CONCRETE AND REINFORCED CONCRETE

A CONDENSED PRACTICAL TREATISE ON THE PROBLEMS OF CONCRETE CONSTRUCTION, INCLUDING CEMENT MIXTURES, TESTS, BEAM AND SLAB DESIGN, CONSTRUCTION WORK, RETAINING WALLS, ETC.

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SPREADING CONCRETE OVER REINFORCING STEEL BY MEANS OF TOWER AND DISTRIBUTING CHUTE

Courtesy of Leonard Construction Company, General Contractors, Chicago

INTRODUCTION

CONCRETE has in a comparatively short span of years become one of the most useful, if not the most useful, substance in the hands of the constructor. Its many forms include the simpler monolithic construction as a substitute for stone as well as the more complicated reinforced concrete types. The interest of the reading public in this subject makes an authoritative yet condensed treatise particularly timely.

¶ In the preparation of this volume the author has endeavored to present the subject in a simple and concise manner suitable for both engineers and students. The composition and treatment of cement, sand, stone, and mortar, the mixtures commonly used, the steel for reinforcing, and the fireproofing qualities of concrete are all sufficiently discussed to give accurate knowledge of their relation to the general subject in hand. The general theory of flexure in reinforced concrete, the author has taken particular pains to make clear and simple. This is also true of the design of the ordinary beam and girder type of floor, special reference being made to the bonding of steel and concrete, vertical shear, and diagonal tension. The text includes tables and diagrams by means of which designs may be made without the use of other portions of the text.

¶ Other important features are the treatment of flat-slab construction, which is complete and in accordance with the best engineering practice; the design of simple and compound footings; gravity and reinforced retaining walls; culverts; girders; and miscellaneous structures. The remainder of the book is devoted to construction work, covering equipment, methods of mixing and transporting concrete, form work for columns, slabs, beams, and walls, and the proper location of construction joints. To drive these suggestions home a number of typical examples of reinforced concrete construction, such as buildings, bridges, large sewers, and tanks, are fully analyzed.

¶ Altogether, the treatise gives just the material the engineer or contractor needs for his work, or that the layman will find interesting for his general reading. It is the hope of the publishers that the book will prove of distinct value in the field of concrete construction.

CHICAGO SERVICE BUILDING OF FORD MOTOR COMPANY, SHOWING MODERN CONCRETE CONSTRUCTION
Courtesy of the Condron Company, Chicago

REINFORCED CONCRETE WORK ON LOWER GATE CHAMBERS FOR ASHOKAN DAM PROJECT

Photo by Brown Brothers, New York City

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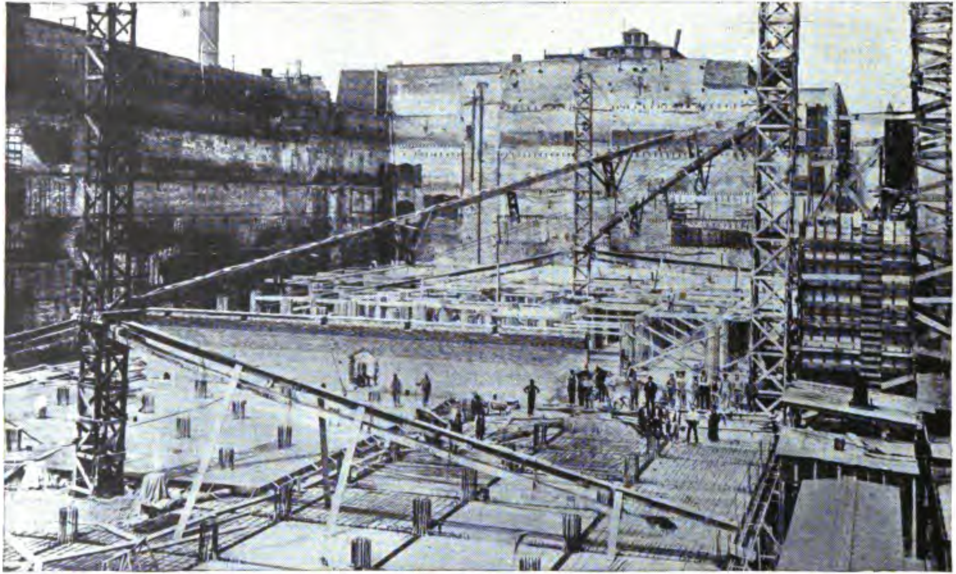


REINFORCED CONCRETE WAREHOUSE BUILT FOR NELSON MORRIS & COMPANY, CHICAGO, ON SITE OF OLD BUILDING WHERE CHIEF HORAN AND A NUMBER OF FIREMEN LOST THEIR LIVES

Courtesy of C. D. Rawstorne, Supervising Engineer for Nelson Morris & Company

CONCRETE AND REINFORCED CONCRETE

Concrete as a substitute for masonry has, in a comparatively few years, earned a very important place for itself in engineering and building work. In the early years of its use, before a proper judgment as to its limitations had been formed, failure of concrete structures created a prejudice against the material, but this has now been dispelled. The evolution of the product has served to standardize concrete as a material, and exhaustive experiments and theoretical deductions supplied by eminent engineers have so standardized the designs of reinforced concrete structures that failures have become practically unknown. The cheapness of concrete and its adaptability to all forms of construction, particularly those of monolithic character, have resulted in wider and wider applications of the material.

Concrete is composed of a mixture of cement, sand, and crushed stone or gravel, which, after being mixed with water, soon sets and obtains a hardness and strength equal to that of a good building stone. A study of the materials composing concrete and of the effect of varying the proportions of these materials will be helpful before taking up the questions of design and erection of concrete structures.

COMPOSITION AND TREATMENT

CEMENT

Cement is manufactured by properly burning, cooling, and grinding a composition of argillaceous and calcareous materials. The burning and cooling require great care, for complicated chemical reactions take place during the process. After cooling, the mass is ground into a powder, most of which will pass a wire sieve having 200 wires per lineal inch. Very fine cements are better than the coarser varieties, other things being equal.

In color, cement varies through the different shades of grays, depending on the color of the stone from which it is made.

Cement, when mixed with water and allowed to set, will harden in a few hours, and should develop a greater part of its ultimate strength in a few days. It is permanent in the respect that no essential change in form or volume will take place, either on account of inherent qualities, or as the result of exterior agencies. There is always a slight shrinkage in the volume of cement or concrete during the process of setting and hardening, but with good cement this shrinkage is not so great as to be objectionable.

Classification of Cement. Portland cement and natural cement are the two principal cements in common use.

Natural Cement. Natural cement is obtained by burning argillaceous limestone, which, in its natural state, has the proper chemical composition. Such cement was formerly, and is sometimes still, called Rosendale cement, from the fact that it was first produced in Rosendale, New York. Rock from which natural cement can be made is found in different sections of the country.

Natural cement is not so uniform in composition as Portland cement, and, therefore, not so reliable. It is quick setting and requires more water in mixing than Portland cement. Since the cement is made wholly from the rock just as the latter is taken out of the quarry, and since the rock is calcined at a much lower temperature than that employed in making Portland cement, natural cement is considerably cheaper than Portland cement. It weighs 300 pounds per barrel of about 4 cubic feet. Natural cement is employed only on account of its cheapness, and only when low stresses are used or quick setting is required. It should never be used for reinforced concrete work.

Portland Cement. Portland cement is obtained from the calcination to incipient fusion of a thorough mixture of proportioned argillaceous and calcareous materials. It is made in many sections of the country and is now in common use; unless natural cement is especially mentioned, Portland cement is the grade which is referred to. It is much stronger than natural cement and more nearly uniform in all of its properties. A barrel of Portland cement usually contains 3.6 cubic feet and weighs 384 pounds net. The cement is generally put up in bags, each containing $\frac{9}{10}$ cubic foot and weighing 96 pounds, that is, one-fourth of a barrel.

Portland cement is extensively used. It is essential in reinforced concrete work and when concrete is laid under water or is subjected to severe and repeated stresses. A preliminary report which was made by the U. S. government shows that approximately 88,514,000 barrels of cement were made in the United States during 1914; the production for 1913 was 92,097,131 barrels. These figures indicate the great importance of cement today in all sorts of construction work.

Standard Cement Specifications

A committee of the American Society for Testing Materials presented to that body the following report on cement testing:*

General Observations

1. These remarks have been prepared with a view to pointing out the pertinent features of the various requirements and the precautions to be observed in the interpretation of the results of the tests.

2. The committee would suggest that the acceptance or rejection under these specifications be based on tests made by an experienced person having the proper means for making the tests.

Specific Gravity

3. Specific gravity is useful in determining adulteration. The results of tests of specific gravity are not necessarily conclusive as an indication of the quality of a cement, but when in combination with the results of other tests may afford valuable indications.

Fineness

4. The sieves should be kept thoroughly dry.

Time of Setting

5. Great care should be exercised to maintain the test pieces under as uniform conditions as possible. A sudden change or a wide range of temperature in the testing room, a very dry or humid atmosphere, and other irregularities vitally affect the rate of setting.

Constancy of Volume

6. The tests for constancy of volume are divided into two classes, the first normal, the second accelerated. The latter should be re-

*Adopted August 16, 1909, except the sections on methods of testing cement referred to in §9 p. 4 which were adopted January 17, 1912.

garded as a precautionary test only, and not infallible. So many conditions enter into the making and interpreting of it that it should be used with extreme care.

7. In making the pats, the greatest care should be exercised to avoid initial strains due to molding or to too rapid drying-out during the first 24 hours. The pats should be preserved under the most uniform conditions possible, and rapid changes of temperature should be avoided.

8. The failure to meet the requirements of the accelerated tests need not be sufficient cause for rejection. The cement, however, may be held for 28 days, and a retest made at the end of that period, using a new sample. Failure to meet the requirements at this time should be considered sufficient cause for rejection, although in the present state of our knowledge it cannot be said that such failure necessarily indicates unsoundness, nor can the cement be considered entirely satisfactory simply because it passes the tests.

General Conditions

1. All cement shall be inspected.

2. Cement may be inspected at the place of manufacture or on the work.

3. In order to allow ample time for inspecting and testing, the cement should be stored in a suitable weather-tight building having the floor properly blocked or raised from the ground.

4. The cement shall be stored in such a manner as to permit easy access for proper inspection and identification of each shipment.

5. Every facility shall be provided by the contractor, and a period of at least 12 days allowed for the inspection and necessary tests.

6. Cement shall be delivered in suitable packages, with the brand and name of manufacturer plainly marked thereon.

7. A bag of cement shall contain 94 pounds of cement net. Each barrel of Portland cement shall contain 4 bags, and each barrel of natural cement shall contain 3 bags of the above net weight.

8. Cement failing to meet the 7-day requirements may be held awaiting the results of the 28-day tests before rejection.

9. All tests shall be made in accordance with the methods proposed by the Special Committee on Uniform Tests of Cement of the American Society of Civil Engineers, presented to the Society on January 17, 1912, with all subsequent amendments thereto.

10. The acceptance or rejection shall be based on the following requirements:

Natural Cement

11. This term shall be applied to the finely pulverized product resulting from the calcination of an argillaceous limestone at a temperature only sufficient to drive off the carbonic acid gas.

Fineness

12. It shall leave by weight a residue of not more than 10 per cent on the No. 100, and 30 per cent on the No. 200 sieve.

Time of Setting

13. It shall not develop initial set in less than 10 minutes, and shall not develop hard set in less than 30 minutes, or more than 3 hours.

Tensile Strength

14. The minimum requirements for tensile strength for briquettes 1 square inch in cross section shall be as follows, and the cement shall show no retrogression in strength within the periods specified:

Neat Cement

AGE	STRENGTH
24 hours in moist air	75 lb.
7 days (1 day in moist air, 6 days in water)	150 "
28 days (1 day in moist air, 27 days in water)	250 "
One Part Cement, Three Parts Standard Ottawa Sand	
7 days (1 day in moist air, 6 days in water)	50 lb.
28 days (1 day in moist air, 27 days in water)	125 "

Constancy of Volume

15. Pats of neat cement about 3 inches in diameter, ½ inch thick at the center, tapering to a thin edge, shall be kept in moist air for a period of 24 hours.

(*a*) A pat is then kept in air at normal temperature.

(*b*) Another is kept in water maintained as near 70° F. as practicable.

16. These pats are observed at intervals for at least 28 days, and, to pass the tests satisfactorily, should remain firm and hard and show no signs of distortion, checking, cracking, or disintegrating.

Portland Cement

17. This term is applied to the finely pulverized product resulting from the calcination to incipient fusion of an intimate mixture of properly proportioned argillaceous and calcareous materials, to which no addition greater than 3 per cent has been made subsequent to calcination.

Specific Gravity

18. The specific gravity of cement shall be not less than 3.10. Should the test of cement as received fall below this requirement, a second test may be made on a sample ignited at a low red heat. The loss in weight of the ignited cement shall not exceed 4 per cent.

Fineness

19. It shall leave by weight a residue of not more than 8 per cent on the No. 100, and not more than 25 per cent on the No. 200 sieve.

Time of Setting

20. It shall not develop initial set in less than 30 minutes; and must develop hard set in not less than 1 hour, nor more than 10 hours.

Tensile Strength

21. The minimum requirements for tensile strength for briquettes

1 square inch in cross section shall be as follows, and the cement shall show no retrogression in strength within the periods specified:

Neat Cement

AGE	STRENGTH
24 hours in moist air	175 lb.
7 days (1 day in moist air, 6 days in water)	500 "
28 days (1 day in moist air, 27 days in water)	600 "
One Part Cement, Three Parts Standard Ottawa Sand	
7 days (1 day in moist air, 6 days in water)	200 lb.
28 days (1 day in moist air, 27 days in water)	275 "

Constancy of Volume

22. Pats of neat cement about 3 inches in diameter, ½ inch thick at the center, and tapering to a thin edge, shall be kept in moist air for a period of 24 hours.

(*a*) A pat is then kept in air at normal temperature and observed at intervals for at least 28 days.

(*b*) Another pat is kept in water maintained as near 70° F. as practicable, and observed at intervals for at least 28 days.

(*c*) A third pat is exposed in any convenient way in an atmosphere of steam, above boiling water, in a loosely closed vessel for 5 hours.

23. These pats, to pass the requirements satisfactorily, shall remain firm and hard, and show no signs of distortion, checking, cracking, or disintegrating.

Sulphuric Acid and Magnesia

24. The cement shall not contain more than 1.75 per cent of anhydrous sulphuric acid (SO_3), nor more than 4 per cent of magnesia (MgO).

SAND

Sand is a constituent part of mortar and concrete. The strength of the masonry is dependent to a considerable extent on the qualities of the sand, and it is therefore important that its desirable and defective qualities be understood, and that these qualities be always investigated as thoroughly as are the qualities of the cement used. There have been many failures of structures due to the use of poor sand.

Value. Sand is required in mortar or concrete for the sake of economy, and to prevent the excessive cracking that would take place in neat lime or cement without it. Mortar made without sand would be expensive, and the neat lime or cement would crack so badly that the increased strength, due to the neat paste, would be of little, if any, value.

Geological Character. Quartz sand is the most durable and unchangeable. Sands that consist largely of grains of feldspar, mica, hornblende, etc., which will decompose upon prolonged exposure to the atmosphere, are less desirable than quartz, although after being made up into the mortar, they are virtually protected against further decomposition.

Essential Qualities. The word *sand* as used above is intended as a generic term to apply to any finely divided material which will not injuriously affect the cement or lime, and which is not subject to disintegration or decay. Sand is almost the only material which is sufficiently cheap and which will fulfil these requirements, although stone screenings (the finest material coming from a stone crusher), powdered slag, and even coal dust have occasionally been used as substitutes. Specifications usually demand that the sand shall be "sharp, clean, and coarse," and these terms have been repeated so often that they are accepted as standard, in spite of the frequent demonstrations that modifications of the terms are not only desirable but would also be economical. These words also ignore other qualities which should be considered, especially when deciding between two or more different sources of sand supply.

Coarseness. A mixture of coarse and fine grains, with the coarse grains predominating, is found very satisfactory, as it makes a denser and stronger concrete with a less amount of cement than when coarse-grained sand is used alone with the same proportion of cement. The small grains of sand fill the voids caused by the coarse grains so that there is not so great a volume of voids to be filled by the cement. Very fine sand may be used alone, but it makes a weaker concrete than either coarse sand or coarse and fine sand mixed. A mortar consisting of very fine sand and cement will not be so dense as one of coarse sand and the same cement, although, when measured or weighed dry, both contain the same proportion of voids and solid matter. In a unit measure of fine sand, there are more grains than in a unit measure of coarse sand, and, therefore, more points of contact. More water is required in gaging a mixture of fine sand and cement than in a mixture of coarse sand and the same cement. The water forms a film and separates the grains, thus producing a larger volume having less density.

The screenings of broken stone are sometimes used instead of sand. Tests frequently show a stronger concrete when screenings are used than when sand is used. This is, perhaps, due to the variable sizes of the screenings, which would have a lower percentage of voids.

Sharpness. The sharpness of sand can be determined approximately by rubbing a few grains in the hand, or by crushing it near the ear and noting if a grating sound is produced; but an examination through a small lens is a better method. Experiments have shown that round grains of sand have fewer voids than do angular grains, and that water-worn sands have from 3 per cent to 5 per cent fewer voids than corresponding sharp grains. In many parts of the country where it is impossible, except at a great expense, to obtain the sharp sand, the round grain is used with very good results. Laboratory tests made under conditions as nearly identical as possible show that the round-grain sand gives as good results as the sharp sand. In consequence of such tests, the requirement that sand shall be *sharp* is now considered useless by many engineers, especially when it leads to additional cost.

Cleanness. In all specifications for concrete work is found the clause: "The sand shall be clean." This requirement is sometimes questioned, as experimenters have found that sand with a small percentage of clay or loam often gives better results than clean sand. Lean mortar may be improved by adding a small percentage of clay or loam, or by using dirty sand, for the fine material increases the density. In rich mortars this is not needed, as the cement furnishes all the fine material necessary, and clay or loam or dirty sand might prove detrimental. Whether it is really a benefit or not depends chiefly upon the richness of the concrete and the coarseness of the sand. Some idea of the cleanness of sand may be obtained by placing it in the palm of one hand and rubbing it with the fingers of the other. If the sand is dirty it will badly discolor the palm of the hand. When it is found necessary to use dirty sand, the strength of the concrete should be tested.

Sand containing loam or earthy material is cleansed by washing with water, either in a machine specially designed for the purpose, or by agitating the sand with water in boxes provided with holes to permit the dirty water to flow away.

Percentage of Voids. As before stated, a mortar is strongest when composed of fine and coarse grains mixed in such proportion that the percentage of voids shall be the least. The simplest method of comparing two sands is to weigh a certain gross volume of each, the sand having been thoroughly shaken down. Assuming that the stone itself of each kind of sand has the same density, then the heavier volume of sand will have the smaller percentage of voids. The percentage of voids in packed sand may be approximately determined by measuring the volume of water which can be added to a given volume of the sand. However, if the water is poured into the sand, it is quite certain that air will remain in the voids in the sand, which will not be dislodged by the water, and the apparent volume of voids will be *less* than the actual.

The precise determination of the percentage of voids involves the measurement of the specific gravity of the stone of which the sand is composed, and the percentage of moisture in the sand, all of which is done with elaborate precautions. Ordinarily such precise determinations are of little practical value, since the product of any one sand bank is quite variable. While it would be theoretically possible to mix fine and coarse sand, varying the ratios according to the coarseness of the grains as obtained from the sand pit, it is quite probable that an over-refinement in this particular would cost more than the possible saving would be worth. Sand usually has from 28 to 40 per cent of voids. An experimental test of sand of various degrees of fineness, 12½ per cent of it passing a No. 100 sieve, showed only 22 per cent of voids; but such a value is of only theoretical interest, and for practical operations we should assume the percentage given above.

BROKEN STONE

Classification of Stones. The term *broken stone* ordinarily signifies the product of a stone crusher or the result of hand-breaking by hammering large blocks of stone; but the term may also include *gravel,* described below.

The best, hardest, and most durable broken stone comes from the *trap rocks*—dark, heavy, close-grained rocks of igneous origin. The term *granite* is usually made to include not only true granite, but also gneiss, mica schist, syenite, etc. These are equally good for concrete work, and are usually less expensive. *Limestone* is suitable for some kinds of concrete work; but its strength is not so great as that of granite or trap rock, and it is more affected by a conflagration. *Conglomerate,* often called *pudding stone,* makes a very good concrete stone. The value of *sandstone* for concrete varies much according to its texture. Some grades are very compact, hard, and tough, and make a good concrete; other grades are friable, and, like shale and slate, are practically unfit for use. *Gravel* consists of pebbles of various sizes, produced from stones which have been broken up and then been worn smooth with rounded corners. The very fact that they have been exposed for indefinite periods to atmospheric disintegration and mechanical wear is a proof of the durability and mechanical strength of the stone.

Sizes of Stones. The size of the broken stone depends altogether upon the use to be made of the concrete. For plain concrete the usual size is 2 inches. The maximum size is never larger than $2\frac{1}{2}$ to 3 inches and the minimum is seldom less than $1\frac{1}{2}$ inches. Between these limits the size is dependent largely upon the massiveness of the work to be constructed. For heavy walls, foundations, abutments, etc., the larger stones may be used, and for the thin walls, arch rings, etc., the smaller stones.

In reinforced concrete work $\frac{3}{4}$-inch and 1-inch stones are generally used. The smaller size is to be preferred. A denser and stronger concrete is secured by using graded stone, for the smaller stones fill in between the larger ones and less mortar is required. For reinforced concrete the stones should vary from $\frac{1}{4}$ inch to $\frac{3}{4}$ or 1 inch, that is, the run of crusher may be used between these limits; for plain concrete, the run of the crusher may be used between the limits of ¼ inch and 2 or 3 inches.

The amount of voids in broken stone of uniform size is about 45 per cent; this is true whether the stone is ¼ inch or 6 inches. To secure dense concrete it is necessary to have these voids filled. A simple method for determining the amount of the voids that is sufficiently accurate for ordinary work has been previously described for obtaining the voids in sand. If the stone is slowly dropped into the water there is much less danger of error due to air bubbles forming on the stone, than if the water is poured into a vessel which contains the stone.

Illustrative Example. A cylinder 10 inches in diameter and 12 inches deep is, say, half full of water, and into it stone is slowly dropped and well settled until the vessel is level full. The stone is then removed and the depth of water measured. Suppose the depth of water is found to be 5½ inches. By dividing the volume of water after the stones have been removed by the volume of the full cylinder we shall determine the percentage of voids. That is:

$5^2 \times 3.1416 \times 12 = 942.48$ cu. in., volume of the full cylinder

$5^2 \times 3.1416 \times 5\frac{1}{2} = 431.97$ cu. in., volume of water with stones removed

$431.97 \div 942.48 = 45.7$, per cent of voids

Broken stone is usually sold by the ton, and its weight varies from 2,200 to 3,200 pounds per cubic yard. Therefore it is necessary to know how much stone will weigh per cubic yard before the cost of concrete can be accurately figured.

Slag. Slag is now being used in some sections of this country for reinforced concrete work on the same basis as trap rock or other approved stone. Such slag must be air-cooled, blast-furnace slag, free from ashes and other débris, and it should weigh at least 2,000 pounds per cubic yard. Water-cooled slag is too porous to be used for the higher grades of reinforced concrete work, but it may be used for reinforced concrete slabs supported on steel beams or walls at a higher value than 1:2:4 cinder concrete. Slag concrete is one of the best fireproofing materials in use.

Cinders. Cinders for concrete should be free from coal and soot. Usually a better mixture can be obtained by screening the fine stuff from the cinders and then mixing in a larger proportion of sand, than by using unscreened material, although if the fine stuff is uniformly distributed through the mass, it may be used without screening and a smaller proportion of sand used.

The strength of cinder concrete, as is shown later, is far less than that of stone concrete; and on this account it cannot be used where high compressive values are necessary. But because of its very low cost compared with broken stone, especially under some conditions, it is used rather commonly for roofs, etc., on which the loads are comparatively small.

One possible objection to the use of cinders often advanced is that they frequently contain sulphur and other chemicals which may corrode the reinforcing steel. The case cited on page 42 would seem to refute this theory. However, in any structure where the strength of the concrete is a matter of importance, cinders should not be used without a thorough inspection, and even then the unit compressive values allowed should be at a very low figure.

MORTARS

The components of mortars are a cementing material, sand, and water. The cementing material may be either lime or cement; often both are used, varying in proportion to suit the character of the work. Mortar made with lime alone as the cementing material is very weak and is seldom used except for the cheapest work. Portland cement is always used when the best mortar is required, either alone or with a small percentage of lime. Good clean sand is always essential in making mortar.

Common Lime Mortar. Lime to be used in mortar of any kind must be well slaked in water-tight boxes, the amount of water being from 2½ to 3 times the volume of the unslaked lime. It is well to mix one part of the slaked lime paste with three parts sand. Lime mortar is used only for cheap work and where low compressive strength is required. Brick work constructed with lime mortar will safely support a load of 8 or 10 tons per square foot.

Natural Cement Mortar. Mortar made with natural cement is little used except in sections of the country where natural cement is made. It has a low compressive strength, and is slow setting. Mortar made with natural cement should never be leaner than one part cement to three parts sand.

Portland Cement Mortar. Mortar made with Portland cement is extensively used. It is usually made in the proportion of one part of Portland cement to three parts sand, when used in retaining walls and other work of a similar character. It is sometimes mixed in the proportions of 1:2, but it should never be made richer than this, for in exposed places the excess of cement will cause it to crack. This mortar is also mixed in the proportions of 1:4 and 1:5, depending on the strength required. Portland cement mortar is the best of all mortars used in building construction, is reliable in all climates, not variable in bulk, and should be used where high compressive strength is required. Brickwork laid up in Portland cement mortar of the proportions 1:3 will safely support a load of 15 tons per square foot, and rubble stone work with the same mortar will support 10 tons per square foot.

Cement Lime Mortar. This mortar varies greatly in its composition. A small amount of lime is often added to a Portland cement mortar to make the mortar work smoothly or a small amount of Portland cement may be added to a lime mortar to give that mortar more strength. As ordinarily used, cement lime mortar is composed of one part Portland cement, one part slaked lime paste, and four parts sand. This mortar has fair adhesive power, is slow to set, works smoothly and easily with the trowel, and is very satisfactory in bonding brick or stone in the walls of a building. Brickwork constructed with this mortar will safely support a load of 12 tons per square foot, and rubble stone, 8 tons per square foot.

CHARACTERISTICS AND PROPERTIES OF CONCRETE MIXTURES

Principles Used in Proportioning Concrete. Theoretically, the proportioning of the sand and cementing material should be done by weight. This is always the method in laboratory testing. The volume of a given weight of cement varies greatly according as it is packed or loosely thrown in a pile. This is also true of sand. The contents of a barrel of Portland cement will increase in volume from 10 to 30 per cent by being merely dumped loosely in a pile and then shoveled into a measuring box. In determining the proportions for concrete, the cement should be measured in the packages in which it comes from the manufacturer, but the sand and stone should be measured loose as they are thrown in the measuring boxes. The volume of sand also depends to a certain extent on its condition. Loose, dry sand occupies a considerably larger volume than wet sand, and this is still more the case when the sand is very fine.

Ideal Conditions. The general principle to be adopted is that the amount of water should be just sufficient to supply that needed for crystallization of the cement paste; that the amount of paste should be just sufficient to fill the voids between the particles of sand; that the mortar thus produced should be just sufficient to fill the voids between the broken stones. If this ideal could be realized the total volume of the mixed concrete would be no greater than that of the broken stone. But no matter how thoroughly and carefully the ingredients are mixed and rammed, the particles of cement will get between the grains of sand and thus cause the volume of the mortar to be greater than that of the sand; the grains of sand will get between the smaller stones and separate them; and the smaller stones will get between the larger stones and separate them. Experiments by Professor I. O. Baker show that, even when the volume of the mortar was only 70 per cent of the volume of the voids in the broken stone, the volume of the rammed concrete was 5 per cent more than that of the broken stone. When the theoretical amount of mortar was added, the volume was 7.5 per cent in excess, which shows that it is practically impossible to ram such concrete and wholly prevent voids. When mortar amounting to 140 per cent of the voids was used, all voids were apparently filled, but the volume of the concrete was 114 per cent of that of the broken stone.

Conditions in Practice. Therefore, on account of the impracticability of securing perfect mixing, the amount of water used is always somewhat in excess (which will do no harm); the cement paste is generally made somewhat in excess of that required to fill the particles in the sand (except in those cases where, for economy, the mortar is purposely made very lean); and the amount of mortar is usually considerably in excess of that required to fill the voids in the stone. Even when we allow some excess in all the ingredients, there is so much variation in the percentage of voids in the sand and broken stone, that not only does the best work require an experimental determination of the voids in the materials used, but, on account of the liability to variation in those percentages, even in materials from the same source of supply, it also requires a constant testing and revision of the proportions as the work proceeds. For less careful work, the proportions ordinarily adopted in practice are considered sufficiently accurate.

Standard Proportions. On the general principle that the voids in ordinary broken stone are somewhat less than half of the volume, it is a very common practice to use one-half as much sand as the volume of the broken stone. The proportion of cement is then varied according to the strength required in the structure, and according to the desire to economize. On this principle we have the familiar ratios 1 : 2 : 4, 1 : 2.5 : 5, 1 : 3 : 6, and 1 : 4 : 8. It should be noted that in each of these cases, in which the numbers give the relative proportions of the cement, sand, and stone, respectively, the ratio of the sand to the broken stone is a constant, and the ratio of the cement is alone variable, for it would be equally correct to express the ratios as follows: 1 : 2 : 4, 0.8 : 2 : 4, 0.67 : 2 : 4, 0.5 : 2 : 4.

Compressive Strength. The compressive strength of concrete is very important, as concrete is used more often in compression than in any other way. To give average values of its

TABLE I

Compressive Strength of Concrete*

(Tests Made at Watertown Arsenal, 1899)

Mixture	Brand of Cement	Strength (Pounds per Square Inch)			
		7 Days	1 Month	3 Months	6 Months
1 :2 :4	Saylor	1,724	2,238	2,702	3,510
	Atlas	1,387	2,428	2,966	3,953
	Alpha	904	2,420	3,123	4,411
	Germania	2,219	2,642	3,082	3,643
	Alsen	1.592	2,269	2,608	3,612
	Average	1,565	2,399	2,896	3,826
1 :3 :6	Saylor	1,625	2.568	2,882	3,567
	Atlas	1,050	1,816	1,538	3,170
	Alpha	892	2,150	2,355	2,750
	Germania	1,550	2,174	2,486	2,930
	Alsen	1,438	2,114	2,349	3,026
	Average	1,311	2,164	2,522	3,088

NOTE.—The values obtained in these tests are exceedingly high, and cannot be safely counted on in practice.

compressive strength is rather difficult, as that is dependent on so many factors. The available aggregates are so varied, and the methods of mixing and manipulation so different, that tests must be studied before any conclusions can be drawn. For extensive work, tests should be made with the materials available under conditions as nearly as possible like those of the actual structure.

A series of experiments made at the Watertown Arsenal for Mr. George A. Kimball, Chief Engineer of the Boston Elevated Railway Company, in 1899, was one of the best sets of tests that have been published, and the results are given in Table I. Portland cement, coarse, sharp sand, and stone up to 2½ inches were used; and when thoroughly rammed, the water barely flushed to the surface.

Tests by Professor A. N. Talbot† on 6-inch cubes of concrete, showed the average values given in Table II. The cubes were about 60 days old when tested.

* From *Tests of Metals*, 1899.

† Bulletin No. 14, University of Illinois.

TABLE II

Compressive Tests of Concrete

(University of Illinois)

No. of Tests	Mixture	Strength (Pounds per Square Inch)
3	1 :2 :4	2,350
6	1 :3 :5.5	1,920
7	1 :3 :6	1,300

With fair conditions as to the character of the materials and workmanship, a mixture of 1 : 2 : 4 concrete should show a compressive strength of 2,000 to 2,300 pounds per square inch in 40 to 60 days; a mixture of 1 : 2.5 : 5 concrete, a strength of 1,800 to 2,000 pounds per square inch; and a mixture of 1 : 3 : 6 concrete, a strength of 1,500 to 1,800 pounds per square inch. The rate of hardening depends upon the consistency and the temperature.

Tensile Strength. The tensile strength of concrete is usually considered about one-tenth of the compressive strength; that is, concrete which has a compressive value of 2,000 pounds per square inch should have a tensile strength of about 200 pounds per square inch. Although there is no fixed relation between the two values, the general law of increase in strength due to increase in the percentage of cement and the density seems to hold in both cases.

Shearing Strength. By shearing is meant the strength of the material against a sliding failure when tested as a rivet would be tested for shear. The shearing strength of concrete is important on account of its intimate relation to the compressive strength and the shearing stresses to which it is subjected in structures reinforced with steel. Only a few tests have been made, as they are rather difficult; but the tests made show that the shearing strength is nearly one-half the crushing strength.

Modulus of Elasticity. The principal use of the modulus of elasticity in designing reinforced concrete is in determining the relative stresses carried by the concrete and the steel. The minimum value used in designing reinforced concrete is usually taken as 2,000,000, and the maximum value as 3,000,000, depending on the richness of the mixture used. For ordinary concrete a value of 2,500,000 is generally taken.

Weight. The weight of stone or gravel concrete will vary from 145 pounds per cubic foot to 155 pounds per cubic foot, depending upon the specific gravity of the materials and the degree of compactness. The weight of a cubic foot is usually considered as 150 pounds.

Cost. The cost of concrete in place ranges from $4.50 per cubic yard to $20, or even $25, per cubic yard, varying chiefly with the character of the work to be done, and the conditions under which it is necessary to do it. The cost of the material, of course, will always have to be considered, but this is not so important as the character of the work. When concrete is laid in large masses, so that the expense for forms is relatively small, the cost ranges from $4.50 per cubic yard to $6 or $7 per cubic yard, depending upon the local conditions and cost of materials. Foundations and heavy walls are good examples of this class of work. For sewers and arches, the cost varies from $7 to $13. In building construction—floors, roofs, and thin walls—the cost ranges from $14 to $20 per cubic yard.

Cement. The cost of Portland cement varies with the demand. As the material is heavy, the freight is often a big item. The price ranges from $1 to $2 per barrel, and to this must be added the cost of handling.

Sand. The cost of sand, including handling and freight, is from $0.75 to $1.50 per cubic yard; a common price for sand delivered in the cities is $1 per cubic yard.

Broken Stone or Gravel. The cost of broken stone delivered in the cities varies from $1.25 to $1.75 per cubic yard. The cost of gravel is usually a little less.

Mixing. Under ordinary conditions and where the concrete must be wheeled only a very short distance, the cost of hand-mixing and placing generally ranges from $0.90 to $1.30 per cubic yard, if done by men skilled in this work. If a mixer is used, the cost is from $0.50 to $0.90 per cubic yard.

Forms. The cost of forms for heavy walls and foundations varies from $0.70 to $1.20 per cubic yard of concrete laid.

These last two items—the cost of forms and mixing—are discussed later.

Variations from Standard Aggregate

In the previous discussions the standard aggregate composed of broken stone or gravel has been assumed. There are two other types of concrete, *cinder* and *rubble,* which are used under certain circumstances.

Cinder Concrete. Cinder concrete has been used to some extent on account of its light weight. The strength of cinder concrete is from one-third to one-half the strength of stone concrete. It weighs about 110 pounds per cubic foot.

Rubble Concrete. *Advantages.* Rubble concrete includes any class of concrete in which large stones are placed. The chief use of this concrete is in constructing dams, lock-walls, breakwaters, retaining walls, and bridge piers.

The cost of rubble concrete in large masses should be less than that of ordinary concrete, as the expense of crushing the stone used as rubble is saved, and as each large stone replaces a portion of cement and aggregate, that portion of cement is saved, as well as the labor of mixing it. The weight of a cubic foot of stone is greater than that of an equal amount of ordinary concrete, because of the pores in the concrete; the rubble concrete is therefore heavier, which increases its value for certain classes of work. Rubble concrete is generally found to be cheaper than rubble masonry, because it requires very little skilled labor, but for walls 3 or 3½ feet thick, the rubble masonry is usually cheaper, owing to the saving in forms.

Proportion and Size of Stone. Usually the proportion of rubble stone is expressed in percentage of the finished work, varying from 20 to 65 per cent. The percentage depends largely on the size of the stone used, as there must be nearly as much space left between small stones as between large ones. The percentage therefore increases with the size of the stones. When "one-man" or "two-man" rubble stone is used, about 20 per cent to 25 per cent of the finished work is composed of these stones. When the stones are large enough to be handled with a derrick, the proportion is increased to about 33 per cent;

and to 55 per cent, or even 65 per cent, when the stones average from 1 to 2½ cubic yards each.

The distance between the stones may vary from 3 inches to 15 or 18 inches. With a very wet mixture of concrete, which is generally used, the stones can be placed much closer than if a dry mixture is used. With the latter mixture, the space must be sufficient to allow the concrete to be thoroughly rammed into all of the crevices. Specifications often state that no rubble stone shall be placed nearer the surface of the concrete than 6 to 12 inches.

Rubble Masonry Faces. The faces of dams are very often built of rubble, ashlar, or cut stone, and the filling between the faces made of rubble concrete. For this style of construction, no forms are required. For rubble concrete, when the faces are not constructed of stone, wooden forms are constructed as for ordinary concrete.

Comparison of Quantities of Materials. The mixture of concrete used for this class of work is often 1 part Portland cement, 3 parts sand, and 6 parts stone. The quantities of materials required for one yard of concrete, according to Table VI, are 1.05 barrels cement, 0.44 cubic yard sand, and 0.88 cubic yard stone. If rubble concrete is used, and if the rubble stone laid averages 0.40 cubic yard for each yard of concrete, then 40 per cent of the cubic contents is rubble, and each of the other materials may be reduced 40 per cent. The proportions for one cubic yard would then be: $1.05 \times 0.60 = 0.63$ barrel of cement; $0.44 \times 0.60 = 0.26$ cubic yard sand; and $0.88 \times 0.60 = 0.53$ cubic yard stone.

A dam on the Quinebaug River is a good example of rubble-concrete construction. The height of the dam varies from 30 to 45 feet above bedrock. In making the concrete there were used the bank sand and gravel excavated from the bars in the bed of the river, and the rock and boulders of varying sizes taken from the site of the dam. Stones containing 2 to 2½ cubic yards were used in the bottom of the dam, but in the upper part of the dam smaller stones were placed. The total amount of concrete used in the dam was about 12,000 cubic yards, there being 1½ cubic yards of concrete for each barrel of cement used. The concrete was mixed wet, and the large stones were so placed that no voids or hollows would exist in the finished work.

MIXING AND LAYING CONCRETE

Methods of Proportioning

Rich Mixture. A rich mixture, proportions 1 : 2 : 4 — that is, 1 barrel (4 bags) packed Portland cement (as it comes from the manufacturer), 2 barrels (7.6 cubic feet) loose sand, and 4 barrels (15.2 cubic feet) loose stone—is used in arches, reinforced concrete floors, beams, and columns for heavy loads; engine and machine foundations subject to vibration; tanks; and for water-tight work.

Medium Mixture. A medium mixture, proportions 1 : 2.5 : 5 —that is, 1 barrel (4 bags) packed Portland cement, 2½ barrels (9.5 cubic feet) loose sand, and 5 barrels (19 cubic feet) loose gravel or stone—may be used in arches, thin walls, floors, beams, sewers, sidewalks, foundations, and machine foundations.

Ordinary Mixture. An ordinary mixture, proportions 1 : 3 : 6—that is, 1 barrel (4 bags) packed Portland cement, 3 barrels (11.4 cubic feet) loose sand, and 6 barrels (22.8 cubic feet) loose gravel or broken stone—may be used for retaining walls, abutments, piers, and machine foundations.

Lean Mixture. A lean mixture, proportions 1 : 4 : 8 — that is, 1 barrel (4 bags) packed Portland cement, 4 barrels (15.2 cubic feet) loose sand, and 8 barrels (30.4 cubic feet) loose gravel or broken stone—may be used in large foundations supporting stationary loads, backing for stone masonry, or where the concrete is subject to a low compressive load.

Tendency toward Richer Mixtures. These proportions must not be taken as being always the most economical, but they represent average practice. Cement is the most expensive ingredient; therefore a reduction of the quantity of cement, by adjusting the proportions of the aggregate so as to produce a concrete with the same density, strength, and impermeability,

TABLE III*

Proportions of Cement, Sand, and Stone in Actual Structures

STRUCTURE	PROPORTIONS	REFERENCE
C. B. & Q. R. R.		
Reinforced Concrete Culverts......	1:3:6	Engr. Cont., Oct. 3, '06
Phila. Rapid Transit Co.		
Floor Elevated Roadway...........	1:3:6	" " Sept. 26, '06
Subway {Walls........................	1:2.5:5	
Subway {Floors......................	1:3:6	
C. P. R. R.		
Arch Rings...........................	1:3:5	
Piers and Abutments...............	1:4:7	Cement Era, Aug. '06
Hudson River Tunnel Caisson........	1:2:4	Eng. Record, Sept. 29, '06
Stand Pipe at Attleboro, Mass.	1:2:4	" " " 29, '06
Height, 106 feet.		
C. C. & St. L. R. R., Danville Arch		
Footings..............................	1:4:8 or 1:9.5	" " March 3, '06
Arch Rings...........................	1:2:4	
Abutments, Piers....................	1:3:6 or 1:6.5	
N. Y. C. & H. R. R. R.		
Ossining Tunnel {Footing.................	1:4:7.5	" " " 3, '06
Ossining Tunnel {Walls....................	1:3:6	
Ossining Tunnel {Coping..................	1:2:4	
American Oak Leather Co.		
Factory at Cincinnati, Ohio........	1:2:4	" " " 3, '03
Harvard University Stadium.........	1:3:6	
New York Subway		
Roofs and Sidewalks...............	1:2:4	
Tunnel Arches.....................	1:2.5:5	
Wet Foundation 2 ft. th. or less.....	1:2:4	
" " exceeding 2 ft.....	1:2.5:5	
Boston Subway......................	1:2.5:4	
P. & R. R. R.		
Arches...............................	1:2:4	" " Oct. 13, '06
Piers and Abutments...............	1:3:6	
Brooklyn Navy Yd. Laboratory		
Columns............................	1:2:3 Trap rock	Eng. News, March 23, '05
Beams and Slabs..................	1:3:5 " "	
Roof Slab..........................	1:3:5 Cinder	
Southern Railway		
Arches...............................	1:2:4	
Piers and Abutments...............	1:2.5:5	

is of great importance. By careful proportioning and workmanship, water-tight concrete has been made of a 1:3:6 mixture.

* Tables III to VII have been taken from Gillette's *Handbook of Cost Data.*

TABLE IV

Barrels of Portland Cement per Cubic Yard of Mortar

(Voids in sand being 35 per cent, and 1 barrel cement yielding 3.65 cubic feet of cement paste)

Proportion of Cement to Sand	1:1	1:1.5	1:2	1:2.5	1:3	1:4
	Bbls.	Bbls.	Bbls.	Bbls.	Bbls.	Bbls.
Bbl. specified to be 3.5 cu. ft.	4.22	3.49	2.97	2.57	2.28	1.76
" " " 3.8 "	4.09	3.33	2.81	2.45	2.16	1.62
" " " 4.0 "	4.00	3.24	2.73	2.36	2.08	1.54
" " " 4.4 "	3.81	3.07	2.57	2.27	2.00	1.40
Cu. yds. sand per cu. yd. mortar	0.6	0.7	0.8	0.9	1.0	1.0

In the last few years the tendency throughout the country has been to use a richer mixture than formerly for reinforced concrete. The 1 : 2 : 4 mixture is now employed for practically all buildings constructed of reinforced concrete, even if low stresses are used, although theoretically a 1 : 2.5 : 5 mixture should have sufficient strength.

In Table III will be found the proportions of the concrete used in various well-known structures and in Tables IV to VII the amounts of materials used per cubic yard for the different proportions.

Proper Proportions Determined by Trial. An accurate and simple method to determine the proportions of concrete is by trial batches. The apparatus consists of a scale, and a cylinder which may be a piece of wrought-iron pipe 10 inches to 12 inches in diameter capped at one end. Measure and weigh the cement, sand, stone, and water, and mix on a piece of sheet steel, the mixture being of the same consistency as that to be used in the work. Place the mixture in the cylinder, carefully tamp it, and note the height to which the pipe is filled. The pipe should be weighed before and after being filled so as to check the weight of the material. The cylinder is then emptied and cleaned. Mix up another batch of the same weight, using the same amount of cement and water, but slightly varying the ratio of the sand and the stone. Put the mixture in the cylinder as before and note its height. Several trials should be made until the mixture is found which gives the least height in the

TABLE V

Barrels of Portland Cement per Cubic Yard of Mortar

(Voids in sand being 45 per cent and 1 barrel cement yielding 3.4 cubic feet of cement paste)

Proportion of Cement to Sand	1:1	1:1.5	1:2	1:2.5	1:3	1:4
	Bbls.	Bbls.	Bbls.	Bbls.	Bbls.	Bbls.
Bbl. specified to be 3.5 cu. ft.	4.62	3.80	3.25	2.84	2.35	1.76
" " " 3.8 "	4.32	3.61	3.10	2.72	2.16	1.62
" " " 4.0 "	4.19	3.46	3.00	2.64	2.05	1.54
" " " 4.4 "	3.94	3.34	2.90	2.57	1.86	1.40
Cu. yds. sand per cu. yds. mortar	0.6	0.8	0.9	1.0	1.0	1.0

cylinder, and at the same time works well while mixing, all the stones being covered with mortar, and which makes a good appearance. This method gives good results, but it does not indicate the various sizes of the sand and stone to use to secure the most economical composition, as would be shown in a thorough mechanical analysis.

There has been much concrete work done where the proportions were selected without any reference to voids, which has given much better results in practice than might be expected. The proportion of cement to the aggregate depends upon the nature of the construction and the required degree of strength and water-tightness, as well as upon the character of the inert

TABLE VI

Ingredients in 1 Cubic Yard of Concrete

(Sand voids, 40 per cent; stone voids, 45 per cent; Portland cement, barrel yielding 3.65 cubic feet paste: barrel specified to be 3.8 cubic feet)

Proportions by Volume	1:2:4	1:2:5	1:2:6	1:2.5:5	1:2.5:6	1:3:4
Bbls. cement per cu. yd. concrete	1.46	1.30	1.18	1.13	1.00	1.25
Cu. yds. sand " "	0.41	0.36	0.33	0.40	0.35	0.53
" stone " "	0.82	0.90	1.00	0.80	0.84	0.71
Proportions by volume	1:3:5	1:3:6	1:3:7	1:4:7	1:4:8	1:4:9
Bbls. cement per cu. yd. concrete	1.13	1.05	0.96	0.82	0.77	0.73
Cu. yds. sand " "	0.48	0.44	0.40	0.46	0.43	0.41
" stone " "	0.80	0.88	0.93	0.80	0.86	0.92

Note.—This table is to be used when cement is measured packed in the barrel, for the ordinary barrel holds 3.8 cubic feet.

TABLE VII

Ingredients in 1 Cubic Yard of Concrete

(Sand voids, 40 per cent; stone voids, 45 per cent; Portland cement, barrel yielding 3.65 cubic feet paste: barrel specified to be 4.4 cubic feet)

Proportions by Volume	1:2:4	1:2:5	1:2:6	1:2.5:5	1:2.5:6	1:3:4
Bbls. cement per cu. yd. concrete..........	1.30	1.16	1.00	1.07	0.96	1.08
Cu. yds. sand " "	0.42	0.38	0.33	0.44	0.40	0.53
" stone " "	0.84	0.95	1.00	0.88	0.95	0.71
Proportions by volume....................	1:3:5	1:3:6	1:3:7	1:4:7	1:4:8	1:4:9
Bbls. cement per cu. yd. concrete..........	0.96	0.90	0.82	0.75	0.68	0.64
Cu. yds. sand " "	0.47	0.44	0.40	0.49	0.44	0.42
" stone " "	0.78	0.88	0.93	0.86	0.88	0.95

NOTE.—This table is to be used when the cement is measured loose, after dumping it into a box, for under such conditions a barrel of cement yields 4.4 cubic feet of loose cement.

materials, both strength and imperviousness being increased with a larger proportion of cement. Richer mixtures are necessary for loaded columns, beams in building construction, and arches, for thin walls subject to water pressure, and for foundations laid under water. The actual measurements of materials as mixed and used usually show leaner mixtures than the nominal proportions specified. This is largely due to the heaping of the measuring boxes.

Wetness of Concrete

In regard to plasticity, or facility for working and molding, concrete may be divided into three classes: dry, medium, and very wet.

Dry Concrete. Dry concrete is used in foundations which may be subjected to severe compression a few weeks after the concrete is laid. It should not be placed in layers of more than 8 inches, and should be thoroughly rammed. In a dry mixture the water will just flush to the surface only when it is thoroughly tamped. A dry mixture sets and will support a load much sooner than will a wetter mixture, and it generally is used only where the load is to be applied soon after the concrete is placed. The mixture requires the exercise of more than ordinary care in ramming, as pockets are likely to form in the concrete. One argument against it is the difficulty of getting a uniform product.

Medium Concrete. Medium concrete will quake when rammed, and has the consistency of liver or jelly. It is adapted for construction work suited to the employment of mass concrete, such as retaining walls, piers, foundations, arches, abutments; sometimes it is also employed for reinforced concrete.

Very Wet Concrete. A very wet mixture of concrete will run off a shovel unless it is handled very quickly, and an ordinary rammer will sink into it of its own weight. This mixture is suitable for reinforced concrete construction, such as thin walls, floors, columns, tanks, and conduits.

Modern Practice. Within the last few years there has been a marked change in the amount of water used in mixing concrete. The dry mixture has been superseded by a medium or very wet mixture, often one so wet as to require no ramming whatever. Experiments have shown that *dry mixtures* give better results in *short time tests* and *wet mixtures* in *long time tests.* Some experiments made on dry, medium, and wet mixtures showed that the medium mixture was the most dense, wet next, and dry the least. The experimenter concluded that the medium mixture is the most desirable, since it will not quake in handling but will quake under heavy ramming. He found medium 1 per cent denser than wet concrete and 9 per cent denser than dry concrete; he considers thorough ramming important.

Concrete is often used so wet that it will not only quake but flow freely, and after setting it appears to be very dense and hard. Some engineers think that the tendency is to use far too much rather than too little water, and that thorough ramming is desirable. In thin walls very wet concrete can be more easily pushed from the surface so that the mortar can get against the forms and give a smooth surface. It has also been found essential that the concrete should be wet enough to flow under and around the steel reinforcement so as to secure a good bond between the steel and concrete.

Following are the specifications (1903) of the American Railway Engineering and Maintenance of Way Association:

The concrete shall be of such consistency that when dumped in place it will not require tamping; it shall be spaded down and tamped sufficiently to level off and will then quake freely like jelly, and be wet enough on top to require the use of rubber boots by workmen.

Methods of Mixing

Characteristics. The method of mixing concrete is immaterial, if a homogeneous mass, containing the cement, sand, and stone in the correct proportion, is secured. The value of the concrete depends greatly upon the thoroughness of the mixing. The color of the mass must be uniform, and each grain of sand and piece of the stone should have cement adhering to every point of its surface.

Two methods are used in mixing concrete—*by hand* and *by machinery*. Good concrete may be made by either method and in both cases the concrete should be carefully watched by a good foreman. If a large quantity of concrete is required, it is cheaper to mix it by machinery. On small jobs where the ratio of the cost of erecting the plant, together with the interest and depreciation, to the number of cubic yards to be made, is large, or if frequent moving is required, it is very often cheaper to mix the concrete by hand. The relative cost of the two methods usually depends upon circumstances, and must be worked out in each individual case.

Mixing by Hand. The placing and handling of materials and the arrangement of the plant are varied by different engineers and contractors. The mixing of concrete is in general a simple operation, but it should be carefully watched by an inspector. He must attend to the following details:

(1) That the exact amount of stone and sand are measured out
(2) That the cement and sand are thoroughly mixed
(3) That the mass is thoroughly mixed
(4) That the proper amount of water is used
(5) That care is taken in dumping the concrete in place
(6) That it is thoroughly rammed

Mixing-Platform. The mixing-platform, which is usually 10 to 20 feet square, is made of 1-inch or 2-inch plank planed on one side and well nailed to stringers; it should be placed as near the work as possible, but so situated that the stone can be dumped on one side of it and the sand on the opposite side. A very convenient way to measure the stone and sand is by the means of bottomless boxes. These boxes are of such a size that they hold the proper proportions of stone or sand to mix a batch of a certain amount. Cement is usually measured by the package, that is, by the barrel or bag, as each contains a definite amount of cement.

Process of Mixing. The method used for mixing the concrete has little effect upon its strength, if the mass has been turned a sufficient number of times thoroughly to mix the ingredients. One of the following five methods is generally used:*

(1) Cement and sand mixed dry and shoveled on the stone or gravel, leveled off, and wet as the mass is turned

(2) Cement and sand mixed dry, the stone measured and dumped on top of it, leveled off, and wet, as turned with shovels

(3) Cement and sand mixed into a mortar, the stone placed on top of it and the mass turned

(4) Cement and sand mixed with water into a mortar which is shoveled on the gravel or stone and the mass turned with shovels

(5) Stone or gravel, sand, and cement spread in successive layers, mixed slightly and shoveled into a mound, water poured into the center, and the mass turned with shovels

The quantity of water is regulated by the appearance of the concrete. The best method of wetting the concrete is by measuring the water in pails. This insures a more nearly uniform mixture than spraying the mass with a hose.

Mixing by Machinery. On large contracts the concrete is generally mixed by machinery. The economy is not only in the mixing itself but in the appliances introduced in handling the raw materials and the final product. If all materials are delivered to the mixer in wheelbarrows, and if the concrete is conveyed away in wheelbarrows, the cost of making concrete is high, even if machine mixers are used. If the materials are fed from bins by gravity into the mixer, and if the concrete

*From Taylor and Thompson's *Concrete.*

TABLE VIII

Tensile Tests of Concrete*

(The mixture tested being composed of 1 part cement and 10.18 parts aggregate)

AGE AND METHOD OF MIXING	STRENGTH (Pounds per Square Inch)		
	High	Low	Average
Age 7 Days			
Machine-mixed sample	260	243	253
Hand-mixed sample	159	113	134
Age 28 Days			
Machine-mixed sample	294	249	274
Hand-mixed sample	231	197	211
Age 6 Months			
Machine-mixed sample	441	345	388
Hand-mixed sample	355	298	324
Age One Year			
Machine-mixed sample	435	367	391
Hand-mixed sample	369	312	343

* From H. A. Reid's *Concrete and Reinforced Concrete Construction.*

is dumped from the mixer into cars and hauled away, the cost of making the concrete should be very low.

Machine vs. Hand Mixing. It has already been stated that good concrete may be produced by either machine or hand mixing, if it is thoroughly mixed.

Tests made by the U. S. government engineers at Duluth, Minnesota, to determine the relative strength of concrete mixed by hand and concrete mixed by machine (a cube mixer), showed that at 7 days, hand-mixed concrete possessed only 53 per cent of the strength of the machine-mixed concrete; at 28 days, 77 per cent; at 6 months, 84 per cent; and at one year, 88 per cent. Details of these tests are given in Table VIII.

It should be noted in this connection, that the variations in strength were greatest in the hand-mixed samples, and that the strength was more nearly uniform in the machine-mixed.

Problems in Laying Concrete

Transporting and Depositing Concrete. Concrete is usually deposited in layers of 6 inches to 12 inches in thickness. In handling and transporting, care must be taken to prevent the separation of the stone from the mortar. The usual method of transportation is by wheelbarrows, although the concrete is often handled by cars and carts, and on small jobs it is sometimes carried in buckets. A common practice is to dump it from a height of several feet into a trench. Many engineers object to this, claiming that the heavy and light portions separate while falling and that the concrete is therefore not uniform through its mass; they insist that the concrete must be gently slid into place. A wet mixture is much more easily handled than a dry mixture, for the stone will not so readily separate from the mass. A very wet mixture has been deposited from the top of forms 43 feet high and the structure was found to be waterproof. On the other hand, the stones in a dry mixture will separate from the mortar on the slightest provocation. Where it is necessary to drop a dry mixture several feet, it should be done by means of a chute or pipe.

Ramming Concrete. Immediately after concrete is placed, it should be rammed or puddled, care being taken to force out the air bubbles. The amount of ramming necessary depends upon the amount of water used in the mixing. If a very wet mixture is used, there is danger of too much ramming, which would result in wedging the stones together and forcing the cement and sand to the surface. The chief object in ramming a very wet mixture is simply to expel the bubbles of air.

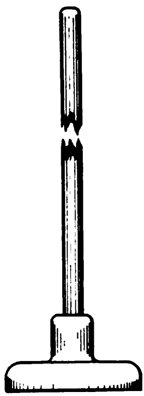


Fig. 1. Rammer for Dry Concrete

The style of rammer ordinarily used depends on whether a dry, medium, or very wet mixture is used. A rammer for dry concrete is shown in Fig. 1; and one for wet concrete, in Fig. 2. In very thin walls, where a wet mixture is used, often the tamping or puddling is done with a part of a reinforcing bar. A common spade is frequently employed for the face of work to push back stones that may have separated from the mass, and also to bring the finer portions of the mass to the face, the method being to work the spade up and down the face until it is thoroughly filled.

Care must be taken not to pry with the spade, as this will spring the forms unless they are very strong.

Depositing Concrete under Water. In depositing concrete under water, some means must be taken to prevent the separation of the materials while passing through the water. The three principal methods are as follows:

(1) By means of closed buckets
(2) By means of cloth or paper bags
(3) By means of tubes

Buckets. For depositing concrete by the first method, special buckets are made with a closed top and hinged bottom. Concrete deposited under water must be disturbed as little as possible, and tipping a bucket is likely to disturb it. Several different types of buckets with hinged bottoms have been devised to open automatically when they reach the place for depositing the concrete. In one type, the latches which fasten the trap-doors are released by the slackening of the rope when the bucket arrives at the bottom, and the doors are open as soon as the bucket begins to ascend. In another type, in which the handle extends down the sides of the bucket to the bottom, the doors are opened by the handles sliding down when the bucket reaches the bottom. The doors are hinged to the sides of the bucket and, when opened, permit the concrete to be deposited in one mass. In depositing concrete by this means, it is found rather difficult to place the layers uniformly and to prevent the formation of mounds.

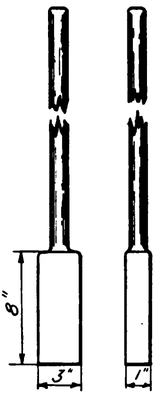


Fig. 2. Rammer for Wet Concrete

Bags. This method of depositing concrete under water is by means of open-woven bags or paper bags, two-thirds to three-quarters full. The bags are sunk in the water and placed in courses, if possible header and stretcher system, each course being arranged as laid. The texture of the bagging is close enough to keep the cement from washing out and, at the same time, open enough to allow the whole to unite into a compact mass. The fact that the bags are crushed into irregular shapes which fit into each other tends to lock them together in a way that makes even an imperfect joint very effective. When the concrete is deposited in paper bags, the water quickly soaks the paper; but the paper retains its strength long enough for the concrete to be deposited properly.

Tubes. The third method of depositing concrete under water is by means of long tubes, 4 to 14 inches in diameter. The tubes extend from the surface of the water to the place where the concrete is to be deposited. If the tube is small—4 to 6 inches in diameter—a cap is placed over the bottom, the tube filled with concrete and lowered. When the bottom is reached the cap is withdrawn, and as fast as the concrete drops out, more is put in at the top of the tube, and there is thus a continuous stream of concrete deposited.

When a large tube is used to deposit concrete in this manner, it will be too heavy to handle conveniently if filled before being lowered, and the empty tube is consequently lowered until the foot of the tube reaches the bottom. The water rises into the chute to the same level as that outside, and into this water the concrete must be dumped until the water is wholly replaced or absorbed by the concrete. This has a tendency to separate the cement from the sand and gravel, and it will take a yard or more of concrete to displace the water in the chute. There is a danger that this amount of badly washed concrete will be deposited whenever it is necessary to charge the chute, a danger which occurs not only when the charge is accidentally lost, but whenever the work is begun, in the morning or at any other time. Each time the work is stopped, the charge must be allowed to run out, or it would set in the tube. The tubes are usually charged by means of wheelbarrows, and a continuous flow of concrete must be maintained. When the chute has been filled, it is raised slowly from the bottom, and a part of the concrete allowed to run out in a conical heap at the foot.

This method has also been employed for grouting stone, a 2-inch pipe, perforated at the bottom, being used. The grout, on account of its great specific gravity, is sufficient to replace the water in the spaces between the stones, and firmly to cement the stones into a mass of concrete. A mixture of one part cement and one part sand is the leanest mixture that can be used for this purpose, as there is a great tendency for the cement and sand to separate.

Bonding Old and New Concrete. To secure a water-tight joint between old and new concrete requires a great deal of care. Where the strain is chiefly compressive, as in foundations, the surface of concrete laid on the previous day should be washed with clean water, no other precautions being necessary. In walls and floors, or where a tensile stress is likely to be applied, the joint should be thoroughly washed and soaked, and then painted with neat cement or a mixture of one part cement and one part sand, made into a very thin mortar.

In the construction of tanks or any other work that is to be water-tight, in which the concrete is not placed in one continuous operation, one or more square or **V**-shaped joints are necessary. These joints are formed by a piece of timber, say 4 inches by 6 inches, imbedded in the surface of the last concrete laid each day. On the following morning, when the timber is removed, the joint is washed and coated with neat cement or 1:1 mortar. The joints may be either horizontal or vertical. The bond between old and new concrete may be aided by roughening the surface, after ramming or before placing the new concrete.

Effects of Freezing of Concrete. Many experiments have been made to determine the effect of freezing of concrete before it has a chance to set. Both from these experiments and from practical experience, it is now generally accepted that the ultimate effect of freezing of Portland cement concrete is to produce only a surface injury. The setting and hardening of the concrete is retarded, and the strength for short periods is lowered; but the ultimate strength appears to be only slightly, if at all, affected. A thin layer about $\frac{1}{16}$ inch in depth is likely to scale off from granolithic or concrete pavements which have been frozen, leaving a rough instead of a troweled wearing surface, and the effect upon concrete walls is often similar;

but there appears to be no other injury. Concrete should not be laid in freezing weather if that can be avoided, as this involves additional expense and requires greater precautions; but with proper care, Portland cement concrete can be laid at almost any temperature.

Preventive Methods. There are three methods which may be used to prevent injury to concrete laid in freezing weather:

(1) Heat the sand and stone, or use hot water in mixing
(2) Add salt, calcium chloride, or other chemicals, to lower the freezing point of the water
(3) Protect the green concrete by enclosing it and keeping the temperature of the enclosure above the freezing point

The first method is perhaps more generally used than either of the others. In heating the aggregate, the frost is driven from it; hot water alone is insufficient to get the frost out of the frozen lumps of sand. If the heated aggregate is mixed with water which is hot but not boiling, experience has shown that a comparatively high temperature can be maintained for several hours, which will usually carry the concrete through the initial set safely. The heating of the materials also hastens the setting of the cement. If the fresh concrete is covered with canvas or other material, that will assist in maintaining a higher temperature. The canvas, however, must not be laid directly on the concrete, but an air space of several inches must be left between the concrete and the canvas.

The aggregate is heated by means of steam pipes laid in the bottom of the bins, or by having pipes of strong sheet iron, about 18 inches in diameter, laid through the bottom of the bins, and fires built in the pipes. The water may be heated by steam jets or other means. It is also well to keep the mixer warm in severe weather, by the use of a steam coil on the outside, and jets of steam on the inside.

The second method—lowering the freezing point of the water by adding salt—has been commonly used. Salt will increase the time of setting and lower the strength of the concrete for short periods. There is a wide difference of opinion as to the amount of salt that may be used without lowering the ultimate strength of the concrete. Specifications for the New York Subway work required 9 pounds of salt to each 100 pounds (12 gallons) of water in freezing weather. A common rule calls for 10 per cent of salt to the weight of water, which is equivalent to about 13 pounds of salt to a barrel of cement.

The third method is the most expensive, and is used only in building construction. It consists in constructing a light wood frame over the site of the work, and covering the frame with canvas or other material. The temperature of the enclosure is maintained above the freezing point by means of stoves.

WATERPROOFING CONCRETE

Concrete Not Generally Water-Tight. Concrete as ordinarily mixed and placed is not water-tight, but experience has shown that where concrete is proportioned to obtain the greatest density practicable and is mixed wet, the resulting concrete is impervious under a moderate pressure. The concrete of the wet mixtures now generally used in engineering work possesses far greater density, and is correspondingly less porous, than the dryer mixtures formerly used. However, it is difficult, on large masses of work, to produce concrete of such close texture as to prevent seepage at all points. It has frequently been observed that when concrete is green there is a considerable seepage through it, but that in a short time all seepage stops. Concrete has been made practically water-tight by forcing through it water containing a small amount of cement, or cement and fine sand.

Effect of Steel Reinforcement. Reinforcing steel properly proportioned and located both horizontally and vertically in long walls, subways, and reservoirs, will greatly assist in rendering the concrete impervious by reducing the cracks so that if they do occur they will be too minute to permit leakage, or will soon fill up with silt.

Waterproofing Methods. Compounds of various kinds have been mixed with concrete, or applied as a wash to the surface to make the concrete water-tight. Many of the compounds are of but temporary value, and in time lose their usefulness as a waterproofing material.

General Considerations. Several successful methods of waterproofing concrete will be given here, most of which will also apply to stone and brickwork. In the operation of waterproofing, a very common mistake is made by applying the waterproofing materials on the wrong side of the wall to be made water-tight. That is, if water finds its way through a cellar wall, it is useless to apply a waterproofing coat on the inside surface of the wall, as the pressure of the water will push it off. (If there is no great pressure behind it, a waterproofing coat applied on the inside of a cellar wall may be successful in keeping moisture out.) To be successful in waterproofing a cellar wall, however, the waterproofing material should be applied on the *outside* surface of the wall; if properly applied, the wall, as well as the cellar, will be entirely free of water.

In tank or reservoir construction, the conditions are different, in that it is generally desired to prevent the escape of water. In these cases, therefore, the waterproofing is applied on the inside surface, and is supported by the materials used in constructing the tank or reservoir. The structure should always be designed so that it can be properly waterproofed, and the waterproofing should always be applied on the side of the wall on which the pressure exists.

Plastering. For cisterns, swimming pools, or reservoirs, two coats of Portland cement grout—1 part cement, 2 parts sand—applied on the inside, have been used to make the concrete water-tight. One inch of rich mortar has usually been found effective under medium pressure.

At Attleboro, Massachusetts, a large reinforced concrete standpipe, 50 feet in diameter, 106 feet high from the inside of the bottom to the top of the cornice, and with a capacity of 1,500,000 gallons, has been constructed, and is in the service of the waterworks of that city. The walls of the standpipe are 18 inches thick at the bottom, and 8 inches thick at the top. A mixture of 1 part cement, 2 parts sand, and 4 parts broken stone, the stone varying from ¼ inch to 1½ inches, was used. The forms were constructed, and the concrete placed, in sections of 7 feet. When the walls of the tank had been completed, there was some leakage at the bottom with a head of water of 100 feet. The inside walls were then thoroughly cleaned and picked, and four coats of plaster applied. The first coat contained 2 per cent of lime to 1 part of cement and 1 part of sand; the remaining three coats were composed of 1 part sand to 1 part cement. Each coat was floated until a hard dense surface was produced; then it was scratched to receive the succeeding coat.

On filling the standpipe after the four coats of plaster had been applied, the standpipe was found to be not absolutely water-tight. The water was drawn out; and four coats of a solution of Castile soap and four of alum were applied alternately; and, under a 100-foot head, only a few leaks then appeared. Practically no leakage occurred at the joints; but in several instances a mixture somewhat wetter than usual was used, with the result that the spading and ramming served to drive the stone to the bottom of the batch being placed, and, as a consequence, in these places porous spots occurred. The joints were obtained by inserting beveled tonguing pieces, and by thoroughly washing the joint and covering it with a layer of thin grout before placing additional concrete.

Alum and Soap. Mortar may be made practically nonabsorbent by the addition of alum and potash soap. One per cent by weight of powdered alum is added to the dry cement and sand, and thoroughly mixed; and about one per cent of any potash soap (ordinary soft soap) is dissolved in the water used in the mortar. A solution consisting of 1 pound of concentrated lye, 5 pounds of alum, and 2 gallons of water, applied while the concrete is green and until it lathers freely, has been successfully used.

Linseed Oil. Coating the surface with boiled linseed oil until the oil ceases to be absorbed is another method that has been tried with satisfaction.

Hydrated Lime. Hydrated lime has been used to render concrete impervious, with favorable results. The very fine particles of the lime fill voids that would otherwise be left, thereby increasing the density of the concrete. For a 1:2:4 concrete the proper amount of hydrated lime would be from 6 to 8 per cent of the weight of the cement used. When the concrete is a leaner mixture the percentage of lime is increased; that is, for a 1:3:6 concrete the quantity of lime is sometimes as much as 16 or 18 per cent.

Sylvester Process. The alternate application of washes of Castile soap and alum, each being dissolved in water, is known as the *Sylvester process* of waterproofing. Castile soap is dissolved in water, ¾ of a pound of soap to a gallon of water, and with a flat brush is applied boiling hot to the concrete surface, care being taken not to form a froth. The alum dissolved in water—1 pound pure alum in 8 gallons of water—is applied 24 hours later, the soap having had time to become dry and hard. The second wash is applied in the same manner as the first, at a temperature of 60 to 70 degrees Fahrenheit. The alternate coats of soap and alum are repeated every 24 hours. Usually four coats will make an impervious coating. The soap and alum combine and form an insoluble compound, filling the pores of the concrete and preventing the seepage of water. The walls should be clean and dry, when the composition is applied, and the temperature of the air not lower than 50 degrees Fahrenheit. The concrete should be still green. This method of waterproofing has been used extensively for years, and has generally given satisfactory results for moderate pressures.

Asphalt. Asphalt as a waterproofing course is laid in thicknesses from ¼ to 1 inch, usually in one or more continuous sheets. It is also used for filling in contraction joints in concrete. The backs of retaining walls, of either concrete, stone, or brick, are often coated with asphalt to make them waterproof, the asphalt being applied hot with a mop. The bottoms of reservoirs have been constructed of concrete blocks 6 to 8 feet square with asphalt joints ⅜ inch to ½ inch in thickness and extending at least halfway through the joint; that is, for a block 6 inches in thickness the asphalt would extend down at least 3 inches.

In the construction of the filter plant at Lancaster, Pennsylvania, in 1905, a pure-water basin and several circular tanks were constructed of reinforced concrete. The pure-water basin is 100 feet wide by 200 feet long and 14 feet deep, with buttresses spaced 12 feet 6 inches center to center. The walls at the bottom are 15 inches thick, and 12 inches thick at the top. Four circular tanks are 50 feet in diameter and 10 feet high, and eight tanks are 10 feet in diameter and 10 feet high. The walls are 10 inches thick at the bottom, and 6 inches at the top. The concrete was a wet mixture of 1 part cement, 3 parts sand, and 5 parts stone. No waterproofing material was used in the construction of the tanks; and when tested, two of the 50-foot tanks were found to be water-tight, and the other two had a few leaks where wires which had been used to hold the forms together had pulled out when the forms were taken down. These holes were stopped up and no further trouble was experienced. In constructing the floor of the pure-water basin, a thin layer of asphalt was used, as shown in Fig. 3, but no waterproofing material was used in the walls, and both were found to be water-tight.

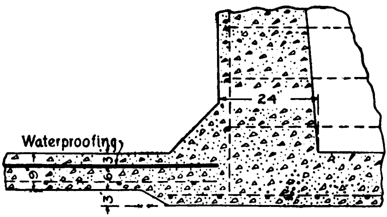


Fig. 3. Floor of Pure-Water Basin

Felt Laid with Asphalt or Coal Tar. Alternate layers of paper or felt laid with asphalt or tar are frequently used to waterproof floors, tunnels, subways, roofs, arches, etc. These materials range from ordinary tar paper laid with coal-tar pitch or asphalt to asbestos or asphalt felt laid in coal tar or asphalt. Coal-tar products have come into very common use for this work, but the coal tar is not satisfactory unless it contains a large percentage of carbon.

In using these materials for rendering concrete water-tight, usually a layer of concrete or brick is first laid. On this is mopped a layer of hot asphalt; felt or paper is then laid on the asphalt, the paper being lapped from 6 to 12 inches. After the first layer of felt is placed, it is mopped over with hot asphalt compound, and another layer of felt or paper is laid, the operation being repeated until the desired thickness is secured, which is usually from 2 to 10 layers—or, in other words, the waterproofing varies from 2-ply to 10-ply. A waterproofing course of this kind, or a course such as was described in the paragraph on asphalt waterproofing, forms a distinct joint, and the strength in bending of the concrete on the two sides of the layer must be considered independently.

When asphalt, or asphalt laid with felt paper, is used for waterproofing the interiors of the walls of tanks, a 4-inch course of brick is required to protect and hold in place the waterproofing materials. Fig. 4* shows a wall section of a reservoir constructed for the New York, New Haven and Hartford Railroad, which illustrates the methods described above. The waterproofing material for the reservoir consisted of 4-ply Hydrex felt, and Hydrex compound was used to cement the layers together.

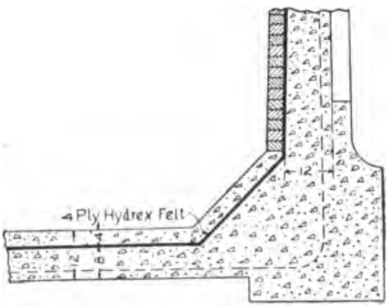


Fig. 4. Method of Waterproofing Reservoirs by Means of Hydrex Felt

Fig. 5 is an illustration of the method used by the Barrett Manufacturing Company in applying their 5-ply coal-tar pitch and felt roofing material. It illustrates in a general way the method used in applying waterproofing. The surfaces to be waterproofed are mopped with pitch or asphalt. While the pitch is still hot, a layer of felt is placed, which is followed by a layer of pitch or asphalt, the alternate layers succeeding each other until the required number of layers of felt has been secured.

* From *Engineering Record*, September 21, 1907.

In no place should one layer of felt be permitted to touch the layer above or below it. When the last layer of felt is laid and thoroughly mopped with the coal tar, something should be placed over the entire surface waterproofed to protect it from injury. For roofing, this protection is gravel, Fig. 5.

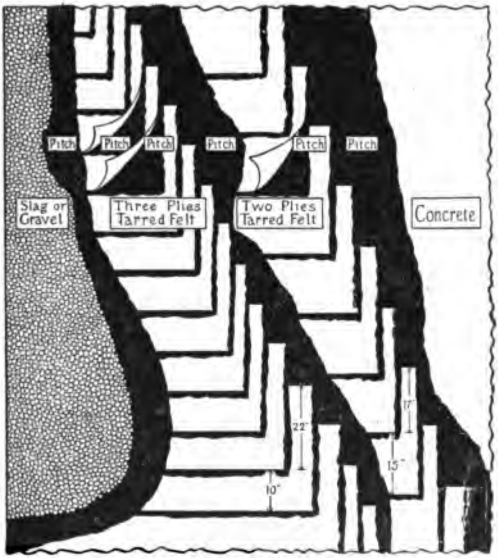


Fig. 5. Section Showing Method of Waterproofing Concrete
Courtesy of Barrett Manufacturing Company

In waterproofing the back of concrete or stone arches usually a layer of brick is placed and then the joints between the bricks are filled with pitch. Brick used in this manner also assist in holding the waterproofing in place. Five layers of felt and pitch should be a sufficient protection against a head of water of ten feet.

PRESERVATION OF STEEL IN CONCRETE

Short-Time Tests. Tests have been made to find the value of Portland cement concrete as a protection for steel or iron against corrosion. Nearly all of the tests have been of short duration (from a few weeks to several months); but they have clearly shown that the steel or iron which has been properly imbedded in concrete, will be found clean and bright, when removed therefrom. Steel removed from concrete containing cracks or voids usually has rust at the points where the voids or cracks occur; but if the steel has been *completely covered* with concrete, there is no corrosion. Tests have also shown that if corroded steel is imbedded in concrete, the concrete will remove the rust. To secure the best results, the concrete should be mixed quite wet, and care should be taken to have the steel thoroughly incased in the concrete.

Cinder vs. Stone Concrete. A compact cinder concrete has proved about as effective a protection for steel as stone concrete. The corrosion found in cinder concrete is mainly due to iron oxide or rust in the cinders, and not to the sulphur. The amount of sulphur in cinders is extremely small, and there seems to be little danger from that source. A steel-frame building erected in New York in 1898 had all its framework, except the columns, imbedded in cinder concrete; when the building was demolished in 1903, the frame showed practically no rust which could be considered as having developed after the material was imbedded.

Illustrations. Cement washes, paints, and plasters have been used for a long time, in both the United States and Europe, for the purpose of protecting iron and steel from rust. The engineers of the Boston Subway, after making careful tests and investigations, adopted Portland cement paint for the protection of the steel work in that structure. The railroad companies of France use cement paint extensively to protect their metal bridges from corrosion. Two coats of the cement paint and sand are applied with leather brushes.

A concrete-steel water main on the Monier system, at Grenoble, France, which was 12 inches in diameter, 1.6 inches thick, and contained a steel framework of ¼-inch and $\frac{1}{16}$-inch steel rods, was taken up after 15 years' use in wet ground. The adhesion was found perfect, and the metal absolutely free from rust.

William Sooy Smith, M. Am. Soc. C. E., states that a bed of concrete at a lighthouse in the Straits of Mackinac, 10 feet below water surface, was removed twenty years after it was laid, and the imbedded iron drift-bolts were free from rust.

An excellent example of the preservation of steel imbedded in concrete is given by Mr. H. C. Turner.* Mr. Turner's company had recently torn down a section of a one-story reinforced concrete building erected by his company in 1902, at New Brighton, Staten Island. The building had a pile foundation, the piles being cut off at mean tide level. The footings, side walls, columns, and roof had all been constructed of reinforced concrete. In concluding his account, Mr. Turner says:

> All steel reinforcement was found in perfect preservation, excepting in a few cases where the hoops were allowed to come closer than ¾ inch to the surface. Some evidence of corrosion was found in such cases, thus demonstrating the necessity of keeping the steel reinforcement at least ¾ inch from the surface. The footings were covered by the tide twice daily. The concrete was extremely hard, and showed no weakness whatever from the action of the salt water. The steel bars in the footings were perfectly preserved, even in cases where the concrete protection was only ¾ inch thick.

FIRE PROTECTIVE QUALITIES OF CONCRETE

High Resisting Qualities. The various tests which have been conducted—including the involuntary tests made as the result of fires—have shown that the fire-resisting qualities of concrete, and even its resistance to a combination of fire and water, are greater than those of any other known type of building construction. Fires and experiments which test buildings of reinforced concrete have proved that where the temperature ranges from 1400 to 1900 degrees Fahrenheit, the surface of the concrete may be injured to a depth of ½ to ¾ inch or even of one inch; but the body of the concrete is not affected, the only repairs required, if any, being a coat of plaster.

Thickness of Concrete Required. Actual fires and tests have shown that 2 inches of concrete will protect an **I**-beam with good assurance of safety. Reinforced concrete beams and girders should have a clear thickness of 1½ inches of concrete

* *Engineering News*, January 16, 1908.

outside the steel on the sides and 2 inches on the bottom; slabs should have at least 1 inch below the slab bars, and columns 2 inches. Structural steel columns should have at least 2 inches of concrete outside of the farthest projecting edge.

Theory. The theory of the fireproofing qualities of Portland cement concrete given by Mr. Spencer B. Newberry is that the capacity of the concrete to resist fire and prevent its transference to steel is due to its *combined water and porosity.* In hardening, concrete takes up 12 to 18 per cent of the water contained in the cement. This water is chemically combined, and not given off at the boiling point. On heating, a part of the water is given off at 500 degrees Fahrenheit, but dehydration does not take place until 900 degrees Fahrenheit is reached. The mass is kept for a long time at comparatively low temperature by the vaporization of water absorbing the heat. A steel beam imbedded in concrete is thus cooled by the volatilization of water in the surrounding concrete.

Resistance to the passage of heat is offered by the porosity of concrete. Air is a poor conductor, and an air space is an efficient protection against conduction. The outside of the concrete may reach a high temperature; but the heat only slowly and imperfectly penetrates the mass, and reaches the steel so gradually that it is given off as fast as it is supplied.

Cinder vs. Stone Concrete. Mr. Newberry says: "Porous substances, such as asbestos, mineral wool, etc., are always used as heat-insulating material. For this same reason, cinder concrete, being highly porous, is a much better non-conductor than a dense concrete made of sand and gravel, or stone, and has the added advantage of being light."

Professor Norton, on the other hand, in comparing the actions of cinder and stone concrete in the great Baltimore fire of February, 1904,* states that there is but little difference in the two concretes. The burning of bits of coal in poor cinder concrete is often balanced by the splitting of stones in the stone concrete. "However, owing to its density, the stone concrete takes longer to heat through."

* Report to the Insurance Engineering Experiment Station.

Results Shown in Baltimore Fire. Engineers and architects, who made reports on the Baltimore fire, generally agree that reinforced concrete construction stood very well—much better than terra cotta. Professor Norton says:

Where concrete floor-arches and concrete-steel construction received the full force of the fire, it appears to have stood well, distinctly better than the terra cotta. The reasons, I believe, are these: The concrete and steel expand at sensibly the same rate, and hence, when heated, do not subject each other to stress; but terra cotta usually expands about twice as fast with increase in temperature as steel, and hence the partitions and floor-arches soon become too large to be contained by the steel members which under ordinary temperature properly enclose them.

STEEL FOR REINFORCING CONCRETE

Quality of Reinforcing Steel. Steel for reinforcing concrete is not usually subjected to such severe treatment as ordinary structural steel, as the impact effect is likely to be a little less; but the quality of the steel should be carefully specified. To reduce the cost of reinforced concrete structures, there has been a tendency to use cheap steel, and this has resulted in bars being rolled from old railroad rails. These bars are known as rerolled bars and they should always be thoroughly tested before being used. If the rails from which the bars are rerolled were of good material, the bars should prove to be satisfactory, but if the rails contained poor materials the bars rolled from them will probably be brittle and easily broken by a sudden blow. Many engineers specify that the bars shall be rolled from billets to avoid using any old material.

The grades of steel used in reinforced concrete range from soft to hard, and may be classified under three heads: soft, medium, and hard.

Soft Steel. Soft steel has an ultimate strength of 50,000 to 58,000 pounds per square inch. It is seldom used in reinforced concrete.

Medium Steel. Medium steel has an ultimate strength of 55,000 to 65,000 pounds per square inch. The elastic limit is from 32,000 to 38,000 pounds per square inch. This grade of steel is extensively used for reinforced concrete work and can be bought in the open market and used with safety.

Hard Steel. Hard steel, better known as ***high-carbon steel,*** should have an ultimate strength of 85,000 to 100,000 pounds per square inch; and the elastic limit should be from 50,000 to 65,000 pounds per square inch. The hard steel has a greater percentage of carbon than the medium steel, and therefore the yield point is higher. This steel is preferred by some engineers for reinforced concrete work, but it should be thoroughly tested to be sure that it is according to specifications. This is the grade of steel into which old rails are rolled, but it is also rolled from billets.

Processes of Making Steel. Reinforcing bars are rolled by both the Bessemer and the open-hearth processes. Bars rolled by either process make good reliable steel, but bars rolled by open-hearth process are generally more uniform in quality.

Types of Bars

The steel bars used in reinforcing concrete usually consist of small bars of such shape and size that they may be easily bent and placed in the concrete so as to form a monolithic structure. To distribute the stress in the concrete, and secure the necessary bond between the steel and concrete, the steel required must be supplied in comparatively small sections. All types of the regularly rolled small bars of square, round, and rectangular section, as well as some of the smaller sections of structural steel, such as angles, T-bars, and channels, and also many special rolled bars, have been used for reinforcing concrete. These bars vary in size from ¼ inch for light construction, up to 1½ inches for heavy beams, and 2 inches for large columns. In Europe plain round bars have been extensively used for many years; they have also been used in the United States, but not to the same extent as in Europe. In America a very much larger percentage of work has been done with *deformed* bars.

Plain Bars. With plain bars, the transmission of stresses is dependent upon the adhesion between the concrete and the steel. Square and round bars show about the same adhesive strength, but the adhesive strength of the flat bars is far below that of the round and square bars. The round bars are more convenient to handle and more easily obtained, and have, therefore, generally been used when plain bars were desirable.

Steel Sections. Small angles, T-bars, and channels have been used to a greater extent in Europe than in this country. They are principally used where riveted skeleton work is prepared for the steel reinforcement; and in this case it is usually desirable to have the steel work self-supporting.

Deformed Bars. There are many forms of reinforcing materials on the market, differing from one another in the manner of forming the irregular projections on their surface. The object of all these special forms of bars is to furnish a bond

Fig. 6. Square Twisted Reinforcing Steel Bar
Courtesy of Inland Steel Company

with the concrete, independent of adhesion. This bond formed between the deformed bar and the concrete is usually called a *mechanical bond.* Some of the most common types of bars used are the *square twisted* bar, the *corrugated,* the *Havemeyer,* and the *Kahn.*

Square Twisted Bar. The twisted bar, shown in Fig. 6, was one of the first steel bars shaped to give a mechanical bond with concrete. This type of bar is a commercial square bar twisted while cold. There are two objects in twisting the bar—*first,* to give the metal a mechanical bond with the concrete; *second,* to increase the elastic limit and ultimate strength of the bar. In twisting the bars, usually one complete turn is given the bar in nine or ten diameters of the bar, with the result that the elastic limit of the bar is increased from 40 to 50 per cent, and the ultimate strength is increased from 25 to 35 per cent. These bars can readily be bought already twisted; or, if it is desired, square bars may be bought and twisted on the site of the work.

Corrugated Bar. The corrugated bar, which has corrugations as shown in Fig. 7, was invented by Mr. A. L. Johnson, M. Am. Soc. C. E. These corrugations, or square shoulders, are placed at right angles to the axis of the bar, and their sides make an angle with the perpendicular to the axis of the bars not exceeding the angle of friction between the bar and con-

Fig. 7. Corrugated Bar for Reinforcement of Concrete
Courtesy of Corrugated Bar Company

crete. These bars are usually rolled from high-carbon steel having an elastic limit of 55,000 to 65,000 pounds per square inch and an ultimate strength of about 100,000 pounds per square inch. They are also rolled from any quality of steel desired. In size they range from ¼ inch to 1¼ inches, their

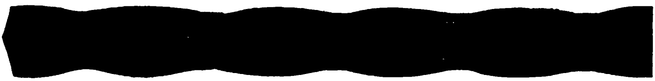


Fig. 8. Havemeyer Bar for Reinforcement of Concrete
Courtesy of Concrete Steel Company

sectional area being the same as that of plain bars of the same size. These bars are rolled in both the common types, round and square.

Havemeyer Bar. The Havemeyer bar, Fig. 8, was invented by Mr. J. F. Havemeyer. This has a uniform cross section throughout its length. The bonding of the bar to the concrete is uniform at all points, and the entire section is available for tensile strength.

Kahn Bar. The Kahn bar, Fig. 9, was invented by Mr. Julius Kahn, Assoc. M. Am. Soc. C. E. This bar is designed

Fig. 9. Kahn Trussed Bar for Reinforcement of Concrete
Courtesy of The Kahn System

with the assumption that the shear members should be rigidly connected to the horizontal members. The bar is rolled with

TABLE IX

Standard Sizes of Expanded Metal

Mesh in Inches	Gage No.	Weight in Lb. per Sq. Ft.	Sectional Area 1 Foot Wide in Sq. In.
3	16	.30	.082
3	10	.625	.177
6	4	.86	.243

a cross section as shown in the figure. The thin edges are cut and turned up, and form the shear members. These bars are manufactured in several sizes.

Expanded Metal. Expanded metal, Fig. 10, is made from plain sheets of steel, slit in regular lines and opened into meshes of any desired size or section of strand. It is commer-

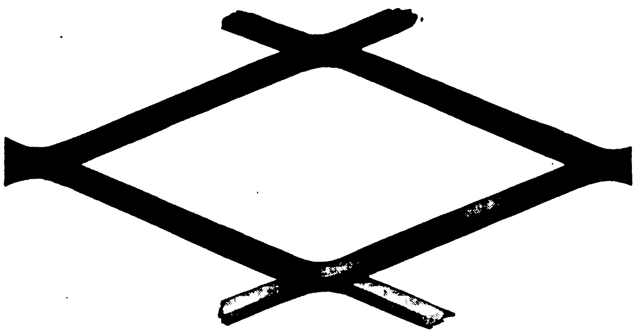


Fig. 10. Example of Expanded Metal Fabric
Courtesy of Northwestern Expanded Metal Company

cially designated by giving the gage of the steel and the amount of displacement between the junctions of the meshes. The most common manufactured sizes are given in Table IX.

Steel Wire Fabric. Steel wire fabric reinforcement consists of a netting of heavy and light wires, usually with rectangular meshes. The heavy wires carry the load, and the light ones are used to space the heavier ones. There are many forms of wire fabric on the market.

Table X is condensed from the handbook of the Cambria

TABLE X

Weights and Areas of Square and Round Bar

(One cubic foot of steel weighs 489.6 pounds)

Thickness or Diameter (Inches)	Weight of Square Bar, 1 Foot Long (Pounds)	Weight of Round Bar, 1 Foot Long (Pounds)	Area of Square Bar, (Sq. In.)	Area of Round Bar, (Sq. In.)	Circum. of Round Bar, (Inches)
$\frac{1}{4}$	.213	.167	.0625	.0491	.7854
$\frac{5}{16}$	.332	.261	.0977	.0767	.9817
$\frac{3}{8}$	.478	.376	.1406	.1104	1.1781
$\frac{7}{16}$	.651	.511	.1914	.1503	1.3744
$\frac{1}{2}$	.850	.668	.2500	.1963	1.5708
$\frac{5}{8}$	1.328	1.043	.3906	.3068	1.9635
$\frac{3}{4}$	1.913	1.502	.5625	.4418	2.3562
1	3.400	2.670	1.0000	.7854	3.1416
$1\frac{1}{8}$	4.303	3.379	1.2656	.9940	3.5343
$1\frac{1}{4}$	5.312	4.173	1.5625	1.2272	3.9270
$1\frac{1}{2}$	7.650	6.008	2.2500	1.7671	4.7124
$1\frac{3}{4}$	10.41	8.178	3.0625	2.4053	5.4978
2	13.60	10.68	4.0000	3.1416	6.2832

Steel Company, and gives the standard weights and areas of plain round and square bars commonly used in reinforced concrete construction.

REINFORCED CONCRETE BEAM DESIGN

GENERAL THEORY OF FLEXURE

The theory of flexure in reinforced concrete is exceptionally complicated. A multitude of simple rules, formulas, and tables for designing reinforced-concrete work have been proposed, some of which are sufficiently accurate and applicable under certain conditions. But the effect of these various conditions should be thoroughly understood. Reinforced concrete should not be designed by "rule-of-thumb" engineers. It is hardly too strong a statement, to say that a man is criminally careless and negligent when he attempts to design a structure on which the safety and lives of people will depend, without thoroughly understanding the theory on which any formula he may use is based. The applicability of all formulas is so dependent on the quality of both the steel and the concrete, as well as on many of the details of the design, that a blind application of a formula is very unsafe. Although the greatest pains will be taken to make the following demonstration as clear and plain as possible, it will be necessary to employ symbols, and to work out several algebraic formulas on which the rules for designing will be based. The full significance of many of the following terms may not be fully understood until several subsequent paragraphs have been studied.

SYMBOLS DEFINED

b = Breadth of concrete beam
d = Depth from compression face to center of gravity of the steel
A = Area of the steel
$p = \frac{A}{bd}$ = Ratio of area of steel to area of concrete above the center of gravity of the steel, generally referred to as *percentage of reinforcement*
E_s = Modulus of elasticity of steel
E_c = *Initial* modulus of elasticity of concrete
$n = \frac{E_s}{E_c}$ = Ratio of the moduli
s = Tensile stress per unit of area in steel
c = Compressive stress per unit of area in concrete at the outer fiber of the beam
ϵ_s = Deformation per unit of length in the steel
ϵ_c = Deformation per unit of length in outer fiber of concrete
k = Ratio of dimension from neutral axis to center of compressive stresses to the total effective depth d
j = Ratio of dimension from steel to center of compressive stresses to the total effective depth d
x = Distance from compressive face to center of compressive stresses
ΣX = Summation of horizontal compressive stresses
M = Resisting moment of a section

Statics of Plain Homogeneous Beams. As a preliminary to the theory of the use of reinforced concrete in beams, a very brief discussion will be given of the statics of an ordinary homogeneous beam, made of a material whose moduli of elasticity in tension and compression are equal. Let *A B*, Fig. 11, represent a beam carrying a uniformly distributed load *W*; then the beam is subjected to transverse stresses. Let us imagine that one-half of the beam is a "free body" in space and is acted on by exactly the same external forces; let us also assume forces *C* and *T* (acting on the exposed section), which are just such forces as are required to keep that half of the beam in equilibrium. These forces and their direction are represented in the lower diagram by arrows. The load *W* is repre-

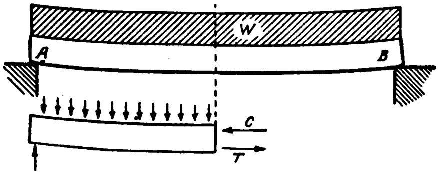


Fig. 11. Diagram of Beam Carrying Uniformly Distributed Load

sented by the series of small, equal, and equally spaced vertical arrows pointing downward. The reaction of the abutment against the beam is an upward force, shown at the left. The forces acting on a section at the center are the equivalent of the two equal forces *C* and *T*.

The force *C*, acting at the top of the section, must act toward the left, and there is therefore compression in that part of the section. Similarly, the force *T* is a force acting toward the right, and the fibers of the lower part of the beam are in tension. For our present purpose we may consider that the forces *C* and *T* are in each case the resultant of the forces acting on a very large number of fibers. The stress in the outer fibers is, of course, greatest. At the center of the height, there is neither tension nor compression. This is called the ***neutral axis,*** Fig. 12.

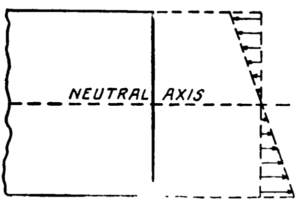


Fig. 12. Diagram Showing Position of Neutral Axis in Beam

Let us consider, for the sake of simplicity, a very narrow portion of the beam, having the full length and depth but so narrow that it includes only one set of fibers, one above the other, as shown in Fig. 13. In the case of a plain rectangular homogeneous beam, the elasticity being assumed equal for tension and compression, the stresses in the fibers would be as given in Fig. 12; the neutral axis would be at the center of the height, and the stress at the bottom and the top would be equal but opposite. If the section were at the center of the beam, with a uniformly distributed load, as indicated in Fig. 11, the shear would be zero.

A beam may be constructed of plain concrete; but its strength will be very small, since the tensile strength of concrete is comparatively insignificant. Reinforced concrete utilizes the great tensile strength of steel in combination with the compressive strength of concrete. It should be realized that two of the most essential qualities are *compression* and *tension*, and, other things being equal, the cheapest method of obtaining the proper compression and tension is the most economical.

Statics of Reinforced Concrete Beams

In a reinforced concrete beam, the steel is placed in the tension side of the beam. Usually it is placed 1 to 2 inches from the outer face, with the double purpose of protecting the steel from corrosion or fire, and of making more certain the union of the concrete and the steel; but the concrete below the steel is not considered in the numerical calculations. The concrete between the steel and the neutral axis performs the very necessary function of transmitting the tension in the steel to the concrete. This stress is called *shear* and is discussed later. Although the concrete in the lower part of the beam is, theoretically, subject to the tension of transverse stress and does actually contribute its share of the tension when the stresses in the beam are small, the proportion of the necessary tension which the concrete can furnish when the beam is heavily loaded is so very little that it is usually ignored, especially since such a policy is on the side of safety, and also since it greatly simplifies the theoretical calculations and yet makes very little difference in the final result. We may, therefore, consider that in a unit section of the beam, Fig. 14, the concrete above the neutral axis

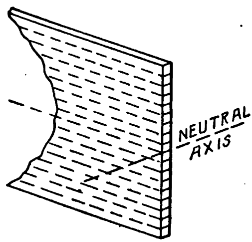


Fig. 13. Diagram Showing Position of Neutral Axis in Narrow Beam

is subject to compression, and that the tension is furnished entirely by the steel.

Elasticity of Concrete in Compression. In computing the transverse stresses in a wood beam or steel I-beam, it is assumed that the modulus of elasticity is uniform for all stresses within the elastic limit. Experimental tests have shown this to be so nearly true that it is accepted as a mechanical law. This means that if a force of 1,000 pounds is required to stretch a bar .001 of an inch, it will require 2,000 pounds to stretch it .002 of an inch. Similar tests have been made with concrete, to determine the law of its elasticity, but unfortunately, concrete is not so nearly uniform in its behavior as steel and the results of the tests are somewhat erratic.

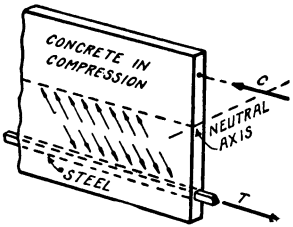


Fig. 14. Diagram Showing Transmission of Tension in Steel to Concrete

It was formerly rather common to base the computation of formulas on the assumption that the curve of compression for concrete is a parabola. The development of the theory is complex, but it has been found that for a compression of 600 or even 800 pounds per square inch, the parabolic curve is not very different from a straight line. A comparison of the results based on the strict parabolic theory with those based on the simpler straight-line formulas shows that the difference is small and often not greater than the uncertainty as to the true strength of the concrete. The straight-line theory will, therefore, be used exclusively.

Theoretical Assumptions. The theory of reinforced concrete beams is based on the usual assumptions that:

(1) The loads are applied at right angles to the axis of the beam. The usual vertical gravity loads supported by a horizontal beam fulfil this condition.

(2) There is no resistance to free horizontal motion. This condition is seldom, if ever, exactly fulfilled in practice. The more rigidly the beam is held at the ends, the greater will be its strength above that computed by the simple theory. Under ordinary conditions the added strength is quite indeterminate and is not allowed for.

(3) The concrete and steel stretch together without breaking the bond between them. This is absolutely essential.

(4) Any section of the beam which is plane before bending is plane after bending.

In Fig. 15 is shown, in a very exaggerated form, the essential meaning of assumption (4). The section *abdc* in the unstrained condition, is changed to the plane *a′b′d′c′* when the load is applied. The compression at the top equals *aa′* equals *bb′*. The neutral axis is unchanged. The concrete at the bottom is stretched an amount equal to *cc′* equals *dd′*, while the stretch in the steel equals *gg′*. The compression in the concrete between

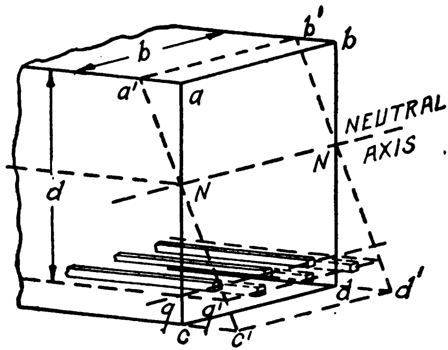


Fig. 15. Exaggerated Diagram Showing Plane Section of Beam before and after Bending

the neutral axis and the top is proportional to the distance from the neutral axis.

In Fig. 16 is given a side view of the beam, with special reference to the deformation of the fibers. Since the fibers between the neutral axis and the compressive face are compressed proportionally, then, if *aa′* represents the lineal compression of the outer fiber, the shaded lines represent, at the same scale, the compression of the intermediate fibers.

Summation of Compressive Forces. The summation of compressive forces evidently equals the sum of all the compressions, varying from zero to the maximum compressive stress *c* at the extreme upper fiber, where the lineal compression is ϵ_c. The average unit compressive stress is, therefore, $\frac{1}{2}c$.

Since k is the ratio of the distance from the neutral axis to the upper fiber to the total effective depth d, that distance equals kd. The breadth of the beam is b; therefore

$$\Sigma X = \frac{1}{2} cbkd \qquad (1)$$

Center of Gravity of Compressive Forces. The center of gravity of compressive forces is sometimes called the *centroid of compression.* It here coincides with the center of gravity of the triangle, which is at one-third the height of the triangle from the upper face. Therefore

$$x = \frac{1}{3} kd \qquad (2)$$

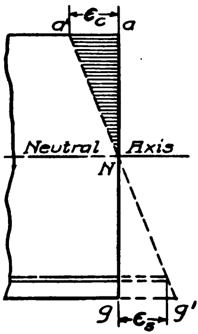


Fig. 16. Diagram Showing Side View of Beam with Reference to Deformation of Fibers

The ratio of the dimension from the steel to the center of the compressive stress to the dimension d equals j and, therefore, the dimension between the centroids of the tensile and the compressive forces equals jd, which equals $(d-x)$.

Position of the Neutral Axis. According to one of the fundamental laws of mechanics, the sum of the horizontal tensile forces must be equal and opposite to the sum of the compressive forces. If the very small amount of tension furnished by the concrete below the neutral axis is ignored, the tension in the steel equals As equals $pbds$ equals the total compression in the concrete which as stated in Equation (1) equals $\frac{1}{2} cbkd$. Therefore

$$pbds = \frac{1}{2} cbkd$$
$$ps = \frac{1}{2} ck \qquad (3)$$

The position of the neutral axis is determined by the value of k, which is a function of the steel ratio p and the ratio of the moduli of elasticities n. We must also eliminate s and c. By definition, c equals $\epsilon_c E_c$ and s equals $\epsilon_s E_s$ and n equals $E_s \div E_c$. Substituting in Equation (3), we have

$$p \epsilon_s E_s = \frac{1}{2} \epsilon_c E_c k \qquad (4)$$

TABLE XI

Value of *k* for Various Values of *n* and *p*
(Straight-Line Formulas)

n	*p*									
	.020	.018	.016	.014	.012	.010	.008	.006	.004	.003
10	.464	.446	.427	.407	.385	.358	.328	.292	.246	.216
12	.493	.476	.457	.436	.412	.385	.353	.314	.266	.235
15	.531	.513	.493	.471	.446	.418	.384	.343	.291	.258
18	.562	.544	.524	.501	.476	.446	.412	.369	.315	.279
20	.580	.562	.542	.519	.493	.463	.428	.384	.328	.292
25	.618	.600	.580	.557	.531	.500	.463	.418	.358	.319
30	.649	.631	.611	.588	.562	.531	.493	.446	.384	.344
40	.698	.679	.659	.637	.611	.579	.542	.493	.428	.384

From the two proportional triangles in Fig. 16, we may write the proportion

$$\frac{\epsilon_c}{kd} = \frac{\epsilon_s}{d - kd} \qquad \text{or } \epsilon_c = \epsilon_s \left(\frac{k}{1-k}\right)$$

Substituting in Equation (4) for the ratio $E_s \div E_c$ its value n, and for ϵ_c, the value just obtained, we have

$$pn = \frac{1}{2}\left(\frac{k^2}{1-k}\right) \tag{5}$$

Solving this quadratic for k, we have

$$k = \sqrt{2\,pn + p^2n^2} - pn \tag{6}$$

Values of Ratio of Moduli of Elasticity. The various values for the ratio of the moduli of elasticity n are discussed in the succeeding paragraphs. The values of k for various values of n and p, have been computed in Table XI. Eight values have been chosen for n, in conjunction with ten values of p, varying by 0.2 per cent and covering the entire practicable range of p, on the basis of which values k has been worked out in the tabular form. Usually the value of k can be determined directly from Table XI. By interpolating between two values in Table XI, any required value within the limits of ordinary practice can be determined with all necessary accuracy.

TABLE XII

Value of *j* for Various Values of *n* and *p*
(Straight-Line Formulas)

n	*p*									
	.020	.018	.016	.014	.012	.010	.008	.006	.004	.003
10	.845	.851	.858	.864	.872	.881	.891	.903	.918	.928
12	.836	.841	.848	.855	.863	.872	.882	.895	.911	.922
15	.823	.829	.836	.843	.851	.861	.872	.886	.903	.914
18	.813	.819	.825	.833	.841	.851	.863	.877	.895	.907
20	.807	.813	.819	.827	.836	.846	.857	.872	.891	.903
25	.794	.800	.807	.814	.823	.833	.846	.861	.881	.894
30	.784	.790	.796	.804	.813	.823	.836	.851	.872	.885
40	.767	.774	.780	.788	.796	.807	.819	.836	.857	.872

The dimension jd from the center of the steel to the centroid of the compression in the concrete equals $(d - x)$. Therefore

$$j = \frac{d-x}{d} = \frac{d - \frac{1}{3}kd}{d} = 1 - \frac{1}{3}k \qquad (7)$$

The corresponding values for j have been computed for the several values of p and n, as shown in Table XII. These several values for k and j which correspond to the various values for p and n are shown in Fig. 17, which is especially useful when the required values of k and j must be obtained by interpolation.

Examples. 1. Assume $n = 15$ and $p = .01$; how much are k and j? *Solution.* Follow up the vertical line on the diagram for the steel ratio, $p = .010$, to the point where it intersects the k curve for $n = 15$; the intersection point is $\frac{9}{10}$ of one of the smallest divisions above the .40 line, as shown on the scale at the left; each small division is .020, and, therefore, the reading is $\frac{9}{10} \times .020 = .018$ plus .400 or .418, the value of k. Similarly, the .010 p line intersects the j curve for $n = 15$ at a point slightly above the .860 line, or at .861.

2. Assume $n = 16$ and $p = .0082$; how much are k and j? *Solution.* One must imagine a vertical line (or perhaps draw one) at $\frac{2}{5}$ of a space between the .0080 and .0085 vertical lines for p. This line would intersect the line for $n = 15$ at about .388; and the line for $n = 18$ at about .416; one-third of the difference (.028) or .009, added to .388 gives .397, the interpolated value. Although this is sufficiently close for practical purposes, the precise value (.398) may be computed from Equation (6). Similarly the value of j may be interpolated as .867. Although the values of these ratios have been computed to three significant figures (thousandths), the uncertainties as to the actual character and strength of the concrete used will make it useless to obtain these ratios closer than the nearest hundredth.

Theoretically, there are an indefinite number of values of *n*, the ratio of the moduli of elasticity of the steel and the concrete. The modulus for steel is fairly constant at about

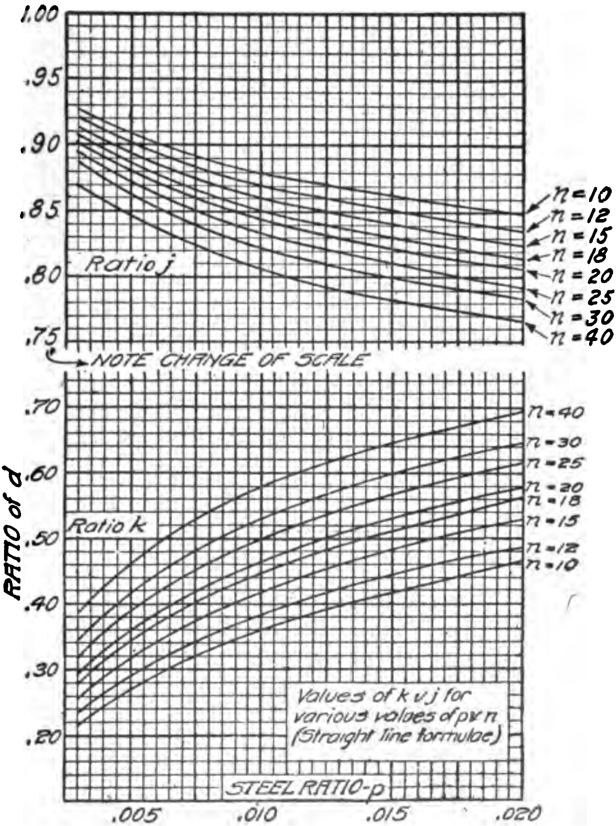


Fig. 17. Curves Giving Values of *k* and *j* for Various Values of *p* and *n*. Values used for these curves will be found in Tables XII and XIII

29,000,000 or 30,000,000. The value of the initial modulus for stone concrete varies, according to the quality of the concrete, from 1,500,000 to 3,000,000. An average value for 1:2:4

TABLE XIII

Modulus of Elasticity of Some Grades of Concrete

KIND OF CONCRETE	AGE (Days)	MIXTURE	E_c	n
Cinder	30	1 :2 :4	1,200,000	25
Broken stone	30	1 :3 :6	2,000,000	15
Broken stone	10	1 :2 :4	2,000,000	15
Broken stone	30	1 :2 :4	2,500,000	12

cinder concrete is about 1,200,000. Some experimental values for stone concrete have fallen somewhat lower than 1,500,000, while others have reached 4,000,000 and even more. We may use the values in table XIII with the constant value of 30,000,000 for the steel.

Percentage of Steel. The previous calculations have been made as if the percentage of the steel might be varied almost indefinitely. While there is considerable freedom of choice, there are limitations beyond which it is useless to pass; and there is always a most economical percentage, depending on the conditions. We must, therefore, determine p in terms of c, s, and n. Substituting in Equation (3), the value of k in Equation (6) we have

$$p = \frac{1}{2} \times \frac{c}{s} \sqrt{2pn + p^2n^2} - \left(\frac{c}{2s}\right) pn$$

which may be reduced to

$$p = \frac{1}{2} \times \frac{c}{s} \times \frac{cn}{(s + cn)} \qquad (8)$$

This equation shows that we cannot select the percentage of steel at random, since it evidently depends on the selected stresses for the steel and concrete and also on the ratio of their moduli. For example, consider a high-grade concrete—1:2:4—whose modulus of elasticity is considered to be 2,500,000, and which has a working compressive stress c of 600 pounds, which we may consider in conjunction with a tensile stress of 16,000 pounds in the steel. The values of c, s, and n are therefore 600, 16,000, and 12, respectively. Substituting these values in Equation (8) we compute p equal to .0058.

This *theoretical* percentage is not, necessarily, the most economical or the most desirable percentage to use. For a beam of given size, some increase of strength may be obtained by using a higher percentage of steel; or for a given strength, or load capacity, the depth may be somewhat decreased by using a higher percentage of steel. The decrease in height, making possible a decrease in the total height of the building for a given clear headroom between floors, *may* justify the increase in the percentage of steel, but that is determined by considerations of economy.

Example. What is the theoretical percentage of steel for ordinary stone concrete when $n = 15$, $c = 650$, and $s = 18{,}000$? *Ans.* .0063

Resisting Moment. The moment which resists the action of the external forces is evidently measured by the product of the distance from the center of gravity of the steel to the centroid of compression of the concrete, times the total compression of the concrete, or times the tension in the steel. As the compression in the concrete and the tension in the steel are equal, it is only a matter of convenience to express this product in terms of the tension in the steel. Therefore, adopting the notation already mentioned, we have the formula

$$M = As\,(jd) \tag{9}$$

But since the computations are frequently made in terms of the dimensions of the concrete and of the percentage of the reinforcing steel, it may be more convenient to write the equation

$$M = (pbds)\,jd \tag{10}$$

From Equation (**1**) we have the total compression in the concrete. Multiplying this by the distance from the steel to the centroid of compression *jd,* we have another equation for the moment

$$M = \frac{1}{2}\,(cbkd)\,jd \tag{11}$$

When the percentage of steel used agrees with that computed from Equation (**8**), then Equations (**10**) and (**11**) will give

TABLE XIV

Value of p for Various Values of $(s \div c)$ and n

Formula: $p = \frac{1}{2} \times \frac{1}{R}\left(\frac{n}{R+n}\right)$, in which $R = (s \div c)$

$(s \div c)$	n							
	10	12	15	18	20	25	30	40
10.0	.0250	.0273	.0300	.0321	.0333	.0357	.0375	.0400
12.5	.0178	.0196	.0218	.0236	.0246	.0267	.0282	.0304
15.0	.0133	.0148	.0167	.0182	.0190	.0208	.0222	.0242
17.5	.0104	.0116	.0132	.0145	.0152	.0168	.0180	.0199
20.0	.0083	.0094	.0107	.0118	.0125	.0139	.0150	.0167
25.0	.0057	.0065	.0075	.0084	.0089	.0100	.0109	.0123
30.0	.0042	.0048	.0056	.0062	.0067	.0076	.0083	.0095
40.0	.0025	.0029	.0034	.0039	.0042	.0048	.0054	.0062
50.0	.0017	.0019	.0023	.0026	.0029	.0033	.0037	.0044

identically the same results; but when the percentage of steel is selected arbitrarily, as is frequently done, then the proposed section should be tested by both equations. When the percentage of steel is larger than that required by Equation (8), the concrete will be compressed more than is intended before the steel attains its normal tension. On the other hand, a lower percentage of steel will require a higher unit tension in the steel before the concrete attains its normal compression. If the discrepancy between the percentage of steel assumed and the true economical value is very great, the stress in the steel, or the concrete, may become dangerously high when the stress in the other element, on which the computation may have been made, is only normal.

Working Values for the Ratio of the Steel Tension to the Concrete Compression. It is often more convenient to obtain working values from tables or diagrams rather than to compute them each time from equations.

If Equation (8) is solved for several combinations of values of $(s \div c)$ and n, we have the values as tabulated in Table XIV. These values are also shown in Fig. 18. For other combinations than those used in Table XIV, the values of p may be obtained with great accuracy provided that $(s \div c)$ corresponds with some curve already on the diagram. If it is necessary to inter-

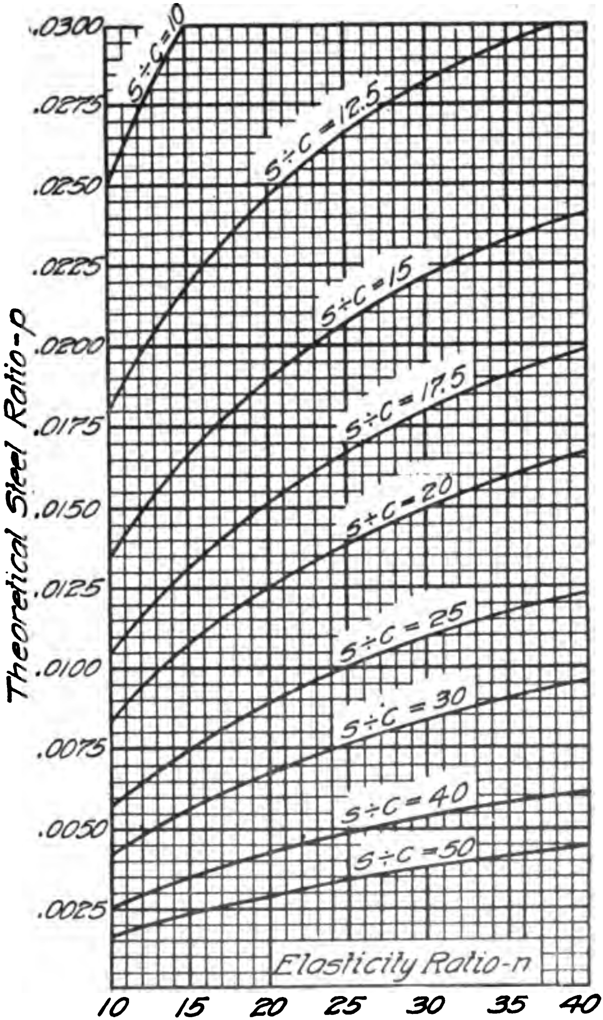


Fig. 18. Curves Showing the Relation of $(s \div c)$ to p and n

polate for some value of $(s \div c)$ of which the curve has not been drawn, it must be recognized that the space between the curves increases rapidly as $(s \div c)$ is smaller. For example, to interpolate for $(s \div c) = 32$, the point must be below the 30 curve by considerably more than 0.2 of the interval between the 30 and 40 curve.

The relative elasticities n of various grades of steel and concrete are usually roughly proportional to the relative working values, as expressed by $(s \div c)$. In other words, if n is large, $(s \div c)$ is correspondingly large unless the working value for s or for c is for some reason made abnormally low. Therefore, there will be little if any use for the values given in the lower left-hand and upper right-hand corners of Table XIV.

Determination of Values for Frequent Use. The moment of resistance of a beam equals the total tension in the steel, or the total compression in the concrete (which are equal) times jd. Therefore, we have the choice of two values, as given in Equations (9) to (11).

$$M_c = \frac{1}{2}(cbkd)\,jd \qquad (12)$$
$$M_s = As\,(jd) = (pbds)\,jd$$

If the theoretical percentage p has already been determined from Equation (8), then either equation may be used, as is most convenient, since the two will give identical results. If the percentage has been arbitrarily chosen, then the least value must be determined, as was described previously. For any given steel ratio and any one grade of concrete, the factors $\frac{1}{2}ckj$ or psj are constant and Equation (12) may be written

$$M_c = R_c bd^2$$
$$M_s = R_s bd^2$$

or, in general,

$$M = Rbd^2$$

when the theoretical percentage of steel is used. Diagrams for quickly determining R are given later.

For 1:2:4 concrete, using $n = 15$, and with a working value for $c = 600$, and $s = 16{,}000$, we find from Equation (8) that the percentage of steel equals

$$p = \frac{1}{2}\,\frac{600}{16{,}000} \times \frac{600 \times 15}{(600 \times 15) + 16{,}000} = .00675$$

From Table XI we find by interpolation that, for $n = 15$ and $p = .00675$, $k = .360$. Then, from Equation (2),

$$x = \frac{1}{3}\,kd = .120\,d \qquad \text{and } j = .880$$

Substituting these values in either formula of Equation (12), we have

$$M = 95\,bd^2$$

The percentage of steel computed from Equation (8) has been called the *theoretical* percentage, because it is the percentage which will develop the maximum allowed stress in the concrete and the steel at the same time, or by the loading of the beam to some definite maximum loading. The real meaning of this is best illustrated by a numerical example with another percentage. Assume that the percentage of steel is exactly doubled, or that p equals $2 \times .00675 = .0135$. From Table XI for $n = 15$, and $p = .0135$ we find $k = .465$; $x = .155\,d$; and $j = .845$. Substituting these values in both forms of Equation (12), we have

$$M_c = 118\,bd^2$$
$$M_s = 183\,bd^2$$

The interpretation of these two equations, and also of the equation found above ($M = 95\,bd^2$), is as follows: Assume a beam of definite dimensions b and d, made of concrete whose modulus of elasticity is $\frac{1}{15}$ that of the modulus of elasticity of the reinforcing steel; assume that it is reinforced with steel having a cross-sectional area equal to $.00675\,bd$. Then, when the beam is loaded with a load which will develop a moment of $95\,bd^2$, the tension in the steel will equal 16,000 pounds per square inch, and the compression in the concrete will equal 600 pounds per square inch at the outer fiber. Assume that the area of the steel is exactly doubled. One effect of this is to lower the neutral axis—k is increased from .360 to .465—and more of the concrete is available for compression. The load may be increased about 24 per cent, or until the moment equals 118 bd^2, before the compression in the concrete reaches 600 pounds per square inch. Under these conditions the steel has a tension of about 10,340 pounds per square inch, and its full strength is not utilized. If the load were increased until the moment were 183 bd^2, then the steel would be stressed to 16,000 pounds per square inch, but the concrete would be compressed to about 930 pounds, which would, of course, be unsafe with such a grade of concrete. If the compression in the concrete is to be limited to 600 pounds per square inch, then the load must be limited to that which will give a moment of 118 bd^2. Even for this the steel is doubled if the load is to be increased 24 per cent. Whether this is justifiable, depends on several circumstances—the relative cost of steel and concrete, the possible necessity for keeping the dimensions of the beam within certain limits, etc. Usually, a much larger ratio of steel than 0.675 per cent is used; 1.0 per cent is far more common; but in the latter case the strength of the steel cannot be fully utilized unless the concrete can stand high compression. A larger value of n will indicate higher values of k, which will indicate higher moments; but n cannot be selected at pleasure. It depends on the character of the concrete used; and, with E_s constant, a large value of n means a small value for E_c, which means also a small value for c, the permissible compression stress. Whenever the percentage of steel is greater than the *theoretical* percentage, as is usual, then the upper of the two formulas of Equation (12) should be used. When in doubt, both should be tested, and that one giving the lower moment should be used.

When $p = .0075$, $n = 15$, $c = 600$, and $s = 16{,}000$, as before, we have $k = .374$, $x = .125\,d$, and $j = .875$. Then, since p is greater than the theoretical value, we use the upper formula of Equation (12) and have

$$M = 98\,bd^2$$

Examples. 1. What is the working moment for a slab with 5-inch thickness to the steel, the concrete having the properties described above?

Solution. Let $b = 12$ inches. Then $M = 98 \times 12 \times 25 = 29{,}400$ inch-pounds, the permissible moment on a section 12 inches wide.

2. A slab having a span of 8 feet is to support a load of 150 pounds per square foot. The concrete is to be as described above, and the percentage of steel is to be 0.75. What is the required thickness d to the steel?

Solution. Allowing 70 pounds per square foot as the estimated weight of the slab itself, the total load is 220 pounds per square foot. A strip 12 inches wide has an area of 8 square feet, and the total load is 1,760 pounds. Assuming the slab as free-ended, the moment is

$$\tfrac{1}{8}\,Wl = \tfrac{1}{8} \times 1{,}760 \times 96 = 21{,}120 \text{ in.-lb.}$$

For a strip 12 inches wide, $b = 12$ inches and

$$M = 98 \times 12 \times d^2 = 1{,}176d^2 = 21{,}120$$
$$d^2 = 17.96$$
$$d = 4.24 \text{ in.}$$

Then, allowing one inch of concrete below the steel, the total thickness of the slab would be 5¼ inches and its weight, allowing 12 pounds per square foot per inch of depth, would be about 63 pounds per square foot, thus agreeing safely with the estimated allowance for dead load. If the computed thickness and weight had proved to be materially more than the original allowance, another calculation would be necessary, assuming a somewhat greater dead load. This increase of dead load would of itself produce a somewhat greater moment, but the increased thickness would develop a greater resisting moment. A little experience will enable one to make the preliminary estimate so close to the final that not more than one trial calculation should be necessary.

PRACTICAL CALCULATION AND DESIGN OF BEAMS AND SLABS

Tables for Slab Computations. The necessity of computing frequently the required thickness of slabs renders very useful the data given in Table XV, which has been worked out on the basis of several combinations of values of c and s. Municipal building laws often specify the unit values which must be used and even the moment formula. For example, slabs are usually continuous over beams and even the wall ends of slabs are so restrained at the wall that the working moment is considerably less than $Wl \div 8$ and, therefore, the formula $Wl \div 10$ is specifically permitted in many municipal regulations. Table XV is computed on that basis, but the tabulated unit loads may be very easily changed to the basis of $Wl \div 8$ or $Wl \div 12$. It should be noted that the unit loads given in Table XV include the slab weight, which must, therefore, be subtracted before the

TABLE XV

Working Loads on Floor Slabs. $M = Wl \div 10$

1 CINDER CONCRETE — $c = 300$; $s = 12{,}000$; $n = 25$; $p = .0048$; $M \div bd^2 = 50$

For $M = Wl \div 8$, subtract 20 per cent from unit loads
For $M = Wl \div 12$, add 20 per cent to unit loads

Total Thickness of Slab (Inches)	Thickness of Concrete Below Steel (Inches)	Effective Thickness "d" (Inches)	Area of Steel in 12-inch Width (Sq. In.)	M for 12-inch Width (In.-Lbs.)	Total Load in Pounds per Square Foot, including Weight of Slab, for the Given Spans in Feet												Weight of Slab per Sq. Ft
					4	5	6	7	8	9	10	11	12	13	14	15	
1 Cinder Concrete																	
3	¾	2¼	.128	3037	158	101	70	51	39	31	...	...	...	...	...	...	27
3½	¾	2¾	.158	4537	236	151	104	77	59	46	37	...	...	...	...	...	32
4	1	3	.173	5400	281	180	125	91	70	55	45	37	...	...	...	...	36
4½	1	3½	.201	7350	383	245	170	125	95	75	61	50	42	...	...	...	41
5	1	4	.230	9600	500	320	222	163	125	98	80	66	55	47	...	..	45
5½	1	4½	.259	12150	633	405	281	206	158	125	101	83	70	59	51	...	50
6	1¼	4¾	.273	13537	705	451	313	230	176	139	113	93	78	66	57	...	54
7	1¼	5¾	.331	19837	...	661	459	337	258	204	165	136	114	97	84	73	63
8	1¼	6¾	.389	27337	...	911	632	465	356	281	228	188	158	135	116	101	72
9	1½	7½	.432	33750	...	...	781	573	439	347	281	232	195	166	143	125	81
10	1½	8½	.489	43350	...	...	...	737	564	446	361	298	251	213	184	160	90
12	1½	10½	.604	66150	...	...	...	...	861	680	551	456	383	326	281	245	108
2 Stone Concrete — $c = 500$; $s = 14{,}000$; $n = 15$; $p = .0062$; $M \div bd^2 = 77$																	
3	¾	2¼	.167	4676	243	156	108	79	60	48	39	...	...	...	..	...	36
3½	¾	2¾	.205	6990	364	233	161	118	91	71	58	48	...	...	...	...	42
4	1	3	.223	8316	433	277	192	141	108	85	69	57	48	...	...	...	48
4½	1	3½	.260	11319	589	377	261	192	147	116	94	78	65	55	..	...	54
5	1	4	.298	14784	769	493	342	251	192	152	123	101	85	72	62	...	60
5½	1	4½	.335	18711	974	623	433	318	243	192	156	129	108	92	79	69	66
6	1¼	4¾	.353	20846	1085	695	482	354	271	214	174	143	120	102	88	77	72
7	1¼	5¾	.428	30550	1591	1018	707	519	397	314	254	210	176	150	129	113	84
8	1¼	6¾	.502	42098	....	1403	974	715	548	437	351	290	243	207	178	156	96
9	1½	7½	.558	51975	....	....	1202	823	677	534	433	351	300	256	216	192	108
10	1½	8½	.632	66759	....	....	1545	1134	868	686	556	460	386	329	283	247	120
12	1½	10½	.780	101871	....	....	....	....	1326	1048	849	702	588	502	433	377	144

TABLE XV (Continued)

Working Loads on Floor Slabs. $M = Wl \div 10$

3 STONE CONCRETE $c=600$; $s=16,000$; $n=15$; $p=.00675$; $M \div bd^2 = 95$

For $M = Wl \div 8$, subtract 20 per cent from unit loads
For $M = Wl \div 12$, add 20 per cent to unit loads

Total Thickness of Slab (Inches)	Thickness of Concrete Below Steel (Inches)	Effective Thickness "d" (Inches)	Area of Steel in 12-inch Width (Sq. In.)	M for 12-inch Width (In.-Lbs.)	Total load in pounds per square foot, including weight of slab, for the given spans in feet: 4	5	6	7	8	9	10	11	12	13	14	15	Weight of Slab per Sq. Ft.
3	¾	2¼	.181	5770	300	192	133	98	75	59	48	39	...	...	...	...	36
3½	¾	2¾	.221	8620	449	287	199	146	112	88	72	59	49	...	...	...	42
4	1	3	.241	10260	535	342	238	174	133	105	85	70	59	50	...	...	48
4½	1	3½	.281	13965	727	465	323	237	181	143	116	96	80	68	59	...	54
5	1	4	.321	18240	950	608	422	310	237	187	152	125	105	90	77	67	60
5½	1	4½	.362	23085	1202	769	534	392	300	237	192	159	133	113	98	85	66
6	1¼	4¾	.382	25720	1340	857	596	437	335	265	214	177	148	127	109	95	72
7	1¼	5¾	.462	37690	....	1256	872	640	490	388	314	259	218	185	160	139	84
8	1¼	6¾	.542	51940	....	....	1202	884	676	535	433	357	300	256	221	192	96
9	1½	7½	.603	64125	....	....	....	1090	834	660	534	442	371	316	272	237	108
10	1½	8½	.683	82365	....	....	....	....	1071	847	686	567	476	406	350	305	120
12	1½	10½	.844	125685	....	....	....	....	....	....	1047	866	726	620	534	465	144

4 STONE CONCRETE $c=650$; $s=16,000$; $n=15$; $p=.0077$; $M \div bd^2 = 107$

Total Thickness of Slab (Inches)	Thickness of Concrete Below Steel (Inches)	Effective Thickness "d" (Inches)	Area of Steel in 12-inch Width (Sq. In.)	M for 12-inch Width (In.-Lbs.)	4	5	6	7	8	9	10	11	12	13	14	15	Weight of Slab per Sq. Ft.
3	¾	2¼	.208	6490	338	216	150	110	84	66	54	44	37	...	...	...	36
3½	¾	2¾	.254	9710	527	323	224	165	126	99	81	66	56	48	...	...	42
4	1	3	.277	11560	602	385	267	196	150	118	96	79	66	57	49	...	48
4½	1	3½	.328	15730	819	524	364	267	204	162	131	108	91	77	66	58	54
5	1	4	.370	20540	1069	684	475	349	267	211	171	141	118	101	87	76	60
5½	1	4½	.416	26000	...	866	602	442	338	267	216	179	150	128	110	96	66
6	1¼	4¾	.439	28880	...	962	668	491	376	297	240	199	167	142	122	106	72
7	1¼	5¾	.531	42450	...	...	982	721	552	436	354	292	245	209	180	157	84
8	1¼	6¾	.624	58500	...	...	...	994	761	602	487	403	338	288	248	217	96
9	1½	7½	.693	72225	...	...	...	1228	940	743	602	498	418	356	307	267	108
10	1½	8½	.785	92770	...	...	...	...	...	954	773	639	536	457	394	343	120
12	1½	10½	.970	141560	...	...	...	...	...	...	...	975	818	697	602	524	144

TABLE XV (Continued)

Working Loads on Floor Slabs. $M = Wl \div 10$

5 STONE CONCRETE — $c = 650$; $s = 18{,}000$; $n = 15$; $p = .0063$; $M \div bd^2 = 100$

For $M = Wl \div 8$, subtract 20 per cent from unit loads
For $M = Wl \div 12$, add 20 per cent to unit loads

Total Thickness of Slab (Inches)	Thickness of Concrete Below Steel (Inches)	Effective Thickness "d" (Inches)	Area of Steel in 12-inch Width (Sq. In.)	M for 12-inch Width (In.-Lbs.)	Total load in pounds per square foot, including weight of slab, for the given spans in feet: 4	5	6	7	8	9	10	11	12	13	14	15	Weight of Slab per Sq. Ft.
3	¾	2¼	.170	6070	316	202	140	103	79	62	50	41	....	....	....	...	36
3½	¾	2¾	.208	9075	472	302	210	154	118	93	75	62	52	44	....	...	42
4	1	3	.227	10800	563	360	250	183	140	111	90	74	62	53	....	...	48
4½	1	3½	.264	14700	765	490	340	250	191	151	122	101	85	72	62	54	54
5	1	4	.302	19200	1000	640	444	326	250	197	160	132	111	94	81	71	60
5½	1	4½	.340	24300	1266	810	562	413	316	250	202	167	140	120	103	90	66
6	1¼	4¾	.359	27035	1408	901	626	459	351	278	225	186	156	133	115	100	72
7	1¼	5¾	.434	39675	....	1322	918	674	516	408	330	274	229	195	168	146	84
8	1¼	6¾	.510	54675	....	....	1265	929	711	563	455	376	316	269	232	202	96
9	1½	7½	.566	67500	....	....	....	1147	878	694	562	465	390	332	287	250	108
10	1½	8½	.642	86700	....	....	....	....	1128	892	722	597	501	427	368	321	120
12	1½	10½	.793	132300	....	....	....	....	....	....	1102	911	764	652	562	490	144

6 STONE CONCRETE — $c = 700$; $s = 18{,}000$; $n = 15$; $p = .0072$; $M \div bd^2 = 113$

Total Thickness of Slab (Inches)	Thickness of Concrete Below Steel (Inches)	Effective Thickness "d" (Inches)	Area of Steel in 12-inch Width (Sq. In.)	M for 12-inch Width (In.-Lbs.)	4	5	6	7	8	9	10	11	12	13	14	15	Weight of Slab per Sq. Ft.
3	¾	2¼	.194	6850	356	228	158	116	89	70	57	47	39	...	...	...	36
3½	¾	2¾	.237	10260	534	342	237	174	133	105	85	70	59	50	43	...	42
4	1	3	.259	12200	635	406	282	207	158	125	101	84	70	60	52	...	48
4½	1	3½	.302	16610	864	553	385	282	216	170	138	114	96	82	70	61	54
5	1	4	.346	21700	1130	723	502	369	282	223	181	149	125	107	92	80	60
5½	1	4½	.389	27460	....	915	636	466	357	282	229	189	158	135	116	101	66
6	1¼	4¾	.410	30470	....	1015	705	518	396	313	254	210	176	150	129	112	72
7	1¼	5¾	.496	44830	....	....	1038	762	583	461	373	309	259	221	190	166	84
8	1¼	6¾	.583	61785	....	....	1430	1050	804	635	515	425	357	304	262	228	96
9	1½	7½	.648	76275	....	....	....	....	993	784	635	525	441	376	324	282	108
10	1½	8½	.734	97970	....	....	....	....	...	1007	816	675	567	485	416	362	120
12	1½	10½	.907	149500	....	....	....	....	..	...	..	1029	865	737	635	553	144

net live load is known. In the last column are shown the unit weights of various slab thicknesses on the basis of 108 pounds per cubic foot for cinder concrete and 144 pounds per cubic foot for stone concrete. These subtractive weights may need to be altered if a concrete of different weight is used, or if an extra top coat of concrete, which cannot be considered as structurally a part of the slab, is laid on afterward. The "thickness of concrete below steel" is such as is approved by good practice, but in case municipal regulations or other reasons should require other thicknesses of concrete below the steel, Table XV may still be used by considering the *effective* thickness d and by varying, as need be, the subtractive weight of the slab to determine the net load. The blanks in the upper right-hand corner of each section of the table indicate that for those spans and slab thicknesses the slabs cannot safely carry their own weight and that even the weights nearest the blanks are so small that, after subtracting the slab weights, the remainders are too small for practical working floor loads, or even roof loads. The blanks in the lower left-hand corner of each section of the table indicate that for these combinations of span, load, and slab thickness, the shearing strength would be insufficient for the load which its transverse strength would enable it to carry and that, therefore, although those slabs would carry a great load, those combinations of span and slab thickness are uneconomical and should not be used.

Examples. 1. Using stone concrete such that $c = 600$, $n = 15$, and $s = 16{,}000$, and with a required working load of 200 pounds per square foot, what span may be chosen?

Solution. This requires Section 3 of Table XV. We note that an 8-inch slab on a span of 12 feet will carry 300 pounds per square foot gross, or 204 pounds net, which is substantially what is required. Another combination would be a 7-inch slab with a span between 10 and 11 feet. To interpolate, subtract 84, the unit slab weight, from 314 and from 259, giving 230 and 175. It should be noted that the difference $388 - 314 = 74$, is greater than the difference $314 - 259 = 55$, which in turn is greater than the difference $259 - 218 = 41$. From this we may know, without precise calculations, that the value for the span 10 feet 6 inches must be such that the difference between 230 (net value) and the net value for 10 feet 6 inches must be greater than the difference between this net value and 175, the net value for an 11-foot span. $230 - 200 = 30$ and $200 - 175 = 25$. Therefore, a span of 10 feet 6 inches is very close to the theoretical value—close enough for practical purposes. Whether an 8-inch slab with 12-foot span or a 7-inch slab with 10-foot-6-inch span is the more economical or desirable depends on other conditions, one of which is the span of the beams. This will be considered later.

2. Find the span, assuming the same data as above, except that municipal regulations require at least 1½ inches of concrete below the steel and also require using the formula $Wl \div 8$.

Solution. An 8-inch slab with 1½ inches of concrete under the steel will be 8¼ inches thick and will weigh 99 pounds per square foot. On the 11-foot span the total load, after subtracting 20 per cent, will be 286 pounds and, after subtracting 99, will leave 187 pounds net. Similarly, the net load on the 10-foot span is 247 pounds. $200 - 187 = 13$, and $247 - 187 = 60$; 13 is nearly one-fourth of 60 and, therefore, the interpolated span is about one-fourth of the interval from 11 feet back to 10 feet, or 10 feet 9 inches. The net effect of adding the extra concrete below the steel and using $Wl \div 8$ instead of $Wl \div 10$, therefore, reduces the span of the 8-inch slab from 12 feet to 10 feet 9 inches. A similar computation could be made for a 7-inch slab—actual thickness 7¼ inches.

3. Assume a slab made of 1 : 2.5 : 5 concrete; the span has been determined already as 6 feet; the floor is to be covered with 2 inches of cinder-concrete fill between the wood sleepers and a wood floor, weighing 23 pounds per square foot; the live load is to be 150 pounds per square foot. Required, the slab thickness.

Solution. For such concrete, use Section 2, Table XV. $150 + 23 = 173$; and adding a trial figure of 50 pounds for the unit weight of the slab, we have 223 as the total load. Under 6-feet span we find 192 for a 4-inch slab and 261 for a 4½-inch slab; 4 inches is too thin and 4½ somewhat needlessly thick. Since 223 is nearer to 192 than to 261, we may economize by cutting the thickness to 4¼ inches. The detail of the interpolation, elaborated in Example 2, shows this to be justifiable. The required area of steel for the 4¼-inch slab is found by interpolation, between .223 and .260, or .242 square inch—the area of steel in 12 inches of width of slab. This is .02 square inch per inch of width; a ⅜-inch square bar has an area of .1406 square inch; therefore, such bars spaced 7 inches apart will fulfil the requirements.

Table for Computation of Simple Beams. In Table XVI has been computed, for convenience, the working total load (including the weight of the beam) on rectangular beams one inch wide and of various depths and spans. For other widths of beams, multiply the tabular load by the width of the beam in inches. Table XVI is based on a grade of concrete such that M equals $100\,bd^2$; for any other grade of concrete, determine the corresponding factor of bd^2, or, in other words, Equation (**12**), compute the value of $\frac{1}{2}\,ckj$, or of psj, whichever is less. Multiply the tabular load by the percentage of that factor

TABLE XVI

Gross Load on Rectangular Beam One Inch Wide. $M=100bd^2$

For other widths, multiply by width of beam.

For any other combination of unit values, multiply by percentage of its formula factor to 100

Effective Depth Depth of Beam "d" (Inches)	Span in Feet																
	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
4	267	213	178	152	133	119	107	97	89	82	76	71	67	....	....	..	. .
5	417	333	278	238	208	186	167	151	139	128	119	111	104	98	93	...	...
6	600	480	400	343	300	267	240	218	200	185	171	160	150	141	133	126	120
7	817	653	544	466	408	363	327	297	272	251	233	218	204	192	181	172	163
8	1067	853	711	609	533	474	427	388	356	328	304	284	267	251	237	224	213
9	1350	1080	900	771	675	600	540	491	450	415	385	360	337	317	300	284	270
10	1667	1333	1111	952	833	741	667	606	556	513	476	444	417	392	370	351	333
11	2017	1613	1344	1151	1008	896	807	733	672	620	575	538	504	474	448	424	403
12	2400	1920	1600	1371	1200	1067	960	872	800	738	685	640	600	564	533	505	480
13	2817	2253	1878	1610	1408	1252	1127	1024	939	867	805	751	704	663	626	593	563
14	3267	2613	2178	1866	1633	1452	1307	1188	1089	1005	933	871	817	768	726	687	653
15	3750	3000	2500	2141	1875	1667	1500	1364	1250	1154	1070	1000	937	882	833	789	750
16	4267	3413	2844	2436	2133	1896	1707	1551	1422	1313	1218	1138	1067	1004	948	898	853
17	4817	3853	3211	2752	2408	2141	1927	1751	1606	1482	1376	1284	1204	1133	1070	1014	963
18	5400	4320	3600	3085	2700	2400	2160	1964	1800	1661	1542	1440	1350	1271	1200	1136	1080
19	6017	4813	4011	3437	3008	2674	2407	2188	2006	1852	1718	1604	1504	1416	1337	1266	1203
20	6667	5333	4444	3809	3333	2963	2667	2422	2222	2050	1904	1778	1667	1569	1481	1404	1333

NOTE. For any beams corresponding to values from the lower left-hand corner of the table, the possible failure by diagonal shear should be carefully tested.

to 100. The concrete of Section **5**, Table XV, has the factor 100 and if such concrete is used, no percentage multiplication is necessary. The blanks in the upper right-hand corner of Table XVI are similar to the corresponding blanks in Table XV—the beams cannot safely carry their own weight. And, as before, the values immediately adjacent to the blanks are of little or no use, since the possible load, after deducting the weight of the beam, would be too small for practical purposes. The values in the lower left-hand corner should be used with great caution. Many of the beams of such relative span and depth would fail from diagonal shear long before the tabulated loads were reached. But, since the liability to failure from diagonal shear is dependent on the nature of the web reinforcement, the line of demarcation is not easily drawn, as was done in Table XV.

Examples. 1. Assume the concrete described in Section **3**, Table XV, which has the factor 95. How much load will be carried by a beam of such concrete, when the beam is 8 inches wide, 16 inches effective depth, and 18 feet span?

Solution. From Table XVI, under 18 feet span and opposite 16 inches effective depth, we find 948, the load for a beam one inch wide. An 8-inch beam will carry 8 times 948, or 7,584 pounds. 95 per cent of 7,584 is 7,205 pounds, the load for that particular grade of concrete. The weight of the concrete, assuming a total depth of 18 inches, is $\frac{8}{12} \times \frac{18}{12} \times 18 \times 144 = 2{,}592$. Deducting this from 7,205, we have the net load as 4,613 pounds.

2. Assume that $c = 500$, $s = 16{,}000$, $n = 12$, and $p = .006$. How much load will be carried by a beam 6 inches wide, 12 inches effective depth, and 14 feet span?

Solution. From the percentage diagram on page 63, we see that for $s \div c = 32$ and for $n = 12$, $p = .0043$; and since this is less than the chosen steel ratio .006, we must use the first part of Equation (**12**). For $n = 12$ and for $p = .006$, $k = .314$ and $j = .895$. Then $\frac{1}{2} c k j = 250 \times .314 \times .895 = 70$, the factor of bd^2. The load on a beam one inch wide, 12 inches effective depth, and 14 feet span is 685 pounds. For 6 inches wide it would be 4,110 pounds. 70 per cent of this is 2,877 pounds. The weight, allowing 2 inches below the steel, is $\frac{6}{12} \times \frac{14}{12} \times 14 \times 144 = 1{,}176$ pounds. The net load is, therefore, $4{,}110 - 1{,}176 = 2{,}934$ pounds.

Bonding Steel and Concrete. *Resistance to Slipping of Steel in Concrete.* The previous discussion has considered merely the tension and compression in the upper and lower sides of the beam. A plain, simple beam resting freely on two end supports has neither tension nor compression in the fibers at the ends of the beam. The horizontal tension and compression, found at or near the center of the beam, entirely disappear by the time the ends of the beam are reached. This is done by means of the intermediate concrete which transfers the tensile stress in the steel at the bottom of the beam to the compression fibers in the top. This is, in fact, the main use of the concrete in the lower part of the beam.

It is, therefore, necessary that the bond between the concrete and the steel shall be sufficiently great to withstand the tendency to slip. The required strength of this bond is evidently equal to the difference in the tension in the steel per unit of length. For example, suppose that we are considering a bar 1 inch square in the middle of the length of a beam. Let the bar be under an actual tension of 15,000 pounds per square inch. Since the bar is 1 inch square, the actual total tension is 15,000 pounds. Suppose that, at a point 1 inch beyond, the moment in the beam is so reduced that the tension in the bar is 14,900 pounds instead of 15,000 pounds. This means that the difference of pull (100 pounds) has been taken up by the concrete. The surface of the bar for that length of 1 inch is 4 square inches. This will require an average adhesion of 25 pounds per square inch between the steel and the concrete in order to take up this difference of tension. The adhesion between concrete and plain bars is usually considerably greater than this, and there is, therefore, but little question about the bond in the center of the beam. But near the ends of the beam, the change in tension in the bar is far more rapid, and the question of the bond then becomes important.

Virtue of Deformed Bars. The fact that the adhesion of the concrete to the steel is a critical feature under some conditions, called attention to the desirability of using deformed bars, which furnish a mechanical bond. Deformed bars have a variety of shapes; and since they are not prismatic, it is evident that, apart from adhesion, they cannot be drawn through the concrete with-

TABLE XVII

Bond Adhesion of Plain and Deformed Bars per Inch of Length

Basis { 75 lb. adhesion per square inch for plain bars
125 lb. adhesion per square inch for deformed bars
For any other unit basis, multiply surface (column 2 or 3) by unit

Size of Bar (Inches)	Surface (Square Inches per Lineal Inch)		Bond Adhesion (Pounds per Lineal Inch)			
			Plain Bars at 75		Deformed Bars at 125	
	Square	Round	Square	Round	Square	Round
$\frac{1}{4}$	1.00	0.785	75	59	125	98
$\frac{5}{16}$	1.25	0.982	94	74	156	123
$\frac{3}{8}$	1.50	1.178	112	88	187	147
$\frac{7}{16}$	1.75	1.375	131	103	219	172
$\frac{1}{2}$	2.00	1.571	150	118	250	196
$\frac{5}{8}$	2.50	1.964	187	147	312	245
$\frac{3}{4}$	3.00	2.356	225	177	375	294
$\frac{7}{8}$	3.50	2.749	262	206	437	344
1	4.00	3.142	300	236	500	393
$1\frac{1}{8}$	4.50	3.534	337	265	562	442
$1\frac{1}{4}$	5.00	3.927	375	324	625	491

out splitting or crushing the concrete immediately around the bars. The choice of form is chiefly a matter of designing a bar which will furnish the greatest resistance, and which at the same time is not unduly expensive to manufacture. Impartial tests have shown that, even under conditions which are most favorable to the plain bars, the deformed bars have an actual hold in the concrete which is from 50 to 100 per cent greater than that of plain bars. It is unquestionable that age will increase rather than diminish the relative inferiority of plain bars.

The specifications of the American Railway Engineering Association, adopted in 1910, allow 80 pounds per square inch of surface for plain bars, 40 for drawn wire, and from 100 to 150 for deformed bars "depending upon form". Municipal regulations frequently limit the adhesion to 75 pounds, without any mention of deformed bars or of any extra allowable adhesion if they are used. The adhesion is of special importance in short but deep, heavily loaded beams. It is frequently difficult to obtain the necessary adhesion with an allowance of only 75 pounds per square inch. Refer to Table XVII.

Computation of Bond Required in Bars. From theoretical mechanics, we learn that the total shear V at any section equals the difference in moment for the ends of a section of infinitesimal length. This may be seen from Fig. 19 where T is tension in steel at left end of section and toward the center of the beam; T' is tension in steel at right end of section; then $T - T'$ is the difference in tension, which is the amount of tension taken up by the concrete in the length x. Then $(T - T')\ jd$ is the difference of moment in the unit distance x. But by taking moments about a, we have the following expression:

$$Vx = (T - T')\ jd$$

from which

$$(T - T') = Vx \div jd$$

If x is considered to be the unit length—say one inch—then the bond adhesion on *all* the bars will be $V \div jd$. If we call v the unit horizontal shear, and the width of the beam b, then

$$v = V \div bjd \qquad \textbf{(13)}$$

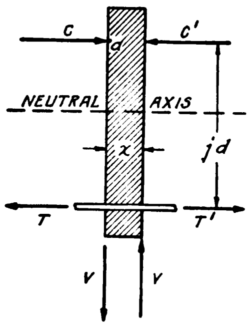


Fig. 19. Diagram for Calculating Moments of Inertia in a Bar

Illustrative Example. Assume an 8-foot beam, uniformly loaded to its capacity, with an effective depth $d =$ 16 inches, width $b =$ 8 inches, $c = 600$, $s = 16{,}000$, and $n = 15$. Then $p = .00675$, $k = .360$, $j = .880$, and $A = 16 \times 8 \times .0067 =$ 0.86 square inch. This area may be obtained from three ⅝-inch round bars, each of which will have a cross-sectional area of .30 square inch and circumference of 1.96 inches, which means an adhesion area of 5.88 square inches per inch of length of the three bars. M equals $95\ bd^2$ or 194,560 inch-pounds equals $Wl \div 8$. Since $l = 96$ inches, then $W = 16{,}213$, and V, the maximum total shear, is one-half of this or 8,107 pounds. At a point one foot from the center the shear will be one-fourth of the maximum shear, or 2,027 pounds; dividing this by *jd,* or .880 × 16, we have 144 pounds, the required bond adhesion at that point. Dividing this by the area, 5.88, we have 24 pounds per square inch, the adhesion stress, which is amply safe.

At the abutment the shear is 8,107 pounds; dividing this by *jd,* or .880 × 16, we have 575 pounds, the required total adhesion. $575 \div 5.88 = 98$, the required unit adhesion. This is greater than the permissible unit adhesion of plain bars, and greater than the uniform figure (75) given in so many municipal building codes, although not greater than that which deformed bars can safely carry.

Another possible solution of the problem, although one with some loss of economy, would be to use four ½-inch square bars, whose total cross-sectional area would be one square inch (instead of 0.86) and whose superficial area per inch of length would be 8 square inches. $578 \div 8 = 72$ pounds per square inch. This is within the specified limit for plain bars. Strictly speaking, this would not be the precise figure, since the added percentage of steel would slightly decrease j and therefore slightly increase the required adhesion, but the effect in this case is very slight, about one pound per square inch.

Since the variation of j is very little for the usual variations in percentage of steel and quality of concrete, it is a common practice to consider that, *as applied to this equation,* j has the uniform value of .875, or ⅞. This would reduce Equation (13) to

$$v = \tfrac{8}{7} V \div bd$$

which means that v, the maximum unit horizontal or vertical shear in a section, is about $\frac{1}{7}$ more than the average shear, found by dividing the total shear by the effective section of the beam.

Vertical Shear and Diagonal Tension. Beams which are tested to destruction frequently fail at the ends long before the transverse strength at the center has been fully developed. Even if the bond between the steel and the concrete is amply strong for the requirements, the beam may fail on account of the shearing or diagonal stresses in the concrete between the steel and the neutral axis. According to the best theory on the subject, supplemented by tests, the unit diagonal stress *may* amount to double the unit vertical shear.

Methods of Guarding against Failure by Shear or by Diagonal Tension. The failure of a beam by actual shear is almost unknown. The failures usually ascribed to shear are generally caused by diagonal tension. A solution of the very simple Equation (**13**) will indicate the intensity of the vertical shear. If a beam is so reinforced that it will safely stand the tests for moment, diagonal shear, and bond adhesion, there is almost no question of its ability to resist vertical shear.

Resistance to Diagonal Tension by Bending Bars or by Use of Stirrups. Resistance to diagonal tension is furnished by bending up the main reinforcing bars, and also by the use of *stirrups.* Unfortunately, it seems impossible to devise any simple, practicable rules (like those for resisting moment) for the precise design of reinforcement to resist diagonal tension.

Since the theory is so uncertain, an empirical method has developed which practice has shown to be safe and which is, fortunately, so inexpensive that any further economies are of little importance. The accepted method may be described as follows: *First,* one or more of the moment bars are bent up at an angle of 30 degrees to 45 degrees near each end of the beam, but one or two bars are always allowed to run straight through. If the beam is very short and there are numerous bars, the bends are made at various distances from the ends, as fast as the moment bars can be spared from their primary work of resisting moment. *Second,* vertical stirrups are placed throughout the length of the beam. Near the ends of the beam, the stirrups should be spaced about one-half the depth of the beam. At the center there is no need for stirrups, except as they keep the other reinforcement in place during construction, and therefore the spacing is gradually widened from the ends toward the center. *Third,* round bars are preferable to square, since they bend more easily into the exact shape desired. The size should vary in general accordance with the size of the beam—$\frac{1}{4}$-inch, $\frac{5}{16}$-inch, and $\frac{3}{8}$-inch are the most common sizes, but $\frac{1}{2}$-inch bars might be used for heavy deep beams of short span.

Calculations by Diagrams of Related Factors. A very large proportion of concrete work is done with a grade of concrete such that we may call the ratio n of the moduli of the steel and the concrete either 12 or 15. The working values of the stresses in the steel and the concrete, s and c, are determined either by public regulation or by the engineer's estimate of the proper values to be used. The diagrams, Figs. 20 and 21, fully cover the whole range of practicable values for steel and for stone concrete. In the previous problems all values have been calculated on the basis of formulas. By means of these diagrams all needed values, on the basis of the other factors, may be read from the diagram with sufficient accuracy for practical work. In addition, the diagrams enable one to note readily the effect of any proposed change in one or more factors.

Illustrative Examples. 1. If a beam, made of concrete such that $n = 15$, is to be so loaded that when the stress in the steel s is 16,000, the stress in the concrete c shall simultaneously be 600, the steel ratio p must be .00675. This is found on the diagram, Fig. 20, for $n = 15$ by following the line $s = 16{,}000$ to its intersection with the line $c = 600$. The intersection point, measured on the steel ratio scale at the bottom of the diagram, reads .00675. Also, running horizontally from the intersection point to the scale at the left, we read $R = 95$, which is the factor for bd^2 in the moment equation, Equation (**12**). Incidentally, the corresponding values of k and j for this steel ratio may be obtained, with greater convenience, from this diagram, although they are also obtainable from the more general diagram, Fig. 17.

2. Assume that, for reasons discussed on page 60, it is decided to increase the steel ratio to 1.2 per cent. Following the vertical line for $p = .012$, we find it intersects the line $c = 600$ at a point where $R = 114$, but the point is about halfway between the lines $s = 10{,}000$ and $s = 12{,}000$, indicating that, using that steel ratio, the stress in the steel for a proper stress in the concrete is far less than the usual working stress, and that it would be about 11,000. If the load were increased so that s would equal 16,000, we can see by estimation that c would probably be over 800, far greater than a proper working value.

3. Assume $p = .004$, $c = 600$, and $n = 15$. How much then are R and s? $R = 79$ and $s = 22,000$, which is impracticably

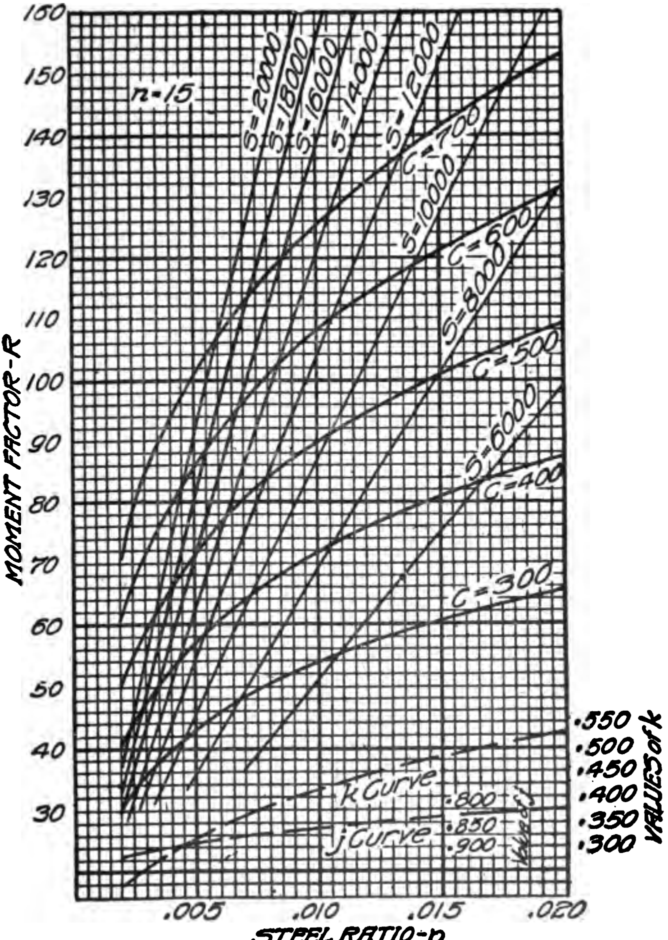


Fig. 20. Curves Showing Values of Moment Factor R for $n = 15$

high. The diagram, Fig. 21, shows plainly that for low steel ratios the values of s are abnormally high for ordinary values

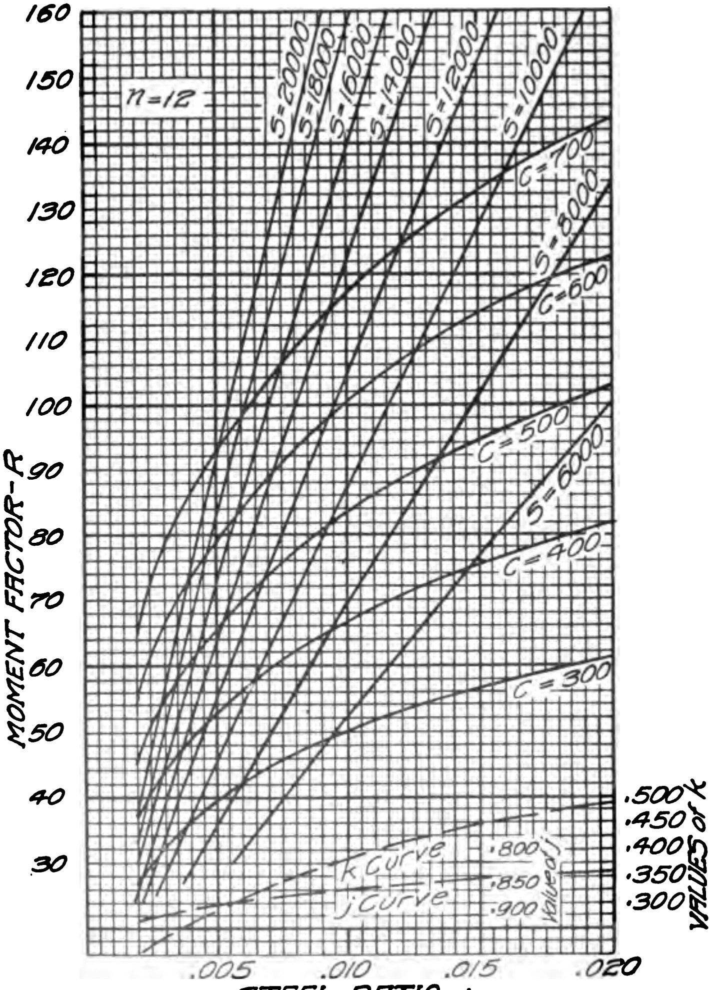


Fig. 21. Curves Showing Values of Moment Factor R for $n = 12$

of c; on the other hand, for high steel ratios, the ordinary values of c cannot utilize the full working strength of the steel.

Slabs on I-Beams. The skeleton framework of buildings, especially if very high, is frequently made of steel, even when the floors have concrete girders, beams, and slabs. But sometimes even the girders and beams are made of steel and only the slab is concrete; steel **I**-beams are used for the floor girders and beams, and the beams are connected by concrete floor slabs, Fig. 22. These are usually computed on the basis of transverse beams which are free at the ends, instead of considering them as continuous beams, which will add about 50 per cent to their strength. Since it would be necessary to move the reinforcing

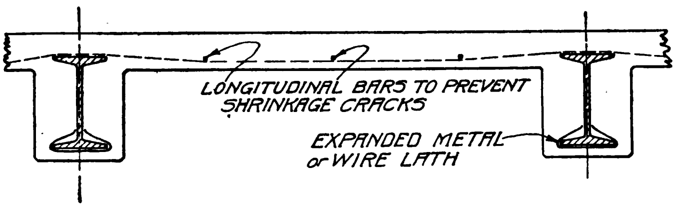


Fig. 22. Diagram Showing Method of Placing Concrete Floor Slabs on I-Beam Girders

steel from the lower part to the upper part of the slab when passing over the floor beams, in order to develop the additional strength which is theoretically possible with continuous beams, and since this is not usually done, it is by far the safest practice to consider all floor slabs as being "free-ended". The additional strength which they undoubtedly have to some extent because they are continuous over the beams merely adds indefinitely to the factor of safety. Usually, the requirement that the **I**-beams shall be fireproofed by surrounding the beam itself with a layer of concrete such that the outer surface is at least 2 inches from the nearest point of the steel beam results in having a shoulder of concrete under the end of each slab, which materially adds to its structural strength. This justifies the frequent practice of using the moment formula $M = Wl \div 10$, which is a compromise between $Wl \div 8$ and $Wl \div 12$. Even this should be done only when the bars are run into the adjoining span far enough so that the bond adhesion, computed at a safe working value, will not exceed the tension in the steel, and also when the steel is raised to a point near the top of the slab over the supports. The fireproofing around the beam must usually be kept in place by wrapping a small sheet of expanded metal or wire lath around the lower part of the beam before the concrete is placed.

Slabs Reinforced in Both Directions. When the floor beams are spaced nearly equally in both directions, so that they form, between the beams, panels which are nearly square, a considerable saving can be made in the thickness of the slab by reinforcing it with bars running in both directions. The theoretical computation of the strength of such slabs is exceedingly complicated. The usual method is to estimate that the total load is divided into two parts such that if l equals the length of a rectangular panel and b equals the breadth (l being greater than, or equal to b), then the ratio of the load carried by the "b" bars is given by the proportion $l^4 \div (l^4 + b^4)$. If the value of this proportion is worked out for several values of the ratio $l : b$, we have the percentages which are given by the tabular form below:

Ratio $l : b$	1.0	1.1	1.2	1.3	1.4	1.5
Proportion of load carried by "b" bars	50%	59%	67%	74%	80%	83%

When l and b are equal, each set of bars takes half the load. When l is only 50 per cent greater than b, the shorter bars take 83 per cent of the load and it is uneconomical to use bars for transverse moment in the longer direction. The lack of economy begins at about 25 per cent excess length, and therefore panels in which the proportion of length to breadth is greater than 125 per cent should be reinforced in the shorter direction only. Strictly speaking, the slab should be thicker by the thickness of one set of reinforcing bars.

Reinforcement against Temperature Cracks. The modulus of elasticity of ordinary concrete is approximately 2,400,000 pounds per square inch, while its ultimate tensional strength is about 200 pounds per square inch. Therefore a pull of about $\frac{1}{12000}$ of the length would nearly, if not quite, rupture the concrete. The coefficient of expansion of concrete has been found to be almost identical with that of steel, or .0000065 for each degree Fahrenheit. Therefore, if a block of concrete were held at the ends with absolute rigidity, while its temperature was lowered about 12 degrees, the stress developed in the concrete would be very nearly, if not quite, at the rupture point. Fortunately, the ends will not usually be held with such rigidity; but, nevertheless, it does generally happen that, unless the entire mass of concrete is permitted to expand and contract freely so that the temperature stresses are small, the stresses will usually localize themselves at the weak point of the cross section, wherever that may be, and will there develop a crack, provided the concrete is not reinforced with steel. If, however, steel is well distributed throughout the cross section of the concrete, it will prevent the concentration of the stresses at local points, and will distribute it uniformly throughout the mass.

Reinforced concrete structures are usually provided with bars running in all directions, so that temperature cracks are prevented, and it is generally unnecessary to make any special provision against them. The most common exception occurs in floor slabs, which structurally require bars in only one direction. It is found that cracks parallel with the bars which reinforce the slab will be prevented, if a few bars are laid perpendicularly to the direction of the main reinforcing bars. Usually, $\frac{1}{2}$-inch or $\frac{3}{8}$-inch bars, spaced about 2 feet apart, will be sufficient.

Retaining walls, the balustrades of bridges, and other similar structures, which may not need any bars for purely structural reasons, should be provided with them in order to prevent temperature cracks. A theoretical determination of the amount of such reinforcing steel is practically impossible, since it depends on assumptions which are themselves very doubtful. It is usually conceded that if there is placed in the concrete an amount of steel whose cross-sectional area equals about ⅓ of 1 per cent of the area of the concrete, the structure will be proof against such cracks. Fortunately, this amount of steel is so small that any great refinement in its determination is of little importance. Moreover, since such bars have a value in tying the structure together, and thus add somewhat to its strength and ability to resist disintegration due to vibrations, the bars are usually worth what they cost.

T-BEAM CONSTRUCTION

When concrete beams are laid in conjunction with overlying floor slabs, the concrete for both the beams and the slabs being laid in one operation, the strength of such beams is very much greater than their strength considered merely as plain beams, even though we compute the depth of the beam as equal to the total depth from the bottom of the beam to the top of the slab. An explanation of this added strength may be made as follows:

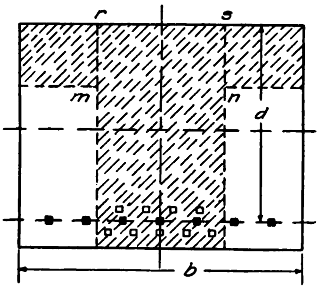


Fig. 23. Diagram of T-Beam in Cross Section

If we construct a very wide beam as shown by the complete rectangle in Fig. 23, there is no hesitation about calculating its strength as that of a plain beam whose width is *b*, and whose effective depth to the reinforcement is *d*. Our previous study in plain beams has shown us that the steel in the bottom of the beam takes care of practically all the tension; that the neutral axis of the beam is somewhat above the center of its height; that the only work of the concrete below the neutral axis is to transfer the stress in the steel to the concrete in the top of the beam; and that even in this work it must be assisted somewhat by stirrups or by bending up the steel bars. If, therefore, we cut out from the lower corners of the beam two rectangles, as shown by the unshaded areas, we are saving a very large part of the concrete, with very little loss in the strength of the beam, provided we can fulfil certain conditions. The steel, instead of being distributed uniformly throughout the bottom of the wide beam, is concentrated into the comparatively narrow portion which we shall hereafter call the rib of the beam. The concentrated tension in the bottom of this rib must be transferred to the compression area at the top of the beam. We must also design the beam so that the shearing stresses in the plane *mn* immediately below the slab shall not exceed the allowable shearing stress in the concrete. We must also provide that failure shall not occur on account of shearing in the vertical planes *mr* and *ns* between the sides of the beam and the flanges.

Resisting Moments of T-Beams. The resisting moments of T-beams will be computed in accordance with straight-line for-

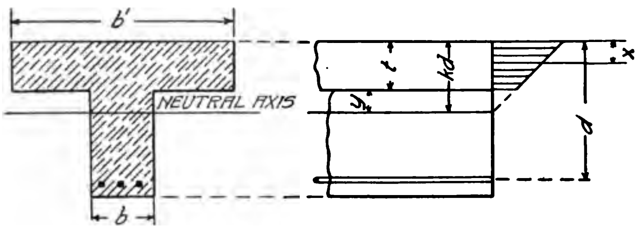


Fig. 24. Compression Stress Diagram for T-Beam

mulas. There are three possible cases, according as the neutral axis is: (1) *below* the bottom of the slab (which is the most common case, and which is illustrated in Fig. 24); (2) *at* the bottom of the slab; or (3) *above* it. All possible effect of tension in the concrete is ignored. For Case I, even the compression furnished by the concrete between the neutral axis and the under side of the slab is ignored. Such compression is, of course, zero at the neutral axis; its maximum value at the bottom of the slab is small; the summation of its compression is evidently small; the lever arm is certainly not more than $\frac{2}{3} y$; therefore, the moment due to such compression is insignificant compared with the resisting moment due to the slab. The computations are much more complicated if it is included;

without it the resulting error is a very small percentage of the true figure, and the error is on the side of safety.

Case I. If c is the maximum compression at the top of the slab, and the stress-strain diagram is rectilinear, as in Fig. 24, then the compression at the bottom of the slab is $c\,\frac{kd-t}{kd}$. The average compression equals

$$\tfrac{1}{2}\left(c+c\,\frac{kd-t}{kd}\right)=\frac{c}{kd}\,(kd-\tfrac{1}{2}t)$$

The total compression C equals the average compression multiplied by the area $b't$; or

$$C=As=b't\left[\frac{c}{kd}(kd-\tfrac{1}{2}t)\right] \tag{14}$$

The center of gravity of the compressive stresses is evidently at the center of gravity of the trapezoid of pressures. The distance x of this center of gravity from the top of the beam is given by the formula

$$x=\frac{t}{3}\times\frac{3\,kd-2\,t}{2\,kd-t} \tag{15}$$

It has already been shown that

$$\frac{\epsilon_s}{\epsilon_c}=\frac{cn}{s}=\frac{kd}{d-kd}$$

Combining this equation with Equation (14), we may eliminate $\frac{c}{s}$, and obtain a value

$$kd=\frac{And+\tfrac{1}{2}\,b't^2}{An+b't} \tag{16}$$

If the percentage of steel is chosen at random, the beam will probably be over-reinforced or under-reinforced. In general, it will therefore be necessary to compute the moment with reference to the steel and also with reference to the concrete, and, as before with plain beams, Equation (12), we shall have a pair of equations

$$M_c = C\,(d-x) = b't\left[\frac{c}{kd}(kd - \tfrac{1}{2}t)\right](d-x) \qquad (17)$$
$$M_s = As\,(d-x) = pb'ds\,(d-x)$$

Case II. If we place $kd = t$ in the equation just above Equation (**16**), and solve for d, we have a relation between d, c, s, n, and t, which holds when the neutral axis is just at the bottom of the slab. The equation becomes

$$d = \frac{t\,(cn+s)}{cn} \qquad (18)$$

A combination of dimensions and stresses which would place the neutral axis *exactly* in this position is improbable, although readily possible; but Equation (**18**) is very useful in determining whether a given numerical problem belongs to Case I or Case III. When the stresses s and c in the steel and concrete, the ratio n of the elasticities, and the thickness t of the slab are all determined, then the solution of Equation (**18**) will give a value of d which would bring the neutral axis at the bottom of the slab. But it should not be forgotten that the compression in the concrete c and the tension in the steel s will not simultaneously have certain definite values—say $c = 500$, and $s = 16{,}000$—unless the percentage of steel has been so chosen as to give those simultaneous values. When, as is usual, some other percentage of steel is used, the equation is not strictly applicable, and it therefore should not be used to determine a value of d which will place the neutral axis at the bottom of the slab and thus simplify somewhat the numerical calculations. For example, for $c = 500$, $s = 16{,}000$, $n = 12$, and $t = 4$ inches, d will equal 14.67 inches. Of course this particular depth may not satisfy the requirements of the problem. If the proper value for d is *less* than that indicated by Equation (**18**), the problem belongs to Case III; if it is *more,* the problem belongs to Case I.

Case III. The diagram of pressure is very similar to that in Fig. 24, except that it is a triangle instead of a trapezoid, the triangle having a base c and a height kd which is less than t. The center of compression is at one-third the height from the base, or $x = \frac{1}{3}kd$. Equations (**9**) to (**12**) are applicable to this case as well as to Case II, which may be considered merely as the limiting case to Case III. But it should be remembered that b' refers to the width of the flange or slab, and not to the width of the stem or rib.

Width of Flange. The width b' of the flange is usually considered as equal to the width between adjacent beams, or as extending from the middle of one panel to the middle of the next. The chief danger in such an assumption lies in the fact that if the beams are very far apart, they must have corresponding strength to carry such a floor load, and the shearing stresses between the rib and the slab will be very great. (The method of calculating such shear will be given later.) It sometimes happens (as illustrated on page 100), that the width of slab on each side of the rib is almost indefinite. In such a case we must arbitrarily assume some limit. Since the unit shear is greater for short beams than for long beams, the slab thickness should bear some relation to the span of the beam. The building code specifications for New York City specify that the width on *each* side of the beam shall not be greater than one-sixth of the beam span, and not greater than six times the slab thickness. If the width of the rib is twice the slab thickness, this rule permits the width of flange b' to be fourteen times the slab thickness, and something over one-third of the beam span, whichever is the less. If the compression is computed for two cases, both of which have the same size of rib, the same steel, and the same thickness of slab, but different slab widths, it is found, as might be expected, that for the narrower slab width the unit compression is greater, the neutral axis is very slightly lower, and even the unit tension in the steel is slightly greater. No demonstration has ever been made to determine any limitation of width of slab beyond which no compression would be developed by the transverse stress in a T-beam rib under it. It is probably safe to assume that compression extends for six times the thickness of the slab on *each* side of the rib. If the beam as a whole is safe on this basis, then it is still safer for any additional width to which the compression may extend.

Width of Rib. Since it is assumed that all of the compression occurs in the slab, the only work done by the concrete in the rib is to transfer the tension in the steel to the slab, to resist the shearing and web stresses, and to keep the bars in their proper places. The width of the rib is to a certain extent determined by the amount of reinforcing steel which must be placed in the rib, and by the number of bars—whether it is desirable to use two or more rows of bars instead of merely one row. As indicated in Fig. 23, the amount of steel required in the base of a T-beam is frequently so great that two rows of bars are necessary in order that the bars may have a sufficient spacing between them so that the concrete between will not split apart. Although it would be difficult to develop any rule for the proper spacing between bars without making assumptions which are perhaps doubtful, the following empirical rule is frequently adopted by designers: The *minimum* spacing between bars, center to center, should be two and one-quarter times the diameter of the bars. Fire insurance and municipal specifications usually require that there shall be one and one-half to two inches clear outside of the steel. This means that the beam shall be three or four inches wider than the net width from out to out of the extreme bars. The data given in Table XVIII will therefore be found very convenient, since, when a certain number of bars of given size are to be used, a glance at the table will show immediately whether it is possible to space them in one row; and, if it is not possible, the necessary arrangement can be very readily designed. For example, assume that six ⅞-inch bars are to be used in a beam. The table shows immediately that, according to the rule, the required width of the beam will be 14.72 inches; but if, for any reason, a beam 11 inches wide is considered preferable, the table shows that four ⅞-inch bars may be placed side by side, leaving two bars to be placed in an upper row. According to the same rule regarding the spacing of the bars in vertical rows, the distance from center to center of the two rows should be $2.25 \times .875 = 1.97$ inches; that is, the rows should be, say, 2 inches apart, center to center. It should also be noted that the

TABLE XVIII

Required Width of Beam, Allowing $2\frac{1}{4} \times d$, for Spacing, Center to Center, and 2 Inches Clear on Each Side

n = number of bars ; d = diameter

Formula : Width $= (n-1)\ 2.25\,d + d + 4 = 2.25\,nd - 1.25\,d + 4$

No. of Bars	Diameter of Bars						
	½ In.	⅝ In.	¾ In.	⅞ In.	1 In.	1⅛ In.	1¼ In.
2	5.62	6.03	6.44	6.84	7.25	7.66	8.06
3	6.75	7.44	8.13	8.81	9.50	8.19	10.87
4	7.87	8.84	9.81	10.78	11.75	12.72	13.68
5	9.00	10.25	11.50	12.75	14.00	15.25	16.50
6	10.12	11.65	13.19	14.72	16.25	17.78	19.31
7	11.25	13.06	14.87	16.68	18.50	20.31	22.12
8	12.37	14.46	16.56	18.65	20.75	22.84	24.94
9	13.50	15.87	18.25	20.62	23.00	25.37	27.75
10	14.62	17.28	19.94	22.59	25.25	27.90	30.56

NOTE.—For side protection of only one and one-half inches, deduct one inch from above figures.

plane of the center of gravity of this steel is at two-fifths of the distance between the bars above the lower row, or that it is .8 inch above the center of the lower row.

Examples. 1. Assume that a 5-inch slab is supporting a load on beams spaced 5 feet apart, the beams having a span of 20 feet. Assume that the moment of the beam has been computed as 900,000 inch-pounds. What will be the dimensions of the beam if the concrete is not to have a compression greater than 600 pounds per square inch and the tension of the steel is not to be greater than 16,000 pounds per square inch?

Solution. There are an indefinite number of solutions to this problem. There are several terms in Equation (**17**) which are mutually dependent; it is, therefore, impracticable to obtain directly the depth of the beam on the basis of assuming the other quantities; therefore, it is only possible to assume figures which experience shows will give approximately accurate results, and then to test these figures to see whether all the conditions are satisfied. Within limitations, we may assume the amount of steel to be used, and determine the depth of beam which will satisfy the other conditions, together with that of the assumed area of steel. For example, we shall assume that six ⅞-inch square bars having an area of 4.59 square inches will be a suitable reinforcement for this beam. We shall also assume as a trial figure that x equals 1.5. Substituting these values in the second formula of Equation (**17**) we may write that formula

$$900{,}000 = 4.59 \times 16{,}000(d - 1.5)$$

Solving for d, we find that $d = 13.75$ inches. If we test this value by means of Equation (**18**) we shall find that, substituting the values of

t, c, n, and s in Equation (18), the resulting value of $d = 16.11$ inches. This shows that if we make the depth of the beam only 13.75, the neutral axis will be within the slab, and the problem comes under Case III, to which we must apply Equation (12). Dividing the area of the steel, 4.59, by $(b' \times d)$, we have the value of $p = .00556$. Interpolating with this value of p in Table XI, we find that when $n = 12$, then $k = .303$; $kd = 4.17$; $x = 1.39$; and $jd = 12.36$. Substituting these values in Equation (12), we find that the moment $900{,}000 = 1.545\,c$, or that $c = 582$ pounds per square inch. This shows that the unit compression of the concrete is safely within the required figure. Substituting the known values in the second part of Equation (12) we find that the stress in the steel s equals about 15,860 pounds per square inch.

2. Assume that a floor is loaded so that the total weight of live and dead load is 200 pounds per square foot; assume that the T-beams are to be 5 feet apart, and that the slab is to be 4 inches thick; assume that the span of the T-beams is 30 feet. Find the dimensions of the beams.

Solution. We have in the case of this floor an area of 150 square feet to be supported by each beam, which will give a total load of 30,000 pounds on each beam. The moment at the center of such a beam will therefore be equal to the total load, multiplied by one-eighth of the span (expressed in inches), and the moment is therefore 1,350,000 inch-pounds. As a trial value, we shall assume that the beam is to be reinforced with six $\frac{3}{4}$-inch square bars, which have an area of 3.375 square inches. Substituting this value of the area in the second part of Equation (17), and assuming that s equals 16,000 pounds per square inch, we find that the approximate value for $(d - x)$ is 25 inches. This is very much greater than the value of d that would be found from substituting the proper values in Equation (18), so that we know at once that the problem must be solved by the methods of Case I. For a 4-inch slab, the value of x must be somewhere between 1.33 and 2.0. As a trial value, we may call it 1.5, and this means that d will equal 26.5 inches. Assuming that this slab is to be made of concrete using a value for n equal to 12, we know all the values in Equation (16), and may solve for kd, which we find equals 5.54 inches. As a check on the approximations made above, we may substitute this value of kd, and also the value of t in Equation (15), and obtain a more precise value of x, which we find equals 1.62. Substituting the value of the moment and the other known quantities in the upper formula of Equation (17), we may solve for the value of c, and obtain the value that c equals 352 pounds per square inch. This value for c is so very moderate that it would probably be economy to assume a lower value for the area of the steel, and increase the unit compression in the concrete; but this solution will not be here worked out.

Shearing Stresses between Beam and Slab. Every solution for T-beam construction should be tested at least to the extent of knowing that there is no danger of failure on account of the shear between the beam and the slab, either on the horizontal plane at the lower edge of the slab, or in the two vertical planes along the two sides of the beam. Let us consider a T-beam such as is illustrated in Fig. 25. In the lower part of the figure is represented one-half of the length of the flange, which is considered as separated from the rib. Following the usual method of regarding this as a free body in space, acted on by external forces and by such internal forces as are necessary to produce equilibrium, we find that it is acted on at the left end by the abutment reaction, which is a vertical force, and also by a vertical load on top. We may consider P' as representing the summation of all compressive forces acting on the flanges at the center of the beam. In order to produce equilibrium, there must be a shearing force acting on the under side of the flange.

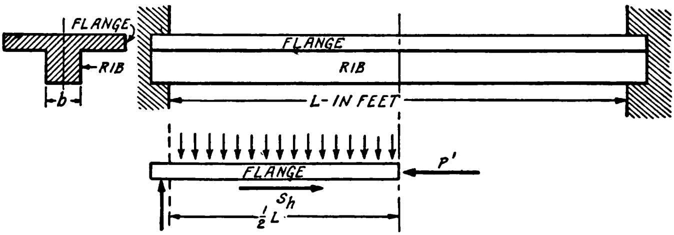


Fig. 25. Diagram Showing Analysis of Stresses in T-Beam

We represent this force by S_h. Since these two forces are the only horizontal forces, or forces with horizontal components, which are acting on this free body in space, P' must equal S_h. Let us consider z as representing the shearing force per unit of area. We know from the laws of mechanics, that, with a uniformly distributed load on the beam, the shearing force is maximum at the ends of the beam, and diminishes uniformly toward the center, where it is zero. Therefore the average value of the unit shear for the half-length of the beam, must equal $\frac{1}{2}z$. As before, we represent the width of the rib by b. For convenience in future computations, we shall consider L as representing the length of the beam, measured in feet. All other dimensions are measured in inches. Therefore the total shearing force along the lower side of the flange, will be

$$S_h = \tfrac{1}{2} z \times b \times \tfrac{1}{2} L \times 12 = 3\,bzL \qquad \textbf{(19)}$$

There is also a possibility that a beam may fail in case the flange, or the slab, is too thin; but the slab is always reinforced by bars which are transverse to the beam, and the slab will be placed on both sides of the beam, giving two shearing surfaces.

Illustrative Example. It is required to test the beam which was computed in Example 1 on page 92. Here the total compressive stress in the flange equals $\frac{1}{2}\,cb'kd = \frac{1}{2} \times 582 \times 60 \times 4.17 = 72{,}808$ pounds. But this compressive stress measures the shearing stress S_h between the flange and the rib. This beam requires six $\frac{7}{8}$-inch bars for the reinforcement. We shall assume that the rib is to be 11 inches wide, that four of the bars are placed in the bottom row, and two bars about 2 inches above them. The effect of this will be to deepen the beam slightly, since d measures the depth of the beam to the center of the reinforcement, and, as already computed numerically on page 91, the center of gravity of this combination will be .8 of an inch above the center of gravity of the lower row of bars. Substituting in Equation (**19**) the values $S_h = 72{,}808$, $b = 11$, and $L = 20$, we find for the unit-value of z, 110 pounds per square inch. This shows that the assumed dimensions of the beam are satisfactory in this respect, since the true shearing stress permissible in concrete is higher than this.

But the beam must be tested also for its ability to withstand shear in vertical planes along the sides of the rib. Since the slab in this case is 5 inches thick and we can count on both surfaces to withstand the shear, we have a width of 10 inches to withstand the shear, as compared with the 11 inches on the underside of the slab. The unit shear would therefore be $\frac{11}{10}$ of the unit shear on the underside of the slab, and would equal 121 pounds per square inch. This is at or beyond the limit, 120, but danger of failure in this respect is avoided by the fact that the slab contains bars which are inserted to reinforce it, and which have such an area that they will effectively prevent any shearing in this way.

Testing Example 2 similarly, we may find the total compression C from Equation (**14**), which here equals $As = 3.375 \times 16{,}000 = 54{,}000$ pounds. The steel reinforcement is six ¾-inch bars, and, from Table XVIII, if the bars are placed side by side, the beam must be 13.19 inches in width, or, in round numbers, 13¼ inches. $S_h = 54{,}000$, $b = 13.25$, $L = 30$; therefore, from Equation (19), $z = 45$ pounds per square inch. Such a value is of course perfectly safe. The shear along the sides of the beam will be considerably greater, since the slab is only 4 inches thick, and twice the thickness is but 8 inches; therefore, the maximum unit shear along the sides will equal 45 times the ratio of 13.25 to 8, or 75 pounds per square inch. Even this would be perfectly safe, to say nothing of the additional shearing strength afforded by the slab bars.

Shear in a T-Beam. The shear here referred to is the shear of the beam as a whole on any vertical section. It does not refer to the shearing stresses between the slab and the rib.

The theoretical computation of the shear of a T-beam is a very complicated problem. Fortunately, it is unnecessary to attempt to solve it exactly. The shearing resistance is certainly far greater in the case of a T-beam than in the case of a plain beam of the same width and total depth and loaded with the same total load. Therefore, if the shearing strength is sufficient, according to the rule, for a plain beam, it is certainly sufficient for the T-beam. In Example 1, above cited, the total load on the beam is 30,000 pounds. Therefore the maximum shear V at the end of the beam is 15,000 pounds. In this particular case, $jd = 12.36$. For this beam, $d = 13.75$ inches, and $b = 11$ inches. Substituting these values in Equation (13), we have

$$v = \frac{V}{b\,(jd)} = \frac{15{,}000}{11 \times 12.36} = 113 \text{ lb. per sq. in.}$$

Although this is probably a very safe stress for direct shearing, it is more than double the allowable direct tension, 40, due to the diagonal stresses; and therefore ample reinforcement must be provided. If only two of the ⅞-inch bars are turned at an angle of 45 degrees at the end, these two bars will have an area of 1.54 square inches, and will have a working tensile strength (at the unit stress of 16,000 pounds) of 24,640 pounds. This is more than the total vertical shear at the ends of the beam, and a pair of turned-up bars would therefore take care of the shear at that point. But considering that stirrups would be used on a beam of 20-foot span, it will be very easy to design these stirrups to provide for this shear, as was explained on a previous page.

Illustration of Slab, Beam, and Girder Construction. Assume a floor construction as outlined in skeleton form in Fig. 26. The columns are spaced 16 feet by 20 feet. Girders which support the alternate rows of beams connect the columns in the 16-foot direction. The live load on the floor is 150 pounds per square foot. The concrete is to be a 1:2:4 mixture, with $n=12$ and $c=600$. Required the proper dimensions for the slab, beams, and girders.

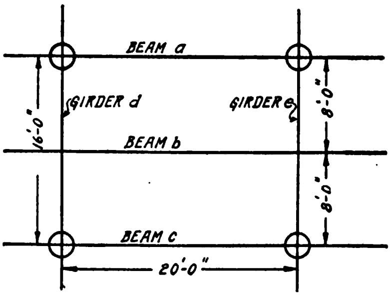


Fig. 26. Skeleton Outline of Floor Panel Showing Slab, Beam, and Girder Construction

Slab. The load on the girders may be computed in either one of two ways, both of which give the same results. We must consider that each beam supports an area of 8 feet by 20 feet. We may therefore consider that girder d supports the load of b (on a floor area 8 feet by 20 feet) as a concentrated load in the center. Or, ignoring the beams, we may consider that the girder supports a uniformly distributed load on an area 16 feet by 20 feet. The moment in either case is the same. Assume that we shall use a 1 per cent reinforcement in the slab. Then, from Table XII, with $n=12$, and $p=.01$, we find that $k=.385$; then $x=.128\,d$, or $jd=.872\,d$. As a trial, we estimate that a 5-inch slab (or $d=4$) will carry the load. This will weigh 60 pounds per square foot, and make a total live and dead load of 210 pounds per square foot. A strip 1 foot wide and 8 feet long will carry a total load of 1,680 pounds, and its moment will be $\frac{1}{8}\times 1,680\times 96=20,160$ inch-pounds.

Using the first half of Equation (**12**), we can substitute the known values, and say that

$$\begin{aligned} 20{,}160 &= \frac{1}{2} \times 600 \times 12 \times .385\,d \times .872\,d \\ &= 1{,}209\,d^2 \\ d^2 &= 16.67 \\ d &= 4.08 \text{ in.} \end{aligned}$$

In this case the span of the slab is considered as the distance from center to center of the beams. This is evidently more nearly exact than to use the net span (which equals 8 feet, less the still unknown width of beam), since the true span is the distance between the centers of pressure on the two beams. It is likely that the true span (really indeterminable) will be somewhat less than 8 feet, which will probably justify using the round value of $d = 4$ inches, and the slab thickness as 5 inches, as first assumed. The area of the steel per inch of width of the slab equals $pbd = .01 \times 1 \times 4.08 = .0408$ square inch. Using ½-inch round bars whose area equals .1963 square inch, the required spacing of the bars will be $.1963 \div .0408 = 4.81$ inches. As shown later, the girder will be 11 inches wide, and the net *width* of the slab is 240 inches less 11 inches or 229 inches. $229 \div 4.81 = 47.6$, call it 48, the number of bars to be spaced equally in one panel.

Beam. The load on a beam is that on an area of 8 feet by 20 feet, and equals $8 \times 20 \times 210 = 33{,}600$ pounds for live and dead load. As a rough trial value, we shall assume that the beam will be 12 inches wide and 15 inches deep below the slab, having, that is, a volume of $1 \times 1.25 \times 20 = 25$ cubic feet, which will weigh 3,600 pounds. Adding this, we have 37,200 pounds as the total live and dead load carried by each beam. The load is uniformly distributed and the moment is

$$M = \frac{1}{8} \times 37{,}200 \times 240 = 1{,}116{,}000 \text{ in.-lb.}$$

We shall assume that the beam is to have a depth d to the reinforcement of 22 inches. Substituting the known quantities in the approximate equation, $M_s = As\,(d - \frac{1}{3}\,t)$, which may be used when we may be sure that the neutral axis is within the slab, we have

$$1{,}116{,}000 = A \times 16{,}000 \times (22 - 1.67)$$
$$A = 3.43 \text{ sq. in.}$$

For T-beams with very wide slabs and great depth of beam, the percentage of steel is always very small. In this case, $p = 3.43 \div (96 \times 22) = .00162$. Such a value is beyond the range of those given in Table XI. We must, therefore, compute the value of k from Equation (6), and we find that $k = .180$, and $kd = 3.96$, which shows that the neutral axis is within the slab; $x = \frac{1}{3}\,kd = 1.32$, and, therefore, $jd = 20.68$. Assume that b' equals fourteen times the slab thickness, or 70 inches. (See page 90.) Substituting these values in the upper part of Equation (12) in order to find the value of c, we find that $c = 390$ pounds per square inch. Substituting the known values in the second half of Equation (12), to obtain a more precise value of s, we find that $s = 15{,}734$ pounds per square inch.

The required area (3.43 square inches) of the bars will be afforded by six $\frac{7}{8}$-inch round bars ($6 \times .60 = 3.60$) with considerable to spare. From Table XVIII we find that six $\frac{7}{8}$-inch bars, either square or round, if placed in one row, would require a beam 14.72 inches wide. This is undesirably wide, and so we shall use two rows, three in each row, and make the beam 9 inches wide. This will add an inch to the depth, and the total depth will be $22 + 3 = 25$ inches. The concrete below the slab is therefore 9 inches wide by 20 inches deep, instead of 12 inches wide by 15 inches deep, as assumed when computing the dead load, but the weight is the same. It should also be noted that the span of these beams was considered as 20 feet, which is the distance from center to center of the columns (or of the girders). This is certainly more nearly correct than to use the net span between the columns—or girders—which is still unknown, since neither columns nor girders are yet designed. Probably a 20-foot span gives some margin of safety.

Girder. The load on one beam is computed above as 37,200 pounds. The load on the girder is, therefore, the equivalent of this load *concentrated* at the center, or of *double* the load (74,400 pounds) uniformly distributed. If for a trial value it is assumed that the girder will be 12 inches by 22 inches below the slab, its weight for sixteen feet will be 4,224 pounds. This gives a total of 78,624 pounds as the equivalent total live and dead load uniformly distributed over the girder. Its moment in the center, therefore, equals $\frac{1}{8} \times 78{,}624 \times 192 = 1{,}886{,}976$ inch-pounds.

The width of the slab in this case is almost indefinite, being 20 feet, or forty-eight times the thickness of the slab. We shall therefore assume that the compression is confined to a width of fourteen times the slab thickness, or that $b' = 70$ inches. Assume for a trial value that $d = 25$ inches; then from the approximate equation $M_s = As\,(d - \frac{1}{3}\,t)$, if $s = 16{,}000$, we find that $A = 5.05$ square inches. Then $p = .00288$; and, from Equation (6), $k = .231$, and $kd = 5.775$. This shows that the neutral axis is below the slab, and that it belongs to Case I, page 88. Checking the computation of kd from Equation (16), we compute $kd = 5.82$, which is probably the more nearly correct value because computed more directly. The discrepancy is due to the dropping of decimals during the computations. From Equation (15), we compute that $x = 1.87$, then $(d - x) = 23.13$. Substituting the value of the moment and of the dimensions in the upper part of Equation (17), we compute c equal to 409 pounds per square inch. Similarly, making substitutions in the lower part of Equation (17), using the more precise value of $(d - x)$ for the lever arm of the steel, we find $s = 16{,}052$ pounds per square inch. (The student should verify in detail all these computations.)

The total required area of 5.08 square inches may be divided into, say, eight round bars $\frac{7}{8}$-inch in diameter. These would have an area of 4.81 square inches. The discrepancy is about 5 per cent. Using the eight round $\frac{7}{8}$-inch bars, the unit stress would be nearly 17,000 pounds. If this is considered undesirable, an area more nearly exact may be obtained by using six round $\frac{7}{8}$-inch bars and two round 1-inch bars. The area would be 5.18 square inches, somewhat in excess of that required. These bars, placed in two rows, would require that the beam be at least 10.78

inches wide. We shall call it 11 inches. The total depth of the beam will be 3 inches greater than *d*, or 28 inches. This means 23 inches below the slab, and the area of concrete below the slab is therefore 11 × 23 = 253 square inches, rather than 12 × 22 = 264 square inches, as assumed for trial.

Shear. The shearing stresses between the rib and slab of the girder are of special importance in this case. The quantity S_h, page 94, equals the total compression in the concrete, which equals the total tension in the steel, which equals, in this case, 16,052 × 5.08 = 81,544 pounds. This equals $3bzL$, in which $b = 11$, $L = 16$ (feet), and z is to be determined.

$$z = 81{,}544 \div (3 \times 11 \times 16) = 154 \text{ lb. per sq. in.}$$

This measures the maximum shearing stress under the slab, and is almost safe, even without the assistance furnished by the stirrups and the bars, which would come up diagonally through the ends of the beam—where this maximum shear occurs—nearly to the top of the slab. The vertical planes on each side of the rib have a combined width of 10 inches, and therefore the *unit stress* is $\frac{11}{10} \times 154 = 169$ pounds per square inch. This is a case of true shear, though it is somewhat larger than the permissible working shear. But there are still other shearing stresses in these vertical planes. In the case of a strip of the slab, say, one foot wide, which is reinforced by slab bars parallel to the girder, the elasticity of such a strip (if disconnected from the girder) would cause it to sag in the center. This must be prevented by the shearing strength of the concrete in the vertical plane along each edge of the girder rib. On account of the combined shearing stresses along these planes, it is usual to specify that when girders are parallel with the slab bars, bars shall be placed across the girder and through the top of the slab for the special purpose of resisting these shearing stresses. Some of the stresses are indefinite, and therefore no precise rules can be computed for the amount of the reinforcement. But since the amount required is evidently very small, no great percentage of accuracy is important. Specifications on this point usually require $\frac{3}{8}$-inch bars, 5 feet long, spaced 12 inches apart.

The shear of the girder, taken as a whole, should be computed as for simple beams, already discussed. Stirrups also should be used.

Another special form of shear must be considered in this problem. Where the beams enter the girder, there is a tendency for the beams to tear their way out through the girder. The total load on the girder by the two beams on each side is of course equal to the total load on one beam, in this case 37,200 pounds. Some of the reinforcing bars of the beam will be bent up diagonally so that they enter the girder near its top, and therefore the beam could not tear out without shearing through the girder from near its top or for a depth of, say, 22 inches (3 inches less than *d*). If there were no reinforcing steel in the girder and enough load were placed on the beam to actually tear it out, the fracture would evidently be in the form of an inverted **V**. The resistance to such tearing out would be chiefly that of the tensile strength of the concrete. If the width of the fracture (or its horizontal projection) is assumed to be 44 inches, and the other dimension, which is the width of the girder rib, 11 inches, there is an area of 484 square inches; and at 40 pounds working tension, it could safely carry a load of 19,360 pounds. But the total load, as shown above, is 37,200 pounds. The steel reinforcement of the girder is, therefore, essential to safety. Although the main reinforcing bars of the girder would have to be torn out before complete failure could take place, the resistance to a small displacement, perpendicular to the bars, is comparatively slight, and therefore these bars should not be depended on to resist this stress. But a pair of ordinary vertical stirrups, passing under the main girder bars, *b b*, Fig. 27, can easily be made of such size as to take any desired portion, or all, of that load. The stirrups should be bent at the upper end so that the strength of the

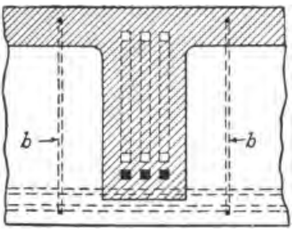


Fig. 27. Details of Reinforcement at Junction of Beam and Girder

bars may be developed without dependence upon bond adhesion. Although precise numerical calculations are impossible without making assumptions which are themselves uncertain, the following calculation is probably safe. $37,200 - 19,360 = 17,840$;

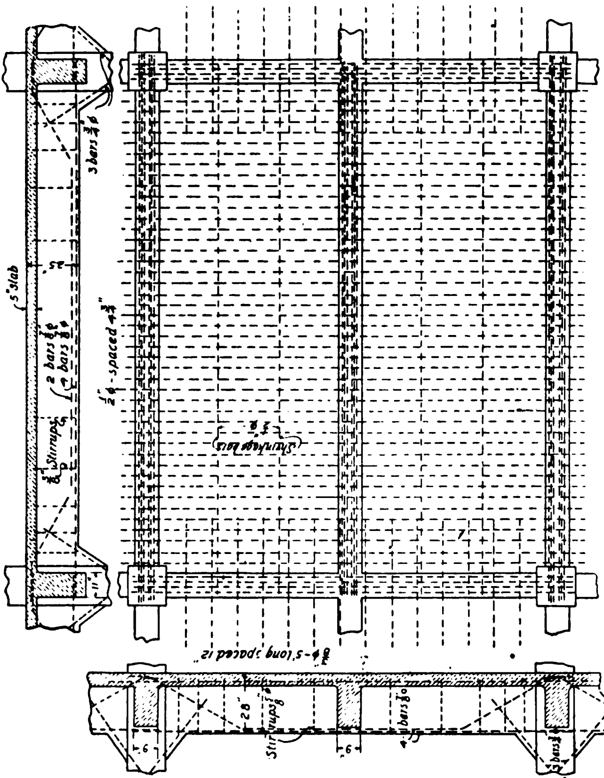


Fig. 28. Detail of Complete Floor Panel

for $s = 16,000$, the required area would be 1.115 square inches. Two pairs of stirrups would give four bar areas which could each be 0.28 square inch, and this amount of reinforcement would be amply provided by ⅝-inch round bars. Fig. 28, which illustrates a complete floor panel, shows nearly all these various details gathered together.

MISCELLANEOUS CONCRETE DESIGNS

SIMPLE FOOTINGS

Effectiveness of Reinforced Concrete Footings. When a definite load, such as a weight carried by a column or wall, is to be supported on a subsoil whose bearing power has been estimated at some definite figure, the required area of the footing becomes a perfectly definite quantity, regardless of the method of construction of the footing. But with the area of the footing once determined, it is possible to effect considerable economy in the construction of the footing, by the use of reinforced concrete. An ordinary footing of masonry is usually made in pyramidal form, although the sides are stepped off instead of being made sloping. It may be stated that the depth of the footing below the base of the column or wall, when ordinary masonry is used, must be practically equal to the width of the footing. The offsets in the masonry cannot ordinarily be made any greater than the heights of the various steps. Such a plan requires an excessive amount of masonry.

Wall Footing. Assume that a 24-inch wall, with a total load of 42,000 pounds per running foot, is to rest on a soil which can safely bear a load of 7,000 pounds per square foot. The required width of footing is 6 feet. The footing will project 2 feet on either side of the wall. For each lineal foot of the wall and on each side, there is an inverted cantilever, with an area 2 feet × 1 foot, and carrying a load of 14,000 pounds. The center of pressure is 12 inches from the wall; the moment about a section through the face of the wall is 12 × 14,000 = 168,000 inch-pounds. Using a grade of concrete such that $M = 95\,bd^2$, $p = .00675$, and $j = .88$, then with $b = 12$, we have

$$\begin{aligned} d^2 &= M \div 95\,b \\ &= 168{,}000 \div 1{,}140 = 147.4 \\ d &= 12.15 \text{ in.} \end{aligned}$$

The amount of steel required per inch of width will equal .00675 × 12.15 = .082 square inch, which may be supplied by $\frac{3}{4}$-inch bars spaced about 7 inches on centers. A total thickness of 15 inches will therefore fulfil the requirements. Theoretically, this thickness could be reduced to 8 or even 6 inches at the outer edge, since there the moment and the shear both reduce to zero. But when the concrete is used very wet and soft, it cannot be laid with an upper surface of even moderate slope without using forms to confine it, and in the case just given such forms would cost more than would be saved in the concrete.

Shear. The shear (V) on a vertical section directly under the face of the wall, and 12 inches long, is 14,000 pounds. Applying Equation (13)

$$\begin{aligned} v &= V \div bjd \\ &= 14{,}000 \div (12 \times .88 \times 12.15) \\ &= 109 \text{ lb. per sq. in.} \end{aligned}$$

This is far greater than a safe working stress and the slab might fail from diagonal tension. When a loaded beam is supported freely at each end, the maximum shear is found at the ends where the moment is minimum, and some of the bars which are not needed there for moment may be bent up so as to resist the shear. Unfortunately, in the case of a cantilever, the maximum moment and maximum shear are found at the same beam section—in this case, at the face of the wall. Therefore, if the concrete itself cannot carry the shear, additional steel must be used to do that work. Bars inclined about 45 degrees serve the purpose most economically, provided they are secured against slipping and can develop their full strength. This may be done by extending them through the column and by bending the free ends. Assume that the concrete alone takes up 40 pounds of the 109 pounds shear, found above, or 37 per cent. This leaves 63 per cent to be taken by the steel bars. $14{,}000 \times .63 = 8{,}820$ pounds per foot or 735 pounds per lineal inch. The only practicable arrangement is to alternate these bars with the moment bars and therefore space them 7 inches apart. Then each bar must take up $7 \times 735 = 5{,}145$ pounds of shear. A $\frac{9}{16}$-inch square bar will safely sustain that stress. Such a bar has a perimeter of 2.25 inches. At 75 pounds per square inch for bond adhesion (plain bars), each lineal inch of the bar would have a working adhesion of 169 pounds. Dividing 5,145 by this gives 30 inches, the required length of bar beyond any point where the stress is as much as 5,145 pounds. Since there is not that length of bar available, bond adhesion cannot be relied on and the bars must be bent, as shown in Fig. 29. Even a deformed bar, although a good type may be used with working adhesion about double that of a plain bar, would need to be longer than space permits, if straight, and it should be hooked.

Bond Adhesion in Moment Bars. The steel required per inch of width is .082 square inch, and in 7 inches, .574 square inch. Since the design calls for a unit tension of 16,000 pounds in the steel, the actual tension in the bar will be 16,000 × .574 = 9,184 pounds. A ¾-inch square bar has a perimeter of 3 inches and, at 75 pounds per square inch, can furnish a working bond adhesion of 225 pounds per lineal inch of bar. But this would need 9,184 ÷ 225 = 41 inches, the required length beyond the face of the wall. If 150 per square inch bond adhesion is allowed for a good type of deformed bar, the required length, computed similarly, would be a little over 20 inches, and as this is less than the 24-inch cantilever, straight deformed bars will do. The designer, therefore, has the choice of using a hook on each end of plain bars, as illustrated in Fig. 29, or using straight deformed bars, which would be cheaper at the usual relative prices.

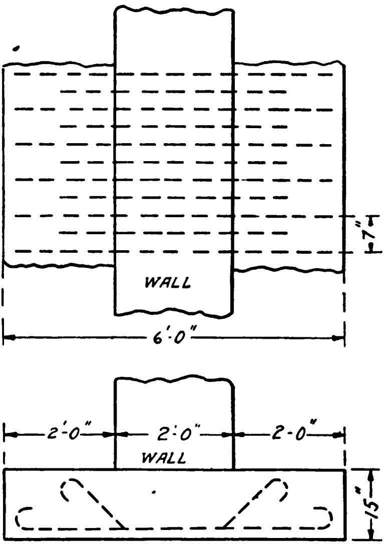


Fig. 29. Diagram of Footing for a Wall

Column Footing. The most common method of reinforcing a simple column footing is shown in Fig. 30. Two sets of the reinforcing bars are at *a-a* and *b-b,* and are placed only under the column. To develop the strength of the corners of the footings, bars are placed diagonally across the footing, as at *c-c* and *d-d*. In designing this footing, the projections of the footing beyond the column are treated as free cantilever beams, or by the method discussed above. The maximum shear occurs near the center; and therefore, if this shear must be taken care of by reinforcement, stirrups or bent bars should be used.

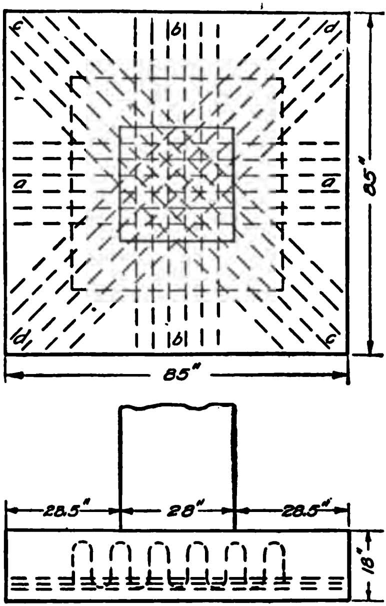


Fig. 30. Diagram of Footing for a Column

Example. Assume that a load of 300,000 pounds is to be carried by a column 28 inches square, on a soil that will safely carry a load of 6,000 pounds per square foot. The reinforcing bars are to run diagonally and directly across the footing, Fig. 30. What should be the dimensions of the footing and the size and spacing of the bars? Also investigate the shear.

Solution. The load of 300,000 pounds will evidently require an area of 50 square feet. The sides of the square footing will evidently be 7.07 feet, or, say, 85 inches; and the offset on each side of the 28-inch column is 28.5 inches. The area of each cantilever wing which is straight out from the column is $28.5 \times 28 = 798$ square inches or 5.54 square feet. The load is, therefore, $5.54 \times 6{,}000 = 33{,}240$ pounds. Its lever arm is one-half of 28.5 inches, or 14.25 inches. The moment is therefore 473,812 inch-pounds. Adopting the straight-line formula, $M = 95\,bd^2$, on the basis that $p = .00675$, we may write the equation

$$473{,}812 = 95 \times 28 \times d^2$$
$$d^2 = 178$$
$$d = 13.3 \text{ in.}$$

Therefore

$$A = p\,b\,d = .00675 \times 28 \times 13.3$$
$$= 2.51 \text{ sq. in.}$$

This area of metal may be furnished by six ¾-inch round bars, and therefore there should be six ¾-inch round bars spaced about 4.5 inches apart under the column in *both* directions, *a-a* and *b-b*.

Corner Sections. The mechanics of the reinforcements of the corner sections, which are each 28.5 inches square, is exceedingly complicated in its precise theory. The following approximation to it is probably sufficiently exact. The area of each corner section is the square of 28.5 inches, or 812.25 square inches. At 6,000 pounds per square foot, the pressure on such a section is 33,844 pounds, and the center of gravity of this section is, of course, at the center of the square, which is $14.25 \times 1.414 = 20.15$ inches from the corner of the column. A bar immediately under this diagonal line would have a lever arm of 20.15 inches. A bar parallel to it would have the same lever arm from the middle of the bar to the point where it passes under the column. Therefore, if we consider that this entire pressure of 33,844 pounds has an average lever arm of 20.15 inches, we would have a moment of 681,957 inch-pounds. Using, as before, the moment equation $M = 95\,bd^2$, we may transpose this equation to read

$$b = \frac{M}{95\,d^2}$$

Then

$$A = p\,b\,d = p\,\frac{M}{95\,d^2}\,d = p\,\frac{M}{95\,d}$$

$$= .00675 \times \frac{681{,}957}{95 \times 14.5}$$

$$= 3.34 \text{ sq. in.}$$

This area of steel will be furnished by six ¾-inch square bars. The diagonal reinforcement will therefore consist of six ¾-inch square bars running diagonally in both directions. These bars should be spaced about 5 inches apart. Those that are nearly under the diagonal lines of the square should be about 9 feet 8 inches long; those parallel to them will each be 10 inches shorter than the next bar.

Bond Adhesion. The total tension in the steel of the *a* and *b* bars is $16{,}000 \times 2.51 = 40{,}160$ pounds, or 6,693 pounds per bar, which is found at a point immediately under the column face. There will be 28.5 inches length of steel in each bar from the column face to the edge of the slab, and this will require a bond adhesion of $6{,}693 \div 28.5 = 235$ pounds per lineal inch. Referring to Table XVII, we see that this unit value is greater than a proper working value for ¾-inch plain round bars but is safe for ¾-inch deformed round bars. Making a similar calculation for the diagonal bars, the stress in each one is $(16{,}000 \times 3.34) \div 6 = 8{,}907$ pounds. The length, practically uniform for all, beyond the face of the column is 40 inches, which will require a bond adhesion of 223 pounds per lineal inch. This is just within the limit for ¾-inch plain square bars.

It should be noted from the solution of this and the previous problem that, on account of the combination of heavy load and small cantilever projection, the bond adhesion of footings is always a critical matter and its investigation should never be neglected. It frequently happens, as above illustrated, that the greater bond resistance of deformed bars will permit the use of a certain bar which is safe for the moment resistance when the same size of plain bar cannot be used. Since smaller bars have a greater surface and a greater adhesion per unit both of area and of strength than larger bars, the requisite adhesion may sometimes be obtained by using a proportionately larger number of smaller bars. When neither method will produce the required adhesion, the bars should be bent into a hook, which should be a full semicircle with a diameter about 8 to 12 times the diameter of the bar.

Shear. The "punching" shear on the slab is measured by the upward pressure on that part of the slab which is outside of the column area. This equals $85^2-28^2=6{,}441$ square inches, or 44.73 square feet. Multiplying by 6,000 we have 268,380 pounds. The resisting area equals the perimeter of the column times *jd,* which here is $4 \times 28 \times .88 \times 13.3 = 1{,}311$ square inches. Dividing 268,380 by this, we have 204 pounds per square inch. If the column and slab were made of plain concrete, this figure would be considered too high for working stress, 120 pounds being usually allowed. In this case, an actual punching of the slab would require that 48 sections of ¾-inch round bars should be sheared off. If the concrete actually takes an average of 120 pounds per square inch on 1,311 square inches of surface, the concrete would take up 157,320 pounds, leaving 111,060 pounds for the 48 bars, or 2,314 pounds for each bar. Dividing by the bar area, we have a shearing stress of 5,237 pounds per square inch of bar section, which is insignificant for the steel and is amply safe, provided that any such shearing stress as 2,314 pounds per bar could be developed before the concrete itself were crushed by the bars. Considering the various forces resisting the punching action, and also that even the 204 pounds per square inch is far short of the ultimate value of *true* shear, the design is probably safe, although the factor of safety is probably low. If further reinforcement were considered necessary, it could be added in the form of bent bars, as in the previous problem.

It is impracticable to develop a true rational formula for the computation of the diagonal tension in slabs which support columns, but several elaborate tests* by Professor Talbot show that the following method gives results which are reasonably consistent and also comparable with the corresponding results for ordinary beams. Consider a section through the slab all the way around the column and at a distance d from the face of the column, and apply Equation (**13**), $v = V \div bjd$. In this case the section would be a square $(2 \times 13.3) + 28 = 54.6$ inches on a side. The area is 2,981 square inches. The area of the whole footing is $85^2 = 7{,}225$ square inches, and the area outside this square is $7{,}225 - 2{,}981 = 4{,}244$ square inches, or 29.5 square feet. $29.5 \times 6{,}000 = 177{,}000$ pounds $= V$; b is the perimeter of the square and equals $4 \times 54.6 = 218.4$; jd is $.88 \times 13.3 = 11.7$. Then $v = 69$. Since this is higher than 40, the usual permissible working stress when taken as a measure of unreinforced diagonal tension, it shows that bent bars or stirrups must be used, but in either case the reinforcement need carry only the extra 29 pounds per square inch. Multiplying this by jd, we have $29 \times 11.7 = 339$, the required assistance in pounds per lineal inch. If a bar is placed every 4.5 inches (corresponding with the main reinforcing bars) the stress per bar will be 1,525 pounds, which at 16,000 pounds unit stress will require .095 square inches or a $\frac{5}{16}$-inch square bar. Perhaps the most convenient form of reinforcement in this case would be a series of stirrups made by a continuous bar, $\frac{5}{16}$ inch square, which zigzags up and down with an amplitude equal to jd or 11.7 inches, and so that there is a bar up or down at every 4.5 inches. This should be located at the "critical section" at a distance d equal to 13.3 inches from the column face. It will require a bar about 16 feet 6 inches long to make the continuous stirrup for each side of the square. Each bar must be bent with about eleven semicircular bends, as shown in Fig. 30, so placed that each downward loop will pass under one of the main reinforcing bars. The loops at the top preclude all possibility of bond failure.

Since the shear decreases to zero at the edge of the slab, and the distance from the stirrup to the edge of the slab is only a

* Bulletin No. 67, University of Illinois.

little more than the thickness of the slab, it is apparent without calculation that no further shear reinforcement is needed.

Continuous Beams. Continuous beams are sometimes used to save the expense of underpinning an adjacent foundation or wall. These footings are designed as simple beams, but the steel is placed in the top of the beams.

Illustrative Example. Assume that the columns on one side of a building are to be supported by a continuous footing; that the columns are 22 inches square, spaced 12 feet on center; and that they support a load of 195,000 pounds each. If the soil will safely support 6,000 pounds per square foot, the area required for a footing will be $195{,}000 \div 6{,}000 = 32.5$ square feet. Since the columns are spaced 12 feet apart, the width of footing will be $32.5 \div 12 = 2.71$ feet, or 2 feet 9 inches. To find the depth and amount of reinforcement necessary for this footing, it is designed as a simple inverted beam supported at both ends (the columns), and loaded with an upward pressure of 6,000 pounds per square foot on a beam 2 feet 9 inches wide. In computing the moment of this beam, the continuous-beam principle may be utilized on all except the end spans, and thus the moment may be reduced and, therefore, the required dimensions of the beam.

COMPOUND FOOTINGS

Conditions Demanding Compound Footings. When a simple footing supports a single column, the center of pressure of the column must pass vertically through the center of gravity of the footing, or there will be dangerous transverse stresses in the column, as is discussed later. It is, however, sometimes necessary to support a column on the edge of a property when it is not permissible to extend the foundations beyond the property line, and in such a case, a simple footing is impracticable. The method of solution to be used is indicated in Fig. 31. The nearest interior column (or even a column on the opposite side of the building, if the building is not too wide) is selected, and a combined footing is constructed under both columns. The weights on the two columns are computed.

If they are equal, the center of gravity is halfway between them; if unequal, the center of gravity is on the line joining their centers, and at a distance from them such that, $x:y::W_2:W_1$, Fig. 31. In this case, evidently W_2 is the greater weight. The area *abdc* must fulfil two conditions:

(1) The area must equal the total loading (W_1+W_2), divided by the allowable loading per square foot.

(2) The center of gravity must be located at *O*.

Practical Treatment of Problem. An analytical solution for all cases of the relative and absolute values of *ab* and *cd* which will fulfil the two conditions is very difficult. Sometimes

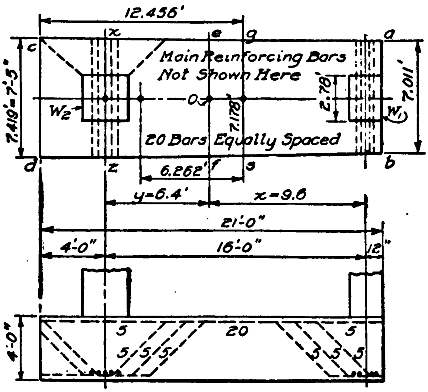


Fig. 31. Combined Footing for Two Columns, One on Edge of Property

the only practicable solution is to obtain, by trial and adjustment, a set of dimensions which will be sufficiently accurate for practical purposes. It usually happens that an inner column of a building carries a greater load than an outer column. This facilitates the solution, for then, as in the example given below, the footing may be extended beyond the inner column and may be made approximately rectangular.

Illustrative Example. A column, W_1, carrying 400,000 pounds, is to be located on the edge of a property and another column, W_2, carrying 600,000 pounds, is located 16 feet from it. Assume that the subsoil can sustain safely 7,000 pounds per square foot. We are to determine the shape and design of the footing.

Assume that the footing slab weighs 400 pounds per square foot of surface; then the *net* effective upward pressure of the subsoil which will support the column equals $7{,}000 - 400 = 6{,}600$ pounds per square foot. For simplicity of calculation, the computations involving soil pressures and slab areas will generally use the units, feet and decimals. The change to feet and inches can be made when the final dimensions have been computed.

The total column load is 1,000,000 pounds; at 6,600 pounds per square foot the area must be 151.515 square feet. Assume that the W_2 column is 2.89 feet square, and that the W_1 column is 2 feet $\times$ 2.78 feet. This means that the net average load is 500 pounds per square inch on each column. In Fig. 31 let *ab* equal *n,* and *cd* equal *m,* both still unknown. The smaller column is on the edge of the property, and the *ab* line is made 1.0 foot from the column center. As a trial solution, assume that the *cd* line is 4.0 feet beyond the other column center. Then the total length of the trapezoid is 21.0, and

$$\tfrac{1}{2}(m+n)\,21.0 = 151.515$$
$$(m+n) = 14.43$$

The center of gravity of the two loads is at $\frac{600{,}000}{1{,}000{,}000}$ of 16 feet, or at 9.6 feet from the smaller column center. This locates *O*. To fulfil condition (2), the dimensions *m* and *n* must be such that the center of gravity of the trapezoid shall be at *O*. In general, the distance *z* of the center of gravity of a trapezoid from its larger base equals one-third of the height *h* times the quotient of the larger base plus twice the smaller base divided by the sum of the bases; or, as an equation

$$z = \tfrac{1}{3}h\frac{m+2n}{m+n}$$

Substituting $z = 10.4$, $h = 21.0$, *m* and *n* being still unknown, we have

$$10.4 = \frac{21.0}{3} \times \frac{m+2n}{m+n}$$

Combining this equation with the equation $(m+n)=14.43$, we may solve and find $m=7.419$ and $n=7.011$. By proportion, we find the dimension ef through $O=7.217$ feet.

The maximum moment is found where the shear is zero, and this must be at the right-hand end of a portion of the slab on which the *net* upward pressure equals 600,000 pounds. That portion must have an area of $600{,}000 \div 6{,}600 = 90.909$ square feet. Similarly, the remaining area is computed to be 60.606 square feet. Let p equal the length of this section (qs in the figure) and h equal its distance from cd. We may write the two equations

$$\tfrac{1}{2}(7.419+p)\,h = 90.909$$

and

$$\tfrac{1}{2}(p+7.011)\,(21-h) = 60.606$$

Solving these two equations for p and h, we have $p=7.178$ and $h=12.456$. It should be noted that this section of maximum moment (on the line qs) is *not* on the line of center of gravity of the whole footing, but is in this case about two feet to the right. The center of gravity of the trapezoid $cdqs$, calculated as above, is at a point 6.262 feet from qs and the *net* upward pressure on this section is 600,000 pounds. Therefore, taking moments about qs, we have

$$\begin{aligned} M = 600{,}000\,(8.456-6.262) &= 1{,}316{,}400 \text{ ft.-lb.} \\ &= 15{,}796{,}800 \text{ in.-lb.} \end{aligned}$$

In this case, $b=7.178$ feet $=86.136$ inches; call it even 86. Then for $M=95\,bd^2$, we have

$$\begin{aligned} 95\,bd^2 = 8{,}170\,d^2 &= 15{,}796{,}800 \\ d^2 &= 1{,}934 \\ d &= 44.0 \text{ in.} \end{aligned}$$

Then

$$A = .00675 \times 86 \times 44 = 25.54 \text{ sq. in.}$$

which may be provided by 20 bars, $1\frac{1}{8}$ inches square.

That portion of the slab between x and z is subject to transverse stress, the parts near x and z tending to bend upward. Although the stresses are not computable with perfect definiteness, being comparable to those in a simple footing (see page 104), we may consider them as approximately measured by the moment of the quadrilateral between the face of the column and x about the face of the column. xz equals 7.34; subtracting the column width and dividing by 2 we have 2.225 feet, or 26.7 inches; the area of the quadrilateral is approximately $\frac{1}{2}(8 + 2.89)\ 2.225 = 12.11$ square feet. The effective upward pressure is $12.11 \times 6{,}600 = 79{,}926$ pounds. The lever arm is approximately $\frac{6}{10}$ of the distance from the face, or $0.6 \times 26.7 = 16$ inches.

$$M = 79{,}926 \times 16 = 1{,}278{,}816 = 95\ bd^2$$

Here d is about one inch less than for the main slab, or, say, 43 inches. Solving, $b = 7.3$ and

$$A = pbd = .00675 \times 43 \times 7.3 = 2.12 \text{ sq. in.}$$

which may be supplied by 4 bars $\frac{3}{4}$ inch square. This calculation shows that a relatively small amount of reinforcement, which should run under the column from x to z, will resist this stress. Increasing the number of bars to 5 or 6 will certainly cover all uncertainties in this part of the calculation. The stresses under the other column are somewhat less and therefore the same reinforcement will be even safer.

The shear around the larger column can be calculated as "punching" shear; b for this case is the perimeter of the column, and equals $4 \times 2.89 = 11.56$ feet, or 138.72 inches; jd equals $.88 \times 44 = 38.72$; V equals $600{,}000 - (2.89^2 \times 6{,}600) = 544{,}890$.

$$v = V \div bjd = 544{,}890 \div (138.72 \times 38.72)$$
$$= 102 \text{ lb. per sq. in.}$$

Since this is a case of true shear, when a working stress of 120 pounds per square inch is allowable, no added reinforcement is necessary.

The other column may be considered similarly, except that it is supported only on three sides. $b = 81$ inches and $bjd = 3{,}136$; $V = 300{,}000 - 36{,}667 = 263{,}333$; then $v = 84$. Since this is only 70 per cent of the allowable stress for true shear, it is probably safe. In addition, the bending down of the main reinforcing bars under each column, as shown in the figure, will add a very large factor of safety.

It is far more difficult, in case the heavier column is next to the property line, to obtain, by the analytical method given above, a trapezoid which will fulfil the two fundamental requirements there given. If the wall column has twice (or more than twice) the load carried by the inner column, no trapezoid is obtainable. In such a case, a figure shaped somewhat like a shovel, the blade being under the heavy column and the handle being a beam which transfers the load of the lighter column to the broad base, may be used, the dimensions and exact shape of which can only be determined by successive trials.

PILES

Advantage of Concrete and Reinforced Concrete Piles. A reinforced concrete pile foundation does not differ essentially in construction from a timber pile foundation. The piles are driven and capped, in the usual manner, with concrete ready for the superstructure. Compared with timber piles, reinforced concrete piles have the advantage of being equally durable in a wet or dry soil, and the disadvantage of being more expensive in first cost. Sometimes their use will effect a saving in the total cost of the foundation by obviating the necessity of cutting the piles off below the water line. The depth of the excavation and the volume of masonry may be greatly reduced, as illustrated in Fig. 32. This figure shows a comparison of the relative amounts of excavation which would be necessary, and also of the concrete which would be required for the piles, thus indicating the economy which is possible in these two items. There is also shown a possible economy in the number of piles required, since concrete piles can readily be made of any desired diameter, while there is a practical limitation to the diameter of wooden piles. Therefore a smaller number of concrete piles will furnish the same resistance as a larger number of wooden piles. In Fig. 32 it is assumed that the three concrete piles not only take the place of the four wooden piles in the width of the foundation, but that there will also be a corresponding reduction in the number of piles in a direction perpendicular to the section shown. The extent of these advantages depends very greatly on the level of the ground-water line. When this level is considerably below the surface of the ground, the excavation and the amount of concrete required, in order that the timber grillage and the tops

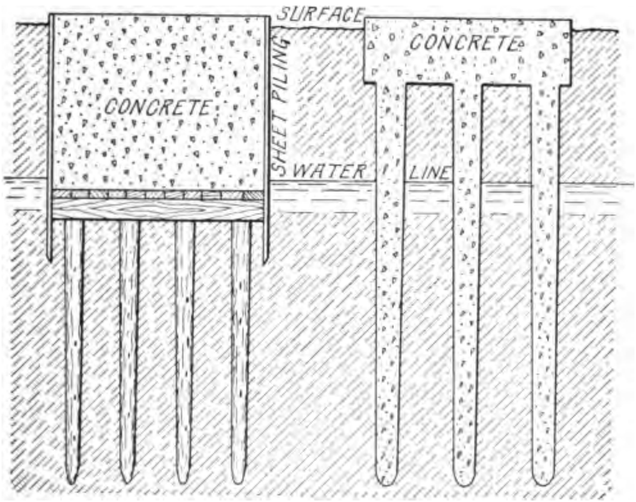


Fig. 32. Comparison of Wooden and Concrete Piles

of the piles shall always be below the water line, will be correspondingly great, and the possible economy of concrete piles will also be correspondingly great.

Capping and Driving. The pile and cap, being of the same material, readily bond together and form a monolithic structure. The capping should be thoroughly reinforced with steel. Reinforced concrete piles can be driven in almost any soil that a timber pile can penetrate, and they are driven in the same manner as the timber piles. A combination of the hammer and water jet has been found to be the most successful manner of driving them. The hammer should be heavy and drop a short distance with rapid blows, rather than a light one dropping a greater distance. For protection while being driven, a hollow cast-iron cap filled with sand is placed on the head of the pile. The cap shown in Fig. 33 has been used successfully in driving concrete piles. A hammer weighing 2,500 pounds was dropped 25 feet, 20 to 30 times per minute, without injury to the head.

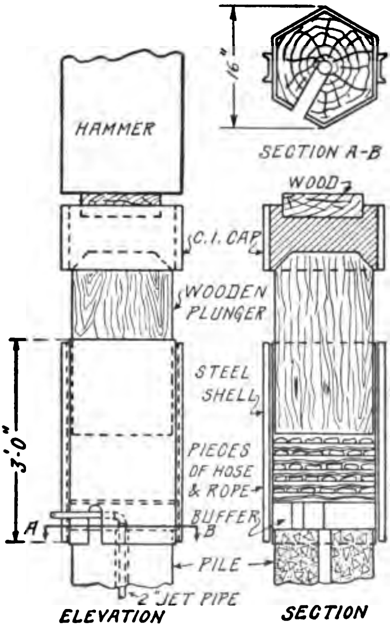


Fig. 33. Cushion Head for Driving Piles

Design. There is no definite way to figure the size or length of a pile to support a given load. Some engineers have determined the size of the piles required for their work by assuming the friction between the soil and concrete to be a given amount per square foot, and then making the pile of sufficient diameter and length so that it will be safely supported by this frictional area; others allow 300 to 500 pounds per square inch of the average cross section. Where it is possible, it is far better to drive the pile through the soft material and a short distance into a firm soil than to depend altogether on the frictional resistance. Borings should always be made at the site of the work to ascertain the nature of the material and, in case a firm stratum is not found within a reasonable depth, a few piles should be made, driven, and tested as to their safe supporting capacity before any definite dimensions are assumed.

Concrete piles that are reinforced with steel bars will resist lateral blows much better than those that are not reinforced. Piles that are made and driven must be reinforced so that they can be handled without breaking. Four bars 1 inch in diameter, with ¼-inch bands 12 inches on centers, will be sufficient reinforcement for piles 14 to 16 inches in diameter.

Loading. A concrete pile 16 to 18 inches at the top, tapering to 8 or 10 inches, and 16 to 20 feet long, should safely support a load of 20 tons in fairly soft, wet soil, and 25 to 30 tons when driven through a soft soil into a firm one. These piles cannot be placed closer than 3 feet on centers.

Types. Concrete and reinforced concrete piles may be classified under two headings: (*a*) those which are formed, hardened, and driven very much as any pile is driven; (*b*) those formed by making a hole in the ground, ramming in the concrete, and letting it harden.

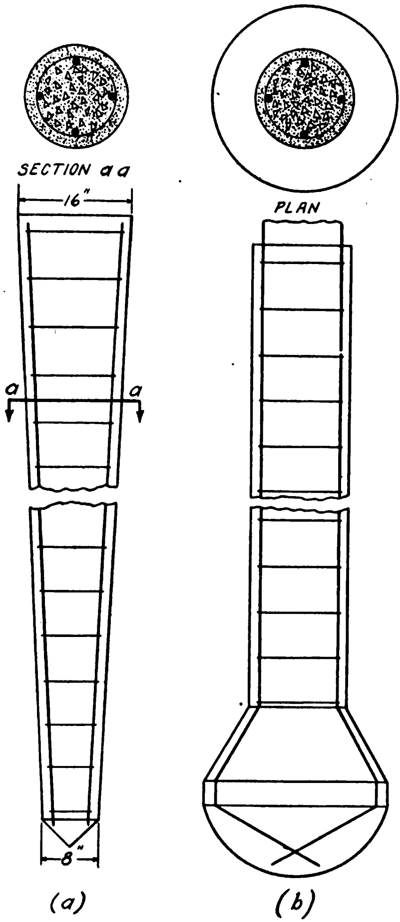


Fig. 34. Reinforced Concrete Piles

Reinforced concrete piles which have been formed on the ground are designed as columns with vertical reinforcement connected at intervals with horizontal bands. These piles are usually round or octagonal, with steel or cast-iron points.

Fig. 34-*a* shows a type commonly used when the piles are constructed in forms, and hardened, and driven just as a wooden pile. These piles must be reinforced with steel so that they can be handled.

Fig. 34-*b* shows the general plan of a type of pile that has been used to some extent along the seashore where piles can be jettied. They are usually molded in a vertical position and as soon as they can be handled are jettied in place. These piles are not dependent on the friction of the surface of the concrete with the sand but can convey the load directly to the sand under the enlarged end. Piles of this type have been used for loads of 50 to 60 tons. They cannot be used in clusters, but each pile must be of sufficient size to support the entire load at any given point.

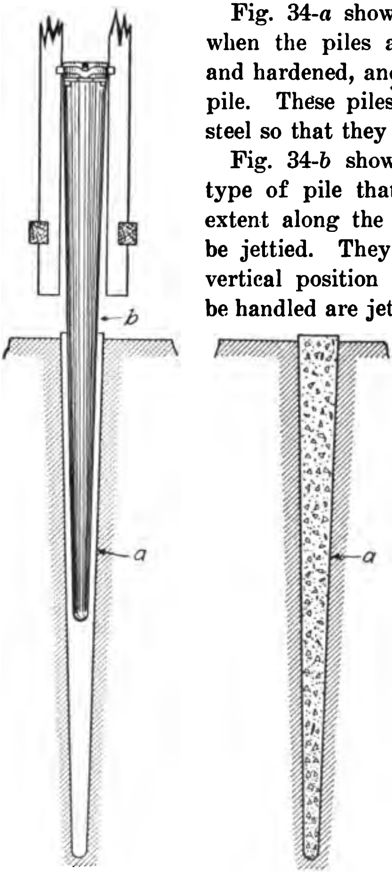


Fig. 35. Raymond Concrete Pile

Raymond Concrete Pile. The Raymond concrete pile, Fig. 35, is constructed in place. A collapsible steel pile core is encased in a thin, closely fitting, sheet-steel shell. The core and shell are driven to the required depth by means of a pile driver. The core is so constructed that when the driving is finished, it is collapsed and withdrawn, leaving the shell in the ground, which acts as a mold for the concrete. When the core is withdrawn, the shell is filled with concrete, which is tamped during the filling process. These piles are usually from 18 to 20 inches in diameter at the top, and from 6 to 8 inches at the point. When it is desirable, the pile can be made larger at

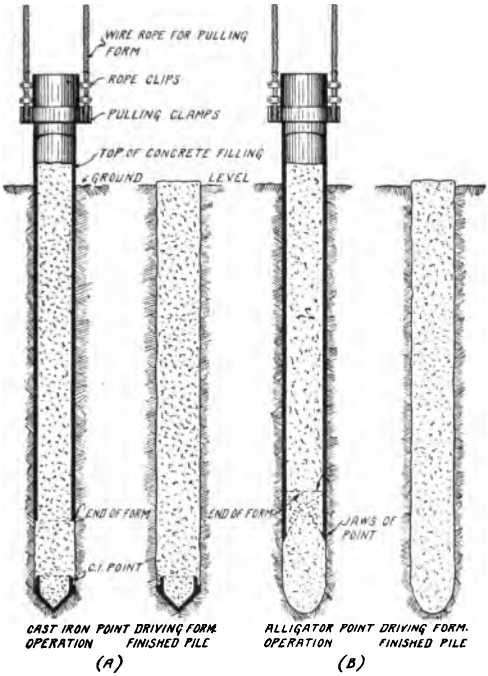


Fig. 36. Standard Simplex Concrete Piles

the small end. The sheet steel used for these piles is usually No. 20 gage. When it is desirable to reinforce these piles, the bars are inserted in the shell after the core has been withdrawn and before the concrete is placed.

Simplex Concrete Pile. The different methods for producing the Simplex pile cover the two general classifications of concrete piles—namely, those molded in place, and those molded above ground and driven with a pile driver. Fig. 36 shows the standard methods of producing the Simplex pile: *A* shows a cast-iron point which has been driven and imbedded in the ground, the concrete deposited, and the form partially withdrawn; while *B* shows the alligator-point driving form. The only difference between the two forms shown in this figure is that the alligator point is withdrawn and the cast-iron point remains in the ground. The concrete in either type is compacted by its own weight. As the form is removed, the con-

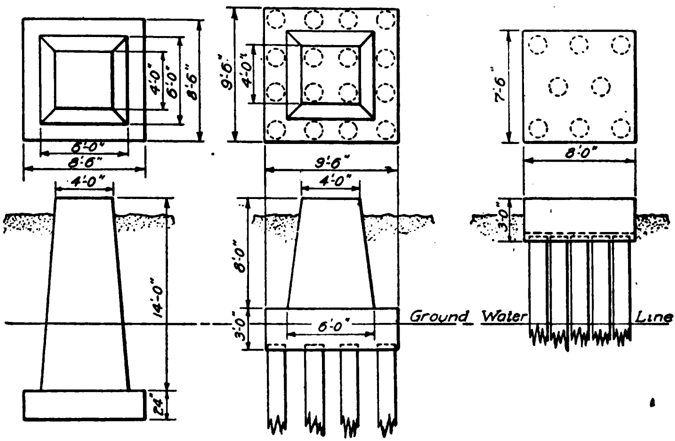


Fig. 37. Types of Foundation for Boston and Maine Railroad

crete comes in contact with the soil and is bonded with it. There is the danger in using this type of pile that, if a stream of water is encountered, the cement may be washed out of the concrete before it has a chance to set.

A shell pile and a molded and driven pile are also produced by the same company which manufactures the Simplex, and are recommended for use under certain conditions. Any of these types of piles can be reinforced with steel. This company has driven piles 20 inches in diameter and 75 feet long.

Cost. Concrete or reinforced piles will cost much more per lineal foot than wood piles, but will support greater loads, and therefore fewer piles are required. As already shown in Fig. 32, it is often more economical to use concrete piles than wood piles. Concrete piles have been driven for $0.70 per lineal foot, but the average price is probably $0.90 to $1.00.

The Boston and Maine Railroad has recently completed the erection of a large group of shop buildings at Billerica, Massachusetts, in which about 3,000 concrete piles were used. These buildings were all supported on concrete piles, as the soil consists of sand and peat on the surface and is underlaid by water-bearing sand. Fig. 37 shows the three designs considered, which were as follows:

(1) Concrete piers carried down in open caissons to firm strata; loading per square foot, 3.2 tons.

(2) Wooden piles cut off below ground-water level and capped with concrete piers; loading per pile, 15 tons.

(3) Concrete piles driven from the surface of the ground and capped with reinforced concrete; loading per pile, 30 tons.

The estimates for these different types of foundations were as follows:

CONCRETE PIERS *

Excavation, pumping, and backfill	$ 90.00
Sheeting and bracing	100.00
Concrete, 18.5 cu. yds., at $7.50	138.75
Total cost	$328.75

WOODEN PILE PIER

Excavation, pumping, and backfill	$ 40.00
Sheeting and bracing	60.00
Concrete, 17.5 cu. yds. at $7.50	131.25
16 wooden piles at $5.00	80.00
Total cost	$311.25

CONCRETE PILE PIER

Reinforced concrete cap (including excavation), 6⅔ cu. yds. at $9.00	$ 60.00
8 concrete pedestal piles at $15.00	120.00
Total cost	$180.00

RETAINING WALLS

Properties of Supported Material Affect Design. A retaining wall is a wall built to sustain the lateral pressure of

* *Concrete-Cement Age*, October, 1914.

earth. The pressure that will be exerted on the wall will depend on the kind of material to be supported, the manner of placing it, and the amount of moisture that it contains. Earth and most other granular masses possess some frictional stability. Loose soil or a hydraulic pressure will exert a full pressure; but a compacted earth, such as clay, may exert only a small pressure due to the cohesion in the materials. This cohesion cannot be depended upon to relieve the pressure against a wall, for the cohesion may be destroyed by vibration due to moving loads or to saturation. In designing a wall the pressure due to a granular or a semifluid mass without cohesion must always be considered.

Failures of Walls. There are three ways in which a masonry wall may fail: (1) by sliding along a horizontal plane; (2) by overturning or rotating; (3) by the crushing of the masonry or its footing. These are the points that must be considered in order to design a wall that will be successful in resisting an embankment. A wall, therefore, must be of sufficient size and weight, to prevent the occurrence of sliding, rotation, or crushing.

Stability of Wall against Sliding. Stability against sliding is secured by making the structure of sufficient weight so that there will be no danger of a movement at the base. In Fig. 38 let E be the horizontal pressure and assume W to be the weight of all materials above the joint. A movement will occur when $E = fW$, where f is the coefficient of friction. Let n be a number greater than unity, the factor of safety; then in order that there be no movement n must be sufficiently large so that $nE = fW$. A common value for n is 2, but sometimes it is taken as low as $1\frac{1}{2}$. Substituting 2 for n,

$$2E = fW$$

$$W = \frac{2E}{f} \qquad (20)$$

An average value of the coefficient of friction for masonry on masonry is 0.65; for masonry on dry clay, 0.50; for masonry on wet clay, 0.33; for masonry on gravel, 0.60; for masonry on wood, 0.50.

Stability of Wall against Rotation. The stability of a wall against rotation is secured by making the wall of such dimension and weight that the resultant *R* of the external forces will pass through the base and well within the base, as shown in Fig. 38. Generally, in designing, the resultant is made to come within or at the edge of the middle third. The nearer the center of the base the resultant comes, the more evenly the pressure will be distributed over the foundation for the wall. When *R* passes through *A*, Fig. 38, the wall will fail by rotation. Methods for finding *R* will be demonstrated in another paragraph.

Stability of Wall against Crushing. The compressive unit stresses in walls built on stone foundations must not be greater than the unit stresses permitted for safe working loads of masonry; but when a wall is built on clay, sand, or gravel, the allowable pressure for such foundations must not be exceeded.

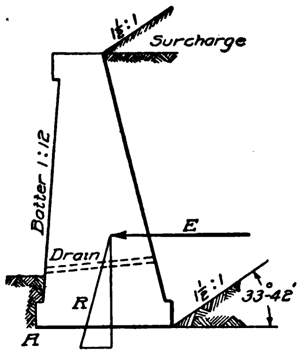


Fig. 38. Section of Retaining Wall

Foundations for Wall. The foundations for a retaining wall must be below the frost line, which is about three feet below the surface in a temperate climate, and deeper in a cold climate. The foundation should be of such a character that it will safely support the wall. If necessary, the soil should be tested to determine if it will safely support the wall.

The foundation should always be well drained. Many failures of walls have occurred owing to the lack of drainage. Water behind a wall greatly increases the stresses in the wall. Water freezing behind a wall usually causes it to bulge out, the first step in the failure of the wall. On a clay foundation the friction is greatly reduced by the clay becoming thoroughly soaked with water. It has just been shown that the difference of the coefficients of friction of masonry on dry clay and wet clay is 0.17. There are different ways of draining a fill behind a retaining wall. Pipes 2 to 4 inches in diameter are often built in the wall, as shown in Fig. 38.

Fill behind Wall. The fill behind the wall is sometimes made horizontal with the top of the wall; at other times the fill is sloped back from the top of the wall, as shown in Fig. 38. When there is a slope to be supported, the wall is said to be surcharged, and the load to be supported is greater than for a horizontal fill.

Design of Wall

Methods of Designing Walls. In designing a retaining wall the dimensions of the section of a wall are generally assumed and then the section investigated graphically to see if the assumed conditions are met. There are theoretical formulas for designing walls which will be given. In designing a wall, the student is advised to make first the section according to the formulas and then to investigate it graphically. All existing walls in that vicinity should be examined to determine their dimensions and to discover if they have been successfully designed. Often, existing walls will give more information to an engineer than he will obtain by a theoretical or graphical study.

In recent years concrete has come into extensive use in building retaining walls. A wall built of a 1:3:6 concrete should be equal in strength to a wall built of cut stone or large-ranged rubble. In heavy walls large stones, 25 to 50 per cent of the volume, are often placed in the concrete. This, usually, greatly reduces the cost of the wall and does not weaken the wall if the stones are properly placed.

Face of Wall. The front or face of a retaining wall is usually built with a batter. This batter often varies from less than an inch per foot in height to more than an inch per foot. The rear face may be built either straight, with a batter, or stepped up. A wall should never be less than 2½ feet to 3 feet wide on top, unless it is a very small one. In that case, probably a width of 12 to 18 inches would be sufficient.

Width of Base. The width of the base of a concrete gravity wall varies from 35 per cent to 50 per cent of the height of the wall. Probably the majority of walls are constructed with a width of base of about 45 per cent of the height. For railroad work this dimension is sometimes made greater than 50 per cent, ranging up to 60 per cent.

Pressure behind Wall. The development of the formulas for finding the pressure behind a wall is a long, complicated theory, and the demonstration will not be given here. The formulas given are those usually found in textbooks. They are based on the Rankine theory, which considers that the earth is a granular mass with an assumed angle of repose of 1.5 to 1, which in degrees is 33° 42′. In applying this method it is immaterial whether the forces representing the earth pressure are considered as acting directly upon the back of the wall, or are considered as acting on a vertical plane passing through the extreme back of the footing. In the latter case, the force representing the lateral earth pressure must be combined (1) with the vertical force representing the weight of the earth prism between the back of the wall and the vertical plane considered; and (2) with the vertical force representing the weight of the wall itself.

In the formulas for determining pressures behind a wall, let E equal total pressure against rear face of wall on a unit length of wall; W equal weight of a unit volume of the earth; h equal height of wall; and ϕ equal angle of repose.

When the upper surface of the earth is horizontal, the equation is

$$E = \tan^2\left(45° - \frac{\phi}{2}\right)\frac{W h^2}{2} \qquad (21)$$

Since the angle of repose for the earth behind the wall has been taken as 33° 42′, equation (21) may be reduced to the following form by substituting the value of the tangent of the angle in the equation

$$E = .286\frac{W h^2}{2} \qquad (21a)$$

When a wall must sustain a surcharge at the slope of 1.5 to 1, the equation is

$$E = \tfrac{1}{2} \cos \phi \, W h^2 \qquad \textbf{(21b)}$$

or

$$E = .833 \frac{W h^2}{2} \qquad \textbf{(21c)}$$

The force E is applied at one-third the height of the wall, measured from the bottom, but for a surcharged wall it is applied at one-third of the height of a plane that passes just

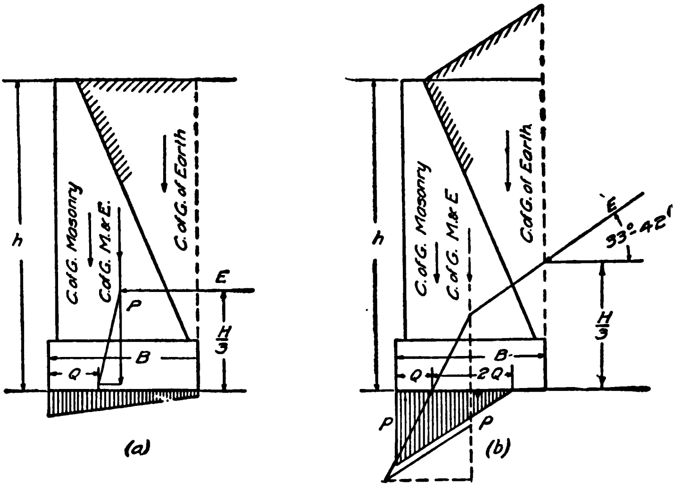


Fig. 39. Diagrams Showing Pressures on Foundations

behind the wall. This is clearly shown in the different figures illustrating retaining walls.

The direction of the center of pressure E is assumed as being parallel to the top of the earth back of the wall. The angle of the surcharge is generally made 1.5 to 1.

Example. What is the pressure per foot of length of a wall 18 feet high, earth weighing 100 pounds per cubic foot, if the fill is level with the top of the wall?

Solution. Substituting in Equation (21a)

$$E = .286 \frac{Wh^2}{2} = .286 \frac{100 \times 18^2}{2} = 4{,}633 \text{ lb.}$$

Pressure on Foundation. The formulas given below for determining the pressure on the foundation are the ones recommended to the American Railway Engineering Association by a committee appointed by that Society to investigate the subject of retaining walls. (See Fig. 39.)

NOTE.—When P equals the vertical component of the resultant pressure on the base, B is the full width of the base in feet, and Q is the distance from the toe to where the force P cuts the base.

When Q is equal to or greater than $\frac{B}{3}$

$$\text{Pressure at the toe} = (4B - 6Q)\frac{P}{B^2} \qquad \textbf{(21d)}$$

$$\text{Pressure at the heel} = (6Q - 2B)\frac{P}{B^2} \qquad \textbf{(21e)}$$

When Q is less than $\frac{B}{3}$

$$\text{Pressure at the toe} = \frac{2P}{3Q} \qquad \textbf{(21f)}$$

Illustrative Example. A retaining wall is to be designed to support an embankment 18 feet high, the top of the fill being level with the top of the wall, the face of the wall to be vertical, the back to slope.

Draw an outline of the proposed section, Fig. 40, and then investigate the section to see if it has sufficient strength to support the embankment. Make the base 45 of the height of the wall.

$$\text{Width of base} = 18 \times .45 = 8.1 \text{ feet}$$

Assume the width at the top to be 2 feet and find the pressure E at the back, substituting in Equation (21a), and apply that pressure at $\frac{H}{3}$.

$$E = .286\,\frac{Wh^2}{2} = .286\,\frac{100 \times \overline{18}^2}{2} = 4{,}633 \text{ lb.}$$

P is found by dividing the wall into rectangles and a triangle and determining the weights and the center of gravity of each, and also of the earth back of the wall, and then finding the combined weights and the center of gravity of the wall and earth. Assume that the weight of the masonry is 150 pounds per cubic foot and the earth 100 pounds per cubic foot, and consider the section of wall as being one foot in length. The details of the computation are given below:

Center of Gravity of Wall

(Moments taken about *A*)

Sections	Area (Sq. Ft.)	Moment Arm	Moment Area
a b c d	24.3	4.05	98.4
e f g h	30.0	2.00	60.0
h i g	27.0	4.20	113.4
	81.3		271.8

Distance from *A* to center of gravity = 271.8 ÷ 81.3 = 3.34 ft.
Weight of wall per lineal foot = 81.3 × 150 = 12,195 lb.
Static moment about *A* = 12,195 × 3.34 = 40,730 ft.-lb.

Center of Gravity of Earth

(Moments taken about *A*)

Section	Area (Sq. Ft.)	Moment Arm	Moment Area
i j g	27.0	5.4	145.8
i j k c	22.5	7.35	165.4
	49.5		311.2

Distance from *A* to center of gravity = 311.2 ÷ 49.5 = 6.3 ft.
Weight of earth per lineal foot = 49.5 × 100 = 4,950 lb.
Static moment about *A* = 4,950 × 6.3 = 31,185 ft.-lb.

The position of the resultant is determined by dividing the sum of the static moments by the sum of the weights:

$$\frac{40{,}730 + 31{,}185}{12{,}195 + \ \ 4{,}950} = \frac{71{,}915}{17{,}145} = 4.2 \text{ ft.}$$

Produce *E* to meet the vertical line passing through the combined centers of gravity. On this vertical line lay off the value of *P*, which is 17,145 pounds, to any convenient scale. At the lower end of P draw a line parallel to line E and on this line lay off the value of E, which is 4,633. Draw line mn, which

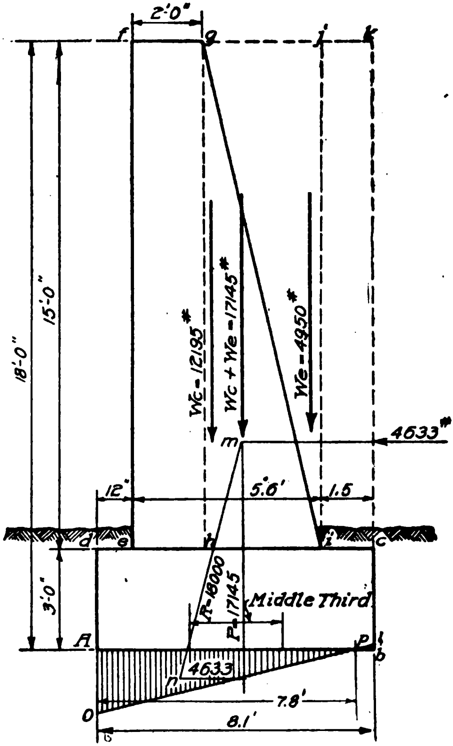


Fig. 40. Design Diagram for Simple Retaining Wall

is the resultant of the two forces. This line cuts the base at a scaled distance of 2.6 feet from the toe, which is about one inch outside the middle third; therefore Q is less than $\frac{B}{3}$.

Substituting in Equation (**21f**) for the condition when Q is less than $\frac{B}{3}$, we have

$$\text{Pressure at toe} = \frac{2}{3} \times \frac{17{,}145}{2.6} = 4{,}397 \text{ lb.}$$

Lay off to any convenient scale the weight 4,397 pounds, and on the base lay off a distance equal to $3\,Q = 7.8$ feet. Through this point draw *Op* and scale the force shown from *l* to the base line *b*, which is less than 200 pounds and need not be further considered.

The pressure at the toe, 4,397 pounds, is easily supported on any ordinary soil and the uplift at the heel, 200 pounds, is too small to be considered. This section should be safe for the conditions given in the problem.

Reinforced Concrete Walls. These are usually made in such shape that advantage is taken of the weight of part of the material supported to increase the stability of the wall against overturning. Fig. 41 shows the outline of such a wall. It consists of a vertical wall *C D*, attached to a floor plate *A B*. To prevent the wall from overturning, the moment of downward forces about the outer edge of the base $M_1 = W_1 l_1 + W_2 l_2$, must be greater than that of the overturning moment, $M_2 = E l_3$. M_1 should be from one and one-half to twice M_2, which would be the factor of safety. In addition to this factor of safety there would be the shearing of the earth along the line *ab*.

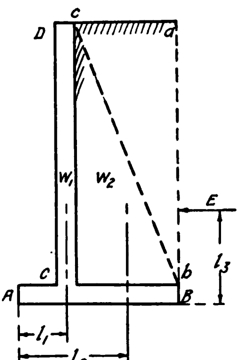


Fig. 41. Outline of Reinforced Concrete Wall

Owing to the skeleton form of these walls it is usually more economical to construct them than solid walls of masonry. The cost per cubic yard of reinforced concrete in the wall will be more than the cost per cubic yard of plain concrete or stone, in a gravity retaining wall, but the quantity of material required will be reduced by 30 to 50 per cent in most cases. There are two forms of these walls. The outline in Fig. 41 shown in solid lines is the simpler to construct and is the more economical of the two types of reinforced concrete walls, up to a height of 18 feet. For higher walls the form shown by the solid lines and heavy dotted line *bc* is used.

Illustrative Example. Suppose a retaining wall is to be designed which is to be 14 feet high to support an earth face with a surcharge at a slope of 1.5 to 1.

The width of the base for reinforced concrete walls is usually made from .4 to .6 of the height. For this wall, with a surcharge, the base will be made one-half of the height, $14 \times \frac{1}{2} = 7$ feet. Assume the weight of the earth at 100 pounds per cubic foot and the reinforced concrete at 150 pounds per cubic foot.

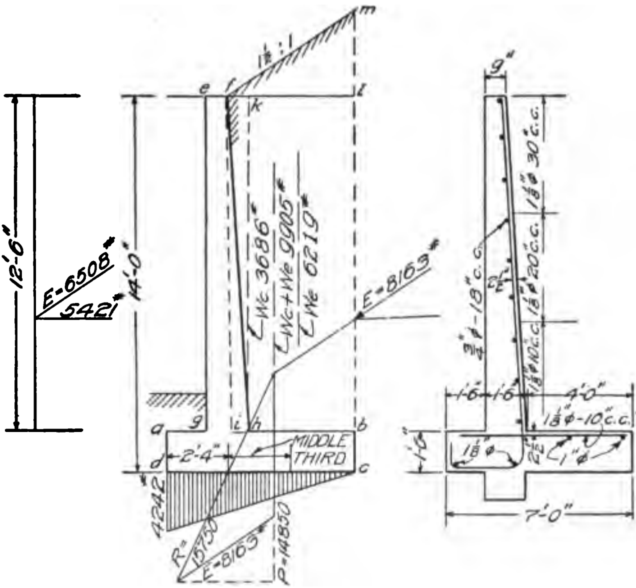


Fig. 42. Design Diagrams for Retaining Wall

Substituting in Equation (**21c**), we have

$$E = .833 \frac{Wh^2}{2} = .833 \times \frac{100 \times \overline{14^2}}{2} = 8{,}163 \text{ lb.}$$

This force is applied on the plane *cm,* Fig. 42, at a point one-third of the height above the base.

It will be necessary to determine the thickness of the vertical wall and the base plate before the stability of the wall can be determined. Assume the base plate to be 18 inches thick; then the vertical slab will be 12 feet 6 inches high and the pressure against this slab will be

$$E = .833 \frac{(100 \times \overline{12.5}^2)}{2} = 6{,}508 \text{ lb.}$$

The horizontal component of this pressure is 6,508 × cos 33° 42′ = 5,421 pounds, as shown diagrammatically in Fig. 42. The bending moment would be

$$M = 5{,}421 \times \frac{12.5}{3} \times 12 = 271{,}050 \text{ in.-lb.}$$

Placing this value of M equal to $95 bd^2$ in which $b = 12$, and solving for d, we have

$$95 \times 12\, d^2 = 271{,}050$$
$$d^2 = 238$$
$$d = 15.4 \text{ in.}$$

With 2.6 inches added for protecting this steel, the total thickness would be 18 inches. The area of the reinforcing steel would be .00675 × 15.4 = .104 square inch of steel per inch of length of wall. Bars 1⅛ inches round (.99 ÷ .10 = 9.9) spaced 10 inches apart, will be required. The bending moment rapidly decreases from the bottom of the slab upwards, and, therefore, it will not be necessary to keep the thickness of 18 inches to the top of the slab or to have all the bars the full length. Make the top 9 inches thick; drop off one-third of the bars at one-third of the height of the slab and one-third at two-thirds of the height. The shear at the bottom of the slab is 5,421 ÷ (12 × 15.4) = 29 pounds per square inch; therefore, as this does not exceed the working stress, no stirrups are needed.

It is very important in a wall of this type not to exceed the bonding stress. The vertical bars must be well anchored in the base plate or they will be of no great value. Since the bars are 1⅛ inches in diameter, the circumference is 3.53 inches. If a bonding stress of 75 pounds per square inch is allowed for, the total bonding per inch of length of bar is $3.53 \times 75 = 265$ pounds. The lever arm is 15.4 inches. As the bars are spaced 10 inches on centers, the stress to be resisted is $\frac{5}{6}$ of 271,050, or 225,875 inch-pounds. Let x be length of anchorage required, then

$$M = 265 \times 15.4 \times x = 225{,}875$$
$$x = 55 \text{ in.}$$

That is, the vertical 1⅛-inch round bars must extend into the footing 55 inches or be anchored in such a way that their strength will be developed.

In designing the footing of a reinforced concrete retaining wall the resultant force should intersect the base within the middle third, as in a masonry wall. The forces acting on the footing are the earth pressure on the plane *mc,* the weight of the earth fill, and the weight of the concrete. The distance from the toe *a* to the point where the resultant acts is obtained as follows: The centers of gravity of the concrete and the earth are found, also the weight of each. The weights are multiplied by the distances from *a,* respectively, which gives the static moment. The sum of the static moments divided by the sum of the weights equals the distance from the toe to the line at which the resultant acts. The detail figures for the problem are given below.

Center of Gravity of Wall

(Moments taken about *a*)

Sections	Area (Sq. Ft.)	Moment Arm	Moment Area
a b c d	10.50	3.50	36.75
e f i g	9.38	1.88	17.63
f i h	4.69	2.50	11.73
	24.57		66.11

Distance from *a* to center of gravity $= 66.11 \div 24.57 = 2.69$ ft.

Weight per lineal foot $= 24.57 \times 150 = 3{,}686 = W_c$

Static moment about $a = 3{,}686 \times 2.69 = 9{,}915$ ft.-lb.

Center of Gravity of Earth

(Moments taken about *a*)

SECTIONS	AREA (Sq. Ft.)	MOMENT ARM	MOMENT AREA
f k h	4.69	2.75	12.90
h b l k	50.00	5.00	250.00
f l m	7.50	5.42	40.65
	62.19		303.55

Distance from *a* to center of gravity $= 303.55 \div 62.19 =$ 4.88 ft.

Weight per lineal foot $= 62.19 \times 100 = 6{,}219 = W_e$

Static moment about $a = 6{,}219 \times 4.88 = 30{,}355$ ft.-lb.

The distance from *a* to the combined center of gravity of the concrete and the earth fill is

$$\frac{9{,}915 + 30{,}355}{3{,}686 + \ 6{,}219} = \frac{40{,}270}{9{,}905} = 4.06 \text{ ft.}$$

To find where the resultant *R* cuts the base, produce *E* to meet the combined center of gravity of the concrete and earth. From their intersection lay off on the vertical line, at any convenient scale, the combined weight 9,905 pounds. At the end of this distance draw a line parallel to the line *E* and lay off the value of *E* which is 8,163 pounds. Draw *R*, which is the resultant and in this case cuts the base at the edge of the middle third, so that the wall will not fall by overturning.

The pressure produced on the foundation is next to be investigated. Since the resultant comes at the edge of the middle third, Equations (**21d**) and (**21e**) are used.

$$\text{Pressure at the toe} = (4B - 6Q)\frac{P}{B^2}$$
$$= [(4 \times 7) - (6 \times 2.33)]\frac{14{,}850}{7^2}$$
$$= 4{,}242 \text{ lb.}$$

$$\text{Pressure at the heel} = (6Q - 2B)\frac{P}{B^2}$$
$$= [(6 \times 2.33) - (2 \times 7)]\frac{14{,}850}{7^2}$$
$$= 0$$

The pressure on the foundation of 4,242 pounds at the top is permissible on most soils.

The stability of a wall of this type must be carefully investigated. Suppose this wall is to be located on a wet clay soil. The coefficient of friction between concrete and wet clay is .33. The horizontal force is 6,800 pounds, and the weight of the concrete and earth acting in a downward direction is 9,915 pounds. With a coefficient of .33 or $\frac{1}{3}$, the resistance to sliding is $9{,}915 \times \frac{1}{3} = 3{,}305$ pounds, which is less than one-half of the horizontal pressure, 6,800. The resistance should be about twice the pressure in order to make the wall safe against sliding, and this would require that the weight should be about four times as much, in order that mere friction should surely prevent sliding. This shows that it will be necessary to construct a projection in the base, as shown in Fig. 42.

The thickness of the base is always made greater than the moment requirements just behind the vertical slab (or at *h*) would demand. If the wall were actually on the point of tipping over, there would cease to be any upward pressure on the base. But there would be a downward pressure on the right cantilever equal to the weight of the earth above it, and the moment in the base at the point *h* would be that produced by that earth pressure and by the weight of the concrete from *h* to *b*. Since the foregoing calculations for the stability of the wall show that the computed lateral pressure cannot produce actual tipping about the toe, no such moment can really be developed, but the calculation of the required thickness to resist such a moment gives a dimension which is certainly more than safe and which, for other reasons, is sometimes made still greater. The weight of the earth is 6,219 pounds and the weight of the concrete is $4 \times 1\frac{1}{2} \times 150 = 900$ pounds. Then $6{,}219 + 900 = 7{,}119$ pounds. Therefore

$$M = 7{,}119 \times 1.90 \times 12 = 162{,}313 \text{ in.-lb.}$$

Placing this moment equal to $M = 95\,bd^2$ and solving for d, we find that d equals 11.9. If 2.5 inches are added for protecting the steel, the total thickness would be 14.4 inches. To anchor properly the bars in the vertical slab the thickness of base plate is seldom made less than the vertical slab. Therefore, we will make $d = 15$ inches, $b = 12$, and solve for the moment factor R.

$$M = 12 \times \overline{15}^2 \times R = 158{,}977$$
$$R = 58.8$$

Fig. 20 shows that when $R = 59$, then $c = 400$, $s = 12{,}000$, and that the percentage of steel required is practically .006. The steel required, therefore, is $12 \times 15 \times .006 = 1.08$ square inches, and bars $1\frac{1}{8}$ inches in diameter, spaced 10 inches, will be needed. The moment in this part of the base plate is negative; therefore the steel must be placed in the top of the concrete.

The vertical shear is $7{,}129 \div (12 \times 15) = 39$ pounds per square inch, which is less than the working value allowed in concrete.

The left cantilever or toe has an upward pressure. At the extreme end it is 4,240 pounds and at the face of the vertical wall it is 3,200 (scaled from Fig. 42). The average pressure is $(4{,}240 + 3{,}200) \div 2 = 3{,}720$ pounds. The moment is, therefore,

$$M = 3{,}720 \times \frac{1.5}{2} \times 12 = 33{,}480 \text{ in.-lb.}$$

Let $d = 15$, $b = 12$, and solve for R.

$$12 \times \overline{15}^2 \times R = 33{,}480$$
$$R = 12.4$$

This value of 12.4 for R is smaller than is found in Fig. 20. Since the bars in the vertical slab are bent in such a shape as to supply this tension, no further consideration of this stress is necessary in this problem.

Some longitudinal bars must be placed in the wall to prevent temperature cracks, and also to tie the concrete together. About .003 per cent of the area above the ground is often used. In this case $\frac{3}{4}$-inch round bars spaced 18 inches on centers will be placed.

Reinforced Concrete Walls with Counterforts. In this type of wall the vertical slab is supported by the counterforts, the principal steel being horizontal. The counterforts act as cantilever beams, being supported by the footing.

Illustrative Example. Design a reinforced concrete wall with counterforts, the wall to be 20 feet high and the fill to be level with the top of the wall.

The spacing of the counterforts is first determined. The economical spacing will vary from 8 feet to 12 feet or more,

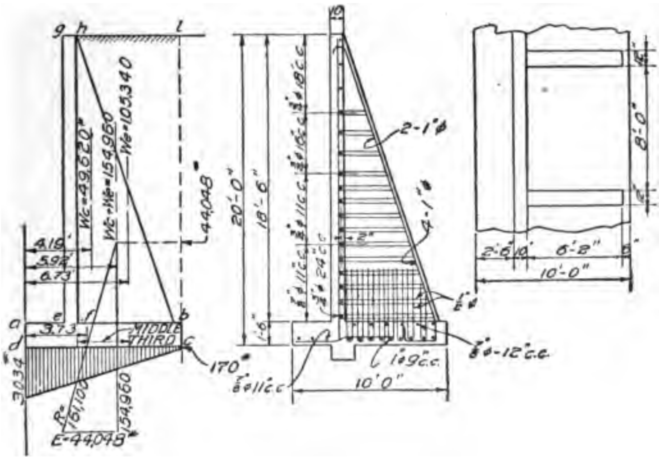


Fig. 43. Design Diagram for Retaining Wall with Counterforts

depending on the height of the wall. A spacing of 9 feet on centers will be used for the counterforts in this case, Fig. 43. The maximum load on the slab is on the bottom unit and it decreases uniformly to zero at the top, when the earth is horizontal with the top of the wall, as in this case. Assume that the base plate will be 18 inches in thickness; then the center of the bottom foot of slab will be 18 feet from the top of the wall. The pressure to be sustained by the lower foot of the slab will then be

$$P = \frac{1}{3} Wh$$

in which P represents the intensity of the horizontal pressure at any depth h, while w represents the weight per cubic foot of the earth.

$$P = \frac{1}{3} \times 100 \times 18$$
$$= 600 \text{ lb. per sq. ft.}$$

Multiplying this value of P by the distance between the centers of the counterforts ($600 \times 9 = 5{,}400$) gives the value of the full load.

$$M = \frac{5{,}400 \times 9 \times 12}{8} = 72{,}900 \text{ in.-lb.}$$

Placing this value of M equal to $95\,bd^2$ in which $b = 12$, and solving for d, we have

$$95 \times 12\,d^2 = 72{,}900$$
$$d^2 = 64$$
$$d = 8 \text{ in.}$$

Adding 2 inches to this for protecting the steel, the total thickness of the wall will be 10 inches. For convenience of construction the slab will be made uniform in thickness. The steel for the bottom inch will be $.00675 \times 18 = .054$ square inch. Round bars $\frac{7}{8}$ inch in diameter may be used and spaced ($.60 \div .054 = 11$) 11 inches on centers. This size of bars and spacing should be used for one-fourth the height of the wall. The next quarter will be reduced twenty-five per cent, and $\frac{3}{4}$-inch round bars, spaced 11 inches, will be used. In the third quarter, the required area will be one-half of that for the first quarter. The steel for this section will be $.054 \div 2 = .027$ square inch, and the bars will be ($.44 \div .027 = 16$) $\frac{3}{4}$-inch round bars spaced 16 inches on centers. In the upper part of the wall $\frac{3}{4}$-inch round bars, spaced 18 inches on centers, should be used.

In order to determine the requirements of the counterforts it will be necessary to determine the horizontal pressure against a section of the wall 9 feet long. Equation (**21**) has already been stated thus:

$$E = \tan^2 (45° - \frac{\phi}{2}) \frac{W h^2}{2}$$

Substituting in the modified form of Equation (**21a**) and multiplying by 9, we have

$$E = .286 \times \frac{100 \times \overline{18.5}^2}{2} \times 9 = 44{,}048 \text{ lb.}$$

This load is applied at one-third of the height of the wall, which is 6.5 feet above the base. The moment in the counterfort is

$$M = 44{,}048 \times 6\tfrac{1}{2} \times 12 = 3{,}435{,}744 \text{ in.-lb.}$$

The width of counterfort must be sufficient to insure rigidity, to resist any unequal pressures, and to imbed thoroughly the reinforcing steel. The width is determined by judgment and in this case will be made 12 inches wide. The counterfort and vertical slab together form a **T**-beam with a depth at the bottom of 84 inches. Let 4 inches be allowed to the center of the steel; then $d = 80$ inches, $jd = .87\,d = .87 \times 80 = 69.6$ inches.

$$M = A_s \times jd \times 16{,}000$$
$$3{,}435{,}744 = As \times 69.6 \times 16{,}000$$
$$A_s = 3.0 \text{ sq. in.}$$

Four 1-inch round bars will give this area. Two of these bars will extend to the top of the wall and two may be dropped off at half the height.

Now that these dimensions have been determined, the wall will be investigated for stability against overturning. Substituting in Equation (**21a**)

$$E = .286 \times \frac{100 \times \overline{20}^2}{2} = 5{,}720 \text{ lb.}$$

To find the center of gravity of the wall, it will be necessary to take a section 9 feet long, that is, center to center of counterforts.

Center of Gravity of Concrete

(Moments taken about *a*)

Section	Volume (Cu. Ft.)	Moment Arm	Volume Moment
a b c d	135.0	5.00	675.0
c f g h	138.8	2.92	405.3
h f b	57.0	5.38	306.7
	330.8		1,387.0

Distance from *a* to center of gravity = 1,387.0 ÷ 330.8 = 4.19 ft.

Weight of 9 feet of wall = 330.8 × 150 = 49,620 lb.

Static moment about *a* for section 9 feet long = 49,620 × 4.19 = 207,908 ft.-lb.

Center of Gravity of Earth

(Moments taken about *a*)

Section	Volume (Cu. Ft.)	Moment Arm	Volume Moment
f b l h	987.0	6.66	6,573.4
b l h	66.4	7.77	515.9
	1,053.4		7,089.3

Distance from *a* to center of gravity = 7,089.3 ÷ 1,053.4 = 6.73 ft.

Weight of earth per 9 feet of wall = 1,053.4 × 100 = 105,340 lb.

Static moment about *a*, for section 9 feet long = 105,340 × 6.73 = 708,930 ft.-lb.

Distance from *a* to the resultant of the concrete and earth

$$\frac{207{,}908 + 708{,}930}{49{,}620 + 105{,}340} = \frac{916{,}838}{154{,}960} = 5.92 \text{ ft.}$$

Drawn the line $W_c + W_e$ at a distance 5.92 feet from *a* and produce the line *E* to meet it. From the intersection of these two lines lay off the sum of the weight of the concrete plus the weight of the earth, at any convenient scale. At the end of this distance draw a line parallel to *E* and lay off on it the value found for E. Draw the resultant R. This line produced on to the base falls within the middle third, and therefore, the wall should be safe against overturning.

Since the resultant cuts the base within the middle third, Q is greater than one-third of the width of the base and Equations (**21d**) and (**21e**) will be applied in finding the pressure on the base. Substituting in Equation (**21d**),

$$\text{Pressure at the toe} = (4\,B - 6\,Q)\frac{P}{B^2}$$
$$= [(4 \times 10) - (6 \times 3.73)]\frac{154{,}960}{\overline{10}^2}$$
$$= 27{,}304 \text{ lb.}$$

Dividing 27,304 by 9 we have 3,034 pounds, which is the weight per foot in length of the wall on the toe.

The pressure at the heel is found by substituting in Equation (**21e**)

$$\text{Pressure at the heel} = (6\,Q - 2\,B)\frac{P}{B^2}$$
$$= [(6 \times 3.73) - (2 \times 10)]\frac{154{,}960}{\overline{10}^2}$$
$$= 3{,}688 \text{ lb.}$$

Dividing 3,688 by 9 gives 410 pounds, which is the weight per lineal foot at the heel.

In designing the toe (left cantilever) there is the average pressure, $(3{,}034 + 2{,}378) \div 2 = 2{,}706$, for which steel must be provided.

$$2{,}706 \times 2.5 = 6{,}765$$

$$M = 6{,}765 \times \frac{2.5}{2} \times 12 = 101{,}475 \text{ in.-lb.}$$

If $b = 12$ and $d = 15$ (the total thickness allowed was 18 inches), solving for R, we have

$$12 \times \overline{15}^2 \times R = 101{,}475$$
$$R = 38$$

Therefore c equals 300 and s equals 12,000 approximately, and p equals .0035. The steel per lineal foot of wall required will be $12 \times 15 \times .0035 = .63$ square inches, and this is equal to ⅞-inch round bars spaced 11 inches on centers. As a precaution against the load being concentrated under the counterforts, three extra bars should be placed in the toe at these places.

The rear portion of the footing is designed as a simple beam between the counterforts. It must have sufficient strength to support the earth above it and also its own weight, although, as explained previously for the L-shaped wall, such a stress cannot be developed unless the wall were just at the point of overturning, and the investigation for stability shows that this cannot happen. The following calculation, therefore, introduces in the design of the base slab an additional factor of safety, of perhaps 2, besides the usual working factor of about 4.

$$\begin{aligned} \text{Weight of earth} &= 105{,}340 \text{ lb.} \\ \text{Weight of base} &= \underline{13.500 \text{ lb.}} \\ & 118{,}840 \text{ lb.} \end{aligned}$$

$$M = \frac{118.840 \times 9 \times 12}{8} = 1{,}604{,}340 \text{ in.-lb.}$$

If $b = 80$ and $d = 15$, solving for R, we have

$$80 \times \overline{15}^2 \times R = 1{,}604{,}340$$
$$R = 89$$

From Fig. 20 we find that with steel stressed to 16,000 pounds the concrete would be stressed to about 575 pounds per square inch and the required percentage of steel would be .0062. The bars required will be ($.0062 \times 80 \times 15 = 7.44$ square inches) nine bars 1 inch round spaced 8 inches apart.

In addition to the steel that has been required to satisfy the different equations, the bars in the vertical slab and those in the rear portion of the footing must be tied to the counterforts. (See Fig. 43.) A few bars should also be placed in the top of the footing, but no definite calculation can be made for them. The vertical slab should be reinforced for temperature stresses. In this wall ⅜-inch round bars spaced 18 inches on centers will be used.

Coping and Anchorages. Retaining walls generally have a coping at the top. This can be made to suit the conditions or to accord with the wish of the designer. When reinforced concrete walls are not stable against sliding, they can be anchored by making a projection of the bottom into the foundation. This is shown in Figs. 42 and 43.

CULVERTS

A flat slab design is generally used for spans up to 20 feet, for both highway and railroad culverts. In highway construction, it is sometimes found more economical to use the girder bridge for spans as short as 14 or 16 feet. The present discussion will be confined to box culverts for highway use. Concrete, and particularly reinforced concrete, is now much used

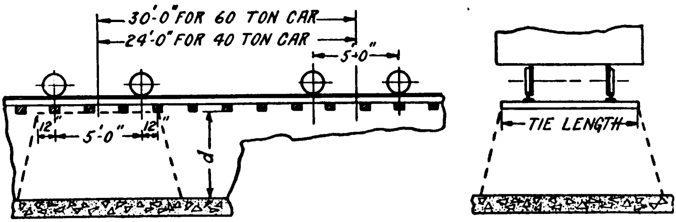


Fig. 44. Load Diagram for 60-Ton and 40-Ton Electric Cars

for culverts and bridges. Its permanence and freedom from maintenance charges, compared with wood and with steel structures, are much in its favor.

Classification by Loadings. Highway structures are usually divided into three classes, as follows:

Class No. 1. Light structures for ordinary country use where the heaviest load may be taken as a 12-ton road roller; the uniform live load, 100 pounds per square foot.

Class No. 2. Heavy structures for use where 20-ton road rollers and electric cars of a minimum weight of 40 tons must be provided for; the uniform distributed load, 125 pounds per square foot.

Class No. 3. City structures for heavy concentrated loads, such as large interurban cars, weighing 60 tons; the uniform distributed load, 150 pounds per square foot.

Load Diagrams. Diagrams representing the loadings for 40- and 60-ton cars, and for road rollers are shown in Figs. 44 and 45 respectively. Since short-span structures are being considered, only one truck of a car will be on the culvert at one time. The truck of a car will be considered as distributing the load over an area which is two feet longer than the center to center of the wheels, and has a width equal to the length of the ties, usually 8 feet. The fill will further distribute this load on a slope of ½ to 1. The fill over a culvert should never be less than one foot. For fast-moving cars the bending moment for the live load should be increased 35 per cent for impact for fills of less than five feet.

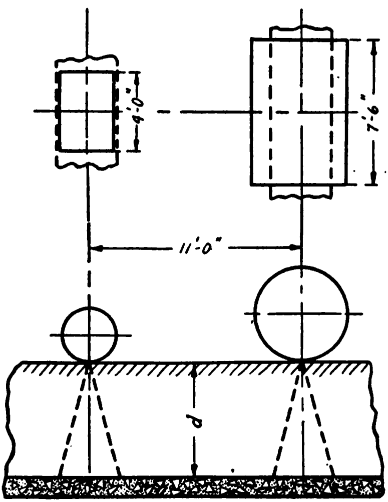


Fig. 45. Load Diagram for Road Roller

Example. Design a flat-slab culvert with a span of 15 feet to support a fill of 4 feet under the ties, a macadam roadway, and a 40-ton car.

Solution. The top will be considered first and a width of one foot will be taken. The fill at 100 pounds per cubic foot will equal $100 \times 4 \times 15 = 6{,}000$ pounds. The macadam will have a thickness of the rail plus the tie, which will be about 12 inches. This material at 125 pounds per cubic foot will equal $125 \times 1 \times 15 = 1{,}875$ pounds for a strip one foot wide. The maximum bending moment for the live load will occur when one of the trucks of a car is at the middle of the span. The load, 20 tons, will be distributed over an area, as shown in Fig. 46, 9 feet by 10 feet $= 90$ square feet. A strip one foot wide then must support $20 \times 2{,}000 \div 10 = 4{,}000$ pounds.

The formula for this bending moment would be

$$M = \left(\frac{Wl}{4} - \frac{Wl_1}{8}\right) 12$$

Substituting in this formula, we have

$$M = \left(\frac{4{,}000 \times 15}{4} - \frac{4{,}000 \times 9}{8}\right) 12 = 126{,}000 \text{ in.-lb.}$$

$$\begin{aligned} &\text{30 per cent added for impact} &&= \underline{37{,}800 \text{ in.-lb.}} \\ &\text{Total moment for live load} &&= 163{,}800 \text{ in.-lb.} \end{aligned}$$

Assume that the slab will be 22 inches thick; then a strip one foot wide weighs $1\frac{5}{6} \times 15 \times 150 = 4{,}125$ pounds. The total weight of the fill, macadam and concrete, is 12,000 pounds. The moment for this load is

$$M = \frac{12{,}000 \times 15 \times 12}{8} = 270{,}000 \text{ in.-lb.}$$

$$\begin{aligned} &\text{Moment for live load} = \underline{163{,}800 \text{ in.-lb.}} \\ &\text{Total moment} \qquad\quad = 433{,}800 \text{ in.-lb.} \end{aligned}$$

Placing this moment equal to $95\,b\,d^2$, where $b = 12$, we have

$$\begin{aligned} 95 \times 12 \times d^2 &= 433{,}800 \\ d^2 &= 380 \\ d &= 19.5 \text{ in.} \end{aligned}$$

If $2\frac{1}{2}$ inches is added for protecting the steel, then the total thickness will be 22 inches. The steel required equals $.00675 \times 12 \times 19.5 = 1.58$ square inches. Round bars 1 inch in diameter, spaced 6 inches on centers, will satisfy this requirement.

The shear at the point of supports will equal one-half the sum of the live and dead loads divided by area of the section, that is, $(4{,}000 + 12{,}000) \div 2 = 8{,}000$.

$$\begin{aligned} v &= 8{,}000 \div b\,j\,d \\ &= 8{,}000 \div (12 \times .87 \times 19.5) \\ &= 39 \text{ lb. per sq. in.} \end{aligned}$$

which is much less than the permissible working load. Even in this case one-third of the bars should be turned up at about 3 feet from the end of the span.

The horizontal pressure on the side walls of the culvert produced by the earth will vary with the depth below the surface. The center of the top foot of the side walls is 7.5 feet and the center of the bottom foot is 12.5 feet below the surface of the roadway. At the top, therefore,

$$P = \frac{Wh}{3} = \frac{100 \times 7.5}{3} = 250 \text{ lb. per sq. ft.}$$

At the bottom it would be

$$P = \frac{100 \times 12.5}{3} = 416 \text{ lb. per sq. ft.}$$

The average pressure equals $(250 + 416) \div 2 = 333$ pounds. This is not strictly accurate but sufficiently so for the side walls. The live load is $4{,}000 \div 9 = 444$ per square foot. It will be assumed that the horizontal pressure from the live load equals $444 \div 3 = 148$ pounds per square foot, this load being independent of the depth of the fill. The total live and dead load is therefore, 333 + 148 = 481 pounds per square foot. The bending moment for this load is

$$M = \frac{Wl^2}{8} \times 12 = \frac{481 \times 6^2}{8} \times 12 = 25{,}974 \text{ in.-lb.}$$

A slab with a thickness of 7 inches would satisfy this equation. Since the side walls must support the top slab as well as the side pressures, they should not be much less in thickness than the top. Make the walls 15 inches thick and reinforce them as shown in Fig. 46.

The bottom is sometimes made the same as the top. This is not necessary unless the foundation is very soft and the load

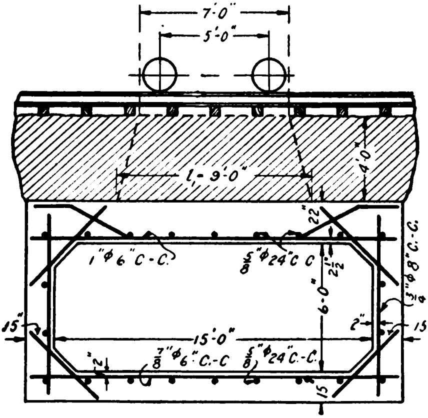


Fig. 46. Design Diagram for Flat-Slab Culvert with 15-Foot Span

must be distributed over the whole area. In this case it will be made the same as the side walls and reinforced as shown.

In designing the culvert, the student will note that while some of the calculations are definite other dimensions must be assumed. The fillets in the corners will assist in stiffening the structure. Wing walls must be provided at the ends. Longitudinal reinforcement also must be provided.

Example. Design a box culvert 5 feet square to support a road roller weighing 12 tons (*Class No. 1*), fill 2 feet deep.

Solution. The maximum load will occur when the rear wheel is at the center of the span, which is two-thirds of 12 tons, or 8 tons, Fig. 47. This will be distributed over an area of 1 foot by 9 feet 6 inches. The live load is therefore 8 × 2,000 ÷ 9.5 = 1,664 pounds for a strip one foot wide. The dead load will be $100 \times 2 = 200$ pounds per square foot for fill and, assuming that the top slab will be 8 inches thick, $12.5 \times 8 = 100$ pounds per square foot. The moments will be as follows:

$$\text{Live load } M = \frac{Wl}{4} \times 12 = \frac{1664 \times 5}{4} \times 12 = 24{,}960 \text{ in.-lb.}$$

$$\text{35 per cent added for impact} = 24{,}960 \times .35 = 8{,}736 \text{ in.-lb.}$$

$$\text{Dead load } M = \frac{Wl^2}{8} \times 12 = \frac{300 \times 5^2}{8} \times 12 = \underline{11{,}250} \text{ in.-lb.}$$

$$\text{Total Moment} = 44{,}946 \text{ in.-lb.}$$

Placing this equal to $95\,b\,d^2$ where $b = 12$,

$$95 \times 12 \times d^2 = 44{,}946$$
$$d^2 = 39.43$$
$$d = 6.28 \text{ in.}$$

Make the total thickness 8 inches. The steel required will be $.00675 \times 6.28 = .04239$ square inch per inch of width, and ¾-inch round bars spaced 10 inches on centers will fulfil the requirements.

The earth pressure on the sides is as follows:

At the top $\quad \frac{Wh}{3} = \frac{100 \times 3.2}{3}$
$= 106$ lb. per sq. ft.

At the bottom $\quad \frac{Wh}{3} = \frac{100 \times 7.2}{3}$
$= 240$ lb. per sq. ft.

Average pressure $\quad (106 + 240) \div 2$
$= 173$ lb. per sq. ft.

Pressure for live load $\quad 1{,}664 \div 3$
$= 555$ lb. per sq. ft.

Total pressure $\quad 173 + 555$
$= 728$ lb.

The bending moment for this load is

$$M = \frac{Wl^2}{8} \times 12 = \frac{728 \times 5^2}{8} \times 12 = 27{,}300 \text{ in.-lb.}$$

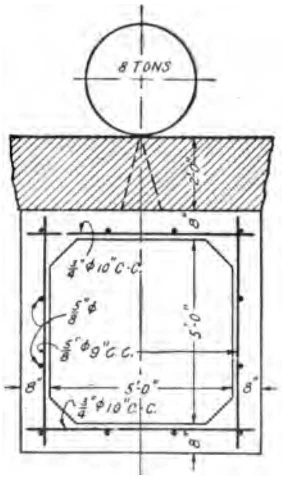


Fig. 47. Design Diagram for Box Culvert 5 Feet Square

A slab 7 inches thick will more than satisfy this equation, but to insure stiffness the sides for a culvert of this size should not be made less than the thickness of the top. Use ⅝-inch round bars, spaced 9 inches on centers, Fig. 47. The bottom will be made 8 inches thick, also, and reinforced with ¾-inch round bars, spaced 10 inches on centers. Temperature bars must also be provided.

GIRDER BRIDGES

Method of Design. Girder bridges are being extensively used for country highways for spans from 20 to 40 feet. They are sometimes used for spans up to 60 feet and often for spans as short as 16 feet. Fig. 48 shows the section of one-half the width of such a bridge. The slab for a bridge of this kind must always be paved or macadamized so that no wheels will come directly on the concrete.

Illustrative Example. Design a girder bridge with a clear span of 26 feet; the width of roadway is 16 feet and there are

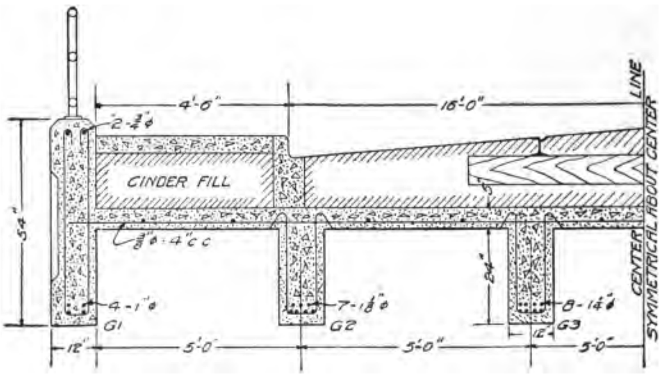


Fig. 48. Design Diagram for Girder Bridge

two sidewalks, each 4 feet 6 inches wide. The loading for this bridge is to be as specified for *Class No. 2,* given on page 145, the car line to be in the center of the bridge, and a fill of six inches to be placed under the ties with a macadam-surfaced roadway.

The slab for such a structure should never be less than 5 inches thick on account of concentrated loads and shear due to road rollers and other such loads. To cover such contingencies the slab will be designed for a live load of 500 pounds per square foot. The slab load and moment, therefore, will be as follows:

Live load	$4 \times 1 \times 500 =$	2,000 lb.
Slab, 5 in.	$\frac{5}{12} \times 150 \times 4 =$	250 lb.
Fill, 20 in.	$1\frac{2}{3} \times 125 \times 4 =$	833 lb.
Total load	$=$	3,083 lb.

$$M = \frac{3{,}100 \times 4 \times 12}{8} = 18{,}600 \text{ in.-lb.}$$

Placing this moment equal to $95\,bd^2$, where $b = 12$, and solving, we find that $d = 4$ inches.

The steel area must, therefore, equal $.00675 \times 4 \times 12 = .32$ square inches per foot of width, which requires $\frac{3}{8}$-inch round bars, spaced 4 inches on centers.

The outside girder (G_1), Fig. 48, supports one-half of the sidewalk load, which is as follows:

Live load 125	$125 \times 2\frac{1}{4} \times 26$	$=$	7,313 lb.
Walk 4 in. thick	$50 \times 2\frac{1}{4} \times 26$	$=$	2,925 lb.
Cinder fill 15 in.	$60 \times 1\frac{1}{4} \times 2\frac{1}{4} \times 26$	$=$	4,388 lb.
Slab 5 in.	$60 \times 2\frac{1}{4} \times 26$	$=$	3,510 lb.
Girder 12 × 54 in.	$150 \times 4\frac{1}{2} \times 26 \times 1$	$=$	17,550 lb.
Total load		$=$	35,686 lb.

$$M = \frac{35{,}686 \times 26 \times 12}{8} = 1{,}391{,}754 \text{ in.-lb.}$$

This moment placed equal to $95\,bd^2$, when $b = 12$, would require a depth of 35 inches to the center of the steel, while the total depth of the beam is 54 inches. Therefore, make b equal 12 and d equal 51, and solve for the moment factor R.

$$12 \times \overline{51}^2 \times R = 1{,}391{,}754$$
$$R = 45$$

By referring to the diagram, Fig. 20, it is at once to be seen that when R equals 45 the compression in the concrete will be low and that a percentage of steel of .005 is more than actually will be required. However, that amount will be used. $12 \times 51 \times .005 = 3.1$ square inches. Four 1-inch round bars will be used, two bars to be straight and two turned up near the ends. The shear per square inch is small, but stirrups should be used.

Girder G_3 will next be designed. For this beam there are three live loads to be considered and the girder will be designed

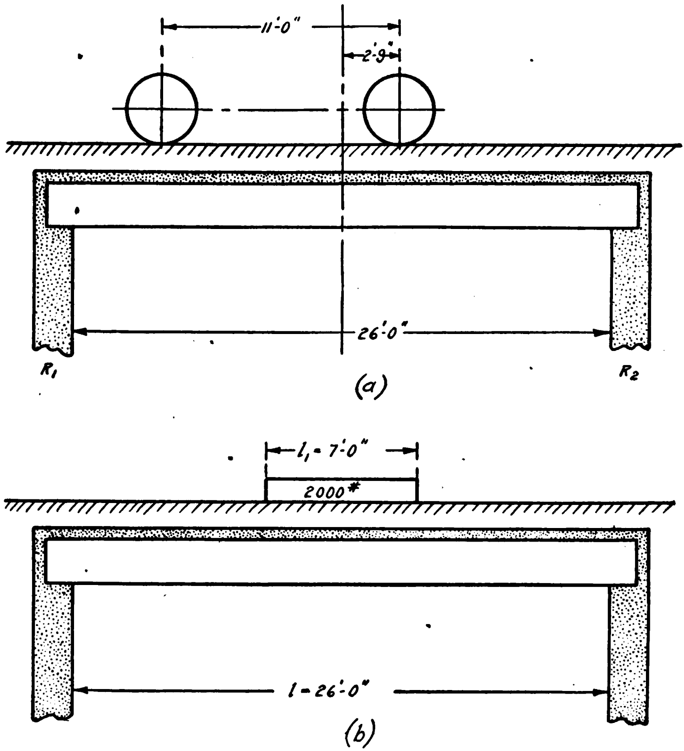


Fig. 49. Diagrams for Loadings for Road Roller and Electric Car

to support the maximum one combined with the dead load. The three live loads are: the uniform load of 125 pounds per square foot, a 20-ton road roller, and a 40-ton electric car.

The dead load and moment for this load will be as follows:

Macadam and fill	$1\frac{2}{3} \times 125 \times 5 \times 26 =$	27,084 lb.
Slab	$\frac{5}{12} \times 150 \times 5 \times 26 =$	8,125 lb.
Beam $12'' \times 24''$ (assumed)	$1 \times 2 \times 150 \times 26 =$	7,800 lb.
Total load	$=$	43,009 lb.

$$M = \frac{43{,}009 \times 26 \times 12}{8} = 1{,}677{,}351 \text{ in.-lb.}$$

The total live load for a uniform loading of 125 pounds per square foot would be $125 \times 5 \times 26 = 16{,}250$ pounds, and its moment would be

$$M = \frac{16{,}250 \times 26 \times 12}{8} = 633{,}750 \text{ in.-lb.}$$

Since the fill is so small the weight of a road roller or car cannot be distributed to any great amount by this means, it will not be considered in the calculations. Each of these beams may be required to support the whole weight of the front wheel and half the weight of the rear wheel. This moment will be a maximum when one wheel is one-fourth of the distance between the center of wheels from the center of the span of the bridge.

The maximum reaction is at the right and is

$$R_1 = \frac{13{,}333 \times 4.75}{26} + \frac{13{,}333 \times 15.75}{26} = 10{,}478$$

Then

$$M = 10{,}478 \times 10.25 \times 12 = 1{,}288{,}794 \text{ in.-lb.}$$

The maximum load produced on girders G_3 by an electric car takes place when one of the trucks is at the center of the span. Each of these girders at that time would be supporting one-fourth of the total weight of 40 tons, or 10 tons. (See Fig. 49.) The moment is, therefore,

$$M = \left(\frac{20{,}000 \times 26}{4} - \frac{20{,}000 \times 7}{8}\right)12 = 1{,}350{,}000 \text{ in.-lb.}$$

35 per cent added for impact	$=$	472,500 in.-lb.
Total	$=$	1,822,500 in.-lb.

The electric car produces a greater bending moment than either of the other live loads and, therefore, will be used together with the dead load. That is, 1,822,500 + 1,677,351 = 3,499,851. Let d equal 25.5; then $25.5 \times .88 = 22.4$ inches. The required amount of steel then is $3,499,851 \div (22.4 \times 16,000) = 9.8$ square inches. Eight bars 1¼ inches in diameter will be used, one-half of which will be turned up in pairs at different points near the ends of the girder.

The shear in this girder will be one-half the sum of 20,000 and 43,000, or 31,500 pounds. Then

$$v = \frac{31,500}{12 \times 23} = 114 \text{ lb. per sq. in.}$$

Therefore stirrups must be used. They should be ⅜ of an inch in diameter, used throughout the length of the girder, and spaced not over 6 inches apart near the ends of the girders.

The bending moment for girder G_2 will be taken as the mean of girders G_1 and G_3, plus the dead load, and will be as follows:

$$G_1 = 1,505,400 \text{ in.-lb.}$$
$$G_3 = 3,499,500 \text{ in.-lb.}$$
$$5,004,900 \text{ in.-lb.}$$
$$\therefore G_2 = 5,004,900 \div 2 = 2,502,450 \text{ in.-lb.}$$

The steel required equals $2,502,450 \div (22.4 \times 16,000) = 7$ square inches. Seven bars 1⅛ inches in diameter will be used, three-eighths of which will be turned up near the ends of the girders. Use ⅜-inch shear bars.

In designing girder bridges the designer must always investigate the shear in the girders and the compression in the T-beams very carefully and see that these stresses are satisfied.

CONCRETE BUILDING BLOCKS

Concrete blocks are sometimes used for the walls of houses, barns, and factory buildings of one to four stories. They are made at a factory or on the site of the work, and are placed in the wall in the same manner as brick or stone. The blocks are made in metal machines, being molded somewhat similarly to brick.

Types. There are two general types of blocks—solid blocks and hollow blocks. The solid blocks are used for heavy work and vary in size according to different classes of work. The hollow blocks are used for the walls of buildings, and this is the type generally referred to when concrete building blocks are mentioned. They are cheaper than the solid blocks and are less easily penetrated by water, cold, and heat. There are also two types of the hollow blocks—the one-piece block and the two-piece block. The one-piece type consists of a block, with hollow cores, making the whole thickness of the wall. In the two-piece type, the front and back of the blocks are made in separate pieces and bonded when laid up in the wall: this bond is secured either by the blocks lapping over each other or by the use of galvanized iron ties. Hollow blocks are also made with two cores. Fig. 50 shows the different types of hollow blocks.

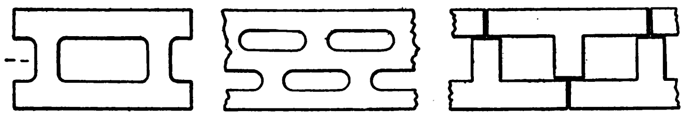


Fig. 50. Types of Hollow Concrete Blocks

There are a great variety of machines in use for the manufacture of concrete hollow blocks. Some types of these machines will be discussed in the section devoted to machinery.

Sizes. Various shapes and sizes of blocks are made. Builders of some of the standard machines have adopted a length of 32 inches and a height of 9 inches for the full-sized blocks, with widths of 8, 10, and 12 inches. Lengths of 8, 12, 16, 20, and 24 inches are made with the same machine, by the use of parting-plates and suitably divided face-plates. Most machines are constructed so that any length between 4 and 32 inches, and any desired height, can be obtained.

The size of the openings (the cores) varies from one-third to one-half of the surface of the top or bottom of the block. The building laws of many cities state that the openings shall amount to only one-third of the surface. For any ordinary purpose, blocks with 50 per cent open space are stronger than necessary.

Materials. The materials for making concrete blocks consist of Portland cement, sand, and crushed stone or gravel. Because of the narrow space to be filled with concrete, the stone and gravel are limited to ½ or ¾ inch; at least one-third of the material, by weight, should be coarser than ⅛ inch.

The proportions of the materials must be such that a dense and water-tight concrete is secured. Cement and a fine sand of uniform size, made into a mortar and used without the addition of any coarse material, will not produce good results. A mixture of 1 part Portland cement and 4 or 6 parts of a coarse sand ranging in size from dust to ½ inch will make good blocks when mixed wet and well tamped. The proportions should never be leaner than 1:2:4 if good blocks are required.

Architectural features often require a special facing for blocks. This can be secured by mixing marble dust with white Portland cement for a white block, or granite chips with Portland cement for a granite finish. The facing-material is made into a mortar and placed against that side of the form which is to make the face of the block. This face may be either a plain face or of various ornamental patterns, as tool-faced, paneled, broken ashlar, etc. The penetration of water may be effectively prevented by this rich coat.

Blocks made with dry concrete will be weak and porous even if they are well sprinkled after being removed from the forms. If the concrete is made too wet it will stick to the sides of the plates, and the blocks will settle out of shape if they are removed promptly from the mold. There should be, therefore, as much water as can be used without causing the block to stick or sag out of shape when removed from the molds. The amount of water is usually from 8 to 12 per cent of the weight of the dry mixture. To secure blocks uniform in strength and color, the same amount of water must be used for every batch.

Mixing and Tamping. Concrete for blocks must be well mixed. This can best be done in a batch mixer, although good results can be attained by hand mixing. Power pressure applied to the concrete is better than hand tamping, because it is more evenly distributed over the whole area of the block.

Curing of Blocks. *Air Curing.* The blocks are removed from the machine on a steel plate, on which they should remain for 24 hours. The blocks should be protected from the sun and dry winds for at least a week, and thoroughly sprinkled frequently. They should be at least four weeks old before they are placed in a wall; if they are built up in a wall while green, shrinkage cracks will be likely to occur in the joints.

Steam Curing. Concrete blocks can be cured much more quickly in a steam chamber than in the open air. They should be left in the steam chamber for 48 hours at a pressure of 80 pounds per square inch. By this method, blocks can be handled and used much more quickly than when air cured, and their strength is much higher than the air-cured blocks when six months old. When a large quantity of blocks is to be made, the steam curing is more economical than the air curing, even considering the much more expensive plant that is required.*

Cost of Making. The following example of the cost of making concrete blocks is quoted from a paper by Mr. N. F. Palmer, C. E.:

Blocks 8 by 9 by 32 inches; gang consisted of five workmen and a foreman; record for one hour, 30 blocks; general average for 10 hours, 200 blocks. The itemized cost was as follows:

1 foreman	@ $2.50	$ 2.50
5 helpers	@ 2.00	10.00
13 bbls. cement	@ 2.00	26.00
10 cu. yds. sand and gravel	@ 1.00	10.00
Interest and depreciation on machine		2.00
Total		$50.50

This is the equivalent of $50.50 ÷ 200, or 25¼ cents per block; or, since the face of the block was 9 by 32 inches, or exactly 2 square feet, the equivalent of 12.6 cents per square foot of an 8-inch wall.

Another illustration, quoted from Gillette, for a 10-inch wall, was itemized as follows, *for each square foot* of wall:

Sand	$ 0.020
Cement @ $1.60 per barrel	.045
Labor @ $1.83 per day	.038
Total per square foot	$ 0.103

* See Technological Papers Bureau of Standards (U. S.), No. 5.

This is apparently considerably cheaper than the first case, even after allowing for the fact that the second case does not provide for interest, depreciation on plant, etc., which in the first case is only 4 per cent of the total. The allowance of 4 per cent is probably too small.

Cement Brick. Cement brick are made of the same proportions of material as concrete blocks. In general, what has been said about concrete blocks applies also to cement brick. They have not been extensively used.

FENCE POSTS

Design. Reinforced concrete fence posts are now being extensively used. They have many of the advantages of wooden posts and few of the disadvantages. In first cost, concrete

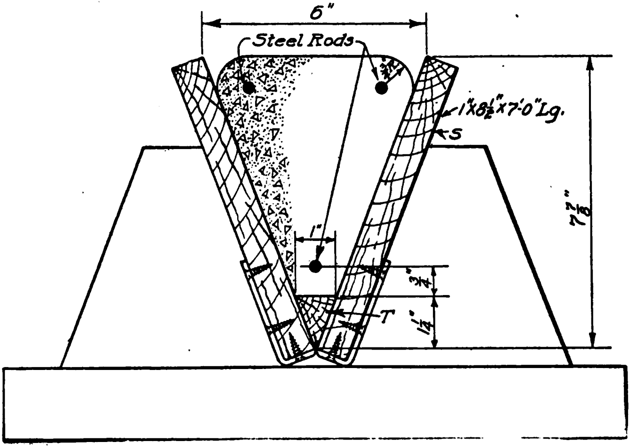


Fig. 51. Cross Section of Triangular Fence Post

posts may be more or less expensive than the wooden posts, depending on the local supply of timber suitable for posts and the local supply of materials for making concrete.

Concrete posts are made in several shapes and sizes. Posts square in cross section and having the same section throughout their length are perhaps the simplest to make, but posts tapering from the bottom to the top on two sides or on all four sides will be more economical in material, lighter to handle, and will look much better. For posts 7 feet long the sections should be 5 inches square at the bottom and taper to 3½ or 4 inches square at the top. A ½-inch bar should be placed in each corner of the post and these bars should be tied together by heavy wire loops spaced 12 inches on centers. Corner and gate posts must be made larger. In Fig. 51 is shown a triangular section illustrated

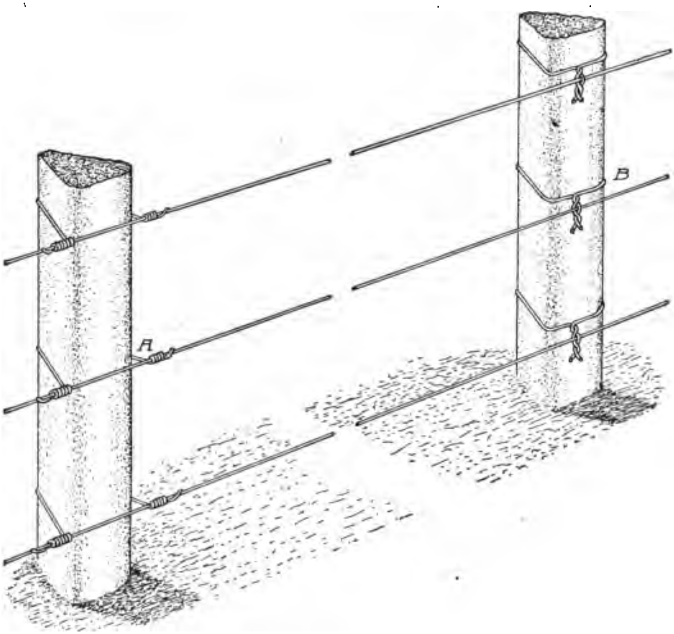


Fig. 52. Methods of Fastening Wire Fencing to Concrete Posts

in Farmers' Bulletin 403, of the U. S. Department of Agriculture; the section is for a post 7 feet long and does not taper. These posts must be set so that the narrow side will support the fencing.

Fastenings. There are several methods of fastening the wire or other fencing to the posts. Galvanized staples or loops may be placed in the green concrete, or small holes may be left in the center of the posts. In Fig. 52 are shown two simple methods of fastening wire fencing given in the Bulletin already mentioned.

Materials and Forms. The concrete should be a 1:2:4 mix in which the stone or gravel should not be larger than ½ inch. It must be a wet mix, well tamped, and the post fully seasoned before being placed. The forms should be so well made that they can be used many times, and they ought to be carefully cleaned and oiled each time.

SILOS

Types. There are two general ways in which silos are constructed of concrete: They may be constructed as a monolith—that is, the concrete poured into the forms—or with blocks. The monolithic walls may be either one solid wall or a double wall with an air space between them. The blocks may also be either solid or hollow. In cold climates it is much better to have the double or hollow walls to retard the freezing of the silage.

Design. In the walls of a silo there is an outward pressure that must be resisted by steel in tension. The amount of this pressure is not so well known as in the case of water pressure in a tank. Some state experiment stations have estimated the silo pressure at 11 pounds per square foot, but to be on the safe side it should be taken as 15 to 20 pounds. There is no definite calculation that can be made to determine the thickness of the concrete walls. For a solid concrete wall the thickness ought never to be less than 6 inches, and for a large silo it should be at least 10 inches. When double walls are used, the inner one should be 5 inches thick and the outer wall 3 or 4 inches thick, with an air space of 4 inches between. The two walls are connected every four feet, making one solid wall.

Blocks for silo walls should be 8 or 10 inches in thickness. There should be a groove made in the top of the block for the reinforcing steel to set in. In this type of construction there is not enough cement mortar used in the joints to develop the strength of the steel reinforcement, and, therefore, the ends of the bars should be fastened together by clips or some other special device. The inside of the wall should be made perfectly smooth so that the silage can slide down with as little resistance from the wall as possible. The roofs can be made of reinforced concrete, but a wooden roof is much cheaper.

Example. Design a silo 16 feet in diameter and 32 feet high, using solid concrete walls.

Solution. For a silo of these dimensions the wall will be made 8 inches thick. The bursting pressure on the bottom foot of the wall will be one-half of the height multiplied by diameter multiplied by the pressure, which is $(31.5 \times 16 \times 20) \div 2 = 5{,}040$. The amount of steel required for the lower foot of the wall will be $5{,}040 \div 16{,}000 = .32$ square inch, which is equivalent to ½-inch round bars spaced 7½ inches on centers. These bars should be used for a height of 5 feet. For the next section of 5 feet the steel required would be $(26.5 \times 16 \times 20) \div 2 = 4{,}240 \div 16{,}000 = .26$ square inch, or round bars spaced 7½ inches on centers. This calculation is repeated, as shown in Fig. 53, but the steel area should never be less than ⅜-inch bars spaced 18 inches. No definite calculation can be made for the vertical bars. Bars ½ inch thick should be spaced 36 inches in the lower part and ⅜-inch bars in the upper part. The foundation under the wall should be made 2 feet wide and 18 inches deep and a 4-inch slab used for the floor.

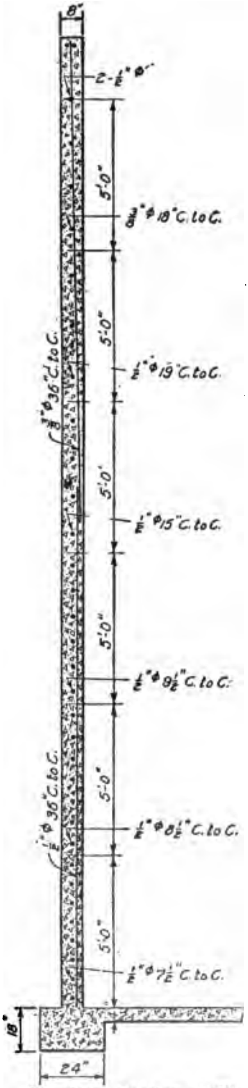


Fig. 53. Section of Silo

CONCRETE WALKS

Drainage of Foundations. The excavation should be made of a sufficient depth to get below the frost line. The ground should be tamped thoroughly, and the excavation filled with cinders, broken stone, gravel, or brickbat, up to within four inches (or whatever thickness of slab is to be used) of the top of the grade. The foundation should be thoroughly rammed, and by using gravel or cinders to make this foundation, a very firm surface can be secured. Side drains should be put in at convenient intervals where outlets can be secured. The foundation is sometimes omitted, even in cold climates, if the soil is porous. Walks laid on the natural soils have proved, in many cases, to be very satisfactory.

At the Convention of the National Cement Users' Association, held at Buffalo, New York, in 1908, the Committee on Sidewalks, Streets, and Floors presented the following specifications for sidewalk foundations:

The ground base shall be made as solid and permanent as possible. Where excavations or fills are made, all wood or other materials which will decompose shall be removed, and replaced with earth or other filling like the rest of the foundation. Fills of clay or other material which will settle after heavy rains or deep frost should be tamped, and laid in layers not more than six inches in thickness, so as to insure a solid embankment which will remain firm after the walk is laid. Embankments should not be less than 2½ feet wider than the walk which is to be laid. When porous materials, such as coal ashes, granulated slag, or gravel, are used, underdrains of tile should be laid to the curb drains or gutters, so as to prevent water accumulating and freezing under the walk and breaking the block.

Fig. 54. Square Tamper

Concrete Base. The concrete for the base of walks is usually composed of 1 part Portland cement, 3 parts sand, and 5 parts stone or gravel. Sometimes, however, a richer mixture is used, consisting of 1 part cement, 2 parts sand, and 4 parts broken stone; but this mixture seems to be richer than what is generally required. The concrete should be thoroughly mixed and rammed, Fig. 54. The broken stone or gravel should not be larger than 1 inch, varying down to ¼ inch, and free from fine screenings or soft stone. All stone or gravel under ⅛ inch is considered sand.

The thickness of the concrete base depends upon the location, the amount of travel, and the danger of being broken by frost. The usual thickness in residence districts is 3 inches, with a wearing thickness of 1 inch, making a total of 4 inches, Fig. 55.

In business sections, the walks vary from 4 to 6 inches in total thickness, in which the finishing coat should not be less than 1¼ inches.

The lines and grades given for walks by the engineer should be carefully followed. The mold strips should be firmly blocked and kept perfectly straight to the height of the grade given. The walks usually are laid with a slope of ¼ inch to the foot toward the curb.

The concrete base is cut into uniform blocks. The blocks are usually from 4 to 6 feet square, but sometimes they are made much larger. The joints made by cutting the concrete should be filled with dry sand, and their exact location marked on the

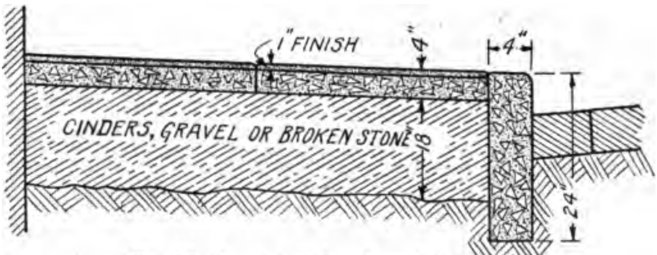


Fig. 55. Concrete Sidewalk and Curb

forms. The cleaver or spud that is used in making the joints should not be less than ⅛ of an inch or over ¼ of an inch in thickness.

Top Surface. The wearing surface usually consists of 1 part Portland cement and 2 parts crushed stone or good, coarse sand—all of which will pass through a ¼-inch mesh screen—thoroughly mixed so that a uniform color is secured. This mixture is then spread over the concrete base to a thickness of one inch, this being done before the concrete of the base has set or become covered with dust. The mortar is leveled off with a straightedge, and smoothed down with a float or trowel after the surface water has been absorbed. The exact time at which the surface should be floated depends upon the setting of the cement, and must be determined by the workmen; but the final floating is not usually performed until the mortar has been in place from two to five hours and is partially set. This final floating is done first with a wooden float, and afterwards with a metal float or trowel. The top surface is then cut directly over the cuts made in the base, care being taken to cut entirely through the top and base all around each block. The joint is then finished with a jointer, Fig. 56, and all edges rounded or beveled. Caution should be observed, in the final floating or finishing, not to overdo it, as too much working will draw the cement to the surface, leaving a thin layer of neat cement, which is likely to peel off. Just before the floating, a very thin layer of *dryer,* consisting of dry cement and sand mixed in the proportion of 1:1, or even richer, is not infrequently spread over the surface; but this is generally undesirable, as it tends to make a glossy walk. A dot roller or line roller, Figs. 57 and 58, may be employed to relieve the smoothness.

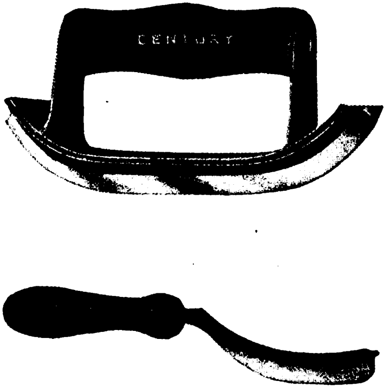


Fig. 56. Jointers

At the meeting of the National Cement Users' Association already referred to, the Committee on Sidewalks, Floors, and Streets recommended the following specifications for the top coat:

> Three parts high-grade Portland cement and five parts clean, sharp sand, mixed dry and screened through a No. 4 sieve. In the top coat, the amount of water used should be just enough so that the surface of the walk can be tamped, struck off, floated, and finished within 20 minutes after it is spread on the bottom coat; and, when finished, it should be solid and not quaky.

In the January, 1907, number of *Cement,* Mr. Albert Moyer, Assoc. M. Am. Soc. C. E., in discussing the subject of cement sidewalk pavements, gives specifications for monolithic slab for paving purposes, and as an example of this construction, he cites the pavement around the Astor Hotel, New York:

As an alternative, and instead of using a top coat, make one slab of selected aggregates for base and wearing surface, filling in between the frames concrete flush with established grade. Concrete to be of selected aggregates, all of which will pass through a ¾-inch mesh sieve; hard, tough stones or pebbles, graded in size; proportions to be 1 part cement, 2½ parts crushed hard stone screenings or coarse sand, all passing a ¾-inch mesh, and all collected on a ¼-inch mesh. Tamped to an even surface, prove surface with straightedge, smooth down with float or trowel; a natural finish can be obtained by scrubbing with a wire brush and water while concrete is "green," but after final set.

Seasoning. During the setting the wearing surface must be protected from the rays of the sun by a covering which is raised a few inches above the pavement so that it does not come in contact with the surface. After the pavement has set hard, sprinkle freely two or three times a day for a week or more.

Fig. 57. Brass Dot Roller

Cost. The cost of concrete sidewalks is variable. The construction at each location usually requires only a few days' work; but the time and expense of transporting the men, tools, and materials make an important item. One of the skilled workmen should be in charge of the men, so that the expense of a foreman will not be necessary. The amount of walk laid per day is limited by the amount of surface that can be floated and troweled in a day. If the surfacers do not work overtime, it will be necessary to stop concreting in the middle of the afternoon, so that the last concrete placed will be in condition for finishing during the regular working hours. The work of concreting may be continued considerably later in the afternoon if a drier concrete is used in mixing the top coat, and only enough water is used so that the surface can be floated and finished soon after being placed. The men who have been mixing, placing, and ramming concrete can complete their day's work by preparing and ramming the foundations for the next day's work.

Fig. 58. Brass Line Roller

The contract price for a well-constructed sidewalk 4 to 5 inches in thickness, with a granolithic finish, will vary from 15 cents to 30 cents per square foot.

CONCRETE CURB

The curb is usually built just in advance of the sidewalk. The foundation is prepared similarly to that of walks; the curb is divided into lengths similar to that of the walk; and the joints between the blocks, and also between the walk and the curb, are made similar to the joints between the blocks of the walk. The concrete is generally composed of 1 part Portland cement, 3 parts sand, and 5 parts stone, although a richer mixture is sometimes used. A facing of mortar or granolithic finish on the exposed part will improve the wearing qualities of the curb.

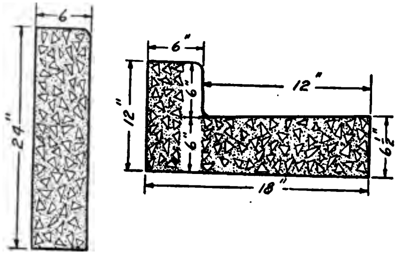


Fig. 59. Typical Curb Sections

Types. There are two general types of curb used—a curb rectangular in section, and a combined curb and gutter; the two types are shown in Fig. 59. The foundations for both are constructed alike. Both kinds of curb are made in place or are molded and set in place like stone curb, but the former method is preferable. A metal corner is sometimes laid in the exposed edge of the curb to protect it from wear.

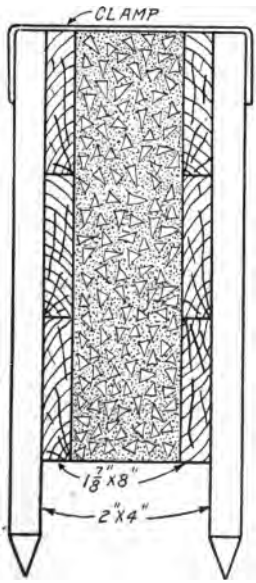


Fig. 60. Forms for Constructing Curb

Construction. The construction of the rectangular section is a simple process, but requires care. The section is usually about 7 inches wide and from 20 to 30 inches deep. After the foundation has been properly prepared, the forms are set in place. Fig. 60 shows the section of a curb 7 inches wide and 24 inches deep, and the forms as they are often used. The forms for the front and back both consist of three planks 1⅞ inches thick and 8 inches wide, and are surfaced on the side next the concrete. They are held in place at the bottom by the two 2- by 4-inch stakes, which at the top are kept from spreading by a clamp.

A sheet-iron plate ¼ inch thick is inserted every 6 feet, or at whatever distance the joints are made. After the concrete has been placed and rammed, and has set hard enough to support itself, the plate and front forms are removed, and the surface and top are finished smooth with a trowel, and with other tools such as shown in Figs. 61, 62, and 63. The joint is usually plastered over, and acts as an expansion joint. The forms on the back are not removed until the concrete is well set. If a mortar or granolithic finish is used, a piece of sheet iron is placed in the form one inch from the facing; the mortar is placed between the sheet iron and the front form, and the coarser concrete is placed back of the sheet iron, Fig. 64. The sheet iron is then withdrawn and the two concretes thoroughly tamped.

Fig. 61. Curb Edger

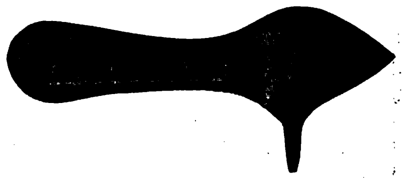


Fig. 62. Radius Tool

Fig. 63. Inside Angle Tool

Fig. 64 shows the section of a combined curb and gutter, and the forms that are necessary for its construction. This combination is often laid with fair results on a porous soil without any special foundation. A 1⅞-inch plank 12 inches wide is used for the back form, and is held in place at the bottom by pegs. The front form consists of a plank 1⅞ by 6 inches, and is held in place by pegs. Before the concrete is placed, two sheet-iron plates, cut as shown in the figure, are inserted in the forms, 6 feet to 8 feet apart. After the concrete for the gutter and the lower part of the curb is placed and rammed, a 1⅞-inch plank is fixed against these plates and held there by screw clamps, Fig. 64. The upper part of the curb is then molded. When the concrete is set sufficiently to stay in place, the front forms and plates are removed, and the surface is treated in the same manner as described for the other type of curb.

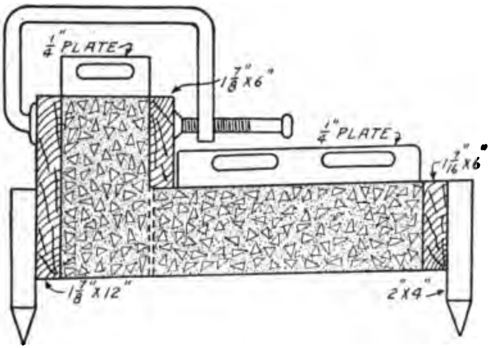


Fig. 64. Forms for Curb and Gutter

Cost. The cost of concrete curb will depend upon the conditions under which it is made. In ordinary circumstances, the contract price will be about 60 cents per lineal foot for rectangular curbing 6 inches wide and 24 inches deep; or 80 cents per lineal foot for curbing 8 inches wide and 24 inches deep. Under favorable conditions on large jobs, 6-inch curbing can be constructed for 40 cents or 45 cents per lineal foot. These prices include the excavation that is required below the street grade.

The cost of the combined curb and gutter is about 10 to 20 per cent more than that of the rectangular curbing. In addition to having a larger surface to finish, the combined curb and gutter requires more material, and therefore more work to construct it.

CONCRETE CONSTRUCTION WORK

MACHINERY FOR CONCRETE WORK

Concrete Plant. No general rule can be given for laying out a plant for concrete work. Every job is, generally, a problem by itself and requires a careful analysis to secure the most economical results. Since it is much easier and cheaper to handle the cement, sand, and stone before they are mixed, the mixing should be done as near the point of installation as possible. All facilities for handling the materials, charging the mixer, and distributing the concrete after it is mixed must be established and maintained. The charging and distributing are often done by wheelbarrows or carts, and economy of operation depends largely upon system and regularity of operation. Simple cycles of operations, the maintenance of proper runways, together with clocklike regularity, are necessary for economy. To shorten the distance of wheeling the concrete, it is very often found, on large buildings, better to have two medium-sized plants located some distance apart, than to have one large plant.

The design of a plant for handling the material and concrete and the selection of a mixer depend upon local conditions, the amount of concrete to be mixed per day, and the total amount required on the contract. It is evident that on large jobs it pays to invest a considerable sum in machinery to reduce the number of men and horses. Even on small jobs, where only 10 to 15 cubic yards are to be mixed daily, it is better to use a mixer than mix the concrete by hand. Mixers are now made in very small sizes, some having a capacity of only 6 or 8 cubic feet. Some of larger sizes have a capacity of 2 to 3 cubic yards.

Concrete Mixers

Types. The best concrete mixer is the one that turns out the maximum of thoroughly mixed concrete at the minimum of cost for power, interest, and maintenance. The type of mixer with a complicated motion gives better and quicker results than one with a simpler motion. There are two general classes of concrete mixers—*continuous* mixers and *batch* mixers. A continuous mixer is one into which the materials are fed constantly, and from which the concrete is discharged constantly. Batch mixers are constructed to receive the cement with its proportionate amount of sand and stone, all at one charge, to mix these ingredients, and then to discharge them in a mass. No

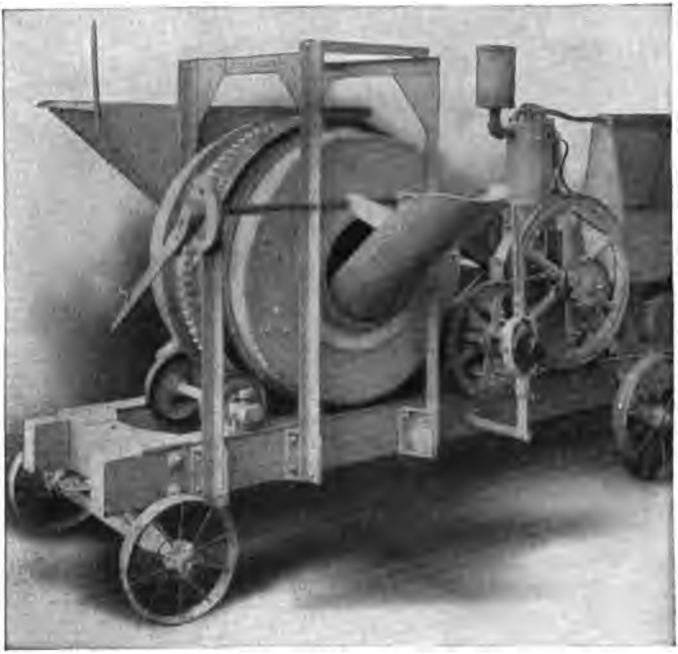


Fig. 65. Ransome Gasoline-Driven Concrete Mixing Outfit with Fixed Batch Hopper. Discharge Chute in Position for Mixing
Courtesy of Ransome Concrete Machinery Company, Chicago, Illinois

very distinct line can be drawn between the two classes, for many of these mixers are adapted to either continuous or batch mixing. Usually, batch mixers are preferred, as it is very difficult to feed the mixers uniformly unless the materials are measured.

Continuous Mixers. These usually consist of a long screw or pug mill that pushes the materials along a drum until they are discharged in a continuous stream of concrete. Where the mixers are fed with automatic measuring devices, the concrete is not regular, as there is no reciprocating motion.

Batch Mixers. Batch mixers differ somewhat in their details; but in general they have a drum, double cone, or a cubical box of steel which is usually fitted up inside with deflector blades. There is a great variety of these mixers on the market.

Fig. 65 represents a Ransome mixer, which is a batch mixer. The concrete is discharged after it is mixed, without tilting the body of the machine. The mixer revolves continuously, even while the concrete is being discharged. Riveted to the inside of the drum are a number of steel scoops or blades. These scoops pick up the material in the bottom of the mixer, and, as the latter revolves, carry the material upward until it slides out from them.

Sources of Power. *General Considerations.* One essential point that must always be considered is the source of power for operating the mixer, conveyors, hoists, derricks, and cableways. If it is possible to run the machinery by electricity, it is often economical to do so, but this will depend a great deal upon the local price for electricity.

Steam Engines. When all the machinery can be supplied with steam from one centrally located boiler, this arrangement will be found, perhaps, the most efficient. A vertical steam engine is generally used to operate the mixer. The smaller sizes of engines and mixers are mounted on the same frame; but on account of the weight it is necessary to mount the larger sizes on separate frames.

Upright tubular boilers are generally used to supply steam for concrete mixers and hoists operated by steam engines when they are isolated. For the smaller sizes of mixers, the boilers are mounted on the same frame as the engine and mixer.

Gasoline Engines. Gasoline engines are used to some extent to operate concrete mixers. Thus far they have been limited chiefly to portable plants such as are employed for street work. The fuel for the gasoline engine is much more easily moved from place to place than the fuel for a steam engine.

There are two types of gasoline engines—the *horizontal* and the *vertical*. The vertical engines occupy much less floor space for a given horsepower than the horizontal. Both are types of the engines commonly known as four-cycle engines. In the operation of them, four strokes of the piston are required to draw in a charge of fuel, compress and ignite it, and discharge the exhaust gases. The quantity of gasoline consumed in 10 hours is, on the average, about 1 gallon for each rated horsepower for any given size of engine. At 15 cents per gallon for gasoline, the hourly expense per horsepower will be 1.5 cents.

Hoisting and Transporting Equipment

General Types. When the concrete requires hoisting, this is done sometimes by the same engine that is used in mixing the concrete, but it is generally considered better practice on large buildings to have a separate engine to do the hoisting. If it is possible to use a standard hoist, it is usually economical to do so. These hoists are equipped with automatic dump buckets.

A standard double-cylinder, double-friction-drum hoisting engine is designed to fulfil the requirements of a general contractor for all classes of derrick work and hoisting. Steam can be supplied from a separate boiler, or from a boiler that supplies various engines with steam. The double-friction drums are independent of each other; therefore, if desired, one or two derricks can be handled at the same time. The hoist is fitted with ratchets and pawls, and winch heads attached to the end of each drum shaft. The winch heads can be used for any hoisting or hauling desired, independent of the drums.

Advantages of Electric Power. Very often the cycle of operation of a hoist is of an intermittent character. The power required is at a maximum only a part of the time, even though the hoist may be operated practically continuously. From an economical point of view, these conditions give the electric-motor-driven hoist special advantages, in that the electric hoist is always ready, but uses power only when in actual operation, and then only in proportion to the load handled. The ease with which a motor is moved, and the simplicity of the connection to the service supply—requiring only that two wires be connected—are also in favor of the electric motor.

Hoisting Lumber and Steel. In constructing large reinforced concrete buildings, usually a separate hoist is used to elevate the steel and the lumber for the forms. It may be equipped with either an electric motor or an engine, depending upon the general arrangement of the plant. These hoists are usually of the single-drum type.

Hoisting Concrete. In building construction, concrete is usually hoisted in automatic dumping buckets, one of which is illustrated in Fig. 66. The bucket is designed to slide up and down a light framework of timber, as shown in Fig. 67, and to dump automatically when it reaches the proper place. The dumping of the buckets is accomplished by the bucket's pitching forward at the point where the front guide in the hoisting tower is cut off; the bucket rights itself automatically as soon as it begins to descend. These buckets are often used for hoisting sand and stone as well as concrete. Their capacity varies from 10 cubic feet to 40 cubic feet.

Fig. 66. Ransome Hoist Buckets for Concrete
Courtesy of Ransome Concrete Machinery Company, Chicago, Illinois.

Charging Mixers. The mixers are usually charged by means of wheelbarrows, although other means are sometimes used. The capacity of the wheelbarrows varies from 2 cubic feet to 4 cubic feet, the former size being the more general one, though with good runways a man can handle 4 cubic feet of stone or sand in a well-constructed wheelbarrow.

In Fig. 68 is shown an automatic loading bucket which has been devised by the Koehring Machine Company for charging the mixers made by them. The bucket is operated by a friction clutch, and is provided with an automatic stop. Wheelbarrows may be used in charging the buckets, unless the materials are close to the mixer.

Transporting Mixed Concrete. Concrete is usually transported by wheelbarrows, carts, cars, or derricks, although other means are frequently employed. It is essential, in handling or transporting concrete, that care be taken to prevent the separation of the stone from the mortar. With a dry mixture, there is not so much danger of the stone separating as with a wet mixture. Owing to the difference in the time of setting of Portland cement and natural cement, the former can be conveyed much farther and with less danger of the initial setting taking place before the concrete is deposited.

Concrete Plant for Street Work. A self-propelling mixing and spreading machine has been found very desirable for laying concrete base for street pavements. A plant of this kind, which has been devised by the Municipal Engineering and Contracting Company, may be described as follows:

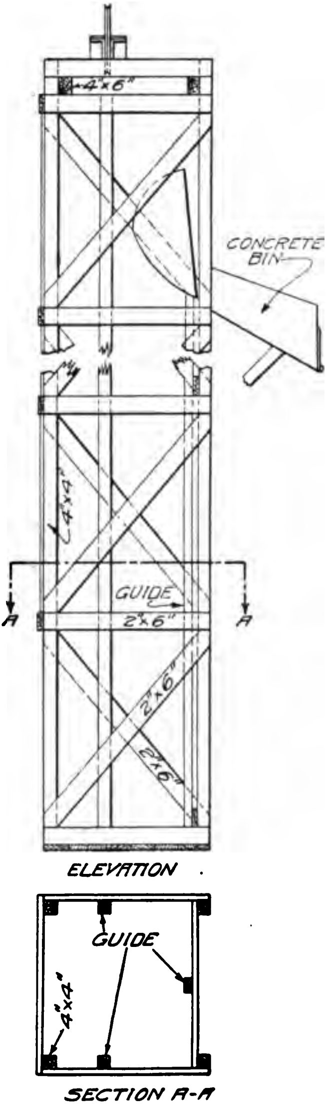


Fig. 67. Details of Hoisting Tower

The mixer is of the improved cube type, mounted on a heavy truck frame. The concrete is discharged into a specially designed bucket, which receives the whole batch and travels to the rear on a truck about 25 feet long. The head of the truck is supported by guys, and also by a pair of small wheels near the middle of the truck, which rest on the graded surface of the street. The truck or boom is pivoted at the end connected to the

Fig. 68. Koehring Steam-Driven Concrete Mixer with Side Loader and Water Measuring Tank
Courtesy of Koehring Machine Company, Milwaukee, Wisconsin

main truck, and has a horizontal swing of about 170 degrees, so that a street 50 feet wide is covered. An inclined track is also constructed, on which a bucket for elevating and charging the mixer is operated. The bucket is loaded while resting on the ground, with the proper ingredients for a batch, from the materials that have been distributed in piles along the street. The bucket is then pulled up the incline, and the contents dumped into the mixer. An automatic water-measuring supply tank, mounted on the upper part of the frame, insures a uniform amount of water for each batch mixed. Power for hoisting, mixing, and distributing the concrete, and propelling the machine is furnished by a 16-horsepower gasoline engine of the automobile type. The machine can be moved backward as well as forward, and is supplied with complete steering gear.

Machinery for Miscellaneous Operations

Concrete-Block Machines. There are two general types of hollow-concrete-block machines on the market—those with a *vertical* face and those with a *horizontal* face. In making blocks

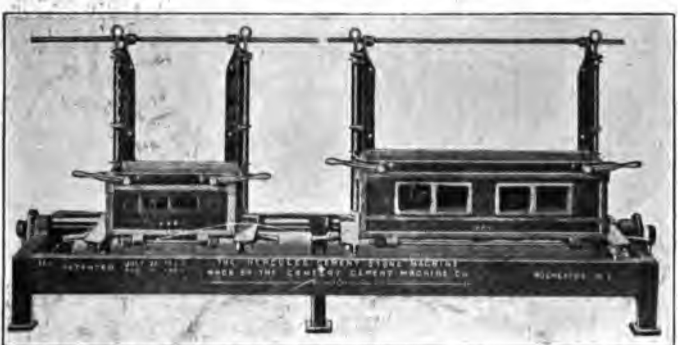


Fig. 69. Hercules Cement Stone Machine
Courtesy of Century Cement Company, Rochester, New York

with the vertical-faced machine, the face of the block is in a vertical position when molded, and the block is simply lifted from the machine on its base plate. In the horizontal-faced type of block machine, the block is made with the face down, the face-plate forming the bottom of the mold. The cores are withdrawn horizontally, or the mold is turned over and the core is taken out vertically; the block is then ready for removal. The principal difference in the two types of machine is that, if a special facing is desired on the block, it is more convenient to do that with a horizontal-faced machine. With the vertical-faced machine, the special facing is put on by the use of a parting-plate. When the parting-plate is removed, the two mixtures of concrete are bonded together by tamping the coarser material into the facing mixture.

Fig. 69 shows a Hercules machine. The foundation parts can be attached for making any length of block up to 6 feet. The illustration shows two molds of different lengths attached. These machines are constructed of iron and steel, except that the pallets (the plates on which the blocks are taken from the machine) may be either wood or steel. This type of machine is the horizontal, or face-down, machine.

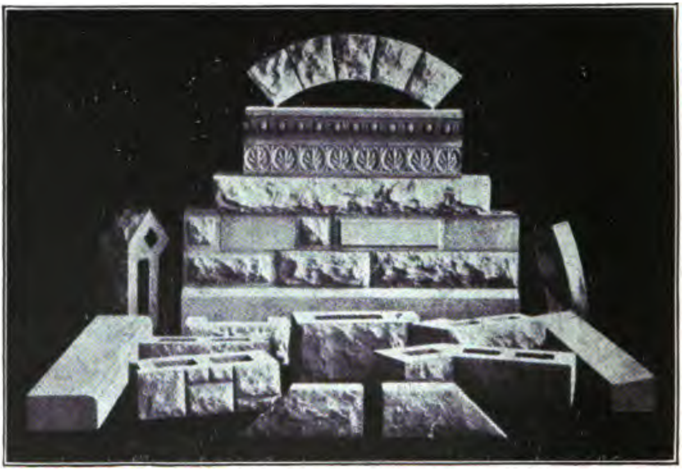


Fig. 70. Blocks Made on Hercules Machine

In Fig. 70 are shown a group of the various forms which may be made. The figure also illustrates the ornamental possibilities of concrete-block construction.

Fig. 71 pictures a Hobbs face-down, wet-process block machine. The front and sides of the machine can be let down, thus facilitating the removal of the blocks. The cores are shown withdrawn in the figure.

Cement-Brick Machines. Fig. 72 shows a machine for making cement brick. Ten bricks, $2\frac{5}{8}$ by $3\frac{7}{8}$ by 8 inches, are made at one operation. By using a machine in which the bricks are made on the side, a wetter mixture of concrete can be used than if they are made on the edge. The concrete usually consists of a mixture of 1 part Portland cement and 4 parts sand. The curing of these bricks is the same as that for concrete blocks. In making the bricks, a number of wood pallets are required, as the brick should not be removed from the pallet until the concrete has set and the bricks are ready to use.

Fig. 71. Hobbs Face-Down, Wet-Process Concrete Block Machine
Courtesy of Hobbs Concrete Machinery Company, Detroit, Michigan

Sand-Washing. Since dirty sand can be easily obtained while clean sand can be secured only at high cost, it sometimes becomes necessary to use dirty sand and to wash it. If only a small quantity is to be washed, it may be done with a hose. A trough should be built about 8 feet wide and 15 feet long, the bottom having a slope of about 19 inches in its entire length. The sides should be approximately 8 inches high at the lower end, and increase gradually to a height of perhaps 36 inches at the upper end. In the lower end of the trough there should be a gate about 6 inches high, sliding in guides so that it can be easily removed. The sand is placed in the upper end of the trough, and a stream of water is played on it. The sand and water flow down the trough, and the dirt passes over the gate with the overflow water. With a trough of the above dimensions, and a stream of water from a ¾-inch hose, 3 cubic yards of sand should be washed in an hour.

Concrete mixers are often used for washing sand. The sand is dumped into the mixer in the usual manner and the water is turned on. When the mixer is filled with water so that it overflows at the discharge end, the mixer is started. The revolving of the mixer enables the water to separate the dirt from the sand, and the dirt is carried off by the overflow of water. When the water runs clear, the washing is complete and the sand is dumped in the usual way. If large quantities of sand require washing special machinery for that purpose should be employed.

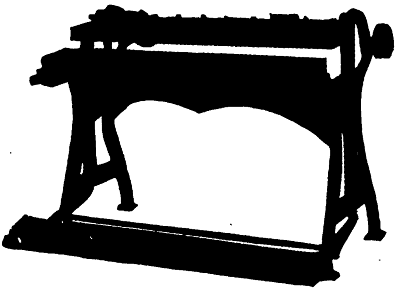


Fig. 72. Century Cement Brick Machine

FORMS

Building Forms

General Requirements. In actual construction work, the cost of forms is a large item of expense and offers the best field for the exercise of ingenuity. For economical work, the design should consist of a repetition of identical units; and the forms should be so devised as to require a minimum of nailing to hold them, and of labor to make and handle them. In constructing a factory building of two or three stories, usually the same set of forms is used for the different floors; but when the building is more than four stories high, two or more sets of forms are specified, so as always to have one set of forms ready to move.

Forms are constructed of the cheaper grades of lumber. To secure a smooth surface, the planks are planed on the side on which the concrete will be placed. Green lumber is preferable to dry, as it is less affected by wet concrete. If the surface of the planks that is placed next to the concrete is well oiled, the planks can be taken down much more easily, and, if kept from the sun, they can be used several times. Crude oil is an excellent and cheap material for greasing forms, and it can be applied with a whitewash brush. The forms should be oiled every time they are used. The object is to fill the pores of the wood rather than to cover it with a film of grease. Thin soft soap, or a paste made from soap and water, is also used.

The forms should be so tight as to prevent the water and thin mortar from running through and thus carrying off the cement. This is accomplished by means of tongued-and-grooved or beveled-edge boards, Fig. 73; but it is often possible to use square lumber, if that is wet thoroughly so as to swell it before the concrete is placed. The beveled-edge boards are often preferred to tongued-and-grooved boards, as the edges tend to crush as the boards swell, and beveling prevents buckling.


Fig. 73. Typical Form of Construction Showing Tongued-and-Grooved and Beveled-Edge Boards

Lumber for forms may be made of 1-inch, 1½-inch, or 2-inch plank. The spacing of studs depends in part upon the thickness of concrete to be supported, and in part upon the thickness of the boards on which the concrete is placed. The size of the studding depends upon the height of the wall and the amount of bracing used. Except in very heavy or high walls, 2- by 4-inch or 2- by 6-inch studs are used. For ordinary floors with 1-inch plank, the supports should be placed about 2 feet apart; with 1½-inch plank, 3 feet apart; and with 2-inch plank, 4 feet apart.

The length of time required for concrete to set depends upon the weather, the consistency of the concrete, and the strain which is to come on it. In good drying weather, and for very light work, it is often possible to remove the forms in 12 to 24 hours after placing the concrete, if there is no load placed on it. The setting of concrete is greatly retarded by cold or wet weather. Forms for concrete arches and beams must be left in place longer than forms in wall work, because of the tendency to fail by rupture across the arch or beam. In small, circular arches, like sewers, the forms may be removed in 18 to 24 hours,

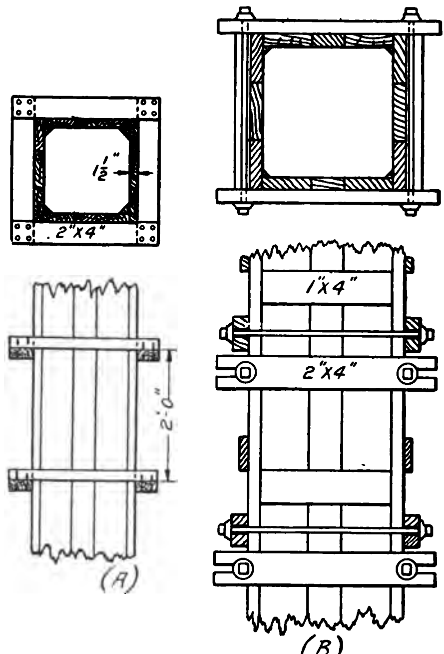


Fig. 74. Forms for Columns. (A) Common Method of Construction; (B) Method in Constructing Harvard University Stadium

if the concrete is mixed dry; but if wet concrete is used, in 24 to 48 hours. Forms for large arch culverts and arch bridges are seldom taken down in less than 28 days. The minimum time for the removal of forms should be:

For bottom of slabs and sides of beams and girders, 7 days; for bottom of beams and girders, 14 days; for columns, 4 days; for walls, not loaded, 1 to 2 days; for bridge arches, 28 days.

The concrete should be thoroughly examined before any forms are removed. Forms must be taken down in such a way as not to deface the structure or to disturb the remaining supports.

Forms for Columns. Column forms for buildings should be so constructed that they will support the ends of the girders and beam forms and also so that they can be taken down before either the girder or beam forms. A pocket should be left at the bottom so that they may be cleaned out before any concrete is poured.

Fig. 74-*A* shows the common way, or some modification of it, of constructing forms for columns. The plank may be 1 inch, 1½ inches, or 2 inches thick; and the cleats are usually 1 by 4 inches and 2 by 4 inches. The spacing of the cleats depends on the size of the columns and the thickness of the vertical plank.

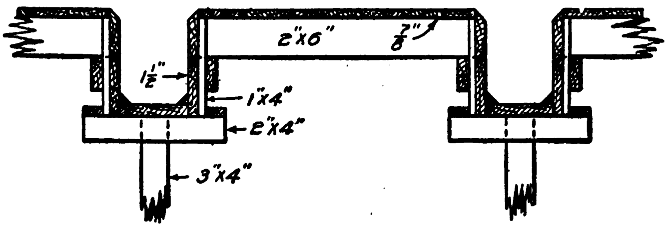


Fig. 75. Forms for Beams and Slabs

Fig. 74-*B* shows column forms similar to those used in constructing the Harvard University stadium. The planks forming each side of the column are fastened together by cleats, and then the four sides are fastened together by slotted cleats and steel tie-rods. These forms can be quickly and easily removed.

Round columns are often desired for the interior columns of buildings. For such columns it is generally cheaper and more satisfactory to use steel forms.

Forms for Beams and Slabs. A very common style of form for beam and slab construction is shown in Fig. 75. The size of the different members of the forms depends upon the size of the beams, the thickness of the slabs, and the relative spacing of some of the members. If the beam is 10 by 20 inches, and the slab is 4 inches thick, then 1-inch plank supported by 2- by 6-inch timbers spaced 2 feet apart will support the slab. The sides and bottom of the beams are enclosed by 1½-inch or 2-inch plank supported by 3- by 4-inch posts spaced 4 feet apart.

Forms for fireproofing I-beams and supporting a reinforced concrete slab are generally made the same as those for reinforced concrete slabs, beams, and girders, except that the forms are suspended from the structural steel work instead of being supported on posts, Fig. 76.

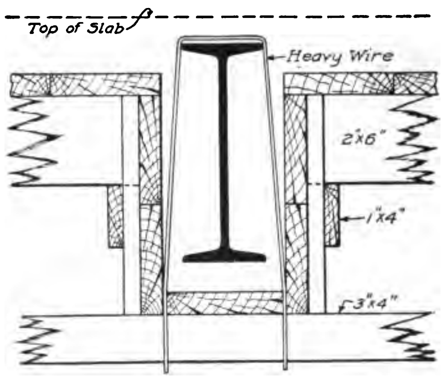


Fig. 76. Forms for Fireproofing I-Beams and Supporting Concrete Slab

Cost of Forms. There are several items that enter into the cost of form work, the principal ones being the cost of the lumber, the number of times that it can be used, and the cost of labor for making, erecting, taking down the forms, and rebuilding them in another location.

For slab forms on I-beams about 1½ feet of lumber are required per square foot of floor. The total cost per square foot of floor for forms, under ordinary conditions, will vary from 8 cents to 11 cents.

For typical reinforced concrete buildings with slabs, beams, girders, and columns, the cost for forms will vary from 9 cents to 12 cents per square foot of surface to be covered. These surfaces include bottom of slab, sides and bottoms of all beams and girders, and sides of all columns.

Forms for Sewers and Walls

Forms for Conduits and Sewers. Forms for conduits and sewers must be strong enough not to give way, or to become deformed, while the concrete is being placed and rammed; and they must be rigid enough not to warp from being alternately wet and dry. They must be constructed so that they can readily be put up and taken down, and can be used several times on the same job. The interior of the sewer or conduit must have a smooth and even finish. This has usually been done by covering the forms with light-weight sheet iron.

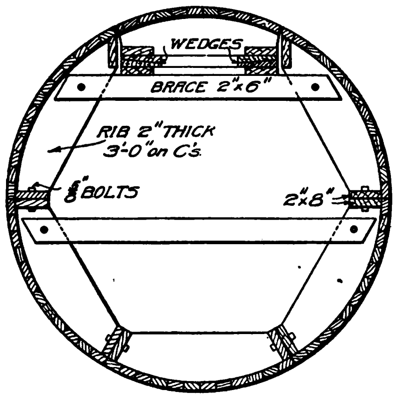


Fig. 77. Center for Round Sewer

These forms are usually built in lengths of 16 feet, with one center at each end, and with three to five—depending on the size of the sewer or conduit—intermediate centers in the lengths of 15 feet. The planks of these forms are made of 2- by 4-inch material, surfaced on the outer side, with the edge beveled to the radius of the conduit. The ribs are bolted together, and are held by wood ties 2 by 4 or 2 by 6 inches.

Forms of Torresdale Filters. In constructing the Torresdale filters for supplying Philadelphia with water, several large sewers and conduits were built of concrete and reinforced with expanded metal. In section, the sewers were round and the conduits were horseshoe-shaped, with a comparatively flat bottom. The sewers were 6 feet and 8 feet 6 inches, respectively, in diameter, and the forms were constructed similarly to the forms shown in Fig. 77, except that at the bottom the lower side ribs were connected to the bottom rib by a horizontal joint, and the spacing of the ribs was 2 feet 6 inches, center to center. Fig. 78 shows the form for the 7-foot 6-inch conduit. The centering for the 9-foot and 10-foot conduits was constructed similarly to the 7-foot 6-inch conduit, except that the ribs were divided into 7 parts instead of 5 parts as shown in Fig. 78. The spacing of the braces depended on the thickness of the lagging. For lagging 1 inch by 2½ inches, the braces were spaced 18 inches, center to center; and for 2- by 3-inch lagging, the spacing of the bracing was 2 feet 6 inches.

These forms were constructed in lengths of 8 feet. The lagging for the smaller sizes of the conduits was 1 inch by 2½ inches, and for the larger sizes 2 by 3 inches; all of this was made of dressed lumber and covered with No. 27 galvanized sheet iron. The bracing of the forms was arranged to permit the centering to be taken apart and brought forward through the sections in front. Three sets of these forms were required for each conduit. The specifications required that the centering be left in place for at least 60 hours after the concrete had been placed. It was also required that this work should be monolithic—that is, the contractor could build as long a section as he could finish in a day, and that the sections should be securely keyed together.

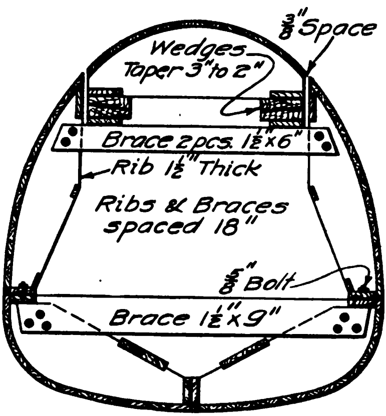


Fig. 78. Form for Construction of Horseshoe-Shaped Conduit

Forms for Walls. The forms for concrete walls should be built strong enough to insure their retaining their correct position while the concrete is being placed and rammed. In high, thin walls, a great deal of care is required to keep the forms in place so that the wall will be true and straight.

Fig. 79 shows a very common method of constructing these forms. The plank against which the concrete is placed is seldom less than 1½ inches thick, and is usually 2 inches thick. One-inch plank is sometimes used for very thin walls, but the supports must be placed close. The planks are generally surfaced on the side against which the concrete is placed. The vertical timbers that hold the planks in place will vary in size from 2 by 4 inches to 4 by 6 inches, or will be even larger, depending on the thickness of the wall, the spacing of these vertical timbers, etc. The vertical timbers are always placed in pairs, and are usually held in place by means of heavy wires.

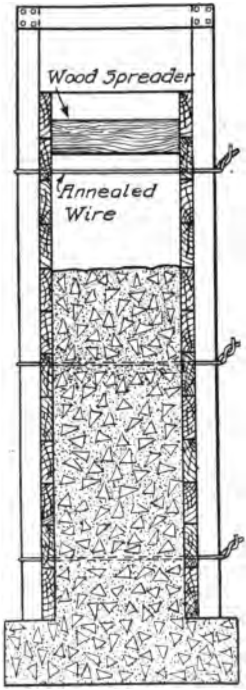


Fig. 79. Typical Wall Form

Forms for Centers of Arches

General Specifications. The centers for stone, plain concrete, and reinforced concrete arches are similar in construction. A reinforced concrete arch of the same span and designed for the same loading will not be so heavy as a plain concrete or stone arch, and the centers need not be constructed so strong as for the other types of arches. One essential difference in the centering for stone arches and that for concrete or reinforced concrete arches is that centering for the latter types serves as a mold for shaping the soffit of the arch ring, the face of the arch ring, and the spandrel walls.

The successful construction of arches depends nearly as much on the centers and their supports as it does on the design of the arch. The centers should be as well constructed and the supports as unyielding as it is possible to make them. When it is necessary to use piles, they should be as well driven as permanent foundation piles, and the load, in most cases, should not be heavier than that on permanent piles.

Classes of Centers. There are two general classes of centers—those which act as a truss, and those in which the support, at the intersection of braces, rests on a pile or footing. Trusses are used when it is necessary to span a stream or roadway. Sometimes the length of the span for the centering is very short, or there is a series of short spans, or the span may be equal to that of the arch. The trusses must be carefully designed, in order that the deflection and deformation due to the changes in the loading will be reduced to a minimum. By placing a temporary load on the centers at the crown, the deformation during construction may be very greatly reduced. This load is removed as the weight of the arches comes on the centers. (For the design of trusses, the reader is referred to the Instruction Papers, or other treatises, on *Bridge Engineering* and on *Roof Trusses.*)

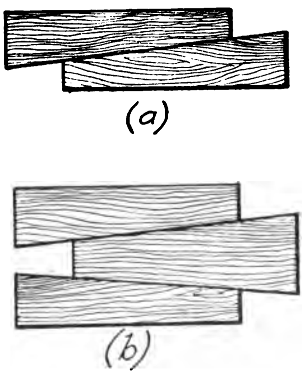


Fig. 80. Wedges Used in Placing and Removing Forms

The lagging for concrete arches usually consists of 2- by 3-inch or 2- by 4-inch plank, either set on edge or laid flat, depending on the thickness of the arch and the spacing of the supports. The side on which the concrete is laid is generally surfaced. The lagging is often supported on ribs constructed of 2- by 12-inch plank, on the back of which is placed a 2-inch plank cut to a curve parallel with the intrados. These 2- by 12-inch planks are set on the timber used to cap the piles, and are usually spaced about 2 feet apart. All the supports should be well braced. The centers should be constructed to give a camber to the arch about equal to the deflection of the arch when under full load. It is, therefore, necessary to make an allowance for the settlement of centering, for the deflection of the arch after the removal of the centering, and for permanent camber.

The centers should be constructed so that they can easily be taken down. To facilitate the striking of centers, the practice is to support them on folding wedges or sand boxes. When the latter method is used, the sand should be fine, clean, and perfectly dry, and the boxes should be sealed around the plunger with cement mortar. Striking forms by means of wedges is the commoner method. The type of wedge generally used is

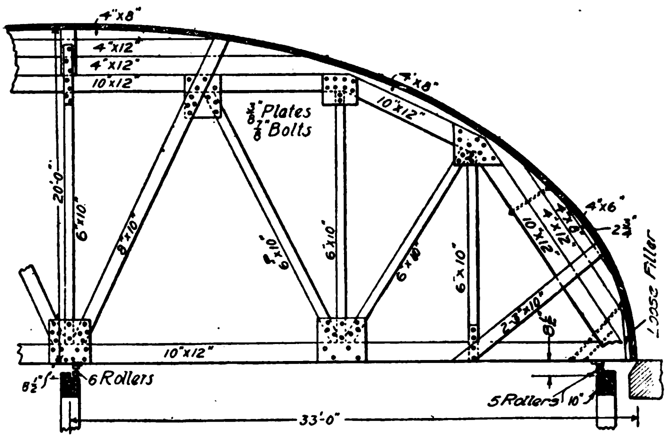


Fig. 81. Typical Arch Form Used at 175th Street, New York City

shown in Fig. 80-*a*, although sometimes three wedges are used, as shown by Fig. 80-*b*. They are from 1 foot to 2 feet long, 6 to 8 inches wide, and have a slope of from 1:6 to 1:10. The centering is lowered by driving back the wedges; and to do this slowly, it is necessary that the wedges have a very slight taper. All wedges should be driven equally when the centering is being lowered. The wedges should be made of hardwood, and are placed on top of the vertical supports or on timbers which rest on the supports. They should be placed at about the same elevation as the springing line of the arch.

Forms for Arch at 175th Street, New York City. In building the 175th Street Arch in New York City, the forms were so built that they could be easily moved. The arch is elliptical and is built of hard-burned brick and faced with granite. The span of the arch is 66 feet; the rise is 20 feet; the thickness of the arch ring is 40 inches and 48 inches, at the crown and the springing line, respectively; and the arch is built on a 9-degree skew. The total length is 800 feet.

The arch is constructed in sections, the centering being supported on 11 trusses placed perpendicular to the axis of the arch and having the form and dimensions shown in Fig. 81. The trusses are placed 5 feet on centers, and are supported at the ends and middle by three lines of 12- by 12-inch yellow-pine caps. The caps are supported by 12- by 12-inch posts, spaced 5 feet center to center, and rest on timber sills on concrete foundations. The upper and lower chord members of the trusses are of long-leaf yellow pine, but the diagonals and verticals are of short-leaf yellow pine. The lagging is 2¾- by 6-inch long-leaf yellow-pine plank. The connections of the timbers are made by means of ⅜-inch steel plates and ⅞-inch bolts, arranged as shown in the illustration. As it was absolutely necessary to have the forms alike, so that they could be moved along the arch and would at all times fit the brickwork, they were built on the ground from the same pattern, and hoisted to their places by two guyed derricks with 70-foot booms.

On the 12- by 12-inch cap was a 3- by 8-inch timber, on which the double wedges were placed. When it was necessary to move the forms, the wedges were removed, the forms rested on the rollers, and there was then a clearance of about 2¼ inches between the brickwork and the lagging. The timber on which the rollers ran was faced with a steel plate ¼ inch by 4 inches in dimensions. The forms were moved forward by means of the derricks. The settlement of the forms under the first section constructed was ¼ inch; and the settlement of the arch ring of that section, after the removal of forms, was ¼ inch.*

Forms for Bridge at Canal Dover, Ohio.† The details of the centering used in erecting one of the spans of a reinforced

* *Engineering Record*, October 5, 1907.
† *Ibid.*, February 9, 1907.

concrete bridge over the Tuscarawas River at Canal Dover, Ohio, are shown in Figs. 82 and 83. This span was 106 feet and 8 inches long; there were two other spans of the same length in the bridge, and a canal span of 70 feet. The centering for the canal span was built in 6 bents, each bent having 7 piles. A clear waterway of 18 feet was required in the canal span by the state canal commissioner, and this passage was arranged under the center of the arch. The piles were driven by means of a scow. The cap for the piles was a 3- by 12-inch timber. Planks 2 inches thick were sawed to the correct curvature, and nailed to the 2- by 12-inch joists, which were spaced about 12 inches apart. The lagging was 1 inch thick, and was nailed to the curved plank. The wedges were made and used as shown.

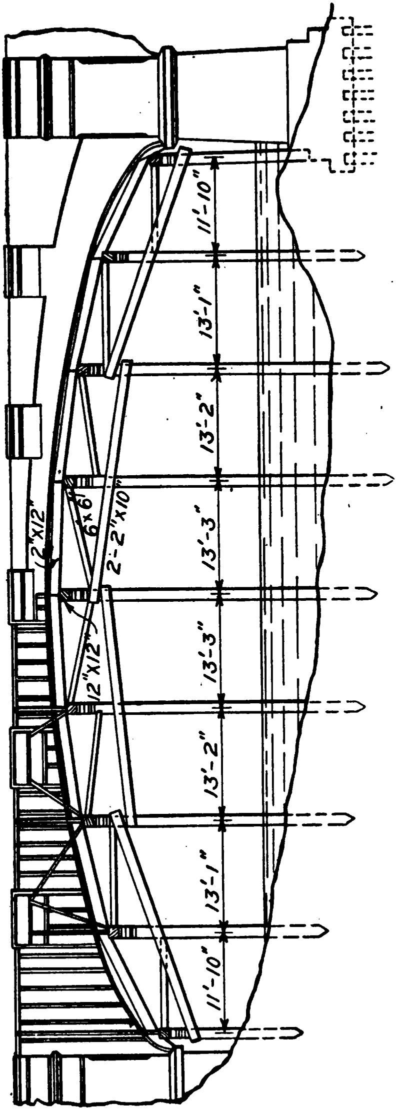


Fig. 82. Centers for Bridge at Canal Dover, Ohio

The centering was constantly checked; this was found important after a strong wind. The centering for the other two of the main arches was constructed as in the arch shown.

After some difficulty had been experienced in keeping the forms in place during the concreting of the first arch, the con-

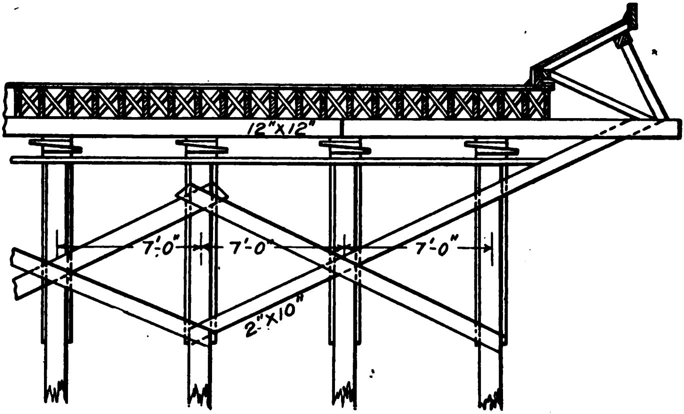


Fig. 83. Centers for Bridge at Canal Dover, Ohio

crete for the other arches was placed in the order shown in Fig. 84, and no other difficulty was encountered. Sections 1 and 1 were first placed, then 2 and 2, etc., section 6 being the last.

The concreting on the canal span was begun in the late fall, and finished in 12 days; the forms were lowered by means of the wedges five weeks later. The deflection at the crown was 0.5

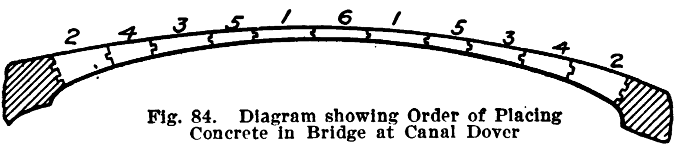


Fig. 84. Diagram showing Order of Placing Concrete in Bridge at Canal Dover

inch, and after the spandrel walls were built and the fill made, there was an additional deflection of 0.4 inch. In building the forms, an allowance of $\frac{1}{800}$ part of the span was made, to allow for this deflection. The deflections at the crown of the other three arches were 0.6 inch, 1.45 inches, and 1.34 inches.

FINISHING SURFACES OF CONCRETE

Imperfections. To give a satisfactory finish to exposed surfaces of concrete is a rather difficult problem. In many instances, when the forms are taken down, the surface shows the joints, knots, and grain of the wood; it has more the appearance of a piece of rough carpentry work than of finished masonry. Moreover, failure to tamp or flat-spade the surfaces next to the forms will result in rough places or *stone pockets.* Lack of homogeneity in the concrete will cause a variation in the surface texture. Diversity of color, or discoloration, is one of the most common imperfections. Old concrete adhering to the forms will

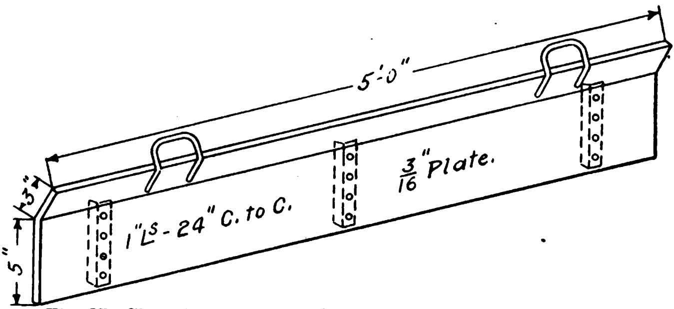


Fig. 85. Sheet-Iron Plate for Giving Finish Surface to Concrete

leave pits in the surface, and the pulling-off of the concrete in spots, as a result of its adhering to the forms when they are removed, will cause a roughness.

To guard against these imperfections, the forms must be well constructed of dressed lumber, and the pores should be carefully filled with soap or paraffin. The concrete should be thoroughly mixed, and, when placed, care should be taken to compact it thoroughly, next to the forms. Differences in color are usually due to the leaching-out of lime, which is deposited in the form of an efflorescence on the surface; or to the use of different cements in adjacent parts of the same work. Variation due to the latter cause can almost always be avoided by using the same brand of cement on the entire work. (The matter of efflorescence is treated later.)

Plastering. Plastering is not usually satisfactory, although there are cases where a mixture of equal parts of cement and sand has, apparently, been successful, and, when finished rough, it did not show any cracks. It is generally considered impossible to apply mortar in thin layers to a concrete surface, and make it adhere for any length of time. When the plastering begins to scale off, the concrete looks worse than with an unfinished surface. This paragraph is intended more as a warning against this manner of finishing concrete surfaces than as a description of it as an approved method of finish.

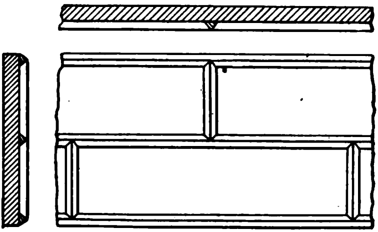


Fig. 86. Diagram Showing Method of Giving Masonry Facing to Concrete

Mortar Facing. A method of placing mortar facing that has been found very satisfactory, and has been adopted extensively in the last few years, is as follows: A sheet-iron plate, 6 or 8 inches wide and about 5 or 6 feet long, has riveted across it on one side, every two feet or so, angles of ¾-inch size, or of such other size as may be necessary to give the desired thickness of mortar facing, Fig. 85. In operation, the ribs of the angles are placed against the forms, and the space between the plate and forms is filled with mortar, mixed in small batches and thoroughly tamped. The concrete back filling is then placed, the mold is withdrawn, and the facing and back filling are rammed together. The mortar facing is mixed in the proportion of 1 part cement, to 1, 2, or 3 parts sand; usually a 1:3 mixture is employed, mixed wet and in small batches as it is needed. As mortar facing shows the roughness

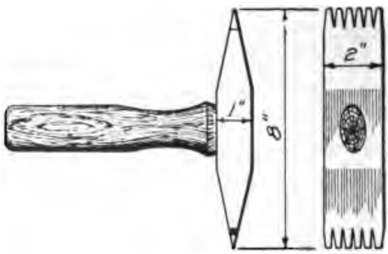


Fig. 87. Typical Facing Hammer

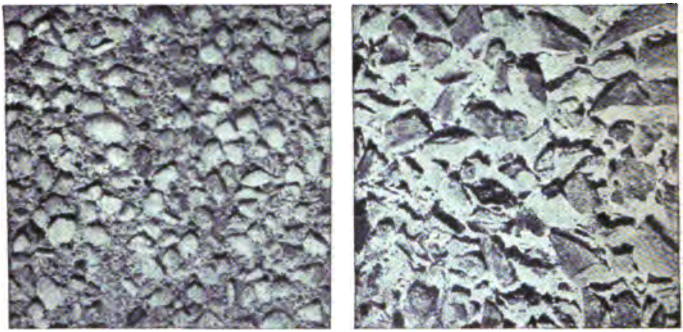
of the forms more readily than concrete does, care is required, in constructing, to secure a smooth finish. When the forms are

Fig. 88. Power-Driven Hand Tool for Surfacing Concrete
Courtesy of "Scientific American"

removed, the face may be treated either by washing or by tool dressing as described in the succeeding paragraphs.

Masonry Facing. Concrete surfaces may be finished to represent ashlar masonry. The process is similar to stone dressing, and any of the forms of finish employed for cut stone can be used for concrete. Very often, when the surface is finished to represent ashlar masonry, vertical and horizontal three-sided pieces of wood are fastened to the forms to make **V**-shaped depressions in the concrete, as shown in Fig. 86.

Hammer Dressing. In constructing the Harvard University stadium, care was taken, after the concrete was placed in the forms, to force the stones back from the face and permit the mortar to cover every stone. When the forms were removed, the surface was picked with the tool shown in Fig. 87. A pneumatic tool has also been devised for this purpose.



(*a*) (*b*)

Fig. 89. Quimby's Finish on Concrete Surfaces. (*a*) Aggregate $\frac{3}{16}$-Inch White Pebbles; (b) Aggregate $\frac{3}{8}$-Inch Screened Stone

The number of square feet to be picked per day depends on the hardness of the concrete. If the picking is performed by hand, it is done by a common laborer, and he is expected to average 50 square feet per day of 10 hours. With a pneumatic tool, a man would cover from 400 to 500 square feet a day.

Recently a motor-driven hand tool, Fig. 88, has been invented. It is driven through a flexible shaft by a motor carried by the operator. Its weight, including the weight of the motor, is only about 20 pounds. The motor may take its actuating current from an ordinary light socket. The concrete is cut by the teeth of a number of wheels revolving at high speed. This machine will dress 700 to 900 square feet a day.

Granolithic Finish. Several concrete bridges in Philadelphia have been finished according to the following specifications and their appearance is very satisfactory:

Granolithic surfacing, where required, shall be composed of 1 part cement, 2 parts coarse sand or gravel, and 2 parts granolithic grit, made into a stiff mortar. Granolithic grit shall be granite or trap rock, crushed to pass a ¼-inch sieve, and screened of dust. For vertical surfaces, the mixture shall be deposited against the face forms to a minimum thickness of 1 inch, by skilled workmen, as the placing of the concrete proceeds; and it thus forms a part of the body of the work. Care must be taken to prevent the occurrence of air space or voids in the surface. The face shall be removed as soon as the concrete has sufficiently hardened; and any voids that may appear shall be filled with the mixture. The surface shall then be immediately washed with water until the grit is exposed and rinsed clean, and shall be protected from the sun and kept moist for three days. For bridge-seat courses and other horizontal surfaces, the granolithic mixture shall be deposited on the concrete to a thickness of at least 1½ inches, immediately after the concrete has been tamped and before it has set, and shall be troweled to an even surface, and, after it has set sufficiently hard, shall be washed until the grit is exposed.

The success of this method depends greatly on the removal of the forms at the proper time. In general, the washing is done the day following that on which the concrete is deposited. The fresh concrete is scrubbed with an ordinary scrubbing brush, removing the film and the impressions of the forms, and exposing the sand and stone of the concrete. If this is done when the material is at the proper degree of hardness, a few rubs of an ordinary house scrubbing brush, with a free flow of water to cut and to rinse clean, are all the work required. The cost of scrubbing is small if done at the right time. A laborer will wash 100 square feet in an hour; but if that same area is permitted to get hard, it may require two men a day, with wire brushes, to secure the desired results. The practicability of removing the forms at the proper time for such treatment depends upon the character of the structure and the conditions under which the work must be done. This method is applicable to vertical walls, but it would not be applicable to the soffit of an arch, Fig. 89.

The Acid Treatment. This process, which has been very successfully used, consists in washing the surface of the concrete with diluted acid, then with an alkaline solution. The diluted acid is applied first, to remove the cement and expose the sand and stone; the alkaline solution is then applied to remove all of the free acid; and, finally, the surface is washed with clear water. The treatment may be applied at any time after the forms are removed; it is simple and effective. Limestone cannot be used in the concrete for any surfaces that are to have this treatment, as the limestone would be affected by the acid.

Fig. 90. Typical Molded Concrete Baluster

Dry Mortar Finish. The dry mortar method consists in using a dry, rich mixture with finely crushed stone. The concrete is usually composed of 1 part cement, 3 parts sand, and 3 parts crushed stone known as the $\frac{1}{4}$-inch size, mixed dry so that no mortar will flush to the surface when well rammed in the forms. The concrete, when placed, is not spaded next to the forms and, since it is dry, there is no smooth mortar surface, but there should be an even-grained, rough surface. With the dry mixture, the imprint of the joints of the forms is hardly noticed, and the grain of the wood is not seen at all. This style of finish has been extensively used in the South Park system of Chicago, and there has been no efflorescence apparent on the surface, which is explained by "the dryness of the mix and the porosity of the surface".

Cast-Slab Veneer. Cast-concrete-slab veneer can be made of any desired thickness or size. It is set in place like stone veneer, with the remainder of the concrete forming the backing. It is usually cast in wood molds, face down. A layer of mortar, 1 part cement, 1 part sand, and 2 or 3 parts fine stone or coarse sand, is placed in the mold to a depth of about 1 inch, and then the mold is filled up with a 1:2:4 concrete. Any steel reinforcement that is desired may be placed in the concrete. Usually, cast-concrete-slab veneer is cheaper than concrete facing cast in place, and gives a better surface finish.

Moldings and Ornamental Shapes. Concrete is now in demand in ornamental shapes for buildings and bridges. The shapes may be either constructed in place, or molded in sections and placed the same as cut stone. Plain cornices or panels are usually constructed in place, but complicated molding or balusters, Fig. 90, are frequently made in sections and erected in separate pieces.

Colors for Concrete Finish. Coloring matter has not been used very extensively with concrete, except in ornamental work. It has not been very definitely determined what coloring matters are detrimental to concrete. Lampblack (boneblack) has been used more than any other coloring matter. It gives different shades of gray, depending on the amount used. Common lampblack and Venetian red should not be used, as they are likely to run or fade. Dry mineral colors, mixed in proportions of 2 to 10 per cent of the cement, give satisfactory results. Red lead should never be used; even 1 per cent is injurious to the concrete. Variations in the color of cement and in the character of the sand used will affect the results obtained in using coloring matter.

Painting Concrete Surfaces. Special paints are made for painting concrete surfaces, since ordinary paints, as a rule, are not satisfactory. Before the paint is applied, the surface of the wall should be washed with dilute sulphuric acid, 1 part acid to 100 parts water.

Finish for Floors. Floors in manufacturing buildings are often finished with a 1-inch coat of cement and sand, mixed in the proportions of 1 part cement to 1 part sand; or 1 part cement to 2 parts sand. This finishing coat must be put on before the concrete base sets, or it will break up and shell off, unless made very thick—from 1½ to 2 inches. A more satisfactory method of finishing such floors is to put 2 inches of cinder concrete on the concrete base, and then put the finishing coat on the cinder concrete. The finish coat and cinder concrete bond together, making a thickness of 3 inches. The cinder concrete may consist of a mixture of 1 part cement, 2 parts sand, and 6 parts cinders, and may be put down at any time—that is, this method of finishing a floor can be used as satisfactorily on an old concrete floor as on one just constructed.

In office buildings, and generally in factory buildings, a wood floor is laid over the concrete. Wood stringers are first laid on the concrete, about 1 foot or 1½ feet apart. The stringers are 2 inches thick and 3 inches wide on top, with sloping edges. The space between the stringers is filled with cinder concrete, as shown in Fig. 91; as a rule this is mixed 1:3:6. When the concrete has set the flooring is nailed to the stringers.

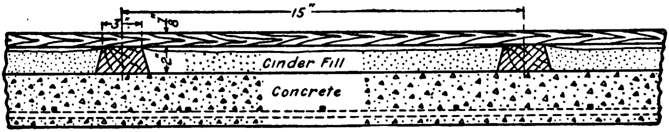


Fig. 91. Diagram Showing Typical Cinder Fill between Stringers

Efflorescence. The white deposit found on the surface of concrete, brick, and stone masonry is called efflorescence. It is caused by the leaching of certain lime compounds, which are deposited on the surface by the evaporation of the water, and this is due, primarily, it is believed, to the variation in the amount of water used in mixing the mortar. An excess of water will cause a segregation of the coarse and fine materials, resulting in a difference of color. In a very wet mixture more lime is set free from the cement and brought to the surface. If great attention is given to the amount of water, and care is taken to prevent the separation of the stone from the mortar when deposited, the concrete will present a fairly uniform color when the forms are removed. The greater danger of the efflorescence at joints than at any other point demands special caution. If the work is to be continued within 24 hours, and care is taken to scrape and remove the laitance, and then if before the next layer is deposited, the scraped surface is coated with a thin cement mortar, the joint should be impervious to moisture, and no trouble with efflorescence should be experienced.

A very successful method of removing efflorescence from a concrete surface consists in applying a wash of dilute hydrochloric acid. The wash consists of 1 part acid to 5 parts water, and is applied with scrubbing brushes. Water is kept constantly playing on the work, by means of a hose, to prevent the penetration of the acid. The cleaning is very satisfactory, and for plain surfaces costs about 20 cents per square yard.

Laitance. Laitance is whitish, spongy material that is washed out of the concrete when it is deposited in water. Before settling on the concrete, it gives the water a milky appearance. It is a semifluid mass, composed of a very fine, flocculent matter in the cement; it generally contains hydrate of lime, stays in a semifluid state for a long time, and acquires very little hardness at its best. Laitance interferes with the bonding of the layers of concrete, and should always be thoroughly cleaned from the surface before another layer of concrete is placed.

BENDING OR TRUSSING BARS

Bending Details. Drawings showing all the bending details of the bars for all reinforced concrete work should be made before the steel is ordered. The designing engineer should detail a few of the typical beams and girders to show, in a general way, what length of bars will be required, the number of turned-up

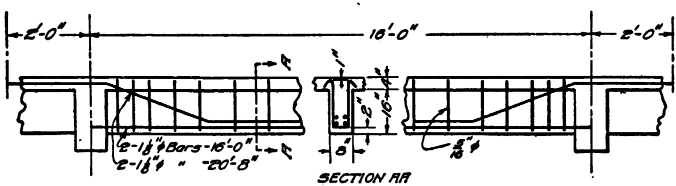


Fig. 92. Details of Beam Construction

bars, the number, size, and spacing of stirrups, and the dimensions of the concrete. This information will be a guide for the construction engineer in making up the details required properly to construct the work. Fig. 92 shows the manner in which the designing engineer should detail a typical beam so that the constructing engineer can develop these details as shown in Fig. 93.

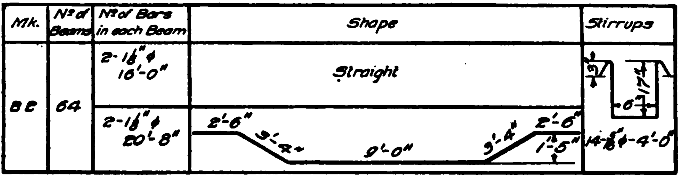


Fig. 93. Bending Details for Beams

Tables for Bending Bars. A simple outfit for bending the bars cold consists of a strong table, the top of which is constructed as shown in Fig. 94. The outline to which the bar is to be bent is laid out on the table, and holes are bored at the points where the bends are to be made. Steel plugs 5 to 6 inches long are then placed in these holes. Short pieces of boards are nailed to the table where necessary, to hold the bar in place while being

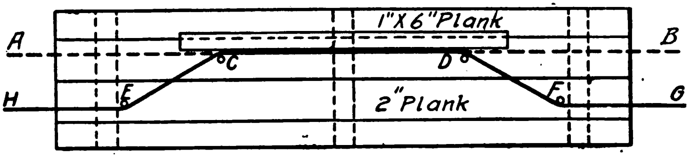


Fig. 94. Plan of Bending Table

bent. The bar is then placed in the position *A-B,* Fig. 94, and bent around the plugs *C* and *D,* and then around the plugs *E* and *F,* until the ends *E H* and *F G* are parallel to *A B.* When bends with short radii are required, the bars are placed in the vise, near the point where the bend is to be made, and the end of the

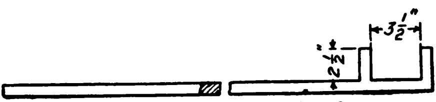


Fig. 95. Type of Lever Bender

bar is pulled around until the required angle is secured. The vise is usually fastened to the table. The lever shown in Fig. 95 is also used in making bends of short radii. This is done by placing the bar between the prongs of the lever and pulling the end of the lever around until the required shape of the bar is obtained.

Bars with Hooked Ends. When plain bars are used for reinforced concrete, architects and engineers very often require that the ends of all the bars in the beams and girders shall be

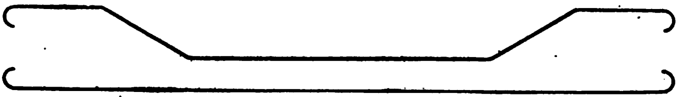


Fig. 96. Bars with Hooked Ends

hooked, as shown in Fig. 96. This is done to prevent the bars from slipping before their tensile strength is fully developed.

Slab Bars. To secure the advantage of a continuous slab, it is very often required that a percentage of the slab bars, usually one-half, shall be turned up over each beam. Construction companies have different methods of bending and holding these bars in place; but the method shown in Fig. 97 will insure good results, as the slab bars are well supported by the two longitudinal bars which are wired to the tops of the stirrups. Fig. 98 shows the bending details of slab bars, the beams being spaced

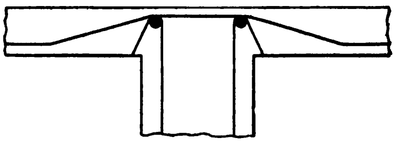


Fig. 97. Slab Bars

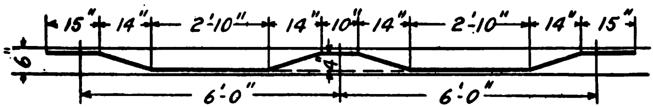


Fig. 98. Diagram Showing Bent Bars for Slabs

six feet, center to center. When slabs are designed as simple beams ($Wl \div 8$) none of the slab bars are bent.

Stirrups. Fig. 99 shows the bending of the bars for stirrups. The ends of the stirrups rest on the forms and support the beam bars, which assist in keeping these bars in place. The ends of the stirrups never show on the bottom of the slab of the finished floor, although the cut ends of the stirrups rest directly on the slab forms. Sufficient mortar seems to get under the ends of the stirrups to cover them. The type of stirrup shown in Fig. 99-*a* is much more extensively used than that in Fig.

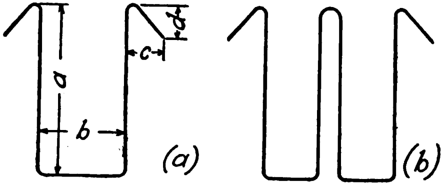


Fig. 99. Diagram Showing Bending Bars for Stirrup

99-*b*. The latter is usually employed when a large amount of steel is required, or if the stirrups are made of very small bars.

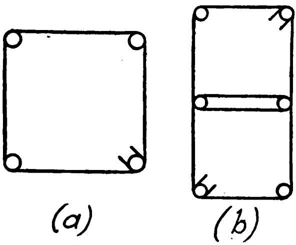


Fig. 100. Column Bands

Column Bands. In Fig. 100 two types of column bands are shown. Fig. 100-*a* shows bands for a square or a round column; and Fig. 100-*b*, bands for a rectangular column. The bar which forms the band is bent close around each vertical bar in the columns, and therefore assists in holding them in place. Two bands of the same size and shape are used for Column *b*.

Spacers. Spacers for holding the bars in place in beams and girders have been successfully used. These spacers, Fig. 101, are made of heavy sheet iron. They are fastened to the stirrups by means of the loops in the spacers. The ends of the

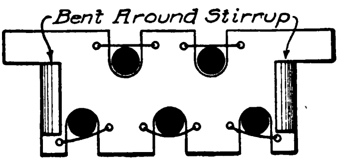


Fig. 101. Typical Spacer for Reinforcing Bars

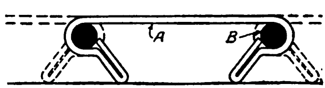


Fig. 102. Spacer for Slab Bars

spacers which project out to the forms of the sides of the beams should be made blunt or rounded. This will prevent the

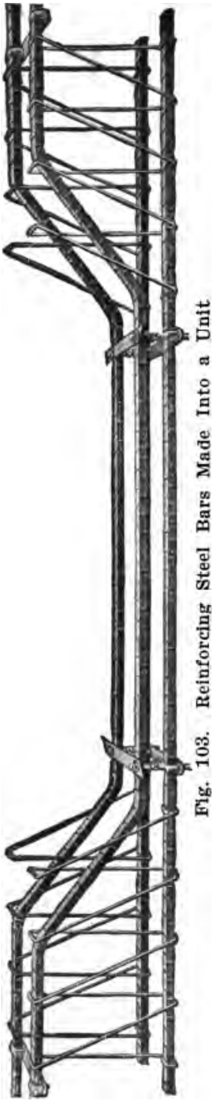


Fig. 103. Reinforcing Steel Bars Made Into a Unit
Courtesy of Corrugated Bar Company, Buffalo, New York

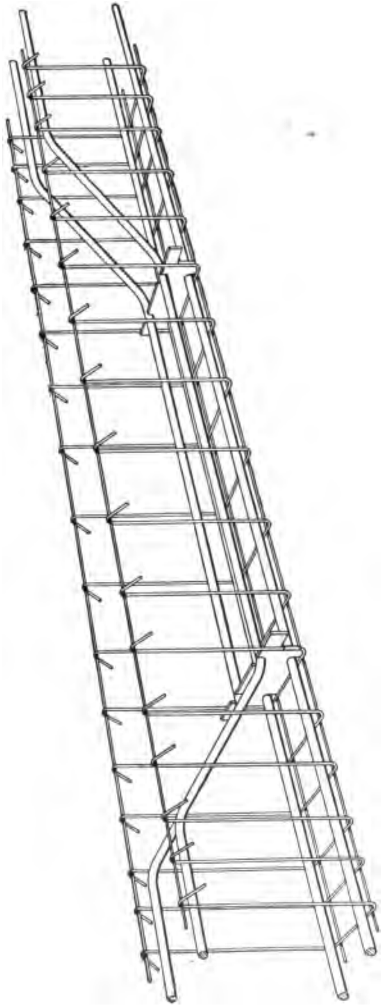


Fig. 104. Design of Collapsible Reinforcing Bar Unit Frame
Courtesy of Concrete Steel Company, Philadelphia, Pennsylvania

ends of the spacers from being driven into the forms when the concrete is being tamped. The number of spacers required (usually 2 to 4) will depend on the lengths of the beams.

Several devices have been manufactured for holding slab bars while the concrete is being poured. Fig. 102 shows a spacer made by the Concrete Steel Company, of Philadelphia.

Unit Frames. Companies making a specialty of supplying reinforcing steel generally have their own methods of making the bars for a beam into a unit. This is accomplished in different ways. The frames are made up at a shop, where there is machinery for doing the work, and shipped to the job as a unit. Fig. 103 shows a unit made by the Corrugated Bar Company.

Fig. 104 shows a collapsible frame made by the Concrete Steel Company. The frame is made up of four small bars, usually ¼ inch round, and the stirrups that are required for the beam are fastened to these bars by clips that permit the frame to be folded up for shipment. When the frame is received on the job it is unfolded, placed in the beam, and then the tension bars are put in the frame and held in place by two or more spacers.

BONDING OLD AND NEW CONCRETE

The place and manner of making breaks or joints in floor construction at the end of a day's work is a subject that has been much discussed by engineers and construction companies. But there has not yet been any general agreement as to the best method and place of constructing these joints. Wherever joints are made, great care should be exercised to secure a bond between the new and the old concrete.

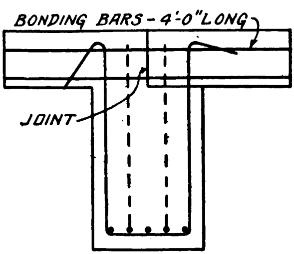


Fig. 105. Method of Bonding Old and New Concrete in Slab

Methods of Making Bonds. (1) Fig. 105 shows a sectional view of one method of making a break at the end of the day's work; this method has been used extensively and successfully. The stirrups and slab bars form the main bond between the old and the new work, if the break is left more than a few hours; short bars in the top of the slab will also assist in making a good bond. An additional number of stirrups should be used where the break is to be made in the beam. Before the new concrete is placed, the old concrete should be well scraped, thoroughly soaked with clean water, and given a thin coat of neat cement grout. An objection to this method of forming a joint is that shrinkage may cause a separation of the concrete placed at the two different times, and that water will thus find a passage. The top coat that is generally placed later greatly assists in overcoming this objection.

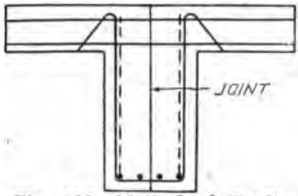


Fig. 106. Method of Bonding Old and New Concrete in Beam

(2) Another method of forming stopping places is by dividing the beam vertically—that is, making two L-beams instead of one T-beam, Fig. 106. Theoretically, this is an excellent way,

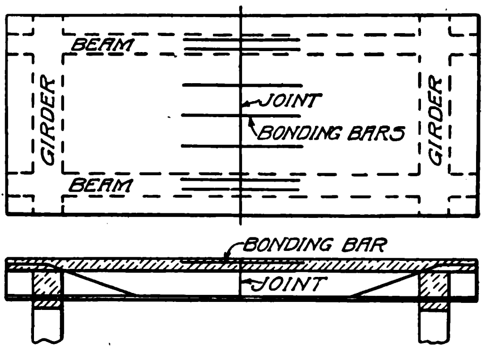


Fig. 107. Method of Bonding Break in Center of Span

but practically, it is found difficult to construct the forms dividing the beam, as the steel is greatly in the way.

(3) The method of stopping the work at the center of the span of the beams and parallel to the girders is the one in general use. Fig. 107 illustrates this method. Theoretically, the

slab is not weakened; and as the maximum bending moment occurs at this point, the shear is zero and, therefore, the beams are not supposed to be weakened, except for the loss of concrete in tension, and this is not considered in the calculation. The bottoms of the beams are tied together by the steel that is placed in the beams to take the tensile stresses; and there should be some short bars in the top of these beams, as well as in the top of the slab, to tie them together. The objection to the first method—that any shrinkage at the joint will permit water to pass through—is greater in the case of second and third methods.

DETAILS OF CONSTRUCTION

Steel Cores. It is often necessary in reinforced concrete buildings to construct columns of some other material than concrete on account of the large space that would be occupied by concrete columns. In such cases steel-core columns are often used. Fig. 108 shows two types of the steel cores. Type (*a*) is used for round columns and the steel consists of four angles, but, when necessary, plates are inserted between the angles to make up the full section. Type (*b*) is used for square columns. In determining the strength

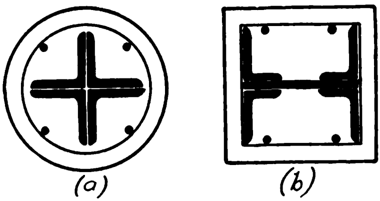


Fig. 108. Typical Sections of Steel-Core Columns

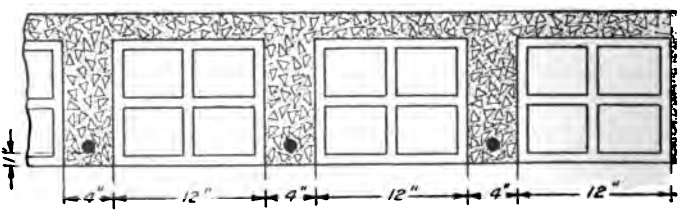


Fig. 109. Tile and Joist Construction

of these columns, the Bureau of Building Inspection of Philadelphia permits the steel to be figured as having a radius of gyration equal to that of the concrete section, which for ordinary story heights makes the permissible loading about 14,000 pounds per square inch, but additional loading is not permitted on the concrete. The steel must be surrounded by at least 2 inches of concrete, in which there must be placed 4 small vertical bars, usually ¾-inch, banded by ¼-inch bars, 12 inches

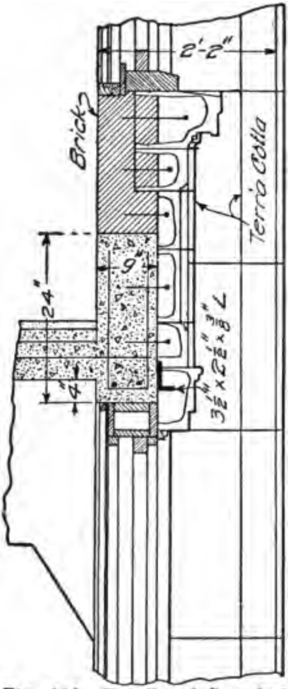


Fig. 110. Details of Spandrel Beams

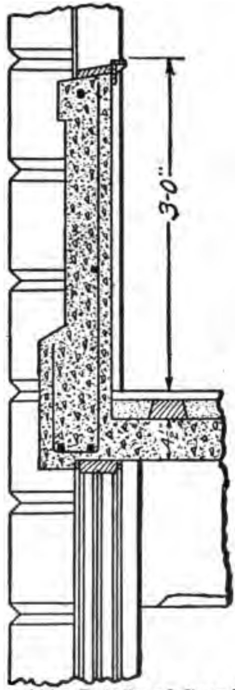


Fig. 111. Details of Spandrel Beams

on centers. The loads are transmitted from the beams and girders to the steel by means of large steel brackets which are riveted to the columns. The work is riveted up in the usual manner for structural steel.

Tile and Joist System. The tile and joist system of constructing fireproof floors is found economical for a certain class of work. It is probably used for apartment houses oftener than anywhere else. The advantage of this construction is that a flat ceiling is secured. The structural frame of the building may be either steel or reinforced concrete. The columns are connected by girders and the space between the girders is filled in with tile and joists. When reinforced concrete girders are used between the columns, a slab of concrete of sufficient width and thickness to take the compression must be constructed.

Fig. 109 shows a section of a tile and joist floor. The terra cotta tile is always 12 inches in width and from 4 inches to 15 inches in depth. The tile is simply a filler between the joists and is so much dead weight to be carried by the joists. The joists are usually 4 inches in width and are designed as T-beams; the slab is usually 2 to 3 inches in thickness. The reinforcing steel in the beam consists of one bar of sufficient area for the tensile stress. The slab should be reinforced with ¼-inch bars, 24 inches center to center each way.

Spandrel Beams. In Figs. 110 and 111 are shown two types of spandrel beams. In each case the head of the window was set up against or near the floor slab so that the maximum amount of light could be secured. In Fig. 110 the spandrel beam and the brickwork above are covered with terra cotta. In Fig. 111 the concrete surface is finished and left exposed, which is often done in factory buildings.

TYPICAL EXAMPLES OF REINFORCED CONCRETE CONSTRUCTION WORK

Allman Building, Philadelphia. The seven-story office building, 24 feet 9½ inches by 122 feet 2¼ inches, was built for Herbert D. Allman, at Seventeenth and Walnut Streets, Philadelphia. Baker and Dallett were the architects. The building is constructed of reinforced concrete, except that steel-core columns are carried up to the sixth floor. Fig. 112 shows the plans of two bays of a floor, the bay windows occurring in alternate bays. The floors are designed for 120 pounds per square foot, live load. The sizes of the different members are given on the plan. Tensile stress in the reinforcing steel is 16,000 pounds per square inch, direct compression in the concrete is 500 pounds per square

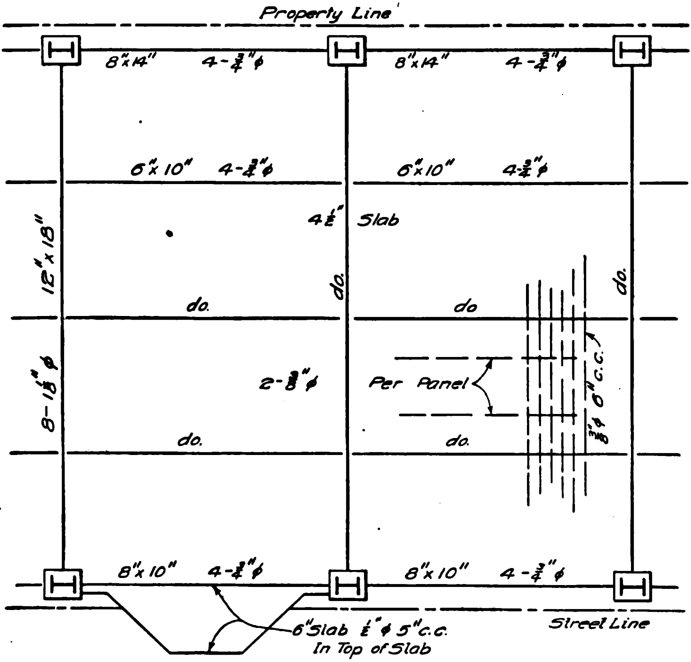


Fig. 112. Plan of Two Bays of a Floor in Allman Building, Philadelphia, Pennsylvania

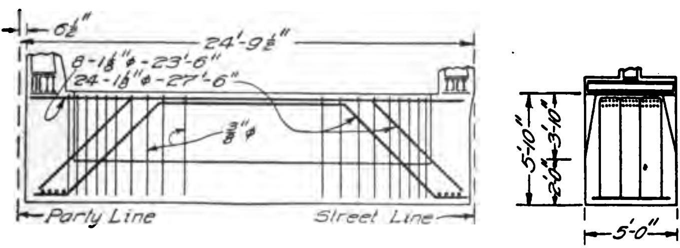


Fig. 113. Footing of Allman Building

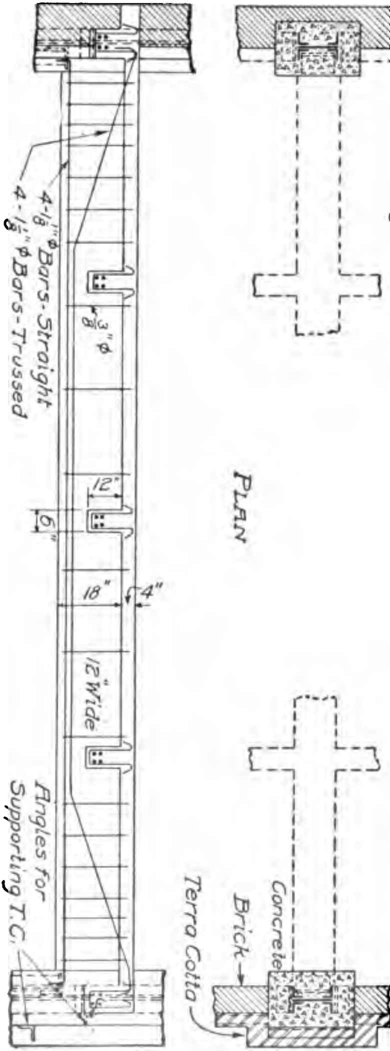


Fig. 114. Detail of Girder for Allman Building

inch, and the transverse stress in compression 600 pounds per square inch, while the shearing stress is 75 pounds per square inch. In designing the columns in which the steel cores occur, the radius of gyration was taken for the whole column; this reduced the working load to 14,000 pounds per square inch for the steel, nothing being allowed for the concrete except the increased radius of gyration. The concrete was a 1:2:4 mixture. The footings used for this building are shown in Fig. 113, and the details of the girders in Fig. 114.

Girder Bridge, Allentown, Pennsylvania. This type of reinforced concrete bridge, Fig. 115, is one that has been found to be economical for short spans. Worn-out wood and steel highway bridges are in general being replaced with reinforced concrete

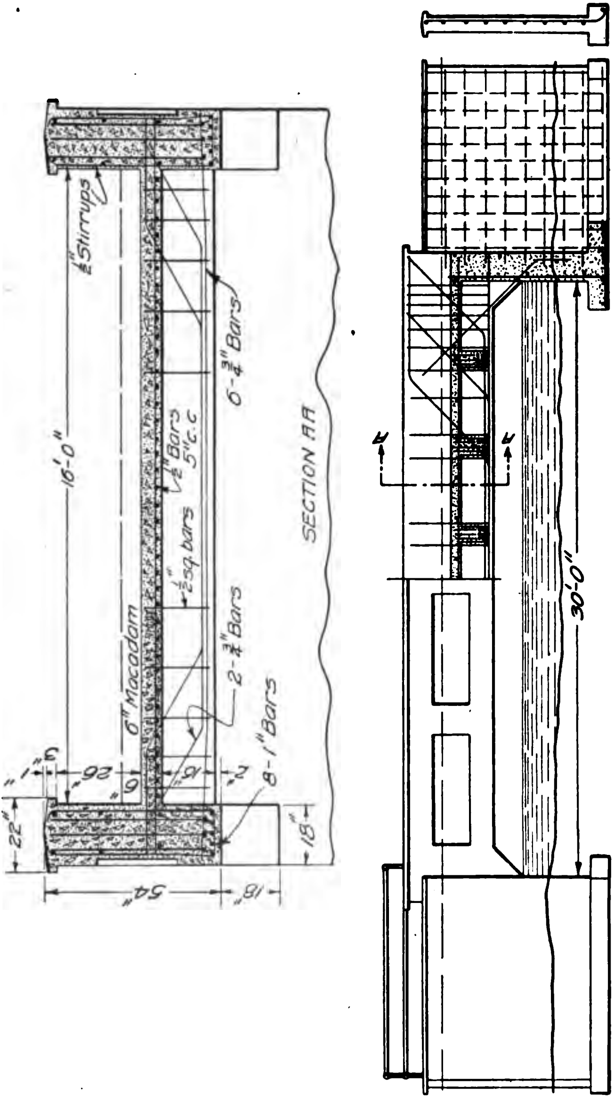


Fig. 115. Details of Girder Bridge near Allentown, Pennsylvania

bridges, usually at a cost less than that of a steel bridge of the same strength. Steel bridges need to be painted every year; and plank floors, commonly used in highway bridges, require almost constant attention and must be entirely renewed several times during the life of a bridge. A reinforced concrete bridge, however, is entirely free of these expenses, and its life should be at least equal to that of a stone arch, with which, architecturally, it compares very favorably.

The bridge shown in Fig. 115 is 16 feet wide, and has a clear span of 30 feet. It is designed to carry a uniformly distributed load of 150 pounds per square foot, or a steel road roller weighing 15 tons and having the following dimensions: width of the front roller, 4 feet, and of each rear roller, 20 inches; distance between the two rear rollers, 5 feet, center to center; distance between front and rear rollers, 11 feet, center to center; weight on front roller, 6 tons; weight on each rear roller, 4.5 tons.

In designing this bridge, the slab was planned to carry a live load of 4.5 tons on a width of 20 inches, when placed at the middle of the span, together with the dead load consisting of the weight of the macadam and the slab. The load considered in designing the crossbeams consisted of the dead load—weight of the macadam, slab, and beam—and a live load of 6 tons placed at the center of the span of the beam, which was designed as a T-beam. In designing each of the longitudinal girders, the live load was taken as a uniformly distributed load of 150 pounds per square foot over one-half of the floor area of the bridge. The live load was increased 20 per cent over the live load given above, to allow for impact. The concrete for the work was composed of 1 part Portland cement, 2 parts sand, and 4 parts 1-inch stone. Corrugated reinforcing bars were used.

In a bridge of this type, longitudinal girders act as a parapet as well as main members of the bridge. When there is sufficient headroom, all the beams can be constructed in the longitudinal direction of the bridge, and are under the slab. The parapet may be constructed of concrete; a cheaper method is to construct a handrailing with 1½-inch or 2-inch pipe.

Circular Tanks. In Fig. 116 is shown the section of the wall of four circular tanks, each of which is 50 feet in diameter. The concrete was a 1:3:5 mix, the materials being carefully graded. The tension in the steel is 12,000 pounds per square inch. Square deformed bars were used. The tanks were made water-tight by the Sylvester process.

Main Intercepting Sewer, Waterbury, Connecticut. In the development of sewage purification work at Waterbury, Connecticut, the construction of a main intercepting sewer was a necessity. This sewer is 3 miles long. It is of horseshoe shape, 4 feet 6 inches by 4 feet 5 inches, and is constructed of reinforced concrete. The details are illustrated in Fig. 117.

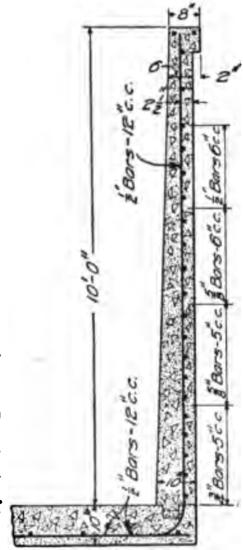


Fig. 116. Typical Section of Tank

The trench excavations were principally through water-bearing gravel, the gravel ranging from coarse to fine. Some rock was encountered in the trench excavations; it was a granite gneiss of irregular fracture, and cost, with labor at 17½ cents per hour, about \$2.00 per cubic yard to remove it. Much of the trench work varied in depth from 20 to 26 feet. To suit different conditions, it was necessary to vary the sewer section somewhat; frequently, the footing course was extended. However, the section shown is the normal one. Very wet concrete was poured into practically water-tight forms. The proportions used were 1 part Atlas Portland cement to 7.5 parts of aggregate, graded to secure a dense concrete. Care was used in placing the concrete, and very smooth surfaces were secured. Plastering of the surfaces was avoided. Any voids were grouted or pointed, and smoothed with a wooden float. Expanded metal and square-twisted bars were used in different parts of the work. Fig. 117 shows the size and spacing of the bars, which were bent to their required shape before they were lowered into the excavation.

The forms in general were constructed as shown in the figure. The inverted section was built as the first operation and, after the surface was thoroughly troweled, the section was allowed to set 36 to 48 hours before the concreting of the arch section was begun. The lagging was ⅞ inch thick, with tongue-and-groove radial joints, and toenailed to the 2-inch plank ribs. The exterior curve was planed and scraped to a true surface. The vertical sides of the inner form are readily removable, and

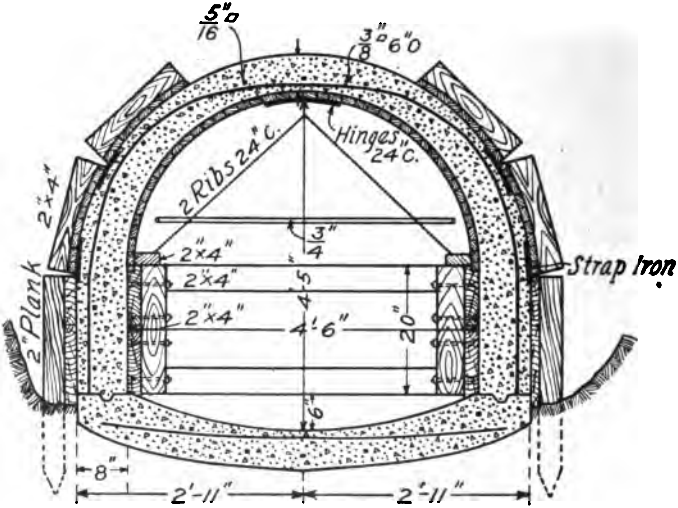


Fig. 117. Section of Intercepting Sewer at Waterbury, Connecticut

the semicircular arch above is hinged at the soffit and is collapsible. The first cost of these forms averaged $18.00 for 10 feet of length, and the cost per foot of sewer built, both first cost and maintenance, averaged 10 cents. Petrolene, a crude petroleum, prevented the concrete from adhering to the forms.

Cost records kept under the several contracts and assembled into a composite form, show what is considered to be the normal cost of this section, under the local conditions. Common labor averaged 17½ cents, sub-foremen 30 cents, and general foremen 50 cents per hour.

Normal Cost per Lineal Foot of 53- by 54-inch Rienforced Concrete Sewer

Steel reinforcement, 17½ lb.	$ .43
Making and placing reinforcement cages	.14
Wood interior forms, cost, maintenance, and depreciation	.12
Wood exterior forms, cost, maintenance, and depreciation	.05
Operation of forms	.16
Coating oil	.01
Mixing concrete	.30
Placing concrete	.27
Screeding and finishing invert	.08
Storage, handling, and cartage of cement	.08
0.482 bbl. cement at $1.53	.74
0.17 cu. yd. sand at $0.50	.09
0.435 cu. yd. broken stone at $1.10	.47
Finishing interior surface	.01
Sprinkling and wetting completed work	.02
Total cost per lineal foot	$2.97

This is equivalent to a cost of $9.02 per cubic yard.

Bronx Sewer, New York City. Fig. 118 shows a section of one of the branch sewers constructed in the Borough of the

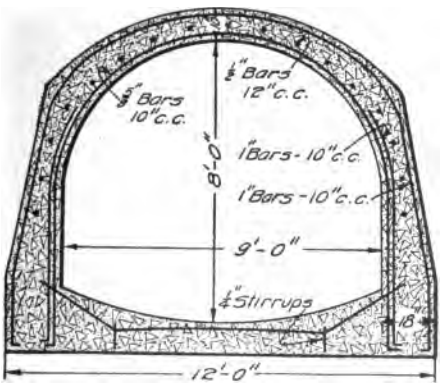


Fig. 118. Section of Bronx Sewer, New York City

Bronx, New York City. A large part of this sewer is located in a salt marsh where water and unstable soil made construction work very difficult. The general elevation of the marsh is 1.5 feet above mean high water. In constructing this sewer in the marsh, it was necessary to build a pile foundation to support the sewer. The foundation was capped with reinforced concrete, and then the sewer, as shown in the section, was constructed on the pile foundation. The concrete for this work was composed of 1 part Portland cement, 2.5 parts sand, and 5 parts trap rock. The rock was crushed to pass a $\frac{3}{4}$-inch screen. Twisted bars were used for the reinforcement in the work.

APPENDIX

FLAT-SLAB CONSTRUCTION

Outline of Method. The so-called "flat-slab method" has the advantages that (*a*) there is a very considerable saving in the required height (and cost) of the building on the basis of a given *net* clear height between floors; (*b*) the architectural appearance is improved by having a flat ceiling surface rather than visible beams and girders; (*c*) there is a saving in the cost of forms, not only in surface area and amount of lumber required but also in simplicity of construction, although this saving is offset by an increase in total volume of concrete used; (*d*) there are no deep ceiling beams to cast shadows and it is possible to extend the windows up to the ceiling, which are important items in the lighting of a factory building. Almost the only disadvantage is the difficulty in making perfectly definite and exact computations of the stresses, as may be done for simple beams and slabs. But methods of computation have been devised which, although admittedly approximate, will produce designs for economical construction, and structures so designed have endured, without distress, test loads considerably greater than the designed working loads.

Consider, first, a simple beam, as in Fig. 119-*a*, the beam being continuous over the supports and uniformly loaded for the distance l between the supports with a load amounting to W. Then the maximum moment is located just over the supports and equals $Wl \div 12$. Another local maximum, equal to $Wl \div 24$, is found at the center. Points of inflection are at $.211l$ from each column.

Assume that a uniformly loaded plate of indefinite extent is supported on four columns, A, B, C, and D, Fig. 119-*b*, the extensions beyond the columns being such that planes tangent to the plate just over the columns will be horizontal. Then

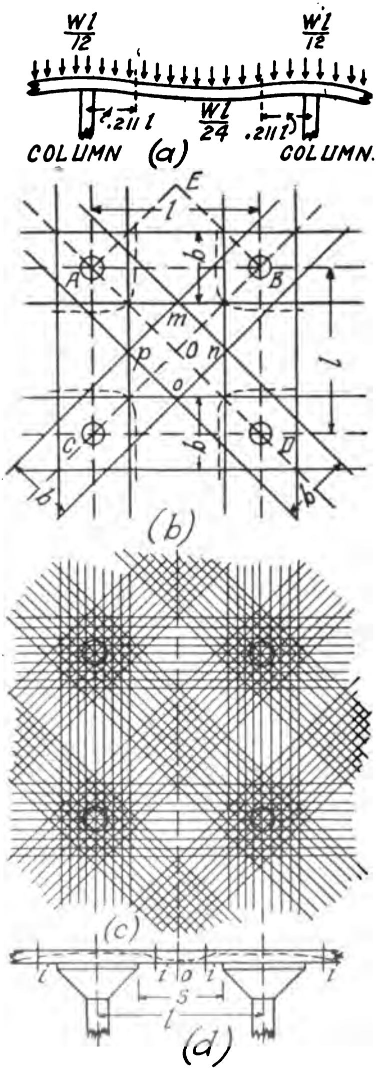


Fig. 119. Flat-Slab Construction

the following conditions may be observed:

(1) The plate will be convex upward over the columns;

(2) The plate will be concave upward at the point O in the center;

(3) There will be curves of inflection, approximately as shown by the dotted curves sketched in around the columns; from the analogy of the simple beam, given above, we may assume that the curves of inflection are approximately at 21 per cent of the span in every direction from the columns.

The columns at the top are made with enlarged sections so as to form a "column head"—which is generally in the form of a frustum of an inverted pyramid or cone, the base being a circle, a square, or a regular polygon.

This device shortens the clear span and decreases the moment. It also increases the size of the hole which the column tends to punch through the plate and hence increases the surface area which resists this punching shear, and thus decreases the unit shear. The diameter of the column head should be about 25 per cent of the span between column centers.

Placing Reinforcing Bars. Various systems of placing the reinforcing bars have been devised, and some of them patented. The methods may be classified as follows: (1) "Four-way" method, in which the bars run not only in lines parallel to the sides of the rectangles joining the column heads, but also parallel to the diagonals; (2) "two-way" method, in which there are no diagonal bars; and (3) designs which have, in addition to the bands of straight bars from column to column, spirals or a series of rings around the column heads for the specific purpose of providing for the "circumferential tension, or moment." This circumferential tension unquestionably exists, but those who use the first two methods claim that the gridiron of bars formed over the column by the two-way method, and still more so by the four-way method, develops plate action, and that the circumferential stress is amply provided for.

It is a simple matter of geometry to prove that if bands of bars of width b, Fig. 119-b, are placed across columns which form square panels with span l, the width b must equal $.414l$, if the bands exactly cover the space without leaving either gaps or overlaps at m, n, o, and p. The bands may be a little narrower than this, say b equals $.4l$, provided the gaps are not much, if any, greater than the spacing of the bars. On the other hand, the bands should not be wider than twice the diameter of the column head. Fig. 119-c shows that, using the four-way system and with b equal to $.414l$, every part of the slab has at least one layer of bars, some parts have two, some three, and that there are four layers of bars over each column. This is where the moment is maximum.

Method of Calculation. One of the simplest methods of calculation, which probably gives a considerable but indeterminate excess of strength, is to consider the bands as so many simple continuous beams, which are wide but shallow. Consider a *direct* band of width b, equal to $.4l$, the word *direct* being used in contradistinction to *diagonal*. If w is the unit dead and live load per square foot, and s the net span between column heads, then the total load on the band is $.4wls$. Computed as a simple continuous beam, the moment in the center would be $(.4wls)\ s \div 24$, and that over the columns would be $(.4wls)\ s \div 12$. By prolonging the steel bars of adjoining bands sufficiently over a column head so that the bond adhesion is sufficient to develop the full tension over the column head, the total effective area of steel in that band over the column head is double what it is in the center. Practically, this means that the steel should extend to the point of inflection beyond the column head or that its length should be 42 per cent longer than the distance between column centers. Then, on the principle of **T**-beam flanges, it is assumed that the concrete above the neutral axis for a width of $(b + 5t)$ may be computed as taking the compression. For the diagonal bands, the load is $w \times .4l \times 1.414s = .565wls$, and then, considering that a considerable part of the area of the diagonal bands includes that already covered by the direct bands, and also that the diagonal bands both support a square in the center which is one-half of the area lying inside of the direct bands, the moment for the central area is divided between the two diagonal bands and that for each is considered to be $(.565wls \times 1.414s) \div 48 = .0166wls^2$. As before, the moment over the columns for these bands is twice as much, but the steel for the double moment may be obtained, as before, by lapping the bars of adjoining diagonal bands over the columns. The area of a panel, outside of the column heads, which are here assumed to be square, is $l^2 - (l - s)^2$. When the column head is 25 per cent of l, then $(l - s) = \frac{1}{4}l$ and the area of the panel is $\frac{15}{16}l^2$, or $.9375l^2$; and the total effective load causing moment on a panel is $W = .9375wl^2$. If we eliminate s and w from the above moment equations, we have

Moment at center, direct band

$$= \frac{(.4wls)s}{24} = \frac{.4wl^3\ \frac{9}{16}}{24} = \frac{3.6}{384}wl^3 = \frac{Wl}{100}$$

Moment over cap, direct band = (double the above) $= Wl \div 50$
Moment at center, diagonal band $= .0166wls^2 = Wl \div 100$
Moment over cap, diagonal band = (double the above) $= Wl \div 50$

Illustrative Example. Assume a live load of 200 pounds per square foot on a square panel 22 feet between column centers. A working rule is that the thickness of the slab should be *at least* $\frac{1}{30}$ of the span; $\frac{1}{30}$ of 22 feet, or 264 inches, is 8.8 inches. We will therefore assume the slab thickness as 10 inches, which will weigh 120 pounds per square foot. Therefore, $w = 320$ and $W = \frac{15}{8} wl^2 = \frac{15}{8} \times 320 \times 22^2 = 145{,}200$. Then the moment at the center of a direct band equals $Wl \div 100 = (145{,}200 \times 264) \div 100 = 383{,}328$ inch-pounds, and the moment for that band over the column is 766,656 inch-pounds. The width of each band b is $.4l = .4 \times 264 = 105.6$ inches. Assume that the steel for one of the bands is placed at 8.5 inches from the compression face, or that $d = 8.5$; if we estimate $j = .91$; then

$$\begin{aligned} M &= 383{,}328 \\ &= pbdsjd \\ &= p \times 105.6 \times 8.5 \times 16{,}000 \times .91 \times 8.5 \end{aligned}$$

from which

$$p = .00345$$

From Table XII, we may note that for $n = 15$ and $p = .00345$, j would be about .91. This checks the assumed value. Then

$$A = pbd = .00345 \times 105.6 \times 8.5 = 3.10 \text{ sq. in.}$$

This may be amply provided by 13 bars $\frac{1}{2}$ inch square. 105.6 $\div$ 12, or about 9 inches, gives the spacing of the bars. Although doubling p changes the value of j and will not *exactly* double the moment, yet it will be sufficiently exact to say that double the moment will be obtained over the cap by prolonging the 13 bars of each of the two direct bands in the same line over the columns as far as the circle of inflection, thus doubling the area of the steel. (The student should work this out as an exercise.) Double p and find the corresponding value of j from Table XII; use the actual area of the 26 bars for the value of A, and compute M from $Asjd$. On account of the slight excess in the area of the 26 bars here used, the moment is a little more than necessary.

Location of Bars. There are four layers of bars over the column head and it is evident that they cannot all lie in the same plane or be at the same distance from the compression face. For the layer of bars considered above, d was assumed at 8.5, the maximum permissible with a 10-inch slab. For the next row deduct ½ inch, the thickness of the bars, and let d equal 8.0. Since the moment is the same, and d is reduced, then p must be increased and j will be less. Assume $j = .90$; then

$$\begin{aligned} M &= 383{,}328 \\ &= pbdsjd \\ &= p \times 105.6 \times 8 \times 16{,}000 \times .9 \times 8 \end{aligned}$$

from which

$$p = .00394$$

This is a little more than for the other band, as was expected. Then $A = pbd = 3.33$ square inches, provided by 14 bars ½ inch square. Similarly, it may be shown that reducing d another half-inch for the next layer will add another bar, making 15 bars for the third layer and 16 bars for the fourth layer. Since the computed moments for the direct and diagonal bands is the same for the center of the band, and since the diagonal bands are the longer, there will be some economy in giving them the advantageous position in the slab (larger values of d) and using 13 and 14 bars for the diagonal bands and 15 and 16 bars for the direct bands. The above variation in the number of bars with the change in d indicates the importance of placing the steel exactly as called for by the plans. The design might be made a little more symmetrical, and more foolproof during construction by using 14 bars in each of the diagonal bands and 16 bars in each of the direct bands, and then being sure that the direct bands are *under* the diagonal bands where they pass over the column heads.

Unit Compression. The unit compression may be computed from the equation

$$M = \tfrac{1}{2}\, cb'kdjd$$

For the concrete compression, we may call $b' = 105.6 + 5t =$

$105.6 + 50 = 155.6$. The critical place is over the column. Here, where the moment is double,

$$p = A \div b'd = 6.5 \div (155.6 \times 8.5) = .00724$$

Then $M = 766{,}656$; $k = .369$; and $j = .88$.

Substituting these values, we find that

$$c = 420 \text{ lb. per sq. in.}$$

But this is a more favorable case than the compression computed for the band whose d is only 7 inches. In this case, $p = A \div bd = 8 \div (155.6 \times 7) = .00734$, which makes $k = .371$ and $j = .88$.

Substituting these values, we find that

$$c = 616 \text{ lb. per sq. in.}$$

This is amply safe, especially in view of the fact that a cube subjected to compression on all six faces, as it is in this case, can stand a far higher unit compression than it can when the compression is only on two faces.

Shear. The cap is a square 66 inches on a side and its perimeter is 264 inches. V in this case equals W and is 145,200 pounds. For this calculation let j equal .88 and d equal 8.5; then

$$v = \frac{V}{bjd} = \frac{145{,}200}{264 \times .88 \times 8.5} = 73.5 \text{ lb. per sq. in.}$$

Since this is a punching shear rather than diagonal tension, this working value is allowable. The usual allowed unit value is 80. At any section farther away from the column head, the total shear is less, and the perimeter, and hence the shearing area, is greater, and therefore the unit shear becomes less and less. The zone around the column head is the critical section and, since it is where the moment is also maximum, no main reinforcing bars can be spared to resist this shear, as is done at the ends of simple beams. A ring of stirrups around each column head is the only practicable method of resisting such shear, if it is excessive.

Wall Panels. The above calculations are virtually for interior panels, or for those where the loads are balanced over the columns. When panels are next to a wall, the bands perpendicular to the wall, and even the diagonal bands, must be anchored by bending them down into the columns. The extra steel is just as necessary, in order to develop the moment at the column head, as if the bands were extended into an adjoining panel. The band along the wall between the wall columns may have part of the usual width cut off. In addition to the floor load, the weight of the wall makes an additional load. This may be most efficiently supported by a "spandrel beam", which is a narrow but deep beam extending up from the floor to the window sill, and which virtually forms that part of the wall, although there may be an outside facing. Sometimes the exterior columns are set in from the building line so as to balance partially, if not entirely, the load on the other side of the columns.

General Constructive Details. The column head should have a considerable thickness at its edge, immediately under the slab, to enable it to withstand shear, as shown in Fig. 119-*d*. If, as is sometimes done, the sloping sides of the head are continued to the slab surface, a considerable deduction should be made in estimating the effective diameter of the head, which means an increase in the net span between columns. The four points marked *i*, Fig. 119-*d*, are at about 20 per cent of the net span between column heads and are the computed points of inflection where there is no moment. The bars should be in about the middle of the slab at these points. They should be at the minimum permissible distance above the bottom of the slab at *O* and similarly near the top of the slab at the edges and across the column heads. There should not be abrupt bends at these points, but the bars should have easy curves through the required positions at *O* and the points of inflection and then, reversing curvature so that it will be concave downward, should again reach a horizontal direction just over the edge of the column head. While no great precision is essential in locating the bars between these specified places, care must be taken to fasten the bars in exact position at the critical points so that they cannot be disturbed. There should always be at least one inch of concrete below the bars in the center of the slab.

Rectangular Panels. The flat-slab method of construction is most economically used when the panels are nearly, if not quite, square, and also when the column spacing can be made about 23 feet. The ratio of length to breadth for rectangular panels should not exceed 4:3. The two pairs of direct bands must then be computed independently and separately. The diagonal bands must be computed according to their actual dimensions, which means that the moment equations given above will not apply, and other equations, computed in the same general manner, must be derived. The quantity b may be considered as 0.4 of the mean of the two column spans. The economy of the flat-slab method is chiefly applicable to heavy floor loadings, such as are required for factories, warehouses, etc.

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