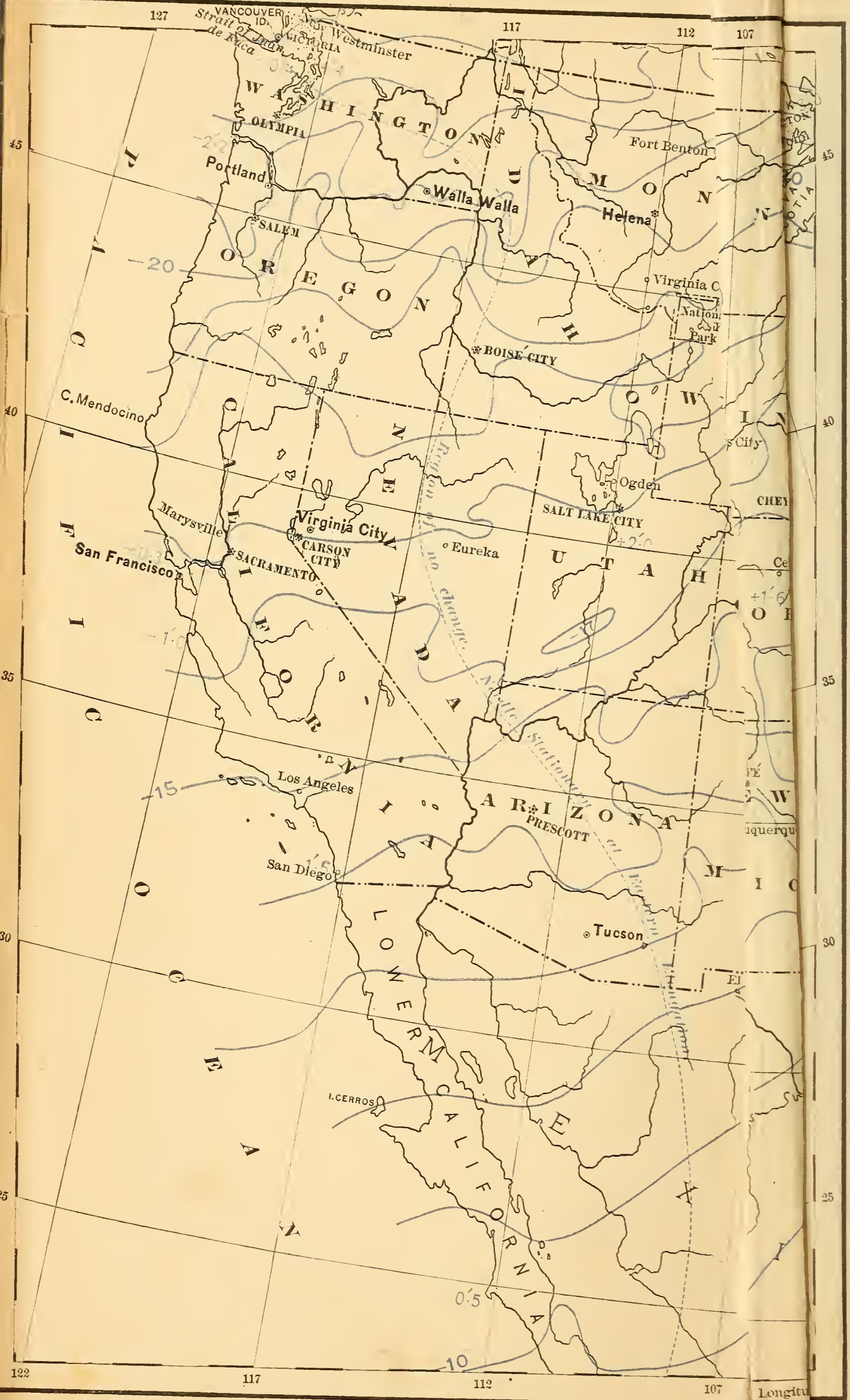

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Strait of Juan de Fuca
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Westminster
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BOISE CITY
MONTANA
Fort Benton
Helena
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National Park
WYOMING
C. Mendocino
CALIFORNIA
Marysville
San Francisco
SACRAMENTO
NEVADA
Virginia City
CARSON CITY
Eureka
Region of no change. Needle Stationary at Equator
UTAH
Ogden
SALT LAKE CITY
City
CHEY
Los Angeles
San Diego
ARIZONA
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MEXICO
LOWER CALIFORNIA
I. CERROS
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A TREATISE

ON

SURVEYING

COMPRISING THE THEORY AND THE PRACTICE

BY

WILLIAM M. GILLESPIE, LL. D.

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REVISED AND ENLARGED BY

CADY STALEY, PH. D.

PRESIDENT OF CASE SCHOOL OF APPLIED SCIENCE

NEW YORK
D. APPLETON AND COMPANY
1887


ISOGONIC CHART
FOR 1885-0.
REDUCED FROM U. S. COAST AND GEODETIC SURVEY REPORT, 1882.
Scale of Miles
0 50 100 150 200 250
NOTE: The isogonics are drawn for differences of one degree, and every fifth curve is indexed with a large number; a + sign indicating west, a — sign east declination.
The annual change at this epoch is given in small numbers for the secular variation stations and is expressed in minutes and decimal, a + sign indicating increasing west declination or decreasing east declination, a — sign indicating increasing east or decreasing west declination.
This annual change may be used for a few years preceding or following 1885-0, without sensible error.
Longitude West from Greenwich
GULF OF MEXICO
PACIFIC OCEAN
ATLANTIC OCEAN

PREFACE.

GILLESPIE'S "Land-Surveying" was first printed in 1851, for use in Professor Gillespie's classes in Union College. It was published in 1855, and very soon became the standard authority on land-surveying.

In the preface to the first edition Professor Gillespie says:

"Land-surveying is perhaps the oldest of the mathematical arts. Indeed, geometry itself, as its name—'land-measuring'—implies, is said to have arisen from the efforts of the Egyptian sages to recover and to fix the landmarks annually swept away by the inundations of the Nile. The art is also one of the most important at the present day, as determining the title to land, the foundation of the whole wealth of the world. It is, besides, one of the most useful as a study, from its striking exemplifications of the practical bearings of abstract mathematics. But, strangely enough, surveying has never yet been reduced to a systematic and symmetric whole. To effect this, by basing the art on a few simple principles and tracing them out into their complicated ramifications and varied applications (which extend from the measurement of 'a mowing-lot' to that of the heavens), has been the earnest endeavor of the present writer.

"The work, in its inception, grew out of the author's own needs. Teaching surveying, as preliminary to a course of civil engineering, he found none of the books in use (though very excellent in many respects) suited to his purpose. He was, therefore, compelled to teach the subject by a combination of familiar lectures on its principles and exemplifications of its practice. His notes continually swelling in bulk, gradually became systematized in nearly their present form. His system has thus been fully tested, and the present volume is the result.

"A double object has been kept in view in its preparation: viz., to produce a very plain introduction to the subject, easy to be mastered by the young scholar or the practical man of little previous acquirement, the only prerequisites being arithmetic and a little geometry; and at the same time to make the instruction of such a character as to lay a foundation broad enough and deep enough for the most complete superstructure which the professional student may subsequently wish to raise upon it."

In the preface to the "Land-Surveying," Professor Gillespie announced that another volume, on "Leveling and Higher Surveying," was to follow. This work was, at the time of his death, in 1868, unfinished.

The same method was pursued in its preparation as for the "Land-Surveying." Parts of it had been printed for class use, and a large part of the book had been given in the form of lectures to the Professor's classes. It was completed by the editor of this volume, and published in 1870.

The two volumes, "Land-Surveying" and "Leveling and Higher Surveying," are now revised and united in this volume.

The best authorities have been consulted, in order to render the work as reliable as possible.

The writer is under obligations to many friends for assistance in the work of revision, and especially to E. P. Dickey, C. E., for a large part of "Mining-Surveying," and to Professor T. W. Wright, C. E., for the formula and table in gradienter measurement, and other valuable assistance.

CADY STALEY.

CASE SCHOOL OF APPLIED SCIENCE,
CLEVELAND, OHIO, *January, 1887.*

GENERAL DIVISION OF THE SUBJECT.

[*A full Analytical Table of Contents is given at the end of the volume.*]

PART I.

LAND-SURVEYING.

PART II.

LEVELING.

PART III.

TOPOGRAPHY.

PART IV.

TRIANGULAR SURVEYING.

PART V.

HYDROGRAPHICAL SURVEYING.

PART VI.

MINING-SURVEYING.

PART I.

LAND-SURVEYING.

CHAPTER I.

GENERAL PRINCIPLES AND FUNDAMENTAL OPERATIONS.

DEFINITIONS AND METHODS.

1. SURVEYING is the art of making such measurements as will determine the relative positions of any points on the surface of the earth ; so that a *Map* of any portion of that surface may be drawn, and its *Content* calculated.

2. The position of a point is said to be *determined*, when it is known how far that point is from one or more given points, and in what direction therefrom ; or how far it is in front of them or behind them, and how far to their right or to their left, etc. ; so that the place of the first point, if lost, could be again found by repeating these measurements in the contrary direction.

The "points" which are to be determined in Surveying are not the mathematical points treated of in Geometry, but the corners of fences, boundary stones, trees, and the like, which are mere points in comparison with the extensive surfaces and areas which they are the means of determining. In strictness, their centers should be regarded as the points alluded to.

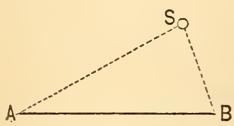
A straight *Line* is "determined," that is, has its length and its position known and fixed, when the points at its extremities are determined ; and a plane *Surface* has its form and dimensions determined when the lines which bound it are determined. Consequently, the determination of the relative positions of *points* is all that is necessary for the principal objects of Surveying ; which are to make a *map* of any surface, such as a field, farm, State, etc., and to calculate its *content* in square feet, acres, or square miles. The former is an application of Drafting, the latter of Mensuration.

The position of a point may be determined by a variety of methods. Those most frequently employed in Surveying are the following—all the points being supposed to be in the same plane :

3. First Method. *By measuring the distances from the required point to two given points.*

Thus, in Fig. 1, the point S is "determined," if it is known to be one inch from A, and half an inch from B ; for its place, if lost, could be found by describing two arcs of circles, from A and B as centers, and with the given distances as radii. The required point would be at the intersection of these arcs.

Fig. 1.



In applying this principle in surveying, S may represent any station, such as a corner of a field, an angle of a fence, a tree, a house, etc. If, then, one corner of a field be 100 feet from a second corner, and 50 feet from a third, the place of the first corner is known and determined with reference to the other two.

There will be two points fulfilling this condition, one on each side of the given line, but it will always be known which of them is the one desired.

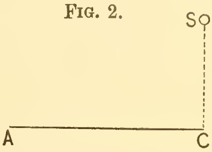
In *Geography,* this principle is employed to indicate the position of a town ; as when we say that Buffalo is distant (in a straight line) 295 miles from New York, and 390 from Cincinnati, and thus convey to a stranger acquainted with only the last two places a correct idea of the position of the first.

In *Analytical Geometry,* the lines A S and B S are known as "*Focal Co-ordinates,*" the general name "co-ordinates" being applied to the lines or angles which determine the position of a point.

4. Second Method. *By measuring the perpendicular distance from the required point to a given line, and the distance thence along the line to a given point.*

Thus, in Fig. 2, if the perpendicular distance S C be half an inch, and C A be one inch, the point S is "determined"; for its place could be again found by measuring one inch from A to C, and half an inch from C, at right angles to A C, which would fix the point S.

Fig. 2.



The public lands of the United States are laid out by this method, as will be explained in Chapter VII.

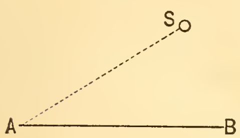
In *Geography,* this principle is employed under the name of Latitude and Longitude.

Thus, Philadelphia is one degree and fifty-two minutes of longitude east of Washington, and one degree and three minutes of latitude north of it.

In *Analytical Geometry,* the lines A C and C S are known as "*Rectangular Co-ordinates.*" The point is there regarded as determined by the intersection of two lines, drawn parallel to two fixed lines, or "*Axes,*" and at a given distance from them. These *Axes,* in the present figure, would be the line A C, and another line, perpendicular to it and passing through A, as the origin.

5. Third Method. *By measuring the angle between a given line and a line drawn from any given point of it to the required point; and also the length of this latter line.*

Fig. 3.



Thus, in Fig. 3, if we know the angle B A S to be a third of a right angle, and A S to be one inch, the point S is determined; for its place could be found by drawing from A, a line making the given angle with A B, and measuring on it the given distance.

In applying this principle in surveying, S, as before, may represent any station, and the line A B may be a fence, or any other real or imaginary line.

In "Compass Surveying," it is a north-and-south line, the direction of which is given by the magnetic needle of the compass.

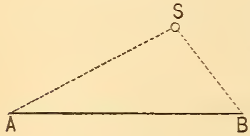
In *Geography,* this principle is employed to determine the relative positions of places, by "bearings and distances"; as when we say that San Francisco is 1,750 miles nearly due west from St.

Louis; the word "west" indicating the *direction*, or angle which the line joining the two places makes with a north-and-south line, and the number of miles giving the *length* of that line.

In *Analytical Geometry,* the line A S, and the angle B A S, are called "*Polar Co-ordinates.*"

6. Fourth Method. *By measuring the angles made with a given line by two other lines starting from given points upon it, and passing through the required point.*

FIG. 4.



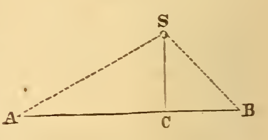
Thus, in Fig. 4, the point S is determined by being in the intersection of the two lines A S and B S, which make respectively angles of a half and of a third of a right angle with the line A B, which is one inch long; for the place of the point could be found, if lost, by drawing from A and B lines making with A B the known angles.

In *Geography,* we might thus fix the position of St. Louis, by saying it lay nearly due north from New Orleans, and due west from Washington.

In *Analytical Geometry,* these two angles would be called "*Angular Co-ordinates.*"

7. In Fig. 5 are shown together all the measurements necessary for determining the same point S, by each of the four preceding methods. In the *First* Method, we measure the distances A S and B S; in the *Second* Method, the distances A C and C S, the latter at right angles to the former; in the *Third* Method, the distance A S, and the angle S A B; and, in the *Fourth* Method, the angles S A B and S B A. In all these methods the point is really determined by the intersection of two lines, either straight lines or arcs of circles. Thus, in the First Method, it is determined by the intersection of two circles; in the Second, by the intersection of two straight lines; in the Third, by the intersection of a straight line and a circle; and, in the Fourth, by the intersection of two straight lines.

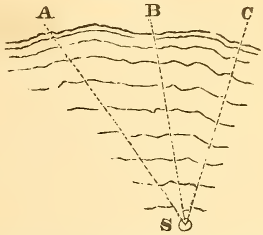
FIG. 5.



8. Fifth Method. *By measuring the angles made with each other by three lines of sight passing from the required point to three points whose positions are known.*

Thus, in Fig. 6, the point S is determined by the angles A S B and B S C, made by the three lines S A, S B, and S C.

Fig. 6.

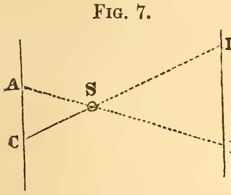


Geographically, the position of Chicago would be determined by three straight lines passing from it to Washington, Cincinnati, and Mobile, and making known angles with each other; that of the first and second lines being about one third, and that of the second and third lines, about one half of a right angle.

From the *three lines* employed, this may be named the Method of *Trilinear Co-ordinates.*

9. The position of a point is sometimes determined by the intersection of two lines, which are themselves determined by their extremities being given. Thus, in Fig. 7, the point S is determined by its being situated in the intersection of A B and C D. This method is sometimes employed to fix the position of a station on a railroad line, etc., when it occurs in a place where a stake can not be driven, such as in a pond, and in a few other cases, but is not used frequently enough to require that it should be called a *sixth* principle of Surveying.

Fig. 7.



10. These five methods of determining the positions of points produce five corresponding systems of Surveying, which may be named as follows:

I. Diagonal Surveying.
II. Perpendicular Surveying
III. Polar Surveying.
IV. Triangular Surveying.
V. Trilinear Surveying.

The above division of Surveying has been made in harmony with the principles involved and the methods employed.

The subject is, however, sometimes divided with reference to the *instruments* employed; as the chain, either alone or with cross-staff; the compass; the transit or theodolite; the sextant; the plane-table, etc.

11. Surveying may also be divided according to its *objects*.

In *Land* Surveying, the content, in acres, etc., of the tract surveyed, is usually the principal object of the survey. A map, showing the shape of the property, may also be required. Certain signs on it may indicate the different kinds of culture, etc. This land may also be required to be divided up in certain proportions; and the lines of division may also be required to be set out on the ground. One or all of these objects may be demanded in Land Surveying.

In *Topographical* Surveying, the measurement and graphical representation of the inequalities of the ground, or its "relief," i. e., its hills and hollows, as determined by the art of "Levelling," is the leading object.

In *Maritime* or *Hydrographical* Surveying, the positions of rocks, shoals, and channels are the chief subjects of examination.

In *Mining* Surveying, the directions and dimensions of the subterranean passages of mines are to be determined.

12. Surveying may also be divided according to the *extent* of the district surveyed into *Plane* and *Geodesic*. Geodesy takes into account the curvature of the earth, and employs Spherical Trigonometry. *Plane* Surveying disregards this curvature, as a needless refinement except in very extensive surveys, such as those of a State, and considers the surface of the earth as plane, which may safely be done in surveys of moderate extent.

13. In all the methods of Land Surveying, there are three stages of operation:

1. *Measuring* certain lines and angles, and recording them;
2. *Drawing them* on paper to some suitable scale;
3. *Calculating* the content of the surface surveyed.

MAKING THE MEASUREMENTS.

14. The *Measurements* which are required in Surveying may be of lines or of angles, or of both, according to the Method employed. Each will be successively considered.

Measuring Straight Lines.

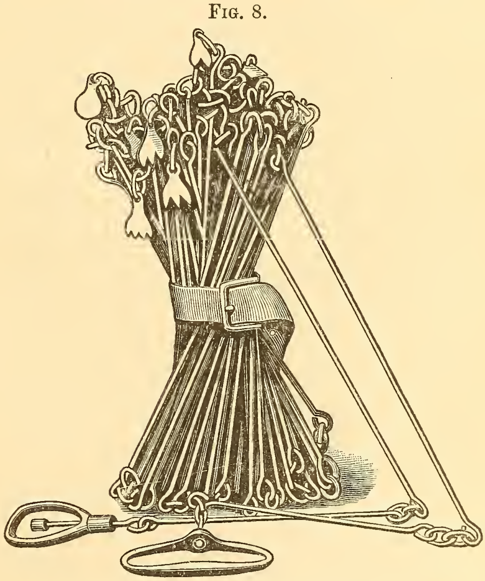
15. The lines, or distances, which are to be measured, may be either actual or visual.

Actual lines are such as really exist on the surface of the land to be surveyed, either bounding it, or crossing it; such as fences, ditches, roads, streams, etc.

Visual lines are imaginary lines of sight, either temporarily measured on the ground, such as those joining opposite corners of a field; or simply indicated by stakes at their extremities or otherwise. If long, they are "ranged out" by methods to be given.

Lines are usually measured with chains, tapes, or rods, divided into yards, feet, links, or some other unit of measurement.

Fig. 8.



16. Gunter's Chain. This is the measure most commonly used in Land Surveying. It is 66 feet, or 4 rods long.* Eighty such chains make one mile.

It is composed of one hundred pieces of iron or steel wire, or links, each bent at the end into a ring, and connected with the

* This length was chosen (by Mr. Edward Gunter) because 10 square chains of 66 feet make one acre, and the computation of areas is thus greatly facilitated. For other surveying purposes, particularly for railroad work, a chain of 100 feet is preferable. On the United States Coast and Geodetic Survey the unit of measurement is the French *Metre*, equal to 3·281 feet nearly.

ring at the end of the next piece by another ring. Sometimes two or three rings are placed between the links. The chain is then less liable to twist and get entangled or "kinked." Two or more swivels are also inserted in the chain, so that it may turn around without twisting. Every tenth link is marked by a piece of brass, having one, two, three, or four points, corresponding to the number of tens which it marks, counting from the nearest end of the chain.* The middle or fiftieth link is marked by a round piece of brass.

The hundredth part of a chain is called a link.† The great advantage of this is that, since links are decimal parts of a chain, they may be so written down, 5 chains and 43 links being 5·43 chains, and all the calculations respecting chains and links can then be performed by the common rules of decimal arithmetic. Each link is 7·92 inches long, being $= 66 \times 12 \div 100$.

The following table will be found convenient:

CHAINS INTO FEET.			
Chains.	Feet.	Chains.	Feet.
0·01	0·66	1·00	66·
0·02	1·32	2·	132·
0·03	1·98	3·	198·
0·04	2·64	4·	264·
0·05	3·30	5·	330·
0·06	3·96	6·	396·
0·07	4·62	7·	462·
0·08	5·28	8·	528·
0·09	5·94	9·	594·
0·10	6 60	10·	660·
0·20	13·20	20·	1320·
0·30	19·80	30·	1980·
0·40	26·40	40·	2640·
0·50	33·00	50·	3300·
0·60	39·60	60·	3960·
0·70	46 20	70·	4620·
0·80	52·80	80·	5280·
0·90	59·40	90·	5940·
1·00	66·00	100·	6600·

FEET INTO LINKS.			
Feet.	Links.	Feet.	Links.
0·10	0·15	10·	15·2
0·20	0·30	15·	22·7
0·25	0·38	20·	30·3
0·30	0·45	25·	37·9
0·40	0·60	30·	45·4
0·50	0·76	33·	50·0
0·60	0·91	35·	53·0
0·70	1·06	40·	60·6
0·75	1·13	45·	68·2
0·80	1·21	50·	75·8
0·90	1 36	55·	83·3
1·00	1·52	60·	90·9
2·	3 0	65·	98·5
3·	4·5	70·	106·1
4·	6·1	75·	113·6
5·	7·6	80·	121·2
6·	9 1	85·	128·8
7·	10·6	90·	136·4
8·	12·1	95·	143·9
9·	13·6	100·	151·5

* To prevent the very common mistake of calling forty, sixty; or thirty, seventy; it has been suggested to make the 11th, 21st, 31st, and 41st links of *brass*, which would at once show on which side of the middle of the chain was the doubtful mark. This would be particularly useful in Mining Surveying.

† This must not be confounded with the pieces of wire which have the same name, since one of them is shorter than the "link" used in calculation by half a ring or more, according to the way in which the chain is made.

To reduce links to feet, subtract from the number of links as many units as it contains hundreds; multiply the remainder by 2 and divide by 3.

To reduce feet to links, add to the given number half of itself, and add one for each hundred (more exactly, for each ninety-nine) in the sum.

The chain is liable to be lengthened by its rings being pulled open, and to be shortened by its links being bent. It should therefore be frequently tested by a carefully measured length of 66 feet, set out by a standard measure on a flat surface, such as the top of a wall, or on smooth level ground between two stakes, their centers being marked by small nails. It may be left a little longer than the true length, since it can seldom be stretched so as to be perfectly horizontal and not hang in a curve, or be drawn out in a perfectly straight line.* Distances measured with a perfectly accurate chain will always and unavoidably be recorded as longer than they really are. To insure the chain being always strained with the same force, a spring, like that of a spring-balance, is sometimes placed between one handle and the rest of the chain.

If a line has been measured with an incorrect chain, the true length of the line will be obtained by multiplying the number of chains and links in the measured distance by 100, and dividing by the length of the standard distance, as given by measurement of it with the incorrect chain. The proportion here employed is this: *As* the length of the standard given by the incorrect chain *is to* the true length of the standard, *so is* the length of the line given by the measurement *to* the true length. Thus, suppose that a line has been measured with a certain chain, and found by it to be ten chains long, and that the chain is afterward found to have been so stretched that the standard distance measured by it appears to be only 99 links long. The measured line is therefore longer than it had been thought to be, and its true length is obtained by multiplying 10 by 100, and dividing by 99.

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* The chain used by the Government surveyors of France, which is ten metres, or about half a Gunter's chain in length, is made from one fifth to two fifths of an inch longer than the standard. An inaccuracy of one five-hundredth of its length (= $1\frac{1}{2}$ inch on a Gunter's chain) is the utmost allowed not to vitiate the survey.

17. Pins. Ten iron pins, or "arrows," usually accompany the chain.* They are about a foot long, and are made of stout iron wire, sharpened at one end, and bent into a ring at the other. Pieces of red and white cloth should be tied to their heads, so that they can be easily found in grass, dead leaves, etc.

They should be strung on a ring, which has a spring-catch to retain them. Their usual form is shown in Fig. 9. Fig. 10 shows another form, made very large, and therefore very heavy near the point, so that, when held by the top and dropped, it may fall vertically. The uses of this will be seen presently.

Fig. 9.

Fig. 10.

On irregular ground, two stout stakes, about six feet long, are needed to put the forward chain-man in line, and to enable whichever of the two is lowest to raise his end of the chain in a truly vertical line, and to strain the chain straight.

A number of long and slender rods are also necessary for "ranging out" lines between distant points.

18. How to Chain. Two men are required—a forward chain-man and a hind chain-man, or leader and follower. The latter takes the handles of the chain in his left hand, and the chain itself in his right hand, and throws it out in the direction in which it is to be drawn. The former takes a handle of the chain and one pin in his right hand, and the other pins (and the staff, if used), in his left hand, and draws out the chain. The follower then walks beside it, examining carefully that it is not twisted or bent. He then returns to its hinder end, which he holds at the beginning of the line to be measured, puts his eye exactly over it and, by the words "Right," "Left," directs the leader how to put his staff, or the pin which he holds up, "in line," so that it may seem to cover and hide the flag-staff, or other object at the end of the line. The leader all the while keeps the chain tightly stretched, and his

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* Eleven pins are sometimes used, one being of brass. Nine of iron, with four or eight of brass, may also be employed. Their uses are explained in Articles 18 and 19.

end of it touching his staff. Every time he moves the chain, he should straighten it by an undulating shake. When the staff (or pin) is at last put "in line," the follower says "Down." The leader then puts in the single pin precisely at the end of the chain, and replies "Down." The follower then (and never before hearing this signal that the point is fixed) loosens his end of the chain, retaining it in his hand. The leader draws on the chain, making a step to one side of the pin just set, to avoid dragging it out. He should keep his eye steadily on the object ahead, or, in a hollow, should line himself approximately by looking back. The follower should count his steps, so as to know where to look for the pin in high grass, etc. As he approaches the pin, he calls "Halt." On reaching it, he holds the handle of the chain against it, pressing his knee against both to keep the pin firm. He then, with his eye over the pin, "lines" the leader as before. When the "Down" has been again called by the follower, and answered by the leader, the former pulls out the pin with the chain-hand, and carries it in his other hand, and they go on as before.* The operation is repeated till the leader has arrived at the end of the line, or has put down all his pins.

When the leader has put down his tenth pin, he draws on the chain its length farther, and, after being lined, puts his foot on the handle to keep it firm, and calls "Tally." The follower then drops his end of the chain, goes up to the leader and gives him back all the pins, both counting them to make sure that none have been lost. One pin is then put down at the forward end of the chain, and they go on as before.

Some surveyors cause the leader to call "tally" at the tenth pin, and then exchange pins ; but then the follower has only the hole made by the pin, or some other indefinite mark, to measure from.

Eleven pins are sometimes preferred, the eleventh being of brass, or otherwise different from the rest, and being used to mark

* When a chain's length would end in a ditch, pool of water, etc., and the chainmen are afraid of wetting their feet, they can measure part of a chain, to the edge of the water, then stretch the chain across it, and then measure another portion of a chain, so that, with the former portion, it may make up a full chain.

the end of the eleventh chain ; another being substituted for it before the leader goes on.

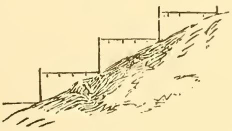
The two chain-men may change duties at each change of pins, if they are of equal skill, but the more careful and intelligent of two laborers should generally be made "follower."

When the leader reaches the end of the line, he stops, and holds his end of the chain against it. The follower drops his end and counts the links beyond the last pin, noting carefully on which side of the "fifty" mark it comes. Each pin now held by the follower, including the one in the ground, represents one chain ; each time "tally" has been called, and the pins exchanged, represents ten chains, and the links just counted make up the total distance.

19. Tallies. In chaining very long distances, there is danger of miscounting the number of "tallies," or tens. To avoid mistakes, pebbles, etc., may be changed from one pocket into another at each change of pins ; or bits of leather on a cord may be slipped from one side to the other ; or knots tied on a string ; but the best plan is the following : Instead of ten iron pins, use nine iron pins, and four, or eight, or ten pins of brass, or very much longer than the rest. At the end of the tenth chain, the iron pins being exhausted, a brass pin is put down by the leader. The follower then comes up, and returns the nine iron pins, but retains the brass one, with the additional advantage of having this pin to measure from. At the end of the twentieth chain, the same operation is repeated ; and so on. When the measurement of the line is completed, each brass pin held by the follower counts ten chains, and each iron pin one, as before.

20. Chaining on Slopes. All the distances employed in Land-surveying must be measured horizontally, or on a level. When the ground slopes, it is therefore necessary to make certain allowances or corrections. If the slope be gentle, hold the up-hill end of the chain on the ground, and raise the down-hill end till the chain is level. To insure the elevated end being exactly over the desired spot, raise it along a staff kept vertical, or drop a pin held by the point with the ring downward (if you have not the heavy pointed ones shown in Fig. 10), or, which is better, use a plumb-line. A person standing beside the chain, and at a little distance from it, can best tell if it be nearly level. If the hill be so steep that a whole chain can not be held up level, use only half or quarter of it at a time. Great care is necessary in this operation. To measure down a steep hill, stretch the whole chain in line. Hold the upper end fast on the ground. Raise up the 20 or 30 link-mark, so that that portion of the chain is level. Drop a plumb-line or pin. Then let the follower come forward and hold down that link on this spot, and the leader hold up another short portion, as before. Chaining down a slope is more accurate than chaining up it, since in the latter case the follower can not easily place his end of the chain exactly over the pin.

Fig. 11.



A more accurate, though more troublesome, method, is to measure the angle of the slope, and make the proper allowance by calculation, or by a table, previously prepared. The correction being found, the chain may be drawn forward the proper number of links, and the correct distance of the various points to be noted will thus be obtained at once, without any subsequent calculation or reduction. If the survey is made with the Transit provided with a vertical circle, the slope of the ground can be measured directly. A "Tangent Scale," for the same purpose, may be formed on the sides of the sights of a Compass. It will be described when the instrument is explained.

In the following table, the first column contains the angle which the surface of the ground makes with the horizon; the second column contains its slope, named by the ratio of the perpendicular to the base; and the third, the correction in links for each chain measured on the slope, i. e., the difference between the hypothenuse, which is the distance measured, and the horizontal base, which is the distance desired.

TABLE FOR CHAINING ON SLOPES.

Angle.	Slope.	Correction in links.	Angle.	Slope.	Correction in links.
3°	1 in 19	0·14	13°	1 in 4½	2·56
4°	1 in 14	0·24	14°	1 in 4	2·97
5°	1 in 11½	0·38	15°	1 in 4	3·41
6°	1 in 9½	0·55	16°	1 in 3¾	3·87
7°	1 in 8	0·75	17°	1 in 3½	4·37
8°	1 in 7	0·97	18°	1 in 3¼	4·89
9°	1 in 6½	1·23	19°	1 in 3	5·45
10°	1 in 6	1·53	20°	1 in 2¾	6·03
11°	1 in 5¼	1·84	25°	1 in 2	9·37
12°	1 in 4¾	2·19	30°	1 in 1¾	13·40

21. Chaining is the fundamental operation in all kinds of Surveying. It has for this reason been very minutely detailed. The "follower" is the most responsible person, and the surveyor will best insure his accuracy by taking that place himself. If he has to employ inexperienced laborers, he will do well to cause them to measure the distance between any two points, and then remeasure it in the opposite direction. The difference of their two results will impress on them the necessity of great carefulness.

To "do up" the chain, take the middle of it in the left hand, and with the right hand take hold of the doubled chain just beyond the second link; double up the two links between your hands, and continue to fold up two double links at a time, laying each pair obliquely across the others, so that when it is all folded up the handles will be on the outside, and the chain will have an hour-glass shape, easy to strap up and to carry.

22. Tape. Though the chain is most usually employed for the principal measurements of Surveying, a *tape-line,* divided on one side into links, and on the other into feet and inches, is more convenient for some purposes. It should be tested very frequently, particularly after getting wet, and the correct length marked on it at every ten feet. A "Metallic Tape," less liable to stretch, is manufactured, in which fine wires form its warp. When the tape is being wound up, it should be passed between two fingers to prevent its twisting in the box, which would make it necessary to unscrew its nut to take it out and untwist it. While in use, it should be made portable by being folded up by arm's lengths, instead of being wound up.

A "Steel Tape," made of a thin ribbon of steel, with the divisions and numbers etched on it, is one of the most accurate measuring instruments. Those intended for accurate measurement have at one end an arrangement for shortening and lengthening the tape to provide for variations in length, due to changes of temperature, and at the other end a level and a spring-balance, so that when measuring the ends of the tape may be held at the same height, and always with the same tension. For methods employed in making accurate measurements, see Part IV.

23. Substitutes for a chain or a tape may be found in leather driving-lines, marked off with a carpenter's rule, or in a cord knotted at the length of every link. A well-made rope (such as a "patent wove line," woven circularly with the strands always straight in the line of the strain), when once well stretched, wetted, and allowed to dry with a moderate strain, will not vary from a chain more than one foot in two thousand, if carefully used.

24. Rods. When unusually accurate measurements are required, rods are employed. They may be of well-seasoned wood, of glass, of iron, etc. They must be placed in line very carefully end to end, or made to coincide in other ways, as will be explained under "Triangular Surveying," in which the peculiarly accurate measurement of one line is required, as all the others are founded upon it.

25. Pacing, sound, and other approximate means, may be used for measuring the length of a line. The *Stadia* and *Gradienter* will be described in Chapter IV.

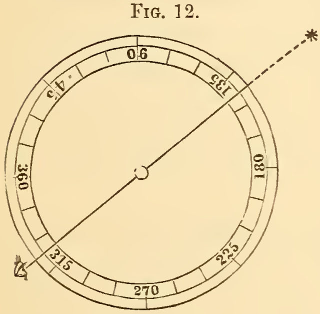
26. A *Perambulator*, or "Measuring-Wheel," is sometimes used for measuring distances, particularly roads. It consists of a wheel which is made to roll over the ground to be measured, and whose motion is communicated to a series of toothed wheels within the machine. These wheels are so proportioned that the index-wheel registers their revolutions, and records the whole distance passed over. If the diameter of the wheel be $31\frac{1}{2}$ inches, the circumference, and therefore each revolution, will be $8\frac{1}{4}$ feet, or half a rod. The roughnesses of the road and the slopes necessarily cause the registered distances to exceed the true measure.

The *Odometer* is an instrument designed to register the number of revolutions of a wagon-wheel. Knowing the circumference of the wheel to which it is attached, and determining the number of revolutions by the odometer, the distance over which the wheel has passed may be approximately determined.

Measuring Angles.

27. The angle made by any two lines—that is, the difference of their directions—may be obtained by a great variety of instruments. All of them are in substance mere modifications of the very simple one which will now be described, and which any one can make for himself:

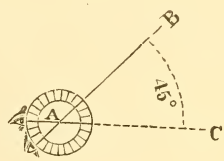
FIG. 12.



Provide a circular piece of wood, and divide its circumference (by any of the methods of Geometrical Drafting) into three hundred and sixty equal parts, or "degrees," and number them as in the figure. The divisions will be like those of a watch-face, but six times as many. These divisions are termed *graduations.* The figure shows only every fifteenth one. In the center of the circle fix a needle, or sharp-pointed wire, and upon this fix a straight stick, or thin ruler placed edgewise (called an *alidade*), so that it may turn freely on this point and nearly touch the graduations of the circle. Fasten the circle on a staff, pointed at the other end, and long enough to bring the alidade to the height of the eyes. The instrument is now complete. It may be called a *Goniometer,* or Angle-measurer.

Now let it be required to measure the angle between the lines A B and A C. Fix the staff in the ground, so that its center shall be exactly over the intersection of the two lines. Turn the alidade so that it points (as determined by sighting along it) to a rod, or other mark at B, a point on one of the lines, and note what degree it covers—i. e., "The Reading." Then, without disturbing the circle, turn the alidade till it points to C, a point on the other line. Note the new reading. The difference of these readings (in the figure, 45 degrees) is the difference in the directions of the two lines, or is the angle which one makes with the other. If the distance from A to C be now measured, the point C is "determined," with respect to the points A and B, on the *Third Principle.* Any number of points may be thus determined.

FIG. 13.



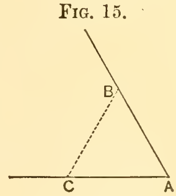
Instead of the very simple and rude alidade, which has been supposed to be used, needles may be fixed on each end of the alidade ; or sights may be added ; or a small straight tube may be used, one end being covered with a piece of pasteboard in which a very small eye-hole is pierced, and threads, called "cross-hairs," being stretched across the other end of it, as in the figure, so that their intersection may give a more precise line for determining the direction of any point.

FIG. 14.

When a telescope is substituted for this tube, and supported in such a way that it can turn over, so as to look both backward and forward, the instrument (with various other additions, which, however, do not affect the principle) is called a *Transit.*

28. Chain Angles. The angle made by any two lines can also be determined without the aid of an angle-measurer. Let it be required to find the angle made by the two lines A B and A C, Fig. 15. Measure off equal distances from A to B and C, and also the "tie-line" B C. It is evident that the tie-line is the chord of the angle to a radius equal to one of the equal distances measured on the sides. Therefore, divide the length of the tie-line by the length of this distance. The quotient will be the chord of the angle to a radius of *one.* In the TABLE OF CHORDS, at the end of this volume, find this quotient, and the number of degrees and minutes corre-

FIG. 15.



sponding to it gives the angle required. Otherwise, since the chord of any angle equals twice the sine of half the angle, we have this rule: Divide half the tie-line by the measured distance, find in a table of natural sines the angle corresponding to the quotient, and multiply this angle by two, to get the angle desired.

Surveying without Instruments.

29. Distances by Pacing. Quite an accurate measurement of a line of ground may be made by walking over it at a uniform pace, and counting the steps taken. But the art of walking in a straight line must first be acquired. To do this, fix the eye on two objects in the desired line, such as two trees, or bushes, or stones, or tufts of grass. Walk forward, keeping the nearest of these objects steadily covering the other. Before getting up to the nearest object, choose a new one in line farther ahead, and then proceed as before, and so on. It is better not to attempt to make each of the paces three feet, but to take steps of the natural length, and to ascertain the value of each by walking over a known distance, and dividing it by the number of paces required to traverse it. Every person should thus determine the usual length of his own steps, repeating the experiment sufficiently often. The French "geographical engineers" accustom themselves to take regular steps of eight tenths of a *metre*, equal to two feet seven and a half inches. The United States military pace is two feet and six inches. This is regarded as a usual average. Quick pacing of 120 such paces per minute gives 3·41 miles per hour. Slow paces, of three feet each and sixty per minute, give 2·04 miles per hour.*

The *Pedometer* is an instrument which counts the steps taken by one wearing it, without any attention on his part. It is made in the form of a watch, and carried in the pocket. The number of the steps given by the pedometer, multiplied by the length of the step, will give approximately any distance walked over. In one form of this instrument the number of steps is registered on a dial up to 2,500.

In another form the instrument is intended to be regulated ac-

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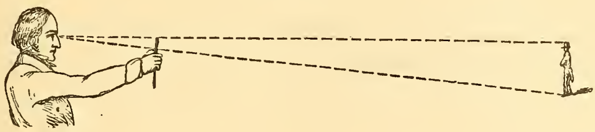
* A horse, on a walk, averages 330 feet per minute, on a trot 650, and on a common gallop 1,040. For longer times, the difference in horses is more apparent.

cording to the length of step of the person carrying it, and then the distance is registered on the dial in miles.

30. Distances by Visual Angles. Prepare a *scale,* by marking off on a pencil what length of it, when it is held off at arm's length, a man's height appears to cover at different distances (previously

Fig. 16.



measured with accuracy) of 100, 500, 1,000 feet, etc. To apply this, when a man is seen at any unknown distance, hold up the pencil at arm's length, making the top of it come in the line from the eye to his head, and placing the thumb-nail in the line from the eye to his feet, as in Fig. 16. The pencil having been previously graduated by the method above explained, the portion of it now intercepted between these two lines will indicate the corresponding distance.

If no previous scale have been prepared, and the distance of a man be required, take a foot-rule, or any measure minutely divided, hold it off at arm's length as before, and see how much a man's height covers. Then, knowing the distance from the eye to the rule, a statement by the rule of three (on the principle of similar triangles) will give the distance required. Suppose a man's height, of 70 inches, covers one inch of the rule. He is then seventy times as far from the eye as the rule, and, if its distance be two feet, that of the man is 140 feet. Instead of a man's height, that of an ordinary house, of an apple-tree, the length of a fence-rail, etc., may be taken as the standard of comparison.

To keep the arm immovable, tie a string of known length to the pencil, and hold between the teeth a knot tied at the other end of the string.

31. Distances by Visibility. The degree of visibility of various well-known objects will indicate approximately how far distant they are. Thus, by ordinary eyes, the windows of a large house can be counted at a distance of about 13,000 feet, or $2\frac{1}{2}$ miles; men and horses will be perceived as points at about half that distance, or $1\frac{1}{4}$ mile; a horse can be clearly distinguished at about 4,000 feet; the movements of men at 2,600 feet, or half a mile; and the head of a man, occasionally, at 2,300 feet, and very plainly at 1,300 feet, or a quarter of a mile. The Arabs of Algeria define a mile as "the distance at which you can no longer distinguish a man from a woman." These distances of visibility will of course vary somewhat with the state of the atmosphere, and still more with individual acuteness of sight, but each person should make a corresponding scale for himself.

32. Distances by Sound. Sound passes through the air with a moderate and known velocity; light passes almost instantaneously. If, then, two distant points be visible from each other, and a gun be fired at night from one of them, an observer at the other, noting by a stop-watch the time at which the flash is seen, and then that at which the report is heard, can tell by the intervening number of seconds how far apart the points are, knowing how far sound travels in a second. Sound moves about 1,098 feet per second in dry air, with the temperature at the freezing-point, 32° Fahr. For higher or lower temperatures add or subtract $1\frac{1}{7}$ foot for each degree of Fahrenheit. If a wind blows with or against the movement of the sound, its velocity must be added or subtracted. If it blows obliquely, the correction will evidently equal its velocity multiplied by the cosine of the angle which the direction of the wind makes with the direction of the sound. If the gun be fired at each end of the base in turn, and the means of the times taken, the effect of the wind will be eliminated.

If a watch is not at hand, suspend a pebble to a string (such as a thread drawn from a handkerchief) and count its vibrations. If it be $39\frac{1}{8}$ inches long, it will vibrate in one second; if $9\frac{3}{4}$ inches long, in half a second, etc. If its length is unknown at the time, still count its vibrations; measure it subsequently; and then will the time of its vibration, in seconds, $= \sqrt{\left(\frac{\text{length of string}}{39\frac{1}{8}}\right)}$.

33. Angles. Right angles are those most frequently required in this kind of survey, and they can be estimated by the eye with much accuracy. If other angles are desired, they will be determined by measuring equal distances along the lines which make the angle, and then the line, or chord, joining the ends of these distances, thus forming chain-angles, explained in Article 28.

Noting the Measurements.

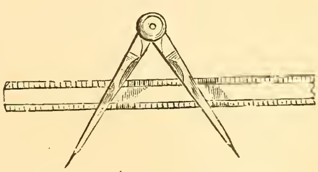
34. The measurements which have been made, whether of lines or of angles, require to be very carefully noted and recorded. Clearness and brevity are the points desired. Different methods of notation are required for each of the systems of surveying which are to be explained, and will therefore be given in their appropriate places.

DRAWING THE MAP.

35. A Map of a survey represents the lines which bound the surface surveyed, and the objects upon it, such as fences, roads, rivers, houses, woods, hills, etc., in their true relative dimensions and positions. It is a miniature copy of the field, farm, etc., as it would be seen by an eye moving over it; or as it would appear, if, from every point of its irregular surface, plumb-lines were dropped to a level surface under it, forming what is called, in geometrical language, its *horizontal projection.*

36. Platting. A *plat* of a survey is a skeleton, or outline map. It is a figure "similar" to the original, having all its angles equal and its sides proportional. Every inch on it represents a foot, a yard, a rod, a mile, or some other length, on the ground; all the measured distances being diminished in exactly the same ratio.

FIG. 17.



PLATTING *is repeating on paper, to a smaller scale, the measurements which have been made on the ground.*

Its various operations may therefore be reduced, in accordance with the principles established in this chapter, to two, viz.: drawing a straight line in a given direction and of a given length;

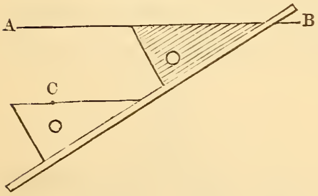
and describing an arc of a circle with a radius whose length is also given. The only instruments absolutely necessary for this are a straight ruler and a pair of "dividers" or "compasses." Others, however, are often convenient, and will be now briefly noticed.

37. Straight Lines. These are usually drawn by the aid of a straight-edged ruler. But to obtain a very long straight line upon paper, stretch a fine silk thread between any two distant points, and mark in its line various points near enough together to be afterward connected by a common ruler. The thread may also be blackened with burned cork and snapped on the paper, as a carpenter snaps his chalk-line; but this is liable to inaccuracies, from not raising the line vertically.

38. Arcs. The arcs of circles used in fixing the position of a point on paper are usually described with compasses, one leg of which carries a pencil-point. A convenient substitute is a strip of pasteboard, through one end of which a fine needle is thrust into the given center, and through a hole in which, at the desired distance, a pencil-point is passed, and can thus describe a circle about the center, the pasteboard keeping it always at the proper distance. A string is a still readier, but less accurate, instrument.

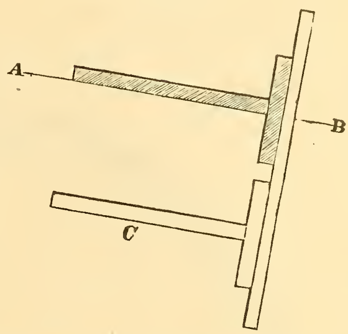
39. Parallels. The readiest mode of drawing parallel lines is by the aid of a triangular piece of wood and a ruler. Let A B be the line to which a parallel is to be drawn, and C the point through which it must pass. Place one side of the triangle against the line, and place the ruler against another side of the triangle. Hold the ruler firm and immovable, and slide the triangle along it till the side of the triangle which had coincided with the given line passes through the given point. This side will then be parallel to that given line, and a line drawn by it will be the line required.

Fig. 18.



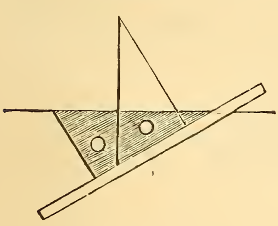
Another easy method of drawing parallels is by means of a T-square, an instrument very valuable for many other purposes. It is nothing but a ruler let into a thicker piece of wood, very truly at right angles to it. For this use of it, one side of the cross-piece must be even or "flush" with the ruler. To use it, lay it on the paper so that one edge of the ruler coincides with the given line A B. Place another ruler against the cross-piece, hold it firm, and slide the T-square along till its edge passes through the given point C, as shown by the lower part of the figure. Then draw by this edge the desired line parallel to the given line.

Fig. 19.



40. Perpendiculars. These may be drawn by the various problems given in Geometry, but more readily by a triangle which has one right angle. Place the longest side of the triangle on the given line, and place a ruler against a second side of the triangle. Hold the ruler fast, and turn the triangle so as to bring its third side against the ruler. Then will the long side be perpendicular to the given line. By sliding the triangle along the ruler, it may be used to draw a perpendicular from any point of the line, or from any point to the line.

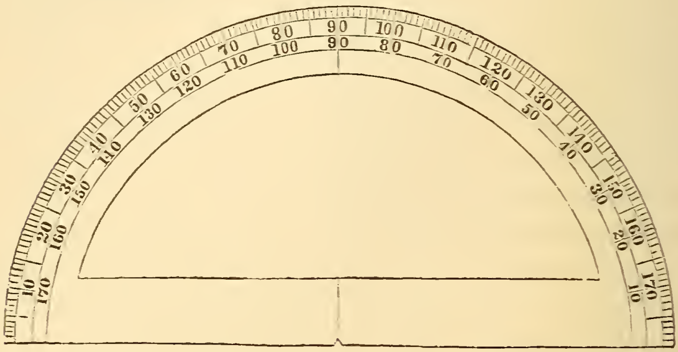
Fig. 20.



41. Angles. These are most easily set out with an instrument called a Protractor. This is usually a semicircle of brass, as in the figure, with its semi-circumference divided into 180 equal parts, or degrees, and numbered in both directions. It is, in fact, a miniature of the instrument (or of half of it) with which the angles have been measured. To lay off any angle at any point of a straight line, place the protractor so that its straight side, the diameter of the semicircle, is on the given line, and the middle of this diameter, which is marked by a notch, is at the given point. With a needle or sharp pencil make a mark on the paper at the required number of degrees, and draw a line from the mark to the given point.

Sometimes the protractor has an arm turning on its center and

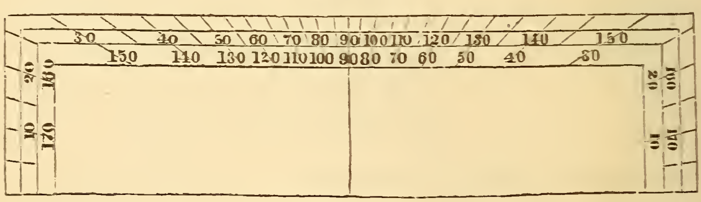
Fig. 21.



extending beyond its circumference, so that a line can be at once drawn by it when it is set to the desired angle. A Vernier scale is sometimes added to it to increase its precision.

A Rectangular Protractor is sometimes used, the divisions of degrees being engraved along three edges of a plane scale. The

Fig. 22.



semicircular one is preferable. The objection to the rectangular protractor is that the division corresponding to a degree is very unequal on different parts of the scale, being usually two or three times as great at its ends as at its middle.

A Protractor embracing an entire circle, with arms carrying verniers, is also sometimes employed, for the sake of greater accuracy.

42. Drawing to Scale. The operation of drawing on paper lines whose length shall be a half, a quarter, a tenth, or any other fraction of the lines measured on the ground, is called "Drawing to Scale."

To set off on a line any given distance to any required scale, determine the number of chains or links which each division of the scale of equal parts shall represent. Divide the given distance by this number. The quotient will be the number of equal parts to be taken in the dividers and to be set off.

For example, suppose the scale of equal parts to be a common carpenter's rule divided into inches and eighths. Let the given distance be twelve chains, which is to be drawn to a scale of two chains to an inch. Then six inches will be the distance to be set off. If the given distance had been twelve chains and seventy-five links, the distance to be set off would have been six inches and three eighths, since each eighth of an inch represents twenty-five links.

If the desired scale were three chains to an inch, each eighth of an inch would represent $37\frac{1}{2}$ links; and the distance of 1,275 links would be represented by thirty-four eighths of an inch, or $4\frac{1}{4}$ inches.

A similar process will give the correct length to be set off for any distance to any scale.

If the scale used had been divided into inches and tenths, as is much the most convenient, the above distances would have become on the former scale $6\frac{37}{100}$ inches, or nearly $6\frac{4}{10}$ inches; and on the latter scale $4\frac{25}{100}$ inches, coming midway between the second and third tenth of an inch.

Conversely, to find the real length of a line drawn on paper to any known scale, reverse the preceding operation. Take the length of the line in the dividers, apply it to the scale, and count how many equal parts it includes. Multiply their number by the number of chains or links which each represents, and the product will be the desired length of the line on the ground.

This operation and the preceding one are greatly facilitated by the use of the scales to be described in Art. 47.

43. Scales. The choice of the scale to which a plat should be drawn—that is, how many times smaller its lines shall be than those which have been measured on the ground—is determined by several considerations. The chief one is that it shall be just large enough to express clearly all the details which it is desirable to know. A Farm Survey would require its plat to show every field and building. A State Survey would show only the towns, rivers, and leading roads. The size of the paper at hand will also limit the scale to be adopted. If the content is to be calculated from the plat, that will forbid it to be less than 3 chains to 1 inch.

Scales are named in various ways. *They should always be expressed fractionally*—i. e., they should be so named as to indicate what fractional part of the real line measured on the ground, the representative line drawn on the paper, actually is. When custom requires a different way of naming the scale, both should be given. It would be still better if the denominator could always be some power of 10, or at least some multiple of 2 and 5, such as $\frac{1}{500}$, $\frac{1}{1000}$, $\frac{1}{2000}$, $\frac{1}{2500}$, etc. For convenience in printing, these may be written thus: 1 : 500, 1 : 1,000, 1 : 2,000, 1 : 2,500, etc.

Plats of *Farm Surveys* are usually named as being so many chains to an inch.

Maps of *Surveys of States* are generally named as being made to a scale of so many miles to an inch.

Maps of *Railroad Surveys* are said to be so many feet to an inch, or so many inches to a mile.

44. Farm Surveys. If these are of small extent, two chains to one inch (which is $= \frac{1}{2 \times 66 \times 12} = \frac{1}{1584} = 1:1,584$) is convenient.

A scale of one chain to one inch (1 : 792) is useful for plans of buildings. Three chains to one inch (1 : 2,376) is suitable for larger farms. It is the scale prescribed by the English Tithe Commissioners for their first-class maps.

In France, the *Cadastre* Surveys are lithographed on a scale about equivalent to this, being 1 : 2,500. The original plans are drawn to a scale of 1 : 5,000. Plans for the division of property are made on the former scale. When the district exceeds 3,000 acres, the scale is 1 : 10,000. When it exceeds 7,500 acres, the scale is 1 : 20,000. A common scale in France for small surveys is 1 : 1,000, about 1½ chain to 1 inch.

45. State Surveys. On these surveys smaller scales are necessarily employed.

On the *United States* Coast and Geodetic Survey all the scales are expressed fractionally and decimally. "The surveys are generally platted originally on a scale of one to ten or twenty thousand, but in some instances the scale is larger or smaller.

"These original surveys are reduced for engraving and publication, and, when issued, are embraced in three general classes: 1, small harbor-charts; 2, charts of bays and sounds; and, 3, the General Coast Charts.

"The scales of the first class vary from 1 : 10,000 to 1 : 60,000, according to the nature of the harbor and the different objects to be represented.

"Where there are many shoals, rocks, or other objects, as in Nantucket Harbor and Hell Gate, or where the importance of the harbor makes it necessary, a larger scale of 1 : 5,000, 1 : 10,000, and 1 : 20,000 is used. But where, from the size of the harbor or its ease of access, a smaller one will point out every danger with sufficient exactness, the scales of 1 : 40,000 and 1 : 60,000 are used, as in the case of New Bedford Harbor, Cat and Ship Island Harbor, New Haven, etc.

"The scale of the second class, in consequence of the large areas to be represented, is usually fixed at 1 : 80,000, as in the case of New York Bay, Delaware Bay and River. Preliminary charts, however, are issued of various scales from 1 : 80,000 to 1 : 200,000.

"Of the third class, the scale is fixed at 1 : 400,000 for the General Chart of the Coast from Gay Head to Cape Henlopen, although considerations of the proximity and importance of points on the coast may change the scales of charts of other portions of our extended coast."

The National Survey of *Great Britain* is called, from the corps employed on it, the "Ordnance Survey."

The "Ordnance Survey" of the southern counties of England was platted on a scale of 2 inches to 1 mile (1 : 31,680), and reduced for publication to that of 1 inch to a mile (1 : 63,360). The scale of 6 inches to a mile (1 : 10,560) was adopted for the northern counties of England and for the southern counties of Scotland. The same scale was employed for platting and engraving in outline the "Ordnance Survey" of Ireland. But a map on a scale of 1 inch to 1 mile (1 : 63,360) is now published, the former scale rendering the maps too unwieldy and cumbrous for consultation.

The Ordnance Survey of Scotland was at first platted on a scale of 6 inches to 1 mile (1 : 10,560). That scale has since been abandoned, and it is now platted on a scale of 2 inches to 1 mile (1 : 31,680), and the general maps are made to only half that scale.

The Ordnance Survey scale for the maps of London and other large towns is 5 feet to 1 mile (1 : 1,056), or 1¼ chain to 1 inch.

In the "Surveys under the Public Health Act" of England, the scale for the general plan is 2 feet to 1 mile (1 : 2,640); and for the detailed plan 10 feet per mile (1 : 528), or ⅛ of a chain per inch.

The Government Survey of *France* is platted to a scale of 1 : 20,000. Copies are made to 1 : 40,000; and the maps are engraved to a scale of 1 : 80,000, or about ¾ of an inch to 1 mile.

Cassini's famous map of France was on a scale of 1 : 86,400.

The French War Department employ the scales of 1 : 10,000, 1 : 20,000, 1 : 40,000, and 1 : 80,000 for the topography of France.

46. Railroad Surveys. For these the New York Railroad Law of 1880 directs the horizontal scale of maps which are to be filed in the State Engineer's Office to be 500 feet to $\frac{1}{10}$ of a foot (= 1 : 5,000), and vertical scale for profiles to be 100 feet to $\frac{1}{10}$ of a foot (= 1 : 1,000).

For the New York Canal Maps a horizontal scale of 2 chains to 1 inch (1 : 1,584), and a vertical scale of 20 feet to 1 inch, are employed.

The parliamentary "standing orders" prescribe the plans of railroads, prepared for parliamentary purposes, to be made on a scale of not less than 4 inches to the mile (1 : 15,840); and the enlarged portions (as of gardens, court-yards, etc.) to be on a scale not smaller than 400 feet to the inch (1 : 4,800). Accordingly, the practice of English railway engineers is to draw the whole plan to a scale of 6 chains, or 396 feet to the inch (1 : 4,752), as being just within the parliamentary limits.

In France, the engineers of "Bridges and Roads" (Corps des Ponts et Chaussées) employ for the general plan of a road a scale of 1 : 5,000, and for appropriations, 1 : 500.

In the United States Engineer Service the following plans are prescribed: General plans of buildings, 1 inch to 10 feet (1 : 120).

Maps of grounds, with horizontal curves one foot apart, 1 inch to 50 feet (1 : 600).

Topographical maps, one mile and a half square, 2 feet to 1 mile (1 : 2,640).

Do., comprising three miles square, 1 foot to one mile (1 : 5,280).

Do., between four and eight miles square, 6 inches to one mile (1 : 10,560).

Do., comprising nine miles square, 4 inches to one mile (1 : 15,840).

Maps not exceeding 24 miles square, 2 inches to one mile (1 : 31,680).

Maps comprising 50 miles square, 1 inch to one mile (1 : 63,360).

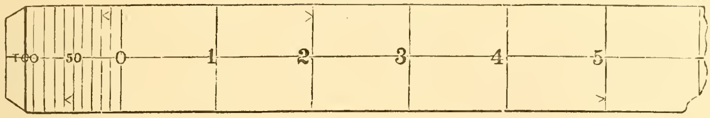
Maps comprising 100 miles square, $\frac{1}{2}$ inch to one mile (1 : 126,720).

Surveys of roads, canals, etc., 1 inch to 50 feet (1 : 600).

47. The most convenient scales of equal parts are those of box-wood, or ivory, which have a *fiducial* or feather edge, along which they are divided, so that distances can be at once marked off from this edge, without requiring to be taken off with the dividers; or the length of a given line can be at once read off. Box-wood is preferable to ivory, as much less liable to warp, or to vary in length with changes in the moisture in the air.

The student can, however, make for himself platting-scales of drawing-paper, or Bristol board. Cut a straight strip of this material, about an inch wide. Draw a line through its middle, and set off on it a number of equal parts, each representing a chain to the desired scale. Subdivide the left-hand division into ten equal

FIG. 23.



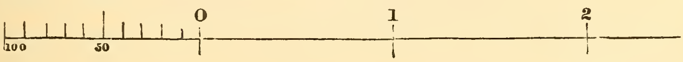
parts, each of which will therefore represent ten links to this scale. Through each point of division on the central line, draw (with the T-square) perpendiculars extending to the edges, and the scale is made. It explains itself. The above figure is a scale of 2 chains to 1 inch. On it the distance 220 links would extend between the arrow-heads above the line in the figure; 560 links extend between the lower arrow-heads, etc.

A paper scale has the great advantage of varying less from a plat which has been made by it, in consequence of changes in the weather, than any other. The mean of many trials showed the difference between such a scale and drawing-paper, when exposed alternately to the damp open atmosphere, and to the air of a warm dry room, to be equal to ·005, while that between box-wood scales and the paper was ·012, or nearly 2½ times as much. The difference with ivory would have been even greater.

Some of the more usual platting-scales are here given in their actual dimensions.

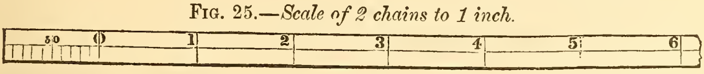
In these five figures, different methods of drawing the scales

FIG. 24.—*Scale of 1 chain to 1 inch.*



have been given, but each method may be applied to any scale. The first and second, being the most simple, are generally the best. In the third the subdivisions are made by a diagonal line: the dis-

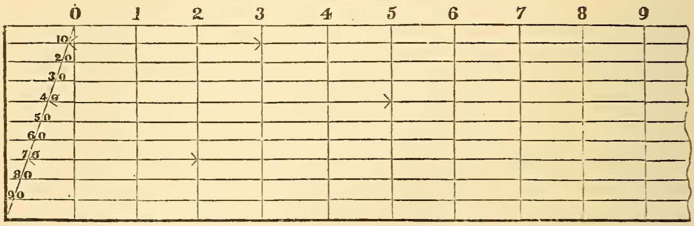
FIG. 25.—*Scale of 2 chains to 1 inch.*



tances between the various pairs of arrow-heads, beginning with the uppermost, are respectively 310, 540, and 270 links.

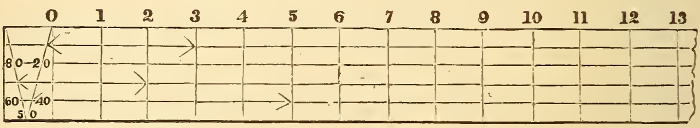
In the fourth figure, the distances between the arrow-heads are respectively 310, 270, and 540 links.

Fig. 26.—*Scale of 3 chains to 1 inch.*



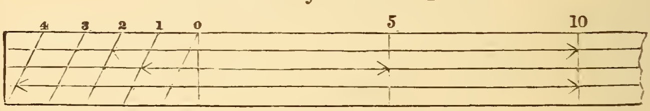
In the fifth figure, the scale of 5 chains to 1 inch is subdivided diagonally to only every quarter-chain, or 25 links. The distance

Fig. 26¹.—*Scale of 4 chains to 1 inch.*



between the upper pair of arrow-heads on it is $12\frac{1}{4}$ chains, or 12·25; between the next pair of arrow-heads it is 6·50; and between the lower pair 14·75.

Fig. 27.—*Scale of 5 chains to 1 inch.*

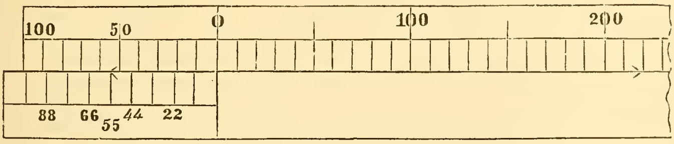


A diagonal scale for dividing an inch, or half an inch, into 100 equal parts, is found on the "plain scale" in every case of instruments.

48. Vernier Scale. This is an ingenious substitute for the diagonal scale. The one given in the following figure divides an inch into 100 equal parts, and, if each inch be supposed to represent a chain, it gives single links.

Make a scale of an inch divided into tenths, as in the upper scale of the above figure. Take in the dividers eleven of these divisions, and set off this distance from the 0 of the scale to the left of it. Divide the distance thus set off into 10 equal parts. Each of them will be one tenth of eleven tenths of one inch, i. e.,

FIG. 28.



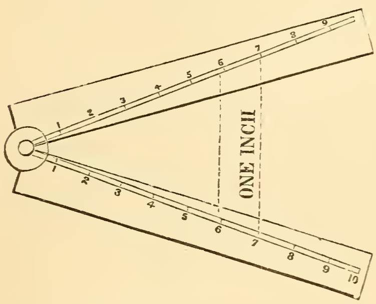
eleven hundredths, or a tenth and a hundredth, and the first division on the short, or vernier scale, will overlap, or be longer than the first division on the long scale, by just *one* hundredth of an inch; the second division will overlap *two* hundredths, and so on. The principle will be more fully developed in treating of "Verniers."

Now, suppose we wish to take off from this scale 275 hundredths of an inch. To get the last figure, we must take five divisions on the lower scale, which will be 55 hundredths, for the reason just given; 220 will remain, which are to be taken from the upper scale, and the entire number will be obtained at once by extending the dividers between the arrow-heads in the figure from 220 on the upper scale (measuring along its lower side) to 55 on the lower scale; 254 would extend from 210 on the upper scale to 44 on the lower; 318 would extend from 230 on the upper scale to 88 on the lower. Always begin then with subtracting 11 times the last figure from the given number; find the remainders on the upper scale, and the number subtracted on the lower scale.

49. A plat is sometimes made by a nominally reduced scale in the following manner: Suppose that the scale of the plat is to be ten chains to one inch, and that a diagonal scale of inches, divided into tenths and hundredths, is the only one at hand. By dividing all the distances by ten, this scale can then be used without any further reduction. But if the content is measured from the plat to the same scale, in the manner explained in the next chapter, the result must be multiplied by 10 times 10. This is called by old surveyors "raising the scale," or "restoring true measure."

50. Sectoral Scales. The *Sector* (called by the French "Compass of Proportion") is an instrument sometimes convenient for obtaining a scale of equal parts. It is in two portions, turning on a hinge, like a carpenter's pocket-rule. It contains a great number of scales, but the one intended for this use is lettered at its ends L in English instruments, and consists of two lines running from the center to the ends of the scale, and each divided into ten equal parts, each of which is again subdivided into ten, so that each leg of the scale contains 100 equal parts. To illustrate its use, suppose that a scale of 7 chains to 1 inch is required. Take 1 inch in the dividers, and open the sector till this distance will just reach from the 7 on one leg to the 7 on the other. The sector is then "*set*" for this scale, and the angle of its opening must not be again changed. Now let a distance of 580 links be required. Open the dividers till they reach from 58 to 58 on the two legs, as in the dotted line in the figure, and it is the required distance. Again, suppose that a scale of $2\frac{1}{2}$ chains to 1 inch is desired. Open the sector so that 1 inch shall extend from 25 to 25. Any other scale may be obtained in the same manner.

FIG. 29.



Conversely, the length of any known line to any desired scale can thus be readily determined.

51. Whatever scale may be adopted for platting the survey, it should be drawn on the map, both for convenience of reference and in order that the contraction and expansion caused by changes in the quantity of moisture in the atmosphere may affect the scale and the map alike. When the drawing-paper has been wet and glued to a board, and cut off when the map is completed, its contractions have been found by many observations to average from one fourth to one half per cent on a scale of 3 chains to an inch

(1 : 2,376), which would therefore require an allowance of from one half perch to one perch per acre.

A scale made as directed in Art. 47, if used to make a plat on unstretched paper, and then kept with the plat, will answer nearly the same purpose.

Such a scale may be attached to a map by slipping it through two or three cuts in the lower part of the sheet, and will be a very convenient substitute for a pair of dividers in measuring any distance upon it.

52. Scale omitted. It may be required to find the unknown scale to which a given map has been drawn, its superficial content being known. Assume any convenient scale, measure the lines of the map by it, and find the content by the methods to be given in the next chapter, proceeding as if the assumed scale were the true one. Then make this proportion, founded on the geometrical principle that the areas of similar figures are as the squares of their corresponding sides : *As* the content found *is* to the given content, *so is* the square of the assumed scale *to* the square of the true scale.

CALCULATING THE CONTENT.

53. The CONTENT of a piece of ground is its superficial *area,* or the number of square feet, yards, acres, or miles which it contains.

54. Horizontal Measurement. All ground, however inclined or uneven its surface may be, should be measured horizontally, or as if brought down to a horizontal plane, so that the surface of a hill, thus measured, would give the same content as the level base on which it may be supposed to stand, or as the figure which would be formed on a level surface beneath it by dropping plumb-lines from every point of it.

This method of procedure is required for both geometrical and social reasons.

Geometrically, it is plain that this horizontal measurement is absolutely necessary for the purpose of obtaining a correct plat. In Fig. 30, let A B C D and B C E F be two square lots of ground, platted horizontally. Suppose the ground to slope in all directions from the point C, which is the summit of a hill. Then the lines B C, D C, measured on the slope, are longer than if measured on a level, and the field A B C D, of Fig. 30, platted with these long lines, would take the shape A B G D in Fig. 31; and the field B C E F, of Fig. 30, would become B H E F, of Fig. 31. The two adjoining fields would thus overlap each other; and the same difficulty would occur in every case of platting any two adjoining fields by the measurements made on the slope.

Fig. 30.

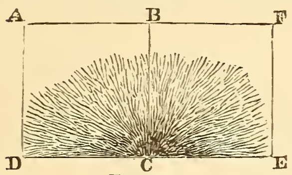
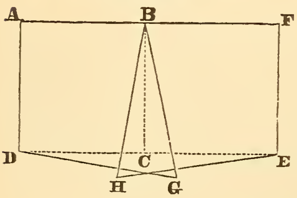


Fig. 31.



Let us suppose another case, more simple than would ever occur in practice, that of a three-sided field, of equal sides, and composed of three portions, each sloping down uniformly (at the rate of one to one) from one point in the center, as in Fig. 32. Each slope being accurately platted, the three could not come together, but would be separated as in Fig. 33.

Fig. 32.

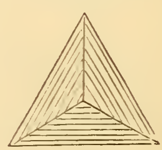
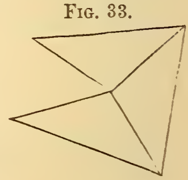


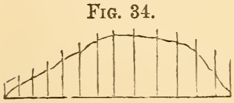
Fig. 33.



We have here taken the most simple cases, those of uniform slopes. But with the common irregularities of uneven ground, to measure its actual surface would not only be improper, but impossible.

In the *social* aspect of this question, the horizontal measurement is justified by the fact that no more houses can be built on a hill than could be built on its flat base; and that no more trees, corn, or other plants, which shoot up vertically, can grow on it; as is represented by the vertical lines in the figure.* Even if a side-hill

Fig. 34.



* This question is more than two thousand years old, for Polybius writes: "Some even of those who are employed in the administration of states, or placed at the head

should produce more of certain creeping plants, the increased difficulty in their cultivation might perhaps balance this. For this reason the surface of the soil thus measured is sometimes called *the productive base* of the ground.

Again, a piece of land containing a hill and a hollow, if measured on the surface, would give a larger content than it would after the hollow had been filled up by the hill, while it would yet really be of greater value than before.

Horizontal measurement is called the "Method of Cultellation," and superficial measurement the "Method of Development." *

An act of the State of New York prescribes that "the acre, for land-measure, shall be measured horizontally."

55. Unit of Content. The *Acre* is the unit of land-measurement. It contains 4 Roods. A *Rood* contains 40 Perches. A *Perch* is a square Rod; otherwise called a Pole. A *Rod* is $5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet.

Hence, 1 Acre = 4 Roods = 160 Perches = 4,840 square yards = 43,560 square feet.

One square mile = 5,280 × 5,280 feet = 640 acres.

Since a chain is 66 feet long, a square chain contains 4,356 square feet; and, consequently, *ten square chains make one acre.*†

The French units of land-measure are the *Are* = 100 square *Metres* = 0·0247 acre = one fortieth of an acre, nearly; and the *Hectare* = 100 *Ares* = 2·47 acres, or nearly two and a half. Their old land-measures were the "Arpent of Paris," containing 36,800 square feet; and the "Arpent of Waters and Woods," containing 55,000 square feet.

56. When the content of a piece of land (obtained by any of the methods to be explained presently) is given in square links, as is

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of armies, imagine that unequal and hilly ground will contain more houses than a surface which is flat and level. This, however, is not the truth. For, the houses, being raised in a vertical line, form right angles, not with the declivity of the ground, but with the flat surface which lies below, and upon which the hills themselves also stand."

* The former from *cultellum*, a knife, as if the hills were sliced off; the latter so named because it strips off or unfolds, as it were, the surface.

† Let the young student beware of confounding 10 square chains with 10 chains square. The former make one acre; the latter space contains ten acres.

customary, cut off four figures on the right (i. e., divide by 10,000) to get it into square chains and decimal parts of a chain; cut off the right-hand figure of the *square chains,* and the remaining figures will be *Acres.* Multiply the remainder by 4, and the figure, if any, outside of the new decimal-point will be *Roods.* Multiply the remainder by 40, and the outside figures will be *Perches.* The nearest round number is usually taken for the Perches; fractions less than a half-perch being disregarded.*

Thus,	86·22 square chains	= 8 Acres	2 Roods	20 Perches.
Also,	64·1818 do.	= 6 A.	1 R.	27 P.
"	43·7564 do.	= 4 A.	1 R.	20 P.

57. Chain Correction. When a survey has been made, and the plat has been drawn, and the content calculated; and afterward the chain is found to have been incorrect, too short or too long, the true content of the land may be found by this proportion: *As* the square of the length of the standard given by the incorrect chain *is to* the square of the true length of the standard, *so is* the calculated content *to* the true content. Thus, suppose that the chain used had been so stretched that the standard distance measured by it appears to be only 99 links long; and that a square field had been measured by it, each side containing 10 of these long chains, and that it had been so platted. This plat, and therefore the content calculated from it, will be smaller than it should be, and the correct content will be found by the proportion $99^2 : 100^2 :: 100$ square chains : 102·03 square chains. If the chain had been stretched so as to be 101 *true* links long, as found by comparing it with a correct chain, the content would be given by this proportion: $100^2 : 101^2 :: 100$ square chains : 102·01 square chains. In the former case, the elongation of the chain was $1\frac{1}{99}$ true links; and $100^2 : (101\frac{1}{99})^2 :: 100$ square chains : 102·03 square chains.

58. Boundary-Lines. The lines which are to be considered as bounding the land to be surveyed are often very uncertain, unless specified by the title-deeds.

* To reduce square yards to acres, instead of dividing by 4,840, it is easier, and very nearly correct, to multiply by 2, cut off four figures, and add to this product one third of one tenth of itself.

If the boundary be a brook, the middle of it is usually the boundary-line. On tide-waters, the land is usually considered to extend to low-water mark.

Where hedges and ditches are the boundaries of fields, as is almost universally the case in England, the dividing line is generally the top edge of the ditch farthest from the hedge, both hedge and ditch belonging to the field on the hedge side. This varies, however, with the customs of the locality. From three to six feet from the roots of the quick-wood of the hedges are allowed for the ditches.

Methods of Calculation.

59. The various methods employed in calculating the content of a piece of ground may be reduced to four, which may be called *Arithmetical, Geometrical, Instrumental,* and *Trigonometrical.*

60. FIRST METHOD.—ARITHMETICALLY. *From direct measurements of the necessary lines on the ground.*

The figures to be calculated by this method may be either the shapes of the fields which are measured, or those into which the fields can be divided by measuring various lines across them.

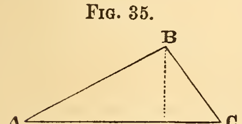
The familiar rules of mensuration for the principal figures which occur in practice will be now briefly enunciated.

61. Rectangles. If the piece of ground be rectangular in shape, its content is found by multiplying its length by its breadth.

62. Triangles. When the given quantities are one side of a triangle and the perpendicular distance to it from the opposite angle, the content of the triangle is equal to half the product of the side and the perpendicular.

When the given quantities are the three sides of the triangle, add together the three sides and divide the sum by 2; from this half sum subtract each of the three sides in turn; multiply together the half sum and the three remainders; take the square root of the product; it is the content required. If the sides of the triangle be designated by a, b, c, and their sum by s, this rule will give its area $= \sqrt{[\frac{1}{2}s\,(\frac{1}{2}s - a)\,(\frac{1}{2}s - b)\,(\frac{1}{2}s - c)]}$.

FIG. 35.



When two sides of a triangle and the included angle are given, its content equals half the product of its sides into the sine of the included angle. Designating the angles of the triangle by the capital letters A, B, C, and the sides opposite them by the corresponding small letters a, b, c, the area $= \frac{1}{2}\, b\, c \sin. A$.

When one side of a triangle and the adjacent angles are given, its content equals the square of the given side multiplied by the sines of each of the given angles, and divided by twice the sine of the sum of these angles. Using the same symbols as before, the area $= a^2 \dfrac{\sin. B \,.\, \sin. C.}{2 \sin. (B + C)}$.

When the three angles of a triangle and its altitude are given, its area, referring to the above figure, $= \frac{1}{2}\, B\, D^2 \,.\, \dfrac{\sin. B}{\sin. A \,.\, \sin. C}$.

63. Parallelograms, or four-sided figures whose opposite sides are parallel. The content of a Parallelogram equals the product of one of its sides by the perpendicular distance between it and the side parallel to it.

64. Trapezoids, or four-sided figures, two opposite sides of which are parallel. The content of a Trapezoid equals half the product of the sum of the parallel sides by the perpendicular distance between them.

If the given quantities are the four sides a, b, c, d, of which b and d are parallel; then, making $q = \frac{1}{2}(a + b + c - d)$, the area of the trapezoid will $= \dfrac{b + d}{b - d} \sqrt{[q\,(q - a)\,(q - c)\,(q - b + d)]}$.

When two parallel sides, b and d, and a third side, a, are given, and also the angle C, which this third side makes with one of the parallel sides, then the content of the trapezoid $= \dfrac{b + d}{2} \,.\, a \,.\, \sin. C$.

65. Trapeziums; four-sided figures, none of whose sides are parallel.

A very gross error, often committed as to this figure, is to take the average, or half sum of its opposite sides, and multiply them together for the area: thus, assuming the trapezium to be equivalent to a rectangle with these averages for sides.

In practical surveying, it is usual to measure a line across it from corner to corner, thus dividing it into two triangles, whose sides are known, and which can therefore be calculated by Art. 62.

When two opposite sides, and all the angles are given, take one side and its adjacent angles (or their supplements, when their sum exceeds 180°), consider them as belonging to a triangle, and find its area by the second formula in Art. 62. Do the same with the other side and its adjacent angles. The difference of the two areas will be the area of the quadrilateral.

When three sides and their two included angles are given, multiply together the sine of one given angle and its adjacent sides. Do the same with the sine of the other given angle and its adjacent sides. Multiply together the two opposite sides and the sine of the supplement of the sum of the given angles. Add together the first two products, and add also the last product, if the sum of the given angles is more than 180°, or subtract it if this sum be less, and take half the result. Calling the given sides p, q, r, and the angle between p and $q = \mathrm{A}$; and the angle between q and $r = \mathrm{B}$; the area of the quadrilateral

$$= \tfrac{1}{2}\,[p\,.\,q \sin.\ \mathrm{A} + q\,.\,r\,.\sin.\ \mathrm{B} \pm p\,.\,r \sin.\ (180^\circ - \mathrm{A} - \mathrm{B})].$$

When the four sides and the sum of any two opposite angles are given, proceed thus: Take half the sum of the four given sides, and from it subtract each side in turn. Multiply together the four remainders, and reserve the product. Multiply together the four sides. Take half their product, and multiply it by the cosine of the given sum of the angles increased by unity. Regard the sign of the cosine. Subtract this product from the reserved product, and take the square root of the remainder. It will be the area of the quadrilateral.

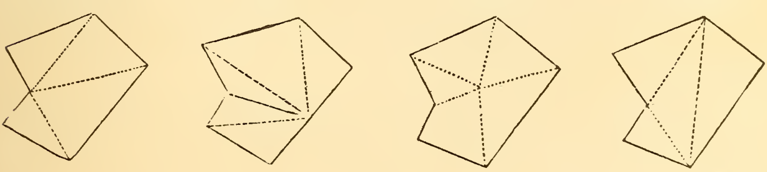
When the four sides and the angle of intersection of the diagonals of the quadrilateral are given, square each side; add together the squares of the opposite sides; take the difference of the two sums; multiply it by the tangent of the angle of intersection, and divide by four. The quotient will be the area.

When the diagonals of the quadrilateral and their included angle are given, multiply together the two diagonals and the sine of their included angle, and divide by two. The quotient will be the area.

66. SECOND METHOD.—GEOMETRICALLY. *From measurements of the necessary lines upon the plat.*

67. Division into Triangles. The plat of a piece of ground having been drawn from the measurements made by any of the methods which will be hereafter explained, lines may be drawn upon the plat so as to divide it into a number of triangles. Four ways of doing this are shown in the figures, viz.: by drawing lines

FIG. 36. FIG. 37. FIG. 38. FIG. 39.



from one corner to the other corners; from a point in one of the sides to the corners; from a point inside of the figure to the corners; and from various corners to other corners. The last method is usually the best. The lines ought to be drawn so as to make the triangles as nearly equilateral as possible.

One side of each of these triangles, and the length of the perpendicular let fall upon it, being then measured, the content of these triangles can be at once obtained by multiplying their base by their altitude, and dividing by two.

The easiest method of getting the length of the perpendicular, without actually drawing it, is to set one point of the dividers at the angle from which a perpendicular is to be let fall, and to open and shut their legs till an arc described by the other point will just touch the opposite side.

Otherwise, a platting scale may be placed so that the zero-point of its edge coincides with the angle, and one of its cross-lines coincides with the side to which a perpendicular is to be drawn. The length of the perpendicular can then at once be read off.

The method of dividing the plat into triangles is the one most commonly employed by surveyors for obtaining the content of a survey, because of the simplicity of the calculations required. Its correctness, however, is dependent on the accuracy of the plat, and on its scale, which should be as large as possible. Three chains to an inch is the smallest scale allowed by the English Tithe Commissioners for plats from which the content is to be determined.

In calculating in this way the content of a farm, and also of its separate fields, the sum of the latter ought to equal the former. A difference of one three-hundredth ($\frac{1}{300}$) is considered allowable.

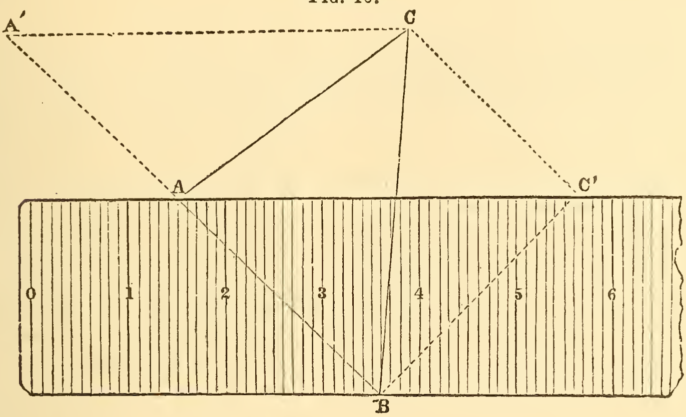
Some surveyors measure the perpendiculars of the triangles by a scale half of that to which the plat is made. Thus, if the scale of the plat be two chains to the inch, the perpendiculars are measured with a scale of one chain to the inch. The product of the base by the perpendicular thus measured, gives the area of the triangle at once, without its requiring to be divided by two.

Another way of attaining the same end, with less danger of mistakes, is to construct a *new* scale of equal parts, longer than those by which the plat was made in the ratio $\sqrt{2} : 1$; or $1{\cdot}414 : 1$. When the base and perpendicular of a triangle are measured by this new scale, and then multiplied together, the product will be the content of the triangle, without any division by two, In this method there is the additional advantage of the greater size and consequent greater distinctness of the scale.

When the measurement of a plat is made some time after it has been drawn, the paper will very probably have contracted or expanded so that the scale used will not exactly apply. In that case a correction is necessary. Measure very precisely the present length of some line on the plat, of known length originally. Then make this proportion : *As* the square of the present length of this line *is to* the square of its original length, *so is* the content obtained by the present measurement *to* the true content.

68. *Graphical Multiplication.* Prepare a strip of drawing-paper, of a width exactly equal to two chains on the scale of the plat; i. e., one inch wide, as in the figure, for a scale of two chains to one inch; two thirds of an inch wide for a scale of three chains; half an inch for four chains, and so on. Draw perpendicular lines across the paper at distances representing one tenth of a chain on the scale of the triangle to be measured, thus making a platting scale. Apply it to the triangle so that one edge of the scale shall pass through one corner, A, of the triangle, and the other edge through another corner, B ; and note very precisely what divisions of the scale are at these points. Then slide the scale in such a way that the points of the scale which had coincided with A and B shall always remain on the line B A produced, till the edge arrives at the point C. Then will A′ C—that is, the distance, or

FIG. 40.



number of divisions on the scale, from the point to which the division A on the scale has arrived, to the third corner of the triangle—express the area of the triangle A B C in square chains.

For, from C draw a parallel to A B, meeting the edge of the scale in C′, and draw C′ B. Then the given triangle A B C = A B C′. But the area of this last triangle = A C′ multiplied by half the width of the scale, i. e., = A C′ × 1 = A C′. But, because of the parallels, A′ C = A C′, therefore the area of the given triangle A B C = A′ C ; i. e., it is equal in square chains to the number of linear chains read off from the scale. This ingenious operation is due to *M. Cousinery*.

69. Division into Trapezoids. A line may be drawn across the field, as in Fig. 41, and perpendiculars drawn to it. The field

FIG. 41.

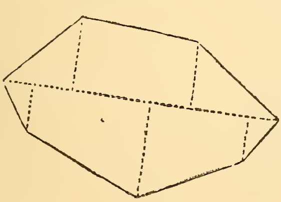
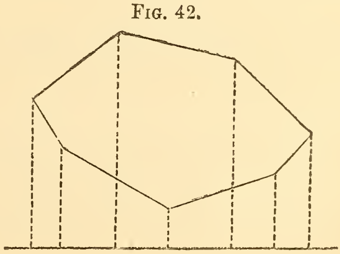


FIG. 42.

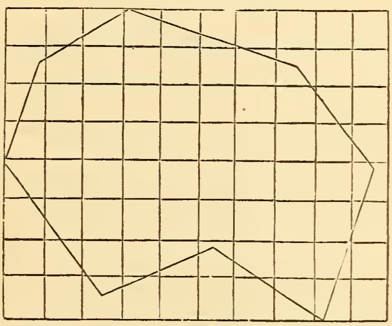


will thus be divided into trapezoids (excepting a triangle at each end), and their content can be calculated by Art. 64.

Otherwise: a line may be drawn outside of the figure, and perpendiculars to it be drawn from each angle. In that case the difference between the trapezoids formed by lines drawn to the outer angles of the figure, and those drawn to the inner angles, will be the content.

70. Division into Squares. Two sets of parallel lines, at right angles to each other, one chain apart (to the scale of the plat) may be drawn over the plat, so as to divide it into squares, as in the figure. The number of squares which fall within the plat represent so many square chains; and the triangles and trapezoids which fall outside of these may then be calculated and added to the entire square chains which have been counted.

Fig. 43.

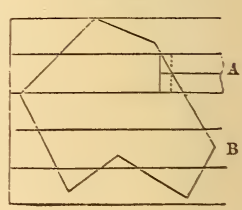


Instead of drawing the parallel lines on the plat, they may better be drawn on a piece of transparent "tracing-paper," which is simply laid upon the plat, and the squares counted as before. The same paper will answer for any number of plats drawn to the same scale. This method is a valuable and easy check on the results of other calculations.

To calculate the fractional parts, prepare a piece of tracing-paper, or glass, by drawing on it one square of the same size as a square of the plat, and subdividing it, by two sets of ten parallels at right angles to each other, into hundredths. This will measure the fractions remaining from the former measurement, as nearly as can be desired.

71. Division into Parallelograms. Draw a series of parallel lines across the plat at equal distances depending on the scale. Thus, for a plat made to a scale of 2 chains to 1 inch, the distance between the parallels should be $2\frac{1}{2}$ inches; for a scale of 3 chains to 1 inch, $1\frac{1}{9}$ inch; for a scale of 4 chains to 1 inch, $\frac{5}{8}$ inch; for a scale of 5 chains to 1 inch, $\frac{4}{10}$ inch; and for any scale, make the distance between the parallels that fraction of an inch which would be expressed by 10 divided by the square of the number of chains to the inch. Then apply a common inch scale, divided on the edge into tenths, to these parallels; and every inch

Fig. 44.



in length of the spaces included between each pair of them will be an acre, and every tenth of an inch will be a square chain.

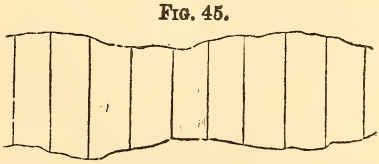
For, calling the number of chains to the inch, $= n$, and making the width between the parallels $\frac{10}{n^2}$ inch, this width will represent $\frac{10}{n^2} \times n = \frac{10}{n}$ chains; and as the inch length represents n chains, their product, $\frac{10}{n} \times n = 10$ square chains $= 1$ acre.

To measure the triangles at the ends of the strips between the parallels, prepare a piece of glass, or stout tracing-paper, of a width equal to the width between the parallels, and draw a line through its middle longitudinally. Apply it to the oblique line at the end of the space between two parallels, and it will bisect the line, and thus reduce the triangle to an equivalent rectangle, as at A in the figure. When an angle occurs between two parallels, as at B in the figure, the fractional part may be measured by any of the preceding methods.

A somewhat similar method is much used by some surveyors, particularly in Ireland—the plat being made on a scale of 5 chains to 1 inch, parallel lines being drawn on it, half an inch apart, and the distances along the parallels being measured by a scale, each large division of which is $\frac{8}{10}$ inch in length. Each division of this scale indicates an acre; for it represents 4 chains, and the distance between the parallels is $2\frac{1}{2}$ chains. This scale is called the "Scale of Acres."

72. Addition of Widths. When the lines of the plat are very irregularly curved, as in the figure, draw across it a number of equidistant lines, as near together as the case may seem to require. Take a straight-edged piece of paper, and apply one edge of it to the middle of the first space, and mark its length from one end; apply the same edge to the middle of the next space, bringing the mark just made to one end, and making another mark at the end of the additional length; so go on, adding the length of each space to the previous ones. When all have been thus measured, the total length, multiplied by the uniform width, will give the content.

FIG. 45.



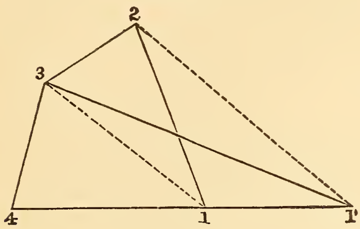
73. THIRD METHOD.—INSTRUMENTALLY. *By performing certain instrumental operations on the plat.*

74. *Reduction of a many-sided figure to a single equivalent triangle.* Any plane figure bounded by straight lines may be reduced to a single triangle, which shall have the same content. This can be done by any instrument for drawing parallel lines.

4

Let the trapezium, or four-sided figure, shown in Fig. 46, be required to be reduced to a single equivalent triangle. Produce one side of the figure, as 4 — 1. Draw a line from the first to the third angle of the figure. From the second angle draw a parallel to the line just drawn, cutting the produced side in a point 1′. From the point 1′ draw a line to the third angle. A triangle (1′— 3 — 4 in the figure) will thus be formed, which will be equivalent to the original trapezium.

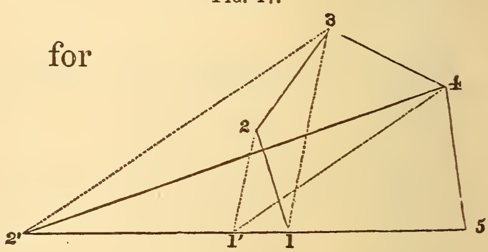
Fig. 46.



For, the triangle 1 — 2 — 3 taken away from the original figure is equivalent to the triangle 1′— 1 — 3 added to it; because both these triangles have the same base and also the same altitude, since the vertices of both lie in the same line parallel to the base.

The content of this final triangle can then be found by measuring its perpendicular, and taking half the product of this perpendicular by the base.

Fig. 47.

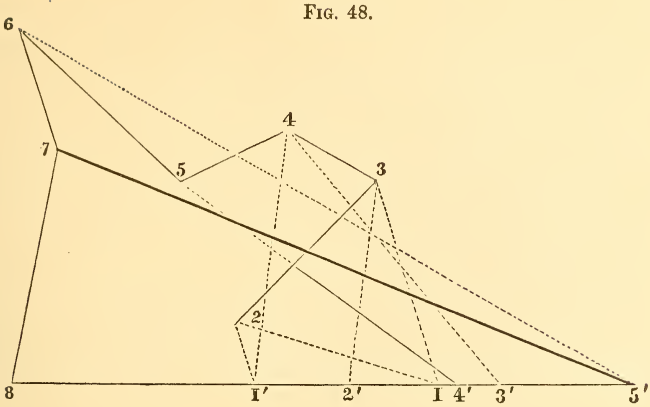


Let the given figure have five sides, as in Fig. 47. For brevity, the angles of the figure will be named as numbered in the engraving. Produce 5 — 1. Join 1 — 3. From 2 draw a parallel to 1 — 3, cutting the produced base in 1′. Join 1′— 4. From 3 draw a parallel to it, cutting the base in 2′. Join 2′— 4. Then will the triangle 2′— 4 — 5 be equivalent to the five-sided figure 1 — 2 — 3 — 4 — 5, for similar reasons to those of the preceding case.

Let the given figure be 1 — 2 — 3 — 4 — 5 — 6 — 7 — 8, as shown in Fig. 48. All the operations are shown by dotted lines, and the finally resulting triangle, 5′— 7 — 8, is equivalent to the original figure of eight sides.

It is best, in choosing the side to be produced, to take one which has a long side adjoining it on the end not produced; so that this long side may form one side of the final triangle, the base of which will therefore be shorter, and will not be cut so acutely by the final line drawn, as to make the point of intersection too indefinite.

FIG. 48.



75. *General Rule.* When the given figure has many sides, with angles sometimes salient and sometimes re-entering, the operations of reduction are very liable to errors if the draughtsman attempts to reason out each step. All difficulties, however, will be removed by the following *General Rule:*

1. Produce one side of the figure, and call it a base. Call one of the angles at the base the first angle, and number the rest in regular succession around the figure.

2. Draw a line from the 1st angle to the 3d angle. Draw a parallel to it from the 2d angle. Call the intersections of this parallel with the base the 1st mark.

3. Draw a line from the 1st mark to the 4th angle. Draw a parallel to it from the 3d angle. Its intersection with the base is the 2d mark.

4. Draw a line from the 2d mark to the 5th angle. Draw a parallel to it from the 4th angle. Its intersection with the base is the 3d mark.

5. In general terms, which apply to every step after the first, draw a line from the last mark obtained to the angle whose number is greater by three than the number of the mark. Draw a parallel to it through the angle whose number is greater by two than that of the mark. Its intersection with the base will be a mark whose number is greater by one than that of the preceding mark.

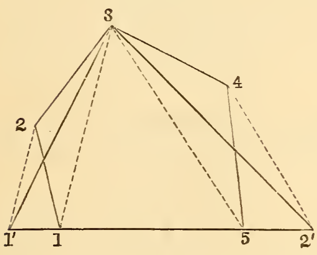
In the concise language of algebra, draw a line from the nth mark to the $n+3$ angle. Draw a parallel to it through the $n+2$ angle, and the intersection with the base will be the $n+1$ mark.

6. Repeat this process for each angle, till you get a mark whose number is such that the angle having a number greater by three is the last angle of the figure—i. e., the angle at the other end of the base. Then join the last mark to the angle which precedes the last angle in the figure, and the triangle thus formed will be the equivalent triangle required.

In practice it is *unnecessary* to actually draw the lines joining the successive angles and marks, but the parallel ruler is merely laid on so as to pass through them, and the points where the parallels cut the base are alone marked.

76. It is generally more convenient to reduce half of the figure on one side and half on the other, as is shown in Fig. 49, which represents the same field as Fig. 47. The equivalent triangle is here 1′— 3 — 2′.

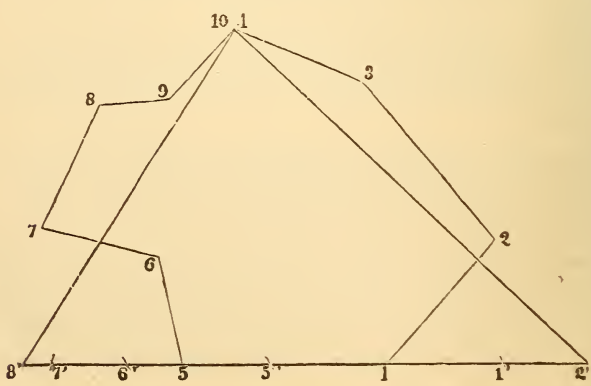
FIG. 49.



When the figure has many angles, they should not be numbered consecutively all the way around, but, after the numbers have gone around as far as the angle where it is intended to have the vertex of the final triangle, the numbers should be continued from the other angle of the base, as is shown in Fig. 50. In it only the intersections are marked.

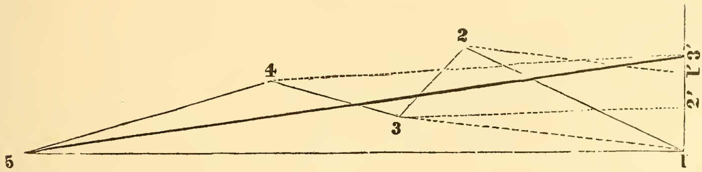
A figure with curved boundaries may be reduced to a triangle in a similar manner. Straight lines must be drawn about the figure, so as to be partly in

FIG. 50.



it and partly out, giving and taking about equal quantities, so that the figure which these lines form shall be about equivalent to the curved figure. This

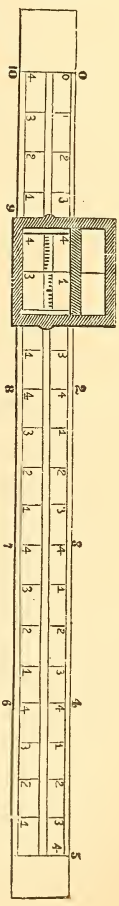
FIG. 51.



having been done, the equivalent straight-lined figure is reduced by the above method.

It is sometimes more convenient not to produce one of the sides of the figure, but to draw at one end of it, as at the point 1 in Fig. 51, an indefinite line, usually a perpendicular, to a line joining two distant angles of the figure, and make this line the base of the equivalent triangle desired. The operation is shown by the dotted lines in the figure. The same General Rule applies to it as to the previous figures.

FIG. 52.



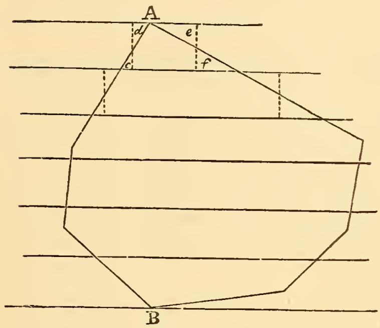
77. Special Instruments. A variety of instruments have been invented for the purpose of determining areas rapidly and correctly.

One of the simplest is the "*Computing Scale*," which is on the same principles as the Method of Art. 71. It is represented in Fig. 52. It consists of a scale divided for its whole length from the zero-point into divisions, each representing $2\frac{1}{2}$ chains to the scale of the plat. The scale carries a slider, which moves along it, and has a wire drawn across its center at right angles to the edges of the scale. On each side of this wire a portion of the slider, equal in length to one of the primary, or $2\frac{1}{2}$ chain, divisions of the scale, is laid off and divided into 40 equal parts.

This instrument is used in connection with a sheet of transparent paper, ruled with parallel lines at distances apart each equal to one chain on the scale of the plat. It is plain that when the instrument is laid on this paper, with its edge on one of the parallel lines, and the slider is moved over one of the divisions of $2\frac{1}{2}$ chains, that *one rood*, or a quarter of an acre, has been measured between two of the parallel lines on the paper (since 10 square chains make one acre); and that one of the smaller divisions measures *one perch* between the same parallels. Four of the larger divisions give one acre. The scale is generally made long enough to measure at once five acres.

To apply this to the plat of a field, or farm, lay the transparent paper over it in such a position that two of the ruled lines shall touch two of the exterior points of the boundaries, as at A and B. Lay the scale, with the slide set to zero, on the paper, in a direction parallel to the ruled lines, and so that the wire of the slide cuts the left-hand oblique line so as to make the spaces *c* and *d* about equal. Hold the scale firm, and move the slider till the wire cuts the right-hand oblique line in such a way as to equalize the spaces *e* and *f*. Without changing the slide, move the scale down the width of a space and to the left-hand end of the next space; begin there again, and proceed as before.

Fig. 53.



So go on, till the whole length of the scale is run out (five acres having been measured), and then begin at the right-hand side and work backward to the left, reading the lower divisions, which run up to 10 acres. By continuing this process, the content of plats of any size can be obtained.

A still simpler substitute for this is a scale similarly divided, but without an attached slide. In place of it there is used a piece of glass, having a line drawn across it and riveted to the end of a short scale of box-wood, divided like the former slide. It is used like the former, except that, at starting, the zero of the short scale and not the line on the glass is made to coincide with the zero of the long scale. The slide is to be held fast to the instrument when this is moved.

78. Planimeters. These determine the area of any figure, whether bounded by straight lines or curved, by merely moving a point around the outline of the surface. This causes motion in a train of wheel-work, which registers the algebraic sum of the product of ordinates to every point in that perimeter, by the increment of their abscissas, and therefore measures the included space.

There are several varieties of these instruments. One of the best of them is Amsler's Polar Planimeter. (For descriptions and theory of Planimeters, see "Mechanical Integrators," by Henry S. H. Shaw.)

79. A purely mechanical means of determining the area of any surface by means of its *weight*, may be placed here. The plat is cut out of paper and weighed by a delicate balance. The weight of a rectangular piece of the same paper containing just one acre is also found; and the "Rule of Three" gives the content. A modification of this is to paste a tracing of the plat on thin sheet-lead, cut out the lead to the proper lines and weigh it.

80. FOURTH METHOD.—TRIGONOMETRICALLY. *By calculating, from the observed angles of the boundaries of the piece of ground, the lengths of the lines needed for calculating the content.*

This method is employed for surveys made with angular instruments, as the compass, etc., in order to obtain the content of the land surveyed, without the necessity of previously making a plat, thus avoiding both that trouble and the inaccuracy of any calculations founded upon it. It is therefore the most accurate method; but will be more appropriately explained in Part I, Chapter III, under the head of "Compass Surveying."

CHAPTER II.

CHAIN-SURVEYING; BY THE FIRST AND SECOND METHODS: OR DIAGONAL AND PERPENDICULAR SURVEYING.

81. THE chain alone is abundantly sufficient, without the aid of any other instrument, for making an accurate survey of any surface, whatever its shape or size, particularly in a district tolerably level and clear. Moreover, since a chain, or some substitute for it, formed of a rope, of leather driving-reins, etc., can be obtained by any one in the most secluded place, this method of surveying deserves more attention than has usually been given to it.

SURVEYING BY DIAGONALS: OR BY THE FIRST METHOD.

82. *Surveying by Diagonals* is an application of the *First Method* of determining the position of a point, given in Art. 3, to which the student should again refer. Each corner of the field or farm which is to be surveyed is "determined" by measuring its distances from two other points. The field is then "platted" by repeating this process on paper, for each corner, in a contrary order, and the "content" is obtained by some of the methods explained in Chapter I.

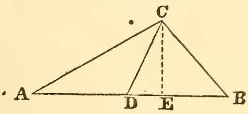
The lines which are measured in order to determine the corners of the field are usually *sides* and *diagonals* of the irregular polygon which is to be surveyed. They therefore divide it up into triangles; whence this mode of surveying is sometimes called "Chain Triangulation."

A few examples will make the principle and practice perfectly clear. Each will be seen to require the three operations of *measuring, platting,* and *calculating.*

A three-sided field; as Fig. 54.

Field-work. Measure the three sides, A B, B C, and C A. Measure also, as a proof-line, the distance from one of the corners, as C, to some point in the opposite side, as D, at which a mark should have been left, when measuring from A to B, at a known distance from A. A stick or twig, with a slit in its top, to receive a piece of paper with the distance from A marked on it, is the most convenient mark.

FIG. 54.



Platting. Choose a suitable scale. Then draw a line equal in length, on the chosen scale, to one of the sides; A B, for example. Take in the compasses the length of another side, as A C, to the same scale, and with one leg in A as a center, describe an arc of a circle. Take the length of the third side, B C, and, with B as a center, describe another arc, intersecting the first arc in a point which will be the third corner C. Draw the lines A C and B C; and A B C will be the *plat*, or miniature copy of the field surveyed.

Instead of describing two arcs to get the point C, two pairs of compasses may be conveniently used. Open them to the lengths, respectively, of the last two sides. Put one foot of each at the ends of the first side, and bring their other feet together, and their point of meeting will mark the desired third point of the triangle.

To "prove" the accuracy of the work, fix the point D, by setting off from A the proper distance, and measure the length of the line D C. If its length on the plat corresponds to its measurement on the ground, the work is correct.

It is a universal principle, in all surveying operations, that the work must be tested by some means independent of the original process, and that the same result must be arrived at by two different methods. The necessary length of this proof-line can also easily be calculated by the principles of trigonometry.

Calculation. The content of the field may now be found, either from the three sides, or more easily though not so accurately, by measuring on the plat, the length of the perpendicular C E, let fall from any angle to the opposite side, and taking half the product of these two lines.

Example 1. Fig. 54 is the plat, on a scale of two chains to one inch, of a field, of which the side A B is 200 links, B C is 100 links, and A C is 150 links. Its content, by the rule of Art. 62, is 0·726 of a square chain, or 0 A. 0 R. 12 P. If the perpendicular C E be accurately measured, it will be found to be $72\frac{1}{2}$ links. Half the product of this perpendicular by the base will be found to give the same content.

Ex. 2. The three sides of a triangular field are respectively 89·39, 54·08, and 45·98. Required its content. *Ans.* 100 A. 0 R. 10 P.

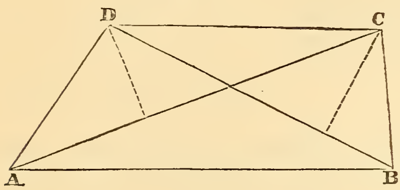
A four-sided field; as Fig. 55.

Field-work. Measure the four sides. Measure also a diagonal, as A C, thus dividing the four-sided field into two triangles. Measure also the other diagonal, or B D, for a "proof-line."

Platting. Draw a line, as A C, equal in length to the diagonal, to any scale. On each side of it construct a triangle with the sides of the field, as directed above.

To prove the accuracy of the work, measure on the plat the length of the "proof-line," B D, and if it agrees with the length of the same line measured on the ground, the field-work and platting are both proved to be correct.

Fig. 55.



Calculation. Find the content of each triangle separately, as in the preceding case, and add them together; or, more briefly, multiply either diagonal (the longer one is preferable) by the sum of the two perpendiculars, and divide the product by two.

Otherwise: reduce the four-sided figure to one triangle, as in Art. 74; or, use any of the methods of the preceding chapter.

Ex. 3. In the field drawn in Fig. 55, on a scale of 3 chains to the inch, A B = 588 links, B C = 210, C D = 430, D A = 274, the diagonal A C = 626, and the proof diagonal B D = 500. The total content will be 1 A. 0 R. 17 P.

Ex. 4. The sides of a four-sided field are A B = 12·41, B C = 5·86, C D = 8·25, D A = 4·24; the diagonal B D = 11·55, and the proof-line A C = 11·04. Required the content. *Ans.* 4 A. 2 R. 38 P.

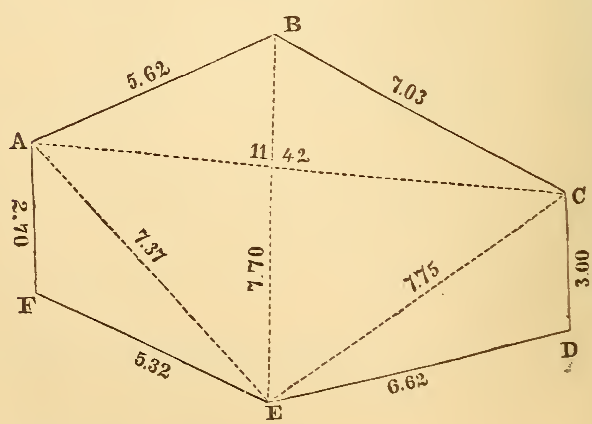
Ex. 5. The sides of a four-sided field are as follows: A B = 8·95, B C = 5·33, C D = 10·10, D A = 6·54; the diagonal from A to C is 11·52; the proof diagonal from B to D is 10·92. Required the content. *Ans.*

Ex. 6. In a four-sided field, A B = 7·68, B C = 4·09, C D = 10·64, D A = 7·24, A C = 10·32, B D = 10·74. Required the content. *Ans.*

A many-sided field, as Fig. 56.

Field-work. Measure all the sides of the field. Measure also diagonals

Fig. 56.



enough to divide the field into triangles, of which there will always be two less than the number of sides. Choose such diagonals as will divide the field into triangles as nearly equilateral as possible. Measure also one or more diagonals for "proof-lines." It is well for the surveyor himself to place stakes in advance at all the corners of the field, as he can then select the best mode of division.

Platting. Begin with any diagonal and plat one triangle. Plat a second triangle adjoining the first one. Plat another adjacent triangle, and so proceed till all have been laid down in their proper places. Measure the proof-lines as before.

Calculation. Proceed to calculate the content of the figure, precisely as directed for the four-sided field, measuring the perpendiculars and calculating the content of each triangle in turn; or taking in pairs those on opposite sides of the same diagonal; or using some of the other methods which have been explained.

Ex. 7. The six-sided field, shown in Fig. 56, has the lengths of its lines, in chains and links, written upon them, and is divided into four triangles, by three diagonals. The diagonal B E is a "proof-line." The figure is drawn to a scale of 4 chains to the inch. The content of the field is 5 A. 3 R. 22 P.

Ex. 8. In a five-sided field, the lengths of the sides are as follows: A B = 2·69, B C = 1·22, C D = 2·32, D E = 3·55, E A = 3·23. The diagonals are A D = 4·81, B D = 3·33. Required its content. *Ans.*

A field may be divided up into triangles, not only by measuring diagonals as in the last figure, but by any of the methods shown in the four figures of Art. 67. The one which we have been employing corresponds to the last of those figures.

Still another mode may be used when the angles can not be seen from one another, or from any one point within. Take two or more convenient points within the field, and measure from them to the corners, and thus form different sets of triangles.

Keeping the Field-Notes.

83. By Sketch. The most simple method is to make a sketch of the field, as nearly correct as the unassisted hand and eye can produce, and note down on it the lengths of all the lines, as in Fig. 56. But when many other points require to be noted, such as where fences, or roads, or streams are crossed in the measurement, or any other additional particulars, the sketch would become confused, and be likely to lead to mistakes in the subsequent platting from it. The following is therefore the usual method of keeping the field-notes. A long, narrow book is most convenient for it.

84. In Columns. Draw two parallel lines, about an inch apart from the bottom to the top of the page of the field-book, as in the margin. This column, or pair of lines, may be conceived to represent the measured line, *split in two,* its two halves being then separated, an inch apart, merely for convenience, so that the distances measured along the line may be written between these halves.

Hold the book in the direction of the measurement. At the *bottom* of the page write down the name, or number, or letter, which represents the station at which the survey is to begin.

A "station" is marked with a triangle or circle, as in the margin. The latter is more easily made.

△

⊙

In the complicated cases, which will be hereafter explained, and in which one long base-line is measured, and also many other subordinate lines, it will be well, as a help to the memory, to mark the stations on the base-line with a triangle, and the stations on the other lines with the ordinary circle.

The station from which the measurements are made is usually put on the left of the column; and the station which is measured to, is put on the right.

	⊙	*to* B
	562	
From A	⊙	

But it is more compact, and avoids interfering with the notes of "offsets" (to be explained hereafter), to write the name or number of the station *in* the column, as in the margin.

B
562
A

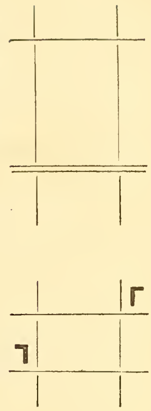
The measurements to different points of a line are written above one another. The numbers all refer to the beginning of the line, and are counted from it.

B
400
250
100
A

The end of a measured line is marked by a line drawn across the page above the numbers which indicate the measurements which have been made.

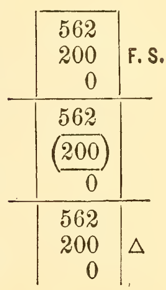
If the chaining does not continue along the adjoining line, but the chain-men go to some other part of the field to begin another measurement, *two* lines are drawn across the page.

When a line has been measured, the marks ┌ or ┐ are made to show whether the following line turns to the right or to the left.



A line is named, either by the names of the stations between which it is measured, as the line A B; or by its length, a line 562 links long, being called the line 562; or it is recorded as Line No. 1, Line No. 2, etc.; or as Line on page 1, 2, etc., of the field-book.

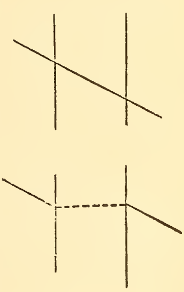
When a mark is left at any point of a line, as at D, in Fig. 49, with the intention of coming back to it again, in order to measure to some other point, the place marked is called a *False Station*, and is marked in the field-book "F. S."; or has a line drawn around it, to distinguish it; or has a station mark △ placed outside of the column, to the right or left, according to the direction in which the measurement from it is to be made. Examples of these three modes are given in the margin.



A false station is named by its position on the line where it belongs; as thus—"200 on 562."

When a gate occurs in a measured line, the distance from the beginning of the line to the side of the gate first reached is the one noted.

When the measured line crosses a fence, brook, road, etc., they are drawn on the field-notes in their true direction, as nearly as possible, but not in a continuous line across the column, as in the first figure in the margin, but as in the second figure, so that the two parts would form a continuous straight line, if the halves of the "split line" were brought together.



It is convenient to name the lines in the margin as being Sides, Diagonals, Proof-lines, etc.

85. The field-notes of the triangular field platted in Fig. 54 are given below, according to both the methods mentioned in the preceding article.

In the field-notes in the column on the right hand, it is not absolutely necessary to repeat the B and C.

PROOF-LINE.	*From* D	89 F. S.	*to* C
SIDE.	*From* C	150 ⊙	*to* A ┐
SIDE.	*From* B	100 ⊙	*to* C ┐
SIDE.	*From* A	200 80 ⊙	*to* B F. S.

PROOF-LINE.	*From*	C 89 (80)	*on* 200
SIDE.	┐	A 150 C	
SIDE.	┐	C 100 B	
SIDE.		B 200 (80) A	

86. The field-notes of the survey platted in Fig. 56 are given below. They begin at the bottom of the left-hand column.

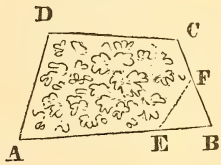
SIDE.		F 532 300 E	*Gate.* ┌
SIDE.		E 662 400 D	*Brook.* ┌
SIDE.		D 300 270 210 80 C	*Road.* ┌
SIDE.		C 703 150 B	*Gate.* ┌
SIDE.		B 562 A	

PROOF-LINE.		E 770 B	
DIAGONAL.	┐	A 1142 C	
DIAGONAL.	┐	C 775 480 420 E	*Road.*
DIAGONAL.		E 737 280 210 A	*Road.*
SIDE.		A 270 130 80 F	*Road.* ┌

SURVEYING BY TIE-LINES.

87. *Surveying by Tie-lines* is a modification of the method explained in the last chapter. It frequently happens that it is impossible to measure the diagonals of a field of many sides, in consequence of obstacles to measurements, such as woods, water, houses, etc. In such cases, "*tie-lines*" (so called because they *tie* the sides together) are employed as substitutes for diagonals.

FIG. 57.



Thus, in the four-sided field shown in the figure, the diagonals can not be measured because of woods intervening. As a substitute, measure off from any convenient corner of the field, as B, any distances, B E, B F, along the sides of the field. Measure also the "tie-line" E F. Measure all the sides of the field as usual.

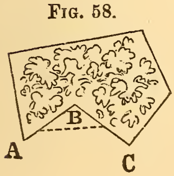
To plat this field, construct the triangle B E F, as in Art. 82. Produce the sides B E and B F, till they become respectively equal to B A and B C, as measured on the ground. Then, with A and C as centers, and with radii respectively equal to A D and C D, describe arcs, whose intersection will be D, the remaining corner of the field.

88. It thus appears that one tie-line is sufficient to determine a four-sided field, two a five-sided field, and so on. But, as a check on errors, it is better to measure a tie-line for each angle, and the agreement, in the plat, of all the measurements will prove the accuracy of the whole work.

Since any inaccuracy in the length of a tie-line is increased in proportion to the greater length of the sides which it fixes, the tie-lines should be measured as far from the point of meeting of these sides as possible—that is, they should be as long as possible.

The radical defect of the system is that it is "working from less to greater" (which is the exact converse of the true principle), thus magnifying inaccuracies at every step.

FIG. 58.



A tie-line may also be employed as a "proof-line," in the place of a diagonal, and tested in the same manner.

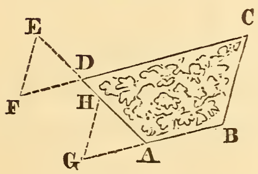
If any angle of the field is re-entering, as at B in the figure, measure a tie-line across the salient angle A B C.

89. Chain-Angles. It is convenient, though not necessary, to measure equal distances along the sides : B E, B F, in Fig. 57, and B A, B C, in Fig. 58. "Chain-angles" are thus formed. To reduce "chain-angles" to degrees and minutes, see Art. 28.

90. Inaccessible Areas. The method of tie-lines can be applied to measuring fields which can not be entered.

Thus, in the figure, A B C D is an inaccessible wooded field, of four sides. To survey it, measure all the sides, and at any corner, as D, measure any distance D E, in the line of A D produced. Measure also another distance D F in the line of C D produced. Measure the tie-line E F, and the figure can be platted as in the case of the field of Fig. 57, the sides of the triangle being produced in the contrary direction.

FIG. 59.



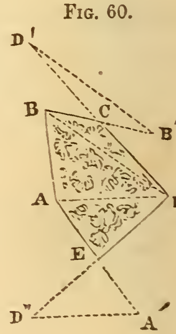
The same end would be attained by prolonging only one side, as shown at the angle A of the same figure, and measuring A G, A H, and G H. It is better, in both cases, to tie *all* the angles in a similar manner.

This method may be applied to a figure of any number of sides by prolonging as many of them as are necessary; all of them, if possible.

91. If the sides C D and A D' were prolonged by their full length, the content of the figure could be calculated without any plat; for the new triangle D E F would equal the triangle D A C, and the sides of the triangle A C B would then be known.

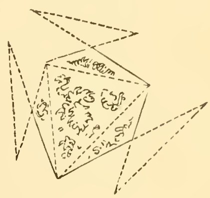
This principle may be extended still further. For a five-sided field, as in Fig. 60, produce two pairs of sides, a distance equal to

FIG. 60.



their length, forming two new triangles, as shown by the dotted lines, and measure the sides B′ D′ and A′ D″. The three sides of each of these triangles will thus be known, and also the three sides of the triangle B A D, since A D = A′ D″, and B D = B′ D′.

Fig. 61.



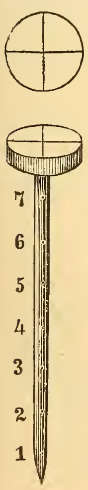
The method of this article may be employed for a figure of six sides, as shown in Fig. 61 (in which the dotted lines within the wooded field have their lengths determined by the triangles formed outside of it), but not for figures of a greater number of sides.

SURVEYING BY PERPENDICULARS: OR BY THE SECOND METHOD.

92. The method of *Surveying by Perpendiculars* is founded on the *Second Method* of determining the position of a point, explained in Art. 4. It is applied in two ways, either to making a complete survey by "*Diagonals and Perpendiculars*," or to measuring a crooked boundary by "*Offsets*." Each will be considered in turn.

The best method of getting perpendiculars on the ground must, however, be first explained.

Fig. 62.



To set out Perpendiculars.

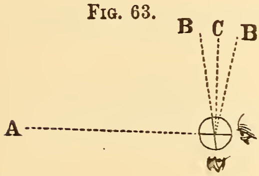
93. Surveyor's Cross. The simplest instrument for this purpose is the *Surveyor's Cross*, or *Cross-Staff*, shown in the figure. It consists of a block of wood, of any shape, having in it two saw-cuts, made very precisely at right angles to each other, about half an inch deep, and with center-bit holes made at the bottom of the cuts to assist in finding the objects. This block is fixed on a pointed staff, on which it can turn freely, and which should be precisely 8 links ($63\frac{1}{3}$ inches) long, for the convenience of short measurements.

To use the cross-staff to erect a perpendicular, set it at the point of the line at which a perpendicular is wanted. Turn its head till, on looking through one saw-cut, you see the ends of the line. Then will the other saw-cut point out the direction of the perpendicular, and thus guide the measurement desired.

To find where a perpendicular to the line, from some object, as a corner of a field, a tree, etc., would meet the line, set up the cross-staff at a point of the line which seems to the eye to be about the spot. Note about how far from the object the perpendicular at this point strikes, and move the cross-staff that distance; and repeat the operation till the correct spot is found.

94. To test the accuracy of the instrument, sight through one slit to some point A, and place a stake B in the line of sight of the other slit. Then turn its head a quarter of the way around, so that the second slit, looked through, points to A. Then see if the other slit covers B again, as it will if correct. If it does not do so, but sights to some other point, as B′, the apparent error is double the real one, for it now points as far to the right of the true point C as it did before to its left.

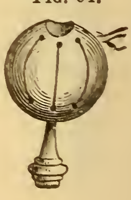
FIG. 63.



This is the first example we have had of the invaluable principle of *Reversion*, which is used in almost every test of the accuracy of surveying and astronomical instruments, its peculiar merit being that it doubles the real error, and thus makes it twice as easy to perceive and correct it.

The instrument, in its most finished form, is made of a hollow brass cylinder, which has two pairs of slits exactly opposite to each other, one of each pair being narrow and the other wide, with a horse-hair stretched from the top to the bottom of the latter. It is also, sometimes, made with eight faces, and two more pairs of slits added, so as to set off half a right angle.

FIG. 64.



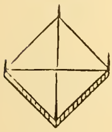
Another form is a hollow brass sphere, as in the figure. This enables the surveyor to set off perpendiculars on very steep slopes.

Another form of the surveyor's cross consists of

Fig. 65.

two pairs of plain "Sights," each shaped as in the figure, placed at the ends of two bars at right angles to each other. The slit, and the opening with a hair stretched from its top to its bottom, are respectively at the top of one sight and at the bottom of the opposite sight.* This is used in the same manner as the preceding form, but is less portable, and more liable to get out of order.

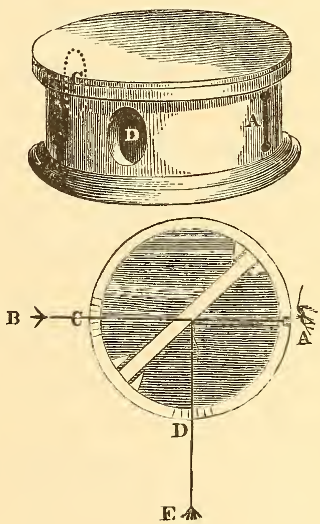
Fig. 66.



A temporary substitute for these instruments may be made by sticking four pins into the corners of a square piece of board, and sighting across them, in the direction of the line and at right angles to it.

95. Optical Square. The most convenient and accurate instrument is, however, the Optical Square. The figures give a perspective view of it, and also a plan with the lid removed. It is a small circular box, containing a strip of looking-glass, from the upper half of which the silvering is removed. This glass is placed so as to make precisely half a right angle with the line of sight, which passes through a slit on one side of the box, and a vertical hair stretched across the opening on the other side, or a mark on the glass. The box is held in the hand over the spot where the perpendicular is desired (a plumb-line in the hand will give perfect accuracy), and the observer applies his eye to the slit A, looking through the upper or unsilvered part of the glass, and turns the box till he sees the other end of the line B, through the opening C. The assistant, with a rod, moves along in the direction where the perpendicular is desired, being seen in the silvered

Fig. 67.



* The French call the narrow opening *œilleton*, and the wide one *croisée*.

parts of the glass, by reflection through the opening D, till his rod, at E, is seen to coincide with, or to be exactly under, the object B. Then is the line D E at right angles to the line A B, by the optical principle of the equality of the angles of incidence and reflection.

To find where a perpendicular from a distant object would strike the line, walk along the line, with the instrument to the eye, till the image of the object is seen, in the silvered part of the glass, to coincide with the direction of the line seen through the unsilvered part.

The instrument may be tested by sighting along the perpendicular, and fixing a point in the original line, on the principle of "reversion."

The surveyor can make it for himself, fastening the glass in the box by four angular pieces of cork, and adjusting it by cutting away the cork on one side, and introducing wedges on the other side. The box should be blackened inside.

Another form of the optical square contains two glasses, fixed at an angle of 45°, and giving a right angle on the principle of the sextant.

Perpendiculars may be set out with the chain alone, by a variety of methods. These methods generally consist in performing on the ground, the operations executed on paper in practical geometry, the chain being used, in the place of the compasses, to describe the necessary arcs.

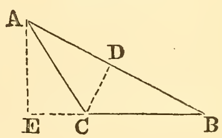
As these operations, however, are less often used for the method of surveying now to be explained, than for overcoming obstacles to measurement, it will be more convenient to consider them in that connection.

Diagonals and Perpendiculars.

96. We have seen in the preceding pages that plats of surveys, made with the chain alone, have their contents most easily determined by measuring, on the plat, the perpendiculars of each of the triangles, into which the diagonals measured on the ground have divided the field. In the *Method of Surveying by Diagonals and Perpendiculars,* now to be explained, the perpendiculars are measured *on the ground.* The content of the field can, therefore, be found at once (by adding together the half products of each perpendicular by the diagonal on which it is let fall), without the necessity of previously making a plat, or of measuring the sides of the field. This is, therefore, the most rapid and easy method of surveying when the content alone is required, and is particularly applicable to the measurement of the ground occupied by crops, for the purpose of determining the number of bushels grown to the acre, the amount to be paid for mowing by the acre, etc.

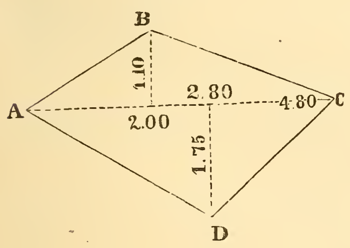
A Three-sided Field. Measure the longest side, as A B, and the perpendicular, C D, let fall on it from the opposite angle C. Then the content is equal to half the product of the side by the perpendicular. If obstacles prevent this, find the point, where a perpendicular let fall from an angle, as A, to the opposite side produced, as B C, would meet it, as at E in the figure. Then half the product of A E by C B is the content of the triangle.

Fig. 68.



A Four-sided Field. Measure the diagonal A C. Leave marks at the points on this diagonal at which perpendiculars from B and from D would meet it, finding these points by trial, as previously directed. The best marks at these "false stations" have been described. Return to these false stations and measure the perpendiculars. When these perpendiculars are measured before finishing the measurement of the diagonal, great care is necessary to avoid making mistakes in the length of the diagonal, when the chainmen return to continue its measurement. One check is to leave at the mark as many pins as have been taken up by the hind-chainman in coming to that point from the beginning of the line.

Fig. 69.



Ex. 9. Required the content of the field of Fig. 69. *Ans.* 0 A. 2 R. 29 P.

The field may be platted from these measurements, if desired, but with more liability to inaccuracy than in the first method, in which the sides are measured. The plat of the figure is three chains to one inch.

The field-notes may be taken by writing the measurements on a sketch, as in the figure; or, in more complicated cases, by the column method, as below. A new symbol may be employed, this mark, ⊢, or ⊣, to show the false station, from which a perpendicular is to be measured.

PERP.	*From* 200 *on* 480	110 F. S.	*to* B ┤
PERP.	*From* 280 *on* 480	175 F. S.	*to* D ├
DIAGONAL.	┤ *From* A	480 280 200 ⊙	*to* C ├

Ex. 10. *Calculation.*

sq. lks.

$$ABC = \tfrac{1}{2} \times 480 \times 110 = 26400$$

$$ADC = \tfrac{1}{2} \times 480 \times 175 = 42000$$

sq. chains 6·8400

Acres 0·684

It is still easier to take the two triangles together; multiplying the diagonal by the sum of the perpendiculars and dividing by two.

A Many-sided Field. Fig. 70 and the accompanying field-notes represent the field which was surveyed by the first method and platted in Fig. 56.

From 5·07 *on* 7·37	1·54 F. S.	*to* F
From 1·60 *on* 7·75	2·53 F. S.	*to* D
From 5·45 *on* 11·42	4·93 F. S.	*to* E
From 4·95 *on* 11·42	2·67 F. S.	*to* B
┤ *From* E	7·37 5·07 ⊙	*to* A ┌
┤ *From* C	7·75 1·60 ⊙	*to* E ┌
┤ *From* A	11·42 5·45 4·95 ⊙	*to* C ├

Ex. 11. *Calculation.* The content of the triangles may be expressed thus:

sq. lks.

$$ABC = \tfrac{1}{2} \times 1142 \times 267 = 152457$$

$$AEC = \tfrac{1}{2} \times 1142 \times 493 = 281503$$

$$CDE = \tfrac{1}{2} \times 775 \times 253 = 98037$$

$$AEF = \tfrac{1}{2} \times 737 \times 154 = 56749$$

sq. chains 58·8746

Acres 5·88746

or, 5 A. 3 R. 22 P.

The first two triangles might have been taken together, as in the previous field.

Content calculated from the perpendiculars will generally vary slightly from that obtained by measuring on the plat.

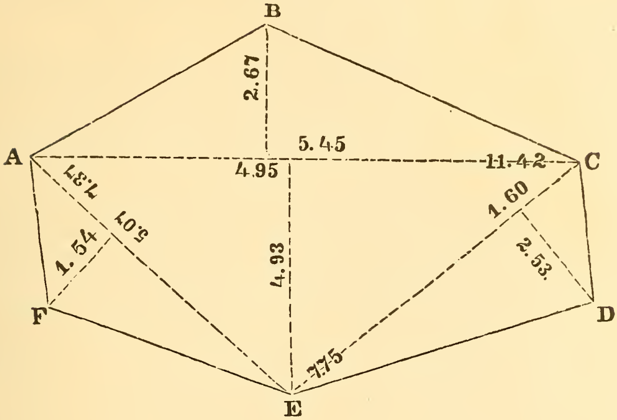
A small field which has many sides may sometimes be conveniently surveyed by taking one diagonal and measuring the perpendiculars let fall on it from each angle of the field, and thus dividing the whole area into triangles and trapezoids, as in Fig. 41.

The line on which the perpendiculars are to be let fall may also be outside of the field, as in Fig. 42.

Such a survey can be platted very readily, but the length of the perpendiculars renders the plat less accurate.

This procedure supplies a transition to the method of "offsets," which is explained in the next article.

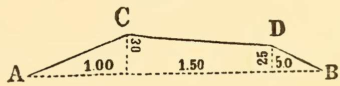
FIG. 70.

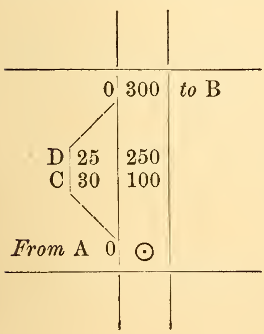


Offsets.

97. Offsets are short perpendiculars, measured from a straight line, to the angles of a crooked or zigzag line near which the straight line runs. Thus, in the figure, let A C D B be a crooked fence, bounding one side of a field. Chain along the straight line A B, which runs from one end of the fence to the other, and, when opposite each corner, note the distance from the beginning, or the point A, and also measure and note the perpendicular distance of each corner C and D from the line. These corners will then be "determined" by the *Second Method, Art. 4.*

FIG. 71.





The field-notes, corresponding to Fig. 71, are as in the margin. The measurements along the line are written in the column, as before, counting from the beginning of the line, and the offsets are written beside it, on the right or left, opposite the distance at which they are taken. A sketch of the crooked line is also usually made in the field-notes, though not absolutely necessary in so simple a case as this. The letters C and D would not be used in practice, but are here inserted to show the connection between the field-notes and the plat.

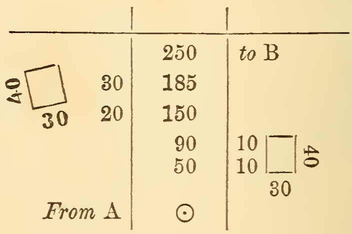
In taking the field-notes, the widths of the offsets should not be drawn proportionally to the distances between them, but the breadths should be greatly exaggerated in proportion to the lengths.

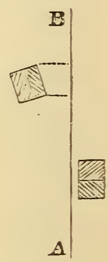
A more extended example, with a little different notation, is given below. In the figure, which is on a scale of eight chains to one inch for the distances along the line, the breadths of the offsets are exaggerated to four times their true proportional dimensions.

FIG. 72.

	B	
	1500	0
	1250	20
0	1000	0
30	750	
50	500	
40	250	
	0	
	A	

FIG. 73.



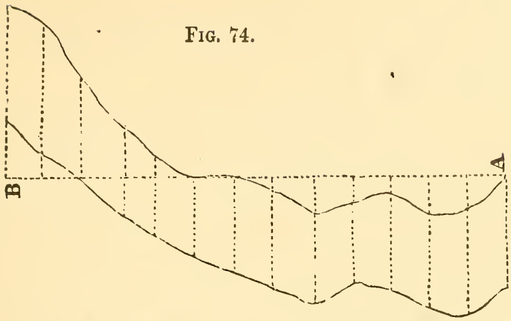


The plat and field-notes of the position of two houses, determined by offsets, are given above on a scale of two chains to one inch :

Double offsets are sometimes convenient ; and sometimes triple and quadruple ones. Below are given the notes and the plat, one chain to one inch, of a road of varying width, both sides of which are determined by double offsets. It will be seen that the line A B crosses one side of the road at 160 links from A, and the other side of it at 220.

Two methods of keeping the field-notes are given. In the first form, the offsets to each side of the road are given separately and connected by the sign +. In the second form, the total distance of the second offset is given, and the two measurements connected by the word "*to.*" This is easier both for measuring and platting.

FIG. 74.



	B			B	
	260	30 + 60		260	30 *to* 90
	240	10 + 70		240	10 *to* 80
0	220	50	0	220	50
20	200	30	20	200	30
40	180	10	40	180	10
45	160	0	45	160	0
50 + 0	140		50 *to* 0	140	
55 + 5	120		60 *to* 5	120	
50 + 20	100		70 *to* 20	100	
45 + 15	80		60 *to* 15	80	
50 + 10	60		60 *to* 10	60	
50 + 20	40		70 *to* 20	40	
55 + 20	20		75 *to* 20	20	
60 + 0	A		60 *to* 0	A	

Offsets may generally be taken with sufficient accuracy by measuring them as nearly at right angles to the base-line as the eye can estimate. The surveyor should stand by the chain, facing the fence, at the place which he thinks opposite to the corner to which he wishes to take an offset, and measure "square" to it by the eye, which a little practice will enable him to do with much correctness.

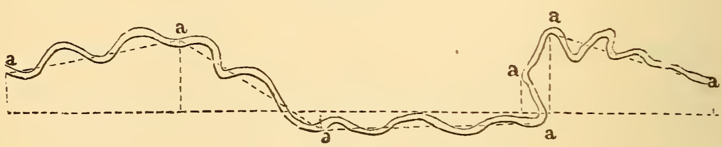
The offsets may be measured, if short, with an *Offset-staff*, a light stick, 10 or 15 links in length, and divided accordingly ; or, if they are long, with a tape. They are generally but a few links in length. A chain's length should be the extreme limit, as laid down by the English "Tithe Commissioners," and that should be employed only in exceptional cases. When the "cross-staff" is in use, its divided length of 8 links renders the offset-staff needless.

When offsets are to be taken, the method of chaining to the end of a line (described in Art. 18) is somewhat modified. After the leader arrives at the end of the line, he should draw on the chain till the follower, with the back end of the chain, reaches the last pin set. This facilitates the counting of the links to the places at which the offsets are taken.

The offsets are to be taken to every angle of the fence or other crooked line; that is, to every point where it changes its direction. These angles or prominent bends can be best found by one of the party walking along the crooked fence and directing another at the chain what points to measure opposite to. If the line which is to be thus determined is *curved,* the offsets should be taken to points so near each other that the portions of the curved line lying between them may, without much error, be regarded as straight. It will be most convenient, for the subsequent calculations, to take the offsets at equal distances apart along the straight line from which they are measured.

In the case of a crooked brook, such as is shown in the figure given below, offsets should be taken to the most prominent angles, such as are marked *a a a* in the figure, and the intermediate bends may be merely sketched by the eye.

Fig. 75.

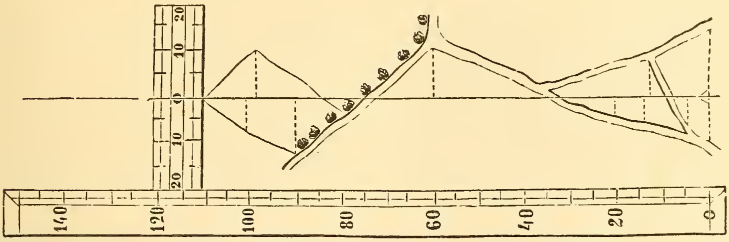


When offsets from lines measured around a field are taken inside of these bounding lines, they are sometimes distinguished as *insets.*

98. Platting. The most rapid method of platting the offsets is by the use of a *Platting Scale* (described in Art. 47) and an *Offset Scale,* which is a short scale divided on its edges like a platting scale, but having its zero in the middle, as in the figure.

The platting scale is placed parallel to the line, with its zero-point opposite to the beginning of the line. The offset scale is slid along the platting scale, till its edge comes to a distance on the latter at which an offset had been taken, the length of which is marked off with a needle-point from the offset scale. This is then slid on to the next distance, and the operation is repeated. If one person reads off the field-notes, and another plats, the operation

FIG. 76.



will be greatly facilitated. The points thus obtained are joined by straight lines, and a miniature copy of the curved line is thus obtained ; all the operations of the platting being merely repetitions of the measurements made on the ground.

If no offset scale is at hand, make one of a strip of thick drawing-paper, or pasteboard ; or use the platting scale itself, turned crossways, having previously marked off from it the points from which the offsets had been taken.

In plats made on a small scale, the shorter offsets are best estimated by the eye.

On the ordnance survey of Ireland, the platting of offsets was facilitated by the use of a combination of the offset scale and the platting scale, the former being made to slide in a groove in the latter, at right angles to it.

99. Calculating Content. When the crooked line determined by offsets is the boundary of a field, the content, inclosed between it and the straight line surveyed, must be determined, that it may be added to, or subtracted from, the content of the field bounded by the straight lines. There are various methods of effecting this.

The area inclosed between the straight and the crooked lines is divided up by the offsets into *triangles and trapezoids*, the content of which may be calculated separately and then added together. The content of the plat on page 65 will, therefore, be $1500 + 4125 + 625 = 6250$ square links $= 0{\cdot}625$ square chain. The content of the plat on page 66 will in like manner be found to be, on the left of the straight line, 30,000 square links, and on its right, 5,000 square links.

100. *When the offsets have been taken at equal distances,* the content may be more easily obtained by adding together half of the first and of the last offset, and all the intermediate ones, and multiplying the sum by one of the equal distances between the offsets. This rule is merely an abbreviation of the preceding one.

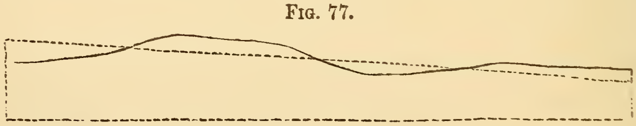
Thus, in the plat of page 66, the distances being equal, the content of the offsets on the left of the straight line will be $120 \times 250 = 30{,}000$ square links, and on the right, $20 \times 250 = 5{,}000$ square links; the same results as before.

When the line determined by the offsets is a curved line, "Simpson's rule" gives the content more accurately. To employ it, an *even* number of *equal* distances must have been measured in the part to be calculated. Then add together the first and last offset, four times the sum of the even offsets (i. e., the 2d, 4th, 6th, etc.), and twice the sum of the odd offsets (i. e., the 3d, 5th, 7th, etc.), not including the first and the last. Multiply the sum by one of the equal distances between the offsets, and divide by 3. The quotient will be the area.

Ex. 12. The offsets from a straight line to a curved fence were 8, 9, 11, 15, 16, 14, 9, links, at equal distances of 5 links. What was the content included between the curved fence and the straight line? *Ans.* 371·666.

101. *Equalizing,* or *giving and taking,* is an approximate mode of calculation much used by practical surveyors. A crooked line,

FIG. 77.



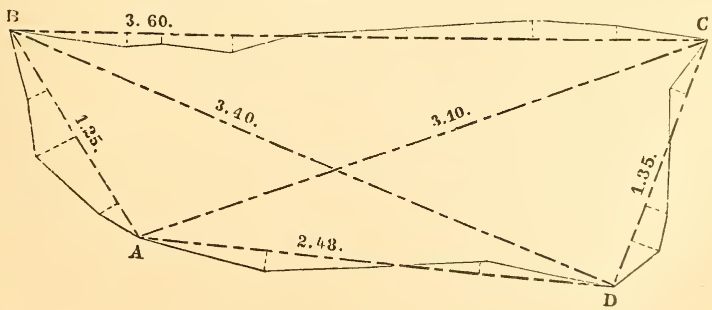
determined by offsets, having been platted, a straight line is drawn on the plat, across the crooked line, leaving as much space outside of the straight line as inside of it, as nearly as can be estimated by the eye, "equalizing" it, or "giving and taking" equal portions. The straight line is best determined by laying across the irregular outline the straight edge of a piece of transparent horn, or tracing-paper, or glass, or a fine thread or horse-hair stretched straight by a light bow of whalebone. In practical hands, this method is sufficiently accurate in most cases. The student will do well to try it on figures, the content of which he has previously ascertained by perfectly accurate methods.

SURVEYING BY THE PRECEDING METHODS COMBINED.

102. All the methods which have been explained in the preceding sections—surveying by *Diagonals*, by *Tie-lines*, and by *Perpendiculars*, particularly in the form of offsets—are frequently required in the same survey. The method by *Diagonals* should be the leading one; in some parts of the survey obstacles to the measurements of diagonal may require the use of *Tie-lines;* and, if the fences are crooked, straight lines are to be measured near them, and their crooks determined by *Offsets.*

Offsets are necessary additions to almost every other method of surveying. In the smallest field surveyed by diagonals, unless all the fences are perfectly straight lines, their bends must be determined by offsets. The plat (scale of one chain to one inch) and field-notes of such a case are given below. A sufficient number of the sides, diagonals, and proof-lines, to prove the work, should be platted before platting the offsets.

Fig. 78.



		C	
	0	360	
	6	315	
	10	275	
SIDE.	5	215	
	0	150	0
		115	10
		80	5
		65	8
		B	0

		B	
	0	125	
	11	90	
SIDE.	23	62	
	12	22	
	0	A	

		B	
PROOF LINE.		340	
		D	

		C	
DIAGONAL.		310	
		A	

		A	
	0	248	
SIDE.	11	180	
	0	105	0
		65	5
		D	0

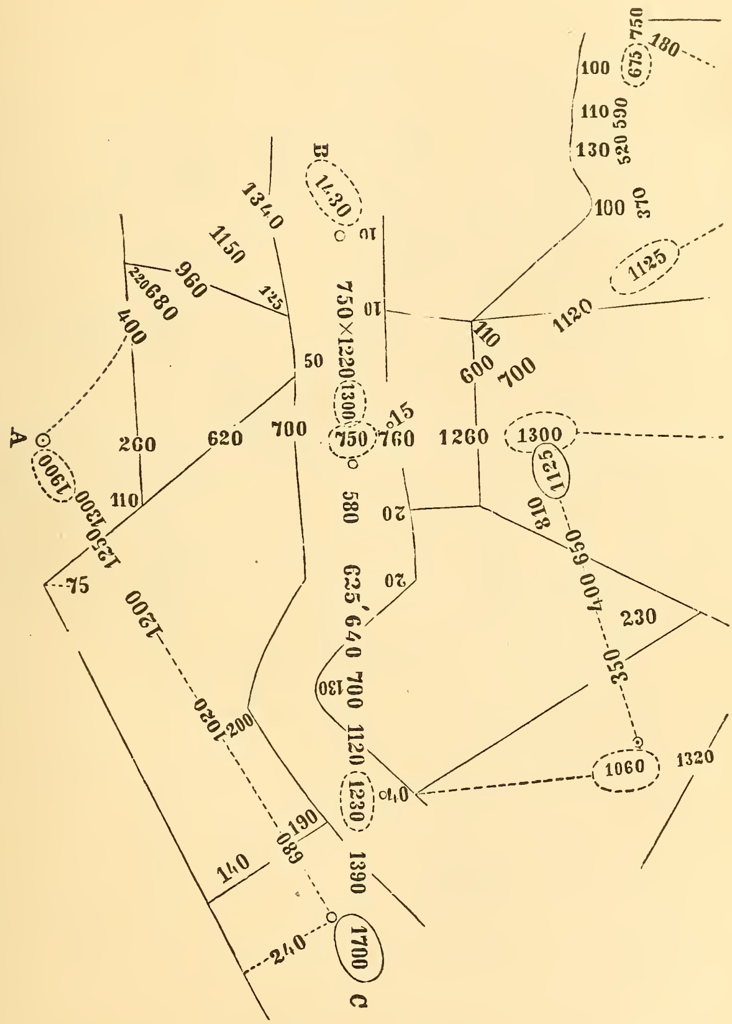
		D	
	0	135	
	15	110	
SIDE.	13	90	
	0	50	0
		30	9
		C	0

Ex. 13. Required the content of the above field. *Ans.*

103. Field-Books. The difficulty and the importance of keeping the field-notes clearly and distinctly increase with each new combination of methods. For this reason, three different methods of keeping the field-notes of the same survey will now be given (from Bourn's "Surveying"), and a careful comparison by the student of the corresponding portions of each will be very profitable to him :

Field-Book No. 1.

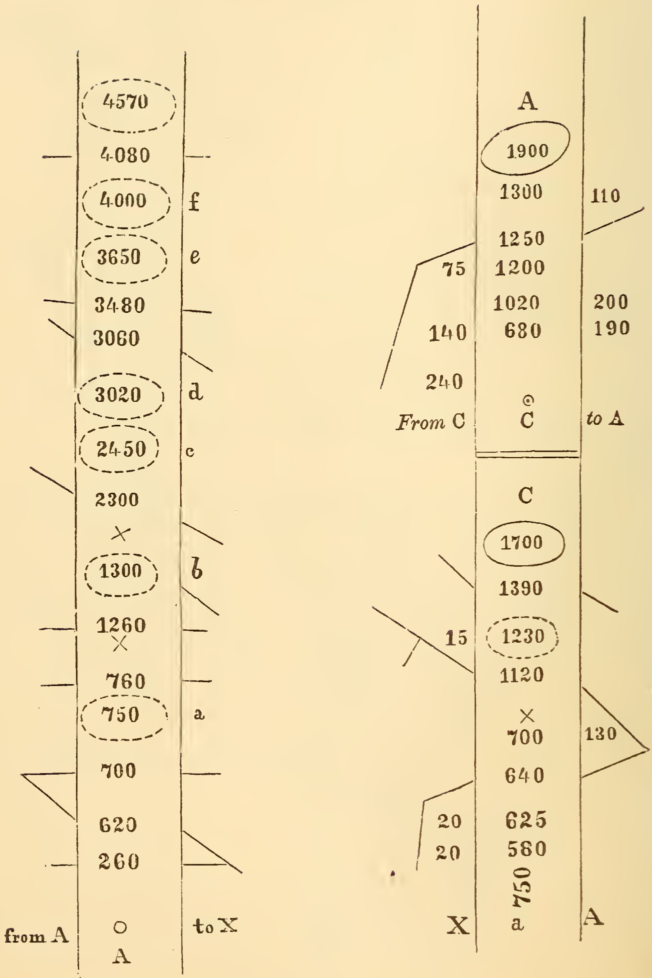
Fig. 79.



Field-Book No. 1 (Fig. 79) shows the Sketch method, explained in Art. 83.

Field-Book No. 2.

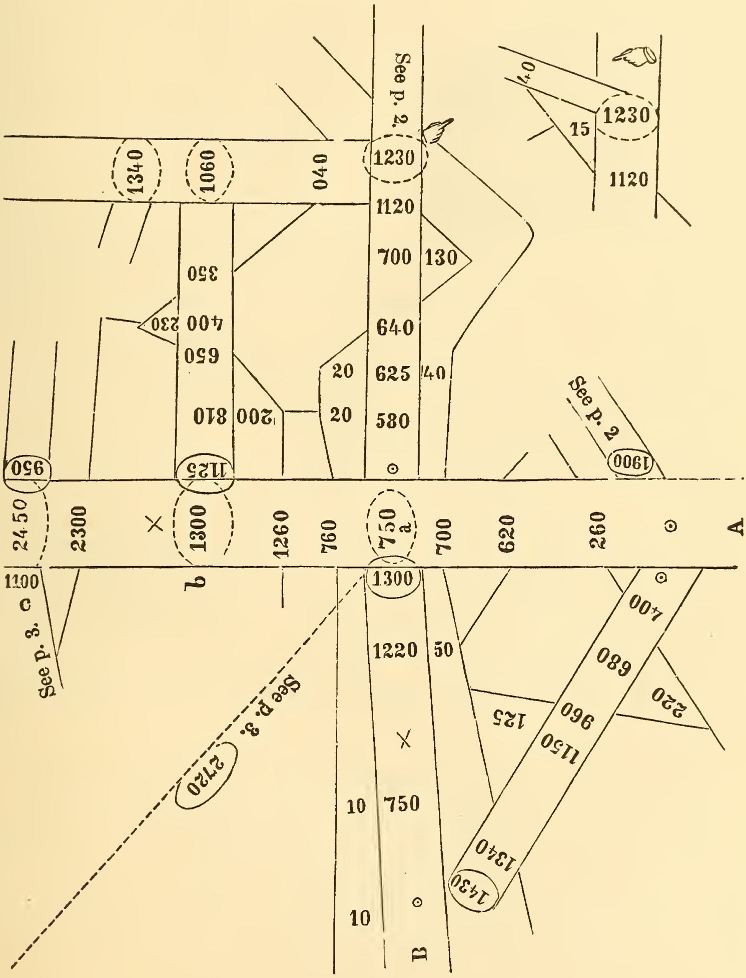
Fig. 80.



Field-Book No. 2 (Fig. 80) shows the Column method, explained in Art. 84.

Field-Book No. 3.

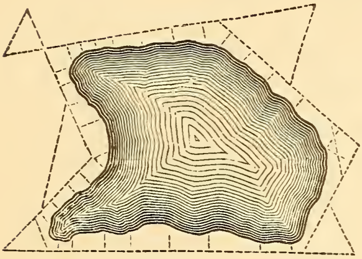
Fig. 81.



Field-Book No. 3 (Fig. 81) is a convenient combination of the two preceding methods. The bottom of the book is at the side of this figure, at A.

104. Inaccessible Areas. A combination of offsets and tie-lines supplies an easy method of surveying an *inaccessible area,* such as a pond, swamp, forest, block of houses, etc., as appears from the figure, in which external bounding lines are taken at will and measured, and tied by "tie-lines" measured between these lines, prolonged when necessary, while offsets from them determine the irregularities of the actual boundaries of the pond, etc.

Fig. 82.

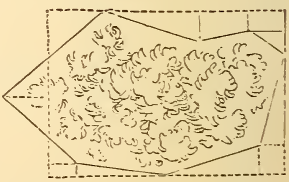


These *offsets* are *insets,* and their content is, of course, to be subtracted from the content of the principal figure.

Even a circular field might thus be approximately measured from the outside.

If the shape of the field admits of it, it will be preferable to measure four lines about the field in such directions as to inclose it in a rectangle, and to measure offsets from the sides of this to the angles of the field.

Fig. 83.



OBSTACLES TO MEASUREMENT IN CHAIN-SURVEYING.

105. In the practice of the various methods of surveying which have been explained, the hills and valleys which are to be crossed, the sheets of water which are to be passed over, the woods and houses which are to be gone through—all these form *obstacles* to the measurement of the necessary lines which are to join certain points, or to be prolonged in the same direction. Many special precautions and contrivances are therefore rendered necessary; and the best methods to be employed, when the chain alone is to be used, will now be given.

These methods for overcoming the various obstacles met with in practice constitute a Land-Geometry. Its problems are performed on the ground instead of on paper; its *compasses* are a chain fixed at one end and free to swing around with the other; its *scale* is the chain itself; and its *ruler* is the same chain stretched tight. Its advantages are that its single instrument (or a substitute for it, such as a tape, a rope, etc.) can be found anywhere; and its only auxiliaries are equally easy to obtain, being a few straight and slender rods, and a plumb-line, for which a pebble suspended by a thread is a sufficient substitute.

Many of these problems require the employment of perpendicular and parallel lines. For this reason we will commence with this class of problems.

The demonstrations of most of these problems will be left as exercises for the student.

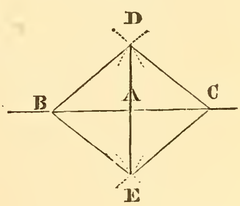
The elegant "Theory of Transversals" (Appendix B) will be an important element in some of these demonstrations.

*Problems on Perpendiculars.**

Problem 1. *To erect a perpendicular at any point of a line.*

106. *First Method.* Let A be the point at which a perpendicular to the line is to be set out. Measure off equal distances A B, A C, on each side of the point. Take a portion of the chain not quite $1\frac{1}{2}$ time as long as A B or A C, fix one end of this at B, and describe an arc with the other end. Do the same from C. The intersection of these arcs will fix a point D. A D will be the perpendicular required. Repeat the operation on the other side of the line.

Fig. 84.



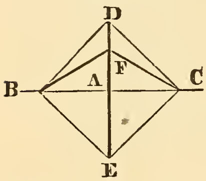
* Many of these methods would seldom be required in practice, but cases sometimes occur, as every surveyor of much experience in field-work has found to his serious inconvenience, in which some peculiarity of the local circumstances forbids any of the usual methods being applied. In such cases the collection here given will be found of great value.

In all the figures, the given and measured lines are drawn with fine full lines, the visual lines, or lines of sight, with broken lines, and the lines of the result with heavy full lines. The points which are centers around which the chain is swung are inclosed in circles. The alphabetical order of the letters attached to the points shows in what order they are taken.

If that is impossible, repeat it on the same side with a different length of chain.

107. *Second Method.* Measure off as before, equal distances A B, A C, but each about only one third of the chain. Fasten the ends of the chain with two pins at B and C. Stretch it out on one side of the line and put a pin at the middle of it, D. Do the same on the other side of the line, and set a pin at E. Then is D E a perpendicular to B C. If it is impossible to perform the operation on both sides of the line, repeat it on the same side with a different length of chain, as shown by the lines B F and C F in the figure, so as to get a second point.

FIG. 85.

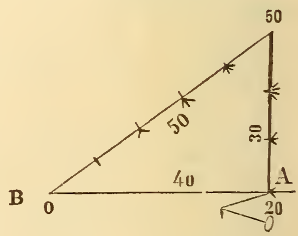


108. *Other Methods.* All the methods to be given for the next problem may be applied to this.

Problem 2. *To erect a perpendicular to a line at a given point, when the point is at or near the end of the line.*

109. *First Method.* Measure 40 links along the line. Let one assistant hold one end of the chain at that point; let a second hold the 20-link mark which is nearest the other end, at the given point A, and let a third take the 50-link mark, and tighten the chain, drawing equally on both portions of it. Then will the 50 link mark be in the perpendicular desired. Repeat the operation on the other side of the line so as to test the work.

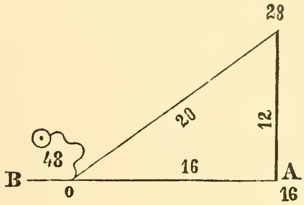
FIG. 86.



The above numbers are the most easily remembered, but the longer the lines measured the better; and nearly the whole chain may be used; thus: Fix down the 36th link from one end at A, and the 4th link from the same end on the line at B. Fix the other end of the chain also at B. Take the 40th-link mark from this last end, and draw the chain tight, and this mark will be in the perpendicular desired. The sides of the triangle formed by the chain will be 24, 32, and 40.

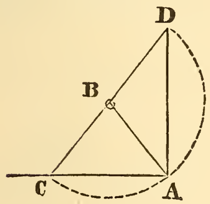
110. Otherwise: using a 50-feet tape, hold the 16-feet mark at A; hold the 48-feet mark and the ring-end of the tape together on the line; take the 28-feet mark of the tape, and draw it tight; then will the 28-feet mark be in the perpendicular desired.

FIG. 87.



111. *Second Method.* Hold one end of the chain at A and fix the other end at a point B, taken at will. Swing the chain around B as a center, till it again meets the line at C. Then carry the same end around (the other end remaining at B) till it comes in the line of C B at D. A D is the perpendicular required.

FIG. 88.



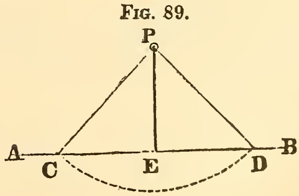
Problem 3. *To erect a perpendicular to an inaccessible line, at a given point of it.*

112. *First Method.* Get points in the direction of the inaccessible line prolonged, and from them set out a parallel to the line, by methods which are given in Art. 121, etc. Find by trial the point in which a perpendicular to this second line (and therefore to the first line) will pass through the required point.

Problem 4. *To let fall a perpendicular from a given point to a given line.*

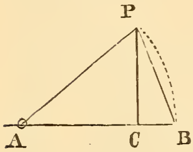
113. *First Method.* Let P be the given point, and A B the given line. Measure some distance, a chain or less, from C to P, and then fix one end of the chain at P, and swing it around till the same distance meets the line at some point D. The middle point E of the distance C D will be the required point, at which the perpendicular from P would meet the line.

FIG. 89.



114. *Second Method.* Stretch a chain, or a portion of it, from the given point P, to some point, as A, of the given line. Hold the end of the distance at A, and swing 'round the other end of the chain from P, so as to set off the same distance along the given line from A to some point B. Measure B P. Then will the distance B C from B to the foot of the desired perpendicular $= \frac{BP^2}{2\,AB}$.

FIG. 90.



Problem 5. *To let fall a perpendicular to a line from a point nearly opposite to the end of the line.*

115. *First Method.* Stretch a chain from the given point P, to some point, as A, of the given line. Fix to the ground the middle point B of the chain A P, and swing around the end which was at P, or at A, till it meets the given line in a point C, which will be the foot of the required perpendicular.

FIG. 91.

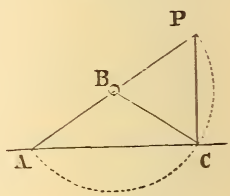
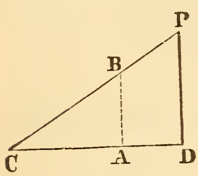
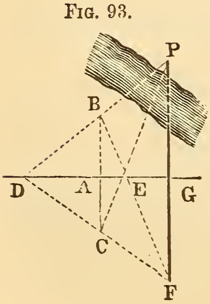


FIG. 92.



116. *Second Method.* At any convenient point, as A of the given line, erect a perpendicular of any convenient length, as A B, and mark a point C on the given line in the line of P and B. Measure C A, C B, and C P. Then the distance from C to the foot of the perpendicular, i. e., $CD = \frac{CA \times CP}{CB}$.

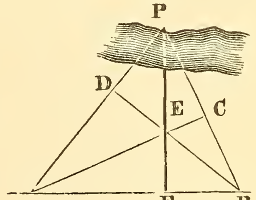
FIG. 93.



Problem 6. *To let fall a perpendicular to a line from an inaccessible point.*

117. *First Method.* Let P be the given point. At any point A, on the given line, set out a perpendicular, A B, of any convenient length. Prolong it on the other side of the line the same distance. Mark on the given line a point D in the line of P B, and a point E in the line of P C. Mark the point F at the intersection of D C and B E prolonged. The line F P is the line required, being perpendicular to the given line at the point G.

FIG. 94.

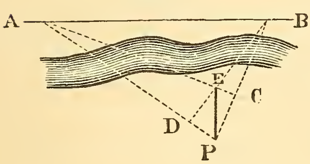


118. *Second Method.* Let A and B be two points of the given line. From A let fall a perpendicular, A C, to the visual line, B P; and from B let fall a perpendicular, B D, to the visual line, A P. Find the point at which these perpendiculars intersect, as at E, and the line P E, prolonged to F, will give the perpendicular required.

Problem 7. *To let fall a perpendicular from a given point to an inaccessible line.*

119. *First Method.* Let P be the given point, and A B the given line. By the preceding problem let fall perpendiculars from A to B P at C; and from B to A P at D; the line P E, passing from the given point to the intersection of these perpendiculars, is the desired perpendicular to the inaccessible line A B.

FIG. 95.



This method will apply when only two points of the line are visible.

The proof of 118 and 119 is found in the "Theory of Transversals," Corollary 3.

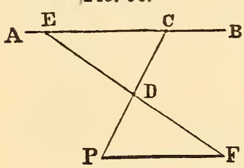
120. *Second Method.* Through the given point set out a line parallel to the inaccessible line. At the given point erect a perpendicular to the parallel line, and it will be the required perpendicular to the inaccessible line.

Problems on Parallels.

Problem 1. *To run a line from a given point parallel to a given line.*

121. *First Method.* Let fall a perpendicular from the point to the line. At another point of the line, as far off as possible, erect a perpendicular equal in length to the one just let fall. The line joining the end of this line to the given point will be the parallel required.

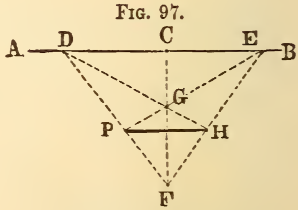
Fig. 96.



122. *Second Method.* Measure from P to any point, as C of the given line, and put a mark at the middle point D of that line. From any point, as E of the given line, measure a line to the point D, and continue it till D F = D E. Then will the line P F be parallel to A B.

123. *Third Method.* From any point, as C of the line, set off equal distances along the line to D and E. Take a point F, in the line of P D. Stake out the lines F C and F E, and also the line E P, crossing the line C F in the point G. Lastly, prolong the line D G till it meets the line E F in the point H. P H is the parallel required.

Fig. 97.

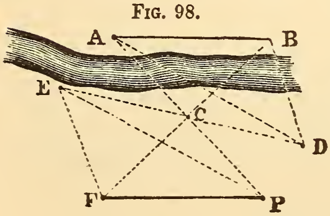


The proof is found in Corollary 4 of "Transversals."

Problem 2. *To run a line from a given point parallel to an inaccessible line.*

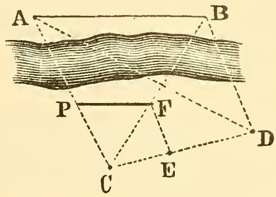
124. *First Method.* Let A B be the given line, and P the given point. Set a stake at C, in the line of P A, and another at any convenient point, D. Through P set out, by the preceding problem, a parallel to D A, and set a stake at the point, as E, where this parallel intersects D C prolonged. Through E set out a parallel to B D, and set a stake at the point F, where this parallel intersects B C prolonged. P F is the parallel required.

Fig. 98.



125. *Second Method.* Set a stake at any point C in the line of A P, and another at any convenient place, as at D. Through P set out a parallel to A D, intersecting C D in E. Through E set out a parallel to D B, intersecting C B in F. The line P F will be the parallel required.

Fig. 99.



126. Alinement and Measurement. We are now prepared, having secured a variety of methods for setting out Perpendiculars and Parallels in every probable case, to take up the general subject of overcoming Obstacles to Measurement.

Before a line can be measured its direction must be determined. This operation is called *Ranging* the line, or *Alining* it, or *Boning* it.* The word *alinement* † will be found very convenient for expressing the direction of a line on the ground, whether between two points or in their direction prolonged.

This branch of our subject naturally divides itself into two parts, the first of which is preliminary to the second, viz. :

I. Of Obstacles to Alinement; *or how to establish the direction of a line in any situation.*

II. Of Obstacles to Measurement; *or how to find the length of a line which can not be actually measured.*

1. *Obstacles to Alinement.*

127. All the cases which can occur under this head may be reduced to two, viz. :

A. To find points in a line *beyond* the given points, i. e., to *prolong* the line.

B. To find points in a line *between* two given points of it, i. e., to *interpolate* points in the line.

A. To Prolong a Line.

128. By ranging with Rods. When two points in a line are given, and it is desired to prolong the line by ranging it out with

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* This word, like many others used in engineering, is derived from a French word, *borner*, to work out or limit, indicating that the Normans introduced the art of surveying into England.

† Slightly modified from the French *alignement.*

rods, three persons are required, each furnished with a straight, slender rod, and with a plumb-line, or other means of keeping their rods vertical. One holds his rod at one of the given points, A in the figure, and another at B. A third, C, goes forward as far as he can without losing sight of the first two rods, and then, looking back, puts himself "in line" with A and B—i. e., so that when his eye is placed at C the rod at B hides or covers the rod at A. This he can do most accurately by holding a plumb-line before his eye, so that it shall cover the first two rods. The lower end of the plumb-bob will then indicate the point where the third rod should be placed, and so with the rest. The first man, at A, is then signaled and comes forward, passes both the others, and puts himself at D, "in line" with C and B. The man at B then goes on to E, and "lines" himself with D and C; and so they proceed, in this "hand-over-hand" operation, as far as is desired. Stakes are driven at each point in the line as soon as it is determined.

Fig. 100.

129. The rods should be perfectly straight, either cylindrical or polygonal, and as slender as they can be without bending. They should be painted in alternate bands of red and white, each a foot or link in length. Their lower ends should be pointed with iron, and a projecting bolt of iron will enable them to be pressed down by the foot into the earth, so that they can stand alone. When this is done, one man can range out a line. A rod can be set perfectly vertical by holding a plumb-line before the eye at some distance from the rod, and adjusting the rod so that the plumb-line covers it from top to bottom, and then repeating the operation in a direction at right angles to the former. A stone dropped from top to bottom of the rods will approximately attain the same end.

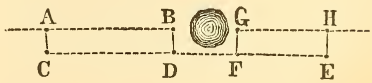
When the lines to be ranged are long, and great accuracy is required, the rods may have attached to them plates of tin with openings cut out of them, and black horse-hairs stretched from top to bottom of the openings.

Fig. 101.

A small telescope must then be used for ranging these hairs in line. In a hasty survey, straight twigs, with their tops split to receive a paper folded as in the figure, may be used.

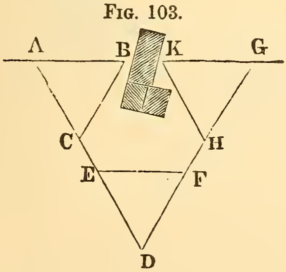
130. By Perpendiculars. The straight line, A B in the figure, is supposed to be stopped by a tree, a house, or other obstacle, and it is desired to prolong the line beyond this obstacle. From any two points, as A and B of the line, set off (by some of the methods which have been given) equal perpendiculars, A C and B D, long enough to pass the obstacle. Prolong this line beyond the obstacle, and from any two points in it, as E and F, measure the perpendiculars E G and F H equal to the first two, but in a contrary direction. Then will G and H be two points in the line A B prolonged which can be continued by the method of the last article. The points A and B should be taken as far apart as possible, as should also the points E and F. Three or more perpendiculars on each side of the obstacle may be set off, in order to increase the accuracy of the operation. The same thing may also be done on the other side of the line, as another confirmation or test of the accuracy of the prolonged line.

Fig. 102.



131. By Equilateral Triangles. The obstacles noticed in the last article may also be overcome by means of three equilateral triangles formed by the chain. Fix one end of the chain, and also the end of the first link from its other end, at B; fix the end of the 33d link at A; take hold of the 66th link and draw the chain tight, pulling equally on each part, and put a pin at the point thus found, C in the figure. An equilateral triangle will thus be formed, each side being 33 links. Prolong the line A C past the obstacle to some

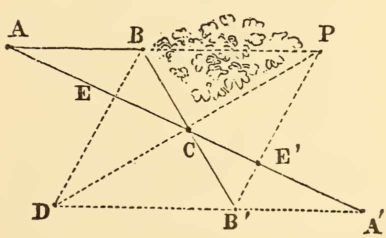
Fig. 103.



point, as D. Make another equilateral triangle, D E F, as before, and thus fix the point F. Prolong D F to a length equal to that of A D, and thus fix a point, G. At G form a third equilateral triangle, G H K, and thus fix a point, K. Then will K G give the direction of A B prolonged.

132. By Symmetrical Triangles. Let A B be the line to be prolonged. Take any convenient point, as C. Range out the line, A C, to a point A′, such that C A′ = C A. Range out C B, so that C B′ = C B. Range backward A′ B′ to some point D, such that D C prolonged will pass the obstacle. Find, by ranging, the intersection at E of D B and A C. From C measure, on C A′, the distance C E′ = C E. Then range out D C and B′ E′ to their intersection in P, which will be a required point in the direction of A B prolonged. The symmetrical points are marked by corresponding letters. Several other points should be obtained in the same manner.

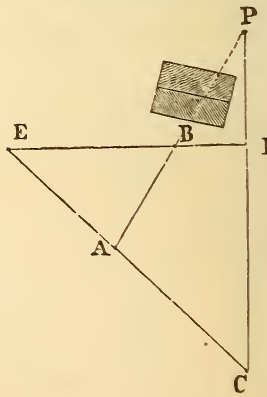
Fig. 104.



In this, as in all similar operations, very acute intersections should be avoided as far as possible.

133. By Transversals. Let A B be the given line. Take any two points C and D, such that the line C D will pass the obstacle. Take another point, E, in the intersection of C A and D B. Measure A E, A C, C D, B D, and B E. Then the distance from D to P, a point in the required prolongation, will be $D P = \dfrac{C D \times B D \times A E}{B E \times A C - B D \times A E}$.

Fig. 105.



Other points in the prolongation may be obtained in the same manner, by merely moving the single point C in the line of E A; in which case the new distances, C A and C D, will alone require to be measured.

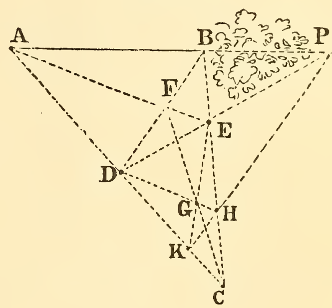
If A E be made equal to A C, then is $DP = \frac{CD \times BD}{BE - BD}$.

If B E be made equal to B D, then is $DP = \frac{CD \times AE}{AC - AE}$.

The *minus* sign in the denominators must be understood as only meaning that the difference of the two terms is to be taken, without regard to which is the greater.

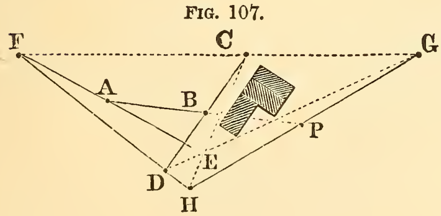
134. By Harmonic Conjugates. Let A B be the given line. Set a stake at any point C. Set stakes at points D, on the line C A, and at E, on the line C B; these points, D and E, being so chosen that the line D E will pass beyond the obstacle. Set a fourth stake, F, at the intersection of the lines A E and D B. Set a fifth stake, G, anywhere in the line C F; a sixth stake, H, at the intersection of C B and D G prolonged; and a seventh, K, at the intersection of C A and E G prolonged. Finally, range out the lines D E and K H, and their intersection at P will be in the line A B prolonged.

FIG. 106.



135. By the Complete Quadrilateral. Let A B be the given line. Take any convenient point C; measure from it to B, and onward, in the same line prolonged, an equal distance to D. Take any other convenient point, E, such that C E and D E produced will clear the obstacle. Measure from E to A, and onward, an equal distance, to F. Range out the lines F C and D E to their intersection in G.

FIG. 107.



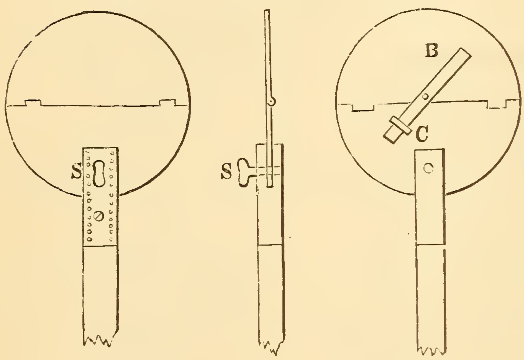
Range out F D and C E to intersect in H. Measure G H. Its middle point, P, is the required point in the line of A B prolonged. The unavoidable acute intersections in this construction are objectionable.

B. To interpolate Points in a Line.

136. The most distant given point of the line must be made as conspicuous as possible by any efficient means, such as placing there a staff bearing a flag : red and white, if seen against woods or other dark background ; and red and green, if seen against the sky.

Fig. 108.

Front View. *Side View.* *Back View.*



A convenient and portable signal is shown in the figure.

The figure represents a disk of tin about six inches in diameter, painted white and hinged in the middle, to make it more portable. It is kept open by the bar, B, being turned into the catch, C. A screw, S, holds the disk in a slit in the top of the pole.

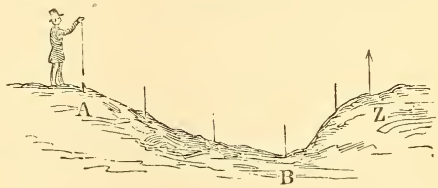
Another contrivance is a strip of tin, which has its ends bent horizontally in contrary directions. As the wind will take strongest hold of the side which is concave toward it, the bent strip will continually revolve, and thus be very conspicuous. Its upper half should be painted red, and its lower half white.

A bright tin cone set on the staff can be seen at a great distance when the sun is shining.

137. Ranging to a point thus made conspicuous is very simple when the ground is level. The surveyor places his eye at the nearest end of the line, or stands a little behind a rod placed on it, and by signs moves an assistant, holding a rod at some point as nearly in the desired line as he can guess, to the right or left, till his rod appears to cover the distant point.

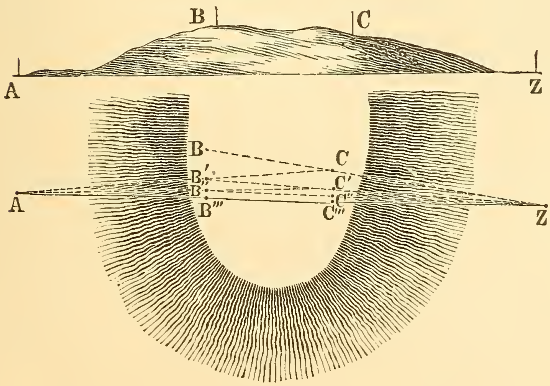
138. Across a Valley. When a valley or low spot intervenes between the two ends of the line, A and Z in the figure, a rod held in the low place, as at B, would seldom be high enough to be seen from A, to cover the distant rod at Z. In such a case, the surveyor at A should hold up a plumb-line over the point, at arm's length, and place his eye so that the plumb-line covers the rod at Z. He should then direct the rod held at B to be moved till it, too, is covered by the plumb-line. The point B is then said to be "in line" between A and Z. In geometrical language, B has now been placed in the vertical plane determined by the vertical plumb-line and the point Z. Any number of intermediate points can thus be "interpolated," or placed in line between A and Z.

FIG. 109.



139. Over a Hill. When a hill rises between two points and prevents one being seen from the other, as in the figure (the upper part of which shows the hill in "elevation," and the lower part in "plan"), two observers, B and C, each holding a rod, may place themselves on the ridge, in the line between the two points, as nearly as they can guess, and so that each can at once see the other and the point

FIG. 110.

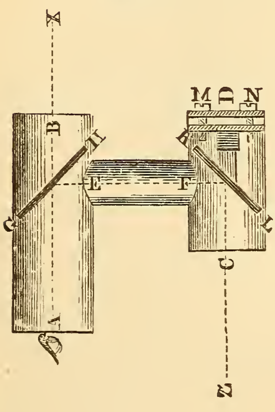


beyond him. B looks to Z, and by signals puts C "in line." C then looks to A, and puts B in line at B′. B repeats his operation from B′, putting C at C′, and is then himself moved to B″, and so they alternately "line" each other, continually approximating to the straight line between A and Z, till they at last find themselves both exactly in it, at B‴ and C‴.

140. A single person may put himself in line between two points, on the same principle, by laying a straight stick on some support, going to each end of it in turn, and making it point successively to each end of the line. The "Surveyor's Cross," Art. 93, is convenient for this purpose, when set up between the two given points and moved again and again, until, by repeated trials, one of its slits sights to the given points when looked through in either direction.

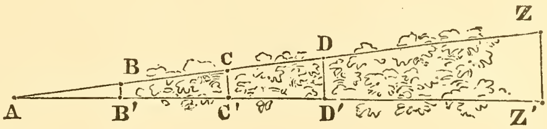
141. On Water. A simple instrument for the same object is represented in the figure. A B and C D are two tubes, about $1\frac{1}{2}$ inch in diameter, connected by a smaller tube, E F. A piece of looking-glass, G H, is placed in the lower part of the tube A B, and another, K L, in the tube C D. The planes of the two mirrors are at right angles to each other. The eye is placed at A, and the tube A B is directed to any distant object, as X, and any other object behind the observer, as Z, will be seen, apparently under the first object in the mirror G H, by reflection from the mirror K L, when the observer has succeeded in getting in line between the two objects. M N are screws by which the mirror K L may be adjusted. The distance between the two tubes will cause a small parallax, which will, however, be insensible except when the two objects are near together.

FIG. 111.



142. Through a Wood. When a wood intervenes between any two given points, preventing one from being seen from the other, as in the figure, in which A and Z are the given points, proceed thus: Hold a rod at some point B′ as nearly in the desired line from A as can be guessed at,

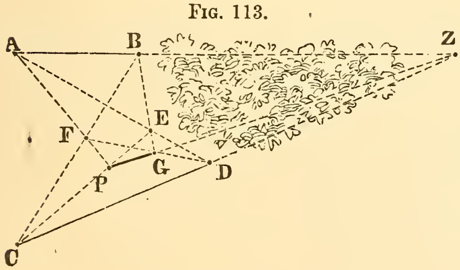
FIG. 112.



and as far from A as possible. To approximate to the proper direction, an assistant may be sent to the other end of the line, and his shouts will indicate the direction ; or a gun may be fired there; or, if very distant, a rocket may be sent up after dark. Then range out the "random line" A B′, by the method given in Art. 128, noting also the distance from A to each point found, till you arrive at a point Z′, opposite to the point Z—i. e., at that point of the line from which a perpendicular there erected would strike the point Z. Measure Z′ Z. Then move each of the stakes, perpendicularly from the line A Z′, a distance proportional to their distances from A. Thus, if A Z′ be 1,000 links, and Z′ Z be 10 links, then a stake B′, 200 links from A, should be moved 2 links to a point B, which will be in the desired straight line A Z; if C′ be 400 links from A, it should be moved 4 links to C, and so with the rest. The line should then be cleared, and the accuracy of the position of these stakes tested by ranging from A to Z.

143. To an Invisible Intersection. Let A B and C D be two lines, which, if prolonged, would meet in a point Z, invisible from either of them; and let P be a point from which a line is required to be set out tending to this invisible intersection. Set stakes at the five

FIG. 113.



given points, A, B, C, D, P. Set a sixth stake at E, in the alinements of A D and C P; and a seventh stake at F, in the alinements of B C and A P. Then set an eighth stake at G, in the alinements of B E and D F. P G will be the required line. This is an application of the "Theory of Transversals."

Otherwise: Through P range out a parallel to the line B D. Note the points where this parallel meets A B and C D, and call these points Q and R. Then the distance from B, on the line B D, to a point which shall be in the required line running from P to the invisible point, will be $= \frac{BD \times QP}{QR}$.

II. Obstacles to Measurement.

144. The cases in which the direct measurement of a line is prevented by various obstacles may be reduced to three:

A. When both ends of the line are accessible.

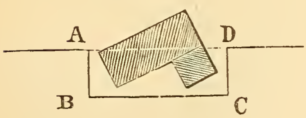
B. When one end of it is inaccessible.

C. When both ends of it are inaccessible.

A. When Both Ends of the Line are accessible.

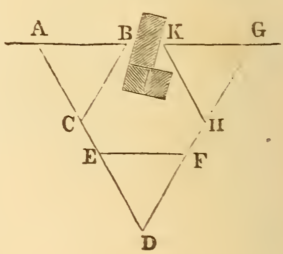
145. By Perpendiculars. On reaching the obstacle, as at A in the figure, set off a perpendicular, A B; turn a second right angle at B, and measure past the obstacle; turn a third right angle at C, and measure to the original line at D. Then will the measured distance, B C, be equal to the desired distance, A D.

Fig. 114.



If the direction of the line is also unknown, it will be most easily obtained by the additional perpendiculars shown in Fig. 102 of Art. 130.

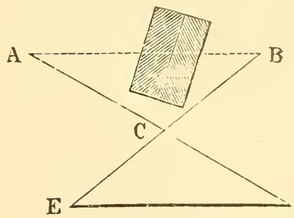
Fig. 115.



146. By Equilateral Triangles. The method given in Art. 131 for determining the direction of a line through an obstacle will also give its length; for in Fig. 115 the desired distance A G is equal to the measured distances A D or D G.

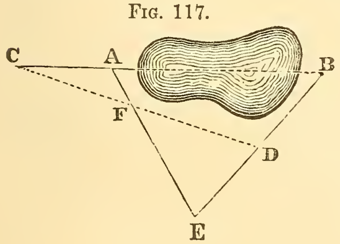
147. By Symmetrical Triangles. Let A B be the distance required. Measure from A obliquely to some point C past the obstacle. Measure onward, in the same line, till C D is as long as A C. Place stakes at C and D. From B measure to C, and from C measure onward, in the same line, till C E is equal to C B. Measure E D, and it will be equal to A B, the distance required. If more convenient, make C D and C E equal, respectively, to half of A C and C B; then will A B be equal to twice D E.

FIG. 116.



148. By Transversals. Let A B be the required distance. Set a stake, C, in the line prolonged; set another stake, D, so that C and B can be seen from it; and a third stake, E, in the line of B D prolonged, and at a distance from D equal to the distance from D to B. Set a fourth stake, F, at the intersection of E A and C D.

FIG. 117.

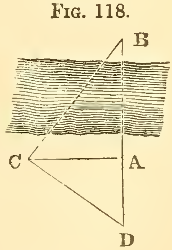


Measure A C, A F, and F E. Then is $AB = \frac{AC}{AF}(FE - AF)$.

B. WHEN ONE END OF THE LINE IS INACCESSIBLE.

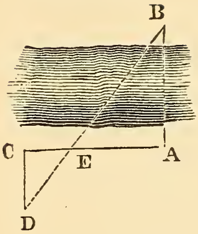
149. By Perpendiculars. This principle may be applied in a variety of ways. In Fig. 118 let A B be the required distance. At the point A set off A C perpendicular to A B, and of any convenient length. At C set off a perpendicular to C B. and continue it to a point, D, in the line of A and B. Measure D A. Then is $AB = \frac{AC^2}{AD.}$

FIG. 118.



150. *Otherwise:* At the point A, in Fig. 119, set off a perpendicular, A C. At C set off another perpendicu-

FIG. 119.

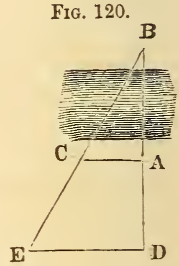


lar, C D. Find a point, E, in the line of A C and B D. Measure A E and E C. Then is $AB = \frac{AE \times CD}{CE}$.

If E C be made equal to A E, and D be set in the line of B E, and also in the perpendicular from C, then will C D be equal to A B.

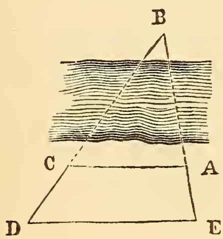
If $EC = \frac{1}{2} AE$, then $CD = \frac{1}{2} AB$.

FIG. 120.



151. *Otherwise:* At A, in Fig. 120, measure a perpendicular, A C, to the line A B; and at any point, as D in this line, set off a perpendicular to D B, and continue it to a point E, in the line of C B. Measure D E and also D A. Then is $AB = \frac{AC \times AD}{DE - AC}$.

FIG. 121.

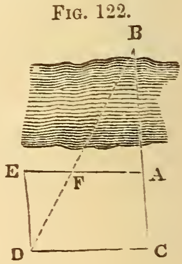


152. By Parallels. From A measure A C in any convenient direction. From a point D, in the line of B C, measure a line parallel to C A, to a point E in the line of A B. Measure also A E.

$$\text{Then is } AB = \frac{AC \times AE}{DE - AC}.$$

153. By a Parallelogram. Set a stake, C, in the line of A and B, and set another stake, D, wherever convenient. With a distance equal to C D, describe from A an arc on the ground; and, with a distance equal to A C, describe another arc from D intersecting the first arc in E. Or, take A C and C D so that together they make one chain; fix the ends of the chain at A and D; take hold of the chain at such a link that one part of it equals A C and the other C D, and draw it tight to fix the point E. Set a stake at F in the intersection

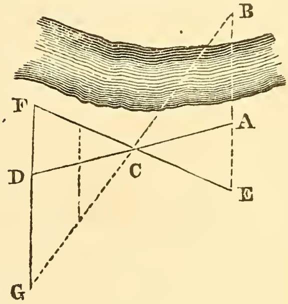
FIG. 122.



of A E and D B. Measure A F and E F. Then is $AB = \frac{AC \times AF}{EF}$; or $CB = \frac{AC \times CD}{EF}$.

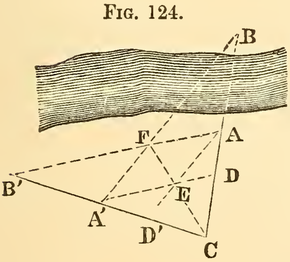
154. By Symmetrical Triangles. Let A B be the required distance. From A measure a line in any convenient direction, as A C, and measure onward, in the same direction, till C D = A C. Take any point E in the line of A and B. Measure from E to C, and onward in the same line, till C F = C E. Then find by trial a point G, which shall be at the same time in the line of C and B, and in the line of D and F. Measure the distance from G to D, and it will be equal to the required distance from A to B. If more convenient, make C D = $\frac{1}{2}$ A C, and C F = $\frac{1}{2}$ C E, as shown by the finely dotted lines in the figure. Then will D G = $\frac{1}{2}$ A B.

Fig. 123.



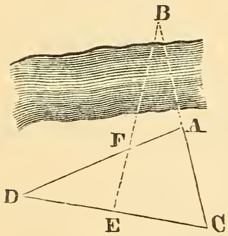
155. *Otherwise:* Prolong B A to some point C. Range out any convenient line C A′, and measure C A′ = C A. The triangle C A′ B is now to be reproduced in a symmetrical triangle situated on the accessible ground. For this object take, on A C, some point D and measure C D′ = C D. Find the point E at the intersection of A D′ and A′ D. Find the point F at the intersection of A′ B and C E. Lastly, find the point B′ at the intersection of A F and C A′. Then will A′ B′ = A B. The symmetrical points have corresponding letters affixed to them.

Fig. 124.



156. By Transversals. Set a stake, C, in the alinement of B A; a second, D, at any convenient point; a third, E, in the line C D; and a fourth, F, at the intersection of the aline-

Fig. 125.



ments of D A and E B. Measure A C, C E, E D, D F, and F A. Then is $A\,B = \dfrac{A\,C \times A\,F \times D\,E}{C\,E \times D\,F - A\,F \times D\,E}$.

If the point E be taken in the middle of C D (as it is in the figure), then $A\,B = \dfrac{A\,C \times A\,F}{D\,F - A\,F}$.

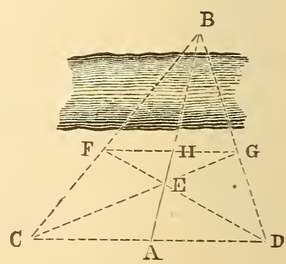
If the point F be taken in the middle of A D, then $A\,B = \dfrac{A\,C \times D\,E}{C\,E - D\,E}$.

The *minus* signs must be interpreted as in Art. 121.

157. By Harmonic Division. Set stakes, C and D, on each side of A, and so that the three are in the same straight line. Set a third stake at any point, E, of the line A B. Set a fourth, F, at the intersection of C B and D E; and a fifth, G, at the intersection of D B and C E. Set a sixth stake, H, at the intersection of A B and F G. Measure A E and E H. Then is

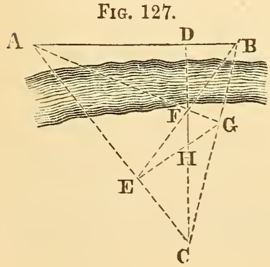
$$A\,B = \frac{A\,E \times A\,H}{A\,E - E\,H}.$$

Fig. 126.



158. To an Inaccessible Line. The shortest distance, C D, from a given point, C, to an inaccessible straight line A B, is required. From C let fall a perpendicular to A B, by the method of Art 119. Then set a stake at any point, E, on the line A C; set a second, F, at the intersection of E B and C D; a third, G, at the intersection of A F and C B; and a fourth, H, at the intersection of E G and C D.

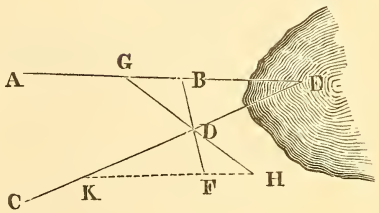
Fig. 127.



Measure C H and H F. Then is $C\,D = \dfrac{C\,H \times C\,F}{C\,H - H\,F}$; or $C\,D = C\,H \cdot \dfrac{C\,H + H\,F}{C\,H - H\,F}$; or $C\,D = \dfrac{C\,H \times C\,F}{2\,C\,H - C\,F}$.

159. To an Inaccessible Intersection. When two lines (as A B, C D, in the figure) meet in a river, a building, or any other inaccessible point, the distance from any point of either to their intersection, D E, for example, may be found thus: From any point B, on one line, measure B D, and continue it till D F = D B. From any other point G of the former line measure G D, and continue the line till D H = G D. Continue H F to meet D C in some point K. Measure K D. K D will be equal to the desired distance D E.

Fig. 128.



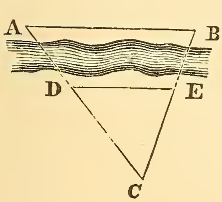
B E can be found by measuring F K, which is equal to it.

If D F and D H be made respectively equal to one half or one third, etc., of D B and D G, then will K D and K F be respectively equal to one half or one third, etc., of D E and B E.

C. When Both Ends of the Line are inaccessible.

160. By Similar Triangles. Let A B be the inaccessible distance. Set a stake at any convenient point C, and find the distances C A and C B by any of the methods just given. Set a second stake at any point, D, on the line C A. Measure a distance equal to $\frac{CB \times CD}{CA}$, from C, on the line C B, to some point E. Measure D E. Then is $AB = \frac{AC \times DE}{CD}$.

Fig. 129.



If more convenient, measure C D in the contrary direction from the river, as in Fig. 130, instead of toward it, and in other respects proceed as before.

161. By Parallels. Let A B be the inaccessible distance. From any point, as C, range out a parallel to A B, as in Art. 124, etc. Find the distance C A by Art. 149, etc. Set a stake at the point E, the

Fig. 130.

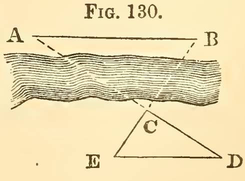
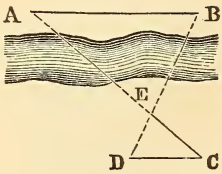


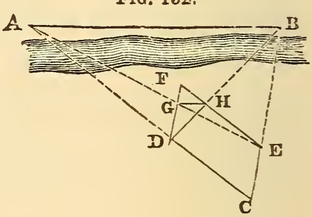
Fig. 131.



intersection of C A and D B, and measure C E. Then is $A B = \frac{C D \times (A C - C E)}{C E}$.

162. By a Parallelogram. Set a stake at any convenient point C. Set stakes D and E anywhere in the alinements C A and C B. With D as a center, and a length of the chain equal to C E, describe an arc; and with E as a center, and a length of the chain equal to C D, describe another arc, intersecting the former one at F. A parallelogram, C D E F, will thus be formed. Set stakes at G and H, where the alinements D B and E A intersect the sides of this parallelogram. Measure C D, D F, G F, F H, and H G. The inaccessible distance $A B = \frac{C D \times D F \times G H}{F G \times F H}$.

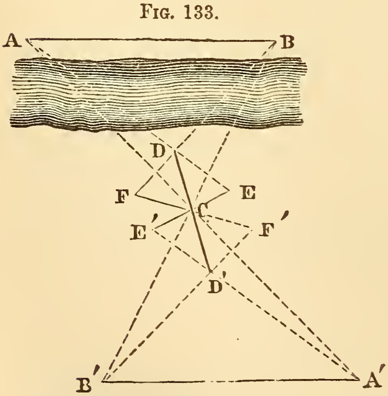
Fig. 132.



If $C D = C E$, then $A B = \frac{C D^2 \times G H}{F G \times F H}$.

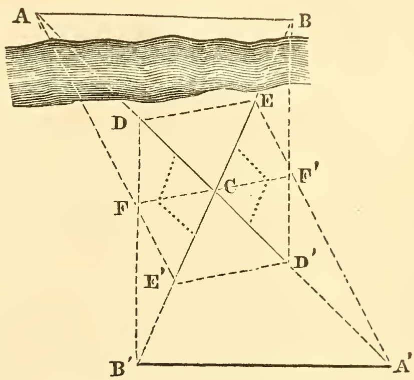
163. By Symmetrical Triangles. Take any convenient point, as C. Set stakes at two other points, D and D′, in the same line, and at equal distances from C. Take a point E, in the line of A D; measure from it to C, and onward till C E′ = C E. Take a point F in the line of B D; measure from it to C, and onward till C F′ = C F. Range out the lines A C and E′ D′, and set a stake at their intersection, A′. Range out the lines B C and F′ D′; and set a stake at their intersection, B′. Measure A′ B′. It will be equal to the desired distance A B.

Fig. 133.



164. *Otherwise:* Take any convenient point, as C, and set off equal distances on each side of it, in the line of C A, to D and D′. Set off the same distances from C, in the line of C B, to E and E′. Through C set out a parallel to D E or D′ E′, and set stakes at the points F and F′ where this parallel intersects A E′ and B D′. Range out the lines A D′ and E F′, and set a stake at their intersection A′. Range out the lines B E′ and D F, and set a stake at their intersection B′. Measure A′ B′, and it will be equal to the desired distance A B.

Fig. 134.



CHAPTER III.

COMPASS-SURVEYING; OR BY THE THIRD METHOD.

165. *Angular Surveying* determines the relative positions of points, and therefore of lines, on the THIRD PRINCIPLE, as explained in Art. 5.

Either the compass or the transit may be employed in angular surveying.

166. Surveying with the compass is a less direct operation than surveying with the transit. But as the use of the compass is much more rapid and easy, for this reason, as well as for its smaller cost, it is the instrument most commonly employed in land-surveying in spite of its imperfections and inaccuracies.

The method of *Polar Surveying* (or surveying by the third method) embraces two minor methods. The most usual one consists in going around the field with the instrument, setting it at each corner, and measuring there the angle which each side makes with its neighbor, as well as the length of each side. This method is called by the French the method of *Cheminement.* It has no special name in English, but may be called (from the American verb, to progress) the *Method of Progression.* The other system, the *Method of Radiation,* consists in setting the instrument at one point and thence measuring the direction and distance of each corner of the field or other object. The corresponding name of what we have called triangular surveying is the *Method of Intersections,* since it determines points by the intersections of straight lines.

167. When the two lines which form an angle lie in the same horizontal or level plain, the angle is called a *horizontal angle.**

When these lines lie in a plane perpendicular to the former, the angle is called a *vertical angle.*

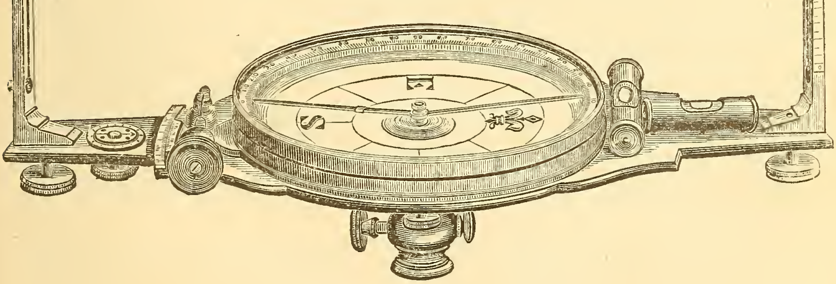
When one of the lines is horizontal, and the other line from the eye of the observer passes above the former, and in the same vertical plane, the angle is called an *angle of elevation.*

When the latter line passes below the horizontal line, and in the same vertical plane, the angle is called an *angle of depression.*

When the two lines which form an angle lie in other planes which make oblique angles with each of the former planes, the angle is called an *oblique angle.*

Horizontal angles are the only angles employed in common land-surveying.

Fig. 135.



THE COMPASS.

168. The Needle. The most essential part of the compass is the magnetic needle. It is a slender bar of steel, usually five or six inches long, strongly magnetized, and balanced on a pivot, so that it may turn freely, and thus be enabled to continue pointing in the same direction (that of the "*magnetic meridian,*" approximately north and south) however much the "compass-box," to which the pivot is attached, may be turned around.

As it is important that the needle should move with the least

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* A plane is said to be *horizontal* or *level* when it is parallel to the surface of standing water, or perpendicular to a plumb-line. A line is horizontal when it lies in a horizontal plane.

possible friction, the pivot should be of the hardest steel ground to a very sharp point; and in the center of the needle, which is to rest on the pivot, should be inserted a cap of agate, or other hard material. Iridium for the pivot, and ruby for the cap, are still better.

If the needle be balanced on its pivot before being magnetized, one end will sink, or "dip," after the needle is magnetized. To bring it to a level, several coils of wire are wound around the needle so that they can be slid along it, to adjust the weight of its two ends and balance it more perfectly.

The north end of the needle is usually cut into a more ornamental form than the south end for the sake of distinction.

The principal requisites of a compass-needle are intensity of directive force and susceptibility. Beyond a certain limit, say five inches, no additional power is gained by increasing the length of the needle. On the contrary, longer ones are apt to have their strength diminished by several consecutive poles being formed. Short needles, made very hard, are therefore to be preferred.

The needle should not come to rest very quickly. If it does, it indicates either that it is weakly magnetized, or that the friction on the pivot is great. Its sensitiveness is indicated by the number of vibrations which it makes in a small space before coming to rest.

A screw, with a milled head, on the under side of the plate which supports the pivot, is used to raise the needle off this pivot when the instrument is carried about, to prevent the point being dulled by unnecessary friction.

169. The Sights. Next after the needle, which gives the direction of the fixed line whose angles with the lines to be surveyed are to be measured, should be noticed the sights, which show the directions of these last lines. At each end of a line passing through the pivot is placed a "sight," consisting of an upright bar of brass, with openings in it of various forms—usually slits, with a circular aperture at their top and bottom; all these arrangements being intended to enable the line of sight to be directed to any desired object with precision.

A telescope which can move up and down in a vertical plane, i. e., a plunging telescope, or one which can turn completely over, is sometimes substituted for the sights. It has the great advantage of giving more distinct vision at long distances, and of admitting of sights up and down very steep slopes. Its accuracy of vision is, however, rendered nugatory by the want of precision in the readings of the needle. If a telescope be applied to the compass, a graduated circle with vernier should be added, thus converting the compass into a "transit."

170. The Divided Circle. We now have the means of indicating the directions of the two lines whose angle is to be measured. The number of degrees contained in it is to be read from a circle divided into degrees, in the center of which is fixed the pivot bearing the needle. The graduations are usually made to half a degree, and a quarter of a degree or less can then be "estimated." The pivot and needle are sunk in a circular box, so that its top may be on a level with the needle. The graduations are usually made on the top of the surrounding rim of the box, but should also be continued down its inside circumference so that it may be easier to see with what division the ends of the needle coincide.

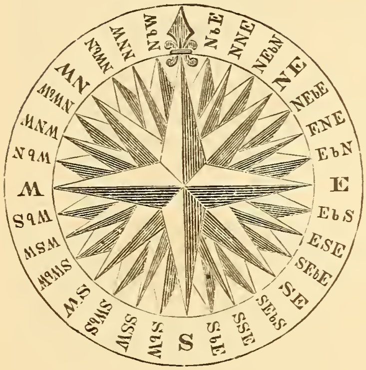
The degrees are not numbered consecutively from 0° around to 360°, but run from 0° to 90°, both ways from the two diametrically opposite points at which a line, passing through the slits in the middle of the sights, would meet the divided circle.

The lettering of the surveyor's compass has one important difference from that of the mariner's compass.

When we stand facing the north, the east is on our right hand, and the west on our left. The graduated card of the mariner's compass, which is fastened to the needle and turns with it, is marked accordingly. But, in the surveyor's compass, one of the 0 points being marked N. or north (or indicated by a *fleur-de-lis*), and the opposite one S. or south, the 90-degrees-point on the *right* of this line, as you stand at the S. end and look toward the N., is marked W. or west; and the *left* hand 90-degrees-point is marked E. or east. The reason of this will be seen when the method of using the compass comes to be explained.

171. The Points. In ordinary land-surveying only four points of the compass have names, viz., north, south, east, and west; the direction of a line being described by the angle which it makes with a north and south line to its east or to its west. But, for nautical purposes, the circle of the compass is divided into thirty-two points, the names of which are shown in the figure.

FIG. 136.



Two rules embrace all the cases: 1. When the letters indicating two points are joined together, the point half-way between the two meant; thus, N. E. is half-way between north and east; and N. N. E. is half-way between north and northeast. 2. When the letters of two points are joined together with the intermediate word *by*, it indicates the point which comes next after the first in going toward the second; thus, N. *by* E. is the point which follows north in going toward the east; S. E. *by* S. is the next point from southeast going toward the south.

172. Eccentricity. The center-pin, or pivot of the needle, ought to be exactly in the center of the graduated circle; the needle ought to be straight, and the line of the sights ought to pass exactly through this center and through the 0 points of the circle. If this is not the case, there will be an error in every observation. This is called the *error of eccentricity.*

When the maker of a compass is about to fix the pivot in place, he is in doubt of two things: whether the needle is perfectly straight, and whether the pivot is exactly in the center. In Figs. 137 and 138 both of these are represented as being excessively in error.

First, to examine if the needle be straight. Fix the pivot temporarily so that the ends of the needle may cut opposite degrees—i. e., degrees differing by 180°. The condition of things at this stage of progress will be represented by Fig. 137. Then turn the compass-box half-way around. The error will now be doubled,

FIG. 137.

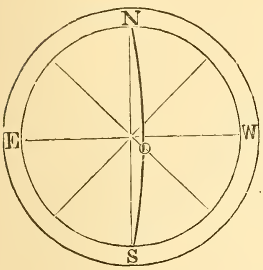


FIG. 138.

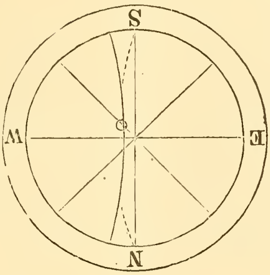
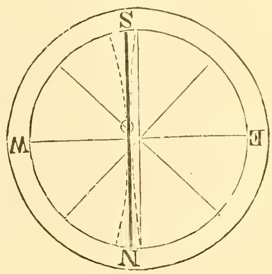


FIG. 139.



as is shown by Fig. 138, in which the former position of the needle is indicated by a dotted line.* Now bend the needle, as in Fig. 139, till it cuts divisions midway between those cut by it in its present and in its former position. This makes it certain that the needle is straight, or that its two ends and its center lie in the same straight line.

Second, to put the pivot in the center. Move it till the straightened needle cuts opposite divisions. It is then certain that the direction of the needle passes through the center. Turn the compass-box one quarter around, and, if the needle does not then cut opposite divisions, move the pivot till it does. Repeat the operation in various positions of the box. It will be a sufficient test if it cuts the opposite divisions of 0°, 45°, and 90°.

To fix the sights precisely in line, draw a hair through their slits and move them till the hair passes over the 0 points on the circle.

The surveyor can also examine for himself, by the principle of reversion, whether the line of the sights passes through the center or not. Sight to any very near object. Read off the number of degrees indicated by one end of the needle. Then turn the compass half around, and sight to the same object. If the two readings do not agree, there is an error of eccentricity, and the arithmetical mean, or half sum of the two readings, is the correct one. In Fig. 140 the line of sight A B is represented as passing to

* This is another example of the fruitful principle of *reversion*.

one side of the center, and the needle as pointing to 46°. In Fig. 141 the compass is supposed to have been turned half around, and

Fig. 140.

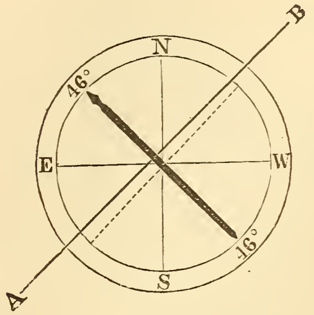
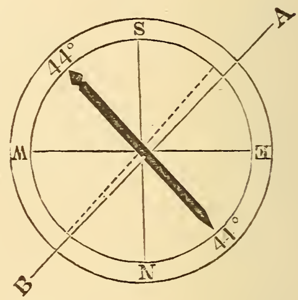


Fig. 141.



the other end of the sights to be directed to the same object. Suppose that the needle would have pointed to 45° if the line of sight had passed through the center; the needle will now point to 44°, the error being doubled by the reversion, and the true reading being the mean.

This does not, however, make it certain that the line of the sights passes through the 0 points, which can only be tested by the hair, as mentioned above.

173. Levels. On the compass-plate are two small spirit-levels. They consist of glass tubes slightly curved upward, and nearly filled with alcohol, leaving a bubble of air within them. They are so adjusted that, when the bubbles are in the centers of the tubes, the plate of the compass shall be level. One of them lies in the direction of the sights, and the other at right angles to this direction.

On the compass-plate, and between the vernier and the left-hand sight in the figure, is the *Outkeeper*, for keeping tally of the chains in any distance.

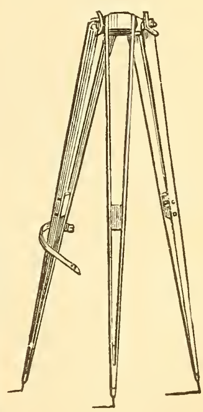
174. Tangent Scale. This is a convenient, though not essential, addition to the compass, for the purpose of measuring the slopes of ground, so that the proper allowance in chaining may be made. In the figure of the compass may be seen, on the edge of the left-hand sight, a small projection of brass with a hole through it. On the edge of the other sight are engraved lines numbered from 0° to 20°, the 0° being of the same height above the compass-plate that the eye-hole is. To use this, set the compass at the bottom of a slope, and at the top set a signal of exactly the height of the eye-hole from the ground. Level the compass very carefully, particularly by the level which lies lengthwise, and, with the eye at the eye-hole, look to the signal and note the number of the division on the farther sight which is cut by the visual ray. That will be the angle of the slope; the distances of the engraved lines from the 0° line being tangents (for the radius equal to the distance between the sights) of the angles corresponding to the numbers of the lines.

175. Vernier. The compass-box is connected with the plate which carries it and the sights, so that it can turn around on this plate. This motion is given to it by a slow motion or tangent screw, shown on the left of the compass-box in the figure. The space through which the compass-box is moved is indicated by a vernier. For description of a vernier, and method of reading it, see subject *Verniers* under Transit-Surveying.

176. Tripod. The compass, like most surveying instruments, is usually supported on a tripod, consisting of three legs, shod with iron, and so connected at top as to be movable in any direction. There are many forms of these. Lightness and stiffness are the qualities desired. The most usual form is shown in the figures of the transit and the level. Of the two represented in Figs. 142 and 143 the first has the advantage of being very easily and cheaply made; and the second that of being light and yet capable of very firmly resisting horizontal torsion.

FIG. 142.

FIG. 143.



The joints by which the instrument is connected with the tripod are also various. Fig. 144 is the "ball-and-socket joint," most usual in this country. It takes its name from the ball in which terminates the covered spindle which enters a corresponding cavity under the compass-plate and the socket in which this ball turns. It admits of motion in any direction, and can be tightened or loosened by turning the upper half of the hollow piece inclosing

Fig. 144.

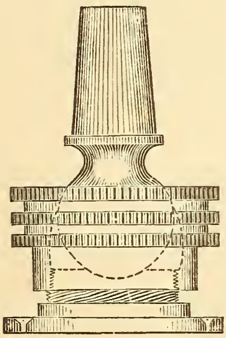


Fig. 145.

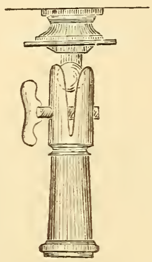
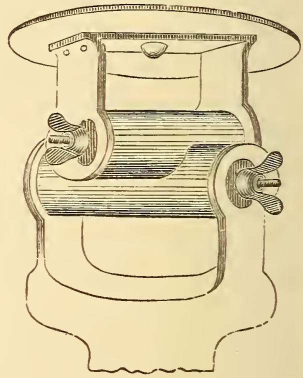


Fig. 146.

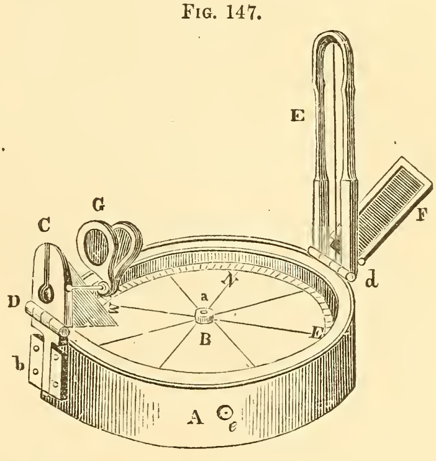


it, which is screwed on the lower half. Fig. 145 is called the "shell-joint." In it the two shell-shaped pieces inclosing the ball are tightened by a thumb-screw. Fig. 146 is "Cugnot's joint." It consists of two cylinders placed at right angles to each other, and through the axes of which pass bolts, which turn freely in the cylinder, and can be tightened or loosened by thumb-screws at their ends. The combination of the two motions which this joint permits enables the instrument which it carries to be placed in any desired direction. This joint is much the most stable of the three.

177. Jacob's Staff. A single leg, called a "Jacob's staff," has some advantages, as it is lighter to carry in the field, and can be made of any wood on the spot where it is to be used, thus saving the expense of a tripod and the trouble of its transportation. Its upper end is fitted into the lower end of a brass head which has a ball-and-socket joint and axis above. Its lower end should be shod with iron, and a spike running through it is useful for pressing it into the ground with the foot. Of course, it can not be conveniently used on frozen ground or on pavements. It may, however, be set before or behind the spot at which the angle is to be measured, provided that it is placed very precisely in the line of direction from that station to the one to which a sight is to be taken.

178. The Prismatic Compass. The peculiarity of this instrument (often called Schmalcalder's) is that a glass triangular prism is substituted for one of the sights. Such a prism has this peculiar property that at the same time it can be seen through, so that a sight can be taken through it, and that its upper surface reflects like a mirror, so that the numbers of the degrees immediately under it can be read off at the same time that a sight to any object is taken. Another peculiarity necessary for profiting by the last one is that the divided circle is not fixed, but is a card fastened to the needle and moving around with it, as in the mariner's compass. The minute description which follows is condensed from Simms.

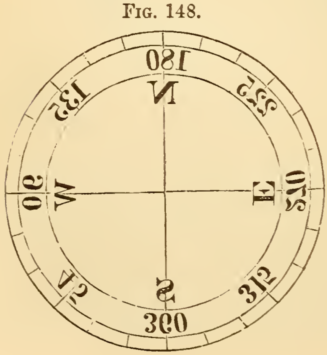
FIG. 147.



In the figure, A represents the compass-box and B the card, which, being attached to the magnetic needle, moves as *it* moves around the agate center *a*, on which it is suspended. The circumference of the card is usually divided to $\frac{1}{4}$ or $\frac{1}{2}$ of a degree. C is a prism which the observer looks through. The perpendicular thread of the sight-vane, E, and the divisions on the card appear *together* on looking through the prism, and the division with which the thread coincides when the needle is at rest, is the "bearing" of whatever object the thread may bisect—i. e., is the angle which the line of sight makes with the direction of the needle. The prism is mounted with a hinge-joint, D. The sight-vane has a fine thread stretched along its opening in the direction of its length, which is brought to bisect any object by turning the box around horizontally. F is a mirror made to slide on or off the sight-vane, E; and it may be reversed at pleasure—that is, turned face downward; it can also be inclined at any angle by means of its joint, *d;* and it will remain stationary on any part of the vane by the friction of its slides. Its use is to reflect the image of an object to the eye of an observer when the object is much above or below the horizontal plane. The colored glasses represented at G are intended for observing the sun. At *e* is shown a spring, which, being pressed by the finger at the time of observation and then released, checks the vibrations of the card, and brings it more speedily to rest. A stop is likewise fixed to the other side of the box, by which the needle may be thrown off its center.

The method of using this instrument is very simple: First raise the prism in its socket, *b,* until you obtain a distinct view of the divisions on the card. Then, standing over the point where the angles are to be taken, hold the instrument to the eye, and, looking through the slit, *C,* turn around till the thread in the sight-vane bisects one of the objects whose bearing is required; then, by touching the spring, *e,* bring the needle to rest, and the division on the card which coincides with the thread on the vane will be the bearing of the object from the north or south points of the magnetic meridian. Then turn to any other object and repeat the operation; the difference between the bearing of this object and that of the former will be the angular distance of the objects in question. Thus, suppose the former bearing to be 40° 30′, and the latter 10° 15′, both east or both west, from the north or south, the angle will be 30° 15′. The divisions are generally numbered 5°, 10°, 15°, etc., around the circle to 360°.

FIG. 148.



The figures on the compass-card are reversed or written upside down, as in the figure (in which only every fifteenth degree is marked), because they are again reversed by the prism.

The prismatic compass is generally held in the hand, the bearing being caught, as it were, in passing; but more accurate readings would, of course, be obtained if it rested on a support, such as a stake cut flat on its top.

In the former mode, the needle never comes completely to rest, particularly in the wind. In such cases, observe the extreme divisions between which the needle vibrates, and take their arithmetical mean.

179. Defects of the Compass. The compass is deficient in both precision and correctness.*

The former defect arises from the indefiniteness of its mode of indicating the part of the circle to which it points. The point of the needle has considerable thickness; it can not quite touch the divided circle; and these divisions are made only to whole or half degrees, though a fraction of a division may be estimated or guessed at. The vernier does not much better this, as we shall see when explaining its use. Now, an inaccuracy of one quarter of a degree in an angle—i. e., in the difference of the directions of two lines—causes them to separate from each other $5\frac{1}{4}$ inches at the end of 100 feet; at the end of 1,000 feet, nearly $4\frac{1}{2}$ feet; and, at the end of a mile, 23 feet. A difference of only one tenth of a degree, or six minutes, would produce a difference of $1\frac{3}{4}$ foot at the end of 1,000 feet; and $9\frac{1}{4}$ feet at the distance of a mile. Such are the differences which may result from the want of *precision* in the indications of the compass.

But a more serious defect is the want of *correctness* in the compass. Its not pointing exactly to the true north does not, indeed, affect the correctness of the angles measured by it. But it does not point in the same or in a parallel direction during even the

* The student must not confound these two qualities. To say that the sun appears to rise in the eastern quarter of the heavens and to set in the western is *correct*, but not *precise*. A watch with a second-hand indicates the time of day *precisely*, but not always *correctly*. The statement that two and two make five is *precise*, but is not usually regarded as *correct*.

same day, but changes its direction between sunrise and noon nearly a quarter of a degree, as will be fully explained hereafter. The effect of such a difference we have just seen. This direction may also be greatly altered in a moment, without the knowledge of the surveyor, by a piece of iron being brought near to the compass, or by some other local attraction, as will be noticed in Art. 186. This is the weak point in the compass.

Notwithstanding these defects, the compass is a very valuable instrument, from its simplicity, rapidity, and convenience in use; and, though never precise, and seldom correct, it is generally not *very* wrong.

THE FIELD-WORK.

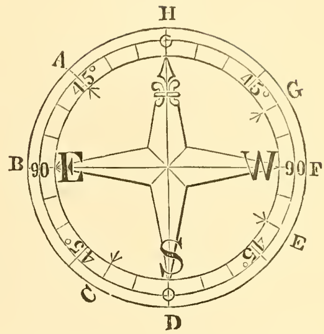
180. Taking Bearings. The "bearing" of a line is the angle which it makes with the direction of the needle. The bearing and length of a line are named collectively the *Course.*

To take the bearing of any line, set the compass exactly over any point of it by a plumb-line suspended from beneath the center of the compass, or, approximately, by dropping a stone. Level the compass by bringing the air-bubbles to the middle of the level tubes. Direct the sights to a rod held truly vertical or "plumb" at another point of the line, the more distant the better. The two ends are usually taken. Sight to the lowest visible point of the rod. When the needle comes to rest, note what division on the circle it points to; taking the one indicated by the north end of the needle, if the north point on the circle is farthest from you, and *vice versa.*

In reading the division to which one end of the needle points, the eye should be placed over the other end, to avoid the error which might result from the "parallax," or apparent change of place of the end read from, when looked at obliquely.

The bearing is read and recorded by noting between what letters the end of the needle comes, and to what number; naming, or writing down, *first,* that letter, N. or S., which is at the 0° point nearest to that end of the needle from which you are reading; *second,* the number of degrees to which it points; and, *third,* the letter E. or W. of the 90° point which is nearest to the

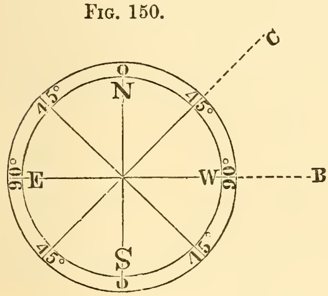
Fig. 149.



same end of the needle. Thus, in the figure, if when the sights were directed along a line (the north point of the compass being most distant from the observer) the north end of the needle was at the point A, the bearing of the line sighted on would be north 45° east; if the end of the needle was at B, the bearing would be *east;* if at C, S. 30° E.; if at D, *south;* if at E, S. 60° W.; if at F, *west;* if at G, N. 60° W.; if at H, *north.*

181. We can now understand why W. is on the right hand of the compass-box and E. on the left. Let the direction from the center of the compass to the point B in the figure be required, and suppose the sights in the first place to be pointing in the direction of the needle, S. N., and the north sight to be ahead. When the sights (and the circle to which they are fastened) have been turned so as to point in the direction of B, the point of the circle marked E. will have come round to the north end of the needle (*since the needle remains immovable*), and the reading will therefore be "east," as it should be. The effect on the reading is the same as if the needle had moved to the left the same distance which the sights have moved to the right, and the left side is therefore properly marked "east," and *vice versa.* So, too, if the bearing of the line to C be desired half-way between north and east—i. e., N. 45° E.; when the sights and the circle have turned 45° to the right, the needle, really standing still, has apparently arrived at the point half-way between N. and E., i. e., N. 45° E.

Fig. 150.



Some surveyors' compasses are marked the reverse of this, the E. on the right and the W. on the left. These letters must then be reversed in the mind before the bearing is noted down.

182. Reading with Vernier. When the needle does not point precisely to one of the division-marks on the circle, the fractional part of the smallest space is usually estimated by the eye, as has been explained. But this fractional part may be measured by the vernier as follows: Suppose the needle to point between N. 31° E. and N. $31\frac{1}{2}$° E. Turn the tangent-screw which moves the compass-box till the smaller division (in this case 31°) has come round to the needle. The vernier will then indicate through what space the compass-box has moved, and therefore how much must be added to the reading of the needle. Suppose it indicates ten minutes of a degree. Then the bearing is N. 31° 10′ E. It is, however, so difficult to move the vernier without disturbing the whole instrument, that this is seldom resorted to in practice. The chief use of the vernier is to set the instrument for running lines and making an allowance for the variation of the needle, as will be explained in the proper place. A vernier arc is sometimes attached to one end of the needle and carried around by it.

183. Practical Hints. Mark every station or spot at which the compass is set by driving a stake, or digging up a sod, or piling up stones, or otherwise, so that it can be found if any error or other cause makes it necessary to repeat the survey.

Very often, when the line of which the bearing is required is a fence, etc., the compass can not be set upon it. In such cases, set the compass so that its center is a foot or two from the line, and set the flag-staff at precisely the same distance from the line at the other end of it. The bearing of the flag-staff from the compass will be the same as that of the fence, the two lines being parallel. The distances should be measured on the real line. If more convenient, the compass may be set at some point on the line prolonged, or at some intermediate point of the line, "in line" between its extremities.

In setting the compass level, it is more important to have it level crosswise of the sights than in their direction; since, if it be not so, on looking up or down hill through the upper part of one sight and the lower part of the other, the line of sight will not be parallel to the N. and S. or zero line on the compass, and an incorrect bearing will therefore be obtained.

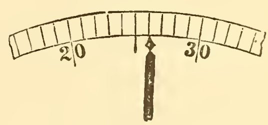
The compass should *not* be leveled by the needle, as some books recommend—i. e., so leveled that the ends of the needle shall be at equal distances below the glass. The needle should be brought so originally by the maker, but, if so adjusted in the morning, it will not be so at noon, owing to the daily variation in the *dip*. If, then, the compass be leveled by it, the lines of sight will generally be more or less oblique, and therefore erroneous. If the needle touches the glass when the compass is leveled, balance it by sliding the coil of wire along it.

The same end of the compass should always go ahead. The north end is preferable. The south end will then be nearest to the observer. Attention to this and to the caution in the next paragraph will prevent any confusion in the bearings.

Always take the readings from the same end of the needle; from the north end, if the north end of the compass goes ahead, and *vice versa*. This is necessary, because the two ends will not always cut opposite degrees. With this precaution, however, the angle of two meeting lines can be obtained correctly from either end, provided the same one is used in taking the bearings of both the lines.

Guard against a very frequent source of error with beginners in reading from the wrong number of the two between which the needle points, such as reading 34° for 26° in a case like that in the figure.

Fig. 151.



Check the vibrations of the needle by gently raising it off the pivot so as to touch the glass, and letting it down again by the screw on the under side of the box.

The compass should be smartly tapped after the needle has settled, to destroy the effect of any adhesion to the pivot or friction of dust upon it.

All iron, such as the chain, etc., must be kept at a distance from the compass, or it will attract the needle, and cause it to deviate from its proper direction.

The surveyor is sometimes troubled by the needle refusing to traverse and adhering to the glass of the compass after he has briskly wiped this off with a silk handkerchief, or it has been carried so as to rub against his clothes. The cause is the electricity excited by the friction. It is at once discharged by applying a wet finger to the glass.

A compass should be carried with its face resting against the side of the surveyor, and one of the sights hooked over his arm.

In distant surveys an extra center-pin should be carried (as it is very liable to injury, and its perfection is most essential), and also an extra needle. When two such are carried they should be placed so that the north pole of one rests against the south pole of the other.

184. When the magnetism of the needle is lessened or destroyed by time, it may be renewed as follows: Obtain two bar magnets. Provide a board with a hole to admit of the axis, so that its collar may fit fairly, and that the needle may rest flat on it without bearing at the center. Place the board before you with the north end of the needle to your right. Take a magnet in each hand, the left holding the north end of the bar, or that which has the mark across, downward, and the right holding the same mark upward. Bring the bars over the axis, about a foot above it, without approaching each other within two inches; bring them down vertically on the needle (the marks as directed) about an inch on each side of its axis; slide them outward to its ends with slight pressure; raise them up; bring them to their former position, and repeat this a number of times.

185. Back-Sights. To test the accuracy of the bearing of a line taken at one end of it, set up the compass at the other end or point sighted to, and look back to a rod held at the first station or point where the compass had been placed originally. The reading of the needle should now be the same as before.

If the position of the sights had been reversed, the reading would be the *Reverse Bearing;* a former bearing of N. 30° E. would then be S. 30° W., and so on.

186. Local Attraction. If the back-sight does not agree with the first or forward sight, this latter must be taken over again. If the same difference is again found, this shows that there is *local attraction* at one of the stations—i. e., some influence, such as a mass of iron-ore, ferruginous rocks, etc., under the surface, which attracts the needle, and makes it deviate from its usual direction. Any high object, such as a house, a tree, etc., has been found to produce a similar effect.

To discover at which station the attraction exists, set the compass at several intermediate points in the line which joins the two stations, and at points in the line prolonged, and take the bearing of the line at each of these points. The agreement of several of these bearings, taken at distant points, will prove their correctness. Otherwise, set the compass at a third station, sight to each of the two doubtful ones, and then from them back to this third station. This will show which is correct.

When the difference occurs in a series of lines, such as around a field or along a road, proceed thus: Let C be the station at which the back-sight to B differs from the fore-sight from B to C. Since the back-sight from B to A is supposed to have agreed with the fore-sight from A to B, the local attraction must be at C, and the forward bearing must be corrected by the difference just found between the fore- and back-sights, adding or subtracting it, according to circumstances. An easy method is to draw a figure for the case, as in Fig. 153. In it, suppose the true bearing of B C, as given by a fore-sight from B to C, to be N. 40° E., but that there is local attraction

FIG. 152.

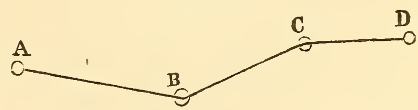
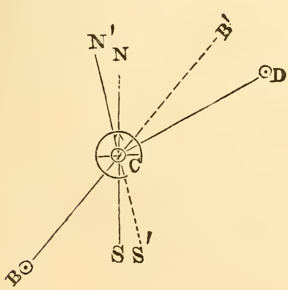


FIG. 153.



at C, so that the needle is drawn aside 10°, and points in the direction S′N′ instead of S N. The back-sight from C to B will then give a bearing of N. 50° E.; a difference or correction for the next fore-sight of 10°. If the next fore-sight, from C to D, be N. 70° E., this 10° must be subtracted from it, making the true fore-sight N. 60° E.

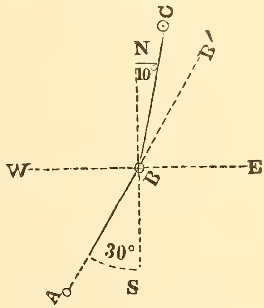
A general rule may also be given. *When the back-sight is greater than the fore-sight,* as in this case, subtract the difference from the next fore-sight, if that course and the preceding one have both their letters the same (as in this case, both being N. and E.), or both their letters different; or add the difference if either the first or last letters of the two courses are different. *When the back-sight is less than the fore-sight,* add the difference in the case in which it has just been directed to subtract it, and subtract it where it was before directed to add it.

187. Angles of Deflection. When the compass indicates much local attraction, the difference between the directions of two meeting lines (or the "*angle of deflection*" of one from the other) can still be correctly measured by taking the difference of the bearings of the two lines, as observed at the same point. For the error caused by the local attraction, whatever it may be, affects both bearings equally, inasmuch as a "bearing" is the angle which a line makes with the direction of the needle, and that here remains fixed in some one direction, no matter what, during the taking of the two bearings. Thus, in Fig. 153, let the true bearing of B C— i. e., the angle which it makes with the line S N—be, as before, N. 40° E., and that of C D, N. 60° E. The true "angle of deflection" of these lines, or the angle B′ C D, is therefore 20°. Now, if local attraction at C causes the needle to point in the direction of S′ N′, 10° to the left of its proper direction, B C will bear N. 50° E., and C D N. 70° E., and the difference of these bearings— i. e., the angle of deflection—will be the same as before.

188. Angles between Courses. To determine the angle of deflection of two courses meeting at any point, the following simple rules, the reasons of which will appear from the accompanying figures, are sufficient:

Case 1. When the first letters of the bearing are alike (i. e., both N. or both S.), and the last letters also alike (i. e., both E. or both W.), take the difference of the bearings. *Example:* If A B bears N. 30° E., and B C bears N. 10° E., the angle of deflection C B B′ is 20°.

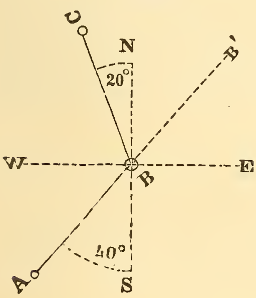
Fig. 154.



Case 2. When the first letters are alike and the last letters different, take the sum of the bearings. *Ex.:* If A B bears N. 40° E. and B C bears N. 20° W., the angle C B B′ is 60°.

Case 3. When the first letters are different and the last letters alike, subtract the sum of the bearings from 180°. *Ex. :* If A B bears N. 30° E. and B C bears S. 40° E., the angle C B B′ is 110°.

Fig. 155.



Case 4. When both the first and last letters are different, subtract the difference of the bearings from 180°. *Ex.:* If A B bears S. 30° W. and B C bears N. 70° E., the angle C B B′ is 140°.

If the angles included between the courses are desired, they will be at once found by reversing one bearing and then applying the above rules; or by subtracting the results obtained as above from 180°; or an analogous set of rules could be formed for them.

Fig. 156.

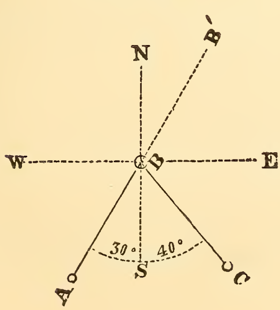
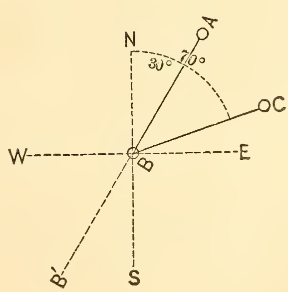


Fig. 157.



189. To change Bearings. It is convenient in certain calculations to suppose one of the lines of a survey to change its direction so as to become due north and south; that is, to become a new meridian line. It is, then, necessary to determine what the bearings of the other lines will be, supposing them to change with it. The subject may be made plain by supposing the survey to be platted in the usual way, with the north uppermost, and the plat to be then turned around till the line to be changed is in the desired direction. The effect of this on the other lines will be readily seen. A *general rule* can also be formed:

Take the *difference* between the original bearing of the side which becomes a meridian, and each of those bearings which have both their letters the same as it, or both different from it. The changed bearings of these lines retain the same letters as before, if they were originally greater than the original bearing of the new meridian line; but, if they were less, they are thrown on the other side of the N. and S. line, and their last letters are changed, E. being put for W., and W. for E.

Take the *sum* of the original bearing of the new meridian line, and each of those bearings which have one letter the same as one letter of the former bearing and one different. If this sum exceeds 90°, this shows that the line is thrown on the other side of the east or west point, and the difference between this sum and 180° will be the new bearing, and the first letter will be changed, N. being put for S. and S. for N.

Example: Let the bearings of the sides of a field be as follows: N. 32° E.; N. 80° E.; S. 48° E.; S. 18° W.; N. 73½° W.; North. Suppose the first side to become due north; the changed bearings will then be as follows: North; N. 48° E.; S. 80° E.; S. 14° E.; S. 74½° W.; N. 32° W.

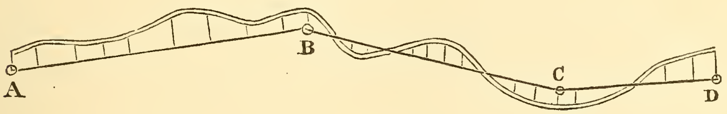
To apply the rule to the "North" course, as above, it must be called N. 0° W.; and then, by the rule, 32° must be added to it.

The true bearings can, of course, be obtained from the changed bearings by reversing the operation, taking the sum instead of the difference, and *vice versa*.

190. Line-Surveying. This name may be given to surveys of lines, such as the windings of a brook, the curves of a road, etc., by way of distinction from *Farm-Surveying*, in which the lines surveyed inclose a space.

To survey a *brook*, or any similar line, set the compass at or near one end of it, and take the bearing of an imaginary or visual

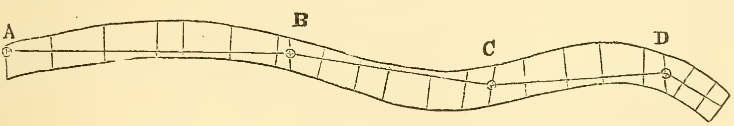
Fig. 158.



line running in the general average direction of the brook, such as A B in the figure. Measure this line, taking offsets to the various bends of the brook, as explained in Art. 97. Then set the compass at B, and take a back-sight to A, and, if they agree, take a fore-sight to C, and proceed as before, noting particularly the points where the line crosses the brook.

To survey a *road*, take the bearings and lengths of the lines

Fig. 159.



which can be most conveniently measured in the road, and measure offsets on each side to the outside of the road.

When the line of a new road is surveyed, the bearings and lengths of the various portions of its intended center-line should be measured, and the distance which it runs through each man's land should be noted. Stones should be set in the ground at recorded distances from each angle of the line, or in each line prolonged a known distance, so as not to be disturbed in making the road.

In surveying a wide river, one bank may be surveyed by the method just given, and points on the opposite banks, as trees, etc., may be fixed by the method of intersections founded on the fourth method of determining the position of a point.

191. Checks by Intersecting Bearings. At each station at which the compass is set take bearings to some remarkable object, such as a church-steeple, a distant house, a high tree, etc. At least three bearings should be taken to each object to make it of any use, since two are necessary to determine it (by our fourth method), and, till thus determined, it can be no check. When the line is platted, by the methods to be explained hereafter, plat also the lines given by these bearings. If those taken to the same object from three different stations intersect in the same point, this proves that there has been no mistake in the survey or platting of those stations.

If any bearing does not intersect a point fixed by previous bearings, it shows that there has been an error, either between the last station and one of those which fixed the point, or in the last bearing to the point. To discover which it was, plat the following line of the survey, and, at its extremity, set off the bearing from it to the point, and, if the line thus platted passes through the point, it proves that there was no error in the line, but only in the bearing to the point. If otherwise, the error was somewhere in the line between the stations from which the bearings to that point were taken.

192. Keeping the Field-Notes. The simplest and easiest method for a beginner is to make a rough sketch of the survey by eye, and write down on the lines their bearings and lengths.

An improvement on this is to actually lay down the precise bearings and lengths of the lines in the field-book in the manner to be explained in the section on Platting, Art. 209.

193. A *second* method is to draw a straight line up the page of the field-book, and to write on it the bearings and lengths of the lines. The only advantage of this method is that the line will not run off the side of the page, as it is apt to do in the preceding method.

194. A *third* method is to represent the line surveyed by a double column, as in Art. 84, which should be now referred to. The bearings are written obliquely up the columns. At the end of each course its length is written in the column, and a line drawn across it. Dotted lines are drawn across the column at any intermediate measurement. Offsets are noted as explained in Art. 97.

The intersection bearings, described in Art. 191, should be entered in the field-book *before* the bearings of the line, in order to avoid mistakes of platting in setting off the measured distances on the wrong line.

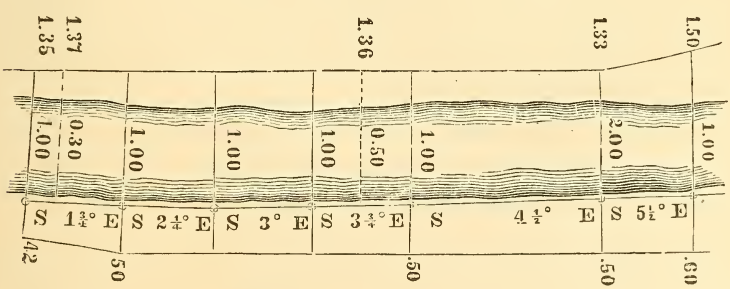
195. A *fourth* method is to write the stations, bearings, and distances in three columns. This is compact, and has the advantage, when applied to farm-surveying, of presenting a form suitable for the subsequent calculations of content, but does not give facilities for noting offsets.

Examples of these four methods are given in Art. 199, which contains the field-notes of the lines bounding a field.

196. New York Canal-Maps. The following is a description of the original maps of the survey of the line of the New York Erie Canal, as published by the Canal Commissioners. The figure represents a portion of such a map, but, necessarily, with all its lines black, red and blue lines being used on the real map:

"The Red Line described along the inner edge of the towing-path is the *base-line,* upon which all the measurements in the di-

Fig. 160.



rection of the length of the canal were made. The *bearings* refer to the magnetic meridian at the time of the survey. The *lengths* of the several portions are inserted at the *end* of each in chains and links. The *offsets* at each station are represented by red lines drawn across the canal in such a direction as to bisect the angles formed by the two contiguous portions of the red or base line upon the towing-path. The intermediate offsets are set off at right angles to the base-line, and the distances on both are given from it in links. The *intermediate offsets* are represented by *red dotted lines*, and the distances to them upon the base-line are reckoned, in each case, from the last preceding station. The same is likewise done with the other distances upon the base-line; those to the *bridges* being taken to the lines joining the nearest angles or corner posts of their abutments; those to the *locks* extending to the lines passing through the centers of the two nearest quoin-posts; and those to the *aqueducts* to the faces of their abutments. The space inclosed by the BLUE LINES represents the portion embraced within the limits of the survey as belonging to the State; and the names of the adjoining proprietors are given as they stood at the time of executing the survey. The distances are projected upon a *scale* of two chains to the inch."

197. Farm-Surveying. A farm or field or other space included within known lines is usually surveyed by the compass thus: Begin by walking around the boundary-lines and setting stakes at all the corners, which the flag-man should specially note, so that he may readily find them again. Then set the compass at any corner, and send the flag-man to the next corner. Take the bearing of the bounding-line running from corner to corner, which is usually a fence. Measure its length, taking offsets if necessary. Note where any other fence, or road, or other line crosses or meets it, and take their bearings. Take the compass to the end of this first bounding-line; sight back, and, if the back-sight agrees, take the bearing and distance of the next bounding-line; and so proceed till you have got back to the point of starting.

198. Where speed is more important than accuracy in a survey, whether of a line or a farm, the compass need be set only at every other station, taking a forward sight from the first station to the second; then, setting the compass at the third station, taking a back-sight to the second station (but with the north point of the compass always ahead), and a fore-sight to the fourth; then going to the fifth, and so on. This is, however, not to be recommended.

199. Field-Notes. The field-notes of a farm-survey may be kept by any of the methods which have been described with reference to a line-survey. Below are given the field-notes of the same field recorded by each of the methods.

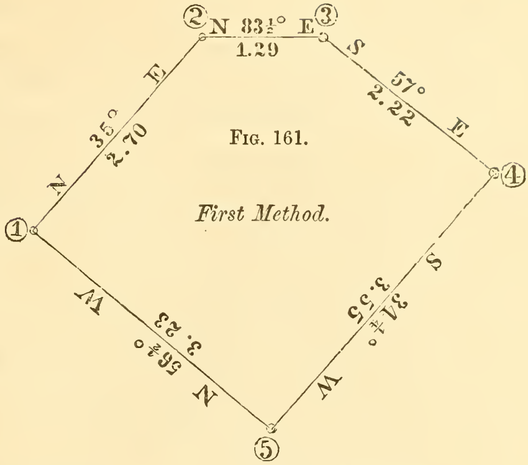


FIG. 161.

First Method.

Second Method.

⊙ (1)
N. 56½° W. 3·23
⊙ (5)
S. 34¼° W. 3·55
⊙ (4)
S. 57° E. 2·22
⊙ (3)
N. 83½° E. 1·29
⊙ (2)
N. 35° E. 2·70
⊙ (1)

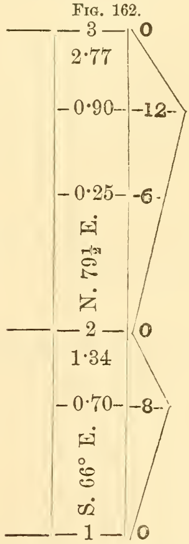
*Third Method.**

—(1)—
3·23
N. 56½ W.
—(5)—
3·55
S. 34¼ W.
—(4)—
2·22
S. 57 E.
—(3)—
1·29
N. 83½ E.
—(2)—
2·70
N. 35 E.
—(1)—

Fourth Method.

STATIONS.	BEARINGS.	DISTANCES.
1	N. 35° E.	2·70
2	N. 83½° E.	1·29
3	S. 57° E.	2·22
4	S. 34¼° W.	3·55
5	N. 56½° W.	3·23

FIG. 162.



* In the "third method" the bearings should be written obliquely upward, as directed in Art. 194, but are not so printed here, from typographical difficulties.

200. The field-notes of a field in which offsets occur may be most easily recorded by the third method, as in Fig. 162.

When the field-notes are recorded by the fourth method, the offsets may be kept in a separate table, in which the first column will contain the stations from which the measurements are made, the second column the distances at which they occur, the third column the lengths of the offsets, and the fourth column the side of the line, "right" or "left," on which they lie.

For calculation, four more columns may be added to the table, containing the intervals between the offsets, the sums of the adjoining pairs, and the products of the numbers in the two preceding columns, separated into right and left, one being additive to the field, and the other subtractive.

201. Tests of Accuracy. 1. The check of intersections described in Art. 191 may be employed to great advantage when some conspicuous object near the center of the farm can be seen from most of its corners.

2. When the survey is platted, if the last course meets the starting-point, it proves the work, and the survey is then said to "close."

3. Diagonal lines running from corner to corner of the farm, like the "proof-lines" in chain-surveying, may be measured and their bearings taken. When these are laid down on the plat, their meeting the points to which they had been measured proves the work.

4. The only certain and precise test is, however, that by "latitudes and departures."

202. Method of Radiation. *A field may be surveyed from one station, either within it or without it,* by taking the bearings and the distances from that point to each of the corners of the field. These corners are then "determined" by the third method, Art. 5. This modification of that method is called the *Method of Radiation.* All our preceding surveys with the compass have been by the *Method of Progression.*

The compass may be set at one corner of the field, or at a point in one of its sides, and the same method of radiation employed.

This method is seldom used, however, since, unlike the method of progression, its operations are not checks upon each other.

203. Method of Intersection. A field may also be surveyed by measuring a *base-line*, either within it or without it, setting the compass at each end of the base-line, and taking from each end the bearings of each corner of the field, which will then be fixed and determined by the fourth method, Art. 6. This mode of surveying is the *Method of Intersections*, noticed in Art. 166.

204. Running out Old Lines. The original surveys of lands in the older States of the American Union were exceedingly deficient in precision. This arose from two principal causes: the small value of land at the period of these surveys, and the want of skill in the surveyors. The effect at the present day is frequent dissatisfaction and litigation. Lots sometimes contain more acres than they were sold for, and sometimes less. Lines which are straight in the deed and on the map are found to be crooked on the ground. The recorded surveys of two adjoining farms often make one overlap the other, or leave a gore between them. The most difficult and delicate duty of the land-surveyor is to run out these old boundary-lines. In such cases, his first business is to find monuments, stones, marked trees, stumps, or any other old "corners" or landmarks. These are his starting-points. The owners whose lands join at these corners should agree on them. Old fences must generally be accepted by right of possession, though such questions belong rather to the lawyer than to the surveyor.* His business is to mark out on the ground the lines given in the deed. When the bounds are given by compass-bearings, the surveyor must be reminded that these bearings are very far from being the same now as originally, having been changing every year. The method of

* "In the description of land conveyed, the rule is that known and fixed monuments control courses and distances. So the certainty of metes and bounds will include and pass all the lands within them, though they vary from the given quantity expressed in the deed. In New York, to remove, deface, or alter landmarks maliciously is an indictable offense."—*Kent's Commentaries*, IV, 515.

determining this important change, and of making the proper allowance, will be found under "Declination of the Magnetic Needle."

PLATTING THE SURVEY.

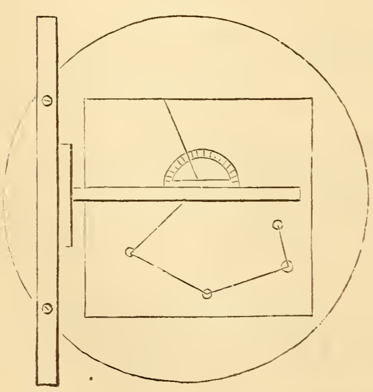
205. The platting of a survey made with the compass consists in drawing on paper the lines and the angles which have been measured on the ground. The angles are laid off and the lines are drawn "to scale," as has been explained in Chapter I.

206. Platting Bearings. Since "bearings" taken with the compass are the angles which the various lines make with the magnetic meridian, or the direction of the compass-needle, which, as we have seen, remains always (approximately) parallel to itself, it is necessary to draw these meridians through each station before laying off the angles of the bearings.

The T-square is the most convenient instrument for this purpose. The paper on which the plat is to be made is fastened on the board so that the intended direction of the north and south line may be parallel to one of the sides of the board. The inner side of the stock of the T-square being pressed against one of the other sides of the board and slid along, the edge of the long blade of the square will always be parallel to itself and to the first-named side of the board, and will thus represent the meridian passing through any station.

If a straight-edged drawing-board or table can not be procured, nail down on a table of any shape a straight-edged ruler, and slide along against it the outside of the stock of a T-square, one side of the stock being flush with the blade.

Fig. 163.

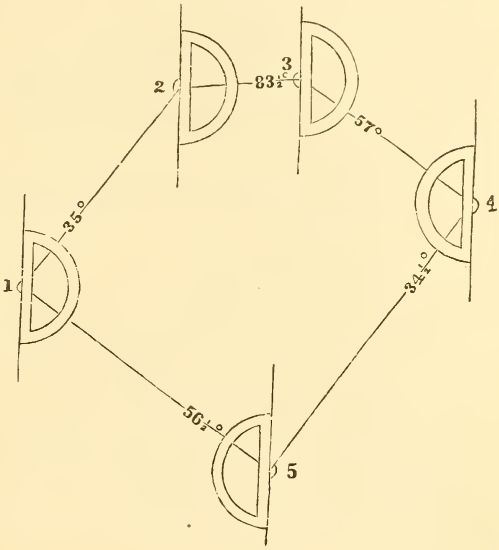


A parallel ruler may also be used, one part of it being screwed down to the board in the proper position.

If none of these means are at hand, approximately parallel meridians may be drawn by the edges of a common ruler at distances apart equal to its width, and the diameter of the protractor made parallel to them by measuring equal distances between it and them.

207. To plat a survey with these instruments, mark with a fine point inclosed in a circle a convenient spot in the paper to represent the first station, 1 in the figure. Its place must be so chosen that the plat may not "run off" the paper. With the T-square draw a meridian through it. The top of the paper is usually, though not necessarily, called north. With the protractor lay off the angle of the first bearing. Set off the length of the first line to the desired scale from 1 to 2. The line 1 - - - - 2 represents the first course.

Fig. 164.



Through 2 draw another meridian, lay off the angle of the second course, and set off the length of this course from 2 to 3.

Proceed in like manner for each course. When the last course is platted, it should end precisely at the starting-point, as the survey did, if it were a closed survey, as of a field. If the plat does not "close" or "come together," it shows some error or inaccuracy either in the original survey, if that have not been "tested" by latitudes and departures, or in the work of platting. The plat here given is the same as that of Fig. 161.

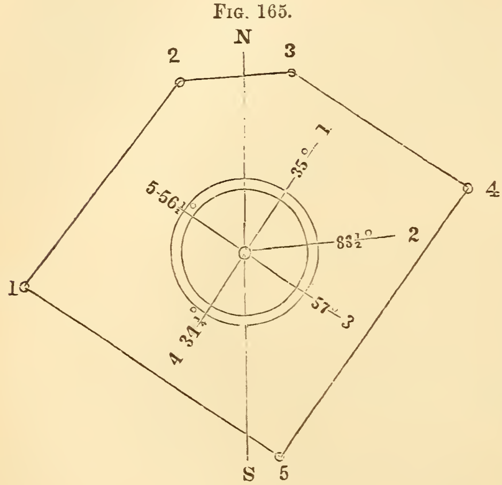
This manner of laying down the directions of lines by the angles which they make with a meridian line has a great advantage, in both accuracy and rapidity, over the method of platting lines by the angles which each makes with the line which comes before it. In the latter method, any error in the direction of one line makes all that follow it also wrong in their directions. In the former, the *direction* of each line is independent of the preceding line, though its *position* would be changed by a previous error.

Instead of drawing a meridian through each station, sometimes only one is drawn, near the middle of the sheet, and all the bearings of the survey are laid off from some one point of it, as shown in the figure, and numbered to correspond with the stations from which these bearings were taken. The circular protractor is convenient for this. They are then transferred to the places where they are wanted by a triangle or other parallel ruler. Fig. 165 represents the same field platted by this method.

A semicircular protractor is sometimes attached to the stock end of the T-square so that its blade may be set at any desired angle with the meridian, and any bearing be thus protracted without drawing a meridian. It has some inconveniences.

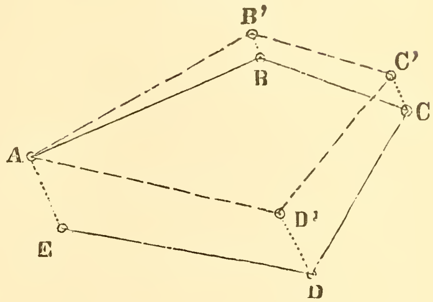
The compass itself may be used to plat bearings. For this purpose it must be attached to a square board so that the N and S line of the compass-box may be parallel to two opposite edges of the board. This is placed on the paper, and the box is turned till the needle points as it did when the first bearing was taken. Then a line drawn by one edge of the board will be in a proper direction. Mark off its length, and plat the next and the succeeding bearings in the same manner.

FIG. 165.



208. When the plat of a survey does not "close," it may be corrected as follows: Let A B C D E be the boundary-lines platted according to the given bearings and distances, and suppose that the last course comes to E instead of ending at A, as it should. Suppose also that there is no reason to suspect any single great error, and that no one of the lines was measured over very rough ground, or was specially uncertain in its direction when observed. The inaccuracy must then be distributed among all the lines *in proportion to their length.* Each point in the figure, B, C, D, E, must be moved in a direction parallel to E A by a certain distance which is obtained thus: Multiply the distance E A by the distance A B, and divide by the sum of all the courses. The quotient will be the distance B B′. To get C C′, multiply E A by A B + B C, and divide the product by the same sum of all the courses. To get D D′, multiply E A by A B + B C + C D, and divide as before. So for any course, multiply by the sum of the lengths of that course and of all those preceding it, and divide as before. Join the points thus obtained, and the closed polygon A B′ C′ D′ A will thus be formed, and will be the most *probable* plat of the given survey.*

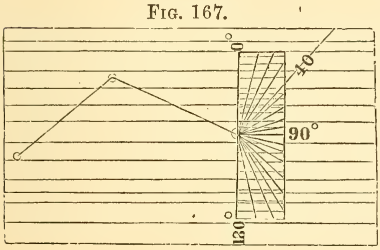
Fig. 166.



The method of latitudes and departures, to be explained hereafter, is, however, the best for effecting this object.

209. Field Platting. It is sometimes desirable to plat the courses of a survey in the field as soon as they are taken, as was mentioned in Art. 192 under the head of "Keeping the Field-Notes." One method of doing this is to have the paper of the field-book ruled with parallel lines at *unequal* distances apart, and to use a rectangular protractor (which may be made of Bristol-board or other stout drawing-paper) with lines ruled across it at equal distances of some fraction of an inch. A bearing having been taken and noted, the protractor is laid on the paper and its center placed at the station where the bearing is to be laid off. It is then turned till one of its cross-lines coincides with some one of the lines on the paper, which represent east and west lines. The long side of the protractor will then be on a meridian, and the proper angle (40° in the figure) can be at once marked off. The length of the course can also be set off by the equal spaces between the cross-lines, letting each space represent any convenient number of links.

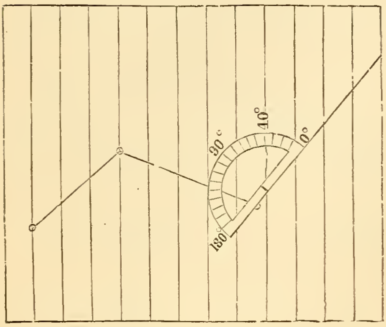
Fig. 167.



* This was demonstrated by Dr. Bowditch in No. 4 of "The Analyst."

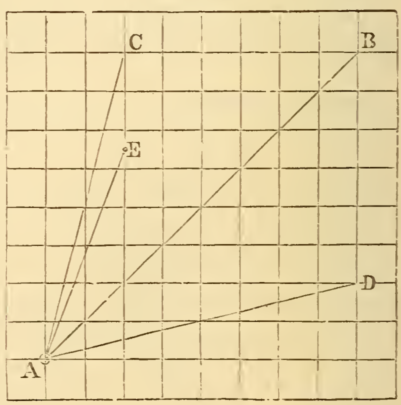
210. A common rectangular protractor without any cross-lines, or a semicircular one, can also be used for the same purpose. The parallel lines on the paper (which, in this method, may be equidistant, as in common ruled writing-paper) will now represent meridians. Place the center of the protractor on the meridian nearest to the station at which the angle is to be laid off, and turn it till the given number of degrees is cut by the meridian. Slide the protractor up or down the meridian (which must continue to pass through the center and the proper degree) till its edge passes through the station, and then draw by this edge a line, which will have the bearing required.

Fig. 168.



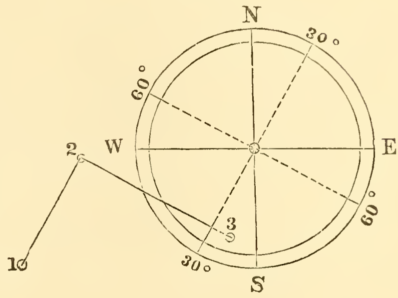
211. *Paper ruled into squares* (as are sometimes the right-hand pages of surveyors' field-books) may be used for platting bearings in the field. The lines running up the page may be called north and south lines, and those running across the page will then be east and west lines. Any course of the survey will be the hypotenuse of a right-angled triangle, and the ratio of its other two sides will determine the angle. Thus, if the ratio of the two sides of the right-angled triangle, of which the line AB in the figure is the hypotenuse, is 1, that line makes an angle of 45° with the meridian. If the ratio of the long to the short side of the right-angled triangle, of which the line A C is the hypotenuse, is 4 to 1, the line A C makes an angle of 14° with the meridian. The line A D, the hypotenuse of an equal triangle which has its long side lying east and west, makes likewise an angle of 14° with that side, and therefore makes an angle of 76° with the meridian.

Fig. 169.



212. With a Paper Protractor. Engraved paper protractors may be obtained from the instrument-makers, and are very convenient. A circle of large size, divided into degrees and quarters, is engraved on copper, and impressions from it are taken on drawing-paper. The divisions are not numbered. Draw a straight line to represent a meridian through the center of the circle in any convenient direction. Number the degrees from 0° to 90° each way from the ends of this meridian, as on the compass-plate. The protractor is now ready for use. Choose a convenient point for the first station. Suppose the first bearing to be N. 30° E. The line passing through the center of the circle and through the opposite points N. 30° E. and S. 30° W. has the bearing required. But it does not pass through the station 1. Transfer it thither by drawing through station 1 a line parallel to it, which will be the course required, its proper length being set off on it from 1 to 2. Now, suppose the bearing from 2 to be S. 60° E. Draw through 2 a line parallel to the line passing through the center of the circle and through the opposite points S. 60° E. and N. 60° W., and it will be the line desired. On it set off the proper length from 2 to 3, and so proceed.

Fig. 170.



When the plat is completed, the engraved sheet is laid on a clean one and the stations "pricked through," and the points thus obtained on the clean sheet are connected by straight lines. The penciled plat is then rubbed off from the engraved sheet, which can be used for a great number of plats.

If the central circle be cut out, the plat, if not too large, can be made directly on the paper where it is to remain.

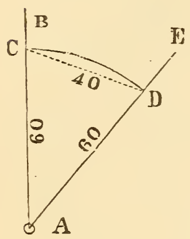
The surveyor can make such a paper protractor for himself with great ease by means of the *Table of Chords* at the end of this volume, the use of which is explained in Art. 215. The engraved ones may have shrunk after being printed.

Such a circle is sometimes drawn on the map itself. This will be particularly convenient if the bearings of any lines on the map not taken on the ground are likely to be required. If the map be very long, more than one may be needed.

213. Drawing-Board Protractor. Such a divided circle as has just been described, or a circular protractor, may be placed on a drawing-board near its center, and so that its 0° and 90° lines are parallel to the sides of the drawing-board. Lines are then to be drawn through the center and opposite divisions by a ruler long enough to reach the edges of the drawing-board on which they are to be cut in and numbered. The drawing-board thus becomes, in fact, a double rectangular protractor. A strip of white paper may have previously been pasted on the edges, or a narrow strip of white wood inlaid. When this is to be used for platting, a sheet of paper is put on the board as usual, and lines are drawn by a ruler laid across the 0° points and the 90° points, and the center of the circle is at once found, and should be marked ⊙. The bearings are then platted as in the last method.

214. With a Scale of Chords. On the plane scale contained in cases of mathematical drawing instruments will be found a series of divisions numbered from 0 to 90, and marked C H or C. This is a scale of chords, and gives the lengths of the chords of any arc for a radius equal in length to the chord of 60° on the scale. To lay off an angle with this scale, as, for example, to draw a line making at A an angle of 40° with A B, take, in the dividers, the distances from 0 to 60 on the scale of chords; with this for radius and A for center, describe an indefinite arc C D. Take the distance from 0 to 40 on the same scale, and set it off on the arc as a chord from C to some point D. Join A D and prolong it. B A E is the angle required.

Fig. 171.



The sector, Fig. 29, supplies a modification of this method sometimes more convenient. On each of its legs is a scale marked C or C H. Open it at pleasure; extend the compass from 60 to 60, one on each leg, and with this radius describe an arc. Then extend the compasses from 40 to 40, and the distance will be the chord of 40° to that radius. It can be set off as above.

The smallness of the scale renders the method with a scale of chords practically deficient in exactness, but it serves to illustrate the next and *best* method.

215. With a Table of Chords. At the end of this volume will be found a table of the lengths of the chords of arcs for every degree and minute of the quadrant calculated for a radius equal to 1.

To use it, take in the compasses one inch, one foot, or any other convenient distance (the longer the better), divided into tenths and hundredths by a diagonal scale or otherwise. With this as radius describe an arc as in the last case. Find in the table of chords the length of the chord of the desired angle. Take it from the scale just used to the nearest decimal part which the scale will give. Set it off as a chord, as in the last figure, and join the point thus obtained to the starting-point. This gives the angle desired.

The superiority of this method to that which employs a protractor is due to the greater precision with which a straight line can be divided than can a circle.

A slight modification of this method is to take in the compasses ten equal parts of any convenient length, inches, half inches, quarter inches, or any other at hand, and with this radius describe an arc as before, and set off a chord ten times as great as the one found in the table—i. e., imagine the decimal-point moved one place to the right.

If the radius be 100 or 1,000 equal parts, imagine the decimal-point moved two or three places to the right.

Whatever radius may be taken or given, the product of that radius into a chord of the table will give the chord for that radius.

This gives an easy and exact method of getting a right angle by describing an arc with a radius of 1, and setting off a chord equal to 1·4142.

If the angle to be constructed is more than 90°, construct on the other side of the given point upon the given line prolonged an angle equal to what the given angle wants of 180°—i. e., its *supplement*, in the language of trigonometry.

This same table gives the means of measuring any angle. With the angular point for a center, and 1 or 10 for a radius, describe an arc. Measure the length of the chord of the arc between the legs of the angle, find this length in the table, and the angle corresponding to it is the one desired.

This table will also serve to find the *natural sine* or *cosine* of any angle. Multiply the given angle by two; find in the table the chord of this double angle; and half of this chord will be the natural sine required. For the chord of any angle is equal to twice the sine of half the angle. To find the *cosine*, proceed as above, with the angle which, added to the given angle, would make 90°.

Another use of this table is to inscribe regular polygons in a circle by setting off the chords of the arcs which their sides subtend.

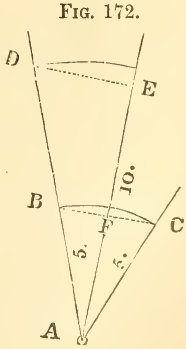
Still another use is to divide an arc or angle into any number of equal parts by setting off the fractional arc or angle.

216. With a Table of Natural Sines. In the absence of a table of chords, heretofore rare, a table of natural sines, which can be found anywhere, may be used as a less convenient substitute. Since the chord of any angle equals twice the sine of half the angle, divide the given angle by two; find in the table the natural sine of this half angle; double it, and the product is the chord of the whole angle. This can then be used precisely as was the chord in the preceding article.

An ingenious modification of this method has been much used. Describe an arc from the given point as center, as in the last two articles, but with a radius of five equal parts. Take from a table the length of the natural sine of half the given angle to a radius of ten. Set off this length as a chord on the arc just described, and join the point thus obtained to the given point.

The reason of this is apparent from the figure. D E is the sine of half the angle B A C to a radius of ten equal parts, and B C is the chord directed to be set off to a radius of five equal parts. B C is equal to D E, for $BC = 2 \cdot BF$ by trigonometry, and $DE = 2 \cdot BF$ by similar triangles; hence $BC = DE$.

Fig. 172.



217. By Latitudes and Departures. When the latitudes and departures of a survey have been obtained and corrected, either to test its accuracy or to obtain its content, they afford the easiest and best means of platting it. The description of this method will be given in Art. 246.

COPYING PLATS.

218. The plat of a survey necessarily has many lines of construction drawn upon it which are not needed in the finished map. These lines and the marks of instruments so disfigure the paper that a fair copy of the plat is usually made before the map is finished. The various methods of copying plats, etc., whether on the same scale, or reduced, or enlarged, will therefore now be described.

219. Stretching the Paper. If the map is to be colored, the paper must first be wetted and stretched, or the application of the wet colors will cause its surface to swell or blister and become uneven. Therefore, with a soft sponge and clean water, wet the back of the paper, working from the center outward in all directions. The "water-mark" reads correctly only when looked at from the front side, which it thus distinguishes. When the paper is thoroughly wet and thus greatly expanded, glue its edges to the drawing-board for half an inch in width, turning them up against a ruler, passing the glue along them, and then turning them down and pressing them with the ruler. Some prefer gluing down opposite edges in succession, and others adjoining edges. The paper must be moderately stretched smooth during the process. Hot glue is best. Paste or gum may be used, if the paper be kept wet by a damp cloth, so that the edges may dry first. "Mouth-glue" may be used by rubbing it (moistened in the mouth or in boiling water) along the turned-up edges, and then rubbing them dry by an ivory folder, a piece of dry paper being interposed. As this is a slower process, the middle of each side should first be fastened down, then the four angles, and lastly the intermediate portions. When the paper becomes dry, the creases and puckerings will have disappeared, and it will be as smooth and tight as a drum-head.

220. Copying by Tracing. Fix a large pane of clear glass in a frame so that it can be supported at any angle before a window, or, at night, in front of a lamp. Place the plat to be copied on this glass, and the clean paper upon it. Connect them by pins, etc. Trace all the desired lines of the original with a sharp pencil as lightly as they can be easily seen. Take care that the paper does not slip. If the plat is larger than the glass, copy its parts successively, being very careful to fix each part in its true relative position. Ink the lines with India ink, making them very fine and pale if the map is to be afterward colored.

221. Copying on Tracing-Paper. A thin transparent paper is prepared expressly for the purpose of making copies of maps and drawings, but it is too delicate for much handling. It may be prepared by soaking tissue-paper in a mixture of turpentine and Canada balsam or balsam of fir (two parts of the former to one of the latter), and drying very slowly. Cold-drawn linseed-oil will answer tolerably, the sheets being hung up for some weeks to dry. Linen is also similarly prepared, and sold under the name of "vellum tracing-paper." It is less transparent than the tracing-paper, but is very strong and durable. Both of these are used rather for preserving duplicates than for finished maps.

222. Copying by Photography. This may be used for copying drawings, and is especially applicable when the drawings are to be very much reduced in size.

223. Copying by Blue Prints. Dissolve one ounce of ferricyanide of potassium in ten ounces of pure water. Also dissolve two ounces of ammonia citrate of iron in ten ounces of water. Mix the two solutions in a cup, and with a brush cover the surface of the paper on which the print is to be made with the mixture.

The surface should be thoroughly covered, but no more of the mixture should be applied than the paper will take up. The paper should become limp and moist but not wet. The work should be done in a room lighted with a lamp, and when the paper is dry it should be kept in a dark place.

To make a blue-print copy, a tracing of the drawing should first be made. Put the tracing over a sheet of the prepared paper and a sheet of glass over the tracing, in order to keep the tracing in contact with the prepared paper. Expose the paper to the sunlight, with the glass toward the sun, until the lines of the drawing are plainly seen on the prepared paper. Wash the paper until the water running off is no longer colored yellow. When dried, the lines of the drawing will be white upon a blue ground. The prepared paper for blue prints can be bought of dealers in engineers' supplies.

There are several similar methods of making prints, differing in the chemicals used, and in the color of the lines and background.

224. Copying by Transfer-Paper. This is thin paper, one side of which is rubbed with black-lead, etc., smoothly spread by cotton. It is laid on the clean paper, the blackened side downward, and the plat is placed upon it. All the lines of the plat are then gone over with moderate pressure by a blunt point, such as the eye-end of a small needle. A faint tracing of these lines will then be found on the clean paper, and can be inked at leisure. If the original can not be thus treated, it may first be copied on tracing-paper, and this copy be thus transferred. If the transfer-paper be prepared by rubbing it with lampblack ground up with hard soap, its lines will be ineffaceable. It is then called "Camp-paper."

225. Copying by Punctures. Fix the clean paper on a drawing-board and the plat over it. Prepare a fine needle with a sealing-wax head. Hold it very truly perpendicular to the board, and prick through every angle of the plat, and every corner and intersection of its other lines, such as houses, fences, etc., or at least the two ends of every line. For circles, the center and one point of the circumference are sufficient. For irregular curves, such as rivers, etc., enough points must be pricked to indicate all their sinuosities. Work with system, finishing up one strip at a time, so as not to omit any necessary points nor to prick through any twice, though the latter is safer. When completed, remove the plat. The copy will present a wilderness of fine points. Select those which determine the leading lines, and then the rest will be easily recognized. A beginner should first pencil the lines lightly, and then ink them. An experienced draughtsman will omit the penciling. Two or three copies may be thus pricked through at once. The holes in the original plat may be made nearly invisible by rubbing them on the back of the sheet with a paper-folder, or the thumb-nail.

226. Copying by Intersections. Draw a line on the clean paper equal in length to some important line of the original. Two starting-points are thus obtained. Take in the dividers the distance from one end of the line on the original to a third point. From the corresponding end on the copy, describe an arc with this distance for radius and about where the point will come. Take the distance on the original from the other end of the line to the point, and describe a corresponding arc on the copy to intersect the former arc in a point which will be that desired. The principle of the operation is that of our "First Method" (Art. 3). Two pairs of dividers may be used, as explained in Art. 82. "Triangular compasses," having three legs, are used by fixing two of their legs on the two given points of the original, and the third leg on the point to be copied, and then transferring them to the copy. All the points of the original can thus be accurately reproduced. The operation is, however, very slow. Only the chief points of a plat may be thus transferred, and the details filled in by the following method:

227. Copying by Squares. On the original plat draw a series of parallel and equidistant lines. The T-square does this most readily. Draw a similar series at right angles to these. The plat will then be covered with squares, as in Fig. 43. On the clean paper draw a similar series of squares. The important points may now be fixed as in the last article, and the rest copied by eye, all the points in each square of the original being properly placed in the corresponding square of the copy, noticing whether they are near the top or bottom of each square, on its right or left side, etc. This method is rapid, and in skillful hands quite accurate.

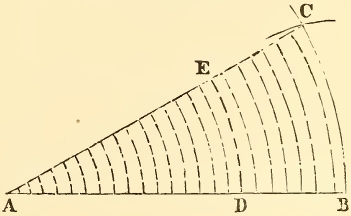
Instead of drawing lines on the original, a sheet of transparent paper containing them may be placed over it; or an open frame with threads stretched across it at equal distances and at right angles.

This method supplies a transition to the *Reduction* and *Enlargement* of plats in any desired ratio; under which head *Copying* by the Pantagraph and Camera Lucida will be noticed.

228. Reducing by Squares. Begin, as in the preceding article, by drawing squares on the original, or placing them over it. Then on the clean paper draw a similar set of squares, but with their sides one half, one third, etc. (according to the desired reduction), of those of the original plat. Then proceed as before to copy into each small square all the points and lines found in the large square of the plat in their true positions relative to the sides and corners of the square, observing to reduce each distance, by eye, or as directed in the following article, in the given ratio.

229. Reducing by Proportional Scales. Many graphical methods of finding the proportionate length of the copy, of any line of the original, may be used. The "angle of reduction" is constructed thus: Draw any line A B. With it for radius and A for center, describe an indefinite arc. With B for center and a radius equal to one half, one third, etc., of A B according to the desired reduction, describe another arc intersecting the former arc in C. Join A C. From A as center describe a series of arcs. Now, to reduce any distance, take it in the dividers, and set it off from A on A B, as to D. Then the distance from D to E, the other end of the arc passing through D, will be the proportionate length to be set off on the copy, in the manner directed in Art. 226.

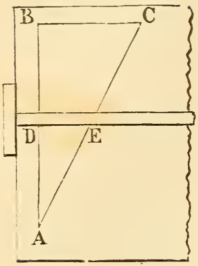
Fig. 173.



The sector, or "compass of proportion," described in Art. 50, presents such an "angle of reduction," always ready to be used in this manner.

10

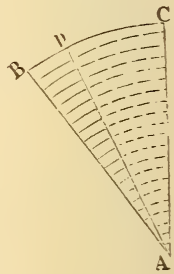
FIG. 174.



The "angle of reduction" may be simplified thus: Draw a line, A B, parallel to one side of the drawing-board, and another, B C, at right angles to it, and one half, etc., of it, as desired. Join A C. Then let A D be the distance required to be reduced. Apply a T-square so as to pass through D. It will meet A C in some point E, and D E will be the reduced length required.

Another arrangement for the same object is shown in Fig. 175. Draw two lines, A B, A C, at any angle, and describe a series of arcs from their intersection, A, as in the figure. Suppose the reduced scale is to be half the original scale. Divide the outermost arc into three equal parts, and draw a line from A to one of the points of division, as D. Then *each* arc will be divided into parts, one of which is twice the other. Take any distance on the original scale, and find by trial which of the arcs on the right-hand side of the figure it corresponds to. The other part of that arc will be half of it, as desired.

FIG. 175.



"Proportional compasses," being properly set, reduce lines in any desired ratio. A simple form of them, known as "wholes and halves," is often useful. It consists of two slender bars, pointed at each end, and united by a pivot which is twice as far from one pair of the points as from the other pair. The long ends being set to any distance, the short ends will give precisely half that distance.

230. Reducing by a Pantagraph. This instrument consists of two long and two short rulers, connected so as to form a parallelogram, and capable of being so adjusted that when a tracing-point attached to it is moved over the lines of a map, etc., a pencil attached to another part of it will mark on paper a precise copy, reduced on any scale desired. It is made in various forms. It is troublesome to use, though rapid in its work.

231. Reducing by a Camera Lucida. This is used in the Coast Survey Office. It can not reduce smaller than one fourth, without losing distinctness, and is very trying to the eyes. Squares drawn on the original are brought to apparently coincide with squares on the reduction, and the details are then filled in with the pencil, as seen through the prism of the instrument.

232. Enlarging Plats. Plats may be enlarged by the principal methods which have been given for reducing them, but this should be done as seldom as possible, since every inaccuracy in the original becomes magnified in the copy. It is better to make a new plat from the original data.

233. Conventional Signs. Various conventional signs or marks have been adopted, more or less generally, to represent on maps the inequalities of the surface of the ground, its different kinds of culture or natural products, and to objects upon it, so as not to encumber and disfigure it with much writing or many descriptive legends. This is the purpose of what is called *Topographical Mapping.* (See Part III, TOPOGRAPHY.)

234. Orientation. The map is usually so drawn that the top of the paper may represent the north. A meridian line should also be drawn, both true and magnetic, as in Fig. 186. The number of degrees and minutes in the variation, if known, should also be placed between the two north points. Sometimes a compass-star is drawn and made very ornamental.

235. Lettering. The style in which this is done very much affects the general appearance of the map. The young surveyor should give it much attention and careful practice. It must all be in imitation of the best printed models. No writing, however beautiful, is admissible. The usual letters are the ordinary ROMAN CAPITALS, Small Roman, *ITALIC CAPITALS*, *Small Italic*, and **GOTHIC OR EGYPTIAN.** This last, when well done, is very effective. For the titles of maps, various fancy letters may be used. For very large letters, those formed only of the shades of the letters regarded as blocks (the body being rubbed out after being penciled as a guide to the placing of the shades) are most easily made to look well. The simplest lettering is generally the best. The sizes of the names of places, etc., should be proportional to their importance. Elaborate tables for various scales have been published. It is better to make the letters too small than too large. They should not be crowded. Pencil-lines should always be ruled as guides. The lettering should be in lines parallel to the bottom of the map, except the names of rivers, roads, etc., whose general course should be followed.

236. Borders. The *Border* may be a single heavy line, inclosing the map in a rectangle, or such a line may be relieved by a finer line drawn parallel and near to it. Time should not be wasted in ornamenting the border. The simplest is the best.

237. Joining Paper. If the map is larger than the sheets of paper at hand, they should be joined with a feather-edge, by proceeding thus: Cut, with a knife guided by a ruler, about one third through the thickness of the paper, and tear off, on the under side, a strip of the remaining thickness, so as to leave a thin, sharp edge. Treat the other sheet in the same way on the other side of it. When these two feather-edges are then put together (with paste, glue, or varnish), they will make a neat and strong joint. The sheet which rests upon the other must be on the right-hand side, if the sheets are joined lengthwise, or below if they are joined in that direction, so that the thickness of the edge may not cast a shadow when properly placed as to the light. The sheets must be joined before lines are drawn across them, or the lines will become distorted. Drawing-paper is now made in rolls of great length, so as to render this operation unnecessary.

238. Mounting Maps. A map is sometimes required to be mounted— i. e., backed with canvas or muslin. To do this, wet the muslin and stretch it strongly on a board by tacks driven very near together. Cover it with strong paste, beating this in with a brush to fill up the pores of the muslin. Then spread paste over the back of the paper, and when it has soaked into it apply it to the muslin, inclining the board, and pasting first a strip, about two inches wide, along the upper side of the paper, pressing it down with clean linen in order to drive out all air-bubbles. Press down another strip in like manner, and so proceed till all is pasted. Let it dry very gradually and thoroughly before cutting the muslin from the board.

Maps may be varnished with picture-varnish, or by applying four or five coats of isinglass-size, letting each dry well before applying the next, and giving a full, flowing coat of Canada balsam diluted with the best oil of turpentine.

LATITUDES AND DEPARTURES.

239. Definitions. The LATITUDE of a point is its distance north or south of some "*Parallel of Latitude,*" or line running east or west. The LONGITUDE of a point is its distance east or west of some "*Meridian,*" or line running north and south. In compass-surveying, the magnetic meridian—i. e., the direction in which the magnetic needle points—is the line from which the longitudes of points are measured or reckoned.

The distance which one end of a line is due north or south of the other end is called the *Difference of Latitude* of the two ends of the line; or its *northing* or *southing;* or simply its *latitude.*

The distance which one end of the line is due east or west of the other is here called the *Difference of Longitude* of the two ends of the line; or its *easting* or *westing;* or its *departure.*

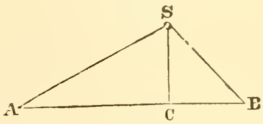
Latitudes and *Departures* are the most usual terms, and will be generally used hereafter, for the sake of brevity.

This subject may be illustrated geographically, by noticing that a traveler, in going from New York to Buffalo in a straight line, would go about one hundred and fifty miles due north, and two hundred and fifty miles due west. These distances would be the differences of latitude and of longitude between the two places, or his northing and westing. Returning from Buffalo to New York, the same distances would be his southing and easting.*

* It should be remembered that the following discussions of the latitudes and longitudes of the points of a survey will not be perfectly applicable to those of distant places, such as the cities just named, in consequence of the surface of the earth not being a plane.

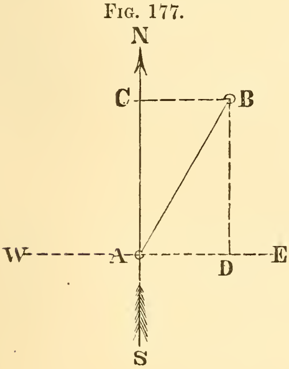
In mathematical language, the operation of finding the latitude and longitude of a line, from its bearing and length, would be called the transformation of Polar Co-ordinates into Rectangular Co-ordinates. It consists in determining, by our *Second Principle,* the position of a point which had originally been determined by the *Third Principle.* Thus, in the figure (which is the same as that of Art. 7), the point S is determined by the angle S A C and by the distance A S. It is also determined by the distances A C and C S, measured at right angles to each other; and then, supposing C S to run due north and south, C S will be the *latitude,* and A C the *departure* of the line A S.

Fig. 176.



240. Calculation of Latitudes and Departures. Let A B be a given line, of which the length A B, and the bearing (or angle, B A C, which it makes with the magnetic meridian), are known. It is required to find the differences of latitude and of longitude between its two extremities A and B—that is, to find A C and C B; or, what is the same thing, B D and D A.

Fig. 177.



It will be at once seen that A B is the hypotenuse of a right-angled triangle, in which the "Latitude" and the "Departure" are the sides about the right angle. We therefore know, from the principles of trigonometry, that

$$A\,C = A\,B \,.\, \cos.\ B\,A\,C,$$
$$B\,C = A\,B \,.\, \sin.\ B\,A\,C.$$

Hence, to find the *latitude* of any course, multiply the natural cosine of the bearing by the length of the course; and to find the *departure* of any course, multiply the natural sine of the bearing by the length of the course.

If the course be northerly, the *latitude* will be north, and will be marked with the algebraic sign $+$, *plus,* or additive; if it be southerly, the latitude will be south, and will be marked with the algebraic sign $-$, *minus*, or subtractive.

If the course be easterly, the *departure* will be east, and marked $+$, or additive; if the course be westerly, the departure will be west, and marked $-$, or subtractive.

241. Formulas. The rules of the preceding article may be expressed thus:

$$\text{Latitude} = \text{distance} \times \text{cos. bearing},$$

$$\text{Departure} = \text{distance} \times \text{sin. bearing.*}$$

From these formulas may be obtained others, by which, when any two of the above four things are given, the remaining two can be found.

When the Bearing and Latitude are given;

$$\text{Distance} = \frac{\text{latitude}}{\text{cos. bearing}} = \text{latitude} \times \text{sec. bearing},$$

$$\text{Departure} = \text{latitude} \times \text{tang. bearing}.$$

When the Bearing and Departure are given;

$$\text{Distance} = \frac{\text{departure}}{\text{sin. bearing}} = \text{departure} \times \text{cosec. bearing}.$$

$$\text{Latitude} = \text{departure} \times \text{cotang. bearing}.$$

When the Distance and Latitude are given;

$$\text{Cos. bearing} = \frac{\text{latitude}}{\text{distance}},$$

$$\text{Departure} = \text{latitude} \times \text{tang. bearing}.$$

When the Distance and Departure are given;

$$\text{Sin. bearing} = \frac{\text{departure}}{\text{distance}},$$

$$\text{Latitude} = \text{departure} \times \text{cotang. bearing}.$$

When the Latitude and Departure are given;

$$\text{Tang. of bearing} = \frac{\text{departure}}{\text{latitude}},$$

$$\text{Distance} = \text{latitude} \times \text{sec. bearing}.$$

Still more simply, any two of these three—distance, latitude, and departure—being given, we have

$$\text{Distance} = \sqrt{(\text{latitude}^2 + \text{departure}^2)}$$

$$\text{Latitude} = \sqrt{(\text{distance}^2 - \text{departure}^2)}$$

$$\text{Departure} = \sqrt{(\text{distance}^2 - \text{latitude}^2)}$$

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* Whenever sines, cosines, tangents, etc., are here named, they mean the natural sines, etc., of an arc described with a radius equal to *one*, or to the unit by which the sines, etc., are measured.

242. Traverse-Tables. The latitude and departure of any distance, for any bearing, could be found by the method given in Art. 240, with the aid of a table of natural sines. But to facilitate these calculations, which are of so frequent occurrence and of so great use, *traverse-tables* have been prepared, originally for navigators (whence the name *traverse*), and subsequently for surveyors.*

The traverse-table at the end of this volume gives the latitude and departure for any bearing, to each quarter of a degree, and for distances from 1 to 9.

To use it, find in it the number of degrees in the bearing, on the left-hand side of the page, if it be less than 45°, or on the right-hand side if it be more. The numbers on the same line, running across the page,† are the latitudes and departures for that bearing, and for the respective distances—1, 2, 3, 4, 5, 6, 7, 8, 9—which are at the top and bottom of the page, and which may represent chains, links, rods, feet, or any other unit. Thus, if the bearing be 15°, and the distance 1, the latitude would be 0·966 and the departure 0·259. For the same bearing, but a distance of 8, the latitude would be 7·727 and the departure 2·071.

Any distance, however great, can have its latitude and departure readily obtained from this table; since, for the same bearing, they are directly proportional to the distance, because of the similar triangles which they form. Therefore, to find the latitude or departure for 60, multiply that for 6 by 10, which merely moves the decimal-point one place to the right; for 500, multiply the numbers found in the table for 5, by 100—i. e., move the decimal-point two places to the right, and so on. Merely moving the decimal-point to the right, one, two, or more places, will therefore enable this table to give the latitude and departure for any decimal multiple of the numbers in the table.

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* The first traverse-table for surveyors seems to have been published in 1791, by John Gale. The most extensive table is that of Captain Boileau, of the British army, being calculated for every minute of bearing, and to five decimal places, for distances from 1 to 10. The table in this volume was calculated for it, and then compared with the one just mentioned.

† In using this or any similar table, lay a ruler across the page, just above or below the line to be followed out. This is a very valuable mechanical assistance.

For compound numbers, such as 873, it is only necessary to find separately the latitudes and departures of 800, of 70, and of 3, and add them together. But this may be done, with scarcely any risk of error, by the following simple rule:

Write down the latitude and departure for the first figure of the given number, as found in the table, neglecting the decimal-point; write under them the latitude and departure of the second figure, setting them one place farther to the right; under them write the latitude and departure of the third figure, setting them one place farther to the right; and so proceed with all the figures of the given number. Add up these latitudes and departures, and cut off the three right-hand figures. The remaining figures will be the latitude and departure of the given number in links, or chains, or feet, or whatever unit it was given in.

For example: Let the latitude and departure of a course having a distance of 873 links, and a bearing of 20°, be required. In the table find 20°, and then take out the latitude and departure for 8, 7, and 3, in turn, placing them as above directed, thus:

Distances.	*Latitudes.*	*Departures.*
800	7518	2736
70	6578	2394
3	2819	1026
873	820·399	298·566

Taking the nearest whole numbers and rejecting the decimals, we find the desired latitude and departure to be 820 and 299.*

When a 0 occurs in the given number, the next figure must be set *two* places to the right, the reason of which will appear from the following example, in which the 0 is treated like any other number:

Given a bearing of 35°, and a distance of 3048 links.

Distances.	*Latitudes.*	*Departures.*
3000	2457	1721
000	0000	0000
40	3277	2294
8	6553	4589
3048	2496·323	1748·529

* It is frequently doubtful, in many calculations, when the final decimal is 5, whether to increase the preceding figure by one or not. Thus, 43·5 may be called 43 or 44 with equal correctness. It is better, in such cases, not to increase the whole number, so as to escape the trouble of changing the original figure, and the increased

Here the latitudes and departures are 2496 and 1749 links.

When the bearing is over 45°, the names of the columns must be read from the bottom of the page, the latitude of any bearing, as 50°, being the departure of the complement of this bearing, or 40°, and the departure of 40° being the latitude of 50°, etc. The reason of this will be at once seen on inspecting Fig. 177, and imagining the east and west line to become a meridian. For, if A C be the magnetic meridian, as before, and therefore B A C be the bearing of the course A B, then is A C the latitude, and C B the departure of that course. But if A E be the meridian and B A D (the complement of B A C) be the bearing, then is A D (which is equal to C B) the latitude, and D B (which is equal to A C) the departure.

As an example of this, let the bearing be 63¼°, and the distance 3,469 links. Proceeding as before, we have—

Distances.	*Latitudes.*	*Departures.*
3000	1350	2679
400	1800	3572
60	2701	5358
9	4051	8037
3469·	1561·061	3097·817

The required latitude and departure are 1561 and 3098 links.

In the few cases occurring in compass-surveying, in which the bearing is recorded as somewhere between the fractions of a degree given in the table, its latitude and departure may be found by interpolation. Thus, if the bearing be 10⅜°, take the half sum of the latitudes and departures for 10¼° and 10½°. If it be 10° 20′, add one third of the difference between the latitudes and departures for 10¼° and for 10½°, to those opposite to 10¼° ; and so in any similar case.

The uses of this table are very varied. The principal applications of it, which will now be explained, are to *testing the accuracy of surveys ;* to *supplying omissions in them ;* to *platting them ;* and to *calculating their content.**

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chance of error. If, however, more than one such case occurs in the same column to be added up, the larger and smaller number should be taken alternately.

* The traverse-table admits of many other minor uses. Thus, it may be used

243. Application to testing a Survey. It is self-evident that, when the surveyor has gone completely around a field or farm, taking the bearings and distances of each boundary-line, till he has got back to the starting-point, he has gone precisely as far south as north, and as far west as east. But the sum of the north latitudes tells how far north he has gone, and the sum of the south latitudes how far south he has gone. Hence these two sums will be equal to each other, if the survey has been correctly made. In like manner, the sums of the east and of the west departures must also be equal to each other.

We will apply this principle to testing the accuracy of the survey of which Fig. 61 is a plat. Prepare seven columns, and head them as below. Find the latitude and departure of each course to the nearest link, and write them in their appropriate columns. Add up these columns. Then will the difference between the sums of the north and south latitudes, and between the sums of the east and west departures, indicate the degree of accuracy of the survey.

STATIONS.	BEARINGS.	DISTANCES.	LATITUDES.		DEPARTURES.	
			N.	S.	E.	W.
1	N. 35° E.	2·70	2·21		1·55	
2	N. 83½° E.	1·29	·15		1·28	
3	S. 57° E.	2·22		1·21	1·86	
4	S. 34¼° W.	3·55		2·93		2·00
5	N. 56½° W.	3·23	1·78			2·69
			4·14	4·14	4·69	4·69

The entire work of the above example is given on the following page.

for solving, approximately, any right-angled triangle by mere inspection, the bearing being taken for one of the acute angles; the latitude being the side adjacent, the departure the side opposite, and the distance the hypotenuse. Any two of these being given, the others are given by the table. The table will therefore serve to show the allowance to be made in chaining on slopes (see Art. 20). Look in the column of bearings for the slope of the ground—i. e., the angle it makes with the horizon, find the given distance, and the latitude corresponding will be the desired horizontal measurement, and the difference between it and the distance will be the allowance to be made.

35°	1638	1147	$34\frac{1}{4}$°	2480	1688
	57340	40150		4133	2814
				4133	2814
270·	221·140	154·850	355·	293·463	199·754
$83\frac{1}{2}$°	113	994	$56\frac{1}{2}$°	1656	2502
	226	1987		1104	1668
	1019	8942		1656	2502
129·	14·579	128·212	323·	178·296	269·382
57°	1089	1677			
	1089	1677			
	1089	1677			
222·	120·879	186·147			

The nearest link is taken to be inserted in the table, and the remaining decimals are neglected.

In the preceding example the respective sums were found to be exactly equal. This, however, will rarely occur in an extensive survey. If the difference be great, it indicates some mistake, and the survey must be repeated with greater care; but if the difference be small it indicates, not absolute errors, but only inaccuracies, unavoidable in surveys with the compass, and the survey may be accepted.

How great a difference in the sums of the columns may be allowed, as not necessitating a new survey, is a dubious point. Some surveyors would admit a difference of 1 link for every 3 chains in the sum of the courses; others only 1 link for every 10 chains. One writer puts the limit at 5 links for each station; another at 25 links in a survey of 100 acres. But every practical surveyor soon learns how near to an equality his instrument and his skill will enable him to come in ordinary cases, and can therefore establish a standard for himself, by which he can judge whether the difference, in any survey of his own, is probably the result of an error, or only of his customary degree of inaccuracy, two things to be very carefully distinguished.*

244. Application to supplying Omissions. Any two omissions in the field-notes can be supplied by a proper use of the method of latitudes and departures; as will be explained in Chapter V, which treats of "Obstacles to Measurement," under which head this

* A French writer fixes the allowable difference in chaining at 1-400 of level lines; 1-200 of lines on moderate slopes; 1-100 of lines on steep slopes.

subject most appropriately belongs. But a knowledge of the fact that any two omissions can be supplied, should not lead the young surveyor to be negligent in making every possible measurement, since an omission renders it necessary to assume all the notes taken to be correct, the means of testing them no longer existing.

245. Balancing a Survey. The subsequent applications of this method require the survey to be previously *balanced.* This operation consists in *correcting* the latitudes and departures of the courses, so that their sums *shall* be equal, and thus "balance." This is usually done by distributing the differences of the sums among the courses in proportion to their length ; saying, *as* the sum of the lengths of all the courses *is* to the whole difference of the latitude, *so is* the length of each course *to* the correction of its latitude. A similar proportion corrects the departures.*

It is not often necessary to make the exact proportion, as the correction can usually be made, with sufficient accuracy, by noting how much per chain it should be, and correcting accordingly.

In the example given below, the differences have purposely been made considerable. The corrected latitudes and departures have been here inserted in four additional columns, but in practice they should be written *in red ink* over the original latitudes and departures, and the latter crossed out with red ink.

STATIONS.	BEARINGS.	DISTANCES.	LATITUDES.		DEPARTURES.		CORRECTED LATITUDES.		CORRECTED DEPARTURES.	
			N. +	S. −	E. +	W. −	N. +	S. −	E. +	W. −
1	N. 52° E.	10·63	6·54		8·38		6·58		8·34	
2	S. 29¾° E.	4·10		3·56	2·03			3·55	2·01	
3	S. 31¾° W.	7·69		6·54		4·05		6·51		4·08
4	N. 61° W.	7·13	3·46			6·24	3·48			6·27
		29·55	10·00	10·10	10·41	10·29	10·06	10·06	10·35	10·35

The corrections are made by the following proportions ; the nearest whole numbers being taken :

* A demonstration of this principle was given by Dr. Bowditch, in No. 4 of "The Analyst."

For the Latitudes.	*For the Departures.*
29·55 : 10·63 :: 10 : 4	29·55 : 10·63 :: 12 : 4
29·55 : 4·10 :: 10 : 1	29·55 : 4·10 :: 12 : 2
29·55 : 7·69 :: 10 : 3	29·55 : 7·69 :: 12 : 3
29·55 : 7·13 :: 10 : 2	29·55 : 7·13 :: 12 : 3
10	12

This rule is not always to be strictly followed. If one line of a survey has been measured over very uneven and rough ground, or if its bearing has been taken with an indistinct sight, while the other lines have been measured over level and clear ground, it is probable that most of the error has occurred on that line, and the correction should be chiefly made on its latitude and departure.

If a slight change of the bearing of a long course will favor the balancing, it should be so changed, since the compass is much more subject to error than the chain. So, too, if shortening any doubtful line will favor the balancing, it should be done, since distances are generally measured too long.

246. Application to Platting. Rule three columns; one for stations, the next for total latitudes, and the third for total departures. Fill the last two columns by beginning at any convenient station (the extreme east or west is best) and adding up (algebraically) the latitudes of the following stations, noticing that the south latitudes are subtractive. Do the same for the departures, observing that the westerly ones are also subtractive.

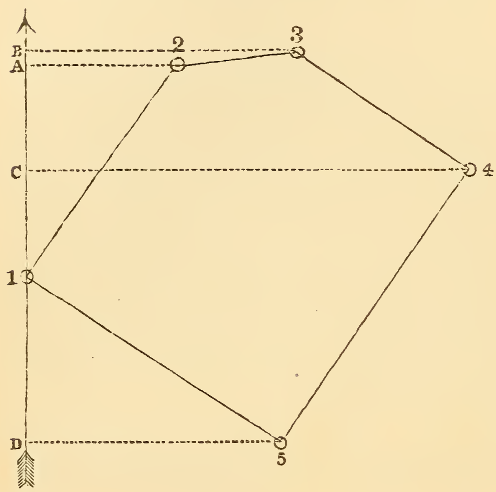
Taking the example given in Art. 243, and beginning with station 1, the following will be the results:

STATIONS.	TOTAL LATITUDES FROM STATION 1.	TOTAL DEPARTURES FROM STATION 1.
1	0·00	0·00
2	+ 2·21 N.	+ 1·55 E.
3	+ 2·36 N.	+ 2·83 E.
4	+ 1·15 N.	+ 4·69 E.
5	− 1·78 S.	+ 2·69 E.
1	0·00	0·00

It will be seen that the work proves itself, by the total latitudes and departures for station 1, again coming out equal to zero.

To use this table, draw a meridian through the point taken for station 1, as in Fig. 178. Set off, upward from this, along the meridian, the latitude, 221 links, to A, and from A, to the right perpendicularly, set off the departure, 155 links.* This gives the point 2. Join 1....2. From 1 again, set off, upward, 236 links, to B, and from B, to the right, perpendicularly, set off 283 links, which will fix the point 3. Join 2....3 ; and so proceed, setting off north latitudes along the meridian upward, and south latitudes along it downward ; east departures perpendicularly to the right, and west departures perpendicularly to the left.

Fig. 178.



The advantages of this method are its rapidity, ease, and accuracy ; the impossibility of any error in platting any one course affecting the following points ; and the certainty of the plat "coming together," if the latitudes and departures have been "balanced."

CALCULATING THE CONTENT.

247. Methods. When a field has been platted, by whatever method it may have been surveyed, its content can be obtained from its plat by dividing it up into triangles, and measuring on the plat their bases and perpendiculars ; or by any of the other means explained in Chapter II.

But these are only approximate methods, their degree of accu-

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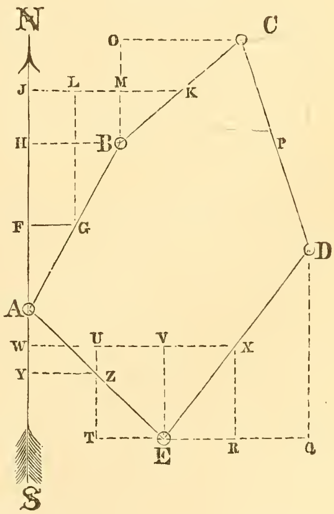
* This is most easily done with the aid of a right-angled triangle, sliding one of the sides adjacent to the right angle along the blade of the square, to which the other side will then be perpendicular.

racy depending on the largeness of scale of the plat and the skill of the draughtsman. The invaluable method of latitudes and departures gives another means, perfectly accurate, and not requiring the previous preparation of a plat. It is sometimes called the rectangular, or the Pennsylvania, or Rittenhouse's method of calculation.*

248. Definitions. Imagine a meridian line to pass through the extreme east or west corner of a field. According to the definitions established in Art. 239 (and here recapitulated for convenience of reference), the perpendicular distance of each station from that meridian is the *Longitude* of that station; additive, or *plus*, if east; subtractive, or *minus*, if west. The distance of the middle of any line, such as the side of the field, from the meridian, is called the *longitude* of that side.† The difference of the longitudes of the two ends of a line is called the *Departure* of that line. The difference of the latitudes of the two ends of a line is called the *Latitude* of the line.

Fig. 179.



249. Longitudes. To give more definiteness to the development of this subject, the figure in the margin will be referred to, and may be considered to represent any space inclosed by straight lines.

Let N S be the meridian passing through the extreme westerly station of the field A B C D E.

* It is, however, substantially the same as Mr. Thomas Burgh's "Method to determine the Areas of Right-lined Figures universally," published nearly a century ago.

† The phrase "meridian distance" is generally used for what is here called "longitude"; but the analogy of "differences of longitude" with "differences of latitude," usually but anomalously united with the word "departure," borrowed from navigation, seems to put beyond all question the propriety of the innovation here introduced.

From the middle and ends of each side draw perpendiculars to the meridian. These perpendiculars will be the *longitudes* and departures of the respective sides. The longitude, F G, of the *first course*, A B, is evidently equal to half its departure, H B. The longitude, J K, of the second course, B C, is equal to J L + L M + M K, or equal to the longitude of the preceding course, plus half its departure, plus half the departure of the course itself. The longitude, Y Z, of some other course, as E A, taken anywhere, is equal to W X − V X − U V, or equal to the longitude of the preceding course, minus half its departure, minus half the departure of the course itself—i. e., equal to the *algebraic* sum of these three parts, remembering that *westerly* departures are negative, and therefore to be subtracted when the directions are to make an *algebraic* addition.

To avoid fractions it will be better to double each of the preceding expressions. We shall then have a

GENERAL RULE FOR FINDING DOUBLE LONGITUDES.

The double longitude of the FIRST COURSE *is equal to its departure.*

The double longitude of the SECOND COURSE *is equal to the double longitude of the first course, plus the departure of that course, plus the departure of the second course.*

The double longitude of the THIRD COURSE *is equal to the double longitude of the second course, plus the departure of that course, plus the departure of the course itself.*

The double longitude of ANY *course is equal to the double longitude of the preceding course, plus the departure of that course, plus the departure of the course itself.**

The double longitude of the *last* course (as well as of the first) is equal to its departure. Its "coming out" so, when obtained by the above rule, proves the accuracy of the calculation of all the preceding double longitudes.

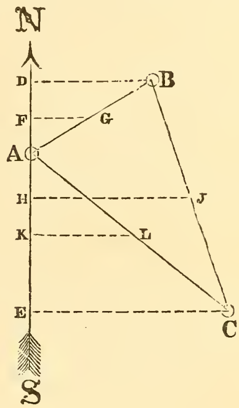
250. Areas. We will now proceed to find the area or content of a field, by means of the "double longitudes" of its sides, which

* The last course is a "preceding course" to the first course, as will appear on remembering that these two courses join each other on the ground.

can be readily obtained by the preceding rule, whatever their number.

251. Beginning with a *three-sided field,* A B C in the figure, draw a meridian through A, and draw perpendiculars to it as in the last figure. It is plain that its content is equal to the difference of the areas of the trapezoid D B C E, and of the triangles A B D and A C E.

FIG. 180.

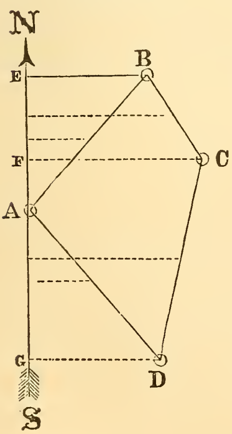


The area of the triangle A B D is equal to the product of A D by half of D B, or to the product of A D by F G; i. e., equal to the product of the latitude of the first course by its longitude.

The area of the trapezoid D B C E is equal to the product of D E by half the sum of D B and C E, or by H J; i. e., to the product of the latitude of the second course by its longitude.

The area of the triangle A C E is equal to the product of A E by half E C, or by K L; i. e., to the product of the latitude of the third course by its longitude.

FIG. 181.



Calling the products in which the latitude was north, *North Products,* and the products in which the latitude was south, *South Products,* we shall find the area of the trapezoid to be a *south product,* and the areas of the triangles to be *north products. The difference of the north products and the south products is therefore the desired area of the three-sided field* A B C.

Using the *double* longitudes (in order to avoid fractions) in each of the preceding products, their difference will be the *double* area of the triangle A B C.

252. Taking now a *four-sided field,* A B C D in the figure, and drawing a meridian and longitudes as before, it is seen, on inspection, that its area would be obtained by taking the two triangles, A B E, A D G, from the figure E B C D G E, or from the sum of the two trapezoids E B C F and F C D G.

The area of the triangle A E B will be found, as in the last article, to be equal to the product of the latitude of the first course by its longitude. The product will be *North.*

The area of the trapezoid E B C F will be found to equal the latitude of the second course by its longitude. The product will be *South.*

The area of the trapezoid F C D G will be found to equal the product of the latitude of the third course by its longitude. The product will be *South.*

The area of the triangle A D G will be found to equal the product of the latitude of the fourth course by its longitude. The product will be *North.*

The difference of the north and south products will therefore be the desired area of the four-sided field A B C D.

Using the *double* longitude as before, in each of the preceding products, their difference will be *double* the area of the field.

253. Whatever the number or directions of the sides of a field, or of any space inclosed by straight lines, its area will always be equal to half of the difference of the north and south products arising from multiplying together the latitude and double longitude of each course or side.

We have, therefore, the following

General Rule for finding Areas.

1. *Prepare ten columns, headed as in the example below, and in the first three write the stations, bearings, and distances.*

2. *Find the latitudes and departures of each course, by the traverse-table, as directed in Art.* 242, *placing them in the four following columns.*

3. *Balance them, as in Art.* 245, *correcting them in red ink.*

4. *Find the double longitudes, as in Art.* 249, *with reference*

to a meridian passing through the extreme east or west station, and place them in the eighth column.

5. *Multiply the double longitude of each course by the corrected latitude of that course, placing the north products in the ninth column, and the south products in the tenth column.*

6. *Add up the last two columns, subtract the smaller sum from the larger, and divide the difference by two. The quotient will be the content desired.*

254. To find the most easterly or westerly station of a survey, without a plat, it is best to make a rough hand-sketch of the survey, drawing the lines in an approximation to their true directions, by drawing a north and south, and east and west lines, and considering the bearings as fractional parts of a right angle, or 90°; a course N. 45° E., for example, being drawn about half-way between a north and an east direction; a course N. 28° W. being not quite one third of the way around from north to west; and so on, drawing them of approximately true proportional lengths.

255. *Example* 1, given below, refers to the five-sided field, of which a plat is given in Fig. 161, and the latitudes and departures of which were calculated in Art. 243. Station 1 is the most westerly station, and the meridian will be supposed to pass through it. The double longitudes are best found by a continual addition and subtraction, as in the margin, where they are marked D. L. The double longitude of the last course comes out equal to its departure, thus proving the work.

STATIONS.	
1	+ 1·55 D. L. + 1·55 + 1·28
2	+ 4·38 D. L. + 1·28 + 1·86
3	+ 7·52 D. L. + 1·86 − 2·00
4	+ 7·38 D. L. − 2·00 − 2·69
5	+ 2·69 D. L.

The double longitudes being thus obtained, are multiplied by the corresponding latitudes, and the content of the field obtained as directed in the General Rule.

This example may serve as a pattern for the most compact manner of arranging the work.

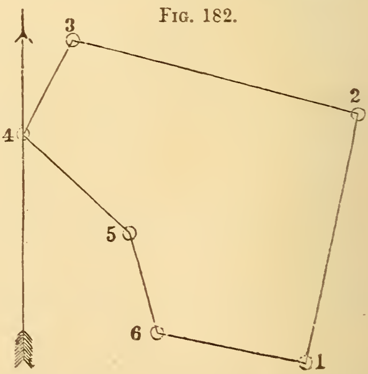
STA-TIONS.	BEARINGS.	DIS-TANCES.	LATITUDES.		DEPARTURES.		DOUBLE LONGI-TUDES.	DOUBLE AREAS.	
			N.+	S.—	E.+	W.—		N.—	S.+
1	N. 35° E.	2·70	2·21		1·55		+1·55	3·4255	
2	N. 83½° E.	1·29	·15		1·28		+4·38	0·6570	
3	S. 57° E.	2·22		1·21	1·86		+7·52		9·0992
4	S. 34¼° W.	3·55		2·93		2·00	+7·38		21·6234
5	N. 56½° W.	3·23	1·78			2·69	+2·69	4·7882	
			4·14	4·14	4·69	4·69		8·8707	30·7226
									8·8707
	Content = 1 A. 0 R. 15 P.								2)21·8519
								square chains,	10·9259

STA-TIONS.	
4	— 2·00 D. L.
	— 2·00
	— 2·69
5	— 6·69 D. L.
	— 2·69
	+ 1·55
1	— 7·83 D. L.
	+ 1·55
	+ 1·28
2	— 5·00 D. L.
	+ 1·28
	+ 1·86
3	— 1·86

256. The meridian might equally well have been supposed to pass through the most easterly station, 4 in the figure. The double longitudes could then have been calculated as in the margin. They will, of course, be all west, or minus. The products being then calculated, the sum of the north products will be found to be 29·9625, and of the south products 8·1106, and their difference to be 21·8519, the same result as before.

257. A number of examples, with and without answers, will now be given as exercises for the student, who should plat them by some of the methods given in the chapter on platting, using each of them at least once. He should then calculate their content by the method just given, and *check* it, by also calculating the area of the plat by some of the geometrical or instrumental methods given in Chapter I; for no single calculation is

FIG. 182.



ever reliable. All the examples (except the last) are from the author's actual surveys.

Example 2, given below, is also fully worked out, as another pattern for the student, who need have no difficulty with any possible case if he strictly follows the directions which have been given. The plat is on a scale of 2 chains to 1 inch (= 1:1584).

STATIONS.	BEARINGS.	DISTANCES.	LATITUDES.		DEPARTURES.		DOUBLE LONGITUDES.	DOUBLE AREAS.	
			N. +	S. −	E. +	W. −		N. +	S. −
1	N. 12¼° E.	2·81	2·75		·60		+6·56	18·0400	
2	N. 76° W.	3·20	·77			3·11	+4·05	3·1185	
3	S. 24½° W.	1·14		1·04		·47	+ ·47		·4888
4	S. 48° E.	1·53		1·02	1·14		+1·14		1·1628
5	S. 12½° E.	1·12		1·09	·24		+2·52		2·7468
6	S. 77° E.	1·64		·37	1·60		+4·36		1·6132
			3·52	3·52	3·58	3·58		21·1585	6·0116
								6·0116	
	Content = 0 A. 3 R. 1 P.							2)15·1469	
							square chains,	7·5734	

Example 3.

STATIONS.	BEARINGS.	DISTANCES.
1	N. 52° E.	10·64
2	S. 29¾° E.	4·09
3	S. 31¾° W.	7·68
4	N. 61° W.	7·24

Ans. 4 A. 3 R. 28 P.

Example 4.

STATIONS.	BEARINGS.	DISTANCES.
1	S. 21° W.	12·41
2	N. 83½° E.	5·86
3	N. 12° E.	8·25
4	N. 47° W.	4·24

Ans. 4 A. 2 R. 37 P.

Example 5.

STATIONS.	BEARINGS.	DISTANCES.
1	N. 34¼° E.	2·73
2	N. 85° E.	1·28
3	S. 56¾° E.	2·20
4	S. 34¼° W.	3·53
5	N. 56½° W.	3·20

Ans. 1 A. 0 R. 14 P.

Example 6.

STATIONS.	BEARINGS.	DISTANCES.
1	N. 35° E.	6·49
2	S. 56¼° E.	14·15
3	S. 34° W.	5·10
4	N. 56° W.	5·84
5	S. 29½° W.	2·52
6	N. 48¼° W.	8·73

Example 7.

STATIONS.	BEARINGS.	DISTANCES.
1	S. 21¼° W.	17·62
2	S. 34° W.	10·00
3	N. 56° W.	14·15
4	N. 34° E.	9·76
5	N. 67° E.	2·30
6	N. 23° E.	7·03
7	N. 18½° E.	4·43
8	S. 76½° E.	12·41

Example 8.

STATIONS.	BEARINGS.	DISTANCES.
1	S. 65½° E.	4·98
2	S. 58° E.	8·56
3	S. 14¼° W.	20·69
4	S. 47° W.	0·60
5	S. 57½° W.	8·98
6	N. 56° W.	12·90
7	N. 34° E.	10·00
8	N. 21¼° E.	17·62

Example 9.

STATIONS.	BEARINGS.	DISTANCES.
1	S. 57° E.	5·77
2	S. 36¾° W.	2·25
3	S. 39¼° W.	1·00
4	S. 70½° W.	1·04
5	N. 68¾° W.	1·23
6	N. 56° W.	2·19
7	N. 33¼° E.	1·05
8	N. 56½° W.	1·54
9	N. 33½° E.	3·18

Ans. 2 A. 0 R. 32 P.

Example 10.

STATIONS.	BEARINGS.	DISTANCES.
1	N. 63° 51′ W.	6·91
2	N. 63° 44′ W.	7·26
3	N. 69° 35′ W.	3·34
4	N. 77° 50′ W.	6·54
5	N. 31° 24′ E.	14·38
6	N. 31° 18′ E.	16·81
7	S. 68° 55′ E.	13·64
8	S. 68° 42′ E.	11·54
9	S. 33° 45′ W.	31·55

Ans. 74 acres.

Example 11.

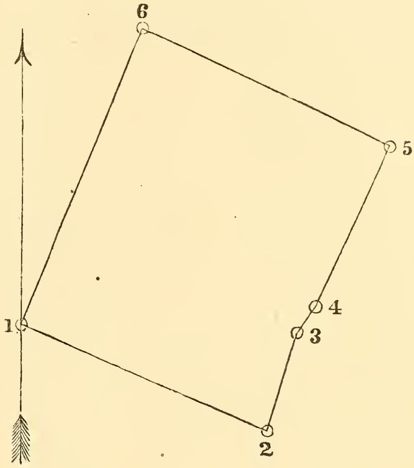
STATIONS.	BEARINGS.	DISTANCES.
1	N. 18¾° E.	1·93
2	N. 9° W.	1·29
3	N. 14° W.	2·71
4	N. 74° E.	0·95
5	S. 48½° E.	1·59
6	S. 14¼° E.	1·14
7	S. 19½° E.	2·15
8	S. 23½° W.	1·22
9	S. 5° W.	1·40
10	S. 30° W.	1·02
11	S. 81½° W.	0·69
12	N. 32½° W.	1·98

Example 12.

STATIONS.	BEARINGS.	DISTANCES.
1	N. 72¾° E.	0·88
2	S. 20½° E.	0·22
3	S. 63° E.	0·75
4	N. 51° E.	2·35
5	N. 44° E.	1·10
6	N. 25½° W.	1·96
7	N. 8½° W.	1·05
8	S. 29° W.	1·63
9	N. 71½° W.	0·81
10	N. 13½° W.	1·17
11	N. 63° W.	1·28
12	West.	1·68
13	N. 49° W.	0·80
14	S. 19½° E.	6·20

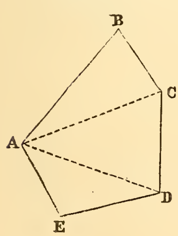
Example 13. A farm is described in an old deed as bounded thus: Beginning at a pile of stones, and running thence twenty-seven chains and seventy links southeasterly sixty-six and a half degrees to a white-oak stump ; thence eleven chains and sixteen links northeasterly twenty and a half degrees to a hickory-tree ; thence two chains and thirty-five links northeasterly thirty-six degrees to the southeasterly corner of the homestead ; thence nineteen chains and thirty-two links northeasterly twenty-six degrees to a stone set in the ground ; thence twenty-eight chains and eighty links northwesterly sixty-six degrees to a pine-stump ; thence thirty-three chains and nineteen links southwesterly twenty-two degrees to the place of beginning, containing ninety-two acres, be the same more or less. Required the exact content.

FIG. 183.



258. Mascheroni's Theorem. *The surface of any polygon is equal to half the sum of the products of its sides (omitting any one side) taken two and two, into the sines of the angles which those sides make with each other.*

FIG. 184.

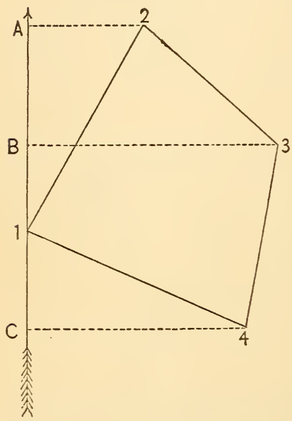


Thus, take any polygon, such as the five-sided one in the figure. Express the angle which the directions of any two sides, as A B, C D, make with each other, thus (A B $\wedge$ C D). Then will the content of that polygon be, as below :

$$= \tfrac{1}{2}[AB \,.\, BC \,.\, \sin(AB \wedge BC) + AB \,.\, CD \,.\, \sin(AB \wedge CD) + AB \,.\, DE \,.\, \sin(AB \wedge DE) + BC \,.\, CD \,.\, \sin(BC \wedge CD) + BC \,.\, DE \,.\, \sin(BC \wedge DE) + CD \,.\, DE \,.\, \sin(CD \wedge DE)]$$

The demonstration consists merely in dividing the polygon into triangles by lines drawn from any angle (as A); then expressing the area of each triangle by half the product of its base and the perpendicular let fall upon it from the above-named angle; and finally separating the perpendicular into parts which can each be expressed by the product of some one side into the sine of the angle made by it with another side. The sum of these triangles equals the polygon.

FIG. 185.



The expressions are simplified by dividing the proposed polygon into two parts by a diagonal, and computing the area of each part separately, making the diagonal the side omitted.

A New Method of calculating Areas.

259. In Fig. 185, let the total latitudes (Art. 246) of the stations 1, 2, 3, and 4 be represented by l_1, l_2, l_3, and l_4, respectively.

Let the departures of each course separately be represented by d_1, d_2, d_3, and d_4, respectively.

The double area of A B 23

$$= \text{A B (A2} + \text{B3)}$$
$$= (l_2 - l_3)\ (d_1 + d_1 + d_2)$$
$$= l_2d_1 + l_2d_1 + l_2d_2 - l_3d_1 - l_3d_1 - l_3d_2. \quad [1.]$$

The double area of C B34

$$= \text{CB (B3} + \text{C4)}$$
$$= (l_3 + l_4)\ (d_4 + d_4 + d_3)$$
$$= l_3d_4 + l_3d_4 + l_3d_3 + l_4d_4 + l_4d_4 + l_4d_3. \quad [2.]$$

The double area of 12 A = A1 (A2) = l_2d_1. [3.]

The double area of 14 C = C1 (C4) = l_4d_4. [4.]

Now, the double area of the figure 1234 is equal to the sum of [1] and [2] – the sum of [3] and [4].

Combining and reducing, we have:

Double area of 1234 = $l_2\ (d_1 + d_2) + l_3\ (d_4 + d_4 + d_3 - d_1 - d_1 - d_2) + l_4\ (d_3 + d_4)$.

Noting that $d_4 + d_3 = d_1 + d_2$, we have,

Double area of $1234 = l_2\,(d_1 + d_2) + l_3\,(d_2 - d_3) + l_4\,(d_3 + d_4)$.

Putting this in the form of a rule, we have : *Multiply the total latitude of each station by the algebraic sum of the departures of the two adjacent courses. One half of the algebraic sum of the products will be the area.*

As an exercise for the student, let him find, by the above method, an expression for the area of figures having five and six sides.

The following example, worked out by the method of double longitudes (on page 158), and below, by the new method, will show the difference between the two methods :

STATIONS.	BEARINGS.	DISTANCES.	LATITUDES.		DEPARTURES.		TOTAL LATITUDES.	ADJACENT DEPARTURES.	DOUBLE AREAS.
			N. +	S. −	E. +	W. −			
1	N. 35° E.	2·70	2·21		1·55				
2	N. 83½° E.	1·29	·15		1·28		2·21	2·83	6·2543
3	S. 57° E.	2·22		1·21	1·86		2·36	3·14	7·4104
4	S. 34¼° W.	3·55		2·93		2·00	1·15	−0·14	−0·1610
5	N. 56½° W.	3·23	1·78			2·69	−1·78	−4·69	8·3482
			4·14	4·14	4·69	4·69			2) 21·8519
							square chains,		10·9259

In computing the total latitudes, if the total latitude of the last station equals the latitude of the last course with sign changed, the total latitudes may be considered correct.

The station through which the meridian of the survey is supposed to pass, and from which the total latitude is reckoned, will have no latitude, and hence the product of its latitude and adjacent departures will be zero. There will therefore be one less product than there are stations.

Any station may be taken as the starting-point.

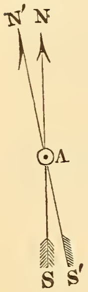
To verify the area obtained in any case, calculate a second time, using a different station as the starting-point.

This method was first published by J. Woodbridge Davis, C. E., Ph. D., in Van Nostrand's "Engineering Magazine," for April, 1879, where a general discussion of the method is given.

THE DECLINATION OF THE MAGNETIC NEEDLE.

260. Definitions. The *magnetic meridian* is the direction indicated by the magnetic needle. The *true meridian* is a true north and south line, which, if produced, would pass through the poles of the earth. The *declination* of the needle is the angle which one of these lines makes with the other.

Fig. 186.



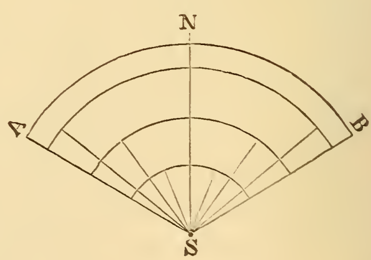
In the figure, if N S represent the direction of the true meridian, and N′ S′ the direction of the magnetic meridian at any place, then is the angle N A N′ the *declination of the needle* at that place.

261. Direction of the Needle. The directions of these two meridians do not generally coincide, but the needle in most places points to the east or to the west of the true north, more or less according to the locality. Observations of the amount and the direction of this declination have been made in nearly all parts of the world. In the United States the declination in the Eastern States is westerly, and in the Western States is easterly, as will be given in detail, after the methods for determining the true meridian, and consequently the declinations, at any place have been explained.

To determine the True Meridian.

262. By Equal Shadows of the Sun. On the south side of any level surface erect an upright staff, shown in horizontal projection at S. Two or three hours before noon, mark the extremity, A, of its shadow. Describe an arc of a circle with S, the foot of the staff, for center, and S A, the distance to the extremity of the shadow, for radius. About as many hours after noon as it had been before noon when the first mark was made, watch for the moment when the end of the shadow

Fig. 187.



touches the arc at another point, B. Bisect the arc A B at N. Draw S N, and it will be the true meridian, or north and south line required.

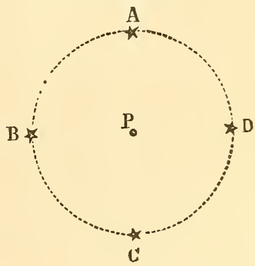
For greater accuracy, describe several arcs beforehand, mark the points in which each of them is touched by the shadow, bisect each, and adopt the average of all. The shadow will be better defined if a piece of tin with a hole through it be placed at the top of the staff, as a bright spot will thus be substituted for the less definite shadow. Nor need the staff be vertical, if from its summit a plumb-line be dropped to the ground, and the point which this strikes be adopted as the center of the arcs.

This method is a very good approximation, though perfectly correct only at the time of the solstices, about June 21st and December 22d. It was employed by the Romans in laying out cities.

To get the declination, set the compass at one end of the true meridian line thus obtained, sight to the other end of it, and take the bearing as of any ordinary line. The number of degrees in the reading will be the desired declination of the needle.

263. By the North Star, when in the Meridian. The north star, or pole star (called by astronomers *Alpha Ursæ Minoris*, or *Polaris*), is not situated precisely at the north pole of the heavens. If it were, the meridian could be at once determined by sighting to it, or placing the eye at some distance behind a plumb-line so that this line should hide the star. But the north star is about $1\frac{1}{2}°$ from the pole. Twice in twenty-four hours, however (more precisely, twenty-three hours fifty-six minutes), it is in the meridian, being then exactly above or below the pole, as at A and C in the figure. To know when it is so, is rendered easy by the aid of another star, easily identified, which at these times is almost exactly above or below the north star—i. e., situated in the same vertical plane. If, then, we watch for the moment at which a suspended plumb-line will cover both these stars, they will then be in the meridian.

FIG. 188.



The other star is in the well-known constellation of the Great Bear, called also the Plow, or the Dipper, or Charles's Wain.

FIG. 189.

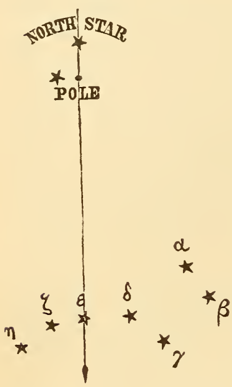
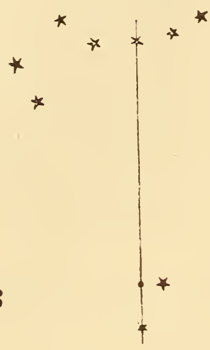


FIG. 190.



Two of its five bright stars (the right-hand ones in Fig. 189) are known as the "Pointers," from their pointing near to the north star, thus assisting in finding it. The star in the tail or handle, nearest to the four which form a quadrilateral, is the star which comes to the meridian at the same time with the north star, twice in twenty-four hours, as in Fig. 189 or 190. It is known as *Alioth*, or *Epsilon Ursæ Majoris.**

To determine the meridian by this method, suspend a long plumb-line from some elevated point, such as a stick projecting from the highest window of a house suitably situated. The plumb-bob may pass into a pail of water to lessen its vibrations. South of this set up the compass, at such a distance from the plumb-line that neither of the stars will be seen above its highest point—i. e., in latitudes of 40° or 50°, not quite as far from the plumb-line as it is long. Or, instead of a compass, place a board on two stakes, so as to form a sort of bench, running east and west, and on it place one of the compass-sights, or anything having a small hole in it to look through. As the time approaches for the north star to be on the meridian (as taken from the table given below) place the compass, or the sight, so that, looking through it, the plumb-line shall seem to cover or hide the north star. As the star moves one way, move the eye and sight the other way, so as to constantly keep the star behind the plumb-line. At last *Alioth*, too, will be

* The north pole is very nearly at the intersection of the line from Polaris to Alioth, and a perpendicular to this line from the small star seen to the left of it in Fig. 189.

covered by the plumb-line. At that moment the eye and the plumb-line are (approximately) in the meridian. Fasten down the sight on the board till morning, or with the compass take the bearing at once, and the reading is the declination.

Instead of one plumb-line and a sight, two plumb-lines may be suspended at the end of a horizontal rod, turning on the top of a pole.

The line thus obtained points to the east of the true line when the north star is above Alioth, and *vice versa*. The north star is exactly in the meridian about twenty-five minutes after it has been in the same vertical plane with Alioth, and may be sighted to, after that interval of time, with perfect accuracy.

Another bright star, which is on the opposite side of the pole, and is known to astronomers as *Gamma Cassiopeiæ*, also comes on the meridian nearly at the same time as the north star, and will thus assist in determining its direction.

264. The time at which the north star passes the meridian above the pole, for every tenth day in the year, is given in the following table, in common clock-time.* The upper transit is the most convenient, since at the other transit Alioth is too high to be conveniently observed :

Times of North Star passing the Meridian.

MONTHS.	1st DAY.	11th DAY.	21st DAY.
	H. M.	H. M.	H. M.
January	6 30 P. M.	5 51 P. M.	5 11 P. M.
February	4 28 "	3 48 "	3 09 "
March	2 37 "	1 58 "	1 18 "
April	0 35 "	11 56 A. M.	11 16 A. M.
May	10 33 A. M.	9 54 "	9 15 "
June	8 32 "	7 52 "	7 13 "
July	6 34 "	5 55 "	5 15 "
August	4 33 "	3 53 "	3 14 "
September	2 31 "	1 52 "	1 12 "
October	0 34 "	11 50 P. M.	11 11 P. M.
November	10 28 P. M.	9 48 "	9 09 "
December	8 30 "	7 50 "	7 11 "

* To calculate the time of the north star passing the meridian at its upper culmination: Find in the "American Ephemeris and Nautical Almanac" the right ascen-

To find the time of the star's passage of the meridian for other days than those given in the table, take from it the time for the day most nearly preceding that desired, and subtract from this time four minutes for each day from the date of the day in the table to that of the desired day ; or, more accurately, interpolate by saying : "*As* the number of days between those given in the table *is to* the number of days from the next preceding day in the table to the desired day, *so is* the difference between the times given in the table for the days next preceding and following the desired day *to* the time to be subtracted from that of the next preceding day."

The north star passes the meridian later every year. In 1890 it will pass the meridian about two minutes later than in 1885 ; in 1895 six minutes, and in 1900 ten minutes later than in 1885, the year for which the preceding table has been calculated.

The times at which the north star passes the meridian *below* the pole in its lower transit can be found by adding eleven hours and fifty-eight minutes to the time of the upper transit, or by subtracting that interval from it.*

265. By the North Star at its Extreme Elongation. When the north star is at its greatest *apparent* angular distance east or west of the pole, as at B or D in Fig. 188, it is said to be at its extreme eastern or extreme western elongation. If it be observed at either of these times, the direction of the meridian can be easily obtained

sion of the star, and from it (increased by twenty-four hours if necessary to render the subtraction possible) subtract the right ascension of the sun at mean noon, or the sidereal time at mean noon, for the given day, as found in the "ephemeris of the sun" in the same almanac. From the remainder subtract the acceleration of sidereal on mean time corresponding to this remainder (3m. 56s. for 24 hours), and the new remainder is the required mean solar time of the upper passage of the star across the meridian, in "astronomical" reckoning, the astronomical day beginning at noon of the common civil day of the same date.

* The north star, which is now about 1° 18′ from the pole, was 12° distant from it when its place was first recorded. Its distance is now diminishing at the rate of about a third of a minute in a year, and will continue to do so till it approaches to within half a degree, when it will again recede. The brightest star in the northern hemisphere, *Alpha Lyræ*, will be the pole-star in about 12,000 years, being then within about 5° of the pole, though now more than 51° distant from it.

from the observation. The great advantage of this method over the preceding is that then the star's motion apparently ceases for a short time.

MEAN TIME OF THE ELONGATIONS OF POLARIS FOR 1885, LATITUDE 40° NORTH.*

DATE.	EASTERN ELONGATION.	WESTERN ELONGATION.
	H. M.	H. M.
January 1, 1885......	12 35·3 P. M.	12 24·6 A. M.
" 15, "	11 36·1 A. M.	11 29·3 P. M.
February 1, "	10 29·0 "	10 22·2 "
" 15, "	9 33·7 "	9 27·0 "
March 1, "	8 38·5 "	8 31·8 "
" 15, "	7 43·4 "	7 36·6 "
April 1, "	6 36·4 "	6 29·7 "
" 15, "	5 41·4 "	5 34·7 "
May 1, "	4 38·6 "	4 31·8 "
" 15, "	3 43·7 "	3 36·9 "
June 1, "	2 37·1 "	2 30·3 "
" 15, "	1 42·2 "	1 35·4 "
July 1, "	12 39·6 "	12 32·8 "
" 15, "	11 44·7 P. M.	11 34·0 A. M.
August 1, "	10 38·2 "	10 27·5 "
" 15, "	9 43·3 "	9 32·6 "
September 1, "	8 36·6 "	8 26·0 "
" 15, "	7 41·7 "	7 31·1 "
October 1, "	6 38·9 "	6 28·2 "
" 15, "	5 43·9 "	5 33·2 "
November 1, "	4 37·0 "	4 26·4 "
" 15, "	3 41·9 "	3 31·3 "
December 1, "	2 38·9 "	2 28·2 "
" 15, "	1 43·6 "	1 33·0 "
January 1, 1886......	12 35·0 "	12 24·3 "

For any other days than those given in the table, interpolate directly, or subtract 3·94 minutes for every day elapsed. For any other year add 0·35 minute for every year. Also add one minute

* To calculate the times of the greatest elongation of the north star: Find in the "American Ephemeris and Nautical Almanac" its polar distance at the given time. Add the logarithm of its tangent to the logarithm of the tangent of the latitude of the place, and the sum will be the logarithm of the cosine of the hour angle before or after the culmination. Reduce the space to time; correct for sidereal acceleration (3m. 56s. for 24 hours) and subtract the result from the time of the star's passing the meridian on that day, to get the time of the eastern elongation, or add it to get the western.

if the year is the second after leap-year; add two minutes if it is the third after leap-year; add three minutes if it is leap-year before March 1st, and subtract one minute if it is leap-year after March 1st.

For any other latitude than 40° north (between 20° and 50°) add 0·14 minute for each degree of latitude south of 40°, or subtract 0·18 minute for each degree of latitude north of 40°.

266. Observations. Knowing from the preceding table the hour and minute of the extreme elongation on any day, a little before that time suspend a plumb-line, precisely as in Art. 263, and place yourself south of it as there directed. As the north star moves one way, move your eye the other, so that the plumb-line shall continually seem to cover the star. At last the star will appear to stop moving for a time, and then begin to move backward. Fix the sight on the board (or the compass, etc.) in the position in which it was when the star ceased moving; for the star was then at its extreme apparent elongation, east or west, as the case may be.

The eastern elongations from October to March, and the western elongations from April to September, occurring in the daytime, they will generally not be visible except with the aid of a powerful telescope.

267. Azimuths. The angle which the line from the eye to the plumb-line makes with the true meridian—i. e., the angle between the meridian plane and the vertical plane passing through the eye and the star—is called the *Azimuth* of the star. It is given in the following table for different latitudes, and for a number of years to come. For the intermediate latitudes it can be obtained by a simple proportion, similar to that explained in detail in Art. 264.*

---

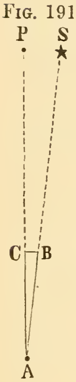
* To calculate this azimuth: From the logarithm of the sine of the polar distance of the star, subtract the logarithm of the cosine of the latitude of the place; the remainder will be the logarithm of the sine of the angle required. The polar distance can be obtained as directed in the last note.

AZIMUTHS (BEARINGS) OF THE NORTH STAR (POLARIS) AT EASTERN AND WESTERN ELONGATIONS.

LAT.	1885.	1886.	1887.	1888.	1889.	1890.	1891.	1892.	1893.	1894.	1895.	1896.	1897.	1900.	1903.	1906.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
20°	1 23·3	1 23·0	1 22·6	1 22·3	1 21·9	1 21·6	1 21·3	1 21·0	1 20·6	1 20·3	1 19·9	1 19·6	1 19·3	1 18·3	1 17·3	1 16·3
21°	23·8	23·5	23·2	22·8	22·5	22·1	21·8	21·5	21·2	20·8	20·5	20·1	19·8	18·8	17·8	16·8
22°	24·4	24·1	23·7	23·4	23·0	22·7	22·4	22·0	21·7	21·4	21·0	20·6	20·3	19·3	18·3	17·3
23°	25·0	24·7	24·3	24·0	23·7	23·3	23·0	22·6	22·3	22·0	21·6	21·2	20·9	19·9	18·9	17·9
24°	25·7	25·3	25·0	24·6	24·3	23·9	23·6	23·3	22·9	22·6	22·2	21·8	21·5	20·5	19·5	18·5
25°	26·4	26·0	25·7	25·3	25·0	24·6	24·3	23·9	23·6	23·2	22·9	22·5	22·2	21·1	20·1	19·1
26°	27·1	26·7	26·4	26·0	25·7	25·3	25·0	24·6	24·3	23·9	23·6	23·2	22·9	21·8	20·8	19·7
27°	27·8	27·5	27·1	26·8	26·4	26·0	25·7	25·4	25·1	24·6	24·3	23·9	23·6	22·5	21·5	20·4
28°	28·6	28·3	27·9	27·6	27·2	26·8	26·5	26·2	25·8	25·4	25·1	24·7	24·4	23·3	22·2	21·2
29°	29·5	29·1	28·8	28·4	28·0	27·6	27·3	27·0	26·6	26·2	25·9	25·5	25·2	24·1	23·0	22·0
30°	1 30·4	1 30·0	1 29·6	1 29·3	1 28·9	1 28·5	1 28·2	1 27·8	1 27·5	1 27·1	1 26·8	1 26·4	1 26·0	1 24·9	1 23·8	1 22·8
31°	31·3	30·9	30·5	30·2	29·8	29·4	29·1	28·8	28·4	28·0	27·6	27·2	26·9	25·8	24·7	23·6
32°	32·3	31·9	31·5	31·2	30·8	30·4	30·1	29·7	29·3	29·0	28·6	28·2	27·8	26·7	25·6	24·5
33°	33·3	33·0	32·6	32·2	31·8	31·4	31·1	30·7	30·3	30·0	29·6	29·2	28·8	27·7	26·6	25·5
34°	34·4	34·0	33·6	33·3	32·9	32·5	32·2	31·8	31·4	31·0	30·6	30·2	29·9	28·7	27·6	26·5
35°	35·5	35·2	34·8	34·4	34·0	33·6	33·3	32·9	32·5	32·1	31·7	31·3	31·0	29·8	28·7	27·6
36°	36·7	36·4	36·0	35·6	35·2	34·8	34·4	34·0	33·6	33·2	32·9	32·5	32·1	30·9	29·8	28·7
37°	38·0	37·6	37·2	36·8	36·4	36·0	35·6	35·2	34·8	34·4	34·1	33·7	33·3	32·1	31·0	29·8
38°	39·3	38·9	38·5	38·1	37·7	37·3	36·9	36·5	36·1	35·7	35·3	34·9	34·6	33·4	32·2	31·0
39°	40·7	40·3	39·9	39·5	39·1	38·7	38·3	37·9	37·5	37·1	36·7	36·3	35·9	34·7	33·5	32·3
40°	1 42·2	1 41·8	1 41·4	1 40·9	1 40·5	1 40·1	1 39·7	1 39·3	1 38·9	1 38·5	1 38·1	1 37·7	1 37·3	1 36·1	1 34·8	1 33·6
41°	43·7	43·3	42·9	42·4	42·0	41·6	41·2	40·8	40·4	40·0	39·6	39·2	38·8	37·5	36·2	35·0
42°	45·3	44·9	44·5	44·0	43·6	43·2	42·8	42·4	42·0	41·5	41·1	40·7	40·3	39·0	37·7	36·5
43°	47·0	46·6	46·1	45·7	45·3	44·9	44·5	44·0	43·6	43·2	42·7	42·3	41·9	40·6	39·3	38·0
44°	48·8	48·4	47·9	47·5	47·1	46·6	46·2	45·8	45·3	44·9	44·4	44·0	43·6	42·3	41·0	39·7
45°	50·7	50·3	49·8	49·4	48·9	48·5	48·0	47·6	47·1	46·7	46·2	45·8	45·4	44·1	42·7	41·4
46°	52·7	52·2	51·8	51·3	50·9	50·4	49·9	49·5	49·0	48·6	48·2	47·7	47·2	45·9	44·5	43·2
47°	54·8	54·3	53·8	53·4	52·9	52·5	52·0	51·5	51·0	50·6	50·2	49·7	49·2	47·8	46·5	45·1
48°	57·0	56·5	56·0	55·6	55·1	54·6	54·2	53·7	53·2	52·7	52·3	51·9	51·4	49·9	48·6	47·1
49°	59·3	58·8	58·3	57·9	57·4	56·9	56·5	56·0	55·5	55·0	54·5	54·1	53·6	52·1	50·8	49·2
50°	2 01·8	2 01·3	2 00·8	2 00·3	1 59·8	1 59·3	1 58·9	1 58·4	1 57·9	1 57·4	1 56·9	1 56·4	1 55·9	1 54·4	1 53·0	1 51·5

268. Setting out a Meridian. When two points in the direction of the north star at its extreme elongation have been obtained, as in Art. 266, the true meridian can be found thus : Let A and B be the two points. Multiply the natural tangent of the azimuth given in the table by the distance A B. The product will be the length of a line which is to be set off from B, perpendicular to A B, to some point C. A and C will then be points in the true meridian. This operation may be postponed till morning.

Fig. 191.



If the directions of both the extreme eastern and extreme western elongations be set out, the line lying midway between them will be the true meridian.

269. Determining the Declination. The declination would, of course, be given by taking the bearing of the meridian thus obtained, but it can also be determined by taking the bearing of the star at the time of the extreme elongation, and applying the following rules :

When the azimuth of the star and its magnetic bearing are one east and the other west, the sum of the two is the magnetic declination, which is of the same name as the azimuth—i. e., east, if that be east, and west, if it be west.

When the azimuth of the star and its magnetic bearing are both east or both west, their difference is the declination, which will be of the same name as the azimuth and bearing, if the azimuth be the greater of the two, or of the contrary name if the azimuth be the smaller.

Fig. 192.

All these cases are presented together in the figure, in which P is the north pole, Z the place of the observer, Z P the true meridian, S the star at its greatest eastern elongation, and Z N, Z N′, Z N″ various supposed directions of the needle.

Call the azimuth of the star—i. e., the angle P Z S—2° east.

Suppose the needle to point to N, and the bearing of the star—i. e., S Z N—to be 5° west of magnetic north. The declination P Z N will evidently be 7° east of true north.

Suppose the needle to point to N′, and the bearing of the star —i. e., N′ Z S—to be $1\frac{1}{4}°$ east of magnetic north. The declination will be $\frac{3}{4}°$ east of true north, and of the same name as the azimuth, because that is greater than the bearing.

Suppose the needle to point to N″, and the bearing of the star —i. e., N″ Z S—to be 10° east of magnetic north. The declination will be 8° west of true north, of the contrary name to the azimuth, because that is the smaller of the two.*

If the star were on the other side of the pole, the rules would apply likewise.

270. Other Methods. Many other methods of determining the true meridian are employed; such as by equal altitudes and azimuths of the sun, or of a star; by one azimuth, knowing the time; by observations of circumpolar stars at equal times before and after their culmination, or before and after their greatest elongation, etc.

All these methods, however, require some degree of astronomical knowledge; and those which have been explained are abundantly sufficient for all the purposes of the ordinary land-surveyor.

"Burt's Solar Compass" is an instrument by which, "when adjusted for the sun's declination and the latitude of the place, the azimuth of any line from the true north and south can be read off, and the difference between it and the bearing by the compass will then be the variation."

271. Magnetic Declination in the United States. The declination in any part of the United States can be approximately obtained by mere inspection of the map at the beginning of this volume.† Through all the places at which the needle, in 1885, pointed to the true north, a line is drawn on the map, and called

* Algebraically, always subtract the bearing from the azimuth, and give the remainder its proper resulting algebraic sign. It will be the declination; east if *plus*, and west if *minus*. Thus, in the first case above, the declination $= +2° - (-5°) = +7° = 7°$ east. In the second case, the declination $= +2° - (+1\frac{1}{4}°) = +\frac{3}{4}° = \frac{3}{4}°$ east. In the third case, the declination $= +2° - (+10°) = -8° = 8°$ west.

† Copied from "United States Coast and Geodetic Survey Report," 1882.

the *line of no declination.* It will be seen to pass a little east of Charleston, South Carolina, thence in a northwesterly direction, passing near Zanesville, Ohio, through the west end of Lake Erie, passing a little west of Detroit, and up through the east end of Lake Superior. This line is now slowly moving westward.

At all places situated to the east of this line (including the New England States, New York, New Jersey, Delaware, Maryland, Pennsylvania, most of Virginia, and the east half of North Carolina and Ohio) the declination is westerly—i. e., the north end of the needle points to the west of the true north. At all places situated to the west of this line (including the Western and Southern States) the declination is easterly—i. e., the north end of the needle points to the east of the true north. This declination increases in proportion to the distance of the place on either side of the line of no variation, reaching 23° of easterly declination in Washington Territory, and 21° of westerly declination in Maine.

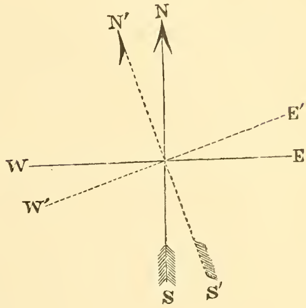
Isogonics, or *lines of equal declination,* are lines drawn through all the places which have the same declination. On the map they are drawn for each degree. All the places situated on the line marked 5°, east or west, have 5° declination ; those on the 10° line have 10° declination, etc. The declination at the intermediate places can be approximately estimated by the eye. These lines all refer to 1885.

The sign + indicates west declination, and the sign — indicates east declination. The annual change in the secular variation for stations is given in minutes and decimals, a + indicating increasing west declination or decreasing east declination, and a — sign indicating increasing east and decreasing west declination.

272. To correct Magnetic Bearings. The declination at any place and time being known, the magnetic bearings taken there and then may be reduced to their true bearings by these rules :

Rule 1. *When the declination is west,* as it is in the Northeastern States, the true bearing will be the *sum* of the declination, and a bearing which is north and west, or south and east ; and the *difference* of the declination and a bearing which is north and east, or south and west. To apply this to the cardinal points, a north bearing must be called N. 0° west, an east bearing N. 90° E., a south bearing S. 0° E., and a west bearing S. 90° W. ; counting around from N′ to N, in the figure, and so onward, "with the sun."

Fig. 193.

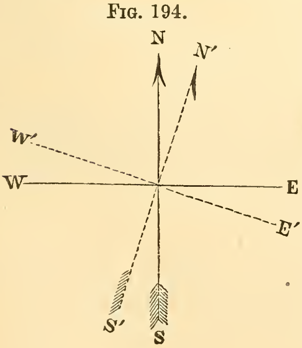


The reasons for these corrections are apparent from the figure, in which the dotted lines and the accented letters represent the direction of the needle, and the full lines and the unaccented letters represent the true north and south and east and west lines.

When the sum of the declination and the bearing is directed to be taken, and comes to more than 90°, the supplement of the sum is to be taken, and the first letter changed. When the difference is directed to be taken, and the declination is greater than the bearing, the last letter must be changed. A diagram of the case will remove all doubts. Examples of all these cases are given below for a declination of 8° west:

MAGNETIC BEARINGS.	TRUE BEARINGS.	MAGNETIC BEARINGS.	TRUE BEARINGS.
North.	N. 8° W.	South.	S. 8° E.
N. 1° E.	N. 7° W.	S. 2° W.	S. 6° E.
N. 40° E.	N. 32° E.	S. 60° W.	S. 52° W.
East.	N. 82° E.	West.	S. 82° W.
S. 50° E.	S. 58° E.	N. 70° W.	N. 78° W.
S. 89° E.	N. 83° E.	N. 83° W.	S. 89° W.

Fig. 194.



Rule 2. *When the declination is east*, as in the Western and Southern States, the preceding directions must be exactly reversed —i. e., the true bearing will be the *difference* of the declination, and a bearing which is north and west or south and east; and the *sum* of the declination and a bearing which is north and east or south and west. A north bearing

must be called N. 0° E., a west bearing N. 90° W., a south bearing S. 0° W., and an east bearing S. 90° E., counting from N′ to N, and so onward, "against the sun." The reasons for these rules are seen in the figure. Examples are given below for a declination of 5° E.:

MAGNETIC BEARINGS.	TRUE BEARINGS.	MAGNETIC BEARINGS.	TRUE BEARINGS.
North.	N. 5° E.	South.	S. 5° W.
N. 40° E.	N. 45° E.	S. 60° W.	S. 65° W.
N. 80° E.	S. 86° E.	S. 87° W.	N. 88° W.
East.	S. 85° E.	West.	N. 85° W.
S. 1° E.	S. 4° W.	N. 70° W.	N. 65° W.
S. 50° E.	S. 45° E.	N. 2° W.	N. 3° E.

273. To survey a Line with True Bearings. The compass may be set, or adjusted, by means of the vernier, according to the declination in any place, so that the bearings of any lines then taken with it will be their true bearings. To effect this, turn aside the compass-plate by means of the tangent-screw which moves the vernier a number of degrees equal to the declination, moving the south end of the compass-box to the *right* (the north end being supposed to go ahead) if the declination be westerly, and *vice versa;* for that moves the north end of the compass-box in the contrary direction, and thus makes a line which before was N. by the needle, now read, as it should truly, north, so many degrees west if the declination was west; and similarly in the reverse case.

Variations of Magnetic Declination.

274. The *variations* of the declination are of more practical importance than its absolute amount. They are of four kinds: Irregular, diurnal, annual, and secular.

275. Irregular Variation. The needle is subject to sudden and violent changes, which have no known law. They are sometimes coincident with a thunder-storm, or an aurora borealis (during which changes of nearly 1° in one minute, $2\frac{1}{2}$° in eight minutes, and 10° in one night, have been observed), but often have no apparent cause, except an otherwise invisible "magnetic storm."

276. The Diurnal Variation. On continuing observations of the direction of the needle throughout an entire day, it will be found, in the northern hemisphere, that the north end of the needle moves westward from about 8 A. M. till about $1\frac{1}{2}$ P. M., over an arc of from 5′ to 15′, and then gradually returns to its former position. A similar but smaller movement takes place during the night. At Philadelphia, the most easterly deflection of the needle is at about $7\frac{3}{4}$ A. M. The north end of the needle then begins to move toward the west, crossing the mean magnetic meridian about $10\frac{1}{2}$ A. M., and reaching its extreme western position about $1\frac{1}{2}$ P. M. The total angular range averages about 8′, being $10\frac{1}{2}$′ in August, and 6′ in November.* The period of this change being a day, it is called the *Diurnal Variation.* Its effect on the permanent variation is necessarily to cause it, in places where it is west, to attain its maximum at about $1\frac{1}{2}$ P. M., and its minimum at about 8 A. M.; and the reverse where the declination is east.

This diurnal variation adds a new element to the inaccuracies of the compass, since the bearings of any line taken on the same day, at a few hours' interval, might vary a quarter of a degree, which would cause a deviation of the end of the line, amounting to nearly half a link at the end of a chain, and to 35 links, or 23 feet, at the end of a mile. The hour of the day at which any important bearing is taken should therefore be noted.

277. The Annual Variation. If the observations be continued throughout an entire year, it will be found that the diurnal changes vary with the seasons, being greater in summer than in winter. The period of this variation being a year, it is called the *Annual Variation.*

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* For table of hourly variation of the declination, see "Report of United States Coast and Geodetic Survey," 1881, p. 136.

278. The Secular Variation. When accurate observations on the declination of the needle in the same place are continued for several years, it is found that there is a continual and tolerably regular increase or decrease of the declination, continuing to proceed in the same direction for so long a period, that it may be called the *Secular Variation* of the declination.

The most ancient observations are those taken in Paris. In the year 1541 the needle pointed 7° east of north ; in 1580 the declination had increased to 11½° east, being its maximum ; the needle then began to move westward, and in 1666 it had returned to the meridian ; the declination then became west, and continued to increase till in 1814 it attained its maximum, being 22° 34′ west of north. It is now decreasing, and, January 1, 1879, it was 16° 56′ west.

In this country the north end of the needle was moving eastward at the earliest recorded observations, and continued to do so till about the year 1810 (variously recorded as from 1765 to 1819), when it began to move westward, which it has ever since continued to do. Thus, in Boston, from 1700 to 1807, the declination changed from 10° west to 6° 5′ west, and, from 1807 to 1879, it changed from 6° 5′ west to 11° 36′ west.

In Philadelphia, from 1701 to 1802, the declination changed from 8° 30′ west to 1° 30′ west, and, from 1802 to 1877, it changed from 1° 30′ west to 6° 2′ west.

Extensive tables of the declination, at more than two thousand stations, in various parts of the United States, are given in the "Report of the United States Coast and Geodetic Survey," 1882, Appendix XIII, by Charles A. Schott. The secular variation is noted on the declination-map in this volume.

An examination of the above-mentioned tables will show that the secular variation often differs greatly in places not far apart, and that it varies in amount at the same place from year to year :

TABLE OF COMPUTED ANNUAL CHANGES IN DECLINATION.

LOCALITY.	ANNUAL CHANGE.		
	1870.	1880.	1885.
Portland, Me	+2·4′	+1·6′	+1·2′
Burlington, Vt	+5·0	+6·0	+5·8
Portsmouth, N. H	+4·4	+3·7	+3·3
Boston, Mass	+3·4	+2·9	+2·5
Hartford, Conn	+3·8	+3·7	+3·6
Albany, N. Y	+4·3	+3·7	+3·4
New York, N. Y	+2·4	+2·5	+2·6
Buffalo, N. Y	+5·1	+5·0	+4·8
Philadelphia, Pa	+4·9	+4·9	+5·3
Baltimore, Md	+3·9	+3·6	+3·2
Washington, D. C	+3·5	+3·2	+3·0
Cleveland, Ohio	+2·8	+2·5	+2·2
Detroit, Mich	+3·4	+3·0	+2·8
St. Louis, Mo	+3·4	+3·2	+3·0
Cape Henry, Va	+3·8	+3·7	+3·6
Charleston, S. C	+3·5	+3·0	+2·7
Savannah, Ga	+3·6	+3·5	+3·3
Key West, Fla	+4·3	+4·2	+4·1
Mobile, Ala	+2·8	+3·4	+3·7
New Orleans, La	+3·1	+3·5	+3·7
San Francisco, Cal	−1·0	−0·5	−0·3
Cape Disappointment, W. Ter.	−3·4	−3·1	−2·7
Sitka, Alaska	+1·0	+2·1	+2·5

279. Determination of the Change, by Interpolation. To determine the change at any place and for any interval not found in the recorded observations, an approximation, sufficient for most purposes of the surveyor, may be obtained by interpolation (by a simple proportion) between the places given on the map, assuming the movements to have been uniform between the given dates, and also assuming the change at any place not found on the map to have been intermediate between those of the lines of equal variation, which pass through the places of recorded observations on each side of it, and to have been in the ratio of its respective distances from those two lines ; for example, taking their arithmetical mean, if the required place is midway between them ; if it be twice as near one as the other, dividing the sum of twice the change of the nearest line, and once the change of the other, by three ; and so in other cases—i. e., giving the change at each place, a "weight" inversely as its distance from the place at which the change is to be found.

280. Determination of the Change by Old Lines. When the former bearing of any old line, such as a farm-fence, etc., is recorded, the change in the declination from the date of the original observation to the present time can be at once found by setting the compass at one end of the line and sighting to the other. The difference of the two bearings is the required change.

If one end of the old line can not be seen from the other, as is often the case when the line is fixed only by a "corner" at each end of it, proceed thus: Run a line from one corner with the old bearing and with its distance. Measure the distance from the end of this line to the other corner, to which it will be opposite. Multiply this distance by 57·3, and divide by the length of the line. The quotient will be the change of variation in degrees.*

For example, a line 63 chains long, in 1827 had a bearing of north 1° east. In 1847 a trial line was run from one end of the former line with the same bearing and distance, and its other end was found to be 125 links to the west of the true corner. The change of declination was therefore $\frac{1{\cdot}25 \times 57{\cdot}3}{63} = 1{\cdot}137° = 1°\ 8'$ westerly.

281. Effects of the Secular Change. These are exceedingly important in the resurvey of farms by the bearings recorded in old deeds. Let S N denote the direction of the needle at the time of the original survey, and S′ N′ its direction at the time of the resurvey, a number of years later. Suppose the change to have been

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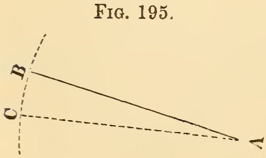
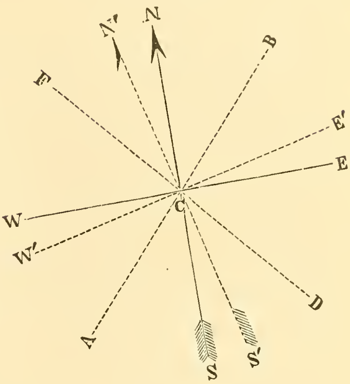
FIG. 195.



* Let A B be the original line; A C the trial line, and B C the distance between their extremities. A B and A C may be regarded as radii of a circle and B C as a chord of the arc which subtends their angle. Assuming the chord and arc to coincide (which they will, nearly, for small angles), we have this proportion: Whole circumference : arc B C : : 360° : B A C : or, 2 × A C × 3·1416 : B C : 360° : B A C, whence $BAC = \frac{BC}{AB} \times 57{\cdot}3$; or, more precisely, 57·29578.

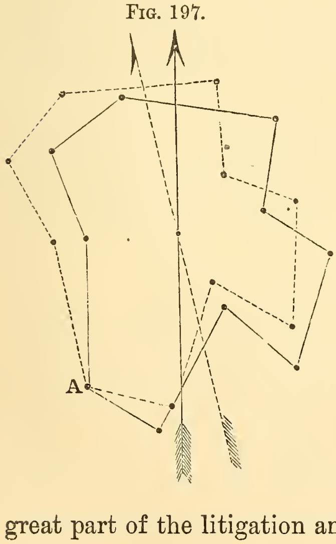
Fig. 196.



3°, the needle pointing so much farther to the west of north. The line S N, which before was due north and south by the needle, will now bear N. 3° E. and S. 3° W. ; the line A B, which before was N. 40° E. will now bear N. 43° E. ; the line D F, which before was N. 40° W., will now bear N. 37° W., and the line W E, which before was due east and west, will now bear S. 87° E. and N. 87° W. Any line is similarly changed. The proof of this is apparent on inspecting the figure.

Suppose, now, that a surveyor, ignorant or neglectful of this change, should attempt to run out a farm by the old bearings of the deed, none of the old fences or corners remaining. The full lines in the figure represent the original bounds of the farm, and the dotted lines those of the *new* piece of land which, starting from A, he would unwittingly run out. It would be of the same size and the same shape as the true one, but it would be in the wrong place. None of its lines would agree with the true ones, and in some places it would encroach on one neighbor, and in other places would leave a gore, which belongs to it, between itself and another neighbor. Yet this is often done, and is the source of a great part of the litigation among farmers respecting their "lines."

Fig. 197.



282. To run out Old Lines. To succeed in retracing old lines, proper allowance must be made for the change in the variation since the date of the original survey. That date must first be accurately ascertained; for the survey may be much older than the deed, into which its bearings may have been copied from an older one. The amount and direction of the change is then to be ascertained by the methods of Art. 279 or 280. The bearings may then be corrected by the following RULES :

When the north end of the needle has been moving westerly, the present bearings will be the *sums* of the change and the old bearings which were northeasterly or southwesterly, and the *differences* of the change and the old bearings which were northwesterly or southeasterly.

If the change has been *easterly*, reverse the preceding rules, subtracting where it is directed to add, and adding where it is directed to subtract.

Run out the lines with the bearings thus corrected.

It will be noticed that the process is precisely the reverse of that in Art. 272. The rules, there given in more detail, may therefore be used : RULE 1. "When the declination is west," being employed when the *change* has been a movement of the N. end of the needle to the east; and RULE 2, "when the declination is east," being employed when the N. end of the needle has been moving to the west.

If the compass has a vernier, it can be set for the change, once for all, precisely as directed in Art. 273, and then the courses can be run out as given in the deed, the correction being made by the instrument.

Example. The following is a remarkable case which came before the Supreme Court of New York: The north line of a large estate was fixed by a royal grant, dated in 1704, as a due east and west line. It was run out in 1715, by a surveyor, whom we will call Mr. A. It was again surveyed in 1765, by Mr. B., who ran a course N. 87° 30′ E. It was run out for a third time in 1789, by Mr. C., who adopted the course N. 86° 18′ E. In 1845 it was surveyed for the fourth time by Mr. D., with a course of N. 88° 30′ E. He found old "corners," and "blazes" of a former survey, on his line. They are also found on another line, south of his. Which of the preceding courses were correct, and where does the true line lie ?

The question was investigated as follows : There were no old records of variation at the precise locality, but it lies between the lines of equal variation which pass through New York and Boston, its distance from the Boston line being about twice its distance from the New York line. The records of those two cities (referred to in Art. 278) could therefore be used in the manner explained in Art. 279. For the later dates, observations at New Haven could serve as a check. Combining all these, the author inferred the variation at the desired place to have been as follows :

In 1715,	variation	8° 02′ west.	
In 1765,	"	5° 32′ "	Decrease since 1715, 2° 30′.
In 1789,	"	5° 05′ "	Decrease since 1765, 0° 27′.
In 1845,	"	7° 23′ "	Increase since 1789, 2° 18′.

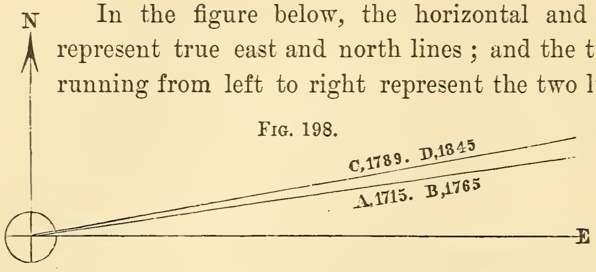
We are now prepared to examine the correctness of the allowances made by the old surveyors.

The course run by Mr. B. in 1765, N. 87° 30′ E., made an allowance of 2° 30′ as the decrease of variation, agreeing precisely with our calculation. The course of Mr. C. in 1789, N. 86° 18′ E., allowed a change of 1° 12′, which was wrong by our calculation, which gives only about 27′, and was deduced from three different records. Mr. D., in 1845, ran a course of N. 88° 30′ E., calling the increase of variation since 1789, 2° 12′. Our estimate was 2° 18′, the difference being comparatively small. Our conclusion, then, is this : The second surveyor retraced correctly the line of the first ; the third surveyor ran out a *new* and incorrect line ; and the fourth surveyor correctly retraced the line of the third, and found his marks, but this line was wrong originally, and therefore wrong now. All the surveyors ran their lines on the supposition that the original "due east and west line" meant east and west as the needle pointed at the time of the original survey.

The preponderance of the testimony as to old landmarks agreed with the results of the above reasoning, and the decision of the court was in accordance therewith.

In the figure below, the horizontal and vertical lines represent true east and north lines; and the two upper lines running from left to right represent the two lines set out by the surveyors, and in the years there named.

Fig. 198.



283. Remedy for the Evils of the Secular Change. The only complete remedy for the disputes, and the uncertainty of bounds, resulting from the continued change in the declination, is this: Let a meridian—i. e., a true north and south line—be established in every town or county, by the authority of the State; monuments, such as stones, set deep in the ground, being placed at each end of it. Let every surveyor be obliged by law to test his compass by this line, at least once in each year, at a given hour in the day. This he could do as easily as in taking the bearing of a fence, by setting his instrument on one monument, and sighting to a staff held on the other. Let the variation thus ascertained be inserted in the notes of the survey, and recorded in the deed. Another surveyor, years or centuries afterward, could test his compass by taking the bearing of the same monuments, and the difference between this and the former bearing would be the change of declination. He could thus determine with entire certainty the proper allowance to be made (as in Art. 282) in order to retrace the original line, no matter how much, or how irregularly, the declination may have changed, or how badly adjusted was the compass of the original survey. Any permanent line employed in the same manner as the meridian line would answer the same purpose, though less conveniently, and every surveyor should have such a line, at least for his own use.*

* This remedy seems to have been first suggested by Rittenhouse. It has since been recommended by T. Sopwith, in 1822; by E. F. Johnson, in 1831, and by W. Roberts, of Troy, in 1839. The errors of resurveys, in which the change is neglected, were noticed in the "Philosophical Transactions," as long ago as 1679. On magnetic declination, see the following "Reports of the United States Coast and Geodetic Survey"; Report of 1881, Appendix IX; Report of 1882, Appendix XII.

CHAPTER IV.

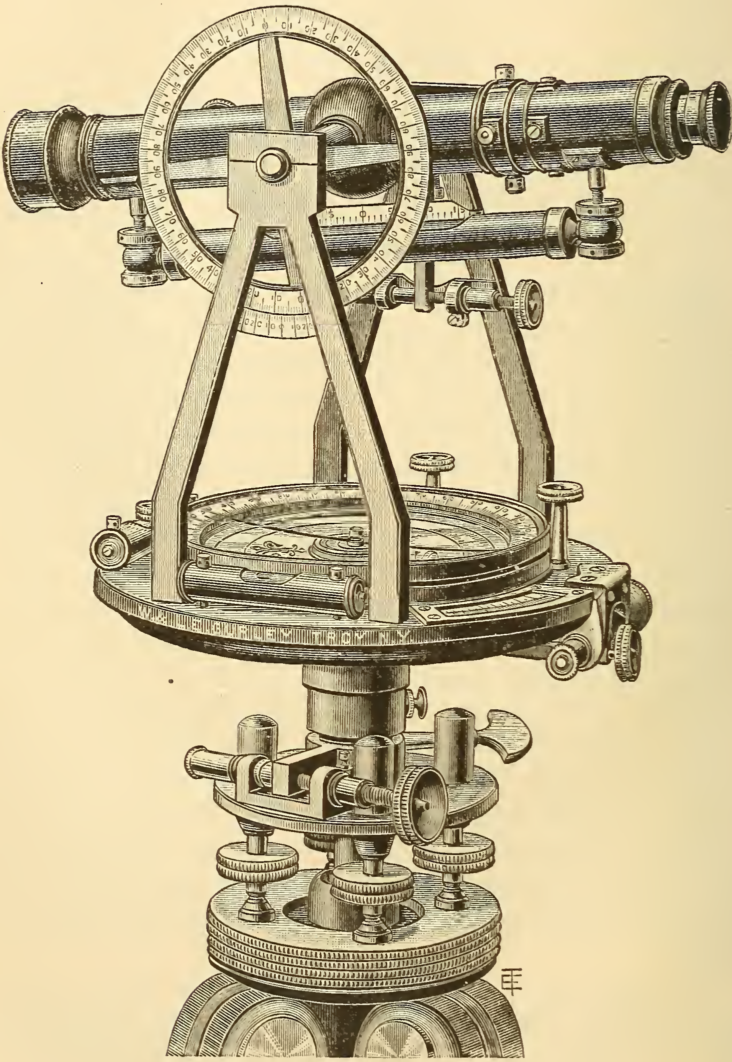
TRANSIT-SURVEYING—BY THE THIRD METHOD.

THE INSTRUMENTS.

284. THE TRANSIT is a *Goniometer*, or Angle-Measurer. It consists, essentially, of a circular plate of metal, supported in such a manner as to be horizontal, and divided on its outer circumference into degrees and parts of degrees. Through the center of this plate passes an upright axis, and on it is fixed a second circular plate, which nearly touches the first plate, and can turn freely around to the right and to the left. This second plate carries a telescope, which rests on upright standards firmly fixed to the plate, and which can be pointed upward and downward. By the combination of this motion and that of the second plate around its axis, the telescope can be directed to any object. The second plate has some mark on its edge, such as an arrow-head, which serves as a pointer or index for the divided circle, like the hand of a clock. When the telescope is directed to one object, and then turned to the right or to the left, to some other object, this index which moves with it, and passes around the divided edge of the other plate, points out the arc passed over by this change of direction, and thus measures the horizontal angle made by the lines imagined to pass from the center of the instrument to the two objects.

The great value of this instrument, and the accuracy of its measurements of angles, are due chiefly to two things: to the *telescope* with its cross-hairs, by which great precision in sighting to a point is obtained; and to the *vernier scale,* which enables minute portions of any arc to be read with ease and correctness. The former assists the eye in directing the line of sight, and the latter aids it in reading off the results. Arrangements for giving slow

FIG. 199.



and steady motion to the movable parts of the instrument add to the value of the above. A contrivance for *repeating* the observation of angles still further lessens the unavoidable inaccuracies of these observations.

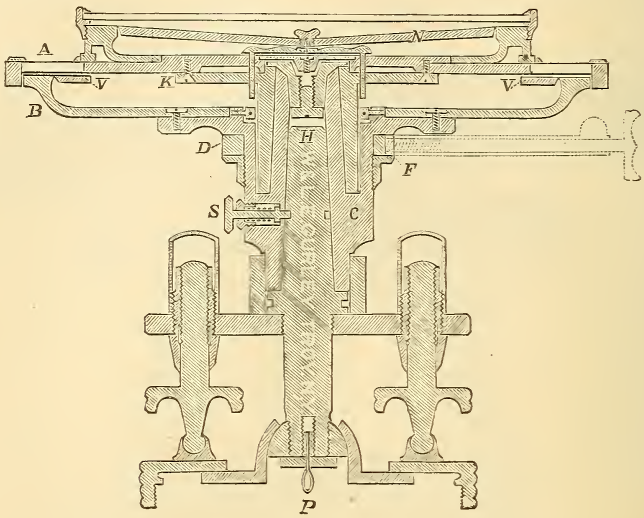
285. The Surveyor's Transit (Fig. 199). In this instrument the telescope takes the place of the plain sights of the surveyor's compass, and the angles are read on the graduated limb to single minutes by the vernier.

A level is attached to the telescope, and a vertical circle is attached to the telescope-axis inside of the left-hand standard. The vertical angles through which the telescope is moved may be read off from the vernier attached to the left-hand standard, and shown below the vertical circle. The slow-motion screw for the vertical circle is shown attached to the right-hand standard. The clamp for the axis is hidden by the telescope. The standards upon which the telescope-axis rests are fastened to the upper plate (the vernier-plate). This plate also carries the compass-circle. The compass-circle with its accessories is similar to that already explained in the Surveyor's Compass. The compass-circle can be turned on its center, so that the declination of the needle can be set off, and lines can be run with their true bearings. The vernier-plate covers the lower plate (the divided limb), so that only two short arcs of the divided limb are seen through openings where the verniers are placed. The screw which clamps the vernier-plate to the divided limb is shown on the right of the plate, together with the slow-motion screw. The lower clamp and the slow-motion screw are attached to the upper parallel plate.

286. As the value of this instrument depends greatly on the accurate fitting and bearings of the two concentric vertical axes, and as their connection ought to be thoroughly understood, a vertical section through the body of the instrument is given in Fig. 200.

The upper plate, or vernier-plate, A, A, carries the verniers, compass-box, and telescope. It is attached to its socket by the flange, K. This socket is fitted to the outside, conical surface of the main socket, C. The main socket, to which is attached the divided limb, B, B, is fitted to the conical spindle H, and held on the spindle by the spring-catch S. A screw holds the conical center, whose upper flange keeps the sockets of the two plates

Fig. 200.



together. The clamp is at F. Two of the four leveling screws are shown in section. The spindle, H, passes through the upper parallel plate, and is attached to a movable section of the lower parallel plate by a ball-and-socket joint. The leveling screws pass through the upper parallel plate, and rest in cups on the lower parallel plate. As the leveling screws are movable on the lower parallel plate, the movable section of this plate enables the upper part of the instrument to be moved from side to side, so as to bring the center of the instrument precisely over any desired point. This arrangement is called a "shifting center." At the lower end of the spindle is a loop, P, from which the plumb-bob is suspended.

287. The Telescope. This is a combination of lenses, placed in a tube, and so arranged, in accordance with the laws of optical science, that an image of any object to which the telescope may be directed, is formed within the tube (by the rays of light coming from the object and bent in passing through the object-glass), and there magnified by an eye-glass, or eye-piece, composed of several lenses. The arrangement of these lenses is very various. Those two combinations, which are preferred for surveying instruments, will be here explained :

Fig. 201.

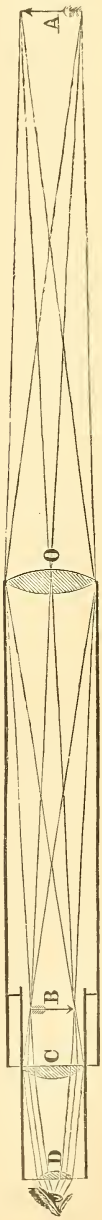


Fig. 201 represents a telescope which inverts objects. Any object is rendered visible by every point of it sending forth rays of light in every direction. In this figure the highest and lowest points of the object, which here is an arrow, A, are alone considered. Those of the rays proceeding from them, which meet the object-glass, O, form a cone. The center line of each cone, and its extreme upper and lower lines, are alone shown in the figure. It will be seen that these rays, after passing through the object-glass, are refracted or bent by it, so as to cross one another, and thus to form at B an inverted image of the object. This would be rendered visible, if a piece of ground-glass, or other semi-transparent substance, were placed at the point B, which is called the *focus* of the object-glass. The rays which form this image continue onward and pass through the two lenses C and D, which act like one magnifying-glass, so that the rays, after being refracted by them, enter the eye at such angles as to form there a magnified and inverted image of the object. This combination of the two plano-convex-lenses, C and D, is known as "Ramsden's Eye-piece."

This telescope, inverting objects, shows them upside down, and the right side on the left. They can be shown erect by adding one or two more lenses, as in the marginal figure. But as these lenses absorb light and lessen the distinctness of vision, the former arrangement is sometimes preferred. A little practice makes it equally convenient for the observer, who soon becomes accustomed to seeing his flagmen standing on their heads, and soon learns to motion them to the right when he wishes them to go to the left, and *vice versa*.

Fig. 202.

Fig. 202 represents a telescope which shows objects erect. Its eye-piece has four lenses. The eye-piece of the common terrestrial telescope, or spy-glass, has three. Many other combinations may be used, all intended to show the object achromatically, or free from false coloring, but the one here shown is that most generally preferred at the present day. It will be seen that an inverted image of the object A is formed at B, as before, but that the rays continuing onward are so refracted in passing through the lens C as to again cross, and thus, after further refraction by the lenses D and E, to form, at F, an erect image, which is magnified by the lens G.

In both these figures, the limits of the page render it necessary to draw the angles of the rays very much out of proportion.

288. Cross-Hairs. Since a considerable field of view is seen in looking through the telescope, it is necessary to provide means for directing the line of sight to the precise point which is to be observed. This could be effected by placing a very fine point, such as that of a needle, within the telescope, at some place where it could be distinctly seen. In practice, this fine point is obtained by the intersection of two very fine lines, placed in the common *focus* of the object-glass and of the eye-piece. These lines are called the *cross-hairs*, or *cross-wires*. Their intersection can be seen through the eye-piece, at the same time, and apparently at the same place, as the image of the distant object. The magnifying powers of the eye-piece will then detect the slightest deviation from perfect coincidence. "This application of the telescope may be considered as completely annihilating that part of the error of observation which might otherwise arise from an erroneous estimation of the direction in which an object lies from the observer's eye, or from the center of the instrument. It is, in fact, the grand source of all the precision of modern astronomy, without which all other refinements in instrumental workmanship would be thrown away." What Sir John Herschel here says of its utility to astronomy is equally applicable to surveying.

The imaginary line which passes through the intersection of the cross-hairs and the optical center of the object-glass is called the *line of collimation* of the telescope.*

The cross-hairs are attached to a ring, or short, thick tube of brass, placed within the telescope - tube, through holes in which pass loosely four screws, whose threads enter and take hold of the ring, behind or in front of the cross-hairs, as shown (in front view and in section) in the two figures in the margin. Their movements will be explained in "ADJUSTMENTS."

FIG. 203.

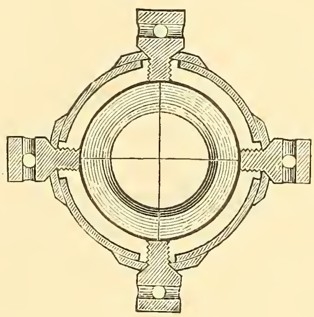
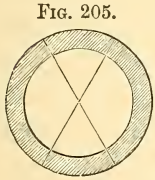


FIG. 204.

Usually, one cross-hair is horizontal, and the other vertical, as in Fig. 203, but sometimes they are arranged as in Fig. 205, which is thought to enable the object to be bisected with more precision. A horizontal hair is sometimes added.

FIG. 205.

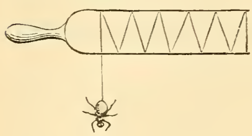


The cross-hairs are best made of platinum wire, drawn out very fine by being previously inclosed in a larger wire of silver, and the silver then removed by nitric acid. Silk threads from a cocoon are sometimes used. Spiders' threads are, however, the most usual. If a cross-hair is broken, the ring must be taken out by removing two opposite screws, and inserting a wire with a screw cut on its end, or a stick of suitable size, into one of the holes thus left open

* From the Latin word *collimo*, or *collineo*, meaning to direct one thing toward another in a straight line, or to aim at. The *line of aim* would express the meaning.

in the ring, it being turned sidewise for that purpose, and then removing the other screws. The spiders' threads are then stretched across the notches seen in the end of the ring, and are fastened by gum, or varnish, or beeswax. The operation is a very delicate one.

Fig. 206.



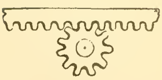
The following plan has been employed : A piece of wire is bent, as in the figure, so as to leave an opening a little wider than the ring of the cross-hairs. A cobweb is chosen, at the end of which a spider is hanging, and it is wound around the bent wire, as in the figure, the weight of the insect keeping it tight and stretching it ready for use, each part being made fast by gum, etc. When a cross-hair is wanted, one of these is laid across the ring and there attached. One method is to draw the thread out of the spider, persuading him to spin, if he sulks, by tossing him from hand to hand. Another method is to unwind the spider-web from the cocoons, frequently to be found in spider-webs. A stock of such threads must be obtained in warm weather for the winter's wants. A piece of thin glass, with a horizontal and a vertical line etched on it, may be made a substitute.

289. Instrumental Parallax. This is an apparent movement of the cross-hairs about the object to which the line of sight is directed, taking place on any slight movement of the eye of the observer. It is caused by the image and the cross-hairs not being precisely in the common focus, or point of distinct vision of the eye-piece and the object-glass. To correct it, move the eye-piece out or in till the cross-hairs are seen clearly and sharply defined against any white object. Then move the object-glass in or out till the object is also distinctly seen. The cross-hairs will then seem to be fixed to the object, and no movement of the eye will cause them to appear to change their place.

290. A milled-headed screw (on the farther side of the telescope, and not shown in the figure) passes into the telescope, and has a pinion at its other end entering a toothed rack (Fig.

207), and is used to move the object-glass, O, out and in, according as the object looked at is nearer or farther than the one last observed. Short distances require a long tube; long distances a short tube.

FIG. 207.



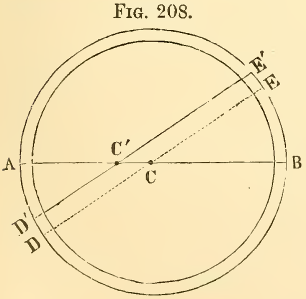
The eye-piece is moved in and out by a similar arrangement to the preceding. This movement is necessary in order to obtain a distinct view of the cross-hairs. Short-sighted persons require the eye-piece to be pushed farther in than persons of ordinary sight, and old or long-sighted persons to have it drawn farther out.

291. Supports. The telescope of the *transit* is supported by a hollow axis at right angles to it, which itself rests, at each end, on two upright pieces, or standards, spreading at their bases so as to increase their stability.

One end of the axis rests upon a movable block, which can be raised or lowered by a capstan-screw. The use of this will be shown in "ADJUSTMENTS."

292. The Indexes. The supports, or standards, of the telescope just described are attached to the upper or index-carrying circle. This, as has been stated, can turn freely on the lower or graduated circle, by means of its conical axis moving in the hollow conical axis of the latter circle. This upper circle carries the index, V, which is an arrow-head or other mark on its edge, or the zero-point of a vernier scale. There are usually two of these, situated exactly opposite to each other, or at the extremities of a diameter of the upper circle, so that the readings on the graduated circle pointed out by them differ, if both are correct, exactly 180°. The object of this arrangement is to correct any error of *eccentricity*, arising from the center of the axis which carries the upper circle (and with which it and its index-pointers

FIG. 208.



turn), not being precisely in the center of the graduated circle. In the figure, let C be the true center of the graduated circle, but C′ the center on which the plate carrying the indexes turns. Let A C′ B represent the direction of a sight taken to one object, and D′ C′ E′ the direction when turned to a second object. The angle subtended by the two objects at the center of the instrument is required. Let D E be a line passing through C, and parallel to D′ E′. The angle A C D equals the required angle, which is therefore truly measured by the arc A D or B E. But if the arc shown by the index is read, it will be A D′ on one side, and B E′ on the other; the first being too small by the arc D D′, and the other too large by the equal arc E E′. If, however, the half-sum of the two arcs A D′ and B E′ be taken, it will equal the true arc, and therefore correctly measure the angle. Thus, if A D′ was 19°, and B E′ 21°, their half-sum, 20°, would be the correct angle.

Three indexes, 120° apart, are sometimes used. They have the advantage of *averaging* the unavoidable inaccuracies and inequalities of graduation on different parts of the limb, and thus diminishing their effect on the resulting angle.

293. The Graduated Circle. This is divided into three hundred and sixty equal parts, or degrees, and each of these is subdivided into two or three parts or more, according to the size of the instrument. In the first case, the smallest division on the circle will of course be 30′; in the second case 20′. More precise reading, to single minutes or even less, is effected by means of the vernier of the index, all the varieties of which will be fully explained under "VERNIERS." The numbers run from 0° around to 360°, which number is necessarily at the same point as the 0, or *zero-point*. In most instruments there is another concentric circle, on which the degrees are also numbered from 0° to 90°, as on the compass-circle. Each tenth degree is usually numbered, each fifth degree is distinguished by a longer line of division, and each degree-division line is longer than those of the subdivisions. A magnifying-glass is needed for reading the divisions with ease. In large instruments it is attached to each vernier.

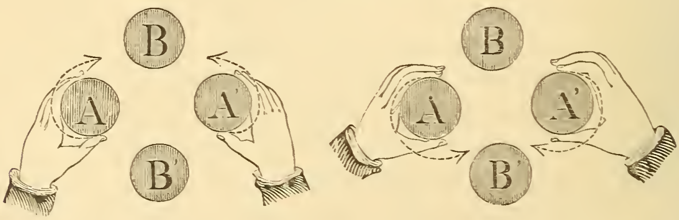
294. Movements. When the line of sight of the telescope is directed to a distant, well-defined point, the unaided hand of the observer can not move it with sufficient delicacy and precision to make the intersection of the cross-hairs exactly cover or "bisect" that point. To effect this, a clamp, and a tangent, or slow-motion, screw are required. This arrangement, as usually applied to the movement of the upper, or vernier plate, consists of a short post of brass, which is attached to the vernier-plate, and through which passes a long and fine-threaded "tangent-screw." The other end of this screw enters into and carries the *clamp*. This consists of two pieces of brass, which, by turning the clamp-screw, which passes through them on the outside, can be made to take hold of and pinch tightly the edge of the lower circle, which lies between them on the inside. The upper circle is now prevented from moving on the lower one, for the tangent-screw keeps them at a fixed distance apart, so that they can not move to or from one another, nor consequently the two circles to which they are respectively made fast. But when this tangent-screw is turned by its milled head, it gives the clamp and with it the upper plate a smooth and slow motion, backward or forward, whence it is called the "slow-motion screw," as well as "tangent-screw," from the direction in which it acts. Another form of clamp is shown in Fig. 200.

A little different arrangement is employed to give a similar motion to the lower circle on the body of the instrument. Its axis is embraced by a brass ring, into which enters a clamp-screw. The clamp-screw causes the ring to pinch and hold immovably the axis of the lower circle, while a turn of the tangent-screw will slowly move the clamp-ring itself, and therefore with it the lower circle. When the clamp is loosened, the lower circle, and with it everything above it, has a perfectly free motion.

295. Levels. Since the object of the instrument is to measure *horizontal* angles, the circular plate on which they are measured must itself be made horizontal. Whether it is so or not is known by means of two small levels placed on the plate at right angles to each other. Each consists of a glass tube, slightly curved upward in its middle, and so nearly filled with alcohol that only a small bubble of air is left in the tube. This always rises to the highest part of the tubes. They are so "adjusted" that when this bubble of air is in the middle of the tubes, or its ends equidistant from the central mark, the plate on which they are fastened shall be level, which way soever it may be turned. One of the levels is sometimes fixed between the standards above one of the verniers, and the other on the plate at the north end of the compass-box.

296. Parallel Plates. To raise or lower either side of the circle, so as to bring the bubbles into the centers of the tubes, requires more gentle and steady movements than the unaided hands can give, and is attained by the parallel plates, and their four milled-headed screws, which hold the plates firmly apart, and, by being turned in or out, raise or lower one side or the other of the upper plate, and thereby of the graduated circle. The two plates are held together by a ball-and-socket joint. To level the instrument, loosen the lower clamp and turn the circle till each level is parallel to the vertical plane passing through a pair of opposite screws. Then take hold of two opposite screws and turn

FIG. 209.



them simultaneously and equally, but in contrary directions, screwing one in and the other out, as shown by the arrows in the figures. A rule easily remembered is that both thumbs must turn in, or both out. The movements represented in the first of these figures would raise the left-hand side of the circle and lower the right-hand side. The movements of the second figure would produce the reverse effect. Care is needed to turn the opposite screws equally, so that they shall not become so loose that the instrument will rock, or so tight as to be cramped. When this last occurs, one of the other pair should be loosened.

Sometimes one of each pair of the screws is replaced by a strong spring, against which the remaining screws act.

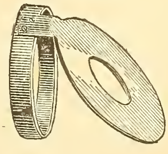
The French and German instruments, and most large instruments, are usually supported by only three screws. In such cases, one level is brought parallel to one pair of screws and leveled by them, and the other level has its bubble brought to its center by the third screw. If there is only one level on the instrument, it is first brought parallel to one pair of screws and leveled, and is then turned one quarter around so as to be perpendicular to them and over the third screw, and the operation is repeated.

297. Watch-Telescope. A second telescope is sometimes attached to the lower part of the instrument. When a number of angles are to be observed from any one station, direct the upper and principal telescope to the first object, and then direct the lower one to any other well-defined point. Then make all the desired observations with the upper telescope, and, when they are finished, look again through the lower one, to see that it and therefore the divided circle have not been moved by the movements of the vernier-plate. The French call this the *Witness-Telescope* (*Lunette témoin*).

298. The Compass. Upon the upper plate is fixed a compass. It has been fully explained in Chapter III. It is little used in connection with the transit, which is so incomparably more accurate, except as a "check," or rough test of the accuracy of the angles taken, which should about equal the difference of the magnetic bearings.

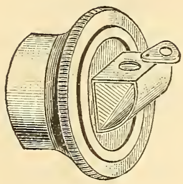
299. The Reflector. In making observations on Polaris at night, or in surveying mines, a reflector (Fig. 210) is used. This is a silvered plate with a hole in it for observing through with the telescope, while a light, held near the silvered surface, illuminates the cross-hairs. The reflector is attached to a ring, fitted to the object-glass slide, and is inclined at an angle of 45° to the ring.

FIG. 210.



300. The Diagonal Prism (Fig. 211). This is a prism attached to the eye-piece of the telescope, so that the rays of light, coming from the object sighted to, and passing through the telescope, are reflected to the eye at an angle of 90° to the line of sight of the telescope. The prism is attached to a movable plate so that it can be turned to suit the position of the observer. This prism enables larger vertical angles to be measured than would be possible without it.

Fig. 211.

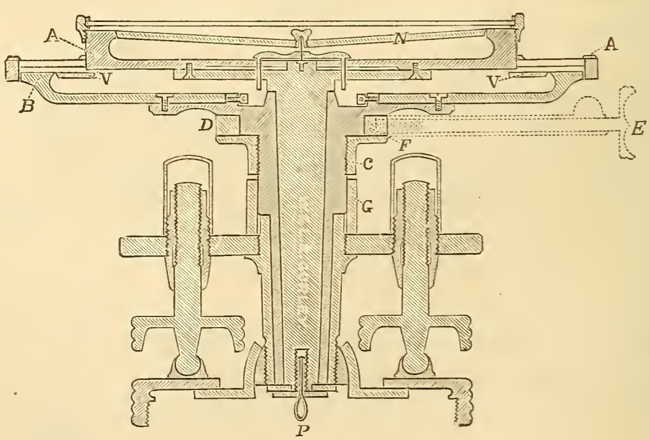


The Transit.

301. The Engineer's Transit (Fig. 213). This instrument is similar in general construction to that shown in Fig. 199, but differs from it in several important particulars. The sockets for the axes of the plates are longer and differently arranged. These are shown in Fig. 212.

Both levels are attached to the upper plate. The verniers, instead of being placed at the sides between the legs of the standards,

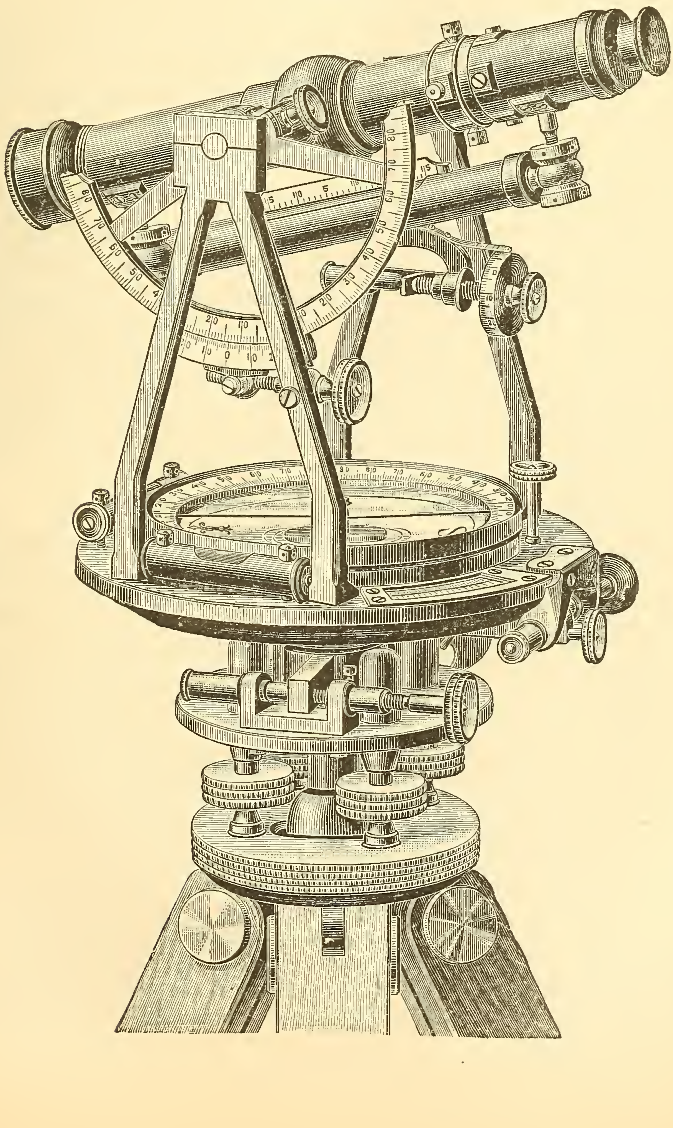
Fig. 212.



as is usual, are placed near the north and south points of the compass-circle, so that the observer can read the vernier without stepping to the side of the instrument. The slow motion, both of the upper and lower plate, is given by one tangent-screw. In each case an opposing spiral spring prevents any shake in the tangent-screw.

The vertical arc is attached to the axis of the telescope by a clamp-screw, shown in the figure. The vernier and the slow-motion screw of the vertical arc are shown below the arc, and are attached to the left-hand standard.

Fig. 213.



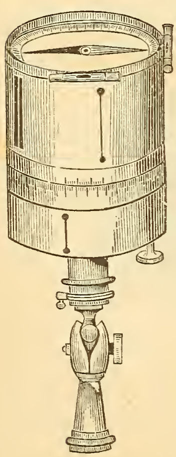
Attached to the right-hand standard is the "Gradienter" (shown in detail in Fig. 245).

302. A vertical section through the body of the engineer's transit is given in Fig. 212. The lower plate, or "divided limb," B, is supported by the hollow socket C. Through this hollow socket passes the conical spindle which supports the upper plate A.

The upper plate carries the telescope, compass-box, and the verniers. The vernier-scales, V, V, are attached to the upper plate, but lie in the same plane as the divisions of the lower plate (so that the two can be viewed together without parallax), and are covered with glass to exclude dust. E is the clamp-screw.

303. The Theodolite. The transit, when furnished with a vertical circle and telescope level, is sometimes called a Theodolite. This name is used almost exclusively in England and on the Continent of Europe. In one form of the theodolite the telescope can not be revolved on its horizontal axis. This form has been almost entirely superseded in this country by that having a reversible telescope. It is then called a Transit Theodolite, or simply a Transit.

Fig. 214.



304. Goniasmometre. A very compact instrument, to which this name has been given in France, where it is much used, is shown in the figure. The upper half of the cylinder is movable on its lower half. The observations may be taken through the slits, as in the surveyor's cross, or a telescope may be added to it. Readings may be taken both from the compass and from the divided edge of the lower half of the cylinder, by means of a vernier on the upper half.*

* The proper care of instruments must not be overlooked. If varnished, they should be wiped gently with fine and clean linen. If polished with oil, they should be rubbed more strongly. The parts neither varnished nor oiled should be cleaned with Spanish-white and alcohol. Varnished wood, when spotted, should be wiped with very soft linen, moistened with a little olive-oil or alcohol. Unpainted wood is

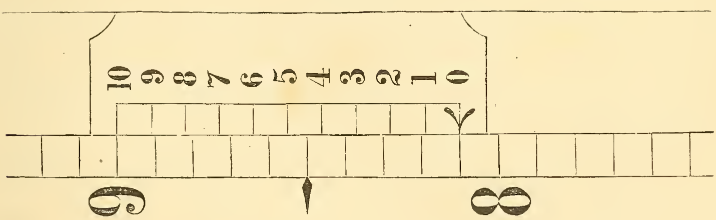
VERNIERS.

305. Definition. A vernier is a contrivance for measuring smaller portions of space than those into which a line is actually divided. It consists of a second line or scale, movable by the side of the first, and divided into equal parts, which are a very little shorter or longer than the parts into which the first line is divided. This small difference is the space which we are thus enabled to measure.*

The vernier scale is usually constructed by taking a length equal to any number of parts on the divided line, and then dividing this length into a number of equal parts, one more or one less than the number into which the same length on the original line is divided.

306. Illustration. The figure represents (to twice the real size) a scale of inches divided into tenths, with a vernier scale beside it, by which hundredths of an inch can be measured. The vernier is

Fig. 215.



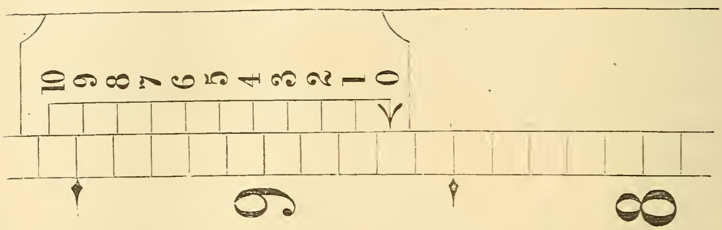
made by setting off on it nine tenths of an inch, and dividing that length into ten equal parts. Each space on the vernier is therefore equal to a tenth of nine tenths of an inch, or to nine hundredths of an inch, and is consequently one hundredth of an inch shorter than one of the divisions of the original scale. The first space of the vernier will therefore fall short of, or be overlapped by, the first

cleaned with sand-paper. Apply olive-oil where steel rubs against brass; and wax softened by tallow where brass rubs against brass. Clean the glasses with kid or buck skin. Wash them, if dirtied, with alcohol.

* The vernier is so named from its inventor, in 1631. The name "Nonius," often improperly given to it, belongs to an entirely different contrivance for a similar object.

space on the scale by this one hundredth of an inch ; the second space of the vernier will fall short by two hundredths of an inch ; and so on. If, then, the vernier be moved up by the side of the original scale, so that the line marked 1 coincides, or forms one straight line, with the line of the scale which was just above it, we know that the vernier has been moved one hundredth of an inch. If the line marked 2 comes to coincide with a line of the scale, the

FIG. 216.



vernier has moved up two hundredths of an inch ; and so for other numbers. If the position of the vernier be as in this figure, the line marked 7 on the vernier corresponding with some line on the scale, the zero-line of the vernier is seven hundredths of an inch above the division of the scale next below this zero-line. If this division be, as in the figure, 8 inches and 6 tenths, the reading will be 8·67 inches.*

A vernier like this is used on some leveling-rods, being engraved on the sides of the opening in the part of the target above its middle line. The rod being divided into hundredths of a foot, this vernier reads to thousandths of a foot. It is also used on some French mountain barometers, which are divided to hundredths of a *metre,* and thus read to thousandths of that unit.

307. General Rules. *To find what any vernier reads to*—i. e., to determine how small a distance it can measure—observe how many parts on the original line are equal to the same number increased or diminished by one on the vernier, and divide the length

* The student will do well to draw such a scale and vernier on two slips of thick paper, and move one beside the other till he can read them in any possible position ; and so with the following verniers.

of a part on the original line by this last number. It will give the required distance.*

For verniers as usually constructed, the following rule will apply : *Divide the value of the smallest division on the original scale by the number of parts on the vernier.*

For example, if the limb of a transit be divided into half degrees, and thirty parts on the vernier are equal to twenty-nine on the limb, then the value of the smallest division on the limb (30 minutes), divided by the number of parts on the vernier (30) equals one minute. This is what the vernier reads to.

To read any vernier, first, look at the zero-line of the vernier (which is sometimes marked by an arrow-head), and if it coincides with any division of the scale, that will be the correct reading, and the vernier divisions are not needed. But if, as usually happens, the zero-line of the vernier comes between any two divisions of the scale, note the nearest next less division on the scale, and then look along the vernier till you come to some line on it which exactly coincides, or forms a straight line, with some line (no matter which) on the fixed scale. The number of this line *on the vernier* (the 7th, in the last figure) tells that so many of the subdivisions which the vernier indicates are to be added to the reading of the entire divisions on the scale.

When several lines on the vernier appear to coincide equally with lines of the scale, take the middle line.

When no line coincides, but one line on the vernier is on one side of a line on the scale, and the next line on the vernier is as far on the other side of it, the true reading is midway between those indicated by these two lines.

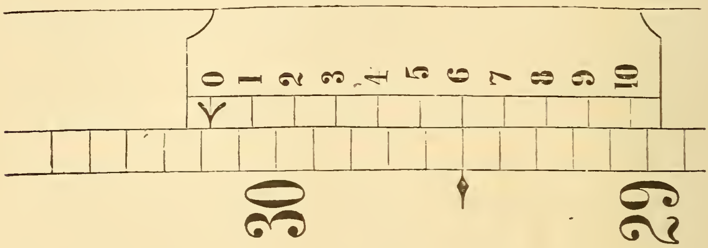
308. Retrograde Verniers. The spaces of the vernier in modern instruments are usually each shorter than those on the scale, a certain number of parts on the scale being divided into a larger number

* In algebraic language, let s equal the length of one part on the original line, and v the unknown length of one part on the vernier. Let m of the former = $m + 1$ of the latter. Then $ms = (m + 1)\,v$. $v = \frac{m}{m+1}s$. $s - v = s$. $\frac{m}{m+1}s = \frac{s}{m+1}$. If $ms = (m - 1)\,v$, then $v - s = \frac{s}{m-1}$.

of parts on the vernier.* In the contrary case,† there is the inconvenience of being obliged to number the lines of the vernier and to count their coincidences with the lines of the scale, in a retrograde or contrary direction to that in which the numbers on the scale run. We will call such arrangements *retrograde* verniers.

309. Illustration. In this figure, the scale, as before, represents (to twice the real size) inches divided into tenths, but the vernier is made by dividing eleven parts of the scale into ten equal

FIG. 217.



parts, each of which is therefore one tenth of eleven tenths of an inch—i. e., eleven hundredths of an inch, or a tenth and a hundredth. Each space of the vernier therefore overlaps a space on the scale by one hundredth of an inch. The manner of reading this vernier is the same as in the last one, except that the numbers run in a reverse direction. The reading of the figure is 30·16.

This vernier is the one generally applied to the common barometer, the zero-point of the vernier being brought to the level of the top of the mercury, whose height it then measures. It is also employed for leveling-rods which read downward from the middle of the target.

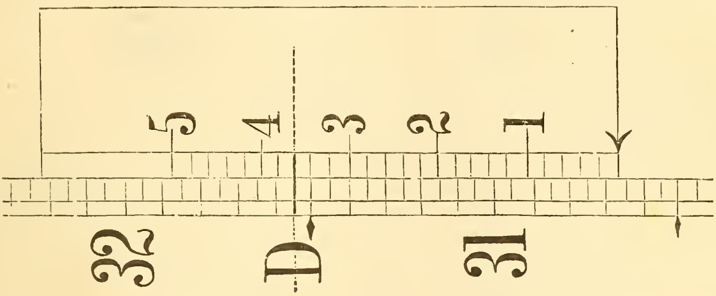
310. Fig. 218 represents (to double size) the usual scale of the English mountain barometer. The scale is first divided into inches. These are subdivided into tenths by the longer

---

* i. e., algebraically, $v = \frac{m}{m+1}$. † i. e., when $v = \frac{m}{m-1} s$.

lines, and the shorter lines again divide these into half tenths, or to 5 hundredths; 24 of these smaller parts are set off on the ver-

Fig. 218.



nier, and divided into 25 equal parts, each of which is therefore $= \frac{24 \times \cdot 05}{25} = \cdot 048$ inch, and is shorter than a division of the scale by $\cdot 050 - \cdot 048 = \cdot 002$, or two thousandths of an inch, a twenty-fifth part of a division on the scale, to which minuteness the vernier can therefore read. The reading in the figure is 30·686 (30·65 by the scale and ·036 by the vernier), the dotted line marked D showing where the coincidence takes place.

311. Circle divided into Degrees. The following illustrations apply to the measurements of angles, the circle being variously divided. In this article, the circle is supposed to be divided into degrees.

If 6 spaces on the vernier are found to be equal to 5 on the circle, the vernier can read to one sixth of a space on the circle—i. e., to 10′.

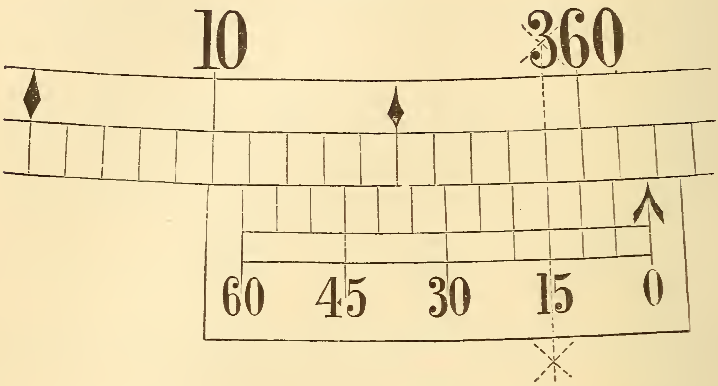
If 10 spaces on the vernier are equal to 9 on the circle, the vernier can read to one tenth of a space on the circle—i. e., to 6′.

If 12 spaces on the vernier are equal to 11 on the circle, the vernier can read to one twelfth of a space on the circle—i. e., to 5′.

Fig. 219 shows such an arrangement. The index, or zero, of the vernier is at a point beyond 358°, a certain distance, which the coincidence of the third line of the vernier (as indicated by the dotted and crossed line) shows to be 15′. The whole reading is therefore 358° 15′.

If 20 spaces on the vernier are equal to 19 on the circle, the vernier can read to one twentieth of a division on the circle—i. e.,

FIG. 219.

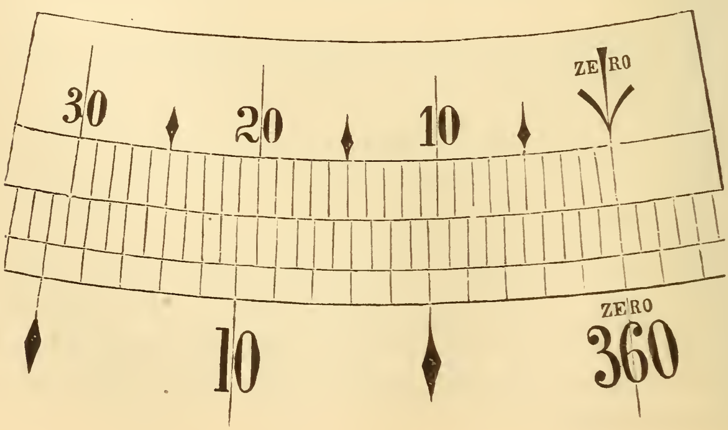


to 3′. English compasses, or "circumferentors," are sometimes thus arranged.

If 60 spaces on the vernier are equal to 59 on the circle, the vernier can read to one sixtieth of a division on the circle—i. e., to 1′.

312. Circle divided to 30′. Such a graduation is a very common one. The vernier may be variously constructed.

FIG. 220.

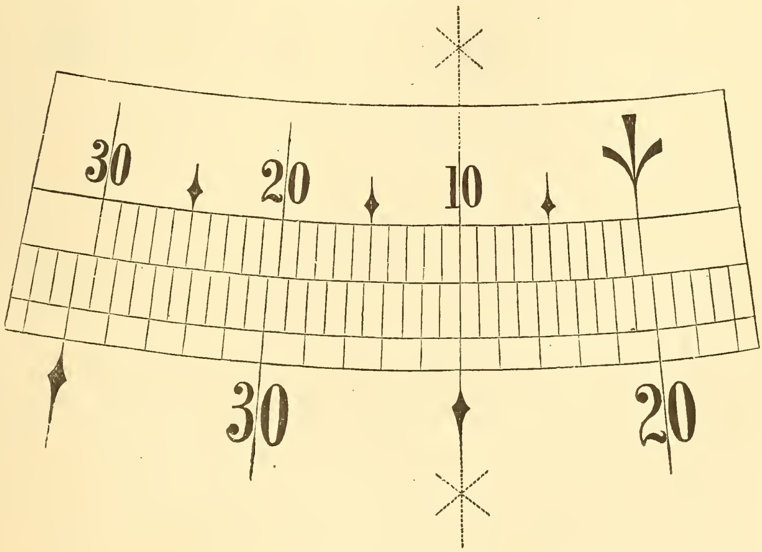


Suppose 30 spaces on the vernier to be equal to 29 on the circle. Each space on the vernier will be $= \frac{29 \times 30'}{30} = 29'$, and will therefore be less than a space of the circle by 1′, to which the vernier will then read.

Fig. 220 shows this arrangement. The reading is 0°, or 360°.

In Fig. 221 the dotted and crossed line shows what divisions coincide, and the reading is 20° 10′; the vernier being the same as in the preceding figure, and its zero being at a point of the circle 10′ beyond 20°.

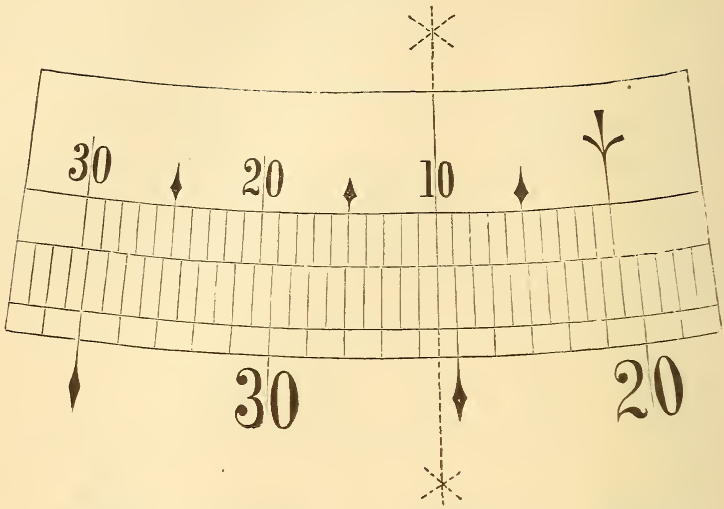
Fig. 221.



In Fig. 222, the reading is 20° 40′, the index being at a point beyond 20° 3′, and the additional space being shown by the vernier to be 10′.

Sometimes 30 spaces on the vernier are equal to 31 on the circle. Each space on the vernier will therefore be $= \frac{31 \times 30'}{30} = 31'$, and will be longer than a space on the circle by 1′, to which it will therefore read, as in the last case, but the vernier will be "retrograde." This is the vernier of the compass. The peculiar manner in which it is applied is shown in Fig. 229.

FIG. 222.

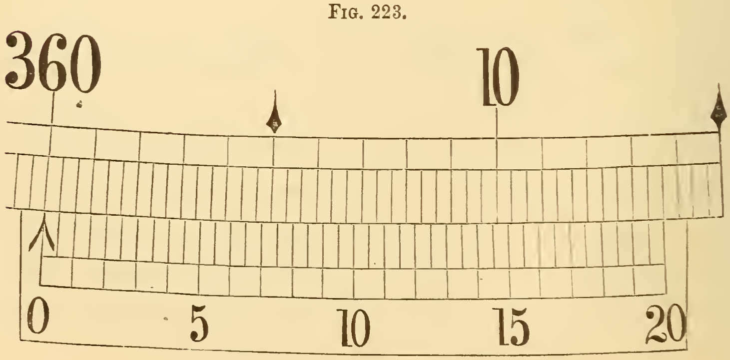


If 15 spaces on the vernier are equal to 16 on the circle, each space on the vernier will be $= \dfrac{16 \times 30'}{15} = 32'$, and the vernier will therefore read to $2'$.

313. Circle divided to 20′. If 20 spaces of the vernier are equal to 19 on the circle, each space of the latter will be $= \dfrac{19 \times 20'}{20} = 19'$, and the vernier will read to $20' - 19' = 1'$.

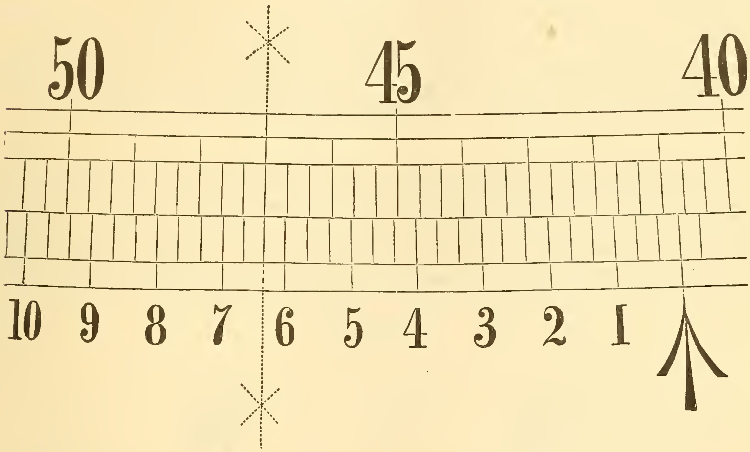
If 40 spaces on the vernier are equal to 41 on the circle, each

FIG. 223.



space on the vernier will be $= \frac{41 \times 20'}{40} = 20\frac{1}{2}'$, and the vernier will therefore read to $20\frac{1}{2}' - 20' = 30''$. It will be retrograde. In Fig. 223 the reading is 360°, or 0° ; and it will be seen that the

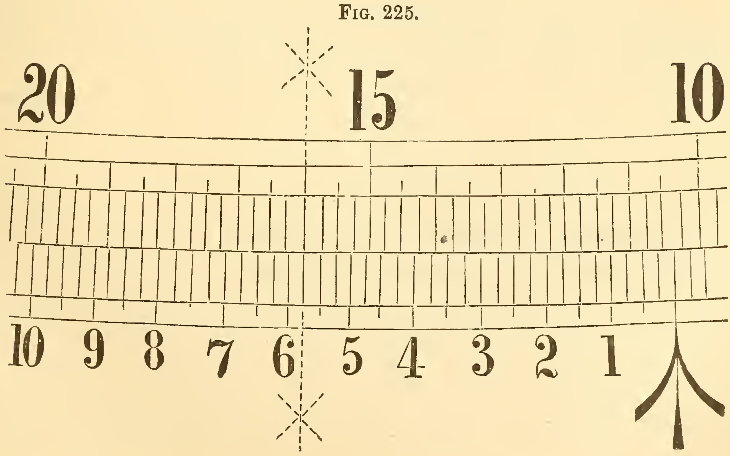
FIG. 224.



40 spaces on the vernier (numbered to whole minutes) are equal to 13° 40′ on the limb—i. e., to 41 spaces, each of 20′.

If 60 spaces on the vernier are equal to 59 on the circle, each of the former will be $= \frac{59 \times 20}{60} = 19' \, 40''$, and the vernier will

FIG. 225.

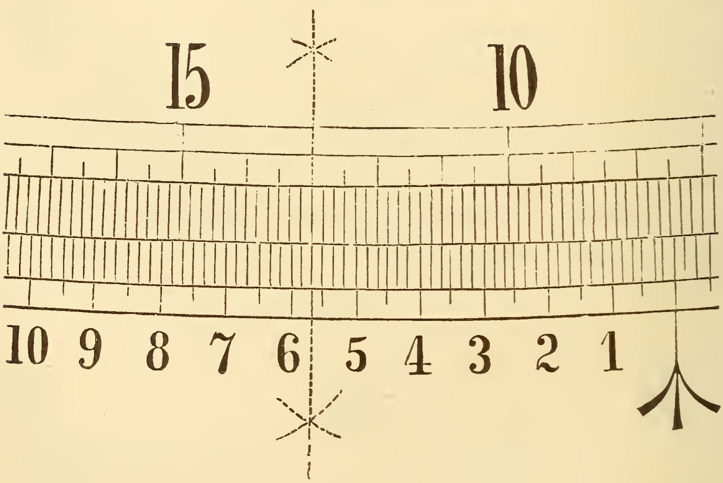


therefore read to $20' - 19'\ 40'' = 20''$. Fig. 224 shows such an arrangement. The reading in that position would be 40° 46′ 20″.

314. Circle divided to 15′. If 60 spaces on the vernier are equal to 59 on the circle, each space on the vernier will be $= \frac{59 \times 15'}{60} = 14'\ 45''$, and the vernier will read to 15″. In Fig. 225 the reading is 10° 20′ 45″, the index pointing to 10° 15′, and something more, which the vernier shows to be 5′ 45″.

315. Circle divided to 10′. If 60 spaces on the vernier be equal to 59 on the limb, the vernier will read to 10″. In Fig. 226 the reading is 7° 25′ 40″, the reading on the circle being 7° 20′, and the vernier showing the remaining space to be 5′ 40″.

FIG. 226.



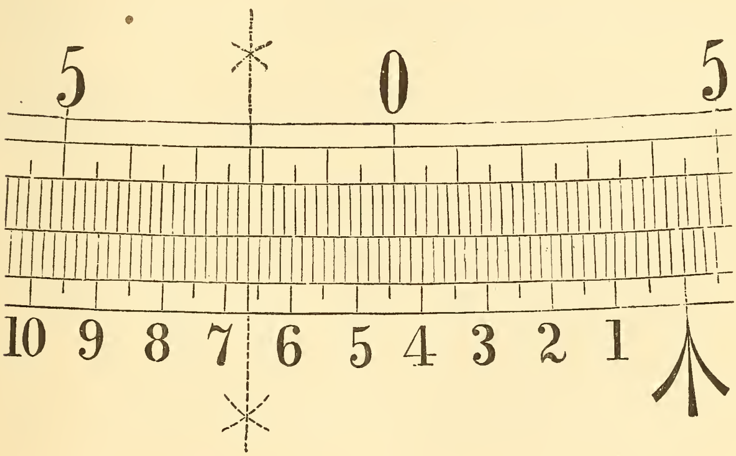
316. Reading backward. When an index carrying a vernier is moved backward, or in a contrary direction to that in which the numbers on the circle run, if we wish to read the space which it has passed over in this direction from the zero-point, the vernier must be read backward (i. e., the highest number be called 0), or its actual reading must be subtracted from the value of the smallest space on the circle. The reason is plain ; for, since the vernier shows how far the index, moving in one direction, has gone past one division-line, the distance which it is from the next division-line (which it may be supposed to have passed, moving in a contrary direction) will be the difference between the reading and the value of one space.

Thus, in Fig. 219, the reading is 358° 15′. But, counting backward from the 360°, or zero-point, it is 1° 45′.

Caution on this point is particularly necessary in using small angles of deflection for railroad-curves.

317. Arc of Excess. On the sextant and similar instruments, the divisions of the limb are carried onward a short distance beyond the zero-point. This portion of the limb is called the "Arc of Excess." When the index of the vernier points to this arc, the

FIG. 227.

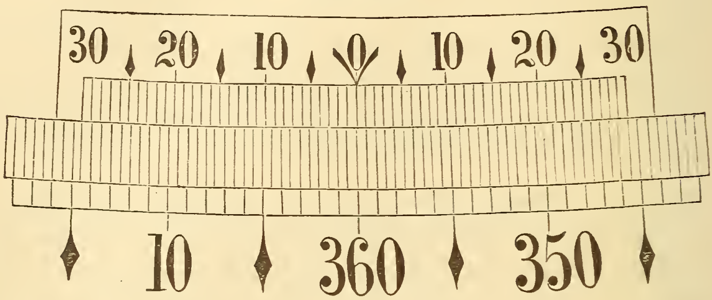


reading must be made as explained in the last article. Thus, in the figure, the reading on the arc from the zero of the limb to the zero of the vernier is 4° 20′, and something more, and the reading of the vernier from 10 toward the right, where the lines coincide, is 3′ 20″ (or it is 10′ − 6′ 40″ = 3′ 20″), and the entire reading is therefore 4° 23′ 20″.

318. Double Verniers. To avoid the inconveniences of reading backward, double verniers are sometimes used. Fig. 228

shows one applied to a transit. Each of the verniers is like the one described in Art. 312, Figs. 220, 221, and 222. When the degrees are counted to the left, or as the numbers run, as is usual,

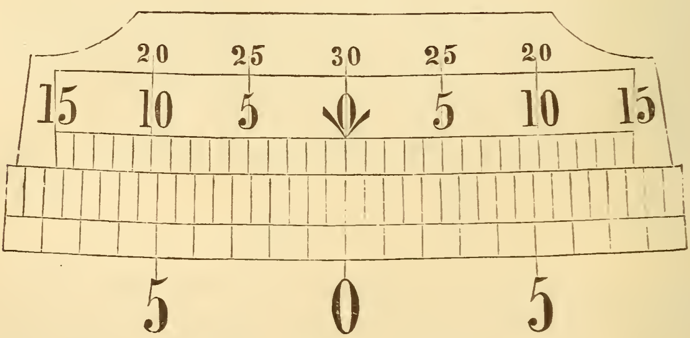
FIG. 228.



the left-hand vernier is to be read, as in Art. 312; but when the degrees are counted to the right, from the 360° line, the right-hand vernier is to be used.

319. Compass-Vernier. Another form of double vernier, often applied to the compass, is shown in Fig. 229. The limb is

FIG. 229.



divided to half-degrees, and the vernier reads to minutes, 30 parts on it being equal to 31 on the limb. But the vernier is only half as long as in the previous case, going only to 15′, the upper figures on one half being a sort of continuation of the lower figures on the other half. Thus, in moving the index to the right, read the *lower* figures on the left-hand vernier (it being retrograde) at any coincidence, when the space passed over is less than 15′; but if it be more, read the *upper* figures on the right-hand vernier, and *vice versa.*

ADJUSTMENTS.

320. The purposes for which the transit (as well as most surveying and astronomical instruments) is to be used, require and presuppose certain parts and lines of the instrument to be placed in certain directions with respect to others; these respective directions being usually parallel or perpendicular. Such arrangements of their parts are called their *Adjustments.* The same word is also applied to placing these lines in these directions. In the following explanations the operations which determine whether these adjustments are correct, will be called their *Verifications;* and the making them right, if they are not so, their *Rectifications.**

321. In observations of horizontal angles with the transit it is required—

1. That the circular plates shall be horizontal in whatever way they may be turned around.

2. That the telescope, when pointed forward, shall look in precisely the reverse of its direction when pointed backward—i. e., that its two lines of sight (or lines of collimation) forward and backward shall lie in the same plane.

3. That the telescope, in turning upward or downward, shall move in a truly vertical plane, in order that the angle measured between a low object and a high one may be precisely the same as would be the angle measured between the low object and a point exactly under the high object, and in the same horizontal plane as the low one.

We shall see that all these adjustments are finally resolvable into these: 1. Making the vertical axis of the instrument perpendicular to the plane of the levels; 2. Making the line of collima-

* It has been well said that, "in the present state of science, it may be laid down as a maxim that every instrument should be so contrived that the observer may easily examine and rectify the principal parts; for, however careful the instrument-maker may be, however perfect the execution thereof, it is not possible that any instrument should long remain accurately fixed in the position in which it came out of the maker's hands." (Adams's "Geometrical and Graphical Essays," 1791.)

tion perpendicular to its axis ; and, 3. Making this axis parallel to the plane of the levels. They are all best tested by the invaluable principle of "reversion."

We have now, first, to examine whether these things *are* so—that is, to "verify" the adjustments; and, second, if we find that they are not so, to *make* them so—i. e., to "rectify" or "adjust" them correctly. The above three requirements produce as many corresponding adjustments.

322. First Adjustment. *To cause the circle to be horizontal in every position.*

Verification. Turn the vernier-plate, which carries the levels, till one of them is parallel to one pair of the parallel plate-screws. The other will then be parallel to the other pair. Bring each bubble to the middle of its tube, by that pair of screws to which it is parallel. Then turn the vernier-plate half-way around—i. e., till the index has passed over 180°. If the bubbles remain in the centers of the tubes, they are in adjustment. If either of them runs to one end of the tube, it requires rectification.

Rectification. The fault which is to be rectified is that the plane of the level (i. e., the plane tangent to the highest point of the level tube) is not perpendicular to the vertical axis on which

Fig. 230.

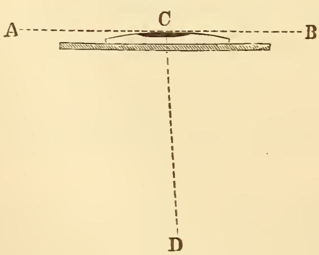
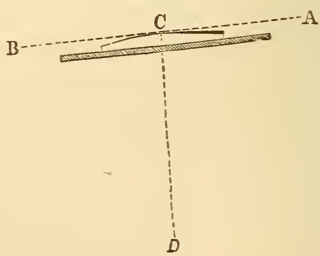


Fig. 231.



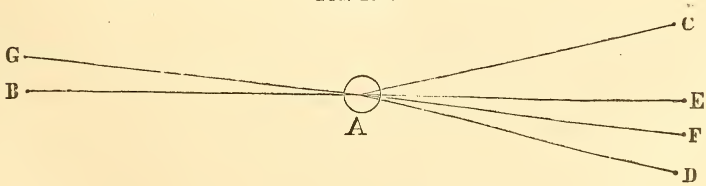
the plate turns. For, let A B represent this plane, seen edgewise, and C D the center line of the vertical axis, which is here drawn as making an acute angle with this plane on the right-hand side. The first figure represents the bubble brought to the center of the tube. The second figure represents the plate turned half around. The center line of the axis is supposed to remain unmoved. The acute angle will now be on the left-hand side, and the plate will no longer be horizontal; consequently, the bubble will run to the higher end of the tube. The rectification necessary is evidently to raise one end of the tube and lower the other. The real error has been doubled to the eye by the reversion. Half of the motion of the bubble was caused by the tangent plane not being perpendicular to the axis, and half by this axis not being vertical. Therefore, raise or lower one end of the level by the screws which fasten it to the plate, till the bubble comes about *half-way* back to the center, and then bring it quite back by turning its pair of parallel plate-screws. Then again reverse the vernier-plate 180°. The bubble should now remain in the center. If not, the operation should be repeated. The same must be done with the other level, if required. Then the bubbles will remain in the center during a complete revolution. This proves that the axis of the vernier-plate is then vertical; and, as it has been fixed by the maker perpendicular to the plate, the latter must then be horizontal.

It is also necessary to examine whether the bubbles remain in the center, when the divided circle is turned round on its axis. If not, the axes of the two plates are not parallel to each other. The defect can be remedied only by the maker; for, if the bubbles be altered so as to be right for this reversal, they will be wrong for the vernier-plate reversal.

323. Second Adjustment. *To cause the line of collimation to revolve in a plane.*

Verification. Set up the transit in the middle of a level piece of ground, as at A in the figure. Level it carefully. Set a stake,

FIG. 232.



with a nail driven into its head, or a chain-pin, as far from the instrument as it is distinctly visible, as at B. Direct the telescope to it, and fix the intersection of the cross-hairs very precisely upon it. Clamp the instrument. Measure from A to B. Then turn over the telescope, and set another stake at an equal distance from the transit, and also precisely in the line of sight. If the line of collimation has *not* continued in the same plane during its half-revolution, this stake will not be at E, but to one side, as at C. To discover the truth, loosen the clamp and turn the vernier-plate half around without touching the telescope. Sight to B, as at first, and again clamp it. Then turn over the telescope, and the line of sight will strike, as at D in the figure, as far to the right of the point as it did before to its left.

Rectification. The fault which is to be rectified is that the line of collimation of the telescope is not perpendicular to the horizontal axis on which the telescope revolves. This will be seen by

Fig. 233.

Fig. 234.

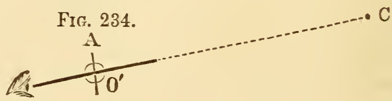


Fig. 235.

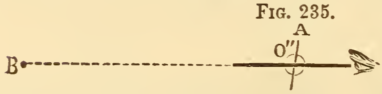
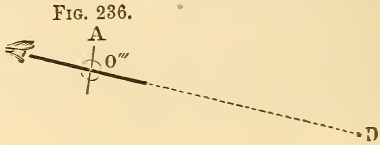


Fig. 236.



the figures, which represent the position of the lines in each of the four observations which have been made. In each of the figures the long, thick line represents the telescope, and the short one the axis on which it turns. In Fig. 233 the line of sight is directed to B. In Fig. 234 the telescope has been turned over, and with it the axis, so that the obtuse angle marked O in the first figure has taken the place, O′, of the acute angle, and the telescope points to C instead of to E. In Fig. 235 the vernier-plate has been turned half around so as to point to B again, and the same obtuse angle has got around to O″. In Fig. 236 the telescope has been turned over, the obtuse angle is at O‴, and the telescope now points to D.

To make the line of collimation perpendicular to the axis, the former must have its direction changed. This is effected by moving the vertical hair the proper distance to one side. By loosening the left-hand screw and tightening the right-hand one, the ring, and with it the cross-hairs, will be drawn to the right, and *vice versa.* Two holes at right angles to each other pass through the outer heads of the screws. Into these holes a stout steel wire is inserted, and the screws can thus be turned around. Screws so made are called "capstan-headed." One of the other pair of screws may need to be loosened to avoid straining the threads. In some French instruments, one of each pair of screws is replaced by a spring.

To find how much to move this vertical hair, measure from C to D, Fig. 232 : Set a stake at the middle point E, and set another at the point F, midway between D and E. Move the vertical hair till the line of sight strikes F. Then the instrument is adjusted ; and, if the line of sight be now directed to E, it will strike B when the telescope is turned over, since the hair is moved half of the doubled error, D E. The operation will generally require to be repeated, not being quite perfected at first.

It should be remembered that, if the telescope used does not invert objects, its eye-piece will do so. Consequently, with such a telescope, if it seems that the vertical hair should be moved to the left, it must be moved to the right, and *vice versa.* An inverting telescope does not invert the cross-hairs.

If the young surveyor has any doubts as to the perfection of his rectification, he may set another stake exactly under the instrument by means of a plumb-line suspended from its center ; and then, in like manner, set his transit over B or E. He will find that the other two stakes, A and the extreme one, *are* in the same straight line with his instrument.

In some instruments, the horizontal axis of the telescope can be taken out of its supports and turned over, end for end. In such a case, the line of sight may be directed to any well-defined point, and the axis then taken out and turned over. If the line of sight again strikes the same point, this line is perpendicular to the axis.

Fig. 237.

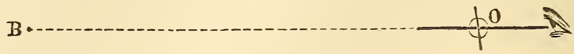
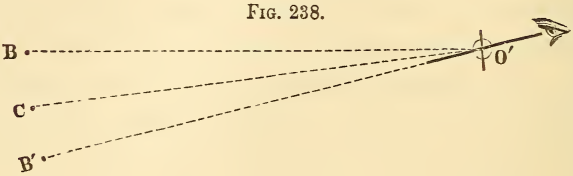


Fig. 238.



If not, the apparent error is double the real error, as appears from the figures, the obtuse angle O coming to O′, and the desired perpendicular line falling at C midway between B and B′. The rectification may be made as before; or, in some large instruments, in which the telescope is supported on Ys, by moving one of the Ys laterally.

324. Third Adjustment. *To cause the line of collimation to revolve in a vertical plane.*

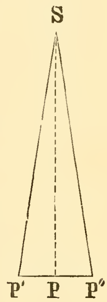
Verification. Suspend a long plumb-line from some high point. Set the instrument near this line, and level it carefully. Direct the telescope to the plumb-line, and see if the intersection of the cross-hairs follows and remains upon this line when the telescope is turned up and down. If it does, it moves in a vertical plane.

The angle of a new and well-built house will form an imperfect substitute for the plumb-line.

Otherwise: The instrument being set up and leveled as above, place a basin of some reflecting liquid (quicksilver being the best, though molasses, or oil, or even water will answer, though less perfectly) so that the top of a steeple, or other point of a high object, can be seen in it through the telescope by reflection. Make the intersection of the cross-hairs cover it. Then turn up the telescope, and, if the intersection of the cross-hairs bisects also the object seen directly, the line of sight has moved in a vertical plane. If a star be taken as the object, the star and its reflection will be equivalent (if it be nearly overhead) to a plumb-line at least fifty million million miles long.

Otherwise: Set the instrument as close as possible to the base of a steeple or other high object; level it, and direct it to the top of the steeple, or to some other elevated and well-defined point. Clamp the plates. Turn down the telescope, and set up a pin in the ground precisely "in line." Then loosen the clamp, turn over the telescope, and turn it half-way around, or so far as to again sight to the high point. Clamp the plates, and again turn down the telescope. If the line of sight again strikes the pin, the telescope has moved in a vertical plane. If not, the apparent error is double the real error. For, let S be the top of the steeple (Fig. 239), and P′ the pin; then the plane in which the telescope moves, seen edgewise, is S P′; and, after being turned around, the line of sight moves in the plane S P″, as far to one side of the vertical plane S P as S P′ was on the other side of it.

FIG. 239.

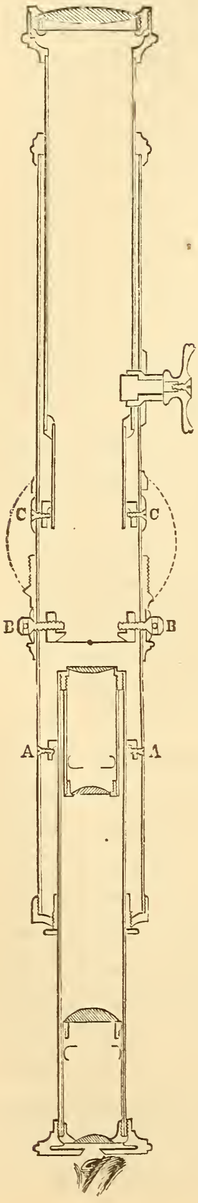


Rectification. Since the second adjustment causes the line of sight to move in a plane perpendicular to the axis on which it turns, it will move in a vertical plane if that axis be horizontal. It can be made so by raising or lowering one end of the axis by means of a screw placed in the standard for that purpose.

325. Centering Eye-Piece. In some instruments, such as that of which a longitudinal section is shown in the margin, the inner end of the eye-piece may be moved so that the cross-hairs shall be seen precisely in the center of its field of view. This is done by means of four screws, arranged in pairs, like those of the cross-hair ring-screws, and capable of moving the eye-piece up and down, and to right or left, by loosening one and tightening the opposite one. Two of them are shown at A, A, in the figure, in which B, B, are two of the cross-hair screws.

326. Centering Object-Glass. In some instruments four screws, similarly arranged, two of which are shown at C, C, can move, in any direction, the inner end of the slide which carries the object-glass. The necessity for such an arrangement arises from the impossibility of drawing a tube perfectly straight. Consequently, the line of collimation, when the tube is drawn in, will not coincide with the same line when the tube is pushed out. If adjusted for one position, it will therefore be wrong for the other. These screws, however, can make it right in both positions. They are used as follows:

Fig. 240.



Sight to some well-defined point as far off as it can be distinctly seen. Then, having the plates firmly clamped, move out the object-glass slide, and fix a point in the line of sight as close to the instrument as can be distinctly seen. Then turn the limb halfway around horizontally, reverse the telescope, and again sight to the near point, by clamping the plates and bringing the vertical cross-hair on the point by means of the tangent-screw. Then draw in the object-glass slide until the distant object is distinctly seen. If the vertical cross-hair bisects it, no adjustment is necessary. If not, correct one half of the apparent error by means of the screws C C in Fig. 240. This may disturb the second adjustment. Try that over again, and again perform the operation of centering the object-glass.

This adjustment is always performed by the maker, and its screws are covered by a short tube.

All the adjustments should be meddled with as little as possible, lest the screws should get loose; and, when once made right, they should be kept so by careful usage.

327. Fourth Adjustment. *To cause the line of collimation of the telescope to be horizontal when the bubble of the level attached to it is in the center of its tube.*

Drive two pegs several hundred feet apart, and set the instrument midway between them. Level, and sight to the rod held on each peg. The difference of the readings will be the true difference of the heights of the pegs, no matter how much the level may be out of adjustment.

Then set the instrument over one peg, and sight to the rod held at the other. Measure the height of the cross-hairs above the first peg. The difference of this height and the reading on the rod *should* equal the difference of the heights of the two points, as previously determined. If it does not, set the target to the sum or difference of the height of the cross-hairs above the first peg and the true difference of height of the points, according as the first point is higher or lower than the second, and hold the rod on the second point. Sight to it, and raise or lower one end of the bubble-tube until the horizontal cross-hair *does* bisect the target when the bubble is in the center.

Instead of setting *over* one peg, it is generally more convenient to set near to it, and sight to a rod held on it, and use this reading instead of the measured height of the cross-hairs.

328. Fifth Adjustment. *To make the vernier of the vertical circle read zero when the bubble of the telescope-level is in the center.*

This is verified in various ways:

1. *By simple inspection.*

2. *By reversion.* Sight to some point. Note the reading on the vertical circle. Turn the telescope half-way around horizontally. Turn over the telescope and again observe the same point, and note the reading. Half the difference (if any) of the two readings is the error.

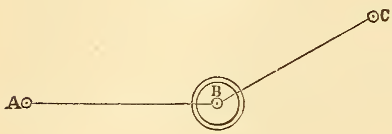
3. *By reciprocal observations.* Observe successively from each of two points to the other. Half the difference of the readings equals the index-error.

When the verification has been made, the error may be rectified on the instrument by moving the vernier-plate, or the circle, or noted as a correction to each observation when the instrument is large and delicate.

THE FIELD-WORK.

329. To measure a Horizontal Angle. Set up the instrument so that its center shall be exactly over the angular point, or in the intersection of the two lines whose difference of direction is to be measured, as at B in the figure. A plumb-line must be suspended from under the center. Dropping a stone is an imperfect substitute for this. Set the instrument so that its *lower* parallel plate may be as nearly horizontal as possible. The levels will serve as guides if the four parallel-plate screws be first so screwed up or down that equal lengths of them shall be above the upper plate. Then level the instrument carefully. Direct the telescope to a rod, stake, or other object, A in the figure, on one of the lines which form the angle. Tighten the clamps, and by the tangent-screw move the telescope so that the intersection of the cross-hairs shall very precisely bisect this object. Note the reading of the vernier. Then loosen the clamp of the vernier, and direct the telescope on the other line (as to C) precisely as before, and again read. The difference of the two readings will be the desired angle, A B C. Thus, if the first reading had been 40° and the last 190°, the angle would be 150°. If the vernier had passed 360° in turning to the second object, 360° should be added to the last reading before subtracting. Thus, if the first reading had been 300°, and the last reading 90°, the angle would be found by calling the last reading, as it really is, $360° + 90° = 450°$, and then subtracting 300°.

FIG. 241.



It is best to sight first to the left-hand object and then to the right-hand one, turning "with the sun" or like the hands of a watch, since the numbering of the degrees usually runs in that direction.

It is convenient, though not necessary, to begin by setting the vernier at zero by the upper movement (that of the vernier-plate on the circle), and then, by means of the lower motion (that of the whole instrument on its axis), to direct the telescope to the first object. Then fasten the lower clamp, and sight to the second object as before. The reading will then be the angle desired. An objection to this is that the two verniers seldom read alike.*

After one or more angles have been observed from one point, the telescope must be directed back to the first object, and the reading to it noted, so as to make sure that it has not slipped. A watch-telescope renders this unnecessary.

The error arising from the instrument not being set precisely over the center of the station will be greater the nearer the object sighted to. Thus, a difference of one inch would cause an error of only 3″ in the apparent direction of an object a mile distant, but one of nearly 3′ at a distance of a hundred feet.

330. Reduction of High and Low Objects. When one of the objects sighted to is higher than the other, the "plunging telescope" of this instrument causes the angle measured to be the true horizontal angle desired—i. e., the same angle as if a point exactly under the high object and on a level with the low object (or *vice versa*) had been sighted to. For the telescope has been caused to move in a vertical plane by the third adjustment, and the angle measured is therefore the angle between the vertical planes which pass through the two objects, and which "project" the two lines of sight on the same horizontal plane.

This constitutes the great practical advantage of these instruments over those which are held in the planes of the two objects observed, such as the sextant and the "circle," much used by the French.

331. Notation of Angles. The angles observed may be noted in various ways. Thus, the observation of the angle A B C, in

* The learner will do well to gauge his own precision and that of the instrument (and he may rest assured that his own will be the one chiefly in fault) by measuring, from any station, the angles between successive points all around him, till he gets back to the first point, beginning at different parts of the circle for each angle. The sum of all these angles *should* exactly equal 360°. He will probably find quite a difference from that.

Fig. 241, may be noted "At B, from A to C, 150°," or, better, "At B, between A and C, 150°." In column form, this becomes

Between A	150°	and C.
At	B	

When the vernier had been set at zero before sighting to the first object, and other objects were then sighted to, those objects, the readings to which were less than 180°, will be on the left of the first line, and those to which the readings were more than 180° will be on its right, looking in the direction in which the survey is proceeding, from A to B, and so on.

In surveying a farm, the angle of deflection at station, or the traverse angle, may be noted, together with the lengths of the courses.

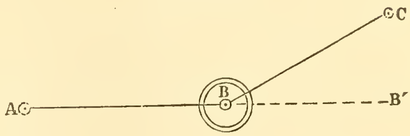
332. To repeat an Angle. Begin as in Art. 329, and measure the angle as there directed. Then unclamp below, and turn the circle around till the telescope is again directed to the first object, and made to bisect it precisely by the lower tangent-screw. Then unclamp above and turn the vernier-plate till the telescope again points to the second object, the first reading remaining unchanged. The angle will now have been measured a second time, but on a part of the circle adjoining that on which it was first measured, the second arc beginning where the first ended. The difference between the first and last readings will therefore be twice the angle.

This operation may be repeated a third, a fourth, or any number of times, always turning the telescope back to the first object by the lower movement (so as to start with the reading at which the preceding observation left off), and turning it to the second object by the upper movement. Take the difference of the first and last readings and divide by the number of observations.

The advantage of this method is that the errors of *observation* (i. e., sighting sometimes to the right and sometimes to the left of the true point) balance each other in a number of repetitions, while the constant error of *graduation* is reduced in proportion to this number. This beautiful principle has some imperfections in practice, probably arising from the slipping and straining of the clamps.

333. Angles of Deflection. The angle of deflection of one line from another is the angle which one line makes with the other line produced. Thus, in the figure, the angle of deflection of B C from A B is B′ B C. It is evidently the supplement of the angle A B C.

FIG. 242.



To measure it with the *Transit*, set the instrument at B, direct the telescope to A, and then turn it over. It will now point in the direction of A B produced, or to B′, if the second adjustment has been performed. Note the reading. Then direct the telescope to C. Note the new reading, and their difference will be the required angle of deflection, B′ B C.

If the vernier be set at zero before taking the first observation, the readings for objects on the right of the first line will be less than 180°, and more than 180° for objects on the left, conversely to Art. 331.

334. Line - Surveying. The survey of a line, such as a road, etc., can be made by the transit with great precision, measuring the angle which each line makes with the preceding line, and noting their lengths, and the necessary offsets on each side.

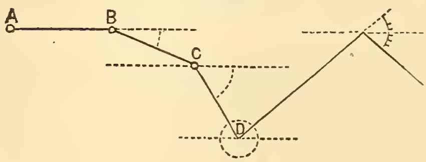
Short lines of sight should be avoided, since a slight inaccuracy in setting the center of the instrument exactly over or under the point previously sighted to would then much affect the angle. Very great accuracy can be obtained by using three tripods. One would be set at the first station and sighted back to from the instrument placed at the second station, and a forward sight be then taken to the third tripod placed at the third station. The instrument would then be set on this third tripod, a back-sight taken to the tripod remaining on the second station, and a fore-sight taken to the tripod brought from the first station to the fourth station, to which the instrument is next taken, and so on. This is especially valuable in surveys of mines.

The field-notes may be taken as directed in compass-surveying, the angles taking the place of the bearings. The "checks by intersecting bearings," before explained, should also be employed. The angles made on each side of the stations may both be measured, and the equality of their sum to 360° would at once prove the accuracy of the work.

If the magnetic bearing of any one of the lines be given, and that of any of the other lines of the series be required, it can be deduced by constructing a diagram, or by modifications of the rules given for the reverse object.

335. Traversing; or, surveying by the Back-Angle. This is a method of observing and recording the different directions of successive portions of a line (such as a road, the boundaries of a farm, etc.), so as to read off on the instrument, at each station, the angle which each line makes—not with the preceding line—but with the first line observed, or some other constant line. This line is, therefore, called the *meridian* of that survey.

FIG. 243.



The operation consists essentially in taking each back-sight by the lower motion (which turns the circle without changing the reading), and taking each forward sight by the upper motion, which moves the vernier over the arc measuring the new angle; and thus adds it to or subtracts it from the previous reading.

Set up the instrument at some station, as B; put the vernier at zero, and, by the lower motion, sight back to A. Tighten the lower clamp, reverse the telescope, loosen the upper clamp, sight to C by the upper motion, and clamp the vernier-plate again. Remove the instrument to C, sight back to B by the lower motion. Then clamp below, reverse the telescope, loosen the upper clamp, and sight to D by the upper motion. Then go to D and proceed as at C; and so on. The reading gives the angles measured to the right or "with the sun," as shown by the arcs in the figure.

Care should be taken to keep the same side of the instrument ahead, and, if only one vernier is read, to read from the same vernier.

The chief advantage of this method is its greater rapidity in the field and in platting, the angles being all laid down from one meridian, as in compass-surveying.

336. Use of the Compass. The chief use of the compass attached to a transit is as a check on the observations; for the difference between the magnetic bearings of any two lines should be the same, approximately, as the angle between them, measured by the more accurate instruments. The bearing also prevents any ambiguity as to whether an angle was taken to the right or to the left.

The instrument may also be used like a simple compass, the telescope taking the place of the sights, and requiring similar tests of accuracy. A more precise way of taking a bearing is to turn the plate to which the compass-box is attached, till the needle points to zero, and note the reading of the vernier; then sight to the object, and again read the vernier. The bearing will thus be obtained more minutely than the divisions on the compass-box could give it.

337. Ranging out Lines. This is the converse of surveying-lines. The instrument is fixed over the first station with great precision, its telescope being very carefully adjusted to move in a vertical plane. A series of stakes, with nails driven in their tops, or otherwise well defined, are then set in the desired line as far as the power of the instrument extends. It is then taken forward to a stake three or four from the last one set, and is fixed over it, first by the plumb and then by sighting backward and forward to the first and last stake. The line is then continued as before. A good object for a long sight is a board painted like a target, with black and white concentric rings, and made to slide in grooves cut in the tops of two stakes set in the ground about in the line. It is moved till the vertical hair bisects the circles (which the eye can determine with great precision), and a plumb-line dropped from their center gives the place of the stake. "Mason and Dixon's Line" was thus ranged.

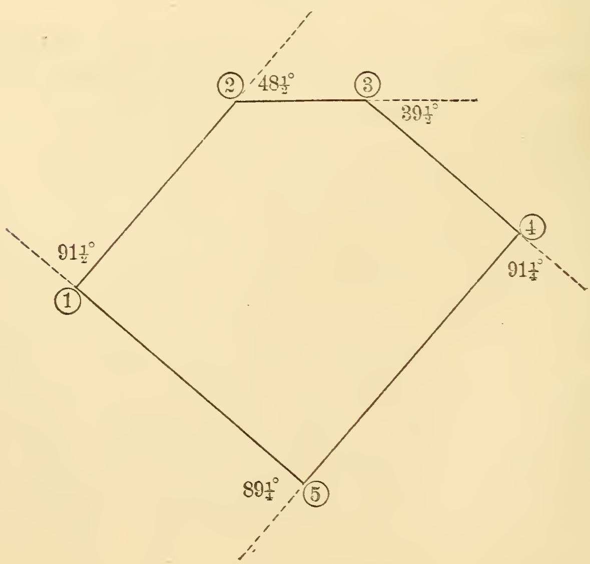
When the transit is used for ranging, its "Second Adjustment" is most important, to insure the accuracy of the reversal of its telescope.

338. Farm-Surveying, etc. A farm can be much more accurately surveyed with the transit than with the compass. The farm should be kept on the right hand, and then the angles measured will be the supplements of the interior angles. If the angles to the right be called *positive,* and those to the left *negative,* their algebraic sum should equal 360°.

If the boundary-lines be surveyed by "Traversing," the reading, on getting back to the last station and looking back to the first line, should be 360°, or 0°.

The content of any surface surveyed by "Traversing" with the transit can be calculated by the traverse-table, by the following modification : When the angle of deflection of any side from the first side, or meridian, is less than 90°, call this angle the bearing, find its latitude and departure, and call them both *plus.* When the angle is between 90° and 180°, call the difference between the

FIG. 244.



angle and 180° the bearing, and call its latitude *minus* and its departure *plus*. When the angle is between 180° and 270°, call its difference from 180° the bearing, and call its latitude *minus* and its departure *minus*. When the angle is more than 270°, call its difference from 360° the bearing, and call its latitude *plus* and its departure *minus*. Then use these as in getting the content of a compass-survey. The signs of the latitudes and departures follow those of the cosines and sines in the successive quadrants.

Fig. 244 is a plat of the survey worked out in Art. 255.

The following table gives the deflection angle at each station, the traverse angle (i. e., the angle which each line makes with the first one), and the reduced bearing, calling the first line (1 to 2) the meridian :

STATIONS.	DEFLECTION ANGLES.	TRAVERSE ANGLES.	BEARINGS.
1	$91\frac{1}{2}°$	0° or 360°	North.
2	$48\frac{1}{2}°$	$48\frac{1}{2}°$	N. $48\frac{1}{2}°$ E.
3	$39\frac{1}{2}°$	88°	N. 88° E.
4	$91\frac{1}{4}°$	$179\frac{1}{4}°$	S. $\frac{3}{4}°$ E.
5	$89\frac{1}{4}°$	$268\frac{1}{2}°$	S. $88\frac{1}{2}°$ W.

If the deflection angle at station 1 ($91\frac{1}{2}°$) be added to the traverse angle at station 5, the sum will be 360°.

Any side may be taken as the meridian of the survey.

If the true bearing of one side be known, the true bearings of the other sides may be determined by Art. 189.

The content is calculated by latitudes and departures, as in compass-surveying.

The latitudes and departures may be taken from the tables, interpolating for minutes as in Art. 242, or they may be calculated with a table of natural sines and cosines, as in Art. 240.

Example. FIELD-BOOK.

STATIONS.	ANGLES OF DEFLECTION.	DISTANCES IN CHAINS.
1	62° 15′	4·64
2	86° 38′	3·60
3	59° 20′	4·15
4	80° 6′	4·22
5	71° 41′	3·25

CALCULATION OF AREAS, CALLING COURSE 1 TO 2 THE MERIDIAN, AND USING SINES AND COSINES INSTEAD OF A TRAVERSE TABLE.

STATIONS.	BEARINGS.	DISTANCES.	SINES.	COSINES.	LATITUDES.		DEPARTURES.		DOUBLE LONGITUDES.	DOUBLE AREAS.	
					+	−	+	−		+	−
1	+00°, 00′+	4·64	·00000	1·00000	4·64		0·00		0·00	0·0000	
2	+86°, 38′+	3·60	·99827	·05873	0·21		3·59		+3·59	·7539	
3	−34°, 2′+	4·15	·55968	·82871		3·44	2·32		+9·50		32·6800
4	−46°, 4′−	4·22	·72015	·69382		2·93		3·04	+8·78		25·7254
5	+62°, 15′−	3·25	·88499	·46561	1·52			2·87	+2·87	4·3624	
					6·37	6·37	5·91	5·91		5·1163	58·4054
											5·1163
											2)53·2891
									Square chains,		26·6445

339. When the lengths of the sides are measured with an engineer's chain, and the distances are determined in feet, the process of calculating the area is the same as for chains and decimals. The area is obtained in square feet instead of square chains, and to reduce it to acres it will be necessary to divide by 43560, the number of square feet in an acre.

340. Platting. Any of these surveys can be platted by any of the methods explained and characterized in Chapter III. A circular protractor may be regarded as a theodolite placed on the paper. "Platting Bearings" can be employed when the survey has been made by "Traversing." But the method of "Latitudes and departures" is by far the most accurate.

THE GRADIENTER.

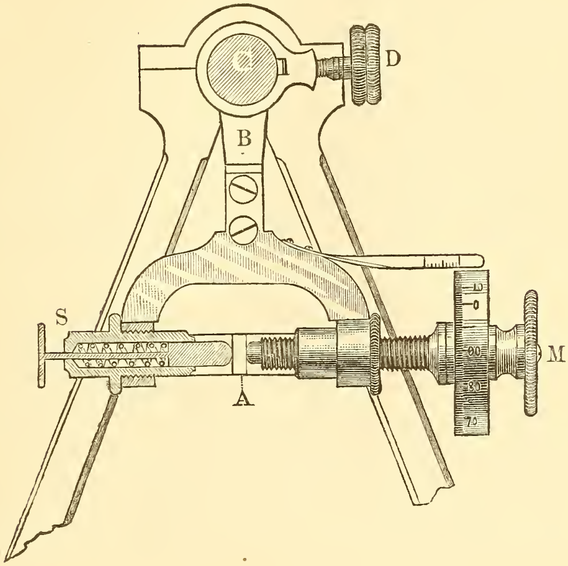
341. This is an attachment to the transit for determining grades and distances. It consists of an arm, attached to the axis of the telescope, and a micrometer-screw, by means of which the movement of the arm, and consequently of the telescope, can be accurately measured.

The arm is placed on the inside of one of the standards, and is attached to the telescope axis by means of a clamp-screw, so that it may be clamped or loosened at pleasure.

The method of measuring the movement of the arm is shown in Fig. 245.

C is a section of the axis of the telescope. B is the arm, which

Fig. 245.



is clamped to the axis by the screw D. M is the micrometer-screw. A is a lip projecting from a plate fastened to the standards.

The screw is accurately cut, so that one revolution of the screw will cause the horizontal cross-hair of the telescope to move over a given space (say one foot) on a rod held at a given distance, as 100 feet. The head of the screw is graduated into equal parts, usually 50 or 100. Above the graduated head is a scale so graduated that one revolution of the screw will move the head over one space on the scale. Thus the number of whole revolutions given to the screw may be read on the scale, and the parts of a revolution read on the graduated head.

The point of the screw presses against the lip, A, and is held firmly against it by the opposing spiral spring, S.

When the arm is made fast to the axis by the clamp-screw, D, and the gradienter-screw, M, is turned, it will turn the telescope vertically on its axis, and the distance which the horizontal cross-hair will pass over on a rod, toward which the telescope is pointed, will vary directly with the distance from the transit to the rod.

342. To establish Grades. Let us suppose that one revolution of the gradienter-screw will move the horizontal cross-hair over a space of one foot, on a rod held at a distance of 100 feet from the transit. Then, to set grades, we have only to level the telescope, clamp the gradienter-arm, and turn the micrometer-screw through as many divisions of the head (graduated into 100 parts) as there are hundredths of a foot rise or fall per hundred feet of horizontal distance; raising the cross-hair for an up-grade, and lowering it for a down-grade. The line of sight will then be on the required grade.

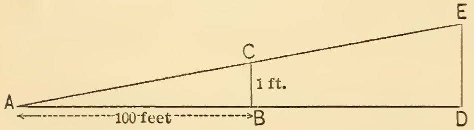
If the transit be set over a point of the required grade-line, set the target on the rod at the height of the center of the telescope-axis above the given point, and then the bottom of the rod, held at any point on the line, will be at a point in the desired grade-line when the horizontal cross-hair bisects the target.

Thus, if the grade is to be 1·64 feet per hundred, turn the screw one entire revolution and 64 of the divisions on the graduated head, and the line of sight will then be on the required grade.

343. To measure Distances. When the ground is level or approximately so, see what space on the rod the horizontal cross-hair moves over for one revolution of the gradienter-screw. Then the distance in feet will be equal to the space on the rod, expressed in feet and decimals, multiplied by 100.

Thus, if the space on the rod, moved over by the cross-hair for one revolution of the gradienter-screw, was 4·27 feet, the distance at which the rod was held was 427 feet.

Fig. 246.



For, in Fig. 246, let A be the position of the transit; C B, the reading on the rod, held at a distance of 100 feet, for one revolution of the screw; and D E the space passed over on the rod for one revolution of the screw when the rod is held at the unknown distance A D. It is evident that the triangles A B C and A D E are similar, and that

C B : A B :: E D : A D,

or, 1 : 100 :: 4·27 : 427.

If the rod sighted to is only graduated to feet—as an ordinary transit-rod—find how many revolutions and parts of revolutions will move the horizontal cross-hair over a whole number of feet on the rod. Then, since one revolution of the screw will move the cross-hair over a space of one foot on the rod at a distance of 100 feet, we have the proportion : *As* the number of revolutions of the screw (whole numbers and decimals) *is to* 100 feet, *so is* the number of feet passed over on the rod by the cross-hair *to* the required distance. For, from Fig. 246 we have, as before :

C B : A B :: D E : A D.

C B now represents what the reading on the rod (in feet and decimals), held at a distance of 100 feet, would be for the given number of revolutions : A B is 100′, D E is the reading on the rod in feet, and A D is the required distance.

Suppose, for example, the gradienter-screw be turned 1·25 time, and the space passed over on the rod by the cross-hair be 3 feet. Then we have :

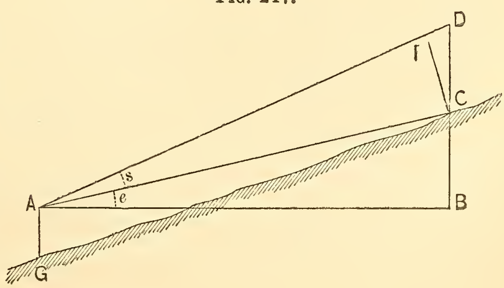
1·25 : 100 :: 3 : 240.

∴ The required distance is 240 feet.

Problem.—When no graduated rod is available, to determine a distance by using, in place of a rod, a stick whose length can afterward be measured.

On sloping ground, the methods given will apply, if the rod be held perpendicular to the line of sight. This, however, is not easily done. It will be better to apply methods specially adapted to sloping ground.

Fig. 247.



344. On Sloping Ground. In Fig. 247, let A be the position of the transit; G the point over which it is set; C where the rod is held; A B a horizontal line through the axis of the telescope; A C the distance from the horizontal axis of the telescope to the foot of the rod; and C D the distance, on a vertical rod, passed over by the horizontal cross-hair for one revolution of the gradienter-screw. Let C F be perpendicular to A C, and D B to A B.

Represent the angle of elevation, B A C, by e, the angle C A D by s, and the distance D C by k. Then we have:

$$DB = DC + CB.$$

$$\therefore AB \tan.(s+e) = k + AB \tan. e,$$

$$\text{and } AB = \frac{k}{\tan.(s+e) - \tan. e}.$$

For convenience of computation, this may be put in another form. Add and subtract $100\,k$, and we have:

$$AB = 100\,k - 100\,k + \frac{k}{\tan.(s+e) - \tan. e}.$$

And, since $\tan. s = \frac{1}{100}$,

$$AB = 100\,k - k\,(100 \sin. e + \cos. e) \sin. e.$$

TABLE FOR GRADIENTER.

ANGLE OF ELEVATION.	(100 SIN. e + COS.e) × SIN.e.
0°	·0
1°	·1
2°	·2
3°	·3
4°	·5
5°	·8
6°	1·2
7°	1·6
8°	2·1
9°	2·6
10°	3·2
11°	3·8
12°	4·5
13°	5·3
14°	6·1
15°	7·0
16°	7·9
17°	8·8
18°	9·8
19°	10·9
20°	12·0

The quantity $(100 \sin. e + \cos. e) \sin. e$, for angles from 1° to 20° will be found in the table for the gradienter. Hence the rule:

Multiply the rod-reading by 100, and deduct the product of the rod-reading by the tabular number corresponding to the angle of elevation, e. The result will be the horizontal distance A B.

Example. Angle of elevation, 4°; rod-reading, 2·63 feet.

$$2{\cdot}63 \times 100 = 263$$
$$2{\cdot}63 \times .5 = 1{\cdot}3$$

Horizontal distance, 261·7

The table for the correction is computed to tenths only, as the unavoidable errors in using the instrument would render any more exact computation useless.

For ordinary cases, when the angle of elevation is small, the computation for the distance and correction can be made mentally.

345. The horizontal distance, A B, is the one almost always required, as all measurements of distances in surveying and engineering should be made horizontally.

The distance from the transit to the point at which the rod is held (i. e., A C) is equal to the horizontal distance, A B, divided by cos. *e*.

The distance G C may be found by solving the triangle C A G, of which the sides A G and A C, and the included angle C A G, are known.

When the angle *e* is an angle of depression, the top of the rod is taken for the point *c*, and the distance C D is measured downward from the top of the rod.

In using the micrometer-screw, care must be taken, when measuring, to always turn the screw in the same direction, in order to avoid any lost motion in the screw. In determining the space passed over by the cross-hair for one revolution of the screw, set the screw back of the first reading, and bring it up by turning the screw in the same direction in which it is to be turned for making the measurement.

THE STADIA OR TELEMETER.

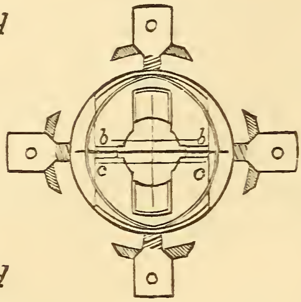
346. On the cross-hair ring of the telescope stretch two more horizontal cross-hairs of spider-web or platinum wire, at equal distances above and below the original one. The two additional wires are called *Stadia Wires.* The stadia wires may be either fixed or adjustable. In the former case they may be attached directly to the cross-hair ring. When they are adjustable, each may be fastened to a separate slide, actuated by a capstan-screw on the outside of the telescope-tube, as shown in Figs. 248 and 249.

The slides to which the stadia wires *b b* and *c c* are attached are held apart by the hoop-spring, shown in the figure, and are adjusted by the capstan-screws *d d.*

It is evident that, in looking through the telescope at a graduated rod, a certain portion of the rod will be intercepted between the stadia wires, and that the greater the distance at which the rod is held, the longer will be the space on the rod intercepted by the stadia wires.

Fig. 248.

Fig. 249.



Referring to Art. 287, Fig. 201, we see that the objective of the telescope forms an image, B, of the arrow, A. A may represent the part of the rod intercepted by the stadia wires, and B the distance between the wires. The farther the rod is carried from the telescope, the nearer the image is formed to the objective. If the rod were at an infinite distance, the image would be formed at the *principal focus* of the objective.

Call the distance of the principal focus from the lens, f; the distance from the lens to the rod held at any point, p; the distance from the lens to the image, q; the space intercepted on the rod by the stadia wires, k; and the distance apart of the stadia wires, a.

As p increases, k increases, q decreases, and a remains constant.

From similar triangles, Fig. 201, we have:

$$p : q :: k : a, \qquad [1.]$$

and from the principles of optics—

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}. \qquad [2.]$$

$$\text{From [1] } \frac{p}{q} = \frac{k}{a}.$$

$$\text{From [2] } \frac{p}{q} = \frac{p}{f} - 1.$$

$$\therefore \frac{p}{f} - 1 = \frac{k}{a}.$$

$$\text{and } p = \frac{f}{a}k + f. \qquad [3.]$$

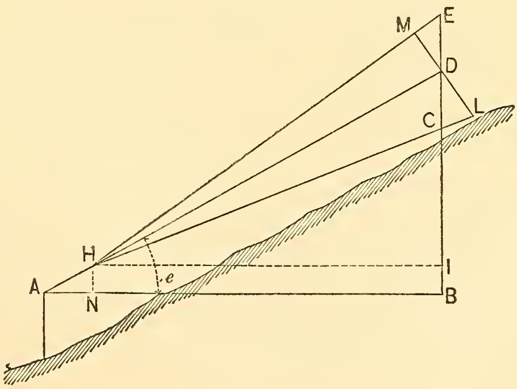
Formula [3] is not perfectly accurate, as p and q are measured from the surface of the lens instead of its center, and the objective of the telescope is not a simple double-convex lens. It is, however, sufficiently exact for this purpose.

We see by the formula [3] that, as f and a are constants, the distance, p, from the objective to the rod is equal to the reading on the rod, multiplied by a constant quantity, plus the principal focal distance of the objective. To obtain the distance from the center of the instrument to the rod, it is also necessary to add the distance from the center of the instrument to the objective. Call this distance c. Then, for the distance from the center of the instrument to the rod, we have:

$$\text{distance} = \frac{f}{a} k + f + c. \qquad [4.]$$

The distance from the objective to the center of the instrument is not precisely the same for all lengths of sight. The farther off the object sighted to is, the nearer the image will be formed to the objective, and hence the objective must be drawn in, in order that the image may be formed at the cross-hairs. When the object sighted to is near, the image is formed farther from the objective, and the objective-slide must be moved out in order that the image may be formed at the cross-hairs. Hence, we see that the quantity c is not rigidly constant. The difference in value, however, is not enough to be taken into consideration. A mean value of c can be determined by sighting to some object at a distance of the mean length of sight (say five hundred feet), and then measuring the distance from the objective to the center of the telescope-axis.

FIG. 250.



347. Formula [4] is for level ground. For sloping ground, this must be modified. In Fig. 250 let A be the center of the telescope-axis; C E, the reading on the rod; D, the point on the rod where the center cross-hair intersects the rod; A B, the horizontal distance; H, a point in front of the object-glass, and at a distance equal to its focal length; *e*, the angle of elevation; M L, perpendicular to the line of sight; *f*, *a*, *c*, and *k* as in [4]. Then we have:

$$\text{M L} = \text{C E} \cos. e = k. \cos. e \text{ and H D} = \frac{f}{a} k \cos. e.$$

$$\text{H I} = \text{H D} \cos. e = \frac{f}{a} k \cos.^2 e.$$

$$\text{A B} = \text{A N} + \text{N B} \; (= \text{H I}).$$

$$\therefore \; \text{A B} = (c + f) \cos. e + \frac{f}{a} k \cos.^2 e. \qquad [5.]$$

The height $\text{B D} = \text{A B} \tan. e$

$$\text{B D} = (c + f) \sin. e + \frac{f}{a} k \frac{\sin. 2 e}{2}. \qquad [6.]$$

To find the value of a in any case, measure off from the point over which the instrument is set a base-line, B (say one thousand feet), and hold the stadia-rod at the farther end. Let the reading on the rod be k'.

$$\text{Then, by [4] } \text{B} = \frac{f}{a} k' + f + c,$$

$$\text{and } a = \frac{f k'}{\text{B} - f - c}.$$

Substituting this value of a in equations [5] and [6], we have:

$$\text{Horizontal distance} = (c + f) \cos. e + \frac{k}{k'} (\text{B} - f - c) \cos.^2 e. \quad [7.]$$

$$\text{Difference of level} = (c + f) \sin. e + \frac{k}{2 k'} (\text{B} - f - c) \sin. 2 e. \quad [8.]$$

348. The Stadia-Tables* given in this volume were calculated from formulas [7] and [8], using the following values:

The measured base, B = 1,000 feet, and k' = the reading on the rod for that distance—i. e., the distance indicated by the stadia-reading is 1,000 feet.

---

* Calculated by Alfred Noble and William T. Casgrain, and used on the United States Lake Survey.

$$(c+f) = 1 \cdot 4 \text{ feet.}$$

The quantities in the columns headed a and b are computed respectively from the expressions $(c+f)$ cos. e, and $(c+f)$ sin. e, in the formulas. They are constant for all readings if the angle e remains the same.

The horizontal distances, and the differences of level, are computed by the tables in a manner similar to that employed in calculating latitudes and departures with a table.

Example. Let $e = 4° \ 27'$, and $k =$ reading corresponding to 1,384 feet when the ground is horizontal.

Take from the table as follows :

HORIZONTAL DISTANCE.		DIFFERENCE OF LEVEL.	
For 1,000........	992·6	For 1,000........	077·2
" 300........	297·78	" 300........	23·17
" 80........	79·407	" 80........	6·180
" 4........	3·9703	" 4........	·3090
" (c+f) cos. e..	1·3958	" (c+f) sin. e..	·1086
	1375·1531		106·9676

The difference of level given by formula [8] is the difference in height between the instrument at A and the point where the central cross-hair strikes the rod at C. The difference between the height of the instrument above the ground, and the height of C above the ground, must be applied as a correction to the difference of level, obtained by the formula, to get the true difference of height of the ground at the instrument, and at the rod.

349. The stadia-wires may be adjusted to use with a rod already graduated to feet and decimals, or, if the wires are fixed, a rod may be graduated to suit the wires.

In the first case the wires are adjusted so that one foot is included between the wires at a given distance (50 or 100 feet) plus the constant c. Suppose the space included between the wires was one foot, at a distance from the center of the instrument of 100 feet $+ c$. Then, if the reading on the rod held at some unknown distance was 3·46 feet, the distance would be 346 feet $+ c$.

If the wires are fixed, measure off from the center of the instrument 500 feet $+ c$, and note the space on the rod, intercepted by the cross-hairs at that distance. Divide this space into five equal parts, subdivide the parts to tenths and hundredths, and graduate the remainder of the rod with similar divisions. This rod can then be used in the same way as the rod, graduated to feet, was in the first case. Suppose, on holding up this rod at an unknown distance, that the stadia-wires intercepted 3·67 of the parts. Then the distance is 367 feet $+ c$.

The rod may be supplied with one or two targets, or may be used as a "speaking-rod"—that is, it may be graduated and marked so as to be read by the observer at the instrument.

For forms of targets, and methods of graduating and marking rods, see subject "RODS," Part II.

350. Several different formulas and methods have been used in stadia-surveying, depending upon the object and extent of the survey, and the degree of accuracy required. Another method is given in the following communication,* together with results in practice:

351. Results of Telemeter Traverse between Triangulation-Points on the Shores of Lake George, New York.

Instrument. Engineer's transit of W. & L. E. Gurley. Focal length = 0·565 feet; distance of cross-wires from center of instrument = 0·13 feet. One extra cross-wire was added to the diaphragm. At 103 feet from the center of the instrument, the distance included between the wires was found to be 1·0253 feet—

by the formula, $t = 0{\cdot}01005\, d - 0{\cdot}01$ feet, [1.]

or, $d = 99{\cdot}48\, t + {\cdot}1$ foot, [2.]

where t = distance included between the wires at any distance, d, from the center of the instrument.

Stadia-Rod or Telemeter. This was graduated especially for the instrument from formula [1], the zero of graduation being displaced 0·01 foot to allow for the constant of the formula. The least reading of the rod was $2\frac{1}{2}$ feet. Distances were estimated and recorded to single feet.

Circumstances of Measurement. Traverse-lines were run between triangulation-points; the distances between the latter were computed from the traverse and compared with the results from triangulation, in nine cases. The aggregate length of these nine lines was about $10\frac{4}{10}$ miles.

Four closed traverses were run around islands, and the errors of closure were obtained.

The lines of sight generally passed over water, which circumstance was favorable to precise reading.

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* From Horace Andrews, C. E., assistant on New York State Survey.

The results of comparison are given below. They indicate that the constants used in graduating the telemeter-rod were not exactly obtained. The error of measurement averaged + 2·2 feet to 1,000. If this allowance had been made in graduating the rod, or this constant error had been allowed for, the purely accidental errors would have been only ± 1·2 foot to the 1,000. The law of propagation of errors of length is favorable to close linear measurements with the telemeter upon traverse-lines, as was found to be actually the case here. In traverse-lines, the larger part of the total error is due to angular errors which overweigh the linear ones, unless exceptional means are taken to avoid this.

(1)	(2)	(3)	(4)	(5)
feet.	feet.	feet.	feet.	
5183·5	+12·8	+2·47	+0·31	9
3988·0	+ 7·5	+1·88	−0·28	7
4925·7	+ 7·6	+1·54	−0·62	9
8427·8	+11·7	+1·39	−0·77	17
2995·0	+15·0	+5·01	+2·85	7
3104·6	+ 9·7	+3·12	+0·96	5
9593·2	+20·2	+2·11	−0·05	15
6987·9	+ 6·0	+0·86	−1·30	20
9850·0	+10·0	+1·02	−1·14	20
55055·7		+2·16	$\pm 1{\cdot}21 = \sqrt{\frac{ss}{n}}$	

(1) = Distances between triangulation-points as computed from traverse.

(2) = Distance by traverse minus distance by triangulation.

(3) = Error to 1,000 feet, including constant error.

(4) = Purely accidental error to 1,000 feet.

(5) = Number of sides to traverse, or number of stadia-readings.

CLOSED TRAVERSES.

LOCALITY.	(1)	(2)	(3)	(4)
	feet.	feet.	feet.	feet.
Mother Bunch Islands.	4061	13·9	14	3·42
Vicar's Island.........	2316	7·1	10	3·06
Harbor Islands.......	5722	1·9	12	0·33
Hatcher Island.......	1610	3·5	6	2·17

(1) = Sum of distances by traverse.

(2) = Closing error.

(3) = Number of sides to closed traverse.

(4) = Error to 1,000 feet, including constant error.

352. In 1881 a stadia-survey for a road was made in Mexico,* from Culiacan to Durango. Two different routes were followed, one in going up the mountains to Durango, and the other on the return to Culiacan. The total distance run was 606 miles, and difference of elevation 11,000 feet. When the entire traverse was closed, the error of closure was found to be 1,100 feet.

* The greater part of the work was done by W. B. Landreth, C. E.

CHAPTER V.

OBSTACLES IN ANGULAR SURVEYING.

353. The obstacles, such as trees, houses, hills, valleys, rivers, etc., which prevent the direct alinement or measurement of any desired course, can be overcome much more easily and precisely with any angular instrument than with the chain, methods for using which were explained in Chapter II. They will, however, be taken up in the same order. As before, the given and measured lines are drawn with fine full lines; the visual lines with broken lines; and the lines of the result with heavy full lines. Part of the demonstrations of the problems are given, and part are left as exercises for the student.

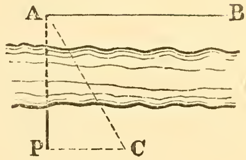
PERPENDICULARS AND PARALLELS.

354. Erecting Perpendiculars. *To erect a perpendicular to a line at a given point,* set the instrument at the given point, and, if it be a *compass,* direct its sights on the line, and then turn them till the new bearing differs 90° from the original one. A convenient approximation is to file notches in the compass-plate, at the 90° points, and stretch over them a thread, sighting across which will give a perpendicular to the direction of the sights.

The *transit* being set as above, note the reading of the vernier, and then turn it till the new reading is 90° more or less than the former one.

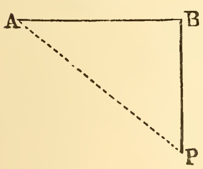
355. *To erect a perpendicular to an inacessible line, at a given point of it.* Let A B be the line and A the point. Calculate the distance from A to any point C, and the angle C A B, by the method of Art. 381. Set the instrument at C, sight to A, turn an angle = C A B, and measure in the direction thus obtained a distance C P = C A . cos. C A B. P A will be the required perpendicular.

Fig. 251.



356. Letting fall Perpendiculars. *To let fall a perpendicular to a line from a given point.* With the *compass,* take the bearing of the given line, and then from the given point run a line, with a bearing differing 90° from the original bearing, till it reaches the given line.

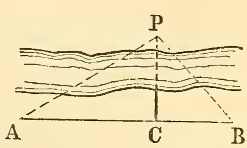
Fig. 252.



With the *transit,* set it at any point of the given line, as A, and observe the angle between this line and a line thence to the given point, P. Then set at P, sight to the former position of the instrument, and turn a number of degrees equal to what the observed angle at A wanted of 90°. The instrument will then point in the direction of the required perpendicular P B.

357. *To let fall a perpendicular to a line from an inaccessible point.* Let A B be the line and P the point. Measure the angles P A B and P B A. Measure A B. The angles A P C and B P C are known, being the complements of the angles measured. Then is $AC = AB \cdot \dfrac{\tan. APC}{\tan. APC + \tan. BPC}$.

Fig. 253.



Proof: A C = P C . tan. A P C; and C B = P C . tan. B P C [Trigonometry, Art. 4].

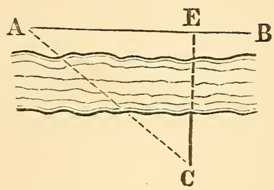
Hence A C : C B :: tan. A P C : tan. B P C; and

A C : A C + C B :: tan. A P C : tan. A P C + tan. B P C.

Consequently, since A C + C B = A B, $AC = AB \cdot \dfrac{\tan. APC}{\tan. APC + \tan. BPC}$.

358. *To let fall a perpendicular to an inaccessible line from a given point.* Let C be the point and A B the line. Calculate the

Fig. 254.

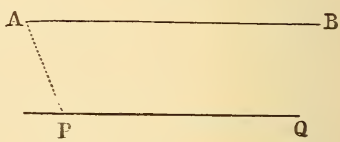


angle C A B by the method of Art. 381. Set the instrument at C, sight to A, and turn an angle = 90 − C A B. It will then point in the direction of the required perpendicular, C E.

359. Running Parallels. *To trace a line through a given point parallel to a given line.* With the *compass,* take the bearing of the given line, and then, from the given point, run a line with the same bearing.

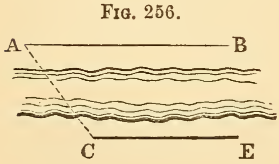
With the *transit* or *theodolite,* set it at any convenient point of the given line, as A, direct it on this line, and note the reading. Then turn the vernier till the cross-hairs bisect the given point, P. Take the instrument to this point and sight back to the former station, by the lower motion, without changing the reading. Then move the vernier till the reading is the same as it was when the telescope was directed on the given line, or 180° different. It will then be directed on P Q, a parallel to A B, since equal angles have been measured at A and P. The manner of reading them is similar to the method of "Traversing."

Fig. 255.



360. *To trace a line through a given point parallel to an inaccessible line.* Let C be the given point and A B the inaccessible line. Find the angle C A B, as in Art. 381. Set the instrument at C, direct it to A, and then turn it so as to make an angle with C A equal to the supplement of the angle C A B. It will then point in a direction, C E, parallel to A B.

Fig. 256.



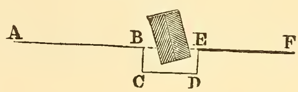
OBSTACLES TO ALINEMENT.

A. To prolong a Line.

361. The instrument being set at the farther end of a line and directed back to its beginning, the sights of the *compass*, if that be used, will at once give the forward direction of the line. A distant point being thus obtained, the compass is taken to it and the process repeated. The use of the *transit* for this purpose has been fully explained.

362. By Perpendiculars. When a tree or house obstructing the line is met with, place the instrument at a point B of the line, and set off there a perpendicular to C; set off another at C to D, a third at D to E, making D E = B C, and a fourth at E, which last will be in the direction of A B prolonged. If perpendiculars can not be conveniently used, let B C and D E make any equal angles with the line A B, so as to make C D parallel to it.

Fig. 257.



363. By an Equilateral Triangle. At B turn aside from the line at an angle of 60°, and measure some convenient distance B C. At C turn 60° in the contrary direction, and measure a distance C D = B C. Then will D be a point in the line A B prolonged. At D turn 60° from C D prolonged, and the new direction will be in the line of A B prolonged. This method requires the measurement of one angle less than the preceding.

Fig. 258.

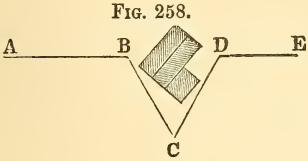
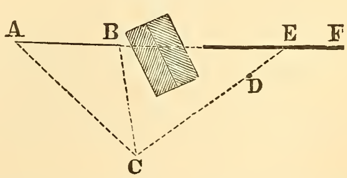


Fig. 259.

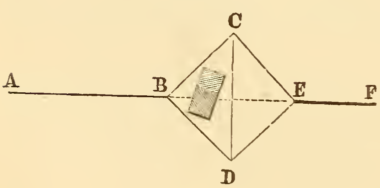


364. By Triangulation. Let A B be the line to be prolonged. Choose some station C, whence can be seen A, B, and a point beyond the obstacle. Measure A B and the angles A and B of the triangle A B C, and thence calculate the side A C. Set the instrument at C, and measure the angle A C D, C D being any line which will clear the obstacle. Let E be the desired point in the lines A B and C D prolonged. Then in the triangle A C E will be known the side A C and its including angles, whence C E can be calculated. Measure the resulting distance on the ground, and its extremity will be the desired point E. Set the instrument at E, sight to C, and turn an angle equal to the supplement of the angle A E C, and you will have the direction, E F, of A B prolonged.

365. When the Line to be prolonged is inaccessible. In this case, before the preceding method can be applied, it will be necessary to determine the lengths of the lines A B and A C, and the angle A, by the method given in Art. 381.

366. To prolong a Line with only an Angular Instrument. This may be done when no means of measuring any distance can be obtained. Let A B be the line to be prolonged. Set the instrument at B and deflect angles of 45° in the directions C and D. Set at some point, C, on one of these lines and deflect from C B 45°, and mark the point D where this direction intersects the direction B D. Also, at C, deflect 90° from B. Then, at D, deflect 90° from D B. The intersections of these last directions will fix a point E. At E deflect 135° from E C or E D, and a line E F, in the direction of A B, will be obtained and may be continued.*

FIG. 260.



B. To interpolate Points in a Line.

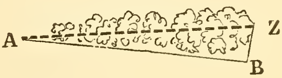
367. The instrument being set at one end of a line and directed to the other, intermediate points can be found, etc. If a valley in-

* This ingenious contrivance is due to Mr. R. Hood, in whose practice, while running an air-line for a railroad, the necessity occurred.

tervenes, the sights of the compass (if the compass-plate be very carefully kept level crosswise), or the telescope of the transit, answer as substitutes for the plumb-line.

368. By a Random Line. When a wood, hill, or other obstacle prevents one end of the line, Z, from being seen from the other, A, run a random line A B with the compass or transit, etc., as nearly in the desired direction as can be guessed, till you arrive opposite the point Z. Measure the error, B Z, at right angles to A B, as an offset. Multiply this error by $57\frac{3}{10}$, and divide the product by the distance A B. The quotient will be the degrees and decimal parts of a degree contained in the angle B A Z. Add or subtract this angle to or from the bearing or reading with which A B was run, according to the side on which the error was, and start from A, with this corrected bearing or reading, to run another line, which will come out at Z, if no error has been committed.

FIG. 261.



Example: A random line was run, by compass, with a bearing of S. 80° E. At twenty chains distance a point was reached opposite to the desired point, and ten links distant from it on its right. Required the correct bearing.

Ans. By the rule, $\frac{10 \times 57^\circ{\cdot}3}{2{,}000} = 0^\circ{\cdot}2865 = 17'$. The correct bearing is therefore S. 80° 17′ E. If the transit had been used, its reading would have been changed for the new line by the same 17′. A simple diagram of the case will at once show whether the correction is to be added to the original bearing or angle, or subtracted from it.

If trigonometrical tables are at hand, the correction will be more precisely obtained from this equation: $\text{Tan. } B\,A\,Z = \frac{B\,Z}{A\,B}$.

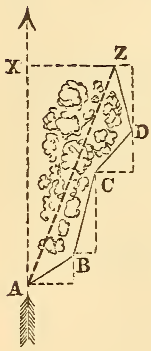
In this example, $\frac{B\,Z}{A\,B} = \frac{10}{2{,}000} = {\cdot}005 = \text{tan. } 17'$.

The 57°·3 rule, as it is sometimes called, may be variously modified. Thus, multiply the error by 86°, and divide by one and a half time the distance; or, to get the correction in minutes, multiply by 3,438 and divide by the distance; or, if the error is given in feet and the distance in four-rod chains, multiply the former by 52 and divide by the distance, to get the correction in minutes.

The correct line may be run with the bearing of the random line by turning the vernier for the correction.

369. By Latitudes and Departures. When a single line, such as A B, can not be run so as to come opposite to the given point Z, proceed thus with the *compass:* Run any number of zigzag courses, A B, B C, C D, D Z, in any convenient direction, so as at last to arrive at the desired point. Calculate the latitude and departure of each of these courses and take their *algebraic* sums. The sum of the latitudes will be equal to A X, and that of the departures to X Z. Then is $\tan. ZAX = \frac{XZ}{XA}$; i. e., the algebraic sum of the departures divided by the algebraic sum of the latitudes is equal to the tangent of the bearing.*

FIG. 262.



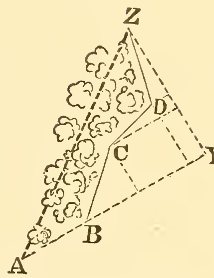
370. When the *transit* is used, any line may be taken as a meridian—i. e., as the line to which the following lines are referred; as in "Traversing," Art. 335, all the successive lines were referred to the first line. In Fig. 263 the same lines as in the preceding figure are represented, but they are referred to the first course, A B, instead of to the magnetic meridian as before, and their latitudes are measured along its produced line, and its departures perpendicular to it. As before, a right-angled triangle will be formed, and the angle Z A Y will be the angle at A between the first line A B and the desired line A Z.

This method of operation has many useful applications, such as in obtaining data for running railroad-curves, etc., and the student should master it thoroughly.

* The length of the line A Z can also be at once obtained, since it is equal to the square root of the sum of the squares of A X and X Z, or to the latitude divided by the cosine of the bearing.

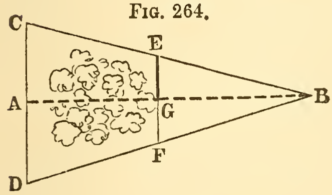
The desired angle (and at the same time the distance) can be obtained, approximately, in this and the preceding case, by finding in a traverse-table the final latitude and departure of the desired line (or a latitude and departure having the same ratio), and the bearing and distance corresponding to these will be the angle and distance desired.

Fig. 263.



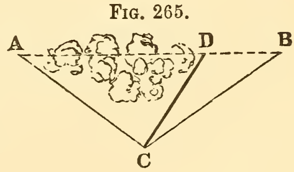
371. By Similar Triangles. Through A measure any line C D. Take a point E, on the line C B, beyond the obstacle, and from it set off a parallel to C D, to some point, F, in the line D B. Measure E F, C D, and C A. Then this proportion, C D : C A : : E F : E G, will give the distance E G, from E to a point in the line A B. So for other points.

Fig. 264.



372. By Triangulation. When obstacles prevent the preceding methods being used, if a point, C, can be found from which A and B are accessible, measure the distances C A, C B, and the angle A C B, and thence calculate the angle C A B. Then observe any angle A C D beyond the obstacle. In the triangle A C D a side and its including angles are known to find C D. Measure it, and a point, D, in the desired line will be obtained.

Fig. 265.

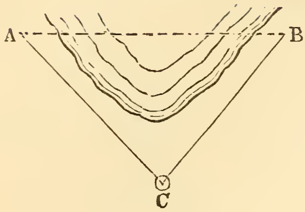


OBSTACLES TO MEASUREMENT.

A. When Both Ends of the Line are accessible.

373. The methods given in the preceding articles for prolonging a line and for interpolating points in it will generally give the length of the line by the same operation. The method of latitudes and departures is very generally applicable. So is the following.

FIG. 266.



374. By Triangulation. Let A B be the inaccessible distance. From any point, C, from which both A and B are accessible, measure C A, C B, and the angle A C B. Then in the triangle A B C two sides and the included angle are known to find the side A B.*

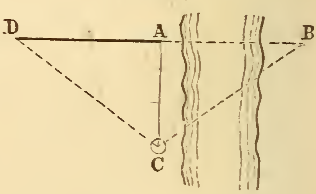
375. By Angles to Known Points. The length of a line, both ends of which are accessible, may also be determined by angles measured at its extremities between it and the directions of two or more known points. But, as the methods of calculation involve subsequent problems, they will be postponed.

B. WHEN ONE END OF THE LINE IS INACCESSIBLE.

376. By Perpendiculars. Many of the methods given for the chain may be still more advantageously employed with angular instruments, which can so much more easily and precisely set off the perpendiculars.

377. By Equal Angles. Let A B be the inaccessible line. At A set off A C perpendicular to A B, and as nearly equal to it, by estimation, as the ground will permit. At C measure the angle A C B, and turn the sights or vernier till A C D = A C B. Find the point, D, at the intersection of the lines C D and B A produced. Then is A D = A B.

FIG. 267.

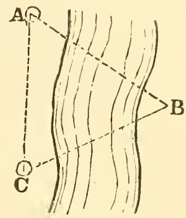


378. By Triangulation. Measure a distance A C, about equal to A B. Measure the angles at A and C. Then, in the triangle A B C, two angles and the included side are known, to find another side, $\text{A B} = \dfrac{\text{A C sin. A C B}}{\text{sin. A B C}}$.

* In this figure and the following ones the angular point inclosed in a circle indicates the place at which the instrument is set.

When the compass is used, the angles between the lines will be deduced from their respective bearings.

FIG. 268.

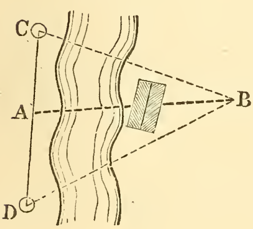


If the angle at A is 90°, A B = A C . tang. A C B.

If the angle A C B = 45°, then A C = A B; but this position could not easily be obtained, except by the use of the sextant, a reflecting instrument, described in Part V.

379. When One Point can not be seen from the other. Choose two points, C and D, in the line of A, and such that from C, A, and B can be seen, and from D, A, and B. Measure A C, A D, and the angles C and D. Then, in the triangle B C D, are known two angles and the included side, to find C B. Then, in the triangle A B C, are known two sides and the included angle, to find the third side, A B.

FIG. 268′.



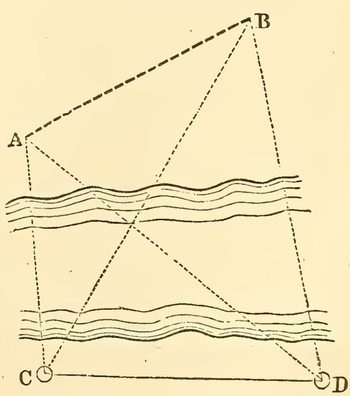
380. To find the Distance from a Given Point to an Inaccessible Line. In Fig. 254, Art. 358, the required distance is C E. The operations therein directed give the line C A and the angle C A B, or C A E. The required distance C E = C A . sin. C A E.

C. WHEN BOTH ENDS OF THE LINE ARE INACCESSIBLE.

381. General Method. Let A B be the inaccessible line. Measure any convenient distance, C D, and the angles A C D, B C D, A D C, B D C.

Then, in the triangle C D A, two angles and the included side are given, to find C A. In the

FIG. 269.



triangle C D B, two angles and the included side are given, to find C B. Then, in the triangle A B C, two sides and the included angle are given, to find A B.

The work may be verified by taking another set of triangles, and finding A B from the triangle A B D instead of A B C.

The following formulas will, however, give the desired distances with less labor:

Find an angle K, such that $\tan. K = \frac{\sin. ADC \,.\, \sin. CBD}{\sin. CAD \,.\, \sin. BDC}$.

Then find the difference of the unknown angles in the triangle C A B from the formula—

$$\text{Tan.} \tfrac{1}{2}(CAB - ABC) = \tan. (45^\circ - K) \,.\, \cot. \tfrac{1}{2} ACB.$$

$$\text{Then is } CAB = \tfrac{1}{2}(CAB - ABC) + \tfrac{1}{2}(CAB + ACB).$$

$$\text{Finally, } AB = CD \frac{\sin. BDC \,.\, \sin. ACB}{\sin. CBD \,.\, \sin. CAB}.$$

Demonstration: In the triangle A B C, designate the angles as A, B, C; and the sides opposite to them as a, b, c. Let C D $= d$. The triangle B C D gives [Trig., Art. 12, Theorem I], $a = d \frac{\sin. BDC}{\sin. CBD}$. The triangle A C D similarly gives $b = d \,.\, \frac{\sin. ADC}{\sin. CAD}$.

In the triangle A B C, we have [Trig., Art. 12, Theorem II],

$$\tan. \tfrac{1}{2}(A - B) : \cot. \tfrac{1}{2} C :: a - b : a + b;$$

whence $$\tan. \tfrac{1}{2}(A - B) = \frac{a - b}{a + b} \,.\, \cot. \tfrac{1}{2} C. \qquad [1.]$$

Let K be an auxiliary angle, such that $b = a \,.\, \tan. K$; whence $\tan. K = \frac{b}{a}$.

Dividing the second member of equation [1], above and below, by a, and substituting tan. K for $\frac{b}{a}$, we get $\tan. \tfrac{1}{2}(A - B) = \frac{1 - \tan. K}{1 + \tan. K} \,.\, \cot. \tfrac{1}{2} C$.

Since $\tan. 45^\circ = 1$, we may substitute it for 1 in the preceding equation, and we get $\tan. \tfrac{1}{2}(A - B) = \frac{\tan. 45^\circ - \tan. K}{\tan. 45^\circ + \tan. K} \,.\, \cot. \tfrac{1}{2} C$.

From the expression for the tangent of the difference of two arcs [Trig., Art. 8], the preceding fraction reduces to $\tan. (45^\circ - K)$; and the equation becomes

$$\tan. \tfrac{1}{2}(A - B) = \tan. (45^\circ - K) \,.\, \cot. \tfrac{1}{2} C. \qquad [2.]$$

In the equation $\tan. K = \frac{b}{a}$, substitute the values of b and a from the formulas at the beginning of this investigation. This gives

$$\tan. K = d \,.\, \frac{\sin. ADC}{\sin. CAD} \div d \,.\, \frac{\sin. BDC}{\sin. CBD} = \frac{\sin. ADC \,.\, \sin. CBD}{\sin. CAD \,.\, \sin. BDC}.$$

(A − B) is then obtained by equation [2]; (A + B) is the supplement of C; therefore, the angle A is known.

Then $c = \text{A B} = \dfrac{a \text{ sin. C}}{\text{sin. A}} = \dfrac{d \text{ . sin. B D C . sin. A C B}}{\text{sin. C B D . sin. C A B}}$.

The use of the auxiliary angle K avoids the calculation of the sides a and b.

Example. Let C D = 7,106·25 feet; A C D = 95° 17′ 20″; B C D = 61° 41′ 50″; A D C = 39° 38′ 40″; B D C = 78° 35′ 10″; required A B.

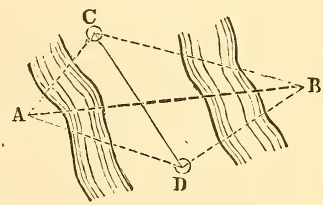
The figure is constructed with these data on a scale of 5,000 feet to 1 inch = 1 : 60000.

By the above formulas, K is found to be 30° 26′ 5″; C A B = 113° 55′ 37″; and, lastly, A B = 6598·32.

Both the methods may be used as mutual checks in any important case.

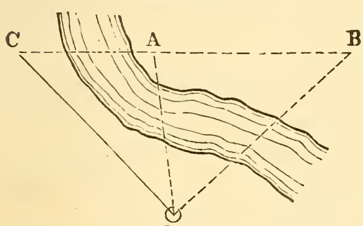
If the lines A B and C D crossed each other, as in Fig. 270, instead of being situated as in the preceding figure, the same method of calculation would apply.

FIG. 270.



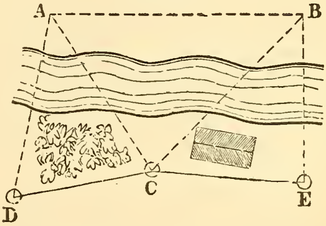
382. Problem. *To measure an inaccessible distance*, A B, *when a point*, C, *in its line can be obtained.* Set the instrument at a point, D, from which A, B, and C can be seen, and measure the angles C D A and A D B. Measure also the line D C and the angle C. Then in the triangle A C D two angles and the included side are given to find A D. In the triangle D A B, the angle D A B is known (being equal to A C D + C DA), and A D having been found, we again have two angles and the included side to find A B.

FIG. 271.



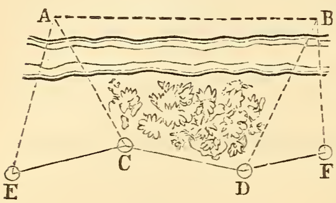
383. Problem. *To measure an inaccessible distance*, A B, *when only one point*, C, *can be found from which both ends of the line can be seen.* Consider C A and C B as distances to be determined, having one end accessible. Determine them as in Art. 378, by choosing a point D, from which C and A are visible, and a point E, from which C and B are visible. At C observe the angles D C A, A C B, and B C E. Measure the distances C D and C E. Observe the angles A D C and B E C. Then in the triangle A D C, two angles and the included side are given, to find C A; and the same in the triangle C B E, to find C B. Lastly, in the triangle A C B two sides and the included angle are known, to find A B.

FIG. 272.



384. Problem. *To measure an inaccessible distance*, A B, *when no point can be found from which the two ends can be seen.* Let C be a point from which A is visible, and D a point from which B is visible, and also C. Measure C D. Find the distances C A and D B, as in the preceding problem, i. e., choose a point E, from which A and C are visible, and another point, F, from which D and B are visible. Measure E C and D F. Observe the angles A E C, E C A, B D F, and D F B; and at the same time the angles A C D and C D B, for the subsequent work. Then C A and D B will be found, as were C A and C B in the last problem. Then in the triangle C D B, two sides and the included angle are known to find C B and the angle D C B; and, lastly, in the triangle A C B, two sides and the included angle (the difference of A C D and D C B) to find A B.

Fig. 273.

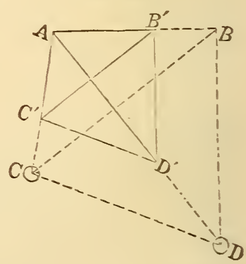


385. Problem. *Given the angles observed, at the ends of a line which can not be measured, between it and the ends of a line of known length but inaccessible, required the length of the former line.* This problem is the converse of that given in Art. 381. Its figure, 269, may represent the case, if the distance A B be regarded as known and C D as that to be found. Use the first and second formulas as before, and invert the last formula, obtaining

$$C D = A B \frac{\sin. C B D \;.\; \sin. C A B}{\sin. B D C \;.\; \sin. A C B}.$$

This problem may also be solved, indirectly, by assuming any length for C D, and thence calculating, as in the first part of Art. 381, the length of A B on this hypothesis. The imaginary figure thus calculated is *similar* to the true one: and the true length of C D will be given by this proportion: Calculated length of A B : true length of A B : : assumed length of C D : true length of C D.

Fig. 274.

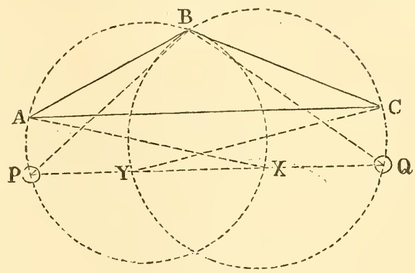


The length of C D can also be obtained graphically. Take a line of any length, as C′ D′, and from C′ and D′ lay off angles equal to those observed at C and D, and thus fix points A, B′. Produce A B′ till it equals the given distance A B, on any desired scale. From B draw a parallel to B′ D′, meeting A D′ produced in D; and from D draw a parallel to D′ C′ meeting A C′ produced in C. Then C D will be the required distance to the same scale as A B.

386. Problem. *Three points*, A B C, *being given by their distances from each other, and two other points*, P *and* Q, *being so situated that from each of them two of the three points can be seen and the angles* A P Q, B P Q. C Q P, B Q P, *be measured, it is required to determine the positions of* P *and* Q.

Construction. Begin by describing a circle passing through A and B, and having the central angle subtended by A B, equal to twice the given angle A P B, and thus containing that angle. The point P will lie somewhere in its circumference. Describe another circle passing through B and C, and having a central angle subtended by B C equal to twice the given angle B Q C. The point Q will lie somewhere in its circumference. From A draw a line making with A B an angle = B P Q, and meeting at X the circle first drawn. From C draw a line making with C B an angle = B Q P, and meeting the second circle in Y. Join X Y and produce it till it cuts the circles in points P and Q, which will be those required; since B P X = B A X = B P Q; and B Q Y = B C Y = B Q P.

Fig. 275.



Calculation. In the triangle A B C, the sides being given, the angle A B C is known. In the triangle A B X, a side and all the angles are known, to find B X. In the triangle C B Y, B Y is similarly found. By subtracting the angle A B C from the sum of the angles A B X and C B Y, the angle X B Y can be obtained. Then in the triangle X B Y, the sides B X, B Y, and the included angle are given to find the other angles. Then in the triangle B P X are known all the angles and the side B X to find B P. In the triangle B Q Y, B Q is found in like manner. Finally, in the triangle B P Q, P Q can then be found.

If desired, we can also obtain A P in the triangle A P B; and C Q in the triangle C B Q.

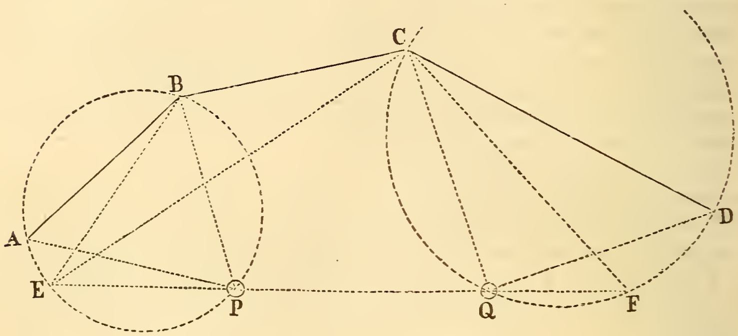
387. Problem. *Four points,* A, B, C, D, *being given in position, by their mutual distances and directions, and two other points,* P *and* Q, *being so situated that from each of them two of the four points can be seen and the angles* A P B, A P Q, P Q C, *and* P Q D *measured, it is required to determine the position of* P *and* Q.

Construction. Begin, as in the last article, by describing on A B the segment of a circle to contain an angle equal to A P B. From B draw a chord B E, making an angle with B A equal to the supplement of the angle A P Q. On C D describe another segment to contain an angle equal to C Q D. From C draw a chord C F, making an angle with C D equal to the supplement of the angle D Q P. Draw the line E F, and it will cut the two circles in the required points P and Q.

For, the angle A P Q in the figure equals the measured angle A P Q, because the supplement of the former, E P A, equals the supplement of the latter, since it is measured by the same arc as the angle A B E, equal to that supplement by construction. So too with the angle D Q P.

CALCULATION. To obtain $PQ = EF - EP - QF$, we proceed to find those three lines thus: In the triangle ABE, we know the side AB, the angle ABE, and the angle $AEB = APB$; whence to find EB. In the same way, the triangle CFD gives FC. In the triangle EBC are known

FIG. 276.



EB and BC, and the angle $EBC = ABC - ABE$; whence EC and the angle ECB are found. In the triangle ECF are known EC, FC, and the angle $ECF = BCD - ECB - FCD$; whence we find EF, and the angles CEF and CFE.

In the triangle BEP, we have EB, the angle $BEP = BEC + CEP$, and the angle $BPE = BPA + APE$; to find EP and PB. In the triangle QCF, we have CF, and the angles CQF and CFQ, to find QC and QF. Then we know $PQ = EF - EP - QF$.

The other distances, if desired, can be easily found from the above data, some of the calculations, not needed for PQ, being made with reference to them. In the triangle ABP, we know AB, BP, and the angle BAP, to find the angle ABP and AP. In the triangle QDC we know QC, CD, and the angle CQD, to find the angle QCD and QD. In the triangle PBC, we know PB, BC, and the angle $PBC = ABC - ABP$, to find PC. Lastly, in the triangle QCB, we know QC, CB, and the angle $QCB = DCB - DCQ$, to find QB.

The solution of this problem includes the two preceding; for, let the line BC be reduced to a point so that its two ends come together and the three lines become two, and we have the problem of Art. 386; and let the line AB be reduced to a point, B, and CD to a point, C, and we have but one line, and the problem becomes that of Art. 385.

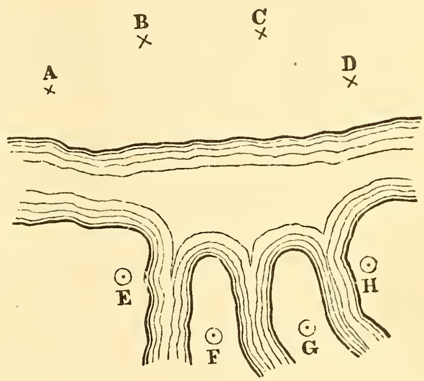
In these three problems, if the two stations lie in a right line with one of the given points, the problem is indeterminate.

388. Problem of the Eight Points. *Four points*, A, B, C, D, *are inaccessible, but visible from four other points*, E, F, G, H; *it is required to find the relative distances of these eight points; the only data being the observa-*

tion, from each of the points of the second system, of the angles under which are seen the points of the first system.

This problem can be solved, but the great length and complication of the investigation and resulting formulas render it more a matter of curiosity than of utility. It may be found in Puissant's "Topographie," page 55; Lefevre's "Trigonométrie," page 90, and Lefevre's "Arpentage," No. 387.

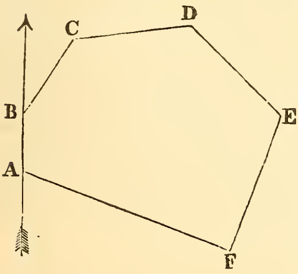
FIG. 277.



TO SUPPLY OMISSIONS.

389. Any two omissions in a closed survey, whether of the direction or of the length, or of both, of one or more of the sides bounding the area surveyed, can always be supplied by a suitable application of the principle of latitudes and departures, although this means should be resorted to only in cases of absolute necessity, since any omission renders it impossible to "test the survey." In the following articles the survey will be considered to have been made with the compass. All the rules will, however, apply to a transit survey, the angles being referred to any line as a meridian, as in "traversing."

FIG. 278.



To save unnecessary labor, the examples in the various cases now to be examined will all be taken from the same survey, a plat of which is given in the margin on the scale of 40 chains to 1 inch (1 : 31,680), and the field-notes of which, with the latitudes and departures carried out to five decimal places, are given on page 258.*

* The teacher can make any number of examples for his own use by taking a tolerably accurate survey, striking out the bearing and distance of any one course, and calculating it precisely as in Case 1, given below. He can then omit any two quantities at will, to be supplied by the student by means of the rules now to be given.

STATIONS.	BEARINGS.	DISTANCE IN LINKS.	LATITUDES.		DEPARTURES.	
			N.	S.	E.	W.
A	North.	1284	1284·00000		0	0
B	N. 32° E.	1782	1511·22171		944·31619	
C	N. 80° E.	2400	416·75568		2363·53872	
D	S. 48° E.	2700		1806·65262	2006·49096	
E	S. 18° W.	2860		2720·02159		883·78862
F	N. 73° 28′ 21″ W.	$4621\frac{1}{2}$	1314·69682			4430·55725
			4526·67421	4526·67421	5314·34587	5314·34587

CASE 1. *When the length and the bearing of any one side are wanting.*

390. Find the latitudes and the departures of the remaining sides. The difference of the north and south latitudes of these lines is the latitude of the omitted line, and the difference of their departures is its departure. This latitude and departure are two sides of a right-angled triangle of which the omitted line is the hypotenuse. Its length is therefore equal to the square root of the sum of their squares, and the quotient of the departure divided by the latitude is the tangent of its bearing.

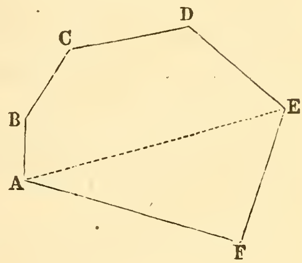
In the above survey, suppose the course from F to A to have been omitted or lost. The difference of the latitudes of the remaining courses will be found to be 1314·69682, and the difference of the departures to be 4430·55725. The square root of the sum of their squares is 4621·5; and the quotient of the departure divided by the latitude is the tangent of 73° 28′ 21″. The deficiencies were in north latitude and west departure, and the omitted course is therefore N. 73° 28′ 21″ W., 4621·5.

CASE 2. *When the length of one side and the bearing of another are wanting.*

391. When the Deficient Sides adjoin Each Other. Find, as in Case 1, the length and bearing of the line joining the ends of the remaining courses. This line and the deficient lines will form a triangle, in which two sides will be known, and the angle between the calculated side and the side whose bearing is given can be found. The parts wanting can then be obtained by the common rules of trigonometry.

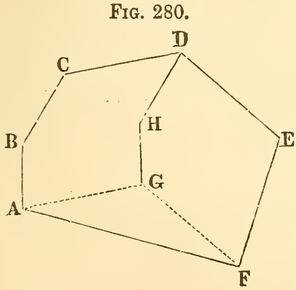
In the figure, let the length of E F and the bearing of F A be the omitted parts. The difference of the sums of the N. and S. latitudes, and the E. and W. departures of the complete courses from A to E, are respectively 1405·32477 north latitude, and 5314·34587 east departure. The course, E A, corresponding to this deficiency, we find, by proceeding as in Case 1, to be S. 75° 11′ 15″ W., 5497·026. The angle A E F is therefore = 75° 11′ 15″ − 18° = 57° 11′ 15″. Then in the triangle A E F are given the sides A E, A F, and the angle A E F to find the remaining parts, viz., the angle A F E = 91° 28′ 21″, whence the bearing of F A = 91° 28′ 21″ − 18° = N. 73° 28′ 21″ W.; and the side E F = 28·60.

Fig. 279.



392. When the Deficient Sides are separated from Each Other. A modification of the preceding method will still apply. In this figure let the omissions be the bearing of F A and the length of C D. Imagine the courses to change places without changing bearings or lengths, so as to bring the deficient lines next to each other by transferring C D to A G, A B to G H, and B C to H D. This will not affect their latitudes or departures. Join G F. Then in the figure D E F G H the latitudes and departures of all the sides but F G are known, whence its length and bearing can be found as in Case 1. Then the triangle A G F may be treated like the triangle A E F in the last article, to obtain the length of A G = C D, and the bearing of F A.

Fig. 280.



Otherwise, by changing the meridian. Imagine the field to turn around till the side of which the distance is unknown becomes the meridian—i. e., comes to be due north and south—all the other sides retaining their *relative* positions, and continuing to make the same angles with each other. Change their bearings accordingly. Find the latitudes and departures of the sides in their new positions. Since the side whose length was unknown has been made the meridian, it has no departure, whatever may be its unknown length; and the difference of the columns of departure will therefore be the departure of the side whose bearing is unknown. The length of this side is given. It is the hypotenuse of a right-angled triangle, of which the departure is one side. Hence the other side, which is the latitude, can be at once found, and also the unknown bearing.

Put this latitude in the table in the blank where it belongs. Then add up the columns of latitude, and the difference of their sums will be the unknown length of the side which had been made a meridian.*

Let the omitted quantities be, as in the last article, the length of C D and the bearing of F A. Make C D the meridian. The changed bearings can then be found to be as in the margin. To aid the imagination, turn the book around till C D points up and down, as north lines are usually placed on a map. Then obtain the latitudes of the courses with their new bearings and old distances, and proceed as has been directed.

STATIONS.	OLD BEARINGS.	NEW BEARINGS.
A	North.	N. 80° W.
B	N. 32° E.	N. 48° W.
C	N. 80° E.	*North.*
D	S. 48° E.	N. 52° E.
E	S. 18° W.	S. 62° E.
F		

CASE 3. *When the lengths of two sides are wanting.*

393. When the Deficient Sides adjoin Each Other. Find the latitudes and departures of the other courses, and then, by Case 1, find the length and bearing of the line joining the extremities of the deficient courses. Then, in the triangle thus formed, are

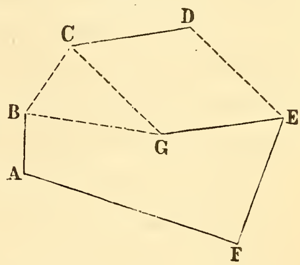
* This conception of thus changing the bearings is stated to be due to Professor Robert Patterson, of Philadelphia, by whom it was communicated to Mr. John Gummere, and published by him, in 1814, in his "Treatise on Surveying."

known one side and all the angles (deduced from the bearings) to find the lengths of the other two sides.

Thus, in Fig. 279, let E F and F A be the sides whose lengths are unknown. E A is then to be calculated, and its length will be found to be 5497·026, and its bearing S. 75° 11′ 15″ W., whence the angle A E F = 75° 11′ 15″ − 18° = 57° 11′ 15″; A F E = 18° + 73° 28′ 21″ = 91° 28′ 21″; and E A F = 31° 20′ 24″; whence can be obtained E F = 28·60 and F A = 46·215.

394. When the Deficient Sides are separated from Each Other. Let the lengths of B C and D E be those omitted. Again imagine the courses to change places, so as to bring the deficient lines together, D E being transferred to C G, and C D to G E. Join B G. Then in the figure A B G E F A are known the latitudes and departures of all the courses except B G, whence its length and bearing can be found, as in Case 1. Then in the triangle B C G, the angle C B G can be found from the bearings of C B and B G, and the angle C G B from the bearings of B G and G C. Then all the angles of the triangle are known and one side, B G, whence to find the required sides, B C = 1,782, and C G = D E = 2,700.

FIG. 281.



Otherwise, by changing the meridian. Imagine the field to turn around till one of the sides whose length is wanting becomes a meridian or due north and south. Change all the bearings correspondingly. Find the latitudes and departures of the changed courses. The difference of the columns of departure will be the departure of the second course of unknown length, since the course made meridian has now no departure. The new bearing of this second course being given in the right-angled triangle formed by this course as an hypotenuse, and its departure and latitude, we know one side, the departure, and the acute angles, which are the bearing and its complement. The length of the course is then readily calculated, and also its latitude. This latitude being inserted in its proper place, the difference of the columns of latitude will be the length of that wanting side which had been made a meridian.

Thus, let the lengths of B C and D E be wanting, as in the preceding example. Make B C a meridian. The other bearings are then changed as in the margin. Calculate new latitudes and departures. The difference of the departures will be the departure of D E, since B C, being a meridian, has no departure. Hence the length and latitude of D E are readily obtained. This latitude being put in the table, and the columns of latitude then added up, their difference will be the length of B C.

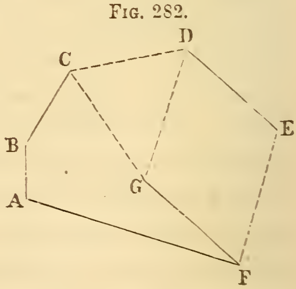
STATIONS.	OLD BEARINGS.	NEW BEARINGS.
A	North.	N. 32° W.
B	N. 32° E.	*North.*
C	N. 80° E.	N. 48° E.
D	S. 48° E.	S. 80° E.
E	S. 18° W.	S. 14° E.
F	N. 73° 28′ 21″ W.	S. 74° 31′ 59″ W.

CASE 4. *When the bearings of two sides are wanting.*

395. When the Deficient Sides adjoin Each Other. Find the latitudes and departures of the other sides, and then, as in Case 1, find the length and bearing of the line joining the extremities of the deficient sides. Then, in the triangle thus formed, we have the three sides to find the angles and thence the bearings.

396. When the Deficient Sides are separated from Each Other. Change the places of the sides so as to bring the deficient ones next to each other. Thus, in the figure, supposing the bearings of C D and E F to be wanting, transfer E F to D G, and D E to G F. Then calculate, as in Case 1, the length and bearing of the line joining the extremities of the deficient sides, C G in the figure. This line and the deficient sides form a triangle in which the three sides are given to determine the angles and thence the required bearings.

FIG. 282.



CHAPTER VI.

LAYING OUT, PARTING OFF, AND DIVIDING UP LAND.

LAYING OUT LAND.

397. Its Nature. This operation is precisely the reverse of those of surveying properly so called. The latter measures certain lines as they are; the former marks them out in the ground where they are required to be, in order to satisfy certain conditions. The same instruments, however, are used as in surveying.

Perpendiculars and parallels are the lines most often employed. Part of the demonstrations of the problems are left as exercises for the student.

398. To lay out Squares. Reduce the desired content to square chains, and extract its square root. This will be the length of the required side, which is to be set out by one of the methods indicated in the preceding article.

An *acre*, laid out in the form of a square, is frequently desired by farmers. Its side must be made $316\frac{1}{4}$ links of a Gunter's chain; or $208\frac{71}{100}$ feet; or $69\frac{57}{100}$ yards. It is often taken at 70 paces.

The number of plants, hills of corn, loads of manure, etc., which an acre will contain at any uniform distance apart, can be at once found by dividing 209 by this distance in feet, and multiplying the quotient by itself, or by dividing 43,560 by the square of the distance in feet. Thus, at 3 feet apart, an acre would contain 4,840 plants, etc.; at 10 feet apart, 436; at a rod apart, 160; and so on. If the distances apart be unequal, divide 43,560 by the product of these distances in feet; thus, if the plants were in rows 6 feet apart, and the plants in the rows were 3 feet apart, 2,420 of them would grow on one acre.

399. To lay out Rectangles. *The content and length being given,* both as measured by the same unit, divide the former by the latter, and the quotient will be the required breadth. Thus, 1 acre or 10 square chains, if 5 chains long, must be 2 chains wide.

The content being given and the length to be a certain number of times the breadth. Divide the content in square chains, etc., by the ratio of the length to the breadth, and the square root of the quotient will be the shorter side desired, whence the longer side is also known. Thus, let it be required to lay out 30 acres in the form of a rectangle 3 times as long as broad; 30 acres = 300 square chains. The desired rectangle will contain 3 squares, each of 100 square chains, having sides of 10 chains. The rectangle will therefore be 10 chains wide and 30 long.

An *acre* laid out in a rectangle twice as long as broad will be 224 links by 448 links, nearly; or, $147\frac{1}{2}$ feet by 295 feet; or, $49\frac{1}{3}$ yards by $98\frac{2}{3}$ yards. Fifty paces by one hundred is often used as an approximation, easy to be remembered.

The content being given, and the difference between the length and breadth. Let c represent this content, and d this difference. Then the longer side $= \frac{1}{2} d + \frac{1}{2} \sqrt{(d^2 + 4c)}$.

Example. Let the content be 6·4 acres, and the difference 12 chains. Then the sides of the rectangle will be respectively 16 chains and 4 chains.

The content being given, and the sum of the length and breadth. Let c represent this content, and s this sum. Then the longer side $= \frac{1}{2} s + \frac{1}{2} \sqrt{(s^2 - 4c)}$.

Example. Let the content be 6·4 acres, and the sum 20 chains. The above formula gives the sides of the rectangle 16 chains and 4 chains as before.

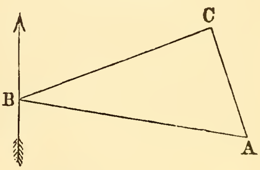
400. To lay out Triangles. The content and the base being given, divide the former by half the latter to get the height. At any point of the base erect a perpendicular of the length thus obtained, and it will be the vertex of the required triangle.

The content being given and the base having to be m times the height, the height will equal the square root of the quotient obtained by dividing twice the given area by m.

The content being given and the triangle to be equilateral, take the square root of the content and multiply it by 1·520. The product will be the length of the side required. This rule makes the sides of an equilateral triangle containing *one acre* to be 480½ links. A quarter of an acre laid out in the same form would have each side 240 links long. An equilateral triangle is very easily set out on the ground, as directed under "Platting," using a rope or chain for compasses.

The content and base being given, and one side having to make a given angle, as B, with the base A B, the length of the side $B\,C = \dfrac{2 \times A\,B\,C}{A\,B\,.\,\sin.\,B}$.

Fig. 283.



Example. Eighty acres are to be laid out in the form of a triangle, on a base, A B, of sixty chains, bearing N. 80° W., the bearing of the side B C being N. 70° E. Here the angle B is found from the bearings (reversing one of them) to be 30°. Hence B C = 53·33. The figure is on a scale of 50 chains to 1 inch = 1 : 39600.

Any right-line figure may be laid out by analogous methods.

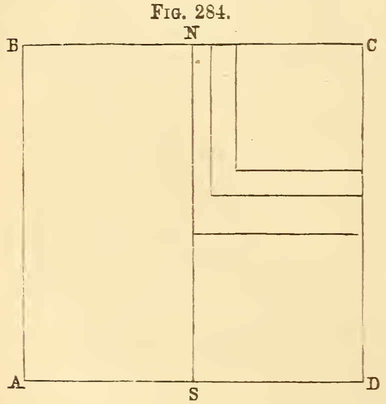
401. To lay out Circles. Multiply the given content by 7, divide the product by 22, and take the square root of the quotient. This will give the radius, with which the circle can be described on the ground with a rope or chain. A circle containing one acre has a radius of 178¼ links. A circle containing a quarter of an acre will have a radius of 89 links.

402. Town-Lots. House-lots in cities are usually laid off as rectangles of 25 feet front and 100 feet depth, variously combined in blocks. Part of New York is laid out in blocks 200 feet by 800, each containing 64 lots, and separated by streets, 60 feet wide, running along their long sides, and avenues, 100 feet wide, on their short sides. The eight lots on each short side of the block front on the avenues, and the remaining forty-eight lots front on the streets. Such a block covers almost precisely 3⅝ acres, and 17½ such lots about make an acre. But, allowing for the streets, land laid out into lots, 25 by 100, arranged as above, would contain only 11·9, or not quite 12 lots per acre.

Lots in small towns and villages are laid out of greater size and less uniformity: 50 feet by 100 is a frequent size for new villages, the blocks being 200 feet by 500, each therefore containing 20 lots.

403. Land sold for Taxes. A case occurring in the State of New York will serve as an application of the modes of laying out squares and rectangles. Land on which taxes are unpaid is sold at auction to the *lowest* bidder—i. e., to him who will accept the smallest portion of it in return for paying the taxes on the whole. The lot in question was originally the east half of the square lot A B C D, containing 500 acres. At a sale for taxes in 1830, 70 acres were bid off, and this area was set off to the purchaser in a square lot, from the northeast corner. Required the side of the square in links. Again, in 1834, 29 acres more were thus sold, to be set off in a strip of equal width around the square previously sold. Required the width of this strip. Once more: in 1839, 42 acres more were sold, to be set off around the preceding piece. Required the dimensions of this third portion. The answer can be proved by calculating if the dimensions of the remaining rectangle will give the content which it should have, viz., 250 — (70 + 29 + 42) = 109 acres.

Fig. 284.



The figure is on a scale of 40 chains to 1 inch = 1 : 31680.

404. New Countries. The operations of laying out land for the purposes of settlers are required on a large scale in new countries, in combination with their survey. There is great difficulty in uniting the necessary precision, rapidity, and cheapness. "Triangular surveying" will insure the first of these qualities, but is deficient in the last two, and leaves the laying out of lots to be subsequently executed. "Compass-surveying" possesses the last two qualities, but not the first. The United States system for surveying and laying out the public lands admirably combines an accurate determination of standard lines (meridians and parallels) with a cheap and rapid subdivision by compass. The subject is so important and extensive that it will be explained by itself.

PARTING OFF LAND.

405. It is often required to part off from a field, or from an indefinite space, a certain number of acres by a fence or other boundary-line, which is also required to run in a particular direction, to start from a certain point, or to fulfill some other condition. The various cases most likely to occur will be here arranged according to these conditions. Both graphical and numerical methods will generally be given.*

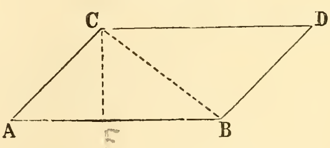
The given content is always supposed to be reduced to square chains and decimal parts, and the lines to be in chains and decimals.

A. By a Line parallel to a Side.

406. To part off a Rectangle. If the sides of the field adjacent to the given side make right angles with it, the figure parted off by a parallel to the given side will be a rectangle, and its breadth will equal the required content divided by that side, as in Art. 398.

If the field be bounded by a curved or zigzag line outside of the given side, find the content between these irregular lines and the given straight side, by the method of offsets, subtract it from the content required to be parted off, and proceed with the *remainder* as above. The same directions apply to the subsequent problems.

Fig. 285.



407. To part off a Parallelogram. If the sides adjacent to the given side be parallel, the figure parted off will be a parallelogram, and its perpendicular width, C E,

* The given lines will be represented by fine full lines, the lines of construction by broken lines, and the lines of the result by heavy full lines.

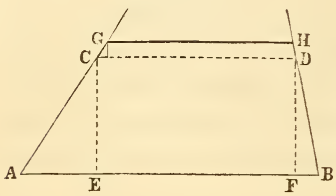
18

will be obtained as above. The length of one of the parallel sides, as $A\,C = \frac{C\,E}{\sin. A} = \frac{A\,B\,D\,C}{A\,B\,.\,\sin. A}$.

408. To part off a Trapezoid. When the sides of a field adjacent to the given side are not parallel, the figure parted off will be a trapezoid.

When the field or figure is given on the ground, or on a plat, begin as if the sides were parallel, dividing the given content by the base A B. The quotient will be an *approximate* breadth, C E, or D F; too small if the sides converge, as in the figure, and *vice versa.* Measure C D. Calculate the content of A B D C. Divide the difference of it and the required content by C D. Set off the quotient perpendicular to C D (in this figure, outside of it), and it will give a new line, G H, a still nearer approximation to that desired. The operation may be repeated, if found necessary.

Fig. 286.



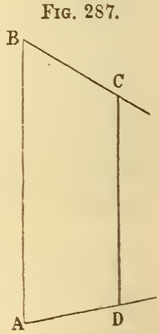
409. When the field is given by bearings, deduce from them the angles at A and B. The required sides will then be given by these formulas:

$$C\,D = \sqrt{\left(A\,B^2 - \frac{2 \times A\,B\,C\,D\,.\,\sin.(A+B)}{\sin. A\,.\,\sin. B}\right)}.$$

$$A\,D = (A\,B - C\,D)\,\frac{\sin. B}{\sin.(A+B)}.$$

$$B\,C = (A\,B - C\,D)\,\frac{\sin. A}{\sin.(A+B)}.$$

Fig. 287.



Demonstration. Produce B C and A D to meet in E. By similar triangles,

$$A\,B\,E : D\,C\,E :: A\,B^2 : D\,C^2.$$

$$A\,B\,E - D\,C\,E : A\,B\,E :: A\,B^2 - D\,C^2 : A\,B^2$$

Now $A\,B\,E - D\,C\,E = A\,B\,C\,D$; also, by Art. 61, note,

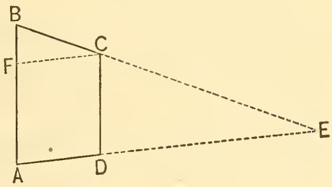
$$A\,B\,E = A\,B^2\,.\,\frac{\sin. A\,.\,\sin. B}{2\,.\,\sin.(A+B)}.$$

The above proportion, therefore, becomes

$$A B C D : A B^2 . \frac{\sin. A . \sin. B}{2 . \sin. (A+B)} :: A B^2 - C D^2 : A B^2.$$

Multiplying extremes and means, canceling, transposing, and extracting the square root, we get $C D = \sqrt{\left[A B^2 - \frac{2 . A B C D . \sin. (A + B)}{\sin. A . \sin. B}\right]}$.

Fig. 288.



When $A + B > 180°$, sin. $(A + B)$ is negative, and therefore the fraction in which it occurs becomes positive.

C F being drawn parallel to D A, we have

$$A D = F C = F B . \frac{\sin. B}{\sin. B C F} = F B . \frac{\sin. B}{\sin. (180° - A - B)}$$

$$= (A B - C D) \frac{\sin. B}{\sin. (A + B)} \quad B C = (A B - C D) \frac{\sin. A}{\sin. (A + B)}.$$

When the sides A D and B C diverge, instead of converging, as in the figure, the negative term, in the expression for C D, becomes positive; and, in the expressions for both A D and B C, the first factor becomes $(C D - A B)$.

The perpendicular breadth of the trapezoid $= A D . \sin. A$; or $= B C . \sin. B$.

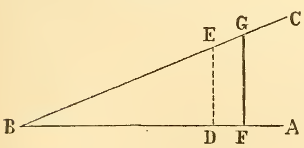
Example. Let A B run north, six chains; A D, N. 80° E.; B C, S. 60° E. Let it be required to part off one acre by a fence parallel to A B. Here $A B = 6{\cdot}00$, $A B C D = 10$ square chains, $A = 80°$, $B = 60°$. *Ans.* $C D = 4{\cdot}57$, $A D = 1{\cdot}92$, $B C = 2{\cdot}18$, and the breadth $= 1{\cdot}89$.

The figure is on a scale of 4 chains to an inch = 1 : 3168.

B. By a Line perpendicular to a Side.

410. To part off a Triangle. Let F G be the required line.

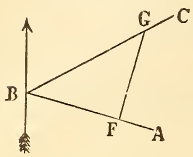
Fig. 289.



When the field is given on the ground, or on a plat, at any point, as D, of the given side A B, set out a "guess-line," D E, perpendicular to A B, and calculate the content of D E B. Then the required distance B F, from the angular point to the foot of the desired perpendicular $= B D \sqrt{\left(\frac{B F G}{B D E}\right)}$.

Since similar triangles are as the squares of their homologous sides, $BDE : BFG :: BD^2 : BF^2$; whence $BF = BD\sqrt{\left(\frac{BFG}{BDE}\right)}$.

FIG. 290.



Example. Let $BD = 30$ chains; $ED = 12$ chains; and the desired area $= 24{\cdot}8$ acres. Then $BF = 35{\cdot}22$ chains.

The scale of the figure is 30 chains to 1 inch $= 1 : 23760$.

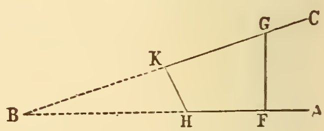
When the field is given by bearings, find the angle B from the bearings; then is $BF = \sqrt{\left(\frac{2 \times BFG}{\tan. B}\right)}$.

Example. Let B A bear S. 75° E., and B C N. 60° E., and let five acres be required to be parted of from the field by a perpendicular to B A. Here the angle $B = 45°$, and $BF = 10{\cdot}00$ chains.

The scale of the figure is 20 chains to 1 inch $= 1 : 15840$.

411. To part off a Quadrilateral. Produce the converging sides to meet at B. Calculate the content of the triangle H K B, whether on the ground or plat, or from bearings. Add it to the content of the quadrilateral required to be parted off, and it will give that of the triangle F G B, and the method of the preceding case can then be applied.

FIG. 291.



412. To part off any Figure. If the field be very irregularly shaped, find by trial any line which will part off a little less than the required area. This trial-line will represent H K in the preceding figure, and the problem is reduced to parting off, according to the required condition, a *quadrilateral,* comprised between the trial-line, two sides of the field, and the required line, and containing the difference between the required content and that parted off by the trial-line.

C. By a Line running in any Given Direction.

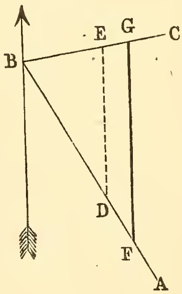
413. To part off a Triangle. By construction, on the ground or the plat, proceed nearly as in Art. 410, setting out a line in the required direction, calculating the triangle thus formed, and obtaining B F by the same formula as in that article.

414. If the field be given by bearings, find from them the angles C B A and G F B; then is $B\,F = \sqrt{\left(\dfrac{2 \times B\,F\,G \sin.\,(B+F)}{\sin.\,B\,.\,\sin.\,F}\right)}$.

Example. Let B A bear S. 30° E.; B C, N. 80° E.; and a fence be required to run from some point in B A, a due north course, and to part off one acre. Required the distance from B to the point F, whence it must start. *Ans.* The angle B = 70°, and F = 30°. Then B F = 6·47.

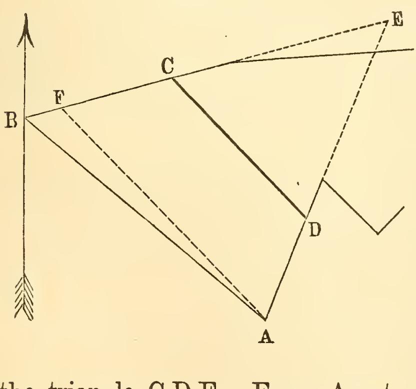
The scale of Fig. 292 is 6 chains to 1 inch = 1 : 4752.

Fig. 292.



415. To part off a Quadrilateral. Let it be required to part off, by a line running in a given direction, a quadrilateral from a field in which are given the side A B, and the directions of the two other sides running from A and from B.

Fig. 293.



On the ground or plat produce the two converging sides to meet at some point E. Calculate the content of the triangle A B E. Measure the side A E. From A B E subtract the area to be cut off, and the remainder will be the content of the triangle C D E. From A set out a line A F parallel to the given direction. Find the content of A B F. Take it from

A B E, and thus obtain A F E. Then this formula, $ED = AE \sqrt{\frac{CDE}{FAE}}$, will fix the point D, since $AD = AE - ED$.

When the field and the dividing line are given by bearings, produce the sides as in the last article. Find all the angles from the bearings. Calculate the content of the triangle A B E, by the formula for one side and its including angles. Take the desired content from this to obtain C D E. Calculate the side $AE = AB \frac{\sin. B}{\sin. E}$. Then is $AD = AE - \sqrt{\left(\frac{2 \times CDE \,.\, \sin. DCE}{\sin. E \,.\, \sin. CDE}\right)}$.

Demonstration. Since triangles which have an angle in each equal, are as the products of the sides about the equal angles, we have

$$ABE : CDE :: AE \times BE : CE \times DE.$$

$$ABE = \tfrac{1}{2} \,.\, AB^2 \,.\, \frac{\sin. A \,.\, \sin. B}{\sin. (A + B)}. \qquad AE = AB \,.\, \frac{\sin. B}{\sin. E}.$$

$$BE = AB \,.\, \frac{\sin. A}{\sin. E}. \qquad CE = DE \,.\, \frac{\sin. CDE}{\sin. DCE}.$$

Substituting these values in the preceding proportion, canceling the common factors, observing that $\sin. (A + B) = \sin. E$, multiplying extremes and means, and dividing, we get $DE = \sqrt{\left(\frac{2 \,.\, CDE \,.\, \sin. DCE}{\sin. E \,.\, \sin. CDE}\right)}$.

Example. Let D A bear S. $20\frac{1}{4}°$ W. ; A B, N. $51\frac{1}{2}°$ W., 8·19 ; B C, N. $73\frac{1}{2}°$ E ; and let it be required to part off two acres by a fence, D C, running N. 45° W. *Ans.* A B E = 32·56 square chains ; whence C D E = 12·56 square chains. Also, A E = 8·37 ; and, finally, A D = 8·37 — 5·51 = 2·86 chains.

The scale of Fig. 293 is 5 chains to 1 inch = 1 : 3960.

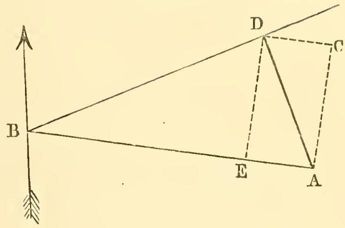
If the sum of the angles at A and B were more than two right angles, the point E would lie on the other side of A B. The necessary modifications are apparent.

416. To part off any Figure. Proceed in a similar manner to that described in Art. 412, by getting a suitable trial-line, producing the sides it intersects, and then applying the method just given.

D. By a Line starting from a Given Point in a Side.

417. To part off a Triangle. Let it be required to cut of from a corner of a field a triangular space of given content, by a line starting from a given point on one of the sides, A in the figure, the base, A B, of the desired triangle being thus given. If the field be given on the ground or on a plat, divide the given content by half the base, and the quotient will be the height of the triangle. Set off this distance from any point of A B, perpendicular to it, as from A to C; from C set out a parallel to A B, and its intersection with the second side, as at D, will be the vertex of the required triangle.

FIG. 294.



Otherwise: Divide the required content by half of the perpendicular distance from A to B D, and the quotient will be B D.

If the field be given by the bearings of two sides and the length of one of them, deduce the angle B (Fig. 294) from the bearings. Then is $BD = \frac{2 \times ABD}{AB \,.\, \sin.\, B}$.

If it is more convenient to fix the point D, by the second method, that of rectangular co-ordinates, we shall have $BE = BD \,.\, \cos.\, B$; and $ED = BD \,.\, \sin.\, B$.

The bearing of A D is obtained from the angle B A D, which is known, since $\frac{ED}{EA} = \frac{ED}{AB - BE} = \tan.\, BAD$.

Example. Eighty acres are to be set off from a corner of a field, the course A B being N. 80° W., sixty chains; and the bearing of B D being N. 70° E. *Ans.* B D = 53·33; B E = 46·19; E D = 26·67; and the bearing of A D, N. 17° 23′ W.

The scale of Fig. 294 is 40 chains to 1 inch = 1 : 31680.

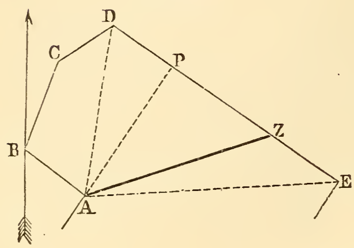
If the field were right-angled at B, of course $DB = \frac{2\,ABD}{AB}$.

418. To part off a Quadrilateral. Imagine the two converging sides of the field produced to meet, as in Art. 415. Calculate the content of the triangle thus formed, and the question will then be reduced to the one explained in the last two articles.

419. To part off any Figure. Proceed as directed in Art. 416. Otherwise, proceed as follows :

The field being given on the ground or on a plat, find on which side of it the required line will end, by drawing or running "guess-lines" from the given point to various angles, and roughly measuring the content thus parted off. If, as in the figure, A being the given point, the guess-line A D parts off less than the required content, and A E parts off more, then the desired division-line A Z will end in the side D E. Subtract the area parted off by A D from the required content, and the difference will be the content of the triangle A D Z. Divide this by half the perpendicular let fall from the given point A to the side D E, and the quotient will be the base, or distance from D to Z.

Fig. 295.



Or, find the content of A D E and make this proportion: A D E : A D Z : : D E : D Z.

The field being given by bearings and distances, find as before, by approximate trials on the plat, or otherwise, which side the desired line of division will terminate in, as D E in the last figure. Draw A D. Find the latitude and departure of this line, and thence its length and bearing. Then calculate the area of the space this line parts off, A B C D in the figure, by the usual method, explained in Part I, Chapter III. Subtract this area from that required to be cut off, and the remainder will be the area of the triangle A D Z. Then, as in Art. 415, $DZ = \frac{2\,ADZ}{AD \,.\, \sin.\, ADZ}$.

This problem may be executed without any other table than that of latitudes and departures, thus: Find the latitude and departure of D A, as before, the area of the space A B C D, and thence the content of A D Z. Then find the latitude and departure of E A, and the content of A D E. Lastly, make this proportion: A D E : A D Z : : D E : D Z.*

* The problem may also be performed by making the side on which the division-

Example. In the field A B C D E, etc., part of which is shown in Fig. 295 (on a scale of 4 chains to 1 inch = 1 : 3168), one acre is to be parted off on the west side, by a line starting from the angle A. Required the distance from D to Z, the other end of this dividing line.*

The only courses needed are these : A B, N. 53° W., 1·55 ; B C, N. 20° E, 2·00 ; C D, N. 53½° E., 1·32 ; D E, S. 57° E., 5·79. A rough measurement will at once show that A B C D is less than an acre, and that A B C D E is more ; hence the desired line will fall on D E. The latitudes and departures of A B, B C, and C D are then found. From them the course A D is found to be N. 8° 1′ 22″ E., 3·634. The content of A B C D will be 3·19 square chains. Subtracting this from one acre, the remainder, 6·81 square chains, is the content of A D Z. A P = 3·63 × sin. 65° = 3·29. Dividing A D Z by half of this, we obtain D Z = 4·14 chains.

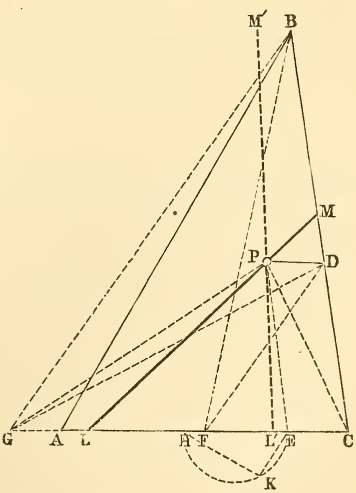
By the second method, the latitude and departure of D A, the area of A B C D, and of A D Z, being found as before, we next find the latitude and departure of E A from those of A D and D E, and thence the area of A D E = 9·53. Lastly, we have the proportion 9·53 : 6·31 : : 5·79 : D Z = 4·14, as before.

E. By a Line passing through a Given Point within the Field.

420. To part off a Triangle. Let P be a point within a field through which it is required to run a line so as to part off from the field a given area in the form of a triangle.

When the field is given on

Fig. 296.



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line is to fall, a meridian, and changing the bearings. The difference of the new departures will be the departure of the division-line. Its position can then be easily determined.

* If the whole field has been surveyed and balanced, the balanced latitudes and departures should be used. We will here suppose the survey to have proved perfectly correct.

the ground or on a plat, the division can be made by construction, thus: Divide the given area by half of the perpendicular distance from P to A C, and set off the quotient from C to G. Bisect G C in H. From P draw P E, parallel to the side B C. On H E describe a semicircle. On it set off E K = E C. Join K H. Set off H L = H K. The line L M, drawn from L through P, will be the division-line required.* If H K be set off in the contrary direction, it will fix another line L′ P M′, meeting C B produced, and thus parting off another triangle of the required content.

Demonstration. By construction, G P C = the required content. Now, G P C = G D C, since they have the same base and equal altitudes. We have now to prove that L M C = G D C. These two triangles have a common angle at C. Hence, they are to each other as the rectangles of the adjacent sides—i. e.,

$$GDC : LMC :: GC \times CD :: LC \times CM.$$

Here C M is unknown, and must be eliminated. We obtain an expression for it by means of the similar triangles L M C and L E P, which give

$$LE : LC :: EP = CD : CM.$$

Hence, $CM = \dfrac{CD \times LC}{LE}$. Substituting this value of C M in the first proportion, and canceling C D in the last two terms, we get

$$GDC : LMC :: GC : \frac{LC^2}{LE}; \text{ or } GDC : LMC :: GC \times LE : LC^2.$$

$$LC^2 = (LH + HC)^2 = LH^2 + 2LH \times HC + HC^2.$$

But, by construction,

$$LH^2 = HK^2 = HE^2 - EK^2 = HE^2 - EC^2 = (HE + EC)(HE - EC) = HC(HE - EC).$$

Also, $\quad GC = 2HC; \text{ and } LE = LH + HE.$

Substituting these values in the last proportion, it becomes

$$\begin{aligned} GDC : LMC :: 2 . HC(LH + HE) &: HC(HE - EC) + 2LH \times HC + HC^2. \\ :: 2LH + 2HE \quad &: HE - EC + 2LH + HC. \\ &: HE - EC + 2LH + HE + EC. \\ &: 2HE + 2LH. \end{aligned}$$

The last two terms of this proportion are thus proved to be equal. Therefore, the first two terms are also equal—i. e., L M C = G D C = the required content.

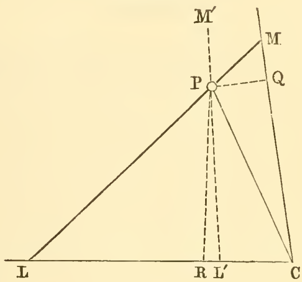
Since $HK = \sqrt{(HE^2 - EK^2)}$, it will have a negative as well as a positive value. It may therefore be set off in the contrary direction from L—i. e., to L′. The line drawn from L′ through P, and meeting C B produced beyond B, will part off *another* triangle of the required content.

Example. Let it be required to part off 31·175 acres by a fence passing through a point P, the distance P D of P from the side

* As some lines in the figure are not used in the construction, though needed for the demonstration, the student should draw it himself to a large scale.

B C, measured parallel to A C, being 6 chains, and D C 18 chains. The angle at C is fixed by a "tie-line" A B = 48·00, B C being 42·00, and C A being 30·00. *Ans.* C L = 27·31 chains, or C L′ = 7·69 chains.

FIG. 297.



The figure is on a scale of 20 chains to 1 inch = 1 : 15840.

If the angle of the field and the position of the point P are given by bearings or angles, proceed thus: Find the perpendicular distances, P Q and P R, from the given point to the sides, by the formulas P Q = P C . sin. P C Q ; and P R = P C . sin. P C R. Let P Q = q, P R = p, and the required content = c. Then $CL = \frac{c}{p} \pm \sqrt{\left(\frac{c^2}{p^2} - \frac{2qc}{p \sin. LCM}\right)}$.

Demonstration. Suppose the line L M drawn. Then, by Art. 61, note, the required content, $c = \frac{1}{2} . CL \times CM . \sin. LCM$. This content will also equal the sum of the two triangles L C P and M C P—i. e., $c = \frac{1}{2} . CL \times p + \frac{1}{2} . CM \times q$. The first of these equations gives $CM = \frac{2c}{CL . \sin. LCM}$. Substituting this in the second equation, we have

$$c = \tfrac{1}{2} . CL \times p + \frac{cq}{CL . \sin. LCM}.$$

Whence, $\frac{1}{2}p . CL^2 . \sin. LCM + cq = c . CL . \sin. LCM.$

Transposing and dividing by the coefficient of CL^2, we get

$$CL^2 - \frac{2c}{p} . CL = - \frac{2cq}{p . \sin. CLM}.$$

$$CL = \frac{c}{p} \pm \sqrt{\left(\frac{c^2}{p^2} - \frac{2cq}{p . \sin. LCM}\right)}.$$

If the given point is *outside* of the lines C L and C M, conceive the desired line to be drawn from it, and another line to join the given point to the corner of the field. Then, as above, get expressions for the two triangles thus formed, and put their sum equal to the expression for the triangle which comprehends them both, and thence deduce the desired distance, nearly as above.

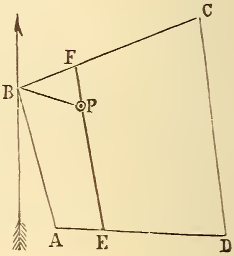
Example. Let the angle L C M = 82°. Let it be required to part off the same area as in the preceding example. Let P C = 19·75, P C Q = 17° 30½′, P C R = 64° 29½′. Required C L. *Ans.* P Q = 5·94, P R = 17·82, and therefore, by the formula, C L =

27·31, or C L′ = 7·69 ; corresponding to the graphical solution. The figure is on the same scale.

If the given point were *without* the field, the division-line could be determined in a similar manner.

421. To part off a Quadrilateral. Conceive the two sides of the field which the division-line will intersect, D A and C B, produced till they meet at a point G, not shown in the figure. Calculate the triangle thus formed outside of the field. Its area, increased by the required area, will be that of the triangle E F G. Then the problem is identical with that in the last article. The following example is that given in Gummere's "Surveying." The figure represents it on a scale of 20 chains to 1 inch = 1 : 15840.

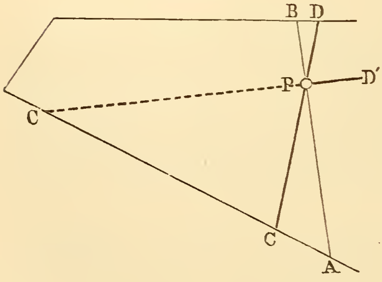
FIG. 298.



Example. A field is bounded thus : N. 14° W., 15·20 ; N. 70½° E., 20·43 ; S. 6° E., 22·79 ; N. 86½° W., 18·00. A spring within it bears from the second corner S. 75° E., 7·90. It is required to cut off 10 acres from the west side of the field by a straight fence through the spring. How far will it be from the first corner to the point at which the division-fence meets the fourth side ? *Ans.* 4·6357 chains.

422. To part off any Figure. Let it be required to part off from a field a certain area by a line passing through a given point P within the field. Run a guess-line A B through P. Calculate the area which it parts off. Call the difference between it and the required area = d. Let C D be the desired line of division, and let P represent the angle, A P C or B P D, which it makes with the given line. Obtain the angles P A C = A, and P B D = B, either by measurement, or by de-

FIG. 299.



duction from bearings. Measure P A and P B. Then the desired angle P will be given by the following formula:

$$\text{Cot. P} = -\tfrac{1}{2}\left(\text{cot. A} + \text{cot. B} - \frac{\text{A P}^2 - \text{B P}^2}{2\,d}\right) \pm$$
$$\left[\sqrt{\frac{\text{A P}^2 \,.\, \text{cot. B} - \text{B P}^2 \,.\, \text{cot. A}}{2\,d} - \text{cot. A} \,.\, \text{cot. B} + \tfrac{1}{4}\left(\text{cot. A} + \text{cot. B} - \frac{\text{A P}^2 - \text{B P}^2}{2\,d}\right)^2}\,\right].$$

If the guess-line be run so as to be perpendicular to one of the sides of the field, at A, for example, the preceding expression reduces to the following simpler form:

$$\text{Cot. P} = -\tfrac{1}{2}\left(\text{cot. B} - \frac{\text{A P}^2 - \text{B P}^2}{2\,d}\right) \pm$$
$$\sqrt{\left[\frac{\text{A P}^2 \,.\, \text{cot. B}}{2\,d} + \tfrac{1}{4}\left(\text{cot. B} - \frac{\text{A P}^2 - \text{B P}^2}{2\,d}\right)^2\right]}.$$

Demonstration. The difference d, between the areas parted off by the guess-line A B, and the required line C D, is equal to the difference between the triangles A P C and B P D.

By Art. 61, note, the triangle $\text{A P C} = \tfrac{1}{2} \,.\, \text{A P}^2 \,.\, \dfrac{\text{sin. A} \,.\, \text{sin. P}}{\text{sin. (A + P)}}$.

Similarly, the triangle $\text{B P D} = \tfrac{1}{2} \,.\, \text{B P}^2 \dfrac{\text{sin. B} \,.\, \text{sin. P}}{\text{sin. (B + P)}}$.

$$\therefore\; d = \tfrac{1}{2} \,.\, \text{A P}^2 \frac{\text{sin. A sin. P}}{\text{sin. (A + P)}} - \tfrac{1}{2}\,\text{B P}^2 \,.\, \frac{\text{sin. B} \,.\, \text{sin. P}}{\text{sin. (B + P)}}.$$

By the expression for sin. $(a + b)$ [Trigonometry, Art. 8], we have

$$d = \tfrac{1}{2}\,\text{A P}^2 \,.\, \frac{\text{sin. A} \,.\, \text{sin. P}}{\text{sin. A} \,.\, \text{cos. P} + \text{sin. P} \,.\, \text{cos. A}} - \tfrac{1}{2}\,\text{B P}^2 \,.\, \frac{\text{sin. B} \,.\, \text{sin. P}}{\text{sin. B} \,.\, \text{cos. P} + \text{sin. P. cos. B}}$$

Dividing each fraction by its numerator, and remembering that $\dfrac{\text{cos. } a}{\text{sin. } a} =$ $= \text{cot. } a$, we have

$$d = \frac{\tfrac{1}{2}\,\text{A P}^2}{\text{cot. P} + \text{cot. A}} - \frac{\tfrac{1}{2}\,\text{B P}^2}{\text{cot. P} + \text{cot. B}}.$$

For convenience, let $p = \text{cot. P}$; $a = \text{cot. A}$; and $b = \text{cot. B}$. The above equation will then read, multiplying both sides by 2,

$$2\,d = \frac{\text{A}p^2}{p + \text{A}} - \frac{\text{B P}^2}{p + b}.$$

Clearing of fractions, we have

$$2\,dp^2 + 2\,d\,ap + 2\,d\,bp + 2\,d\,ab = p \,.\, \text{A P}^2 + b \,.\, \text{A P}^2 - p \,.\, \text{B P}^2 - a \,.\, \text{B P}^2.$$

Transposing, dividing through by $2\,d$, and separating into factors, we get

$$p^2 + \left(a + b - \frac{\text{A P}^2 - \text{B P}^2}{2\,d}\right)p = \frac{b \,.\, \text{A P}^2 - a \,.\, \text{B P}^2}{2\,d} - ab.$$

$$\because\; p = -\tfrac{1}{2}\left(a + b - \frac{\text{A P}^2 - \text{B P}^2}{2\,d}\right) \pm \sqrt{\left[\frac{b \,.\, \text{A P}^2 - a \,.\, \text{B P}^2}{2\,d}\right.}$$

$$- ab + \tfrac{1}{4}\left(a + b - \frac{AP^2 - BP^2}{2d}\right)^2\Big].$$

If $A = 90°$, cot. $A = 0$; and the expression reduces to the simpler form given in the article.

Example. It was required to cut off from a field twelve acres by a line passing through a spring P. A guess-line, A B, was run making an angle with one side of the field, at A, of 55°, and with the opposite side, at B, of 81°. The area thus cut off was found to be 13·10 acres. From the spring to A was 9·30 chains, and to B 3·30 chains. Required the angle which the required line, C D, must make with the guess-line, A B, at P. *Ans.* 20° 45′; or − 86° 25′. The heavy broken line, C′ D′, shows the latter.

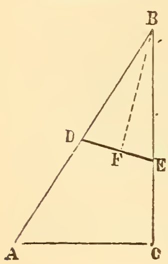
The scale of the figure is 10 chains to 1 inch = 1 : 7920.

If the given point were outside of the field, the calculations would be similar.

F. By the Shortest Possible Line.

423. To part off a Triangle. Let it be required to part off a triangular space, B D E, of given content, from the corner of a field, A B C, by the shortest possible line, D E.

Fig. 300.



From B set off B D and B E each equal to $\sqrt{\left(\frac{2\,BDE}{\sin. B}\right)}$. The line D E thus obtained will be perpendicular to the line, B F, which bisects the angle B. The length of $DE = \frac{\sqrt{(2\,.\,DBE\,.\,\sin. B)}}{\cos. \frac{1}{2} B}$.

Demonstration Conceive a perpendicular, B F, to be let fall from B to the required line D E. Let B represent the angle D B E, and β the unknown angle D B F. The angle $BDF = 90° - \beta$; and the angle $BEF = 90° - (B - \beta) = 90° - B + \beta$. By Art. 61, note, the area of the triangle D B E

$$= \tfrac{1}{2}\,DE^2 . \frac{\sin. BDE . \sin. BED}{\sin. (BDE + BED)} = \tfrac{1}{2} . DE^2 . \frac{\sin. (90° - \beta)\ \sin. (90° - B + \beta)}{\sin. B}.$$

$$\text{Hence, } DE^2 = \frac{2 \times DBE \times \sin. B}{\sin. (90° - \beta) . (\sin. (90° - B + \beta)} = \frac{2 \times DBE \times \sin. B}{\cos. \beta\ .\ \cos. (B - \beta)}.$$

Now, in order that D E may be the least possible, the denominator of the last fraction must be the greatest possible. It may be transformed, by the formula, cos. a . cos. $b = \frac{1}{2}$ cos. $(a + b) + \frac{1}{2}$. cos. $(a - b)$ [Trigonometry, Art. 8], into $\frac{1}{2}$ cos. B + $\frac{1}{2}$. cos. (B − 2 β). Since B is constant, the value of this expression depends on its second term, and that will be the greatest possible when $B - 2\beta = 0$, in which case $\beta = \frac{1}{2} B$.

It hence appears that the required line D E is perpendicular to the line, B F, which bisects the given angle B. This gives the *direction* in which D E is to be run.

Its starting-point, D or E, is found thus: The area of the triangle D B E $= \frac{1}{2}$ B D . B E . sin. B. Since the triangle is isosceles, this becomes

$$DBE = \tfrac{1}{2} BD^2 . \sin. B\text{; whence } BD = \sqrt{\left(\frac{2\,DBE}{\sin. B}\right)}.$$

D E is obtained from the expression for DE^2, which becomes, making $\beta = \frac{1}{2} B$,

$$DE^2 = \frac{2 \times DBE \times \sin. B}{\cos. \frac{1}{2} B . \cos. \frac{1}{2} B}\text{, whence, } DE = \frac{\sqrt{(2 . DBE . \sin. B)}}{\cos. \frac{1}{2} B}.$$

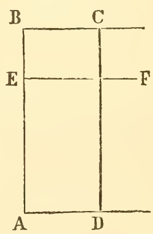
Example. Let it be required to part off 1·3 acre from the corner of a field, the angle, B, being 30°. *Ans.* B D = B E = 7·21; and D E = 3·73.

The scale of the figure is 10 chains to 1 inch = 1 : 7920.

G. Land of Variable Value.

424. Let the figure represent a field in which the land is of two qualities and values, divided by the "quality-line" E F. It is required to part off from it a quantity of land worth a certain sum, by a straight fence parallel to A B.

Fig. 301.



Multiply the value per acre of each part by its length (in chains) on the line A B, add the products, multiply the value to be set off by 10, divide by the above sum, and the quotient will be the desired breadth, B C or A D, in chains.

Demonstration. Let a = value per acre of one portion of the land, and b that of the other portion. Let x = the width required, B C or A D. Then the value of B C F E $= a \times \frac{x \times BE}{10}$, and the value of A D F E $= b \times \frac{x \times AE}{10}$.

Putting the sum of these equal to the value required to be parted off, we obtain $x = \frac{\text{value required} \times 10}{a \times BE + b \times AE}$.

Example. Let the land on one side of E F be worth \$200 per acre, and on the other side \$100. Let the length of the former, B E, be 10 chains, and E A be 30 chains. It is required to part off a quantity of land worth $7,500. *Ans.* The width of the desired strip will be 15 chains.

The scale of the figure is 40 chains to 1 inch = 1 : 31680.

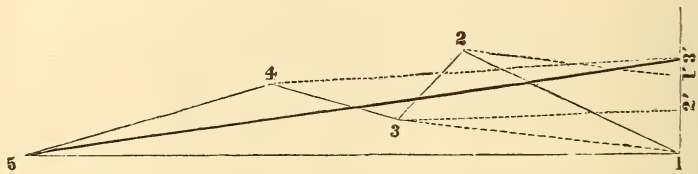
If the "quality-line" be not perpendicular to A B, it may be made so by "giving and taking," or as in the article following this one.

The same method may be applied to land of any number of different qualities; and a combination of this method with the preceding problems will solve any case which may occur.

H. Straightening Crooked Fences.

425. It is often required to substitute a straight fence for a crooked one, so that the former shall part off precisely the same quantity of land as did the latter. This can be done on a plat by

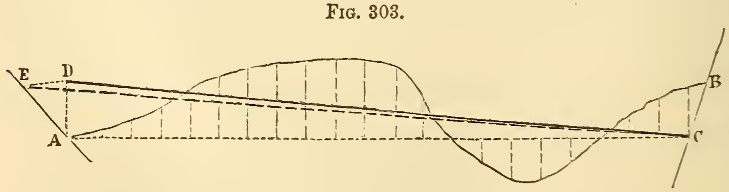
Fig. 302.



the method given in Art. 76, by which the irregular figure 1...2...3...4...5 is reduced to the equivalent triangle 1...5...3′, and the straight line 5...3′ therefore parts off the same quantity of land on either side as did the crooked one. The distance from 1 to 3′, as found on the plat, can then be set out on the ground and the straight fence be then ranged from 3′ to 5.

The work may be done on the ground more accurately by run-

Fig. 303.



ning a guess-line, A C, Fig. 303, across the bends of the fence which crooks from A to B, measuring offsets to the bends on each side of the guess-line, and calculating their content. If the sums of these areas on each side of A C chanced to be equal, that would be the line desired; but if, as in the figure, it passes too far on one side, divide the difference of the areas by half of A C, and set off the quotient at right angles to A C, from A to D. D C will then be a line parting off the same quantity of land as did the crooked fence. If the fence at A was not perpendicular to A C, but oblique, as A E, then from D run a parallel to A C, meeting the fence at E, and E C will be the required line.

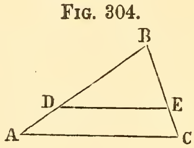
DIVIDING UP LAND.

426. Most of the problems for "dividing up" land may be brought under the cases in the preceding articles, by regarding one of the portions into which the figure is to be divided as an area to be "parted off" from it. Many of them, however, can be most neatly executed by considering them as independent problems, and this will be here done. They will be arranged, first, according to the simplicity of the figure to be divided up, and then sub-arranged, according to the manner of the division.

Division of Triangles.

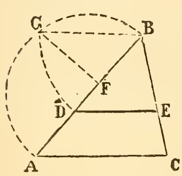
427. By Lines parallel to a Side. Suppose that the triangle A B C is to be divided into two equivalent parts by a line parallel to A C. The desired point, D, from which this line is to start, will be obtained by measuring $BD = AB\sqrt{\frac{1}{2}}$. So, too, E is fixed by $BE = BC\sqrt{\frac{1}{2}}$.

FIG. 304.



Generally, to divide the triangle into two parts, B D E and A C E D, which shall have to each other a ratio $= m : n$, we have $BD = AB\sqrt{\dfrac{m}{m+n}}$.

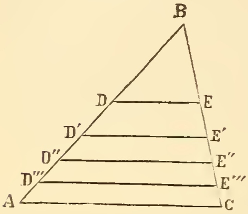
FIG. 305.



This may be constructed thus: Describe a semicircle on A B as a diameter. From B set off $BF = \dfrac{m}{m+n} \cdot BA$. At F erect a perpendicular meeting the semicircle at G. Set off B G from B to D. D is the starting-point of the division-line required. In the figure, the two parts are as 2 to 3, and B F is therefore $= \frac{2}{5}$ B A.

To divide the triangle A B C into five equivalent parts, we should have, similarly, $BD = AB\sqrt{\frac{1}{5}}$; $BD' = AB\sqrt{\frac{2}{5}}$; $BD'' = AB\sqrt{\frac{3}{5}}$; $BD''' = AB\sqrt{\frac{4}{5}}$.

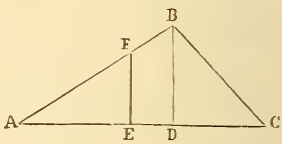
FIG. 306.



The same method will divide the triangle into any desired number of parts having any ratios to each other.

428. By Lines perpendicular to a Side. Suppose that A B C is to be divided into two parts having a ratio $= m : n$, by a line perpendicular to A C. Let E F be the dividing line whose position is required. Let B D be a perpendicular let fall from B to A C. Then is $AE = \sqrt{\left(AC \times AD \times \frac{m}{m+n}\right)}$. In this figure, A F E : E F B C $:: m : n :: 1 : 2$.

FIG. 307.



If the triangle had to be divided into two equivalent parts, the above expression would become $AE = \sqrt{(\frac{1}{2} AC \times AD)}$.

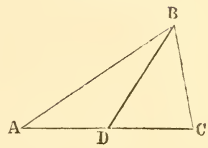
Demonstration. By hypothesis, A E F : E F B C $:: m : n$; whence A E F : A B C $:: m : m + n$; and $AEF = ABC\,\frac{m}{m+n} = \frac{AC \times DB}{2} \cdot \frac{m}{m+n}$. Also, $AEF = \frac{1}{2} \cdot AE \times EF$.

The similar triangles A E F and A B D give A D : D B :: A E : E F $= \frac{DB \times AE}{AD}$. The second expression for A E F then becomes $AEF = \frac{1}{2} AE \cdot \frac{DB \times AE}{AD}$. Equating this with the other value of A E F, we have $\frac{AC \times DB}{2} \cdot \frac{m}{m+n} = \frac{AE^2 \times DB}{2 \cdot AD}$; whence $AE = \sqrt{\left(AC \times AD \times \frac{m}{m+n}\right)}$.

429. By Lines running in any Given Direction. Let a triangle, A B C, be given to be divided into two parts, having a ratio $= m : n$, by a line making a given angle with a side. Part off, as in Art. 413 or 414, Fig. 292, an area $BFG = \frac{m}{m+n} \cdot ABC$.

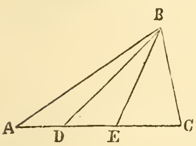
430. By Lines starting from an Angle. Divide the side opposite to the given angle into the required number of parts, and draw lines from the angle to the points of division. In the figure the triangle is represented as being thus divided into two equivalent parts.

Fig. 308.



If the triangle were required to be divided into two parts, having to each other a ratio $= m : n$, we should have $A D = A C \frac{m}{m+n}$, and $D C = A C \frac{n}{m+n}$.

Fig. 309.

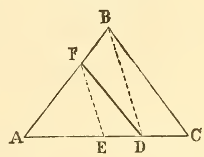


If the triangle had to be divided into three parts which should be to each other $: : m : n : p$, we should have $A D = A C \frac{m}{m+n+p}$, $D E = A C \frac{m}{m+n+p}$, and $E C = A C \frac{p}{m+n+p}$.

Suppose that a triangular field, A B C, had to be divided among five men, two of them to have a quarter each, and three of them each a sixth. Divide A C into two equal parts, one of these again into two equal parts, and the other one into three equal parts. Run the lines from the four points thus obtained to the angle B.

431. By Lines starting from a Point in a Side. Suppose that the triangle A B C is to be divided into *two* equivalent parts by a line starting from a point D in the side A C. Take a point E in the middle of A C. Join B D, and from E draw a parallel to it, meeting A B in F. D F will be the dividing line required.

Fig. 310.



The point F will be most easily obtained on the ground by the proportion $A D : A B : : A E = \frac{1}{2} A C : A F$.

The altitude of A F D of course equals $\frac{1}{2} A B C \div \frac{1}{2} A D$.

If the triangle is to be divided into two parts having any other ratio to each other, divide A C in that ratio, and then proceed as before. Let this ratio $= m : n$, then $A F = \frac{A B \times A C}{A D} \cdot \frac{m}{m+n}$.

Demonstration. In Fig. 310, conceive the line E B to be drawn. The triangle A E B $= \frac{1}{2}$ A B C, having the same altitude and half the base; and A F D = A E B, because of the equivalency of the triangles E F D and E F B, which, with A E F, make up A F D and A E B.

The point F is fixed by the similar triangles A D B and A E F.

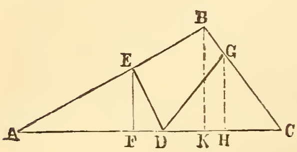
The expression for A F, in the last paragraph, is given by the proportion,

$$ABC : ADF :: AB \times AC : AD \times AF;$$

whence, $$AF = \frac{AB \times AC}{AD} \cdot \frac{ADF}{ABC} = \frac{AB \times AC}{AD} \cdot \frac{m}{m+n}.$$

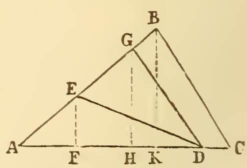
Next suppose that the triangle A B C is to be divided into *three* equivalent parts, meeting at D. The altitudes, E F and G H, of the parts A D E and D C G, will be obtained by dividing $\frac{1}{3}$ A B C, by half of the respective bases A D and D C.

Fig. 311.



If one of these quotients gives an altitude greater than that of the triangle A B C, it will show that the two lines D E and D G would both cut the same side, as in Fig. 312, in which E F is obtained as above, and G H $= \frac{2}{3}$ A B C $\div \frac{1}{2}$ A D.

Fig. 312.



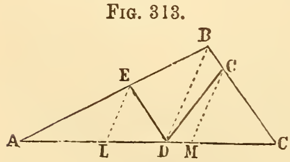
In practice it is more convenient to determine the points F and G, by these proportions:

B K : A K : : E F : A F; and B K : A K : : G H : A H.

The division of a triangle into a greater number of parts, having any ratios, may be effected in a similar manner.

This problem admits of a more elegant solution, analogous to that given for the division into two parts, graphically. Divide A C into three equal parts at L and M. Join B D, and from L and M draw parallels to it, meeting A B and B C in E and G. Draw E D and G D, which will be the desired lines of division. The figure is the same triangle as Fig. 311.

Fig. 313.

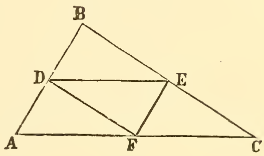


The points E and G can be obtained on the ground by measuring A D and A B, and making the proportion A D : A B : : $\frac{1}{3}$ A C : A E. The point G is similarly obtained.

The same method will divide a triangle into a greater number of parts.

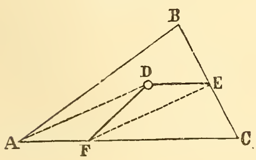
To divide a triangle into *four* equivalent triangles by lines terminating in the sides, is very easy. From D, the middle point of A B, draw D E parallel to A C, and from F, the middle of A C, draw F D and F E. The problem is now solved.

Fig. 314.



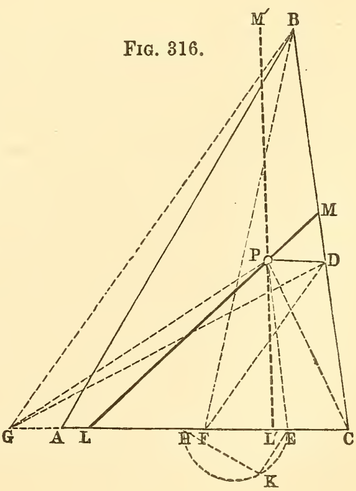
432. By Lines passing through a Point within the Triangle. Let D be a given point (such as a well, etc.) within a triangular field A B C, from which fences are to run so as to divide the triangle into *two* equivalent parts. Join A D. Take E in the middle of B C, and from it draw a parallel to D A, meeting A C in F. E D F is the fence required.

Fig. 315.



If it be required to divide a triangle into two equivalent parts by a straight line passing through a point within it, proceed thus: Let P be the given point. From P draw P D parallel to A C, and P E parallel to B C. Bisect A C at F. Join F D. From B draw B G parallel to D F. Then bisect G C in H. On H E describe a semicircle. On it set off E K = E C. Join K H. Set off H L = H K. The line L M drawn from L, through P, will be the division-line required.

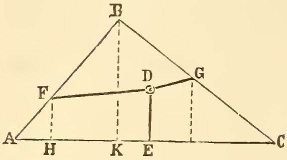
Fig. 316.



This figure is the same as that of Art. 416. The triangle A B C contains 62·35 acres, and the distance C L = 27·31 chains, as in the example in that article.

433. Next suppose that the triangle A B C is to be divided into *three* equivalent parts by lines starting from a point D, within the triangle, given by the rectangular co-ordinates A E and E D. Let E D be one of the lines of division, and F and G the other points required. The point F will be determined if A H is known; A H and H F being its rectangular co-ordinates. From B let fall the perpendicular B K on A C.

FIG. 317.



Then is $AH = \dfrac{AK\,(\frac{2}{3}ABC - AE \times ED)}{AE \times BK - ED \times AK}$. The position of the other point, G, is determined in a similar manner.

Demonstration. Let $AE = x$, $ED = y$, $AH = x'$, $HF = y'$, $AK = a$, $KB = b$.

The quadrilateral A F D E, equivalent to $\frac{1}{3}$ A B C, but which we will represent generally by m^2, is made up of the triangle A F H and the trapezoid F H E D.

$$AFH = \tfrac{1}{2} . x'y'. \qquad FHED = \tfrac{1}{2}(x - x')(y + y').$$

$$\therefore AFDE = m^2 = \tfrac{1}{2} . x'y' + \tfrac{1}{2}(x - x')(y + y') = \tfrac{1}{2}x(y + y) - \tfrac{1}{2}x'y.$$

The similar triangles, A H F and A K B, give

$$a : b :: x' : y' = \frac{b\,x'}{a}.$$

Substituting this value of y' in the expression for m^2, we have

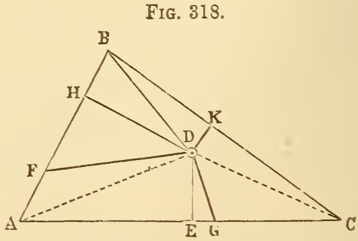
$$m^2 = \tfrac{1}{2}x\left(y + \frac{b\,x'}{a}\right) - \tfrac{1}{2}x'y;$$

whence, $$x' = \frac{a\,(2\,m^2 - x\,y)}{b\,x - a\,y} = \frac{AK\,(\frac{2}{3}ABC - AE \times ED)}{KB \times AE - AK \times ED}.$$

The formula is general, whatever may be the ratio of the area m^2 to that of the triangle A B C.

Let D B, instead of D E, be one of the required lines of division. Divide $\frac{1}{3}$ A B C by half of the perpendicular D H, let fall from D to A B, and the quotient will be the distance B F. To find G, if, as in this figure, the triangle B D C ($=$ B C $\times$ $\frac{1}{2}$ D K) is less than $\frac{1}{3}$ A B C, divide the excess of the

FIG. 318.



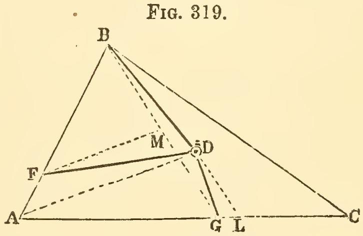
latter (which will be C D G) by $\frac{1}{2}$ D E, and the quotient will be C G.

Example. Let A B = 30·00 ; B C = 45·00 ; C A = 50·00. Let the perpendiculars from D to the sides be these : D E = 10·00 ; D H = 20·00 ; D K = 5·17$\frac{1}{3}$. The content of the triangle A B C will be 666·6 square chains. Each of the small triangles must therefore contain 222·2 square chains, B D being one division-line. We shall therefore have B F = 222·2 ÷ $\frac{1}{2}$ D H = 22·2 chains. B D C = 45 × $\frac{1}{2}$ × 5·17$\frac{1}{3}$ = 116·4 square chains, not enough for a second portion, but leaving 105·8 square chains for C D G ; whence C G = 21·16 chains. To prove the work, calculate the content of the remaining portion, G D F A. We shall find D G A = 144·2 square chains, and A D F = 78·0 square chains, making together 222·2 square chains, as required.

The scale of Fig. 318 is 30 chains to 1 inch = 1 : 23760.

434. The preceding case may be also solved graphically, thus: Take C L = $\frac{1}{3}$ A C. Join D L, and from B draw B G parallel to D L. Join D G. It will be a second line of division. Then take a point, M, in the middle of B G, and from it draw a line, M F, parallel to D A. D F will be the third line of division. This method is neater on paper than the preceding, but less convenient on the ground.

Fig. 319.

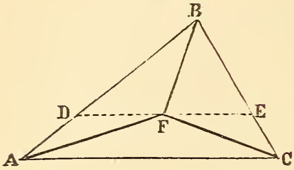


Demonstration. In Fig. 319 D G is a second line of division, because, drawing B L, the triangle B L C = $\frac{1}{3}$ A B C; and B D G C is equivalent to B L C, because of the common part B C L D, and the equivalency of the triangles D L G and D L B.

To prove that D F is a third line of division, join M D and M A. Then B M A = $\frac{1}{2}$ B G A. From B M A take M F A and add its equivalent M F D, and we have M D F B = $\frac{1}{2}$ B G A = $\frac{1}{2}$ (A B D G − B D G) = $\frac{1}{2}$ ($\frac{2}{3}$ A B C − B D G) = $\frac{1}{3}$ A B C − $\frac{1}{2}$ B D G. To M D F B add M D B, and add its equivalent, $\frac{1}{2}$ B D G, to the other side of the equation, and we have
M D F B + M D B = $\frac{1}{3}$ A B C − $\frac{1}{2}$ B D G + $\frac{1}{2}$ B D G; or, B D F = $\frac{1}{3}$ A B C.

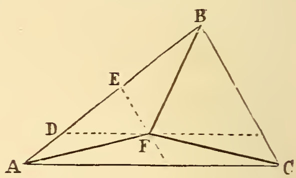
435. Let it be required to divide the triangle A B C into three equivalent triangles, by lines drawn from the three angular points to some *unknown* point within the triangle. This point is now to be found. On any side, as A B, take A D = $\frac{1}{3}$ A B. From D draw D E parallel to A C. The middle, F, of D E, is the point required.

Fig. 320.



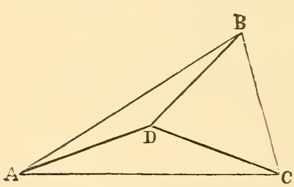
If the three small triangles are not to be equivalent, but are to have to each other the ratios : : $m : n : p$, divide a side, A B, into parts having these ratios, and through each point of division, D, E, draw a parallel to the side nearest to it. The intersection of these parallels, in F, is the point required. In the figure the parts A C F, A B F, B C F, are as 2 : 3 : 4.

Fig. 321.



436. Let it be required to find the position of a point, D, situated within a given triangle, A B C, and equally distant from the points, A, B, C; and to determine the ratios to each other of the three triangles into which the given triangle is divided.

Fig. 322.

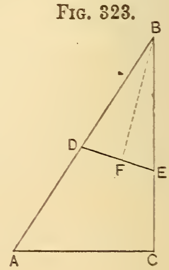


By construction, find the center of the circle passing through A, B, C. This will be the required point.

By calculation, the distance $D A = D B = D C = \frac{AB \times BC \times CA}{4 \times \text{area } ABC}$.

The three small triangles will be to each other as the sines of their angles at D—i. e., A D B : A D C : B D C : : sin. A D B : sin. A D C : sin. B D C. These angles are readily found, since the sine of half of each of them equals the opposite side divided by twice one of the equal distances.

Fig. 323.



437. By the Shortest Possible Line. Let it be required to divide the triangle A B C by the shortest possible line, D E, into two parts, which shall be to each other : : $m : n$; or D B E : A B C : : $m : m + n$.

From the smallest angle, B, of the triangle, measure along the sides, B A and B C, a distance $BD = BE = \sqrt{\left(\frac{m}{m+n} \times AB \times BC\right)}$.

D E is the line required. It is perpendicular to the line B F which bisects the angle A B C ; and it is

$$= \frac{\sin. B}{\cos. \frac{1}{2} B} \sqrt{\left(\frac{m}{m+n} \times AB \times BC\right)}.$$

The formulas are obtained from Art. 419.

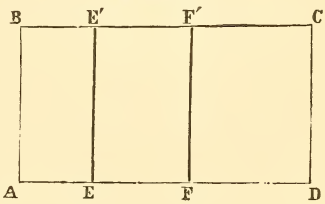
Division of Rectangles.

438. By Lines parallel to a Side. Divide two opposite sides into the required number of parts, either equal or in any given ratio to each other, and the lines joining the points of division will be the lines desired.

The same method is applicable to any parallelogram.

Example. A rectangular field A B C D, measuring 15·00 chains by 8·00, is bought by three men, who pay respectively \$300, \$400, and \$500. It is to be divided among them in that proportion. *Ans.* The portion of the first, A E E′B, is obtained by making the proportion 300 + 400 + 500 : 300 : : 15·00 : A E = 3·75. E F is in like manner found to be 5·00 ; and F D = 6·25. B E′ is made equal to A E ; E′ F′ to E F ; and F′ C to F D. Fences from E to E′, and from F to F′, will divide the land as required.

Fig. 324.



The scale of the figure is 10 chains to 1 inch = 1 : 7920.

The other modes of dividing up rectangles will be given under the head of "Quadrilaterals," Art. 443, etc.

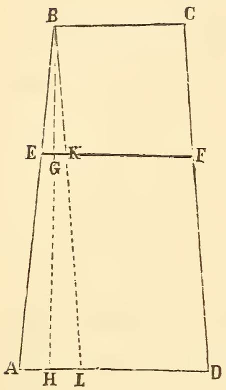
Division of Trapezoids.

439. By Lines parallel to the Bases. Given the bases and a third side of the trapezoid, A B C D, to be divided into two parts, such that B C F E : E F D A : : $m : n$.

The length of the desired dividing line,

$$EF = \sqrt{\left(\frac{m \times AD^2 + n \times BC^2}{m+n}\right)}.$$

FIG. 325.



The distance $BE = \frac{AB\,(EF - BC)}{AD - BC}$.

Demonstration. In Fig. 325, conceive the sides A B and D C, produced, to meet in some point P. Then, by reason of the similar triangles, $ADP : BCP :: AD^2 : BC^2$, whence, by "division," $ADP - BCP = ABCD : BCP :: AD^2 - BC^2 : BC^2$.

In like manner, comparing E F P and B C P, we get $EBCF : BCP : EF^2 - BC^2 : BC^2$. Combining these two proportions, we have $ABCD : EBCF :: AD^2 - BC^2 : EF^2 - BC^2$; or, $m + n : m :: AD^2 - BC^2 : EF^2 - BC^2$. Whence, $(m+n)\,EF^2 - m\,.\,BC^2 - nBC^2 = m\,.\,AD^2 - m\,.\,BC^2$;

$$\therefore EF = \sqrt{\left(\frac{m \times AD^2 + n \times BC^2}{m+n}\right)}.$$

Also, from the similar triangles formed by drawing B L parallel to C D, we have

$$AL : EK :: BA : BE = \frac{BA \times EK}{AL} = \frac{AB\,(EF - BC)}{AD - BC}.$$

Example. Let A D = 30 chains; B C = 20 chains; and A B = $54\frac{1}{3}$ chains; and the parts to be as 1 to 2; required E F and B E. *Ans.* E F = 23·80; and B E = 20·65.

The figure is on a scale of 30 chains to 1 inch = 1: 23760.

440. Given the bases of a trapezoid, and the perpendicular distance, B H, between them; it is required to divide it as before, and to find E F, and the altitude, B G, of one of the parts. Let $BCFE : EFDA :: m : n$. Then $BG = -\frac{BC \times BH}{AD - BC} +$

$$\sqrt{\left[\frac{m}{m+n} \times \frac{2 \times ABCD \times BH}{AD - BC} + \left(\frac{BC \times BH}{AD - BC}\right)^2\right]}.$$

$$EF = BC + BG \times \frac{AD - BC}{BH}.$$

Demonstration. Let $BEFC = \frac{m}{m+n}\,.\,ABCD = a$; let $BC = b$; $BH = h$; and $AD - BC = c$. Also, let $BG = x$; and $EF = y$. Draw B L parallel to C D. By similar triangles, $AL : EK :: BA : BE :: BH :$

B G; or, A D − B C : E F − B C : : B H : B G; i. e., $c : y - b :: h : x$; whence $x = \frac{h(y - b)}{c}$.

Also, the area B E F C $= a = \frac{1}{2}$. B G (E F + B C) $= \frac{1}{2} x (y + b)$; whence $y = \frac{2a}{x} - b$.

Substituting this value of y in the expression for x, and reducing, we obtain

$$x^2 + \frac{2bh}{c}x = \frac{2ah}{c}; \text{ whence we have } x = -\frac{bh}{c} \pm \sqrt{\left(\frac{2ah}{c} + \frac{b^2h^2}{c^2}\right)}.$$

The second proportion above gives $y - b = \frac{cx}{h}$; whence $y = b + \frac{c}{h} \cdot x$.

Replacing the symbols by their lines, we get the formulas in the text.

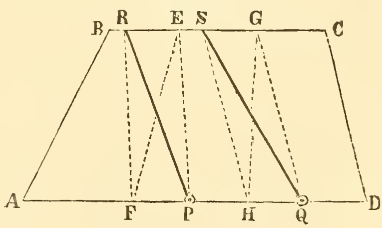
Example. Let A D = 30·00; B C = 20·00; B H = 54·00; and the two parts to be to each other : : 46 : 89.

The above data give the content of A B C D = 1,350 square chains. Substituting these numbers in the above formula, we obtain B G = 20·96, and E F = 23·88.

441. By Lines starting from Points in a Side. To divide a trapezoid into parts equivalent, or having any ratios, divide its parallel sides in the same ratios, and join the corresponding points.

If it be also required that the division-lines shall start from *given* points on a side, proceed thus: Let it be required to divide the trapezoid A B C D into three equivalent parts by fences starting from P and Q. Divide the trapezoid, as above directed, into three equivalent trapezoids by the lines E F and G H. These three trapezoids must now be transformed, thus: Join E P, and from F draw F R parallel to it. Join P R, and it will be one of the division-lines required.

Fig. 326.



The other division-line, Q S, is obtained similarly.

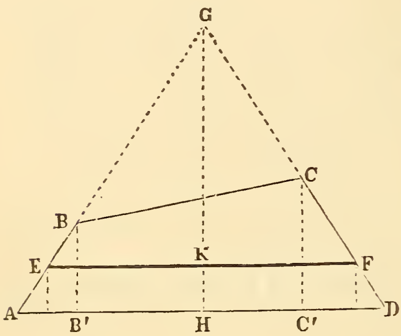
442. Other Cases. For other cases of dividing trapezoids, apply those for quadrilaterals in general, given in the following articles.*

* If a line be drawn joining the middle points of the parallel bases of a trape-

Division of Quadrilaterals.

443. By Lines parallel to a Side. Let A B C D be a quadrilateral which it is required to divide, by a line E F, parallel to A D, into two parts, B E F C and E F D A, which shall be to each other as $m : n$. Prolong A B and C D to intersect in G. Let a be the area of the triangle A D G, obtained by any method, graphical or trigonometrical, and a' = the area of the triangle B C G, obtained by subtracting the area of the given quadrilateral from that of the triangle A D G. Then $GK = GH\sqrt{\left(\frac{m\,a + n\,a'}{(m+n)\,a}\right)}$. Having measured this length of G K from G on G H, set off at K a perpendicular to G K, and it will be the required line of division.

FIG. 327.



Demonstration. In Fig. 327, since E F is parallel to A D, we have $ADG : EGF :: GH^2 : GK^2$. E G F is made up of the triangle $BCG = a'$, and the quadrilateral $BEFC = \frac{m}{m+n} \cdot ABCD = \frac{m}{m+n} \cdot (a - a')$. Hence the above proportion becomes

$$a : a' + \frac{m}{m+n}(a - a') :: GH^2 : GK^2;\ \text{or,}$$

$$(m+n)\,a : m\,a + n\,a' :: GH^2 : GK^2;\ \text{whence}\ GK = GH\sqrt{\left(\frac{m\,a+n\,a'}{(m+n)\,a}\right)}.$$

G E is given by the proportion $GH : GK :: GA : GE = GA \cdot \frac{GK}{GH}$.

In Fig. 328, the division into p parts is founded on the same principle. The triangle $EFG = GBC + EFCB = a' + \frac{Q}{p}$. Now $ADG : EFG :: AG^2 : EG^2$; or, $a' + Q : a' + \frac{Q}{p} :: AG^2 : EG^2$;

$$\text{whence}\ GE = AG\sqrt{\left(\frac{a' + \frac{Q}{p}}{a' + Q}\right)}.$$

---

zoid, *any line* drawn through the middle of the first line will divide the trapezoid into two equivalent parts.

G L is obtained by taking the triangle L M G $= a' + \frac{2\,Q}{p}$; and so for the rest.

Otherwise, take G E $=$ G A $\sqrt{\left(\frac{m\,a + n\,a'}{(m+n)\,a}\right)}$; and from E run a parallel to A D.

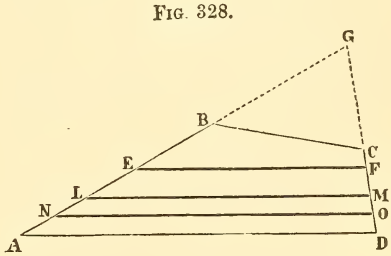
If the two parts of the quadrilateral were to be equivalent, $m = n$, and we have G K $=$ G H $\sqrt{\left(\frac{a + a'}{2\,a}\right)}$; and consequently G E to G A in the same ratio.

Example. Let a quadrilateral, A B C D, be required to be thus divided, and let its angles, B and C, be given by rectangular co-ordinates, viz., A B′ = 6·00 ; B′ B = 9·00 ; D C′ = 8·00 ; C′ C = 13·00 ; B′ C′ = 24·00. Here G H is readily found to be 29·64 ; A D G = 563·16 square chains ; and B G C = 220·16 square chains. Hence, by the formula, G K = 24·72 ; whence K H = G H − G K = 4·92 ; and the abscissas for the points E and F can be obtained by a simple proportion.

The scale of the figure is 20 chains to 1 inch = 1 : 15840.

If the quadrilateral be given by bearings, part off the desired area $= \frac{n}{m+n}$. A B C D, by the formulas of Art. 403.

FIG. 328.



Suppose now that a quadrilateral, A B C D, is to be divided into p equivalent parts, by lines parallel to A D. Measure, or calculate by trigonometry, A G. Let Q be the quadrilateral A B C D, and, as before, a' = B C G. Then

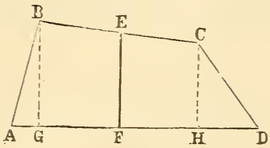
$$G\,E = A\,G\sqrt{\left\{\frac{a' + \frac{Q}{p}}{a' + Q}\right\}};\quad G\,L = A\,G\sqrt{\left\{\frac{a' + \frac{2\,Q}{p}}{a' + Q}\right\}};$$

$$G\,N = A\,G\sqrt{\left\{\frac{a' + \frac{3\,Q}{p}}{a' + Q}\right\}};\ \text{etc.}$$

If the quadrilateral be given by bearings, part off $\frac{1}{p}$. A B C D, then part off $\frac{2}{p}$. A B C D, etc. ; so in any similar case.

444. By Lines perpendicular to a Side. Let A B C D be a quadrilateral which is to be divided, by a line perpendicular to A D, into two parts having a ratio $= m : n$. By hypothesis, $ABEF = \frac{m}{m+n} . ABCD$.

Fig. 329.

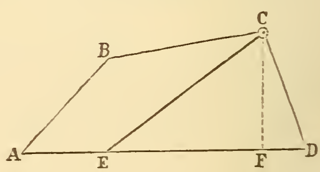


Taking away the triangle A B G, the remainder, G B E F, will be to the rest of the figure in a known ratio, and the position of E F, parallel to B G, will be found as in the last article.

445. By Lines running in any Given Direction. To divide a quadrilateral A B C D into two parts $: : m : n$, part off from it an area $= \frac{m}{m+n} . ABCD$, by the methods of Art. 407 or 408, if the area parted off is to be a triangle, or Art. 409 if the area parted off is to be a quadrilateral.

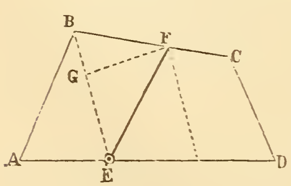
446. By Lines starting from an Angle. A B C D is to be divided, by the line C E, into two parts having the ratio $m : n$. Since the area of the triangle $CDE = \frac{m}{m+n} . ABCD$, D E will be obtained by dividing this area by half of the altitude C F.

Fig. 330.



447. By Lines starting from Points in a Side. Let it be required to divide A B C D into two parts $: : m : n$, by a line starting from the point E. The area A B F E is known (being $= \frac{m}{m+n} . ABCD$) as also A B E; AB, B E, and E A being given on the ground. B E F will then be known $= ABFE - ABE$. Then $GF = \frac{BEF}{\frac{1}{2} BE}$, and the point F is obtained by

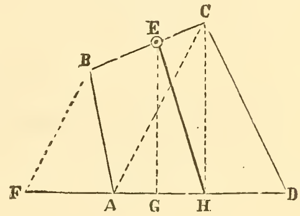
Fig. 331.



running a parallel to B E, at a perpendicular distance from it = G F.

To divide a quadrilateral, A B C D, graphically, into *two equivalent* parts by a line from a point, E, on a side, proceed thus: Draw the diagonal C A, and from B draw a parallel to it, meeting D A prolonged in F. Mark the middle point, G, of F D. Join G E. From C draw a parallel to E G, meeting D A in H. E H is the required line. The quadrilateral could also be divided in any ratio $= m : n$, by dividing F D in that ratio.

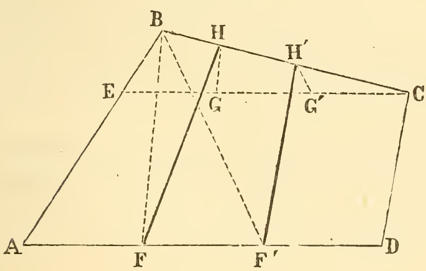
Fig. 332.



If the quadrilateral be given by bearings, proceed to part off the desired area, as in Art. 412 or 413.

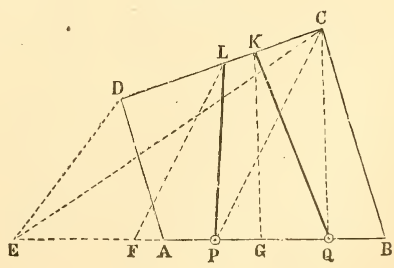
448. Let it be required to divide a quadrilateral, A B C D, into *three equivalent* parts. From any angle, as C, draw C E, parallel to D A. Divide A D and E C, each into three equal parts, at F, F′, and G, G′. Draw B F, B F′. From G draw G H, parallel to F B, and from G′ draw G′ H′, parallel to F′ B. F H and F′ H′ are the required lines of division.

Fig. 333.



Let it be required to make the above division by *lines starting from two given points*, P and Q. Reduce the quadrilateral to an equivalent triangle C B E. Divide E B into three equal parts at F and G. Join C Q, and, from G, draw G K parallel

Fig. 334.



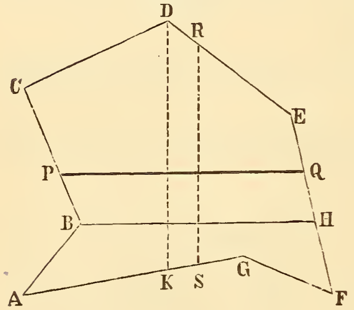
to it. Join C P, and from F draw F L parallel to it. Join P L and Q K, and they will be the division-lines required.

449. By Lines passing through a Point within the Figure. Proceed to part off the desired area as in Art. 416 or 417, according to the circumstances of the case.

Division of Polygons.

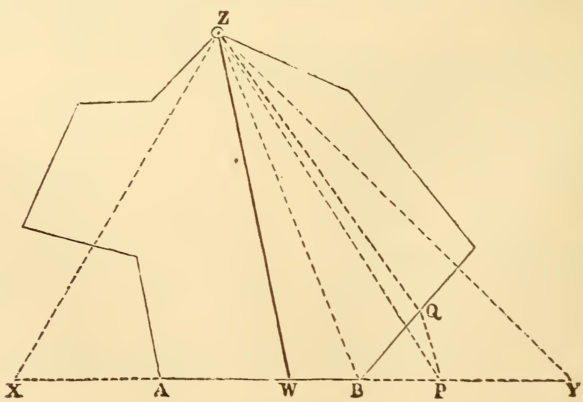
450. By Lines running in any Direction. Let A B C D E F G be a given polygon, and B H the direction parallel to which is to be drawn a line P Q, dividing the polygon into two parts in any desired ratio $= m : n$. The area P C D E Q $= \frac{m}{m+n}$. A B C D E F G. Taking it from the area B C D E H, the remainder will be the area B P Q H. The quadrilateral B C E H, C E being supposed to be drawn, can then be divided by the method of Art. 443 into two parts, B P Q H and P Q E C, having to each other a known relation.

FIG. 335.



If D K were the given direction, at right angles to the former, the position of a dividing line R S could be similarly obtained.

FIG. 336.



451. By Lines starting from an Angle. Produce one side, A B, of the given polygon, both ways, and reduce the polygon to a single equivalent triangle, X Y Z. Then divide the base, X Y, in the required ratio, as at W, and draw Z W, which will be the division-line desired. In this figure the polygon is divided into two equivalent parts.

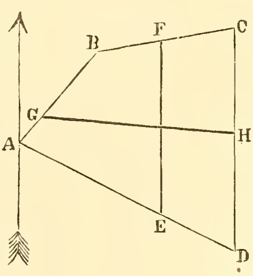
If the division-line should pass outside of the polygon, as does Z P, through P draw a parallel to B Z, meeting the adjacent side of the polygon in Q, and Z Q will be the division-line desired.

452. By Lines starting from a Point on a Side. See Articles 414 and 415.

453. By Lines passing through a Point within the Figure. Part off, as in Art. 416 or 418, if a straight line be required, or by guess-lines and the addition of triangles, as in Art. 433, if the lines have merely to start from the point, such as a spring or well.

454. Other Problems. The following is from Gummere's "Surveying": *Question.* A tract of land is bounded thus: N. $35\frac{1}{4}°$ E., 23·00; N. $75\frac{1}{2}°$ E., 30·50; S. $3\frac{1}{4}°$ E., 46·49; N. $66\frac{1}{4}°$ W., 49·64. It is to be divided into four equivalent parts by two straight lines, one of which is to run parallel to the third side; required the distance of the parallel division-line from the first corner, measured on the fourth side; also the bearing of the other division-line, and its distance from the same corner measured on the first side. *Ans.* Distance of the parallel division-line from the first corner, 32·50; the bearing of the other, S. 88° 22′ E.; and its distance from the same corner 5·99.

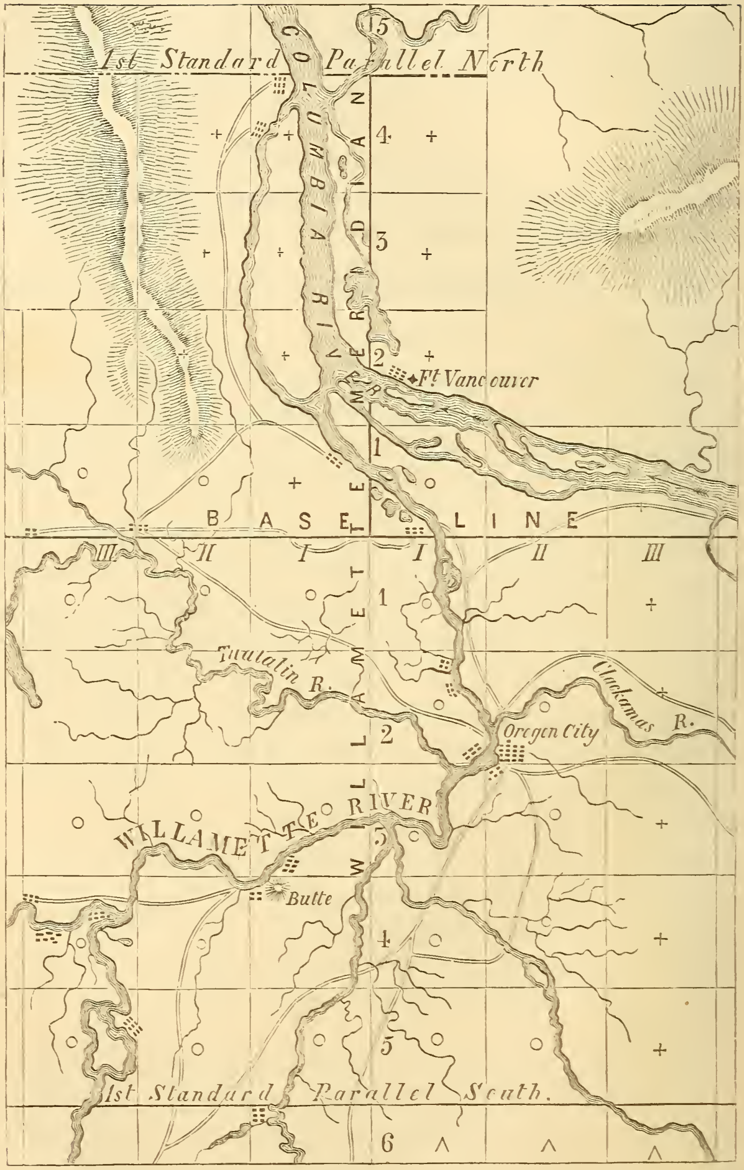
Fig. 337.



The scale of the figure is 40 chains to 1 inch = 1 : 31680.

An indefinite number of problems on this subject might be proposed, but they would be matters of curiosity rather than of utility, and exercises in geometry and trigonometry rather than in surveying.

FIG. 338.


1st Standard Parallel North
COLUMBIA RIVER
MERIDIAN
WILLAMETTE MERIDIAN
Ft Vancouver
BASE LINE
III
II
I
I
II
III
Tualatin R.
Clackamas R.
Oregon City
WILLAMETTE RIVER
Butte
1st Standard Parallel South.
5
4
3
2
1
1
2
3
4
5
6

CHAPTER VII.

*THE PUBLIC LANDS OF THE UNITED STATES.**

455. General System. The public lands of the United States of America are generally divided and laid out into squares (approximately), the sides of which run truly north and south, or east and west.

This is effected by means of meridian lines and parallels of latitude, established six miles apart. The squares thus formed are called TOWNSHIPS. They contain 36 square miles, or 23,040 acres, "as nearly as may be." A *principal meridian*, running due north and south, and a *base-line*, running due east and west, are first established astronomically, and the half-mile, mile, and six-mile corners are permanently marked on them. These two lines form the basis of all the subsequent subdivision into townships and sections. All of the lines on the public surveys, except these two and the *standard parallels*, are run with compass and chain.

The map, Fig. 338, represents a portion of the State of Oregon thus laid out. The scale is 10 miles to 1 inch = 1 : 633600. On it will be seen the "Willamette meridian," running truly north and south, and a "base-line," which is a "parallel of latitude," running truly east and west. Parallel to these, and six miles from them, are other lines, forming townships. All the townships, situated north or south of each other, form a RANGE. The ranges are named by their number east or west of the principal meridian. In the figure are seen three ranges east and west of

* Arts. 455 to 462 of this chapter are mainly taken from "Instructions to the Surveyor-General of Oregon, being a Manual for Field Operations," prepared, in March, 1851, by John M. Moore, Principal Clerk of Surveys.

the Willamette meridian. They are noted as R. I. E., R. I. W., etc. The townships in each range are named by their number north or south of the base-line. In the figure, along the principal meridian, are seen four north and five south of the base-line. They are noted as T. 1 N., T. 2 N., T 1 S., etc.*

Each township is divided into 36 SECTIONS, each one mile square, and therefore containing, "as nearly as may be," 640 acres. The sections in each township are numbered, as in the margin, from 1 to 36, beginning at the northeast angle of the township, and going west from 1 to 6, then east from 7 to 12, and so on alternately to section 36, which will be in the southeast angle of the township. The sections are subdivided into quarter-sections, half a mile square, and containing 160 acres, and sometimes into half-quarter-sections of 80 acres, and quarter-quarter-sections of 40 acres.

N

	6	5	4	3	2	1	
	7	8	9	10	11	12	
W	18	17	16	15	14	13	E
	19	20	21	22	23	24	
	30	29	28	27	26	25	
	31	32	33	34	35	36	

S

By this beautiful system, the smallest subdivision of land can be at once designated ; such as the northeast quarter of section 31, in township *two* south, in range *two* east of Willamette meridian.

456. Difficulty. "The law requires that the lines of the public surveys shall be governed by the true meridian, and that the townships shall be *six miles square*—two things involving in connection a mathematical impossibility—for, strictly to conform to the meridian, necessarily throws the township out of square, by reason of *the convergency of meridians ;* hence, adhering to the true meridian renders it necessary to depart from the strict requirements of law as respects the precise area of townships, and the subdivisional parts thereof, the township assuming something of a trapezoidal form, which inequality develops itself, more and more as such, the higher the latitude of the surveys. In view of these circumstances, the law provides that the sections of a mile square shall contain

* The marks O, +, and ∧, merely refer to the dates of the surveys. They are sometimes used to point out lands offered for sale, or reserved, etc.

the quantity of 640 acres, *as nearly as may be;* and, moreover, provides that, 'in all cases where the exterior lines of the townships, thus to be subdivided into sections or half-sections, shall exceed, or shall not exceed, six miles, the excess or deficiency shall be specially noted, and added to or deducted from the *western* or *northern* ranges of sections or half-sections in such township, according as the error may be in running the lines from east to west, or from south to north.'"

"In order to throw the excesses or deficiencies, as the case may be, on the *north* and on the *west* sides of a township, according to law, it is necessary to survey the *section-lines* from *south* to north on a true meridian, leaving the result in the northern line of the township to be governed by the convexity of the earth and the convergency of meridians."

Thus, suppose the land to be surveyed lies between 46° and 47° of north latitude. The length of a degree of longitude in latitude 46° N. is taken as 48·0705 statute miles, and in latitude 47° N. as 47·1944. The difference, or convergency per square degree = 0·8761 = 70·08 chains. The convergency per range (8 per degree of longitude) equals one eighth of this, or 8·76 chains; and per township (11½ per degree of latitude) equals the above divided by 11½—i. e., 0·76 chain. We therefore know that the width of the townships along their northern line is 76 links less than on their southern line. The townships north of the base-line therefore become narrower and narrower than the six-mile width with which they start, by that amount.

"STANDARD PARALLELS (usually called *correction-lines*) are established at stated intervals of 30 miles,* to provide for or counteract the error that otherwise would result from the convergency of meridians; and, because the public surveys have to be governed by the true meridian, such lines serve also to arrest errors arising from inaccuracies in measurements. Such lines, when lying north of the principal base, themselves constitute a *base* to the surveys on the north of them."

The convergency or divergency above noticed is taken up on

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* Until 1866 they were either 24 or 30 miles apart.

these correction-lines, from which the townships start again with their proper widths. On these, therefore, there are found *double corners,* both for townships and sections, one set being the *closing corners* of the surveys ending there, and the other set being the *standard corners* for the surveys starting there.

AUXILIARY MERIDIANS. These are run north and south from the base-line, at intervals of twenty-four miles, or four townships.

457. Running Township-Lines. "The principal meridian, the base-line, and the standard parallels, having been first astronomically run, measured, and marked, according to instructions, on true meridians, and true parallels of latitude, the process of running, measuring, and marking the exterior lines of townships will be as follows :

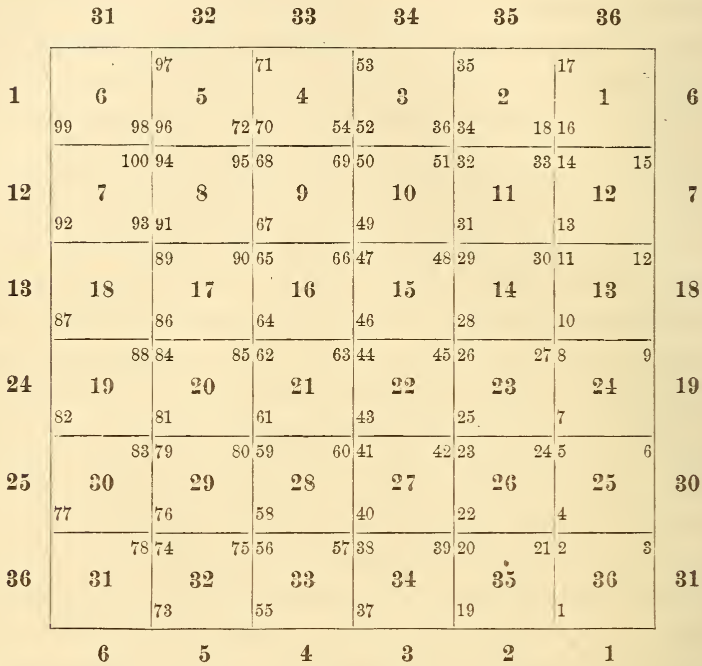
Townships situated NORTH *of the base-line and* WEST *of the principal meridian.** Commence at Station No. 1, being the southwest corner of T. 1 N.—R. 1 W., as established on the base-line ; thence run *north,* on a true meridian line, 480 chains, establishing the mile and half-mile corners thereon, as per instructions, to No. 2 (the northwest corner of the same township), whereat establish the corner of Tps. 1 and 2 N.—Rs. 1 and 2 W. ; thence *east,* on a random or trial line, setting *temporary* mile and half-mile stakes to No. 3 (the northeast corner of the same township), where measure and note the distance at which the line intersects the eastern boundary, north or south of the *true* or established corner. Run and measure *westward,* on the true line (taking care to note all the land and water crossings, etc., as per instructions), to No. 4, which is identical with No. 2, establishing the mile and half-mile PERMANENT CORNERS on said line, the last half-mile of which will fall short of being forty chains, by about the amount of the calculated convergency per township, 76 links in the case above supposed. Should it ever happen, however, that such random line materially falls short, or overruns in length, or intersects the eastern boundary of the township at any considerable distance from the *true* corner thereon (either of which would indicate an im-

* The surveyor should prepare a diagram of the townships, with the numbers here referred to, in their proper places, as here indicated.

portant error in the surveying), the lines must be *retraced,* even if found necessary to remeasure the meridional boundaries of the township (especially the western boundary), so as to discover and correct the error ; in doing which, the *true corners* must be established and marked, and the *false ones* destroyed and obliterated, to prevent confusion in future ; and *all the facts* must be distinctly set forth in the notes. Thence proceed in a similar manner north, from No. 4 to No. 5 (the N. W. corner of T. 2 N.—R. 1 W.), east from No. 5 to No. 6 (the N. E. corner of the same township), west from No. 6 to No. 7 (the same as No. 5), north from No. 7 to No. 8 (the N. W. corner of T. 3 N., R. 1 W.), east from No. 8 to No. 9 (the N. E. corner of the same township, and thence west to No. 10 (the same as No. 8), or the southwest corner T. 4 N.—R. 1 W. Thence north, still on a true meridian line establishing the mile and half-mile corners, until reaching the STANDARD PARALLEL or correction-line (which is here four townships north of the base-line) ; throwing the *excess* over, or *deficiency* under, *four hundred and eighty chains,* on the *last* half-mile, according to law, and at the intersection establishing the "CLOSING CORNER," the distance of which *from* the standard corner must be measured and noted as required by the instructions. But should it ever so happen that some impassable barrier will have prevented or delayed the extension of the standard parallel along and above the field of present survey, then the surveyor will plant, in place, the corner for the township, subject to correction thereafter, should such parallel be extended.

Townships situated NORTH *of the base-line, and* EAST *of the principal meridian.* Commence at No. 1, being the *southeast* corner of T. 1 N.—R. 1 E., and proceed as with townships situated "north and west," except that the *random* or trial lines will be run and measured *west,* and the *true* lines east, throwing the excess over or deficiency under four hundred and eighty chains on the *west end* of the line, as required by law ; wherefore, the surveyor will commence his measurement with the length of the deficient or excessive half-section boundary on the west of the township, and thus the remaining measurements will all be *even* miles and half-miles.

458. Running Section-Lines. The interior or sectional lines of all townships, however situated in reference to the BASE and MERIDIAN lines, are laid off and surveyed as below:



In the above diagram, the squares and large figures represent sections, and the small figures at their corners are those referred to in the following directions:

"Commence at No. 1 (see *small* figures on the diagram), the corner established on the township boundary for sections 1, 2, 35, and 36; thence run *north* on a true meridian; at 40 chains setting the half-mile or quarter-section post, and at 80 chains (No. 2) establishing and marking the corner of sections 25, 26, 35, and 36. Thence *east*, on a *random* line, to No. 3, setting the temporary quarter-section post at 40 chains, noting the measurement to No. 3, and the measured distance of the random's intersection *north* or *south* of the true or established corner of sections 25, 36, 30, and 31, on the township boundary. Thence *correct, west,* on the *true* line to No. 4, setting the quarter-section post on this line exactly at the *equidistant point,* now known, between the section corners indicated by the small figures Nos. 3 and 4. Proceed, in like manner, from No. 4 to No. 5, 5 to 6, 6 to 7, and so on to No. 16, the corner to sections 1, 2, 11, and 12. Thence *north* on a random line, to No. 17, setting a temporary quarter-section post at 40 chains, noting the length of the whole line, and the measured distance of the random's intersection *east* or *west* of the true corner of sections 1, 2, 35, and 36, established on the township boundary; thence *southwardly* from the latter, on a true line, noting the course and distance to No. 18, the established corner to sections 1, 2, 11, and 12, taking care to establish the quarter-section corner on the true line, at the distance of 40 chains from said section corner, so as to throw the *excess* or *deficiency* on the northern half-mile, according to law. Proceed in like manner through all the intervening tiers of sections to No. 73, the corner to sections 31, 32, 5, and 6; thence *north,* on a true meridian line, to No. 74, establishing the quarter-section corner at 40 chains, and at 80 chains the corner to sections 29, 30, 31, and 32; thence *east,* on a random line to No. 75, setting a temporary quarter-section post at 40 chains, noting the measurement to No. 75, and the distance of the random's intersection *north* or *south* of the established corner of sections 28, 29, 32, and 33; thence *west* from said corner, on the true line, setting the quarter-section post at the equidistant point, to No. 76, which is identical with 74; thence *west,* on a random line, to No. 77, and setting a temporary quarter-section post at 40 chains, noting the measurement to No. 77, and the distance of the random's intersection with the western boundary, *north* or *south* of the established corner of sections 25, 36, 30, and 31; and from No. 77, correct, *eastward,* on the true line, *giving its course,* but establishing the quarter-section post, on this line so as to retain the distance of 40 chains from the corner of sections 29, 30, 31, and 32; thereby throwing the *excess* or *deficiency* of measurement on the most *western* half-mile. Proceed *north,* in a similar manner, from No. 78 to 79, 79 to 80, 80 to 81, and so on to 96, the southeast corner of section 6, where having established the corner for sections 5, 6, 7, and 8, run thence, successively, on random line *east* to 95, *north* to 97, and *west* to 99; and by reverse courses *correct on true lines back* to said *southeast* corner of section 6, establishing the quarter-section corners, and noting the courses, distances, etc., as before described.

"In townships contiguous to standard parallels, the above method will be varied as follows: In every township SOUTH of the principal base-line, which *closes* on a standard parallel, the surveyor will begin at the *southeast* corner of the township, and measure *west* on the standard, establishing thereon the *mile* and *half-mile* corners, and noting their distances from the pre-established corners. He then will proceed to subdivide, as directed under the above head.

"In the townships NORTH of the principal base-line, which *close* on the standard parallel, the sectional lines must be closed on the standard by true meridians, instead of by course-lines, as directed under the above head for townships otherwise situated; and the connections of the closing corners with the pre-established standard corners are to be ascertained and noted. Such procedure does away with any necessity for running the randoms. But in case he is unable to close the lines on account of the standard not having been run, from some inevitable necessity, as heretofore mentioned, he will plant a *temporary* stake, or mound, at the end of the *sixth* mile, thus leaving the lines and their connections to be finished, and the *permanent* corners to be planted, at such time as the standard shall be extended."

459. Exceptional Methods. Departures from the general system of subdividing public lands have been authorized by law in certain cases, particularly on water-fronts.

Thus, an act of Congress, March 3, 1811, authorized the surveyors of Louisiana, "in surveying and dividing such of the public lands in the said Territory, which are or may be authorized to be surveyed and divided, as are adjacent to any river, lake, creek, bayou, or water-course, to lay out the same into tracts, as far as practicable, of fifty-eight poles in front, and four hundred and sixty-five poles in depth, of such shape and bounded by such lines, as the nature of the country will render practicable and most convenient." Another act, of May 24, 1824, authorizes lands similarly situated "to be surveyed in tracts of two acres in width, fronting on any river, bayou, lake, or water-course, and running back the depth of forty acres; which tracts of land, so surveyed, shall be offered for sale entire, instead of in half-quarter-sections."

The "Instructions" from which we have quoted say: "In those localities where it would best subserve the interests of the people to have fronts on the navigable streams, and to run back into the uplands for quantity and timber, the principles of the act of May 24, 1824, may be adopted, and you are authorized to enlarge the quantity, so as to embrace four acres front by forty in depth, forming tracts of one hundred and sixty acres. But in so doing it is designed only to survey the lines *between every four lots* (or 640 acres), but to establish the boundary posts, or mounds, *in front* and *in rear*, at the distances requisite to secure the quantity of 160 acres to each lot, either rectangularly, when practicable, or at oblique angles, when otherwise. The angle is not important, so that the principle be maintained, as far as practicable, of making the work to square in the rear with the regular sectioning.

"The numbering of all anomalous lots will commence with No. 37, to avoid the possibility of conflict with the numbering of the regular sections."

The act of September 27, 1850, authorizes the Department, should it deem expedient, to cause the Oregon surveys to be executed according to the principles of what is called the "Geodetic Method."

The complete adoption of this has not been thought to be expedient; but "it was deemed useful to institute on the principal base and meridian lines of the public surveys in Oregon, ordered to be established by the act referred to, a system of triangulations from the recognized legal stations, to all prominent objects within the range of the theodolite; by means of which the relative distances of such objects, in respect to those main lines, and also to each other, might be observed, calculated, and protracted, with the view of contributing to the knowledge of the topography of the country in advance of the progressing linear surveys, and to obtain the elements for estimating the areas of valleys intervening between the spurs of the mountains."

"Meandering" is a name given to the usual mode of surveying with the compass, particularly as applied to navigable streams. The "Instructions" for this are, in part, as follows:

"Both banks of *navigable* rivers are to be meandered by taking the courses and distances of their sinuosities, and the same are to be entered in the '*meander* field-book.' At those points where either the township or section lines intersect the banks of a navigable stream, POSTS, or, where necessary, MOUNDS of *earth* or *stone* (as noted in Art. 461), are to be established at the time of running these lines. These are called 'meander corners'; and in meandering you are to commence at one of those corners on the township-line, coursing the banks, and measuring the distance of each course from your commencing corner to the next 'meander corner,' upon the same or another boundary of the same township; carefully noting your intersection with all intermediate meander corners. By the same method you are to meander the opposite bank of the same river.

"The crossing distance *between* the MEANDER CORNERS, on the same line, is to be ascertained by triangulation, in order that the river may be protracted with entire accuracy. The particulars to be given in the field-notes.

"The courses and distances on meandered navigable streams govern the calculations wherefrom are ascertained the true areas of the tracts of land (sections, quarter-sections, etc.) known to the law as *fractional*, and bounding on such streams.

"You are also to meander, in manner aforesaid, all *lakes* and deep ponds of the area of twenty-five acres and upward; also navigable bayous.

"The precise relative position of islands, in a township made fractional by the river in which the same are situated, is to be determined trigonometrically. Sighting to a flag or other fixed object on the island, from a special and carefully measured base-line, connected with the surveyed lines, on or near the river-bank, you are to form connection between the meander corners on the river to points corresponding thereto, in direct line, on the bank of the island, and there establish the proper meander corners, and calculate the distance across."

460. Marking-Lines. "All lines on which are to be established the legal corner boundaries are to be marked after this method, viz.: Those trees which may intercept your line must have two chops or notches cut on each side of them, without any other marks whatever. These are called '*sight-trees,*' or '*line-trees.*'

"A sufficient number of other trees standing nearest to your line, on either side of it, are to be *blazed* on two sides, diagonally or quartering toward the line, in order to render the line conspicuous, and readily to be traced, the blazes to be opposite each other, coinciding in direction with the line where the trees stand very near it, and to approach nearer each other, the farther the line passes from the blazed trees. Due care must ever be taken to have the lines so well marked as to be readily followed."

461. Marking-Corners. "After a true coursing, and most exact measurements, the corner boundary is the consummation of the work, for which all the previous pains and expenditure have been incurred. A boundary corner, in a timbered country, is to be a *tree,* if one be found at the precise spot; and if not, a *post* is to be planted thereat; and the position of the corner post is to be indicated by trees adjacent (called bearing-trees), the angular bearings and distances of which from the corner are facts to be ascertained and registered in your field-book.

"In a region where stone abounds, the corner boundary will be a small *monument of stones* alongside of a single marked stone, for a township corner—and *a single stone* for all other corners.

"In a region where timber is not near, nor stone, the corner will be a *mound of earth,* of prescribed size, varying to suit the case.

"Corners are to be fixed, for township boundaries, at intervals of every six miles; for section boundaries, at intervals of every mile, or 80 chains; and, for quarter-section boundaries, at intervals of every half-mile, or 40 chains.

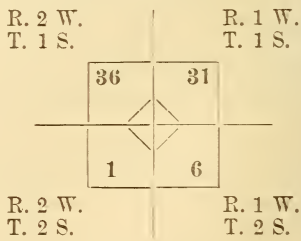
"MEANDER CORNER POSTS are to be planted at all those points where the township or section lines intersect the banks of such rivers, lakes, or islands, as are by law directed to be meandered," as explained in Art. 459.

"When *posts* are used, their length and size must be proportioned to the importance of the corner, whether township, section, or quarter-section, the first being at least twenty-four inches above-ground, and three inches square.

"Where a township post is a corner common to *four* townships, it is to be set in the earth *diagonally*, thus: N, W  E, S, and the cardinal points of the compass are to be indicated thereon by a cross-line, or wedge (one eighth of an inch deep at least), cut or sawed out of its top, as in the figure. On each surface of the post is to be marked the number of the particular township, and its range, which it *faces*. Thus, if the post be a common boundary to four townships, say *one* and *two*, south of the base-line, of range *one*, west of the meridian; also to townships *one* and *two*, south of the base-line, of range *two*, west of the meridian—it is to be marked thus:

From N. to E.	R. 1 W. T. 1 S. S. 31
From N. to W.	2 W. 1 S. 36
From E. to S.	1 W. 2 S. 6
From W. to S.	2 W. 2 S. 1

The position of the post, which is here taken as an example, is shown in the following diagram:



"These marks are to be distinctly and neatly chiseled into the wood, at least the eighth of an inch deep; and to be also marked with *red chalk*. The *number* of the *sections* which they respectively *face* will *also be marked* on the township post.

"*Section* or mile posts, being corners of sections, when they are common to *four* sections, are to be set *diagonally* in the earth (in the manner provided for township corner posts), and with a similar cross cut in the top, to indicate the cardinal points of the compass; and on each side of the squared surfaces is to be marked the appropriate *number* of the particular one of the *four sections*, respectively, which such side *faces;* also on one side thereof are to be marked the numbers of its *township* and *range;* and, to make such marks yet more conspicuous (in manner aforesaid), a streak of red chalk is to be applied.

"In the case of an *isolated* township, subdivided into thirty-six sections, there are twenty-five interior sections, the south-west corner boundary of each of which will be *common* to *four* sections. On all the extreme sides of an isolated township, the outer tiers of sections have corners *common* only to *two* sections then surveyed. The posts, however, must be planted precisely like the former, but presenting two *vacant* surfaces to receive the appropriate marks when the adjacent survey may be made.

"A quarter-section or half-mile post is to have no other mark on it than ¼ S., to indicate what it stands for.

"Township corner posts are to be NOTCHED with *six* notches on each of the four angles of the squared part set to the cardinal points.

"All mile-posts *on township lines* must have as many notches on them, on two opposite *angles* thereof, as they are miles distant from the township corners, respectively. Each of the posts at the corners of sections in the *interior* of a township must indicate, by a number of notches on each of its four corners directed to the cardinal points, the corresponding number of miles that it stands from the *outlines* of the township. The four sides of the post will indicate the number of the section they respectively *face.* Should a tree be found at the place of any corner, it will be marked and notched, as aforesaid, and answer for the corner in lieu of a post; the kind of tree and its diameter being given in the field-notes.

"The position of all corner posts, or corner trees of whatever description, which may be established, is to be perpetuated in the following manner, viz.: From such post or tree the courses shall be taken, and the distances measured, to two or more adjacent trees, in opposite directions, as nearly as may be, which are called '*bearing-trees,*' and are to be blazed near the ground, with a large blaze facing the post, and having one notch in it, neatly and plainly made with an axe, square across, and a little below the middle of the blaze. The kind of tree and the diameter of each are facts to be distinctly set forth in the field-book.

"On each bearing-tree the letters B. T. must be distinctly cut into the wood, in the blaze, a little above the notch, or on the bark, with the number of the range, township, and section.

"At all township corners, and at all section corners, on range or township lines, *four* bearing-trees are to be marked in this manner, one in each of the adjoining sections.

"At interior section corners *four* trees, one to stand within each of the four sections to which such corner is common, are to be marked in the manner aforesaid, if such be found.

"From quarter-section and meander corners two bearing-trees are to be marked, one within each of the adjoining sections.

"Stones at township corners (a small monument of stones being alongside thereof) must have *six* notches cut with a pick or chisel on each edge or side toward the cardinal points; and where used as section corners on the range and township lines, or as section corners in the interior of a township, they will also be notched by a pick or chisel, to correspond with the directions given for notching posts similarly situated.

"Stones, when used as quarter-section corners, will have ¼ cut on them; on the west side on north and south lines, and on the north side on east and west lines.

"Whenever bearing-trees are not found, MOUNDS of earth, or stone, are to be raised *around posts* on which the corners are to be marked in the manner aforesaid. Wherever a mound of earth is adopted, the same will present a conical shape; but at its base, on the earth's surface, a *quadrangular trench* will be dug; a *spade-deep* of earth being thrown up from the four sides of the line, *outside* the trench, so as to form a *continuous elevation along* *its outer edge.* In mounds of earth, common to *four* townships or to *four* sections, they will present the *angles* of the quadrangular trench (*diagonally*) toward the cardinal points. In mounds common only to *two* townships or *two* sections, the *sides* of the quadrangular trench will *face* the cardinal points.

"Prior to piling up the earth to construct a mound, in a *cavity* formed at the corner boundary point is to be deposited a *stone*, or a portion of *charcoal*, or a *charred stake* is to be driven twelve inches down into such center point, to be a *witness* for the future.

"The surveyor is further specially enjoined to plant, *midway* between each pit and the trench, seeds of some tree, those of fruit-trees adapted to the climate being always to be preferred.

"DOUBLE CORNERS are to be found nowhere except on the standard parallels or correction-lines, whereon are to appear both the corners which mark the intersection of the lines which close thereon, and those from which the surveys start in the opposite direction.

"The corners which are established on the standard parallel, at the time of running it, are to be known as '*Standard Corners*,' and, in addition to all the *ordinary* marks (as herein prescribed), they will be marked with the letters S. C. The '*closing corners*' will be marked C. C."

462. Field-Books. There should be several distinct and separate field-books, viz. :

"1. Field-notes of the MERIDIAN and BASE LINES, showing the establishment of the *township*, *section*, or mile, and *quarter-section* or half-mile, boundary corners thereon ; with the crossings of streams, ravines, hills, and mountains ; character of soil, timber, minerals, etc. These notes will be arranged, in series, by *mile-stations*, from number *one* to number ——.

"2. Field-notes of the 'STANDARD PARALLELS, or correction-lines,' showing the establishment of the township, section, and quarter-section corners, besides exhibiting the topography of the country on line, as required on the base and meridian lines.

"3. Field-notes of the EXTERIOR lines of TOWNSHIPS, showing the establishment of the corners on line, and the topography, as aforesaid.

"4. Field-notes of the SUBDIVISIONS of TOWNSHIPS into sections and quarter-sections ; at the close whereof will follow the notes of the MEANDERS of navigable streams. These notes will also show, by *ocular* observation, the estimated rise and fall of the land on the line. A description of the timber, undergrowth, surface, soil, and minerals, upon each section-line, is to follow the notes thereof, and not to be mixed up with them."

5. The "Geodetic Field-Book," comprising all triangulations, angles of elevation and depression, leveling, etc.

The examples on the next two pages, taken from the "Instructions" which we have followed throughout, will show what is required.

The ascents and descents are recorded in the right-hand columns.

For full details of public-land surveying, see "System of Rectangular Surveying," by J. H. Hawes.

"Instructions" are issued from the General Land-Office from time to time, giving any changes in methods of work, or of marking-points.

FIELD-NOTES OF

THE EXTERIOR LINES

OF AN ISOLATED TOWNSHIP.

Field-notes of the Survey of Township 25 north, of Range 2 west, of the Willamette meridian, in the Territory of OREGON, *by Robert Acres, Deputy-Surveyor, under his contract No. 1, bearing date the 2d day of January, 1851.*

	Chs. lks.		Feet.
Random tp. S. boundary.		TOWNSHIP LINES commenced January 20, 1851.	
		Southern boundary variation 18° 41′ E.	
	East.	On a *random* line on the south boundaries of sections 31, 32, 33, 34, 35, and 36. Set temporary mile and half-mile posts, and intersected the eastern boundary 2 chains 20 links north of the true corner 5 miles 74 chains 53 links.	
		Therefore the correction will be 5 chains 47 links W., 37·1 links S. per mile.	
Timbered corners.		TRUE SOUTHERN BOUNDARY variation 18° 41′ E.	
	West.	On the southern boundary of sec. 36, Jan. 24, 1851.	
	40·00	Set qr. sec. post from which	*a* 10
		a beech 24 in. dia. bears N. 11 E. 38 links dist.	
		a do. 9 do. do. S. 9 E. 17 do.	
	62·50	a brook 8 l. wide, course N. W.	*d* 10
	80·00	Set post cor. of secs. 35 & 36, 1 & 2, from which	*a* 5
		a beech 9 in. dia. bears S. 46 E. 8 l. dist.	
		a do. 8 do. do. S. 62 W. 7 do.	
		a w. oak 10 do. do. N. 19 W. 14 do.	
		a b. oak 14 do. do. N. 29 E. 16 do.	
		Land level, part wet and swampy; timber, beech, oak, ash, hickory, etc.	
Deficient timbered corners.	West.	On the S. boundary of sec. 35—	
	40·00	Set qr. sec. post, with trench, from which	*a* 10
		a beech 6 in. dia. bears N. 80 E. 8 l. dist.	
		planted S. W. a yellow-locust secd.	
	65·00	To beginning of hill	*a* 5
	80·00	Set post, with trench, cor. of secs. 34 & 35, 2 & 3, from which	*a* 20
		a beech 10 in. dia. bears S. 51 E. 13 l. dist.	
		a do. 10 do. do. N. 56 W. 9 do.	
		Planted S. W. a white-oak acorn,	
		N. E. a beechnut.	
		Land level, rich, and good for farming; timber same.	
Mound section corner.	West.	On the S. boundary of sec. 34—	
	40·00	Set qr. sec. post, with trench, from which	*a* 5
		a black oak 10 in. dia. bears N. 2 E. 635 l. dist.	
		Planted S. W. a beechnut.	
	80·00	To corner of sections 33, 34, 3 and 4, drove charred stakes; raised mound, with trench, as per instructions, and	*a* 10
		Planted N. E. a white-oak acorn; N. W. a yellow-locust seed; S. E. a butternut; S. W. a beechnut.	
		Land level, rich, and good for farming; some scattering oak and walnut.	
		Etc., etc., etc.	

FIELD-NOTES OF THE

SUBDIVISIONAL OR SECTIONAL LINES,

AND MEANDERS.

Township 25 N., Range 2 W., Willamette Mer.

	Chs. lks.		Feet.
		SUBDIVISIONS. Commenced February 1, 1851.	
True line.	North.	Between secs. 35 and 36—	
	9·19	A beech 30 in. dia	*d* 10
	29·97	A beech 30 in. dia	*d* 5
	40·00	Set qr. sec. post, from which	*d* 5
		a beech 15 in. dia. bears S. 48 E. 12 l. dist.	
		a do. 8 do. do. N. 23 W. 45 do.	
	51·90	A beech 18 in. dia	*d* 5
	76·73	A sugar 30 in. dia	*d* 8
	80·00	Set a post cor. of secs. 25, 26, 35, 36, from which	*d* 2
		a beech 24 in. dia. bears N. 62 W. 17 l. dist.	
		a poplar 36 do. do. S. 66 E. 34 do.	
		a do. 20 do. do. S. 70 W. 50 do.	
		a beech 28 do. do. N. 60 E. 45 do.	
		Land level, second rate; timber, beech, poplar, sugar, and und'gr. spice, etc.	
Random.	East.	On random line between secs. 25 and 36—	
	9·00	A brook 30 l. wide, course N	*d* 10
	15·00	To foot of hill	*d* 10
	40·00	Set temporary qr. sec. post	*a* 60
	55·00	To opposite foot of hill	*d* 40
	72·00	A brook 15 l. wide, course N	*d* 20
	80·00	Intersect E. boundary at post	*a* 10
		Land level, second rate; timber, beech, oak, ash, etc.	
		Etc., etc., etc.	

MEANDERS OF CHICKEELES RIVER.

Beginning at a meander post in the northern township boundary, and thence on the left bank down-stream. *Commenced February 11, 1851.*

Courses.	Distances. Chs. lks.	REMARKS.
S. 76 W.	18·46	In section 4 bearing to corner sec. 4 on right bank N. 70° W.
S. 61 W.	10·00	Bearing to cor. sec. 4 and 5, right bank N. 52° W.
S. 61 W.	8·18	To post in line between sections 4 and 5, breadth of river by triangulation 9 chains 51 links.
S. 54 W.	10·69	In section 5.
S. 40 W.	5·59	
S. 50 W.	8·46	
S. 37 W.	16·50	To upper corner of John Smith's claim, course E.
S. 44 W.	21·96	
S. 36 W.	27·53	To post in line between sections 5 and 8, breadth of river by triangulation 8 chains 78 links.
		Etc., etc., etc.

THE SOLAR COMPASS.

463. Nearly all of the lines required in the public-land surveys are meridians and parallels of latitude. Meridians may be located by the methods given in Chapter III, but the easiest method is with the *Solar Compass.*

There are several varieties of this instrument, all of which are constructed on the same principle, and are modifications of the instrument invented by William A. Burt, and patented by him in 1836.

Before describing the solar compass, it will be necessary to define the terms to be used.

464. Definitions. The *axis of the earth* is the imaginary line about which it revolves. The points in which the axis meets the surface of the earth are called the *poles* of the earth.

Meridians are great circles of the earth's surface, passing through the poles. The equator is a great circle of the earth's surface, 90° from the poles. *Parallels of latitude* are small circles of the earth's surface parallel to the equator. *Latitude* is the distance north or south from the equator, and is measured on a meridian circle. *Longitude* is distance east or west from some established meridian. The meridian of Greenwich, England, is usually taken as the *prime meridian*, from which longitude is reckoned.

Astronomical Terms. Conceive all of the heavenly bodies projected upon the concave surface of a sphere, of which the earth is the center, and whose radius is infinitely great when compared with that of the earth. This is called the *Celestial Sphere.*

If the axis of the earth be prolonged, the points in which it meets the celestial sphere are called the north and south poles of the heavens, and the line joining them is called the axis of the celestial sphere. The apparent revolution of the heavenly bodies about the axis of the celestial sphere is due to the rotation of the earth on its axis once in twenty-four hours.

A plane passed tangent to the earth at the feet of an observer is the *sensible horizon;* and a plane passed, parallel to this, through the center of the earth, is the *rational horizon.* Since the radius of the earth is infinitely small in comparison with that of the celestial sphere, if the planes of the rational horizon and sensible horizon be extended in every direction indefinitely, they will meet the celestial sphere in one great circle, called the *celestial horizon.* If the plane of the earth's equator be extended indefinitely, it will meet the celestial sphere in a great circle, called the *celestial equator*, or *equinoctial.*

If through any place a line be passed, perpendicular to the plane of the horizon, the point in which it meets the celestial sphere above the observer is called the *zenith;* and the point in which it meets the celestial sphere below the observer, the *nadir.*

Great circles passing through the zenith and nadir are *vertical circles.*

The *zenith distance* of a *heavenly body* is its angular distance from the zenith, and is measured on a vertical circle. The *altitude* of a body is its angular distance above the celestial horizon, and is measured on a vertical circle. Altitude and zenith distance are complements of each other.

Great circles passing through the poles of the celestial sphere are called *circles of declination*, or *hour-circles.* The *declination* of a heavenly body is its angular distance north or south from the equinoctial, and is measured on a circle of declination.

The celestial meridian of any place is a great circle passing through the zenith, and through the poles of the celestial sphere. The line in which the plane of the celestial meridian meets the plane of the horizon is the *terrestrial meridian*, or true north and south line.

The *hour-angle* of a heavenly body is the angle at the pole between the meridian and the declination circle passing through the *body.*

The *parallactic angle* is the angle at the body between the declination circle and vertical passing through the body.

The *azimuth* of a heavenly body is the angle between the celestial meridian and a vertical circle passing through the body, and is measured on the celestial horizon.

If an observer be at the equator, the celestial horizon will pass through the poles of the heavens, and the celestial equator through the zenith. For each degree which the observer travels northward on the earth, the north pole of the heavens will appear to rise *one degree* above the horizon, and the celestial equator will appear to move *one degree* southward from the zenith. The latitude of a place, then, is equal to the altitude of the elevated pole, or to the declination of the zenith. In the northern hemisphere the north pole of the heavens is the elevated pole.

The earth revolves around the sun in an elliptical orbit once in a year. This gives the sun an apparent motion around the earth. The path of the earth, or the apparent path of the sun in the heavens, is called the *ecliptic.* It is a great circle on the celestial sphere, making an angle with the celestial equator of about 23° 27′. The two points in which the ecliptic meets the equinoctial are called the *equinoxes.* The sun is on the equinoctial the 21st of March. This is the *vernal equinox.* It then moves north of the equator, increasing constantly in northern declination, until the 21st of June, when its declination is about 23° 27′ north. This is the *northern summer solstice.* It then decreases in declination until September 21st, when it is again on the equinoctial. This is the *autumnal equinox.* It then moves south of the equator, increasing in southern declination until December 21st, when its declination is about 23° 27′ south. This is the *northern winter solstice.* It then decreases in declination until March 21st, when it again arrives at the vernal equinox. The declination of the sun is given in the "Nautical Almanac" for every day in the year.

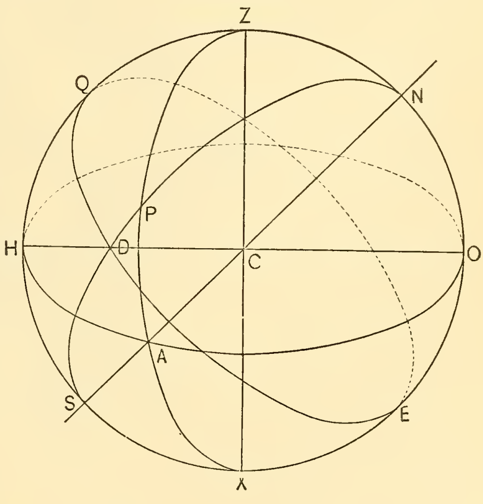
The *transit* of a heavenly body is its passage across the celestial meridian.

A *sidereal day* is the interval of time between two successive transits of the vernal equinox. A *solar day* is the interval of time between two successive transits of the sun. The apparent motion of the sun is not uniform, and hence use is made of a fictitious, or mean sun, moving on the equinoctial with a uniform motion, and keeping *mean solar time.* This is the time kept by clocks and watches. The time indicated by the true sun is called *apparent solar time.* This is the time given by sun-dials. The difference between apparent solar time and mean solar time is called the *equation of time.* The equation of time is zero four times in a year, and its maximum value is about sixteen minutes. It is given in the "Nautical Almanac" for every day in the year.

A ray of light, passing from a rarer to a denser medium, is bent, or refracted, toward a perpendicular to the surface of the second medium at the point where the ray enters. The atmosphere surrounding the earth varies in density, being denser as we approach the surface of the earth. The light coming from a heavenly body, and passing through the atmosphere, will be constantly bent toward a perpendicular to the surface of the earth, and its path will be a curve, and not a straight line. The apparent direction of a heavenly body will be tangent to this curve where it meets the eye of the observer. The difference between the apparent and the true positions of a heavenly body is called *refraction.* It is zero at the zenith, and about 33′ at the horizon; 45° from the zenith it is about 57″.

Refraction increases the altitude of a heavenly body and decreases the zenith distance.

FIG. 339.



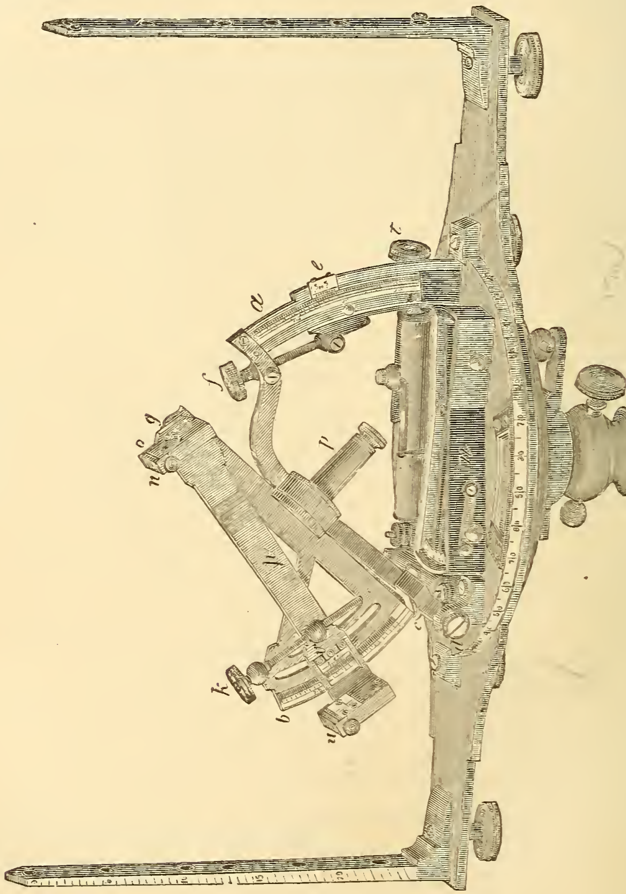
In Fig. 339, N S represents the axis of the celestial sphere, N the north pole, and S the south pole. E D Q is the equinoctial, H A O the horizon, and H Z O X the meridian. Z A X is a vertical circle, N D S a declination-circle. C (the position of the earth) is the center of the celestial sphere. Z is the zenith and X the nadir. Let P be any point on the celestial sphere. A P is its altitude, P Z its zenith distance, and P D its declination; Z N P its hour-angle, Z P N its parallactic angle, and N Z P its azimuth.

465. The solar compass differs from the ordinary compass, Fig. 135, in having a solar apparatus, instead of a magnetic needle, for determining the meridian.

In the figure, a is the *latitude-arc*, whose center of motion is in two pivots, one of which is shown at d. It is furnished with a clamp, slow-motion screw, f, and vernier, e.

The *declination-arc* is shown at b. The movable arm, h, has its center of motion in a pivot at g, and is furnished with a clamp, vernier, v, and a slow-motion screw, k.

Fig. 340.



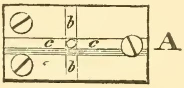
Burt's Solar Compass.

The plane of the *hour-arc*, c, is at right angles to the latitude-arc, and its center is in the *polar axis* p.

The declination-arc and latitude-arc are read to minutes by the verniers. The hour-arc is graduated to half-degrees, and is figured both for hours and degrees.

Attached to each end of the arm h is a rectangular block of brass, in which is set a convex lens, whose focus is on a silver plate attached to the face of the opposite block. The silver plate is marked by two sets of parallel lines, at right angles to each other, as shown in Fig. 341 ; $b\ b$ are called the *hour-lines,* and $c\ c$ the equatorial lines. The distance between the hour-lines and between the equatorial lines is equal to the diameter of the image of the sun, formed by the lens in the opposite block.

Fig. 341.



The needle-box n contains a magnetic needle, and is furnished with an arc of about 36° in extent, graduated to half-degrees. The needle-box can be moved about its center by the slow-motion screw t.

The sight and levels are similar to those of the ordinary compass.

The *equatorial sights,* u and n, attached to the upper side of the rectangular lens-blocks, are used in the adjustments.

The *adjuster,* also used in adjusting the instrument, is kept in the instrument-box, and is not shown in the figure.

The compass-sights are attached to the lower plate, and the solar apparatus, levels, and needle-box to the upper plate. The horizontal limb is read to single minutes by the vernier.

Suppose the instrument to be set up and leveled, with the latitude-arc toward the south. If, now, the latitude-arc be set to the latitude of the place of observation (that is, so that the plane of the hour-arc makes an angle with the vertical equal to the latitude of the place), the plane of the hour-arc will then be in the plane of the celestial equator, and the polar axis will be parallel to the axis of the earth, and will point toward the north pole of the heavens. If the sun be on the celestial equator, the declination-arm, h, may be set at zero on the declination-arc, and it will then lie in the plane in which the sun appears to move. If the declination-arc be turned so as to point toward the sun, the lens in the block toward the sun will form an image on the silver plate attached to the opposite block. By means of the polar axis, p, the declination-arm may be turned so as to follow the sun all day.

When the sun is not at the equinoxes, set off its declination on the declination-arc, and the declination-arm, when turned about on the axis, p, will still turn in the plane in which the sun appears to move. When the sun is in south declination, turn the declination-arc away from the sun ; and when the sun is in north declination, turn the declination-arc toward the sun.

When the instrument is in perfect adjustment, and is properly set up and leveled, the image of the sun can not be brought between the equatorial lines, unless the sights are in the plane of the meridian.

Adjustments.

466. The adjustments will be given in the order in which they should be made. In describing each adjustment, it will be supposed that the instrument has been properly set up and leveled, and the latitude-arc turned toward the south.

467. First Adjustment. *To cause the level-bubbles to remain in the center of the tubes when the instrument is turned around on its vertical axis.* The verification and rectification are the same as those given for the common compass.

468. Second Adjustment. *To adjust the equatorial lines and solar lenses.* Detach the declination-arm, h, by removing the necessary screws, and attach in its place the *adjuster,* replacing the screws of the pivot, and also of the clamp.

Place the arm h on the adjuster, with the same side against the declination-arc as before it was detached. Then, by means of the vertical axis of the instrument, the declination and latitude arcs, and the leveling-screws, turn the arm in the direction of the sun, and bring the image of the sun between the equatorial lines. Then turn the arm half over, bringing the opposite faces of the blocks in contact with the adjuster.

If the sun's image remains between the equatorial lines, the silver plate is in its proper position. If not, loosen the screws which hold the plate, and move the plate so as to correct half of the apparent error. Verify the work by repeating the above operation, until the image remains between the lines in both positions of the arm.

To adjust the other plate, turn the arm end for end on the adjuster, and then proceed as for the first plate.

When both plates have been properly adjusted, remove the adjuster, and replace the declination-arm and its attachments.

469. Third Adjustment. *To adjust the vernier of the declination-arc.* Set the vernier of the declination-arc at zero. Turn the declination-arm *h* so as to point toward the sun. Bring the sun's image between the equatorial lines, by means of the slow-motion screw of the latitude-arc and the parallel plate-screws, as in the second adjustment. Then revolve the arm so as to bring the opposite solar lens toward the sun. If the sun's image now comes between the equatorial lines, no adjustment is necessary. If not, correct half of the apparent error by means of the slow-motion screw *k*. Verify the work by repeating the above operation until the image comes between the lines in both positions of the arm. The zero of the vernier will now not coincide with the zero of the arc. Make it do so by loosening the screws which hold the vernier, and moving the vernier.

470. Fourth Adjustment. *To adjust the Solar Apparatus to the Compass-Sights.* Set the vernier of the horizontal limb at zero. Raise the latitude-arc until the polar axis is horizontal, and set the vernier of the declination-arc at zero. Direct the equatorial sights at some distant point. If the same point is seen through the sights, no adjustment is necessary. If not, the sights must be changed, or some equivalent adjustment made, which can only be done by an instrument-maker.

Field-Work.

471. Before the instrument can be used in the field, it is necessary to determine what angles are to be set off on the declination-arc and on the latitude-arc.

On the declination-arc, both the declination of the sun and the correction for refraction must be provided for.

472. Declination. The declination of the sun at noon at Greenwich, England, is given in the "Nautical Almanac" for every day in the year, together with the hourly change in declination.

To determine the declination at any place for any time, a correction will need to be applied for difference of declination due to the difference of time corresponding to difference of longitude, and also for change of declination for different hours of the day.

For example, suppose we wish to find the declination of the sun at Schenectady, New York, for the different hours of the day on May 1, 1885. The longitude of Schenectady is 73° 55′ 50″ west. This in time is 4 h. 55 m. 43 sec., or approximately (and near enough for this purpose) 5 hours. From the "Nautical Almanac" we find that the declination of the sun at Greenwich, noon on May 1st, to be 15° 12′ 37·5″ north, and the hourly difference is 45″.

When it is noon at Greenwich, it is 7 o'clock in the morning at Schenectady, and at that time the declination of the sun is 15° 12′ 37″.

For the successive hours of the day we have only to add the hourly difference in declination, 55″ (the sun at that time having a motion northward from the equator).

473. Refraction. Tables of refraction have been calculated, giving the amount of refraction for different altitudes from the horizon. These tables, however, give the refraction in a vertical plane, and are not directly applicable for use as a correction in declination. It is evident that, in revolving the declination-arc around the polar axis, the declination-arc will not lie in the plane of a vertical circle, except when it is placed in the plane of the meridian. The correction for refraction, to be set off on the declination-arc, will not, therefore, be equal to the refraction given in the tables except at noon.

The proper correction for refraction to be set off on the declination-arc varies with the latitude, declination of the sun, and hour-angle of the sun.

From Chauvenet's "Astronomy," Art. 120, we have:

$$\text{Refraction in declination} = k' \,.\, \tan.\, z \,.\, \cos.\, q.$$

The value of k' may be taken from Table II, Chauvenet's "Astronomy." Its mean value is about 57″, and this may be employed when very precise results are not required.

z is the zenith distance, and q the parallactic angle.

From Art. 15, Chauvenet's "Astronomy," we have:

$$\tan.\, z \,.\, \cos.\, q = \cot.\, (\delta + N),$$

in which $\delta =$ declination of the sun, and N is an auxiliary quantity. Tan. N equals cot. ϕ . cos. t, in which ϕ is the latitude of the place, and t the hour-angle of the sun.

The tables of Refraction in Declination * are calculated by the above formulas.

In the tables the hour-angle denotes the distance of the sun from the meridian in hours. Thus, at 7 o'clock A. M. the value of the hour-angle is five hours. The north declinations are indicated by + and the south declinations by —.

When the sun is in north declination, the refraction in declination given by the tables is additive. When the sun is in south declination, it is subtractive.

No tables of refraction can be relied upon for altitudes of less than five degrees.

To use the tables, suppose the declination, corrected for refraction, be required for each hour of the day, May 1, 1885, at Schenectady, New York.

By Art. 472 we found that the declination at 7 o'clock in the morning was 15° 12′ 37″. The latitude of Schenectady is 42° 49′. (Take tabular values for 42° 30′.)

In the tables we find that the refraction in declination for latitude 42° 30′, when the sun's declination is 15°, and hour-angle 5 hours, is 1′ 36″. Adding this to 15° 12′ 37″, we have 15° 14′ to be set off on the declination-arc.

474. To determine the Latitude. Set off on the declination-arc the declination of the sun at noon on the given day (corrected for refraction).

A few minutes before noon, set up and level the instrument, set the declination-arc at 12 o'clock on the hour-arc, and turn the instrument horizontally until the declination-arm is directed toward the sun. Move the latitude-arc vertically so as to bring the sun's image between the equatorial lines. As the sun moves toward the meridian, turn the instrument horizontally so as to keep the image between the hour-lines, and move the latitude-arc so as to keep the

* These tables were calculated by Edward W. Arms, C. E., for W. & L. E. Gurley.

image between the equatorial lines. So long as the sun is ascending, the image will move downward on the plate. When the sun has passed the meridian, and begins to descend, the image will move upward. When the image begins to move upward, the reading on the latitude-arc will give the latitude of the place.

475. To determine the "Meridian," or true North and South Line. Set off on the latitude-arc the latitude of the place, and on the declination-arc the declination of the sun at the time, corrected for refraction. Level the instrument, clamp the horizontal plates at zero, turn the latitude-arc approximately south, and direct the declination-arm toward the sun. Then with one hand turn the instrument horizontally, and with the other revolve the declination-arm on the polar axis, until the image of the sun is brought between the equatorial lines. The sights will then point north and south.

476. Running Lines. The meridian being given by the solar compass, it can be used for determining the bearing of lines in the same way as an ordinary compass, but with greater precision, as the meridian is more accurately determined, and the angles are read by the vernier to single minutes.

477. Use of the Magnetic Needle. Since the solar compass gives the true meridian, and the magnetic needle the "magnetic meridian," the declination of the magnetic needle can be read off directly from the magnetic needle. If the needle be kept at zero of the compass-box arc, by turning the box with its tangent-screw, the declination of the needle can be read to minutes on the arc which shows the movement of the compass-box.

By constantly noting the declination of the needle, or by moving the needle-box so as to keep the needle reading zero, lines may be run by the needle, while the sun is obscured, or at such times as for any reason the solar apparatus is not reliable, as when the sun is near the horizon or the meridian.

478. Solar Attachment.* The solar apparatus may be attached to a transit, as shown in Fig. 342.

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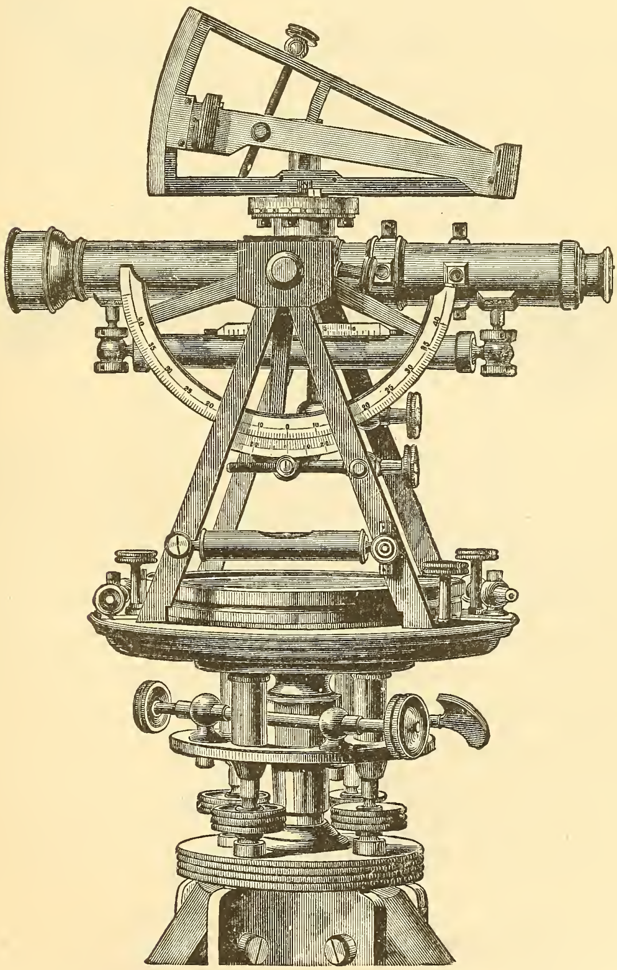
* This attachment, shown in Fig. 342, is manufactured by W. & L. E. Gurley, Troy, New York.

The "polar axis" of the solar apparatus is attached to the horizontal axis of the telescope, and projects upward. The "hour-circle" is the small graduated circle, shown above the telescope.

Fig. 342.



Engineer's Transit, with Solar Attachment.

On the "polar axis" rests the frame, which carries the "declination-arc," and the "arm" with its slow-motion attachments, "solar lenses," and "equatorial lines," as before described.

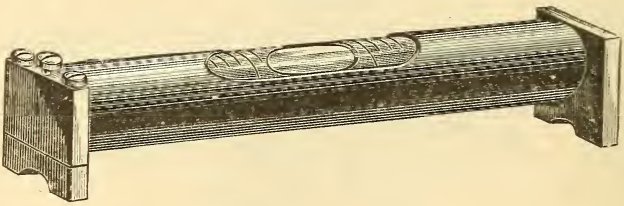
The vertical circle, or arc, of the transit, is used for a "latitude-arc."

Adjustments.

479. The first, second, and third adjustments are similar to those of the solar compass, already explained.

480. To adjust the Polar Axis. Level the instrument carefully, and then level the telescope by means of the level attached to it. Set the arm of the declination-arc at zero, and bring it parallel to the telescope. Place an adjusting level, shown in Fig. 343, on the

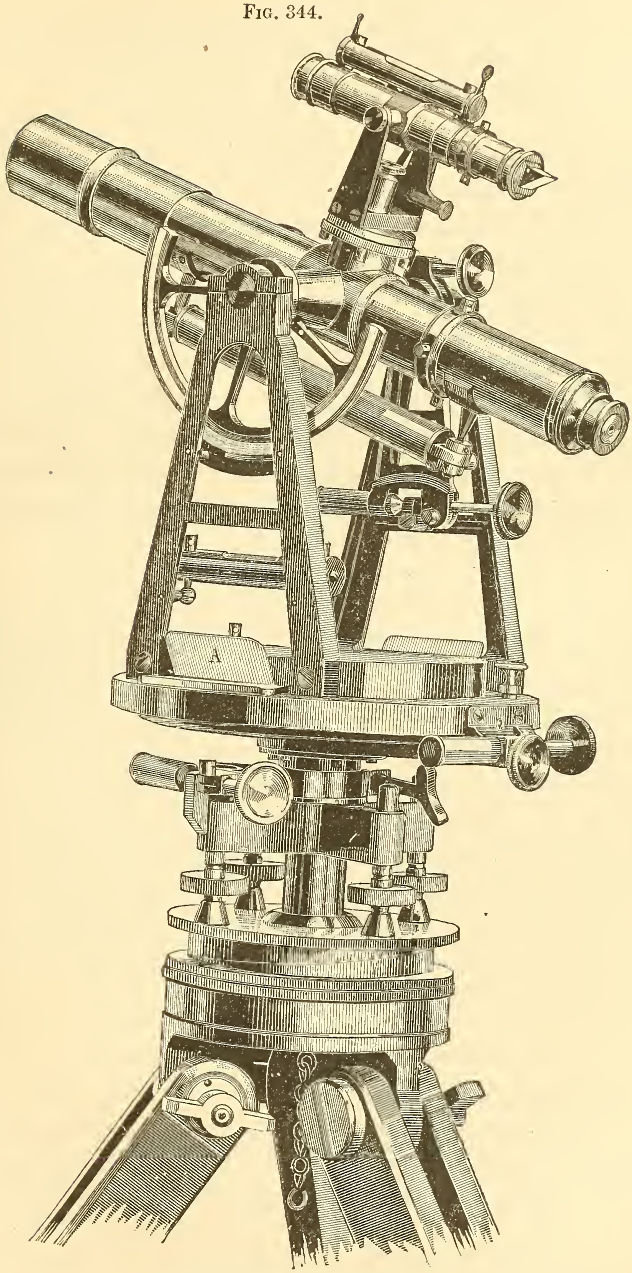
FIG. 343



rectangular blocks attached to the declination-arm. If the bubble remains in the center, the polar axis needs no adjustment in the plane of the axis of the telescope. If not, bring the bubble to the center by means of the two capstan-head screws under the hour-circle, and in line with the telescope. Then turn the declination-arm on the polar axis until it is parallel to the telescope axis, and at right angles to its former position. If the bubble now remains in the center, no adjustment is necessary. If not, bring the bubble to the center by means of the pair of capstan-head screws under the hour-circle and in line with the telescope axis. Verify, and repeat the above operations until the bubble of the adjusting level will remain in the center while the declination-arm is revolved horizontally on the polar axis.

481. To adjust the Hour-Arc. When the telescope is in the plane of the meridian, the index of the hour-circle should give apparent solar time—that is, mean solar time ± the equation of time. If the index does not point to the proper division, it can be made to do so by loosening the screws on the top of the hour-circle, and turning it until the correct time is indicated by the index.

FIG. 344.



Transit, with Solar Attachment.

482. The method of using the solar apparatus on the transit is so nearly the same as that on the compass, already given, that no separate directions will be necessary.

483. Fig. 344 represents a transit with another form of solar attachment.* It consists essentially of a small telescope and level, the telescope being mounted in standards, in which it can be elevated or depressed. The standard revolves around an axis, called the polar axis, which is fastened to the telescope axis of the transit instrument. The telescope, called the "solar telescope," can thus be moved in altitude and azimuth. It is provided with shade-glasses to subdue the glare of the sun, as well as a prism to observe with greater ease when the declination is far north. Two pointers attached to the telescope to approximately set the instrument are so adjusted that when the shadow of the one is thrown on the other the sun will appear in the field of view.

Adjustment of the Apparatus.

First. Attach the "polar axis" to the main telescope axis in the center at right angles to the line of collimation. The base of this axis is provided with three adjusting-screws for this purpose; by means of the level on the solar telescope this condition can be readily and accurately tested.

Second. Point the transit telescope—which instrument we assume to be in adjustment—exactly horizontal, and bisect any distant object. The transit level will then be in the middle of the scale. Point the "solar telescope" also horizontally by observing the same object, and adjust its level to read zero, for which purpose the usual adjusting-screws are provided.

Directions for using the Attachment.

First. Take the declination of the sun as given in the "Nautical Almanac" for the given day and hour, and correct it for refraction and hourly change. Incline the *transit telescope* until this amount is indicated by its vertical arc. If the declination of the

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* Invented by G. N. Saegmüller, and manufactured by Fauth & Co., Washington, D. C., from whose catalogue the description is taken.

sun is north, depress it; if south, elevate it. Without disturbing the position of the transit telescope, bring the solar telescope to a horizontal position by means of its level. The two telescopes will now form an angle which equals the amount of the declination.

Second. Without disturbing the *relative* positions of the two telescopes, incline them and set the vernier to the latitude of the place.

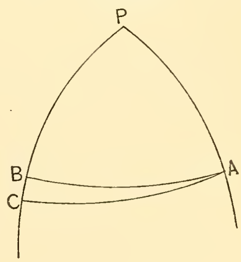
The vertical axis of the "solar attachment" will then point to the pole, the apparatus being in fact a small equatorial.

By moving the transit and the "solar attachment" around their respective *vertical* axes, the image of the sun will be brought into the field of the solar telescope, and after actually bisecting it the *transit telescope must be in the meridian, and the compass-needle indicates its deviation at that place.*

To locate a Parallel of Latitude.

484. In Fig. 345, let P be the pole of the earth, P A and P B the meridians, and A B the desired parallel.

FIG. 345.



First Method. If from A a line, A C, be run perpendicular to the meridian A P, it is evident that, owing to the convergence of the meridians, the perpendicular will not coincide with the parallel of latitude through A. In north latitudes, as in the United States, the perpendicular, A C, will run to the south of the parallel, A B.

To find the distance C B, when the latitude of the starting-point A, and the distance A C are known.

In the triangle P A C, right-angled at A:

$$\cos. PC = \cos. AP \times \cos. AC.$$

$$BC = PC - PB, \text{ and } AP = BP = \text{co-latitude}.$$

$$\therefore \cos. PC = \sin. \text{latitude} \times \cos. AC. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [1.]$$

A C, being a measured distance on an arc of a great circle, must be reduced to the corresponding angle.

$$\text{Angle of } any \text{ arc in minutes} = \frac{\text{length of arc} \times 3437{\cdot}7468}{\text{radius}}.$$

$(3437 \cdot 7468 = 57 \cdot 29598 \times 60)$. Art. 280.

Treating the earth as a sphere, this becomes:

$$\text{Angle of arc in minutes} = \text{length of arc} \, \frac{3437 \cdot 7468}{20912405}.$$

$$\text{Log. arc in minutes} = \text{log. length} - 3 \cdot 7941301 \quad . \quad . \quad . \quad [2.]$$

Then use the value obtained by [2] in formula [1].

B C is found as an angle. To reduce it to feet, we have:

$$\text{Length in feet} = \frac{\text{angle in minutes} \times \text{radius}}{3437 \cdot 7468}.$$

$$\text{Length in feet} = \frac{\text{angle in seconds} \times \text{radius}}{60 \times 3437 \cdot 7468}.$$

$$\text{Log. length in feet} = \text{log. angle in seconds} + 2 \cdot 0059789 \quad . \quad . \quad [3.]$$

485. *Otherwise.* Find the length of an arc subtending one second at the center.

$$\frac{2\,\pi \times 20912405}{360 \times 60 \times 60} = 101 \cdot 386 \text{ feet};$$

i. e., 101·386 feet subtends an angle of one second at the center of the earth. Then, angle in seconds $= \frac{\text{distance in feet}}{101 \cdot 386}$, and distance = angle in seconds $\times$ 101·386 . . . . [4.]

486. *Approximately,*

$$\text{B C in seconds} = \tfrac{1}{4}\,\text{P}^2 \text{ (in seconds)} \times \sin.\, 2\,\text{P A} \times \sin.\, 1''. \quad . \quad [5.]$$

$$\text{To find P.} \qquad \tan.\, \text{P} = \frac{\tan.\, \text{A B}}{\sin.\, \text{A P}}.$$

487. *Example.* Latitude 45° north, and distance 6 miles, required the offset B C.

$$\begin{array}{lrl}
 & \text{6 miles} = & \text{31680 feet.} \\
\text{By [2]} & \text{log. } 31680 = & 4 \cdot 5007852 \\
 & & -\,3 \cdot 7941301 \\
 & \text{log. } 5' \cdot 089265 = & \cdot 7066551 \\
 & 5' \cdot 089265 = & 5'\, 5'' \cdot 356 \\
\text{By [1]} & \text{log. sin. } 45° = & 9 \cdot 8494850 \\
 & \text{log. cos. } 5'\, 5'' \cdot 356 = & 9 \cdot 9999995 \\
\multicolumn{2}{r}{\text{log. cos. P C} = \text{log. cos. } 45°\, 0'\, 0'' \cdot 237 =} & 9 \cdot 8494845 \\
 & \therefore\ \text{B C} = & 0'' \cdot 237
\end{array}$$

To reduce to feet by [3], $\quad \log. 0''{\cdot}237 = \bar{1}{\cdot}3747483$

$$+\ 2{\cdot}0059789$$

$$\log. \text{B C in feet} = \log. 24{\cdot}029 \text{ feet} = 1{\cdot}3807272$$

Second Method:

$$\text{Angle} = \frac{31680}{101{\cdot}386} = 312'' = 5'\ 12''{\cdot}468.$$

Then, as above, we find B C = $0''{\cdot}237$ of arc.

$$\text{B C in feet} = 0''{\cdot}237 \times 101{\cdot}386 = 24{\cdot}0289 \text{ feet.}$$

Approximate Method:

Solving by formula [5], we find B C = 24·3 feet.

488. Spheroidal Formula. The preceding methods suppose the earth to be a sphere. Treating it as a spheroid, the following formula is without material error for distances within 100 miles:

$$\text{C B} = \tfrac{1}{2}\, k^2 \tan.\ \text{L}\ \frac{(1 - [e^2\ .\ \sin.^2\ \text{L}])^{\frac{1}{2}}}{a}.$$

k = distance in feet, L = latitude of initial point.

a equatorial radius = 20926062 feet.

e = ·08169683.

Example. Latitude 45° N. Distance 6 miles.

$$\begin{array}{rl}
\log.\ e^2 = & \bar{3}{\cdot}8244104. \\
\log.\ \sin.^2 45^\circ = & \begin{cases} 9{\cdot}8494850 \\ 9{\cdot}8494850 \end{cases} \\
\log.\ {\cdot}0033718 = & \bar{3}{\cdot}5233804 \\
1 - {\cdot}0033718 = & {\cdot}9966283 \\
\log.\ 0{\cdot}9966283 = & \bar{1}{\cdot}9992666 = \log.\ \text{numerator.} \\
\log.\ \tfrac{1}{2} = & \bar{1}{\cdot}6989700 \\
\log.\ k^2 = & \begin{cases} 4{\cdot}5007852 \\ 4{\cdot}5007852 \end{cases} \\
\log.\ \tan.\ 45 = & 10{\cdot} \\
\log.\ \text{numerator} = & \bar{1}{\cdot}9992666 \\
 & 8{\cdot}6998070 \\
\log.\ a = & 7{\cdot}3206875 \\
\log.\ 23{\cdot}939 \text{ feet} = & 1{\cdot}3791195
\end{array}$$

489. Length of Parallels. The radius of any parallel of latitude equals the radius at the equator multiplied by the cos. latitude.

Then length in feet of $1° = \frac{\pi}{180}$. radius in feet $\times$ cos. latitude.

Then length in feet of $1° = \frac{\pi}{180} \times 20912405 \times$ cos. latitude.

log. length in feet of $1° =$ log. cos. latitude $+ 5·5622814$.

Example. To find the length of a degree on the 45° parallel.

$$\begin{array}{rl} \text{log. cos. } 45 = & 9·8494855. \\ & 5·5622814 \\ \hline \text{log. } 258087 = & 5·4117669. \end{array}$$

Conversely. The angle, in minutes, subtended by *any* arc $= \frac{\text{length of arc} \times 3437·7468}{\text{radius} \times \text{cos. latitude}}$.

log. angle in minutes = log. arc in feet $-$ 3·7841301 $-$ cos. latitude.

Example. Latitude 45° N. and distance 6 miles.

$$\begin{array}{rr} \text{log. } 31680 = & 4·5007852 \\ & -\ 3·7841301 \\ \hline & ·7166551 \\ \text{co-log. cos. } 45° = & ·1505150 \\ \hline \text{log. } 7'\ 21''·897 & ·8671701 \end{array}$$

490. The difference of lengths of any two parallels is called the convergence of the meridians between those parallels. This may be obtained more easily, since the distances between the meridians are as the cosines of the latitudes.

Example. Two "range-lines" (meridians) are 6 miles (480 chains) apart on the base-line of 46°.

Required their convergence at 47° north.

Length at $47° = 480 \frac{\text{cos. } 47°}{\text{cos. } 46°} = 471·252$.

$480 - 471·252 = 8$ chains 74·8 links.

PART II.

LEVELING.

INTRODUCTION.

491. Leveling in General. *A level surface* is one which is everywhere perpendicular to the direction of gravity, as indicated by a plumb-line, etc., and consequently parallel to the surface of standing water. It is, therefore, spherical (more precisely, spheroidal), but, for a small extent, may be considered as plane. Any line lying in it is *a level line.*

A vertical line is one which coincides with the direction of gravity.

The *height* of a point is its distance from a given level surface, measured perpendicularly to that surface, and therefore in a vertical line.

LEVELING is the art of determining the difference of the heights of two or more points.

To obtain a level surface or line, usually the latter, is the first thing required in leveling.

When this has been obtained, by any of the methods to be hereafter described, the desired height of a point may be determined *directly* or *indirectly.*

492. Direct Leveling. In this method of leveling, a level line is so directed and prolonged, either actually or visually, as to pass exactly over or under the point in question—i. e., so as to be in the same vertical plane with it—and the height (or depth) of the point above (or below) this level line is measured by a vertical rod, or by some similar means. The height of any other point being determined in the same manner, the difference of the two will be the height of one of the points above the other. So on, for any number of points.

Direct Leveling is the method most commonly employed. It will form Chapter I of this part.

493. Indirect Leveling. In this method of leveling the desired height is obtained by calculation from certain co-ordinate measured lines or angles, which fix the place of the point.

Thus, the horizontal distance from any point to a tree being known, and also the angle with the horizon made by a straight line passing from the point to the top of the tree, its height above the point can be readily calculated. This is the most simple and most usual form of this method, though many others may be employed.

Indirect Leveling will be developed in Chapter II.

494. Barometric Leveling. This determines the difference of the heights of two points by the difference of the weights of the portions of the atmosphere which are above each of them, as indicated by a barometer. It is explained in Chapter III.

CHAPTER I.

DIRECT LEVELING.

GENERAL PRINCIPLES.

495. Leveling Instruments. The instruments employed to obtain a level line may be arranged in three classes, depending on these three principles :

1. That a line perpendicular to a vertical line is a horizontal or level line.

2. That the surface of a liquid in repose is horizontal.

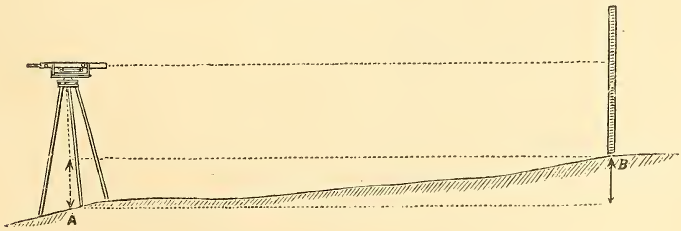
3. That a bubble of air, confined in a vessel otherwise full of a liquid, will rise to the highest point of that liquid.

They will be described in the following pages.

496. Methods of Operation. When a level line has been obtained, by any means, the difference of heights of any two points may be found by either of these two methods :

First Method. Set the leveling instrument over one of the

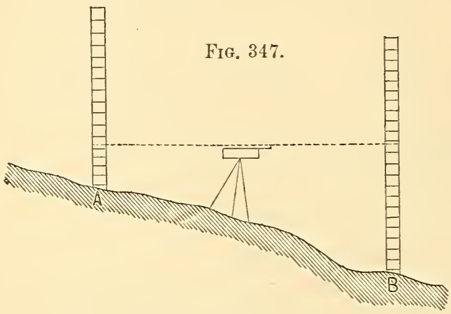
FIG. 346.



points, as A, in Fig. 346. Measure the height of the level line above the point. Then direct this line to a rod held on the other point, and note the reading. The difference of the two measurements at A and B will be the difference of their heights.

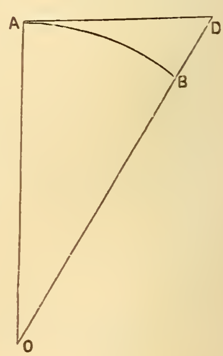
Second Method. Let A and B, Fig. 347, represent the two points. Set the instrument on any spot from which both the points can be seen, and at such a height that the level line will pass above the highest one. Sight to a rod held at A, and note the reading. Then turn the instrument toward B, and note the height observed on the rod held at that point. The difference of the two readings will be the difference of the heights required. The *absolute* height of the level line itself is a matter of indifference.

FIG. 347.



497. Curvature. The level line given by an instrument is tangent to the surface of the earth. Therefore, the line of *true level* is always below the line of *apparent level.* In Fig. 348, A D represents the line of apparent level, and A B the line of true level. D B is the correction for the earth's curvature. By geometry we have : $AD^2 = DB \times (DB + 2\,BO)$. But D B, being very small, compared with the diameter of the earth, may be dropped from the quantity in the parenthesis, and we have :

FIG. 348.



$$DB = \frac{AD^2}{2\,BO};$$

i. e., the correction equals the square of the distance divided by the diameter of the earth.

The difference of height for a distance of

$$1 \text{ mile} = \frac{1}{7916} = \frac{5280 \times 12}{7916} = 8 \text{ inches.}$$

This varies as the square of the distance. The effect, if neglected, is to make distant objects appear lower than they really are.

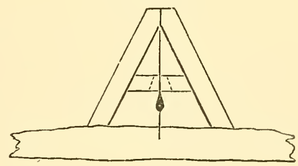
The effect is destroyed by setting the instrument midway between the two points.

498. Refraction. Rays of light coming through the air are curved downward. The effect is, to make objects look higher than they really are. Its amount is about one seventh that of curvature, and it operates in a contrary direction.

PERPENDICULAR LEVELS.

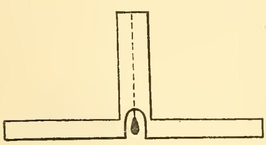
499. Principle. The principle upon which these are constructed is, that a line perpendicular to the direction of gravity is a level line.

FIG. 349.



500. Plumb-line Levels. The A level, Fig. 349, is so adjusted that, when the plumb-line coincides with the mark on the cross-piece, the feet of the level shall be at the same height. It is adjusted by reversion thus: Place its feet on any two points. Mark on the cross-bar the place of the plumb-line. Turn the instrument end for end, resting it on the same points, and mark the new place of the plumb-line. The point midway between the two is the right one.

FIG. 350.

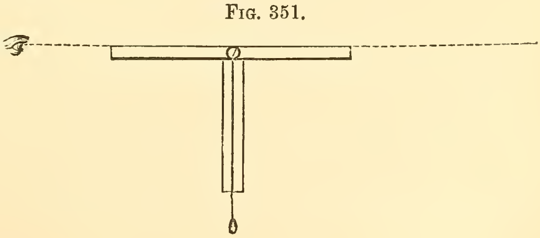


Another form is shown in Fig. 350.

The above forms are not convenient for prolonging a level line. To do this, invert the preceding form, as in Fig. 351.

7

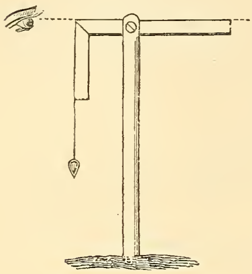
FIG. 351.



To test and adjust this, sight to some distant point nearly on a level, and mark where the plumb-line comes to on the bottom of the rod. Turn the instrument around and sight again, and note the place of the plumb-line. The midway point is the right one.

A modification of the last form is to fasten a common carpenter's square in a slit in the top of a staff, by means of a screw, and then tie a plumb-line at the angle so that it may hang beside one arm. When it has been brought to do so, by turning the square, then the other arm will be level.

FIG. 352.

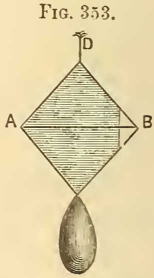


501. Reflecting Levels. In these, the perpendicular to the direction of gravity is not an actual line, but an imaginary reflected line.

It depends on the optical principle that a ray of light which meets a reflecting plane at right angles is reflected back in the same line.

When the eye sees itself in a plane mirror, the imaginary line which passes from the eye to its image is perpendicular to the mirror. Therefore, if the mirror be vertical, the line will be horizontal. It may therefore be used like any other line of sight for determining points at the same height as itself.

FIG. 353.



The first form, Fig. 353 (Colonel Burel's), consists of a rhomb of lead, of about two inches on a side, and one inch thick.

One side (the shaded part of the figure) is faced with a mirror. The right-hand corner of the rhomb is cut off, as seen in the figure, and a wire, A B, is stretched across the mirror.

To use this, hold up the instrument, with the mirror opposite the eye, by the string D, so that the eye seems bisected in the mirror by the wire A B. Then glance through the opening at B, and any point in the line of the eye and wire will be in the same horizontal plane with them.

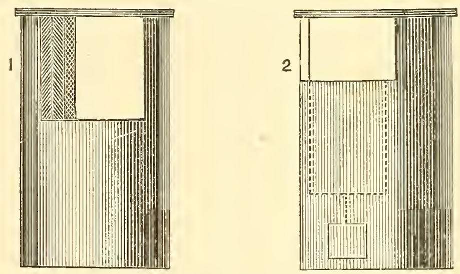
The correctness of the instrument may be verified in the following manner : Hold up the instrument before any plane surface, as a wall, and determine the height of some point, as previously directed. Then, without changing the height of the instrument, turn it half around, place yourself between it and the wall, and note the point of the wall which is seen in the mirror to coincide with the image of the eye.

If the two points on the wall coincide, the instrument is correct. If they do not, the mirror does not hang plumb, and the point midway between the two is the true one.

The instrument is rectified, or made to hang plumb, by means of the pear-shaped piece of lead seen attached to the lower corner of the rhomb.

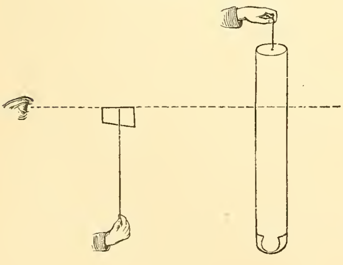
The second form consists of a hollow brass cylinder, with an opening at the upper end, as seen in Fig. 354. At the opening is a small mirror, whose vertical plane makes an angle with the vertical plane of section by which the cylinder was cut in forming the aperture. The edge of the mirror is marked thus (x) in the first half of Fig. 354. The mirror is made to hang plumb by means of a one-sided weight within the cylinder.

FIG. 354.



This is used by setting it on a stake driven into the ground, or by holding it in the hand, making the lower edge of the opening answer the same purpose as the wire in the other case.

FIG. 355.



The same methods of verification and rectification are used as with the first form of the instrument.

The instrument, in its third form, is simply a small steel cylinder, 4″ or 5″ long, and ½″ in diameter, highly polished, and suspended from the center of one end by a fine thread.

To use this, hold it up by the thread with one hand, and with the other hand hold a card between the eye and instrument, using the upper edge of the card, as seen reflected in the mirror, the same as the wire in the first form.

This instrument is the invention of M. Cousinery.

WATER-LEVELS.

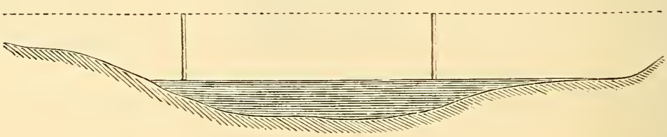
502. Continuous Water-Levels. These may consist of a channel connecting the two points, and filled with water; or of a tube, usually flexible, with the ends turned up, and extending from one point to the other.

By measuring up or down, from the surface of the water at each end, the relative heights of the two points may be determined.

503. Visual Water-Levels. The simplest one is a short surface of water prolonged by sights at equal distances above it, as in Fig. 356.

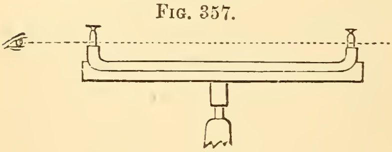
A portable form is a tube bent up at each end, and nearly filled

Fig. 356.



with water. The surface of the water in one end will always be at the same height as that in the other, however the position of the tube may vary. It may be easily constructed with a tube of tin, lead, copper, etc., by bending up, at right angles, an inch or two of each end, and supporting the tube, if

Fig. 357.



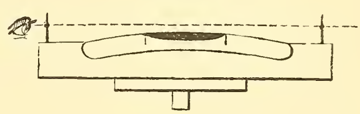
too flexible, on a wooden bar. In these ends, cement (with putty, twine dipped in white-lead, etc.) thin vials, with their bottoms broken off, so as to leave a free communication between them. Fill the tube and the vials, nearly to their top, with colored water. Blue vitriol or cochineal may be used for coloring it. Cork their mouths, and fit the instrument, by a steady but flexible joint, to a tripod.

To use it, set it in the desired spot, place the tube by eye nearly level, remove the corks, and the surfaces of the water in the two vials will come to the same level. Stand about a yard behind the nearest vial, and let one eye, the other being closed, glance along the right-hand side of one vial, and the left-hand side of the other. Raise or lower the head till the two surfaces seem to coincide, and this line of sight, prolonged, will give the level line desired. Sights of equal height, floating on the water, and rising above the tops of the vials, would give a better-defined line.

AIR-BUBBLE OR SPIRIT LEVELS.

504. The "*spirit-level*" consists essentially of a curved glass tube nearly filled with alcohol, but with a bubble of air left within, which always seeks the highest spot in the tube, and will therefore, by its movements, indicate any change in the position of the tube. Whenever the bubble, by raising or lowering one end, has been brought to stand between two marks on the tube, or, in case of expansion or contraction, to extend an equal distance on either side of them, the bottom of the block (if the tube be in one), or sights at each end of the tube, previously properly adjusted, will be on the same level line. It may be placed on a board fixed to the top of a staff or tripod.

Fig. 358.



When, instead of the sights, a telescope is made parallel to the level, and various contrivances to increase its delicacy and accuracy are added, the instrument becomes the engineer's spirit-level.

The upper surface of the tube is usually the arc of a circle, and, when we speak of lines parallel to a "level," we mean parallel to the tangent of this arc at its highest point, as indicated by the middle of the bubble.

505. Sensibility. This is estimated by the distance which the bubble moves for any change of inclination. It is directly proportional to the radius of curvature of the tube. To determine the radius, proceed thus :

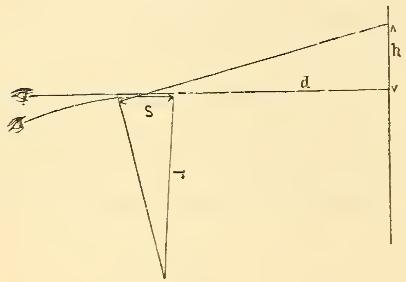
Let S = length of the arc over which the bubble moves for an inclination of 1 second (1″).

Let R = its radius of curvature.

$$\text{Then } S : 2\pi R :: 1'' : 360^\circ,$$
$$\text{whence } R = 206265 \times S,$$
$$\text{or } S = \frac{R}{206265}.$$

S may be found by trial, the level being attached to a finely divided vertical circle. The radius may also be found without this, thus: Bring the bubble to center, and sight to a divided rod. Raise or lower one end of the level, and again sight to the rod. Call the difference of the readings h, the distance of the rod d, and the space which the bubble moved S. Then we have two approximately similar triangles; whence $r = \frac{d\,S}{h}$.

Fig. 359.

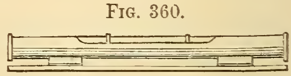


Example. At 100 feet distance, the difference of readings was 0·02 foot, and the bubble moved 0·01 foot. Then the radius was $\frac{100 \times 0{\cdot}01}{0{\cdot}02} = 50$ feet.

The sensibility of an air-bubble level equals that of a plumb-line level having a plumb-line of the same length as the radius of curvature.

506. Block-Level. If this is marked by the maker, and the bubble does not come to the center, when turned end for end, plane or grind off one end of the bottom until it does.

Fig. 360.

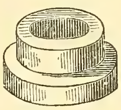


Otherwise, if the bubble-tube is capable of movement, raise or lower one end of it until it will verify, bringing the bubble half-way back to the middle by this means, and the other half by raising or lowering one end of the block, because the reversion has doubled the error.

Repeat this, if necessary.

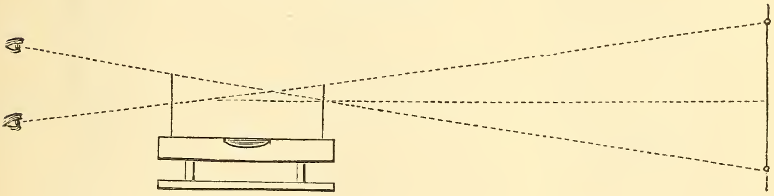
Circular Level. The upper surface of this is spherical. It will therefore indicate a level in *every* direction, instead of only one, as does the preceding. It is adjusted like the last one, but in two directions, at right angles to each other.

FIG. 361.



507. Level with Sights. The line of sight is made parallel to the tangent of the level. It may be tested thus :

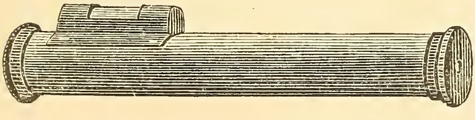
FIG. 362.



Bring the bubble to the center of the tube and make a mark, in the line of sight, as far off as can be seen. Then turn the level end for end, and sight again. If the bubble remains in the same place, "all right." If not, rectify it by altering the sights, or by altering the marks for the bubble to come to, bringing the bubble half-way back, and trying it again.

508. Hand-Reflected Level. This consists of a brass tube, about six inches long, and one inch in diameter. To the inside of the upper portion of the tube is attached a small level. A small mirror is placed at an angle in the lower side of the tube, so that it will reflect the point to which the bubble must come, in order to have the instrument level, to the eye.

FIG. 363.



A small hole at one end, and a horizontal cross-hair at the other, give the desired level line. It is used by holding it in the hand.

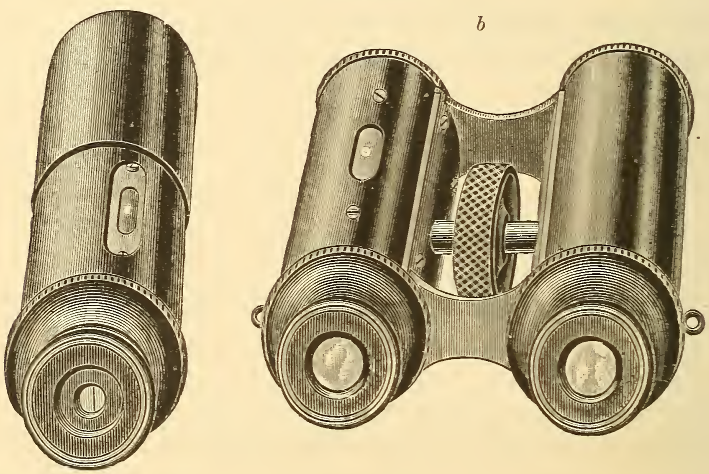
Fig. 363 is an approved form, made by Young, of Philadelphia. The improvement consists in the patent "Locke sight," which enables the near cross-hair to be distinctly seen at the same time as the distant object.

509. Gurley's Telescopic Hand-Level (Fig. 363′, *a*). "This consists of a tube to which are fitted the lenses of a single opera-

FIG. 363′.

a

b



glass, and containing in addition thereto a reflecting prism, cross-wire, and small spirit-level, the last being shown in the open part of the tube.

"The eye-lens, as indicated in the cut, is made of two separate pieces, the larger one being the usual concave eye-lens of the opera-glass, the smaller one a segment of a plano-convex lens having its focus in a cross-wire under the level-vial and above the reflecting prism.

"The observer holds the tube horizontal, with the level opening uppermost, and with the same eye sees the object toward which the instrument is directed, and observes the position of the bubble. When the level is truly horizontal, the cross-wire will bisect the bubble, and will also determine the level of any object seen through the telescope.

"In the binocular form of this level (Fig. 363′, *b*) the tube on the right incloses the usual lenses of the opera-glass, while that on the left contains only the prism, level-vial, and cross-wire. The binocular hand-level gives a clearer view of an object than is possible with a single tube, there being no light lost by the interference of the prism and level-vial."

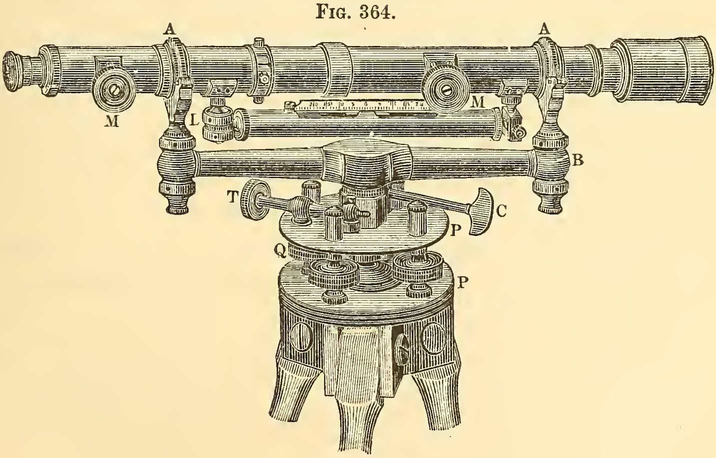
510. The Telescope-Level. In this the line of collimation of the telescope corresponds to the sights of Fig. 362, and is made parallel to the level—i. e., this line is so adjusted as to be horizontal when the bubble of its level is in the center.

There are many different forms of the telescope-level, of which the most important ones will now be given.

511. The Y-Level. This is so named from the shape of the supports of the telescope. It is the variety most used by American engineers.

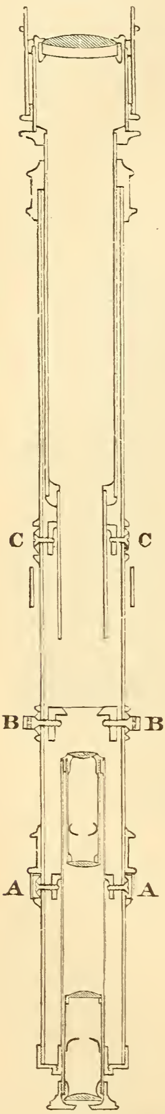
Fig. 364 represents a Y-level of the usual form. The telescope is held in the wyes by the clips, A A, which are fastened to the

Fig. 364.



wyes by tapering pins, so that the telescope can be clamped in any position. The milled-headed screws at M and M are used to move the object-glass and eye-piece in and out, so as to adjust them for long and short sights, and for short-sighted and long-sighted persons. L is a spirit-level ; P and P are parallel plates ; C is the clamp-screw, which fastens the spindle on which the level-bar, B, which supports the wyes, turns ; T is the tangent-screw, by which the telescope may be slowly turned around horizontally.

Fig. 365.



512. The Telescope. The arrangement of the parts of the telescope is shown in Fig. 365. O is the object-glass, by which an image of any object, toward which the telescope may be directed, is formed within the tube. E E is the eye-piece—a combination of lenses, so arranged as to magnify the small image formed by the object-glass. The cross-hairs are at X. They are moved by means of the screws shown at B B. A A are screws used for centering the eye-piece. C C are screws used for centering the object-glass. At D D are rings, or collars, of exactly the same diameter, turned very truly, by which the telescope revolves in the wyes.

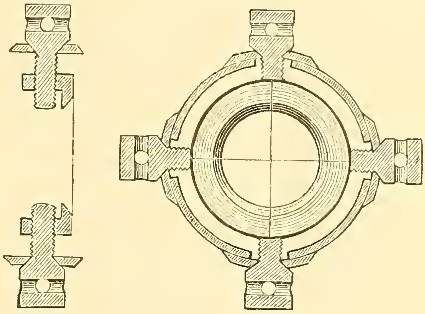
The telescope shown in the figure forms the image erect. Other combinations of lenses are used, some of which invert the image ; but the one here shown is generally preferred.

513. The Cross-Hairs. These are made of very fine platinum wire or of spider-threads. They are attached to a short, thick tube, placed within the telescope-tube, through which pass loosely four screws whose threads enter and take hold of the cross-hair ring, as shown in Fig. 366.

In some instruments, one of each pair of opposite screws is replaced by a spring ; and the screws, instead of being capstan-headed, and moved by an "adjusting-pin," have square heads, and are moved by a "key," like a watch-key.

The line of collimation (or *line of aim*) is the imaginary line passing through the intersection of the cross-hairs and the optical center of the object-glass.

Fig. 366.



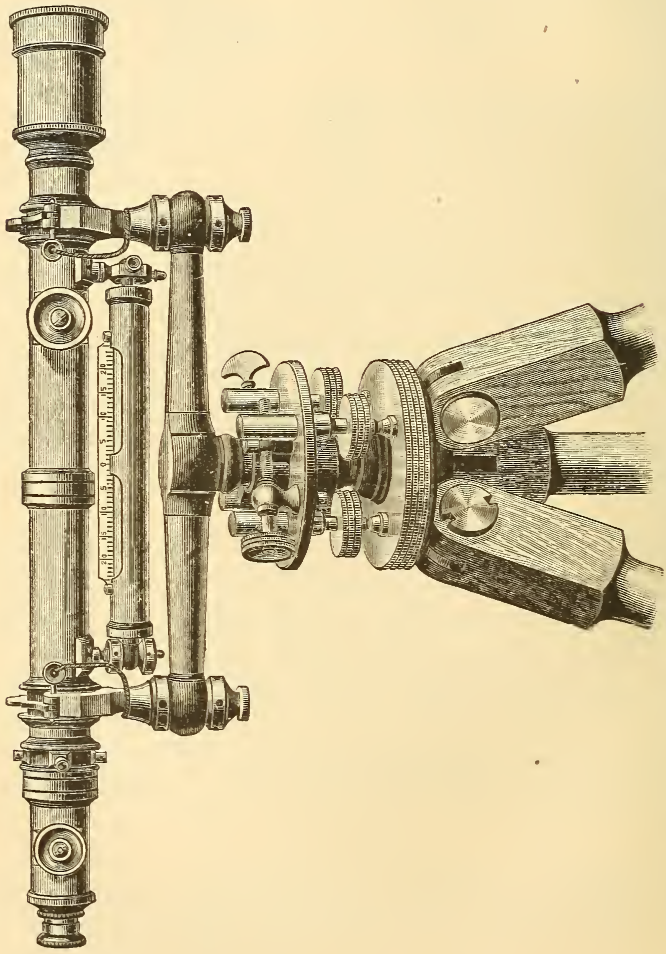
The image formed by the object-glass should coincide precisely with the cross-hairs. When this is not the case, there will be an apparent movement of the cross-hairs, about the objects sighted to, on moving the eye of the observer. This is called *instrumental parallax*. To correct it, move the eye-piece out or in, till the cross-hairs are sharply defined against any white object. Then move the object-glass in or out, till the object is also distinctly seen. The image is now formed where the cross-hairs are, and no movement of the eye will cause any apparent motion of the cross-hairs.

514. The Level. This consists of a thick glass tube, slightly curved upward, and so nearly filled with alcohol that only a small bubble of air remains in the tube. This always rises to the highest part. The brass case, in which this is inclosed, is attached to the under side of the telescope, and is furnished with the means of moving, at one end vertically, and at the other horizontally. Over the aperture, in the case, through which the bubble-vial is seen, is a graduated level-scale, numbered each way from zero at the center.

515. Supports. The wyes in which the telescope rests are supported by the level-bar, B, and fastened to it by two nuts at each end (one above, one below the bar), which may be moved with an adjusting-pin. The use of these nuts will be explained under "Adjustments." Attached to the center of the level-bar is a steel spindle, made so as to turn smoothly and firmly in a hollow cylinder of bell-metal ; this, again, is fitted to the main socket of the upper parallel plate.

516. Parallel Plates. It is by the aid of these that the instrument is leveled. The plates are united by a ball-and-socket joint,

Fig. 367.



Twenty-inch Y-Level.

and are held apart by the four plate-screws, Q Q Q Q, which pass through the upper one, and press against the lower one.

To level the instrument, turn the telescope till it is brought over a pair of opposite parallel plate-screws. Then turn the pair of screws, to which the telescope has been made parallel, equally in opposite directions, screwing one in and the other out, till the bubble is brought to the center. Then turn the telescope so as to bring it over the other pair of opposite screws, and bring the bubble to the center, as before.

Repeat the operation, as moving one pair of screws may affect the other.

Sometimes one of each pair of opposite screws is replaced by a strong spring, and in some instruments only three screws are used.

The lower plate is screwed on to the tripod-head.

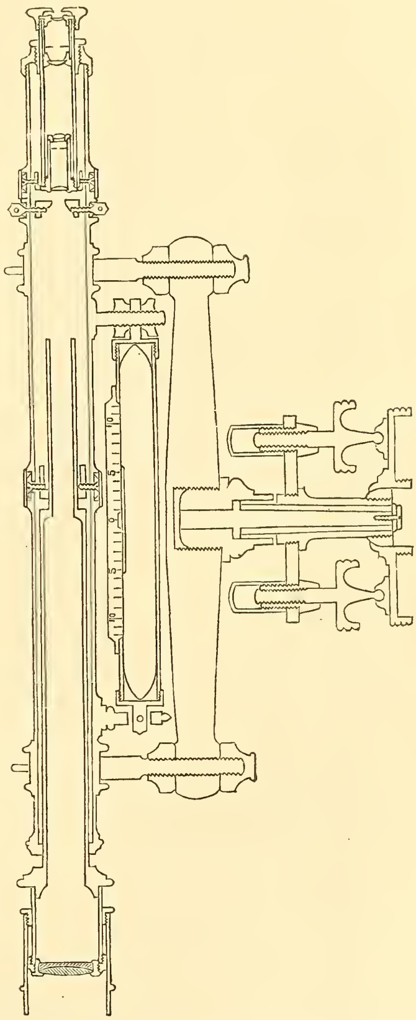


FIG. 368.

Section of Twenty-inch Y-Level.

517. Fig. 367 is a twenty-inch Y-level, and Fig. 368 is a longitudinal section of it, showing its construction.

In Fig. 368, B B are the screws attached to the cross-hair ring. At A are four screws holding a ring through which the inner end of the eye-piece passes. At C are four screws holding a ring, through which the inner end of the object-glass slide passes. The use of these sets of screws will be explained under "Adjustments."

The interior spindle, D, which supports the instrument, and on which it turns, is made of steel, and is carefully fitted to the interior of a hollow socket of bell-metal, which has its exterior surface fitted to the main socket, E, of the tripod-head. The hollow bell-metal socket is held in place by a washer and screw, shown at D.

A screw, passing through the main socket, E, enters a groove in the exterior of the bell-metal socket, and fastens the instrument to the tripod-head.

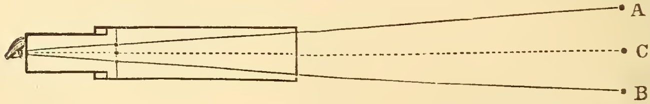
ADJUSTMENTS.

518. The line of collimation of the telescope should be horizontal when the bubble is in the center of the tube ; which will be the case when this line is parallel to the plane of the level. But both this line and this plane are imaginary, and can not be compared together directly. They are therefore compared indirectly. The line of collimation is made parallel to the bottom of the collars, and the plane of the level is then made parallel to them.

519. First Adjustment. *To make the line of collimation parallel to the bottoms of the collars.*

Sight to some well-defined point, as far off as it can be dis-

Fig. 369.



tinctly seen. Then revolve the telescope half around in its supports—i. e., turn it upside down. If the line of collimation was not in the imaginary axis of the rings, or collars, on which the telescope rests, it will now no longer bisect the object sighted to. Thus, if the horizontal hair was too high, as in Fig. 369, this line of collimation would point at first to A, and, after being turned over, it would point to B. The error is doubled by the reversion, and it should point to C, midway between A and B. Make it do so, by unscrewing the upper capstan-headed screw, and screwing in the lower one, till the horizontal hair is brought half-way back to the point B. Remember that, in an erecting telescope, the cross-hairs are reversed, and *vice versa.* Bring it the rest of the way by means of the parallel plate-screws. Then revolve it in the wyes back to its original position, and see if the intersection of the cross-hairs now bisects the point, as it should. If not, again revolve, and repeat the operation till it is perfected. If the vertical hair passes to the right or to the left of the point when the telescope is turned half around, it must be adjusted in the same manner by the other pair of cross-hair screws. One of these adjustments may disturb the other, and they should be repeated alternately. When they are perfected, the intersection of the cross-hairs, when once fixed on a point, will not move from it when the telescope is revolved in its supports. This double operation is called *adjusting the line of collimation.*

It has now been brought into the center line, or axis, of the collars, and is therefore parallel to their bottoms, or the points on which they rest, if they are of equal diameters. We have to assume this as having been effected by the maker.

In making this adjustment, the level should be clamped, but need not be leveled.

520. Second Adjustment. *To make the bottoms of the collars parallel to the plane of the level*—i. e., *to insure their being horizontal when the bubble is in the center.*

Clamp the instrument, and bring the bubble to the center by the parallel plate-screws. Take the telescope out of the wyes, and turn it end for end. If the bubble returns to the center, "all right." If not, rectify it, by bringing the bubble half-way back, by means of the nuts which are above and below one end of the bubble-tube, and which work on a screw. Bring it the rest of the way by the plate-screws, and again turn end for end. Repeat the operation, if necessary.

If, in revolving the telescope (as in the first adjustment), the bubble runs toward either end, it must be adjusted sidewise, by means of two screws which press horizontally against the other end of the bubble-tube. This part of the adjustment may derange the preceding part, which must, therefore, be tried again.

521. Third Adjustment. *To cause the bubble to remain in the center of the tube when the telescope is turned around horizontally.*

To verify this, bring the bubble to the center of the tube, and then turn the telescope half-way around horizontally. If the bubble does not remain in the center, adjust it by bringing it half-way back by means of the nuts at the end of the level-bar. Test it by bringing it the rest of the way back by the parallel plate-screws, and again turning half-way around.

The cause of the difficulty is, that the plane of the level is not perpendicular to the axis about which it turns, and that this axis is not vertical. The above operations correct both these faults.

This adjustment is mainly for convenience, and not for accuracy, except in a very small degree.

Some instruments have no means of making the third adjustment. They must be treated thus:

Use the screws at the end of the bubble-tube, to cause the bubble to remain in the center when the level is turned around horizontally. Then make the line of collimation parallel to the level by raising or lowering the cross-hairs.

522. When levels are provided with the means of centering the eye-piece and object-glass, these operations should precede the first three which we have just explained.

Centering the Object-Glass. After adjusting the line of collimation for a distant object (as explained in the "First Adjustment") move out the slide, which carries the object-glass, until a point ten or fifteen feet distant can be distinctly seen. Then turn the telescope half over, as before, and see if the intersection of the cross-hairs bisects the point. If not, bring it half-way back by the screws C C, Fig. 365, moving only one pair of screws at a time. Repeat the operation for a distant point, and then again for a near one, if necessary. We have now adjusted the line of collimation for long and short sights, and may assume it to be in adjustment for intermediate ones, since the bearings of the slides are supposed to be true, and their planes parallel to each other.

Centering the Eye-Piece. This is to enable the observer to see the intersection of the cross-hairs precisely in the center of the field of view of the eye-piece. It is adjusted by means of four screws, two of which are shown at A A.

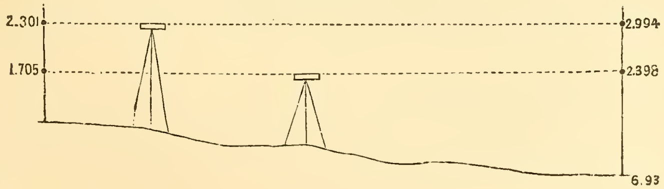
These operations are performed by the maker so permanently as to need no further attention from the engineer, and the heads of the screws, by which these adjustments are made, are covered by a thin ring which protects them from disturbance.

523. Adjustment by setting between two points, or the "*Peg-Method.*" Drive two pegs several hundred feet apart, and set the instrument midway between them. Level, and sight to the rod held on each peg. The difference of the readings will be the true difference of the heights of the pegs, no matter how much the level may be out of adjustment.

Then set the level over one peg, and sight to the rod at the other. Measure the height of the cross-hairs above the first peg. The difference of this and the reading on the rod *should* equal the difference of the heights of the two points, as previously determined. If it does not, set the target to the sum or difference of the height of the cross-hairs above the first peg, and the true difference of height of the points, according as the first point is higher or lower than the second, and hold the rod on the second point. Sight to it, and raise or lower one end of the bubble-tube until the horizontal cross-hair *does* bisect the target when the bubble is in the center. Then perform the "third adjustment."

Instead of setting *over* one peg, it is generally more convenient

FIG. 370.



to set near to it, and sight to a rod held on it, and use this reading instead of the measured height of the cross-hairs.

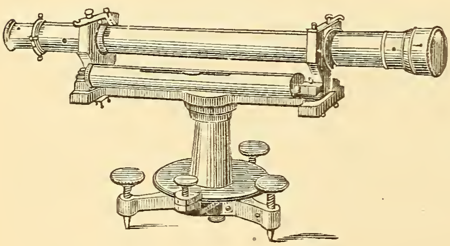
N. B.—This verification should *always* be used for every level, even after the three usual adjustments have been made; for it is independent of the equality of the collars.

In running a long line of levels, let the last sight at night be taken midway between the last two "turning-point" pegs, and in the morning try their difference by setting close to the last one. This tests the level every day with very little extra labor.

524. Egault's Level. In this level the bubble-tube is not connected with the telescope. It is used thus:

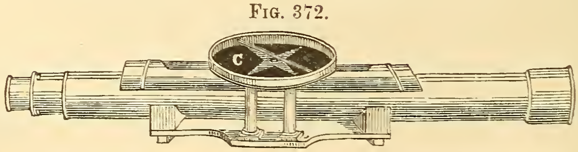
Level and sight as usual. Then turn the telescope upside down, end for end, and half-way around horizontally, and sight again. Half the sum of the two readings is the correct one, no matter how much the instrument is out of adjustment (assuming the collars to be of equal size); for the errors then cancel each other. This is the one used principally in France.

Fig. 371.



The rod used with it is marked with numbers only half the real heights above its bottom. Then the *sum* of the readings is the true one. Thus the rod itself takes the mean of the readings.

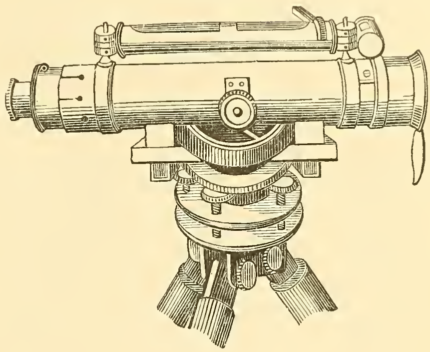
Fig. 372.



525. Troughton's Level. In this the bubble-tube is permanently fastened in the top of the telescope-tube. It is adjusted by the "peg method," or some similar one, the cross-hair being moved up or down until the observation gives the true difference of height of the pegs when the bubble is in the center. Then make the "third adjustment," by means of the screws under the telescope.

526. Gravatt's Level, or the "Dumpy Level." Its diameter is very great, thus giving more light. Its bubble is on the top, and can be seen in a small inclined mirror, by the observer. It also has a cross-level.

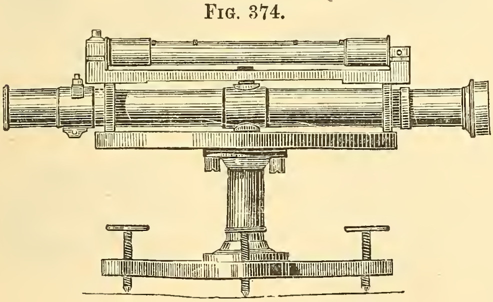
Fig. 373.



527. Lenoir's Level. In this, the telescope carries, at each end, a steel block, whose upper and lower faces are made perfectly parallel. They are placed on a brass circle, which is made level by reversing a level placed upon the upper surface of the steel blocks.

528. Tripods. These consist of three legs, shod with iron, and connected by joints at the top. There are many different forms, the most common of which is given in Fig. 367. Other forms are given in Art. 476. Lightness and stiffness are the desired qualities.

Fig. 374.



Stephenson's tripod has a ball-and-socket joint below the parallel plates, so as to admit of being at once set *nearly* level on very steep slopes.

"Quick-leveling" tripod-heads, for quickly setting the leveling-plates nearly level, are made of various patterns.

Extension tripods are manufactured which provide for lengthening and shortening the legs of the tripod.

529. Rods. These should be made of light, well-seasoned wood. A plumb or level attached to them will show when they are held vertically. To detect whether the rod leans to or from the instrument, its front may be angular or curved. If angular, when held leaning toward the instrument, the lines of division will appear as in Fig. 375. When leaning from the instrument, they will appear as in Fig. 376. They are usually divided to feet, tenths, and hundredths.

FIG. 375.

FIG. 376.

530. Target. This is a plate of iron or brass, attached to the rod in such a way that it may be moved up and down the rod and clamped in any position. The face of the target should be painted of such a pattern that, when sighting to it, it may be very precisely bisected by the horizontal cross-hair. Some of the many varieties are given in Figs. 377–385.

Those represented in Figs. 377, 378, and 379 are bad, because

FIG. 377.

FIG. 378.

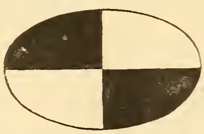


FIG. 379.

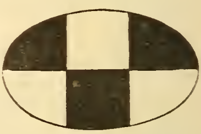


FIG. 380.

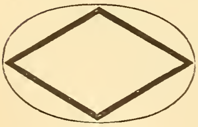


FIG. 381.

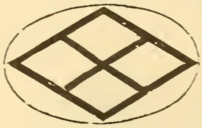


FIG. 382.

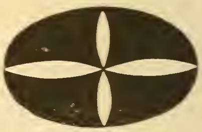


FIG. 383.

FIG. 384.

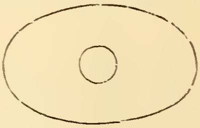
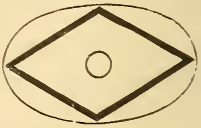


FIG. 385.



the cross-hair may be above or below the middle of the target by its full thickness, as magnified by the eye-piece of the telescope without the error being perceptible. The next three, Figs. 380, 381, and 382, depend upon the nicety with which the eye can determine if a line bisects an angle. Fig. 383 depends upon the accuracy with which the eye can bisect a space. Fig. 384 depends upon the accuracy with which the eye can bisect a circle. Figs. 381, 382, and 385 are the best forms for use. Red and white are the best colors.

Fig. 386.

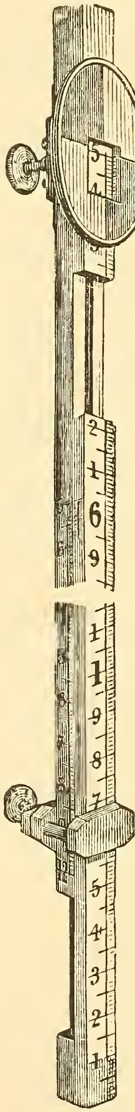


Fig. 387.

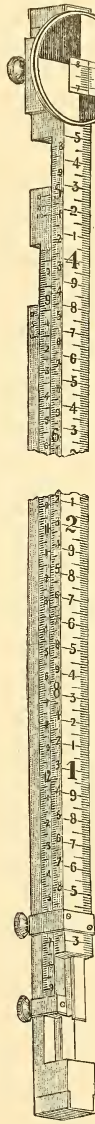
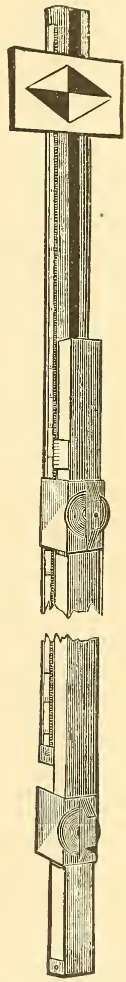


Fig. 388.



531. Vernier. The target carries a vernier, by which smaller spaces may be measured than those into which the rod is divided. It may be placed on the side of an aperture, in the face of the target, through which the divisions on the rod can be seen, or carried on the back or side of the rod by the target-clamp.

532. The New York Rod (Fig. 386). This is usually in two pieces, sliding one upon the other, and connected by a tongue. It is graduated to tenths and hundredths of a foot, and can be read to thousandths by the vernier. Up to six feet and a half the target is used as on other rods. For greater heights, the target is fixed at six and a half feet, and the back part of the rod, which carries the target, is shoved up (Fig. 386) until the target is bisected by the cross-hairs. Its height is then read off on the side of the rod, on which the numbers run downward, and on which is a second vernier, which gives the precise reading. It is convenient for its portability, but apt to be too tight or too loose, as the weather is moist or dry. Sometimes it is in three pieces, as in Fig. 387.

Fig. 389.

533. The Boston Rod (Fig. 388). This is usually in two parts, like the New York rod. The target is rectangular, and is fastened to one of the pieces near its extremity. For heights less than six feet, the rod is held with the target-end down, and the target is moved up by sliding up the piece which carries it. For heights above six feet, the rod is turned end for end, bringing the target-end up, and then sliding up the piece which carries the target.

534. The Philadelphia Rod (Fig. 389). This is in two parts, held together by brass clamps, and is furnished with a *target.* It is graduated and painted so as to be used as a "speaking-rod," or with a target. When the target is used, the vernier on the target is read for height up to seven feet. For greater heights, the target is clamped at seven feet, and the part to which the target is clamped is slid up, and the vernier on the upper clamp is used.

535. Speaking-Rods. These are rods which are read without targets, the divisions and subdivisions being painted on the face of the rod. They produce great saving of time and increase of accuracy.

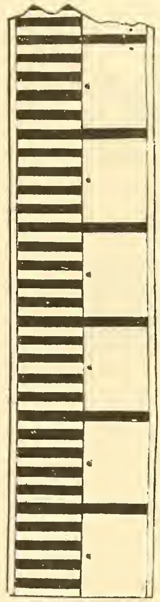
In one form (Fig. 390) the face of the rod is divided into tenths of feet, and smaller divisions estimated. In Bourdaloue's rod the divisions are each four centimetres

(1·6 inch), and are numbered at half their value. He arranges them as in Fig. 391.

Fig. 390.

Fig. 391.

Fig. 392.



Gravatt's Rod (Fig. 392). This is divided to 0·01 foot. The upper hundredth of each tenth extends across the rod. Each half-tenth is marked by a dot; each half-foot by two dots. Every other tenth is numbered, and the numbers are each 0·1 high. It is in three parts, which slide into each other like a telescope.

Barlow's Rod (Fig. 393). In this the divisions are marked by triangles, each 0·02 foot high, so that it reads to hundredths, and less by estimation. This is based on the power the eye has in bisecting angles.

Fig. 393.

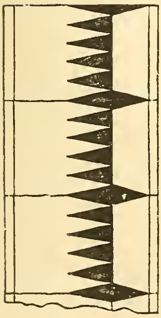
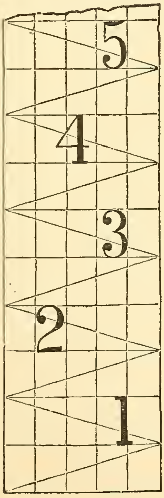


Fig. 394.



Stephenson's Rod (Fig. 394). This is based upon the principle of the diagonal scale. Each tenth is bisected by a horizontal line, and the diagonals enable the observer to read to hundredths.

Conybeare's Rod (Fig. 395). It reads to hundredths of a foot by means of the cross-hair bisecting the tops and bottoms and angles of hexagons. The odd tenths are made white and the even ones black. The figures are placed so that their centers are opposite the divisions they refer to.

Fig. 395.

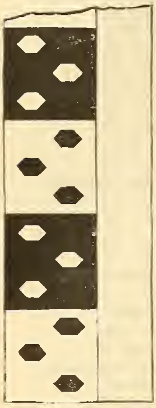
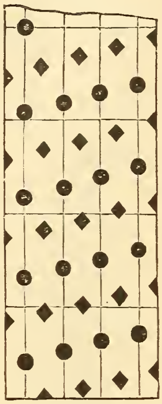


Fig. 396.



Pemberton's Rod (Fig. 396). This is on the principle of nine verniers placed side by side. It reads to hundredths, which are given by counting up from the dot which the hair bisects, to the dot in the same vertical line which is bisected by one of the horizontal lines which mark the tenths. The inventor claims that it can be read nine times as far as Gravatt's.

On all speaking-rods, to avoid confounding numbers, such as 3 and 8, they may be marked thus :

1 . 2 . III . 4 . V . 6 . 7 . 8 . IX . X . 11 . XII.

The French, who go by tenths, use the following :

1 . 2 . T . 4 . V . 6 . 7 . 8 . N . X.

The figures are sometimes placed with their tops on a level with the tops of the dimensions they mark—e. g., feet ; and sometimes with their middles on the dividing line.

THE PRACTICE.

536. Field Routine ; or, how to start and go on :

1. The rodman holds the rod on the starting-point, which may be a peg, a door-sill, or other "bench-mark." He stands square behind his rod, and holds it as nearly vertical as possible.

2. The leveler sets up the instrument, somewhere in the direction in which he is going, but not necessarily, or usually, in the precise line. He then levels the instrument by the parallel plate-screws, sights to the rod, and notes the reading, whether of target or speaking-rod, as a "back-sight" (B. S.), or + (plus) sight ; entering it in the proper column of one of the tabular forms of field-book, given in the following articles.

3. The rodman is then sent ahead about as far as he was behind, and he there drives a "level-peg" nearly to the surface of the ground, or finds a hard, well-defined point, and holds the rod upon it.

4. The leveler then again sights to the rod, and notes the reading as a "fore-sight" (F. S.), or — (minus) sight. The difference of the two readings is the difference of the heights of the points.

5. He then takes up the instrument, goes beyond the rod, any convenient distance, sets up again, and proceeds as in paragraph 2; and so on for any number of points, which will form a series of pairs. The successive observations of each pair give their difference of heights, and the combination of all these gives the difference of heights of the first and last points of the series.

6. If the vertical cross-hair be strictly vertical, it will determine whether the rod leans to the right or left. To know whether the cross-hair is vertical or not, try whether it coincides with a plumb-line, or sight to some fixed point, turn the telescope from side to side horizontally, and see if the horizontal cross-hair continues to cover the spot. If it does not, turn the telescope around in the wyes till it does; then it is truly horizontal, and the other hair, being perpendicular to it, is truly vertical. To know whether the rod leans forward or backward, have the rodman move it from and to himself. If the line bisected by the cross-hair descends in both motions, the rod was vertical; if the line rises, the rod was leaning. The lowest reading is the true one.

7. When a target is used, signals are made by the leveler with the hand, "up" and "down," to indicate in which direction to move the target. Drawing the hand to the side signifies "stop," and both hands brought together above the head signifies "all right." The rodman should move the target fast at first, and slowly after having passed the right point. When signaled "all right," he should clamp the target and show again. Then call out the reading before moving, and show it to the leveler, as either passes the other.

8. We have thus far supposed that only the difference of heights of the two extreme points is desired. But when a section or profile of the ground is required, the rod must be held and observed, at

each change of slope of the ground, or at regular distances; usually, for railroad-work, at every hundred feet, and also at any change of slope between those points.

Any number of points, within sight, may have their relative heights determined at one setting of the level.

The names back-sight (B. S.) and fore-sight (F. S.) do not necessarily mean sights taken looking forward or backward (though they are generally so for turning-points), but the first sight taken, after setting up the instrument, is a B. S. or + (plus) sight, and all following ones, taken before removing the instrument, are F. S.'s, or — (minus) sights. The full meaning of this will appear in considering the forms of field-book.

All but the first and last points sighted to are called *intermediate points*, or "*intermediates.*" The last point sighted to before moving the instrument is called a *turning-point*, or *changing-point.*

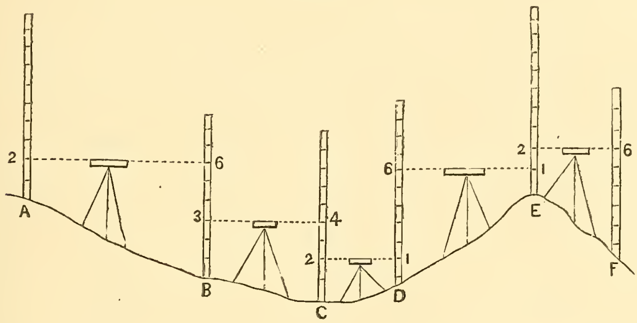
The first and last sights, taken at any one setting of the instrument, require the greatest possible accuracy. The intermediate points may be taken only to the nearest tenth, or hundredth at most; because any error in them will not affect the final result, but only the height of that single point at which it was taken.

Two rodmen are often used to save the time of the leveler. Then it is well to use a target-rod for the "turning-points," which are often distant and need most precision, and a speaking-rod for the intermediate points. Where one rod is used, the rodman should keep notes of the readings at the turning-points.

537. Field-Notes. The beginner may sketch the heights and distances measured, in a profile or side view, as in Fig. 397. But when the observations are numerous, they should be placed in one of the tabular forms given on the following pages.

538. First Form of Field-Book. In this, the names of the points or "stations," whose heights are demanded, are placed in the first column, and their heights, as finally ascertained, in reference to the first point, in the last column. The heights above the starting-point are marked +, and those below it are marked —. The back-sight to any station is placed on the line below the point to which it refers. When a back-sight exceeds a fore-sight, their difference

Fig. 397.



is placed in the column of "Rise"; when it is less, their difference is a "Fall." The following table represents the same observations as the last figure, and their careful comparison will explain any obscurities in either:

STATIONS.	DISTANCES.	BACK-SIGHTS.	FORE-SIGHTS.	RISE.	FALL.	TOTAL HEIGHTS.
A						0·00
B	100	2·00	6·00		—4·00	—4·00
C	60	3·00	4·00		—1·00	—5·00
D	40	2·00	1·00	+1·00		—4·00
E	70	6·00	1·00	+5·00		+1·00
F	50	2·00	6·00		—4·00	—3·00
		15·00	18·00		—3·00	

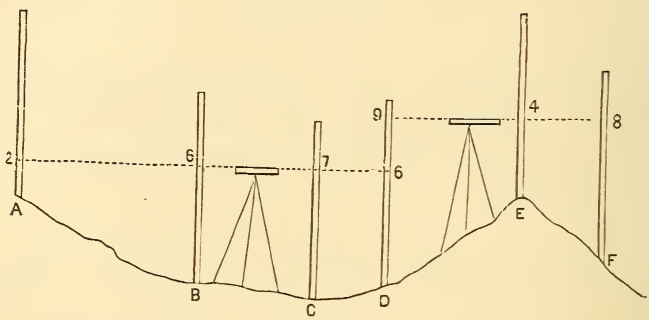
The above table shows that B is 4 feet below A; that C is 5 feet below A; that E is 1 foot above A; and so on. To test the calculations, add up the back-sights and fore-sights. The difference of the sums should equal the last "total height."

An objection to this form is that the back-sights come on the line *below* the station to which they are taken, which is embarrassing to a beginner.

When "intermediate" observations are taken, the "fore-sights" taken to these intermediate points are put down in their proper column, and are also set down in the column of "back-sights"; so that, when the two columns are added up, any error in these intermediate sights (which are usually not taken very accurately) will be canceled, and will not affect the final result. The effect is the same as if, after the fore-sight to the intermediate point had been taken, the instrument had been taken up and set down again at precisely the same height as before, and a back-sight had then been taken to the same point. Hence, in this form, the "turning-points" are those stations which have different back-sights and fore-sights, while those which have them the same are "intermediates."

The following figure and table represent the same ground as the

Fig. 398.



preceding one, but with only two settings of the instrument. D is the turning-point :

STATIONS.	DISTANCES.	BACK-SIGHTS. +	FORE-SIGHTS. —	RISE.	FALL.	TOTAL HEIGHTS.
A						0·00
B		2·00	6·00		4·00	—4·00
C		6·00	7·00		1·00	—5·00
D		7·00	6·00	1·00		—4·00
E		9·00	4·00	5·00		+1·00
F		4·00	8·00		4·00	—3·00
		+28·00	—31·00		3 00	

In leveling for "sections," the distances between the points leveled must be recorded. They are usually put down after the stations *to* which they are measured; although in surveying with the compass, etc., they are put down after the stations *from* which they are measured. In the following notes, which contain intermediate stations, they are put down *before* the stations *to* which they are measured. It should be remembered that these distances are measured between the points at which the rod is held, and have no reference to the points at which the instrument is set up :

DISTANCES.	STATIONS.	BACK-SIGHTS. +	FORE-SIGHTS. —	RISE.	FALL.	TOTAL HEIGHTS.
	260					91·397
100	261	4·576	3·726	0·850		92·247
100	262	5·420	4·500	0·920		93·167
100	263	4·500	3·170	1·330		94·497
40	263·40	4·910	4·938		0·028	94·469
60	264	4·938	6·386		1·448	93·021
100	265	3·380	4·640		1·260	91·761
100	266	4·640	5·400		0·760	91·001
70	266·70	2·760	3·070		0·310	90·691
30	267	3·070	3·750		0·680	90·011
100	268	6·750	5·925		3·175	86·836
		41·944	46·505		—4·561	
			41·944		+91·397	
			—4·561		86·836	

539. Second Form of Field-Book. This is presented below. It refers to the same stations and levels noted in the first table, and shown in Fig. 397 :

STATIONS.	DISTANCES.	BACK-SIGHTS.	HEIGHT OF INSTRUMENT ABOVE DATUM.	FORE-SIGHTS.	TOTAL HEIGHTS.
A					0·00
B	100	2·00	+2·00	6·00	—4·00
C	60	3·00	—1·00	4·00	—5·00
D	40	2·00	—3·00	1·00	—4·00
E	70	6·00	+2·00	1·00	+1·00
F	50	2·00	+3·00	6·00	—3·00
		15·00		18·00	—3·00

In the preceding form it will be seen that a new column is introduced, containing the height of the instrument—i. e., of its line of sight—not above the ground where it stands, but above the *Datum*, or starting-point, of the levels. The former columns of "rise" and "fall" are omitted. The preceding notes are taken thus : The height of the starting-point, or "datum," at A, is 0·00. The instrument being set up and leveled, the rod is held at A.

The back-sight upon it is 2·00 ; therefore the height of the instrument is also 2·00. The rod is next held at B. The fore-sight to it is 6·00. That point is therefore 6·00 below the instrument, or $2·00 - 6·00 = -4·00$ below the datum. The instrument is now moved, and again set up, and the back-sight to B, being 3·00, the height of the instrument is $-4·00 + 3·00 = -1·00$, and so on ; the height of the instrument being always obtained by adding the back-sight to the height of the peg on which the rod is held, and the height of the next peg being obtained by subtracting the fore-sight to the rod held on that peg, from the height of the instrument.

This form is better than the first form, in leveling for a section of the ground to make a profile ; or when several observations are to be made at one setting of the level ; or when points of desired heights are to be established, as in "leveling-location."

This form may be modified by putting the back-sights on the same line with the stations to which they are taken. This avoids the defect of the first form, but introduces the new defect of writing them down after the number which they precede, in a back-handed way, which may be a source of error.

This modification is shown in the following table, which corresponds to Fig. 398. In the column of fore-sights, the "turning-points" (T. P.), and "intermediate points" (Int.), are put in separate columns ; so that, to prove the work, the difference of the sum of the back-sights and of the sum of the turning-point fore-sights, is the number which should equal the difference of the heights of the first and last points :

STATIONS.	DISTANCES.	BACK-SIGHTS. +	HEIGHT OF INSTRUMENT.	FORE-SIGHTS. —		TOTAL HEIGHTS.
				T. P.	INT.	
A		2·00				0·00
B			+2·00		6·00	−4·00
C					7·00	−5·00
D		9·00		6·00		−4·00
E			+5·00		4·00	+1·00
F				8·00		−3·00
		+11·00		−14·00		
				+11·00		
				−3·00		

When a line is divided up into stations of 100 feet each, as on railroad-work, the number of the station indicates its distance from the starting-point. When an observation is taken at a point between these hundred-feet stations, it is noted as a decimal, thus : Station 4·60 is 460 feet from the starting point. In the field-notes of such work, the column of distances may be omitted, as in the following table. The heights and distances are the same as in the last table under Art. 538 :

STATIONS.	BACK-SIGHTS.	HEIGHT OF INSTRUMENT.	FORE-SIGHTS.		TOTAL HEIGHTS.
			T. P.	INT.	
260	4·576	95·973			91·397
261	5·420	97·667	3·726		92·247
262				4·500	93·167
263	4·910	99·407	3·170		94·497
263·40				4·938	94·469
264	3·380	96·401	6·386		93·021
265				4·640	91·761
266	2·760	93·761	5·400		91·001
266·70				3·070	90·691
267				3·750	90·011
268			6·925		86·836
	+21·046		−25·607		
			+21·046		
			− 4·561		
			+91·397		
			+86·836		

540. Third Form of Field-Book. In this the back-sights are placed directly under the height of the station to which they are taken, which lessens the chance of making mistakes in adding to get the height of instrument. The height of instrument is distinguished by being included between two horizontal lines. The following table refers to the same ground as the preceding one :

STATIONS.	FORE-SIGHTS.	HEIGHTS.	REMARKS.
260		91·397	
		4·576	
		95·973	
261	3·726	92·247	
		5·420	
		97·667	
262	4·500	93·167	
263	3·170	94·497	
		4·910	
		99·407	
+40	4·938	94·469	
264	6·386	93·021	
		3·380	
		96·401	
265	4·640	91·761	
266	5·400	91·001	
		2·760	
		93·761	
+70	3·070	90·691	
267	3·750	90·011	
268	6·925	86·836	

541. Best Length of Sights. There are two classes of inaccuracies. With very long sights, the errors of imperfect adjustment and curvature are greatest; the former varying as the length, and the latter as the square of the length. With very short sights, and therefore more numerous, the errors of inaccurate sighting at the target are greatest. The best usual mean is from 200 feet to 300 feet, or more if equal distances for back-sights and fore-sights to turning-points can be obtained.

542. Equal Distances of Sight. They are always very desirable. They are most easily determined, when no stakes have been previously set, by "stadia" cross-hairs in the telescope of the level.

543. Datum-Level. This is the plane of reference, from which, above it or below it, usually the former, the heights of all points of the line are reckoned.

It may be taken as the height of the starting-point. If the line descends, it is better to call the starting-point 10 feet or 100 feet above some imaginary plane, so that points below the starting-point may not have minus-signs.

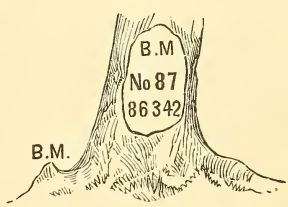
It is desirable to refer all levels in a country to some one datum. This is usually the surface of the sea, and, for general purposes, *mean tide* is best. *Low-water* mark should be the datum when the levelings are connected with harbor-surveys, whose soundings always refer to low water. *High-water* mark should be used when the levelings relate to the drainage of a country.

544. Bench-Marks (B. M.). These are permanent objects, natural or artificial, whose heights above the datum are determined and recorded for future reference.

Good objects are these: Pointed tops of rocks, tops of milestones, stone door-sills, tops of gate-posts or hinges, and generally any object not easily disturbed, and easily described and found.

A knob made on the spreading root of a tree is good. A nail may be driven in it, and the tree "blazed" and marked, as in Fig. 399. A stake will do till frost.

Fig. 399.



Bench-marks should be made near the starting-point of a line of levels; near where the line crosses a road; on each side of a river crossed by it; at the top and bottom of any high hill passed over; and *always* at every half-mile or mile.

The precise location and description of every bench-mark should be noted very fully and precisely, and in such a way that an entire stranger could find it, with the aid of the notes.

545. Check-Levels, or Test-Levels. No *single* set of levels is to be trusted; but they must be tested by another set, run between the bench-marks (B. M.'s), though not necessarily over the same ground.

A set of levels will verify themselves if they come around to the starting-point again.

546. Limits of Precision. Errors and inaccuracies should be carefully distinguished. For the latter, every leveler must make a standard for himself, so as to be able, in testing his work, to distinguish any *real error* from his *usual inaccuracy.*

The result of four sets of levelings, in France, of from 45 to 140 miles, averaged a difference of $\frac{1}{10}$ foot in 43 miles, and the greatest error was $\frac{1}{3}$ foot in 56 miles.

A French leveler, M. Bourdaloue, contracts to level the benchmarks of a railroad survey to within 0·002 foot per mile, or $\frac{1}{10}$ foot per 50 miles.

In Scotland, the difference of two sets of levels of 26 miles was 0·02 foot.

547. Trial-Levels, or Flying-Levels. Their object is to get a general approximate idea of the comparative heights of a portion of the country, as a guide in choosing lines to be leveled more accurately. More rapidity is required, and less precision is necessary. The distances may be measured at the same time by stadia-hairs.

548. Leveling for Sections. The object of this is to measure all the ascents and descents of the line, and the distances between the points at which the slope changes ; so that a section or profile of it can be made from the observations taken.

The line of a railroad is usually set out by a party with compass or transit, who drive at every hundred feet a large stake with the number of the station on it, and beside it a small level-peg, even with the surface of the ground. On this the rod is held for the observations. The level-peg is set in "line," and the large stake a foot or two to one side.

549. Profiles. A profile is a section of ground by a vertical plane or cylindrical surface,* passing through the line along which a profile is desired. It represents to any desired scale the heights and distances of the various points of a line, its ascents and descents, as seen in a side view. It is made thus : Any point on the

* A cylindrical surface is here understood to mean that formed by a line moving parallel to itself along *any* line, instead of only a circle, as in elementary geometry.

paper being assumed for the first station, a horizontal line is drawn through it; the distance to the next station is measured along it, to the required scale; at the termination of this distance a vertical line is drawn; and the given height of the second station above or below the first is set off on this vertical line. The point thus fixed determines the second station, and a line joining it to the first station represents the slope of the ground between the two. The process is repeated for the next station, etc.

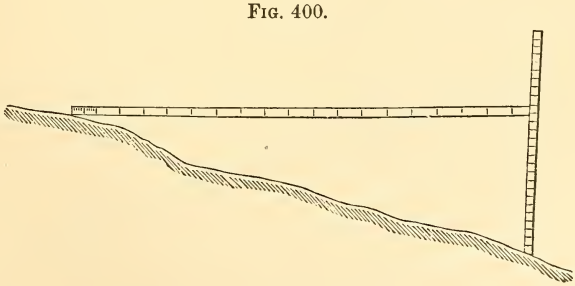
But the rises and falls of a line are always very small in proportion to the distances passed over, even mountains being merely as the roughnesses of the rind of an orange. If the distances and the heights were represented on a profile to the same scale, the latter would be hardly visible. To make them more apparent, it is usual to "exaggerate the vertical scale" tenfold, or more—i. e., to make the representation of a foot of height ten times as great as that of a foot of length, as in Fig. 397, in which one inch represents one hundred feet for the distances, and ten feet for the heights.

In practice, engraved profile-paper is generally used, which is ruled in squares or rectangles, to which any arbitrary values may be assigned.

When the line leveled over is not straight, the profile, whose length is that of the line straightened out, will extend beyond the "plan" when both are on the same sheet.

550. Cross-Levels. These show the heights of the ground on a line at right angles to the main line. They give "cross-sections"

Fig. 400.



of it. In the note-book they are put on the right-hand page. They may be taken at the same time with the other levels, or independently. In taking cross-levels where the slopes are quite steep, as in mountain districts, frequent settings of the instrument are necessary.

A much more rapid method is by the use of "cross-section rods." These are two rods, one of which is about ten or twelve feet long, provided with a bubble-tube near each end, so as to be held level, and graduated to feet, tenths, and hundredths. The other is simply a graduated rod. The manner of using them is shown in Fig. 400.

A slope-level is sometimes used. (See "Angular Surveying.")

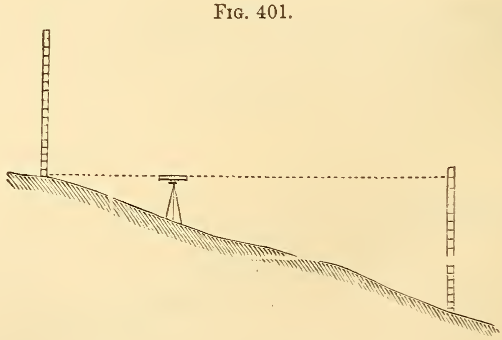
DIFFICULTIES.

551. Steep Slopes. In descending or ascending a hill, the instrument and the rod should be so placed that the sight should strike as near as possible to the bottom of the rod on the up-hill side, and the top of the rod on the down-hill side.

Try this by leveling over two screws, setting the instrument so that one pair of opposite plate-screws shall point in the direction of the line, but do not be too particular; it is a waste of time.

Doing this produces sights of unequal length. The rod being about three times as high as the instrument, the down-hill sights will be about double the length of the up-hill ones, as shown in Fig. 401. Then set to one side of the line. This is necessary on

FIG. 401.

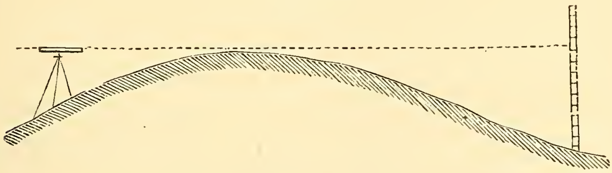


slopes so steep that the rod is too near the level to be read. If this be impossible, keep notes of the lengths of the sights to the turning-points, backward and forward, and as soon as possible take sights unequal in the contrary direction till the differences of lengths balance the former ones. When approaching a long ascent or descent, make these compensations in advance.

In leveling over a line of stakes already set, as on a railroad, at every 100 feet, if the line of sight strikes not quite up to one, drive a peg as high as you can see it, and make it a turning-point, noting it "peg" in the field-book.

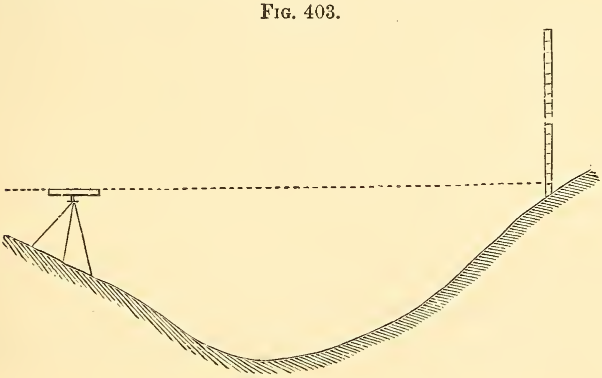
In leveling across a hill or hollow, instead of setting the instru-

FIG. 402.



ment on the top of the hill or bottom of the hollow, time will be saved by the method represented in Figs. 402 and 403.

FIG. 403.



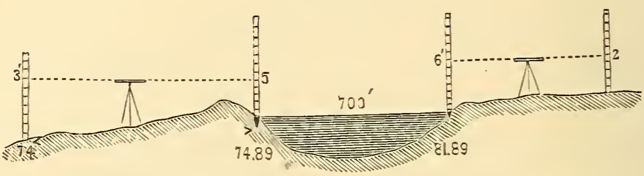
552. *When the rod is a little too low,* raise it alongside of a stake, or the body, and put the top of the rod "right"; then measure down from the bottom of the rod, and add it to its length.

553. *When the rod is a little too high,* so that the line of sight strikes the peg below the bottom of the rod, measure down from the top of the peg, and put down the sight with a contrary sign to what it would have had—i. e., if a back-sight make it minus, and if a fore-sight make it plus.

554. *When the rod is too near.* When no figure is visible, raise the rod slowly till a figure comes in sight. If too near to read, and there is no target, use a field-book as target. If the instrument is exactly over the peg, measure up to the height of the cross-hairs, as given by the side-screws.

555. WATER. A.—*A pond too wide to be sighted across.* Drive a peg to the level of the water, on the first side, and observe its height, as an F. S. Then drive a peg on the other side of the pond, also to the surface of the water. Hold the rod on it. Set

FIG. 404.



up the level beyond it, and sight to it as a B. S., and put down the observation as if it had been taken to the first peg.

FORE-SIGHTS.	STATIONS.	HEIGHTS.	BACK-SIGHTS.	⊖
	74	50·00	3·00	53·00
5·0	74·89 }	48·00		
	81·89 }		6·00	54·00

There must be no wind in the direction of the line of level.

B.—*For leveling across a running stream.* Set the two pegs in a line at right angles to the current, although the line to be leveled may cross it obliquely.

If a profile or section of the ground under the water be required, find the height of the surface, and measure the depths below this at a sufficient number of points, measuring the distances also, and put these depths down as fore-sights.

556. A Swamp, or Marsh. This can not be treated like a pond, for the water may seem nearly stagnant while its surface has considerable slope, its flow being retarded by vegetation. If only slightly "shaky," have an observer at each end of the level. If more so, push the legs down as far as they will go, and let both observers lie down on their sides. If still more "shaky," drive three stakes or piles, to support the legs of the tripod, and stand the tripod on them.

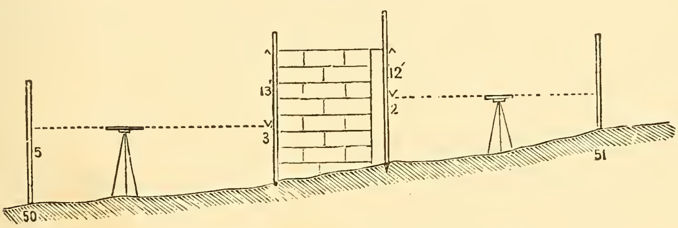
A water-level will level itself. Use that for intermediate points on the swamp, and test the result by leveling *around* the swamp with the spirit-level.

557. Underwood. If it can not be cut away, set the instrument on some eminence, natural or artificial.

558. Board Fence. Run a knife-blade through one of the boards, and hold the rod upon it on each side of the fence, as if it were a peg, keeping the blade in the same horizontal position while the rod and instrument are taken over.

559. A Wall. *First Method.* Drive a peg at the bottom of the wall, on the first side, and observe on it. Measure the height of the wall above the peg, and put this down as a B. S. Drive another peg on the other side of the wall; measure down to it from the top of the wall, and put that down as an F. S., just as if the level had been set in the air at the height of the top of the

Fig. 405.



wall, and this B. S. and F. S. had been really taken. Set up the instrument beyond the wall, take a B. S. to this peg, and go on as usual.

FORE-SIGHTS.	STATIONS.	HEIGHTS.	BACK-SIGHTS.	⊕
	50	74·00	5·00	79·00
3·00	Peg.	76·00	13·00	89·00
12·00	Peg.	77·00	2·00	79·00
1·00	51	78·00		

Second Method. Mark where the line of sight strikes the wall; measure up to the top of the wall, and put this down as an F. S., with a plus-sign, as in 553, where the line of sight struck below the top of the peg.

On the other side of the wall, sight back to it, and mark where the line of sight strikes. Measure to the top of the wall, and put this down as a B. S., with a minus-sign, and then go on as usual.

560. House. First try to find some place for the instrument from which you can see through, by opening doors or windows. Or, find some place in the house where you can set the instrument and see both ways, or hold the rod at some point inside, and look to it from front and back. A straight stick may be used if the rod can not be held upright, and the height measured on the rod.

561. The Sun. It often causes the leveler much difficulty—

1. By shining in the object-glass. If the instrument has a shade on it, draw it out. If not, shade the glass with your hand or hat, or set the instrument to one side of the line.

2. By heating the level unequally in all its parts. Holding an umbrella over it will remedy this.

3. By causing irregular refraction. Some parts of the ground become heated more than others, and therefore rarefy the air at those places. This can not be avoided nor corrected.

562. Wind. Watch for lulls of wind, and observe then several times, and take the mean. The least wind is at daybreak.

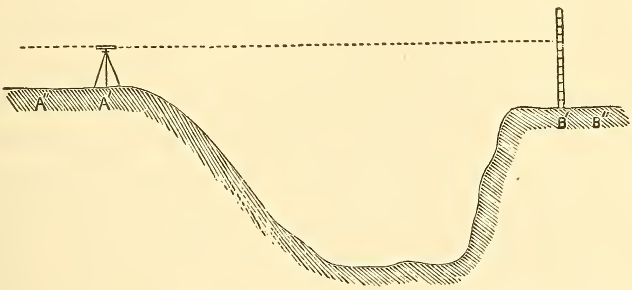
563. Idiosyncrasies. Different persons do not see things precisely alike. Each individual may have an inaccuracy peculiar to himself. One may read an observation higher or lower than another equal in skill. Also, a person's right and left eye may differ. This difference in individuals is termed their "personal equation."

To test the accuracy of your eye, turn the head so as to bring the eyes in the same vertical line, and sight to the rod held horizontally. Note where the vertical hair strikes. Then turn the head to the other side, so as to invert the position of the eyes, and then sight again. As before, the mean of the two readings is the correct one.

564. Reciprocal Leveling. This is to be used when it is impossible to set midway between the two points, and the distance can not be readily determined.

Set the instrument over A, and sight to a rod at B, and note

FIG. 406.



reading. The difference of the reading and of the height of the cross-hairs gives *a* difference of height of A and B. Then set up at B, and observe to A, similarly. A new difference of height is obtained. The mean of these two is the correct one.

Ht. of cross-hairs above peg at A $=4{\cdot}3'$	Ht. of cross-hairs above peg at B $=4{\cdot}9'$
Observation to B $=7{\cdot}0'$	Observation to A $=4{\cdot}2'$
Diff. of height $=2{\cdot}7'$	Diff. of height $=0{\cdot}7'$

True difference $= \frac{1}{2}\,(2{\cdot}7' + 0{\cdot}7') = 1{\cdot}7'$.

Otherwise, set the instrument at an equal distance from each point, as A$'$ and B$'$, and observe to each in turn. The mean of the two differences of height obtained will be the true difference, as before.

LEVELING LOCATION.

565. Its Nature. It is the converse of the general problem of leveling, which is to find the difference of heights of two given points. *This* consists in determining the place of a point of any *required* height above or below any given point.

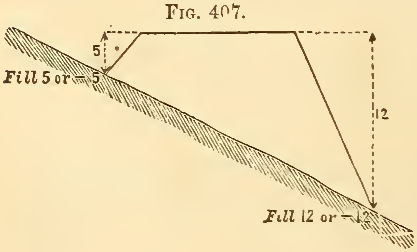
To do this, hold the rod on some point of known height above the datum-level; sight to it, and thus determine the height of the cross-hairs. Subtract from this the desired height of the required point, and set the target at the difference. Hold the rod at the place where the height is desired, and raise or lower it till the cross-hair bisects the target. Then the bottom of the rod is at the desired height. Usually, a peg is driven till its top is at the given height above the datum.

566. Difficulties. If the difference of height be too much to be measured at one setting of the instrument, take a series of levels up or down to the desired point. So, too, if they be far apart; and thus find a place where, the instrument having a known height of cross-hairs, the target can finally be set, as before.

If the ground be so low or so high that a peg can not be set with its top at the required height, drive a peg till its top is just above the surface of the ground. Observe to the rod on it, determine its height above or below the desired point, and note this on a large stake driven beside it; or, place its top a whole number of feet above or below the required height, and mark the difference on it, or on a stake beside it.

567. Staking out Work. When embankments and excavations are to be made for roads, etc., side-stakes are set at points in their intended outside edges—i. e., where their slopes will meet the surface of the ground; and the height which the ground at those points is above or below the required height or depth of the top or bottom of the finished work, is marked on these stakes with the words "cut," or "fill," or the signs + or —.

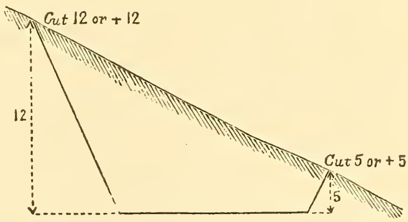
Fig. 407.



The places of the stakes are found by trial. (See Gillespie's "Road-Making," page 145.) These stakes are set to prepare the work for contractors. When the work is nearly finished, other stakes are set at the exact required height.

In staking out *foundation-pits,* set temporary stakes exactly above the intended bottom angles of the completed pit, thus marking out on the surface of the ground its intended shape. Take the heights of each of these stakes and move them outward such distances that cutting down from them with the proper depth and slope will bring you to the desired bottom angle.

FIG. 408.



568. To locate a Level-Line. This consists in determining on the surface of the ground a series of points which are at the same level—i. e., at the same height above some datum. Set one peg at the desired height, as in Art. 565. Sight to the rod held thereon, and make fast the target when bisected. Then send on the rod in the desired direction, and have it moved up or down along the slope of the ground, until the target is again bisected. This gives a second point. So go on as far as sights can be correctly taken, keeping unchanged the instrument and target. Make the last point sighted to a "turning-point." Carry the instrument beyond it, set up again, take a B. S., and proceed as at first.

The rod should be held and pegs driven at points so near together that the level-line between them will be approximately straight.

569. Applications. One use of this operation is to mark out the line which will be the edge of the water of a pond to be formed by a dam. In that case, a point of a height equal to that of the top of the proposed dam, *plus* the height which the water will stand on it (to be determined by hydraulic formulas), will be the starting-point. Then proceed to set stakes as directed in the last article.

The line from stake to stake may then be surveyed like the sides of a field, and the area to be overflowed thus determined.

Strictly, the surface of the water behind a dam is not level, but is curved concavely upward, and is therefore higher and sets back farther than if level. The backing up of the water is called *Remous.*

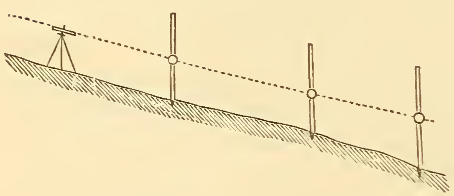
Another important application of this problem is to obtain "contour-lines" for topography.

570. To run a Grade-Line. This consists in setting a series of pegs so that their tops shall be points in a line which shall have any required slope, ascending or descending.

When a grade-line is to be run straight between two given points, set the level over one point, set the target at the height of the cross-hairs, hold the rod on the other point, and raise or lower one end of the instrument till the cross-hair bisects the target. Then send the rod along the line, and drive pegs to such heights that when the rod is held on them the cross-hair will bisect the target. A stake may be driven at the extreme point to the height of the target.

Another Method. Knowing the horizontal distance between the two given points, and their difference of level, determine the rise or fall per hundred feet. Then drive stakes at every hundred feet, so that the top of each succeeding one is the given grade per hundred feet higher or lower, according as the grade is ascending or descending.

Fig. 409.



For example, suppose the horizontal distance from A to B is 1,200 feet, and that B is 16·8 feet higher than A. The rise per hundred feet from A is 1·4 foot. Beginning at A, set stakes at every hundred feet, so that the top of each one is 1·4 foot higher than the preceding one.

A line of uniform grade or slope is not a straight line. Calling the globe spherical, this line, when traced in the plane of a great circle, would be a logarithmic spiral. On a length of six miles, the difference in the middle between it and its straight chord would be six feet.

CHAPTER II.

INDIRECT LEVELING.

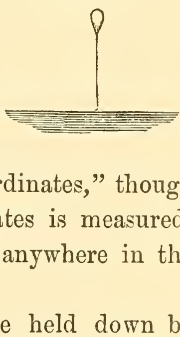
METHODS AND INSTRUMENTS.

571. Vertical Surveying. Leveling may be named VERTICAL SURVEYING, or *Up-and-down Surveying ;* Land-Surveying being HORIZONTAL SURVEYING, or *Right-and-left* and *Fore-and-aft Surveying.*

All the methods of determining the position of a point in horizontal surveying may be used in vertical surveying.

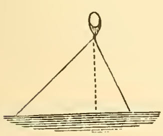
The point may be determined by co-ordinates situated in a vertical plane, as in any of the systems employed in a horizontal plane.

FIG. 410.



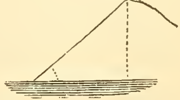
Thus, if a balloon be held down by a single rope attached to a point in a level surface, its height above that surface is found by measuring the length of the rope. This is the direct method. It resembles that of "rectangular co-ordinates," though here only one of the co-ordinates is measured. The other might be situated anywhere in the surface.

FIG. 411.



If, however, the balloon be held down by two cords, its height can be determined by measuring the length of the cords and the distance between their lower ends. They correspond to the "focal co-ordinates." The required vertical height can be calculated by trigonometry. So in the following other indirect methods :

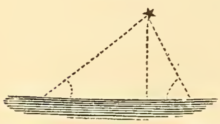
FIG. 412.



The length of the string of a kite, and the angle which this string makes with the horizon, are the "polar co-ordinates" of the kite.

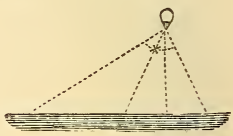
The "angles of elevation" of a meteor, observed by two persons in the same vertical plane with it, and at known distances apart, are its "angular co-ordinates."

Fig. 413.



Finally, an aëronaut could determine his own height by observing the angles subtended by three given objects situated on the earth's surface, at known distances, and in the same vertical plane with him. These would be "trilinear co-ordinates."

Fig. 414.



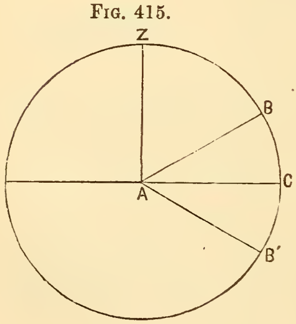
Many other systems of co-ordinate lines and angles, variously combined, may be employed.

The desired heights may also be determined by various other methods, analogous to those given for "inaccessible distances."

Combinations of measurements not in the same vertical plane may also be used, as will be shown in this chapter.

572. Vertical Angles. The vertical angles measured may be those made—either with a level line, or with a vertical line—by the line passing from one point to the other.

Fig. 415.



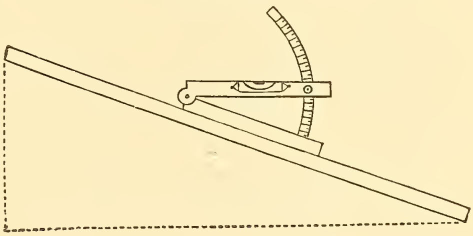
The angle B A C is called an "angle of elevation," and the angle B′ A C an "angle of depression." The former angle may be called positive, and the latter negative.

The angle B A Z or B′ A Z is called the zenith-distance of the object. It is the complement of the former angle—i. e., = 90° — that angle taken with its proper algebraic sign. An angle of elevation, B A C = 10°, would be a zenith-distance of 80°. An angle of depression, B′ A C = — 10°, would be a zenith-distance of 100°. The zenith-distance is preferable in important and complicated operations, as avoiding the ambiguity of the other mode of notation.

573. Instruments. All contain a divided circle, or arc, placed vertically, and a level or plumb line. By these is measured the desired vertical angle made by the inclined line with either a level line or vertical line.

This inclined line may be an actual line or a visual line. In the former case, it may be a rod, or cord, or wire, as shown in Figs. 416–418.

FIG. 416.



This last arrangement of a cord or wire (Fig. 418) is used in mine-surveying. A light surveyor's chain may be similarly used, with the advantage of giving, at the same time, difference of heights and distance.

FIG. 417.

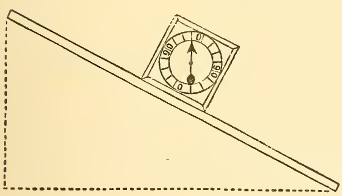
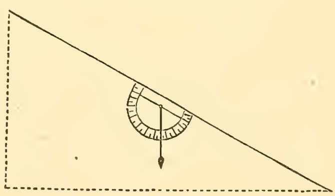


FIG. 418.



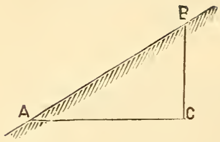
$$\text{Difference of heights} = \text{length of chain} \times \text{sin. angle.}$$

$$\text{Horizontal distance} = \text{length of chain} \times \text{cos. angle.}$$

These instruments are all "slope-measurers." They are also called *Clinometers, Clisimeters, Eclimeters,* etc., all meaning the same thing.

574. Slopes. These may be designated by their angles with the horizon, or by the relations of their bases and heights. The French engineers name a slope by the ratio of its height to its base—i. e.,

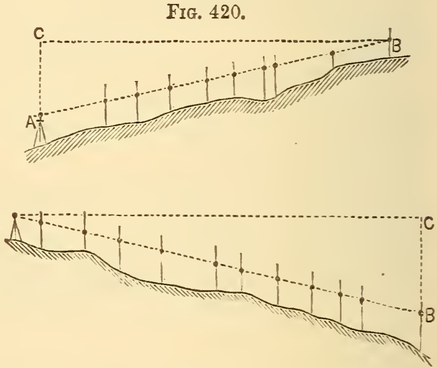
Fig. 419.



$\frac{BC}{AC}$; which is the tangent of the angle BAC. The English and Americans use the ratio of the base to the height—i. e., $\frac{AC}{BC}$, and make the height the unit, so that if $AC = 2\,CB$, the slope is called 2 to 1; and so on.

When the inclined line is a visual line, such as the line of sight of a telescope, whose angular movements are measured on a vertical circle beside it, and when with these is combined a horizontal circle for measuring horizontal angles, the instrument is called a "transit."

Fig. 420.

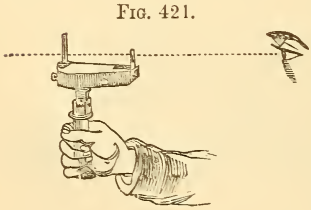


575. Angular Profiles A section or profile of a tolerably uniform slope is most easily obtained, as shown in the figures, by measuring the heights or depths below an inclined line, instead of below a level line.

A cross-section for a road may be taken in the same way.

576. Burnier's Level. It is a pear-shaped instrument, having two graduated circles: one vertical, having a weight attached so as to keep it in the same vertical position when in use; and the other, a horizontal graduated circle, made light and carried around by a magnetic needle, so that the instrument can be used as a compass as well as a slope or angular level. It has a convex-glass, or lens, in the smaller end, through which can be seen a hair which covers, on the circle, the number of the degrees of the angle of inclination, or of the horizontal angle.

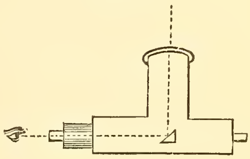
Fig. 421.



The sights are on the top or sides, according as it is used as a compass or slope-measurer. It is used by sighting to the object, and at the same time reading off the angle, the hair covering the zero-mark when the instrument is level.

577. German Universal Instrument. Its use is to enable the observer to sight to an object nearly or quite overhead. It consists of a telescope having the part which carries the eye-piece at right angles to the part carrying the object-glass, instead of being in the same straight line, as in an ordinary telescope. The part containing the eye-piece is connected with the other part at the axis, and is in the same line with the axis.

Fig. 422.



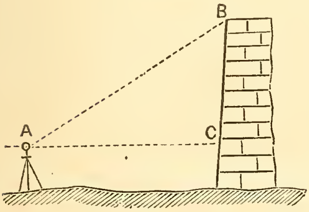
In the telescope is placed a small mirror, or reflector, or (what is still better) a triangular prism of glass, at an angle of 45° to the line of sight. Thus the observer can keep his eye at the same place at any inclination of the telescope.

SIMPLE ANGULAR LEVELING.

A. For Short Distances.

578. Principle. For short distances, curvature and refraction may be neglected. Thus, if the height of a wall, house, tree, etc., be desired, note the point where the horizontal line strikes the wall, etc., and add its height above the ground to that calculated by the formula :

Fig. 423.



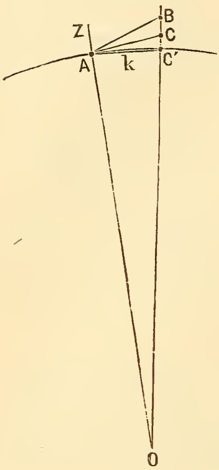
$$BC = AC \cdot \text{tang.}\ BAC. \quad . \quad . \quad [1.]$$

579. The "best-condition" angle for observation is 45°. Hence, in setting the instrument, we should, where practicable, have the distance about equal to the height of the point whose height we wish to ascertain.

B. For Greater Distances.

580. Correction for Curvature. A C is the line of apparent level, as given by the instrument, and A C′ is the line of true level. Calling the angle A C B = 90° (which it is approximately for moderately great distances), formula [1] gives B C as the height of B above A. But B C′ is the true difference of heights of A and B.

Fig. 424.



A correction for the curvature of the earth must therefore be made. It may be done in two ways: either by calculating C C′, and adding it to B C, obtained by formula [1], or by calculating the angle C A C′, adding it to B A C, and then applying the formula [1] to the angle B A C′.

581. Correcting the Result. Expressing the distance by k, we have, by Art. 497:

$$\text{In feet C C}' = \frac{k^2}{2\,\text{R}} = \frac{k^2}{2 \times 20912405} = 0{\cdot}000000023909k^2.$$

Then, calling A C B a right angle, we have:

$$\text{B C}' = k \times \text{tang. B A C} + 0{\cdot}000000023909k^2 \text{ in feet.} \quad . \quad . \quad [2.]$$

The arc A C′ and the straight lines A C′ and A C are all three approximately equal.

582. Correcting the Angle. The angle C A C′ = ½ A O C′, the central angle, which is measured by the arc A C′, or k.

The length of the arc subtending one minute

$$= \frac{2\,\pi \times 20912405}{360 \times 60} = 6083 \text{ feet.}$$

Then for any arc, k, the angle O in minutes

$$= \frac{k}{6083} = 0{\cdot}00016438k\,;$$

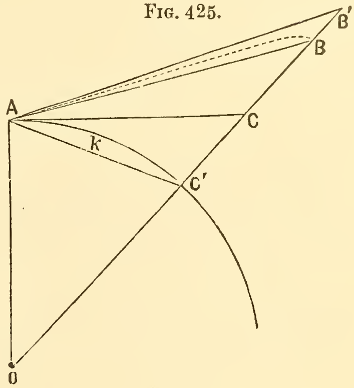
and the angle C A C′ (in minutes) = $0{\cdot}000082193k$.

Adding this to the observed angle, B A C, and calling A C′ B a right angle, we have, by [1]:

$$\text{B C}' = k \text{ tang. (B A C} + 0{\cdot}000082193k). \quad . \quad . \quad . \quad [3.]$$

583. Correction for Refraction. The effect of refraction causes the angle actually observed to be, not C A B, but C A B′, which will be designated by $a°$. For small distances, B and B′ sensibly coincide. The correction for refraction may be made in two ways, as for curvature.

Fig. 425.



To correct the result by finding B B′. It varies very irregularly, with wind, barometer, temperature, etc.; but is usually taken, as an average, $BB' = 0{\cdot}16\,CC'$.

Subtracting this from the value of B C′, in formula [2], it becomes $BC' = k\,.\,\text{tang.}\ B'AC + 0{\cdot}000000022k^2$. . . . . . [4.]

To correct the observed angle. Subtract from it the angle B A B′, which is about 0·16 of the angle C A C′.

This changes formula [3] to

$$BC' = k\,.\,\text{tang.}\ (B'AC + 0{\cdot}00006844k). \quad . \quad . \quad . \quad . \quad . \qquad [5.]$$

C. For Very Great Distances.

584. Correction for Curvature. As before, there are two methods of making the correction.

For these distances we can not consider the angle at C′ a right angle. The triangle A B C gives

$$BC = k\,.\,\frac{\sin.\ BAC}{\sin.\ B}.$$

To find the angle B, we have, in the triangle B A O,

$$B = 180° - (O + BAO),$$
$$B = 180° - (O + 90° + BAC),$$
$$B = \ \ 90° - (O + BAC)\,;$$

Hence, $\sin.\ B = \cos.\ (O + BAC)$.

Then, $BC = k\,.\,\dfrac{\sin.\ BAC}{\cos.\ (O + BAC)}$,

and $BC' = BC + CC' = k\,.\,\dfrac{\sin.\ BAC}{\cos.\ (O + BAC)} + 0{\cdot}000000023909k^2$.

$$BC' = k\frac{\sin.\ BAC}{\cos.\ (BAC + 0{\cdot}0001646k)} + 0{\cdot}000000023909k^2 \quad . \qquad [6.]$$

Correcting the Angle. In the triangle A B C′, getting expressions for the angles, and using the sine proportion, as before, in A B C, we have :

$$\mathrm{B\,C'} = k\,.\,\frac{\sin.\ (\mathrm{B\,A\,C} + \frac{1}{2}\,\mathrm{O})}{\cos.\ (\mathrm{B\,A\,C} + \mathrm{O})}.$$

$$\mathrm{B\,C'} = k\,.\,\frac{\sin.\ (\mathrm{B\,A\,C} + 0{\cdot}000082193k)}{\cos.\ (\mathrm{B\,A\,C} + 0{\cdot}000164387k)}.\quad .\quad [7.]$$

585. Correction for Refraction. Formula [6] becomes

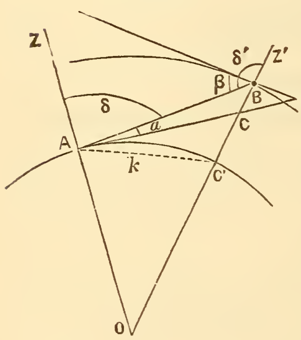
$$\mathrm{B\,C'} = k\,.\,\frac{\sin.\ (\mathrm{B'\,A\,C} - 0{\cdot}00001375k)}{\cos.\ (\mathrm{B'\,A\,C} + 0{\cdot}000150636k)} + 0{\cdot}000000023909k^2.\quad [8.]$$

Formula [7] becomes, diminishing B A C in both numerator and denominator by 0·08 of O,

$$\mathrm{B\,C'} = k\,.\,\frac{\sin.\ (\mathrm{B'\,A\,C} + 0{\cdot}000068442k)}{\cos.\ (\mathrm{B'\,A\,C} + 0{\cdot}000150636k)}.\quad .\quad .\quad [9.]$$

586. Reciprocal Observations for canceling Refraction. Observe the reciprocal zenith-distances from each point to the other. Call these angles Δ and Δ'.

FIG. 426.



The angle Z A B is the observed zenith-distance (Δ) of β, plus the refraction ρ—i. e., Z A B $= \Delta + \rho$, and Z′ B A $= \Delta' + \rho'$.

Let $\delta = \Delta + \rho$ and $\delta' = \Delta' + \rho'$,

Then $\delta + \delta' = \Delta + \Delta' + \rho + \rho' = 180 + \mathrm{O}$.

The observations should be simultaneous as well as reciprocal. When this is the case, we may take $\rho = \rho'$.

$$\text{Then } \rho = 90 + \tfrac{1}{2}\,\mathrm{O} - \tfrac{1}{2}\,(\Delta + \Delta'),$$

$$\delta' = \Delta' + \rho = 90 + \tfrac{1}{2}\,\mathrm{O} + \tfrac{1}{2}\,(\Delta' - \Delta),$$

$$\mathrm{Z\,A\,C'} = 90 + \tfrac{1}{2}\,\mathrm{O}.$$

In the triangle B A C′, B C′ : A C′ ($= k$) : : sin. B A C′ : sin. A B C′.

$$\therefore\ \mathrm{B\,C'} = k\,\frac{\sin.\ \mathrm{B\,A\,C'}}{\sin.\ \mathrm{A\,B\,C'}} = k\,\frac{\sin.\ (\mathrm{Z\,A\,B} + \mathrm{C'\,A\,O})}{\sin.\ \mathrm{Z'\,B\,A}},$$

$$\mathrm{B\,C'} = k\,\frac{\sin.\ [180^\circ - \frac{1}{2}\,(\Delta' - \Delta)]}{\sin.\ [90^\circ + \frac{1}{2}\,\mathrm{O} + \frac{1}{2}\,(\Delta' - \Delta)]},$$

$$B\,C' = k\,\frac{\sin.\,\frac{1}{2}\,(\Delta' - \Delta)}{\cos.\,\frac{1}{2}\,(\Delta' - \Delta + O)}.$$

When the angle O is *very* small compared with the other angles, this becomes : $B\,C' = k\,.\,\tan.\,\frac{1}{2}\,(\Delta' - \Delta)$.

Or, using angles of elevation and depression (α and β) we have :

$$B\,C' = k\,.\,\frac{\sin.\,\frac{1}{2}\,(\alpha + \beta)}{\cos.\,\frac{1}{2}\,(\alpha + \beta + O)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad [10.]$$

NOTE.—Angle O, in minutes $= 0{\cdot}000164387k$.
Log. $0{\cdot}000164387 = \bar{4}{\cdot}2158699$.

When O is very small, compared with the other angles, by neglecting it we have :

$$B\,C' = k\,.\,\text{tang.}\,\frac{1}{2}\,(\alpha + \beta). \quad . \quad . \quad . \quad . \quad [11.]$$

The following is from the "New York State Survey Report," 1882 :

The formula employed in deducing differences of height from reciprocal zenith-distance observations is

$$H' - H = K\,\tan.\,\frac{Z' - Z}{2}\left(1 + \frac{H + H'}{2\,r}\right),$$

where H′ and H are the heights of the stations above sea-level, K is the distance between the stations in metres, as given by the triangulation, and consequently reduced to sea-level, Z′ and Z are the observed zenith-distances ; r is the mean radius of the earth in metres ; its logarithm is 6·80454 for latitude 43°, according to Bessel's determination. This mean value may be safely taken as constant throughout the area of New York State without any practical error in the resulting differences of height.

The factor $\left(1 + \dfrac{H + H'}{2\,r}\right)$ will never in this State affect $H' - H$ by more than $\frac{1}{4000}$ part of its value ; it is usual, therefore, to compute the difference of height from the formula $H' - H = K\,\tan.\,\dfrac{Z' - Z}{2}$; and if by inspection of a short table of values of the omitted factor it is seen that its effect will be appreciable, it is then introduced.

For computing differences of height from zenith-distances observed at one station only, the formula

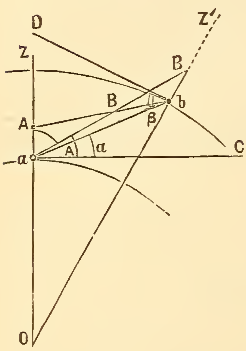
$$H' - H = K\,\cot.\,Z\left(1 + \frac{H + H'}{2\,r}\right) + \frac{1 - 2\,m}{2\,r}\,K^2$$

is employed. The symbols here have the same significance as before, and $2\,m$ is the ratio of the radius of the earth to the radius of the curve of light. The value of m may be approximately determined by means of reciprocal zenith-distance observations. From 137 of such observations the State Survey has found $m = 0{\cdot}0730$; its value is liable to considerable fluctuation, but it may be considered constant within the hours to which the observations are confined on the survey without any material error.

The factor $\left(1 + \frac{H + H'}{2r}\right)$ is treated as before. The logarithm of the coefficient $\frac{1 - 2m}{2r}$ is 2 . 82589. The quantity $\frac{1 - 2m}{2r} K^2$ has been tabulated for values of K up to 18,000 metres for office use.

587. Reduction to the Summits of the Signals. Stations a and b can not be seen from each other. Signals are erected at each point, and from a the angle $B\,a\,C = A$ is observed; and from b the angle $A\,b\,D = B$. The heights of the signals above the instrument at a and b are h and h'. The distance between the signals is k.

FIG. 427.



Required the reduced angles $\alpha = c\,a\,b$ and $\beta = D\,b\,a$.

$$\left.\begin{aligned} \alpha &= A - \frac{h \,.\, \cos.\, A}{k \,.\, \sin.\, 1''} \\ \beta &= B + \frac{h' \,.\, \cos.\, B}{k \,.\, \sin.\, 1''} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \quad [12.]$$

The difference is in seconds.

Usually, in such cases, zenith-distances are taken, and the observed angles are called Δ and Δ'. The reduced angles are δ and δ'.

Draw a line in the figure from A to B. Then in the triangle $A\,B\,a$ we have:

$$\sin.\, A\,B\,\alpha : \sin.\, \Delta :: h : k.$$

$$\text{or, } \sin.\, A\,B\,\alpha = \frac{h \sin.\, \Delta}{k \sin.\, 1''},$$

$$\text{and } \alpha\,B\,A = \frac{h \sin.\, \Delta'}{k \sin.\, 1''}.$$

$$\therefore\; \delta = \Delta + \frac{h \,.\, \sin.\, \Delta}{k \,.\, \sin.\, 1''}, \text{ and } \delta' = \Delta' + \frac{h' \,.\, \sin.\, \Delta'}{k \,.\, \sin.\, 1''}. \quad . \; . \; . \quad [13.]$$

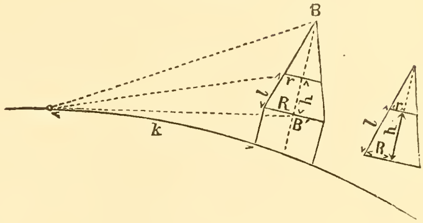
The difference is seconds.

Instead of h and h', some writers use d H and d H′; or d A and d A′, meaning difference of height, and difference of altitude.

For great exactness, instead of using the mean radius of the earth to get O, the radius at the point of observation is used.

588. When the height of the signal above the instrument can not be measured, if the signal be conical, like a spire, etc., to find B B′ we measure two diameters, 2 R and 2 r, and the distance apart, h.

FIG. 428.

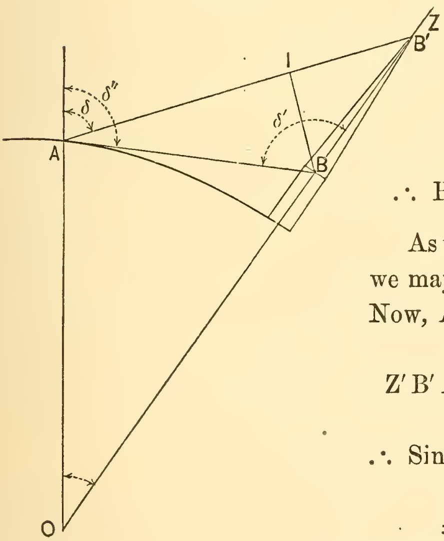


Then, $BB' = \dfrac{R\,h}{R - r}$. [14.]

If the oblique distance l be measured instead of h, then

$$BB' = \frac{R}{R - r}\sqrt{[l + (R - r)]\,[l - (R - r)]}. \quad . \quad . \quad . \qquad [15.]$$

FIG. 429.



When a Spire is very acute. From B let fall B I perpendicular to A B′. Then $BI = BB' . \sin. AB'B = AI . \tan. (\delta'' - \delta)$,

$$\therefore\; BB' = \frac{AI . \tan. (\delta'' - \delta)}{\sin. AB'B}.$$

As the angle $(\delta'' - \delta)$ is so small, we may take $AI = AB = K$.

Now, $AB'B = 180° - Z'B'A$, and from (586)

$$Z'B'A = \delta' = 90 + \tfrac{1}{2}O + \tfrac{1}{2}(\Delta' - \Delta).$$

$$\therefore\; \text{Sin.}\, AB'B = \sin.\{180° - [90° + \tfrac{1}{2}O + \tfrac{1}{2}(\Delta' - \Delta)]\}$$
$$= \cos. \tfrac{1}{2}(\Delta' - \Delta + O).$$

$$BB' = \frac{K. \tan. (\delta'' - \delta)}{\cos. \tfrac{1}{2}(\Delta' - \Delta + O)}.$$

589. Leveling by the Horizon of the Sea. Owing to refraction, the apparent zenith-distance will be Z B A′.

Let R = radius of the earth ; H H′ = horizon.

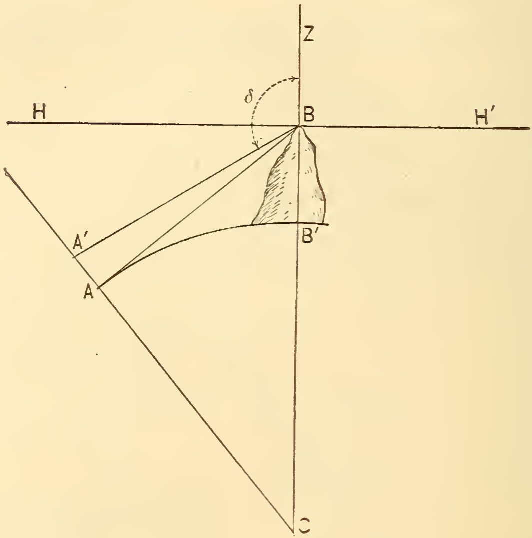
$$\text{Then } R + BB' = \frac{R}{\cos. C}.$$

26

$$\therefore\ B\,B' = R\,\frac{(1 - \cos.\ C)}{\cos.\ C}. \quad [16.]$$

Now, $(1 - \cos.\ C) = 2 \sin.^2 \tfrac{1}{2} C$. Transposing, we have $\cos.\ C$

FIG. 430.



$= \cos.^2 \tfrac{1}{2} C - \sin.^2 \tfrac{1}{2} C$. Substituting these values in equation (1), we get $B\,B' = \dfrac{R\,(2 \sin.^2 \frac{1}{2} C)}{\cos.^2 \frac{1}{2} C - \sin.^2 \frac{1}{2} C} = 2\,R \dfrac{\sin.^2 \frac{1}{2} C}{\cos.^2 \frac{1}{2} C - \sin.^2 \frac{1}{2} C}$.

(Developing by the binomial formula)—

$$= 2\,R \tan.^2 \tfrac{1}{2} C\ (1 + \tan.^2 \tfrac{1}{2} C - \tan.^4 \tfrac{1}{2} C +, \text{etc.})$$

Using the first two terms of the series, we have

$$B\,B' = 2\,R \tan^2 \tfrac{1}{2} C\ (1 + \tan.^2 \tfrac{1}{2} C).$$

As the angle C is very small, we may express the tangent as an arc in terms of the radius, without greater error than one foot in an altitude of 45,000.

Then we have $B\,B' = \dfrac{R}{2} C^2 \left(1 + \dfrac{C^2}{4}\right)$. [17.]

The angle $C = H\,B\,A' + A\,B\,A = \delta - 90° - n\,C$, n being the coefficient of refraction. $\therefore\ C = \dfrac{\delta - 90°}{1 - n}$.

In order to introduce the value of C into equation (2), we multiply it by the sine of 1″, to reduce arc to linear measure.

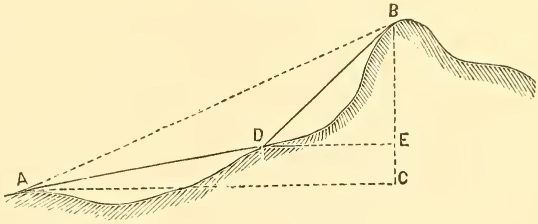
Then we have

$$B\,B' = \tfrac{1}{2}\,R\left(\frac{\sin.\,1''}{1-n}\right)^2 (\delta - 90^\circ)^2 \left\{ 1 + \tfrac{1}{4}\left(\frac{\sin.\,1''}{1-n}\right)^2 (\delta - 90^\circ) \right\}. \quad [18.]$$

COMPOUND ANGULAR LEVELING.

590. The following problems may mostly be reduced to a combination of: first, determining the inaccessible distance to a point immediately under (or over) the point whose height is desired, and then using this distance to obtain that height.

FIG. 431.

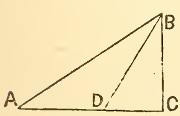


591. By Angular Co-ordinates in one Plane. Take two stations, A and D, in the same vertical plane with B. At A observe the angles of elevation of B and D. Measure A D. At D observe the angle A D B. Then, in the triangle A B D we get A B, and in the triangle B A C we get B C.

$$B\,C = A\,D\,.\,\frac{\sin.\ B\,D\,A\,.\ \sin.\ B\,A\,C}{\sin.\ A\,B\,D}. \quad . \quad . \quad . \quad . \quad . \quad . \quad [19.]$$

For great distances, the corrections for curvature and refraction are to be made as in the preceding articles.

FIG. 432.



If A D be horizontal, the same formula applies; but there is one angle less to measure, since B A C = B A D. Formula [19] gives the height of B above A.

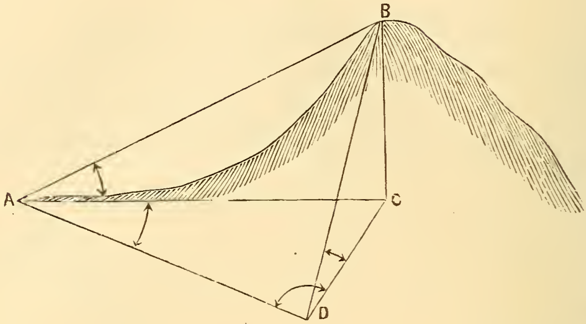
If the height of B above D, in Fig. 432, be desired, find BD in the triangle B A D, observe the angle of elevation of B from D, and then the desired height equals

$$B\,D\,.\ \sin.\ B\,D\,E.$$

Otherwise, find height of D above A, and subtract it from B C.

592. By Angular Co-ordinates in Several Planes. On irregular ground, when the distance between the two points is unknown, the operations for finding it by the various methods already given may be combined with the observation of vertical angles, thus:

FIG. 433.



At A measure the vertical angle of elevation, B A C. Also measure the horizontal angle, C A D, to some point, D, and measure horizontally the distance, A D. At D measure the horizontal angle, A D C. Then,

$$A\,C = A\,D\,\frac{\sin.\,A\,D\,C}{\sin.\,A\,C\,D}. \quad B\,C = A\,C\,.\,\text{tang.}\,B\,A\,C.$$

$$B\,C = A\,D\,\frac{\sin.\,A\,D\,C\,.\,\text{tang.}\,B\,A\,C}{\sin.\,A\,C\,D} \quad .\;.\;.\;.\;.\;.\;. \quad [20.]$$

593. *Conversely.* The distance may be obtained when the height is known.

Let C B be a known height. Then, A C = C B . tan. A B C.

B C is a known height, and D E an inaccessible line in the same

FIG. 434.

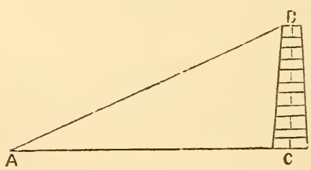
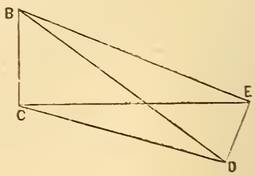


FIG. 435.



horizontal plane as C. Find C D and C E by the last method, and measure the horizontal angle E C D subtended at C by E D.

Then the two sides and the included angle of a triangle are known, to find the third side.

CHAPTER III.

BAROMETRIC LEVELING.

PRINCIPLES AND FORMULAS.

594. Principles. The difference of the heights of two places may be determined by finding the difference of their depths below the top of the atmosphere in the same way as the comparative heights of ground under water are determined by the difference of the depths below the top of the water. The desired height of the atmosphere above any point, such as the top of a mountain, or the bottom of a valley, is determined by weighing it. This is done by trying how high a column of mercury or other liquid the column of air above it will balance; or what pressure it will exert against an elastic box containing a vacuum, etc. Such instruments are called *Barometers.*

595. Applications. Since the column of mercury in the barometer is supported by the column of air above it, the mercury sinks when the barometer is carried higher, and *vice versa.*

The weight of any portion of air decreases from the surface of the earth to the assumed surface of the atmosphere. It has been found that, as the heights to which the barometer is carried increase in arithmetical progression, the weights of the column of air above the barometer, and consequently its readings, decrease in geometrical progression. Consequently, the difference of the heights of any two not very distant points on the earth's surface is proportional to the difference of the logarithms of the readings of the barometer at those points—i. e., equal to this latter difference multiplied by some constant coefficient. This is found by experiment to be 60159, at the freezing-point, or temperature of 32° Fahr., the readings of the mercury being in inches, and the product, which is the difference of height, being in feet.

Several corrections are necessary.

596. Correction for Temperature of the Mercury. If the temperature of the mercury be different at the two stations, it is expanded at the one, and contracted at the other, to a height different from that which is due to the mere weight of the air above it.

Mercury expands about $\frac{1}{10000}$ of its bulk for each degree of F. Therefore, this fraction of the height of the column is to be added to the height of the colder column, or subtracted from the height of the warmer one, in order to reduce them to the same standard. A thermometer is therefore attached to the instrument in such a manner as to give the temperature of the mercury.

If a brass scale is used, the correction is $\frac{9}{100000}$ for each degree F.

597. Correction for Temperature of the Air. The warmer the air is, the lighter it is ; so that a column of warm air of any height will weigh less than when it is colder. Consequently, the mercury in warm air falls less in ascending any height, and is higher at the place than it otherwise would be. Hence the height given by the preceding approximate result will be too small, and must be increased by $\frac{1}{491}$ part for each degree F. that the temperature of the air is above 32°. The effect of moisture in the air changes this fraction to $\frac{1}{450}$.

598. Other Corrections. For *very* great accuracy, we should allow for the variation of gravity, corresponding to the variation of latitude on either side of the mean. So, too, we should allow for the decrease of gravity corresponding to any increase of height of the place.

599. Rules for calculating Heights by the Mercurial Barometer.

1. At each station read the barometer ; note its temperature by the attached thermometer, and note the temperature of the air by a detached thermometer.

2. Multiply the height of the upper column by the difference

of readings of the attached thermometer, and that by $\frac{9}{100000}$, and add the product to the upper column, if that be the colder, or subtract it, if that be the warmer. This gives the corrected height of the mercury.

3. Multiply the difference of the logarithms of the corrected heights of the mercury—i. e., the corrected upper one and the lower one—by 60159, and the product is the *approximate* difference of heights of the places in feet for the temperature of 32°.

4. Subtract 32° from the arithmetical mean of the temperatures of the detached thermometer; multiply the approximate altitude by this difference; divide the product by 450; add the quotient to the approximate altitude, and the sum is the corrected altitude.

600. Formulas. The rules just given are best expressed in formulas, thus:

	AT LOWER STATION.	AT UPPER STATION.
Height of mercury..........	H	h'
Temperature of mercury.....	T	T′
Temperature of air..........	t	t'

Calling the reduced height of mercury at the upper station h, we have, by Rule 2:

$$h = h' + 0{\cdot}00009\,(\mathrm{T} - \mathrm{T}')\,h' \quad . \quad . \quad . \quad . \quad . \quad [1.]$$

N. B.—If T′ is more than T, the product will be subtractive.

Then, by Rule 3, we have:

$$\text{Approximate height} = 60159\,(\log.\ \mathrm{H} - \log.\ h).$$

By Rule 4, the correction for temperature of air

$$= \text{approximate height} \times \frac{t + t' - 64}{900}.$$

Adding this correction to the approximate height, and factoring the sum, we get:

$$\text{Corrected ht.} = 60159\,(\log.\ \mathrm{H} - \log.\ h)\left(1 + \frac{t + t' - 64}{900}\right). \quad [2.]$$

601. To correct for Latitude. Multiply the preceding result by 0·00265 . cos. 2 L (L being the latitude), and add (algebraically) the product to the preceding result.

At 45°, correction is zero. At equator it is $+ 0{\cdot}00265$. At pole it is $- 0{\cdot}00265$.

To correct for Elevation of the Place. Call the last corrected height x', and the height of the lower place above the level of the sea, S, and add to x' this quantity:

$$\frac{x' + 52251}{20912405} + \frac{S}{10456203}.$$

602. Final English Formula. Combining the previous results into one formula, we get:

$$\text{Ht.} = 60159\,(\log. H - \log. h) \left\{ \begin{array}{l} \left(1 + \dfrac{t + t' - 64}{900}\right), \\ (1 + 0{\cdot}00265 \,.\, \cos. 2\,L), \\ \left(1 + \dfrac{x' + 52251}{20912405} + \dfrac{S}{10456203}\right) \end{array} \right\} \quad [3.]$$

In this formula, the three quantities under each other are three factors.

Usually, only the first factor is required, and then we have formula [2]. Using the second, also, we correct for latitude; and, using the third, for the elevation.

603. French Formulas. French barometers are graduated in French millimetres, each $= 0{\cdot}03937$ inch, and the thermometer is centigrade, in which the freezing-point is zero, and boiling-point 100°:

$$a^\circ \text{ cent.} = (\tfrac{9}{5}a + 32)^\circ \text{ F.}$$

Then, the French formula corresponding to [3] is the following (H and h' being in millimetres, and the temperatures centigrade):

$$h = h'\left(1 + \frac{T - T'}{6200}\right).$$

And the difference of heights in metres

$$= 18336\,(\log. H - \log. h) \left\{ \begin{array}{l} \left(1 + \dfrac{2\,(t + t')}{1000}\right), \\ (1 + 0{\cdot}00265 \,.\, \cos. 2\,L), \\ \left(\dfrac{1 + x' + 15926}{6372481}\right) + \dfrac{S}{3186241} \end{array} \right\} \quad [4.]$$

604. Babinet's Simplified Formula, without Logarithms.

h' is reduced to h, as before, viz. : $h = h'\left(1 + \frac{T - T'}{6200}\right)$.

Then, the difference of heights in metres

$$= 16000 \,.\, \frac{H - h}{H + h}\left(1 + \frac{2\,(t + t')}{1000}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [5.]$$

The heights are in millimetres and the temperatures centigrade.

Example. $H = 755. \quad h = 745$

$t = 15° \quad t' = 10°.$

$$\text{Ht.} = 16000\,\frac{10}{1500}\left(1 + \frac{50}{1000}\right) = 112 \text{ m.}$$

Correct result is 111·6 m.

This formula is a very near approximation for moderate heights.

Babinet's formula in English measures (the heights being in inches, and temperatures Fahrenheit) is in feet :

$$52494\left(\frac{H - h}{H + h}\right)\left(1 + \frac{t + t' - 64}{900}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad [6.]$$

Leslie's formula is :

$$\text{height in feet} = 55000\,\frac{B - b}{B + b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [7.]$$

In which B = upper reading, and b = lower reading. This is for a temperature of 55° Fahr.

605. Tables. These shorten the operations greatly. The most *portable* are in "Annuaire du Bureau des Longitudes." The most *complete* are Professor Guyot's, published by the Smithsonian Institution at Washington.

606. Approximations. One tenth of an inch difference of readings in two places corresponds to about ninety feet difference of elevation. One millimetre difference of readings corresponds to about ten and a half metres difference of height, or about thirty-four feet.

This is correct near the freezing-point, and near the level of the sea. The height corresponding to any given difference of readings increases, however, with the temperature and with the height of the station. Thus, at 70° F., $\frac{1}{10}$ of an inch corresponds to an elevation of 95 feet; and one millimetre at 30° cent. corresponds to $11\frac{3}{4}$ metres, or about 40 feet.

Instruments.

607. Barometers made for leveling are called *Mountain Barometers.* They are either *cistern* barometers or *siphon* barometers.

Fig. 436.

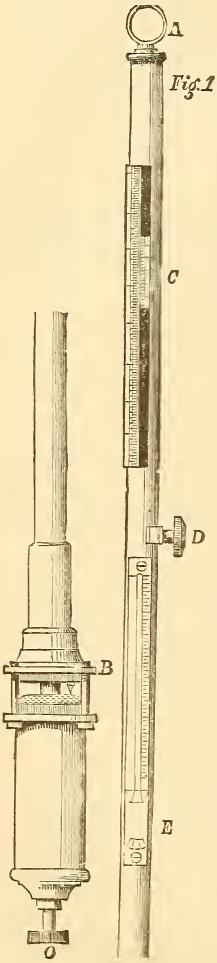


Fig. 436 is a cistern barometer.* This consists of a column of mercury, contained in a glass tube, whose lower end is placed in a cistern of mercury. The tube is covered with a brass case, terminating at the top in a ring, A, for suspension, and at the bottom in a flange, B, to which the cistern is attached.

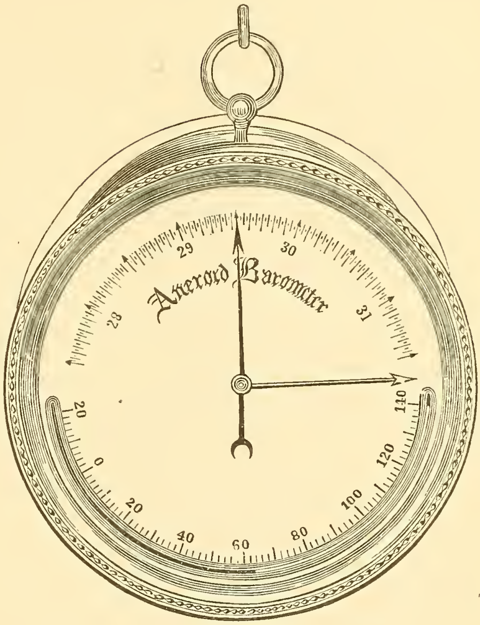
At C is a vernier, by which the height of the mercury is read off. The vernier is moved by means of a rack, worked by the milled head shown at D.

The zero of the scale is a small ivory point, shown below the flange B. The mercury in the cistern is raised or lowered, by means of the milled-headed screw O, till its surface is just in contact with the ivory point. The upper part of the cistern is of glass, so that the surface of the mercury in the cistern, and the ivory point, may be readily seen. At E is the attached thermometer which indicates the temperature of the mercury. When it is carried, the mercury is screwed up to prevent breaking the glass.

608. The Aneroid Barometer. This is a thin box of corrugated copper, exhausted of air. When the air grows heavier, the box is compressed; and when the air grows lighter, it is expanded by a spring inside. This motion is communicated

* Made by Henry J. Green, 771 Broadway, New York.

FIG. 437.



by suitable levers to the index-hand, on the face, which indicates the pressure of the atmosphere, the face being graduated to correspond with a common barometer.

There are several varieties of this instrument, differing principally in the method of determining the movement of the corrugated box due to changes in the density of the atmosphere.

They are made in sizes varying from two to six inches in diameter. They are much used on account of their portability, but are not as reliable as the mercurial barometer.

Approximately, a difference of reading of $\frac{1}{100}$ of an inch corresponds to a difference of height of nine feet. The following table is more nearly accurate:

MEAN TEMPERATURE.	30°	40°	50°	60°	70°	80°
Mean pressure, 27 inches.......	9·7	9·9	10·1	10·3	10·5	10·8
" 28 "	9·3	9·5	9·8	10·0	10·2	10·4
" 29 "	9·0	9·2	9·4	9·6	9·8	10·
" 30 "	8·7	8·9	9·1	9·3	9·5	9·7

609. The Hypsometer. The temperature at which water boils varies with the pressure of the atmosphere, and therefore decreases in ascending heights. Then a thermometer becomes a substitute for a barometer.

Approximately, each degree of difference (Fahr.) corresponds to about 550 feet difference of elevation, subject to the usual barometric corrections for the temperature of the air. (For minute tables, see Guyot's.)

TEMPERATURE OF BOILING WATER.	CORRESPONDING BAROMETER READINGS.
213°	30″·522
212°	29″·922
211°	29″·331
210°	28″·751
209°	28″·180
208°	27″·618

610. Accuracy of Barometric Observations. This increases with the number of repetitions of them, the mean being taken. With great skill and experience they may be depended upon to a very few feet.

PROFESSOR GUYOT'S RESULTS.

HEIGHTS FOUND BY THE BAROMETER.	CORRESPONDING HEIGHTS FOUND BY THE SPIRIT-LEVEL.
6707 feet.	6711 feet.
2752 "	2752 "
6291 "	6285 " 6293 "

611. The observations at the two places, whose difference of heights is to be determined, should be taken simultaneously at a series of intervals previously agreed upon, the distance apart of the places being as short as possible. Distant places should be connected by a series of intermediate ones.

PART III.

TOPOGRAPHY.

INTRODUCTION.

612. Definition. Topography is the complete determination and representation of any portion of the surface of the earth, embracing the relative position and heights of its inequalities; its hills and hollows, its ridges and valleys, level plains, slopes, etc., telling precisely where any point is, and how high it is.

It therefore determines the three co-ordinates of any point; the horizontal ones by surveying, and the vertical ones by leveling.

The results of these determinations are represented in a conventional manner, which is called "topographical mapping."

The difficulty is, that we see hills and hollows in *elevation*, while we have to represent them in *plan*.

613. Systems. Hills are represented by various systems:

1. By level contour-lines, or horizontal sections.
2. By lines of greatest slope, perpendicular to the former.
3. By shades from vertical light.
4. By shades from oblique light.

The most usual method is a combination of the first, second, and third systems.

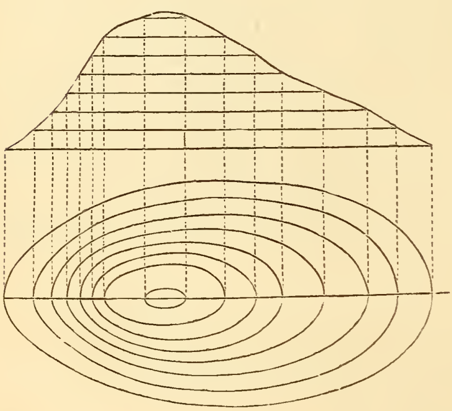
CHAPTER I.

FIRST SYSTEM.

BY HORIZONTAL CONTOUR-LINES.

614. General Ideas. Imagine a hill to be sliced off by a number of equidistant horizontal planes, and their intersections with it to be drawn as they would be seen from above, or horizontally projected on the map, as in Fig. 438. These are "contour-lines."

Fig. 438.



They are the same lines as would be formed by water surrounding the hill, and rising one foot (or any other height) at a time till it reached the top of the hill. The edge of the water, or its shore, at each successive rise, would be one of these horizontal contour-lines. It is plain that their nearness or distance on the map would indicate the steepness or gentleness of the slopes. A

Fig. 439.

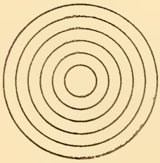


Fig. 440.

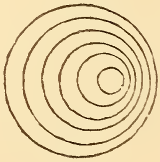
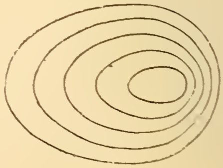


Fig. 441.



right cone would thus be represented by a series of concentric circles, as in Fig. 439; an oblique cone, by circles not concentric, but nearer to each other on the steep side than on the other, as in Fig. 440; and by a half-egg, somewhat as in Fig. 441.

615. Plane of Reference. The horizontal plane on which the contour-lines are projected, and to which they are referred, is called the "plane of reference." This plane may be assumed in any position, and the distance of the contour-lines above or below it is noted on them. It is usually best to assume the position of the plane of reference lower than any point to be represented; so that all the contour-lines will be above it, and none of them have minus signs.

616. Vertical Distances of the Horizontal Sections. These depend on the object of the survey, the population of the country, the irregularity of the surface, and the scale of the map. In mountainous districts they may be 100 feet apart. On the United States Coast Survey they are twenty feet; for engineering purposes, five feet, or less. One rule is to make the distance in feet equal to the denominator of the ratio of the scale of the map, divided by 600.

617. Methods for determining Contour-Lines. They are of two classes: 1. Determining them on the ground at once; 2. Determining the highest and lowest points, and thence deducing the contour-lines.

FIRST METHOD.

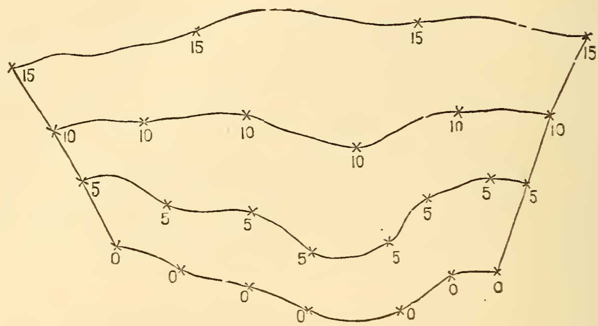
618. General Method. Determine one point at the desired height of one line, and then "locate" a line at that level.

The "reflected hand-level," or "reflecting-level," or "water-level," are sufficiently accurate between "bench-marks" not very distant.

One such line having been determined, a point in the next higher or the next lower one is fixed, and the preceding operations repeated.

619. On a Long, Narrow Strip of Ground, such as that required for locating a road: Run a section across it at every quarter or half mile, about in the line of greatest slope. Set stakes on these sections at the heights of the desired contour-lines, and then get inter-

FIG. 442.



mediate points at these heights between the stakes. These sections *check* the levels.

620. On a Broad Surface. Level around it setting-stakes, at points of the desired height, and then run sections across it, and from them obtain the contour-lines as before.

The external lines here serve as checks to the cross-lines.

621. Surveying the Contour-Lines. The contour-lines thus found may be surveyed by any method. If they are long, and not very much curved, the compass and chain and the method of "progression" is best. If they are curved irregularly, the method of radiation is best. When straight lines exist among them, such as fences, etc., or can conveniently be established, then rectangular co-ordinates are most convenient.

SECOND METHOD.

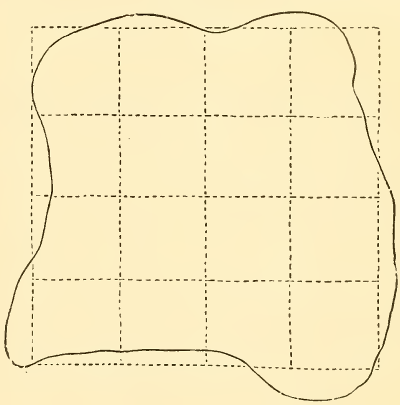
622. General Nature. This method consists in determining the heights and positions of the principal points, where the surface of the ground changes its slope in degree or in direction—i. e., determining all the highest and lowest points and lines, the tops of the hills and bottoms of the hollows, ridges and valleys, etc., and then, by proportion or interpolation, obtaining the places of the points which are at the same desired level. The heights of the principal points are found by common leveling, and their places fixed as in Art. 621.

The first method is more *accurate;* the second is more *rapid.*

623. Irregular Ground. When the ground has no very marked features, run lines across it in various directions, and level along them, taking heights at each change of slope, just as in taking sections for profiles.

Otherwise, thus: Set stakes on four sides of the field, so as to inclose it in a rectangle, if possible, as in Fig. 443. Place the stakes equidistant, so that the imaginary visual lines connecting them would divide the surface into rectangles. Send the rod along one of these lines till it gets in the range of a cross-one, and observe to it there. Put down the observed heights of these points at the corresponding points on the plat, on which these lines have been drawn. The contour-lines are determined as in Art. 626.

Fig. 443.



624. On a Single Hill. Proceed thus: From its top, range lines down the hill, in various directions, and take their bearings. Set stakes on them at each change of slope, and note the heights and distances of these stakes from the starting-point, and plat their places. The contour-lines are then put in as in Art. 626.

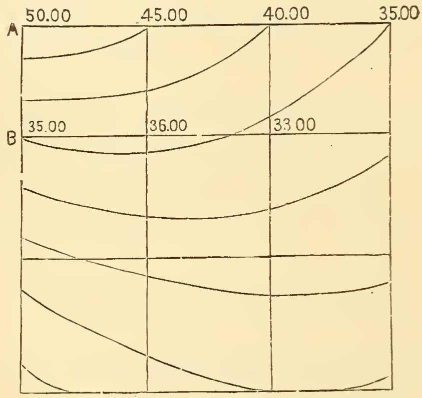
With a transit, the heights of the points could be determined by vertical angles; and also their distances with stadia-hairs, their directions being given by the horizontal circle of the transit. The French use for this purpose a "leveling-compass."

625. For an Extensive Topographical Survey. Proceed thus: Set up and get the height of the cross-hairs from some bench-mark, and get the heights of high and low prominent points all around. Then go beyond these points and set up again. Sight to one of these known points as a "turning-point," and get the heights of all the points now in sight, as before. Then go beyond these again, and so on. The places of these new points are fixed as before.

626. Interpolation. The heights and the places of the principal points being determined, by either of the preceding methods, points of any intermediate height, corresponding to any desired contour-curve, are obtained by proportion.

If, in Fig. 444, the heights of the intersection of the lines being found, as in Art. 623, and their distance apart being 100 feet, it is required to construct contour-curves whose difference of heights is 5 feet: Taking, for example, the one whose height is 45 feet, we see it must fall between the points A and B, whose heights are 50 feet and 35 feet; and its distance from A will be found by the proportion, as 15 is to 5 so is 100 to the required distance. So on for any number of points. To save the labor of continually calculating the fourth proportional, a scale of proportion may be constructed.

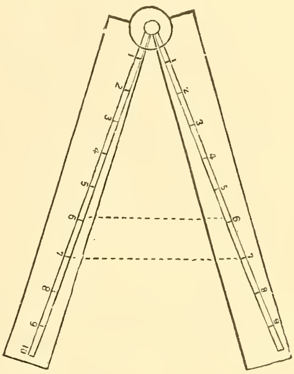
Fig. 444.



627. Interpolating with the Sector. This is one of the easiest ways. The problem is: having given on a plat two points of known height, to interpolate between them a point of any desired intermediate height.

Take in the dividers the distance between the given points on the plat; open the sector so that this distance shall just reach between numbers, on the scale marked L, corresponding to the difference of the heights of the two given points—i. e., from 6 to 6, or 7 to 7, and so on. The sector is then *set* for all the interpolations between these two points.

Fig. 445.



Then note the difference of height between the desired point and one of the given points, and extend the dividers between the corresponding numbers on the scale. This opening will be the distance to be set off on the plat from the given point to the desired point.

628. Ridges and Thalwegs. The general character of the surface of a country is given by two sets of lines: the *ridge-lines,* or *water-shed lines;* and *the "thalwegs,"* or *"lowest lines of valleys."*

The former are lines which divide the water falling upon them, and from which it passes off on contrary sides. They are the lines of least slope when looking along them from above downward; and they are the lines of greatest slope when looking from below upward. They can therefore be readily determined by the slope-level, etc. They are the lines of *least* zenith-distances when viewed from either direction.

On these lines are found all the projecting or protruding bends of the contour-lines, convex toward the lower ground, as shown in Fig. 396.

The second set of lines, or the "thalwegs," are the converse of the former. They are indicated by the water-courses which follow them or occupy them. They are the lines of greatest slope when looked at from above, and of least slope when looked at from below. They are the lines of *greatest* zenith-distance when viewed from either direction.

On these lines are the receding or re-entering points of the contour-curves, concave toward the lower ground.

The general system of the surface of a country is most easily characterized by putting down these two sets of lines, and marking the changes of slope, especially the beginning and the end.

Fig. 446.

The most important points to be determined are :

1. At the top and bottom of slopes.

2. At the changes of slopes in degree.

3. On the water-shed lines, and on the thalwegs.

4. On "cols," or culminating points of passes.

629. Forms of Ground. It will be found, on the inspection of a "contour-map" (which shows ground much more plainly to the eye than does the ground itself), that its infinite variety of form may, for the purposes of the engineer, be reduced to five :

1. Sloping down on all sides—i. e., a hill (Fig. 447).

Fig. 447.

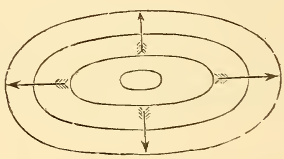
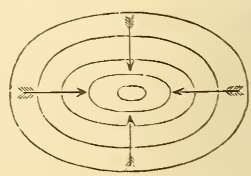


Fig. 448.



2. Sloping up on all sides—i. e., a hollow (Fig. 448).

3. Sloping down on three sides and up on one—i. e., a *croupe,*

Fig. 449.

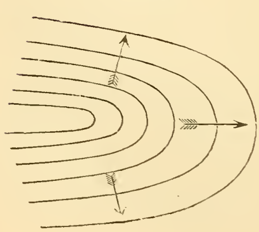
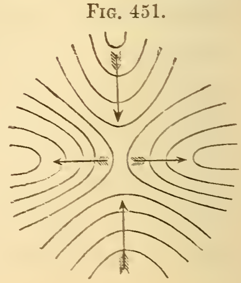


Fig. 450.

Fig. 451.



or shoulder, or promontory, the end of a ridge or water-shed line (Fig. 449).

4. Sloping up on three sides and down on one—i. e., a valley, or thalweg (Fig. 450).

5. Sloping up on two sides and sloping down on two, alternately—i. e., a "pas," or "col," or "saddle" (Fig. 451).

[NOTE.—The arrows in the figures indicate the direction in which water would run.]

630. Sketching Ground by Contours. A valuable guide is, the observation that the contour-lines are perpendicular to the water-shed lines and thalwegs. Note especially the contour-lines at the bottoms of hills and ridges, and at the tops of hollows and valleys, putting them down, in their true relative positions and distances, to an estimated scale.

On a long slope or hill, draw first the bottom contour-line, and the top one; then the middle one; and afterward interpolate others. Remember that two of them can never meet, except on a perpendicular face; and that, if one of them passes entirely around a hill or hollow, it will come back to its starting-point. Hold the field-book so that the lines on it have their true direction. As far as possible, all of the work should be done in the field with the ground in sight, and not trust to finishing from memory.

631. Ambiguity. In contour-maps of ground, if the heights of the contour-lines are not written upon them, it may be doubtful which are the highest and lowest; which are ridges and which valleys, etc.

1. Numbers remove this.

2. The water-courses show the slopes. If there are none, put some in, in the thalwegs of a rough sketch.

3. Put hatchings on the lower sides of the contour-lines, as if water were draining off.

4. Tint the valleys and low places.

632. Conventionalities. Sometimes the spaces between contour-lines are colored with tints of Indian-ink, sepia, etc., increasing in darkness as the depth increases.

Ground under water is commonly so represented, beginning at the low-water line and covering the space to the six-feet-deep contour-line with a dark shade of Indian-ink ; then a lighter shade from 6 to 12 ; a still lighter from 12 to 18 ; and the lightest from 18 to 24.

Greater depths are noted in fathoms and fractions.

633. Applications of Contour-Lines. They have many important uses besides their representation of ground :

1. To obtain vertical sections—i. e., profiles.
2. To obtain oblique sections.
3. To locate roads.
4. To calculate excavation and embankment. Consider the contour-lines to represent sections of the mass by horizontal planes. Then each slice between them will have its contents equal, approximately, to half the sum of its upper and lower surfaces multiplied by the vertical distance apart of the sections. This method is used to get the cubic contents of a hill to be cut away ; of a hollow to be filled up ; of a great reservoir in a valley, either only projected, or full of water, etc.

634. Sections by Oblique Planes. This method was much used by the old military topographers. It is picturesque, but not precise. The cutting-planes are parallel, and may make any angle with the horizon.

CHAPTER II.

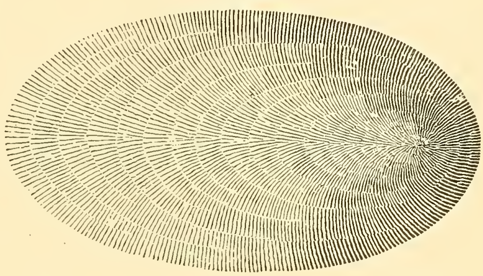
SECOND SYSTEM.

BY LINES OF GREATEST SLOPE.

635. Their Direction. It is that which water would take in running down a slope. They are drawn perpendicularly to the contour-lines, and are the "lines of greatest slope." They are called "hatchings."

Fig. 452 represents an oval hill by this system.

FIG. 452.



636. Sketching Ground by this System. This is rapid and effective, but not precise. In doing this, hold the book to correspond with your position on the ground, and always draw toward you. If at the top of a hill, begin by drawing lines from the bottom, and *vice versa.* The hatchings are guided by contour-lines lightly sketched in.

637. Details of Hatchings. They must be drawn very truly perpendicular to the contour-lines. But if the contour-lines are not parallel, the hatchings must curve. In finishing drawings, sketch in the curved hatchings with a pencil at some distance apart as guides. When the contours are very far apart, as on nearly level ground, pencil in intermediate ones.

Hatchings in adjoining rows should not be continuous, but "break joints," to indicate the places of the contour-lines, which are usually penciled in to guide the hatchings, and then rubbed out. The rows of hatchings must neither overlap nor separate, and the lines should be made slightly tremulous. When they are put in without contour-lines to guide them, take care never to let two rows run into one; for the breaks between the rows represent contour-lines, and two contour-lines of different heights can never meet except on a vertical surface.

In drawing a hill begin at the top. When hatchings diverge very much, as on hill-tops, put in alternate short ones. When the formation is very convex or concave, short auxiliary contours may be used.

CHAPTER III.

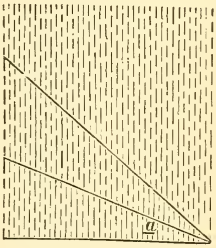
THIRD SYSTEM.

BY SHADES FROM VERTICAL LIGHT.

638. Degree of Shade. The steeper the slope is, the less light it receives, in the inverse ratio of its length—i. e., inversely as the secant of the angle a which it makes with the horizon, or directly as cos. a. Then the ratio of the black to the white is,

$$: : 1 - \cos. a : \cos. a.$$

Fig. 453.



In practice, the difference of shade is much exaggerated.

Tables have been prepared by various nations, establishing the ratio of black and white.

The proper degree of shade may be given to the hills and hollows on the map by various means.

639. Shades by Tints. Indian-ink, or sepia, is used. The shades are put on with proper darkness, according to a previously prepared "diapason of tints." The tints are made light for gentle slopes, and dark for steep slopes, in a constant ratio, a slope of 60° being quite black, one of 30° a tint midway between that and white, and so on. The edges at the top and bottom are softened off with a clean brush. This is rapid and effective, but not very definite or precise, except in combination with contour-lines.

640. Shades by Contour-Lines. This is done by making the contour-lines more numerous—i. e., interpolating new ones between those first determined. One objection to this is confusion of these lines with roads.

641. Shades by Lines of Greatest Slope. The lines of steepest slope—i. e., the hatchings between the contours — have their thickness and distance apart made proportional to the steepness of the slope, in some definite ratio. This is the most usual method.

The tints may be produced by varying the thickness of the hatchings, or their distance apart. Both are usually combined.

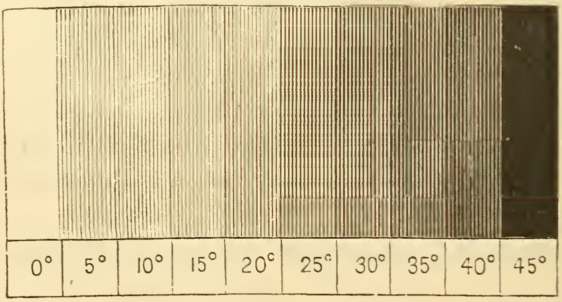
642. The French Method. In this the degree of inclination is indicated by varying the distances between the centers of the hatchings. The rule is : *the distance between the centers of the lines shall equal* $\frac{2}{100}$ *of an inch, plus* $\frac{1}{4}$ *of the denominator of the fraction denoting the declivity*—i. e., *tangent of the angle made by the surface of the ground with the plane of reference—expressed in hundredths of an inch.*

The lines are made heavier as the slope is steeper, being fine for the most gentle slopes, and increasing in breadth till the blank space between them equals $\frac{1}{2}$ the breadth of the lines.

Only slopes of from $\frac{1}{1}$ to $\frac{1}{64}$ inclusive are represented by this method.

643. The German, or Lehmann's Method. He uses nine grades for slopes from 0° to 45°, the first being white and the last black.

FIG. 454.



For the intermediate slopes, he makes the white to the black in the following proportion :

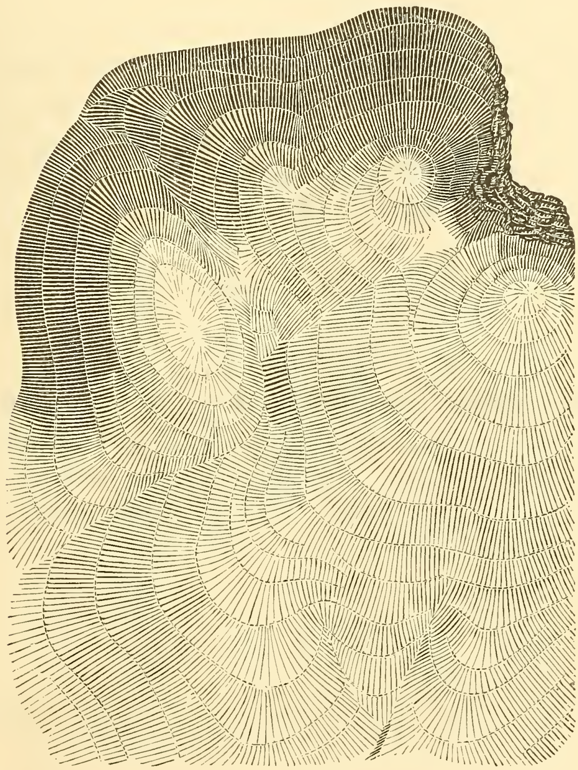
The white : *the black* : : 45° — *angle of slope* : *angle of slope.*

For example, for 30° :

light : *shade* : : 45° — 30° : 30° : : 1 : 2.

Hence, the space between the strokes is to their thickness, as 45° minus the angle of the slope is to the angle of the slope. Slopes

Fig. 455.



steeper than 45° are represented by short, heavy lines, parallel to the contour-lines, as shown in the upper right-hand corner of Fig. 455—a hill drawn by Lehmann's method.

644. Another Diapason of Tints:

Slope......	$2\frac{1}{2}°$	5°	10°	15°	25°	35°	45°	60°	75°
Black	1	2	3	4	5	6	7	8	9
White	10	9	8	7	6	5	4	3	2

This distinguishes gentle slopes better. It makes them darker, and the steeper slopes lighter, and provides for slopes beyond 45°. To use this standard, make it on the edge of a strip of paper, and apply that to the map in various parts, and draw a few lines corresponding to the slope of those parts; then fill up the intervening portions with suitable gradations. The angle of the slope is known from the map, since its tangent equals the vertical distance between the contours, divided by the horizontal distance. A scale can be made for any given vertical distance.

FOURTH SYSTEM.

BY SHADES PRODUCED BY OBLIQUE LIGHT.

645. Light is supposed to fall from the upper left-hand corner, as in drawing an "elevation," although the map is in plan. Then slopes facing the light will have a light tint, and those on the opposite side a dark tint.

This is picturesque, but not precise. It gives apparent "relief" to the ground, but does not show the degree of steepness.

The shades may be produced, as in the last method, by any means—tints, contours, or hatchings.

By making a map with contour-lines, and shaded obliquely, it will be both effective and precise.

CHAPTER IV.

CONVENTIONAL SIGNS.

646. Signs for Natural Surface. *Sand* is represented by fine dots made with the point of the pen ; *gravel,* by coarser dots. *Rocks* are drawn in their proper places, in irregular angular forms, imitating their true appearance as seen from above. The nature of the rocks, or the *geology* of the country, may be shown by applying the proper colors, as agreed on by geologists, to the back of the map, so that they may be seen by holding it up against the light, while they will thus not confuse the usual details.

647. Signs for Vegetation. *Woods* are represented by scalloped circles, irregularly disposed, imitating trees seen "in plan," and closer or farther apart according to the thickness of the forest (Fig. 456). It is usual to shade their lower and right-hand sides, and to represent their shadows, as in the figure, though, in strictness, this is inconsistent with the hypothesis of vertical light,

FIG. 456.

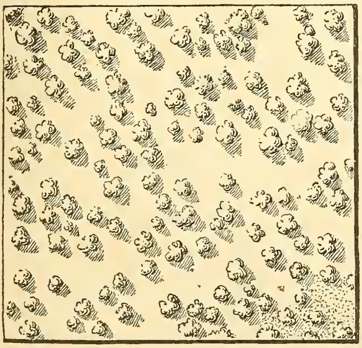
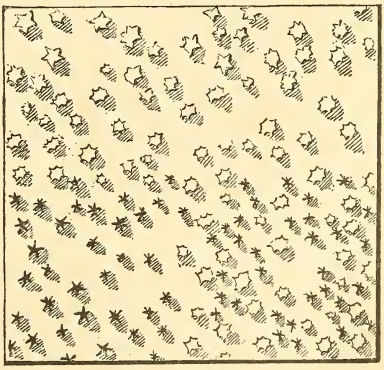


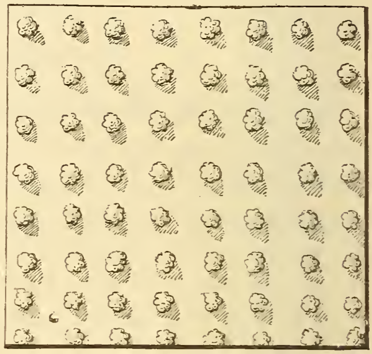
FIG. 457.



usually adopted for "hill-drawing." For pine and similar forests, the signs may have a star-like form, as in the lower part of Fig. 457. When it is desired to distinguish deciduous trees, they are represented as in the upper part of Fig. 457. Trees are sometimes drawn "in elevation," or sidewise, as usually seen (Fig. 458). This makes them more easily recognized, but is in

Fig. 458.

Fig. 459.



utter violation of the principles of mapping in horizontal projection, though it may be defended as a pure convention. *Orchards* are represented by trees arranged in rows (Fig. 459). *Bushes* may be drawn like trees, but smaller. Fig. 460 represents trees and bushes intermingled.

Grass-land is drawn with irregularly scattered groups of short lines, as in Fig. 461, the lines being arranged in odd numbers, and

Fig. 460.

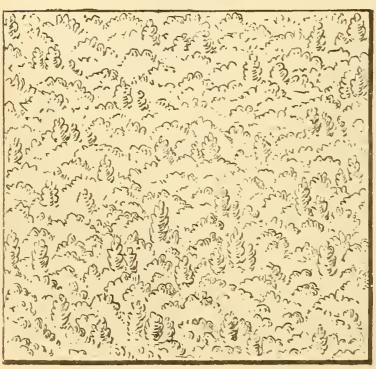


Fig. 461.

so that the top of each group is convex, and its bottom horizontal or parallel to the base of the drawing. *Meadows* are sometimes represented by pairs of diverging lines which may be regarded as tall blades of grass. *Uncultivated* land is indicated by appropriately intermingling the signs for grass-land, bushes, sand, and rocks. *Cultivated* land is shown by parallel rows of broken and dotted lines, as in the figure, representing furrows. In Fig. 462 is represented on the right cultivated land with fences, and on the

Fig. 462.

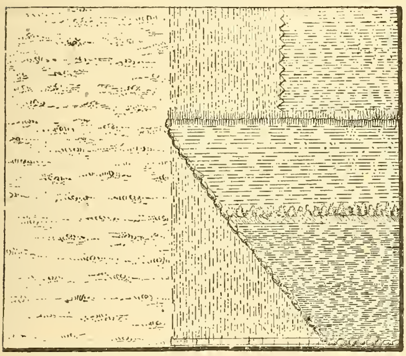
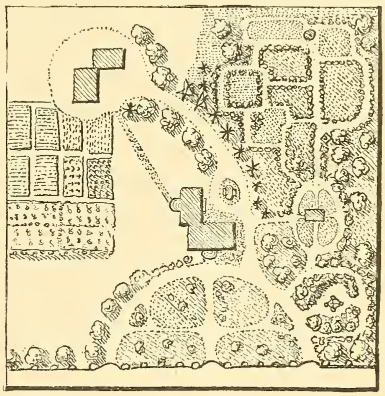


Fig. 463.



left, uncultivated land or "common." *Crops* are so temporary that signs for them are unnecessary, though often used. They are usually imitative, as for cotton, sugar, tobacco, rice, vines, hops,

Fig. 464.

Fig. 465.

Fig. 466.

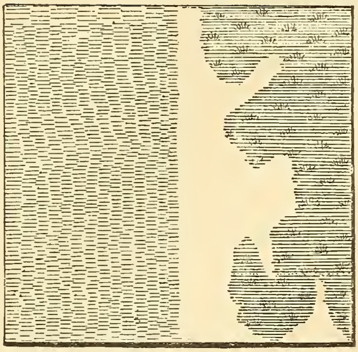
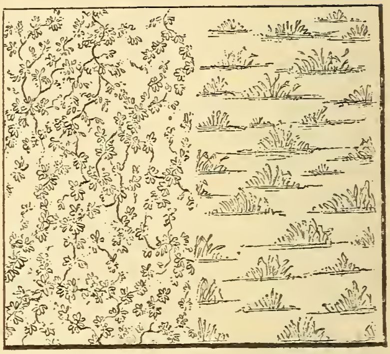


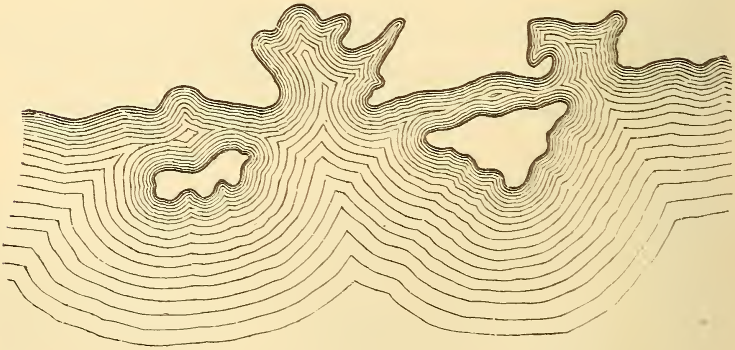
Fig. 467.



etc. *Gardens* are drawn with circular and other beds and walks. Fig. 463 represents a house with grounds.

648. Signs for Water. The *sea-coast* is represented by drawing a line parallel to the shore, following all its windings and indentations, and as close to it as possible ; then another parallel line a little more distant ; then a third still more distant, and so on, as in Fig. 468. If these lines are drawn from the low-tide mark, a

Fig. 468.

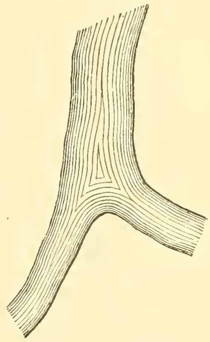


similar set may be drawn between that and the high-tide mark, and dots, for sand, be made over the included space. Fig. 464 represents a sea-coast with rocks and reefs.

Rivers have each shore treated like the sea-shore, as in Fig. 469.

Brooks would be shown by only two lines, or one, according to their magnitude. *Ponds* may be drawn like sea-shores, or represented by parallel horizontal lines ruled across them. *Marshes* and *swamps* are represented by an irregular intermingling of the preceding sign with that for grass and bushes. Fig. 465 represents a fresh-water marsh. Fig. 466 represents a salt marsh on the right and mud on the left. Fig. 467 represents osier-beds on the right, and mangrove on the left.

FIG. 469.



649. Colored Topography. The conventional signs which have been described, as made with the pen, require much time and labor. Colors are generally used by the French as substitutes for them, and combine the advantages of great rapidity and effectiveness. Only three colors (besides Indian-ink) are required, viz., gamboge (yellow), indigo (blue), and lake (scarlet); sepia, burnt sienna, yellow ochre, red-lead, and vermilion, are also sometimes used. The last three are difficult to work with. To use these paints, moisten the end of a cake and rub it up with a drop of water, afterward diluting this to the proper tint, which should always be light and delicate. To cover any surface with a uniform flat tint, use a large camel's-hair or sable brush, keep it always moderately full, incline the board toward you, previously moisten the paper with clean water if the outline is very irregular, begin at the top of the surface, apply a tint across the upper part, and continue it downward, *never letting the edge dry.* This last is the secret of a smooth tint. It requires rapidity in returning to the beginning of a tint to continue it, and dexterity in following the outline. *Marbling,* or variegation, is produced by having a brush at each end of a stick, one for each color, and applying first one, and then the other, beside it before it dries, so that they may blend, but not mix, and produce an irregularly clouded appearance. Scratched parts of the paper may be painted over by first applying strong alum-water to the place.

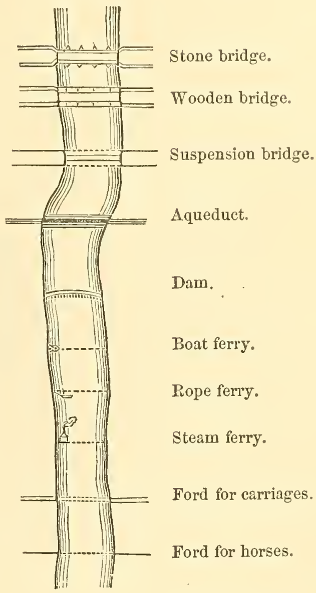
The conventions for colored topography, adopted by the French military engineers, are as follows: WOODS, *yellow;* using gamboge and a very little indigo. GRASS-LAND, *green;* made of gamboge and indigo. CULTIVATED LAND, *brown;* lake, gamboge, and a little Indian-ink; "burnt sienna" will answer. Adjoining fields should be slightly varied in tint. Sometimes furrows are indicated by strips of various colors. GARDENS are represented by small rectangular patches of brighter *green* and *brown.* UNCULTIVATED LAND, marbled *green* and light *brown.* BRUSH, BRAMBLES, etc., marbled *green* and *yellow.* HEATH, FURZE, etc., marbled *green* and *pink.* VINEYARDS, *purple;* lake and indigo. SANDS, a light *brown;* gamboge and lake; "yellow ochre" will do. LAKES and RIVERS, light *blue,* with a darker tint on their upper and left-hand sides. SEAS, dark *blue,* with a little yellow added. MARSHES, the *blue* of water, with spots of grass, *green,* the touches all lying horizontally. ROADS, *brown;* between the tints for sand and cultivated ground, with more Indian-ink. HILLS, *greenish-brown;* gamboge, indigo, lake, and Indian-ink. WOODS may be finished up by drawing the trees and coloring them green, with touches of gamboge toward the light (the upper and left-hand side), and of indigo on the opposite side.

650. Signs for Miscellaneous Objects. Too great a number of these will cause confusion. A few leading ones will be given:

Signal of survey,		FIG. 470	*Saw-mill,*		FIG. 479
Telegraph,		" 471	*Wind-mill,*		" 480
Court-house,		" 472	*Steam-mill,*		" 481
Post-office,		" 473	*Furnace,*		" 482
Tavern,		" 474	*Woolen-factory,*		" 483
Blacksmith's shop,		" 475	*Cotton-factory,*		" 484
Guide-board,		" 476	*Glass-works,*		" 485
Quarry,		" 477	*Church,*		" 486
Grist-mill,		" 478	*Graveyard,*		" 487

An ordinary house is drawn in its true position and size, and the ridge of its roof shown, if the scale of the map is large enough. On a very small scale, a small shaded rectangle represents it. If colors are used, buildings of masonry are tinted a deep crimson (with lake), and those of wood with Indian-ink. Their lower and right-hand sides are drawn with heavier lines. Fences of stone or wood, and hedges, may be drawn in imitation of the realities; and, if desired, colored appropriately.

FIG. 488.



Mines may be represented by the signs of the planets, which were anciently associated with the various metals. The signs here given represent respectively:

Gold.	Silver.	Iron.	Copper.	Tin.	Lead	Quicksilver.
☉	☽	♂	♀	♃	♄	☿

A large black circle, ●, may be used for coal.

Boundary-lines, of private properties, of townships, of counties, and of States, may be indicated by lines formed of various combinations of short lines, dots, and crosses, as below:

— —

. . . .

—.—.—.—.—.—.—.—.—.—.—.—.—.—.—.—.—.—.

+ +

651. Scales. The scale to which a topographical map should be drawn depends on several considerations. The principal ones are these: It should be large enough to express all necessary details, and yet not so large as to be unwieldy. The scale should be so chosen that the dimensions measured on the ground can be easily

Fig. 489.

A Topographical Drawing of Eagle Cliff, by E. Hergesheimer, Assistant, United States Coast Geodetic Survey. Scale $\frac{1}{10000}$.

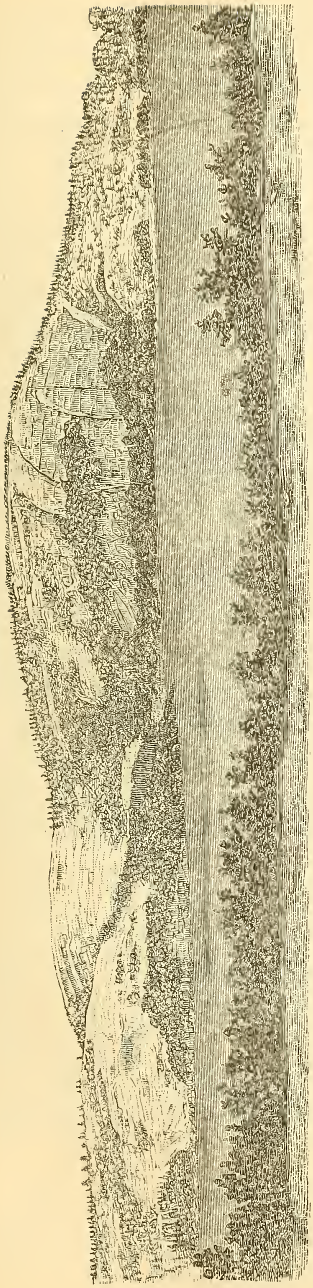


FIG. 490.

A View of Eagle Cliff, Mt. Desert Island (looking West).

converted, without calculation, into the corresponding dimensions on the map. (See "Scales," Part I.)

For specimens of topographical drawing, see Enthoffer's "Topography," and "United States Coast and Geodetic Survey Reports."

THE PLANE-TABLE.

652. The Plane-Table is in substance merely a drawing-board fixed on a tripod, so that lines may be drawn on it by a ruler placed so as to point to any object in sight. All its parts are mere additions to render this operation more convenient and precise.*

Such an arrangement may be applied to any kind of "Angular Surveying," such as the Third Method, "Polar Surveying," in its two modifications of *Radiation* and *Progression*, and the Fourth Method, by *Intersections*. Each of these will be successively explained. The instrument is very convenient for filling in the details of a survey, when the principal points have been determined by the more

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* The Plane-Table is not a *Goniometer*, or *Angle-measurer*, like the compass, transit, etc., but a *Gonigraph*, or *Angle-drawer*.

precise method of "Triangular Surveying," and can then be platted on the paper in advance. It has the great advantage of dispensing with all notes and records of the measurements, since they are platted as they are made. It thus saves time and lessens mistakes, but is wanting in precision.

653. The Table. It is usually a rectangular board of well-seasoned pine, about twenty inches wide and thirty long. The paper to be drawn upon may be attached to it by drawing-pins, or by clamping-plates fixed on its sides for that purpose, or by springs pressed upon it, or it may be held between rollers at opposite sides of the table. Tinted paper is less dazzling in the sun. Cugnot's joint, or a pair of parallel plates, like those of the transit, may be used for connecting it with its tripod. A detached level is placed on the board to test its horizontality; though a smooth ball, as a marble, will answer the same purpose approximately.

A pair of sights, like those of the compass, are sometimes placed under the board, serving, like a "watch-telescope," to detect any movement of the instrument. To find what point on the lower side of the board is exactly under a point on the upper side, so that by suspending a plumb-line from the former the latter may be exactly over any desired point of ground, a large pair of "callipers," or dividers with curved legs, may be used, one of their points being placed on the upper point of the board, and their other point then determining the corresponding under point; or a frame forming three sides of a rectangle, like a slate-frame, may be placed so that one end of one side of it touches the upper point, and the end of the corresponding side is under the table precisely below the given point, so that from this end a plumb-line can be dropped. A compass is sometimes attached to the table, or a detached compass, consisting of a needle in a narrow box (called a Declinator), is placed upon it, as desired. The edges of the table are sometimes divided into degrees, like the "Drawing-board Protractor." It then becomes a sort of goniometer.

654. The Alidade. The ruler has a fiducial or feather edge, which may be divided into inches, tenths, etc. At each end it carries a sight like those of the compass. Two needles would be tolerable substitutes. The sights project beyond its edge so that their center lines shall be precisely in the same vertical plane as this edge, in order that the lines drawn by it may correspond to the lines sighted on by them. To test this, fix a needle in the board, place the alidade against it, sight to some near point, draw a line by the ruler, turn it end for end, again place it against the needle, again sight to the same point, and draw a new line. If it coincides with the former line, the above condition is satisfied. The ruler and sights together take the name of *Alidade.* If a point should be too high or too low to be seen with the alidade, a plumb-line, held between the eye and the object, will remove the difficulty.

A telescope is sometimes substituted for the sights, being supported above the ruler by a standard, and capable of pointing upward or downward. It admits of adjustments similar in principle to the second and third adjustments of the transit.

But even without these adjustments, whether of the sights or of the telescope, a survey could be made which would be perfectly correct as to the relative position of its parts, however far the line of sight might be from lying in the same vertical plane as the edge of the ruler, or even from being parallel to it ; just as in the transit or theodolite the index or vernier need not to be exactly under the vertical hair of the telescope, since the angular deviation affects all the observed directions equally.

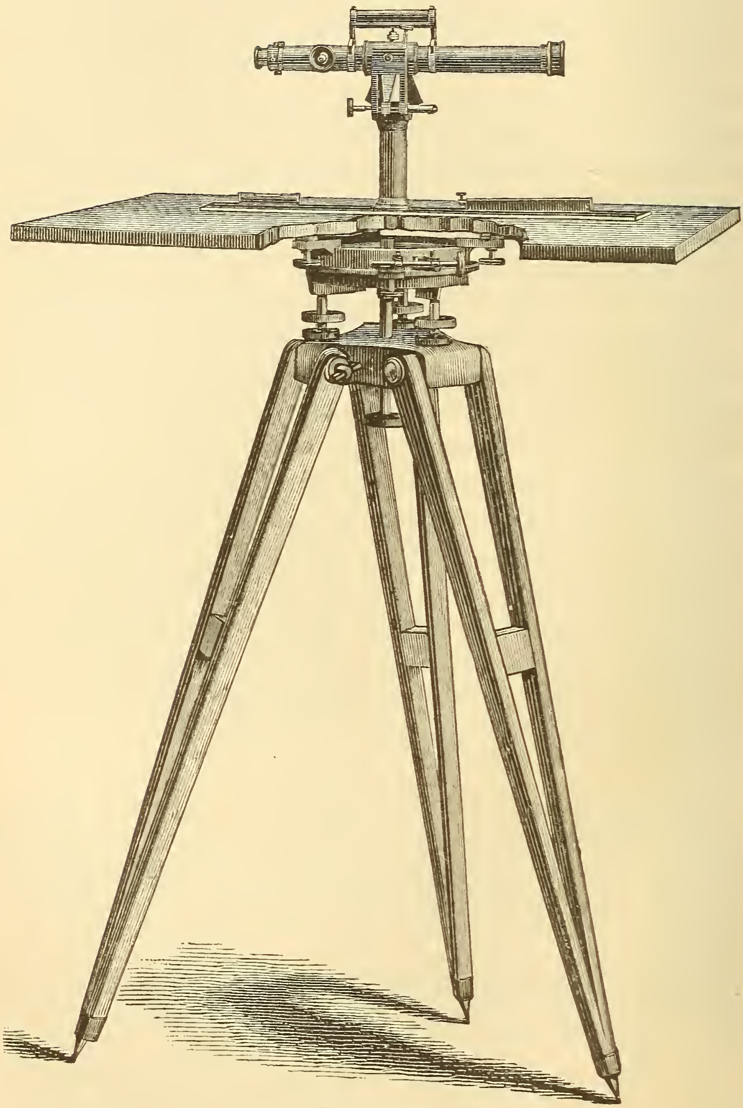
655. The plane-table shown in Fig. 491 is one of the standard forms.* The table is leveled by means of three leveling-screws, and tested by a spirit-level on the alidade. The telescope of the alidade is "transit-mounted"—that is, has both ends of the axis supported.

Distances may be determined by means of stadia-wires placed in the telescope, and heights by means of the vertical arc.

656. Method of Radiation. This is the simplest, though not the best, method of surveying with the plane-table. It is especially

* Manufactured by Fauth & Co., Washington, D. C.

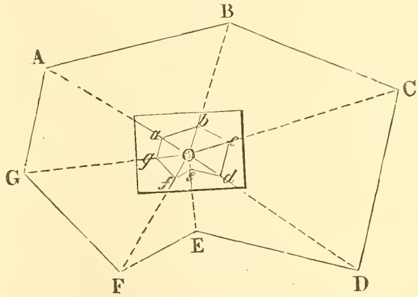
Fig. 491.



Plane-Table.

applicable to surveying a field, as in the figure. In it and the following figures, the size of the table is much exaggerated. Set the instrument at any convenient point, as O; level it, and fix a needle (having a head of sealing-wax) in the board to represent the station. Direct the alidade to any corner of the field, as A, the fiducial edge of the ruler touching the needle, and draw an indefinite line by it. Measure O A, and set off the distance, to any desired scale, from the needle-point, along the line just drawn, to *a*. The line O A is thus platted on the paper of the table as soon as determined in the field. Determine and plat in the same way, O B, O C, etc., to *b*, *c*, etc. Join *a b*, *b c*, etc., and a complete plat of the field is obtained. Trees, houses, hills, bends of rivers, etc., may be determined in the same manner. The corresponding method with the compass or transit has been described. The table may be set at one of the angles of the field, if more convenient. If the alidade has a telescope, the method of measuring distances with a stadia may be here applied with great advantage.

Fig. 492.



657. Method of Progression. Let A B C D, etc., be the line to be surveyed. Fix a needle at a convenient point of the plane-table, near a corner so as to leave room for the plat, and set up the table at B, the second angle of the line, so that the needle, whose point represents B, and which should be named *b*, shall be exactly over that station. Sight to A, pressing the fiducial edge of the ruler against the needle, and draw a line by it. Measure B A, and set off its length, to the desired scale, on the line just drawn, from *b* to a point *a*, representing A. Then sight to C, draw an indefinite line by the ruler, and on it set off the length of B C from *b* to *c*. Fix the needle at *c*. Set up at C, the point *c* being over this station, and make the line *c b* of the plat coincide in direction with

C B on the ground, by placing the edge of the ruler on *c b*, and turning the table till the sights point to B. The compass, if the

FIG. 493.

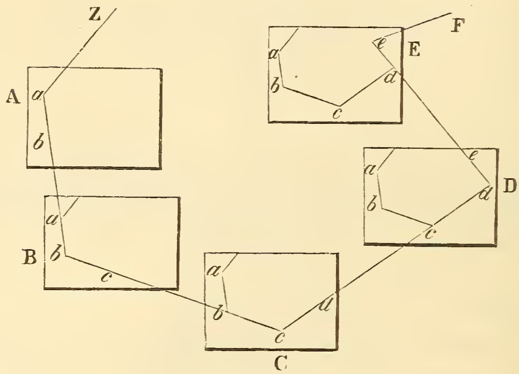


table have one, will facilitate this. Then sight forward from C to D, and fix C D, *c d* on the plat, as *b c* was fixed. Set up at D, make *d c* coincide with D C, and proceed as before. The figure shows the lines drawn at each successive station. The table drawn at A shows how the survey might be commenced there.

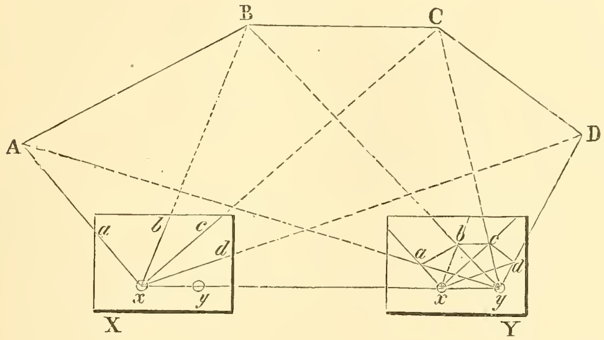
In going around a field, the work would be proved by the last line "closing" at the starting-point; and, during the progress of the survey, by any direction, as from C to A on the ground, coinciding with the corresponding line, *c a*, on the plat.

This method is substantially the same as the method of surveying a line with the transit. It requires all the points to be accessible. It is especially suited to the survey of a road, a brook, a winding path through woods, etc. The offsets required may often be sketched in by the eye with sufficient precision.

When the paper is filled, put on a new sheet, and begin by fixing on it two points, such as C and D, which were on the former sheet, and from them proceed as before. The sheets can then be afterward united, so that all the points on both shall be in their true relative positions.

658. Method of Intersection. This is the most usual and the most rapid method of using the plane-table. Set up the instrument at any convenient point, as X in the figure, and sight to all the desired points, A, B, C, etc., which are visible, and draw in-

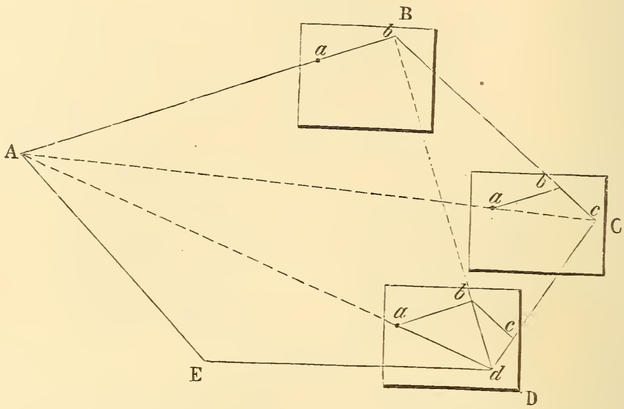
Fig. 494.



definite lines in their directions. Measure any line X Y, Y being one of the points sighted to, and set off this line on the paper to any scale. Set up at Y, and turn the table till the line X Y on the paper lies in the direction of X Y, on the ground, as at C in the last method. Sight to all the former points and draw lines in their directions, and the intersections of the two lines of sight to each point will determine them, by the Fourth Method. Points on the other side of the line X Y could be determined at the same time. In surveying a field, one side of it may be taken for the base X Y. Very acute or obtuse intersections should be avoided—30° and 150° should be the *extreme* limits. The impossibility of always doing this renders this method often deficient in precision. When the paper is filled, put on a new sheet, by fixing on it two known points, as in the preceding method.

659. Method of Resection. This method (called by the French *Recoupement*) is a modification of the preceding method of intersection. It requires the measurement of only one distance, but all the points must be accessible. Let A B be the measured distance. Lay it off on the paper as *a b*. Set the table up at B, and turn it till the line *b a* on the paper coincides with B A on the ground, as in the Method of Progression. Then sight to C, and draw an indefinite line by the ruler. Set up at C, and turn the line last drawn so as to point to B. Fix a needle at a on the table, place the alidade against the needle and turn it till it sights to A. Then

FIG. 495.



the point in which the edge of the ruler cuts the line drawn from B will be the point c on the table. Next sight to D, and draw an indefinite line. Set up at D, and make the line last drawn point to C. Then fix the needle at a or b, and by the alidade, as at the last station, get a new line back from either of them, to cut the last-drawn line at a point which will be d. So proceed as far as desired.

660. To orient the Table.* The operation of orientation consists in placing the table at any point so that its lines shall have the same directions as when it was at previous stations in the same survey.

With a compass this is very easily effected by turning the table till the needle of the attached compass, or that of the declinator, placed in a fixed position, points to the same degree as when at the previous station.

Without a compass the table is oriented, when set at one end of a line previously determined, by sighting back on this line, as at C in the Method of Progression.

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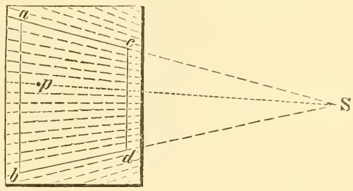
* The French phrase, to "*orient* one's self," meaning to determine one's position, usually with respect to the four quarters of the heavens, of which the Orient is the leading one, well deserves naturalization in our language.

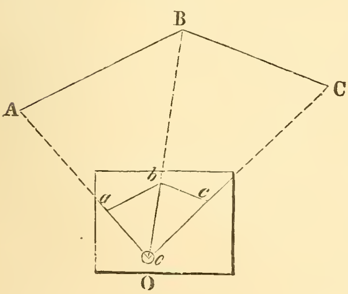
To orient the table, when at a station unconnected with others, is more difficult. It may be effected thus: Let *a b* on the table represent a line A B on the ground. Set up at A, make *a b* coincide with A B, and draw a line from *a* directed toward a steeple, or other conspicuous object, as S. Do the same at B. Draw a line *c d*, parallel to *a b*, and intercepted between *a* S, and *b* S. Divide *a b* and *c d* into the same number of equal parts. The table is then prepared. Now let there be a station, P, *p* on the table, at which the table is to be oriented. Set the table, so that *p* is over P, apply the edge of the ruler to *p*, and turn it till this edge cuts *c d* in the division corresponding to that in which it cuts *a b*. Then turn the table till the sights point to S, and the table will be oriented.

Fig. 496.



661. To Find One's Place on the Ground. This problem may be otherwise expressed as interpolating a point in a plat. It is most easily performed by reversing the Method of Intersection. Set up the table over the station, O in the figure, whose place on the plat already on the table is desired, and *orient* it, by one of the means described in the last article. Make the edge of the ruler pass through some point, *a* on the table, and turn it till the sights point to the corresponding station, A on the ground. Draw a line by the ruler. The desired point is somewhere in this line. Make the ruler pass through another point, *b* on the table, and make the sights point to B on the ground. Draw a second line, and its intersection with the first will be the point desired. Using C in the same way would give a third line to prove the work. This operation may be used as a

Fig. 497.



new method of surveying with the plane-table, since any number of points can have their places fixed in the same manner.

This problem may also be executed without orientation on the principle of trilinear surveying. Three points being given on the table, lay on it a piece of transparent paper, fix a needle anywhere on this, and with the alidade sight and draw lines toward each of these three points on the ground. Then use this paper to find the desired point, precisely as directed in the last sentence of Art. 720, page 487.

When it is desired to set up the plane-table at some undetermined point, not connected by known lines with any other point in the survey, and the table can be readily only approximately oriented, the table may be accurately oriented and the point be determined by means of the "three-point problem." For the solution of this problem, and for treatise on the plane-table, see "United States Coast and Geodetic Survey Report," 1880, Appendix XIII.

662. Inaccessible Distances. Many of the problems in Part I, Chapter V, can be at once solved on the ground by the plane-table, since it is at the same time a goniometer and a protractor. Thus, the Problem of Art. 385 may be solved as follows, on the principle of the construction in the last paragraph of that article: Set the table at C. Mark on it a point, c', to represent C, placing c' vertically over C. Sight to A, B, and D, and draw corresponding lines from c'. Set up at D, mark any point on the line drawn from c' toward D, and call it d'. Let d' be exactly over D, and direct $d'c'$ toward C. Then sight to A and B, and draw corresponding lines, and their intersections with the lines before drawn toward A and B will fix points a' and b'. Then on the line joining a and b, given on the paper to represent A and B, $a\,b$ being equal to A B on any scale, construct a figure, $a\,b\,c\,d$, similar to $a'\,b'\,c'\,d'$, and the line $c\,d$ thus determined will represent C D on the same scale as A B.

663. Contouring with the Plane-Table. It is used to map the points on the contour-lines as soon as obtained, thus: Range out an approximately level line, and on it set equidistant stakes. At these stakes range out perpendiculars to the line, and set up several stakes on them for the alignment of the rodman. Draw these lines on the plane-table. Set up and "orient" the table on the ground. Send the rod along one of the perpendiculars till it comes to a point of the right height. Then sight to it with the alidade, and its edge will cut the corresponding line on the table at the correct place on the plat. So for the other perpendiculars.

PART IV.

TRIANGULAR SURVEYING:

OR

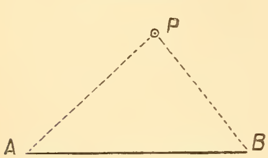
By the Fourth Method.

CHAPTER I.

PLANE SURFACES.

664. TRIANGULAR SURVEYING is founded on the method of determining the position of a point by the intersection of two known lines. Thus, the point P is determined by knowing the length of the line A B, and the angles P B A and P A B, which the lines P A and P B make with A B. By an extension of the principle, a field, a farm, or a country, can be surveyed by measuring only one line, and calculating all the other desired distances, which are made sides of a connected series of imaginary *triangles*, whose angles are carefully measured. The district surveyed is covered with a sort of network of such triangles, whence the name given to this kind of surveying. It is more commonly called "Trigonometrical Surveying," and sometimes "Geodesic Surveying," but improperly, since it does not necessarily take into account the curvature of the earth, though always adopted in the great surveys in which that is considered.

FIG. 498.



665. Outline of Operations. A *base-line*, as long as possible (five or ten miles in surveys of countries), is measured with extreme accuracy.

From its extremities, angles are taken to the most distant objects visible, such as steeples, signals on mountain-tops, etc.

The distances to these and between these are then calculated by the rules of trigonometry.

The instrument is then placed at each of these new stations, and angles are taken from them to still more distant stations, the calculated lines being used as new base-lines.

This process is repeated and extended till the whole district is embraced by these "primary triangles" of as large sides as possible.

One side of the last triangle is so located that its length can be obtained by measurement as well as by calculation, and the agreement of the two proves the accuracy of the whole work.

Within these primary triangles, *secondary* or smaller triangles are formed, to fix the position of the minor local details, and to serve as starting-points for common surveys with chain and compass, etc. Tertiary triangles may also be required.

The larger triangles are first formed, and the smaller ones based on them, in accordance with the important principle in all surveying operations, always to work from the whole to the parts, and from greater to less.

666. Measuring a Base. Extreme accuracy in this is necessary, because any error in it will be *multiplied* in the subsequent work. The ground on which it is located must be smooth and nearly level, and its extremities must be in sight of the chief points in the neighborhood. Its point of beginning must be marked by a stone set in the ground with a bolt let into it. Over this a theodolite or transit is to be set, and the line "ranged out." The measurement may be made with chains, steel tapes, etc., or with rods.

667. Measuring a Base with Rods. We will notice, in turn, their *materials*, *supports*, *alignment*, *leveling*, and *contact*.

As to *materials*, iron, brass, and other metals, have been used, but are greatly lengthened and shortened by changes of temperature. Wood is affected by moisture. Glass rods and tubes are preferable on both these accounts; but wood is the most convenient. Wooden rods should be straight-grained white pine, etc., well seasoned, baked, soaked in boiling oil, painted, and varnished. They may be trussed, or framed like a mason's plumb-line level, to prevent their bending. Ten or fifteen feet is a convenient length. Three are required, which may be of different colors, to prevent mistakes in recording. They must be very carefully compared with a standard measure.

Supports must be provided for the rods, in accurate work. Posts, set in line at distances equal to the length of the rods, may be driven or sawed to a uniform line, and the rods laid on them, either directly or on beams a little shorter. Tripods or trestles, with screws in their tops to raise or lower the ends of the rods resting on them, or blocks with three long screws passing through them and serving as legs, may also be used. Staves, or legs, for the rods have been used, these legs bearing pieces which can slide up and down them, and on which the rods themselves rest.

The *alignment* of the rods can be effected if they are laid on the ground, by strings, two or three hundred feet long, stretched between the stakes set in the line, a notched peg being driven when the measurement has reached the end of one string, which is then taken on to the next pair of stakes; or, if the rods rest on supports, by projecting points on the rods being aligned by the instrument.

The *leveling* of the rods can be performed with a common mason's level; or their angle measured, if not horizontal, by a "slope-level."

The *contacts* of the rods may be effected by bringing them end to end. The third rod must be applied to the second before the first has been removed, to detect any movement. The ends must be protected by metal, and should be rounded (with radius equal to length of rod), so as to touch in only one point. Round-headed nails will answer tolerably. Better are small steel cylinders, horizontal on one end and vertical on the other. Sliding ends, with verniers, have been used. If one rod be higher than the next one, one must be brought to touch a plumb-line which touches the other, and its thickness be added. To prevent a shock from contact, the rods may be brought not quite in contact, and a wedge be let down between them till it touches both at known points on its graduated edges. The rods may be laid side by side, and lines drawn across the end of each be made to coincide or form one line. This is more accurate. Still better is a "visual contact," a double microscope with cross-hairs being used, so placed that one tube bisects a dot at the end of one rod, and the other tube bisects a dot at the end of the next rod. The rods thus never touch. The distance between the two sets of cross-hairs is of course to be added.

A base could be measured over very uneven ground, or even water, by suspending a series of rods from a stretched rope by rings in which they can move, and leveling them and bringing them into contact as above.

668. Measuring a Base with a Steel Tape. The tape should be from two hundred to five hundred feet long, furnished at one end with a spring-balance for determining the pull on the tape when measuring. It should be tested under the same conditions in which it is to be used—that is, supported at points from ten to twenty-five feet apart, and subjected to a pull of from ten to twenty pounds. The temperature at which the test is made should be noted.

Let us suppose that the tape was tested, resting on supports twenty feet apart, and under a pull of fifteen pounds.

To measure the base, drive stakes along the base-line twenty feet apart, and with one face in line. Drive nails in the lined face of the stakes at the same level, or on an even grade if the ground is not level.

Set a post solidly in the ground at each tape-length along the line, so that the top of the post shall be at the height at which the tape is to be held.

Place the tape on the nails in the stakes, or, better still, on hooks swinging from the nails, and apply a pull of fifteen pounds, bringing the ends of the tape over the posts. Hold the first graduation of the tape over the starting-point on the first post, and mark where the last graduation comes on the second post, by making a line on the head of a copper tack driven into the post, or on a piece of metal fastened on the top of the post. Bring the first graduation on the tape to the mark on the second post, and mark the place of the last graduation on the third post. So proceed for the whole length of the line.

A steel tape will expand ·000007 of its length for each degree (Fahr.) of rise in temperature. The temperature should be carefully noted when the measurement is made, and the proper correction applied.

The measurement is best made on a still, cloudy day.

If the measured line be on a slope, its measured length must be multiplied by the cosine of the angle of inclination, to reduce it to the horizontal distance between its extremities.

669. Corrections of Base. If the rods were not level, their length must be reduced to its horizontal projection. This would be the square root of the difference of the squares of the length of the rod (or of the base) and of the height of one end above the other; or the product of the same length by the cosine of the angle which it makes with the horizon.*

If the rods were metallic, they would need to be corrected for temperature. Thus, if an iron bar expands $\frac{7}{1000000}$ of its length for 1° Fahrenheit, and had been tested at 32°, and a base had been measured at 72° with such a bar ten feet long, and found to contain 3,000 of them, its apparent length would be 30,000 feet, but its real length would be 8·4 feet more. An iron and a brass bar can be so combined that the difference of their expansion causes two points attached to their ends to remain at the same distance at all temperatures. Such a combination is used on the United States Coast Survey.

Expansion for 1° *Fahrenheit.*

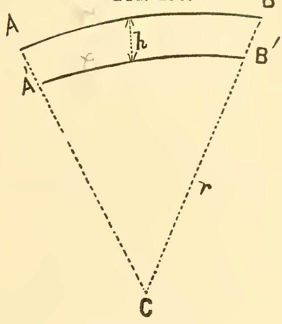
Brass bar = 0·00001050903;
Iron bar = 0·000006963535;
Platinum = 0·0000051344;
Glass = 0·0000043119;
White-pine = 0·0000022685.

670. Reducing the Base to the Level of the Sea. Let A B = *a*

* More precisely, A being this angle, and not more than 2° or 3°, the difference between the inclined and horizontal lengths equals the inclined or real length multiplied by the square of the minutes in A, and that by the decimal 0·00000004231.

be the measured base, and $A' B' = x$, the base reduced to the level of the sea, h the height of the measured base above the level of the sea, and r the radius of the earth to the level of the sea. Then we have :

Fig. 499.



$$r + h : r :: a : x.$$

$$\therefore\ x = a \frac{r}{r+h}.$$

$$a - x = \frac{a\,h}{r+h} = \frac{\frac{a\,h}{r}}{1+\frac{h}{r}} = \frac{a\,h}{r}\left(1+\frac{h}{r}\right)^{-1}$$

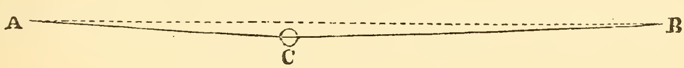
Developing by the binomial formula, we get :

$$a - x = a\frac{h}{r} - a\frac{h^2}{r^2} + a\frac{h^3}{r^3} -, \text{ etc.}$$

As h is very small in comparison with r, the first term of the correction is generally sufficient.

671. A Broken Base. When the angle C is very obtuse, the lines A C and C B being measured, and forming nearly a straight

Fig. 500.



line, the length of the line A B is found thus : Naming the lines, as is usual in trigonometry, by small letters corresponding to the capital letters at the angles to which they are opposite, and letting K = the number of minutes in the supplement of the angle C, we shall have :

$$A\,B = c = a + b - 0{\cdot}000000042308 \times \frac{a\,b\,K^2}{a+b}.$$

$$\text{Log. } 0{\cdot}000000042308 = 2{\cdot}6264222 - 10.$$

Proof. Art. 12, Theorem III [Trigonometry, Appendix A], gives, $\cos. C = \frac{a^2 + b^2 - c^2}{2\,a\,b}$; or $c^2 = a^2 + b^2 - 2\,a\,b\,.\cos. C$. This becomes [Trig., Art. 6], K being the supplement of C, $c^2 = a^2 + b^2 + 2\,a\,b\,.\cos. K$. The series [Trig., Art. 5] for the length of a cosine gives, taking only its first two terms, since K is very small, cos. $K = 1 - \frac{1}{2}K^2$. Hence,

$$c^2 = a^2 + b^2 + 2ab - abK^2 = (a+b)^2 - abK^2 = (a+b)^2\left(1 - \frac{abK^2}{(a+b)^2}\right);$$

whence, $$c = (a+b)\sqrt{\left(1 - \frac{abK^2}{(a+b)^2}\right)}.$$

Developing the quantity under the radical sign by the binomial theorem, and neglecting the terms after the second, it becomes

$$1 - \tfrac{1}{2}\,.\,\frac{abK^2}{(a+b)^2} +, \text{ etc.}$$

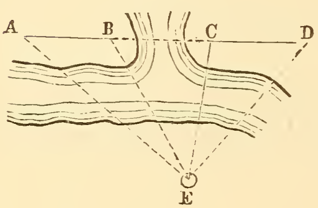
Substituting for K minutes, K . sin. 1′ [Trig., Art. 5], and performing the multiplication by $a + b$, we obtain

$$c = a + b - \frac{abK^2\,.\,(\text{sin. }1')^2}{2\,(a+b)}. \quad \text{Now, } \frac{(\text{sin. }1')^2}{2} = 0{\cdot}0000000423079;$$

whence the formula, $c = a + b - 0{\cdot}000000042308 \times \dfrac{abK^2}{a+b}$.

672. Problem to interpolate a Base. *Four inaccessible objects,* A, B, C, D, *being in a right line, and visible from only one point,* E, *it is required to determine the distance between the middle points,* B *and* C, *the exterior distances,* A B *and* C D, *being known.*

Fig. 501.



Let $AB = a$, $CD = b$, $BC = x$; $AEB = P$, $AEC = Q$, $AED = R$.

Calculate an auxiliary angle, K, such that

$$\text{tang.}^2\,K = \frac{4ab}{(a-b)^2}\,.\,\frac{\text{sin. }Q\,.\,\text{sin. }(R-P)}{\text{sin. }P\,.\,\text{sin. }(R-Q)}.$$

$$\text{Then is } x = -\frac{a+b}{2} \pm \frac{a-b}{2\,.\,\text{cos. }K}.$$

Of the two values of x, the positive one is alone to be taken.

This problem is used when a portion of a base-line passes over water, etc.

Proof. In Fig. 501, produce A D to some point F. The exterior angles, $EBC = A + P$; $ECD = A + Q$; $EDF = A + R$. The triangle A B E gives $\dfrac{BE}{a} = \dfrac{\text{sin. }A}{\text{sin. }P}$. The triangle A C E gives $\dfrac{CE}{a+x} = \dfrac{\text{sin. }A}{\text{sin. }Q}$.

Dividing member by member, we get $\dfrac{BE}{CE} = \dfrac{a\,.\,\text{sin. }Q}{(a+x)\,\text{sin. }P}$.

In the same way the triangle B E D and C E D give $\frac{\text{B E}}{b + x} = \frac{\sin.(\text{A} + \text{R})}{\sin.(\text{R} - \text{P})}$; and $\frac{\text{C E}}{b} = \frac{\sin.(\text{A} + \text{R})}{\sin.(\text{R} - \text{Q})}$. Whence as before, $\frac{\text{B E}}{\text{C E}} = \frac{(b + x)\sin.(\text{R} - \text{Q})}{b \,.\, \sin.(\text{R} - \text{P})}$.

Equating these two values of the same ratio, we get

$$\frac{a \,.\, \sin.\text{Q}}{(a + x)\sin.\text{P}} = \frac{(b + x)\sin.(\text{R} - \text{Q})}{b \,.\, \sin.(\text{R} - \text{P})}; \text{ and thence}$$

$$\frac{a\,b \,.\, \sin.\text{Q} \,.\, \sin.(\text{R} - \text{P})}{\sin.\text{P} \,.\, \sin.(\text{R} - \text{Q})} = (a + x)(b + x) = a\,b + (a + b)x + x^2.$$

To solve this equation of the second degree, with reference to x, make

$$\tan.^2 \text{K} = \frac{4\,a\,b}{(a - b)^2} \cdot \frac{\sin.\text{Q}(\sin.\text{R} - \text{P})}{\sin.\text{P}(\sin.\text{R} - \text{Q})}.$$

Then the first member of the preceding equation $= \frac{1}{4} \,.\, (a - b)^2 \times \tan.^2 \text{K}$, and we get $x^2 + (a + b)\,x = \frac{1}{4}(a - b)^2 \,.\, \tan{}^2 \text{K} - a\,b$,

and

$$\begin{aligned} x &= -\tfrac{1}{2}(a + b) \pm \sqrt{[\tfrac{1}{4}(a - b)^2 \,.\, \tan.^2 \text{K} - a\,b + \tfrac{1}{4}(a + b)^2]}, \\ &= -\tfrac{1}{2}(a + b) \pm \sqrt{[\tfrac{1}{4}(a - b)^2 \,.\, \tan.^2 \text{K} + \tfrac{1}{4}(a - b)^2]}, \\ &= -\tfrac{1}{2}(a + b) \pm \tfrac{1}{2}(a - b)\sqrt{(\tan.^2 \text{K} + 1)}. \end{aligned}$$

Or, since $\sqrt{(\tan.^2 \text{K} + 1)} = \text{secant K} = \frac{1}{\cos.\text{K}}$, we have $x = -\frac{a + b}{2} \pm \frac{a - b}{2 \,.\, \cos.\text{K}}$.

When $a = b$, or when the two known parts are equal to each other, the above solution is indeterminate. For this case put

$$\tan.^2 \text{K}' = \frac{a\,b\sin.\text{Q}\sin.(\text{R} - \text{P})}{\sin.\text{P} \,.\, \sin.(\text{R} - \text{Q})},$$

and the solution gives:

$$x = -\tfrac{1}{2}(a + b) \pm \sqrt{\tan.^2 \text{K}' + \frac{(a - b)^2}{4}}.$$

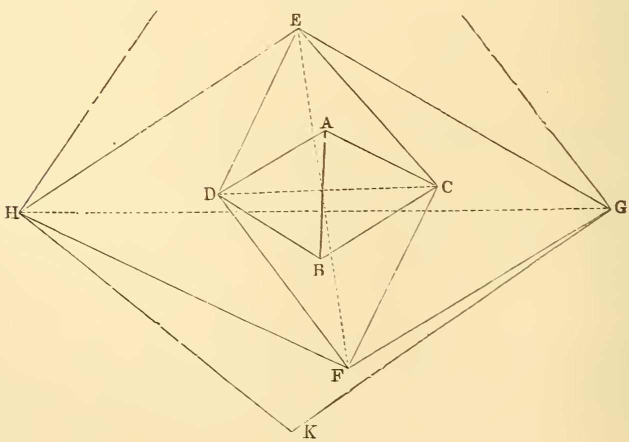
If $a = b$, this becomes:

$$x = -\tfrac{1}{2}(a + b) \pm \tan.\text{K}'.$$

673. Base of Verification. As mentioned in Art. 665, a side of the last triangle is so located that it can be measured, as was the first base. If the measured and calculated lengths agree, this proves the accuracy of all the previous work of measurement and calculation, since the whole is a chain of which this is the last link, and any error in any previous part would affect the very last line, except by some improbable compensation. How near the agreement should be, will depend on the nicety desired and attained in the previous operations. Two bases, 60 miles distant, differed on one great English survey 28 inches; on another, 1 inch; and on a French triangulation extending over 500 miles, the difference was less than 2 feet. Results of equal or greater accuracy are obtained on the United States Coast Survey. "The Fire Island base, on the south side of Long Island, and the Kent Island base in Chesapeake Bay, are connected by a primary triangulation. This Kent Island base is 5 miles and 4 tenths long, and the original Fire Island base is 8 miles and 7 tenths. The shortest distance between them is 208 miles, but the distance through the triangulation is 320. The number of intervening triangles is 32, yet the computed and measured lengths of the Kent Island base exhibit a discrepancy no greater than 4 inches."

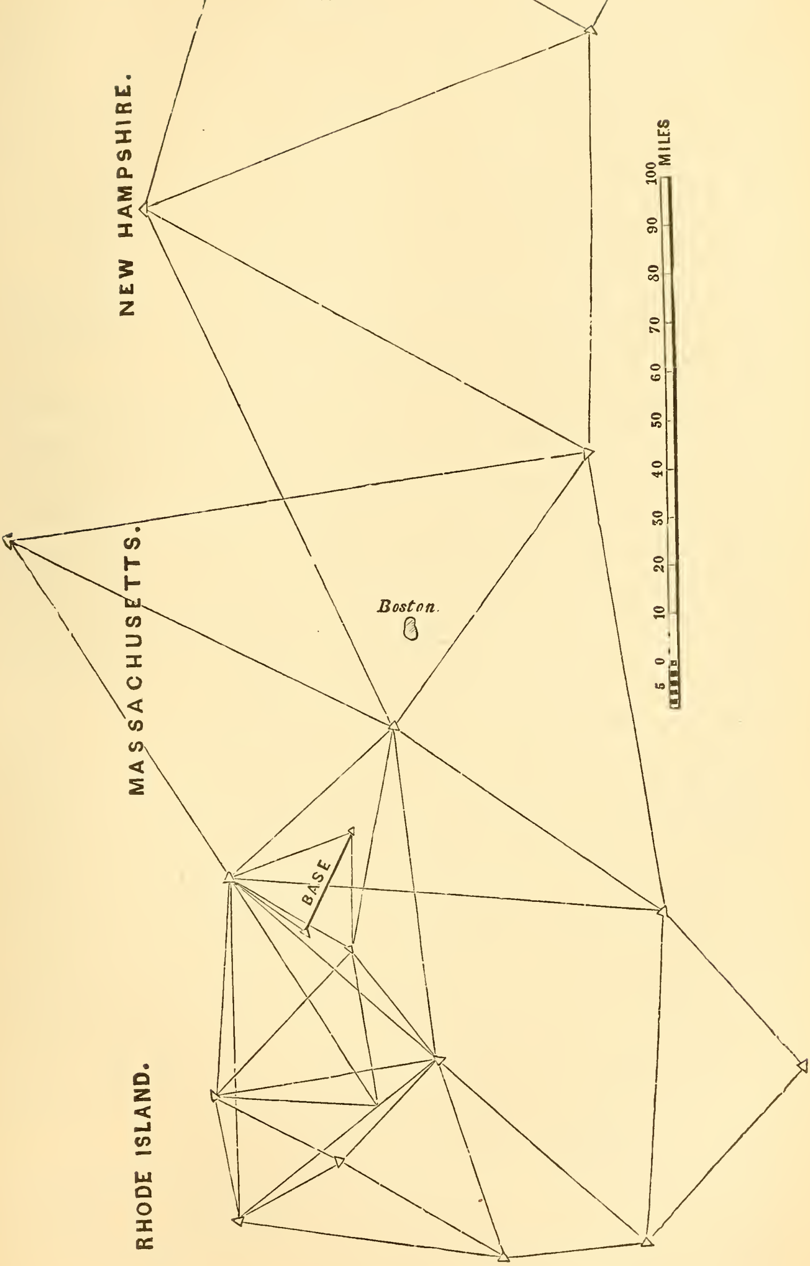
674. Choice of Stations. The stations, or "trigonometrical points," which are to form the vertices of the triangles, and to be observed to and from, must be so selected that the resulting triangles may be "well-conditioned"—i. e., may have such sides and angles that a small error in any of the measured quantities will cause the least possible errors in the quantities calculated from them. The higher calculus shows that the triangles should be as nearly equilateral as possible. This is seldom attainable, but no angle should be admitted less than 30°, or more than 120°. When two angles only are observed, as is often the case in the secondary

Fig. 502.



triangulation, the unobserved angle ought to be nearly a right angle.

FIG. 503.


NEW HAMPSHIRE.
MASSACHUSETTS.
RHODE ISLAND.
Boston.
BASE
MILES
100 90 80 70 60 50 40 30 20 10 0 5

To extend the triangulation, by continually increasing the sides of the triangles, without introducing "ill-conditioned" triangles, may be effected as in Fig. 502. A B is the measured base, C and D are the nearest stations. In the triangles A B C and A B D, all the angles being observed, and the side A B known, the other sides can be readily calculated. Then, in each of the triangles D A C and D B C, two sides and the contained angles are given to find D C, one calculation checking the other. D C then becomes a base to calculate E F, which is then used to find G H, and so on.

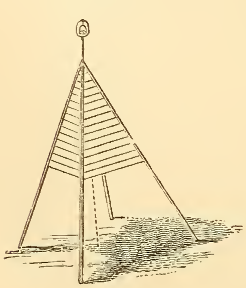
The fewer primary stations used the better, both to prevent confusion and because the smaller number of triangles makes the correctness of the results more "probable."

The United States Coast and Geodetic Survey displays some fine illustrations of these principles, and of the modifications they may undergo to suit various localities. Fig. 503 represents part of the scheme of the primary triangulation resting on the Massachusetts base, and including some remarkably well-conditioned triangles, as well as the system of quadrilaterals, which is a valuable feature of the scheme when the sides of the triangles are extended to considerable lengths, and quadrilaterals, with both diagonals determined, take the place of simple triangles.

The engraving is on a scale of 1 : 1,200,000.

675. Signals. They must be high, conspicuous, and so made that the instrument can be placed precisely under them.

Fig. 504.



Three or four timbers framed into a pyramid, as in Fig. 504, with a long mast projecting above, fulfill the first and last conditions. The mast may be made vertical by directing two theodolites to it, and adjusting it so that their telescopes follow it up and down, their lines of sight being at right angles to each other. Guy ropes may be used to keep it vertical.

Another form of signal is represented in the three following figures. It consists merely of three stout sticks, which form a tripod, framed with the signal-staff, by a bolt passing through their ends and its middle. Fig. 505 represents the signal as framed on the ground; Fig. 506 shows it erected and ready for observation, its base being steadied

Fig. 505.

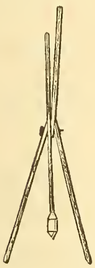


Fig. 506.

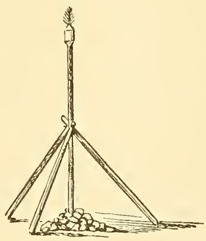
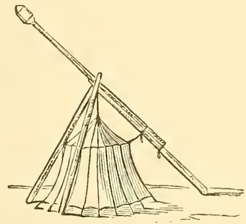


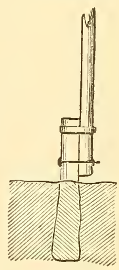
Fig. 507.



with stones; and Fig. 507 shows it with the staff turned aside, to make room for the theodolite and its protecting tent. The heights of these signals varied between fifteen and eighty feet.

Another good signal consists of a stout post let into the ground, with a mast fastened to it by a bolt below and a collar above. By opening the collar, the mast can be turned down and the theodolite set exactly under the former summit of the signal, i. e., in its vertical axis.

Fig. 508.

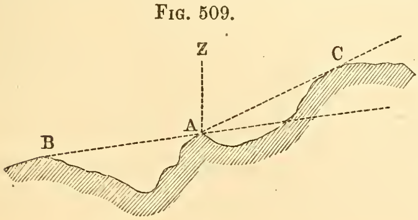


A tripod of gas-pipe has been used to support the signal in positions exposed to the sea, as on shoals. It is taken to the desired spot in pieces, and there screwed together and set up.

Signals should have a height equal to at least $\frac{1}{7000}$ of their distance, so as to subtend an angle of half a minute, which experience has shown to be the least allowable.

To make the tops of the signal-masts conspicuous, flags may be attached to them: white and red, if to be seen against the ground; and red and green, if to be seen against the sky.* The motion of

Fig. 509.



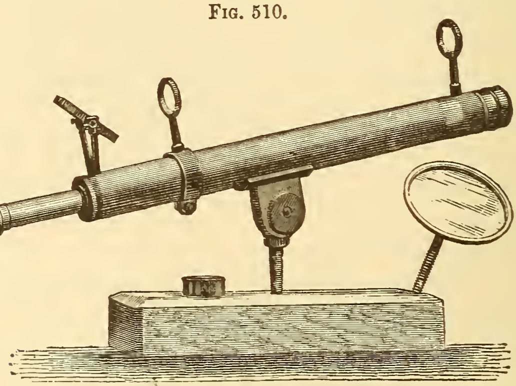
* To determine at a station A, whether its signal can be seen from B, projected against the sky or not, measure the vertical angles B A Z and Z A C. If their sum equals or exceeds 180°, A will be thus seen from B. If not, the signal at A must be raised till this sum equals 180°.

flags renders them visible, when much larger motionless objects are not; but they are useless in calm weather. A disk of sheet-iron, with a hole in it, is very conspicuous. It should be arranged so as to be turned to face each station. A barrel, formed of muslin sewed together, four or five feet long, with two hoops in it two feet apart, and its loose ends sewed to the signal-staff, which passes through it, is a cheap and good arrangement. A tuft of pine-boughs fastened to the top of the staff will be well seen against the sky.

In sunshine a number of pieces of tin, nailed to the staff at different angles, will be very conspicuous. A truncated cone of burnished tin will reflect the sun's rays to the eye in almost every situation.

The most perfect arrangement is the "heliotrope." This consists of a mirror a few inches in diameter, so mounted on a tele-

FIG. 510.



scope, near the eye-end, that the reflection of the sun may be thrown in any desired direction. They have been observed on at a distance of nearly two hundred miles, when the outlines of the mountains on which they were placed were invisible. A man, called a "heliotroper," is stationed at the instrument. He directs the telescope toward the station at which the transit is placed for observation, and keeps the mirror turned so as to reflect the sun in a direction parallel to the axis of the instrument. This he accomplishes by causing the reflection to pass through two perforated disks, mounted on the telescope, one near the object-end, and the other near the mirror.

For night-signals, an Argand lamp has been used ; or, better still, a Drummond light, or a magnesium-light. The distinctness of the light is exceedingly increased by a parabolic reflector behind it, or a lens in front of it.

676. Observations of the Angles. These should be repeated as often as possible. In extended surveys, three sets, of ten each, are recommended. They should be taken on different parts of the circle. In ordinary surveys, it is well to employ the method of "traversing." In long sights, the state of the atmosphere has a very remarkable effect on both the visibility of the signals and on the correctness of the observations.

When many angles are taken from one station, it is important to record them by some uniform system. The form given below is convenient. It will be noticed that only the minutes and seconds of the second vernier are employed, the degrees being all taken from the first :

Observations at ————————.

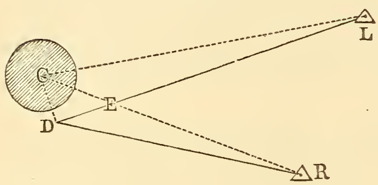
STATIONS OBSERVED TO.	READINGS.		MEAN READINGS.	RIGHT OR LEFT OF PRECEDING OBJECT.	REMARKS.
	VERNIER A.	VERNIER B.			
	° ′ ″	′ ″	° ′ ″		
A	70 19 0	18 40	70 18 50		
B	103 32 20	32 40	103 32 30	R.	
C	115 14 20	14 50	115 14 35	R.	

When the angles are "repeated," the multiple arcs will be registered under each other, and the mean of the seconds shown by all the verniers at the first and last readings be adopted.

When the country over which the triangulation extends is flat, it has been found necessary to elevate the transit some distance from the surface of the ground, the stratum of air near the surface being so disturbed by exhalations and inequalities of temperature and density as to render accurate observation impossible. The plan adopted on the Coast Survey is as follows : On the top of a signal-tripod, forty-three feet high, is placed a cap-block, into which is mortised a square hole to receive the signal-pole. Around the tripod, but not touching it, is erected a rectangular scaffold, forty feet high. On the top of it is a platform, from which the observations are taken, the signal-pole being removed from the cap-block, and the transit placed so that its center shall be precisely over the station-point.

677. Reduction to the Center. It is often impossible to set the instrument precisely at or under the signal which has been observed. In such cases proceed thus: Let C be the center of the signal, and R C L the desired angle, R being the right-hand object and L the left-hand one. Set the instrument at D, as near as possible to C, and measure the angle R D L. It may be less than R C L, or greater than it, or equal to it, according as D lies without the circle passing through C, L, and R, or within it, or in its circumference. The instrument should be set as nearly as possible in this last position. To find the proper correction for the observed angle, observe also the angle L D C (called the angle of direction), counting it from 0° to 360°, going from the left-hand object toward the left, and measure the distance D C. Calculate the distances C R and C L with the angle R D L, instead of R C L, since they are sufficiently nearly equal. Then,

Fig. 511.



$$R\,C\,L = R\,D\,L + \frac{C\,D\,.\sin.(R\,D\,L + L\,D\,C)}{C\,R\,.\sin.1''} - \frac{C\,D\,.\sin.L\,D\,C}{C\,L\,.\sin.1''}$$

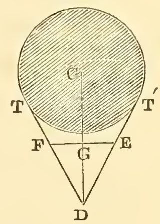
The last two terms will be the number of seconds to be added or subtracted. The trigonometrical signs of the sines must be attended to. The log. sin. 1″ = 4·6855749. Instead of dividing by sin. 1″, the correction without it, which will be a very small fraction, may be reduced to seconds by multiplying it by 206265.

Example. Let R D L = 32° 20′ 18·06″; L D C 101° 15′ 32·4″; C D = 0·9; C R = 35845·12; C L = 29783·1.

The first term of the correction will be + 3·750″, and the second term − 6·113″. Therefore, the observed angle R D L must be diminished by 2·363″, to reduce it to the desired angle R C L.

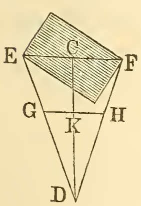
Much calculation may be saved by taking the station D so that all the signals to be observed can be seen from it. Then only a single distance and angle of direction need be measured.

Fig. 512.



It may also happen that the center, C, of the signal can not be seen from D. Thus, if the signal be a solid circular tower, set the theodolite at D, and turn its telescope so that its line of sight becomes tangent to the tower at T, T′; measure on these tangents equal distances, D E, D F, and direct the telescope to the middle, G, of the line E F. It will then point to the center, C; and the distance D C will equal the distance from D to the tower plus the radius obtained by measuring the circumference.

Fig. 513.



If the signal be rectangular, measure D E, D F. Take any point G on D E, and on D F set off D H $= DG\frac{DF}{DE}$. Then is G H parallel to E F (since D G : D H : : D E : D F), and the telescope directed to its middle, K, will point to the middle of the diagonal E F. We shall also have $DC = DK\frac{DE}{DG}$.

Any such case may be solved by similar methods.

The "*phase*" of objects is the effect produced by the sun shining on only one side of them, so that the telescope will be directed from a distant station to the middle of that bright side instead of to the true center. It is a source of error to be guarded against. Its effect may, however, be calculated.

When the signal is a tin cone:

Let r = radius of the signal;

Z = angle at the point of observation between the sun and the signal;

D = the distance.

Then, the correction $= \pm \frac{r \cos.^2 \frac{1}{2} Z}{D \sin. 1''}$.

678. Correction of the Angles. When all the angles of any triangle can be observed, their sum should equal 180°.* If not, they must be corrected. If all the observations are considered equally accurate, one third of the difference of their sum from 180° is to be added to, or subtracted from, each of them. But if the angles are the means of unequal numbers of observations, their errors may be considered to be inversely as those numbers, and they may be corrected by this proportion ; *As* the sum of the reciprocals of each of the three numbers of observations *is to* the whole error, *so is* the reciprocal of the number of observations of one of the angles *to* its correction. Thus, if one angle was the mean of three observations, another of four, and the third of ten, and the sum of all the angles was 180° 3′, the first-named angle must be diminished by the fourth term of this proportion ; $\frac{1}{3} + \frac{1}{4} + \frac{1}{10} : 3' : : \frac{1}{3} : 1'\ 27{\cdot}8''$. The second angle must in like manner be diminished by 1′ 5·9″ ; and the third by 26·3″. Their corrected sum will then be 180°.

It is still more accurate, but laborious, to apportion the total error, or difference from 180°, among the angles inversely as the "*weights.*" On the United States Coast Survey, in six triangles measured in 1844 by Professor Bache, the *greatest* error was six tenths of a second.

678¹. Calculation and Platting. The lengths of the sides of the triangles should be calculated with extreme accuracy, in two ways if possible, and by at least two persons. Plane trigonometry may be used for even large surveys ; for, though these sides are really arcs and not straight lines, the difference will be only one twentieth of a foot in a distance of 11½ miles ; half a foot in 23 miles ; a foot in 34½ miles, etc.

The platting is most correctly done by constructing the triangles, by means of the calculated lengths of their sides. If the measured angles are platted, the best method is that of chords. If many triangles are successively based on one another, they will be platted most accurately by referring all their sides to some one

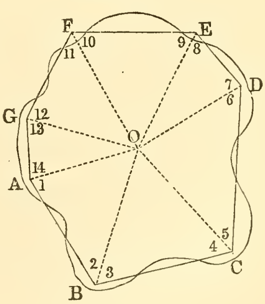
* If the triangles were very large, they would have to be regarded as spherical, and the sum of their angles would be more than 180°; but this "spherical excess" would be only 1″ for a triangle containing 76 square miles, 1′ for 4,500 square miles, etc.; and may therefore be neglected in all ordinary surveying operations.

meridian line by means of "Rectangular Co-ordinates." In the survey of a country, this meridian would be the true north and south line passing through some well-determined point.

679. Interior Filling up. The stations whose positions have been determined by the triangulation are so many fixed points, from which more minute surveys may start and interpolate any other points. The trigonometrical points are like the observed latitudes and longitudes which the mariner obtains at every opportunity, so as to take a new departure from them and determine his course in the intervals by the less precise methods of his compass and log. The chief interior points may be obtained by "Secondary Triangulation," and the minor details be then filled in by any of the methods of surveying, with chain, compass, or transit, already explained, or by the plane-table. With the transit, "Traversing" is the best mode of surveying, the instrument being set at zero, and being then directed from one of the trigonometrical points to another, which line therefore becomes the "meridian" of that survey. On reaching this second point, in the course of the survey, and sighting back to the first, the reading should of course be 0°.

680. Radiating Triangulation. This name may be given to a method shown in the figure. Choose a conspicuous point, O, nearly in the center of the field or farm to be surveyed. Find other points, A, B, C, D, etc., such that the signal at O can be seen from all of them, and that the triangles A B O, B C O, etc., shall be as nearly equilateral as possible. Measure one side, A B for example. At A measure the angles O A B and O A G ; at B measure the angles O B A and O B C ; and so on, around the polygon. The correctness of these measurements may be tested by the sum of the angles. It may also be tested by the trigonometrical principle that the product of the sines of every alternate angle,

Fig. 514.



or the odd numbers in the figure, should equal the product of the sines of the remaining angles, the even numbers in the figure.

The triangles A O B, B O C, C O D, etc., give the following proportions [Trigonometry, Art. 12, Theorem I] : A O : O B : : sin. (2) : sin. (1); O B : O C : : sin. (4) : sin. (3); O C : O D : : sin. (6) : sin. 5; and so on around the polygon. Multiplying together the corresponding terms of all the proportions, the sides will all be canceled, and there will result

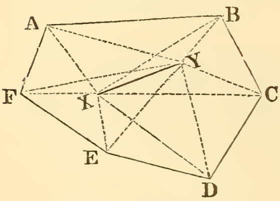
1 : 1 : : sin. (2) × sin. (4) × sin. (6) × sin. (8) × sin. (10) × sin. (12) × sin. (14);
sin. (1) × sin. (3) × sin. (5) × sin. (7) × sin. (9) × sin. (11) × sin. (13).

Hence the equality of the last two terms of the proportion.

The calculations of the unknown sides are readily made. In the triangle A B O, one side and all the angles are given to find A O and B O. In the triangle B C O, B O and all the angles are given to find B C and C O; and so with the rest. Another proof of the accuracy of the work will be given by the calculation of the length of the side A O in the last triangle, agreeing with its length as obtained in the first triangle.

681. Farm Triangulation. A farm or field may be surveyed by the previous methods, but the following plan will often be more convenient: Choose a base, as X Y, within the field, and from its ends measure the angles between it and the direction of each corner of the field, if the theodolite or transit be used, or take the bearing of each, if the compass be used. Consider first the triangles which have X Y for a base, and the corners of the field, A, B, C, etc., for vertices. In each of them one side and the angles will be known to find the other sides, X A, X B, etc. Then consider the field as made up of triangles which have their vertices at X. In each of them two sides and the included angle will be given to find its content. If Y be then taken for the common vertex, a test of the former work will be obtained.

Fig. 515.

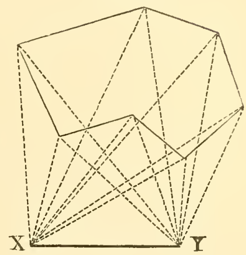


The operation will be somewhat simplified by taking for the base-line a diagonal of the field, or one of its sides.

682. Inaccessible Areas. A field or farm may be surveyed, by this "Fourth Method," without entering it. Choose a base-line

X Y, from which all the corners of the field can be seen. Take their bearings, or the angles between the base-line and their directions. The distances from X and Y to each of them can be calculated as in the last article. The figure will then show in what manner the content of the field is the difference between the contents of the triangles, having X (or Y) for a vertex, which lie outside of it, and those which lie partly within the field and partly outside of it. Their contents can be calculated as in the last article, and their difference will be the desired content. If the figure be regarded as generated by the revolution of a line one end of which is at X, while its other end passes along the boundaries of the field, shortening and lengthening accordingly, and if those triangles generated by its movement in one direction be called *plus* and those generated by the contrary movement be called *minus*, their algebraic sum will be the content.

Fig. 516.



683. Inversion of the Fourth Method. In all the operations which have been explained, the position of a point has been determined, as in Art. 6, by taking the angles, or bearings, of two lines passing from the two ends of a base-line to the unknown point. But the same determination may be effected inversely, by taking from the point the bearings, by compass, of the two ends of the base-line, or of any two known points. The unknown point will then be fixed by platting from the two known points the *opposite* bearings, for it will be at the intersection of the lines thus determined.

684. Defects of the Method of Intersection. The determination of a point by the Fourth Method, founded on the intersection of lines, has the serious defect that the point sighted to will be very indefinitely determined if the lines which fix it meet at a very acute or a very obtuse angle, which the relative positions of the points observed from and to often render unavoidable. Intersections at right angles should therefore be sought for, so far as other considerations will permit.

CHAPTER II.

SPHERICAL SURVEYING, OR GEODESY.

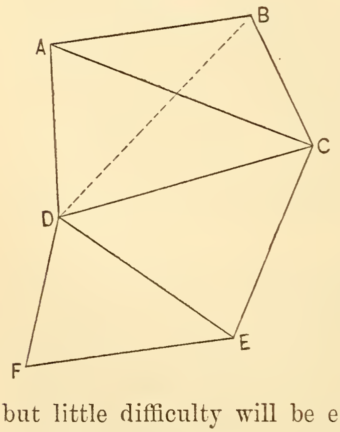
685. Nature. It comprises the methods of surveying areas of such extent that the curvature of the earth can not be neglected.

The general method is the same as that given in Chapter I, but more precise methods of measurement and of computation are required, since the triangles into which the surface is divided are spherical triangles.

The United States Coast and Geodetic Survey, the Lake Survey, and the State Surveys organized by several of the States, are works of this character.

The subject is too extensive to be properly treated within the limits of this work. Only a general sketch of it will be given, with references to such authorities as will enable the student to further investigate the subject.

Fig. 517.



Field-Work.

686. Reconnaissance. The first step in making a geodetic survey is the selection of a series of points, A, B, C, etc. (Fig. 517), as the basis of a system of triangulation. In case the country is broken or open, but little difficulty will be experienced in locating these points, and often lines of great length may be secured. Thus, in the triangulation of California,* the line Mount Helena–Mount Shasta

* See "Report of Coast and Geodetic Survey," 1868, 1876, 1880, 1882.

is 192 miles in length. It is in general advisable to choose the points so that the resulting triangle sides are as nearly equal as possible. To do this, it may be necessary to build towers or scaffolds at the stations A, B, etc., on which to place the instrument. Signals must also be placed at the stations sighted at, their general character depending on the length of the lines of sight.

687. The Base. In order to compute a triangulation, we must have at least one side measured. This measured side is called the base-line, or simply the base. In geodetic work the base must be measured with great accuracy, though it is more important that many bases occur in a system, and these be measured with moderate precision, than that only a few occur, and these be measured with great precision. The reason is, that a check can be more frequently had of the character of the work.

Several different forms of base-measuring apparatus* have been designed, of which probably the simplest and best consists of a steel bar packed in melting ice. The bar will remain of the same length throughout the measurement, as its temperature is always 32° Fahr.

688. The Angles. Suppose the observer at any station, as D for example. The angles to be measured would be A D C, C D E, E D F. Each of these angles should be measured *independently* a number of times, depending on the quality of the instrument used, and the mean of the results taken. As a check against mistakes and accidental errors of various kinds, combinations of these angles should be measured, as A D E, A D F, C D F. On the method of measuring an angle with a theodolite, see Wright's "Adjustment of Observations," pp. 253, 254.

Office-Work.

689. Computation of the Sides of the Triangles. The triangles observed are supposed to have sides of such length that the sum of

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* For descriptions of various forms of base apparatus, see "Report of United States Coast and Geodetic Survey," 1854, 1857, 1880, 1882; "Report of Primary Triangulation of the United States Lake Survey"; Wright's "Adjustment of Observations," Chapter VII.

the three angles exceeds 180° by a certain sensible quantity called the *spherical excess.* This is usually only a few seconds. For a triangle containing about 76 square miles, which, if equilateral, would have sides 13 miles long, the spherical excess is only one second. For a triangle with sides of 102 miles it is one minute. It must be determined before we can know how much the error of closure is, and therefore what the correction to each angle should be.

690. Spherical Excess. Calling the earth a sphere, the spherical excess e (in seconds) of a triangle is found from the relation

$$e = \frac{\text{area of triangle}}{R^2 \sin. 1''},$$

when R = the radius of the earth.

The triangle surface being small, compared with R^2, may be obtained with sufficient accuracy by treating it as if it were plane. Thus, when two sides and the contained angle are given, we have:

$$\text{area} = \tfrac{1}{2}\, a\, b \sin. C\,;$$

and therefore
$$e = \frac{a\, b \sin. C}{2\, R^2 \sin. 1''}.$$

The earth, however, instead of being spherical, is spheroidal in form; and since a spheroidal triangle may be computed as a spherical triangle on a sphere whose radius is $\sqrt{R\,N}$, when R and N are the radii of curvature of the meridian and of the section normal to the meridian at the mean of the latitudes of the triangle vertices, we replace R^2 in the above value of e by R N. We have then:

$$\text{excess in seconds} = \frac{a\, b \sin C.}{2\, R\, N \text{ arc } 1''}.$$

Writing this in the form

$$e = m\, a\, b \sin. C,$$

the values of m may be taken from the following table, the argument being the mean latitude of the triangle vertices. The metre is the unit of length:

LAT.	LOG. *m*.	LAT.	LOG. *m*.	LAT.	LOG. *m*.	LAT.	LOG. *m*.
°		°		°		°	
10	1·40675	25	1·40589	40	1·40451	55	1·40299
11	1·40672	26	1·40581	41	1·40441	56	1·40289
12	1·40668	27	1·40573	42	1·40431	57	1·40280
13	1·40663	28	1·40564	43	1·40420	58	1·40271
14	1·40659	29	1·40555	44	1·40410	59	1·40262
15	1·40654	30	1·40547	45	1·40400	60	1·40253
16	1·40649	31	1·40537	46	1·40390	61	1·40244
17	1 40643	32	1·40528	47	1·40380	62	1·40235
18	1·40637	33	1·40519	48	1·40369	63	1·40226
19	1·40631	34	1·40509	49	1·40359	64	1·40218
20	1·40625	35	1·40500	50	1·40349	65	1·40210
21	1·40618	36	1·40491	51	1·40339	66	1·40202
22	1·40611	37	1·40481	52	1·40329	67	1·40195
23	1·40604	38	1·40471	53	1·40319	68	1·40188
24	1·40597	39	1·40461	54	1·40309	69	1·40181
						70	1·40174

Example. In a spherical triangle, given $a = 122755$, $b = 94616^{m}$, angle $C = 50° 10' 20''$, mean latitude of vertices, A, B, C = 45° 15′ ; required the spherical excess.

$$\begin{aligned} \log.\ a, &\quad 5{\cdot}08904 \\ \log.\ b, &\quad 4{\cdot}97596 \\ \log.\ \sin.\ C, &\quad 9{\cdot}88535 \\ \log.\ m, &\quad 1{\cdot}40398 \\ \hline \log.\ 22{\cdot}61, &\quad 1{\cdot}35433 \end{aligned}$$

whence excess $e = 22''{\cdot}61$.

691. Having found the spherical excess, if the sum of the angles of the triangle is not equal to 180° plus this excess, the difference is distributed among them, and each angle is corrected by one third of this difference. The angles are then said to be "adjusted."

Example.

STATIONS.	OBSERVED ANGLES.	ADJUSTED ANGLES.
	° ′ ″	″
Prince...........	41 47 41·79	41·19
Buck.............	81 13 13·78	13·18
Hill	56 59 07·39	06·79
°	180 00 02·96	01·16 check.
180 + e =	180 00 01·16	
	″ 1·80 ÷ 3 = 0·60	

The difference between the sum of the observed angles and 180° plus the spherical excess (1″·16) is 1″·80, which will make a correction for each angle of 0″·60. Subtracting this from the observed angles, we get the corrected or adjusted spherical angles as in the table.

692. Having now the length of one side (or base), and the adjusted values of the three angles of a triangle, the other sides might be computed by the principles of spherical trigonometry. This would be very laborious, but by the help of Legendre's theorem the triangle may be computed as if it were a plane one, and the work be greatly shortened. The theorem is as follows :

Legendre's Theorem. "In any spherical triangle, the sides of which are small compared with the radius of the sphere, if each of the angles be diminished by one third of the spherical excess, the sines of these angles will be proportional to the lengths of the opposite sides."

Example.

STATIONS.	SPHERICAL ANGLES.	PLANE ANGLES AND DISTANCES.	LOGARITHMS.
	Buck to Hill.	m. 19189·80	4·2830705
	° ′ ″	″	
Prince.............	41 47 41·19	40·80	0·1762239
Buck	81 13 13·18	12·79	9·9948811
Hill	56 59 6·79	06·41	9 9235180
	1·16		
	Prince to Hill.	m. 28456·10	4·4541755
	Prince to Buck.	24144·18	4·3828124

One third of the spherical excess is subtracted from the spherical angles to reduce them to plane angles, which are placed in the third column. Using these plane angles, and the given side, and applying the sine proportion, we have :

To find b.

Log. a	= 4·2830705
Log. sin. B	= 9·9948811
Co-log. sin. A	= 0·1762239
Log. b	= 4·4541755
Prince to Hill	= 28456·10

To find c.

Log. a	= 4·2830705
Log. sin. C	= 9·9235180
Co-log. sin. A	= 0·1762239
Log. c	= 4·3828124
Prince to Buck	= 24144·18

The logarithms of the sides and of the sines of the plane angles are placed in the last column. For convenience in calculation, the co-log. of angle opposite the given side is taken.

693. In this manner, starting from the base A B (Fig. 517), a *single chain* of spherical triangles may be computed. If another base were measured at E F, a check of the accuracy of the work would be afforded by comparing the computed and measured values of E F. In the Lake-Survey triangulation of Lake Erie, the measured value of the Sandusky base differed from the value computed from the Buffalo base through a chain of thirty-six triangles intervening, by about one inch and a half.

694. Adjustment of a Triangulation. We have considered the measurement and computation of a single chain of triangles proceeding from a single measured base A B. Suppose now that the observer while at station B had sighted over the line B D, measuring the angles A B D, C B D, and while at D had measured the angles A D B, C D B. We should then have been able to compute C D from A B, by using any one of the pairs of triangles A B C, B C D : A B C, A C D : A B D, B C D : A B D, A C D. A contradiction is to be expected, as the measurements are not perfect, and therefore before beginning the computation of the sides, an "adjustment" of the angles must be made, so that their most probable values alone enter, and no contradiction will appear in the computed lengths.

The question becomes more complicated when bases are measured at intervals. Thus, suppose the triangulation adjusted from A B as base and E F computed. Another adjustment is needed to harmonize this value with the measured value of E F.

Still further contradictions arise from the introduction of the astronomical determination of the direction of a line (or azimuth), which must be adjusted for before the work is ready for mapping.

Consult "Report of the United States Coast and Geodetic Survey," 1854, 1864 ; Wright, "Adjustment of Observations," chaps. v to ix. On mapping, see "Report United States Coast and Geodetic Survey," 1880.

695. Co-ordinates of the Points. The *polar spherical co-ordinates* of a point with respect to another point are these : the length of the arc of the great circle passing through the points, and its azimuth, i. e., the angle it makes with the meridian passing through one of its points.

The *rectangular spherical co-ordinates* of a point have for axes the meridian passing through the origin, and a perpendicular to it. For short distances these may be regarded as in one plane. For greater distances new meridians must be taken—say, not farther apart than fifty miles.

Within that limit the successive triangles may be conceived to be turned down into the same plane.

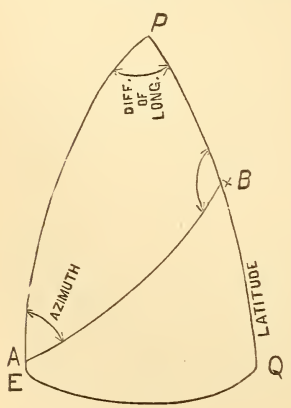
The astronomical co-ordinates of a point are its latitude and longitude. These are determined by practical astronomy.

See "Report of the United States Coast and Geodetic Survey," 1866, 1868, 1872, 1876, 1880 ; Chauvenet's "Astronomy," vol. ii ; Brunnow's "Astronomy" ; Doolittle's "Astronomy."

The methods of transformation from one system of co-ordinates to another are of great importance in practice. Two problems of common occurrence are the following :

696. *Problem.* Given the latitude and longitude of A, and the azimuth and distance from A to B. Required the latitude and longitude of B, and the azimuth from B to A.

FIG. 518.



AE = latitude.

When the triangle sides do not exceed fifteen miles, the geodetic latitudes, longitudes, and azimuths required are computed as follows :

Let K = distance in metres between two stations, the latitude and longitude of one of which are known.

L = latitude of first station.

M = longitude of first.

Z = azimuth of second station from first, counted from the south around by the west, from 0° to 360°. The algebraic signs of the sine and cosine of this angle must be carefully attended to.

L', M', Z', the same things at second station, or quantities required.

$e =$ the eccentricity.

$R =$ the radius of curvature of the meridian, in metres.

$N =$ the radius of curvature of a section perpendicular to the meridian, in metres.

Then we have

$$L' = L - \frac{K \cos. Z}{R \text{ arc } 1''} - K^2 \sin.^2 Z \frac{\tan. L}{2 R N \text{ arc } 1''} - \frac{3}{4} \frac{K^2 e^2 \sin. 2 L \cos.^2 Z}{R^2 (1 - e^2 \sin.^2 L)^{\frac{3}{2}} \text{ arc } 1''}$$

$$= L - K B \cos. Z - K^2 C \sin.^2 Z - K^2 B^2 D \cos.^2 Z$$

$$M' = M + \frac{K \sin. Z}{N' \cos. L' \text{ arc } 1''}$$

$$= M + \frac{A' K \sin. Z}{\cos. L'}$$

$$Z' = Z + 180 - (M' - M) \frac{\sin. \frac{1}{2} (L' + L)}{\cos. \frac{1}{2} (L' - L)}$$

when the quantities B, C, D, A′ may be tabulated for given values of the latitude entering. Tables for this purpose will be found in "Report of the United States Coast and Geodetic Survey," 1884.

Example. Given latitude and longitude of station Victory and length and azimuth of line Victory–Oswego, to find latitude and longitude of Oswego and azimuth of line Oswego–Victory.

The computation may be conveniently arranged in the following tabular form :

		°	′	″
Z	Victory to Oswego............	196	39	39·23
Z′ − Z			3	48·46
180°		180		
Z′	Oswego to Victory............	16	43	27·69

	°	′	″			°	′	″
L	43	13	06·82	Victory	M	76	36	22·13
L′ − L		13	30·48		M′ − M	—	5	32·92
L′	43	26	37·30	Oswego	M′	76	30	49·21

K	4·4168423	K^2	8·83368		
Cos. Z	9·9813739$_n$	Sin.^2 Z	8·91488	$(\text{K B cos. Z})^2$	5·8176
B	8 5106052	C	1·37716	D	2·3924
K B cos. Z	2·9088214$_n$		9·12572		8·2100
	″				
1st term.	− 810·63				
2d term.	0·13				
3d term.	0·02				
	° ′ ″			A′	8·5090305
− (L′−L)	−13 30·48	M′ − M	2·5223459$_n$	K	4·4168423
½ (L′+L)	43 19 52·06	Sin. ½ (L′+L)	9·8364593	Sin. Z	9·4574399$_n$
½ (L′−L)	6 45	Cos. ½ (L′−L) ar. co.	0·	Cos. L′ ar. co.	0·1390332
			2·3588052$_n$		2·5223459$_n$
		Z′ − Z	−228·46′	M′ − M	−332″.94

697. *Problem.* Given latitude and longitude of two stations, to find the distance between them and the azimuth from each to the other.

This is the inverse problem of the preceding. It is solved by dividing

$$M' - M = A' K \sin. Z \sec. L'$$

by the first term for L′ − L, namely,

$$L' - L = B K \cos. Z,$$

whence

$$\tan. Z = \frac{(M' - M) B}{(L' - L) A'} \cos. L',$$

which would give us the azimuth at once if we knew L′ − L. We therefore seek to compute the smaller terms for the difference of latitude in order to obtain K B cos. Z. by subtracting them from the known difference of latitude.

698. In addition to the authorities already quoted, and which give the methods in use in the United States, the following list may be of service: "Ordnance Survey of Great Britain"; "Great Trigonometrical Survey of India"; "Die Preussische Landestriangulation"; Bessel, "Gradmessung in Ostpreussen"; Jordan, "Handbuch der Vermessungskunde"; Helmert, "Geodäsie"; Puissant, "Géodésie."

PART V.

MARITIME OR HYDROGRAPHICAL SURVEYING.

INTRODUCTION.

699. The object of this is to fix the positions of the deep and shallow points in harbors, rivers, etc., and thus to discover and record the shoals, rocks, channels, and other important features of the locality.

The relative positions of prominent points on the shore are first very precisely determined by "Trigonometrical Surveying," Part IV. These form the basis of operations, and afford the means of correcting the results obtained by the less accurate methods employed for filling in the details.

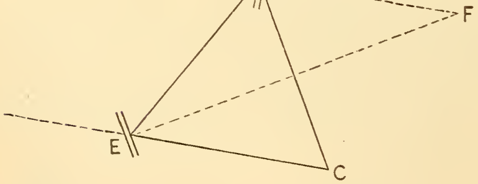
In addition to the surveying-instruments already described, the sextant is much used in hydrographical surveying. When the sextant is used for determining the position of a point, the angles are measured between three lines, passing from the required point to three known points. The required point is thus determined by trilinear co-ordinates, or by the *fifth method*, as explained in Art. 8.

CHAPTER I.

THE SEXTANT.

700. Principle. The angle subtended at the eye by lines passing from it to two distant objects, may be measured by so arranging two mirrors that one object is looked at directly, and the other object is seen by its image, reflected from one mirror to the second, and from the second mirror to the eye. If the first mirror be moved so that the doubly reflected image of the second object be made to cover or coincide with the object seen directly, then is the desired angle equal to twice the angle which the mirrors make with each other.

FIG. 519.



Proof. In Fig. 519, let D and E be two mirrors, perpendicular to the plane of the paper. Let a ray of light from the object A be reflected from the mirrors D and E to the eye at C, and B be the other object, looked at directly. Erect perpendiculars to the mirrors, and prolong them until they meet at F. Prolong the line A D until it meets the line B E at C. The angle D F E is equal to the angle which the two mirrors make with each other.

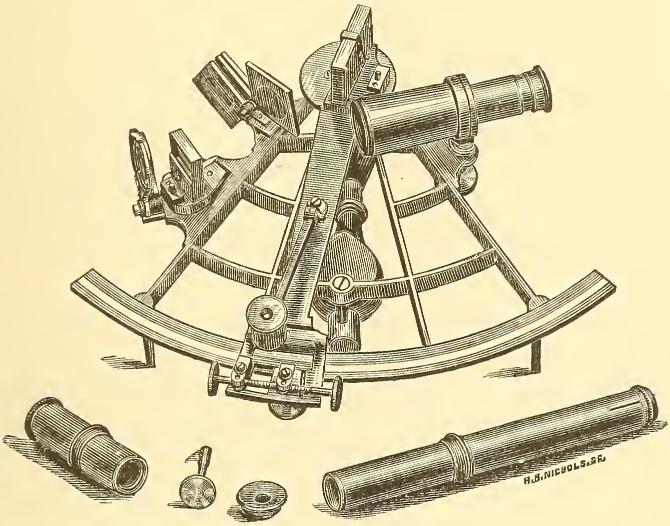
Since the angle of incidence equals the angle of reflection, A D G = G D E, and D E F = F E C, then we have : $D C E = A D E - D E C$

$$D C E = 2(G D E - D E F)$$

$$D C E = 2 D F E.$$

701. Description of the Sextant (Fig. 520). The frame is usually of brass, constructed so as to combine strength with lightness. The

Fig. 520.



Sextant.

handle by which it is held is of wood. The index-arm is movable about a pivot in the center of the graduated arc. The index-glass is a small mirror, attached to the index-arm at the pivot, so as to be perpendicular to the plane of the graduated arc. The horizon-glass on the left in the figure is attached perpendicularly to the plane of the instrument, and parallel to the index-glass when the index is at zero. The lower half of this glass is silvered, to make it a reflector, and the upper half is transparent. The telescope is attached so as to point toward the horizon-glass. Sets of colored glasses are used to moderate the light of the sun, when that body is observed.

The sextant has an arc of one sixth of a circle, and measures angles up to 120°, the divisions of the graduated arc being numbered with twice their real value, so that the true desired angle, subtended by the two objects, is read off at once. The arc is usually graduated to 10′ and read by a vernier to 10″.

702. The box or pocket sextant has the same glasses as the larger sextant, inclosed in a circular box, about three inches in diameter. The lower part, which answers for a handle when in use, screws off and is used for a cover.

The octant has an arc of one eighth of a circumference, and measures angles to 90°.

703. The Reflecting Circle. This is an instrument constructed on the same principle, and used for the same purposes, as the sextant. In it the graduated arc extends to the whole circumference, and more than one vernier may be used by producing the index-arm to meet the circumference in one or two more points.

704. Adjustments of the Sextant. 1. *To make the index-glass perpendicular to the plane of the arc:*

Bring the index near the center of the arc and place the eye near the index-glass, and nearly in the plane of the arc. See if the part of the arc reflected in the mirror appears to be a continuation of the part seen directly. If so, the glass is perpendicular to the plane of the arc. If not, adjust it by the screws behind it.

2. *To make the horizon-glass perpendicular to the plane of the arc:*

The index-glass having been adjusted, sight to some well-defined object, as a star, and if, in moving the index-arm, one image seems to separate from or overlap the other, then the horizon-glass is not perpendicular to the plane of the arc. It must be made so by the screws attached to it.

Another method of testing the perpendicularity of the horizon-glass is as follows: Hold the instrument vertically, and bring the direct and reflected images of a smooth portion of the distant horizon into coincidence. Then turn the instrument until it makes an angle with the vertical. If the two images still coincide, the glasses are parallel; and, as the index-glass has been made perpendicular to the plane of the arc, the horizon-glass is in adjustment.

3. *To make the line of collimation of the telescope parallel to the plane of the arc:*

The line of collimation of the telescope is an imaginary line, passing through the optical center of the object-lens, and a point midway between the two parallel wires. These wires are made parallel to the plane of the sextant by revolving the tube in which they are placed.

To see whether the line of collimation of the telescope is in adjustment, bring the images of two objects, such as the sun and moon, into contact at the wire nearest the instrument, and then, by moving the instrument, bring them to the other wire. If the contact remains perfect, the line of collimation is parallel to the plane of the arc; if it does not, the adjustment must be made by the screws in the collar of the telescope.

4. *To see if the two mirrors are parallel when the index is at zero:*

Bring the direct and reflected images of a star into coincidence. If the index is at zero, then no correction is necessary; if not, the reading is the "*index-error,*" and is positive or negative, according as the index is to the right or left of the zero.

The "index-error" may be rectified by moving the horizon-glass until the images do coincide when the index is at zero, but it is usually merely noted, and used as a correction, being added to each reading if the error is positive, or subtracted from each reading if the error is negative.

705. How to observe. Hold the instrument so that its plane is in the plane of the two objects to be observed, and hold it loosely. Look through the eye-hole, or plain tube, or telescope, at the left-hand or lower object, by direct vision, through the unsilvered part of the horizon-glass. Then move the index-arm till the other object is seen in the silvered part of the horizon-glass, and the two are brought to apparently coincide. Then the reading of the vernier is the angle desired.

If one object be brighter than the other, look at the former by reflection. If the brighter object be to the left or below, hold the instrument upside down.

If the angular distance of the object be more than the range of the sextant (about 120°), observe from one of them to some intermediate object, and thence to the other.

A good rest for a sextant is an ordinary telescope-clamp, through which is passed a stick, one end of which is fitted into a hole made in the sextant-handle, and the other end of which is weighted for a counterpoise.

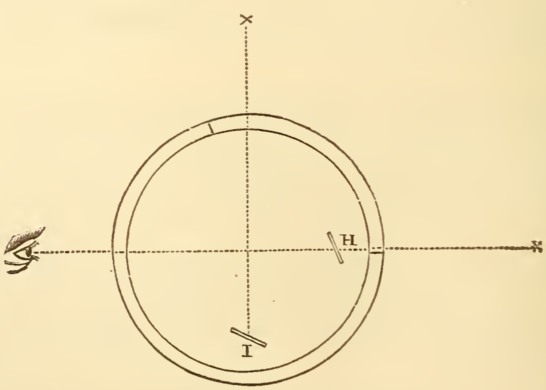
THE PRACTICE.

706. To set out Perpendiculars. Set the index at 90°. Hold the instrument over the given point by a plumb-line, and look along the line by direct vision. Send a rod in about the desired direction, and when it is seen by reflection to coincide with the point on the line looked at directly, it will be in a line perpendicular to the given line at the desired point.

Conversely, to find where a perpendicular from a given point would strike a line :

Set the index at 90°, and walk along the line, looking directly at a point on it, until the given point is seen by reflection to coincide with the point on the line. A plumb-line let fall from the eye will give the desired point.

FIG. 521.

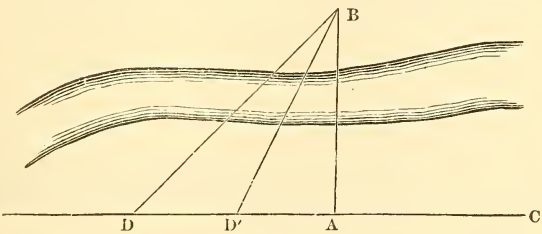


707. The Optical Square (Fig. 521). This is a box containing two mirrors, fixed at an angle of 45° to each other, and therefore giving an angle of 90°, as does the sextant with its glasses fixed at that angle. It is used to set out perpendiculars.

708. To measure a Line, One End being inaccessible. Let A B be the required line, and B the inaccessible point.

At A set off a perpendicular, A C, by Art. 706. Then set the index at 45°, and walk backward from A in the line of C A pro-

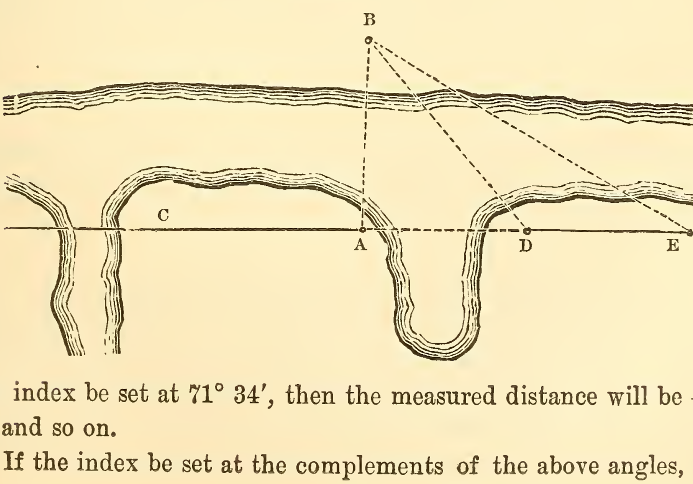
FIG. 522.



longed, looking by direct vision at C, until you arrive at some point, D, from which B is seen by reflection to coincide with C. Then is A D = A B.

If more convenient, after setting off the right angle, set the index at 63° 26′, and then proceed as before. The objects will be seen to coincide when at some point, D′. Then A D′ = ½ A B. If

FIG. 523.



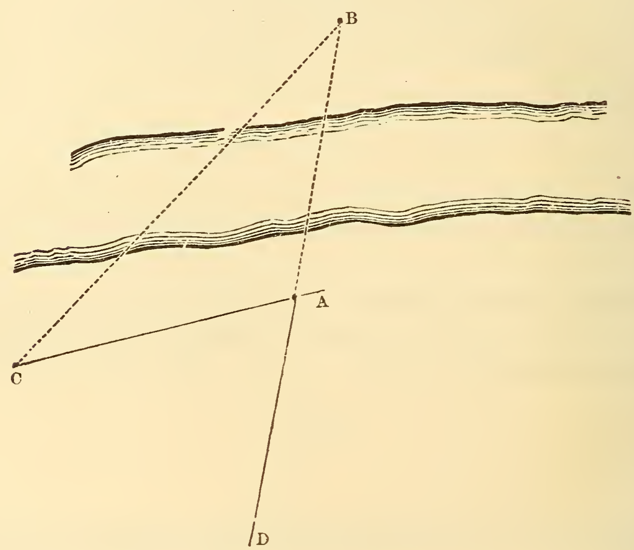
the index be set at 71° 34′, then the measured distance will be ⅓ A B, and so on.

If the index be set at the complements of the above angles, the distance measured will be, in the first case, twice, and in the second case three times the desired one.

When the distance A D can not be measured, as in Fig. 523, fix D as before. Set the index at 26° 34′, and go along the line to E, where the objects are seen to coincide with each other; then is A E twice A B, and hence E D = A B.

709. *Otherwise.* At A set off an angle, as C A D (A D being a prolongation of A B). Then walk along the line A C with the index

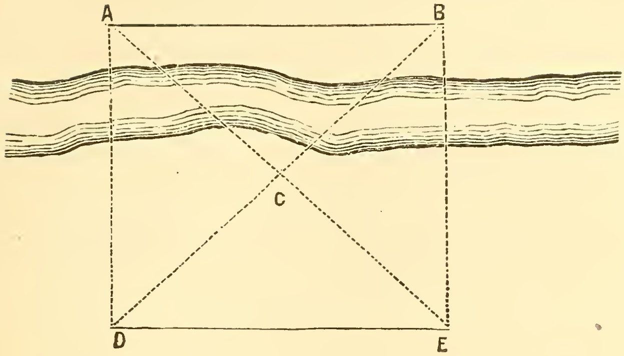
Fig. 524.



set to half that angle, looking at A directly, and B by reflection, till you come to some point, C, at which they coincide. Then is C A = A B.

710. To measure a Line when Both Ends are inaccessible. Let A B be the required line. At any point, C, measure the angle A C B. Set the sextant to half that angle, and walk back in the line B C prolonged till at some point, D, A, and B are seen to coincide, as in last problem; thus making A C = C D. Do the same on A C produced to some point, E. Then is D E = A B.

Fig. 525.



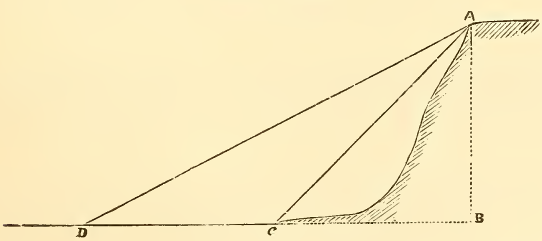
711. All the methods for overcoming obstacles to measurement, determining inaccessible distances, etc. (Part I, Chapter V), with the transit or theodolite, can be executed with the sextant.

712. To measure Heights. Measure the vertical angle between the top of the object and a mark at the height of the eye, as with a theodolite or transit, and then calculate the height as in Part II, Art. 578.

Otherwise. Set the index at 45°, and walk backward till the mark and the top of the object are brought to coincide. Then the horizontal *distance* equals the *height.*

So, too, if the index is set at 63° 26′, the height equals twice the distance, and so on. The ground is supposed to be level.

Fig. 526.

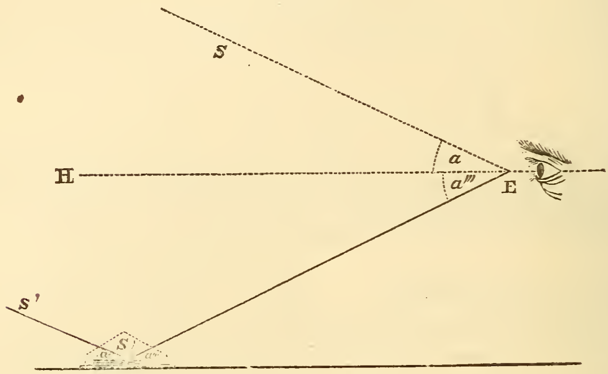


When the base is inaccessible: Make C = 45°, and D = 26° 34′. Then C D = A B. So, too, if C = 26° 34′, and D = 18° 26′.

This may be used when a river flows along the base of a hill whose height is desired, or in any other like circumstance.

713. To observe Altitudes in an Artificial Horizon. In this method we measure the angle subtended at the eye between the object and its image reflected from an artificial horizon of mercury, molasses, oil, or water. The image of the object in the mercury is looked at directly, and the object itself is viewed by reflection. The object observed is supposed to be so distant that the rays from it,

FIG. 527.



which strike respectively the index-glass and the artificial horizon, are parallel; i. e., S and S′, Fig. 527, are the same point.

Then will the observed angle S E S″ be double the required angle S E H.

Demonstration.

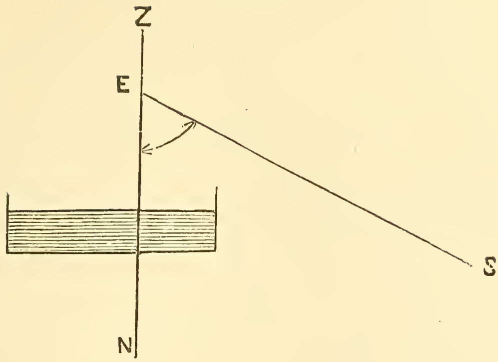
$$a = a', \; a' = a'', \text{ and } a'' = a'''. \quad \text{Hence } a''' = a.$$
$$\text{S E S}'' = a + a''' = 2\,a = 2\,\text{S E H}.$$

714. When the sun is the object observed, to determine whether it is his upper or lower limb whose altitude has been observed, proceed thus:

Having brought two limbs to touch, push the index-arm from you. If one image passes over the other, so that the other two limbs come together, then you had the lower limb at first. If they separate, you had the upper limb.

In the forenoon, with an inverting telescope, the lower limbs are parting, and the upper limbs are approaching; and *vice versa* in the afternoon.

Fig. 528.



715. To observe very small altitudes and depressions with the artificial horizon :

Stretch a string over the artificial horizon. Place your head so that you see the string cover its image in the mercury. Then the eye and string determine a vertical plane.

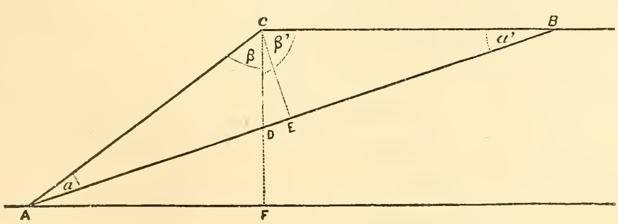
Then observe, looking at the string by direct vision, and seeing the object by reflection, and you have the angle S E N, in Fig. 528, the supplement of the zenith-distance.

Otherwise. Fix behind the horizon-glass a piece of white paper with a small hole in it, and with a black line on it perpendicular to the plane of the arc.

Then look into the mercury, so as to see in it the image of the line. Your line of sight is then vertical, and the angle to the object seen by reflection is measured as before.

716. To measure Slopes with the Sextant and Artificial Horizon. Let A B be the surface of the ground, and A F a horizontal

Fig. 529.



line. Mark two points equally distant from the eye. Measure, by the preceding method, the angles β and β', which C A and C B make with the vertical C D. Then will half the difference of these angles equal the angle which the slope makes with the horizon.

Demonstration. Continue the vertical line C D to meet the horizontal line in F, and draw C E perpendicular to A B. Then the triangles C D E and A D F are similar, being right-angled and having the acute angles at D equal. Consequently, the angle D C E = D A F, which is the angle of the slope with the horizon. But D C E $= \frac{1}{2}(\beta' - \beta)$, hence $\frac{1}{2}(\beta' - \beta) =$ the angle which the slope of the ground makes with the horizon.

If the points A and B be not equally distant from C, but yet far apart, this method will still give a very near approximation, the error, which is additive, being $\frac{1}{2}(a' - a)$.

Demonstration.

$$
\begin{aligned}
\mathrm{D\,C\,E} &= \beta' + a' - 90^\circ, \\
\mathrm{D\,C\,E} &= -\beta - a + 90^\circ, \\
\hline
2\,\mathrm{D\,C\,E} &= \beta' - \beta + a' - a, \\
\mathrm{D\,C\,E} &= \tfrac{1}{2}(\beta' - \beta) + \tfrac{1}{2}(a' - a).
\end{aligned}
$$

717. Oblique Angles. When the plane of two objects, observed by the sextant, is very oblique to the horizon, the observed angle will differ much from the horizontal angle which is its horizontal projection, and which is the angle needed for platting. The projected angle may be larger or smaller than the observed angle.

This difficulty may be obviated in various ways :

1. Observe the angular distance of each object from some third object, very far to the right or left of both. The difference of these angles will be nearly equal to the desired angle.

2. Note, if possible, some point above or below one of the objects, and on the same level with the other, and observe to it and the other object.

3. Suspend two plumb-lines, and place the eye so that these lines cover the two objects. Then observe the horizontal angle between the plumb-lines.

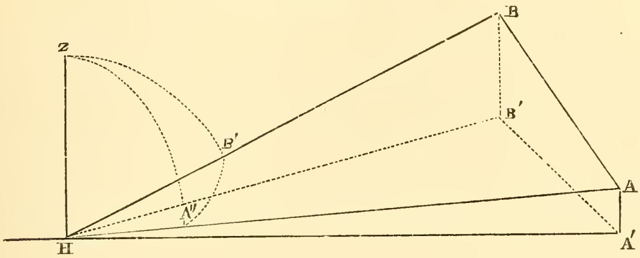
4. For perfect precision, observe the oblique angle itself, and

also the angle of elevation or depression of each of the objects. With these data the oblique angle can be reduced to its horizontal projection, either by descriptive geometry or more precisely by calculation, thus :

Let A H B be the observed angle, and A′ H B′ the required horizontal angle.

Conceive a vertical H Z, and a spherical surface, of which H, the vertex of the angle, is the center. Then will the vertical

FIG. 530.



planes, A H A′ and B H B′, and the oblique plane A H B, cut this sphere in arcs of great circles, Z A″, Z B″, and A″ B″, thus forming a spherical triangle, A″ Z B″, in which A″ B″ $= h$ measures the observed angle ; Z A″ = Z measures the zenith-distance of the point A ; and Z B″ = Z′ measures the zenith-distance of the point B.

These zenith-distances are observed directly, or given by the observed angles of elevation or depression. Then we have the three sides of the triangle to find the angle B = A′ H B′.

Calling P the half sum of the three sides, we have :

$$\text{Sin. } \tfrac{1}{2}\,\text{B} = \sqrt{\frac{\sin.\ (\text{P} - \text{Z}) \sin.\ (\text{P} - \text{Z}')_1}{\sin.\ \text{Z}\ .\ \sin.\ \text{Z}'}}\,.$$

An approximate correction, when the zenith-distances do not differ from 90° by more than 2° or 3°, is this :

$$\left(90^\circ - \frac{\text{Z} + \text{Z}'}{2}\right)^2 \text{tang. } \tfrac{1}{2} h\ .\ \sin.\ 1'' - \left(\frac{\text{Z} - \text{Z}'}{2}\right)^2 \cot.\ \tfrac{1}{2} h\ .\ \sin.\ 1''.$$

The quantities in the parentheses are to be taken in seconds. The answer is in seconds, and additive.

717[1]. The advantages of the sextant over the theodolite are these :

1. It does not require a fixed support, but can be used while the observer is on horseback, or on a surface in motion, as at sea.

2. It can take simultaneous observations on two moving bodies, as the moon and a star.

It can also do all that the theodolite can. Its only defect is in observing oblique angles in some cases. By these properties it determines distances, heights, time, latitude, longitude, and true meridian, and thus is a portable observatory.

CHAPTER II.

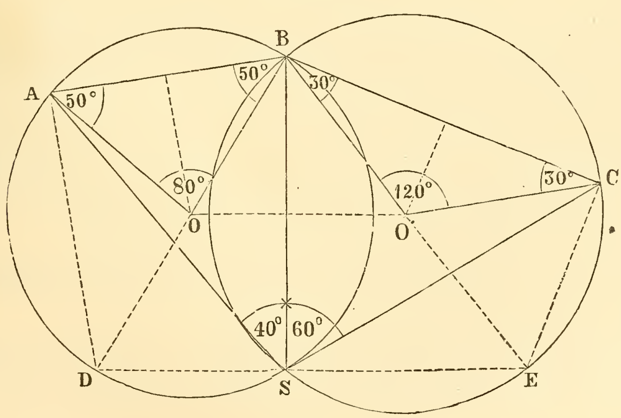
TRILINEAR SURVEYING.

718. Trilinear Surveying is founded on the fifth method of determining the position of a point, by measuring the angles between three lines conceived to pass from the required point to three known points, as illustrated in Art. 8.

To fix the place of the point from these data is much more difficult than in the preceding methods, and is known as the "Problem of the three points." It will be here solved geometrically, instrumentally, and analytically.

719. Geometrical Solution. Let A, B, and C be the known objects observed from S, the angles A S B and B S C being there

Fig. 531.



measured. To fix this point, S, on the plat containing A, B, and C, draw lines from A and B, making angles with A B each equal to 90° — A S B. The intersection of these lines at O will be the center of a circle passing through A and B, in the circumference of which the point S will be situated.* Describe this circle. Also draw lines from B and C, making angles with B C, each equal to 90° — B S C. Their intersection, O′, will be the center of a circle passing through B and C. The point S will lie somewhere in its circumference, and therefore in its intersection with the former circumference. The point is thus determined.

In the figure the observed angles, A S B and B S C, are supposed to have been respectively 40° and 60°. The angles set off are therefore 50° and 30°. The central angles are consequently 80° and 120°, twice the observed angles.

The dotted lines refer to the checks explained in the latter part of this article.

When one of the angles is obtuse, set off its difference from 90° on the opposite side of the line joining the two objects to that on which the point of observation lies.

When the angle A B C is equal to the supplement of the sum of the observed angles, the position of the point will be indeterminate, for the two centers obtained will coincide, and the circle described from this common center will pass through the three points, and any point of the circumference will fulfill the conditions of the problem.

A third angle, between one of the three points and a fourth point, should always be observed, if possible, and used like the others, to serve as a check.

Many tests of the correctness of the position of the point determined may be employed. The simplest one is that the centers of the circles, O and O′, should lie in the perpendiculars drawn through the middle points of the lines A B and B C.

Another is that the line B S should be bisected perpendicularly by the line O O′.

A third check is obtained by drawing at A and C perpendiculars to A B and C B, and producing them to meet B O and B O′,

* For the arc A B measures the angle A O B at the center, which angle = 180° — 2 (90° — A S B) = 2 A S B. Therefore, any angle inscribed in the circumference and measured by the same arc is equal to A S B.

produced, in D and E. The line D E should pass through S; for, the angles B S D and B S E being right angles, the lines D S and S E form one straight line.

The figure shows these three checks by its dotted lines.

720. Instrumental Solution. The preceding process is tedious where many stations are to be determined. They can be more readily found by an instrument called a *Station-pointer*, or *Chorograph.* It consists of three arms, or straight-edges, turning about a common center, and capable of being set so as to make with each other any angles desired. This is effected by means of graduated arcs carried on their ends, or by taking off with their points (as with a pair of dividers) the proper distance from a scale of chords constructed to a radius of their length. Being thus set so as to make the two observed angles, the instrument is laid on a map containing the three given points, and is turned about till the three edges pass through these points. Then their center is at the place of the station, for the three points there subtend on the paper the angles observed in the field.

A simple and useful substitute is a piece of transparent paper, or ground glass, on which three lines may be drawn at the proper angles and moved about on the paper as before.

721. Analytical Solution. The distances of the required point from each of the known points may be obtained analytically. Let $AB = c$; $BC = a$; $ABC = B$; $ASB = S$; $BSC = S'$. Also, make $T = 360° - S - S' - B$. Let $BAS = U$; $BCS = V$. Then we shall have:

$$\text{Cot. } U = \text{cot. } T\left(\frac{c \,.\, \text{sin. } S'}{a \,.\, \text{sin. } S \,.\, \text{cos. } T} + 1\right),$$

$$V = T - U,$$

$$SB = \frac{c \,.\, \text{sin. } U}{\text{sin. } S}; \text{ or, } = \frac{a \,.\, \text{sin. } V}{\text{sin. } S'},$$

$$SA = \frac{c \,.\, \text{sin. } ABS}{\text{sin. } S}, \quad SC = \frac{a \,.\, \text{sin. } CBS}{\text{sin. } S'}.$$

Proof. In the triangle A B S, we have

$$\text{sin. } ASB : \text{sin. } BAS :: AB : SB = \frac{AB \,.\, \text{sin. } BAS}{\text{sin. } ASB} = \frac{c \,.\, \text{sin. } U}{\text{sin. } S}. \quad [1.]$$

In the triangle C B S, we have

$$\sin. \text{B S C} : \sin. \text{B C S} :: \text{B C} : \text{S B} = \frac{\text{B C} . \sin. \text{B C S}}{\sin. \text{B S C}} = \frac{a . \sin \text{V}}{\sin. \text{S}'}. \qquad [2.]$$

Hence, $\frac{c . \sin. \text{U}}{\sin. \text{S}} = \frac{a . \sin. \text{V}}{\sin. \text{S}'}$; whence, $c . \sin. \text{S}' . \sin. \text{U} - a . \sin. \text{S} . \sin. \text{V} = 0$ . . . . [3.]

In the quadrilateral A B C S, we have

B C S = 360° − A S B − B S C − A B C − B A S; or V = 360° − S − S′ − B − U.

Let T = 360° − S − S′ − B, and we have V = T − U. . . . . . [4.]

Substituting this value of V, in equation [3], we get [Trig., Art. 8],

$$c . \sin. \text{S}' \sin. \text{U} - a . \sin. \text{S} (\sin. \text{T} . \cos. \text{U} - \cos. \text{T} . \sin. \text{U}) = 0.$$

Dividing by sin. U, we get

$$c . \sin. \text{S}' - a . \sin. \text{S} \left(\sin. \text{T} . \frac{\cos. \text{U}}{\sin. \text{U}} - \cos. \text{T}\right) = 0.$$

Whence we have

$$\frac{\cos. \text{U}}{\sin. \text{U}} = \cot. \text{U} = \frac{c . \sin. \text{S}' + a . \sin. \text{S} . \cos. \text{T}}{a . \sin. \text{S} . \sin. \text{T}}.$$

Separating this expression into two parts, and canceling, we get

$$\cot. \text{U} = \frac{c . \sin. \text{S}'}{a . \sin. \text{S} . \sin. \text{T}} + \frac{\cos. \text{T}}{\sin. \text{T}}.$$

Separating the second member into factors, we get

$$\cot. \text{U} = \frac{\cos. \text{T}}{\sin. \text{T}} \left(\frac{c . \sin. \text{S}'}{a . \sin. \text{S} . \cos. \text{T}} + 1\right); \text{ or}$$

$$\cot. \text{U} = \cot. \text{T} \left(\frac{c . \sin. \text{S}'}{a . \sin. \text{S} . \cos. \text{T}} + 1\right).$$

Having found U, equation [4] gives V; and either [1] or [2] gives S B; and S A and S C are then given by the familiar "Sine proportion" [Trig., Art. 12].

Attention must be given to the algebraic signs of the trigonometrical functions.

Example. A S B = 33° 45′; B S C = 22° 30′; A B = 600 feet; B C = 400 feet; A C = 800 feet. Required the distances and directions of the point S from each of the stations.

In the triangle A B C, the three sides being known, the angle A B C is found to be 104° 28′ 39″. The formula then gives the angle B A S = U = 105° 8′ 10″; whence B C S is found to be 94° 8′ 11″; and S B = 1042·51; S A = 710·193; and S C = 934·291.

CHAPTER III.

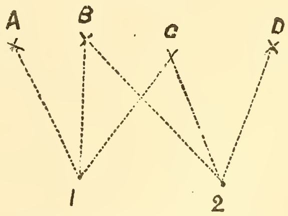
SURVEYING THE SHORE-LINE.

722. The High-water Line. The principal points on the high-water line are determined by triangulating. The sections between these points are surveyed with the compass and chain, by running a series of straight lines so as to follow, approximately, the shore-line, and taking offsets from the straight lines of the survey to the bends in the shore-line. The straight lines can be more accurately determined by "traversing" with the transit.

723. The Low-water Line. In "tidal-waters" this is more difficult, because low and bare for only a short time. The survey is best made with the sextant, observing from prominent points to three signals, by the trilinear method, and sketching, by the eye, bends of the shore between the stations observed from.

There should be one to observe and one to record. Let 1 and 2, Fig. 532, be two points on the low-water line, whose position it is desired to determine. The observations taken will be as follows :

FIG. 532.



(1.)	A and B . . .	18°
	B and C . . .	20°
(2.)	B and C . . .	15°
	C and D . . .	45°

When the shore is inaccessible, a base-line must be measured on the water, and points on the shore fixed by angles from its ends, as in Art. 729.

724. Measuring a Base on the Water. 1. By sound. Sound travels at the rate of 1,090 feet per second, with the temperature at 30° Fahr. For higher or lower temperatures, add or subtract $1\frac{1}{7}$ foot for each degree. If the wind blows with or against the movement of the sound, its velocity must be added or subtracted. If it blows obliquely, the correction will be its velocity multiplied by the cosine of the angle which the direction of the wind makes with the direction of the sound.

2. By measuring with the sextant the angular height of the mast of a vessel, then we have:

$$\text{Distance} = \text{height of mast} \div \text{tan. of the angle.}$$

CHAPTER IV.

SOUNDINGS.

725. In sounding, the object is to determine the contour of the bottom of any river, lake, bay, etc., so that a chart of it may be drawn, showing the depth of water at all points covered by the survey. The heights of the points on the bottom are referred to the surface of the water as a "datum-plane," and contour-lines may be determined in the manner described in "Topography."

For the same extent of surface, however, if the same degree of accuracy is required, it will be necessary to measure the height of more points in sounding than in topographical surveying, as the surface between the points, whose heights are measured, can not be seen and sketched.

726. For depths up to eighteen feet a sounding-rod, graduated to feet and tenths, may be used. For greater depths, a lead-line marked to fathoms and half-fathoms will be necessary. The size of the line and the weight of the lead will depend upon the depth of the water. A lead weighing ten pounds will be sufficient for depths up to twenty fathoms. Before using a lead-line it should be thoroughly wet and stretched, and the length of the line should be frequently tested.

727. Before commencing the soundings, stations should be erected on all of the principal points on the shore, such as headlands, bights of bays, etc.

A good station-mark is a post, set in the ground about three feet, leaving about one foot above the surface. The flag-pole is placed in an auger-hole made in the top of the post. The flag-pole can readily be lifted out, and the transit set over the center of the station. The number of the station should be marked on each post, and it should be distinguished by the combination of colors on the flag, or by the number and arrangement of cross-pieces on the staff.

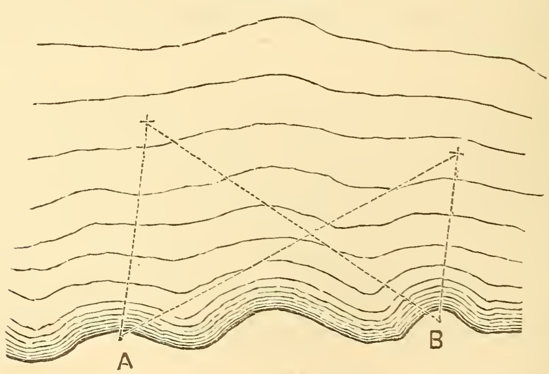
A permanent "bench-mark" must be established, and the height of the water, when the soundings are made, noted and recorded.

Stations on the water are marked by buoys. A buoy may be made of a light wood float, in which is a hole for the flag-pole. The float is anchored with a stone, or by some other means.

728. The position of the station-buoys, and of the boat when sounding, is determined in various ways.

729. From the Shore. A point on the water may be determined by observing to it with a transit from two stations on the shore, at a given signal or fixed time. In Fig. 533, the length of the line

Fig. 533.



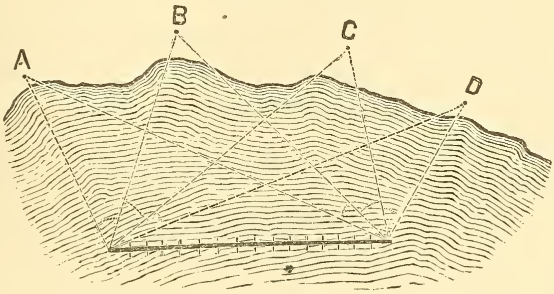
A B, and the angles which the lines of sight make with it would then be known, and its place would be fixed by angular co-ordinates. Two observers are necessary.

730. From the Boat with a Compass. Observe from the boat with a prismatic compass, or a Burnier's compass, to two signals on shore. The place of the boat is then determined, and may be fixed on the map by drawing, from the two known points, lines having the *opposite* bearings, and their intersection will be the required point. This is rapid and easy, but not precise.

731. From the Boat with the Sextant. Observe with the sextant to three signals on shore, noting the two angles. Two observers, or one observer with two sextants, are necessary. This is the trilinear method, given in Chapter II of this part.

732. Between Stations. Positions of the boat are thus determined only at considerable distances apart, and the boat is rowed

FIG. 534.



from one of these points to a second one, and soundings taken at regular intervals of time between them.

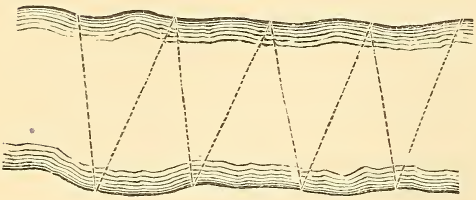
The distance apart of the soundings depends on the regularity of the bottom, the depth of the water, and the object of the survey. Care should be taken to leave no spot unexplored.

For great accuracy, anchor at some point, and determine its place as above, and then proceed to another point, paying out a line, fastened to the anchor, and sounding at regular distances. Cast anchor at the second point, go back to the first, take up the anchor, go on to the second, and then proceed as before.

733. In a river or narrow water, the soundings may be taken in zigzag lines, from shore to shore, at equal intervals of time, as in Fig. 535.

Where soundings can be made through the ice, the position of all the points can be determined by any of the methods of surveying. This is the most accurate method of sounding.

Fig. 535.



734. On the sea-coast the soundings must all be reduced to mean low spring-tides.

735. Tide-Gauges. Tidal observations consist in recording the heights of the water at stated times. In order to determine this, tide-gauges are necessary. The simplest form is a stick of timber, graduated to feet and inches, or tenths, and either set up in the water, or fastened to the face of a dock, or pier, so that the rise of the tide may be noted upon it. The zero-point of each gauge is taken at or below the lowest tide, and is referred to a permanent "bench-mark" on the shore. On account of the difficulty of sustaining a timber of considerable height against the force of the wind and waves, several successive gauges are sometimes used—the bottom mark on each gauge higher up being on a level with the top line of the next lower. Such an arrangement is required on gentle slopes.

On the sea-coast, where the waves make the reading of the staff difficult, the staff may be attached to a float, inclosed in an upright tube, pierced with holes. The holes in the tube should be of such a size as to allow the water to find the mean height inside, and yet reduce the oscillations to very small limits. Permanent tide-gauges should be self-registering. For a description of a self-registering tide-gauge, see "United States Coast Survey Report," 1853.

736. "Establishment of the Port." Owing to the obstructions which the tidal wave meets with from the formation of the sea-bed as it approaches the shore, and the character and direction of the channels, the time of high water will differ for different ports in the same vicinity. In order that navigators, entering a port, may be able to find the time of high water, a standard tide-time is established—i. e., the number of hours at which high water occurs after the moon's transit over the meridian. This is called the "Establishment of the Port." This time varies with the age of the moon. When observed on the days of full or change, it is the "Vulgar Establishment of the Port." The "Corrected Establishment of the Port" is the mean of the intervals between the times of the transit of the moon and the times of high tide for half a month. This is used for finding the time of high water on any given day, and tables are constructed, from observations at the principal ports, for finding the correction for semi-monthly inequality.

737. In rivers, a number of tide-gauges are necessary, at moderate distances apart, especially at the bends, because the tidal lines of high and low water are not parallel to one another.

The soundings are to be reduced by the nearest gauge, or by the mean of the two between which they may be taken.

738. Beacons and Buoys. Beacons are permanent objects, such as piles of stones with signals on them, usually on shoals and dangerous rocks.

Buoys are floating objects, such as barrels, or hollow iron spheres or cylinders, anchored by a chain, and variously painted, to indicate either dangers or channels.

Those placed by the United States Coast Survey are so colored and numbered that, in entering a bay, harbor, or channel, red buoys with even numbers shall be passed on the starboard or right hand, black buoys with odd numbers on the port hand or left hand, and buoys with red and black stripes on either hand. Buoys in channel-ways are colored with alternate white and black vertical stripes.

CHAPTER V.

THE CHART.

739. HAVING determined the lines of high and low water, the position of the channels, rocks, shoals, etc., and the soundings, a chart must be made, on which all these are laid down in their proper places. For scales, see Arts. 43–45.

FIG. 536.

The high-water line is platted like the bounding lines of a farm. The points determined in the low-water line, and the positions of the boat, determined by the method given in Arts. 728–731, are fixed on the chart by one of the methods given in Arts. 719–721. Contour curves are drawn as in land topography (Part III), for the first four fathoms. These may be indicated by dotted lines, as

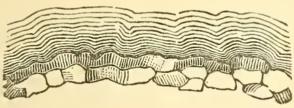
FIG. 537.

in Fig. 536, or they may be shaded with Indian-ink, as in Fig. 537.

Beyond four fathoms, the depths are noted in fathoms and vulgar fractions.

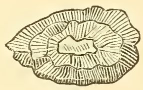
740. Various conventional signs are used; some of the principal ones are given in Figs. 538–558.

Fig. 538.



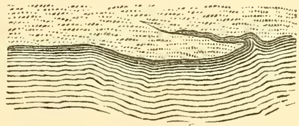
Rocky shore.

Fig. 539.



Rocks always bare.

Fig. 540.



Low, swampy shore.

Fig. 541.

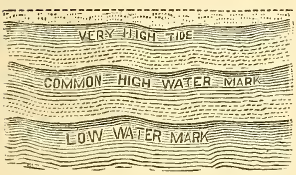
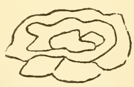
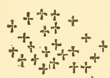


Fig. 542.



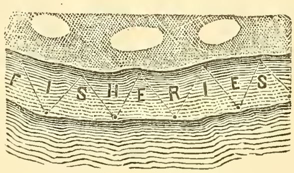
Rocks sometimes bare.

Fig. 544.



Reef of rocks.

Fig. 543.



Sandy shore, with hillocks.

Fig. 545.

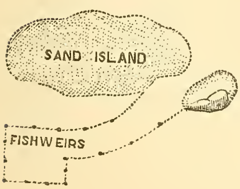


Fig. 546.

Fig. 547.

Fig. 548.

Fig. 549.

Fig. 550.

Anchorage for ships.

Fig. 551.

Buoys.

Fig. 552.

Light-house.

Fig. 553.

Anchorage for coasters.

Fig. 554.

Wrecks.

Fig. 555.

Signal-house.

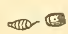
Fig. 556.

Rocks always covered.

Fig. 557.

Harbors.

Fig. 558.



Channel-marks.

PART VI.

UNDERGROUND OR MINING SURVEYING.

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741. It has three objects:

1. To determine the directions and extent of the present workings of a mine.

2. To find a point on the surface of the ground from which to sink a shaft, to meet a desired spot of the underground workings.

3. To direct the underground workings to meet a shaft or any other desired point.

It attains these objects by a combination of surveying and leveling.

CHAPTER I.

SURVEYING AND LEVELING OLD LINES.

742. First Object. To determine the direction and extent of the present workings of a mine.

We have to measure:

1. Azimuths, or directions right and left.

2. Lengths or distances.

3. Heights, or distances up and down, either by perpendicular or angular leveling; usually the latter.

This being done, the relative positions of all the points are known by their three rectangular co-ordinates.

They are referred, first, to a vertical plane (which may be either north and south, or pass through the first line of the survey); second, to another vertical plane, perpendicular to the preceding one; and, third, to a horizontal datum-plane.

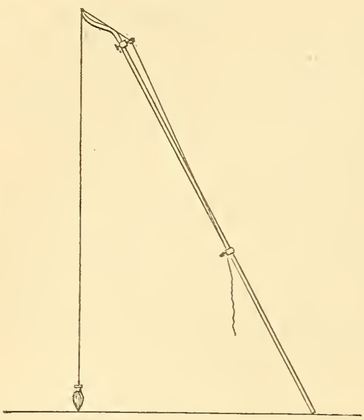
743. In making an underground survey, the same rules and principles apply as to work on the surface. Some differences in methods and detail are necessary, on account of the entire dependence upon artificial light, and the circumscribed limits within which the surveyor is obliged to work.

As the headings and air-ways of a mine are generally driven far in advance of the other workings, it is essential that they should be surveyed with great accuracy, in order to give an intelligent idea of the territory about to be mined. It is also essential, in order that they may serve as a base from which to continue and check the surveys of the interior portions of the mine.

744. Stations. The work may often be much simplified by a careful selection of the stations. See that the average distance between them is as long as possible; that they are convenient for future use; and are so chosen that the instrument can be easily set over them. It is also important to locate them where they can be easily and permanently marked. Frequently a station may be so chosen that several different sights can be taken from it—thus economizing much time.

745. Marking the Stations. Whenever possible, all stations should be plainly marked with white paint, and given some distinguishing number or letter. This is necessary for use in extending the surveys at some future time, and also to make the map of use when wishing to identify some particular locality in the mine. The precise point may be indicated by an iron spud like a horseshoe nail, with a hole through the head large enough to take the line of a plumb-bob or plummet-lamp. The spud is driven in a crack in the roof, or in a wooden plug which is driven in a hole that has been previously drilled. The objections to this method are, the length of time it takes to get the spuds in the roof, and also the difficulty in using them when the roof is high. Another objection is that mischievous workmen will drive the spuds up in the plugs out of sight with the ends of their drills. Probably, as satisfactory a way as any to mark the point is to drill a shallow hole, about one eighth of an inch in diameter, in the center of a painted +, or a circle about six inches in diameter. Fig. 559 shows a very convenient device for marking the stations, and plumbing down from them when the roof is high. It is made of light gas-pipe, about half an inch in diameter, and of any convenient length. At one end is a drill; the other end is bent about three inches out of line, and tapered at the end to fit into the hole made with the drill. There is also a notch in the end large enough to hold the line of a plumb-bob. Attached to the pipe are two rings with shanks about an inch in length. The lower one is fixed, the other is adjustable with a clamp-screw. The upper ring is split in the back wide enough to take a plumb-line easily. To use this device in marking the stations, first strike the drill against the roof, then twist it around a few times. This will generally make a mark large enough to be easily identified. Then reverse the instrument, put the handle of the paint-brush in the upper ring, adjust to the proper height, and clamp it fast. Put the claw, or notch, in the drill-hole and describe a circle, and also paint the number or letter. To plumb down from the point in the roof, remove the brush, put the plumb-line in the small notch, and through the upper ring, which can be easily done through the split. Hold the claw with the plumb-line in it against the roof at the proper point, then pay out the plumb-line until the plumb-bob reaches the bottom, when the point can be fixed. When not in use, bring the two rings together, gripping the plumb-bob between them, and clamp fast. Wrap the cord around the shanks of the rings, and fasten with a half-hitch.

FIG. 559.



746. Points for setting the Transit over. These may be made in a variety of ways, as a nail in a tie, a chalk × on a rail or stone, a × scratched with a measuring-pin, a speck of paint, or a spot of white paint with a speck of coal in the center. If the chalked × is too coarse, rub away a portion of it with the finger. Special cases may arise where it would be advisable to carry along weights of lead with a short piece of brass wire projecting above the surface, to give a precise point. A center-mark on the top of the telescope will afford the means of placing the transit in position under a plumb-bob suspended from the roof.

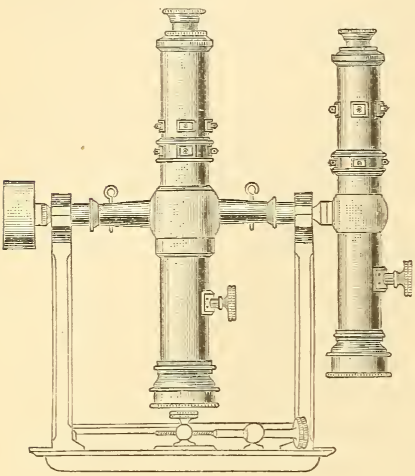
747. Giving the Sights. A measuring-pin, if held plumb, with a lamp in front, and a little to one side, makes a very good sight. The pin should be whitened with chalk to make a background for the cross-hair. The cord of a plumb-bob can be seen distinctly up to three or four hundred feet, if a piece of white paper is held behind it and a light is held in front. Care must be taken not to mistake the shadow of the line for the line itself. It is difficult to hold the plumb-bob steady unless it can be hung in the iron spuds mentioned in Art. 745, or the device shown in Fig. 559 is used. Where the mine is smoky, or the sights are very long, sight to the center of the blaze of the lamp, which must be carefully plumbed over the point. To meet cases of this kind, the plummet-lamp has been devised (Fig. 560). It consists of a brass lamp hung in gimbals and supported by two chains. The lamp terminates below in a conical plummet. A shield at the top prevents the flame from burning the string. The sight is taken to the center of the flame. These lamps are generally used in pairs, for back-and-forward sights. They are inconvenient to use, as they require the iron spuds with a hole through the head to support them from the top. Where the roof is high, it is difficult to get up to the station to put the string through the hole.

FIG. 560.

If care is taken not to make them too heavy, they can be supported with the device mentioned in Art. 745. Another objection is the additional load they impose upon the party to carry.

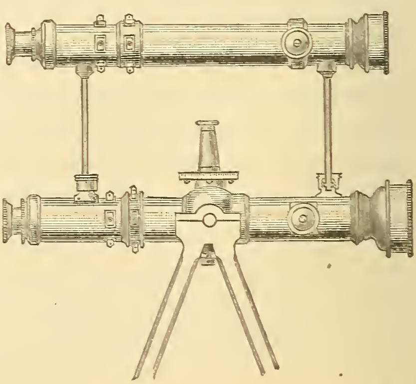
748. The Transit. The essential features of a transit to be used for surveys in mines are that the verniers should be so placed as to be easily read by lamp-light, and that the marking should be very distinct, on account of the imperfect light available. Again, the instrument should not be too heavy, as there is often difficult climbing to be done over fallen rock and other mine *débris*. If the instrument be easily detached from its tripod, it will often be found a convenience, as thereby the load may be lightened and the instrument itself more carefully carried and more fully protected.

Fig. 561.



Graduations on solid silver are apt to be tarnished by the powder-smoke of the mines. Some makers claim to obviate this by making the graduations on platinum.

Fig. 562.



If the telescope has a level attached, see that the lamp is not held under it for any length of time, as the heat may explode it. Accidents of this kind have occurred, producing serious results.

In one form of mining

transit an extra telescope is attached on one side, as shown in Fig. 561, and is balanced by a weight on the opposite side. The advantage of this form is, that sights may be taken vertically up or down, as is sometimes necessary in connecting the underground surveys with those on the surface.

In another form, the extra telescope is attached to the transit-telescope, as shown in Fig. 562.

The diagonal prism, shown in Fig. 211, may be used with advantage on the extra telescope.

749. Taking the Sights. The beginner will at first have some trouble in catching the light through the telescope. A little practice will overcome this. Hold a lamp a little above the instrument, sight over the top of the telescope, and turn it until it points to the light which it is desired to observe. Now sight through the telescope, and turn it a little each way, until the eye catches the light. Clamp the instrument, and move the object-glass until the light looks like a large round blur. This will form a background on which the cross-hairs can be plainly seen. "Bisect" the blur, then focus the object-glass, and the cross-hairs will be so near the right place that there will be no trouble to find them in bisecting a plumb-line, or whatever else is sighted to. Some instruments have a reflector for illuminating the cross-hairs by throwing a light into the telescope (Fig. 210). The same result can be accomplished by holding a lamp two or three feet in front of the object-glass, and a little to one side, so as to be out of the line of sight.

750. Measuring the Angles. Proceed as in making a traverse on the surface, noting whether the angles are to the right or left. It is generally more satisfactory to put the vernier at zero every time rather than to survey or traverse by the back-angle. The instrument gets some hard usage, and when the surveyor reviews the angle, after having moved to the next station preparatory to measuring a new angle, he has the unsatisfied feeling of not knowing whether the upper motion has slipped, or that he read the angle wrong before. It is also more troublesome to set the vernier at odd degrees and minutes than at 0, in case there should be a slip of the upper motion. The surveyor should never omit to check the reading of his angles, either by noting whether the sum of the two readings on each side of the 0 of the vernier is equal to 180° or by repeating the angle. The latter method is the most satisfactory. If the graduated circle has a double row of figures reading 180° each way, and the deflection should be greater than 90°, it is only necessary to read the supplement or smaller angle, noting at the same time whether it reads to the right or left on the limb.

The needle-readings, which should always be taken, will prevent the gross error of getting into the wrong quadrant.

	BACK-SIGHTS.	ANGLES.	FORE-SIGHTS.	
Thus,	S. 30° 00′ W.	165° 00′ L	N. 45° 00′ E.	is the same
as	S. 30° 00′ W.	15° 00′ R	N. 45° 00′ E.	the needle,

showing that the last course should be N. E. instead of S. W., as the angle would seem to indicate.

The advantage of this method is that it is a little more convenient to use in working out the courses. It also relieves the surveyor of the inquiry as to whether his vernier has passed the 90°, and he should use the larger or smaller angle. He reads the vernier as it stands, and lets the needle determine the quadrant. It is almost impossible to set up an instrument so solidly that when the cross-hairs are put on a given point they will remain there for any length of time. For this reason it is best not to begin to measure the angle until everything is all ready; then measure and check by doubling it as quickly as can be done with accuracy. Occasions sometimes arise in which a surveyor has but a few hours in which to make an extended survey. For a necessity of this kind the use of three transits will be found to expedite the work very greatly. This prevents loss of time in setting the instrument over a given point, the work being carried on from the plumb-line of one instrument to that of the next.

751. Plumbing the Shaft. In order that the lines underground may be worked from the same meridian as those on the surface, they must be deflected from some line whose azimuth is known. Should it not be considered justifiable to depend upon the needle to determine the azimuth, and should it be impossible to enter the mine by a *slope* or a tunnel, the surveyor will be obliged to resort to plumbing the shaft. Two plumb-lines are carefully put into some known line on the surface, and their direction, which will be in the same line, is again taken at the foot of the shaft, as a meridian from which all the lines underground are deflected. As the two plumb-lines are necessarily but a few feet apart, and as the integrity of all the subsequent work depends upon the accuracy with which the direction of the line on the surface is reproduced by the plumb-lines at the foot of the shaft, it is necessary that extreme care should be exercised in doing the work. Much time will be saved by studying the local conditions of the shaft, and making thorough preparations before beginning the work. In the selection of wires, iron and steel are excellent, when new, as their strength enables a fine wire to support a heavy weight. The objection is that they rust and become treacherous, breaking at most inopportune times. Hard-rolled brass wire, though free from this objection, has to be very carefully used, as it is liable to kink, and then break. If it slips out of the hands while attaching the weights at the bottom, it will fly up the shaft in an almost inextricable tangle. Copper stretches and the weights have to be carefully watched to see that they do not touch the bottom of the vessel in which they are suspended. On the whole, however, it seems to give the best satisfaction. Have the wire wound on two strong reels, set in frames which can be securely anchored. The reels should have stops, so that the weights can be held at any point that may be desired.

752. Suspending the Wires. Nail two boards on the sides of the head-frame, at right angles to the line of sight, and about four feet from the ground. Place on each of these boards a scantling about twelve feet long, letting one end rest on the ground a little out of the line of sight. The upper end should project over the shaft far enough to clear the sides. Put the reels in position, about twenty feet back from the shaft, and also a little out of the line of sight, and anchor them securely. Fasten weights of about five pounds each to the ends of the wires, and pass them over the ends of the scantlings. Then pay out the wires until the bottom of the shaft is reached. Bring the wires approximately into line by tapping the scantlings with a hammer. In the mean time the assistants at the foot of the shaft will attach the large weights and place them in pails of water. When the signal is given that all is right below, the wires are brought precisely into line, putting in the wire farthest from the instrument first, then bringing the other to it. This can be very easily and accurately done by tapping the scantling gently with a hammer. Examine the wires from the top to the bottom of the shaft to be sure they touch no projecting points. Make all secure at the surface, and, before taking up the instrument to go below, review the work, to be sure that all is correct. Be very careful that no work is done over the head of the shaft while men are at work in the shaft at the foot, lest accidents should occur. At the bottom of the shaft nail two boards across the foot-frame, the same as at the surface. On these place two other boards, about ten inches wide and one quarter of an inch apart, and reaching across the shaft so that the wires will swing freely in the crack between them. These boards serve as a rest for the hand in steadying the vibrations of the wires. They also prevent drops of water from falling into the pails and producing currents which will move the weights. Take a small piece of board and bevel one edge slightly with a knife. Then lay it across the crack between the boards, and bring the beveled edge slowly up to one of the wires until it almost touches. Make a mark on the edge where it bisects the wire, then watch to see if the wire is perfectly still. In deep shafts the oscillations of the wire are very slow, and it is trying to the eye to watch them through the telescope until they are perfectly still.

Sometimes wires may be steadied by uniting them with a thread or string slightly shorter than the distance between them. The weights are also sometimes placed in oil or mercury. Molasses has also been suggested. If it is impossible to perfectly steady the wires, fasten them at the mean of the oscillations.

753. Getting the instrument into line is not an easy task for the beginner, owing to the difficulty in distinguishing between the lines when looking through the telescope. This is overcome by an assistant holding a white paper with a light alternately in front of and behind the wire farthest away. Another method is to put a couple of round rings in the first wire, and then the second wire can be seen through the openings in the rings. Another very good way is to tack a piece of sheet-iron, of about eight by ten inches, to a piece of board of the same size. Make a hole about one sixteenth of an inch in diameter in the center of the sheet-iron, and at the height of the center of the blaze of a mine-lamp above the board. Bend the sheet-iron so that it will be slightly convex with the bend at the hole. Place this contrivance behind and as close as possible to the rear wire, with the small hole bisecting it. Place a lighted lamp behind the sheet-iron so that the blaze will cover the hole. Put a small piece of board with white paper tacked on it behind the first wire; also a lighted lamp in front. The instrument can now very quickly be brought into line with the first wire, and the point of light at the second. Verify by holding white paper, with a light, behind the second wire, and noting whether it is entirely concealed by the other wire.

If possible, use two transits, placed on opposite sides of the shaft, then verify by seeing if they bisect each other's plumb-lines. Do not try to set up the instrument too far away, as it increases the difficulty of getting a clear sight of the wires. Watch, also, that the shadow of the wire is not mistaken for the wire itself. When all is completed, mark the line permanently for future use. Where great accuracy is required, plumb the shaft several times, and take the mean, depending also upon which of the several plumbings has been done with the least probability of error.

754. Second Method. When there are two shafts convenient to each other, let a plumb-line down each shaft; then connect them by a careful survey, both on the surface and underground. Calculate the course between the lines on the surface. Calculate also the course between the wires underground from an assumed meridian. The difference between the two courses will be the correction to be applied to the underground courses to make them correspond with the azimuth assumed on the surface.

755. Third Method. Use a transit with a telescope outside the standards (Fig. 561). Place the instrument in line directly over the shaft, then produce the line to the foot of the shaft by revolving the telescope so as to sight directly down the shaft. Get two points as far apart as possible at the foot of the shaft, then stretch a fine wire carefully over them, producing the line far enough to make a convenient station over which the transit can be set. In shallow shafts, where communication between the top and bottom is easy, the wire may be lined in directly with the instrument.

756. Fourth Method. If no local attraction exists, and extreme accuracy is not required, use the needle. The needle can be read to within five minutes, and the errors have the probability of correcting each other in the different courses taken. If there is only time and means to do ordinary work, it is better to depend exclusively upon the needle than upon plumbing and deflections poorly done.

The beginner should remember that the greatest care is necessary, and that, when his best has been done, there are possibilities of error. A surveyor who appreciates these errors will not fail to verify his work by repetitions at a later date; as, by making a connection with other openings to the surface, such as a drill-hole, an opening for air, or a connection through a neighboring mine, should such an opportunity present itself.

757. Keeping the Notes. These will depend very much upon the character of the work to be done. Some surveyors prefer to use two note-books. In one are recorded all the instrumental work done with the transit, together with the stations, and whatever explanatory remarks may be necessary. In another, made especially for the purpose, are kept all measurements and references, accompanied with a sketch showing where they were taken. Where the party is large enough, it may be divided so that both of these kinds of work may be kept going at the same time. Another method, much used, is to keep all the work in one book, where everything will be all together when it is wanted. By having the figures represent certain things when in particular places, and the use of a few symbols and small sketches in special localities, a note-book kept in this manner can generally be made to convey all needed information. Below will be found the right- and left-hand pages of a note-book kept in this manner; also a map showing the portion of the mine included in the survey of which the notes are a part.

In the first column are the numbers of the stations; also P ×, indicating that the station is marked, and in what manner. In the second column are the needle-courses of the back-sights. The third column shows the angles, with R. and L. for right and left. Fourth column, the needle-courses of the fore-sights; the corrected courses can afterward be placed above them in red ink. Fifth column, distances. Sixth column, slopes, and whether ±. Seventh column, height to roof. On the right-hand page, station 1 would be called out by the chairman as follows: Produce 1 and 2 back. At 12, 4 right; at 20, pillar 7 right; at 25, 2 left; at 50, leave point for future reference; at 0, 5 right and 9 left; at 25, 3 right and 8 left; at 58, 1 right and 10 left; at 58, entrance right, 8 wide and walled; at 100, 9 right and 3 left; at 119, entrance right, 8 wide and walled; at distance, 8 right and 2 left, etc.

There will occur to the surveyor, in practice, various symbols and abbreviations which he can use to lessen the labor of recording.

March 4, 1886.—NEAR FOOT OF SHAFT 14.

Set up at point on line of × 52 and × 51, produced 39·6 from × 51. B. S. on × 52.

		BACK-SIGHTS.	ANGLES.	FORE-SIGHTS.	DISTANCES.	SLOPE ±.	HEIGHT TO ROOF.
		° ′	° ′	° ′		° ′	
P. × 70.	0	N. 55–30· W. N. 56–50· W.	40–15 L.		39·2	−0–45	Rail. 7·42
P. × 71.	1			S. 84–15 W. S. 83–00· W.	121·0	+2–05	Rail. 10·25
P. × 72.	2	S. 85–30· W.	22–06· R.	N. 73–39 W. N. 72–10· W.	126·0	+0–55	Pave. 9·73
From 2.		N. 73–00· W.	84–25· R.	S. 10–46 W. S. 12–15· W.	93·0	+7–35	Pave. 5·21
	3	N. 73–00· W.	12–29· L.	N. 86–08 W. N. 86–05· W.	104·3	+1–02	Pave. 11·43
P. × V. 1.	A.	N. 86–00· W.	74–27· L.	S. 19–25 W. S. 19–30· W.	84·5	+10–02	Tie. 4·23
P. × 74.	4	N. 86–00· W.	89–55· L.	N. 3–57· E. N. 4 00· E.	41·8	−3–01	Tie. 6·75
P. × 75.	5	N. 4–00· E.	88–39· L.	S. 84–42· E. S. 84 40 E.	78·3	−0–32	Tie. 7·21
P. × 76.	6	S. 84–45· E.	10· 09· R.	S. 74–33· E. S. 74–25· E.	125·7	−0–22	Rail. 7·35
P. × 77.	7	S. 74–30· E.	16–59· L.	N. 88–28· E. N. 89–00· E.	144·9	−0–08	Pave. 14·12
From 77.		S. 86–35· E.	3–07· L.	N. 85–21· E. N. 89–55· E.	217·0	−0–15	Pave. 7·52
P. × H. From 77		S. 86–35· E.	43–17· R.	N. 48–15· W. N. 50–30· W.	73·6	−4–12	Pave. 6·25
	8	S. 86–35· E.	33–34· R.	S. 57–58· E. S. 53–35· E.	99·3	−0–30	Rock. 7·15
	9	S. 53–35· E.	37 16· L.	N. 85–46. E. N. 89–10· E.	N. 85°–47′ E. Error 0°–01′.		

Begin at P. × V. 1 above to run short chambers.

P. × V. 2.	1	S. 80–45· E.	81–44· R.		36·40	−2·01	Pave. 4·92
P. × V. 3.	2	S. 1–00· W.			20·00	+15 08	Pave. 5·23
P. × V. 5.	3	N. 85 15· E.	81–24· L		79·80	−6·30	Pave. 5·21
	4	S. 3–50· W.			43·00	+25·00	Rail. 8·20

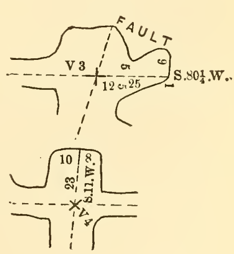
Set up at ⊕ 53 on line between × 74 and × 75. B. S. on × 74.

	S. 84–42· E. S. 84–40· E.	80 03 L.				
P. N.			N. 15–15· E.	77·20	−10–12	8·7

R. = right. ⊕ = mark for future use. O = width of place is put in circle.
L. = left.] = face and stopped. P. = pillar, also = painted.

	18, door 21· P. R./10·5 39·2/8·11
12/4— 20· P. R./7—	25/—2 — 50 ⊕ { (8)/58· R (8)/119· R 0/8·11 25/3 8 50/1·10 100/9·3 dist./8·2
	15/4·6 34· P. L./3–8 50/2·— 60/6·— 77· F. R. L./5·6 80·2 ⊕/5·6 81· P. R./5 6 100/4·7 dist./5—
(11) 40 L. (10) 50· R. (16) 95· R.	7· P. R./3·10 40/10·10 60/10·16 75/10·18 93 ⊕/10·12
(8)/30· R. (7) 102· R.	57, door. 20/8·3 (20) 30· ch. L./5–7 50/9·2 (23) 65· ch. L./8–5 100/8·2 (18) 103· ch. L./10·3 124·3 P. × 73/7–3 + 10 face]
(10) 20· R. (10) 50 R. (8) 48. L.	3· P. L./15·3 20/12·5 48/12·10 83/10·— + 10 face. Up ch.
	10 rb./3·4 32 N. R./4·3 + 6· F. R.
— 13/12·4 face] }	53 ⊕ (7)/70 R. 25/10·6 50 P. L./8·8 dist./9·6
	25/11 3 50/10·6 65/12·2 80/22·4 85· P. R./8·6 100/9·7 dist./7·7
(10)/15· R. (8)/7·8· R.	20/8·4 50/10·3 75/13·3 100/14·2 120/15·1 dist./14·3 138· P. L./15·3
Close on P. × 40·	15/12·5 25 P. R./6·5 50/8·3 70/10·3 100/9·3 125/10·3 150/10·3 170/11·4 200/9·7 207/7·7 dist./10·3
	5 P. L./12·1 50/10·3 73/8·7 + 10 stopped by water.
To P. × 50.	10/6·— 15/2·— 25 door & P. R. & L./2–9 35/—10 56·9 to ⊕ 50 Course 1. 50/4·8 · 75/3·7 · 85/8·8 · 97/8·11 ·
To P. × 49.	

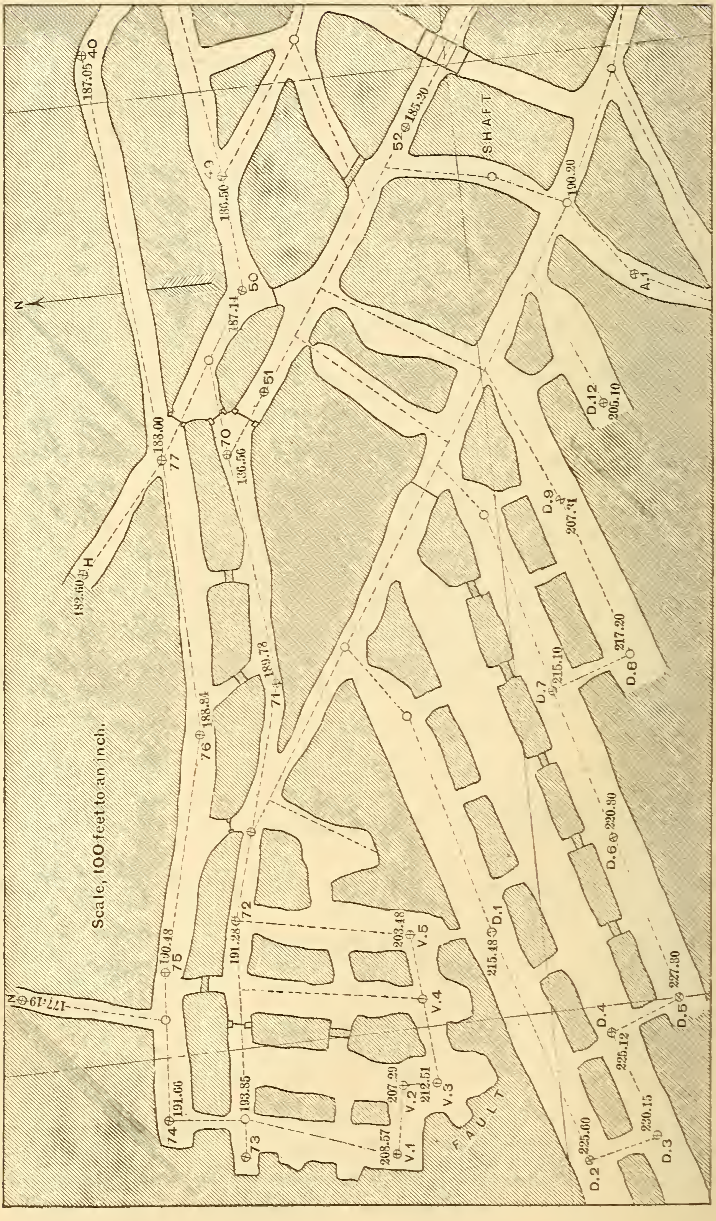
	10· rb./12·5 25· N. R./—·7 + 11 fr. R. 20/12·5
12· L.	12/15·12 30/10·17 40/8·18 50/0·2 face]
Close on ⊕ 93· Sta. 2 top of page }	18 rb./8·10 44 P. × V. 4/face 23 R.] 73· rb./8·7 + 12 fr. R.
Close on × D. 1 in Evan Jones's cha.	10/5·11 15/3·3 25/3·3 N. R. ch. 40 + 15· fr.



	10 rb./5·4 20/7·4 30/7·3 50/8·3 dist./8·3 + 7 face.

F. R. = far rib. ⌐ = blind entrance. rb. = rib. hdg. = heading.
N. R. = near rib. dist. = distance. ch. = chamber. Pave. = pavement.

FIG. 563.


Scale, 100 feet to an inch.
SHAFT
FAULT
N
40
187.05
49
186.50
50
187.14
51
52
185.20
190.20
A.1
77
183.00
70
186.56
H
182.60
71
189.78
76
188.84
75
190.48
72
191.28
N
177.19
74
191.66
73
193.85
V.1
208.57
V.2
207.29
V.3
212.51
V.4
V.5
203.48
D.1
215.48
D.2
225.60
D.3
230.15
D.4
225.12
D.5
227.30
D.6
220.80
D.7
215.10
D.8
217.20
D.9
207.71
D.12
205.10

758. Tabling the Survey. On pages 514 and 515 will be found a form and the tabling of the above field-notes for office use and record. It is best to have a specially prepared book already ruled to the required form. All the work of tabling can then be done in this book. Should there ever be an occasion to review the work, it can easily be found.

The two double columns headed 1 and 2 are for convenience in taking down the numbers as they are called off from Gurden's "Traverse Tables," which are to single minutes, and distances to one hundred feet. For convenience in description, we will suppose two persons, A and B, to be tabling the above survey. A will take the sheet or book on which have been recorded the stations, corrected courses, distances, and slopes, and call out the angle, which in the present case we will suppose to be N. 55° 30′ W., distance 39·19. B finds this in the book of tables, and on the edge of a sheet of blank paper checks the heavy line on the center of the page; also, the two minute columns. A then calls out the distance, 39·19, which B sets down on his sheet of paper, and then, using his paper as a straight-edge, slides it down the page until he comes to 39, taking care to keep the check on the center line. He will then call out the numbers under the checks for the minute columns, *always* reading the left-hand one first, to A, who will record them as he receives them in columns 1 and 2. The same operation is repeated for the 19. A will then call out the next angle, and, while B is searching for it, he will add the numbers given, and, if he has time, carry the results out to the proper columns of N., S., E., and W. A glance at the course, noting whether it is greater or less than 45°, will tell him whether the larger number should be put in the column of Latitude or Departure. The same operation is repeated for all the courses.

For convenience in plotting and calculations, the latitudes and departures should all be referred to a common origin of co-ordinates. In this survey the origin is taken at the west plumb-line of the shaft. Station 51 has been found by previous work to have latitude north + 112, and departure west 159. In like manner, 51 has been found to have a + elevation of 187·70. The slopes and distances should be reduced first, then the

STATIONS.	COURSE.	DISTANCE.	SLOPE + OR —.	SLOPE DISTANCES REDUCED.		COURSE AND DISTANCE REDUCED.	
				1st.	2d.	1st.	2d.
70.............	° ' N. 55·30 W.	*39·19* 39·20	° ' —0·45	38·99 ·20 39·19	0·51 ·00 —0·51	32·14 ·16 32 30	22·09 ·11 22·20
71............	S. 84·15 W.	*120·96* 121·00	+1·31	99·97 20·99 120·96	2·66 ·56 +3·22	119·40 ·96 120·36	12·02 ·09 12·11
72............	N. 73 39 W.	*125·99* 126·00	+0·41	99·99 26·00 125·99	1·19 ·31 +1·50	95·96 23·99 ·95 120·90	28·15 7·04 ·28 35·47
73 is 20 beyond station......	N. 86 08 W.	*104·28* 104·30	+1–02	99·98 4·30 104·28	1·80 ·77 +2·57	99·77 3 99 ·28 104·04	6·74 ·27 ·02 7·03
74....	N. 3 57 E.	*41·74* 41·80	—3 01	40·94 ·80 41·74	2·15 ·04 —2·19	40·90 ·73 41·63	2·82 ·05 2·87
75............	S. 84 42 E.	*78·30* 78·30	—0 52	78·00 ·30 78·30	1·18 ·00 —1·18	77·67 ·30 77·97	7·20 ·03 7·23
76............	S. 74 33 E.	*125·70* 125·70	—0 45	120·00 5·70 125·70	1·57 ·07 —1 64	115·65 5·49 121·14	31·97 1·51 33·48
77............	N. 88 28 E.	*144·90* 144·90	—0 20	140·00 4·90 144·90	0·82 ·02 —0·84	139·95 4·90 144·85	3·75 ·12 3·87
Point on line between 77 & 50	S. 57–58 E.	*56·89* 56·90	—1 00	55·99 ·90 56·89	0·98 ·00 —0·98	47·47 ·75 48·22	29·70 ·47 30·17
Close on 70....	S. 84 15 W.	*50·00* 50·00	—0 32	50·00	—0·46	49·75	5·01
From 72 to V. 5.	S. 10 46 W.	*91·65* 92·46	+7·35	91·20 ·45 91·65	12·14 ·06 + 12 20	89·40 ·64 90·04	17·00 ·13 17·13
From 20 back of 73 to V. 1....	S. 19 25 W.	*82·21* 84·50	+10 02	82·72 ·49 83·21	14·63 ·09 +14·72	78·28 ·20 78·48	27·59 ·07 27·66
Old sta. From 77 to 40...	N. 85 21 E.	*216·99* 217·00	—0 15	209·99 7·00 216·99	·92 ·03 —0·95	209·31 6·97 216·28	17·02 ·57 17 59
From 77 to H...	N. 48 15 W.	*73·49* 73·68	—4 12	72·81 ·68 73·49	5·35 ·05 —5·40	54·46 ·37 54·83	48·61 ·33 48·94
Old sta. From 77 to 50...	S. 57 58 E.	*99·30* 99·30	—0 30	99·00 ·30 99·30	·86 —·86	83·93 ·25 84·18	52·51 ·16 52·67
From V. 1 to V. 2......	S. 80 45 E.	*36·38* 36·40	—2 01	35·98 ·40 36·38	1·27 ·01 —1·28	35·53 ·37 35·90	5·79 ·06 5·85
V. 3......	S. 1 00 W.	*19·30* 20·00	+15·08	19·30	+5 22	19·00 ·30 19·30	0·33 ·01 0·44
V. 5...... Not deflected.	N. 85 15 E.	*79·28* 79·80	— 6 30	78·49 ·79 79·29	8·94 ·09 —9·03	78·73 28 79·01	6·54 ·02 6·56
Close to D. 1.	S. 3 50 W.	*43·00* 44·70	+15 46	42·33 ·67 43·00	11·96 ·19 +12·15	42·90	2·87
From 74 to ⊕ 53.	S. 84–42 E.	53·00	—0·52	53·00	—0·80	52·77	4·90
From ⊕ 53 to N.	N. 15–15 E.	*75·97* 77·20	—10·12	75·78 ·19 75·97	13·63 ·04 —13·67	72·36 ·94 73·30	19·73 ·25 19·98

N.	S.	E.	W.	ALGEBRAIC SUM OF LATITUDES.	ALGEBRAIC SUM OF DEPARTURES.	ALGEBRAIC SUM OF SLOPES.	HEIGHT OF ROOF.
				+112·00	−159·00	+187·07	See sta.50.
22·20			32·30	+134·20	−191·30	+186·56	Rail. 7·42
	12·11		120·36	+122·09	−311·66	+169·78	Rail. 10·25
35·47			120·90	+157·56	−432 56	+191·28	Pave. 9·73
7·03			104·04	+164·59	−536·60	+193·85	Pave. 11·43
41·63		2·87		+206·22	−533·73	+191·66	Tie. 6·75
	7·23	77·97		+198·99	−455·76	+190·48	Tie. 7·21
	33·48	121·14		+165·51	−334·62	+188·84	Rail. 7·35
3·87		144·85		+169·38	−189·77	+188·00	Pave. 14·12
	30·17	48·22		+139·21	−141·55	+187·02	
	5·01		49·75	+134·20	−191·30	+186·56	
88·00	88·00	395·05	395·05				
	90·04		17·13	+67·52	−449·69	+203·48	Pave. 5·21
	78·48		27·66	+86·11	−564·26	+208·57	Rock. 4·23
17·59		216·28		+186·97	+26·51	+187·05	Pave. 7·52
48 83			48·94	+218·42	−244·60	+182·60	Pave. 6·25
	52·67	84·18		+116·71	−105·59	+187·14	Rock. 7·15
	5·85	35·90		+80·26	−528·36	+207·29	Pave. 4·92
	19·30		0·34	+60·96	−528·70	+212·51	Pave. 5·23
6·56		79·01		+67·52	−449·69	+203·48	Pave. 5·21
	42·90		2·87	+24·62	−452·56	+215·63	Rail. 8·20
	4·90	52·77		−201·30	−180·96	+190·86	Pave. 7·50
73·30		19·98		+274·60	−460·98	+177·19	Rail. 8·45

corrected horizontal distances placed over the others in red ink.

Problem. It is desired to drive the heading from H so that it will intersect the slope at N. Required, the course and distance. From the columns of total latitudes and departures in the sheet of calculations take :

	Latitude.	Departure.
N =	+ 274·60	− 460·98
H =	+ 218·42	− 244·60
	+ 56·18	− 216·38

Tangent, of course, equal departure divided by latitude.

$$\begin{aligned} \log.\ 216{\cdot}38 &= 2{\cdot}3352171 \\ \log.\ 56{\cdot}18 &= 1{\cdot}7495817 \\ \text{tem. } 75 - 27 &= 10{\cdot}5856354 = 75^\circ - 27' = \text{course.} \\ \log.\ 56{\cdot}18 &= 1{\cdot}7495817 \\ \cos.\ 75^\circ\,27' &= 9{\cdot}4000625 \\ & \quad 2{\cdot}3495192 = 223{\cdot}62 = \text{distance.} \end{aligned}$$

N, being north and west of H, shows the course to be N. W., or N. 75° − 26 W. 223·36.

Unless in special cases where great accuracy is required, the more common method of solving this and similar problems is to take the course and distance from the map with a protractor and scale, this being sufficiently accurate for all practical purposes.

759. Making the Map. If the map is to be much handled, use the best quality of cloth-backed paper. The edges should be bound with linen tape, which, if sewed, should be double-stitched, with about three stitches to the inch. If the stitches are made closer than this, the binding will break off in the line of the needle-holes. Ascertain from existing maps, or whatever data may be at hand, the most advantageous direction for the meridian of the survey to assume on the map. Fix also upon a point for the origin of coordinates. Begin at the origin and rule the paper into five- or ten-inch squares, parallel with the meridian of the survey. Very great care is required in doing this work, in order to make all the squares check precisely with the scale and be rectangular. Owing to the expansion and contraction of the paper, the work of laying out the squares should be concluded on the same day it is started. In addition to the underground workings, the map should show all land-lines, dwellings, roads, streams, ponds of water, and any other features of the surface that may have a bearing on an intelligent working of the mine. Both surveys should be referred to the same origin of co-ordinates. In plotting an underground traverse, it is generally more convenient to locate only every fifth or tenth station by its co-ordinates, and use a protractor for filling in the balance.

Take a paper protractor, and letter it N. S. E. W., and fix it at any convenient place on the paper, so its N. and S. points will correspond with the meridian of the survey. Fasten with weights; then transfer the courses from the protractor to where they are wanted on the map, scaling off the distances as required. The stations that have been located by ordinates will check the slight errors in the plotting from the protractor. Having plotted all the courses, proceed to fill in the interior work from the references and sketches shown on the right-hand page of the note-book.

In inking the map, use only colors that will wash. A diluted solution of bichromate of potash mixed with India-ink will prevent spreading of the lines when touched with a wet tinting-brush.

The map should show all the survey-stations, stoppings of entrances, inclination of strata, and elevation of the stations above tide or other datum.

When different "levels" are to be represented, with their connecting shafts, etc., "isometrical projection" has been used, but "military or cavalier projection" is best.

CHAPTER II.

LOCATING NEW LINES.

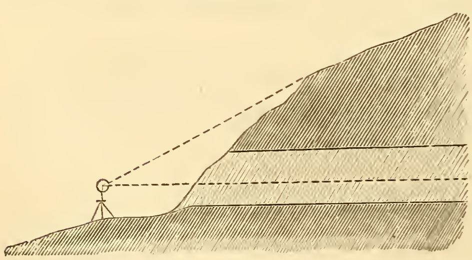
760. Second Object. To determine, on the surface of the ground, where to sink a shaft to meet a desired point in the underground workings.

To do this, repeat on the surface of the ground the survey made under it—i. e., trace on it the courses and distances of the galleries, or their equivalents (Art. 764).

The chief difficulty is to get a starting-point, and to determine the direction of the first line.

761. When the Mine is entered by an Adit (Fig. 564). Set the transit at the entrance, and get the direction of the adit, and prolong it up the hill—i. e., in the same vertical plane. The third adjustment is here important.

Fig. 564.



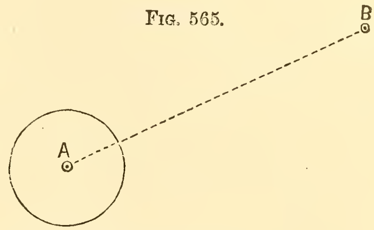
If the line has to be prolonged by setting the instrument farther on, the second adjustment is important.

762. When the Mine is entered by a Shaft. Get the magnetic bearing of the first underground line, at the bottom of the shaft, with great care. Bring up the end of the line through the shaft by a plumb-line, and set the compass over this point. Set out a line with the same bearing and length as the first underground line, and repeat the succeeding courses.

WHEN THE COMPASS CAN NOT BE SET OVER THE POINT, proceed thus :

1. Find, by trial, a spot, as B (Fig. 565), which is in the correct course, and measure off a distance equal to the length of the first underground course, and then proceed as before.

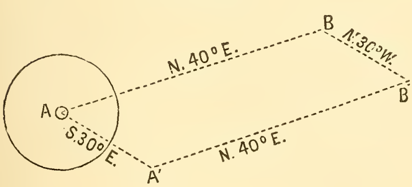
FIG. 565.



2. *Otherwise.* Set up anywhere, as at A′ (Fig. 566), take the bearing and distance of A from A′; run a line corresponding with the one underground, from A′ to B′. Repeat the course A′A from B′ B; then A B is the desired line.

763. To dispense with the Magnetic Needle. *First Method.* Let down two plumb-lines on opposite sides of the shaft, so that their lower ends shall be very precisely in the underground line (see Art. 751).

FIG. 566.



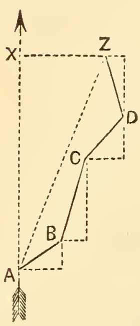
Second Method. Set, by repeated trials, two transits on opposite sides of the shaft, so that they shall at the same time point to one another, and each, also, to one of two points in the underground line. They will then give the direction of the line above-ground.

Third Method. If the telescope of the transit be eccentric, as in Fig. 561, set the instrument on a platform over the mouth of the shaft, so that the line of collimation of the telescope shall be in the same vertical plane with two points in the underground line, on opposite sides of the shaft. When the instrument is so placed that, in turning the telescope, the intersection of the cross-hairs strikes the two points in the underground line, the line of sight, when directed along the surface, will give the required line.

764. Having determined the first line, the courses of the underground survey may be repeated on the surface; or the bearing and length of a single line be calculated, which shall arrive at the desired point.

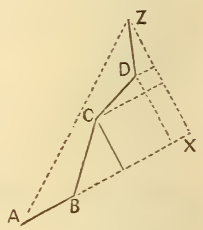
Let the zigzag line, A B, B C, C D, D Z (Fig. 567), be the courses surveyed underground, A being an adit, or at the bottom of a shaft, and Z the point to which it is desired to sink a shaft. It is required to find the direction and length of the straight line A Z.

Fig. 567.



When the compass is used, calculate the latitude and departure of each of the courses, A B, B C, etc. The algebraic sum of their latitudes will be equal to A X, and the algebraic sum of their departures will be equal to X Z. Then is tan. $ZAX = \frac{XZ}{XA}$; that is, the algebraic sum of the departures divided by the algebraic sum of the latitudes is equal to the tangent of the bearing. The length of the line A Z equals the square root of the sum of the squares of A X and X Z; or equals the latitude divided by the cosine of the bearing.

Fig. 568.



When the transit is used, instead of referring all of the lines to the magnetic meridian, as in the preceding case, any line of the survey may now be taken as the meridian, as in "traversing."

In Fig. 568 all of the courses are referred to the first line of the survey. As before, a right-angled triangle will be formed.

Tan. $ZAX = \frac{XZ}{XA}$, and the length of $AZ = \sqrt{\overline{AX}^2 + \overline{XZ}^2}$; or $AX \div \cos. XAZ$.

Two or more lines may be substituted for the single line in the two preceding cases; the condition being, that the algebraic sums of their latitudes and of their departures shall be equal to those of the underground survey.

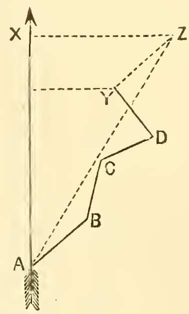
765. Third Object. To direct the workings of a mine to any desired point.

This is the converse of the second object. We repeat under the ground the courses run above-ground; or their equivalents, as in Art. 764.

In Fig. 569, let A B, B C, C D, D Y, be the present workings of a mine, and Z the shaft to which the workings are to be directed.

Find the latitude and departure of A Z. Then the difference between the algebraic sum of the latitudes of the underground courses already run, and the latitude of A Z, is the latitude of the required course; and the difference between the algebraic sum of the departures of the underground lines, and the departure of A Z, is the departure of the required course.

Fig. 569.

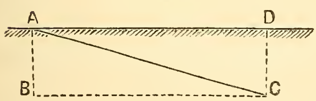


The length of Y Z equals the square root of the sum of the squares of its latitude and departure.

766. Problems. Most of the problems which arise in mining-surveying can be solved by an application of the familiar principles of geometry and trigonometry:

1. Given the angle which a vein makes with the horizon, and the place where it meets the surface, to find how deep a shaft at D will be required to strike the vein:

Fig. 570.



$$D\,C = A\,D \,.\, \tan.\, D\,A\,C.$$

2. Given the depth of the shaft D C, and the "dip" of the vein, to find where it crops out:

$$A\,D = D\,C \,.\, \cot.\, D\,A\,C.$$

3. Given the depth of a shaft when the vein "crops out," and the "dip" of the vein, to find the distance from the bottom of the shaft to the vein:

$$B\,C = A\,B \,.\, \cot.\, A\,C\,B.$$

If the ground makes an angle with the horizon, then the problems involve oblique-angled triangles instead of right-angled triangles, as in the preceding cases. Their solution, however, is quite as simple.

In the more difficult problems, the measurement of lines is required, one or both ends of which are inaccessible. (For a full investigation of this subject, see Part I, Chapter V.)

APPENDIX.

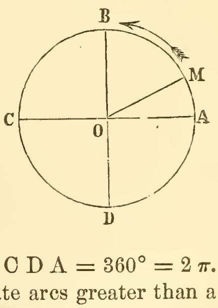
APPENDIX A.

*SYNOPSIS OF PLANE TRIGONOMETRY.**

1. Definition. Plane Trigonometry is that branch of mathematical science which treats of the relations between the sides and angles of plane triangles. It teaches how to find any three of these six parts, when the other three are given, and one of them, at least, is a side.

2. Angles and Arcs. The *angles* of a triangle are measured by the *arcs* described, with any radius, from the angular points as centers, and intercepted between the legs of the angles. These arcs are measured by comparing them with an entire circumference, described with the same radius. Every circumference is regarded as being divided into 360 equal parts, called *degrees.* Each degree is divided into 60 equal parts, called *minutes*, and each minute into 60 *seconds.* These divisions are indicated by the marks $^\circ\ '\ ''$. Thus 28 degrees, 17 minutes, and 49 seconds, are written $28^\circ\ 17'\ 49''$ Fractions of a second are best expressed decimally. An arc, including a quarter of a circumference and measuring a right angle, is therefore 90°. A semicircumference comprises 180°. It is often represented by π, which equals 3·14159, etc., or $3\frac{1}{7}$ approximately, the radius being unity.

Fig. 571.



The length of 1° in parts of radius $= 0{\cdot}01745329$; that of $1' = 0{\cdot}00029089$; and that of $1'' = 0{\cdot}00000485$.

The length of the radius of a circle in degrees, or 360ths of the circumference $= 57{\cdot}29578^\circ = 57^\circ\ 17'\ 24{\cdot}8'' = 3437{\cdot}747' = 206264{\cdot}8''$. †

An arc may be regarded as generated by a point, M, moving from an origin, A, around a circle, in the direction of the arrow. The point may thus describe arcs of any lengths, such as AM; $AB = 90^\circ = \frac{1}{2}\pi$; $ABC = 180^\circ = \pi$; $ABCD = 270^\circ = \frac{3}{2}\pi$; $ABCDA = 360^\circ = 2\pi$.

The point may still continue its motion, and generate arcs greater than a

---

* For merely solving triangles, only Articles 1, 2, 3, 5, 6, 10, 11, and 12 are needed.

† The number of seconds in any arc which is given in parts of radius, radius being unity, equals the length of the arc so given divided by the length of the arc of one second; or multiplied by the number of seconds in radius.

circumference, or than two circumferences, or than three; or even infinite in length.

While the point, M, describes these arcs, the radius, O M, indefinitely produced, generates corresponding angles.

If the point, M, should move from the origin, A, in the contrary direction to its former movement, the arcs generated by it are regarded as *negative*, or *minus;* and so too, of necessity, the angles measured by the arcs.

Arcs and angles may therefore vary in length from 0 to $+\infty$ in one direction, and from 0 to $-\infty$ in the contrary direction.

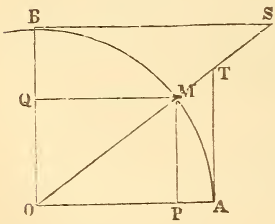
The *Complement* of an arc is the arc which would remain after subtracting the arc from a quarter of the circumference, or from 90°. If the arc be more than 90°, its complement is necessarily negative.

The *Supplement* of an arc is what would remain after subtracting it from half the circumference, or from 180°. If the arc be more than 180°, its supplement is necessarily negative.

3. Trigonometrical Lines. The relations of the sides of a triangle to its angles are what is required; but it is more convenient to replace the angles by arcs; and, once more, to replace the arcs by certain straight lines depending upon them, and increasing and decreasing with them, or, conversely, in such a way that the length of the lines can be found from that of the arcs, and *vice versa.* It is with these lines that the sides of a triangle are compared.* These lines are called *Trigonometrical Lines*, or *Circular Functions*, because their length is a function of that of the circular arcs. The principal trigonometrical lines are *Sines*, *Tangents*, and *Secants.* Chords and versed sines are also used.

The SINE of an arc, A M, is the perpendicular, M P, let fall, from one extremity of the arc, upon the diameter which passes through the other extremity.

FIG. 572.



The TANGENT of an arc, A M, is the distance, A T, intercepted, on the tangent drawn at one extremity of the arc, between that extremity and the prolongation of the radius which passes through the other extremity.

The SECANT of an arc, A M, is the part, O T, of the prolonged radius, comprised between the center and the tangent.

The sine, tangent, and secant of the complement of an arc are called the Co-SINE, Co-TANGENT, and Co-SECANT of that arc. Thus, M Q is the cosine of A M, B S its cotangent, and O S its cosecant. The cosine M Q is equal to O P, the part of the radius comprised between the center and the foot of the sine.

The *chord* of an arc is equal to twice the sine of half that arc.

The *versed-sine* of an arc, A M, is the distance, A P, comprised between the origin of the arc and the foot of the sine. It is consequently equal to the difference between the radius and the sine.

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* For the great value of this indirect mode of comparing the sides and angles of triangles, see Comte's "Philosophy of Mathematics" (Harper's, 1857), page 225.

The trigonometrical lines are usually written in an abbreviated form. Calling the arc A M $= a$, we write,

M P $=$ sin. a. A T $=$ tan. a. O T $=$ sec. a.

M Q $=$ cos. a. B S $=$ cot. a. O S $=$ cosec. a.

The period after sin., tan., etc., indicating abbreviation, is frequently omitted.

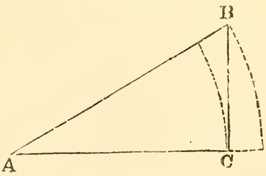
The arcs whose sines, tangents, etc., are equal to a line $= a$, are written,

$$\text{sin.}^{\overline{\quad}}\, a, \text{ or arc (sin.} = a);$$

$$\text{tan.}^{\overline{\quad}}\, a, \text{ or arc (tan.} = a); \text{ etc.}$$

4. The Lines as Ratios. The ratios between the trigonometrical lines and the radius are the same for the same angles, or number of degrees in an arc, whatever the length of the radius or arc. Consequently, radius being unity, these lines may be expressed as simple ratios. Thus, in the right-angled triangle A B C, we would have

Fig. 573.



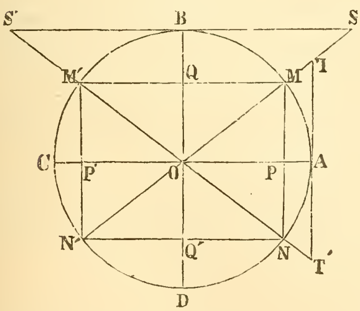
$$\text{sin. } A = \frac{B\,C}{A\,B} = \frac{\text{opposite side}}{\text{hypotenuse}}, \qquad \text{cos. } A = \frac{A\,C}{A\,B} = \frac{\text{adjacent side}}{\text{hypotenuse}},$$

$$\text{tan. } A = \frac{B\,C}{A\,C} = \frac{\text{opposite side}}{\text{adjacent side}}, \qquad \text{cot. } A = \frac{A\,C}{B\,C} = \frac{\text{adjacent side}}{\text{opposite side}},$$

$$\text{sec. } A = \frac{A\,B}{A\,C} = \frac{\text{hypotenuse}}{\text{adjacent side}}, \qquad \text{cosec. } A = \frac{A\,B}{B\,C} = \frac{\text{hypotenuse}}{\text{opposite side}}.$$

When the radius of the arcs which measure the angles is unity, these ratios may be used for the lines. If the radius be any other length, the results which have been obtained by the above supposition must be modified by dividing each of the trigonometrical lines in the result by radius, and thus rendering the equations of the results "homogeneous." The same effect would be produced by multiplying each term in the expression by such a power of radius as would make it contain a number of *linear* factors equal to the greatest number in any term. The radius is usually represented by r, or R.

Fig. 574.



5. Their Variations in Length. As the point M moves around the circle, and the arc thus increases, the sines, tangents, and secants, starting from zero, also increase; till, when the point M has arrived at B, and the arc has become 90°, the sine has become equal to radius, or unity, and the tangent and secant have become infinite. The complementary lines have decreased, the cosine being equal to radius or unity at starting and becoming zero, and the cotangent and cosecant passing from infinity to zero.

When the point M has passed the first quadrant at B, and is proceeding toward C, the sines, tangents, and secants begin to decrease, till, when the point has reached C, they have the same values as at A. They then begin to increase again, and so on. The table on page 527 indicates these variations.

The sines and tangents of very small arcs may be regarded as sensibly proportional to the arcs themselves; so that for sin. a'', we may write a . sin. $1''$; and similarly, though less accurately, for sin. a', we may write a . sin. $1'$.

The sines and tangents of very small arcs may similarly be regarded as sensibly of the same length as the arcs themselves.*

a being the length of any arc expressed in parts of radius, the lengths of its sine and cosine may be obtained by the following series:

$$\sin. a = a - \frac{a^3}{2 \,.\, 3} + \frac{a^5}{2 \,.\, 3 \,.\, 4 \,.\, 5} - \frac{a^7}{2 \,.\, 3 \ldots . 7} +, \text{etc.}$$

$$\cos. a = 1 - \frac{a^2}{2} + \frac{a^4}{2 \,.\, 3 \,.\, 4} - \frac{a^6}{2 \ldots . 6} +, \text{etc.}$$

Let it be required to find cos. 30°, by the above series.

$$30^\circ = \frac{30}{180}\pi = \tfrac{1}{6} \times 3{\cdot}1416 = {\cdot}5236.$$

Substituting this number for a, the series becomes, taking only three terms of it,

$$1 - \frac{({\cdot}5236)^2}{2} + \frac{({\cdot}5236)^4}{24} -, \text{etc.} = 1 - 0{\cdot}137078 + 0{\cdot}003130 = {\cdot}866052;$$

which is the correct value of cos. 30° for the first four places of decimals.

The lengths of the other lines can be obtained from the mutual relations given in Art. 7. Some particular values are given below:

sin. $30^\circ = \frac{1}{2}$	sin. $45^\circ = \frac{1}{2}\sqrt{2}$.	sin. $60^\circ = \frac{1}{2}\sqrt{3}$.
tan. $30^\circ = \frac{1}{3}\sqrt{3}$.	tan. $45^\circ = 1$.	tan. $60^\circ = \sqrt{3}$.
sec. $30^\circ = \frac{2}{3}\sqrt{3}$.	sec. $45^\circ = \sqrt{2}$.	sec. $60^\circ = 2$.

6. Their Changes of Sign. Lines measured in contrary directions from a common origin usually receive contrary algebraic signs. If, then, all the lines in the first quadrant are called positive, their signs will change in some of the other quadrants. Thus the *sines* in the first quadrant being all measured upward, when they are measured downward, as they are in the third and fourth quadrants, they will be negative. The *cosines* in the first quadrant are measured from left to right, and when they are measured from right to left, as in the second and third quadrants, they will be negative. The *tangents* and *secants* follow similar rules.

The variations in length and the changes of sign are all indicated in the following table, radius being unity. The terms "increasing" and "decreasing" apply to the lengths of the lines without any reference to their signs:

* Consequently, the note on page 523 may read thus: The number of seconds in any very small arc given in parts of radius, radius being unity, is equal to the length of the arc so given divided by sin. 1.

Lengths and Signs of the Trigonometrical Lines for Arcs from 0° to 360°.

Arcs.	0°	Between 0° and 90°.	90°	Between 90° and 180°.	180°
Sine.......	0	+, and increasing,	+1	+, and decreasing,	0
Tangent.....	0	+, and increasing,	$\pm\infty$	—, and decreasing,	0
Secant......	+1	+, and increasing,	$\pm\infty$	—, and decreasing,	—1
Cosine......	+1	+, and decreasing,	0	—, and increasing,	—1
Cotangent....	$\pm\infty$	+, and decreasing,	0	—, and increasing,	$\mp\infty$
Cosecant....	$\pm\infty$	+, and decreasing,	+1	+, and increasing,	$\pm\infty$

Arcs.	180°	Between 180° and 270°.	270°	Between 270° and 360°.	360°
Sine.......	0	—, and increasing,	—1	—, and decreasing,	0
Tangent.....	0	+, and increasing,	$\pm\infty$	—, and decreasing,	0
Secant......	—1	—, and increasing,	$\mp\infty$	+, and decreasing,	+1
Cosine......	—1	—, and decreasing,	0	+, and increasing,	+1
Cotangent...	$\mp\infty$	+, and decreasing,	0	—, and increasing,	$\mp\infty$
Cosecant....	$\pm\infty$	—, and decreasing,	—1	—, and increasing,	$\mp\infty$

From this table, and Fig. 574, we see that *an arc and its supplement* have the same sine; and that their tangents, secants, cosines, and cotangents are of equal length but of contrary signs; while the cosecants are the same in both length and sign.

We also deduce from the figure the following consequences:

$$\sin.\ (a^\circ + 180^\circ) = -\sin.\ a^\circ. \qquad \cos.\ (a^\circ + 180^\circ) = -\cos.\ a^\circ.$$
$$\tan.\ (a^\circ + 180^\circ) = \tan.\ a^\circ. \qquad \cot.\ (a^\circ + 180^\circ) = \cot.\ a^\circ.$$
$$\sec.\ (a^\circ + 180^\circ) = -\sec.\ a^\circ. \qquad \text{cosec.}\ (a^\circ + 180^\circ) = -\text{cosec.}\ a^\circ.$$
$$\sin.\ (-a^\circ) = -\sin.\ a^\circ \qquad \cos.\ (-a^\circ) = \cos.\ a^\circ.$$
$$\tan.\ (-a^\circ) = -\tan.\ a^\circ \qquad \cot.\ (-a^\circ) = -\cot.\ a^\circ.$$
$$\sec.\ (-a^\circ) = \sec.\ a^\circ. \qquad \text{cosec.}\ (-a^\circ) = -\text{cosec.}\ a^\circ.$$

An infinite number of arcs have the same trigonometrical lines; for, an arc a, the same arc plus a circumference, the same arc plus two circumferences, and so on, would have the same sine, etc.

"To bring back to the first quadrant" the trigonometrical lines of any large arc, proceed thus: Let 1029° be an arc the sine of which is desired. Take from it as many times 360° as possible. The remainder will be 309°. Then we shall have $\sin.\ 309^\circ = \sin.\ (180^\circ - 309^\circ) = \sin. -129^\circ = -\sin.\ 129^\circ = -\sin.\ (180^\circ - 129^\circ) = -\sin.\ 51^\circ$.

7. Their Mutual Relations. Radius being unity,

$$\tan.\ a^\circ = \frac{\sin.\ a^\circ}{\cos.\ a^\circ}. \qquad \cot.\ a^\circ = \frac{\cos.\ a^\circ}{\sin.\ a^\circ}.$$
$$\sec.\ a^\circ = \frac{1}{\cos.\ a^\circ} \qquad \text{cosec.}\ a^\circ = \frac{1}{\sin.\ a^\circ}.$$
$$\tan.\ a^\circ \times \cot.\ a^\circ = 1. \qquad (\sin.\ a^\circ)^2 + (\cos.\ a^\circ)^2 = 1.^*$$
$$1 + (\tan.\ a^\circ)^2 = (\sec.\ a^\circ)^2. \qquad 1 + (\cot.\ a^\circ)^2 = (\text{cosec.}\ a^\circ).$$

* The square, etc., of the sine, etc., of an arc, is often expressed by placing the exponent between the abbreviation of the name of the trigonometrical line and the

Hence, any one of the trigonometrical lines being given, the rest can be found from some of these equations.

8. Two Arcs. Let a and b represent any two arcs, a being the greater. Then the following formulas apply:

$$\sin.(a+b) = \sin. a \,.\, \cos. b + \cos. a \,.\, \sin. b.$$
$$\sin.(a-b) = \sin. a \,.\, \cos. b - \cos. a \,.\, \sin. b.$$
$$\cos.(a+b) = \cos. a \,.\, \cos. b - \sin. a \,.\, \sin. b.$$
$$\cos.(a-b) = \cos. a \,.\, \cos. b + \sin. a \,.\, \sin. b.$$
$$\tan.(a+b) = \frac{\tan. a + \tan. b}{1 - \tan. a \,.\, \tan. b}.$$
$$\tan.(a-b) = \frac{\tan. a - \tan. b}{1 + \tan. a \,.\, \tan. b}.$$
$$\cot.(a+b) = \frac{\cot. a \,.\, \cot. b - 1}{\cot. b + \cot. a}.$$
$$\cot.(a-b) = \frac{\cot. a \,.\, \cot. b + 1}{\cot. b - \cot. a}.$$
$$\sin. a \,.\, \sin. b = \tfrac{1}{2} \,.\, \cos.(a-b) - \tfrac{1}{2}\cos.(a+b).$$
$$\cos. a \,.\, \cos. b = \tfrac{1}{2} \,.\, \cos.(a+b) + \tfrac{1}{2}\cos.(a-b).$$
$$\sin. a \,.\, \cos. b = \tfrac{1}{2} \,.\, \sin.(a+b) + \tfrac{1}{2}\sin.(a-b).$$
$$\cos. a \,.\, \sin. b = \tfrac{1}{2} \,.\, \sin.(a+b) - \tfrac{1}{2}\sin.(a-b).$$
$$\sin. a + \sin. b = 2 \sin. \tfrac{1}{2}(a+b) \cos. \tfrac{1}{2}(a-b).$$
$$\cos. a + \cos. b = 2 \cos. \tfrac{1}{2}(a+b) \cos. \tfrac{1}{2}(a-b).$$
$$\sin. a - \sin. b = 2 \sin. \tfrac{1}{2}(a-b) \cos. \tfrac{1}{2}(a+b).$$
$$\cos. b - \cos. a = 2 \sin. \tfrac{1}{2}(a-b) \sin. \tfrac{1}{2}(a+b).$$
$$\tan. a + \tan. b = \frac{\sin.(a+b)}{\cos. a \,.\, \cos. b}.$$
$$\tan. a - \tan. b = \frac{\sin.(a-b)}{\cos. a \,.\, \cos. b}.$$
$$\cot. b + \cot. a = \frac{\sin.(a+b)}{\sin. a \,.\, \sin. b}.$$
$$\cot. b - \cot. a = \frac{\sin.(a-b)}{\sin. a \,.\, \sin. b}.$$

9. Double and Half Arcs. Letting a represent any arc, as before, we have the following formulas:

$$\sin. 2a = 2 \sin. a \,.\, \cos. a.$$
$$\cos. 2a = (\cos. a)^2 - (\sin. a)^2 = 2(\cos. a)^2 - 1 = 1 - 2(\sin. a)^2.$$
$$\tan. 2a = \frac{2 \tan. a}{1 - (\tan. a)^2} = \frac{2 \cot. a}{(\cot. a)^2 - 1} = \frac{2}{\cot. a - \tan. a}.$$
$$\cot. 2a = \frac{(\cot. a)^2 - 1}{2 \cot. a} = \tfrac{1}{2}(\cot. a - \tan. a).$$

---

number of the degrees in the arc, thus: $\text{Sin.}^2 a°$, $\tan.^2 a°$, etc. But the notation given above places the index as used by Gauss, Delambre, Arbogast, etc., though the first two omit the parentheses.

$$\sin.\tfrac{1}{2}a = \sqrt{[\tfrac{1}{2}(1-\cos.a)]}.$$
$$\cos.\tfrac{1}{2}a = \sqrt{[\tfrac{1}{2}(1+\cos.a)]}.$$
$$\tan.\tfrac{1}{2}a = \frac{\sin.a}{1+\cos.a} = \frac{1-\cos.a}{\sin.a} = \sqrt{\left(\frac{1-\cos.a}{1+\cos.a}\right)}.$$
$$\cot.\tfrac{1}{2}a = \frac{1+\cos.a}{\sin.a} = \frac{\sin.a}{1-\cos.a} = \sqrt{\left(\frac{1+\cos.a}{1-\cos.a}\right)}.$$

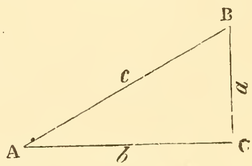
10. Trigonometrical Tables. In the usual tables of the natural trigonometrical lines, the degrees from 0° to 45° are found at the top of the table, and those from 45° to 90° at the bottom; the latter being complements of the former. Consequently, the columns which have *Sine* and *Tangent* at top have *Cosine* and *Cotangent* at bottom, since the cosine or cotangent of any arc is the same thing as the sine or tangent of its complement. The minutes to be added to the degrees are found in the left-hand column, when the number of degrees at the top of the page are used, and in the right-hand column for the degrees when at the bottom of the page. The lines for arcs intermediate between those in the tables are found by proportion. The lines are calculated for a radius equal unity. Hence, the values of the sines and cosines are decimal fractions, though the point is usually omitted. So too are the tangents from 0° to 45°, and the cotangents from 90° to 45°. Beyond those points they are integers and decimals.

The calculations, like all others involving large numbers, are shortened by the use of logarithms, which substitute addition and subtraction for multiplication and division; but the young student should avoid the frequent error of regarding logarithms as a necessary part of trigonometry.

SOLUTION OF TRIANGLES.

11. Right-angled Triangles. Let A B C be any right-angled triangle. Denote the sides opposite the angles by the corresponding small letters. Then any one side and one acute angle, or any two sides being given, the other parts can be obtained by one of the following equations:

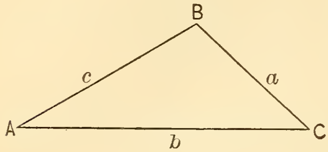
Fig. 575.



GIVEN.	REQUIRED.	FORMULAS.
a, b	c, A, B	$c = \sqrt{(a^2+b^2)}$; $\tan.A = \frac{a}{b}$; $\cot.B = \frac{a}{b}$.
a, c	b, A, B	$b = \sqrt{(c^2-a^2)}$; $\sin.A = \frac{a}{c}$; $\cos.B = \frac{a}{c}$.
a, A	b, c, B	$b = a.\cot.A$; $c = \frac{a}{\sin.A}$; $B = 90° - A$.
b, A	a, c, B	$a = b.\tan.A$; $c = \frac{b}{\cos.A}$; $B = 90° - A$.
c, A	a, b, B	$a = c.\sin.A$; $b = c\cos.A$; $B = 90° - A$.

12. Oblique-angled Triangles. Let A B C be any oblique-angled triangle, the angles and sides being noted as in the figure. Then any three of its six parts being given, and one of them being a side, the other parts can be obtained by one of the following methods, which are founded on these three theorems:

FIG. 576.



THEOREM I.—*In every plane triangle, the sines of the angles are to each other as the opposite sides.*

THEOREM II.—*In every plane triangle, the sum of two sides is to their difference as the tangent of half the sum of the angles opposite those sides is to the tangent of half their difference.*

THEOREM III.—*In every plane triangle, the cosine of any angle is equal to a fraction whose numerator is the sum of the squares of the sides adjacent to the angle,* minus *the square of the side opposite to the angle, and whose denominator is twice the product of the sides adjacent to the angle.*

All the cases for solution which can occur may be reduced to four:

CASE 1.—*Given a side and two angles.* The third angle is obtained by subtracting the sum of the two given angles from 180°. Then either unknown side can be obtained by Theorem I.

Calling the given side a, we have $b = a . \dfrac{\text{sin. B}}{\text{sin. A}}$; and $c = a \dfrac{\text{sin. C}}{\text{sin. A}}$.

CASE 2.—*Given two sides and an angle opposite one of them.* The angle opposite the other given side is found by Theorem I. The third angle is obtained by subtracting the sum of the other two from 180°. The remaining side is then obtained by Theorem I.

Calling the given sides a and b, and the given angle A, we have sin. B = sin. A . $\dfrac{b}{a}$.

Since an angle and its supplement have the same sine, the result is ambiguous; for the angle B may have either of the two supplementary values indicated by the sine, if $b > a$, and A is an acute angle.

$$C = 180° - (A + B). \qquad c = \text{sin. C} \frac{a}{\text{sin. A}}.$$

CASE 3.—*Given two sides and their included angle.* Applying Theorem II (obtaining the sum of the angles opposite the given sides by subtracting the given included angle from 180°), we obtain the difference of the unknown angles. Adding this to their sum we obtain the greater angle, and subtracting it from their sum we get the less. Then Theorem I will give the remaining side.

Calling the given sides a and b, and the included angle C, we have A + B = 180° − C. Then

$$\text{tan. } \tfrac{1}{2} (A - B) = \text{tan. } \tfrac{1}{2} (A + B) . \frac{a - b}{a + b}.$$

$$\tfrac{1}{2} (A + B) + \tfrac{1}{2} (A - B) = A. \quad \tfrac{1}{2} (A + B) - \tfrac{1}{2} (A - B) = B. \quad c = a \frac{\text{sin C.}}{\text{sin. A}}.$$

In the first equation cot. $\frac{1}{2}$ C may be used in the place of tan. $\frac{1}{2}$ (A + B).

CASE 4.—*Given the three sides.* Let s represent half the sum of the three sides $= \frac{1}{2}(a + b + c)$. Then any angle, as A, may be obtained from either of the following formulas, founded on Theorem III:

$$\sin. \tfrac{1}{2} A = \sqrt{\left[\frac{(s-b)(s-c)}{bc}\right]}.$$

$$\cos. \tfrac{1}{2} A = \sqrt{\left[\frac{s(s-a)}{bc}\right]}.$$

$$\tan. \tfrac{1}{2} A = \sqrt{\left[\frac{(s-b)(s-c)}{s(s-a)}\right]}.$$

$$\sin. A = \frac{2\sqrt{[s(s-a)(s-b)(s-c)]}}{bc}.$$

$$\cos. A = \frac{b^2 + c^2 - a^2}{2bc}.$$

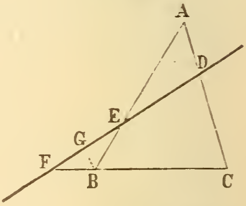
The first formula should be used when $A < 90°$, and the second when $A > 90°$. The third should not be used when A is nearly 180°; nor the fourth when A is nearly 90; nor the fifth when A is very small. The third is the most convenient when all the angles are required.

APPENDIX B.

TRANSVERSALS.

THEOREM I.—*If a straight line be drawn so as to cut any two sides of a triangle, and the third side prolonged, thus dividing them into six parts (the prolonged side and its prolongation being two of the parts), then will the product of any three of those parts, whose extremities are not contiguous, equal the product of the other three parts.*

FIG. 577.



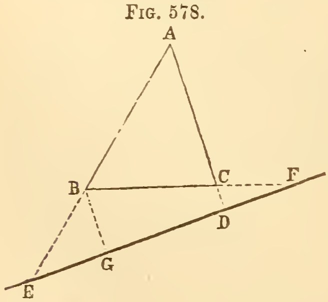
That is, in Fig. 577, A B C being the triangle, and D F the transversal, $BE \times AD \times CF = EA \times DC \times BF$.

To prove this, from B draw B G, parallel to C A. From the similar triangles B E G and A E D, we have $BG : BE :: AD : AE$. From the similar triangles B F G and C F D, we have $CD : CF :: BG : BF$. Multiplying these proportions together, we have $BG \times CD : BE \times CF :: AD \times BG : AE \times BF$. Multiplying extremes and means, and suppressing the common factor B G, we have $BE \times AD \times CF = EA \times DC \times BF$.

These six parts are sometimes said to be *in involution*.

If the transversal passes entirely outside of the triangle and cuts the prolongations of all three sides, as in Fig. 578, the theorem still holds good. The same demonstration applies without any change.*

FIG. 578.



THEOREM II.—Conversely: *If three points be taken on two sides of a triangle, and on the third side prolonged, or on the prolongations of the three sides, dividing them into six parts, such that the product of three non-consecutive parts equals the product of the other three parts, then will these three points lie in the same straight line.*

This theorem is proved by a *reductio ad absurdum.*

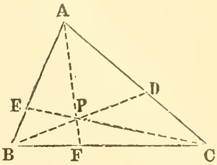
THEOREM III.—*If, from the summits*

---

* This theorem may be extended to polygons.

of a triangle, lines be drawn, to a point situated either within or without the triangle, and prolonged to meet the sides of the triangle, or their prolongations, thus dividing them into six parts, then will the product of any three non-consecutive parts be equal to the product of the other three parts.

Fig. 579.



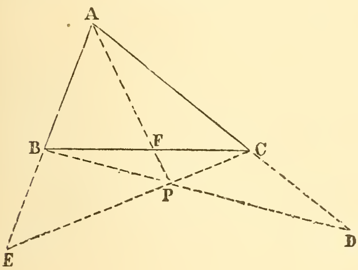
That is, in Fig. 579, or Fig. 580,

$$AE \times BF \times CD = EB \times FC \times DA.$$

For, the triangle A B F, being cut by the transversal E C, gives the relation (Theorem I).

$$AE \times BC \times FP = EB \times FC \times PA.$$

Fig. 580.



The triangle A C F, being cut by the transversal D B, gives

$$DC \times FB \times PA = AD \times CB \times FP.$$

Multiplying these equations together, and suppressing the common factors P A, C B, and F P, we have $AE \times BF \times CD = EB \times FC \times DA$.

Theorem IV. — Conversely: *If three points are situated on the three sides of a triangle, or on their prolongations (either one, or three, of these points being on the sides), so that they divide these lines in such a way that the product of any three non-consecutive parts equals the product of the other three parts, then will lines drawn from these points to the opposite angles meet in the same point.*

This theorem can be demonstrated by a *reductio ad absurdum.*

COROLLARIES OF THE PRECEDING THEOREMS.

Corollary 1.—*The* MEDIANS *of a triangle* (i. e., the lines drawn from its summits to the middles of the opposite sides) *meet in the same point.*

For, supposing, in Fig. 579, the points D, E, and F to be the middles of the sides, the products of the non-consecutive parts will be equal—i. e., $AE \times BF \times CD = DA \times EB \times FC$; since $AE = EB$, $BF = FC$, $CD = DA$. Then Theorem IV applies.

Cor. 2.—*The* BISSECTRICES *of a triangle* (i. e., the lines bisecting its angles) *meet in the same point.*

For, in Fig. 579, supposing the lines A F, B D, C E to be bissectrices, we have (Legendre, IV, 17):

$$\left.\begin{array}{l} BF : FC :: AB : AC \\ CD : DA :: BC : BA \\ AE : EB :: CA : CB \end{array}\right\} \text{ whence } \left\{\begin{array}{l} BF \times AC = FC \times AB, \\ CD \times BA = DA \times BC, \\ AE \times CB = EB \times CA. \end{array}\right.$$

Multiplying these equations together, and omitting the common factors, we have $BF \times CD \times AE = FC \times DA \times EB$. Then Theorem IV applies.

Cor. 3.—*The* ALTITUDES *of a triangle* (i. e., the lines drawn from its summits perpendicular to the opposite sides) *meet in the same point.*

For, in Fig. 579, supposing the lines A F, B D, and C E to be altitudes, we have three pairs of similar triangles, B C D and F C A, C A E and D A B, A B F and E B C, by comparing which we obtain relations from which it is easy to deduce $BF \times CD \times AE = EB \times FC \times DA$; and then Theorem IV again applies.

Cor. 4.—*If, in Fig.* 579, *or Fig.* 580, *the point* F *be taken in the middle of* B C, *then will the line* E D *be parallel to* B C.

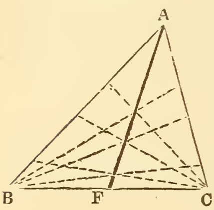
For, since $BF = FC$, the equation of Theorem III reduces to $AE \times CD = EB \times DA$; whence $AE : EB :: AD : DC$; consequently E D is parallel to B C.

Cor. 5.—Conversely: *If* E D *be parallel to* B C, *then is* $BF = FC$.

For, since $AE : EB :: AD : DC$, we have $AE \times DC = EB \times AD$; whence, in the equation of Theorem III, we must have $BF = FC$.

Cor. 6.—From the preceding corollary, we derive the following:

Fig. 581.



If two sides of a triangle are divided proportionally, starting from the same summit, as A, *and lines are drawn from the extremities of the third side to the points of division, the intersections of the corresponding lines will all lie in the same straight line joining the summit* A, *and the middle of the base.*

Cor. 7.—A particular case of the preceding corollary is this:

In any trapezoid, the straight line which joins the intersection of the diagonals and the point of meeting of the non-parallel sides produced, passes through the middle of the two parallel bases.

Cor. 8.—*If the three lines drawn through the corresponding summits of two triangles cut each other in the same point, then the three points in which the corresponding sides, produced if necessary, will meet, are situated in the same straight line.*

This corollary may be otherwise enunciated, thus:

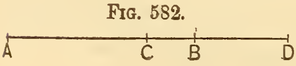
If two triangles have their summits situated, two and two, on three lines which meet in the same point, then, etc.

This is proved by obtaining by Theorem I three equations, which, being multiplied together, and the six common factors canceled, give an equation to which Theorem II applies.

Triangles thus situated are called *homologic;* the common point of meeting of the lines passing through their summits is called the *center of homology;* and the one on which the sides meet, the *axis of homology.*

HARMONIC DIVISION.

Fig. 582.



Definitions.—A straight line, A B, is said to be *harmonically divided* at the points C and D, when these points determine two additive segments, A C, B C, and two subtractive segments, A D, B D, proportional to one another; so that A C : B C :: A D : B D. It will be seen that A C must be more than B C, since A D is more than B D.*

This relation may be otherwise expressed, thus: The product of the whole line by the middle part equals the product of the extreme parts.

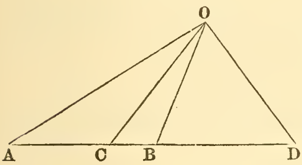
Reciprocally, the line D C is harmonically divided at the points B and A, since the preceding proportion may be written D B : C B :: D A : C A.

The four points, A, B, C, D, are called *harmonics.* The points C and D are called *harmonic conjugates.* So are the points A and B.

When a straight line, as A B, is divided harmonically, its half is a mean proportional between the distance from the middle of the line to the two points, C and D, which divide it harmonically.

If, from any point, O, lines be drawn so as to divide a line harmonically, these lines are called an *harmonic pencil.* The four lines which compose it, O A, O C, O B, O D, in the figure, are called its *radii*, and the pairs which pass through the conjugate points are called *conjugate radii.*

FIG. 583.



THEOREM V.—*In any harmonic pencil, a line drawn parallel to any one of the radii is divided by the three other radii into two equal parts.*

Let E F be the line, drawn parallel to O A. Through B draw G H, also parallel to O A. We have,

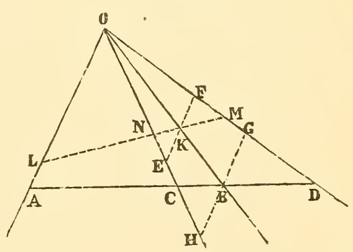
G B : O A :: B D : A D; and

B H : O A :: B C : A C.

But, by hypothesis, A C : B C :: A D : B D.

Hence, the first two proportions reduce to G B = B H; and, consequently, E K = K F.

FIG. 584.



The *reciprocal* is also true—i. e.,

If four lines radiating from a point are such that a line drawn parallel to one of them is divided into two equal parts by the other three, the four lines form an harmonic pencil.

THEOREM VI.—*If any transversal to an harmonic pencil be drawn, it will be divided harmonically.*

Let L M be the transversal. Through K, where L M intersects O B, draw E F parallel to O A. It is bisected at K by the preceding theorem; and the

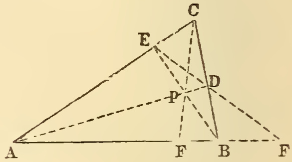
* Three numbers, m, n, p, arranged in decreasing order of size, form an *harmonic proportion*, when the difference of the first and the second is to the difference of the second and the third, as the first is to the third. Such are the numbers 6, 4, and 3; or 6, 3, and 2; or 15, 12, and 10; etc. So, in Fig. 582, are the lines A D, A B, and A C, which thus give B D : C B :: A D : A C; or A C : C B :: A D : B D. The series of fractions, $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc., is called an *harmonic progression*, because any consecutive three of its terms form an *harmonic proportion.*

similar triangles, F M K and L M O, E K N and L N O, give the proportions

L M : K M : : O L : F K, and L N : N K : : O L : E K; whence,
since F K = E K, we have L N : N K : : L M : K M.

Corollary.—*The two sides of any angle, together with the bissectrices of the angle and of its supplement, form an harmonic pencil.*

Fig. 585.



Theorem VII.—*If, from the summits of any triangle,* A B C, *through any point,* P, *there be drawn the transversals* A D, B E, C F, *and the transversal* E D *be drawn to meet* A B *prolonged in* F′, *the points* F *and* F′ *will divide the base* A B *harmonically.*

This may be otherwise expressed, thus:

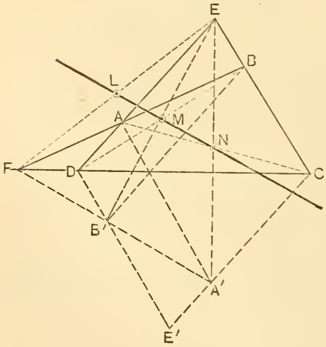
The line, C P, *which joins the intersection of the diagonals of any quadrilateral,* A B D E, *with the point of meeting,* C, *of two opposite sides prolonged, cuts the side* A B *in a point* F, *which is the harmonic conjugate of the point of meeting,* F′ *of the other two sides,* E D *and* A B, *prolonged.*

For, by Theorem I, A F′ × B D × C E = F′ B × D C × E A; and
by Theorem III, A F × B D × C E = F B × D C × E A;
whence A F : F B : : A F′ : F′ B.

THE COMPLETE QUADRILATERAL.

A *Complete Quadrilateral* is formed by drawing any four straight lines, so that each of them shall cut each of the other three, so as to give six different points of intersection. It is so called because in the figure thus formed are found three quadrilaterals; viz., in Fig. 586, A B C D, a common *convex* quadrilateral; E A F C, a *uni-concave* quadrilateral; and E B A F D, a *bi-concave* quadrilateral, composed of two opposite triangles.

Fig. 586.



The complete quadrilateral, A E B C D F, has three diagonals; viz., two interior, A C, B D; and one exterior, E F.

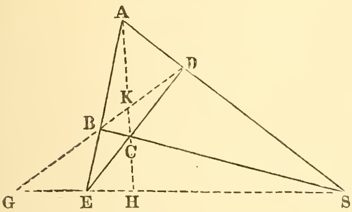
Theorem VIII. — *In every* complete quadrilateral *the middle points of its three diagonals lie in the same straight line.*

A E B C D F is the quadrilateral, and L M N the middle points of its three diagonals. From A and D draw parallels to B C, and from B and C draw parallels to A D. The triangle E D C being cut by the transversal B F, we have (Theorem I), D F × C B × E A = C F × E B × D A. From the equality of parallels between parallels, we have C B = E′ B′, E A = C A′, E B = D B′, D A = E′ A′. Hence, the above equation becomes D F × E′ B′ × C A′

= C F × D B′ × E′ A′; therefore, by Theorem II, the points, F, B′, A′, lie in the same straight line. Now, since the diagonals of the parallelogram E C A′ A bisect each other at N, and those of the parallelogram E B B′ D at M, we have E N : N A′ : : E M : M B′. Then M N is parallel to F A′, and we have E N : N A′ : : E L : L F, or E L = L F, so that L is the middle of E F, and the same straight line passes through L, M, and N.

THEOREM IX.—*In every complete quadrilateral each of the three diagonals is divided harmonically by the two others.*

FIG. 587.



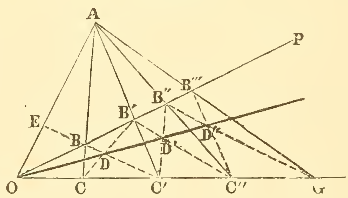
C E B A D F is the complete quadrilateral. The diagonal E F is divided harmonically at G and H by D B and A C produced; since A H, D E, and F B are three transversals drawn from the summits of the triangle A E F through the same point C; and therefore, by Theorem VII, D B G and A C H divide E F harmonically.

So too, in the triangle A B D, C B, C A, C D, are the three transversals passing through C; and G and K therefore divide the diagonal B D harmonically.

So, too, in the triangle, A B C, D A, D B, D C are the transversals, and H and K the points which divide the diagonal A C harmonically.

THEOREM X.—*If from a point*, A, *any number of lines be drawn, cutting the sides of an angle* P O Q, *the intersections of the diagonals of the quadrilaterals thus formed will all lie in the same straight line passing through the summit of the angle.*

FIG. 588.



By the preceding theorem, the diagonal B C′ of the complete quadrilateral, B A B′ C′ C O, is divided harmonically at D and E. Hence, O A, O P, O D, and O Q, form an harmonic pencil. So do O A, O P, O D′, and O Q. Therefore, the lines O D, O D′, coincide. So for the other intersections.

If the point A moves on O A, the line O D is not displaced. If, on the contrary, O A is displaced, O D turns around the point O. Hence, the point A is said to be a *pole* with respect to the line O D, which is itself called the *polar* of the point A. Similarly, D is a pole of O A, which is the polar of D. O D is likewise the polar of any other point on the line O A; and this property is necessarily reciprocal for the two conjugate radii O A, O D, with respect to the lines O P, O Q, which are also conjugate radii. Hence: in every harmonic pencil, each of the radii is a *polar* with respect to each point of its conjugate; and each point of this latter line is a *pole* with respect to the former.

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LATITUDES AND DEPARTURES.

Bearing.	1		2		3		4		5	Bearing.
	Lat.	Dep.	Lat	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
0°	1·000	0·000	2·000	0·000	3·000	0·000	4·000	0·000	5·000	**90°**
0¼	1·000	0·004	2·000	0·009	3·000	0·013	4·000	0·017	5·000	89¾
0½	1·000	0·009	2·000	0·017	3·000	0·026	4·000	0·035	5·000	89½
0¾	1·000	0·013	2·000	0·026	3·000	0·039	4·000	0·052	5·000	89¼
1°	1·000	0·017	2·000	0·035	3·000	0·052	3·999	0·070	4·999	**89°**
1¼	1·000	0·022	2·000	0·044	2·999	0·065	3·999	0·087	4·999	88¾
1½	1·000	0·026	1·999	0·052	2·999	0·079	3·999	0·105	4·998	88½
1¾	1·000	0·031	1·999	0·061	2·999	0·092	3·998	0·122	4·998	88¼
2°	0·999	0·035	1·999	0·070	2·998	0·105	3·998	0·140	4·997	**88°**
2¼	0·999	0·039	1·998	0·079	2·998	0·118	3·997	0·157	4·996	87¾
2½	0·999	0·044	1·998	0·087	2·997	0·131	3·996	0·174	4·995	87½
2¾	0·999	0·048	1·998	0·096	2·997	0·144	3·995	0·192	4·994	87¼
3°	0·999	0·052	1·997	0·105	2·996	0·157	3·995	0·209	4·993	**87°**
3¼	0·998	0·057	1·997	0·113	2·995	0·170	3·994	0·227	4·992	86¾
3½	0·998	0·061	1·996	0·122	2·994	0·183	3·993	0·244	4·991	86½
3¾	0·998	0·065	1·996	0·131	2·994	0·196	3·991	0·262	4·989	86¼
4°	0·998	0·070	1·995	0·140	2·993	0·209	3·990	0·279	4·988	**86°**
4¼	0·997	0·074	1·995	0·148	2·992	0·222	3·989	0·296	4·986	85¾
4½	0·997	0·078	1·994	0·157	2·991	0·235	3·988	0·314	4·985	85½
4¾	0·997	0·083	1·993	0·166	2·990	0·248	3·986	0·331	4·983	85¼
5°	0·996	0·087	1·992	0·174	2·989	0·261	3·985	0·349	4·981	**85°**
5¼	0·996	0·092	1·992	0·183	2·987	0·275	3·983	0·366	4·979	84¾
5½	0·995	0·096	1·991	0·192	2·986	0·288	3·982	0·383	4·977	84½
5¾	0·995	0·100	1·990	0·200	2·985	0·301	3·980	0·401	4·975	84¼
6°	0·995	0·105	1·989	0·209	2·984	0·314	3·978	0·418	4·973	**84°**
6¼	0·994	0·109	1·988	0·218	2·982	0·327	3·976	0·435	4·970	83¾
6½	0·994	0·113	1·987	0·226	2·981	0·340	3·974	0·453	4·968	83½
6¾	0·993	0·118	1·986	0·235	2·979	0·353	3·972	0·470	4·965	83¼
7°	0·993	0·122	1·985	0·244	2·978	0·366	3·970	0·487	4·963	**83°**
7¼	0·992	0·126	1·984	0·252	2·976	0·379	3·968	0·505	4·960	82¾
7½	0·991	0·131	1·983	0·261	2·974	0·392	3·966	0·522	4·957	82½
7¾	0·991	0·135	1·982	0·270	2·973	0·405	3·963	0·539	4·954	82¼
8°	0·990	0·139	1·981	0·278	2·971	0·418	3·961	0·557	4·951	**82°**
8¼	0·990	0·143	1·979	0·287	2·969	0·430	3·959	0·574	4·948	81¾
8½	0·989	0·148	1·978	0·296	2·967	0·443	3·956	0·591	4·945	81½
8¾	0·988	0·152	1·977	0·304	2·965	0·456	3·953	0·608	4·942	81¼
9°	0·988	0·156	1·975	0·313	2·963	0·469	3·951	0·626	4·938	**81°**
9¼	0·987	0·161	1·974	0·321	2·961	0·482	3·948	0·643	4·935	80¾
9½	0·986	0·165	1·973	0·330	2·959	0·495	3·945	0·660	4·931	80½
9¾	0·986	0·169	1·971	0·339	2·957	0·508	3·942	0·677	4·928	80¼
10°	0·985	0·174	1·970	0·347	2·954	0·521	3·939	0·695	4·924	**80°**
10¼	0·984	0·178	1·968	0·356	2·952	0·534	3·936	0·712	4·920	79¾
10½	0·983	0·182	1·967	0·364	2·950	0·547	3·933	0·729	4·916	79½
10¾	0·982	0·187	1·965	0·373	2·947	0·560	3·930	0·746	4·912	79¼
11°	0·982	0·191	1·963	0·382	2·945	0·572	3·927	0·763	4·908	**79°**
11¼	0·981	0·195	1·962	0·390	2·942	0·585	3·923	0·780	4·904	78¾
11½	0·980	0·199	1·960	0·399	2·940	0·598	3·920	0·797	4·900	78½
11¾	0·979	0·204	1·958	0·407	2·937	0·611	3·916	0·815	4·895	78¼
12°	0·978	0·208	1·956	0·416	2·934	0·624	3·913	0·832	4·891	**78°**
12¼	0·977	0·212	1·954	0·424	2·932	0·637	3·909	0·849	4·886	77¾
12½	0·976	0·216	1·953	0·433	2·929	0·649	3·905	0·866	4·881	77½
12¾	0·975	0·221	1·951	0·441	2·926	0·662	3·901	0·883	4·877	77¼
13°	0·974	0·225	1·949	0·450	2·923	0·675	3·897	0·900	4·872	**77°**
13¼	0·973	0·229	1·947	0·458	2·920	0·688	3·894	0·917	4·867	76¾
13½	0·972	0·233	1·945	0·467	2·917	0·700	3·889	0·934	4·862	76½
13¾	0·971	0·238	1·943	0·475	2·914	0·713	3·885	0·951	4·857	76¼
14°	0·970	0·242	1·941	0·484	2·911	0·726	3·881	0·968	4·851	**76°**
14¼	0·969	0·246	1·938	0·492	2·908	0·738	3·877	0·985	4·846	75¾
14½	0·968	0·250	1·936	0·501	2·904	0·751	3·873	1·002	4·841	75½
14¾	0·967	0·255	1·934	0·509	2·901	0·764	3·868	1·018	4·835	75¼
15°	0·966	0·259	1·932	0·518	2·898	0·776	3·864	1·035	4·830	**75°**
Bearing.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Bearing.
	1		2		3		4		5	

Bearing.	5	6		7		8		9		Bearing.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
0°	0·000	6·000	0·000	7·000	0·000	8·000	0·000	9·000	0·000	**90°**
0¼	0·022	6·000	0·026	7·000	0·031	8·000	0·035	9·000	0·039	89¾
0½	0·044	6·000	0·052	7·000	0·061	8·000	0·070	9·000	0·079	89½
0¾	0·065	5·999	0·079	6·999	0·092	7·999	0·105	8·999	0·118	89¼
1°	0·087	5·999	0·105	6·999	0·122	7·999	0·140	8·999	0·157	**89°**
1¼	0·109	5·999	0·131	6·998	0·153	7·998	0·175	8·998	0·196	88¾
1½	0·131	5·998	0·157	6·998	0·183	7·997	0·209	8·997	0·236	88½
1¾	0·153	5·997	0·183	6·997	0·214	7·996	0·244	8·996	0·275	88¼
2°	0·174	5·996	0·209	6·996	0·244	7·995	0·279	8·995	0·314	**88°**
2¼	0·196	5·995	0·236	6·995	0·275	7·994	0·314	8·993	0·353	87¾
2½	0·218	5·994	0·262	6·993	0·305	7·992	0·349	8·991	0·393	87½
2¾	0·240	5·993	0·288	6·992	0·336	7·991	0·384	8·990	0·432	87¼
3°	0·262	5·992	0·314	6·990	0·366	7·989	0·419	8·988	0·471	**87°**
3¼	0·283	5·990	0·340	6·989	0·397	7·987	0·454	8·986	0·510	86¾
3½	0·305	5·989	0·366	6·987	0·427	7·985	0·488	8·983	0·549	86½
3¾	0·327	5·987	0·392	6·985	0·458	7·983	0·523	8·981	0·589	86¼
4°	0·349	5·985	0·419	6·983	0·488	7·981	0·558	8·978	0·628	**86°**
4¼	0·371	5·984	0·445	6·981	0·519	7·978	0·593	8·975	0·667	85¾
4½	0·392	5·982	0·471	6·978	0·549	7·975	0·628	8·972	0·706	85½
4¾	0·414	5·979	0·497	6·976	0·580	7·973	0·662	8·969	0·745	85¼
5°	0·436	5·977	0·523	6·973	0·610	7·970	0·697	8·966	0·784	**85°**
5¼	0·458	5·975	0·549	6·971	0·641	7·966	0·732	8·962	0·824	84¾
5½	0·479	5·972	0·575	6·968	0·671	7·963	0·767	8·959	0·863	84½
5¾	0·501	5·970	0·601	6·965	0·701	7·960	0·802	8·955	0·902	84¼
6°	0·523	5·967	0·627	6·962	0·732	7·956	0·836	8·951	0·941	**84°**
6¼	0·544	5·964	0·653	6·958	0·762	7·952	0·871	8·947	0·980	83¾
6½	0·566	5·961	0·679	6·955	0·792	7·949	0·906	8·942	1·019	83½
6¾	0·588	5·958	0·705	6·951	0·823	7·945	0·940	8·938	1·058	83¼
7°	0·609	5·955	0·731	6·948	0·853	7·940	0·975	8·933	1·097	**83°**
7¼	0·631	5·952	0·757	6·944	0·883	7·936	1·010	8·928	1·136	82¾
7½	0·653	5·949	0·783	6·940	0·914	7·932	1·044	8·923	1·175	82½
7¾	0·674	5·945	0·809	6·936	0·944	7·927	1·079	8·918	1·214	82¼
8°	0·696	5·942	0·835	6·932	0·974	7·922	1·113	8·912	1·253	**82°**
8¼	0·717	5·938	0·861	6·928	1·004	7·917	1·148	8·907	1·291	81¾
8½	0·739	5·934	0·887	6·923	1·035	7·912	1·182	8·901	1·330	81½
8¾	0·761	5·930	0·913	6·919	1·065	7·907	1·217	8·895	1·369	81¼
9°	0·782	5·926	0·939	6·914	1·095	7·902	1·251	8·889	1·408	**81°**
9¼	0·804	5·922	0·964	6·909	1·125	7·896	1·286	8·883	1·447	80¾
9½	0·825	5·918	0·990	6·904	1·155	7·890	1·320	8·877	1·485	80½
9¾	0·847	5·913	1·016	6·899	1·185	7·884	1·355	8·870	1·524	80¼
10°	0·868	5·909	1·042	6·894	1·216	7·878	1·389	8·863	1·563	**80°**
10¼	0·890	5·904	1·068	6·888	1·246	7·872	1·424	8·856	1·601	79¾
10½	0·911	5·900	1·093	6·883	1·276	7·866	1·458	8·849	1·640	79½
10¾	0·933	5·895	1·119	6·877	1·306	7·860	1·492	8·842	1·679	79¼
11°	0·954	5·890	1·145	6·871	1·336	7·853	1·526	8·835	1·717	**79°**
11¼	0·975	5·885	1·171	6·866	1·366	7·846	1·561	8·827	1·756	78¾
11½	0·997	5·880	1·196	6·859	1·396	7·839	1·595	8·819	1·794	78½
11¾	1·018	5·874	1·222	6·853	1·425	7·832	1·629	8·811	1·833	78¼
12°	1·040	5·869	1·247	6·847	1·455	7·825	1·663	8·803	1·871	**78°**
12¼	1·061	5·863	1·273	6·841	1·485	7·818	1·697	8·795	1·910	77¾
12½	1·082	5·858	1·299	6·834	1·515	7·810	1·732	8·787	1·948	77½
12¾	1·103	5·852	1·324	6·827	1·545	7·803	1·766	8·778	1·986	77¼
13°	1·125	5·846	1·350	6·821	1·575	7·795	1·800	8·769	2·025	**77°**
13¼	1·146	5·840	1·375	6·814	1·604	7·787	1·834	8·760	2·063	76¾
13½	1·167	5·834	1·401	6·807	1·634	7·779	1·868	8·751	2·101	76½
13¾	1·188	5·828	1·426	6·799	1·664	7·771	1·902	8·742	2·139	76¼
14°	1·210	5·822	1·452	6·792	1·693	7·762	1·935	8·733	2·177	**76°**
14¼	1·231	5·815	1·477	6·785	1·723	7·754	1·969	8·723	2·215	75¾
14½	1·252	5·809	1·502	6·777	1·753	7·745	2·003	8·713	2·253	75½
14¾	1·273	5·802	1·528	6·769	1·782	7·736	2·037	8·703	2·291	75¼
15°	1·294	5·796	1·553	6·761	1·812	7·727	2·071	8·693	2·329	**75°**
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
Bearing.	5	6		7		8		9		Bearing.

LATITUDES AND DEPARTURES.

Bearing.	1 Lat.	1 Dep.	2 Lat.	2 Dep.	3 Lat.	3 Dep.	4 Lat.	4 Dep.	5 Lat.	Bearing.
15°	0·966	0·259	1·932	0·518	2·898	0·776	3·864	1·035	4·830	**75°**
15¼	0·965	0·263	1·930	0·526	2·894	0·789	3·859	1·052	4·824	74¾
15½	0·964	0·267	1·927	0·534	2·891	0·802	3·855	1·069	4·818	74½
15¾	0·962	0·271	1·925	0·543	2·887	0·814	3·850	1·086	4·812	74¼
16°	0·961	0·276	1·923	0·551	2·884	0·827	3·845	1·103	4·806	**74°**
16¼	0·960	0·280	1·920	0·560	2·880	0·839	3·840	1·119	4·800	73¾
16½	0·959	0·284	1·918	0·568	2·876	0·852	3·835	1·136	4·794	73½
16¾	0·958	0·288	1·915	0·576	2·873	0·865	3·830	1·153	4·788	73¼
17°	0·956	0·292	1·913	0·585	2·869	0·877	3·825	1·169	4·782	**73°**
17¼	0·955	0·297	1·910	0·593	2·865	0·890	3·820	1·186	4·775	72¾
17½	0·954	0·301	1·907	0·601	2·861	0·902	3·815	1·203	4·769	72½
17¾	0·952	0·305	1·905	0·610	2·857	0·915	3·810	1·220	4·762	72¼
18°	0·951	0·309	1·902	0·618	2·853	0·927	3·804	1·236	4·755	**72°**
18¼	0·950	0·313	1·899	0·626	2·849	0·939	3·799	1·253	4·748	71¾
18½	0·948	0·317	1·897	0·635	2·845	0·952	3·793	1·269	4·742	71½
18¾	0·947	0·321	1·894	0·643	2·841	0·964	3·788	1·286	4·735	71¼
19°	0·946	0·326	1·891	0·651	2·837	0·977	3·782	1·302	4·728	**71°**
19¼	0·944	0·330	1·888	0·659	2·832	0·989	3·776	1·319	4·720	70¾
19½	0·943	0·334	1·885	0·668	2·828	1·001	3·771	1·335	4·713	70½
19¾	0·941	0·338	1·882	0·676	2·824	1·014	3·765	1·352	4·706	70¼
20°	0·940	0·342	1·879	0·684	2·819	1·026	3·759	1·368	4·698	**70°**
20¼	0·938	0·346	1·876	0·692	2·815	1·038	3·753	1·384	4·691	69¾
20½	0·937	0·350	1·873	0·700	2·810	1·051	3·747	1·401	4·683	69½
20¾	0·935	0·354	1·870	0·709	2·805	1·063	3·741	1·417	4·676	69¼
21°	0·934	0·358	1·867	0·717	2·801	1·075	3·734	1·433	4·668	**69°**
21¼	0·932	0·362	1·864	0·725	2·796	1·087	3·728	1·450	4·660	68¾
21½	0·930	0·367	1·861	0·733	2·791	1·100	3·722	1·466	4·652	68½
21¾	0·929	0·371	1·858	0·741	2·786	1·112	3·715	1·482	4·644	68¼
22°	0·927	0·375	1·854	0·749	2·782	1·124	3·709	1·498	4·636	**68°**
22¼	0·926	0·379	1·851	0·757	2·777	1·136	3·702	1·515	4·628	67¾
22½	0·924	0·383	1·848	0·765	2·772	1·148	3·696	1·531	4·619	67½
22¾	0·922	0·387	1·844	0·773	2·767	1·160	3·689	1·547	4·611	67¼
23°	0·921	0·391	1·841	0·781	2·762	1·172	3·682	1·563	4·603	**67°**
23¼	0·919	0·395	1·838	0·789	2·756	1·184	3·675	1·579	4·594	66¾
23½	0·917	0·399	1·834	0·797	2·751	1·196	3·668	1·595	4·585	66½
23¾	0·915	0·403	1·831	0·805	2·746	1·208	3·661	1·611	4·577	66¼
24°	0·914	0·407	1·827	0·813	2·741	1·220	3·654	1·627	4·568	**66°**
24¼	0·912	0·411	1·824	0·821	2·735	1·232	3·647	1·643	4·559	65¾
24½	0·910	0·415	1·820	0·829	2·730	1·244	3·640	1·659	4·550	65½
24¾	0·908	0·419	1·816	0·837	2·724	1·256	3·633	1·675	4·541	65¼
25°	0·906	0·423	1·813	0·845	2·719	1·268	3·625	1·690	4·532	**65°**
25¼	0·904	0·427	1·809	0·853	2·713	1·280	3·618	1·706	4·522	64¾
25½	0·903	0·431	1·805	0·861	2·708	1·292	3·610	1·722	4·513	64½
25¾	0·901	0·434	1·801	0·869	2·702	1·303	3·603	1·738	4·503	64¼
26°	0·899	0·438	1·798	0·877	2·696	1·315	3·595	1·753	4·494	**64°**
26¼	0·897	0·442	1·794	0·885	2·691	1·327	3·587	1·769	4·484	63¾
26½	0·895	0·446	1·790	0·892	2·685	1·339	3·580	1·785	4·475	63½
26¾	0·893	0·450	1·786	0·900	2·679	1·350	3·572	1·800	4·465	63¼
27°	0·891	0·454	1·782	0·908	2·673	1·362	3·564	1·816	4·455	**63°**
27¼	0·889	0·458	1·778	0·916	2·667	1·374	3·556	1·831	4·445	62¾
27½	0·887	0·462	1·774	0·923	2·661	1·385	3·548	1·847	4·435	62½
27¾	0·885	0·466	1·770	0·931	2·655	1·397	3·540	1·862	4·425	62¼
28°	0·883	0·469	1·766	0·939	2·649	1·408	3·532	1·878	4·415	**62°**
28¼	0·881	0·473	1·762	0·947	2·643	1·420	3·524	1·893	4·404	61¾
28½	0·879	0·477	1·758	0·954	2·636	1·431	3·515	1·909	4·394	61½
28¾	0·877	0·481	1·753	0·962	2·630	1·443	3·507	1·924	4·384	61¼
29°	0·875	0·485	1·749	0·970	2·624	1·454	3·498	1·939	4·373	**61°**
29¼	0·872	0·489	1·745	0·977	2·617	1·466	3·490	1·954	4·362	60¾
29½	0·870	0·492	1·741	0·985	2·611	1·477	3·481	1·970	4·352	60½
29¾	0·868	0·496	1·736	0·992	2·605	1·489	3·473	1·985	4·341	60¼
30°	0·866	0·500	1·732	1·000	2·598	1·500	3·464	2·000	4·330	**60°**
Bearing.	1 Dep.	1 Lat.	2 Dep.	2 Lat.	3 Dep.	3 Lat.	4 Dep.	4 Lat.	5 Dep.	Bearing.

Bearing.	5	6		7		8		9		Bearing.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
15°	1·294	5·796	1·553	6·761	1·812	7·727	2·071	8·693	2·329	**75°**
15¼	1·315	5·789	1·578	6·754	1·841	7·718	2·104	8·683	2·367	74¾
15½	1·336	5·782	1·603	6·745	1·871	7·709	2·138	8·673	2·405	74½
15¾	1·357	5·775	1·629	6·737	1·900	7·700	2·172	8·662	2·443	74¼
16°	1·378	5·768	1·654	6·729	1·929	7·690	2·205	8·651	2·481	**74°**
16¼	1·399	5·760	1·679	6·720	1·959	7·680	2·239	8 640	2·518	73¾
16½	1·420	5·753	1·704	6·712	1·988	7·671	2·272	8·629	2·556	73½
16¾	1·441	5·745	1·729	6·703	2·017	7·661	2·306	8·618	2·594	73¼
17°	1·462	5·738	1·754	6·694	2·047	7·650	2·339	8·607	2·631	**73°**
17¼	1·483	5·730	1·779	6·685	2·076	7·640	2·372	8·595	2·669	72¾
17½	1·504	5·722	1·804	6·676	2·105	7·630	2·406	8·583	2·706	72½
17¾	1·524	5·714	1·829	6·667	2·134	7·619	2·439	8·572	2·744	72¼
18°	1·545	5·706	1·854	6·657	2·163	7·608	2·472	8·560	2·781	**72°**
18¼	1·566	5·698	1·879	6·648	2·192	7·598	2·505	8·547	2·818	71¾
18½	1·587	5·690	1·904	6·638	2·221	7·587	2·538	8·535	2·856	71½
18¾	1·607	5·682	1·929	6·629	2·250	7·575	2·572	8·522	2·893	71¼
19°	1·628	5·673	1·953	6·619	2·279	7·564	2·605	8·510	2·930	**71°**
19¼	1·648	5·665	1·978	6·609	2·308	7·553	2·638	8·497	2·967	70¾
19½	1·669	5·656	2·003	6·598	2·337	7·541	2·670	8·484	3·004	70½
19¾	1·690	5·647	2·028	6·588	2·365	7·529	2·703	8·471	3·041	70¼
20°	1·710	5·638	2·052	6·578	2·394	7·518	2·736	8·457	3·078	**70°**
20¼	1·731	5·629	2·077	6·567	2·423	7·506	2·769	8·444	3·115	69¾
20½	1·751	5·620	2·101	6·557	2·451	7·493	2·802	8·430	3·152	69½
20¾	1·771	5·611	2·126	6·546	2·480	7·481	2·834	8·416	3·189	69¼
21°	1·792	5·601	2·150	6·535	2·509	7·469	2·867	8·402	3·225	**69°**
21¼	1·812	5·592	2·175	6·524	2·537	7·456	2·900	8·388	3·262	68¾
21½	1·833	5·582	2·199	6·513	2·566	7·443	2·932	8·374	3·299	68½
21¾	1·853	5·573	2·223	6·502	2·594	7·430	2·964	8·359	3·335	68¼
22°	1·873	5·563	2·248	6·490	2·622	7·417	2·997	8·345	3·371	**68°**
22¼	1·893	5·553	2·272	6·479	2·651	7·404	3·029	8·330	3·408	67¾
22½	1·913	5·543	2·296	6·467	2·679	7·391	3·061	8·315	3·444	67½
22¾	1·934	5·533	2·320	6·455	2·707	7·378	3·094	8·300	3·480	67¼
23°	1·954	5·523	2·344	6·444	2·735	7·364	3·126	8·285	3·517	**67°**
23¼	1·974	5·513	2·368	6·432	2·763	7·350	3·158	8·269	3·553	66¾
23½	1·994	5·502	2·392	6·419	2·791	7·336	3·190	8·254	3·589	66½
23¾	2·014	5·492	2·416	6·407	2·819	7·322	3·222	8·238	3·625	66¼
24°	2·034	5·481	2·440	6·395	2·847	7·308	3·254	8·222	3·661	**66°**
24¼	2·054	5·471	2·464	6·382	2·875	7·294	3·286	8·206	3·696	65¾
24½	2·073	5·460	2·488	6·370	2·903	7 280	3·318	8·190	3·732	65½
24¾	2·093	5·449	2·512	6·357	2·931	7·265	3·349	8·173	3·768	65¼
25°	2·113	5·438	2·536	6·344	2·958	7·250	3·381	8·157	3·804	**65°**
25¼	2·133	5·427	2·559	6·331	2·986	7·236	3·413	8·140	3·839	64¾
25½	2·153	5·416	2·583	6·318	3·014	7·221	3·444	8·123	3·875	64½
25¾	2·172	5·404	2·607	6·305	3·041	7·206	3·476	8·106	3·910	64¼
26°	2·192	5·393	2·630	6·292	3·069	7·190	3·507	8·089	3·945	**64°**
26¼	2·211	5·381	2·654	6·278	3·096	7·175	3·538	8·072	3·981	63¾
26½	2·231	5·370	2·677	6·265	3·123	7·160	3·570	8·054	4·016	63½
26¾	2·250	5·358	2·701	6·251	3·151	7·144	3·601	8·037	4·051	63¼
27°	2·270	5·346	2·724	6·237	3·178	7·128	3·632	8·019	4·086	**63°**
27¼	2·289	5·334	2·747	6·223	3·205	7·112	3·663	8·001	4·121	62¾
27½	2·309	5·322	2·770	6·209	3·232	7·096	3·694	7·983	4·156	62½
27¾	2·328	5·310	2·794	6·195	3·259	7·080	3·725	7·965	4·190	62¼
28°	2·347	5·298	2·817	6·181	3·286	7·064	3·756	7·947	4·225	**62°**
28¼	2 367	5·285	2·840	6·166	3·313	7·047	3·787	7·928	4·260	61¾
28½	2·386	5·273	2·863	6·152	3·340	7·031	3·817	7·909	4·294	61½
28¾	2·405	5·260	2·886	6·137	3·367	7·014	3·848	7·891	4·329	61¼
29°	2·424	5·248	2·909	6·122	3·394	6·997	3·878	7·872	4·363	**61°**
29¼	2·443	5·235	2·932	6·107	3·420	6·980	3·909	7·852	4·398	60¾
29½	2·462	5·222	2·955	6·093	3·447	6·963	3·939	7·833	4·432	60½
29¾	2·481	5·209	2·977	6·077	3·474	6·946	3·970	7·814	4·466	60¼
30°	2·500	5·196	3·000	6·062	3·501	6·928	4·000	7·794	4·500	**60°**
Bearing.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Bearing.
	5	6		7		8		9		

LATITUDES AND DEPARTURES.

Bearing.	1 Lat.	1 Dep.	2 Lat.	2 Dep.	3 Lat.	3 Dep.	4 Lat.	4 Dep.	5 Lat.	Bearing.
30°	0·866	0·500	1·732	1·000	2·598	1·500	3·464	2·000	4·330	**60°**
30¼	0·864	0·504	1·728	1·008	2·592	1·511	3·455	2·015	4·319	59¾
30½	0·862	0·508	1·723	1·015	2·585	1·523	3·447	2·030	4·308	59½
30¾	0·859	0·511	1·719	1·023	2·578	1·534	3·438	2·045	4·297	59¼
31°	0·857	0·515	1·714	1·030	2·572	1·545	3·429	2·060	4·286	**59°**
31¼	0·855	0·519	1·710	1·038	2·565	1·556	3·420	2·075	4·275	58¾
31½	0·853	0·522	1·705	1·045	2·558	1·567	3·411	2·090	4·263	58½
31¾	0·850	0·526	1·701	1·052	2·551	1·579	3·401	2·105	4·252	58¼
32°	0·848	0·530	1·696	1·060	2·544	1·590	3·392	2·120	4·240	**58°**
32¼	0·846	0·534	1·691	1·067	2·537	1·601	3·383	2·134	4·229	57¾
32½	0·843	0·537	1·687	1·075	2·530	1·612	3·374	2·149	4·217	57½
32¾	0·841	0·541	1·682	1·082	2·523	1·623	3·364	2·164	4·205	57¼
33°	0·839	0·545	1·677	1·089	2·516	1·634	3·355	2·179	4·193	**57°**
33¼	0·836	0·548	1·673	1·097	2·509	1·645	3·345	2·193	4·181	56¾
33½	0·834	0·552	1·668	1·104	2·502	1·656	3·336	2·208	4·169	56½
33¾	0·831	0·556	1·663	1·111	2·494	1·667	3·326	2·222	4·157	56¼
34°	0·829	0·559	1·658	1·118	2·487	1·678	3·316	2·237	4·145	**56°**
34¼	0·827	0·563	1·653	1·126	2·480	1·688	3·306	2·251	4·133	55¾
34½	0·824	0·566	1·648	1·133	2·472	1·699	3·297	2·266	4·121	55½
34¾	0·822	0·570	1·643	1·140	2·465	1·710	3·287	2·280	4·108	55¼
35°	0·819	0·574	1·638	1·147	2·457	1·721	3·277	2·294	4·096	**55°**
35¼	0·817	0·577	1·633	1·154	2·450	1·731	3·267	2·309	4·083	54¾
35½	0·814	0·581	1·628	1·161	2·442	1·742	3·257	2·323	4·071	54½
35¾	0·812	0·584	1·623	1·168	2·435	1·753	3·246	2·337	4·058	54¼
36°	0·809	0·588	1·618	1·176	2·427	1·763	3·236	2·351	4·045	**54°**
36¼	0·806	0·591	1·613	1·183	2·419	1·774	3·226	2·365	4·032	53¾
36½	0·804	0·595	1·608	1·190	2·412	1·784	3·215	2·379	4·019	53½
36¾	0·801	0·598	1·603	1·197	2·404	1·795	3·205	2·393	4·006	53¼
37°	0·799	0·602	1·597	1·204	2·396	1·805	3·195	2·407	3·993	**53°**
37¼	0·796	0·605	1·592	1·211	2·388	1·816	3·184	2·421	3·980	52¾
37½	0·793	0·609	1·587	1·218	2·380	1·826	3·173	2·435	3·967	52½
37¾	0·791	0·612	1·581	1·224	2·372	1·837	3·163	2·449	3·953	52¼
38°	0·788	0·616	1·576	1·231	2·364	1·847	3·152	2·463	3·940	**52°**
38¼	0·785	0·619	1·571	1·238	2·356	1·857	3·141	2·476	3·927	51¾
38½	0·783	0·623	1·565	1·245	2·348	1·868	3·130	2·490	3·913	51½
38¾	0·780	0·626	1·560	1·252	2·340	1·878	3·120	2·504	3·899	51¼
39°	0·777	0·629	1·554	1·259	2·331	1·888	3·109	2·517	3·886	**51°**
39¼	0·774	0·633	1·549	1·265	2·323	1·898	3·098	2·531	3·872	50¾
39½	0·772	0·636	1·543	1·272	2·315	1·908	3·086	2·544	3·858	50½
39¾	0·769	0·639	1·538	1·279	2·307	1·918	3·075	2·558	3·844	50¼
40°	0·766	0·643	1·532	1·286	2·298	1·928	3·064	2·571	3·830	**50°**
40¼	0·763	0·646	1·526	1·292	2·290	1·938	3·053	2·584	3·816	49¾
40½	0·760	0·649	1·521	1·299	2·281	1·948	3·042	2·598	3·802	49½
40¾	0·758	0·653	1·515	1·306	2·273	1·958	3·030	2·611	3·788	49¼
41°	0·755	0·656	1·509	1·312	2·264	1·968	3·019	2·624	3·774	**49°**
41¼	0·752	0·659	1·504	1·319	2·256	1·978	3·007	2·637	3·759	48¾
41½	0·749	0·663	1·498	1·325	2·247	1·988	2·996	2·650	3·745	48½
41¾	0·746	0·666	1·492	1·332	2·238	1·998	2·984	2·664	3·730	48¼
42°	0·743	0·669	1·486	1·338	2·229	2·007	2·973	2·677	3·716	**48°**
42¼	0·740	0·672	1·480	1·345	2·221	2·017	2·961	2·689	3·701	47¾
42½	0·737	0·676	1·475	1·351	2·212	2·027	2·949	2·702	3·686	47½
42¾	0·734	0·679	1·469	1·358	2·203	2·036	2·937	2·715	3·672	47¼
43°	0·731	0·682	1·463	1·364	2·194	2·046	2·925	2·728	3·657	**47°**
43¼	0·728	0·685	1·457	1·370	2·185	2·056	2·913	2·741	3·642	46¾
43½	0·725	0·688	1·451	1·377	2·176	2·065	2·901	2·753	3·627	46½
43¾	0·722	0·692	1·445	1·383	2·167	2·075	2·889	2·766	3·612	46¼
44°	0·719	0·695	1·439	1·389	2·158	2·084	2·877	2·779	3·597	**46°**
44¼	0·716	0·698	1·433	1·396	2·149	2·093	2·865	2·791	3·582	45¾
44½	0·713	0·701	1·427	1·402	2·140	2·103	2·853	2·804	3·566	45½
44¾	0·710	0·704	1·420	1·408	2·131	2·112	2·841	2·816	3·551	45¼
45°	0·707	0·707	1·414	1·414	2·121	2·121	2·828	2·828	3·536	**45°**
Bearing.	1 Dep.	1 Lat.	2 Dep.	2 Lat.	3 Dep.	3 Lat.	4 Dep.	4 Lat.	5 Dep.	Bearing.

Bearing.	5	6		7		8		9		Bearing.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
30°	2·500	5·196	3·000	6·062	3·500	6·928	4·000	7·794	4·500	**60°**
30¼	2·519	5·183	3·023	6·047	3·526	6·911	4·030	7·775	4·534	59¾
30½	2·538	5·170	3·045	6·031	3·553	6·893	4·060	7·755	4·568	59½
30¾	2·556	5·156	3·068	6·016	3·579	6·875	4·090	7·735	4·602	59¼
31°	2·575	5·143	3·090	6·000	3·605	6·857	4·120	7·715	4·635	**59°**
31¼	2·594	5·129	3·113	5·984	3·631	6·839	4·150	7·694	4·669	58¾
31½	2·612	5·116	3·135	5·968	3·657	6·821	4 80	7·674	4·702	58½
31¾	2·631	5·102	3·157	5·952	3 683	6·803	4·210	7·653	4·736	58¼
32°	2·650	5·088	3·180	5·936	3·709	6·784	4·239	7·632	4·769	**58°**
32¼	2·668	5·074	3·202	5·920	3·735	6·766	4·269	7·612	4·802	57¾
32½	2·686	5·060	3·224	5·904	3·761	6·747	4·298	7·591	4·836	57½
32¾	2·705	5·046	3·246	5·887	3·787	6·728	4·328	7·569	4·869	57¼
33°	2·723	5·032	3·268	5·871	3·812	6·709	4·357	7·548	4·902	**57°**
33¼	2·741	5·018	3·290	5·854	3·838	6·690	4·386	7·527	4·935	56¾
33½	2·760	5·003	3·312	5·837	3·864	6·671	4·416	7·505	4·967	56½
33¾	2·778	4·989	3·333	5·820	3·889	6·652	4·445	7·483	5·000	56¼
34°	2·796	4·974	3·355	5·803	3·914	6·632	4·474	7·461	5·033	**56°**
34¼	2·814	4·960	3·377	5·786	3·940	6·613	4·502	7·439	5·065	55¾
34½	2·832	4·945	3·398	5·769	3·965	6·593	4·531	7·417	5·098	55½
34¾	2·850	4·930	3·420	5·752	3·990	6·573	4·560	7·395	5·130	55¼
35°	2·868	4·915	3·441	5·734	4·015	6·553	4·589	7·372	5·162	**55°**
35¼	2·886	4·900	3·463	5·716	4·040	6·533	4·617	7 350	5·194	54¾
35½	2·904	4·885	3·484	5·699	4·065	6·513	4·646	7·327	5·226	54½
35¾	2·921	4·869	3·505	5·681	4·090	6·493	4·674	7·304	5·258	54¼
36°	2·939	4·854	3·527	5·663	4·115	6·472	4·702	7·281	5·290	**54°**
36¼	2·957	4·839	3·548	5·645	4·139	6·452	4·730	7·258	5·322	53¾
36½	2·974	4·823	3·569	5·627	4·164	6·431	4·759	7·235	5·353	53½
36¾	2·992	4·808	3·590	5·609	4·188	6·410	4·787	7·211	5·385	53¼
37°	3·009	4·792	3·611	5·590	4·213	6·389	4·815	7·188	5·416	**53°**
37¼	3·026	4·776	3·632	5·572	4·237	6·368	4·842	7·164	5·448	52¾
37½	3·044	4·760	3·653	5·554	4·261	6·347	4·870	7·140	5·479	52½
37¾	3·061	4·744	3·673	5·535	4·286	6·326	4·898	7·116	5·510	52¼
38°	3·078	4·728	3·694	5·516	4·310	6·304	4·925	7·092	5·541	**52°**
38¼	3·095	4·712	3·715	5·497	4·334	6·283	4·953	7·068	5·572	51¾
38½	3·113	4·696	3·735	5·478	4·358	6·261	4·980	7·043	5·603	51½
38¾	3·130	4·679	3·756	5·459	4·381	6·239	5·007	7·019	5·633	51¼
39°	3·147	4·663	3·776	5·440	4·405	6·217	5·035	6·994	5·664	**51°**
39¼	3·164	4·646	3·796	5·421	4·429	6·195	5·062	6·970	5·694	50¾
39½	3·180	4·630	3·816	5·401	4·453	6·173	5·089	6·945	5·725	50½
39¾	3·197	4·613	3·837	5·382	4·476	6·151	5·116	6·920	5·755	50¼
40°	3·214	4·596	3·857	5·362	4·500	6·128	5·142	6·894	5·785	**50°**
40¼	3·231	4·579	3·877	5·343	4·523	6·106	5·169	6·869	5·815	49¾
40½	3·247	4·562	3·897	5·323	4·546	6·083	5·196	6·844	5·845	49½
40¾	3·264	4·545	3·917	5·303	4·569	6·061	5·222	6·818	5·875	49¼
41°	3·280	4·528	3·936	5·283	4·592	6·038	5·248	6·792	5·905	**49°**
41¼	3·297	4·511	3·956	5·263	4·615	6·015	5·275	6·767	5·934	48¾
41½	3·313	4·494	3·976	5·243	4·638	5·992	5·301	6·741	5·964	48½
41¾	3·329	4·476	3·995	5·222	4·661	5·968	5·327	6·715	5·993	48¼
42°	3·346	4·459	4·015	5·202	4·684	5·945	5·353	6·688	6·022	**48°**
42¼	3·362	4·441	4·034	5·182	4·707	5·922	5·379	6·662	6·051	47¾
42½	3·378	4·424	4·054	5·161	4·729	5·898	5·405	6·635	6·080	47½
42¾	3·394	4·406	4·073	5·140	4·752	5·875	5·430	6·609	6·109	47¼
43°	3·410	4·388	4·092	5·119	4·774	5·851	5·456	6·582	6·138	**47°**
43¼	3·426	4·370	4·111	5·099	4·796	5·827	5·481	6·555	6·167	46¾
43½	3·442	4·352	4·130	5·078	4·818	5·803	5·507	6·528	6·195	46½
43¾	3·458	4·334	4·149	5·057	4·841	5·779	5·532	6·501	6·224	46¼
44°	3·473	4·316	4·168	5·035	4·863	5·755	5·557	6·474	6·252	**46°**
44¼	3·489	4·298	4·187	5·014	4·885	5·730	5·582	6·447	6·280	45¾
44½	3·505	4·280	4·206	4·993	4·906	5·706	5·607	6·419	6·308	45½
44¾	3·520	4·261	4·224	4·971	4·928	5·681	5·632	6·392	6·336	45¼
45°	3·536	4·243	4·243	4·950	4·950	5·657	5·657	6·364	6·364	**45°**
Bearing.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Bearing.
	5	6		7		8		9		

TABLE OF CHORDS: [Radius = 1.0000].

M.	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	M.
0′	·0000	·0175	·0349	·0524	·0698	·0872	·1047	·1221	·1395	·1569	·1743	0′
1	·0003	·0177	·0352	·0526	·0701	·0875	·1050	·1224	·1398	·1572	·1746	1
2	·0006	·0180	·0355	·0529	·0704	·0878	·1053	·1227	·1401	·1575	·1749	2
3	·0009	·0183	·0358	·0532	·0707	·0881	·1055	·1230	·1404	·1578	·1752	3
4	·0012	·0186	·0361	·0535	·0710	·0884	·1058	·1233	·1407	·1581	·1755	4
5	·0015	·0189	·0364	·0538	·0713	·0887	·1061	·1235	·1410	·1584	·1758	5
6	·0017	·0192	·0366	·0541	·0715	·0890	·1064	·1238	·1413	·1587	·1761	6
7	·0020	·0195	·0369	·0544	·0718	·0893	·1067	·1241	·1415	·1589	·1763	7
8	·0023	·0198	·0372	·0547	·0721	·0896	·1070	·1244	·1418	·1592	·1766	8
9	·0026	·0201	·0375	·0550	·0724	·0899	·1073	·1247	·1421	·1595	·1769	9
10	·0029	·0204	·0378	·0553	·0727	·0901	·1076	·1250	·1424	·1598	·1772	10
11	·0032	·0207	·0381	·0556	·0730	·0904	·1079	·1253	·1427	·1601	·1775	11
12	·0035	·0209	·0384	·0558	·0733	·0907	·1082	·1256	·1430	·1604	·1778	12
13	·0038	·0212	·0387	·0561	·0736	·0910	·1084	·1259	·1433	·1607	·1781	13
14	·0041	·0215	·0390	·0564	·0739	·0913	·1087	·1262	·1436	·1610	·1784	14
15	·0044	·0218	·0393	·0567	·0742	·0916	·1090	·1265	·1439	·1613	·1787	15
16	·0047	·0221	·0396	·0570	·0745	·0919	·1093	·1267	·1442	·1616	·1789	16
17	·0049	·0224	·0398	·0573	·0747	·0922	·1096	·1270	·1444	·1618	·1792	17
18	·0052	·0227	·0401	·0576	·0750	·0925	·1099	·1273	·1447	·1621	·1795	18
19	·0055	·0230	·0404	·0579	·0753	·0928	·1102	·1276	·1450	·1624	·1798	19
20	·0058	·0233	·0407	·0582	·0756	·0931	·1105	·1279	·1453	·1627	·1801	20
21	·0061	·0236	·0410	·0585	·0759	·0933	·1108	·1282	·1456	·1630	·1804	21
22	·0064	·0239	·0413	·0588	·0762	·0936	·1111	·1285	·1459	·1633	·1807	22
23	·0067	·0241	·0416	·0590	·0765	·0939	·1114	·1288	·1462	·1636	·1810	23
24	·0070	·0244	·0419	·0593	·0768	·0942	·1116	·1291	·1465	·1639	·1813	24
25	·0073	·0247	·0422	·0596	·0771	·0945	·1119	·1294	·1468	·1642	·1816	25
26	·0076	·0250	·0425	·0599	·0774	·0948	·1122	·1296	·1471	·1645	·1818	26
27	·0079	·0253	·0428	·0602	·0776	·0951	·1125	·1299	·1473	·1647	·1821	27
28	·0081	·0256	·0430	·0605	·0779	·0954	·1128	·1302	·1476	·1650	·1824	28
29	·0084	·0259	·0433	·0608	·0782	·0957	·1131	1305	·1479	·1653	·1827	29
30	·0087	·0262	·0436	·0611	·0785	·0960	·1134	·1308	·1482	·1656	·1830	30
31	·0090	·0265	·0439	·0614	·0788	·0962	·1137	·1311	·1485	·1659	·1833	31
32	·0093	·0268	·0442	·0617	·0791	·0965	·1140	·1314	·1488	·1662	·1836	32
33	·0096	·0271	·0445	·0619	·0794	·0968	·1143	·1317	·1491	·1665	·1839	33
34	·0099	·0273	·0448	·0622	·0797	·0971	·1145	·1320	·1494	·1668	·1842	34
35	·0102	·0276	·0451	·0625	·0800	·0974	·1148	·1323	·1497	·1671	·1845	35
36	·0105	·0279	·0454	·0628	·0803	·0977	·1151	·1325	·1500	·1674	·1847	36
37	·0108	·0282	·0457	·0631	·0806	·0980	·1154	·1328	·1502	·1676	·1850	37
38	·0111	·0285	·0460	·0634	·0808	·0983	·1157	·1331	·1505	1679	·1853	38
39	·0113	·0288	·0462	·0637	·0811	·0986	·1160	·1334	·1508	·1682	·1856	39
40	·0116	·0291	·0465	·0640	·0814	·0989	·1163	·1337	·1511	·1685	·1859	40
41	·0119	·0294	·0468	·0643	·0817	·0992	·1166	·1340	·1514	·1688	·1862	41
42	·0122	·0297	·0471	·0646	·0820	·0994	·1169	·1343	·1517	·1691	·1865	42
43	·0125	·0300	·0474	·0649	·0823	·0997	·1172	·1346	·1520	·1694	·1868	43
44	·0128	·0303	·0477	·0651	·0826	·1000	·1175	·1349	·1523	·1697	·1871	44
45	·0131	·0305	·0480	·0654	·0829	·1003	·1177	·1352	·1526	·1700	·1873	45
46	·0134	·0308	·0483	·0657	·0832	·1006	·1180	·1355	·1529	·1703	·1876	46
47	·0137	·0311	·0486	·0660	·0835	·1009	·1183	·1357	·1531	·1705	·1879	47
48	·0140	·0314	·0489	·0663	·0838	·1012	·1186	·1360	·1534	·1708	·1882	48
49	·0143	·0317	·0492	·0666	·0840	·1015	·1189	·1363	·1537	·1711	·1885	49
50	·0145	·0320	·0494	·0669	·0843	·1018	·1192	·1366	·1540	·1714	·1888	50
51	·0148	·0323	·0497	·0672	·0846	·1021	·1195	·1369	·1543	·1717	·1891	51
52	·0151	·0326	·0500	·0675	·0849	·1023	·1198	·1372	·1546	·1720	·1894	52
53	·0154	·0329	·0503	·0678	·0852	·1026	·1201	·1375	·1549	·1723	·1897	53
54	·0157	·0332	·0506	·0681	·0855	·1029	·1204	·1378	·1552	·1726	·1900	54
55	·0160	·0335	·0509	·0683	·0858	·1032	·1206	·1381	·1555	·1729	·1902	55
56	·0163	·0337	·0512	·0686	·0861	·1035	·1209	·1384	·1558	·1732	·1905	56
57	·0166	·0340	·0515	·0689	·0864	·1038	·1212	·1386	·1560	·1734	·1908	57
58	·0169	·0343	·0518	·0692	·0867	·1041	·1215	·1389	·1563	·1737	·1911	58
59	·0172	0346	·0521	·0695	·0869	·1044	·1218	·1392	·1566	·1740	·1914	59
60	·0175	·0349	·0524	·0698	·0872	·1047	·1221	·1395	·1569	·1743	·1917	60

TABLE OF CHORDS: [Radius = 1.0000].

M.	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	21°	M.
0′	·1917	·2091	·2264	·2437	·2611	·2783	·2956	·3129	·3301	·3473	·3645	0
1	·1920	·2093	·2267	·2440	·2613	·2786	·2959	·3132	·3304	·3476	·3648	1
2	·1923	·2096	·2270	·2443	·2616	·2789	·2962	·3134	·3307	·3479	·3650	2
3	·1926	·2099	·2273	·2446	·2619	·2792	·2965	·3137	·3310	·3482	·3653	3
4	·1928	·2102	·2276	·2449	·2622	·2795	·2968	·3140	·3312	·3484	·3656	4
5	·1931	·2105	·2279	·2452	·2625	·2798	·2971	·3143	·3315	·3487	·3659	5
6	·1934	·2108	·2281	·2455	·2628	·2801	·2973	·3146	·3318	·3490	·3662	6
7	·1937	·2111	·2284	·2458	·2631	·2804	·2976	·3149	·3321	·3493	·3665	7
8	·1940	·2114	·2287	·2460	·2634	·2807	·2979	·3152	·3324	·3496	·3668	8
9	·1943	·2117	·2290	·2463	·2636	·2809	·2982	·3155	·3327	·3499	·3670	9
10	·1946	·2119	·2293	·2466	·2639	·2812	·2985	·3157	·3330	·3502	·3673	10
11	·1949	·2122	·2296	·2469	·2642	·2815	·2988	·3160	·3333	·3504	·3676	11
12	·1952	·2125	·2299	·2472	·2645	·2818	2991	·3163	·3335	·3507	·3679	12
13	·1955	·2128	·2302	·2475	·2648	·2821	·2994	·3166	·3338	·3510	·3682	13
14	·1957	·2131	·2305	·2478	·2651	·2824	·2996	·3169	·3341	·3513	·3685	14
15	·1960	·2134	·2307	·2481	·2654	·2827	·2999	·3172	·3344	·3516	·3688	15
16	·1963	·2137	·2310	·2484	·2657	·2830	·3002	·3175	·3347	·3519	·3690	16
17	·1966	·2140	·2313	·2486	·2660	·2832	·3005	·3178	·3350	·3522	·3693	17
18	·1969	·2143	·2316	·2489	·2662	·2835	·3008	·3180	·3353	·3525	·3696	18
19	·1972	·2146	·2319	·2492	·2665	·2838	·3011	3183	·3355	·3527	·3699	19
20	·1975	·2148	·2322	·2495	·2668	·2841	·3014	·3186	·3358	·3530	·3702	20
21	·1978	·2151	·2325	·2498	·2671	·2844	·3017	·3189	·3361	·3533	·3705	21
22	·1981	·2154	·2328	·2501	·2674	·2847	·3019	·3192	·3364	·3536	·3708	22
23	·1983	·2157	·2331	·2504	·2677	·2850	·3022	·3195	·3367	·3539	·3710	23
24	·1986	·2160	·2333	·2507	·2680	·2853	·3025	·3198	·3370	·3542	·3713	24
25	·1989	·2163	·2336	·2510	·2683	·2855	·3028	·3200	·3373	·3545	·3716	25
26	·1992	·2166	·2339	·2512	·2685	·2858	·3031	·3203	·3376	·3547	·3719	26
27	·1995	·2169	·2342	·2515	·2688	·2861	·3034	·3206	·3378	·3550	·3722	27
28	·1998	·2172	·2345	·2518	·2691	·2864	·3037	·3209	·3381	·3553	·3725	28
29	·2001	·2174	·2348	·2521	·2694	·2867	·3040	·3212	·3384	·3556	·3728	29
30	·2004	·2177	·2351	·2524	·2697	·2870	·3042	·3215	·3387	·3559	·3730	30
31	·2007	·2180	·2354	·2527	·2700	·2873	·3045	·3218	·3390	·3562	·3733	31
32	·2010	·2183	·2357	·2530	·2703	·2876	·3048	·3221	·3393	·3565	·3736	32
33	·2012	·2186	·2359	·2533	·2706	·2878	·3051	·3223	·3396	·3567	·3739	33
34	·2015	·2189	·2362	·2536	·2709	·2881	·3054	·3226	·3398	·3570	·3742	34
35	·2018	·2192	·2365	·2538	·2711	·2884	·3057	·3229	·3401	·3573	·3745	35
36	·2021	·2195	·2368	·2541	·2714	·2887	·3060	·3232	·3404	·3576	·3748	36
37	·2024	·2198	·2371	·2544	·2717	·2890	·3063	·3235	·3407	·3579	·3750	37
38	·2027	·2200	·2374	·2547	·2720	·2893	·3065	·3238	·3410	·3582	·3753	38
39	·2030	·2203	·2377	·2550	·2723	·2896	·3068	·3241	·3413	·3585	·3756	39
40	·2033	·2206	·2380	·2553	·2726	·2899	·3071	·3244	·3416	·3587	·3759	40
41	·2036	·2209	·2383	·2556	·2729	·2902	·3074	·3246	·3419	·3590	·3762	41
42	·2038	·2212	·2385	·2559	·2732	·2904	·3077	·3249	·3421	·3593	·3765	42
43	·2041	·2215	·2388	·2561	·2734	·2907	·3080	·3252	·3424	·3596	·3768	43
44	·2044	·2218	·2391	·2564	·2737	·2910	·3083	·3255	·3427	·3599	·3770	44
45	·2047	·2221	·2394	·2567	·2740	·2913	·3086	·3258	·3430	·3602	·3773	45
46	·2050	·2224	·2397	·2570	·2743	·2916	·3088	·3261	·3433	·3605	·3776	46
47	·2053	·2226	·2400	·2573	·2746	·2919	·3091	·3264	·3436	·3608	·3779	47
48	·2056	·2229	·2403	·2576	·2749	·2922	·3094	·3267	·3439	·3610	·3782	48
49	·2059	·2232	·2406	·2579	·2752	·2925	·3097	·3269	·3441	·3613	·3785	49
50	·2062	·2235	·2409	·2582	·2755	·2927	·3100	·3272	·3444	·3616	·3788	50
51	·2065	·2238	·2411	·2585	·2758	·2930	·3103	·3275	·3447	·3619	·3790	51
52	·2067	·2241	·2414	·2587	2760	·2933	·3106	·3278	·3450	·3622	·3793	52
53	·2070	·2244	·2417	·2590	·2763	·2936	·3109	·3281	·3453	·3625	·3796	53
54	·2073	·2247	·2420	·2593	·2766	·2939	·3111	·3284	·3456	·3628	·3799	54
55	·2076	·2250	·2423	·2596	·2769	·2942	·3114	·3287	·3459	·3630	·3802	55
56	·2079	·2253	·2426	·2599	·2772	·2945	·3117	·3289	·3462	·3633	·3805	56
57	·2082	·2255	·2429	·2602	·2775	·2948	·3120	·3292	·3464	·3636	·3808	57
58	·2085	·2258	·2432	·2605	·2778	·2950	·3123	·3295	·3467	·3639	·3810	58
59	·2088	·2261	·2434	·2608	·2781	·2953	·3126	·3298	·3470	·3642	·3813	59
60	·2091	·2264	·2437	·2611	·2783	·2956	·3129	·3301	·3473	3645	·3816	60

TABLE OF CHORDS: [Radius = 1.0000].

M.	22°	23°	24°	25°	26°	27°	28°	29°	30°	31°	32°	M.
0'	.3816	.3987	.4158	.4329	.4499	.4669	.4838	.5008	.5176	.5345	.5513	0'
1	.3819	.3990	.4161	.4332	.4502	.4672	.4841	.5010	.5179	.5348	.5516	1
2	.3822	.3993	.4164	.4334	.4505	.4675	.4844	.5013	.5182	.5350	.5518	2
3	.3825	.3996	4167	.4337	.4508	.4677	.4847	.5016	.5185	.5353	.5521	3
4	.3828	.3999	.4170	.4340	.4510	.4680	.4850	.5019	.5188	.5356	.5524	4
5	.3830	.4002	.4172	.4343	.4513	.4683	.4853	.5022	.5190	.5359	.5527	5
6	.3833	.4004	.4175	.4346	.4516	.4686	.4855	.5024	.5193	.5362	.5530	6
7	.3836	.4007	.4178	.4349	.4519	.4689	.4858	.5027	.5196	.5364	.5532	7
8	.3839	.4010	.4181	.4352	.4522	.4692	.4861	.5030	.5199	.5367	.5535	8
9	.3842	.4013	.4184	.4354	.4525	.4694	.4864	.5033	.5202	.5370	.5538	9
10	.3845	.4016	.4187	.4357	.4527	.4697	.4867	.5036	.5204	.5373	.5541	10
11	.3848	.4019	.4190	.4360	.4530	.4700	.4869	.5039	.5207	.5376	.5543	11
12	.3850	.4022	.4192	.4363	.4533	.4703	.4872	.5041	.5210	.5378	.5546	12
13	.3853	.4024	.4195	.4366	.4536	.4706	.4875	.5044	.5213	.5381	.5549	13
14	.3856	.4027	.4198	.4369	.4539	.4708	.4878	.5047	.5216	.5384	.5552	14
15	.3859	.4030	.4201	.4371	.4542	.4711	.4881	.5050	.5219	.5387	.5555	15
16	.3862	.4033	.4204	.4374	.4544	.4714	.4884	.5053	.5221	.5390	.5557	16
17	.3865	.4036	.4207	.4377	.4547	.4717	.4886	.5055	.5224	.5392	.5560	17
18	.3868	.4039	.4209	.4380	.4550	.4720	.4889	.5058	.5227	.5395	.5563	18
19	.3870	.4042	.4212	.4383	.4553	.4723	.4892	.5061	.5230	.5398	.5566	19
20	.3873	.4044	.4215	.4386	.4556	.4725	.4895	.5064	.5233	.5401	.5569	20
21	.3876	.4047	.4218	.4388	.4559	.4728	.4898	.5067	.5235	.5404	.5571	21
22	.3879	.4050	.4221	.4391	.4561	.4731	.4901	.5070	.5238	.5406	.5574	22
23	.3882	.4053	.4224	.4394	.4564	.4734	.4903	.5072	.5241	.5409	.5577	23
24	.3885	.4056	.4226	.4397	.4567	.4737	.4906	.5075	.5244	.5412	.5580	24
25	.3888	.4059	.4229	.4400	.4570	.4740	.4909	.5078	.5247	.5415	.5583	25
26	.3890	.4061	.4232	.4403	.4573	.4742	.4912	.5081	.5249	.5418	.5585	26
27	.3893	.4064	.4235	.4405	.4576	.4745	.4915	.5084	.5252	.5420	.5588	27
28	.3896	.4067	.4238	.4408	.4578	.4748	.4917	.5086	.5255	.5423	.5591	28
29	.3899	.4070	.4241	.4411	.4581	.4751	.4920	.5089	.5258	.5426	.5594	29
30	.3902	.4073	.4244	.4414	.4584	.4754	.4923	.5092	.5261	.5429	.5597	30
31	.3905	.4076	.4246	.4417	.4587	.4757	.4926	.5095	.5263	.5432	.5599	31
32	.3908	.4079	.4249	.4420	.4590	.4759	.4929	.5098	.5266	.5434	.5602	32
33	.3910	.4081	.4252	.4422	.4593	.4762	.4932	.5100	.5269	.5437	.5605	33
34	.3913	.4084	.4255	.4425	.4595	.4765	.4934	.5103	.5272	.5440	.5608	34
35	.3916	.4087	.4258	.4428	.4598	.4768	.4937	.5106	.5275	.5443	.5611	35
36	.3919	.4090	.4261	.4431	.4601	.4771	.4940	.5109	.5277	.5446	.5613	36
37	.3922	.4093	.4263	.4434	.4604	.4773	.4943	.5112	.5280	.5448	.5616	37
38	.3925	.4096	.4266	.4437	.4607	.4776	.4946	.5115	.5283	.5451	.5619	38
39	.3927	.4098	.4269	.4439	.4609	.4779	.4948	.5117	.5286	.5454	.5622	39
40	.3930	.4101	.4272	.4442	.4612	.4782	.4951	.5120	.5289	.5457	.5625	40
41	.3933	.4104	.4275	.4445	.4615	.4785	.4954	.5123	.5291	.5460	.5627	41
42	3936	.4107	.4278	.4448	.4618	.4788	.4957	.5126	.5294	.5462	.5630	42
43	3939	.4110	.4280	.4451	.4621	.4790	.4960	.5129	.5297	.5465	.5633	43
44	.3942	.4113	.4283	.4454	.4624	.4793	.4963	.5131	.5300	.5468	.5636	44
45	.3945	.4116	.4286	.4456	.4626	.4796	.4965	.5134	.5303	.5471	.5638	45
46	.3947	.4118	.4289	.4459	.4629	.4799	.4968	.5137	.5306	.5474	5641	46
47	.3950	.4121	.4292	.4462	.4632	.4802	.4971	.5140	.5308	.5476	.5644	47
48	.3953	.4124	.4295	.4465	.4635	.4805	.4974	.5143	.5311	.5479	.5647	48
49	3956	.4127	.4298	.4468	.4638	.4807	.4977	.5145	.5314	.5482	.5650	49
50	.3959	.4130	.4300	.4471	.4641	.4810	.4979	.5148	.5317	.5485	.5652	50
51	.3962	.4133	.4303	.4474	.4643	.4813	.4982	.5151	.5320	.5488	.5655	51
52	.3965	.4135	.4306	.4476	.4646	.4816	.4985	.5154	.5322	.5490	.5658	52
53	.3967	.4138	.4309	.4479	.4649	.4819	.4988	.5157	.5325	.5493	.5661	53
54	.3970	.4141	.4312	.4482	.4652	.4822	.4991	.5160	.5328	.5496	.5664	54
55	.3973	.4144	.4315	.4485	.4655	.4824	.4994	.5162	.5331	.5499	.5666	55
56	.3976	.4147	.4317	.4488	.4658	.4827	.4996	.5165	.5334	.5502	.5669	56
57	.3979	.4150	.4320	.4491	.4660	.4830	.4999	.5168	.5336	.5504	.5672	57
58	.3982	.4153	.4323	.4493	.4663	.4833	.5002	.5171	.5339	.5507	.5675	58
59	.3985	.4155	.4326	.4496	.4666	.4836	.5005	.5174	.5342	.5510	.5678	59
60	.3987	.4158	.4329	.4499	.4669	.4838	.5008	.5176	.5345	5513	.5680	60

TABLE OF CHORDS: [Radius = 1.0000].

M.	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	43°	M.
0'	·5680	·5847	·6014	·6180	·6346	·6511	·6676	·6840	·7004	·7167	·7330	0
1	·5683	·5850	·6017	·6183	·6349	·6514	·6679	·6843	7007	·7170	·7333	1
2	·5686	·5853	·6020	·6186	·6352	·6517	·6682	·6846	7010	7173	·7335	2
3	·5689	·5856	·6022	·6189	·6354	·6520	·6684	·6849	·7012	·7176	·7338	3
4	·5691	·5859	·6025	·6191	·6357	·6522	·6687	·6851	·7015	7178	·7341	4
5	·5694	·5861	·6028	·6194	·6360	·6525	·6690	·6854	·7018	·7181	·7344	5
6	·5697	·5864	·6031	·6197	·6363	·6528	·6693	·6857	·7020	·7184	·7346	6
7	·5700	·5867	·6034	·6200	·6365	·6531	·6695	·6860	·7023	·7186	·7349	7
8	·5703	·5870	·6036	·6202	·6368	·6533	·6698	·6862	·7026	·7189	·7352	8
9	·5705	·5872	·6039	·6205	·6371	·6536	·6701	·6865	·7029	·7192	·7354	9
10	·5708	·5875	·6042	·6208	·6374	·6539	·6704	·6868	·7031	·7195	·7357	10
11	·5711	·5878	·6045	·6211	·6376	·6542	·6706	·6870	·7034	·7197	·7360	11
12	·5714	·5881	·6047	·6214	·6379	·6544	·6709	·6873	·7037	·7200	·7362	12
13	·5717	·5884	·6050	·6216	·6382	·6547	·6712	·6876	·7040	·7203	·7365	13
14	·5719	·5886	·6053	·6219	·6385	·6550	·6715	·6879	·7042	·7205	·7368	14
15	·5722	·5889	·6056	·6222	·6387	·6553	·6717	·6881	·7045	·7208	·7371	15
16	·5725	·5892	·6058	·6225	·6390	·6555	·6720	·6884	·7048	·7211	·7373	16
17	·5728	·5895	·6061	·6227	·6393	·6558	·6723	·6887	·7050	·7214	·7376	17
18	·5730	·5897	·6064	·6230	·6396	·6561	·6725	·6890	·7053	·7216	·7379	18
19	·5733	·5900	·6067	·6233	·6398	·6564	·6728	·6892	·7056	·7219	·7381	19
20	·5736	·5903	·6070	·6236	·6401	·6566	·6731	·6895	·7059	·7222	·7384	20
21	·5739	·5906	·6072	·6238	·6404	·6569	·6734	·6898	·7061	·7224	·7387	21
22	·5742	·5909	·6075	·6241	·6407	·6572	·6736	·6901	·7064	·7227	·7390	22
23	·5744	·5911	·6078	·6244	·6410	·6575	·6739	·6903	·7067	·7230	·7392	23
24	·5747	·5914	·6081	·6247	·6412	·6577	·6742	·6906	·7069	·7232	·7395	24
25	·5750	·5917	·6083	·6249	·6415	·6580	·6745	·6909	·7072	·7235	·7398	25
26	·5753	·5920	·6086	·6252	·6418	·6583	·6747	·6911	·7075	·7238	·7400	26
27	·5756	·5922	·6089	·6255	·6421	·6586	·6750	·6914	·7078	·7241	·7403	27
28	·5758	·5925	·6092	·6258	·6423	·6588	·6753	·6917	·7080	·7243	·7406	28
29	·5761	·5928	·6095	·6260	·6426	·6591	·6756	·6920	·7083	·7246	·7408	29
30	·5764	·5931	·6097	·6263	·6429	·6594	·6758	6922	·7086	·7249	·7411	30
31	·5767	·5934	·6100	·6266	·6432	·6597	·6761	·6925	·7089	·7251	·7414	31
32	·5769	·5936	·6103	·6269	·6434	·6599	·6764	·6928	·7091	·7254	·7417	32
33	·5772	·5939	·6106	·6272	·6437	·6602	·6767	·6931	·7094	·7257	·7419	33
34	·5775	·5942	·6108	·6274	·6440	·6605	·6769	·6933	·7097	·7260	·7422	34
35	·5778	·5945	·6111	·6277	·6443	·6608	·6772	·6936	·7099	·7262	·7425	35
36	·5781	·5947	·6114	·6280	·6445	·6610	·6775	·6939	·7102	·7265	·7427	36
37	·5783	·5950	·6117	·6283	·6448	·6613	·6777	·6941	·7105	·7268	·7430	37
38	·5786	·5953	·6119	·6285	·6451	·6616	·6780	·6944	·7108	·7270	·7433	38
39	·5789	·5956	·6122	·6288	·6454	·6619	·6783	·6947	·7110	·7273	·7435	39
40	·5792	·5959	·6125	·6291	·6456	·6621	·6786	·6950	·7113	·7276	·7438	40
41	·5795	·5961	·6128	·6294	·6459	·6624	·6788	·6952	·7116	·7279	·7441	41
42	·5797	·5964	·6130	·6296	·6462	·6627	·6791	·6955	·7118	·7281	·7443	42
43	·5800	·5967	·6133	·6299	·6465	·6630	·6794	·6958	·7121	·7284	·7446	43
44	·5803	·5970	·6136	·6302	·6467	·6632	·6797	·6961	·7124	·7287	·7449	44
45	·5806	·5972	·6139	·6305	·6470	·6635	·6799	·6963	·7127	·7289	·7452	45
46	·5808	·5975	·6142	·6307	·6473	·6638	·6802	·6966	·7129	·7292	·7454	46
47	·5811	·5978	·6144	·6310	·6476	·6640	·6805	·6969	·7132	·7295	·7457	47
48	·5814	·5981	·6147	·6313	·6478	·6643	·6808	·6971	·7135	·7298	·7460	48
49	·5817	·5984	·6150	·6316	·6481	·6646	·6810	·6974	·7137	·7300	·7462	49
50	·5820	·5986	·6153	·6318	·6484	·6649	·6813	·6977	·7140	·7303	·7465	50
51	·5822	·5989	·6155	·6321	·6487	·6651	·6816	·6980	·7143	·7306	·7468	51
52	·5825	·5992	·6158	·6324	·6489	·6654	·6819	·6982	·7146	·7308	·7471	52
53	·5828	·5995	·6161	·6327	·6492	·6657	·6821	·6985	·7148	·7311	·7473	53
54	·5831	·5997	·6164	·6330	·6495	·6660	·6824	·6988	·7151	·7314	·7476	54
55	·5834	·6000	·6166	·6332	·6498	·6662	·6827	·6991	·7154	·7316	·7479	55
56	·5836	·6003	·6169	·6335	·6500	·6665	·6829	·6993	·7156	·7319	·7481	56
57	·5839	·6006	·6172	·6338	·6503	·6668	·6832	·6996	·7159	·7322	·7484	57
58	·5842	·6009	·6175	·6341	·6506	·6671	·6835	·6999	·7162	·7325	·7487	58
59	·5845	·6011	·6178	·6343	·6509	·6673	·6838	·7001	·7165	·7327	·7489	59
60	·5847	·6014	·6180	·6346	·6511	·6676	·6840	·7004	·7167	·7330	·7492	60

TABLE OF CHORDS: [Radius = 1.0000].

M.	44°	45°	46°	47°	48°	49°	50°	51°	52°	53°	54°	M.
0′	.7492	.7654	.7815	.7975	.8135	.8294	.8452	.8610	.8767	.8924	.9080	0′
1	.7495	.7656	.7817	.7978	.8137	.8297	.8455	.8613	.8770	.8927	.9082	1
2	.7498	.7659	.7820	.7980	.8140	.8299	.8458	.8615	.8773	.8929	.9085	2
3	.7500	.7662	.7823	.7983	.8143	.8302	.8460	.8618	.8775	.8932	.9088	3
4	.7503	.7664	.7825	.7986	.8145	.8304	.8463	.8621	.8778	.8934	.9090	4
5	.7506	.7667	.7828	.7988	.8148	.8307	.8466	.8623	.8780	.8937	.9093	5
6	.7508	.7670	.7831	.7991	.8151	.8310	.8468	.8626	.8783	.8940	.9095	6
7	.7511	.7672	.7833	.7994	.8153	.8312	.8471	.8629	.8786	.8942	.9098	7
8	.7514	.7675	.7836	.7996	.8156	.8315	.8473	.8631	.8788	.8945	.9101	8
9	.7516	.7678	.7839	.7999	.8159	.8318	.8476	.8634	.8791	.8947	.9103	9
10	.7519	.7681	.7841	.8002	.8161	.8320	.8479	.8636	.8794	.8950	.9106	10
11	.7522	.7683	.7844	.8004	.8164	.8323	.8481	.8639	.8796	.8953	.9108	11
12	.7524	.7686	.7847	.8007	.8167	.8326	.8484	.8642	.8799	.8955	.9111	12
13	.7527	.7689	.7849	.8010	.8169	.8328	.8487	.8644	.8801	.8958	.9113	13
14	.7530	.7691	.7852	.8012	.8172	.8331	.8489	.8647	.8804	.8960	.9116	14
15	.7533	.7694	.7855	.8015	.8175	.8334	.8492	.8650	.8807	.8963	.9119	15
16	.7535	.7697	.7857	.8018	.8177	.8336	.8495	.8652	.8809	.8966	.9121	16
17	.7538	.7699	.7860	.8020	.8180	.8339	.8497	.8655	.8812	.8968	.9124	17
18	.7541	.7702	.7863	.8023	.8183	.8341	.8500	.8657	.8814	.8971	.9126	18
19	.7543	.7705	.7865	.8026	.8185	.8344	.8502	.8660	.8817	.8973	.9129	19
20	.7546	.7707	.7868	.8028	.8188	.8347	.8505	.8663	.8820	.8976	.9132	20
21	.7549	.7710	.7871	.8031	.8190	.8349	.8508	.8665	.8822	.8979	.9134	21
22	.7551	.7713	.7873	.8034	.8193	.8352	.8510	.8668	.8825	.8981	.9137	22
23	.7554	.7715	.7876	.8036	.8196	.8355	.8513	.8671	.8828	.8984	.9139	23
24	.7557	.7718	.7879	.8039	.8198	.8357	.8516	.8673	.8830	.8986	.9142	24
25	.7560	.7721	.7882	.8042	.8201	.8360	.8518	.8676	.8833	.8989	.9145	25
26	.7562	.7723	.7884	.8044	.8204	.8363	.8521	.8678	.8835	.8992	.9147	26
27	.7565	.7726	.7887	.8047	.8206	.8365	.8523	.8681	.8838	.8994	.9150	27
28	.7568	.7729	.7890	.8050	.8209	.8368	.8526	.8684	.8841	.8997	.9152	28
29	.7570	.7731	.7892	.8052	.8212	.8371	.8529	.8686	.8843	.8999	.9155	29
30	.7573	.7734	.7895	.8055	.8214	.8373	.8531	.8689	.8846	.9002	.9157	30
31	.7576	.7737	.7898	.8058	.8217	.8376	.8534	.8692	.8848	.9005	.9160	31
32	.7578	.7740	.7900	.8060	.8220	.8378	.8537	.8694	.8851	.9007	.9163	32
33	.7581	.7742	.7903	.8063	.8222	.8381	.8539	.8697	.8854	.9010	.9165	33
34	.7584	.7745	.7906	.8066	.8225	.8384	.8542	.8699	.8856	.9012	.9168	34
35	.7586	.7748	.7908	.8068	.8228	.8386	.8545	.8702	.8859	.9015	.9170	35
36	.7589	.7750	.7911	.8071	.8230	.8389	.8547	.8705	.8861	.9018	.9173	36
37	.7592	.7753	.7914	.8074	.8233	.8392	.8550	.8707	.8864	.9020	.9176	37
38	.7595	.7756	.7916	.8076	.8236	.8394	.8552	.8710	.8867	.9023	.9178	38
39	.7597	.7758	.7919	.8079	.8238	.8397	.8555	.8712	.8869	.9025	.9181	39
40	.7600	.7761	.7922	.8082	.8241	.8400	.8558	.8715	.8872	.9028	.9183	40
41	.7603	.7764	.7924	.8084	.8244	.8402	.8560	.8718	.8874	.9031	.9186	41
42	.7605	.7766	.7927	.8087	.8246	.8405	.8563	.8720	.8877	.9033	.9188	42
43	.7608	.7769	.7930	.8090	.8249	.8408	.8566	.8723	.8880	.9036	.9191	43
44	.7611	.7772	.7932	.8092	.8251	.8410	.8568	.8726	.8882	.9038	.9194	44
45	.7613	.7774	.7935	.8095	.8254	.8413	.8571	.8728	.8885	.9041	.9196	45
46	.7616	.7777	.7938	.8098	.8257	.8415	.8573	.8731	.8887	.9044	.9199	46
47	.7619	7780	.7940	.8100	.8259	.8418	.8576	.8734	.8890	.9046	.9201	47
48	.7621	.7782	.7943	.8103	.8262	.8421	.8579	.8736	.8893	.9049	.9204	48
49	.7624	.7785	.7946	.8105	.8265	.8423	.8581	.8739	.8895	.9051	.9207	49
50	.7627	.7788	.7948	.8108	.8267	.8426	.8584	.8741	.8898	.9054	.9209	50
51	.7629	.7791	.7951	.8111	.8270	.8429	.8587	.8744	.8900	.9056	.9212	51
52	.7632	.7793	.7954	.8113	.8273	.8431	.8589	8747	.8903	.9059	.9214	52
53	.7635	.7796	.7956	.8116	.8275	.8434	.8592	.8749	.8906	.9062	.9217	53
54	.7638	.7799	.7959	8119	.8278	.8437	.8594	.8752	.8908	.9064	.9219	54
55	.7640	.7801	7962	.8121	.8281	.8439	.8597	.8754	.8911	.9067	.9222	55
56	.7643	.7804	.7964	.8124	.8283	.8442	.8600	.8757	.8914	.9069	.9225	56
57	.7646	.7807	.7967	.8127	.8286	.8444	.8602	.8760	.8916	.9072	.9227	57
58	.7648	.7809	.7970	.8129	.8289	.8447	.8605	.8762	.8919	.9075	.9230	58
59	7651	.7812	.7972	.8132	.8291	.8450	.8608	.8765	.8921	.9077	.9232	59
60	.7654	.7815	.7975	.8135	.8294	.8452	.8610	.8767	.8924	.9080	.9235	60

TABLE OF CHORDS: [Radius = 1.0000].

M.	55°	56°	57°	58°	59°	60°	61°	62°	63°	64°	M.
0′	·9235	·9389	·9543	·9696	·9848	1·0000	1·0151	1·0301	1·0450	1·0598	0′
1	·9238	·9392	·9546	·9699	·9851	1·0003	1·0153	1·0303	1·0452	1·0601	1
2	·9240	·9395	·9548	·9701	·9854	1·0005	1·0156	1·0306	1·0455	1·0603	2
3	·9243	·9397	·9551	·9704	·9856	1·0008	1·0158	1·0308	1·0457	1·0606	3
4	·9245	·9400	·9553	·9706	·9859	1·0010	1·0161	1·0311	1·0460	1·0608	4
5	·9248	·9402	·9556	·9709	·9861	1·0013	1·0163	1·0313	1·0462	1·0611	5
6	·9250	·9405	·9559	·9711	·9864	1·0015	1·0166	1·0316	1·0465	1·0613	6
7	·9253	·9407	·9561	·9714	·9866	1·0018	1·0168	1·0318	1·0467	1·0616	7
8	·9256	·9410	·9564	·9717	·9869	1·0020	1·0171	1·0321	1·0470	1·0618	8
9	·9258	·9413	·9566	·9719	·9871	1·0023	1·0173	1·0323	1·0472	1·0621	9
10	·9261	·9415	·9569	·9722	·9874	1·0025	1·0176	1·0326	1·0475	1·0623	10
11	·9263	·9418	·9571	·9724	·9876	1·0028	1·0178	1·0328	1·0477	1·0626	11
12	·9266	·9420	·9574	·9727	·9879	1·0030	1·0181	1·0331	1·0480	1·0628	12
13	·9268	·9423	·9576	·9729	·9881	1·0033	1·0183	1·0333	1·0482	1·0630	13
14	·9271	·9425	·9579	·9732	·9884	1·0035	1·0186	1·0336	1·0485	1·0633	14
15	·9274	·9428	·9581	·9734	·9886	1·0038	1·0188	1·0338	1·0487	1·0635	15
16	·9276	·9430	·9584	·9737	·9889	1·0040	1·0191	1·0341	1·0490	1·0638	16
17	·9279	·9433	·9587	·9739	·9891	1·0043	1·0193	1·0343	1·0492	1·0640	17
18	·9281	·9436	·9589	·9742	·9894	1·0045	1·0196	1·0346	1·0495	1·0643	18
19	·9284	·9438	·9592	·9744	·9897	1·0048	1·0198	1·0348	1·0497	1·0645	19
20	·9287	·9441	·9594	·9747	·9899	1·0050	1·0201	1·0351	1·0500	1·0648	20
21	·9289	·9443	·9597	·9750	·9902	1·0053	1·0203	1·0353	1·0502	1·0650	21
22	·9292	·9446	·9599	·9752	·9904	1·0055	1·0206	1·0356	1·0504	1·0653	22
23	·9294	·9448	·9602	·9755	·9907	1·0058	1·0208	1·0358	1·0507	1·0655	23
24	·9297	·9451	·9604	·9757	·9909	1·0060	1·0211	1·0361	1·0509	1·0658	24
25	·9299	·9454	·9607	·9760	·9912	1·0063	1·0213	1·0363	1·0512	1·0660	25
26	·9302	·9456	·9610	·9762	·9914	1·0065	1·0216	1·0366	1·0514	1·0662	26
27	·9305	·9459	·9612	·9765	·9917	1·0068	1·0218	1·0368	1·0517	1·0665	27
28	·9307	·9461	·9615	·9767	·9919	1·0070	1·0221	1·0370	1·0519	1·0667	28
29	·9310	·9464	·9617	·9770	·9922	1·0073	1·0223	1·0373	1·0522	1·0670	29
30	·9312	·9466	·9620	·9772	·9924	1·0075	1·0226	1·0375	1·0524	1·0672	30
31	·9315	·9469	·9622	·9775	·9927	1·0078	1·0228	1·0378	1·0527	1·0675	31
32	·9317	·9472	·9625	·9778	·9929	1·0080	1·0231	1·0380	1·0529	1·0677	32
33	·9320	·9474	·9627	·9780	·9932	1·0083	1·0233	1·0383	1·0532	1·0680	33
34	·9323	·9477	·9630	·9783	·9934	1·0086	1·0236	1·0385	1·0534	1·0682	34
35	·9325	·9479	·9633	9785	·9937	1·0088	1·0238	1·0388	1·0537	1·0685	35
36	·9328	·9482	·9635	·9788	·9939	1·0091	1·0241	1·0390	1·0539	1·0687	36
37	·9330	·9484	·9638	·9790	·9942	1·0093	1·0243	1·0393	1·0542	1·0690	37
38	·9333	·9487	·9640	·9793	·9945	1·0096	1·0246	1·0395	1·0544	1·0692	38
39	·9335	·9489	·9643	·9795	·9947	1·0098	1·0248	1·0398	1·0547	1·0694	39
40	·9338	·9492	·9645	·9798	·9950	1·0101	1·0251	1·0400	1·0549	1·0697	40
41	·9341	·9495	·9648	·9800	·9952	1·0103	1·0253	1·0403	1·0551	1·0699	41
42	·9343	·9497	·9650	·9803	·9955	1·0106	1·0256	1·0405	1·0554	1·0702	42
43	·9346	·9500	·9653	·9805	·9957	1·0108	1·0258	1·0408	1·0556	1·0704	43
44	·9348	·9502	·9655	·9808	·9960	1·0111	1·0261	1·0410	1·0559	1·0707	44
45	·9351	·9505	·9658	·0810	·9962	1·0113	1·0263	1·0413	1·0561	1·0709	45
46	·9353	·9507	·9661	·9813	·9965	1·0116	1·0266	1·0415	1·0564	1·0712	46
47	·9356	·9510	·9663	·9816	·9967	1·0118	1·0268	1·0418	1·0566	1·0714	47
48	·9359	·9512	·9666	·9818	·9970	1·0121	1·0271	1·0420	1·0569	1·0717	48
49	·9361	·9515	·9668	·9821	·9972	1·0123	1·0273	1·0423	1·0571	1·0719	49
50	·9364	·9518	·9671	·9823	·9975	1·0126	1·0276	1·0425	1·0574	1·0721	50
51	·9366	·9520	·9673	·9826	·9977	1·0128	1·0278	1·0428	1·0576	1·0724	51
52	·9369	·9523	·9676	·9828	·9980	1·0131	1·0281	1·0430	1·0579	1·0726	52
53	·9371	·9525	·9678	·9831	·9982	1·0133	1·0283	1·0433	1·0581	1·0729	53
54	·9374	·9528	·9681	·9833	·9985	1·0136	1·0286	1·0435	1·0584	1·0731	54
55	·9377	·9530	·9683	·9836	·9987	1·0138	1·0288	1·0438	1·0586	1·0734	55
56	·9379	·9533	·9686	·9838	·9990	1·0141	1·0291	1·0440	1·0589	1·0736	56
57	·9382	·9536	·9689	·9841	·9992	1·0143	1·0293	1·0443	1·0591	1·0739	57
58	·9384	·9538	·9691	·9843	·9995	1·0146	1·0296	1·0445	1·0593	1·0741	58
59	·9387	·9541	·9694	·9846	·9998	1·0148	1·0298	1·0447	1·0596	1·0744	59
60	·9389	·9543	·9696	·9848	10000	1·0151	1·0301	1·0450	1·0598	1·0746	60

TABLE OF CHORDS: [Radius = 1.0000].

M.	65°	66°	67°	68°	69°	70°	71°	72°	73°	M.
0′	1·0746	1·0893	1·1039	1·1184	1·1328	1·1472	1·1614	1·1756	1·1896	0
1	1·0748	1·0895	1·1041	1·1186	1·1331	1·1474	1·1616	1·1758	1·1899	1
2	1·0751	1·0898	1·1044	1·1189	1·1333	1·1476	1·1619	1·1760	1·1901	2
3	1·0753	1·0900	1·1046	1·1191	1·1335	1·1479	1·1621	1·1763	1·1903	3
4	1·0756	1·0903	1·1048	1·1194	1·1338	1·1481	1·1624	1·1765	1·1906	4
5	1·0758	1·0905	1·1051	1·1196	1·1340	1·1483	1·1626	1·1767	1·1908	5
6	1·0761	1·0907	1·1053	1 1198	1·1342	1 1486	1·1628	1·1770	1·1910	6
7	1·0763	1·0910	1·1056	1·1201	1·1345	1·1488	1·1631	1·1772	1·1913	7
8	1·0766	1·0912	1·1058	1·1203	1·1347	1·1491	1·1633	1·1775	1·1915	8
9	1·0768	1·0915	1·1061	1·1206	1·1350	1·1493	1·1635	1·1777	1·1917	9
10	1·0771	1·0917	1·1063	1·1208	1·1352	1·1495	1·1638	1·1779	1·1920	10
11	1·0773	1·0920	1·1065	1·1210	1·1354	1·1498	1·1640	1·1782	1·1922	11
12	1·0775	1·0922	1·1068	1·1213	1·1357	1·1500	1·1642	1·1784	1·1924	12
13	1·0778	1·0924	1·1070	1·1215	1·1359	1·1502	1·1645	1·1786	1·1927	13
14	1·0780	1·0927	1·1073	1·1218	1·1362	1·1505	1·1647	1·1789	1·1929	14
15	1·0783	1·0929	1·1075	1·1220	1·1364	1·1507	1·1650	1·1791	1·1931	15
16	1·0785	1·0932	1·1078	1·1222	1·1366	1·1510	1·1652	1·1793	1·1934	16
17	1·0788	1·0934	1·1080	1·1225	1·1369	1·1512	1·1654	1·1796	1·1936	17
18	1·0790	1·0937	1·1082	1·1227	1·1371	1·1514	1·1657	1·1798	1·1938	18
19	1·0793	1·0939	1·1085	1·1230	1·1374	1·1517	1·1659	1·1800	1·1941	19
20	1·0795	1·0942	1·1087	1·1232	1·1376	1·1519	1·1661	1·1803	1·1943	20
21	1·0797	1·0944	1·1090	1·1234	1·1378	1·1522	1·1664	1·1805	1·1946	21
22	1·0800	1·0946	1·1092	1·1237	1·1381	1·1524	1·1666	1·1807	1·1948	22
23	1·0802	1·0949	1·1094	1·1239	1·1383	1·1526	1·1668	1·1810	1·1950	23
24	1·0805	1·0951	1·1097	1·1242	1·1386	1·1529	1·1671	1·1812	1·1952	24
25	1·0807	1·0954	1·1099	1·1244	1·1388	1·1531	1·1673	1·1814	1·1955	25
26	1·0810	1·0956	1·1102	1·1246	1·1390	1·1533	1·1676	1·1817	1·1957	26
27	1·0812	1·0959	1·1104	1·1249	1·1393	1·1536	1·1678	1·1819	1·1959	27
28	1·0815	1·0961	1·1107	1·1251	1·1395	1·1538	1·1680	1·1821	1·1962	28
29	1·0817	1·0963	1·1109	1·1254	1·1398	1·1541	1·1683	1·1824	1·1964	29
30	1·0820	1·0966	1·1111	1·1256	1·1400	1·1543	1·1685	1·1826	1·1966	30
31	1·0822	1·0968	1·1114	1·1258	1·1402	1·1545	1·1687	1·1829	1·1969	31
32	1·0824	1·0971	1·1116	1·1261	1·1405	1·1548	1·1690	1·1831	1·1971	32
33	1·0827	1·0973	1·1119	1·1263	1·1407	1·1550	1·1692	1·1833	1·1973	33
34	1·0829	1·0976	1·1121	1·1266	1·1409	1·1552	1·1694	1·1836	1·1976	34
35	1·0832	1·0978	1·1123	1·1268	1·1412	1·1555	1·1697	1·1838	1·1978	35
36	1·0834	1·0980	1·1126	1·1271	1·1414	1·1557	1·1699	1·1840	1·1980	36
37	1·0837	1·0983	1·1128	1·1273	1·1417	1·1560	1·1702	1·1843	1·1983	37
38	1·0839	1·0985	1·1131	1·1275	1·1419	1·1562	1·1704	1·1845	1·1985	38
39	1·0841	1·0988	1·1133	1·1278	1·1421	1·1564	1·1706	1·1847	1·1987	39
40	1·0844	1·0990	1·1136	1·1280	1·1424	1·1567	1·1709	1·1850	1·1990	40
41	1·0846	1·0993	1·1138	1·1283	1·1426	1·1569	1·1711	1·1852	1·1992	41
42	1·0849	1·0995	1·1140	1·1285	1·1429	1·1571	1·1713	1·1854	1·1994	42
43	1·0851	1·0997	1·1143	1·1287	1·1431	1·1574	1·1716	1·1857	1·1997	43
44	1·0854	1·1000	1·1145	1·1290	1·1433	1·1576	1·1718	1·1859	1·1999	44
45	1·0856	1·1002	1·1148	1·1292	1·1436	1·1579	1·1720	1·1861	1·2001	45
46	1·0859	1·1005	1·1150	1·1295	1·1438	1·1581	1·1723	1·1864	1·2004	46
47	1·0861	1·1007	1·1152	1·1297	1·1441	1·1583	1·1725	1·1866	1·2006	47
48	1·0863	1·1010	1·1155	1·1299	1·1443	1·1586	1·1727	1·1868	1·2008	48
49	1·0866	1·1012	1·1157	1·1302	1·1445	1·1588	1·1730	1·1871	1·2011	49
50	1·0868	1·1014	1·1160	1·1304	1·1448	1·1590	1·1732	1·1873	1·2013	50
51	1·0871	1·1017	1·1162	1·1307	1·1450	1·1593	1·1735	1·1875	1·2015	51
52	1·0873	1·1019	1·1165	1·1309	1·1452	1·1595	1·1737	1·1878	1·2018	52
53	1·0876	1·1022	1·1167	1·1311	1·1455	1·1598	1·1739	1·1880	1·2020	53
54	1·0878	1·1024	1·1169	1·1314	1·1457	1·1600	1·1742	1·1882	1·2022	54
55	1·0881	1·1027	1·1172	1·1316	1·1460	1·1602	1·1744	1·1885	1·2025	55
56	1·0883	1·1029	1·1174	1·1319	1·1462	1·1605	1·1746	1·1887	1·2027	56
57	1·0885	1·1031	1·1177	1·1321	1·1464	1·1607	1·1749	1·1889	1·2029	57
58	1·0888	1·1034	1·1179	1·1323	1·1467	1·1609	1·1751	1·1892	1·2032	58
59	1·0890	1·1036	1·1181	1·1326	1·1469	1·1612	1·1753	1·1894	1·2034	59
60	1·0893	1·1039	1·1184	1·1328	1·1472	1·1614	1·1756	1·1896	1·2036	60

TABLE OF CHORDS: [Radius = 1.0000].

M.	74°	75°	76°	77°	78°	79°	80°	81°	82°	M.
0′	1·2036	1·2175	1·2313	1·2450	1·2586	1·2722	1·2856	1·2989	1·3121	0′
1	1·2039	1·2178	1·2316	1·2453	1·2589	1·2724	1·2858	1·2991	1·3123	1
2	1·2041	1·2180	1·2318	1·2455	1·2591	1·2726	1·2860	1·2993	1·3126	2
3	1·2043	1·2182	1·2320	1·2457	1·2593	1·2728	1·2862	1·2996	1·3128	3
4	1·2046	1·2184	1·2322	1·2459	1·2595	1·2731	1·2865	1·2998	1·3130	4
5	1·2048	1·2187	1·2325	1·2462	1·2598	1·2733	1·2867	1·3000	1·3132	5
6	1·2050	1·2189	1·2327	1·2464	1·2600	1·2735	1·2869	1·3002	1·3134	6
7	1·2053	1·2191	1·2329	1·2466	1·2602	1·2737	1·2871	1·3004	1·3137	7
8	1·2055	1·2194	1·2332	1·2468	1·2604	1·2740	1·2874	1·3007	1·3139	8
9	1·2057	1·2196	1·2334	1·2471	1·2607	1·2742	1·2876	1·3009	1·3141	9
10	1·2060	1·2198	1·2336	1·2473	1·2609	1·2744	1·2878	1·3011	1·3143	10
11	1·2062	1·2201	1·2338	1·2475	1·2611	1·2746	1·2880	1·3013	1·3145	11
12	1·2064	1·2203	1·2341	1·2478	1·2614	1·2748	1·2882	1·3015	1·3147	12
13	1·2066	1·2205	1·2343	1·2480	1·2616	1·2751	1·2885	1·3018	1·3150	13
14	1·2069	1·2208	1·2345	1·2482	1·2618	1·2753	1·2887	1·3020	1·3152	14
15	1·2071	1·2210	1·2348	1·2484	1·2620	1·2755	1·2889	1·3022	1·3154	15
16	1·2073	1·2212	1·2350	1·2487	1·2623	1·2757	1·2891	1·3024	1·3156	16
17	1·2076	1·2214	1·2352	1·2489	1·2625	1·2760	1·2894	1·3027	1·3158	17
18	1·2078	1·2217	1·2354	1·2491	1·2627	1·2762	1·2896	1·3029	1·3161	18
19	1·2080	1·2219	1·2357	1·2493	1·2629	1·2764	1·2898	1·3031	1·3163	19
20	1·2083	1·2221	1·2359	1·2496	1·2632	1·2766	1·2900	1·3033	1·3165	20
21	1·2085	1·2224	1·2361	1·2498	1·2634	1·2769	1·2903	1·3035	1·3167	21
22	1·2087	1·2226	1·2364	1·2500	1·2636	1·2771	1·2905	1·3038	1·3169	22
23	1·2090	1·2228	1·2366	1·2503	1·2638	1·2773	1·2907	1·3040	1·3172	23
24	1·2092	1·2231	1·2368	1·2505	1·2641	1·2775	1·2909	1·3042	1·3174	24
25	1·2094	1·2233	1·2370	1·2507	1·2643	1·2778	1·2911	1·3044	1·3176	25
26	1·2097	1·2235	1·2373	1·2509	1·2645	1·2780	1·2914	1·3046	1·3178	26
27	1·2099	1·2237	1·2375	1·2512	1·2648	1·2782	1·2916	1·3049	1·3180	27
28	1·2101	1·2240	1·2377	1·2514	1·2650	1·2784	1·2918	1·3051	1·3183	28
29	1·2104	1·2242	1·2380	1·2516	1·2652	1·2787	1·2920	1·3053	1·3185	29
30	1·2106	1·2244	1·2382	1·2518	1·2654	1·2789	1·2922	1·3055	1·3187	30
31	1·2108	1·2247	1·2384	1·2521	1·2656	1·2791	1·2925	1·3057	1·3189	31
32	1·2111	1·2249	1·2386	1·2523	1·2659	1·2793	1·2927	1·3060	1·3191	32
33	1·2113	1·2251	1·2389	1·2525	1·2661	1·2795	1·2929	1·3062	1·3193	33
34	1·2115	1·2254	1·2391	1·2528	1·2663	1·2798	1·2931	1·3064	1·3196	34
35	1·2117	1·2256	1·2393	1·2530	1·2665	1·2800	1·2934	1·3066	1·3198	35
36	1·2120	1·2258	1·2396	1·2532	1·2668	1·2802	1·2936	1·3068	1·3200	36
37	1·2122	1·2260	1·2398	1·2534	1·2670	1·2804	1·2938	1·3071	1·3202	37
38	1·2124	1·2263	1·2400	1·2537	1·2672	1·2807	1·2940	1·3073	1·3204	38
39	1·2127	1·2265	1·2402	1·2539	1·2674	1·2809	1·2942	1·3075	1·3207	39
40	1·2129	1·2267	1·2405	1·2541	1·2677	1·2811	1·2945	1·3077	1·3209	40
41	1·2131	1·2270	1·2407	1·2543	1·2679	1·2813	1·2947	1·3079	1·3211	41
42	1·2134	1·2272	1·2409	1·2546	1·2681	1·2816	1·2949	1·3082	1·3213	42
43	1·2136	1·2274	1·2412	1·2548	1·2683	1·2818	1·2951	1·3084	1·3215	43
44	1·2138	1·2277	1·2414	1·2550	1·2686	1·2820	1·2954	1·3086	1·3218	44
45	1·2141	1·2279	1·2416	1·2552	1·2688	1·2822	1·2956	1·3088	1·3220	45
46	1·2143	1·2281	1·2418	1·2555	1·2690	1·2825	1·2958	1·3090	1·3222	46
47	1·2145	1·2283	1·2421	1·2557	1·2692	1·2827	1·2960	1·3093	1·3224	47
48	1·2148	1·2286	1·2423	1·2559	1·2695	1·2829	1·2962	1·3095	1·3226	48
49	1·2150	1·2288	1·2425	1·2562	1·2697	1·2831	1·2965	1·3097	1·3228	49
50	1·2152	1·2290	1·2428	1·2564	1·2699	1·2833	1·2967	1·3099	1·3231	50
51	1·2154	1·2293	1·2430	1·2566	1·2701	1·2836	1·2969	1·3101	1·3233	51
52	1·2157	1·2295	1·2432	1·2568	1·2704	1·2838	1·2971	1·3104	1·3235	52
53	1·2159	1·2297	1·2434	1·2571	1·2706	1·2840	1·2973	1·3106	1·3237	53
54	1·2161	1·2299	1·2437	1·2573	1·2708	1·2842	1·2976	1·3108	1·3239	54
55	1·2164	1·2302	1·2439	1·2575	1·2710	1·2845	1·2978	1·3110	1·3242	55
56	1·2166	1·2304	1·2441	1·2577	1·2713	1·2847	1·2980	1·3112	1·3244	56
57	1·2168	1·2306	1·2443	1·2580	1·2715	1·2849	1·2982	1·3115	1·3246	57
58	1·2171	1·2309	1·2446	1·2582	1·2717	1·2851	1·2985	1·3117	1·3248	58
59	1·2173	1·2311	1·2448	1·2584	1·2719	1·2854	1·2987	1·3119	1·3250	59
60	1·2175	1·2313	1·2450	1·2586	1·2722	1·2856	1·2989	1·3121	1·3252	60

TABLE OF CHORDS: [Radius = 1.0000].

M.	83°	84°	85°	86°	87°	88°	89°	M.
0'	1·3252	1·3383	1·3512	1·3640	1·3767	1·3893	1·4018	0'
1	1·3255	1·3385	1·3514	1·3642	1·3769	1·3895	1·4020	1
2	1·3257	1·3387	1·3516	1·3644	1·3771	1·3897	1·4022	2
3	1·3259	1·3389	1·3518	1·3646	1·3773	1·3899	1·4024	3
4	1·3261	1·3391	1·3520	1·3648	1·3776	1·3902	1·4026	4
5	1·3263	1·3393	1·3523	1·3651	1·3778	1·3904	1·4029	5
6	1·3265	1·3396	1·3525	1·3653	1·3780	1·3906	1·4031	6
7	1·3268	1·3398	1·3527	1·3655	1·3782	1·3908	1·4033	7
8	1·3270	1·3400	1·3529	1·3657	1·3784	1·3910	1·4035	8
9	1·3272	1·3402	1·3531	1·3659	1·3786	1·3912	1·4037	9
10	1·3274	1·3404	1·3533	1·3661	1·3788	1·3914	1·4039	10
11	1·3276	1·3406	1·3535	1·3663	1·3790	1·3916	1·4041	11
12	1·3279	1·3409	1·3538	1·3665	1·3792	1·3918	1·4043	12
13	1·3281	1·3411	1·3540	1·3668	1·3794	1·3920	1·4045	13
14	1·3283	1·3413	1·3542	1·3670	1·3797	1·3922	1·4047	14
15	1·3285	1·3415	1·3544	1·3672	1·3799	1·3925	1·4049	15
16	1·3287	1·3417	1·3546	1·3674	1·3801	1·3927	1·4051	16
17	1·3289	1·3419	1·3548	1·3676	1·3803	1·3929	1·4053	17
18	1·3292	1·3421	1·3550	1·3678	1·3805	1·3931	1·4055	18
19	1·3294	1·3424	1·3552	1·3680	1·3807	1·3933	1·4058	19
20	1·3296	1·3426	1·3555	1·3682	1·3809	1·3935	1·4060	20
21	1·3298	1·3428	1·3557	1·3685	1·3811	1·3937	1·4062	21
22	1·3300	1·3430	1·3559	1·3687	1·3813	1·3939	1·4064	22
23	1·3302	1·3432	1·3561	1·3689	1·3816	1·3941	1·4066	23
24	1·3305	1·3434	1·3563	1·3691	1·3818	1·3943	1·4068	24
25	1·3307	1·3437	1·3565	1·3693	1·3820	1·3945	1·4070	25
26	1·3309	1·3439	1·3567	1·3695	1·3822	1·3947	1·4072	26
27	1·3311	1·3441	1·3570	1·3697	1·3824	1·3950	1·4074	27
28	1·3313	1·3443	1·3572	1·3699	1·3826	1·3952	1·4076	28
29	1·3315	1·3445	1·3574	1·3702	1·3828	1·3954	1·4078	29
30	1·3318	1·3447	1·3576	1·3704	1·3830	1·3956	1·4080	30
31	1·3320	1·3449	1·3578	1·3706	1·3832	1·3958	1·4082	31
32	1·3322	1·3452	1·3580	1·3708	1·3834	1·3960	1·4084	32
33	1·3324	1·3454	1·3582	1·3710	1·3837	1·3962	1·4086	33
34	1·3326	1·3456	1·3585	1·3712	1·3839	1·3964	1·4089	34
35	1·3328	1·3458	1·3587	1·3714	1·3841	1·3966	1·4091	35
36	1·3331	1·3460	1·3589	1·3716	1·3843	1·3968	1·4093	36
37	1·3333	1·3462	1·3591	1·3718	1·3845	1·3970	1·4095	37
38	1·3335	1·3465	1·3593	1·3721	1·3847	1·3972	1·4097	38
39	1·3337	1·3467	1·3595	1·3723	1·3849	1·3975	1·4099	39
40	1·3339	1·3469	1·3597	1·3725	1·3851	1·3977	1·4101	40
41	1·3341	1·3471	1·3599	1·3727	1·3853	1·3979	1·4103	41
42	1·3344	1·3473	1·3602	1·3729	1·3855	1·3981	1·4105	42
43	1·3346	1·3475	1·3604	1·3731	1·3858	1·3983	1·4107	43
44	1·3348	1·3477	1·3606	1·3733	1·3860	1·3985	1·4109	44
45	1·3350	1·3480	1·3608	1·3735	1·3862	1·3987	1·4111	45
46	1·3352	1·3482	1·3610	1·3738	1·3864	1·3989	1·4113	46
47	1·3354	1·3484	1·3612	1·3740	1·3866	1·3991	1·4115	47
48	1·3357	1·3486	1·3614	1·3742	1·3868	1·3993	1·4117	48
49	1·3359	1·3488	1·3617	1·3744	1·3870	1·3995	1·4119	49
50	1·3361	1·3490	1·3619	1·3746	1·3872	1·3997	1·4122	50
51	1·3363	1·3492	1·3621	1·3748	1·3874	1·3999	1·4124	51
52	1·3365	1·3495	1·3623	1·3750	1·3876	1·4002	1·4126	52
53	1·3367	1·3497	1·3625	1·3752	1·3879	1·4004	1·4128	53
54	1·3370	1·3499	1·3627	1·3754	1·3881	1·4006	1·4130	54
55	1·3372	1·3501	1·3629	1·3757	1·3883	1·4008	1·4132	55
56	1·3374	1·3503	1·3631	1·3759	1·3885	1·4010	1·4134	56
57	1·3376	1·3505	1·3634	1·3761	1·3887	1·4012	1·4136	57
58	1·3378	1·3508	1·3636	1·3763	1·3889	1·4014	1·4138	58
59	1·3380	1·3510	1·3638	1·3765	1·3891	1·4016	1·4140	59
60	1·3383	1·3512	1·3640	1·3767	1·3893	1·4018	1·4142	60

TABLE I.

OF

LOGARITHMS OF NUMBERS

FROM

1 TO 10000.

N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0·000000	26	1·414973	51	1·707570	76	1·880814
2	0·301030	27	1·431364	52	1·716003	77	1·886491
3	0·477121	28	1·447158	53	1·724276	78	1·892095
4	0·602060	29	1·462398	54	1·732394	79	1·897627
5	0·698970	30	1·477121	55	1·740363	80	1·903090
6	0·778151	31	1·491362	56	1·748188	81	1·908485
7	0·845098	32	1·505150	57	1·755875	82	1·913814
8	0·903090	33	1·518514	58	1·763428	83	1·919078
9	0·954243	34	1·531479	59	1·770852	84	1·924279
10	1·000000	35	1·544068	60	1·778151	85	1·929419
11	1·041393	36	1·556303	61	1·785330	86	1·934498
12	1·079181	37	1·568202	62	1·792392	87	1·939519
13	1·113943	38	1·579784	63	1·799341	88	1·944483
14	1·146128	39	1·591065	64	1·806180	89	1·949390
15	1·176091	40	1·602060	65	1·812913	90	1·954243
16	1·204120	41	1·612784	66	1·819544	91	1·959041
17	1·230449	42	1·623249	67	1·826075	92	1·963788
18	1·255273	43	1·633468	68	1·832509	93	1·968483
19	1·278754	44	1·643453	69	1·838849	94	1·973128
20	1·301030	45	1·653213	70	1·845098	95	1·977724
21	1·322219	46	1·662758	71	1·851258	96	1·982271
22	1·342423	47	1·672098	72	1·857333	97	1·986772
23	1·361728	48	1·681241	73	1·863323	98	1·991226
24	1·380211	49	1·690196	74	1·869232	99	1·995635
25	1·397940	50	1·698970	75	1·875061	100	2·000000

N. B. In the following table, in the last nine columns of each page, where the first or leading figures change from 9's to 0's, the character ⬥ is introduced instead of the 0's, to catch the eye, and to indicate that from thence the annexed first two figures of the Logarithm in the second column stand in the next lower line directly under the *asterisk*.

N.	0	1	2	3	4	5	6	7	8	9	D.
100	00 0000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	*8600	9026	9451	9876	◆300	0724	1147	1570	1993	2415	424
103	01 2837	3259	3680	4100	4521	4940	5360	5779	6197	6616	419
104	*7033	7451	7868	8284	8700	9116	9532	9947	◆361	0775	416
105	02 1189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
107	*9384	9789	◆195	0600	1004	1408	1812	2216	2619	3021	404
108	03 3424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	*7426	7825	8223	8620	9017	9414	9811	◆207	0602	0998	396
110	04 1393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	389
112	*9218	9606	9993	◆380	0766	1153	1538	1924	2309	2694	386
113	05 3078	3463	3846	4230	4613	4996	5378	5760	6142	6524	382
114	*6905	7286	7666	8046	8426	8805	9185	9563	9942	◆320	379
115	06 0698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	372
117	*8186	8557	8928	9298	9668	◆038	0407	0776	1145	1514	369
118	07 1882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	*9181	9543	9904	◆266	0626	0987	1347	1707	2067	2426	360
121	08 2785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
123	*9905	◆258	0611	0963	1315	1667	2018	2370	2721	3071	351
124	09 3422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
125	*6910	7257	7604	7951	8298	8644	8990	9335	9681	◆026	346
126	10 0371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	340
128	*7210	7549	7888	8227	8565	8903	9241	9579	9916	◆253	338
129	11 0590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	3943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
131	*7271	7603	7934	8265	8595	8926	9256	9586	9915	◆245	330
132	12 0574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
134	*7105	7429	7753	8076	8399	8722	9045	9368	9690	◆012	323
135	13 0334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	315
138	*9879	◆194	0508	0822	1136	1450	1763	2076	2389	2702	314
139	14 3015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
141	*9219	9527	9835	◆142	0449	0756	1063	1370	1676	1982	307
142	15 2288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
144	*8362	8664	8965	9266	9567	9868	◆168	0469	0769	1068	301
145	16 1368	1667	1967	2266	2564	2863	3161	3460	3758	4055	299
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
147	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
148	17 0262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
150	6091	6381	6670	6959	7248	7536	7825	8113	8401	8689	289
151	*8977	9264	9552	9839	◆126	0413	0699	0985	1272	1558	287
152	18 1844	2129	2415	2700	2985	3270	3555	3839	4123	4407	285
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
154	*7521	7803	8084	8366	8647	8928	9209	9490	9771	◆051	281
155	19 0332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	*8657	8932	9206	9481	9755	◆029	0303	0577	0850	1124	274
159	20 1397	1670	1943	2216	2488	2761	3033	3305	3577	3848	272
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
160	20 4120	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
161	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
162	*9515	9783	◆051	0319	0586	0853	1121	1388	1654	1921	267
163	21 2188	2454	2720	2986	3252	3518	3783	4049	4314	4579	266
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
165	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
166	22 0108	0370	0631	0892	1153	1414	1675	1936	2196	2456	261
167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
169	*7887	8144	8400	8657	8913	9170	9426	9682	9938	◆193	256
170	23 0449	0704	0960	1215	1470	1724	1979	2234	2488	2742	254
171	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
173	*8046	8297	8548	8799	9049	9299	9550	9800	◆050	0300	250
174	24 0549	0799	1048	1297	1546	1795	2044	2293	2541	2790	249
175	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
176	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	*7973	8219	8464	8709	8954	9198	9443	9687	9932	◆176	245
178	25 0420	0664	0908	1151	1395	1638	1881	2125	2368	2610	243
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
180	5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	26 0071	0310	0548	0787	1025	1263	1501	1739	1976	2214	238
183	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
185	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
186	*9513	9746	9980	◆213	0446	0679	0912	1144	1377	1609	233
187	27 1842	2074	2306	2538	2770	3001	3233	3464	3696	3927	232
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
189	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
190	*8754	8982	9211	9439	9667	9895	◆123	0351	0578	0806	228
191	28 1033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	29 0035	0257	0480	0702	0925	1147	1369	1591	1813	2034	222
196	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
197	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
198	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
199	*8853	9071	9289	9507	9725	9943	◆161	0378	0595	0813	218
200	30 1030	1247	1464	1681	1898	2114	2331	2547	2764	2980	217
201	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
202	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
203	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
204	*9630	9843	◆056	0268	0481	0693	0906	1118	1330	1542	212
205	31 1754	1966	2177	2389	2600	2812	3023	3234	3445	3656	211
206	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
207	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
209	32 0146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	2219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
211	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
212	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
213	*8380	8583	8787	8991	9194	9398	9601	9805	◆008	0211	203
214	33 0414	0617	0819	1022	1225	1427	1630	1832	2034	2236	202
215	2438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
216	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
217	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
218	*8456	8656	8855	9054	9253	9451	9650	9849	◆047	0246	199
219	34 0444	0642	0841	1039	1237	1435	1632	1830	2028	2225	198
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
220	34 2423	2620	2817	3014	3212	3409	3606	3802	3999	4196	197
221	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
223	*8305	8500	8694	8889	9083	9278	9472	9666	9860	•054	194
224	35 0248	0442	0636	0829	1023	1216	1410	1603	1796	1989	193
225	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
226	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
228	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
229	*9835	•025	0215	0404	0593	0783	0972	1161	1350	1539	189
230	36 1728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
231	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
234	*9216	9401	9587	9772	9958	•143	0328	0513	0698	0883	185
235	37 1068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
239	*8398	8580	8761	8943	9124	9306	9487	9668	9849	•030	181
240	38 0211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
241	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
242	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
243	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
244	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
245	*9166	9343	9520	9698	9875	•051	0228	0405	0582	0759	177
246	39 0935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
247	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
248	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
249	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	7940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
251	*9674	9847	•020	0192	0365	0538	0711	0883	1056	1228	173
252	40 1401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
253	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
254	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
255	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
256	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
257	*9933	•102	0271	0440	0609	0777	0946	1114	1283	1451	169
258	41 1620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
259	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
260	4973	5140	5307	5474	5641	5808	5974	6141	6308	6474	167
261	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
262	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
263	*9956	•121	0286	0451	0616	0781	0945	1110	1275	1439	165
264	42 1604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
265	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
266	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
267	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
268	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
269	*9752	9914	•075	0236	0398	0559	0720	0881	1042	1203	161
270	43 1364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
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272	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
273	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
274	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
275	*9333	9491	9648	9806	9964	•122	0279	0437	0594	0752	158
276	44 0909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
277	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
278	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
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291	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
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295	* 9822	9969	•116	0263	0410	0557	0704	0851	0998	1145	147
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305	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
306	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997	142
307	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
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309	* 9958	•099	0239	0380	0520	0661	0801	0941	1081	1222	140
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338	* 8917	9045	9174	9302	9430	9559	9687	9815	9943	•072	128
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411	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
412	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
413	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
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419	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
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423	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
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470	2098	2190	2283	2375	2467	2560	2652	2744	2836	2929	92
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473	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
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475	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
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487	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
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489	*9309	9398	9486	9575	9664	9753	9841	9930	*019	0107	89
490	69 0196	0285	0373	0462	0550	0639	0728	0816	0905	0993	89
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494	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
495	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
496	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
497	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
498	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
499	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	8970	9057	9144	9231	9317	9404	9491	9578	9664	9751	87
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502	70 0704	0790	0877	0963	1050	1136	1222	1309	1395	1482	86
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507	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
508	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
509	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
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512	*9270	9355	9440	9524	9609	9694	9779	9863	9948	*033	85
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514	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
515	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
516	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
517	3491	3575	3650	3742	3826	3910	3994	4078	4162	4246	84
518	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
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522	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
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524	*9331	9414	9497	9580	9663	9745	9828	9911	9994	*077	83
525	72 0159	0242	0325	0407	0490	0573	0655	0738	0821	0903	83
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527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
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529	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
530	4276	4358	4440	4522	4604	4685	4767	4849	4931	5013	82
531	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
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533	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
534	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
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536	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
537	*9974	*055	0136	0217	0298	0378	0459	0540	0621	0702	81
538	73 0782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
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540	2394	2474	2555	2635	2715	2796	2876	2956	3037	3117	80
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547	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
549	*9572	9651	9731	9810	9889	9968	*047	0126	0205	0284	79
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552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2646	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
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555	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
556	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
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565	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
566	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
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568	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
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571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
572	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
573	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
574	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
575	*9668	9743	9819	9894	9970	*045	0121	0196	0272	0347	75
576	76 0422	0498	0573	0649	0724	0799	0875	0950	1025	1101	75
577	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
578	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
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588	*9377	9451	9525	9599	9673	9746	9820	9894	9968	◆042	74
589	77 0115	0189	0263	0336	0410	0484	0557	0631	0705	0778	74
590	0852	0926	0999	1073	1146	1220	1293	1367	1440	1514	74
591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
597	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
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605	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
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609	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	5330	5401	5472	5543	5615	5686	5757	5828	5899	5970	71
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614	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
615	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
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618	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
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620	2392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
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622	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
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624	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
625	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
626	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
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633	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
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635	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
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645	*9560	9627	9694	9762	9829	9896	9964	*031	0098	0165	67
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647	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
648	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
649	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
650	2913	2980	3047	3114	3181	3247	3314	3381	3448	3514	67
651	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
652	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
653	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
654	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
655	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
656	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
657	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
658	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
659	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	*9544	9610	9676	9741	9807	9873	9939	*004	0070	0136	66
661	82 0201	0267	0333	0399	0464	0530	0595	0661	0727	0792	66
662	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
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664	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
665	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
666	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
667	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	6075	6140	6204	6269	6334	6399	6464	6528	6593	6658	65
671	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
672	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
673	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
674	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
675	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
676	*9947	*011	0075	0139	0204	0268	0332	0396	0460	0525	64
677	83 0589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
678	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
679	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
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683	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
684	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
685	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
686	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
687	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
688	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
689	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
690	8849	8912	8975	9038	9101	9164	9227	9289	9352	9415	63
691	*9478	9541	9604	9667	9729	9792	9855	9918	9981	*043	63
692	84 0106	0169	0232	0294	0357	0420	0482	0545	0608	0671	63
693	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
694	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
695	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
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702	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
703	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
705	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
707	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
708	85 0033	0095	0156	0217	0279	0340	0401	0462	0524	0585	61
709	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	1258	1320	1381	1442	1503	1564	1625	1686	1747	1809	61
711	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
712	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
713	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
714	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
715	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
716	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
717	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
718	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
719	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
720	7332	7393	7453	7513	7574	7634	7694	7755	7815	7875	60
721	7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
722	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
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724	*9739	9799	9859	9918	9978	•038	0098	0158	0218	0278	60
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726	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
727	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
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731	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
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733	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
734	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
735	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
736	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
737	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
738	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
739	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
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741	*9818	9877	9935	9994	•053	0111	0170	0228	0287	0345	59
742	87 0404	0462	0521	0579	0638	0696	0755	0813	0872	0930	58
743	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
744	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
745	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
746	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
747	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
748	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
749	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
750	5061	5119	5177	5235	5293	5351	5409	5466	5524	5582	58
751	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
752	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
753	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
754	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
755	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
756	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
757	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
758	*9669	9726	9784	9841	9898	9956	•013	0070	0127	0185	57
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762	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
763	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
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767	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
768	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
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776	*9862	9918	9974	*030	0086	0141	0197	0253	0309	0365	56
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783	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
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785	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
786	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
787	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
788	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
789	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	7627	7682	7737	7792	7847	7902	7957	8012	8067	8122	55
791	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
792	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
793	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
794	*9821	9875	9930	9985	*039	0094	0149	0203	0258	0312	55
795	90 0367	0422	0476	0531	0586	0640	0695	0749	0804	0859	55
796	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
797	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
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800	3090	3144	3199	3253	3307	3361	3416	3470	3524	3578	54
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802	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
803	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
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809	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
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812	*9556	9610	9663	9716	9770	9823	9877	9930	9984	*037	53
813	91 0091	0144	0197	0251	0304	0358	0411	0464	0518	0571	53
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815	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
816	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
817	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
818	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
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822	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
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831	*9601	9653	9706	9758	9810	9862	9914	9967	*019	0071	52
832	92 0123	0176	0228	0280	0332	0384	0436	0489	0541	0593	52
833	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
834	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
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838	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
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843	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
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849	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
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851	*9930	9981	*032	0083	0134	0185	0236	0287	0338	0389	51
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853	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
854	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
855	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
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857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
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874	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
875	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
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882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	9390	9439	9488	9536	9585	9634	9683	9731	9780	9829	49
891	*9878	9926	9975	*024	0073	0121	0170	0219	0267	0316	49
892	95 0365	0414	0462	0511	0560	0608	0657	0706	0754	0803	49
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
895	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
898	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
899	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
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901	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
902	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
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906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
908	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
909	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	9041	9089	9137	9185	9232	9280	9328	9375	9423	9471	48
911	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
912	*9995	*042	0090	0138	0185	0233	0280	0328	0376	0423	48
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914	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	47
915	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
916	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
917	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
918	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
919	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	3788	3835	3882	3929	3977	4024	4071	4118	4165	4212	47
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923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
924	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
925	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
926	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
929	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
930	8483	8530	8576	8623	8670	8716	8763	8810	8856	8903	47
931	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
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934	97 0347	0393	0440	0486	0533	0579	0626	0672	0719	0765	46
935	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
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942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	7724	7769	7815	7861	7906	7952	7998	8043	8089	8135	46
951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
955	98 0003	0049	0094	0140	0185	0231	0276	0322	0367	0412	45
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	45
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	6772	6817	6861	6906	6951	6996	7040	7085	7130	7175	45
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
975	9005	9049	9004	9138	9183	9227	9272	9316	9361	9405	45
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
977	* 9895	9939	9983	◆028	0072	0117	0161	0206	0250	0294	44
978	99 0339	0383	0428	0472	0516	0561	0605	0650	0694	0738	44
979	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
980	1226	1270	1315	1359	1403	1448	1492	1536	1580	1625	44
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
985	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	5635	5679	5723	5767	5811	5854	5898	5942	5986	6030	44
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
995	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
N.	0	1	2	3	4	5	6	7	8	9	D.

TABLE II.

LOGARITHMIC SINES AND TANGENTS.

FOR

EVERY DEGREE AND MINUTE OF THE QUADRANT.

If the logarithms of the values in Table III. be each increased by 10, the results will be the values of this table.

The logarithmic Secants and Cosecants are not given. They may be readily obtained, as follows:—Subtract the logarithmic Cosine from 20, and the remainder will be the logarithmic Secant; subtract the logarithmic Sine from 20, and the remainder will be the logarithmic Cosecant.

0°								179°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	Inf. Neg.		10·000000		Inf. Neg.		Infinite.	60
1	6·463726	501717	000000	00	6·463726	501717	13·536274	59
2	764756	293485	000000	00	764756	293483	235244	58
3	940847	208231	000000	00	940847	208231	059153	57
4	7·065786	161517	000000	00	7·065786	161517	12·934214	56
5	162696	131968	000000	00	162696	131969	837304	55
6	241877	111575	9·999999	01	241878	111578	758122	54
7	308824	96653	999999	01	308825	99653	691175	53
8	366816	85254	999999	01	366817	85254	633183	52
9	417968	76263	999999	01	417970	76263	582030	51
10	463726	68988	999998	01	463727	68988	536273	50
11	7·505118	62981	9·999998	01	7·505120	62981	12·494880	49
12	542906	57936	999997	01	542909	57933	457091	48
13	577668	53641	999997	01	577672	53642	422328	47
14	609853	49938	999996	01	609857	49939	390143	46
15	639816	46714	999996	01	639820	46715	360180	45
16	667845	43881	999995	01	667849	43882	332151	44
17	694173	41372	999995	01	694179	41373	305821	43
18	718997	39135	999994	01	719003	39136	280997	42
19	742478	37127	999993	01	742484	37128	257516	41
20	764754	35315	999993	01	764761	35136	235239	40
21	7·785943	33672	9·999992	01	7·785951	33673	12·214049	39
22	806146	32175	999991	01	806155	32176	193845	38
23	825451	30805	999990	01	825460	30806	174540	37
24	843934	29547	999989	02	843944	29549	156056	36
25	861662	28388	999989	02	861674	28390	138326	35
26	878695	27317	999988	02	878708	27318	121292	34
27	895085	26323	999987	02	895099	26325	104901	33
28	910879	25399	999986	02	910894	25401	089106	32
29	926119	24538	999985	02	926134	24540	073866	31
30	940842	23733	999983	02	940858	23735	059142	30
31	7·955082	22980	9·999982	02	7·955100	22981	12·044900	29
32	968870	22273	999981	02	968889	22275	031111	28
33	982233	21608	999980	02	982253	21610	017747	27
34	995198	20981	999979	02	995219	20983	004781	26
35	8·007787	20390	999977	02	8·007809	20392	11·992191	25
36	020021	19831	999976	02	020044	19833	979956	24
37	031919	19302	999975	02	031945	19305	968055	23
38	043501	18801	999973	02	043527	18803	956473	22
39	054781	18325	999972	02	054809	18327	945191	21
40	065776	17872	999971	02	065806	17874	934194	20
41	8·076500	17441	9·999969	02	8·076531	17444	11·923469	19
42	086965	17031	999968	02	086997	17034	913003	18
43	097183	16639	999966	02	097217	16642	902783	17
44	107167	16265	999964	03	107203	16268	892797	16
45	116926	15908	999963	03	116963	15910	883037	15
46	126471	15566	999961	03	126510	15568	873490	14
47	135810	15238	999959	03	135851	15241	864149	13
48	144953	14924	999958	03	144996	14927	855004	12
49	153907	14622	999956	03	153952	14627	846048	11
50	162681	14333	999954	03	162727	14336	837273	10
51	8·171280	14054	9·999952	03	8·171328	14057	11·828672	9
52	179713	13786	999950	03	179763	13790	820237	8
53	187985	13529	999948	03	188036	13532	811964	7
54	196102	13280	999946	03	196156	13284	803844	6
55	204070	13041	999944	03	204126	13044	795874	5
56	211895	12810	999942	04	211953	12814	788047	4
57	219581	12587	999940	04	219641	12590	780359	3
58	227134	12372	999938	04	227195	12376	772805	2
59	234557	12164	999936	04	234621	12168	765379	1
60	241855	11963	999934	04	241921	11967	758079	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
90°								89°

1° 178°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	8·241855	11963	9·999934	04	8·241921	11967	11·758079	60
1	249033	11768	999932	04	249102	11772	750898	59
2	256094	11580	999929	04	256165	11584	743835	58
3	263042	11398	999927	04	263115	11402	736885	57
4	269881	11221	999925	04	269956	11225	730044	56
5	276614	11050	999922	04	276691	11054	723309	55
6	283243	10883	999920	04	283323	10887	716677	54
7	289773	10721	999918	04	289856	10726	710144	53
8	296207	10565	999915	04	296292	10570	703708	52
9	302546	10413	999913	04	302634	10418	697366	51
10	308794	10266	999910	04	308884	10270	691116	50
11	8·314954	10122	9·999907	04	8·315046	10126	11·684954	49
12	321027	9982	999905	04	321122	9987	678878	48
13	327016	9847	999902	04	327114	9851	672886	47
14	332924	9714	999899	05	333025	9719	666975	46
15	338753	9586	999897	05	338856	9590	661144	45
16	344504	9460	999894	05	344610	9465	655390	44
17	350181	9338	999891	05	350289	9343	649711	43
18	355783	9219	999888	05	355895	9224	644105	42
19	361315	9103	999885	05	361430	9108	638570	41
20	366777	8990	999882	05	366895	8995	633105	40
21	8·372171	8880	9·999879	05	8·372292	8885	11·627708	39
22	377499	8772	999876	05	377622	8777	622378	38
23	382762	8667	999873	05	382889	8672	617111	37
24	387962	8564	999870	05	388092	8570	611908	36
25	393101	8464	999867	05	393234	8470	606766	35
26	398179	8366	999864	05	398315	8371	601685	34
27	403199	8271	999861	05	403338	8276	596662	33
28	408161	8177	999858	05	408304	8182	591696	32
29	413068	8086	999854	05	413213	8091	586787	31
30	417919	7996	999851	06	418068	8002	581932	30
31	8·422717	7909	9·999848	06	8·422869	7914	11·577131	29
32	427462	7823	999844	06	427618	7830	572382	28
33	432156	7740	999841	06	432315	7745	567685	27
34	436800	7657	999838	06	436962	7663	563038	26
35	441394	7577	999834	06	441560	7583	558440	25
36	445941	7499	999831	06	446110	7505	553890	24
37	450440	7422	999827	06	450613	7428	549387	23
38	454893	7346	999824	06	455070	7352	544930	22
39	459301	7273	999820	06	459481	7279	540519	21
40	463665	7200	999816	06	463849	7206	536151	20
41	8·467985	7129	9·999813	06	8·468172	7135	11·531828	19
42	472263	7060	999809	06	472454	7066	527546	18
43	476498	6991	999805	06	476693	6998	523307	17
44	480693	6924	999801	06	480892	6931	519108	16
45	484848	6859	999797	07	485050	6865	514950	15
46	488963	6794	999794	07	489170	6801	510830	14
47	493040	6731	999790	07	493250	6738	506750	13
48	497078	6669	999786	07	497293	6676	502707	12
49	501080	6608	999782	07	501298	6615	498702	11
50	505045	6548	999778	07	505267	6555	494733	10
51	8·508974	6489	9·999774	07	8·509200	6496	11·490800	9
52	512867	6431	999769	07	513098	6439	486902	8
53	516726	6375	999765	07	516961	6382	483039	7
54	520551	6319	999761	07	520790	6326	479210	6
55	524343	6264	999757	07	524586	6272	475414	5
56	528102	6211	999753	07	528349	6218	471651	4
57	531828	6158	999748	07	532080	6165	467920	3
58	535523	6106	999744	07	535779	6113	464221	2
59	539186	6055	999740	07	539447	6062	460553	1
60	542819	6004	999735	07	543084	6012	456916	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

91° 88°

2°								177°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	8·542819	6004	9·999735	07	8·543084	6012	11·456916	60
1	546422	5955	999731	07	546691	5962	453309	59
2	549995	5906	999726	07	550268	5914	449732	58
3	553539	5858	999722	08	553817	5866	446183	57
4	557054	5811	999717	08	557336	5819	442664	56
5	560540	5765	999713	08	560828	5773	439172	55
6	563999	5719	999708	08	564291	5727	435709	54
7	567431	5674	999704	08	567727	5682	432273	53
8	570836	5630	999699	08	571137	5638	428863	52
9	574214	5587	999694	08	574520	5595	425480	51
10	577566	5544	999689	08	577877	5552	422123	50
11	8·580892	5502	9·999685	08	8·581208	5510	11·418792	49
12	584193	5460	999680	08	584514	5468	415486	48
13	587469	5419	999675	08	587795	5427	412205	47
14	590721	5379	999670	08	591051	5387	408949	46
15	593948	5339	999665	08	594283	5347	405717	45
16	597152	5300	999660	08	597492	5308	402508	44
17	600332	5261	999655	08	600677	5270	399323	43
18	603489	5223	999650	08	603839	5232	396161	42
19	606623	5186	999645	09	606978	5194	393022	41
20	609734	5149	999640	09	610094	5158	389906	40
21	8·612823	5112	9·999635	09	8·613189	5121	11·386811	39
22	615891	5076	999629	09	616262	5085	383738	38
23	618937	5041	999624	09	619313	5050	380687	37
24	621962	5006	999619	09	622343	5015	377657	36
25	624965	4972	999614	09	625352	4981	374648	35
26	627948	4938	999608	09	628340	4947	371660	34
27	630911	4904	999603	09	631308	4913	368692	33
28	633854	4871	999597	09	634256	4880	365744	32
29	636776	4839	999592	09	637184	4848	362816	31
30	639680	4806	999586	09	640093	4816	359907	30
31	8·642563	4775	9·999581	09	8·642982	4784	11·357018	29
32	645428	4743	999575	09	645853	4753	354147	28
33	648274	4712	999570	09	648704	4722	351296	27
34	651102	4682	999564	09	651537	4691	348463	26
35	653911	4652	999558	10	654352	4661	345648	25
36	656702	4622	999553	10	657149	4631	342851	24
37	659475	4592	999547	10	659928	4602	340072	23
38	662230	4563	999541	10	662689	4573	337311	22
39	664968	4535	999535	10	665433	4544	334567	21
40	667689	4506	999529	10	668160	4526	331840	20
41	8·670393	4479	9·999524	10	8·670870	4488	11·329130	19
42	673080	4451	999518	10	673563	4461	326437	18
43	675751	4424	999512	10	676239	4434	323761	17
44	678405	4397	999506	10	678900	4417	321100	16
45	681043	4370	999500	10	681544	4380	318456	15
46	683665	4344	999493	10	684172	4354	315828	14
47	686272	4318	999487	10	686784	4328	313216	13
48	688863	4292	999481	10	689381	4303	310619	12
49	691438	4267	999475	10	691963	4277	308037	11
50	693998	4242	999469	10	694529	4252	305471	10
51	8·696543	4217	9·999463	11	8·697081	4228	11·302919	9
52	699073	4192	999456	11	699617	4203	300383	8
53	701589	4168	999450	11	702139	4170	297861	7
54	704090	4144	999443	11	704646	4155	295354	6
55	706577	4121	999437	11	707140	4132	292860	5
56	709049	4097	999431	11	709618	4108	290382	4
57	711507	4074	999424	11	712083	4085	287917	3
58	713952	4051	999418	11	714534	4062	285466	2
59	716383	4029	999411	11	716972	4040	283028	1
60	718800	4006	999404	11	719396	4017	280604	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
92°								87°

3°								176°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	8·718800	4006	9·999404	11	8·719396	4017	11·280604	60
1	721204	3984	999398	11	721806	3995	278194	59
2	723595	3962	999391	11	724204	3974	275796	58
3	725972	3941	999384	11	726588	3952	273412	57
4	728337	3919	999378	11	728959	3930	271041	56
5	730688	3898	999371	11	731317	3909	268683	55
6	733027	3877	999364	12	733663	3889	266337	54
7	735354	3857	999357	12	735996	3868	264004	53
8	737667	3836	999350	12	738317	3848	261683	52
9	739969	3816	999343	12	740626	3827	259374	51
10	742259	3796	999336	12	742922	3807	257078	50
11	8·744536	3776	9·999329	12	8·745207	3787	11·254793	49
12	746802	3756	999322	12	747479	3768	252521	48
13	749055	3737	999315	12	749740	3749	250260	47
14	751297	3717	999308	12	751989	3729	248011	46
15	753528	3698	999301	12	754227	3710	245773	45
16	755747	3679	999294	12	756453	3692	243547	44
17	757955	3661	999287	12	758668	3673	241332	43
18	760151	3642	999279	12	760872	3655	239128	42
19	762337	3624	999272	12	763065	3636	236935	41
20	764511	3606	999265	12	765246	3618	234754	40
21	8·766675	3588	9·999257	12	8·767417	3600	11·232583	39
22	768828	3570	999250	13	769578	3583	230422	38
23	770970	3553	999242	13	771727	3565	228273	37
24	773101	3535	999235	13	773866	3548	226134	36
25	775223	3518	999227	13	775995	3531	224005	35
26	777333	3501	999220	13	778114	3514	221886	34
27	779434	3484	999212	13	780222	3497	219778	33
28	781524	3467	999205	13	782320	3480	217680	32
29	783605	3451	999197	13	784408	3464	215592	31
30	785675	3431	999189	13	786486	3447	213514	30
31	8·787736	3418	9·999181	13	8·788554	3431	11·211446	29
32	789787	3402	999174	13	790613	3414	209387	28
33	791828	3386	999166	13	792662	3399	207338	27
34	793859	3370	999158	13	794701	3383	205299	26
35	795881	3354	999150	13	796731	3368	203269	25
36	797894	3339	999142	13	798752	3352	201248	24
37	799897	3323	999134	13	800763	3337	199237	23
38	801892	3308	999126	13	802765	3322	197235	22
39	803876	3293	999118	13	804758	3307	195242	21
40	805852	3278	999110	13	806742	3292	193258	20
41	8·807819	3263	9·999102	13	8·808717	3278	11·191283	19
42	809777	3249	999094	14	810683	3262	189317	18
43	811726	3234	999086	14	812641	3248	187359	17
44	813667	3219	999077	14	814589	3233	185411	16
45	815599	3205	999069	14	816529	3219	183471	15
46	817522	3191	999061	14	818461	3205	181539	14
47	819436	3177	999053	14	820384	3191	179616	13
48	821343	3163	999044	14	822298	3177	177702	12
49	823240	3149	999036	14	824205	3163	175795	11
50	825130	3135	999027	14	826103	3150	173897	10
51	8·827011	3122	9·999019	14	8·827992	3136	11·172008	9
52	828884	3108	999010	14	829874	3123	170126	8
53	830749	3095	999002	14	831748	3110	168252	7
54	832607	3082	998993	14	833613	3096	166387	6
55	834456	3069	998984	14	835471	3083	164529	5
56	836297	3056	998976	14	837321	3070	162679	4
57	838130	3043	998967	15	839163	3057	160837	3
58	839956	3030	998958	15	840998	3045	159002	2
59	841774	3017	998950	15	842825	3032	157175	1
60	843585	3000	998941	15	844644	3019	155356	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
93°								86°

4° 175°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	8·843585	3005	9·998941	15	8·844644	3019	11·155356	60
1	845387	2992	998932	15	846455	3007	153545	59
2	847183	2980	998923	15	848260	2995	151740	58
3	848971	2967	998914	15	850057	2982	149943	57
4	850751	2955	998905	15	851846	2970	148154	56
5	852525	2943	998896	15	853628	2958	146372	55
6	854291	2931	998887	15	855403	2946	144597	54
7	856049	2919	998878	15	857171	2935	142829	53
8	857801	2907	998869	15	858932	2923	141068	52
9	859546	2896	998860	15	860686	2911	139314	51
10	861283	2884	998851	15	862433	2900	137567	50
11	8·863014	2873	9·998841	15	8·864173	2888	11·135827	49
12	864738	2861	998832	15	865906	2877	134094	48
13	866455	2850	998823	16	867632	2866	132368	47
14	868165	2839	998813	16	869351	2854	130649	46
15	869868	2828	998804	16	871064	2843	128936	45
16	871565	2817	998795	16	872770	2832	127230	44
17	873255	2806	998785	16	874469	2821	125531	43
18	874938	2795	998776	16	876162	2811	123838	42
19	876615	2786	998766	16	877849	2800	122151	41
20	878285	2773	998757	16	879529	2789	120471	40
21	8·879949	2763	9·998747	16	8·881202	2779	11·118798	39
22	881607	2752	998738	16	882869	2768	117131	38
23	883258	2742	998728	16	884530	2758	115470	37
24	884903	2731	998718	16	886185	2747	113815	36
25	886542	2721	998708	16	887833	2737	112167	35
26	888174	2711	998699	16	889476	2727	110524	34
27	889801	2700	998689	16	891112	2717	108888	33
28	891421	2690	998679	16	892742	2707	107258	32
29	893035	2680	998669	17	894366	2697	105634	31
30	894643	2670	998659	17	895984	2687	104016	30
31	8·896246	2660	9·998649	17	8·897596	2677	11 102404	29
32	897842	2651	998639	17	899203	2667	100797	28
33	899432	2641	998629	17	900803	2658	099197	27
34	901017	2631	998619	17	902398	2648	097602	26
35	902596	2622	998609	17	903987	2638	096013	25
36	904169	2612	998599	17	905570	2629	094430	24
37	905736	2603	998589	17	907147	2620	092853	23
38	907297	2593	998578	17	908719	2610	091281	22
39	908853	2584	998568	17	910285	2601	089715	21
40	910404	2575	998558	17	911846	2592	088154	20
41	8·911949	2566	9·998548	17	8·913401	2583	11·086599	19
42	913488	2556	998537	17	914951	2574	085049	18
43	915022	2547	998527	17	916495	2565	083505	17
44	916550	2538	998516	18	918034	2556	081966	16
45	918073	2529	998506	18	919568	2547	080432	15
46	919591	2520	998495	18	921096	2538	078904	14
47	921103	2512	998485	18	922619	2530	077381	13
48	922610	2503	998474	18	924136	2521	075864	12
49	924112	2494	998464	18	925649	2512	074351	11
50	925609	2486	998453	18	927156	2503	072844	10
51	8·927100	2477	9·998442	18	8·928658	2495	11·071342	9
52	928587	2469	998431	18	930155	2486	069845	8
53	930068	2460	998421	18	931647	2478	068353	7
54	931544	2452	998410	18	933134	2470	066866	6
55	933015	2443	998399	18	934616	2461	065384	5
56	934481	2435	998388	18	936093	2453	063907	4
57	935942	2427	998377	18	937565	2445	062435	3
58	937398	2419	998366	18	939032	2437	060968	2
59	938850	2411	998355	18	940494	2430	059506	1
60	940296	2403	998344	18	941952	2421	058048	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

94° 85°

5°								174°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	8·940296	2403	9·998344	19	8·941952	2421	11·058048	60
1	941738	2394	998333	19	943404	2413	056596	59
2	943174	2387	998322	19	944852	2405	055148	58
3	944606	2379	998311	19	946295	2397	053705	57
4	946034	2371	998300	19	947734	2390	052266	56
5	947456	2363	998289	19	949168	2382	050832	55
6	948874	2355	998277	19	950597	2374	049403	54
7	950287	2348	998266	19	952021	2366	047979	53
8	951696	2340	998255	19	953441	2360	046559	52
9	953100	2332	998243	19	954856	2351	045144	51
10	954499	2325	998232	19	956267	2344	043733	50
11	8·955894	2317	9·998220	19	8·957674	2337	11·042326	49
12	957284	2310	998209	19	959075	2329	040925	48
13	958670	2302	998197	19	960473	2323	039527	47
14	960052	2295	998186	19	961866	2314	038134	46
15	961429	2288	998174	19	963255	2307	036745	45
16	962801	2280	998163	19	964639	2300	035361	44
17	964170	2273	998151	19	966019	2293	033981	43
18	965534	2266	998139	20	967394	2286	032606	42
19	966893	2259	998128	20	968766	2279	031234	41
20	968249	2252	998116	20	970133	2271	029867	40
21	8·969600	2244	9·998104	20	8·971496	2265	11·028504	39
22	970947	2238	998092	20	972855	2257	027145	38
23	972289	2231	998080	20	974209	2251	025791	37
24	973628	2224	998068	20	975560	2244	024440	36
25	974962	2217	998056	20	976906	2237	023094	35
26	976293	2210	998044	20	978248	2230	021752	34
27	977619	2203	998032	20	979586	2223	020414	33
28	978941	2197	998020	20	980921	2217	019079	32
29	980259	2190	998008	20	982251	2210	017749	31
30	981573	2183	997996	20	983577	2204	016423	30
31	8·982883	2177	9·997984	20	8·984899	2197	11·015101	29
32	984189	2170	997972	20	986217	2191	013783	28
33	985491	2163	997959	20	987532	2184	012468	27
34	986789	2157	997947	20	988842	2178	011158	26
35	988083	2150	997935	21	990149	2171	009851	25
36	989374	2144	997922	21	991451	2165	008549	24
37	990660	2138	997910	21	992750	2158	007250	23
38	991943	2131	997897	21	994045	2152	005955	22
39	993222	2125	997885	21	995337	2146	004663	21
40	994497	2119	997872	21	996624	2140	003376	20
41	8·995768	2112	9·997860	21	8·997908	2134	11·002092	19
42	997036	2106	997847	21	999188	2127	000812	18
43	998299	2100	997835	21	9·000465	2121	10·999535	17
44	999560	2094	997822	21	001738	2115	998262	16
45	9·000816	2087	997809	21	003007	2109	996993	15
46	002069	2082	997797	21	004272	2103	995728	14
47	003318	2076	997784	21	005534	2097	994466	13
48	004563	2070	997771	21	006792	2091	993208	12
49	005805	2064	997758	21	008047	2085	991953	11
50	007044	2058	997745	21	009298	2080	990702	10
51	9·008278	2052	9·997732	21	9·010546	2074	10·989454	9
52	009510	2046	997719	21	011790	2068	988210	8
53	010737	2040	997706	21	013031	2062	986969	7
54	011962	2034	997693	22	014268	2056	985732	6
55	013182	2029	997680	22	015502	2051	984498	5
56	014400	2023	997667	22	016732	2045	983268	4
57	015613	2017	997654	22	017959	2040	982041	3
58	016824	2012	997641	22	019183	2033	980817	2
59	018031	2006	997628	22	020403	2028	979597	1
60	019235	2000	997614	22	021620	2023	978380	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
95°								84°

6° 173°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·019235	2000	9·997614	22	9·021620	2023	10·978380	60
1	020435	1995	997601	22	022834	2017	977166	59
2	021632	1989	997588	22	024044	2011	975956	58
3	022825	1984	997574	22	025251	2006	974749	57
4	024016	1978	997561	22	026455	2000	973545	56
5	025203	1973	997547	22	027655	1995	972345	55
6	026386	1967	997534	23	028852	1990	971148	54
7	027567	1962	997520	23	030046	1985	969954	53
8	028744	1957	997507	23	031237	1979	968763	52
9	029918	1951	997493	23	032425	1974	967575	51
10	031089	1947	997480	23	033609	1969	966391	50
11	9·032257	1941	9·997466	23	9·034791	1964	10·965209	49
12	033421	1936	997452	23	035969	1958	964031	48
13	034582	1930	997439	23	037144	1953	962856	47
14	035741	1925	997425	23	038316	1948	961684	46
15	036896	1920	997411	23	039485	1943	960515	45
16	038048	1915	997397	23	040651	1938	959349	44
17	039197	1910	997383	23	041813	1933	958187	43
18	040342	1905	997369	23	042973	1928	957027	42
19	041485	1899	997355	23	044130	1923	955870	41
20	042625	1894	997341	23	045284	1918	954716	40
21	9·043762	1889	9·997327	24	9·046434	1913	10·953566	39
22	044895	1884	997313	24	047582	1908	952418	38
23	046026	1879	997299	24	048727	1903	951273	37
24	047154	1875	997285	24	049869	1898	950131	36
25	048279	1870	997271	24	051008	1893	948992	35
26	049400	1865	997257	24	052144	1889	947856	34
27	050519	1860	997242	24	053277	1884	946723	33
28	051635	1855	997228	24	054407	1879	945593	32
29	952749	1850	997214	24	055535	1874	944465	31
30	053859	1845	997199	24	056659	1870	943341	30
31	9·054966	1841	9·997185	24	9·057781	1865	10·942219	29
32	056071	1836	997170	24	058900	1869	941100	28
33	057172	1831	997156	24	060016	1855	939984	27
34	058271	1827	997141	24	061130	1851	938870	26
35	059367	1822	997127	24	062240	1846	937760	25
36	060460	1817	997112	24	063348	1842	936652	24
37	061551	1813	997098	24	064453	1837	935547	23
38	062639	1808	997083	25	065556	1833	934444	22
39	063724	1804	997068	25	066655	1828	933345	21
40	064806	1799	997053	25	067752	1824	932248	20
41	9·065885	1794	9·997039	25	9·068846	1819	10·931154	19
42	066962	1790	997024	25	069938	1815	930062	18
43	068036	1786	997009	25	071027	1810	928973	17
44	069107	1781	996994	25	072113	1806	927887	16
45	070176	1777	996979	25	073197	1802	926803	15
46	071242	1772	996964	25	074278	1797	925722	14
47	072306	1768	996949	25	075356	1793	924644	13
48	073366	1763	996934	25	076432	1789	923568	12
49	074424	1759	996919	25	077505	1784	922495	11
50	075480	1755	996904	25	078576	1780	921424	10
51	9·076533	1750	9·996889	25	9·079644	1776	10·920356	9
52	077583	1746	996874	25	080710	1772	919290	8
53	078631	1742	996858	25	081773	1767	918227	7
54	079676	1738	996843	25	082833	1763	917167	6
55	080719	1733	996828	25	083891	1759	916109	5
56	081759	1729	996812	26	084947	1755	915053	4
57	082797	1725	996797	26	086000	1751	914000	3
58	083832	1721	996782	26	087050	1747	912950	2
59	084864	1717	996766	26	088098	1743	911902	1
60	085894	1713	996751	26	089144	1738	910856	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

96° 83°

7°								172°
′	Sine.	D.	Cosine.	D	Tang.	D.	Cotang.	′
0	9·085894	1713	9·996751	26	9·089144	1738	10·910856	60
1	086922	1709	996735	26	090187	1734	909813	59
2	087947	1704	996720	26	091228	1730	908772	58
3	088970	1700	996704	26	092266	1727	907734	57
4	089990	1696	996688	26	093302	1722	906698	56
5	091008	1692	996673	26	094336	1719	905664	55
6	092024	1688	996657	26	095367	1715	904633	54
7	093037	1684	996641	26	096395	1711	903605	53
8	094047	1680	996625	26	097422	1707	902578	52
9	095056	1676	996610	26	098446	1703	901554	51
10	096062	1673	996594	26	099468	1699	900532	50
11	9·097065	1668	9·996578	27	9·100487	1695	10·899513	49
12	098066	1665	996562	27	101504	1691	898496	48
13	099065	1661	996546	27	102519	1687	897481	47
14	100062	1657	996530	27	103532	1684	896468	46
15	101056	1653	996514	27	104542	1680	895458	45
16	102048	1649	996498	27	105550	1676	894450	44
17	103037	1645	996482	27	106556	1672	893444	43
18	104025	1641	996465	27	107559	1669	892441	42
19	105010	1638	996449	27	108560	1665	891440	41
20	105992	1634	996433	27	109559	1661	890441	40
21	9·106973	1630	9·996417	27	9·110556	1658	10·889444	39
22	107951	1627	996400	27	111551	1654	888449	38
23	108927	1623	996384	27	112543	1650	887457	37
24	109901	1619	996368	27	113533	1646	886467	36
25	110873	1616	996351	27	114521	1643	885479	35
26	111842	1612	996335	27	115507	1639	884493	34
27	112809	1608	996318	27	116491	1636	883509	33
28	113774	1605	996302	28	117472	1632	882528	32
29	114737	1601	996285	28	118452	1629	881548	31
30	115698	1597	996269	28	119429	1625	880571	30
31	9·116656	1594	9·996252	28	9·120404	1622	10·879596	29
32	117613	1590	996235	28	121377	1618	878623	28
33	118567	1587	996219	28	122348	1615	877652	27
34	119519	1583	996202	28	123317	1611	876683	26
35	120469	1580	996185	28	124284	1607	875716	25
36	121417	1576	996168	28	125249	1604	874751	24
37	122362	1573	996151	28	126211	1601	873789	23
38	123306	1569	996134	28	127172	1597	872828	22
39	124248	1566	996117	28	128130	1594	871870	21
40	125187	1562	996100	28	129087	1591	870913	20
41	9·126125	1559	9·996083	29	9·130041	1587	10·869959	19
42	127060	1556	996066	29	130994	1584	869006	18
43	127993	1552	996049	29	131944	1581	868056	17
44	128925	1549	996032	29	132893	1577	867107	16
45	129854	1545	996015	29	133839	1574	866161	15
46	130781	1542	995998	29	134784	1571	865216	14
47	131706	1539	995980	29	135726	1567	864274	13
48	132630	1535	995963	29	136667	1564	863333	12
49	133551	1532	995946	29	137605	1561	862395	11
50	134470	1529	995928	29	138542	1558	861458	10
51	9·135387	1525	9·995911	29	9·139476	1555	10·860524	9
52	136303	1522	995894	29	140409	1551	859591	8
53	137216	1519	995876	29	141340	1548	858660	7
54	138128	1516	995859	29	142269	1545	857731	6
55	139037	1512	995841	29	143196	1542	856804	5
56	139944	1509	995823	29	144121	1539	855879	4
57	140850	1506	995806	29	145044	1535	854956	3
58	141754	1503	995788	29	145966	1532	854034	2
59	142655	1500	995771	29	146885	1529	853115	1
60	143555	1496	995753	29	147803	1526	852197	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
97°								82°

8° 171°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·143555	1496	9·995753	30	9·147803	1526	10·852197	60
1	144453	1493	995735	30	148718	1523	851282	59
2	145349	1490	995717	30	149632	1520	850368	58
3	146243	1487	995699	30	150544	1517	849456	57
4	147136	1484	995681	30	151454	1514	848546	56
5	148026	1481	995664	30	152363	1511	847637	55
6	148915	1478	995646	30	153269	1508	846731	54
7	149802	1475	995628	30	154174	1505	845826	53
8	150686	1472	995610	30	155077	1502	844923	52
9	151569	1469	995591	30	155978	1499	844022	51
10	152451	1466	995573	30	156877	1496	843123	50
11	9·153330	1463	9·995555	30	9·157775	1493	10·842225	49
12	154208	1460	995537	30	158671	1490	841329	48
13	155083	1457	995519	30	159565	1487	840435	47
14	155957	1454	995501	31	160457	1484	839543	46
15	156830	1451	995482	31	161347	1481	838653	45
16	157700	1448	995464	31	162236	1479	837764	44
17	158569	1445	995446	31	163123	1476	836877	43
18	159435	1442	995427	31	164008	1473	835992	42
19	160301	1439	995409	31	164892	1470	835108	41
20	161164	1436	995390	31	165774	1467	834226	40
21	9·162025	1433	9·995372	31	9·166654	1464	10·833346	39
22	162885	1430	995353	31	167532	1461	832468	38
23	163743	1427	995334	31	168409	1458	831591	37
24	164600	1424	995316	31	169284	1455	830716	36
25	165454	1422	995297	31	170157	1453	829843	35
26	166307	1419	995278	31	171029	1450	828971	34
27	167159	1416	995260	31	171899	1447	828101	33
28	168008	1413	995241	32	172767	1444	827233	32
29	168856	1410	995222	32	173634	1442	826366	31
30	169702	1407	995203	32	174499	1439	825501	30
31	9·170547	1405	9·995184	32	9·175362	1436	10·824638	29
32	171389	1402	995165	32	176224	1433	823776	28
33	172230	1399	995146	32	177084	1431	822916	27
34	173070	1396	995127	32	177942	1428	822058	26
35	173908	1394	995108	32	178799	1425	821201	25
36	174744	1391	995089	32	179655	1423	820345	24
37	175578	1388	995070	32	180508	1420	819492	23
38	176411	1386	995051	32	181360	1417	818640	22
39	177242	1383	995032	32	182211	1415	817789	21
40	178072	1380	995013	32	183059	1412	816941	20
41	9·178900	1377	9·994993	32	9·183907	1409	10·816093	19
42	179726	1374	994974	32	184752	1407	815248	18
43	180551	1372	994955	32	185597	1404	814403	17
44	181374	1369	994935	32	186439	1402	813561	16
45	182196	1366	994916	33	187280	1399	812720	15
46	183016	1364	994896	33	188120	1396	811880	14
47	183834	1361	994877	33	188958	1393	811042	13
48	184651	1359	994857	33	189794	1391	810206	12
49	185466	1356	994838	33	190629	1389	809371	11
50	186280	1353	994818	33	191462	1386	808538	10
51	9·187092	1351	9·994798	33	9·192294	1384	10·807706	9
52	187903	1348	994779	33	193124	1381	806876	8
53	188712	1346	994759	33	193953	1379	806047	7
54	189519	1343	994739	33	194780	1376	805220	6
55	190325	1341	994720	33	195606	1374	804394	5
56	191130	1338	994700	33	196430	1371	803570	4
57	191933	1336	994680	33	197253	1369	802747	3
58	192734	1333	994660	33	198074	1366	801926	2
59	193534	1330	994640	33	198894	1364	801106	1
60	194332	1328	994620	33	199713	1361	800287	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

98° 81°

9°								170°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·194332	1328	9·994620	33	9·199713	1361	10·800287	60
1	195129	1326	994600	33	200529	1359	799471	59
2	195925	1323	994580	33	201345	1356	798655	58
3	196719	1321	994560	34	202159	1354	797841	57
4	197511	1318	994540	34	202971	1352	797029	56
5	198302	1316	994519	34	203782	1349	796218	55
6	199091	1313	994499	34	204592	1347	795408	54
7	199879	1311	994479	34	205400	1345	794600	53
8	200666	1308	994459	34	206207	1342	793793	52
9	201451	1306	994438	34	207013	1340	792987	51
10	202234	1304	994418	34	207817	1338	792183	50
11	9·203017	1301	9·994398	34	9·208619	1335	10·791381	49
12	203797	1299	994377	34	209420	1333	790580	48
13	204577	1296	994357	34	210220	1331	789780	47
14	205354	1294	994336	34	211018	1328	788982	46
15	206131	1292	994316	34	211815	1326	788185	45
16	206906	1289	994295	34	212611	1324	787389	44
17	207679	1287	994274	35	213405	1321	786595	43
18	208452	1285	994254	35	214198	1319	785802	42
19	209222	1282	994233	35	214989	1317	785011	41
20	209992	1280	994212	35	215780	1315	784220	40
21	9·210760	1278	9·994191	35	9·216568	1312	10·783432	39
22	211526	1275	994171	35	217356	1310	782644	38
23	212291	1273	994150	35	218142	1308	781858	37
24	213055	1271	994129	35	218926	1305	781074	36
25	213818	1268	994108	35	219710	1303	780290	35
26	214579	1266	994087	35	220492	1301	779508	34
27	215338	1264	994066	35	221272	1299	778728	33
28	216097	1261	994045	35	222052	1297	777948	32
29	216854	1259	994024	35	222830	1294	777170	31
30	217609	1257	994003	35	223607	1292	776393	30
31	9·218363	1255	9·993982	35	9·224382	1290	10·775618	29
32	219116	1253	993960	35	225156	1288	774844	28
33	219868	1250	993939	35	225929	1286	774071	27
34	220618	1248	993918	35	226700	1284	773300	26
35	221367	1246	993897	36	227471	1281	772529	25
36	222115	1244	993875	36	228239	1279	771761	24
37	222861	1242	993854	36	229007	1277	770993	23
38	223606	1239	993832	36	229773	1275	770227	22
39	224349	1237	993811	36	230539	1273	769461	21
40	225092	1235	993789	36	231302	1271	768698	20
41	9·225833	1233	9·993768	36	9·232065	1269	10·767935	19
42	226573	1231	993746	36	232826	1267	767174	18
43	227311	1228	993725	36	233586	1265	766414	17
44	228048	1226	993703	36	234345	1262	765655	16
45	228784	1224	993681	36	235103	1260	764897	15
46	229518	1222	993660	36	235859	1258	764141	14
47	230252	1220	993638	36	236614	1256	763386	13
48	230984	1218	993616	36	237368	1254	762632	12
49	231715	1216	993594	37	238120	1252	761880	11
50	232444	1214	993572	37	238872	1250	761128	10
51	9·233172	1212	9·993550	37	9·239622	1248	10·760378	9
52	233899	1209	993528	37	240371	1246	759629	8
53	234625	1207	993506	37	241118	1244	758882	7
54	235349	1205	993484	37	241865	1242	758135	6
55	236073	1203	993462	37	242610	1240	757390	5
56	236795	1201	993440	37	243354	1238	756646	4
57	237515	1199	993418	37	244097	1236	755903	3
58	238235	1197	993396	37	244839	1234	755161	2
59	238953	1195	993374	37	245579	1232	754421	1
60	239670	1193	993351	37	246319	1230	753681	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
99°								80°

10° · 169°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·239670	1193	9·993351	37	9·246319	1230	10·753681	60
1	240386	1191	993329	37	247057	1228	752943	59
2	241101	1189	993307	37	247794	1226	752206	58
3	241814	1187	993284	37	248530	1224	751470	57
4	242526	1185	993262	37	249264	1222	750736	56
5	243237	1183	993240	37	249998	1220	750002	55
6	243947	1181	993217	38	250730	1218	749270	54
7	244656	1179	993195	38	251461	1217	748539	53
8	245363	1177	993172	38	252191	1215	747809	52
9	246069	1175	993149	38	252920	1213	747080	51
10	246775	1173	993127	38	253648	1211	746352	50
11	9·247478	1171	9·993104	38	9·254374	1209	10·745626	49
12	248181	1169	993081	38	255100	1207	744900	48
13	248883	1167	993059	38	255824	1205	744176	47
14	249583	1165	993036	38	256547	1203	743453	46
15	250282	1163	993013	38	257269	1201	742731	45
16	250980	1161	992990	38	257990	1200	742010	44
17	251677	1159	992967	38	258710	1198	741290	43
18	252373	1158	992944	38	259429	1196	740571	42
19	253067	1156	992921	38	260146	1194	739854	41
20	253761	1154	992898	38	260863	1192	739137	40
21	9·254453	1152	9·992875	38	9·261578	1190	10·738422	39
22	255144	1150	992852	38	262292	1189	737708	38
23	255834	1148	992829	39	263005	1187	736995	37
24	256523	1146	992806	39	263717	1185	736283	36
25	257211	1144	992783	39	264428	1183	735572	35
26	257898	1142	992759	39	265138	1181	734862	34
27	258583	1141	992736	39	265847	1179	734153	33
28	259268	1139	992713	39	266555	1178	733445	32
29	259951	1137	992690	39	267261	1176	732739	31
30	260633	1135	992666	39	267967	1174	732033	30
31	9·261314	1133	9·992643	39	9·268671	1172	10·731329	29
32	261994	1131	992619	39	269375	1170	730625	28
33	262673	1130	992596	39	270077	1169	729923	27
34	263351	1128	992572	39	270779	1167	729221	26
35	264027	1126	992549	39	271479	1165	728521	25
36	264703	1124	992525	39	272178	1164	727822	24
37	265377	1122	992501	39	272876	1162	727124	23
38	266051	1120	992478	40	273573	1160	726427	22
39	266723	1119	992454	40	274269	1158	725731	21
40	267395	1117	992430	40	274964	1157	725036	20
41	9·268065	1115	9·992406	40	9·275658	1155	10·724342	19
42	268734	1113	992382	40	276351	1153	723649	18
43	269402	1111	992359	40	277043	1151	722957	17
44	270069	1110	992335	40	277734	1150	722266	16
45	270735	1108	992311	40	278424	1148	721576	15
46	271400	1106	992287	40	279113	1147	720887	14
47	272064	1105	992263	40	279801	1145	720199	13
48	272726	1103	992239	40	280488	1143	719512	12
49	273388	1101	992214	40	281174	1141	718826	11
50	274049	1099	992190	40	281858	1140	718142	10
51	9·274708	1098	9·992166	40	9·282542	1138	10·717458	9
52	275367	1096	992142	40	283225	1136	716775	8
53	276025	1094	992118	41	283907	1135	716093	7
54	276681	1092	992093	41	284588	1133	715412	6
55	277337	1091	992069	41	285268	1131	714732	5
56	277991	1089	992044	41	285947	1130	714053	4
57	278645	1087	992020	41	286624	1128	713376	3
58	279297	1086	991996	41	287301	1126	712699	2
59	279948	1084	991971	41	287977	1125	712023	1
60	280599	1082	991947	41	288652	1123	711348	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

100° 79°

11°									168°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.		′
0	9·280599	1082	9·991947	41	9·288652	1123	10·711348		60
1	281248	1081	991922	41	289326	1122	710674		59
2	281897	1079	991897	41	289999	1120	710001		58
3	282544	1077	991873	41	290671	1118	709329		57
4	283190	1076	991848	41	291342	1117	708658		56
5	283836	1074	991823	41	292013	1115	707987		55
6	284480	1072	991799	41	292682	1114	707318		54
7	285124	1071	991774	42	293350	1112	706650		53
8	285766	1069	991749	42	294017	1111	705983		52
9	286408	1067	991724	42	294684	1109	705316		51
10	287048	1066	991699	42	295349	1107	704651		50
11	9·287688	1064	9·991674	42	9·296013	1106	10·703987		49
12	288326	1063	991649	42	296677	1104	703323		48
13	288964	1061	991624	42	297339	1103	702661		47
14	289600	1059	991599	42	298001	1101	701999		46
15	290236	1058	991574	42	298662	1100	701338		45
16	290870	1056	991549	42	299322	1098	700678		44
17	291504	1054	991524	42	299980	1096	700020		43
18	292137	1053	991498	42	300638	1095	699362		42
19	292768	1051	991473	42	301295	1093	698705		41
20	293399	1050	991448	42	301951	1092	698049		40
21	9·294029	1048	9·991422	42	9·302607	1090	10·697393		39
22	294658	1046	991397	42	303261	1089	696739		38
23	295286	1045	991372	43	303914	1087	696086		37
24	295913	1043	991346	43	304567	1086	695433		36
25	296539	1042	991321	43	305218	1084	694782		35
26	297164	1040	991295	43	305869	1083	694131		34
27	297788	1039	991270	43	306519	1081	693481		33
28	298412	1037	991244	43	307168	1080	692832		32
29	299034	1036	991218	43	307816	1078	692184		31
30	299655	1034	991193	43	308463	1077	691537		30
31	9·300276	1032	9·991167	43	9·309109	1075	10·690891		29
32	300895	1031	991141	43	309754	1074	690246		28
33	301514	1029	991115	43	310399	1073	689601		27
34	302132	1028	991090	43	311042	1071	688958		26
35	302748	1026	991064	43	311685	1070	688315		25
36	303364	1025	991038	43	312327	1068	687673		24
37	303979	1023	991012	43	312968	1067	687032		23
38	304593	1022	990986	43	313608	1065	686392		22
39	305207	1020	990960	43	314247	1064	685753		21
40	305819	1019	990934	44	314885	1062	685115		20
41	9·306430	1017	9·990908	44	9·315523	1061	10·684477		19
42	307041	1016	990882	44	316159	1060	683841		18
43	307650	1014	990855	44	316795	1058	683205		17
44	308259	1013	990829	44	317430	1057	682570		16
45	308867	1011	990803	44	318064	1055	681936		15
46	309474	1010	990777	44	318697	1054	681303		14
47	310080	1008	990750	44	319330	1053	680670		13
48	310685	1007	990724	44	319961	1051	680039		12
49	311289	1005	990697	44	320592	1050	679408		11
50	311893	1004	99067[illegible]	44	321222	1048	678778		10
51	9·312495	1003	9·990645	44	9·321851	1047	10·678149		9
52	313097	1001	990618	44	322479	1045	677521		8
53	313698	1000	990591	44	323106	1044	676894		7
54	314297	998	990565	44	323733	1043	676267		6
55	314897	997	990538	44	324358	1041	675642		5
56	315495	996	990511	45	324983	1040	675017		4
57	316092	994	990485	45	325607	1039	674393		3
58	316689	993	990458	45	326231	1037	673769		2
59	317284	991	990431	45	326853	1036	673147		1
60	317879	990	990404	45	327475	1035	672525		0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.		′
101°									78°

12°								167°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·317879	990	9·990404	45	9·327475	1035	10·672525	60
1	318473	988	990378	45	328095	1033	671905	59
2	319066	987	990351	45	328715	1032	671285	58
3	319658	986	990324	45	329334	1030	670666	57
4	320249	984	990297	45	329953	1029	670047	56
5	320840	983	990270	45	330570	1028	669430	55
6	321430	982	990243	45	331187	1026	668813	54
7	322019	980	990215	45	331803	1025	668197	53
8	322607	979	990188	45	332418	1024	667582	52
9	323194	977	990161	45	333033	1023	666967	51
10	323780	976	990134	45	333646	1021	666354	50
11	9·324366	975	9·990107	46	9·334259	1020	10·665741	49
12	324950	973	990079	46	334871	1019	665129	48
13	325534	972	990052	46	335482	1017	664518	47
14	326117	970	990025	46	336093	1016	663907	46
15	326700	969	989997	46	336702	1015	663298	45
16	327281	968	989970	46	337311	1013	662689	44
17	327862	966	989942	46	337919	1012	662081	43
18	328442	965	989915	46	338527	1011	661473	42
19	329021	964	989887	46	339133	1010	660867	41
20	329599	962	989860	46	339739	1008	660261	40
21	9·330176	961	9·989832	46	9·340344	1007	10·659656	39
22	330753	960	989804	46	340948	1006	659052	38
23	331329	958	989777	46	341552	1004	658448	37
24	331903	957	989749	47	342155	1003	657845	36
25	332478	956	989721	47	342757	1002	657243	35
26	333051	954	989693	47	343358	1000	656642	34
27	333624	953	989665	47	343958	999	656042	33
28	334195	952	989637	47	344558	998	655442	32
29	334767	950	989610	47	345157	997	654843	31
30	335337	949	989582	47	345755	996	654245	30
31	9·335906	948	9·989553	47	9·346353	994	10·653647	29
32	336475	946	989525	47	346949	993	653051	28
33	337043	945	989497	47	347545	992	652455	27
34	337610	944	989469	47	348141	991	651859	26
35	338176	943	989441	47	348735	990	651265	25
36	338742	941	989413	47	349329	988	650671	24
37	339307	940	989385	47	349922	987	650078	23
38	339871	939	989356	47	350514	986	649486	22
39	340434	937	989328	47	351106	985	648894	21
40	340996	936	989300	47	351697	983	648303	20
41	9·341558	935	9·989271	47	9·352287	982	10·647713	19
42	342119	934	989243	47	352876	981	647124	18
43	342679	932	989214	47	353465	980	646535	17
44	343239	931	989186	47	354053	979	645947	16
45	343797	930	989157	47	354640	977	645360	15
46	344355	929	989128	48	355227	976	644773	14
47	344912	927	989100	48	355813	975	644187	13
48	345469	926	989071	48	356398	974	643602	12
49	346024	925	989042	48	356982	973	643018	11
50	346579	924	989014	48	357566	971	642434	10
51	9·347134	922	9·988985	48	9·358149	970	10·641851	9
52	347687	921	988956	48	358731	969	641269	8
53	348240	920	988927	48	359313	968	640687	7
54	348792	919	988898	48	359893	967	640107	6
55	349343	917	988869	48	360474	966	639526	5
56	349893	916	988840	48	361053	965	638947	4
57	350443	915	988811	49	361632	963	638368	3
58	350992	914	988782	49	362210	962	637790	2
59	351540	913	988753	49	362787	961	637213	1
60	352088	911	988724	49	363364	960	636636	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
102°								77°

13°								166°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·352088	911	9·988724	49	9·363364	960	10·636636	60
1	352635	910	988695	49	363940	959	636060	59
2	353181	909	988666	49	364515	958	635485	58
3	353726	908	988636	49	365090	957	634910	57
4	354271	907	988607	49	365664	955	634336	56
5	354815	905	988578	49	366237	954	633763	55
6	355358	904	988548	49	366810	953	633190	54
7	355901	903	988519	49	367382	952	632618	53
8	356443	902	988489	49	367953	951	632047	52
9	356984	901	988460	49	368524	950	631476	51
10	357524	899	988430	49	369094	949	630906	50
11	9·358064	898	9·988401	49	9·369663	948	10·630337	49
12	358603	897	988371	49	370232	946	629768	48
13	359141	896	988342	49	370799	945	629201	47
14	359678	895	988312	50	371367	944	628633	46
15	360215	893	988282	50	371933	943	628067	45
16	360752	892	988252	50	372499	942	627501	44
17	361287	891	988223	50	373064	941	626936	43
18	361822	890	988193	50	373629	940	626371	42
19	362356	889	988163	50	374193	939	625807	41
20	362889	888	988133	50	374756	938	625244	40
21	9·363422	887	9·988103	50	9·375319	937	10·624681	39
22	363954	885	988073	50	375881	935	624119	38
23	364485	884	988043	50	376442	934	623558	37
24	365016	883	988013	50	377003	933	622997	36
25	365546	882	987983	50	377563	932	622437	35
26	366075	881	987953	50	378122	931	621878	34
27	366604	880	987922	50	378681	930	621319	33
28	367131	879	987892	50	379239	929	620761	32
29	367659	877	987862	50	379797	928	620203	31
30	368185	876	987832	51	380354	927	619646	30
31	9·368711	875	9·987801	51	9·380910	926	10·619090	29
32	369236	874	987771	51	381466	925	618534	28
33	369761	873	987740	51	382020	924	617980	27
34	370285	872	987710	51	382575	923	617425	26
35	370808	871	987679	51	383129	922	616871	25
36	371330	870	987649	51	383682	921	616318	24
37	371852	869	987618	51	384234	920	615766	23
38	372373	867	987588	51	384786	919	615214	22
39	372894	866	987557	51	385337	918	614663	21
40	373414	865	987526	51	385888	917	614112	20
41	9·373933	864	9·987496	51	9·386438	915	10·613562	19
42	374452	863	987465	51	386987	914	613013	18
43	374970	862	987434	51	387536	913	612464	17
44	375487	861	987403	52	388084	912	611916	16
45	376003	860	987372	52	388631	911	611369	15
46	376519	859	987341	52	389178	910	610822	14
47	377035	858	987310	52	389724	909	610276	13
48	377549	857	987279	52	390270	908	609730	12
49	378063	856	987248	52	390815	907	609185	11
50	378577	854	987217	52	391360	906	608640	10
51	9·379089	853	9·987186	52	9·391903	905	10·608097	9
52	379601	852	987155	52	392447	904	607553	8
53	380113	851	987124	52	392989	903	607011	7
54	380624	850	987092	52	393531	902	606469	6
55	381134	849	987061	52	394073	901	605927	5
56	381643	848	987030	52	394614	900	605386	4
57	382152	847	986998	52	395154	899	604846	3
58	382661	846	986967	52	395694	898	604306	2
59	383168	845	986936	52	396233	897	603767	1
60	383675	844	986904	52	396771	896	603229	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
103°								76°

14°								165°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·383675	844	9·986904	52	9·396771	896	10·603229	60
1	384182	843	986873	53	397309	896	602691	59
2	384687	842	986841	53	397846	895	602154	58
3	385192	841	986809	53	398383	894	601617	57
4	385697	840	986778	53	398919	893	601081	56
5	386201	839	986746	53	399455	892	600545	55
6	386704	838	986714	53	399990	891	600010	54
7	387207	837	986683	53	400524	890	599476	53
8	387709	836	986651	53	401058	889	598942	52
9	388210	835	986619	53	401591	888	598409	51
10	388711	834	986587	53	402124	887	597876	50
11	9·389211	833	9·986555	53	9·402656	886	10·597344	49
12	389711	832	986523	53	403187	885	596813	48
13	390210	831	986491	53	403718	884	596282	47
14	390708	830	986459	53	404249	883	595751	46
15	391206	828	986427	53	404778	882	595222	45
16	391703	827	986395	53	405308	881	594692	44
17	392199	826	986363	54	405836	880	594164	43
18	392695	825	986331	54	406364	879	593636	42
19	393191	824	986299	54	406892	878	593108	41
20	393685	823	986266	54	407419	877	592581	40
21	9·394179	822	9·986234	54	9·407945	876	10·592055	39
22	394673	821	986202	54	408471	875	591529	38
23	395166	820	986169	54	408996	874	591004	37
24	395658	819	986137	54	409521	874	590479	36
25	396150	818	986104	54	410045	873	589955	35
26	396641	817	986072	54	410569	872	589431	34
27	397132	817	986039	54	411092	871	588908	33
28	397621	816	986007	54	411615	870	588385	32
29	398111	815	985974	54	412137	869	587863	31
30	398600	814	985942	54	412658	868	587342	30
31	9·399088	813	9·985909	55	9·413179	867	10·586821	29
32	399575	812	985876	55	413699	866	586301	28
33	400062	811	985843	55	414219	865	585781	27
34	400549	810	985811	55	414738	864	585262	26
35	401035	809	985778	55	415257	864	584743	25
36	401520	808	985745	55	415775	863	584225	24
37	402005	807	985712	55	416293	862	583707	23
38	402489	806	985679	55	416810	861	583190	22
39	402972	805	985646	55	417326	860	582674	21
40	403455	804	985613	55	417842	859	582158	20
41	9·403938	803	9·985580	55	9·418358	858	10·581642	19
42	404420	802	985547	55	418873	857	581127	18
43	404901	801	985514	55	419387	856	580613	17
44	405382	800	985480	55	419901	855	580099	16
45	405862	799	985447	55	420415	855	579585	15
46	406341	798	985414	56	420927	854	579073	14
47	406820	797	985381	56	421440	853	578560	13
48	407299	796	985347	56	421952	852	578048	12
49	407777	795	985314	56	422463	851	577537	11
50	408254	794	985280	56	422974	850	577026	10
51	9·408731	794	9·985247	56	9·423484	849	10·576516	9
52	409207	793	985213	56	423993	848	576007	8
53	409682	792	985180	56	424503	848	575497	7
54	410157	791	985146	56	425011	847	574989	6
55	410632	790	985113	56	425519	846	574481	5
56	411106	789	985079	56	426027	845	573973	4
57	411579	788	985045	56	426534	844	573466	3
58	412052	787	985011	56	427041	843	572959	2
59	412524	786	984978	56	427547	843	572453	1
60	412996	785	984944	56	428052	842	571948	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
104°								75°

15°								164°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·412996	785	9·984944	57	9·428052	842	10·571948	60
1	413467	784	984910	57	428558	841	571442	59
2	413938	783	984876	57	429062	840	570938	58
3	414408	783	984842	57	429566	839	570434	57
4	414878	782	984808	57	430070	838	569930	56
5	415347	781	984774	57	430573	838	569427	55
6	415815	780	984740	57	431075	837	568925	54
7	416283	779	984706	57	431577	836	568423	53
8	416751	778	984672	57	432079	835	567921	52
9	417217	777	984638	57	432580	834	567420	51
10	417684	776	984603	57	433080	833	566920	50
11	9·418150	775	9·984569	57	9·433580	832	10·566420	49
12	418615	774	984535	57	434080	832	565920	48
13	419079	773	984500	57	434579	831	565421	47
14	419544	773	984466	57	435078	830	564922	46
15	420007	772	984432	58	435576	829	564424	45
16	420470	771	984397	58	436073	828	563927	44
17	420933	770	984363	58	436570	828	563430	43
18	421395	769	984328	58	437067	827	562933	42
19	421857	768	984294	58	437563	826	562437	41
20	422318	767	984259	58	438059	825	561941	40
21	9·422778	767	9·984224	58	9·438554	824	10·561446	39
22	423238	766	984190	58	439048	823	560952	38
23	423697	765	984155	58	439543	823	560457	37
24	424156	764	984120	58	440036	822	559964	36
25	424615	763	984085	58	440529	821	559471	35
26	425073	762	984050	58	441022	820	558978	34
27	425530	761	984015	58	441514	819	558486	33
28	425987	760	983981	58	442006	819	557994	32
29	426443	760	983946	58	442497	818	557503	31
30	426899	759	983911	58	442988	817	557012	30
31	9·427354	758	9·983875	58	9·443479	816	10·556521	29
32	427809	757	983840	59	443968	816	556032	28
33	428263	756	983805	59	444458	815	555542	27
34	428717	755	983770	59	444947	814	555053	26
35	429170	754	983735	59	445435	813	554565	25
36	429623	753	983700	59	445923	812	554077	24
37	430075	752	983664	59	446411	812	553589	23
38	430527	752	983629	59	446898	811	553102	22
39	430978	751	983594	59	447384	810	552616	21
40	431429	750	983558	59	447870	809	552130	20
41	9·431879	749	9·983523	59	9·448356	809	10·551644	19
42	432329	749	983487	59	448841	808	551159	18
43	432778	748	983452	59	449326	807	550674	17
44	433226	747	983416	59	449810	806	550190	16
45	433675	746	983381	59	450294	806	549706	15
46	434122	745	983345	59	450777	805	549223	14
47	434569	744	983309	59	451260	804	548740	13
48	435016	744	983273	60	451743	803	548257	12
49	435462	743	983238	60	452225	802	547775	11
50	435908	742	983202	60	452706	802	547294	10
51	9·436353	741	9·983166	60	9·453187	801	10·546813	9
52	436798	740	983130	60	453668	800	546332	8
53	437242	740	983094	60	454148	799	545852	7
54	437686	739	983058	60	454628	799	545372	6
55	438129	738	983022	60	455107	798	544893	5
56	438572	737	982986	60	455586	797	544414	4
57	439014	736	982950	60	456064	796	543936	3
58	439456	736	982914	60	456542	796	543458	2
59	439897	735	982878	60	457019	795	542981	1
60	440338	734	982842	60	457496	794	542504	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
105°								74°

16° 163°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·440338	734	9·982842	60	9·457496	794	10·542504	60
1	440778	733	982805	60	457973	793	542027	59
2	441218	732	982769	61	458449	793	541551	58
3	441658	731	982733	61	458925	792	541075	57
4	442096	731	982696	61	459400	791	540600	56
5	442535	730	982660	61	459875	790	540125	55
6	442973	729	982624	61	460349	790	539651	54
7	443410	728	982587	61	460823	789	539177	53
8	443847	727	982551	61	461297	788	538703	52
9	444284	727	982514	61	461770	788	538230	51
10	444720	726	982477	61	462242	787	537758	50
11	9·445155	725	9·982441	61	9·462715	786	10·537285	49
12	445590	724	982404	61	463186	785	536814	48
13	446025	723	982367	61	463658	785	536342	47
14	446459	723	982331	61	464128	784	535872	46
15	446893	722	982294	61	464599	783	535401	45
16	447326	721	982257	61	465069	783	534931	44
17	447759	720	982220	62	465539	782	534461	43
18	448191	720	982183	62	466008	781	533992	42
19	448623	719	982146	62	466477	780	533523	41
20	449054	718	982109	62	466945	780	533055	40
21	9·449485	717	9·982072	62	9·467413	779	10·532587	39
22	449915	716	982035	62	467880	778	532120	38
23	450345	716	981998	62	468347	778	531653	37
24	450775	715	981961	62	468814	777	531186	36
25	451204	714	981924	62	469280	776	530720	35
26	451632	713	981886	62	469746	775	530254	34
27	452060	713	981849	62	470211	775	529789	33
28	452488	712	981812	62	470676	774	529324	32
29	452915	711	981774	62	471141	773	528859	31
30	453342	710	981737	62	471605	773	528395	30
31	9·453768	710	9·981700	63	9·472069	772	10·527931	29
32	454194	709	981662	63	472532	771	527468	28
33	454619	708	981625	63	472995	771	527005	27
34	455044	707	981587	63	473457	770	526543	26
35	455469	707	981549	63	473919	769	526081	25
36	455893	706	981512	63	474381	769	525619	24
37	456316	705	981474	63	474842	768	525158	23
38	456739	704	981436	63	475303	767	524697	22
39	457162	704	981399	63	475763	767	524237	21
40	457584	703	981361	63	476223	766	523777	20
41	9·458006	702	9·981323	63	9·476683	765	10·523317	19
42	458427	701	981285	63	477142	765	522858	18
43	458848	701	981247	63	477601	764	522399	17
44	459268	700	981209	63	478059	763	521941	16
45	459688	699	981171	63	478517	763	521483	15
46	460108	698	981133	64	478975	762	521025	14
47	460527	698	981095	64	479432	761	520568	13
48	460946	697	981057	64	479889	761	520111	12
49	461364	696	981019	64	480345	760	519655	11
50	461782	695	980981	64	480801	759	519199	10
51	9·462199	695	9·980942	64	9·481257	759	10·518743	9
52	462616	694	980904	64	481712	758	518288	8
53	463032	693	980866	64	482167	757	517833	7
54	463448	693	980827	64	482621	757	517379	6
55	463864	692	980789	64	483075	756	516925	5
56	464279	691	980750	64	483529	755	516471	4
57	464694	690	980712	64	483982	755	516018	3
58	465108	690	980673	64	484435	754	515565	2
59	465522	689	980635	64	484887	753	515113	1
60	465935	688	980596	64	485339	753	514661	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

106° 73°

17°								162°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·465935	688	9·980596	64	9·485339	755	10·514661	60
1	466348	688	980558	64	485791	752	514209	59
2	466761	687	980519	65	486242	751	513758	58
3	467173	686	980480	65	486693	751	513307	57
4	467585	685	980442	65	487143	750	512857	56
5	467996	685	980403	65	487593	749	512407	55
6	468407	684	980364	65	488043	749	511957	54
7	468817	683	980325	65	488492	748	511508	53
8	469227	683	980286	65	488941	747	511059	52
9	469637	682	980247	65	489390	747	510610	51
10	470046	681	980208	65	489838	746	510162	50
11	9·470455	680	9·980169	65	9·490286	746	10·509714	49
12	470863	680	980130	65	490733	745	509267	48
13	471271	679	980091	65	491180	744	508820	47
14	471679	678	980052	65	491627	744	508373	46
15	472086	678	980012	65	492073	743	507927	45
16	472492	677	979973	65	492519	743	507481	44
17	472898	676	979934	66	492965	742	507035	43
18	473304	676	979895	66	493410	741	506590	42
19	473710	675	979855	66	493854	740	506146	41
20	474115	674	979816	66	494299	740	505701	40
21	9·474519	674	9·979776	66	9·494743	740	10·505257	39
22	474923	673	979737	66	495186	739	504814	38
23	475327	672	979697	66	495630	738	504370	37
24	475730	672	979658	66	496073	737	503927	36
25	476133	671	979618	66	496515	737	503485	35
26	476536	670	979579	66	496957	736	503043	34
27	476938	669	979539	66	497399	736	502601	33
28	477340	669	979499	66	497841	735	502159	32
29	477741	668	979459	66	498282	734	501718	31
30	478142	667	979420	66	498722	734	501278	30
31	9·478542	667	9·979380	66	9·499163	733	10·500837	29
32	478942	666	979340	66	499603	733	500397	28
33	479342	665	979300	67	500042	732	499958	27
34	479741	665	979260	67	500481	731	499519	26
35	480140	664	979220	67	500920	731	499080	25
36	480539	663	979180	67	501359	730	498641	24
37	480937	663	979140	67	501797	730	498203	23
38	481334	662	979100	67	502235	729	497765	22
39	481731	661	979059	67	502672	728	497328	21
40	482128	661	979019	67	503109	728	496891	20
41	9·482525	660	9·978979	67	9·503546	727	10·496454	19
42	482921	659	978939	67	503982	727	496018	18
43	483316	659	978898	67	504418	726	495582	17
44	483712	658	978858	67	504854	725	495146	16
45	484107	657	978817	67	505289	725	494711	15
46	484501	657	978777	67	505724	724	494276	14
47	484895	656	978737	67	506159	724	493841	13
48	485289	655	978696	68	506593	723	493407	12
49	485682	655	978655	68	507027	722	492973	11
50	486075	654	978615	68	507460	722	492540	10
51	9·486467	653	9·978574	68	9·507893	721	10·492107	9
52	486860	653	978533	68	508326	721	491674	8
53	487251	652	978493	68	508759	720	491241	7
54	487643	651	978452	68	509191	719	490809	6
55	488034	651	978411	68	509622	719	490378	5
56	488424	650	978370	68	510054	718	489946	4
57	488814	650	978329	68	510485	718	489515	3
58	489204	649	978288	68	510916	717	489084	2
59	489593	648	978247	68	511346	716	488654	1
60	489982	648	978206	68	511776	716	488224	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
107°								72°

18° 161°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·489982	648	9·978206	68	9·511776	716	10·488224	60
1	490371	648	978165	68	512206	716	487794	59
2	490759	647	978124	68	512635	715	487365	58
3	491147	646	978083	69	513064	714	486936	57
4	491535	646	978042	69	513493	714	486507	56
5	491922	645	978001	69	513921	713	486079	55
6	492308	644	977959	69	514349	713	485651	54
7	492695	644	977918	69	514777	712	485223	53
8	493081	643	977877	69	515204	712	484796	52
9	493466	642	977835	69	515631	711	484369	51
10	493851	642	977794	69	516057	710	483943	50
11	9·494236	641	9·977752	69	9·516484	710	10·483516	49
12	494621	641	977711	69	516910	709	483090	48
13	495005	640	977669	69	517335	709	482665	47
14	495388	639	977628	69	517761	708	482239	46
15	495772	639	977586	69	518186	708	481814	45
16	496154	638	977544	70	518610	707	481390	44
17	496537	637	977503	70	519034	706	480966	43
18	496919	637	977461	70	519458	706	480542	42
19	497301	636	977419	70	519882	705	480118	41
20	497682	636	977377	70	520305	705	479695	40
21	9·498064	635	9·977335	70	9·520728	704	10·479272	39
22	498444	634	977293	70	521151	703	478849	38
23	498825	634	977251	70	521573	703	478427	37
24	499204	633	977209	70	521995	703	478005	36
25	499584	632	977167	70	522417	702	477583	35
26	499963	632	977125	70	522838	702	477162	34
27	500342	631	977083	70	523259	701	476741	33
28	500721	631	977041	70	523680	701	476320	32
29	501099	630	976999	70	524100	700	475900	31
30	501476	629	976957	70	524520	699	475480	30
31	9·501854	629	9·976914	70	9·524940	699	10·475060	29
32	502231	628	976872	71	525359	698	474641	28
33	502607	628	976830	71	525778	698	474222	27
34	502984	627	976787	71	526197	697	473803	26
35	503360	626	976745	71	526615	697	473385	25
36	503735	626	976702	71	527033	696	472967	24
37	504110	625	976660	71	527451	696	472549	23
38	504485	625	976617	71	527868	695	472132	22
39	504860	624	976574	71	528285	695	471715	21
40	505234	623	976532	71	528702	694	471298	20
41	9·505608	623	9·976489	71	9·529119	693	10·470881	19
42	505981	622	976446	71	529535	693	470465	18
43	506354	622	976404	71	529951	693	470049	17
44	506727	621	976361	71	530366	692	469634	16
45	507099	620	976318	71	530781	691	469219	15
46	507471	620	976275	71	531196	691	468804	14
47	507843	619	976232	72	531611	690	468389	13
48	508214	619	976189	72	532025	690	467975	12
49	508585	618	976146	72	532439	689	467561	11
50	508956	618	976103	72	532853	689	467147	10
51	9·509326	617	9·976060	72	9·533266	688	10·466734	9
52	509696	616	976017	72	533679	688	466321	8
53	510065	616	975974	72	534092	687	465908	7
54	510434	615	975930	72	534504	687	465496	6
55	510803	615	975887	72	534916	686	465084	5
56	511172	614	975844	72	535328	686	464672	4
57	511540	613	975800	72	535739	685	464261	3
58	511907	613	975757	72	536150	685	463850	2
59	512275	612	975714	72	536561	684	463439	1
60	512642	612	975670	72	536972	684	463028	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

108° 71°

19°								160°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·512642	612	9·975670	73	9·536972	684	10·463028	60
1	513009	611	975627	73	537382	683	462618	59
2	513375	611	975583	73	537792	683	462208	58
3	513741	610	975539	73	538202	682	461798	57
4	514107	609	975496	73	538611	682	461389	56
5	514472	609	975452	73	539020	681	460980	55
6	514837	608	975408	73	539429	681	460571	54
7	515202	608	975365	73	539837	680	460163	53
8	515566	607	975321	73	540245	680	459755	52
9	515930	607	975277	73	540653	679	459347	51
10	516294	606	975233	73	541061	679	458939	50
11	9·516657	605	9·975189	73	9·541468	678	10·458532	49
12	517020	605	975145	73	541875	678	458125	48
13	517382	604	975101	73	542281	677	457719	47
14	517745	604	975057	73	542688	677	457312	46
15	518107	603	975013	73	543094	676	456906	45
16	518468	603	974969	74	543499	676	456501	44
17	518829	602	974925	74	543905	675	456095	43
18	519190	601	974880	74	544310	675	455690	42
19	519551	601	974836	74	544715	674	455285	41
20	519911	600	974792	74	545119	674	454881	40
21	9·520271	600	9·974748	74	9·545524	673	10·454476	39
22	520631	599	974703	74	545928	673	454072	38
23	520990	599	974659	74	546331	672	453669	37
24	521349	598	974614	74	546735	672	453265	36
25	521707	598	974570	74	547138	671	452862	35
26	522066	597	974525	74	547540	671	452460	34
27	522424	596	974481	74	547943	670	452057	33
28	522781	596	974436	74	548345	670	451655	32
29	523138	595	974391	74	548747	669	451253	31
30	523495	595	974347	75	549149	669	450851	30
31	9·523852	594	9·974302	75	9·549550	668	10·450450	29
32	524208	594	974257	75	549951	668	450049	28
33	524564	593	974212	75	550352	667	449648	27
34	524920	593	974167	75	550752	667	449248	26
35	525275	592	974122	75	551153	666	448847	25
36	525630	591	974077	75	551552	666	448448	24
37	525984	591	974032	75	551952	665	448048	23
38	526339	590	973987	75	552351	665	447649	22
39	526693	590	973942	75	552750	665	447250	21
40	527046	589	973897	75	553149	664	446851	20
41	9·527400	589	9·973852	75	9·553548	664	10·446452	19
42	527753	588	973807	75	553946	663	446054	18
43	528105	588	973761	75	554344	663	445656	17
44	528458	587	973716	76	554741	662	445259	16
45	528810	587	973671	76	555139	662	444861	15
46	529161	586	973625	76	555536	661	444464	14
47	529513	586	973580	76	555933	661	444067	13
48	529864	585	973535	76	556329	660	443671	12
49	530215	585	973489	76	556725	660	443275	11
50	530565	584	973444	76	557121	659	442879	10
51	9·530915	584	9·973398	76	9·557517	659	10·442483	9
52	531265	583	973352	76	557913	659	442087	8
53	531614	582	973307	76	558308	658	441692	7
54	531963	582	973261	76	558703	658	441297	6
55	532312	581	973215	76	559097	657	440903	5
56	532661	581	973169	76	559491	657	440509	4
57	533009	580	973124	76	559885	656	440115	3
58	533357	580	973078	76	560279	656	439721	2
59	533704	579	973032	77	560673	655	439327	1
60	534052	578	972986	77	561066	655	438934	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
109°								70°

20° 159°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·534052	578	9·972986	77	9·561066	655	10·438934	60
1	534399	577	972940	77	561459	654	438541	59
2	534745	577	972894	77	561851	654	438149	58
3	535092	577	972848	77	562244	653	437756	57
4	535438	576	972802	77	562636	653	437364	56
5	535783	576	972755	77	563028	653	436972	55
6	536129	575	972709	77	563419	652	436581	54
7	536474	574	972663	77	563811	652	436189	53
8	536818	574	972617	77	564202	651	435798	52
9	537163	573	972570	77	564593	651	435407	51
10	537507	573	972524	77	564983	650	435017	50
11	9·537851	572	9·972478	77	9·565373	650	10·434627	49
12	538194	572	972431	78	565763	649	434237	48
13	538538	571	972385	78	566153	649	433847	47
14	538880	571	972338	78	566542	649	433458	46
15	539223	570	972291	78	566932	648	433068	45
16	539565	570	972245	78	567320	648	432680	44
17	539907	569	972198	78	567709	647	432291	43
18	540249	569	972151	78	568098	647	431902	42
19	540590	568	972105	78	568486	646	431514	41
20	540931	568	972058	78	568873	646	431127	40
21	9·541272	567	9·972011	78	9·569261	645	10·430739	39
22	541613	567	971964	78	569648	645	430352	38
23	541953	566	971917	78	570035	645	429965	37
24	542293	566	971870	78	570422	644	429578	36
25	542632	565	971823	78	570809	644	429191	35
26	542971	565	971776	78	571195	643	428805	34
27	543310	564	971729	79	571581	643	428419	33
28	543649	564	971682	79	571967	642	428033	32
29	543987	563	971635	79	572352	642	427648	31
30	544325	563	971588	79	572738	642	427262	30
31	9·544663	562	9·971540	79	9·573123	641	10·426877	29
32	545000	562	971493	79	573507	641	426493	28
33	545338	561	971446	79	573892	640	426108	27
34	545674	561	971398	79	574276	640	425724	26
35	546011	560	971351	79	574660	639	425340	25
36	546347	560	971303	79	575044	639	424956	24
37	546683	559	971256	79	575427	639	424573	23
38	547019	559	971208	79	575810	638	424190	22
39	547354	558	971161	79	576193	638	423807	21
40	547689	558	971113	79	576576	637	423424	20
41	9·548024	557	9·971066	80	9·576959	637	10·423041	19
42	548359	557	971018	80	577341	636	422659	18
43	548693	556	970970	80	577723	636	422277	17
44	549027	556	970922	80	578104	636	421896	16
45	549360	555	970874	80	578486	635	421514	15
46	549693	555	970827	80	578867	635	421133	14
47	550026	554	970779	80	579248	634	420752	13
48	550359	554	970731	80	579629	634	420371	12
49	550692	553	970683	80	580009	634	419991	11
50	551024	553	970635	80	580389	633	419611	10
51	9·551356	552	9·970586	80	9·580769	633	10·419231	9
52	551687	552	970538	80	581149	632	418851	8
53	552018	552	970490	80	581528	632	418472	7
54	552349	551	970442	80	581907	632	418093	6
55	552680	551	970394	80	582286	631	417714	5
56	553010	550	970345	81	582665	631	417335	4
57	553341	550	970297	81	583044	630	416956	3
58	553670	549	970249	81	583422	630	416578	2
59	554000	549	970200	81	583800	629	416200	1
60	554329	548	970152	81	584177	629	415823	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

110° 69°

21° 158°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·554329	548	9·970152	81	9·584177	629	10·415823	60
1	554658	548	970103	81	584555	629	415445	59
2	554987	547	970055	81	584932	628	415068	58
3	555315	547	970006	81	585309	628	414691	57
4	555643	546	969957	81	585686	627	414314	56
5	555971	546	969909	81	586062	627	413938	55
6	556299	545	969860	81	586439	627	413561	54
7	556626	545	969811	81	586815	626	413185	53
8	556953	544	969762	81	587190	626	412810	52
9	557280	544	969714	81	587566	625	412434	51
10	557606	543	969665	81	587941	625	412059	50
11	9·557932	543	9·969616	82	9·588316	625	10·411684	49
12	558258	543	969567	82	588691	624	411309	48
13	558583	542	969518	82	589066	624	410934	47
14	558909	542	969469	82	589440	623	410560	46
15	559234	541	969420	82	589814	623	410186	45
16	559558	541	969370	82	590188	623	409812	44
17	559883	540	969321	82	590562	622	409438	43
18	560207	540	969272	82	590935	622	409065	42
19	560531	539	969223	82	591308	622	408692	41
20	560855	539	969173	82	591681	621	408319	40
21	9·561178	538	9·969124	82	9·592054	621	10·407946	39
22	561501	538	969075	82	592426	620	407574	38
23	561824	537	969025	82	592799	620	407201	37
24	562146	537	968976	82	593171	619	406829	36
25	562468	536	968926	83	593542	619	406458	35
26	562790	536	968877	83	593914	618	406086	34
27	563112	536	968827	83	594285	618	405715	33
28	563433	535	968777	83	594656	618	405344	32
29	563755	535	968728	83	595027	617	404973	31
30	564075	534	968678	83	595398	617	404602	30
31	9·564396	534	9·968628	83	9·595768	617	10·404232	29
32	564716	533	968578	83	596138	616	403862	28
33	565036	533	968528	83	596508	616	403492	27
34	565356	532	968479	83	596878	616	403122	26
35	565676	532	968429	83	597247	615	402753	25
36	565995	531	968379	83	597616	615	402384	24
37	566314	531	968329	83	597985	615	402015	23
38	566632	531	968278	83	598354	614	401646	22
39	566951	530	968228	84	598722	614	401278	21
40	567269	530	968178	84	599091	613	400909	20
41	9·567587	529	9·968128	84	9·599459	613	10·400541	19
42	567904	529	968078	84	599827	613	400173	18
43	568222	528	968027	84	600194	612	399806	17
44	568539	528	967977	84	600562	612	399438	16
45	568856	528	967927	84	600929	611	399071	15
46	569172	527	967876	84	601296	611	398704	14
47	569488	527	967826	84	601663	611	398337	13
48	569804	526	967775	84	602029	610	397971	12
49	570120	526	967725	84	602395	610	397605	11
50	570435	525	967674	84	602761	610	397239	10
51	9·570751	525	9·967624	84	9·603127	609	10·396873	9
52	571066	524	967573	84	603493	609	396507	8
53	571380	524	967522	85	603858	609	396142	7
54	571695	523	967471	85	604223	608	395777	6
55	572009	523	967421	85	604588	608	395412	5
56	572323	523	967370	85	604953	607	395047	4
57	572636	522	967319	85	605317	607	394683	3
58	572950	522	967268	85	605682	607	394318	2
59	573263	521	967217	85	606046	606	393954	1
60	573575	521	967166	85	606410	606	393590	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

111° 68°

22° 157°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·573575	521	9·967166	85	9·606410	606	10·393590	60
1	573888	520	967115	85	606773	606	393227	59
2	574200	520	967064	85	607137	605	392863	58
3	574512	519	967013	85	607500	605	392500	57
4	574824	519	966961	85	607863	604	392137	56
5	575136	519	966910	85	608225	604	391775	55
6	575447	518	966859	85	608588	604	391412	54
7	575758	518	966808	85	608950	603	391050	53
8	576069	517	966756	86	609312	603	390688	52
9	576379	517	966705	86	609674	603	390326	51
10	576689	516	966653	86	610036	602	389964	50
11	9·576999	516	9·966602	86	9·610397	602	10·389603	49
12	577309	516	966550	86	610759	602	389241	48
13	577618	515	966499	86	611120	601	388880	47
14	577927	515	966447	86	611480	601	388520	46
15	578236	514	966395	86	611841	601	388159	45
16	578545	514	966344	86	612201	600	387799	44
17	578853	513	966292	86	612561	600	387439	43
18	579162	513	966240	86	612921	600	387079	42
19	579470	513	966188	86	613281	599	386719	41
20	579777	512	966136	86	613641	599	386359	40
21	9·580085	512	9·966085	87	9·614000	598	10·386000	39
22	580392	511	966033	87	614359	598	385641	38
23	580699	511	965981	87	614718	598	385282	37
24	581005	511	965928	87	615077	597	384923	36
25	581312	510	965876	87	615435	597	384565	35
26	581618	510	965824	87	615793	597	384207	34
27	581924	509	965772	87	616151	596	383849	33
28	582229	509	965720	87	616509	596	383491	32
29	582535	509	965668	87	616867	596	383133	31
30	582840	508	965615	87	617224	595	382776	30
31	9·583145	508	9·965563	87	9·617582	595	10·382418	29
32	583449	507	965511	87	617939	595	382061	28
33	583754	507	965458	87	618295	594	381705	27
34	584058	506	965406	87	618652	594	381348	26
35	584361	506	965353	88	619008	594	380992	25
36	584665	506	965301	88	619364	593	380636	24
37	584968	505	965248	88	619720	593	380280	23
38	585272	505	965195	88	620076	593	379924	22
39	585574	504	965143	88	620432	592	379568	21
40	585877	504	965090	88	620787	592	379213	20
41	9·586179	503	9·965037	88	9·621142	592	10·378858	19
42	586482	503	964984	88	621497	591	378503	18
43	586783	503	964931	88	621852	591	378148	17
44	587085	502	964879	88	622207	590	377793	16
45	587386	502	964826	88	622561	590	377439	15
46	587688	501	964773	88	622915	590	377085	14
47	587989	501	964720	88	623269	589	376731	13
48	588289	501	964666	89	623623	589	376377	12
49	588590	500	964613	89	623976	589	376024	11
50	588890	500	964560	89	624330	588	375670	10
51	9·589190	499	9·964507	89	9·624683	588	10·375317	9
52	589489	499	964454	89	625036	588	374964	8
53	589789	499	964400	89	625388	587	374612	7
54	590088	498	964347	89	625741	587	374259	6
55	590387	498	964294	89	626093	587	373907	5
56	590686	497	964240	89	626445	586	373555	4
57	590984	497	964187	89	626797	586	373203	3
58	591282	497	964133	89	627149	586	372851	2
59	591580	496	964080	89	627501	585	372499	1
60	591878	496	964026	89	627852	585	372148	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

112° 67°

23° 156°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·591878	496	9·964026	89	9·627852	585	10·372148	60
1	592176	495	963972	89	628203	585	371797	59
2	592473	495	963919	89	628554	585	371446	58
3	592770	495	963865	90	628905	584	371095	57
4	593067	494	963811	90	629255	584	370745	56
5	593363	494	963757	90	629606	583	370394	55
6	593659	493	963704	90	629956	583	370044	54
7	593955	493	963650	90	630306	583	369694	53
8	594251	493	963596	90	630656	583	369344	52
9	594547	492	963542	90	631005	582	368995	51
10	594842	492	963488	90	631355	582	368645	50
11	9·595137	491	9·963434	90	9·631704	582	10·368296	49
12	595432	491	963379	90	632053	581	367947	48
13	595727	491	963325	90	632402	581	367598	47
14	596021	490	963271	90	632750	581	367250	46
15	596315	490	963217	90	633099	580	366901	45
16	596609	489	963163	90	633447	580	366553	44
17	596903	489	963108	91	633795	580	366205	43
18	597196	489	963054	91	634143	579	365857	42
19	597490	488	962999	91	634490	579	365510	41
20	597783	488	962945	91	634838	579	365162	40
21	9·598075	487	9·962890	91	9·635185	578	10·364815	39
22	598368	487	962836	91	635532	578	364468	38
23	598660	487	962781	91	635879	578	364121	37
24	598952	486	962727	91	636226	577	363774	36
25	599244	486	962672	91	636572	577	363428	35
26	599536	485	962617	91	636919	577	363081	34
27	599827	485	962562	91	637265	577	362735	33
28	600118	485	962508	91	637611	576	362389	32
29	600409	484	962453	91	637956	576	362044	31
30	600700	484	962398	92	638302	576	361698	30
31	9·600990	484	9·962343	92	9·638647	575	10·361353	29
32	601280	483	962288	92	638992	575	361008	28
33	601570	483	962233	92	639337	575	360663	27
34	601860	482	962178	92	639682	574	360318	26
35	602150	482	962123	92	640027	574	359973	25
36	602439	482	962067	92	640371	574	359629	24
37	602728	481	962012	92	640716	573	359284	23
38	603017	481	961957	92	641060	573	358940	22
39	603305	481	961902	92	641404	573	358596	21
40	603594	480	961846	92	641747	572	358253	20
41	9·603882	480	9·961791	92	9·642091	572	10·357909	19
42	604170	479	961735	92	642434	572	357566	18
43	604457	479	961680	92	642777	572	357223	17
44	604745	479	961624	93	643120	571	356880	16
45	605032	478	961569	93	643463	571	356537	15
46	605319	478	961513	93	643806	571	356194	14
47	605606	478	961458	93	644148	570	355852	13
48	605892	477	961402	93	644490	570	355510	12
49	606179	477	961346	93	644832	570	355168	11
50	606465	476	961290	93	645174	569	354826	10
51	9·606751	476	9·961235	93	9·645516	569	10·354484	9
52	607036	476	961179	93	645857	569	354143	8
53	607322	475	961123	93	646199	569	353801	7
54	607607	475	961067	93	646540	568	353460	6
55	607892	474	961011	93	646881	568	353119	5
56	608177	474	960955	93	647222	568	352778	4
57	608461	474	960899	93	647562	567	352438	3
58	608745	473	960843	94	647903	567	352097	2
59	609029	473	960786	94	648243	567	351757	1
60	609313	473	960730	94	648583	566	351417	0
′	Cosine.	D.	Sine.	D	Cotang.	D.	Tang.	′

113° 66°

24°								155°
′	Sine.	D	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·609313	473	9·960730	94	9·648583	566	10·351417	60
1	609597	472	960674	94	648923	566	351077	59
2	609880	472	960618	94	649263	566	350737	58
3	610164	472	960561	94	649602	566	350398	57
4	610447	471	960505	94	649942	565	350058	56
5	610729	471	960448	94	650281	565	349719	55
6	611012	470	960392	94	650620	565	349380	54
7	611294	470	960335	94	650959	564	349041	53
8	611576	470	960279	94	651297	564	348703	52
9	611858	469	960222	94	651636	564	348364	51
10	612140	469	960165	94	651974	563	348026	50
11	9·612421	469	9·960109	95	9·652312	563	10·347688	49
12	612702	468	960052	95	652650	563	347350	48
13	612983	468	959995	95	652988	563	347012	47
14	613264	467	959938	95	653326	562	346674	46
15	613545	467	959882	95	653663	562	346337	45
16	613825	467	959825	95	654000	562	346000	44
17	614105	466	959768	95	654337	561	345663	43
18	614385	466	959711	95	654674	561	345326	42
19	614665	466	959654	95	655011	561	344989	41
20	614944	465	959596	95	655348	561	344652	40
21	9·615223	465	9·959539	95	9·655684	560	10·344316	39
22	615502	465	959482	95	656020	560	343980	38
23	615781	464	959425	95	656356	560	343644	37
24	616060	464	959368	95	656692	559	343308	36
25	616338	464	959310	96	657028	559	342972	35
26	616616	463	959253	96	657364	559	342636	34
27	616894	463	959195	96	657699	559	342301	33
28	617172	462	959138	96	658034	558	341966	32
29	617450	462	959080	96	658369	558	341631	31
30	617727	462	959023	96	658704	558	341296	30
31	9·618004	461	9·958965	96	9·659039	558	10·340961	29
32	618281	461	958908	96	659373	557	340627	28
33	618558	461	958850	96	659708	557	340292	27
34	618834	460	958792	96	660042	557	339958	26
35	619110	460	958734	66	660376	557	339624	25
36	619386	460	958677	96	660710	556	339290	24
37	619662	459	958619	96	661043	556	338957	23
38	619938	459	958561	96	661377	556	338623	22
39	620213	459	958503	97	661710	555	338290	21
40	620488	458	958445	97	662043	555	337957	20
41	9·620763	458	9·958387	97	9·662376	555	10·337624	19
42	621038	457	958329	97	662709	554	337291	18
43	621313	457	958271	97	663042	554	336958	17
44	621587	457	958213	97	663375	554	336625	16
45	621861	456	958154	97	663707	554	336293	15
46	622135	456	958096	97	664039	553	335961	14
47	622409	456	958038	97	664371	553	335629	13
48	622682	455	957979	97	664703	553	335297	12
49	622956	455	957921	97	665035	553	334965	11
50	623229	455	957863	97	665366	552	334634	10
51	9·623504	454	9·957804	97	9·665698	552	10·334302	9
52	623774	454	957746	98	666029	552	333971	8
53	624047	454	957687	98	666360	551	333640	7
54	624319	453	957628	98	666691	551	333309	6
55	624591	453	957570	98	667021	551	332979	5
56	624863	453	957511	98	667352	551	332648	4
57	625135	452	957452	98	667682	550	332318	3
58	625406	452	957393	98	668013	550	331987	2
59	625677	452	957335	98	668343	550	331657	1
60	625948	451	957276	98	668673	550	331327	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
114°								65°

25°								154°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·625948	451	9·957276	98	9·668673	550	10·331327	60
1	626219	451	957217	98	669002	549	330998	59
2	626490	451	957158	98	669332	549	330668	58
3	626760	450	957099	98	669661	549	330339	57
4	627030	450	957040	98	669991	548	330009	56
5	627300	450	956981	98	670320	548	329680	55
6	627570	449	956921	99	670649	548	329351	54
7	627840	449	956862	99	670977	548	329023	53
8	628109	449	956803	99	671306	547	328694	52
9	628378	448	956744	99	671635	547	328365	51
10	628647	448	956684	99	671963	547	328037	50
11	9·628916	457	9·956625	99	9·672291	547	10·327709	49
12	629185	447	956566	99	672619	546	327381	48
13	629453	447	956506	99	672947	546	327053	47
14	629721	446	956447	99	673274	546	326726	46
15	629989	446	956387	99	673602	546	326398	45
16	630257	446	956327	99	673929	545	326071	44
17	630524	446	956268	99	674257	545	325743	43
18	630792	445	956208	100	674584	545	325416	42
19	631059	445	956148	100	674911	544	325089	41
20	631326	445	956089	100	675237	544	324763	40
21	9·631593	444	9·956029	100	9·675564	544	10·324436	39
22	631859	444	955969	100	675890	544	324110	38
23	632125	444	955909	100	676217	543	323783	37
24	632392	443	955849	100	676543	543	323457	36
25	632658	443	955789	100	676869	543	323131	35
26	632923	443	955729	100	677194	543	322806	34
27	633189	442	955669	100	677520	542	322480	33
28	633454	442	955609	100	677846	542	322154	32
29	633719	442	955548	100	678171	542	321829	31
30	633984	441	955488	100	678496	542	321504	30
31	9·634249	441	9·955428	101	9·678821	541	10·321179	29
32	634514	440	955368	101	679146	541	320854	28
33	634778	440	955307	101	679471	541	320529	27
34	635042	440	955247	101	679795	541	320205	26
35	635306	439	955186	101	680120	540	319880	25
36	635570	439	955126	101	680444	540	319556	24
37	635834	439	955065	101	680768	540	319232	23
38	636097	438	955005	101	681092	540	318908	22
39	636360	438	954944	101	681416	539	318584	21
40	636623	438	954883	101	681740	539	318260	20
41	9·636886	437	9·954823	101	9·682063	539	10·317937	19
42	637148	437	954762	101	682387	539	317613	18
43	637411	437	954701	101	682710	538	317290	17
44	637673	437	954640	101	683033	538	316967	16
45	637935	436	954579	101	683356	538	316644	15
46	638197	436	954518	102	683679	538	316321	14
47	638458	436	954457	102	684001	537	315999	13
48	638720	435	954396	102	684324	537	315676	12
49	638981	435	954335	102	684646	537	315354	11
50	639242	435	954274	102	684968	537	315032	10
51	9·639503	434	9·954213	102	9·685290	536	10·314710	9
52	639764	434	954152	102	685612	536	314388	8
53	640024	434	954090	102	685934	536	314066	7
54	640284	433	954029	102	686255	536	313745	6
55	640544	433	953968	102	686577	535	313423	5
56	640804	433	953906	102	686898	535	313102	4
57	641064	432	953845	102	687219	535	312781	3
58	641324	432	953783	102	687540	535	312460	2
59	641583	432	953722	103	687861	534	312139	1
60	641842	431	953660	103	688182	534	311818	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
115°								64°

26° | 153°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·641842	431	9·953660	103	9·688182	534	10·311818	60
1	642101	431	953599	103	688502	534	311498	59
2	642360	431	953537	103	688823	534	311177	58
3	642618	430	953475	103	689143	533	310857	57
4	642877	430	953413	103	689463	533	310537	56
5	643135	430	953352	103	689783	533	310217	55
6	643393	430	953290	103	690103	533	309897	54
7	643650	429	953228	103	690423	533	309577	53
8	643908	429	953166	103	690742	532	309258	52
9	644165	429	953104	103	691062	532	308938	51
10	644423	428	953042	103	691381	532	308619	50
11	9·644680	428	9·952980	104	9·691700	531	10·308300	49
12	644936	428	952918	104	692019	531	307981	48
13	645193	427	952855	104	692338	531	307662	47
14	645450	427	952793	104	692656	531	307344	46
15	645706	427	952731	104	692975	531	307025	45
16	645962	426	952669	104	693293	530	306707	44
17	646218	426	952606	104	693612	530	306388	43
18	646474	426	952544	104	693930	530	306070	42
19	646729	425	952481	104	694248	530	305752	41
20	646984	425	952419	104	694566	529	305434	40
21	9·647240	425	9·952356	104	9·694883	529	10·305117	39
22	647494	424	952294	104	695201	529	304799	38
23	647749	424	952231	104	695518	529	304482	37
24	648004	424	952168	105	695836	529	304164	36
25	648258	424	952106	105	696153	528	303847	35
26	648512	423	952043	105	696470	528	303530	34
27	648766	423	951980	105	696787	528	303213	33
28	649020	423	951917	105	697103	528	302897	32
29	649274	422	951854	105	697420	527	302580	31
30	649527	422	951791	105	697736	527	302264	30
31	9·649781	422	9·951728	105	9·698053	527	10·301947	29
32	650034	422	951665	105	698369	527	301631	28
33	650287	421	951602	105	698685	526	301315	27
34	650539	421	951539	105	699001	526	300999	26
35	650792	421	951476	105	699316	526	300684	25
36	651044	420	951412	105	699632	526	300368	24
37	651297	420	951349	106	699947	526	300053	23
38	651549	420	951286	106	700263	525	299737	22
39	651800	419	951222	106	700578	525	299422	21
40	652052	419	951159	106	700893	525	299107	20
41	9·652304	419	9·951096	106	9·701208	524	10·298792	19
42	652555	418	951032	106	701523	524	298477	18
43	652806	418	950968	106	701837	524	298163	17
44	653057	418	950905	106	702152	524	297848	16
45	653308	418	950841	106	702466	524	297534	15
46	653558	417	950778	106	702781	523	297219	14
47	653808	417	950714	106	703095	523	296905	13
48	654059	417	950650	106	703409	523	296591	12
49	654309	416	950586	106	703722	523	296278	11
50	654558	416	950522	107	704036	522	295964	10
51	9·654808	416	9·950458	107	9·704350	522	10·295650	9
52	655058	416	950394	107	704663	522	295337	8
53	655307	415	950330	107	704976	522	295024	7
54	655556	415	950266	107	705290	522	294710	6
55	655805	415	950202	107	705603	521	294397	5
56	656054	414	950138	107	705916	521	294084	4
57	656302	414	950074	107	706228	521	293772	3
58	656551	414	950010	107	706541	521	293459	2
59	656799	413	949945	107	706854	521	293146	1
60	657047	413	949881	107	707166	520	292834	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

116° | 63°

27°								152°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·657047	413	9·949881	107	9·707166	520	10·292834	60
1	657295	413	949816	107	707478	520	292522	59
2	657542	412	949752	107	707790	520	292210	58
3	657790	412	949688	108	708102	520	291898	57
4	658037	412	949623	108	708414	519	291586	56
5	658284	412	949558	108	708726	519	291274	55
6	658531	411	949494	108	709037	519	290963	54
7	658778	411	949429	108	709349	519	290651	53
8	659025	411	949364	108	709660	519	290340	52
9	659271	410	949300	108	709971	518	290029	51
10	659517	410	949235	108	710282	518	289718	50
11	9·659763	410	9·949170	108	9·710593	518	10·289407	49
12	660009	409	949105	108	710904	518	289096	48
13	660255	409	949040	108	711215	518	288785	47
14	660501	409	948975	108	711525	517	288475	46
15	660746	409	948910	108	711836	517	288164	45
16	660991	408	948845	108	712146	517	287854	44
17	661236	408	948780	109	712456	517	287544	43
18	661481	408	948715	109	712766	516	287234	42
19	661726	407	948650	109	713076	516	286924	41
20	661970	407	948584	109	713386	516	286614	40
21	9·662214	407	9·948519	109	9·713696	516	10·286304	39
22	662459	407	948454	109	714005	516	285995	38
23	662703	406	948388	109	714314	515	285686	37
24	662946	406	948323	109	714624	515	285376	36
25	663190	406	948257	109	714933	515	285067	35
26	663433	405	948192	109	715242	515	284758	34
27	663677	405	948126	109	715551	514	284449	33
28	663920	405	948060	109	715860	514	284140	32
29	664163	405	947995	110	716168	514	283832	31
30	664406	404	947929	110	716477	514	283523	30
31	9·664648	404	9·947863	110	9 716785	514	10·283215	29
32	664891	404	947797	110	717093	513	282907	28
33	665133	403	947731	110	717401	513	282599	27
34	665375	403	947665	110	717709	513	282291	26
35	665617	403	947600	110	718017	513	281983	25
36	665859	402	947533	110	718325	513	281675	24
37	666100	402	947467	110	718633	512	281367	23
38	666342	402	947401	110	718940	512	281060	22
39	666583	402	947335	110	719248	512	280752	21
40	666824	401	947269	110	719555	512	280445	20
41	9·667065	401	9·947203	110	9·719862	512	10·280138	19
42	667305	401	947136	111	720169	511	279831	18
43	667546	401	947070	111	720476	511	279524	17
44	667786	400	947004	111	720783	511	279217	16
45	668027	400	946937	111	721089	511	278911	15
46	668267	400	946871	111	721396	511	278604	14
47	668506	399	946804	111	721702	510	278298	13
48	668746	399	946738	111	722009	510	277991	12
49	668986	399	946671	111	722315	510	277685	11
50	669225	399	946604	111	722621	510	277379	10
51	9·669464	398	9·946538	111	9·722927	510	10·277073	9
52	669703	398	946471	111	723232	509	276768	8
53	669942	398	946404	111	723538	509	276462	7
54	670181	397	946337	111	723844	509	276156	6
55	670419	397	946270	112	724149	509	275851	5
56	670658	397	946203	112	724454	509	275546	4
57	670896	397	946136	112	724760	508	275240	3
58	671134	396	946069	112	725065	508	274935	2
59	671372	396	946002	112	725370	508	274630	1
60	671609	396	945935	112	725674	508	274326	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
117°								62°

28°								151°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·671609	396	9·945935	112	9·725674	508	10·274326	60
1	671847	395	945868	112	725979	508	274021	59
2	672084	395	945800	112	726284	507	273716	58
3	672321	395	945733	112	726588	507	273412	57
4	672558	395	945666	112	726892	507	273108	56
5	672795	394	945598	112	727197	507	272803	55
6	673032	394	945531	112	727501	507	272499	54
7	673268	394	945464	113	727805	506	272195	53
8	673505	394	945396	113	728109	506	271891	52
9	673741	393	945328	113	728412	506	271588	51
10	673977	393	945261	113	728716	506	271284	50
11	9·674213	393	9·945193	113	9·729020	506	10·270980	49
12	674448	392	945125	113	729323	505	270677	48
13	674684	392	945058	113	729626	505	270374	47
14	674919	392	944990	113	729929	505	270071	46
15	675155	392	944922	113	730233	505	269767	45
16	675390	391	944854	113	730535	505	269465	44
17	675624	391	944786	113	730838	504	269162	43
18	675859	391	944718	113	731141	504	268859	42
19	676094	391	944650	113	731444	504	268556	41
20	676328	390	944582	114	731746	504	268254	40
21	9·676562	390	9·944514	114	9·732048	504	10·267952	39
22	676796	390	944446	114	732351	503	267649	38
23	677030	390	944377	114	732653	503	267347	37
24	677264	389	944309	114	732955	503	267045	36
25	677498	389	944241	114	733257	503	266743	35
26	677731	389	944172	114	733558	503	266442	34
27	677964	388	944104	114	733860	502	266140	33
28	678197	388	944036	114	734162	502	265838	32
29	678430	388	943967	114	734463	502	265537	31
30	678663	388	943899	114	734764	502	265236	30
31	9·678895	387	9·943830	114	9 735066	502	10·264934	29
32	679128	387	943761	114	735367	502	264633	28
33	679360	387	943693	115	735668	501	264332	27
34	679592	387	943624	115	735969	501	264031	26
35	679824	386	943555	115	736269	501	263731	25
36	680056	386	943486	115	736570	501	263430	24
37	680288	386	943417	115	736870	501	263130	23
38	680519	385	943348	115	737171	500	262829	22
39	680750	385	943279	115	737471	500	262529	21
40	680982	385	943210	115	737771	500	262229	20
41	9·681213	385	9·943141	115	9·738071	500	10·261929	19
42	681443	384	943072	115	738371	500	261629	18
43	681674	384	943003	115	738671	499	261329	17
44	681905	384	942934	115	738971	499	261029	16
45	682135	384	942864	115	739271	499	260729	15
46	682365	383	942795	116	739570	499	260430	14
47	682595	383	942726	116	739870	499	260130	13
48	682825	383	942656	116	740169	499	259831	12
49	683055	383	942587	116	740468	498	259532	11
50	683284	382	942517	116	740767	498	259233	10
51	9·683514	382	9·942448	116	9·741066	498	10·258934	9
52	683743	382	942378	116	741365	498	258635	8
53	683972	382	942308	116	741664	498	258336	7
54	684201	381	942239	116	741962	497	258038	6
55	684430	381	942169	116	742261	497	257739	5
56	684658	381	942099	116	742559	497	257441	4
57	684887	380	942029	116	742858	497	257142	3
58	685115	380	941959	116	743156	497	256844	2
59	685343	380	941889	117	743454	497	256546	1
60	685571	380	941819	117	743752	496	256248	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
118°								61°

29°								150°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·685571	380	9·941819	117	9·743752	496	10·256248	60
1	685799	379	941749	117	744050	496	255950	59
2	686027	379	941679	117	744348	496	255652	58
3	686254	379	941609	117	744645	496	255355	57
4	686482	379	941539	117	744943	496	255057	56
5	686709	378	941469	117	745240	496	254760	55
6	686936	378	941398	117	745538	495	254462	54
7	687163	378	941328	117	745835	495	254165	53
8	687389	378	941258	117	746132	495	253868	52
9	687616	377	941187	117	746429	495	253571	51
10	687843	377	941117	117	746726	495	253274	50
11	9·688069	377	9·941046	118	9·747023	494	10·252977	49
12	688295	377	940975	118	747319	494	252681	48
13	688521	376	940905	118	747616	494	252384	47
14	688747	376	940834	118	747913	494	252087	46
15	688972	376	940763	118	748209	494	251791	45
16	689198	376	940693	118	748505	493	251495	44
17	689423	375	940622	118	748801	493	251199	43
18	689648	375	940551	118	749097	493	250903	42
19	689873	375	940480	118	749393	493	250607	41
20	690098	375	940409	118	749689	493	250311	40
21	9·690323	374	9·940338	118	9·749985	493	10·250015	39
22	690548	374	940267	118	750281	492	249719	38
23	690772	374	940196	118	750576	492	249424	37
24	690996	374	940125	119	750872	492	249128	36
25	691220	373	940054	119	751167	492	248833	35
26	691444	373	939982	119	751462	492	248538	34
27	691668	373	939911	119	751757	492	248243	33
28	691892	373	939840	119	752052	491	247948	32
29	692115	372	939768	119	752347	491	247653	31
30	692339	372	939697	119	752642	491	247358	30
31	9·692562	372	9·939625	119	9·752937	491	10·247063	29
32	692785	371	939554	119	753231	491	246769	28
33	693008	371	939482	119	753526	491	246474	27
34	693231	371	939410	119	753820	490	246180	26
35	693453	371	939339	119	754115	490	245885	25
36	693676	370	939267	120	754409	490	245591	24
37	693898	370	939195	120	754703	490	245297	23
38	694120	370	939123	120	754997	490	245003	22
39	694342	370	939052	120	755291	490	244709	21
40	694564	369	938980	120	755585	489	244415	20
41	9·694786	369	9·938908	120	9·755878	489	10·244122	19
42	695007	369	938836	120	756172	489	243828	18
43	695229	369	938763	120	756465	489	243535	17
44	695450	368	938691	120	756759	489	243241	16
45	695671	368	938619	120	757052	489	242948	15
46	695892	368	938547	120	757345	488	242655	14
47	696113	368	938475	120	757638	488	242362	13
48	696334	367	938402	121	757931	488	242069	12
49	696554	367	938330	121	758224	488	241776	11
50	696775	367	938258	121	758517	488	241483	10
51	9·696995	367	9·938185	121	9·758810	488	10·241190	9
52	697215	366	938113	121	759102	487	240898	8
53	697435	366	938040	121	759395	487	240605	7
54	697654	366	937967	121	759687	487	240313	6
55	697874	366	937895	121	759979	487	240021	5
56	698094	365	937822	121	760272	487	239728	4
57	698313	365	937749	121	760564	487	239436	3
58	698532	365	937676	121	760856	486	239144	2
59	698751	365	937604	121	761148	486	238852	1
60	698970	364	937531	121	761439	486	238561	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
119°								60°

30° 149°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·698970	364	9·937531	121	9·761439	486	10·238561	60
1	699189	364	937458	122	761731	486	238269	59
2	699407	364	937385	122	762023	486	237977	58
3	699626	364	937312	122	762314	486	237686	57
4	699844	363	937238	122	762606	485	237394	56
5	700062	363	937165	122	762897	485	237103	55
6	700280	363	937092	122	763188	485	236812	54
7	700498	363	937019	122	763479	485	236521	53
8	700716	363	936946	122	763770	485	236230	52
9	700933	362	936872	122	764061	485	235939	51
10	701151	362	936799	122	764352	484	235648	50
11	9·701368	362	9·936725	122	9·764643	484	10·235357	49
12	701585	362	936652	123	764933	484	235067	48
13	701802	361	936578	123	765224	484	234776	47
14	702019	361	936505	123	765514	484	234486	46
15	702236	361	936431	123	765805	484	234195	45
16	702452	361	936357	123	766095	484	233905	44
17	702669	360	936284	123	766385	483	233615	43
18	702885	360	936210	123	766675	483	233325	42
19	703101	360	936136	123	766965	483	233035	41
20	703317	360	936062	123	767255	483	232745	40
21	9·703533	359	9·935988	123	9·767545	483	10·232455	39
22	703749	359	935914	123	767834	483	232166	38
23	703964	359	935840	123	768124	482	231876	37
24	704179	359	935766	124	768414	482	231586	36
25	704395	359	935692	124	768703	482	231297	35
26	704610	358	935618	124	768992	482	231008	34
27	704825	358	935543	124	769281	482	230719	33
28	705040	358	935469	124	769571	482	230429	32
29	705254	358	935395	124	769860	481	230140	31
30	705469	357	935320	124	770148	481	229852	30
31	9·705683	357	9·935246	124	9·770437	481	10·229563	29
32	705898	357	935171	124	770726	481	229274	28
33	706112	357	935097	124	771015	481	228985	27
34	706326	356	935022	124	771303	481	228697	26
35	706539	356	934948	124	771592	481	228408	25
36	706753	356	934873	124	771880	480	228120	24
37	706967	356	934798	125	772168	480	227832	23
38	707180	355	934723	125	772457	480	227543	22
39	707393	355	934649	125	772745	480	227255	21
40	707606	355	934574	125	773033	480	226967	20
41	9·707819	355	9·934499	125	9·773321	480	10·226679	19
42	708032	354	934424	125	773608	479	226392	18
43	708245	354	934349	125	773896	479	226104	17
44	708458	354	934274	125	774184	479	225816	16
45	708670	354	934199	125	774471	479	225529	15
46	708882	353	934123	125	774759	479	225241	14
47	709094	353	934048	125	775046	479	224954	13
48	709306	353	933973	125	775333	479	224667	12
49	709518	353	933898	126	775621	478	224379	11
50	709730	353	933822	126	775908	478	224092	10
51	9·709941	352	9·933747	126	9·776195	478	10·223805	9
52	710153	352	933671	126	776482	478	223518	8
53	710364	352	933596	126	776768	478	223232	7
54	710575	352	933520	126	777055	478	222945	6
55	710786	351	933445	126	777342	478	222658	5
56	710997	351	933369	126	777628	477	222372	4
57	711208	351	933293	126	777915	477	222085	3
58	711419	351	933217	126	778201	477	221799	2
59	711629	350	933141	126	778488	477	221512	1
60	711839	350	933066	126	778774	477	221226	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

120° 59°

31°								148°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·711839	350	9·933066	126	9·778774	477	10·221226	60
1	712050	350	932990	127	779060	477	220940	59
2	712260	350	932914	127	779346	476	220654	58
3	712469	349	932838	127	779632	476	220368	57
4	712679	349	932762	127	779918	476	220082	56
5	712889	349	932685	127	780203	476	219797	55
6	713098	349	932609	127	780489	476	219511	54
7	713308	349	932533	127	780775	476	219225	53
8	713517	348	932457	127	781060	476	218940	52
9	713726	348	932380	127	781346	475	218654	51
10	713935	348	932304	127	781631	475	218369	50
11	9·714144	348	9·932228	127	9·781916	475	10·218084	49
12	714352	347	932151	127	782201	475	217799	48
13	714561	347	932075	128	782486	475	217514	47
14	714769	347	931998	128	782771	475	217229	46
15	714978	347	931921	128	783056	475	216944	45
16	715186	347	931845	128	783341	475	216659	44
17	715394	346	931768	128	783626	474	216374	43
18	715602	346	931691	128	783910	474	216090	42
19	715809	346	931614	128	784195	474	215805	41
20	716017	346	931537	128	784479	474	215521	40
21	9·716224	345	9·931460	128	9·784764	474	10·215236	39
22	716432	345	931383	128	785048	474	214952	38
23	716639	345	931306	128	785332	473	214668	37
24	716846	345	931229	129	785616	473	214384	36
25	717053	345	931152	129	785900	473	214100	35
26	717259	344	931075	129	786184	473	213816	34
27	717466	344	930998	129	786468	473	213532	33
28	717673	344	930921	129	786752	473	213248	32
29	717879	344	930843	129	787036	473	212964	31
30	718085	343	930766	129	787319	472	212681	30
31	9·718291	343	9·930688	129	9·787603	472	10·212397	29
32	718497	343	930611	129	787886	472	212114	28
33	718703	343	930533	129	788170	472	211830	27
34	718909	343	930456	129	788453	472	211547	26
35	719114	342	930378	129	788736	472	211264	25
36	719320	342	930300	130	789019	472	210981	24
37	719525	342	930223	130	789302	471	210698	23
38	719730	342	930145	130	789585	471	210415	22
39	719935	341	930067	130	789868	471	210132	21
40	720140	341	929989	130	790151	471	209849	20
41	9·720345	341	9·929911	130	9·790434	471	10·209566	19
42	720549	341	929833	130	790716	471	209284	18
43	720754	340	929755	130	790999	471	209001	17
44	720958	340	929677	130	791281	471	208719	16
45	721162	340	929599	130	791563	470	208437	15
46	721366	340	929521	130	791846	470	208154	14
47	721570	340	929442	130	792128	470	207872	13
48	721774	339	929364	131	792410	470	207590	12
49	721978	339	929286	131	792692	470	207308	11
50	722181	339	929207	131	792974	470	207026	10
51	9·722385	339	9·929129	131	9·793256	470	10·206744	9
52	722588	339	929050	131	793538	469	206462	8
53	722791	338	928972	131	793819	469	206181	7
54	722994	338	928893	131	794101	469	205899	6
55	723197	338	928815	131	794383	469	205617	5
56	723400	338	928736	131	794664	469	205336	4
57	723603	337	928657	131	794946	469	205054	3
58	723805	337	928578	131	795227	469	204773	2
59	724007	337	928499	131	795508	468	204492	1
60	724210	337	928420	131	795789	468	204211	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
121°								58°

32° 147°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·724210	337	9·928420	132	9·795789	468	10·204211	60
1	724412	337	928342	132	796070	468	203930	59
2	724614	336	928263	132	796351	468	203649	58
3	724816	336	928183	132	796632	468	203368	57
4	725017	336	928104	132	796913	468	203087	56
5	725219	336	928025	132	797194	468	202806	55
6	725420	335	927946	132	797474	468	202526	54
7	725622	335	927867	132	797755	468	202245	53
8	725823	335	927787	132	798036	467	201964	52
9	726024	335	927708	132	798316	467	201684	51
10	726225	335	927629	132	798596	467	201404	50
11	9·726426	334	9·927549	132	9·798877	467	10·201123	49
12	726626	334	927470	133	799157	467	200843	48
13	726827	334	927390	133	799437	467	200563	47
14	727027	334	927310	133	799717	467	200283	46
15	727228	334	927231	133	799997	466	200003	45
16	727428	333	927151	133	800277	466	199723	44
17	727628	333	927071	133	800557	466	199443	43
18	727828	333	926991	133	800836	466	199164	42
19	728027	333	926911	133	801116	466	198884	41
20	728227	333	926831	133	801396	466	198604	40
21	9·728427	332	9·926751	133	9·801675	466	10·198325	39
22	728626	332	926671	133	801955	466	198045	38
23	728825	332	926591	133	802234	465	197766	37
24	729024	332	926511	134	802513	465	197487	36
25	729223	331	926431	134	802792	465	197208	35
26	729422	331	926351	134	803072	465	196928	34
27	729621	331	926270	134	803351	465	196649	33
28	729820	331	926190	134	803630	465	196370	32
29	730018	330	926110	134	803909	465	196091	31
30	730217	330	926029	134	804187	465	195813	30
31	9·730415	330	9·925949	134	9·804466	464	10·195534	29
32	730613	330	925868	134	804745	464	195255	28
33	730811	330	925788	134	805023	464	194977	27
34	731009	329	925707	134	805302	464	194698	26
35	731206	329	925626	134	805580	464	194420	25
36	731404	329	925545	135	805859	464	194141	24
37	731602	329	925465	135	806137	464	193863	23
38	731799	329	925384	135	806415	463	193585	22
39	731996	328	925303	135	806693	463	193307	21
40	732193	328	925222	135	806971	463	193029	20
41	9·732390	328	9·925141	135	9·807249	463	10·192751	19
42	732587	328	925060	135	807527	463	192473	18
43	732784	328	924979	135	807805	463	192195	17
44	732980	327	924897	135	808083	463	191917	16
45	733177	327	924816	135	808361	463	191639	15
46	733373	327	924735	136	808638	462	191362	14
47	733569	327	924654	136	808916	462	191084	13
48	733765	327	924572	136	809193	462	190807	12
49	733961	326	924491	136	809471	462	190529	11
50	734157	326	924409	136	809748	462	190252	10
51	9·734353	326	9·924328	136	9·810025	462	10·189975	9
52	734549	326	924246	136	810302	462	189698	8
53	734744	325	924164	136	810580	462	189420	7
54	734939	325	924083	136	810857	462	189143	6
55	735135	325	924001	136	811134	461	188866	5
56	735330	325	923919	136	811410	461	188590	4
57	735525	325	923837	136	811687	461	188313	3
58	735719	324	923755	137	811964	461	188036	2
59	735914	324	923673	137	812241	461	187759	1
60	736109	324	923591	137	812517	461	187483	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

122° 57°

33°									146°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′	
0	9·736109	324	9·923591	137	9·812517	461	10·187483	60	
1	736303	324	923509	137	812794	461	187206	59	
2	736498	324	923427	137	813070	461	186930	58	
3	736692	323	923345	137	813347	460	186653	57	
4	736886	323	923263	137	813623	460	186377	56	
5	737080	323	923181	137	813899	460	186101	55	
6	737274	323	923098	137	814176	460	185824	54	
7	737467	323	923016	137	814452	460	185548	53	
8	737661	322	922933	137	814728	460	185272	52	
9	737855	322	922851	137	815004	460	184996	51	
10	738048	322	922768	138	815280	460	184720	50	
11	9·738241	322	9·922686	138	9·815555	459	10·184445	49	
12	738434	322	922603	138	815831	459	184169	48	
13	738627	321	922520	138	816107	459	183893	47	
14	738820	321	922438	138	816382	459	183618	46	
15	739013	321	922355	138	816658	459	183342	45	
16	739206	321	922272	138	816933	459	183067	44	
17	739398	321	922189	138	817209	459	182791	43	
18	739590	320	922106	138	817484	459	182516	42	
19	739783	320	922023	138	817759	459	182241	41	
20	739975	320	921940	138	818035	458	181965	40	
21	9·740167	320	9·921857	139	9·818310	458	10·181690	39	
22	740359	320	921774	139	818585	458	181415	38	
23	740550	319	921691	139	818860	458	181140	37	
24	740742	319	921607	139	819135	458	180865	36	
25	740934	319	921524	139	819410	458	180590	35	
26	741125	319	921441	139	819684	458	180316	34	
27	741316	319	921357	139	819959	458	180041	33	
28	741508	318	921274	139	820234	458	179766	32	
29	741699	318	921190	139	820508	457	179492	31	
30	741889	318	921107	139	820783	457	179217	30	
31	9·742080	318	9·921023	139	9·821057	457	10·178943	29	
32	742271	318	920939	140	821332	457	178668	28	
33	742462	317	920856	140	821606	457	178394	27	
34	742652	317	920772	140	821880	457	178120	26	
35	742842	317	920688	140	822154	457	177846	25	
36	743033	317	920604	140	822429	457	177571	24	
37	743223	317	920520	140	822703	457	177297	23	
38	743413	316	920436	140	822977	456	177023	22	
39	743602	316	920352	140	823251	456	176749	21	
40	743792	316	920268	140	823524	456	176476	20	
41	9·743982	316	9·920184	140	9·823798	456	10·176202	19	
42	744171	316	920099	140	824072	456	175928	18	
43	744361	315	920015	140	824345	456	175655	17	
44	744550	315	919931	141	824619	456	175381	16	
45	744739	315	919846	141	824893	456	175107	15	
46	744928	315	919762	141	825166	456	174834	14	
47	745117	315	919677	141	825439	455	174561	13	
48	745306	314	919593	141	825713	455	174287	12	
49	745494	314	919508	141	825986	455	174014	11	
50	745683	314	919424	141	826259	455	173741	10	
51	9·745871	314	9·919339	141	9·826532	455	10·173468	9	
52	746060	314	919254	141	826805	455	173195	8	
53	746248	313	919169	141	827078	455	172922	7	
54	746436	313	919085	141	827351	455	172649	6	
55	746624	313	919000	141	827624	455	172376	5	
56	746812	313	918915	142	827897	454	172103	4	
57	746999	313	918830	142	828170	454	171830	3	
58	747187	312	918745	142	828442	454	171558	2	
59	747374	312	918659	142	828715	454	171285	1	
60	747562	312	918574	142	828987	454	171013	0	
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′	
123°									56°

34° 145°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·747562	312	9·918574	142	9·828987	454	10·171013	60
1	747749	312	918489	142	829260	454	170740	59
2	747936	312	918404	142	829532	454	170468	58
3	748123	311	918318	142	829805	454	170195	57
4	748310	311	918233	142	830077	454	169923	56
5	748497	311	918147	142	830349	453	169651	55
6	748683	311	918062	142	830621	453	169379	54
7	748870	311	917976	143	830893	453	169107	53
8	749056	310	917891	143	831165	453	168835	52
9	749243	310	917805	143	831437	453	168563	51
10	749429	310	917719	143	831709	453	168291	50
11	9·749615	310	9·917634	143	9·831981	453	10·168019	49
12	749801	310	917548	143	832253	453	167747	48
13	749987	309	917462	143	832525	453	167475	47
14	750172	309	917376	143	832796	453	167204	46
15	750358	309	917290	143	833068	452	166932	45
16	750543	309	917204	143	833339	452	166661	44
17	750729	309	917118	144	833611	452	166389	43
18	750914	308	917032	144	833882	452	166118	42
19	751099	308	916946	144	834154	452	165846	41
20	751284	308	916859	144	834425	452	165575	40
21	9·751469	308	9·916773	144	9·834696	452	10·165304	39
22	751654	308	916687	144	834967	452	165033	38
23	751839	308	916600	144	835238	452	164762	37
24	752023	307	916514	144	835509	452	164491	36
25	752208	307	916427	144	835780	451	164220	35
26	752392	307	916341	144	836051	451	163949	34
27	752576	307	916254	144	836322	451	163678	33
28	752760	307	916167	145	836593	451	163407	32
29	752944	306	916081	145	836864	451	163136	31
30	753128	306	915994	145	837134	451	162866	30
31	9·753312	306	9·915907	145	9·837405	451	10·162595	29
32	753495	306	915820	145	837675	451	162325	28
33	753679	306	915733	145	837946	451	162054	27
34	753862	305	915646	145	838216	451	161784	26
35	754046	305	915559	145	838487	450	161513	25
36	754229	305	915472	145	838757	450	161243	24
37	754412	305	915385	145	839027	450	160973	23
38	754595	305	915297	145	839297	450	160703	22
39	754778	304	915210	145	839568	450	160432	21
40	754960	304	915123	146	839838	450	160162	20
41	9·755143	304	9·915035	146	9·840108	450	10·159892	19
42	755326	304	914948	146	840378	450	159622	18
43	755508	304	914860	146	840648	450	159352	17
44	755690	304	914773	146	840917	449	159083	16
45	755872	303	914685	146	841187	449	158813	15
46	756054	303	914598	146	841457	449	158543	14
47	756236	303	914510	146	841727	449	158273	13
48	756418	303	914422	146	841996	449	158004	12
49	756600	303	914334	146	842266	449	157734	11
50	756782	302	914246	147	842535	449	157465	10
51	9·756963	302	9·914158	147	9·842805	449	10·157195	9
52	757144	302	914070	147	843074	449	156926	8
53	757326	302	913982	147	843343	449	156657	7
54	757507	302	913894	147	843612	449	156388	6
55	757688	301	913806	147	843882	448	156118	5
56	757869	301	913718	147	844151	448	155849	4
57	758050	301	913630	147	844420	448	155580	3
58	758230	301	913541	147	844689	448	155311	2
59	758411	301	913453	147	844958	448	155042	1
60	758591	301	913365	147	845227	448	154773	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

124° 55°

35° 144°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·758591	301	9·913365	147	9·845227	448	10·154773	60
1	758772	300	913276	147	845496	448	154504	59
2	758952	300	913187	148	845764	448	154236	58
3	759132	300	913099	148	846033	448	153967	57
4	759312	300	913010	148	846302	448	153698	56
5	759492	300	912922	148	846570	447	153430	55
6	759672	299	912833	148	846839	447	153161	54
7	759852	299	912744	148	847108	447	152892	53
8	760031	299	912655	148	847376	447	152624	52
9	760211	299	912566	148	847644	447	152356	51
10	760390	299	912477	148	847913	447	152087	50
11	9·760569	298	9·912388	148	9·848181	447	10·151819	49
12	760748	298	912299	149	848449	447	151551	48
13	760927	298	912210	149	848717	447	151283	47
14	761106	298	912121	149	848986	447	151014	46
15	761285	298	912031	149	849254	447	150746	45
16	761464	298	911942	149	849522	447	150478	44
17	761642	297	911853	149	849790	446	150210	43
18	761821	297	911763	149	850057	446	149943	42
19	761999	297	911674	149	850325	446	149675	41
20	762177	297	911584	149	850593	446	149407	40
21	9·762356	297	9·911495	149	9·850861	446	10·149139	39
22	762534	296	911405	149	851129	446	148871	38
23	762712	296	911315	150	851396	446	148604	37
24	762889	296	911226	150	851664	446	148336	36
25	763067	296	911136	150	851931	446	148069	35
26	763245	296	911046	150	852199	446	147801	34
27	763422	296	910956	150	852466	446	147534	33
28	763600	295	910866	150	852733	445	147267	32
29	763777	295	910776	150	853001	445	146999	31
30	763954	295	910686	150	853268	445	146732	30
31	9·764131	295	9·910596	150	9·853535	445	10·146465	29
32	764308	295	910506	150	853802	445	146198	28
33	764485	294	910415	150	854069	445	145931	27
34	764662	294	910325	151	854336	445	145664	26
35	764838	294	910235	151	854603	445	145397	25
36	765015	294	910144	151	854870	445	145130	24
37	765191	294	910054	151	855137	445	144863	23
38	765367	294	909963	151	855404	445	144596	22
39	765544	293	909873	151	855671	444	144329	21
40	765720	293	909782	151	855938	444	144062	20
41	9·765896	293	9·909691	151	9·856204	444	10·143796	19
42	766072	293	909601	151	856471	444	143529	18
43	766247	293	909510	151	856737	444	143263	17
44	766423	293	909419	151	857004	444	142996	16
45	766598	292	909328	152	857270	444	142730	15
46	766774	292	909237	152	857537	444	142463	14
47	766949	292	909146	152	857803	444	142197	13
48	767124	292	909055	152	858069	444	141931	12
49	767300	292	908964	152	858336	444	141664	11
50	767475	291	908873	152	858602	443	141398	10
51	9·767649	291	9·908781	152	9·858868	443	10·141132	9
52	767824	291	908690	152	859134	443	140866	8
53	767999	291	908599	152	859400	443	140600	7
54	768173	291	908507	152	859666	443	140334	6
55	768348	290	908416	153	859932	443	140068	5
56	768522	290	908324	153	860198	443	139802	4
57	768697	290	908233	153	860464	443	139536	3
58	768871	290	908141	153	860730	443	139270	2
59	769045	290	908049	153	860995	443	139005	1
60	769219	290	907958	153	861261	443	138739	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

125° 54°

36° 143°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·769219	290	9·907958	153	9·861261	443	10·138739	60
1	769393	289	907866	153	861527	443	138473	59
2	769566	289	907774	153	861792	442	138208	58
3	769740	289	907682	153	862058	442	137942	57
4	769913	289	907590	153	862323	442	137677	56
5	770087	289	907498	153	862589	442	137411	55
6	770260	288	907406	153	862854	442	137146	54
7	770433	288	907314	154	363119	442	136881	53
8	770606	288	907222	154	863385	442	136615	52
9	770779	288	907129	154	863650	442	136350	51
10	770952	288	907037	144	863915	442	136085	50
11	9·771125	288	9·906945	154	9·864180	442	10·135820	49
12	771298	287	906852	154	864445	442	135555	48
13	771470	287	906760	154	864710	442	135290	47
14	771643	287	906667	154	864975	441	135025	46
15	771815	287	906575	154	865240	441	134760	45
16	771987	287	906482	154	865505	441	134495	44
17	772159	287	906389	155	865770	441	134230	43
18	772331	286	906296	155	866035	441	133965	42
19	772503	286	906204	155	866300	441	133700	41
20	772675	286	906111	155	866564	441	133436	40
21	9·772847	286	9·906018	155	9·866829	441	10·133171	39
22	773018	286	905925	155	867094	441	132906	38
23	773190	286	905832	155	867358	441	132642	37
24	773361	285	905739	155	867623	441	132377	36
25	773533	285	905645	155	867887	441	132113	35
26	773704	285	905552	155	868152	440	131848	34
27	773875	285	905459	155	868416	440	131584	33
28	774046	285	905366	156	868680	440	131320	32
29	774217	285	905272	156	868945	440	131055	31
30	774388	284	905179	156	869209	440	130791	30
31	9·774558	284	9·905085	156	9·869473	440	10·130527	29
32	774729	284	904992	156	869737	440	130263	28
33	774899	284	904898	156	870001	440	129999	27
34	775070	284	904804	156	870265	440	129735	26
35	775240	284	904711	156	870529	440	129471	25
36	775410	283	904617	156	870793	440	129207	24
37	775580	283	904523	156	871057	440	128943	23
38	775750	283	904429	157	871321	440	128679	22
39	775920	283	904335	157	871585	440	128415	21
40	776090	283	904241	157	871849	439	128151	20
41	9·776259	283	9·904147	157	9·872112	439	10·127888	19
42	776429	282	904053	157	872376	439	127624	18
43	776598	282	903959	157	872640	439	127360	17
44	776768	282	903864	157	872903	439	127097	16
45	776937	282	903770	157	873167	439	126833	15
46	777106	282	903675	157	873430	439	126570	14
47	777275	281	903581	167	873694	439	126306	13
48	777444	281	903487	157	873957	439	126043	12
49	777613	281	903392	158	874220	439	125780	11
50	777781	281	903298	158	874484	439	125516	10
51	9·777950	281	9·903203	158	9·874747	439	10·125253	9
52	778119	281	903108	158	875010	439	124990	8
53	778287	280	903014	158	875273	438	124727	7
54	778455	280	902919	158	875537	438	124463	6
55	778624	280	902824	158	875800	438	124200	5
56	778792	280	902729	158	876063	438	123937	4
57	778960	280	902634	158	876326	438	123674	3
58	779128	280	902539	159	876589	438	123411	2
59	779295	279	902444	159	876852	438	123148	1
60	779463	279	902349	159	877114	438	122886	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

126° 53°

37°								142°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·779463	279	9·902349	159	9·877114	438	10·122886	60
1	779631	279	902253	159	877377	438	122623	59
2	779798	279	902158	159	877640	438	122360	58
3	779966	279	902063	159	877903	438	122097	57
4	780133	279	901967	159	878165	438	121835	56
5	780300	278	901872	159	878428	438	121572	55
6	780467	278	901776	159	878691	438	121309	54
7	780634	278	901681	159	878953	437	121047	53
8	780801	278	901585	159	879216	437	120784	52
9	780968	278	901490	159	879478	437	120522	51
10	781134	278	901394	160	879741	437	120259	50
11	9·781301	277	9·901298	160	9·880003	437	10·119997	49
12	781468	277	901202	160	880265	437	119735	48
13	781634	277	901106	160	880528	437	119472	47
14	781800	277	901010	160	880790	437	119210	46
15	781966	277	900914	160	881052	437	118948	45
16	782132	277	900818	160	881314	437	118686	44
17	782298	276	900722	160	881577	437	118423	43
18	782464	276	900626	160	881839	437	118161	42
19	782630	276	900529	160	882101	437	117899	41
20	782796	276	900433	161	882363	436	117637	40
21	9·782961	276	9·900337	161	9·882625	436	10·117375	39
22	783127	276	900240	161	882887	436	117113	38
23	783292	275	900144	161	883148	436	116852	37
24	783458	275	900047	161	883410	436	116590	36
25	783623	275	899951	161	883672	436	116328	35
26	783788	275	899854	161	883934	436	116066	34
27	783953	275	899757	161	884196	436	115804	33
28	784118	275	899660	161	884457	436	115543	32
29	784282	274	899564	161	884719	436	115281	31
30	784447	274	899467	162	884980	436	115020	30
31	9·784612	274	9·899370	162	9·885242	436	10·114758	29
32	784776	274	899273	162	885504	436	114496	28
33	784941	274	899176	162	885765	436	114235	27
34	785105	274	899078	162	886026	436	113974	26
35	785269	273	898981	162	886288	436	113712	25
36	785433	273	898884	162	886549	435	113451	24
37	785597	273	898787	162	886811	435	113189	23
38	785761	273	898689	162	887072	435	112928	22
39	785925	273	898592	162	887333	435	112667	21
40	786089	273	898494	163	887594	435	112406	20
41	9·786252	272	9·898397	163	9·887855	435	10·112145	19
42	786416	272	898299	163	888116	435	111884	18
43	786579	272	898202	163	888378	435	111622	17
44	786742	272	898104	163	888639	435	111361	16
45	786906	272	898006	163	888900	435	111100	15
46	787069	272	897908	163	889161	435	110839	14
47	787232	271	897810	163	889421	435	110579	13
48	787395	271	897712	163	889682	435	110318	12
49	787557	271	897614	163	889943	435	110057	11
50	787720	271	897516	163	890204	434	109796	10
51	9·787883	271	9·897418	164	9·890465	434	10·109535	9
52	788045	271	897320	164	890725	434	109275	8
53	788208	271	897222	164	890986	434	109014	7
54	788370	270	897123	164	891247	434	108753	6
55	788532	270	897025	164	891507	434	108493	5
56	788694	270	896926	164	891768	434	108232	4
57	788856	270	896828	164	892028	434	107972	3
58	789018	270	896729	164	892289	434	107711	2
59	789180	270	896631	164	892549	434	107451	1
60	789342	269	896532	164	892810	434	107190	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
127°								52°

38° 141°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·789342	269	9·896532	164	9·892810	434	10·107190	60
1	789504	269	896433	165	893070	434	106930	59
2	789665	269	896335	165	893331	434	106669	58
3	789827	269	896236	165	893591	434	106409	57
4	789988	269	896137	165	893851	434	106149	56
5	790149	269	896038	165	894111	434	105889	55
6	790310	268	895939	165	894372	434	105628	54
7	790471	268	895840	165	894632	433	105368	53
8	790632	268	895741	165	894892	433	105108	52
9	790793	268	895641	165	895152	433	104848	51
10	790954	268	895542	165	895412	433	104588	50
11	9·791115	268	9·895443	166	9·895672	433	10·104328	49
12	791275	267	895343	166	895932	433	104068	48
13	791436	267	895244	166	896192	433	103808	47
14	791596	267	895145	166	896452	433	103548	46
15	791757	267	895045	166	896712	433	103288	45
16	791917	267	894945	166	896971	433	103029	44
17	792077	267	894846	166	897231	433	102769	43
18	792237	266	894746	166	897491	433	102509	42
19	792397	266	894646	166	897751	433	.02249	41
20	792557	266	894546	166	898010	433	101990	40
21	9·792716	266	9·894446	167	9·898270	433	10·101730	39
22	792876	266	894346	167	898530	433	101470	38
23	793035	266	894246	167	898789	433	101211	37
24	793195	265	894146	167	899049	432	100951	36
25	793354	265	894046	167	899308	432	100692	35
26	793514	265	893946	167	899568	432	100432	34
27	793673	265	893846	167	899827	432	100173	33
28	793832	265	893745	167	900087	432	099913	32
29	793991	265	893645	167	900346	432	099654	31
30	794150	264	893544	167	900605	432	099395	30
31	9·794308	264	9·893444	168	9·900864	432	10·099136	29
32	794467	264	893343	168	901124	432	098876	28
33	794626	264	893243	168	901383	432	098617	27
34	794784	264	893142	168	901642	432	098358	26
35	794942	264	893041	168	901901	432	098099	25
36	795101	264	892940	168	902160	432	097840	24
37	795259	263	892839	168	902420	432	097580	23
38	795417	263	892739	168	902679	432	097321	22
39	795575	263	892638	168	902938	432	097062	21
40	795733	263	892536	168	903197	431	096803	20
41	9·795891	263	9·892435	169	9·903456	431	10·096544	19
42	796049	263	892334	169	903714	431	096286	18
43	796206	263	892233	169	903973	431	096027	17
44	796364	262	892132	169	904232	431	095768	16
45	796521	262	892030	169	904491	431	095509	15
46	796679	262	891929	169	904750	431	095250	14
47	796836	262	891827	169	905008	431	094992	13
48	796993	262	891726	169	905267	431	094733	12
49	797150	261	891624	169	905526	431	094474	11
50	797307	261	891523	170	905785	431	094215	10
51	9·797464	261	9·891421	170	9·906043	431	10·093957	9
52	797621	261	891319	170	906302	431	093698	8
53	797777	261	891217	170	906560	431	093440	7
54	797934	261	891115	170	906819	431	093181	6
55	798091	261	891013	170	907077	431	092923	5
56	798247	261	890911	170	907336	431	092664	4
57	798403	260	890809	170	907594	431	092406	3
58	798560	260	890707	170	907853	431	092147	2
59	798716	260	890605	170	908111	430	091889	1
60	798872	260	890503	170	908369	430	091631	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

128° 51°

39°								140°
′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·798872	260	9·890503	170	9·908369	430	10·091631	60
1	799028	260	890400	171	908628	430	091372	59
2	799184	260	890298	171	908886	430	091114	58
3	799339	259	890195	171	909144	430	090856	57
4	799495	259	890093	171	909402	430	090598	56
5	799651	259	889990	171	909660	430	090340	55
6	799806	259	889888	171	909918	430	090082	54
7	799962	259	889785	171	910177	430	089823	53
8	800117	259	889682	171	910435	430	089565	52
9	800272	258	889579	171	910693	430	089307	51
10	800427	258	889477	171	910951	430	089049	50
11	9·800582	258	9·889374	172	9·911209	430	10·088791	49
12	800737	258	889271	172	911467	430	088533	48
13	800892	258	889168	172	911725	430	088275	47
14	801047	258	889064	172	911982	430	088018	46
15	801201	258	888961	172	912240	430	087760	45
16	801356	257	888858	172	912498	430	087502	44
17	801511	257	888755	172	912756	430	087244	43
18	801665	257	888651	172	913014	429	086986	42
19	801819	257	888548	172	913271	429	086729	41
20	801973	257	888444	173	913529	429	086471	40
21	9·802128	257	9·888341	173	9·913787	429	10·086213	39
22	802282	256	888237	173	914044	429	085956	38
23	802436	256	888134	173	914302	429	085698	37
24	802589	256	888030	173	914560	429	085440	36
25	802743	256	887926	173	914817	429	085183	35
26	802897	256	887822	173	915075	429	084925	34
27	803050	256	887718	173	915332	429	084668	33
28	803204	256	887614	173	915590	429	084410	32
29	803357	255	887510	173	915847	429	084153	31
30	803511	255	887406	174	916104	429	083896	30
31	9·803664	255	9·887302	174	9·916362	429	10·083638	29
32	803817	255	887198	174	916619	429	083381	28
33	803970	255	887093	174	916877	429	083123	27
34	804123	255	886989	174	917134	429	082866	26
35	804276	254	886885	174	917391	429	082609	25
36	804428	254	886780	174	917648	429	082352	24
37	804581	254	886676	174	917906	429	082094	23
38	804734	254	886571	174	918163	428	081837	22
39	804886	254	886466	174	918420	428	081580	21
40	805039	254	886362	175	918677	428	081323	20
41	9·805191	254	9·886257	175	9·918934	428	10·081066	19
42	805343	253	886152	175	919191	428	080809	18
43	805495	253	886047	175	919448	428	080552	17
44	805647	253	885942	175	919705	428	080295	16
45	805799	253	885837	175	919962	428	080038	15
46	805951	253	885732	175	920219	428	079781	14
47	806103	253	885627	175	920476	428	079524	13
48	806254	253	885522	175	920733	428	079267	12
49	806406	252	885416	175	920990	428	079010	11
50	806557	252	885311	176	921247	428	078753	10
51	9·806709	252	9·885205	176	9·921503	428	10·078497	9
52	806860	252	885100	176	921760	428	078240	8
53	807011	252	884994	176	922017	428	077983	7
54	807163	252	884889	176	922274	428	077726	6
55	807314	252	884783	176	922530	428	077470	5
56	807465	251	884677	176	922787	428	077213	4
57	807615	251	884572	176	923044	428	076956	3
58	807766	251	884466	176	923300	428	076700	2
59	807917	251	884360	176	923557	427	076443	1
60	808067	251	884254	177	923814	427	076186	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′
129°								50°

40° | 139°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·808067	251	9·884254	177	9·923814	427	10·076186	60
1	808218	251	884148	177	924070	427	075930	59
2	808368	251	884042	177	924327	427	075673	58
3	808519	250	883936	177	924583	427	075417	57
4	808669	250	883829	177	924840	427	075160	56
5	808819	250	883723	177	925096	427	074904	55
6	808969	250	883617	177	925352	427	074648	54
7	809119	250	883510	177	925609	427	074391	53
8	809269	250	883404	177	925865	427	074135	52
9	809419	249	883297	178	926122	427	073878	51
10	809569	249	883191	178	926378	427	073622	50
11	9·809718	249	9·883084	178	9·926634	427	10·073366	49
12	809868	249	882977	178	926890	427	073110	48
13	810017	249	882871	178	927147	427	072853	47
14	810167	249	882764	178	927403	427	072597	46
15	810316	248	882657	178	927659	427	072341	45
16	810465	248	882550	178	927915	427	072085	44
17	810614	248	882443	178	928171	427	071829	43
18	810763	248	882336	179	928427	427	071573	42
19	810912	248	882229	179	928684	427	071316	41
20	811061	248	882121	179	928940	427	071060	40
21	9·811210	248	9·882014	179	9·929196	427	10·070804	39
22	811358	247	881907	179	929452	427	070548	38
23	811507	247	881799	179	929708	427	070292	37
24	811655	247	881692	179	029964	426	070036	36
25	811804	247	881584	179	930220	426	069780	35
26	811952	247	881477	179	930475	426	069525	34
27	812100	247	881369	179	930731	426	069269	33
28	812248	247	881261	180	930987	426	069013	32
29	812396	246	881153	180	931243	426	068757	31
30	812544	246	881046	180	931499	426	068501	30
31	9·812692	246	9·880938	180	9·931755	426	10·068245	29
32	812840	246	880830	180	932010	426	067990	28
33	812988	246	880722	180	932266	426	067734	27
34	813135	246	880613	180	932522	426	067478	26
35	813283	246	880505	180	932778	426	067222	25
36	813430	245	880397	180	933033	426	066967	24
37	813578	245	880289	181	933289	426	066711	23
38	813725	245	880180	181	933545	426	066455	22
39	813872	245	880072	181	933800	426	066200	21
40	814019	245	879963	181	934056	426	065944	20
41	9·814166	245	9·879855	181	9·934311	426	10·065689	19
42	814313	245	879746	181	934567	426	065433	18
43	814460	244	879637	181	934822	426	065178	17
44	814607	244	879529	181	935078	426	064922	16
45	814753	244	879420	181	935333	426	064667	15
46	814900	244	879311	181	935589	426	064411	14
47	815046	244	879202	182	935844	426	064156	13
48	815193	244	879093	182	936100	426	063900	12
49	815339	244	878984	182	936355	426	063645	11
50	815485	243	878875	182	936611	426	063389	10
51	9·815631	243	9·878766	182	9·936866	425	10·063134	9
52	815778	243	878656	182	937121	425	062879	8
53	815924	243	878547	182	937377	425	062623	7
54	816069	243	878438	182	937632	425	062368	6
55	816215	243	878328	182	937887	425	062113	5
56	816361	243	878219	183	938142	425	061858	4
57	816507	242	878109	183	938398	425	061602	3
58	816652	242	877999	183	938653	425	061347	2
59	816798	242	877890	183	938908	425	061092	1
60	816943	242	877780	183	939163	425	060837	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

130° | 49°

41° 138°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·816943	242	9·877780	183	9·939163	425	10·060837	60
1	817088	242	877670	183	939418	425	060582	59
2	817233	242	877560	183	939673	425	060327	58
3	817379	242	877450	183	939928	425	060072	57
4	817524	241	877340	183	940183	425	059817	56
5	817668	241	877230	184	940439	425	059561	55
6	817813	241	877120	184	940694	425	059306	54
7	817958	241	877010	184	940949	425	059051	53
8	818103	241	876899	184	941204	425	058796	52
9	818247	241	876789	184	941459	425	058541	51
10	818392	241	876678	184	941713	425	058287	50
11	9·818536	240	9·876568	184	9·941968	425	10·058032	49
12	818681	240	876457	184	942223	425	057777	48
13	818825	240	876347	184	942478	425	057522	47
14	818969	240	876236	185	942733	425	057267	46
15	819113	240	876125	185	942988	425	057012	45
16	819257	240	876014	185	943243	425	056757	44
17	819401	240	875904	185	943498	425	056502	43
18	819545	239	875793	185	943752	425	056248	42
19	819689	239	875682	185	944007	425	055993	41
20	819832	239	875571	185	944262	425	055738	40
21	9·819976	239	9·875459	185	9·944517	425	10·055483	39
22	820120	239	875348	185	944771	424	055229	38
23	820263	239	875237	185	945026	424	054974	37
24	820406	239	875126	186	945281	424	054719	36
25	820550	238	875014	186	945535	424	054465	35
26	820693	238	874903	186	945790	424	054210	34
27	820836	238	874791	186	946045	424	053955	33
28	820979	238	874680	186	946299	424	053701	32
29	821122	238	874568	186	946554	424	053446	31
30	821265	238	874456	186	946808	424	053192	30
31	9·821407	238	9·874344	186	9·947063	424	10·052937	29
32	821550	238	874232	187	947318	424	052682	28
33	821693	237	874121	187	947572	424	052428	27
34	821835	237	874009	187	947827	424	052173	26
35	821977	237	873896	187	948081	424	051919	25
36	822120	237	873784	187	948335	424	051665	24
37	822262	237	873672	187	948590	424	051410	23
38	822404	237	873560	187	948844	424	051156	22
39	822546	237	873448	187	949099	424	050901	21
40	822688	236	873335	187	949353	424	050647	20
41	9·822830	236	9·873223	187	9·949608	424	10·050392	19
42	822972	236	873110	188	949862	424	050138	18
43	823114	236	872998	188	950116	424	049884	17
44	823255	236	872885	188	950371	424	049629	16
45	823397	236	872772	188	950625	424	049375	15
46	823539	236	872659	188	950879	424	049121	14
47	823680	235	872547	188	951133	424	048867	13
48	823821	235	872434	188	951388	424	048612	12
49	823963	235	872321	188	951642	424	048358	11
50	824104	235	872208	188	951896	424	048104	10
51	9·824245	235	9·872095	189	9·952150	424	10·047850	9
52	824386	235	871981	189	952405	424	047595	8
53	824527	235	871868	189	952659	424	047341	7
54	824668	234	871755	189	952913	424	047087	6
55	824808	234	871641	189	953167	423	046833	5
56	824949	234	871528	189	953421	423	046579	4
57	825090	234	871414	189	953675	423	046325	3
58	825230	234	871301	189	953929	423	046071	2
59	825371	234	871187	189	954183	423	045817	1
60	825511	234	871073	190	954437	423	045563	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

131° 48°

42° 137°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·825511	234	9·871073	190	9·954437	423	10·045563	60
1	825651	233	870960	190	954691	423	045309	59
2	825791	233	870846	190	954946	423	045054	58
3	825931	233	870732	190	955200	423	044800	57
4	826071	233	870618	190	955454	423	044546	56
5	826211	233	870504	190	955708	423	044292	55
6	826351	233	870390	190	955961	423	044039	54
7	826491	233	870276	190	956215	423	043785	53
8	826631	233	870161	190	956469	423	043531	52
9	826770	232	870047	191	956723	423	043277	51
10	826910	232	869933	191	956977	423	043023	50
11	9·827049	232	9·869818	191	9·957231	423	10·042769	49
12	827189	232	869704	191	957485	423	042515	48
13	827328	232	869589	191	957739	423	042261	47
14	827467	232	869474	191	957993	423	042007	46
15	827606	232	869360	191	958247	423	041753	45
16	827745	232	869245	191	958500	423	041500	44
17	827884	231	869130	191	958754	423	041246	43
18	828023	231	869015	192	959008	423	040992	42
19	828162	231	868900	192	959262	423	040738	41
20	828301	231	868785	192	959516	423	040484	40
21	9·828439	231	9·868670	192	9·959769	423	10·040231	39
22	828578	231	868555	192	960023	423	039977	38
23	828716	231	868440	192	960277	423	039723	37
24	828855	230	868324	192	960530	423	039470	36
25	828993	230	868209	192	960784	423	039216	35
26	829131	230	868093	192	961038	423	038962	34
27	829269	230	867978	193	961292	423	038708	33
28	829407	230	867862	193	961545	423	038455	32
29	829545	230	867747	193	961799	423	038201	31
30	829683	230	867631	193	962052	423	037948	30
31	9·829821	229	9·867515	193	9·962306	423	10·037694	29
32	829959	229	867399	193	962560	423	037440	28
33	830097	229	867283	193	962813	423	037187	27
34	830234	229	867167	193	963067	423	036933	26
35	830372	229	867051	193	963320	423	036680	25
36	830509	229	866935	194	963574	423	036426	24
37	830646	229	866819	194	963828	423	036172	23
38	830784	229	866703	194	964081	423	035919	22
39	830921	228	866586	194	964335	423	035665	21
40	831058	228	866470	194	964588	422	035412	20
41	9·831195	228	9·866353	194	9·964842	422	10·035158	19
42	831332	228	866237	194	965095	422	034905	18
43	831469	228	866120	194	965349	422	034651	17
44	831606	228	866004	195	965602	422	034398	16
45	831742	228	865887	195	965855	422	034145	15
46	831879	228	865770	195	966109	422	033891	14
47	832015	227	865653	195	966362	422	033638	13
48	832152	227	865536	195	966616	422	033384	12
49	832288	227	865419	195	966869	422	033131	11
50	832425	227	865302	195	967123	422	032877	10
51	9·832561	227	9·865185	195	9·967376	422	10·032624	9
52	832697	227	865068	195	967629	422	032371	8
53	832833	227	864950	195	967883	422	032117	7
54	832969	226	864833	196	968136	422	031864	6
55	833105	226	864716	196	968389	422	031611	5
56	833241	226	864598	196	968643	422	031357	4
57	833377	226	864481	196	968896	422	031104	3
58	833512	226	864363	196	969149	422	030851	2
59	833648	226	864245	196	969403	422	030597	1
60	833783	226	864127	196	969656	422	030344	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

132° 47°

43° 136°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·833783	226	9·864127	196	9·969656	422	10·030344	60
1	833919	225	864010	196	969909	422	030091	59
2	834054	225	863892	197	970162	422	029838	58
3	834189	225	863774	197	970416	422	029584	57
4	834325	225	863656	197	970669	422	029331	56
5	834460	225	863538	197	970922	422	029078	55
6	834595	225	863419	197	971175	422	028825	54
7	834730	225	863301	197	971429	422	028571	53
8	834865	225	863183	197	971682	422	028318	52
9	834999	224	863064	197	971935	422	028065	51
10	835134	224	862946	198	972188	422	027812	50
11	9·835269	224	9·862827	198	9·972441	422	10·027559	49
12	835403	224	862709	198	972695	422	027305	48
13	835538	224	862590	198	972948	422	027052	47
14	835672	224	862471	198	973201	422	026799	46
15	835807	224	862353	198	973454	422	026546	45
16	835941	224	862234	198	973707	422	026293	44
17	836075	223	862115	198	973960	422	026040	43
18	836209	223	861996	198	974213	422	025787	42
19	836343	223	861877	198	974466	422	025534	41
20	836477	223	861758	199	974720	422	025280	40
21	9·836611	223	9·861638	199	9·974973	422	10·025027	39
22	836745	223	861519	199	975226	422	024774	38
23	836878	223	861400	199	975479	422	024521	37
24	837012	222	861280	199	975732	422	024268	36
25	837146	222	861161	199	975985	422	024015	35
26	837279	222	861041	199	976238	422	023762	34
27	837412	222	860922	199	976491	422	023509	33
28	837546	222	860802	199	976744	422	023256	32
29	837679	222	860682	200	976997	422	023003	31
30	837812	222	860562	200	977250	422	022750	30
31	9·837945	222	9·860442	200	9·977503	422	10·022497	29
32	838078	221	860322	200	977756	422	022244	28
33	838211	221	860202	200	978009	422	021991	27
34	838344	221	860082	200	978262	422	021738	26
35	838477	221	859962	200	978515	422	021485	25
36	838610	221	859842	200	978768	422	021232	24
37	838742	221	859721	201	979021	422	020979	23
38	838875	221	859601	201	979274	422	020726	22
39	839007	221	859480	201	979527	422	020473	21
40	839140	220	859360	201	979780	422	020220	20
41	9·839272	220	9·859239	201	9·980033	422	10·019967	19
42	839404	220	859119	201	980286	422	019714	18
43	839536	220	858998	201	980538	422	019462	17
44	839668	220	858877	201	980791	421	019209	16
45	839800	220	858756	202	981044	421	018956	15
46	839932	220	858635	202	981297	421	018703	14
47	840064	219	858514	202	981550	421	018450	13
48	840196	219	858393	202	981803	421	018197	12
49	840328	219	858272	202	982056	421	017944	11
50	840459	219	858151	202	982309	421	017691	10
51	9·840591	219	9·858029	202	9·982562	421	10·017438	9
52	840722	219	857908	202	982814	421	017186	8
53	840854	219	857786	202	983067	421	016933	7
54	840985	219	857665	203	983320	421	016680	6
55	841116	218	857543	203	983573	421	016427	5
56	841247	218	857422	203	983826	421	016174	4
57	841378	218	857300	203	984079	421	015921	3
58	841509	218	857178	203	984332	421	015668	2
59	841640	218	857056	203	984584	421	015416	1
60	841771	218	856934	203	984837	421	015163	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

133° 46°

44° 135°

′	Sine.	D.	Cosine.	D.	Tang.	D.	Cotang.	′
0	9·841771	218	9·856934	203	9·984837	421	10·015163	60
1	841902	218	856812	203	985090	421	014910	59
2	842033	218	856690	204	985343	421	014657	58
3	842163	217	856568	204	985596	421	014404	57
4	842294	217	856446	204	985848	421	014152	56
5	842424	217	856323	204	986101	421	013899	55
6	842555	217	856201	204	986354	421	013646	54
7	842685	217	856078	204	986607	421	013393	53
8	842815	217	855956	204	986860	421	013140	52
9	842946	217	855833	204	987112	421	012888	51
10	843076	217	855711	205	987365	421	012635	50
11	9·843206	216	9·855588	205	9·987618	421	10·012382	49
12	843336	216	855465	205	987871	421	012129	48
13	843466	216	855342	205	988123	421	011877	47
14	843595	216	855219	205	988376	421	011624	46
15	843725	216	855096	205	988629	421	011371	45
16	843855	216	854973	205	988882	421	011118	44
17	843984	216	854850	205	989134	421	010866	43
18	844114	215	854727	206	989387	421	010613	42
19	844243	215	854603	206	989640	421	010360	41
20	844372	215	854480	206	989893	421	010107	40
21	9·844502	215	9·854356	206	9·990145	421	10·009855	39
22	844631	215	854233	206	990398	421	009602	38
23	844760	215	854109	206	990651	421	009349	37
24	844889	215	853986	206	990903	421	009097	36
25	845018	215	853862	206	991156	421	008844	35
26	845147	215	853738	206	991409	421	008591	34
27	845276	214	853614	207	991662	421	008338	33
28	845405	214	853490	207	991914	421	008086	32
29	845533	214	853366	207	992167	421	007833	31
30	845662	214	853242	207	992420	421	007580	30
31	9·845790	214	9·853118	207	9·992672	421	10·007328	29
32	845919	214	852994	207	992925	421	007075	28
33	846047	214	852869	207	993178	421	006822	27
34	846175	214	852745	207	993431	421	006569	26
35	846304	214	852620	207	993683	421	006317	25
36	846432	213	852496	208	993936	421	006064	24
37	846560	213	852371	208	994189	421	005811	23
38	846688	213	852247	208	994441	421	005559	22
39	846816	213	852122	208	994694	421	005306	21
40	846944	213	851997	208	994947	421	005053	20
41	9·847071	213	9·851872	208	9·995199	421	10·004801	19
42	847199	213	851747	208	995452	421	004548	18
43	847327	213	851622	208	995705	421	004295	17
44	847454	212	851497	209	995957	421	004043	16
45	847582	212	851372	209	996210	421	003790	15
46	847709	212	851246	209	996463	421	003537	14
47	847836	212	851121	209	996715	421	003285	13
48	847964	212	850996	209	996968	421	003032	12
49	848091	212	850870	209	997221	421	002779	11
50	848218	212	850745	209	997473	421	002527	10
51	9·848345	212	9·850619	209	9·997726	421	10·002274	9
52	848472	211	850493	210	997979	421	002021	8
53	848599	211	850368	210	998231	421	001769	7
54	848726	211	850242	210	998484	421	001516	6
55	848852	211	850116	210	998737	421	001263	5
56	848979	211	849990	210	998989	421	001011	4
57	849106	211	849864	210	999242	421	000758	3
58	849232	211	849738	210	999495	421	000505	2
59	849359	211	849611	210	999747	421	000253	1
60	849485	211	849485	210	10·000000	421	10·000000	0
′	Cosine.	D.	Sine.	D.	Cotang.	D.	Tang.	′

134° 45°

TABLE III.,

OF

NATURAL SINES AND TANGENTS;

TO

EVERY DEGREE AND MINUTE OF THE QUADRANT.

If the given angle is less than 45°, look for the degrees and the title of the column, at the *top* of the page; and for the minutes on the *left*. But if the angle is between 45° and 90°, look for the degrees and the title of the column, at the *bottom*; and for the minutes on the *right*.

The *Secants and Cosecants*, which are not inserted in this table, may be easily supplied. If 1 be divided by the cosine of an arc, the quotient will be the secant of that arc. And if 1 be divided by the sine, the quotient will be the cosecant.

The values of the Sines and Cosines are less than a unit, and are given in decimals, although the decimal point is not printed. So also, the tangents of arcs less than 45°, and cotangents of arcs greater than 45°, are less than a unit and are expressed in decimals with the decimal point omitted.

′	0° Sine.	0° Cosine.	1° Sine.	1° Cosine.	2° Sine.	2° Cosine.	3° Sine.	3° Cosine.	4° Sine.	4° Cosine.	′
0	00000	Unit.	01745	99985	03490	99939	05234	99863	06976	99756	60
1	00029	Unit.	01774	99984	03519	99938	05263	99861	07005	99754	59
2	00058	Unit.	01803	99984	03548	99937	05292	99860	07034	99752	58
3	00087	Unit.	01832	99983	03577	99936	05321	99858	07063	99750	57
4	00116	Unit.	01862	99983	03606	99935	05350	99857	07092	99748	56
5	00145	Unit.	01891	99982	03635	99934	05379	99855	07121	99746	55
6	00175	Unit.	01920	99982	03664	99933	05408	99854	07150	99744	54
7	00204	Unit.	01949	99981	03693	99932	05437	99852	07179	99742	53
8	00233	Unit.	01978	99980	03723	99931	05466	99851	07208	99740	52
9	00262	Unit.	02007	99980	03752	99930	05495	99849	07237	99738	51
10	00291	Unit.	02036	99979	03781	99929	05524	99847	07266	99736	50
11	00320	99999	02065	99979	03810	99927	05553	99846	07295	99734	49
12	00349	99999	02094	99978	03839	99926	05582	99844	07324	99731	48
13	00378	99999	02123	99977	03868	99925	05611	99842	07353	99729	47
14	00407	99999	02152	99977	03897	99924	05640	99841	07382	99727	46
15	00436	99999	02181	99976	03926	99923	05669	99839	07411	99725	45
16	00465	99999	02211	99976	03955	99922	05698	99838	07440	99723	44
17	00495	99999	02240	99975	03984	99921	05727	99836	07469	99721	43
18	00524	99999	02269	99974	04013	99919	05756	99834	07498	99719	42
19	00553	99998	02298	99974	04042	99918	05785	99833	07527	99716	41
20	00582	99998	02327	99973	04071	99917	05814	99831	07556	99714	40
21	00611	99998	02356	99972	04100	99916	05844	99829	07585	99712	39
22	00640	99998	02385	99972	04129	99915	05873	99827	07614	99710	38
23	00669	99998	02414	99971	04159	99913	05902	99826	07643	99708	37
24	00698	99998	02443	99970	04188	99912	05931	99824	07672	99705	36
25	00727	99997	02472	99969	04217	99911	05960	99822	07701	99703	35
26	00756	99997	02501	99969	04246	99910	05989	99821	07730	99701	34
27	00785	99997	02530	99968	04275	99909	06018	99819	07759	99699	33
28	00814	99997	02560	99967	04304	99907	06047	99817	07788	99696	32
29	00844	99996	02589	99966	04333	99906	06076	99815	07817	99694	31
30	00873	99996	02618	99966	04362	99905	06105	99813	07846	99692	30
31	00902	99996	02647	99965	04391	99904	06134	99812	07875	99689	29
32	00931	99996	02676	99964	04420	99902	06163	99810	07904	99687	28
33	00960	99995	02705	99963	04449	99901	06192	99808	07933	99685	27
34	00989	99995	02734	99963	04478	99900	06221	99806	07962	99683	26
35	01018	99995	02763	99962	04507	99898	06250	99804	07991	99680	25
36	01047	99995	02792	99961	04536	99897	06279	99803	08020	99678	24
37	01076	99994	02821	99960	04565	99896	06308	99801	08049	99676	23
38	01105	99994	02850	99959	04594	99894	06337	99799	08078	99673	22
39	01134	99994	02879	99959	04623	99893	06366	99797	08107	99671	21
40	01164	99993	02908	99958	04653	99892	06395	99795	08136	99668	20
41	01193	99993	02938	99957	04682	99890	06424	99793	08165	99666	19
42	01222	99993	02967	99956	04711	99889	06453	99792	08194	99664	18
43	01251	99992	02996	99955	04740	99888	06482	99790	08223	99661	17
44	01280	99992	03025	99954	04769	99886	06511	99788	08252	99659	16
45	01309	99991	03054	99953	04798	99885	06540	99786	08281	99657	15
46	01338	99991	03083	99952	04827	99883	06569	99784	08310	99654	14
47	01367	99991	03112	99952	04856	99882	06598	99782	08339	99652	13
48	01396	99990	03141	99951	04885	99881	06627	99780	08368	99649	12
49	01425	99990	03170	99950	04914	99879	06656	99778	08397	99647	11
50	01454	99989	03199	99949	04943	99878	06685	99776	08426	99644	10
51	01483	99989	03228	99948	04972	99876	06714	99774	08455	99642	9
52	01513	99989	03257	99947	05001	99875	06743	99772	08484	99639	8
53	01542	99988	03286	99946	05030	99873	06773	99770	08513	99637	7
54	01571	99988	03316	99945	05059	99872	06802	99768	08542	99635	6
55	01600	99987	03345	99944	05088	99870	06831	99766	08571	99632	5
56	01629	99987	03374	99943	05117	99869	06860	99764	08600	99630	4
57	01658	99986	03403	99942	05146	99867	06889	99762	08629	99627	3
58	01687	99986	03432	99941	05175	99866	06918	99760	08658	99625	2
59	01716	99985	03461	99940	05205	99864	06947	99758	08687	99622	1
60	01745	99985	03490	99939	05234	99863	06976	99756	08716	99619	0
′	89° Cosine.	89° Sine.	88° Cosine.	88° Sine.	87° Cosine.	87° Sine.	86° Cosine.	86° Sine.	85° Cosine.	85° Sine.	′

′	5° Sine	5° Cosine.	6° Sine.	6° Cosine.	7° Sine.	7° Cosine.	8° Sine.	8° Cosine.	9° Sine.	9° Cosine.	′
0	08716	99619	10453	99452	12187	99255	13917	99027	15643	98769	60
1	08745	99617	10482	99449	12216	99251	13946	99023	15672	98764	59
2	08774	99614	10511	99446	12245	99248	13975	99019	15701	98760	58
3	08803	99612	10540	99443	12274	99244	14004	99015	15730	98755	57
4	08831	99609	10569	99440	12302	99240	14033	99011	15758	98751	56
5	08860	99607	10597	99437	12331	99237	14061	99006	15787	98746	55
6	08889	99604	10626	99434	12360	99233	14090	99002	15816	98741	54
7	08918	99602	10655	99431	12389	99230	14119	98998	15845	98737	53
8	08947	99599	10684	99428	12418	99226	14148	98994	15873	98732	52
9	08976	99596	10713	99424	12447	99222	14177	98990	15902	98728	51
10	09005	99594	10742	99421	12476	99219	14205	98986	15931	98723	50
11	09034	99591	10771	99418	12504	99215	14234	98982	15959	98718	49
12	09063	99588	10800	99415	12533	99211	14263	98978	15988	98714	48
13	09092	99586	10829	99412	12562	99208	14292	98973	16017	98709	47
14	09121	99583	10858	99409	12591	99204	14320	98969	16046	98704	46
15	09150	99580	10887	99406	12620	99200	14349	98965	16074	98700	45
16	09179	99578	10916	99402	12649	99197	14378	98961	16103	98695	44
17	09208	99575	10945	99399	12678	99193	14407	98957	16132	98690	43
18	09237	99572	10973	99396	12706	99189	14436	98953	16160	98686	42
19	09266	99570	11002	99393	12735	99186	14464	98948	16189	98681	41
20	09295	99567	11031	99390	12764	99182	14493	98944	16218	98676	40
21	09324	99564	11060	99386	12793	99178	14522	98940	16246	98671	39
22	09353	99562	11089	99383	12822	99175	14551	98936	16275	98667	38
23	09382	99559	11118	99380	12851	99171	14580	98931	16304	98662	37
24	09411	99556	11147	99377	12880	99167	14608	98927	16333	98657	36
25	09440	99553	11176	99374	12908	99163	14637	98923	16361	98652	35
26	09469	99551	11205	99370	12937	99160	14666	98919	16390	98648	34
27	09498	99548	11234	99367	12966	99156	14695	98914	16419	98643	33
28	09527	99545	11263	99364	12995	99152	14723	98910	16447	98638	32
29	09556	99542	11291	99360	13024	99148	14752	98906	16476	98633	31
30	09585	99540	11320	99357	13053	99144	14781	98902	16505	98629	30
31	09614	99537	11349	99354	13081	99141	14810	98897	16533	98624	29
32	09642	99534	11378	99351	13110	99137	14838	98893	16562	98619	28
33	09671	99531	11407	99347	13139	99133	14867	98889	16591	98614	27
34	09700	99528	11436	99344	13168	99129	14896	98884	16620	98609	26
35	09729	99526	11465	99341	13197	99125	14925	98880	16648	98604	25
36	09758	99523	11494	99337	13226	99122	14954	98876	16677	98600	24
37	09787	99520	11523	99334	13254	99118	14982	98871	16706	98595	23
38	09816	99517	11552	99331	13283	99114	15011	98867	16734	98590	22
39	09845	99514	11580	99327	13312	99110	15040	98863	16763	98585	21
40	09874	99511	11609	69324	13341	99106	15069	98858	16792	98580	20
41	09903	99508	11638	99320	13370	99102	15097	98854	16820	98575	19
42	09932	99506	11667	99317	13399	99098	15126	98849	16849	98570	18
43	09961	99503	11696	99314	13427	99094	15155	98845	16878	98565	17
44	09990	99500	11725	99310	13456	99091	15184	98841	16906	98561	16
45	10019	99497	11754	99307	13485	99087	15212	98836	16935	98556	15
46	10048	99494	11783	99303	13514	99083	15241	98832	16964	98551	14
47	10077	99491	11812	99300	13543	99079	15270	98827	16992	98546	13
48	10106	99488	11840	99297	13572	99075	15299	98823	17021	98541	12
49	10135	99485	11869	99293	13600	99071	15327	98818	17050	98536	11
50	10164	99482	11898	99290	13629	99067	15356	98814	17078	98531	10
51	10192	99479	11927	99286	13658	99063	15385	98809	17107	98526	9
52	10221	99476	11956	99283	13687	99059	15414	98805	17136	98521	8
53	10250	99473	11985	99279	13716	99055	15442	98800	17164	98516	7
54	10279	99470	12014	99276	13744	99051	15471	98796	17193	98511	6
55	10308	99467	12043	99272	13773	99047	15500	98791	17222	98506	5
56	10337	99464	12071	99269	13802	99043	15529	98787	17250	98501	4
57	10366	99461	12100	99265	13831	99039	15557	98782	17279	98496	3
58	10395	99458	12129	99262	13860	99035	15586	98778	17308	98491	2
59	10424	99455	12158	99258	13889	99031	15615	98773	17336	98486	1
60	10453	99452	12187	99255	13917	99027	15643	98769	17365	98481	0
′	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	17365	98481	19081	98163	20791	97815	22495	97437	24192	97030	60
1	17393	98476	19109	98157	20820	97809	22523	97430	24220	97023	59
2	17422	98471	19138	98152	20848	97803	22552	97424	24249	97015	58
3	17451	98466	19167	98146	20877	97797	22580	97417	24277	97008	57
4	17479	98461	19195	98140	20905	97791	22608	97411	24305	97001	56
5	17508	98455	19224	98135	20933	97784	22637	97404	24333	96994	55
6	17537	98450	19252	98129	20962	97778	22665	97398	24362	96987	54
7	17565	98445	19281	98124	20990	97772	22693	97391	24390	96980	53
8	17594	98440	19309	98118	21019	97766	22722	97384	24418	96973	52
9	17623	98435	19338	98112	21047	97760	22750	97378	24446	96966	51
10	17651	98430	19366	98107	21076	97754	22778	97371	24474	96959	50
11	17680	98425	19395	98101	21104	97748	22807	97365	24503	96952	49
12	17708	98420	19423	98096	21132	97742	22835	97358	24531	96945	48
13	17737	98414	19452	98090	21161	97735	22863	97351	24559	96937	47
14	17766	98409	19481	98084	21189	97729	22892	97345	24587	96930	46
15	17794	98404	19509	98079	21218	97723	22920	97338	24615	96923	45
16	17823	98399	19538	98073	21246	97717	22948	97331	24644	96916	44
17	17852	98394	19566	98067	21275	97711	22977	97325	24672	96909	43
18	17880	98389	19595	98061	21303	97705	23005	97318	24700	96902	42
19	17909	98383	19623	98056	21331	97698	23033	97311	24728	96894	41
20	17937	98378	19652	98050	21360	97692	23062	97304	24756	96887	40
21	17966	98373	19680	98044	21388	97686	23090	97298	24784	96880	39
22	17995	98368	19709	98039	21417	97680	23118	97291	24813	96873	38
23	18023	98362	19737	98033	21445	97673	23146	97284	24841	96866	37
24	18052	98357	19766	98027	21474	97667	23175	97278	24869	96858	36
25	18081	98352	19794	98021	21502	97661	23203	97271	24897	96851	35
26	18100	98347	19823	98016	21530	97655	23231	97264	24925	96844	34
27	18138	98341	19851	98010	21559	97648	23260	97257	24954	96837	33
28	18166	98336	19880	98004	21587	97642	23288	97251	24982	96829	32
29	18195	98331	19908	97998	21616	97636	23316	97244	25010	96822	31
30	18224	98325	19937	97992	21644	97630	23345	97237	25038	96815	30
31	18252	98320	19965	97987	21672	97623	23373	97230	25066	96807	29
32	18281	98315	19994	97981	21701	97617	23401	97223	25094	96800	28
33	18309	98310	20022	97975	21729	97611	23429	97217	25122	96793	27
34	18338	98304	20051	97969	21758	97604	23458	97210	25151	96786	26
35	18367	98299	20079	97963	21786	97598	23486	97203	25179	96778	25
36	18395	98294	20108	97958	21814	97592	23514	97196	25207	96771	24
37	18424	98288	20136	97952	21843	97585	23542	97189	25235	96764	23
38	18452	98283	20165	97946	21871	97579	23571	97182	25263	96756	22
39	18481	98277	20193	97940	21899	97573	23599	97176	25291	96749	21
40	18509	98272	20222	97934	21928	97566	23627	97169	25320	96742	20
41	18538	98267	20250	97928	21956	97560	23656	97162	25348	96734	19
42	18567	98261	20279	97922	21985	97553	23684	97155	25376	96727	18
43	18595	98256	20307	97916	22013	97547	23712	97148	25404	96719	17
44	18624	98250	20336	97910	22041	97541	23740	97141	25432	96712	16
45	18652	98245	20364	97905	22070	97534	23769	97134	25460	96705	15
46	18681	98240	20393	97899	22098	97528	23797	97127	25488	96697	14
47	18710	98234	20421	97893	22126	97521	23825	97120	25516	96690	13
48	18738	98229	20450	97887	22155	97515	23853	97113	25545	96682	12
49	18767	98223	20478	97881	22183	97508	23882	97106	25573	96675	11
50	18795	98218	20507	97875	22212	97502	23910	97100	25601	96667	10
51	18824	98212	20535	97869	22240	97496	23938	97093	25629	96660	9
52	18852	98207	20563	97863	22268	97489	23966	97086	25657	96653	8
53	18881	98201	20592	97857	22297	97483	23995	97079	25685	96645	7
54	18910	98196	20620	97851	22325	97476	24023	97072	25713	96638	6
55	18938	98190	20649	97845	22353	97470	24051	97065	25741	96630	5
56	18967	98185	20677	97839	22382	97463	24079	97058	25769	96623	4
57	18995	9817[illegible]	20706	97833	22410	97457	24108	97051	25798	96615	3
58	19024	98174	20734	97827	22438	97450	24136	97044	25826	96608	2
59	19052	98168	20763	97821	22467	97444	24164	97037	25854	96600	1
60	19081	98163	20791	97815	22495	97437	24192	97030	25882	96593	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	79°		78°		77°		76°		75°		′

′	15° Sine.	15° Cosine.	16° Sine.	16° Cosine.	17° Sine.	17° Cosine.	18° Sine.	18° Cosine.	19° Sine.	19° Cosine.	′
0	25882	96593	27564	96126	29237	95630	30902	95106	32557	94552	60
1	25910	96585	27592	96118	29265	95622	30929	95097	32584	94542	59
2	25938	96578	27620	96110	29293	95613	30957	95088	32612	94533	58
3	25966	96570	27648	96102	29321	95605	30985	95079	32639	94523	57
4	25994	96562	27676	96094	29348	95596	31012	95070	32667	94514	56
5	26022	96555	27704	96086	29376	95588	31040	95061	32694	94504	55
6	26050	96547	27731	96078	29404	95579	31068	95052	32722	94495	54
7	26079	96540	27759	96070	29432	95571	31095	95043	32749	94485	53
8	26107	96532	27787	96062	29460	95562	31123	95033	32777	94476	52
9	26135	96524	27815	96054	29487	95554	31151	95024	32804	94466	51
10	26163	96517	27843	96046	29515	95545	31178	95015	32832	94457	50
11	26191	96509	27871	96037	29543	95536	31206	95006	32859	94447	49
12	26219	96502	27899	96029	29571	95528	31233	94997	32887	94438	48
13	26247	96494	27927	96021	29599	95519	31261	94988	32914	94428	47
14	26275	96486	27955	96013	29626	95511	31289	94979	32942	94418	46
15	26303	96479	27983	96005	29654	95502	31316	94970	32969	94409	45
16	26331	96471	28011	95997	29682	95493	31344	94961	32997	94399	44
17	26359	96463	28039	95989	29710	95485	31372	94952	33024	94390	43
18	26387	96456	28067	95981	29737	95476	31399	94943	33051	94380	42
19	26415	96448	28095	95972	29765	95467	31427	94933	33079	94370	41
20	26443	96440	28123	95964	29793	95459	31454	94924	33106	94361	40
21	26471	96433	28150	95956	29821	95450	31482	94915	33134	94351	39
22	26500	96425	28178	95948	29849	95441	31510	94906	33161	94342	38
23	26528	96417	28206	95940	29876	95433	31537	94897	33189	94332	37
24	26556	96410	28234	95931	29904	95424	31565	94888	33216	94322	36
25	26584	96402	28262	95923	29932	95415	31593	94878	33244	94313	35
26	26612	96394	28290	95915	29960	95407	31620	94869	33271	94303	34
27	26640	96386	28318	95907	29987	95398	31648	94860	33298	94293	33
28	26668	96379	28346	95898	30015	95389	31675	94851	33326	94284	32
29	26696	96371	28374	95890	30043	95380	31703	94842	33353	94274	31
30	26724	96363	28402	95882	30071	95372	31730	94832	33381	94264	30
31	26752	96355	28429	95874	30098	95363	31758	94823	33408	94254	29
32	26780	96347	28457	95865	30126	95354	31786	94814	33436	94245	28
33	26808	96340	28485	95857	30154	95345	31813	94805	33463	94235	27
34	26836	96332	28513	95849	30182	95337	31841	94795	33490	94225	26
35	26864	96324	28541	95841	30209	95328	31868	94786	33518	94215	25
36	26892	96316	28569	95832	30237	95319	31896	94777	33545	94206	24
37	26920	96308	28597	95824	30265	95310	31923	94768	33573	94196	23
38	26948	96301	28625	95816	30292	95301	31951	94758	33600	94186	22
39	26976	96293	28652	95807	30320	95293	31979	94749	33627	94176	21
40	27004	96285	28680	95799	30348	95284	32006	94740	33655	94167	20
41	27032	96277	28708	95791	30376	95275	32034	94730	33682	94157	19
42	27060	96269	28736	95782	30403	95266	32061	94721	33710	94147	18
43	27088	96261	28764	95774	30431	95257	32089	94712	33737	94137	17
44	27116	96253	28792	95766	30459	95248	32116	94702	33764	94127	16
45	27144	96246	28820	95757	30486	95240	32144	94693	33792	94118	15
46	27172	96238	28847	95749	30514	95231	32171	94684	33819	94108	14
47	27200	96230	28875	95740	30542	95222	32199	94674	33846	94098	13
48	27228	96222	28903	95732	30570	95213	32227	94665	33874	94088	12
49	27256	96214	28931	95724	30597	95204	32254	94656	33901	94078	11
50	27284	96206	28959	95715	30625	95195	32282	94646	33929	94068	10
51	27312	96198	28987	95707	30653	95186	32309	94637	33956	94058	9
52	27340	96190	29015	95698	30680	95177	32337	94627	33983	94049	8
53	27368	96182	29042	95690	30708	95168	32364	94618	34011	94039	7
54	27396	96174	29070	95681	30736	95159	32392	94609	34038	94029	6
55	27424	96166	29098	95673	30763	95150	32419	94599	34065	94019	5
56	27452	96158	29126	95664	30791	95142	32447	94590	34093	94009	4
57	27480	96150	29154	95656	30819	95133	32474	94580	34120	93999	3
58	27508	96142	29182	95647	30846	95124	32502	94571	34147	93989	2
59	27536	96134	29209	95639	30874	95115	32529	94561	34175	93979	1
60	27564	96126	29237	95630	30902	95106	32557	94552	34202	93969	0
′	Cosine. 74°	Sine 74°	Cosine. 73°	Sine. 73°	Cosine. 72°	Sine. 72°	Cosine. 71°	Sine. 71°	Cosine. 70°	Sine. 70°	′

′	20°		21°		22°		23°		24°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	34202	93969	35837	93358	37461	92718	39073	92050	40674	91355	60
1	34229	93959	35864	93348	37488	92707	39100	92039	40700	91343	59
2	34257	93949	35891	93337	37515	92697	39127	92028	40727	91331	58
3	34284	93939	35918	93327	37542	92686	39153	92016	40753	91319	57
4	34311	93929	35945	93316	37569	92675	39180	92005	40780	91307	56
5	34339	93919	35973	93306	37595	92664	39207	91994	40806	91295	55
6	34366	93909	36000	93295	37622	92653	39234	91982	40833	91283	54
7	34393	93899	36027	93285	37649	92642	39260	91971	40860	91272	53
8	34421	93889	36054	93274	37676	92631	39287	91959	40886	91260	52
9	34448	93879	36081	93264	37703	92620	39314	91948	40913	91248	51
10	34475	93869	36108	93253	37730	92609	39341	91936	40939	91236	50
11	34503	93859	36135	93243	37757	92598	39367	91925	40966	91224	49
12	34530	93849	36162	93232	37784	92587	39394	91914	40992	91212	48
13	34557	93839	36190	93222	37811	92576	39421	91902	41019	91200	47
14	34584	93829	36217	93211	37838	92565	39448	91891	41045	91188	46
15	34612	93819	36244	93201	37865	92554	39474	91879	41072	91176	45
16	34639	93809	36271	93190	37892	92543	39501	91868	41098	91164	44
17	34666	93799	36298	93180	37919	92532	39528	91856	41125	91152	43
18	34694	93789	36325	93169	37946	92521	39555	91845	41151	91140	42
19	34721	93779	36352	93159	37973	92510	39581	91833	41178	91128	41
20	34748	93769	36379	93148	37999	92499	39608	91822	41204	91116	40
21	34775	93759	36406	93137	38026	92488	39635	91810	41231	91104	39
22	34803	93748	36434	93127	38053	92477	39661	91799	41257	91092	38
23	34830	93738	36461	93116	38080	92466	39688	91787	41284	91080	37
24	34857	93728	36488	93106	38107	92455	39715	91775	41310	91068	36
25	34884	93718	36515	93095	38134	92444	39741	91764	41337	91056	35
26	34912	93708	36542	93084	38161	92432	39768	91752	41363	91044	34
27	34939	93698	36569	93074	38188	92421	39795	91741	41390	91032	33
28	34966	93688	36596	93063	38215	92410	39822	91729	41416	91020	32
29	34993	93677	36623	93052	38241	92399	39848	91718	41443	91008	31
30	35021	93667	36650	93042	38268	92388	39875	91706	41469	90996	30
31	35048	93657	36677	93031	38295	92377	39902	91694	41496	90984	29
32	35075	93647	36704	93020	38322	92366	39928	91683	41522	90972	28
33	35102	93637	36731	93010	38349	92355	39955	91671	41549	90960	27
34	35130	93626	36758	92999	38376	92343	39982	91660	41575	90948	26
35	35157	93616	36785	92988	38403	92332	40008	91648	41602	90936	25
36	35184	93606	36812	92978	38430	92321	40035	91636	41628	90924	24
37	35211	93596	36839	92967	38456	92310	40062	91625	41655	90911	23
38	35239	93585	36867	92956	38483	92299	40088	91613	41681	90899	22
39	35266	93575	36894	92945	38510	92287	40115	91601	41707	90887	21
40	35293	93565	36921	92935	38537	92276	40141	91590	41734	90875	20
41	35320	93555	36948	92924	38564	92265	40168	91578	41760	90863	19
42	35347	93544	36975	92913	38591	92254	40195	91566	41787	90851	18
43	35375	93534	37002	92902	38617	92243	40221	91555	41813	90839	17
44	35402	93524	37029	92892	38644	92231	40248	91543	41840	90826	16
45	35429	93514	37056	92881	38671	92220	40275	91531	41866	90814	15
46	35456	93503	37083	92870	38698	92209	40301	91519	41892	90802	14
47	35484	93493	37110	92859	38725	92198	40328	91508	41919	90790	13
48	35511	93483	37137	92849	38752	92186	40355	91496	41945	90778	12
49	35538	93472	37164	92838	38778	92175	40381	91484	41972	90766	11
50	35565	93462	37191	92827	38805	92164	40408	91472	41998	90753	10
51	35592	93452	37218	92816	38832	92152	40434	91461	42024	90741	9
52	35619	93441	37245	92805	38859	92141	40461	91449	42051	90729	8
53	35647	93431	37272	92794	38886	92130	40488	91437	42077	90717	7
54	35674	93420	37299	92784	38912	92119	40514	91425	42104	90704	6
55	35701	93410	37326	92773	38939	92107	40541	91414	42130	90692	5
56	35728	93400	37353	92762	38966	92096	40567	91402	42156	90680	4
57	35755	93389	37380	92751	38993	92085	40594	91390	42183	90668	3
58	35782	93379	37407	92740	39020	92073	40621	91378	42209	90655	2
59	35810	93368	37434	92729	39046	92062	40647	91366	42235	90643	1
60	35837	93358	37461	92718	39073	92050	40674	91355	42262	90631	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	69°		68°		67°		66°		65°		′

′	25°		26°		27°		28°		29°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	42262	90631	43837	89879	45399	89101	46947	88295	48481	87462	60
1	42288	90618	43863	89867	45425	89087	46973	88281	48506	87448	59
2	42315	90606	43889	89854	45451	89074	46999	88267	48532	87434	58
3	42341	90594	43916	89841	45477	89061	47024	88254	48557	87420	57
4	42367	90582	43942	89828	45503	89048	47050	88240	48583	87406	56
5	42394	90569	43968	89816	45529	89035	47076	88226	48608	87391	55
6	42420	90557	43994	89803	45554	89021	47101	88213	48634	87377	54
7	42446	90545	44020	89790	45580	89008	47127	88199	48659	87363	53
8	42473	90532	44046	89777	45606	88995	47153	88185	48684	87349	52
9	42499	90520	44072	89764	45632	88981	47178	88172	48710	87335	51
10	42525	90507	44098	89752	45658	88968	47204	88158	48735	87321	50
11	42552	90495	44124	89739	45684	88955	47229	88144	48761	87306	49
12	42578	90483	44151	89726	45710	88942	47255	88130	48786	87292	48
13	42604	90470	44177	89713	45736	88928	47281	88117	48811	87278	47
14	42631	90458	44203	89700	45762	88915	47306	88103	48837	87264	46
15	42657	90446	44229	89687	45787	88902	47332	88089	48862	87250	45
16	42683	90433	44255	89674	45813	88888	47358	88075	48888	87235	44
17	42709	90421	44281	89662	45839	88875	47383	88062	48913	87221	43
18	42736	90408	44307	89649	45865	88862	47409	88048	48938	87207	42
19	42762	90396	44333	89636	45891	88848	47434	88034	48964	87193	41
20	42788	90383	44359	89623	45917	88835	47460	88020	48989	87178	40
21	42815	90371	44385	89610	45942	88822	47486	88006	49014	87164	39
22	42841	90358	44411	89597	45968	88808	47511	87993	49040	87150	38
23	42867	90346	44437	89584	45994	88795	47537	87979	49065	87136	37
24	42894	90334	44464	89571	46020	88782	47562	87965	49090	87121	36
25	42920	90321	44490	89558	46046	88768	47588	87951	49116	87107	35
26	42946	90309	44516	89545	46072	88755	47614	87937	49141	87093	34
27	42972	90296	44542	89532	46097	88741	47639	87923	49166	87079	33
28	42999	90284	44568	89519	46123	88728	47665	87909	49192	87064	32
29	43025	90271	44594	89506	46149	88715	47690	87896	49217	87050	31
30	43051	90259	44620	89493	46175	88701	47716	87882	49242	87036	30
31	43077	90246	44646	89480	46201	88688	47741	87868	49268	87021	29
32	43104	90233	44672	89467	46226	88674	47767	87854	49293	87007	28
33	43130	90221	44698	89454	46252	88661	47793	87840	49318	86993	27
34	43156	90208	44724	89441	46278	88647	47818	87826	49344	86978	26
35	43182	90196	44750	89428	46304	88634	47844	87812	49369	86964	25
36	43209	90183	44776	89415	46330	88620	47869	87798	49394	86949	24
37	43235	90171	44802	89402	46355	88607	47895	87784	49419	86935	23
38	43261	90158	44828	89389	46381	88593	47920	87770	49445	86921	22
39	43287	90146	44854	89376	46407	88580	47946	87756	49470	86906	21
40	43313	90133	44880	89363	46433	88566	47971	87743	49495	86892	20
41	43340	90120	44906	89350	46458	88553	47997	87729	49521	86878	19
42	43366	90108	44932	89337	46484	88539	48022	87715	49546	86863	18
43	43392	90095	44958	89324	46510	88526	48048	87701	49571	86849	17
44	43418	90082	44984	89311	46536	88512	48073	87687	49596	86834	16
45	43445	90070	45010	89298	46561	88499	48099	87673	49622	86820	15
46	43471	90057	45036	89285	46587	88485	48124	87659	49647	86805	14
47	43497	90045	45062	89272	46613	88472	48150	87645	49672	86791	13
48	43523	90032	45088	89259	46639	88458	48175	87631	49697	86777	12
49	43549	90019	45114	89245	46664	88445	48201	87617	49723	86762	11
50	43575	90007	45140	89232	46690	88431	48226	87603	49748	86748	10
51	43602	89994	45166	89219	46716	88417	48252	87589	49773	86733	9
52	43628	89981	45192	89206	46742	88404	48277	87575	49798	86719	8
53	43654	89968	45218	89193	46767	88390	48303	87561	49824	86704	7
54	43680	89956	45243	89180	46793	88377	48328	87546	49849	86690	6
55	43706	89943	45269	89167	46819	88363	48354	87532	49874	86675	5
56	43733	89930	45295	89153	46844	88349	48379	87518	49899	86661	4
57	43759	89918	45321	89140	46870	88336	48405	87504	49924	86646	3
58	43785	89905	45347	89127	46896	88322	48430	87490	49950	86632	2
59	43811	89892	45373	89114	46921	88308	48456	87476	49975	86617	1
60	43837	89879	45399	89101	46947	88295	48481	87462	50000	86603	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	64°		63°		62°		61°		60°		′

′	30°		31°		32°		33°		34°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	50000	86603	51504	85717	52992	84805	54464	83867	55919	82904	60
1	50025	86588	51529	85702	53017	84789	54488	83851	55943	82887	59
2	50050	86573	51554	85687	53041	84774	54513	83835	55968	82871	58
3	50076	86559	51579	85672	53066	84759	54537	83819	55992	82855	57
4	50101	86544	51604	85657	53091	84743	54561	83804	56016	82839	56
5	50126	86530	51628	85642	53115	84728	54586	83788	56040	82822	55
6	50151	86515	51653	85627	53140	84712	54610	83772	56064	82806	54
7	50176	86501	51678	85612	53164	84697	54635	83756	56088	82790	53
8	50201	86486	51703	85597	53189	84681	54659	83740	56112	82773	52
9	50227	86471	51728	85582	53214	84666	54683	83724	56136	82757	51
10	50252	86457	51753	85567	53238	84650	54708	83708	56160	82741	50
11	50277	86442	51778	85551	53263	84635	54732	83692	56184	82724	49
12	50302	86427	51803	85536	53288	84619	54756	83676	56208	82708	48
13	50327	86413	51828	85521	53312	84604	54781	83660	56232	82692	47
14	50352	86398	51852	85506	53337	84588	54805	83645	56256	82675	46
15	50377	86384	51877	85491	53361	84573	54829	83629	56280	82659	45
16	50403	86369	51902	85476	53386	84557	54854	83613	56305	82643	44
17	50428	86354	51927	85461	53411	84542	54878	83597	56329	82626	43
18	50453	86340	51952	85446	53435	84526	54902	83581	56353	82610	42
19	50478	86325	51977	85431	53460	84511	54927	83565	56377	82593	41
20	50503	86310	52002	85416	53484	84495	54951	83549	56401	82577	40
21	50528	86295	52026	85401	53509	84480	54975	83533	56425	82561	39
22	50553	86281	52051	85385	53534	84464	54999	83517	56449	82544	38
23	50578	86266	52076	85370	53558	84448	55024	83501	56473	82528	37
24	50603	86251	52101	85355	53583	84433	55048	83485	56497	82511	36
25	50628	86237	52126	85340	53607	84417	55072	83469	56521	82495	35
26	50654	86222	52151	85325	53632	84402	55097	83453	56545	82478	34
27	50679	86207	52175	85310	53656	84386	55121	83437	56569	82462	33
28	50704	86192	52200	85294	53681	84370	55145	83421	56593	82446	32
29	50729	86178	52225	85279	53705	84355	55169	83405	56617	82429	31
30	50754	86163	52250	85264	53730	84339	55194	83389	56641	82413	30
31	50779	86148	52275	85249	53754	84324	55218	83373	56665	82396	29
32	50804	86133	52299	85234	53779	84308	55242	83356	56689	82380	28
33	50829	86119	52324	85218	53804	84292	55266	83340	56713	82363	27
34	50854	86104	52349	85203	53828	84277	55291	83324	56736	82347	26
35	50879	86089	52374	85188	53853	84261	55315	83308	56760	82330	25
36	50904	86074	52399	85173	53877	84245	55339	83292	56784	82314	24
37	50929	86059	52423	85157	53902	84230	55363	83276	56808	82297	23
38	50954	86045	52448	85142	53926	84214	55388	83260	56832	82281	22
39	50979	86030	52473	85127	53951	84198	55412	83244	56856	82264	21
40	51004	86015	52498	85112	53975	84182	55436	83228	56880	82248	20
41	51029	86000	52522	85096	54000	84167	55460	83212	56904	82231	19
42	51054	85985	52547	85081	54024	84151	55484	83195	56928	82214	18
43	51079	85970	52572	85066	54049	84135	55509	83179	56952	82198	17
44	51104	85956	52597	85051	54073	84120	55533	83163	56976	82181	16
45	51129	85941	52621	85035	54097	84104	55557	83147	57000	82165	15
46	51154	85926	52646	85020	54122	84088	55581	83131	57024	82148	14
47	51179	85911	52671	85005	54146	84072	55605	83115	57047	82132	13
48	51204	85896	52696	84989	54171	84057	55630	83098	57071	82115	12
49	51229	85881	52720	84974	54195	84041	55654	83082	57095	82098	11
50	51254	85866	52745	84959	54220	84025	55678	83066	57119	82082	10
51	51279	85851	52770	84943	54244	84009	55702	83050	57143	82065	9
52	51304	85836	52794	84928	54269	83994	55726	83034	57167	82048	8
53	51329	85821	52819	84913	54293	83978	55750	83017	57191	82032	7
54	51354	85806	52844	84897	54317	83962	55775	83001	57215	82015	6
55	51379	85792	52869	84882	54342	83946	55799	82985	57238	81999	5
56	51404	85777	52893	84866	54366	83930	55823	82969	57262	81982	4
57	51429	85762	52918	84851	54391	83915	55847	82953	57286	81965	3
58	51454	85747	52943	84836	54415	83899	55871	82936	57310	81949	2
59	51479	85732	52967	84820	54440	83883	55895	82920	57334	81932	1
60	51504	85717	52992	84805	54464	83867	55919	82904	57358	81915	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	59°		58°		57°		56°		55°		′

′	35°		36°		37°		38°		39°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	57358	81915	58779	80902	60182	79864	61566	78801	62932	77715	60
1	57381	81899	58802	80885	60205	79846	61589	78783	62955	77696	59
2	57405	81882	58826	80867	60228	79829	61612	78765	62977	77678	58
3	57429	81865	58849	80850	60251	79811	61635	78747	63000	77660	57
4	57453	81848	58873	80833	60274	79793	61658	78729	63022	77641	56
5	57477	81832	58896	80816	60298	79776	61681	78711	63045	77623	55
6	57501	81815	58920	80799	60321	79758	61704	78694	63068	77605	54
7	57524	81798	58943	80782	60344	79741	61726	78676	63090	77586	53
8	57548	81782	58967	80765	60367	79723	61749	78658	63113	77568	52
9	57572	81765	58990	80748	60390	79706	61772	78640	63135	77550	51
10	57596	81748	59014	80730	60414	79688	61795	78622	63158	77531	50
11	57619	81731	59037	80713	60437	79671	61818	78604	63180	77513	49
12	57643	81714	59061	80696	60460	79653	61841	78586	63203	77494	48
13	57667	81698	59084	80679	60483	79635	61864	78568	63225	77476	47
14	57691	81681	59108	80662	60506	79618	61887	78550	63248	77458	46
15	57715	81664	59131	80644	60529	79600	61909	78532	63271	77439	45
16	57738	81647	59154	80627	60553	79583	61932	78514	63293	77421	44
17	57762	81631	59178	80610	60576	79565	61955	78496	63316	77402	43
18	57786	81614	59201	80593	60599	79547	61978	78478	63338	77384	42
19	57810	81597	59225	80576	60622	79530	62001	78460	63361	77366	41
20	57833	81580	59248	80558	60645	79512	62024	78442	63383	77347	40
21	57857	81563	59272	80541	60668	79494	62046	78424	63406	77329	39
22	57881	81546	59295	80524	60691	79477	62069	78405	63428	77310	38
23	57904	81530	59318	80507	60714	79459	62092	78387	63451	77292	37
24	57928	81513	59342	80489	60738	79441	62115	78369	63473	77273	36
25	57952	81496	59365	80472	60761	79424	62138	78351	63496	77255	35
26	57976	81479	59389	80455	60784	79406	62160	78333	63518	77236	34
27	57999	81462	59412	80438	60807	79388	62183	78315	63540	77218	33
28	58023	81445	59436	80420	60830	79371	62206	78297	63563	77199	32
29	58047	81428	59459	80403	60853	79353	62229	78279	63585	77181	31
30	58070	81412	59482	80386	60876	79335	62251	78261	63608	77162	30
31	58094	81395	59506	80368	60899	79318	62274	78243	63630	77144	29
32	58118	81378	59529	80351	60922	79300	62297	78225	63653	77125	28
33	58141	81361	59552	80334	60945	79282	62320	78206	63675	77107	27
34	58165	81344	59576	80316	60968	79264	62342	78188	63698	77088	26
35	58189	81327	59599	80299	60991	79247	62365	78170	63720	77070	25
36	58212	81310	59622	80282	61015	79229	62388	78152	63742	77051	24
37	58236	81293	59646	80264	61038	79211	62411	78134	63765	77033	23
38	58260	81276	59669	80247	61061	79193	62433	78116	63787	77014	22
39	58283	81259	59693	80230	61084	79176	62456	78098	63810	76996	21
40	58307	81242	59716	80212	61107	79158	62479	78079	63832	76977	20
41	58330	81225	59739	80195	61130	79140	62502	78061	63854	76959	19
42	58354	81208	59763	80178	61153	79122	62524	78043	63877	76940	18
43	58378	81191	59786	80160	61176	79105	62547	78025	63899	76921	17
44	58401	81174	59809	80143	61199	79087	62570	78007	63922	76903	16
45	58425	81157	59832	80125	61222	79069	62592	77988	63944	76884	15
46	58449	81140	59856	80108	61245	79051	62615	77970	63966	76866	14
47	58472	81123	59879	80091	61268	79033	62638	77952	63989	76847	13
48	58496	81106	59902	80073	61291	79016	62660	77934	64011	76828	12
49	58519	81089	59926	80056	61314	78998	62683	77916	64033	76810	11
50	58543	81072	59949	80038	61337	78980	62706	77897	64056	76791	10
51	58567	81055	59972	80021	61360	78962	62728	77879	64078	76772	9
52	58590	81038	59995	80003	61383	78944	62751	77861	64100	76754	8
53	58614	81021	60019	79986	61406	78926	62774	77843	64123	76735	7
54	58637	81004	60042	79968	61429	78908	62796	77824	64145	76717	6
55	58661	80987	60065	79951	61451	78891	62819	77806	64167	76698	5
56	58684	80970	60089	79934	61474	78873	62842	77788	64190	76679	4
57	58708	80953	60112	79916	61497	78855	62864	77769	64212	76661	3
58	58731	80936	60135	79899	61520	78837	62887	77751	64234	76642	2
59	58755	80919	60158	79881	61543	78819	62909	77733	64256	76623	1
60	58779	80902	60182	79864	61566	78801	62932	77715	64279	76604	0
	Cosine	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	54°		53°		52°		51°		50°		′

′	40°		41°		42°		43°		44°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	64279	76604	65606	75471	66913	74314	68200	73135	69466	71934	60
1	64301	76586	65628	75452	66935	74295	68221	73116	69487	71914	59
2	64323	76567	65650	75433	66956	74276	68242	73096	69508	71894	58
3	64346	76548	65672	75414	66978	74256	68264	73076	69529	71873	57
4	64368	76530	65694	75395	66999	74237	68285	73056	69549	71853	56
5	64390	76511	65716	75375	67021	74217	68306	73036	69570	71833	55
6	64412	76492	65738	75356	67043	74198	68327	73016	69591	71813	54
7	64435	76473	65759	75337	67064	74178	68349	72996	69612	71792	53
8	64457	76455	65781	75318	67086	74159	68370	72976	69633	71772	52
9	64479	76436	65803	75299	67107	74139	68391	72957	69654	71752	51
10	64501	76417	65825	75280	67129	74120	68412	72937	69675	71732	50
11	64524	76398	65847	75261	67151	74100	68434	72917	69696	71711	49
12	64546	76380	65869	75241	67172	74080	68455	72897	69717	71691	48
13	64568	76361	65891	75222	67194	74061	68476	72877	69737	71671	47
14	64590	76342	65913	75203	67215	74041	68497	72857	69758	71650	46
15	64612	76323	65935	75184	67237	74022	68518	72837	69779	71630	45
16	64635	76304	65956	75165	67258	74002	68539	72817	69800	71610	44
17	64657	76286	65978	75146	67280	73983	68561	72797	69821	71590	43
18	64679	76267	66000	75126	67301	73963	68582	72777	69842	71569	42
19	64701	76248	66022	75107	67323	73944	68603	72757	69862	71549	41
20	64723	76229	66044	75088	67344	73924	68624	72737	69883	71529	40
21	64746	76210	66066	75069	67366	73904	68645	72717	69904	71508	39
22	64768	76192	66088	75050	67387	73885	68666	72697	69925	71488	38
23	64790	76173	66109	75030	67409	73865	68688	72677	69946	71468	37
24	64812	76154	66131	75011	67430	73846	68709	72657	69966	71447	36
25	64834	76135	66153	74992	67452	73826	68730	72637	69987	71427	35
26	64856	76116	66175	74973	67473	73806	68751	72617	70008	71407	34
27	64878	76097	66197	74953	67495	73787	68772	72597	70029	71386	33
28	64901	76078	66218	74934	67516	73767	68793	72577	70049	71366	32
29	64923	76059	66240	74915	67538	73747	68814	72557	70070	71345	31
30	64945	76041	66262	74896	67559	73728	68835	72537	70091	71325	30
31	64967	76022	66284	74876	67580	73708	68857	72517	70112	71305	29
32	64989	76003	66306	74857	67602	73688	68878	72497	70132	71284	28
33	65011	75984	66327	74838	67623	73669	68899	72477	70153	71264	27
34	65033	75965	66349	74818	67645	73649	68920	72457	70174	71243	26
35	65055	75946	66371	74799	67666	73629	68941	72437	70195	71223	25
36	65077	75927	66393	74780	67688	73610	68962	72417	70215	71203	24
37	65100	75908	66414	74760	67709	73590	68983	72397	70236	71182	23
38	65122	75889	66436	74741	67730	73570	69004	72377	70257	71162	22
39	65144	75870	66458	74722	67752	73551	69025	72357	70277	71141	21
40	65166	75851	66480	74703	67773	73531	69046	72337	70298	71121	20
41	65188	75832	66501	74683	67795	73511	69067	72317	70319	71100	19
42	65210	75813	66523	74664	67816	73491	69088	72297	70339	71080	18
43	65232	75794	66545	74644	67837	73472	69109	72277	70360	71059	17
44	65254	75775	66566	74625	67859	73452	69130	72257	70381	71039	16
45	65276	75756	66588	74606	67880	73432	69151	72236	70401	71019	15
46	65298	75738	66610	74586	67901	73413	69172	72216	70422	70998	14
47	65320	75719	66632	74567	67923	73393	69193	72196	70443	70978	13
48	65342	75700	66653	74548	67944	73373	69214	72176	70463	70957	12
49	65364	75680	66675	74528	67965	73353	69235	72156	70484	70937	11
50	65386	75661	66697	74509	67987	73333	69256	72136	70505	70916	10
51	65408	75642	66718	74489	68008	73314	69277	72116	70525	70896	9
52	65430	75623	66740	74470	68029	73294	69298	72095	70546	70875	8
53	65452	75604	66762	74451	68051	73274	69319	72075	70567	70855	7
54	65474	75585	66783	74431	68072	73254	69340	72055	70587	70834	6
55	65496	75566	66805	74412	68093	73234	69361	72035	70608	70813	5
56	65518	75547	66827	74392	68115	73215	69382	72015	70628	70793	4
57	65540	75528	66848	74373	68136	73195	69403	71995	70649	70772	3
58	65562	75509	66870	74353	68157	73175	69424	71974	70670	70752	2
59	65584	75490	66891	74334	68179	73155	69445	71954	70690	70731	1
60	65606	75471	66913	74314	68200	73135	69466	71934	70711	70711	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	49°		48°		47°		46°		45°		′

′	0° Tangent.	0° Cotang.	1° Tangent.	1° Cotang.	2° Tangent.	2° Cotang.	3° Tangent.	3° Cotang.	′
0	00000	Infinite.	01746	57·2900	03492	28·6363	05241	19·0811	60
1	00029	3437·75	01775	56·3506	03521	28·3994	05270	18·9755	59
2	00058	1718·87	01804	55·4415	03550	28·1664	05299	18·8711	58
3	00087	1145·92	01833	54·5613	03579	27·9372	05328	18·7678	57
4	00116	859·436	01862	53·7086	03609	27·7117	05357	18·6656	56
5	00145	687·549	01891	52·8821	03638	27·4899	05387	18·5645	55
6	00175	572·957	01920	52·0807	03667	27·2715	05416	18·4645	54
7	00204	491·106	01949	51·3032	03696	27·0566	05445	18·3655	53
8	00233	429·718	01978	50·5485	03725	26·8450	05474	18·2677	52
9	00262	381·971	02007	49·8157	03754	26·6367	05503	18·1708	51
10	00291	343·774	02036	49·1039	03783	26·4316	05533	18·0750	50
11	00320	312·521	02066	48·4121	03812	26·2296	05562	17·9802	49
12	00349	286·478	02095	47·7395	03842	26·0307	05591	17·8863	48
13	00378	264·441	02124	47·0853	03871	25·8348	05620	17·7934	47
14	00407	245·552	02153	46·4489	03900	25·6418	05649	17·7015	46
15	00436	229·182	02182	45·8294	03929	25·4517	05678	17·6106	45
16	00465	214·858	02211	45·2261	03958	25·2644	05708	17·5205	44
17	00495	202·219	02240	44·6386	03987	25·0798	05737	17·4314	43
18	00524	190·984	02269	44·0661	04016	24·8978	05766	17·3432	42
19	00553	180·932	02298	43·5081	04046	24·7185	05795	17·2558	41
20	00582	171·885	02328	42·9641	04075	24·5418	05824	17·1693	40
21	00611	163·700	02357	42·4335	04104	24·3675	05854	17·0837	39
22	00640	156·259	02386	41·9158	04133	24·1957	05883	16·9990	38
23	00669	149·465	02415	41·4106	04162	24·0263	05912	16·9150	37
24	00698	143·237	02444	40·9174	04191	23·8593	05941	16·8319	36
25	00727	137·507	02473	40·4358	04220	23·6945	05970	16·7496	35
26	00756	132·219	02502	39·9655	04250	23·5321	05999	16·6681	34
27	00785	127·321	02531	39·5059	04279	23·3718	06029	16·5874	33
28	00814	122·774	02560	39·0568	04308	23·2137	06058	16·5075	32
29	00844	118·540	02589	38·6177	04337	23·0577	06087	16·4283	31
30	00873	114·589	02619	38·1885	04366	22·9038	06116	16·3499	30
31	00902	110·892	02648	37·7686	04395	22·7519	06145	16·2722	29
32	00931	107·426	02677	37·3579	04424	22·6020	06175	16·1952	28
33	00960	104·171	02706	36·9560	04454	22·4541	06204	16·1190	27
34	00989	101·107	02735	36·5627	04483	22·3081	06233	16·0435	26
35	01018	98·2179	02764	36·1776	04512	22·1640	06262	15·9687	25
36	01047	95·4895	02793	35·8006	04541	22·0217	06291	15·8945	24
37	01076	92·9085	02822	35·4313	04570	21·8813	06321	15·8211	23
38	01105	90·4633	02851	35·0695	04599	21·7426	06350	15·7483	22
39	01135	88·1436	02881	34·7151	04628	21·6056	06379	15·6762	21
40	01164	85·9398	02910	34·3678	04658	21·4704	06408	15·6048	20
41	01193	83·8435	02939	34·0273	04687	21·3369	06437	15·5340	19
42	01222	81·8470	02968	33·6935	04716	21·2049	06467	15·4638	18
43	01251	79·9434	02997	33·3662	04745	21·0747	06496	15·3943	17
44	01280	78·1263	03026	33·0452	04774	20·9460	06525	15·3254	16
45	01309	76·3900	03055	32·7303	04803	20·8188	06554	15·2571	15
46	01338	74·7292	03084	32·4213	04832	20·6932	06584	15·1893	14
47	01367	73·1390	03114	32·1181	04862	20·5691	06613	15·1222	13
48	01396	71·6151	03143	31·8205	04891	20·4465	06642	15·0557	12
49	01425	70·1533	03172	31·5284	04920	20·3253	06671	14·9898	11
50	01455	68·7501	03201	31·2416	04949	20·2056	06700	14·9244	10
51	01484	67·4019	03230	30·9599	04978	20·0872	06730	14·8596	9
52	01513	66·1055	03259	30·6833	05007	19·9702	06759	14·7954	8
53	01542	64·8580	03288	30·4116	05037	19·8546	06788	14·7317	7
54	01571	63·6567	03317	30·1446	05066	19·7403	06817	14·6685	6
55	01600	62·4992	03346	29·8823	05095	19·6273	06847	14·6059	5
56	01629	61·3829	03376	29·6245	05124	19·5156	06876	14·5438	4
57	01658	60·3058	03405	29·3711	05153	19·4051	06905	14·4823	3
58	01687	59·2659	03434	29·1220	05182	19·2959	06934	14·4212	2
59	01716	58·2612	03463	28·8771	05212	19·1879	06963	14·3607	1
60	01746	57·2900	03492	28·6363	05241	19·0811	06993	14·3007	0
′	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	′
	89°		88°		87°		86°		

′	4°		5°		6°		7°		′
	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	
0	06993	14·3007	08749	11·4301	10510	9·51436	12278	8·14435	60
1	07022	14·2411	08778	11·3919	10540	9·48781	12308	8·12481	59
2	07051	14·1821	08807	11·3540	10569	9·46141	12338	8·10536	58
3	07080	14·1235	08837	11·3163	10599	9·43515	12367	8·08600	57
4	07110	14·0655	08866	11·2789	10628	9·40904	12397	8·06674	56
5	07139	14·0079	08895	11·2417	10657	9·38307	12426	8·04756	55
6	07168	13·9507	08925	11·2048	10687	9·35724	12456	8·02848	54
7	07197	13·8940	08954	11·1681	10716	9·33154	12485	8·00948	53
8	07227	13·8378	08983	11·1316	10746	9·30599	12515	7·99058	52
9	07256	13·7821	09013	11·0954	10775	9·28058	12544	7·97176	51
10	07285	13·7267	09042	11·0594	10805	9·25530	12574	7·95302	50
11	07314	13·6719	09071	11·0237	10834	9·23016	12603	7·93438	49
12	07344	13·6174	09101	10·9882	10863	9·20516	12633	7·91582	48
13	07373	13·5634	09130	10·9529	10893	9·18028	12662	7·89734	47
14	07402	13·5098	09159	10·9178	10922	9·15554	12692	7·87895	46
15	07431	13·4566	09189	10·8829	10952	9·13093	12722	7·86064	45
16	07461	13·4039	09218	10·8483	10981	9·10646	12751	7·84242	44
17	07490	13·3515	09247	10·8139	11011	9·08211	12781	7·82428	43
18	07519	13·2996	09277	10·7797	11040	9·05789	12810	7·80622	42
19	07548	13·2480	09306	10·7457	11070	9·03379	12840	7·78825	41
20	07578	13·1969	09335	10·7119	11099	9·00983	12869	7·77035	40
21	07607	13·1461	09365	10·6783	11128	8·98598	12899	7·75254	39
22	07636	13·0958	09394	10·6450	11158	8·96227	12929	7·73480	38
23	07665	13·0458	09423	10·6118	11187	8·93867	12958	7·71715	37
24	07695	12·9962	09453	10·5789	11217	8·91520	12988	7·69957	36
25	07724	12·9469	09482	10·5462	11246	8·89185	13017	7·68208	35
26	07753	12·8981	09511	10·5136	11276	8·86862	13047	7·66466	34
27	07782	12·8496	09541	10·4813	11305	8·84551	13076	7·64732	33
28	07812	12·8014	09570	10·4491	11335	8·82252	13106	7·63005	32
29	07841	12·7536	09600	10·4172	11364	8·79964	13136	7·61287	31
30	07870	12·7062	09629	10·3854	11394	8·77689	13165	7·59575	30
31	07899	12·6591	09658	10·3538	11423	8·75425	13195	7·57872	29
32	07929	12·6124	09688	10·3224	11452	8·73172	13224	7·56176	28
33	07958	12·5660	09717	10·2913	11482	8·70931	13254	7·54487	27
34	07987	12·5199	09746	10·2602	11511	8·68701	13284	7·52806	26
35	08017	12·4742	09776	10·2294	11541	8·66482	13313	7·51132	25
36	08046	12·4288	09805	10·1988	11570	8·64275	13343	7·49465	24
37	08075	12·3838	09834	10·1683	11600	8·62078	13372	7·47806	23
38	08104	12·3390	09864	10·1381	11629	8·59893	13402	7·46154	22
39	08134	12·2946	09893	10·1080	11659	8·57718	13432	7·44509	21
40	08163	12·2505	09923	10·0780	11688	8·55555	13461	7·42871	20
41	08192	12·2067	09952	10·0483	11718	8·53402	13491	7·41240	19
42	08221	12·1632	09981	10·0187	11747	8·51259	13521	7·39616	18
43	08251	12·1201	10011	9·98930	11777	8·49128	13550	7·37999	17
44	08280	12·0772	10040	9·96007	11806	8·47007	13580	7·36389	16
45	08309	12·0346	10069	9·93101	11836	8·44896	13609	7·34786	15
46	08339	11·9923	10099	9·90211	11865	8·42795	13639	7·33190	14
47	08368	11·9504	10128	9·87338	11895	8·40705	13669	7·31600	13
48	08397	11·9087	10158	9·84482	11924	8·38625	13698	7·30018	12
49	08427	11·8673	10187	9·81641	11954	8·36555	13728	7·28442	11
50	08456	11·8262	10216	9·78817	11983	8·34496	13758	7·26873	10
51	08485	11·7853	10246	9·76009	12013	8·32446	13787	7·25310	9
52	08514	11·7448	10275	9·73217	12042	8·30406	13817	7·23754	8
53	08544	11·7045	10305	9·70441	12072	8·28376	13846	7·22204	7
54	08573	11·6645	10334	9·67680	12101	8·26355	13876	7·20661	6
55	08602	11·6248	10363	9·64935	12131	8·24345	13906	7·19125	5
56	08632	11·5853	10393	9·62205	12160	8·22344	13935	7·17594	4
57	08661	11·5461	10422	9·59490	12190	8·20352	13965	7·16071	3
58	08690	11·5072	10452	9·56791	12219	8·18370	13995	7·14553	2
59	08720	11·4685	10481	9·54106	12249	8·16398	14024	7·13042	1
60	08749	11·4301	10510	9·51436	12278	8·14435	14054	7·11537	0
	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	
′	85°		84°		83°		82°		′

′	8°		9°		10°		11°		′
	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	
0	14054	7·11537	15838	6·31375	17633	5·67128	19438	5·14455	60
1	14084	7·10038	15868	6·30189	17663	5·66165	19468	5·13658	59
2	14113	7·08546	15898	6·29007	17693	5·65205	19498	5·12862	58
3	14143	7·07059	15928	6·27829	17723	5·64248	19529	5·12069	57
4	14173	7·05579	15958	6·26655	17753	5·63295	19559	5·11279	56
5	14202	7·04105	15988	6·25486	17783	5·62344	19589	5·10490	55
6	14232	7·02637	16017	6·24321	17813	5·61397	19619	5·09704	54
7	14262	7·01174	16047	6·23160	17843	5·60452	19649	5·08921	53
8	14291	6·99718	16077	6·22003	17873	5·59511	19680	5·08139	52
9	14321	6·98268	16107	6·20851	17903	5·58573	19710	5·07360	51
10	14351	6·96823	16137	6·19703	17933	5·57638	19740	5·06584	50
11	14381	6·95385	16167	6·18559	17963	5·56706	19770	5·05809	49
12	14410	6·93952	16196	6·17419	17993	5·55777	19801	5·05037	48
13	14440	6·92525	16226	6·16283	18023	5·54851	19831	5·04267	47
14	14470	6·91104	16256	6·15151	18053	5·53927	19861	5·03499	46
15	14499	6·89688	16286	6·14023	18083	5·53007	19891	5·02734	45
16	14529	6·88278	16316	6·12899	18113	5·52090	19921	5·01971	44
17	14559	6·86874	16346	6·11779	18143	5·51176	19952	5·01210	43
18	14588	6·85475	16376	6·10664	18173	5·50264	19982	5·00451	42
19	14618	6·84082	16405	6·09552	18203	5·49356	20012	4·99695	41
20	14648	6·82694	16435	6·08444	18233	5·48451	20042	4·98940	40
21	14678	6·81312	16465	6·07340	18263	5·47548	20073	4·98188	39
22	14707	6·79936	16495	6·06240	18293	5·46648	20103	4·97438	38
23	14737	6·78564	16525	6·05143	18323	5·45751	20133	4·96690	37
24	14767	6·77199	16555	6·04051	18353	5·44857	20164	4·95945	36
25	14796	6·75838	16585	6·02962	18383	5·43966	20194	4·95201	35
26	14826	6·74483	16615	6·01878	18414	5·43077	20224	4·94460	34
27	14856	6·73133	16645	6·00797	18444	5·42192	20254	4·93721	33
28	14886	6·71789	16674	5·99720	18474	5·41309	20285	4·92984	32
29	14915	6·70450	16704	5·98646	18504	5·40429	20315	4·92249	31
30	14945	6·69116	16734	5·97576	18534	5·39552	20345	4·91516	30
31	14975	6·67787	16764	5·96510	18564	5·38677	20376	4·90785	29
32	15005	6·66463	16794	5·95448	18594	5·37805	20406	4·90056	28
33	15034	6·65144	16824	5·94390	18624	5·36936	20436	4·89330	27
34	15064	6·63831	16854	5·93335	18654	5·36070	20466	4·88605	26
35	15094	6·62523	16884	5·92283	18684	5·35206	20497	4·87882	25
36	15124	6·61219	16914	5·91235	18714	5·34345	20527	4·87162	24
37	15153	6·59921	16944	5·90191	18745	5·33487	20557	4·86444	23
38	15183	6·58627	16974	5·89151	18775	5·32631	20588	4·85727	22
39	15213	6·57339	17004	5·88114	18805	5·31778	20618	4·85013	21
40	15243	6·56055	17033	5·87080	18835	5·30928	20648	4·84300	20
41	15272	6·54777	17063	5·86051	18865	5·30080	20679	4·83590	19
42	15302	6·53503	17093	5·85024	18895	5·29235	20709	4·82882	18
43	15332	6·52234	17123	5·84001	18925	5·28393	20739	4·82175	17
44	15362	6·50970	17153	5·82982	18955	5·27553	20770	4·81471	16
45	15391	6·49710	17183	5·81966	18986	5·26715	20800	4·80769	15
46	15421	6·48456	17213	5·80953	19016	5·25880	20830	4·80068	14
47	15451	6·47206	17243	5·79944	19046	5·25048	20861	4·79370	13
48	15481	6·45961	17273	5·78938	19076	5·24218	20891	4·78673	12
49	15511	6·44720	17303	5·77936	19106	5·23391	20921	4·77978	11
50	15540	6·43484	17333	5·76937	19136	5·22566	20952	4·77286	10
51	15570	6·42253	17363	5·75941	19166	5·21744	20982	4·76595	9
52	15600	6·41026	17393	5·74949	19197	5·20925	21013	4·75906	8
53	15630	6·39804	17423	5·73960	19227	5·20107	21043	4·75219	7
54	15660	6·38587	17453	5·72974	19257	5·19293	21073	4·74534	6
55	15689	6·37374	17483	5·71992	19287	5·18480	21104	4·73851	5
56	15719	6·36165	17513	5·71013	19317	5·17671	21134	4·73170	4
57	15749	6·34961	17543	5·70037	19347	5·16863	21164	4·72490	3
58	15779	6·33761	17573	5·69064	19378	5·16058	21195	4·71813	2
59	15809	6·32566	17603	5·68094	19408	5·15256	21225	4·71137	1
60	15838	6·31375	17633	5·67128	19438	5·14455	21256	4·70463	0
	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	
′	81°		80°		79°		78°		′

′	12°		13°		14°		15°		′
	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	
0	21256	4·70463	23087	4·33148	24933	4·01078	26795	3·73205	60
1	21286	4·69791	23117	4·32573	24964	4·00582	26826	3·72771	59
2	21316	4·69121	23148	4·32001	24995	4·00086	26857	3·72338	58
3	21347	4·68452	23179	4·31430	25026	3·99592	26888	3·71907	57
4	21377	4·67786	23209	4·30860	25056	3·99099	26920	3·71476	56
5	21408	4·67121	23240	4·30291	25087	3·98607	26951	3·71046	55
6	21438	4·66458	23271	4·29724	25118	3·98117	26982	3·70616	54
7	21469	4·65797	23301	4·29159	25149	3·97627	27013	3·70188	53
8	21499	4·65138	23332	4·28595	25180	3·97139	27044	3·69761	52
9	21529	4·64480	23363	4·28032	25211	3·96651	27076	3·69335	51
10	21560	4·63825	23393	4·27471	25242	3·96165	27107	3·68909	50
11	21590	4·63171	23424	4·26911	25273	3·95680	27138	3·68485	49
12	21621	4·62518	23455	4·26352	25304	3·95196	27169	3·68061	48
13	21651	4·61868	23485	4·25795	25335	3·94713	27201	3·67638	47
14	21682	4·61219	23516	4·25239	25366	3·94232	27232	3·67217	46
15	21712	4·60572	23547	4·24685	25397	3·93751	27263	3·66796	45
16	21743	4·59927	23578	4·24132	25428	3·93271	27294	3·66376	44
17	21773	4·59283	23608	4·23580	25459	3·92793	27326	3·65957	43
18	21804	4·58641	23639	4·23030	25490	3·92316	27357	3·65538	42
19	21834	4·58001	23670	4·22481	25521	3·91839	27388	3·65121	41
20	21864	4·57363	23700	4·21933	25552	3·91364	27419	3·64705	40
21	21895	4·56726	23731	4·21387	25583	3·90890	27451	3·64289	39
22	21925	4·56091	23762	4·20842	25614	3·90417	27482	3·63874	38
23	21956	4·55458	23793	4·20298	25645	3·89945	27513	3·63461	37
24	21986	4·54826	23823	4·19756	25676	3·89474	27545	3·63048	36
25	22017	4·54196	23854	4·19215	25707	3·89004	27576	3·62636	35
26	22047	4·53568	23885	4·18675	25738	3·88536	27607	3·62224	34
27	22078	4·52941	23916	4·18137	25769	3·88068	27638	3·61814	33
28	22108	4·52316	23946	4·17600	25800	3·87601	27670	3·61405	32
29	22139	4·51693	23977	4·17064	25831	3·87136	27701	3·60996	31
30	22169	4·51071	24008	4·16530	25862	3·86671	27732	3·60588	30
31	22200	4·50451	24039	4·15997	25893	3·86208	27764	3·60181	29
32	22231	4·49832	24069	4·15465	25924	3·85745	27795	3·59775	28
33	22261	4·49215	24100	4·14934	25955	3·85284	27826	3·59370	27
34	22292	4·48600	24131	4·14405	25986	3·84824	27858	3·58966	26
35	22322	4·47986	24162	4·13877	26017	3·84364	27889	3·58562	25
36	22353	4·47374	24193	4·13350	26048	3·83906	27920	3·58160	24
37	22383	4·46764	24223	4·12825	26079	3·83449	27952	3·57758	23
38	22414	4·46155	24254	4·12301	26110	3·82992	27983	3·57357	22
39	22444	4·45548	24285	4·11778	26141	3·82537	28015	3·56957	21
40	22475	4·44942	24316	4·11256	26172	3·82083	28046	3·56557	20
41	22505	4·44338	24347	4·10736	26203	3·81630	28077	3·56159	19
42	22536	4·43735	24377	4·10216	26235	3·81177	28109	3·55761	18
43	22567	4·43134	24408	4·09699	26266	3·80726	28140	3·55364	17
44	22597	4·42534	24439	4·09182	26297	3·80276	28172	3·54968	16
45	22628	4·41936	24470	4·08666	26328	3·79827	28203	3·54573	15
46	22658	4·41340	24501	4·08152	26359	3·79378	28234	3·54179	14
47	22689	4·40745	24532	4·07639	26390	3·78931	28266	3·53785	13
48	22719	4·40152	24562	4·07127	26421	3·78485	28297	3·53393	12
49	22750	4·39560	24593	4·06616	26452	3·78040	28329	3·53001	11
50	22781	4·38969	24624	4·06107	26483	3·77595	28360	3·52609	10
51	22811	4·38381	24655	4·05599	26515	3·77152	28391	3·52219	9
52	22842	4·37793	24686	4·05092	26546	3·76709	28423	3·51829	8
53	22872	4·37207	24717	4·04586	26577	3·76268	28454	3·51441	7
54	22903	4·36623	24747	4·04081	26608	3·75828	28486	3·51053	6
55	22934	4·36040	24778	4·03578	26639	3·75388	28517	3·50666	5
56	22964	4·35459	24809	4·03075	26670	3·74950	28549	3·50279	4
57	22995	4·34879	24840	4·02574	26701	3·74512	28580	3·49894	3
58	23026	4·34300	24871	4·02074	26733	3·74075	28612	3·49509	2
59	23056	4·33723	24902	4·01576	26764	3·73640	28643	3·49125	1
60	23087	4·33148	24933	4·01078	26795	3·73205	28675	3·48741	0
′	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	′
	77°		76°		75°		74°		

′	16°		17°		18°		19°		′
	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	
0	28675	3·48741	30573	3·27085	32492	3·07768	34433	2·90421	60
1	28706	3·48359	30605	3·26745	32524	3·07464	34465	2·90147	59
2	28738	3·47977	30637	3·26406	32556	3·07160	34498	2·89873	58
3	28769	3·47596	30669	3·26067	32588	3·06857	34530	2·89600	57
4	28800	3·47216	30700	3·25729	32621	3·06554	34563	2·89327	56
5	28832	3·46837	30732	3·25392	32653	3·06252	34596	2·89055	55
6	28864	3·46458	30764	3·25055	32685	3·05950	34628	2·88783	54
7	28895	3·46080	30796	3·24719	32717	3·05649	34661	2·88511	53
8	28927	3·45703	30828	3·24383	32749	3·05349	34693	2·88240	52
9	28958	3·45327	30860	3·24049	32782	3·05049	34726	2·87970	51
10	28990	3·44951	30891	3·23714	32814	3·04749	34758	2·87700	50
11	29021	3·44576	30923	3·23381	32846	3·04450	34791	2·87430	49
12	29053	3·44202	30955	3·23048	32878	3·04152	34824	2·87161	48
13	29084	3·43829	30987	3·22715	32911	3·03854	34856	2·86892	47
14	29116	3·43456	31019	3·22384	32943	3·03556	34889	2·86624	46
15	29147	3·43084	31051	3·22053	32975	3·03260	34922	2·86356	45
16	29179	3·42713	31083	3·21722	33007	3·02963	34954	2·86089	44
17	29210	3·42343	31115	3·21392	33040	3·02667	34987	2·85822	43
18	29242	3·41973	31147	3·21063	33072	3·02372	35019	2·85555	42
19	29274	3·41604	31178	3·20734	33104	3·02077	35052	2·85289	41
20	29305	3·41236	31210	3·20406	33136	3·01783	35085	2·85023	40
21	29337	3·40869	31242	3·20079	33169	3·01489	35117	2·84758	39
22	29368	3·40502	31274	3·19752	33201	3·01196	35150	2·84494	38
23	29400	3·40136	31306	3·19426	33233	3·00903	35183	2·84229	37
24	29432	3·39771	31338	3·19100	33266	3·00611	35216	2·83965	36
25	29463	3·39406	31370	3·18775	33298	3·00319	35248	2·83702	35
26	29495	3·39042	31402	3·18451	33330	3·00028	35281	2·83439	34
27	29526	3·38679	31434	3·18127	33363	2·99738	35314	2·83176	33
28	29558	3·38317	31466	3·17804	33395	2·99447	35346	2·82914	32
29	29590	3·37955	31498	3·17481	33427	2·99158	35379	2·82653	31
30	29621	3·37594	31530	3·17159	33460	2·98868	35412	2·82391	30
31	29653	3·37234	31562	3·16838	33492	2·98580	35445	2·82130	29
32	29685	3·36875	31594	3·16517	33524	2·98292	35477	2·81870	28
33	29716	3·36516	31626	3·16197	33557	2·98004	35510	2·81610	27
34	29748	3·36158	31658	3·15877	33589	2·97717	35543	2·81350	26
35	29780	3·35800	31690	3·15558	33621	2·97430	35576	2·81091	25
36	29811	3·35443	31722	3·15240	33654	2·97144	35608	2·80833	24
37	29843	3·35087	31754	3·14922	33686	2·96858	35641	2·80574	23
38	29875	3·34732	31786	3·14605	33718	2·96573	35674	2·80316	22
39	29906	3·34377	31818	3·14288	33751	2·96288	35707	2·80059	21
40	29938	3·34023	31850	3·13972	33783	2·96004	35740	2·79802	20
41	29970	3·33670	31882	3·13656	33816	2·95721	35772	2·79545	19
42	30001	3·33317	31914	3·13341	33848	2·95437	35805	2·79289	18
43	30033	3·32965	31946	3·13027	33881	2·95155	35838	2·79033	17
44	30065	3·32614	31978	3·12713	33913	2·94872	35871	2·78778	16
45	30097	3·32264	32010	3·12400	33945	2·94590	35904	2·78523	15
46	30128	3·31914	32042	3·12087	33978	2·94309	35937	2·78269	14
47	30160	3·31565	32074	3·11775	34010	2·94028	35969	2·78014	13
48	30192	3·31216	32106	3·11464	34043	2·93748	36002	2·77761	12
49	30224	3·30868	32139	3·11153	34075	2·93468	36035	2·77507	11
50	30255	3·30521	32171	3·10842	34108	2·93189	36068	2·77254	10
51	30287	3·30174	32203	3·10532	34140	2·92910	36101	2·77002	9
52	30319	3·29829	32235	3·10223	34173	2·92632	36134	2·76750	8
53	30351	3·29483	32267	3·09914	34205	2·92354	36167	2·76498	7
54	30382	3·29139	32299	3·09606	34238	2·92076	36199	2·76247	6
55	30414	3·28795	32331	3·09298	34270	2·91799	36232	2·75996	5
56	30446	3·28452	32363	3·08991	34303	2·91523	36265	2·75746	4
57	30478	3·28109	32396	3·08685	34335	2·91246	36298	2·75496	3
58	30509	3·27767	32428	3·08379	34368	2·90971	36331	2·75246	2
59	30541	3·27426	32460	3·08073	34400	2·90696	36364	2·74997	1
60	30573	3·27085	32492	3·07768	34433	2·90421	36397	2·74748	0
′	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	′
	73°		72°		71°		70°		

′	20°		21°		22°		23°		′
	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	
0	36397	2·74748	38386	2·60509	40403	2·47509	42447	2·35585	60
1	36430	2·74499	38420	2·60283	40436	2·47302	42482	2·35395	59
2	36463	2·74251	38453	2·60057	40470	2·47095	42516	2·35205	58
3	36496	2·74004	38487	2·59831	40504	2·46888	42551	2·35015	57
4	36529	2·73756	38520	2·59606	40538	2·46682	42585	2·34825	56
5	36562	2·73509	38553	2·59381	40572	2·46476	42619	2·34636	55
6	36595	2·73263	38587	2·59156	40606	2·46270	42654	2·34447	54
7	36628	2·73017	38620	2·58932	40640	2·46065	42688	2·34258	53
8	36661	2·72771	38654	2·58708	40674	2·45860	42722	2·34069	52
9	36694	2·72526	38687	2·58484	40707	2·45655	42757	2·33881	51
10	36727	2·72281	38721	2·58261	40741	2·45451	42791	2·33693	50
11	36760	2·72036	38754	2·58038	40775	2·45246	42826	2·33505	49
12	36793	2·71792	38787	2·57815	40809	2·45043	42860	2·33317	48
13	36826	2·71548	38821	2·57593	40843	2·44839	42894	2·33130	47
14	36859	2·71305	38854	2·57371	40877	2·44636	42929	2·32943	46
15	36892	2·71062	38888	2·57150	40911	2·44433	42963	2·32756	45
16	36925	2·70819	38921	2·56928	40945	2·44230	42998	2·32570	44
17	36958	2·70577	38955	2·56707	40979	2·44027	43032	2·32383	43
18	36991	2·70335	38988	2·56487	41013	2·43825	43067	2·32197	42
19	37024	2·70094	39022	2·56266	41047	2·43623	43101	2·32012	41
20	37057	2·69853	39055	2·56046	41081	2·43422	43136	2·31826	40
21	37090	2·69612	39089	2·55827	41115	2·43220	43170	2·31641	39
22	37124	2·69371	39122	2·55608	41149	2·43019	43205	2·31456	38
23	37157	2·69131	39156	2·55389	41183	2·42819	43239	2·31271	37
24	37190	2·68892	39190	2·55170	41217	2·42618	43274	2·31086	36
25	37223	2·68653	39223	2·54952	41251	2·42418	43308	2·30902	35
26	37256	2·68414	39257	2·54734	41285	2·42218	43343	2·30718	34
27	37289	2·68175	39290	2·54516	41319	2·42019	43378	2·30534	33
28	37322	2·67937	39324	2·54299	41353	2·41819	43412	2·30351	32
29	37355	2·67700	39357	2·54082	41387	2·41620	43447	2·30167	31
30	37388	2·67462	39391	2·53865	41421	2·41421	43481	2·29984	30
31	37422	2·67225	39425	2·53648	41455	2·41223	43516	2·29801	29
32	37455	2·66989	39458	2·53432	41490	2·41025	43550	2·29619	28
33	37488	2·66752	39492	2·53217	41524	2·40827	43585	2·29437	27
34	37521	2·66516	39526	2·53001	41558	2·40629	43620	2·29254	26
35	37554	2·66281	39559	2·52786	41592	2·40432	43654	2·29073	25
36	37588	2·66046	39593	2·52571	41626	2·40235	43689	2·28891	24
37	37621	2·65811	39626	2·52357	41660	2·40038	43724	2·28710	23
38	37654	2·65576	39660	2·52142	41694	2·39841	43758	2·28528	22
39	37687	2·65342	39694	2·51929	41728	2·39645	43793	2·28348	21
40	37720	2·65109	39727	2·51715	41763	2·39449	43828	2·28167	20
41	37754	2·64875	39761	2·51502	41797	2·39253	43862	2·27987	19
42	37787	2·64642	39795	2·51289	41831	2·39058	43897	2·27806	18
43	37820	2·64410	39829	2·51076	41865	2·38862	43932	2·27626	17
44	37853	2·64177	39862	2·50864	41899	2·38668	43966	2·27447	16
45	37887	2·63945	39896	2·50652	41933	2·38473	44001	2·27267	15
46	37920	2·63714	39930	2·50440	41968	2·38279	44036	2·27088	14
47	37953	2·63483	39963	2·50229	42002	2·38084	44071	2·26909	13
48	37986	2·63252	39997	2·50018	42036	2·37891	44105	2·26730	12
49	38020	2·63021	40031	2·49807	42070	2·37697	44140	2·26552	11
50	38053	2·62791	40065	2·49597	42105	2·37504	44175	2·26374	10
51	38086	2·62561	40098	2·49386	42139	2·37311	44210	2·26196	9
52	38120	2·62332	40132	2·49177	42173	2·37118	44244	2·26018	8
53	38153	2·62103	40166	2·48967	42207	2·36925	44279	2·25840	7
54	38186	2·61874	40200	2·48758	42242	2·36733	44314	2·25663	6
55	38220	2·61646	40234	2·48549	42276	2·36541	44349	2·25486	5
56	38253	2·61418	40267	2·48340	42310	2·36349	44384	2·25309	4
57	38286	2·61190	40301	2·48132	42345	2·36158	44418	2·25132	3
58	38320	2·60963	40335	2·47924	42379	2·35967	44453	2·24956	2
59	38353	2·60736	40369	2·47716	42413	2·35776	44488	2·24780	1
60	38386	2·60509	40403	2·47509	42447	2·35585	44523	2·24604	0
′	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	′
	69°		68°		67°		66°		

′	24°		25°		26°		27°		′
	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	
0	44523	2·24604	46631	2·14451	48773	2·05030	50953	1·96261	60
1	44558	2·24428	46666	2·14288	48809	2·04879	50989	1·96120	59
2	44593	2·24252	46702	2·14125	48845	2·04728	51026	1·95979	58
3	44627	2·24077	46737	2·13963	48881	2·04577	51063	1·95838	57
4	44662	2·23902	46772	2·13801	48917	2·04426	51099	1·95698	56
5	44697	2·23727	46808	2·13639	48953	2·04276	51136	1·95557	55
6	44732	2·23553	46843	2·13477	48989	2·04125	51173	1·95417	54
7	44767	2·23378	46879	2·13316	49026	2·03975	51209	1·95277	53
8	44802	2·23204	46914	2·13154	49062	2·03825	51246	1·95137	52
9	44837	2·23030	46950	2·12993	49098	2·03675	51283	1·94997	51
10	44872	2·22857	46985	2·12832	49134	2·03526	51319	1·94858	50
11	44907	2·22683	47021	2·12671	49170	2·03376	51356	1·94718	49
12	44942	2·22510	47056	2·12511	49206	2·03227	51393	1·94579	48
13	44977	2·22337	47092	2·12350	49242	2·03078	51430	1·94440	47
14	45012	2·22164	47128	2·12190	49278	2·02929	51467	1·94301	46
15	45047	2·21992	47163	2·12030	49315	2·02780	51503	1·94162	45
16	45082	2·21819	47199	2·11871	49351	2·02631	51540	1·94023	44
17	45117	2·21647	47234	2·11711	49387	2·02483	51577	1·93885	43
18	45152	2·21475	47270	2·11552	49423	2·02335	51614	1·93746	42
19	45187	2·21304	47305	2·11392	49459	2·02187	51651	1·93608	41
20	45222	2·21132	47341	2·11233	49495	2·02039	51688	1·93470	40
21	45257	2·20961	47377	2·11075	49532	2·01891	51724	1·93332	39
22	45292	2·20790	47412	2·10916	49568	2·01743	51761	1·93195	38
23	45327	2·20619	47448	2·10758	49604	2·01596	51798	1·93057	37
24	45362	2·20449	47483	2·10600	49640	2·01449	51835	1·92920	36
25	45397	2·20278	47519	2·10442	49677	2·01302	51872	1·92782	35
26	45432	2·20108	47555	2·10284	49713	2·01155	51909	1·92645	34
27	45467	2·19938	47590	2·10126	49749	2·01008	51946	1·92508	33
28	45502	2·19769	47626	2·09969	49786	2·00862	51983	1·92371	32
29	45537	2·19599	47662	2·09811	49822	2·00715	52020	1·92235	31
30	45573	2·19430	47698	2·09654	49858	2·00569	52057	1·92098	30
31	45608	2·19261	47733	2·09498	49894	2·00423	52094	1·91962	29
32	45643	2·19092	47769	2·09341	49931	2·00277	52131	1·91826	28
33	45678	2·18923	47805	2·09184	49967	2·00131	52168	1·91690	27
34	45713	2·18755	47840	2·09028	50004	1·99986	52205	1·91554	26
35	45748	2·18587	47876	2·08872	50040	1·99841	52242	1·91418	25
36	45784	2·18419	47912	2·08716	50076	1·99695	52279	1·91282	24
37	45819	2·18251	47948	2·08560	50113	1·99550	52316	1·91147	23
38	45854	2·18084	47984	2·08405	50149	1·99406	52353	1·91012	22
39	45889	2·17916	48019	2·08250	50185	1·99261	52390	1·90876	21
40	45924	2·17749	48055	2·08094	50222	1·99116	52427	1·90741	20
41	45960	2·17582	48091	2·07939	50258	1·98972	52464	1·90607	19
42	45995	2·17416	48127	2·07785	50295	1·98828	52501	1·90472	18
43	46030	2·17249	48163	2·07630	50331	1·98684	52538	1·90337	17
44	46065	2·17083	48198	2·07476	50368	1·98540	52575	1·90203	16
45	46101	2·16917	48234	2·07321	50404	1·98396	52613	1·90069	15
46	46136	2·16751	48270	2·07167	50441	1·98253	52650	1·89935	14
47	46171	2·16585	48306	2·07014	50477	1·98110	52687	1·89801	13
48	46206	2·16420	48342	2·06860	50514	1·97966	52724	1·89667	12
49	46242	2·16255	48378	2·06706	50550	1·97823	52761	1·89533	11
50	46277	2·16090	48414	2·06553	50587	1·97680	52798	1·89400	10
51	46312	2·15925	48450	2·06400	50623	1·97538	52836	1·89266	9
52	46348	2·15760	48486	2·06247	50660	1·97395	52873	1·89133	8
53	46383	2·15596	48521	2·06094	50696	1·97253	52910	1·89000	7
54	46418	2·15432	48557	2·05942	50733	1·97111	52947	1·88867	6
55	46454	2·15268	48593	2·05790	50769	1·96969	52984	1·88734	5
56	46489	2·15104	48629	2·05637	50806	1·96827	53022	1·88602	4
57	46525	2·14940	48665	2·05485	50843	1·96685	53059	1·88469	3
58	46560	2·14777	48701	2·05333	50879	1·96544	53096	1·88337	2
59	46595	2·14614	48737	2·05182	50916	1·96402	53134	1·88205	1
60	46631	2·14451	48773	2·05030	50953	1·96261	53171	1·88073	0
	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	
′	65°		64°		63°		62°		′

′	28° Tangent.	28° Cotang.	29° Tangent.	29° Cotang.	30° Tangent.	30° Cotang.	31° Tangent.	31° Cotang.	′
0	53171	1·88073	55431	1·80405	57735	1·73205	60086	1·66428	60
1	53208	1·87941	55469	1·80281	57774	1·73089	60126	1·66318	59
2	53246	1·87809	55507	1·80158	57813	1·72973	60165	1·66209	58
3	53283	1·87677	55545	1·80034	57851	1·72857	60205	1·66099	57
4	53320	1·87546	55583	1·79911	57890	1·72741	60245	1·65990	56
5	53358	1·87415	55621	1·79788	57929	1·72625	60284	1·65881	55
6	53395	1·87283	55659	1·79665	57968	1·72509	60324	1·65772	54
7	53432	1·87152	55697	1·79542	58007	1·72393	60364	1·65663	53
8	53470	1·87021	55736	1·79419	58046	1·72278	60403	1·65554	52
9	53507	1·86891	55774	1·79296	58085	1·72163	60443	1·65445	51
10	53545	1·86760	55812	1·79174	58124	1·72047	60483	1·65337	50
11	53582	1·86630	55850	1·79051	58162	1·71932	60522	1·65228	49
12	53620	1·86499	55888	1·78929	58201	1·71817	60562	1·65120	48
13	53657	1·86369	55926	1·78807	58240	1·71702	60602	1·65011	47
14	53694	1·86239	55964	1·78685	58279	1·71588	60642	1·64903	46
15	53732	1·86109	56003	1·78563	58318	1·71473	60681	1·64795	45
16	53769	1·85979	56041	1·78441	58357	1·71358	60721	1·64687	44
17	53807	1·85850	56079	1·78319	58396	1·71244	60761	1·64579	43
18	53844	1·85720	56117	1·78198	58435	1·71129	60801	1·64471	42
19	53882	1·85591	56156	1·78077	58474	1·71015	60841	1·64363	41
20	53920	1·85462	56194	1·77955	58513	1·70901	60881	1·64256	40
21	53957	1·85333	56232	1·77834	58552	1·70787	60921	1·64148	39
22	53995	1·85204	56270	1·77713	58591	1·70673	60960	1·64041	38
23	54032	1·85075	56309	1·77592	58631	1·70560	61000	1·63934	37
24	54070	1·84946	56347	1·77471	58670	1·70446	61040	1·63826	36
25	54107	1·84818	56385	1·77351	58709	1·70332	61080	1·63719	35
26	54145	1·84689	56424	1·77230	58748	1·70219	61120	1·63612	34
27	54183	1·84561	56462	1·77110	58787	1·70106	61160	1·63505	33
28	54220	1·84433	56500	1·76990	58826	1·69992	61200	1·63398	32
29	54258	1·84305	56539	1·76869	58865	1·69879	61240	1·63292	31
30	54296	1·84177	56577	1·76749	58904	1·69766	61280	1·63185	30
31	54333	1·84049	56616	1·76630	58944	1·69653	61320	1·63079	29
32	54371	1·83922	56654	1·76510	58983	1·69541	61360	1·62972	28
33	54409	1·83794	56693	1·76390	59022	1·69428	61400	1·62866	27
34	54446	1·83667	56731	1·76271	59061	1·69316	61440	1·62760	26
35	54484	1·83540	56769	1·76151	59101	1·69203	61480	1·62654	25
36	54522	1·83413	56808	1·76032	59140	1·69091	61520	1·62548	24
37	54560	1·83286	56846	1·75913	59179	1·68979	61561	1·62442	23
38	54597	1·83159	56885	1·75794	59218	1·68866	61601	1·62336	22
39	54635	1·83033	56923	1·75675	59258	1·68754	61641	1·62230	21
40	54673	1·82906	56962	1·75556	59297	1·68643	61681	1·62125	20
41	54711	1·82780	57000	1·75437	59336	1·68531	61721	1·62019	19
42	54748	1·82654	57039	1·75319	59376	1·68419	61761	1·61914	18
43	54786	1·82528	57078	1·75200	59415	1·68308	61801	1·61808	17
44	54824	1·82402	57116	1·75082	59454	1·68196	61842	1·61703	16
45	54862	1·82276	57155	1·74964	59494	1·68085	61882	1·61598	15
46	54900	1·82150	57193	1·74846	59533	1·67974	61922	1·61493	14
47	54938	1·82025	57232	1·74728	59573	1·67863	61962	1·61388	13
48	54975	1·81899	57271	1·74610	59612	1·67752	62003	1·61283	12
49	55013	1·81774	57309	1·74492	59651	1·67641	62043	1·61179	11
50	55051	1·81649	57348	1·74375	59691	1·67530	62083	1·61074	10
51	55089	1·81524	57386	1·74257	59730	1·67419	62124	1·60970	9
52	55127	1·81399	57425	1·74140	59770	1·67309	62164	1·60865	8
53	55165	1·81274	57464	1·74022	59809	1·67198	62204	1·60761	7
54	55203	1·81150	57503	1·73905	59849	1·67088	62245	1·60657	6
55	55241	1·81025	57541	1·73788	59888	1·66978	62285	1·60553	5
56	55279	1·80901	57580	1·73671	59928	1·66867	62325	1·60449	4
57	55317	1·80777	57619	1·73555	59967	1·66757	62366	1·60345	3
58	55355	1·80653	57657	1·73438	60007	1·66647	62406	1·60241	2
59	55393	1·80529	57696	1·73321	60046	1·66538	62446	1·60137	1
60	55431	1·80405	57735	1·73205	60086	1·66428	62487	1·60033	0
′	61° Cotang.	61° Tangent.	60° Cotang.	60° Tangent.	59° Cotang.	59° Tangent.	58° Cotang.	58° Tangent.	′

′	32°		33°		34°		35°		′
	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	
0	62487	1·60033	64941	1·53986	67451	1·48256	70021	1·42815	60
1	62527	1·59930	64982	1·53888	67493	1·48163	70064	1·42726	59
2	62568	1·59826	65023	1·53791	67536	1·48070	70107	1·42638	58
3	62608	1·59723	65065	1·53693	67578	1·47977	70151	1·42550	57
4	62649	1·59620	65106	1·53595	67620	1·47885	70194	1·42462	56
5	62689	1·59517	65148	1·53497	67663	1·47792	70238	1·42374	55
6	62730	1·59414	65189	1·53400	67705	1·47699	70281	1·42286	54
7	62770	1·59311	65231	1·53302	67748	1·47607	70325	1·42198	53
8	62811	1·59208	65272	1·53205	67790	1·47514	70368	1·42110	52
9	62852	1·59105	65314	1·53107	67832	1·47422	70412	1·42022	51
10	62892	1·59002	65355	1·53010	67875	1·47330	70455	1·41934	50
11	62933	1·58900	65397	1·52913	67917	1·47238	70499	1·41847	49
12	62973	1·58797	65438	1·52816	67960	1·47146	70542	1·41759	48
13	63014	1·58695	65480	1·52719	68002	1·47053	70586	1·41672	47
14	63055	1·58593	65521	1·52622	68045	1·46962	70629	1·41584	46
15	63095	1·58490	65563	1·52525	68088	1·46870	70673	1·41497	45
16	63136	1·58388	65604	1·52429	68130	1·46778	70717	1·41409	44
17	63177	1·58286	65646	1·52332	68173	1·46686	70760	1·41322	43
18	63217	1·58184	65688	1·52235	68215	1·46595	70804	1·41235	42
19	63258	1·58083	65729	1·52139	68258	1·46503	70848	1·41148	41
20	63299	1·57981	65771	1·52043	68301	1·46411	70891	1·41061	40
21	63340	1·57879	65813	1·51946	68343	1·46320	70935	1·40974	39
22	63380	1·57778	65854	1·51850	68386	1·46229	70979	1·40887	38
23	63421	1·57676	65896	1·51754	68429	1·46137	71023	1·40800	37
24	63462	1·57575	65938	1·51658	68471	1·46046	71066	1·40714	36
25	63503	1·57474	65980	1·51562	68514	1·45955	71110	1·40627	35
26	63544	1·57372	66021	1·51466	98557	1·45864	71154	1·40540	34
27	63584	1·57271	66063	1·51370	68600	1·45773	71198	1·40454	33
28	63625	1·57170	66105	1·51275	68642	1·45682	71242	1·40367	32
29	63666	1·57069	66147	1·51179	68685	1·45592	71285	1·40281	31
30	63707	1·56969	66189	1·51084	68728	1·45501	71329	1·40195	30
31	63748	1·56868	66230	1·50988	68771	1·45410	71373	1·40109	29
32	63789	1·56767	66272	1·50893	68814	1·45320	71417	1·40022	28
33	63830	1·56667	66314	1·50797	68857	1·45229	71461	1·39936	27
34	63871	1·56566	66356	1·50702	68900	1·45139	71505	1·39850	26
35	63912	1·56466	66398	1·50607	68942	1·45049	71549	1·39764	25
36	63953	1·56366	66440	1·50512	68985	1·44958	71593	1·39679	24
37	63994	1·56265	66482	1·50417	69028	1·44868	71637	1·39593	23
38	64035	1·56165	66524	1·50322	69071	1·44778	71681	1·39507	22
39	64076	1·56065	66566	1·50228	69114	1·44688	71725	1·39421	21
40	64117	1·55966	66608	1·50133	69157	1·44598	71769	1·39336	20
41	64158	1·55866	66650	1·50038	69200	1·44508	71813	1·39250	19
42	64199	1·55766	66692	1·49944	69243	1·44418	71857	1·39165	18
43	64240	1·55666	66734	1·49849	69286	1·44329	71901	1·39079	17
44	64281	1·55567	66776	1·49755	69329	1·44239	71946	1·38994	16
45	64322	1·55467	66818	1·49661	69372	1·44149	71990	1·38909	15
46	64363	1·55368	66860	1·49566	69416	1·44060	72034	1·38824	14
47	64404	1·55269	66902	1·49472	69459	1·43970	72078	1·38738	13
48	64446	1·55170	66944	1·49378	69502	1·43881	72122	1·38653	12
49	64487	1·55071	66986	1·49284	69545	1·43792	72166	1·38568	11
50	64528	1·54972	67028	1·49190	69588	1·43703	72211	1·38484	10
51	64569	1·54873	67071	1·49097	69631	1·43614	72255	1·38399	9
52	64610	1·54774	67113	1·49003	69675	1·43525	72299	1·38314	8
53	64652	1·54675	67155	1·48909	69718	1·43436	72344	1·38229	7
54	64693	1·54576	67197	1·48816	69761	1·43347	72388	1·38145	6
55	64734	1·54478	67239	1·48722	69804	1·43258	72432	1·38060	5
56	64775	1·54379	67282	1·48629	69847	1·43169	72477	1·37976	4
57	64817	1·54281	67324	1·48536	69891	1·43080	72521	1·37891	3
58	64858	1·54183	67366	1·48442	69934	1·42992	72565	1·37807	2
59	64899	1·54085	67409	1·48349	69977	1·42903	72610	1·37722	1
60	64941	1·53986	67451	1·48256	70021	1·42815	72654	1·37638	0
′	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	′
	57°		56°		55°		54°		

′	36°		37°		38°		39°		′
	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	
0	72654	1·37638	75355	1·32704	78129	1·27994	80978	1·23490	60
1	72699	1·37554	75401	1·32624	78175	1·27917	81027	1·23416	59
2	72743	1·37470	75447	1·32544	78222	1·27841	81075	1·23343	58
3	72788	1·37386	75492	1·32464	78269	1·27764	81123	1·23270	57
4	72832	1·37302	75538	1·32384	78316	1·27688	81171	1·23196	56
5	72877	1·37218	75584	1·32304	78363	1·27611	81220	1·23123	55
6	72921	1·37134	75629	1·32224	78410	1·27535	81268	1·23050	54
7	72966	1·37050	75675	1·32144	78457	1·27458	81316	1·22977	53
8	73010	1·36967	75721	1·32064	78504	1·27382	81364	1·22904	52
9	73055	1·36883	75767	1·31984	78551	1·27306	81413	1·22831	51
10	73100	1·36800	75812	1·31904	78598	1·27230	81461	1·22758	50
11	73144	1·36716	75858	1·31825	78645	1·27153	81510	1·22685	49
12	73189	1·36633	75904	1·31745	78692	1·27077	81558	1·22612	48
13	73234	1·36549	75950	1·31666	78739	1·27001	81606	1·22539	47
14	73278	1·36466	75996	1·31586	78786	1·26925	81655	1·22467	46
15	73323	1·36383	76042	1·31507	78834	1·26849	81703	1·22394	45
16	73368	1·36300	76088	1·31427	78881	1·26774	81752	1·22321	44
17	73413	1·36217	76134	1·31348	78928	1·26698	81800	1·22249	43
18	73457	1·36133	76180	1·31269	78975	1·26622	81849	1·22176	42
19	73502	1·36051	76226	1·31190	79022	1·26546	81898	1·22104	41
20	73547	1·35968	76272	1·31110	79070	1·26471	81946	1·22031	40
21	73592	1·35885	76318	1·31031	79117	1·26395	81995	1·21959	39
22	73637	1·35802	76364	1·30952	79164	1·26319	82044	1·21886	38
23	73681	1·35719	76410	1·30873	79212	1·26244	82092	1·21814	37
24	73726	1·35637	76456	1·30795	79259	1·26169	82141	1·21742	36
25	73771	1·35554	76502	1·30716	79306	1·26093	82190	1·21670	35
26	73816	1·35472	76548	1·30637	79354	1·26018	82238	1·21598	34
27	73861	1·35389	76594	1·30558	79401	1·25943	82287	1·21526	33
28	73906	1·35307	76640	1·30480	79449	1·25867	82336	1·21454	32
29	73951	1·35224	76686	1·30401	79496	1·25792	82385	1·21382	31
30	73996	1·35142	76733	1·30323	79544	1·25717	82434	1·21310	30
31	74041	1·35060	76779	1·30244	79591	1·25642	82483	1·21238	29
32	74086	1·34978	76825	1·30166	79639	1·25567	82531	1·21166	28
33	74131	1·34896	76871	1·30087	79686	1·25492	82580	1·21094	27
34	74176	1·34814	76918	1·30009	79734	1·25417	82629	1·21023	26
35	74221	1·34732	76964	1·29931	79781	1·25343	82678	1·20951	25
36	74267	1·34650	77010	1·29853	79829	1·25268	82727	1·20879	24
37	74312	1·34568	77057	1·29775	79877	1·25193	82776	1·20808	23
38	74357	1·34487	77103	1·29696	79924	1·25118	82825	1·20736	22
39	74402	1·34405	77149	1·29618	79972	1·25044	82874	1·20665	21
40	74447	1·34323	77196	1·29541	80020	1·24969	82923	1·20593	20
41	74492	1·34242	77242	1·29463	80067	1·24895	82972	1·20522	19
42	74538	1·34160	77289	1·29385	80115	1·24820	83022	1·20451	18
43	74583	1·34079	77335	1·29307	80163	1·24746	83071	1·20379	17
44	74628	1·33998	77382	1·29229	80211	1·24672	83120	1·20308	16
45	74674	1·33916	77428	1·29152	80258	1·24597	83169	1·20237	15
46	74719	1·33835	77475	1·29074	80306	1·24523	83218	1·20166	14
47	74764	1·33754	77521	1·28997	80354	1·24449	83268	1·20095	13
48	74810	1·33673	77568	1·28919	80402	1·24375	83317	1·20024	12
49	74855	1·33592	77615	1·28842	80450	1·24301	83366	1·19953	11
50	74900	1·33511	77661	1·28764	80498	1·24227	83415	1·19882	10
51	74946	1·33430	77708	1·28687	80546	1·24153	83465	1·19811	9
52	74991	1·33349	77754	1·28610	80594	1·24079	83514	1·19740	8
53	75037	1·33268	77801	1·28533	80642	1·24005	83564	1·19669	7
54	75082	1·33187	77848	1·28456	80690	1·23931	83613	1·19599	6
55	75128	1·33107	77895	1·28379	80738	1·23858	83662	1·19528	5
56	75173	1·33026	77941	1·28302	80786	1·23784	83712	1·19457	4
57	75219	1·32946	77988	1·28225	80834	1·23710	83761	1·19387	3
58	75264	1·32865	78035	1·28148	80882	1·23637	83811	1·19316	2
59	75310	1·32785	78082	1·28071	80930	1·23563	83860	1·19246	1
60	75355	1·32704	78129	1·27994	80978	1·23490	83910	1·19175	0
	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	
′	53°		52°		51°		50°		′

′	40°		41°		42°		43°		′
	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	
0	83910	1·19175	86929	1·15037	90040	1·11061	93252	1·07237	60
1	83960	1·19105	86980	1·14969	90093	1·10996	93306	1·07174	59
2	84009	1·19035	87031	1·14902	90146	1·10931	93360	1·07112	58
3	84059	1·18964	87082	1·14834	90199	1·10867	93415	1·07049	57
4	84108	1·18894	87133	1·14767	90251	1·10802	93469	1·06987	56
5	84158	1·18824	87184	1·14699	90304	1·10737	93524	1·06925	55
6	84208	1·18754	87236	1·14632	90357	1·10672	93578	1·06862	54
7	84258	1·18684	87287	1·14565	90410	1·10607	93633	1·06800	53
8	84307	1·18614	87338	1·14498	90463	1·10543	93688	1·06738	52
9	84357	1·18544	87389	1·14430	90516	1·10478	93742	1·06676	51
10	84407	1·18474	87441	1·14363	90569	1·10414	93797	1·06613	50
11	84457	1·18404	87492	1·14296	90621	1·10349	93852	1·06551	49
12	84507	1·18334	87543	1·14229	90674	1·10285	93906	1·06489	48
13	84556	1·18264	87595	1·14162	90727	1·10220	93961	1·06427	47
14	84606	1·18194	87646	1·14095	90781	1·10156	94016	1·06365	46
15	84656	1·18125	87698	1·14028	90834	1·10091	94071	1·06303	45
16	84706	1·18055	87749	1·13961	90887	1·10027	94125	1·06241	44
17	84756	1·17986	87801	1·13894	90940	1·09963	94180	1·06179	43
18	84806	1·17916	87852	1·13828	90993	1·09899	94235	1·06117	42
19	84856	1·17846	87904	1·13761	91046	1·09834	94290	1·06056	41
20	84906	1·17777	87955	1·13694	91099	1·09770	94345	1·05994	40
21	84956	1·17708	88007	1·13627	91153	1·09706	94400	1·05932	39
22	85006	1·17638	88059	1·13561	91206	1·09642	94455	1·05870	38
23	85057	1·17569	88110	1·13494	91259	1·09578	94510	1·05809	37
24	85107	1·17500	88162	1·13428	91313	1·09514	94565	1·05747	36
25	85157	1·17430	88214	1·13361	91366	1·09450	94620	1·05685	35
26	85207	1·17361	88265	1·13295	91419	1·09386	94676	1·05624	34
27	85257	1·17292	88317	1·13228	91473	1·09322	94731	1·05562	33
28	85307	1·17223	88369	1·13162	91526	1·09258	94786	1·05501	32
29	85358	1·17154	88421	1·13096	91580	1·09195	94841	1·05439	31
30	85408	1·17085	88473	1·13029	91633	1·09131	94896	1·05378	30
31	85458	1·17016	88524	1·12963	91687	1·09067	94952	1·05317	29
32	85509	1·16947	88576	1·12897	91740	1·09003	95007	1·05255	28
33	85559	1·16878	88628	1·12831	91794	1·08940	95062	1·05194	27
34	85609	1·16809	88680	1·12765	91847	1·08876	95118	1·05133	26
35	85660	1·16741	88732	1·12699	91901	1·08813	95173	1·05072	25
36	85710	1·16672	88784	1·12633	91955	1·08749	95229	1·05010	24
37	85761	1·16603	88836	1·12567	92008	1·08686	95284	1·04949	23
38	85811	1·16535	88888	1·12501	92062	1·08622	95340	1·04888	22
39	85862	1·16466	88940	1·12435	92116	1·08559	95395	1·04827	21
40	85912	1·16398	88992	1·12369	92170	1·08496	95451	1·04766	20
41	85963	1·16329	89045	1·12303	92223	1·08432	95506	1·04705	19
42	86014	1·16261	89097	1·12238	92277	1·08369	95562	1·04644	18
43	86064	1·16192	89149	1·12172	92331	1·08306	95618	1·04583	17
44	86115	1·16124	89201	1·12106	92385	1·08243	95673	1·04522	16
45	86166	1·16056	89253	1·12041	92439	1·08179	95729	1·04461	15
46	86216	1·15987	89306	1·11975	92493	1·08116	95785	1·04401	14
47	86267	1·15919	89358	1·11909	92547	1·08053	95841	1·04340	13
48	86318	1·15851	89410	1·11844	92601	1·07990	95897	1·04279	12
49	86368	1·15783	89463	1·11778	92655	1·07927	95952	1·04218	11
50	86419	1·15715	89515	1·11713	92709	1·07864	96008	1·04158	10
51	86470	1·15647	89567	1·11648	92763	1·07801	96064	1·04097	9
52	86521	1·15579	89620	1·11582	92817	1·07738	96120	1·04036	8
53	86572	1·15511	89672	1·11517	92872	1·07676	96176	1·03976	7
54	86623	1·15443	89725	1·11452	92926	1·07613	96232	1·03915	6
55	86674	1·15375	89777	1·11387	92980	1·07550	96288	1·03855	5
56	86725	1·15308	89830	1·11321	93034	1·07487	96344	1·03794	4
57	86776	1·15240	89883	1·11256	93088	1·07425	96400	1·03734	3
58	86827	1·15172	89935	1·11191	93143	1·07362	96457	1·03674	2
59	86878	1·15104	89988	1·11126	93197	1·07299	96513	1·03613	1
60	86929	1·15037	90040	1·11061	93252	1·07237	96569	1·03553	0
	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	Cotang.	Tangent.	
′	49°		48°		47°		46°		′

′	44° Tangent.	44° Cotang.	′	′	44° Tangent.	44° Cotang.	′
0	96569	1·03553	60	31	98327	1·01702	29
1	96625	1·03493	59	32	98384	1·01642	28
2	96681	1·03433	58	33	98441	1·01583	27
3	96738	1·03372	57	34	98499	1·01524	26
4	96794	1·03312	56	35	98556	1·01465	25
5	96850	1·03252	55	36	98613	1·01406	24
6	96907	1·03192	54	37	98671	1·01347	23
7	96963	1·03132	53	38	98728	1·01288	22
8	97020	1·03072	52	39	98786	1·01229	21
9	97076	1·03012	51	40	98843	1·01170	20
10	97133	1·02952	50	41	98901	1·01112	19
11	97189	1·02892	49	42	98958	1·01053	18
12	97246	1·02832	48	43	99016	1·00994	17
13	97302	1·02772	47	44	99073	1·00935	16
14	97359	1·02713	46	45	99131	1·00876	15
15	97416	1·02653	45	46	99189	1·00818	14
16	97472	1·02593	44	47	99247	1·00759	13
17	97529	1·02533	43	48	99304	1·00701	12
18	97586	1·02474	42	49	99362	1·00642	11
19	97643	1·02414	41	50	99420	1·00583	10
20	97700	1·02355	40	51	99478	1·00525	9
21	97756	1·02295	39	52	99536	1·00467	8
22	97813	1·02236	38	53	99594	1·00408	7
23	97870	1·02176	37	54	99652	1·00350	6
24	97927	1·02117	36	55	99710	1·00291	5
25	97984	1·02057	35	56	99768	1·00233	4
26	98041	1·01998	34	57	99826	1·00175	3
27	98098	1·01939	33	58	99884	1·00116	2
28	98155	1·01879	32	59	99942	1·00058	1
29	98213	1·01820	31	60	Unit.	Unit.	0
30	98270	1·01761	30				
′	45° Cotang.	45° Tangent.	′	′	45° Cotang.	45° Tangent.	′

TABLE OF CONSTANTS.

Base of Napier's system of logarithms = ... ε = 2·718281828459

Mod. of common syst. of logarithms = ... com. log. ε = M = 0·434294481903

Ratio of circumference to diameter of a circle =........... π = 3·14159265359O

log. π = 0·497149872694

π^2 = 9·869604401089............ $\sqrt{\pi}$ = 1·772453850906

Arc of same length as radius = ...180° ÷ π = 10800′ ÷ π = 648000″ ÷ π

180° ÷ π = 57°·2957795130,............................log. = 1·758122632409

10800′ ÷ π = 3437′·7467707849,........................log. = 3·536273882793

648000″ ÷ π = 206264″·8062470964, ...log. = 5·314425133176

Tropical year = 365d. 5h. 48m. 47s. ·588 = 365d. ·242217456, log. = 2·5625810

Sidereal year = 365d. 6h. 9m. 10s. ·742 = 365d. ·256374332, log. = 2·5625978

24h. sol. t.=24h. 3m. 56s. ·555335 sid. t.=24h.×1·00273791, log. 1·002=0·0011874

24h. sid. t.=24h. −(3m.55s.·90944) sol. t.=24h.×0·9972696, log. 0·997=9·9988126

British imperial gallon = 277·274 cubic inches,...............log. = 2·4429091

Length of sec. pend., in inches, at London, 39·13929; Paris, 39·1285; New York, 39·1285.

French *metre* = 3·2808992 English *feet* = 39·3707904 *inches.*

1 cubic inch of water (bar. 30 inches, Fahr. therm. 62°) = 252·458 Troy grains.

Refraction in Declination.

For Latitude 15°.

Hour Angle.	+ 20°	+ 15°	+ 10°	+ 5°	0°	— 5°	— 10°	— 15°	— 20°
0 h.	—05″	0″	+05″	10″	15″	21″	27″	33″	40″
2	—03	+02	07	12	18	23	29	36	43
3	+01	05	11	16	22	28	34	41	49
4	08	12	19	24	30	37	44	53	1′04
5	29	34	41	49	59	1′10	1′24	1′43	2′08

For Latitude 17° 30′.

Hour Angle.	+ 20°	+ 15°	+ 10°	+ 5°	0°	— 5°	— 10°	— 15°	— 20°
0 h.	—02″	+02″	08″	13″	18″	24″	30″	36″	44″
2	0	05	10	15	21	27	33	40	48
3	+02	10	15	21	27	33	40	48	57
4	13	18	23	29	35	43	51	1′01	1′13
5	34	41	49	58	1′10	1′23	1′41	2 06	2 42

For Latitude 20°.

Hour Angle.	+ 20°	+ 15°	+ 10°	+ 5°	0°	— 5°	— 10°	— 15°	— 20°
0 h.	0″	05″	10″	15″	21″	27″	33″	40″	48″
2	03	07	13	18	24	30	36	44	52
3	06	13	18	24	30	36	44	52	1′02
4	17	22	28	35	42	50	1′00	1′11	1 26
5	39	47	57	1′07	1′20	1′37	2 00	2 32	3 25

For Latitude 22° 30′.

Hour Angle.	+ 20°	+ 15°	+ 10°	+ 5°	0°	— 5°	— 10°	— 15°	— 20°
0 h.	02″	08″	13″	18″	24″	30″	36″	44″	52″
2	06	11	15	21	27	33	40	48	57
3	11	15	21	27	33	40	48	57	1′08
4	20	26	32	39	46	56	1′07	1′19	1 37
5	45	53	1′03	1′16	1′31	1′52	2 21	3 07	4 28

For Latitude 25°.

Hour Angle.	+ 20°	+ 15°	+ 10°	+ 5°	0°	— 5°	— 10°	— 15°	— 20°
0 h.	05″	10″	15″	21″	27″	33″	40″	48″	57″
2	08	14	19	25	31	38	46	54	1′05
3	12	18	24	30	37	44	53	1′04	1 18
4	23	29	35	45	53	1′03	1′16	1 31	1 52
5	49	59	1′10	1′24	1′52	2 07	2 44	3 46	5 43

For Latitude 27° 30′.

Hour Angle.	+ 20°	+ 15°	+ 10°	+ 5°	0°	— 5°	— 10°	— 15°	— 20°
0 h.	08″	13″	18″	24″	30″	36″	44″	52″	1′02″
2	11	16	22	28	34	41	49	1′00	1 10
3	17	22	28	35	42	50	1′00	1 11	1 26
4	28	35	42	50	1′00	1′11	1 26	1 43	2 09
5	54	1′05	1′18	1′34	1 54	2 24	3 11	4 38	8 15

For Latitude 30°.

Hour Angle.	+ 20°	+ 15°	+ 10°	+ 5°	0°	— 5°	— 10°	— 15°	— 20°
0 h.	10″	15″	21″	27″	33″	40″	48″	57″	1′08″
2	14	19	25	31	38	46	54	1′05	1 18
3	20	26	32	39	47	55	1′06	1 19	1 36
4	32	39	46	52	1′06	1′19	1 35	1 57	2 29
5	1′00	1′10	1′24	1′52	2 07	2 44	3 46	5 43	13 06

For Latitude 32° 30′.

Hour Angle.	+ 20°	+ 15°	+ 10°	+ 5°	0°	— 5°	— 10°	— 15°	— 20°
0 h.	13″	18″	24″	30″	36″	44″	52″	1′02″	1′14″
2	17	22	28	35	42	50	1′00	1 11	1 26
3	23	29	35	43	51	1′01	1 13	1 28	1 47
4	35	43	51	1′01	1′13	1 27	1 46	2 13	2 54
5	1′03	1′15	1′31	1 53	2 20	3 05	4 25	7 36	

For Latitude 35°.

Hour Angle.	+ 20°	+ 15°	+ 10°	+ 5°	0°	— 5°	— 10°	— 15°	— 20°
0 h.	15″	21″	27″	33″	40″	48″	57″	1′08″	1′21″
2	20	25	32	38	46	55	1′05	1 18	1 35
3	26	33	39	47	56	1′07	1 21	1 38	2 00
4	39	47	56	1′07	1′20	1 36	1 59	2 32	3 25
5	1′07	1′20	1′38	2 00	2 34	3 29	5 14	10 16	

Hour Angle.	+ 20°	+ 15°	+ 10°	+ 5°	0°	− 5°	− 10°	− 15°	− 20°
				For Latitude 37° 30′.					
0 h.	18″	24″	30″	36″	44″	52″	1′ 02″	1′ 14″	1′ 29″
2	22	28	35	42	50	1′ 00	1 12	1 26	1 45
3	29	36	43	52	1′ 02	1 14	1 29	1 49	2 16
4	43	51	1′ 01	1′ 13	1 27	1 49	2 14	2 54	4 05
5	1′ 11	1′ 26	1 54	2 10	2 49	3 55	6 15	14 58	
				For Latitude 40°.					
0 h.	21″	27″	33″	40″	48″	57″	1′ 08″	1′ 21″	1′ 39″
2	25	32	39	46	52	1′ 06	1 19	1 35	1 57
3	33	40	48	57	1′ 08	1 21	1 38	2 02	2 36
4	47	55	1′ 06	1′ 19	1 36	1 58	2 30	3 21	4 59
5	1′ 15	1′ 31	1 51	2 20	3 05	4 25	7 34	25 18	
				For Latitude 42° 30′.					
0 h.	24″	30″	36″	44″	52″	1′ 02″	1′ 14″	1′ 29″	1′ 49″
2	28	35	39	50	1′ 00	1 12	1 26	1 45	2 11
3	36	43	52	1′ 02	1 13	1 29	1 49	2 17	2 59
4	50	1′ 00	1′ 11	1 26	1 44	2 10	2 49	3 55	6 16
5	1′ 16	1 36	1 58	2 30	3 22	5 00	9 24		
				For Latitude 45°.					
0 h.	27″	33″	40″	48″	57″	1′ 08″	1′ 21″	1′ 39″	2′ 02″
2	32	39	46	52	1′ 06	1 19	1 35	1 57	2 29
3	40	47	56	1′ 07	1 21	1 38	2 00	2 34	3 29
4	54	1′ 04	1′ 16	1 33	1 54	2 24	3 11	4 38	8 15
5	1′ 23	1 41	2 05	2 41	3 40	5 40	12 02		
				For Latitude 47° 30′.					
0 h.	30″	36″	44″	52″	1′ 02″	1′ 14″	1′ 29″	1′ 49″	2′ 18″
2	35	42	50	1′ 00	1 12	1 26	1 45	2 01	2 51
3	43	51	1′ 01	1 13	1 28	1 47	2 15	2 56	4 08
4	56	1′ 09	1 23	1 40	2 05	2 40	3 39	5 37	11 18
5	1′ 27	1 46	2 12	2 52	4 01	6 30	16 19		
				For Latitude 50°.					
0 h.	33″	40″	48″	57″	1′ 08″	1′ 21″	1′ 39″	2′ 02″	2′ 36″
2	38	46	55	1′ 06	1 18	1 35	1 57	2 28	3 19
3	47	56	1′ 06	1 19	1 36	2 29	2 31	3 23	5 02
4	1′ 02	1′ 14	1 29	1 48	2 16	2 58	4 18	6 59	19 47
5	1 30	1 51	2 19	3 04	4 22	7 28	24 10		
				For Latitude 52° 30′.					
0 h.	36″	44″	52″	1′ 02″	1′ 14″	1′ 29″	1′ 49″	2′ 18″	3′ 05″
2	43	50	59	1 11	1 26	1 42	2 23	2 49	3 55
3	50	1′ 00	1′ 11	1 26	1 45	2 11	2 51	2 58	6 22
4	1′ 05	1 18	1 35	2 10	2 28	3 19	4 53	8 42	
5	1 34	1 56	2 27	3 16	4 47	8 52			
				For Latitude 55°.					
0 h.	40″	48″	57″	1′ 08″	1′ 21″	1′ 39″	2′ 02″	2′ 36″	3′ 33″
2	46	55	1′ 05	1 18	1 34	1 56	2 30	3 15	4 47
3	55	1′ 06	1 19	1 35	1 58	2 30	3 21	4 58	9 19
4	1′ 10	1 23	1 42	2 06	2 43	3 44	5 49	12 41	
5	1 37	2 01	2 34	3 28	5 15	10 18			
				For Latitude 57° 30′.					
0 h.	44″	52″	1′ 02″	1′ 14″	1′ 29″	1′ 49″	2′ 18″	3′ 05″	4′ 37″
2	50	59	1 11	1 25	1 43	2 09	2 47	3 51	6 04
3	58	1′ 10	1 24	1 42	2 07	2 43	3 45	5 50	12 47
4	1′ 11	1 25	1 43	2 10	2 50	3 55	6 14	14 49	
5	1 41	2 06	2 42	3 42	5 46	12 26			
				For Latitude 60°.					
0 h.	48″	57″	1′ 08″	1′ 21″	1′ 39″	2′ 02″	2′ 36″	3′ 33″	5′ 23″
2	54	1′ 04	1 17	1 33	1 54	2 24	3 12	4 38	8 15
3	1′ 03	1 15	1 30	1 51	2 20	3 04	4 24	7 31	24 44
4	1 18	1 34	1 56	2 28	3 18	4 50	8 53		
5	1 45	2 11	2 50	3 57	6 21	15 32			

TABLES

FOR OBTAINING

HORIZONTAL DISTANCES

AND

DIFFERENCES OF LEVEL,

FROM

STADIA READINGS.

′	1	2	3	4	5	6	7	8	9	a
00	0.9986	1.9972	2.9958	3.9944	4.9930	5.9916	6.9902	7.9888	8.9874	1.4000
01	0.9986	1.9972	2.9958	3.9944	4.9930	5.9916	6.9902	7.9888	8.9874	1.4000
02	0.9986	1.9972	2.9958	3.9944	4.9930	5.9916	6.9902	7.9888	8.9874	1.4000
03	0.9986	1.9972	2.9958	3.9944	4.9930	5.9916	6.9902	7.9888	8.9874	1.4000
04	0.9986	1.9972	2.9958	3.9944	4.9930	5.9916	6.9902	7.9888	8.9874	1.4000
05	0.9986	1.9972	2.9958	3.9944	4.9930	5.9916	6.9902	7.9888	8.9874	1.4000
06	0.9986	1.9972	2.9958	3.9944	4.9930	5.9916	6.9902	7.9888	8.9874	1.4000
07	0.9986	1.9972	2.9958	3.9944	4.9930	5.9916	6.9902	7.9888	8.9873	1.4000
08	0.9986	1.9972	2.9958	3.9944	4.9930	5.9916	6.9902	7.9888	8.9873	1.4000
09	0.9986	1.9972	2.9958	3.9944	4.9930	5.9916	6.9902	7.9887	8.9873	1.4000
10	0.9986	1.9972	2.9958	3.9944	4.9930	5.9916	6.9901	7.9887	8.9873	1.4000
11	0.9986	1.9972	2.9958	3.9944	4.9930	5.9915	6.9901	7.9887	8.9873	1.4000
12	0.9986	1.9972	2.9958	3.9943	4.9929	5.9915	6.9901	7.9887	8.9873	1.4000
13	0.9986	1.9972	2.9958	3.9943	4.9929	5.9915	6 9901	7.9887	8.9873	1.4000
14	0.9986	1.9972	2.9957	3.9943	4.9929	5.9915	6.9901	7.9887	8.9872	1.4000
15	0.9986	1.9972	2.9957	3.9943	4.9929	5.9915	6.9901	7.9886	8.9872	1.4000
16	0.9986	1.9972	2.9957	3.9943	4.9929	5.9915	6.9900	7.9886	8.9872	1.4000
17	0.9986	1.9972	2.9957	3.9943	4.9929	5.9915	6.9900	7.9886	8.9872	1.4000
18	0.9986	1.9971	2.9957	3.9943	4.9929	5.9914	6.9900	7.9886	8.9872	1.4000
19	0.9986	1.9971	2.9957	3.9943	4.9929	5.9914	6.9900	7.9886	8.9871	1.4000
20	0.9986	1.9971	2.9957	3.9943	4.9928	5.9914	6.9900	7.9885	8.9871	1.4000
21	0.9986	1.9971	2.9957	3.9943	4.9928	5.9914	6.9899	7.9885	8.9871	1.3999
22	0.9986	1.9971	2.9957	3.9942	4.9928	5.9913	6.9899	7.9885	8.9870	1.3999
23	0.9986	1.9971	2.9957	3.9942	4.9928	5.9913	6.9899	7.9884	8.9870	1.3999
24	0.9985	1.9971	2 9956	3.9942	4.9927	5.9913	6.9898	7.9884	8.9869	1.3999
25	0.9985	1.9971	2.9956	3.9942	4.9927	5.9913	6.9898	7.9884	8.9869	1.3999
26	0.9985	1.9971	2.9956	3.9942	4.9927	5.9912	6.9898	7.9883	8.9869	1.3999
27	0.9985	1.9971	2.9956	3.9941	4.9927	5.9912	6.9898	7.9883	8.9868	1.3999
28	0.9985	1.9971	2.9956	3.9941	4.9927	5.9912	6.9897	7.9883	8 9868	1.3999
29	0.9985	1.9971	2.9956	3.9941	4.9926	5.9912	6.9897	7.9882	8.9868	1.3999
30	0.9985	1.9970	2.9956	3.9941	4.9926	5.9911	6.9897	7.9882	8.9867	1.3999
31	0.9985	1.9970	2.9956	3.9941	4.9926	5.9911	6.9896	7.9881	8.9867	1.3999
32	0.9985	1.9970	2.9955	3.9940	4.9026	5.9911	6.9896	7.9881	8.9866	1.3999
33	0.9985	1.9970	2.9955	3.9940	4.9925	5.9910	6.9895	7.9880	8.9866	1.3999
34	0.9985	1.9970	2.9955	3.9940	4.9925	5.9910	6.9895	7.9880	8.9865	1.3999
35	0.9985	1.9970	2.9955	3.9940	4.9925	5.9910	6.9895	7.9880	8.9865	1.3999
36	0.9985	1.9970	2.9955	3.9940	4.9924	5.9909	6.9894	7.9879	8.9864	1.3999
37	0.9985	1.9970	2.9954	3.9939	4.9924	5.9909	6.9894	7.9879	8.9863	1.3999
38	0.9985	1.9970	2.9954	3.9939	4.9924	5.9909	6.9893	7.9878	8.9863	1.3999
39	0.9985	1.9969	2.9954	3.9939	4.9924	5.9908	6.9893	7.9878	8.9862	1.3999
40	0.9985	1.9969	2.9954	3.9939	4.9923	5.9908	6.9893	7.9877	8.9862	1.3999
41	0.9985	1.9969	2.9954	3.9938	4.9923	5.9907	6.9892	7.9877	8.9861	1.3998
42	0.9984	1.9969	2.9953	3.9938	4.9922	5.9907	6.9891	7.9876	8.9860	1.3998
43	0.9984	1.9969	2.9953	3.9938	4.9922	5.9907	6.9891	7.9875	8.9860	1.3998
44	0.9984	1.9969	2.9953	3.9937	4.9922	5.9906	6.9890	7.9875	8.9859	1.3998
45	0.9984	1.9969	2.9953	3.9937	4.9921	5.9906	6.9890	7.9874	8.9858	1.3998
46	0.9984	1.9968	2.9953	3.9937	4.9921	5.9905	6.9889	7.9874	8.9858	1.3998
47	0.9984	1.9968	2.9952	3.9936	4.9921	5.9905	6.9889	7.9873	8.9857	1.3998
48	0.9984	1.9968	2.9952	3.9936	4.9920	5.9904	6.9888	7.9872	8.9856	1.3998
49	0.9984	1.9968	2.9952	3.9936	4.9920	5.9904	6.9888	7.9872	8.9856	1.3998
50	0.9984	1.9968	2.9952	3.9936	4.9919	5.9903	6.9887	7.9871	8.9855	1.3998
51	0.9984	1.9968	2.9951	3.9935	4.9919	5.9903	6.9887	7.9870	8.9854	1.3998
52	0.9984	1.9967	2.9951	3.9935	4.9919	5.9902	6.9886	7.9870	8.9853	1.3998
53	0.9984	1.9967	2.9951	3.9934	4.9918	5.9902	6.9885	7.9869	8.9852	1.3998
54	0.9984	1.9967	2.9951	3'9934	4.9918	5.9901	6.9885	7.9868	8.9852	1.3998
55	0.9983	1.9967	2.9950	3.9934	4.9917	5.9901	6.9884	7.9867	8.9851	1.3998
56	0.9983	1.9967	2.9950	3.9933	4.9917	5.9900	6.9883	7.9867	8.9850	1.3998
57	0.9983	1.9966	2.9950	3.9933	4.9916	5.9899	6.9883	7.9866	8.9849	1.3998
58	0.9983	1.9966	2.9949	3.9933	4.9916	5.9899	6.9882	7.9865	8.9848	1.3998
59	0.9983	1.9966	2.9949	3.9932	4.9915	5.9898	6.9881	7.9864	8.9847	1.3998
60	0.9983	1.9966	2.9949	3.9932	4.9915	5.9898	6.9881	7.9864	8.9847	1.3998

1	2	3	4	5	6	7	8	9	b	′
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	00
0.0003	0.0006	0.0009	0.0012	0.0015	0.0017	0.0020	0.0023	0.0026	0.0004	01
0.0006	0.0012	0.0017	0.0023	0.0029	0.0035	0.0041	0.0046	0.0052	0.0008	02
0.0009	0.0017	0.0026	0.0035	0.0044	0.0052	0.0061	0.0070	0.0078	0.0012	03
0.0012	0.0023	0.0035	0.0046	0.0058	0.0070	0.0081	0.0093	0.0105	0.0016	04
0.0015	0.0029	0.0044	0.0058	0.0073	0.0087	0.0102	0.0116	0.0131	0.0020	05
0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0139	0.0157	0.0024	06
0.0020	0.0041	0.0061	0.0081	0.0102	9.0122	0.0142	0.0163	0.0183	0.0029	07
0.0023	0.0046	0.0070	0.0093	0.0116	0.0139	0.0163	0.0186	0.0209	0.0033	08
0.0026	0.0052	0.0078	0.0105	0.0131	0.0157	0.0183	0.0209	0.0235	0.0037	09
0.0029	0.0058	0.0087	0.0116	0.0145	0.0174	0.0203	0.0232	0.0261	0.0041	10
0.0032	0.0064	0.0096	0.0128	0.0160	0.0192	0.0224	0.0256	0.0288	0.0045	11
0.0035	0.0070	0.0105	0.0139	0.0174	0.0209	0.0244	0.0279	0.0314	0.0049	12
0.0038	0.0076	0.0113	0.0151	0.0189	0.0227	0.0264	0.0302	0.0340	0.0053	13
0.0041	0.0081	0.0122	0.0163	0.0203	0.0244	0.0285	0.0325	0.0366	0.0057	14
0.0044	0.0087	0.0131	0.0174	0.0218	0.0261	0.0305	0.0349	0.0392	0.0061	15
0.0046	0.0093	0.0139	0.0186	0.0232	0.0279	0.0325	0.0372	0.0418	0.0065	16
0.0049	0.0099	0.0148	0.0198	0.0247	0.0296	0.0346	0.0395	0.0444	0.0069	17
0.0052	0.0105	0.0157	0.0209	0.0261	0.0314	0.0366	0.0418	0.0471	0.0073	18
0.0055	0.0110	0.0166	0.0221	0.0276	0.0331	0.0386	0.0442	0.0497	0.0077	19
0.0058	0.0116	0.0174	0.0232	0.0290	0.0349	0.0407	0.0465	0.0523	0.0081	20
0.0061	0.0122	0.0183	0.0244	0.0305	0.0366	0.0427	0.0488	0.0549	0.0086	21
0.0064	0.0128	0.0192	0.0256	0.0320	0.0383	0.0447	0.0511	0.0575	0.0090	22
0.0067	0.0134	0.0200	0.0267	0.0334	0.0401	0.0468	0.0534	0.0601	0.0094	23
0.0070	0.0139	0.0209	0.0279	0.0349	0.0418	0.0488	0.0558	0.0627	0.0098	24
0.0073	0.0145	0.0218	0.0290	0.0363	0.0436	0.0508	0.0581	0.0654	0.0102	25
0.0076	0.0151	0.0227	0.0302	0.0378	0·0453	0.0529	0.0604	0.0680	0.0106	26
0.0078	0.0157	0.0235	0.0314	0.0392	0.0471	0.0549	0.0627	0.0706	0.0110	27
0.0081	0.0163	0.0244	0.0325	0.0407	0.0488	0.0569	0.0651	0.0732	0.0114	28
0.0084	0.0168	0.0253	0.0337	0.0421	0.0505	0.0590	0.0674	0.0758	0.0118	29
0.0087	0.0174	0.0261	0.0349	0.0436	0.0523	0.0610	0.0697	0.0784	0.0122	30
0.0090	0.0180	0.0270	0.0360	0.0450	0.0540	0.0630	0.0720	0.0810	0.0126	31
0.0093	0.0186	0.0279	0.0372	0.0465	0.0558	0.0651	0.0744	0.0837	0.0130	32
0.0096	0.0192	0.0288	0.0383	0.0479	0.0575	0.0671	0.0767	0.0863	0.0134	33
0.0099	0.0198	0.0296	0.0395	0.0494	0.0593	0.0691	0.0790	0.0889	0.0138	34
0.0102	0.0203	0.0305	0.0407	0.0508	0.0610	0.0712	0.0813	0.0915	0.0143	35
0.0105	0.0209	0.0314	0.0418	0.0523	0.0627	0.0732	0.0836	0.0941	0.0147	36
0.0107	0.0215	0.0322	0.0430	0.0537	0.0645	0.0752	0.0860	0.0967	0.0151	37
0.0110	0.0221	0.0331	0.0441	0.0552	0.0662	0.0773	0.0883	0.0993	0.0155	38
0.0113	0.0227	0.0340	0.0453	0.0566	0.0680	0.0793	0.0906	0.1019	0.0159	39
0.0116	0.0232	0.0349	0.0465	0.0581	0.0697	0.0813	0.0929	0.1046	0.0163	40
0.0119	0.0238	0.0357	0.0476	0.0595	0.0715	0.0834	0.0953	0.1072	0.0167	41
0.0122	0.0244	0.0366	0.0488	0.0610	0.0732	0.0854	0.0976	0.1098	0.0171	42
0.0125	0.0250	0.0375	0.0500	0.0624	0.0749	0.0874	0.0999	0.1124	0.0175	43
0.0128	0.0256	0.0383	0.0511	0.0639	0.0767	0.0895	0.1022	0.1150	0.0179	44
0.0131	0.0261	0.0392	0.0523	0.0654	0.0784	0.0915	0.1046	0.1176	0.0183	45
0.0134	0.0267	0.0401	0.0534	0.0668	0.0802	0.0935	0.1069	0.1202	0.0187	46
0.0137	0.0273	0.0410	0.0546	0.0683	0.0819	0.0956	0.1092	0.1229	0.0191	47
0.0139	0.0279	0.0418	0.0558	0.0697	0.0836	0.0976	0.1115	0.1255	0.0195	48
0.0142	0.0285	0.0427	0.0569	0.0712	0.0854	0.0996	0.1138	0.1281	0.0200	49
0.0145	0.0290	0.0436	0.0581	0.0726	0.0871	0.1017	0.1162	0.1307	0.0204	50
0.0148	0.0296	0.0444	0.0592	0.0741	0.0889	0.1037	0.1185	0.1333	0.0208	51
0.0151	0.0302	0.0453	0.0604	0.0755	0.0906	0.1057	0.1208	0.1359	0.0212	52
0.0154	0.0308	0.0462	0.0616	0.0770	0.0923	0.1077	0.1231	0.1385	0.0216	53
0.0157	0.0314	0.0470	0.0627	0.0784	0.0941	0.1098	0.1254	0.1411	0.0220	54
0.0160	0.0319	0.0479	0.0639	0.0799	0.0958	0.1118	0.1278	0.1437	0.0224	55
0.0163	0.0325	0.0488	0.0650	0.0813	0.0976	0.1138	0.1301	0.1463	0.0228	56
0.0166	0.0331	0.0497	0.0662	0.0828	0.0993	0.1159	0.1324	0.1490	0.0232	57
0.0168	0.0337	0.0505	0.0674	0.0842	0.1011	0.1179	0.1348	0.1516	0.0236	58
0.0171	0.0343	0.0514	0.0685	0.0857	0.1028	0.1199	0.1371	0.1542	0.0240	59
0.0174	0.0349	0.0523	0.0697	0.0871	0.1046	0.1220	0.1394	0.1568	0.0244	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9983	1.9966	2.9949	3.9932	4.9915	5.9898	6.9881	7.9864	8.9847	1.3998
01	0.9983	1.9966	2.9949	3.9931	4.9914	5.9897	6.9880	7.9863	8.9846	1.3997
02	0.9983	1.9965	2.9948	3.9931	4.9914	5.9896	6.9879	7.9862	8.9845	1.3997
03	0.9983	1.9965	2.9948	3.9931	4.9913	5.9896	6.9878	7.9861	8.9844	1.3997
04	0.9983	1.9965	2.9948	3.9930	4.9913	5.9895	6.9878	7.9860	8.9843	1.3997
05	0.9982	1.9965	2.9947	3.9930	4.9912	5.9894	6.9877	7.9859	8.9842	1.3997
06	0.9982	1.9965	2.9947	3.9929	4.9912	5.9894	6.9876	7.9858	8.9841	1.3997
07	0.9982	1.9964	2.9947	3.9929	4.9911	5.9893	6.9875	7.9858	8.9840	1.3997
08	0.9982	1.9964	2.9946	3.9928	4.9910	5.9892	6.9875	7.9857	8.9839	1.3997
09	0.9982	1.9964	2.9946	3.9928	4.9910	5.9892	6.9874	7.9856	8.9838	1.3997
10	0.9982	1.9964	2.9946	3.9927	4.9909	5.9891	6.9873	7.9855	8.9837	1.3997
11	0.9982	1.9963	2.9945	3.9927	4.9909	5.9890	6.9872	7.9854	8.9836	1.3997
12	0.9982	1.9963	2.9945	3.9926	4.9908	5.9890	6.9871	7.9853	8.9834	1.3997
13	0.9981	1.9963	2.9944	3.9926	4.9907	5.9889	6.9870	7.9852	8.9833	1.3997
14	0.9981	1.9963	2.9944	3.9925	4.9907	5.9888	6.9870	7.9851	8.9832	1.3996
15	0.9981	1.9962	2.9944	3.9925	4.9906	5.9887	6.9869	7.9850	8.9831	1.3996
16	0.9981	1.9962	2.9943	3.9924	4.9906	5.9887	6.9868	7.9849	8.9830	1.3996
17	0.9981	1.9962	2.9943	3.9924	4.9905	5.9886	6.9867	7.9848	8.9829	1.3996
18	0.9981	1.9962	2.9943	3.9923	4.9904	5.9885	6.9866	7.9847	8.9828	1.3996
19	0.9981	1.9962	2.9942	3.9923	4.9904	5.9884	6.9865	7.9846	8.9827	1.3996
20	0.9981	1.9961	2.9942	3.9922	4.9903	5.9884	6.9864	7.9845	8.9825	1.3996
21	0.9980	1.9961	2.9941	3.9922	4.9902	5.9883	6.9863	7.9844	8.9824	1.3996
22	0.9980	1.9961	2.9941	3.9921	4.9902	5.9882	6.9862	7.9842	8.9823	1.3996
23	0.9980	1.9960	2.9941	3.9921	4.9901	5.9881	6.9861	7.9841	8.9822	1.3996
24	0.9980	1.9960	2.9940	3.9920	4.9900	5.9880	6.9860	7.9840	8.9820	1.3996
25	0.9980	1.9960	2.9940	3.9920	4.9899	5.9879	6.9859	7.9839	8.9819	1.3995
26	0.9980	1.9959	2.9939	3.9919	4.9899	5.9878	6.9858	7.9838	8.9818	1.3995
27	0.9980	1.9959	2.9939	3.9918	4.9898	5.9878	6.9857	7.9837	8.9816	1.3995
28	0.9979	1.9959	2.9938	3.9918	4.9897	5.9877	6.9856	7.9836	8.9815	1.3995
29	0.9979	1.9959	2.9938	3.9917	4.9897	5.9876	6.9855	7.9834	8.9814	1.3995
30	0.9979	1.9958	2.9937	3.9917	4.9896	5.9875	6.9854	7.9833	8.9812	1.3995
31	0.9979	1.9958	2.9937	3.9916	4.9895	5.9874	6.9853	7.9832	8.9811	1.3995
32	0.9979	1.9958	2.9937	3.9915	4.9894	5.9873	6.9852	7.9831	8.9810	1.3995
33	0.9979	1.9957	2.9936	3.9915	4.9893	5.9872	6.9851	7.9829	8.9808	1.3995
34	0.9979	1.9957	2.9936	3.9914	4.9893	5.9871	6.9850	7.9828	8.9807	1.3995
35	0.9978	1.9957	2.9935	3.9913	4.9892	5.9870	6.9849	7.9827	8.9805	1.3995
36	0.9978	1.9956	2.9935	3.9913	4.9891	5.9869	6.9847	7.9826	8.9804	1.3994
37	0.9978	1.9956	2.9934	3.9912	4.9890	5.9868	6.9846	7.9824	8.9802	1.3994
38	0.9978	1.9956	2.9934	3.9911	4.9889	5.9867	6.9845	7.9823	8.9801	1.3994
39	0.9978	1.9955	2.9933	3.9911	4.9889	5.9866	6.9844	7.9822	8.9799	1.3994
40	0.9978	1.9955	2.9933	3.9910	4.9888	5.9865	6.9843	7.9820	8.9798	1.3994
41	0.9977	1.9955	2.9932	3.9909	4.9887	5.9864	6.9842	7.9819	8.9796	1.3994
42	0.9977	1.9954	2.9932	3.9909	4.9886	5.9863	6.9840	7.9818	8.9795	1.3994
43	0.9977	1.9954	2.9931	3.9908	4.9885	5.9862	6.9839	7.9816	8.9793	1.3994
44	0.9977	1.9954	2.9931	3.9907	4.9884	5.9861	6.9838	7.9815	8.9792	1.3994
45	0.9977	1.9953	2.9930	3.9907	4.9883	5.9860	6.9837	7.9813	8.9790	1.3994
46	0.9976	1.9953	2.9929	3.9906	4.9882	5.9859	6.9835	7.9812	8.9788	1.3993
47	0.9976	1.9953	2.9929	3.9905	4.9882	5.9858	6.9834	7.9810	8.9787	1.3993
48	0.9976	1.9952	2.9928	3.9905	4.9881	5.9857	6.9833	7.9809	8.9785	1.3993
49	0.9976	1.9952	2.9928	5.9904	4.9880	5.9856	6.9832	7.9808	8.9784	1.3993
50	0.9976	1.9952	2.9927	3.9903	4.9879	5.9855	6.9830	7.9806	8.9782	1.3993
51	0.9976	1.9951	2.9927	3.9902	4.9878	5.9854	6.9829	7.9805	8.9780	1.3993
52	0.9975	1.9951	2.9926	3.9902	4.9877	5.9852	6.9828	7.9803	8.9779	1.3993
53	0.9975	1.9950	2.9926	3.9901	4.9876	5.9851	6.9826	7.9802	8.9777	1.3993
54	0.9975	1.9950	2.9925	3.9900	4.9875	5.9850	6.9825	7.9800	8.9775	1.3993
55	0.9975	1.9950	2.9924	3.9899	4.9874	5.9849	6.9824	7.9798	8.9773	1.3992
56	0.9975	1.9949	2.9924	3.9898	4.9873	5.9848	6.9822	7.9797	8.9772	1.3992
57	0.9974	1.9949	2.9923	3.9898	4.9872	5.9847	6.9821	7.9795	8.9770	1.3992
58	0.9974	1.9948	2.9923	3.9897	4.9871	5.9845	6.9820	7.9794	8.9768	1.3992
59	0.9974	1.9948	2.9922	3.9896	4.9870	5.9844	6.9818	7.9792	8.9766	1.3992
60	0.9974	1.9948	2.9922	3.9895	4.9869	5.9843	6.9817	7.9791	8.9765	1.3992

1	2	3	4	5	6	7	8	9	b	′
0.0174	0.0349	0.0523	0.0697	0.0871	0.1046	0.1220	0.1394	0.1568	0.0244	00
0.0177	0.0354	0.0531	0.0708	0.0886	0.1063	0.1240	0.1417	0.1594	0.0248	01
0.0180	0.0360	0.0540	0.0720	0.0900	0.1080	0.1260	0.1440	0.1620	0.0253	02
0.0183	0.0366	0.0549	0.0732	0.0915	0.1098	0.1281	0.1464	0.1647	0.0257	03
0.0186	0.0372	0.0558	0.0743	0.0929	0.1115	0.1301	0.1487	0.1673	0.0261	04
0.0189	0.0378	0.0566	0.0755	0.0944	0.1133	0.1321	0.1510	0.1699	0.0265	05
0.0192	0.0383	0.0575	0.0767	0.0958	0.1150	0.1342	0.1533	0.1725	0.0269	06
0.0195	0.0389	0.0584	0.0778	0.0973	0.1167	0.1362	0.1557	0.1751	0.0273	07
0.0197	0.0395	0.0592	0.0790	0.0987	0.1185	0.1382	0.1580	0.1777	0.0277	08
0.0200	0.0401	0.0601	0.0802	0.1002	0.1202	0.1403	0.1603	0.1803	0.0281	09
0.0203	0.0407	0.0610	0.0813	0.1016	0.1220	0.1423	0.1626	0.1830	0.0285	10
0.0206	0.0412	0.0619	0.0825	0.1031	0.1237	0.1443	0.1649	0.1856	0.0289	11
0.0209	0.0418	0.0627	0.0836	0.1045	0.1255	0.1464	0.1673	0.1882	0.0293	12
0.0212	0.0424	0.0636	0.0848	0.1060	0.1272	0.1484	0.1696	0.1908	0.0297	13
0.0215	0.0430	0.0645	0.0860	0.1075	0.1289	0.1504	0.1719	0.1934	0.0301	14
0.0218	0.0436	0.0653	0.0871	0.1089	0.1307	0.1525	0.1742	0.1960	0.0305	15
0.0221	0.0441	0.0662	0.0883	0.1104	0.1324	0.1545	0.1766	0.1986	0.0309	16
0.0224	0.0447	0.0671	0.0894	0.1118	0.1342	0.1565	0.1789	0.2012	0.0314	17
0.0227	0.0453	0.0680	0.0906	0.1133	0.1359	0.1586	0.1812	0.2039	0.0318	18
0.0229	0.0459	0.0688	0.0918	0.1147	0.1376	0.1606	0.1835	0.2065	0.0322	19
0.0232	0.0465	0.0697	0.0929	0.1162	0.1394	0.1626	0.1858	0.2091	0.0326	20
0.0235	0.0470	0.0706	0.0941	0.1176	0.1411	0.1646	0.1882	0.2117	0.0330	21
0.0238	0.0476	0.0714	0.0952	0.1191	0.1429	0.1667	0.1905	0.2143	0.0334	22
0.0241	0.0482	0.0723	0.0964	0.1205	0.1446	0.1687	0.1928	0.2169	0.0338	23
0.0244	0.0488	0.0732	0.0976	0.1220	0.1463	0.1707	0.1951	0.2195	0.0342	24
0.0247	0.0494	0.0740	0.0987	0.1234	0.1481	0.1728	0.1974	0.2221	0.0346	25
0.0250	0.0499	0.0749	0.0999	0.1249	0.1498	0.1748	0.1998	0.2247	0.0350	26
0.0253	0.0505	0.0758	0.1010	0.1263	0.1516	0.1768	0.2021	0.2273	0.0354	27
0.0256	0.0511	0.0767	0.1022	0.1278	0.1533	0.1789	0.2044	0.2300	0.0358	28
0.0258	0.0517	0.0775	0.1034	0.1292	0.1550	0.1809	0.2067	0.2326	0.0362	29
0.0261	0.0523	0.0784	0.1045	0.1307	0.1568	0.1829	0.2090	0.2352	0.0366	30
0.0264	0.0528	0.0793	0.1057	0.1321	0.1585	0.1849	0.2114	0.2378	0.0371	31
0.0267	0.0534	0.0801	0.1068	0.1336	0.1603	0.1870	0.2137	0.2404	0.0375	32
0.0270	0.0540	0.0810	0.1080	0.1350	0.1620	0.1890	0.2160	0.2430	0.0379	33
0.0273	0.0546	0.0819	0.1092	0.1365	0.1637	0.1910	0.2183	0.2456	0.0383	34
0.0276	0.0552	0.0827	0.1103	0.1379	0.1655	0.1931	0.2206	0.2482	0.0387	35
0.0279	0.0557	0.0836	0.1115	0.1394	0.1672	0.1951	0.2230	0.2508	0.0391	36
0.0282	0.0563	0.0845	0.1126	0.1408	0.1690	0.1971	0.2253	0.2534	0.0395	37
0.0285	0.0569	0.0854	0.1138	0.1423	0.1707	0.1992	0.2276	0.2561	0.0399	38
0.0287	0.0575	0.0862	0.1150	0.1437	0.1724	0.2012	0.2299	0.2587	0.0403	39
0.0290	0.0581	0.0871	0.1161	0.1452	0.1742	0.2032	0.2322	0.2613	0.0407	40
0.0293	0.0586	0.0880	0.1173	0.1466	0.1759	0.2052	0.2346	0.2639	0.0411	41
0.0296	0.0592	0.0888	0.1184	0.1481	0.1777	0.2073	0.2369	0.2665	0.0415	42
0.0299	0.0598	0.0897	0.1196	0.1495	0.1794	0.2093	0.2392	0.2691	0.0419	43
0.0302	0.0604	0.0906	0.1208	0.1510	0.1811	0.2113	0.2415	0.2717	0.0423	44
0.0305	0.0610	0.0914	0.1219	0.1524	0.1829	0.2134	0.2438	0.2743	0.0428	45
0.0308	0.0615	0.0923	0.1231	0.1539	0.1846	0.2154	0.2462	0.2769	0.0432	46
0.0311	0.0621	0.0932	0.1242	0.1553	0.1864	0.2174	0.2485	0.2795	0.0436	47
0.0314	0.0627	0.0941	0.1254	0.1568	0.1881	0.2195	0.2508	0.2822	0.0440	48
0.0316	0.0633	0.0949	0.1266	0.1582	0.1898	0.2215	0.2531	0.2848	0.0444	49
0.0319	0.0639	0.0958	0.1277	0.1597	0.1916	0.2235	0.2554	0.2874	0.0448	50
0.0322	0.0644	0.0967	0.1289	0.1611	0.1933	0.2255	0.2578	0.2900	0.0452	51
0.0325	0.0650	0.0975	0.1300	0.1626	0.1951	0.2276	0.2601	0.2926	0.0456	52
0.0328	0.0656	0.0984	0.1312	0.1640	0.1968	0.2296	0.2624	0.2952	0.0460	53
0.0331	0.0662	0.0993	0.1324	0.1655	0.1985	0.2316	0.2647	0.2978	0.0464	54
0.0334	0.0668	0.1001	0.1335	0.1669	0.2003	0.2336	0.2670	0.3004	0.0468	55
0.0337	0.0673	0.1010	0.1347	0.1684	0.2020	0.2357	0.2694	0.3030	0.0472	56
0.0340	0.0679	0.1019	0.1358	0.1698	0.2038	0.2377	0.2717	0.3056	0.0476	57
0.0342	0.0685	0.1027	0.1370	0.1712	0.2055	0.2397	0.2740	0.3082	0.0480	58
0.0345	0.0691	0.1036	0.1382	0.1727	0.2072	0.2418	0.2763	0.3109	0.0485	59
0.0348	0.0697	0.1045	0.1393	0.1742	0.2090	0.2438	0.2786	0.3135	0.0489	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9974	1.9948	2.9922	3.9895	4.9869	5.9843	6.9817	7.9791	8.9765	1.3992
01	0.9974	1.9947	2.9921	3.9895	4.9868	5.9842	6.9815	7.9789	8.9763	1.3992
02	0.9973	1.9947	2.9920	3.9894	4.9867	5.9841	6.9814	7.9787	8.9761	1.3992
03	0.9973	1.9946	2.9920	3.9893	4.9866	5.9839	6.9812	7.9786	8.9759	1.3992
04	0.9973	1.9946	2.9919	3.9892	4.9865	5.9838	6.9811	7.9784	8.9757	1.3991
05	0.9973	1.9946	2.9918	3.9891	4.9864	5.9837	6.9810	7.9782	8.9755	1.3991
06	0.9973	1.9945	2.9918	3.9890	4.9863	5.9835	6.9808	7.9781	8.9753	1.3991
07	0.9972	1.9945	2.9917	3.9889	4.9862	5.9834	6.9807	7.9779	8.9751	1.3991
08	0.9972	1.9944	2.9916	3.9889	4.9861	5.9833	6.9805	7.9777	8.9749	1.3991
09	0.9972	1.9944	2.9916	3.9888	4.9860	5.9832	6.9804	7.9776	8.9747	1.3991
10	0.9972	1.9943	2.9915	3.9887	4.9859	5.9830	6.9802	7.9774	8.9746	1.3991
11	0.9972	1.9943	2.9915	3.9886	4.9858	5.9829	6.9801	7.9772	8.9744	1.3990
12	0.9971	1.9943	2.9914	3.9885	4.9856	5.9828	6.9799	7.9770	8.9741	1.3990
13	0.9971	1.9942	2.9913	3.9884	4.9855	5.9826	6.9797	7.9768	8.9739	1.3990
14	0.9971	1.9942	2.9912	3.9883	4.9854	5.9825	6.9796	7.9767	8.9737	1.3990
15	0.9971	1.9941	2.9912	3.9882	4.9853	5.9824	6.9794	7.9765	8.9735	1.3990
16	0.9970	1.9941	2.9911	3.9881	4.9852	5.9822	6.9793	7.9763	8.9733	1.3990
17	0.9970	1.9940	2.9910	3.9881	4.9851	5.9821	6.9791	7.9761	8.9731	1.3989
18	0.9970	1.9940	2.9910	3.9880	4.9850	5.9819	6.9789	7.9759	8.9729	1.3989
19	0.9970	1.9939	2.9909	3.9879	4.9848	5.9818	6.9788	7.9757	8.9727	1.3989
20	0.9969	1.9939	2.9908	3.9378	4.9847	5.9817	6.9786	7.9756	8.9725	1.3989
21	0.9969	1.9938	2.9908	3.9877	4.9846	5.9815	6.9784	7.9754	8.9723	1.3989
22	0.9969	1.9938	2.9907	3.9876	4.9845	5.9814	6.9783	7.9752	8.9721	1.3989
23	0.9969	1.9937	2.9906	3.9875	4.9844	5.9812	6.9781	7.9750	8.9718	1.3988
24	0.9968	1.9937	2.9905	3.9874	4.9842	5.9811	6.9779	7.9748	8.9716	1.3988
25	0.9968	1.9936	2.9905	3.9873	4.9841	5.9809	6.9778	7.9746	8.9714	1.3988
26	0.9968	1.9936	2.9904	3.9872	4.9840	5.9808	6.9776	7.9744	8.9712	1.3988
27	0.9968	1.9935	2.9903	3.9871	4.9839	5.9806	6.9774	7.9742	8.9710	1.3988
28	0.9967	1.9935	2.9902	3.9870	4.9837	5.9805	6.9772	7.9740	8.9707	1.3987
29	0.9967	1.9934	2.9902	3.9869	4.9836	5.9803	6.9771	7.9738	8.9705	1.3987
30	0.9967	1.9934	2.9901	3.9868	4.9835	5.9802	6.9769	7.9736	8.9703	1.3987
31	0.9967	1.9933	2.9900	3.9867	4.9834	5.9800	6.9767	7.9734	8.9701	1.3987
32	0.9966	1.9933	2.9899	3.9866	4.9832	5.9799	6.9765	7.9732	8.9698	1.3987
33	0.9966	1.9932	2.9899	3.9865	4.9831	5.9797	6.9764	7.9730	8.9696	1.3987
34	0.9966	1.9932	2.9898	3.9864	4.9830	5.9796	6.9762	7.9728	8.9694	1.3986
35	0.9966	1.9931	2.9897	3.9863	4.9828	5.9794	6.9760	7.9726	8.9691	1.3986
36	0.9965	1.9931	2.9896	3.9862	4.9827	5.9793	6.9758	7.9723	8.9689	1.3986
37	0.9965	1.9930	2.9896	3.9861	4.9826	5.9791	6.9756	7.9721	8.9687	1.3986
38	0.9965	1.9930	2.9895	3.9860	4.9825	5.9789	6.9754	7.9719	8.9684	1.3986
39	0.9965	1.9929	2.9894	3.9859	4.9823	5.9788	6.9753	7.9717	8.9682	1.3985
40	0.9964	1.9929	2.9893	3.9858	4.9822	5.9786	6.9751	7.9715	8.9680	1.3985
41	0.9964	1.9928	2.9892	3.9856	4.9821	5.9785	6.9749	7.9713	8.9677	1.3985
42	0.9964	1.9928	2.9891	3.9855	4.9819	5.9783	6.9747	7.9711	8.9674	1.3985
43	0.9964	1.9927	2.9891	3.9854	4.9818	5.9781	6.9745	7.9708	8.9672	1.3985
44	0.9963	1.9927	2.9890	3.9853	4.9816	5.9780	6.9743	7.9706	8.9669	1.3984
45	0.9963	1.9926	2.9889	3.9852	4.9815	5.9778	6.9741	7.9704	8.9667	1.3984
46	0.9963	1.9925	2.9888	3.9851	4.9814	5.9776	6.9739	7.9702	8.9664	1.3984
47	0.9962	1.9925	2.9887	3.9850	4.9812	5.9775	6.9737	7.9700	8.9662	1.3984
48	0.9962	1.9924	2.9886	3.9849	4.9811	5.9773	6.9735	7.9697	8.9659	1.3984
49	0.9962	1.9924	2.9886	3.9848	4.9809	5.9771	6.9733	7.9695	8.9657	1.3983
50	0.9962	1.9923	2.9885	3.9846	4.9808	5.9770	6.9731	7.9693	8.9654	1.3983
51	0.9961	1.9923	2.9884	3.9845	4.9807	5.9768	6.9729	7.9690	8.9652	1.3983
52	0.9961	1.9922	2.9883	3.9844	4.9805	5.9766	6.9727	7.9688	8.9649	1.3983
53	0.9961	1.9921	2.9882	3.9843	4.9804	5.9764	6.9725	7.9686	8.9646	1.3983
54	0.9960	1.9921	2.9881	3.9842	4.9802	5.9763	6.9723	7.9683	8.9644	1.3982
55	0.9960	1.9920	2.9880	3.9841	4.9801	5.9761	6.9721	7.9681	8.9641	1.3982
56	0.9960	1.9920	2.9879	3.9839	4.9799	5.9759	6.9719	7.9679	8.9638	1.3982
57	0.9960	1.9919	2.9879	3.9838	4.9798	5.9757	6.9717	7.9676	8.9636	1.3982
58	0.9959	1.9918	2.9878	3.9837	4.9796	5.9756	6.9715	7.9674	8.9633	1.3981
59	0.9959	1.9918	2.9877	3.9836	4.9795	5.9754	6.9713	7.9672	8.9631	1.3981
60	0.9959	1.9917	2.9876	3.9835	4.9793	5.9752	6.9711	7.9669	8.9628	1.3981

1	2	3	4	5	6	7	8	9	b	′
0.0348	0.0697	0.1045	0.1393	0.1742	0.2090	0.2438	0.2786	0.3135	0.0489	00
0.0351	0.0702	0.1054	0.1405	0.1756	0.2107	0.2458	0.2810	0.3161	0.0493	01
0.0354	0.0708	0.1062	0.1416	0.1771	0.2125	0.2479	0.2833	0.3187	0.0497	02
0.0357	0.0714	0.1071	0.1428	0.1785	0.2142	0.2499	0.2856	0.3213	0.0501	03
0.0360	0.0720	0.1080	0.1440	0.1800	0.2159	0.2519	0.2879	0.3239	0.0505	04
0.0363	0.0726	0.1088	0.1451	0.1814	0.2177	0.2540	0.2902	0.3265	0.0509	05
0.0366	0.0731	0.1097	0.1463	0.1828	0.2194	0.2560	0.2926	0.3291	0.0513	06
0.0369	0.0737	0.1106	0.1474	0.1843	0.2212	0.2580	0.2949	0.3317	0.0517	07
0.0371	0.0743	0.1114	0.1486	0.1857	0.2229	0.2600	0.2972	0.3343	0.0521	08
0.0374	0.0749	0.1123	0.1498	0.1872	0.2246	0.2621	0.2995	0.3370	0.0525	09
0.0377	0.0755	0.1132	0.1509	0.1886	0.2264	0.2641	0.3018	0.3396	0.0529	10
0.0380	0.0760	0.1141	0.1521	0.1901	0.2281	0.2661	0.3042	0.3422	0.0533	11
0.0383	0.0766	0.1149	0.1532	0.1915	0.2299	0.2682	0.3065	0.3448	0.0537	12
0.0386	0.0772	0.1158	0.1544	0.1930	0.2316	0.2702	0.3088	0.3474	0.0541	13
0.0389	0.0778	0.1167	0.1556	0.1944	0.2333	0.2722	0.3111	0.3500	0.0546	14
0.0392	0.0783	0.1175	0.1567	0.1959	0.2350	0.2742	0.3134	0.3526	0.0550	15
0.0395	0.0789	0.1184	0.1578	0.1973	0.2368	0.2762	0.3157	0.3552	0.0554	16
0.0398	0.0795	0.1193	0.1590	0.1988	0.2385	0.2783	0.3180	0.3578	0.0558	17
0.0400	0.0801	0.1201	0.1602	0.2002	0.2402	0.2803	0.3203	0.3604	0.0562	18
0.0403	0.0807	0.1210	0.1613	0.0017	0.2420	0.2823	0.3226	0.3630	0.0566	19
0.0406	0.0812	0.1219	0.1625	0.2031	0.2437	0.2843	0.3250	0.3656	0.0570	20
0.0409	0.0818	0.1227	0.1636	0.2046	0.2455	0.2864	0.3273	0.3682	0.0574	21
0.0412	0.0824	0.1236	0.1648	0.2060	0.2472	0.2884	0.3296	0.3708	0.0578	22
0.0415	0.0830	0.1245	0.1660	0.2075	0.2489	0.2904	0.3319	0.3734	0.0582	23
0.0418	0.0836	0.1253	0.1671	0.2089	0.2507	0.2925	0.3342	0.3760	0.0586	24
0.0421	0.0841	0.1262	0.1683	0.2103	0.2524	0.2945	0.3366	0.3786	0.0590	25
0.0424	0.0847	0.1271	0.1694	0.2118	0.2542	0.2965	0.3389	0.3812	0.0594	26
0.0426	0.0853	0.1279	0.1706	0.2132	0.2559	0.2985	0.3412	0.3838	0.0598	27
0.0429	0.0859	0.1288	0.1718	0.2147	0.2576	0.3006	0.3435	0.3865	0.0602	28
0.0432	0.0865	0.1297	0.1729	6.2161	0.2594	0.3026	0.3458	0.3891	0.0607	29
0.0435	0.0870	0.1306	0.1741	0.2176	0.2611	0.3046	0.3482	0.3917	0.0611	30
0.0438	0.0876	0.1314	0.1752	0.2190	0.2629	0.3067	0.3505	0.3943	0.0615	31
0.0441	0.0882	0.1323	0.1764	0.2205	0.2646	0.3087	0.3528	0.3969	0.0619	32
0.0444	0.0888	0.1331	0.1775	0.2219	0.2663	0.3107	0.3551	0.3995	0.0623	33
0.0447	0.0893	0.1340	0.1787	0.2234	0.2680	0.3127	0.3574	0.4021	0.0627	34
0.0450	0.0899	0.1349	0.1798	0.2248	0.2698	0.3147	0.3597	0.4047	0.0631	35
0.0453	0.0905	0.1358	0.1810	0.2263	0.2715	0.3168	0.3620	0.4073	0.0635	36
0.0455	0.0911	0.1366	0.1822	0.2277	0.2732	0.3188	0.3643	0.4099	0.0639	37
0.0458	0.0917	0.1375	0.1833	0.2292	0.2750	0.3208	0.3666	0.4125	0.0643	38
0.0461	0.0922	0.1384	0.1845	0.2306	0.2767	0.3228	0.3690	0.4151	0.0647	39
0.0464	0.0928	0.1392	0.1856	0.2321	0.2785	0.3249	0.3713	0.4177	0.0651	40
0.0467	0.0934	0.1401	0.1868	0.2335	0.2802	0.3269	0.3736	0.3203	0.0655	41
0.0470	0.0940	0.1410	0.1880	0.2350	0.2819	0.3289	0.3759	0.4229	0.0659	42
0.0473	0.0946	0.1418	0.1891	0.2364	0.2837	0.3310	0.3782	0.4255	0.0664	43
0.0476	0.0951	0.1427	0.1903	0.2378	0.2854	0.3330	0.3806	0.4281	0.0668	44
0.0479	0.0957	0.1436	0.1914	0.2393	0.2872	0.3350	0.3829	0.4307	0.0672	45
0.0481	0.0963	0.1444	0.1926	0.2407	0.2889	0.3370	0.3852	0.4333	0.0676	46
0.0484	0.0969	0.1453	0.1937	0.2422	0.2906	0.3390	0.3875	0.4359	0.0680	47
0.0487	0.0974	0.1462	0.1949	0.2436	0.2923	0.3410	0.3898	0.4385	0.0684	48
0.0490	0.0980	0.1470	0.1960	0.2451	0.2941	0.3431	0.3921	0.4411	0.0688	49
0.0493	0.0986	0.1479	0.1972	0.2465	0.2958	0.3451	0.3944	0.4437	0.0692	50
0.0496	0.0992	0.1488	0.1984	0.2480	0.2975	0.3471	0.3967	0.4463	0.0696	51
0.0499	0.0998	0.1496	0.1995	0.2494	0.2993	0.3492	0.3990	0.4489	0.0700	52
0.0502	0.1003	0.1505	0.2007	0.2508	0.3010	0.3512	0.4014	0.4515	0.0704	53
0.0505	0.1009	0.1514	0.2018	0.2523	0.3028	0.3532	0.4037	0.4541	0.0708	54
0.0507	0.1015	0.1522	0.2030	0.2537	0.3045	0.3552	0.4060	0.4567	0.0712	55
0.0510	0.1021	0.1531	0.2042	0.2552	0.3062	0.3573	0.4083	0.4593	0.0716	56
0.0513	0.1026	0.1540	0.2053	0.2566	0.3079	0.3593	0.4106	0.4619	0.0721	57
0.0516	0.1032	0.1548	0.2064	0.2581	0.3097	0.3613	0.4129	0.4645	0.0725	58
0.0519	0.1038	0.1557	0.2076	0.2595	0.3114	0.3633	0.4152	0.4671	0.0729	59
0.0522	0.1044	0.1566	0.2088	0.2610	0.3131	0.3653	0.4175	0.4697	0.0733	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9959	1.9917	2.9876	3.9835	4.9793	5.9752	6.9711	7.9669	8.9628	1.3981
01	0.9958	1.9917	2.9875	3.9833	4.9792	5.9750	6.9708	7.9667	8.9625	1.3981
02	0.9958	1.9916	2.9874	3.9832	4.9790	5.9748	6.9706	7.9664	8.9622	1.3981
03	0.9958	1.9915	2.9873	3.9831	4.9789	5.9746	6.9704	7.9662	8.9619	1.3980
04	0.9957	1.9915	2.9872	3.9830	4.9787	5.9744	6.9702	7.9659	8.9617	1.3980
05	0.9957	1.9914	2.9871	3.9828	4.9785	5.9743	6.9700	7.9657	8.9614	1.3980
06	0.9957	1.9914	2.9870	3.9827	4.9784	5.9741	6.9697	7.9654	8.9611	1.3980
07	0.9956	1.9913	2.9869	3.9826	4.9782	5.9739	6.9695	7.9652	8.9608	1.3980
08	0.9956	1.9912	2.9868	3.9825	4.9781	5.9737	6.9693	7.9649	8.9605	1.3979
09	0.9956	1.9912	2.9868	3.9823	4.9779	5.9735	6.9691	7.9647	8.9603	1.3979
10	0.9956	1.9911	2.9867	3.9822	4.9778	5.9733	6.9689	7.9644	8.9600	1.3979
11	0.9955	1.9910	2.9866	3.9821	4.9776	5.9731	6.9686	7.9642	8.9597	1.3979
12	0.9955	1.9910	2.9865	3.9819	4.9774	5.9729	6.9684	7.9639	8.9594	1.3978
13	0.9955	1.9909	2.9864	3.9818	4.9773	5.9727	6.9682	7.9636	8.9591	1.3978
14	0.9954	1.9908	2.9863	3.9817	4.9771	5.9725	6.9679	7.9634	8.9588	1.3978
15	0.9954	1.9908	2.9862	3.9816	4.9769	5.9723	6.9677	7.9631	8.9585	1.3978
16	0.9954	1.9907	2.9861	3.9814	4.9768	5.9721	6.9675	7.9628	8.9582	1.3977
17	0.9953	1.9906	2.9860	3.9813	4.9768	5.9719	6.9673	7.9626	8.9579	1.3977
18	0.9953	1.9906	2.9859	3.9812	4.9764	5.9717	6.9670	7.9623	8.9576	1.3977
19	0.9953	1.9905	2.9858	3.9810	4.9763	5.9715	6.9668	7.9621	8.9573	1.3977
20	0.9952	1.9904	2.9857	3.9809	4.9761	5.9713	6.9666	7.9618	8.9570	1.3976
21	0.9952	1.9904	2.9856	3.9808	4.9759	5.9711	6.9663	7.9615	8.9567	1.3976
22	0.9952	1.9903	2.9855	3.9806	4.9758	5.9709	6.9661	7.9612	8.9564	1.3976
23	0.9951	1.9902	2.9854	3.9805	4.9756	5.9707	6.9658	7.9610	8.9561	1.3976
24	0.9951	1.9902	2.9853	3.9803	4.9754	5.9705	6.9656	7.9607	8.9558	1.3975
25	0.9951	1.9901	2.9852	3.9802	4.9753	5.9703	6.9654	7.9604	8.9555	1.3975
26	0.9950	1.9900	2.9850	3.9801	4.9751	5.9701	6.9651	7.9601	8.9551	1.3975
27	0.9950	1.9900	2.9849	3.9799	4.9749	5.9699	6.9649	7.9599	8.9548	1.3975
28	0.9949	1.9899	2.9848	3.9798	4.9747	5.9697	6.9646	7.9596	8.9545	1.3974
29	0.9949	1.9898	2.9847	3.9797	4.9746	5.9695	6.9644	7.9593	8.9542	1.3974
30	0.9949	1.9898	2.9846	3.9795	4.9744	5.9693	6.9641	7.9590	8.9539	1.3974
31	0.9948	1.9897	2.9845	3.9794	4.9742	5.9691	6.9639	7.9587	8.9536	1.3973
32	0.9948	1.9896	2.9844	3.9792	4.9740	5.9688	6.9636	7.9584	8.9533	1.3973
33	0.9948	1.9895	2.9843	3.9791	4.9738	5.9686	6.9634	7.9582	8.9529	1.3973
34	0.9947	1.9895	2.9842	3.9789	4.9737	5.9684	6.9631	7.9579	8.9526	1.3973
35	0.9947	1.9894	2.9841	3.9788	4.9735	5.9682	6.9629	7.9576	8.9523	1.3972
36	0.9947	1.9893	2.9840	3.9786	4.9733	5.9680	6.9626	7.9573	8.9519	1.3972
37	0.9946	1.9893	2.9839	3.9785	4.9731	5.9678	6.9624	7.9570	8.9516	1.3972
38	0.9946	1.9892	2.9838	3.9784	4.9729	5.9675	6.9621	7.9567	8.9513	1.3972
39	0.9946	1.9891	2.9837	3.9782	4.9728	5.9673	6.9619	7.9564	8.9510	1.3971
40	0.9945	1.9890	2.9835	3.9781	4.9726	5.9671	6.9616	7.9561	8.9506	1.3971
41	0.9945	1.9890	2.9834	3.9779	4.9724	5.9669	6.9613	7.9558	8.9503	1.3971
42	0.9944	1.9889	2.9833	3.9778	4.9722	5.9666	6.9611	7.9555	8.9500	1.3971
43	0.9944	1.9888	2.9832	3.9776	4.9720	5.9664	6.9608	7.9552	8.9496	1.3970
44	0.9944	1.9887	2.9831	3.9775	4.9718	5.9662	6.9605	7.9549	8.9493	1.3970
45	0.9943	1.9887	2.9830	3.9773	4.9716	5.9660	6.9603	7.9546	8.9489	1.3970
46	0.9943	1.9886	2.9829	3.9772	4.9714	5.9657	6.9600	7.9543	8.9486	1.3969
47	0.9943	1.9885	2.9828	3.9770	4.9713	5.9655	6.9598	7.9540	8.9483	1.3969
48	0.9942	1.9884	2.9826	3.9769	4.9711	5.9653	6.9595	7.9537	8.9479	1.3969
49	0.9942	1.9884	2.9825	3.9767	4.9709	5.9651	6.9592	7.9534	8.9476	1.3969
50	0.9941	1.9883	2.9824	3.9765	4.9707	5.9648	6.9590	7.9531	8.9472	1.3968
51	0.9941	1.9882	2.9823	3.9764	4.9705	5.9646	6.9587	7.9528	8.9469	1.3968
52	0.9941	1.9881	2.9822	3.9762	4.9703	5.9643	6.9584	7.9525	8.9465	1.3968
53	0.9940	1.9880	2.9821	3.9761	4.9701	5.9641	6.9581	7.9521	8.9462	1.3968
54	0.9940	1.9880	2.9819	3.9759	4.9699	5.9639	6.9579	7.9518	8.9458	1.3967
55	0.9939	1.9879	2.9818	3.9758	4.9697	5.9636	6.9576	7.9515	8.9455	1.3967
56	0.9939	1.9878	2.9817	3.9756	4.9695	5.9634	6.9573	7.9512	8.9451	1.3967
57	0.9939	1.9877	2.9816	3.9754	4.9693	5.9632	6.9570	7.9509	8.9447	1.3967
58	0.9938	1.9876	2.9815	3.9753	4.9691	5.9629	6.9567	7.9506	8.9444	1.3966
59	0.9938	1.9876	2.9813	3.9751	4.9689	5.9627	6.9565	7.9502	8.9440	1.3966
60	0.9937	1.9875	2.9812	3.9750	4.9687	5.9624	6.9562	7.9499	8.9437	1.3966

1	2	3	4	5	6	7	8	9	b	′
0.0522	0.1044	0.1566	0.2088	0.2610	0.3131	0.3653	0.4175	0.4697	0.0733	00
0.0525	0.1050	0.1574	0.2099	0.2624	0.3149	0.3674	0.4198	0.4723	0.0737	01
0.0528	0.1055	0.1583	0.2111	0.2638	0.3166	0.3694	0.4222	0.4749	0.0741	02
0.0531	0.1061	0.1592	0.2122	0.2653	0.3184	0.3714	0.4245	0.4775	0.0745	03
0.0533	0.1067	0.1600	0.2134	0.2667	0.3201	0.3734	0.4268	0.4801	0.0749	04
0.0536	0.1073	0.1609	0.2145	0.2682	0.3218	0.3754	0.4291	0.4827	0.0753	05
0.0539	0.1078	0.1618	0.2157	0.2696	0.3235	0.3774	0.4314	0.4853	0.0757	06
0.0542	0.1084	0.1626	0.2168	0.2711	0.3253	0.3795	0.4337	0.4879	0.0761	07
0.0545	0.1090	0.1635	0.2180	0.2725	0.3270	0.3815	0.4360	0.4905	0.0765	08
0.0548	0.1096	0.1644	0.2192	0.2739	0.3287	0.3835	0.4383	0.4931	0.0769	09
0.0551	0.1102	0.1652	0.2293	0.2754	0.3305	0.3856	0.4406	0.4957	0.0773	10
0.0554	0.1107	0.1661	0.2215	0.2768	0.3322	0.3876	0.4430	0.4983	0.0777	11
0.0557	0.1113	0.1670	0.2226	0.2783	0.3340	0.3896	0.4453	0.5009	0.0781	12
0.0559	0.1119	0.1678	0.2238	0.2797	0.3356	0.3916	0.4475	0.5035	0.0786	13
0.0562	0.1125	0.1687	0.2249	0.2812	0.3374	0.3936	0.4498	0.5061	0.0790	14
0.0565	0.1130	0.1696	0.2261	0.2826	0.3391	0.3956	0.4522	0.5087	0.0794	15
0.0568	0.1136	0.1704	0.2272	0.2841	0.3409	0.3977	0.4545	0.5113	0.0798	16
0.0571	0.1142	0.1713	0.2284	0.2855	0.3426	0.3997	0.4568	0.5139	0.0802	17
0.0574	0.1148	0.1722	0.2296	0.2869	0.3443	0.4017	0.4591	0.5165	0.0806	18
0.0577	0.1154	0.1730	0.2307	0.2884	0.3461	0.4038	0.4614	0.5191	0.0810	19
0.0580	0.1159	0.1739	0.2319	0.2898	0.3478	0.4058	0.4638	0.5217	0.0814	20
0.0583	0.1165	0.1748	0.2330	0.2913	0.3495	0.4078	0.4660	0.5243	0.0818	21
0.0585	0.1171	0.1756	0.2342	0.2927	0.3512	0.4098	0.4683	0.5269	0.0822	22
0.0588	0.1177	0.1765	0.2353	0.2942	0.3530	0.4118	0.4706	0.5295	0.0826	23
0.0591	0.1182	0.1774	0.2365	0.2956	0.3547	0.4138	0.4730	0.5321	0.0830	24
0.0594	0.1188	0.1782	0.2376	0.2971	0.3565	0.4159	0.4753	0.5347	0.0834	25
0.0597	0.1194	0.1791	0.2388	0.2985	0.3582	0.4179	0.4776	0.5373	0.0838	26
0.0600	0.1200	0.1799	0.2399	0.2999	0.3599	0.4199	0.4799	0.5399	0.0842	27
0.0603	0.1205	0.1808	0.2411	0.3014	0.3616	0.4219	0.4822	0.5425	0.0847	28
0.0606	0.1211	0.1817	0.2422	0.3028	0.3634	0.4239	0.4845	0.5451	0.0851	29
0.0608	0.1217	0.1825	0.2434	0.3042	0.3651	0.4259	0.4868	0.5477	0.0855	30
0.0611	0.1223	0.1834	0.2446	0.3057	0.3668	0.4280	0.4891	0.5503	0.0859	31
0.0614	0.1229	0.1843	0.2457	0.3071	0.3686	0.4300	0.4914	0.5529	0.0863	32
0.0617	0.1234	0.1851	0.2468	0.3086	0.3703	0.4320	0.4937	0.5554	0.0867	33
0.0620	0.1240	0.1860	0.2480	0.3100	0.3720	0.4340	0.4960	0.5580	0.0871	34
0.0623	0.1246	0.1869	0.2492	0.3115	0.3737	0.4360	0.4983	0.5606	0.0875	35
0.0626	0.1252	0.1877	0.2503	0.3129	0.3755	0.4381	0.5006	0.5632	0.0879	36
0.0629	0.1257	0.1886	0.2515	0.3143	0.3772	0.4401	0.5030	0.5658	0.0883	37
0.0632	0.1263	0.1895	0.2526	0.3158	0.3789	0.4421	0.5053	0.5684	0.0887	38
0.0634	0.1269	0.1903	0.2538	0.3172	0.3806	0.4441	0.5075	0.5710	0.0891	39
0.0637	0.1275	0.1912	0.2549	0.3187	0.3824	0.4461	0.5098	0.5736	0.0895	40
0.0640	0.1280	0.1921	0.2561	0.3201	0.3841	0.4481	0.5122	0.5762	0.0899	41
0.0643	0.1286	0.1929	0.2572	0.3215	0.3859	0.4502	0.5145	0.5788	0.0903	42
0.0646	0.1292	0.1938	0.2584	0.3230	0.3876	0.4522	0.5168	0.5814	0.0908	43
0.0649	0.1298	0.1946	0.2595	0.3244	0.3893	0.4542	0.5190	0.5839	0.0912	44
0.0652	0.1303	0.1955	0.2607	0.3259	0.3910	0.4562	0.5214	0.5865	0.0916	45
0.0655	0.1309	0.1964	0.2618	0.3273	0.3928	0.4582	0.5237	0.5891	0.0920	46
0.0657	0.1315	0.1972	0.2630	0.3287	0.3945	0.4602	0.5260	0.5917	0.0924	47
0.0660	0.1321	0.1981	0.2642	0.3302	0.3962	0.4622	0.5283	0.5943	0.0928	48
0.0663	0.1326	0.1990	0.2653	0.3316	0.3979	0.4642	0.5306	0.5969	0.0932	49
0.0666	0.1332	0.1998	0.2664	0.3331	0.3997	0.4663	0.5329	0.5995	0.0936	50
0.0669	0.1338	0.2007	0.2676	0.3345	0.4014	0.4683	0.5352	0.6021	0.0940	51
0.0672	0.1344	0.2016	0.2688	0.3359	0.4031	0.4703	0.5375	0.6047	0.0944	52
0.0675	0.1349	0.2024	0.2699	0.3374	0.4048	0.4723	0.5398	0.6073	0.0948	53
0.0678	0.1355	0.2033	0.2710	0.3388	0.4066	0.4743	0.5421	0.6099	0.0952	54
0.0681	0.1361	0.2042	0.2722	0.3403	0.4083	0.4764	0.5444	0.6125	0.0956	55
0.0683	0.1367	0.2050	0.2734	0.3417	0.4100	0.4784	0.5467	0.6151	0.0961	56
0.0686	0.1373	0.2059	0.2745	0.3431	0.4118	0.4804	0.5490	0.6177	0.0965	57
0.0689	0.1378	0.2067	0.2756	0.3446	0.4135	0.4824	0.5513	0.6202	0.0969	58
0.0692	0.1384	0.2076	0.2768	0.3460	0.4152	0.4844	0.5536	0.6228	0.0973	59
0.0695	0.1390	0.2085	0.2780	0.3474	0.4169	0.4864	0.5559	0.6254	0.0977	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9937	1.9875	2.9812	3.9750	4.9687	5.9624	6.9562	7.9499	8.9437	1.3966
01	0.9937	1.9874	2.9811	3.9748	4.9685	5.9622	6.9559	7.9496	8.9433	1.3966
02	0.9937	1.9873	2.9810	3.9746	4.9683	5.9619	6.9556	7.9493	8.9429	1.3965
03	0.9936	1.9872	2.9809	3.9745	4.9681	5.9617	6.9553	7.9489	8.9426	1.3965
04	0.9936	1.9872	2.9807	3.9743	4.9679	5.9615	6.9550	7.9486	8.9422	1.3965
05	0.9935	1.9871	2.9806	3.9741	4.9677	5.9612	6.9547	7.9483	8.9418	1.3965
06	0.9935	1.9870	2.9805	3.9740	4.9675	5.9610	6.9545	7.9479	8.9414	1.3964
07	0.9935	1.9869	2.9804	3.9738	4.9673	5.9607	6.9542	7.9476	8.9411	1.3964
08	0.9934	1.9868	2.9802	3.9736	4.9671	5.9605	6.9539	7.9473	8.9407	1.3964
09	0.9934	1.9867	2.9801	3.9735	4.9668	5.9602	6.9536	7.9470	8.9403	1.3963
10	0.9933	1.9867	2.9800	3.9734	4.9666	5.9600	6.9533	7.9466	8.9400	1.3963
11	0.9933	1.9866	2.9799	3.9731	4.9664	5.9597	6.9530	7.9463	8.9396	1.3963
12	0.9932	1.9865	2.9797	3.9730	4.9662	5.9595	6.9527	7.9459	8.9392	1.3963
13	0.9932	1.9864	2.9796	3.9728	4.9660	5.9592	6.9524	7.9456	8.9388	1.3962
14	0.9932	1.9863	2.9795	3.9726	4.9658	5.9589	6.9521	7.9452	8.9384	1.3962
15	0.9931	1.9862	2.9793	3.9725	4.9656	5.9587	6.9518	7.9449	8.9380	1.3962
16	0.9931	1.9861	2.9792	3.9723	4.9654	5.9584	6.9515	7.9446	8.9376	1.3962
17	0.9930	1.9861	2.9791	3.9721	4.9651	5.9582	6.9512	7.9442	8.9373	1.3961
18	0.9930	1.9860	2.9790	3.9719	4.9649	5.9579	6.9509	7.9439	8.9369	1.3961
19	0.9929	1.9859	2.9788	3.9718	4.9647	5.9577	6.9506	7.9435	8.9365	1.3961
20	0.9929	1.9858	2.9787	3.9716	4.9645	5.9574	6.9503	7.9432	8.9361	1.3960
21	0.9929	1.9857	2.9786	3.9714	4.9643	5.9571	6.9500	7.9428	8.9357	1.3960
22	0.9928	1.9856	2.9784	3.9712	4.9641	5.9569	6.9497	7.9425	8.9353	1.3960
23	0.9928	1.9855	2.9783	3.9711	4.9638	5.9566	6.9494	7.9421	8.9349	1.3959
24	0.9927	1.9854	2.9782	3.9709	4.9636	5.9563	6.9490	7.9418	8.9345	1.3959
25	0.9927	1.9854	2.9780	3.9707	4.9634	5.9561	6.9487	7.9414	8.9341	1.3959
26	0.9926	1.9853	2.9779	3.9705	4.9632	5.9558	6.9484	7.9410	8.9337	1.3958
27	0.9926	1.9852	2.9778	3.9703	4.9629	5.9555	6.9481	7.9407	8.9333	1.3958
28	0.9925	1.9851	2.9776	3.9702	4.9627	5.9553	6.9478	7.9403	8.9329	1.3958
29	0.9925	1.9850	2.9775	3.9700	4.9625	5.9550	6.9475	7.9400	8.9325	1.3958
30	0.9925	1.9849	2.9774	3.9698	4.9623	5.9547	6.9472	7.9396	8.9321	1.3957
31	0.9924	1.9848	2.9772	3.9696	4.9620	5.9544	6.9468	7.9393	8.9317	1.3957
32	0.9924	1.9847	2.9771	3.9694	4.9618	5.9542	6.9465	7.9389	8.9312	1.3957
33	0.9923	1.9846	2.9769	3.9693	4.9616	5.9539	6.9462	7.9385	8.9308	1.3956
34	0.9923	1.9845	2.9768	3.9691	4.9613	5.9536	6.9459	7.9381	8.9304	1.3956
35	0.9922	1.9844	2.9767	3.9689	4.9611	5.9533	6.9456	7.9378	8.9300	1.3956
36	0.9922	1.9844	2.9765	3.9687	4.9609	5.9531	6.9452	7.9374	8.9296	1.3955
37	0.9921	1.9843	2.9764	3.9685	4.9606	5.9528	6.9449	7.9370	8.9292	1.3955
38	0.9921	1.9842	2.9762	3.9683	4.9604	5.9525	6.9446	7.9367	8.9287	1.3955
39	0.9920	1.9841	2.9761	3.9681	4.9602	5.9522	6.9443	7.9363	8.9283	1.3954
40	0.9920	1.9840	2.9760	3.9680	4.9600	5.9519	6.9439	7.9359	8.9279	1.3954
41	0.9919	1.9839	2.9758	3.9678	4.9597	5.9517	6.9436	7.9355	8.9275	1.3954
42	0.9919	1.9838	2.9757	3.9676	4.9595	5.9514	6.9433	7.9352	8.9270	1.3953
43	0.9918	1.9837	2.9755	3.9674	4.9592	5.9511	6.9429	7.9348	8.9266	1.3953
44	0.9918	1.9836	2.9754	3.9672	4.9590	5.9508	6.9426	7.9344	8.9262	1.3953
45	0.9918	1.9835	2.9753	3.9670	4.9588	5.9505	6.9423	7.9340	8.9258	1.3952
46	0.9917	1.9834	2.9751	3.9668	4.9585	5.9502	6.9419	7.9336	8.9253	1.3952
47	0.9917	1.9833	2.9750	3.9566	4.9583	5.9499	6.9416	7.9332	8.9249	1.3952
48	0.9916	1.9832	2.9748	3.9664	4.9580	5.9496	6.9412	7.9329	8.9245	1.3951
49	0.9916	1.9831	2.9747	3.9662	4.9578	5.9494	6.9409	7.9325	8.9240	1.3951
50	0.9915	1.9830	2.9745	3.9660	4.9576	5.9491	6.9406	7.9321	8.9236	1.3951
51	0.9915	1.9829	2.9744	3.9658	4.9573	5.9488	6.9402	7.9317	8.9231	1.3950
52	0.9914	1.9828	2.9742	3.9656	4.9571	5.9485	6.9399	7.9313	8.9227	1.3950
53	0.9914	1.9827	2.9741	3.9654	4.9568	5.9482	6.9395	7.9309	8.9223	1.3950
54	0.9913	1.9826	2.9739	3.9653	4.9566	5.9479	6.9392	7.9305	8.9218	1.3949
55	0.9913	1.9825	2.9738	3.9651	4.9563	5.9476	6.9388	7.9301	8.9214	1.3949
56	0.9912	1.9824	2.9736	3.9649	4.9561	5.9473	6.9385	7.9297	8.9209	1.3949
57	0.9912	1.9823	2.9735	3.9647	4.9558	5.9470	6.9381	7.9293	8.9205	1.3948
58	0.9911	1.9822	2.9733	3.9645	4.9556	5.9467	6.9378	7.9289	8.9100	1.3948
59	0.9911	1.9821	2.9732	3.9643	4.9553	5.9464	6.9375	7.9285	8.9196	1.3948
60	0.9910	1.9820	2.9730	3.9641	4.9551	5.9461	6.9371	7.9281	8.9191	1.3947

1	2	3	4	5	6	7	8	9	b	'
0.0695	0.1390	0.2085	0.2780	0.3474	0.4169	0.4864	0.5559	0.6254	0.0977	00
0.0698	0.1396	0.2093	0.2791	0.3489	0.4187	0.4884	0.5582	0.6280	0.0981	01
0.0701	0.1401	0.2102	0.2802	0.3503	0.4204	0.4904	0.5605	0.6306	0.0985	02
0.0704	0.1407	0.2111	0.2814	0.3518	0.4221	0.4925	0.5628	0.6332	0.0989	03
0.0706	0.1413	0.2119	0.2826	0.3532	0.4238	0.4945	0.5651	0.6358	0.0993	04
0.0709	0.1419	0.2128	0.2837	0.3546	0.4256	0.4965	0.5674	0.6384	0.0997	05
0.0712	0.1424	0.2136	0.2848	0.3561	0.4273	0.4985	0.5697	0.6409	0.1001	06
0.0715	0.1430	0.2145	0.2860	0.3575	0.4290	0.5005	0.5720	0.6435	0.1005	07
0.0718	0.1436	0.2154	0.2872	0.3589	0.4307	0.5025	0.5743	0.6461	0.1009	08
0.0721	0.1442	0.2162	0.2883	0.3604	0.4325	0.5045	0.5766	0.6487	0.1013	09
0.0724	0.1447	0.2171	0.2894	0.3618	0.4342	0.5065	0.5789	0.6513	0.1017	10
0.0727	0.1453	0.2180	0.2906	0.3633	0.4359	0.5086	0.5812	0.6539	0.1021	11
0.0729	0.1459	0.2188	0.2918	0.3647	0.4376	0.5106	0.5835	0.6565	0.1025	12
0.0732	0.1465	0.2197	0.2929	0.3661	0.4394	0.5126	0.5858	0.6591	0.1029	13
0.0735	0.1470	0.2205	0.2940	0.3676	0.4411	0.5146	0.5881	0.6616	0.1033	14
0.0738	0.1476	0.2214	0.2952	0.3690	0.4428	0.5166	0.5904	0.6642	0.1037	15
0.0741	0.1482	0.2223	0.2964	0.3704	0.4445	0.5186	0.5927	0.6668	0.1041	16
0.0744	0.1488	0.2231	0.2975	0.3719	0.4463	0.5206	0.5950	0.6694	0.1046	17
0.0747	0.1493	0.2240	0.2986	0.3733	0.4480	0.5226	0.5973	0.6720	0.1050	18
0.0749	0.1499	0.2248	0.2998	0.3747	0.4497	0.5246	0.5996	0.6746	0.1054	19
0.0752	0.1505	0.2257	0.3010	0.3762	0.4514	0.5266	0.6019	0.6772	0.1058	20
0.0755	0.1510	0.2266	0.3021	0.3776	0.4531	0.5286	0.6042	0.6797	0.1062	21
0.0758	0.1516	0.2274	0.3032	0.3791	0.4549	0.5307	0.6065	0.6823	0.1066	22
0.0761	0.1522	0.2283	0.3044	0.3805	0.4566	0.5327	0.6088	0.6849	0.1070	23
0.0764	0.1528	0.2292	0.3056	0.3819	0.4583	0.5347	0.6111	0.6875	0.1074	24
0.0767	0.1533	0.2300	0.3067	0.3834	0.4600	0.5367	0.6134	0.6900	0.1078	25
0.0770	0.1539	0.2309	0.3078	0.3848	0.4618	0.5387	0.6157	0.6926	0.1082	26
0.0772	0.1545	0.2317	0.3090	0.3862	0.4635	0.5407	0.6180	0.6952	0.1086	27
0.0775	0.1551	0.2326	0.3101	0.3877	0.4652	0.5427	0.6203	0.6978	0.1090	28
0.0778	0.1556	0.2335	0.3113	0.3891	0.4669	0.5447	0.6226	0.7004	0.1094	29
0.0781	0.1562	0.2343	0.3124	0.3905	0.4687	0.5467	0.6249	0.7030	0.1098	30
0.0784	0.1568	0.2352	0.3136	0.3920	0.4703	0.5487	0.6271	0.7055	0.1102	31
0.0787	0.1574	0.2360	0.3147	0.3934	0.4721	0.5508	0.6294	0.7081	0.1107	32
0.0790	0.1579	0.2369	0.3159	0.3948	0.4738	0.5528	0.6318	0.7107	0.1111	33
0.0793	0.1585	0.2378	0.3170	0.3963	0.4755	0.5548	0.6340	0.7133	0.1115	34
0.0795	0.1591	0.2386	0.3182	0.3977	0.4772	0.5568	0.6363	0.7159	0.1119	35
0.0798	0.1597	0.2395	0.3193	0.3991	0.4790	0.5588	0.6386	0.7185	0.1123	36
0.0801	0.1602	0.2403	0.3204	0.4007	0.4807	0.5608	0.6409	0.7210	0.1127	37
0.0804	0.1608	0.2412	0.3216	0.4020	0.4824	0.5628	0.6432	0.7236	0.1131	38
0.0807	0.1614	0.2421	0.3228	0.4034	0.4841	0.5648	0.6455	0.7262	0.1135	39
0.0810	0.1620	0.2429	0.3239	0.4049	0.4859	0.5668	0.6478	0.7288	0.1139	40
0.0813	0.1625	0.2438	0.3250	0.4063	0.4876	0.5688	0.6501	0.7313	0.1143	41
0.0815	0.1631	0.2446	0.3262	0.4077	0.4893	0.5708	0.6524	0.7339	0.1147	42
0.0818	0.1637	0.2455	0.3273	0.4092	0.4910	0.5728	0.6546	0.7365	0.1151	43
0.0821	0.1642	0.2464	0.3285	0.4106	0.4927	0.5748	0.6570	0.7391	0.1155	44
0.0824	0.1648	0.2472	0.3296	0.4120	0.4945	0.5768	0.6593	0.7417	0.1159	45
0.0827	0.1654	0.2481	0.3308	0.4135	0.4961	0.5788	0.6615	0.7442	0.1163	46
0.0830	0.1660	0.2489	0.3319	0.4149	0.4979	0.5809	0.6638	0.7468	0.1167	47
0.0833	0.1665	0.2498	0.3331	0.4163	0.4996	0.5829	0.6662	0.7494	0.1171	48
0.0836	0.1671	0.2507	0.3342	0.4178	0.5013	0.5849	0.6684	0.7520	0.1176	49
0.0838	0.1677	0.2515	0.3354	0.4192	0.5030	0.5869	0.6707	0.7546	0.1180	50
0.0841	0.1683	0.2524	0.3365	0.4206	0.5048	0.5889	0.6730	0.7572	0.1184	51
0.0844	0.1688	0.2532	0.3376	0.4221	0.5065	0.5909	0.6753	0.7597	0.1188	52
0.0847	0.1694	0.2541	0.3388	0.4235	0.5082	0.5929	0.6776	0.7623	0.1192	53
0.0850	0.1700	0.2549	0.3399	0.4249	0.5099	0.5949	0.6798	0.7648	0.1196	54
0.0853	0.1705	0.2558	0.3411	0.4264	0.5116	0.5969	0.6822	0.7674	0.1200	55
0.0856	0.1711	0.2567	0.3422	0.4278	0.5134	0.5989	0.6845	0.7700	0.1204	56
0.0858	0.1717	0.2575	0.3434	0.4292	0.5150	0.6009	0.6867	0.7726	0.1208	57
0.0861	0.1723	0.2584	0.3445	0.4306	0.5168	0.6029	0.6890	0.7752	0.1212	58
0.0864	0.1728	0.2593	0.3457	0.4321	0.5185	0.6049	0.6914	0.7778	0.1216	59
0.0867	0.1734	0.2601	0.3468	0.4335	0.5202	0.6069	0.6936	0.7803	0.1220	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9910	1.9820	2.9730	3.9641	4.9551	5.9461	6.9371	7.9281	8.9191	1.3947
01	0.9910	1.9819	2.9729	3.9639	4.9548	5.9458	6.9367	7.9277	8.9187	1.3947
02	0.9909	1.9818	2.9727	3.9636	4.9546	5.9455	6.9364	7.9273	8.9182	1.3946
03	0.9909	1.9817	2.9726	3.9634	4.9543	5.9452	6.9360	7.9269	8.9177	1.3946
04	0.9908	1.9816	2.9724	3.9632	4.9541	5.9449	6.9357	7.9265	8.9173	1.3946
05	0.9908	1.9815	2.9723	3.9630	4.9538	5.9446	6.9353	7.9261	8.9168	1.3945
06	0.9907	1.9814	2.9721	3.9628	4.9535	5.9442	6.9349	7.9257	8.9164	1.3945
07	0.9907	1.9813	2.9720	3.9626	4.9533	5.9439	6.9346	7.9252	8.9159	1.3944
08	0.9906	1.9812	2.9718	3.9624	4.9530	5.9436	6.9342	7.9248	8.9154	1.3944
09	0.9906	1.9811	2.9717	3.9622	4.9528	5.9433	6.9339	7.9244	8.9150	1.3944
10	0.9905	1.9810	2.9715	3.9620	4.9525	5.9430	6.9335	7.9240	8.9145	1.3943
11	0.9904	1.9809	2.9713	3.9618	4.9522	5.9427	6.9331	7.9236	8.9140	1.3943
12	0.9904	1.9808	2.9712	3.9616	4.9520	5.9424	6.9328	7.9232	8.9136	1.3942
13	0.9903	1.9807	2.9710	3.9614	4.9517	5.9421	6.9324	7.9227	8.9131	1.3942
14	0.9903	1.9806	2.9709	3.9612	4.9515	5.9417	6.9320	7.9223	8.9126	1.3941
15	0.9902	1.9805	2.9707	3.9610	4.9512	5.9414	6.9317	7.9219	8.9121	1.3941
16	0.9902	1.9804	2.9706	3.9607	4.9509	5.9411	6.9313	7.9215	8.9117	1.3941
17	0.9901	1.9803	2.9704	3.9605	4.9507	5.9408	6.9309	7.9211	8.9112	1.3940
18	0.9901	1.9802	2.9702	3.9603	4.9504	5.9405	6.9306	7.9206	8.9107	1.3940
19	0.9900	1.9801	2.9701	3.9601	4.9501	5.9402	6.9302	7.9202	8.9102	1.3940
20	0.9900	1.9799	2.9699	3.9599	4.9499	5.9398	6.9298	7.9198	8.9098	1.3939
21	0.9899	1.9798	2.9698	3.9597	4.9496	5.9395	6.9294	7.9193	8.9093	1.3939
22	0.9899	1.9797	2.9696	3.9595	4.9493	5.9392	6.9290	7.9189	8.9088	1.3938
23	0.9898	1.9796	2.9694	3.9592	4.9490	5.9389	6.9287	7.9185	8.9083	1.3938
24	0.9898	1.9795	2.9693	3.9590	4.9488	5.9385	6.9283	7.9180	8.9078	1.3938
25	0.9897	1.9794	2.9691	3.9588	4.9485	5.9382	6.9279	7.9176	8.9073	1.3937
26	0.9896	1.9793	2.9689	3.9586	4.9482	5.9379	6.9275	7.9172	8.9068	1.3937
27	0.9896	1.9792	2.9688	3.9584	4.9480	5.9375	6.9271	7.9167	8.9063	1.3936
28	0.9895	1.9791	2.9686	3.9581	4.9477	5.9372	6.9268	7.9163	8.9058	1.3936
29	0.9895	1.9790	2.9684	3.9579	4.9474	5.9369	6.9264	7.9159	8.9053	1.3936
30	0.9894	1.9789	2.9683	3.9577	4.9471	5.9366	6.9260	7.9154	8.9048	1.3935
31	0.9894	1.9787	2.9681	3.9575	4.9469	5.9362	6.9256	7.9150	8.9043	1.3935
32	0.9893	1.9786	2.9679	3.9573	4.9466	5.9359	6.9252	7.9145	8.9038	1.3934
33	0.9893	1.9783	2.9678	3.9570	4.9463	5.9355	6.9248	7.9141	8.9033	1.3934
34	0.9892	1.9784	2.9676	3.9568	4.9460	5.9352	6.9244	7.9136	8.9028	1.3934
35	0.9891	1.9783	2.9674	3.9566	4.9457	5.9349	6.9240	7.9132	8.9023	1.3933
36	0.9891	1.9782	2.9673	3.9564	4.9454	5.9345	6.9236	7.9127	8.9018	1.3933
37	0.9890	1.9781	2.9671	3.9561	4.9452	5.9342	6.9232	7.9123	8.9013	1.3932
38	0.9890	1.9780	2.9669	3.9559	4.9449	5.9339	6.9228	7.9118	8.9008	1.3932
39	0.9889	1.9778	2.9668	3.9557	4.9446	5.9335	6.9224	7.9114	8.9003	1.3932
40	0.9889	1.9777	2.9666	3.9555	4.9443	5.9332	6.9220	7.9109	8.8998	1.3931
41	0.9888	1.9776	2.9664	3.9552	4.9440	5.9328	6.9216	7.9104	8.8993	1.3931
42	0.9887	1.9775	2.9662	3.9550	4.9437	5.9325	6.9212	7.9100	8.8987	1.3930
43	0.9887	1.9774	2.9661	3.9548	4.9435	5.9321	6.9208	7.9095	8.8982	1.3930
44	0.9886	1.9773	2.9659	3.9545	4.9432	5.9318	6.9204	7.9091	8.8977	1.3930
45	0.9886	1.9772	2.9657	3.9543	4.9429	5.9315	6.9200	7.9086	8.8972	1.3929
46	0.9885	1.9770	2.9656	3.9541	4.9426	5.9311	6.9196	7.9081	8.8967	1.3929
47	0.9885	1.9769	2.9654	3.9538	4.9423	5.9308	6.9192	7.9077	8.8961	1.3928
48	0.9884	1.9768	2.9652	3.9536	4.9420	5.9304	6.9188	7.9072	8.8956	1.3928
49	0.9883	1.9767	2.9650	3.9534	4.9417	5.9300	6.9184	7.9067	8.8951	1.3928
50	0.9883	1.9766	2.9649	3.9531	4.9414	5.9297	6.9180	7.9063	8.8946	1.3927
51	0.9882	1.9765	2.9647	3.9529	4.9411	5.9294	6.9176	7.9058	8.8940	1.3927
52	0.9882	1.9763	2.9645	3.9527	4.9408	5.9290	6.9172	7.9053	8.8935	1.3926
53	0.9881	1.9762	2.9643	3.9524	4.9405	5.9286	6.9167	7.9048	8.8930	1.3926
54	0.9880	1.9761	2.9641	3.9522	4.9402	5.9283	6.9163	7.9044	8.8924	1.3926
55	0.9880	1.9760	2.9640	3.9519	4.9399	5.9279	6.9159	7.9039	8.8919	1.3925
56	0.9879	1.9759	2.9638	3.9517	4.9396	5.9276	6.9155	7.9034	8.8913	1.3925
57	0.9879	1.9757	2.9636	3.9515	4.9393	5.9272	6.9151	7.9029	8.8908	1.3924
58	0.9878	1.9756	2.9634	3.9512	4.9390	5.9268	6.9147	7.9025	8.8903	1.3924
59	0.9877	1.9755	2.9632	3.9510	4.9387	5.9265	6.9142	7.9020	8.8897	1.3924
60	0.9877	1.9754	2.9631	3.9508	4.9384	5.9261	6.9138	7.9015	8.8892	1.3923

1	2	3	4	5	6	7	8	9	b	′
0.0867	0.1734	0.2601	0.3468	0.4335	0.5202	0.6069	0.6936	0.7803	0.1220	00
0.0870	0.1740	0.2610	0.3480	0.4349	0.5219	0.6089	0.6959	0.7829	0.1224	01
0.0873	0.1745	0.2618	0.3491	0.4364	0.5236	0.6109	0.6982	0.7854	0.1228	02
0.0876	0.1751	0.2627	0.3502	0.4378	0.5254	0.6129	0.7005	0.7880	0.1232	03
0.0878	0.1757	0.2635	0.3514	0.4392	0.5271	0.6149	0.7028	0.7906	0.1236	04
0.0881	0.1763	0.2644	0.3525	0.4407	0.5288	0.6169	0.7050	0.7932	0.1240	05
0.0884	0.1768	0.2653	0.3537	0.4421	0.5305	0.6189	0.7074	0.7958	0.1244	06
0.0887	0.1774	0.2661	0.3548	0.4435	0.5322	0.6209	0.7096	0.7983	0.1248	07
0.0890	0.1780	0.2670	0.3560	0.4450	0.5539	0.6229	0.7119	0.8009	0.1253	08
0.0893	0.1786	0.2678	0.3571	0.4464	0.5357	0.6249	0.7142	0.8035	0.1257	09
0.0896	0.1791	0.2687	0.3582	0.4478	0.5374	0.6269	0.7165	0.8060	0.1261	10
0.0898	0.1797	0.2695	0.3594	0.4492	0.5391	0.6289	0.7188	0.8086	0.1265	11
0.0901	0.1803	0.2704	0.3605	0.4507	0.5408	0.6309	0.7211	0.8112	0.1269	12
0.0904	0.1808	0.2713	0.3617	0.4521	0.5425	0.6329	0.7234	0.8138	0.1273	13
0.0907	0.1814	0.2721	0.3628	0.4535	0.5442	0.6349	0.7256	0.8163	0.1277	14
0.0910	0.1820	0.2730	0.3640	0.4550	0.5459	0.6369	0.7279	0.8189	0.1281	15
0.0913	0.1826	0.2738	0.3651	0.4564	0.5477	0.6389	0.7302	0.8215	0.1285	16
0.0916	0.1831	0.2747	0.3662	0.4578	0.5494	0.6409	0.7325	0.8240	0.1289	17
0.0918	0.1837	0.2755	0.3674	0.4592	0.5511	0.6429	0.7348	0.8266	0.1293	18
0.0921	0.1843	0.2764	0.3685	0.4607	0.5528	0.6449	0.7371	0.8292	0.1297	19
0.0924	0.1848	0.2773	0.3697	0.4621	0.5545	0.6469	0.7394	0.8318	0.1301	20
0.0927	0.1854	0.2781	0.3708	0.4635	0.5562	0.6489	0.7416	0.8343	0.1305	21
0.0930	0.1860	0.2790	0.3720	0.4649	0.5579	0.6509	0.7439	0.8369	0.1309	22
0.0933	0.1865	0.2798	0.3731	0.4664	0.5596	0.6529	0.7462	0.8394	0.1313	23
0.0936	0.1871	0.2807	0.3742	0.4678	0.5614	0.6549	0.7485	0.8420	0.1317	24
0.0938	0.1877	0.2815	0.3754	0.4692	0.5631	0.6569	0.7507	0.8446	0.1321	25
0.0941	0.1883	0.2824	0.3765	0.4706	0.5648	0.6589	0.7530	0.8472	0.1326	26
0.0944	0.1888	0.2833	0.3777	0.4721	0.5665	0.6609	0.7553	0.8498	0.1330	27
0.0947	0.1894	0.2841	0.3788	0.4735	0.5682	0.6629	0.7576	0.8523	0.1334	28
0.0950	0.1900	0.2850	0.3800	0.4749	0.5699	0.6649	0.7599	0.8549	0.1338	29
0.0953	0.1905	0.2858	0.3811	0.4764	0.5716	0.6669	0.7622	0.8574	0.1342	30
0.0956	0.1911	0.2867	0.3822	0.4778	0.5734	0.6689	0.7645	0.8600	0.1346	31
0.0958	0.1917	0.2875	0.3834	0.4792	0.5751	0.6709	0.7667	0.8626	0.1350	32
0.0961	0.1923	0.2884	0.3845	0.4806	0.5768	0.6729	0.7690	0.8652	0.1354	33
0.0964	0.1928	0.2892	0.3856	0.4820	0.5785	0.6749	0.7713	0.8677	0.1358	34
0.0967	0.1934	0.2901	0.3868	0.4835	0.5802	0.6769	0.7736	0.8703	0.1362	35
0.0970	0.1940	0.2909	0.3879	0.4849	0.5819	0.6789	0.7759	0.8728	0.1366	36
0.0973	0.1945	0.2918	0.3891	0.4863	0.5836	0.6809	0.7782	0.8754	0.1370	37
0.0976	0.1951	0.2927	0.3902	0.4878	0.5853	0.6829	0.7804	0.8780	0.1374	38
0.0978	0.1957	0.2935	0.3914	0.4892	0.5870	0.6849	0.7827	0.8806	0.1378	39
0.0981	0.1962	0.2944	0.3925	0.4906	0.5887	0.6869	0.7850	0.8831	0.1382	40
0.0984	0.1968	0.2952	0.3936	0.4920	0.5905	0.6889	0.7873	0.8857	0.1389	41
0.0987	0.1974	0.2961	0.3948	0.4935	0.5921	0.6908	0.7895	0.8882	0.1390	42
0.0990	0.1979	0.2969	0.3959	0.4948	0.5938	0.6928	0.7918	0.8907	0.1395	43
0.0993	0.1985	0.2978	0.3970	0.4963	0.5956	0.6948	0.7941	0.8933	0.1399	44
0.0995	0.1991	0.2986	0.3982	0.4977	0.5973	0.6968	0.7963	0.8959	0.1403	45
0.0998	0.1997	0.2995	0.3993	0.4991	0.5990	0.6988	0.7986	0.8985	0.1407	46
0.1001	0.2002	0.3003	0.4004	0.5006	0.6007	0.7008	0.8009	0.9010	0.1411	47
0.1004	0.2008	0.3012	0.4016	0.5020	0.6024	0.7028	0.8032	0.9036	0.1415	48
0.1007	0.2014	0.3020	0.4027	0.5034	0.6041	0.7048	0.8054	0.9061	0.1419	49
0.1010	0.2019	0.3029	0.4039	0.5049	0.6058	0.7068	0.8078	0.9087	0.1423	50
0.1013	0.2025	0.3038	0.4050	0.5063	0.6075	0.7088	0.8100	0.9113	0.1427	51
0.1015	0.2031	0.3046	0.4062	0.5077	0.6092	0.7108	0.8123	0.9139	0.1431	52
0.1018	0.2036	0.3055	0.4073	0.5091	0.6109	0.7127	0.8146	0.9164	0.1435	53
0.1021	0.2042	0.3063	0.4084	0.5105	0.6126	0.7147	0.8168	0.9189	0.1439	54
0.1024	0.2048	0.3072	0.4096	0.5119	0.6143	0.7167	0.8191	0.9215	0.1443	55
0.1027	0.2053	0.3080	0.4107	0.5134	0.6160	0.7187	0.8214	0.9240	0.1447	56
0.1030	0.2059	0.3089	0.4118	0.5148	0.6177	0.7207	0.8237	0.9266	0.1451	57
0.1032	0.2065	0.3097	0.4130	0.5162	0.6194	0.7227	0.8259	0.9292	0.1455	58
0.1035	0.2071	0.3106	0.4141	0.5176	0.6212	0.7247	0.8282	0.9318	0.1459	59
0.1038	0.2076	0.3114	0.4153	0.5191	0.6229	0.7267	0.8305	0.9343	0.1463	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9877	1.9754	2.9631	3.9508	4.9384	5.9261	6.9138	7.9015	8.8892	1.3923
01	0.9876	1.9753	2.9629	3.9505	4.9381	5.9258	6.9134	7.9010	8.8887	1.3923
02	0.9876	1.9751	2.9627	3.9503	4.9378	5.9254	6.9130	7.9005	8.8881	1.3922
03	0.9875	1.9750	2.9625	3.9500	4.9375	5.9250	6.9125	7.9000	8.8875	1.3922
04	0.9874	1.9749	2.9623	3.9498	4.9372	5.9247	6.9121	7.8996	8.8870	1.3921
05	0.9874	1.9748	2.9621	3.9495	4.9369	5.9243	6.9117	7.8991	8.8864	1.3921
06	0.9873	1.9746	2.9620	3.9493	4.9366	5.9239	6.9113	7.8986	8.8859	1.3921
07	0.9873	1.9745	2.9618	3.9490	4.9363	5.9236	6.9108	7.8981	8.8853	1.3920
08	0.9872	1.9744	2.9616	3.9488	4.9360	5.9232	6.9104	7.8976	8.8848	1.3920
09	0.9871	1.9743	2.9614	3.9486	4.9357	5.9228	6.9100	7.8971	8.8842	1.3919
10	0.9871	1.9742	2.9612	3.9483	4.9354	5.9225	6.9095	7.8966	8.8837	1.3919
11	0.9870	1.9740	2.9610	3.9481	4.9351	5.9221	6.9091	7.8961	8.8831	1.3919
12	0.9870	1.9739	2.9609	3.9478	4.9348	5.9217	6.9087	7.8956	8.8826	1.3918
13	0.9869	1.9738	2.9607	3.9476	4.9344	5.9213	6.9082	7.8951	8.8820	1.3918
14	0.9868	1.9737	2.9605	3.9473	4.9341	5.9210	6.9078	7.8946	8.8814	1.3917
15	0.9868	1.9735	2.9603	3.9471	4.9338	5.9206	6.9073	7.8941	8.8809	1.3917
16	0.9867	1.9734	2.9601	3.9468	4.9335	5.9202	6.9069	7.8936	8.8803	1.3917
17	0.9866	1.9733	2.9599	3.9465	4.9332	5.9198	6.9065	7.8931	8.8797	1.3916
18	0.9866	1.9732	2.9597	3.9463	4.9329	5.9195	6.9060	7.8926	8.8792	1.3916
19	0.9865	1.9730	2.9595	3.9460	4.9326	5.9191	6.9056	7.8921	8.8786	1.3915
20	0.9864	1.9729	2.9593	3.9458	4.9322	5.9187	6.9051	7.8916	8.8780	1.3915
21	0.9864	1.9728	2.9592	3.9455	4.9319	5.9183	6.9047	7.8911	8.8775	1.3915
22	0.9863	1.9726	2.9590	3.9453	4.9316	5.9179	6.9042	7.8906	8.8769	1.3914
23	0.9863	1.9725	2.9588	3.9450	4.9313	5.9175	6.9038	7.8900	8.8763	1.3914
24	0.9862	1.9724	2.9586	3.9448	4.9310	5.9171	6.9033	7.8895	8.8757	1.3913
25	0.9861	1.9723	2.9584	3.9445	4.9306	5.9168	6.9029	7.8890	8.8751	1.3913
26	0.9861	1.9721	2.9582	3.9442	4.9303	5.9164	6.9024	7.8885	8.8745	1.3913
27	0.9860	1.9720	2.9580	3.9440	4.9300	5.9160	6.9020	7.8880	8.8740	1.3912
28	0.9859	1.9719	2.9578	3.9437	4.9297	5.9156	6.9015	7.8875	8.8734	1.3912
29	0.9859	1.9717	2.9576	3.9435	4.9293	5.9152	6.9011	7.8869	8.8728	1.3911
30	0.9858	1.9716	2.9574	3.9432	4.9290	5.9148	6.9006	7.8864	8.8722	1.3911
31	0.9857	1.9715	2.9572	3.9429	4.9287	5.9144	6.9002	7.8859	8.8716	1.3910
32	0.9857	1.9713	2.9570	3.9427	4.9284	5.9140	6.8997	7.8854	8.8710	1.3910
33	0.9856	1.9712	2.9568	3.9424	4.9280	5.9136	6.8992	7.8848	8.8704	1.3910
34	0.9855	1.9711	2.9566	3.9422	4.9277	5.9132	6.8988	7.8843	8.8698	1.3909
35	0.9855	1.9709	2.9564	3.9419	4.9274	5.9128	6.8983	7.8838	8.8692	1.3909
36	0.9854	1.9708	2.9562	3.9416	4.9270	5.9124	6.8978	7.8832	8.8687	1.3908
37	0.9853	1.9707	2.9560	3.9414	4.9267	5.9120	6.8974	7.8827	8.8681	1.3908
38	0.9853	1.9705	2.9558	3.9411	4.9264	5.9116	6.8969	7.8822	8.8675	1.3907
39	0.9852	1.9704	2.9556	3.9408	4.9260	5.9112	6.8964	7.8817	8.8669	1.3907
40	0.9851	1.9703	2.9554	3.9406	4.9257	5.9108	6.8960	7.8811	8.8663	1.3906
41	0.9851	1.9701	2.9552	3.9403	4.9254	5.9104	6.8955	7.8806	8.8657	1.3906
42	0.9850	1.9700	2.9550	3.9400	4.9250	5.9100	6.8950	7.8800	8.8650	1.3905
43	0.9849	1.9699	2.9548	3.9398	4.9247	5.9096	6.8946	7.8795	8.8644	1.3905
44	0.9849	1.9697	2.9546	3.9395	4.9244	5.9092	6.8941	7.8790	8.8638	1.3904
45	0.9848	1.9696	2.9544	3.9392	4.9240	5.9088	6.8936	7.8784	8.8632	1.3904
46	0.9847	1.9695	2.9542	3.9389	4.9237	5.9084	6.8931	7.8779	8.8626	1.3903
47	0.9847	1.9693	2.9540	3.9387	4.9233	5.9080	6.8927	7.8773	8.8620	1.3903
48	0.9846	1.9692	2.9538	3.9384	4.9230	5.9076	6.8922	7.8768	8.8614	1.3902
49	0.9845	1.9691	2.9536	3.9381	4.9227	5.9072	6.8917	7.8762	8.8608	1.3902
50	0.9845	1.9689	2.9534	3.9379	4.9223	5.9068	6.8912	7.8757	8.8602	1.3901
51	0.9844	1.9688	2.9532	3.9376	4.9220	5.9064	6.8908	7.8752	8.8595	1.3901
52	0.9843	1.9686	2.9530	3.9373	4.9216	5.9059	6.8903	7.8746	8.8589	1.3900
53	0.9843	1.9685	2.9528	3.9370	4.9213	5.9055	6.8898	7.8740	8.8583	1.3900
54	0.9842	1.9684	2.9526	3.9367	4.9209	5.9051	6.8893	7.8735	8.8577	1.3899
55	0.9841	1.9682	2.9523	3.9365	4.9206	5.9047	6.8888	7.8729	8.8570	1.3899
56	0.9840	1.9681	2.9521	3.9362	4.9202	5.9043	6.8883	7.8724	8.8564	1.3898
57	0.9840	1.9680	2.9519	3.9359	4.9199	5.9039	6.8878	7.8718	8.8558	1.3898
58	0.9839	1.9678	2.9517	3.9356	4.9195	5.9034	6.8874	7.8713	8.8552	1.3897
59	0.9838	1.9677	2.9515	3.9354	4.9192	5.9030	6.8869	7.8707	8.8545	1.3897
60	0.9838	1.9675	2.9513	3.9351	4.9188	5.9026	6.8864	7.8702	8.8539	1.3896

1	2	3	4	5	6	7	8	9	b	′
0.1038	0.2076	0.3114	0.4153	0.5191	0.6229	0.7267	0.8305	0.9343	0.1463	00
0.1041	0.2082	0.3123	0.4164	0.5205	0.6246	0.7287	0.8327	0.9368	0.1467	01
0.1044	0.2088	0.3131	0.4175	0.5219	0.6263	0.7307	0.8350	0.9394	0.1471	02
0.1047	0.2093	0.3140	0.4186	0.5233	0.6280	0.7326	0.8373	0.9419	0.1476	03
0.1049	0.2099	0.3148	0.4198	0.5247	0.6297	0.7346	0.8396	0.9445	0.1480	04
0.1052	0.2105	0.3157	0.4209	0.5262	0.6314	0.7366	0.8418	0.9471	0.1484	05
0.1055	0.2110	0.3165	0.4220	0.5276	0.6331	0.7386	0.8441	0.9496	0.1488	06
0.1058	0.2116	0.3174	0.4232	0.5290	0.6348	0.7406	0.8464	0.9522	0.1492	07
0.1061	0.2122	0.3182	0.4243	0.5304	0.6365	0.7426	0.8486	0.9547	0.1496	08
0.1064	0.2127	0.3191	0.4255	0.5318	0.6382	0.7446	0.8509	0.9573	0.1500	09
0.1067	0.2133	0.3200	0.4266	0.5333	0.6399	0.7466	0.8532	0.9599	0.1504	10
0.1069	0.2139	0.3208	0.4277	0.5347	0.6416	0.7485	0.8554	0.9624	0.1508	11
0.1072	0.2144	0.3217	0.4389	0.5361	0.6433	0.7505	0.8577	0.9650	0.1512	12
0.1075	0.2150	0.3225	0.43c0	0.5375	0.6450	0.7525	0.8600	0.9675	0.1516	13
0.1078	0.2156	0.3233	0.4311	0.5389	0.6467	0.7545	0.8622	c.97c0	0.1520	14
0.1081	0.2161	0.3242	0.4323	c.5403	0.6484	0.7565	0.8645	0.9726	0.1524	15
0.1084	0.2167	0.3251	0.4334	0.5418	0.6501	0.7585	0.8668	0.9752	0.1528	16
0.1086	0.2173	0.3259	0.4346	c.5432	0.6518	0.7605	0.8691	0.9778	0.1532	17
0.1089	0.2178	0.3268	0.4357	0.5446	0.6535	0.7624	0.8714	0.9803	0.1536	18
c.1092	0.2184	0.3276	0.4368	0.5460	0.6552	0.7644	0.8736	0.9828	0.1540	19
0.1095	0.2190	0.3285	0.4380	0.5474	0.6569	0.7664	0.8759	0.9854	0.1544	20
0.1098	0.2195	0.3293	0.4391	0.5488	0.6586	0.7684	0.8782	0.9879	0.1548	21
0.1101	0.2201	0.3302	0.4402	0.5503	0.6603	0.7704	0.8804	0.9905	0.1552	22
0.1103	0.2207	0.3310	0.4414	0.5517	0.6620	0.7724	0.8827	0.9931	0.1556	23
0.1106	0.2212	0.3319	0.4425	0.5531	0.6637	0.7743	0.8850	0.9956	0.1561	24
0.1109	0.2218	0.3327	0.4436	0.5545	0.6654	0.7763	0.8872	0.9981	0.1565	25
0.1112	0.2224	0.3336	0.4448	0.5559	0.6671	0.7783	0.8889	1.0007	0.1569	26
0.1115	0.2229	0.3344	0.4459	0.5573	0.6688	0.7803	c.8918	1.0032	0.1573	27
0.1118	0.2235	0.3353	0.4470	0.5588	0.6705	0.7823	0.8940	1.c058	0.1577	28
0.1120	0.2241	0.3361	0.4481	0.5602	0.6722	0.7842	0.8963	1.0083	0.1581	29
0.1123	c.2246	0.3370	0.4493	0.5616	0.6739	0.7862	0·8986	1.01c9	0.1585	30
0.1126	0.2252	0.3378	0.4504	0.5630	0.6756	0.7882	0.9008	1.0134	0.1589	31
0.1129	0.2258	0.3386	0.4515	0.5644	0.6773	0.7902	0.9031	1.0159	0.1593	32
0.1132	0.2263	0.3395	0.4527	0.5659	0.6790	c.7922	0.9054	1.0185	0.1597	33
0.1134	0.2269	0.3403	0.4538	0.5673	0.6807	0.7941	0.9076	1.0210	0.1601	34
0.1137	0.2275	0.3412	0.4549	0.5687	0.6824	0.7961	0.9098	1.0236	0.1605	35
0.1140	0.2280	0.3421	0.4561	0.5701	0.6841	0.8081	0.9121	1.0262	0.16c9	36
0.1143	0.2286	0.3429	0.4572	0.5715	c.6858	0.8001	0.9144	1.0287	0.1613	37
0.1146	0.2292	0.3437	0.4583	0.5729	0.6875	0.8021	0.9166	1.0312	0.1617	38
0.1149	0.2297	0.3446	0.4594	0.5743	0.6892	0.8040	0.9189	1.0337	0.1621	39
0.1151	0.2303	0.3454	0.4606	0.5757	0.6909	0.8060	0.9212	1.0363	0.1625	40
0.1154	0.2309	0.3463	0.4617	0.5771	0.6926	0.8080	0.9234	1.0389	0.1629	41
0.1157	0.2314	0.3471	0.4628	0.5786	0.6943	0.8100	0.9257	1.0414	0.1633	42
0.1160	0.2320	0.3480	0.4640	0.5800	0.6960	0.8119	0.9279	1.0439	0.1637	43
0.1163	0.2326	0.3488	0.4651	0.5814	0.6977	0.8139	0.9302	1.0465	0.1641	44
0.1166	0.2331	0.3497	0.4662	0.5828	0.6994	0.8159	0.9325	1.0490	0.1645	45
0.1168	0.2337	0.3505	0.4674	0.5842	0.7010	0.8179	0.9347	1.0516	0.1650	46
0.1171	0.2342	0.3514	0.4685	0.5856	0.7027	0.8199	0.9370	1.0541	0.1654	47
0.1174	0.2348	0.3522	0.4696	0.5870	0.7045	0.8219	0.9393	1.0567	0.1658	48
0.1177	0.2354	0.3531	0.4708	0.5884	0.7061	0.8238	0.9415	1.0592	0.1662	49
0.1180	0.2359	0.3539	0.4719	0.5899	0.7078	0.8258	0.9438	1.0617	0.1666	50
0.1183	0.2365	0.3548	0.4730	0.5913	0.7095	0.8278	0.9460	1.0643	0.1670	51
0.1185	0.2371	0.3556	0.4742	0.5927	0.7112	0.8298	0.9483	1.0669	0.1674	52
0.1188	0.2376	0.3565	0.4753	0.5941	0.7129	0.8317	0.9506	1.0694	0.1678	53
0.1191	0.2382	0.3573	0.4764	0.5955	0.7146	0.8337	0.9528	1.0719	0.1682	54
0.1194	0.2388	0.3581	0.4775	0.5969	0.7163	0.8357	0.9550	1.0744	0.1686	55
0.1197	0.2393	0.3590	0.4786	0.5983	0.7180	0.8376	0.9573	1.0769	0.1690	56
0.1199	0.2399	0.3598	0.4798	0.5997	0.7197	0.8396	0.9596	1.0795	0.1694	57
0.1202	0.2405	0.3607	0.4809	0.6011	0.7214	0.8416	0.9618	1.0821	0.1698	58
0.1205	0.2410	0.3615	0.4820	0.6025	0.7231	0.8436	0.9641	1.0846	0.1702	59
0.1208	0.2416	0.3624	0.4832	0.6040	0.7247	0.8455	0.9663	1.0871	0.1706	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9838	1.9675	2.9513	3.9351	4.9188	5.9026	6.8864	7.8702	8.8539	1.3896
01	0.9837	1.9674	2.9511	3.9348	4.9185	5.9022	6.8859	7.8696	8.8533	1.3896
02	0.9836	1.9673	2.9509	3.9345	4.9181	5.9018	6.8854	7.8690	8.8526	1.3895
03	0.9836	1.9671	2.9507	3.9342	4.9178	5.9013	6.8849	7.8684	8.8520	1.3895
04	0.9835	1.9670	2.9505	3.9339	4.9174	5.9009	6.8844	7.8679	8.8514	1.3894
05	0.9834	1.9668	2.9502	3.9337	4.9171	5.9005	6.8839	7.8673	8.8507	1.3894
06	0.9833	1.9667	2.9500	3.9334	4.9167	5.9001	6.8834	7.8667	8.8501	1.3893
07	0.9833	1.9665	2.9498	3.9331	4.9164	5.8996	6.8829	7.8662	8.8494	1.3893
08	0.9832	1.9664	2.9496	3.9328	4.9160	5.8992	6.8824	7.8656	8.8488	1.3892
09	0.9831	1.9663	2.9494	3.9325	4.9156	5.8988	6.8819	7.8650	8.8482	1.3892
10	0.9831	1.9661	2.9492	3.9322	4.9153	5.8983	6.8814	7.8645	8.8475	1.3891
11	0.9830	1.9660	2.9490	3.9319	4.9149	5.8979	6.8809	7.8639	8.8469	1.3891
12	0.9829	1.9658	2.9487	3.9316	4.9146	5.8975	6.8804	7.8633	8.8462	1.3890
13	0.9828	1.9657	2.9485	3.9314	4.9142	5.8970	6.8799	7.8627	8.8456	1.3890
14	0.9828	1.9655	2.9483	3.9311	4.9138	5.8966	6.8794	7.8621	8.8449	1.3889
15	0.9827	1.9654	2.9481	3.9308	4.9135	5.8962	6.8789	7.8616	8.8442	1.3889
16	0.9826	1.9652	2.9479	3.9305	4.9131	5.8957	6.8783	7.8610	8.8436	1.3888
17	0.9825	1.9651	2.9476	3.9302	4.9127	5.8953	6.8778	7.8604	8.8429	1.3888
18	0.9825	1.9650	2.9474	3.9299	4.9124	5.8949	6.8773	7.8598	8.8423	1.3887
19	0.9824	1.9648	2.9472	3.9296	4.9120	5.8944	6.8768	7.8592	8.8416	1.3887
20	0.9823	1.9647	2.9470	3.9293	4.9117	5.8940	6.8763	7.8586	8.8410	1.3886
21	0.9823	1.9645	2.9468	3.9290	4.9113	5.8935	6.8758	7.8580	8.8403	1.3886
22	0.9822	1.9644	2.9465	3.9287	4.9109	5.8931	6.8753	7.8574	8.8396	1.3885
23	0.9821	1.9642	2.9463	3.9284	4.9105	5.8926	6.8747	7.8569	8.8390	1.3885
24	0.9820	1.9641	2.9461	3.9281	4.9102	5.8922	6.8742	7.8563	8.8383	1.3884
25	0.9820	1.9639	2.9459	3.9278	4.9098	5.8918	6.8737	7.8557	8.8376	1.3884
26	0.9819	1.9638	2.9457	3.9275	4.9094	5.8913	6.8732	7.8551	8.8370	1.3883
27	0.9818	1.9636	2.9454	3.9272	4.9091	5.8909	6.8727	7.8545	8.8363	1.3883
28	0.9817	1.9635	2.9452	3.9269	4.9087	5.8904	6.8722	7.8539	8.8356	1.3882
29	0.9817	1.9633	2.9450	3.9266	4.9083	5.8900	6.8716	7.8533	8.8349	1.3882
30	0.9816	1.9632	2.9448	3.9263	4.9079	5.8895	6.8711	7.8527	8.8343	1.3881
31	0.9815	1.9630	2.9445	3.9260	4.9076	5.8891	6.8706	7.8521	8.8336	1.3881
32	0.9814	1.9629	2.9443	3.9257	4.9072	5.8886	6.8700	7.8515	8.8329	1.3880
33	0.9814	1.9627	2.9441	3.9254	4.9068	5.8882	6.8695	7.8509	8.8322	1.3880
34	0.9813	1.9626	2.9438	3.9251	4.9064	5.8877	6.8690	7.8503	8.8315	1.3879
35	0.9812	1.9624	2.9436	3.9248	4.9060	5.8872	6.8684	7.8497	8.8309	1.3879
36	0.9811	1.9623	2.9434	3.9245	4.9057	5.8868	6.8679	7.8490	8.8302	1.3878
37	0.9811	1.9621	2.9432	3.9242	4.9053	5.8863	6.8674	7.8484	8.8295	1.3878
38	0.9810	1.9620	2.9429	3.9239	4.9049	5.8859	6.8669	7.8478	8.8288	1.3877
39	0.9809	1.9618	2.9427	3.9236	4.9045	5.8854	6.8663	7.8472	8.8281	1.3876
40	0.9808	1.9617	2.9425	3.9233	4.9041	5.8850	6.8658	7.8466	8.8274	1.3876
41	0.9807	1.9615	2.9422	3.9230	4.9037	5.8845	6.8652	7.8460	8.8267	1.3875
42	0.9807	1.9613	2.9420	3.9227	4.9034	5.8840	6.8647	7.8454	8.8260	1.3875
43	0.9806	1.9612	2.9418	3.9224	4.9030	5.8836	6.8642	7.8448	8.8253	1.3874
44	0.9805	1.9610	2.9416	3.9221	4.9026	5.8831	6.8636	7.8441	8.8247	1.3874
45	0.9804	1.9609	2.9413	3.9218	4.9022	5.8826	6.8631	7.8435	8.8240	1.3873
46	0.9804	1.9607	2.9411	3.9214	4.9018	5.8822	6.8625	7.8429	8.8233	1.3872
47	0.9803	1.9606	2.9409	3.9211	4.9014	5.8817	6.8620	7.8423	8.8226	1.3872
48	0.9802	1.9604	2.9406	3.9208	4.9010	5.8812	6.8614	7.8416	8.8219	1.3871
49	0.9801	1.9603	2.9404	3.9205	4.9006	5.8808	6.8609	7.8410	8.8212	1.3871
50	0.9801	1.9601	2.9402	3.9202	4.9003	5.8803	6.8604	7.8404	8.8205	1.3870
51	0.9800	1.9599	2.9399	3.9199	4.8999	5.8798	6.8598	7.8398	8.8197	1.3870
52	0.9799	1.9598	2.9397	3.9196	4.8995	5.8794	6.8592	7.8391	8.8190	1.3869
53	0.9798	1.9596	2.9394	3.9193	4.8991	5.8789	6.8587	7.8385	8.8183	1.3868
54	0.9797	1.9595	2.9392	3.9189	4.8987	5.8784	6.8581	7.8379	8.8176	1.3868
55	0.9797	1.9593	2.9390	3.9186	4.8983	5.8779	6.8576	7.8372	8.8169	1.3867
56	0.9796	1.9592	2.9387	3.9183	4.8979	5.8775	6.8570	7.8366	8.8162	1.3866
57	0.9795	1.9590	2.9385	3.9180	4.8975	5.8770	6.8565	7.8360	8.8155	1.3866
58	0.9794	1.9588	2.9383	3.9177	4.8971	5.8765	6.8559	7.8353	8.8148	1.3865
59	0.9793	1.9587	2.9380	3.9173	4.8967	5.8760	6.8554	7.8347	8.8140	1.3865
60	0.9793	1.9585	2.9378	3.9170	4.8963	5.8755	6.8548	7.8341	8.8133	1.3864

1	2	3	4	5	6	7	8	9	b	′
0.1208	0.2416	0.3624	0.4832	0.6040	0.7247	0.8455	0.9663	1.0871	0.1706	00
0.1211	0.2421	0.3632	0.4843	0.6054	0.7264	0.8475	0.9686	1.0896	0.1710	01
0.1214	0.2427	0.3641	0.4854	0.6068	0.7281	0.8495	0.9709	1.0922	0.1714	02
0.1216	0.2433	0.3649	0.4866	0.6082	0.7298	0.8515	0.9731	1.0948	0.1718	03
0.1219	0.2438	0.3658	0.4877	0.6196	0.7315	0.8534	0.9754	1.0973	0.1722	04
0.1222	0.2444	0.3666	0.4888	0.6110	0.7332	0.8554	0.9776	1.0998	0.1726	05
0.1225	0.2450	0.3674	0.4899	0.6124	0.7349	0.8574	0.9798	1.1023	0.1730	06
0.1228	0.2455	0.3683	0.4910	0.6138	0.7366	0.8593	0.9821	1.1048	0.1734	07
0.1230	0.2461	0.3691	0.4922	0.6152	0.7382	0.8613	0.9843	1.1074	0.1738	08
0.1233	0.2467	0.3700	0.4933	0.6166	0.7400	0.8633	0.9866	1.1100	0.1743	09
0.1236	0.2472	0.3708	0.4944	0.6180	0.7417	0.8653	0.9889	1.1125	0.1747	10
0.1239	0.2478	0.3717	0.4956	0.6194	0.7433	0.8672	0.9911	1.1150	0.1751	11
0.1242	0.2483	0.3725	0.4967	0.6209	0.7450	0.8692	0.9934	1.1175	0.1755	12
0.1245	0.2489	0.3734	0.4978	0.6223	0.7467	0.8712	0.9956	1.1201	0.1759	13
0.1247	0.2495	0.3742	0.4989	0.6237	0.7484	0.8731	0.9978	1.1226	0.1763	14
0.1250	0.2500	0.3750	0.5000	0.6251	0.7501	0.8751	1.0001	1.1251	0.1767	15
0.1253	0.2506	0.3759	0.5012	0.6265	0.7518	0.8771	1.0024	1.1277	0.1771	16
0.1256	0.2512	0.3767	0.5023	0.6279	0.7535	0.8791	1.0046	1.1302	0.1775	17
0.1259	0.2517	0.3776	0.5034	0.6293	0.7552	0.8810	1.0069	1.1327	0.1779	18
0.1261	0.2523	0.3784	0.5046	0.6307	0.7568	0.8830	1.0091	1.1353	0.1783	19
0.1264	0.2528	0.3793	0.5057	0.6321	0.7585	0.8849	1.0114	1.1378	0.1787	20
0.1267	0.2534	0.3801	0.5068	0.6335	0.7602	0.8869	1.0136	1.1403	0.1791	21
0.1270	0.2540	0.3809	0.5079	0.6349	0.7619	0.8889	1.0158	1.1428	0.1795	22
0.1273	0.2545	0.3818	0.5090	0.6363	0.7636	0.8908	1.0181	1.1453	0.1799	23
0.1275	0.2551	0.3826	0.5102	0.6377	0.7652	0.8928	1.0203	1.1479	0.1803	24
0.1278	0.2556	0.3835	0.5113	0.6391	0.7669	0.8947	1.0226	1.1504	0.1807	25
0.1281	0.2562	0.3843	0.5124	0.6405	0.7680	0.8967	1.0248	1.1529	0.1811	26
0.1284	0.2568	0.3852	0.5136	0.6419	0.7703	0.8987	1.0271	1.1555	0.1815	27
0.1287	0.2573	0.3860	0.5147	0.6433	0.7720	0.9007	1.0294	1.1580	0.1819	28
0.1289	0.2579	0.3868	0.5158	0.6447	0.7737	0.9026	1.0316	1.1605	0.1823	29
0.1292	0.2585	0.3877	0.5169	0.6461	0.7754	0.9046	1.0338	1.1631	0.1827	30
0.1295	0.2590	0.3885	0.5180	0.6475	0.7771	0.9066	1.0361	1.1656	0.1831	31
0.1298	0.2596	0.3894	0.5192	0.6489	0.7787	0.9085	1.0383	1.1681	0.1835	32
0.1301	0.2601	0.3902	0.5203	0.6503	0.7804	0.9105	1.0406	1.1706	0.1839	33
0.1303	0.2607	0.3910	0.5214	0.6517	0.7821	0.9124	1.0428	1.1731	0.1843	34
0.1306	0.2613	0.3919	0.5225	0.6532	0.7838	0.9144	1.0450	1.1757	0.1847	35
0.1309	0.2618	0.3927	0.5236	0.6546	0.7855	0.9164	1.0473	1.1782	0.1852	36
0.1312	0.2624	0.3936	0.5248	0.6560	0.7871	0.9183	1.0495	1.1807	0.1856	37
0.1315	0.2629	0.3944	0.5259	0.6574	0.7888	0.9203	1.0518	1.1832	0.1860	38
0.1318	0.2635	0.3953	0.5270	0.6588	0.7905	0.9223	1.0540	1.1858	0.1864	39
0.1320	0.2641	0.3961	0.5281	0.6602	0.7922	0.9242	1.0562	1.1883	0.1868	40
0.1323	0.2646	0.3969	0.5292	0.6616	0.7939	0.9262	1.0585	1.1908	0.1872	41
0.1326	0.2652	0.3978	0.5304	0.6630	0.7955	0.9281	1.0607	1.1933	0.1876	42
0.1329	0.2657	0.3986	0.5315	0.6644	0.7972	0.9301	1.0630	1.1958	0.1880	43
0.1332	0.2663	0.3995	0.5326	0.6658	0.7989	0.9321	1.0652	1.1984	0.1884	44
0.1334	0.2669	0.4003	0.5337	0.6672	0.8006	0.9340	1.0674	1.2009	0.1888	45
0.1337	0.2674	0.4011	0.5348	0.6686	0.8023	0.9360	1.0697	1.2034	0.1892	46
0.1340	0.2680	0.4020	0.5360	0.6700	0.8039	0.9379	1.0719	1.2059	0.1896	47
0.1343	0.2685	0.4028	0.5371	0.6714	0.8056	0.9399	1.0742	1.2084	0.1900	48
0.1346	0.2691	0.4037	0.5382	0.6728	0.8073	0.9419	1.0764	1.2110	0.1904	49
0.1348	0.2697	0.4045	0.5393	0.6742	0.8090	0.9438	1.0786	1.2135	0.1908	50
0.1351	0.2702	0.4053	0.5404	0.6756	0.8107	0.9458	1.0809	1.2160	0.1912	51
0.1354	0.2708	0.4062	0.5416	0.6770	0.8123	0.9477	1.0831	1.2185	0.1916	52
0.1357	0.2713	0.4070	0.5427	0.6783	0.8140	0.9497	1.0854	1.2210	0.1920	53
0.1359	0.2719	0.4078	0.5438	0.6797	0.8157	0.9516	1.0876	1.2235	0.1924	54
0.1362	0.2725	0.4087	0.5449	0.6811	0.8174	0.9536	1.0898	1.2261	0.1928	55
0.1365	0.2730	0.4095	0.5460	0.6825	0.8191	0.9556	1.0921	1.2286	0.1932	56
0.1368	0.2736	0.4104	0.5472	0.6839	0.8207	0.9575	1.0943	1.2311	0.1936	57
0.1371	0.2741	0.4112	0.5483	0.6853	0.8224	0.9595	1.0966	1.2336	0.1940	58
0.1374	0.2747	0.4121	0.5494	0.6867	0.8241	0.9615	1.0988	1.2362	0.1944	59
0.1376	0.2753	0.4129	0.5505	0.6881	0.8258	0.9634	1.1010	1.2387	0.1948	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9793	1.9585	2.9378	3.9170	4.8963	5.8755	6.8548	7.8341	8.8133	1.3864
01	0.9792	1.9584	2.9375	3.9167	4.8959	5.8751	6.8542	7.8334	8.8126	1.3863
02	0.9791	1.9582	2.9373	3.9164	4.8955	5.8746	6.8537	7.8328	8.8119	1.3862
03	0.9790	1.9580	2.9370	3.9161	4.8951	5.8741	6.8531	7.8321	8.8111	1.3862
04	0.9789	1.9579	2.9368	3.9157	4.8947	5.8736	6.8525	7.8315	8.8104	1.3861
05	0.9789	1.9577	2.9366	3.9154	4.8943	5.8731	6.8520	7.8308	8.8097	1.3861
06	0.9788	1.9575	2.9363	3.9151	4.8939	5.8726	6.8514	7.8302	8.8090	1.3860
07	0.9787	1.9574	2.9361	3.9148	4.8935	5.8722	6.8508	7.8295	8.8082	1.3860
08	0.9786	1.9572	2.9358	3.9144	4.8931	5.8717	6.8503	7.8289	8.8075	1.3859
09	0.9785	1.9571	2.9356	3.9141	4.8927	5.8712	6.8497	7.8282	8.8068	1.3859
10	0.9785	1.9569	2.9354	3.9138	4.8923	5.8707	6.8492	7.8276	8.8061	1.3858
11	0.9784	1.9567	2.9351	3.9135	4.8918	5.8702	6.8486	7.8269	8.8053	1.3858
12	0.9783	1.9566	2.9349	3.9131	4.8914	5.8697	6.8480	7.8263	8.8046	1.3857
13	0.9782	1.9564	2.9346	3.9128	4.8910	5.8692	6.8474	7.8256	8.8038	1.3856
14	0.9781	1.9562	2.9344	3.9125	4.8906	5.8687	6.8468	7.8250	8.8031	1.3856
15	0.9780	1.9561	2.9341	3.9122	4.8902	5.8682	6.8463	7.8243	8.8023	1.3855
16	0.9780	1.9559	2.9339	3.9118	4.8898	5.8677	6.8457	7.8236	8.8016	1.3854
17	0.9779	1.9557	2.9336	3.9115	4.8894	5.8672	6.8451	7.8230	8.8009	1.3854
18	0.9778	1.9556	2.9334	3.9112	4.8890	5.8667	6.8445	7.8223	8.8001	1.3853
19	0.9777	1.9554	2.9331	3.9108	4.8885	5.8662	6.8440	7.8217	8.7994	1.3853
20	0.9776	1.9553	2.9329	3.9105	4.8881	5.8657	6.8434	7.8210	8.7986	1.3852
21	0.9775	1.9551	2.9326	3.9102	4.8877	5.8652	6.8428	7.8203	8.7979	1.3852
22	0.9775	1.9549	2.9324	3.9098	4.8873	5.8647	6.8422	7.8197	8.7971	1.3851
23	0.9774	1.9547	2.9321	3.9095	4.8869	5.8642	6.8416	7.8190	8.7964	1.3850
24	0.9773	1.9546	2.9319	3.9092	4.8864	5.8637	6.8410	7.8183	8.7956	1.3850
25	0.9772	1.9544	2.9316	3.9088	4.8860	5.8632	6.8404	7.8176	8.7948	1.3849
26	0.9771	1.9542	2.9314	3.9085	4.8856	5.8627	6.8398	7.8170	8.7941	1.3849
27	0.9770	1.9541	2.9311	3.9081	4.8852	5.8622	6.8393	7.8163	8.7933	1.3848
28	0.9770	1.9539	2.9309	3.9078	4.8848	5.8617	6.8387	7.8156	8.7926	1.3847
29	0.9769	1.9537	2.9306	3.9075	4.8843	5.8612	6.8381	7.8149	8.7918	1.3847
30	0.9768	1.9536	2.9304	3.9071	4.8839	5.8607	6.8375	7.8143	8.7911	1.3846
31	0.9767	1.9534	2.9301	3.9068	4.8835	5.8602	6.8369	7.8136	8.7903	1.3846
32	0.9766	1.9532	2.9298	3.9064	4.8831	5.8597	6.8363	7.8129	8.7895	1.3845
33	0.9765	1.9531	2.9296	3.9061	4.8826	5.8592	6.8357	7.8122	8.7887	1.3844
34	0.9764	1.9529	2.9293	3.9058	4.8822	5.8586	6.8351	7.8115	8.7880	1.3844
35	0.9764	1.9527	2.9291	3.9054	4.8818	5.8581	6.8345	7.8108	8.7872	1.3843
36	0.9763	1.9525	2.9288	3.9051	4.8813	5.8576	6.8339	7.8102	8.7864	1.3843
37	0.9762	1.9524	2.9285	3.9047	4.8809	5.8571	6.8333	7.8095	8.7856	1.3842
38	0.9761	1.9522	2.9283	3.9044	4.8805	5.8566	6.8327	7.8088	8.7849	1.3841
39	0.9760	1.9520	2.9280	3.9040	4.8801	5.8561	6.8321	7.8081	8.7841	1.3841
40	0.9759	1.9519	2.9278	3.9037	4.8796	5.8556	6.8315	7.8074	8.7833	1.3840
41	0.9758	1.9517	2.9275	3.9034	4.8792	5.8550	6.8309	7.8067	8.7826	1.3840
42	0.9758	1.9515	2.9273	3.9030	4.8788	5.8545	6.8303	7.8060	8.7818	1.3839
43	0.9757	1.9513	2.9270	3.9027	4.8783	5.8540	6.8296	7.8053	8.7810	1.3838
44	0.9756	1.9512	2.9267	3.9023	4.8779	5.8535	6.8290	7.8046	8.7802	1.3838
45	0.9755	1.9510	2.9265	3.9020	4.8774	5.8529	6.8284	7.8039	8.7794	1.3837
46	0.9754	1.9508	2.9262	3.9016	4.8770	5.8524	6.8278	7.8032	8.7786	1.3837
47	0.9753	1.9506	2.9259	3.9013	4.8766	5.8519	6.8272	7.8025	8.7778	1.3836
48	0.9752	1.9505	2.9257	3.9009	4.8761	5.8514	6.8266	7.8018	8.7770	1.3835
49	0.9751	1.9503	2.9254	3.9006	4.8757	5.8508	6.8260	7.8011	8.7763	1.3835
50	0.9751	1.9501	2.9252	3.9002	4.8753	5.8503	6.8254	7.8004	8.7755	1.3834
51	0.9750	1.9499	2.9249	3.8999	4.8748	5.8498	6.8247	7.7997	8.7747	1.3834
52	0.9749	1.9497	2.9246	3.8995	4.8744	5.8492	6.8241	7.7990	8.7739	1.3833
53	0.9748	1.9496	2.9244	3.8991	4.8739	5.8487	6.8235	7.7983	8.7731	1.3832
54	0.9747	1.9494	2.9241	3.8988	4.8735	5.8482	6.8229	7.7976	8.7723	1.3832
55	0.9746	1.9492	2.9238	3.8984	4.8730	5.8476	6.8222	7.7969	8.7715	1.3831
56	0.9745	1.9490	2.9236	3.8981	4.8726	5.8471	6.8216	7.7961	8.7707	1.3831
57	0.9744	1.9489	2.9233	3.8977	4.8721	5.8466	6.8210	7.7954	8.7699	1.3830
58	0.9743	1.9487	2.9230	3.8974	4.8717	5.8460	6.8204	7.7947	8.7691	1.3829
59	0.9743	1.9485	2.9228	3.8970	4.8713	5.8455	6.8198	7.7940	8.7683	1.3829
60	0.9742	1.9483	2.9225	3.8966	4.8708	5.8450	6.8191	7.7933	8.7675	1.3828

1	2	3	4	5	6	7	8	9	b	′
0.1376	0.2753	0.4129	0.5505	0.6881	0.8258	0.9634	1.1010	1.2387	0.1948	00
0.1380	0.2758	0.4137	0.5516	0.6895	0.8275	0.9654	1.1033	1.2412	0.1952	01
0.1382	0.2764	0.4145	0.5527	0.6909	0.8291	0.9673	1.1055	1.2437	0.1956	02
0.1385	0.2769	0.4154	0.5538	0.6923	0.8308	0.9692	1.1077	1.2462	0.1960	03
0.1387	0.2775	0.4162	0.5550	0.6937	0.8324	0.9712	1.1099	1.2487	0.1965	04
0.1390	0.2780	0.4171	0.5561	0.6951	0.8341	0.9731	1.1122	1.2512	0.1969	05
0.1393	0.2786	0.4179	0.5572	0.6965	0.8358	0.9751	1.1144	1.2537	0.1973	06
0.1396	0.2792	0.4187	0.5583	0.6979	0.8375	0.9771	1.1166	1.2562	0.1977	07
0.1399	0.2797	0.4196	0.5594	0.6993	0.8392	0.9790	1.1189	1.2587	0.1981	08
0.1401	0.2803	0.4204	0.5606	0.7007	0.8408	0.9810	1.1211	1.2613	0.1985	09
0.1404	0.2808	0.4213	0.5617	0.7021	0.8425	0.9829	1.1234	1.2638	0.1989	10
0.1407	0.2814	0.4221	0.5628	0.7035	0.8441	0.9849	1.1256	1.2663	0.1993	11
0.1410	0.2819	0.4229	0.5639	0.7049	0.8458	0.9868	1.1278	1.2688	0.1997	12
0.1413	0.2825	0.4238	0.5650	0.7063	0.8475	0.9888	1.1300	1.2713	0.2001	13
0.1415	0.2831	0.4246	0.5661	0.7077	0.8492	0.9907	1.1322	1.2738	0.2005	14
0.1418	0.2836	0.4254	0.5672	0.7091	0.8509	0.9927	1.1345	1.2763	0.2009	15
0.1421	0.2842	0.4263	0.5684	0.7104	0.8525	0.9946	1.1367	1.2788	0.2013	16
0.1424	0.2847	0.4271	0.5695	0.7118	0.8542	0.9966	1.1390	1.2813	0.2017	17
0.1426	0.2853	0.4279	0.5706	0.7132	0.8558	0.9985	1.1412	1.2838	0.2021	18
0.1429	0.2858	0.4288	0.5717	0.7146	0.8575	1.0005	1.1434	1.2863	0.2025	19
0.1432	0.2864	0.4296	0.5728	0.7160	0.8592	1.0024	1.1456	1.2888	0.2029	20
0.1435	0.2870	0.4304	0.5739	0.7174	0.8609	1.0044	1.1478	1.2913	0.2033	21
0.1438	0.2875	0.4313	0.5750	0.7188	0.8626	1.0063	1.1501	1.2938	0.2037	22
0.1440	0.2881	0.4321	0.5762	0.7202	0.8642	1.0083	1.1523	1.2963	0.2041	23
0.1443	0.2886	0.4329	0.5773	0.7216	0.8659	1.0102	1.1545	1.2988	0.2045	24
0.1446	0.2892	0.4338	0.5784	0.7230	0.8675	1.0121	1.1567	1.3013	0.2049	25
0.1449	0.2897	0.4346	0.5795	0.7243	0.8692	1.0141	1.1590	1.3038	0.2053	26
0.1451	0.2903	0.4354	0.5806	0.7257	0.8709	1.0160	1.1612	1.3063	0.2057	27
0.1454	0.2909	0.4363	0.5817	0.7271	0.8726	1.0180	1.1634	1.3088	0.2061	28
0.1457	0.2914	0.4371	0.5828	0.7285	0.8742	1.0199	1.1656	1.3113	0.2065	29
0.1460	0.2920	0.4379	0.5839	0.7299	0.8759	1.0219	1.1678	1.3138	0.2069	30
0.1463	0.2925	0.4388	0.5850	0.7313	0.8776	1.0238	1.1701	1.3163	0.2073	31
0.1465	0.2931	0.4396	0.5862	0.7327	0.8792	1.0258	1.1723	1.3188	0.2077	32
0.1468	0.2936	0.4404	0.5873	0.7341	0.8809	1.0277	1.1745	1.3213	0.2081	33
0.1471	0.2942	0.4413	0.5884	0.7355	0.8825	1.0296	1.1767	1.3238	0.2085	34
0.1474	0.2947	0.4421	0.5895	0.7368	0.8842	1.0316	1.1790	1.3263	0.2089	35
0.1476	0.2953	0.4429	0.5906	0.7382	0.8859	1.0335	1.1812	1.3288	0.2093	36
0.1479	0.2958	0.4438	0.5917	0.7396	0.8875	1.0355	1.1834	1.3313	0.2097	37
0.1482	0.2964	0.4446	0.5928	0.7410	0.8892	1.0374	1.1856	1.3338	0.2101	38
0.1485	0.2970	0.4454	0.5939	0.7424	0.8909	1.0394	1.1878	1.3363	0.2105	39
0.1488	0.2975	0.4463	0.5950	0.7438	0.8926	1.0413	1.1901	1.3388	0.2110	40
0.1490	0.2981	0.4471	0.5961	0.7452	0.8942	1.0432	1.1923	1.3413	0.2114	41
0.1493	0.2986	0.4479	0.5972	0.7466	0.8959	1.0452	1.1945	1.3438	0.2118	42
0.1496	0.2992	0.4488	0.5984	0.7479	0.8975	1.0471	1.1967	1.3463	0.2122	43
0.1499	0.2997	0.4496	0.5995	0.7493	0.8992	1.0491	1.1989	1.3488	0.2126	44
0.1501	0.3003	0.4504	0.6006	0.7507	0.9008	1.0510	1.2011	1.3513	0.2130	45
0.1504	0.3008	0.4513	0.6017	0.7521	0.9025	1.0529	1.2034	1.3538	0.2134	46
0.1507	0.3014	0.4521	0.6028	0.7535	0.9042	1.0549	1.2056	1.3563	0.2138	47
0.1510	0.3019	0.4529	0.6039	0.7549	0.9058	1.0568	1.2078	1.3588	0.2142	48
0.1513	0.3025	0.4538	0.6050	0.7563	0.9075	1.0588	1.2100	1.3613	0.2146	49
0.1515	0.3031	0.4546	0.6061	0.7576	0.9092	1.0607	1.2122	1.3638	0.2150	50
0.1518	0.3036	0.4554	0.6072	0.7590	0.9108	1.0626	1.2144	1.3662	0.2154	51
0.1521	0.3042	0.4562	0.6083	0.7604	0.9125	1.0646	1.2166	1.3687	0.2158	52
0.1524	0.3047	0.4571	0.6094	0.7618	0.9142	1.0665	1.2189	1.3712	0.2162	53
0.1526	0.3053	0.4579	0.6105	0.7632	0.9158	1.0684	1.2211	1.3737	0.2166	54
0.1529	0.3058	0.4587	0.6116	0.7646	0.9175	1.0704	1.2233	1.3762	0.2170	55
0.1532	0.3064	0.4596	0.6128	0.7660	0.9191	1.0723	1.2255	1.3787	0.2174	56
0.1535	0.3069	0.4604	0.6139	0.7673	0.9208	1.0742	1.2277	1.3812	0.2178	57
0.1537	0.3075	0.4612	0.6150	0.7687	0.9224	1.0762	1.2299	1.3837	0.2182	58
0.1540	0.3080	0.4621	0.6161	0.7701	0.9241	1.0781	1.2321	1.3862	0.2186	59
0.1543	0.3086	0.4629	0.6172	0.7715	0.9257	1.0800	1.2343	1.3886	0.2190	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9742	1.9483	2.9225	3.8966	4.8708	5.8450	6.8191	7.7933	8.7675	1.3828
01	0.9741	1.9481	2.9222	3.8963	4.8704	5.8444	6.8185	7.7926	8.7666	1.3827
02	0.9740	1.9480	2.9219	3.8959	4.8699	5.8439	6.8179	7.7918	8.7658	1.3826
03	0.9739	1.9478	2.9217	3.8956	4.8695	5.8433	6.8172	7.7911	8.7650	1.3826
04	0.9738	1.9476	2.9214	3.8952	4.8690	5.8428	6.8166	7.7904	8.7642	1.3825
05	0.9737	1.9474	2.9211	3.8948	4.8686	5.8423	6.8160	7.7897	8.7634	1.3825
06	0.9736	1.9472	2.9209	3.8945	4.8681	5.8417	6.8153	7.7890	8.7626	1.3824
07	0.9735	1.9471	2.9206	3.8941	4.8676	5.8412	6.8147	7.7882	8.7618	1.3824
08	0.9734	1.9469	2.9203	3.8938	4.8672	5.8406	6.8141	7.7875	8.7609	1.3823
09	0.9733	1.9467	2.9200	3.8934	4.8667	5.8401	6.8134	7.7868	8.7601	1.3822
10	0.9733	1.9465	2.9198	3.8930	4.8663	5.8395	6.8128	7.7861	8.7593	1.3821
11	0.9732	1.9463	2.9195	3.8927	4.8658	5.8390	6.8122	7.7853	8.7585	1.3821
12	0.9731	1.9461	2.9192	3.8923	4.8654	5.8384	6.8115	7.7846	8.7577	1.3820
13	0.9730	1.9460	2.9189	3.8919	4.8649	5.8379	6.8109	7.7838	8.7568	1.3819
14	0.9729	1.9458	2.9187	3.8916	4.8644	5.8373	6.8102	7.7831	8.7560	1.3819
15	0.9728	1.9456	2.9184	3.8912	4.8640	5.8368	6.8096	7.7824	8.7552	1.3818
16	0.9727	1.9454	2.9181	3.8908	4.8635	5.8362	6.8089	7.7816	8.7543	1.3818
17	0.9726	1.9452	2.9178	3.8904	4.8631	5.8357	6.8083	7.7809	8.7535	1.3817
18	0.9725	1.9450	2.9176	3.8901	4.8626	5.8351	6.8076	7.7802	8.7527	1.3816
19	0.9724	1.9449	2.9173	3.8897	4.8621	5.8346	6.8070	7.7794	8.7518	1.3816
20	0.9723	1.9447	2.9170	3.8893	4.8617	5.8340	6.8063	7.7787	8.7510	1.3815
21	0.9722	1.9445	2.9167	3.8890	4.8612	5.8334	6.8057	7.7779	8.7502	1.3814
22	0.9721	1.9443	2.9164	3.8886	4.8607	5.8329	6.8050	7.7772	8.7493	1.3814
23	0.9721	1.9441	2.9162	3.8882	4.8603	5.8323	6.8044	7.7764	8.7485	1.3813
24	0.9720	1.9439	2.9159	3.8878	4.8598	5.8318	6.8037	7.7757	8.7476	1.3813
25	0.9719	1.9437	2.9156	3.8875	4.8593	5.8312	6.8031	7.7749	8.7468	1.3812
26	0.9718	1.9435	2.9153	3.8871	4.8589	5.8306	6.8024	7.7742	8.7460	1.3811
27	0.9717	1.9434	2.9150	3.8867	4.8584	5.8301	6.8018	7.7734	8.7451	1.3811
28	0.9716	1.9432	2.9148	3.8863	4.8579	5.8295	6.8011	7.7727	8.7443	1.3810
29	0.9715	1.9430	2.9145	3.8860	4.8575	5.8290	6.8004	7.7719	8.7434	1.3810
30	0.9714	1.9428	2.9142	3.8856	4.8570	5.8284	6.7998	7.7712	8.7426	1.3809
31	0.9713	1.9426	2.9139	3.8852	4.8565	5.8278	6.7991	7.7704	8.7417	1.3808
32	0.9712	1.9424	2.9136	3.8848	4.8560	5.8272	6.7984	7.7697	8.7409	1.3808
33	0.9711	1.9422	2.9133	3.8844	4.8556	5.8267	6.7978	7.7689	8.7400	1.3807
34	0.9710	1.9420	2.9130	3.8841	4.8551	5.8261	6.7971	7.7681	8.7391	1.3806
35	0.9709	1.9418	2.9128	3.8837	4.8546	5.8255	6.7964	7.7674	8.7383	1.3806
36	0.9708	1.9417	2.9125	3.8833	4.8541	5.8250	6.7958	7.7666	8.7374	1.3805
37	0.9707	1.9415	2.9122	3.8829	4.8537	5.8244	6.7951	7.7658	8.7366	1.3804
38	0.9706	1.9413	2.9119	3.8825	4.8532	5.8238	6.7944	7.7651	8.7357	1.3804
39	0.9705	1.9411	2.9116	3.8822	4.8527	5.8232	6.7938	7.7643	8.7349	1.3803
40	0.9704	1.9409	2.9113	3.8818	4.8522	5.8227	6.7931	7.7636	8.7340	1.3802
41	0.9703	1.9407	2.9110	3.8814	4.8517	5.8221	6.7924	7.7628	8.7331	1.3802
42	0.9703	1.9405	2.9108	3.8810	4.8513	5.8215	6.7918	7.7620	8.7323	1.3801
43	0.9702	1.9403	2.9105	3.8806	4.8508	5.8209	6.7911	7.7612	8.7314	1.3800
44	0.9701	1.9401	2.9102	3.8802	4.8503	5.8203	6.7904	7.7604	8.7305	1.3799
45	0.9700	1.9399	2.9099	3.8798	4.8498	5.8198	6.7897	7.7597	8.7296	1.3799
46	0.9699	1.9397	2.9096	3.8794	4.8493	5.8192	6.7890	7.7589	8.7288	1.3798
47	0.9698	1.9395	2.9093	3.8791	4.8488	5.8186	6.7884	7.7581	8.7279	1.3797
48	0.9697	1.9393	2.9090	3.8787	4.8483	5.8180	6.7877	7.7573	8.7270	1.3797
49	0.9696	1.9391	2.9087	3.8783	4.8479	5.8174	6.7870	7.7566	8.7261	1.3796
50	0.9695	1.9389	2.9084	3.8779	4.8474	5.8168	6.7863	7.7558	8.7253	1.3795
51	0.9694	1.9388	2.9081	3.8775	4.8469	5.8163	6.7856	7.7550	8.7244	1.3795
52	0.9693	1.9386	2.9078	3.8771	4.8464	5.8157	6.7849	7.7542	8.7235	1.3794
53	0.9692	1.9384	2.9075	3.8767	4.8459	5.8151	6.7843	7.7534	8.7226	1.3793
54	0.9691	1.9382	2.9072	3.8763	4.8454	5.8145	6.7836	7.7526	8.7217	1.3792
55	0.9690	1.9380	2.9069	3.8759	4.8449	5.8139	6.7829	7.7519	8.7208	1.3792
56	0.9689	1.9378	2.9066	3.8755	4.8444	5.8133	6.7822	7.7511	8.7199	1.3791
57	0.9688	1.9376	2.9064	3.8751	4.8439	5.8127	6.7815	7.7503	8.7191	1.3790
58	0.9687	1.9374	2.9061	3.8747	4.8434	5.8121	6.7808	7.7495	8.7182	1.3789
59	0.9686	1.9372	2.9058	3.8744	4.8429	5.8115	6.7801	7.7487	8.7173	1.3789
60	0.9685	1.9370	2.9055	3.8740	4.8424	5.8109	6.7794	7.7479	8.7164	1.3788

1	2	3	4	5	6	7	8	9	b	′
0.1543	0.3086	0.4629	0.6172	0.7715	0.9257	1.0800	1.2343	1.3886	0.2190	00
0.1546	0.3091	0.4637	0.6183	0.7729	0.9274	1.0820	1.2365	1.3911	0.2194	01
0.1548	0.3097	0.4645	0.6194	0.7742	0.9290	1.0839	1.2387	1.3936	0.2198	02
0.1551	0.3102	0.4654	0.6205	0.7756	0.9307	1.0858	1.2410	1.3961	0.2202	03
0.1554	0.3108	0.4662	0.6216	0.7770	0.9324	1.0878	1.2432	1.3986	0.2206	04
0.1557	0.3113	0.4670	0.6227	0.7784	0.9340	1.0897	1.2454	1.4010	0.2210	05
0.1559	0.3119	0.4678	0.6238	0.7797	0.9357	1.0916	1.2476	1.4035	0.2214	06
0.1562	0.3124	0.4687	0.6249	0.7811	0.9373	1.0936	1.2498	1.4060	0.2218	07
0.1565	0.3130	0.4695	0.6260	0.7825	0.9390	1.0955	1.2520	1.4085	0.2222	08
0.1568	0.3136	0.4703	0.6271	0.7839	0.9407	1.0975	1.2542	1.4110	0.2226	09
0.1571	0.3141	0.4712	0.6282	0.7853	0.9423	1.0994	1.2564	1.4135	0.2230	10
0.1573	0.3147	0.4720	0.6293	0.7866	0.9440	1.1013	1.2586	1.4160	0.2234	11
0.1576	0.3152	0.4728	0.6304	0.7880	0.9456	1.1032	1.2608	1.4184	0.2238	12
0.1579	0.3158	0.4736	0.6315	0.7894	0.9473	1.1052	1.2630	1.4209	0.2242	13
0.1582	0.3163	0.4745	0.6326	0.7908	0.9489	1.1071	1.2652	1.4234	0.2246	14
0.1584	0.3169	0.4753	0.6337	0.7922	0.9506	1.1090	1.2674	1.4259	0.2250	15
0.1587	0.3174	0.4761	0.6348	0.7935	0.9523	1.1110	1.2697	1.4284	0.2254	16
0.1590	0.3180	0.4769	0.6359	0.7949	0.9539	1.1129	1.2719	1.4308	0.2258	17
0.1593	0.3185	0.4778	0.6370	0.7963	0.9556	1.1148	1.2741	1.4333	0.2262	18
0.1595	0.3191	0.4786	0.6381	0.7977	0.9572	1.1167	1.2763	1.4358	0.2266	19
0.1598	0.3196	0.4794	0.6392	0.7991	0.9589	1.1187	1.2785	1.4383	0.2270	20
0.1601	0.3202	0.4802	0.6403	0.8004	0.9605	1.1206	1.2807	1.4407	0.2274	21
0.1604	0.3207	0.4811	0.6414	0.8018	0.9622	1.1225	1.2829	1.4432	0.2278	22
0.1606	0.3213	0.4819	0.6425	0.8032	0.9638	1.1244	1.2851	1.4457	0.2282	23
0.1609	0.3218	0.4827	0.6436	0.8046	0.9655	1.1264	1.2873	1.4482	0.2287	24
0.1612	0.3224	0.4835	0.6447	0.8059	0.9671	1.1283	1.2895	1.4506	0.2291	25
0.1615	0.3229	0.4844	0.6458	0.8073	0.9688	1.1302	1.2917	1.4531	0.2295	26
0.1617	0.3235	0.4852	0.6469	0.8087	0.9704	1.1321	1.2939	1.4556	0.2299	27
0.1620	0.3240	0.4860	0.6480	0.8100	0.9721	1.1341	1.2961	1.4581	0.2303	28
0.1623	0.3246	0.4868	0.6491	0.8114	0.9737	1.1360	1.2983	1.4605	0.2307	29
0.1626	0.3251	0.4877	0.6502	0.8128	0.9754	1.1379	1.3005	1.4630	0.2311	30
0.1628	0.3257	0.4885	0.6513	0.8142	0.9770	1.1398	1.3027	1.4655	0.2315	31
0.1631	0.3262	0.4893	0.6524	0.8155	0.9787	1.1418	1.3049	1.4680	0.2319	32
0.1634	0.3268	0.4901	0.6535	0.8169	0.9803	1.1437	1.3071	1.4704	0.2323	33
0.1637	0.3273	0.4910	0.6546	0.8183	0.9819	1.1456	1.3092	1.4729	0.2327	34
0.1639	0.3279	0.4918	0.6557	0.8196	0.9836	1.1475	1.3114	1.4754	0.2331	35
0.1642	0.3284	0.4926	0.6568	0.8210	0.9852	1.1494	1.3136	1.4778	0.2335	36
0.1645	0.3290	0.4934	0.6579	0.8224	0.9869	1.1514	1.3158	1.4803	0.2339	37
0.1648	0.3295	0.4943	0.6590	0.8238	0.9885	1.1533	1.3180	1.4828	0.2343	38
0.1650	0.3301	0.4951	0.6601	0.8251	0.9902	1.1552	1.3202	1.4853	0.2347	39
0.1653	0.3306	0.4959	0.6612	0.8265	0.9918	1.1571	1.3224	1.4877	0.2351	40
0.1656	0.3311	0.4967	0.6623	0.8279	0.9934	1.1590	1.3246	1.4901	0.2355	41
0.1658	0.3317	0.4975	0.6634	0.8292	0.9951	1.1609	1.3268	1.4926	0.2359	42
0.1661	0.3322	0.4984	0.6645	0.8306	0.9967	1.1629	1.3290	1.4951	0.2363	43
0.1664	0.3328	0.4992	0.6656	0.8320	0.9984	1.1648	1.3312	1.4976	0.2367	44
0.1667	0.3333	0.5000	0.6667	0.8334	1.0000	1.1667	1.3334	1.5000	0.2371	45
0.1669	0.3339	0.5008	0.6678	0.8347	1.0016	1.1686	1.3355	1.5025	0.2375	46
0.1672	0.3344	0.5017	0.6689	0.8361	1.0033	1.1705	1.3378	1.5050	0.2379	47
0.1675	0.3350	0.5025	0.6700	0.8375	1.0049	1.1724	1.3399	1.5074	0.2383	48
0.1678	0.3355	0.5033	0.6711	0.8388	1.0066	1.1743	1.3421	1.5098	0.2387	49
0.1680	0.3361	0.5041	0.6722	0.8402	1.0082	1.1763	1.3443	1.5123	0.2391	50
0.1683	0.3366	0.5049	0.6732	0.8416	1.0099	1.1782	1.3465	1.5148	0.2395	51
0.1686	0.3372	0.5057	0.6743	0.8429	1.0115	1.1801	1.3487	1.5172	0.2399	52
0.1689	0.3377	0.5066	0.6754	0.8443	1.0132	1.1820	1.3509	1.5197	0.2403	53
0.1691	0.3383	0.5074	0.6765	0.8457	1.0148	1.1839	1.3531	1.5222	0.2407	54
0.1694	0.3388	0.5082	0.6776	0.8470	1.0165	1.1859	1.3553	1.5247	0.2411	55
0.1697	0.3394	0.5090	0.6787	0.8484	1.0181	1.1878	1.3574	1.5271	0.2415	56
0.1700	0.3399	0.5099	0.6798	0.8498	1.0197	1.1897	1.3596	1.5296	0.2419	57
0.1702	0.3404	0.5107	0.6809	0.8511	1.0213	1.1916	1.3618	1.5320	0.2423	58
0.1705	0.3410	0.5115	0.6820	0.8525	1.0230	1.1935	1.3640	1.5345	0.2427	59
0.1708	0.3415	0.5123	0.6831	0.8539	1.0246	1.1954	1.3662	1.5369	0.2431	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9685	1.9370	2.9055	3.8740	4.8424	5.8109	6.7794	7.7479	8.7164	1.3788
01	0.9684	1.9368	2.9052	3.8736	4.8419	5.8103	6.7787	7.7471	8.7155	1.3787
02	0.9683	1.9366	2.9049	3.8732	4.8414	5.8097	6.7780	7.7463	8.7146	1.3786
03	0.9682	1.9364	2.9046	3.8728	4.8409	5.8091	6.7773	7.7455	8.7137	1.3786
04	0.9681	1.9362	2.9043	3.8724	4.8404	5.8085	6.7766	7.7447	8.7128	1.3785
05	0.9680	1.9360	2.9040	3.8720	4.8399	5.8079	6.7759	7.7439	8.7119	1.3784
06	0.9679	1.9358	2.9037	3.8716	4.8394	5.8073	6.7752	7.7431	8.7110	1.3783
07	0.9678	1.9356	2.9034	3.8712	4.8389	5.8067	6.7745	7.7423	8.7101	1.3783
08	0.9677	1.9354	2.9031	3.8707	4.8384	5.8061	6.7738	7.7415	8.7092	1.3782
09	0.9676	1.9352	2.9028	3.8703	4.8379	5.8055	6.7731	7.7407	8.7083	1.3781
10	0.9675	1.9350	2.9025	3.8699	4.8374	5.8049	6.7724	7.7399	8.7074	1.3780
11	0.9674	1.9348	2.9022	3.8695	4.8369	5.8043	6.7717	7.7391	8.7065	1.3780
12	0.9673	1.9346	2.9019	3.8691	4.8364	5.8037	6.7710	7.7383	8.7056	1.3779
13	0.9672	1.9344	2.9015	3.8687	4.8359	5.8031	6.7703	7.7375	8.7046	1.3778
14	0.9671	1.9342	2.9012	3.8683	4.8354	5.8025	6.7696	7.7366	8.7037	1.3777
15	0.9670	1.9340	2.9009	3.8679	4.8349	5.8019	6.7689	7.7358	8.7028	1.3776
16	0.9669	1.9338	2.9006	3.8675	4.8344	5.8011	6.7681	7.7350	8.7019	1.3776
17	0.9668	1.9336	2.9003	3.8671	4.8339	5.8007	6.7674	7.7342	8.7010	1.3775
18	0.9667	1.9333	2.9000	3.8667	4.8334	5.8000	6.7667	7.7334	8.7001	1.3774
19	0.9666	1.9331	2.8997	3.8663	4.8329	5.7994	6.7660	7.7326	8.6991	1.3773
20	0.9665	1.9329	2.8994	3.8659	4.8324	5.7988	6.7653	7.7318	8.6982	1.3773
21	0.9664	1.9327	2.8991	3.8655	4.8318	5.7982	6.7646	7.7309	8.6973	1.3772
22	0.9663	1.9325	2.8988	3.8651	4.8313	5.7976	6.7638	7.7301	8.6964	1.3771
23	0.9662	1.9323	2.8985	3.8646	4.8308	5.7970	6.7631	7.7293	8.6954	1.3770
24	0.9661	1.9321	2.8982	3.8642	4.8303	5.7963	6.7624	7.7285	8.6945	1.3769
25	0.9660	1.9319	2.8979	3.8638	4.8298	5.7957	6.7617	7.7276	8.6936	1.3769
26	0.9659	1.9317	2.8976	3.8634	4.8293	5.7951	6.7610	7.7268	8.6927	1.3768
27	0.9657	1.9315	2.8972	3.8630	4.8287	5.7945	6.7602	7.7260	8.6917	1.3767
28	0.9656	1.9313	2.8969	3.8626	4.8282	5.7939	6.7595	7.7252	8.6908	1.3766
29	0.9655	1.9311	2.8966	3.8622	4.8277	5.7932	6.7588	7.7243	8.6899	1.3765
30	0.9654	1.9309	2.8963	3.8617	4.8272	5.7926	6.7581	7.7235	8.6889	1.3765
31	0.9653	1.9307	2.8960	3.8613	4.8267	5.7920	6.7573	7.7227	8.6880	1.3764
32	0.9652	1.9305	2.8957	3.8609	4.8261	5.7914	6.7566	7.7218	8.6870	1.3763
33	0.9651	1.9302	2.8954	3.8605	4.8256	5.7907	6.7559	7.7210	8.6861	1.3762
34	0.9650	1.9300	2.8951	3.8601	4.8251	5.7901	6.7551	7.7201	8.6852	1.3761
35	0.9649	1.9298	2.8947	3.8597	4.8246	5.7895	6.7544	7.7193	8.6842	1.3761
36	0.9648	1.9296	2.8944	3.8592	4.8240	5.7888	6.7537	7.7185	8.6833	1.3760
37	0.9647	1.9294	2.8941	3.8588	4.8235	5.7882	6.7529	7.7176	8.6823	1.3759
38	0.9646	1.9292	2.8938	3.8584	4.8230	5.7876	6.7522	7.7168	8.6814	1.3759
39	0.9645	1.9290	2.8935	3.8580	4.8225	5.7870	6.7515	7.7159	8.6804	1.3758
40	0.9644	1.9288	2.8932	3.8576	4.8219	5.7863	6.7507	7.7151	8.6795	1.3757
41	0.9643	1.9286	2.8928	3.8571	4.8214	5.7857	6.7500	7.7143	8.6785	1.3756
42	0.9642	1.9284	2.8925	3.8567	4.8209	5.7851	6.7492	7.7134	8.6776	1.3755
43	0.9641	1.9281	2.8922	3.8563	4.8203	5.7844	6.7485	7.7126	8.6766	1.3755
44	0.9640	1.9279	2.8919	3.8558	4.8198	5.7838	6.7477	7.7117	8.6757	1.3754
45	0.9639	1.9277	2.8916	3.8554	4.8193	5.7831	6.7470	7.7108	8.6747	1.3753
46	0.9638	1.9275	2.8912	3.8550	4.8187	5.7825	6.7462	7.7100	8.6737	1.3752
47	0.9636	1.9273	2.8909	3.8546	4.8182	5.7819	6.7455	7.7091	8.6728	1.3752
48	0.9635	1.9271	2.8906	3.8541	4.8177	5.7812	6.7448	7.7083	8.6718	1.3751
49	0.9634	1.9269	2.8903	3.8537	4.8172	5.7806	6.7440	7.7074	8.6709	1.3750
50	0.9633	1.9266	2.8900	3.8533	4.8166	5.7799	6.7433	7.7066	8.6699	1.3749
51	0.9632	1.9264	2.8896	3.8529	4.8161	5.7793	6.7425	7.7057	8.6689	1.3748
52	0.9631	1.9262	2.8893	3.8524	4.8155	5.7786	6.7417	7.7049	8.6680	1.3748
53	0.9630	1.9260	2.8890	3.8520	4.8150	5.7780	6.7410	7.7040	8.6670	1.3747
54	0.9629	1.9258	2.8887	3.8516	4.8145	5.7773	6.7402	7.7031	8.6660	1.3746
55	0.9628	1.9256	2.8883	3.8511	4.8139	5.7767	6.7395	7.7023	8.6650	1.3745
56	0.9627	1.9254	2.8880	3.8507	4.8134	5.7761	6.7387	7.7014	8.6641	1.3744
57	0.9626	1.9251	2.8877	3.8503	4.8128	5.7754	6.7380	7.7005	8.6631	1.3744
58	0.9625	1.9249	2.8874	3.8498	4.8123	5.7748	6.7372	7.6997	8.6621	1.3743
59	0.9624	1.9247	2.8871	3.8494	4.8118	5.7741	6.7365	7.6988	8.6612	1.3742
60	0.9622	1.9245	2.8867	3.8490	4.8112	5.7735	6.7357	7.6979	8.6602	1.3742

1	2	3	4	5	6	7	8	9	b	′
0.1708	0.3415	0.5123	0.6831	0.8539	1.0246	1.1954	1.3662	1.5369	0.2431	00
0.1710	0.3421	0.5131	0.6842	0.8552	1.0262	1.1973	1.3683	1.5394	0.2435	01
0.1713	0.3426	0.5140	0.6853	0.8566	1.0279	1.1992	1.3705	1.5419	0.2439	02
0.1716	0.3432	0.5148	0.6864	0.8580	1.0295	1.2011	1.3727	1.5443	0.2443	03
0.1719	0.3437	0.5156	0.6875	0.8593	1.0312	1.2030	1.3749	1.5468	0.2447	04
0.1721	0.3443	0.5164	0.6886	0.8607	1.0328	1.2050	1.3771	1.5493	0.2451	05
0.1724	0.3448	0.5173	0.6896	0.8620	1.0345	1.2069	1.3793	1.5517	0.2455	c6
c.1727	0.3454	0.5180	0.6907	0.8634	1.0361	1.2088	1.3814	1.5541	0.2459	07
0.1730	0.3459	0.5189	0.6918	0.8648	1.0377	1.2107	1.3836	1.5566	0.2463	08
0.1732	0.3464	0.5197	0.6929	0.8661	1.0393	1.2126	1.3858	1.5590	0.2467	09
0.1735	0.3470	0.5205	0.6940	0.8675	1.0410	1.2145	1.3880	1.5615	0.2471	10
0.1738	0.3475	c.5213	0.6951	0.8688	1.0426	1.2164	1.3902	1.5639	0.2475	11
0.1740	0.3481	0.5221	0.6962	0.8702	1.0442	1.2183	1.3923	1.5664	0.2479	12
0.1743	0.3486	0.5229	0.6973	0.8716	1.0459	1.2202	1.3945	1.5688	0.2483	13
0.1746	0.3492	0.5238	0.6984	0.8729	1.0475	1.2221	1.3967	1.5713	0.2487	14
0.1749	0.3497	0.5246	0.6994	0.8743	1.0492	1.2240	1.3989	1.5737	0.2491	15
0.1751	0.3503	0.5254	0.7005	0.8757	1.0508	1.2259	1.4010	1.5762	c.2495	16
0.1754	0.3508	0.5262	0.7016	c.8770	1.0524	1.2278	1.4032	1.5786	0.2499	17
0.1757	0.3513	0.5270	0.7027	c.8784	1.0540	1.2297	1.4054	1.5810	0.2503	18
0.1759	0.3519	0.5278	0.7038	0.8797	1.0557	1.2316	1.4076	1.5835	0.2507	19
0.1762	0.3524	0.5287	0.7049	0.8811	1.0573	1.2335	1.4098	1.5860	0.2511	20
0.1765	0.3530	0.5295	0.7060	0.8824	1.0589	1.2354	1.4119	1.5884	0.2515	21
0.1768	0.3535	0.5303	0.7070	0.8838	1.0606	1.2373	1.4141	1.5908	0.2519	22
0.1770	0.3541	0.5311	0.7081	0.8852	1.0622	1.2392	1.4162	1.5933	0.2523	23
0.1773	0.3546	0.5319	0.7092	0.8865	1.0638	1.2411	1.4184	1.5957	0.2527	24
0.1776	0.3552	0.5327	0.7103	0.8879	1.0655	1.2430	1.4206	1.5982	0.2531	25
0.1778	0.3557	0.5335	0.7114	0.8892	1.0671	1.2449	1.4228	1.6006	0.2535	26
0.1781	0.3562	0.5344	0.7125	0.8906	1.0687	1.2468	1.4250	1.6031	0.2539	27
0.1784	0.3568	0.5352	0.7136	0.8920	1.0703	1.2487	1.4271	1.6055	0.2543	28
0.1787	0.3573	0.5360	0.7146	0.8933	1.0720	1.2506	1.4293	1.6079	0.2547	29
0.1789	0.3579	0.5368	0.7157	0.8947	1.0736	1.2525	1.4314	1.6104	0.2551	30
0.1792	0.3584	0.5376	0.7168	0.8960	1.0752	1.2544	1.4336	1.6128	0.2555	31
0.1795	0.3590	0.5384	0.7179	0.8974	1.0769	1.2563	1.4358	1.6153	0.2559	32
0.1797	0.3595	0.5392	0.7190	0.8987	1.0785	1.2582	1.4380	1.6177	0.2563	33
0.1800	0.3600	0.5401	0.7201	0.9001	1.0801	1.2601	1.4402	1.6202	0.2567	34
0.1803	0.3606	c.5409	0.7212	0.9014	1.0817	1.2620	1.4423	1.6226	0.2571	35
0.1806	0.3611	0.5417	0.7222	0.9028	1.0834	1.2639	1.4445	1.6250	0.2575	36
0.1808	0.3617	0.5425	0.7233	0.9041	1.0850	1.2658	1.4466	1.6275	0.2579	37
0.1811	0.3622	0.5433	0.7244	0.9055	1.0866	1.2677	1.4488	1.6299	0.2583	38
0.1814	0.3627	0.5441	0.7255	c.9069	1.c882	1.2696	1.4510	1.6323	0.2587	39
0.1816	0.3633	0.5449	0.7266	0.9082	1·0898	1.2715	1.4531	1.6348	0.2591	40
0.1819	0.3638	0.5457	0.7276	0.9096	1.c915	1.2734	1.4553	1.6372	0.2595	41
0.1822	0.3644	0.5465	0.7287	0.9109	1.0931	1.2753	1.4574	1.6396	0.2599	42
0.1825	0.3649	0.5474	0.7298	0.9123	1.0947	1.2772	1.4596	1.6421	0.2603	43
0.1827	0.3654	0.5482	0.7309	0.9136	1.0963	1.2790	1.4618	1.6445	0.2607	44
0.1830	0.3660	0.5490	0.7320	0.9150	1.0979	1.2809	1.4639	1.6469	0.2611	45
0.1833	0.3665	0.5498	0.7330	0.9163	1.c996	1.2828	1.4661	1.6493	0.2615	46
0.1835	0.3671	0.5506	0.7341	0.9177	1.1012	1.2847	1.4682	1.6518	0.2619	47
0.1838	0.3676	0.5514	0.7352	0.9190	1.1028	1.2866	1.4704	1.6542	0.2623	48
0.1841	0.3681	0.5522	0.7363	0.9204	1.1044	1.2885	1.4726	1.6566	0.2627	49
0.1843	0.3687	0.5530	0.7374	0.9217	1.1061	1.2904	1.4748	1.6591	0.2631	50
0.1846	0.3692	0.5538	0.7384	0.9231	1.1077	1.2923	1.4769	1.6615	0.2635	51
0.1849	0.3698	0.5546	0.7395	0.9244	1.1093	1.2942	1.4790	1.6639	0.2639	52
0.1852	0.3703	0.5555	0.7406	0.9258	1.1109	1.2961	1.4812	1·6664	0.2643	53
0.1854	0.3708	0.5563	0.7417	0.9271	1.1125	1.2979	1.4834	1.6688	0.2647	54
0.1857	0.3714	0.5571	0.7428	0.9285	1.1141	1.2998	1.4855	1.6712	0.2651	55
0.1860	0.3719	0.5579	0.7438	0.9298	1.1158	1.3017	1.4877	1.6736	0.2655	56
0.1862	0.3725	0.5587	0.7449	0.9312	1.1174	1.3036	1.4898	1.6761	0.2659	57
0.1865	0.3730	0.5595	0.7460	0.9325	1.1190	1.3055	1.4920	1.6785	0.2663	58
0.1868	0.3735	0.5603	0.7471	0.9339	1.1206	1.3074	1.4942	1.6809	0.2667	59
0.1870	0.3741	0.5611	0.7482	0.9352	1.1222	1.3093	1.4963	1.6834	0.2671	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9622	1.9245	2.8867	3.8490	4.8112	5.7735	6.7357	7.6979	8.6602	1.3742
01	0.9621	1.9243	2.8864	3.8485	4.8107	5.7728	6.7349	7.6971	8.6592	1.3741
02	0.9620	1.9240	2.8861	3.8481	4.8101	5.7721	6.7342	7.6962	8.6582	1.3740
03	0.9619	1.9238	2.8857	3.8477	4.8096	5.7715	6.7334	7.6953	8.6572	1.3739
04	0.9618	1.9236	2.8854	3.8472	4.8090	5.7708	6.7326	7.6944	8.6562	1.3738
05	0.9617	1.9234	2.8851	3.8468	4.8085	5.7702	6.7319	7.6936	8.6553	1.3738
06	0.9616	1.9232	2.8848	3.8463	4.8079	5.7695	6.7311	7.6927	8.6543	1.3737
07	0.9615	1.9230	2.8844	3.8459	4.8074	5.7689	6.7303	7.6918	8.6533	1.3736
08	0.9614	1.9227	2.8841	3.8455	4.8068	5.7682	6.7296	7.6909	8.6523	1.3735
09	0.9613	1.9225	2.8838	3.8450	4.8063	5.7675	6.7288	7.6901	8.6513	1.3734
10	0.9611	1.9223	2.8834	3.8446	4.8057	5.7669	6.7280	7.6892	8.6503	1.3734
11	0.9610	1.9221	2.8831	3.8441	4.8052	5.7662	6.7273	7.6883	8.6493	1.3733
12	0.9609	1.9218	2.8828	3.8437	4.8046	5.7655	6.7265	7.6874	8.6483	1.3732
13	0.9608	1.9216	2.8824	3.8433	4.8041	5.7649	6.7257	7.6865	8.6473	1.3731
14	0.9607	1.9214	2.8821	3.8428	4.8035	5.7642	6.7249	7.6856	8.6463	1.3730
15	0.9606	1.9212	2.8818	3.8424	4.8030	5.7635	6.7241	7.6847	8.6453	1.3730
16	0.9605	1.9210	2.8814	3.8419	4.8024	5.7629	6.7234	7.6838	8.6443	1.3729
17	0.9604	1.9207	2.8811	3.8415	4.8018	5.7622	6.7226	7.6830	8.6433	1.3728
18	0.9603	1.9205	2.8808	3.8410	4.8013	5.7615	6.7218	7.6821	8.6423	1.3727
19	0.9601	1.9203	2.8804	3.8406	4.8007	5.7609	6.7210	7.6812	8.6413	1.3726
20	0.9600	1.9201	2.8801	3.8401	4.8002	5.7602	6.7202	7.6803	8.6403	1.3726
21	0.9599	1.9198	2.8798	3.8397	4.7996	5.7595	6.7195	7.6794	8.6393	1.3725
22	0.9598	1.9196	2.8794	3.8392	4.7990	5.7589	6.7187	7.6785	8.6383	1.3724
23	0.9597	1.9194	2.8791	3.8388	4.7985	5.7582	6.7179	7.6776	8.6373	1.3723
24	0.9596	1.9192	2.8788	3.8383	4.7979	5.7575	6.7171	7.6767	8.6363	1.3722
25	0.9595	1.9189	2.8784	3.8379	4.7974	5.7568	6.7163	7.6758	8.6352	1.3722
26	0.9594	1.9187	2.8781	3.8374	4.7968	5.7562	6.7155	7.6749	8.6342	1.3721
27	0.9592	1.9185	2.8777	3.8370	4.7962	5.7555	6.7147	7.6740	8.6332	1.3720
28	0.9591	1.9183	2.8774	3.8365	4.7957	5.7548	6.7139	7.6731	8.6322	1.3719
29	0.9590	1.9180	2.8771	3.8361	4.7951	5.7541	6.7131	7.6722	8.6312	1.3718
30	0.9589	1.9178	2.8767	3.8356	4.7945	5.7534	6.7124	7.6713	8.6302	1.3718
31	0.9588	1.9176	2.8764	3.8352	4.7940	5.7528	6.7116	7.6704	8.6291	1.3717
32	0.9587	1.9174	2.8760	3.8347	4.7934	5.7521	6.7108	7.6694	8.6281	1.3716
33	0.9586	1.9171	2.8757	3.8343	4.7928	5.7514	6.7100	7.6685	8.6271	1.3715
34	0.9585	1.9169	2.8754	3.8338	4.7923	5.7507	6.7092	7.6676	8.6261	1.3714
35	0.9583	1.9167	2.8750	3.8333	4.7917	5.7500	6.7084	7.6667	8.6250	1.3714
36	0.9582	1.9164	2.8747	3.8329	4.7911	5.7493	6.7076	7.6658	8.6240	1.3713
37	0.9581	1.9162	2.8743	3.8324	4.7905	5.7487	6.7068	7.6649	8.6230	1.3712
38	0.9580	1.9160	2.8740	3.8320	4.7900	5.7480	6.7060	7.6640	8.6219	1.3711
39	0.9579	1.9158	2.8736	3.8315	4.7894	5.7473	6.7052	7.6630	8.6209	1.3710
40	0.9578	1.9155	2.8733	3.8311	4.7888	5.7466	6.7044	7.6621	8.6199	1.3710
41	0.9577	1.9153	2.8730	3.8306	4.7883	5.7459	6.7036	7.6612	8.6189	1.3709
42	0.9575	1.9151	2.8726	3.8301	4.7877	5.7452	6.7027	7.6603	8.6178	1.3708
43	0.9574	1.9148	2.8723	3.8297	4.7871	5.7445	6.7019	7.6593	8.6168	1.3707
44	0.9573	1.9146	2.8719	3.8292	4.7865	5.7438	6.7011	7.6584	8.6157	1.3706
45	0.9572	1.9144	2.8716	3.8287	4.7859	5.7431	6.7003	7.6575	8.6147	1.3706
46	0.9571	1.9141	2.8712	3.8283	4.7854	5.7424	6.6995	7.6566	8.6136	1.3705
47	0.9570	1.9139	2.8709	3.8278	4.7848	5.7417	6.6987	7.6556	8.6126	1.3704
48	0.9568	1.9137	2.8705	3.8274	4.7842	5.7410	6.6979	7.6547	8.6116	1.3703
49	0.9567	1.9134	2.8702	3.8269	4.7836	5.7403	6.6971	7.6538	8.6105	1.3702
50	0.9566	1.9132	2.8698	3.8264	4.7830	5.7396	6.6962	7.6529	8.6095	1.3702
51	0.9565	1.9130	2.8695	3.8260	4.7824	5.7389	6.6954	7.6519	8.6084	1.3701
52	0.9564	1.9127	2.8691	3.8255	4.7819	5.7382	6.6946	7.6510	8.6073	1.3700
53	0.9563	1.9125	2.8688	3.8250	4.7813	5.7375	6.6938	7.6500	8.6063	1.3699
54	0.9561	1.9123	2.8684	3.8245	4.7807	5.7368	6·6930	7.6491	8.6052	1.3698
55	0.9560	1.9120	2.8681	3.8241	4.7801	5.7361	6.6921	7.6482	8.6042	1.3698
56	0.9559	1.9118	2.8677	3.8236	4.7795	5.7354	6.6913	7.6472	8.6031	1.3697
57	0.9558	1.9116	2.8674	3.8231	4.7789	5.7347	6.6905	7.6463	8.6021	1.3696
58	0.9557	1.9113	2.8670	3.8227	4.7783	5.7340	6.6897	7.6453	8.6010	1.3695
59	0.9556	1.9111	2.8667	3.8222	4.7778	5.7333	6.6889	7.6444	8.6000	1.3694
60	0.9554	1.9109	2.8663	3.8217	4.7772	5.7326	6.6880	7.6435	8.5989	1.3694

1	2	3	4	5	6	7	8	9	b	′
0.1870	0.3741	0.5611	0.7482	0.9352	1.1222	1.3093	1.4963	1.6834	0.2671	00
0.1873	0.3746	0.5619	0.7492	0.9366	1.1239	1.3112	1.4985	1.6858	0.2675	01
0.1876	0.3752	0.5627	0.7503	0.9379	1.1255	1.3131	1.5006	1.6882	0.2679	02
0.1878	0.3757	0.5635	0.7514	0.9392	1.1271	1.3149	1.5028	1.6906	0.2683	03
0.1881	0.3762	0.5644	0.7525	0.9406	1.1287	1.3168	1.5050	1.6931	0.2687	04
0.1884	0.3768	0.5652	0.7536	0.9420	1.1303	1.3187	1.5071	1.6955	0.2691	05
0.1887	0.3773	0.5660	0.7546	0.9433	1.1319	1.3206	1.5093	1.6979	0.2695	06
0.1889	0.3778	0.5668	0.7557	0.9446	1.1335	1.3224	1.5114	1.7003	0.2699	07
0.1892	0.3784	0.5676	0.7568	0.9460	1.1351	1.3243	1.5135	1.7027	0.2703	08
0.1895	0.3789	0.5684	0.7578	0.9473	1.1368	1.3262	1.5157	1.7051	0.2707	09
0.1897	0.3795	0.5692	0.7589	0.9487	1.1384	1.3281	1.5178	1.7076	0.2711	10
0.1900	0.3800	0.5700	0.7600	0.9500	1.1400	1.3300	1.5200	1.7100	0.2715	11
0.1903	0.3805	0.5708	0.7611	0.9513	1.1416	1.3319	1.5222	1.7124	0.2719	12
0.1905	0.3811	0.5716	0.7622	0.9527	1.1432	1.3338	1.5243	1.7149	0.2723	13
0.1908	0.3816	0.5724	0.7632	0.9540	1.1448	1.3357	1.5265	1.7173	0.2727	14
0.1911	0.3821	0.5732	0.7643	0.9554	1.1464	1.3375	1.5286	1.7197	0.2731	15
0.1913	0.3827	0.5740	0.7654	0.9567	1.1480	1.3394	1.5307	1.7221	0.2735	16
0.1916	0.3832	0.5748	0.7664	0.9580	1.1497	1.3413	1.5329	1.7245	0.2739	17
0.1919	0.3838	0.5756	0.7675	0.9594	1.1513	1.3432	1.5350	1.7269	0.2743	18
0.1921	0.3843	0.5764	0.7686	0.9607	1.1529	1.3450	1.5372	1.7293	0.2747	19
0.1924	0.3848	0.5773	0.7697	0.9621	1.1545	1.3469	1.5394	1.7317	0.2751	20
0.1927	0.3854	0.5781	0.7707	0.9634	1.1561	1.3488	1.5415	1.7341	0.2755	21
0.1930	0.3859	0.5789	0.7718	0.9648	1.1577	1.3507	1.5436	1.7366	0.2759	22
0.1932	0.3864	0.5797	0.7729	0.9661	1.1593	1.3525	1.5458	1.7390	0.2763	23
0.1935	0.3870	0.5805	0.7740	0.9674	1.1609	1.3544	1.5479	1.7414	0.2767	24
0.1938	0.3875	0.5813	0.7750	0.9688	1.1625	1.3563	1.5500	1.7438	0.2771	25
0.1940	0.3880	0.5821	0.7761	0.9701	1.1641	1.3581	1.5522	1.7462	0.2775	26
0.1943	0.3886	0.5829	0.7772	0.9714	1.1657	1.3600	1.5543	1.7486	0.2779	27
0.1946	0.3891	0.5837	0.7782	0.9728	1.1674	1.3619	1.5565	1.7510	0.2783	28
0.1948	0.3896	0.5845	0.7793	0.9741	1.1689	1.3637	1.5586	1.7534	0.2787	29
0.1951	0.3902	0.5853	0.7804	0.9755	1.1705	1.3656	1.5607	1.7558	0.2791	30
0.1954	0.3907	0.5861	0.7814	0.9768	1.1722	1.3675	1.5629	1.7582	0.2795	31
0.1956	0.3913	0.5869	0.7825	0.9781	1.1738	1.3694	1.5650	1.7607	0.2799	32
0.1959	0.3918	0.5877	0.7836	0.9795	1.1753	1.3712	1.5671	1.7630	0.2803	33
0.1962	0.3923	0.5885	0.7846	0.9808	1.1770	1.3731	1.5693	1.7654	0.2807	34
0.1964	0.3929	0.5893	0.7857	0.9821	1.1786	1.3750	1.5714	1.7679	0.2811	35
0.1967	0.3934	0.5901	0.7868	0.9835	1.1802	1.3769	1.5736	1.7703	0.2815	36
0.1970	0.3939	0.5909	0.7878	0.9848	1.1818	1.3787	1.5757	1.7727	0.2819	37
0.1972	0.3945	0.5917	0.7889	0.9861	1.1834	1.3806	1.5778	1.7751	0.2823	38
0.1975	0.3950	0.5925	0.7900	0.9875	1.1850	1.3825	1.5800	1.7775	0.2827	39
0.1978	0.3955	0.5933	0.7910	0.9888	1.1866	1.3843	1.5821	1.7798	0.2831	40
0.1980	0.3961	0.5941	0.7921	0.9901	1.1882	1.3862	1.5842	1.7823	0.2835	41
0.1983	0.3966	0.5949	0.7932	0.9915	1.1898	1.3881	1.5864	1.7847	0.2839	42
0.1986	0.3971	0.5957	0.7942	0.9928	1.1914	1.3899	1.5885	1.7871	0.2843	43
0.1988	0.3977	0.5965	0.7953	0.9941	1.1930	1.3918	1.5906	1.7895	0.2847	44
0.1991	0.3982	0.5973	0.7964	0.9955	1.1946	1.3937	1.5928	1.7919	0.2851	45
0.1994	0.3987	0.5981	0.7974	0.9968	1.1962	1.3955	1.5949	1.7942	0.2855	46
0.1996	0.3993	0.5989	0.7985	0.9981	1.1978	1.3974	1.5970	1.7966	0.2859	47
0.1999	0.3998	0.5997	0.7996	0.9994	1.1993	1.3992	1.5991	1.7990	0.2863	48
0.2002	0.4003	0.6005	0.8006	1.0008	1.2010	1.4011	1.6013	1.8014	0.2867	49
0.2004	0.4009	0.6013	0.8017	1.0021	1.2026	1.4030	1.6034	1.8038	0.2871	50
0.2007	0.4014	0.6021	0.8028	1.0034	1.2041	1.4048	1.6055	1.8062	0.2875	51
0.2010	0.4019	0.6029	0.8038	1.0048	1.2057	1.4067	1.6077	1.8086	0.2879	52
0.2012	0.4024	0.6037	0.8049	1.0061	1.2073	1.4085	1.6098	1.8110	0.2883	53
0.2015	0.4030	0.6045	0.8060	1.0075	1.2089	1.4104	1.6119	1.8134	0.2887	54
0.2018	0.4035	0.6053	0.8070	1.0088	1.2105	1.4123	1.6141	1.8158	0.2891	55
0.2020	0.4040	0.6061	0.8081	1.0101	1.2121	1.4141	1.6162	1.8182	0.2895	56
0.2023	0.4046	0.6069	0.8092	1.0114	1.2137	1.4160	1.6183	1.8206	0.2899	57
0.2026	0.4051	0.6077	0.8102	1.0128	1.2153	1.4179	1.6204	1.8230	0.2903	58
0.2028	0.4056	0.6085	0.8113	1.0141	1.2169	1.4197	1.6226	1.8254	0.2907	59
0.2031	0.4062	0.6092	0.8123	1.0154	1.2185	1.4216	1.6247	1.8278	0.2911	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9554	1.9109	2.8663	3.8217	4.7772	5.7326	6.6880	7.6435	8.5989	1.3694
01	0.9553	1.9106	2.8659	3.8213	4.7766	5.7319	6.6872	7.6425	8.5978	1.3693
02	0.9552	1.9104	2.8656	3.8208	4.7760	5.7312	6.6864	7.6416	8.5968	1.3693
03	0.9551	1.9102	2.8652	3.8203	4.7754	5.7305	6.6855	7.6406	8.5957	1.3692
04	0.9550	1.9099	2.8649	3.8198	4.7748	5.7297	6.6847	7.6397	8.5946	1.3691
05	0.9548	1.9097	2.8645	3.8194	4.7742	5.7290	6.6839	7.6387	8.5936	1.3690
06	0.9547	1.9094	2.8642	3.8189	4.7736	5.7283	6.6830	7.6378	8.5925	1.3689
07	0.9546	1.9092	2.8638	3.8184	4.7730	5.7276	6.6822	7.6368	8.5914	1.3688
08	0.9545	1.9090	2.8634	3.8179	4.7724	5.7269	6.6814	7.6359	8.5903	1.3687
09	0.9544	1.9087	2.8631	3.8175	4.7718	5.7262	6.6805	7.6349	8.5893	1.3687
10	0.9542	1.9085	2.8627	3.8170	4.7712	5.7255	6.6797	7.6340	8.5882	1.3686
11	0.9541	1.9082	2.8624	3.8165	4.7706	5.7247	6.6789	7.6330	8.5871	1.3685
12	0.9540	1.9080	2.8620	3.8160	4.7700	5.7240	6.6780	7.6320	8.5860	1.3684
13	0.9539	1.9078	2.8616	3.8155	4.7694	5.7233	6.6772	7.6311	8.5849	1.3683
14	0.9538	1.9075	2.8613	3.8150	4.7688	5.7226	6.6763	7.6301	8.5839	1.3682
15	0.9536	1.9073	2.8609	3.8146	4.7682	5.7219	6.6755	7.6291	8.5828	1.3681
16	0.9535	1.9070	2.8606	3.8141	4.7676	5.7211	6.6747	7.6282	8.5817	1.3681
17	0.9534	1.9068	2.8602	3.8136	4.7670	5.7204	6.6738	7.6272	8.5806	1.3680
18	0.9533	1.9066	2.8598	3.8131	4.7664	5.7197	6.6730	7.6262	8.5795	1.3679
19	0.9532	1.9063	2.8595	3.8126	4.7658	5.7190	6.6721	7.6253	8.5784	1.3678
20	0.9530	1.9061	2.8591	3.8122	4.7652	5.7182	6.6713	7.6243	8.5774	1.3677
21	0.9529	1.9058	2.8588	3.8117	4.7646	5.7175	6.6704	7.6233	8.5763	1.3676
22	0.9528	1.9056	2.8584	3.8112	4.7640	5.7168	6.6696	7.6224	8.5752	1.3675
23	0.9527	1.9053	2.8580	3.8107	4.7634	5.7160	6.6687	7.6214	8.5741	1.3674
24	0.9526	1.9051	2.8577	3.8102	4.7628	5.7153	6.6679	7.6204	8.5730	1.3673
25	0.9524	1.9049	2.8573	3.8097	4.7622	5.7146	6.6670	7.6194	8.5719	1.3672
26	0.9523	1.9046	2.8569	3.8092	4.7615	5.7138	6.6662	7.6185	8.5708	1.3672
27	0.9522	1.9044	2.8566	3.8087	4.7609	5.7131	6.6653	7.6175	8.5697	1.3671
28	0.9521	1.9041	2.8562	3.8083	4.7603	5.7124	6.6644	7.6165	8.5686	1.3670
29	0.9519	1.9039	2.8558	3.8078	4.7597	5.7117	6.6636	7.6155	8.5675	1.3669
30	0.9518	1.9036	2.8555	3.8073	4.7591	5.7109	6.6627	7.6146	8.5664	1.3668
31	0.9517	1.9034	2.8551	3.8068	4.7585	5.7102	6.6619	7.6136	8.5653	1.3667
32	0.9516	1.9031	2.8547	3.8063	4.7579	5.7094	6.6610	7.6126	8.5642	1.3667
33	0.9514	1.9029	2.8543	3.8058	4.7572	5.7087	6.6601	7.6116	8.5630	1.3666
34	0.9513	1.9027	2.8540	3.8053	4.7566	5.7080	6.6593	7.6106	8.5619	1.3665
35	0.9512	1.9024	2.8536	3.8048	4.7560	5.7072	6.6584	7.6096	8.5608	1.3664
36	0.9511	1.9022	2.8532	3.8043	4.7554	5.7065	6.6575	7.6086	8.5597	1.3663
37	0.9510	1.9019	2.8529	3.8038	4.7548	5.7057	6.6567	7.6076	8.5586	1.3662
38	0.9508	1.9017	2.8525	3.8033	4.7542	5.7050	6.6558	7.6066	8.5575	1.3661
39	0.9507	1.9014	2.8521	3.8028	4.7535	5.7042	6.6550	7.6057	8.5564	1.3660
40	0.9506	1.9012	2.8518	3.8023	4.7529	5.7035	6.6541	7.6047	8.5553	1.3660
41	0.9505	1.9009	2.8514	3.8018	4.7523	5.7028	6.6532	7.6037	8.5541	1.3659
42	0.9503	1.9007	2.8510	3.8013	4.7517	5.7020	6.6523	7.6027	8.5530	1.3658
43	0.9502	1.9004	2.8506	3.8008	4.7510	5.7013	6.6515	7.6017	8.5519	1.3657
44	0.9501	1.9002	2.8503	3.8003	4.7504	5.7005	6.6506	7.6007	8.5508	1.3656
45	0.9500	1.8999	2.8499	3.7998	4.7498	5.6998	6.6497	7.5997	8.5496	1.3655
46	0.9498	1.8997	2.8495	3.7993	4.7492	5.6990	6.6488	7.5987	8.5485	1.3654
47	0.9497	1.8994	2.8491	3.7988	4.7485	5.6983	6.6480	7.5977	8.5474	1.3653
48	0.9496	1.8992	2.8488	3.7983	4.7479	5.6975	6.6471	7.5967	8.5463	1.3652
49	0.9495	1.8989	2.8484	3.7978	4.7473	5·6968	6.6462	7.5957	8.5451	1.3651
50	0.9493	1.8987	2.8480	3.7973	4.7467	5.6960	6.6453	7.5947	8.5440	1.3651
51	0.9492	1.8984	2.8476	3.7968	4.7460	5.6952	6.6444	7.5937	8.5429	1.3650
52	0.9491	1.8982	2.8472	3.7963	4.7454	5.6945	6.6436	7.5926	8.5417	1.3649
53	0.9490	1.8979	2.8469	3.7958	4.7448	5.6937	6.6427	7.5916	8.5406	1.3648
54	0.9488	1.8977	2.8465	3.7953	4.7441	5.6930	6.6418	7.5906	8.5395	1.3647
55	0.9487	1.8974	2.8461	3.7948	4.7435	5.6922	6.6409	7.5896	8.5383	1.3646
56	0.9486	1.8971	2.8457	3.7943	4.7429	5.6914	6.6400	7.5886	8.5372	1.3645
57	0.9484	1.8969	2.8453	3.7938	4.7422	5.6907	6.6391	7.5876	8.5360	1.3644
58	0.9483	1.8966	2.8450	3.7933	4.7416	5.6899	6.6382	7.5866	8.5349	1.3643
59	0.9482	1.8964	2.8446	3.7928	4.7410	5.6892	6.6374	7.5856	8.5338	1.3642
60	0.9481	1.8961	2.8442	3.7923	4.7403	5.6884	6.6365	7.5845	8.5326	1.3641

1	2	3	4	5	6	7	8	9	b	′
0.2031	0.4062	0.6092	0.8123	1.0154	1.2185	1.4216	1.6247	1.8278	0.2911	00
0.2033	0.4067	0.6100	0.8134	1.0167	1.2201	1.4234	1.6268	1.8302	0.2915	01
0.2036	0.4072	0.6108	0.8144	1.0181	1.2217	1.4253	1.6289	1.8325	0.2919	02
0.2039	0.4078	0.6116	0.8155	1.0194	1.2233	1.4272	1.6310	1.8349	0.2923	03
0.2041	0.4083	0.6124	0.8166	1.0207	1.2248	1.4290	1.6331	1.8373	0.2927	04
0.2044	0.4088	0.6132	0.8176	1.0221	1.2264	1.4309	1.6353	1.8397	0.2931	05
0.2047	0.4093	0.6140	0.8187	1.0234	1.2280	1.4327	1.6374	1.8420	0.2935	06
0.2049	0.4099	0.6148	0.8198	1.0247	1.2296	1.4346	1.6395	1.8444	0.2939	07
0.2052	0.4104	0.6156	0.8208	1.0260	1.2312	1.4364	1.6416	1.8468	0.2943	08
0.2055	0.4109	0.6164	0.8219	1.0273	1.2328	1.4383	1.6438	1.8492	0.2947	09
0.2057	0.4115	0.6172	0.8229	1.0287	1.2344	1.4401	1.6459	1.8516	0.2951	10
0.2060	0.4120	0.6180	0.8240	1.0300	1.2360	1.4420	1.6480	1.8540	0.2955	11
0.2063	0.4125	0.6188	0.8250	1.0313	1.2376	1.4438	1.6501	1.8564	0.2959	12
0.2065	0.4131	0.6196	0.8261	1.0326	1.2392	1.4457	1.6522	1.8588	0.2962	13
0.2068	0.4136	0.6204	0.8272	1.0340	1.2408	1.4475	1.6543	1.8611	0.2966	14
0.2071	0.4141	0.6212	0.8282	1.0353	1.2424	1.4494	1.6565	1.8635	0.2970	15
0.2073	0.4146	0.6220	0.8293	1.0366	1.2439	1.4512	1.6586	1.8659	0.2974	16
0.2076	0.4152	0.6227	0.8303	1.0379	1.2455	1.4531	1.6607	1.8682	0.2978	17
0.2078	0.4157	0.6235	0.8314	1.0392	1.2471	1.4549	1.6628	1.8706	0.2982	18
0.2081	0.4162	0.6243	0.8324	1.0406	1.2487	1.4568	1.6649	1.8730	0.2986	19
0.2084	0.4168	0.6251	0.8335	1.0419	1.2503	1.4587	1.6670	1.8754	0.2990	20
0.2086	0.4173	0.6259	0.8346	1.0432	1.2518	1.4605	1.6691	1.8778	0.2994	21
0.2089	0.4178	0.6267	0.8356	1.0445	1.2534	1.4623	1.6712	1.8801	0.2998	22
0.2092	0.4183	0.6275	0.8367	1.0458	1.2550	1.4642	1.6734	1.8825	0.3002	23
0.2094	0.4189	0.6283	0.8377	1.0472	1.2566	1.4660	1.6755	1.8849	0.3006	24
0.2097	0.4194	0.6291	0.8388	1.0485	1.2582	1.4679	1.6776	1.8873	0.3010	25
0.2100	0.4199	0.6299	0.8398	1.0498	1.2598	1.4697	1.6797	1.8896	0.3014	26
0.2102	0.4204	0.6307	0.8409	1.0511	1.2613	1.4715	1.6818	1.8920	0.3018	27
0.2105	0.4210	0.6315	0.8420	1.0524	1.2629	1.4734	1.6839	1.8944	0.3022	28
0.2107	0.4215	0.6322	0.8430	1.0537	1.2645	1.4752	1.6860	1.8967	0.3026	29
0.2110	0.4220	0.6330	0.8440	1.0551	1.2661	1.4771	1.6881	1.8991	0.3030	30
0.2113	0.4226	0.6338	0.8451	1.0564	1.2677	1.4790	1.6902	1.9015	0.3034	31
0.2115	0.4231	0.6346	0.8462	1.0577	1.2692	1.4808	1.6923	1.9039	0.3038	32
0.2118	0.4236	0.6354	0.8472	1.0590	1.2708	1.4826	1.6944	1.9062	0.3042	33
0.2121	0.4241	0.6362	0.8483	1.0603	1.2724	1.4845	1.6965	1.9086	0.3046	34
0.2123	0.4247	0.6370	0.8493	1.0616	1.2740	1.4863	1.6986	1.9110	0.3050	35
0.2126	0.4252	0.6378	0.8504	1.0630	1.2755	1.4881	1.7007	1.9133	0.3054	36
0.2129	0.4257	0.6386	0.8514	1.0643	1.2771	1.4900	1.7028	1.9157	0.3058	37
0.2131	0.4262	0.6394	0.8525	1.0656	1.2787	1.4918	1.7049	1.9181	0.3062	38
0.2134	0.4268	0.6401	0.8535	1.0669	1.2803	1.4937	1.7070	1.9204	0.3066	39
0.2136	0.4273	0.6409	0.8546	1.0682	1.2818	1.4955	1.7091	1.9228	0.3070	40
0.2139	0.4278	0.6417	0.8556	1.0695	1.2834	1.4973	1.7112	1.9251	0.3074	41
0.2142	0.4283	0.6425	0.8567	1.0708	1.2850	1.4992	1.7133	1.9275	0.3078	42
0.2144	0.4289	0.6433	0.8577	1.0721	1.2866	1.5010	1.7154	1.9299	0.3082	43
0.2147	0.4294	0.6441	0.8588	1.0735	1.2881	1.5028	1.7175	1.9322	0.3086	44
0.2150	0.4299	0.6449	0.8598	1.0748	1.2897	1.5047	1.7196	1.9346	0.3090	45
0.2152	0.4304	0.6457	0.8609	1.0761	1.2913	1.5065	1.7217	1.9370	0.3094	46
0.2155	0.4310	0.6464	0.8619	1.0774	1.2929	1.5084	1.7238	1.9393	0.3098	47
0.2157	0.4315	0.6472	0.8630	1.0787	1.2944	1.5102	1.7259	1.9417	0.3102	48
0.2160	0.4320	0.6480	0.8640	1.0800	1.2960	1.5120	1.7280	1.9440	0.3106	49
0.2163	0.4325	0.6488	0.8651	1.0813	1.2976	1.5138	1.7301	1.9464	0.3110	50
0.2165	0.4331	0.6496	0.8661	1.0826	1.2992	1.5157	1.7322	1.9487	0.3114	51
0.2168	0.4336	0.6504	0.8671	1.0839	1.3007	1.5175	1.7343	1.9511	0.3118	52
0.2170	0.4341	0.6511	0.8682	1.0852	1.3023	1.5193	1.7364	1.9534	0.3121	53
0.2173	0.4346	0.6519	0.8692	1.0866	1.3039	1.5212	1.7385	1.9558	0.3125	54
0.2176	0.4351	0.6527	0.8703	1.0879	1.3054	1.5230	1.7406	1.9581	0.3129	55
0.2178	0.4357	0.6535	0.8713	1.0892	1.3070	1.5248	1.7427	1.9605	0.3133	56
0.2181	0.4362	0.6543	0.8724	1.0905	1.3186	1.5267	1.7448	1.9629	0.3137	57
0.2184	0.4367	0.6551	0.8734	1.0918	1.3101	1.5285	1.7468	1.9652	0.3141	58
0.2186	0.4372	0.6559	0.8745	1.0931	1·3117	1.5303	1.7489	1.9676	0.3145	59
0.2189	0.4378	0.6566	0.8755	1.0944	1.3133	1.5322	1.7510	1.9699	0.3149	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9481	1.8961	2.8442	3.7923	4.7403	5.6884	6.6365	7.5845	8.5320	1.3641
01	0.9479	1.8959	2.8438	3.7918	4.7397	5.6876	6.6356	7.5835	8.5315	1.3640
02	0.9478	1.8956	2.8434	3.7912	4.7391	5.6869	6.6347	7.5825	8.5303	1.3639
03	0.9477	1.8954	2.8431	3.7907	4.7384	5.6861	6.6338	7.5815	8.5292	1.3638
04	0.9476	1.8951	2.8427	3.7902	4.7378	5.6853	6.6329	7.5804	8.5280	1.3637
05	0.9474	1.8949	2.8423	3.7897	4.7371	5.6846	6.6320	7.5794	8.5269	1.3636
06	0.9473	1.8946	2.8419	3.7892	4.7365	5.6838	6.6311	7.5784	8.5257	1.3635
07	0.9472	1.8943	2.8415	3.7887	4.7359	5.6830	6.6302	7.5774	8.5245	1.3634
08	0.9470	1.8941	2.8411	3.7882	4.7352	5.6823	6.6293	7.5763	8.5234	1.3634
09	0.9469	1.8938	2.8407	3.7877	4.7346	5.6815	6.6284	7.5753	8.5222	1.3633
10	0.9468	1.8936	2.8404	3.7871	4.7339	5.6807	6.6275	7.5743	8.5211	1.3632
11	0.9467	1.8933	2.8400	3.7866	4.7333	5.6799	6.6266	7.5733	8.5199	1.3631
12	0.9465	1.8931	2.8396	3.7861	4.7326	5.6792	6.6257	7.5722	8.5188	1.3630
13	0.9464	1.8928	2.8392	3.7856	4.7320	5.6784	6.6248	7.5712	8.5176	1.3629
14	0.9463	1.8925	2.8388	3.7851	4.7313	5.6776	6.6239	7.5702	8.5164	1.3628
15	0.9461	1.8923	2.8384	3.7846	4.7307	5.6708	6.6230	7.5691	8.5153	1.3627
16	0.9460	1.8920	2.8380	3.7840	4.7300	5.6761	6.6221	7.5681	8.5141	1.3626
17	0.9459	1.8918	2.8376	3.7835	4.7294	5.6753	6.6212	7.5670	8.5129	1.3625
18	0.9458	1.8915	2.8373	3.7830	4.7288	5.6745	6.6203	7.5660	8.5118	1.3624
19	0.9456	1.8912	2.8369	3.7825	4.7281	5.6737	6.6193	7.5650	8.5106	1.3623
20	0.9455	1.8910	2.8365	3.7820	4.7275	5.6729	6.6184	7.5639	8.5094	1.3622
21	0.9454	1.8907	2.8361	3.7814	4.7268	5.6722	6.6175	7.5629	8.5082	1.3621
22	0.9452	1.8905	2.8357	3.7809	4.7261	5.6714	6.6166	7.5618	8.5071	1.3620
23	0.9451	1.8902	2.8353	3.7804	4.7255	5.6706	6.6157	7.5608	8.5059	1.3619
24	0.9450	1.8899	2.8349	3.7799	4.7248	5.6698	6.6148	7.5597	8.5047	1.3618
25	0.9448	1.8897	2.8345	3.7793	4.7242	5.6690	6.6139	7.5587	8.5035	1.3618
26	0.9447	1.8894	2.8341	3.7788	4.7235	5.6682	6.6129	7.5576	8.5023	1.3617
27	0.9446	1.8891	2.8337	3.7783	4.7229	5.6674	6.6120	7.5566	8.5012	1.3616
28	0.9444	1.8889	2.8333	3.7778	4.7222	5.6667	6.6111	7.5555	8.5000	1.3615
29	0.9443	1.8886	2.8329	3.7772	4.7216	5.6659	6.6102	7.5545	8.4988	1.3614
30	0.9442	1.8884	2.8325	3.7767	4.7209	5.6651	6.6093	7.5534	8.4976	1.3613
31	0.9440	1.8881	2.8321	3.7762	4.7202	5.6643	6.6083	7.5524	8.4964	1.3612
32	0.9439	1.8878	2.8317	3.7757	4.7196	5.6635	6.6074	7.5513	8.4952	1.3611
33	0.9438	1.8876	2.8313	3.7751	4.7189	5.6627	6.6065	7.5503	8.4940	1.3610
34	0.9436	1.8873	2.8309	3.7746	4.7182	5.6619	6.6055	7.5492	8.4928	1.3609
35	0.9435	1.8870	2.8306	3.7741	4.7176	5.6611	6.6046	7.5481	8.4917	1.3608
36	0.9434	1.8868	2.8302	3.7735	4.7169	5.6603	6.6037	7.5471	8.4905	1.3607
37	0.9433	1.8865	2.8298	3.7730	4.7163	5.6595	6.6028	7.5460	8.4893	1.3606
38	0.9431	1.8862	2.8294	3.7725	4.7156	5.6587	6.6018	7.5449	8.4881	1.3605
39	0.9430	1.8860	2.8290	3.7719	4.7149	5.6579	6.6009	7.5439	8.4869	1.3604
40	0.9429	1.8857	2.8286	3.7714	4.7143	5.6571	6.6000	7.5428	8.4857	1.3603
41	0.9427	1.8854	2.8282	3.7709	4.7136	5.6563	6.5990	7.5418	8.4845	1.3602
42	0.9426	1.8852	2.8278	3.7703	4.7129	5.6555	6.5981	7.5407	8.4833	1.3602
43	0.9425	1.8849	2.8274	3.7698	4.7123	5.6547	6.5972	7.5396	8.4821	1.3601
44	0.9423	1.8846	2.8270	3.7693	4.7116	5.6539	6.5962	7.5385	8.4809	1.3600
45	0.9422	1.8844	2.8265	3.7687	4.7109	5.6531	6.5953	7.5375	8.4796	1.3599
46	0.9420	1.8841	2.8261	3.7682	4.7102	5.6523	6.5943	7.5364	8.4784	1.3598
47	0.9419	1.8838	2.8257	3.7677	4.7096	5.6515	6.5934	7.5353	8.4772	1.3597
48	0.9418	1.8836	2.8253	3.7671	4.7089	5.6507	6.5925	7.5342	8.4760	1.3596
49	0.9416	1.8833	2.8249	3.7666	4.7082	5.6499	6.5915	7.5332	8.4748	1.3595
50	0.9415	1.8830	2.8245	3.7660	4.7076	5.6491	6.5906	7.5321	8.4736	1.3594
51	0.9414	1.8828	2.8241	3.7655	4.7069	5.6483	6.5896	7.5310	8.4724	1.3593
52	0.9412	1.8825	2.8237	3.7650	4.7062	5.6474	6.5887	7.5299	8.4712	1.3592
53	0.9411	1.8822	2.8233	3.7644	4.7055	5.6466	6.5877	7.5288	8.4699	1.3591
54	0.9410	1.8819	2.8229	3.7639	4.7048	5.6458	6.5868	7.5278	8.4687	1.3590
55	0.9408	1.8817	2.8225	3.7633	4.7042	5.6450	6.5858	7.5267	8.4675	1.3589
56	0.9407	1.8814	2.8221	3.7628	4.7035	5.6442	6.5849	7.5256	8.4663	1.3588
57	0.9406	1.8811	2.8217	3.7623	4.7028	5.6434	6.5839	7.5245	8.4651	1.3587
58	0.9404	1.8809	2.8213	3.7617	4.7021	5.6426	6.5830	7.5234	8.4638	1.3586
59	0.9403	1.8806	2.8209	3.7612	4.7015	5.6418	6.5820	7.5223	8.4626	1.3585
60	0.9402	1.8803	2.8205	3.7606	4.7008	5.6409	6.5811	7.5212	8.4614	1.3584

1	2	3	4	5	6	7	8	9	b	′
0.2189	0.4378	0.6566	0.8755	1.0944	1.3133	1.5322	1.7510	1.9699	0.3149	00
0.2191	0.4383	0.6574	0.8766	1.0957	1.3148	1.5340	1.7531	1.9723	0.3153	01
0.2194	0.4388	0.6582	0.8776	1.0970	1.3164	1.5358	1.7552	1.9746	0.3157	02
0.2197	0.4393	0.6590	0.8786	1.0983	1.3180	1.5376	1.7573	1.9769	0.3161	03
0.2199	0.4398	0.6598	0.8797	1.0996	1.3195	1.5394	1.7594	1.9793	0.3165	04
0.2202	0.4404	0.6605	0.8807	1.1009	1.3211	1.5413	1.7614	1.9816	0.3169	05
0.2204	0.4409	0.6613	0.8818	1.1022	1.3226	1.5431	1.7635	1.9840	0.3173	06
0.2207	0.4414	0.6621	0.8828	1.1035	1.3242	1.5449	1.7656	1.9863	0.3177	07
0.2210	0.4419	0.6629	0.8838	1.1048	1.3258	1.5467	1.7677	1.9886	0.3181	08
0.2212	0.4425	0.6637	0.8849	1.1061	1.3274	1.5486	1.7698	1.9911	0.3185	09
0.2215	0.4430	0.6645	0.8860	1.1074	1.3289	1.5504	1.7719	1.9934	0.3189	10
0.2217	0.4435	0.6652	0.8870	1.1087	1.3305	1.5522	1.7740	1.9957	0.3193	11
0.2220	0.4440	0.6660	0.8880	1.1100	1.3321	1.5541	1.7761	1.9981	0.3197	12
0.2223	0.4445	0.6668	0.8891	1.1113	1.3336	1.5559	1.7782	2.0004	0.3201	13
0.2225	0.4451	0.6676	0.8901	1.1126	1.3352	1.5577	1.7802	2.0028	0.3205	14
0.2228	0.4456	0.6684	0.8912	1.1139	1.3367	1.5595	1.7823	2.0051	0.3209	15
0.2230	0.4461	0.6691	0.8922	1.1152	1.3383	1.5613	1.7844	2.0074	0.3213	16
0.2233	0.4466	0.6699	0.8932	1.1165	1.3399	1.5632	1.7865	2.0098	0.3217	17
0.2236	0.4471	0.6707	0.8943	1.1178	1.3414	1.5650	1.7886	2.0121	0.3221	18
0.2238	0.4477	0.6715	0.8953	1.1191	1.3430	1.5668	1.7906	2.0145	0.3225	19
0.2241	0.4482	0.6723	0.8964	1.1204	1.3445	1.5686	1.7927	2.0168	0.3229	20
0.2243	0.4487	0.6730	0.8974	1.1217	1.3460	1.5704	1.7947	2.0191	0.3232	21
0.2246	0.4492	0.6738	0.8984	1.1230	1.3476	1.5722	1.7968	2.0214	0.3236	22
0.2249	0.4497	0.6746	0.8994	1.1243	1.3492	1.5740	1.7989	2.0237	0.3240	23
0.2251	0.4502	0.6754	0.9005	1.1256	1.3507	1.5758	1.8010	2.0261	0.3244	24
0.2254	0.4508	0.6761	0.9015	1.1269	1.3523	1.5777	1.8030	2.0284	0.3248	25
0.2256	0.4513	0.6769	0.9026	1.1282	1.3538	1.5795	1.8051	2.0308	0.3252	26
0.2259	0.4518	0.6777	0.9036	1.1295	1.3554	1.5813	1.8072	2.0331	0.3256	27
0.2262	0.4523	0.6785	0.9046	1.1308	1.3570	1.5831	1.8093	2.0354	0.3260	28
0.2264	0.4528	0.6793	0.9057	1.1321	1.3585	1.5849	1.8114	2.0378	0.3264	29
0.2267	0.4534	0.6800	0.9067	1.1334	1.3601	1.5868	1.8134	2.0401	0.3268	30
0.2269	0.4539	0.6808	0.9078	1.1347	1.3616	1.5886	1.8155	2.0424	0.3272	31
0.2272	0.4544	0.6816	0.9088	1.1360	1.3631	1.5904	1.8175	2.0447	0.3276	32
0.2275	0.4549	0.6824	0.9098	1.1373	1.3647	1.5922	1.8196	2.0471	0.3280	33
0.2277	0.4554	0.6831	0.9108	1.1385	1.3663	1.5940	1.8217	2.0494	0.3284	34
0.2280	0.4559	0.6839	0.9119	1.1398	1.3678	1.5958	1.8238	2.0517	0.3288	35
0.2282	0.4565	0.6847	0.9129	1.1411	1.3694	1.5976	1.8258	2.0541	0.3292	36
0.2285	0.4570	0.6855	0.9140	1.1424	1.3709	1.5994	1.8279	2.0564	0.3296	37
0.2287	0.4575	0.6862	0.9150	1.1437	1.3725	1.6012	1.8300	2.0587	0.3300	38
0.2290	0.4580	0.6870	0.9160	1.1450	1.3740	1.6030	1.8320	2.0610	0.3304	39
0.2293	0.4585	0.6878	0.9170	1.1463	1.3756	1.6048	1.8341	2.0633	0.3308	40
0.2295	0.4590	0.6886	0.9181	1.1476	1.3771	1.6066	1.8361	2.0657	0.3312	41
0.2298	0.4596	0.6993	0.9191	1.1489	1.3787	1.6085	1.8382	2.0680	0.3316	42
0.2300	0.4601	0.6901	0.9202	1.1502	1.3802	1.6103	1.8403	2.0703	0.3320	43
0.2303	0.4606	0.6909	0.9212	1.1515	1.3817	1.6121	1.8423	2.0726	0.3324	44
0.2306	0.4611	0.6917	0.9222	1.1528	1.3833	1.6139	1.8444	2.0750	0.3328	45
0.2308	0.4616	0.6924	0.9232	1.1541	1.3849	1.6157	1.8465	2.0773	0.3331	46
0.2311	0.4621	0.6932	0.9243	1.1554	1.3864	1.6175	1.8486	2.0796	0.3335	47
0.2313	0.4626	0.6940	0.9253	1.1566	1.3879	1.6193	1.8506	2.0819	0.3339	48
0.2316	0.4632	0.6947	0.9263	1.1579	1.3895	1.6211	1.8527	2.0842	0.3343	49
0.2318	0.4637	0.6955	0.9274	1.1592	1.3910	1.6229	1.8547	2.0866	0.3347	50
0.2321	0.4642	0.6963	0.9284	1.1605	1.3926	1.6247	1.8568	2.0889	0.3351	51
0.2324	0.4647	0.6971	0.9294	1.1618	1.3941	1.6265	1.8588	2.0912	0.3355	52
0.2326	0.4652	0.6978	0.9304	1.1630	1.3957	1.6283	1.8609	2.0935	0.3359	53
0.2329	0.4657	0.6986	0.9315	1.1643	1.3972	1.6301	1.8630	2.0958	0.3363	54
0.2331	0.4662	0.6994	0.9325	1.1656	1.3987	1.6319	1.8650	2.0981	0.3367	55
0.2334	0.4668	0.7001	0.9335	1.1669	1.4003	1.6337	1.8670	2.1004	0.3371	56
0.2336	0.4673	0.7009	0.9346	1.1682	1.4018	1.6355	1.8691	2.1028	0.3375	57
0.2339	0.4678	0.7017	0.9356	1.1695	1.4033	1.6373	1.8711	2.1050	0.3379	58
0.2342	0.4683	0.7025	0.9366	1.1708	1.4049	1.6391	1.8732	2.1074	0.3383	59
0.2344	0.4688	0.7032	0.9376	1.1720	1.4065	1.6409	1.8753	2.1097	0.3387	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9402	1.8803	2.8205	3.7606	4.7008	5.6409	6.5811	7.5212	8.4614	1.3584
01	0.9400	1.8800	2.8201	3.7601	4.7001	5.6401	6.5801	7.5202	8.4602	1.3583
02	0.9399	1.8798	2.8196	3.7595	4.6994	5.6393	6.5792	7.5191	8.4589	1.3582
03	0.9397	1.8795	2.8192	3.7590	4.6987	5.6385	6.5782	7.5180	8.4577	1.3581
04	0.9396	1.8792	2.8188	3.7584	4.6980	5.6376	6.5773	7.5169	8.4565	1.3580
05	0.9395	1.8789	2.8184	3.7579	4.6974	5.6368	6.5763	7.5158	8.4552	1.3579
06	0.9393	1.8787	2.8180	3.7573	4.6967	5.6360	6.5753	7.5147	8.4540	1.3578
07	0·9392	1.8784	2.8176	3.7568	4.6960	5.6352	6.5744	7.5136	8.4528	1.3577
08	0.9391	1.8781	2.8172	3.7562	4.6953	5.6344	6.5734	7.5125	8.4515	1.3576
09	0.9389	1.8778	2.8168	3.7557	4.6946	5.6335	6.5725	7.5114	8.4503	1.3575
10	0.0388	1.8776	2.8164	3.7551	4.6939	5.6327	6.5715	7.5103	8.4491	1.3574
11	0.9386	1.8773	2.8159	3.7546	4.6932	5.6319	6.5705	7.5092	8.4478	1.3573
12	0.9385	1.8770	2.8155	3.7540	4.6925	5.6310	6.5696	7.5081	8.4466	1.3572
13	0.9384	1.8767	2.8151	3.7535	4.6918	5.6302	6.5686	7.5070	8.4453	1.3571
14	0.9382	1.8765	2.8147	3.7529	4.6912	5.6294	6.5676	7.5058	8.4441	1.3570
15	0.9381	1.8762	2.8143	3.7524	4.6905	5.6286	6.5666	7.5047	8.4428	1.3569
16	0.9380	1.8759	2.8139	3.7518	4.6898	5.6277	6.5657	7.5036	8.4416	1.3568
17	0.9378	1.8756	2.8134	3.7513	4.6891	5.6269	6.5647	7.5025	8.4403	1.3567
18	0.9377	1.8754	2.8130	3.7507	4.6884	5.6261	6.5637	7.5014	8.4391	1.3566
19	0.9375	1.8751	2.8126	3.7501	4.6877	5.6252	6.5628	7.5003	8.4378	1.3565
20	0.9374	1.8748	2.8122	3.7496	4.6870	5.6244	6.5618	7.4992	8.4366	1.3564
21	0.9373	1.8745	2.8118	3.7490	4.6863	5.6235	6.5608	7.4981	8.4353	1.3563
22	0.9371	1.8742	2.8114	3.7485	4.6856	5.6227	6.5598	7.4969	8.4341	1.3562
23	0.9370	1.8740	2.8109	3.7479	4.6849	5.6219	6.5588	7.4958	8.4328	1.3561
24	0.9368	1.8737	2.8105	3.7474	4.6842	5.6210	6.5579	7.4947	8.4315	1.3560
25	0.9367	1.8734	2.8101	3.7468	4.6835	5.6202	6.5569	7.4936	8.4303	1.3559
26	0.9366	1.8731	2.8097	3.7462	4.6828	5.6193	6.5559	7.4925	8.4290	1.3558
27	0.9364	1.8728	2.8093	3.7457	4.6821	5.6185	6.5549	7.4913	8.4278	1.3557
28	0.9363	1.8726	2.8088	3.7451	4.6814	5.6177	6.5539	7.4902	8.4265	1.3556
29	0.9361	1.8723	2.8084	3.7445	4.6807	5.6168	6.5530	7.4891	8.4252	1.3555
30	0.9360	1.8720	2.8080	3.7440	4.6800	5.6160	6.5520	7.4880	8.4240	1.3554
31	0.9359	1.8717	2.8076	3.7434	4.6793	5.6151	6.5510	7.4868	8.4227	1.3553
32	0.9357	1.8714	2.8071	3.7429	4.6786	5.6143	6.5500	7.4857	8.4214	1.3552
33	0.9356	1.8711	2.8067	3.7423	4.6779	5.6134	6.5490	7.4846	8.4202	1.3551
34	0.9354	1.8709	2.8063	3.7417	4.6772	5.6126	6.5480	7.4834	8.4189	1.3550
35	0.9353	1.8706	2.8059	3.7412	4.6764	5.6117	6.5470	7.4823	8.4176	1.3549
36	0.9351	1.8703	2.8054	3.7406	4.6757	5.6109	6.5460	7.4812	8.4163	1.3548
37	0.9350	1.8700	2.8050	3.7400	4.6750	5.6100	6.5450	7.4801	8.4151	1.3547
38	0.9349	1.8697	2.8046	3.7395	4.6743	5.6092	6.5441	7.4789	8.4138	1.3546
39	0.9347	1.8694	2.8042	3.7389	4.6736	5.6083	6.5431	7.4778	8.4125	1.3545
40	0.9346	1.8692	2.8037	3.7383	4.6729	5.6075	6.5421	7.4767	8.4112	1.3544
41	0.9344	1.8689	2.8033	3.7378	4.6722	5.6066	6.5411	7.4755	8.4100	1.3543
42	0.9343	1.8686	2.8029	3.7372	4.6715	5.6058	6.5401	7.4744	8.4087	1.3542
43	0.9342	1.8683	2.8025	3.7366	4.6708	5.6049	6.5391	7.4732	8.4074	1.3541
44	0.9340	1.8680	2.8020	3.7360	4.6701	5.6041	6.5381	7.4721	8.4061	1.3540
45	0.9339	1.8677	2.8016	3.7355	4.6693	5.6032	6.5371	7.4709	8.4048	1.3539
46	0.9337	1.8674	2.8012	3.7349	4.6686	5.6023	6.5361	7.4698	8.4035	1.3538
47	0.9336	1.8672	2.8007	3.7343	4.6679	5.6015	6.5351	7.4686	8.4022	1.3537
48	0.9334	1.8669	2.8003	3.7338	4.6672	5.6006	6.5341	7.4675	8.4009	1.3536
49	0.9333	1.8666	2.7999	3.7332	4.6665	5.5998	6.5331	7.4664	8.3997	1.3535
50	0.9332	1.8663	2.7995	3.7326	4.6658	5.5989	6.5321	7.4652	8.3984	1.3534
51	0.9330	1.8660	2.7990	3.7320	4.6650	5.5980	6.5311	7.4641	8.3971	1.3533
52	0.9329	1.8657	2.7986	3.7315	4.6643	5.5972	6.5300	7.4629	8.3958	1.3531
53	0.9327	1.8654	2.7982	3.7309	4.6636	5.5963	6.5290	7.4618	8.3945	1.3530
54	0.9326	1.8651	2.7977	3.7303	4.6629	5.5954	6.5280	7.4606	8.3932	1.3529
55	0.9324	1.8649	2.7973	3.7297	4.6621	5.5946	6.5270	7.4594	8.3919	1.3528
56	0.9323	1.8646	2.7969	3.7291	4.6614	5.5937	6.5260	7.4583	8.3906	1.3527
57	0.9321	1.8643	2.7964	3.7286	4.6607	5.5928	6.5250	7.4571	8.3893	1.3526
58	0.9320	1.8640	2.7960	3.7280	4.6600	5.5920	6.5240	7.4560	8.3880	1.3525
59	0.9319	1.8637	2.7956	3.7274	4.6593	5.5911	6.5230	7.4548	8.3867	1.3524
60	0.9317	1.8634	2.7951	3.7268	4.6585	5.5902	6.5219	7.4537	8.3854	1.3523

1	2	3	4	5	6	7	8	9	b	′
0.2344	0.4688	0.7032	0.9376	1.1720	1.4065	1.6409	1.8753	2.1097	0.3387	00
0.2347	0.4693	0.7040	0.9386	1.1733	1.4080	1.6426	1.8773	2.1120	0.3391	01
0.2349	0.4698	0.7048	0.9397	1.1746	1.4095	1.6444	1.8794	2.1143	0.3395	02
0.2352	0.4704	0.7055	0.9407	1.1759	1.4111	1.6462	1.8814	2.1166	0.3399	03
0.2354	0.4709	0.7063	0.9417	1.1772	1.4126	1.6480	1.8834	2.1189	0.3403	04
0.2357	0.4714	0.7071	0.9428	1.1785	1.4141	1.6498	1.8855	2.1212	0.3407	05
0.2359	0.4719	0.7078	0.9438	1.1797	1.4156	1.6516	1.8875	2.1235	0.3411	06
0.2362	0.4724	0.7086	0.9448	1.1810	1.4172	1.6534	1.8896	2.1258	0.3414	07
0.2365	0.4729	0.7094	0.9458	1.1823	1.4188	1.6552	1.8917	2.1281	0.3418	08
0.2367	0.4734	0.7101	0.9468	1.1836	1.4203	1.6570	1.8937	2.1304	0.3422	09
0.2370	0.4739	0.7109	0.9479	1.1848	1.4218	1.6588	1.8958	2.1327	0.3426	10
0.2372	0.4744	0.7117	0.9489	1.1861	1.4233	1.6606	1.8978	2.1350	0.3430	11
0.2375	0.4750	0.7124	0.9499	1.1874	1.4249	1.6624	1.8998	2.1373	0.3434	12
0.2377	0.4755	0.7132	0.9509	1.1887	1.4264	1.6641	1.9018	2.1396	0.3438	13
0.2380	0.4760	0.7140	0.9520	1.1899	1.4279	1.6659	1.9039	2.1419	0.3442	14
0.2382	0.4765	0.7147	0.9530	1.1912	1.4295	1.6677	1.9060	2.1442	0.3446	15
0.2385	0.4770	0.7155	0.9540	1.1925	1.4310	1.6695	1.9080	2.1465	0.3450	16
0.2388	0.4775	0.7163	0.9550	1.1938	1.4326	1.6713	1.9101	2.1488	0.3454	17
0.2390	0.4780	0.7170	0.9560	1.1951	1.4341	1.6731	1.9121	2.1511	0.3458	18
0.2393	0.4785	0.7178	0.9571	1.1963	1.4356	1.6749	1.9142	2.1534	0.3462	19
0.2395	0.4790	0.7186	0.9581	1.1976	1.4371	1.6767	1.9162	2.1557	0.3466	20
0.2398	0.4796	0.7193	0.9591	1.1989	1.4387	1.6785	1.9182	2.1580	0.3470	21
0.2400	0.4801	0.7201	0.9601	1.2002	1.4402	1.6802	1.9202	2.1603	0.3474	22
0.2403	0.4806	0.7209	0.9611	1.2014	1.4417	1.6820	1.9223	2.1626	0.3478	23
0.2405	0.4811	0.7216	0.9622	1.2027	1.4432	1.6838	1.9243	2.1648	0.3482	24
0.2408	0.4816	0.7224	0.9632	1.2040	1.4448	1.6856	1.9264	2.1671	0.3485	25
0.2411	0.4821	0.7232	0.9642	1.2053	1.4463	1.6874	1.9284	2.1694	0.3489	26
0.2413	0.4826	0.7239	0.9652	1.2065	1.4478	1.6891	1.9304	2.1717	0.3493	27
0.2416	0.4831	0.7247	0.9662	1.2078	1.4494	1.6909	1.9325	2.1740	0.3497	28
0.2418	0.4836	0.7254	0.9672	1.2091	1.4509	1.6927	1.9345	2.1763	0.3501	29
0.2421	0.4841	0.7262	0.9683	1.2103	1.4524	1.6945	1.9366	2.1786	0.3505	30
0.2423	0.4846	0.7270	0.9693	1.2116	1.4539	1.6962	1.9386	2.1809	0.3509	31
0.2425	0.4851	0.7277	0.9703	1.2129	1.4554	1.6980	1.9406	2.1831	0.3513	32
0.2428	0.4857	0.7285	0.9713	1.2141	1.4570	1.6998	1.9426	2.1855	0.3517	33
0.2431	0.4862	0.7292	0.9723	1.2154	1.4585	1.7016	1.9446	2.1877	0.3521	34
0.2433	0.4867	0.7299	0.9733	1.2166	1.4600	1.7033	1.9466	2.1900	0.3525	35
0.2436	0.4872	0.7307	0.9743	1.2179	1.4615	1.7051	1.9487	2.1923	0.3529	36
0.2438	0.4877	0.7315	0.9754	1.2192	1.4630	1.7069	1.9507	2.1946	0.3533	37
0.2441	0.4882	0.7323	0.9764	1.2205	1.4646	1.7087	1.9528	2.1969	0.3537	38
0.2443	0.4847	0.7330	0.9774	1.2217	1.4661	1.7104	1.9548	2.1991	0.3541	39
0.2446	0.4892	0.7338	0.9784	1.2230	1.4676	1.7122	1.9568	2.2014	0.3545	40
0.2449	0.4897	0.7346	0.9794	1.2243	1.4691	1.7140	1.9588	2.2037	0.3549	41
0.2451	0.4902	0.7353	0.9804	1.2255	1.4706	1.7157	1.9608	2.2059	0.3553	42
0.2454	0.4907	0.7361	0.9814	1.2268	1.4722	1.7175	1.9629	2.2082	0.3556	43
0.2456	0.4912	0.7368	0.9824	1.2281	1.4737	1.7193	1.9649	2.2105	0.3560	44
0.2459	0.4917	0.7376	0.9835	1.2294	1.4752	1.7211	1.9670	2.2128	0.3564	45
0.2461	0.4922	0.7384	0.9845	1.2306	1.4767	1.7228	1.9690	2.2151	0.3568	46
0.2464	0.4927	0.7391	0.9855	1.2319	1.4782	1.7246	1.9710	2.2173	0.3572	47
0.2466	0.4932	0.7399	0.9865	1.2331	1.4797	1.7263	1.9730	2.2196	0.3576	48
0.2469	0.4938	0.7406	0.9875	1.2344	1.4813	1.7281	1.9750	2.2219	0.3580	49
0.2471	0.4943	0.7414	0.9885	1.2357	1.4828	1.7299	1.9770	2.2242	0.3584	50
0.2474	0.4948	0.7421	0.9895	1.2369	1.4843	1.7317	1.9790	2.2264	0.3588	51
0.2476	0.4953	0.7429	0.9905	1.2382	1.4858	1.7334	1.9810	2.2287	0.3592	52
0.2479	0.4958	0.7436	0.9915	1.2394	1.4873	1.7352	1.9831	2.2310	0.3596	53
0.2481	0.4963	0.7444	0.9926	1.2407	1.4888	1.7370	1.9851	2.2333	0.3600	54
0.2484	0.4968	0.7452	0.9936	1.2420	1.4903	1.7387	1.9871	2.2355	0.3604	55
0.2486	0.4973	0.7459	0.9946	1.2432	1.4918	1.7405	1.9891	2.2378	0.3608	56
0.2489	0.4978	0.7467	0.9956	1.2445	1.4933	1.7422	1.9911	2.2400	0.3612	57
0.2491	0.4983	0.7474	0.9966	1.2457	1.4949	1.7440	1.9932	2.2423	0.3616	58
0.2494	0.4988	0.7482	0.9976	1.2470	1.4964	1.7458	1.9952	2.2446	0.3620	59
0.2497	0.4993	0.7490	0.9986	1.2483	1.4979	1.7476	1.9972	2.2469	0.3623	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9317	1.8634	2.7951	3.7268	4.6585	5.5902	6.5219	7.4537	8.3854	1.3523
01	0.9316	1.8631	2.7947	3.7262	4.6578	5.5894	6.5209	7.4525	8.3840	1.3522
02	0.9314	1.8628	2.7942	3.7257	4.6571	5.5885	6.5199	7.4513	8.3827	1.3521
03	0.9313	1.8625	2.7938	3.7251	4.6563	5.5876	6.5189	7.4502	8.3814	1.3520
04	0.9311	1.8622	2.7934	3.7245	4.6556	5.5867	6.5179	7.4490	8.3801	1.3519
05	0.9310	1.8620	2.7929	3.7239	4.6549	5.5859	6.5168	7.4478	8.3788	1.3518
06	0.9308	1.8617	2.7925	3.7233	4.6542	5.5850	6.5158	7.4466	8.3775	1.3517
07	0.9307	1.8614	2.7921	3.7227	4.6534	5.5841	6.5148	7.4455	8.3762	1.3516
08	0.9305	1.8611	2.7916	3.7222	4.6527	5.5832	6.5138	7.4443	8.3749	1.3515
09	0.9304	1.8608	2.7912	3.7216	4.6520	5.5824	6.5128	7.4432	8.3736	1.3514
10	0.9302	1.8605	2.7907	3.7210	4.6512	5.5815	6.5117	7.4420	8.3722	1.3513
11	0.9301	1.8602	2.7903	3.7204	4.6505	5.5806	6.5107	7.4408	8.3709	1.3512
12	0.9300	1.8599	2.7899	3.7198	4.6498	5.5797	6.5097	7.4396	8.3696	1.3511
13	0.9298	1.8596	2.7894	3.7192	4.6490	5.5788	6.5086	7.4384	8.3682	1.3510
14	0.9297	1.8593	2.7890	3.7186	4.6483	5.5779	6.5076	7.4373	8.3669	1.3509
15	0.9295	1.8590	2.7885	3.7180	4.6476	5.5771	6.5066	7.4361	8.3656	1.3508
16	0.9294	1.8587	2.7881	3.7175	4.6468	5.5762	6.5055	7.4349	8.3643	1.3507
17	0.9292	1.8584	2.7876	3.7169	4.6461	5.5753	6.5045	7.4337	8.3629	1.3506
18	0.9291	1.8581	2.7872	3.7163	4.6453	5.5744	6.5035	7.4325	8.3616	1.3504
19	0.9289	1.8578	2.7868	3.7157	4.6446	5.5735	6.5024	7.4314	8.3603	1.3503
20	0.9288	1.8575	2.7863	3.7151	4.6439	5.5726	6.5014	7.4302	8.3590	1.3502
21	0.9286	1.8572	2.7859	3.7145	4.6431	5.5717	6.5004	7.4290	8.3576	1.3501
22	0.9285	1.8570	2.7854	3.7139	4.6424	5.5709	6.4993	7.4278	8.3563	1.3500
23	0.9283	1.8567	2.7850	3.7133	4.6416	5.5700	6.4983	7.4266	8.3549	1.3499
24	0.9282	1.8564	2.7845	3.7127	4.6409	5.5691	6.4972	7.4254	8.3536	1.3498
25	0.9280	1.8561	2.7841	3.7121	4.6401	5.5682	6.4962	7.4242	8.3523	1.3497
26	0.9279	1.8558	2.7836	3.7115	4.6394	5.5673	6.4952	7.4230	8.3509	1.3496
27	0.9277	1.8555	2.7832	3.7109	4.6387	5.5664	6.4941	7.4218	8.3496	1.3495
28	0.9276	1.8552	2.7827	3.7103	4.6379	5.5655	6.4931	7.4207	8.3482	1.3494
29	0.9274	1.8549	2.7823	3.7097	4.6372	5.5646	6.4920	7.4195	8.3469	1.3493
30	0.9273	1.8546	2.7819	3.7091	4.6364	5.5637	6.4910	7.4183	8.3456	1.3491
31	0.9271	1.8543	2.7814	3.7085	4.6357	5.5628	6.4899	7.4171	8.3442	1.3490
32	0.9270	1.8540	2.7809	3.7079	4.6349	5.5619	6.4889	7.4159	8.3428	1.3489
33	0.9268	1.8537	2.7805	3.7073	4.6342	5.5610	6.4878	7.4147	8.3415	1.3488
34	0.9267	1.8534	2.7800	3.7067	4.6334	5.5601	6.4868	7.4135	8.3401	1.3487
35	0.9265	1.8531	2.7796	3.7061	4.6327	5.5592	6.4857	7.4123	8.3388	1.3486
36	0.9264	1.8528	2.7791	3.7055	4.6319	5.5583	6.4847	7.4111	8.3374	1.3485
37	0.9262	1.8525	2.7787	3.7049	4.6312	5.5574	6.4836	7.4098	8.3361	1.3484
38	0.9261	1.8522	2.7782	3.7043	4.6304	5.5565	6.4826	7.4086	8.3347	1.3483
39	0.9259	1.8519	2.7778	3.7037	4.6297	5.5556	6.4815	7.4074	8.3334	1.3482
40	0.9258	1.8516	2.7773	3.7031	4.6289	5.5547	6.4805	7.4062	8.3320	1.3480
41	0.9256	1.8513	2.7769	3.7025	4.6281	5.5538	6.4794	7.4050	8.3307	1.3479
42	0.9255	1.8510	2.7764	3.7019	4.6274	5.5529	6.4783	7.4038	8.3293	1.3478
43	0.9253	1.8506	2.7760	3.7013	4.6266	5.5519	6.4773	7.4026	8.3279	1.3477
44	0.9252	1.8503	2.7755	3.7007	4.6259	5.5510	6.4762	7.4014	8.3266	1.3476
45	0.9250	1.8500	2.7751	3.7001	4.6251	5.5501	6.4751	7.4002	8.3252	1.3475
46	0.9249	1.8497	2.7746	3.6995	4.6243	5.5492	6.4741	7.3990	8.3238	1.3474
47	0.9247	1.8494	2.7742	3.6989	4.6236	5.5483	6.4730	7.3977	8.3225	1.3473
48	0.9246	1.8491	2.7737	3.6983	4.6228	5.5474	6.4720	7.3965	8.3211	1.3472
49	0.9244	1.8488	2.7732	3.6977	4.6221	5.5465	6.4709	7.3953	8.3197	1.3470
50	0.9243	1.8485	2.7728	3.6970	4.6213	5.5456	6.4698	7.3941	8.3184	1.3469
51	0.9241	1.8482	2.7723	3.6964	4.6205	5.5447	6.4688	7.3929	8.3170	1.3468
52	0.9240	1.8479	2.7719	3.6958	4.6198	5.5437	6.4677	7.3916	8.3156	1.3467
53	0.9238	1.8476	2.7714	3.6952	4.6190	5.5428	6.4666	7.3904	8.3142	1.3466
54	0.9236	1.8473	2.7709	3.6946	4.6182	5.5419	6.4655	7.3892	8.3128	1.3465
55	0.9235	1.8470	2.7705	3.6940	4.6175	5.5410	6.4645	7.3880	8.3115	1.3464
56	0.9233	1.8467	2.7700	3.6934	4.6167	5.5401	6.4634	7.3867	8.3101	1.3463
57	0.9232	1.8464	2.7696	3.6928	4.6159	5.5391	6.4623	7.3855	8.3087	1.3461
58	0.9230	1.8461	2.7691	3.6921	4.6152	5.5382	6.4613	7.3843	8.3073	1.3460
59	0.9228	1.8458	2.7686	3.6915	4.6144	5.5373	6.4602	7.3831	8.3059	1.3459
60	0.9227	1.8455	2.7982	3.6909	4.6137	5.5364	6.4591	7.3818	8.3046	1.3458

1	2	3	4	5	6	7	8	9	h	′
0.2497	0.4993	0.7490	0.9986	1.2483	1.4979	1.7476	1.9972	2.2469	0.3623	00
0.2409	0.4998	0.7497	0.9996	1.2495	1.4994	1.7493	1.9992	2.2491	0.3627	01
0.2502	0.5003	0.7505	1.0006	1.2508	1.5009	1.7511	2.0012	2.2514	0.3631	02
0.2504	0.5008	0.7512	1.0016	1.2520	1.5024	1.7528	2.0032	2.2536	0.3635	03
0.2507	0.5013	0.7520	1.0026	1.2533	1.5040	1.7546	2.0053	2.2559	0.3639	04
0.2509	0.5018	0.7527	1.0036	1.2545	1.5055	1.7564	2.0073	2.2582	0.3643	05
0.2512	0.5023	0.7535	1.0046	1.2558	1.5070	1.7581	2.0093	2.2604	0.3647	06
0.2515	0.5028	0.7542	1.0056	1.2570	1.5084	1.7599	2.0113	2.2627	0.3651	07
0.2517	0.5033	0.7550	1.0066	1.2583	1.5100	1.7616	2.0133	2.2649	0.3655	08
0.2519	0.5038	0.7557	1.0076	1.2596	1.5115	1.7634	2.0153	2.2672	0.3659	09
0.2522	0.5043	0.7565	1.0086	1.2608	2.5130	1.7651	2.0173	2.2694	0.3663	10
0.2524	0.5048	0.7572	1.0096	1.2621	1.5145	1.7669	2.0193	2.2717	0.3667	11
0.2527	0.5053	0.7580	1.0106	1.2633	1.5160	1.7686	2.0213	2.2739	0.3671	12
0.2529	0.5058	0.7587	1.0116	1.2646	1.5175	1.7704	2.0233	2.2762	0.3674	13
0.2532	0.5063	0.7595	1.0126	1.2658	1.5190	1.7721	2.0253	2.2784	0.3678	14
0.2534	0.5068	0.7602	1.0136	1.2671	1.5205	1.7739	2.0273	2.2807	0.3682	15
0.2537	0.5073	0.7610	1.0146	1.2683	1.5220	1.7756	2.0293	2.2829	0.3686	16
0.2539	0.5078	0.7617	1.0156	1.2696	1.5235	1.7774	2.0313	2.2852	0.3690	17
0.2542	0.5083	0.7625	1.0166	1.2708	1.5250	1.7791	2.0333	2.2874	0.3694	18
0.2544	0.5088	0.7632	1.0176	1.2721	1.5265	1.7809	2.0353	2.2897	0.3698	19
0.2547	0.5093	0.7640	1.0186	1.2733	1.5280	1.7826	2.0373	2.2919	0.3702	20
0.2549	0.5098	0.7647	1.0196	1.2746	1.5295	1.7844	2.0393	2.2942	0.3706	21
0.2552	0.5103	0.7655	1.0206	1.2758	1.5310	1.7861	2.0413	2.2964	0.3710	22
0.2554	0.5108	0.7662	1.0216	1.2771	1.5325	1.7879	2.0433	2.2987	0.3714	23
0.2557	0.5113	0.7670	1.0226	1.2783	1.5340	1.7896	2.0453	2.3009	0.3718	24
0.2559	0.5118	0.7677	1.0236	1.2796	1.5355	1.7914	2.0473	2.3032	0.3722	25
0.2562	0.5123	0.7685	1.0246	1.2808	1.5370	1.7931	2.0493	2.3054	0.3725	26
0.2564	0.5128	0.7692	1.0256	1.2821	1.5385	1.7949	2.0513	2.3077	0.3729	27
0.2567	0.5133	0.7700	1.0266	1.2833	1.5400	1.7966	2.0533	2.3099	0.3733	28
0.2569	0.5138	0.7707	1.0276	1.2845	1.5415	1.7984	2.0553	2.3122	0.3737	29
0.2572	0.5143	0.7715	1.0286	1.2858	1.5430	1.8001	2.0573	2.3144	0.3741	30
0.2574	0.5148	0.7722	1.0296	1.2870	1.5445	1.8019	2.0593	2.3167	0.3745	31
0.2577	0.5153	0.7730	1.0306	1.2883	1.5460	1.8036	2.0613	2.3189	0.3749	32
0.2579	0.5158	0.7737	1.0316	1.2895	1.5474	1.8053	2.0632	2.3211	0.3753	33
0.2582	0.5163	0.7745	1.0326	1.2908	1.5489	1.8071	2.0652	2.3234	0.3757	34
0.2584	0.5168	0.7752	1.0336	1.2920	1.5504	1.8088	2.0672	2.3256	0.3761	35
0.2587	0.5173	0.7760	1.0346	1.2933	1.5519	1.8106	2.0692	2.3279	0.3765	36
0.2589	0.5178	0.7767	1.0356	1.2945	1.5534	1.8123	2.0712	2.3301	0.3769	37
0.2591	0.5183	0.7774	1.0366	1.2957	1.5549	1.8140	2.0732	2.3323	0.3773	38
0.2594	0.5188	0.7782	1.0376	1.2970	1.5564	1.8158	2.0752	2.3346	0.3776	39
0.2596	0.5193	0.7789	1.0386	1.2982	1.5578	1.8175	2.0771	2.3368	0.3780	40
0.2599	0.5198	0.7797	1.0396	1.2995	1.5593	1.8192	2.0791	2.3390	0.3784	41
0.2601	0.5203	0.7804	1.0406	1.3007	1.5608	1.8210	2.0811	2.3413	0.3788	42
0.2604	0.5208	0.7811	1.0416	1.3019	1.5623	1.8227	2.0831	2.3435	0.3792	43
0.2606	0.5213	0.7819	1.0426	1.3032	1.5638	1.8245	2.0851	2.3458	0.3796	44
0.2609	0.5218	0.7826	1.0435	1.3044	1.5653	1.8262	2.0870	2.3479	0.3800	45
0.2611	0.5223	0.7834	1.0445	1.3057	1.5668	1.8279	2.0890	2.3502	0.3804	46
0.2614	0.5228	0.7841	1.0455	1.3069	1.5683	1.8297	2.0910	2.3524	0.3808	47
0.2616	0.5233	0.7849	1.0465	1.3081	1.5698	1.8314	2.0930	2.3547	0.3812	48
0.2619	0.5237	0.7856	1.0475	1.3094	1.5712	1.8331	2.0950	2.3569	0.3816	49
0.2621	0.5242	0.7864	1.0485	1.3106	1.5727	1.8348	2.0970	2.3591	0.3820	50
0.2624	0.5247	0.7871	1.0495	1.3118	1.5742	1.8366	2.0990	2.3613	0.3824	51
0.2626	0.5252	0.7879	1.0505	1.3131	1.5757	1.8383	2.1010	2.3636	0.3827	52
0.2629	0.5257	0.7886	1.0514	1.3143	1.5772	1.8400	2.1029	2.3658	0.3831	53
0.2631	0.5262	0.7893	1.0524	1.3155	1.5787	1.8418	2.1049	2.3680	0.3835	54
0.2634	0.5267	0.7901	1.0534	1.3168	1.5802	1.8435	2.1069	2.3702	0.3839	55
0.2636	0.5272	0.7908	1.0544	1.3180	1.5816	1.8452	2.1088	2.3724	0.3843	56
0.2638	0.5277	0.7915	1.0554	1.3192	1.5831	1.8469	2.1108	2.3746	0.3847	57
0.2641	0.5282	0.7923	1.0564	1.3205	1.5846	1.8487	2.1128	2.3769	0.3851	58
0.2643	0.5287	0.7930	1.0574	1.3217	1.5860	1.8504	2.1147	2.3791	0.3855	59
0.2646	0.5292	0.7938	1.0584	1.3230	1.5875	1.8521	2.1167	2.3813	0.3859	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9227	1.8455	2.7682	3.6909	4.6137	5.5364	6.4591	7.3818	8.3046	1.3458
01	0.9226	1.8452	2.7677	3.6903	4.6129	5.5355	6.4580	7.3806	8.3032	1.3457
02	0.9224	1.8448	2.7673	3.6897	4.6121	5.5345	6.4569	7.3794	8.3018	1.3456
03	0.9223	1.8445	2.7668	3.6891	4.6113	5.5336	6.4559	7.3781	8.3004	1.3455
04	0.9221	1.8442	2.7663	3.6884	4.6006	5.5327	6.4548	7.3769	8.2990	1.3454
05	0.9220	1.8439	2.7659	3.6878	4.6098	5.5317	6.4537	7.3757	8.2976	1.3453
06	0.9218	1.8436	2.7654	3.6872	4.6090	5.5308	6.4526	7.3744	8.2962	1.3451
07	0.9216	1.8433	2.7649	3.6866	4.6082	5.5299	6.4515	7.3732	8.2948	1.3450
08	0.9215	1.8430	2.7645	3.6860	4.6075	5.5290	6.4505	7.3719	8.2934	1.3449
09	0.9213	1.8427	2.7640	3.6854	4.6067	5.5280	6.4494	7.3707	8.2921	1.3448
10	0.9212	1.8424	2.7636	3.6847	4.6059	5.5271	6.4483	7.3695	8.2907	1.3447
11	0.9210	1.8421	2.7631	3.6841	4.6051	5.5262	6.4472	7.3682	8.2893	1.3446
12	0.9209	1.8417	2.7626	3.6835	4.6044	5.5252	6.4461	7.3670	8.2878	1.3445
13	0.9207	1.8414	2.7621	3.6829	4.6036	5.5243	6.4450	7.3657	8.2864	1.3444
14	0.9206	1.8411	2.7617	3.6822	4.6028	5.5234	6.4439	7.3645	8.2850	1.3442
15	0.9204	1.8408	2.7612	3.6816	4.6020	5.5224	6.4428	7.3632	8.2836	1.3441
16	0.9202	1.8405	2.7607	3.6810	4.6012	5.5215	6.4417	7.3620	8.2822	1.3440
17	0.9201	1.8402	2.7603	3.6804	4.6005	5.5205	6.4406	7.3607	8.2808	1.3439
18	0.9199	1.8399	2.7598	3.6797	4.5997	5.5196	6.4395	7.3595	8.2794	1.3438
19	0.9198	1.8396	2.7593	3.6791	4.5989	5.5187	6.4385	7.3582	8.2780	1.3437
20	0.9196	1.8392	2.7589	3.6785	4.5981	5.5177	6.4374	7.3570	8.2766	1.3436
21	0.9195	1.8389	2.7584	3.6779	4.5973	5.5168	6.4363	7.3557	8.2752	1.3435
22	0.9193	1.8386	2.7579	3.6772	4.5965	5.5158	6.4352	7.3545	8.2738	1.3433
23	0.9192	1.8383	2.7575	3.6766	4.5958	5.5149	6.4341	7.3532	8.2724	1.3432
24	0.9190	1.8380	2.7570	3.6760	4.5950	5.5140	6.4330	7.3519	8.2709	1.3431
25	0.9188	1.8377	2.7565	3.6753	4.5942	5.5130	6.4319	7.3507	8.2695	1.3430
26	0.9187	1.8374	2.7560	3.6747	4.5934	5.5121	6.4307	7.3494	8.2681	1.3429
27	0.9185	1.8370	2.7556	3.6741	4.5926	5.5111	6.4296	7.3482	8.2667	1.3428
28	0.9184	1.8367	2.7551	3.6735	4.5918	5.5102	6.4285	7.3469	8.2653	1.3427
29	0.9182	1.8364	2.7546	3.6728	4.5910	5.5092	6.4274	7.3456	8.2639	1.3425
30	0.9180	1.8361	2.7541	3.6722	4.5902	5.5083	6.4263	7.3444	8.2624	1.3424
31	0.9179	1.8358	2.7537	3.6716	4.5894	5.5073	6.4252	7.3431	8.2610	1.3423
32	0.9177	1.8355	2.7532	3.6709	4.5887	5.5064	6.4241	7.3418	8.2596	1.3422
33	0.9176	1.8351	2.7527	3.6703	4.5879	5.5054	6.4230	7.3406	8.2581	1.3420
34	0.9174	1.8348	2.7522	3.6696	4.5871	5.5045	6.4219	7.3393	8.2567	1.3419
35	0.9173	1.8345	2.7518	3.6690	4.5863	5.5035	6.4208	7.3380	8.2553	1.3418
36	0.9171	1.8342	2.7513	3.6684	4.5855	5.5026	6.4197	7.3368	8.2539	1.3417
37	0.9169	1.8339	2.7508	3.6677	4.5847	5.5016	6.4186	7.3355	8.2524	1.3416
38	0.9168	1.8336	2.7503	3.6671	4.5839	5.5007	6.4174	7.3342	8.2510	1.3415
39	0.9166	1.8332	2.7499	3.6665	4.5831	5.4997	6.4163	7.3329	8.2496	1.3413
40	0.9165	1.8329	2.7494	3.6658	4.5823	5.4988	6.4152	7.3317	8.2481	1.3412
41	0.9163	1.8326	2.7489	3.6652	4.5815	5.4978	6.4141	7.3304	8.2467	1.3411
42	0.9161	1.8323	2.7484	3.6646	4.5807	5.4968	6.4130	7.3291	8.2452	1.3410
43	0.9160	1.8320	2.7479	3.6639	4.5799	5.4959	6.4118	7.3278	8.2438	1.3409
44	0.9158	1.8316	2.7475	3.6633	4.5791	5.4949	6.4107	7.3265	8.2424	1.3407
45	0.9157	1.8313	2.7470	3.6626	4.5783	5.4939	6.4096	7.3253	8.2409	1.3406
46	0.9155	1.8310	2.7465	3.6620	4.5775	5.4930	6.4085	7.3240	8.2395	1.3405
47	0.9153	1.8307	2.7460	3.6613	4.5767	5.4920	0.4074	7.3227	8.2380	1.3404
48	0.9152	1.8304	2.7455	3.6607	4.5759	5.4911	6.4062	7.3214	8.2366	1.3403
49	0.9150	1.8300	2.7450	3.6601	4.5751	5.4901	6.4051	7.3201	8.2351	1.3402
50	0.9149	1.8297	2.7446	3.6594	4.5743	5.4891	6.4040	7.3188	8.2337	1.3400
51	0.9147	1.8294	2.7441	3.6588	4.5735	5.4882	6.4029	7.3176	8.2322	1.3399
52	0.9145	1.8291	2.7436	3.6581	4.5727	5.4872	6.4017	7.3163	8.2308	1.3398
53	0.9144	1.8287	2.7431	3.6575	4.5719	5.4862	6.4006	7.3150	8.2293	1.3397
54	0.9142	1.8284	2.7426	3.6568	4.5710	5.4853	6.3995	7.3137	8.2279	1.3395
55	0.9140	1.8281	2.7421	3.6562	4.5702	5.4843	6.3983	7.3124	8.2264	1.3394
56	0.9139	1.8278	2.7417	3.6555	4.5694	5.4833	6.3972	7.3111	8.2250	1.3393
57	0.9137	1.8274	2.7412	3.6549	4.5686	5.4823	6.3961	7.3098	8.2235	1.3392
58	0.9136	1.8271	2.7407	3.6542	4.5678	5.4814	6.3949	7.3085	8.2221	1.3390
59	0.9134	1.8268	2.7402	3.6536	4.5670	5.4804	6.3938	7.3072	8.2206	1.3389
60	0.9132	1.8265	2.7397	3.6530	4.5662	5.4794	6.3927	7.3059	8.2192	1.3388

1	2	3	4	5	6	7	8	9	b	′
0.2646	0.5292	0.7938	1.0584	1.3230	1.5875	1.8521	2.1167	2.3813	0.3859	00
0.2648	0.5297	0.7945	1.0594	1.3242	1.5890	1.8539	2.1187	2.3836	0.3863	01
0.2651	0.5302	0.7952	1.0603	1.3254	1.5905	1.8556	2.1206	2.3857	0.3867	02
0.2653	0.5307	0.7960	1.0613	1.3266	1.5920	1.8573	2.1226	2.3880	0.3871	03
0.2656	0.5311	0.7967	1.0623	1.3279	1.5934	1.8590	2.1246	2.3902	0.3875	04
0.2658	0.5316	0.7975	1.0633	1.3291	1.5949	1.8607	2.1266	2.3924	0.3878	05
0.2661	0.5321	0.7982	1.0642	1.3303	1.5964	1.8624	2.1285	2.3946	0.3882	06
0.2663	0.5326	0.7989	1.0652	1.3316	1.5979	1.8642	2.1305	2.3968	0.3886	07
0.2666	0.5331	0.7997	1.0662	1.3328	1.5994	1.8659	2.1325	2.3990	0.3890	08
0.2668	0.5336	0.8004	1.0672	1.3340	1.6008	1.8676	2.1344	2.4012	0.3894	09
0.2670	0.5341	0.8011	1.0682	1.3352	1.6023	1.8693	2.1364	2.4034	0.3898	10
0.2673	0.5346	0.8019	1.0692	1.3365	1.6037	1.8710	2.1383	2.4056	0.3902	11
0.2675	0.5351	0.8026	1.0702	1.3377	1.6052	1.8728	2.1403	2.4078	0.3906	12
0.2678	0.5356	0.8033	1.0711	1.3389	1.6067	1.8745	2.1422	2.4100	0.3910	13
0.2680	0.5361	0.8041	1.0721	1.3401	1.6082	1.8762	2.1442	2.4123	0.3914	14
0.2683	0.5365	0.8048	1.0731	1.3414	1.6096	1.8779	2.1462	2.4145	0.3917	15
0.2685	0.5370	0.8056	1.0741	1.3426	1.6111	1.8796	2.1482	2.4167	0.3921	16
0.2688	0.5375	0.8063	1.0750	1.3438	1.6126	1.8813	2.1501	2.4189	0.3925	17
0.2690	0.5380	0.8070	1.0760	1.3450	1.6141	1.8831	2.1521	2.4211	0.3929	18
0.2693	0.5385	0.8078	1.0770	1.3463	1.6155	1.8848	2.1540	2.4233	0.3933	19
0.2695	0.5390	0.8085	1.0780	1.3475	1.6170	1.8865	2.1560	2.4255	0.3937	20
0.2697	0.5395	0.8092	1.0790	1.3487	1.6184	1.8882	2.1579	2.4277	0.3941	21
0.2700	0.5400	0.8100	1.0800	1.3499	1.6199	1.8899	2.1599	2.4299	0.3945	22
0.2702	0.5405	0.8107	1.0809	1.3512	1.6214	1.8916	2.1618	2.4321	0.3949	23
0.2705	0.5409	0.8114	1.0819	1.3524	1.6228	1.8933	2.1638	2.4343	0.3953	24
0.2707	0.5414	0.8122	1.0829	1.3536	1.6243	1.8950	2.1658	2.4365	0.3957	25
0.2710	0.5419	0.8129	1.0838	1.3548	1.6258	1.8967	2.1677	2.4387	0.3960	26
0.2712	0.5424	0.8136	1.0848	1.3560	1.6273	1.8985	2.1697	2.4409	0.3964	27
0.2715	0.5429	0.8144	1.0858	1.3573	1.6287	1.9002	2.1716	2.4431	0.3968	28
0.2717	0.5434	0.8151	1.0868	1.3585	1.6302	1.9019	2.1736	2.4453	0.3972	29
0.2719	0.5439	0.8158	1.0878	1.3597	1.6316	1.9036	2.1755	2.4475	0.3976	30
0.2722	0.5444	0.8165	1.0887	1.3609	1.6331	1.9053	2.1775	2.4496	0.3980	31
0.2724	0.5448	0.8173	1.0897	1.3621	1.6345	1.9070	2.1794	2.4518	0.3984	32
0.2727	0.5453	0.8180	1.0907	1.3634	1.6361	1.9087	2.1814	2.4540	0.3988	33
0.2729	0.5458	0.8187	1.0916	1.3646	1.6375	1.9104	2.1833	2.4562	0.3992	34
0.2732	0.5463	0.8195	1.0926	1.3658	1.6390	1.9121	2.1853	2.4584	0.3996	35
0.2734	0.5468	0.8202	1.0936	1.3670	1.6404	1.9138	2.1872	2.4606	0.3999	36
0.2736	0.5473	0.8209	1.0946	1.3682	1.6418	1.9155	2.1891	2.4628	0.4003	37
0.2739	0.5478	0.8216	1.0955	1.3694	1.6433	1.9172	2.1911	2.4650	0.4007	38
0.2741	0.5483	0.8224	1.0965	1.3706	1.6448	1.9189	2.1930	2.4672	0.4011	39
0.2744	0.5487	0.8231	1.0975	1.3719	1.6462	1.9206	2.1950	2.4693	0.4015	40
0.2746	0.5492	0.8238	1.0984	1.3731	1.6477	1.9223	2.1969	2.4715	0.4019	41
0.2749	0.5497	0.8246	1.0994	1.3743	1.6491	1.9240	2.1988	2.4737	0.4023	42
0.2751	0.5502	0.8253	1.1004	1.3755	1.6506	1.9257	2.2008	2.4759	0.4027	43
0.2753	0.5507	0.8260	1.1014	1.3767	1.6520	1.9274	2.2027	2.4781	0.4031	44
0.2756	0.5512	0.8267	1.1023	1.3779	1.6535	1.9291	2.2046	2.4802	0.4035	45
0.2758	0.5516	0.8275	1.1033	1.3791	1.6549	1.9308	2.2066	2.4824	0.4039	46
0.2761	0.5521	0.8282	1.1043	1.3803	1.6564	1.9325	2.2085	2.4846	0.4042	47
0.2763	0.5526	0.8289	1.1052	1.3815	1.6579	1.9342	2.2105	2.4868	0.4046	48
0.2766	0.5531	0.8297	1.1062	1.3828	1.6593	1.9359	2.2124	2.4890	0.4050	49
0.2768	0.5536	0.8304	1.1072	1.3840	1.6607	1.9375	2.2143	2.4911	0.4054	50
0.2770	0.5541	0.8311	1.1081	1.3852	1.6622	1.9392	2.2163	2.4933	0.4058	51
0.2773	0.5546	0.8318	1.1091	1.3864	1.6637	1.9409	2.2182	2.4955	0.4062	52
0.2775	0.5550	0.8326	1.1101	1.3876	1.6651	1.9426	2.2202	2.4977	0.4066	53
0.2778	0.5555	0.8333	1.1110	1.3888	1.6666	1.9443	2.2221	2.4998	0.4070	54
0.2780	0.5560	0.8340	1.1120	1.3900	1.6680	1.9460	2.2240	2.5020	0.4074	55
0.2782	0.5565	0.8347	1.1130	1.3912	1.6695	1.9477	2.2259	2.5042	0.4078	56
0.2785	0.5570	0.8354	1.1139	1.3924	1.6709	1.9494	2.2278	2.5063	0.4081	57
0.2787	0.5574	0.8362	1.1149	1.3936	1.6723	1.9510	2.2298	2.5085	0.4085	58
0.2790	0.5579	0.8369	1.1158	1.3948	1.6738	1.9527	2.2317	2.5107	0.4089	59
0.2792	0.5584	0.8376	1.1168	1.3960	1.6752	1.9544	2.2337	2.5129	0.4093	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9132	1.8265	2.7397	3.6530	4.5662	5.4794	6.3927	7.3059	8.2192	1.3388
01	0.9131	1.8262	2.7392	3.6523	4.5654	5.4785	6.3915	7.3046	8.2177	1.3387
02	0.9129	1.8258	2.7387	3.6517	4.5646	5.4775	6.3904	7.3033	8.2162	1.3385
03	0.9127	1.8255	2.7382	3.6510	4.5637	5.4765	6.3892	7.3020	8.2147	1.3384
04	0.9126	1.8252	2.7378	3.6503	4.5629	5.4755	6.3881	7.3007	8.2133	1.3383
05	0.9124	1.8248	2.7373	3.6497	4.5621	5.4745	6.3870	7.2994	8.2118	1.3382
06	0.9123	1.8245	2.7368	3.6490	4.5613	5.4736	6.3858	7.2981	8.2103	1.3381
07	0.9121	1.8242	2.7363	3.6484	4.5605	5.4726	6.3847	7.2968	8.2089	1.3379
08	0.9119	1.8239	2.7358	3.6477	4.5597	5.4716	6.3835	7.2955	8.2074	1.3378
09	0.9118	1.8235	2.7353	3.6471	4.5589	5.4706	6.3824	7.2942	8.2059	1.3377
10	0.9116	1.8232	2.7348	3.6464	4.5580	5.4696	6.3812	7.2929	8.2045	1.3376
11	0.9114	1.8229	2.7343	3.6458	4.5572	5.4687	6.3801	7.2915	8.2030	1.3375
12	0.9113	1.8226	2.7338	3.6451	4.5564	5.4677	6.3789	7.2902	8.2015	1.3373
13	0.9111	1.8222	2.7333	3.6445	4.5556	5.4667	6.3778	7.2889	8.2000	1.3372
14	0.9109	1.8219	2.7328	3.6438	4.5547	5.4657	6.3766	7.2876	8.1985	1.3371
15	0.9108	1.8216	2.7324	3.6431	4.5539	5.4647	6.3755	7.2863	8.1971	1.3370
16	0.9106	1.8212	2.7319	3.6425	4.5531	5.4637	6.3743	7.2850	8.1956	1.3368
17	0.9105	1.8209	2.7314	3.6418	4.5523	5.4627	6.3732	7.2836	8.1941	1.3367
18	0.9103	1.8206	2.7309	3.6412	4.5515	5.4617	6.3720	7.2823	8.1926	1.3366
19	0.9101	1.8203	2.7304	3.6405	4.5506	5.4608	6.3709	7.2810	8.1911	1.3365
20	0.9100	1.8199	2.7299	3.6398	4.5498	5.4598	6.3697	7.2797	8.1897	1.3364
21	0.9098	1.8196	2.7294	3.6392	4.5490	5.4588	6.3686	7.2784	8.1882	1.3362
22	0.9096	1.8193	2.7289	3.6385	4.5482	5.4578	6.3674	7.2770	8.1867	1.3361
23	0.9095	1.8189	2.7284	3.6379	4.5473	5.4568	6.3662	7.2757	8.1852	1.3360
24	0.9093	1.8186	2.7279	3.6372	4.5465	5.4558	6.3651	7.2744	8.1837	1.3359
25	0.9091	1.8183	2.7274	3.6365	4.5457	5.4548	6.3639	7.2731	8.1822	1.3358
26	0.9090	1.8179	2.7269	3.6359	4.5448	5.4538	6.3628	7.2717	8.1807	1.3357
27	0.9088	1.8176	2.7264	3.6352	4.5440	5.4528	6.3616	7.2704	8.1792	1.3355
28	0.9086	1.8173	2.7259	3.6345	4.5432	5.4518	6.3604	7.2691	8.1777	1.3354
29	0.9085	1.8169	2.7254	3.6339	4.5423	5.4508	6.3593	7.2678	8.1762	1.3353
30	0.9083	1.8166	2.7249	3.6332	4.5415	5.4498	6.3581	7.2664	8.1747	1.3352
31	0.9081	1.8163	2.7244	3.6325	4.5407	5.4488	6.3570	7.2651	8.1732	1.3350
32	0.9080	1.8159	2.7239	3.6319	4.5398	5.4478	6.3558	7.2637	8.1717	1.3349
33	0.9078	1.8156	2.7234	3.6312	4.5390	5.4468	6.3546	7.2624	8.1702	1.3348
34	0.9076	1.8153	2.7229	3.6305	4.5382	5.4458	6.3534	7.2611	8.1687	1.3347
35	0.9075	1.8149	2.7224	3.6299	4.5373	5.4448	6.3523	7.2597	8.1672	1.3346
36	0.9073	1.8146	2.7219	3.6292	4.5365	5.4438	6.3511	7.2584	8.1657	1.3344
37	0.9071	1.8143	2.7214	3.6285	4.5357	5.4428	6.3499	7.2571	8.1642	1.3343
38	0.9070	1.8139	2.7209	3.6279	4.5348	5.4418	6.3488	7.2557	8.1627	1.3342
39	0.9068	1.8136	2.7204	3.6272	4.5340	5.4408	6.3476	7.2544	8.1612	1.3341
40	0.9066	1.8133	2.7199	3.6265	4.5332	5.4398	6.3464	7.2530	8.1597	1.3339
41	0.9065	1.8129	2.7194	3.6258	4.5323	5.4388	6.3452	7.2517	8.1581	1.3338
42	0.9063	1.8126	2.7189	3.6252	4.5315	5.4378	6.3441	7.2503	8.1566	1.3337
43	0.9061	1.8122	2.7184	3.6245	4.5306	5.4367	6.3429	7.2490	8.1551	1.3336
44	0.9060	1.8119	2.7179	3.6238	4.5298	5.4357	6.3417	7.2476	8.1536	1.3335
45	0.9058	1.8116	2.7174	3.6231	4.5289	5.4347	6.3405	7.2463	8.1521	1.3333
46	0.9056	1.8112	2.7169	3.6225	4.5281	5.4337	6.3393	7.2449	8.1506	1.3332
47	0.9054	1.8109	2.7163	3.6218	4.5272	5.4327	6.3381	7.2436	8.1490	1.3331
48	0.9053	1.8106	2.7158	3.6211	4.5264	5.4317	6.3370	7.2422	8.1475	1.3330
49	0.9051	1.8102	2.7153	3.6204	4.5256	5.4307	6.3358	7.2409	8.1460	1.3329
50	0.9049	1.8099	2.7148	3.6198	4.5247	5.4297	6.3346	7.2395	8.1445	1.3327
51	0.9048	1.8095	2.7143	3.6191	4.5239	5.4286	6.3334	7.2382	8.1430	1.3326
52	0.9046	1.8092	2.7138	3.6184	4.5230	5.4276	6.3322	7.2368	8.1414	1.3325
53	0.9044	1.8089	2.7133	3.6177	4.5222	5.4266	6.3310	7.2355	8.1399	1.3324
54	0.9043	1.8085	2.7128	3.6171	4.5213	5.4256	6.3298	7.2341	8.1384	1.3323
55	0.9041	1.8082	2.7123	3.6164	4.5205	5.4246	6.3287	7.2327	8.1368	1.3321
56	0.9039	1.8078	2.7118	3.6157	4.5196	5.4235	6.3275	7.2314	8.1353	1.3320
57	0.9038	1.8075	2.7113	3.6150	4.5188	5.4225	6.3263	7.2300	8.1338	1.3319
58	0.9036	1.8072	2.7107	3.6143	4.5179	5.4215	6.3251	7.2287	8.1322	1.3318
59	0.9034	1.8068	2.7102	3.6137	4.5171	5.4205	6.3239	7.2273	8.1307	1.3316
60	0.9032	1.8065	2.7097	3.6130	4.5162	5.4195	6.3227	7.2259	8.1292	1.3315

1	2	3	4	5	6	7	8	9	b	′
0.2792	0.5584	0.8376	1.1168	1.3960	1.6752	1.9544	2.2337	2.5129	0.4093	00
0.2794	0.5589	0.8383	1.1178	1.3972	1.6767	1.9561	2.2356	2.5150	0.4097	01
0.2797	0.5594	0.8391	1.1187	1.3984	1.6781	1.9578	2.2375	2.5172	0.4101	02
0.2799	0.5599	0.8398	1.1197	1.3996	1.6796	1.9595	2.2394	2.5193	0.4105	03
0.2802	0.5603	0.8405	1.1207	1.4008	1.6810	1.9612	2.2414	2.5215	0.4109	04
0.2804	0.5608	0.8412	1.1216	1.4020	1.6825	1.9629	2.2433	2.5237	0.4113	05
0.2806	0.5613	0.8419	1.1226	1.4032	1.6839	1.9645	2.2452	2.5258	0.4116	06
0.2809	0.5618	0.8427	1.1236	1.4044	1.6853	1.9662	2.2471	2.5280	0.4120	07
0.2811	0.5623	0.8434	1.1245	1.4056	1.6868	1.9679	2.2490	2.5302	0.4124	08
0.2814	0.5627	0.8441	1.1255	1.4068	1.6882	1.9696	2.2510	2.5323	0.4128	09
0.2816	0.5632	0.8448	1.1264	1.4080	1.6897	1.9713	2.2529	2.5345	0.4132	10
0.2818	0.5637	0.8455	1.1274	1.4092	1.6911	1.9729	2.2548	2.5366	0.4136	11
0.2821	0.5642	0.8463	1.1284	1.4104	1.6925	1.9746	2.2567	2.5388	0.4140	12
0.2823	0.5647	0.8470	1.1293	1.4116	1.6940	1.9763	2.2586	2.5409	0.4144	13
0.2826	0.5651	0.8477	1.1303	1.4128	1.6954	1.9780	2.2606	2.5431	0.4148	14
0.2828	0.5656	0.8484	1.1312	1.4140	1.6969	1.9797	2.2625	2.5453	0.4151	15
0.2830	0.5661	0.8491	1.1322	1.4152	1.6983	1.9813	2.2644	2.5474	0.4155	16
0.2833	0.5666	0.8499	1.1332	1.4164	1.6997	1.9830	2.2663	2.5496	0.4159	17
0.2835	0.5670	0.8506	1.1341	1.4176	1.7011	1.9847	2.2682	2.5517	0.4163	18
0.2838	0.5675	0.8513	1.1351	1.4188	1.7026	1.9863	2.2701	2.5539	0.4167	19
0.2840	0.5680	0.8520	1.1360	1.4200	1.7040	1.9880	2.2720	2.5560	0.4171	20
0.2842	0.5685	0.8527	1.1370	1.4212	1.7054	1.9097	2.2739	2.5582	0.4175	21
0.2845	0.5690	0.8534	1.1379	1.4224	1.7069	1.9914	2.2758	2.5603	0.4179	22
0.2847	0.5694	0.8542	1.1389	1.4236	1.7083	1.9930	2.2778	2.5625	0.4183	23
0.2850	0.5699	0.8549	1.1398	1.4248	1.7098	1.9947	2.2797	2.5646	0.4186	24
0.2852	0.5704	0.8556	1.1408	1.4260	1.7112	1.9964	2.2816	2.5668	0.4190	25
0.2854	0.5709	0.8563	1.1417	1.4272	1.7126	1.9980	2.2834	2.5689	0.4194	26
0.2857	0.5713	0.8570	1.1427	1.4284	1.7140	1.9997	2.2854	2.5710	0.4198	27
0.2859	0.5718	0.8577	1.1436	1.4296	1.7155	2.0014	2.2873	2.5732	0.4202	28
0.2861	0.5723	0.8584	1.1446	1.4307	1.7169	2.0030	2.2892	2.5753	0.4208	29
0.2864	0.5728	0.8592	1.1456	1.4319	1.7183	2.0047	2.2911	2.5775	0.4210	30
0.2866	0.5732	0.8599	1.1465	1.4331	1.7197	2.0063	2.2930	2.5796	0.4214	31
0.2869	0.5737	0.8606	1.1474	1.4343	1.7212	2.0080	2.2949	2.5818	0.4217	32
0.2871	0.5742	0.8613	1.1484	1.4355	1.7226	2.0097	2.2968	2.5839	0.4221	33
0.2873	0.5747	0.8620	1.1494	1.4367	1.7240	2.0114	2.2987	2.5861	0.4225	34
0.2876	0.5752	0.8627	1.1503	1.4379	1.7255	2.0131	2.3006	2.5882	0.4229	35
0.2878	0.5756	0.8634	1.1512	1.4390	1.7269	2.0147	2.3025	2.5903	0.4233	36
0.2880	0.5761	0.8641	1.1522	1.4402	1.7283	2.0163	2.3044	2.5924	0.4237	37
0.2883	0.5766	0.8649	1.1532	1.4414	1.7297	2.0180	2.3063	2.5946	0.4241	38
0.2885	0.5770	0.8656	1.1541	1.4426	1.7311	2.0196	2.3082	2.5967	0.4245	39
0.2888	0.5775	0.8663	1.1550	1.4438	1.7326	2.0213	2.3101	2.5988	0.4249	40
0.2890	0.5780	0.8670	1.1560	1.4450	1.7340	2.0230	2.3120	2.6010	0.4252	41
0.2892	0.5785	0.8677	1.1569	1.4462	1.7354	2.0246	2.3139	2.6031	0.4256	42
0.2895	0.5789	0.8684	1.1579	1.4474	1.7368	2.0263	2.3158	2.6052	0.4260	43
0.2897	0.5794	0.8691	1.1588	1.4485	1.7383	2.0280	2.3177	2.6074	0.4264	44
0.2899	0.5799	0.8698	1.1598	1.4497	1.7397	2.0296	2.3196	2.6095	0.4268	45
0.2902	0.5804	0.8705	1.1607	1.4509	1.7411	2.0313	2.3215	2.6116	0.4272	46
0.2904	0.5808	0.8713	1.1617	1.4521	1.7425	2.0329	2.3233	2.6138	0.4276	47
0.2907	0.5813	0.8720	1.1626	1.4533	1.7439	2.0345	2.3252	2.6159	0.4280	48
0.2909	0.5818	0.8727	1.1636	1.4544	1.7453	2.0362	2.3271	2.6180	0.4283	49
0.2911	0.5823	0.8734	1.1645	1.4556	1.7468	2.0379	2.3290	2.6202	0.4287	50
0.2914	0.5827	0.8741	1.1654	1.4568	1.7482	2.0395	2.3309	2.6223	0.4291	51
0.2916	0.5832	0.8748	1.1664	1.4580	1.7496	2.0412	2.3328	2.6244	0.4295	52
0.2918	0.5837	0.8755	1.1673	1.4591	1.7510	2.0428	2.3346	2.6265	0.4299	53
0.2921	0.5841	0.8762	1.1683	1.4603	1.7524	2.0445	2.3365	2.6286	0.4303	54
0.2923	0.5846	0.8769	1.1692	1.4615	1.7538	2.0461	2.3384	2.6307	0.4307	55
0.2925	0.5851	0.8776	1.1702	1.4627	1.7552	2.0478	2.3403	2.6329	0.4311	56
0.2928	0.5856	0.8783	1.1711	1.4639	1.7567	2.0495	2.3422	2.6350	0.4315	57
0.2930	0.5860	0.8790	1.1720	1.4651	1.7581	2.0511	2.3441	2.6371	0.4318	58
0.2932	0.5865	0.8797	1.1730	1.4662	1.7595	2.0527	2.3460	2.6392	0.4322	59
0.2935	0.5870	0.8804	1.1739	1.4674	1.7609	2.0544	2.3478	2.6413	0.4326	60

′	1	2	3	4	5	6	7	8	9	a
00	0.9032	1.8065	2.7097	3.6130	4.5162	5.4195	6.3227	7.2259	8.1292	1.3315
01	0.9031	1.8061	2.7092	3.6123	4.5154	5.4184	6.3215	7.2246	8.1276	1.3314
02	0.9029	1.8058	2.7087	3.6116	4.5145	5.4174	6.3203	7.2232	8.1261	1.3312
03	0.9027	1.8055	2.7082	3.6109	4.5136	5.4164	6.3191	7.2218	8.1246	1.3311
04	0.9026	1.8051	2.7077	3.6102	4.5128	5.4153	6.3179	7.2205	8.1230	1.3310
05	0.9024	1.8048	2.7072	3.6095	4.5119	5.4143	6.3167	7.2191	8.1215	1.3309
06	0.9022	1.8044	2.7066	3.6089	4.5111	5.4133	6.3155	7.2177	8.1199	1.3308
07	0.9020	1.8041	2.7061	3.6082	4.5102	5.4123	6.3143	7.2163	8.1184	1.3306
08	0.9019	1.8037	2.7056	3.6075	4.5094	5.4112	6.3131	7.2150	8.1168	1.3305
09	0.9017	1.8034	2.7051	3.6068	4.5085	5.4102	6.3119	7.2136	8.1153	1.3304
10	0.9015	1.8031	2.7046	3.6061	4.5076	5.4092	6.3107	7.2122	8.1138	1.3302
11	0.9014	1.8027	2.7041	3.6054	4.5068	5.4081	6.3095	7.2108	8.1122	1.3301
12	0.9012	1.8024	2.7035	3.6047	4.5059	5.4071	6.3083	7.2095	8.1106	1.3300
13	0.9010	1.8020	2.7030	3.6040	4.5050	5.4061	6.3071	7.2081	8.1091	1.3298
14	0.9008	1.8017	2.7025	3.6033	4.5042	5.4050	6.3059	7.2067	8.1075	1.3297
15	0.9007	1.8013	2.7020	3.6027	4.5033	5.4040	6.3046	7.2053	8.1060	1.3296
16	0.9005	1.8010	2.7015	3.6020	4.5025	5.4029	6.3034	7.2039	8.1044	1.3294
17	0.9003	1.8006	2.7010	3.6013	4.5016	5.4019	6.3022	7.2025	8.1029	1.3293
18	0.9001	1.8003	2.7004	3.6006	4.5007	5.4009	6.3010	7.2012	8.1013	1.3292
19	0.9000	1.7999	2.6999	3.5999	4.4999	5.3998	6.2998	7.1998	8.0998	1.3291
20	0.8998	1.7996	2.6994	3.5992	4.4990	5.3988	6.2986	7.1984	8.0982	1.3289
21	0.8996	1.7993	2.6989	3.5985	4.4981	5.3978	6.2974	7.1970	8.0966	1.3288
22	0.8995	1.7989	2.6984	3.5978	4.4973	5.3967	6.2962	7.1956	8.0951	1.3287
23	0.8993	1.7986	2.6978	3.5971	4.4964	5.3957	6.2949	7.1942	8.0935	1.3285
24	0.8991	1.7982	2.6973	3.5964	4.4955	5.3946	6.2937	7.1928	8.0919	1.3284
25	0.8989	1.7979	2.6968	3.5957	4.4946	5.3936	6.2925	7.1914	8.0904	1.3283
26	0.8988	1.7975	2.6963	3.5950	4.4938	5.3925	6.2913	7.1900	8.0888	1.3281
27	0.8986	1.7972	2.6957	3.5943	4.4929	5.3915	6.2901	7.1886	8.0872	1.3280
28	0.8984	1.7968	2.6952	3.5936	4.4920	5.3904	6.2888	7.1873	8.0857	1.3279
29	0.8982	1.7965	2.6947	3.5929	4.4912	5.3894	6.2876	7.1859	8.0841	1.3278
30	0.8981	1.7961	2.6942	3.5922	4.4903	5.3884	6.2864	7.1845	8.0825	1.3276
31	0.8979	1.7958	2.6937	3.5915	4.4894	5.3873	6.2852	7.1831	8.0810	1.3275
32	0.8977	1.7954	2.6931	3.5908	4.4885	5.3862	6.2840	7.1817	8.0794	1.3274
33	0.8975	1.7951	2.6926	3.5901	4.4877	5.3852	6.2827	7.1803	8.0778	1.3272
34	0.8974	1.7947	2.6921	3.5894	4.4868	5.3841	6.2815	7.1789	8.0762	1.3271
35	0.8972	1.7944	2.6915	3.5887	4.4859	5.3831	6.2803	7.1775	8.0746	1.3269
36	0.8970	1.7940	2.6910	3.5880	4.4850	5.3820	6.2790	7.1760	8.0731	1.3268
37	0.8968	1.7937	2.6905	3.5873	4.4842	5.3810	6.2778	7.1746	8.0715	1.3267
38	0.8967	1.7933	2.6900	3.5866	4.4833	5.3799	6.2766	7.1732	8.0699	1.3265
39	0.8965	1.7930	2.6894	3.5859	4.4824	5.3789	6.2754	7.1718	8.0683	1.3264
40	0.8963	1.7926	2.6889	3.5852	4.4815	5.3778	6.2741	7.1704	8.0667	1.3263
41	0.8961	1.7923	2.6884	3.5845	4.4806	5.3768	6.2729	7.1690	8.0651	1.3262
42	0.8960	1.7919	2.6879	3.5838	4.4798	5.3757	6.2717	7.1676	8.0636	1.3260
43	0.8958	1.7915	2.6873	3.5831	4.4789	5.3746	6.2704	7.1662	8.0620	1.3259
44	0.8956	1.7912	2.6868	3.5824	4.4780	5.3736	6.2692	7.1648	8.0604	1.3258
45	0.8954	1.7908	2.6863	3.5817	4.4771	5.3725	6.2679	7.1634	8.0588	1.3257
46	0.8952	1.7905	2.6857	3.5810	4.4762	5.3715	6.2667	7.1619	8.0572	1.3255
47	0.8951	1.7901	2.6852	3.5803	4.4753	5.3704	6.2655	7.1605	8.0556	1.3254
48	0.8949	1.7898	2.6847	3.5796	4.4744	5.3693	6.2642	7.1591	8.0540	1.3253
49	0.8947	1.7894	2.6841	3.5789	4.4736	5.3683	6.2630	7.1577	8.0524	1.3251
50	0.8945	1.7891	2.6836	3.5781	4.4727	5.3672	6.2618	7.1563	8.0508	1.3250
51	0.8944	1.7887	2.6831	3.5774	4.4718	5.3661	6.2605	7.1549	8.0492	1.3249
52	0.8942	1.7884	2.6825	3.5767	4.4709	5.3651	6.2593	7.1534	8.0476	1.3247
53	0.8940	1.7880	2.6820	3.5760	4.4700	5.3640	6.2580	7.1520	8.0460	1.3246
54	0.8938	1.7876	2.6815	3.5753	4.4691	5.3629	6.2568	7.1506	8.0444	1.3245
55	0.8936	1.7873	2.6809	3.5746	4.4682	5.3619	6.2555	7.1492	8.0428	1.3243
56	0.8935	1.7869	2.6804	3.5739	4.4673	5.3608	6.2543	7.1477	8.0412	1.3242
57	0.8933	1.7866	2.6799	3.5732	4.4664	5.3597	6.2530	7.1463	8.0396	1.3241
58	0.8931	1.7862	2.6793	3.5724	4.4656	5.3587	6.2518	7.1449	8.0380	1.3239
59	0.8929	1.7859	2.6788	3.5717	4.4647	5.3576	6.2505	7.1435	8.0364	1.3238
60	0.8928	1.7855	2.6783	3.5710	4.4638	5.3565	6.2493	7.1420	8.0348	1.3237

1	2	3	4	5	6	7	8	9	b	′
0.2935	0.5870	0.8804	1.1739	1.4674	1.7609	2.0544	2.3478	2.6413	0.4326	00
0.2937	0.5874	0.8811	1.1749	1.4686	1.7623	2.0560	2.3497	2.6434	0.4330	01
0.2940	0.5879	0.8819	1.1758	1.4698	1.7637	2.0577	2.3516	2.6456	0.4334	02
0.2942	0.5884	0.8826	1.1768	1.4709	1.7651	2.0593	2.3535	2.6477	0.4338	03
0.2944	0.5888	0.8833	1.1777	1.4721	1.7665	2.0609	2.3554	2.6498	0.4342	04
0.2947	0.5893	0.8840	1.1786	1.4733	1.7679	2.0626	2.3573	2.6519	0.4346	05
0.2949	0.5898	0.8847	1.1796	1.4744	1.7693	2.0642	2.3591	2.6540	0.4349	06
0.2951	0.5902	0.8854	1.1805	1.4756	1.7707	2.0659	2.3610	2.6561	0.4353	07
0.2954	0.5907	0.8861	1.1814	1.4768	1.7721	2.0675	2.3629	2.6582	0.4357	08
0.2956	0.5912	0.8868	1.1824	1.4780	1.7735	2.0691	2.3647	2.6603	0.4361	09
0.2958	0.5917	0.8875	1.1833	1.4791	1.7750	2.0708	2.3666	2.6625	0.4365	10
0.2961	0.5921	0.8882	1.1842	1.4803	1.7764	2.0724	2.3685	2.6645	0.4369	11
0.2963	0.5926	0.8889	1.1852	1.4815	1.7778	2.0740	2.3703	2.6666	0.4373	12
0.2965	0.5931	0.8896	1.1861	1.4826	1.7792	2.0757	2.3722	2.6688	0.4377	13
0.2968	0.5935	0.8903	1.1870	1.4838	1.7806	2.0773	2.3741	2.6709	0.4380	14
0.2970	0.5940	0.8910	1.1880	1.4850	1.7820	2.0790	2.3760	2.6730	0.4384	15
0.2972	0.5945	0.8917	1.1889	1.4861	1.7834	2.0806	2.3778	2.6751	0.4388	16
0.2975	0.5949	0.8924	1.1898	1.4873	1.7848	2.0822	2.3797	2.6771	0.4392	17
0.2977	0.5954	0.8931	1.1908	1.4885	1.7862	2.0838	2.3815	2.6792	0.4396	18
0.2979	0.5959	0.8938	1.1917	1.4896	1.7876	2.0855	2.3834	2.6813	0.4400	19
0.2982	0.5963	0.8945	1.1926	1.4908	1.7890	2.0871	2.3853	2.6834	0.4404	20
0.2984	0.5968	0.8952	1.1936	1.4920	1.7904	2.0887	2.3871	2.6855	0.4407	21
0.2986	0.5973	0.8959	1.1945	1.4931	1.7918	2.0904	2.3890	2.6876	0.4411	22
0.2989	0.5977	0.8966	1.1954	1.4943	1.7932	2.0920	2.3909	2.6897	0.4415	23
0.2991	0.5982	0.8973	1.1964	1.4955	1.7946	2.0936	2.3927	2.6918	0.4419	24
0.2993	0.5987	0.8980	1.1973	1.4966	1.7960	2.0953	2.3946	2.6939	0.4423	25
0.2996	0.5991	0.8987	1.1982	1.4978	1.7974	2.0969	2.3965	2.6960	0.4427	26
0.2998	0.5996	0.8994	1.1992	1.4989	1.7987	2.0985	2.3983	2.6981	0.4431	27
0.3000	0.6000	0.9001	1.2001	1.5001	1.8001	2.1001	2.4002	2.7002	0.4435	28
0.3003	0.6005	0.9008	1.2010	1.5013	1.8015	2.1018	2.4020	2.7023	0.4438	29
0.3005	0.6010	0.9015	1.2020	1.5024	1.8029	2.1034	2.4039	2.7044	0.4442	30
0.3007	0.6014	0.9022	1.2029	1.5036	1.8043	2.1050	2.4058	2.7065	0.4446	31
0.3009	0.6019	0.9028	1.2038	1.5047	1.8057	2.1066	2.4076	2.7085	0.4450	32
0.3012	0.6024	0.9035	1.2047	1.5059	1.8071	2.1083	2.4094	2.7106	0.4454	33
0.3014	0.6028	0.9042	1.2056	1.5071	1.8085	2.1099	2.4113	2.7127	0.4458	34
0.3016	0.6033	0.9049	1.2066	1.5082	1.8099	2.1115	2.4132	2.7148	0.4462	35
0.3019	0.6038	0.9056	1.2075	1.5094	1.8113	2.1132	2.4150	2.7169	0.4466	36
0.3021	0.6042	0.9063	1.2084	1.5105	1.8127	2.1148	2.4169	2.7190	0.4469	37
0.3023	0.6047	0.9070	1.2094	1.5117	1.8140	2.1164	2.4187	2.7211	0.4473	38
0.3026	0.6051	0.9077	1.2103	1.5128	1.8154	2.1180	2.4206	2.7231	0.4477	39
0.3028	0.6056	0.9084	1.2112	1.5140	1.8168	2.1196	2.4214	2.7252	0.4481	40
0.3030	0.6061	0.9091	1.2121	1.5152	1.8182	2.1212	2.4242	2.7273	0.4485	41
0.3033	0.6065	0.9098	1.2130	1.5163	1.8196	2.1228	2.4261	2.7294	0.4489	42
0.3035	0.6070	0.9105	1.2140	1.5175	1.8210	2.1245	2.4280	2.7315	0.4493	43
0.3037	0.6074	0.9112	1.2149	1.5186	1.8223	2.1261	2.4298	2.7335	0.4496	44
0.3040	0.6079	0.9119	1.2158	1.5198	1.8237	2.1277	2.4316	2.7356	0.4500	45
0.3042	0.6084	0.9125	1.2167	1.5209	1.8251	2.1293	2.4334	2.7376	0.4504	46
0.3044	0.6088	0.9132	1.2176	1.5221	1.8265	2.1309	2.4353	2.7397	0.4508	47
0.3046	0.6093	0.9139	1.2186	1.5232	1.8279	2.1325	2.4372	2.7418	0.4512	48
0.3049	0.6098	0.9146	1.2195	1.5244	1.8293	2.1341	2.4390	2.7439	0.4516	49
0.3051	0.6102	0.9153	1.2204	1.5255	1.8306	2.1357	2.4408	2.7459	0.4520	50
0.3053	0.6107	0.9160	1.2214	1.5267	1.8320	2.1374	2.4427	2.7480	0.4524	51
0.3056	0.6111	0.9167	1.2223	1.5278	1.8334	2.1390	2.4446	2.7501	0.4527	52
0.3058	0.6116	0.9174	1.2232	1.5290	1.8348	2.1406	2.4464	2.7521	0.4531	53
0.3060	0.6120	0.9181	1.2241	1.5301	1.8361	2.1422	2.4482	2.7542	0.4535	54
0.3063	0.6125	0.9188	1.2250	1.5313	1.8375	2.1438	2.4500	2.7563	0.4539	55
0.3065	0.6130	0.9194	1.2259	1.5324	1.8389	2.1454	2.4518	2.7583	0.4543	56
0.3067	0.6134	0.9201	1.2268	1.5336	1.8403	2.1470	2.4537	2.7604	0.4547	57
0.3069	0.6139	0.9208	1.2278	1.5347	1.8416	2.1486	2.4555	2.7625	0.4551	58
0.3072	0.6143	0.9215	1.2287	1.5358	1.8430	2.1502	2.4574	2.7645	0.4555	59
0.3074	0.6148	0.9222	1.2296	1.5370	1.8444	2.1518	2.4592	2.7666	0.4558	60

′	1	2	3	4	5	6	7	8	9	a
00	0.8928	1.7855	2.6783	3.5710	4.4638	5.3565	6.2493	7.1420	8.0348	1.3237
01	0.8926	1.7851	2.6777	3.5703	4.4629	5.3554	6.2480	7.1406	8.0332	1.3236
02	0.8924	1.7848	2.6772	3.5696	4.4620	5.3544	6.2468	7.1392	8.0316	1.3234
03	0.8922	1.7844	2.6766	3.5689	4.4611	5.3533	6.2455	7.1377	8.0299	1.3233
04	0.8920	1.7841	2.6761	3.5681	4.4602	5.3522	6.2443	7.1363	8.0283	1.3232
05	0.8919	1.7837	2.6756	3.5674	4.4593	5.3511	6.2430	7.1349	8.0267	1.3230
06	0.8917	1.7834	2.6750	3.5667	4.4584	5.3501	6.2417	7.1334	8.0251	1.3229
07	0.8915	1.7830	2.6745	3.5660	4.4575	5.3490	6.2405	7.1320	8.0235	1.3228
08	0.8913	1.7826	2.6740	3.5653	4.4566	5.3479	6.2392	7.1305	8.0219	1.3226
09	0.8911	1.7823	2.6734	3.5646	4.4557	5.3468	6.2380	7.1291	8.0203	1.3225
10	0.8910	1.7819	2.6729	3.5638	4.4548	5.3458	6.2367	7.1277	8.0186	1.3224
11	0.8908	1.7816	2.6723	3.5631	4.4539	5.3447	6.2354	7.1262	8.0170	1.3222
12	0.8906	1.7812	2.6718	3.5624	4.4530	5.3436	6.2342	7.1248	8.0154	1.3221
13	0.8904	1.7808	2.6712	3.5617	4.4521	5.3425	6.2329	7.1233	8.0137	1.3220
14	0.8902	1.7805	2.6707	3.5609	4.4512	5.3414	6.2316	7.1219	8.0121	1.3218
15	0.8901	1.7801	2.6702	3.5602	4.4503	5.3403	6.2304	7.1204	8.0105	1.3217
16	0.8899	1.7797	2.6696	3.5595	4.4494	5.3392	6.2291	7.1190	8.0089	1.3215
17	0.8897	1.7794	2.6691	3.5588	4.4485	5.3382	6.2279	7.1175	8.0072	1.3214
18	0.8895	1.7790	2.6685	3.5580	4.4476	5.3371	6.2266	7.1161	8.0056	1.3213
19	0.8893	1.7787	2.6680	3.5573	4.4467	5.3360	6.2253	7.1146	8.0040	1.3211
20	0.8892	1.7783	2.6675	3.5566	4.4458	5.3349	6.2241	7.1132	8.0024	1.3210
21	0.8890	1.7779	2.6669	3.5559	4.4448	5.3338	6.2228	7.1117	8.0007	1.3209
22	0.8888	1.7776	2.6664	3.5551	4.4439	5.3327	6.2215	7.1103	7.9991	1.3207
23	0.8886	1.7772	2.6658	3.5544	4.4430	5.3316	6.2202	7.1088	7.9974	1.3206
24	0.8884	1.7768	2.6653	3.5537	4.4421	5.3305	6.2190	7.1074	7.9958	1.3205
25	0.8882	1.7765	2.6647	3.5530	4.4412	5.3294	6.2177	7.1059	7.9942	1.3203
26	0.8881	1.7761	2.6642	3.5522	4.4403	5.3283	6.2164	7.1045	7.9925	1.3202
27	0.8879	1.7758	2.6636	3.5515	4.4394	5.3273	6.2151	7.1030	7.9909	1.3201
28	0.8877	1.7754	2.6631	3.5508	4.4385	5.3262	6.2139	7.1015	7.9892	1.3199
29	0.8875	1.7750	2.6625	3.5500	4.4376	5.3251	6.2126	7.1001	7.9876	1.3198
30	0.8873	1.7747	2.6620	3.5493	4.4366	5.3240	6.2113	7.0986	7.9860	1.3197
31	0.8871	1.7743	2.6614	3.5486	4.4357	5.3229	6.2100	7.0972	7.9843	1.3195
32	0.8870	1.7739	2.6609	3.5478	4.4348	5.3218	6.2087	7.0957	7.9827	1.3194
33	0.8868	1.7736	2.6603	3.5471	4.4339	5.3207	6.2075	7.0942	7.9810	1.3193
34	0.8866	1.7732	2.6598	3.5464	4.4330	5.3196	6.2062	7.0928	7.9794	1.3191
35	0.8864	1.7728	2.6592	3.5456	4.4321	5.3185	6.2049	7.0913	7.9777	1.3190
36	0.8862	1.7725	2.6587	3.5449	4.4311	5.3174	6.2036	7.0898	7.9761	1.3189
37	0.8860	1.7721	2.6581	3.5442	4.4302	5.3163	6.2023	7.0884	7.9744	1.3187
38	0.8859	1.7717	2.6576	3.5434	4.4293	5.3152	6.2010	7.0869	7.9727	1.3186
39	0.8857	1.7714	2.6570	3.5427	4.4284	5.3141	6.1997	7.0854	7.9711	1.3185
40	0.8855	1.7710	2.6565	3.5420	4.4275	5.3130	6.1985	7.0840	7.9694	1.3183
41	0.8853	1.7706	2.6559	3.5412	4.4265	5.3119	6.1972	7.0825	7.9678	1.3182
42	0.8851	1.7702	2.6554	3.5405	4.4256	5.3107	6.1959	7.0810	7.9661	1.3181
43	0.8849	1.7699	2.6548	3.5398	4.4247	5.3096	6.1946	7.0795	7.9645	1.3179
44	0.8848	1.7695	2.6543	3.5390	4.4238	5.3085	6.1933	7.0780	7.9628	1.3178
45	0.8846	1.7691	2.6537	3.5383	4.4229	5.3074	6.1920	7.0766	7.9611	1.3177
46	0.8844	1.7688	2.6532	3.5375	4.4219	5.3063	6.1907	7.0751	7.9595	1.3175
47	0.8842	1.7684	2.6526	3.5368	4.4210	5.3052	6.1894	7.0736	7.9578	1.3174
48	0.8840	1.7680	2.6520	3.5361	4.4201	5.3041	6.1881	7.0721	7.9561	1.3173
49	0.8838	1.7677	2.6515	3.5353	4.4192	5.3030	6.1868	7.0706	7.9545	1.3171
50	0.8836	1.7673	2.6509	3.5346	4.4182	5.3019	6.1855	7.0692	7.9528	1.3170
51	0.8835	1.7669	2.6504	3.5338	4.4173	5.3008	6.1842	7.0677	7.9511	1.3169
52	0.8833	1.7665	2.6498	3.5331	4.4164	5.2996	6.1829	7.0662	7.9495	1.3167
53	0.8831	1.7662	2.6493	3.5324	4.4154	5.2985	6.1816	7.0647	7.9478	1.3166
54	0.8829	1.7658	2.6487	3.5316	4.4145	5.2974	6.1803	7.0632	7.9461	1.3165
55	0.8827	1.7654	2.6481	3.5309	4.4136	5.2963	6.1790	7.0617	7.9444	1.3163
56	0.8825	1.7651	2.6476	3.5301	4.4127	5.2952	6.1777	7.0602	7.9428	1.3162
57	0.8823	1.7647	2.6470	3.5294	4.4117	5.2941	6.1764	7.0588	7.9411	1.3160
58	0.8822	1.7643	2.6465	3.5286	4.4108	5.2929	6.1751	7.0573	7.9394	1.3159
59	0.8820	1.7639	2.6459	3.5279	4.4099	5.2918	6.1738	7.0558	7.9377	1.3158
60	0.8818	1.7636	2.6454	3.5271	4.4089	5.2907	6.1725	7.0543	7.9361	1.3156

1	2	3	4	5	6	7	8	9	b	′
0.3074	0.6148	0.9222	1.2296	1.5370	1.8444	2.1518	2.4592	2.7666	0.4558	00
0.3076	0.6153	0.9229	1.2305	1.5381	1.8458	2.1534	2.4610	2.7687	0.4562	01
0.3079	0.6157	0.9236	1.2314	1.5393	1.8472	2.1550	2.4629	2.7707	0.4566	02
0.3081	0.6162	0.9243	1.2324	1.5404	1.8485	2.1566	2.4647	2.7728	0.4570	03
0.3083	0.6166	0.9249	1.2333	1.5416	1.8499	2.1582	2.4665	2.7748	0.4573	04
0.3085	0.6171	0.9256	1.2342	1.5427	1.8512	2.1598	2.4683	2.7769	0.4577	05
0.3088	0.6175	0.9263	1.2351	1.5439	1.8526	2.1614	2.4702	2.7789	0.4581	06
0.3090	0.6180	0.9270	1.2360	1.5450	1.8540	2.1630	2.4720	2.7810	0.4585	07
0.3092	0.6185	0.9277	1.2369	1.5461	1.8554	2.1646	2.4738	2.7831	0.4589	08
0.3095	0.6189	0.9284	1.2378	1.5473	1.8568	2.1662	2.4757	2.7851	0.4593	09
0.3097	0.6194	0.9290	1.2387	1.5484	1.8581	2.1678	2.4775	2.7871	0.4596	10
0.3099	0.6198	0.9297	1.2396	1.5495	1.8595	2.1694	2.4793	2.7892	0.4600	11
0.3101	0.6203	0.9304	1.2406	1.5507	1.8608	2.1710	2.4811	2.7913	0.4604	12
0.3104	0.6207	0.9311	1.2415	1.5518	1.8622	2.1726	2.4830	2.7933	0.4608	13
0.3106	0.6212	0.9318	1.2424	1.5530	1.8636	2.1742	2.4848	2.7953	0.4612	14
0.3108	0.6216	0.9325	1.2433	1.5541	1.8649	2.1758	2.4866	2.7974	0.4616	15
0.3110	0.6221	0.9331	1.2442	1.5552	1.8663	2.1773	2.4884	2.7994	0.4619	16
0.3113	0.6226	0.9338	1.2451	1.5564	1.8677	2.1789	2.4902	2.8015	0.4623	17
0.3115	0.6230	0.9345	1.2460	1.5575	1.8690	2.1805	2.4920	2.8035	0.4627	18
0.3117	0.6235	0.9352	1.2469	1.5587	1.8704	2.1821	2.4938	2.8056	0.4631	19
0.3120	0.6239	0.9359	1.2478	1.5598	1.8718	2.1837	2.4957	2.8076	0.4635	20
0.3122	0.6244	0.9365	1.2487	1.5609	1.8731	2.1853	2.4975	2.8096	0.4639	21
0.3124	0.6248	0.9372	1.2496	1.5620	1.8745	2.1869	2.4993	2.8117	0.4642	22
0.3126	0.6253	0.9379	1.2505	1.5632	1.8758	2.1885	2.5011	2.8137	0.4646	23
0.3129	0.6257	0.9386	1.2514	1.5643	1.8772	2.1900	2.5029	2.8157	0.4650	24
0.3131	0.6262	0.9393	1.2524	1.5654	1.8785	2.1916	2.5047	2.8178	0.4654	25
0.3133	0.6266	0.9399	1.2533	1.5666	1.8799	2.1932	2.5065	2.8198	0.4658	26
0.3135	0.6271	0.9406	1.2542	1.5677	1.8812	2.1948	2.5083	2.8219	0.4662	27
0.3138	0.6275	0.9413	1.2551	1.5688	1.8826	2.1964	2.5101	2.8239	0.4665	28
0.3140	0.6280	0.9420	1.2560	1.5700	1.8839	2.1979	2.5119	2.8259	0.4669	29
0.3142	0.6284	0.9427	1.2569	1.5711	1.8853	2.1995	2.5138	2.8280	0.4673	30
0.3144	0.6289	0.9433	1.2578	1.5722	1.8867	2.2011	2.5156	2.8300	0.4677	31
0.3147	0.6293	0.9440	1.2587	1.5734	1.8880	2.2027	2.5174	2.8320	0.4681	32
0.3149	0.6298	0.9447	1.2596	1.5745	1.8894	2.2043	2.5192	2.8341	0.4685	33
0.3151	0.6302	0.9454	1.2605	1.5756	1.8907	2.2058	2.5210	2.8361	0.4689	34
0.3153	0.6307	0.9460	1.2614	1.5767	1.8921	2.2074	2.5228	2.8381	0.4692	35
0.3156	0.6311	0.9467	1.2623	1.5779	1.8934	2.2090	2.5246	2.8401	0.4696	36
0.3158	0.6316	0.9474	1.2632	1.5790	1.8948	2.2106	2.5264	2.8422	0.4700	37
0.3160	0.6320	0.9481	1.2641	1.5801	1.8961	2.2122	2.5282	2.8442	0.4704	38
0.3162	0.6325	0.9487	1.2650	1.5812	1.8975	2.2137	2.5300	2.8462	0.4708	39
0.3165	0.6329	0.9494	1.2659	1.5824	1.8988	2.2153	2.5318	2.8482	0.4712	40
0.3167	0.6334	0.9501	1.2668	1.5835	1.9002	2.2169	2.5336	2.8503	0.4715	41
0.3169	0.6338	0.9508	1.2677	1.5846	1.9015	2.2184	2.5354	2.8523	0.4719	42
0.3171	0.6343	0.9514	1.2686	1.5857	1.9029	2.2200	2.5372	2.8543	0.4723	43
0.3174	0.6347	0.9521	1.2695	1.5868	1.9042	2.2216	2.5390	2.8563	0.4727	44
0.3176	0.6352	0.9528	1.2704	1.5880	1.9055	2.2231	2.5407	2.8583	0.4731	45
0.3178	0.6356	0.9535	1.2713	1.5891	1.9069	2.2247	2.5425	2.8604	0.4735	46
0.3180	0.6361	0.9541	1.2722	1.5902	1.9082	2.2263	2.5443	2.8624	0.4739	47
0.3183	0.6365	0.9548	1.2731	1.5913	1.9096	2.2279	2.5461	2.8644	0.4742	48
0.3185	0.6370	0.9555	1.2740	1.5924	1.9109	2.2294	2.5479	2.8664	0.4746	49
0.3187	0.6374	0.9561	1.2748	1.5936	1.9123	2.2310	2.5497	2.8684	0.4750	50
0.3189	0.6379	0.9568	1.2757	1.5947	1.9136	2.2326	2.5515	2.8704	0.4754	51
0.3192	0.6383	0.9575	1.2766	1.5958	1.9150	2.2341	2.5533	2.8724	0.4758	52
0.3194	0.6388	0.9581	1.2775	1.5969	1.9163	2.2357	2.5551	2.8744	0.4761	53
0.3196	0.6392	0.9588	1.2784	1.5980	1.9177	2.2373	2.5569	2.8765	0.4765	54
0.3198	0.6397	0.9595	1.2793	1.5991	1.9190	2.2388	2.5586	2.8785	0.4769	55
0.3201	0.6401	0.9602	1.2802	1.6003	1.9203	2.2404	2.5604	2.8805	0.4773	56
0.3203	0.6405	0.9608	1.2811	1.6014	1.9216	2.2419	2.5622	2.8825	0.4777	57
0.3205	0.6410	0.9615	1.2820	1.6025	1·9230	2.2435	2.5640	2.8845	0.4781	58
0.3207	0.6414	0.9622	1.2829	1.6036	1.9243	2.2450	2.5658	2.8865	0.4784	59
0.3209	0.6419	0.9628	1.2838	1.6047	1.9257	2.2465	2.5675	2.8885	0.4788	60