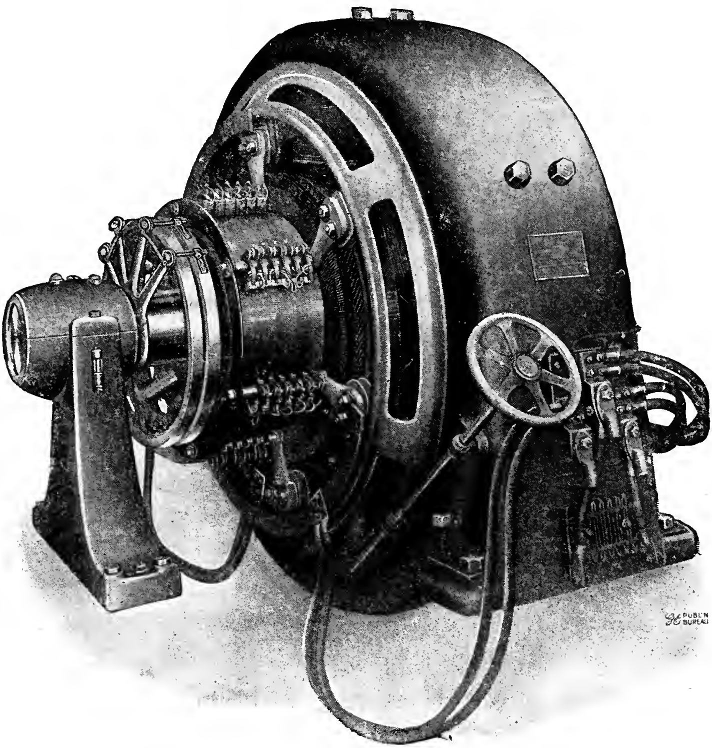


Fig. 110.

connected to the brushes *B*. The reactor has a low resistance, but a large inductance. The electromotive force across the terminals *C* is an alternating *E.M.F.*; and, because of the large inductance of the coil, very little current flows through it when the loads on the two sides of the system are equal. If the system be unbalanced, the current flowing in the neutral wire, since it is direct current, will suffer little impedance in passing through the reactor.

A 150-K.W., 6-pole General Electric Company generator, provided with two slip rings for connection to a reactor as just described, is shown in Fig. 110. These machines are usually wound for 250 volts, so that 125 volts can be obtained on either side of the three-wire system. They may be flat- or over-compounded to compensate for line drop.

Three-wire generators having a single slip ring for connection to the neutral wire are manufactured by the Burke Electric Company. The reactor forms a part of the armature and revolves with it, the middle point of the reactor being connected to the slip ring.

Three-wire generators are extensively used in isolated plants for electric lighting and light power service.

82. Homopolar Dynamos. — A type of direct-current generator in which the armature conductors move in a unidirectional and uniform magnetic field, and therefore have induced in them electromotive forces of constant direction and magnitude, is known as the *homopolar* dynamo, sometimes also as the *acyclic* or *unipolar* dynamo. Fig. 111 shows a cross-section of a simple machine of this type, with one conductor, *A*, connected to two slip rings *B*. The magnetic field of the generator is set up by the current flowing in the field coils *C*; the paths of the lines of force being represented by the dotted lines. The current is led from the machine by means of brushes which slide upon the slip rings and are connected to wires projecting through apertures in the field-magnet yoke.

Single-conductor homopolar dynamos are suitable for supplying a large current at low voltage, and even then the magnetic flux traversing the air-gap must be large and the armature must be run at high speed. As there is little demand for such low-voltage generators, homopolar machines

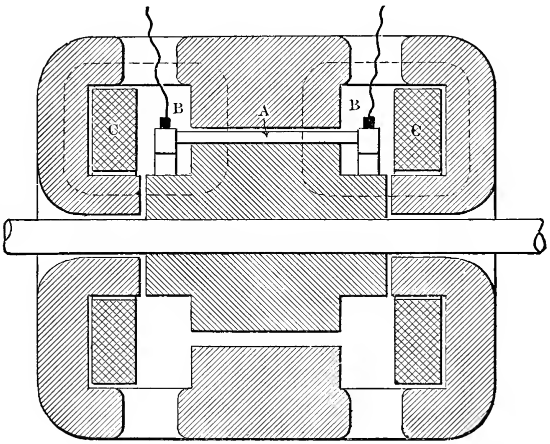


Fig. 111.

for practical purposes must be designed to generate higher voltages. This may be accomplished by increasing the number of armatures mounted together to form one machine, or by employing several conductors insulated from one another and connected in series. The end of one conductor must be joined to the beginning of another, but this end connection must not cut the lines of force, otherwise the resultant *E.M.F.* would be zero. Consequently the end connections must be stationary, and this means that two slip rings must be provided for each conductor. Limitations to increasing the number of conductors are the available space for the slip rings and the increased brush friction.

The electromotive force generated by a multi-conductor homopolar dynamo is

$$E_{av} = N \Phi \frac{V}{60} 10^{-8} \text{ volts,}$$

where N = number of conductors in series,

Φ = total flux entering armature,

and V = rev. per min.

The armature of a 300-K.W., 500-volt, 3000 rev. per min., turbine-driven homopolar generator is shown in Fig. 112. It consists of 12 copper conductors mounted on a

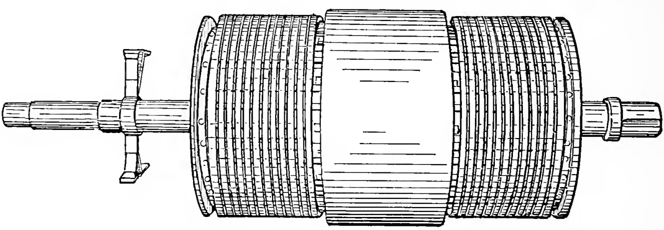


Fig. 112.

cast-steel core, the ends of the conductors being connected to 12 cast-steel slip rings at either end of the armature. One copper brush is provided for each ring, and access is obtained thereto through apertures in the cast-steel field-magnet frame. Compounding is effected by utilizing the *M.M.F.* of the currents in the stationary leads which are connected to the brushes, or that of the currents in the slip rings from the connection points to the brushes, to aid the *M.M.F.* of the field current. The degree of compounding may be varied by shifting the brushes. Homopolar generators may be separately- or self-excited.

As the air-gap of homopolar machines may be very small, the field current need not be great in order to set up the required magnetic flux through the armature. This fact indicates a low excitation loss. There are practically no iron losses in this type of generator because of the constancy of flux density. The brush losses, however, are large. The efficiency curve of the 300-K.W. generator is shown in Fig. 113. The voltage regulation is from 6 to 12 per cent.

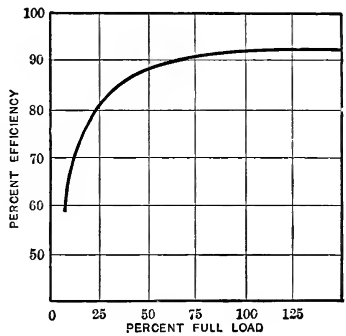


Fig. 113.

Constant-Current Generators.

83. Characteristic Curves of Series-Wound Generators. — Fig. 114 shows the characteristic curves of a 15 K.W. series-wound generator at a speed of 1000 rev. per min. Curve E indicates the terminal voltage of the machine when delivering various currents, and is called the *external characteristic.* This curve shows that the *E.M.F.* of the generator at first increases in proportion to the current output, but as the load increases the resistance drop of the field and armature windings and armature reaction cause the curve to bend back. This is also due to the fact that, as the magnetic circuit approaches saturation, the magnetic flux does not increase proportionally to the increase of field current. To obtain the external characteristic of a series-wound generator experimentally the machine is run at a definite and constant speed and observations of terminal voltage and current output are made at different loads. At constant load the terminal voltage will vary directly with the speed.

The total characteristic of a series-wound generator may be obtained from the external characteristic and the resistance drop of the windings, § 74. Thus in Fig. 114, curve

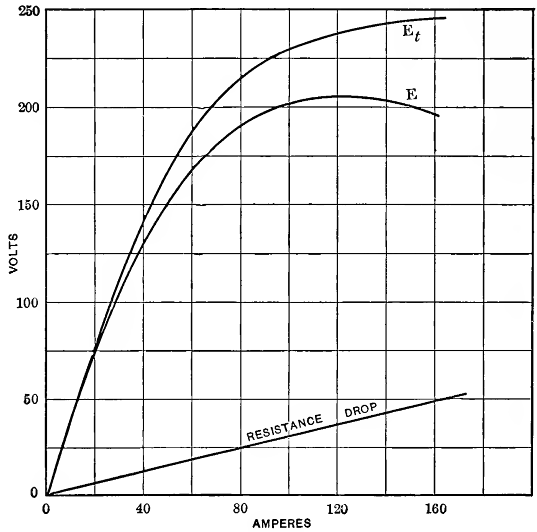


Fig. 114.

E_t is plotted by adding the ordinates of the curves of terminal voltage and resistance drop. The curves of both E and E_t start above zero because of residual magnetism in the cores of the field magnets.

The total characteristic resembles the *magnetization curve* of series-wound generators. The latter is a curve which shows the terminal voltage of a machine at no load for different values of field current. The difference existing between these curves is due solely to the demagnetizing effect of the armature current on the magnetic circuit.

84. Power Lines. — Where volts and amperes are used as ordinates and abscissæ, lines can be drawn connecting points of constant product of the two, representing watts or power. Fig. 115 shows such lines drawn for one, two,

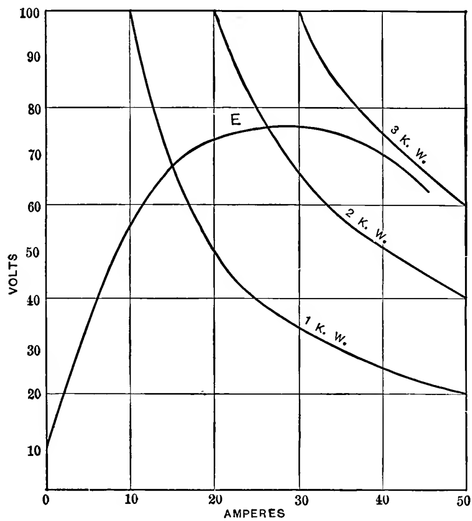


Fig. 115.

and three kilowatts. If E be the external characteristic of a dynamo, then the curves make it apparent that the machine cannot generate 3 K.W., but that for most values under 3 K.W. there will be two loads under which the generator can run and yield the same voltage.

85. Series-Wound Generators. — The advantage of constant-current distribution for arc lighting lies in the saving of conductor material. In this system, as the load increases the voltage must increase a corresponding amount, so that the current flowing will be unchanged. An ordinary series arc lamp, as it is trimmed and adjusted for general use, requires between 45 and 50 volts to force its rated current through it. A generator supplying a circuit of say 2000 candle-power lamps with n such lamps in the circuit must be capable of generating a constant current of 9.8 amperes. It must be able to regulate its pressure between the limits of 50 and 50 n volts. This is necessary in order that it may operate all the lamps or any part of the whole number at proper illumination.

The current of an arc-light machine must not exceed nor fall below its normal value, no matter how suddenly the load is varied; for the slightest change affects the intensity of the light at the lamps. It is obvious that some mechanical device could be applied to an ordinary shunt-wound generator to cause it to give constant current, either by changing the position of the brushes or by varying the ampere-turns of the field coils. However, any such device would be slow of operation, and a sudden short-circuit would cause a destructive current to flow before the regulator completed its action. It is therefore necessary to rely on the armature reactions for regulation, since they vary simultaneously with the current. All successful constant-current machines are constructed on this principle. The machine is designed with a magnetic field of great intensity, the armature reactions are very great, and thus the total flux effective in producing *E.M.F.* is reduced. A slight increase of current in the armature materially increases the armature reactions. The effective flux is thus reduced, and the pressure falls until the current returns to its normal value. Thus the machine is completely and instantly self-regulating. The field coils are series wound on all arc-light generators, and the cores of the field magnets are worked at a very high magnetic density, since the magnets are then less sensitive to slight changes in the magnetizing force. In commercial machines the densities in the field cores are from 17,000 to 18,000 lines per square centimeter for wrought iron or steel, and from 9000 to 11,000 lines for cast iron.

In the armature high magnetic density is also required to prevent a sudden rise of voltage when the circuit is broken. With no current in the armature, the total magnetomotive force of the field magnets would be effective in producing *E.M.F.*, and a destructive rise of pressure would result, since the total *M.M.F.* of the field magnets is much greater than the normal effective *M.M.F.* But a high magnetic density in the armature core leaves the latter incapable of receiving such an increase of flux, and therefore destructive voltages are avoided. In practice the armature core is designed to have a density of from 15,000 to 20,000 lines per square centimeter at its minimum cross-section.

A consideration of the foregoing theory of regulation shows that the following conditions should obtain more or less completely in a successful constant-current generator: (*a*) since the current is small, there must be a great number of armature turns; (*b*) the magnetic field of the machine must be much distorted; (*c*) the path of the lines of force of the field coils must be long and of small area, so the *M.M.F.* cannot be readily changed; (*d*) the path of the lines of force due to armature magnetization must be short and of great area, so that the *M.M.F.* of the armature will change with the slightest change of current; and (*e*) the pole pieces must be worked at a high flux density.

Evidently extreme difficulty is found in so designing the different parts of the machine as to give proper consideration to each of the conditions and yet produce a machine that will regulate for constant current at all loads. This leads to the introduction of automatic mechanical devices for aiding in the regulation. These devices must not be considered as being the sole regulators, for in every case they are secondary to the natural self-regulating tendency of the armature. In general they regulate for the gradual and greater changes of load, while the armature reactions take care of the smaller and more sudden fluctuations.

There are two general systems of regulating arc-light dynamos. The first method is to cause the machine to develop an *E.M.F.* in excess of that required for the load, and then to collect an *E.M.F.* just sufficient for the load in hand. This is done by shifting the brushes from the neutral plane (§ 52). In a closed-coil armature this causes a counter pressure to be developed in those conductors lying between the commutating plane and a similar plane in the other direction making an equal angle with the neutral axis. This reduces the pressure to the desired amount. In an open-coil armature the brushes, when in the maximum position, connect to the circuit those coils of the armature which at that instant have the maximum *E.M.F.* generated in them. By shifting the brushes either way, coils can be connected to the circuit which have some *E.M.F.* lower than the total *E.M.F.* generated in them, and the amount of shifting regulates the pressure on the line.

The second method of arc-light dynamo regulation is to vary the magnetizing force in the field magnets just enough to put the required pressure on the line. Since the magnetizing force is dependent on the ampere-turns of the field coils, it can be varied either by cutting out or short-circuiting some of the turns or by changing the current in them by means of a variable resistance which is shunted across the field terminals. In practice both these methods have been used.

Whether regulation is effected by changing the position of the brushes, or by changing the field excitation, sparking will occur at the points of collection of the current if means are not provided to avoid it. Sparkless collection could be obtained were the magnetic field perfectly uniform all around the armature. In general this condition is impracticable, since it requires almost the whole armature to be covered by the pole faces, and it requires the density in the gap beneath them to be uniform. Considerations of magnetic leakage and armature reaction render almost impossible the satisfying of these conditions. Another and more practical method is to employ for current collection at one terminal of the machine two brushes connected in parallel. These are moved in opposite directions, thus giving the effect of a single brush of varying circumferential contact, the center of which can always be kept in the neutral plane. This device avoids excessive sparking, and is used quite successfully in practice. There is, however, some question as to the advisability of resorting to it.

86. The Brush Machine. — Fig. 116 shows a standard 160-light Brush arc-light generator, made by the General Electric Company. The armature revolves between the pole faces of two sets of field magnets. Like poles are opposed to each other. The flux, therefore, takes a path out of the opposing pole faces into the armature core, and then circumferentially through the core and out into the next pair of opposing pole faces.

The armature is of the open-coil type and consists of a number of coils or bobbins placed on a ring core of greater radial depth than breadth, and the pole faces cover the sides

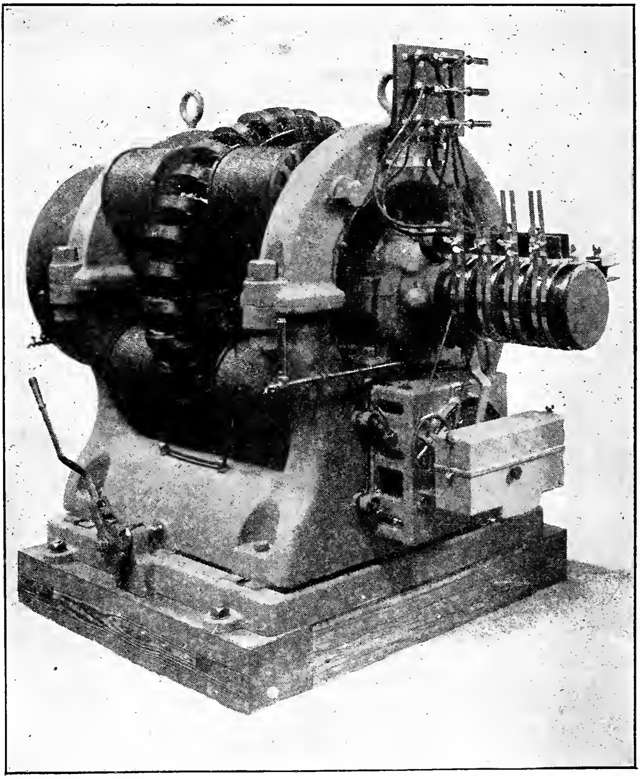


Fig. 116.

instead of the circumference. The individual bobbins are protected by an insulating box, but are not surrounded by any masses of metal. This fact, together with the fact that the armature is of such shape as to cause great air disturbances, insures exceptionally good ventilation of the armature. This machine is of relatively slow speed, the larger sizes running at only 500 rev. per min.

At a given instant of time, the different coils on the moving armature have *E.M.F.*'s of widely different magnitudes induced in them. The commutator, Fig. 117, is so

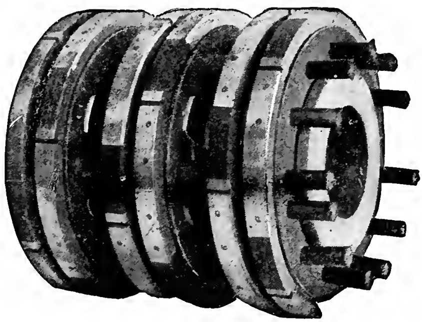


Fig. 117.

designed that it connects the coils of highest *E.M.F.* in series with one another to the external circuit, and connects the coils of medium *E.M.F.* in multiple with one another to the external circuit, while those of smallest *E.M.F.* are cut out entirely from the circuit.

The bearings are self-lubricated by means of rings. Since the poles are on the sides of the armature, side play in the bearings must be prevented. The commutator end of the shaft is turned with thrust collars which are engaged by corresponding annular recesses in the brasses.

Voltage regulation on these machines is effected by a variable resistance in shunt with the field coils ; and as the field current is changed the position of the brushes is also changed, not to collect current at a lower voltage, as described in § 85, but to obtain sparkless collection. These two operations are performed by a regulator, shown in Fig. 118, which is attached directly to the frame of the machine. The mechanism consists of a rotary oil-pump driven by a belt from the armature shaft, a balance valve of the piston

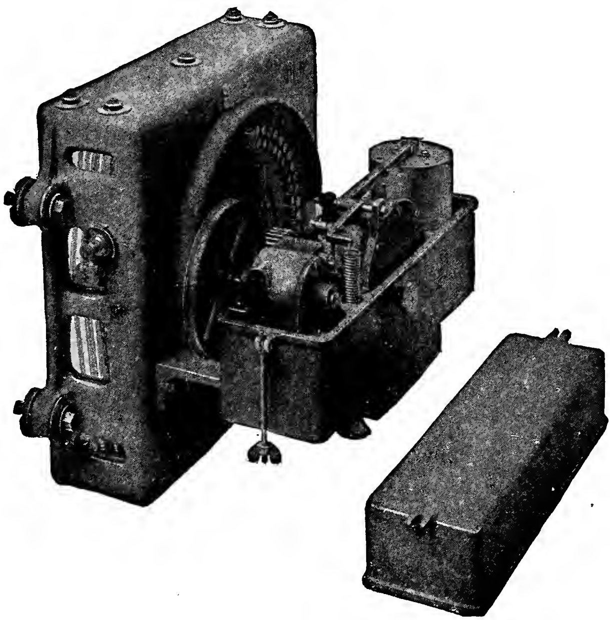


Fig. 118.

type, and a rotary piston in a short cylinder, which is directly connected to an arm moving over the contacts of the field-shunt rheostat. The valve is operated by a lever actuated by a controlling electro-magnet which is energized by the whole generator current. At normal current the valve is centrally placed, and the oil from the pump flows around the overlapping ports into the reservoir without effect. If the current rises above the normal, the armature of the controlling magnet is attracted, the balance valve moves up, and oil enters the cylinder, moving the rotary piston in a clockwise direction. The shaft of this piston moves the arm of the rheostat, cutting out resistance and thus lowering the field exciting current. At the same

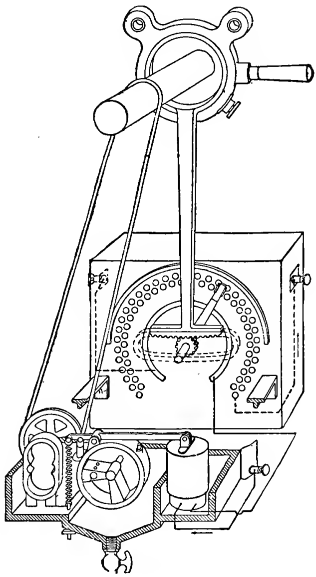


Fig. 119.

time a pinion on the shaft, seen in Fig. 119, actuates a rocker arm which moves the brush holders to a position such that the collection of current by the brushes will be sparkless. When the current returns to its normal value the adjusting spring returns the lever and balance valve to the central position. If the current falls below normal value, these operations are reversed. It is claimed for this regulator that it can bring the current back to normal from a complete short-circuit in from $3\frac{1}{2}$ to 4 seconds. The tension of the adjusting spring can be regulated from the outside of the dust-proof case by a hard rubber knob.

87. The Excelsior Arc-Light Generator. — This machine, Fig. 120, is a closed-coil ring-armature generator, having pole faces that cover both the sides and the circumference of the armature. The interesting feature of this machine is the method of regulation. The proper voltage is supplied to the line by using both methods of control in conjunction; that is, sections of the field windings are cut in or out of the circuit, and at the same time the position of the brushes is shifted. The proper motion of the field regulator arm and of the brush holder is obtained by means of a small motor whose field is "sneaked" from the main magnets of the machine. This motor is operated by a device shown in Fig. 121. The whole device is inserted in series with one of the mains from the generator. The right-hand lever is of insulating material, with the contact blocks *a* and *b* properly placed upon it. The left-hand lever is of conducting material, and is capable of being attracted by the electro-magnet which is excited by the main current.

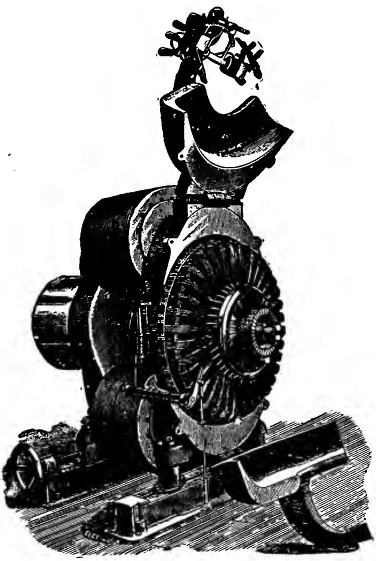


Fig. 120.

The magnet and spring are so adjusted that when the normal current is flowing, both a and b are in contact with the left lever, and the current flows in the three shunt paths, R, R_1, and R_2. There will be no current in the armature of the regulating motor, since the potential at brush x is

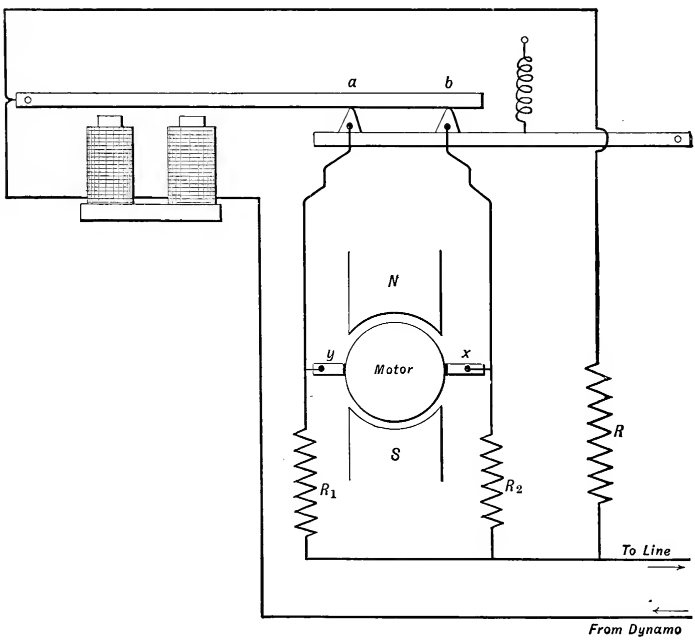


Fig. 121.

equal to the potential at brush y. If now the line current becomes too strong the magnet attracts the left lever to it and the contact at a is broken. Immediately the current flowing through b divides at the brush x, part going through R_2 and part through the motor armature and R_1. The motor will then revolve in a given direction, and by simple mechanical devices will cut out sections of the field windings, and will shift the brushes until the normal current is flowing, when contact is again made at *a* and the controlling motor stops. If the line current drops below normal, the spring pulls the lever away from the magnet and the contact at *b* is broken. Part of the current then flows from *y* to *x* through the motor armature. It therefore revolves in a direction opposite to that which it had before. The brushes on the dynamo are shifted back again, and more sections of field winding are put into circuit.

When the current is broken at *a* or *b*, there is no serious sparking, since there are always two circuits in shunt with the break. The whole current of the dynamo does not exceed ten amperes; and the resistances R, R_1, and R_2 are so proportioned that only a small portion of this current flows through *a* or *b*.

In practice the levers and the magnet are mounted on the wall or the switchboard, while the regulating motor is mounted on the dynamo frame.

88. The Thomson-Houston Dynamo. — The Thomson-Houston arc-light generator is of a type entirely different from the other machines here described, not only in appearance, but also in method of armature winding and of regulation. A view of this machine is given in Fig. 122. Each field coil has for its core an iron tube, flanged exteriorly at each end to form a recess for the windings, and fitted at the armature end with a concave iron piece that surrounds part of the armature. This tube, with the flanges and the cup-shaped end, is cast in one piece. The farthermost flange of each field core is bolted to a number of wrought-iron connecting-rods which hold the magnets in place, protect the field windings, and take the place of the yoke of other machines in completing the magnetic circuit. The magnets are mounted on a frame, including legs and bearing supports for the armature shaft.

The armatures of the older machines of this type are spheroidal in shape, while the more recent ones have ring

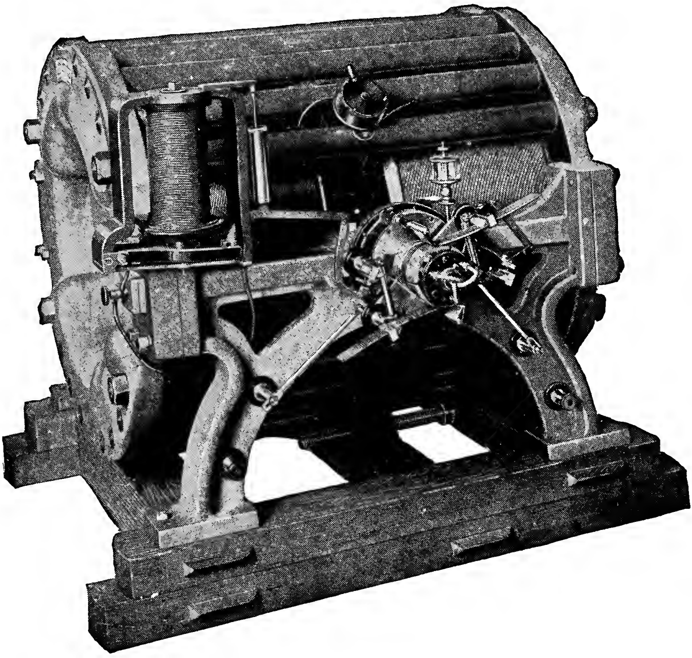


Fig. 122.

armatures; these are more readily repaired or rewound. The winding of either form of armature is peculiar in that only three coils are employed, set with an angular displacement from one another of 120 degrees. The inner ends of the three coils are joined to each other, and are not attached to any other conductor, an arrangement unique in direct-current dynamos. The outside ends are connected to the segments of a three-bar commutator, from which the current is collected by four copper brushes connected in multiple.

Regulation is obtained by shifting the brushes in the following manner. Fig. 123 shows the two possible relations between brushes and commutator that may exist at any instant. Both brushes of each set may rest on one commutator bar, or the brushes of one set may span the gap between the other two bars. These conditions are repeated three times at each brush for each revolution. If

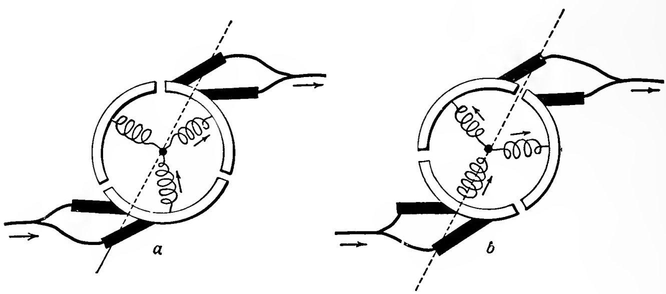


Fig. 123.

the dotted line shows the position where the maximum *E.M.F.* is generated in the coils, then in Fig. 123 *a* the two most active coils are connected in series with the outside circuit, while the coil near the position of least activity is out of circuit. In Fig. 123 *b* the two less active coils are in multiple with themselves and in series with the most active coil and the external circuit. In practice the brushes of a set are 60 degrees apart, leaving 120 degrees between the leading brush of one set and the following brush of the other set; and since 120 degrees is the angular measure of the length of a commutator bar, there is no coil out of circuit at normal load, two being always in parallel and in series with the third. If the current rise above the normal the leading brushes move a small angle forward, while the following brushes recede through three times that angle. This will shorten the time that a single coil gives its whole *E.M.F.* to the circuit, and will place it more quickly in parallel with a comparatively inactive coil. But such a movement will reduce the angular distance between the nearest brushes of the opposite sets to less than 120 degrees, hence the machine will be short-circuited six times per revolution, since one brush of each set will touch one segment of the commutator at the same time. If the current in the line falls below normal, then the brushes close together, and the time that a coil is in series is lengthened, and the time that it is in parallel with an inactive one is lessened.

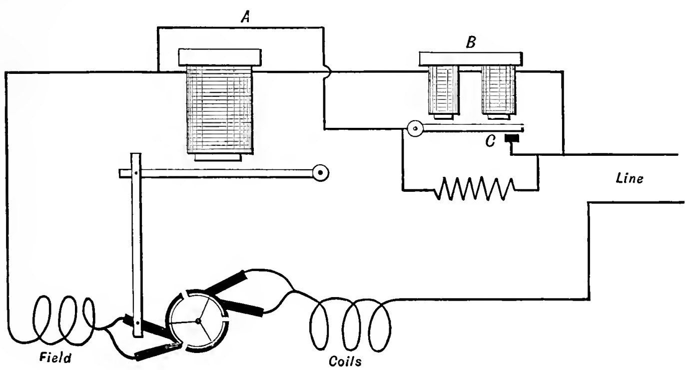


Fig. 124.

The arrangement for moving the brushes is shown in Fig. 124. The leading brushes are shifted forward on an increase of current merely to help avoid sparking. The brushes are moved by levers actuated by a series magnet *A*. This magnet is normally short-circuited by the by-pass circuit. On an undue rise of current this circuit is broken by the series magnet *B*. *A* then becomes more powerful, and the levers separate the brushes. While the machine is in operation the circuit-breaker *C* is constantly vibrating, the brushes adjusting to suit the load. A high carbon resistance is shunted across *C* to prevent sparking at that point.

As might be expected, with but three parts to the commutator and collection made with small regard to the neutral point, the sparking of this machine is such as speedily to ruin the commutator and the brushes, if means are not taken to suppress it. A rotary blower is mounted on the shaft, and is arranged to give intermittent puffs of air, which at the right moment blow out the spark. The insulation between the segments is air, considerable gap being left between them, and through these gaps the sparks are blown.

89. Western Electric Arc-Light Dynamo. — Fig. 125 represents a Western Electric Company generator, which is regulated by means of shifting the brushes. The brush and rocker are connected by means of a link and a ball-and-socket joint with a long screw, the latter being held in position by a nut. When the current is normal, both the nut and screw revolve at the same rate, and consequently there is no axial movement of the screw, and the brush, therefore, remains stationary. An electro-magnet, energized by a coil which is in series with the main circuit, attracts an armature whose movement toward the magnet is opposed by the action of a spring which is susceptible of adjustment. When the current has too high a value, the electro-magnet attracts its armature more strongly than ordinarily. The latter moves toward the magnet, and by its movement catches a stop on the revolving nut, and thereby prevents the revolution of the nut until the resulting longitudinal movement of the screw has shifted the brushes sufficiently

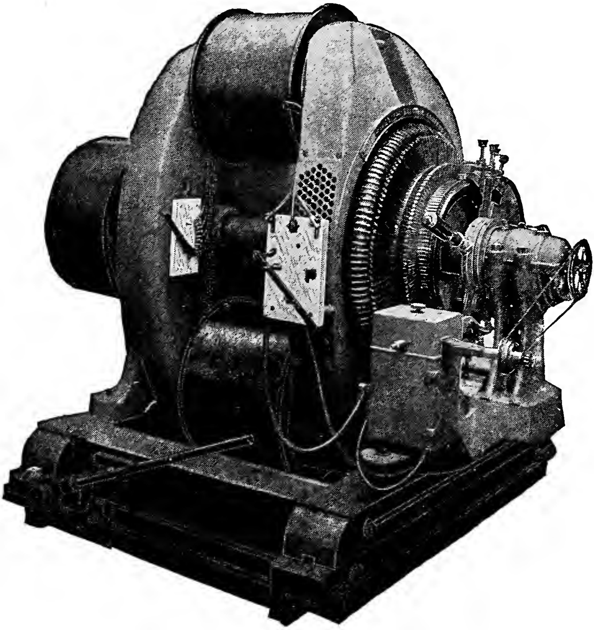


Fig. 125.

to bring the current to its normal value. If the current be too weak, the spring which is attached to the electro-magnet armature verpowers the magnetic attraction. The resulting movement of the armature stops the rotation of the screw and permits the rotation of the nut. This results in a longitudinal movement of the screw and a shifting of the

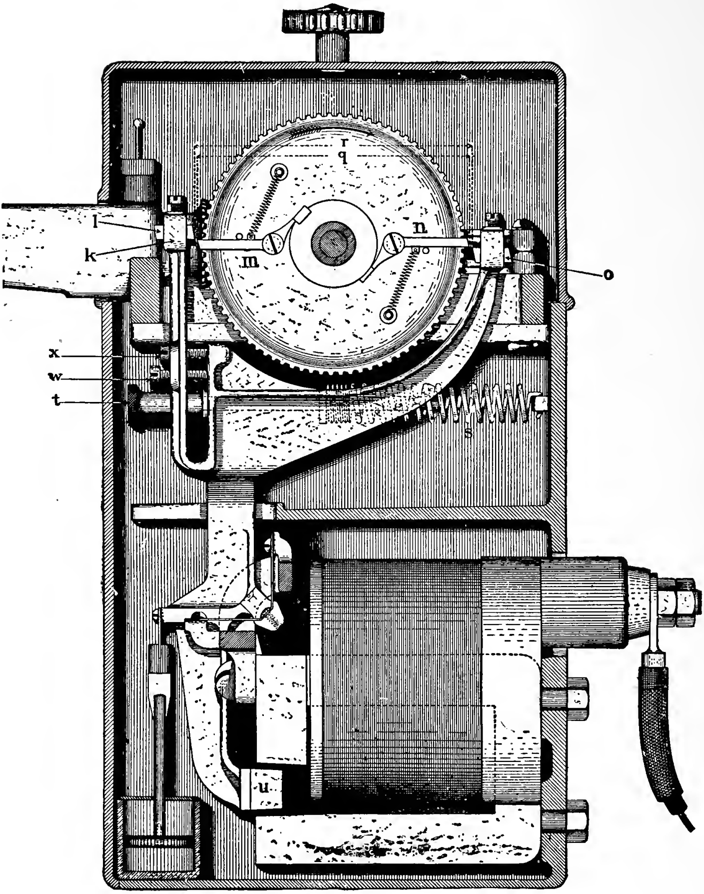


Fig. 126.

brushes in the opposite direction. The stopping and starting of the nut and screw are accomplished through the medium of small triggers controlled by the armature of the series magnet. The triggers are fastened to the gear rotated from the main shaft by a belt, and they engage with stops on the nut and screw respectively. Fig. 126 gives a sectional view of the regulator. The trigger which engages with the screw is shown at *n*, and the one which engages with the nut is shown at *m*.

PROBLEMS.

1. The resistance of the field winding of a generator which has been standing idle for a considerable time in an engine-room at a temperature of 25° C., is 22.1 ohms. The resistance of this winding when the generator is in operation at full load for several hours is 25 ohms. Determine the temperature elevation of the field coils.

2. Estimate the output of a generator the armature core of which is 18 inches in diameter and 13 inches in length; the speed of the machine being 500 rev. per min.

3. From the following data of a 350-K.W., 250-volt, 90 rev. per min., 16-pole, shunt-wound generator, determine the armature core losses:

Armature diameter	108 inches
Gross length of armature	15 inches
Net length of armature	12 inches
Number of slots (open type)	576
Depth of slot	1.0 inch
Width of slot	0.3 inch
Radial core depth back of slots	7 inches
Conductors per slot	2
Size of armature conductors	0.3 × 0.2 inch
Flux per pole at full load	16 megamaxwells

Type of armature winding	simplex; lap
Peripheral length of pole face	15 inches
Radial length of air-gap	0.3 inch
Number of field turns per pole	600
Mean length of a field turn	59 inches
Cross-section of field conductor	0.03 sq. in.
Drop at the carbon brushes	2.4 volts
Current density at brushes	40 amperes per sq. in.
Diameter of commutator	72 inches
Axial length of commutator	9 inches
Exposed surface per field coil	1250 sq. in.

Field pole shoes are of laminated steel.

4. What is the armature copper loss at full load of the generator of the foregoing problem?

5. Compute the pole-face loss of the 350-K.W. generator, the data of which are given under Prob. 3.

6. Calculate the excitation loss of the generator of Prob. 3 at full load.

7. Find the total commutator losses at full load of the 350-K.W. generator of Prob. 3.

8. Determine the temperature elevations of the armature, field coils, and commutator of the generator discussed in the foregoing problems, when operating continuously at full load.

9. A motor-generator set consists of a direct-current generator coupled to an alternating-current synchronous motor [power-factor = 1]. When the generator delivers a current of 600 amperes with 250 volts across the machine terminals, the motor takes 26.4 amperes at 6600 volts. Determine the efficiency of the motor-generator set.

10. The *E.M.F.* of a shunt-wound railway generator rises from 550 volts at full load to 645 volts upon disconnecting the load. What is the percentage regulation of the machine?

11. A shunt-wound generator, rated at 50 K.W., supplies current to an external circuit with 550 volts across the machine

terminals. To produce this voltage 6280 ampere-turns per pole are required at no load, and 7640 ampere-turns at full load. How many series field turns per pole must be provided for flat compounding?

12. A three-wire system supplies current to 110-volt lamps and to a 220-volt motor which is connected to the outside wires. It is found that the lamps on one side of the system burn more brightly than those on the other, while the motor operates as usual. What is the trouble?

13. Find the flux density in the air-gap of the 300-K.W., 500-volt homopolar generator mentioned in § 82; the armature diameter being taken as 20 inches and its net axial length as 12 inches.

14. When a series-wound generator is driven at 1200 rev. per min. its terminal voltage is 150 volts, with a current output of 20 amperes. Compute the terminal voltage of the machine when it delivers a current of 50 amperes at a speed of 1500 rev. per min.; the resistance of armature and field windings together being 0.125 ohm. The increase in magnetic flux accompanying the increased current is 60 %.

CHAPTER VII.

MOTORS.

90. Principle of Action of a Motor. — Any direct-current generator will operate as a motor and deliver mechanical energy if supplied with current from some external source. This source may be a constant-potential system or a constant-current system of electrical distribution. Structurally generators and motors are identical, but as motors are generally placed as near as possible to their loads, they may frequently be exposed to severe weather conditions, dirt, etc., and for this reason motors for electric railways, for rolling mills and for machine tools are of the enclosed type.

When a current flows through a conductor which is situated in a magnetic field, a force will be exerted upon that conductor tending to move it perpendicularly to itself and to the magnetic flux; the magnitude of this force in dynes being

$$F = \frac{Il\mathcal{B}}{10}, \quad (\S\ 22)$$

where I = current flowing in conductor in amperes,

l = length of conductor in centimeters,

and $\mathcal{B}$ = flux density of magnetic field in gausses.

Irrespective of the multipolarity of the field magnets or of the method of armature winding, the force actions between the magnetic field and all the currents in the inductors will conspire to produce rotation in one direction.

Consider a single armature conductor, Fig. 127, to carry a current flowing away from the observer. The lines of force which surround the conductor due to the current in it will have a clockwise direction. Thus, to the right of the conductor these lines will have the same direction as the lines of force from the field magnet N, and to the left of the conductor they will be opposed to the latter.

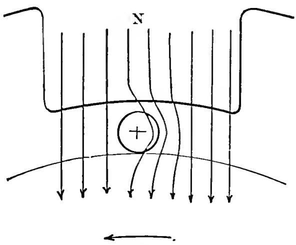


Fig. 127.

The resultant field, therefore, will be stronger on the right-hand side, as shown; and consequently the armature carrying that conductor will be pushed to the left and will rotate counter-clockwise.

91. Direction of Rotation. — To determine the direction of movement of a conductor carrying a current of definite direction in a magnetic field of known direction, one may

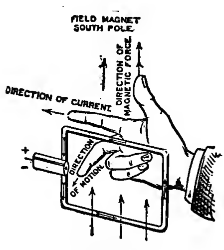


Fig. 128.

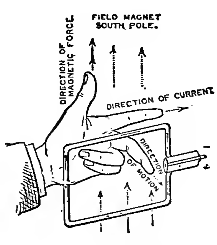


Fig. 129.

employ a modification of Fleming's rule. Thus in a generator the thumb and first two fingers of the *right* hand determine the direction of the induced $E.M.F.$ as shown in Fig. 128. But in a motor the thumb and first two fingers of the *left* hand may determine the direction of rotation as shown in Fig. 129.

If in a dynamo the direction of the field flux remain unaltered, and the armature be supplied with a current flowing in the same direction as when the machine was operated as a generator, then the direction of rotation will be opposite to that while driven as a generator. Thus, if the positive brush of the generator be connected to the positive terminal of an external source of supply, and if the negative brush be connected to the negative terminal, then the direction of current flow in the armature will be reversed. The connections of shunt-wound and series-

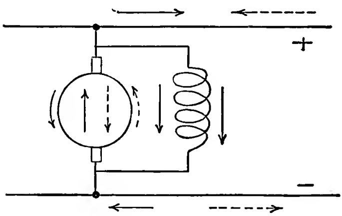


Fig. 130.

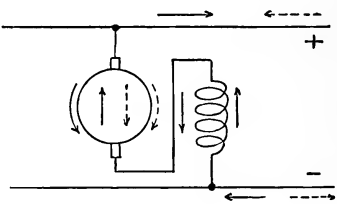


Fig. 131.

wound dynamos are shown respectively in Figs. 130 and 131, in which the full arrows represent generator conditions and the dotted arrows represent motor conditions, the connections to the circuit remaining unchanged. In shunt-wound, separately excited and magneto machines, since the magnetic fields in these dynamos are not reversed, the direction of rotation will be *unaltered*. The direction of rotation of the armature of series-wound dynamos, since the field flux also has its direction changed, will be *reversed*. Compound-wound machines will have the *same* or *reversed* direction of rotation, depending upon whether the magnetizing effect of the shunt coils is stronger or weaker than that of the series coils. In a compound-wound generator the actions of the shunt coils and the series coils are cumulative, i.e., in the same direction ; but when used as a motor the actions are differential, i.e., opposed to each other. Motors are also wound so as to have cumulatively acting series coils.

To reverse the direction of rotation of a motor one must not change the connections with the supply mains, for this would reverse the current directions in both armature and field windings, and thus leave the direction of rotation unaltered. It is necessary to change the connections of either field or armature winding, but not of both.

92. Torque Exerted by a Motor. — The force which is exerted upon each conductor carrying a current I and situated in a uniform magnetic field of flux density $\mathcal{B}$ is $\frac{Il\mathcal{B}}{10}$ dynes, § 90. The total number of conductors on the armature which are under the $2p$ poles may be represented as

$$kSq,$$

where k is the ratio of the circumferential length of the pole face to the pole pitch, S is the number of conductors in series between brushes, and q is the number of current paths through the armature between brushes. Let I_t be the total or external armature current in amperes, and D be the diameter of the armature in centimeters. Then the total torque exerted by the armature in dyne-cm. is

$$T = \frac{D}{2} \cdot \frac{I_t}{q} \cdot l\mathcal{B} \cdot \frac{kSq}{10} = 0.05\ kDl\mathcal{B}SI_t.$$

But the total flux per pole is

$$\Phi = \frac{k\pi Dl\mathcal{B}}{2p}.$$

Therefore the total torque in dyne-cm. is

$$T = \frac{2}{\pi} .05\, pS\Phi I_t = \frac{p}{10\pi} S\Phi I_t,$$

which shows that the torque exerted by the motor is proportional to the magnetic flux and to the armature current. Since there are $980 \times 453.6 \times 30.48$ or 13,549,000 dyne-centimeters in one pound-foot, the torque in pound-feet may be expressed as

$$T = 2.35\, pS\Phi I_t\, 10^{-9}.$$

The effective torque available at the pulley of the motor is somewhat less than that given by the foregoing equation, due to the mechanical and iron losses.

When load is placed upon a motor, such as machinery in one form or another, a certain torque must be exerted which is equal to the torque-reaction of the load. With greater load more torque must be exerted, and therefore the product ΦI_t must become larger. As a result a motor takes more current when operating under heavy load than when running light.

93. Counter Electromotive Force. — The armature of a motor revolving in a magnetic field under the influence of supplied electrical energy differs in no respect from the same armature revolving in a magnetic field under the influence of supplied mechanical energy. There is an *E.M.F.* generated in it which is determined by the speed and quantity of flux. For the same speed and the same flux there would be generated the same *E.M.F.* in the case of a motor as in the case of a generator. The direction of this *E.M.F.* is, however, such as to tend to send a current in a direction opposite to that of the current flowing under the influence of the external supply of *E.M.F.*, according to § 91.

Therefore this pressure which is induced in the armature of a motor is called *counter electromotive force.* The current which will flow through the inductors of an armature is therefore equal to the difference between the supply *E.M.F.* and the counter *E.M.F.* divided by the resistance of the armature, or

$$I_a = \frac{E - E_c}{R_a}.$$

For example, an unloaded 1-K.W. shunt-wound motor having an armature resistance of 1 ohm, when connected to a source of constant-potential supply of 100 volts, would not take a current of 100 amperes as dictated by Ohm's law, unless its armature were clamped so as to prevent rotation. If unclamped, its armature would assume such a speed that it would have induced in it a counter *E.M.F.* of say 97.5 volts. The current then flowing in the armature would be

$$\frac{100 - 97.5}{1} = 2.5 \text{ amperes.}$$

The power represented by this current, viz., 2.5×100 watts, would all be expended in overcoming the losses of the machine.

The magnitude of this counter *E.M.F.* in volts is

$$E_c = 2\, p\Phi S \frac{V}{60} 10^{-8}, \ (\S\ 39)$$

where Φ is the flux per pole in maxwells, and V is the speed in rev. per min.

If the load upon a motor be increased, its torque is no longer sufficient to overcome the load and consequently its speed drops. A lowering of speed implies the generation of a lower counter *E.M.F.*, and thus permits a greater current to flow through the armature. The greater current results in a greater torque.

94. Armature Reactions. — Since in a motor, for a given direction of rotation and of flux, the current in the armature flows in a direction contrary to that which it would have as a generator, therefore the effect of the motor armature cross turns is to distort the magnetic field against the direction of rotation, as in Fig. 132. This increases the flux density in the leading pole tips, and decreases it in the trailing tips. This necessitates, for sparkless operation, a *backward lead*, or a *lag*, of the brushes. If the brushes were in the same place as when the machine was operated as a generator, the direction of armature current having been reversed, then the demagnetizing or back turns of the generator would become magnetizing turns for the motor; but with the brushes shifted to a position of lag, then the motor has also demagnetizing or back turns.

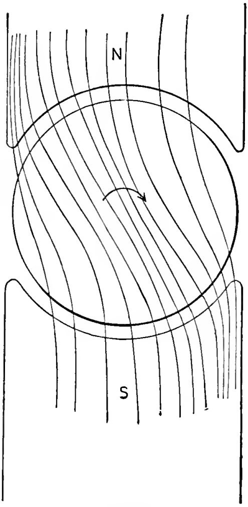


Fig. 132.

95. Power of Motors. — The mechanical power of a motor when running at V rev. per min. and exerting a torque T dyne-centimeters is equal to the product of the angular velocity in radians per second into the torque

$$P = \omega T = \frac{2\pi V}{60} T.$$

But the torque exerted, in dyne-centimeters, is

$$T = \frac{p}{10\pi} S \Phi I_t. \qquad (\S\ 92)$$

Therefore $$P = \left[2\,p\Phi S\frac{V}{60}\right]\frac{I_t}{10}.$$

But the quantity in the brackets is equal to $10^8\,E_c$. (§ 93) Whence

$$P = E_c I_t\,10^7\,\frac{\text{dyne-cm.}}{\text{sec.}} = E_c I_t \text{ watts.}$$

Thus, the rate at which a motor does mechanical work is equal to the product of the counter electromotive force generated, in volts, into the total current flowing through the armature in amperes.

Shunt Motors.

96. Speed of Shunt Motors. — In shunt-wound motors connected to constant-potential supply circuits the field current is constant and consequently the magnetic field is of unvarying intensity. Solving the equations of § 93 for speed, there results

$$V = \frac{(E - I_a R_a)\,60 \cdot 10^8}{2\,p\Phi S},$$

and therefore if Φ is constant the speed of the motor will be practically constant. It will not be absolutely constant because the small resistance drop occasions a slight lowering of speed with increased load on the machine. On the other hand the effect of the armature current is to weaken the magnetic field, if the brushes be displaced backward from the neutral plane, and thereby tend to increase the speed. This partially counteracts the lowering of speed due to resistance drop. The speed variation of shunt motors from no load to full load ranges from 2 to 10 per cent of the speed at no load, the lower value representing that for large machines.

An inspection of the foregoing equation suggests the following possible ways of controlling the speed of a motor: (1) changing the exciting current in order to change the magnetic flux passing through the armature, (2) changing the resistance of the armature circuit, and (3) changing the impressed electromotive force. A slight change of speed can be effected by shifting the position of the brushes, for at a given load the speed is a minimum with the brushes in the neutral plane, and it will be increased by a lag of the brushes; commutation difficulties limit the speed variation by this method.

(1) A rheostat placed in the field circuit of a shunt motor may be used to vary its speed, Fig. 133. By increasing the amount of resistance in this rheostat the current in the field coils will be decreased; this results in a weaker magnetic field, and consequently the motor will run at a higher speed. If the iron of the magnetic circuit is well saturated, a considerable change in resistance is necessary materially to alter the field intensity. A large exciting current is then required to increase the magnetic flux, and this may occasion excessive heating of the field coils. Again, if an attempt be made to reduce the flux to a considerable extent by introducing more resistance to obtain a high speed, the demagnetizing effect of the armature current will be greater upon the weakened magnetic field, and consequently serious sparking will result. Thus

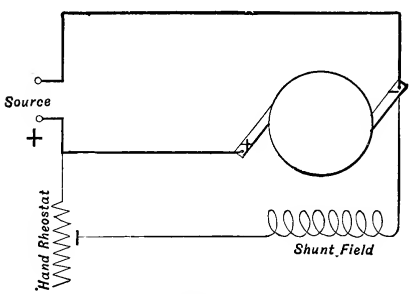


Fig. 133.

armature reaction limits speed variation. A shunt motor of the usual type which operates at a speed of V rev. per min. when the iron of its magnetic circuit is near saturation will operate satisfactorily at any speed up to say 2 V revolutions per minute. Field rheostats are described in § 77.

In order to vary the speed of a shunt motor over a wide range by this method it is necessary to neutralize the effect of armature reaction. This neutralization is accomplished by the provision of a reversing magnetic field obtained by the insertion of auxiliary poles, called *commutating-poles* or *inter-poles*, between the field-magnet poles. The coils on these auxiliary poles are connected in series with the armature, as shown in Fig. 134, and therefore the magnetic

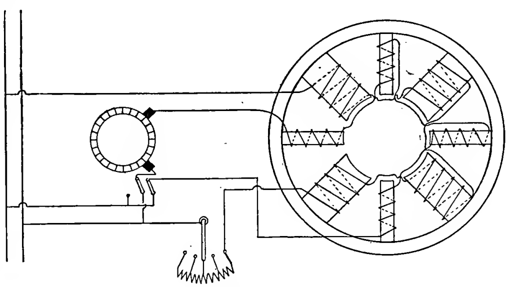


Fig. 134.

flux from them is practically proportional to the armature current. The reactance voltage (§ 57) generated in a short-circuited armature coil due to its rotation in the main magnetic field is also proportional to the current flowing in it. The $M.M.F.$ of the interpoles is adjusted so that a magnetic field is produced in the commutating zone of such magnitude that an $E.M.F.$ is generated in the short-circuited coils by their rotation which is equal but opposite to the reactance voltage. The action of the interpoles is therefore entirely automatic and enables sparkless commutation at all loads and speeds. Interpole motors are particularly adapted for individual motor drive of machine tools and for elevator operation, where large speed variations are essential. The Electro-Dynamic Company manufactures such motors, Fig.

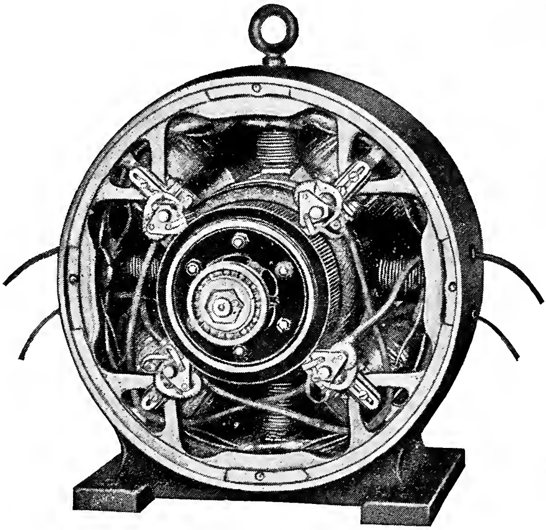


Fig. 135.

135, which operate at a speed of from 100 % to 600 % of the minimum speed.

The data of a 5-H.P. interpole motor follow:

Resistance of shunt field	175 ohms,
Resistance of armature	1.18 ohms,
Resistance of interpolar windings	0.21 ohm,
Armature current at full load	22 to 24 amperes,
Field current	0.15 to 1.26 amperes,
Speed	200 to 1200 rev. per min.,
Weight	1200 pounds.

A change in magnetic flux can also be accomplished by varying the reluctance of the magnetic circuit, the field current remaining unaltered. The reluctance may be increased by lengthening the air-gap of the motor; this decreases the flux and consequently produces a higher speed. A variable-speed motor depending upon change of reluctance for speed control is shown in Fig. 136,

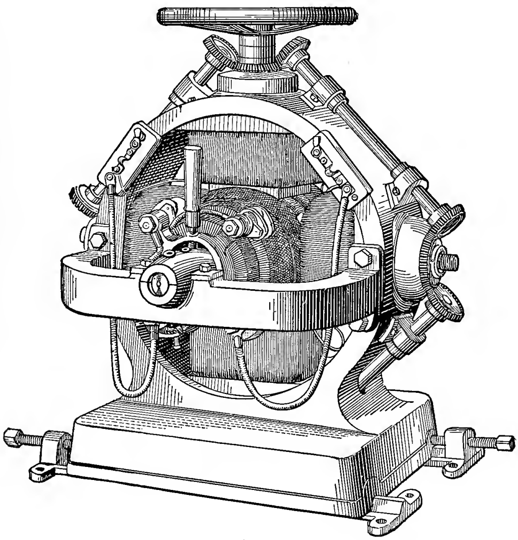


Fig. 136.

which depicts a 4-pole machine of the Stow Manufacturing Company. The field cores are hollow and are provided with movable iron poles, the positions of which are simultaneously shifted by means of hand wheel and gears. Large air-gaps are conducive to sparkless commutation.

(2) The speed of a shunt-wound motor with constant excitation may be varied by introducing a variable resistance

in the armature circuit. The use of this method of speed control is not to be advised save for experimental purposes, since it is very wasteful of energy. The I^2R loss in the regulating resistance at certain speeds is considerably more than the power required by the motor. Further, the speed, when reduced in this way, changes very considerably when the load on the motor is altered.

(3) Changing the electromotive force impressed upon the armature of a shunt motor will cause a corresponding change in speed. Speed control by this method may be accomplished by subdividing the generator voltage into two or more components, and by supplying current at these different voltages over a number of line wires to the motor. A controller is provided by means of which the motor armature may be connected to any pair of supply mains, the field winding of the machine being always connected to a definite pair of them. The main generator voltage is subdivided by a set of generators, called a *balancer;* Fig. 181 illustrates a three-element balancer for a 4-wire multivoltage distribution system. The connections of a motor to such a system through a controller are shown in Fig. 137. Six different voltages are obtainable, namely, 40, 80, 120, 160, 200 and 240 volts, by moving the controller handle. The motor speeds under these voltages are approximately proportional to the voltages themselves, so that this method of speed control gives a number of definite and widely different speeds. Intermediate speeds may be obtained by weakening the magnetic fields of the motors using field rheostats as described. Controllers designed to perform both of these functions are also employed. This system is extensively used in machine shops for driving lathes, planers, and similar machines.

Speed control of shunt motors by varying the voltage impressed upon the armature is also the principle of the Ward Leonard system, which differs from the foregoing multivoltage system in that a finely graded variation of speed is made possible by the employment of a separate generator which supplies current to the motor. This generator, *G*, Fig. 138, is driven at constant speed by any type

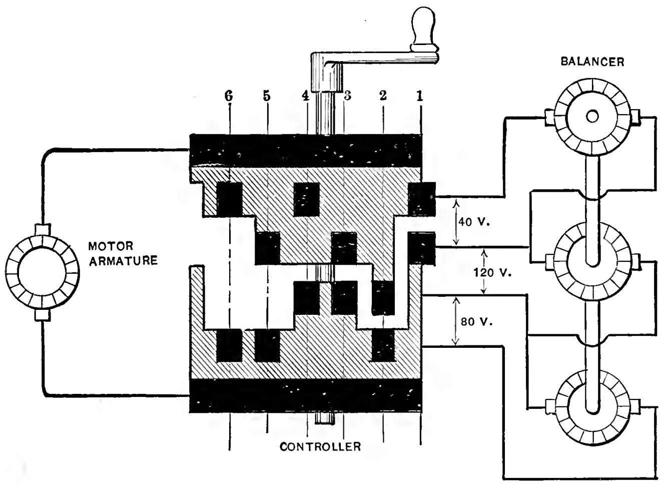


Fig. 137.

of prime mover, *S*, or by an auxiliary shunt motor which takes current from the supply mains. An adjustable resistance in the generator field circuit regulates the voltage which is impressed upon the motor armature, *M*, from practically zero to its maximum value. The field winding of the motor is connected directly to the supply circuit, so that the intensity of its magnetic field is constant.

When it is desired to start the motor, the rheostat is adjusted so that a high resistance is in circuit with the field winding of the generator; thus current at low voltage will be supplied to the motor. The latter then starts to revolve slowly. To accelerate the motor, more resistance is cut out of the generator field circuit and consequently the voltage across the motor armature terminals increases, thus resulting in higher speed.

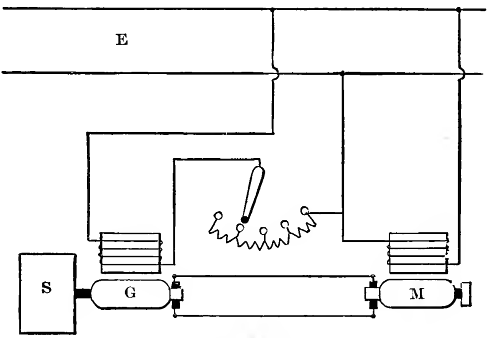


Fig. 138.

This system of motor control is especially advocated for operating guns and turrets on battleships, where thorough control is essential. For the latter, the field rheostats are designed to yield seventy or more speeds, the maximum speed being usually 100 degrees per minute. The turret-turning motors are rated at 25 and 15 H.P. respectively for 12-inch and 8-inch turrets. The gun-elevating motors are rated at 8 and 5 H.P. for 12-inch and 8-inch guns respectively.

Of the various methods of speed control just described, the field rheostat method is perhaps the most used. It is simple, cheap, and enables the speed to be kept at definite value under changes of load. Its range is limited in shunt motors of the usual type, but the interpole motor removes this difficulty. The change of reluctance method does not require a field rheostat, but this is offset by the increased cost of construction of such motors. Both the multivoltage and Ward Leonard systems are very practical but expensive, since the former requires a balancer, a number of live wires, and individual controllers; and the latter system requires a motor-generator set and rheostat for each motor.

97. Starting of Shunt Motors. When the armature of a motor is at rest there is no counter *E.M.F.*; and at the instant of closing the circuit a destructive current would flow if a resistance were not first inserted in the circuit, except in the case of very small motors whose armatures have small moments of inertia. As the speed rises the counter electromotive force increases and the current is reduced, thus permitting the resistance to be gradually lessened without causing an excessive current to flow through the armature. When the speed approaches its ultimate value this resistance is entirely cut out of circuit. In order that the counter *E.M.F.* may be generated the shunt field circuit must be closed, so that the armature conductors cut lines of force. An arrangement for conveniently performing these functions is called a *starting box* or *starting rheostat.*

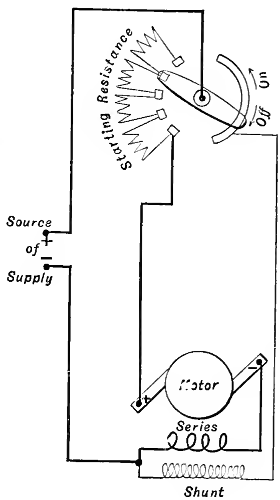


Fig. 139.

The connections of a simple starting rheostat are shown in Fig. 139. Its main feature is a contact arm capable of rotation about its center so that one end moves over a series of contact studs while the other makes contact with the segment which is connected to the field winding. As the arm is slowly turned around, it first completes the field circuit, then the other end touches the first contact stud, thereby closing the armature circuit through all the resistance of the rheostat. As the speed increases the revolving arm cuts out more and more of the resistance, until finally the armature is operating on the full voltage of the supply circuit.

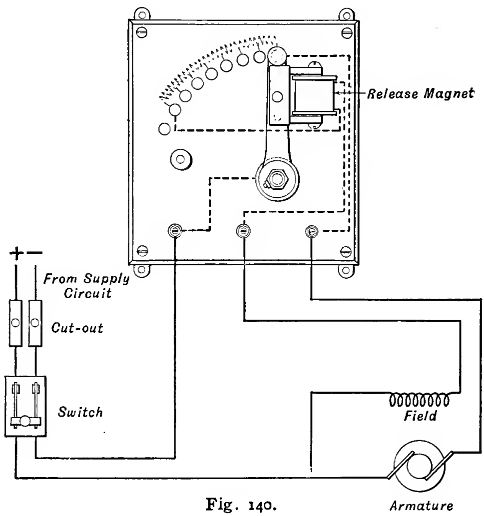


Fig. 140.

A shunt motor may have its armature coils destroyed by an excessive rush of current resulting from a dropping or interruption of the supply voltage followed by a sudden renewal after the speed of the armature has fallen. These conditions may arise through accidents to supply mains or because of an extremely heavy load on mains of insufficient cross-section. An armature may also be burned out by an excessive current due to overloading the motor. The resulting lowering of its speed is accompanied by a corresponding lowering of the counter *E.M.F.* Again, an abnormal voltage, which might result from some cross or other accident, might cause a destructive rush of current. To meet these conditions, starting rheostats are often provided with attachments for opening the circuit on *no voltage* or low voltage, and others with attachments for opening the circuit on *overload.* Some have both attachments, but it is modern practice to place the overload device upon the switchboard rather than on the starting rheostat. Fig. 140 is a wiring diagram of a shunt-motor starting box with automatic release and no-voltage attachment.

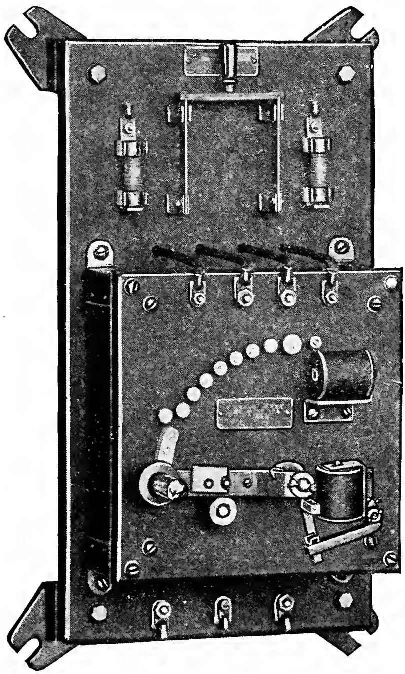


Fig. 141.

A view of a motor-starting panel with both no-voltage and overload attachments is given in Fig. 141. When the handle is placed in the "on" position, the magnet in the field circuit holds it there, although a spring tends to throw it back. If now, because of low voltage, the current in field winding and magnet becomes low, the magnet is no longer able to retain the handle, and the spring throws it to the "off" position, where it stays until the motor is again turned on by an attendant. The overload coil is connected in series with the motor armature, and on overload becomes strong enough to attract an iron piece. This operation places a short-circuit on the release magnet, which therefore permits the starting arm to spring back to the "off" position. This panel is provided with a main switch and enclosed fuses, although the latter are frequently replaced by circuit breakers. Self-contained motor-starting panels avoid considerable external wiring, thereby increasing reliability, are quickly installed, and add to the neatness of the equipment.

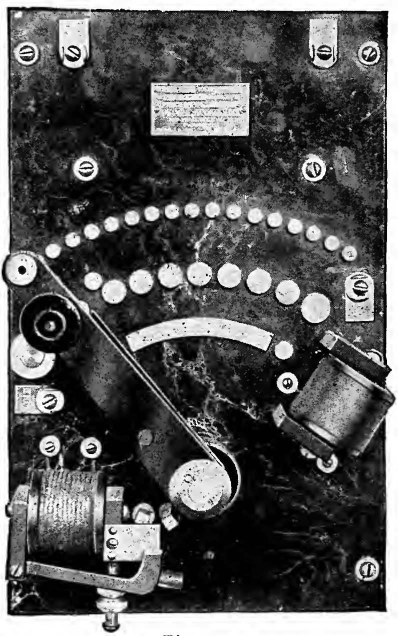


Fig. 142.

A combined starting and field-regulating rheostat made by the Cutler-Hammer Manufacturing Company is shown in Fig. 142. This type of apparatus is designed for 2 to 1 up to 5 to 1 speed variation. The movable arm consists of two parts which wipe over separate sets of contacts. To start the motor the handle is moved to the extreme right, in which position the magnet will hold the lower portion of the arm. The upper arm is then free to move back and

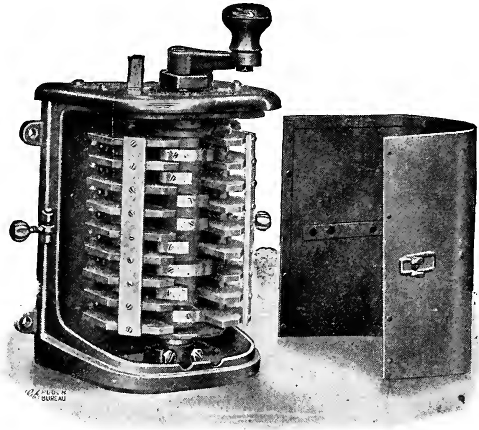


Fig. 143.

make contact with the studs joined to the field-regulating resistance.

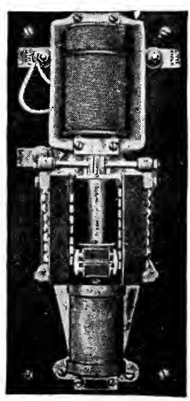


Fig. 144.

Fig. 143 depicts a General Electric Company controller for 5-H.P. shop-tool motors. There are three starting points, 21 forward and 11 reverse running points. Speed control is effected by field regulation.

A self-starter for shunt motors made by the Ward Leonard Electric Company is shown in Fig. 144. It consists of an electromagnet with a movable core carrying wipers which make contact with a series of studs as the core is attracted by the magnet. The rapidity with which this operation may be performed is controlled by a dash-pot. Thus the starting and stopping of the motor is accomplished simply and effectively by means of a main line switch.

98. Design of Starting Rheostats. — The design of a starting-box for a shunt motor under constant excitation is governed by the permissible starting current through the motor armature. This maximum current value is usually specified in terms of the full-load current of the machine. Let γ be the ratio of maximum starting current under load to the full-load current; this ratio is always greater than unity. Let $r_1, r_2, r_3, \ldots, r$ be the resistances respectively of the starting-box when the rheostat arm is on contact

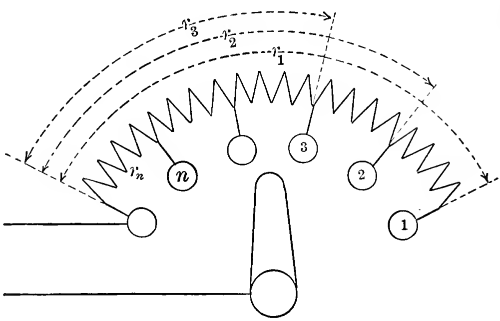


Fig. 145.

studs 1, 2, 3, . . . , n, Fig. 145. At the instant when the arm touches stud 1, the current flowing through the armature is

$$\gamma I = \frac{E}{r_1 + R_a}, \tag{1}$$

where E is the line voltage and R_a is the resistance of the motor armature. When the motor runs at constant speed with this rheostat setting, the current flowing through the armature is

$$I = \frac{E - E_{c1}}{r_1 + R_a}, \tag{2}$$

where E_{c_1} is the counter *E.M.F.* generated at this speed. The rheostat arm may then be turned to contact stud 2, and this results in a momentary increase of current,

$$\gamma I = \frac{E - E_{c1}}{r_2 + R_a}. \tag{3}$$

This causes the motor to exert a greater torque than that necessary to overcome the load and consequently the motor is accelerated and will assume some higher speed. The current will then diminish to

$$I = \frac{E - E_{c_2}}{r_2 + R_a}, \tag{4}$$

where E_{c_2} is the *E.M.F.* generated at the increased speed. Similarly, when the arm makes contact with stud 3, the current flowing through the motor armature will again increase to

$$\gamma I = \frac{E - E_{c2}}{r_3 + R_a}. \tag{5}$$

From equations (2) and (3) and equations (4) and (5) there result respectively

$$\gamma = \frac{r_1 + R_a}{r_2 + R_a} \quad \text{and} \quad \gamma = \frac{r_2 + R_a}{r_3 + R_a}. \tag{6}$$

There are n such equations, the last one being

$$\gamma = \frac{r_n + R_a}{R_a}.$$

The number of steps into which the total resistance r_1 is to be divided, so that the starting current shall not exceed the specified value of γI amperes, may be determined from the product of these n equations, which is

$$\gamma^n = \frac{r_1 + R_a}{R_a}\,.$$

But from (1)

$$r_1 = \frac{E}{\gamma I} - R_a\,; \tag{7}$$

consequently

$$n = \frac{\log \dfrac{E}{\gamma I R_a}}{\log \gamma}\,.$$

If the motor is to be accelerated from rest to full speed without any load on it, fewer steps are required, because the no-load current is much less than the full-load current.

The resistance of each of the various steps may then be computed; thus for the first portion between studs 1 and 2 the resistance is, from (6),

$$r_1 - r_2 = (R_a + r_1)\left(1 - \frac{1}{\gamma}\right),$$

and similarly the resistance of the next part is

$$r_2 - r_3 = (R_a + r_2)\left(1 - \frac{1}{\gamma}\right), \text{ etc.}$$

The resistances of the various steps of a starting rheostat will be found in examples to differ from one another. Sometimes additional steps are provided, so that the maximum permissible current will not flow through the armature when the rheostat arm touches the first stud, but only when contact is made with the second or third stud.

99. Speed Regulation. — A shunt-wound motor under constant impressed terminal voltage will have an approximately constant speed. It will decrease somewhat as the load on the machine increases. The principal cause of this speed variation under varying load is the change of the armature resistance drop, and it is therefore desirable that the resistance of the armature be small. The change of speed, with a fixed setting of the field rheostat, from full load to no load, expressed in terms of the speed at full load, is called the *speed regulation* of the motor. For example, the speeds of a shunt motor at no load and at full load are 860 and 825 rev. per min. respectively. Consequently its speed regulation is

$$\frac{860 - 825}{825} = .0425, \text{ or } 4\tfrac{1}{4} \text{ per cent.}$$

The maintenance of a strictly constant speed necessitates the manipulation of a field rheostat, that is, the adjustment of a device external to the motor itself. Speed regulation is to be distinguished from speed control. The former indicates the speed changes inherent in the machine, whereas the latter means adjustment for various desired speeds.

100. Characteristic Curves of Shunt Motors. — The characteristic curves of a motor include curves of speed, efficiency, current input, and torque, in terms of the H.P. output of the machine. Such curves for a 7.5-H.P., 230-volt General Electric Company Type CQ motor are shown in Fig. 146.

A shunt motor when started cold on no load quickly arrives at a speed which then *gradually* rises to a maximum. The gradual heating of the field coils increases their resistance. This allows less current to flow in them, and the resulting magnetic flux is less. Therefore the armature must rotate faster to generate the same counter *E.M.F.*

The efficiency of a motor is the ratio of the mechanical output to the electrical input. The determination of the output may be made directly by experiment, or it may be

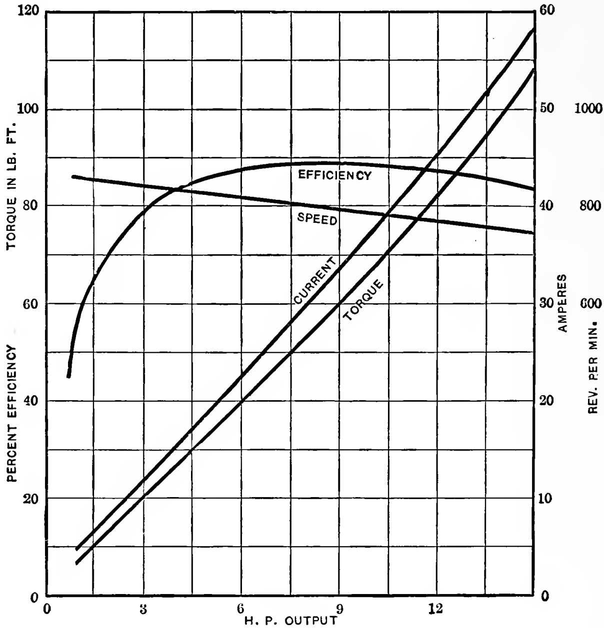


Fig. 146.

found from the measurement of the losses. The latter method is to be preferred because of its greater accuracy. The various losses may be obtained as in Chap. VI. Thus, the efficiency is

$$\varepsilon = \frac{746 \text{ H.P.}}{746 \text{ H.P.} + (P_h + P_e + P_a + P_p + P_f + P_{fw} + P_c)}. \quad (\S 69)$$

The efficiency at any load should be determined at the ultimate temperature assumed under continuous operation at that load, referred to the standard engine-room temperature of 25° C.

In the direct determination of motor output, any convenient load may be placed on the machine. For the smaller motors a Prony brake may be used, the strap-brake being a convenient form, Fig. 147. The power absorbed in watts is expressed as

$$\text{Output} = \frac{2\pi r V(P - P')\,746}{33000},$$

where r is the radius of the pulley in *feet*, V is the speed in rev. per min., and $(P - P')$ is the difference of the two scale readings in *pounds*.

For large motors the load usually consists of generators, the output of which may be absorbed by suitable resistances. If the generator be of proper voltage the current therefrom may be returned to the supply circuit. This method of loading a motor, called the *loading-back* method, results in a considerable saving of power, since the net power taken from the mains is only that required to supply the losses of the two machines. The amount of load is regulated by changing the field current of the generator.

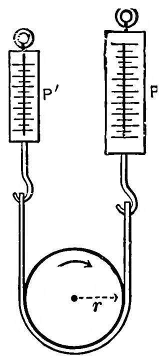


Fig. 147.

The efficiency of a shunt motor at full load can be estimated from the data stamped on the name plate of the machine. Thus, for the following data:

H.P. — 20	Amp. — 150
Volts — 120	R.P.M. — 925

the efficiency at full load is

$$\varepsilon = \frac{746 \times 20}{120 \times 150} = .83, \text{ or } 83 \text{ per cent.}$$

101. Industrial Applications of Shunt Motors. — The design of motors differs frequently in many details from that of generators, especially if the motors are to be directly coupled to the machines they drive. These points of difference are principally in the construction of the frame, bearing supports and shafts. Often motors must be placed out of doors or where they are exposed to dust, chips, etc.; in such cases they should be of the enclosed type.

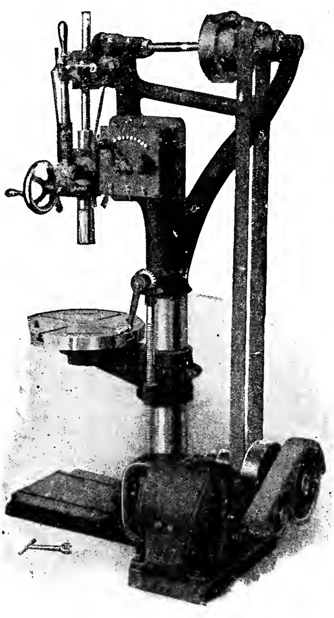


Fig. 148.

Fig. 148 depicts a back-geared motor driving a Hamilton drill press. This type of motor is desirable for slowly moving machines, since it permits of the usual high armature speeds. A protecting guard surrounding the gear and pinion is usually furnished to prevent accidents.

A Crocker-Wheeler adjustable-speed motor directly geared to a 36-inch lathe is shown in Fig. 149. Motors so situated are usually of the open type, but they are

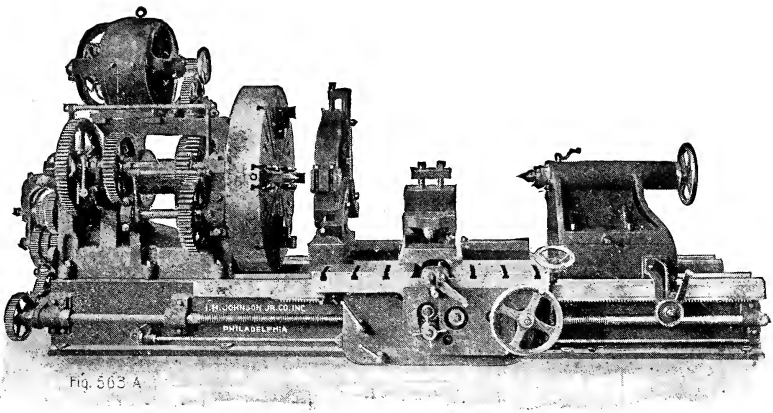


Fig. 149.

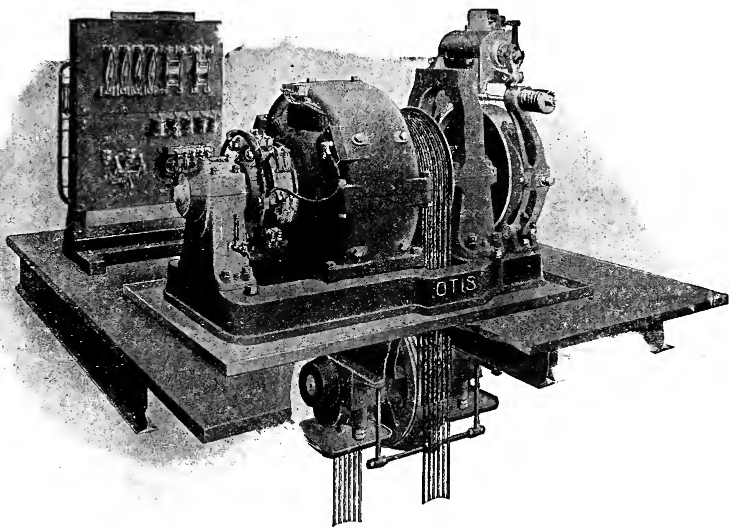


Fig. 150.

sometimes provided with gridiron covers and gauze to give better protection against dirt.

The costs per hour of operating machine tools driven by individual motors are given in the following table, the data representing conditions such as obtain in large machine shops. Fixed charges include interest and insurance on investment in buildings and equipment, variable charges include repairs and renewals, and salaries include cost of management, engineering, labor, etc.; these charges are apportioned among the various machines.

Type of Machine.	Size.	Hourly Operating Expense in Dollars.						
		Fixed Charges.	Variable Charges.	Salaries.	Interest on Cost of Tool.	Depreciation of Tool.	Cost of Power.	Total.
Vertical Boring Mills	40″–60″	.02	.25	.15	.05	.05	.01	0.53
	72″–100″	.04	.45	.25	.08	.08	.01	0.91
	10′–14′	.05	.80	.40	.15	.15	.02	1.57
	16′–24′	.08	2.00	1.00	.30	.30	.03	3.71
Radial Drills	5′	.02	.30	.20	.03	.03	.01	0.59
	10′	.04	.60	.35	.09	.09	.01	1.18
Engine Lathes	30″–40″	.02	.25	.12	.04	.04	.01	0.48
	40″–60″	.03	50	.25	.10	.10	.01	0.99
Planers	36″–56″	.04	.55	.30	.05	.05	.01	1.00
	7′–10′	.06	1.10	.60	.15	.15	.02	2.08
	12′–14′	.15	2.60	1.40	.25	.25	.03	4.68

Power for machine tool operation may be furnished either by individual motors or from a line shaft. The initial investment for line-shaft drive is usually less than for individual motor drive, but the latter is conducive to increased production. Heavier cuts are possible and the time for a given operation is shorter with individual motors.

Fig. 150 illustrates the operating mechanism of the Otis

Traction Elevator, which consists essentially of a slow-speed shunt-wound motor, a sheave, and a brake pulley, the latter enveloped by a pair of powerful spring-actuated and electrically released brake shoes, all compactly grouped on a heavy iron bedplate. The armature shaft serves as a support for the elevator car and counterweight, and on it are mounted the sheave and brake pulley, the drive between the armature spider and sheave being effected through the engagement of projecting arms on each cushioned by rubber buffers. A controller is used for accelerating and retarding the car. The control equipment is so designed that the cars are automatically and gradually retarded and brought to rest at the upper and lower terminals of travel, an operation which is entirely independent of the position of the car controller. Apparatus of this kind is installed in the Singer and Metropolitan towers in New York City, and enables one to reach the fortieth floors of these buildings from the street level in less than one minute.

Series Motors.

102. Series Motors. — As the current traversing the field windings of a series-wound motor is the same as that which flows through its armature, the field strength of such a machine will vary with the load placed upon it. Torque, being proportional to the product of the magnetic flux and the armature current, § 92, will vary approximately as the square of the current taken by the motor. This is true for a series motor with an unsaturated magnetic circuit, but in practice the magnetic circuit is designed to approach saturation near the rated output, and consequently the torque exerted varies to a smaller extent than the square of the current.

The speed of a series motor in revolutions per minute is

$$V = \frac{(E - IR)\, 60}{2\, p\Phi S}\, 10^8, \qquad (\S\ 93)$$

where E is the impressed *E.M.F.*, R is the combined resistance of armature and field windings, p is the number of pairs of poles, Φ is the total flux per pole, and S is the number of armature conductors in series between brushes. An inspection of this expression shows that with increased load the numerator will be but slightly altered because R is small, and that the denominator will be considerably increased since Φ varies with the current. Consequently the speed of the motor decreases as the load increases.

The speed of the armature of a series motor will be such that the counter *E.M.F.* generated at that speed will reduce the current to a proper value, so that the total power consumed will be equal to the sum of the motor output and the losses. In a shunt-wound motor, a very small variation of speed is sufficient to compensate for a wide variation of load. With decrease of load both shunt and series motors speed up and generate a higher counter electromotive force. The resulting decrease of current causes, in the series machine, a weakening of the magnetic field, and as a consequence additional speed is required to maintain this *E.M.F.* Thus a small variation in load on a series machine results in a wide change of speed.

The exertion of a large torque at low speeds and a small torque at high speeds results in a rather uniform energy consumption, for power output equals the product of torque and angular velocity. For this reason the series motor is particularly suitable for traction and for the operation of cranes and of rolling mills. A series-wound machine can be used on either a constant-current circuit or on a constant-potential circuit; but a series motor is seldom run on a constant-potential circuit unless it is directly or very solidly coupled with its load. If connected by means of a belt, and if the belt should break or slip off, the motor would speed up indefinitely and cause the armature to fly to pieces. The series motor, therefore, cannot be run at no load and rated voltage. This difficulty does not present itself when series motors are operated on constant-current circuits, a practice no longer in vogue.

103. Characteristic Curves of Series Motors. — The characteristic curves of a 5-H.P., 220-volt, back-geared, series-wound motor are shown in Fig. 151, and include curves of current input, torque, speed and efficiency, plotted in terms of the horse-power output. The high speeds attained when the motor is under light loads are clearly indicated by the speed-output curve. Frequently curves of torque in terms of speed are used, especially in the selection of motor capacity for electric cars or locomotives. Fig. 152 depicts such a curve for the 5-H.P. motor mentioned above.

If a series motor be at rest and the circuit be closed, an enormous rush of current will occur, giving a tremendous torque. Destructive heating and sparking would probably result. To prevent damage it is therefore necessary, in the operation of these motors, to insert at the start a series resistance which may be cut out after the speed has risen enough to give a sufficient counter *E.M.F.* In practice *controllers* are used for this purpose.

104. Railway Motors. — Series motors operating on constant-potential circuits of from 500 to 600 volts furnish a very satisfactory motive power for the propulsion of trolley street cars and electric railway motor cars. This type of

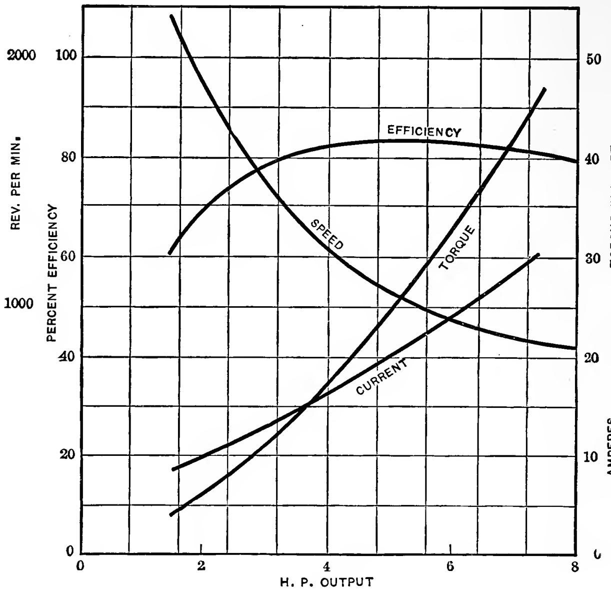


Fig. 151.

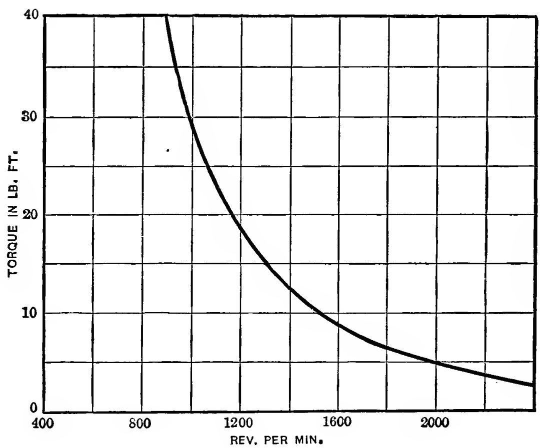


Fig. 152.

motor has been developed to a high degree of perfection during recent years, and is reasonably well fitted to meet the many and severe requirements of railway service. Recent improvements are directed to reliability rather than to increased efficiency. It is not unusual for modern railway motors to be in service for a year or more or to have traveled 60,000 miles without overhauling.

A railway motor must be mechanically strong to withstand the continual strains to which it will be subjected when in service. Poor roadbed, defective switches, snow-covered tracks, etc., are conditions met with in railway service. Railway motors are also subject to abuse at the hands of the motormen. The series resistance is often cut out too rapidly, before the car has an opportunity to accelerate. As a result there is an enormous current flow and a large torque exerted with little speed. This severely strains the motor and is particularly liable to disturb the armature windings. Railway motors are of weatherproof construction, being totally enclosed to guard against the intrusion of water, slush and mud.

Fig. 153 illustrates the box-type frame of a No. 134 Westinghouse railway motor for the heavier class of interurban service. There are four poles built up of soft steel punchings assembled and riveted together between wrought-iron end plates and bolted to the motor frame. The field coils are straight and are formed of copper strap wound on edge, the individual turns being insulated from each other by asbestos. The coils are insulated by several tapings and impregnated with an insulating compound to render them impervious to moisture. They are held in place independently of the poles by brass hangers which are bolted to the motor frame.

The armature bearings of this motor are carried in housings which are firmly clamped into bored seats in the frame. The bearing at the commutator end is $3\frac{3}{8}$ inches in diameter and 10 inches long. One end of the motor frame contains bearings which run on the wheel axle and keep the

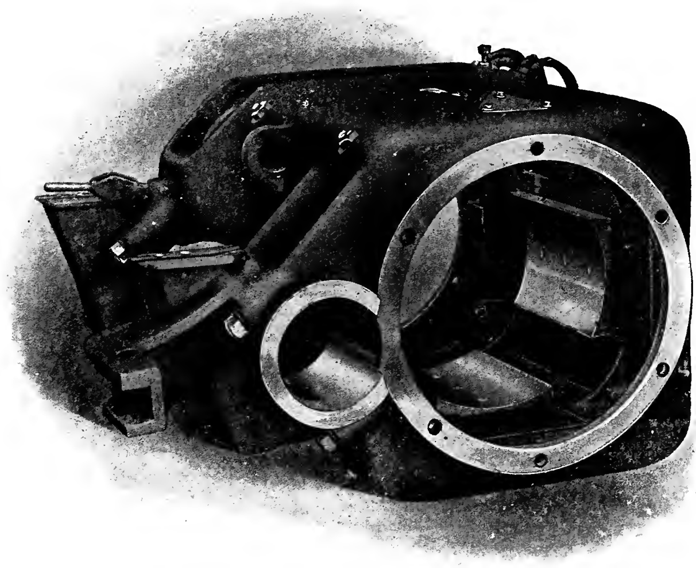


Fig. 153.

pitch circle of the armature shaft pinion always tangent to the pitch circle of the gear which is mounted on the axle. These bearings are $11\frac{1}{4}$ inches long and are furnished for a maximum axle diameter of 6 inches. The bearings, both for armature shaft and for axle, consist of solid bronze shells lined with babbitt metal soldered to the bronze, and are arranged for oil-saturated waste as lubricant. Large pockets are provided for the waste which is in contact with the shaft, and the oil is led up to the waste from below. The openings in the bearing shells are usually 60 % of the total length of the shell and 80 degrees wide.

The armature is built up of thin soft-steel laminations mounted on a spider together with the commutator. Openings in the laminations and spaces between groups of them provide for thorough ventilation by means of the air drawn in at the ends and passing out against the field windings. The winding consists of formed single-turn coils made in two parts, the lower and upper halves being connected at the rear of the armature by soldered copper clips. The winding is insulated with mica and sealed by linen tape followed by dipping in varnish to render it oil-proof. The winding is firmly secured in place by steel wire wound around

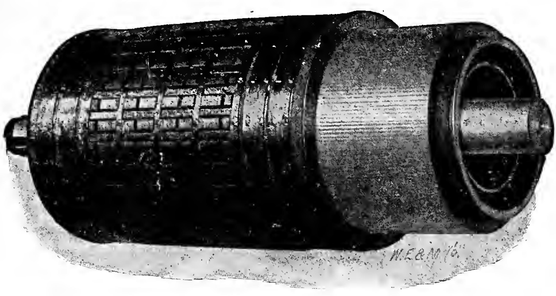


Fig. 154.

the core and over the end connections. Fig. 154 shows the armature and commutator mounted on the shaft. The diameter of the armature is $17\frac{1}{2}$ inches and that of the commutator is $14\frac{3}{4}$ inches. Brush holders, Fig. 155, for this motor are supported by two steel studs which are secured to the motor frame by means of clamps, as shown.

The railway motor described in the foregoing has a nominal rating of 160 H.P. based on a one-hour run with a temperature rise not exceeding 75° C., as thermometrically measured, in any part of the winding above the surrounding air taken at 25° C. An equipment comprising

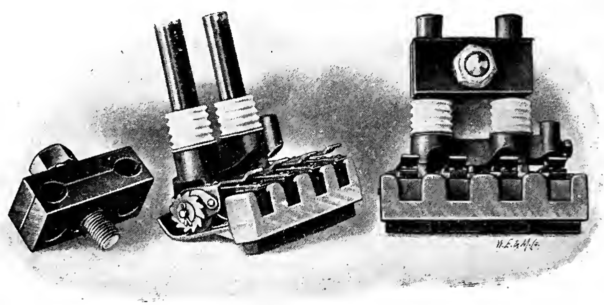


Fig. 155.

two such motors would propel a car weighing, without passengers and electrical equipment, 25 tons, over a level track, and maintain a schedule speed of 30 miles per hour with stops two miles apart. These figures are based upon a gear ratio of 24–53 and 33-inch car wheels.

The performance curves of this railway motor at 500 volts are given in Fig. 156. The usual torque and rev.-per-min. curves of motors are replaced in railway work by curves of tractive effort (i.e., force exerted at the base of the car wheels) and speed in miles per hour. Knowing the gear ratio, gear efficiency, β, and car-wheel diameter,

D inches, this conversion can be effected by means of the following expressions :

Lbs. Tractive Effort =

$$\frac{\text{no. gear teeth}}{\text{no. pinion teeth}} \cdot \frac{24\beta}{D} \times \text{Torque in lbs.-ft.}$$

$$\text{Miles per Hour} = \frac{2\,\pi\,60 \text{ Rev. per min.} \times \text{Torque in lbs.ft.}}{5280 \times \text{Lbs. Tractive Effort}}.$$

The continuous capacity of this motor is given as 120 amperes at 300 volts and 110 amperes at 400 volts.

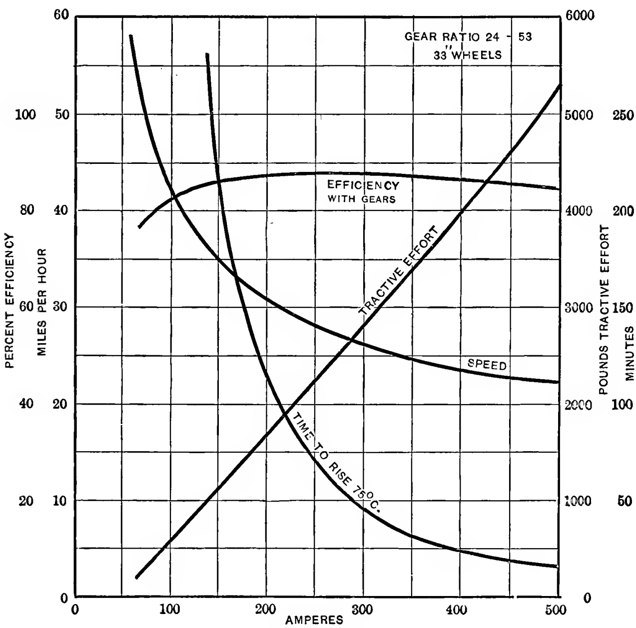


Fig. 156.

A motor for railway service, very similar in design to the one just described, is the GE-216 made by the Gen-

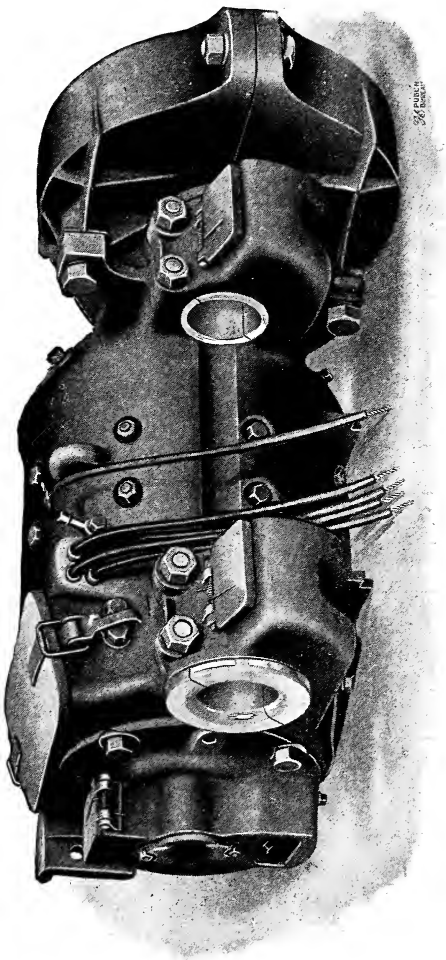


Fig. 157.

eral Electric Company, and shown in Fig. 157. It is a 4-pole motor provided with an equal number of commutating poles, the latter being conducive to better commutation during the acceleration period. The one-hour rating of this motor is 50 H.P. at 600 volts.

The gear case rides with the motor and is fastened to the magnet frame at three points in order to eliminate vibration. The case is made of malleable iron and constructed with strengthening ribs to prevent cracking.

Some operating characteristics and constructive data of 550-volt railway motors are embodied in the curves of Fig. 158. Curve A represents the efficiency of the various sizes

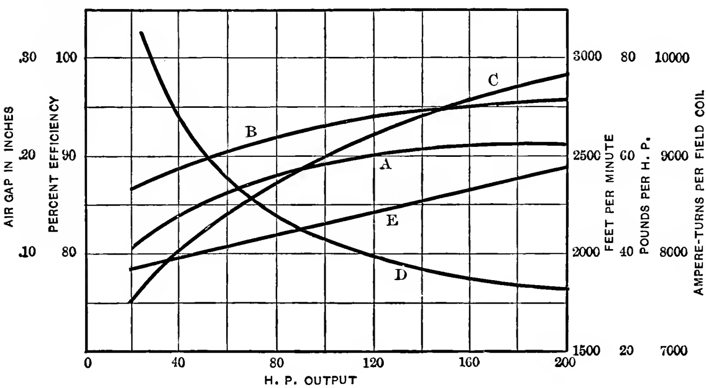


Fig. 158.

of motors at normal load, curve B shows the radial length of the air-gap between armature and field poles, curve C gives the peripheral speed of the armature in feet per minute, curve D indicates the weight of the motor per horse-power, and curve E shows the number of ampere-turns per field coil at normal load current.

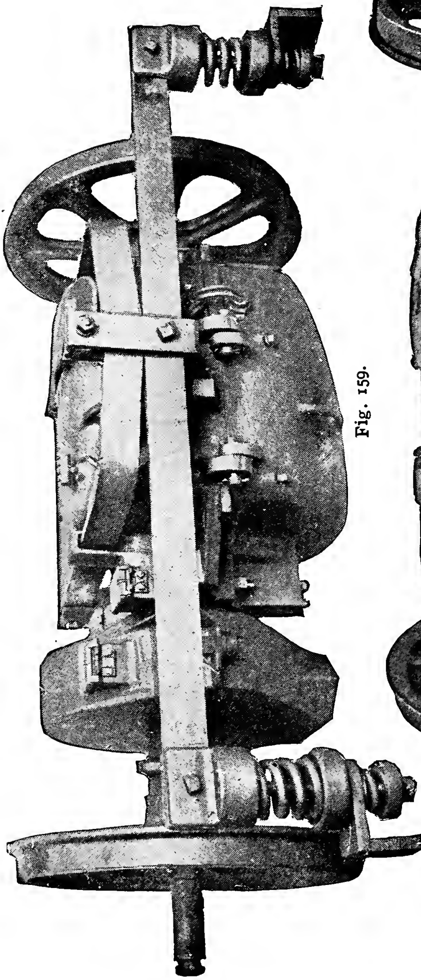


Fig. 159.

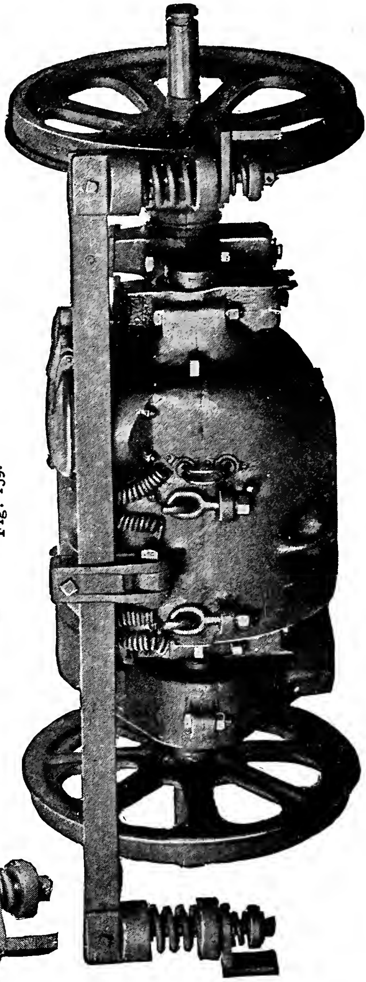


Fig. 160.

The manner of suspending the motors from the trucks is a matter of considerable importance. One end of the motor frame contains bearings which run on the wheel axle, and the other end or the sides are provided with lugs for attachment to a heavy bar which is supported by springs on the truck frame. Figs. 159 and 160 show two methods of motor suspension.

At present a few interurban railways are in operation in this country upon which 1200- and 1400-volt series motors are used. The design of these motors is not materially different from that of the 600-volt type, but particular attention is directed to the avoidance of commutation difficulties.

105. Railway Motor Control. — The function of a railway controller is to allow the motors to start from rest and to accelerate to full speed, this operation being performed with moderate uniformity, due consideration being given to the durability of the apparatus and to the comfort of passengers. Two general methods for accomplishing this result are in use : the rheostatic, and the series-parallel method.

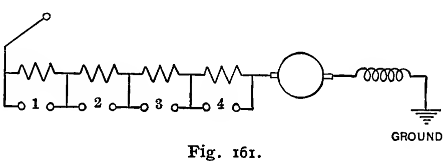


Fig. 161.

In the rheostatic method for use with one or more motors, resistance is placed in the motor circuits, which can be adjusted to regulate the impressed electromotive force. A scheme of connection for a rheostatic railway controller is given in Fig. 161. The change of the resistance in the motor circuit is accomplished by short-circuiting successive portions of it by closing switches 1, 2, 3 and 4 in the order named. This method is infrequently employed although simple, because the loss in the regulating resistance does not permit of economical operation.

The series-parallel method of motor control is extensively used for equipments with two (or any multiple of two) motors. The speed of the car is regulated by first placing the two motors and a resistance in series, and then cutting out the resistance step by step until the motors are operating in series on full voltage. Since, with all the resistance cut out, there is no unnecessary I^2R loss, this is called a running connection, and the controlling mechanism is said to be upon a *running point.* To further increase the speed, the motors are placed in parallel with a resistance in series with both. This resistance is then cut out step by step until the motors are each operating on full voltage. This, again, constitutes a running connection.

The connections of a series-parallel controller are more complex than those of the rheostatic type, since the matter of transition from the series to the parallel positions demands attention. During this period one motor may be shunted or short-circuited, the motor circuit may be opened, or the full current may be maintained through all motors. A scheme of connections illustrating the latter type of series-parallel control is shown in Fig. 162. The controller performs the following operations: switches A and B are closed, thus placing both motors and all the resistance in series; switches 1 to 7 are closed consecutively and then switch C is closed; followed by the opening of switches 2 to 7 and B; switches a and b are closed; thus two currents will flow through switch C in opposite directions, one from the trolley through the motors to ground and the other through the resistance to ground. If the resistance be properly proportioned no current will pass through switch *C*, and this may be opened, thus placing both motors with resistance directly across the line. Switches 2 to 7 are then closed progressively as before, after which the motors are operating in parallel without resistance. This scheme is therefore desirable in that no motor is subjected to sudden increased voltage nor is the circuit opened at any time.

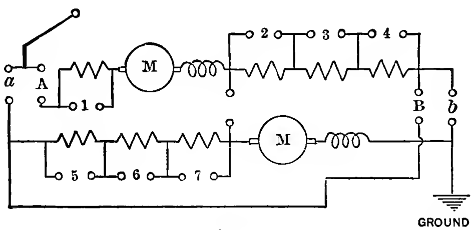


Fig. 162.

When four motors are installed on a car, they may first be connected in series, then each pair in parallel with the two groups in series, and finally all connected in parallel; this is known as the series, series-parallel, parallel method.

The manipulation of the various switches may be accomplished directly by hand or through the intervention of an auxiliary control. In the former system the connections are made by a motorman who moves a handle at the top of the controller on the car platform. This causes the rotation of a vertical cylinder and permits of the successive connection of various contact studs thereon with stationary fingers. Such a controller, made by the Westinghouse Electric and Manufacturing Company, is shown, with the cover removed, in Fig. 163. In this controller for series-parallel operation there are seven controlling points in the series position and six in the parallel position; during the transition from one position to the other one of the motors is short-circuited. The wires from the trolley, from the

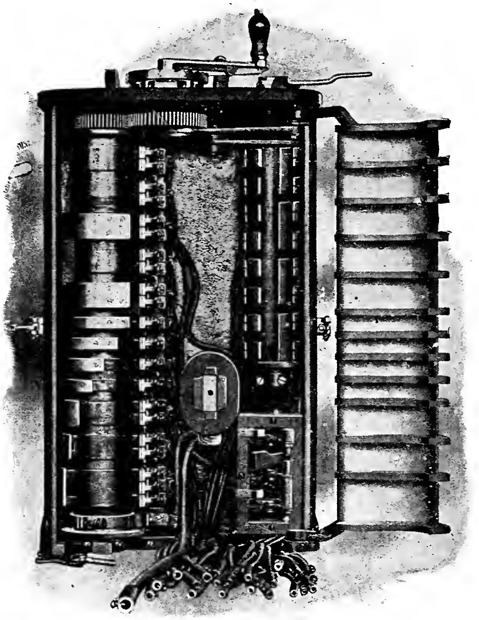


Fig. 163.

motors, and from the different terminals of the resistance grids are brought up under the car to the proper fingers. A smaller cylinder, moved by a reversing lever, is situated to the right of the main cylinder. This has contact pieces which are arranged so as to enable the motorman to reverse the direction of rotation of both motors or to cut them out entirely. Interlocking devices are supplied so that the reversing handle cannot be moved unless the controlling handle is in such a position that connection with the trolley is broken. The controlling handle also cannot be moved if the reversing handle is not properly set either to go forward or to go backward. The reversing handle cannot be removed from the controller, save when the smaller cylinder is in the position that cuts out both motors. As serious arcs are liable to develop upon breaking a circuit of 500 volts, the contact pieces and fingers are separated from adjacent ones by strips of insulating material which are fastened to the inside of a separate cover. Such arcs are effectively disrupted by the field of an electromagnet, which is an essential part of controllers for motors of large size.

In operating an electric car, the power should never be turned off by a slow reverse movement of the controller handle, as destructive arcs are liable to occur upon a slow break. To lower the speed of a car, the power should be completely and suddenly shut off. Before the car has slackened its speed too much the controller handle can be brought up to the proper point.

The system of motor car control in which the various switches are operated by an auxiliary circuit is called the multiple-unit control, since it is designed primarily for the operation of several cars coupled together in a train from any controller on it. The control apparatus for each motor car consists essentially of a series-parallel motor controller and two master controllers. The motor controller is composed of a number of electrically operated switches or *contactors* which close and open the various motor and resistance circuits, and a separate electrically operated reversing switch which governs the direction of movement of the car. This apparatus is usually placed underneath the car. Both the contactors and the *reverser* are operated by solenoids, the current to which is varied by the master controller. The latter is considerably smaller than the ordinary street-car controller, but is similar in appearance and method of operation. A cable which connects each master controller with the motor controllers runs the entire length of the train, the connections between cars being made by suitable couplers. Current for the master control is taken from the line through whichever controller the motorman operates, the amount being about $2\frac{1}{2}$ amperes for each equipment of 400 H.P. As the motor controller is connected to the train auxiliary circuit, any master controller on the train will simultaneously operate corresponding contactors on all the motor cars and establish similar motor connections on them. The connections of the Sprague-General Electric multiple-unit control system are shown in Fig. 164.

If the current supply be momentarily interrupted, the motor control switches automatically return to the "off" position, and upon the restoration of the power supply the same connections are again progressively made that existed immediately preceding the interruption. To avoid accidents which may occur through the physical disability of a motorman, master controllers are sometimes arranged with a button on the handle which must be kept down in order to keep the auxiliary control circuit closed.

The multiple-unit control system of the Westinghouse Electric Company differs from the preceding in the method of actuating the contactors and reversers. Compressed air is used for this purpose, the necessary valves being

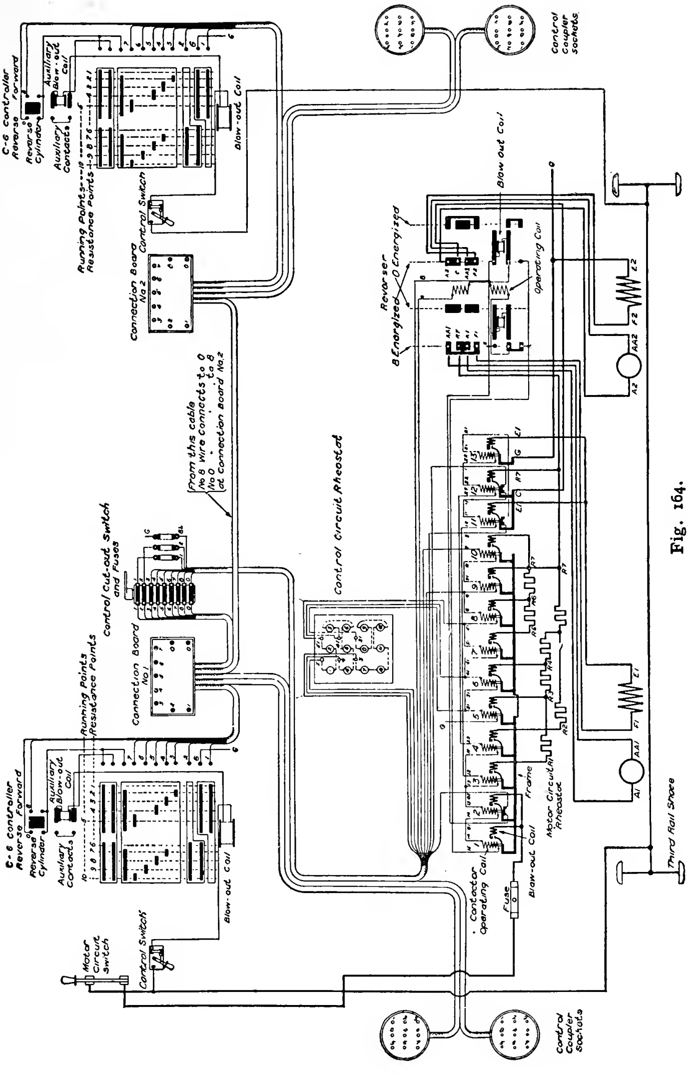

C-6 Controller
Reverse Forward
Reverse Cylinder
Auxiliary Blow-out Coil
Auxiliary Contacts
Running Points
Resistance Points
Blow-out Coil
Control Switch
Connection Board No 2
Connection Board No 1
Control Coupler Sockets
From this cable No 8 wire connects to 0 No 0 " " to 8 at connection board No 2
Control Cut-out Switch and Fuses
Control Circuit Rheostat
Reverser
B Energized
O Energized
Blow Out Coil
Operating Coil
Motor Circuit Switch
Contactor Operating Coil
Fuse
Blow-out Coil
Frame
Motor Circuit Rheostat
Third Rail Shoe

Fig. 164.

operated electrically by a master controller from a storage battery.

106. Motors for Automobiles. — For electric automobiles the series-wound motor is invariably employed. A storage battery of 40 or 44 cells is the customary source of power for these motors. The use of these cells affords a convenient and economical means of speed control. In the case of a single motor, for the first controller notch, the cells may be connected in four-series groups of 10 or 11 each, giving about 22 volts, the four groups being connected in parallel. Other notches would correspond to other series-parallel combinations, and finally the last and highest speed notch would correspond to a connection of all the cells in series. By this arrangement one cell is used just as much as any other, and they are discharged at equal rates. As the voltage supplied to the motor is varied without recourse to a series regulating resistance, there is no resistance loss in starting or running at less than full speed.

Frequently two motors are used, one on each of the two driving wheels; this arrangement allows independent rotation of the wheels on turning curves, while if only one motor be used some form of differential gear must be employed to allow the vehicle to make sharp turns. But the efficiency of one motor is in general greater than the efficiency of two motors of half the rating, and the gain in efficiency by using one motor more than balances the cost and complication of a differential gear in the case of light vehicles.

It is general practice to rate automobile motors at 75 volts, or at $37\frac{1}{2}$ volts if two are used in series and controlled as a single motor. Since the voltage of 40 or 44 cells of battery in series can fall to 75 volts without injury, this is the lowest pressure on which the motors will be expected to run for any length of time at full speed. Hence this voltage is used as the basis for rating. For the best motors the rating is such that a temperature rise of 50° C. will not be exceeded on an indefinite run. A motor so rated will carry 100 per cent overload for a half hour without overheating or damaging the insulation.

Although the voltage of these motors is somewhat low for the use of carbon brushes, the necessity of reversal of direction and the liability of sparking on overload make their use desirable. Soft carbon brushes of low resistance can, however, be obtained, and they are to be recommended.

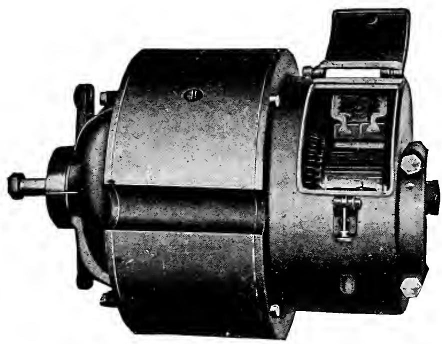


Fig. 165.

Fig. 165 illustrates a motor which is used on automobiles and manufactured by the Eddy Electric Manufacturing Company. The armature winding consists of formed coils which are cross-connected, and therefore only two brushes are required. These brushes are made accessible by providing a window in the end plate. A pinion which is mounted upon the armature shaft meshes with an internal gear on the wheel of the vehicle.

107. Motors for Rolling Mills. — For many kinds of mill work requiring great torque, reversibility, and wide variation of speed, the series motor is well adapted. The shocks and jars which such motors must withstand are very severe because the load conditions are heavy and intermittent, and therefore they must be of particularly strong construction. Mill motors must be totally enclosed to guard against dust and small particles of metal, and consequently must be amply designed so that their temperature elevation will not be excessive. These motors usually operate in both directions and therefore the brushes can have no lead. Sparkless running is insured by employing large air-gaps.

The Crocker-Wheeler Company manufacture form W motors for rolling mills in sizes ranging from $7\frac{1}{2}$ to 200 H.P. for 220 volts, one of which is shown in Fig. 166. They are four-pole machines, and the frames are divided horizontally, the upper half being provided with two hand holes for access to the commutator and brushes.

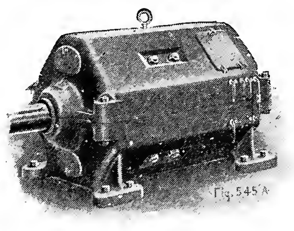


Fig. 166.

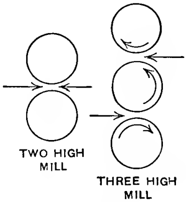


Fig. 167.

A rolling mill may be two high or three high, as indicated in Fig. 167. In the latter the center roll rotates constantly in one direction, while the other two rolls revolve constantly in the opposite direction. Thus a piece of steel can pass through the lower set to the right, then be raised on a table and pass through the upper set of rolls to the left, and continuing in this way. In the two-high mill the steel must pass through the rolls in both directions and consequently the motor driving such a mill must be capable of reversal. Difficulty is encountered in designing motors which are to reverse their direction quickly because of the large moments of inertia of the armatures and rolls. Sometimes two or three armatures of smaller diameter than an equivalent single armature are placed upon one shaft, thus obtaining a smaller radius of gyration.

A mill motor has been built by Messrs. Siemens weighing 235 tons, the armature weighing 74 tons, and it is capable of exerting a torque of 650,000 lbs.-ft. up to a speed of 60 rev. per min., thus corresponding to over 7000 H.P. output. This mill has been in operation for some time, and it is found possible to reverse its direction 28 times per minute from a speed of 60 rev. per min. in one direction to an equal speed in the other direction.

The coupling between a motor and a rolling mill should be such that if the roll breaks obliquely the resulting end thrust will not damage the motor. Some form of shell coupling should be used between the spindle and the motor shaft.

108. Crane Motors.—Series motors for operating cranes or hoists are generally equipped with a brake attachment so that the load may be held after raising it. Brakes are of two types, friction brakes and dynamic brakes. Friction brakes are made in a number of ways, one type of which is depicted in Fig. 168. This shows a spring-actuated shoe brake which is kept normally in engagement but is released when current is supplied to the solenoid. Another form is the band brake, but this is mostly used with non-reversing motors, although some forms are applicable to reversing motors.

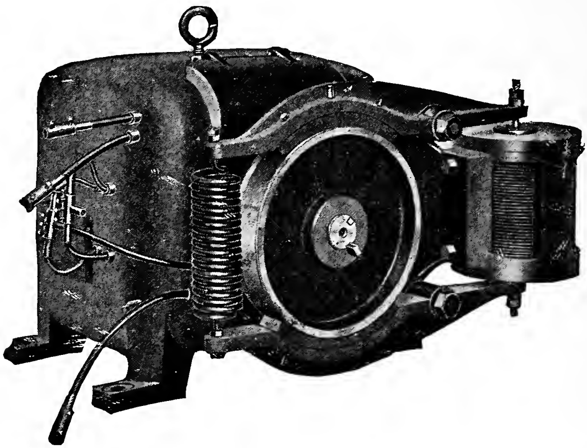


Fig. 168.

Dynamic braking is accomplished by connecting the motor to operate as a generator which will deliver energy to some local circuit or return it to the supply circuit. Such brakes are generally supplemented by friction brakes which become operative when the motor comes to rest and the dynamic braking ceases. The controller for dynamic braking is arranged to connect the armature in a closed circuit containing an adjustable resistance, or to the line. It is desirable first to connect the series field with resistance across the line wires to insure the motor building up as a series generator. Then the motor is disconnected from the line, leaving the motor armature and field in circuit with the resistance. In Fig. 169 these connections are shown respectively at *a* and *b*.

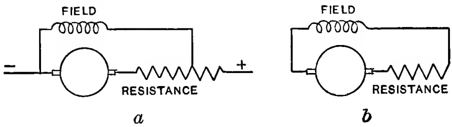


Fig. 169.

A controller for crane motors made by the Cutler-Hammer Manufacturing Company, intended for either hoist or travel duty, is shown in Fig. 170. The resistances are of the cast-iron grid type and are supported by iron rods over which mica tubes are previously placed. Controllers for hoist duty are provided with higher resistance than those for travel duty so as to insure good speed control under light loads.

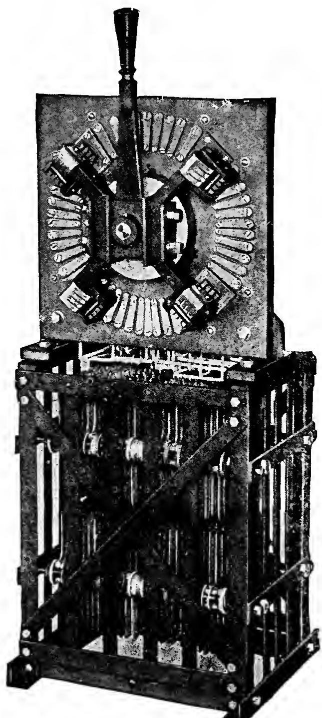


Fig. 170.

109. Compound-wound Motors. — In a compound-wound motor the magnetomotive force of the shunt winding may assist or oppose the magnetomotive force of the series winding, depending upon the manner of connection. In the first case the machine is called a *compound motor* and in the latter a *differential motor*.

The magnetic field of a compound motor becomes more intense with increasing load, and consequently the speed decreases; the amount of speed decrease will depend upon the relative magnetomotive forces of the two windings. At light load there is always a definite field strength, due to the shunt winding, and therefore the speed of the motor cannot exceed a predetermined value. For heavy intermittent loads, such as in operating rolling mills, hoists, elevators, etc., compound motors are much used, because they can exert a powerful starting torque and yet the speed will not be excessively variable under changes of load. Such motors may be safely belted to machine tools in operating them.

Differential motors may be designed to run at almost constant speed by properly proportioning the series and shunt windings so that the magnetic field becomes weaker as the load increases. A powerful starting torque cannot be obtained from this motor inasmuch as the large starting current in the series winding greatly decreases the field strength. If such motors be suddenly started under load their direction of rotation may reverse because the series winding has a lower inductance than the shunt winding; hence in starting differential motors the series winding at first should be automatically short-circuited. These motors are rarely used in practice, as improvements in the design of shunt motors have given the latter good speed regulation, so there is no need of resorting to differential motors with their objectionable features.

PROBLEMS.

1. The armature core of a 4-pole motor has 41 slots, each containing 24 conductors. At full load the wave-wound armature takes 20 amperes at 500 volts, and a flux of 2,100,000 maxwells enters the armature core per pole. What torque is developed at full load?

2. If the armature mentioned in the preceding problem revolve at 640 rev. per min, determine the horse-power developed at full load.

3. The armature resistance, including that of brushes and brush contacts, of a 250-volt, 6-pole motor is .0079 ohm. The armature is lap-wound and has 570 conductors. What is the flux per pole entering the armature when the latter runs at 400 rev. per min. and takes 660 amperes at full load?

4. A 25-H.P., 220-volt shunt-wound motor takes a current of 98 amperes at full load. The armature resistance is 0.090 ohm. With a maximum allowable current intake of 140 amperes, calculate the number of steps required in a starting rheostat for this motor.

5. What are the resistance values of the various steps of the starting box mentioned in the previous problem?

6. From the following name-plate data of a shunt motor determine its regulation and full-load efficiency:

H.P. = 50, Volts = 250, Amperes = 170,
R.P.M. = 420 at no load, R.P.M. = 400 at full load.

7. What is the average pull in pounds on each of the 372 armature conductors of the motor cited in the foregoing problem at full load if the conductors are situated 9 inches from the axis?

8. The motor of an electric car having 33-inch wheels, when traveling at 25 miles per hour, exerts a torque of 550 pounds (at one foot radius from the center of the armature shaft). If the gear ratio is 26 to 60, and the efficiency of the gears is 97%, determine the tractive effort at the base of the car wheels, the horse-power, and the number of revolutions of the motor per minute.

CHAPTER VIII.

DYNAMOTORS, MOTOR-GENERATORS, BOOSTERS, AND STORAGE BATTERIES.

110. **Dynamotors.** — Dynamotors are transforming devices combining both motor and generator action in one magnetic field, with two armatures or with an armature having two separate windings. They are generally supplied with commutators, one at each end, which are connected to the two windings respectively. Either winding of the armature may be used as a motor winding, and the other as the generator winding. These machines occupy the same position as regards direct-current practice as is occupied by transformers in alternating-current practice. That is, they enable one to take electrical energy from a system of supply at one voltage, and deliver it at another voltage to a circuit where it is to be utilized. They cannot, however, be constructed so as to operate with the same high efficiency as a transformer does. As the currents in the two armatures flow in opposite directions, and the machines are so designed as to have practically the same number of armature ampere-turns when in operation, there is practically no armature reaction. The field, therefore, is not distorted so as to require a shifting of the brushes upon a change of load. These machines are more efficient than motor-generators, which will be described later, as they have but a single field magnet. They cannot be compounded so as to yield a constant *E.M.F.* at the dynamo end, for, while a cumulative series coil would tend to raise

the *E.M.F.* at the generator end, there would be a corresponding decrease of the speed of the armature, due to the increase of magnetic flux. To generate the same counter *E.M.F.* in the motor winding as previously existed without the series coil, requires a lower armature speed.

Dynamotors are used in the so-called Teaser system of variable speed control of motors which are intended to receive energy from supply circuits, that are at the same time giving energy to lighting and similar load circuits. For instance, they are used in connection with large printing presses whose parts have large moments of inertia, and which demand an unusually large starting torque to be exerted by the driving motor. Sometimes this is as much as six times the torque which the motor is called upon to exert at normal speed and full load. Since the torque which is exerted by a motor is dependent upon the current which flows through its armature, while the speed at which this torque is applied is dependent upon the pressure impressed upon it, it is desirable that the large starting current should be taken at a low voltage from some transforming device, such as a dynamotor, rather than at the higher and constant voltage of the supply mains. The dynamotor for instance may be so designed that the counter *E.M.F.* of its motor winding is five times the *E.M.F.* induced in its generator winding. Consider the dynamotor and main motor to be connected to the supply mains as indicated in Fig. 171, the field winding of the former being excited by current taken directly from the mains, and the negative brush of the motor end being connected with the positive brush of the generator end. The two armature windings are connected in series with a regulating resistance to the supply mains.

At starting, the main motor, which drives the press and which is generally a cumulatively compound-wound motor, is supplied with current from the generator end of the dynamotor. The voltage with which it is supplied is somewhat less than one-fifth that of the main supply, depending upon the magnitude of the resistance in series with the dynamotor. This low voltage permits of the application of a proper amount of torque at a low speed. Furthermore, the drain of current from the supply mains is about one-fifth that which passes through the main motor. By manipulating the dynamo regulating resistance, the electromotive force supplied to the main motor is raised, and with it the speed. The highest speed of the main motor which can be attained by this arrangement is such that, when attained, the motor's connections may be transferred to the supply mains through another series regulating resistance without any excessive drain of current from those mains. The amount of current which is taken by the main motor as compared with the amount of current which is drawn from the supply mains may be represented as in Fig. 172. Regulation of the resistances and changes of connection are accomplished through the aid of a controller. The different speeds are secured by the manipulation of a single hand-wheel on the controller, and thus the pressman has at his command a simple means of manipulating the printing press.

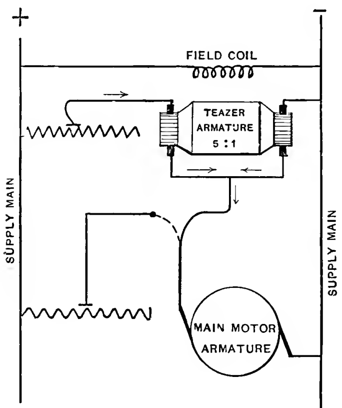


Fig. 171.

Dynamotors are also used as *equalizers*, in connection with 3-wire constant-potential distribution circuits, to

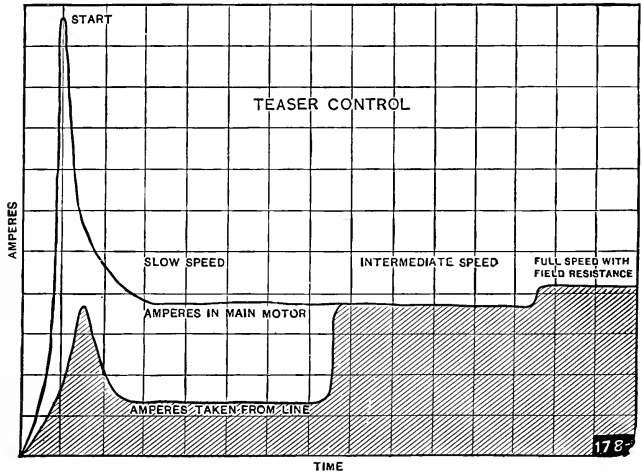


Fig. 172.

equalize the potential differences between the two outer and the neutral wire when one side of the system is carrying a larger load than the other. For instance, with simi-

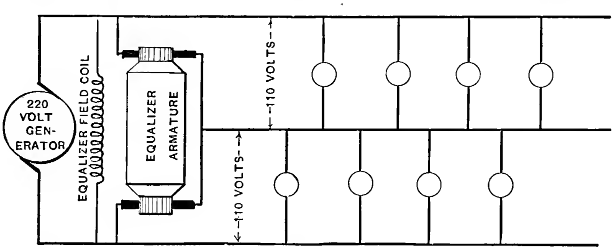


Fig. 173.

larly wound windings upon its armature, the dynamotor would be connected as indicated in Fig. 173 to a system supplied by a 220-volt generator. When the system was unbalanced the side having the smaller load would have the lesser drop and therefore the higher pressure. The winding of the dynamotor which is connected with this side would act as a motor winding and drive the armature, while the other winding would be the seat of generated *E.M.F.*'s which would tend to raise the pressure of the more heavily loaded side.

Consider that each of the equalizer armature windings has a resistance of R ohms and generates E_e volts, and that the power losses due to hysteresis, eddy currents, and friction are P_l watts. Then, with E volts between the outer wires, and the load on the two sides balanced, that is, no current flowing through the neutral wire,

$$E = 2\,E_e + 2\,I_0 R, \qquad (1)$$

and

$$P_l = 2\,E_e I_0, \qquad (2)$$

where I_0 represents the current in each armature winding. If, however, the load be not balanced, and the neutral wire carry a current I_n, and if the armature windings carry

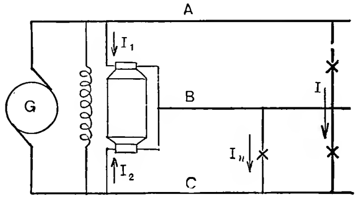


Fig. 174.

currents I_1 and I_2 respectively, their directions being as indicated in Fig. 174, then

$$E = 2\,E_e + I_1 R - I_2 R, \qquad (3)$$

(§ 5)

$$I_n = I_1 + I_2, \qquad (4)$$

and, as the iron and friction losses remain unaltered upon the changed conditions of load, and since the power exerted by the motor element equals the power output of the generator element plus the frictional and iron losses,

$$E_e I_1 = E_e I_2 + P_l. \tag{5}$$

Dividing by E_e and using (2),

$$I_1 = I_2 + \frac{P_l}{E_e} = I_2 + 2\,I_0. \tag{6}$$

Substituting (6) in (4) and solving

$$I_2 = \frac{I_n}{2} - I_0, \tag{7}$$

and

$$I_1 = \frac{I_n}{2} + I_0. \tag{8}$$

From (3), (7), and (8)

$$E = 2\,E_e + 2\,I_0 R, \tag{9}$$

which is the same as equation (1). Therefore the *E.M.F.* generated by each armature winding is not altered by the application of the load, under the conditions assumed. Half of the unbalanced portion of the load I_n is supplied by one equalizer armature winding, while the other half comes from the main generator after passing through the other equalizer armature winding, which at the time is operating as a motor winding. The *E.M.F.*'s generated in the equalizer windings are not altered by the application of the load, nor is the speed of the armature. The capacity of the equalizer armature, if the losses be disregarded, should be equal to the power represented by the maximum unbalanced load, for each armature supplies a voltage approximately equal to that of either side of the distribution circuit, and carries a maximum current of approximately half the maximum carried by the neutral wire. The employment of an equalizer enables one to dispense with the neutral wire between the point where it is located and the main generator, and makes it possible to use a 3-wire distributing system in connection with a single generator instead of a 2-wire system.

Assuming the normal pressure per side to be half that between outer wires, and since with the maximum unbalanced load the side pressures are respectively

$$E_{AB} = E_e + I_1R,$$

and

$$E_{BC} = E_e - I_2R,$$

then the pressure regulations, under this condition, are $+I_1R/E_e$ and $-I_2R/E_e$. The pressure on the loaded side of the system therefore drops and that on the unloaded side rises, as is the case with two main generators.

Dynamotors are used in telegraph stations, the motor windings being designed for connections to lighting circuits, while the generator windings yield pressures suitable for telegraphic service. This is often different in the case of different machines. These machines are designed to take the place of batteries of a large number of gravity cells such as were used, in large quantities, a few years ago. The cost of operation of a dynamotor for this service is about one-fifth of what it is in the case of the gravity cells. The space which the machine occupies is much less than that by the cells. Dynamotors are to be preferred to batteries also on the ground of cleanliness. Their reliability, when supplied with electric energy from large city service mains, is equal to that of the cells, but this cannot be said in the case of small towns. The telephone companies sometimes employ dynamotors for the purpose of charging storage cells. Such a machine is shown in Fig. 175. With some forms, the charging of the cells can go on continuously, they being at the same time used for telephonic communication.

Dynamotors also furnish a convenient and satisfactory

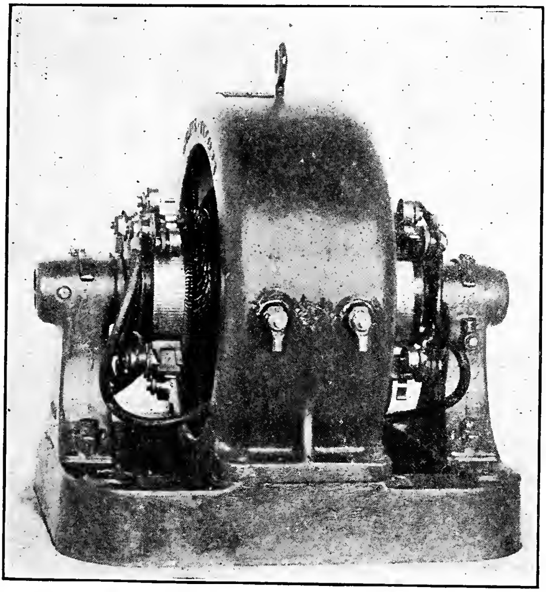


Fig. 175.

means of heating surgeons' electro-cauteries. Cautery knives take from 3 to 8 amperes at 5 volts, while dome cauteries take from 15 to 20 amperes at the same voltage.

111. **Motor-Generators.** — A motor-generator is a transforming device consisting of a motor mechanically connected to one or more generators.

A form in which both the motor and the generator are direct-current machines is extensively used in connection with 3-wire distributing systems, under the name of *balancer*. If the shunt field coils of the two halves of the balancer be connected in series with each other and to the outer conductors of the system, and if the armature windings be alike, the balancer will operate exactly as would a dynamotor when used for this purpose and in the manner described in the preceding section. The balancer, however, as well as the motor-generator in general, is to be preferred to the dynamotor from an operating viewpoint, because the absolute independence of the two field magnets and of their exciting coils makes possible a considerable variation in the speed of the motor element as well as in the voltage of the generator element. If the field windings of the two elements of the balancer and their armatures be reciprocally connected with opposite sides of the system, as indicated in Fig. 176, the regulation of the voltage on the two sides of the system will be improved. The *E.M.F.*'s produced in the two armatures when one side of the system is carrying a greater load than the other will be unlike, the motor flux will be reduced and the generator flux will be increased. The motor will therefore increase its speed and the generator will produce a greater *E.M.F.* not only because of the increased speed of the armature but also because of the increased flux from its field magnets. With such reciprocal field excitation perfect voltage regulation cannot be obtained, for the method postulates a

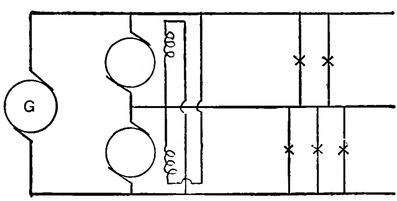


Fig. 176.

voltage difference as a basis for its operativeness. Perfect regulation may, however, be obtained by the use of compound-wound elements in the balancer. The directions of the currents in the system and in the exciting coils are indicated in Fig. 177, where the motor and generator armatures of the balancer are marked respectively with *M* and *G*. The iron of the magnetic circuits of the two elements of the compound balancer, under normal operation, should not be fluxed to near the point of saturation.

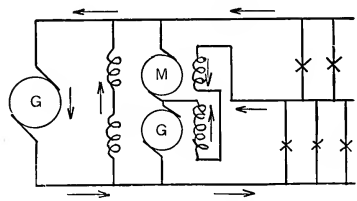


Fig. 177.

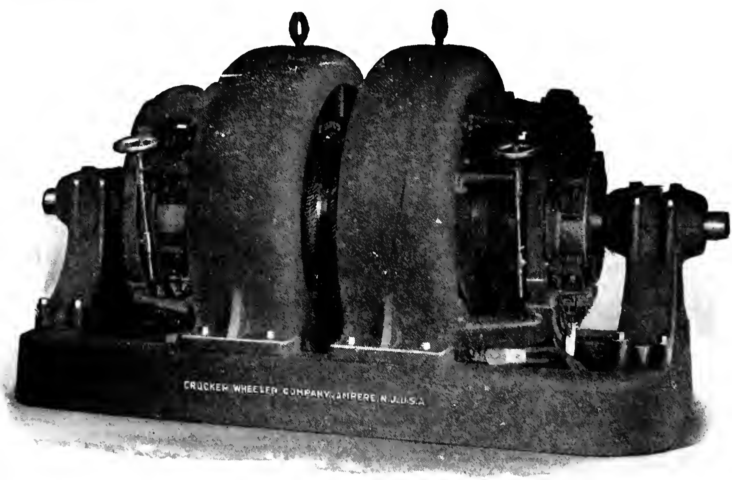


Fig. 178.

A balancer, for use on 3-wire circuits having a pressure of 125 volts per side is shown in Fig. 178.

Motor-generators having the armatures of their elements wound for unlike voltages are used for maintaining constancy between the various conductors of multivoltage circuits as used in connection with the operation of variable-speed shunt motors. Consider such a balancer to be connected to a circuit and the currents to be as indicated in Fig. 179 and the armature of the lower element normally to generate an $E.M.F.$ of E' volts, to have a resistance of R ohms,

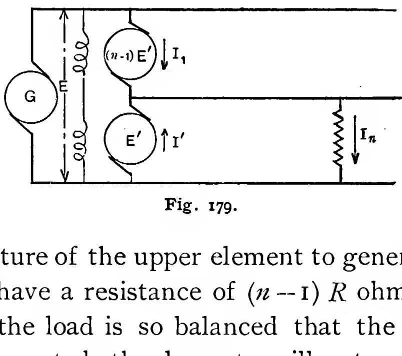


Fig. 179.

and the armature of the upper element to generate $(n-1)E'$ volts and to have a resistance of $(n-1)R$ ohms resistance. Then when the load is so balanced that the middle wire carries no current, both elements will act as motors and will carry a current I_0, and the following relations will hold:

$$E - nE' - I_0 nR = 0. \qquad (1)$$

$$\therefore \quad E' = \frac{E}{n} - I_0 R. \qquad (2)$$

When, again, the middle wire carries a current I_n, as indicated in the figure,

$$E - nE' - I_1(n - 1)R + I'R = 0, \qquad (3)$$

and (§5)

$$I_1 + I' = I_n, \qquad (4)$$

and, since motor-element output equals the sum of power generated and of the frictional losses,

$$(n - 1)E'I_1 = E'I' + EI_0; \qquad (5)$$

but, since nE' is practically equal to E, dividing by E', solving for I', and substituting in (4), there results

$$I_1 = \frac{I_n}{n} + I_0. \qquad (6)$$

Substituting (6) in (4),

$$I' = \left(1 - \frac{1}{n}\right)I_n - I_0, \qquad (7)$$

and therefore, as in (2),

$$E' = \frac{E}{n} - I_0 R. \qquad (8)$$

Therefore the main generator supplies approximately $1/n$th the unbalanced load current at full voltage, whereas the local mains supply this load current at E/nths the pressure between outer conductors.

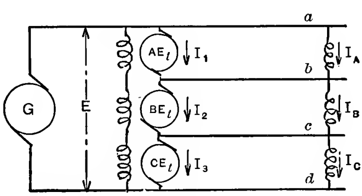


Fig. 180.

In the case of a 4-wire multivoltage distribution circuit, using a 3-element motor-generator as a balancer, with currents and voltages as marked and directed in Fig. 180 the preceding equations apply when the symbols are properly interpreted.

Let E_t equal the sum of the voltages generated in the armature windings of all the elements of the balancer, and assume that the different armatures generate AE_t, BE_t, and CE_t volts and that their windings have resistances of AR, BR, and CR ohms, where $A+B+C=1$. Then all the armatures carry the no-load current I_0, and each armature carries a portion of the unbalanced load-current of every other branch than the one to which it is directly connected, as well as a portion of the unbalanced load-current of the branch to which it is directly connected. The last will flow in a direction opposite to that indicated in the figure. According to the foregoing discussion, (6) and (7), the apportioning is as follows: —

$$\begin{aligned} I_1 &= I_0 + (A-1)I_A + BI_B + CI_C, \\ \text{and } I_2 &= I_0 + AI_A + (B-1)I_B + CI_C, \\ I_3 &= I_0 + AI_A + BI_B + (C-1)I_C. \end{aligned} \tag{9}$$

The pressures between successive distribution conductors are, therefore,

$$\begin{aligned} E_{ab} &= A(E_t + I_1R), \\ E_{bc} &= B(E_t + I_2R), \\ \text{and} \quad E_{cd} &= C(E_t + I_3R). \end{aligned} \tag{10}$$

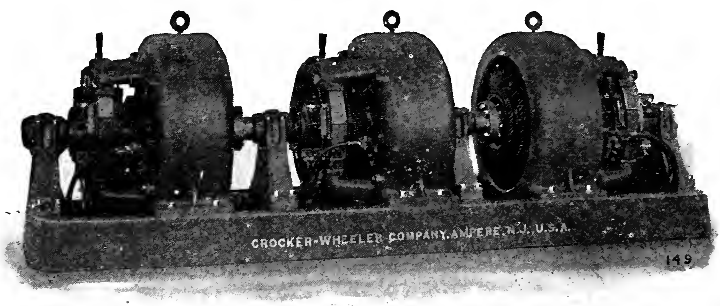


Fig. 181.

The equations of (9) and (10) can be extended to embrace any number of branches. Balancers having three elements, however, satisfactorily meet the requirements of multiple-voltage systems for speed control of motors. Common branch pressures are 40, 80, and 120 volts respectively. By use of such a system a very wide range in speed is economically possible. A four-wire balancer for this purpose is shown in Fig. 181.

112. **Boosters.** — A booster is a machine inserted in series in a circuit to change its voltage. It may be driven by an electric motor, in which case it is termed a *motor-booster*, or otherwise.

In the distribution of electric energy from a central station, at constant potential, it often happens that excessive currents must be supplied at a considerable distance from the station or that currents of ordinary magnitude must be furnished at very distant points. If the supply potential is to be the same at the distant points as at those near by, and if the current at all points is to come from the same generators, then the cross-section of a feeder to a distant point needs to be very great, unless some means be taken to compensate for the drop in pressure caused by its resistance. *Series boosters* are frequently employed for this purpose. The generator element of the booster, in such a case, is connected, at the station or at any other point, in series with the feeder, the voltage-drop in which is to be compensated. As the drop is equal to the current, I, carried by the feeder times its resistance R_f, the voltage generated in the booster, E_B, should be such that

$$\frac{E_B}{I} = \text{constant},$$

a condition which is satisfied by a generator whose characteristic is a straight line, Fig. 182, making an angle α with the abscissæ whose tangent is E_B/I. Such a characteristic cannot be obtained, but, if the iron of the magnetic circuit be not fluxed above the knee of the magnetization curve by the maximum current to be carried by the feeder, a curved characteristic may be obtained which lies sufficiently close to the straight line to yield satisfactory results. The curve lies above the straight line and is concave towards it if the compensation is perfect at full load. With boosters to be used on ordinary railroad circuits, the maximum variation of voltage from that indicated by the straight line, is generally considered to be permissible, if it does not exceed 10% of the maximum voltage supplied by the booster. As the field flux increases with the load, sparkless commutation is easily obtained at full load, and therefore copper brushes may advantageously be employed for the purpose of increasing the efficiency.

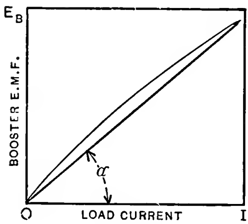


Fig. 182.

The series booster is also frequently used in electric railway systems, which use the grounded rails for returning the propulsion current to the generating station. It is used to reduce the portion of the return current which would otherwise pass through the earth and its substructures. It is then termed a *negative* or *track-return booster.* Consider a point on the track rail to be at a potential above the grounded terminal of the generator at the station and that the propulsion current is returned to this terminal, under this potential, by three paths connected in parallel, namely the track rails, the earth, and a *negative feeder* in series with a negative booster. Representing the currents flowing in the respective paths by I_t, I_e, and I_f, and the resistances by R_t, R_e, and R_f, then, if the voltage generated by the booster be E_B and if it be properly directed, the difference of potential will be

$$R_t I_t = R_e I_e = I_f\left(R_f - \frac{E_B}{I_f}\right).$$

The expression in parentheses represents the apparent resistance of the negative feeder. If the booster be so designed that the slope of its characteristic is approximately R_f the apparent resistance of the negative feeder becomes zero, nearly all the current will return through it, and the earth currents will be reduced.

Sometimes the field coil of the negative booster is connected in series with the outgoing feeders while the armature is connected in series with the return feeder as above.

Boosters are extensively used in connection with storage batteries for regulating the voltage between conductors of constant-potential systems.

A so-called *shunt booster*, with its armature connected, in series with a battery, between the station bus-bars, and with its shunt field coils also connected through a regulating rheostat to the same bus-bars, is often used to increase the voltage impressed upon the battery above that between bus-bars during the charging of the battery. Since the charging pressure per cell varies from 1.8 volts at the start to 2.65 volts at heavy over-charge, if there be n cells and E volts between bus-bars, the booster will be required to furnish at a maximum an *E.M.F.*

$$E_B = n(2.65 - 1.8) = 0.85\, n \text{ volts},$$

and the current capacity must be the same as the maximum charging current of the battery. When such an arrangement is used the booster is employed only during the charging of the cells and not all the cells are used during the whole time of the discharge of the battery. A few cells, called *end-cells*, are cut out of circuit at the beginning of the discharge and are successively cut in again as the voltage per cell decreases due to the discharge. At the close of the discharge all the cells are in circuit and their number is

$$n = \frac{E}{1.8}.$$

The *E.M.F.* capacity of the booster as well as its normal volt-ampere capacity accordingly amounts to about 40 % that of the battery.

Frequently *differentially-wound boosters*, connected as shown in Fig. 183, are used in connection with generators

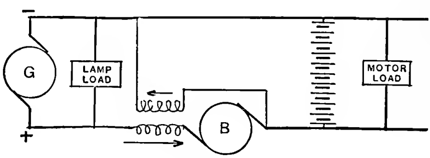


Fig. 183.

supplying energy both for lighting as well as for motors. This arrangement is especially suited to cases where the motor load fluctuates much while its average value is small and where a reduction of pressure upon the motor circuits under load is advantageous. Such cases are found in office buildings, apartment houses, and hotels, where a single generator supplies energy for lamps as well as for elevator and pump motors. The motor bus-bars generally have 15 volts greater potential difference than the lighting bus-bars. At average motor load, the shunt ampere-turns predominate, the series coil carries the average motor-load current, the booster *E.M.F.*, E_B, is added to the generator *E.M.F.*, and the battery neither charges nor discharges. With heavy motor load, the series excitation falls slightly with a consequent fall of E_B, of motor bus-bar pressure, and of shunt excitation. The battery, therefore, supplies the excess above the average of the motor-load current, while the main generator supplies, as before, this average. On light motor load E_B increases slightly, the motor bus-bar pressure rises, and the battery takes a charging current equal to the difference between the motor-load current and its average value. The current in the booster varies in practice but a few per cent from the average motor-load current, and its direction is always the same. Such a machine is therefore called a *non-reversible* or a *constant-current* booster.

In electric railway systems where there is a large average current the battery charge and discharge rates are moderate and it is desirable that the voltage should not decrease with increase of load. In such cases the differentially-wound booster may be employed, with connections as indicated in Fig. 184, where the direction of the current

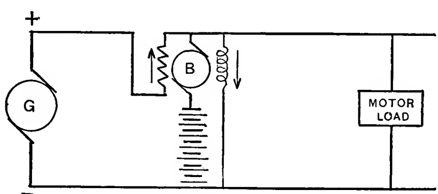


Fig. 184.

through the armature alters with change from charge to discharge of the battery. It is therefore called a *reversible* booster. At normal load, the series and the shunt ampere-turns are equal and opposed to each other, and hence the booster *E.M.F.*, E_B, is zero, the battery is neither charging nor discharging and its open-circuit voltage, E_s, is equal to E, that of the generator and system. With heavy loads the series ampere-turns predominate, E_B is added to E_s, and the battery discharges. With light loads the shunt ampere-turns predominate, E_B opposes E_s, and the battery charges. When the battery is discharged its *E.M.F.* falls, but the booster compensates therefor by taking more current in the series coil. The load on the generator is practically constant and the battery takes up the variations. The booster has to carry the maximum battery current and it must at the same time give its maximum *E.M.F.*; and these values, therefore, determine its capacity.

The series coils of the two differentially-wound boosters must be of sufficient cross-section to carry very large currents, and this again requires such large magnet frames as to make the cost of the machines excessive. Means have therefore been devised for making use of shunt-wound boosters, the current in the shunt coil being changed by and in accordance with variations in the generator current.

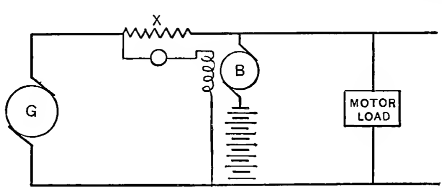


Fig. 185.

The *Hubbard* booster system makes use of an auxiliary generator, X, for furnishing exciting current for the booster, the connections being as indicated in Fig. 185. In practice the exciter and driving motor are on one shaft. At average load the voltage of the exciter, E_x, is the same as that of the system, E, and opposed to it, the booster voltage, E_B, is zero, and the battery neither charges nor discharges. With heavy load $E_x > E$, E_B is added to E_s, and the battery discharges. With light load, $E_x < E$, E_B is reversed and is opposed to E_s, and the battery charges. This system is sometimes called the *counter E.M.F. system* because the exciter *E.M.F.* opposes that of the system.

Another much-used booster system employs the *Entz carbon-plate regulator* to control the exciting current in an auxiliary generator used as an exciter for the booster. The

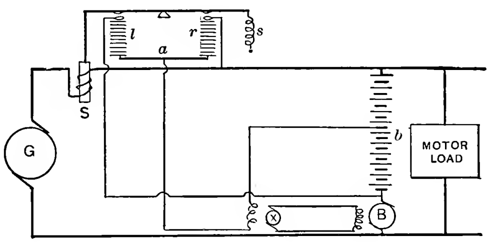


Fig. 186.

connections are as indicated in Fig. 186. Two piles of carbon plates l and r have variable resistances, which are reduced under pressure, and whose magnitudes are controlled by the combined action of the solenoid S and the spring s. The resistance R_l is reduced and R_r increased upon increase of current in S. At average load R_l and R_r are equal and therefore the point a is at the same potential as b, and there is no current in the field coil of the exciter X. Hence $E_x = E_B = 0$, $E_s = E$, and the battery neither charges nor discharges. With heavy load $R_l < R_r$, $E_x > 0$, E_B has the same direction as E_s, and the battery discharges. With light load $R_r < R_l$, $E_x > 0$, E_B is opposed to

E_s, and the battery charges. The appearance of the regulator is shown in Fig. 187.

Booster armatures are generally lap-wound because of the large currents which they must carry. The current density in the windings can be made high because they are

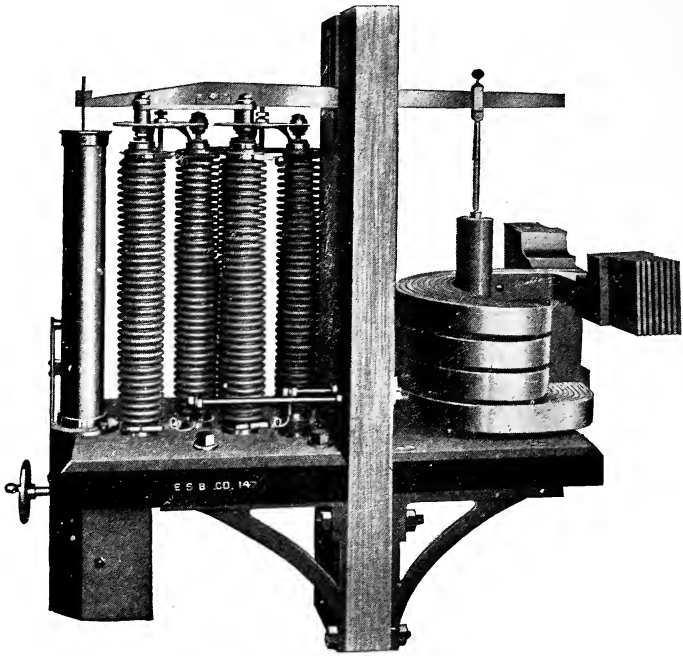


Fig. 187.

rarely called upon to give their rated output. The reversible boosters should have laminated field-magnet cores in order to avoid a sluggish behavior. Sparking is liable to occur, unless the armature coils are of low reactance, because of the weak fluxing of the iron of the magnetic circuit. Safety relay devices should be used to prevent racing of the generator element in case of the accidental opening of the shunt exciting circuit of the motor element.

113. Storage Batteries. — Storage batteries are reversible electrolytic cells, whose electrodes are chemically modified by the passage of current through the cells and which thereby absorb and store energy when the current flows in one direction and give it up when the direction is reversed. The commercial forms make use of lead for electrodes and dilute sulphuric acid for an electrolyte. When charged, and energy has been absorbed, the positive electrode, that is the one of higher potential, is modified so as to contain an amount of lead peroxide (PbO_2), while the other or negative electrode contains a corresponding amount of sponge lead.

Before a commercial cell can be considered as ready to receive its first or factory charge, the electrodes must have been materially modified from the condition of ordinary reguline lead. Their surfaces may have been rendered porous by mechanical and electrochemical treatment, or lead oxides may have been conductively united with them through mechanical means. The plates constituting the former electrodes are termed *Planté plates*, while the latter are known as *pasted plates*. The purpose of the preliminary treatment or *formation* of the electrodes is eventually to expose a large surface to the electrolyte, so that, under the limitations as to the velocities of the chemical reactions, a relatively high rate and large amount of energy may be absorbed during charge and be liberated during discharge. When fully charged, the porous portion of the positive electrode is peroxide of lead and of the negative is sponge lead. At all other times there is some lead sulphate present in the porous or *active material* of both electrodes. As lead sulphate is a non-conductor of electricity, an excessive amount of it will interfere with the functioning of the active material. When the open-circuit potential of a cell sinks to 1.8 volts the amount has increased to the permissible limit. The chemical changes taking place during charge and discharge are represented by the formula

$$PbO_2 + Pb + 2\,H_2SO_4 \underset{\xrightarrow[\text{Discharge}]{}}{\overset{\xleftarrow{\text{Charge}}}{=}} 2\,PbSO_4 + 2\,H_2O.$$

Planté plates are larger, heavier, more expensive, and more likely to be injured by impurities in the electrolyte than pasted plates, although they are more efficient, durable, and dependable. They are best fitted for use in connection with central stations. Pasted plates are to be preferred for motor-car propulsion. The electrochemical action does not penetrate much more than a millimeter below the surface of the electrode, because the active material has so much greater conductivity than the electrolyte and, at that depth, most of the current is confined to the active material, and there is, accordingly, no appreciable release of ions from the electrolyte.

The acid of the electrolyte should be made from sulphur and not from pyrites and at full charge should be diluted to have a specific gravity of 1.20. During discharge the electrolyte gives up to the electrodes SO_3 and its specific gravity therefore falls from 1.13 to 1.19 depending upon the amount of electrolyte. Calculations of the resistance offered by the electrolyte can be based upon the assumption that its resistivity is 4/3 ohm per cubic centimeter at 18° C. with a negative temperature coefficient of 0.016 per degree Centigrade.

The *E.M.F.* of a cell depends upon the concentration of the electrolyte and varies in accordance with the curve shown in Fig. 188. It also depends upon the condition of

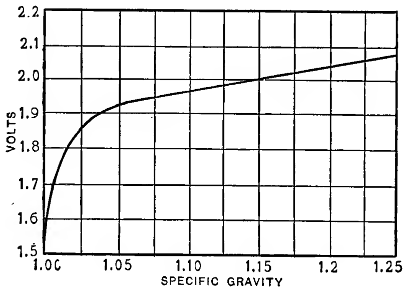


Fig. 188.

charge. The terminal voltages during the hours of charge and discharge as a function of the time are shown in Fig. 189.

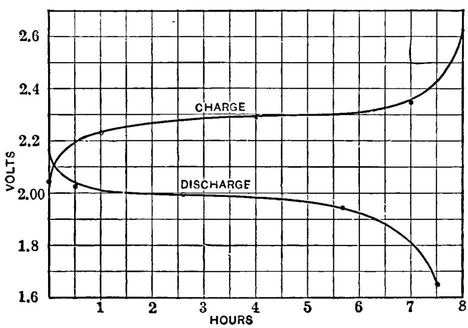


Fig. 189.

The *E.M.F.* of a fully charged cell is 2.5 volts. If an auxiliary cadmium electrode be inserted in the electrolyte its potential should be 2.3 volts below that of the positive plate and 0.2 volt above that of the negative plate. When fully discharged the *E.M.F.* of a cell is 1.8 volts and cadmium should have a potential 2.05 below the positive electrode and 0.25 below the negative. By means of such a cadmium test the condition of either electrode can be determined. It is common to take the average *E.M.F.* of a cell as 2 volts. To obtain a battery of greater *E.M.F.* a plurality of cells, connected in series, is employed.

The capacity rating of a storage cell is expressed by the number of ampere-hours which it will furnish in discharging itself at constant current from a fully charged condition to a point where its potential on open circuit is 1.8 volts, the discharge being completed after the expiration of 8 hours. The actual ampere-hour capacity decreases with an increase of discharge rate, that is if made in less than 8 hours. It is reduced to one-half if made in one hour. The capacity is from 40 to 60 ampere-hours per square foot of exposed area of positive electrode, counting both sides but taking no account of increase of surface due to porosity. The normal current rate of charge or discharge is therefore from 5 to 8 amperes per square foot. A continuous discharge should not exceed 25 amperes per square foot. Double this rate is permissible for 30 seconds or less. If it be desired to charge the cell rapidly the charging current should not be kept constant. To charge in three hours, for example, the current for the first hour should be 4 times the 8-hour rate, for the next 2.5 times, and for the last 1.5 times. Theoretically the amount of lead chemically modified per ampere-hour on either electrode is 0.135 oz. Practically from 0.5 to 0.9 oz. is required. There should be at least $\frac{1}{10}$ lb. of electrolyte for each ampere-hour.

It is customary to connect numbers of positive plates by a lead lug to form the positive electrode of a cell and to use one or more negative plates for the negative electrode. These are assembled in the electrolyte so that successive plates are of opposite polarity and both sides of each positive plate are exposed to a negative. Containing jars are

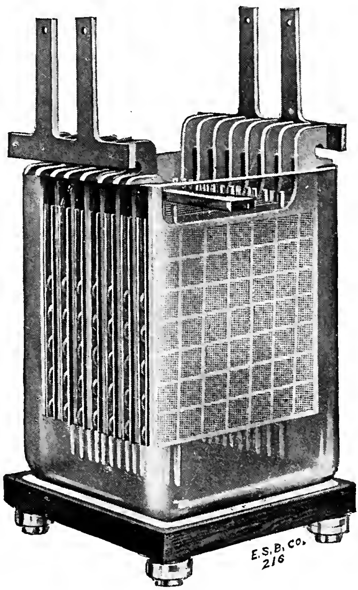


Fig. 190.

made of glass or hard rubber, and for large cells lead-lined wooden tanks are used. Fig. 190 shows a 560-ampere-hour cell in a glass jar. The power output per pound of complete cell is from 8 to 14 watts with pasted plates and from 3 to 7 watts with Planté plates.

PROBLEMS.

1. A dynamotor, the resistances of whose armature windings are each 0.1 ohm, is used as an equalizer on a 3-wire equivoltage system with 200 volts between outside conductors. One ampere flows through the armatures when the system is balanced. Find the power expended in frictions and the counter *E.M.F.* of each armature winding.

2. Find the regulation of each side of the system of the preceding problem if the maximum unbalanced load be 100 amperes.

3. A motor-generator, with one armature having a resistance two-thirds as great as the other and designed to generate at the same speed two-thirds the *E.M.F.* generated by the other, is used as a balancer between outer wires having a potential difference of 500 volts. The power required to overcome frictions is 400 watts and the resistance of the two armatures in series is 1.2 ohms. A current of 150 amperes flows through the middle wire. Determine the capacity of each unit of the balancer.

4. A 3-element four-wire balancer is used on a multivoltage variable-speed motor system with 40, 120, and 80 volts between successive wires. The armature resistances are proportional to the *E.M.F.* generated in them, and together amount to 0.48 ohm. On balanced load the armatures take 10 amperes. If the current between successive line wires be 30, 75, and 10 amperes respectively, what are the respective currents in the armatures of the motor-generator elements? What is the pressure between successive wires?

5. From a point one mile distant there returns to the generating station for a single-track railway, that uses 70-lb. (per yard) rails, 200 amperes by way of the rails. Two parallel No. 0000 copper wires in series with a negative booster and returning 300 amperes to the station, will reduce to zero the drop between this point and the station with what booster voltage? The resistivity of the rails, including bonds, is 110 ohms per mil-foot.

6. In Fig. 191 the curve represents a maximum daily load-curve, on a 500-volt system, variations in which are to be provided for by a storage battery. Determine the current I_{av} such that the charging ampere-hours shall exceed the quantity of discharge by 10%, and then obtain the ampere-hour capacity and the exposed area of positive plates per cell and the number of cells in an appropriate battery having end-cell regulation.

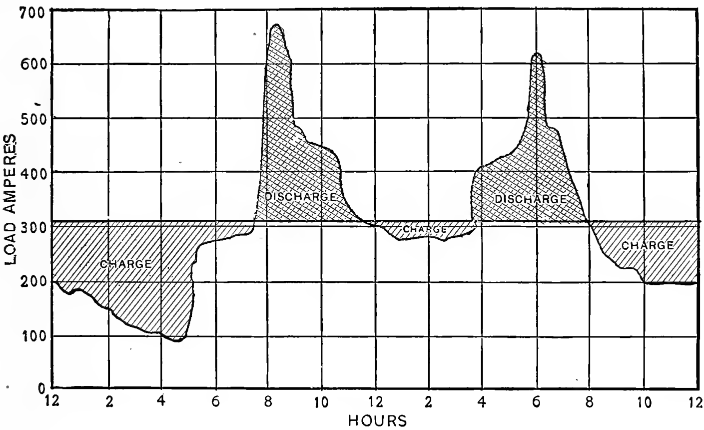


Fig. 191.

7. At 500 volts with battery arranged as in Fig. 184, how many cells would be required in the battery?

8. Plot a power output-time curve of the reversible booster of the preceding problem, the load curve being that shown in Fig. 191; each cell having an ampere-hour capacity as in problem 6, the voltage-hour curves of charge and discharge being those shown in Fig. 189, and the cell being considered as fully charged at the instant when the load first assumes its average value.

9. If the booster and its driving motor each have an overload capacity of 25% for two hours and have an efficiency of 90% at full load, what is the capacity of each in kilowatts? What is the maximum motor input?

CHAPTER IX.

CENTRAL-STATION EQUIPMENT.

114. Paralleling of Generators. — In general a generator is much more efficient when operated at its full load than when operated at one-half or one-quarter load. It is usual to install in central stations, which, as a rule, must supply different quantities of electrical energy at different times of the day, a number of smaller units rather than one unit large enough to supply the total energy. By this means any load can be handled by a machine or by a number of machines all operating at about their maximum efficiency. It is necessary, therefore, to consider the methods of combining two or more machines to supply energy to a single load.

The simplest and most usual method of connecting generators is that employed in incandescent light generating stations, where a number of constant-pressure machines are connected in parallel, the positive and negative terminals of each generator being connected respectively to positive and negative common conductors, called *bus-bars*, which are located on the rear of the operating switchboard. The connections of two shunt-wound machines with their regulating rheostats are shown in Fig. 192. The various load external circuits are connected in parallel to the bus-bars. This practice is frequently modified by separating those machines which supply energy to the more distant loads from those that supply the shorter circuits. This is because the maintaining of a constant and uniform pressure at all distributing points requires a higher pressure on the station ends of the longer mains than that on the shorter. When a machine is to be connected to bus-bars to which other operating machines are already connected, it is first brought

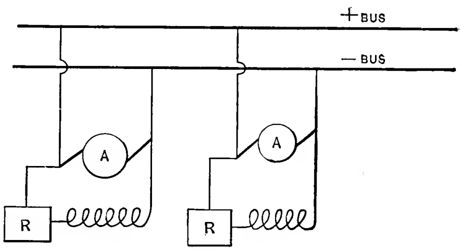


Fig. 192.

up to speed; the field magnetization is then adjusted till the machine generates the same voltage as that of the bus-bars, and the main switch is then closed, which puts the machine in circuit. The proper voltage at which to connect in the new machine may be roughly determined by comparing the relative brightness of its pilot lamp with that of the lamps operating on the circuit. A more exact way is to compare the readings of a voltmeter across the terminals of a machine with one across the bus-bars. Another method is to connect the generator to the bus-bars through a high resistance and a galvanometer indicator. When the latter indicates no deflection, the voltages of the machine and the bus-bars are identical, and the machine may be connected in. Sometimes the differential indicator is used for this purpose.

When shunt machines are connected in parallel, their voltages should be maintained the same so that the total load may be properly apportioned among them. If this equality of voltage be not maintained, no serious damage will occur, since the machine which generates the lower voltage merely fails to take its full share of the load. Even if the voltage of one machine falls so low that it is overpowered and run as a motor, still no damage will result, save perhaps the blowing of a fuse, since the direction of rotation for a shunt machine is the same whether it be run as a generator or as a motor. If it be desired to regulate a number of machines simultaneously by one regulator, it may be accomplished by bringing the positive ends of the field coils to one terminal of the regulator and connecting the other terminal to the negative bus.

Shunt machines may be operated in series by connecting the positive brush of one machine to the negative brush of the next, and connecting the extreme outside brushes with the line wires. Each machine can be regulated separately to generate any portion of the pressure, or, if it be desired to regulate all the machines thus connected uniformly and as a unit, the field coils of all the machines may be joined in series with one regulating rheostat, and shunted across the line wires. Fig. 193 illustrates such an arrangement for two 115-volt generators.

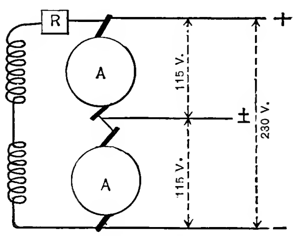


Fig. 193.

Series-wound generators may be operated in series, as in the Thury system of direct-current high-potential power transmission, which is in use on a number of lines in Europe. The aggregate capacity of the 15 plants at present employing this system is 25,000 horse-power, the line from Montiers to Lyons being the longest (112 miles) and employing the highest voltage (57,000 volts).

There are a number of groups of generators at the power house connected in series, each group being driven by a water turbine or other prime mover; the voltage per machine being less than 4000. Each generator is insulated from ground and from the other machines of the group, the middle point of the system being grounded to limit the required insulation. The line current is maintained constant by several complicated auxiliary devices. These automatically regulate the speed of all the prime movers, so as to keep the line voltage proportional to the load; cut in or out of circuit one or more machines if there be large changes in load; and short-circuit any disabled machine. In the substations, a number of series-wound motors are connected in series across the line, the motors being arranged in groups, each group driving a generator. The generators, which may deliver either direct or alternating current, are connected in multiple for distribution. The motors and generators in the substations are insulated from each other and from ground, just as are the machines in the generating station. The current taken by the motors is kept constant by an automatic shifting of the brushes. The Thury system is adapted only to undertakings where the power is to be transmitted over a long distance and the load is to be concentrated at few points, since at every tap a complete substation must be provided containing motors having an aggregate voltage equal to the line voltage. If the line alone be considered, direct-current power transmission is far superior to alternating-current transmission, which is employed exclusively in this country. Less conductor material is required and none of the disturbing influences met with in alternating-current transmission are encountered in the Thury system.

Difficulty is experienced if it be attempted to operate series generators in parallel. If the machines start with a proper distribution of load among them, and if one generates slightly less than its full voltage, then this machine does not continue to take its full share of the load; and, since it is series wound, the magnetic field becomes weakened, thus resulting in a still lower voltage. The conditions continually grow more uneven until the machine is overpowered and it becomes a motor. Since the direction of rotation of a series-wound motor is opposite to its direction when run as a generator, serious results may occur. One way in which this difficulty may be overcome is to arrange the field coils so that the magnetization in any one machine will remain the same as in the other machines, even though its pressure falls below that of the others. To accomplish this result the series fields must all be placed in parallel. This may be done by means of an *equalizer*, which is a wire of small resistance connecting all of the brushes of one polarity, and

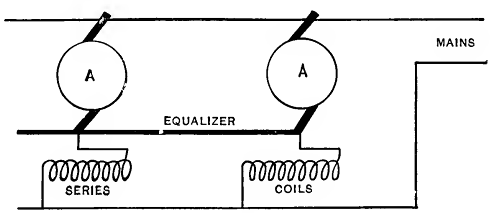


Fig. 194.

placing the field in parallel, as shown in Fig. 194. Two series dynamos may be run in parallel without an equalizer by resorting to *mutual excitation*, that is, by letting the current of one armature excite the field of the other. In this case, if the pressure of one machine falls and its load therefore decreases, the magnetization of the other is reduced, compelling the first to maintain its share of the load. Series dynamos are never operated in parallel in practice, but this discussion is introduced because of its application to the parallel operation of compound-wound dynamos.

Compound-wound generators are extensively used for constant-potential distribution. Since these machines have series field coils as well as shunt field coils, the parallel connection thereof for combined output should involve an equalizing bus, as in Fig. 195. Any number of compound generators may be operated in parallel regardless of the size of the units, provided their voltages are the same and that the resistances of their series field coils are inversely proportional to the currents supplied by the individual machines. Overcompounded generators which are to be operated in parallel must yield exactly the same voltage increase at full load or at any other load. This may be adjusted by interposing additional resistance in the series field coil of that generator which carries more than its share of the load.

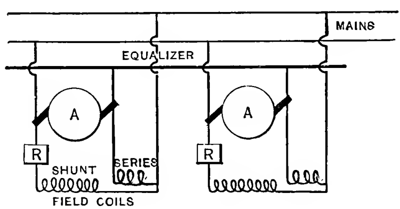


Fig. 195.

115. Parallel Operation of Motors. — Any number of shunt motors may be placed in parallel across constant-pressure mains, and their operation will be satisfactory whether each has a separate load or whether they be connected through suitable devices to a common shaft. Shunt motors will operate in series on a constant-pressure circuit when positively coupled together; but if connected to the same shaft by belts, and one belt slips or comes off, that motor will race, and receive more than its proper portion of the voltage. This arrangement is not common.

Series motors will operate satisfactorily on constant-pressure circuits if rigidly coupled to their loads. Series motors connected in series on constant-pressure mains will operate satisfactorily, dividing up the total voltage between them according to the load each is carrying. If it be desired to make them share a load equally they must be geared together so that each rotates at the speed corresponding to its share of the voltage. Series motors only are used on constant-current circuits. Any number of these may be placed in series on such a circuit individually or connected to a common shaft. A series motor on a constant-current circuit may be overloaded until it stops without harm, since a constant current flows at any speed.

Compound-wound motors are coming into quite general use, and they are invariably operated on constant-pressure circuits, and each machine has its own load.

116. **Switches.** — Switches are devices inserted in a circuit to facilitate its establishment or interruption. They are generally of the form known as knife switches, and may be single-pole, double-pole, etc., according to the number of circuit interruptions simultaneously effected. Fig. 196 shows a 3000-ampere 125-volt double-throw back-connected multiple-blade knife switch made by the Anderson Manufacturing Company.

The metal parts of such switches consist of copper hinges, blades, and clips, and these must be properly designed to have sufficient current-carrying capacity and contact surface.

It is usual to allow one square inch of cross-sectional area for every 800 to 1000 amperes, and to provide one square inch of contact surface for every 60 to 75 amperes of current. The distances between metal parts of opposite polarity and the break distances of approved switches, in

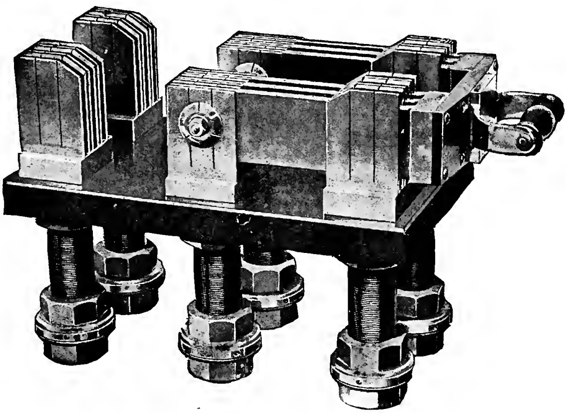


Fig. 196.

inches, are given in the following table. Frequently switches are provided with fuse connections, and also lugs into which the ends of the leads are soldered.

SIZE OF SWITCH	MINIMUM SEPARATION OF METAL PARTS OF OPPOSITE POLARITY			MINIMUM BREAK DISTANCE		
	125 V. OR LESS	125 V. TO 250 V.	250 V. TO 600 V.	125 V. OR LESS	125 V. TO 250 V.	250 V. TO 600 V.
10 amp. or less	1	1½	3½	¾	1¼	3
10–35 amperes	1¼	1¾	4	1	1½	3½
35–100 "	1½	2¼	4½	1¼	2	4
100–300 "	2¼	2½		2	2¼	
300–600 "	2¾	2¾		2½	2½	
600–1000 "	3	3		2¾	2¾	

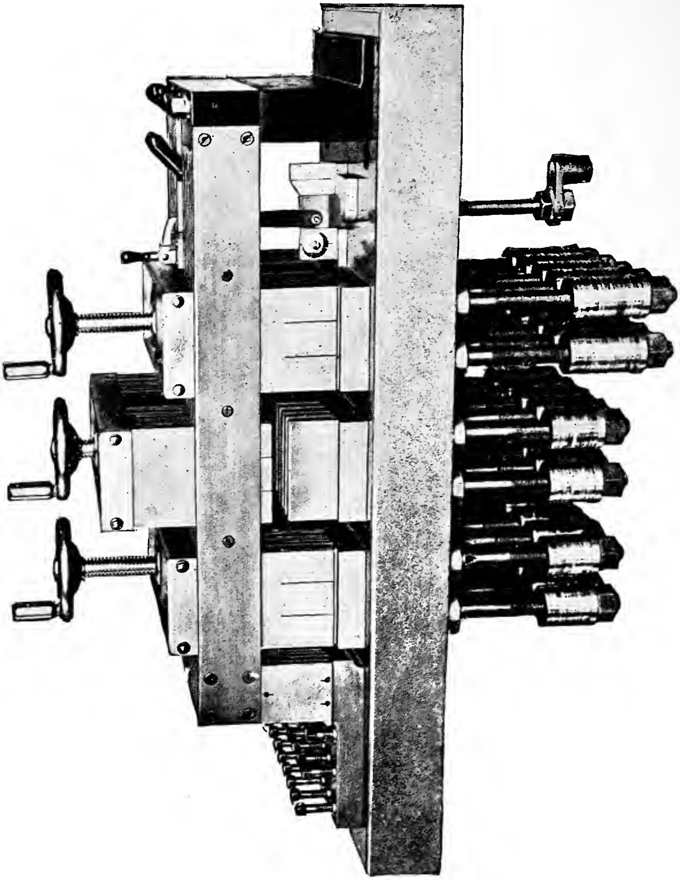


Fig. 197.

A 10000 ampere 500 volt "rotary" switch for use on central-station switchboards is shown in Fig. 197.

117. Fuses. — Fuses are devices intended to protect circuits from destruction or damage which might result from an excessive flow of current through them. They are made of fusible material, generally of lead or an alloy of tin and lead, and take the form of wire or strips provided at each end with a copper terminal which is slotted to fit into fuse receptacles.

The magnitude of the current which will melt a fuse depends upon the length of the wire. Short lengths of a wire of given cross-section and given material will carry larger currents than longer lengths. The heat which is generated in the short ones escapes more rapidly, owing to the proximity of large masses of metal which commonly form the terminals of the fuse. Fuses are rated at 80 per cent of the greatest current they can carry indefinitely without melting. This rating enables the fuse to carry a current 25 per cent greater than the normal current for which the fuse is designed.

The "blowing" of a fuse is accompanied by a flash and a spattering of the fused metal; this may ignite near-by combustible materials. Therefore link fuses should be placed in separate porcelain or other fireproof receptacles. The better practice is to use enclosed fuses, in which the fusible conductor is surrounded by a finely divided powder contained in an insulating casing.

118. Circuit Breakers.—In central-station practice the fuse with its uncertainties has been superseded by the electro-magnetic circuit breaker. This device acts promptly and definitely, and has the advantage that a circuit once opened by it, due to an excessive current therein, may be instantly reëstablished.

A circuit breaker consists of a switch which may be closed against the action of a strong spring and kept closed by means of a latch. This latch is controlled by the plunger of a solenoid which is connected in series with the line. When an abnormal current flows through the circuit, the plunger is attracted and strikes against a trigger which releases the latch. The spring then becomes operative and the circuit breaker is opened. The opening of a circuit carrying an excessive current is accompanied by an arc across the break. To avoid arcing across the metal contacts of a circuit breaker, this device is provided with an auxiliary set of carbon contacts so arranged that the latter are opened an instant later than the main metal contacts. Thus all the sparking takes place at the renewable carbon contacts.

A Westinghouse single-pole 500-volt circuit breaker for use with compound-wound generators is shown in Fig. 198.

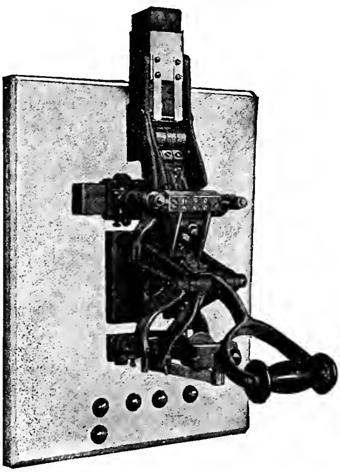


Fig. 198.

The solenoid is a massive copper coil located on the rear of the marble panel. The movable member is built up of thin sheets of spring copper which contact edgewise against solid copper terminal blocks. This instrument may be adjusted to open the circuit which it protects for any predetermined current strength between the limits of 20 % less than, and 50 % in excess of, the normal current.

When generators are operated in parallel, each machine should be protected by a *reverse-current* circuit breaker, which will open when a reverse current of predetermined value flows. The usual *overload* circuit breaker, just described, may be used for this purpose when provided with a relay; or, better still, a circuit breaker equipped with two solenoids may be used, one of which operates on the passage of an excessive current and the other on the flowing of a reverse current. A circuit breaker especially suitable for storage-battery work to prevent the current from flowing back through the generator is the *underload* circuit breaker. It is usually adjusted to open the circuit when the current falls below one-tenth of its normal rated value. Another type of circuit breaker is that having a *no-voltage release;* this instrument is particularly adapted for the protection of motor circuits from dangers accruing from the circuit remaining closed while the line is idle.

119. Measuring Instruments. — The instruments employed in direct-current work are ammeters, voltmeters and wattmeters, for measuring respectively current, voltage and power. Every instrument has some movable part to which a pointer, passing over a divided scale, is attached. Two forces act upon this moving element, one causing a deflection thereof from its zero position, and the other, opposing the first, limits the deflection so that the position of the moving element when in equilibrium gives a proper indication of the magnitude of the deflecting force.

Ammeters. There are four types of commercial ammeters: (1) those in which the force-action between two coils carrying current serves as a measure of that current, (2) those in which the force-action between a permanent magnet and a coil carrying current is utilized as a measure of the current, (3) those in which the amount of attraction of a soft-iron core or vane by a coil carrying current serves as an indication of the current strength, and (4) those in which the expansion of a wire heated by the passage of a current through it is utilized as a measure of the current.

The electrodynamometer is an ammeter of the first type, and consists of two coils connected in series, one coil being fixed and the other movable. The planes of these coils are normally at right angles to each other, but when a current flows through them they tend to place themselves in the same plane. This tendency of rotation of the movable coil is resisted by a torsional spring. The angle through which the spring is turned in order to restore the coil while carrying the current to its original position is measured by means of a pointer and a dial. If α be the angle turned through, then the current strength is

$$I = k\sqrt{\alpha},$$

where k is a constant determined by calibration.

Ammeters of the second type may have the coil fixed and the permanent magnet movable, but the moving-coil instruments, such as the Weston meter, are more generally used. The Weston instrument consists of a coil composed of a large number of turns of fine insulated copper wire wound on a light rectangular frame of copper or aluminum, this coil being pivoted in jeweled bearings and mounted in an annular space between the poles of a permanent magnet and a soft-iron core at the center, as shown in Fig. 199. Two spiral springs serve to carry the current to and from the moving coil, and also control the amount of deflection. The current strength is directly proportional to the angle of deflection, and therefore the scales of such instruments are uniform over the entire range.

Ammeters in which a soft-iron piece is attracted by an electromagnet carrying the current to be indicated are most generally used for the measurement of alternating currents, but serve equally well for direct currents. In the most approved form of this type of instrument the current to be measured passes through a fixed coil and thereby magnetizes a soft-iron vane which is pivoted and controlled by a spring. Magnetizing the vane causes it to move, the amount of movement indicating the current strength on the properly calibrated scale.

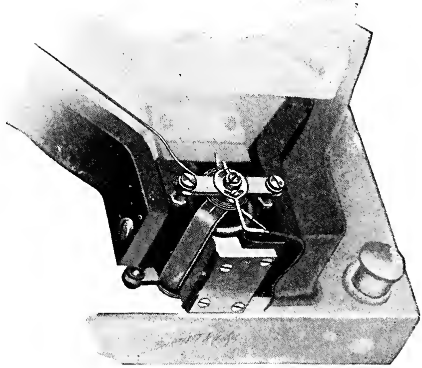


Fig. 199.

The principle of operation of the hot-wire type of ammeter is that the heat produced by a current which traverses a wire having a negligible temperature coefficient of resistance is proportional to the square of that current, and, since the temperature rise of the wire is proportional to the heating, and the linear expansion of the wire is proportional to the temperature rise, it follows that the linear expansion is directly proportional to the square of the current value. The expansion of the wire may be observed by a pointer, P, which passes over a calibrated scale, the arrangement being as shown in Fig. 200.

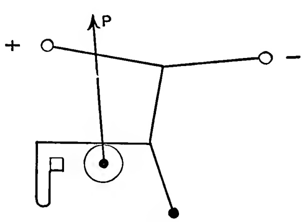


Fig. 200.

Frequently only a small part of the whole current to be measured passes through the ammeter coil, the remainder flowing through a by-path of low resistance, called a *shunt.* The resistance of a shunt for a given instrument is so proportioned that a full-scale deflection will be produced when a specified current flows through the shunt. The size of the shunt is designated by this current value. If it be proposed to use a moving-coil instrument, which gives a full-scale deflection on E volts, to indicate a maximum current of I amperes, then the resistance of the shunt must be $\frac{E}{I}$ ohm.

Voltmeters. Most voltmeters are simply ammeters of very high resistance. They may therefore be connected to supply mains without causing more than a slight flow of current through the instrument. The resistance of voltmeters is in the neighborhood of 100 ohms per volt of maximum scale deflection. The appearance of switchboard voltmeters and ammeters is shown in Fig. 201.

Fig. 201.

Another type of voltmeter, depending for its operation on the attraction between two electrically charged bodies, is the electrostatic voltmeter. Low-voltage instruments of this type consist of a number of thin plates horizontally suspended between corresponding quadrants, and fitted with a pointer which plays over a divided scale.

In order to vary the range of the usual type of voltmeters, resistance is connected in series with the instruments, such resistances being known as *multipliers*. If R be the resistance of a low-reading voltmeter, and r be that of the multiplier, then the range of the instrument has been increased $\frac{r + R}{R}$ times. Multipliers should be wound non-inductively with wire having a negligible temperature coefficient of resistance.

Wattmeters. The power delivered to direct-current receiving circuits may be determined from voltmeter and ammeter readings, or may be measured directly by means of a wattmeter. This instrument consists of a fixed coil, which is connected in series with the load circuit, and a movable coil, which is connected in series with a high resistance across the supply mains, as shown in Fig. 202.

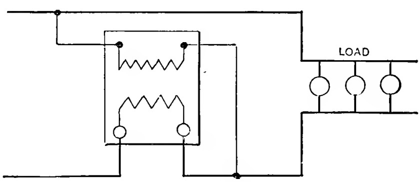


Fig. 202.

The deflecting force is proportional to the product of the currents flowing in the two windings, but the current flowing in the movable coil is proportional to the voltage of the circuit; therefore the deflecting force is proportional to the product of the current supplied and the voltage, or to the power delivered to the load. Wattmeters are used principally in connection with alternating-current measurements.

To measure the energy delivered to a circuit, *watt-hour* meters are employed. In order that an instrument may record the time element, some part of the meter must move constantly through unit distance for each unit of energy delivered, and this movement must be permanently recorded by a suitable device such as a dial train. The Thomson watt-hour meter, shown in Fig. 203, consists of a spherical armature rotating within two circular field coils, one on either side of the armature. This instrument is connected to the circuit in the same manner as the indicating watt-meter. The armature is carried by a vertical spindle, the lower end of which rests in a jeweled bearing, and the upper end is provided with a worm which meshes with a chain of wheels constituting the counting mechanism. The torque produced is proportional to the product of the field flux and the armature current. As there is no iron in the magnetic circuit and since the speed of rotation is low, little or no counter *E.M.F.* will be induced in the armature coil; thus the armature current is independent of the speed, and is directly proportional to the line voltage. For the same reason the field flux is proportional to the main current. Therefore the torque is proportional to the power consumed by the load. In order that the meter shall record correctly it is only necessary to provide some means for making the

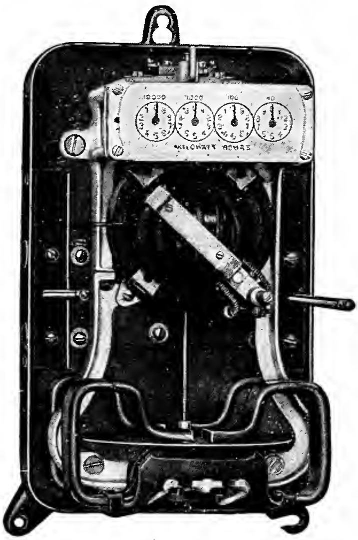


Fig. 203.

speed of rotation proportional to the torque. This is accomplished by applying a magnetic drag, in the form of an aluminum disk fastened to the armature spindle and passing between the poles of permanent magnets. The electromotive forces induced in the disk are proportional to the number of lines of force cut in a given time, and, since the resistance of the disk is constant, the strength of the eddy currents will be proportional to the rate of cutting lines of force, and consequently will vary with the speed of rotation. The drag or counter torque, being proportional to the product of the constant flux and the eddy currents, will vary directly with the speed. To overcome mechanical friction an auxiliary field coil or starting coil is provided which consists of a few turns of fine wire connected in series with the armature.

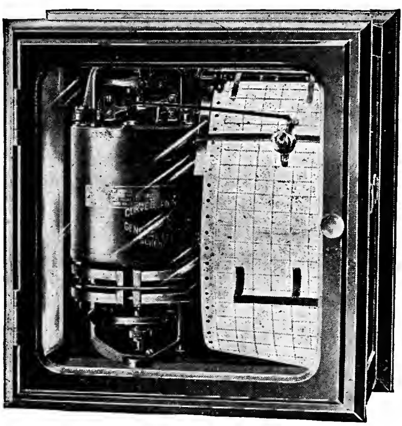


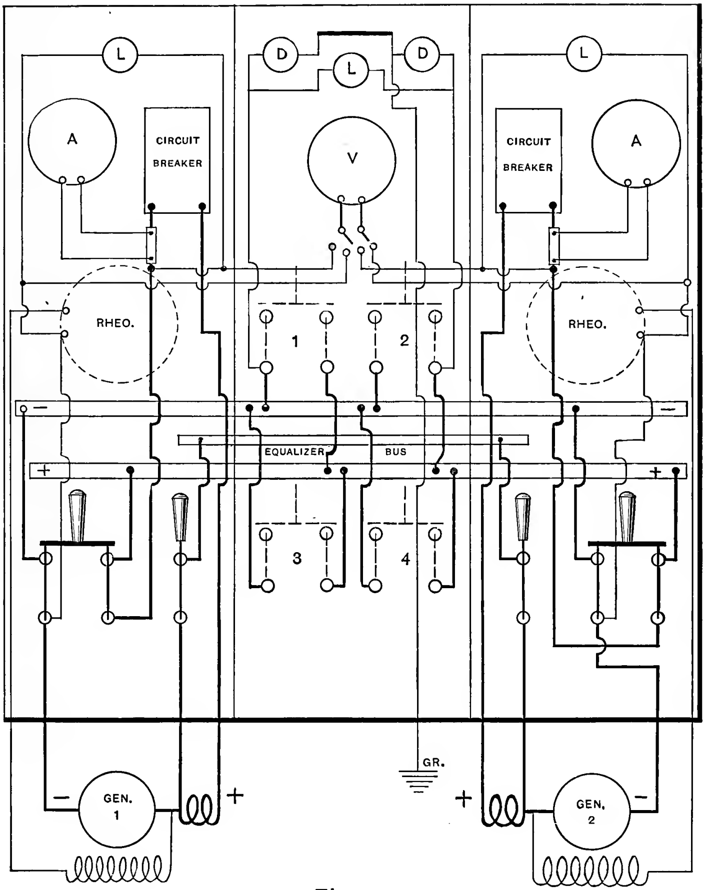
Fig. 204.

Instruments for recording successive instantaneous values of current, voltage or power are called *recording instruments*. They operate on the same principles as indicating instruments, and are made recording by fitting the pointer with an ink pen which presses against a paper chart wound on a drum, the latter rotating slowly at constant speed by clockwork. A curve-drawing wattmeter is shown in Fig. 204.

120. Switchboards. — The object of a central-station switchboard is to group the necessary devices for controlling, distributing and measuring the current received or delivered, particular attention being directed toward locating these devices for convenient operation. Safety apparatus for protecting generators or the lines against abnormal conditions are sometimes placed upon the switchboard.

Switchboards are designed with a view to a symmetrical arrangement of the apparatus and instruments, and it is usual to place all similar devices in the same horizontal row. As a rule circuit breakers are located at the top of the board and recording instruments at the bottom. Measuring instruments are placed at such height as to be conveniently read by the switchboard attendant. Rheostats may be placed above or below the switchboard at the rear, but the handles controlling them, through the agency of chains and sprocket wheels, must be located so that the attendant can manipulate them and note the instrument indications simultaneously. The bus-bars and the connections from them to the switches, circuit breakers, rheostats and to the instruments are mounted on the rear of the switchboard.

Switchboards are constructed so that each panel controls the apparatus for a single generator or controls a definite number of feeders. Fig. 205 illustrates a switchboard for two compound-wound generators, and consisting of two generator panels and one feeder panel for four feeders. Each generator may be connected to the bus-bars by means of a main switch and a circuit breaker. The current supplied to the load by each generator is measured by a separate ammeter and shunt, but the voltage of both machines is measured alternately by a single voltmeter which may be connected to either machine by a voltmeter switch


L
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CIRCUIT BREAKER
A
RHEO.
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2
RHEO.
−
−
EQUALIZER
BUS
+
+
3
4
GR.
−
GEN. 1
+
+
GEN. 2
−

Fig. 205.

on the feeder panel. The rheostats on the generator panels regulate the excitation of the generator shunt fields. In order that these compound-wound machines may be operated in parallel, equalizer switches and an equalizer bus-bar are necessary. A lamp on each panel provides illumination for the scales of the measuring instruments; the lamps on the generator panels also serve as pilot lamps. The outer lamps on the feeder panel constitute a *ground detector*, and are used for indicating grounds. Should a partial ground occur on one line the corresponding lamp would burn dimly and the other brightly, thus indicating by their relative brightness the extent of the fault.

121. **Works Cost.** — The costs of the various items which are involved in the manufacture of dynamo-electric machines are so dependent upon the types of the specific designs that a comprehensive discussion of them cannot be given in this book. Commercial designers, however, are responsible for the manufacture of machines which are as inexpensive as is consistent with satisfactory operation. Although, with a machine of given speed and output, the detail costs for labor, iron, copper and insulating material may vary widely in different cases, the total works costs are not very different. If the diameter and over-all length of the armature be D and l inches respectively, then the works cost in dollars may be expressed as

$$\text{Cost} = KDl,$$

where K is a function of the speed and output of the machine. This function is fairly constant for pressures between 100 and 500 volts, for the extra costs of insulation in the construction of the machines of higher voltage are compensated for by the reduced costs of commutator copper.

The values of K may be obtained from the curves shown in Fig. 206, which correspond to peripheral velocities, v, of the armature of 200, 150, and 100 or less feet per second. The increased values for velocities above 100 are due to increased costs of labor and of suitable material for withstanding the higher centrifugal forces.

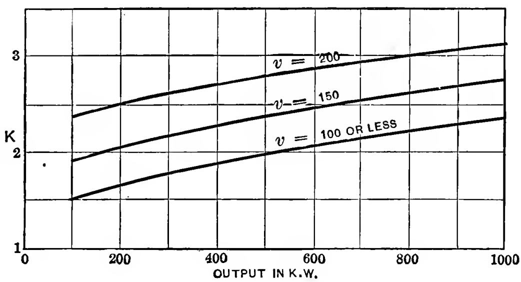

Fig. 206.

122. **Selling Prices.**—The selling price of a machine must exceed the works cost by an amount sufficient to include the profit and such expenses as litigation, advertising, and sales commissions or expenses. It is customary for manufacturing concerns to print in their commercial publications list-prices of machines which considerably exceed the total costs of them when delivered. Substantial discounts are allowed to purchasers, amounting to as much as 50 % in some cases, and depending upon the extent of purchase, date of payment and many other complex conditions. The actual selling price per K.W. is dependent upon the speed, output and type of machine. To give a general idea as to the selling prices of compound-wound generators the curves of Figs. 207 and 208 are given. The prices are not far from those which would be paid by unfavored pur-

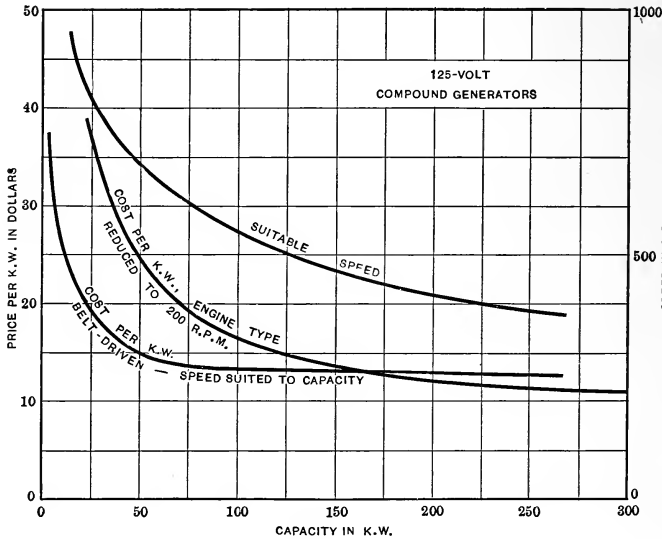

Fig. 207.

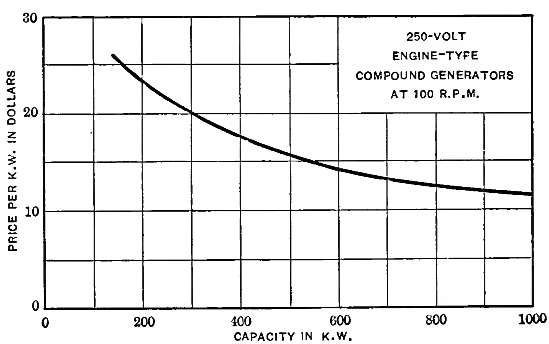

Fig. 208.

chasers in 1910. In Fig. 209 are given the selling prices of 230-volt shunt or series motors for normal speeds of 1000 revolutions per minute, and of 125-volt balancers.

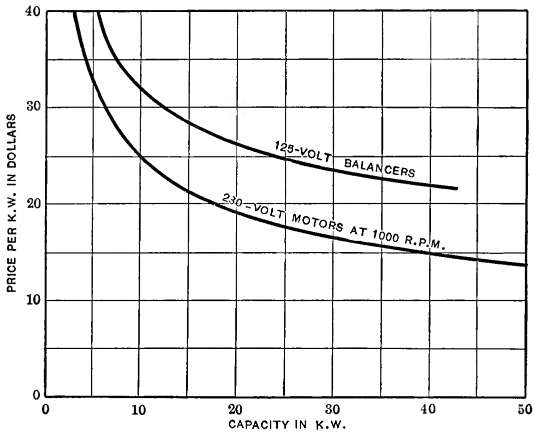

Fig. 209.

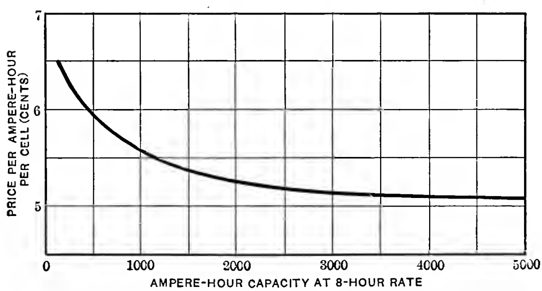

Fig. 210.

The selling prices of lead-lead-sulphuric acid storage cells per ampere-hour of capacity at an 8-hour rate are embodied in the curve of Fig. 210 as functions of the capacities.

123. Plant Costs. — A fair average cost per kilowatt for a steam-driven power plant is $100, although with steam-turbine plants it may be placed at $80. For hydraulic plants the cost is greater than these, and has been estimated at $200 per kilowatt. These prices are applicable to central stations of reasonably large capacity. In such stations alternating-current generators are usually employed. Direct-current generators are more generally used in isolated plants, such as in office buildings or in manufactories, where steam is generated for heating or power purposes and its use for driving electrical generators is subsidiary and incidental. In such cases the cost of delivery and of erection of generators may be estimated as 60 cents per K.W. The various installation costs may be estimated from the data given by Stott in the following table:

POWER PLANT COST PER K.W.

	Minimum	Maximum
1. Real Estate	$3.00	$7.00
2. Excavation	.75	1.25
3. Foundations, Reciprocating Engines	2.00	3.00
4. Foundations, Turbines	.50	.75
5. Iron and Steel Structure	8.00	10.00
6. Building (Roof and Main Floor)	8.00	10.00
7. Floors, Galleries and Platforms	1.50	2.50
8. Tunnels, Intake and Discharge	1.40	2.80
9. Ash Storage Pocket, etc.	.70	1.50
10. Coal-hoisting Tower	1.20	2.00
11. Cranes	.40	.60
12. Coal and Ash Conveyors	2.00	2.75
13. Ash Cars, Locomotives and Tracks	.15	.30
14. Coal and Ash Chutes, etc.	.40	1.00
15. Water Meters, Storage Tanks and Mains	.50	1.00
16. Stacks	1.25	2.00
17. Boilers	9.50	11.50
18. Boiler Setting	1.25	1.75

	Minimum	Maximum
19. Stokers	$1.30	$2.20
20. Economizers	1.30	2.25
21. Flues, Dampers and Regulators	.60	.90
22. Forced Draught Blowers, Air Ducts, etc.	1.25	1.65
23. Boiler Feed and Other Pumps	.40	.75
24. Feed-water Heaters, etc.	.20	.35
25. Steam and Water Piping, Traps, Separators, High and Low Pressure	3.00	5.00
26. Pipe Covering	.60	1.00
27. Valves	.60	1.00
28. Main Engines, Reciprocating	22.00	30.00
29. Exciter Engines, Reciprocating	.40	.70
30. Condensers, Barometric or Jet	1.00	2.50
31. Condensers, Surface	6.00	7.50
32. Electric Generators	16.00	22.00
33. Exciters	.60	.80
34. Steam Turbine Units Complete	22.00	32.00
35. Rotaries, Transformers, Blowers, etc.	.60	1.00
36. Switchboards Complete	3.00	3.90
37. Wiring for Lights, Motors, etc.	.20	.30
38. Oiling System Complete	.15	.35
39. Compressed Air System and Other Small Auxiliaries	.20	.30
40. Painting, Labor, etc.	1.25	1.75
41. Extras	2.00	2.00
42. Engineering Expenses and Inspection	4.00	6.00

Costs of Excavation near New York City

Earth	$2.44 per cu. yd.
Rock	6.00 per cu. yd.
Brickwork	11.30 per cu. yd.

124. **Operating Expenses.** — There are many items which make up the expenses for operating a plant and for making repairs which are essential for maintaining the machinery in good running condition. The relative costs of these items have been given by Stott as listed in the following table:

DISTRIBUTION OF MAINTENANCE AND OPERATING CHARGES.

	Reciprocating Engines	Steam Turbines	Reciprocating Engines and Steam Turbines	Gas Engine Plant	Gas Engines and Steam Turbines
MAINTENANCE					
1. Engine Room Mechanical	2.57	0.51	1.54	2.57	1.54
2. Boiler Room or Producer Room	4.61	4.30	3.52	1.15	1.95
3. Coal- and Ash-handling Apparatus	0.58	0.54	0.44	0.29	0.29
4. Electrical Apparatus	1.12	1.12	1.12	1.12	1.12
OPERATION					
5. Coal- and Ash-handling Labor	2.26	2.11	1.74	1.13	1.13
6. Removal of Ashes	1.06	0.94	0.80	0.53	0.50
7. Dock Rental	0.74	0.74	0.74	0.74	0.74
8. Boiler-room Labor	7.15	6.68	5.46	1.79	3.03
9. Boiler-room Oil, Waste, etc.	0.17	0.17	0.17	0.17	0.17
10. Coal	61.30	57.30	46.87	26.31	25.77
11. Water	7.14	0.71	5.46	3.57	2.14
12. Engine-room Mechanical Labor	6.71	1.35	4.03	6.71	4.03
13. Lubrication	1.77	0.35	1.01	1.77	1.06
14. Waste, etc.	0.30	0.30	0.30	0.30	0.30
15. Electrical Labor	2.52	2.52	2.52	2.52	2.52
Relative Cost of Maintenance and Operation	100.00	79.64	75.72	50.67	46.32
Relative Investment in per cent	100.00	82.50	77.00	100.00	91.20

125. Cost of Electrical Energy. — The cost attendant upon the delivery of electrical energy at the bus-bars of a central station is made up of two factors.

The first is constant in magnitude, is independent of the amount of delivered energy, and appears in the station records as a *fixed charge*. It includes such items as interest on the investment, insurance, taxes, depreciation, and obsolescence. The last item is one which is due to the frequent improvements in central station apparatus, which make it desirable at times to cast aside operative machines before they are worn out, in order to take advantage of the greater efficiency afforded by newer types. Dr. C. T. Hutchinson gives the following approximate values for the items of fixed charges expressed as percentages of the cost of the plant:

FIXED CHARGES OF GENERATING PLANTS.

	STEAM	WATER
Interest	6.0 %	6.0 %
Insurance	0.5	
Taxes	0.5	0.5
Depreciation	5.0	1.0
Obsolescence	5.0	1.5
	17.0 %	9.0 %

The second factor is the *maintenance and operating expense*, which depends upon the amount of electrical energy delivered to the bus-bars and varies with it. Although this expense per kilowatt-hour varies somewhat with the ratio of the load to the maximum capacity of the plant, it may safely be estimated at 0.5 cent.

The average cost throughout a day or year of a unit of delivered electrical energy, therefore, depends upon the ratio of the average power to the maximum power. This ratio is termed the *load factor* of the plant. Inasmuch as all generators have an overload capacity, due to their ability to store heat for a limited time without an excessive resultant rise of temperature, the maximum power is really taken as the average power for say an hour during the period of maximum load. It is common for load factors to be as low as 10 %, although that of the Interborough Rapid Transit Company varies between 50 % and 55 %. The influence of the load factor upon the cost of a kilowatt-hour of electrical energy is shown in the following table, the calculations being based upon a 1000-K.W. steam-turbine plant costing $80 per kilowatt and operating continuously throughout the year, viz. for 8760 hours.

EFFECT OF LOAD FACTOR ON COST OF POWER.

COSTS	LOAD FACTORS				
	0.2	0.4	0.6	0.8	1.0
Fixed Charges	0.775	0.388	0.258	0.194	0.155
Operating Expenses	0.500	0.500	0.500	0.500	0.500
Total per Kilowatt-hour in Cents .	1.275	0.888	0.758	0.694	0.655

PROBLEMS.

1. Design a good 500-ampere, 110-volt, double-pole, single-throw, back-connected switch, but use no more material than necessary. How many pounds of copper are required if the terminal studs be 5 inches long?

2. A moving-coil instrument gives a full-scale deflection on 0.068 volt. What is the resistance of a 200-ampere shunt therefor?

3. The resistance of a 150-volt voltmeter is 14,000 ohms. What must be the resistance of a multiplier for this instrument so that it can measure *E.M.F.*'s up to 750 volts?

4. A watt-hour meter without a starting coil reads correctly when run on a 1-K.W. load. On light load a power consumption of 100 watts will just start the armature rotating. What will be the meter indication after running $2\frac{1}{2}$ hours on a constant load of 600 watts, assuming running friction as one-half of starting friction?

5. Lay out the connections of a suitable switchboard for an isolated plant having a single compound-wound generator, which is intended to supply three feeder circuits.

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