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ON

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ANSWERS TO QUESTIONS

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A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

CONTAINED IN THE

QUESTION PAPERS INCLUDED IN THE PREVIOUS VOLUMES

The various Keys composing this volume have been given the same section numbers as the Question Papers to which they refer; and the answers and solutions have been numbered to correspond with the questions contained in the Question Papers. In many instances the answer to a question would involve a repetition of statements given in the Instruction Papers; hence, in all such cases, the student has been referred to an article in the Instruction Paper, the reading of which will enable him to answer the question himself.

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ARITHMETIC.

(PART 1.)

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Arts. **5** and **6.**

(4) See Arts. **10** and **11.**

(5) 980 = Nine hundred eighty.

605 = Six hundred five.

28,284 = Twenty-eight thousand, two hundred eighty-four.

9,006,042 = Nine million, six thousand and forty-two.

850,317,002 = Eight hundred fifty million, three hundred seventeen thousand and two.

700,004 = Seven hundred thousand and four.

(6) Seven thousand six hundred = 7,600.

Eighty-one thousand four hundred two = 81,402.

Five million, four thousand and seven = 5,004,007.

One hundred and eight million, ten thousand and one = 108,010,001.

Eighteen million and six = 18,000,006.

Thirty thousand and ten = 30,010.

(7) In adding whole numbers, place the numbers to be added directly under each other so that the extreme right-hand figures will stand in the same column, regardless of the position of those at the left. Add the first column of figures at the extreme right, which equals 19 units, or 1 ten and 9 units. We place 9 units under the units column, and reserve 1 ten for the column

```
  3290
   504
865403
  2074
    81
     7
------
871359 Ans.
```

of tens. $1 + 8 + 7 + 9 = 25$ tens, or 2 hundreds and 5 tens. Place 5 tens under the tens column, and reserve 2 hundreds for the hundreds column. $2 + 4 + 5 + 2 = 13$ hundreds, or 1 thousand and 3 hundreds. Place 3 hundreds under the hundreds column, and reserve the 1 thousand for the thousands column. $1 + 2 + 5 + 3 = 11$ thousands, or 1 ten-thousand and 1 thousand. Place the 1 thousand in the column of thousands, and reserve the 1 ten-thousand for the column of ten-thousands. $1 + 6 = 7$ ten-thousands. Place this seven ten-thousands in the ten-thousands column. There is but one figure 8 in the hundreds of thousands place in the numbers to be added, so it is placed in the hundreds of thousands column of the sum.

A simpler (though less scientific) explanation of the same problem is the following: $7 + 1 + 4 + 3 + 4 + 0 = 19$; write the nine and reserve the 1. $1 + 8 + 7 + 0 + 0 + 9 = 25$; write the 5 and reserve the 2. $2 + 0 + 4 + 5 + 2 = 13$; write the 3 and reserve the 1. $1 + 2 + 5 + 3 = 11$; write the 1 and reserve 1. $1 + 6 = 7$; write the 7. Bring down the 8 to its place in the sum.

(8)

$$
\begin{array}{r}
709 \\
8304725 \\
391 \\
100302 \\
300 \\
909 \\
\hline
8407336
\end{array}
\quad \text{Ans.}
$$

(9) (*a*) In subtracting whole numbers, place the subtrahend or smaller number under the minuend or larger number, so that the right-hand figures stand directly under each other. Begin *at the right* to subtract. We can not subtract 8 units from 2 units, so we take 1 ten from the 6 tens and add it to the 2 units. As 1 *ten* = 10 *units*, we have 10 units + 2 units = 12 units. Then, 8 units from 12 units leaves 4 units. We took 1 ten from 6 tens, so only 5 tens remain. 3 tens from 5 tens leaves 2 tens. In the hundreds column we have 3 hundreds from 9 hundreds leaves 6 hundreds. We can not subtract 3 thousands from 0 thousands, so we take 1 ten-thousand from 5 ten-thousands and add it to the 0 thousands. 1 *ten-thousand* = 10 *thousands*, and 10 thousands + 0 thousands = 10 thousands. Subtracting, we have 3 thousands from 10 thousands leaves 7 thousands. We took 1 ten-thousand from 5 ten-thousands and have 4 ten-thousands remaining. Since there are no ten-thousands in the subtrahend, the 4 in the ten-thousands column in the minuend is brought down into the same column in the remainder, because 0 from 4 leaves 4.

```
 50962
  3338
 -----
 47624  Ans.
```

```
(b) 15339
    10001
    -----
     5338  Ans.
```

```
(10) (a) 70968          (b) 100000
         32975               98735
         -----              ------
         37993  Ans.          1265  Ans.
```

(11) We have given the minuend or greater number (1,004) and the difference or remainder (49). Placing these in the usual form of subtraction we have

```
1004
----
  49
```

in which the dash (——) represents the number sought. This number is evidently *less* than 1,004 by the difference 49, hence, 1,004 − 49 = 955, the smaller number. For the sum of the two numbers we then have

```
1004 larger
 955 smaller
----
1959 sum.  Ans.
```

Or, this problem may be solved as follows: If the greater of two numbers is 1,004, and the difference between them is 49, then it is evident that the smaller number must be equal to the difference between the greater number (1,004) and the difference (49); or, 1,004 − 49 = 955, the smaller number. Since the greater number equals 1,004 and the smaller number equals 955, their sum equals 1,004 + 955 = 1,959 sum. Ans.

(12) The numbers connected by the plus (+) sign must first be added. Performing these operations we have

```
 5962          3874
 8471          2039
 9023          ----
-----          5913 sum.
23456 sum.
```

Subtracting the smaller number (5,913) from the greater (23,456) we have

```
 23456
  5913
 -----
 17543 difference.   Ans.
```

(13) $44675 = amount willed to his son.
  26380 = amount willed to his daughter.
$71055 = amount willed to his two children.
$125000 = amount willed to his wife and two children.
  71055 = amount willed to his two children.
$53945 = amount willed to his wife. Ans.

(14) In the multiplication of whole numbers, place the multiplier under the multiplicand, and multiply each term of the multiplicand by each term of the multiplier, writing the right-hand figure of each product obtained under the term of the multiplier which produces it.

```
(a)
526387
     7
-------
3684709  Ans.
```

7 × 7 units = 49 units, or 4 tens and 9 units. We write the 9 units and reserve the 4 tens. 7 times 8 tens = 56 tens; 56 tens + 4 tens reserved = 60 tens or 6 hundreds and 0 tens. Write the 0 tens and reserve the 6 hundreds. 7 × 3 hundreds = 21 hundreds; 21 + 6 hundreds reserved = 27 hundreds, or 2 thousands and 7 hundreds. Write the 7 hundreds and reserve

the 2 thousands. 7 × 6 thousands = 42 thousands; 42 + 2 thousands reserved = 44 thousands or 4 ten-thousands and 4 thousands. Write the 4 thousands and reserve the 4 ten-thousands. 7 × 2 ten-thousands = 14 ten-thousands; 14 + 4 ten-thousands reserved = 18 ten-thousands, or 1 hundred-thousand and 8 ten-thousands. Write the 8 ten-thousands and reserve the 1 hundred-thousand. 7 × 5 hundred-thousands = 35 hundred-thousands; 35 + 1 hundred-thousand reserved = 36 hundred-thousands. Since there are no more figures in the multiplicand to be multiplied, we write the 36 hundred-thousands in the product. This completes the multiplication.

A simpler (though less scientific) explanation of the same problem is the following:

7 times 7 = 49; write the 9 and reserve the 4. 7 times 8 = 56; 56 + 4 reserved = 60; write the 0 and reserve the 6. 7 times 3 = 21; 21 + 6 reserved = 27; write the 7 and reserve the 2. 7 × 6 = 42; 42 + 2 reserved = 44; write the 4 and reserve 4. 7 × 2 = 14; 14 + 4 reserved = 18; write the 8 and reserve the 1. 7 × 5 = 35; 35 + 1 reserved = 36; write the 36.

In this case the multiplier is 17 *units*, or 1 *ten* and 7 *units*, so that the product is obtained by adding two partial products, namely, 7 × 700,298 and 10 × 700,298. The actual operation is performed as follows:

(*b*)	700298	
	17	
	4902086	
	700298	
	11905066	Ans.

7 times 8 = 56; write the 6 and reserve the 5. 7 times 9 = 63; 63 + 5 reserved = 68; write the 8 and reserve the 6. 7 times 2 = 14; 14 + 6 reserved = 20; write the 0 and reserve the 2. 7 times 0 = 0; 0 + 2 reserved = 2; write the 2. 7 times 0 = 0; 0 + 0 reserved = 0; write the 0. 7 times 7 = 49; 49 + 0 reserved = 49; write the 49.

To multiply by the 1 ten we say 1 times 700298 = 700298, and write 700298 under the first partial product, as shown, with the right-hand figure 8 under the multiplier 1. Add the two partial products; their sum equals the entire product.

(*c*)

```
    217
    103
    651
   2170
  22351
     67
 156457
134106
1497517  Ans.
```

Multiply any two of the numbers together and multiply their product by the third number.

(**15**) If your watch ticks every second, then to find how many times it ticks in one week it is necessary to find the number of seconds in 1 week.

```
    60 seconds = 1 minute.
    60 minutes = 1 hour.
  3600 seconds = 1 hour.
    24 hours = 1 day.
 14400
 7200
 86400 seconds = 1 day.
     7 days = 1 week.
604800 seconds in 1 week or the number of times that
  Ans.       your watch ticks in 1 week.
```

(**16**) If a monthly publication contains 24 pages, a yearly volume will contain 12×24 or 288 pages, since there are 12 months in one year; and eight yearly volumes will contain 8×288, or 2,304 pages.

```
  24
  12
 288
   8
2304  Ans.
```

(**17**) If an engine and boiler are worth $3,246, and the building is worth 3 times as much, plus $1,200, then the building is worth

```
       $3246
           3
        9738
plus    1200
      $10938 = value of building.
```

If the tools are worth twice as much as the building, plus $1,875, then the tools are worth

$$\begin{array}{rr} & \$10938 \\ & 2 \\ \hline & 21876 \\ \text{plus} & 1875 \\ \hline & \$23751 \end{array} = \text{value of tools.}$$

$$\begin{array}{lr} \text{Value of building} = & \$10938 \\ \text{Value of tools} \quad = & 23751 \\ \hline & \$34689 \end{array} = \text{value of the building and tools. (a) Ans.}$$

$$\begin{array}{lr} \text{Value of engine and boiler} = & \$\ \ 3246 \\ \text{Value of building and tools} = & 34689 \\ \hline & \$37935 \end{array} = \text{value of the whole plant. (b) Ans.}$$

(18) (*a*) $(72 \times 48 \times 28 \times 5) \div (96 \times 15 \times 7 \times 6)$.
Placing the numerator over the denominator the problem becomes

$$\frac{72 \times 48 \times 28 \times 5}{96 \times 15 \times 7 \times 6} = ?$$

The 5 in the *dividend* and 15 in the *divisor* are both *divisible* by 5, since 5 divided by 5 equals 1, and 15 divided by 5 equals 3. *Cross off* the 5 and write the 1 *over* it; also *cross off* the 15 and write the 3 *under* it. Thus,

$$\frac{72 \times 48 \times 28 \times \overset{1}{\not 5}}{96 \times \underset{3}{\not 1\not 5} \times 7 \times 6} =$$

The 5 and 15 are *not* to be considered any longer, and, in fact, may be erased entirely and the 1 and 3 placed in their stead, and treated as if the 5 and 15 *never* existed. Thus,

$$\frac{72 \times 48 \times 28 \times 1}{96 \times 3 \times 7 \times 6} =$$

72 in the *dividend* and 96 in the *divisor* are *divisible* by 12, since 72 divided by 12 equals 6, and 96 divided by 12 equals 8. *Cross off* the 72 and write the 6 *over* it; also, *cross off* the 96 and write the 8 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times 28 \times 1}{\underset{8}{\cancel{96}} \times 3 \times 7 \times 6} =$$

The 72 and 96 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 6 and 8 placed in their stead, and treated as if the 72 and 96 *never* existed. Thus,

$$\frac{6 \times 48 \times 28 \times 1}{8 \times 3 \times 7 \times 6} =$$

Again, 28 in the *dividend* and 7 in the *divisor* are *divisible* by 7, since 28 divided by 7 equals 4, and 7 divided by 7 equals 1. *Cross off* the 28 and write the 4 *over* it; also, cross off the 7 and write the 1 *under* it. Thus,

$$\frac{6 \times 48 \times \overset{4}{\cancel{28}} \times 1}{8 \times 3 \times \underset{1}{\cancel{7}} \times 6} =$$

The 28 and 7 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 4 and 1 placed in their stead, and treated as if the 28 and 7 *never* existed. Thus,

$$\frac{6 \times 48 \times 4 \times 1}{8 \times 3 \times 1 \times 6} =$$

Again, 48 in the *dividend* and 6 in the *divisor* are *divisible* by 6, since 48 divided by 6 equals 8, and 6 divided by 6 equals 1. *Cross off* the 48 and write the 8 *over* it; also, cross off the 6 and write the 1 *under* it. Thus,

$$\frac{6 \times \overset{8}{\cancel{48}} \times 4 \times 1}{8 \times 3 \times 1 \times \underset{1}{\cancel{6}}} =$$

The 48 and 6 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 8 and 1 placed in their stead, and treated as if the 48 and 6 *never* existed. Thus,

$$\frac{6 \times 8 \times 4 \times 1}{8 \times 3 \times 1 \times 1} =$$

Again, 6 in the *dividend* and 3 in the *divisor* are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and write the 2 *over* it; also, cross off the 3 and write the 1 *under* it. Thus,

$$\frac{\overset{2}{\not{6}} \times 8 \times 4 \times 1}{8 \times \underset{1}{\not{3}} \times 1 \times 1} =$$

The 6 and 3 are *not* to be considered any longer, and, in fact, may be *erased* entirely and the 2 and 1 placed in their stead, and treated as if the 6 and 3 *never* existed. Thus,

$$\frac{2 \times 8 \times 4 \times 1}{8 \times 1 \times 1 \times 1} =$$

Canceling the 8 in the dividend and the 8 in the divisor, the result is

$$\frac{2 \times \overset{1}{\not{8}} \times 4 \times 1}{\underset{1}{\not{8}} \times 1 \times 1 \times 1} = \frac{2 \times 1 \times 4 \times 1}{1 \times 1 \times 1 \times 1}.$$

Since there are *no two remaining numbers* (one in the dividend and one in the divisor) *divisible* by *any number* except 1, without a remainder, it is *impossible* to cancel further. *Multiply* all the *uncanceled numbers* in the *dividend* together, and divide their *product* by the *product* of all the *uncanceled* numbers in the divisor. The *result* will be the *quotient*. The *product* of all the *uncanceled numbers* in the *dividend* equals $2 \times 1 \times 4 \times 1 = 8$; the product of all the *uncanceled* numbers in the *divisor* equals $1 \times 1 \times 1 \times 1 = 1$.

Hence, $$\frac{2 \times 1 \times 4 \times 1}{1 \times 1 \times 1 \times 1} = \frac{8}{1} = 8. \quad \text{Ans.}$$

Or, $$\frac{\overset{\overset{2}{\not{6}}}{\not{72}} \times \overset{\not{8}}{\not{48}} \times \overset{4}{\not{28}} \times \overset{1}{\not{5}}}{\underset{\underset{1}{\not{8}}}{\not{96}} \times \underset{\underset{1}{\not{3}}}{\not{15}} \times \underset{1}{\not{7}} \times \underset{1}{\not{6}}} = \frac{8}{1} = 8. \quad \text{Ans.}$$

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Placing the numerator over the denominator, the problem becomes

$$\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$$

The 50 in the *dividend* and 70 in the *divisor* are both *divisible* by 10, since 50 divided by 10 equals 5, and 70 divided by 10 equals 7. *Cross off* the 50 and write the 5 *over* it; also, *cross off* the 70 and write the 7 *under* it. Thus,

$$\frac{80 \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times 20} =$$

The 50 and 70 are not to be considered any longer, and, in fact, may be erased entirely and the 5 and 7 placed in their stead, and treated as if the 50 and 70 *never* existed. Thus,

$$\frac{80 \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times 20} =$$

Also, 80 in the *dividend* and 20 in the *divisor* are *divisible* by 20, since 80 divided by 20 equals 4, and 20 divided by 20 equals 1. Cross off the 80 and write the 4 *over* it; also, cross off the 20 and write the 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times \underset{1}{\cancel{20}}} =$$

The 80 and 20 are *not* to be considered any longer, and, in fact, may be erased entirely and the 4 and 1 placed in their stead, and treated as if the 80 and 20 *never* existed. Thus,

$$\frac{4 \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times 1} =$$

Again, 16 in the *dividend* and 24 in the *divisor* are *divisible* by 8, since 16 divided by 8 equals 2, and 24 divided by 8 equals 3. *Cross off* the 16 and write the 2 *over* it; also cross off the 24 and write the 3 *under* it. Thus,

$$\frac{4 \times 60 \times 5 \times \overset{2}{\cancel{16}} \times 14}{7 \times 50 \times \underset{3}{\cancel{24}} \times 1} =$$

The 16 and 24 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 3 placed in their stead, and treated as if the 16 and 24 *never* existed. Thus,

$$\frac{4 \times 60 \times 5 \times 2 \times 14}{7 \times 50 \times 3 \times 1} =$$

Again, 60 in the *dividend* and 50 in the *divisor* are *divisible* by 10, since 60 divided by 10 equals 6, and 50 divided by 10 equals 5. *Cross off* the 60 and write the 6 *over* it; also, cross off the 50 and write the 5 *under* it. Thus,

$$\frac{4 \times \overset{6}{\cancel{60}} \times 5 \times 2 \times 14}{7 \times \underset{5}{\cancel{50}} \times 3 \times 1} =$$

The 60 and 50 are not to be considered any longer, and, in fact, may be erased entirely and the 6 and 5 placed in their stead, and treated as if the 60 and 50 *never* existed. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times 14}{7 \times 5 \times 3 \times 1} =$$

The 14 in the *dividend* and 7 in the *divisor* are *divisible* by 7, since 14 divided by 7 equals 2, and 7 divided by 7 equals 1. *Cross off* the 14 and write the 2 *over* it; also, cross off the 7 and write the 1 *under* it. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}} \times 5 \times 3 \times 1} =$$

The 14 and 7 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 1 placed in their stead, and treated as if the 14 and 7 *never* existed. Thus,

$$\frac{4 \times 6 \times 5 \times 2 \times 2}{1 \times 5 \times 3 \times 1} =$$

The 5 in the *dividend* and 5 in the *divisor* are *divisible* by 5, since 5 divided by 5 equals 1. *Cross off* the 5 of the *dividend* and write the 1 *over* it; also, cross off the 5 of the *divisor* and write the 1 *under* it. Thus,

$$\frac{4 \times 6 \times \overset{1}{\not{5}} \times 2 \times 2}{1 \times \underset{1}{\not{5}} \times 3 \times 1} =$$

The 5 in the *dividend* and 5 in the *divisor* are not to be considered any longer, and, in fact, may be erased entirely and 1 and 1 placed in their stead, and treated as if the 5 and 5 *never* existed. Thus,

$$\frac{4 \times 6 \times 1 \times 2 \times 2}{1 \times 1 \times 3 \times 1} =$$

The 6 in the *dividend* and 3 in the *divisor* are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and place 2 *over* it; also, cross off the 3 and place 1 *under* it. Thus,

$$\frac{4 \times \overset{2}{\not{6}} \times 1 \times 2 \times 2}{1 \times 1 \times \underset{1}{\not{3}} \times 1} =$$

The 6 and 3 are not to be considered any longer, and, in fact, may be erased entirely and 2 and 1 placed in their stead, and treated as if the 6 and 3 *never* existed. Thus,

$$\frac{4 \times 2 \times 1 \times 2 \times 2}{1 \times 1 \times 1 \times 1} = \frac{32}{1} = 32. \quad \text{Ans.}$$

Hence,

$$\frac{\overset{4}{\not{80}} \times \overset{\overset{2}{\not{6}}}{\not{60}} \times \overset{\overset{1}{\not{5}}}{\not{50}} \times \overset{2}{\not{16}} \times \overset{2}{\not{14}}}{\underset{\underset{1}{\not{7}}}{\not{70}} \times \underset{\underset{1}{\not{5}}}{\not{50}} \times \underset{\underset{1}{\not{3}}}{\not{24}} \times \underset{1}{\not{20}}} = \frac{4 \times 2 \times 1 \times 2 \times 2}{1 \times 1 \times 1 \times 1} = \frac{32}{1} = 32. \quad \text{Ans.}$$

(19) 28 acres of land at \$133 an acre would cost $28 \times \$133 = \$3,724$.

```
   28
 1064
 266
$3724
```

If a mechanic earns $1,500 a year and his expenses are $968 per year, then he would save $1500 − $968, or $532 per year.

```
 $1500
   968
 $532
```

If he saves $532 in 1 year, to save $3,724 it would take as many years as $532 is contained times in $3,724, or 7 years.

```
532 ) 3724 ( 7 years.   Ans.
      3724
```

(20) If the freight train ran 365 miles in one week, and 3 times as far lacking 246 miles the next week, then it ran (3 × 365 miles) − 246 miles, or 849 miles the second week. Thus,

```
              365
                3
             1095
              246
difference    849 miles.   Ans.
```

(21) The distance from Philadelphia to Pittsburg is 354 miles. Since there are 5,280 feet in one mile, in 354 miles there are 354 × 5,280 feet, or 1,869,120 feet. If the driving wheel of the locomotive is 16 feet in circumference, then in going from Philadelphia to Pittsburg, a distance of 1,869,120 feet, it will make 1,869,120 ÷ 16, or 116,820 revolutions.

```
16)1869120(116820 rev.   Ans.
   16
    26
    16
    109
     96
     131
     128
       32
       32
        0
```

(22) (*a*)

```
576)589824(1024  Ans.
    576
    ----
    1382
    1152
    ----
     2304
     2304
     ----
```

(*b*)

```
43911)369730620(8420  Ans.
      351288
      ------
       184426
       175644
       ------
         87822
         87822
         -----
             0
```

(*c*)

```
505)2527525(5005  Ans.
    2525
    ----
       2525
       2525
       ----
```

(*d*)

```
1234)4961794302(4020903  Ans.
     4936
     ----
       2579
       2468
       ----
        11143
        11106
        -----
           3702
           3702
           ----
```

(23) The harness evidently cost the difference between \$444 and the amount which he paid for the horse and wagon.

Since \$264 + \$153 = \$417, the amount paid for the horse and wagon, \$444 − \$417 = \$27, the cost of the harness.

```
$264          $444
 153           417
----          ----
$417           $27  Ans.
```

(24) (*a*)

```
  1024
   576
  ----
  6144
 7168
5120
------
589824  Ans.
```

(*b*)

```
   5005
    505
  -----
  25025
250250
-------
2527525  Ans.
```

(*c*)

```
    43911
     8420
   ------
   878220
 175644
351288
---------
369730620  Ans.
```

(25) Since there are 12 months in a year, the number of days the man works is $25 \times 12 = 300$ days. As he works 10 hours each day, the number of hours that he works in one year is $300 \times 10 = 3{,}000$ hours. Hence, he receives for his work $3{,}000 \times 30 = 90{,}000$ cents, or $90{,}000 \div 100 = \$900$. Ans.

(26) See Art. **71.**

(27) See Art. **77.**

(28) See Art. **73.**

(29) See Art. **73.**

(30) See Art. **75.**

(31) $\frac{13}{8}$ is an improper fraction, since its numerator 13 is greater than its denominator 8.

(32) $4\frac{1}{2}$; $14\frac{3}{10}$; $85\frac{4}{19}$.

(33) To reduce a fraction to its lowest terms means to change its form without changing its value. In order to do this, we must divide both numerator and denominator by the same number until we can no longer find any number (except 1) which will divide both of these terms without a remainder.

To reduce the fraction $\frac{4}{8}$ to its lowest terms we divide both numerator and denominator by 4, and obtain as a result the fraction $\frac{1}{2}$. Thus, $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$; similarly, $\frac{4 \div 4}{16 \div 4} = \frac{1}{4}$; $\frac{8 \div 4}{32 \div 4} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$; $\frac{32 \div 8}{64 \div 8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$. Ans.

(34) When the denominator of any number is not expressed, it is understood to be 1, so that $\frac{6}{1}$ is the same as $6 \div 1$, or 6. To reduce $\frac{6}{1}$ to an improper fraction whose denominator is 4, we must multiply both numerator and denominator by some number which will make the denominator of 6 equal to 4. Since this denominator is 1, by multiplying both terms of $\frac{6}{1}$ by 4 we shall have $\frac{6 \times 4}{1 \times 4} = \frac{24}{4}$, which has the *same value* as 6, but has a *different form*. Ans.

(35) In order to reduce a mixed number to an improper fraction, we must *multiply the whole number by the denominator of the fraction* and *add the numerator of the fraction to that product.* This *result* is the *numerator of the improper fraction*, of which the *denominator is the denominator of the fractional part of the mixed number.*

$7\frac{7}{8}$ means the same as $7 + \frac{7}{8}$. In 1 there are $\frac{8}{8}$, hence in 7 there are $7 \times \frac{8}{8} = \frac{56}{8}$; $\frac{56}{8}$ plus the $\frac{7}{8}$ of the mixed number $= \frac{56}{8} + \frac{7}{8} = \frac{63}{8}$, which is the required improper fraction.

$$13\frac{5}{16} = \frac{(13 \times 16) + 5}{16} = \frac{213}{16};\quad 10\frac{3}{4} = \frac{(10 \times 4) + 3}{4} = \frac{43}{4}.$$

(36) The value of a fraction is obtained by dividing the numerator by the denominator.

To obtain the value of the fraction $\frac{13}{2}$ we divide the numerator 13 by the denominator 2. 2 is contained in 13 six times, with 1 remaining. This 1 remaining is written over the denominator 2, thereby making the fraction $\frac{1}{2}$, which is annexed to the whole number 6, and we obtain $6\frac{1}{2}$ as the mixed number. The reason for performing this operation is the following: In 1 there are $\frac{2}{2}$ (two halves), and in $\frac{13}{2}$ (thirteen halves) there are as many units (1) as 2 is contained times in 13, which is 6, and $\frac{1}{2}$ (one-half) unit remaining. Hence, $\frac{13}{12} = 6 + \frac{1}{2} = 6\frac{1}{2}$, the required mixed number. Ans.

$\frac{17}{4} = 4\frac{1}{4}$. Ans. $\frac{69}{16} = 4\frac{5}{16}$. Ans. $\frac{16}{8} = 2$. Ans. $\frac{67}{64} = 1\frac{3}{64}$. Ans.

(37) In division of fractions, *invert the divisor* (or, in other words, turn it upside down) *and proceed as in multiplication.*

(*a*) $35 \div \frac{5}{16} = \frac{35}{1} \times \frac{16}{5} = \frac{35 \times 16}{1 \times 5} = \frac{560}{5} = 112$. Ans.

(*b*) $\frac{9}{16} \div 3 = \frac{9}{16} \div \frac{3}{1} = \frac{9}{16} \times \frac{1}{3} = \frac{9 \times 1}{16 \times 3} = \frac{9}{48} = \frac{3}{16}$. Ans.

(*c*) $\frac{17}{2} \div 9 = \frac{17}{2} \div \frac{9}{1} = \frac{17}{2} \times \frac{1}{9} = \frac{17 \times 1}{2 \times 9} = \frac{17}{18}$. Ans.

(*d*) $\frac{113}{64} \div \frac{7}{16} = \frac{113}{64} \times \frac{16}{7} = \frac{113 \times 16}{64 \times 7} = \frac{1,808}{448} = \frac{452}{112} = \frac{113}{28}$

$$28)\,113\,(4\tfrac{1}{28}. \quad \text{Ans.}$$
$$\underline{112}$$
$$1$$

(*e*) $15\frac{3}{4} \div 4\frac{3}{8} = ?$ Before proceeding with the division, reduce both of the mixed numbers to improper fractions. Thus, $15\frac{3}{4} = \frac{(15 \times 4) + 3}{4} = \frac{60 + 3}{4} = \frac{63}{4}$, and $4\frac{3}{8} = \frac{(4 \times 8) + 3}{8} = \frac{32 + 3}{8} = \frac{35}{8}$. The problem is now $\frac{63}{4} \div \frac{35}{8} = ?$ As before, invert the divisor and multiply; $\frac{63}{4} \div \frac{35}{8} = \frac{63}{4} \times \frac{8}{35} = \frac{63 \times 8}{4 \times 35} = \frac{504}{140} = \frac{252}{70} = \frac{126}{35} = \frac{18}{5}$.

$$\frac{18}{5})\,18\,(3\frac{3}{5} \quad \text{Ans.}$$
$$\underline{15}$$
$$3$$

(38) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = \frac{1 + 2 + 5}{8} = \frac{8}{8} = 1.$ Ans.

When the *denominators* of the fractions to be added *are alike*, we know that the units are divided into the *same number of parts* (in this case *eighths*); we, therefore, *add the numerators* of the fractions to find the number of parts (eighths) taken or considered, thereby obtaining $\frac{8}{8}$ or 1 as the sum.

(39) When the *denominators* are *not alike* we know that the units are divided into *unequal parts*, so before adding them we must find a common denominator for the denominators of all the fractions. Reduce the fractions to fractions having this common denominator, add the numerators and write the sum over the common denominator.

In this case, the least common denominator, or the least number that will contain all the denominators, is 16; hence, we must reduce all these fractions to sixteenths and then add their numerators.

$\frac{1}{4} + \frac{3}{8} + \frac{5}{16} = ?$ To reduce the fraction $\frac{1}{4}$ to a fraction having 16 for a denominator, we must multiply both terms of the fraction by some number which will make the denominator 16. This number evidently is 4, hence, $\frac{1 \times 4}{4 \times 4} = \frac{4}{16}$.

Similarly, both terms of the fraction $\frac{3}{8}$ must be multiplied by 2 to make the denominator 16, and we have $\frac{3 \times 2}{8 \times 2} = \frac{6}{16}$. The fractions now have a common denominator 16; hence, we find their sum by adding the numerators and placing their sum over the common denominator, thus: $\frac{4}{16} + \frac{6}{16} + \frac{5}{16} = \frac{4 + 6 + 5}{16} = \frac{15}{16}$. Ans.

(40) When mixed numbers and whole numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction.

$42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Reducing $\frac{5}{8}$ to a fraction having a denominator of 16, we have $\frac{5}{8} \times \frac{2}{2} = \frac{10}{16}$. Adding the two fractional parts of the mixed numbers we have $\frac{10}{16} + \frac{7}{16} = \frac{10 + 7}{16} = \frac{17}{16} = 1\frac{1}{16}$.

The problem now becomes $42 + 31 + 9 + 1\frac{1}{16} = ?$

$$\begin{array}{r} 42 \\ 31 \\ 9 \\ 1\frac{1}{16} \\ \hline 83\frac{1}{16} \end{array}$$ Ans.

Adding all the whole numbers and the number obtained from adding the fractional parts of the mixed numbers, we obtain $83\frac{1}{16}$ as their sum.

(41) $29\frac{3}{4} + 50\frac{5}{8} + 41 + 69\frac{3}{16} = ?$ $\frac{3}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16}.$

$\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}.$ $\frac{12}{16} + \frac{10}{16} + \frac{3}{16} = \frac{12 + 10 + 3}{16} = \frac{25}{16} = 1\frac{9}{16}.$

The problem now becomes $29 + 50 + 41 + 69 + 1\frac{9}{16} = ?$

29	square inches.
50	square inches.
41	square inches.
69	square inches.
$1\frac{9}{16}$	square inches.
$190\frac{9}{16}$	square inches. Ans.

(42) (*a*) $\dfrac{7}{\frac{3}{16}} = 7 \div \frac{3}{16} = 7 \times \frac{16}{3} = \frac{7 \times 16}{3} = \frac{112}{3} = 37\frac{1}{3}.$ Ans.

The line between 7 and $\frac{3}{16}$ means that 7 is to be divided by $\frac{3}{16}$.

(*b*) $\dfrac{\frac{15}{32}}{\frac{5}{8}} = \frac{15}{32} \div \frac{5}{8} = \frac{15}{32} \times \frac{8}{5} = \frac{\overset{3}{\cancel{15}} \times \cancel{8}}{\underset{4}{\cancel{32}} \times \cancel{5}} = \frac{3}{4}.$ Ans.

(*c*) $\dfrac{\frac{4+3}{2+6}}{5} = \dfrac{\frac{7}{8}}{5} = \frac{7}{8 \times 5} = \frac{7}{40}.$ (See Art. **131.**) Ans.

(43) $\frac{7}{8}$ = value of the fraction, and 28 = the numerator. We find that 4 multiplied by 7 = 28, so multiplying 8, the denominator of the fraction, by 4, we have 32 for the required denominator, and $\frac{28}{32} = \frac{7}{8}$. Hence, 32 is the required denominator. Ans.

(44) (*a*) $\frac{7}{8} - \frac{7}{16} = ?$ When the *denominators* of fractions are *not alike* it is evident that the units are divided into *unequal parts*, therefore, before subtracting, ***reduce the***

fractions to fractions having a common denominator. Then, *subtract the numerators, and place the remainder over the common denominator.*

$$\frac{7 \times 2}{8 \times 2} = \frac{14}{16}. \quad \frac{14}{16} - \frac{7}{16} = \frac{14 - 7}{16} = \frac{7}{16}. \quad \text{Ans.}$$

(*b*) $13 - 7\frac{7}{16} = ?$ This problem may be solved in two ways:

First: $13 = 12\frac{16}{16}$, since $\frac{16}{16} = 1$, and $12\frac{16}{16} = 12 + \frac{16}{16} = 12 + 1 = 13$.

$$\begin{array}{r} 12\frac{16}{16} \\ 7\frac{7}{16} \\ \hline 5\frac{9}{16} \end{array}$$

We can now subtract the whole numbers separately, and the fractions separately, and obtain $12 - 7 = 5$ and $\frac{16}{16} - \frac{7}{16} = \frac{16 - 7}{16} = \frac{9}{16}$. $5 + \frac{9}{16} = 5\frac{9}{16}$. Ans.

Second: By reducing both numbers to improper fractions having a denominator of 16.

$$13 = \frac{13}{1} = \frac{13 \times 16}{1 \times 16} = \frac{208}{16}. \quad 7\frac{7}{16} = \frac{(7 \times 16) + 7}{16} = \frac{112 + 7}{16} = \frac{119}{16}.$$

Subtracting, we have $\frac{208}{16} - \frac{119}{16} = \frac{208 - 119}{16} = \frac{89}{16}$ and $\frac{89}{16} = 16)89(5\frac{9}{16}$ the same result that was obtained by the first method.

$$\begin{array}{r} 16)89(5\frac{9}{16} \\ 80 \\ \hline 9 \\ \hline 16 \end{array}$$

(*c*) $312\frac{9}{16} - 229\frac{5}{32} = ?$ We first reduce the fractions of the two mixed numbers to fractions having a common denominator. Doing this we have $\frac{9}{16} = \frac{9 \times 2}{16 \times 2} = \frac{18}{32}$. We can now subtract the whole numbers and fractions separately, and have $312 - 229 = 83$ and $\frac{18}{32} - \frac{5}{32} = \frac{18 - 5}{32} = \frac{13}{32}$.

$$\begin{array}{r} 312\frac{18}{32} \\ 229\frac{5}{32} \\ \hline 83\frac{13}{32} \end{array} \quad 83 + \frac{13}{32} = 83\frac{13}{32}. \quad \text{Ans.}$$

(45) The man evidently traveled $85\frac{5}{12}+78\frac{9}{15}+125\frac{17}{35}$ miles.

Adding the fractions separately in this case,

$$\frac{5}{12}+\frac{9}{15}+\frac{17}{35}=\frac{5}{12}+\frac{3}{5}+\frac{17}{35}=\frac{175+252+204}{420}=\frac{631}{420}=1\frac{211}{420}.$$

Adding the whole numbers and the mixed number representing the sum of the fractions, the sum is $289\frac{211}{420}$ miles. Ans.

$$\begin{array}{r} 85 \\ 78 \\ 125 \\ 1\frac{211}{420} \\ \hline 289\frac{211}{420} \end{array}$$

To find the least common denominator, we have

$$\begin{array}{r|lll} 5 & 12, & 5, & 35 \\ \hline 7 & 12, & 1, & 7 \\ \hline & 12, & 1, & 1, \end{array} \text{ or } 5\times7\times12=420.$$

(46)

$$\begin{array}{rl} 573\frac{4}{5} & \text{tons.} \\ 216\frac{5}{8} & \text{tons.} \\ \hline \textit{difference } 357\frac{7}{40} & \text{tons. Ans.} \end{array} \qquad \begin{array}{l} \frac{4}{5}=\frac{32}{40} \\ \frac{5}{8}=\frac{25}{40} \\ \hline \frac{7}{40}=\textit{difference.} \end{array}$$

(47) Reducing $9\frac{1}{4}$ to an improper fraction, it becomes $\frac{37}{4}$. Multiplying $\frac{37}{4}$ by $\frac{3}{8}$, $\frac{37}{4}\times\frac{3}{8}=\frac{111}{32}=3\frac{15}{32}$ dollars. Ans.

(48) Referring to Arts. **114** and **116**,

$\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{11}$ of $\frac{19}{20}$ of 11 multiplied by $\frac{7}{8}$ of $\frac{5}{6}$ of 45 =

$$\frac{\not{2}\times\not{3}\times7\times19\times\not{11}\times7\times5\times\overset{\overset{3}{\not{15}}}{\not{45}}}{\not{3}\times4\times\not{11}\times\underset{4}{\not{20}}\times1\times8\times\underset{\not{3}}{\not{6}}\times1}=\frac{7\times19\times7\times5\times3}{4\times4\times8}=\frac{13,965}{128}=$$

$109\frac{13}{128}$. Ans.

(49) $\frac{3}{4}$ of $16=\frac{3}{\not{4}}\times\frac{\overset{4}{\not{16}}}{1}=12.$ $12\div\frac{2}{3}=\frac{\overset{6}{\not{12}}}{1}\times\frac{3}{\not{2}}=18.$ Ans.

(50) $211\frac{1}{4}\times1\frac{7}{8}=\frac{845}{4}\times\frac{15}{8}$, reducing the mixed numbers to improper fractions. $\frac{845}{4} \times \frac{15}{8} = \frac{12,675}{32}$ cents = amount paid for the lead. The number of pounds sold is evidently $\frac{12,675}{32} \div 2\frac{1}{2} = \frac{12,675}{32} \times \frac{2}{5} = \frac{2,535}{16} = 158\frac{7}{16}$ pounds. The amount remaining is $211\frac{1}{4} - 158\frac{7}{16} = \frac{845}{4} - \frac{2,535}{16} = \frac{3,380}{16} - \frac{2,535}{16} = \frac{845}{16} = 52\frac{13}{16}$ pounds. Ans.

(51)

	tenths.	hundredths.	
.	0	8	= *Eight* **hundredths.**

	tenths.	hundredths.	thousandths.	
.	1	3	1	= *One hundred thirty-one* **thousandths.**

	tenths.	hundredths.	thousandths.	ten-thousandths.	
.	0	0	0	1	= *One* **ten-thousandth.**

	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	
.	0	0	0	0	2	7	= *Twenty-seven* **millionths.**

	tenths.	hundredths.	thousandths.	ten-thousandths.	
.	0	1	0	8	= *One hundred eight* **ten-thousandths.**

tenths.	hundredths.	thousandths.	ten-thousandths.	
93.0	1	0	1	= Ninety-three, and *one hundred one* **ten-thousandths.**

In reading decimals, read the number just as you would if there were no ciphers before it. Then count from the decimal point towards the right, beginning with tenths, to as many places as there are figures, and the *name* of the last figure must be annexed to the previous reading of the figures to give the decimal reading. Thus, in the first example above, the simple reading of the figure is *eight*, and the name of its position in the decimal scale is **hundredths,** so that the decimal reading is *eight* **hundredths.** Similarly, the figures in the fourth example are ordinarily read *twenty-seven;* the name of the position of the figure 7 in the decimal scale is **millionths,** giving, therefore, the decimal reading as *twenty-seven* **millionths.**

If there should be a whole number before the decimal point, read it as you would read any whole number, and read the decimal as you would if the whole number were not there; or, read the whole number and then say, "and" so many hundredths, thousandths, or whatever it may be, as "ninety-three, *and* one hundred one ten thousandths."

(52) See Art. **139.**

(53) See Art. **153.**

(54) See Art. **160.**

(55) A fraction is one or more of the equal parts of a unit, and is expressed by a numerator and a denominator, while a decimal fraction is a number of *tenths*, *hundredths*, *thousandths*, etc., of a unit, and is expressed by placing a period (.), called a decimal point, to the left of the figures of the number, and omitting the denominator.

(56) See Art. **165.**

(57) To reduce the fraction $\frac{1}{2}$ to a decimal, we annex one cipher to the numerator, which makes it 1.0. Dividing 1.0, the numerator, by 2, the denominator, gives a quotient of .5, the decimal point being placed before the *one* figure of the quotient, or .5, since only *one* cipher was annexed to the numerator. Ans.

```
 7
 - 
 8)7.000
   .875  Ans.
```

```
 5
 --
 32)5.00000(.15625  Ans.
    32
    180
    160
     200
     192
      80
      64
      160
      160
```

Since $.65 = \frac{65}{100}$, then, $\frac{65}{100}$ must equal .65. Or, when the denominator is 10, 100, 1000, etc., point off as many places in the numerator as there are ciphers in the denominator. Doing so, $\frac{65}{100} = .65$. Ans.

$\frac{125}{1000} = .125$. Ans.

(58) (*a*) This example, written in the form of a fraction, means that the numerator (32.5 + .29 + 1.5) is to be divided by the denominator (4.7 + 9). The operation is as follows:

$$\frac{32.5 + .29 + 1.5}{4.7 + 9} = ?$$

```
            32.5
         +    .29
         +   1.5
 13.7 ) 34.29000 ( 2.5029  Ans.
        274
  4.7    689
+ 9.0    685
 13.7      400
           274
           1260
           1233
             27
```

Since there are 5 decimal places in the dividend and 1 in the divisor, there are 5 − 1 or 4 places to be pointed off in the quotient. The fifth figure of the decimal is evidently less than 5.

(*b*) Here again the problem is to divide the numerator, which is $(1.283 \times \overline{8+5})$, by the denominator, which is 2.63. The operation is as follows:

$$\frac{1.283 \times \overline{8+5}}{2.63} = ? \quad \overline{8+5} = 13.$$

```
          1.283
        ×    13
          3849
         1283
2.63)16.679000(6.3418  Ans.
     1578
       899              480
       789              263
      1100             2170
      1052             2104
       480               66
```

(*c*) $$\frac{\overline{589+27} \times \overline{163-8}}{25+39} = ?$$

```
  589               163
+  27             −   8
  616               155
                  × 616
                    930
                   155
                  930
   25        64)95480.000(1491.875
 + 39           64                Ans.
   64           314
                256
                 588
                 576
                  120
                   64
                   560
                   512
                    480
                    448
                     320
                     320
```

There are three decimal places in the quotient, since three ciphers were annexed to the dividend.

(*d*) $\dfrac{\overline{40.6 + 7.1} \times (3.029 - 1.874)}{6.27 + 8.53 - 8.01} = ?$

$$\begin{array}{r} 40.6 \\ +\ \ 7.1 \\ \hline 47.7 \end{array} \qquad \begin{array}{r} 3.029 \\ -\ 1.874 \\ \hline 1.155 \\ \times\ \ 47.7 \\ \hline 8085 \\ 8085 \\ 4620 \\ \hline 55.093500 \end{array}$$

$$\begin{array}{r} 6.27 \\ +\ 8.53 \\ \hline 14.80 \\ -\ \ 8.01 \\ \hline 6.79 \end{array}$$

$$\begin{array}{l} 6.79)\,55.093500\,(\,8.1139. \text{ Ans} \\ 5432 \\ \hline 773 \\ 679 \\ \hline 945 \\ 679 \\ \hline 2660 \\ 2037 \\ \hline 6230 \\ 6111 \\ \hline 119 \end{array}$$

6 decimal places in the dividend — 2 decimal places in the divisor = 4 decimal places to be pointed off in the quotient.

(59) $.875 = \dfrac{875}{1{,}000} = \dfrac{175}{200} = \dfrac{7}{8}$ of a foot.

1 foot = 12 inches.

$$\frac{7}{8} \text{ of 1 foot} = \frac{7}{\underset{2}{\cancel{8}}} \times \frac{\overset{3}{\cancel{12}}}{1} = \frac{21}{2} = 10\frac{1}{2} \text{ inches.} \quad \text{Ans.}$$

(60) 12 inches = 1 foot.

$$\frac{3}{16} \text{ of an inch} = \frac{3}{16} \div 12 = \frac{\cancel{3}}{16} \times \frac{1}{\underset{4}{\cancel{12}}} = \frac{1}{64} \text{ of a foot.}$$

Point off 6 decimal places in the quotient, since we annexed six ciphers to the dividend, the divisor containing no decimal places; hence, 6 − 0 = 6 places to be pointed off.

```
  1
 ---) 1.000000(.015625  Ans.
 64     64
        ---
        360
        320
        ---
         400
         384
         ---
          160
          128
          ---
           320
           320
           ---
```

(61) If 1 cubic inch of water weighs .03617 of a pound, the weight of 1,500 cubic inches will be .03617 × 1,500 = 54.255 lb.

```
 .03617 lb.
   1500
 -------
1808500
3617
-------
54.25500 lb.   Ans.
```

(62) 72.6 feet of fencing at $.50 a foot would cost

```
    72.6 × .50, or $36.30.
     .50
 --------
 $36.300
```

If, by selling a carload of coal at a profit of $1.65 per ton, I make $36.30, then there must be as many tons of coal in the car as 1.65 is contained times in 36.30, or 22 tons.

```
1.65 ) 36.30 ( 22 tons.   Ans.
       330
       ---
        330
        330
        ---
```

(63) 231) 17892.00000 (77.45454, or 77.4545 to four decimal places. Ans.

```
231 ) 17892.00000 ( 77.45454
      1617
      ----
       1722
       1617
       ----
        1050
         924
        ----
         1260
         1155
         ----
          1050
           924
          ----
          1260
          1155
          ----
          1050
```

(64)
$$\frac{\overset{37.13}{\cancel{74.26}} \times \overset{2}{\cancel{24}} \times \overset{.0952}{\cancel{3.1416}} \times 19 \times 19 \times 350}{\underset{1,000}{\cancel{33,000}} \times \underset{2}{\cancel{12}} \times \cancel{4}} =$$

$$\frac{37.13 \times .0952 \times 19 \times 19 \times 350}{1,000} = \frac{446,618.947600}{1,000} =$$

446.619 to three decimal places. Ans.

```
   37.13        19         361           3.534776
   .0952        19         350             126350
  ------       ---       -----         ----------
    7426       171       18050          176738800
  18565        19        1083          10604328
 33417         ---       ------       21208656
 --------      361       126350       7069552
 3.534776                            3534776
                                     --------------
                                     446618.947600
```

(65) See Art. **174.** Applying rule in Art. **175,**

(*a*) $.7928 \times \frac{64}{64} = \frac{50.7392}{64} = \frac{51}{64}.$ Ans.

(*b*) $.1416 \times \frac{32}{32} = \frac{4.5312}{32} = \frac{5}{32}.$ Ans.

(*c*) $.47915 \times \frac{16}{16} = \frac{7.6664}{16} = \frac{8}{16} = \frac{1}{2}.$ Ans.

(66) In subtraction of decimals, *place the decimal points directly under each other*, and proceed as in the subtraction of whole numbers, placing the *decimal point in the remainder directly under the decimal points above.*

(*a*)	709.6300	
	.8514	
	708.7786	Ans.

In the above example we proceed as follows: We can not subtract 4 ten-thousandths from 0 ten-thousandths, and, as there are no thousandths, we take 1 hundredth from the three hundredths. 1 *hundredth* = 10 *thousandths* = 100 *ten-thousandths.* 4 ten-thousandths from 100 ten-thousandths leaves 96 ten-thousandths. 96 ten-thousandths = 9 *thousandths* + 6 *ten-thousandths.* Write the 6 ten-thousandths in the ten-thousandths place in the remainder. The next figure in the subtrahend is 1 thousandth. This must be subtracted from the 9 thousandths which is a part of the 1 hundredth taken previously from the 3 hundredths. Subtracting, we have 1 thousandth from 9 thousandths leaves 8 thousandths, the 8 being written in its place in the remainder. Next we have to subtract 5 hundredths from 2 hundredths (1 hundredth having been taken from the 3 hundredths makes it but 2 hundredths now). Since we can not do this, we take 1 tenth from 6 tenths. 1 tenth (= 10 hundredths) + 2 hundredths = 12 hundredths. 5 hundredths from 12 hundredths leaves 7 hundredths. Write the 7 in the hundredths place in the remainder. Next we have to subtract 8 tenths from 5 tenths (5 tenths now, because 1 tenth was taken from the 6 tenths). Since this can not be done, we take 1 unit from the 9 units. 1 *unit* = 10 *tenths;* 10 tenths + 5 tenths = 15 tenths, and 8 tenths from 15 tenths leaves 7 tenths. Write the 7 in the tenths place in the remainder. In the minuend we now have 708 units (one unit having been taken away) and 0 units in the subtrahend. 0 units from 708 units leaves 708 units; hence, we write 708 in the remainder.

(*b*)	81.963		(*c*)	18.00		(*d*)	1.000
	1.700			.18			.001
	80.263	Ans.		17.82	Ans.		.999 Ans.

(*e*) $872.1 - (.8721 + .008) = ?$ In this problem we are to subtract $(.8721 + .008)$ from 872.1. First perform the operation as indicated by the sign between the decimals enclosed by the parenthesis.

```
.8721
.008
-----
.8801 sum.
```

Subtracting the sum (obtained by adding the decimals enclosed within the parenthesis) from the number 872.1 (as required by the minus sign before the parenthesis), we obtain the required remainder.

```
872.1000
   .8801
--------
871.2199 Ans.
```

(*f*) $(5.028 + .0073) - (6.704 - 2.38) = ?$ First perform the operations as indicated by the signs between the numbers enclosed by the parentheses. The first parenthesis shows that 5.028 and .0073 are to be added. This gives 5.0353 as their sum.

```
5.0280
 .0073
------
5.0353 sum.
```

The second parenthesis shows that 2.38 is to be subtracted from 6.704. The difference is found to be 4.324.

```
6.704
2.380
-----
4.324 difference.
```

The sign between the parentheses indicates that the quantities obtained by performing the above operations, are to be subtracted, namely, that 4.324 is to be subtracted from 5.0353. Performing this operation we obtain .7113 as the final result.

```
5.0353
4.324
------
 .7113 Ans.
```

(**67**) In subtracting a decimal from a fraction, or subtracting a fraction from a decimal, either reduce the fraction to a decimal before subtracting, or reduce the decimal to a fraction and then subtract.

(*a*) $\frac{7}{8} - .807 = ?$ $\frac{7}{8}$ reduced to a decimal becomes

```
7
8)7.000
  .875
```

Subtracting .807 from .875 the remainder is .068, as shown.

```
.875
.807
----
.068 Ans.
```

(*b*) $.875 - \frac{3}{8} = ?$ Reducing .875 to a fraction we have $875 = \frac{875}{1,000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; hence, $\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8} = \frac{1}{2}$. Ans.

Or, by reducing $\frac{3}{8}$ to a decimal, $8\overline{)3.000}$ giving $.375$, and then subtracting, we obtain $.875 - .375 = .5 = \frac{5}{10} = \frac{1}{2}$, the same answer as above.

$$\begin{array}{r} .875 \\ .375 \\ \hline .500 \end{array} \text{ Ans.}$$

(*c*) $\left(\frac{5}{32} + .435\right) - \left(\frac{21}{100} - .07\right) = ?$ We first perform the operations as indicated by the signs between the numbers enclosed by the parentheses. Reduce $\frac{5}{32}$ to a decimal and we obtain $\frac{5}{32} = .15625$ (see example 57).

Adding .15625 and .435,

$$\begin{array}{r} .15625 \\ .435 \\ \hline \textit{sum}\ \ .59125 \end{array}$$

$\frac{21}{100} = .21$; subtracting,

$$\begin{array}{r} .21 \\ .07 \\ \hline \textit{difference}\ \ .14 \end{array}$$

We are now prepared to perform the operation indicated by the minus sign between the parentheses, which is,

$$\begin{array}{r} .59125 \\ .14 \\ \hline \textit{difference}\ \ .45125 \end{array} \text{ Ans.}$$

(*d*) This problem means that 33 millionths and 17 thousandths are to be added. Also, that 53 hundredths and 274 thousandths are to be added, and the smaller of these sums is to be subtracted from the larger sum. Thus, $(.53 + .274) - (.000033 + .017) = ?$

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred thousandths.	millionths.	
.0	0	0	0	3	3	
.0	1	7				
.0	1	7	0	3	3	*sum.*

tenths.	hundredths.	thousandths.	
.5	3		
.2	7	4	
.8	0	4	*sum.*

$$\begin{array}{rl} .804 & \textit{larger sum.} \\ .017033 & \textit{smaller sum,} \\ \hline \textit{difference}\ \ .786967 & \text{Ans.} \end{array}$$

(68) In addition of decimals the *decimal points must be placed directly under each other*, so that *tenths* will come *under tenths*, *hundredths under hundredths*, *thousandths under thousandths*, etc. The addition is then performed as in whole numbers, *the decimal point of the sum being placed directly under the decimal points above.*

```
  .125
  .7
  .089
  .4005
  .9
  .000027
 --------
 2.214527   Ans.
```

(69)

```
   927.416
     8.274
   372.6
    62.07938
 -----------
  1370.36938   Ans.
```

(70) tenths. hundredths. thousandths. ten-thousandths. hundred-thousandths. millionths.

```
 .017
 .2
 .000047
 --------
 .217047 = Ans.
```

(71) (*a*)

```
    .107
    .013
    ----
     321
    107
 -------
 .001391   Ans.
```

There are 3 decimal places in the multiplicand and 3 in the multiplier; hence, there are 3 + 3 or 6 decimal places in the product. Since the product contains but four figures, we prefix two ciphers in order to obtain the necessary six decimal places.

(*b*)

```
     203
     2.03
     ----
      609
    4060
   ------
   412.09
     .203
   ------
   123627
  824180
  --------
  83.65427   Ans.
```

There are two decimal places in the multiplier and none in the multiplicand; hence, there are 2 + 0 or two decimal places in the first product.

Since there are 2 decimal places in the multiplicand and 3 decimal places in the multiplier, there are 3 + 2 or 5 decimal places in the second product.

(*c*) First perform the operations indicated by the signs between the numbers enclosed by the parenthesis, and then perform whatever may be required by the sign before the parenthesis.

Multiply together the numbers 2.7 and 31.85.

```
  31.85
    2.7
  -----
  22295
  6370
  -----
 85.995
```

The parenthesis shows that .316 is to be taken from 3.16.

```
 3.160
  .316
 -----
 2.844
```

The product obtained by the first operation is now multiplied by the remainder obtained by performing the operation indicated by the signs within the parenthesis.

```
     85.995
      2.844
   --------
     343980
    343980
   687960
  171990
 ----------
 244.569780   Ans.
```

(*d*) $(107.8 + 6.541 - 31.96) \times 1.742 = ?$

```
     107.8
 +     6.541
   ---------
     114.341
 -    31.96
   ---------
      82.381
 ×     1.742
   ---------
      164762
     329524
    576667
    82381
 -----------
 143.507702   Ans.
```

(72) (*a*) $\left(\frac{7}{16} - .13\right) \times \overline{.625 + \frac{5}{8}} = ?$

First perform the operation indicated by the parenthesis.

$\frac{7}{16} = 16)\,7.0000\,(.4375$

```
16)7.0000(.4375
   64
   --
    60
    48
    --
    120
    112
    ---
      80
      80
      --
```

We point off four decimal places since we annexed four ciphers.

```
                              .4375
                              .13
                              -----
Subtracting, we obtain        .3075
```

The vinculum has the same meaning as the parenthesis; hence, we perform the operation indicated by it. We point off three decimal places, since three ciphers were annexed to the 5.

$\frac{5}{8} = 8\,)\,\underline{5.000}$
$\quad .625$

```
Adding the terms in-          .625
cluded by the vinculum,       .625
                              -----
we obtain                     1.250
```

The final operation is to perform the work indicated by the sign between the parenthesis and the vinculum, thus,

```
      .3075
       1.25
     ------
      15375
      6150
     3075
    -------
    .384375   Ans.
```

(*b*) $\left(\frac{19}{32} \times .21\right) - \left(.02 \times \frac{3}{16}\right) = ?$

$.21 = \frac{21}{100}$. $\frac{19}{32} \times \frac{21}{100} = \frac{399}{3200}$. $.02 = \frac{2}{100}$. $\frac{2}{100} \times \frac{3}{16} = \frac{6}{1600} = \frac{3}{800}$.

$\frac{3}{800} = \frac{3 \times 4}{800 \times 4} = \frac{12}{3200}$. $\frac{399}{3200} - \frac{12}{3200} = \frac{399 - 12}{3200} = \frac{387}{3200}$.

Reducing $\frac{387}{3200}$ to a decimal, we obtain

$$\frac{387}{3200}\; 3200)\,387.0000000\,(.1209375 \quad \text{Ans.}$$

```
3200 ) 387.0000000 ( .1209375   Ans.
       3200
        6700
        6400
         30000
         28800
          12000
           9600
           24000
           22400
            16000
            16000
```

Point off seven decimal places, since seven ciphers were annexed to the dividend.

(*c*) $\left(\frac{13}{4} + .013 - 2.17\right) \times \overline{13\frac{1}{4} - 7\frac{5}{16}} = ?$

$\frac{13}{4} = \frac{13}{4)\,13.00}$
$\quad 3.25$

Point off two decimal places, since two ciphers were annexed to the dividend.

```
   3.25
+  .013
  3.263
- 2.17
  1.093
```

$\frac{5}{16}$ reduced to a decimal is .3125, since

```
 5
16 ) 5.0000 ( .3125
     48
      20
      16
       40
       32
        80
        80
```

Point off four decimal places, since four ciphers were annexed to the dividend.

Then, $7\frac{5}{16} = 7.3125$, and $13\frac{1}{4} = 13.25$, since $\frac{1}{4} = \frac{1}{4)\,1.00}$
$\quad .25$

$$\begin{array}{r} 13.25 \\ -\ \ 7.3125 \\ \hline 5.9375 \end{array} \qquad \begin{array}{r} 5.9375 \\ \times\ \ 1.093 \\ \hline 178125 \\ 534375\ \\ 593750\ \ \ \ \\ \hline 6.4896875 \end{array} \text{ Ans.}$$

(73) (*a*) $.875 \div \frac{1}{2} = .875 \div .5 \left(\text{since } \frac{1}{2} = .5\right) = 1.75.$ Ans.

Another way of solving this is to reduce .875 to its equivalent common fraction and then divide.

$.875 = \frac{7}{8}$, since $.875 = \frac{875}{1{,}000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; then, $\frac{7}{8} \div \frac{1}{2} = \frac{7}{\underset{4}{\not 8}} \times \frac{\not 2}{1} = \frac{7}{4} = 1\frac{3}{4}$. Since $\frac{3}{4} = $

$$\begin{array}{r} 4)\ 3.00\ (.75 \\ 2\,8\ \ \ \ \ \ \ \\ \hline 2\,0\ \ \ \ \ \ \\ 2\,0\ \ \ \ \ \ \\ \hline \end{array}$$

$1\frac{3}{4} = 1.75$, the same answer as above.

(*b*) $\frac{7}{8} \div .5 = \frac{7}{8} \div \frac{1}{2} \left(\text{since } .5 = \frac{1}{2}\right) = \frac{7}{\underset{4}{\not 8}} \times \frac{\not 2}{1} = \frac{7}{4} = 1\frac{3}{4}$, or 1.75. Ans.

This can also be solved by reducing $\frac{7}{8}$ to its equivalent decimal and dividing by .5; $\frac{7}{8} = .875$; $.875 \div .5 = 1.75$. Since there are three decimal places in the dividend and one in the divisor, there are 3 — 1, or 2 decimal places in the quotient.

(*c*) $\dfrac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125} = ?$ We shall solve this problem by first reducing the decimals to their equivalent common fractions.

$.375 = \frac{375}{1{,}000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$. $\frac{3}{8} \times \frac{1}{4} = \frac{3}{32}$, or the value of the numerator of the fraction.

$.125 = \frac{125}{1{,}000} = \frac{25}{200} = \frac{1}{8}$. Reducing $\frac{1}{8}$ to sixteenths, we have $\frac{1 \times 2}{8 \times 2} = \frac{2}{16}$. Then, $\frac{5}{16} - \frac{2}{16} = \frac{3}{16}$, or the value of the denominator of the fraction. The problem is now reduced to

$$\frac{\frac{3}{32}}{\frac{3}{16}} = ? \quad \frac{\frac{3}{32}}{\frac{3}{16}} = \frac{3}{32} \div \frac{3}{16} = \frac{\not 3}{\underset{2}{\not 32}} \times \frac{\not 16}{\not 3} = \frac{1}{2} \text{ or } .5. \quad \text{Ans.}$$

(**74**) $\frac{1.25 \times 20 \times 3}{\frac{87 + (11 \times 8)}{459 + 32}} = ?$ In this problem $1.25 \times 20 \times 3$ constitutes the numerator of the complex fraction.

$$\begin{array}{r} 1.25 \\ \times \quad 20 \\ \hline 25.00 \\ \times \quad 3 \\ \hline 75 \end{array}$$

Multiplying the factors of the numerator together, we find their product to be 75.

The fraction $\frac{87 + (11 \times 8)}{459 + 32}$ constitutes the denominator of the complex fraction. The value of the numerator of this fraction equals $87 + 88 = 175$.

The numerator is combined as though it were written $87 + (11 \times 8)$, and its result is

$$\begin{array}{r} 11 \\ 8 \text{ times} \\ \hline 88 \\ +87 \\ \hline 175 \end{array}$$

The value of the denominator of this fraction is equal to $459 + 32 = 491$. The problem then becomes

$$\frac{75}{\frac{175}{491}} = \frac{75}{1} \div \frac{175}{491} = \frac{75}{1} \times \frac{491}{175} = \frac{\overset{3}{\not 75} \times 491}{\underset{7}{\not 175}} = \frac{1,473}{7} = 210\frac{3}{7}. \quad \text{Ans.}$$

(**75**) 1 plus .001 = 1.001. .01 plus .000001 = .010001. And 1.001 − .010001 =

$$\begin{array}{r} 1.001 \\ .010001 \\ \hline .990999 \end{array} \quad \text{Ans.}$$

(76) $.49175 \times 30 = 14.7525$ pounds. Ans.

(77) If the cars at the shaft were the same size as those at the slope, the number of cars from the shaft would then be three times the number from the slope, its output being 3 times that of the slope. But the capacity of the shaft cars is double that of the slope cars; hence, instead of there being 3 times the number of cars coming from the shaft as from the slope, there are only $\frac{3}{2}$ times as many cars. Or $500 \times \frac{3}{2} = 750$ cars. Ans.

(78) $\frac{26}{5.2} = 5$ inches. Ans.

(79) $133.68 \times 1{,}728 = 230{,}999.04$ cu. in.

$\frac{230{,}999.04}{231} = 1{,}000$ gallons, nearly. Ans.

(80)

Cage,	1,850 pounds.
Car,	967 pounds.
Coal,	1,235 pounds.
Total load,	4,052 pounds.
Friction, $\frac{4{,}052}{4}$ =	1,013 pounds.
Rope, $.88 \times 250$ =	220 pounds.
Total strain,	5,285 pounds. Ans.

(81) $800 \times 1\frac{1}{2} = 1{,}200$ gallons pumped before water is turned on.

800 gallons flowing in per hr.

$(5 + 3) \times 60 = 480$ gallons flowing out per hr.

Difference $= 320$ gallons, net quantity flowing in per hr.

$\frac{2{,}000 - 1{,}200}{320} = 2\frac{1}{2}$ hours. $1\frac{1}{2} + 2\frac{1}{2} = 4$ hours. Ans.

(82) $900 \times \frac{2}{3} = 600$ tons, to railroad. Ans.

$900 \times \frac{1}{4} = 225$ tons, to private trade. Ans.

$600 + 225 = 825$ tons.

$900 - 825 = 75$ tons, to team sales. Ans.

(83) $720 \times \frac{2}{3} = 480$ tons, lump.
$720 \times \frac{1}{3} = 240$ tons, screenings.
$240 \times \frac{2}{3} = 160$ tons, nut.
$240 \times \frac{1}{3} = 80$ tons, steam coal.

$$\begin{array}{rrl} 480 \times \$1.50 = & \$720 \\ 160 \times 1.00 = & 160 \\ 80 \times .25 = & 20 \\ \hline \text{Total sales} = & \$900 & \text{Ans.} \end{array}$$

(84) $\$7{,}230 \div 328 = \$22.04$ per foot. Ans.

(85) $62.5 \times 1.27 = 79.375$ pounds. Ans.

(86) $2{,}000 \div 93.5 = 21.39$ cu. ft. Ans.

(87) $.49175 \div .03617 = 13.5955$ cu. in. Ans.

(88)
$$1 - \frac{3}{8} = \frac{5}{8}$$
$$270 = \frac{3}{8} \text{ of the whole.}$$
$$\frac{270}{3} = 90 = \frac{1}{8} \text{ of the whole.}$$

$90 \times 5 = 450 = \frac{5}{8}$ of the whole, or the negroes employed.
Ans. 450 negroes.

(89) Pounds of coal mined $= 5{,}000 \times 344 = 1{,}720{,}000$ lb.
Each car contains $2{,}000 \times 20 = 40{,}000$ lb.
Number of cars $= 1{,}720{,}000 \div 40{,}000 = 43$. Ans.

(90)
$$\begin{array}{lrr} \text{Lump,} & 25 \times 20 = 500 \text{ tons.} & \\ \text{Lump,} & 20 \times 16 = 320 \text{ tons.} & \\ \hline \text{Total,} & 820 \times 1.40 = & \$1{,}148 \\ \text{Nut,} & 15 \times 16 = 240 \times .90 = & 216 \\ \text{Steam,} & 12 \times 16 = 192 \times .25 = & 48 \\ \hline \text{Total receipts,} & & \$1{,}412 \quad \text{Ans.} \end{array}$$

(91)
$$\begin{array}{lrr} \text{Props,} & 1{,}000 \times 4 = 4{,}000 \text{ feet.} & \\ \text{Props,} & 1{,}200 \times 5 = 6{,}000 \text{ feet.} & \\ \hline \text{Total,} & 10{,}000 \times .01 = & \$100.00 \\ \text{Caps,} & \dfrac{2{,}640 \times 7.50}{1{,}000} = & 19.80 \\ \hline \text{Total cost,} & & \$119.80 \quad \text{Ans.} \end{array}$$

ARITHMETIC.

(PART 2.)

(92) A certain per cent. of a number means so many hundredths of that number.

25% of 8,428 lb. means 25 hundredths of 8,428 lb. Hence, 25% of 8,428 lb. = .25 × 8,428 lb. = 2,107 lb. Ans.

(93) Here $100 is the base and 1% = .01 is the rate. Then, .01 × $100 = $1. Ans.

(94) $\frac{1}{2}$% means one-half of one per cent. Since 1% is .01, $\frac{1}{2}$% is .005, for, $\dfrac{2\,).010}{.005}$. And .005 × $35,000 = $175. Ans.

(95) Here 50 is the base, 2 is the percentage, and it is required to find the rate. Applying rule, Art. **193**,

rate = percentage ÷ base;
rate = 2 ÷ 50 = .04 or 4%. Ans.

(96) By Art. **193**, rate = percentage ÷ base.*
As percentage = 10 and base = 10, we have rate = 10 ÷ 10 = 1 = 100%. Hence, 10 is 100% of 10. Ans.

(97) (*a*) Rate = percentage ÷ by base. Art. **193**.
As percentage = $176.54 and base = $2,522, we have

rate = 176.54 ÷ 2,522 = .07 = 7%. Ans.

$$\begin{array}{r} 2522\,)\,\underline{176.54} \\ .07 \end{array}$$

---

* Remember that an expression of this form means that the first term is to be *divided by* the second term. Thus, as above, it means percentage *divided by* base.

(*b*) Base = percentage ÷ rate. Art. **192.**

As percentage = 16.96 and rate = 8% = .08, we have

$$\text{base} = 16.96 \div .08 = 212. \quad \text{Ans.}$$

```
.08 ) 16.96
      -----
       212
```

(*c*) Amount is the sum of the base and percentage; hence, the percentage = amount minus the base.

Amount = 216.7025 and base = 213.5; hence, percentage = 216.7025 − 213.5 = 3.2025.

Rate = percentage ÷ base. Art. **193.**

Therefore, rate = 3.2025 ÷ 213.5 = .015 = $1\frac{1}{2}$%. Ans.

```
213.5 ) 3.2025 ( .015 = 1½%
        2135
        -----
        10675
        10675
        -----
```

(*d*) The difference is the remainder found by subtracting the percentage from the base; hence, base − the difference = the percentage. Base = 207 and difference = 201.825, hence percentage = 207 − 201.825 = 5.175.

Rate = percentage ÷ base. Art. **193.**

$$\text{Therefore, rate} = 5.175 \div 207 = .025 = .02\frac{1}{2} = 2\frac{1}{2}\%. \quad \text{Ans.}$$

```
207 ) 5.175 ( .025
      414
      ----
      1035
      1035
      ----
```

(**98**) In this problem $5,500 is the amount, since it equals what he paid for the farm + what he gained; 15% is the rate, and the cost (to be found) is the base. Applying rule, Art. **197,**

base = amount ÷ (1 + rate); hence,
base = $5,500 ÷ (1 + .15) = $4,782.61. Ans.

$$
\begin{array}{l}
1.15)5500.0000(4782.61 \\
\underline{460} \\
\quad 900 \\
\quad \underline{805} \\
\quad\quad 950 \\
\quad\quad \underline{920} \\
\quad\quad\quad 300 \\
\quad\quad\quad \underline{230} \\
\quad\quad\quad\quad 700 \\
\quad\quad\quad\quad \underline{690} \\
\quad\quad\quad\quad\quad 100 \\
\quad\quad\quad\quad\quad \underline{115}
\end{array}
$$

The example can also be solved as follows: 100% = cost; if he gained 15%, then 100 + 15 = 115% = $5,500, the selling price. If 115% = $5,500, 1% = $\frac{1}{115}$ of $5,500 = $47.8261, and 100%, or the cost, = 100 × $47.8261 = $4,782.61. Ans.

(99)

$$
\begin{array}{lll}
24\ \% \text{ of } \$950 = .24 \times 950 & = & \$228 \\
12\frac{1}{2}\% \text{ of } \$950 = .125 \times 950 & = & 118.75 \\
\underline{17\ \%} \text{ of } \$950 = .17 \times 950 & = & \underline{161.50} \\
53\frac{1}{2}\% \text{ of } \$950 & = & \$508.25
\end{array}
$$

The total amount of his yearly expenses, then, is $508.25, hence his savings are $950 − $508.25 = $441.75. Ans.

Or, as above, 24% + 12$\frac{1}{2}$% + 17% = 53$\frac{1}{2}$%, the total percentage of expenditures; hence, 100% − 53$\frac{1}{2}$% = 46$\frac{1}{2}$% = per cent. saved. And $950 × .465 = $441.75 = his yearly savings. Ans.

(100) The percentage is 961.38, and the rate is .37$\frac{1}{2}$. By Art. **192,**

Base = percentage ÷ rate
= 961.38 ÷ .375 = 2,563.68, the number. Ans.

Another method of solving is the following:

If $37\frac{1}{2}\%$ of a number is 961.38, then $.37\frac{1}{2}$ times the number = 961.38 and the number = $961.38 \div .37\frac{1}{2}$, which, as above = 2,563.68. Ans.

```
.375)961.38000(2563.68
     750
     ----
     2113
     1875
     ----
      2388
      2250
      ----
       1380
       1125
       ----
        2550
        2250
        ----
         3000
         3000
         ----
```

(101) Here \$1,125 is 30% of some number; hence, \$1,125 = the percentage, 30% = the rate, and the required number is the base. Applying rule, Art. **192,**

Base = percentage ÷ rate = \$1,125 ÷ .30 = \$3,750.

Since \$3,750 is $\frac{3}{4}$ of the property, one of the fourths is $\frac{1}{3}$ of \$3,750 = \$1,250, and $\frac{4}{4}$ or the entire property, is 4 × \$1,250 = \$5,000. Ans.

(102) Here \$4,810 is the difference and 35% the rate. By Art. **198,**

Base = difference ÷ (1 − rate)
= \$4,810 ÷ (1 − .35) = \$4,810 ÷ .65 = \$7,400. Ans.

```
.65)4810.00(7400.
    455
    ---
     260            1.00
     260             .35
     ---            ----
      00             .65
```

Solution can also be effected as follows: 100% = the sum diminished by 35%, then (1 − .35) = .65, which is \$4,810.

If 65% = \$4,810, 1% = $\frac{1}{65}$ of 4,810 = \$74, and 100% = 100 × \$74 = \$7,400. Ans.

(103) In this example the sales on Monday amounted to \$197.55, which was $12\frac{1}{2}$% of the sales for the entire week; i. e., we have given the percentage, \$197.55, and the rate, $12\frac{1}{2}$%, and the required number (or the amount of sales for the week) equals the base. By Art. **192**,

Base = percentage ÷ rate = \$197.55 ÷ .125;

or,

```
.125)197.5500(1580.4  Ans.
     125
     ---
      725
      625
      ---
      1005
      1000
      ----
         500
         500
         ---
```

Therefore, base = \$1,580.40, which also equals the sales for the week.

(104) 16.5 miles = $12\frac{1}{2}$% of the entire length of the road. We wish to find the *entire* length.

16.5 miles is the percentage, $12\frac{1}{2}$% is the rate, and the entire length will be the base. By Art. **192**,

Base = percentage ÷ rate = 16.5 ÷ .12$\frac{1}{2}$.

```
.125)16.500(132 miles.  Ans.
     125
     ---
      400
      375
      ---
       250
       250
       ---
```

(105) Here we have given the difference, or $35, and the rate, or 60%, to find the base. We use the rule in Art. **198**,

Base = difference ÷ (1 − rate)

= $35 ÷ (1 − .60) = $35 ÷ .40 = $87.50. Ans.

```
.40)35.000(87.5
    320
    ---
     300
     280
     ---
      200
      200
      ---
```

Or, 100% = whole debt; 100% − 60% = 40% = $35.

If 40% = $35, then $1\% = \frac{1}{40}$ of $35 = $\frac{35}{40}$, and 100% = $\frac{35}{40} \times 100 =$ $87.50. Ans.

(106) 28 rd. 4 yd. 2 ft. 10 in. to inches.

```
 ×  5½
   ----
   154
 +   4
   ----
   158 yards
 ×   3
   ----
   474
 +   2
   ----
   476 feet
×   12
  -----
  5712
+   10
  -----
  5722 inches. Ans.
```

Since there are 5½ yards in one rod, in 28 rods there are 28 × 5½ or 154 yards; 154 yards plus 4 yards = 158 yards. There are 3 feet in one yard; therefore, in 158 yards there are 3 × 158 or 474 feet; 474 feet + 2 feet = 476 feet. There are 12 inches in one foot, and in 476 feet there are 12 × 476 or 5,712 inches; 5,712 inches + 10 inches = 5,722 inches. Ans.

(107)

```
12 ) 5722 inches.
 3 ) 476 + 10 inches.
5½ ) 158 + 2 feet.
      28 + 4 yards.
```

Ans. = 28 rd. 4 yd. 2 ft. 10 in.

EXPLANATION.—There are 12 inches in 1 foot; hence, in 5,722 inches there are as many feet as 12 is contained times in 5,722 inches, or 476 ft. and 10 inches remaining. Write these 10 inches as a remainder. There are 3 feet in 1 yard; hence, in 476 feet there are as many yards as 3 is contained times in 476 feet, or 158 yards and 2 feet remaining. There are $5\frac{1}{2}$ yards in one rod; hence, in 158 yards there are 28 rods and 4 yards remaining. Then, in 5,722 inches there are 28 rd. 4 yd. 2 ft. 10 in.

(108)

$$\begin{array}{rl} 5 & \text{weeks } 3.5 \text{ days.} \\ \times\ 7 & \\ \hline 35 & \text{days in 5 weeks.} \\ +\ 3.5 & \\ \hline 38.5 & \text{days.} \end{array}$$

Then, we find how many seconds there are in 38.5 days.

$$\begin{array}{rl} 38.5 & \text{days} \\ \times\ 24 & \text{hours in one day.} \\ \hline 1540 & \\ 770 & \\ \hline 924.0 & \text{hours in 38.5 days.} \\ \times\ 60 & \text{minutes in one hour.} \\ \hline 55440 & \text{minutes in 38.5 days.} \\ \times\ 60 & \text{seconds in one minute.} \\ \hline 3326400 & \text{seconds in 38.5 days. Ans.} \end{array}$$

(109) Since there are 24 gr. in 1 pwt., in 13,750 gr. there are as many pennyweights as 24 is contained times in 13,750, or 572 pwt. and 22 gr. remaining. Since there are 20 pwt. in 1 oz., in 572 pwt. there are as many ounces as 20 is contained times in 572, or 28 oz. and 12 pwt. remaining. Since there are 12 oz. in 1 lb. (Troy), in 28 oz. there are as many pounds as 12 is contained times in 28, or 2 lb. and 4 oz. remaining. We now have the pounds and ounces required by the problem; therefore, in 13,750 gr. there are 2 lb. 4 oz. 12 pwt. 22 gr.

```
24 ) 13750 gr.
     20 ) 572 pwt. + 22 gr.
          12 ) 28 oz. + 12 pwt.
                2 lb. + 4 oz.
```

Ans. = 2 lb. 4 oz. 12 pwt. 22 gr.

(110)

```
100 ) 4763254 li.
      80 ) 47632 + 54 li.
              595 + 32 ch.
```

Ans. = 595 mi. 32 ch. 54 li.

Explanation.—There are 100 li. in one chain; hence, in 4,763,254 li. there are as many chains as 100 is contained times in 4,763,254 li., or 47,632 ch. and 54 li. remaining. Write the 54 li. as a remainder. There are 80 ch. in one mile; hence, in 47,632 ch. there are as many miles as 80 is contained times in 47,632 ch., or 595 miles and 32 ch. remaining. Then, in 4,763,254 li. there are 595 mi. 32 ch. 54 li.

(111)

```
1728 ) 764325 cu. in.
       27 ) 442 + 549 cu. in.
             16 cu. yd. + 10 cu. ft.
```

Ans. = 16 cu. yd. 10 cu. ft. 549 cu. in.

Explanation.—There are 1,728 cu. in. in one cubic foot; hence, in 764,325 cu. in. there are as many cubic feet as 1,728 is contained times in 764,325, or 442 cu. ft. and 549 cu. in. remaining. Write the 549 cu. in. as a remainder. There are 27 cu. ft. in one cubic yard; hence, in 442 cu. ft. there are as many cubic yards as 27 is contained times in 442 cu. ft., or 16 cu. yd. and 10 cu. ft. remaining. Then, in 764,325 cu. in. there are 16 cu. yd. 10 cu. ft. 549 cu. in.

(112) We must arrange the different terms in columns, taking care to have like denominations in the same column.

	rd.	yd.	ft.	in.	
	2	2	2	3	
		4	1	9	
			2	7	
	3	$2\frac{1}{2}$	0	7	
or	3	2	2	1	Ans.

Explanation.—We begin to add at the right-hand column. $7 + 9 + 3 = 19$ in.; as 12 in. make one foot, 19 in. = 1 ft. and 7 in. Place the 7 in. in the inches column, and reserve the 1 ft. to add to the next column.

1 (reserved) $+ 2 + 1 + 2 = 6$ ft. Since 3 ft. make 1 yard. 6 ft. = 2 yd. and 0 ft. remaining. Place the cipher in the column of feet and reserve the 2 yd. for the next column.

2 (reserved) $+ 4 + 2 = 8$ yd. Since $5\frac{1}{2}$ yd. = 1 rod, 8 yd. = 1 rd. and $2\frac{1}{2}$ yd. Place $2\frac{1}{2}$ yd. in the yards column, and reserve 1 rd. for the next column; 1 (reserved) $+ 2 = 3$ rd.

Ans. =	3 rd.	$2\frac{1}{2}$ yd.	0 ft.	7 in.	
or,	3 rd.	2 yd.	1 ft.	13 in.	
or,	3 rd.	2 yd.	2 ft.	1 in.	Ans.

(113) We write the compound numbers so that the units of the same denomination shall stand in the same column. Beginning to add with the lowest denomination, we find that the sum of the gills is $1 + 2 + 3 = 6$. Since there are 4 gi. in 1 pint, in 6 gi. there are as many pints as 4 is contained times in 6, or 1 pt. and 2 gi. We place 2 gi. under the gills column and reserve the 1 pt. for the pints column; the sum of the pints is 1 (reserved) $+ 5 + 1 + 1 = 8$. Since there are 2 pt. in 1 quart, in 8 pt. there are as many quarts as 2 is contained times in 8, or 4 qt. and 0 pt. We place the cipher under the column of pints and reserve the 4 for the quarts column. The sum of the quarts is 4 (reserved) $+ 8 + 3 = 15$. Since there are 4 qt. in 1 gallon, in 15 qt. there are as many gallons as 4 is contained times in 15, or 3 gal. and 3 qt. remaining. We now place the 3 under the quarts column and reserve the 3 gal. for the gallons column. The sum of the gallons column is 3 (reserved) $+ 4 + 6 + 3 = 16$ gal. Since we can not reduce 16 gal. to any higher denomination, we have 16 gal. 3 qt. 0 pt. and 2 gi. for the answer.

gal.	qt.	pt.	gi.
3	3	1	3
6	0	1	2
4	0	0	1
	8	5	0
16 gal.	3 qt.	0 pt.	2 gi.

(114) Reduce the grains, pennyweights, and ounces to higher denominations.

24) 240 gr.	20) 125 pwt.	12) 50 oz.
10 pwt.	6 oz. 5 pwt.	4 lb. 2 oz.

Then, 3 lb. + 4 lb. 2 oz. + 6 oz. 5 pwt. + 10 pwt. =

lb.	oz.	pwt.	
3			
4	2		
	6	5	
		10	
7 lb.	8 oz.	15 pwt.	Ans.

(115) Since "seconds" is the lowest denomination in this problem, we find their sum first, which is 11 + 29 + 25 + 30 + 12, or 107 seconds. Since there are 60 seconds in 1 minute, in 107" there are as many minutes as 60 is contained times in 107, or 1 minute and 47 seconds remaining. We place the 47 under the seconds column and reserve the 1 for the minutes column. The sum of the minutes is 1 (reserved) + 17 + 26 + 19 + 16, or 79. Since there are 60 minutes in 1 degree, in 79 minutes there are as many degrees as 60 is contained times in 79, or 1 degree and 19 minutes remaining. We place the 19 under the minutes column and reserve the 1 degree for the degrees column. The sum of the degrees is 1 (reserved) + 10 + 20 + 13 + 11, or 55 degrees. Since we can not reduce 55 degrees to any higher denominations, we have 55° 19' 47" for the answer.

deg.	min.	sec.
11	16	12
13	19	30
20	0	25
0	26	29
10	17	11
55°	19'	47"

(116) Since "inches" is the lowest denomination in this problem, we find their sum first, which is 11 + 8 + 6, or 25 inches. Since there are 12 inches in 1 foot, in 25 inches there are as many feet as 12 is contained times in 25, or 2 feet and 1 inch remaining. Place the 1 inch under the inches column, and reserve the 2 feet to add to the column of feet. The sum of the feet is 2 feet (reserved) + 2 + 1 = 5 feet. Since there are 3 feet in 1 yard, in 5 feet there are as many yards as 3 is contained times in 5 feet, or 1 yard and 2 feet remaining. Place the 2 feet under the column of feet, and reserve the 1 yard to add to the column of yards. The sum of the yards is 1 yard (reserved) + 4 + 5 = 10 yards. Since there are $5\frac{1}{2}$ yards in 1 rod, in 10 yards there are as many rods as $5\frac{1}{2}$ is contained times in 10, or 1 rod and $4\frac{1}{2}$ yards remaining. Place the $4\frac{1}{2}$ yards under the column of yards, and reserve the 1 rod for the column of rods. The sum of the rods is 1 (reserved) + 304 + 215 + 130 = 650 rods. Place 650 rods under the column of rods. Therefore, the sum is 650 rd. $4\frac{1}{2}$ yd. 2 ft. 1 in. Or, since $\frac{1}{2}$ a yard = 1 ft. 6 in., and since there are 320 rods in 1 mile, the sum may be expressed as 2 mi. 10 rd. 5 yd. 0 ft. 7 in. Ans.

mi.	rd.	yd.	ft.	in.
	130	5	1	6
	215	0	2	8
	304	4	0	11
	650	$4\frac{1}{2}$	2	1
or, 2	10	5	0	7 Ans.

(117) Since "square links" is the lowest denomination in this problem, we find their sum first, which is 21 + 23 + 16 + 18 + 23 + 21, or 122 square links. Place 122 square links under the column of square links. The sum of the square rods is 2 + 3 + 2 + 2 + 2 + 3, or 14 square rods. Place 14 square rods under the column of square rods. The sum of the square chains is 323 square chains. Since there are 10 square chains in 1 acre, in 323 square chains there are as many acres as 10 is

A.	sq. ch.	sq. rd.	sq. li.
21	67	3	21
28	78	2	23
47	6	2	18
56	59	2	16
25	38	3	23
46	75	2	21
255	3	14	122

contained times in 323 square chains, or 32 acres and 3 square chains remaining. Place 3 square chains under the column of square chains, and reserve the 32 acres to add to the column of acres. The sum of the acres is 32 acres (reserved) + 46 + 25 + 56 + 47 + 28 + 21, or 255 acres. Place 255 acres under the column of acres. Therefore, the sum is 255 A. 3 sq. ch. 14 sq. rd. 122 sq. li. Ans.

(118) Before we can subtract 300 ft. from 20 rd. 2 yd. 2 ft. and 9 in., we must reduce the 300 ft. to higher denominations.

Since there are 3 feet in 1 yard, in 300 feet there are as many yards as 3 is contained times in 300, or 100 yards. There are $5\frac{1}{2}$ yards in 1 rod, hence in 100 yards there are as many rods as $5\frac{1}{2}$ or $\frac{11}{2}$ is contained times in $100 = 18\frac{2}{11}$ rods.

$$100 \div \frac{11}{2} = 100 \times \frac{2}{11} = \frac{100 \times 2}{11} = \frac{200}{11}$$

$$\begin{array}{r} 11\,)\,200\,(\,18\frac{2}{11}\text{ rd.} \\ \underline{11} \\ 90 \\ \underline{88} \\ 2 \end{array}$$

Since there are $5\frac{1}{2}$ or $\frac{11}{2}$ yards in 1 rod, in $\frac{2}{11}$ rods there are $\frac{2}{11} \times \frac{11}{2}$, or one yard, so we find that 300 feet equals 18 rods and 1 yard. The problem now is as follows: From 20 rd. 2 yd. 2 ft. and 9 in. take 18 rd. and 1 yd.

We place the smaller number under the larger one, so that units of the same denomination fall in the same column. Beginning with the lowest denomination, we see that 0 inches from 9 inches leaves 9 inches. Going to the next higher denomination, we see that 0 feet from 2 feet leaves 2 feet. Subtracting 1 yard from 2

rd.	yd.	ft.	in.
20	2	2	9
18	1	0	0
2	1	2	9

yards, we have 1 yard remaining, and 18 rods from 20 rods leaves 2 rods. Therefore, the difference is 2 rd. 1 yd. 2 ft. 9 in. Ans.

(119)

A.	sq. rd.	sq. yd.	
114	80	25	
75	70	30	
39	9	$25\frac{1}{4}$	Ans.

EXPLANATION.—Place the subtrahend under the minuend so that like denominations are under each other. Then begin at the right with the lowest denomination. We can not subtract 30 from 25, so we take one square rod ($= 30\frac{1}{4}$ square yards) from 80 square rods, leaving 79 square rods; adding $30\frac{1}{4}$ square yards to 25 square yards, we have $55\frac{1}{4}$ square yards; subtracting 30 from $55\frac{1}{4}$ square yards leaves $25\frac{1}{4}$ square yards; we now subtract 70 square rods from 79 square rods, which leaves 9 square rods; next, we subtract 75 acres from 114 acres, which leaves 39 acres, which we place under the column of acres.

(120) If 10 gal. 2 qt. and 1 pt. of molasses are sold from a hogshead at one time, and 26 gal. 3 qt. are sold at another time, then the total amount of molasses sold equals 10 gal. 2 qt. 1 pt. plus 26 gal. 3 qt.

Since the pint is the lowest denomination, we add the pints first, which equal $0 + 1$, or 1 pint. We can not reduce 1 pint to any higher denomination, so we place it under the pint column. The number of quarts is $3 + 2$, or 5. Since there are 4 quarts in 1 gallon, in 5 quarts there are as many gallons as 4 is contained times in 5, or 1 gallon and 1 quart remaining. We place the 1 quart under the quart column, and reserve the 1 gallon to add to the column of

gal.	qt.	pt.
10	2	1
26	3	0
37 gal.	1 qt.	1 pt.

gallons. The number of gallons equals 1 (reserved) + 26 + 10, or 37 gallons.

If 37 gal. 1 qt. and 1 pt. are sold from a hogshead of molasses (63 gal.), there remains the difference between 63 gal. and 37 gal. 1 qt. 1 pt., or 25 gal. 2 qt. and 1 pt.

63 gal. is the same as 62 gal. 3 qt. 2 pt., since 1 gal. equals 4 qt. and 1 qt. = 2 pt.

Beginning with the lowest denomination, 1 pt. from the 2 pt. 1 pint from 2 pints leaves 1 pint. One quart from 3 quarts leaves 2 quarts, and 37 gallons from 62 gallons leaves 25 gallons. Therefore, there are 25 gal. 2 qt. and 1 pt. of molasses remaining in the hogshead. Ans.

gal.	qt.	pt.
62	3	2
37	1	1
25	2	1

(121) If a person were born June 19, 1850, in order to find how old he would be on Aug. 3, 1892, subtract the earlier date from the later date.

On August 3, 7 mo. and 3 da. have elapsed from the beginning of the year, and on June 19, 5 mo. and 19 da.

Beginning with the lowest denomination, we find that 19 days can not be taken from 3 days, so we take 1 month from 7 months. The 1 month which we took equals 30 days, for in all cases 30 days are allowed to a month. Adding 30 days to the 3 days, we have 33 days; subtracting 19 days from 33 days, we have 14 days remaining. Since we borrowed 1 month from the months column, we have 7 − 1, or 6 months remaining; subtracting 5 months from 6 months, we have 1 month remaining. 1850 from 1892 leaves 42 years. Therefore, he would be 42 years 1 month and 14 days old. Ans.

yr.	mo.	da.
1892	7	3
1850	5	19
42	1	14

(122) If a note given Aug. 5, 1890, were paid June 3, 1892, in order to find the length of time it was due, subtract the earlier date from the later date.

Beginning with the lowest denomination, we find that 5 can not be subtracted from 3, so we take a unit from the next

yr.	mo.	da.
1892	5	3
1890	7	5
1	9	28

higher denomination, which is months. The 1 month which we take equals 30 days. Adding the 30 days to the 3 days, we have 33 days. 5 days from 33 days leaves 28 days. Since we took 1 month from the months column, only 4 months remain. 7 months cannot be taken from 4 months, so we take 1 year from the years column, which equals 12 months. 12 months + 4 months = 16 months. 7 months from 16 months = 9 months. Since we took 1 year from the years column, we have 1892 − 1, or 1891 remaining. 1890 from 1891 leaves 1 year. Hence, the note ran 1 year 9 months and 28 days. Ans.

(123) Write the number of the year, month, day, hour, and minute of the earlier date under the year, month, day, hour, and minute of the later date, and subtract.

22 minutes before 8 o'clock is the same as 38 minutes after 7 o'clock. 7 o'clock P. M. is 19 hours from the beginning of the day, as there are 12 hours in the morning and 7 in the afternoon. December is 11 months from the beginning of the year.

10 o'clock A. M. is 10 hours from the beginning of the day. July is 6 months from the beginning of the year. The minuend would be the later date, or 1,888 years, 11 months, 11 days, 19 hours, and 38 minutes.

The subtrahend would be the earlier date, or 1,883 years, 6 months, 3 days, 10 hours, and 16 minutes.

Subtracting, we have

yr.	mo.	da.	hr.	min.
1888	11	11	19	38
1883	6	3	10	16
5	5	8	9	22

or, 5 yr. 5 mo. 8 da. 9 hr. and 22 min. Ans.

16 minutes subtracted from 38 minutes leaves 22 minutes; 10 hours from 19 hours leaves 9 hours; 3 days from 11 days leaves 8 days; 6 months subtracted from 11 months leaves 5 months; 1,883 from 1,888 leaves 5 years.

(124) In multiplication of denominate numbers, we place the multiplier under the lowest denomination of the multiplicand, as

17 ft.	3 in.
	51
879 ft.	9 in.

and begin at the right to multiply. $51 \times 3 = 153$ in. As there are 12 inches in 1 foot, in 153 in. there are as many feet as 12 is contained times in 153, or 12 feet and 9 inches remaining. Place the 9 inches under the inches, and reserve the 12 feet. 51×17 ft. $= 867$ ft. 867 ft. $+ 12$ ft. (reserved) $= 879$ ft.

879 feet can be reduced to higher denominations by dividing by 3 feet to find the number of yards, and by $5\frac{1}{2}$ yards to find the number of rods.

3) 879 ft. 9 in.
5.5) 293 yd.
53 rd. $1\frac{1}{4}$ yd.

Then, answer = 53 rd. $1\frac{1}{4}$ yd. 0 ft. 9 in.; or 53 rd. 1 yd 2 ft. 3 in

(125)

	qt.	pt.	gi.	
	3	1	3	
			4.7	
	18.2 qt.	0	.1	
or,	18 qt.	0 pt.	1.7 gi.	
or, 4 gal.	2 qt.	0 pt.	1.7 gi.	Ans.

Place the multiplier under the lowest denomination of the multiplicand, and proceed to multiply. 4.7×3 gi. $= 14.1$ gi. As 4 gi. = 1 pt., there are as many pints in 14.1 gi. as 4 is contained times in $14.1 = 3.5$ pt. and .1 gi. over. Place .1 under gills and carry the 3.5 pt. forward. 4.7×1 pt. $= 4.7$ pt.; $4.7 + 3.5$ pt. $= 8.2$ pt. As 2 pt. = 1 qt., there are as many quarts in 8.2 pt. as 2 is contained times in $8.2 = 4.1$ qt. and no pints over. Place a cipher under the pints, and carry the 4.1 qt. to the next product. 4.7×3 qt. $= 14.1$; $14.1 + 4.1 = 18.2$ qt. The answer now is 18.2 qt. 0 pt. .1

gi. Reducing the fractional part of a quart, we have 18 qt. 0 pt. 1.7 gi. (.2 qt. = .2 × 8 = 1.6 gi.; 1.6 + .1 gi. = 1.7 gi.). Then, we can reduce 18 qt. to gallons (18 ÷ 4 = 4 gal. and 2 qt.) = 4 gal. 2 qt. 1.7 gi. Ans.

The answer may be obtained in another and much easier way by reducing all to gills, multiplying by 4.7, and then changing back to quarts and pints. Thus,

```
  3 qt.
× 2 pt.
-------
  6 pt.
+ 1 pt.
-------
  7 pt.
× 4 gi.
-------
 28 gi.
+ 3 gi.
-------
 31 gi.
```

3 qt. 1 pt. 3 gi. = 31 gi.
31 gi. × 4.7 = 145.7 gi.

```
4)145.7 gi.
 2)36     pt. + 1.7 gi.
   18     qt. +   0 pt.
```

Ans. = 18 qt. 1.7 gi.;
or, 4 gal. 2 qt. 1.7 gi.

(126) (3 lb. 10 oz. 13 pwt. 12 gr.) × 1.5 = ?

```
      3 lb. 10 oz. 13 pwt. 12 gr.
   ×12
    36   oz.
   +10
    46   oz.
   × 20
    920 pwt.
   + 13
    933 pwt.
   × 24
  22392 gr.
   + 12
  22404 gr.
```

22,404 gr. × 1.5 = 33,606 gr.

```
24)33606 gr.
  20)1400 pwt. + 6 gr
     12)70 oz. + 0 pwt
         5 lb. + 10 oz.
```

Since there are 24 gr. in 1 pwt., in 33,606 gr. there are as many pwt. as 24 is contained times in 33,606, or 1,400 pwt. and 6 gr. remaining. This gives us the number of grains in the answer. We now reduce 1,400 pwt. to higher denominations. Since there are 20 pwt. in 1 oz., in 1,400 pwt. there are as many ounces as 20 is contained times in 1,400, or 70 oz. and 0 pwt. remaining; therefore, there are 0 pwt. in the answer. We reduce 70 oz. to higher denominations. Since there are 12 oz. in 1 lb., in 70 oz. there are as many pounds as 12 is contained times in 70, or 5 lb. and 10 oz. remaining. We can not reduce 5 lb. to any higher denominations. Therefore, our answer is 5 lb. 10 oz. 6 gr.

Another but more complicated way of working this problem is as follows:

lb.	oz.	pwt.	gr.	
3	10	13	12	
			1.5	
4.5	15	19.5	18	
or, 4	21	19	30	
or, 5	10	0	6	Ans.

To get rid of the decimal in the pounds, reduce .5 of a pound to ounces. Since 1 lb. = 12 oz., .5 of a pound equals .5 lb. × 12 = 6 oz. 6 oz. + 15 oz. = 21 oz. We now have 4 lb. 21 oz. 19.5 pwt. and 18 gr., but we still have a decimal in the column of pwt., so we reduce .5 pwt. to grains to get rid of it. Since 1 pwt. = 24 gr., .5 pwt. = .5 pwt. × 24 = 12 gr. 12 gr. + 18 gr. = 30 gr. We now have 4 lb. 21 oz. 19 pwt. and 30 gr. Since there are 24 gr. in 1 pwt., in 30 gr. there is 1 pwt. and 6 gr. remaining. Place 6 gr. under the column of grains and add 1 pwt. to the pwt. column. Adding 1 pwt., we have 19 + 1 = 20 pwt. Since there are 20 pwt. in 1 oz., we have 1 oz. and 0 pwt. remaining. Write the 0 pwt. under the pwt. column, and reserve the 1 oz. to the oz. column. 21 oz. + 1 oz. = 22 oz. Since there are 12 oz. in 1 lb., in 22 oz. there is 1 lb. and 10 oz. remaining. Write the 10 oz. under the ounce column, and reserve the 1 lb. to add to the lb. column. 4 lb. + 1 lb. (reserved) = 5 lb. Hence, the answer equals 5 lb. 10 oz. 6 gr.

(127) If each barrel of apples contains 2 bu. 3 pk. and 6 qt., then 9 bbl. will contain 9 × (2 bu. 3 pk. 6 qt.).

We write the multiplier under the lowest denomination of the multiplicand, which is quarts in this problem. 9 times 6 qt. equals 54 qt. There are 8 qt. in 1 pk., and in 54 qt. there are as many pecks as 8 is contained times in 54, or 6 pk. and 6 qt. We write the 6 qt. under the column of quarts, and reserve the 6 pk. to add to the product of the pecks. 9 times 3 pk. equals 27 pk.; 27 pk. plus the 6 pk. reserved equals 33 pk. Since there are 4 pk. in 1 bu., in 33 pk. there are as many bushels as 4 is contained times in 33, or 8 bu. and 1 pk. remaining. We write the 1 pk. under the column of pecks, and reserve the 8 bu. for the product of the bushels. 9 times 2 bu. plus the 8 bu. reserved equals 26 bu. Therefore, we find that 9 bbl. contain 26 bu. 1 pk. 6 qt. of apples. Ans.

	bu.	pk.	qt.
	2	3	6
			9
	18	27	54
or,	26	1	6

(128) (7 T. 15 cwt. 10.5 lb.) × 1.7 = ? When the multiplier is a decimal, instead of multiplying the denominate numbers as in the case when the multiplier is a whole number, it is much easier to reduce the denominate numbers to the lowest denomination given; then, multiply that result by the decimal, and, lastly, reduce the product to higher denominations. Although the correct answer can be obtained by working examples involving decimals in the manner as in the last example, it is much more complicated than this method.

$$
\begin{array}{rl}
 & 7\text{ T. } 15\text{ cwt. } 10.5\text{ lb.} \\
\times & 20 \\
\hline
 & 140\text{ cwt.} \\
 & 15 \\
\hline
 & 155\text{ cwt.} \\
\times & 100 \\
\hline
 & 15500\text{ lb.} \\
 & 10.5 \\
\hline
 & 15510.5\text{ lb.}
\end{array}
$$

15,510.5 lb. × 1.7 = 26,367.85 lb.

There are 100 lb. in 1 cwt., and in 26,367.85 lb. there are as many cwt. as 100 is contained times in 26,367.85, which equals 263 cwt. and 67.85 lb. remaining. Since we have the number of pounds for our answer, we reduce 263 cwt. to higher denominations.

$$\begin{array}{r} 100\,\underline{)\,2\,6\,3\,6\,7.8\,5 \text{ lb.}} \\ 20\,\underline{)\,2\,6\,3 \text{ cwt.} + 67.85 \text{ lb.}} \\ 1\,3 \text{ T.} + 3 \text{ cwt.} \end{array}$$

There are 20 cwt. in 1 ton, and in 263 cwt. there are as many tons as 20 is contained times in 263, or 13 tons and 3 cwt. remaining. Since we cannot reduce 13 tons any higher, our answer is 13 T. 3 cwt. 67.85 lb. Or, since .85 lb. = .85 lb. × 16 = 13.6 oz., the answer may be written 13 T. 3 cwt. 67 lb. 13.6 oz.

(129)
$$\begin{array}{rrrrl} 7\,)\,358 \text{ A.} & 57 \text{ sq. rd.} & 6 \text{ sq. yd.} & 2 \text{ sq. ft.} & \\ \hline 51 \text{ A.} & 31 \text{ sq. rd.} & 0 \text{ sq. yd.} & 8 \text{ sq. ft.} & \text{Ans.} \end{array}$$

We begin with the highest denomination, and divide each term in succession by 7.

7 is contained in 358 A. 51 times and 1 A. remaining. We write the 51 A. under the 358 A. and reduce the remaining 1 A. to square rods = 160 sq. rd.; 160 sq. rd. + the 57 sq. rd. in the dividend = 217 sq. rd. 7 is contained in 217 sq. rd. 31 times and 0 sq. rd. remaining. 7 is not contained in 6 sq. yd., so we write 0 under the sq. yd. and reduce 6 sq. yd. to square feet. 9 sq. ft. × 6 = 54 sq. ft. 54 sq. ft. + 2 sq. ft. in the dividend = 56 sq. ft. 7 is contained in 56 sq. ft. 8 times. We write 8 under the 2 sq. ft. in the dividend.

(130)
$$\begin{array}{rrrrl} 12\,)\,282 \text{ bu.} & 3 \text{ pk.} & 1 \text{ qt.} & 1 \text{ pt.} & \\ \hline 23 \text{ bu.} & 2 \text{ pk.} & 2 \text{ qt.} & \frac{1}{4} \text{ pt.} & \text{Ans.} \end{array}$$

12 is contained in 282 bu. 23 times and 6 bu. remaining. We write 23 bu. under the 282 bu. in the dividend, and reduce the remaining 6 bu. to pecks = 24 pk. + the 3 pk. in the dividend = 27 pk. 12 is contained in 27 pk. 2 times and 3 pk. remaining. We write 2 pk. under the 3 pk. in the dividend, and reduce the remaining 3 pk. to quarts. 3 pk. = 24 qt.; 24 qt. + the 1 qt. in the dividend = 25 qt. 12 is contained in 25 qt. 2 times and 1 qt. remaining. We write

2 qt. under the 1 qt. in the dividend, and reduce 1 qt. to pints = 2 pt. + the 1 pt. in the dividend = 3 pt. $3 \div 12 = \frac{3}{12}$ or $\frac{1}{4}$ pt.

(131) We must first reduce 23 miles to feet before we can divide by 30 feet. 1 mi. contains 5,280 ft.; hence, 23 mi. contain 5,280 × 23 = 121,440 ft.

121,440 ft. ÷ 30 ft. = 4,048 rails for 1 side of the track.

The number of rails for 2 sides of the track = 2 × 4,048, or 8,096 rails. Ans.

(132) In this case where both dividend and divisor are compound, reduce each to the lowest denomination mentioned in either and then divide as in simple numbers.

```
  1 bu. 1 pk. 7 qt.          3 5 6 bu. 3 pk. 5 qt.
 × 4                              × 4
 ----                        ---------
   4 pk.                     1 4 2 4 pk.
 + 1                              + 3
 ----                        ---------
   5 pk.                     1 4 2 7 pk.
 × 8                              × 8
 ----                        ---------
  4 0 qt.                   1 1 4 1 6 qt.
 + 7                              + 5
 ----                        ---------
  4 7 qt.                   1 1 4 2 1 qt.
47 ) 1 1 4 2 1 ( 243
     9 4
     -----                  11,421 qt. ÷ 47 qt. = 243 boxes.
     2 0 2                                              Ans.
     1 8 8
     -----
       1 4 1
       1 4 1
       -----
```

(133) We must first reduce 16 square miles to acres.

In 1 sq. mi. there are 640 A., and in 16 sq. mi. there are 16 × 640 A. = 10,240 A.

```
62 ) 1 0 2 4 0 A.
     ------------------------------------------------
       1 6 5 A. 25 sq.rd. 24 sq.yd. 3 sq.ft. 80+sq. in.   Ans.
```

62 is contained in 10,240 A. 165 times and 10 A. remaining. We write 165 A. under the 10,240 A. in the dividend and reduce 10 A. to sq. rd. In 1 A. there are 160 sq. rd., and in 10 A. there are $10 \times 160 = 1,600$ sq. rd. 62 is contained in 1,600 sq. rd. 25 times and 50 sq. rd. remaining. We write 25 sq. rd. in the quotient and reduce 50 sq. rd. to sq. yd. In 1 sq. rd. there are $30\frac{1}{4}$ sq. yd., and in 50 sq. rd. there are 50 times $30\frac{1}{4}$ sq. yd. $= 1,512\frac{1}{2}$ sq. yd. 62 is contained in $1,512\frac{1}{2}$ sq. yd. 24 times and $24\frac{1}{2}$ sq. yd. remaining. In 1 sq. yd. there are 9 sq. ft., and in $24\frac{1}{2}$ sq. yd. there are $24\frac{1}{2} \times 9 = 220\frac{1}{2}$ sq. ft. 62 is contained in $220\frac{1}{2}$ sq. ft. 3 times and $34\frac{1}{2}$ sq. ft. remaining. We write 3 sq. ft. in the quotient and reduce $34\frac{1}{2}$ sq. ft. to sq. in. In 1 sq. ft. there are 144 sq. in., and in $34\frac{1}{2}$ sq. ft. there are $34\frac{1}{2} \times 144 = 4,968$ sq. in. 62 is contained in 4,968 sq. in. 80 times and 8 sq. in. remaining.

We write 80 sq. in. in the quotient.

It should be borne in mind that it is only for the purpose of illustrating the method that this problem is carried out to square inches. It is not customary to reduce any lower than square rods in calculating the area of a farm.

(**134**) To square a number, we must multiply the number by itself once, that is, use the number twice as a factor. Thus, the second power of 108 is $108 \times 108 = 11,664$. Ans.

$$\begin{array}{r} 108 \\ 108 \\ \hline 864 \\ 108 \\ \hline 11664 \end{array}$$

(135)

```
       181.25
       181.25
       ------
       90625
      36250
     18125
    145000
    18125
    -----------
    32851.5625
       181.25
    -----------
   1642578125
   657031250
  328515625
 2628125000
 328515625
 -------------
 5954345.703125
```

The third power of 181.25 equals the number obtained by using 181.25 as a factor three times. Thus, the third power of 181.25 is 181.25 × 181.25 × 181.25 = 5,954,345.703125. Ans.

Since there are 2 decimal places in the multiplier, and 2 in the multiplicand, there are 2 + 2 = 4 decimal places in the first product.

Since there are 4 decimal places in the multiplicand, and 2 in the multiplier, there are 4 + 2 = 6 decimal places in the final product.

(136)

```
            27.61
            27.61
            -----
            2761
           16566
          19327
          5522
          ---------
          762.3121
             27.61
          ---------
          7623121
        45738726
       53361847
      15246242
      -------------
      21047.437081
             27.61
      -------------
      21047437081
    126284622486
   147332059567
   42094874162
   ---------------
   581119.73780641
```

The fourth power of 27.61 is the number obtained by using 27.61 as a factor four times. Thus, the fourth power of 27.61 is 27.61 × 27.61 × 27.61 × 27.61 = 581,119.73780641. Ans.

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are 2 + 2 = 4 decimal places in the first product.

Since there are 4 decimal places in the multiplicand and 2 in the multiplier, there are 4 + 2 = 6 decimal places in the second product.

Since there are 6 decimal places in the multiplicand and 2 in the multiplier, there are 6 + 2 = 8 decimal places in the final product.

(137) (*a*) $106^2 = 106 \times 106 = 11{,}236$. Ans.

$$\begin{array}{r} 106 \\ 106 \\ \hline 636 \\ 1060 \\ \hline 11236 \end{array}$$

(*b*) $\left(182\frac{1}{8}\right)^2 = 182\frac{1}{8} \times 182\frac{1}{8} = 33{,}169.515625$. Ans.

$\frac{1}{8} = 8\overline{)1.000}$, giving $.125$

$$\begin{array}{r} 182.125 \\ 182.125 \\ \hline 910625 \\ 364250 \\ 182125 \\ 364250 \\ 1457000 \\ 182125 \\ \hline 33169.515625 \end{array}$$

Since there are 3 decimal places in the multiplier and 3 in the multiplicand, there are 3 + 3 = 6 decimal places in the product.

(*c*) $.005^2 = .005 \times .005 = .000025$. Ans.

$$\begin{array}{r} .005 \\ .005 \\ \hline .000025 \end{array}$$ Ans.

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are 3 + 3 = 6 decimal places in the product.

(*d*) $.0063^2 = .0063 \times .0063 = .00003969$. Ans.

$$\begin{array}{r} .0063 \\ .0063 \\ \hline 189 \\ 378 \\ \hline .00003969 \end{array}$$ Ans.

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are 4 + 4 = 8 decimal places in the product.

(*e*) $10.06^2 = 10.06 \times 10.06 = 101.2036$. Ans.

$$\begin{array}{r} 10.06 \\ 10.06 \\ \hline 6036 \\ 100600 \\ \hline 101.2036 \end{array}$$

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, there are 2 + 2 = 4 decimal places in the product.

(138) (*a*) $753^3 = 753 \times 753 \times 753 = 426{,}957{,}777$. Ans.

```
      753
      753
     2259
    3765
   5271
   567009
      753
  1701027
 2835045
3969063
426957777
```

(*b*) $987.4^3 = 987.4 \times 987.4 \times 987.4 = 962{,}674{,}279.624$. Ans.

```
      987.4
      987.4
      39496
     69118
    78992
   88866
   974958.76
       987.4
  389983504
 682471132
779967008
877462884
962674279.624
```

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, there are $1 + 1 = 2$ decimal places in the first product.

Since there are 2 decimal places in the multiplicand and one in the multiplier, there are $2 + 1 = 3$ decimal places in the final product.

(*c*) $.005^3 = .005 \times .005 \times .005 = .000000125$. Ans.

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3 + 3 = 6$ decimal places in the first product; but, as there are only 2 figures in the product, we prefix four ciphers to make the six decimal places.

```
      .005
      .005
   .000025
      .005
.000000125
```

Since there are six decimal places in the multiplicand and 3 in the multiplier, there are $6 + 3 = 9$ decimal places in the final product. In this case we prefix six ciphers to form the nine decimal places.

(*d*) $.4044^3 = .4044 \times .4044 \times .4044 = .066135317184$. Ans.

```
        .4044
        .4044
     --------
        16176
       16176
     161760
    ---------
    .16353936
        .4044
    ---------
     65415744
    65415744
  654157440
 ------------
.066135317184
```

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4 + 4 = 8$ decimal places in the first product.

Since there are 8 decimal places in the second multiplicand and 4 in the multiplier, there are $8 + 4 = 12$ decimal places in the final product; but, as there are only 11 figures in the product, we prefix 1 cipher to make 12 decimal places.

(139) $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$. Ans.

(140) $3^4 = 3 \times 3 \times 3 \times 3 = 81$. Ans.

(141) (*a*) $67.85^2 = 67.85 \times 67.85 = 4{,}603.6225$. Ans.

```
     67.85
     67.85
   -------
     33925
    54280
   47495
  40710
  ---------
  4603.6225  Ans.
```

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2 + 2 = 4$ decimal places in the product.

(*b*) $967{,}845^2 = 967{,}845 \times 967{,}845 = 936{,}723{,}944{,}025$. Ans.

```
      967845
      967845
     -------
     4839225
    3871380
   7742760
  6774915
 5807070
8710605
------------
936723944025
```

(*c*) A fraction may be raised to any power by raising both numerator and denominator to the required term.

Thus, $\left(\frac{3}{8}\right)^2 = \frac{3}{8} \times \frac{3}{8} = \frac{3 \times 3}{8 \times 8} = \frac{9}{64}$. Ans.

(*d*) $\left(\frac{1}{4}\right)^2 = \frac{1}{4} \times \frac{1}{4} = \frac{1 \times 1}{4 \times 4} = \frac{1}{16}$. Ans.

(142) (*a*) $5^{10} = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 =$ 9,765,625. Ans.

(*b*) $9^5 = 9 \times 9 \times 9 \times 9 \times 9 = 59{,}049$. Ans.

```
      5          9
      5          9
     --         --
     25         81
      5          9
    ---        ---
    125        729
      5          9
    ---       ----
    625       6561
      5          9
   ----      -----
   3125      59049
      5
  -----
  15625
      5
  -----
  78125
      5
 ------
 390625
      5
-------
1953125
      5
-------
9765625
```

(143) (*a*) $1.2^4 = 1.2 \times 1.2 \times 1.2 \times 1.2 = 2.0736$. Ans. Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we must point off $1 + 1 = 2$ decimal places in the first product.

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we must point off $2 + 1 = 3$ decimal places in the second product.

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we must point off $3 + 1 = 4$ decimal places in the final product.

```
     1.2
     1.2
     ---
     2 4
   1 2
  -----
   1.4 4
     1.2
  -----
   2 8 8
 1 4 4
 -------
 1.7 2 8
     1.2
 -------
 3 4 5 6
1 7 2 8
---------
2.0 7 3 6
```

(*b*) $11^6 = 11 \times 11 \times 11 \times 11 \times 11 \times 11 = 1{,}771{,}561$. Ans

```
     11
     11
    ---
    121
     11
   ----
   1331
     11
  -----
  14641
     11
 ------
 161051
     11
-------
1771561
```

(*c*) $1^7 = 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$. Ans.

(*d*) $.01^4 = .01 \times .01 \times .01 \times .01 = .00000001$. Ans.

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, we must point off $2 + 2 = 4$ decimal places in the first product; but, as there is only 1 figure in the product, we prefix 3 ciphers to make the 4 necessary decimal places.

$$\begin{array}{r} .01 \\ .01 \\ \hline .0001 \\ .01 \\ \hline .000001 \\ .01 \\ \hline .00000001 \end{array}$$

Since there are 4 decimal places in the second multiplicand and 2 in the multiplier, we must point off $4 + 2 = 6$ decimal places in the second product. It is necessary to prefix 5 ciphers to make 6 decimal places.

Since there are 6 decimal places in the third multiplicand and 2 in the multiplier, we must point off $6 + 2 = 8$ decimal places in the product. It is necessary to prefix 7 ciphers to make 8 decimal places in the final product.

(*e*) $.1^5 = .1 \times .1 \times .1 \times .1 \times .1 = .00001$. Ans.

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we must point off $1 + 1 = 2$ decimal places in the first product. It is necessary to prefix 1 cipher to the product.

$$\begin{array}{r} .1 \\ .1 \\ \hline .01 \\ .1 \\ \hline .001 \\ .1 \\ \hline .0001 \\ .1 \\ \hline .00001 \end{array}$$

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we must point off $2 + 1 = 3$ decimal places in the second product. It is necessary to prefix 2 ciphers to the second product.

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we must point off $3 + 1 = 4$ decimal places in the third product. It is necessary to prefix 3 ciphers to this product.

Since there are 4 decimal places in the fourth multiplicand and 1 in the multiplier, we must point off $4 + 1$ or 5 decimal places in the final product. It is necessary to prefix 4 ciphers to this product.

(144) (*a*) $.0133^3 = .0133 \times .0133 \times .0133 = .000002352637$. Ans.

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, we must point off $4 + 4 = 8$ decimal places in the product; but, as there are only 5 figures in the product, we prefix three ciphers to form the eight necessary decimal places in the first product.

Since there are 8 decimal places in the multiplicand and 4 in the multiplier, we must point off $8 + 4 = 12$ decimal places in the product; but, as there are only 7 figures in the product, we prefix 5 ciphers to make the 12 necessary decimal places in the final product.

```
      .0133
      .0133
      -----
        399
       399
      133
  ---------
  .00017689
      .0133
  ---------
      53067
     53067
    17689
-------------
.000002352637
```

(*b*) $301.011^3 = 301.011 \times 301.011 \times 301.011 =$ 27,273,890.942264331. Ans.

```
            301.011
            301.011
            -------
            301011
           301011
          3010110
        9030330
        ------------
        90607.622121
             301.011
        ------------
        90607622121
       90607622121
      906076221210
  2718228663630
  ------------------
  27273890.942264331
```

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, we must point off $3 + 3 = 6$ decimal places in the first product.

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, we must point off $6 + 3 = 9$ decimal places in the final product.

(*c*) $\left(\frac{1}{8}\right)^3 = \frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} = \frac{1 \times 1 \times 1}{8 \times 8 \times 8} = \frac{1}{512}$. Ans.

(*d*) To find any power of a mixed number, first reduce it to an improper fraction, and then multiply the numerators together for the numerator of the answer, and multiply the denominators together for the denominator of the answer.

$$\left(3\frac{3}{4}\right)^3 = \frac{15}{4} \times \frac{15}{4} \times \frac{15}{4} = \frac{15 \times 15 \times 15}{4 \times 4 \times 4} = \frac{3,375}{64} = 52.734+.$$

Ans.

$$3\frac{3}{4} = \frac{3 \times 4 + 3}{4} = \frac{12 + 3}{4} = \frac{15}{4}.$$

```
   15        64)3375.000(52.734+
   15           320
   --           ----
   75            175
  15             128
  ---            ---
  225             470
   15             448
 ----             ---
 1125              220
 225               192
 ----              ---
 3375               280
                    256
                    ---
                     24
```

Since *three* ciphers were annexed to the dividend, *three* decimal places must be pointed off in the quotient. It is easy to see that the next figure will be a 3; hence, write the sign +, as shown.

(145) Evolution is the reverse of involution. In involution we find the *power* of a number by multiplying the number by itself one or more times, while in evolution we find the *number* or *root* which was multiplied by itself one or more times to make the power.

(146) (*a*)

```
1
1
---
20
 8
---
28
 8
---
360
  6
---
366
  6
----
3720
   7
----
3727
   7
----
3734
```

$\sqrt{3'48'67'84.40'10} = 1867.29+$ Ans.

```
1
-----
248
224
-----
 2467
 2196
------
  27184
  26089
```

```
3734 ) 1095.000 (.293 or .29+
         7468
        -----
         34820
         33606
         -----
          12140
```

Explanation.—Applying the short method described in Art. **272,** we extract the root by the regular method to four figures, since there are six figures in the answer, and $6 \div 2 + 1 = 4$. The last remainder is 1095, and the last trial divisor (with the cipher omitted) is 3734. Dividing 1095 by 3734, as shown, the quotient is .293+, or .29+ using two figures. Annexing to the root, gives 1,867.29+. Ans.

```
(b)  (a)  3      √9'00'00'99.40'09'00 = 3000.0165 + Ans.
          3  (b)  9
          —      ——————————————
     (d) 60 (c)   0000994009
          0            600001
         ——           ————————
         600           39400800
           0           36000156
         ———          ————————
         6000           3400644
            0
         ————
         60000
             0
         —————
         600000
              1
         ——————
         600001
              1
         ———————
         6000020
               6
         ———————
         6000026
```

EXPLANATION.—Beginning at the decimal point we point off the whole number into periods of *two* figures each, proceeding from *right* to *left*; also, point off the decimal into periods of *two* figures each, proceeding from *left* to *right*. The largest number whose square is contained in the first period, 9, is 3; hence, 3 is the first figure of the root. Place 3 at the left, as shown at (*a*), and multiply it by the first figure in the root, or 3. The result is 9. Write 9 under the first period, 9, as at (*b*), subtract, and there is no remainder. Bring down the next period, which is 00, as shown at (*c*). Add the root already found to the 3 at (*a*), obtaining 6, and annex a cipher to this 6, thus making it 60, which is the *trial divisor*, as shown at (*d*). Divide the dividend (*c*) by the trial divisor, and obtain 0 as the next figure in the root. Write 0 in the root, as shown, and also add it to the trial divisor, 60, and annex a cipher, thereby making the next trial divisor 600. Bring down the next period, 00, annex it to the dividend already obtained, and divide it by the trial divisor. 600 is contained in 0000, 0 times, so we place another cipher in the root. Write 0 in the root, as shown, and also add it to the trial divisor, 600, and annex a cipher, thereby making the next trial divisor 6,000. Bring down the next period, 99. The trial divisor 6,000 is contained in 000099, 0 times, so we place 0 as the next figure in the root, as shown, and also add it to the trial divisor 6,000, and annex a cipher, thereby making the next trial divisor 60,000. Bring down the next period, 40, and annex it to the dividend already obtained to form the new dividend, 00009940, and divide it by the trial divisor 60,000. 60,000 is contained in 00009940, 0 times, so we place another cipher in the root, as shown, and also add it to the trial divisor 60,000, and annex one cipher, thereby making the next trial divisor 600,000. Bring down the next period, 09, and annex it to the dividend already obtained to form the new dividend, 0000994009, and divide it by the trial divisor 600,000. 600,000 is contained in 0000994009 once, so we place 1 as the next figure in the root, and also add it to the trial divisor 600,000, thereby making the complete divisor 600,001. Multiply the complete divisor, 600,001, by 1, the sixth figure in the root, and subtract the result obtained from the dividend. The remainder is 394,008, to which we annex the next period, 00, to form the next new dividend, or 39,400,800. Add the sixth figure of the root, or 1, to the divisor 600,001, and annex a cipher, thus obtaining 6,000,020 as the next trial divisor. Dividing 39,400,800 by 6,000,020, we find 6 to be the next figure of the root. Adding this last figure, 6, to the trial divisor, we obtain 6,000,026 for our next complete divisor, which, multiplied by the last figure of the root, or 6, gives 36,000,156, which write under 39,400,800 and subtract. Since there is a remainder, it is clearly evident that the given power is not a perfect square, so we place + after the root. Since the next figure is 5, the answer is 3,000.017 –.

In this problem there are *seven* periods—four in the whole number and three in the decimal—hence, there will be *seven* figures in the root, *four* figures constituting the whole number, and three figures the decimal of the root. Hence, $\sqrt{9,000,099.4009} = 3,000.017 -$.

(*c*)

```
3                 √.00'12'25 = .035.   Ans.
3                  00
—                  ——
60                  12
 5                   9
—                   ———
65                   325
                     325
                     ———
```

Pointing off periods, we find that the first period is composed of ciphers; hence, the first figure of the root will be a cipher. No further explanation is necessary, since this problem is solved in a manner exactly similar to the problem solved in Art. **264.** Since there are *three* decimal periods in the power, there will be three decimal figures in the root.

(147) (*a*)

```
1          √1'07'95.21 = 103.9   Ans.
1           1
—           ————
20           0795
 0            609
———          —————
200           18621
  3           18621
———           —————
203
  3
————
2060
   9
————
2069
```

(*b*)

```
2          √7'30'08.04 = 270.2   Ans.
2           4
—           ———
40          330
 7          329
—           —————
47            10804
 7            10804
————          —————
5400
   2
————
5402
```

(*c*)

```
   9
   9
 ----
 180
   4
 ----
 184
   4
 -----
 1880
    8
 -----
 1888
    8
 -----
 1896
```

$\sqrt{90.00'00'00} = 9.487 -$ Ans.

```
   81
   ---
   900
   736
   -----
   16400
   15104
   -----
1896 ) 1296.00 (.68 + or .7 -
       11376
       -----
        15840
        15168
        -----
```

Having found the first three figures, we find the fourth by division, as shown.

(*d*) $\sqrt{.09} = .3$. Ans.

(148) (*a*)

```
   6        36
   6        72
  ---     -------
  12       10800
   6        1504
  ---     -------
  180      12304
    8       1568
  ---     -------
  188     1387200
    8       18441
  ---     -------
  196     1405641
    8       18522
  ----    -------
  2040    1424163
     9
  ----
  2049
     9
  ----
  2058
     9
  ----
  2067
```

$\sqrt[3]{.327'680'000} = .6894 +$ Ans.

```
     216
     ------
     111680
      98432
     --------
     13248000
     12650769
     --------
1424163 ) 597231.00(.41+or.4+
          5696652
          -------
           2756580
           1424163
           -------
```

Here we find the first three figures in the regular way, and the fourth figure by the short method. See Art. **284.**

EXPLANATION.—(1) When extracting the *cube* root we divide the power into periods of three figures each. Always begin at the decimal point, and proceed to the *left* in pointing off the whole number, and to the *right* in pointing off the decimal. In this power $\sqrt[3]{.32768}$, a cipher must be annexed to 68 to complete the second decimal period. Cipher periods may now be annexed until the root has as many figures as desired.

(2) We find by trial that the largest number whose cube is contained in the first period, 327, is 6. Write 6 as the first figure of the root, also at the extreme left at the head of column (1). Multiply the 6 in column (1) by the first figure of the root, 6, and write the product 36 at the head of column (2). Multiply the number in column (2) by the first figure of the root, 6, and write the product 216 under the figures in the first period. Subtract and bring down the next period 680; annex it to the remainder 111, thereby obtaining 111,680 for a new dividend. Add the first figure of the root, 6, to the number in column (1), obtaining 12, which we call the *first correction;* multiply the first correction 12 by the first figure of the root, and we obtain 72 as the product, which, added to 36 of column (2), gives 108. Annexing two ciphers to 108, we have 10,800 for the trial divisor. Dividing the dividend by the trial divisor, we see that it is contained about 8 times, so we write 8 as the second figure of the root. Add the first figure of the root to the first correction, and we obtain 18 as the *second correction.* To this annex *one* cipher, and add the second figure of the root, and we obtain 188. This, multiplied by the second figure of the root, 8, equals 1,504, which, added to the trial divisor 10,800, forms the *complete divisor* 12,304. Multiplying the complete divisor 12,304 by 8, the second figure of the root, the result is 98,432. Write 98,432 under the dividend 111,680; subtract, and there is a remainder of 13,248. To this remainder annex the next period 000, thereby obtaining 13,248,000 for the next new dividend.

(3) Adding the second figure of the root, 8, to the number in column (1), 188, we have 196 for the *first new*

correction. This, multiplied by the second figure of the root, 8, gives 1,568. Adding this product to the last complete divisor, and annexing two ciphers, gives 1,387,200 for the next trial divisor. Adding the second figure of the root, 8, to the first new correction, 196, we obtain 204 for the *new second correction.* Dividing the dividend by the trial divisor 1,387,200, we see that it is contained about 9 times. Write 9 as the third figure of the root. Annex *one* cipher to the *new second correction*, and to this add the third figure of the root, 9, thereby obtaining 2,049. This, multiplied by 9, the third figure of the root, equals 18,441, which, added to the trial divisor, 1,387,200, forms the complete divisor 1,405,641. Multiplying the complete divisor by the third figure of the root, 9, and subtracting, we have a remainder of 597,231. We then find the fourth figure by division, as shown.

(*b*)

4	16	$\sqrt[3]{74'088} = 42$ Ans.
4	32	64
8	4800	10088
4	244	10088
120	5044	
2		
122		

(*c*)

4	16	$\sqrt[3]{92'416} = 45.212-$ Ans.
4	32	64
8	4800	28416
4	625	27125
120	5425	1291000
5	650	1220408
125	607500	612912)70592.000(.115
5	2704	612912
130	610204	930080
5	2708	612912
1350	612912	3171680
2		3064560
1352		107120
2		
1354		

(*d*)

```
   7          4 9       ∛.373'248 = .72  Ans.
   7          9 8        343
 ---        -----       -----
  14        14700        30248
   7          424        30248
 ---        -----       -----
 210        15124
   2
 ---
 212
```

(149)

```
1      1            ∛2.000'000'000 = 1.259921 +  Ans.
1      2             1
-      -----        -----
2      300           1000
1       64            728
--     -----        ------
30     364            272000
 2      68            225125
--     -----        --------
32     43200            46875000
 2      1825            42491979
--     -----        -----------
34     45025        4755243)4383021.000(.9217 or .922–
 2      1850             42797187
---    -------          ---------
360    4687500           10330230
  5      33831            9510486
---    -------          ---------
365    4721331             8197440
  5      33912             4755243
---    -------          ---------
370    4755243             34421970
  5
----
3750
   9
----
3759
   9
----
3768
```

This example shows what a great saving of figures is effected by using the short method. The figures obtained by the division are 9217, thus making the last figures of the answer 922, according to Art. **272.** This is not correct in this case; the true answer to eight decimal places being 1.25992104 +; hence, the first three figures found by division should be used in this case. The reason for the apparent failure of the method in this case to give the seventh figure of the root correctly is because the fifth figure (the first obtained by division) is 9. Whenever the first figure obtained by division is 8 or 9, it is better to carry the root process one place further, before applying Art. **272,** if it is desired to obtain absolutely correct results.

(150) (*a*)

1	1	$\sqrt[3]{1'758.416'743} = 12.07$ Ans.
1	2	1
2	300	758
1	64	728
30	364	30416743
2	68	30416743
32	4320000	
2	25249	
34	4345249	
2		
3600		
7		
3607		

(*b*)

1	1	$\sqrt[3]{1'191'016} = 106$ Ans.
1	2	1
2	30000	191016
1	1836	191016
300	31836	
6		
306		

(*c*) $\sqrt[3]{\frac{4}{32}} = \sqrt[3]{\frac{1}{8}} = \frac{\sqrt[3]{1}}{\sqrt[3]{8}} = \frac{1}{2}$. Ans.

(*d*) $\sqrt[3]{\frac{27}{512}} = \frac{\sqrt[3]{27}}{\sqrt[3]{512}} = \frac{3}{8}$. Ans.

(151)

1
1
2
1
30
4
34
4
38
4
420
4
424
4
428
4
4320
2
4322
2
4324

1
2
300
136
436
152
58800
1696
60496
1712
6220800
8644
6229444
8648
6238092

$\sqrt[3]{3.000'000'000} = 1.442250 -$ Ans.
1
2000
1744
256000
241984
14016000
12458888
6238092) 1557112.000 (.2496 or .250 −
12476184
30949360
24952368
59969920
56142828
3827092

(152) (*a*)

1
1
20
1
21
1
220
1
221

$\sqrt{1'23.21} = 11.1$ Ans.
1
23
21
221
221

(*b*)

1
1
200
7
207
7
2140
2
2142

$\sqrt{1'14.92'10} = 10.72 +$ Ans.
1
1492
1449
4310
4284
26

(*c*)

7
7
140
0
1400
9
1409

$\sqrt{50'26'81} = 709$ Ans.
49
12681
12681

(*d*)

2
2
400
3
403

$\sqrt{.00'04'12'09} = .0203$ Ans.
00
04
4
1209
1209

(153) (*a*)

1	1	$\sqrt[3]{.006'500'000} = .18663 -$ Ans
1	2	1
2	300	5500
1	304	4832
30	604	668000
8	368	602856
38	97200	103788) 65144.00 (.627 or .63 −
8	3276	622728
46	100476	287120
8	3312	207576
540	103788	79544
6		
546		
6		
552		

(*b*)

2	4	$\sqrt[3]{.021'000'000} = .2759 -$ Ans.
2	8	8
4	1200	13000
2	469	11683
60	1669	1317000
7	518	1113875
67	218700	226875) 203125.0 (.89 or .9−
7	4075	1815000
74	222775	216250
7	4100	
810	226875	
5		
815		
5		
820		

(*c*)

2	4	$\sqrt[3]{8'036'054'027} = 2{,}003$ Ans.
2	8	8
4	12000000	036054027
2	18009	36054027
6000	12018009	
3		
6003		

(*d*)

1	1	$\sqrt[3]{.000'004'096} = .016$ Ans.
1	2	000
2	300	004
1	216	1
30	516	3096
6		3096
36		

(*e*)

2	4	$\sqrt[3]{17.000'000} = 2.5713-$ Ans.
2	8	8
4	1200	9000
2	325	7625
60	1525	1375000
5	350	1349593
65	187500	198147) 25407.00 (.128 or **.13**−
5	5299	198147
70	192799	559230
5	5348	396294
750	198147	162936
7		
757		
7		
764		

(154) (*a*) In this example the index is 4, and equals 2 × 2. The root indicated is the fourth root, hence the square root must be extracted twice. Thus, $\sqrt[4]{\ } = \sqrt{\ }$ of the $\sqrt{\ }$ and $\sqrt[4]{6561} = \sqrt{\sqrt{6561}} = \sqrt{81} = 9$. Ans.

8	$\sqrt{65\,61} = 81$	$\sqrt{81} = 9$ Ans.
8	64	
160	161	
1	161	
161		

(*b*) In this example the index is 6, and 6 equals 3 × 2 or 2 × 3. The root indicated is the sixth root; hence, extract both the square and cube root, it making no particular difference as to which root is extracted first. Thus,

$$\sqrt[6]{\ } = \sqrt[3]{\ } \text{ of the } \sqrt{\ }, \text{ or } \sqrt{\ } \text{ of the } \sqrt[3]{\ }.$$

Hence, $\sqrt[6]{117{,}649} = \sqrt[3]{\sqrt{117{,}649}} = \sqrt[3]{343} = 7$. Ans.

3	$\sqrt{11'76'49} = 343$	$\sqrt[3]{343} = 7$ Ans.
3	9	
60	276	
4	256	
64	2049	
4	2049	
680		
3		
683		

(*c*) $\sqrt[6]{.000064} = \sqrt[3]{\sqrt{.000064}} = .2$. Ans.

$\sqrt{.000064} = .008$. $\sqrt[3]{.008} = .2$. Hence, $\sqrt[6]{.000064} = .2$.
Ans

(*d*) $\sqrt[3]{\frac{3}{8}} = ?$ $\frac{3}{8} = .375$, since $8\overline{)3.000}$
.375

7	49	$\sqrt[3]{.375'000'000} = .72112+$ Ans.
7	98	343
14	14700	32000
7	424	30248
210	15124	1752000
2	428	1557361
212	1555200	1559523) 19463 9.00 (.124 or .12 +
2	2161	1559523
214	1557361	3868670
2	2162	3119046
2160	1559523	749624
1		
2161		
1		
2162		

Hence, $\sqrt[3]{\frac{3}{8}} = .72112+$. Ans.

(155) (*a*) $\sqrt{\frac{1225}{5476}} = \frac{\sqrt{1225}}{\sqrt{5476}}$.

3	$\sqrt{12'25} = 35$
3	9
60	325
5	325
65	

7	$\sqrt{54'76} = 74$
7	49
140	576
4	576
144	

Hence, $\sqrt{\frac{1225}{5476}} = \frac{35}{74}$. Ans.

(*b*)

5	$\sqrt{.33'64} = .58$ Ans.
5	25
100	864
8	864
108	

(*c*)

3	$\sqrt{.10'00'00'00} = .31623-$ Ans.
3	9
60	100
1	61
61	3900
1	3756
620	632) 144.00 (.227 or .23 −
6	1264
626	1760
6	1264
632	496

(*d*) $25.0\frac{3}{4} = 25.075.$

5	$\sqrt{25.07'50'00'00'00} = 5.00749+$ Ans.
5	25
10000	075000
7	70049
10007	495100
7	400576
100140	9452400
4	9013401
100144	438999
4	
1001480	
9	
1001489	

(*e*) $.000\frac{4}{9} = .0004444444+.$

2	$\sqrt{.00'04'44'44'44} = .02108+$ Ans
2	00
40	04
1	4
41	44
1	41
4200	34444
8	33664
4208	780

(156) (*a*) $\sqrt[4]{2} = \sqrt{\sqrt{2}}.$

```
1            √2.0 0'0 0'0 0'0 0 = 1.41421356 +
1             1
---          ---
20           100
 4            96
---          -----
24             400
 4             281
---            -----
280            11900
  1            11296
---            ------
281              60400
  1              56564
----           -----------
2820      28284 ) 3836.0000 ( .13562 or .1356 +
   4                28284
----                ------
2824                100760
   4                 84852
-----                ------
28280                159080
    2                141420
-----                ------
28282                 176600
    2                 169704
-----                 ------
28284                   6896
```

```
1            √1.4 1'4 2'1 3'5 6  = 1.1892 +    Ans.
1             1
---          ---
20            41
 1            21
---           ----
21            2042
 1            1824
---           -----
220            21813
  8            21321
---            -----
228              49256
  8              47564
----             -----
2360              1692
   9
----
2369
   9
-----
23780
    2
-----
23782
```

It is required in this problem to extract the fourth root of 2 to four decimal places; hence, we must extract the square root twice, since $\sqrt[4]{} = \sqrt{}$ of the $\sqrt{}$. In the first operation we carry the root to 8 decimal places, in order to carry the root in the second operation to 4 decimal places.

(*b*) $\sqrt[6]{6} = \sqrt[3]{\sqrt{6}}$

2	$\sqrt{6.00'00'00'00'00'00} = 2.4494897428+$
2	4
40	200
4	176
44	2400
4	1936
480	46400
4	44001
484	239900
4	195936
4880	4396400
9	3919104
4889	489896) 477296.000000 (.974280 or .97428 +
9	4409064
48980	3638960
4	3429272
48984	2096880
4	1959584
489880	1372960
8	979792
489888	3931680
8	3919168
489896	12512

It is required in this problem to find the sixth root of 6; hence it is necessary to extract both the square and cube roots in succession, since the index, 6, equals 2×3 or 3×2. It makes no particular difference as to which root we extract first, but it will be more convenient to extract the square root first. The result has been carried to 10 decimal places; since the answer requires but 5 decimal places, the remaining decimals will not affect the cube root in the fifth decimal place, as the student can see for himself if he will continue the operation.

1
1
2
1
30
3
33
3
36
3
390
4
394
4
398
4
4020
8
4028
8
4036

1
2
300
99
399
108
50700
1576
52276
1592
5386800
32224
5419024
32288
5451312

$\sqrt[3]{2.449'489'742'800} = 1.34801 -$ Ans.
1
1449
1197
252489
209104
43385742
43352192
5451312) 33550.000 (.006 or .01 −
32707872
842128

(157) (*a*)

1
1
20
7
27
7
340
7
347
7
354

$\sqrt{3.14'16} = 1.7725 -$ Ans.
1
214
189
2516
2429
354) 87.00 (.245 + or .25 −
708
1620
1416
204

(*b*)

```
   8
   8
 ---
 160
   8
 ---
 168
   8
 ----
 1760
    6
 ----
 1766
    6
 ----
 1772
```

```
√.78'54'00 = .8862 +   Ans.
  64
 ----
  1454
  1344
  -----
   11000
   10596
   -----
1772 ) 404.0 (.22 or .2+
       3544
       ----
        496
```

(158) (*a*)

```
1        1
1        2
-        ----
2        300
1        136
--       ---
30       436
 4       152
--       -----
34       58800
 4        2556
--       -----
38       61356
 4        2592
---      -------
420      6394800
  6        17536
---      -------
426      6412336
  6        17552
---      -------
432      6429888
  6
----
4380
   4
----
4384
   4
----
4388
```

```
∛3.141'600'000 = 1.4646 −
 1                     Ans
 ----
 2141
 1744
 ------
  397600
  368136
  --------
   29464000
   25649344
   --------
6429888 ) 38146656.0 (.59 or .6−
          32149440
          --------
           5997120
```

(*b*)

.8	64	$\sqrt[3]{.523'600'000} = .80599 +$ or $.8060 -$ Ans.
8	128	512
16	1920000	11600000
8	12025	9660125
2400	1932025	1944075)1939875.00(.99
5	12050	17496675
2405	1944075	1902075
5		
2410		

(159) $11.7 : 13 :: 20 : x$.

$11.7x = 13 \times 20$

$11.7x = 260$

The product of the means equals the product of the extremes.

$$x = \frac{260}{11.7}$$

```
11.7 ) 260.000 ( 22.22+   Ans.
       234
        260
        234
         260
         234
          260
          234
           26
```

(160) (*a*) $20 + 7 : 10 + 8 :: 3 : x$.

$27 : 18 :: 3 : x$

$27x = 18 \times 3$

$27x = 54$

$$x = \frac{54}{27} = 2. \quad \text{Ans.}$$

(*b*) $12^2 : 100^2 :: 4 : x$.

$144 : 10{,}000 :: 4 : x$

$144x = 10{,}000 \times 4$

$144x = 40{,}000$

$$x = \frac{40,000}{144}$$

```
144 ) 40000.0 ( 277.7 +   Ans.
      288
      1120
      1008
       1120
       1008
        1120
        1008
         112
```

(161) (*a*) $\frac{4}{x} = \frac{7}{21}$, is equivalent to $4 : x :: 7 : 21$. The product of the means equals the product of the extremes. Hence,

$$7x = 4 \times 21$$
$$7x = 84$$
$$x = \frac{84}{7} \text{ or } 12. \quad \text{Ans.}$$

(*b*) In like manner,

$\frac{x}{24} = \frac{8}{16}$ is equivalent to $x : 24 :: 8 : 16$.

$$16x = 24 \times 8$$
$$16x = 192$$
$$x = \frac{192}{16} = 12. \quad \text{Ans.}$$

(*c*) $\frac{2}{10} = \frac{x}{100}$ is equivalent to $2 : 10 :: x : 100$.

$$10x = 2 \times 100$$
$$10x = 200$$
$$x = \frac{200}{10} = 20. \quad \text{Ans.}$$

(*d*) $\frac{15}{45} = \frac{60}{x}$ is equivalent to $15 : 45 :: 60 : x$.

$$15x = 45 \times 60$$
$$15x = 2,700$$
$$x = \frac{2,700}{15} = 180. \quad \text{Ans.}$$

(*e*) $\frac{10}{150} = \frac{x}{600}$ is equivalent to $10 : 150 :: x : 600$.

$$150x = 10 \times 600$$
$$150x = 6,000$$
$$x = \frac{6,000}{150} = 40. \quad \text{Ans.}$$

(162) $x : 5 :: 27 : 12.5.$

$$\begin{array}{r} 5 \\ \hline 12.5\,)\,135.0\,(\,10\frac{4}{5} \text{ Ans.} \\ 125 \\ \hline \frac{100}{125} = \frac{4}{5} \end{array}$$

(163) $45 : 60 :: x : 24$

$60x = 45 \times 24$

$60x = 1{,}080$

$x = \frac{1{,}080}{60} = 18.$ Ans

(164) $x : 35 :: 4 : 7.$

$7x = 35 \times 4$

$7x = 140$

$x = \frac{140}{7} = 20.$ Ans.

(165) $9 : x :: 6 : 24.$

$6x = 9 \times 24$

$6x = 216$

$x = \frac{216}{6} = 36.$ Ans.

(166)

$\sqrt[3]{1{,}000} : \sqrt[3]{1{,}331} :: 27 : x.$

$10 : 11 :: 27 : x.$

$10x = 297.$

$x = \frac{297}{10} = 29.7.$ Ans.

$\sqrt[3]{1{,}000} = 10.$

$\sqrt[3]{1{,}331} = 11.$

$$\begin{array}{r} 1 \\ 1 \\ \hline 2 \\ 1 \\ \hline 30 \\ 1 \\ \hline 31 \end{array} \qquad \begin{array}{r} 1 \\ 2 \\ \hline 300 \\ 31 \\ \hline 331 \end{array} \qquad \begin{array}{r} 1'331\,(\,11 \\ 1 \\ \hline 331 \\ 331 \\ \hline \end{array}$$

(167) $64 : 81 = 21^2 : x^2.$

Extracting the square root of each term of any proportion does not change its value, so we find that $\sqrt{64} : \sqrt{81} = \sqrt{21^2} : \sqrt{x^2}$ is the same as

$8 : 9 = 21 : x$

$8x = 189$

$x = 23.625.$ Ans.

(168) $7 + 8 : 7 = 30 : x$ is equivalent to

$15 : 7 = 30 : x.$

$15x = 7 \times 30$

$15x = 210$

$x = \frac{210}{15} = 14.$ Ans.

(169) 2 ft. 5 in. = 29 in.; 2 ft. 7 in. = 31 in. Stating as a direct proportion, 29 : 31 = 2,480 : x. Now, it is easy to see that x will be greater than 2,480. But x should be less than 2,480, since, when a man lengthens his steps, the number of steps required for the same distance is less: hence, the proportion is an inverse one, and

$$29 : 31 = x : 2,480,$$
or, $$31x = 71,920;$$
whence, $$x = 71,920 \div 31 = 2,320 \text{ steps. Ans.}$$

(170) This is evidently a direct proportion. 1 hr 36 min. = 96 min.; 15 hr. = 900 min. Hence,

$$96 : 900 = 12 : x,$$
or, $$96x = 10,800;$$
whence, $$x = 10,800 \div 96 = 112.5 \text{ mi. Ans.}$$

(171) This is also a direct proportion; hence,

$$27.63 : 29.4 = .76 : x,$$
or, $$27.63x = 29.4 \times .76 = 22.344;$$
whence, $$x = 22.344 \div 27.63 = .808 + \text{lb. Ans.}$$

(172) 2 gal. 3 qt. 1 pt. = 23 pt.; 5 gal. 3 qt. = 46 pt. Hence,

$$23 : 46 = 5 : x,$$
or, $$23x = 46 \times 5 = 230;$$
whence, $$x = 230 \div 23 = 10 \text{ days. Ans.}$$

(173) First cause, 5 men and 8 hours; second cause, x men, 10 hours. The effect is the amount of work, which is the same in each case.

~~5~~ ~~8~~ 4	x ~~10~~ = work ~~2~~	work

$$x = 4 \text{ men. Ans.}$$

(174) Taking the times as the causes,

$$\begin{array}{c|c} \cancel{20} & \cancel{25} \\ & \cancel{5} \\ & \\ \cancel{40} & x \\ 2 & \end{array} = \begin{array}{c|l} & 14 \\ & \cancel{70} \\ \cancel{540} & \cancel{630} \\ \cancel{27} & \\ 3 & \end{array}$$

hence, $3x = 2 \times 14 = 28$, or $x = 9\frac{1}{3}$ hr.

(175) $10,000^2 : 20,000^2 = 8 : x$;

or $1^2 : 2^2 = 8 : x$;

or $1 : 4 = 8 : x$;

hence, $x = \frac{4 \times 8}{1} = 32$ pounds. Ans.

(176) $\frac{10}{320} : \frac{8}{600} = 400 : x$;

hence, $x = \frac{8 \times 600 \times 400}{10 \times 320} = 600$ hoists. Ans.

(177) $20,000^2 : 30,000^2 = 16 : x$;

or $2^2 : 3^2 = 16 : x$;

or $8 : 27 = 16 : x$;

hence, $x = \frac{27 \times 16}{8} = 54$ horsepower. Ans.

FORMULAS.

(178) Substituting for D, x, B, and i their values,

$$C = \frac{D - x}{B + i} = \frac{120 - 12}{10 + 3.5} = \frac{108}{13.5} = 8. \quad \text{Ans.}$$

A line between two numbers signifies that the one above the line, or numerator, is to be divided by the one below the line, or denominator.

(179) Substituting for A, h, D, and x their values,

$$\frac{A\,h + D}{2\,x + 6} = \frac{(5 \times 200) + 120}{(2 \times 12) + 6} = \frac{1{,}000 + 120}{24 + 6} = \frac{1{,}120}{30} = 37\tfrac{1}{3}.$$

$$37\tfrac{1}{3} + D = 37\tfrac{1}{3} + 120 = 157\tfrac{1}{3}. \quad \text{Ans.}$$

When there is no sign between the letters, multiplication is understood.

(180) Substituting for B, h, A, x, and i their values

$$r = \frac{3.246 \times B \times h}{\dfrac{A\,x + h}{A\,i - B}} = \frac{3.246 \times 10 \times 200}{\dfrac{(5 \times 12) + 200}{(5 \times 3.5) - 10}} = \frac{6{,}492}{\dfrac{260}{7.5}} =$$

$$6{,}492 \div \frac{260}{7.5} = 6{,}492 \times \frac{7.5}{260} = 187.269 +. \quad \text{Ans.}$$

(181) Substituting for A, D, i, and B their values,

$$v = \sqrt{\frac{A\,D}{i\,B + 1.5}} = \sqrt{\frac{5 \times 120}{(3.5 \times 10) + 1.5}} = \sqrt{\frac{600}{36.5}} =$$

$$\sqrt{16.4383} = 4.05 +. \quad \text{Ans.}$$

The square root sign extends over both numerator and denominator, thus indicating that the square root of the entire fraction is to be extracted.

(182) Substituting for B, x, h, and A their values,

$$u = \sqrt[3]{\frac{B x}{.00018\, h\, (A^2 - x)}} = \sqrt[3]{\frac{10 \times 12}{.00018 \times 200 \times (5^2 - 12)}} =$$

$$\sqrt[3]{\frac{120}{.036 \times (25 - 12)}} = \sqrt[3]{\frac{120}{.036 \times 13}} = \sqrt[3]{\frac{120}{.468}} =$$

$$\sqrt[3]{256.41 +} = 6.35 +. \quad \text{Ans.}$$

(183) Substituting for h, D, and A their values,

$$f = \frac{10\,(h - D)^2}{\sqrt[3]{D + A}} = \frac{10\,(200 - 120)^2}{\sqrt[3]{120 + 5}} = \frac{10 \times 80^2}{\sqrt[3]{125}} = \frac{64{,}000}{5} = 12{,}800. \quad \text{Ans.}$$

(184) Substituting for B, A, and D their values,

$$g = \frac{(B - A)^2 - \sqrt[3]{D + A}}{A^2 - (1 + D)} = \frac{(10 - 5)^2 - \sqrt[3]{120 + 5}}{5^2 - (1 + 120)} =$$

$$\frac{5^2 - \sqrt[3]{125}}{125 - 121} = \frac{25 - 5}{4} = \frac{20}{4} = 5. \quad \text{Ans.}$$

(185) Substituting for A, B, and h their values,

$$k = \sqrt{\frac{A\,B^2}{\sqrt[3]{A\,h}}} = \sqrt{\frac{5 \times 10^2}{\sqrt[3]{5 \times 200}}} = \sqrt{\frac{5 \times 100}{\sqrt[3]{1{,}000}}} = \sqrt{\frac{500}{10}} =$$

$$\sqrt{50} = 7.071 +. \quad \text{Ans.}$$

(186) Substituting for A, h, D, x, and B their values,

$$T = \sqrt{\frac{A^2\left[490 + \frac{(h\,x)^2}{D^2}\right]}{h + \frac{x}{D}\,(A^2 - B)^2}} = \sqrt{\frac{5^2\left[490 + \frac{(200 \times 12)^2}{120^2}\right]}{200 + \frac{12}{120}\,(5^2 - 10)^2}} =$$

$$\sqrt{\frac{25\,(490 + 400)}{200 + (\frac{1}{10} \times 225)}} = \sqrt{\frac{25 \times 890}{200 + 22.5}} = \sqrt{\frac{22{,}250}{222.5}} =$$

$$\sqrt{100} = 10. \quad \text{Ans.}$$

GEOMETRY AND TRIGONOMETRY.

(273) When one straight line meets another straight line at a point between the ends, the sum of the two adjacent angles equals two right angles. Therefore, since one of the angles equals $\frac{4}{5}$ of a right angle, then, the other angle equals $\frac{10}{5}$, or two right angles, minus $\frac{4}{5}$. We have, then, $\frac{10}{5} - \frac{4}{5} = \frac{6}{5}$, or $1\frac{1}{5}$ right angles.

(274) The size of one angle is $\frac{1}{6}$ of two right angles, or $\frac{1}{3}$ of a right angle.

(275) The pitch being 4, the number of teeth in the wheel equals 4×12, or 48. The angle formed by drawing lines from the center to the middle points of two adjacent teeth equals $\frac{1}{48}$ of 4 right angles, or $\frac{1}{12}$ of a right angle.

(276) It is an isosceles triangle, since the sides opposite the equal angles are equal.

(277) An equilateral heptagon has seven equal sides; therefore, the length of the perimeter equals 7×3, or 21 inches.

(278) A regular decagon has 10 equal sides; therefore, the length of one side equals $\frac{40}{10}$, or 4 inches.

(279) The sum of all the interior angles of any polygon equals two right angles, multiplied by the number of sides in the polygon, less two. As a regular dodecagon has 12 equal sides, the sum of the interior angles equals two right angles × 10 (= 12 − 2), or 20 right angles. Since there are 12 equal angles, the size of any one of them equals 20 ÷ 12, or $1\frac{2}{3}$ right angles.

(280) Equilateral triangle.

(281) No, since the sum of the two smaller sides is not greater than the third side.

(282) No, since the sum of the three smaller sides is not greater than the fourth side.

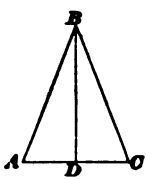

FIG. 1.

(283) Since the two angles A and C, Fig. 1, are equal, the triangle is isosceles, and a line drawn from the vertex B will bisect the line $A\,C$, the length of which is 7 inches; therefore,

$$A\,D = D\,C = \frac{7}{2} = 3\frac{1}{2} \text{ in.} \quad \text{Ans.}$$

(284) The length of the line $= \sqrt{12^2 - 9^2} + \sqrt{15^2 - 9^2}$, or 19.94 inches.

(285) The sum of the three angles is equal to $\frac{8}{4}$, or 2 right angles; therefore, since the sum of two of them equals $\frac{5}{4}$ of a right angle, the third angle must equal $\frac{8}{4} - \frac{5}{4}$, or $\frac{3}{4}$ of a right angle.

(286) One of the angles of an equiangular octagon is equal to $\frac{1}{8}$ of 12 right angles, or $1\frac{1}{2}$ right angles, since the sum of the interior angles of the equiangular octagon equals 12 right angles.

(287) The sum of the acute angles of a right-angled triangle equals one right angle; therefore, if one of them equals $\frac{5}{8}$ of a right angle, the other equals $\frac{8}{8} - \frac{5}{8}$, or $\frac{3}{8}$ of a right angle.

(288) (See Art. **734.**)

(289) In Fig. 2, $AB = 4$ inches, and $AO = 6$ inches. We first find the length of DO. $DO = \sqrt{\overline{OA}^2 - \overline{DA}^2}$; but $\overline{OA}^2 = 6^2$, or 36, and $\overline{DA}^2 = \left(\frac{4}{2}\right)^2$ or 4; therefore, $DO = \sqrt{36 - 4}$, or 5.657.

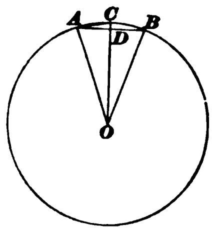

FIG. 2.

$DC = CO - DO$, or $DC = 6 - 5.657$, or .343 inch. In the right-angled triangle ADC, we have AC, which is the chord of one-half the arc ACB, equals $\sqrt{2^2 + .343^2}$, or 2.03 inches.

(290) The method of solving this is similar to the last problem.

$$DO = \sqrt{9 - 4}, \text{ or } 2.236. \quad DC = 3 - 2.236 = .764.$$

$$AC = \sqrt{2^2 + .764^2}, \text{ or } 2.14 \text{ inches.}$$

(291) Let HK of Fig. 3 be the section; then, $BI = 2$ inches, and $HK = 6$ inches, to find AB. HI ($= 3$ inches) being a mean proportional between the segments AI and IB, we have

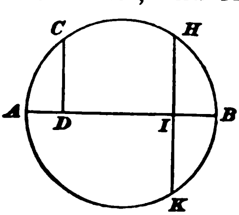

FIG. 3.

$$BI : HI :: HI : IA,$$

$$\text{or} \quad 2 : 3 :: 3 : IA.$$

$$\text{Therefore,} \quad IA = 4\frac{1}{2}.$$

$AB = AI + IB$; therefore, $AB = 4\frac{1}{2} + 2$, or $6\frac{1}{2}$ inches.

(292) Given $OC = 5\frac{3}{4}$ inches, and $OA = \frac{17}{2}$, or $8\frac{1}{2}$ inches, to find AB (see Fig. 4). CA, which is one-half the chord AB, equals

$$\sqrt{\overline{OA}^2 - \overline{OC}^2};$$

therefore, $CA = \sqrt{(8\frac{1}{2})^2 - (5\frac{3}{4})^2}$, or 6.26 inches. Now, $AB = 2 \times CA$; therefore, $AB = 2 \times 6.26$, or 12.52 inches.

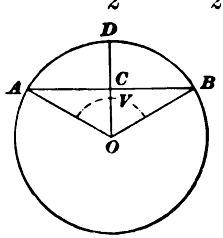

FIG. 4.

(293) The arc intercepted equals $\frac{3}{4}$ of 4, or 3 quadrants. As the inscribed angle is measured by one-half the intercepted arc, we have $\frac{3}{2} = 1\frac{1}{2}$ quadrants as the size of the angle.

(294) Four right angles $\div \frac{2}{7} = 4 \times \frac{7}{2}$, or 14 equal sectors.

(295) Since 24 inches equals the perimeter, we have $\frac{24}{8}$, or 3 inches, as the length of each side or chord.

Then, $2 \times \sqrt{\left(\frac{3}{2}\right)^2 + 3.62^2} = 7.84$ inches diameter.

(296) Given, $AC = \frac{AB}{2} = \frac{10.5}{2}$, or 5.25 inches. AO and $AP = 13$ inches. (See Fig. 5.)

The required distance between the arcs DD' is equal to $OA + AP - OP$. In the right-angled triangle ACO, we have

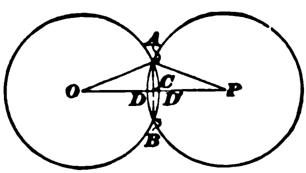

FIG. 5.

$$OC = \sqrt{\overline{AO}^2 - \overline{AC}^2},$$

or $OC = \sqrt{169 - 27.5625} = 11.9$ inches.

Likewise, $CP = \sqrt{\overline{AP}^2 - \overline{AC}^2} = 11.9$. $OP = OC + CP = 11.9 + 11.9 = 23.8$ inches. $OA + AP = 13 + 13 = 26$ inches. $26 - 23.8 = 2.2$ inches. Ans.

(297) Given $AP = 13$ inches, $OA = 8$ inches, and $AC = 5.25$ inches. Fig. 6.

$OC = \sqrt{\overline{AO}^2 - \overline{AC}^2} = \sqrt{8^2 - 5.25^2} =$ 6.03 inches.

$CP = \sqrt{\overline{AP}^2 - \overline{AC}^2} = 11.9$ inches.

$OP = OC + CP = 6.03 + 11.9 =$ 17.93 inches.

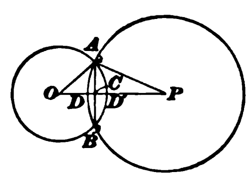

FIG. 6.

$DD' = OA + AP - OP = 8 + 13 - 17.93 = 3.07$ inches. Ans.

(298) $AB = 14$ inches, and $AE = 3\frac{1}{4}$ inches, Fig. 7. $CE = ED$ is a mean proportional between the segments

AE and EB. Then,

$$AE : CE :: CE : EB,$$

or $$3\frac{1}{4} : CE :: CE : 10\frac{3}{4},$$

or $$\overline{CE}^2 = 3\frac{1}{4} \times 10\frac{3}{4} = 34.9375.$$

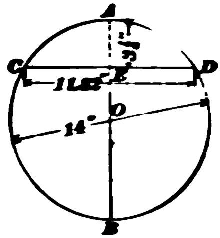

FIG. 7.

Extracting the square root, we have

$$CE = 5.91.$$

$2 \times CE = CD = 2 \times 5.91$, or 11.82 inches. Ans.

(299) In 19° 19′ 19″ there are 69,559 seconds, and in 360°, or a circle, there are 1,296,000 seconds. Therefore, 69,559 seconds equal $\frac{69,559}{1,296,000}$, or .053672 part of a circle. Ans.

(300) In an angle measuring 19° 19′ 19″ there are 69,559 seconds, and in a quadrant, which is $\frac{1}{4}$ of 360°, or 90°, there are 324,000 seconds. Therefore, 69,559 seconds equal $\frac{69,559}{324,000}$, or .214688 part of a quadrant. Ans.

(301) Given, $OB = OA = \frac{23}{2}$, or $11\frac{1}{2}$ inches, and angle $AOB = \frac{1}{10}$ of 360°, or 36°. (See Fig. 8.) In the right-angled triangle COB, we have

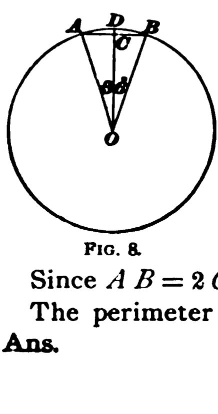

FIG. 8.

$$\sin COB = \frac{CB}{OB}, \text{ or } CB = OB \times \sin COB.$$

Substituting the values of OB and sin COB, we have

$$CB = 11\frac{1}{2} \times \sin 18°,$$

or $$CB = 11\frac{1}{2} \times .30902 = 3.55.$$

Since $AB = 2\,CB$, $AB = 2 \times 3.55 = 7.1$ inches.

The perimeter then equals $10 \times 7.1 = 71$ inches, nearly. **Ans.**

(302)

$$\begin{array}{rrr} 90° = 89° & 59' & 60'' \\ 35° & 24' & 25.8'' \\ \hline 54° & 35' & 34.2'' \end{array} \text{ Ans.}$$

(303) The side $BC = \sqrt{\overline{AB}^2 - \overline{AC}^2}$, or $BC = \sqrt{17.69^2 - 9.75^2} = \sqrt{217.8736} = 14$ ft. 9 in To find the angle BAC, we have $\cos BAC = \frac{AC}{AB}$, or $\cos BAC = \frac{9.75}{17.69} =$ .55116.

.55116 equals the cos of 56° 33′ 12.5″.

Angle $ABC = 90° -$ angle BAC, or $90° - 56° 33' 12.5'' =$ 33° 26′ 47.5″.

(304)

$$\begin{array}{rrr} 159° & 27' & 34.6'' \\ 25° & 16' & 8.7'' \\ 3° & 48' & 53\ \ '' \\ \hline 188° & 32' & 36.3'' \end{array}$$

(305)

$$\sin 17° 28' = .30015.$$
$$\sin 17° 27' = .29987.$$

$.30015 - .29987 = .00028$, the difference for 1′.

$.00028 \times \frac{37}{60} = .00017$, difference for 37″.

$.29987 + .00017 = .30004 = \sin 17° 27' 37''$.

$$\cos 17° 27' = .95398.$$
$$\cos 17° 28' = .95389.$$

$.95398 - .95389 = .00009$, difference for 1′.

$.00009 \times \frac{37}{60} = .00006$, difference for 37″.

$.95398 - .00006 = .95392 = \cos 17° 27' 37''$.

$$\tan 17° 28' = .31466.$$
$$\tan 17° 27' = .31434.$$

$.31466 - .31434 = .00032$, difference for 1′.

$.00032 \times \frac{37}{60} = .00020$, difference for 37″.

$31434 + .0002 = .31454 = \tan 17° 27' 37''$.

$$\left.\begin{array}{l} \sin 17° 27' 37'' = .30004 \\ \cos 17° 27' 37'' = .95392 \\ \tan 17° 27' 37'' = .31454 \end{array}\right\} \text{Ans.}$$

(306) From the vertex B, draw BD perpendicular to AC, forming the right-angled triangles ADB and BDC. In the right-angled triangle ADB, AB is known, and also the angle A. Hence, $BD = 26.583 \times \sin 36° 20' 43'' = 26.583 \times .59265 = 15.754$ feet. $AD = 26.583 \times \cos 36° 20' 43'' = 26.583 \times .80546 = 21.411$. $AC - AD = 40 - 21.411 = 18.589$ feet $= DC$. In the right-angled triangle BDC, the two sides BD and DC are known; hence, $\tan C = \frac{BD}{DC} = \frac{15.754}{18.589} = .84749$, and angle $C = 40° 16' 52''$. Ans.

$$BC = \frac{BD}{\sin C} = \frac{15.754}{\sin 40° 16' 52''} = \frac{15.754}{.64654} = 24.37, \text{ or } 24 \text{ ft. } 4.4 \text{ in.}$$
Ans.

Angle $B = 180° - (36° 20' 43'' + 40° 16' 52'') = 180° - 76° 37' 35'' = 103° 22' 25''$. Ans.

(307) This problem is solved exactly like problem No. 305.

Sin of $63° 4' 51.8'' = .89165$.
Cos of $63° 4' 51.8'' = .45273$.
Tan of $63° 4' 51.8'' = 1.96949$.

(308)
$.27038 = \sin\ 15° 41' 12.9''$.
$.27038 = \cos\ 74° 18' 47.1''$.
$2.27038 = \tan\ 66° 13' 43.2''$.

(309) The angle formed by drawing radii to the extremities of one of the sides equals $\frac{360°}{11}$, or $32° 43' 38.2''$. Ans. The length of one side of the undecagon equals $\frac{4 \text{ ft. } 3 \text{ in.}}{11}$, or 4.6364 inches. The radius of the circle equals

$$\frac{\frac{1}{2} \text{ of } 4.6364}{\sin \text{ of } \frac{1}{2}(32° 43' 38.2'')} = \frac{2.3182}{.28173} = 8.23 \text{ inches. Ans.}$$

(310) If one of the angles is twice the given one, then it must be $2 \times (47° 13' 29'')$, or $94° 26' 58''$. Since there are two right angles, or $180°$, in the three angles of a triangle, the third angle must be $180 - (47° 13' 29'' + 94° 26' 58'')$, or $38° 19' 33''$.

(311) If one of the angles is one-half as large as the given angle, then it must be $\frac{1}{2}$ of 75° 48′ 17″, or 37° 54′ 8.5″. The third angle equals 180° − (75° 48′ 17″ + 37° 54′ 8.5″), or 66° 17′ 34.5″.

(312) From the vertex B, draw BD perpendicular to AC, forming the two right-angled triangles ADB and BDC. In the right-angled triangle ADB, AB is known, and also the angle A. Hence, $BD = \sin A \times AB = \sin 54° 54' 54'' \times 16\frac{5}{12} = .81830 \times 16\frac{5}{12} = 13.434$ feet.

Sine of angle $C = \frac{BD}{BC} = \frac{13.434}{13.542} = .99202$, and, hence, angle $C = 82° 45' 30''$. Ans.

Angle $B = 180° - (54° 54' 54'' + 82° 45' 30'') = 180 - 137° 40' 24'' = 42° 19' 36''$. Ans.

$AD = AB \times \cos A = 16\frac{5}{12} \times \cos 54° 54' 54'' = 16\frac{5}{12} \times .57479 = 9.43613$ ft.

$CD = BC \times \cos C = BC \times \cos 82° 45' 30'' = 13\frac{13}{24} \times .12605 = 1.70692$ ft.

$AC = AD + CD = 9.43613 + 1.70692 = 11.143 = 11$ ft. $1\frac{3}{4}$ in. Ans.

(313) If one-third of a certain angle equals 14° 47′ 10″, then the angle must be 3 × 14° 47′ 10″, or 44° 21′ 30″. $2\frac{1}{2}$ × 44° 21′ 30″, or 110° 53′ 45″, equals one of the other two angles. The third angle equals 180° − (110° 53′ 45″ + 44° 21′ 30″), or 24° 44′ 45″.

(314) Given, $BC = 437$ feet and $AC = 792$ feet, to find the hypotenuse AB and the angles A and B.

$AB = \sqrt{\overline{AC}^2 + \overline{BC}^2} = \sqrt{792^2 + 437^2} = \sqrt{818,233} = 904$ ft $6\frac{3}{4}$ in. Ans.

Tan $A = \frac{437}{792} = .55177$; therefore, $A = 28° 53' 19''$. Ans.

Angle $B = 90° - 28° 53' 19'' = 61° 6' 41''$. Ans.

(315) In Fig. 9, angle $A\,O\,B = \frac{1}{8}$ of 360°, or 45°. Angle $m\,O\,B = \frac{1}{2}$ of 45°, or $22\frac{1°}{2}$. Side $A\,B = \frac{1}{8}$ of 56 feet, or 7 feet. Now, in the triangle $m\,O\,B$, we have the angle $m\,O\,B = 22\frac{1°}{2}$, and $m\,B = \frac{7}{2}$, or $3\frac{1}{2}$ feet, given, to find $O\,B$ and the angle $m\,B\,O$.

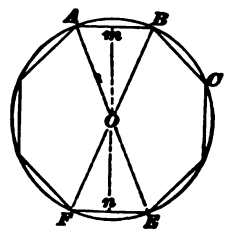

FIG. 9.

$$\text{Sin } m\,O\,B = \frac{m\,B}{O\,B}, \text{ or } O\,B = \frac{m\,B}{\sin m\,O\,B}.$$

Substituting their values, $O\,B = \frac{3.5}{\sin 22\frac{1}{2}°} = \frac{3.5}{.38268} = 9.146$ feet.

$B\,F$, the diameter of the circle, equals $2 \times B\,O$; therefore, $B\,F = 2 \times 9.146 = 18.292$ feet $= 18$ feet $3\frac{1}{2}$ inches.

$$\text{Angle } B\ O\ m = 22° 30'.$$
$$B\ O\ m + O\ B\ m = 90°.$$

Therefore, $O\ B\ m = 90° - B\ O\ m = 90° - 22° 30'$.
$$= 67° 30'.$$
$$A\ B\ C = 2\ O\ B\ m = 2\ (67° 30') = 135°.$$

Ans.

By Art. **703,** the sum of the interior angles of an octagon is $2\,(8 - 2) = 12$ right angles. Since the octagon is regular, the interior angles are equal, and since there are eight of them, each one is $\frac{12}{8} = 1\frac{1}{2}$ right angles. $1\frac{1}{2} \times 90° = 135°$.

(316) Lay off with a protractor the angle $A\ O\ C$ equal to 67° 8′ 49″, Fig. 10. Tangent to the circle at A, draw the line $A\ T$. Through the point C, draw the line $O\ C$, and continue it until it intersects the line $A\ T$ at T. From C

draw the lines CD and CB perpendicular, respectively, to the radii OE and OA. CB is the sine, CD the cosine, and AT the tangent.

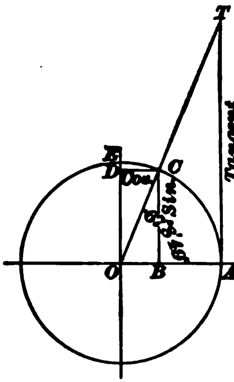

FIG. 10.

(317) Suppose that in Fig. 10, the line AT has been drawn equal to 3 times the radius OA. From T draw TO; then, the tangent of $TOA = \frac{TA}{OA} = 3$. Where TO cuts the circle at C, draw CD and CB perpendicular, respectively, to OE and OA. CD is the cosine and CB the sine. The angle corresponding to tan 3 is found by the table to equal 71° 33′ 54″; therefore, sin 71° 33′ 54″ = .94868 and cos 71° 33′ 54″ = .31623.

(318) The angle whose cos is .39278 = 66° 52′ 20″.
Sin of 66° 52′ 20″ = .91963.
Tan of 66° 52′ 20″ = 2.34132.

For a circle with a diameter $4\frac{3}{4}$ times as large, the values of the above cos, sin, and tan will be

$$\left.\begin{array}{l} 4\frac{3}{4} \times .39278 = 1.86570 \text{ cos.} \\ 4\frac{3}{4} \times .91963 = 4.36824 \text{ sin.} \\ 4\frac{3}{4} \times 2.34132 = 11.12127 \text{ tan.} \end{array}\right\} \text{Ans.}$$

(319) See Fig. 11. Angle B = 180° − (29° 21′ + 76° 44′ 18″) = 180° − 106° 5′ 18″ = 73° 54′ 42″.

From C, draw CD perpendicular to AB.

$AD = AC \cos A = 31.833 \times \cos 29° 21′ = 31.833 \times .87164 = 27.747$ ft. $CD = AC \sin A = 31.833 \times \sin 29° 21′ = 31.833 \times .49014 = 15.603$.

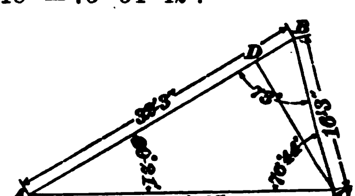

FIG. 11.

$$B C = \frac{C D}{\sin B} = \frac{15.603}{\sin 73° 54' 42''} = 16.24 \text{ feet} = 16 \text{ ft. } 3 \text{ in.}$$

$$B D = \frac{D C}{\tan B} = \frac{15.603}{\tan 73° 54' 42''} = 4.5 \text{ feet.}$$

$$A B = A D + D B = 27.747 + 4.5 = 32.247 = 32 \text{ ft. } 3 \text{ in.}$$

$$\text{Ans.} \begin{cases} A B = 32 \text{ ft. } 3 \text{ in.} \\ B C = 16 \text{ ft. } 3 \text{ in.} \\ B = 73° 54' 42''. \end{cases}$$

(320) In Fig. 8, problem 301, $A B$ is the side of a regular decagon; then, the angle $C O B = \frac{1}{20}$ of 360°, or 18°. To find the side CB, we have $CB = O B \times \sin 18°$, or $CB = 9.75 \times .30902 = 3.013$ inches. Since $A B = 2 \times CB$, $A B = 2 \times 3.013$, or 6.026 inches, which multiplied by 10, the number of sides, equals 60.26 inches. Ans.

(321) Perimeter of circle equals $2 \times 9.75 \times 3.1416$, or 61.26 inches. $61.26 - 60.26 = 1$ inch, the difference in their perimeters. Ans.

In order to find the area of the decagon, we must first find the length of the perpendicular $C O$ (see Fig. 8 in answer to question 301); $C O = O B \times \cos 18°$, or $C O = 9.75 \times .95106 = 9.273$. Area of triangle $A O B = \frac{1}{2} \times 9.273 \times 6.026$, or 27.939, which multiplied by 10, the number of triangles in the decagon, equals 279.39 square inches. Area of the circle $= 3.1416 \times 9.75 \times 9.75$, or 298.65 square inches.

298.65 – 279 39 = 19.26 square inches difference. Ans.

(322) The diameter of the circle equals $\sqrt{\frac{89.42}{.7854}} = \sqrt{113.8528}$, or 10.67 inches. Ans.

The circumference equals 10.67×3.1416, or 33.52 inches. Ans.

In a regular hexagon inscribed in a circle, each side is equal to the radius of the circle; therefore, $\frac{10.67}{2} = 5.335$ inches is the length of a side. Ans.

(323) Angle $m\,O\,B = \frac{1}{16}$ of 360°, or $22\frac{1}{2}°$. $m\,O = \frac{1}{2}$ of $m\,n = \frac{1}{2}$ of 2, or 1 inch. (See Fig. 12).

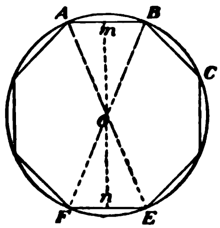

FIG. 12.

Side $m\,B = O\,m \times \tan 22\frac{1}{2}°$, or $m\,B = 1 \times .41421 = .41421$.

$A\,B = 2\,m\,B$; therefore, $A\,B = .82842$ inch.

Area of $A\,O\,B = \frac{1}{2} \times .82842 \times 1 = .41421$ square inch, which, multiplied by 8, the number of equal triangles, equals 3.31368 square inches.

Wt. of bar equals $3.31368 \times 10 \times 12 \times .282$, or 112 pounds 2 ounces. Ans.

(324) $16 \times 16 \times 16 \times \frac{1}{6} \times 3.1416 = 2{,}144.66$ cu. in. equals the volume of a sphere 16 inches in diameter.

$12 \times 12 \times 12 \times \frac{1}{6} \times 3.1416 = 904.78$ cu. in. equals the volume of a sphere 12 inches in diameter.

The difference of the two volumes equals the volume of the spherical shell, and this multiplied by the weight per cubic inch equals the weight of the shell. Hence, we have $(2{,}144.66 - 904.78) \times .261 = 323.61$ lb. Ans.

(325) The circumference of the circle equals $\frac{5\frac{13}{32} \times 360}{27}$, or 72.0833 inches. The diameter, therefore, equals $\frac{72.0833}{3.1416}$, or 22.95 inches.

(326) The number of square inches in a figure 7 inches square equals 7×7, or 49 square inches. $49 - 7 = 42$ square inches difference in the two figures.

$\sqrt{7} = 2.64$ inches is the length of side of square containing 7 square inches. The length of one side of the other square equals 7 inches.

(327) (*a*) $17\frac{1}{64}$ inches = 17.016 inches.

Area of circle = $17.016^2 \times .7854 = 227.41$ sq. in. Ans.
Circumference = $17.016 \times 3.1416 = 53.457$ inches.

$$16° \; 7' \; 21'' = 16.1225°.$$

(*b*) Length of the arc = $\frac{16.1225 \times 53.457}{360} = 2.394$ inches.
Ans.

(328) Area = $12 \times 8 \times .7854 = 75.4$ sq. in. Ans.
Perimeter = $(12 \times 1.82) + (8 \times 1.315) = 32.36$ in. Ans.

(329) Area of base = $\frac{1}{4} \times 3.1416 \times 7 \times 7 = 38.484$ sq. in.

The slant height of the cone equals $\sqrt{11^2 + 3\frac{1}{2}^2}$, or 11.5434 in.

Circumference of base = $7 \times 3.1416 = 21.9912$.

Convex area of cone = $21.9912 \times \frac{11.5434}{2} = 126.927$.

Total area = $126.927 + 38.484 = 165.41$ square inches.
Ans.

(330) Volume of sphere equals $10 \times 10 \times 10 \times \frac{1}{6} \times 3.1416 = 523.6$ cu. in.

Area of base of cone = $\frac{1}{4} \times 3.1416 \times 10 \times 10 = 78.54$ sq. in.

$\frac{3 \times 523.6}{78.54} = 20$ inches, the altitude of the cone. Ans.

(331) Volume of sphere = $\frac{1}{6} \times 3.1416 \times 12 \times 12 \times 12 = 904.7808$ cu. in.

Area of base of cylinder = $\frac{1}{4} \times 3.1416 \times 12 \times 12 = 113.0976$ sq. in.

Height of cylinder = $\frac{904.7808}{113.0976} = 8$ inches. Ans.

(332) (*a*) Area of the triangle equals $\frac{1}{2} A\,C \times B\,D$, or $\frac{1}{2} \times 9\frac{1}{2} \times 12 = 57$ square inches. Ans.

(*b*) See Fig. 13. Angle $BAC = 79° 22'$; angle $ABD = 90° - 79° 22' = 10° 38'$. Side $AB = BD \div \sin 79° 22' = 12 \div .98283 = 12.209$ inches.

Side $AD = BD \times \tan 10° 38' = 12 \times .18775 = 2.253$ inches.

Side $DC = AC - AD = 9.5 - 2.253 = 7.247$ inches.

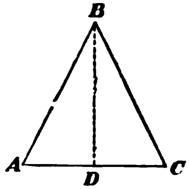

FIG. 13

Side $BC = \sqrt{\overline{DB}^2 + \overline{DC}^2} = \sqrt{12^2 + 7.247^2} = \sqrt{196.519} = 14.018$ inches.

Perimeter of triangle equals $AB + BC + AC = 12.209 + 14.018 + 9.5 = 35.73$ inches. Ans.

(333) The diagonal divides the trapezium into two triangles; the sum of the areas of these two triangles equals the area of the trapezium, which is, therefore,

$$\frac{11 \times 7}{2} + \frac{11 \times 4\frac{1}{4}}{2} = 61\frac{7}{8} \text{ square inches. Ans.}$$

(334) Referring to Fig. 17, problem 350, we have OA or $OB = \frac{10}{2}$ or 5 inches, and $AB = 6\frac{3}{4}$ inches.

$\sin COB = \frac{CB}{OB} = \frac{6\frac{3}{4} \div 2}{5} = .675$; therefore, angle $COB = 42° 27' 14.3''$.

Angle $AOB = (42° 27' 14.3'') \times 2 = 84° 54' 28.6''$. Ans.

$CO = OB \times \cos COB = 5 \times .73782 = 3.6891$.

Area of sector $= 10^2 \times .7854 \times \frac{84° 54' 28.6''}{360°} = 18.524$ sq. in

Area of triangle $= \frac{6.75 \times 3.6891}{2} = 12.450$ sq. in.

$18.524 - 12.450 = 6.074$ sq. in., the area of the segment. Ans.

(335) Convex area $=$

$$\frac{\text{perimeter of base} \times \text{slant height}}{2} = \frac{63 \times 17}{2} = 535.5 \text{ square inches. Ans.}$$

(336) See Fig. 14. Area of lower base $= 18^2 \times .7854 = 254.469$ sq. in.

Area of upper base $= 12^2 \times .7854 = 113.0976$ sq. in.

$GE = BG - AF = 9 - 6$, or 3 inches.

Slant height $FG = \sqrt{\overline{GE}^2 + \overline{EF}^2} = \sqrt{3^2 + 14^2} = 14.32$ inches.

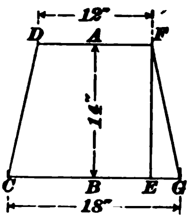

FIG. 14.

Convex area =

$$\frac{\text{circumference of upper base} + \text{circumference of lower base}}{2} \times$$

slant height, or convex area $= \dfrac{37.6992 + 56.5488}{2} \times 14.32 = 674.8156$ sq. in.

Total area $= 674.8156 + 254.469 + 113.0976 = 1,042.38$ sq. in. Ans.

Volume = (area of upper base + area of lower base + $\sqrt{\text{area of upper base} \times \text{area of lower base}}$) $\times \dfrac{1}{3}$ of the altitude =

$(113.0976 + 254.4696 + \sqrt{113.0976 \times 254.4696}) \dfrac{14}{3} = 2,506.997$ cubic inches. Ans.

(337) Area of surface of sphere 27 inches in diameter $= 27^2 \times 3.1416 = 2,290.2$ sq. in. Ans.

(338) Volume of each ball $= \dfrac{10}{.261} = 38.3142$ cu. in.

Diameter of ball $= \sqrt[3]{\dfrac{38.3142}{.5236}} = 4.18$ inches. Ans.

(339) Area of end $= 19^2 \times .7854 = 283.5294$ sq. in. Volume $= 283.5294 \times 24 = 6,804.7056$ cubic inches $= 3.938$ cubic feet. Ans.

(340) Given, $BI = 2$ inches and $HI = IK = \dfrac{14}{2} = 7$ inches to find the radius.

$BI : HI :: HI : AI$, or $2 : 7 :: 7 : AI$;

therefore, $AI = \dfrac{49}{2} = 24\dfrac{1}{2}$ inches.

$$AB = AI + BI = 24\frac{1}{2} + 2 = 26\frac{1}{2} \text{ inches.}$$

$$\text{Radius} = \frac{AB}{2} = \frac{26\frac{1}{2}}{2} = 13\frac{1}{4} \text{ inches. Ans.}$$

(341) (*a*) Area of piston $= 19^2 \times .7854 = 283.529$ sq. in., or 1.9689 square feet.

Length of stroke plus the clearance $= 1.14 \times 2$ ft. (24 in. = 2 ft.) = 2.28 ft.

$1.9689 \times 2.28 = 4.489$ cubic feet, or the volume of steam in the small cylinder. Ans.

(*b*) Area of piston $= 31^2 \times .7854 = 754.7694$ sq. in., or 5.2414 square feet.

Length of stroke plus the clearance $= 1.08 \times 2 = 2.16$ ft.

$5.2414 \times 2.16 = 11.321$ cubic feet, or the volume of steam in the large cylinder. Ans.

$$(c)\quad \text{Ratio} = \frac{11.321}{4.489}, \text{ or } 2.522:1. \quad \text{Ans.}$$

(342) (*a*) Area of cross-section of pipe $= 8^2 \times .7854 =$ 50.2656 sq. in.

$$\text{Volume of pipe} = \frac{50.2656 \times 7}{144} = 2.443 \text{ cu. ft.} \quad \text{Ans.}$$

(*b*) Ratio of volume of pipe to volume of small cylinder $= \frac{2.443}{4.489}$, or 0.544:1. Ans.

(343) (*a*) In Fig. 15, given, $OB = \frac{16}{2}$, or 8 inches, and $OA = \frac{13}{2}$, or $6\frac{1}{2}$ inches, to find the volume, area and weight:

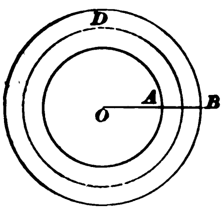

FIG. 15.

Radius of center circle equals $\frac{8 + 6.5}{2}$ or $7\frac{1}{4}$ inches. Length of center line $= 2 \times 3.1416 \times 7\frac{1}{4} = 45.5532$ inches.

The radius of the inner circle is $6\frac{1}{2}$ inches, and of the outer circle 8 inches; therefore, the diameter of the cross-section on the line AB is $1\frac{1}{2}$ inches. Then, the area of the ring is $1\frac{1}{2} \times 3.1416 \times 45.553 = 214.665$ square inches. Ans.

Diameter of cross-section of ring = $1\frac{1}{2}$ inches.

Area of cross-section of ring $= \left(1\frac{1}{2}\right)^2 \times .7854 = 1.76715$ sq. in. Ans.

Volume of ring $= 1.76715 \times 45.553 = 80.499$ cu. in. Ans.

(*b*) Weight of ring $= 80.499 \times .261 = 21$ lb. Ans.

(344) The problem may be solved like the one in Art. **790.** A quicker method of solution is by means of the principle given in Art. **826.**

(345) The convex area $= 4 \times 5\frac{1}{4} \times 18 = 378$ sq. in. Ans.

Area of the bases $= 5\frac{1}{4} \times 5\frac{1}{4} \times 2 = 55.125$ sq. in.

Total area $= 378 + 55.125 = 433.125$ sq. in. Ans.

Volume $= \left(5\frac{1}{4}\right)^2 \times 18 = 496.125$ cu. in. Ans.

(346) In Fig. 16, $OC = \frac{AC}{\tan 30°}$. $\left(\frac{1}{6}\text{ of } 360° = 60°, \text{ and since } AOC = \frac{1}{2} \text{ of } AOB,\ AOC = 30°\right)$.

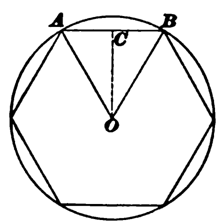

FIG. 16.

$OC = \frac{6}{.57735} = 10.392.$

Area of $AOB = \frac{12 \times 10.392}{2} = 62.352$ square feet.

Since there are 6 equal triangles in a hexagon, then the area of the base $= 6 \times 62.352$, or 374.112 square feet.

Perimeter $= 6 \times 12$, or 72 feet.

Convex area $= \frac{72 \times 37}{2} = 1{,}332$ sq. ft. Ans.

Total area $= 1{,}332 + 374.112 = 1{,}706.112$ sq. ft. Ans.

(347) Area of the base = 374.112 square feet, and altitude = 37 feet. Since the volume equals the area of the base multiplied by $\frac{1}{3}$ of the altitude, we have

$$\text{Volume} = 374.112 \times \frac{37}{3} = 4{,}614 \text{ cubic feet.}$$

(348) Area of room = 15 × 18 or 270 square feet.

One yard of carpet 27 inches wide will cover $3 \times 2\frac{1}{4}$ (27 inches $= 2\frac{1}{4}$ ft.) $= 6\frac{3}{4}$ sq. ft. To cover 270 sq. ft., it will take $\frac{270}{6\frac{3}{4}}$, or 40 yards. Ans.

(349)

Area of ceiling = 16 × 20 =	320 square feet.
Area of end walls = 2(16 × 11) =	352 square feet.
Area of side walls = 2(20 × 11) =	440 square feet.
Total area =	1,112 square feet.

From the above number of square feet, the following deductions are to be made:

Windows = 4(7 × 4) = 112 square feet.
Doors = 3(9 × 4) = 108 square feet.

Baseboard less the width of the three doors

equals $(72' - 12') \times \frac{6}{12} = 30$ square feet.

Total No. of feet to be deducted = 250 square feet.

Number of square feet to be plastered, then, equals 1,112 − 250, or 862 square feet, or $95\frac{7}{9}$ square yards. Ans.

(350) Given $AB = 6\frac{7}{8}$ inches, and $OB = OA = \frac{10}{2}$ or 5 inches, Fig. 17, to find the area of the sector.

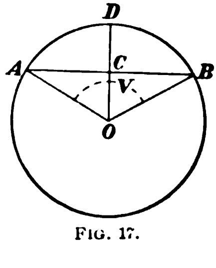

FIG. 17.

Area of circle $= 10^2 \times .7854 = 78.54$ square inches.

$\text{Sin } AOC = \frac{AC}{OA} = \frac{6\frac{7}{8} \div 2}{5} = .6875$; therefore, $AOC = 43° 26'$.

$AOB = 2 \times AOC = 2 \times 43° 26' = 86° 52' = 86.8666°$.

$\frac{86.8666}{360} \times 78.54 = 18.95$ square inches.

Ans.

(351) Area of parallelogram equals

$$7 \times 10\frac{3}{4}\ (129 \text{ inches} = 10\frac{3}{4} \text{ ft.}) = 75\frac{1}{4} \text{ sq. ft.} \quad \text{Ans.}$$

(352) (*a*) See Art. **778.**

Area of the trapezoid $= \dfrac{15\frac{7}{12} + 21\frac{11}{12}}{2} \times 7\frac{2}{3} = 143.75$ sq. ft.

Ans.

(*b*) In the equilateral triangle ABC, Fig. 18, the area, 143.75 square feet, is given to find a side. Since the triangle is equilateral all the angles are equal to $\frac{1}{3}$ of 180° or 60°. In the triangle $ABD = ADC$, we have $AD = AB \times \sin 60°$. The area of any triangle is equal to one-half the product of the base by the altitude, therefore, $\dfrac{BC \times AD}{2} = 143.75$.

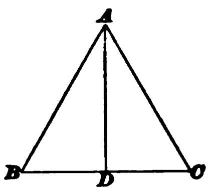

FIG. 18.

$BC = AB$ and $AD = AB \times \sin 60°$; then, the above becomes

$$\frac{AB \times AB \sin 60°}{2} = 143.75,$$

$$\text{or} \quad \frac{AB^2 \times .86603}{2} = 143.75,$$

$$\text{or} \quad AB^2 = \frac{2 \times 143.75}{.86603}.$$

$$\text{Therefore,} \quad AB = \sqrt{\frac{287.50}{.86603}} = 18 \text{ ft. } 2.64 \text{ in.} \quad \text{Ans.}$$

(353) (*a*) Side of square having an equivalent area $= \sqrt{143.75} = 11.99$ feet. Ans.

(*b*) Diameter of circle having an equivalent area $= \sqrt{\dfrac{143.75}{.7854}} = \sqrt{183.0277} = 13\frac{1}{2}$ feet. Ans.

(*c*) Perimeter of square $= 4 \times 11.99 = 47.96$ ft.

Circumference or perimeter of circle $= 13\frac{1}{2} \times 3.1416 = 42.41$ ft.

Difference of perimeter $= 5.55$ ft. $=$

5 feet 6.6 inches. Ans.

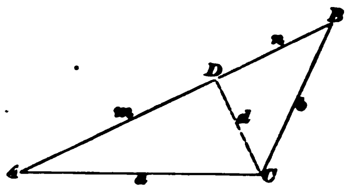

FIG. 19.

(354) In the triangle ABC, Fig. 19,

$AB = 24$ feet,
$BC = 11.25$ feet, and
$AC = 18$ feet.

$m + n : a + b :: a - b : m - n$,
or $24 : 29.25 :: 6.75 : m - n$.

$$m - n = \frac{29.25 \times 6.75}{24} = 8.226562.$$

Solving for m and n (see Art. **761**),

$$m = \frac{(m+n)+(m-n)}{2} = \frac{24 + 8.226562}{2} = 16.113281 \text{ ft.};$$

$$n = \frac{(m+n)-(m-n)}{2} = \frac{24 - 8.226562}{2} = 7.886719 \text{ ft.}$$

In the triangle ADC, side $AC = 18$ feet, side $AD = 16.113281$; hence, according to Rule **3**, Art. **754**, $\cos A = \frac{16.113281}{18} = .89518$, or angle $A = 26° 28' 5''$. In the triangle BDC, side $BD = 7.886719$, and side $BC = 11.25$ ft. Hence, $\cos B = \frac{7.886719}{11.25} = .70104$, or angle $B = 45° 29' 23''$. Angle $C = 180° - (45° 29' 23'' + 26° 28' 5'') = 108° 2' 32''$.

Ans. $\begin{cases} A = 26° 28' 5''. \\ B = 45° 29' 23''. \\ C = 108° 2' 32''. \end{cases}$

GASES MET WITH IN MINES.

(355) See Art. **828.**

(356) See Art. **834.**

(357) See Art. **841.**

(358) See Art. **849.**

(359) Carbonic acid gas, because it contains more matter per unit of volume, and is more compact and heavier than marsh-gas. See Art. **830** and Table 19 (Art. **865**).

(360) See Art. **831.**

(361) Applying formula **2,** we have

$$\text{Sp. Gr.} = \frac{175}{62.5} = 2.8. \quad \textbf{Ans.}$$

(362) See Arts. **835** and **836.**

(363) It drives or tends to drive the molecules apart. See Art. **843.**

(364) See Arts. **846, 847,** and **848.**

(365) See Art. **860.**

(366) See Art. **864.**

(367) See Art. **881.**

(368) The amount of matter in a body, regardless of the space it occupies, is called mass, while the space which the body occupies, regardless of the amount of matter, is called the volume. See Art. **829.**

(369) Applying formula **4,** we have $62.5 \times 1.5 = 93.75$ lb., the weight of 1 cu. ft. of anthracite coal. Hence, the weight of 1 cu. yd. or 27 cu. ft. = 27 × 93.75 lb. = 2,531.25 lb. Ans.

(370) See Art. **836.**

(371) See Art. **838.**

(372) See Art. **840.**

(373) A compound substance is a substance formed of molecules which are unlike in their nature. See Arts. **836** and **842.**

(374) See Art. **844.**

(375) See Art. **847.**

(376) See Art. **850.**

(377) See Art. **851.**

(378) Applying formula **8,**

$$v_1 = \frac{6 \times 8}{80} = \frac{3}{5} = .6 \text{ cu. ft.} \quad \text{Ans.}$$

(379) See Art. **853.**

(380) See Art. **859.**

(381) See Art. **860.**

(382) See Art. **866.**

(383) Applying formula **21,**

$$W = \frac{1.3253 \times 300 \times 30 \times 1.5291}{459 + 70} = 34.48 \text{ lb.} \quad \text{Ans.}$$

(384) See Art. **877.**

(385) See Art. **887.**

(386) See Art. **897.**

(387) See Arts. **902, 903,** and **904.**

(388) See Art. **871.**

(389) See Art. **861.**

(390) See Art. **898.**

(391) See Art. **895.**

(392) See Art. **899.**

(393) Applying formula **12,**

$$20 : W_1 :: 12 : .0763, \text{ or } W_1 = \frac{20 \times .0763}{12} = .1272 \text{ lb. Ans}$$

(394) See Art. **889.**

(395) Applying formula **7,**

$$p_1 = \frac{3 \times 36}{4} = 27 \text{ lb. per square inch. Ans.}$$

(396) See Art. **870.**

(397) See Arts. **882** and **883.**

(398) See Art. **876.**

(399) See Art. **848.**

(400) See Art. **845.**

(401) See Art. **874.**

(402) One pound of bituminous coal when burned, furnishes 14,400 B. T. U. (see Table 18); hence, 2,000 pounds will furnish 2,000 × 14,400 = 28,800,000 B. T. U. Ans.

(403) See Arts. **878** and **879.**

(404) See Art. **873.**

(405) See Art. **856.**

(406) See Art. **839.**

(407) See Art. **841.**

(408) The carbon in the coal = .88 × 300 lb. = 264 lb., and since the molecular weight of carbonic acid gas (CO_2) is 12 + 32 = 44, the carbon in the gas must be $\frac{12}{44}$ of the weight of the gas. Therefore, if 264 lb. of carbon be used to produce carbonic acid gas, 264 lb. will represent $\frac{12}{44}$ of the resulting product. Hence, $\frac{44}{44}$, or the whole of the gas formed, $= \frac{264 \times 44}{12} = 968$ lb. Ans.

See Art. **838** and Table 17.

(409) Dissociation is the disunion of the elements forming a compound. See Art. **837.**

(410) See Art. **834.**

(411) Applying formula **1,** we have

$$\text{Sp. Gr.} = \frac{10}{10 - 8.6} = 7.14. \quad \text{Ans.}$$

(412) To gases only. See Art. **841.**

(413) One molecule of CH_4 yields one molecule of CO_2, and since they are both compound gases, a molecule of each occupies the same volume. Hence, 1,200 cu. ft. of CH_4 will yield 1,200 cu. ft. of CO_2. Ans. See Art. **841.**

(414) Gases, liquids, and solids. See Art. **833.**

(415) See Art. **863.**

(416) See Art. **869.**

(417) See Art. **900.**

(418) See Art. **907.**

(419) See Art. **922.**

(420) See Art. **916.**

(421) Applying formula **20,**

$$V = \frac{15 \times 20 + 9 \times 15}{21} = 20.71 \text{ cu. ft.} \quad \text{Ans.}$$

(422) See Art. **918.**

(423) See Art. **922.**

(424) See Art. **862.**

(425) See Art. **923.**

(426) See Art. **919.**

(427) See Arts. **906** and **915.**

(428) See Art. **890.**

(429) Applying formula **14,** we have

$$p_1 = 14.7\left(\frac{459 + 212}{459 + 50}\right) = 19.38 \text{ lb. per square inch.} \quad \text{Ans.}$$

(**430**) See Art. **901.**

(**431**) See Arts. **907** and **910.**

(**432**) See Art. **913.**

(**433**) See Art. **915.**

(**434**) See Art. **891.**

(**435**) Applying formula **15,** we have

$$P = \frac{.37052 \times 6.5 \times (459 + 84)}{76} = 17.21 \text{ lb. per square inch.}$$

Ans.

(**436**) See Art. **911.**

(**437**) See Arts. **912** and **914.**

(**438**) See Art. **892.**

(**439**) See Arts. **893** and **894.**

(**440**) See Art. **900.**

(**441**) See Art. **896.**

(**442**) Weight $= .0766 \times .559 \times 100 = 4.28$ lb. Ans. See Art. **832.**

(**443**) Because the square root of the density of carbonic acid gas is greater than that of marsh-gas. See Art. **865.**

(**444**) The specific gravity of marsh-gas is .559. Using formula **21,**

$$W = \frac{1.3253 \times 650 \times 29.5 \times .559}{459 + 60} = 26.75 \text{ lb.} \quad \text{Ans.}$$

MINE VENTILATION.

(PART 1.)

(445) See Arts. **925, 927,** and **932.**

(446) (*a*) Using formula **27,**

$$t = \frac{v}{g} = \frac{1,876}{32.16} = 58.333 \text{ sec.},$$

the time the ball would require to reach the highest point. Hence, $58.333 \times 2 = 116.66$ seconds, or 1.944 min. Ans.

(*b*) By using formula **28,**

$$h = \frac{v^2}{2g} = \frac{1,876^2}{2 \times 32.16} = 54,716.66 \text{ ft.},$$

the height the ball will rise. Hence, $54,716.66 \times 2 = 109,433.3$ ft., the total distance over which the ball will pass. Ans.

(447) See Art. **937.**

(448) See Art. **936.**

(449) Using formula **43,**

$$q = a\,v = 7 \times 7 \times 300 = 14,700 \text{ cu. ft. per min. Ans.}$$

(450) Since the water-gauge is equivalent to a certain pressure, law **(3)**, Art. **980,** may be used. Hence, substituting W and W_1 for p and p_1, respectively,

$$W : W_1 :: q^2 : q_1^2, \text{ or } 2 : 8 :: 15,000^2 : p_1;$$

whence, $p_1 = 30,000$ cu. ft. Ans.

(451) Law **(15)** evidently applies to this case. Calling the original quantity 1,

$$H : H_1 :: q^2 : q_1^2, \text{ or } 2 : H_1 :: 1^2 : 2^2;$$

whence, $H_1 = 16$ horsepower. Ans

(452) Since $p = 5.2\,W$, $p = 5.2 \times 1.5 = 7.8$ lb. per sq. ft. Applying formula **36**,

$$P = p\,a = 7.8 \times 6 \times 7 = 327.6 \text{ lb.} \quad \text{Ans.}$$

(453) The perimeter of the 6 ft. × 6 ft. airway is $6 \times 4 = 24$ ft.; of the 8 ft. × $4\frac{1}{2}$ ft., $8 \times 2 + 4\frac{1}{2} \times 2 = 25$ ft. Since both airways have the same length, the 6 ft. × 6 ft. airway has less rubbing surface than the 8 ft. × $4\frac{1}{2}$ ft., its perimeter being less. Hence, the 6 ft. × 6 ft. airway will pass the greater quantity of air.

(454) Applying the method illustrated in Art. **992**,

$$\sqrt{\frac{a_1^3}{s_1}} = \sqrt{\frac{36^3}{36{,}000}} = 1.1384, \text{ since } a_1 = 6 \times 6 = 36, \text{ and } s_1 = 4 \times 6 \times 1{,}500 = 36{,}000.$$

$$\sqrt{\frac{a_2^3}{s_2}} = \sqrt{\frac{42^3}{46{,}800}} = 1.2582, \text{ since } a_2 = 6 \times 7 = 42, \text{ and } s_2 = (2 \times 6 + 2 \times 7) \times 1{,}800 = 46{,}800.$$

$$\sqrt{\frac{a_3^3}{s_3}} = \sqrt{\frac{30^3}{29{,}700}} = .9535, \text{ since } a_3 = 6 \times 5 = 30, \text{ and } s_3 = (2 \times 6 + 2 \times 5) \times 1{,}350 = 29{,}700.$$

$$\sqrt{\frac{a_4^3}{s_4}} = \sqrt{\frac{25^3}{30{,}000}} = .7217, \text{ since } a_4 = 5 \times 5 = 25, \text{ and } s_4 = 4 \times 5 \times 1{,}500 = 30{,}000.$$

$$sum = 4.0718$$

$$q_1 = \frac{1.1384}{4.0718} \times 45{,}000 = 12{,}582 \text{ cu. ft. per min. for (1).}$$

$$q_2 = \frac{1.2582}{4.0718} \times 45{,}000 = 13{,}905 \text{ cu. ft. per min. for (2).}$$

$$q_3 = \frac{.9535}{4.0718} \times 45{,}000 = 10{,}539 \text{ cu. ft. per min. for (3).}$$

$$q_4 = \frac{.7217}{4.0718} \times 45{,}000 = 7{,}974 \text{ cu. ft. per min. for (4)}$$

$$sum = 45{,}000$$

Ans.

(455) Apply law (**22**). Since $p = 5.2\,W$, $p = 5.2 \times 1 = 5.2$ lb. per square foot. Then, $p : p_1 :: d_1^5 : d^5$, or $5.2 : p_1 :: 5^5 : 6^5$; whence, $p_1 = 12.94$ lb. per sq. ft. Ans.

(456) Since quantity and velocity are directly proportional, we may substitute v for q in law (**15**), obtaining $u : u_1 :: v^3 ; v_1^3$, or, calling the power originally required 1,

$1 : u_1 :: 4^3 : 8^3$; whence, $u_1 = 8$; i. e., the ratio of increase will be 8 : 1. Ans.

(457) Applying law (**5**), and calling the original pressure and length each 1,

$$p : p_1 :: l : l_1, \text{ or } 1 : p_1 :: 1 : 2;$$

whence, $p_1 = 2$, and the ratio is 2 : 1. Ans.

(458) Applying law (**3**),

$$p : p_1 :: q^2 : q_1^2, \text{ or } 1 : p_1 :: 1^2 : 2^2;$$

whence, $p_1 = 4$, and the ratio is 4 : 1. Ans.

(459) Applying law (**15**),

$$u : u_1 :: q^3 : q_1^3, \text{ or } 1 : u_1 :: 1^3 : 2^3;$$

whence, $u_1 = 8$, and the ratio is 8 : 1. Ans.

(460) Since the volumes are proportional to the absolute temperatures, we may write $v : v_1 :: T : T_1$, T being $459 + 30 = 489$, and T_1 being $459 + 70 = 529$. Hence, $10{,}000 : v_1 :: 489 : 529$, or $v_1 = 10{,}818$ cu. ft. per min. Ans.

(461) Since $p = 5.2\,W$, $p = 5.2 \times 2 = 10.4$ lb. per sq. ft. Applying formula **48**,

$$H = \frac{pq}{33{,}000} = \frac{10.4 \times 120{,}000}{33{,}000} = 37.82 \text{ horsepower, nearly.}$$

Ans.

(462) Substituting W and W_1 for p and p_1 in law (**3**),

$$W : W_1 :: q^2 : q_1^2, \text{ or } 3 : W_1 :: 20{,}000^2 : 30{,}000^2;$$

whence, $W_1 = 6\frac{3}{4}$ in. Ans.

(463) 5 ft. per sec. $= 5 \times 60 = 300$ ft. per min. Applying formula **45**,

$$a = \frac{q}{v} = \frac{8{,}000}{300} = 26\tfrac{2}{3} \text{ sq. ft.} = \text{area of No. 1 split.} \quad \text{Ans.}$$

$$a = \frac{10{,}000}{300} = 33\tfrac{1}{3} \text{ sq. ft.} = \text{area of No. 2 split.} \quad \text{Ans.}$$

$$a = \frac{12{,}000}{300} = 40 \text{ sq. ft.} = \text{area of No. 3 split.} \quad \text{Ans.}$$

$$a = \frac{14{,}000}{300} = 46\tfrac{2}{3} \text{ sq. ft.} = \text{area of No. 4 split.} \quad \text{Ans.}$$

$$a = \frac{16{,}000}{300} = 53\tfrac{1}{3} \text{ sq. ft.} = \text{area of No. 5 split.} \quad \text{Ans.}$$

(**464**) Applying formula **50**,

$$p = \frac{33{,}000\,H}{q} = \frac{33{,}000 \times 40}{112{,}000} = 11.79 \text{ lb. per sq. ft., nearly.}$$

Hence, $W = \frac{11.79}{5.2} = 2.27$ in., nearly. Ans.

(**465**) Applying the method described in Art. **992**,

$$\sqrt{\frac{a_1^{\,3}}{s_1}} = \sqrt{\frac{48^3}{192{,}000}} = .75894$$

$$\sqrt{\frac{a_2^{\,3}}{s_2}} = \sqrt{\frac{48^3}{280{,}000}} = .62846$$

$$sum = 1.38740$$

Then, $q_1 = \frac{.75894}{1.3874} \times 10{,}000 = 5{,}470$ cu. ft. per min. in A.

$q_2 = \frac{.62846}{1.3874} \times 10{,}000 = 4{,}530$ cu. ft. per min. in B. } Ans.

(**466**) (*a*) The easiest way to work this example is to calculate the ventilating pressure for each split; if all are equal, no regulators will be required, but if some, or all, are different, regulators must be introduced into those splits having the lesser values. The pressure may be calculated by using formula **44** to find the velocity, and then using formula **38**; but an easier way is to use the following formula, which is obtained by transposing terms in formula **q**, Art. **979**:

$$p = \frac{k s q^2}{a^3}.$$

Applying this formula, we have

$p = \frac{k s q^2}{a^3} = \frac{.0000000217 \times 280{,}000 \times 5{,}000^2}{48^3} = 1.374$ lb. per sq. ft. for A.

$p = \frac{k s q^2}{a^3} = \frac{.0000000217 \times 150{,}000 \times 10{,}000^2}{50^3} = 2.604$ lb. per sq. ft. for B.

$$p = \frac{k s q^2}{a^3} = \frac{.0000000217 \times 360,000 \times 20,000^2}{72^3} = 8.372 \text{ lb. per}$$
sq. ft. for *C*.

$$p = \frac{k s q^2}{a^3} = \frac{.0000000217 \times 160,000 \times 15,000^2}{48^3} = 7.064 \text{ lb. per}$$
sq. ft. for *D*.

Hence, to distribute the air as required by the example, regulators must be placed at *A*, *B*, and *D*. Ans.

(*b*) After placing the regulators, the pressure will be 8.372 lb. per sq. ft. in all the splits. Therefore, applying formula **48**,

$$H = \frac{p q}{33,000} = \frac{8.372 \times 50,000}{33,000} = 12.685 \text{ horsepower. Ans.}$$

(**467**) Using formula **21**,

$$W = \frac{1.3253\, V B D}{T}, \; W = \frac{1.3253 \times 30 \times 1 \times 1}{459 + 62} = .076313 \text{ lb.}$$

Now, applying formula **34**,

$$M = \frac{5.2\, G}{W} = \frac{5.2 \times .4}{.076313} = 27.256 \text{ ft. Ans.}$$

(**468**) In the last example, the weight of a cubic foot of air at 62° F. and 30 inches barometer was found to be .076313 lb.

Hence, applying formula **32**,

$$v = \sqrt{\frac{2 g F}{w}} = \sqrt{\frac{2 \times 32.16 \times 2.08}{.076313}} = 41.873 \text{ ft. per sec.} =$$
2,512.38 ft. per min. Ans.

(**469**) None of the laws will apply to this case, but the example may be worked as follows: Denoting the quantity passed with 15 horsepower by 1, we have for the quantity passed with 36 horsepower [applying law (**15**)],

$$H : H_1 :: q^3 : q_1^3, \text{ or } 15 : 36 :: 1^3 : q_1^3;$$

whence, $q_1^3 = 2.4$, and $q_1 = \sqrt[3]{2.4}$.

Now, applying law (**3**) and substituting W and W_1 for p and p_1, respectively,

$$W : W_1 :: q^2 : q_1^2, \text{ or } .6 : W_1 :: 1^2 : \sqrt[3]{2.4^2};$$

whence, $W_1 = 1.076$ in., nearly. Ans.

(**470**) The rubbing surfaces are $(2 \times 6 + 2 \times 8) \times 8{,}000 = 224{,}000$ sq. ft., and $(2 \times 6 + 2 \times 8) \times 10{,}000 = 280{,}000$ sq. ft. Hence, applying law (**10**), Art. **980**,

$$q : q_1 :: \sqrt{s_1} : \sqrt{s}, \text{ or } 10{,}000 : q_1 :: \sqrt{280{,}000} : \sqrt{224{,}000};$$

whence, $q_1 = 8{,}945$ cu. ft. per min., nearly. Ans.

(**471**) See Arts. **975** and **976.** Since the airways have similar sections,

$$10 : x :: \sqrt[5]{5{,}000} : \sqrt[5]{8{,}000}, \text{ or } x = 10.99 \text{ ft.};$$

also, $10 : 10.99 :: 8 : x$, or $x = 8.792$ ft.

Hence, the required section is 8.792 ft. × 10.99 ft., say 8.8 ft. × 11 ft. Ans.

(**472**) Since the airway is square and $a = 64$ sq. ft., the length of a side $= d = \sqrt{64} = 8$ ft. Representing the pressure by 1, the units of power required would be $u = pq = 1 \times 15{,}000 = 15{,}000$ ft.-lb. Since the power is to remain the same, the pressure for the new airway must be less (the length remaining the same), since the quantity is greater. Hence, $u = p_1 q_1 = p_1 \times 20{,}000 = 15{,}000$, or $p_1 = \frac{15{,}000}{20{,}000} = .75$; i. e., the new pressure is .75 of the original pressure. By using formula **55**, $q = \sqrt{\frac{p d^5}{4 k l}}$, whence, $q^2 = \frac{p d^5}{4 k l}$; also, $q_1 = \sqrt{\frac{p_1 d_1^5}{4 k l}}$, whence, $q_1^2 = \frac{p_1 d_1^5}{4 k l}$. Dividing the first by the second, $\frac{q^2}{q_1^2} = \frac{p d^5}{p_1 d_1^5}$, the denominators canceling out, being equal, or $q^2 : q_1^2 :: p d^5 : p_1 d_1^5$.

Substituting the values of q, q_1, p, d, and p_1,

$$15{,}000^2 : 20{,}000^2 :: 1 \times 8^5 : .75 d_1^5;$$

whence, $$d_1 = \sqrt[5]{\frac{20{,}000^2 \times 8^5}{.75 \times 15{,}000^2}} = 9.507 \text{ ft.}$$

Hence, the area $= 9.507^2 = 90.38$ sq. ft. Ans.

(473) Applying formula **56**,

$$A = \frac{.0004 q}{\sqrt{W}} = \frac{.0004 \times 8{,}000}{\sqrt{\frac{1}{4}}} = 6.4 \text{ sq. ft.} \quad \text{Ans.}$$

(474) (*a*) Applying the method described in Art. **992**,

$$\sqrt{\frac{a_1^3}{s_1}} = \sqrt{\frac{30^3}{19{,}800}} = 1.1678$$

$$\sqrt{\frac{a_2^3}{s_2}} = \sqrt{\frac{36^3}{19{,}800}} = 1.5350$$

$$\sqrt{\frac{a_3^3}{s_3}} = \sqrt{\frac{24^3}{16{,}800}} = .9071$$

$$\sqrt{\frac{a_4^3}{s_4}} = \sqrt{\frac{20^3}{12{,}960}} = .7857$$

$$sum = 4.3956$$

$$\left.\begin{aligned} q_1 &= \frac{1.1678}{4.3956} \times 60{,}000 = 15{,}942 \text{ cu. ft. for 1st split.} \\ q_2 &= \frac{1.5350}{4.3956} \times 60{,}000 = 20{,}952 \text{ cu. ft. for 2d split.} \\ q_3 &= \frac{.9071}{4.3956} \times 60{,}000 = 12{,}381 \text{ cu. ft. for 3d split.} \\ q_4 &= \frac{.7857}{4.3956} \times 60{,}000 = 10{,}725 \text{ cu. ft. for 4th split.} \end{aligned}\right\} \text{Ans.}$$

$$sum = 60{,}000$$

(*b*) Velocity in main split = 60,000 ÷ 80 = 750 ft. per min., since sectional area = 8 × 10 = 80 sq. ft. Applying formula **38** to find the pressure,

$$p = \frac{k s v^2}{a} = \frac{.0000000217 \times 54{,}000 \times 750^2}{80} =$$

8.24 lb. per sq. ft., nearly.

To find the pressure necessary to force the air through the splits, consider split No. 1.

$$\text{Velocity} = \frac{15{,}942}{30} = 531.4 \text{ ft. per min.}$$

Applying formula **38**,

$$p_1 = \frac{k s_1 v_1^2}{a_1} = \frac{.0000000217 \times 19{,}800 \times 531.4^2}{30} =$$

4.04 lb. per sq. ft., nearly.

Total pressure = 8.24 + 4.04 = 12.28 lb. per sq. ft.

Hence, water-gauge $= \frac{12.28}{5.2} = 2.36$ in., nearly. Ans.

(475) According to law **(15)**,

$$u : u_1 :: q^3 : q_1^3, \text{ or } 1 : u_1 :: 1^3 : 2^3; \text{ whence, } u_1 = 8.$$

That is, the power must be increased to 8 times its original amount in order to double the quantity. Ans.

(476) It is evident, from the conditions of the example, that one airway is 5 times the length of the other. Calling the length of the short airway 1, and the quantity passing through it 1; and the length of the long airway 5, and the quantity passing through it q_1, we have, applying law **(20)**, Art. **980**,

$$l : l_1 :: q_1^2 : q^2, \text{ or } 1 : 5 :: q_1^2 : 1; \text{ whence, } q_1 = \sqrt{\tfrac{1}{5}} = .4472.$$

Since $q = 1$, $q + q_1 = 1 + .4472 = 1.4472$. Hence,

$q = \frac{1}{1.4472} \times 100{,}000 = 69{,}099$ cu. ft. per min. in short airway.

$q_1 = \frac{.4472}{1.4472} \times 100{,}000 = 30{,}901$ cu. ft. per min. in long airway.

} Ans.

(477) (*a*) Applying formula **35**,

$$M = \frac{D(t - t_1)}{459 + t} = \frac{300\,(130 - 50)}{459 + 130} = 40.75 \text{ ft. Ans.}$$

(*b*) Applying formula **35**,

$$M = \frac{300\,(150 - 50)}{459 + 150} = 49.26 \text{ ft.}$$

The weight of a cubic foot of air at 50° F. and 30 inches barometer is

$$W = \frac{1.3253 \times 30}{459 + 50} = .07811 \text{ lb.}$$

Hence, the pressure per square foot = .07811 × 49.26 = 3.85 lb. Ans.

(478) (*a*) Applying the method illustrated in Art. **992**,

$$\sqrt{\frac{a_1^3}{s_1}} = \sqrt{\frac{36^3}{96,000}} = .69714$$

$$\sqrt{\frac{a_2^3}{s_2}} = \sqrt{\frac{30^3}{66,000}} = .63960$$

$$\sqrt{\frac{a_3^3}{s_3}} = \sqrt{\frac{25^3}{80,000}} = .44194$$

$$sum = 1.77868$$

$q_1 = \frac{.69714}{1.77868} \times 50,000 = 19,597$ cu. ft. per min. for 1st split.

$q_2 = \frac{.63960}{1.77868} \times 50,000 = 17,980$ cu. ft. per min. for 2d split.

$q_3 = \frac{.44194}{1.77868} \times 50,000 = 12,423$ cu. ft. per min. for 3d split.

} Ans.

$$sum = 50,000$$

(*b*) Applying formula used in solving example 466,

$$p = \frac{k s q^2}{a^3} = \frac{.0000000217 \times 400,000 \times 50,000^2}{100^3} =$$

21.7 lb. per sq. ft.

Therefore, $H = \frac{pq}{33,000} = \frac{21.7 \times 50,000}{33,000} =$

32.88 horsepower for first case. Ans.

$$p_1 = \frac{k s_1 q_1^2}{a_1^3} = \frac{.0000000217 \times 96,000 \times 19,597^2}{36^3} =$$

17.15 lb. per sq. ft.

Therefore, $H = \frac{pq}{33,000} = \frac{17.15 \times 50,000}{33,000} =$

25.98 horsepower for second case. Ans.

(479) Applying formula **41**,

$$l = \frac{s}{o} = \frac{54,000}{2 \times 9 + 2 \times 6} = 1,800 \text{ ft.} \quad \text{Ans.}$$

(480) 7 ft. 3 in. = $7\frac{1}{4}$ ft.; 11 ft. 9 in. = $11\frac{3}{4}$ ft. Hence, applying formula **43**,

$q = av = 7\frac{1}{4} \times 11\frac{3}{4} \times 434 = 36{,}971$ cu. ft. per min., nearly. Ans.

(481) Applying law **(3)**,

$p : p_1 :: q^2 : q_1^2$, or $1 : p_1 :: 120{,}000^2 : 180{,}000^2$; whence, $p_1 = 2\frac{1}{4}$.

Therefore, the original pressure must be increased $2\frac{1}{4}$ times. Ans.

(482) Applying law **(3)**,

$$p : p_1 :: q^2 : q_1^2, \text{ or } 19.2 : p_1 :: 160{,}000^2 : 120{,}000^2;$$

whence, $p_1 = 10.8$ lb. per sq. ft. Ans.

(483) For a water-gauge of .9 in., $p = 5.2 \times .9 = 4.68$ lb. per sq. ft. Applying formula **48**,

$$H = \frac{pq}{33{,}000} = \frac{4.68 \times 70{,}000}{33{,}000} = 9.927 \text{ horsepower. Ans.}$$

(484) Applying formula **35**,

$$M = \frac{D(t - t_1)}{459 + t} = \frac{300\,(120 - 45)}{459 + 120} = 38.86 \text{ ft. Ans.}$$

(485) For a water-gauge of .6 in., $p = 5.2 \times .6 = 3.12$ lb. per sq. ft. Applying law **(3)**,

$$p : p_1 :: q^2 : q_1^2, \text{ or } 3.12 : p_1 :: 12{,}000^2 : 24{,}000^2;$$

whence, $p_1 = 12.48$ lb. per sq. ft. Ans.

(486) Quantity passing in first case is $6 \times 8 \times 300 =$ 14,400 cu. ft. per min. Applying law **(3)**,

$$p : p_1 :: q^2 : q_1^2, \text{ or } 4 : p_1 :: 14{,}400^2 : 24{,}000^2;$$

whence, $p_1 = 11\frac{1}{9}$ lb. per sq. ft. Ans.

(487) Applying formula **48**,

$$H = \frac{pq}{33{,}000} = \frac{2.5 \times 20{,}000}{33{,}000} = 1\tfrac{17}{33} \text{ horsepower.}$$

Applying law **(15)**,

$$H : H_1 :: q^3 : q_1^3, \text{ or } 1\tfrac{17}{33} : H_1 :: 20{,}000^3 : 25{,}000^3;$$

whence, $H_1 = 2.959$. Ans.

(488) (*a*) Applying formula **45**,

$$a = \frac{q}{v} = \frac{30,000}{500} = 60 \text{ sq. ft.} \quad \text{Ans.}$$

(*b*) If the current be divided equally, $30,000 \div 2 = 15,000$ cu. ft. per min. must pass in each split, and area of either split $= \frac{15,000}{500} = 30$ sq. ft. Ans.

(*c*) Perimeter of large airway $= \sqrt{60} \times 4 = 31$ ft., nearly. Sum of perimeters of two small airways $= \sqrt{30} \times 4 \times 2 = 44$ ft., nearly.

Since the lengths of all the airways are equal, it is evident that the two small airways together have more rubbing surface than the large one; hence, they offer more resistance and require greater power in the proportion of 44 : 31, or 1.4194 : 1. Ans.

(489) The easiest method of solving this example is as follows:

By formula **r**, Art. **979**, $s = \frac{u}{k v^3}$ for the first airway, and $s_1 = \frac{u}{k v_1^3}$ for the second airway, since u, the power, is the same for both airways. By transposing the terms v^3 and v_1^3 in their respective equations, $s v^3 = \frac{u}{k}$ and $s_1 v_1^3 = \frac{u}{k}$. Since $s v^3$ and $s_1 v_1^3$ equal the same thing, i. e., $\frac{u}{k}$, they are equal to each other; in other words, $s v^3 = s_1 v_1^3$. For, if $5 \times 6 = 30$ and $2 \times 15 = 30$, it is clearly evident that $5 \times 6 = 2 \times 15$. Since the lengths of the airways are the same, the rubbing surfaces are proportional to the perimeters, and o and o_1 may be substituted for s and s_1. Hence, $o v^3 = o_1 v_1^3$. Now, $o = 2 \times 6 + 2 \times 10 = 32$ ft.; $o_1 = 2 \times 5 + 2 \times 6 = 22$ ft., and $v = \frac{24,000}{6 \times 10} = 400$ ft. per min. Therefore, $o v^3 = o_1 v_1^3$, or $32 \times 400^3 = 22 \times v_1^3$; whence,

$$v_1 = \sqrt[3]{\frac{32 \times 400^3}{22}} = 453.21 \text{ ft. per min.} =$$

velocity in small airway.

Applying formula **43**,

$q_1 = a_1 v_1 = 5 \times 6 \times 453.21 = 13,596$ cu. ft. per min., nearly. Ans.

(490) (*a*) The rubbing surface $= 8 \times 3.1416 \times 1,800 =$ 45,239 sq. ft. Applying the formula,

$$p = \frac{k s q^2}{a^2},$$

$$p = \frac{.0000000217 \times 45,239 \times 40,000^2}{(.7854 \times 8^2)^2} =$$

12.36 lb. per sq. ft., nearly. Ans.

(*b*) Applying formula **48**,

$$H = \frac{pq}{33,000} = \frac{12.36 \times 40,000}{33,000} = 14.982 \text{ horsepower. Ans.}$$

(491) The velocity before putting in regulator $= \frac{35,000}{70} = 500$ ft. per min.

The velocity after putting in regulator $= \frac{21,000}{70} = 300$ ft. per min.

Then, as in Art. **998**, $p : p_1 :: v^2 : v_1^2$, or (substituting W and W_1 for p and p_1) $.75 : W_1 :: 500^2 : 300^2$; whence, $W_1 = .27$ in. Therefore, $W - W_1 = .75 - .27 = .48$ in.

Applying formula **56**,

$$A = \frac{.0004 q}{\sqrt{W}} = \frac{.0004 \times 21,000}{\sqrt{.48}} = 12.12 \text{ sq. ft., nearly. Ans.}$$

(492) (*a*) See Art. **941**.
(*b*) See Art. **943**.
(*c*) See Art. **985**.

(493) (*a*) See Art. **997**.
(*b*) See Art. **993**.
(*c*) See Arts. **985** to **987** and Art. **999**.

(494) In Art. **995**, it is shown that $p : p_1 :: s v^2 : s_1 v_1^2$; substituting in this proportion the values given, and replacing p and p_1 by W and W_1,

$.7 : W_1 :: 1 \times 8^2 : 3 \times 10^2$; whence, $W_1 = 3.28$ in. Ans.

(495) (*a*) The rubbing surfaces of the splits are $(2 \times 7 + 2 \times 6) \times 2,000 \times 3 = 156,000$ sq. ft., and $(2 \times 7 + 2 \times 6) \times 5,000 \times 3 = 390,000$ sq. ft. Apply formula **q**, Art **979**, to the short split.

Since $p = 5.2 \times 2.5 = 13$ lb. per sq. ft., $q = a\sqrt{\frac{p\,a}{k\,s}} = \sqrt{\frac{p\,a^3}{k\,s}} = \sqrt{\frac{13 \times 42^3}{.0000000217 \times 156,000}} = 16,868$ cu. ft. per min. Ans.

Applying the same formula to the long split,

$$q = \sqrt{\frac{p\,a^3}{k\,s}} = \sqrt{\frac{13 \times 42^3}{.0000000217 \times 390,000}} =$$
10,668 cu. ft. per min. Ans.

(*b*) The total quantity = 16,868 + 10,668 = 27,536 cu. ft. per min. Applying formula **48**,

$$H = \frac{p\,q}{33,000} = \frac{13 \times 27,536}{33,000} = 10.85 \text{ horsepower. Ans.}$$

(*c*) As in example 491, $W : W_1 :: v^2 : v_1^2$. But $v = \frac{16,868}{42} = 401.6$ ft. per min., nearly; hence, $v_1 = \frac{401.6}{2} = 200.8$ ft. per min. Therefore, $2.5 : W_1 :: 401.6^2 : 200.8^2$; whence, $W_1 = .625$ in. Applying formula **56**, we have, since $16,868 \div 2 = 8,434$, and $2.5 - .625 = 1.875$,

$$A = \frac{.0004\,q}{\sqrt{W}} = \frac{.0004 \times 8,434}{\sqrt{1.875}} = 2.46 \text{ sq. ft., nearly. Ans.}$$

(496) According to formula **53**, $u = k\,s\,v^2$. Since $v = \frac{q}{a}$, $v^2 = \frac{q^2}{a^2}$ and $u = k\,s\frac{q^2}{a^3}$, or $\frac{s\,q^2}{a^3} = \frac{u}{k}$. Now, as the power is the same for both airways, we have $\frac{s_1 q_1^2}{a_1^3} = \frac{u}{k}$. Hence, $\frac{s\,q^2}{a^3} = \frac{s_1 q_1^2}{a_1^3}$. Assuming that both airways have the same length, we can substitute o and o_1 for s and s_1. Therefore, $\frac{o\,q^2}{a^3} = \frac{o_1 q_1^2}{a_1^3}$. Substituting the values given,

$$\frac{3.1416 \times 18 \times 50,000^2}{(.7854 \times 18^2)^3} = \frac{3.1416 \times 6 \times q_1^2}{(.7854 \times 6^2)^3};$$

whence, $q_1 = 8,012.5$ cu. ft. per min. Ans.

(497) (*a*) See Art. **973.**
(*b*) See Art. **958.**
(*c*) See Art. **968.**

(498) (*a*) The solution is similar to that given in Art **1000,** $2^5 = 32$; hence, the first figure is 2.

(3) $35 - 32 = 3$. Annexing five ciphers gives 300,000.

(4) $2^4 \times 5$ with four ciphers annexed = 800,000; 2^3 with four ciphers annexed = 80,000; the sum = 880,000.

(5) Since 300,000 will not contain 880,000, the second figure of the root is 0, and the first two figures are 20.

(6) $20^5 = 3,200,000$; $3,500,000 - 3,200,000 = 300,000$.

(7) $20^4 \times 5 = 800,000$; $300,000 \div 800,000 = .37$, or .4.

(8) 20^3 with a cipher annexed = 80,000; $80,000 \times .4 = 32,000$, and $800,000 + 32,000 = 832,000$.

(9) $300,000 \div 832,000 = .36$, the fourth and fifth figures. Hence, $\sqrt[5]{35} = 2.036$. Ans.

(*b*) (1) Pointing off into periods gives 642'68937.

(2) $3^5 = 243$; $4^5 = 1,024$; hence, the first figure of the root is 3.

(3) $642 - 243 = 399$; annexing the second period gives 39,968,937.

(4) $3^4 \times 5$ with four ciphers annexed = 4,050,000; 3^3 with four ciphers annexed = 270,000; the sum = $4,050,000 + 270,000 = 4,320,000$.

(5) $39,968,937 \div 4,320,000 = 9 +$. But 9 is evidently much too large; hence, try 6, making the first two figures of the root 36.

(6) $36^5 = 60,466,176$; $64,268,937 - 60,466,176 = 3,802,761$.

(7) $36^4 \times 5 = 8,398,080$; $3,802,761 \div 8,398,080 = .452$, or .4.

(8) 36^3 with a cipher annexed = 466,560; $466,560 \times .4 = 186,624$, and $8,398,080 + 186,624 = 8,584,704$.

(9) $3,802,761 \div 8,584,704 = .443$, say .44.

Hence, $\sqrt[5]{64,268,937} = 36.44$. Ans.

MINE VENTILATION.

(PART 2.)

(499) See Art. **1002.**

(500) See Art. **1004.**

(501) See Art. **1005.**

(502) See Art. **1006.**

(503) Using formula **57,**

$$W = \frac{1.3253 \times 30.25}{459 + 350} = .0495.$$

$$.0495 \times 500 \times 3 = 74.25 \text{ lb.} \quad \text{Ans.}$$

(504) Using formula **57,**

$$W = \frac{1.3253 \times 29.3}{459 + 32} = .0791 \text{ lb., nearly.} \quad \text{Ans.}$$

(505) Using formula **43,** $q = a\,v = 8 \times 10 \times 800 =$ 64,000 cu. ft. of air at 32° F. Then, using formula **58,** and substituting for T, t, and v their numerical values as given, we have

$$459 + 60 = \frac{V}{64{,}000} \times (459 + 32),$$

and $$519 = \frac{491}{64{,}000} V = .007672\,V,$$

and $$V = \frac{519}{.007672} = 67{,}650 \text{ cu. ft., nearly.} \quad \text{Ans.}$$

(506) (*a*) Using formula **57,**

$$W = \frac{1.3253 \times 29.8}{459 + 0} = .086043 \text{ lb.},$$

average weight of 1 cu. ft. downcast air;

$$W = \frac{1.3253 \times 29.8}{459 + 300} = .052034 \text{ lb.},$$

average weight of 1 cu. ft. upcast air.

$.086043 \times 540 = 46.46322$ lb. per sq. ft., pressure in down cast column.
$.052034 \times 540 = 28.09836$ lb. per sq. ft., pressure in upcast column.

18.36486 lb. per sq. ft., difference. Ans.

(*b*) Using formula **60,** in which we substitute for .077, the average weight of 1 cu. ft. of the downcast air, as obtained in (*a*), we have

$$p = \frac{300 - 0}{459 + 300} \times .086043 \times 540 = 18.365 \text{ lb.} \quad \text{Ans.}$$

(507) Using formula **59,**

$$M = \frac{360 - 60}{459 + 360} \times 200 \times 3 = 219.78 \text{ ft.} \quad \text{Ans.}$$

(508) See Art. **1009.** The isolation of the ribs by side drifts. The isolation of the roof by an air-space between the two arches. The grate area must be proportioned to its work, or vary inversely as the square root of the depth of the shaft. The sectional area over and around the furnace must be proportional to the quantity of air required.

(509) Using formula **61,**

$$s = \frac{34}{\sqrt{250}} = 2.15 \text{ sq. ft. per horsepower.}$$

Using formula **48,** the horsepower is

$$H = \frac{50{,}000 \times 2 \times 5.2}{33{,}000} = 15.76 \text{ H. P., nearly.}$$

$$2.15 \times 15.76 = 33.884 \text{ sq. ft.} \quad \text{Ans.}$$

(510) (*a*) See Art. **1011.** Its object is to isolate the return air of a gaseous mine from the flaming gases and sparks of the furnace.

(*b*) 150 feet.

(511) See Arts. **1012** and **1013.**

(512) (*a*) Any mechanical device for producing an air-current.

(*b*) The centrifugal fan and the steam-jet are familiar examples.

(513) The most prominent types of centrifugal ventilators now in use are represented by the Waddle, Schiele, Guibal, and Capell fans. See Art. **1044.**

(514) (*a*) By blowing and by exhausting.

(*b*) The efficiencies of each method are practically the same. See Art. **1043.**

(515) Blades curve backwards from the direction of their motion, and are so tapered that the breadths of the blades at different distances from the center vary inversely as their distances from the center. See Art. **1045.**

(516) The Schiele fan consists of a central disk provided with duplicate sets of blades upon its two sides. Air enters upon each side. The fan is surrounded by a spiral casing, which conducts the air to an evase chimney. See Art. **1046.**

(517) It provides a uniformly increasing sectional area about the fan, which gives a uniform velocity to the air-current all around the circumference. See Art. **1046.**

(518) To reduce the velocity of discharge and loss of energy. See Art. **1042.**

(519) See Art. **1047.**

(520) See Art **1048.**

(521) (1) It is safer and (2) it has a uniform efficiency in deep and shallow mines alike.

(522) The *furnace* rarefies the air of one shaft by heat, thus causing a difference in pressure, which is the ventilating pressure; the *fan*, by exhaustion or compression, creates the difference in pressure between the intake and discharge openings of a mine. See Arts. **1021** and **1022.**

(523) Using formula **64,**

$$v = 18\sqrt{8.41} = 52.2 \text{ ft. per sec. Ans.}$$

(524) The velocity of the air entering the fan should not exceed 18 feet per second. See Art. **1030.**

(525) See Art. **1030.**

$$\frac{175,000}{2} = 87,500 \text{ cu. ft. on each side.}$$

Using formula **66,**

$$d = .0343\sqrt{87,500} = 10.146 \text{ ft. Ans.}$$

(526) (*a*) It is the surface of the imaginary cylinder whose diameter is the diameter of the port of entry of the fan, and whose length is the breadth of the fan-blades. See Art. **1031.**

(*b*) The diameter of the port of entry and the width of the blades.

(527) See Art. **1030.**

$$\frac{250,000}{2} = 125,000 \text{ cu. ft. on each side.}$$

Using formula **66,** $d = .0343\sqrt{125,000} = 12.127$ ft. Now, using formula **67,** second case, $b = \frac{1}{2}d = \frac{1}{2} \times 12.127 = 6.06$ ft. Ans.

(528) See Art. **1031.**

$$\sqrt{\frac{153.9384}{.7854}} = 14.0 \text{ ft., diameter of port of entry.}$$

Using formula **67,**

$$b = \frac{1}{4}d = \frac{1}{4} \times 14 = 3.5 \text{ ft. Ans.}$$

(529) See Art. **1032.**

(530) See Art. **1050.**

(531) See Art. **1051.**

(532) That the velocity is not too low. See Art. **1052.**

(533) (*a*) Too high a velocity of the air-current renders a safety-lamp unsafe, as the flame may be blown through the gauze of the lamp.

(*b*) 450 feet per minute. See Art. **1053.**

(534) (*a*) Doors, stoppings, brattices, curtains, regulators, and overcasts, or bridges.

(*b*) See Art. **1054.**

(535) The water-gauge or manometer, and the anemometer. The water-gauge is used to measure the difference of pressure between the intake and return airways. The manometer is used for the same purpose. The anemometer is used to determine the velocity of the current. See Art. **1057.**

(536) See Arts. **1058, 1059,** and **1060.**

(537) See Arts. **1061, 1062,** and **1063.**

(538) Density of air depends mainly upon two factors, barometric pressure measured by the barometer, and temperature measured by the thermometer. See Art. **1064.**

(539) (*a*) Freezing-point of water = 0° C. and 32° F.; boiling-point of water = 100° C. and 212° F.

(*b*) 212° − 32° = 180° F. = 100° C. See Arts. **1066** and **1067.**

(540) Using formula **76,**

(*a*) $F = \frac{9}{5} \times 350 + 32 = 630 + 32 = 662°$ F. Ans.

(*b*) $F = \frac{9}{5}(-10) + 32 = -18 + 32 = 14°$ F. Ans.

(*c*) $F = \frac{9}{5}(-25) + 32 = -45 + 32 = -13°$ F. Ans.

(541) Using formula **77,**

(*a*) $C = \frac{5}{9}(365 - 32) = \frac{5}{9}(333) = 185°$ C. Ans.

(*b*) $C = \frac{5}{9}(5 - 32) = \frac{5}{9}(-27) = -15°$ C. Ans.

(*c*) $C = \frac{5}{9}(-49 - 32) = \frac{5}{9}(-81) = -45°$ C. Ans.

(542) See Art. **1068.**

(543) See Art. **1069.**

(544) Dip workings, because, as a rule, the *intake* air is cooler than the return air from the workings, and it is natural for the heavier, cool air to flow to the dip, and the lighter air to the rise. See Art. **1071.**

(545) (*a*) Positive air columns are those whose weight acts in the direction in which the current moves.

(*b*) Negative air columns are those whose weight is opposed to the direction of the current. See Art. **1070.**

(546) By ascensional ventilation we understand such a method of ventilation that the general course of the current will be towards the rise.

(547) The algebraic sum of the weights of the positive and negative air columns in any mine is always equal to the weight of the *motive* column, or the pressure per square foot producing the flow of air. See Art. **1072.**

(548) The main feature in such a case is the manner of splitting the air at each pair of cross-entries, when these entries are sufficiently developed to warrant the expense. See Art. **1074.**

(549) The velocity of the divided current, which must not fall below 3 or 4 feet per second in non-gaseous mines and 5 or 6 feet per second where gas is given off. See Art. **1074.**

(550) In non-gaseous mines the haulage roads should be made the return airways for two reasons. See Art. **1075.** In gaseous mines, haulage is done upon the intake airway in order to lessen, as far as possible, liability to explosion. See Art. **1077.**

(551) That the ventilation shall be ascensional.

(552) (1) The establishment of the air-current. (2) The direction of the current should not be altered, except upon most urgent demand. (3) Repairs of stoppings, doors, and brattices must be made rapidly, as no great advance can be made ahead of the air. See Art. **1081.**

(553) See Art. **1083.**

(554) Care must be taken to begin sealing off a fire at its side next to the return air, and work towards the intake, as then there is less opportunity for the entrapping of pure air, which would give rise to an explosion under certain conditions.

MINE SURVEYING AND MAPPING.

(PART 1.)

(555) See Art. **1087.**

(556) See Arts. **1091** and **1093.**

(557) See Art. **1098.**

(558) As 11.55 feet. See Art. **1093.**

(559) The distance could be measured by the ordinary method of measuring up or down hill, as explained in Art. **1098,** or by measuring the actual length of the slope and multiplying this length by the cosine of the angle of dip or rise. See Art. **1100.**

(560) See Art. **1099.**

(561) See Arts. **1101** to **1103.**

(562) See Art. **1101.**

(563) See Arts. **1104** and **1105.**

(564) The order of the letters on the face of the compass is N-W-S-E taken in a clockwise direction, the E and W being transposed to facilitate the taking of the bearings. See Art. **1107.**

(565) It is graduated to half-degrees. See Art. **1106.**

(566) See Arts. **1109** and **1110.**

(567) See Art. **1111.**

(568) See Arts. **1112** and **1113.**

(569) The zero of the vernier has moved towards the right more than one whole degree division, and lies between

1° 00′ and 1° 30′. Reading towards the left from the zero of the vernier, we reach the 15′ mark without having found a coinciding line. We therefore begin at the extreme right of the vernier and read towards its zero. The twelfth mark from the right end, or the twenty-seventh from the zero, according to the order followed, and which is shown by the top row of figures, is the coinciding line. Consequently, the reading is 1° 00′ + 27′ = 1° 27′. See Art. **1114.**

(570) Observe, the zero of the vernier has been moved to the left a little more than two whole degree divisions on the limb. Reading the vernier in the direction opposite to its motion, we observe that the first line on the vernier which exactly coincides with a line on the limb is the 01′ line. The reading is, therefore, 2° 01′. See Art. **1115.**

(571) The zero of the vernier has moved to the left over three and one-half whole degree divisions, and lies between 3° 30′ and 4°. Reading the vernier in the direction opposite to its motion, we observe that 15′ is the first line on the vernier which exactly coincides with a line on the limb. The reading is, therefore, 3° 30′ + 15′ = 3° 45′. See Art. **1115.**

(572) The zero of the vernier has moved to the right not quite two whole degree divisions, and lies between 1° 30′ and 2° 00′. Reading towards the left from the zero of the vernier, we reach the 15′ mark without having found a coinciding line. We therefore begin at the extreme right of the vernier and read towards its zero. The fourth mark from the right end, or the nineteenth mark from the zero in the order followed (shown by the top row of figures), is found to coincide; consequently, the reading is 1° 30′ + 19′ = 1° 49′. See Art. **1114.**

(573) See Arts. **1116** and **1117.**

(574) See Art. **1118.**

(**575**) For putting up points by which rooms, or chambers, can be driven very approximately on the proper course. See Arts. **1101** and **1122.**

(**576**) See Art. **1121.**

(**577**) Since the butt entry runs N 30° E, and the rooms run N 20° W, the center line of the entry and that of a room make an angle of 30° + 20° = 50°. Hence, applying formula **78,**

$$D = \frac{54}{\sin 50^\circ} = \frac{54}{.76604} = 70.49 \text{ ft. Ans.}$$

(**578**) See Art. **1131.**

(**579**) The total latitude of a course is the distance its end (not beginning) is north or south of some station, usually the first, to which all the courses of the survey are referred. The total departure of the course is the distance its end is east or west of the same station. See Arts. **1135** to **1137.**

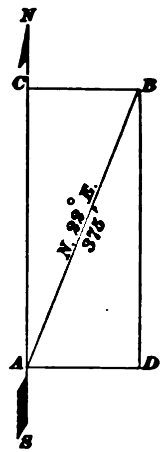

FIG. 20.

(**580**) Let *NS*, Fig. 20, be a meridian, and *A B* be the course; then, its latitude = *DB* = 375 × cos 22° = 347.7 feet, and its departure = *CB* = 375 × sin 22° = 140.5 ft. Ans. See Art. **1132.**

(**581**)

Station.	Bearing.	Distance.	Cosine.	Sine.	Latitude. North.	Latitude. South.	Departure. East.	Departure. West.
1–2	S 46° 30′ E	207.6	.68835	.72537		142.90	150.59	
2–3	S 74° 30′ E	309.5	.26724	.96363		82.71	298.24	
3–4	N 33° 15′ E	188.0	.83629	.54829	157.22		103.08	
4–5	N 56° 00′ W	276.0	.55919	.82904	154.34			228.82
5–6	Due West	213.5	.00000	1.00000				213.50
6-1	S 51° 54′ W	139.3	.61704	.78676		85.95		109.59
					311.56	311.56	551.91	551.91
					311.56		551.91	

(582)

Station.	Total.			
	Latitude.		Departure.	
	North.	South.	East.	West.
1–2		142.90	150.59	
2–3		225.61	448.83	
3–4		68.39	551.91	
4–5	85.95		323.09	
5–6	85.95		109.59	
6–1				

(583) Station 3 has a total south latitude of 225.61 feet and a total east departure of 448.83 feet. Hence, its bearing will equal the angle whose tangent is $\frac{448.83}{225.61} = 1.98941$. By looking in a table of natural tangents, it will be found that 1.98941 corresponds very nearly to the tangent of 63° 19′. Therefore, the bearing from 1–3 is S 63° 19′ E. Ans.

(584) Care must be exercised in calculating the total latitudes and the total departures where more than one station has been located from the same station. See Art. **1138.**

(585)

Station.	Bearing.	Distance.	Cosine.	Sine.	Latitude.		Departure.	
					North.	South.	East.	West.
1–2	Due E	130	.00000	1.00000			130.00	
2–3	N 8 E	137	.99027	.13917	135.67		19.07	
3–4	N 81° W	186	.15643	.98769	29.10			183.71
4–5	Due S	54	1.00000	.00000		54.00		
5–6	S 36 W	125	.80902	.58779		101.13		73.47
6–7	S 45° E	89	.70711	.70711		62.93	62.93	
7–1	N 40 18′ E	69.86	.76267	.64679	53.28		45.18	
					218.05	218.06	257.18	257.18

(586)

Station.	Bearing.	Distance.	Cosine.	Sine.	Latitude.		Departure.	
					North.	South.	East.	West.
1–2	N 37° 13′ E	413.6	.79635	.60483	329.37		250.16	
2–3	N 10° 56′ E	246.7	.98185	.18967	242.22		46.79	
3–4	S 17° 23′ E	253.0	.95433	.29876		241.45	75.59	
4–5	S 43° 37′ E	216.0	.72397	.68983		156.38	149.00	
5–6	S 33° 43′ W	789.0	.83179	.55509		656.28		437.97
					571.59	1054.11	521.54	437.97
						571.59	437.97	
						482.52	83.57	

Station 6 is, therefore, 482.52′ south, and 83.57′ east of Station 1.

Hence, $\frac{83.57}{482.52}$ = tangent A = .17319. See Fig. 21.

The angle A is, therefore, equal to 9° 49′+, and the bearing from 1–6 is S 9° 49′ E.

The distance from 1–6 = $\sqrt{482.52^2 + 83.57^2}$ = 489.7 ft. Ans.

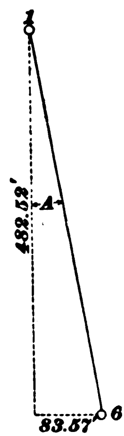

FIG. 21.

(587) This disagreement in the readings shows that the magnetic variation has changed as much as the difference between the two readings, or 30′. The variation has been from west to east; therefore, turn the vernier 30′ to the right, and the reading will be the same.

(588) They facilitate the calculating of latitudes and departures. See Art. **1133.**

(589) See Art. **1140.**

(590) The actual distance between the two points on the surface = 4.378 × 200 = 875.6 ft. Ans. See Art. **1141.**

(591) The distance between them on the map = $\frac{537.8}{150}$ = 3.585 in. Ans. See Art. **1141.**

(592) (*a*) See Art. **1146.**
(*b*) See Art. **1152.**
(*c*) See Art. **1148.**

(593) See note, Art. **1146.**

(594) Platting by bearing and platting by latitude and departure; the latter method is the better, particularly where the total latitudes and the total departures are employed. See Arts. **1155, 1156,** and **1162.**

(595) The closing line of a survey is the straight line between two points connected by a survey. Its bearing and length are determined as explained in Arts. **1164** and **1165.**

(596) See Art. **1156.**

(597) See Art. **1166.**

(598) To find the area of the survey, we must calculate first the area enclosed by the outside lines *M*, *N*, *O*, *P*, Fig. 22,

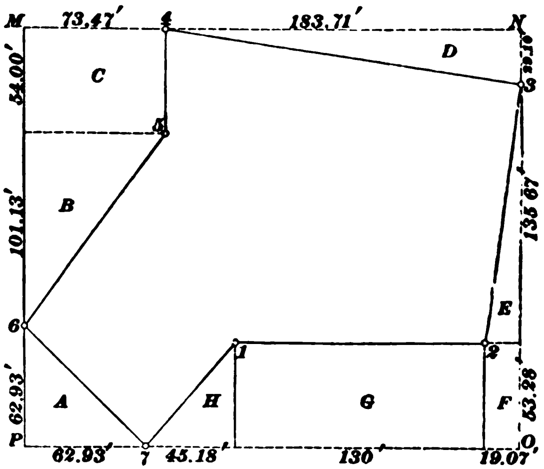

Fig. 22.

and from it subtract the combined areas of *A*, *B*, *C*, *D*, *E*, *F*, *G*, and *H*. This remainder will be the area of the survey.

		Square Feet.
Area of rectangle $MNOP$	$= 218.05 \times 257.18 =$	56,078.10
Area of triangle A	$= \dfrac{62.93 \times 62.93}{2} =$	1,980.09
Area of triangle B	$= \dfrac{101.13 \times 73.47}{2} =$	3,715.01
Area of rectangle C	$= 73.47 \times 54 =$	3,967.38
Area of triangle D	$= \dfrac{183.71 \times 29.10}{2} =$	2,672.98
Area of triangle E	$= \dfrac{135.67 \times 19.07}{2} =$	1,293.61
Area of rectangle F	$= 53.28 \times 19.07 =$	1,016.05
Area of rectangle G	$= 130 \times 53.28 =$	6,926.40
Area of triangle H	$= \dfrac{53.28 \times 45.18}{2} =$	1,203.60
Area of $A+B+C+D+E+F+G+H$	$=$	22,775.12

The area of the survey is, therefore, 56,078.10−22,775.12= 33,302.98 sq. ft. Ans.

(599) In order to calculate the area of the survey

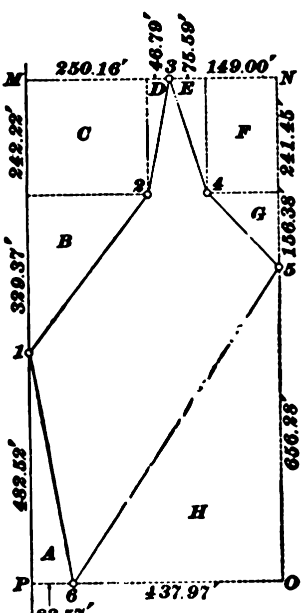

FIG. 23.

shown in Fig. 23, it will be necessary first to determine the

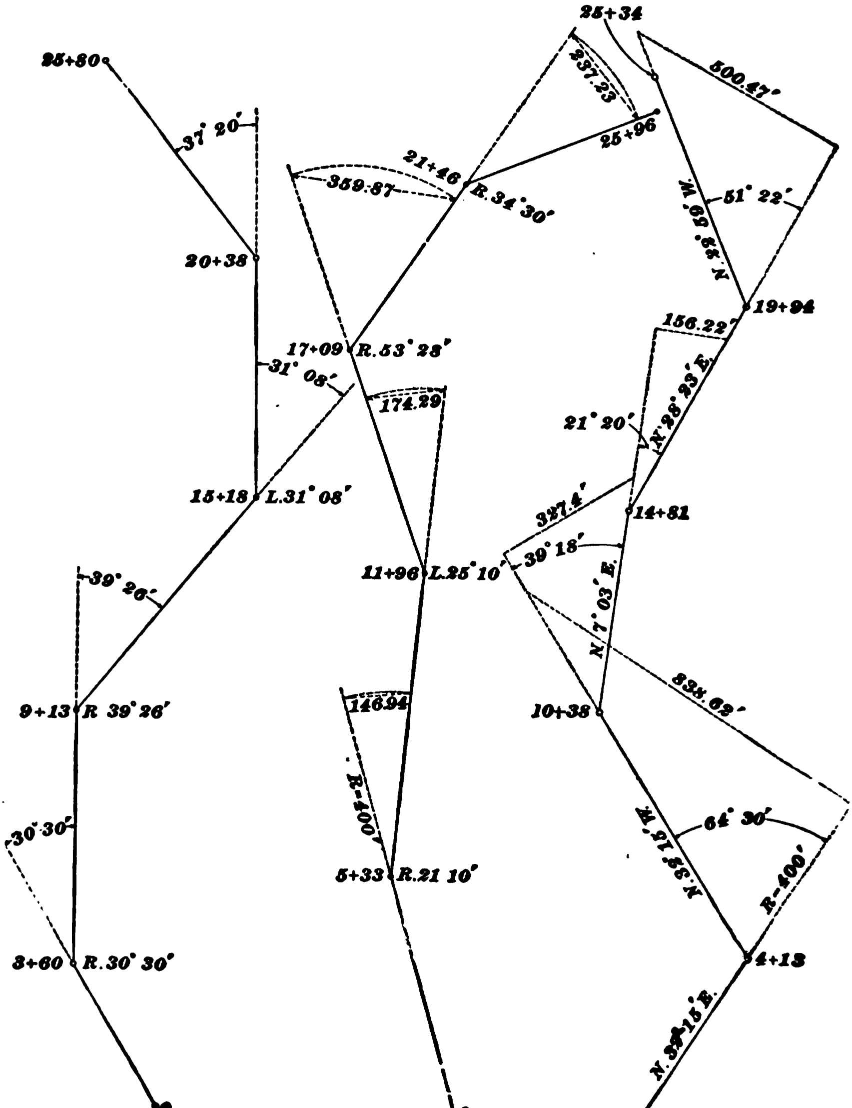

FIG. 24. FIG. 25. FIG. 26.

area enclosed by the outside lines MN, NO, OP, and PM, and then subtract from it the combined areas of A, B, C, D, E, F, G, and H.

		Square Feet.
The area of $MNOP$ =	$1,054.11 \times 521.54$ =	549,760.53
The area of A =	$\frac{482.52 \times 83.57}{2}$ =	20,162.10
The area of B =	$\frac{329.37 \times 250.16}{2}$ =	41,197.60
The area of C =	250.16×242.22 =	60,593.76
The area of D =	$\frac{242.22 \times 46.79}{2}$ =	5,666.74
The area of E =	$\frac{241.45 \times 75.59}{2}$ =	9,125.60
The area of F =	241.45×149.00 =	35,976.05
The area of G =	$\frac{156.38 \times 149.00}{2}$ =	11,650.31
The area of H =	$\frac{437.97 \times 656.28}{2}$ =	143,715.48
Total area of $A+B+C+D+E+F+G+H$ =		328,087.64

The area of the survey is, therefore, $549,760.53 - 328,087.64 = 221,672.89$ sq. ft. Ans.

(600) Fig. 24 shows the required plat. For details of platting a line by means of the protractor, see Arts. **1143** to **1145**.

(601) Fig. 25 shows the required plat. For method of platting a broken line by means of chords, see Art. **1147**.

(602) Fig. 26 shows the required line platted by tangents, as explained in Art. **1152**.

MINE SURVEYING AND MAPPING.

(PART 2.)

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(603) See Art. **1167.**

(604) The advantages of the transit over the vernier compass are mainly due to the telescope on the transit, and its vertical arc, by means of which vertical angles can be measured. See Art. **1167.**

(605) A direct vernier. See Art. **1168.**

(606) There are four adjustments of the transit, which should be made whenever the instrument is to be used. The first adjustment is made to remove any lack of level that may be in the limb; the second is to bring the intersection of the cross-wires into the optical axis of the telescope; the third is to correct any derangement which the standards may have suffered; and the fourth adjustment is to make the line of sight of the telescope a level line when the bubble in the attached bubble-tube is in the center of the tube. These adjustments are made as explained in Arts. **1171** to **1174.**

(607) The reading is $70° 30' + 21'$, or $70° 51'$. For an explanation of the method of taking this reading and those of the following three questions, see Arts. **1176** to **1179.**

(608) The reading is $263° + 13' = 263° 13'$.

(609) The readings are $54° + 23' = 54° 23'$, and $305° + 37' = 305° 37'$.

(610) The readings are $60° 30' + 13' = 60° 43'$, and $119° + 17' = 119° 17'$.

(611) (*a*) The horizontal limb of the instrument is the horizontal plate carrying the small level tubes.

(*b*) The axis of the instrument is the vertical line passing through the center of the instrument perpendicular to the line of collimation.

(*c*) The line of collimation is the optical axis of the telescope.

(*d*) The standards are the supports for the horizontal axes of the telescope with its attached level, and they rest on and are made fast to the horizontal limb. See Arts. **1168** and **1169.**

(612) A horizontal angle measured from any point to two objects is the angle included between two vertical planes passing through the point and the objects. It may also be defined as the horizontal projection of the angle formed by drawing lines from one point to two other points. The method of measuring horizontal angles is explained in Art. **1180.**

(613) See Art. **1181.**

(614) (*a*) N E. (*b*) E. (*c*) S E. (*d*) S. (*e*) S W. (*f*) W. (*g*) N W. (*h*) N. See Art. **1182.**

(615) See Art. **1183.**

Stations.	Azimuths with *A B*.	Bearings with *A B*.
A	0°	Due North
B	35° 30′	N 35° 30′ E
C	110° 30′	S 69° 30′ E
D	270° 00′	Due West
E	330° 45′	N 29° 15′ W

(616) If the given first course *A B* is not really a north and south line, its magnetic bearing must be obtained by the compass, and then the magnetic bearings of the succeeding courses can be calculated by adding the magnetic bearing of $A B$ to, or subtracting it from (as the case may be), the azimuths of the given courses, according as the magnetic bearing of $A B$ is N E or N W of the meridian or north and south line. See Art. **1183.**

(617) See Art. **1181.**

(618) See Art. **1200.**

(619) See Art. **1209.**

(620) See Art. **1190.**

(621) The magnetic readings are taken merely to check the azimuth readings. See Art. **1191.**

(622) (*a*) A one-degree curve is one on which a 100-foot chord will subtend a central angle of one degree.

(*b*) A five-degree curve is one on which a 100-foot chord will subtend a central angle of five degrees. See Art. **1199.**

(623) See Arts. **1218** to **1223.**

(624) See Arts. **1225** to **1230.**

(625) There is not only danger of the timber being more or less displaced by the overlying weight, but it is possible that the timbermen may, in repairing the cross-bar or collar, so displace the station that the displacement would not be noticed. In this case, all work run from such a station would be wrong. It is better to have a station destroyed entirely than to have it displaced in such a way that it would not be noticed. See Art. **1228.**

(626) See Arts. **1184** and **1185.**

(627) They are technically called tangents. See Art. **1194.**

(628) The degree of a curve is determined by the central angle subtended by a chord of 100 feet. See Art. **1199.**

(629) See Art. **1207.**

(630) A right-angled glass prism placed at the end of the telescope. See Art. **1222.**

(631) See Art. **1205.**

(632) $24° \ 30' = 24.5°$. Hence, there will be as many hundred-foot lengths in the curve as 5 is contained in 24.5°, or $\frac{24.5°}{5} = 4.9$ lengths = 490 ft. Ans. See Art. **1205.**

(633) See Art. **1201.**

(634) Applying formula **80,**

$$T = R \tan \tfrac{1}{2} I = 819.02 \times \tan \frac{36°}{2} = 819.02 \times .32492 = 266.12 \text{ ft. Ans.}$$

The value of R for a 7° curve is found in the table of Radii and Deflections.

(635) See Art. **1188.**

(636) See Art. **1195.**

(637) See Arts. **1186** and **1189.**

(638) See Arts. **1215** and **1216.**

(639) See Art. **1192.**

(640) Applying formula **81,**

$$d = \frac{c^2}{R} = \frac{9^2}{45} = \frac{81}{45} = 1.8 \text{ ft. Ans.}$$

(641) The deflection angle for 100 feet on a 6° 30′ curve is $\frac{6° \ 30'}{2} = 3° \ 15' = 195'$; hence, the deflection angle for 1 foot on the same curve is $\frac{195'}{100} = 1.95'$, and for 48 feet it is $1.95' \times 48 = 93.6' = 1° \ 33' \ 36''$. Ans. See Art. **1205.**

(642) They are usually designated as curves oi so many feet radius. See Art. **1215.**

(643) See Art. **1203.**

(644) See Art. **1205.**

(645) Since the deflection angle for a 10° curve is 5°, 5 stations 100 feet apart could be located with the transit at the P. C. of the curve, because $\frac{25}{5} = 5$. To locate other stations, the transit should be moved up to the last station, and the vernier plate firmly clamped at a reading of 360° − 25° = 335°. A backsight should then be taken to the P. C. station, being careful to keep the vernier plate clamped, and the line of sight of the telescope made to cut the P. C. station by means of the lower tangent screw. The vernier clamp is loosened and the instrument made to read 0°, at which reading the line of sight of the telescope will be tangent to the curve. From this point the operation is the same as starting from the P. C. of the curve. See Art. **1206.**

(646) A 9° curve has a radius of 637.27 ft. See Table 27. Hence, applying formula **81,**

$$d = \frac{c^2}{R} = \frac{120^2}{637.27} = \frac{14,400}{637.27} = 22.6 \text{ ft., nearly.} \quad \text{Ans.}$$

Also, applying formula **82,**

$$f = \frac{c^2}{2R} = \frac{120^2}{2 \times 637.27} = \frac{14,400}{1,374.54} = 11.3 \text{ ft., nearly.} \quad \text{Ans.}$$

(647) See Art. **1231.**

(648) See Art. **1237.**

(649) It is best to use large numbers for stations in the mine; for if a station is partly obliterated, it is more easily deciphered when marked with a large number than when marked with a small number. See Art. **1232.**

(650) See Art. **1233.**

(651) See Art. **1235.**

(652) Fig. 29 shows the platted survey and a vertical section through Sta. 1 and the face of the tunnel.

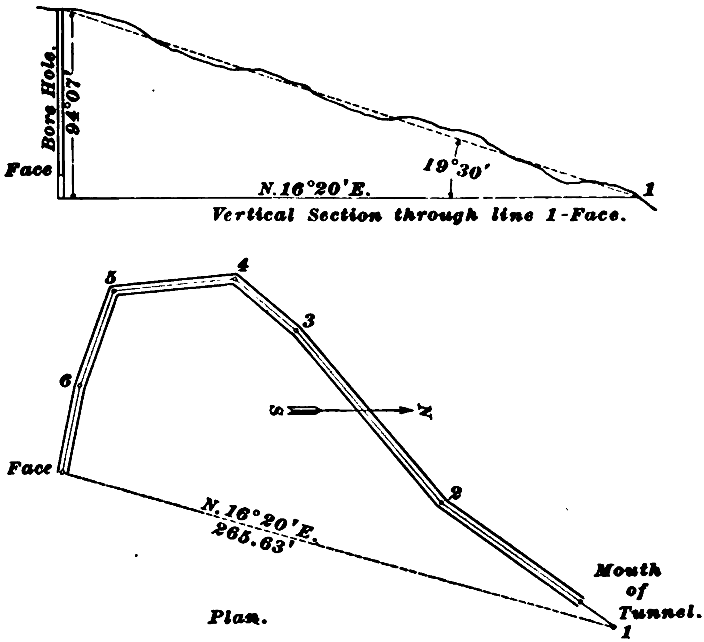

FIG. 29.

Station.	Bearing.	Distance.	Cosine.	Sine.	Latitude.		Departure.	
					North.	South.	East.	West.
1–2	S 36° 50′ W	99.1	.80038	.59949		79.32		59.41
2–3	S 49° 47′ W	104.2	.64568	.76361		67.28		79.57
3–4	S 40° 00′ W	37.1	.76604	.64279		28.42		23.85
4–5	S 4° 55′ E	56.5	.99632	.08571		56.29	4.84	
5-6	S 71° 15′ E	46.0	.32144	.94693		14.79	43.56	
6–Face	S 77° 30′ E	40.7	.21644	.97630		8.81	39.74	
					000.00	254.91	88.14	162.83
						000.00		88.14
						254.91		74.69

(*a*) The face, therefore, is 254.91 feet south, and 74.69 feet west of Sta. 1. Hence, its bearing from Sta. 1 is found thus:

$$\frac{74.69}{254.91} = \text{tangent of bearing} = .29300 = \tan 16° \; 20'.$$

Hence, the bearing is S 16° 20′ W from Sta. 1 to face of tunnel, or N 16° 20′ E from face of tunnel to Sta. 1.

The distance from Sta. 1 to face $= \sqrt{254.91^2 + 74.69^2} =$ 265.63 feet. Ans.

Station.	Distance.	In the Tunnel.		On the Surface.	
		Vertical Angle.	Vertical Height.	Vertical Angle.	Vertical Height.
1–2	99.1	+ 1° 18′	+ 2.25	+ 10° 35′	+ 18.52
2–3	104.2	+ 0° 31′	+ 0.94	+ 15° 43′	+ 29.32
3–4	37.1	+ 0° 45′	+ 0.49	+ 14° 27′	+ 9.56
4–5	56.5	− 0° 34′	− 0.56	+ 16° 17′	+ 16.50
5–6	46.0	+ 3° 37′	+ 2.91	+ 12° 21′	+ 10.07
6–Face	40.7	+ 3° 30′	+ 2.49	+ 13° 56′	+ 10.10
	Total,		+ 8.52		+ 94.07
					8.52

(*b*) Depth of face below surface = 85.55 ft. Ans.

(*c*) The vertical angle from Sta. 1 to a point on the surface vertically over the breast is found thus:

$$\frac{94.07}{265.63} = 0.35414 = \tan 19° \; 30';$$

hence, 19° 30′ is the vertical angle. Ans.

MINE SURVEYING AND MAPPING.

(PART 3.)

(655) See Art. **1238.**

(656) See Art. **1239.**

(657) The best method of keeping notes in leveling is the method known as "height of instrument." For explanation of this method, see Art. **1248.**

(658) See Art. **1258.**

(659) When only the elevation between two points is required, the operation of leveling is very simple. The level is set up near one of the points and a reading taken on the rod placed upon the point of starting, which is assumed to have a certain height above the datum line. The height of the instrument is then determined, and a sight taken to the rod placed as far in the direction in which it is decided to run the line of levels between the two points as it will be possible to take a reading upon the rod. The level is then moved in the proper direction as far past this point as it will be possible to get a reading upon the rod still held upon the point last located. The height of the instrument is again determined, and a sight taken to the rod moved beyond the level as before. This process is continued until a reading is obtained on the rod placed upon the last point. The last reading taken from the last height of instrument will be the elevation of the last point above the datum line; then the difference in the elevation of the two points will simply be the difference of their heights above the datum line. Throughout the entire operation readings are taken on turning points only. See Art. **1249.**

(660) See Arts. **1251** and **1252.**

(661) See Art. **1241.**

(662) See Art. **1247.**

(663) See Art. **1240.**

(664) The slight curvature in the bubble-tube. The less the curvature the more sensitive the level. See Art. **1241.**

(665) This is done for uniformity, or so that by adding algebraically the backsight to the elevation of the turning point, the height of instrument is determined, or by adding algebraically the foresight to the height of instrument, the elevation of the last point is determined. See Art. **1248.**

(666) See Art. **1238.**

(667) See Arts. **1242** to **1244.**

(668) See Art. **1257.**

(669) The object of a topographical survey is to determine accurately the irregularities of the surface for the purpose of making a map on which such irregularities will be plainly shown. See Art. **1258.**

(670) See Art. **1258.**

(671) Level notes are checked by adding algebraically the foresights to the backsights. This sum should equal the difference in elevation of the first and the last station. See Art. **1248.**

(672) See Art. **1254.**

(673) See Art. **1259.**

(674) The leveling operations can only be checked by repeating the work. See Art. **1250.**

(675) See Art. **1246.**

(676) See Art. **1241.**

(677) The stations which are at regular distances apart are numbered consecutively, beginning at 0 and running up. A station between two regular stations is marked with a plus sign between the number of the regular station immediately preceding it and the number of feet beyond the same station. Thus, a station between Stations

3 and 4, and 35 feet beyond Station 3, would be marked as Station 3 + 35. See Art. **1247.**

(678) A general map of the mine, the property on which it is located, and the surface arrangements enables one to see at a glance the relative positions of the entries and rooms of the mine to the property lines, buildings, or bodies of water on the surface. See Art. **1264.**

(679) Buildings or other objects are located either by taking rights and lefts from some established line, or by sighting to them with the transit, and measuring the distances over the lines of sight. See Art. **1268.**

(680) It enables the engineer to determine the probable area underlaid with coal, as well as to determine the relative positions of the entries, room, etc., with the outcrop. See Art. **1267.**

(681) See Art. **1266.**

(682) See Art. **1267.**

(683) See Art. **1251.**

(684) See Art. **1266.**

(685) Referring to Fig. 30, $AM = AG \times \sin 43° =$

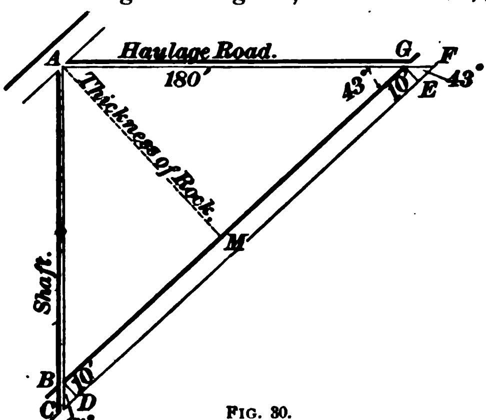

FIG. 30.

$180 \times .682 = 122.76$ feet, which is the thickness of rock between the seams measured at right angles to the pitch. Ans.

(686) (*a*) Referring to Fig. 30, it will be seen that $AB + BC =$ the depth of the shaft.

$$AB = 180 \times \tan 43^\circ = 180 \times .93252 = 167.85 \text{ ft.}$$

$$BC = \frac{10}{\sin 47^\circ} = \frac{10}{.73135} = 13.67 \text{ ft.}$$

Hence, the depth of the shaft $= 167.85 + 13.67 = 181.52$ ft. Ans.

(*b*) The distance on the pitch from the foot of the shaft to the level of the haulage-road in the underlying seam is equal to CF.

$$CF = \sqrt{\overline{AC}^2 + \overline{AF}^2} = \sqrt{(AC)^2 + (AG + GF)^2}.$$

$$GF = \frac{10}{\sin 43^\circ} = \frac{10}{.682} = 14.66 \text{ ft.}$$

Therefore, $AF = AG + GF = 180 + 14.66 = 194.66$ ft.;

hence, $CF = \sqrt{181.52^2 + 194.66^2} = 266.16$ ft. Ans.

(687) In the right-angled triangle ABC, Fig. 31, we have given the angle $CAB = 30°$. To find the length of AB, we have

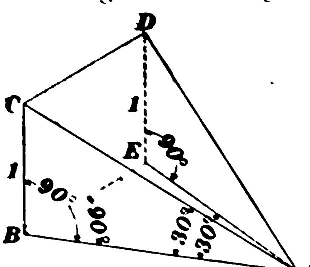

FIG. 31.

$$\tan CAB = \frac{CB}{AB}, \text{ or } AB = \frac{CB}{\tan CAB}.$$

Assuming CB to equal 1 foot, and substituting,

$$AB = \frac{1}{\tan 30^\circ} = \frac{1}{.57735} = 1.732.$$

In the right-angled triangle ABE, we know the length of AB, and also the horizontal angle $BAE = 30°$. To find the length of AE, we have

$$\cos BAE = \frac{AB}{AE}, \text{ or } AE = \frac{AB}{\cos BAE}.$$

Substituting, we have

$$AE = \frac{1.732}{\cos 30^\circ} = \frac{1.732}{.866} = 2.$$

In the triangle AED, $AE = 2$ feet, and $DE = CB =$ 1 foot. Hence, to find the angle DAE, which is the pitch of the rooms, we have

$$\tan DAE = \frac{DE}{AE} = \frac{1}{2} = .5,$$

which corresponds to 26° 34′, nearly. Ans.

(688) The dimensions of a claim, as allowed by the United States Mineral Laws, are 1,500 feet in the direction, or on the strike, of the vein, and 300 feet on each side of the middle of the vein at the surface. See Art. **1269.**

(689) The general principles in surveying metalliferous mines are the same as in surveying coal mines. The differences are only in the details. See Art. **1274.**

(690) Four drawings are necessary to represent the workings of a metalliferous mine: (1) the surface plan; (2) the working plan; (3) a longitudinal section; (4) a transverse section. See Art. **1274.**

(691) See Art. **1274.**

(692) See Art. **1274.**

(693) See Art. **1274.**

(694) See Art. **1274.**

(695) When the lode is very nearly flat, the longitudinal section is made along the lode. See Art. **1274.**

(696) The datum line is assumed to be 91.397 feet below the first bench-mark.

The level must be in perfect adjustment. It is then set up in some convenient place, and the reading of the rod is taken on the bench-mark. It is equal to 4.576 feet, and is recorded in the notes in the plus or backsight column opposite B. M. As the B. M. is assumed to be 91.397 feet above the datum line, the height of the instrument (or line of collimation) above this datum will be $91.397 + 4.576 =$ 95.973 feet.

The unit of measurement in the column of distances is 100 feet. Readings are taken at intermediate points (as at

340 and 670 feet in this example) where there are any abrupt changes in the inclination of the surface.

Station 1 is a turning point, T. P. The reading of the rod, held vertically on it, is 3.726 feet. This reading is recorded as a minus sight, and the surface height is the difference between 95.973 and 3.726 = 92.247 feet.

The levelman now goes forward as before, sets up his instrument in a convenient place, levels it and takes a backsight upon the rod, the target reading being taken to thousandths. The height of instrument will be the elevation of the T. P. plus this reading, which is on that account recorded, as in the notes, as a *plus* sight.

The height of instrument is, therefore, 92.247 + 5.420 = 97.667 feet.

The rodman now goes forward with the rod, and sets it upon Station 2, while the levelman sights to it without changing his former position. This reading is recorded as a minus sight, and by subtracting it from the height of instrument at the last turning point, we obtain the surface height; thus, 97.667 – 4.5 = 93.167 feet.

Station 3 is a turning point, T. P. The reading of the rod on it is 3.170 feet, whence the surface height = 97.667 – 3.170 = 94.497 feet. The levelman goes forward, backsights upon the rod, and has the target set so that the reading can be taken to thousandths. The height of the instrument is, therefore, 94.497 + 4.910 = 99.407 feet.

The rodman goes forward with the rod, sets it upon Station 3 + 40, and the levelman sights to it from his former position. The reading 4.9 feet is recorded as a minus sight, and the surface height is found to equal 99.407 – 4.9 = 94.507 feet.

Station 4 is a turning point, T. P. The reading of the rod upon it is 6.386 feet, whence the surface height = 99.407 – 6.386 = 93.021 feet.

The levelman goes forward, backsights upon the rod, and has the target set so that the reading can be taken to thousandths. The height of the instrument is, therefore, 93.021 + 3.380 = 96.401 feet.

(697)

Distances.	B. S. +	Height of Instrument.	F. S. – T. P.	F. S. – Intermediate Sights.	Elevations.
B. M.	4.576	95.973			91.397
100 T. P.	5.420	97.667	3.726		92.247
200				4.5	93.167
300 T. P.	4.910	99.407	3.170		94.497
340				4.9	94.507
400 T. P.	3.380	96.401	6.386		93.021
500				4.6	91.801
600 T. P.	2.760	93.761	5.400		91.001
670				3.1	90.661
700				3.8	89.961
800 B. M.			6.925		86.836

Total, + 21.046   – 25.607
+ 21.046
Difference = – 4.561
+ 91.397
+ 86.836

} Proof of the correctness.

(698) Fig. 32 is a profile made from the notes in example 696, after calculating the elevation of each station.

(699) See Arts. **1262** and **1263.**

(700) The best method of platting the notes for a mine map is to plat the main passages by means of total latitude and total departure, and the rooms or chambers by means of the protractor. This method possesses both the advantage of accuracy and of rapidity. See Art. **1268.**

(701) See Art. **1251.**

(702) Its proper location is determined by a careful examination of the notes. See Art. **1268.**

(703) See Art. **1255.**

(704) Begin traversing the transit notes by ruling eleven columns, and head them as shown in Art. **1137.**

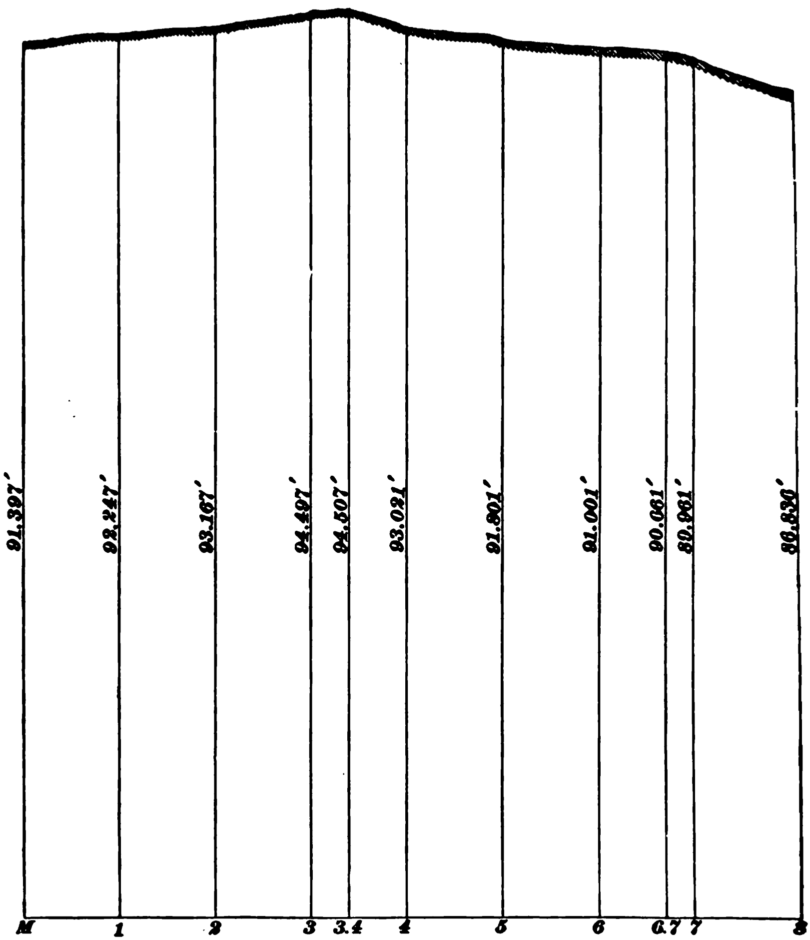

FIG. 82.

Fill in the stations and courses as given in the question, and then reduce all districts measured on an inclination to horizontals by multiplying their measured length by the cosine of the angle of inclination; then, fill in the column of distances.

Next, multiply each horizontal distance by the cosine and the sine of its course (bearing), the products being the latitude and the departure, respectively. Place these in their proper columns.

Calculate the *total* latitudes and the *total* departures with reference to Sta. 302 as the initial station.

Begin the platting by drawing two lines at right angles, their intersection being the position of Sta. 302. Measure on the meridian line a distance north equal to the total latitude of Sta. 303, which is 17.12 feet, and mark the point. Measure due east from this point a distance of 241.74 feet, and mark the point, which, if no error has been made, is the proper location of Sta. 303. To check the correctness of this location, measure the distance from the point to Sta. 302, and if it scales 242.35 feet, as nearly as can be judged, the work is correct.

Plat and check each of the other stations in precisely the same manner, joining consecutive stations by straight lines, thus forming a skeleton upon which the side notes are built.

Before platting these side notes reduce the distances on the tape, where the side measurements were taken, to horizontal lengths. For this purpose a traverse table is very convenient, as it is sufficiently exact to take the vertical angle to the nearest even quarter degree and the distance to the nearest foot. For example, the sight from Sta. 321 to face was on an inclination of 16° 40′. The nearest quarter degree is 16° 45′, and a traverse table gives the horizontal distance for the side-note distances of 48, 56, 66, 144, 152, 160, and 171 feet, as 46, 54, 63, 138, 146, 153, and 164 feet, respectively.

Plat the gangway side notes first, beginning at Sta. 302. On the line joining 302 with 303, mark off to scale points 51, 91, 122, 141, etc., feet from Sta. 302. At these points measure to scale right and left the distances given in the notes. Where "chutes" are indicated, sketch them in at once, and from their corners draw lines (free-hand) through the other points located to the next corner of a chute. Locate all side-note distances at *right angles* to the line of sight. When all the side notes (gangway and chambers) have been platted, the work is complete, as shown in Fig. 33.

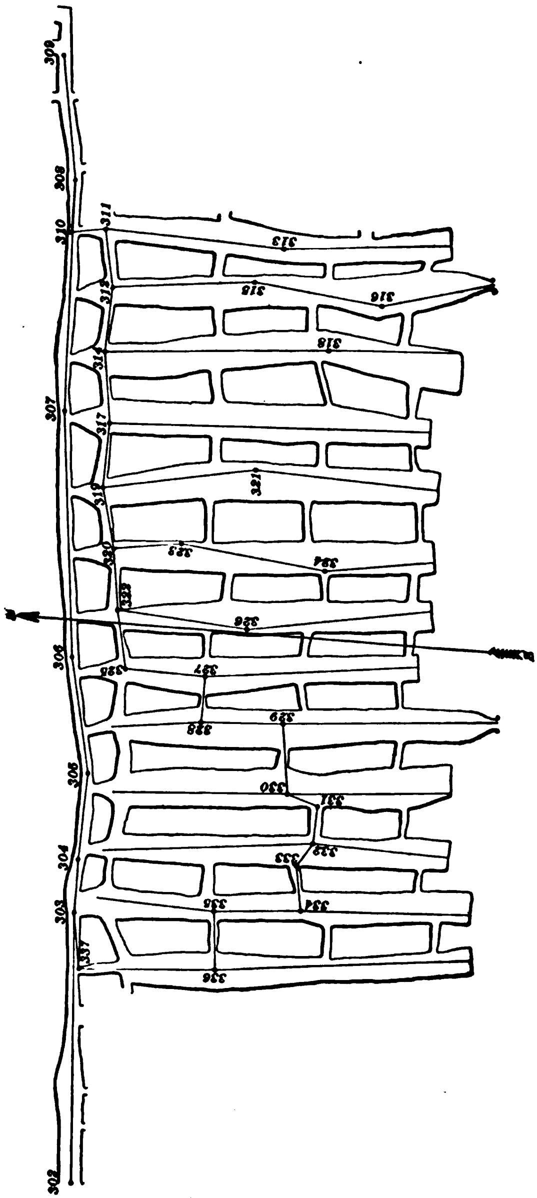
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FIG. 33.

TRAVERSED SURVEY.

Stations.	Bearings.	Distances.	N.	S.	E.	W.	Total.			
							N.	S.	E.	W.
302							0.0	0.0	0.0	0.0
302–303	N 85° 57′ E	242.35	17.12		241.74		17.12		241.74	
303–304	N 89° 33′ E	47.47	.37		47.47		17.49		289.21	
304–305	S 87° 45′ E	77.06		3.03	77.00		14.46		366.21	
305–306	N 77° 51′ E	104.69	22.03		102.34		36.49		468.55	
306–307	N 84° 02′ E	220.76	22.95		219.56		59.44		688.11	
307–308	N 88° 21′ E	206.18	5.94		206.10		65.38		894.21	
308–309	N 80° 36′ E	113.44	18.53		111.92		83.91		1,006.13	
308–310	S 89° 35′ W	46.87		.34		46.87	65.04		847.34	
310–311	S 8° 55′ E	30.16		29.80	4.67		35.24		852.01	
311–312	S 78° 14′ W	52.78		10.76		51.67	24.48		800.34	
311–313	S 2° 12′ W	161.26		161.14		6.19		125.90	845.82	
312–314	N 86° 48′ W	57.40	3.20			57.31	27.68		743.03	
312–315	S 6° 33′ E	127.05		126.22	14.49			101.74	814.83	
315–316	S 6° 38′ W	115.22		114.45		13.31		216.19	801.52	
314–317	S 81° 47′ W	63.86		9.13		63.20	18.55		679.83	
314–318	S 4° 30′ E	200.46		199.84	15.73			172.16	758.76	
317–319	N 88° 27′ W	58.08	1.57			58.06	20.12		621.77	
319–320	S 75° 39′ W	55.52		13.76		53.79	6.36		567.98	

TRAVERSED SURVEY—Continued.

Stations	Bearings	Distances	N.	S.	E.	W.	Total N.	Total S.	Total E.	Total W.
319–321	S 10° 51′ E	137.76		135.30	25.93			[illegible]	[illegible]	
320–322	S 82° 05′ W	54.78		7.54		54.26		[illegible]	[illegible]	
320–323	S 8° 14′ E	60.24		59.63	8.63			[illegible]	[illegible]	
323–324	S 6° 12′ W	130.84		130.07		14.13		[illegible]	[illegible]	
322–325	S 78° 07′ W	52.67		10.85		51.54		[illegible]	[illegible]	
322–326	S 4° 08′ W	117.42		117.11		8.46		[illegible]	[illegible]	
325–327	S 1° 24′ W	72.08		72.06		1.76		[illegible]	[illegible]	
327–328	N 88° 43′ W	40.97	.92			40.96		[illegible]	[illegible]	
328–329	S 3° 21′ E	73.16		73.03	4.28			[illegible]	[illegible]	
329–330	S 81° 42′ W	62.61		9.04		61.95		[illegible]	[illegible]	
330–331	S 17° 50′ W	28.38		27.02		8.69		[illegible]	[illegible]	
331–332	N 87° 20′ W	33.52	1.56			33.48		[illegible]	[illegible]	
332–333	N 58° 15′ W	24.52	12.90			20.85		[illegible]	[illegible]	
333–334	S 79° 31′ W	40.10		7.30		39.43		[illegible]	[illegible]	
334–335	N 4° 53′ W	77.79	77.51			6.62		[illegible]	[illegible]	
335–336	S 84° 15′ W	51.80		5.19		51.54		[illegible]	[illegible]	
336–337	N 4° 17′ W	122.52	122.18			9.15	0.40		102.03	
337–303	N 80° 34′ E	50.50	8.28		49.82		17.08		241.85	

ECONOMIC GEOLOGY OF COAL.

(705) See Art. **1279.**

(706) See Art. **1297.**

(707) See Art. **1302.**

(708) (*a*) and (*b*) See Art. **1332.**

(709) Deposits of coal in the Sub-Carboniferous period are called false coal measures. (Art. **1340.**)

(710) (*a*) and (*b*) See Art. **1341.**

(711) See Art. **1351.**

(712) See Arts. **1315** and **1352.**

(713) See Art. **1325.**

(714) See Art. **1308.**

(715) (*a*) See Art. **1338.** (*b*) No; there were no materials for the formation of coal during the Silurian Age on the American continent.

(716) See Art. **1346.**

(717) See Arts. **1345** and **1347.**

(718) No. 21, Fig. 375, is a trilobite.

(719) No rule can be given for determining the displacement of a fault. (Art. **1320.**)

(720) (*a*) and (*b*) See Art. **1305.**

(721) (*a*) and (*b*) See Art. **1341.**

(722) See Art. **1350.**

(723) The Acadian epoch. (See Art. **1336.**)

(724) Pennine fault. (See Art. **1324.**)

(725) See Art. **1311.**

(726) The Silurian. (Art. **1339.**)

(727) Using formula **83,** Art. **1287,**

$$T = 50.68 + \frac{900 - 19.68}{67.2} = 63.78°. \quad \text{Ans.}$$

(728) (*a*) See Art. **1330.** (*b*) Dawn of animal life; old life; middle life; recent life; era of mind. (See Art. **1331.**)

(729) (*a*) and (*b*) See Art. **1349.**

(730) The Corniferous. (See Geological Chart for North America.)

(731) (*a*) The mountain limestone belongs to the Lower Carboniferous epoch.

(*b*) The millstone grit belongs to the coal measures. (See Geological Chart.)

(732) (*a*) and (*b*) See Arts. **1340** and **1355.**

(733) See Arts. **1296** and **1307.**

(734) See Art. **1309** and glossary, Art. **1382.**

(735) No. (See Art. **1300.**)

(736) We should follow the life system rather than the rock system. (See Art. **1330.**)

(737) See Art. **1350.**

(738) No. (See Art. **1365.**)

(739) See Art. **1339.**

(740) No.

(741) The Silurian. (Art. **1336.**)

(742) (*a*) and (*b*) See Art. **1288.**

(743) See Art. **1304.**

(744) See Art. **1340.**

(745) See Art. **1316.**

(746) See Art. **1343.**

(747) Thickness $b\,c = a\,b \times \sin 70° = 1{,}000 \times .93969 =$ 939.69 ft. Ans.

(748) See Art. **1311.**

(749) See Arts. **1344** and **1345.**

(750) (*a*) See Art. **1339.** (*b*) No.

(751) Fossils of fishes. (Art. **1339.**)

(752) See Art. **1359.**

(753) No. (Art. **1339.**)

(754) See Art. **1325.**

(755) See Art. **1320.**

(756) See Art. **1296.**

(757) Yes. (See Art. **1307.**)

(758) 2,000. (See Art. **1353.**)

(759) Pennsylvania, Virginia, Kentucky, and Indiana. (See Geological Chart.)

(760) (*a*) and (*b*) See Arts. **1281, 1282,** and **1283.**

(761) Anticlinal axis. (Art. **1300.**)

(762) See Art. **1303.**

(763) See Art. **1312.**

(764) (*a*) and (*b*) See Art. **1327.**

(765) See Art. **1353.**

(766) See Art. **1341.**

(767) See Art. **1307.**

(768) No. The dip may be inclined to either side of the line of strike. (See Art. **1299.**)

(769) See Art. **1291.**

(770) See Art. **1316.**

(771) See Art. **1364.**

(772) See Arts. **1308** and **1310.**

(773) (*a*) and (*b*) See Art. **1300.**

(774) The rocks of the Cretaceous period are less frequently metamorphic than the older rocks. (See Art. **1363.**)

PROSPECTING FOR COAL AND LOCATION OF OPENINGS.

(775) See Art. **1383.**

(776) See Art. **1390.**

(777) See Arts. **1424, 1433,** and **1434.**

(778) See Art. **1447.**

(779) See Art. **1434.**

(780) The presence of coal is determined by boring. (See Art. **1397.**)

(781) (*a*) From No. 3 to No. 2 rise = 275 ft. − 260 ft. = 15 ft. 15 ft. in 540 ft. or 1 in $\frac{540}{15}$ = 36 ft. From No. 2 rise to No. 4 will be $\frac{960}{36} = 26\frac{2}{3}$ ft.; 260 ft. − $26\frac{2}{3}$ ft. = $233\frac{1}{3}$ ft. Ans.

(*b*) From No. 2 to No. 1 rise = 260 ft. − 180 ft. = 80 ft. 80 ft. in 1,500 ft., or 1 in $\frac{1,500}{80} = 18.75$ ft. From No. 2 vein rises towards No. 1, 1 in 18.75, and it will rise to level of No. 4, or $26\frac{2}{3}$ ft., in going $18.75 \times 26\frac{2}{3} = 500$ ft. Ans.

(*c*) From No. 2 vein will fall in the opposite direction at same rate, 1 ft. in 18.75 ft., and it will fall to the level of No. 3, or 15 ft., in going $18.75 \times 15 = 281.25$ ft. Ans.

(*a*′) True dip is at right angles to the line of strike. We first find the length of the line $x\,y$, in which the vein falls 260 ft. − $233\frac{1}{3}$ ft. = $26\frac{2}{3}$ ft. Thus, $\tan a = \frac{500}{960} = .52083$; hence, angle $a = 27°\ 30'$; then, $x\,y = 960 \times \sin a = 960 \times$

sin 27° 30′ = 960 × .46175 = 443.28 ft. And true dip = $\frac{443.28}{26\frac{2}{3}}$ = 16.63, or dip = 1 ft. in 16.63 ft. Ans.

(*e*) See Fig. 34.

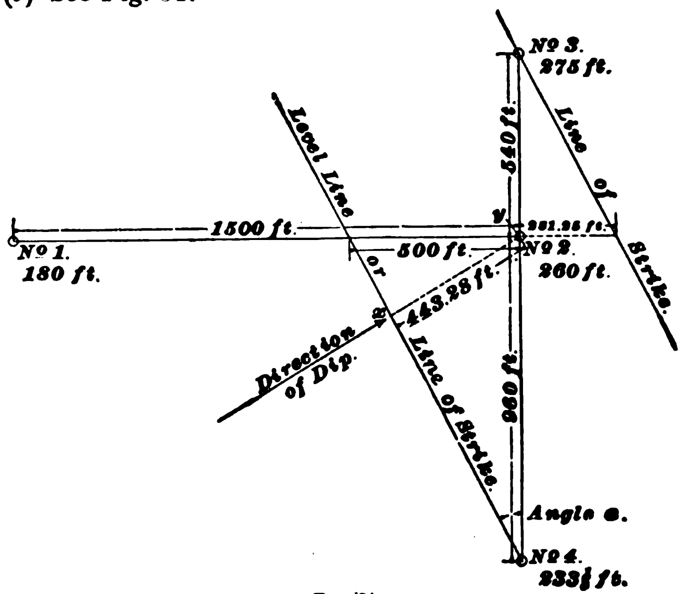

FIG. 34.

(782) See Art. **1386.**

(783) See Art. **1441.**

(784) See Art. **1437.**

(785) See Art. **1405.**

(786) See Art. **1394.**

(787) Specific gravity = $\frac{\text{Weight in air}}{\text{Difference}} = \frac{530}{530 - 105} = \frac{530}{425}$ = 1.247. Ans.

(788) Depth of bore hole = 550 × tan 46° 20′ = 550 × 1.04766 = 576.2 ft. Ans.

(789) See Art. **1384.**

(**790**) See Arts. **1398** and **1399.**

(**791**) See Art. **1437.**

(**792**) See Art. **1392.**

(**793**) See Arts. **1424** and **1434.**

(**794**) See Arts. **1449** and **1452.**

(**795**) See Art. **1436.**

(**796**) See Art. **1396.**

(**797**) 30° and 120° to 150°. (See Art. **1410.**)

(**798**) See Art. **1418.**

(**799**) See Art. **1442.**

(**800**) See Art. **1398.**

(**801**) See Arts. **1385** and **1389.**

(**802**) See Art. **1396.**

(**803**) See Art. **1421.**

(**804**) See Arts. **1408** and **1412.**

(**805**) See Art. **1393.**

(**806**) See Art. **1432.**

(**807**) (1) One acre = 43,560 sq. ft.

$43,560 \times 4\frac{1}{2} = 196,020$ cu. ft.

Weight of coal $= 196,020 \times 62.355 \times 14 = 17,111,958$ lb.

$17,111,958 \div 2,000 = 8,556$ short tons. Ans.

(2) Tons per inch per acre is 141, since specific gravity is 1.4; $4\frac{1}{2}$ ft. = 54 in.

$141 \times 54 = 7,614$ tons. Ans.

(**808**) See Art. **1388.**

(**809**) See Art. **1396.**

(**810**) See Art. **1448.**

(**811**) See Art. **1417.**

(**812**) See Arts. **1451** and **1452.**

SHAFTS, SLOPES AND DRIFTS.

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(813) See Art. **1453.**

(814) The shaft should be placed on the lowest boundary of the field, or at such a distance from it as will provide for a sump on the low side of the shaft bottom, and the workings driven to the rise. This will ensure good drainage for the working places, and the most advantageous grade for haulage purposes. (See Art. **1456.**)

(815) See Art. **1454.**

(816) See Art. **1457.**

(817) See Arts. **1462** and **1463.**

(818) See Art. **1470.**

(819) It is carried down to the bed rock, or "hard pan." (See Art. **1464.**)

(820) See Art. **1464.**

(821) See Art. **1478.**

(822) See Art. **1474.**

(823) See Art. **1485.**

(824) See Art. **1486.**

(825) See Art. **1489.**

(826) Substituting values in formula **87**, Art. **1487**, we have

$$N = \frac{(16^2 - 15^2) \times .7854 \times 65}{.25 \times .3333 \times .6666} = 28,486.458.$$

Less 10% = 25,638 bricks. Ans.

(827) See Art. **1453.**

(828) Steam. (See Art. **1498.**)

(829) See Art. **1472.**

(830) See Art. **1457.**

(831) The water should be shut off by building a coffer dam, as shown at *K*, Fig. 425. (See Art. **1466.**)

(832) Buntons are timbers placed horizontally across the shaft to carry the cage guides, column pipes, etc.; also to strengthen the timbering. They are set into the rock and keyed firmly. (See Art. **1467.**)

(833) The size of the shaft will depend upon what it is used for; i. e., it may be used for hoisting, ventilating (return and intake airways), and pumping, which require a shaft of greater dimensions than when an air-shaft is sunk near by. (See Arts. **1458** and **1459.**)

(834) The circular form should be used, shutting off the water by means of iron tubbing. (See Arts. **1457** and **1476.**)

(835) See Art. **1482.**

(836) See Art. **1475.**

(837) See Art. **1488.**

(838) A wedging curb is a permanent curb which must be used with any form of walling that is required to be water tight. (See Art. **1490.**)

(839) See Art. **1457.**

(840) See Art. **1461.**

(841) See Art. **1454.**

(842) See Art. **1456.**

(843) Substituting the values in formula **86, Art. 1480,** we have

$$t = \frac{1,800 \times 15 - 6 \times 15\sqrt{90,000 - 6.944 \times 375}}{\sqrt{90,000 - 6.944 \times 375}} = 1.333 \text{ in.}$$

Adding $\frac{1}{8}$ in., thickness = 1.333 + .125 = 1.458 in. Ans.

(844) See Art. **1468.**

(845) See Art. **1473.**

(846) See Art. **1483.**

(847) See Art. **1491.**

(848) By the use of cement, by freezing, and by pumping off the water. (See Arts. **1494, 1523,** and **1518.**)

(849) See Art. **1498.**

(850) They are the holes bored in the middle of the shaft bottom for the purpose of blasting it out, before the lateral holes are fired to trim up the shaft to the proper dimensions. (See Art. **1499.**)

(851) See Art. **1503.**

(852) See Art. **1513.**

(853) A number of detonators are used in the same hole. (See Arts. **1505** and **1506.**)

(854) See Art. **1500.**

(855) See Art. **1496.**

(856) See Art. **1497.**

(857) Dynamite is exploded by means of a shock produced by an exploder, or detonator, while powder is exploded by a spark or flame. (See Arts. **1501** and **1502.**)

(858) By incandescence in the bridge *E D* (Fig. 446). (See Art. **1505.**)

(859) See Art. **1506.**

(860) See Art. **1502.**

(861) Shots are said to be fired (1) in series, when they are connected as shown in Fig. 447; (2) in parallel, when connected as shown in Fig. 448; (3) in multiple series, when connected as shown in Fig. 449. (See Art. **1508.**)

(862) The method shown in Fig. 452 should be used (See Art. **1516.**)

(863) See Arts. **1519, 1520,** and **1521.**

(864) See Arts. **1523** to **1527.**

(865) The method shown in Fig. 453 should be employed. (See Art. **1517.**)

(866) See Art. **1528.**

(867) See Art. **1521.**

(868) See Art. **1539.**

(869) Great difficulty is found in keeping the holes exactly vertical. (See Art. **1540.**)

(870) The Kind-Chaudron method should be used. (See Arts. **1531** to **1538.**)

(871) The water in the shaft will prevent the inflow of quicksand or other soft material, and assist in lowering the tubbing, and no pumping will be required while sinking. (See Arts. **1537** and **1538.**)

(872) See Art. **1541.**

(873) See Art. **1542.**

(874) The widening of a shaft should begin at the top, and the process requires that the mine be idle while the work is being done. (See Art. **1543.**)

(875) Wooden guides and iron guides (T-iron, round-iron), and wire-rope guides. (See Arts. **1545** to **1551.**)

(876) By turnbuckles and weights hung upon their lower ends. (See Arts. **1548** and **1551.**)

(877) See Art. **1552.**

(878) The point at which the face of the slope will have a vertical height greater than the timbers. (See Art. **1553.**)

(879) Where the top is very soft and falls in as quickly as the excavation takes place. (See Art. **1553.**)

(880) See Art. **1555.**

(881) They are set so that their tops lean a few degrees up the pitch from the perpendicular to the slope. (See Art. **1561.**)

(882) See Arts. **1561** and **1563.**

(883) See Art. **1560.**

(884) See Art. **1564.**

(885) The face is arranged in two lifts, the top one being carried forwards in advance of the lower one. (See Art. **1567.**)

(886) See Art. **1569.**

(887) The vertical distance in feet of the outcrop above the level of the parting $= 600 \tan 30' = 600 \times .00873 =$ 5.238 feet. The vertical distance in feet that the drift mouth will be below the level of the parting $= 600 \times \frac{1.5}{100} =$ 9 feet Therefore, $5.238 + 9 = 14.238$ feet. Ans.

(888) By substituting the values in formula **85,** Art. **1459,** we have

$$L = \frac{1,200 \times 2,000 \times 600}{36,000 \times 8 \times 50 \times 4 \times 3} + 1 = 9 \text{ feet 4 inches.}$$

Total length of shaft = width of hoisting compartments + width of pumpway + width of buntons $= 2(5 + 2) + 6 + 2 \times \frac{10}{12} =$ 21 ft. 8 in.

The shaft is 21 ft. 8 in. × 9 ft. 4 in. in the clear. Ans.

METHODS OF WORKING COAL MINES.

(PART 1.)

(889) See Art. **1575.**

(890) The practice in the locality in which the shaft or slope is to be sunk. (See Art. **1576.**)

(891) $\frac{625}{4} + 125 = 281\frac{1}{4}$ feet, the radius of shaft pillar. (See Art. **1576.**)

(892) Because the plane of fracture is nearly perpendicular to the slope, while it is nearly parallel to the line of the shaft. (See Art. **1578.**)

(893) Because the overlying strata increase as the slope advances downwards. (See Art. **1579.**)

(894) A hard top and a soft bottom. (See Art. **1580.**)

(895) Because the pillars do not increase in strength (width) as the slope is driven downwards. (See Art. **1579.**)

(896) The pillars in the upper seam should be formed vertically over those in the lower seam. (See Art. **1616.**)

(897) Shafts should be sunk so that the tracks on the cages are parallel with the strike of the seam, and slopes should be sunk on the full dip. (See Art. **1582.**)

(898) See Arts. **1583, 1584,** and **1585.**

(899) They should not be less than 100 feet wide. (See Art. **1581.**)

(900) From 1 to 2 per cent. grades. (See Art. **1585.**)

(901) They should be formed so that their longer sides will be parallel to the line of dip. (See Art. **1593.**)

(902) See Art. **1595.**

(903) See Art. **1596.**

(904) See Art. **1601.**

(905) See Art. **1602.**

(906) See Art. **1592.**

(907) See Art. **1594.**

(908) They are turned off both butt entries when the seam is comparatively flat, or when the seam is inclined and the butt headings run to the rise or dip; but when the seam is inclined and the productive headings run along the strike, the rooms are turned off on one side only. (See Art. **1606.**)

(909) The former; because when the rooms are turned off both of the butt headings, one group of rooms is driven to meet another group coming from the next pair of butt entries, necessitating only one-half the amount of productive or butt entry-driving. (See Art. **1606.**)

(910) By the panel system. (See Art. **1612.**)

(911) See Art. **1650.**

(912) It is arranged in sections, each of which is made level by the refuse of the seam. (See Art. **1625.**)

(913) From 15° to 30°. (See Art. **1626.**)

(914) See Art. **1630,** and Figs. 499 and 500.

(915) See Art. **1629,** and Fig. 498.

(916) See Art. **1631.**

(917) See Arts. **1635** and **1639.**

(918) The danger is that, in the case of a collapse of a manway, the entire ventilation of all the breasts of the section in which the collapse occurred is destroyed. (See Art. **1638.**)

(919) By leaving a pillar at the mouth of the breast to protect the airway, as shown at *A*, Fig. 503.

(920) The air is conducted around the breast in which the accident occurred through the airway and the small passages leading from the manway chutes to the airway. These passages are shown in Fig. 504 as *c* and *d*, respectively.

(921) They are batteries placed in chutes to prevent the air-current from taking a short cut from the gangway to the breast airways. (See Art. **1637.**)

(922) It is that system of mining by which the coal from the upper seam is run through rock chutes to the seam below. (See Art. **1643.** For the undetermined points, see Art. **1645.**)

(923) See Art. **1646.**

(924) It is a breast driven over the gangway for the purpose of getting a large portion of the gangway pillar, which would otherwise be lost. (See Art. **1641.**)

(925) See Art. **1642.**

(926) One bore hole should be drilled straight ahead, and flank bore holes should be drilled on each side. (See Art. **1653.**)

(927) The thickness and character of the parting between them. (See Art. **1649.**)

(928) See Art. **1621.**

(929) It should be opened up in a manner similar to that shown in Fig. 504, Art. **1639.**

(930) They should be driven parallel to the face cleats, so that the rooms turned perpendicularly off them will be driven on the face cleats. (See Art. **1613.**)

(931) It should be set so that it leans from 2° to 6° up the pitch from the perpendicular to the seam. Theoretically, the post should be set perpendicular to the seam, or strata; because, when the weight of the overlying strata which is to be supported is resolved into two components, one along the roof and the other perpendicular to it, the post must support that component of the weight which is perpendicular to the seam, while the roof itself holds in equilibrium the other. The post is inclined slightly up the pitch, in order that it will tighten rather than fall out in case the roof slides down the pitch. (See Arts. **1654**, **1655**, and **1656**.)

(932) See Art. **1657**.

(933) By rounding their bottoms. (See Art. **1658**.)

(934) See Art. **1658**.

(935) By placing the "sights" near one side of the heading. (See Art. **1661**.)

(936) It gives, in general, a very crooked or circuitous road, which is very hard on the cars and makes mechanical haulage difficult. (See Art. **1661**.)

(937) See Art. **1660**.

(938) The pitch and thickness of the seam. (See Art. **1662**.)

(939) By lifting the bottom on the rise side. (See Fig. 515, Art. **1664**.)

(940) See Fig. 518, Art. **1667**.

(941) See Figs. 523, 524, and 525, Art. **1672**.

(942) See Art. **1675**.

(943) From 25 to 45 pounds per yard. (See Art. **1676**.)

(944) Wooden rails are laid outside the **T** rails to obtain a greater friction when the wheel is spragged or the brake is applied. (See Art. **1676**.)

(945) The spikes have not the proper relative position in the ties. (See Art. **1676**.)

(946) One that gives an unbroken main road, such as that shown in Fig. 527, Art. **1678**.

(947) They are used at turnouts or landings, or where two tracks come together, one of which is for the loaded cars and the other for the empties. (See Arts. **1583, 1588,** and **1693.**)

(948) There should be a difference in the relative heights of the lead and follower rails. (See Art. **1680.**)

(949) See Fig. 532.

(950) See Art. **1687.**

(951) See Art. **1694.**

(952) By means of a slope carriage, or gunboat. (See Art. **1695.**)

(953) If both sides of the mine do not produce the same amount of coal, caging becomes proportionately more difficult. (See Art. **1696.**)

(954) The size of the mine cars. (See Art. **1697.**)

(955) One bore hole should be drilled straight ahead, and flank bore holes should be drilled on the high side only. (See Art. **1653.**)

(956) $(15 \times 4) + (15 \times 5) = 135$ ft. Ans. (See Art. **1651.**)

(957) $[(6 \times 3) + (6 \times 5)] \times 2 = 96$ ft. Ans. (See Art. **1651.**)

(958) See Art. **1704.**

(959) As water exerts a pressure of 0.434 lb. per sq. in. per foot of depth, the pressure per sq. in. under a 200-foot head of water equals $200 \times .434 = 86.8$ lb. Then by formula **88** the thickness of the dam is

$$T = 360 \times \left(1 - \sqrt{1 - \frac{20 \times 86.8}{8,000}}\right) =$$

$$41.44 \text{ in.} = 3.45 \text{ ft. } 6 \text{ in., nearly.}$$

By doubling the calculated thickness to ensure safety, we have 3 ft. 6 in. $\times 2 = 7$ ft. Ans. (See Art. **1710.**)

(960) Using formula **88**,

$$T = 84 \times \left(1 - \sqrt{1 - \frac{20 \times 108.5}{2,500}}\right) =$$

53.48 in. = 4 ft. 6 in., nearly.

4 ft. 6 in. × 2 = 9 feet. Ans. (See Art. **1710.**)

(961) Applying formula **89**,

$$T = 96 \times \left(1 - \sqrt[3]{1 - \frac{15 \times 130.2}{2,500}}\right) =$$

38.15 in. = 3 ft. 2 in., nearly.

3 ft. 2 in. × 2 = 6 ft. 4 in. Ans. (See Art. **1710.**)

(962) It prevents leakage, and transmits the pressure from one concentric arch to the other. (See Art. **1708.**)

(963) The best Portland cement. (See Art. **1709.**)

METHODS OF WORKING COAL MINES.

(PART 2.)

(964) See Art. **1711.**

(965) See Art. **1713.**

(966) If longwall advancing be used, the method should be similar to that of Fig. 555 or 559; but if longwall retreating be used, the method shown in Fig. 580 should be used. The combined method (Fig. 582) may also be used.

(967) See Art. **1717.**

(968) By first bearing in a short distance at one side of his room or allotted portion of the working face and continuing the shallow mining across it, by which time the weighing action will have softened the coal just beyond the mining and make it comparatively easy to repeat the operation and deepen the mining. (See Art. **1735.**)

(969) See Art. **1730.**

(970) Because it is otherwise a difficult matter to maintain a continuous line of working face, which is essential to the best working of longwall. (See Art. **1750.**)

(971) See Art. **1769.**

(972) Where the roof is brittle sufficient breadth can not be maintained to take the car along the face, making it necessary to approach the face by a greater number of roads. (See Art. **1712.**)

(973) See Art. **1755.**

(974) Shafts are protected either by pillars left at the bottom of the shaft or by carefully stowing up the space made vacant by taking out all the coal. (See Art. **1714.**)

(975) The face should advance parallel to the principal or face cleats, i. e., the "end on" plan should be adopted. (See Art. **1725.**)

(976) See Arts. **1735** and **1736.**

(977) There is danger of throwing excessive weight upon the face and destroying the timber. (See Art. **1730.**)

(978) It is so arranged that each miner will have a long rise side and a short dip side with reference to the road leading to the face. This saves him the work of shoveling coal up hill. (See Art. **1763.**)

(979) To take advantage of gravity. (See Art. **1727.**)

(980) Where the roof is bad and roads 10 or 12 yards apart approach the face, the coal is shoveled to the road-heads; but where the seam is low, a buggy is used to convey the coal along the face to the road-heads. (See Arts. **1712** and **1755.**)

(981) See Art. **1716.**

(982) See Art. **1752.**

(983) See Art. **1743.**

(984) See Art. **1712.**

(985) See Art. **1765.**

(986) The inclination of the seam and the condition of the packwalls. (See Art. **1766.**)

(987) To properly regulate the weight upon the working face. (See Art. **1729.**)

(988) See Art. **1721.**

(989) By being able to make a deep holing or mining with a very low front. (See Art. **1736.**)

(990) The steps are made between the pairs of roads. (See Art. **1745.**)

(991) The seam is divided into layers or lifts which are worked independently, or so that one lift is a short distance ahead of the other. (See Art. **1771.**)

(992) At the road-head. (See Art. **1759.**)

(993) If the rise be too steep for the mule to enter the room with the car, the coal is conveyed to the level either by chutes or incline planes. (See Art. **1745.**)

(994) By taking a proper width of coal out of both the high and low side of the level. (See Art. **1754.**)

(995) It should advance perpendicularly to the face cleats. (See Art. **1723.**)

(996) The weighting action of the roof. (See Art. **1751.**)

(997) From the roof, the floor, the seam itself, and from the surface. (See Art. **1711.**)

(998) They impede ventilation, increase the cost of production, and cause the coal to be crushed more than when the continuous face is employed. (See Art. **1716.**)

(999) See Art. **1731.**

(1000) So as to get the line of fracture, which takes place along the rib side, below the level, in order to secure good roof in the level and support to the lower edge of the loosened mass, whereby it will be prevented from slipping down hill and destroying the packwalls. (See Art. **1752** and Fig. 561.)

(1001) The pressure of the gas in the coal greatly assists in the work of extraction or in loosening the coal at the working face. (See Art. **1738.**)

(1002) The principal difficulties are found in the tendency of the roof to gravitate away from the working face, and in keeping the packwalls in good condition. (See Art. **1729.**)

(1003) See Art. **1740.**

(1004) See Art. **1792.**

(1005) The cleats may have any direction with reference to the line of dip. (See Art. **1728.**)

(1006) See Art. **1742.**

(1007) Upon the height of the seam and amount of stowage. (See Art. **1746.**)

(1008) A pair of entries (slopes) should be driven in the lower seam from the outcrop to the boundary, if possible, and directly on the line of maximum dip. Then a cross-heading should be driven connecting the three seams, which are finally opened out by driving headings or levels to the right and left in each seam from the cross-heading and not more than 600 feet away from it. These headings are connected at their far ends, and sometimes in the middle, for the purpose of ventilation. On the rise side of each level longwall faces should be started, giving in all 6 places. The faces should be worked in the ascending order and lead each other about 30 feet. The cross-heading or slant should be moved forwards about every 60 feet. (See Art. **1777** and Fig. 581.)

(1009) By systematic and efficient support of the roof near the face by means of props, nogs, chocks, etc. (See Art. **1734.**)

(1010) When the coal is situated at a shallow depth, and it is cheaper to sink extra shafts than to maintain permanent haulways through the gob. (See Art. **1746.**)

(1011) By the escape of the gas in advance of the working face. (See Art. **1738.**)

(1012) The dip and the tendency of the roof to gravitate away from the working face is lessened. (See Art. **1733.**)

(1013) See Art. **1784.**

(1014) See Arts. **1794** to **1797.**

(1015) Special attention must be given to the first packwalls, because more settling takes place at that time than in the ordinary working. (See Art. **1749.**)

(1016) See Art. **1734.**

(1017) By taking down top and lifting bottom. (See Art. **1746.**)

(1018) By leaving a portion of the solid coal on the roof over them. (See Art. **1750.**)

(1019) See Art. **1785.**

(1020) By the escape of the gas through the adjoining strata and by excessive weight upon the working face. (See Arts. **1738** and **1798.**)

(1021) See Art. **1728.**

(1022) "Blind pits" are used to lower coal from one seam to another. (See Art. **1756.**)

(1023) They may have any relative position, but in new coal fields they should be reasonably close together so as to speedily secure a permanent return airway. (See Art. **1783.**)

(1024) The overlying weight is distributed over a greater portion of the bottom. (See Art. **1788.**)

(1025) See Art. **1739.**

(1026) When the top has settled completely. (See Art. **1746.**)

(1027) When the mining or holing is done at the top or near the middle of the seam. (See Art. **1756.**)

(1028) By the use of dynamite. (See Art. **1795.**)

(1029) A constant output is secured. (See Art. **1783.**)

(1030) By driving main headings to the boundary and commencing the necessary narrow work there and bringing it back in advance of the working face. (See Art. **1775** and Fig. 580.)

(1031) See Art. **1797.**

(1032) Longwall retreating. (See Art. **1782.**)

(1033)

Let $r =$ radius, in feet, of pillar to support shaft only.

Then $r + 96 =$ radius, in feet, of pillar to support shaft and buildings.

Hence, $r + 96 = \frac{135 \times 3}{2} = 202.5$, or $r = 202.5 - 96 = 106.5$.

But depth of shaft $D = 4\,r = 426$ feet. Ans. (See Art. **1576.**)

MECHANICS.

(PART 1.)

(1034) See Arts. **1800** and **1801**.

(1035) Reducing 14 minutes to seconds, $14 \times 60 =$ 840 seconds.

$$840 \times 40 = 33{,}600 \text{ ft.} \doteq 6\tfrac{4}{11} \text{ miles.} \quad \text{Ans}$$

(1036) See Art. **1839**.

(1037) Using formula **94**,

$$P a = W b, \text{ or } P \times F c = W \times F b.$$

Hence,

$$W \times 3\tfrac{1}{2} = 85 \times 21;$$

$$W = \frac{85 \times 21}{3\frac{1}{2}} = 510 \text{ lb.} \quad \text{Ans.}$$

(1038) Applying formula **99**,

$$N = \frac{80 \times 28}{21} = 106\tfrac{2}{3} \text{ rev. per min.} \quad \text{Ans.}$$

(1039) (*a*) Applying formula **102**,

$$D = \frac{1\frac{1}{2} \times 50}{3.1416} = 23.87 \text{ in.} \quad \text{Ans.}$$

(*b*) See Art. **1872**. Addendum = .3 of the pitch. 1.5 in. × .3 = .45 in. .45 in. × 2 = .9 in. = difference between the diameter of the pitch circle and the outside diameter Hence, outside diameter = 23.87 + .9 = 24.77 in. Ans

(1040) Apply formula **107**.

$$r = \frac{45 \times 212}{180} = 53 \text{ R. P. M.} \quad \text{Ans.}$$

(1041) Apply formula **110.**

Pitch = $\frac{1}{13}$ in.; therefore,

$$W = \frac{6.2832 \times 24 \times 11}{\frac{1}{13}} = 21,563.94 \text{ lb.} \quad \text{Ans.}$$

(1042) The pull on the support equals the centrifugal force of the ball. Hence, applying formula **112,**

$$F = .00034 \times 5 \times \tfrac{36}{12} \times 350^2 = 555\tfrac{1}{3} \text{ lb.} \quad \text{Ans.}$$

(1043) Apply formula **113.**

$$K = \frac{2 \times 600^2}{64.32} = 11,194 \text{ ft.-lb.} \quad \text{Ans.}$$

(1044) 7 ft. = 84 in. Arc of contact = $\frac{84}{63 \times 3.1416} \times 360° = 153°$. $800 + 3(180 - 153) = 881$. Applying formula **115,**

$$W = \frac{881 \times 150}{3,000} = 44.05 \text{ in.}$$

Using formula **117,**

$$W_1 = 44.05 \times \tfrac{2}{3} = 29.37 \text{ in., or say } 29.5 \text{ in.} \quad \text{Ans.}$$

(1045) See Arts. **1803** to **1823.**

(1046) There are 1,760 yd. in 1 mile. If a man can run 100 yd. in 12 seconds, in 1 second he can run $\frac{100}{12}$ yd., that is, his velocity is $\frac{100}{12}$ yd. per sec. Applying formula **92,**

$$t = 1,760 \div \tfrac{100}{12} = 1,760 \times \tfrac{12}{100} = 211.2 \text{ sec.} = 3 \text{ min. } 31.2 \text{ sec.} \quad \text{Ans.}$$

(1047) See Art. **1840.**

(1048) See Arts. **1860** and **1861.**

(1049) 13 ft. = 156 in. Applying formula **99,**

$$N = \frac{91 \times 108}{156} = 63 \text{ rev. per min., the speed of the engine.} \quad \text{Ans.}$$

(1050) Applying formula **102,**

$$D = \frac{2\tfrac{1}{2} \times 192}{3.1416} = 152.79 \text{ in.} \quad \text{Ans.}$$

(1051) Apply formula **108.**

$$R = \frac{81 \times 80}{18} = 360 \text{ rev. per min.} \quad \text{Ans.}$$

(1052) Pitch = $\frac{1}{8}$ in. Using formula **110,**

$$W = \frac{6.2832 \times 60 \times 26}{\frac{1}{8}} = 78{,}414.336 = \text{the theoretical pressure.}$$

Since the efficiency is but 40%, the actual pressure is 78,414.336 × .40 = 31,365.7 lb. Ans.

(1053) See Art. **1899.**

(1054) First determine the speed of the center of gravity of the section in feet per second. This point revolves in a circle whose diameter is 6 ft. $1\frac{3}{4}$ in. × 2 = 12 ft. $3\frac{1}{2}$ in. = 12.2917 ft. Distance traveled in one revolution = 12.2917 × 3.1416 = 38.6156 ft. Distance traveled in one second = $\frac{38.6156 \times 150}{60}$ = 96.539 ft. Hence, applying formula **113,**

$$K = \frac{13{,}000 \times 96.539^2}{64.32} = 1{,}883{,}661.7 \text{ ft.-lb.} \quad \text{Ans.}$$

(1055) Arc of contact = $\frac{18}{14 \times 3.1416} \times 360° = 147°$. $800 + 3\,(180 - 147) = 899$. Applying formula **115,**

$$W = \frac{899 \times 2.5}{2{,}000} = 1.12 \text{ in., say 1 in.} \quad \text{Ans.}$$

(1056) (*a*) See Arts. **1810** and **1835.**

(*b*) and (*c*) See Art. **1809.**

(1057) $\frac{48}{12} \times 3.1416 = 12.5664$ ft. = circumference of pulley. $\frac{3{,}000}{12.5664} = 238.73$ revolutions in 1 minute, or 60 seconds. To make 100 revolutions will require $\frac{100}{238.73} \times 60 = 25.13$ sec., nearly. Ans.

(1058) 4 ft. 6 in. = 54 in. $54 \times 2 \times \frac{3}{4} \times .261 = 21.141$ lb. = weight of lever. Considering the weight of the lever to be concentrated at its center of gravity, we have three weights of 47, 21.141, and 71 lb., with the smaller weight, $\frac{54}{2} = 27$ in., from the other two. To find the center of gravity of the two large weights, apply formula **93.** $l_1 = \frac{47 \times 54}{71 + 47} = 21.508$ in. = the distance bc in Fig. 35. Consider both weights to be concentrated at b; that is, imagine both

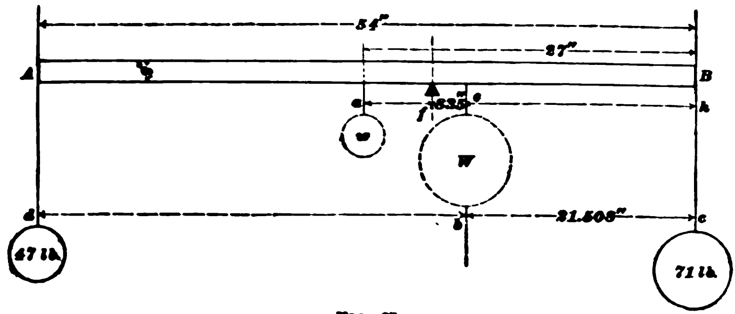

FIG. 35.

weights removed and to be replaced by the dotted weight W, equal to $71 + 47 = 118$ lb. The dotted circle w represents the weight of the bar. The distance $ae = 27 - 21.508 = 5.492$ in. Distance of balancing point f from e is found by means of formula **93** to be $\frac{21.141 \times 5.492}{118 + 21.141} = .834$ in. Finally, $fh =$

$21.508 + .834 = 22.342$ in. = short arm. Ans.
$54 - 22.342 = 31.658$ in. = long arm. Ans.

(1059) See Art. **1862.**

(1060) Apply formula **99,** after reducing the 2 ft. to inches.

$$N = \frac{32 \times 63}{24} = 84 \text{ per min.} \quad \text{Ans.}$$

(1061) Apply formula **103.**

$$T = \frac{11.48 \times 3.1416}{1\frac{1}{8}} = 32 \text{ teeth.} \quad \textbf{Ans.}$$

(1062) Apply formula **108** to find the number of revolutions of the driving gear. $R = \frac{75 \times 88}{44} = 150$ rev. per min. of the driving gear and also of the 8-inch pulley. Using formula **96** to find the diameter of pulley on the shaft,

$$D = \frac{8 \times 150}{200} = 6 \text{ in. Ans.}$$

(1063) (*b*) Using formula **110**,

$$W = \frac{6.2832 \times 25 \times 15}{\frac{1}{4}} = 9{,}424.8 \text{ lb.} = \text{theoretical pressure.}$$

Ans.

(*a*) 9,424.8 − 5,000 = 4,424.8 lb. Ans.

(1064) See Art. **1899.** $\frac{51}{62.5} = .816$, the specific gravity. Ans.

(1065) 660 ft. per min. $= \frac{660}{60} = 11$ ft. per sec.

Applying formula **113**,

$$K = \frac{325 \times 11^2}{64.32} = 611.4 \text{ ft.-lb., nearly. Ans.}$$

(1066) Applying formula **109**,

$$\text{H. P.} = .01 \times 1^2 \times 1{,}200 = 12. \text{ Ans.}$$

(1067) See Arts. **1824** and **1826.**

(1068) See Art. **1830.**

(1069) In Fig. 36, *ABC* represents the triangle. The center of gravity is found as explained in Art. **1845.** The distance of the center of gravity from the side $AC = 1\frac{3}{32}$ in. Ans.

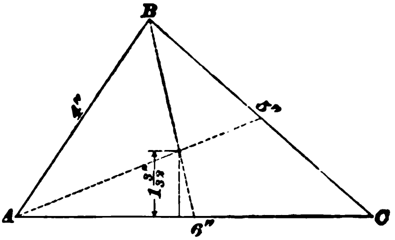

FIG. 36.

(1070) Apply formula **96.**

$$D = \frac{36 \times 60}{40} = 54 \text{ in. Ans.}$$

(1071) (*a*) Applying formula **98**,

$$n = \frac{12 \times 80}{8} = 120 \text{ R. P. M.} \quad \text{Ans.}$$

(*b*) Applying formula **98**,

$$n = \frac{120 \times 20}{6} = 400 \text{ R. P. M.} \quad \text{Ans.}$$

(*c*) Applying formula **98** again,

$$n = \frac{400 \times 6}{4} = 600 \text{ R. P. M.} \quad \text{Ans.}$$

(1072) Applying formula **103**,

$$T = \frac{3.1416 \times 34.15}{1\frac{3}{8}} = 78 \text{ teeth.} \quad \text{Ans.}$$

(1073) Since there are two parts to the rope, the pulleys will raise a load of 225 × 2 = 450 lb. Ans.

(1074) 5 ft. 6 in. = 66 in.

(*a*) 66 ÷ 6 = 11 = velocity ratio. Ans.

(*b*) 11 × 5 = 55 lb. Ans.

(1075) See Art. **1896.** 55 × .65 = 35.75 lb. Ans.

(1076) Referring to the table of Weights per Cubic Foot, a cubic foot of platinum weighs 1,343.8 lb. Therefore, 1 cu. in. weighs $\frac{1,343.8}{1,728}$ lb., and 10 lb. will contain $10 \div \frac{1,343.8}{1,728} = 10 \times \frac{1,728}{1,343.8} = 12.86$ cu. in., nearly. Ans.

(1077) See Art. **1907.**

(1078) Speed of a point on the pitch circle in feet per minute = $\frac{30}{12} \times 3.1416 \times 100 = 785.4$ ft. per min. Apply formula **109.**

$$\text{H. P.} = .01 \times 785.4 \times 1.57^2 = 19.36. \quad \text{Ans.}$$

(1079) See Art. **1826.**

(1080) See Art. **1831.**

(1081) Volume of sphere $= .5236 \times 5^3 = 65.45$ cu. in. 1 cu. in. of cast iron weighs .261 lb.; hence, weight of ball $= 65.45 \times .261 = 17.08$ lb. Weight of a cu. in. of steel is .284 lb.; hence, weight of handle $= (\frac{7}{8})^2 \times .7854 \times 40 \times .284 = 6.83$ lb. Distance of center of gravity of rod from center of ball $= \frac{40}{2} + \frac{5}{2} = 22\frac{1}{2}$ in. Apply formula **93.** Distance of center of gravity of both ball and rod from center of ball $= \frac{6.83 \times 22.5}{17.08 + 6.83} = 6.427$ in. Ans.

(1082) Applying formula **96,**

$$D = \frac{180 \times 30}{240} = 22\frac{1}{2} \text{ in.} \quad \text{Ans.}$$

(1083) Applying formula **100,**

$$P = \frac{6{,}000 \times 6 \times 5 \times 8 \times 3}{18 \times 12 \times 15 \times 12} = 111\frac{1}{9} \text{ lb.}$$

Since there is a loss of 20%, $111\frac{1}{9}$ represents 80% of the total force. Hence, the force actually required $= 111\frac{1}{9} \div .80 = 138\frac{8}{9}$ lb. Ans.

(1084) Apply formula **104.**

$$P = \frac{3.1416 \times 24.16}{38} = 1.9974 \text{ in.} \quad \text{Ans.}$$

(1085) See Art. **1858.** Since there are eight parts of the rope, the force required $= 1{,}890 \div 8 = 236\frac{1}{4}$ lb. Ans.

(1086) (*a*) Velocity ratio $= \frac{1{,}000}{50} = 20.$ Ans.

(*b*) Efficiency $= \frac{10}{19} = .5263 = 52.63\%.$ Ans.

(1087) Volume $= (\frac{1}{2})^2 \times .7854 \times 10 = 1.963$ cu. in. One cu. in. of lead weighs .411 lb. (see table of Weights per Cubic Foot); consequently, $1.963 \times .411 = .807$ lb. $= 12.91$ oz. Ans.

(1088) Using formula **114,**

$$E = 3\frac{1}{4} \times \frac{11 + 7}{2} + 2 \times 38 = 105\frac{1}{4} \text{ ft.} = 105 \text{ ft. } 3 \text{ in.} \quad \text{Ans.}$$

(1089) (*a*) $18 \times 60 \times 60 = 64,800$ miles per hour. Ans.
(*b*) $64,800 \times 24 = 1,555,200$ miles per day. Ans.

(1090) See Art. **1825.**

(1091) See Art. **1832.**

(1092) Length of power arm = 4 ft. − 4 in. = 48 in. − 4 in. = 44 in. According to formula **94,** $P \times 44 = 1,500 \times 4 = 6,000$; hence, $P = \frac{6,000}{44} = 136\frac{4}{11}$ lb. Ans.

(1093) Length of power arm = 4 ft. = 48 in. Hence, as in the preceding question, $P = \frac{6,000}{48} = 125$ lb. Ans.

(1094) Apply formula **97.**

$$d = \frac{10 \times 88}{110} = 8 \text{ ft., the diameter of the pulley.} \quad \text{Ans.}$$

(1095) See Arts. **1869** and **1870.**

(1096) Apply formula **104.**

$$P = \frac{3.1416 \times 36.56}{42} = 2.7347 \text{ in.} \quad \text{Ans.}$$

(1097) See Art. **1885.** $4,000 \times 45 = 400 \times$ the force. Hence, force $= \frac{4,000 \times 45}{400} = 450$ lb. Ans.

(1098) See Arts. **1889** and **1895.**

(1099) One foot of the wire will weigh $(\frac{1}{16})^2 \times .7854 \times 12 \times .303 = .011155$ lb. (See table of Weights per Cubic Foot.) Consequently, 10 lb. will contain $\frac{10}{.011155} = 896$ ft., nearly. Ans.

(1100) 14 ft. = 168 in. Applying formula **114,**

$$B = 3\tfrac{1}{4} \times \frac{18 + 14}{2} + 2 \times 168 = 388 \text{ in.} = 32 \text{ ft. } 4 \text{ in.} \quad \text{Ans.}$$

(1101) 30 miles per hour = $5,280 \times 30 = 158,400$ ft. per hour $= \frac{158,400}{60} = 2,640$ ft. per min. $= \frac{2,640}{60} = 44$ ft. per sec. Ans.

(1102) $\frac{28}{12} \times 1{,}500 = 3{,}500$ ft. in 6 min. $= \frac{3{,}500}{360} = 9\frac{13}{18}$ ft. per sec., since 6 min. $= 6 \times 60 = 360$ sec. **Ans.**

(1103) See Art. **1833.**

(1104) Apply formula **97.**

$$d = \frac{40 \times 120}{160} = 30 \text{ in.} \quad \text{Ans.}$$

(1105) See Arts. **1871** and **1872.**

(1106) See Arts. **1876** and **1877.**

(1107) The weight which comes on the block and tackle is the same as the force required to pull the body up the plane, or is equal to $\frac{50{,}000 \times 125}{1{,}200} = 5{,}208\frac{1}{3}$ lb. Since there are 12 parts to the rope, the force required to be exerted on the free end is $5{,}208 \div 12 = 434$ lb. **Ans.**

(1108) See Art. **1898.**

(1109) See Arts. **1901** to **1905.**

(1110) 19 ft. 3 in. = 231 in. Applying formula **114,**

$$B = 3\tfrac{1}{4} \times \frac{20 + 8}{2} + 2 \times 231 = 507\tfrac{1}{2} \text{ in.} = 42 \text{ ft. } 3\tfrac{1}{2} \text{ in.} \quad \text{Ans.}$$

(1111) (*a*) 15 miles per hour $= \frac{15 \times 5{,}280}{60 \times 60} = 22$ ft. per sec. Since the bodies are moving in opposite directions, they are moving *away* from each other, and their distance apart is increasing at the constant rate of $11 + 22 = 33$ ft. per sec. In 8 min. the distance between them will be $\frac{33 \times 8 \times 60}{5{,}280} = 3$ miles. **Ans.**

(*b*) $825 \div 33 = 25$ sec. **Ans.**

(1112) 2 min. 10 sec. = 130 sec. 2 miles = 10,560 ft. Applying formula **90,**

$$v = \frac{10{,}560}{130} = 81.23 \text{ ft. per sec.} \quad \text{Ans.}$$

(1113) See Art. **1838.**

(1114) Applying formula **95,** letting P represent the required force,

$$P \times 30 \times 20 \times 10 \times 15 = 1{,}250 \times 6 \times 5 \times 4 \times 7,$$

or $$P = \frac{1{,}250 \times 6 \times 5 \times 4 \times 7}{30 \times 20 \times 10 \times 15} = 11\tfrac{2}{3} \text{ lb. Ans.}$$

(1115) Applying formula **98,**

$$n = \frac{20 \times 150}{16} = 187\tfrac{1}{2} \text{ rev. per min. Ans.}$$

(1116) See Art. **1872.**

(1117) Apply formula **105.**

$$T = \frac{60 \times 40}{100} = 24, \text{ the number of teeth. Ans.}$$

(1118) See Art. **1885.** $\frac{750 \times 50}{80} = 468\tfrac{3}{4}$ lb. Ans.

(1119) Substituting in formula **112,**

$$F = .00034 \times 128 \times \frac{8\tfrac{3}{4}}{12} \times 180^2 = 1{,}028.16 \text{ lb. Ans.}$$

(1120) One cubic foot of water weighs 62.5 lb.; hence, 20 cu. ft. weigh 62.5 × 20 = 1,250 lb. The work done = 1,250 × 50 = 62,500 ft.-lb. Ans.

(1121) Arc of contact $= \frac{21}{15 \times 3.1416} \times 360° = 160°$. $800 + 3(180 - 160) = 860$. Applying formula **116,**

$$H = \frac{5 \times 1{,}960}{860} = 11.4 \text{ H. P. Ans.}$$

(1122) 18,000 + 10,000 = 28,000 lb. = the load which the screw must overcome.

Using formula **111,**

$$P = \frac{\tfrac{1}{3} \times 28{,}000}{6.2832 \times 15} = 99 \text{ lb., nearly. Ans.}$$

(1123) $\frac{9 \times 3.1416 \times 100 \times 60}{5,280} = 32.13$ miles per hour = the velocity. Applying formula **91**,

$$s = 32.13 \times 1\frac{1}{4} = 40.16\frac{1}{4} \text{ miles. Ans.}$$

(1124) If the ball fitted the gun loosely and the gun was held horizontally, the ball would roll out and fall to the floor, since, according to the first law of motion, every body tends to preserve its velocity unless acted upon by some force. The ball has a velocity due to the train of 100 ft. per sec. When the gun is fired, the force applied to the ball apparently gives it a velocity of 100 ft. per sec. in the opposite direction, but it really stops the ball and brings it to rest relatively to a point on the earth. The gun and car keep up their motion and draw away from the ball, which is stationary with respect to a point on the earth, and the ball falls to the ground.

(1125) $30 \times 14\frac{1}{2} \times 2 = 870$. $870 \div 5 = 174$ lb. Ans.

(1126) Substituting in formula **98**,

$n = \frac{42 \times 108}{36} = 126$ revolutions per minute of the countershaft. Ans.

(1127) See Art. **1872.**

(1128) Apply formula **106.**

$$t = \frac{34 \times 360}{170} = 72 \text{ teeth. Ans.}$$

(1129) Answer from your own observation.

(1130) The number of foot-pounds of work done in 1 minute is

$$10^2 \times .7854 \times 41.38 \times \frac{14}{12} \times 450 = 1,949,991 \text{ ft.-lb.}$$

Dividing by 33,000 to obtain the horsepower,

$$\frac{1,949,991}{33,000} = 59.091 \text{ horsepower, nearly. Ans.}$$

(1131) Since the width of a double belt is but $\frac{2}{3}$ of that of a single belt to transmit the same horsepower, a single belt doing the same work as the 20-inch double belt in this example must be $20 \div \frac{2}{3} = 20 \times \frac{3}{2} = 30$ inches wide.

$$\text{Arc of contact} = \frac{5.75}{4 \times 3.1416} \times 360^\circ = 165^\circ.$$

$$800 + 3\,(180 - 165) = 845.$$

Applying formula **116,**

$$H = \frac{30 \times 2{,}800}{845} = 99.4 \text{ H. P.} \quad \text{Ans.}$$

MECHANICS.

(PART 2.)

(1132) That force which will produce the same final effect upon a body as all the other forces acting separately or together.

(1133) This example is solved by the parallelogram of forces, as in Art. **1917.** Measuring the diagonal, the total pressure on the shaft is found to be $7\frac{1}{4}$ tons, nearly. Ans.

(1134) See Arts. **1932** and **1933.**

(1135) Applying formula **122,**

$$W = 12{,}000 \times (\tfrac{3}{8})^2 = 1{,}687.5 \text{ lb.} \quad \text{Ans.}$$

(1136) Apply formula **125,** and use 1,000 instead of 600, as the rope is of steel.

$$W = 1{,}000 \times (5\tfrac{1}{4})^2 = 27{,}562.5 \text{ lb.} \quad \text{Ans.}$$

(1137) Applying formula **132,**

$$\text{force} = 6^2 \times .7854 \times 60{,}000 = 1{,}696{,}464 \text{ lb.} \quad \text{Ans.}$$

(1138) (*a*) If a 5-inch line = 20 lb., a 1-inch line = 4 lb. $1 \div 4 = \frac{1}{4}$ inch = 1 lb. Ans.

(*b*) $6\frac{1}{4} \div 4 = 1.5625$ inches $= 6\frac{1}{4}$ lb. Ans.

(1139) See Art. **1964.**

(1140) The method of obtaining the resultant is shown in Fig. 37. The forces are laid off to scale to form a polygon, and the closing line gives the direction and magnitude of the resultant. See Art. **1918.**

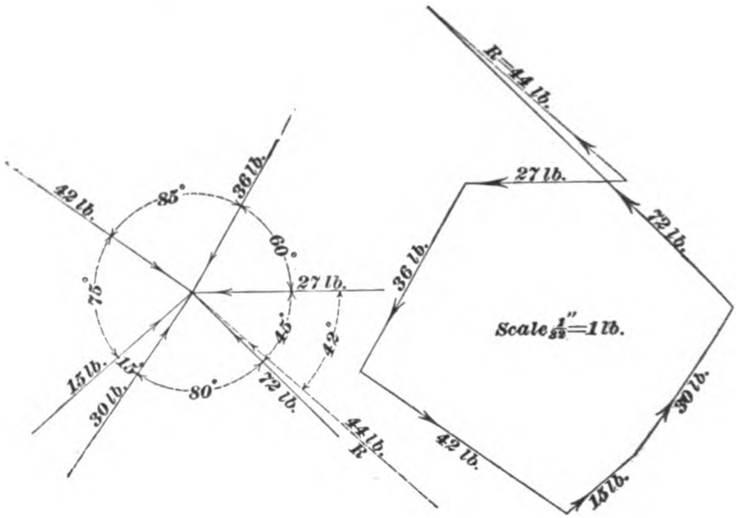

FIG. 87.

(1141) Apply formula **120.**

$$S = \frac{12,400}{3.5} = 3,543 \text{ lb. per sq. in., nearly.} \quad \text{Ans.}$$

(1142) Apply formula **123.**

$$W = 100 \times 4^2 = 1,600 \text{ lb.} \quad \text{Ans.}$$

(1143) Area of cross-section $= 8^2 \times .7854 = 50.2656$ sq. in. 10 ft. = 120 in. = L. Crushing strength = 3.5 tons per sq. in. (see Table 33). $a = 187.5$ (see Table 36). Substituting these values in formula **127,**

$$W = \frac{3.5 \times 50.2656}{\frac{120^2}{187.5 \times 8^2} + 1} = 80 \text{ tons, very nearly.}$$

Hence, $80 \div 6 = 13\frac{1}{3}$ tons = safe load. Ans.

(1144) Those forces by which the given force may be replaced, and which will produce the same effect on a body as the given force.

(1145) Apply formula **119.**

$$A = \frac{12{,}000}{5{,}000} = 2.4 \text{ sq. in., the area of the bolt.}$$

$$\text{Diameter} = \sqrt{\frac{2.4}{.7854}} = 1.74 \text{ in.} +. \quad \text{Ans.}$$

(1146) Using formula **125,** with a constant of 1,000 for steel wire,

$$W = 1{,}000 \times 4.75^2 = 22{,}562.5 \text{ lb.} \quad \text{Ans.}$$

(1147) First calculate the load it will sustain in the middle by means of formula **130.**

$$\text{Load in middle} = \frac{4 \times 10^2 \times 8 \times 30}{28} = 3{,}428\tfrac{4}{7} \text{ lb.}$$

$$\text{Uniform load} = 3{,}428\tfrac{4}{7} \times 2 = 6{,}857\tfrac{1}{7} \text{ lb.} \quad \text{Ans.}$$

(1148) Apply formula **133.** From Table 40, the proper constant is 70.

$$\text{Horsepower} = \frac{10^3 \times 200}{70} = 2{,}857\tfrac{1}{7}. \quad \text{Ans.}$$

(1149) Southeast in the direction of the diagonal of a square. See Fig. 38.

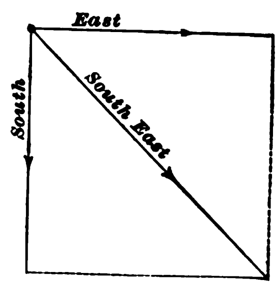

FIG. 38.

(1150) Total pressure on the head $= 19^2 \times .7854 \times 180 = 51{,}035$ lb.

Tension in each stud $= \frac{51{,}035}{14} = 3{,}645$ lb. Applying formula **119,**

$$A = \frac{3{,}645}{5{,}000} = .729 \text{ sq. in., the area of the bolt.} \quad \text{Ans.}$$

(1151) Apply formula **124.**

$$\text{Circumference} = .1\sqrt{4{,}200} = 6.48 \text{ in., say } 6\tfrac{1}{2} \text{ in.} \quad \text{Ans.}$$

(1152) Substitute in formula **127.** For this case, $C = 18$, $S = 6 \times 2\frac{1}{2} = 15$ sq. in., $L = 10 \times 12 = 120$ in., $a = 1{,}500$, and $d = 2\frac{1}{2}$ in. Consequently,

$$W = \frac{18 \times 15}{\dfrac{120^2}{1{,}500 \times 2.5^2} + 1} = 106.467 \text{ tons.}$$

$$\frac{106.467}{6} = 17.7445 \text{ tons} = 35{,}489 \text{ lb. Ans.}$$

(1153) $2\frac{7}{16}$-inch shafting. See Art. **1964.**

(1154) See Fig. 39.

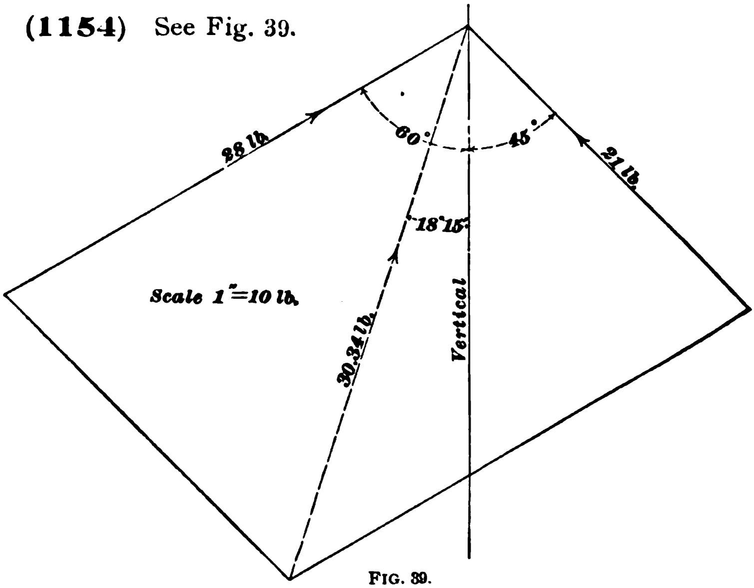

FIG. 39.

(1155) See Fig. 40. By trigonometry, $bc = 87 \times \sin 23° = 87 \times .39073 = 33.994$ lb. $ac = 87 \times \cos 23° = 87 \times .92050 = 80.084$ lb.

(1156) Apply formula **121.**

$W = 18{,}000 \times .5^2 = 4{,}500$ lb., the load. Ans.

(1157) Applying formula **126,**

$C = .0408\sqrt{14{,}000} = 4.83$ in., the circumference, nearly. Ans.

(1158) Applying formula **131**,

$$\text{Load} = \frac{4 \times 2^2 \times .6 \times 150}{6} = 480 \text{ lb. Ans.}$$

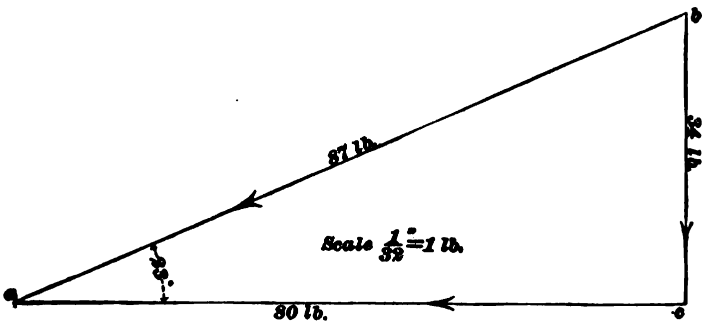

FIG. 40.

(1159) Apply formula **135.** The constant for cast iron is 90 (see Table 40).

$$\text{Diameter} = \sqrt[3]{\frac{90 \times 1{,}000}{80}} = 10.4 \text{ in. Ans.}$$

(1160) See Fig. 41.

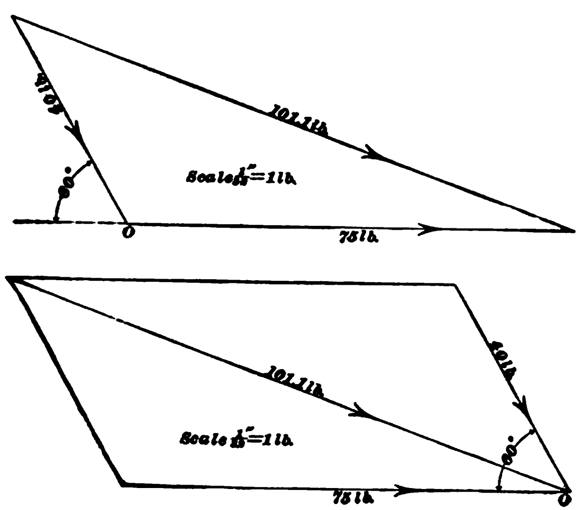

FIG. 41.

(1161) See Fig. 42.

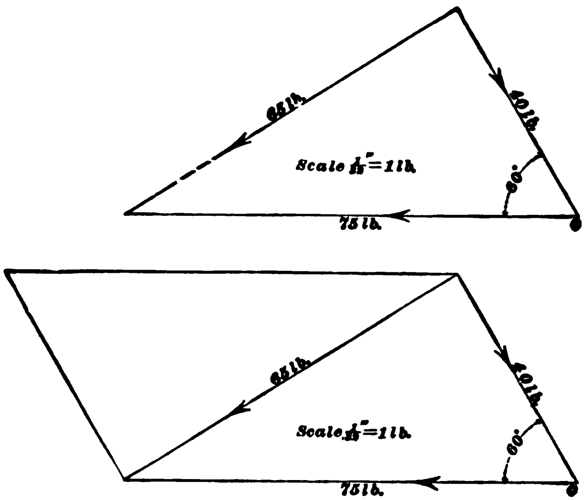

FIG. 42.

(1162) $46 - 27 = 19$ lb., acting in the direction of the force of 46 lb. Ans.

(1163) Area of cross-section $= 1\frac{3}{4} \times 3 = 5.25$ sq. in. Applying formula **118**,

$$W = 5.25 \times 6{,}000 = 31{,}500 \text{ lb., the safe load.} \quad \text{Ans.}$$

(1164) Apply formula **124.**

$$\text{Circumference} = .1\sqrt{W} = .1\sqrt{2{,}400} = 4.9 \text{ in.} \quad \text{Ans.}$$

(1165) See Fig. 43.

(1166) The graphical construction is clearly shown in Fig. 44.

(1167) See Arts. **1926** to **1928.**

(1168) Apply formula **121.**

$W = 18{,}000 \times (\frac{13}{16})^2 = 11{,}883$ lb., the greatest safe load. Ans.

(1169) Apply formula **126,** and use .0316 instead of .0408, since the rope is of steel.

$$C = .0316\sqrt{8{,}000} = 2.83 \text{ in.} \quad \text{Ans.}$$

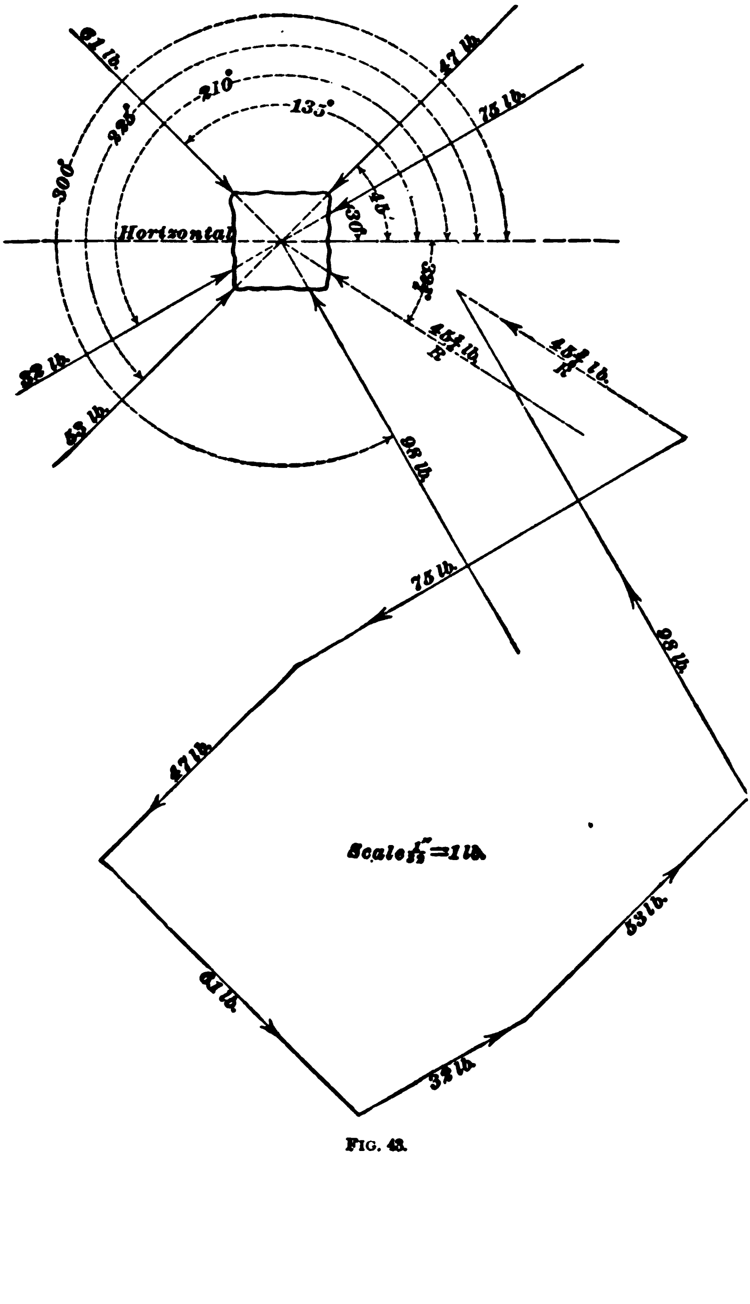
61 lb.
47 lb.
75 lb.
210°
135°
225°
300°
45°
30°
Horizontal
32 lb.
53 lb.
98 lb.
75 lb.
98 lb.
47 lb.
Scale $\frac{1}{32}''$ = 1 lb.
53 lb.
61 lb.
32 lb.

FIG. 43.

(1170) Apply formula **130,** and multiply the result by 2.

$$W = \frac{4 \times 6^2 \times 2 \times 160}{20} \times 2 = 4{,}608 \text{ lb., the load. Ans.}$$

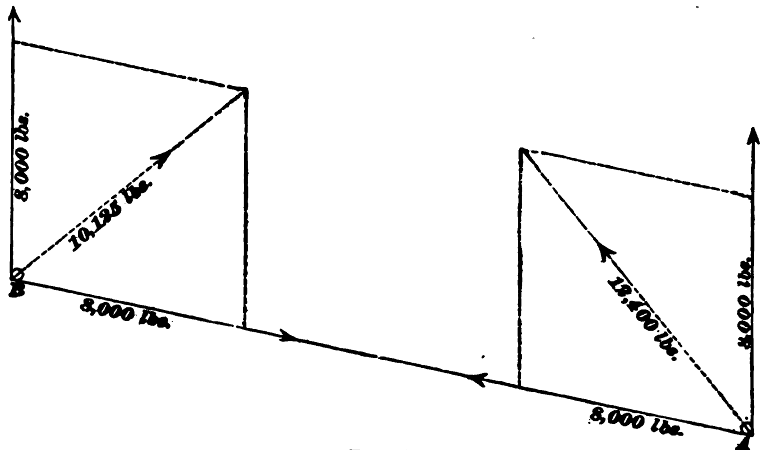

FIG. 44.

(1171) Apply formula **134.**

$$R = \frac{85 \times 80}{4^2} = 106\tfrac{1}{4} \text{ revolutions per minute. Ans.}$$

(1172) See Arts. **1929** to **1931.**

(1173) Apply formula **122.**

$$W = 12{,}000 \times (\tfrac{5}{8})^2 = 4{,}687.5 \text{ lb. Ans.}$$

(1174) Apply formula **125.**

$$W = 600 \times 6^2 = 21{,}600 \text{ lb. Ans.}$$

(1175) 4 ft. = 48 in. Area to be sheared = $48 \times \frac{1}{2} =$ 24 sq. in. Applying formula **132,**

$$W = 24 \times 40{,}000 = 960{,}000 \text{ lb., the force required. Ans.}$$

(1176) Applying formula **134,**

$$\frac{70 \times 200}{7^2} = 40.8 \text{ revolutions per minute, nearly. Ans.}$$

(1177) See Art. **1963.** Area to be punched $= 1 \times 3.1416 \times \frac{7}{16} = 1.37445$ sq. in. Applying formula **132,**

$$\text{force} = 1.37445 \times 40{,}000 = 54{,}978 \text{ lb. Ans.}$$

(1178) Apply formula **133.**

$$H = \frac{(1\frac{3}{4})^2 \times 180}{95} = 12.49. \quad \text{Ans.}$$

(1179) Total pressure against the head $= 44^2 \times .7854 \times 100 = 152{,}053.44$ lb. Applying formula **119,**

$$\text{area of studs} = \frac{152{,}053.44}{5{,}000} = 30.41 \text{ sq. in., nearly.}$$

$$30.41 \div 1.057 = 29 \text{ studs. Ans.}$$

(1180) Apply formula **125.**

$$\text{Load} = 600 \times 4^2 = 9{,}600 \text{ lb. Ans.}$$

(1181) Apply formula **128.**

$$\text{Load} = \frac{2.5^2 \times 1.5 \times 100}{4\frac{8}{12}} = 201 \text{ lb., nearly. Ans.}$$

(1182) Apply formula **133.**

$$\text{Horsepower} = \frac{(2\frac{7}{16})^3 \times 120}{85} = 20.445. \quad \text{Ans.}$$

(1183) Area of cross-section $= (1\frac{1}{2})^2 \times .7854 = 1.7671$ sq. in. Apply formula **118.**

$$\text{Safe steady load} = 12{,}000 \times 1.7671 = 21{,}205.2 \text{ lb. Ans.}$$

(1184) Apply formula **123.**

$$W = 100 \times 6^2 = 3{,}600 \text{ lb., the safe load. Ans.}$$

(1185) Substituting the values of $C = 40$, $S = 14^2 \times .7854 - 11.5^2 \times .7854 = 50.0693$, $L = 20 \times 12 = 240$, $a = 562.5$, and $d = 14$ in formula **127,** we have

$$W = \frac{40 \times 50.0693}{\frac{240^2}{562.5 \times 14^2} + 1} = \frac{2{,}002.772}{1.5225} = 1{,}315.45 \text{ tons.}$$

$$\frac{1{,}315.45}{6} = 219.24 \text{ tons. Ans.}$$

(1186) See Art. **1963.** Area punched $= 1\frac{1}{2} \times 3.1416 \times \frac{3}{4} = 3.5343$ sq. in. Force $= 3.5343 \times 60{,}000 = 212{,}058$ lb. Ans.

(1187) See Fig. 45.

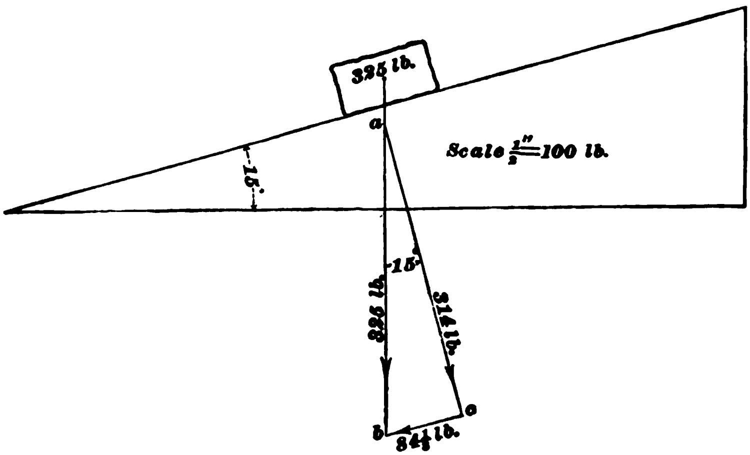

FIG. 45.

(*a*) $ac = 325 \times \cos 15° = 325 \times .96593 = 313.93$ lb. Ans.

(*b*) $bc = 325 \times \sin 15° = 325 \times .25882 = 84.12$ lb. Ans.

STEAM AND STEAM-BOILERS.

(1188) See Arts. **1970** to **1977.**

(1189) See Arts. **1972** to **1974.**

(1190) Less. See Art. **1982.**

(1191) See Arts. **1975** to **1977.**

(1192) (*a*) See Art. **1979.**

(*b*) and (*c*) See Art. **1978.**

(*d*) See Art. **1982.**

(1193) See Arts. **1984** to **1986.**

(1194) (*a*) and (*b*) See Arts. **1980** and **1981.**

(*c*) 1 B. T. U. = 778 ft.-lb.

$30\frac{1}{2}$ B. T. U. = $30\frac{1}{2} \times 778 = 23,729$ ft.-lb. Ans.

(1195) 35 H. P. = $35 \times 33,000$ ft.-lb. per min. = $35 \times 33,000 \times 60$ ft.-lb. per hour = $\frac{35 \times 33,000 \times 60}{778}$ B. T. U. per hour = 89,074.5 B. T. U. per hour.

But this is the heat actually used, or 20% of the whole. Hence, the heat required is $89,074.5 \div .20 = 445,372.5$ B.T.U. per hour. Ans.

(1196) One horsepower = $33,000 \times 60$ ft.-lb. per hour = $\frac{33,000 \times 60}{778}$ B. T. U. per hour.

Each pound of coal gives 14,000 B. T. U., of which 8%, or $14,000 \times .08 = 1,120$ B. T. U., is utilized. Hence, the coal required per hour per H. P. is

$$\frac{33,000 \times 60}{778} \div 1,120 = 2.27 \text{ lb.} \quad \text{Ans.}$$

(1197) The specific heat of sulphur is .2026. (See Table 41.) By formula **136**, $U = c\,W(t_1 - t) = .2026 \times 22\frac{1}{2} \times (68 - 44) = 109.4$ B. T. U. Ans.

(1198) (*a*) See Art. **1984.**

(*b*) To raise the ice from 17° to 32° requires for each pound $.504 \times (32 - 17) = 7.56$ B. T. U. To melt it requires 144 B. T. U. Hence, 1 lb. requires $144 + 7.56 = 151.56$ B. T. U. 11 lb. requires $11 \times 151.56 = 1{,}667.16$ B. T. U. Ans.

(1199) By formula **136**, $U = c\,W(t_1 - t) = .4805 \times 6 \times (342 - 310) = 92.256$ B. T. U. Ans.

(1200) Using formula **137**,

$$T = \frac{w\,c\,t + w_1\,c_1\,t_1 + w_2\,c_2\,t_2 + \ldots}{w\,c + w_1\,c_1 + w_2\,c_2 + \ldots} =$$

$$\frac{18 \times .0951 \times 305 + 13 \times .1138 \times 278 + 32 \times 1 \times 56}{18 \times .0951 + 13 \times .1138 + 32 \times 1} = 77.45°. \quad \text{Ans.}$$

(1201) (*a*) 966 B. T. U. Ans.

(*b*) To raise a pound of water from 63° to 212° requires $212 - 63 = 149$ B. T. U. To change it into steam requires 966.069 more B. T. U. $966 + 149 = 1{,}115$ B. T. U. for 1 lb. Hence, $8 \times 1{,}115 = 8{,}920$ B. T. U. are required. Ans.

(1202) To change 1 lb. of ice from 23° to 32° requires $(32 - 23) \times .504 = 4.536$ B. T. U. To melt the ice requires 144 B. T. U. To change the water at 32° to water at 212° requires 180 B. T. U. per pound. To change the water at 212° into steam at 212° requires 966 B. T. U. per pound. $4.536 + 144 + 180 + 966 = 1{,}294.536$ B. T. U. per pound. For 2.2 pounds, $1{,}294.536 \times 2.2 = 2{,}847.98$ B. T. U., as required. Ans.

(1203) See Arts. **1991** to **1993.**

(1204) See Art. **2024** and Arts. **2027** to **2030.**

(1205) In the return-tubular boiler the one or two large flues are replaced by a large number of small tubes. In other respects, the boilers are quite similar in principle.

(1206) See Art. **2024.**

(1207) See Arts. **2017, 2023, 2025,** and **2028.**

(1208) See Art. **2024.**

(1209) See Art. **2023.**

(1210) See Art. **2011.**

(1211) See Art. **2011.**

(1212) (*a*) See Art. **2004.**

(*b*) The temperature at which combustion takes place is always the same for the same substance. The nitrogen reduces the temperature of the furnace, since a portion of the heat given off by combustion is required to heat the nitrogen.

(1213) No. See definition of combustion, Art. **2003.**

(1214) See Art. **2007.**

(1215) See Art. **2008.**

(1216) The number of heat units required to convert a pound of water at 32° into steam at 400° may be found by means of formula **140.**

$$H = 1{,}081.4 + .305 \times 400 = 1{,}203.4 \text{ B. T. U.} \quad \text{Ans.}$$

(1217) In order to use formula **140,** the temperature must be known. This can be found when the pressure is known, by means of formula **138.** Applying the formula, $t = 14\sqrt{175} + 199 = 384.2°$, the temperature of saturated steam having a pressure of 175 pounds per square inch. Now, using formula **140,**

$$H = 1{,}081.4 + .305 \times 384.2 = 1{,}198.6 \text{ B. T. U.} \quad \text{Ans.}$$

(1218) Since the expansion follows Mariotte's law, the final pressure may be found by the formula $p_1 = \frac{p v}{v_1}$. Substituting, $p_1 = \frac{60 \times 5}{5 \times 2.5} = 24$ lb. per sq. in. above vacuum. $24 - 14.7 = 9.3$ lb. per sq. in. above atmosphere. Ans.

(1219) From Table 42, column 5, the total heat of combustion of one pound of coal is found to be 14,133 B. T. U.

$11 \times 13 \times 5 = 715$ pounds of coal burned in 5 hours.

$14,133 \times 715 = 10,105,095$ B. T. U. generated by the combustion of the coal. Ans.

(1220) According to Table 42, the amount of air required for the complete combustion under a blast draft is found to be 14 pounds. Hence, the amount of air required for combustion of the coal in Question 1219 is

$$715 \times 14 = 10,010 \text{ pounds. Ans.}$$

(1221) The number of pounds of water having a temperature of 62° which can be converted into steam having a temperature of 212° is found, from Table 42, column 6, to be 12.67 pounds. Hence, the total quantity of water which could be evaporated under the above conditions by the combustion of 715 pounds of coal is

$$12.67 \times 715 = 9,059.05 \text{ pounds. Ans.}$$

(1222) Since the pressure is 3,600 pounds per *square foot* above a vacuum, and there are 144 square inches in a square foot, the pressure above a vacuum is $\frac{3,600}{144} = 25$ pounds per *square inch*. Consequently, the pressure per square inch above the atmosphere is $25 - 14.7 = 10.3$ pounds. Ans.

(1223) See Art. **2001.**

(1224) According to formula **138,** the required temperature is

$$t = 199 + 14 \times \sqrt{152} = 371.62° \text{ F. Ans.}$$

(1225) Applying formula **139,** we have for the required pressure $p = \left(\frac{232 - 199}{14}\right)^2 = 5.56$ pounds per square inch gauge-pressure. Ans.

(1226) 132 tons equal $132 \times 2,000 = 264,000$ pounds. $264,000 \times 296 = 78,144,000$ = foot-pounds of work necessary to raise the coal to the top of the shaft. Since 1 B. T. U. = 778 foot-pounds, the heat supplied is

$$\frac{78,144,000}{778} = 100,442.15 \text{ B. T. U.} \quad \text{Ans.}$$

(1227) $277,160 \times 778 = 215,630,480$ foot-pounds of work done by the engine in two hours.

Hence, $\frac{215,630,480}{2} = 107,815,240$ ft.-lb. done in one hour. Ans.

(1228) The strength of any construction is always that of its weakest part. In the present example the diameter and thickness of the steam and water drums only are given, the thickness of the flues, mud-drum, and boiler-shell, and the diameter of the boiler-shell being omitted. Such being the case, we must confine ourselves to the strength of the steam and water drums, assuming that the other parts of the boiler have been made correspondingly strong. The pressure which the steam-drum will safely sustain is found by formula **141** to be $\frac{16,608 \times \frac{5}{16}}{24} = 216.25$ pounds per square inch, and the pressure which the water-drum will safely sustain is found by the same formula to be $\frac{16,608 \times \frac{5}{16}}{20} = 259.5$ pounds per square inch. Since the safe pressure upon the steam-drum is less than that which can be sustained by the water-drum, the pressure on the boiler must not exceed the safe pressure which can be sustained by the steam-drum; that is, 216.25 pounds per square inch. Ans.

(1229) From Table 43, it is seen that from 14 to 18 square feet of water-heating surface are required to produce one horsepower with a return-tubular boiler. Using 16 square feet as a mean, we obtain

$$\frac{1,620}{16} = 101\frac{1}{4} \text{ H. P.} \quad \text{Ans.}$$

(1230) In the same manner as in example 1229, it is found that about 11 square feet of heating-surface are required to produce 1 horsepower with a water-tube boiler. Hence,

$$\text{H. P.} = \frac{3{,}025}{11} = 275 \text{ horsepower. Ans.}$$

(1231) Applying formula **142,** the height of the chimney is found to be

$$h = \left[\frac{348}{3.33 \times 12 - 2\sqrt{12}}\right]^2 = \left[\frac{348}{3.33 \times 12 - (2 \times 3.464)}\right]^2 =$$

111 ft. Ans.

(1232) The dome and the dry-pipe. See Arts. **2022** and **2023.**

(1233) See Art. **2019.**

(1234) Blow-off pipes are provided to remove the collected sediment. The boiler is also provided with manholes or handholes for cleaning purposes.

(1235) See Art. **2013.**

(1236) To avoid overheating and burning out the upper plates of the furnace. So long as the water is in contact with the plates which are next to the fire, they can not be overheated or burned.

(1237) See Art. **2023.**

(1238) See Art. **2017.**

(1239) See Art. **2018.**

(1240) Answer from the result of your own observations.

(1241) The steam-pipe conveys the steam after it is generated from the boiler to the place where it is used. The feed-water pipe is the one through which the water is introduced to the boiler. A blow-off pipe is one attached to the lower part of the boiler or to a mud-drum. It is used to empty the boiler of the whole or a part of its contents.

(1242) See Art. **2012.**

(1243) The arm of the safety-valve is a lever in which the power is the total steam-pressure on the valve, $6 \times 81 =$ 486 pounds. The power arm is 2 inches, and the weight is 54 pounds. Calling the weight arm b, we have, from formula **94**,

$$Pa = Wb, \text{ or } 486 \times 2 = 54 \times b.$$

Hence, $$b = \frac{486 \times 2}{54} = 18 \text{ in. Ans.}$$

(1244) According to formula **141**, $p = \frac{10,224 \times \frac{3}{8}}{30} =$ 127.8 pounds per square inch, the greatest pressure under which it would be safe to operate a boiler of these dimensions.

(1245) See Art. **2033.**

(1246) (*a*) See Art. **2035.**

(*b*) The top and sides of the furnace, and the tubes.

(1247) Using formula **141**, $p = \frac{13,152 \times \frac{5}{16}}{45} = 91\frac{1}{3}$ lb. per sq. in., the safe working pressure. Therefore it would be unsafe to use 110 lb. pressure.

(1248) (*b*) According to Table 43, a vertical boiler has from 15 to 20 square feet of heating-surface per horsepower. Assuming 18 sq. ft. per H. P., the heating-surface will be $35 \times 18 = 630$ square feet. Ans.

(*a*) Since the heating-surface is 25 to 30 times the grate area, the latter must lie between $\frac{630}{25} = 25.2$ sq. ft. and $\frac{630}{30} =$ 21 sq. ft.; say about 23 sq. ft. Ans.

(*c*) One horsepower is equivalent to an evaporation of 30 pounds of water per hour, the feed being at 100° and the steam-pressure at 70 pounds. The evaporation per hour is, therefore, $35 \times 30 = 1,050$ lb. Ans.

STEAM-ENGINES.

(1249) The stationary parts of a plain slide-valve engine are the steam-cylinder, steam-chest, supply-pipe, exhaust-pipe, guide-bars, shaft-bearings, and the bed or frame of the engine.

(1250) The expansion curve of steam on an indicator-card represents the decrease of pressure of the steam after cut-off, corresponding to the increase of volume.

(1251) It passes its central position during the interval between the point of release of the steam from the head end of the cylinder, and the point of compression of the steam in the crank end of the cylinder, during the forward stroke of the piston, and conversely for the backward stroke.

(1252) Plain slide-valves usually cut off between one-half and full stroke.

(1253) The points of cut-off and release are marked, as shown in Figs. 46, 47, 48, and 49. The perpendicular distances from these points to the atmospheric line are measured. Multiplying the lengths of these perpendiculars by 45, the scale of the spring, we obtain

1.3750 in. × 45 = 61.8750 lb. for cut-off .5625 in. × 45 = 25.3125 lb. for release	Fig. 46.
1.3750 in. × 45 = 61.8750 lb. for cut-off .6800 in. × 45 = 30.6000 lb. for release	Fig. 47.
1.3800 in. × 45 = 62.1000 lb. for cut-off .1200 in. × 45 = 5.4000 lb. for release	Fig. 48.
1.3700 in. × 45 = 61.650 lb. for cut-off .1200 in. × 45 = 5.4000 lb. for release	Fig. 49.

To find the back-pressure, in each case, find the perpendicular distance between the lowest point of diagram and

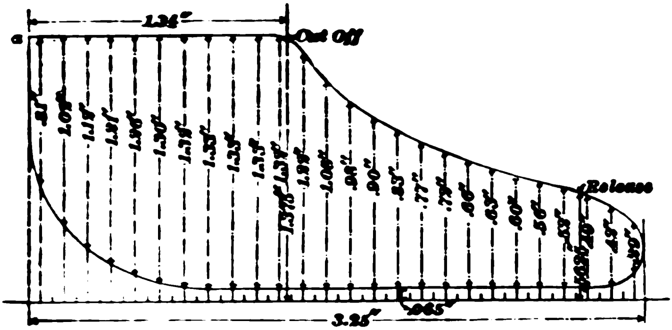

FIG. 46.

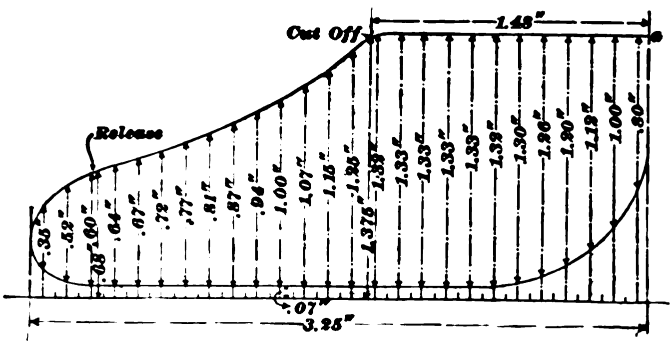

FIG. 47.

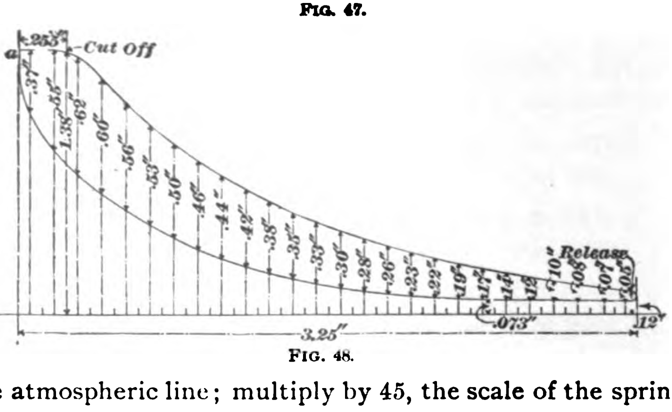

FIG. 48.

the atmospheric line; multiply by 45, the scale of the spring, and the products will be as follows:

.065 in. × 45 = 2.925 lb. back-pressure for Fig. 46.

.070 in. × 45 = 3.15 lb. back-pressure for Fig. 47.

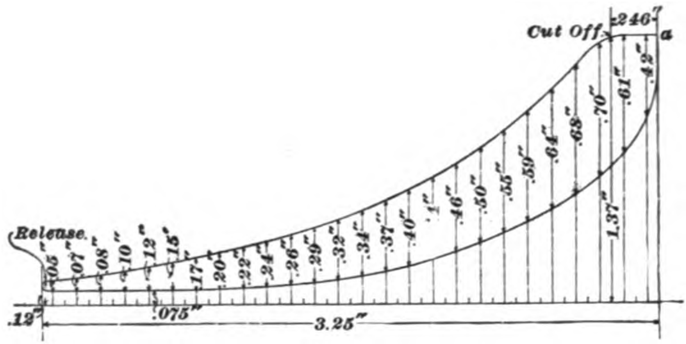

FIG. 49.

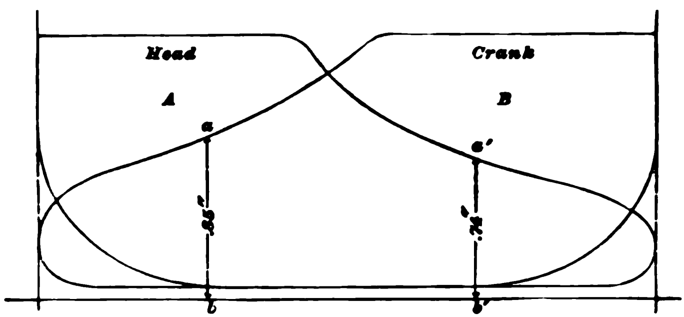

FIG. 50.

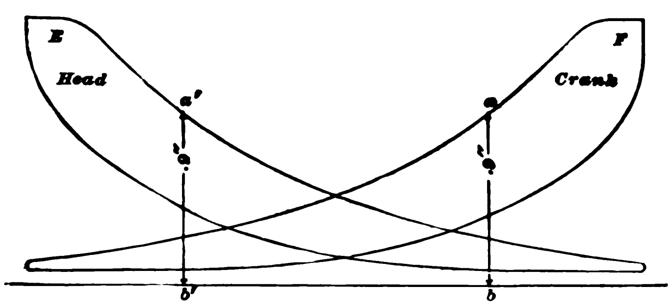

FIG. 51.

.073 in. × 45 = 3.285 lb. back-pressure for Fig. 48.

.075 in. × 45 = 3.375 lb. back-pressure for Fig. 49.

To determine the steam-pressure in the cylinder at the point of compression, we must combine diagrams *A* and *B* and *E* and *F*. These diagrams are combined by placing *B* upon *A* and *F* upon *E*, the atmospheric and extreme right and left hand lines coinciding. The height *a b* of the diagram *B*, in Fig. 50, represents the pressure of the steam in the crank end of the cylinder at the point of compression of the diagram *A*. This is as it should be, since the compression curve is drawn by the pencil of the indicator when the piston is making its return stroke. In a similar manner, the pressure of the steam in the head end of the cylinder at the point of compression in the crank end is the height *a' b'* of *A*. In Fig. 51 the height *ab* represents the pressure at compression for *E*, and *a' b'* the same for *F*. These results tabulated are as follows:

.85 in. × 45 = 38.25 lb. for *A*.

.74 in. × 45 = 33.30 lb. for *B*.

.90 in. × 45 = 40.50 lb. for *E*.

.90 in. × 45 = 40.50 lb. for *F*.

(1254) Project the extreme right and left hand points of the indicator-diagrams upon the atmospheric line; divide the distance between them into any number of equal spaces—26 in this case—and through the centers of these spaces draw lines across the diagram perpendicular to the atmospheric line. Now measure the length in inches of each of these perpendicular lines (the lengths are given in all the figures), and take their sum; divide this sum by the number of the equal spaces into which the atmospheric line is divided, and multiply the quotient by the scale of the spring.

Sum of the perpendiculars of the diagram of Fig. 46 = 24.02 in.; then,

$$\frac{24.02}{26} \times 45 = 41.573 \text{ lb., M. E. P.}$$

Sum of the perpendiculars of the diagram, Fig. 47, = 26 in.; then,

$$\tfrac{26}{26} \times 45 = 45 \text{ lb., M. E. P.}$$

The average M. E. P. for both diagrams is

$$\frac{41.573 + 45}{2} = 43.29 \text{ lb. per sq. in. Ans.}$$

(1255) Sum of the perpendiculars of the diagram, Fig. 48, = 8.32 in.; then,

$$\frac{8.32}{26} \times 45 = 14.40 \text{ lb., M. E. P.}$$

Sum of the perpendiculars of the diagram, Fig. 49, = 8.97; then,

$$\frac{8.97}{26} \times 45 = 15.525 \text{ lb., M. E. P.}$$

The average M. E. P. for the two diagrams is

$$\frac{14.40 + 15.525}{2} = 14.96 \text{ lb. per sq. in. Ans.}$$

(1256) Area of 15-inch piston $= 15^2 \times .7854 = 176.715$ square inches.

Using formula **143**,

$$\text{I. H. P.} = \frac{43.29 \times 2 \times 176.715 \times 175}{33{,}000} = 81.14 \text{ I. H. P. Ans.}$$

(1257) Proceeding as in example 1256,

$$\text{I. H. P.} = \frac{14.96 \times 2 \times 176.715 \times 175}{33{,}000} = 28.04 \text{ I. H. P. Ans.}$$

(1258) The actual horsepower is $81.14 - 28.04 = 53.1$ H. P. Ans.

Applying formula **146**, the efficiency is

$$\frac{53.1}{81.14} = .654 = 65.4 \text{ per cent. Ans.}$$

(1259) The force of gravity and the centrifugal force.

(1260) See Art. **2098.**

(1261) The piston, piston-rod, cross-head, connecting-rod, crank, crank-shaft, eccentric, eccentric-rod, slide-valve, and fly-wheel.

(1262) In order that the energy stored in them may be utilized in carrying the crank over its dead-center position, and also to cause the engine to run at a more uniform speed.

(1263) Compression is taking place. See Figs. 50 and 51.

(1264) Any portion added to the length of a valve more than is absolutely necessary, in order to cover the outside edges of the steam-ports when the valve is in its central position, is called the outside lap of the valve. It is added to enable the valve to cut off the live steam before the piston reaches the end of its stroke.

(1265) Apply rule, Art. **2059.** Cut-off in the diagram, Fig. 807, takes place at a point 1.34 inches from *a*. See Fig. 46.

Therefore, cut-off equals $\frac{1.34}{3.25}$, or 41% of stroke.

Cut-off in the diagram, Fig. 808, takes place at a point 1.48 inches from *a*. See Fig. 47.

Therefore, cut-off equals $\frac{1.48}{3.25}$, or 46% of stroke, nearly.

Cut-off in the diagram, Fig. 809, takes place at a point .255 inch from *a*. See Fig. 48.

Therefore, cut-off equals $\frac{.255}{3.25}$, or 7.8% of stroke.

Cut-off in the diagram, Fig. 810, takes place at a point .246 inch from *a*. See Fig. 49.

Therefore, cut-off equals $\frac{.246}{3.25}$, or 7.6% of stroke.

In each case the length of the diagram is 3.25 inches.

(1266) The indicated horsepower of this engine will be about one-half greater than the actual horsepower, or $\frac{65}{2} + 65 = 97.5$ horsepower. See example, Art. **2077.**

A fair piston speed is 500 feet per minute.

Assume the cut-off to be taken at $\frac{3}{4}$ and the boiler-pressure to be 70 pounds per square inch. Applying formula **144**, the M. E. P. $= .9\,[.937\,(70 + 14.7) - 17] = 56.13$ pounds per square inch. Letting d = diameter of cylinder,

$$\text{I. H. P.} = \frac{d^2 \times .7854 \times 56.13 \times 500}{33{,}000} = 97.5, \text{ or}$$

$$d = \sqrt{\frac{97.5 \times 33{,}000}{.7854 \times 56.13 \times 500}} = 12.08 \text{ inches, or say } 12 \text{ inches.}$$

Taking the ratio of stroke to diameter of cylinder as $1\frac{1}{2}$, we have stroke $= 12 \times 1\frac{1}{2} = 18$ inches. The number of revolutions of the crank is

$$\frac{500 \times 6}{18} = 166\frac{2}{3} \text{ revolutions per minute.}$$

(1267) A combination of two single-cylinder engines of exactly the same description and dimensions, which have their cranks rigidly connected to a common crank-shaft and take the steam at the same pressure, is called a *duplex* engine.

Compound engines are those having two cylinders, of which the working lengths are usually the same, but the diameter of one, the high-pressure cylinder, is less than that of the other, the low-pressure cylinder, and the steam, instead of entering both cylinders at boiler-pressure, enters first the high-pressure cylinder, and is exhausted from there into the low-pressure cylinder

(1268) One in which the cylinder is in a vertical or upright position.

(1269) The stroke of an engine is the distance passed over by the piston when moving from one end of the cylinder to the other end, and is equal to the *throw of the crank*, or to the diameter of the circle described by the center of the crank-pin.

(1270) An eccentric is a disk, or wheel, so arranged upon a shaft that the center of the wheel and that of the shaft do not coincide. It is equivalent to a crank having the same throw, and is used to give motion to the slide-valve.

(1271) It is the period during which the steam remaining in the cylinder after the exhaust-valve has closed is compressed as the piston continues the return stroke. It begins at the instant that the valve closes the port to the exhaust-steam.

(1272) It shortens the period of release and lengthens both the period of expansion and compression.

(1273) It permits an earlier cut-off, together with a greater range and more perfect steam distribution.

(1274) Using formula **143**,

$$\text{I. H. P.} = \frac{62.4 \times .7854 \times 18^2 \times \frac{24}{12} \times 2 \times 175}{33{,}000} =$$

336.825 I. H. P. Ans.

(1275) By setting the cranks at right angles, both engines can not be on a dead-center at the same time.

(1276) See Arts. **2097** and **2098.**

(1277) By the *bore* of a cylinder is meant its diameter.

(1278) Steam is called live steam when it leaves the boiler and before doing any work in the cylinder. The energy stored in the live steam is potential energy.

(1279) The fly-wheel supplies the force necessary to overcome the retarding effect of compression.

(1280) (*a*) The dead-center positions occur when the piston reaches the end of its stroke, and the centers of the cross-head pin, crank-pin, and crank-shaft are in the same straight line.

(*b*) Twice.

(1281) A steam-engine indicator is an instrument which draws a diagram showing the pressure of the steam in the cylinder at every point of the stroke. See Fig. 679 for method of fastening to cylinder.

(1282) See Art. **2097.**

(1283) See Art. **2039.**

(1284) It is the resistance against being pushed into the condenser or the atmosphere which the exhaust-steam exerts on the piston.

(1285) By period of release is meant that period during which the steam is escaping into the atmosphere or condenser. The point of compression marks the end of release.

(1286) See Art. **2049.**

(1287) Two. One spring is to resist any upward motion of the indicator-piston, and the other is to carry the drum back to its first position when the pull on the cord is discontinued.

(1288) The back-pressure line would then fall below its present position a distance represented by a pressure of $\frac{5}{6} \times 14.7 = 12\frac{1}{4}$ pounds $= \frac{12\frac{1}{4}}{45} = .27$ inch, nearly. Then, for the same mean effective pressure, the cut-off would be earlier.

(1289) See Art. **2091.**

(1290) See Art. **2039.**

(1291) See Art. **2042.**

(1292) Release is taking place.

(1293) The varying pressures of the steam while being compressed.

(1294) See Art. **2050.**

(1295) See Art. **2055.**

(1296) See Art. **2055.**

(1297) See Art. **2078.**

(1298) See Art. **2039.**

(1299) See Art. **2050.**

(1300) At the end. See Art. **2045.**

(1301) See Arts. **2048** and **2050.**

(1302) See Art. **2055.**

(1303) Using formula **144** and the constants in Table 44, the M. E. P. for $\frac{3}{10}$ cut-off is

$$.9[.708(75+14.7)-17]=41.86 \text{ lb. per sq. in.} \quad \text{Ans.}$$

For cut-off at $\frac{1}{2}$ stroke,

$$\text{M. E. P.}=.9[.847(75+14.7)-17]=53.16 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(1304) See Art. **2080.**

(1305) See Art. **2092.**

(1306) See Art. **2039.**

(1307) See Art. **2044.**

(1308) Closed. See Art. **2045** and Fig. 670 (*a*).

(1309) See Art. **2045.**

(1310) Using formula **145,**

$$S=\frac{lR}{6} \text{ or } l=\frac{6S}{R}=\frac{350\times 6}{175}=12 \text{ inches.} \quad \text{Ans.}$$

(1311) See Art. **2080.**

(1312) See Arts. **2094** and **2098.**

(1313) See Art. **2039.**

(1314) See Art. **2043.**

(1315) See Art. **2052.**

(1316) Applying the rule, Art. **2056,**

$$\text{length}=\frac{96\times 3}{12}=24 \text{ inches.} \quad \text{Ans.}$$

(1317) Applying formula **145,**

$$S=\frac{lR}{6}=\frac{48\times 50}{6}=400 \text{ feet per minute.} \quad \text{Ans}$$

(1318) See Art. **2085.**

(1319) See Arts. **2096** and **2097.**

(1320) See Art. **2044.**

(1321) See Art. **2044.**

(1322) Formula **143** gives

$$\text{I. H. P.} = \frac{PLAN}{33{,}000} = \frac{43.4 \times \frac{18}{12} \times 22^2 \times .7854 \times 2 \times 200}{33{,}000} =$$

300 H. P., nearly. Ans.

(1323) See Art. **2061.**

(1324) Applying formula **145,**

$$S = \frac{lR}{6}, \text{ or } R = \frac{6S}{l};$$

therefore,

$$R = \frac{6 \times 750}{60} = 75 \text{ rev. per min.} \quad \text{Ans.}$$

(1325) See Art. **2081.**

(1326) See Art. **2095.**

AIR AND AIR COMPRESSION.

(1327) See Art. **2101.**

(1328) (*a*) 4 ft. = 48 in. A cubic inch of mercury weighs .49 lb.; hence, the pressure exerted by 48 inches of mercury = $48 \times .49 = 23.52$ lb. per sq. in. Ans.

(*b*) Since the pressure of 1 atmosphere is 14.7 lb. per sq. in., a pressure of 23.52 lb. per sq. in. is equivalent to $23.52 \div 14.7 = 1.6$ atmospheres. Ans.

(1329) A pressure of 1 atmosphere will support a column of water 34 ft. high. Since the column of water is 15 ft. high, the height of the confined air is $34 - 15 = 19$ ft., or, in other words, the tension of the confined air in pounds per square inch is equal to the weight of a column of water 1 in. square and 19 ft. high. The pressure exerted by a column of water 1 ft. high and having a cross-section of 1 sq. in. = $12 \times .03617 = .434$ lb. Hence the tension of the confined air = $.434 \times 19 = 8.246$ lb. per sq. in. Ans.

(1330) See Arts. **2117** and **2118.**

(1331) See Art. **2118.**

(1332) There would be no loss, because the air would have no opportunity to lose heat by radiation in the pipes. The heat stored in the air during compression would be available for useful work.

(1333) See Arts. **2121** and **2126.**

(1334) See Art. **2131.**

(1335) See Art. **2136.**

(1336) See Art. **2145.**

(1337) (*a*) See Art. **2113.** (*b*) More work is required to compress air adiabatically.

(1338) Applying formula **157** and substituting,

$$p_1 = p\left(\frac{459 + t_1}{459 + t}\right) = 40 \times \left(\frac{459 + 55}{459 + 120}\right) =$$

$$40 \times \frac{514}{579} = \frac{20,560}{579} = 35.51 \text{ lb. per sq. in. Ans.}$$

(1339) (*a*) 14.7 + 9 = 23.7 lb. per sq. in., the tension. Since the area of the piston remains constant, the volume at any point of the stroke will be proportional to the distance passed over by the piston; hence, we may substitute the latter for the former in formula **151.**

$v_1 = \frac{p v}{p_1} = \frac{14.7 \times 80}{23.7} = 49.62$ in., distance between piston and end of stroke. The distance passed over by piston = 80 − 49.62 = 30.38 in. Ans.

(*b*) Area of piston = $80^2 \times .7854$ sq. in.

Volume of air at point of discharge = $80^2 \times .7854 \times 49.62 =$ 249,417.91 cu. in.

$$249,417.91 \div 1,728 = 144.34 \text{ cu. ft. Ans.}$$

(1340) Since the required horsepower is 25 and the loss is 35%, the horsepower of the engine must be 25 ÷ (100% − 35%) = 25 ÷ .65 = 38.46 H. P.

To calculate the M. E. P., formula **144** may be used.

$$\text{M. E. P.} = .9\,[.904\,(92 + 14.7) - 17] = 71.5 \text{ lb. per sq. in.}$$

To find the diameter of the steam-cylinder, substitute in formula **148.**

$$D = 79.6\sqrt[3]{\frac{38.46}{1\frac{3}{8} \times 71.5 \times 340}} = 8.34;\ \text{or say } 8\tfrac{3}{8} \text{ in.}$$

Length of stroke = $8\frac{3}{8} \times 1\frac{3}{8} = 11\frac{1}{2}$ in. Consequently, the steam-cylinder should be $8\frac{3}{8} \times 11\frac{1}{2}$ in. Ans.

(1341) The steam-cylinder will show the greater I. H. P. The difference represents the horsepower required to overcome the friction of the moving parts of the compressor.

(1342) See Art. **2149.**

(1343) (*a*) Volume of cylinder $= \frac{20^2 \times .7854 \times 32}{1,728} =$ 5 8178 cu. ft.

$32 - 26 = 6$ in., length of stroke unfinished.

The volume at discharge is $\frac{6}{32}$ of volume at beginning of stroke, or $\frac{6}{32} \times 5.8178 = 1.0908$ cu. ft. Ans.

(*b*) To calculate the weight, substitute in formula **161,** taking the values of P, V, and T at beginning of stroke.

$$W = \frac{PV}{.37052\,T} = \frac{14.7 \times 5.8178}{.37052 \times (459 + 76)} = .43143 \text{ lb.} \quad \text{Ans.}$$

(1344) Since the area of the cylinder remains constant, any variation in the volume will be proportional to the distance between the piston and end of stroke; hence, we may substitute the latter for the volume in formula **150.**

$$p_1 = \frac{pv}{v_1} = \frac{14.7 \times 32}{6} = 78.4 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(1345) Using formula **150,**

$$p_1 = \frac{pv}{v_1} = \frac{(3 \times 14.7) \times 1}{2.5} = 17.64 \text{ lb.} \quad \text{Ans.}$$

(1346) Applying formula **159** and substituting,

$$V = \frac{.37052\,W\,T}{P} = \frac{.37052 \times 7.14 \times (459 + 75)}{1.5 \times 14.7} =$$

64.068 cu. ft. Ans.

(1347) $p = 3\frac{1}{2}$ atmospheres $= 3\frac{1}{2} \times 14.7 = 51.45$ lb. per sq. in.

Applying formula **154** and substituting,

$$p : W :: p_1 : W_1.$$

$$51.45 : 13 :: p_1 : 2.$$

$$p_1 = \frac{2 \times 51.45}{13} = 7.915 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(1348) Volume at beginning of stroke =

$$\frac{48^2 \times .7854 \times 60}{1,728} = 62.832 \text{ cu. ft.}$$

Substituting in formula **161** to obtain the weight of the air,

$$W = \frac{PV}{.37052\,T} = \frac{14.7 \times 62.832}{.37052 \times (459 + 60)} = 4.8031 \text{ lb.}$$

Volume at time of discharge =

$$\frac{48^2 \times .7854 \times (60 - 50)}{1,728} = 10.472 \text{ cu. ft.}$$

To calculate the tension, substitute in formula **158**, taking the values of T and V at the time of discharge and the value of W as 4.8031.

$$P = \frac{.37052\,W\,T}{V} = \frac{.37052 \times 4.8031 \times (459 + 130)}{10.472} =$$

100.096 lb. per sq. in. Ans.

(1349) Applying formula **159** and substituting,

$$V = \frac{.37052\,W\,T}{P} = \frac{.37052 \times 1 \times (459 + 127)}{27} = 8.042 \text{ cu. ft.}$$

Ans.

(1350) A pressure of 4,000 lb. per sq. ft. is equivalent to $\frac{4,000}{144}$, or 27.777 lb. per sq. in. Using formula **159**,

$$V = \frac{.37052\,W\,T}{P} = \frac{.37052 \times .5 \times 559}{27.777} = 3.728 \text{ cu. ft.} \quad \text{Ans.}$$

(1351) Applying formula **156** and substituting,

$$v_1 = v\left(\frac{459 + t_1}{459 + t}\right) = 4 \times \left(\frac{459 + 115}{459 + 40}\right) = 4.6012 \text{ cu. ft.} \quad \text{Ans.}$$

(1352) See Art. **2147.**

(1353) Since the ordinary temperature is given in each case, we add 459° to obtain the corresponding absolute temperatures.

$459° + 32° = 491°$; $459° + 212° = 671°$; $459° + 62° = 521°$; $459° + 0° = 459°$; $459° - 40° = 419°$.

(1354) $P = 10$ atmospheres $= 10 \times 14.7 = 147$ lb. per sq. in. Applying formula **160** and substituting,

$$T = \frac{PV}{.37052\,W} = \frac{147 \times 4}{.37052 \times 3.5} = 453.417^\circ.$$

$453.417^\circ - 459^\circ = -5.583^\circ$, or 5.583° below zero. Ans.

(1355) See Art. **2134.**

(1356) Applying formula **150** and substituting,

$$p_1 = \frac{p\,v}{v_1} = \frac{130 \times 11.798}{75} = 20.45 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(1357) Applying formula **160,** $T = \frac{PV}{.37052\,W}$.

Substituting, $T = \frac{18 \times 14}{.37052 \times 1.2} = 566.77^\circ$. $566.77^\circ - 459^\circ = 107.77^\circ$. Ans.

(1358) Applying formula **156** and substituting,

$$v_1 = v\left(\frac{459 + t_1}{459 + t}\right) = 21 \times \left(\frac{459 + 420}{459 + 60}\right) = 35.57 \text{ cu. ft.} \quad \text{Ans.}$$

(1359) To obtain absolute pressure, 1 atmosphere must be added to the gauge-pressure. $6 + 1 = 7$ atmospheres. Substituting in formula **161,**

$$W = \frac{PV}{.37052\,T} = \frac{14.7 \times 7 \times 12}{.37052 \times (459 + 90)} = 6.07033 \text{ lb.,}$$

weight of 12 cubic feet. $6.07033 \div 12 = .50586$ lb., weight per cubic foot. Ans.

(1360) .5 lb. = 8 oz. 1 lb. 6 oz. = 22 oz.

Applying formula **154** and substituting,

$$p : W :: p_1 : W_1.$$
$$14.7 : 8 :: p_1 : 22.$$

$$p_1 = \frac{14.7 \times 22}{8} = 40.425 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(1361) Apply formula **156.** $v_1 = v\left(\frac{459 + t_1}{459 + t}\right)$.
Substituting,

$$v_1 = 4{,}516\left(\frac{459 + 80}{459 + 260}\right) = 4{,}516 \times \frac{539}{719} = 3{,}385.42 \text{ cu. in.}$$

$3{,}385.42 \div 1{,}728 = 1.96$ cu. ft. Ans.

(1362) $P = 1\frac{1}{4} \times 14.7$ lb. per sq. in.

Applying formula **161** and substituting,

$$W = \frac{PV}{.37052\,T} = \frac{1\frac{1}{4} \times 14.7 \times 55}{.37052 \times (459 + 88)} = \frac{1{,}010.625}{202.67444} = 4.986 \text{ lb.}$$ Ans.

(1363) Since the temperature and volume in both vessels are the same, the pressure of the mixture will be equal to the sum of the pressures.

2 atmospheres $= 2 \times 14.7 = 29.4$ lb. per sq. in.

$29.4 + 40 = 69.4$ lb. per sq. in. Ans. See Art. **2167.**

(1364) We would understand that the mercury had fallen 7 inches, and that there was enough air in the condenser to produce a pressure of $\frac{30 - 23}{30} \times 14.7$, or $\frac{7}{30} \times 14.7 = 3.43$ lb. per sq. in. Ans. See Art. **2155.**

(1365) $144 \times 14.7 = 2{,}116.8$ lb. per sq. ft. Ans.

(1366) If the weight of 3 cu. ft. under a pressure of 30 lb. per sq. in. is .27 lb., the weight per cu. ft. $= \frac{.27}{3} = .09$ lb.

Applying formula **154** and substituting,

$$p : W :: p_1 : W_1, \text{ or } 30 : .09 :: 65 : W_1.$$

$$W_1 = \frac{.09 \times 65}{30} = .195 \text{ lb.} \quad \text{Ans.}$$

(1367) To find the absolute temperature, we substitute in formula **160,** the values of P, V, and W given in Question 1366.

$$T = \frac{PV}{.37052\,W} = \frac{30 \times 3}{.37052 \times .27} = 899.64°.$$

Ordinary temperature $= 899.64° - 459° = 440.64°$. Ans.

(1368) Since the pressures and volumes are unequal, we apply formula **162** in order to obtain the tension of the mixture.

$$P = \frac{p\,v + p_1\,v_1}{V}.$$

Substituting,

$$P = \frac{14.7 \times 12 + 3 \times 14.7 \times 8}{20} = \frac{176.4 + 352.8}{20} =$$

26.46 lb. per sq. in. Ans.

(1369) Applying formula **163** and substituting,

$$V = \frac{p\,v + p_1\,v_1}{P} = \frac{14.7 \times 12 + 3 \times 14.7 \times 8}{24} = 22.05 \text{ cu. ft.}$$

Ans.

(1370) See Art. **2155.**

(1371) Since a cubic inch of mercury weighs .49 lb., $\frac{1}{40}$ of a cubic inch weighs $\frac{1}{40} \times .49 = \frac{.49}{40}$ lb. Consequently, a height of $\frac{1}{40}$ in. of mercury is equivalent to a pressure of $\frac{.49}{40}$ lb. per sq. in. 1 sq. ft. = 144 sq. in. The equivalent pressure upon a sq. ft. $= \frac{.49}{40} \times 144 = 1.764$ lb. Ans.

(1372) (*a*) See Art. **2155.**

(*b*) The height of the mercury in the tube shows the number of inches of vacuum.

Since the mercury column is $4\frac{1}{2}$ inches shorter than the barometer column, the gauge will show $30 - 4\frac{1}{2} = 25\frac{1}{2}$ inches vacuum. Ans.

(1373) Since a column of mercury 30 in. high will support a column of water 34 ft., 1 in. of mercury will support a column of water of $\frac{1}{30} \times 34$, or $\frac{34}{30}$ ft. in height.

Hence, 27 inches of mercury will support $27 \times \frac{34}{30} = 30.6$ ft. of water. Ans.

(1374) See Art. **2142.**

(1375) See Art. **2140.** Each rock-drill requires a receiver volume of 10 cubic feet; therefore, to supply 8 rock-drills, the volume of the receiver should be $8 \times 10 = 80$ cu. ft. Ans.

(1376) See Art. **2119.** (1) There is a loss due to useless heating of the air during the compression; this is reduced by the water-jacket or by water injection (2) The loss due to the friction of the engine and compressor can only be reduced by careful workmanship and design in the building of the compressor. (3) There is a slight loss due to leakage and friction of air in pipes. The loss due to friction becomes large when the pipe is very long and of small diameter; therefore, this loss is reduced to a minimum by using large pipes for conveying the air.

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(1377) (*b*) To obtain the discharge in cubic feet per second, apply formula **180.**

$$Q_a = .41\, b \sqrt{2g}\,[\sqrt{h^3} - \sqrt{h_1^{\,3}}] =$$

$$.41 \times \tfrac{30}{12} \times \sqrt{2 \times 32.16}\,[\sqrt{(5\tfrac{1}{2})^3} - \sqrt{(3\tfrac{1}{2})^3}] = 52.21 \text{ cu. ft. per sec.}$$

Ans

(*a*) Area of weir $= b\,d = 2.5 \times 2 = 5$ sq. ft.

Apply formula **179.**

$$v_m = \frac{Q_a}{b\,d} = \frac{52.21}{5} = 10.44 \text{ ft. per sec.} \quad \text{Ans.}$$

(*c*) To get the discharge in gallons per hour, multiply (*b*) by 60 × 60 (seconds in an hour) and by 7.48 (gallons in a cubic foot). Thus, 52.21 × 60 × 60 × 7.48 = 1,405,910.9 gal. per hour. Ans.

(1378) First find the coefficient of friction by using formula **182** and Table 45.

$$v_m = 2.315 \sqrt{\frac{h\,d}{f\,l}} = 2.315 \sqrt{\frac{76 \times 7.5}{.025 \times 12{,}000}} = 3.19 \text{ ft. per sec.}$$

From the table, $f = .0243$ for $v_m = 3$, and .023 for $v_m = 4$; the difference is .0013 = difference for a difference of velocity of 1 ft. per sec. Then, .0013 × .19 ft. per sec. = .000247, or say .0002, using but four decimal places = difference for a difference of velocity of .19 ft. per sec. Therefore, $f = .0243 - .0002 = .0241$, or say .024.

Use formula **186.** Substitute in it the value of f here found, and multiply by 60 to get the discharge per minute.

$$Q = .09445\, d^2 \sqrt{\frac{h\,d}{f\,l + .125\,d}} \times 60 =$$

$$.09445 \times 7.5^2 \sqrt{\frac{76 \times 7.5}{.024 \times 12{,}000 + .125 \times 7.5}} \times 60 = 447.7 \text{ gal. per min.} \quad \text{Ans.}$$

(1379) (*a*) Use formula **181.**

$$v_m = 2.315 \sqrt{\frac{h\,d}{f\,l + .125\,d}} \times 60 =$$

$$2.315 \sqrt{\frac{76 \times 7.5}{.024 \times 12{,}000 + .125 \times 7.5}} \times 60 = 195.08 \text{ ft. per min.} \quad \text{Ans.}$$

(*b*) 447.7 gal. per min. ÷ 60 = $7.46\frac{1}{8}$ gal. per sec. = 1 cu. ft. per sec., nearly. Ans.

(1380) Use formula **167.**

$$v = \sqrt{2\,g\,h} = \sqrt{2 \times 32.16 \times 10} = 25.36 \text{ ft. per sec.} \quad \text{Ans.}$$

(1381) Use formulas **185** and **183.**

$$v_m = \frac{24.51\,Q}{d^2} = \frac{24.51 \times 42{,}000}{6.5^2 \times 60 \times 60} = 6.768 \text{ ft. per sec.}$$

From Table 45, $f = .021$ for $v_m = 6.768$; hence,

$$h = \frac{f\,l\,v_m^2}{5.36\,d} + .0233\,v_m^2 =$$

$$\frac{.021 \times 1{,}500 \times 6.768^2}{5.36 \times 6.5} + .0233 \times 6.768^2 = 42.48 \text{ ft.} \quad \text{Ans.}$$

(1382) Area of top or bottom of cylinder equals $20^2 \times .7854 = 314.16$ sq. in. Area of cross-section of pipe = $(\frac{3}{8})^2 \times .7854 = .1104$ sq. in. 25 lb. 10 oz. = 25.625 lb. 25.625 ÷ .1104 = 232.11 lb. pressure per sq. in. on top or bottom exerted by the weight and piston.

Pressure due to a head of 10 ft. = .434 × 10 = 4.34 lb. per sq. in.

Pressure due to a head of 13 ft. = .434 × 13 = 5.642 lb. per sq. in.

(Since a column of water 1 ft. high exerts a pressure of .434 lb. per sq. in. See Art. **2289.**)

(*a*) Pressure on bottom = pressure due to weight + pressure due to head of 13 ft. = 232.11 + 5.64 = 237.75 lb. per sq. in. Ans.

(*b*) Pressure on the top = pressure due to weight + pressure due to head of 10 feet = 232.11 + 4.34 = 236.45 lb. per sq. in. Ans.

(*c*) Total pressure, or equivalent weight on the bottom, = 237.752 × 314.16 = 74,692.17 lb. Ans.

(1383) .434 × 1½ = .651 lb., pressure due to the head of water in the cylinder above the orifice.

236.45, pressure on top per sq. in. + .651 = 237.1, total pressure per sq. in. Area of orifice = $1^2 \times .7854 = .7854$ sq. in.

$$.7854 \times 237.1 = 186.22 \text{ lb. Ans.}$$

(1384) First find the coefficient of friction by formula **182** and Table 45.

$$v_m = 2.315\sqrt{\frac{hd}{.025\,l}} = 2.315\sqrt{\frac{120 \times 4}{.025 \times 4{,}000}} = 5.072 \text{ ft. per sec.,}$$

or say 5 ft. per sec.

From the table, $f = .023$ for $v_m = 4$ and .0214 for $v_m = 6$.

$$\frac{.023 - .0214}{6 - 4} = .0008. \quad .023 - .0008 = .0222 = f \text{ for } v_m = 5.$$

Use formula **182**, because the pipe is longer than 10,000 times its diameter.

$$\text{Hence, } v_m = 2.315\sqrt{\frac{120 \times 4}{.0222 \times 4{,}000}} = 5.38 \text{ ft. per sec. Ans.}$$

(1385) Use formulas **182** and **181**.

$$v_m = 2.315\sqrt{\frac{120 \times 4}{.025 \times 2{,}000}} = 7.17 \text{ ft. per sec.}$$

From the table, $f = .0214$ for $v_m = 6$ and .0205 for $v_m = 8$.

$$\frac{.0214 - .0205}{8 - 6} = .00045$$ decrease for an increase of velocity of 1 ft. per sec. 7.17 − 6 = 1.17. .00045 × 1.17 = .0005265, total decrease. $f = .0214 - .0005265 = .0208735$, or .0209, using four figures.

Hence, the velocity of discharge =

$$v_m = 2.315\sqrt{\frac{120 \times 4}{.0209 \times 2{,}000 + .125 \times 4}} = 7.79 \text{ ft. per sec.}$$

Ans.

(1386) (*a*) $f = .0205$ for $v_m = 8$. Therefore, using formula **183**,

$$h = \frac{.0205 \times 5{,}280 \times 8^2}{5.36 \times 10} + .0233 \times 8^2 = 130.73 \text{ ft.} \quad \text{Ans.}$$

(*b*) Using formula **184,** $Q = .0408\,d^2 v_m = .0408 \times 10^2 \times 8 = 32.64$ gal. per sec. $32.64 \times 60 \times 60 = 117{,}504$ gallons per hour. Ans.

(1387) A column of water 1 inch square and 2.304 ft. high weighs 1 lb.; hence, to produce a pressure of 30 lb. per sq. in., it will require a column of water $2.304 \times 30 = 69.12$ ft. high = head. Using formula **172,**

$$v = .98\sqrt{2gh} = .98\sqrt{2 \times 32.16 \times 69.12} = 65.34 \text{ ft. per sec. Ans.}$$

(1388) (*a*) 36 in. = 3 ft. A column of water 1 inch square and 1 ft. long weighs .43403 lb. $.43403 \times 43 = 18.6633$ lb. per sq. in. on the bottom of the cylinder. $.43403 \times 40 = 17.3612$ lb. per sq. in. on the top of the cylinder. Area of base of cylinder $= 20^2 \times .7854 = 314.16$. $314.16 \times 18.6633 = 5{,}863.26$ lb., the total pressure on the bottom. Ans.

(*b*) $314.16 \times 17.3612 = 5{,}454.19$ lb., total pressure on top. Ans.

(1389) Use formula **168.** $h = \frac{33^2}{64.32} = 16.931$ ft. per sec. Ans.

(1390) (*a*) Use formula **178** and multiply by $7.48 \times 60 \times 60$ to reduce cubic feet per second to gallons per hour.

$$Q_a = .41 \times \tfrac{24}{12} \times \sqrt{2 \times 32.16 \times (\tfrac{15}{12})^3} \times 7.48 \times 60 \times 60 = 216{,}551 \text{ gal. per hr. Ans.}$$

(*b*) By formula **179,**

$$v_m = \frac{Q_a}{bd} = \frac{.615 \times \frac{2}{3} \times \frac{21}{12} \times \sqrt{2 \times 32.16 \times (\frac{15}{12})^3}}{\frac{21}{12} \times \frac{15}{12}} = 3.676 \text{ ft. per sec. Ans.}$$

(1391) $f = .0193$ for $v_m = 12$. Therefore, using formula **183**,

$$h = \frac{f l v_m^2}{5.36 d} + .0233 v_m^2 = \frac{.0193 \times 6,000 \times 12^2}{5.36 \times 3} + (.0233 \times 12^2) =$$

1,040.37 ft. Ans.

(1392) $\frac{5^2 \times .7854}{144}$ = area of pipe in sq. ft. Using formula **165**,

$$Q = A v = \frac{5^2 \times .7854}{144} \times 7.2 = \text{discharge in cu. ft. per sec.}$$

$\frac{5^2 \times .7854}{144} \times 7.2 \times 7.48 \times 60 \times 60 \times 24 = 634,478$ gal. discharged in 1 day. Ans.

(1393) 38,000 gal. per hour $= \frac{38,000}{60 \times 60}$ gal. per sec. $= Q$. Using formula **185**,

$$v_m = \frac{24.51\, Q}{d^2} = \frac{24.51 \times 38,000}{5.5^2 \times 60 \times 60} = 8.5526 \text{ ft. per sec.} \quad \text{Ans.}$$

(1394) Use formula **178**.

(*a*) $Q_a = .41 \times b \sqrt{2 g d^3} = .41 \times \frac{27}{12} \times \sqrt{2 \times 32.16 \times (\frac{36}{12})^3} =$ 38.44 ft. per sec. Ans.

(*b*) $Q = \frac{Q_a}{.615} = \frac{38.44}{.615} = 62.5$ cu. ft. per sec. Ans.

(1395) Use formulas **167** and **169**.

(*a*) $v = \sqrt{2 g h} = \sqrt{2 \times 32.16 \times 45} = 53.8$ ft. per sec. Ans.

(*b*) $2.304 \times 10 = 23$ ft., nearly = height of a column of water which will give a pressure of 10 lb. per sq. in. $45 + 23 = 68$ ft.

$$v = \sqrt{2 g (h_1 + h)} = \sqrt{2 \times 32.16 \times 68} = 66.153 \text{ ft. per sec.}$$

Ans.

(1396) Use formula **184**.

$Q = .0408\, d^2 v_m = .0408 \times 6^2 \times 7.5 = 11.016$ gal. per sec. Ans.

(1397) Head = 41 ÷ .434 = 94.47 ft.

Using formula **172,**

$$v = .98\sqrt{2gh} = .98\sqrt{2 \times 32.16 \times 94.47} = 76.39 \text{ ft. per sec.}$$

Ans.

(1398) Divide by 60 × 60 to get the discharge in gal. per sec., and by 7.48 to obtain the discharge in cu. ft. per sec.

$$\text{Area in sq. ft.} = \frac{4^2 \times .7854}{144} = .087267.$$

$$v_m = \frac{Q}{A} = \frac{12{,}000}{60 \times 60 \times 7.48 \times .087267} = 5.106 \text{ ft. per sec. Ans.}$$

(1399) (*a*) Area of pump-piston = $(\frac{1}{2})^2 \times .7854 = .19635$ sq. in.

Area of plunger = $10^2 \times .7854 = 78.54$ sq. in.

Pressure per sq. in. exerted by piston = $\frac{100}{.19635}$ pounds.

Hence, according to Pascal's law, the pressure on the plunger is $\frac{100}{.19635} \times 78.54 = 40{,}000$ lb. Ans.

(*b*) According to the principle given in Art. **2181,** $D \times 1\frac{1}{2}$ inches = $W \times$ distance moved by plunger, or $100 \times 1\frac{1}{2} = 40{,}000 \times$ required distance; hence, the required distance = $\frac{100 \times 1\frac{1}{2}}{40{,}000} = .00375$ in. Ans.

(1400) (*a*) Use formula **180,** and multiply by 7.48 and 60 to reduce the discharge from cu. ft. per sec. to gal. per min.

$$Q_a = .41\, b\sqrt{2g}\left[\sqrt{h^3} - \sqrt{h_1^3}\right] \times 60 \times 7.48 =$$

$$.41 \times \tfrac{14}{12} \times \sqrt{64.32}\left[\sqrt{(9 + \tfrac{20}{12})^3} - \sqrt{9^3}\right] \times 60 \times 7.48 =$$

13,491.22 gallons per minute. Ans.

(*b*) In the second case,

$$Q_a = .41 \times \tfrac{20}{12} \times \sqrt{64.32}\left[\sqrt{(9 + \tfrac{14}{12})^3} - \sqrt{9^3}\right] \times 60 \times 7.48 =$$

13,322.47 gallons per minute. Ans.

(1401) Area of weir = 14 × 20 ÷ 144 sq. ft. Use formula **166,** and divide by 60 × 7.48 to reduce gal. per min. to cu. ft. per sec.

(*a*) $v_m = \frac{Q}{A} = \frac{13,491.22 \times 144}{60 \times 7.48 \times 14 \times 20} = 15.46$ ft. per sec. Ans.

(*b*) $\frac{13,322.47 \times 144}{60 \times 7.48 \times 14 \times 20} = 15.264$ ft. per sec. Ans.

(1402) In Art. **2197** it is stated that the theoretical mean velocity is $\frac{2}{3}\sqrt{2gh}$. Hence,

$$v_m = \tfrac{2}{3}\sqrt{2 \times 32.16 \times 3} = 9.26 \text{ ft. per sec. Ans.}$$

(1403) (*a*) 4 ft. 9 in. = 4.75 ft. 19 − 4.75 = 14.25. Using formula **170,**

$$R = \sqrt{4hy} = \sqrt{4 \times 4.75 \times 14.25} = 16.454 \text{ ft. Ans.}$$

(*b*) 19 − 4.75 = 14.25 ft. Ans.

(*c*) 19 ÷ 2 = 9.5. Greatest range = $\sqrt{4 \times 9.5^2} = 19$ ft. Ans.

(1404) Use formulas **182** and **186.**

$$v_m = 2.315\sqrt{\frac{hd}{fl}} = 2.315\sqrt{\frac{25 \times 5}{.025 \times 1,300}} = 4.54 \text{ ft. per sec.}$$

From the table, $f = .023$ for $v_m = 4$, and .0214 for $v_m = 6$.

$\frac{.023 - .0214}{2} \times .54 = .000432.$ $f = .023 - .000432 = .022568.$

$$Q = 60 \times 60 \times .09445 \times 5^2 \times \sqrt{\frac{25 \times 5}{.022568 \times 1,300 + .125 \times 5}} =$$

17,368.95 gal. per hr. Ans.

(1405) Obtain the values by approximating to those given in Table 45. Thus, for $v_m = 2$, $f = .0265$, and for $v_m = 3$, $f = .0243$.

Difference = .0022. .0022 × .37 = .000814, or say .0008. .0265 − .0008 = .0257 = f for $v_m = 2.37$. Ans.

.0243 − .023 = .0013. .0013 × .19 = .000247, or say .0002. .0243 − .0002 = .0241 = f for $v_m = 3.19$. Ans.

$.023 - .0214 = .0016.$ $\frac{.0016}{2} \times 1.8 = .00144$, or say .0014. $.023 - .0014 = .0216 = f$ for $v_m = 5.8$. Ans.

$.0214 - .0205 = .0009.$ $\frac{.0009}{2} \times 1.4 = .00063$, or say .0006. $.0214 - .0006 = .0208 = f$ for $v_m = 7.4$.

$.0205 - .0193 = .0012.$ $\frac{.0012}{4} \times 1.83 = .000549$, or say .0005. $.0205 - .0005 = .02 = f$ for $v_m = 9.83$. Ans.

$.0205 - .0193 = .0012.$ $\frac{.0012}{4} \times 3.5 = .00105$, or say .0011. $.0205 - .0011 = .0194 = f$ for $v_m = 11.5$. Ans.

(1406) The specific gravity of sea-water is 1.026 (see table of Specific Gravity); hence, the weight of a cubic foot of sea-water $= 1.026 \times 62.5 = 64.1$ lb.

Total area of cube $= \frac{10.5^2 \times 6}{144}$ sq. ft. 1 mile = 5,280 ft.

Hence, total pressure on cube $= \frac{10.5^2 \times 6}{144} \times 5{,}280 \times 3.5 \times 64.1 = 5{,}441{,}609.25$ lb. Ans.

(1407) (*b*) The pressure per square inch equals the weight of a volume of water 1 in. square and 12 in. high; that is, it equals

$$1 \times 1 \times .03617 \times 12 = .434 \text{ lb., nearly.} \quad \text{Ans.}$$

(*a*) Total pressure on the bottom = area of bottom in square inches multiplied by the pressure per square inch $= 8^2 \times .7854 \times .434 = 21.82$ lb. Ans.

(1408) 8,000 gal. per hr. $= \frac{8{,}000}{60}$, or $133\frac{1}{3}$ gal. per min.

Plunger speed per min. $= 7 \times 10 = 70$ ft. Applying formula **190** and substituting,

$$d = 5.535\sqrt{\frac{G}{S}} = 5.535\sqrt{\frac{133\frac{1}{3}}{70}} = 7\frac{5}{8} \text{ in.} \quad \text{Ans.}$$

(1409) See Art. **2266.**

(1410) The height to which a pump will lift water is directly proportional to the atmospheric pressure; that is, proportional to the length of the mercury column.

Letting x represent the height to which the pump will lift the water on top of the mountain, we have the proportion, $30 : 22 :: 25.5 : x$, or $30x = 22 \times 25.5$; whence, $x =$ 18.7 ft. Ans.

(1411) Area of dam $= 40 \times 12 = 480$ sq. ft.

$\frac{1}{2} \times 12 = 6$ ft., depth of center of gravity below the level of the liquid.

The total pressure on the dam $= 40 \times 12 \times 6 \times 62\frac{1}{2} =$ 180,000 lb. Ans.

(1412) (*a*) Apply formula **190.**

$$d = 5.535\sqrt{\tfrac{750}{100}} = 15 \text{ inches. Ans.}$$

(*b*) Use formula **195.**

$$d_1 = .35\sqrt{G} = .35\sqrt{750} = 10 \text{ inches. Ans.}$$

(*c*) Use formula **196.**

$$d_2 = .25\sqrt{G} = .25\sqrt{750} = 7 \text{ inches. Ans.}$$

(1413) First obtain the coefficient of friction from formula **182** and Table 45.

$$v_m = 2.315\sqrt{\frac{h\,d}{f\,l}} = 2.315\sqrt{\frac{40 \times 6}{.025 \times 840}} = 7.82 \text{ ft. per sec.}$$

From the table, $f = .0214$ for $v_m = 6$ and .0205 for $v_m = 8$.

$\frac{.0214 - .0205}{2} = .00045$ decrease for an increase of velocity of 1 ft. per sec. $7.82 - 6 = 1.82$.

$.00045 \times 1.82 = .0008$, nearly, total decrease. $f = .0214 - .0008 = .0206$.

To obtain the discharge in gal. per sec., substitute this value of f in formula **186,** and multiply by 60×60 to get the discharge in gal. per hr.

$$Q = .09445\,d^2\sqrt{\frac{h\,d}{f\,l + .125\,d}} \times 60 \times 60 =$$

$$.09445 \times 6^2 \times \sqrt{\frac{40 \times 6}{.0206 \times 840 + .125 \times 6}} \times 60 \times 60 =$$

44,553.6 gal. per hr. Ans.

(1414) If the area of the tube is $\frac{1}{2}$ sq. in., and that of the cylinder 80 sq. in., a force of 80 lb. on the small piston will raise a weight of $\frac{80}{\frac{1}{2}} \times 80 = 12{,}800$ lb. on the large piston. Since the length between the hand and the fulcrum is $7\frac{1}{2}$ times the distance between the piston-rod and the fulcrum, a force of 80 lb. on the end of the lever will raise a weight of $7\frac{1}{2} \times 12{,}800 = 96{,}000$ lb. Ans.

(1415) (*a*) Using formula **190,**

$$d = 5.535\sqrt{\frac{200}{2 \times 150}} = 4\tfrac{1}{2} \text{ in.} \quad \text{Ans.}$$

(*b*) Use formula **195.**

$$d_1 = .35\sqrt{G} = .35\sqrt{200} = 5 \text{ in.} \quad \text{Ans.}$$

(*c*) Use formula **196.**

$$d_2 = .25\sqrt{G} = .25\sqrt{200} = 3\tfrac{1}{2} \text{ in.} \quad \text{Ans.}$$

(*d*) Applying formula **192** and substituting,

$$H = .00038\,G h = .00038 \times 200 \times 250 = 19 \text{ H. P.} \quad \text{Ans.}$$

(1416) (*a*) Since the pressure exerted by a column of water 1 foot high = .434 lb. per sq. in., the pressure exerted by a column of water 210 ft. high = $210 \times .434 = 91.14$ lb. per sq. in. Ans.

(*b*) Applying formula **167** and substituting,

$$v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 210} = 116.22 \text{ ft. per sec.} \quad \text{Ans.}$$

(1417) To calculate the diameter of the steam-cylinder, we apply formula **194.** But we must first obtain the value of H, or the horsepower, by formula **192.** $H = .00038\,G h$. Substituting, $H = .00038 \times \frac{27{,}000}{60} \times 240 = 41.04$ H. P. for both sides of the pump. $\frac{41.04}{2} = 20.52$ H. P. for each side. Substituting in formula **194,**

$$D = 205\sqrt{\frac{20.52}{90 \times 85}} = 10\tfrac{5}{8} \text{ inches.} \quad \text{Ans.}$$

Apply formula **190.**

27,000 gal. per hr. $= \frac{27,000}{60}$, or 450 gal. per min. for both sides. $\frac{450}{2} = 225$ gal. for one side $= G$.

$$d = 5.535\sqrt{\frac{G}{S}} = 5.535\sqrt{\frac{225}{90}} = 8\tfrac{3}{4} \text{ inches. Ans.}$$

(1418) (*a*) A column of water 1 foot high and having a cross-section of 1 sq. in. weighs .434 lb. Hence, the pressure per sq. in. at the bottom of the stand-pipe $= .434 \times 70 = 30.38$ lb. per sq. in. Ans.

(*b*) At a distance of 30 ft. from the top of the water the pressure is $.434 \times 30 = 13.02$ lb. per sq. in. Ans.

(1419) See Art. **2216.**

(1420) (*a*) Apply formula **191.**

$G = .03264\,d^2 S = .03264 \times 14^2 \times 100 = 639.744$ gal. per min. due to one side of pump. $639.744 \times 2 = 1,279.488$ gal., total discharge per minute. $1,279.488 \times 60 = 76,769.28$ gal. per hour. Ans.

(*b*) To obtain the height to which water can be raised, we apply formula **193;** but, before we can substitute in this formula, we must obtain the horsepower by applying the formula $H = \frac{PLAN}{33,000}$. Remembering that $L \times N =$ piston speed, we have

$$H = \frac{45 \times 22^2 \times .7854 \times 100}{33,000} = 51.8364 \text{ H. P.}$$

Substituting in formula **193,**

$$h = \frac{H}{.00038\,G} = \frac{51.8364}{.00038 \times 639.744} = 213.22 \text{ feet. Ans.}$$

(1421) $307 \times .434 = 133.238$ lb. per sq. in. Ans.

(1422) The time of making the stroke depends simply on the acceleration of the pit-work, which in turn depends solely on the difference between the weight of the pit-work and water column minus the frictional resistances. Now, if this difference is too great, the stroke will be made too quickly for safety and convenience, and, to obviate this, the weight of the descending pit-work must be made less or the weight of the ascending water column greater. This is accomplished by balancing the pit-work, as explained in Arts. **2247** to **2249.**

(1423) First find the value of f from Table 45.

$$f = .0243 \text{ for } v_m = 3 \text{ and } .023 \text{ for } v_m = 4.$$

Difference $= .0013$. $3.3 - 3 = .3$. Then, $.0013 \times .3 = .00039$, total decrease. $f = .0243 - .00039 = .02391$.

Substituting in formula **183,**

$$h = \frac{f l v_m^2}{5.36 d} + .0233 v_m^2 =$$

$$\frac{.02391 \times 2{,}000 \times (3.3)^2}{5.36 \times 2.5} + .0233 \times (3.3)^2 = 39.12 \text{ ft. Ans.}$$

(1424) (*b*) 80,000 gal. per hr. $= \frac{80{,}000}{60} = 1{,}333\frac{1}{3}$ gal. per min.

To obtain the actual horsepower, apply formula **192.**

$$H = .00038\, G h = .00038 \times 1{,}333\tfrac{1}{3} \times 420 = 212.8 \text{ H. P. Ans.}$$

(*a*) The theoretical horsepower $= \frac{2}{3} \times 212.8 = 141.87$ H. P. Ans.

(1425) Applying formula **187** and substituting,

$$D = \frac{835.5\, G h}{W} = \frac{835.5 \times 30{,}000 \times 290}{600} = 12{,}114{,}750 \text{ ft.-lb. Ans.}$$

(1426) We first calculate the value of f from formula **182** and the table.

$$v_m = 2.315 \sqrt{\frac{h d}{f l}} = 2.315 \sqrt{\frac{220 \times 6}{.025 \times 6{,}500}} = 6.597 \text{ ft. per sec.}$$

From the table, $f = .0214$ for $v_m = 6$ and $.0205$ for $v_m = 8$.

$\frac{.0214 - .0205}{2} = .00045 =$ decrease for an increase of velocity of 1 ft. per sec. .00045 × .597 ft. per sec. = 000268, total decrease. .0214 − .000268 = .02113 = f.

Substituting in formula **182,**

$$v_m = 2.315\sqrt{\frac{220 \times 6}{.02113 \times 6{,}500}} = 7.17 \text{ ft. per sec.} \quad \text{Ans.}$$

(1427) (*a*) See Art. **2290.**
Head = 45 × 2.304 = 103.68 ft. Ans.

(*b*) 2.304 × 86 = 198.144 ft. Ans.

(*c*) 2.304 × 108 = 248.832 ft. Ans.

(1428) (*b*) Applying formula **191** and substituting, $G = .03264d^2S = .03264 \times 15^2 \times 100 = 734.4$ gal. per min. Ans.

(*a*) To calculate the diameter of the steam-cylinder, we first obtain the horsepower from formula **192,** then substitute in formula **194.**

$$H = .00038\,G\,h = .00038 \times 734.4 \times 310 = 86 \text{ H. P.}$$

$$D = 205\sqrt{\frac{H}{PS}} = 205\sqrt{\frac{86}{50 \times 100}} = 27 \text{ in.} \quad \text{Ans.}$$

(1429) Applying formula **167** and substituting,
$v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 13.7} = 29.685$ ft. per sec. Ans.

(1430) According to Pascal's law, the pressure per square inch on each piston is the same. In order that the weights shall balance, they must be proportional to the area of the piston. Hence, we have the proportion

$$5 : 73 :: 22 : x.$$

$$x = \frac{22 \times 73}{5}, \text{ or } 321.2 \text{ lb.} \quad \text{Ans.}$$

(1431) As the lips are applied to the tube and the breath drawn in, the air in the tube above the surface of the water is drawn into the mouth, and a partial vacuum in the tube is the result of the operation. Now, as there is very little pressure on the water in the tube, and as the water outside the tube is exposed to the pressure of the atmosphere, 14.7 lb. per sq. in., the water must be forced up the tube by the greater pressure of the atmosphere. The action known as suction is, therefore, only a manifestation of atmospheric pressure.

(1432) (*a*) Applying formula **191** and substituting,
$G = .03264\,d^2 S = .03264 \times 11^2 \times 100 = 394.944$ gal. per min.

$394.944 \times 60 = 23{,}696.64$ gal. per hour. Ans.

(*c*) Use formula **192.**
$H = .00038\,G h = .00038 \times 394.944 \times 300 = 45.024$ H. P. Ans.

(*b*) Applying formula **194,**

$$D = 205\sqrt{\frac{H}{PS}} = 205\sqrt{\frac{45.024}{50 \times 100}} = 19\tfrac{1}{2} \text{ inches. Ans.}$$

(1433) Because the water helps to fill up the pores in the flat surface and the glass, thus creating a partial vacuum between the surfaces.

(1434) First finding the value of f by formula **182** and Table 45, we have

$$v_m = 2.315\sqrt{\frac{hd}{fl}} = 2.315\sqrt{\frac{15 \times 3.5}{.025 \times 88}} =$$
11.3 ft. per sec., nearly.

From the table, $f = .0205$ for $v_m = 8$ and $.0193$ for $v_m = 12$.
$\frac{.0205 - .0193}{4} = .0003$, decrease for an increase of 1 ft. per sec.

$.0003 \times 3.3 = .00099$, total decrease.

$f = .0205 - .00099 = .01951$.

To obtain the discharge in gal. per sec., substitute this value of f in formula **186.**

$$Q = .09445\,d^2\sqrt{\frac{hd}{fl + .125\,d}} =$$

$$.09445 \times (3.5)^2\sqrt{\frac{15 \times 3.5}{.01951 \times 88 + .125 \times 3.5}} = 5.711 \text{ gal. per sec}$$

$5.711 \times 60 = 342.66$ gal. per min. Ans.

(1435) (*a*) To obtain the diameter of the steam-cylinder, we calculate the horsepower from formula **192**, then substitute in formula **194**.

$$H = .00038\,G\,h = .00038 \times 300 \times 225 = 25.65 \text{ H. P.}$$

$$D = 205\sqrt{\frac{H}{PS}} = 205\sqrt{\frac{25.65}{110 \times 50}} = 14 \text{ in.} \quad \text{Ans.}$$

(*b*) Applying formula **190** and substituting,

$$d = 5.535\sqrt{\frac{G}{S}} = 5.535\sqrt{\frac{300}{110}} = 9\tfrac{1}{8} \text{ in.} \quad \text{Ans.}$$

(*c*) Assume the number of strokes per minute to be 110; then, stroke $= \frac{110}{110} = 1$ ft. $= 12$ in. Ans.

(*d*) Use formula **195**.

$$d_1 = .35\sqrt{G} = .35\sqrt{300} = 6 \text{ in.} \quad \text{Ans.}$$

(*e*) Apply formula **196**.

$$d_2 = .25\sqrt{G} = .25\sqrt{300} = 4\tfrac{1}{2} \text{ in.} \quad \text{Ans.}$$

(1436) (*a*) To obtain the theoretical discharge, apply formula **167**. $v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 15.75} = 31.83$ ft. per sec., since 15 ft. 9 in. = 15.75 ft.

31.83 × 60 = 1,909.8 ft. per min.

Area of orifice = 11.2 sq. in. = $\frac{11.2}{144}$ sq. ft.

To calculate the theoretical quantity in cu. ft. per min., substitute in formula $Q = A\,v$.

$$Q = \frac{11.2}{144} \times 1{,}909.8 = 148.54 \text{ cu. ft. per min.} \quad \text{Ans.}$$

(*b*) Applying formula **174** and substituting,

$$Q_a = .615 \times \frac{11.2}{144}\sqrt{2 \times 32.16 \times 15.75} = 1.5224 \text{ cu. ft. per sec.}$$

$$1.5224 \times 60 = 91.344 \text{ cu. ft. per min.} \quad \text{Ans.}$$

(1437) Because if any air be left in the siphon, it will exert a pressure on the water in the arms of the siphon that will exactly balance the atmospheric pressure on the surface of the water outside, which tends to force the water up the arms.

Therefore, the water in each arm is in equilibrium, and no motion can take place. As soon, however, as the air is expelled, either by filling the siphon with water or by pumping the air out, the water is no longer in equilibrium, and will begin to flow.

(1438) Piston speed per minute = 9 × 5 = 45 ft.

(*a*) Applying formula **191**,

$G = .03264 d^2 S = .03264 \times 19^2 \times 45 = 530.24$ gal. per min. Ans.

(*b*) 530.24 × 60 = 31,814.4 gal. per hr. Ans.

(1439) Applying formula **187** and substituting,

$$D = \frac{835.5 G h}{W} = \frac{835.5 \times 80{,}000 \times 340}{400} = 56{,}814{,}000 \text{ ft.-lb.} \quad \text{Ans.}$$

(1440) See Art. **2290.**

(*a*) 2.304 × 80 = 184.32 ft. Ans.

(*b*) 2.304 × 30.5 = 70.272 ft. Ans.

(*c*) 2.304 × 108 = 248.832 ft. Ans.

(*d*) 2.304 × 215 = 495.36 ft. Ans.

(1441) Applying formula **191** and substituting,

$G = .03264 d^2 S = .03264 \times 14^2 \times 100 = 639.744$ gal. per min.

639.744 × 60 = 38,384.64 gal. per hr., the delivery for one side.

Total delivery = 38,384.64 × 2 = 76,769.28 gal. per hr. Ans.

(1442) $f = .0205$ for $v_m = 8$.

Substituting in formula **183**,

$$h = \frac{f l v_m^2}{5.36 d} + .0233 v_m^2 = \frac{.0205 \times 5{,}000 \times 8^2}{5.36 \times 4} + .0233 \times 8^2 =$$

$$\frac{6{,}560}{21.44} + 1.49 = 307.46 \text{ ft.} \quad \text{Ans.}$$

(1443) Since the area of the orifice is greater than $\frac{1}{16}$ of the area of the cross-section of the vessel, we use formula **171.**

$$v = \sqrt{\frac{2gh}{1-\frac{a^2}{A^2}}}. \quad \sqrt{\frac{2 \times 32.16 \times 12}{1-\frac{(11 \times 11)^2}{(36^2 \times .7854)^2}}} = \sqrt{\frac{771.84}{1-.0141}} =$$

27.979 ft. per sec. Ans.

(1444) Applying formula **187** and substituting,

$$D = \frac{835.5\,Gh}{W} = \frac{835.5 \times 4{,}000{,}000 \times 125}{7{,}460} = 55{,}998{,}660 \text{ ft.-lb.}$$

Ans.

(1445) Force available to accelerate the moving mass = $20-(12+3) = 5$ tons $= F$. Weight to be accelerated $= 20 + 12 = 32$ tons $= W$. By formula **188,** acceleration $= f = \frac{gF}{W} = \frac{32.16 \times 5}{32} = 5.025$ ft. per sec.

By formula **189,** $t = \sqrt{\frac{2s}{f}} = \sqrt{\frac{2 \times 10}{5.025}} = 1.995$ sec. = time occupied in passing over 10 feet. This is at the rate of $\frac{10}{1.995} \times 60 = 300$ ft. per min.

Since the speed must not exceed 200 ft. per minute, the pit-work must be counterbalanced. Suppose a counterweight of 2 tons be tried, assuming that the frictional resistances are not increased.

Then, force $= F = 20 - (12 + 3 + 2) = 3$ tons.

Weight $= 20 + 12 + 2 = 34$ tons.

$$f = \frac{gF}{W} = \frac{32.16 \times 3}{34} = 2.84 \text{ ft. per sec.}$$

$$t = \sqrt{\frac{2s}{f}} = \sqrt{\frac{2 \times 10}{2.84}} = 2.65 \text{ sec., nearly.}$$

$$\frac{10}{2.65} \times 60 = 226.42 \text{ ft. per sec.}$$

This speed is also too great, so we will try a counterbalance of $2\frac{1}{2}$ tons.

$$\text{Force} = F = 20 - (12 + 3 + 2\tfrac{1}{2}) = 2\tfrac{1}{2} \text{ tons.}$$

$$\text{Weight} = W = 20 + 12 + 2\tfrac{1}{2} = 34\tfrac{1}{2} \text{ tons.}$$

$$f = \frac{gF}{W} = \frac{32.16 \times 2\frac{1}{2}}{34\frac{1}{2}} = 2.33 \text{ ft. per sec.}$$

$$t = \sqrt{\frac{2s}{f}} \sqrt{\frac{2 \times 10}{2.33}} = 2.93 \text{ sec., nearly.}$$

$$\frac{10}{2.93} \times 60 = 204.78 \text{ ft. per min.}$$

This is near enough for practical purposes, but if a counterweight of 2.6 tons be tried, it will reduce the acceleration so that the speed of the pit-work is almost exactly 200 ft. per min.

(1446) By the difference of cylinder volumes. The steam is admitted into the high-pressure cylinder and exhausted into the low-pressure cylinder.

(1447) See Art. **2269.**

(1448) Apply formulas **195** and **196.**

(*a*) $d_1 = .35\sqrt{G} = .35\sqrt{\frac{70,000}{60}} = 12$ in. Ans.

(*b*) $d_2 = .25\sqrt{G} = .25\sqrt{\frac{70,000}{60}} = 8\frac{1}{2}$ in. Ans.

(1449) 100,000 gal. per hr. $= \frac{100,000}{60}$ gal. per min.

Applying formula **192,**

$$H = .00038\, G h = .00038 \times \frac{100,000}{60} \times 480 = 304 \text{ H. P.} \quad \text{Ans.}$$

(1450) Apply formula **184.**

$$Q = .0408\, d^2 v_m = .0408 \times 7^2 \times .721 = 14.414232 \text{ gal. per sec.}$$

$14.414232 \times 60 \times 60 = 51,891.24$ gal. per hr. Ans.

(1451) See Art. **2260.**

(1452) See Art. **2271.**

(1453) See Arts. **2225, 2226, 2259, 2271,** and **2280.**

(1454) 200 ft. per min.; 400 ft. per min.; 100 ft. per min.

(1455) Applying formula **191** and substituting,

$G = .03264\, d^2 S = .03264 \times 15^2 \times 95 =$ number of gal. per min.

$.03264 \times 15^2 \times 95 \times 60 = 41{,}860.8$ gal. per hr. Ans.

(1456) See Arts. **2250** to **2254.**

(1457) Applying formula **172** and substituting,

$v = .98\sqrt{2gh} = .98\sqrt{2 \times 32.16 \times 69.12} = 65.34$ ft. per sec. Ans.

(1458) (*a*) Area of piston $= (\frac{7}{8})^2 \times .7854 = .6013$ sq. in.

Pressure per sq. in. exerted by piston $= \frac{50}{.6013} = 83.15$ lb.

A column of water 1 foot high and of 1 sq. in. cross-section weighs .434 pound, and therefore exerts a pressure of .434 pound per sq. in. The height of a column of water to exert a pressure of 83.15 lb. per sq. in. must be $\frac{83.15}{.434} = 191.6$ feet. Consequently, the water will rise 191.6 feet.

The diameter of the hole in the squirt-gun has nothing to do with the height of the water, since the pressure per square inch will remain the same, no matter what may be the diameter.

(*b*) Using formula **170,**

$R = \sqrt{4hy} = \sqrt{4 \times 10 \times 191.6} = 87.54$ ft. Ans.

MINE HAULAGE.

(1459) See Art. **2298.**

(1460) See Art. **2299.**

(1461) See Art. **2300.**

(1462) See Art. **2301.**

(1463) See Art. **2303.**

(1464) See Art. **2304.**

(1465) See Art. **2305.**

(1466) See Arts. **2305** and **2311.**

(1467) See Art. **2306.**

(1468) See Art. **2307.**

(1469) See Art. **2308.**

(1470) See Art. **2309.**

(1471) See Art. **2310.**

(1472) See Art. **2311.**

(1473) See Art. **2312.**

(1474) See Art. **2312.**

(1475) See Art. **2312.**

(1476) See Art. **2313.**

(1477) See Art. **2314.**

(1478) As the hold or grip increases directly as the square of the number of coils, the proportion of grip the latter will have compared with the former is as $2^2 : 4^2$, or as 4 : 16; that is, the rope turned four times around the drum will have 4 times the grip or hold that the rope coiled twice around the drum has. See Art. **2315.**

(1479) It would equal two complete coils on one wheel, and the grip or haulage power would be four times that of a single coil around one wheel. See Art. **2315.**

(1480) See Arts. **2317** and **2318.**

(1481) See Art. **2319.**

(1482) See Art. **2320.**

(1483) See Art. **2321.**

(1484) See Art. **2322.**

(1485) 50 lb. See Art. **2324.**

(1486) See Art. **2324.**

(1487) See Art. **2325.**

(1488) Applying formula **197,** we have

$$.12 \times 3,500 + \frac{3,500}{40} = 507.5 \text{ lb.} \quad \text{Ans.}$$

(1489) See Art. **2327.** By adding the weight of the empty car to that of the loaded car and dividing the sum by the coefficient of friction, or 40.

(1490) Applying formula **197,** we have $F_2 = .10 \times 4,200 + \frac{4,200}{40} = 525$ lb., the force required to move the rope. Applying formula **199,** we have $F = .10 \times (4,000 - 1,800) - \frac{4,000 + 1,800}{40} = 75$ lb., the available gravity force due to one pair of cars. Therefore, the number of cars that must run in a train is $525 \div 75 = 7$ cars. Ans.

(1491) See Art. **2328.**

(1492) The rope weighs $250 \times 1.5 = 375$ lb. Applying formula **197,** we have $F_2 = .25 \times 375 + \frac{375}{40} = 103.125$ lb., the power necessary to raise the rope. Applying formula **199,** and assuming that the loaded car is at the bottom of the jig and the balance car at the bottom of the plane, we have $F = .25\,(4{,}000 - 2{,}900) - \frac{4{,}000 + 2{,}900}{40} = 102.5$ lb., the available gravity force at the descent of the full car. Now, as it requires 103.125 lb. to move the rope, and there is only 102.5 of gravity force available, it is plain that this jig will not operate.

(1493) See Art. **2328.**

(1494) See Art. **2329.**

(1495) See Art. **2329.**

$$\frac{(2 \times 1{,}800) + 1{,}200}{40} + .25\,(2 \times 1{,}800 + 1{,}200) = 1{,}320 \text{ lb.}$$

Ans.

(1496) See Arts. **2330** and **2331.**

(1497) See Arts. **2332** and **2333.**

(1498) See Art. **2337.**

(1499) See Art. **2339.**

(1500) See Art. **2340.**

(1501) See Art. **2342.**

(1502) See Art. **2343.**

(1503) See Art. **2343.**

(1504) See Art. **2344.**

(1505) See Art. **2345.**

(1506) See Art. **2346.**

(1507) See Art. **2347.**

(1508) Applying formula **200,**

$$T = \frac{(18 \times 4{,}000) + (5{,}000 \times .88)}{40} + .05\,(18 \times 4{,}000 + 5{,}000 \times .88) =$$

5,730 lb. Ans.

(1509) The velocity of the train is $\frac{10 \times 5,280}{60} = 880$ ft. per min. $880 \times 5,730 = 5,042,400$ = foot-pounds of work per minute required of the engine. $5,042,400 \div 33,000 = 152.8$ H. P. Ans.

(1510) See Art. **2349.**

(1511) See Art. **2350.**

(1512) See Art. **2351.**

(1513) The weight of the rope $= 3,000 \times 2 \times .88 = 5,280$ lb. The weight of 25 empty cars weighing 1,500 lb. each $= 1,500 \times 25 = 37,500$ lb.; therefore, the resistance due to friction $= \frac{5,280 + 37,500}{40} = 1,069.5$ lb. The resistance due to gravity $= 37,500 \times .05 = 1,875$ lb. Then, $1,069.5 + 1,875 = 2,944.5$ lb., the tension on the rope. 12 miles per hour $= \frac{5,280 \times 12}{60} = 1,056$ ft. per minute.

$$\frac{1,056 \times 2,944.5}{33,000} = 94.2 \text{ H. P. Ans.}$$

Or, by formula **201**, the tension can be found as follows:

$$T = \frac{37,500 + 5,280}{40} + .05 \times 37,500 = 2,944.5 \text{ lb.}$$

The H. P. can be found by formula **202**, as follows:

$$H = \frac{2,944.5 \times 1,056}{33,000} = 94.2 \text{ H. P. Ans.}$$

(1514) See Art. **2354.**

(1515) See Art. **2355.**

(1516) See Art. **2356.**

(1517) See Arts. **2357** and **2358.**

(1518) See Art. **2358.**

(1519) See Art. **2359.**

(1520) See Art. **2360.**

(1521) See Art. **2365.**

(1522) As the roads are level, there is no tension due to grade, and formula **201** becomes simply $T = \frac{W + w}{40} = \frac{90,000 + 9,100}{40} = 2,477.5$ lb., the tension in the main rope. Ans.

To find the tension in the tail-rope, the weight of the train of empty cars is found.

Then, $T_1 = \frac{30,000 + 9,100}{40} = 977.5$ lb. Ans.

As the conditions of the problem require the maximum tension on the rope, we take that on the main rope, or 2,477.5 lb., and applying formula **202,**

$$H = \frac{2,477.5 \times \frac{(10 \times 5,280)}{60}}{33,000} = \frac{2,180,200}{33,000} = 66.1 \text{ H. P.} \quad \text{Ans.}$$

(1523) Applying formula **201,** we have

$$T = \frac{90,000 + 12,480}{40} + .03 \times 90,000 = 5,262 \text{ lb.} \quad \text{Ans.}$$

(1524) Applying formula **203,** we have

$$P_1 = \frac{90\,(6,000 + 60)}{6,000} = 90.9 \text{ H. P.}$$

(1525) As the gravity force due to the pitch of the incline reduces the tension on the main rope, it must be treated negatively. Then, formula **201** becomes

$$T = \frac{W + w}{40} - a \times W = \frac{100,000 + 7,040}{40} - .04 \times 100,000 =$$

$-$ 1,324 lb., or the negative tension on the main rope. Ans.

This means that there is not only no tension on the main rope, but an excess of gravity force equal to 1,324 lb. The gravity force in the case of hauling the train of empty cars is positive, and can be found by use of formula **201.**

$$T = \frac{40,000 + 7,040}{40} + .04 \times 40,000 = 2,776 \text{ lb.,}$$

the tension of the tail-rope. Ans.

No horsepower is exerted on the main rope, because, as shown previously, the tension is negative. By using formula **202**, the horsepower exerted over the tail-rope is

$$H = \frac{2,776 \times \frac{(5,280 \times 11)}{60}}{33,000} = 81.4 \text{ H. P. Ans.}$$

(1526) By the use of formula **204**, we have $T = \frac{5,000 \times 20 + 4,000\,(3.65 + .6)}{40} - .04 \times (100,000 - 12,200) = (2,925 - 3,512) = -587$ lb., the negative tension. For the tension in the tail-rope, formula **205** is used. $T_1 = \frac{40,000 + 4,000\,(3.65 + 6)}{40} + .04\,(40,000 - 12,200) = 1,425 + 1,112 = 2,537$ lb., the tension on the tail-rope. Now, as the tension on the main rope is negative, there is no power applied to it; on the tail-rope, however, in which there is a tension of 2,537 lb. with the trains running 11 miles per hour, we have $\text{H.P.} = \frac{2,537 \times \frac{(5,280 \times 11)}{60}}{33,000} = 74.4$ H.P. Ans.

(1527) See Art. **2372.**

(1528) See Art. **2372.**

(1529) See Art. **2373.**

(1530) $6,000 + 4,800 + 2,500 + 7,000 + 3,000 = 23,300$; $\frac{23,300}{5} = 4,660$, the mean length. $\frac{5,280 \times 12 \times 10}{4,660 \times 3} = 45.3$, practically 46 trains. Ans.

(1531) $\frac{2,500}{46 \times 2.5} = 21.7$, say 22 cars. Ans.

(1532) Allowing $\frac{1}{3}$ of the time for stoppage, the rope travels for $\frac{2}{3}$ of $10 = 6\frac{2}{3}$ hours, and hauls coal for but $\frac{1}{2}$ this time, or $3\frac{1}{3}$ hours. Hence, the distance the rope travels while hauling coal is $5,280 \times 11 \times 3\frac{1}{3} = 193,600$ feet, and since the mean length of the haulage roads, which is found by dividing their total length by 4, is

$$\frac{4{,}250 + 3{,}012 + 756 + 514}{4} = 2{,}133 \text{ feet.}$$

the number of loaded trains is

$$\frac{193{,}600}{2{,}133} = 90.76, \text{ or } 91. \quad \text{Ans.}$$

The number of cars in each train is

$$\frac{2{,}500}{91 \times 2.5} = 11. \quad \text{Ans.}$$

The weight of the rope is equal to its weight per foot multiplied by twice the maximum haul, or $4{,}250 \times 2 \times 1.5 = 12{,}750$ pounds, and the weight of a loaded car is $2{,}000 + 2.5 \times 2{,}000 = 7{,}000$ pounds.

Substituting in formula **201**,

$$T = \frac{7{,}000 \times 11 + 12{,}750}{40} + .03 \times 77{,}000 = 4{,}553.75 \text{ pounds.}$$

The speed of the train is equal to

$$\frac{5{,}280 \times 11}{60} = 968 \text{ feet per minute.}$$

Applying formula **202**,

$$H = \frac{4{,}553.75 \times 968}{33{,}000} = 133.6 \text{ horsepower, nearly.} \quad \text{Ans.}$$

(1533) See Art. **2374.**

(1534) See Art. **2375.**

(1535) See Art. **2375.**

(1536) See Art. **2376.**

(1537) See Art. **2377.**

(1538) See Art. **2377.**

(1539) See Art. **2377.**

(1540) See Art. **2378.**

(1541) See Art. **2379.**

(1542) See Art. **2379.**

(1543) See Art. **2380.**

(1544) See Arts. **2384** and **2385.**

(1545) See Arts. **2384** and **2386.**

(1546) See Art. **2382.**

(1547) See Art. **2382.**

(1548) See Art. **2387.**

(1549) See Art. **2387.**

(1550) See Arts. **2387** and **2388.**

(1551) See Art. **2388.**

(1552) See Arts. **2390** to **2392.**

(1553) See Art. **2395.**

(1554) See Art. **2395.**

(1555) See Arts. **2397** and **2398.**

(1556) See Art. **2401.**

(1557) By formula **207,** the number of cars on the rope is $\frac{2,500 \times 5,230}{2 \times 5,280 \times 8 \times 1.5} = 103.18$, say 103. Ans.

And, by formula **208,** the distance the cars are apart is $\frac{5,230}{103.18} = 50.68$ ft. Ans.

(1558) See Art. **2406.**

(1559) See Art. **2407.**

(1560) See Arts. **2408** and **2409.**

(1561) See Art. **2411.**

(1562) By formula **207,** the number of loaded cars on the rope at one time is $\frac{976 \times 4,720}{2.5 \times 5,280 \times 10 \times 1.25} = 27.919$. The weight of the loaded cars on one side of the rope will then be $4,000 \times 27.919 = 111,676$ pounds. Taking the weight of an empty car at 1,200 pounds, the weight of the empty cars on the rope will be $1,200 \times 27.919 = 33,502.8$. The weight of the rope is $4,720 \times 2 \times 3 = 28,320$ pounds. Then, substituting in formula **210,** $T = \frac{(111,676 + 33,502.8 + 28,320)}{40} =$ 4,337.47 pounds, the tension on the rope. Ans.

A velocity of $2\frac{1}{2}$ miles an hour is equal to $\frac{2.5 \times 5,280}{60} =$ 220 feet per minute. Using formula **202,** the horsepower is

$$H = \frac{4,337.47 \times 220}{33,000} = 28.92 \text{ H. P.} \quad \text{Ans.}$$

(1563) As the two sides of the rope balance each other and the cars balance each other, only the weight of the coal is subject to the gravity of the grade. Substituting in formula **210,** we have

$$T = \frac{(111,676 + 33,502.8 + 28,320)}{40} + .025(111,676 - 33,502.8) =$$

6,291.8 pounds tension on the rope. Ans.

The velocity is $\frac{2.5 \times 5,280}{60} = 220$ feet per minute. The horsepower is, therefore, $\frac{6,291.8 \times 220}{33,000} = 41.9$ horsepower. Ans.

(1564) Substituting in formula **210,** we have

$$T = \frac{(111,676 + 33,502.8 + 28,320)}{40} - .025(111,676 - 33,502.8) =$$

2,383.14 lb., the tension on the rope. Ans.

The horsepower required is $\frac{2,383.14 \times 220}{33,000} = 15.89$ horse power. Ans.

(1565) See Art. **2414.**

(1566) See Arts. **2414** and **2415.**

(1567) See Art. **2418.**

(1568) See Art. **2420.**

(1569) See Art. **2422.**

(1570) See Arts. **2428** to **2430.**

(1571) See Art. **2431.**

(1572) See Arts. **2432** and **2433.**

(1573) See Art. **2435.**

(1574) See Art. **2437.**

(1575) See Art. **2439.**

(1576) See Art. **2440.**

(1577) See Art. **2443.**

(1578) See Art. **2444.**

(1579) See Art. **2446.**

(1580) See Art. **2446.**

(1581) See Art. **2449.**

(1582) See Art. **2450.**

(1583) See Art. **2450.**

(1584) See Art. **2316.**

HOISTING AND HOISTING APPLIANCES.

(1585) See Art. **2452.**

(1586) Electric motors and steam or compressed-air engines.

(1587) See Art. **2463.**

(1588) See Arts. **2454** and **2455.**

(1589) See Arts. **2458** and **2459.**

(1590) See Art. **2460.**

(1591) See Art. **2462.**

(1592) See Art. **2464.**

(1593) See Art. **2464.**

(1594) See Arts. **2465** to **2469.**

(1595) See Art. **2465.**

(1596) See Art. **2467.**

(1597) See Arts. **2466** and **2467.**

(1598) See Art. **2468.**

(1599) See Art. **2469.**

(1600) See Art. **2470.**

(1601) See Art. **2470.**

(1602) See Art. **2471.**

(1603) The minimum diameter of drum is 60 times the diameter of the rope.

(1604) (*a*), (*b*), and (*c*) See Art. **2471.**
(*d*) See Art. **2473.**

(1605) (*a*) Assume the weight of the rope to be 2,000 lb. Then, the load on the rope is

Material	3,000 lb.
Car	1,800 lb.
Cage	2,400 lb.
Rope	2,000 lb.
Total	9,200 lb.

Using a factor of safety of 10, the breaking load is 92,000 lb. = 46 tons. Referring to Table 46, a 1-inch plow-steel rope with 19 wires to the strand has a breaking load of 47 tons; its weight is 1.58 lb. per ft. In this case, the weight is 1,200 × 1.58 = 1,896 lb., which is quite close to the assumed weight. Therefore, a 1-inch rope should be used. Ans.

(*b*) The smallest allowable drum has a diameter 60 times that of the rope, or 60 in. = 5 ft. Ans.

(1606) Using two cages, the gross load is

Material	3,000 lb.
2 cars	3,600 lb.
2 cages	4,800 lb.
Rope	1,896 lb.
Total	13,296 lb.

The net load is

Material	3,000 lb.
Rope	1,896 lb.
Total	4,896 lb.

Actual load = net load + 10% of gross load = 4,896 + .10 × 13,296 = 6,225.6 lb. Ans.

(1607) See Art. **2474.**

(1608) The working diameter of the drum is 60 + 1 = 61 in. = $\frac{61}{12}$ ft.

$$6,225.6 \times \frac{61}{12} \times 3.1416 = 99,421.6 \text{ ft.-lb.} \quad \text{Ans.}$$

(1609) Using formula **211**,

$$D = 1.97\sqrt[3]{\frac{99{,}421.6}{48.76 \times 1.5}} = 21.82, \text{ say } 22 \text{ in. Ans.}$$

Stroke = 22 × 1.5 = 33 in. Ans.

(1610) Area of piston = $12^2 \times .7854 = 113.1$ sq. in. The piston travels per revolution $\frac{24}{12} \times 2 = 4$ ft. Total pressure on piston = 113.1 × 40. Work = total pressure × distance traveled by piston = 113.1 × 40 × 4 = 18,096 ft.-lb. Ans.

(1611) See Arts. **2471** and **2472**.

(1612) (*a*) Using formula **211**,

$$D = 1.97\sqrt[3]{\frac{36{,}000}{40 \times 2.5}} = 14 \text{ in. Ans.}$$

Stroke = 14 × 2.5 = 35 in. Ans.

(*b*) Area of piston = $14^2 \times .7854 = 153.938$ sq. in.
Length of crank = $\frac{35}{2} = 17\frac{1}{2}$ in.
Turning moment = total pressure on piston × length of crank = $153.938 \times 40 \times 17\frac{1}{2} = 107{,}757$ in.-lb. Ans.

(1613) See Art. **2476**.

(1614) It may be smaller. See Art. **2477**.

(1615) Larger. See Art. **2478**.

(1616) See Art. **2480**.

(1617) See Art. **2482**.

(1618) See Art. **2484**.

(1619) Least diameter of drum = $1\frac{1}{2} \times 60 = 90$ in.
Effective diameter = $90 + 1\frac{1}{2} = 91\frac{1}{2}$ in. = $7\frac{5}{8}$ ft.
Circumference = $7\frac{5}{8} \times 3.1416 = 24$ ft., nearly.

$$\text{Number of turns} = \frac{1{,}800}{24} = 75.$$

Adding 5 turns for friction and for possible overwinding, the number of turns is 80.

Width for each turn = $1\frac{1}{2} + \frac{1}{4} = 1\frac{3}{4}$ in.

$80 \times 1\frac{3}{4} = 140$ in. = 11 ft. 8 in. Ans.

(1620) See Art. **2488.**

(1621) See Arts. **2488** and **2491.**

(1622) See Arts. **2485** and **2526.**

(1623) Assume, first, that the rope weighs 2,000 lb. Then, the load on the rope is

Material	4,000 lb.
Car	3,000 lb.
Cage	3,200 lb.
Rope	2,000 lb.
Total	12,200 lb.

Using a factor of safety of 10, the breaking load is 61 tons, which, from Table 46, requires a $1\frac{3}{8}$-inch rope, the weight of which is 3 lb. per foot. The weight of rope is $3 \times 800 = 2,400$ lb., which adds 400 lb. to the previous total weight, making it 12,600 lb., or 6.3 tons. The breaking load is, therefore, 63 tons, and the $1\frac{3}{8}$-inch rope is correct. The minimum diameter is $1\frac{3}{8} \times 60 = 82\frac{1}{2}$ in. $= 7$ ft., nearly. Ans.

Using formula **213**,

$$D = \frac{7\,(4,000 + 2 \times 6,200 + 2 \times 2,400)}{4,000 + 2 \times 6,200} = 9.05 \text{ ft., say 9 ft.}$$

Ans.

(1624) See Art. **2492.**

(1625) See Arts. **2498** to **2503.**

(1626) See Art. **2505.**

(1627) See Arts. **2507** and **2510.**

(1628) See Art. **2511.**

(1629) See Art. **2512.**

(1630) See Art. **2516.**

(1631) See Art. **2514.**

(1632) See Art. **2516.**

(1633) See Art. **2516.**

(1634) See Arts. **2518** and **2520.**

(1635) See Art. **2497.**

(1636) See Art. **2529.**

(1637) See Arts. **2530** and **2531.**

(1638) See Art. **2532.**

(1639) See Art. **2535.**

(1640) See Art. **2535.**

(1641) See Art. **2537.**

(1642) See Arts. **2538** and **2539.**

(1643) See Art. **2540.**

(1644) See Art. **2543.**

(1645) See Art. **2546.**

(1646) See Art. **2547.**

(1647) See Art. **2549.**

(1648) See Art. **2549.**

(1649) See Art. **2559.**

(1650) See Art. **2560.**

(1651) See Arts. **2563** and **2564.**

SURFACE ARRANGEMENTS
OF
BITUMINOUS MINES.

(1652) By limiting the size of the opening through which the cars are hauled Small seams necessitate the use of low cars, and a bad roof necessitates narrow headings, and, consequently, comparatively narrow cars. See Art. **2568.**

(1653) See Art. **2569** and Figs. 947 and 948.

(1654) See Arts. **2572** to **2575** and Fig. 947.

(1655) See Arts. **2576** to **2579** and Fig. 948.

(1656) See Art. **2611.**

(1657) See Art. **2583.**

(1658) From 1 to 1.3 times the vertical height of the center of the sheaves above the center of the drum, or from 60 to 78 feet. See Art. **2586.**

(1659) See Art. **2589.**

(1660) See Art. **2589.**

(1661) See Art. **2596.**

(1662) See Art. **2597.**

(1663) See Art. **2600.**

(1664) See Art. **2601.**

(1665) See Figs. 947 and 948 and Art. **2602.**

(1666) See Art. **2606.**

(1667) See Art. **2607.**

(1668) See Art. **2610.**

(1669) See Arts. **2613** and **2620.**

(1670) See Art. **2614.**

(1671) See Art. **2616.**

(1672) See Arts. **2618** and **2619.**

(1673) See Art. **2622.**

(1674) $\frac{8}{2} = 4$ trips for each car per day. $4 \times 2 = 8$ tons for each car per day. Hence, $\frac{1,500}{8} = 187.5$, or 188 cars. Ans.

(1675) See Figs. 963 and 964 and accompanying description.

(1676) See Art. **2625.**

(1677) See Art. **2630.**

(1678) See Arts. **2630** and **2640.**

(1679) See Figs. 965 and 966 and accompanying descriptions.

(1680) See Art. **2631.**

(1681) See Art. **2631.**

(1682) See Art. **2634.**

(1683) See Art. **2634.**

(1684) See Art. **2634.**

(1685) See Art. **2638.**

(1686) See Arts. **2639** and **2640.**

(1687) See Art. **2644.**

(1688) See Art. **2647.**

(1689) See Art. **2649.**

(1690) See Art. **2650.**

(1691) See Art. **2654.**

(1692) See Art. **2654.**

(1693) See Art. **2660.**

(1694) See Art. **2657.**

(1695) See Art. **2655.**

(1696) See Art. **2655.**

(1697) See Art. **2669.**

(1698) See Art. **2670.**

(1699) See Art. **2663.**

(1700) See Art. **2664.**

(1701) See Art. **2664.**

(1702) See Art. **2664.**

(1703) See Art. **2665.**

(1704) See Art. **2665.**

(1705) See Art. **2665.**

(1706) See Art. **2666.**

(1707) See Art. **2667.**

(1708) See Art. **2669.**

(1709) See Art. **2672.**

(1710)

	Tons.
Output of lump coal $= 1{,}500 \times .70 =$	1,050
Output of nut coal $= 1{,}500 \times .15 =$	225
Output of pea coal $= 1{,}500 \times .08 =$	120
Output of slack coal $= 1{,}500 \times .07 =$	105

The lengths of sidings are:

$$\left.\begin{array}{ll} \text{Lump,} & \dfrac{1{,}050 \times 34}{30} = 1{,}190 \text{ ft.} \\ \text{Nut,} & \dfrac{225 \times 34}{30} = 255 \text{ ft.} \\ \text{Pea,} & \dfrac{120 \times 34}{30} = 136 \text{ ft.} \\ \text{Slack,} & \dfrac{105 \times 34}{30} = 119 \text{ ft.} \end{array}\right\} \text{Ans.}$$

See Art. **2673.**

(1711) See Art. **2674.**

(1712) See Arts. **2675** and **2676.**

(1713) See Art. **2675.**

(1714) See Art. **2676.**

(1715) 8 hr. = 8 × 60 = 480 min.
480 ÷ 20 = 24 trips.
1,500 ÷ 24 = 62, the number of tons hauled per trip.
62 ÷ 2 = 31, number of cars per smallest trip.
31 × 8 = 248 ft., smallest length of siding.
248 ft. × 2 = 496 ft., proper length. Ans. See Art. **2677.**

(1716) See Art. **2684.**

(1717) See Arts. **2695** to **2698.**

SURFACE ARRANGEMENTS
OF
ANTHRACITE MINES.

(1718) A drift is driven in the coal seam, while a tunnel is driven across the measures. See Arts. **2720** and **2721**.

(1719) See Art. **2747.**

(1720) See Art. **2797.**

(1721) See Art. **2862.**

(1722) See Art. **2802.**

(1723) See Arts. **2738** and **2739.**

(1724) (*a*) Since the inclination of the dump chute is $3\frac{1}{2}$ inches per foot, or $\frac{7}{24}$, the tower end is $\frac{7}{24} \times 200 = 58\frac{1}{3}$ feet higher than the breaker end. Therefore, the tower end of the chute is $90 + 58\frac{1}{3} = 148\frac{1}{3}$ feet above the wall or the breaker, or $148\frac{1}{3} - 12 = 136.33$ feet above the wall of tower. Ans.

(*b*) The length of the chute is the length of the hypotenuse of a right-angled triangle of which the base is 200 feet and the altitude is the rise, $58\frac{1}{3}$ feet. The length is, therefore, $\sqrt{200^2 + 58\frac{1}{3}^2} = 208.33$ ft. Ans.

(1725) See Arts. **2731** and **2904.**

(1726) See Art. **2724.**

(1727) See Arts. **2778** to **2792.**

(1728) See Art. **2858.**

(1729) See Arts. **2793** and **2794.**
(1730) See Art. **2826.**
(1731) See Art. **2868.**
(1732) See Art. **2740.**
(1733) See Art. **2893.**
(1734) See Art. **2743.**
(1735) See Art. **2806.**
(1736) See Art. **2875.**
(1737) See Art. **2731.**
(1738) See Art. **2877.**
(1739) See Art. **2803.**
(1740) See Art. **2729.**
(1741) At right angles. See Art. **2895.**
(1742) See Art. **2745.**
(1743) See Art. **2803.**
(1744) See Art. **2867.**
(1745) See Art. **2840.**
(1746) See Art. **2749.**
(1747) See Art. **2800.**
(1748) See Art. **2823.**
(1749) See Arts. **2866** to **2874.**
(1750) See Art. **2750.**
(1751) See Art. **2799.**
(1752) See Art. **2835.**
(1753) See Art. **2731.**
(1754) See Art. **2842.**
(1755) See Art. **2770.**
(1756) See Art. **2896.**

(1757) See Art. **2735.**

(1758) See Art. **2758.**

(1759) See Art. **2833.**

(1760) See Arts. **2829** and **2924.**

(1761) See Art. **2830.**

(1762) See Art. **2856.**

(1763) See Art. **2771.**

(1764) See Art. **2889.**

(1765) See Art. **2854.**

(1766) See Art. **2925.**

(1767) See Art. **2817.**

(1768) See Art. **2764.**

(1769) See Art. **2822.**

(1770) See Art. **2828.**

(1771) See Art. **2924.**

(1772) See Art. **2763.**

(1773) The Guibal. See Art. **2766.**

(1774) See Art. **2829.**

(1775) See Art. **2767.**

(1776) See Art. **2866.**

(1777) The description is given in Art. **2783**

(1778) See Art. **2723.**

(1779) See Art. **2866.**

(1780) See Art. **2851.**

(1781) See Art. **2762.**

(1782) See Art. **2782.**

(1783) See Art. **2883.**

(1784) See Art. **2773.**

(1785) See Art. **2849.**

(1786) See Art. **2830.**

(1787) See Art. **2884.**

(1788) See Arts. **2778** and **2792.**

(1789) See Art. **2846.**

(1790) See Art. **2857.**

(1791) See Arts. **2831** and **2896.**

PERCUSSIVE AND ROTARY BORING.

(1) See Art. **2.**

(2) See Art. **4.**

(3) See Art. **84.**

(4) See Art. **5.**

(5) See Art. **95.**

(6) See Art. **21.**

(7) See Arts. **92** and **94.**

(8) See Art. **11.**

(9) See Art. **87.**

(10) See Art. **22.**

(11) See Arts. **105–107.**

(12) See Art. **133.**

(13) See Art. **24.**

(14) See Art. **118.**

(15) See Art. **41.**

(16) See Art. **147.**

(17) See Art. **88.**

(18) See Art. **36.**

(19) See Arts. **52** and **53.**

(20) See Art. **56.**

(21) See Art. **69.**

(22) See Art. **81.**

(23) See Art. **56.**

(24) See Art. **91.**

(25) See Art. **102.**

(26) See Art. **54.**

(27) See Art. **42.**

(28) See Art. **72.**

(29) See Art. **60.**

(30) See Arts. **7** and **8.**

(31) See Art. **4.**

(32) See Art. **89.**

(33) See Art. **100.**

(34) See Art. **10.**

(35) See Art. **50.**

(36) See Art. **9.**

(37) See Art. **12.**

(38) See Art. **44.**

(39) See Art. **46.**

(40) See Art. **55.**

(41) See Art. **13.**

(42) See Art. **16.**

(43) See Art. **121.**

(44) See Arts. **113** and **114.**

(45) See Art. **124.**

(46) See Art. **135.**

(47) See Art. **15.**

(48) See Art. **152.**

(49) See Art. **17.**

(50) See Art. **160.**

(51) See Art. **110.**

(52) See Art. **15.**

(53) See Art. **22.**

(54) See Art. **148.**

(55) See Art. **18.**

(56) See Art. **15.**

(57) See Art. **39.**

(58) See Art. **40.**

(59) See Art. **120.**

(60) See Art. **37.**

(61) See Art. **64.**

(62) See Art. **73.**

(63) See Art. **142.**

(64) See Art. **67.**

(65) See Arts. **116-118.**

(66) See Art. **66.**

(67) See Art. **40.**

(68) See Arts. **128-130.**

(69) See Art. **36.**

(70) See Art. **74.**

(71) See Art. **65.**

(72) See Art. **139.**

(73) See Arts. **33** and **34.**

(74) See Art. **26.**

(75) See Art. **150.**

(76) See Art. **169.**

(77) See Art. **35.**

(78) See Art. **141.**

(79) See Art. **23.**

(80) See Art. **25.**

(81) See Art. **19.**

(82) See Art. **14.**

(83) See Art. **24.**

(84) See Art. **28.**

(85) See Art. **30.**

(86) See Art. **28.**

(87) See Art. **31.**

(88) See Art. **63.**

(89) See Art. **143.**

(90) See Art. **149.**

COMPRESSED-AIR COAL-CUTTING MACHINERY.

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Art. **7.**

(4) See Arts. **11** and **12.**

(5) See Art. **20.**

(6) See Art. **26.**

(7) See Art. **31.**

(8) See Art. **37.**

(9) See Art. **1.**

(10) See Arts. **4** and **9.**

(11) See Art. **10.**

(12) See Art. **14.**

(13) See Art. **22.**

(14) Substituting in formula **3,**

$$V = \sqrt{\frac{2 \times 32.16 \times 2^2 \times 3.1416 \times 80 \times 12}{250 \times 12}} = 16.08 \text{ ft. per sec.}$$

Ans.

(15) See Art. **32.**

(16) See Art. **37.**

(17) See Art. **40.**

(18) See Art. **38.**

(19) See Art. **43.**

(20) See Arts. **2** and **21.**

(21) See Art. **5.**

(22) See Art. **13.**

(23) See Art. **28.**

(24) See Art. **34.**

(25) See Art. **39.**

(26) See Art. **45.**

(27) See Art. **2.**

(28) See Art. **5.**

(29) See Art. **6.**

(30) See Art. **16.**

(31) See Art. **23.**

(32) See Art. **47.**

(33) See Arts. **29** and **30.**

(34) See Art. **33.**

(35) See Art. **39.**

(36) Substituting in formula **4,**

$$F = \frac{18 \times 33{,}000}{330} = 1{,}800 \text{ lb.} \quad \text{Ans.}$$

(37) See Art. **5.**

(38) See Art. **9.**

(39) See Art. **18.**

(40) Using formula **1,**

$$U = \frac{6 \times 33,000}{200} = 990 \text{ ft.-lb.} \quad \text{Ans.}$$

(41) See Art. **36.**

(42) See Art. **6.**

(43) See Art. **17.**

(44) See Art. **19.**

(45) Using formula **2,**

$$F = \frac{22^2 \times 175 \times 12}{2 \times 32.16 \times 2.5} = 6,320.9 \text{ lb.} \quad \text{Ans.}$$

(46) See Art. **19.**

(47) See Arts. **46** and **47.**

(48) See Art. **46.**

(49) See Arts. **30** and **31.**

(50) No key.

DYNAMOS AND MOTORS.

(PART 1.)

(1) The end b; because, in looking at that end, the current circulates around the helix in an opposite direction to the hands of a watch. (Art. **29**.)

(2) (*a*) Negative. (Art. **7.**)
(*b*) Negative. (Art. **7.**)
(*c*) Positive. (Art. **7.**)

(3) By formula **6,** $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total generated E. M. F. in volts, and R is the total resistance in ohms of the circuit. In this example, $E = 20$ volts and $R = 30 + 80 = 110$ ohms; hence, $C = \frac{E}{R} = \frac{20}{110} = .1818$ ampere. Ans.

(4) Let A represent the first branch and B the second; then, $r_1 = 16.2$ ohms, $r_2 = 14.1$ ohms, and $C = 6.37$ amperes.

The current c_1 in branch A is found by using formula **10;** substituting, gives $c_1 = \frac{C r_2}{r_1 + r_2} = \frac{6.37 \times 14.1}{16.2 + 14.1} = 2.9643$ amperes. Ans.

The current c_2 in branch B is found by using formula **11;** substituting, gives

$$c_2 = \frac{C r_1}{r_1 + r_2} = \frac{6.37 \times 16.2}{16.2 + 14.1} = 3.4057 \text{ amperes. Ans.}$$

(5) From formula **23,** $W =$ H. P. $\times$ 746, where H. P. is the horsepower and W is the power in watts. In this example, H. P. = 2.33 horsepower; hence, $W =$ H. P. $\times 746 = 2.33 \times 746 = 1{,}738.18$ watts. Ans.

(6) (*a*) From Art. **64** and formula **6**, $C = \frac{E'}{R'}$, where C is the current in amperes, E' is the difference of potential in volts between two points, and R' is the resistance in ohms between them. In this example, $E' = 58.4$ volts and $R' = 2.3$ ohms; hence, $C = \frac{E'}{R'} = \frac{58.4}{2.3} = 25.3913$ amperes. Ans.

(*b*) From formula **21**, $W = \frac{E^2}{R}$, where W is the power in watts, E is the E. M. F., or difference of potential in volts, and R is the resistance in ohms. In this example, $E = 58.4$ volts and $R = 2.3$ ohms; hence,

$$W = \frac{E^2}{R} = \frac{58.4^2}{2.3} = \frac{3,410.56}{2.3} = 1,482.8521 \text{ watts. Ans.}$$

(*c*) By formula **22**, H. P. $= \frac{W}{746}$; by formula **21**, $W = \frac{E^2}{R}$; therefore (see Art. **81**), H. P. $= \frac{E^2}{746\,R}$, where H. P. is the horsepower, E the E. M. F., or difference of potential in volts, and R the resistance in ohms.

Hence, H. P. $= \frac{58.4^2}{746 \times 2.3} = \frac{3,410.56}{1,715.8} = 1.9877$ horsepower. Ans.

(7) Platinum, as it follows zinc in the list (Art. **13**).

(8) Towards the *east* (Art. **26**).

(9) By formula **8**, $E = C\,R$, where E is the total E.M.F. in volts developed in a closed circuit, C is the current in amperes flowing, and R is the total resistance in ohms of the circuit. In this example, $C = .75$ ampere and $R = 17.2 + 8.2 + 11.3 = 36.7$ ohms; hence, $E = C\,R = .75 \times 36.7 = 27.525$ volts, the total E. M. F. developed in the battery.

By derivation from formula **8**, $E' = C\,R'$, where E' is the difference of potential in volts between two points, C is the current in amperes flowing, and R' is the resistance in ohms between the two points.

Between a and b, $R' = 11.3$ ohms and $C = .75$ ampere; hence, $E' = C R' = .75 \times 11.3 = 8.475$ volts. Ans.

Between b and c, $R' = 8.2$ ohms and $C = .75$ ampere; hence, $E' = C R' = .75 \times 8.2 = 6.15$ volts. Ans.

Between a and c, the difference of potential is the difference of potential between a and b plus that between b and c, which is $6.15 + 8.475 = 14.625$ volts. Or, since the difference of potential between a and c is the available E. M. F. of the battery, when a current of .75 ampere is flowing, it can be calculated by using formula **9**, $E' = E - C r_i$, where E' is the available E. M. F., E is the total E. M. F. developed in the battery, C is the current flowing, and r_i is the internal resistance of the battery. In this case, $E = 27.525$ volts, $C = .75$ ampere, and $r_i = 17.2$ ohms; therefore, $E' = E - C r_i = 27.525 - (.75 \times 17.2) = 14.625$ volts. Ans.

(10) The sectional area of a wire .2 in. in diameter is $.7854 \times .2 \times .2 = .031416$ sq. in., or, nearly, .0314 sq. in.

Reduce the specific resistance in microhms to the resistance in ohms by dividing by 1,000,000 (Art. **44**), which gives $\frac{.5921}{1,000,000} = .0000005921$ ohm; or, in other words, the resistance of a piece of silver one inch long, and whose sectional area is one square inch, is .0000005921 ohm. Next, from this resistance and length calculate the resistance of 1,000 feet of the silver with a sectional area of 1 sq. in., by using formula **1**, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance, r_2 is the resistance after the length of the conductor is changed, l_1 is the original length of the conductor, and l_2 is the changed length. In this example, $r_1 = .0000005921$ ohm, $l_1 = 1$ inch, and $l_2 = 1,000$ feet, or 12,000 inches. Hence, by substituting, $r_2 = \frac{r_1 l_2}{l_1} = .0000005921 \times \frac{12,000}{1} = .0071052$ ohm; that is, the resistance of 1,000 feet of silver having a sectional area of 1 sq. in. is .0071052 ohm. From this result calculate the resistance of 1,000 feet of the silver when its sectional area is .0314 sq. in., by using formula **2**, $r_2 = \frac{r_1 a_1}{a_2}$, where r_1 is the original resistance, r_2 is the resistance after the sectional area has been changed, a_1 is the original area, and a_2 is the changed sectional area. At this stage of the example, $r_1 = .0071052$ ohm, $a_1 = 1$ sq. in., and $a_2 = .0314$ sq. in. Hence,

$$r_2 = \frac{.0071052 \times 1}{.0314} = .2262 \text{ ohm.} \quad \text{Ans.}$$

(11) By formula **8**, $E = C R$, where E is the total E. M. F. in volts developed in a closed circuit, C is the current in amperes flowing, and R is the total resistance in ohms of the circuit. In this example, $C = .127$ ampere and $R = 36.2 + 21.7 = 57.9$ ohms. Hence, by substituting, $E = C R = .127 \times 57.9 = 7.3533$ volts. Ans.

(12) By formula **14**, $Q = C t$, where Q is the quantity of electricity in coulombs which passes through a circuit, C is the current in amperes flowing in that circuit, and t is the time in seconds during which the current flows. In this example, $C = 8.32$ amperes and $t = 2.25 \times 60 \times 60 = 8{,}100$ seconds. Hence, by substituting, $Q = C t = 8.32 \times 8{,}100 = 67{,}392$ coulombs. Ans.

(13) By formula **19**, $W = C E$, where W is the power in watts, E is the E. M. F. in volts, and C is the current in amperes. In this example, $E = 112.5$ volts and $C = 12.2$ amperes. Hence, by substituting, $W = C E = 12.2 \times 112.5 = 1{,}372.5$ watts. Ans.

(14) By formula **12**, the joint resistance of a derived circuit of two branches in parallel $R'' = \frac{r_1 r_2}{r_1 + r_2}$. In this case, $r_1 = 2.4$ and $r_2 = 987.3$; then their joint resistance in parallel $R'' = \frac{2.4 \times 987.3}{2.4 + 987.3} = \frac{2{,}369.52}{989.7} = 2.3941$ ohms. Ans.

(15) By formula **4**, $r_2 = r_1 (1 + t k)$, where r_1 is the original resistance of a conductor, r_2 is the resistance after a rise in temperature, k is the temperature coefficient, and t is the rise of temperature in degrees F. In this example,

$r_1 = 43.2$ ohms, $t = 85 - 60 = 25°$ F., and $k = .002155$, from Table 1. Hence, by substituting, $r_2 = r\ (1 + t\,k) = 43.2\ (1 + 25 \times .002155) = 43.2 \times 1.053875 = 45.5274$ ohms. Ans.

(16) By formula **13,** the joint resistance of three conductors in parallel $R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}$, where r_1, r_2, and r_3 are the separate resistances of the three conductors, respectively. In this example, let $r_1 = 37$ ohms, the resistance of A; $r_2 = 45$ ohms, the resistance of B; and $r_3 = 72$ ohms, the resistance of C. Substituting, we have

$$\frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} = \frac{37 \times 45 \times 72}{45 \times 72 + 37 \times 72 + 37 \times 45} = \frac{119,880}{7,569} =$$

15.8383 ohms, the joint resistance of the three conductors A, B, and C connected in parallel. Ans.

(17) By formula **8,** $E = C\,R$, where E is the total E. M. F. in volts developed within a closed circuit, C is the current in amperes, and R is the total resistance in ohms of the circuit. In this example, $C = 2.73$ amperes and $R = 49.3$ ohms; hence, by substituting, $E = C\,R = 2.73 \times 49.3 = 134.589$ volts. Ans.

(18) From Art. **43,** the joint resistance of several conductors connected in series is equal to the sum of their separate resistances; hence, in this example, the joint resistance of the four conductors A, B, C, and D, in series, is $3 + 19 + 72 + 111 = 205$ ohms. Ans.

(19) (*a*) By formula **7,** $R = \frac{E}{C}$, where R is the total resistance in ohms of a closed circuit, E is the total E. M. F. in volts developed, and C is the current in amperes flowing in the circuit. In this example, $E = 28.2$ volts and $C = 5.2$ amperes; hence, $R = \frac{28.2}{5.2} = 5.423$ ohms. Ans.

(*b*) The total resistance of a closed circuit, Art. **60,** is equal to the sum of the internal and external resistances. Since in this example the external resistance is 7 times the internal, and their sum is 5.423, let $\frac{1}{8}$ of the total resistance represent the internal, and, therefore, $\frac{7}{8}$ of the total resistance represents the external resistance. Hence, $\frac{1}{8} \times 5.423 =$ .677875 ohm, the internal resistance. Ans.

And $\frac{7}{8} \times 5.423 = 4\ 745125$ ohms, the external resistance. Ans.

(20) We here use formula **16**, $J = C^2 R t$, where J is the work in joules, C is the current in amperes, R is the resistance in ohms, and t is the time in seconds. In this case, $C = 14.2$ amperes, $R = 8$ ohms, $t = 4{,}500$ seconds. Then, the work done $= 14.2 \times 14.2 \times 8 \times 4{,}500 = 7{,}259{,}040$ joules. Ans.

(21) By formula **5**, $r_2 = \frac{r_1}{1 + t k}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its temperature has fallen, t is the fall of temperature in degrees F., and k is the temperature coefficient. In this example, $r_1 = 214$ ohms, $t = 82 - 50 = 32°$ F., and $k = .002094$, from Table 1. Hence,

$$r_2 = \frac{r_1}{1 + t k} = \frac{214}{1 + 32 \times .002094} = \frac{214}{1.067008} = 200.5608 \text{ ohms.}$$

Ans.

(22) From Art. **75**, the separate resistance of any branch of a derived circuit is equal to the difference of potential between where all the branches divide and where they unite, divided by the current in that branch.

Hence, the separate resistance of branch A is $\frac{11.6}{6.7} =$ 1.7313 ohms. Ans.

The separate resistance of branch B is $\frac{11.6}{4.9} = 2.3673$ ohms. Ans.

(23) By formula **7**, $R = \frac{E}{C}$, where R is the total resistance in ohms of a closed circuit, E is the total E. M. F. in volts developed in the circuit, and C is the current in amperes flowing in the circuit. In this example, $E = 22.4$ volts and $C = .43$ ampere; hence, $R = \frac{E}{C} = \frac{22.4}{.43} = 52.093$

ohms, the total resistance of the circuit. Since the total resistance of a closed circuit is equal to the sum of the external and internal resistances, the external resistance must be the difference between the total resistance and the internal resistance. Hence, the external resistance $= 52.093 - 13.4 = 38.693$ ohms. Ans.

(24) By transposition of terms in formula **14,** $C = \frac{Q}{t}$, where C is the current in amperes, Q is the quantity of electricity in coulombs, and t is the time in seconds. In this example, $Q = 368{,}422$ coulombs and $t = 4.5 \times 60 \times 60 = 16{,}200$ seconds; hence, $C = \frac{Q}{t} = \frac{368{,}422}{16{,}200} = 22.7421$ amperes. Ans.

(25) By formula **16,** $J = C^2 R t$, where J is the work done in joules, C is the current in amperes, R is the resistance in ohms, and t is the time in seconds. In this example, $C = 2.4$ amperes, $R = 45$ ohms, and $t = 3{,}000$ seconds. Then the electrical work done $= 2.4 \times 2.4 \times 45 \times 3{,}000 = 777{,}600$ joules. By formula **18,** the mechanical work done $=$ F. P. $= .7373\ J = .7373 \times 777{,}600 = 573{,}324.48$ foot-pounds. Ans.

(26) By formula **22,** H. P. $= \frac{W}{746}$; by formula **19,** $W = C E$; therefore (see Art. **81**), H. P. $= \frac{E\,C}{746}$, where H. P. is the horsepower, E is the E. M. F. in volts, and C is the current in amperes. In this example, $E = 525$ volts and $C = 12.5$ amperes; hence,

$$\text{H. P.} = \frac{E\,C}{746} = \frac{525 \times 12.5}{746} = 8.7969 \text{ horsepower. Ans.}$$

(27) (*a*) By formula **20,** $W = C^2 R$, where W is the power in watts, C is the current in amperes, and R is the resistance in ohms. In this example, $C = 110$ amperes and $R = 4.2$ ohms; hence, $W = C^2 R = 110^2 \times 4.2 = 50{,}820$ watts. Ans.

(*b*) By formula **22,** H. P. $= \frac{W}{746}$, where H. P. is the horsepower and W is the power in watts. In this example, $W = 50,820$ watts; hence,

$$\text{H. P.} = \frac{W}{746} = \frac{50,820}{746} = 68.1233 \text{ horsepower. Ans.}$$

(28) The diagram, Fig. 1, shows the connections of the battery and galvanometer circuits to the circular type of resistance-box for measuring unknown resistances by the Wheatstone-bridge method. The upper balance arm (Art. **53**) of the bridge includes the resistance coils from c to a, the lower balance arm includes the coils from a to d, and the adjustable arm includes the coils from d to b. One pole of the battery B is connected to the junction of the two balance arms, the other to the junction of the adjustable arm and the unknown resistance X. One terminal of the galvanometer G is connected to the junction of the lower balance arm and the adjustable arm, the other to the junction of the upper balance arm and the unknown resistance.

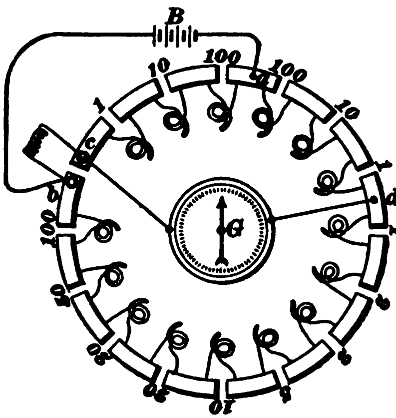

FIG. 1.

(29) By formula **1**, the changed resistance for variation in length $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance, l_1 is the original length, and l_2 is the changed length. In this case, $r_1 = 1$ ohm, $l_1 = 1,000$ feet, and $l_2 = 2,000$ feet. Then, the changed resistance $r_2 = \frac{1 \times 2,000}{1,000} = 2$ ohms. The next operation is to determine the resistance of the wire when its sectional area is changed. A round wire .1″ in diameter has a sectional area of $.1^2 \times .7854 = .007854$ sq. in., and a square wire .1″ on a side has a sectional area of $.1 \times .1 = .01$ sq. in. By formula **2**, $r_2 = \frac{r_1 a_1}{a_2}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its sectional area is changed, a_1 is the original sectional area, and a_2 is the changed sectional area. At this stage of the example, $r_1 = 2$ ohms, $a_1 = .007854$ sq. in., and $a_2 = .01$ sq. in. Hence, $r_2 = \frac{r_1 a_1}{a_2} = \frac{2 \times .007854}{.01} = 1.5708$ ohms. Ans.

(30) By formula **22**, H. P. $= \frac{W}{746}$, where H. P. is the horsepower and W is the power in watts. In this example, $W = 54{,}200$ watts; hence, H. P. $= \frac{W}{746} = \frac{54{,}200}{746} = 72.6541$ horsepower. Ans.

(31) The sectional area of a round column .04 in. in diameter is $.04^2 \times .7854 = .00125664$ sq. in., or .001257 sq. in., nearly.

Reduce the specific resistance in microhms to the resistance in ohms by dividing by 1,000,000, Art. **44**, which gives $\frac{37.15}{1{,}000{,}000} = .00003715$ ohm; or, in other words, the resistance of a quantity of mercury 1 in. long, and whose sectional area is 1 sq. in., is .00003715 ohm. Next, from this resistance and length, calculate the resistance of a column of mercury 72.3″ high, with a sectional area of 1 sq. in. by using formula **1**, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its length has been changed, l_1 is its original length, and l_2 is its changed length. In this example, $r_1 = .00003715$ ohm, $l_1 = 1''$, and $l_2 = 72.3$ inches. Hence, $r_2 = \frac{r_1 l_2}{l_1} = \frac{.00003715 \times 72.3}{1} = .002685945$, or .002686 ohm, nearly; or, in other words, the resistance of a column of mercury 72.3″ high, having a sectional area of 1 sq. in., is .002686 ohm. From this result calculate the resistance of the column when its sectional area is .001257 sq. in., by using formula **2**, $r_2 = \frac{r_1 a_1}{a_2}$, where r_1 is the original resistance,

r_2 is the resistance after the sectional area has been changed, a_1 is the original sectional area, and a_2 is the changed sectional area. At this stage of the example, $r_1 = .002686$ ohm, $a_1 = 1$ sq. in., and $a_2 = .001257$ sq. in. Hence,

$$r_2 = \frac{r_1 a_1}{a_2} = \frac{.002686 \times 1}{.001257} = 2.1368 \text{ ohms.} \quad \text{Ans.}$$

(32) By formula **6**, $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total E. M. F. in volts generated, and R is the total resistance in ohms of the circuit. Since the total resistance of a closed circuit is the sum of the external and internal circuits, $R = 33 + 30 = 63$ ohms and $E = 45$ volts; hence, $C = \frac{E}{R} = \frac{45}{63} = .7143$ ampere. Ans.

(33) In Fig. 5, question 33, the reading of the voltmeter gives the total E. M. F. of the battery. Hence, after the connections are made, as shown in Fig. 6, question 33, there is a closed circuit in which the total E. M. F. developed is 24.4 volts, and through which a current of .8 ampere is flowing. By formula **7**, $R = \frac{E}{C}$, where R is the total resistance in ohms of a closed circuit, E is the total E. M. F. in volts developed, and C is the current in amperes flowing. In this example, $E = 24.4$ volts and $C = .8$ ampere; hence, $R = \frac{E}{C} = \frac{24.4}{.8} = 30.5$ ohms, the total resistance of the circuit.

From the reading of the voltmeter, after the connections are made as shown in Fig. 6, it will be seen that when a current of .8 ampere flows through the external resistance from b to a through R, there is a drop or loss of potential of 18 volts. By Art. **64** and formula **7**, $R' = \frac{E'}{C}$, where R' is the resistance of a conductor, E' the drop, or loss, of potential in that conductor, and C is the current in amperes flowing through it. In this case, $E' = 18$ volts and $C = .8$ ampere; hence, $R' = \frac{E'}{C} = \frac{18}{.8} = 22.5$ ohms, the resistance of the external circuit from b to a through the resistance R. Ans.

Since the total resistance of a closed circuit is the sum of the external and internal resistances, Art. **60**, the internal resistance must be the difference between the total and external resistances. Hence, $30.5 - 22.5 = 8$ ohms, the internal resistance of the battery B. Ans.

(34) By formula **19**, $W = CE$, where W is the power in watts, E is the E. M. F., or difference of potential in volts, and C is the current in amperes. In this example, $E = 510$ volts and $C = 24.3$ amperes; hence, $W = 510 \times 24.3 = 12,393$ watts. Ans.

(35) Referring to Art. **56**, the total E. M. F. developed by connecting several cells in series is equal to the E. M. F. of one cell multiplied by the number of cells; hence, the E. M. F. of one of the groups of 6 cells is $6 \times 1.5 = 9$ volts. In the same article it is stated that connecting cells in multiple, or parallel, does not change the E. M. F. between the main conductors. In this case, each group of six cells can be considered as one large cell developing an E. M. F. of 9 volts, and, consequently, the E. M. F. of the four groups connected in multiple, or parallel, is 9 volts, which would be the E. M. F. indicated by a voltmeter connected to the main conductors c and c', as shown in Fig. 7, question 35. Ans.

(36) By formula **22**, H. P. $= \frac{W}{746}$; by formula **19**, $W = CE$; therefore (see Art. **81**), H. P. $= \frac{E\,C}{746}$, where H. P. is the horsepower, E is the E. M. F. in volts, and C is the current in amperes. In this example, $E = 250$ volts and $C = 65.7$ amperes; hence, H. P. $= \frac{E\,C}{746} =$

$$\frac{250 \times 65.7}{746} = \frac{16,425}{746} = 22.0174 \text{ horsepower. Ans.}$$

(**37**) In Art. **26** it is stated that when a compass is placed under a conductor in which an electric current is flowing from the south to the north, the north pole of the compass needle tends to point towards the west, and if the direction of the current in the conductor is reversed, the north pole will point towards the east. Since, in this example, the north pole of the needle tends to point towards the east, the current must be flowing *from* the north *to* the south.

(**38**) End *a*, since (Art. **29**), in looking at the face of the end *a*, the current circulates around the core in the same direction as the movement of the hands of a watch.

(**39**) Attract one another; since (Art. **7**) a positive charge is developed upon the ivory when rubbed with silk and a negative charge upon sealing-wax when rubbed with fur; and from Art. **6**, electrified bodies with dissimilar charges are mutually attractive.

(**40**) The exposed end of the iron; since, from Art. **13**, the iron forms the positive element of the cell, and, from Art. **12**, the pole, or electrode, attached to the exposed end of a voltaic element is always of opposite sign to the element itself.

(**41**) From Art. **21**, iron and its alloys, nickel, cobalt, manganese, oxygen, cerium, and chromium.

(**42**) Towards the south pole, since, from Art. **20**, unlike poles attract one another.

(**43**) Towards the north pole, since, from Art. **20**, unlike poles attract one another.

(**44**) *From* the north *to* the south, since, from Art. **26**, the north pole of a compass needle tends to point towards the east when the compass is placed over a conductor in which the current is flowing from the south to the north; and by reversing the direction of the current in the conductor, the north pole of the needle tends to point towards the west and the south pole towards the east.

(**45**) The current should enter the wire at end *b*; since (Art. **29**), in looking at the face of the south pole of the magnet, the current circulates around the core in the direction of the motion of the hands of a watch.

(46) By formula **1**, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its length has been changed, l_1 is the original length, and l_2 is its changed length. In this example, $r_1 = 100.8$ ohms, $l_1 = (112 \times 12) + 6 = 1{,}350$ inches, and $l_2 = 11.7$ inches. Hence,

$$r_2 = \frac{r_1 l_2}{l_1} = \frac{100.8 \times 11.7}{1{,}350} = .8736 \text{ ohm. Ans.}$$

(47) By formula **3**, $r_2 = \frac{r_1 D^2}{d^2}$, where r_1 is the original resistance of a round conductor, r_2 is the resistance after its diameter has been changed, D is its original diameter, and d is its changed diameter. In this example, $r_1 = 86.5$ ohms, $D = .1$ inch, and $d = .02$ inch; hence, $r_2 = \frac{r_1 D^2}{d^2} = \frac{86.5 \times .1^2}{.02^2} = \frac{86.5 \times .01}{.0004} = 2{,}162.5$ ohms. Ans.

(48) By formula **4**, $r_2 = r_1 (1 + t k)$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its temperature has risen, k is the temperature coefficient, and t is the number of degrees rise Fahrenheit. In this example, $r_1 = 91.8$ ohms, $t = 72 - 45 = 27$ degrees, and $k = .000244$, from Table 1. Hence, $r_2 = r_1 (1 + t k) = 91.8 (1 + 27 \times .000244) = 91.8 \times 1.006588 = 92.4048$ ohms. Ans.

(49) By formula **5**, $r_2 = \frac{r_1}{1 + t k}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its temperature has fallen, t is the number of degrees fall Fahrenheit, and k is the temperature coefficient of the material. In this example, $r_1 = .144$ ohm, $t = 87 - 41 = 46$ degrees Fahrenheit, and $k = .002155$, from Table 1. Hence,

$$r_2 = \frac{r_1}{1 + t k} = \frac{.144}{1 + 46 \times .002155} = \frac{.144}{1.09913} = .131 \text{ ohm.}$$

Ans

(50) First reduce the specific resistance in microhms to the resistance in ohms by dividing by 1,000,000, Art. **44**, which gives $\frac{3.565}{1,000,000} = .000003565$ ohm; or, in other words, the resistance of a block of platinum one inch long, and whose sectional area is one square inch, is .000003565 ohm. Next, from this resistance and length, calculate the resistance of 126 feet of platinum with a sectional area of 1 sq. in., by using formula **1**, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its length has been changed, l_1 is the original length of the conductor, and l_2 is its changed length. In this example, $r_1 = .000003565$ ohm, $l_1 = 1$ inch, and $l_2 = 126 \times 12 = 1,512$ inches. Hence,

$$r_2 = \frac{r_1 l_2}{l_1} = \frac{.000003565 \times 1,512}{1} = .00539028 \text{ ohm};$$

that is, the resistance of 126 feet of platinum having a sectional area of 1 sq. in. is .00539 ohm, nearly. From this result calculate the resistance of 126 feet of platinum when its sectional area $= .1^2 \times .7854 = .007854$ sq. in., by using formula **2**, $r_2 = \frac{r_1 a_1}{a_2}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its sectional area is changed, a_1 is its original sectional area, and a_2 is its changed sectional area. At this stage of the example, $r_1 = .00539$ ohm, $a_1 = 1$ sq. in., $a_2 = .007854$ sq. in. Hence, $r_2 = \frac{.00539 \times 1}{.007854} = .686$ ohm. Ans.

(51) From Art. **53**, the fundamental equation of the Wheatstone bridge is $X = \frac{M}{N} \times P$, where X is the unknown resistance, M is the resistance of the upper balance arm, N is the resistance of the lower balance arm, and P is the resistance of the adjustable arm. It will be seen from the connections of the battery and galvanometer circuits in the diagram that the coils lying between c and a form the upper balance arm of the bridge, and hence, in this example, $M = 1$ ohm; the coils between *a* and *d* form the lower balance arm, and hence $N = 100$ ohms; the coils between *d* and *b* form the adjustable arm, and hence $P = 500 + 200 + 20 + 2 + 1 = 723$ ohms. Substituting these values in the fundamental equation gives

$$X = \frac{M}{N} \times P = \frac{1}{100} \times 723 = 7.23 \text{ ohms. Ans.}$$

(52) By formula **6**, $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total E. M. F. in volts developed in the circuit, and R is the total resistance in ohms of the circuit. In this example, $E = 36$ volts and $R = 24 + 18 = 42$ ohms; since, Art. **60**, the total resistance of a closed circuit is the sum of the internal and external resistances. Hence, $C = \frac{E}{R} = \frac{36}{42} = .8571$ ampere. Ans.

(53) By formula **7**, $R = \frac{E}{C}$, where R is the total resistance in ohms of a closed circuit, E is the total E. M. F. in volts developed in the circuit, and C is the current in amperes flowing through the circuit. In this example, $E = 12.6$ volts and $C = 2.7$ amperes; hence, $R = \frac{E}{C} = \frac{12.6}{2.7} = 4.6667$ ohms. Ans.

(54) By formula **8**, $E = CR$, where E is the total E. M. F. in volts developed in a closed circuit, C is the current in amperes flowing through the circuit, and R is the total resistance of the circuit. In this example, $C = .8$ ampere and $R = 31.5 + 11 = 42.5$ ohms, since, Art. **60**, the total resistance of a closed circuit is the sum of the internal and external resistances. Therefore, $E = CR = .8 \times 42.5 = 34$ volts. Ans.

(55) By Art. **64** and formula **8**, $E' = CR'$, where E' is the difference of potential in volts between two points in a circuit, C the current in amperes flowing through the circuit, and R' the resistance of the circuit between the two points. In this example, $C = .12$ ampere and $R' = 204$ ohms; hence, $E' = C R' = .12 \times 204 = 24.48$ volts. Ans.

(56) (*a*) By Art. **64** and formula **7**, $R' = \frac{E'}{C}$, where R' is the total resistance in ohms between two points in a circuit, E' the drop, or loss, of potential in volts between the two points, and C the current in amperes flowing in the circuit. In this example, the two conductors leading to and from the receptive device can be considered as in series, forming one single conductor 1,200 feet in length, in which the drop, or loss, of potential is 10% of 250 volts, or $.10 \times 250 = 25$ volts; that is, $E' = 25$ volts. Since $C = 80$ amperes, then $R' = \frac{E'}{C} = \frac{25}{80} = .3125$ ohm; or, in other words, the sum of the resistances of two conductors which transmit a current of 80 amperes to and from the receptive device with a loss of 25 volts is .3125 ohm. Ans.

(*b*) The resistance per foot of any conductor is found by dividing its total resistance by its length in feet. Assume the two conductors leading to and from the receptive device to be one single conductor 1,200 feet in length and offering a resistance of .3125 ohm; hence, its resistance per foot is

$$\frac{.3125}{1,200} = .00026 \text{ ohm. Ans.}$$

(57) By formula **6**, $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total E. M. F. in volts developed in the circuit, and R is the total resistance in ohms of the circuit. In this example, $E = 24$ volts and $R = 8.1 + 15.9 = 24$ ohms, since, Art. **60**, the total resistance of a closed circuit is the sum of the internal and external resistances. Hence,

$$C = \frac{E}{R} = \frac{24}{24} = 1 \text{ ampere.}$$

By formula **9**, $E' = E - C r_i$, where E' is the available, or external, E. M. F. in volts of a battery or other electric source in a closed circuit, E is the total E. M. F. in volts developed in the source, C is the current in amperes flowing through the circuit, and r_i is the internal resistance of the battery or electric source. In this example, $E = 24$ volts, $C = 1$ ampere, and $r_i = 8.1$ ohms. Hence, $E' = E - C r_i = 24 - (8.1 \times 1) = 15.9$ volts. Ans.

(58) Let A represent the first branch and B the second; then, $r_1 = 1.2$ ohms, $r_2 = 2.2$ ohms, and $C = 45$ amperes.

The current c_1 in branch A will then be found by substituting these values in formula **10,** which gives

$$c_1 = \frac{C r_2}{r_1 + r_2} = \frac{45 \times 2.2}{1.2 + 2.2} = \frac{99}{3.4} = 29.1176 \text{ amperes. Ans.}$$

Since the sum of the currents in the two branches is 45 amperes, the current in branch B is, therefore, the difference between 45 amperes and the current in branch A, or $45 - 29.1176 = 15.8824$ amperes. Ans.

(59) By formula **12,** the joint resistance of two conductors connected in parallel is equal to the product of their separate resistances divided by their sum, or $\frac{r_1 r_2}{r_1 + r_2}$, where r_1 and r_2 are the separate resistances of the two branches. In this example, $r_1 = 45$ ohms and $r_2 = 63$ ohms. Hence, $\frac{45 \times 63}{45 + 63} = 26.25$ ohms, the joint resistance of the two conductors connected in parallel.

(60) From Art. **72,** the joint resistance of three conductors connected in parallel is given by formula **13,** $R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}$, where r_1, r_2, and r_3 are the separate resistances of the three conductors. In this example, let $r_1 = 414$ ohms, $r_2 = 810$ ohms, and $r_3 = 1,206$ ohms. Then, the joint resistance of the three conductors A, B, and C when connected in parallel is

$$\frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} = \frac{414 \times 810 \times 1,206}{810 \times 1,206 + 414 \times 1,206 + 414 \times 810} =$$

$$\frac{404,420,040}{976,860 + 499,284 + 335,340} = \frac{404,420,040}{1,811,484} = 223.2534 \text{ ohms.}$$

Ans.

DYNAMOS AND MOTORS.

(PART 2.)

(**1**) By formula **1**, $E = \frac{2NSn}{10^8}$. In this example, $N = 6{,}250{,}000$ lines of force, $S = 100$ outside, or face, wires in series, for if 200 turns were wound around the core, there would be 200 outside, or face, wires, and, from Art. **23**, one-half would be connected in series, and $n = \frac{1200}{60}$ revolutions per second. Substituting these values in above formula gives $E = \frac{2NSn}{10^8} = \frac{2 \times 6{,}250{,}000 \times 100 \times 1{,}200}{100{,}000{,}000 \times 60} =$ 250 volts. Ans.

(**2**) From Art. **36**, it will be seen that the current in the shunt field of a dynamo is equal to the difference of potential between the brushes divided by the resistance of the shunt-field circuit, or $C_s = \frac{E_e}{R_s}$. In this example, $E_e =$ 220 volts and $R_s = 440$ ohms; hence, $C_s = \frac{E_e}{R_s} = \frac{220}{440} = .5$ ampere. Ans.

(**3**) See Arts. **13** and **14**.

(**4**) In Art. **1**, it is stated that a current will be induced in a closed coil or circuit when there is a change in the number of lines of force passing through that coil or circuit. In this case, as the magnetic field is uniform, there is no change in the number of lines of force passing through the coil C when it is moved from its original position to the position C', as shown by the dotted outlines; and, hence, no current will flow around the ring.

(5) In order to determine the result in watts, it is necessary to reduce all quantities to the same units; hence, the first operation is to reduce the input from horsepower to watts. From Art. **81**, Part 1, one horsepower is equivalent to 746 watts; therefore, the input $= 18 \times 746 = 13.428$ watts. Then, by formula **5**, the output

$$O = \frac{13,428 \times 88}{100} = 11,816.64 \text{ watts. Ans.}$$

(6) In this example, it is first necessary to determine the input in watts. By formula **4**, the input $I = \frac{100 \times 17,500}{87.5} = 20,000$ watts. According to Art. **64**, it will be seen that the watts lost in the field coils are equal to the input in watts multiplied by the per cent. loss and divided by 100. Hence, the loss $= \frac{20,000 \times 2.6}{100} = 520$ watts. Ans.

(7) In Art. **67**, $C_s = \frac{E_e}{r_s}$. In this example, $E_e = \frac{110}{55} = 2$ amperes. By formula **19**, Part 1, the watts lost in the shunt circuit are equal to the difference of potential between the terminals of that circuit multiplied by the current in amperes flowing through the circuit; or, $W = E \times C$. Substituting, we have $W = 110 \times 2 = 220$ watts. Ans.

(8) See Arts. **14** and **19.**

(9) From Ohm's law, the resistance of the field circuit is equal to the difference of potential between the brushes divided by the current in the field circuit. Let E_e be the difference of potential between the brushes of the dynamo, C_1 be the strength of current when the resistance is all in circuit, and C_2 be the strength of current when the resistance is cut out, or short-circuited. If r_1 represents the resistance of the field circuit, including that of the rheostat, then $r_1 = \frac{E_e}{C_1} = \frac{360}{1.5} = 240$ ohms; and if r_2 represents the resistance of the field circuit when the resistance of the rheostat has been cut out, or short-circuited, then $r_s = \frac{E_e}{C_s} = \frac{360}{1.8} =$ 200 ohms. Hence, the amount of resistance which was cut out, or short-circuited, in the rheostat is the difference between these two resistances, or $240 - 200 = 40$ ohms. Ans.

(10) Use formula **4.** In this example, the input $= \frac{100 \times 65,000}{90.5} = 71,823.2044$ watts. Since one horsepower equals 746 watts, then 71,823.2044 watts equal $\frac{71,823.2044}{746} =$ 96.2778 horsepower. Ans.

(11) If the current circulates in the direction indicated by the arrow-heads, *neither* pole-piece will be a north pole; for, by applying the rule given in Art. **29,** Part 1, it will be seen that the north pole of one field coil is opposite the south pole of the other, and the lines of force circulate around the magnets without passing through the armature. If the winding of the right-hand coil were reversed, its top would be a north pole, and the top of the left-hand coil being also north, the pole-piece *P* would become a north consequent pole.

(12) See Arts. **19** and **21.**

(13) In this example, it is first necessary to change the input from horsepower to watts. Since one horsepower is equivalent to 746 watts, then 10 horsepower is equivalent to $10 \times 746 = 7,460$ watts. Then, by formula **2,** the efficiency $\mathbf{E} = \frac{6,341 \times 100}{7,460} = 85$ per cent. Ans.

(14) From *b* to *a* through the conductor; for, by applying the thumb-and-finger rule given in Art. **8,** it will be seen that the middle finger points towards *a* from *b*.

(15) In this example, it is first necessary to find the input in watts. The output $= 11,900$ watts and the per cent. efficiency = 85. Then, by formula **4,** the input = $\frac{100 \times 11{,}900}{85} = 14{,}000$ watts. According to Art. **64,** the watts lost are found by multiplying the input by the per cent. loss and dividing by 100. Hence, $\frac{14{,}000 \times 1.8}{100} = 252$ watts lost in core by eddy currents and hysteresis. Ans.

(16) (*a*) See Art. **28.**
(*b*) See Art. **25.**

(17) According to Art. **70** and formula **20,** Part 1, $W_t = C^2 r_t$. In this example $C = 120$ amperes and $r_t = .040$ ohm; hence, $W_t = 120^2 \times .040 = 576$ watts. Ans.

(18) From example 17, the normal output from the dynamo is 120 amperes at 125 volts, or $120 \times 125 = 15{,}000$ watts. The next step is to determine the input at this output when the efficiency is 75%. By formula **4,** the input in this case is $\frac{100 \times 15{,}000}{75} = 20{,}000$ watts. From example 17, there are 576 watts lost in the armature due to its resistance, and, from Art. **72,** the loss in the armature due to its resistance is $\frac{576 \times 100}{20{,}000} = 2.88\%$. Ans.

(19) (*a*) and (*b*) See Art. **77.**

(20) Under "Field Losses," in Art. **66,** the watts lost in the series coils are found by using formula **20,** Part 1, where $W = C^2 R$. In this example, $C = 40$ amperes and $R = .04$ ohm; hence, $W = 40^2 \times .04 = 40 \times 40 \times .04 = 64$ watts, which represents the loss in the series coils. The watts lost in the shunt coil are given by formula **21,** Part 1, where $W = \frac{E^2}{R}$. In this case, $E = 550$ volts and $R = 550$ ohms; hence, $W = \frac{E^2}{R} = \frac{550 \times 550}{550} = 550$ watts, which is the loss in the shunt field. The total loss in the fields of a compound dynamo is equal to the sum of the losses in the series and shunt coils. Hence, the total loss in this case is $64 + 550 = 614$ watts. Ans.

(21) P' is the south consequent pole of the field, since, from the rule in Art. **29,** Part 1, in looking through the coils *c* and *d* from a position near P', the current is circulating around the field cores in the same direction as the movements of the hands of a watch; while, on the contrary, in looking through the coils *a* and *b* from a position near *P*, the current is circulating around the upper field cores in a direction opposite to the movements of the hands of a watch.

(22) See Art. **29.**

(23) From Art. **64,** the total loss in a dynamo is the sum of the separate losses; hence, in this example, the total loss in watts is $356 + 178 + 263 + 423 + 50 = 1{,}270$ watts. From Art. **59,** the input to the dynamo in this case is $15{,}000 + 1{,}270 = 16{,}270$ watts. By formula **2,** the efficiency $\mathsf{E} = \frac{15{,}000 \times 100}{16{,}270} = 92.1942\%$ at this output. Ans.

(24) (*a*) From example 23, the loss in mechanical friction is 356 watts, and the input is 16,270 watts; hence (see Art. **72**), the per cent. loss is $\frac{356 \times 100}{16{,}270} = 2.1881\%$. Ans.

(*b*) From example 23, the loss in the core by eddy currents and hysteresis is 178 watts, and the input is 16,270 watts; hence (see Art. **72**), the per cent. loss is

$$\frac{178 \times 100}{16{,}270} = 1.094\%. \quad \text{Ans.}$$

(*c*) From example 23, the loss in the field coils is 263 watts, and the input is 16,270 watts; hence, the per cent. loss is $\frac{263 \times 100}{16{,}270} = 1.6165\%$. Ans.

(*d*) From example 23, the loss in the armature ($C^2 r$) = 423 watts, and the input is 16,270 watts; hence, the per cent. loss is $\frac{423 \times 100}{16{,}270} = 2.5999\%$. Ans.

(*e*) From example 23, the sum of the separate losses is 1,270 watts, and this is the difference between the input and the output; the input is 16,270 watts. Hence, by formula **3,** the total per cent. loss $L = \frac{100 \times 1,270}{16,270} = 7.8058\%$. Ans.

(25) From Art. **22,** it will be seen that the electromotive force generated in an armature is proportional to the speed, other conditions and quantities remaining unchanged. Hence, in this example, if E represents the electromotive force which is generated when the armature is driven at 1,400 revolutions per minute, then, by proportion, $440 : E = 1,200 : 1,400$, or $E \times 1,200 = 440 \times 1,400$; therefore, $E = \frac{440 \times 1,400}{1,200} = 513\frac{1}{3}$ volts. Ans.

(26) See Art. **73** and those following.

(27) In Art. **6,** under "Mutual Induction," it is stated that when the current in the primary circuit tends to increase in strength, the induced current in the secondary coil will tend to circulate around the core which forms the magnetic circuit of both coils, in the opposite direction to that of the current in the primary circuit. In this case, the current in the primary circuit flows from the positive terminal n of the battery when the circuit is closed, around the coil to the negative terminal m; or, in other words, the current circulates around the core in an opposite direction to the movements of the hands of a watch, as viewed by a person looking through the coil from a position near C. Consequently, the momentary current induced in the secondary coil S would tend to circulate around the core in the reverse direction, that is, in the same direction as the movements of the hands of a watch. The direction of the current in the secondary circuit would, therefore, be from the terminal x through the coil to y, and then through the resistance R to x again.

(28) In Art. **6,** under "Mutual Induction," it is stated that when the strength of the current in the primary circuit suddenly decreases, the momentary current induced in the secondary coil will circulate around the core which forms the magnetic circuit of both coils in the same direction to that of the current in the primary coil. In this case, when the strength of the current in the primary circuit suddenly decreases, it continues to flow in the same direction as in example 27, that is, from the terminal *n* through the primary coil *P* to *m*, and completing the circuit to *n* through the battery *B*. Consequently, the current in the secondary coil *S* circulates around the core in the same direction, that is, from *y* through the secondary coil *S* to *x*, completing the circuit to *y* again through the resistance *R*.

(29) See Arts. **78** and **79.**

(30) Yes; because (Art. **1**) a change takes place in the number of lines which pass through the coil. From the rule given in Art. **7,** it will be seen that the current will circulate around the ring in the same direction as the movements of the hands of a watch; for the effect of the motion is to diminish the number of lines of force which pass through the coil, and the observer is looking along the magnetic field in the direction of the lines of force.

(31) See Art. **42.**

(32) From Art. **11,** it will be seen that the *rate of cutting* lines of force is found by dividing the number cut by the time required to cut them; hence, in this case, the rate of cutting is $\frac{8,000,000}{.25} = 32,000,000$ lines of force per second.

(33) Because the solid iron core would act as a large conductor cutting lines of force at an angle, and thereby producing *local*, or *eddy*, currents in the core, heating it badly, and uselessly dissipating a large amount of energy. (Art. **16.**)

(34) By formula **1,** $E = \frac{2NSn}{10^8}$. In this example, if 150 complete turns of wire are wound upon the drum core, there will be 300 outside, or face, wires, and one-half of these will be connected in series, as explained in Art. **21**; then, $S = 150$ outside wires connected in series, $N = 2{,}500{,}000$ lines of force, and $n = \frac{1020}{60}$. Hence,

$$E = \frac{2NSn}{10^8} = \frac{2 \times 2{,}500{,}000 \times 150 \times 1{,}020}{100{,}000{,}000 \times 60} = 127.5 \text{ volts. Ans.}$$

(**35**) From Art. **22**, it will be seen that the electromotive force generated in an armature is proportional to the number of lines of force passing through the core. Let E represent the electromotive force which is generated when 1,250,000 lines of force are passing through the core; then, by proportion, $200 : E = 750{,}000 : 1{,}250{,}000$, or $E \times 750{,}000 = 200 \times 1{,}250{,}000$; therefore, $E = \frac{200 \times 1{,}250{,}000}{750{,}000} = 333\frac{1}{3}$ volts. Ans.

(**36**) (*a*) See Art. **65.**
(*b*) See Art. **70.**
(*c*) See Art. **66.**

(**37**) Towards the side *a*; for by applying the thumb-and-finger rule given in Art. **26**, and making the forefinger point in the direction of the lines of force and the middle finger in the direction of the current, the thumb will point towards the side *a*.

(**38**) Use the formula given under "Field Losses" in Art. **67**, $C_s = \frac{E_e}{r_s}$, which is a modification of formula **6**, Part 1. In this example, $E_e = 525$ volts and $r_s = 650$ ohms; hence, $C_s = \frac{E_e}{r_s} = \frac{525}{650} = .8076$ ampere. Ans.

(**39**) The increase in voltage from no load to full load is $124.2 - 115 = 9.2$ volts, which is $\frac{9.2 \times 100}{115} = 8\%$ of the normal voltage. Therefore, the over-compounding is 8%. Ans.

(40) See Art. **20.**

(41) First change the input from horsepower to watts. Since 1 horsepower is equivalent to 746 watts, 44 horsepower is equivalent to $44 \times 746 = 32{,}824$ watts. By formula **2,** the efficiency

$$E = \frac{100 \times 29{,}820}{32{,}824} = 90.8481\%. \quad \text{Ans.}$$

(42) In this example, the input $I = 20{,}000$ watts and the output $O = 17{,}500$ watts. Then, by formula **3,** the per cent. loss $L = \frac{100 \times (20{,}000 - 17{,}500)}{20{,}000} = 12.5\%$. Ans.

(43) In this example, the output is 12,500 watts and the efficiency is 92.5%. Consequently, by formula **4,** the input $I = \frac{100 \times 12{,}500}{92.5} = 13{,}513.5135$ watts. Reducing this input from watts to horsepower gives

$$\frac{13{,}513.5135}{746} = 18.1146 \text{ horsepower.} \quad \text{Ans.}$$

(44) See Art. **73.**

(45) In this example, it is first necessary to change the input from horsepower to watts. Since one horsepower is equivalent to 746 watts, fifty-five horsepower is equivalent to $55 \times 746 = 41{,}030$ watts. The output of the dynamo, by formula **5,** $= \frac{41{,}030 \times 88.5}{100} = 36{,}311.55$ watts. Ans.

(46) In the same manner as shown in Art. **64,** it will be seen that the loss in watts in the field coils is equal to the input multiplied by the per cent. loss and divided by 100. In this example, it is first necessary to change the input from horsepower to watts. Since one horsepower is equivalent to 746 watts, forty-five horsepower is equivalent to $45 \times 746 = 33{,}570$ watts. Consequently, the loss in the field coils is $\frac{33{,}570 \times 2}{100} = 671.4$ watts. Ans.

(47) See Arts. **34, 36,** and **39.**

(**48**) From Art. **72**, the per cent. loss in the core is found by dividing the watts lost in the core by the input and multiplying by 100. Reducing 64 horsepower to watts gives $64 \times 746 = 47,744$ watts. Consequently, the loss in the core is $\frac{800 \times 100}{47,744} = 1.6756\%$. Ans.

(**49**) See Art. **76**.

(**50**) See Art. **43**.

DYNAMOS AND MOTORS.

(PART 3.)

(1) (*a*) and (*b*) See Art. **40.**

(2) See Art. **54.**

(3) See Art. **94.**

(4) Drum. See Art. **27.**

(5) See Art. **1.**

(6) See Art. **37.**

(7) See Art. **31.**

(8) An open circuit in the armature winding, probably in the lead to the burned commutator segment. See Art. **97.**

(9) (*a*) See Art. **67.**

(*b*) See Art. **56.**

(10) (*a*) and (*b*) See Art. **24.**

(11) See Art. **21.**

(12) See Art. **36.**

(13) (*a*) The length of the arm of the brake being 36 inches, or 3 feet, the torque of the motor is $27 \times 3 = 81$ foot-pounds (Arts. **62** and **63**). The revolutions per minute being 900, the H. P. output of the motor is, from formula **3**,

$$\text{H. P.} = \frac{2 \times 3.1416\, TS}{33{,}000} = \frac{6.2832 \times 81 \times 900}{33{,}000} = 13.88 \text{ H. P.} \quad \text{Ans.}$$

(*b*) To find the efficiency, it is first necessary to find the input and reduce the input and the output to the same units (Art. **63**). In this case, the input is 25 × 480 = 12,000 watts. Reducing 13.88 H. P. to watts, 13.88 × 746 = 10,354 watts. Then, by formula **2,** Part 2, the efficiency

$$E = \frac{100 \times 10,354}{12,000} = 86.3\%. \quad \text{Ans.}$$

(**14**) See Art. **68.**

(**15**) (*a*) and (*b*) See Art. **99.**

(**16**) The connections would be about as shown in Fig. 1, in which *F* is the field circuit, *B*, B_1 the brushes of the motor, *R* the starting resistance, *R. S.* the reversing-switch, *F. B.* the fuse boxes, and *S* the main switch.

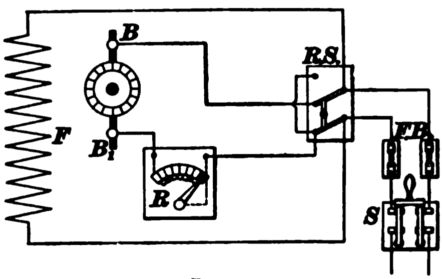

FIG. 1.

(**17**) By varying the applied E. M. F. or the strength of the field. Art. **57.**

(**18**) See Art. **39.**

(**19**) Because the self-induction of the coil having the lesser E. M. F. prevents the flow of current. Art. **14.**

(**20**) Circuit No. *4* (Fig. 48, Art. **115**) would first be short-circuited by plugging in a cable from terminal −*3* to terminal −*4*. The cable from terminal −*3* to terminal +*4* may now be removed from terminal +*4* and connected to terminal +*1*, and a cable plugged in from terminal −*1* to terminal −*B*. Then, by pulling out the cable from terminal −*3* to terminal −*4*, circuit No. *1* is connected in series with circuit No. *3*, on dynamo *B*, as required. The cable from terminal −*4* to terminal −*B* should be removed to make circuit No. *4* "dead."

(21) (*a*) See Art. **103.**

(*b*) See Arts. **92** and **93.**

(22) (*a*) See Art. **3.**

(*b*) See Art. **5.**

(23) (*a*) 29 amperes flowing out.

(*b*) See Art. **35.**

(24) See Art. **69.**

(25) The input to the motor being $33 \times 230 = 7{,}590$ watts, and the efficiency being 85%, the output, by formula **5,** Part 2, is $\frac{7{,}590 \times 85}{100} = 6{,}451.5$ watts. This is equal to $\frac{6{,}451.5}{746} = 8.65$ H. P. The arm of the brake being 2 feet long and the pressure on the scale platform being 20 lb., the torque of the motor must be 40 foot-pounds $= T$. Knowing the H. P. and the torque, the speed may be found from formula **4,** $S = \frac{33{,}000 \text{ H. P.}}{2 \times 3.1416\, T}$. Substituting the above values for H. P. and T,

$$S = \frac{33{,}000 \times 8.65}{2 \times 3.1416 \times 40} = \frac{285{,}450}{251.328} = 1{,}136 \text{ rev. per min.} \quad \text{Ans.}$$

(26) The frequency is equal to the number of revolutions per second multiplied by the number of pairs of poles (Art **26**). In this case, $\frac{1080}{60} \times \frac{14}{2} = 126$ cycles per second. Ans.

(27) (*a*) and (*b*) See Art. **118.**

(28) See Art. **58.**

(29) See Art. **78.**

(30) See Art. **101.**

(31) (*a*) See Art. **32.**

(*b*) See Art. **31.**

(**32**) See Arts. **29** and **30.**

(**33**) See Art. **85.**

(**34**) See Arts. **114** and **132.**

(**35**) See Art. **17** and Fig. 6.

(**36**) See Art. **50.**

(**37**) See Art. **124.**

(**38**) See Art. **80.**

(**39**) See Art. **12.**

(**40**) See Art. **127.**

(**41**) See Art. **90.**

(**42**) See Art. **55.**

(**43**) See Art. **106.**

(**44**) See Art. **109.**

(**45**) See Art. **65.**

(**46**) See Art. **128.**

(**47**) See Art. **23.**

(**48**) See Art. **41.**

(**49**) See Art. **131.**

(**50**) When in the position of least action, a coil is momentarily disconnected from the external circuit, then is thrown in parallel with the coil ahead of it, then in series with the other two coils which are then in parallel, then in parallel with the coil behind it, and then disconnected from the circuit again. See Art **20,** also Fig. 7.

(**51**) (*a*) See Art. **68.**

(*b*) See Arts. **80** and **84.**

(**52**) (*a*) Of the 5 amperes input, by Ohm's law, $\frac{125}{62.5} =$ 2 amperes go to the field, the loss being, therefore, $2 \times 125 = 250$ watts. The rest, or $3 \times 125 = 375$ watts, make up the friction and core losses of the machine (Art. **65**).

When taking an input of 77 amperes at 125 volts, or 9,625 watts, there would still be required 250 watts for the field and 375 watts for the core losses and friction. Of the 77 amperes, 75 flow through the armature, and as this has a resistance of .04 ohm, the armature $C^2 r$ would be $75^2 \times .04 = 225$ watts. The total losses would then be $250 + 375 + 225 = 850$ watts, and the output would, therefore, be $9,625 - 850 = 8,775$ watts, or $\frac{8775}{746} = 11.76$ H. P. Ans.

(*b*) The output being 8,775 watts and the input 9,625, the efficiency is, by formula **2,** Part 2, $\frac{100 \times 8,775}{9,625} = 91.17$ per cent. Ans.

(53) See Art. **104.**

(54) (*a*) and (*b*) See Art. **26.**

(55) See Art. **10.**

(56) See Art. **54.**

(57) (*a*) and (*b*) See Art. **121** and Figs. 51 and 52.

(58) From Art. **79,** the speed of the field would be $\frac{72}{5} =$ 14.4 revolutions per second, or $14.4 \times 60 = 864$ revolutions per minute. With 2.5% slip, the speed of the armature would be $864 - (864 \times .025) = 842.4$ revolutions per minute. Ans.

(59) See Art. **92.**

(60) (*a*) See Art. **52.**

(*b*) and (*c*) See Art. **53.**

(61) When the whole of the coil is directly under one pole-piece. See Art. **111.**

(62) See Art. **73.**

(63) See Art. **26.**

(64) See Art. **130.**

(65) See Art. **59.**

(66) See Art. **94.**

(67) (*a*) and (*b*) See Art. **56.**
(*c*) See Arts. **55** and **56.**

(68) See Arts. **21** and **22.**

(69) See Art. **96.**

(70) See Art. **79.**

(71) See Art. **52.**

(72) See Art. **41.**

(73) See Art. **79.**

(74) There is no definite answer for this problem, as a great number of different arrangements is possible. See Art. **129.** By comparing it with Fig. 54, Art. **127,** it will be seen, if the connections are correctly made and all the necessary instruments in place, the exact arrangement is a matter of taste and judgment.

(75) See Art. **63.**

(76) See Art. **27.**

(77) The frequency being 132 and there being 11 pairs of poles, the motor will run at $\frac{132}{11} = 12$ revolutions per second, or $60 \times 12 = 720$ revolutions per minute. See Art. **68.**

(78) (*a*) and (*b*) See Art. **80.**

(79) See Art. **6.**

(80) See Art. **57.**

(81) See Art. **97.**

(82) See Art. **6.**

(83) The effective voltage of the alternating current is obtained by using formula **2,** Art. **47,**

$$\bar{E} = .612 \times 200 = 122.4 \text{ volts. Ans.}$$

(84) (*a*) The strength of the direct current would be the same as the *effective* strength of the alternating current.

which is .707 of its maximum strength. See Art. **36.** Then, $.707 \times 12 = 8.48$ amperes. Ans.

(*b*) See Art. **36.**

(85) The counter E. M. F. E' depends on the field strength and on the speed of rotation of the armature. Hence, if the field is weakened, E' will decrease and allow a larger current to flow through the armature. This will result in an increase in speed, till at the new speed and with the weakened field, the counter E. M. F. is raised sufficiently to cut down the current to its proper value, or so that the torque will again balance the resistance to rotation. See Art. **57.**

(86) See Art. **83.**

(87) See Art. **71.**

(88) See Art. **81.**

(89) See Art. **83.**

(90) See Art. **46.** The E. M. F. of each phase will be given by formula **1,**

$$\bar{E} = .707 \times 220 = 155.5 \text{ volts. Ans.}$$

(91) See Art. **48.**

(92) See Art. **34.**

ELECTRIC HOISTING AND HAULAGE.

(1) See Art. **3.**

(2) See Art. **4.**

(3) See Art. **5.**

(4) See Art. **8.**

(5) See Art. **16.**

(6) See Arts. **20** and **24.**

(7) See Arts. **21** and **22.**

(8) See Art. **26.**

(9) See Art. **27.**

(10) See Art. **28.**

(11) See Art. **40.**

(12) See Arts. **41** and **45.**

(13) See Art. **42.**

(14) See Art. **44.**

(15) See Art. **50.**

(16) See Art. **51.**

(17) See Art. **54.**

(18) See Art. **1.**

(19) See Art. **5.**

(20) See Art. **5.**

ELECTRIC PUMPING, SIGNALING, AND LIGHTING.

(1) See Fig. 19, Art. **32.**

(2) See Fig. 20, Art. **34.**

(3) (*a*) See Art. **23.**

(*b*) By connecting the two terminals of the electromagnet directly to the binding-posts of the bell. See Art. **24.**

(4) The Edison-Lalande or Gordon type. See Art. **36.**

(5) See Fig. 21, Art. **35.**

(6) (*a*) About $3\frac{1}{2}$ to 4 watts.

(*b*) About 200.

(*c*) $\frac{1}{2}$ ampere. See Arts. **47** and **48.**

(7) (*a*) The lamps would be connected two in series, as shown in Fig. 29, Art. **52.**

(*b*) For the 550-volt circuit, 5 lamps would have to be connected in series; hence, the sketch required is similar to that shown in Fig. 29 except that there will be 5 lamps in series instead of 2.

(8) (*a*) A short circuit is a path of low resistance established between two points of a system. It usually refers to a connection of low resistance between the two sides of a constant-potential system, such as might be caused, for example, by the wires becoming crossed or accidentally connected together in some way.

(*b*) Because the pressure is capable of setting up a very large current that may be sufficient to burn the wire and set fire to surrounding material. See Art. **62.**

(9) (*a*) See Arts. **49** and **50.**

(*b*) Carbons last about 10 hours in open-arc lamps and about 150 hours in the enclosed-arc lamps. See Art. **50.**

(10) See Arts. **4** and **5.**

(11) A motor-driven pump runs at a nearly constant speed, regardless of the load; hence, with the electric pump, there is not as much danger from hammering as with steam or compressed-air pumps. See Art. **6.**

(12) See Art. **9.**

(13) (*a*) 450 watts.

(*b*) 300 watts. See Art. **49.**

(14) (*a*) From 65 to 70 per cent.

(*b*) From 70 to 75 per cent. See Art. **9.**

(15) See Art. **7.**

(16) (*a*) The drop is the falling off in pressure from the dynamo to the point where the current is utilized. It is equal to the E. M. F. necessary to force the current through the line, or, in other words, to overcome the line resistance.

(*b*) The drop depends on the amount of current to be forced through the line and the resistance of the line. It is equal in amount to the product of the current and the resistance. See Art. **59.**

(17) (*a*) The bells do not all vibrate in unison, and the result is that the circuit is broken in one part when it is closed at another; hence, the bells either do not ring at all or else they ring with a very weak sound.

(*b*) Make all the bells single-stroke but one, and allow this one bell to do the interrupting of the current. See Art. **21.**

(18) See Art. **22.**

(19) The current C will be 75 amperes, because each lamp will take $\frac{1}{2}$ ampere. The total length of line will be 3,000 feet, because the distance to the lamps is 1,500 feet. We may then obtain the size of wire by substituting in formula **2,**

$$A = \frac{10.8 \times 3{,}000 \times 75 \times 100}{110 \times 10} = 220{,}909. \quad \text{Ans.}$$

This is a little larger than a No. 0000 wire, as will be seen by referring to Table 2, but a No. 0000 would probably be used.

(20) .125 inch = 125 mils, hence circular mils = 125^2 = 15,625. Ans. See Art. **55.**

(21) See Fig. 13, Art. **26.**

(22) See Fig. 14, Art. **27.**

(23) (*a*) See Art. **53.**

(*b*) Because it permits the use of high line pressures, thus keeping down the line current for a given amount of power transmitted and reducing the amount of copper required in the line.

(24) (*a*) See Arts. **62** and **63.**

(*b*) If no fuse were provided, the heavy flow of current might be sufficient to burn the insulation off the wire or even fuse the wire itself.

(25) Since the primary pressure is 2,000 volts, each ampere in the primary will be equivalent to 40 lights on the secondary. See Art. **53.** The line current required for 1,200 lamps will be $\frac{1200}{40}$ = 30 amperes. The total length of line is 4 miles, or 21,120 feet. Applying formula **2,** we find the area of cross-section to be

$$A = \frac{10.8 \times 21,120 \times 30 \times 100}{2,000 \times 10} = 34,214. \quad \text{Ans.}$$

A No. 5 B. & S. wire would likely be used.

(26) See Fig. 40, Art. **71.**

(27) See Art. **6.**

(28) See Fig. 33, Art. **61.**

(29) Each set of five lamps will take $\frac{1}{2}$ ampere; hence, the 200 lamps will take 20 amperes. See Art. **52.**

ELECTRIC COAL-CUTTING MACHINERY.

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(1) See Arts. **1** and **2.**

(2) See Art. **1.**

(3) See Arts. **11** and **27.**

(4) See Art. **16.**

(5) See Art. **18.**

(6) See Art. **18.**

(7) See Arts. **21** and **22.**

(8) See Art. **23.**

(9) See Art. **25.**

(10) See Art. **26.**

(11) See Art. **31.**

(12) See Art. **28.**

(13) See Art. **5.**

(14) See Arts. **5** to **8.**

(15) See Art. **13.**

(16) See Arts. **16** and **17.**

(17) See Art. **22.**

(18) See Art. **26.**

BLOWPIPING.

(1) See Art. **2.**

(2) See Arts. **3** and **4.**

(3) See Arts. **6** and **7.**

(4) See Arts. **8** and **9.**

(5) See Art. **10.**

(6) $Al_2(OH)_6$; $Cr_2(SO_4)_3$.

(7) See Art. **15.**

(8) See Arts. **16** to **19.**

(9) See Art. **22.**

(10) See Art. **12.**

(11) See Art. **13.**

(12) See Art. **14.**

(13) See Art. **13.**

(14) See Art. **31.**

(15) See Arts. **33** and **34.**

(16) See Arts. **44** to **46.**

(17) See Art. **48.**

(18) See Arts. **49** to **53.**

(19) See Art. **54.**

(20) See Art. **36.**

(21) See Art. **37.**

(22) See Arts. **41** to **43.**

(23) See Art. **94.**

(24) See Arts. **96** to **98.**

(25) See Arts. **96** and **98.**

(26) See Arts. **96** and **98.**

(27) See Arts. **96** and **98.**

(28) See Arts. **99** and **100.**

(29) See Arts. **58** and **59.**

(30) See Art. **100.**

(31) See Arts. **105** and **106.**

(32) See Arts. **101** and **102.**

(33) See Art. **103.**

(34) See Art. **104.**

(35) See Arts. **107** to **111.**

(36) See Tables **III** and **IV.**

(37) See Tables **V** and **VI.**

(38) See Arts. **112** to **114.**

(39) See Art. **115.**

(40) See Table VII.

(41) See Art. **96.**

(42) See Art. **117.**

(43-50) No Key.

MINERALOGY.

(1) See Art. **1.**

(2) See Art. **2.**

(3) See Arts. **9** and **10.**

(4) See Art. **10.**

(5) See Art. **4.**

(6) See Art. **12.**

(7) See Arts. **5** and **6.**

(8) See Art. **11.**

(9) See Arts. **1** to **17.**

(10) See Arts. **14** to **16.**

(11) See Art. **8.**

(12) See Arts. **18** and **19.**

(13) See Art. **19.**

(14) See Art. **21.**

(15) See Art. **22.**

(16) See Art. **29.**

(17) See Art. **36.**

(18) See Art. **36.**

(19) See Art. **80.**

(20) See Art. **81.**

(21) See Arts. **39** to **45.**

(22) See Art **38.**

(23) See Arts. **39** to **45.**

(24) See Art. **44.**

(**25**) See Art. **42.**

(**26**) See Art. **59.**

(**27**) See Art. **60.**

(**28**) See Arts. **65** to **71.**

(**29**) See Art. **70.**

(**30**) See Art. **72.**

(**31**) See Art. **78.**

(**32**) See Art. **47.**

(**33**) See Art. **55.**

(**34**) See Art. **53.**

(**35**) See Art. **49.**

(**36**) See Table **I.**

(**37**) See Table **I.**

(**38**) See Art. **66.**

(**39**) See Art. **85.**

(**40**) See Arts. **85** to **90.**

(**41**) See Table **II** and Arts. **101** and **104.**

(**42**) See Art. **92.**

(**43**) See Table **I** and Art. **94.**

(**44**) See Table **II** and Arts. **101** and **102.**

(**45**) See Art. **98.**

(**46**) See Art. **98.**

(**47**) See Art. **98.**

(**48**) See Art. **98.**

(**49**) See Art. **98.**

(**50**) See Table **II.**

(**51**) See Arts. **44** and **49.**

(**52**) See Art. **64.**

(**53**) See Arts. **99** to **104.**

ASSAYING.

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(1) (*a*) See Art. **1.**

(*b*) See Art. **3.**

(2) See Art. **2.**

(3) See Art. **6.**

(4) See Arts. **107** to **111.**

(5) (*a*) See Arts. **134** to **135.**

(*b*) See Art. **138.**

(6) (*a*) See Art. **26.**

(*b*) See Art. **28.**

(7) (*a*) See Art. **36.**

(*b*) See Art. **36.**

(8) (*a*) In weighing the first button, place the button on one pan and it will be found that a 10-mg. and a 5-mg. weight in the other pan will not quite balance the button, while the addition of a 1-mg. weight will overbalance it; remove the 1-mg. weight, close the scale case, and move the rider along the beam until the button is balanced. This will occur with the rider 38 divisions from the center of the beam on the side opposite to that in which the button is placed; hence, the weight of the button will be

$$10 + 5 + .38 \times 2 = 15.76 \text{ mg.}$$

Now remove the button which has been weighed, leaving the weights in the scale-pan and the rider in its position. Place the other button in the scale-pan and it will be found that the weights slightly overbalance the button. By removing the rider 1 division of a beam towards the center, it will be found that a balance is established, and hence the second button weighs

$$10 + 5 + .37 \times 2 = 15.74 \text{ mg.}$$

Of course, the weight of the second button could be obtained by simply subtracting the amount the rider was moved from the first reading, or, as it was moved 1 division, it would represent .02 mg.

In weighing the gold, place the gold in one pan and it will be found that the 2-mg. weight slightly overbalances, while a 1-mg. weight does not nearly balance it. It may be weighed by leaving the 1-mg. weight in place and moving the rider out 49 divisions to the side opposite to that on which the gold has been placed; hence, the weight will be

$$1 + .49 \times 2 = 1.98 \text{ mg.}$$

This weight may also be obtained by leaving the 2-mg. weight on the side opposite the gold and carrying the rider across to the side on which the gold is being weighed. In this case, a balance will be established when the rider has been moved 1 division towards the pan containing the gold, and the weight will be

$$2 - .01 \times 2 = 1.98 \text{ mg.}$$

(For detailed description of weighing see Art. **32.**)

(*b*) As ½ A. T. was taken, adding the two weights will give the amount contained in 1 A. T.; hence,

$$\begin{array}{r} 15.76 \\ \underline{15.74} \\ 31.50 \end{array} \text{ mg.} =$$

combined weight of gold and silver, or the total number of ounces per ton of precious metals contained in the ore; the weight of the gold equals the number of ounces per ton of gold in the ore, and

31.50
1.98 mg., or the number of ounces of gold per ton.
29.52 mg., or the number of ounces of silver per ton.

(9) (*a*) See Art. **140.**

(*b*) See Art. **141.**

(*c*) See Art. **144.**

(10) (*a*) See Art. **181.**

(*b*) See Art. **182.**

(11) See Art. **229.**

(12) (*a*) and (*b*) See Art. **4.**

(13) (*a*) and (*b*) See Art. **24.**

(14) (*a*) See Art. **60.**

(*b*) See Arts. **64** to **76.**

(15) (*a*) See Table I.

(*b*) See Table III.

(16) From Table IV, by looking in the first column we find 300, and opposite in the second column, 305 to 310 as the number of milligrams of silver to be used in the proof assay when 300 mg. were obtained from the preliminary assay. As there is 15% copper in the bullion, we turn to the fourth column of Table IV, and opposite 15% find in the fifth column 75 mg. as the amount of c. p. copper to be added to the proof assay. The sum of the weights of the silver and the copper contained in the preliminary assay is as follows:

$$\begin{array}{r} 75 \\ \underline{300} \\ 375 \end{array} \text{ and }$$

$$\begin{array}{r} 500 \\ \underline{375} \\ 125 \end{array} \text{ mg.,}$$

or the amount of test lead which must be used in making up the proof assay. In column 3 of Table IV, opposite 300 in the first column we find 15 as the number of milligrams of

c. p. lead foil to be used for wrapping the proof assay; hence, the proof assay will consist of

305 to 310 mg. of silver,
75 mg. of copper,
125 mg. of test lead,

all to be wrapped in 15 mg. of c. p. lead foil.

(17) In order to obtain the average of 2 titrations, add the results from both and divide the sum by 2.

$$\begin{array}{r} 48.3 \\ \underline{48.2} \\ 2)\overline{96.5} \\ 48.25 \end{array} = \text{average number of c. c.}$$

As the standard of the iron solution is 0.00995, we must multiply the average number of c. c. by it.

48.25 × 0.00995 = .48008, or practically 48.01% of iron.

(18) (*a*), (*b*), and (*c*) See Art. **6.**

(19) (*a*) See Arts. **29** to **31.**
(*b*) See Art. **29.**

(20) (*a*), (*b*), and (*c*) See Art. **69.**

(21) (*a*) See Arts. **100** to **103.**
(*b*) See Art. **105.**

(22) As the standard for iron is normal and would read directly into per cent. of iron, it is simply necessary to multiply the number of c. c. used by the factor for manganese, or

20.5 × .2946 = 6.039, or practically 6.04% manganese.

(23) See Arts. **232** to **235.**

(24) (*a*) See Art. **8.**
(*b*) See Arts. **8** and **9.**

(25) (*a*) See Arts. **40** to **43.**
(*b*) See Art. **44.**

(26) As 2 A.T. of litharge were employed when testing the litharge, it will be necessary to divide the result by 2 to obtain the amount of silver contained in 1 A. T.; hence, $1.5 \div 2 = .75$ mg., or the amount of silver in 1 assay ton of litharge; and as 5.75 mg. were obtained as the total weight of silver, the weight of silver in the ore can be found by subtracting the amount of silver in the litharge from this, or 5.75 mg. $-$ .75 $=$ 5.0 mg. $=$ the number of ounces of silver per ton in the ore.

(27) See Arts. **229** to **236.**

(28) As 500 mg. of fine bullion were used, it will be necessary to multiply the results by 2; hence, $398.5 \times 2 = 797.0$, or the number of milligrams of silver in 1,000 mg. of bullion, and $16 \times 2 = 32$, or the number of milligrams of gold in 1,000 mg. of bullion; hence, the bullion would be reported as 797.0 fine in silver and 32.0 fine in gold, or, as the bar would be stamped,

Silver.......... .797,
Gold........... .032,

omitting the word fine.

(29) (*a*) As the iron solution is normal and as the results in CaO are just ½ of those in iron, we will simply have to divide the number of c. c. obtained by 2, or $30.4 \div 2 = 15.2$, the percentage of CaO in the ore.

(*b*) As CaO contains 71.43% of Ca, the percentage contained in the ore will be found by multiplying 15.2 by this factor, or $15.2 \times .7143 = .10857$, or practically 10.86% Ca.

(30) See Art. **213.**

(31) (*a*) and (*b*) See Art. **30.**

(32) As the same amount of test lead was employed in the assay and in obtaining the amount of silver in the test lead, we will have simply to subtract the amount obtained from the results of the assay, or 32.5 — .3 = 32.2 mg. As the number of milligrams of silver contained in .1 A.T. of ore and as .1 of ore was employed, the results must be multiplied by 10 to obtain the amount contained in 1 A.T. Hence, 32.2 × 10 = 322 mg., or the number of ounces of silver per ton contained in the ore.

(33) (*a*) See Arts. **145** to **147.**

(*b*) See Art. **148.**

(34) (*a*) and (*b*) See Art. **225.**

(35) (*a*) See Arts. **11** to **13.**

(*b*) See Art. **14.**

(36) (*a*) See Arts. **50** to **52.**

(*b*) See Art. **56.**

(37) (*a*) First, adding the two weights so as to obtain the average, we have

3.45
3.47
6.92 = total number mg. from $\frac{1}{5}$ A.T.
2.12 = number of mg. of gold from $\frac{1}{5}$ A.T.
4.80 = number of mg. of silver from $\frac{1}{5}$ A. T.

On account of the fact that only $\frac{1}{5}$ A. T. has been taken for obtaining the results in milligrams, it will be necessary to multiply by 5, or

2.12 × 5 = 10.6 ounces of gold per ton,
4.80 × 5 = 24.00 ounces of silver per ton.

(*b*) The value of the precious metals is obtained by multiplying the number of ounces of each by its value per ton, or

10.6 × 20.67 =	$219.10
24.00 × .60 =	14.40
Total,	$233.50

(38) (*a*) Owing to the fact that two 5-gram charges of ore were taken, the amount in one 10-gram charge can be obtained by adding the two results, or

$$\begin{array}{l} 3.25 \\ \underline{3.27} \\ 6.52 \text{ g. of lead in 10 g. of ore; hence,} \end{array}$$

$6.52 \div 10 = .652$, or the ore contains 65.2% lead.

(*b*) If 10-gram charges had been taken in place of 5, and the same amount of lead obtained from each charge, the ore would have contained just half the amount of lead given above, or

$$.652 \div 2 = .326, \text{ or } 32.6\%.$$

(39) See Art. **226.**

(40) As the potassium-cyanide solution is a $\frac{1}{2}$-normal solution, it will take 2 c. c. of it to equal 1% of copper when 1 gram of ore is employed, and hence 1 gram of ore containing 15% of copper would require 15×2 or 30 c. c. of the standard solution to titrate the copper.

(41) See Art. **13.**

(42) See Art. **120.**

(43) (*a*) and (*b*) See Art. **78.**

(44) (*a*) As $\frac{1}{2}$ A. T. charges of ore were taken, the number of milligrams of the precious metal per A. T. will be found by adding the two results, or

$$\begin{array}{l} 4.76 \\ \underline{4.75} \\ 9.51 \text{ mg.} = \text{ounces precious metals per ton.} \end{array}$$

$$\begin{array}{r} 9.51 \\ \underline{.75} \text{ mg.} = \text{ounces gold per ton.} \\ 8.76 \text{ mg.} = \text{ounces silver per ton.} \end{array}$$

(*b*) $.75 \times 20.67 = \$15.50$, value of gold per ton.
$8.76 \times .59 = \quad 5.17$, value of silver per ton.
$\$20.67$, total value per ton.

(45) (*a*), (*b*), and (*c*) See Art. **170.**

(46) See Arts. **230** and **231.**

(47) See Arts. **16** and **17.**

(48) See Arts. **32** and **33.**

(49) (*a*) and (*b*) See Art. **83.**

(50) See Arts. **85** to **89.**

(51) (*a*) See Art. **128.**

(*b*) See Art. **129.**

(52) If 1 A. T. of $10.00 ore had been taken, the weight of the button obtained from the assay would be $10.00 \div 20.67 = .48379$, or practically .4838. As ½ A. T. was taken, the weight of the button would be ½ of this amount, or $.4838 \div 2 = .2419$ mg.

NOTE.—Most button balances would not determine the weight closer than .24 mg.

(53) (*a*) and (*b*) See Art. **104.**

(54) (*a*), (*b*), and (*c*) See Arts. **11** and **18.**

(55) (*a*) See Art. **56.**

(*b*) See Art. **55.**

(56) (*a*) and (*b*) See Art. **89.**

(57) (*a*) $\frac{AC + 29.166\,D}{A + B}$ = ounces per ton.

(*b*) As ½ A. T. was taken for the determination of gold and silver in pulp, the results from 1 A. T. can be obtained as follows:

$$\begin{array}{r} 98.51 \\ 98.53 \\ \hline 197.04 \end{array}$$

197.04 = mg. from 1 A. T.

1.75 = mg. gold in the A. T. = oz. per ton.

195.29 = mg. silver in the A. T. = oz. per ton.

Now, substituting these factors in the equation, we obtain for silver

$$\frac{248.5 \times 195.29 + 29.166 \times 853.51}{248.5 + 1.98} = 293.129 \text{ oz. silver per ton.}$$

The value of gold per ton is obtained by substituting as follows:

$$\frac{248.5 \times 1.75 + 29.166 \times 5.62}{248.5 + 1.98} = 2.39 \text{ oz. gold per ton.}$$

(*c*) The total value of the ore is as follows:

$2.39 \times 20.67 =$ \$ 49.4013, value of gold.
$293.129 \times .60 =$ 175.8774, value of silver.

\$225.2787, or \$225.28, total value per ton.

(58) See Arts. **172** and **175** to **180.**

(59) (*a*) and (*b*) See Art. **95.**

(60) (*a*) See Art. **22.**
(*b*) See Art. **23.**

(61) To obtain the weight of the barium sulphate, subtract the weight of the filter ash from the total weight obtained, or

$$\begin{array}{r} .423 \\ .004 \\ \hline .419 \end{array}$$

This multiplied by .13734 = .057545, or practically 5.75% sulphur.

(62) (*a*) See Arts. **90** to **96.**
(*b*) See Art. **90.**
(*c*) See Art. **94.**

(63) (*a*), (*b*), and (*c*) See Art. **133.**

(64) As the piano wire contains only 99.7% iron, it will be necessary to determine the amount of iron in the samples used, and

$0.105 \times .997 = .104685$, or the number of grams of iron in the first sample, and

$0.112 \times .997 = .111664$, or the number of grams of iron in the second sample.

Dividing each weight of iron by the number of c. c. of solution which it required to titrate it, we will obtain the standard for each case, or

$0.104685 \div 20.9 = .005008$, for the first standard, and
$0.111664 \div 22.4 = .004985$, for the second standard.

Adding these two, we obtain .009993, and dividing the result by 2, we have .004996 g., equals the amount of iron in grams which each c. c. of the standard solution is equal to, or the standard of the solution in iron.

(65) For the gold and silver determination, as ½ A. T. charges were taken, hence, it will be necessary to add the combined weight of the buttons to obtain the amount in 1 A. T., or

8.63
8.62
17.25 mg. = weight of gold and silver.
2.35 mg. = ounces of gold per ton.
14.90 mg. = ounces of silver per ton.

As 25 c. c. of the normal solution was employed in the iron determination and 1 gram of ore was taken, each c. c. of the solution will be equal to 1% iron, or the sample will contain 25% iron.

As the same solution was employed in titrating 1 gram of the ore in the determination of CaO, and as the standard for CaO is just ½ that of iron, the result will be

$18.4 \div 2 = 9.2 =$ the percentage of CaO in the sample.

For obtaining the percentage of copper, we multiply the number of c. c. employed by the factor for the solution, or

$26.3 \times .0049 = .12887$, or practically
12.89 per cent. of copper.

In like manner, in the determination of zinc, we multiply the number of c. c. employed by the factor for the solution, or

$$4.5 \times .0099 = .04455, \text{ or practically } 4.46\% \text{ zinc.}$$

For the determination of silica in soluble matter, we would simply divide the weight obtained by the weight taken, or

$$.125 \div 1.00 = .125 = 12.5\% \text{ silica.}$$

Hence the ore contains

2.35 ounces gold per ton,
14.9 ounces silver per ton,
25% iron,
9.2% *CaO*,
12.89% copper,
4.46% zinc,
12.5% silica.

GEOLOGY

(1) See Art. **1.**

(2) See Art. **2.**

(3) See Art. **4.**

(4) See Art. **13.**

(5) See Art. **8.**

(6) See Art. **14.**

(7) See Art. **14.**

(8) See Art. **21.**

(9) See Art. **23.**

(10) See Art. **26.**

(11) See Art. **27.**

(12) See Arts. **28** to **30.**

(13) See Arts. **31** and **32.**

(14) See Art. **35.**

(15) See Art. **38.**

(16) See Art. **40.**

(17) See Art. **42.**

(18) See Art. **45.**

(19) See Art. **46.**

(20) See Art. **48.**

(21) See Art. **49.**

(22) See Art. **50.**

(23) See Art. **52.**

(24) See Arts. **53** and **54**.

(25) See Art. **55.**

(26) See Art. **56.**

(27) See Art. **59.**

(28) See Art. **63.**

(29) See Art. **66.**

(30) See Art. **68**

(31) See Art. **69.**

(32) See Art. **70.**

(33) See Arts. **73** to **80.**

(34) See Art. **81.**

(35) See Arts. **82** to **89.**

(36) See Arts. **79** and **82.**

(37) See Art. **92.**

(38) See Art. **94.**

(39) See Art. **98.**

(40) See Art. **99.**

(41) See Arts. **109** to **113.**

(42) See Art. **116.**

(43) See Art. **118.**

(44) See Arts. **121** to **124.**

(45) See Art. **124.**

(46) See Art. **127.**

(47) See Arts. **128** and **129.**

(48) See Art. **132.**

(49) See Art. **140.**

(50) See Art. **141.**

(51) See Art. **147.**

(52) See Arts. **148** and **149.**

(53) See Art. **152.**

(54) See Art. **156.**

(55) See Art. **160.**

(56) See Art. **169.**

(57) See Art. **171.**

(58) See Art. **174.**

(59) See Art. **175.**

(60) See Art. **154.**

(61) See Arts. **181** and **182.**

(62) See Arts. **186** and **187.**

(63) See Art. **189.**

(64) See Arts. **190** to **192.**

(65) See Art. **195.**

(66) See Arts. **203** to **207.**

(67) See Arts. **208** and **209.**

(68) See Arts. **216** to **218.**

(69) See Art. **227.**

(70) See Arts. **244** to **249.**

(71) See Art. **250.**

(72) See Art. **260.**

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(**2**) See Art. **81.**

(**3**) See Arts. **35** and **36.**

(**4**) See Arts. **37** to **39.**

(**5**) See Art. **40.**

(**6**) (*a*) See Art. **41.**
(*b*) See Art. **55.**

(**7**) See Arts. **42** and **56.**

(**8**) (*a*) See Art. **45.**
(*b*) See Art. **57.**

(**9**) See Art. **47.**

(**10**) (*a*) See Art. **48.**
(*b*) See Art. **49.**
(*c*) See Art. **50.**

(**11**) See Arts. **53** to **60.**

(**12**) See Art. **51.**

(**13**) See Art. **13.**

(**14**) See Arts. **12** to **16.**

(**15**) (*a*) See Arts. **18** and **19.**
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(**16**) (*a*) See Art. **21.**
(*b*) See Art. **33.**

(17) See Art. **54.**

(18) See Art. **73.**

(19) See Art. **73.**

(20) (*a*) See Art. **86.**
(*b*) See Art. **87.**

(21) See Art. **93.**

(22) See Arts. **92** to **95.**

(23) See Art. **99.**

(24) See Arts. **67** and **68.**

(25) See Arts. **61** and **62.**

(26) See Arts. **64** and **65.**

(27) See Arts. **61** to **63.**

(28) See Arts. **123** to **125.**

(29) See Arts. **123** to **125.**

(30) See Arts. **112** and **114.**

(31) (*a*) See Art. **111.**
(*b*) See Art. **110.**

(32) See Art. **107.**

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(2) See Arts. **4** to **8.**

(3) See Arts. **15** to **24** and Art. **142.**

(4) See Arts. **41** to **50.**

(5) See Art. **128.**

(6) See Arts. **128** to **130.**

(7) See Art. **128.**

(8) See Arts. **13** and **74.**

(9) See Art. **14.**

(10) See Arts. **118** and **119.**

(11) See Art. **37.**

(12) See Art. **65.**

(13) See Art. **61.**

(14) See Art. **41.**

(15) See Art. **74.**

(16) See Art. **76.**

(17) See Art. **78.**

(18) See Art. **85.**

(19) See Arts. **86** and **87.**

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(21) See Arts. **91** to **93.**

(22) See Arts. **108** to **110.**

(23) See Art. **112.**

(24) See Art. **115.**

(25) See Art. **115.**

(26) See Arts. **120** to **122.**

(27) See Arts. **120** and **121.**

(28) See Art. **125.**

(29) See Arts. **126** and **128.**

(30) See Art. **134.**

(31) See Art. **133.**

(32) See Art. **132.**

(33) See Arts. **47** and **49.**

(34) See Art. **17.**

(35) See Art. **21.**

(36) See Art. **22.**

(37) See Arts. **25** and **36.**

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(41) See Art. **34.**

(42) See Art. **66.**

(43) See Arts. **61** and **64.**

(44) See Art. **38.**

(45) See Arts. **67, 70,** and **71.**

(46) See Art. **72.**

(47) See Art. **51.**

(48) See Art. **142.**

(49) See Art. **56.**

(50) See Art. **135.**

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(2) See Art. **14.**

(3) See Arts. **16** and **146** to **148.**

(4) See Art. **18.**

(5) See Art. **35.**

(6) See Art. **36.**

(7) See Arts. **41** and **42.**

(8) See Art. **48.**

(9) See Art. **59.**

(10) See Arts. **61** and **62.**

(11) See Art. **66.**

(12) See Art. **70.**

(13) See Arts. **71** and **72.**

(14) See Arts. **73** and **74.**

(15) See Art. **81.**

(16) See Art. **82.**

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(2) See Arts. **10** and **11.**

(3) See Arts. **13** and **14.**

(4) See Arts. **18** to **21.**

(5) See Art. **22.**

(6) See Arts. **25** to **27.**

(7) See Art. **31.**

(8) See Arts. **39** to **44.**

(9) See Arts. **45** to **53.**

(10) See Arts. **59** to **66.**

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(13) See Arts. **78** to **81.**

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(**27**) See Art. **155.**

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(5) See Arts. **23** and **24.**

(6) See Art. **26.**

(7) See Art. **32.**

(8) See Art. **37.**

(9) See Art. **38.**

(10) See Art. **43.**

(11) See Art. **45.**

(12) See Art. **47.**

(13) See Art. **49.**

(14) See Art. **53.**

(15) See Arts. **61** and **62.**

(16) See Art. **69.**

(17) See Art. **73.**

(18) See Art. **74.**

(19) See Art. **76.**

(20) See Art. **78.**

(21) See Art. **82.**

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(12) See Art. **18.**

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(18) See Art. **33.**

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(27) See Arts. **65** to **67.**

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(31) See Art. **92.**

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(55) See Art. **218.**

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(59) See Art. **250.**

A TEXTBOOK

ON

MINING ENGINEERING

INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

TABLES AND FORMULAS

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A-2

TABLES AND FORMULAS.

This volume contains all the principal Tables, Rules, and Formulas occurring in the Instruction Papers of the Course. They have been collected and placed in this volume in order to make them convenient for ready reference, so that the student will not be obliged to search the Instruction Papers to find them.

The various Rules and Formulas are here grouped under the titles of the Instruction Papers in which they occur. Following each rule and formula are its number and also that of the article of the Instruction Paper in which it is discussed. These numbers may be readily found in the text by noting the section number on the headline at the top of the page. Thus: on page 161 following, we see that the formula for "Discharge of Pump" is followed by "(**191**)," the number of the formula, and by "Art. **2292**," the article in which it occurs; and that "§ 21" is on the head-line. We then find § 21 and look through this section until Art. **2292** is found.

TABLES

OF

NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS

GIVING THE VALUES OF THE FUNCTIONS FOR ALL DEGREES AND MINUTES FROM

0° TO 90°

′	0°		1°		2°		3°		4°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.00000	1.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	1.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	1.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	1.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	1.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	1.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	1.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	1.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	1.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	1.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	1.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	89°		88°		87°		86°		85°		

	[illegible]		[illegible]		[illegible]		[illegible]		9°		'
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.15643	.98769	60
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	59
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	58
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	57
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.98751	56
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.98746	55
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.98741	54
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.98737	53
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.98732	52
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	51
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.98723	50
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.98718	49
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	38
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	37
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	36
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	34
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	33
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.16533	.98624	29
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	24
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.98595	23
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	18
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	17
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	16
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.98551	14
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	13
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.17021	[illegible]	12
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.17078	.98531	10
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.17107	.98526	9
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.15414	[illegible]	.17136	.98521	8
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.15442	[illegible]	.17164	.98516	7
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.15471	[illegible]	.17193	.98511	6
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.17222	.98506	5
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.17250	.98501	4
57	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.17279	.98496	3
58	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.17308	.98491	2
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.17336	.98486	1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.17365	.98481	0
'	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	'
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94204	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31051	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32109	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	[illegible]	[illegible]	[illegible]	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	[illegible]	[illegible]	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	[illegible]	[illegible]	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	[illegible]	[illegible]	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	[illegible]	[illegible]	.95124	.32502	.94571	.34147	.93989	2
59	[illegible]	.96134	[illegible]	[illegible]	[illegible]	.95115	.32529	.94561	.34175	.93979	1
60	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.32557	.94552	.34202	.93969	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	74°		73°		72°		71°		70°		

′	20°		21°		22°		23°		24°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	69°		68°		67°		66°		65°		

′	25° Sine	25° Cosine	26° Sine	26° Cosine	27° Sine	27° Cosine	28° Sine	28° Cosine	29° Sine	29° Cosine	′
0	[illegible]	[illegible]	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	[illegible]	[illegible]	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	[illegible]	[illegible]	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	[illegible]	[illegible]	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	[illegible]	[illegible]	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	[illegible]	[illegible]	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	[illegible]	[illegible]	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	[illegible]	[illegible]	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	[illegible]	[illegible]	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87340	52
9	[illegible]	[illegible]	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	[illegible]	[illegible]	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	[illegible]	[illegible]	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	[illegible]	[illegible]	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	[illegible]	[illegible]	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	[illegible]	[illegible]	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	[illegible]	[illegible]	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	[illegible]	[illegible]	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	[illegible]	[illegible]	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	[illegible]	[illegible]	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	[illegible]	[illegible]	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	[illegible]	[illegible]	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	[illegible]	[illegible]	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	[illegible]	[illegible]	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	[illegible]	[illegible]	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	[illegible]	[illegible]	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	[illegible]	[illegible]	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	[illegible]	[illegible]	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	[illegible]	[illegible]	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	[illegible]	[illegible]	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	[illegible]	[illegible]	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	[illegible]	[illegible]	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	[illegible]	[illegible]	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	[illegible]	[illegible]	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	[illegible]	[illegible]	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	[illegible]	[illegible]	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	[illegible]	[illegible]	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	[illegible]	[illegible]	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	[illegible]	[illegible]	[illegible]	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	[illegible]	[illegible]	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	[illegible]	[illegible]	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	[illegible]	[illegible]	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	[illegible]	[illegible]	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	[illegible]	[illegible]	[illegible]	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	[illegible]	[illegible]	[illegible]	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	[illegible]	[illegible]	[illegible]	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	[illegible]	[illegible]	[illegible]	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	[illegible]	[illegible]	[illegible]	[illegible]	.46587	.88485	.48124	.87659	.49647	.86805	14
47	[illegible]	[illegible]	[illegible]	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	[illegible]	[illegible]	[illegible]	[illegible]	.46639	.88458	.48175	.87631	.49697	.86777	12
49	[illegible]	[illegible]	.45114	[illegible]	.46664	.88445	.48201	.87617	.49723	.86762	11
50	[illegible]	[illegible]	[illegible]	[illegible]	.46690	.88431	.48226	.87603	.49748	.86748	10
51	[illegible]	[illegible]	[illegible]	[illegible]	.46716	.88417	.48252	.87589	.49773	.86733	9
52	[illegible]	[illegible]	[illegible]	[illegible]	.46742	.88404	.48277	.87575	.49798	.86719	8
53	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.48303	.87561	.49824	.86704	7
54	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.88377	.48328	.87546	.49849	.86690	6
55	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.48354	.87532	.49874	.86675	5
56	[illegible]	[illegible]	[illegible]	[illegible]	.46844	[illegible]	.48379	.87518	.49899	.86661	4
57	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.48405	.87504	.49924	.86646	3
58	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.88322	.48430	.87490	.49950	.86632	2
59	[illegible]	[illegible]	[illegible]	[illegible]	.46921	[illegible]	.48456	.87476	.49975	.86617	1
60	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.48481	.87462	.50000	.86603	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	64°		63°		62°		61°		60°		

′	30°		31°		32°		33°		34°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	59°		58°		57°		56°		55°		

′	35° Sine	35° Cosine	36° Sine	36° Cosine	37° Sine	37° Cosine	38° Sine	38° Cosine	39° Sine	39° Cosine	′
0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.77715	60
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	59
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	58
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	57
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.77641	56
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	55
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	54
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.63090	[illegible]	53
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.63113	[illegible]	52
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	51
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.63158	.77531	50
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.63180	.77513	49
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	48
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	47
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.63248	[illegible]	46
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	45
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.77421	44
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	43
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.77384	42
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.77366	41
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.78442	.63383	.77347	40
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.78424	[illegible]	.77329	39
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.63428	.77310	38
23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.63451	.77292	37
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.79441	.62115	[illegible]	[illegible]	.77273	36
25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.79424	[illegible]	[illegible]	[illegible]	.77255	35
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.63518	[illegible]	34
27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.78315	.63540	[illegible]	33
28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.63563	[illegible]	32
29	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.78279	.63585	.77181	31
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.62251	.78261	[illegible]	.77162	30
31	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.78243	[illegible]	.77144	29
32	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.78225	[illegible]	.77125	28
33	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.63675	.77107	27
34	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	26
35	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.78170	[illegible]	.77070	25
36	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.78152	.63742	.77051	24
37	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.62411	[illegible]	[illegible]	.77033	23
38	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.63787	.77014	22
39	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	21
40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.76977	20
41	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.63854	.76959	19
42	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.63877	[illegible]	18
43	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.76921	17
44	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	16
45	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
46	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	14
47	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.76847	13
48	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.76828	12
49	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.76810	11
50	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.76791	10
51	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.76772	9
52	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.76754	8
53	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7
54	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.64145	[illegible]	6
55	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5
56	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4
57	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3
58	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2
59	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1
60	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	54°		53°		52°		51°		50°		

′	40° Sine	40° Cosine	41° Sine	41° Cosine	42° Sine	42° Cosine	43° Sine	43° Cosine	44° Sine	44° Cosine	′
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	49°		48°		47°		46°		45°		

′	0°		1°		2°		3°		4°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	28.6363	.05241	19.0811	.06993	14.3007	60
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.05328	18.7678	.07080	14.1235	57
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.05357	18.6656	.07110	14.0655	56
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.05387	18.5645	.07139	14.0079	55
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.05416	18.4645	.07168	13.9507	54
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.05708	17.5205	.07461	13.4039	44
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.05737	17.4314	.07490	13.3515	43
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.05766	17.3432	.07519	13.2996	42
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.05795	17.2558	.07548	13.2480	41
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	24.5418	.05824	17.1693	.07578	13.1969	40
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.05854	17.0837	.07607	13.1461	39
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.05883	16.0990	.07636	13.0958	38
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	22.9038	.06116	16.3499	.07870	12.7062	30
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	22.4541	.06204	16.1190	.07958	12.5660	27
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06233	16.0435	.07987	12.5199	26
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06262	15.9687	.08017	12.4742	25
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06350	15.7483	.08104	12.3390	22
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06379	15.6762	.08134	12.2946	21
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06408	15.6048	.08163	12.2505	20
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06437	15.5340	.08192	12.2067	19
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06467	15.4638	.08221	12.1632	18
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06496	15.3943	.08251	12.1201	17
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06525	15.3254	.08280	12.0772	16
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06554	15.2571	.08309	12.0346	15
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06584	15.1893	.08339	11.9923	14
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06613	15.1222	.08368	11.9504	13
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06642	15.0557	.08397	11.9087	12
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06671	14.9898	.08427	11.8673	11
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06700	14.9244	.08456	11.8262	10
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06730	14.8596	.08485	11.7853	9
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06759	14.7954	.08514	11.7448	8
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06788	14.7317	.08544	11.7045	7
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06817	14.6085	.08573	11.6645	6
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06847	14.6059	.08602	11.6248	5
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06876	14.5438	.08632	11.5853	4
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06905	14.4823	.08661	11.5461	3
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06934	14.4212	.08690	11.5072	2
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06963	14.3607	.08720	11.4685	1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.06993	14.3007	.08749	11.4301	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		85°		

′	5° Tang	5° Cotang	6° Tang	6° Cotang	7° Tang	7° Cotang	8° Tang	8° Cotang	9° Tang	9° Cotang	′
0	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	60
1	.08778	11.3919	.10540	9.48781	.12308	8.12481	.14084	7.10038	.15868	6.30189	59
2	.08807	11.3540	.10569	9.46141	.12338	8.10536	.14113	7.08546	.15898	6.29007	58
3	.08837	11.3163	.10599	9.43515	.12367	8.08600	.14143	7.07059	.15928	6.27829	57
4	.08866	11.2789	.10628	9.40904	.12397	8.06674	14173	7.05579	.15958	6.26655	56
5	.08895	11.2417	.10657	9.38307	.12426	8.04756	.14202	7.04105	.15988	6.25486	55
6	.08925	11.2048	.10687	9.35724	.12456	8.02848	.14232	7.02637	.16017	6.24321	54
7	.08954	11.1681	.10716	9.33155	.12485	8.00948	.14262	7.01174	.16047	6.23160	53
8	.08983	11.1316	.10746	9.30599	.12515	7.99058	.14291	6.99718	.16077	6.22003	52
9	.09013	11.0954	.10775	9.28058	.12544	7.97176	.14321	6.98268	.16107	6.20851	51
10	.09042	11.0594	.10805	9.25530	.12574	7.95302	.14351	6.96823	.16137	6.19703	50
11	.09071	11.0237	.10834	9.23016	.12603	7.93438	.14381	6.95385	.16167	6.18559	49
12	.09101	10.9882	.10863	9.20516	.12633	7.91582	.14410	6.93952	.16196	6.17419	48
13	.09130	10.9529	.10893	9.18028	.12662	7.89734	.14440	6.92525	.16226	6.16283	47
14	.09159	10.9178	.10922	9.15554	.12692	7.87895	.14470	6.91104	.16256	6.15151	46
15	.09189	10.8829	.10952	9.13093	.12722	7.86064	.14499	6.89688	.16286	6.14023	45
16	.09218	10.8483	.10981	9.10646	.12751	7.84242	.14529	6.88278	.16316	6.12899	44
17	.09247	10.8139	.11011	9.08211	.12781	7.82428	.14559	6.86874	.16346	6.11779	43
18	.09277	10.7797	.11040	9.05789	.12810	7.80622	.14588	6.85475	.16376	6.10664	42
19	.09306	10.7457	.11070	9.03379	.12840	7.78825	.14618	6.84082	.16405	6.09552	41
20	.09335	10.7119	.11099	9.00983	.12869	7.77035	.14648	6.82694	.16435	6.08444	40
21	.09365	10.6783	.11128	8.98598	.12899	7.75254	.14678	6.81812	.16465	6.07340	39
22	.09394	10.6450	.11158	8.96227	.12929	7.73480	.14707	6.79936	.16495	6.06240	38
23	.09423	10.6118	.11187	8.93867	.12958	7.71715	.14737	6.78564	.16525	6.05143	37
24	.09453	10.5789	.11217	8.91520	.12988	7.69957	.14767	6.77199	.16555	6.04051	36
25	.09482	10.5462	.11246	8.89185	.13017	7.68208	.14796	6.75838	.16585	6.02962	35
26	.09511	10.5136	.11276	8.86862	.13047	7.66466	.14826	6.74483	.16615	6.01878	34
27	.09541	10.4813	.11305	8.84551	.13076	7.64732	.14856	6.73133	.16645	6.00797	33
28	.09570	10.4491	.11335	8.82252	.13106	7.63005	.14886	6.71789	.16674	5.99720	32
29	.09600	10.4172	.11364	8.79964	.13136	7.61287	.14915	6.70450	.16704	5.98646	31
30	.09629	10.3854	.11394	8.77689	.13165	7.59575	.14945	6.69116	.16734	5.97576	30
31	.09658	10.3538	.11423	8.75425	.13195	7.57872	.14975	6.67787	.16764	5.96510	29
32	.09688	10.3224	.11452	8.73172	.13224	7.56176	.15005	6.66463	.16794	5.95448	28
33	.09717	10.2913	.11482	8.70931	.13254	7.54487	.15034	6.65144	.16824	5.94390	27
34	.09746	10.2602	.11511	8.68701	.13284	7.52806	.15064	6.63831	.16854	5.93335	26
35	.09776	10.2294	.11541	8.66482	.13313	7.51132	.15094	6.62523	.16884	5.92283	25
36	.09805	10.1988	.11570	8.64275	.13343	7.49465	.15124	6.61219	.16914	5.91236	24
37	.09834	10.1683	.11600	8.62078	.13372	7.47806	.15153	6.59921	.16944	5.90191	23
38	.09864	10.1381	.11629	8.59893	.13402	7.46154	.15183	6.58627	.16974	5.89151	22
39	.09893	10.1080	.11659	8.57718	.13432	7.44509	.15213	6.57339	.17004	5.88114	21
40	.09923	10.0780	.11688	8.55555	.13461	7.42871	.15243	6.56055	.17033	5.87080	20
41	.09952	10.0483	.11718	8.53402	.13491	7.41240	.15272	6.54777	.17063	5.86051	19
42	.09981	10.0187	.11747	8.51259	.13521	7.39616	.15302	6.53503	.17093	5.85024	18
43	.10011	9.98931	.11777	8.49128	.13550	7.37999	.15332	6.52234	.17123	5.84001	17
44	.10040	9.96007	.11806	8.47007	.13580	7.36389	.15362	6.50970	.17153	5.82982	16
45	.10069	9.93101	.11836	8.44896	.13609	7.34786	.15391	6.49710	.17183	5.81966	15
46	.10099	9.90211	.11865	8.42795	.13639	7.33190	.15421	6.48456	.17213	5.80953	14
47	.10128	9.87338	.11895	8.40705	.13669	7.31600	.15451	6.47206	.17243	5.79944	13
48	.10158	9.84482	.11924	8.38625	.13698	7.30018	.15481	6.45961	.17273	5.78938	12
49	.10187	9.81641	.11954	8.36555	.13728	7.28442	.15511	6.44720	.17303	5.77936	11
50	.10216	9.78817	.11983	8.34496	.13758	7.26873	.15540	6.43484	.17333	5.76937	10
51	.10246	9.76009	.12013	8.32446	.13787	7.25310	.15570	6.42253	.17363	5.75941	9
52	.10275	9.73217	.12042	8.30406	.13817	7.23754	.15600	6.41026	.17393	5.74949	8
53	.10305	9.70441	.12072	8.28376	.13846	7.22204	.15630	6.39804	.17423	5.73960	7
54	.10334	9.67680	.12101	8.26355	.13876	7.20661	.15660	6.38587	.17453	5.72974	6
55	.10363	9.64935	.12131	8.24345	.13906	7.19125	.15689	6.37374	.17483	5.71992	5
56	.10393	9.62205	.12160	8.22344	.13935	7.17594	.15719	6.36165	.17513	5.71013	4
57	.10422	9.59490	.12190	8.20352	.13965	7.16071	.15749	6.34961	.17543	5.70037	3
58	.10452	9.56791	.12219	8.18370	.13995	7.14553	.15779	6.33761	.17573	5.69064	2
59	.10481	9.54106	.12249	8.16398	.14024	7.13042	.15809	6.32566	.17603	5.68094	1
60	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	.17633	5.67128	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	84°		83°		82°		81°		80°		

14° Tang	14° Cotang	′
[illegible]	4.01078	60
[illegible]	4.00582	59
[illegible]	[illegible]	58
[illegible]	[illegible]	57
[illegible]	[illegible]	56
[illegible]	[illegible]	55
[illegible]	[illegible]	54
[illegible]	[illegible]	53
[illegible]	[illegible]	52
[illegible]	[illegible]	51
[illegible]	[illegible]	50
[illegible]	[illegible]	49
[illegible]	[illegible]	48
[illegible]	[illegible]	47
[illegible]	[illegible]	46
[illegible]	[illegible]	45
[illegible]	[illegible]	44
[illegible]	[illegible]	43
[illegible]	[illegible]	42
[illegible]	[illegible]	41
[illegible]	[illegible]	40

′	15° Tang	15° Cotang	16° Tang	16° Cotang	17° Tang	17° Cotang	18° Tang	18° Cotang	19° Tang	19° Cotang	′
0	.26795	3.73205	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.26826	3.72771	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.26857	3.72338	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.26888	3.71907	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.26920	3.71476	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.26951	3.71046	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.26982	3.70616	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.27013	3.70188	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.27044	3.69761	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.27076	3.69335	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.27107	3.68909	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.27138	3.68485	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.27169	3.68061	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.27201	3.67638	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.27232	3.67217	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.27263	3.66796	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.27294	3.66376	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.27326	3.65957	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.27357	3.65538	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.27388	3.65121	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.27419	3.64705	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.27451	3.64289	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.27482	3.63874	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.27513	3.63461	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.27545	3.63048	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.27576	3.62636	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.27607	3.62224	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.27638	3.61814	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.27670	3.61405	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.27701	3.60996	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.27732	3.60588	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.27764	3.60181	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.27795	3.59775	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.27826	3.59370	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.27858	3.58966	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.27889	3.58562	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.27921	3.58160	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.27952	3.57758	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.27983	3.57357	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.28015	3.56957	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.28046	3.56557	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.28077	3.56159	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.28109	3.55761	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.28140	3.55364	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.28172	3.54968	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.28203	3.54573	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.28234	3.54179	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.28266	3.53785	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.28297	3.53393	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.28329	3.53001	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.28360	3.52609	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.28391	3.52219	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.28423	3.51829	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.28454	3.51441	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.28486	3.51053	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.28517	3.50666	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.28549	3.50279	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.28580	3.49894	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.28612	3.49509	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.28643	3.49125	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	74°		73°		72°		71°		70°		

′	23° Tang	23° Cotang	24° Tang	24° Cotang	′
0	[illegible]	[illegible]	.44523	2.24604	60
1	[illegible]	[illegible]	.44558	2.24428	59
2	[illegible]	[illegible]	.44593	2.24252	58
3	[illegible]	[illegible]	.44627	2.24077	57
4	[illegible]	[illegible]	.44662	2.23902	56
5	[illegible]	[illegible]	.44697	2.23727	55
6	[illegible]	[illegible]	.44732	2.23553	54
7	[illegible]	[illegible]	.44767	2.23378	53
8	[illegible]	[illegible]	.44802	2.23204	52
9	[illegible]	[illegible]	.44837	2.23030	51
10	[illegible]	[illegible]	.44872	2.22857	50
11	[illegible]	[illegible]	.44907	2.22683	49
12	[illegible]	[illegible]	.44942	2.22510	48
13	[illegible]	[illegible]	.44977	2.22337	47
14	[illegible]	[illegible]	.45012	2.22164	46
15	[illegible]	[illegible]	.45047	2.21992	45
16	[illegible]	[illegible]	.45082	2.21819	44
17	[illegible]	[illegible]	.45117	2.21647	43
18	[illegible]	[illegible]	.45152	2.21475	42
19	[illegible]	[illegible]	.45187	2.21304	41
20	[illegible]	[illegible]	.45222	2.21132	40
21	[illegible]	[illegible]	.45257	2.20961	39
22	[illegible]	[illegible]	.45292	2.20790	38
23	[illegible]	[illegible]	.45327	2.20619	37
24	[illegible]	[illegible]	.45362	2.20449	36
25	[illegible]	[illegible]	.45397	2.20278	35
26	[illegible]	[illegible]	.45432	2.20108	34
27	[illegible]	[illegible]	.45467	2.19938	33
28	[illegible]	[illegible]	.45502	2.19769	32
29	[illegible]	[illegible]	.45538	2.19599	31
30	[illegible]	[illegible]	.45573	2.19430	30
31	[illegible]	[illegible]	.45608	2.19261	29
32	[illegible]	[illegible]	.45643	2.19092	28
33	[illegible]	[illegible]	.45678	2.18923	27
34	[illegible]	[illegible]	.45713	2.18755	26
35	[illegible]	[illegible]	.45748	2.18587	25
36	[illegible]	[illegible]	.45784	2.18419	24
37	[illegible]	[illegible]	.45819	2.18251	23
38	[illegible]	[illegible]	.45854	2.18084	22
39	[illegible]	[illegible]	.45889	2.17916	21
40	[illegible]	[illegible]	.45924	2.17749	20
41	[illegible]	[illegible]	.45960	2.17582	19
42	[illegible]	[illegible]	.45995	2.17416	18
43	[illegible]	[illegible]	.46030	2.17249	17
44	[illegible]	[illegible]	.46065	2.17083	16
45	[illegible]	[illegible]	.46101	2.16917	15
46	[illegible]	[illegible]	.46136	2.16751	14
47	[illegible]	[illegible]	.46171	2.16585	13
48	[illegible]	[illegible]	.46206	2.16420	12
49	[illegible]	[illegible]	.46242	2.16255	11
50	[illegible]	[illegible]	.46277	2.16090	10
51	[illegible]	[illegible]	.46312	2.15925	9
52	[illegible]	[illegible]	.46348	2.15760	8
53	[illegible]	[illegible]	.46383	2.15596	7
54	[illegible]	[illegible]	.46418	2.15432	6
55	[illegible]	[illegible]	.46454	2.15268	5
56	[illegible]	[illegible]	.46489	2.15104	4
57	[illegible]	[illegible]	.46525	2.14940	3
58	[illegible]	[illegible]	.46560	2.14777	2
59	[illegible]	[illegible]	.46595	2.14614	1
60	[illegible]	[illegible]	.46631	2.14451	0
′	Cotang	Tang	Cotang	Tang	′
	66°		65°		

′	25° Tang	25° Cotang	26° Tang	26° Cotang	27° Tang	27° Cotang	28° Tang	28° Cotang	29° Tang	29° Cotang	′
0	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	60
1	.46666	2.14288	.48809	2.04879	.50989	1.96120	.53208	1.87941	.55469	1.80281	59
2	.46702	2.14125	.48845	2.04728	.51026	1.95979	.53246	1.87809	.55507	1.80158	58
3	.46737	2.13963	.48881	2.04577	.51063	1.95838	.53283	1.87677	.55545	1.80034	57
4	.46772	2.13801	.48917	2.04426	.51099	1.95698	.53320	1.87546	.55583	1.79911	56
5	.46808	2.13639	.48953	2.04276	.51136	1.95557	.53358	1.87415	.55621	1.79788	55
6	.46843	2.13477	.48989	2.04125	.51173	1.95417	.53395	1.87283	.55659	1.79665	54
7	.46879	2.13316	.49026	2.03975	.51209	1.95277	.53432	1.87152	.55697	1.79542	53
8	.46914	2.13154	.49062	2.03825	.51246	1.95137	.53470	1.87021	.55736	1.79419	52
9	.46950	2.12993	.49098	2.03675	.51283	1.94997	.53507	1.86891	.55774	1.79296	51
10	.46985	2.12832	.49134	2.03526	.51319	1.94858	.53545	1.86760	.55812	1.79174	50
11	.47021	2.12671	.49170	2.03376	.51356	1.94718	.53582	1.86630	.55850	1.79051	49
12	.47056	2.12511	.49206	2.03227	.51393	1.94579	.53620	1.86499	.55888	1.78929	48
13	.47092	2.12350	.49242	2.03078	.51430	1.94440	.53657	1.86369	.55926	1.78807	47
14	.47128	2.12190	.49278	2.02929	.51467	1.94301	.53694	1.86239	.55964	1.78685	46
15	.47163	2.12030	.49315	2.02780	.51503	1.94162	.53732	1.86109	.56003	1.78563	45
16	.47199	2.11871	.49351	2.02631	.51540	1.94023	.53769	1.85979	.56041	1.78441	44
17	.47234	2.11711	.49387	2.02483	.51577	1.93885	.53807	1.85850	.56079	1.78319	43
18	.47270	2.11552	.49423	2.02335	.51614	1.93746	.53844	1.85720	.56117	1.78198	42
19	.47305	2.11392	.49459	2.02187	.51651	1.93608	.53882	1.85591	.56156	1.78077	41
20	.47341	2.11233	.49495	2.02039	.51688	1.93470	.53920	1.85462	.56194	1.77955	40
21	.47377	2.11075	.49532	2.01891	.51724	1.93332	.53957	1.85333	.56232	1.77834	39
22	.47412	2.10916	.49568	2.01743	.51761	1.93195	.53995	1.85204	.56270	1.77713	38
23	.47448	2.10758	.49604	2.01596	.51798	1.93057	.54032	1.85075	.56309	1.77592	37
24	.47483	2.10600	.49640	2.01449	.51835	1.92920	.54070	1.84946	.56347	1.77471	36
25	.47519	2.10442	.49677	2.01302	.51872	1.92782	.54107	1.84818	.56385	1.77351	35
26	.47555	2.10284	.49713	2.01155	.51909	1.92645	.54145	1.84689	.56424	1.77230	34
27	.47590	2.10126	.49749	2.01008	.51946	1.92508	.54183	1.84561	.56462	1.77110	33
28	.47626	2.09969	.49786	2.00862	.51983	1.92371	.54220	1.84433	.56501	1.76990	32
29	.47662	2.09811	.49822	2.00715	.52020	1.92235	.54258	1.84305	.56539	1.76869	31
30	.47698	2.09654	.49858	2.00569	.52057	1.92098	.54296	1.84177	.56577	1.76749	30
31	.47733	2.09498	.49894	2.00423	.52094	1.91962	.54333	1.84049	.56616	1.76629	29
32	.47769	2.09341	.49931	2.00277	.52131	1.91826	.54371	1.83922	.56654	1.76510	28
33	.47805	2.09184	.49967	2.00131	.52168	1.91690	.54409	1.83794	.56693	1.76390	27
34	.47840	2.09028	.50004	1.99986	.52205	1.91554	.54446	1.83667	.56731	1.76271	26
35	.47876	2.08872	.50040	1.99841	.52242	1.91418	.54484	1.83540	.56769	1.76151	25
36	.47912	2.08716	.50076	1.99695	.52279	1.91282	.54522	1.83413	.56808	1.76032	24
37	.47948	2.08560	.50113	1.99550	.52316	1.91147	.54560	1.83286	.56846	1.75913	23
38	.47984	2.08405	.50149	1.99406	.52353	1.91012	.54597	1.83159	.56885	1.75794	22
39	.48019	2.08250	.50185	1.99261	.52390	1.90876	.54635	1.83033	.56923	1.75675	21
40	.48055	2.08094	.50222	1.99116	.52427	1.90741	.54673	1.82906	.56962	1.75556	20
41	.48091	2.07939	.50258	1.98972	.52464	1.90607	.54711	1.82780	.57000	1.75437	19
42	.48127	2.07785	.50295	1.98828	.52501	1.90472	.54748	1.82654	.57039	1.75319	18
43	.48163	2.07630	.50331	1.98684	.52538	1.90337	.54786	1.82528	.57078	1.75200	17
44	.48198	2.07476	.50368	1.98540	.52575	1.90203	.54824	1.82402	.57116	1.75082	16
45	.48234	2.07321	.50404	1.98396	.52613	1.90069	.54862	1.82276	.57155	1.74964	15
46	.48270	2.07167	.50441	1.98253	.52650	1.89935	.54900	1.82150	.57193	1.74846	14
47	.48306	2.07014	.50477	1.98110	.52687	1.89801	.54938	1.82025	.57232	1.74728	13
48	.48342	2.06860	.50514	1.97966	.52724	1.89667	.54975	1.81899	.57271	1.74610	12
49	.48378	2.06706	.50550	1.97823	.52761	1.89533	.55013	1.81774	.57309	1.74492	11
50	.48414	2.06553	.50587	1.97681	.52798	1.89400	.55051	1.81649	.57348	1.74375	10
51	.48450	2.06400	.50623	1.97538	.52836	1.89266	.55089	1.81524	.57386	1.74257	9
52	.48486	2.06247	.50660	1.97395	.52873	1.89133	.55127	1.81399	.57425	1.74140	8
53	.48521	2.06094	.50696	1.97253	.52910	1.89000	.55165	1.81274	.57464	1.74022	7
54	.48557	2.05942	.50733	1.97111	.52947	1.88867	.55203	1.81150	.57503	1.73905	6
55	.48593	2.05790	.50769	1.96969	.52985	1.88734	.55241	1.81025	.57541	1.73788	5
56	.48629	2.05637	.50806	1.96827	.53022	1.88602	.55279	1.80901	.57580	1.73671	4
57	.48665	2.05485	.50843	1.96685	.53059	1.88469	.55317	1.80777	.57619	1.73555	3
58	.48701	2.05333	.50879	1.96544	.53096	1.88337	.55355	1.80653	.57657	1.73438	2
59	.48737	2.05182	.50916	1.96402	.53134	1.88205	.55393	1.80529	.57696	1.73321	1
60	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	.57735	1.73205	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	64°		63°		62°		61°		60°		

′	30° Tang	30° Cotang	31° Tang	31° Cotang	32° Tang	32° Cotang	33° Tang	33° Cotang	34° Tang	34° Cotang	′
0	.57735	1.73205	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	60
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.64982	1.53888	.67493	1.48163	59
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65023	1.53791	.67536	1.48070	58
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65065	1.53693	.67578	1.47977	57
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65106	1.53595	.67620	1.47885	56
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65148	1.53497	.67663	1.47792	55
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65189	1.53400	.67705	1.47699	54
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65231	1.53302	.67748	1.47607	53
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65272	1.53205	.67790	1.47514	52
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65314	1.53107	.67832	1.47422	51
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65355	1.53010	.67875	1.47330	50
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65397	1.52913	.67917	1.47238	49
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65438	1.52816	.67960	1.47146	48
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65480	1.52719	.68002	1.47053	47
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65521	1.52622	.68045	1.46962	46
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65563	1.52525	.68088	1.46870	45
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65604	1.52429	.68130	1.46778	44
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65646	1.52332	.68173	1.46686	43
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65688	1.52235	.68215	1.46595	42
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65729	1.52139	.68258	1.46503	41
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65771	1.52043	.68301	1.46411	40
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65813	1.51946	.68343	1.46320	39
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65854	1.51850	.68386	1.46229	38
23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65896	1.51754	.68429	1.46137	37
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65938	1.51658	.68471	1.46046	36
25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.65980	1.51562	.68514	1.45955	35
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66021	1.51466	.68557	1.45864	34
27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66063	1.51370	.68600	1.45773	33
28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66105	1.51275	.68642	1.45682	32
29	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66147	1.51179	.68685	1.45592	31
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66189	1.51084	.68728	1.45501	30
31	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66230	1.50988	.68771	1.45410	29
32	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66272	1.50893	.68814	1.45320	28
33	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66314	1.50797	.68857	1.45229	27
34	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66356	1.50702	.68900	1.45139	26
35	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66398	1.50607	.68942	1.45049	25
36	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66440	1.50512	.68985	1.44958	24
37	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66482	1.50417	.69028	1.44868	23
38	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66524	1.50322	.69071	1.44778	22
39	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66566	1.50228	.69114	1.44688	21
40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66608	1.50133	.69157	1.44598	20
41	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66650	1.50038	.69200	1.44508	19
42	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66692	1.49944	.69243	1.44418	18
43	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66734	1.49851	.69286	1.44329	17
44	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66776	1.49755	.69329	1.44239	16
45	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66818	1.49661	.69372	1.44149	15
46	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66860	1.49566	.69416	1.44060	14
47	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66902	1.49472	.69459	1.43970	13
48	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66944	1.49378	.69502	1.43881	12
49	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.66986	1.49284	.69545	1.43792	11
50	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.67028	1.49190	.69588	1.43703	10
51	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.67071	1.49097	.69631	1.43614	9
52	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.67113	1.49003	.69675	1.43525	8
53	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.67155	1.48909	.69718	1.43436	7
54	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.67197	1.48816	.69761	1.43347	6
55	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.67239	1.48722	.69804	1.43258	5
56	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.67282	1.48629	.69847	1.43169	4
57	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.67324	1.48536	.69891	1.43080	3
58	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.67366	1.48442	.69934	1.42992	2
59	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.67409	1.48349	.69977	1.42903	1
60	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.67451	1.48256	.70021	1.42815	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.70021	1.42815	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.70064	1.42726	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.70107	1.42638	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.70151	1.42550	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.70194	1.42462	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.70238	1.42374	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.70281	1.42286	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.70325	1.42198	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.70368	1.42110	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.70412	1.42022	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.70455	1.41934	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.70499	1.41847	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.70542	1.41759	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.70586	1.41672	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.70629	1.41584	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.70673	1.41497	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.70717	1.41409	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.70760	1.41322	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.70804	1.41235	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.70848	1.41148	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.70891	1.41061	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.70935	1.40974	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.70979	1.40887	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.71023	1.40800	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.71066	1.40714	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.71110	1.40627	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.71154	1.40540	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.71198	1.40454	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.71242	1.40367	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.71285	1.40281	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.71329	1.40195	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.71373	1.40109	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.71417	1.40022	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.71461	1.39936	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.71505	1.39850	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.71549	1.39764	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.71593	1.39679	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.71637	1.39593	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.71681	1.39507	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.71725	1.39421	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.71769	1.39336	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.71813	1.39250	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.71857	1.39165	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.71901	1.39079	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.71946	1.38994	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.71990	1.38909	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.72034	1.38824	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.72078	1.38738	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.72122	1.38653	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.72167	1.38568	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.72211	1.38484	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.72255	1.38399	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.72299	1.38314	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.72344	1.38229	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.72388	1.38145	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.72432	1.38060	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.72477	1.37976	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.72521	1.37891	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.72565	1.37807	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.72610	1.37722	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	.96625	1.03493	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	.96681	1.03433	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	.96738	1.03372	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	.96794	1.03312	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	.96850	1.03252	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	.96907	1.03192	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	.96963	1.03132	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	.97020	1.03072	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	.97076	1.03012	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	.97133	1.02952	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	.97189	1.02892	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	.97246	1.02832	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	.97302	1.02772	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	.97359	1.02713	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	.97416	1.02653	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	.97472	1.02593	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	.97529	1.02533	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	.97586	1.02474	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	.97643	1.02414	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	.97700	1.02355	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	.97756	1.02295	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	.97813	1.02236	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	.97870	1.02176	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	.97927	1.02117	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	.97984	1.02057	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	.98041	1.01998	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	.98098	1.01939	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	.98155	1.01879	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	.98213	1.01820	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	.98270	1.01761	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	.98327	1.01702	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	.98384	1.01642	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	.98441	1.01583	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	.98499	1.01524	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	.98556	1.01465	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	.98613	1.01406	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	.98671	1.01347	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	.98728	1.01288	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	.98786	1.01229	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	.98843	1.01170	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	.98901	1.01112	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	.98958	1.01053	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	.99016	1.00994	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	.99073	1.00935	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	.99131	1.00876	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	.99189	1.00818	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	.99247	1.00759	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	.99304	1.00701	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	.99362	1.00642	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	.99420	1.00583	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	.99478	1.00525	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	.99536	1.00467	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	.99594	1.00408	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	.99652	1.00350	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	.99710	1.00291	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	.99768	1.00233	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	.99826	1.00175	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	.99884	1.00116	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	.99942	1.00058	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	1.00000	1.00000	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	49°		48°		47°		46°		45°		

TRAVERSE TABLES

OR

LATITUDES AND DEPARTURES OF COURSES

CALCULATED TO

THREE DECIMAL PLACES

FOR

EACH QUARTER DEGREE OF BEARING.

Bearing.	1 Lat.	1 Dep.	2 Lat.	2 Dep.	3 Lat.	3 Dep.	4 Lat.	4 Dep.	5 Lat.	Bearing.
0	1.000	0.000	2.000	0.000	3.000	0.000	4.000	0.000	5.000	90°
0¼	1.000	0.004	2.000	0.009	3.000	0.013	4.000	0.017	5.000	89¾
0½	1.000	0.009	2.000	0.017	3.000	0.026	4.000	0.035	5.000	89½
0¾	1.000	0.013	2.000	0.026	3.000	0.039	4.000	0.052	5.000	89¼
1	1.000	0.017	2.000	0.035	3.000	0.052	3.999	0.070	4.999	89
1¼	1.000	0.022	2.000	0.044	2.999	0.065	3.999	0.087	4.999	88¾
1½	1.000	0.026	1.999	0.052	2.999	0.079	3.999	0.105	4.998	88½
1¾	1.000	0.031	1.999	0.061	2.999	0.092	3.998	0.122	4.998	88¼
2	0.999	0.035	1.999	0.070	2.998	0.105	3.998	0.140	4.997	88
2¼	0.999	0.039	1.998	0.079	2.998	0.118	3.997	0.157	4.996	87¾
2½	0.999	0.044	1.998	0.087	2.997	0.131	3.996	0.174	4.995	87½
2¾	0.999	0.048	1.998	0.096	2.997	0.144	3.995	0.192	4.994	87¼
3	0.999	0.052	1.997	0.105	2.996	0.157	3.995	0.209	4.993	87
3¼	0.998	0.057	1.997	0.113	2.995	0.170	3.994	0.227	4.992	86¾
3½	0.998	0.061	1.996	0.122	2.994	0.183	3.993	0.244	4.991	86½
3¾	0.998	0.065	1.996	0.131	2.994	0.196	3.991	0.262	4.989	86¼
4	0.998	0.070	1.995	0.140	2.993	0.209	3.990	0.279	4.988	86
4¼	0.997	0.074	1.995	0.148	2.992	0.222	3.989	0.296	4.986	85¾
4½	0.997	0.078	1.994	0.157	2.991	0.235	3.988	0.314	4.985	85½
4¾	0.997	0.083	1.993	0.166	2.990	0.248	3.986	0.331	4.983	85¼
5	0.996	0.087	1.992	0.174	2.989	0.261	3.985	0.349	4.981	85
5¼	0.996	0.092	1.992	0.183	2.987	0.275	3.983	0.366	4.979	84¾
5½	0.995	0.096	1.991	0.192	2.986	0.288	3.982	0.383	4.977	84½
5¾	0.995	0.100	1.990	0.200	2.985	0.301	3.980	0.401	4.975	84¼
6	0.995	0.105	1.989	0.209	2.984	0.314	3.978	0.418	4.973	84
6¼	0.994	0.109	1.988	0.218	2.982	0.327	3.976	0.435	4.970	83¾
6½	0.994	0.113	1.987	0.226	2.981	0.340	3.974	0.453	4.968	83½
6¾	0.993	0.118	1.986	0.235	2.979	0.353	3.972	0.470	4.965	83¼
7	0.993	0.122	1.985	0.244	2.978	0.366	3.970	0.487	4.963	83
7¼	0.992	0.126	1.984	0.252	2.976	0.379	3.968	0.505	4.960	82¾
7½	0.991	0.131	1.983	0.261	2.974	0.392	3.966	0.522	4.957	82½
7¾	0.991	0.135	1.982	0.270	2.973	0.405	3.963	0.539	4.954	82¼
8	0.990	0.139	1.981	0.278	2.971	0.418	3.961	0.557	4.951	82
8¼	0.990	0.143	1.979	0.287	2.969	0.430	3.959	0.574	4.948	81¾
8½	0.989	0.148	1.978	0.296	2.967	0.443	3.956	0.591	4.945	81½
8¾	0.988	0.152	1.977	0.304	2.965	0.456	3.953	0.608	4.942	81¼
9	0.988	0.156	1.975	0.313	2.963	0.469	3.951	0.626	4.938	81
9¼	0.987	0.161	1.974	0.321	2.961	0.482	3.948	0.643	4.935	80¾
9½	0.986	0.165	1.973	0.330	2.959	0.495	3.945	0.660	4.931	80½
9¾	0.986	0.169	1.971	0.339	2.957	0.508	3.942	0.677	4.928	80¼
10°	0.985	0.174	1.970	0.347	2.954	0.521	3.939	0.695	4.924	80
10¼	0.984	0.178	1.968	0.356	2.952	0.534	3.936	0.712	4.920	79¾
10½	0.983	0.182	1.967	0.364	2.950	0.547	3.933	0.729	4.916	79½
10¾	0.982	0.187	1.965	0.373	2.947	0.560	3.930	0.746	4.912	79¼
11	0.982	0.191	1.963	0.382	2.945	0.572	3.927	0.763	4.908	79
11¼	0.981	0.195	1.962	0.390	2.942	0.585	3.923	0.780	4.904	78¾
11½	0.980	0.199	1.960	0.399	2.940	0.598	3.920	0.797	4.900	78½
11¾	0.979	0.204	1.958	0.407	2.937	0.611	3.916	0.815	4.895	78¼
12	0.978	0.208	1.956	0.416	2.934	0.624	3.913	0.832	4.891	78
12¼	0.977	0.212	1.954	0.424	2.932	0.637	3.909	0.849	4.886	77¾
12½	0.976	0.216	1.953	0.433	2.929	0.649	3.905	0.866	4.881	77½
12¾	0.975	0.221	1.951	0.441	2.926	0.662	3.901	0.883	4.877	77¼
13	0.974	0.225	1.949	0.450	2.923	0.675	3.897	0.900	4.872	77°
Bearing.	1 Dep.	1 Lat.	2 Dep.	2 Lat.	3 Dep.	3 Lat.	4 Dep.	4 Lat.	5 Dep.	Bearing.

Bearing.	5	6		7		8		9		Bearing.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
0°	0.000	6.000	0.000	7.000	0.000	8.000	0.000	9.000	0.000	90°
0¼	0.022	6.000	0.026	7.000	0.031	8.000	0.035	9.000	0.039	89¾
0½	0.044	6.000	0.052	7.000	0.061	8.000	0.070	9.000	0.079	89½
0¾	0.065	5.999	0.079	6.999	0.092	7.999	0.105	8.999	0.118	89¼
1	0.087	5.999	0.105	6.999	0.122	7.999	0.140	8.999	0.157	89
1¼	0.109	5.999	0.131	6.998	0.153	7.998	0.175	8.998	0.196	88¾
1½	0.131	5.998	0.157	6.998	0.183	7.997	0.209	8.997	0.236	88½
1¾	0.153	5.997	0.183	6.997	0.214	7.996	0.244	8.996	0.275	88¼
2	0.174	5.996	0.209	6.996	0.244	7.995	0.279	8.995	0.314	88
2¼	0.196	5.995	0.236	6.995	0.275	7.994	0.314	8.993	0.353	87¾
2½	0.218	5.994	0.262	6.993	0.305	7.992	0.349	8.991	0.393	87½
2¾	0.240	5.993	0.288	6.992	0.336	7.991	0.384	8.990	0.432	87¼
3	0.262	5.992	0.314	6.990	0.366	7.989	0.419	8.988	0.471	87
3¼	0.283	5.990	0.340	6.989	0.397	7.987	0.454	8.986	0.510	86¾
3½	0.305	5.989	0.366	6.987	0.427	7.985	0.488	8.983	0.549	86½
3¾	0.327	5.987	0.392	6.985	0.458	7.983	0.523	8.981	0.589	86¼
4	0.349	5.985	0.419	6.983	0.488	7.981	0.558	8.978	0.628	86
4¼	0.371	5.984	0.445	6.981	0.519	7.978	0.593	8.975	0.667	85¾
4½	0.392	5.982	0.471	6.978	0.549	7.975	0.628	8.972	0.706	85½
4¾	0.414	5.979	0.497	6.976	0.580	7.973	0.662	8.969	0.745	85¼
5	0.436	5.977	0.523	6.973	0.610	7.970	0.697	8.966	0.784	85
5¼	0.458	5.975	0.549	6.971	0.641	7.966	0.732	8.962	0.824	84¾
5½	0.479	5.972	0.575	6.968	0.671	7.963	0.767	8.959	0.863	84½
5¾	0.501	5.970	0.601	6.965	0.701	7.960	0.802	8.955	0.902	84¼
6	0.523	5.967	0.627	6.962	0.732	7.956	0.836	8.951	0.941	84
6¼	0.544	5.964	0.653	6.958	0.762	7.952	0.871	8.947	0.980	83¾
6½	0.566	5.961	0.679	6.955	0.792	7.949	0.906	8.942	1.019	83½
6¾	0.588	5.958	0.705	6.951	0.823	7.945	0.940	8.938	1.058	83¼
7	0.609	5.955	0.731	6.948	0.853	7.940	0.975	8.933	1.097	83
7¼	0.631	5.952	0.757	6.944	0.883	7.936	1.010	8.928	1.136	82¾
7½	0.653	5.949	0.783	6.940	0.914	7.932	1.044	8.923	1.175	82½
7¾	0.674	5.945	0.809	6.936	0.944	7.927	1.079	8.918	1.214	82¼
8	0.696	5.942	0.835	6.932	0.974	7.922	1.113	8.912	1.253	82
8¼	0.717	5.938	0.861	6.928	1.004	7.917	1.148	8.907	1.291	81¾
8½	0.739	5.934	0.887	6.923	1.035	7.912	1.182	8.901	1.330	81½
8¾	0.761	5.930	0.913	6.919	1.065	7.907	1.217	8.895	1.369	81¼
9	0.782	5.926	0.939	6.914	1.095	7.902	1.251	8.889	1.408	81
9¼	0.804	5.922	0.964	6.909	1.125	7.896	1.286	8.883	1.447	80¾
9½	0.825	5.918	0.990	6.904	1.155	7.890	1.320	8.877	1.485	80½
9¾	0.847	5.913	1.016	6.899	1.185	7.884	1.355	8.870	1.524	80¼
10	0.868	5.909	1.042	6.894	1.216	7.878	1.389	8.863	1.563	80
10¼	0.890	5.904	1.068	6.888	1.246	7.872	1.424	8.856	1.601	79¾
10½	0.911	5.900	1.093	6.883	1.276	7.866	1.458	8.849	1.640	79½
10¾	0.933	5.895	1.119	6.877	1.306	7.860	1.492	8.842	1.679	79¼
11	0.954	5.890	1.145	6.871	1.336	7.853	1.526	8.835	1.717	79
11¼	0.975	5.885	1.171	6.866	1.366	7.846	1.561	8.827	1.756	78¾
11½	0.997	5.880	1.196	6.859	1.396	7.839	1.595	8.819	1.794	78½
11¾	1.018	5.874	1.222	6.853	1.425	7.832	1.629	8.811	1.833	78¼
12	1.040	5.869	1.247	6.847	1.455	7.825	1.663	8.803	1.871	78
12¼	1.061	5.863	1.273	6.841	1.485	7.818	1.697	8.795	1.910	77¾
12½	1.082	5.858	1.299	6.834	1.515	7.810	1.732	8.787	1.948	77½
12¾	1.103	5.852	1.324	6.827	1.545	7.803	1.766	8.778	1.986	77¼
13	1.125	5.846	1.350	6.821	1.575	7.795	1.800	8.769	2.025	77
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
Bearing.	5	6		7		8		9		Bearing.

Bearing.	1 Lat.	1 Dep.	2 Lat.	2 Dep.	3 Lat.	3 Dep.	4 Lat.	4 Dep.	5 Lat.	Bearing.
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.900	4.872	77°
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.917	4.867	76¾
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.934	4.862	76½
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.951	4.857	76¼
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.968	4.851	76°
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.985	4.846	75¾
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.002	4.841	75½
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4.835	75¼
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4.830	75°
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4.824	74¾
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4.818	74½
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4.812	74¼
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4.806	74°
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4.800	73¾
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	73½
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	73¼
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	73°
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	72¾
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	72½
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	72¼
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	72°
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	71¾
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	71½
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	71¼
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	71°
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	70°
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	69°
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Bearing.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	Bearing.
	1		2		3		4		5	

Bearing.	5	6		7		8		9		Bearing.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
13°	1.125	5.846	1.350	6.821	1.575	7.795	1.800	8.769	2.025	77°
13¼	1.146	5.840	1.375	6.814	1.604	7.787	1.834	8.760	2.063	76¾
13½	1.167	5.834	1.401	6.807	1.634	7.779	1.868	8.751	2.101	76½
13¾	1.188	5.828	1.426	6.799	1.664	7.771	1.902	8.742	2.139	76¼
14	1.210	5.822	1.452	6.792	1.693	7.762	1.935	8.733	2.177	76
14¼	1.231	5.815	1.477	6.785	1.723	7.754	1.969	8.723	2.215	75¾
14½	1.252	5.809	1.502	6.777	1.753	7.745	2.003	8.713	2.253	75½
14¾	1.273	5.802	1.528	6.769	1.782	7.736	2.037	8.703	2.291	75¼
15	1.294	5.796	1.553	6.761	1.812	7.727	2.071	8.693	2.329	75
15¼	1.315	5.789	1.578	6.754	1.841	7.718	2.104	8.683	2.367	74¾
15½	1.336	5.782	1.603	6.745	1.871	7.709	2.138	8.673	2.405	74½
15¾	1.357	5.775	1.629	6.737	1.900	7.700	2.172	8.662	2.443	74¼
16	1.378	5.768	1.654	6.729	1.929	7.690	2.205	8.651	2.481	74
16¼	1.399	5.760	1.679	6.720	1.959	7.680	2.239	8.640	2.518	73¾
16½	1.420	5.753	1.704	6.712	1.988	7.671	2.272	8.629	2.556	73½
16¾	1.441	5.745	1.729	6.703	2.017	7.661	2.306	8.618	2.594	73¼
17°	1.462	5.738	1.754	6.694	2.047	7.650	2.339	8.607	2.631	73°
17¼	1.483	5.730	1.779	6.685	2.076	7.640	2.372	8.595	2.669	72¾
17½	1.504	5.722	1.804	6.676	2.105	7.630	2.406	8.583	2.706	72½
17¾	1.524	5.714	1.829	6.667	2.134	7.619	2.439	8.572	2.744	72¼
18	1.545	5.706	1.854	6.657	2.163	7.608	2.472	8.560	2.781	72°
18¼	1.566	5.698	1.879	6.648	2.192	7.598	2.505	8.547	2.818	71¾
18½	1.587	5.690	1.904	6.638	2.221	7.587	2.538	8.535	2.856	71½
18¾	1.607	5.682	1.929	6.629	2.250	7.575	2.572	8.522	2.893	71¼
19	1.628	5.673	1.953	6.619	2.279	7.564	2.605	8.510	2.930	71°
19¼	1.648	5.665	1.978	6.609	2.308	7.553	2.638	8.497	2.967	70¾
19½	1.669	5.656	2.003	6.598	2.337	7.541	2.670	8.484	3.004	70½
19¾	1.690	5.647	2.028	6.588	2.365	7.529	2.703	8.471	3.041	70¼
20	1.710	5.638	2.052	6.578	2.394	7.518	2.736	8.457	3.078	70°
20¼	1.731	5.629	2.077	6.567	2.423	7.506	2.769	8.444	3.115	69¾
20½	1.751	5.620	2.101	6.557	2.451	7.493	2.802	8.430	3.152	69½
20¾	1.771	5.611	2.126	6.546	2.480	7.481	2.834	8.416	3.189	69¼
21°	1.792	5.601	2.150	6.535	2.509	7.469	2.867	8.402	3.225	69°
21¼	1.812	5.592	2.175	6.524	2.537	7.456	2.900	8.388	3.262	68¾
21½	1.833	5.582	2.199	6.513	2.566	7.443	2.932	8.374	3.299	68½
21¾	1.853	5.573	2.223	6.502	2.594	7.430	2.964	8.359	3.335	68¼
22	1.873	5.563	2.248	6.490	2.622	7.417	2.997	8.345	3.371	68°
22¼	1.893	5.553	2.272	6.479	2.651	7.404	3.029	8.330	3.408	67¾
22½	1.913	5.543	2.296	6.467	2.679	7.391	3.061	8.315	3.444	67½
22¾	1.934	5.533	2.320	6.455	2.707	7.378	3.094	8.300	3.480	67¼
23	1.954	5.523	2.344	6.444	2.735	7.364	3.126	8.285	3.517	67°
23¼	1.974	5.513	2.368	6.432	2.763	7.350	3.158	8.269	3.553	66¾
23½	1.994	5.502	2.392	6.419	2.791	7.336	3.190	8.254	3.589	66½
23¾	2.014	5.492	2.416	6.407	2.819	7.322	3.222	8.238	3.625	66¼
24	2.034	5.481	2.440	6.395	2.847	7.308	3.254	8.222	3.661	66°
24¼	2.054	5.471	2.464	6.382	2.875	7.294	3.286	8.206	3.696	65¾
24½	2.073	5.460	2.488	6.370	2.903	7.280	3.318	8.190	3.732	65½
24¾	2.093	5.449	2.512	6.357	2.931	7.265	3.349	8.173	3.768	65¼
25	2.113	5.438	2.536	6.344	2.958	7.250	3.381	8.157	3.804	65
25¼	2.133	5.427	2.559	6.331	2.986	7.236	3.413	8.140	3.839	64¾
25½	2.153	5.416	2.583	6.318	3.014	7.221	3.444	8.123	3.875	64½
25¾	2.172	5.404	2.607	6.305	3.041	7.206	3.476	8.106	3.910	64¼
26	2.192	5.393	2.630	6.292	3.069	7.190	3.507	8.089	3.945	64
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
Bearing.	5	6		7		8		9		Bearing.

Bearing	1 Lat.	1 Dep.	2 Lat.	2 Dep.	3 Lat.	3 Dep.	4 Lat.	4 Dep.	5 Lat.	Bearing
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	64°
26¼	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	63¾
26½	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	63½
26¾	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	63¼
27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	63°
27¼	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	62¾
27½	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	62½
27¾	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	62¼
28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	62°
28¼	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	61¾
28½	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	61½
28¾	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	61¼
29	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4.373	61°
29¼	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.954	4.362	60¾
29½	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.970	4.352	60½
29¾	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.473	1.985	4.341	60¼
30	0.866	0.500	1.732	1.000	2.598	1.500	3.464	2.000	4.330	60°
30¼	0.864	0.504	1.728	1.008	2.592	1.511	3.455	2.015	4.319	59¾
30½	0.862	0.508	1.725	1.015	2.585	1.523	3.447	2.030	4.308	59½
30¾	0.860	0.511	1.719	1.023	2.579	1.534	3.438	2.045	4.297	59¼
31	0.857	0.515	1.714	1.030	2.572	1.545	3.429	2.060	4.286	59°
31¼	0.855	0.519	1.710	1.038	2.565	1.556	3.420	2.075	4.275	58¾
31½	0.853	0.522	1.706	1.045	2.557	1.567	3.411	2.090	4.263	58½
31¾	0.850	0.526	1.701	1.052	2.551	1.579	3.401	2.105	4.252	58¼
32	0.848	0.530	1.696	1.060	2.544	1.590	3.392	2.120	4.240	58°
32¼	0.846	0.534	1.691	1.067	2.537	1.601	3.383	2.134	4.229	57¾
32½	0.843	0.537	1.687	1.075	2.530	1.612	3.374	2.149	4.217	57½
32¾	0.841	0.541	1.682	1.082	2.523	1.623	3.364	2.164	4.205	57¼
33	0.839	0.545	1.677	1.089	2.516	1.634	3.355	2.179	4.193	57°
33¼	0.836	0.548	1.673	1.097	2.509	1.645	3.345	2.193	4.181	56¾
33½	0.834	0.552	1.668	1.104	2.502	1.656	3.336	2.208	4.169	56½
33¾	0.831	0.556	1.663	1.111	2.494	1.667	3.326	2.222	4.157	56¼
34	0.829	0.559	1.658	1.118	2.487	1.678	3.316	2.237	4.145	56°
34¼	0.827	0.563	1.653	1.126	2.480	1.688	3.306	2.251	4.133	55¾
34½	0.824	0.566	1.648	1.133	2.472	1.699	3.297	2.266	4.121	55½
34¾	0.822	0.570	1.643	1.140	2.465	1.710	3.287	2.280	4.108	55¼
35	[illegible]	[illegible]	[illegible]	1.147	2.457	1.721	3.277	2.294	4.096	55°
35¼	[illegible]	[illegible]	[illegible]	1.154	2.450	1.731	3.267	2.309	4.083	54¾
35½	[illegible]	[illegible]	[illegible]	1.161	2.442	1.742	3.257	2.323	4.071	54½
35¾	[illegible]	[illegible]	[illegible]	1.168	2.435	1.753	3.246	2.337	4.058	54¼
36	[illegible]	[illegible]	[illegible]	1.176	2.427	1.763	3.236	2.351	4.045	54°
36¼	[illegible]	[illegible]	[illegible]	[illegible]	2.419	1.774	3.226	2.365	4.032	53¾
36½	[illegible]	[illegible]	[illegible]	[illegible]	2.412	1.784	3.215	2.379	4.019	53½
36¾	[illegible]	[illegible]	[illegible]	[illegible]	2.404	1.795	3.205	2.393	4.006	53¼
37	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.805	3.195	2.407	3.993	53°
37¼	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.816	3.184	2.421	3.980	52¾
37½	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.826	3.173	2.435	3.967	52½
37¾	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.837	3.163	2.449	3.953	52¼
38	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.847	3.152	2.463	3.940	52°
38¼	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.857	3.141	2.476	3.927	51¾
38½	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.868	3.130	2.490	3.913	51½
38¾	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.878	3.120	2.504	3.899	51¼
39	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.888	3.109	2.517	3.886	51°
Bearing	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Bearing
	1		2		3		4		5	

Bearing.	5	6		7		8		9		Bearing.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
26°	2.192	5.393	2.630	6.292	3.069	7.190	3.507	8.089	3.945	**64°**
26¼	2.211	5.381	2.654	6.278	3.096	7.175	3.538	8.072	3.981	63¾
26½	2.231	5.370	2.677	6.265	3.123	7.160	3.570	8.054	4.016	63½
26¾	2.250	5.358	2.701	6.251	3.151	7.144	3.601	8.037	4.051	63¼
27°	2.270	5.346	2.724	6.237	3.178	7.128	3.632	8.019	4.086	**63°**
27¼	2.289	5.334	2.747	6.223	3.205	7.112	3.663	8.001	4.121	62¾
27½	2.309	5.322	2.770	6.209	3.232	7.096	3.694	7.983	4.156	62½
27¾	2.328	5.310	2.794	6.195	3.259	7.080	3.725	7.965	4.190	62¼
28°	2.347	5.298	2.817	6.181	3.286	7.064	3.756	7.947	4.225	**62°**
28¼	2.367	5.285	2.840	6.166	3.313	7.047	3.787	7.928	4.260	61¾
28½	2.386	5.273	2.863	6.152	3.340	7.031	3.817	7.909	4.294	61½
28¾	2.405	5.260	2.886	6.137	3.367	7.014	3.848	7.891	4.329	61¼
29	2.424	5.248	2.909	6.122	3.394	6.997	3.878	7.872	4.363	**61°**
29¼	2.443	5.235	2.932	6.107	3.420	6.980	3.909	7.852	4.398	60¾
29½	2.462	5.222	2.955	6.093	3.447	6.963	3.939	7.833	4.432	60½
29¾	2.481	5.209	2.977	6.077	3.474	6.946	3.970	7.814	4.466	60¼
30	2.500	5.196	3.000	6.062	3.500	6.928	4.000	7.794	4.500	**60**
30¼	2.519	5.183	3.023	6.047	3.526	6.911	4.030	7.775	4.534	59¾
30½	2.538	5.170	3.045	6.031	3.553	6.893	4.060	7.755	4.568	59½
30¾	2.556	5.156	3.068	6.016	3.579	6.875	4.090	7.735	4.602	59¼
31°	2.575	5.143	3.090	6.000	3.605	6.857	4.120	7.715	4.635	**59°**
31¼	2.594	5.129	3.113	5.984	3.631	6.839	4.150	7.694	4.669	58¾
31½	2.612	5.116	3.135	5.968	3.657	6.821	4.180	7.674	4.702	58½
31¾	2.631	5.102	3.157	5.952	3.683	6.803	4.210	7.653	4.736	58¼
32°	2.650	5.088	3.180	5.936	3.709	6.784	4.239	7.632	4.769	**58°**
32¼	2.668	5.074	3.202	5.920	3.735	6.766	4.269	7.612	4.802	57¾
32½	2.686	5.060	3.224	5.904	3.761	6.747	4.298	7.591	4.836	57½
32¾	2.705	5.046	3.246	5.887	3.787	6.728	4.328	7.569	4.869	57¼
33°	2.723	5.032	3.268	5.871	3.812	6.709	4.357	7.548	4.902	**57°**
33¼	2.741	5.018	3.290	5.854	3.838	6.690	4.386	7.527	4.935	56¾
33½	2.760	5.003	3.312	5.837	3.864	6.671	4.416	7.505	4.967	56½
33¾	2.778	4.989	3.333	5.820	3.889	6.652	4.445	7.483	5.000	56¼
34°	2.796	4.974	3.355	5.803	3.914	6.632	4.474	7.461	5.033	**56**
34¼	2.814	4.960	3.377	5.786	3.940	6.613	4.502	7.439	5.065	55¾
34½	2.832	4.945	3.398	5.769	3.965	6.593	4.531	7.417	5.098	55½
34¾	2.850	4.930	3.420	5.752	3.990	6.573	4.560	7.395	5.130	55¼
35°	2.868	4.915	3.441	5.734	4.015	6.553	4.589	7.372	5.162	**55**
35¼	2.886	4.900	3.463	5.716	4.040	6.533	4.617	7.350	5.194	54¾
35½	2.904	4.885	3.484	5.699	4.065	6.513	4.646	7.327	5.226	54½
35¾	2.921	4.869	3.505	5.681	4.090	6.493	4.674	7.304	5.258	54¼
36	2.939	4.854	3.527	5.663	4.115	6.472	4.702	7.281	5.290	**54**
36¼	2.957	4.839	3.548	5.645	4.139	6.452	4.730	7.258	5.322	53¾
36½	2.974	4.823	3.569	5.627	4.164	6.431	4.759	7.235	5.353	53½
36¾	2.992	4.808	3.590	5.609	4.188	6.410	4.787	7.211	5.385	53¼
37	3.009	4.792	3.611	5.590	4.213	6.389	4.815	7.188	5.416	**53**
37¼	3.026	4.776	3.632	5.572	4.237	6.368	4.842	7.164	5.448	52¾
37½	3.044	4.760	3.653	5.554	4.261	6.347	4.870	7.140	5.479	52½
37¾	3.061	4.744	3.673	5.535	4.286	6.326	4.898	7.116	5.510	52¼
38°	3.078	4.728	3.694	5.516	4.310	6.304	4.925	7.092	5.541	**52**
38¼	3.095	4.712	3.715	5.497	4.334	6.283	4.953	7.068	5.572	51¾
38½	3.113	4.696	3.735	5.478	4.358	6.261	4.980	7.043	5.603	51½
38¾	3.130	4.679	3.756	5.459	4.381	6.239	5.007	7.019	5.633	51¼
39	3.147	4.663	3.776	5.440	4.405	6.217	5.035	6.994	5.664	**51**
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
Bearing.	5	6		7		8		9		Bearing.

Bearing.	1		2		3		4		5	Bearing.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	
39°	0.777	0.629	1.554	1.259	2.331	1.888	3.109	2.517	3.886	51°
39¼	0.774	0.633	1.549	1.265	2.323	1.898	3.098	2.531	3.872	50¾
39½	0.772	0.636	1.543	1.272	2.315	1.908	3.086	2.544	3.858	50½
39¾	0.769	0.639	1.538	1.279	2.307	1.918	3.075	2.558	3.844	50¼
40°	0.766	0.643	1.532	1.286	2.298	1.928	3.064	2.571	3.830	50°
40¼	0.763	0.646	1.526	1.292	2.290	1.938	3.053	2.584	3.816	49¾
40½	0.760	0.649	1.521	1.299	2.281	1.948	3.042	2.598	3.802	49½
40¾	0.758	0.653	1.515	1.306	2.273	1.958	3.030	2.611	3.788	49¼
41°	0.755	0.656	1.509	1.312	2.264	1.968	3.019	2.624	3.774	49°
41¼	0.752	0.659	1.504	1.319	2.256	1.978	3.007	2.637	3.759	48¾
41½	0.749	0.663	1.498	1.325	2.247	1.988	2.996	2.650	3.745	48½
41¾	0.746	0.666	1.492	1.332	2.238	1.998	2.984	2.664	3.730	48¼
42°	0.743	0.669	1.486	1.338	2.229	2.007	2.973	2.677	3.716	48°
42¼	0.740	0.672	1.480	1.345	2.221	2.017	2.961	2.689	3.701	47¾
42½	0.737	0.676	1.475	1.351	2.212	2.027	2.949	2.702	3.686	47½
42¾	0.734	0.679	1.469	1.358	2.203	2.036	2.937	2.715	3.672	47¼
43°	0.731	0.682	1.463	1.364	2.194	2.046	2.925	2.728	3.657	47°
43¼	0.728	0.685	1.457	1.370	2.185	2.056	2.913	2.741	3.642	46¾
43½	0.725	0.688	1.451	1.377	2.176	2.065	2.901	2.753	3.627	46½
43¾	0.722	0.692	1.445	1.383	2.167	2.075	2.889	2.766	3.612	46¼
44°	0.719	0.695	1.439	1.389	2.158	2.084	2.877	2.779	3.597	46°
44¼	0.716	0.698	1.433	1.396	2.149	2.093	2.865	2.791	3.582	45¾
44½	0.713	0.701	1.427	1.402	2.140	2.103	2.853	2.804	3.566	45½
44¾	0.710	0.704	1.420	1.408	2.131	2.112	2.841	2.816	3.551	45¼
45°	0.707	0.707	1.414	1.414	2.121	2.121	2.828	2.828	3.536	45°
B'ring	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	B'ring

Bearing.	5	6		7		8		9		Bearing.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
39°	3.147	4.663	3.776	5.440	4.405	6.217	5.035	6.994	5.664	51°
39¼	3.164	4.646	3.796	5.421	4.429	6.195	5.062	6.970	5.694	50¾
39½	3.180	4.630	3.816	5.401	4.453	6.173	5.089	6.945	5.725	50½
39¾	3.197	4.613	3.837	5.382	4.476	6.151	5.116	6.920	5.755	50¼
40°	3.214	4.596	3.857	5.362	4.500	6.128	5.142	6.894	5.785	50°
40¼	3.231	4.579	3.877	5.343	4.523	6.106	5.169	6.869	5.815	49¾
40½	3.247	4.562	3.897	5.323	4.546	6.083	5.196	6.844	5.845	49½
40¾	3.264	4.545	3.917	5.303	4.569	6.061	5.222	6.818	5.875	49¼
41°	3.280	4.528	3.936	5.283	4.592	6.038	5.248	6.792	5.905	49°
41¼	3.297	4.511	3.956	5.263	4.615	6.015	5.275	6.767	5.934	48¾
41½	3.313	4.494	3.976	5.243	4.638	5.992	5.301	6.741	5.964	48½
41¾	3.329	4.476	3.995	5.222	4.661	5.968	5.327	6.715	5.993	48¼
42°	3.346	4.459	4.015	5.202	4.684	5.945	5.353	6.688	6.022	48°
42¼	3.362	4.441	4.034	5.182	4.707	5.922	5.379	6.662	6.051	47¾
42½	3.378	4.424	4.054	5.161	4.729	5.898	5.405	6.635	6.080	47½
42¾	3.394	4.406	4.073	5.140	4.752	5.875	5.430	6.609	6.109	47¼
43°	3.410	4.388	4.092	5.119	4.774	5.851	5.456	6.582	6.138	47°
43¼	3.426	4.370	4.111	5.099	4.796	5.827	5.481	6.555	6.167	46¾
43½	3.442	4.352	4.130	5.078	4.818	5.803	5.507	6.528	6.195	46½
43¾	3.458	4.334	4.149	5.057	4.841	5.779	5.532	6.501	6.224	46¼
44°	3.473	4.316	4.168	5.035	4.863	5.755	5.557	6.474	6.252	46°
44¼	3.489	4.298	4.187	5.014	4.885	5.730	5.582	6.447	6.280	45¾
44½	3.505	4.280	4.206	4.993	4.906	5.706	5.607	6.419	6.308	45½
44¾	3.520	4.261	4.224	4.971	4.928	5.681	5.632	6.392	6.336	45¼
45°	3.536	4.243	4.243	4.950	4.950	5.657	5.657	6.364	6.364	45°
B'ring	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	B'ring

TABLE OF

RADII AND CHORD AND TANGENT DEFLECTIONS.

The formulas used in the computation of the following table are as follows:

For Radii, $R = \frac{50}{\sin D}$. **(79.)** Art. **1200.** § 9.

For Chord Deflections,

$$d = \frac{c^2}{R}.$$ **(81.)** Art. **1208.** § 9.

For Tangent Deflections,

$$\text{tan deflection} = \frac{c^2}{2R}.$$ **(82.)** Art. **1208.** § 9.

TABLE OF RADII AND DEFLECTIONS OF CURVES.

Degree.	Radii.	Chord Deflection.	Tangent Deflection.
° ′			
0 5	68754.94	.145	.073
10	34377.48	.291	.145
15	22918.33	.436	.218
20	17188.76	.582	.291
25	13751.02	.727	.364
30	11459.19	.873	.436
35	9822.18	1.018	.509
40	8594.41	1.164	.582
45	7639.49	1.309	.654
50	6875.55	1.454	.727
55	6250.51	1.600	.800
1 0	5729.65	1.745	.873
5	5288.92	1.891	.945
10	4911.15	2.036	1.018
15	4583.75	2.182	1.091
20	4297.28	2.327	1.164
25	4044.51	2.472	1.236
30	3819.83	2.618	1.309
35	3618.80	2.763	1.382
40	3437.87	2.909	1.454
45	3274.17	3.054	1.527
50	3125.36	3.200	1.600
55	2989.48	3.345	1.673
2 0	2864.93	3.490	1.745
5	2750.35	3.636	1.818
10	2644.58	3.781	1.891
15	2546.64	3.927	1.963
20	2455.70	4.072	2.036
25	2371.04	4.218	2.109
30	2292.01	4.363	2.181
35	2218.09	4.508	2.254
40	2148.79	4.654	2.327
45	2083.68	4.799	2.400
50	2022.41	4.945	2.472
55	1964.64	5.090	2.545
3 0	1910.08	5.235	2.618
5	1858.47	5.381	2.690
10	1809.57	5.526	2.763
15	1763.18	5.672	2.836
20	1719.12	5.817	2.908
25	1677.20	5.962	2.981
30	1637.28	6.108	3.054
35	1599.21	6.253	3.127
40	1562.88	6.398	3.199
45	1528.16	6.544	3.272
50	1494.95	6.689	3.345
55	1463.16	6.835	3.417
4 0	1432.69	6.980	3.490
5	1403.46	7.125	3.563
10	1375.40	7.271	3.635
15	1348.45	7.416	3.708
20	1322.53	7.561	3.781
25	1297.58	7.707	3.853
30	1273.57	7.852	3.926
35	1250.42	7.997	3.999
40	1228.11	8.143	4.071
45	1206.57	8.288	4.144
50	1185.78	8.433	4.217
55	1165.70	8.579	4.289
5 0	1146.28	8.724	4.362
5	1127.50	8.869	4.435
10	1109.33	9.014	4.507

Degree.	Radii.	Chord Deflection.	Tangent Deflection.
° ′			
5 15	1091.73	9.160	4.580
20	1074.68	9.305	4.653
25	1058.16	9.450	4.725
30	1042.14	9.596	4.798
35	1026.60	9.741	4.870
40	1011.51	9.886	4.943
45	996.87	10.031	5.016
50	982.64	10.177	5.088
55	968.81	10.322	5.161
6 0	955.37	10.467	5.234
5	942.29	10.612	5.306
10	929.57	10.758	5.379
15	917.19	10.903	5.451
20	905.13	11.048	5.524
25	893.39	11.193	5.597
30	881.95	11.339	5.669
35	870.79	11.484	5.742
40	859.92	11.629	5.814
45	849.32	11.774	5.887
50	838.97	11.919	5.960
55	828.88	12.065	6.032
7 0	819.02	12.210	6.105
5	809.40	12.355	6.177
10	800.00	12.500	6.250
15	790.81	12.645	6.323
20	781.84	12.700	6.305
25	773.07	12.936	6.468
30	764.49	13.081	6.540
35	756.10	13.226	6.613
40	747.80	13.371	6.685
45	739.86	13.516	6.758
50	732.01	13.661	6.831
55	724.31	13.806	6.903
8 0	716.78	13.951	6.976
5	709.40	14.096	7.048
10	702.18	14.241	7.121
15	695.09	14.387	7.193
20	688.16	14.532	7.266
25	681.35	14.677	7.338
30	674.69	14.822	7.411
35	668.15	14.967	7.483
40	661.74	15.112	7.556
45	655.45	15.257	7.628
50	649.27	15.402	7.701
55	643.22	15.547	7.773
9 0	637.27	15.692	7.846
5	631.44	15.837	7.918
10	625.71	15.982	7.991
15	620.09	16.127	8.063
20	614.56	16.272	8.136
25	609.14	16.417	8.208
30	603.80	16.562	8.281
35	598.57	16.707	8.353
40	593.42	16.852	8.426
45	588.36	16.996	8.498
50	583.38	17.141	8.571
55	578.49	17.286	8.643
10 0	573.69	17.431	8.716
10	564.31	17.721	8.860
20	555.23	18.011	9.005
30	546.44	18.300	9.150
40	537.02	18.590	9.295

Degree.	Radii.	Chord Deflection.	Tangent Deflection.
° ′			
10 50	529.67	18.880	9.440
11 0	521.67	19.169	9.585
10	513.91	19.459	9.729
20	506.38	19.748	9.874
30	499.06	20.038	10.019
40	491.96	20.327	10.164
50	485.05	20.616	10.308
12 0	478.34	20.906	10.453
10	471.81	21.195	10.597
20	465.46	21.484	10.742
30	459.28	21.773	10.887
40	453.26	22.063	11.031
50	447.40	22.352	11.176
13 0	441.68	22.641	11.320
10	436.12	22.930	11.465
20	430.69	23.219	11.609
30	425.40	23.507	11.754
40	420.23	23.796	11.898
50	415.19	24.085	12.043
14 0	410.28	24.374	12.187
10	405.47	24.663	12.331
20	400.78	24.951	12.476
30	396.20	25.240	12.620
40	391.72	25.528	12.764
50	387.34	25.817	12.908
15 0	383.06	26.105	13.053
10	378.88	26.394	13.197
20	374.79	26.682	13.341
30	370.78	26.970	13.485
40	366.86	27.258	13.629
50	363.02	27.547	13.773
16 0	359.26	27.835	13.917
10	355.59	28.123	14.061
20	351.98	28.411	14.205
30	348.45	28.699	14.349
40	344.99	28.986	14.493
50	341.60	29.274	14.637
17 0	338.27	29.562	14.781
10	335.01	29.850	14.925
20	331.82	30.137	15.068
30	328.68	30.425	15.212
40	325.60	30.742	15.356
50	322.59	31.000	15.500
18 0	319.62	31.287	15.643
10	316.71	31.574	15.787
20	313.86	31.861	15.931
30	311.06	32.149	16.074
40	308.30	32.436	16.218
50	305.60	32.723	16.361
19 0	302.94	33.010	16.505
10	300.33	33.296	16.648
20	297.77	33.583	16.792
30	295.25	33.870	16.935
40	292.77	34.157	17.078
50	290.33	34.443	17.222
20 0	287.94	34.730	17.365

SPECIFIC GRAVITIES AND WEIGHTS PER CUBIC FOOT.

METALS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Osmium	23.00	1,437.5
Platinum	21.50	1,343.8
Gold	19.50	1,218.8
Mercury	13.60	850.0
Lead (cast)	11.35	709.4
Silver	10.50	656.3
Copper (cast)	8.79	549.4
Brass	8.38	523.8
Wrought Iron	7.68	480.0
Cast Iron	7.21	450.0
Steel	7.84	490.0
Tin (cast)	7.29	455.6
Zinc (cast)	6.86	428.8
Antimony	6.71	419.4
Aluminum	2.50	156.3

WOODS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Ash	.845	52.80
Beech	.852	53.25
Cedar	.561	35.06
Cork	.240	15.00
Ebony (American)	1.331	83.19
Lignum-vitæ	1.333	83.30
Maple	.750	46.88
Oak (old)	1.170	73.10
Spruce	.500	31.25
Pine (yellow)	.660	41.20
Pine (white)	.554	34.60
Walnut	.671	41.90

LIQUIDS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Acetic Acid	1.062	66.4
Nitric Acid	1.217	76.1
Sulphuric Acid	1.841	115.1
Muriatic Acid	1.200	75.0
Alcohol	.800	50.0
Turpentine	.870	54.4
Sea Water (ordinary)	1.026	64.1
Milk	1.032	64.5

GASES.

At 32° F., and under a Pressure of One Atmosphere.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Atmospheric Air	1.0000	.08073
Carbonic Acid	1.5290	.12344
Carbonic Oxide	.9674	.07810
Chlorine	2.4400	.19700
Oxygen	1.1056	.08925
Nitrogen	.9736	.07860
Smoke (bituminous coal)	.1020	.00815
Smoke (wood)	.0900	.00727
*Steam at 212° F.	.4700	.03790
Hydrogen	.0692	.00559

* The specific gravity of steam at any temperature and pressure compared with air at the same temperature and pressure is 0.622.

MISCELLANEOUS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Emery	4.00	250
Glass (average)	2.80	175
Chalk	2.78	174
Granite	2.65	166
Marble	2.70	169
Stone (common)	2.52	158
Salt (common)	2.13	133
Soil (common)	1.98	124
Clay	1.93	121
Brick	1.90	118
Plaster of Paris (average)	2.00	125
Sand	1.80	113

SPECIFIC HEAT OF VARIOUS SUBSTANCES.

Substance.	Sp. Heat.	Substance.	Sp. Heat.
Water	1.0000	Ice	.5040
Sulphur	.2026	Steam (superheated)	.4805
Iron	.1138	Air	.2375
Copper	.0951	Oxygen	.2175
Silver	.0570	Hydrogen	3.4090
Tin	.0562	Nitrogen	.2438
Mercury	.0333	Carbon Monoxide	.2479
Lead	.0314	Carbon Dioxide	.2170

ANALYSES OF COAL.

	Carbon.	Hydro-Carbons.			Sulph.	Ash.
		Hydro.	Oxyg.	Nitro.		
Pocahontas, Va.	72.71	18.81			0.79	5.19
Connellsville, Pa.	59.79	31.80			0.53	7.16
Anthracite, S. Wales	92.56	3.33	2.53	......	......	1.58
Anthracite, Pa.	92.59	2.63	1.61	0.92	......	2.25
Anthracite, Pa.	90.45	2.43	2.45	......	......	4.67
Caking Coal, Kentucky	74.45	4.93	13.08	1.03	0.91	5.00
Caking Coal, Nelsonville, O.	73.80	5.79	16.58	1.52	0.41	1.90
Caking Coal, South Wales	82.56	5.36	8.22	1.65	0.75	1.46
Caking Coal, Northumberland	78.69	6.00	10.07	2.37	1.51	1.36
Non-Caking, Kentucky	77.89	5.42	12.57	1.82	3.00	2.00
Non-Caking, Block Coal, Ind.	82.70	4.77	9.39	1.62	0.45	1.07
Non-Caking, Briar Hill, O.	78.94	5.92	11.50	1.58	0.56	1.45
Non-Caking, S. Staffordshire.	76.40	4.62	17.43	......	0.55	1.55
Non-Caking, Scotland	76.08	5.31	13.33	2.09	1.23	1.96
Cannel Coal, Breckenridge, Ky.	68.13	6.49	5.83	2.27	2.48	12.30
Cannel Coal, Wigan, Eng.	80.07	5.53	8.10	2.12	1.50	2.70
Cannel Coal, "Torbanite"	64.02	8.90	5.66	0.55	0.50	20.32
Albertite, Nova Scotia	86.04	8.96	1.97	2.93	trace	0.10
Brown Coal, Bovey	66.31	5.63	22.86	0.57	2.36	2.27
Brown Coal, Wittenburg	64.07	5.03	27.55	......	......	3.85
Brown Coal, Carbon, Wy.	75.20	4.74	10.37	1.37	1.11	7.20
Peat, light brown (imperfect)	50.86	5.80	42.57	0.77	......	......
Peat, dark brown	59.47	6.52	31.51	2.51	......	......
Peat, black	59.70	5.70	33.04	1.56	......	......
Peat, black	59.71	5.27	32.07	2.59	......	......
Non-Coking, Bon-Air, Tenn.	58.00	Vola. matter 37%			0.75	4.00
Coking, Tracy City, Tenn.	61.00	Vola. matter 29.30%			trace	7.00

COKING AND NON-COKING COALS.

	Moist.	Volatile Matter.	Fixed Carbon.	Ash.	Sulphur.	Phos.	Remarks.	Formation Mined in.
Connellsville, Pa.	1.25	31.80	59.79	7.16	0.53	0.024	Best Coking Coal	Carboniferous
Pocahontas, Va.	1.011	18.812	72.708	5.191	0.787		Good Coking Coal	Carboniferous
Broad Top, Pa.	1.28	18.40	71.12	5.50	1.70	traces	Good Coking Coal	Carboniferous
Bennington, Pa.	1.20	23.68	68.77	5.73	0.62	0.017	Good Coking Coal	Carboniferous
Johnstown, Pa	0.72	16.49	73.84	7.97	1.97		Dry Coking Coal	Carboniferous
Greensburg, Pa.	1.02	33.50	61.34	3.28	0.86		Good Coking Coal	Carboniferous
Armstrong Co., Pa.	0.96	38.20	52.03	5.14	3.66		Pitchy Coking Coal	Carboniferous
Mt. Carbon, Ill.	2.08	38.20	53.47	8.02	0.63	0.027	Pitchy Coking Coal	Carboniferous
El Moro, Col.	0.95	29.82	56.41	12.82	0.41		Good Coking Coal	Cretaceous
Crested Butte, Col.	0.72	23.44	71.91	3.93	0.36		Good Coking Coal	Cretaceous
Sand Coulee, Mont.	2.26	33.60	54.47	7.82	1.86	0.009	Non-Coking Coal	Cretaceous
Beet Mountain, Mont.	2.98	28.71	53.31	13.34	1.65	0.012	Non-Coking Coal	Cretaceous
Coahuila, Mex.	1.60	15.00	67.64	12.01	0.86		Coking Coal	?

CHEMICAL ELEMENTS.

Name.	Symbol	Atomic Weight.	Name.	Symbol.	Atomic Weight.
* **Aluminum,**	*Al.*	27.4	**Mercury,**	*Hg.*	200.0
* Antimony,	*Sb.*	122.0	* Molybdenum,	*Mo.*	96.0
* Arsenic,	*As.*	75.0	Nickel,	*Ni.*	58.0
Barium,	*Ba.*	137.0	Niobium (Columbium, Cb.),	*Nb.*	94.0
Beryllium (Glucinum, Gl.),	*Be.*	9.2	***Nitrogen,***	*N.*	14.0
Bismuth,	*Bi.*	210.0	Osmium,	*Os.*	200.0
Boron,	*B.*	11.0	***Oxygen,***	*O.*	16.0
Bromine,	*Br.*	80.0	Palladium,	*Pd.*	106.0
Cadmium,	*Cd.*	112.0	* *Phosphorus,*	*P.*	31.0
Cæsium,	*Cs.*	133.0	Platinum,	*Pt.*	197.4
Calcium,	*Ca.*	40.0	**Potassium,**	*K.*	39.1
* ***Carbon,***	*C.*	12.0	Rhodium,	*Ro.*	104.0
Cerium,	*Ce.*	91.3	Rubidium,	*Rb.*	85.4
Chlorine,	*Cl.*	35.5	Ruthenium,	*Ru.*	104.0
* Chromium,	*Cr.*	52.2	Samarium,	*Sm.*	150.0
Cobalt,	*Co.*	60.0	Scandium,	*Sc.*	44.9
* Columbium (Niobium, Nb.),	*Cb.*	94.0	* Selenium,	*Se.*	79.0
Copper,	*Cu.*	63.4	* ***Silicon,***	*Si.*	28.0
Decipium,	*Dp.*	159.0	**Silver,**	*Ag.*	108.0
Didymium,	*D.*	95.0	**Sodium,**	*Na.*	23.0
Erbium,	*E.*	112.6	Strontium,	*Sr.*	88.0
Fluorine,	*F.*	19.0	* ***Sulphur,***	*S.*	32.0
Gallium,	*Ga.*	69.8	* Tantalum,	*Ta.*	182.0
Glucinum (Beryllium, Be.),	*Gl.*	9.2	* Tellurium,	*Te.*	128.0
* **Gold,**	*Au.*	197.0	Terbium,	*Tb.*	75.4
* **Hydrogen,**	*H.*	1.0	Thallium,	*Tl.*	204.0
Indium,	*In.*	113.4	Thorium,	*Th.*	118.4
Iodine,	*I.*	127.0	* Tin,	*Sn.*	118.0
Iridium,	*Ir.*	198.0	* Titanium,	*Ti.*	50.0
Iron,	*Fe.*	56.0	* Tungsten,	*W.*	184.0
Lanthanum,	*La.*	92.0	* Uranium,	*U.*	120.0
Lead,	*Pb.*	207.0	Vanadium,	*V.*	51.3
Lithium,	*Li.*	7.0	Ytterbium,	*Yb.*	173.0
Magnesium,	*Mg.*	24.0	Yttrium,	*Y.*	89.0
* Manganese,	*Mn.*	55.0	Zinc,	*Zn.*	65.0
			Zirconium,	*Zr.*	89.6

* Sometimes basic, sometimes acid.

NOTE.—Heavy-faced type indicates the elements of most importance to the student of this subject. Basic elements are printed in common type; acid elements in Italics.

MISCELLANEOUS TABLES.

HEAT OF COMBUSTION OF VARIOUS SUBSTANCES.

Substance.	British Thermal Units (per Pound).
Hydrogen Gas (H)	62,000
Marsh-gas (CH_4)	23,500
Carbonic Oxide Gas (CO)	4,300
Anthracite Coal	15,230
Bituminous Coal	14,400
Coke	12,600
Wood (ordinary)	5,000

RATES OF DIFFUSION OF GASES.

Gas.	Density, or Specific Gravity.	Square Root of Density.	$\frac{1}{\sqrt{\text{Density}}}$.	Velocity of Diffusion. Air = 1.
Hydrogen	.0693	.2632	3.7987	3.830
Marsh-gas	.5590	.7477	1.3375	1.344
Carbonic Oxide	.9670	.9834	1.0169	1.015
Nitrogen	.9713	.9855	1.0147	1.014
Oxygen	1.1057	1.0515	.9510	.949
Sulphureted Hydrogen	1.1912	1.0914	.9163	.950
Carbonic Acid	1.5291	1.2366	.8087	.812

EXPLOSIVES.

Explosive.	Temperature of the Explosion (F.).	Products of Explosion.		Ruptive Pressure. (Pounds per Sq. In.)
		Combustible.	Incombustible.	
Blasting Powder..	2,000° to 3,600°	42 per cent.	58 per cent.	12,400 to 20,500
Nitroglycerine ...	5,740°	0 per cent.	100 per cent.	152,640
Dynamite	5,280°	0 per cent.	100 per cent.	48,000
Blasting-gelatine..	5,830°	46 per cent.	54 per cent.	....
Gun-cotton	4,800°	61 per cent.	39 per cent.	90,000 to 100,000
Tonite	4,800°	8 per cent.	92 per cent.	....
Roburite	3,800°	0 per cent.	100 per cent.	....
Ammonite	......	0 per cent.	100 per cent.	....
Securite..........	......	0 per cent.	100 per cent.	....
Carbonite	......	41 per cent.	59 per cent.	....

ILLUMINATING POWER OF SAFETY-LAMPS.

Name of Lamp.	Illuminating Power of Lamp, with a Candle Taken as 1, or Unity.
Davy	.16
Geordy.................................	.10
Clanny...............	.20
Mueseler.............................	.35
Evan Thomas........................	.45
Marsaut, 3 gauzes..................	.45
Marsaut, 2 gauzes..................	.55
Howat's Deflector..................	.65
Ashworth-Hepplewhite-Gray...........	.65 (about)

TEMPERATURE OF COMBUSTION OF VARIOUS GASES.

Gases.	Temperature of Combustion (F.).
Marsh-Gas................	1,220°
Ordinary Illuminating Gas..	1,198°
Carbonic Oxide Gas........	1,184°
Hydrogen.................	1,148°

HEAT, VOLUME OF GAS, AND EXPLOSIVE FORCE OF PROMINENT EXPLOSIVES.

Substance.	Heat.	Volume of Gas.	Estimated Explosive Force.
Blasting powder..........	510	0.173 liter*	88
Artillery powder..........	608	0.225 liter	137
Sporting powder..........	641	0.216 liter	139
Powder, nitrate of soda for its base...............	764	0.248 liter	190
Powder, chlorate of potash for its base............	972	0.318 liter	309
Gun-cotton...............	590	0.801 liter	472
Picric acid	687	0.780 liter	536
Picrate of potash	578	0.585 liter	337
Gun-cotton, mixed with chlorate of potash	1,420	0.484 liter	680
Picric acid, mixed with chlorate of potash	1,424	0.408 liter	582
Picrate, mixed with chlorate of potash..........	1,420	0.337 liter	478
Nitroglycerin.............	1,320	0.710 liter	939

*A *liter* is equal to 61.027051 cubic inches.

COMBUSTION OF FUELS.

One Pound of Combustible.	Theoretical Weight of Gas, in Pounds, Required to Effect the Complete Combustion of One Pound of Combustible.		Actual Weight of Air, in Pounds, Required to Effect the Complete Combustion of One Pound of Combustible.		Total Heat of Combustion of One Pound of Combustible in B. T. U.	The Equivalent of the Total Heat of Combustion, Expressed in the Number of Pounds of Water under Atmospheric Pressure it Would Evaporate.	
	Oxygen.	Air.	With Chimney Draft, and Initial Temperature of Air at 62° F.	With Blast Air-Supply at 62° F., and Waste Gases at 320° F.		From 62° F. and at 212° F.	From and at 212° F.
	1	2	3	4	5	6	7
Hydrogen	8.00	34.80	70	47	62,032	55.60	64.00
Carbon (completely burned)	2.66	11.60	22	15	14,500	13.00	15.00
Coal (of average composition)	2.46	10.70	21	14	14,133	12.67	14.63
Coke	2.50	10.90	22	15	13,550	12.14	14.02
Wood (average kiln-dried)	1.40	6.10	12	18	7,792	6.98	8.07
Petroleum	3.54	11.90	31	21	20,408	18.83	21.18

HEATING SURFACE, GRATE AREA, AND HORSEPOWER.

Type of Boiler.	Heating Surface / Horsepower.	Heating Surface / Grate Area.
Plain Cylindrical	6 to 10	12 to 15
Flue....................	8 to 12	20 to 25
Return-Tubular.........	14 to 18	25 to 35
Vertical..	15 to 20	25 to 30
Water-Tube	10 to 12	35 to 40
Locomotive............	1 to 2	50 to 100

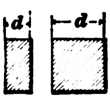
COEFFICIENTS OF FRICTION.

Description of Surfaces in Contact.	Disposition of Fibers.	State of the Surfaces.	Coefficient of Friction.
Oak on Oak	Parallel	Dry	.48
Oak on Oak	Parallel	Soaped	.16
Wrought Iron on Oak	Parallel	Dry	.62
Wrought Iron on Oak	Parallel	Soaped	.21
Cast Iron on Oak	Parallel	Dry	.49
Cast Iron on Oak	Parallel	Soaped	.19
Wrought Iron on Cast Iron .	—	Slightly Unctuous	.18
Wrought Iron on Bronze....	—	Slightly Unctuous	.18
Cast Iron on Cast Iron.....	—	Slightly Unctuous	.15

CONSTANTS USED IN DETERMINING M. E. P.

Cut-off.	Constant.	Cut-off.	Constant.	Cut-off.	Constant.
1/6	.566	3/8	.771	2/3	.917
1/5	.603	.4	.789	.7	.926
1/4	.659	1/2	.847	3/4	.937
.3	.708	.6	.895	.8	.944
1/3	.743	5/8	.904	7/8	.951

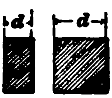
CONSTANTS FOR CAST-IRON PILLARS.

Cross-section of Pillar.	When Both Ends of the Pillar are Flat or Fixed.	When One End of the Pillar is Flat or Fixed, and the Other Round or Movable.	When Both Ends of the Pillar are Round or Movable.
 Round.	281.25	187.5	140.625
 Square or Rectangle.	375.00	250.0	187.500
 Thin Square Tube.	750.00	500.0	375.000
 Thin Round Tube.	562.50	375.0	281.250
 Angle with Equal Sides.	187.50	125.0	93.750
 Cross with Equal Arms.	187.50	125.0	93.750
 I Beam.	$375 \times \frac{A}{A+B}$	$250 \times \frac{A}{A+B}$	$125 \times \frac{A}{A+B}$

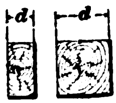
CONSTANTS FOR LINE SHAFTING.

Material of Shaft.	No Pulleys Between Bearings.	Pulleys Between Bearings.
Steel or Cold-Rolled Iron	65	85
Wrought Iron....	70	95
Cast Iron..............	90	120

CONSTANTS FOR WROUGHT-IRON PILLARS.

Cross-section of Pillar.	When Both Ends of the Pillar are Flat or Fixed.	When One End of the Pillar is Flat or Fixed, and the Other Round or Movable.	When Both Ends of the Pillar are Round or Movable.
 Round.	2,250	1,500	1,125
 Square or Rectangle.	3,000	2,000	1,500
 Thin Square Tube.	6,000	4,000	3,000
 Thin Round Tube.	4,500	3,000	2,250
 Angle with Equal Sides.	1,500	1,000	750
 Cross with Equal Arms	1,500	1,000	750
 I Beam.	$3{,}000 \times \frac{A}{A+B}$	$2{,}000 \times \frac{A}{A+B}$	$1{,}500 \times \frac{A}{A+B}$

CONSTANTS FOR WOODEN PILLARS.

Cross-section of Pillar.	When Both Ends of the Pillar are Flat or Fixed.	When One End of the Pillar is Flat or Fixed, and the Other Round or Movable.	When Both Ends of the Pillar are Round or Movable.
 Round.	187.5	125.00	93.75
 Square or Rectangle.	250.0	166.66	125.00
 Hollow Square Made of Boards.	500.0	333.33	250.00

CONSTANTS FOR TRANSVERSE STRENGTH OF BEAMS.

Materials.	Safe Transverse Strength in Pounds.	Materials.	Safe Transverse Strength in Pounds.
Metals:		Woods:	
Cast Iron.......	100	Birch..........	35
Wrought Iron...	150	Elm...........	25
Structural Steel	160	Ash............	45
Copper.........	50	Beech..........	30
Brass...........	55	Hickory........	50
		Maple..........	60
		Oak (American)	45
		Pine (Pitch)....	40
		Pine (White)...	30

TENSILE STRENGTHS OF MATERIALS.

Materials.	Breaking Stress in Pounds per Square Inch.	Working Stress in Pounds per Square Inch.
Timber...........	10,000	600 to 1,200
Cast Iron.........	16,000	1,500 to 3,500
Wrought Iron.....	50,000	5,000 to 12,000
Steel........	70,000	6,000 to 13,000

CRUSHING STRENGTHS OF MATERIALS.

Materials.	Crushing Strength in Tons per Square Inch.
Cast Iron....................	40.0
Wrought Iron	18.0
Mild Steel....................	26.0
Cast Copper.................	5.0
Cast Brass...................	4.5
Timber (dry).................	3.5
Brick	1.0
Stone	3.0

SHEARING STRENGTHS OF MATERIALS.

Materials.	Greatest Shearing Stress in Pounds per Square Inch.	Safe Shearing Stress in Pounds per Square Inch.
Cast Iron.........	18,000	1,500 to 3,000
Wrought Iron.....	40,000	4,000 to 10,000
Steel.............	60,000	5,000 to 12,000

COMPOSITION OF FUELS.

	Carbon.	Hydrogen.	Oxygen.	*Disposable Hydrogen.
Wood (mean of several analyses)	100	12.18	83.07	1.80
Peat	100	9.85	55.67	2.89
Lignite	100	8.37	42.42	3.07
Ten-yard seam of S. Staffordshire (Bituminous)	100	6.12	21.23	3.47
Steam Coal	100	5.91	18.32	3.62
Anthracite	100	2.84	1.74	2.63

* Disposable Hydrogen is that portion of hydrogen available for heating purposes in fuel, which is in excess of the quantity required to form water with the oxygen contained in the coal.

SPACING OF LINE-SHAFT BEARINGS.

Diameter of Shaft in Inches.	Distance Between Bearings in Feet.	
	Wrought-Iron Shaft.	Steel Shaft.
2	11	11.50
3	13	13.75
4	15	15.75
5	17	18.25
6	19	20.00
7	21	22.25
8	23	24.00
9	25	26.00

TABLE FOR FINDING THE FIFTH ROOT OF A NUMBER.

(Art. **1000.** § 6.)

1	2	3	4	1	2	3	4
1.0	.100	.5000	1.0000	5.6	17.6	491.7	5,507.3
1.1	.133	.7321	1.6105	5.7	18.5	527.8	6,016.9
1.2	.173	1.037	2.4883	5.8	19.5	565.8	6,563.6
1.3	.220	1.428	3.7129	5.9	20.5	605.9	7,149.2
1.4	.274	1.921	5.3782	6.0	21.6	648.0	7,776.0
1.5	.338	2.531	7.5938	6.1	22.7	692.3	8,446.0
1.6	.410	3.277	10.486	6.2	23.8	738.8	9,161.3
1.7	.491	4.176	14.199	6.3	25.0	787.6	9,924.4
1.8	.583	5.249	18.896	6.4	26.2	838.9	10,737
1.9	.686	6.516	24.761	6.5	27.5	892.5	11,603
2.0	.800	8.000	32.000	6.6	28.7	948.7	12,523
2.1	.926	9.724	40.841	6.7	30.1	1,007	13,501
2.2	1.06	11.71	51.536	6.8	31.4	1,069	14,539
2.3	1.22	13.99	64.363	6.9	32.9	1,133	15,640
2.4	1.38	16.59	79.626	7.0	34.3	1,201	16,807
2.5	1.56	19.53	97.656	7.1	35.8	1,271	18,042
2.6	1.76	22.85	118.81	7.2	37.3	1,344	19,349
2.7	1.97	26.57	143.49	7.3	38.9	1,420	20,731
2.8	2.20	30.73	172.10	7.4	40.5	1,499	22,190
2.9	2.44	35.36	205.11	7.5	42.2	1,582	23,730
3.0	2.70	40.50	243.00	7.6	43.9	1,668	25,355
3.1	2.98	46.18	286.29	7.7	45.7	1,758	27,068
3.2	3.28	52.43	335.54	7.8	47.5	1,851	28,872
3.3	3.59	59.30	391.35	7.9	49.3	1,948	30,771
3.4	3.93	66.82	454.35	8.0	51.2	2,048	32,768
3.5	4.29	75.03	525.22	8.1	53.1	2,152	34,868
3.6	4.67	83.98	604.66	8.2	55.1	2,261	37,074
3.7	5.07	93.71	693.44	8.3	57.2	2,373	39,390
3.8	5.49	104.3	792.35	8.4	59.3	2,489	41,821
3.9	5.93	115.7	902.24	8.5	61.4	2,610	44,371
4.0	6.40	128.0	1,024.0	8.6	63.6	2,735	47,043
4.1	6.89	141.3	1,158.6	8.7	65.9	2,864	49,842
4.2	7.41	155.6	1,306.9	8.8	68.1	2,998	52,773
4.3	7.95	170.9	1,470.1	8.9	70.5	3,137	55,841
4.4	8.52	187.4	1,649.2	9.0	72.9	3,281	59,049
4.5	9.11	205.0	1,845.3	9.1	75.4	3,429	62,403
4.6	9.73	223.9	2,059.6	9.2	77.9	3,582	65,908
4.7	10.4	244.0	2,293.5	9.3	80.4	3,740	69,569
4.8	11.1	265.4	2,548.0	9.4	83.1	3,904	73,390
4.9	11.8	288.2	2,824.8	9.5	85.7	4,073	77,378
5.0	12.5	312.5	3,125.0	9.6	88.5	4,247	81,537
5.1	13.3	338.3	3,450.3	9.7	91.3	4,426	85,873
5.2	14.1	365.6	3,802.1	9.8	94.1	4,612	90,392
5.3	14.9	394.5	4,182.0	9.9	97.0	4,803	95,099
5.4	15.7	425.2	4,591.7	10.0	100.0	5,000	100,000
5.5	16.6	457.5	5,032.8				

WEIGHT AND BREAKING STRENGTH OF IRON AND STEEL WIRE ROPES.

WIRE ROPES, 19 WIRES TO THE STRAND.

Diameter in Inches.	Weight in Lb. per Foot.	Breaking Load in Tons of 2,000 Lb.		
		Iron.	Cast Steel.	Plow Steel.
1/2	0.35	3.48	7.00	10.00
9/16	0.44	4.27	9.00	13.00
5/8	0.60	5.13	12.00	18.00
3/4	0.88	8.64	18.00	27.00
7/8	1.20	11.50	25.00	37.00
1	1.58	16.00	33.00	47.00
1 1/8	2.00	20.00	42.00	60.00
1 1/4	2.50	27.00	52.00	75.00
1 3/8	3.00	33.00	63.00	90.00
1 1/2	3.65	39.00	77.00	110.00
1 5/8	4.10	44.00	86.00	123.00
1 3/4	5.25	54.00	106.00	157.00
2	6.30	65.00	125.00	189.00

WIRE ROPES, 7 WIRES TO THE STRAND.

Diameter in Inches.	Weight in Lb. per Foot.	Breaking Load in Tons of 2,000 Lb.		
		Iron.	Cast Steel.	Plow Steel.
1/2	0.31	2.83	6.00	9.00
9/16	0.41	4.10	8.00	12.00
5/8	0.57	5.80	11.00	16.00
11/16	0.70	7.60	14.00	21.00
3/4	0.88	8.80	17.00	25.00
7/8	1.12	12.30	22.00	33.00
1	1.50	16.00	30.00	45.00
1 1/8	1.82	20.00	36.00	
1 1/4	2.28	25.00	44.00	
1 3/8	2.77	30.00	52.00	
1 1/2	3.37	36.00	62.00	

FLAT-STEEL ROPES.

Size. Inches.	Weight per Ft. Pounds.	Breaking Strength. Pounds.	Size. Inches.	Weight per Ft. Pounds	Breaking Strength. Pounds.
3/8 × 2	1.27	38,500	1/2 × 3	2.57	76,500
3/8 × 2 1/2	1.52	46,000	1/2 × 3 1/2	3.00	89,500
3/8 × 3	2.02	61,000	1/2 × 4	3.43	102,000
3/8 × 3 1/2	2.28	69,000	1/2 × 4 1/2	3.86	115,000
3/8 × 4	2.53	76,500	1/2 × 5	4.29	127,500
3/8 × 4 1/2	3.04	92,000	1/2 × 6	5.15	153,000
3/8 × 5	3.29	99,500	1/2 × 7	6.01	178,500
3/8 × 6	4.05	122,500	1/2 × 8	6.86	204,000
5/8 × 4	3.96	143,000	3/4 × 5	5.81	183,500
5/8 × 4 1/2	4.62	166,500	3/4 × 6	7.74	245,000
5/8 × 5	5.28	190,500	3/4 × 7	8.71	275,500
5/8 × 6	6.60	238,000	3/4 × 8	9.68	306,000
5/8 × 7	7.26	262,000	3/4 × 9	11.62	367,000
5/8 × 8	8.58	309,500	3/4 × 10	12.58	398,000
5/8 × 9	9.24	333,000	3/4 × 11	13.55	428,500
5/8 × 10	10.56	381,000	3/4 × 12	15.49	489,500

RELATIVE SIZES OF SPIKES AND RAILS.

Size of Spike in Inches.	Weight in Pounds per Yard of Rails Used.	Average Number of Spikes per Keg of 200 Lb.
4 × 1/2	25	600
4 1/2 × 1/2	35	525
5 × 1/2	35 to 45	448

COST OF DIAMOND DRILLING.

The cost of drilling 2,084 feet of hole in prospecting the ground through which the Croton Aqueduct Tunnel was to pass is given as follows:

814 ft. of soft rock (decomposed gneiss), in which an average of 23.1 ft. per day was drilled, at a cost of $1.15 per ft.

347 ft. of hard rock (gneiss), in which an average of 11.1 ft. per day was drilled, at a cost of $3.97 per ft.

923 ft. of clay, gravel, and boulders, in which from 6¼ to 9 ft. per day were drilled, at a cost of $4.07 per ft.

The average progress per day in drilling the entire 2,084 ft. was 10.2 ft. per day.

In the Minnesota Iron Co.'s mines at Soudan, Minnesota, the diamond drill is used for drilling holes from 10 to 40 feet in depth in the back of the stopes, practically all the work being done in iron ore. The average cost per foot of drilling 13,512 feet of hole was $.7703, which was divided as follows:

Carbons	$.3400
Supplies, oil, etc	.0700
Fuel	.0400
Repairs	.0500
Labor	.2703
Total	$.7703

Tables A and B give the cost of boring at two Michigan mines:

TABLE A.

Ishpeming, Michigan.

			Total cost.	Cost per ft.
Labor	400¼ days setter at $3.00	$1200.75	$2,506.10	$.669
	372 " runner " 2.25	837.00		
	230½ " " " 2.00	460.50		
	4½ " laborer " 1.75	7.85		
Carbon, 68⅜ carats, at $15.144			$1,035.47	.276
Bits, lifters, shells, barrels, and repairs			433.81	.115
Oil, candles, waste, and supplies			128.09	.035
Estimated cost compressed air			374.60	.100
Total			$4,478.07	$1.195

Number holes drilled	28
Drilled in hematite	193 feet.
" " jasper	646 "
" " mixed ore	986 "
" " dioritic schist	1,921 "
Total drilling	3,746 feet.

No. of 10-hour shifts drill was running, including moving and setting up	603
Amount drilling per 10-hour shift	6.2 feet.

TABLE B.

Underground drilling	6,075 feet.
Surface drilling	1,414 "
Stand-pipe sunk	470 "
Total distance run	7,959 feet.

Actual drilling time underground	672 shifts.
" " " on surface	165 "
Time of foreman, setter, moving, and stand-piping	1,314 "
Total time worked	2,151 shifts.

Av. progress per man per shift	3.70 feet.
" " " drill " " actually running	8.95 "
Weight of carbon consumed	111.00 carats.
Dist. drilled per carat of carbon consumed	67.38 feet.

	Amount.	Per Foot.
Cost of carbon	$1,887.00	$.237
" " supplies and oils	134.13	.017
" " fuel	360.73	.045
" " shop material, etc.	663.36	.083
Pay roll	4,000.03	.502
Total cost	$7,045.25	$.884

RECORDS OF COST PER FOOT IN DIAMOND DRILLING.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Labor..........	.707	1.040	2.483	1.150	.581	1.615	1.030	1.720	1.189	1.284	.721	1.200	.939	.812	.984
Fuel...........	.094	.270	.256	.019	.000	.216	.090	.214	.157	.339	.419	.329	.126	.182	.251
Camp acct......	.373	.559	.789	.538	.295	.621	.384	.549	.516	.495	.519	.595	.644	.722	.636
Repairs........	.139	.110	.294	.171	.135	.144	.103	.185	.154	.165	.040	.087	.138	.126	.116
Supplies.......	.034	.065	.039	.074	.023	.032	.011	.039	.048	.097	.020	.092	.076	.097	.088
Carbon.........	.263	.658	.859	.860	.843	1.587	.934	.684	.684	.733	.227	.209	.553	.239	.330
Supt...........	.239	.322	.628	.040	.063	.192	.140	.305	.259	.172	.347	.220	.106	.196	.199
Total..........	1.849	3.024	5.348	2.852	1.940	4.407	2.692	3.696	3.007	3.285	2.293	2.732	2.582	2.374	2.604

A 5 holes, 1,066 ft. Sandstone and marble.

B 1 hole, 1,293 ft. Black slate and jasper.

C 3 holes, 478 ft. Jasper, very hard.

D 5 holes, 780 ft. Jasper, hard.

E 1 hole, 216 ft. Iron slates.

F 1 hole, 174 ft. Jasper and slate.

G 2 holes, 267 ft. Jasper and slate.

H 3 holes, 410 ft. Jasper.

I Average cost of total work of drilling 21 holes. Total of 4,684 ft.

J 2 holes, 634 ft. Iron slates.

K 2 holes, 360 ft. Schist and jasper.

L 6 holes, 1,350 ft. Iron slates.

M 2 holes, 611 ft. Schist, jasper, and quartzite.

N 6 holes, 2,091 ft. Quartzite.

O Average cost of drilling 18 holes, 5,046 ft.

DIMENSIONS AND WEIGHTS OF BORING TOOLS.

Name.	Length.	Diameter.	Weight.
Rope-socket,	3 ft. 6 in.		80 lb.
Sinker-bar,	18 ft.	3½ in.	540 lb.
Jars;	7 ft. 4 in.	5½ in.	320 lb.
Auger-stem,	30 ft.		1,020 lb.
Bit,	3 ft. 3 in.		140 lb.

REQUIREMENTS OF A GOOD PERCUSSIVE MACHINE ROCK-DRILL.

André sums up the requirements of a good percussive machine rock-drill as follows:

1. It should be simple in construction and strong in every part.

2. It should consist of few parts, and especially of few moving parts.

3. It should be as light as possible, consistent with strength.

4. It should occupy as little space as possible.

5. The striking part should be of relatively great weight, and should strike the rock directly.

6. No parts except the piston and bit should be subject to violent shocks.

7. The piston should have a variable stroke.

8. The sudden removal of the resistance should not cause any injury to any part.

9. The drill should be rotated automatically.

10. The feed, if automatic, should be regulated by the advance of the piston as the cutting advances.

COLORS IMPARTED TO THE FLAME BY VARIOUS MINERALS

[illegible]	[illegible]	
Violet	Potassium and all its compounds except the phosphate [illegible] Very small quantities of sodium and lithium will obscure the reaction. In such cases by viewing the flame through a sufficiently thick blue glass, the potash flame becomes apparent, the glass having the power of intercepting the yellow light of sodium and modifying the red of lithium to violet, while the potassium flame appears purplish red.	
Red.	*Carmine Red.* Lithium and its compounds. The reaction is obscured by much sodium, but not by potassium. In visible through green glass. *Crimson.* Strontium and its salts. Reaction is masked when much barium is present. *Yellowish-Red.* Calcium (lime) and most of its salts. Reaction masked by barium flame. Not to be confounded with strontium flame.	The strontium flame is intensified when the heated assay is moistened with *HCl* and again ignited. The calcium flame is intensified as above.

Green.	*Yellowish-Green.* Barium and its compounds, silicates excepted. Reaction is not prevented by the presence of lime. *Yellowish-Green.* Molybdic acid, and also oxide and sulphide of molybdenum. *Green.* Metallic copper and its salts. Tellurous acid. *Bluish-Green.* Phosphoric acid. Many phosphates give this reaction.* *Yellowish-* (or light-) *Green.* Boracic acid and borates.† *Whitish-Green* (intense). Metallic zinc.	*Other phosphates must be powdered and then moistened with H_2SO_4. The coloration is often but momentary. †Moistened with H_2SO_4, when powdered, gives an intensely green flame which lasts but a short time and then changes back to its yellowish-green color.	Barium flame intensified when substance is moistened with HCl.
Blue.	*Light-Blue.* Metallic arsenic, arsenates, and arsenous acid, and arsenides of bases which do not themselves color the flame. *Greenish-Blue.* Metallic antimony, and the sublimate of antimonous acid on charcoal. *Azure Blue.* Metallic lead, when fused in the R. F.; also selenium; likewise metallic copper and its salts, when moistened with hydrochloric acid (copper chloride).		Copper and its salts moistened with HCl give an intensely azure-blue flame.

COLORS IMPARTED TO A BORAX BEAD BY THE METALLIC OXIDES IN THE OXIDIZING FLAME (O. F.)

Oxides of	Amount of Oxide	Color Hot	Color Cold	Remarks
Silicon, Aluminum, and Tin.	Little or much.	Colorless.	Colorless.	
Barium, Strontium, Calcium, Magnesium, Glucinum, Thorium, Yttrium, Zirconium, Lanthanum, Tantalum, Niobium, Tellurium, Zinc, and Silver.	Little or much.	Colorless.	Colorless and opaque white according to the amount of oxide.	Can be flamed opaque white.
Antimony and Bismuth.	Much.	Pale yellow.	Colorless or white as above.	Opaque or turbid by flaming.
Titanium, Lead, Cadmium, Molybdenum, and Tungsten.	Medium.			
Iron and Uranium.	Little.	Yellow.	Nearly colorless.	
Cerium.	Little.	Yellow.	Pale yellow.	
Vanadium.	Little.	Yellow.	Yellowish green, almost colorless.	

Chromium.	Little.	Yellow.	Yellowish-green.	
Iron, Uranium, and Cerium.	Medium to saturated.	Deep yellow to orange-red.	Yellow.	Uranium and Cerium enamel-yellow by flaming.
Chromium.	Medium to saturated.	Deep yellow to orange-red.	Yellowish-green.	
Iron with Manganese.	Little.	Red to brown.	Yellowish-red.	Iron recognized by R. F. test.
Cobalt.	Little to medium.	Blue.	Blue.	With manganese, violet hot and cold.
Copper.	Little to medium.	Green.	Blue.	
Mixtures of Copper, Iron, Nickel, and Cobalt.	Medium.	Green.	Various shades of yellow, green, and blue, according to the mixture.	
Nickel.	Little to medium.	Violet (amethystine).	Reddish-brown.	With Cobalt, brownish cold; violet with much Cobalt.
Manganese.	Little.	Violet (amethystine).	Reddish-violet.	With Cobalt, violet hot and cold.
Didymium.	Much.	Pale rose.	Pale rose.	

COLORS IMPARTED TO A BORAX BEAD BY THE METALLIC OXIDES IN THE REDUCING FLAME (R. F.)

[illegible]	[illegible]	[illegible]	[illegible]	[illegible] changed by flaming
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible] Cerium, [illegible] Zirconium, Lanthanum, Tantalum, Niobium	Little to much	Colorless	Colorless and opaque white according to amount of oxide	Opaque white by flaming
Silver, Lead, Zinc, Bismuth, Antimony, Cadmium, and Tellurium	Much	Pale yellow to colorless.	Colorless after long blowing. After short blast, gray.	Flamed opaque gray Antimony and Bismuth, on charcoal with tin, black; colorless after long blowing.
Nickel.	Little to medium.			

Titanium and Tungsten.	Medium.	Yellow.	Yellow to yellowish-brown.	Much Titanium, brown, hot and cold; enamel-blue by flaming.
Molybdenum.	Much.	Brown.	Brown.	Separated binoxide of Molybdenum is seen if bead is pressed flat.
Iron.	Little.	Pale green.	Nearly colorless.	Uranium bead, when highly saturated, becomes black by flaming.
Uranium.	Little to much.			
Iron.	Medium to much.	Bottle-green.	Pale bottle-green.	Distinctive test for iron.
Vanadium.	Little.	Dirty green.	Fine chrome-green.	
Chromium.	Medium to much.	Green.	Green.	With much, dark, emerald-green.
Copper.	Little to medium.	Colorless to green.	Colorless with little; opaque red with much oxide.*	*Succeeds best on coal, with tin.
Cobalt.	Little to medium.	Blue.	Blue.	
Didymium.	Much.	Pale rose.	Pale rose.	

COLORS IMPARTED TO A SALT OF PHOSPHORUS BEAD BY THE METALLIC OXIDES IN THE OXIDIZING FLAME (O. F.)

Oxides of	Amount of Oxide	Colors: Hot	Colors: Cold	Remarks
Silicon, Aluminum, and Tin.	Little.	Colorless.	Colorless.	Soluble with difficulty in small quantities; silica almost insoluble. If much silica or alumina be added the undissolved part then forms a semi-transparent skeleton.
Barium, Strontium, Calcium, Magnesium, Glucinum, Thorium, Yttrium, Zirconium, Lanthanum, Tellurium.	Little or much.	Colorless.	Colorless.	Saturated beads, opaque white by flaming.
Antimony, Bismuth, Cadmium, Lead, Niobium, Tantalum, Zinc, Silver.	Much.	Very pale yellow.	Colorless.	Beads containing oxide of silver, opalescent when cold.
Titanium, Tungsten.	Little to medium.	Pale yellow.	Colorless.	
Iron.	Little.	Yellow.	Colorless.	
Cerium.	Medium.	Yellow.	Colorless.	

Uranium.	Medium.	Yellow	Pale greenish-yellow.	
Molybdenum.	Medium.	Yellowish-green.	Colorless.	
Vanadium.	Little to medium.	Yellow to deep yellow.	Yellow.	
Iron.	Medium to much.	Deep yellow to brownish-red.	Yellow to almost colorless.	
Nickel.	Little to medium.	Reddish to brownish-red.	Yellow to reddish-yellow.	Same in R. F.
Copper.	Little to medium.	Green.	Blue.	Greenish-blue when highly saturated.
Mixtures of Copper, Iron, Nickel, and Cobalt.	Medium.	Green.	Various shades of yellow, green, and blue, according to mixture.	
Chromium.	Little to medium.	Dirty green.	Fine emerald-green.	
Cobalt.	Little to medium.	Blue.	Blue.	With much appears black. Same in R. F.
Manganese.	Medium.	Grayish or brownish violet.	Violet.	Nearly same as borax, but requires more oxide.
Didymium.	Much.	Pale rose.	Pale rose.	

COLORS IMPARTED TO A SALT OF PHOSPHORUS BEAD BY THE METALLIC OXIDES IN THE REDUCING FLAME (R. F.).

Oxides of	Amount of Oxide.	COLOR.		Remarks.
		Hot.	Cold.	
Silicon, Aluminum, and Tin.	Little.	Colorless.	Colorless.	Same as O. F.
Barium, Strontium, Calcium, Magnesium, Glucinum, Thorium, Yttrium, Zirconium, Lanthanum.	Little or much.	Colorless.	Colorless.	Saturated beads, opaque white by flaming.
Manganese, Cerium, and Didymium.	Medium.	Colorless.	Colorless.	Manganese beads, apt to become a pale rose-color on cooling, as with borax.
Silver, Lead, Zinc, Bismuth, Antimony, Cadmium, Tellurium, Tantalum.	Medium to much.	Colorless to very pale yellow or gray.	Colorless after long blowing; after short blast, gray.	Cadmium and Tantalum volatilize as fast as reduced, leaving colorless beads. Other metals volatilize more slowly, or collect into a globule.
Titanium.	Little to medium.	Yellow.	Violet	

Iron.	Little.	Very pale yellowish-green.	Colorless.	
Uranium.	Medium.	Pale, dirty green.	Fine green.	
Chromium and Vanadium.	Little to medium.	Dirty green.	Fine green.	
Molybdenum.	Medium.			
Iron.	Medium to much.	Yellowish-green or yellow to red.	Almost colorless to pale red-violet.	
Copper.	Little.	Pale yellowish-green.	Pale blue, nearly colorless; sometimes ruby-red.	
Nickel.	Little to medium.	Reddish to brownish-red.	Yellow to reddish-yellow.	Same in O. F.
Copper.	Medium.	Brownish-green.	Opaque red.	
Cobalt.	Little to medium.	Blue.	Blue.	Same in O. F.
Didymium.	Much.	Pale rose.	Pale rose.	

CHARACTERISTIC REACTIONS FOR THE MORE COMMON METALLIC OXIDES.

Oxides of	Alone on Charcoal and in the Forceps.	With Borax.	With Salt of Phosphorus.	With Soda on Charcoal.	With Cobalt Solution in O. F.	Special Tests.
Aluminum. (Alumina.)	Unaltered.	Dissolves slowly to a clear glass, becoming opaque neither by flaming nor saturation. When much is added in fine powder, the glass is cloudy and scarcely fusible, and shows a crystalline surface on cooling.	Dissolves slowly to a clear glass that is always clear. When much is added the undissolved part becomes semi-transparent.	Swells a little, forms an infusible compound, and the excess of soda goes into the coal.	After a strong blast, assumes a fine blue color, the intensity of which is properly apparent only on cooling.	
Silicon. (Silica.)	Unaltered.	Dissolves slowly to a clear, difficultly fusible glass, that can not be made opaque by flaming.	Dissolves in very small quantities to a clear glass. The undissolved portion becomes semi-transparent.	Dissolves with lively effervescence to a clear glass.	With a little of the solution assumes a feeble bluish color, becoming black or dark gray with more. The thinnest edges can be fused to a reddish-blue glass in a very hot flame.	

Antimony. (Antimonous Acid.)	O. F. Is displaced and spreads partly over another spot. R. F. Is reduced and volatilized, tingeing the flame a greenish-blue.	O. F. Dissolves largely to a clear glass, yellowish while hot, colorless on cooling. R. F. The glass treated only for a short time with the O. F. becomes on charcoal grayish and clouded from reduced *Sb*, but this afterwards volatilizes, leaving a clear glass. With tin the glass becomes gray or black, according to the degree of saturation.	O. F. Dissolves with ebullition to a clear glass, only slightly yellowish while hot. R. F. The saturated glass becomes cloudy on coal at first, afterwards clear as the *Sb* is reduced and volatilizes. Treated with tin the glass becomes cloudy from reduced antimony, but on longer blowing, clear again. Tin produces a grayish cloudiness, even if very little *Sb* is present.	Is reduced very easily on coal in either flame, but the metal volatilizes immediately, forming a white coat of oxide of antimony.	A coat of *Sb* moistened with cobalt solution and ignited in the O. F., volatilizes in part, but the remainder is more highly oxidized, and when quite cool appears dirty dark green.	The coat of oxide is moistened with hydriodic acid, and heated with a pure blue flame. The coat becomes a brilliant red; this color disappears if exposed to ammonia fumes. Tests in open and closed tubes (Arts. **96** and **97**) are also very characteristic.

CHARACTERISTIC REACTIONS FOR THE MORE COMMON METALLIC OXIDES *Continued*

Oxides of	Alone on Charcoal and in the Forceps	With Borax	With Salt of Phosphorus	With Soda on Charcoal	Special Tests
Arsenic. (Arsenous Acid.)	Volatilizes below a red heat giving off dense fumes with a characteristic garlic odor.	---		Is reduced on coal with evolution of arsenical fumes, recognized by their garlic odor.	The coat tested with hydriodic acid becomes orange-yellow when heated. Tube tests.
Bismuth.	On coal is reduced in O. F. and R. F. to metal, and is gradually volatilized, forming a coat of yellow oxide; beyond this coat is another thinner white coat. In the R. F. these coats disappear without coloring the flame.	O. F. Dissolves easily to a clear, yellow glass, which, when a little is present, is colorless on cooling. With more, the glass is yellowish-red while hot, becomes yellow on cooling, and when quite cold is opalescent. R. F. The glass on coal is at first gray and clouded, then the oxide is reduced to metal with effervescence, and the glass becomes clear. Tin accelerates this reduction.	O.F. Dissolves easily to a clear, yellow glass, colorless when cold. With much, the glass can be flamed to an enamel, and with still more it becomes enamel-white of itself, on cooling. R.F. On coal, especially with tin, the glass changes, becoming clear and colorless while hot, but blackish-gray and opaque on cooling.	On charcoal is reduced immediately to metallic bismuth.	The coat tested with hydriodic acid becomes reddish-brown. With bismuth-flux, on plaster, chocolate-brown coat, changing to brilliant red if exposed to ammonia fumes.

Cadmium.	On coal, in R.F., it shortly disappears and coats the surrounding coal with a reddish-brown to dark-yellow coat. The proper color is seen when cold. The coal beyond the coat shows a variegated tarnish.	O. F. Dissolves very largely to a clear, yellowish glass, almost colorless on cooling. When strongly saturated, the glass becomes milk-white by flaming, and with still more, becomes enamel-white of itself, on cooling. R. F. The glass containing oxide boils on coal; the cadmium is reduced, and immediately volatilizes, coating the coal with a dark-yellow coat.	O. F. As with borax. R. F. The oxide dissolved in the bead is slowly and imperfectly reduced on coal, forming a very slight dark-yellow coat, showing its proper color only when cold. Tin accelerates the reduction.	O. F. Insoluble. R.F. On coal is immediately reduced and volatilizes, coating the coal with reddish-brown to dark-yellow oxide. The more remote portion of the coal assumes a variegated tarnish.	With hydriodic acid the coat becomes white when heated.
Cobalt.	O.F. Unchanged. R. F. Shrinks somewhat, and is reduced, without fusing, to metal, which is magnetic and assumes a metallic luster when rubbed in the mortar.	O. F. Colors very strongly; the glass is pure smalt-blue, both hot and cold. The strongly saturated glass is so dark-blue it appears black. R.F. As in O.F.	O. F. As with borax, but the same quantity will not give as intense a blue. The color is best observed on cooling. R.F. As in O.F.	On coal is reduced to a gray magnetic powder, assuming a metallic luster by friction.	In the presence of copper or nickel, the bead must be treated on charcoal with tin before the blue color is distinctly observed.

CHARACTERISTIC REACTIONS FOR THE MORE COMMON METALLIC OXIDES—*Continued.*

Oxides of	Alone on Charcoal and in the Forceps.	With Borax.	With Salt of Phosphorus.	With Soda on Charcoal.	Special Tests.
Chromium.	Unaltered in O. F. and R. F.	O. F. Dissolves slowly, but colors strongly. With little, the hot glass appears yellow (chromic acid), but when cold is yellowish-green. With more, it is dark red when hot, becomes yellow on cooling, and when cold is fine yellowish-green. R. F. The slightly saturated glass has a fine green color, both hot and cold (*Cr*). With more, it becomes darker, or pure emerald-green. Tin causes no change.	O. F. Soluble to a clear glass, reddish while hot, but dirty green while cooling, and when quite cold, a fine green. R. F. As in O. F., but the colors are somewhat darker; the same with tin.	Can not be reduced to metal on charcoal, but remains as green chromium, while the soda sinks into the coal.	When very little of the oxide of chromium is associated with other metals which also color the beads, we proceed by fusing the substance on platinum foil with a mixture of equal parts of soda and niter. The mass is heated for some time in O. F., thus forming chromic acid. The mass is dissolved in water and filtered, and afterwards a drop or two of acetic acid and a crystal of acetate of lead are added to the solution, whereupon a lemon-yellow precipitate of *lead chromate* is formed. This precipitate may be collected on a filter and then tested B. B.

Copper.	O. F. Fuses to a black globule, soon spreads out, and is reduced to metallic copper on the lower side. R. F. Reduced below the melting point of copper, the reduced particles show a copper luster, but when the blast is stopped, oxidize again on the surface, becoming black or brown. Strongly heated, it fuses to a button of copper.	O. F. Colors rather strongly; a little causes a green glass while hot, and a blue glass when cold. With more, it is dark green to opaque when hot, but greenish-blue on cooling. R. F. Saturated to a certain degree, the glass soon becomes colorless, but on cooling is red and opaque. On coal the copper is reduced to metal, and the cold glass is quite colorless. A glass containing oxide treated on coal with tin becomes brownish-red and opaque on cooling.	O.F. The colors are the same as with borax, but not so strong for the same amount of oxide. R. F. A rather strongly saturated glass becomes dark green, and on cooling changes, at the moment of solidifying, to an opaque brownish-red. A glass containing but little, if treated on coal with tin, appears colorless while hot, but becomes brownish-red and opaque when cold.	On coal is easily reduced to metal, which can be fused into a button by sufficient heat.	The numerous tests before mentioned for copper will generally serve to detect it. When, however, it is combined with nickel, cobalt, iron, and arsenic, the greater part of the iron and arsenic may be separated by treating the borax bead on charcoal. The remaining metallic globule is fused with a small quantity of pure lead, and then boracic acid is added, which dissolves the lead and the rest of the cobalt and iron, while most of the arsenic volatilizes. The remaining globule, an alloy of copper and nickel, is treated in a salt of phosphorus bead in O. F. The bead will be dark green while hot and clear green when cold.

CHARACTERISTIC REACTIONS FOR THE MORE COMMON METALLIC OXIDES—*Continued.*

Oxides of	Alone on Charcoal and in the Forceps.	With Borax.	With Salt of Phosphorus.	With Soda on Charcoal.	Special Tests.
Iron.	O.F. Unchanged. R.F. Becomes black and magnetic.	O. F. With little, the glass is yellow when hot, colorless on cooling. With more, it is red while hot, yellow when cold; with still more, dark-red while hot, and dark-yellow on cooling. R. F. The glass becomes bottle-green. On coal, with tin, it becomes, at first, bottle-green, but on longer blowing, vitriol-green.	O. F. With a certain amount, the hot glass is yellowish-red, but becomes first yellow on cooling, then greenish and finally colorless. With very much, it is dark red when hot, brownish-red, then dirty green while cooling, and finally a less brownish-red. The colors disappear on cooling much sooner than with borax. R.F. The glass seems unchanged with a small amount of oxide; with more, it is red while hot and on cooling, becomes first yellow, then greenish, and finally reddish. With tin, on coal, becomes green on cooling and finally colorless.	O. F. Insoluble. R. F. Is reduced on coal, yielding a gray, magnetic, metallic powder, when the particles of coal are washed away.	In the presence of lead, tin, bismuth, antimony, or zinc, iron may be detected by treating the borax bead on charcoal until all have been reduced except the iron, which gives the glass a bottle-green color. In the presence of cobalt, nickel, and copper, the two latter will be reduced by tin on charcoal, while the cobalt will color the bead blue. By adding more fresh borax and treating in O. F., the bead will be green while hot, blue on cooling. If the assay is added to a borax bead colored blue by the oxide of copper, and the whole is heated for a few seconds, the protoxide of iron will be further oxidized at the expense of the oxygen of the oxide of copper, and opaque red spots of oxide of copper will appear in the glass when it cools.

Lead.	On coal is immediately reduced, with effervescence, by either flame, and the metal gradually volatilizes, coating the coal with yellow oxide; behind this is another thinner coat of white. The coats disappear under the R. F., tingeing the flame azure blue.	O. F. Dissolves easily to a clear yellow glass, colorless on cooling; with a larger quantity, becomes opaque by flaming; with still more, becomes opaque and enamel-yellow of itself, on cooling. R. F. The glass spreads out on coal and becomes cloudy; on continuing the blast, the oxide reduces with effervescence.	O.F. As with borax, but requiring more of the oxide to obtain a glass; yellow when hot. R.F. The glass becomes grayish and cloudy on coal. With more oxide, a yellow coat is formed on the coal. With tin the glass becomes cloudier and darker gray, but never quite opaque.	The oxide is immediately reduced to metal, which afterwards coats the coal with oxide.	The coat treated on coal with hydriodic acid, and heated, becomes chrome-yellow, or greenish-yellow in very thin coats. In solutions of the salts of lead, sulphuric acid gives an almost insoluble precipitate of lead sulphate, which can be separated and further tested B. B.

CHARACTERISTIC REACTIONS FOR THE MORE COMMON METALLIC OXIDES *Continued*

Oxide	[illegible]	Borax bead	Salt of [illegible] phorus [illegible]	[illegible]	Special tests
Manganese	O.F. Infusible. In a hot enough flame yields oxygen, and turns brownish red. R.F. The same.	O.F. Colors very intensely. The hot glass is violet [illegible], but on cooling becomes violet red. Large amounts render the glass quite black and opaque, the coloring being visible only by pinching the still soft glass with the forceps. R.F. The glass becomes colorless. With a very dark glass this succeeds better on coal with tin. May be pale rose-color on cooling, from incipient oxidation.	O.F. A considerable addition is requisite to render the glass, which then has a brownish violet color while hot and a reddish violet on cooling, but never opaque. When the glass contains so little as to be colorless, the color can be produced by the addition of a small fragment of niter. A glass containing oxide bubbles and yields gas at a high temperature. R.F. The glass very soon becomes colorless and then remains perfectly quiet.	Not reducible to metal on coal. The soda sinks into the coal, leaving the oxide behind.	The tests given are most characteristic, and generally serve to distinguish manganese even when in very small quantities.

Nickel.	O.F. Unchanged. R.F. On charcoal is reduced to metal. The coherent metallic powder can not be fused; strongly rubbed in a mortar, it assumes a metallic luster, and is highly magnetic.	O. F. Colors intensely. In small quantities it colors the glass violet while hot, which becomes pale red-brown on cooling; with more, the colors are darker. R. F. The glass becomes gray and turbid from separation of metallic nickel. By long blowing the metallic particles cohere, and leave the glass colorless. On charcoal, with tin especially, the reduction is more rapid.	O.F. Dissolves to a reddish glass which becomes yellow on cooling; with larger quantity the hot glass is brownish-red and becomes reddish-yellow on cooling. R.F. The glass from the O.F. is unaltered. On charcoal, with tin, all the *Ni* is reduced after continued blowing and the glass becomes colorless.	O.F. Insoluble. R. F. On charcoal easily reduced to small, brilliant, metallic particles which are highly magnetic.	The usual tests for nickel are sufficient, unless it is present with much cobalt, in which case a saturated bead is fused on charcoal with about one grain of fine gold. The oxide of nickel, with a little cobalt, is reduced to metal and unites with the gold. The button is then freed from the bead, and treated again on charcoal in O. F. with salt of phosphorus. The first bead will probably be blue, since cobalt is more easily oxidized, in which case the globule is successively treated in pure beads until the nickel reaction is plainly visible.

CHARACTERISTIC REACTIONS FOR THE MORE COMMON METALLIC OXIDES (continued)

[illegible] of	Alone on Charcoal and in the Forceps	With Borax	With Salt of Phosphorus	[illegible]	[illegible]	[illegible]
Mercury.	Is instantly reduced and volatilized.					[illegible]

Oxides of	Alone on Charcoal and in the Forceps.	With Borax.	With Salt of Phosphorus.	With Soda on Charcoal.	Special Tests.
Molybdenum. (Molybdic Acid.)	O. F. Fuses, spreads out, volatilizes, and forms, at a certain distance, a yellowish, pulverulent coat, consisting of small crystals nearest the assay. The coat becomes white on cooling, and the crystals colorless. Beyond this a thin, non-volatile coat of oxide forms, which, on cooling, is dark copper-red, and has a metallic luster. R. F. The greater part sinks into the coal, and can be reduced to metal with a hot flame. It appears as a gray powder.	O. F. Dissolves easily and largely to a clear glass, yellow while hot, and colorless on cooling. A very large addition produces a glass, dark-yellow to dark-red while hot, and opaline to bluish-green enamel when cold. R. F. The glass produced in the O. F. becomes brown with a certain degree of saturation, and with more, is opaque. In a good flame black flocks of *Mo* separate and can be seen in the yellowish glass when mashed flat.	O. F. Dissolves easily to a clear glass, which is yellowish-green while hot, if a moderate amount is added, but becomes nearly colorless on cooling. On coal becomes quite dark, and on cooling is fine green. R. F. The glass becomes quite dirty green, but purer green on cooling. The same on coal. With tin a somewhat darker green.	On coal fuses with effervescence at first, but afterwards is absorbed by the coal and the greater part is reduced to metallic *Mo* which can be obtained as a steel-gray powder by washing away the particles of coal.	Substances containing Molybdenum, if finely powdered and heated in a porcelain dish with concentrated H_2SO_4, give, upon the addition of alcohol, a fine azure-blue color, especially on the sides of the dish.

CHARACTERISTIC REACTIONS FOR THE MORE COMMON METALLIC OXIDES *Continued*

Oxides of	Alone on Charcoal and in the Forceps	With Borax	With Salt of Phosphorus	With Soda on Charcoal	With Cobalt Solution in O. F.	Special Tests
Tin.	O. F. The protoxide of tin takes fire and burns like tinder to a higher oxide, called the binoxide, which glows strongly and is yellowish while hot; on cooling becomes dirty yellowish-white. R. F. By long blowing the oxide may be reduced to metallic tin, with formation of a slight sublimate of oxide, which coats the charcoal near the assay.	O. F. Very slowly soluble in small quantities to a colorless bead, which remains clear after cooling, and is not made turbid by flaming. A bead saturated with oxide, allowed to become perfectly cool, and then heated gently becomes turbid, loses its round form and manifests indistinct crystallization.	O. F. Very slowly soluble in small quantity to a colorless glass, that remains clear on cooling. R. F. The glass from the O. F. is unaltered either on the wire or on charcoal.	On charcoal is reduced to metallic tin.	Assumes a bluish-green which must be observed after the assay is perfectly cold.	Oxides of tin are best reduced on charcoal with soda or cyanide of potassium. If much iron is present, borax is added. Some compounds of tin, when treated with nitric acid, separate oxide of tin as a white precipitate which can be tested in the usual way.

Oxides of	Alone on Charcoal and in the Forceps.	With Borax.	With Salt of Phosphorus.	With Soda on Charcoal.	Special Tests.
Tellurium. (Tellurous Acid.)	O. F. Fuses and is reduced with effervescence. The reduced metal votalizes immediately, and a white coating of tellurous acid deposits on coal. The edges of the sublimate are commonly red or dark yellow. R. F. As in O. F. The outer flame is tinged bluish-green.	O. F. Soluble to a clear, colorless glass, which becomes gray from separation of *Te* when heated on coal. R. F. The clear glass from O. F., heated on coal, becomes first gray and finally colorless, all the tellurium being reduced and volatilized and coating the coal with tellurous acid.	As with borax.	On charcoal is reduced and volatilized with the formation of a coat of tellurous acid.	Substances containing tellurium may be triturated (ground by rubbing) with soda and charcoal dust, and fused in a closed tube; then allowed to cool, and a little hot water dropped into the tube. The water will assume a purple color from the dissolved telluride of sodium. Tellurium compounds, heated with concentrated H_2SO_4, impart to it a purple color, which disappears on cooling or on the addition of water, and a black-gray precipitate is formed.
Silver.	Easily reduced to metallic silver, which fuses to globules.	O. F. Is partly dissolved, and partly reduced to metal. The glass on cooling becomes opaline or milk-white, according to the degree of saturation. R. F. The glass from the O. F. becomes at first gray from separation of metal, then clear and colorless, all the silver separating and fusing to a globule.	O.F. Both the oxide and the metal yield a yellowish glass. A highly saturated bead appears opaline on cooling. Its color is yellow by daylight, and red by candle-light. R. F. As with borax.	Is immediately reduced; fuses to metallic globules, while the soda is absorbed by the charcoal.	When silver is combined with large quantities of lead or bismuth, etc., it had best be fused to a button with the addition of some test lead and borax glass, and the resulting button cupelled in a bone-ash cupel.

CHARACTERISTIC REACTIONS FOR THE MORE COMMON METALLIC OXIDES—Concluded.

Oxides of	Alone on Charcoal and in the Forceps.	With Borax.	With Salt of Phosphorus	With Soda on Charcoal	With Cobalt Solution in O. F.	Special Tests
Titanium. (Titanic Acid.)	In both flames becomes yellow; on cooling resumes its white color. Is not otherwise changed.	O. F. Easily soluble to a clear glass. Yellow while hot; colorless when cold. May be rendered turbid by flaming if much is present. If very much is present the glass becomes opaque white on cooling. R.F. Dissolved in small quantity, the glass is yellow; with more, it becomes dark yellow to brown. A saturated glass is made enamel-blue by flaming.	O.F. Easily soluble to a clear glass. If much is present, the glass is yellow while hot, colorless on cooling. R. F. The glass from the O. F. is yellow while hot, but on cooling, reddens and finally assumes a fine violet color. If the assay contains iron, the glass is brown-yellow or brown-red on cooling. On coal, with tin, the glass becomes violet if not too much iron is present.	O.F. On charcoal soluble with effervescence to a dark yellow glass, which, on cooling, crystallizes, with the production of so much heat that the globule becomes again, of itself, white hot. When fully cold the glass is white or grayish. R. F. As in O. F. No reduction to metal is accomplished.	Assumes a yellowish-green color similar to oxide of zinc but less fine.	A substance containing titanium, if fused with soda and dissolved in HCl and then heated with metallic tin or zinc, the titanic acid is reduced and the liquid is temporarily colored violet, and finally the hydrated sesquioxide of titanium is precipitated.

Zinc.	O.F. Becomes yellow on heating, but resumes its white color when cold. It is infusible, and glows vividly on strong ignition. R. F. Gradually reduces, and disappears; the metal volatilizing and reoxidizing is for the greater part deposited as oxide on the charcoal, forming a coat, yellow while hot, white when cold.	O. F. Easily soluble to a clear glass, which is yellowish while hot, colorless when cold. When considerably saturated may be made opaque by flaming, and when more highly saturated becomes opaque on cooling. R.F. The saturated glass when first heated becomes turbid and grayish (separation of a part of the oxide); by longer blowing becomes clear again. On charcoal the oxide is gradually reduced, the metal volatilizing and coating the surrounding charcoal with oxide.	As with borax.	O. F. Insoluble. R. F. On charcoal is reduced. The metal, however, volatilizes immediately, and if the heat be strong, burns with a greenish-white flame, while the charcoal is coated with oxide.	Assumes a fine yellowish-green color, best observed when cold.	Zinc is difficult to detect when a small quantity of it is combined with large quantities of lead, bismuth, or antimony. In such cases the assay should be fused with a mixture of two parts soda and one part borax. The zinc will be volatilized, and in the moment of coming in contact with the air, is oxidized and coats the coal. If a large amount of lead is present, this also oxidizes and forms a coat, but on moistening with cobalt solution and heating in the O. F., the lead coating is reduced by charcoal, and the zinc coat becomes green on cooling. In the presence of much tin and antimony it is almost impossible to detect small quantities of zinc B. B.

MINERALS CONSTITUTING METALLIC ORES OF SECONDARY IMPORTANCE

Metal	Ores	Color, Streak and Luster	H.	Sp. Gr.	Composition	Blowpipe Tests	Distinguishing Characteristics
Manganese (*Mn*)	Pyrolusite (oxide)	Color: Iron black. Streak: Iron black. Luster: Sub-metallic.	2-2.5	4.8	MnO_2. Mn 63.2.	B.B. infusible. [illegible]	[illegible]
	Psilomelane (oxide)	Color: Iron black. Streak: Brownish black. Luster: Sub-metallic.	5-6	4.2	Doubtful composition, essentially MnO_2, barium, potassium and water.	B.B. same as above; also in closed tube gives water. Reaction for barium and potassium (flame tests).	Botryoidal structure. Hardness, color and streak, and blowpipe reaction.
	Rhodonite (silicate)	Color: Pink to brown. Streak: White. Luster: Vitreous.	5.5-6.5	3.5	$MnSiO_3$. MnO 54.1, SiO_2 45.9.	B.B. blackens, fuses with slight intumescence (puffing or swelling) at 2.5, to the beads, reactions for manganese. Slightly acted upon by acids. Reaction for silica in salt of phosphorus bead.	Usually massive structure. Color, streak, hardness, and blowpipe reactions.

Manganese (Mn)	Rhodochrosite (carbonate)	Color: Pink to brown. Streak: White. Luster: Vitreous.	3.5—4.5	3.5	$MnCO_3$ MnO—61.4% CO_2—38.6%	B. B. changes color; decrepitates strongly; infusible; manganese reaction in beads; dissolves with effervescence in warm HCl.	Usually massive. Color, perfect cleavage, hardness, and blowpipe reactions.
Tin (Sn)	Cassiterite (oxide)	Color: Nearly white to black. Streak: White. Luster: Adamantine.	6—7	6.6	SnO_2 Sn—78.67% O—21.33%	B. B. unaltered; on charcoal with soda, reduced to metallic tin, and gives a white coating which reacts characteristically with cobalt solution. In beads, gives reactions for manganese and iron. Only slightly acted upon by acids.	High gravity, hardness, infusibility, and blowpipe characteristics. Structure granular; often reniform or botryoidal shapes, concentric in structure; also crystalline.
Mercury (Hg)	Cinnabar (sulphide)	Color: Bright to dark red. Streak: Scarlet. Luster: Adamantine.	2—2.5	9	HgS Hg—86.2% S—13.8%	B. B. on charcoal, wholly volatile if pure. In closed tube, black sublimate. In open tube, sulphurous fumes and metallic globules of mercury.	Usually massive. Color, streak, high gravity, blowpipe reactions.

MINERALS CONSTITUTING METALLIC ORES OF SECONDARY IMPORTANCE—*Continued*

Metal	Ores	Color, Streak and Luster	H	Sp. Gr.	Composition	Before the Blowpipe	Distinguishing Characteristics
Nickel (Ni) and Cobalt (Co)	Niccolite (arsenide)	Color: Copper-red. Streak: Brownish-red. Luster: Metallic.	5–5.5	7.5	$NiAs$. Ni 44, As 56.	B. B. [illegible] fuses and [illegible] a globule, which treated with borax affords reactions for nickel, cobalt and iron which occur as impurities. In the closed and open tubes characteristic reactions for arsenic, the assay becoming yellowish-green. Soluble in aqua regia.	Color and blowpipe reactions. Brownish-red streak.
	Garnierite (silicate)	Color: Apple-green. Streak: Greenish-white. Luster: Resinous to earthy.	2.5	2.8	Doubtful. A hydrated silicate of nickel and magnesium, of indefinite composition.	B. B. Infusible. In closed tube, blackens and gives off water. In the beads, reacts for nickel. Decomposed by HCl without gelatinizing.	Never crystallized; occurs in crusts. Resembles chrysocolla. Falls to pieces in water; may be polished under the nail. Color and blowpipe reactions.

Nickel (Ni) and Cobalt (Co)	Millerite (sulphide)	Color: Brass-yellow. Streak: Yellow, bright. Luster: Metallic.	3—3.5	4.6—5.6	NiS Ni—64.4 S—33.6	B. B. on charcoal, fuses to a globule. When roasted, gives reactions for nickel in the beads, and is magnetic. In open tube, sulphurous fumes. Soluble in nitric acid.	Usually in capillary, or hair-like, crystals radiating from a center. Color often bronze-yellow to iridescent; blowpipe reactions.
	Smaltite (arsenide)	Color: Tin-white. Streak: Grayish-black. Luster: Metallic.	5.5—6	6.4—7.2	$(CoNi)As_2$ Co—0 to 23.5% Essentially cobalt arsenide, graduating into nickel arsenide (chloanthite).	B. B. on charcoal, gives arsenical odor, fuses to a globule, which treated in the beads gives reactions for nickel, cobalt, and iron. In closed and open tubes, reacts for arsenic. Soluble in nitric acid, giving pink solution.	Color sometimes grayish or iridescent. Streak and blowpipe reactions.
	Cobaltite (sulphide and arsenide)	Color: Silver-white to reddish. Streak: Grayish-black. Luster: Metallic.	5.5	6.2	$CoS_2 + CoAs_2$ Co—35.5% As—45.2% S—19.3%	B. B. on charcoal, gives off sulphur and arsenic and fuses to a globule, which gives reactions in the beads for cobalt. Unaltered in closed tube; in open tube, gives reactions for sulphur and arsenic. Soluble in warm nitric acid, with separation of arsenous oxide and sulphur.	Reddish color or tarnish, and blowpipe reactions.

MINERALS CONSTITUTING METALLIC ORES OF SECONDARY IMPORTANCE—*Continued.*

Metal.	Ores.	Color, Streak, and Luster.	H.	Sp. Gr.	Composition.	Before the Blowpipe.	Distinguishing Characteristics.
Antimony (Sb)	Stibnite (sulphide)	Color: Lead-gray to blackish-gray. Streak: Same as color. Luster: Metallic, shining.	2	4.6	Sb_2S_3 Sb—71.8% S—28.2%	B. B. on charcoal, fuses at 1, spreads out and gives sulphurous and antimonous fumes, coating the coal white; this coating, treated in the R. F., tinges the flame greenish-blue. In open tube, gives sulphurous and antimonous fumes, the latter condensing in a non-volatile white sublimate. When pure, perfectly soluble in HCl.	Perfect cleavage and often needle-like or bladed crystals of a steel-gray color. Extreme fusibility and blowpipe reactions.
Aluminum (Al)	Bauxite (hydrous oxide)	Color: White to brown. Streak: Same as color. Luster: Sub-metallic to earthy.	5—5.5	3.6—4	Doubtful: Probably (when pure) Al_2O_3+2Aq.	B. B. acts like limonite. Bauxite is really a limonite in which most or all of the iron has been replaced by aluminum. Gives characteristic reactions for aluminum, iron, silica, and water.	Blowpipe reactions.

Aluminum (Al)	Cryolite (fluoride)	Color: White to brownish-red. Streak: White. Luster: Vitreous and greasy.	2.5	2.9	$Al_2F_6 6 NaF$ or Na_3AlF_6 Al—13.0% Na—32.8% F—54.2%	B.B. in the open tube, heated so that the flame enters the tube, gives off hydrofluoric acid, which etches the glass; the water which condenses in the upper part of the tube reacts for fluorine with Brazil-wood paper. Fuses in the flame of a candle. Reacts for sodium in a colorless flame. On charcoal, fuses to a clear bead, which on cooling becomes opaque; after long blowing the assay spreads out, the sodium fluoride is absorbed by the coal, a suffocating odor of fluorine is given off, and a crust of alumina remains which reacts for alumina with cobalt solution. Soluble in sulphuric acid, with evolution of hydrofluoric acid fumes.	Extreme fusibility and its yielding hydrofluoric acid in the open tube.

MINERALS CONSTITUTING METALLIC ORES OF SECONDARY IMPORTANCE—*Concluded.*

Metal.	Ores.	Color, Streak, and Luster.	H.	Sp. Gr.	Composition.	Before the Blowpipe.	Distinguishing Characteristics.
Aluminum (Al)	Corundum (oxide)	Color: Red, blue, green, yellow; colorless when pure. Streak: White or colorless. Luster: Adamantine to vitreous.	9	4	Al_2O_3 Al—53% O—47% (Pure alumina)	B. B. unaltered; dissolves slowly in borax and salt of phosphorus to a clear glass, which is colorless when free from iron. Not acted upon by soda. The finely pulverized mineral when heated with cobalt solution gives a beautiful blue color. Friction excites electricity, and in polished specimens the electrical attraction continues for a considerable length of time.	Hardness, it scratches quartz and topaz, its infusibility and high gravity, the peculiar barrel shape of well-developed crystals. The varieties of corundum are named according to their color, sapphire (blue), oriental ruby (red), oriental topaz (yellow); oriental emerald (green), oriental amethyst (purple); emery (black).

MINERALS SENT WITH THE BLOWPIPE OUTFIT. (Subject to change.)

Name.	Color, Streak, and Luster.	H.	Sp. Gr.	Composition.	Before the Blowpipe.	Distinguishing Characteristics.
Arsenopyrite	Color: Silver-white, inclining to steel-gray. Streak: Dark grayish-black. Luster: Metallic.	5.5—6	6.0—6.4	*FeAsS* (Iron-arsenic sulphide)	On charcoal, gives odor of arsenic. Varieties containing cobalt give blue borax bead in O. F.	Reactions B. B.
Asbestos*	Color: White to green and wood-brown. Silky luster.			Essentially a silicate or magnesium, or of calcium and magnesium.	Infusible. Some varieties give off water in closed tube.	Usually in fibers; infusibility.
Barite*	Color: White, inclining to yellow, gray, blue, red, brown. Streak: White. Luster: Vitreous, resinous, pearly.	2.5—3.5	4.3—4.72	$BaSO_4$ (Barium sulphate)	Decrepitates and fuses at 3, fused mass turning red litmus paper blue. On charcoal, reduces to a sulphide, and with soda, gives sulphur reaction. Insoluble in acids.	High specific gravity; crystals usually tabular; perfect basal cleavage; insolubility; green coloration of blowpipe flame.

* Of commercial importance.

MINERALS SENT WITH THE BLOWPIPE OUTFIT—*Continued.*

Name.	Color, Streak, and Luster.	H.	Sp. Gr.	Composition.	Before the Blowpipe.	Distinguishing Characteristics.
Celestite	Color: White to faint bluish; sometimes reddish. Streak: White. Luster: Vitreous, inclining to pearly.	3—3.5	3.9—3.97	$SrSO_4$ (Strontium sulphate)	Frequently decrepitates. Fuses at 3, fused mass being alkaline. Colors the flame strontia-red. Insoluble in acids.	From carbonate by its failure to effervesce with acids; from barite by its lower specific gravity; by color imparted to flame.
Dolomite	Color: White, reddish or greenish white; also rose-red, green, brown, gray, and black. Luster: Vitreous, inclining to pearly.	3.5—4	2.8—2.9	$(CaMg)CO_3$ (Calcium-magnesium carbonate)	(See Calcite.) Like calcite. Attacked but little by cold acids. Powder dissolves with effervescence in warm acids.	Failure to effervesce in cold acid when in lumps; rhombohedral form and cleavage; curved crystal and cleavage faces.
Graphite*	**Color**: Iron-black to dark steel-gray. **Streak**: Black and shining. **Luster**: Metallic, greasy.	1—2	2.1—2.2	Pure carbon, with some iron sesquioxide mixed mechanically at times.	Volatilizes at extreme temperatures without flame or smoke. Infusible and unaltered by acids.	Color and streak; infusibility; from molybdenite by giving no sulphur reaction.

Halite* (Rock salt)	Color: Colorless or white; sometimes yellowish, reddish, bluish, or purplish. Streak: White. Luster: Vitreous.	2.5	2.1—2.25	$NaCl$ (Sodium chloride)	Fuses in closed tube, often with decrepitation. Fused on platinum foil, colors flame deep yellow.	Taste, solubility, and perfect cleavage into cubes.
Kaolinite*	Color: White, grayish-white, yellowish, reddish, brownish, bluish. Luster of plates: pearly; of mass: pearly to dull earthy.	1—2.5	2.4—2.63	$Al_2Si_2O_7 + 2Aq.$ (Hydrous aluminum silicate)	Yields water. Infusible, and insoluble in acids. With cobalt solution, gives blue color.	Soapy feel, and alumina reaction B. B.

* Of commercial importance.

MINERALS SENT WITH THE BLOWPIPE OUTFIT—*Continued.*

Name.	Color, Streak, and Luster.	H.	Sp. Gr.	Composition.	Before the Blowpipe.	Distinguishing Characteristics.
Mica*	Color: White, colorless, yellowish, brownish, green, violet, rose-red, and black. Streak: Colorless; grayish-green. Luster: Adamantine, vitreous, pearly, submetallic.	2—3.1	2.7—3.1	Silicates of alumina, iron, magnesia, soda, potash, and other substances; often contains fluorine.	Some varieties give water in closed tube. Fusibility varies with the composition from 1 in one variety to difficultly fusible and almost infusible (fusible only on thin edges) in the commercial mica. Most varieties are decomposed by acids.	Cleavage into thin plates; luster.
Natrolite	Color: White, colorless, gray, yellow to red. Streak: Uncolored. Luster: Vitreous to pearly.	5—5.5	2.17—2.25	$Na_2Al_2Si_3O_{10}+2Aq.$ (Hydrous sodium-aluminum silicate)	Loses water in closed tube, becoming white and opaque. Fuses at 2 in flame of candle to colorless glass. Becomes jelly-like with acids.	Reactions. B. B.
Actinolite	Color: Bright green, grayish-green. Streak: Uncolored. Luster: Vitreous to silky.	5—6	3—3.2	$(CaMgFe)SiO_3$ (Silicate of calcium, magnesium, and iron)	Fuses at 4 to a black or green glass.	Color; columnar structure, fibrous, and frequently radiating; luster.

Orthoclase	Color: White, gray, and flesh-red; sometimes greenish to bright green. Streak: Uncolored. Luster: Vitreous to pearly.	6—6.5	2.4—2.6	$K_2Al_2Si_6O_{16}$ (Potassium aluminum silicate)	Fuses at 5 and is unaltered by acids.	Crystal form (see Fig. 7); basal and pinacoidal cleavages at right angles; luster.
Serpentine*	Color: Various shades of green; also brownish-red, brownish-yellow, sometimes nearly white. Streak: White, slightly shining. Luster: Resinous, greasy, pearly, earthy.	2.5—4	2.5—2.65	$Mg_3Si_2O_7 + 2Aq.$ (Hydrous magnesium silicate)	Yields water in closed tube. Difficultly fusible on edges. Decomposed by HCl and H_2SO_4.	Softness (can be cut with a knife), low specific gravity, and resinous luster.

* Of commercial importance.

MINERALS SENT WITH THE BLOWPIPE OUTFIT—*Continued.*

Name.	Color, Streak, and Luster.	H.	Sp. Gr.	Composition.	Before the Blowpipe.	Distinguishing Characteristics.
Talc*	Color: White, various shades of green; sometimes reddish. Streak: White or lighter than color. Luster: Pearly.	1—1.5	2.56—2.8	$H_2Mg_3Si_4O_{12}$ (Hydrous magnesium silicate)	Most varieties yield water in closed tube at high heat. Difficultly fusible on thin edges to a white enamel. Gives pale red color when moistened with cobalt solution and heated. Not decomposed by acids.	Extreme softness; greasy feel; foliated structure; inelastic, though flexible; yielding water only on intense ignition.
Gypsum*	Color: Usually white; gray, reddish, yellow, blue, black. Streak: White. Luster: Pearly to earthy.	1.5—2	2.3	$CaSO_4 + 2Aq.$ (Hydrous calcium sulphate)	Gives off water and becomes opaque in closed tube. Soluble in hydrochloric acid and in 400 to 500 parts of water. When water is driven off gypsum becomes plaster of Paris.	Softness; does not effervesce; does not become jelly-like with acids.
Calcite*	Color: White, colorless, gray, red, green, blue, violet, yellow, brown, and black. Streak: White or grayish. Luster: Vitreous to earthy.	3	2.5—2.8	$CaCO_3$ (Calcium carbonate)	Infusible when pure. Moistened with HCl, colors the flame red (lime). Fragments dissolve with brisk effervescence in cold acids.	Crystal form (see Art. **34**); perfect rhombohedral cleavage; so soft as to be scratched with a knife; effervescence in cold dilute acid; infusibility.

Fluorite*	Color: White, yellow, green, rose, crimson-red, violet, sky-blue, brown. Streak: White. Luster: Vitreous, splendent, glimmering.	4	3—3.25	CaF_2 (Calcium fluoride)	Decrepitates and phosphoresces in closed tube. B. B. fuses at 3 and reacts alkaline. Gives fluorine reactions.	Perfect octahedral cleavage. Etching power of hydrofluoric acid.
Apatite*	Color: Sea-green, bluish-green, violet-blue, white, yellow, gray, red, and brown. Streak: White. Luster: Vitreous.	5	2.9—3.2	$3Ca_3P_2O_8 + CaCl_2$ or $3Ca_3P_2O_8 + CaF_2$ (Essentially calcium phosphate with chloride or fluoride of calcium, or both.)	Difficultly fusible on edges. Gives pale, bluish-green flame when moistened with H_2SO_4 and heated. Dissolves in HCl and HNO_3. Some varieties phosphoresce when heated.	Hexagonal form (usually prismatic). Does not effervesce with acids. Distinguished from beryl by its softness; from pyromorphite by its giving no lead reaction.

* Of commercial importance.

MINERALS SENT WITH THE BLOWPIPE OUTFIT (*Concluded*)

Name.	Color, Streak, and Luster	H	Sp. Gr.	Composition	Blowpipe Reactions	Distinctive Features
Albite	Color: White, bluish, gray, reddish, green. Streak: Uncolored. Luster: Pearly, vitreous.	6–7	2.6	$NaAlSi_3O_8$ (Sodium aluminum silicate)	Fuses at 4 to a white glass, coloring the flame intensely yellow. Not acted upon by acid.	Luster, color, [illegible] H. B.
Quartz	Color: Colorless, yellow, brown, red, green, blue, black. Streak: White, or paler than color. Luster: Vitreous, resinous to dull.	7	2.5–2.8	SiO_2 (Oxide of silicon, silica)	Unaltered, with borax dissolves to clear glass, with soda dissolves with effervescence; with salt of phosphorus unaltered. Insoluble in acids.	Hexagonal crystals and massive; no cleavage; hardness, infusibility, insolubility. Reaction with soda.

PRECIOUS STONES.

Name.	Color, Streak, and Luster.	H.	Sp. Gr.	Composition.	Before the Blowpipe.	Distinguishing Characteristics.
Diamond	Color: Colorless; occasionally yellow, red, green, blue, and even black. Streak: Colorless. Luster: Adamantine (brilliant).	10	3.5	Pure carbon.	Burns at very high temperatures, and is wholly consumed, producing carbon dioxide, CO_2. It is not acted upon by acids or alkalies.	Extreme hardness, brilliant luster, etc.
Sapphire (Corundum)	Color: Generally blue; also red, yellow, green, and purple, as described under corundum. Streak: Colorless. Luster: Adamantine to vitreous.	9	3.9—4	Al_2O_3 (Oxide of aluminum = alumina)	Same reactions as for corundum. Infusible, and insoluble in acids.	Same as for corundum. From spinel by crystallizing in rhombohedral-hexagonal system, usually in barrel-shaped, hexagonal, prismatic crystals, with perfect cleavage, sometimes basal and sometimes rhombohedral.

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Ruby (Spinel)	Color: Clear red, blue-green blue, yellow, brown, and black. Streak: Colorless. Luster: Vitreous to splendent.	8	3.5–4	[illegible]	[illegible]	[illegible]
Topaz	Color: Colorless to yellow. Streak: Colorless. Luster: Vitreous.	8	3.5	Al_2SiO_5 (Aluminum silicate)	B.B. infusible. Some varieties take a wine yellow or pink tinge when heated. Part of the oxygen is replaced by fluorine; when fused in the open tube with salt of phosphorus, gives reaction for fluorine. With cobalt solution, when heated, gives reaction for alumina. Reacts for silica in salt of phosphorus bead. Insoluble in acids.	Hardness, notably quartz. Infusibility; cracks at high heat; B.B. yields fluorine. Orthorhombic prisms with perfect basal cleavage; crystals are usually differently modified on the top and bottom.

Emerald (Aquamarine; Beryl)	Color: Green; also pale-blue and yellow. Streak: Colorless. Luster: Vitreous to resinous.	7.5—8	2.65	$Be_3Al_2Si_6O_{18}$ (Beryllium-aluminum silicate)	B. B. alone, unchanged or becomes clouded; at a high temperature the edges are rounded (fusibility = 5.5). Reaction for chromium in borax bead, due to the presence of chromium as an impurity which imparts to the mineral its green color. Slowly soluble in salt of phosphorus, without leaving a silicious residue. Unacted upon by acids.	Distinguished from apatite by its hardness, not being scratched with a knife; from topaz by its imperfect basal cleavage and its fusibility. It never occurs massive; crystallizes in hexagonal crystals. Distinguished from oriental emerald by crystal form, imperfect cleavage, inferior hardness, and fusibility.

[illegible] *Continued*

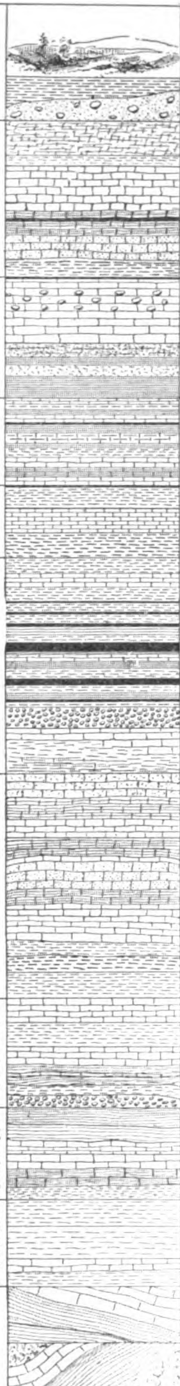
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Zircon	Color: Colorless, brown, and brown, pink or green. Streak: Colorless. Luster: Adamantine.	7.5	4.7	Zirconium silicate	[illegible] ... with melting the platinum wire, and if the powder is moistened in dilute HCl it gives the orange color characteristic of zirconia when tested with turmeric paper. In powder, slightly acted upon by sulphuric acid. Decomposed by fusion with the alkaline carbonates and bisulphates.	Adamantine luster, [illegible] and [illegible]. [illegible] in square [illegible] tetragonal system; also granular.

Turquoise	Color: Sky-blue, bluish-green, apple-green. Streak: White or greenish. Luster: Somewhat waxy.	6	2.6—2.8	$Al_4P_2O_{11} + 5Aq$. or $2Al_2O_3 + P_2O_5 + 5Aq$. (Hydrous aluminum phosphate)	B. B. in the forceps, becomes brown and assumes a glassy appearance, but does not fuse. Colors the flame green; moistened with hydrochloric acid, the color is azure-blue, which is due to some copper which exists as an impurity. In the beads, reacts for copper. With salt of phosphorus and tin on charcoal, gives an opaque red bead (copper). In the closed tube, decrepitates, yields water, and turns brown or black; by adding a small fragment of metallic magnesium or sodium to the assay, phosphoreted hydrogen is given off, recognizable by its disagreeable odor. Soluble in hydrochloric acid.	Color and luster; absence of cleavage; blowpipe reactions; infusibility; and solubility in *HCl*.

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Garnet	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Opal	Color: White, yellow, red, brown, green, gray, generally pale; frequently an iridescent play of colors. Streak: White. Luster: Vitreous to resinous.	5.5	6.5	1.9	2.3	$SiO_2 + nH_2O$ (usually) (Oxide of silicon, amorphous silica)	B. B. Infusible, but becomes opaque. Some yellow varieties containing iron turn red. In closed tube, yields water.	Is not crystallized. Has beautiful opalescent appearance and color. Amorphous variety of quartz, having a somewhat inferior hardness and gravity.

HISTORICAL GEOLOGICAL CHART.



Ages	Eras	American Periods	Foreign Equivalents	
Cenozoic	Quaternary	Recent Champlain Glacial	Recent Pleistocene	Age of Man
	Tertiary	Pliocene	Pliocene	Age of Mammals
		Miocene	Miocene	
		Eocene	Eocene	
Mesozoic	Cretaceous	(Laramie Series) Upper Cretaceous	Upper Cretaceous	Age of Reptiles
		Lower Cretaceous (Dakota Group) (Comanche Group)	Lower Cretaceous or Neocomian	
	Jurassic	Alantosaurus Beds.	Oölite. Lias.	
	Triassic	Connecticut River Beds	Keuper & Rhætic Muschelkalk Bunter Sandstein	
Paleozoic	Carboniferous	Permian	Permian	Age of Amphibians or Age of Acrogens (plants of the coal period)
		Carboniferous or Coal Measures.	Carboniferous or Coal Measures	
		Subcarboniferous	Mountain Limestone	
	Devonian	Catskill. Chemung.	Old Red Sandstone	Age of Fishes
		Hamilton		
		Corniferous		
		Oriskany		
	Upper Silurian	Helderberg	Ludlow	
		Onondaga. Salina		
		Niagara	Wenlock Llandovery	
	Lower Silurian	Trenton	Bala or Caradoc Llandeilo Flags	Age of Invertebrates
		Canadian	Arenig	
	Cambrian or Primordial	Potsdam	Tremadoc Slates Lingula Flags	
		Acadian	Menevian Solva	
		Georgian	Caerfai	
Archæan or Eozoic		Huronian	Archæan.	No Distinct Organic Remains
		Laurentian		

GANGUE MATERIALS.

[illegible]	Basic.
[illegible]	Metallic oxides and carbonates, [illegible] iron, calcium [illegible] magnesium, and manganese.
[illegible]	[illegible] or fluor-spar (calcium fluoride).
[illegible]	Barite or heavy-spar (barium sulphate).
[illegible]	Generally speaking, all of the metallic elements and their common salts, with the exception of silicates, act as bases.

FLUXES.

LEAD FLUXES.

[illegible] calculated for the fire [illegible] all good general fluxes, however [illegible] as the basis of a gold-silver [illegible] charges:

[illegible]	4 parts.*
[illegible]	4 parts.
[illegible]	2 parts.
[illegible]	1 part.
[illegible]	13 parts.
[illegible]	11 parts.
[illegible]	5 parts.
[illegible]	[illegible] 4 parts.

[illegible] the proportion of flour may [illegible] the flour may be omitted [illegible] should be added to [illegible] salt or borax cover.

* [illegible] the proportions of the con- [illegible]

GOLD AND SILVER CRUCIBLE FLUXES.

III.	Sodium bicarbonate	5 parts.
	Potassium carbonate	4 parts.
	Borax	2 parts.
	Flour	1 part.
	Litharge	8 parts.
IV.	Sodium bicarbonate	1 part.
	Borax glass	1 part.
	Litharge	5 parts.
	Ore	1 part.
V.	Sodium bicarbonate	3 parts.
	Litharge	5 parts.
	Borax	2 parts.
	Reducer or oxidizer, as in IV.	Salt cover.

TABLES USED IN ASSAYING.

PROOF ASSAY CHARGES.

Weight in mg. of Silver from Preliminary Assay of 500 mg. Bullion.	Weight of Silver in mg. To Be Used in Proof.	Weight in Grams of c. p. Lead Foil To Be Used in Proof and in Regular Assay.	Per Cent. of Copper in Bullion as Determined by Analysis.	Weight in mg. of c.p. Copper Foil To Be Used in Proof.
475	480	5	2.5	12.5
450	455–460	7	5.0	25.0
425	430–435	8	10.0	50.0
400	405–410	10	15.0	75.0
375	380–385	11	20.0	100.0
350	355–360	12	25.0	125.0
325	330–335	13	30.0	150.0
300	305–310	15	35.0	175.0
250	255–260	17	40.0	200.0
200	205–210	19	45.0	225.0
150	155–160	20	50.0	250.0

[illegible]

Ore	Description									Cover	Remarks
[illegible]	[illegible]	[illegible]								[illegible]	[illegible]
[illegible]	[illegible]	[illegible]								[illegible]	Special method. [illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]					[illegible]	
Oxidized	[illegible] lead	½	[illegible]		[illegible]		15			Borax	If pyrite is present or carbonate of iron, add 2 or 3 g argol.
Oxidized	Basic with barite ($BaSO_4$)	½	40	20	5		15	1		Borax	Borax glass may be substituted for part of the silica.
Galena.	Lead, 84%. (Concentrates.)	½	20			10				Salt.	Heat gradually until mass subsides.

Galena.	Silicious.	½	15	20	20		5					Salt.	Litharge is added, according to the lead contents of the ore.
Lead carbonate.	Neutral.	½	30	10	15							Borax.	Litharge is added, according to the lead contents of the ore.
Iron pyrites.	None. (Concentrates.)	½		35	20		5	15		3		Borax.	Collect matte, if any forms, and scorify with lead button.
Copper pyrites.	Iron pyrites. (Concentrates.)	½		35	30		5	15		3		Borax.	Collect matte, if any forms, and scorify with lead button.
Tellurides.	Silicious.	½	30	30	40–80							Salt.	If button is hard or brittle, scorify with lead. Scorifier assay preferable.
Tellurides.	Silicious.	½			80				2			Salt.	Special method.
Arsenical.		½		15	30	17						Salt.	Scorify button. Scorifier assay preferable.
Slags.		1	20	40	10						10	Salt.	If slag contains matte, add a nail.

SCORIFIER CHARGES.

Ore.	Grams of Test Lead.	Grams of Borax Glass.	Remarks.
Galena	15–18	Up to 0.5	
Galena, with zinc-blende and pyrite	20–35	0.4–0.8	
Iron pyrites	30–45	0.3–0.8	
Arsenical pyrites	45–50	0.3–1.5	High temperature. Addition of litharge helps assay.
Gray copper	35–48	0.3–0.5	Low temperature.
Zinc-blende	30–45	0.3–0.6	High temperature. Addition of oxide of iron helps assay.
Copper ores and mattes	35–40	0.3–0.5	Low temperature. If necessary, the button should be rescorified with lead.
Tellurides	50	0.3	Add cover of litharge, and rescorify the button.
Silicious	25–30		
Basic	25–30	0.5–2.0	If the ore contains much lime or magnesia, the addition of sodium carbonate helps the assay.
Basic with barium sulphate	25–30	0.5–1.5	Addition of sodium carbonate helps assay.
Lead carbonate	10–15	Up to 0.5	

POWER OF REDUCING AGENTS.

APPROXIMATE REDUCING POWER (IN TERMS OF PARTS OF METALLIC LEAD REDUCED FROM LITHARGE BY 1 PART OF THE REDUCER).

1 part of *charcoal*	will reduce	22 to 32	parts of lead.
1 part of *hard coal*	will reduce	25	parts of lead.
1 part of *coke*	will reduce	24	parts of lead.
1 part of *soft coal*	will reduce	22	parts of lead.
1 part of *wheat flour*	will reduce	15	parts of lead.
1 part of *white sugar*	will reduce	14½	parts of lead.
1 part of *starch*	will reduce	11½ to 13	parts of lead.
1 part of *gum arabic*	will reduce	11	parts of lead.
1 part of *crude argol*	will reduce	5½ to 8½	parts of lead.
1 part of *cream of tartar*	will reduce	4½ to 6½	parts of lead.

PREPARATION OF REAGENTS.

The following table of proportions for the preparation of reagents may be found useful. The concentrated acids have not been included in the table on account of the fact that they are used as received from the supply houses.

Dilute hydrochloric acid (*HCl*).	One portion of *HCl* to 8 portions of water by volume.
Dilute nitric acid (NHO_3).	One portion of concentrated HNO_3 to 8 portions of water by volume.
Nitro-hydrochloricacid (aqua regia).	One volume of concentrated nitric acid added to 3 volumes of hydrochloric acid forms aqua regia, which should be prepared only as required. It may be used either concentrated or dilute.
Dilute sulphuric acid (H_2SO_4).	One portion of concentrated H_2SO_4 to 4 portions of water by volume. Note.—Always pour the concentrated acid into the water, and never water into the concentrated acid. The union of sulphuric acid and water produces heat, and if water were poured into the acid, an explosion might result. Note.—Some chemists use 1 portion of concentrated acid to 4 portions of water for all the dilute reagents.

Dilute acetic acid ($HC_2H_3O_2$).	One portion of 33% acid to 1 portion of water by volume, or 1 portion of glacial to 4 of water.
Oxalic acid ($H_2C_2O_4$).	One gram of the crystals to 10 c. c. of water, which makes a practically saturated solution.
Tartaric acid ($H_2C_4H_4O_6$).	One gram of crystals to 3 c. c. of water.
Hydric-sulphide or sulphureted hydrogen (H_2S).	This is formed by treating iron sulphide (FeS) with sulphuric acid. If iron sulphide can not be obtained, it may be prepared by fusing iron nails with sulphur, in the proportion of about 1 part by weight of iron to 2 parts by weight of sulphur. H_2S gas may be led into water until the water is saturated and the saturated water used as a reagent. The water should be kept in colored-glass bottles, as it is quickly decomposed when exposed to the light. When it is desired to precipitate any substance from the solution by means of H_2S, it will be better to conduct the gas itself into the solution than to employ water charged with the gas, on account of the fact that in order to add a sufficient amount of gas, it would be necessary to add a very large amount of water, thus unnecessarily increasing the bulk of the solution.
Chlorine or chlorine water (Cl).	Chlorine may be generated by treating bleaching powder (chloride of lime, $CaOCl_2$) with sulphuric acid, and the gas may be absorbed in water, but chlorine water must be kept in a colored-glass bottle or in the dark, for in the light the chlorine will decompose water and form hydrochloric acid (HCl). Note.—Chlorine gas may also be prepared by mixing 50 grams of coarse salt and 40 grams of powdered black oxide of manganese, and adding to it when cold a mixture of 125 grams of concentrated sulphuric acid and 60 grams of water; shake well together and warm, gently collecting the gas as it comes over in water contained in a black-glass bottle.

Ammonium chloride (NH_4Cl).	One gram of the crystallized salt to 8 c. c. of water.
Ammonium carbonate ($(NH_4)_2CO_3$).	The ordinary commercial carbonate (known as sesqui-carbonate) produces in solution a mixture of the neutral and acid carbonates. This is objectionable when the neutral carbonate is to be used, and hence the reagent should be made up as follows: Dissolve the crystallized sesqui-carbonate in the proportion of 1 gram of the sesqui-carbonate to 4 c. c. of water and then add 1 c. c. of concentrated ammonium hydrate (NH_4OH).
Ammonium oxalate ($(NH_4)_2C_2O_4$.	One gram of the crystallized salt to 20 c. c. of water.
Plumbic or lead acetate $Pb(C_2H_3O_2)_2$.	One gram of salt to 10 c. c. of water.
Potassium chromate (K_2CrO_4).	One gram of salt to 10 c. c. of water.
Potassium cyanide (KCN).	One gram of salt to 4 c. c. of water. Note.—Great care should be taken in handling potassium cyanide, as it is extremely poisonous.
Potassium hydrate (KOH).	One gram of salt to 10 c. c. of water.
Potassium iodide (KI).	One gram of the salt to 25 c. c. of water.
Potassium ferricyanide $K_3Fe(CN)_6$.	One gram of the salt to 10 c. c. of water.
Potassium sulphocyanate ($KCNS$).	Dissolve 1 gram of the salt to 10 c. c. of water.
Potassium ferrocyanide $K_4Fe(CN)_6$.	One gram of the salt to 10 c. c. of water.
Sodium carbonate (Na_2CO_3).	When dry sodium carbonate is employed, 1 gram of the material to 5 c. c. of water makes a practically concentrated solution, while if the crystals are employed, it would require 2.7 grams to 5 c. c. of water. This is on account of the fact that the crystals contain water of crystallization.
Sodium hydrate ($NaOH$).	One gram of salt to 10 c. c. of water.

Ammonium sulphide $(NH_4)_2S$.	Conduct hydrogen sulphide gas (H_2S) into a bottle two-thirds full of concentrated ammonia hydrate (NH_4OH) until it is saturated, which is indicated by the bubbles coming from the liquid undiminished in size. Fill the bottle with concentrated ammonia and mix it thoroughly. This stock solution should be kept in full, tightly-stoppered bottles, and the bottles should be colored, as light decomposes the ammonia sulphide. Before using, the stock solution should be diluted with twice its volume of water, and this diluted solution should be kept in the ordinary colored-glass reagent bottle.
Yellow ammonium sulphide $(NH_4)_2S_2$.	This is made by adding a small quantity of flower of sulphur to common ammonia sulphide and shaking until the sulphur is dissolved. Enough sulphur should be added to give the solution an amber color.
Barium chloride $(BaCl_2)$.	One gram of the crystallized salt to 10 c. c. of water.
Barium carbonate $(BaCO_3)$.	Barium carbonate may be prepared by precipitating a pure barium chloride solution with ammonium carbonate; then wash on the filter until all the ammonia salts have been removed. The wet precipitate should be stirred into the water so as to form a thin cream or emulsion. It should be thoroughly mixed before using.
Barium hydrate $Ba(OH)_2$.	Barium hydrate may be prepared by dissolving salt in the proportion of 1 gram of salt to 10 c. c. of water. This should be digested or heated for several hours and then the pure liquid filtered off and kept in a well-stoppered bottle.
Bromine water $(Br+H_2O)$.	Bromine water may be formed by making a saturated solution of bromine in distilled water. It should be kept in a tightly-stoppered colored-glass bottle and in a cool place. When opening the bromine water in warm weather, care should be taken, for there is liable to be a sudden rush of vapor upon withdrawing the stopper, and this vapor is not only disagreeable, but somewhat poisonous.

Calcium hydrate or lime-water $Ca(OH)_2$.	This may be prepared by slacking fresh quicklime and adding a large quantity of water placed in a large glass bottle, and shake well several times; then allow to settle. The clear solution can be decanted off and used as a reagent. It contains 1 part of lime and several hundred parts of water.
Sodium acetate ($NaC_2H_3O_2$).	One gram of salt to 10 c. c. of water.
Argenic or silver nitrate ($AgNO_3$).	One gram of salt to 25 c. c. of water.
Stannous chloride ($SnCl_2$).	One gram of the salt to 3 c. c. of HCl and 8 c. c. of water. Metallic tin should be kept in solution and should be kept from the air, to prevent the formation of oxides.

WEIGHTS AND MEASURES.

(English and Metric Systems.)

AVOIRDUPOIS WEIGHT.

16 drams (*dr.*)	=	1 ounce	*oz.*	=	28.3495 g.
16 ounces	=	1 pound	*lb.*	=	453.5920 g.
100 pounds	=	1 hundredweight	*cwt.*	=	45.359 Kg.
20 cwt., or 2,000 lb.	=	1 ton	*T.*	=	907.184 Kg.

TROY WEIGHT.

24 grains (*gr.*)	=	1 pennyweight	*pwt.*	=	1.5552 g.
20 pennyweights	=	1 ounce	*oz.*	=	31.1035 g.
12 ounces	=	1 pound	*lb.*	=	373.2419 g.

MEASURES OF LENGTH (*Metric*).

The **meter** is the *unit of length*, and is equal to 39.37 inches, nearly.

10 millimeters (*mm.*)	=	1 centimeter *cm.*	=	0.3937 in.
10 centimeters	=	1 decimeter *dm.*	=	3.937 in.
10 decimeters	=	1 meter *m.*	=	3.28 ft.
10 meters	=	1 dekameter *Dm.*	=	32.8 ft.
10 dekameters	=	1 hektometer *Hm.*	=	328.09 ft.
10 hektometers	=	1 kilometer *Km.*	=	0.62137 mi.
10 kilometers	=	1 myriameter *Mm.*	=	6.2137 mi.

MEASURES OF WEIGHT (*Metric*).

The **gram** is the *unit of weight*, and is equal to 15.432 grains, or the weight of a cube of pure distilled water at 4° C., the edge of which is *one one-hundredth* ($\frac{1}{100}$) of a meter.

10 milligrams (*mg.*)	=	1 centigram *cg.*	=	0.15 gr.
10 centigrams	=	1 decigram *dg.*	=	1.54 gr.
10 decigrams	=	1 gram *g.*	=	15.432 gr.
10 grams	=	1 dekagram *Dg.*	=	154.32 gr.
10 dekagrams	=	1 hektogram *Hg.*	=	3.53 oz., avoir.
10 hektograms	=	1 kilogram or kilo *Kg.* or *K.*	=	2.20 lb., avoir.
10 kilograms	=	1 myriagram *Mg.*	=	22.05 lb., avoir.

CUBIC MEASURE (*Metric*).

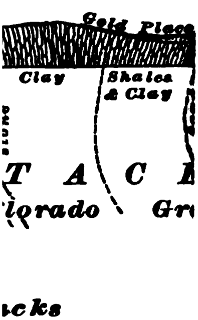
1,000 cubic centimeters (c. c. or *cm.*^{3}) = 1 cubic decimeter, or liter (*l.*)
1 liter of water at 4° C. weighs 2.2 lb., avoirdupois.
1,000 cubic decimeters = 1 cubic meter (*cu. m.*), or kiloliter (*Kl.*).
1 kiloliter of water at 4° C. weighs 22.04 cwt.

ASSAY-TON WEIGHTS.

Multiples: 4 assay tons = 116.66666 grams. 2 assay tons = 58.33333 grams.

Unit: The assay ton (*A. T.*) is equal to 29.16666 grams.

Subdivisions:
- $\frac{1}{3}$ assay ton = 9.7222 grams.
- $\frac{1}{6}$ assay ton = 4.8611 grams.
- $\frac{1}{10}$ assay ton = 2.9166 grams.
- $\frac{1}{20}$ assay ton = 1.4583 grams.


Clay
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FLOW OF WATER.

DISCHARGE OF WATER THROUGH A RIGHT-ANGLED V NOTCH.

h Head, Inches.	*Q* Quant. per Min., Cu. Ft.	*h* Head, Inches.	*Q* Quant. per Min., Cu. Ft.	*h* Head, Inches.	*Q* Quant. per Min., Cu. Ft.	*h* Head, Inches.	*Q* Quant. per Min., Cu. Ft.	*h* Head, Inches.	*Q* Quant. per Min., Cu. Ft.
1.05	0.3457	3.25	5.827	5.45	21.22	7.65	49.53	9.85	93.18
1.10	0.3884	3.30	6.054	5.50	21.71	7.70	50.34	9.90	94.37
1.15	0.4340	3.35	6.285	5.55	22.20	7.75	51.16	9.95	95.56
1.20	0.4827	3.40	6.523	5.60	22.70	7.80	51.99	10.00	96.77
1.25	0.5345	3.45	6.765	5.65	23.22	7.85	52.83	10.05	97.98
1.30	0.5896	3.50	7.012	5.70	23.74	7.90	53.67	10.10	99.20
1.35	0.6480	3.55	7.266	5.75	24.26	7.95	54.53	10.15	100.43
1.40	0.7096	3.60	7.524	5.80	24.79	8.00	55.39	10.20	101.67
1.45	0.7747	3.65	7.788	5.85	25.33	8.05	56.26	10.25	102.92
1.50	0.8432	3.70	8.058	5.90	25.87	8.10	57.14	10.30	104.18
1.55	0.9153	3.75	8.332	5.95	26.42	8.15	58.03	10.35	105.45
1.60	0.9909	3.80	8.613	6.00	26.98	8.20	58.92	10.40	106.73
1.65	1.0700	3.85	8.899	6.05	27.55	8.25	59.82	10.45	108.02
1.70	1.1530	3.90	9.191	6.10	28.12	8.30	60.73	10.50	109.31
1.75	1.2400	3.95	9.489	6.15	28.70	8.35	61.65	10.55	110.62
1.80	1.3300	4.00	9.792	6.20	29.28	8.40	62.58	10.60	111.94
1.85	1.4240	4.05	10.100	6.25	29.88	8.45	63.51	10.65	113.26
1.90	1.5220	4.10	10.410	6.30	30.48	8.50	64.45	10.70	114.60
1.95	1.6250	4.15	10.730	6.35	31.09	8.55	65.41	10.75	115.94
2.00	1.7310	4.20	11.060	6.40	31.71	8.60	66.37	10.80	117.29
2.05	1.8410	4.25	11.390	6.45	32.33	8.65	67.34	10.85	118.65
2.10	1.9550	4.30	11.730	6.50	32.96	8.70	68.32	10.90	120.02
2.15	2.0740	4.35	12.070	6.55	33.60	8.75	69.30	10.95	121.41
2.20	2.1960	4.40	12.420	6.60	34.24	8.80	70.30	11.00	122.81
2.25	2.3230	4.45	12.780	6.65	34.89	8.85	71.30	11.05	124.21
2.30	2.4550	4.50	13.140	6.70	35.56	8.90	72.31	11.10	125.61
2.35	2.5900	4.55	13.510	6.75	36.23	8.95	73.33	11.15	127.03
2.40	2.7300	4.60	13.890	6.80	36.89	9.00	74.36	11.20	128.45
2.45	2.8750	4.65	14.270	6.85	37.58	9.05	75.40	11.25	129.90
2.50	3.0240	4.70	14.650	6.90	38.27	9.10	76.44	11.30	131.35
2.55	3.1770	4.75	15.040	6.95	38.96	9.15	77.49	11.35	132.81
2.60	3.3350	4.80	15.440	7.00	39.67	9.20	78.55	11.40	134.27
2.65	3.4980	4.85	15.850	7.05	40.38	9.25	79.63	11.45	135.75
2.70	3.6660	4.90	16.260	7.10	41.10	9.30	80.71	11.50	137.23
2.75	3.8380	4.95	16.680	7.15	41.83	9.35	81.80	11.55	138.73
2.80	4.0140	5.00	17.110	7.20	42.56	9.40	82.90	11.60	140.23
2.85	4.1960	5.05	17.540	7.25	43.30	9.45	84.01	11.65	141.75
2.90	4.3820	5.10	17.970	7.30	44.06	9.50	85.12	11.70	143.28
2.95	4.5740	5.15	18.420	7.35	44.82	9.55	86.24	11.75	144.82
3.00	4.7700	5.20	18.870	7.40	45.58	9.60	87.37	11.80	146.36
3.05	4.9710	5.25	19.320	7.45	46.36	9.65	88.52	11.85	147.91
3.10	5.1780	5.30	19.790	7.50	47.14	9.70	89.67	11.90	149.48
3.15	5.3880	5.35	20.260	7.55	47.92	9.75	90.83	11.95	151.05
3.20	5.6050	5.40	20.730	7.60	48.72	9.80	92.00	12.00	152.64

1 cubic foot contains 7.48 U. S. gallons; 1 U. S. gallon weighs 8.34 pounds.

VALUES OF THE COEFFICIENT OF DISCHARGE FOR WEIRS WITH END CONTRACTIONS.

Effective Head in Feet.	Length of Weir in Feet.						
	0.66	1	2	3	5	10	19
0.10	0.632	0.639	0.646	0.652	0.653	0.655	0.656
0.15	0.619	0.625	0.634	0.638	0.640	0.641	0.642
0.20	0.611	0.618	0.626	0.630	0.631	0.633	0.634
0.25	0.605	0.612	0.621	0.624	0.626	0.628	0.629
0.30	0.601	0.608	0.616	0.619	0.621	0.624	0.625
0.40	0.595	0.601	0.609	0.613	0.615	0.618	0.620
0.50	0.590	0.596	0.605	0.608	0.611	0.615	0.617
0.60	0.587	0.593	0.601	0.605	0.608	0.613	0.615
0.70		0.590	0.598	0.603	0.606	0.612	0.614
0.80			0.595	0.600	0.604	0.611	0.613
0.90			0.592	0.598	0.603	0.609	0.612
1.00			0.590	0.595	0.601	0.608	0.611
1.20			0.585	0.591	0.597	0.605	0.610
1.40			0.580	0.587	0.594	0.602	0.609
1.60				0.582	0.591	0.600	0.607

VALUES OF THE COEFFICIENT OF DISCHARGE FOR WEIRS WITHOUT END CONTRACTIONS.

Effective Head in Feet.	Length of Weir in Feet.						
	19	10	7	5	4	3	2
0.10	0.657	0.658	0.658	0.659			
0.15	0.643	0.644	0.645	0.645	0.647	0.649	0.652
0.20	0.635	0.637	0.637	0.638	0.641	0.642	0.645
0.25	0.630	0.632	0.633	0.634	0.636	0.638	0.641
0.30	0.626	0.628	0.629	0.631	0.633	0.636	0.639
0.40	0.621	0.623	0.625	0.628	0.630	0.633	0.636
0.50	0.619	0.621	0.624	0.627	0.630	0.633	0.637
0.60	0.618	0.620	0.623	0.627	0.630	0.634	0.638
0.70	0.618	0.620	0.624	0.628	0.631	0.635	0.640
0.80	0.618	0.621	0.625	0.629	0.633	0.637	0.643
0.90	0.619	0.622	0.627	0.631	0.635	0.639	0.645
1.00	0.619	0.624	0.628	0.633	0.637	0.641	0.648
1.20	0.620	0.626	0.632	0.636	0.641	0.646	
1.40	0.622	0.629	0.634	0.640	0.644		
1.60	0.623	0.631	0.637	0.642	0.647		

WEIR TABLE GIVING CUBIC FEET DISCHARGED PER MINUTE FOR EACH INCH IN LENGTH OF WEIR FOR DEPTHS FROM 1-8 INCH TO 25 INCHES.

Inches.		⅛	¼	⅜	½	⅝	¾	⅞
		0.01	0.05	0.09	0.14	0.20	0.26	0.33
1	0.40	0.47	0.55	0.65	0.74	0.83	0.93	1.03
2	1.14	1.24	1.36	1.47	1.59	1.71	1.83	1.96
3	2.09	2.23	2.36	2.50	2.63	2.78	2.92	3.07
4	3.22	3.37	3.52	3.68	3.83	3.99	4.16	4.32
5	4.50	4.67	4.84	5.01	5.18	5.36	5.54	5.72
6	5.90	6.09	6.28	6.47	6.65	6.85	7.05	7.25
7	7.44	7.64	7.84	8.05	8.25	8.45	8.66	8.86
8	9.10	9.31	9.52	9.74	9.96	10.18	10.40	10.62
9	10.86	11.08	11.31	11.54	11.77	12.00	12.23	12.47
10	12.71	13.95	13.19	13.43	13.67	13.93	14.16	14.42
11	14.67	14.92	15.18	15.43	15.67	15.96	16.20	16.46
12	16.73	16.99	17.26	17.52	17.78	18.05	18.32	18.58
13	18.87	19.14	19.42	19.69	19.97	20.24	20.52	20.80
14	21.09	21.37	21.65	21.94	22.22	22.51	22.79	23.08
15	23.38	23.67	23.97	24.26	24.56	24.86	25.16	25.46
16	25.76	26.06	26.36	26.66	26.97	27.27	27.58	27.89
17	28.20	28.51	28.82	29.14	29.45	29.76	30.08	30.39
18	30.70	31.02	31.34	31.66	31.98	32.31	32.63	32.96
19	33.29	33.61	33.94	34.27	34.60	34.94	35.27	35.60
20	35.94	36.27	36.60	36.94	37.28	37.62	37.96	38.31
21	38.65	39.00	39.34	39.69	40.04	40.39	40.73	41.09
22	41.43	41.78	42.13	42.49	42.84	43.20	43.56	43.92
23	44.28	44.64	45.00	45.38	45.71	46.08	46.43	46.81
24	47.18	47.55	47.91	48.28	48.65	49.02	49.39	49.76

PROPERTIES OF COPPER WIRE.—AMERICAN OR BROWN & SHARPE GAUGE.

Number. B. & S Gauge.	Diameter in Mils.	Area in Circular Mils. $C.M. = d^2$.	Weights.		Resistance per 1,000 Ft. International Ohms. 75° F.	Current Capacity (Amperes) National Board Fire Underwriters.	
			Per 1,000 Ft.	Per Mile.		Open.	Concealed.
0000	460.0	211,600	641	3,382	.04966	312	218
000	409.6	167,805	509	2,687	.06251	262	181
00	364.8	133,079	403	2,129	.07887	220	150
0	324.8	105,534	320	1,688	.09948	185	125
1	289.3	83,694	253	1,335	.1258	156	105
2	257.6	66,373	202	1,064	.1579	131	88
3	229.4	52,634	159	838	.2004	110	75
4	204.3	41,742	126	665	.2525	92	63
5	181.9	33,102	100	529	.3172	77	53
6	162.0	26,250	79	419	.4104	65	45
7	144.2	20,816	63	331	.5067		
8	128.4	16,509	50	262	.6413	46	33
9	114.4	13,094	39	208	.8085		
10	101.8	10,381	32	166	1.010	32	25
11	90.7	8,234	25	132	1.269		
12	80.8	6,529	20	105	1.601	23	17
13	71.9	5,178	15.7	83	2.027		
14	64.0	4,106	12.4	65	2.565	16	12
15	57.0	3,256	9.8	52	3.234		
16	50.8	2,582	7.9	42	4.040	8	6
17	45.2	2,048	6.1	32	5.189		
18	40.3	1,624	4.8	25.6	6.567	5	3
19	35.8	1,288	3.9	20.7	8.108		
20	31.9	1,021	3.1	16.4	10.260		
21	28.5	810.1	2.5	13.0	12.940		
22	25.3	642.4	1.9	10.2	16.41		
23	22.6	509.4	1.5	8.2	20.57		
24	20.1	404.0	1.2	6.5	26.01		
25	17.9	320.4	.97	5.1	32.79		
26	15.9	254.1	.77	4.0	41 56		
27	14.2	201.5	.61	3.2	52.11		
28	12.6	159.8	.48	2.5	66.18		
29	11.3	126.7	.39	2.0	82.29		
30	10 0	100.5	.30	1.6	105.10		

CURRENT REQUIRED FOR INCANDESCENT LAMPS.

Volts.	Candle-power.	Amperes.
52	16	1.0
52	32	2.0
52	100	6.0
110	16	.5
110	32	1.0
110	100	3.0

CARRYING CAPACITY OF FUSES.

Diam. in Mils.	B. & S. Gauge (Approx.).	Amperes.
.017	25	3
.020	24	4
.032	20	7
.042	18–17	10
.056	15	15
.065	14	18
.075	13–12	25
.085	12–11	28
.096	11–10	31
.111	9	36
.130	8	50
.150	7–6	70

CARRYING CAPACITY OF CABLES.

Area Circular Mils.	Current Amperes.		Area Circular Mils.	Current Amperes.	
	Exposed.	Concealed.		Exposed.	Concealed.
200,000	299	200	1,200,000	1,147	715
300,000	405	272	1,300,000	1,217	756
400,000	503	336	1,400,000	1,287	796
500,000	595	393	1,500,000	1,356	835
600,000	682	445	1,600,000	1,423	873
700,000	765	494	1,700,000	1,489	910
800,000	846	541	1,800,000	1,554	946
900,000	924	586	1,900,000	1,618	981
1,000,000	1,000	630	2,000,000	1,681	1,015
1,100,000	1,075	673			

SPECIAL METHODS OF SHAFT SINKING.

Quicksand.	"Forepoling."
	"Metal Linings." (Forced down without the use of compressed air.)
	"Pneumatic" Method. (Limited to about 100 feet in depth.)
Quicksand; Rock (hard or soft, but very wet).	"Poetsch" Process. (Freezing Method.)
Rock (hard or soft, but very wet).	"Kind-Chaudron" Method.
Rock (hard or soft, but not very wet).	"Continuous" or "Long-Hole" Method.

APPROXIMATE MAXIMUM PRESSURES OF EXPLOSIVES.

Mercury fulminate	193 tons per sq. in.
Nitroglycerin	86 tons per sq. in.
Guncotton	71 tons per sq. in.
Blasting-powder	43 tons per sq. in.

RELATIVE VALUES OF EXPLOSIVES.

Gunpowder containing 61% saltpeter	1.0
Dynamite containing 75% nitroglycerin	2.2
Blasting gelatine containing 92% nitroglycerin.	3.2
Nitroglycerin	3.3

RULES AND FORMULAS.

RULES USED IN TRIGONOMETRY.

THE TRIGONOMETRIC FUNCTIONS.

Art. **754.**

Rule 1.—*Sine* $= \dfrac{\textit{side opposite}}{\textit{hypotenuse}}$.

Rule 2.—*Side opposite = hypotenuse × sine.*

Rule 3.—*Cosine* $= \dfrac{\textit{side adjacent}}{\textit{hypotenuse}}$.

Rule 4.—*Side adjacent = hypotenuse × cosine.*

Rule 5.—*Tangent* $= \dfrac{\textit{side opposite}}{\textit{side adjacent}}$.

Rule 6.—*Side opposite = side adjacent × tangent.*

Rule 7.—*Cotangent* $= \dfrac{\textit{side adjacent}}{\textit{side opposite}}$.

Rule 8.—*Side adjacent = cotangent × side opposite.*

Rule 9.—*Hypotenuse* $= \dfrac{\textit{side opposite}}{\textit{sine}}$.

Rule 10.—*Hypotenuse* $= \dfrac{\textit{side adjacent}}{\textit{cosine}}$.

RULES FOR USING TRIGONOMETRIC TABLES.

Given an angle, to find its sine, cosine, tangent, and cotangent:

Rule 11.—*Find in the table the sine, cosine, tangent, or cotangent corresponding to the degrees and minutes of the angle.*

For the seconds, find the difference of the values of the sine, cosine, tangent, or cotangent, taken from the table between which the seconds of the angle fall; multiply this difference by a fraction whose numerator is the number of seconds in the given angle, and whose denominator is 60.

If sine or tangent, add this correction to the value first found; if cosine or cotangent, subtract the correction. Art. **756.**

To find the angle corresponding to a given sine, cosine, tangent, or cotangent, whose exact value is not contained in the table:

Rule 12.—*Find the difference of the two numbers in the table between which the given sine, cosine, or tangent falls, and use the number of parts in this difference as the denominator of a fraction.*

Find the difference between the number belonging to the **smaller angle,** *and the given sine, cosine, tangent, or cotangent, and use the number of parts in the difference just found as the numerator of the fraction mentioned above. Multiply this fraction by 60, and the result will be the number of seconds to be added to the* **smaller angle.** Art. **758.**

RULES USED IN MENSURATION.

THE TRIANGLE.

Rule.—*The area of any triangle equals one-half the product of the base and the altitude.* Art. **766.**

THE QUADRILATERAL.

To find the area of a parallelogram:

Rule.—*The area of any parallelogram equals the product of the base and the altitude.* Art. **777.**

To find the area of a trapezoid:

Rule.—*The area of a trapezoid equals one-half the sum of the parallel sides multiplied by the altitude.* Art. **778.**

To find the area of an irregular figure bounded by straight lines:

Rule.—*Divide the figure into triangles, and find the area of each triangle separately. The sum of the areas of all the triangles will be the area of the figure.* Art. **779.**

THE CIRCLE.

To find the circumference or diameter of a circle:

Rule.—*The circumference of a circle equals the diameter multiplied by 3.1416.* Art. **780.**

Rule.—*The diameter of a circle equals the circumference divided by 3.1416.* Art. **780.**

To find the length of an arc of a circle :

Rule.—*The length of an arc of a circle equals the circumference of the circle of which the arc is a part, multiplied by the number of degrees in the arc, and divided by 360.* Art. **781.**

To find the area of a circle:

Rule.—*Square the diameter, and multiply by .7854.* Art. **782.**

Given the area of a circle, to find its diameter:

Rule.—*Divide the area by .7854 and extract the square root of the quotient.* Art. **783.**

To find the area of a sector:

Rule.—*Divide the number of degrees in the arc of a sector by 360. Multiply the result by the area of the circle of which the sector is a part.* Art. **784.**

To find the area of a segment of a circle:

Rule.—*Draw radii from the center of the circle to the extremities of the arc of the segment; find the area of the sector thus formed, subtract from this the area of the triangle formed by the radii and the chord of the arc of the segment, and the result is the area of the segment.* Art. **785.**

THE ELLIPSE.

To find the perimeter of an ellipse: There is no exact method, but the following is close enough for most cases:

Rule.—*Multiply the major axis by 1.82, and the minor axis by 1.315. The sum of the results will be the perimeter.* Art. **788.**

To find the exact area of an ellipse:

Rule.—*The area of an ellipse is equal to the product of its two diameters multiplied by .7854.* Art. **789.**

ANY PLANE FIGURE.

Rule.—*The area of any plane figure may be found by dividing it into triangles, quadrilaterals, circles or parts of circles, and ellipses, finding the area of each part separately and adding them together.* Art. **790.**

THE PRISM AND CYLINDER.

To find the area of the convex surface of any right prism, or right cylinder:

Rule.—*Multiply the perimeter of the base by the altitude.* Art. **803.**

To find the volume of a right prism, or cylinder:

Rule.—*The volume of any right prism or cylinder equals the area of the base multiplied by the altitude.* Art. **804.**

THE PYRAMID AND CONE.

To find the area of a right pyramid or right cone:

Rule.—*The convex area of a right pyramid or cone equals the perimeter of the base multiplied by one-half the slant height.* Art. **809.**

To find the volume of a right pyramid or cone:

Rule.—*The volume of a right pyramid or cone equals the area of the base multiplied by one-third of the altitude.* Art. **810.**

THE FRUSTUM OF A PYRAMID OR CONE.

To find the convex area of a frustum of a right pyramid or right cone:

Rule.—*The convex area of a frustum of a right pyramid or right cone equals one-half the sum of the perimeters of its bases multiplied by the slant height of the frustum.* Art. **814.**

To find the volume of the frustum of a pyramid or cone:

Rule.—*Add the areas of the upper base, the lower base, and the square root of the product of the areas of the two bases; multiply this sum by one-third of the altitude.* Art. **815.**

THE SPHERE.

To find the area of the surface of a sphere:

Rule.—*The area of the surface of a sphere equals the square of the diameter multiplied by 3.1416.* Art. **817.**

To find the volume of a sphere:

Rule.—*The volume of a sphere equals the cube of the diameter multiplied by .5236.* Art. **818.**

THE CYLINDRICAL RING.

To find the volume of a cylindrical ring:

Rule.—*Multiply the area of the cross-section of the ring by the length of the center line.* Art. **822.**

FORMULAS USED IN GASES MET WITH IN MINES.

SPECIFIC GRAVITY.

Let W = weight of the substance in air;
W_1 = weight of the substance in water;
Sp. Gr. = specific gravity of the substance;
w = weight of one cubic foot of the substance.

Then, $\text{Sp. Gr.} = \frac{W}{W - W_1}$. **(1.)** Art. **831.**

For solids or liquids,

$$\text{Sp. Gr.} = \frac{w}{62.5}. \quad \textbf{(2.)} \text{ Art. } \textbf{831.}$$

$$w = 62.5 \times \text{Sp. Gr.} \quad \textbf{(4.)} \text{ Art. } \textbf{832.}$$

For gases,

$$\text{Sp. Gr.} = \frac{w}{.0766} \quad \textbf{(3.)} \text{ Art. } \textbf{831.}$$

$$w = .0766 \times \text{Sp. Gr.} \quad \textbf{(5.)} \text{ Art. } \textbf{832.}$$

PRESSURE, VOLUME, DENSITY, AND WEIGHT OF AIR WHEN THE TEMPERATURE IS CONSTANT.

Mariotte's Law.—*The temperature remaining the same, the volume of a given quantity of gas varies inversely as the pressure.*

Let p = pressure for one position of the piston;
p_1 = pressure for any other position of the piston;
v = volume corresponding to the pressure p;
v_1 = volume corresponding to the pressure p_1.

Then, $p v = p_1 v_1$; **(6.)**

also, $p_1 = \frac{p v}{v_1}$; **(7.)** Art. **852.**

and, $v_1 = \frac{p v}{p_1}$. **(8.)**

Let D be the density corresponding to the pressure p and volume v, and D_1 be the density corresponding to the pressure p_1 and volume v_1; then,

$p : D :: p_1 : D_1$, or $p D_1 = p_1 D$, **(9.)**
and $v : D_1 :: v_1 : D$, or $v D = v_1 D_1$. **(10.)** Art. **852.**

Thus, let W be the weight of a quantity of air or other gas whose volume is v and pressure is p; let W_1 be the weight of the same quantity when the volume is v_1 and pressure is p_1; then,

$p : W :: p_1 : W_1$, or $p W_1 = p_1 W$. **(11.)**
$v : W_1 :: v_1 : W$, or $v W = v_1 W_1$. **(12.)** Art. **852.**

PRESSURE AND VOLUME OF A GAS WITH VARIABLE TEMPERATURE.

Gay-Lussac's Law.—*If the pressure remains constant, every increase of temperature of 1° F. produces in a given quantity of gas an expansion of $\frac{1}{491}$ of its volume at 32° F.*

If the pressure remains constant, it will also be found that every decrease of temperature of 1° F. will cause a decrease of $\frac{1}{491}$ of the volume at 32° F.

Let v = volume of gas before heating;
v_1 = volume of gas after heating;
t = temperature corresponding to volume v;
t_1 = temperature corresponding to volume v_1.

Then, $v_1 = v\left(\frac{459 + t_1}{459 + t}\right)$. **(13.)** Art. **854.**

Let p = the original tension;
t = the corresponding temperature;
t_1 = any higher or lower temperature;
p_1 = corresponding tension.

Then, $p_1 = p\left(\frac{459 + t_1}{459 + t}\right)$. **(14.)** Art. **854.**

Let P = pressure in pounds per square inch;
V = volume of air in cubic feet;
T = absolute temperature;
W = weight.

Then, $P = \frac{.37052\, W T}{V}$. **(15.)**

$V = \frac{.37052\, W T}{P}$. **(16.)**

$T = \frac{P V}{.37052\, W}$. **(17.)**

$W = \frac{P V}{.37052\, T}$. **(18.)**

(15.)–(18.) Art. **855.**

MIXTURE OF TWO GASES HAVING UNEQUAL VOLUMES AND PRESSURES.

Let v and p be the volume and pressure, respectively, of one of the gases.

Let v_1 and p_1 be the volume and pressure, respectively, of the other gas.

Let V and P be the volume and pressure, respectively, of the mixture.

Then, if the temperature remains the same,

$P = \frac{p v + p_1 v_1}{V}$. **(19.)**

$V = \frac{p v + p_1 v_1}{P}$. **(20.)**

(19.)–(20.) Art. **857.**

CALCULATION OF THE WEIGHT OF A GAS.

The weight of any gas, at a given pressure and temperature, is equal to the weight of an equal volume of air, at the same pressure and temperature, multiplied by the specific gravity of the gas.

Let W = weight in pounds;
V = volume in cubic feet;
B = barometric pressure in inches of mercury;
D = specific gravity of the gas—found in table of Rates of Diffusion of Gases;
T = absolute temperature.

Then, $W = \frac{1.3253 \times V \times B \times D}{T}$. **(21.)** Art. **872.**

TO FIND THE WEIGHT OF A DYNAMITE CARTRIDGE.

Rule.—*Multiply the square of the diameter of the cartridge by its length, all in inches, and take $\frac{5}{8}$ of the product; the result will be the weight of the cartridge in ounces.*

Let W = weight of cartridge (ounces);
d = diameter of cartridge (inches);
l = length of cartridge (inches).

Then, $W = \frac{5}{8} l d^2$. **(22.)** Art. **891.**

RULES AND FORMULAS USED IN MINE VENTILATION.

GRAVITATION.

Law of Gravitation.—*The force of attraction by which one body tends to draw another body towards it is directly proportional to its mass, and inversely proportional to the square of the distance between their centers.* Art. **928.**

Laws of Weight.—*Bodies weigh most at the surface of the earth. Below the surface, the weight decreases as the distance to the center decreases.*

Above the surface, the weight decreases as the square of the distance increases. Art. **929.**

FORMULAS FOR GRAVITY PROBLEMS.

Let W = weight of body at the surface;
w = weight of a body at a given distance above or below the surface;
m = mass of the body;
d = distance between the center of the earth and the center of the body;
R = radius of the earth = 4,000 miles;
g = force of gravity where body is weighed.

$$\text{Mass} = \frac{\text{weight of body}}{\text{force of gravity}}, \text{ or } m = \frac{W}{g}. \qquad \textbf{(23.)} \quad \text{Art. } \mathbf{927.}$$

Formula for weight when the body is below the surface:

$$wR = dW. \qquad \textbf{(24.)} \quad \text{Art. } \mathbf{930.}$$

Formula for weight when the body is above the surface:

$$wd^2 = WR^2. \qquad \textbf{(25.)} \quad \text{Art. } \mathbf{930.}$$

FORMULAS FOR FALLING BODIES.

Let g = force of gravity = constant accelerating force due to the attraction of the earth;
t = number of seconds the body falls;
v = velocity at the end of the time t;
h = distance that a body falls during the time t.

$$v = gt. \qquad \textbf{(26.)} \quad \text{Art. } \mathbf{933.}$$

That is, *the velocity acquired by a freely falling body at the end of t seconds equals 32.16 multiplied by the time in seconds.*

$$t = \frac{v}{g}. \qquad \textbf{(27.)} \quad \text{Art. } \mathbf{933.}$$

That is, *the number of seconds during which a body must have fallen to acquire a given velocity equals the given velocity in feet per second divided by 32.16.*

$$h = \frac{v^2}{2g}. \qquad \textbf{(28.)} \quad \text{Art. } \mathbf{933.}$$

That is, *the height from which a body must fall to acquire a given velocity equals the square of the given velocity divided by 2 × 32.16.*

$$v = \sqrt{2gh}. \qquad \textbf{(29.)} \text{ Art. } \mathbf{933.}$$

That is, *the velocity that a body will acquire in falling through a given height equals the square root of the product of twice 32.16, and the given height.*

$$h = \tfrac{1}{2}gt^2. \qquad \textbf{(30.)} \text{ Art. } \mathbf{933.}$$

That is, *the distance a body will fall in a given time equals 32.16 ÷ 2, multiplied by the square of the number of seconds.*

$$t = \sqrt{\frac{2h}{g}}. \qquad \textbf{(31.)} \text{ Art. } \mathbf{933.}$$

That is, *the time it will take a body to fall through a given height equals the square root of twice the height divided by 32.16.*

THEORETICAL VELOCITY OF AIR.

Let v = velocity of air in feet per second;
F = the constant force represented by difference of pressure in pounds per square foot;
w = weight of a cubic foot of the air;
g = acceleration due to gravity = 32.16 ft.

Then, $$v = \sqrt{\frac{2gF}{w}}. \qquad \textbf{(32.)} \text{ Art. } \mathbf{940.}$$

THE MOTIVE COLUMN.

Let W = the weight of a cubic foot of air in the downcast shaft;
w = the weight of a cubic foot of air in the upcast shaft;
p = the pressure of the downcast shaft;
p_1 = the pressure in the upcast shaft;
t_1 = the average temperature of the air in the downcast shaft;

t = the average temperature of the air in the upcast shaft;
D = the depth of the upcast shaft in feet;
M = the length of the motive column in feet;
G = the water-gauge in inches.

Then,

$$M = \frac{p - p_1}{W}. \quad (33.)$$

$$M = \frac{5.2\,G}{W}. \quad (34.)$$

$$M = \frac{D(t - t_1)}{459 + t}. \quad (35.)$$

Art. **942.**

THE THREE LAWS OF FRICTION.

As the result of many experiments, the truth of the three following laws, called the **three laws of friction**, has been firmly established.

First Law.—*When the velocity remains the same, the total pressure required to overcome friction varies directly as the extent of the rubbing surface.* Art. **946.**

Second Law.—*When the velocities and rubbing surfaces remain the same, the pressures required to force air through the passages of a mine increase and decrease inversely as the sectional areas of the passages increase or decrease.* Art. **952.**

Third Law.—*The pressure required to overcome friction in an airway varies as the squares of the velocities when the rubbing surface and the areas of section are the same; and the pressures required to overcome friction vary as the squares of the velocities multiplied by the rubbing surfaces per square foot of section in all airways.* Art. **955.**

FORMULAS FOR VENTILATION.

Let a = sectional area of airway in square feet;
H = horsepower;
k = coefficient of friction = .0000000217;
l = length of airway in feet;

o = perimeter of airway in feet;
p = ventilating pressure in pounds per square foot;
P = total ventilating pressure in pounds;
q = quantity of air in cubic feet per minute;
s = rubbing surface in square feet;
u = units of power in foot-pounds per minute;
v = velocity in feet per minute;
W = water-gauge in inches of water.

Then,

$$P = p a. \qquad (36.)$$

$$P = k s v^2. \qquad (37.)$$

$$p = \frac{k s v^2}{a}. \qquad (38.)$$

$$s = \frac{P}{k v^2} = \frac{p a}{k v^2}. \qquad (39.)$$

$$v = \sqrt{\frac{p a}{k s}}. \qquad (40.)$$

$$l = \frac{s}{o}. \qquad (41.)$$

$$s = l o. \qquad (42.)$$

(36.)–(42.) Art. **962.**

$$q = a v. \qquad (43.)$$ Art. **963.**

$$v = \frac{q}{a}. \qquad (44.)$$ Art. **965.**

$$a = \frac{q}{v}. \qquad (45.)$$ Art. **966.**

$$u = P v. \qquad (46.)$$

$$u = p q. \qquad (47.)$$

(46.)–(47.) Art. **968.**

$$H = \frac{u}{33{,}000} = \frac{P v}{33{,}000} = \frac{p a v}{33{,}000} = \frac{p q}{33{,}000}. \qquad (48.)$$ Art. **968.**

$$p = \frac{33{,}000\,H}{a v}. \qquad (49.)$$

$$p = \frac{33{,}000\,H}{q}. \qquad (50.)$$

(49.)–(50.) Art. **969.**

$$q = \frac{33,000\,H}{p}. \quad \textbf{(51.)}$$

$$v = \frac{33,000\,H}{P} = \frac{33,000\,H}{p\,a}. \quad \textbf{(52.)}$$

Art. **970.**

$$u = k\,s\,v^2. \quad \textbf{(53.)}$$

$$q = \frac{k\,s\,v^2}{p}. \quad \textbf{(54.)}$$

Art. **971.**

$$q = \sqrt{\frac{p\,d^2}{4\,k\,l}}. \quad \textbf{(55.)} \quad \text{Art. } \textbf{974.}$$

The above formulas, with others less important, are given below; they are so arranged that all the formulas used in obtaining the value of a given quantity are grouped together.

To find the area:

$$a = \frac{P}{p}.$$

$$a = \frac{k\,s\,v^2}{p}.$$

$$a = \frac{q}{v}.$$

$$a = \frac{u}{p\,v}.$$

$$a = \frac{33,000\,H}{p\,v}.$$

$$a = \frac{k\,s\,v^2\,q}{u}.$$

To find the horsepower:

$$H = \frac{u}{33,000}.$$

$$H = \frac{P\,v}{33,000}.$$

$$H = \frac{p\,q}{33,000}.$$

$$H = \frac{p\,a\,v}{33,000}.$$

To find the coefficient of friction:

$$k = \frac{P}{s v^2}.$$

$$k = \frac{p a}{s v^2}.$$

$$k = \frac{u}{s v^3}.$$

$$k = \frac{p q}{s v^3}.$$

To find the length of the airway:

$$l = \frac{s}{o}.$$

To find the perimeter of the airway:

$$o = \frac{s}{l}.$$

To find the total pressure:

$$P = p a.$$

$$P = k s v^2.$$

$$P = \frac{u}{v}.$$

$$P = \frac{33{,}000 H}{v}.$$

$$P = \frac{k s q^2}{a^2}.$$

To find the pressure in pounds per square foot:

$$p = \frac{P}{a}.$$

$$p = \frac{k s v^2}{a}.$$

$$p = \frac{u}{q}.$$

$$p = \frac{33{,}000 H}{a v}.$$

$$p = \frac{33{,}000\,H}{q}.$$

$$p = \frac{k s v^3}{q}.$$

$$p = 5.2\,W.$$

To find the quantity of air passing in cubic feet per minute:

$$q = a v.$$

$$q = \frac{u}{p}.$$

$$q = \frac{33{,}000\,H}{p}.$$

$$q = \frac{k s v^3}{p}.$$

$$q = a\sqrt{\frac{p a}{k s}}.$$

To find the rubbing surface in square feet:

$$s = \frac{P}{k v^2}.$$

$$s = \frac{p a}{k v^2}.$$

$$s = l o.$$

$$s = \frac{u}{k v^3}.$$

$$s = \frac{p q}{k v^3}.$$

To find the units of power in foot-pounds per minute:

$$u = P v.$$

$$u = p q.$$

$$u = 33{,}000\,H.$$

$$u = p a v.$$

$$u = k s v^3.$$

To find the velocity in feet per minute:

$$v = \sqrt{\frac{p\,a}{k\,s}}.$$

$$v = \sqrt{\frac{P}{k\,s}}.$$

$$v = \frac{q}{a}.$$

$$v = \frac{u}{P}.$$

$$v = \frac{u}{p\,a}.$$

$$v = \frac{33{,}000\,H}{P}.$$

$$v = \frac{33{,}000\,H}{p\,a}.$$

$$v = \sqrt[3]{\frac{u}{k\,s}}.$$

$$v = \sqrt[3]{\frac{p\,q}{k\,s}}.$$

To find the water-gauge:

$$W = \frac{p}{5.2}.$$

LAWS OF VENTILATION.

Art. **980.**

In order to ascertain the effects produced by varying the airway, or by varying the quantity, velocity, etc., of the air, it is generally easier to make use of one of the following *laws* than to solve by means of one of the foregoing formulas. The laws are also useful for comparing the results obtained from two airways. Letting p, q, v, s, etc., represent, respectively, the pressure, quantity, velocity, rubbing surface, etc., before the change, and p_1, q_1, v_1, s_1, etc., the same things after the change, the laws may be stated as follows:

(1) The pressure varies directly as the extent of the rubbing surface; i. e., $p : p_1 :: s : s_1$, or $P : P_1 :: s : s_1$.

(2) The pressure varies directly as the density* of the air; i. e., $p : p_1 :: w : w_1$, or $P : P_1 :: w : w_1$.

* By density is meant the *weight of a cubic foot in pounds.*

(3) The pressure varies directly as the square of the quantity; i. e., $p : p_1 :: q^2 : q_1^2$, or $P : P_1 :: q^2 : q_1^2$.

(4) The pressure varies directly as the square of the velocity; i. e., $p : p_1 :: v^2 : v_1^2$, or $P : P_1 :: v^2 : v_1^2$.

(5) The pressure varies directly as the length of the airway; i. e., $p : p_1 :: l : l_1$, or $P : P_1 :: l : l_1$.

(6) The pressure varies directly as the length of the perimeter; i. e., $p : p_1 :: o : o_1$, or $P : P_1 :: o : o_1$.

(7) The pressure per square foot varies inversely as the area of the airway; i. e., $p : p_1 :: a_1 : a$.

(8) The quantity varies directly as the square root of the pressure; i. e., $q : q_1 :: \sqrt{p} : \sqrt{p_1}$, or $q : q_1 :: \sqrt{P} : \sqrt{P_1}$.

(9) The quantity varies directly as the cube root of the power; i. e., $q : q_1 :: \sqrt[3]{u} : \sqrt[3]{u_1}$, or $q : q_1 :: \sqrt[3]{H} : \sqrt[3]{H_1}$.

(10) The quantity varies inversely as the square root of the rubbing surface; i.e., $q : q_1 :: \sqrt{s_1} : \sqrt{s}$.

(11) The velocity varies directly as the square root of the pressure; i. e., $v : v_1 :: \sqrt{p} : \sqrt{p_1}$, or $v : v_1 :: \sqrt{P} : \sqrt{P_1}$.

(12) The velocity varies directly as the square root of the area; i. e., $v : v_1 :: \sqrt{a} : \sqrt{a_1}$.

(13) The velocity varies inversely as the square root of the length of the airway; i. e., $v : v_1 :: \sqrt{l_1} : \sqrt{l}$.

(14) The velocity varies inversely as the square root of the rubbing surface; i. e., $v : v_1 :: \sqrt{s_1} : \sqrt{s}$.

(15) The power varies directly as the cube of the quantity; i. e., $u : u_1 :: q^3 : q_1^3$, or $H : H_1 :: q^3 : q_1^3$.

(16) The rubbing surface varies inversely as the square of the quantity; i. e., $s : s_1 :: q_1^2 : q^2$.

(17) The rubbing surface varies inversely as the square of the velocity; i. e., $s : s_1 :: v_1^2 : v^2$.

(18) The sectional area varies directly as the square of the velocity; i. e., $a : a_1 :: v^2 : v_1^2$.

(19) The length of the airway varies inversely as the square of the velocity; i. e., $l : l_1 :: v_1^2 : v^2$.

(20) The length of the airway varies inversely as the square of the quantity; i. e., $l : l_1 :: q_1^2 : q^2$.

For similar airways, let d equal the length of a side; then,

(21) The quantity varies directly as the square root of the fifth power of the length of the side; i. e., $q : q_1 :: \sqrt{d^5} : \sqrt{d_1^5}$.

(22) The pressure varies inversely as the fifth power of the length of the side; i. e., $p : p_1 :: d_1^5 : d^5$.

(23) The length of the side varies inversely as the fifth root of the pressure; i. e., $d : d_1 :: \sqrt[5]{p_1} : \sqrt[5]{p}$.

(24) The length of the side varies directly as the fifth root of the square of the quantity; i. e., $d : d_1 :: \sqrt[5]{q^2} : \sqrt[5]{q_1^2}$.

To the above laws may also be added another:

(25) If equal quantities of air pass through two airways, the velocities will vary inversely as the areas; i. e., $v : v_1 :: a_1 : a$.

AREA OF REGULATOR OPENING.

Let A = area of opening in square feet;
q = quantity of air in cubic feet per minute which it is desired to pass through the opening;
W = difference of pressure in inches of water on the two sides of the regulator.

Then, $$A = \frac{.0004\,q}{\sqrt{W}}. \qquad \textbf{(56.)}\ \text{Art. } \textbf{998}.$$

WEIGHT OF AIR.

Let W = weight of a cubic foot of air;
B = height of barometer, inches of mercury;
t = temperature of air, Fahrenheit.

$$W = \frac{1.3253\,B}{459 + t}. \qquad \textbf{(57.)} \quad \text{Art. } \mathbf{1006.}$$

EFFECT OF TEMPERATURE ON VOLUME OF AIR.

Let T = required absolute temperature;
t = given temperature of air, Fahrenheit;
V = volume of air at absolute temperature, T;
v = volume of air at Fahrenheit temperature, t.

$$T = \frac{V}{v} \times (459 + t). \qquad \textbf{(58.)} \quad \text{Art. } \mathbf{1007.}$$

VENTILATING PRESSURE.

Let p = ventilating pressure in pounds per square foot;
t = higher temperature of air;
t_1 = lower temperature;
D = depth of shaft.

$$p = \frac{(t - t_1)}{(459 + t)} \times .077 \times D. \qquad \textbf{(60.)} \quad \text{Art. } \mathbf{1008.}$$

GRATE AREA OF VENTILATING FURNACE.

Let D = depth of furnace in feet;
s = grate area per horsepower in square feet.

$$s = \frac{84}{\sqrt{D}}. \qquad \textbf{(61.)} \quad \text{Art. } \mathbf{1010.}$$

RELATION BETWEEN WEIGHT OF AIR AND VENTILATING PRESSURE.

Let w = weight of one cubic foot of air in the upcast;
W = weight of one cubic foot of air in the downcast;
p = ventilating pressure in pounds per square foot;
P = atmospheric pressure in pounds per square foot = 2,116.

$$w = \frac{P - p}{P} \times W. \qquad \textbf{(62.)} \quad \text{Art. } \mathbf{1022.}$$

FLOW OF AIR.

Let v = velocity of air in feet per second;
p = difference of pressure producing the flow.

$$v = 18\sqrt{p}. \qquad (64.) \text{ Art. } \mathbf{1027.}$$

$$p = \left(\frac{v}{18}\right)^2. \qquad (65.) \text{ Art. } \mathbf{1027.}$$

AREA OF AIR-PASSAGES.

Let d = diameter of port of entry in feet;
b = width of fan-blades in feet;
q = quantity of air in cubic feet passing through *one* port.

$$d = .0343\sqrt{q}. \qquad (66.) \text{ Art. } \mathbf{1030.}$$

$$b = \tfrac{1}{4}d. \qquad (67.) \text{ Art. } \mathbf{1031.}$$

MANOMETRIC EFFICIENCY.

Let A = area of port of entry of fan;
a = area of port of discharge;
O = pressure required to blow air out of fan;
I = depression required for air to enter fan;
M = mine resistance in pounds per sq. ft. as measured with the water-gauge.
C = manometric efficiency.

$$O = \frac{A^2 I}{a^2}. \qquad (68.) \text{ Art. } \mathbf{1032.}$$

$$C = \frac{100\,M}{M + I + O}. \qquad (69.) \text{ Art. } \mathbf{1032.}$$

CENTRIFUGAL FANS.

Centrifugal Force of a Body Moving in a Circle.

Let w = weight of body in pounds;
v = velocity in feet per second;
R = radius of circle in feet;
g = 32.16 = acceleration due to gravity,
f = centrifugal force in pounds.

$$f = \frac{w v^2}{R g}. \qquad (70.) \text{ Art. } \mathbf{1033.}$$

If the body is free to move outward, instead of being constrained to move in a circle, the formula becomes

$$f = \frac{w v^2}{3.1416 g}. \qquad \textbf{(71.)} \text{ Art. } \mathbf{1033.}$$

Centrifugal Force Developed by a Fan.

Let l = radial length of the fan-blade;
a = angle between blade and radius.

$$f = \frac{w l v^2}{3.1416 g}. \qquad \textbf{(72.)} \text{ Art. } \mathbf{1035.}$$

$$f = \frac{w l v^2 (\cos a)^2}{3.1416 g}. \qquad \textbf{(75.)} \text{ Art. } \mathbf{1037.}$$

Fan Dimensions.

Let D = diameter of fan;
d = diameter of port of entry;
r = radius of gyration of blade;
l = radial length of blade.

$$l = \frac{D - d}{2}. \qquad \textbf{(73.)} \text{ Art. } \mathbf{1035.}$$

$$r = \frac{d}{2} + .6 l. \qquad \textbf{(74.)} \text{ Art. } \mathbf{1036.}$$

THERMOMETERS.

Let F = temperature in degrees Fahrenheit;
C = temperature in degrees Centigrade.

To change Centigrade readings to Fahrenheit:

$$F = \tfrac{9}{5} C + 32. \qquad \textbf{(76.)} \text{ Art. } \mathbf{1067.}$$

To change Fahrenheit readings to Centigrade:

$$C = \tfrac{5}{9} (F - 32). \qquad \textbf{(77.)} \text{ Art. } \mathbf{1067.}$$

FORMULAS USED IN MINE SURVEYING AND MAPPING.

TO FIND THE DISTANCE BETWEEN CHAMBERS OR ROOMS, MEASURED ALONG THE ENTRY.

Let D = required distance;
p = perpendicular distance between chambers;
A = angle between center lines of entry and chamber.

$$D = \frac{p}{\sin A}. \qquad \textbf{(78.)} \text{ Art. } \mathbf{1130.}$$

LATITUDES AND DEPARTURES.

Latitude = distance × cosine of bearing.
Departure = distance × sine of bearing. } Art. **1132.**

CURVES.

Let R = radius of curve;
D = deflection angle of curve;
I = angle of intersection;
T = tangent distance;
c = chord;
d = chord deflection;
f = tangent deflection.

Then, $R = \frac{50}{\sin D}$. **(79.)** Art. **1200.**

$T = R \tan \frac{1}{2} I$. **(80.)** Art. **1204.**

$d = \frac{c^2}{R}$. **(81.)** Art. **1208.**

$f = \frac{c^2}{2R}$. **(82.)** Art. **1208.**

ECONOMIC GEOLOGY OF COAL.

INCREASE OF TEMPERATURE WITH INCREASE OF DEPTH BELOW THE EARTH'S SURFACE.

Let T = temperature in degrees Fahrenheit;
D = depth below surface in feet.

$T = 50.68 + \frac{D - 19.68}{67.2}$. **(83.)** Art. **1287.**

PROSPECTING FOR COAL AND LOCATION OF OPENINGS.

THE ANGLE OF CORRECTED DIP FOR AN OBLIQUE SECTION.

Let a = tangent of angle of corrected dip;
b = angle of dip at right angles to strike;
c = angle at which the section lies to right or left of the full dip.

Then, $a = \tan b \times \cos c$. **(84.)** Art. **1423.**

The correction for the angles most used, as calculated from the above formula, are given in the following table:

OBLIQUE SECTION TABLE.

b =		5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°
		Corrected Angles.											
c = 5°	...	4° 59′	9° 58′	14° 57′	19° 56′	24° 55′	29° 50′	34° 54′	39° 51′	44° 53′	49° 54′	54° 54′	59° 54′
c = 10°	...	4° 55′	9° 51′	14° 47′	19° 43′	24° 40′	29° 37′	34° 35′	39° 34′	44° 34′	49° 34′	54° 35′	59° 37′
c = 15°	...	4° 50′	9° 40′	14° 31′	19° 22′	24° 15′	29° 09′	34° 04′	39° 02′	44° 00′	49° 01′	54° 04′	59° 08′
c = 20°	...	4° 42′	9° 25′	14° 08′	18° 53′	23° 40′	28° 29′	33° 21′	38° 15′	43° 13′	48° 14′	53° 18′	58° 26′
c = 25°	...	4° 32′	9° 05′	13° 39′	18° 15′	22° 54′	27° 37′	32° 24′	37° 15′	42° 11′	47° 12′	52° 19′	57° 04′
c = 30°	a =	4° 20′	8° 41′	13° 04′	17° 30′	22° 00′	26° 34′	31° 14′	36° 00′	40° 54′	45° 54′	51° 03′	56° 18′
c = 35°	...	4° 06′	8° 13′	12° 23′	16° 36′	20° 54′	25° 19′	29° 50′	34° 30′	39° 19′	44° 56′	50° 20′	54° 49′
c = 40°	...	3° 50′	7° 42′	11° 36′	15° 35′	19° 39′	23° 51′	28° 16′	32° 44′	37° 27′	42° 25′	47° 34′	53° 00′
c = 45°	...	3° 32′	7° 06′	10° 44′	14° 26′	18° 15′	22° 12′	26° 21′	30° 41′	35° 16′	40° 07′	45° 17′	50° 46′
c = 50°	...	3° 13′	6° 28′	9° 47′	13° 10′	17° 28′	20° 22′	24° 14′	28° 20′	32° 44′	37° 27′	42° 33′	48° 05′
c = 55°	...	2° 52′	5° 46′	8° 44′	11° 47′	14° 58′	18° 19′	21° 53′	25° 42′	29° 50′	34° 21′	39° 19′	44° 59′
c = 60°	...	2° 30′	5° 02′	7° 38′	10° 19′	13° 07′	16° 06′	19° 17′	22° 45′	26° 34′	30° 27′	35° 32′	40° 54′

SHAFTS, SLOPES, AND DRIFTS.

TO FIND THE LENGTH OF THE WINDING COMPARTMENTS OF A RECTANGULAR SHAFT.

Let S = output speed;
D = depth of shaft;
T = tonnage expressed in pounds;
N = number of working hours;
W = weight of a cubic foot of broken coal;
B = average inside width of car;
d = inside depth of car;
L = length of compartment;
f = clearance in shaft at ends of cage = 1 foot.

Then, $L = \frac{TD}{SNWBd} + f$. **(85.)** Art. **1459.**

THICKNESS OF CAST-IRON TUBBING FOR CIRCULAR SHAFTS.

Let t = thickness of tubbing in inches;
d = diameter of shaft in feet;
D = depth in feet;
G = the crushing load of cast iron per square inch.

$$t = \frac{6d\sqrt{G} - 6d\sqrt{G - 6.944D}}{\sqrt{G - 6.944D}} \quad \textbf{(86.)} \text{ Art. } \textbf{1480.}$$

$$= \frac{1,800d - 6d\sqrt{90,000 - 6.944D}}{\sqrt{90,000 - 6.944D}}, \text{ when } G = 90,000.$$

The upper course of tubbing should in all cases be at least ½ an inch thick in the plate, even in shafts of very small diameter; and ⅝ of an inch thick in shafts of large diameter, to prevent liability to fracture. It is also desirable to add a constant, usually ⅛ of an inch, to the thickness obtained by the formula, to allow for wear and tear, and for corrosion or other chemical action on the metal.

In this formula no allowance is made for the extra strength given the segments by the flanges and ribs. Theoretically, each set of segments should have a different thickness, but in practice they are calculated for every 25 or 30 feet.

NUMBER OF BRICKS REQUIRED TO LINE A CIRCULAR SHAFT.

Let N = number of bricks required;
D = outer diameter of the shaft;
d = inner diameter of the shaft;
t = thickness of brick;
b = breadth of brick;
l = length of brick;
x = depth of shaft.

All dimensions must be in feet or all in inches.

$$N = \frac{.7854\,x\,(D^2 - d^2)}{t\,b\,l}. \qquad \textbf{(87.)} \quad \text{Art. } \textbf{1487.}$$

METHODS OF WORKING COAL MINES.

RADIUS OF SHAFT PILLAR.

Let R = required radius of pillar;
D = depth of shaft;
t = thickness of seam.

Then, $$R = 3\sqrt{D\,t}.$$

THICKNESS OF CYLINDRICAL OR SPHERICAL DAMS TO RESIST A GIVEN PRESSURE.

Let T = thickness in inches;
R = short radius in inches;
U = ultimate crushing strength in pounds per square inch, which is, for timber, 8,000; for stone, 6,000; and for brick, 2,500;
P = head of water in pounds per square inch.

Then, for a cylindrical dam,

$$T = R\left\{1 - \sqrt{1 - \frac{20P}{U}}\right\}. \qquad \textbf{(88.)} \quad \text{Art. } \textbf{1710.}$$

For a spherical dam,

$$T = R\left\{1 - \sqrt[3]{1 - \frac{15P}{U}}\right\}. \qquad \textbf{(89.)} \quad \text{Art. } \textbf{1710.}$$

These formulas give very small thicknesses for dams to resist comparatively slight pressures. In no case, when a water head of over 10 feet is to be resisted, is it good practice to make the dam less than 3 feet thick. For heavy pressures the formulas are safe, provided their results exceed 36 inches, after being multiplied by 2.

FORMULAS USED IN MECHANICS.

MOTION AND VELOCITY.

Let s = distance traveled by moving body;
v = uniform velocity of body;
t = the time.

Then, $v = \frac{s}{t}$. **(90.)** Art. **1827.**

$s = v\,t$. **(91.)** Art. **1828.**

$t = \frac{s}{v}$. **(92.)** Art. **1829.**

CENTER OF GRAVITY.

Let w = weight of smaller body;
W = weight of larger body;
l = distance between centers of gravity of the two bodies;
l_1 = distance from the center of gravity of the two to center of gravity of larger body.

Then, $l_1 = \frac{w\,l}{W + w}$. **(93.)** Art. **1842.**

THE LEVER.

Let P = the power;
W = the weight;
a = perpendicular distance of power from fulcrum = power arm;
b = perpendicular distance of weight from fulcrum = weight arm;
$a_1, a_2, a_3, \ldots$ = power arms of compound lever;
$b_1, b_2, b_3, \ldots$ = weight arms of compound lever.

Then, $P a = W b$. **(94.)** Art. **1851.**

$P \times a_1 \times a_2 \times a_3 \times \ldots = W \times b_1 \times b_2 \times b_3 \times \ldots$ **(95.)** Art. **1852.**

RELATION BETWEEN SPEED AND DIAMETER OF PULLEYS.

Let D = diameter of the driver;
d = diameter of the driven;
N = number of revolutions of the driver;
n = number of revolutions of the driven.

Then, $D = \frac{dn}{N}$. **(96.)** Art. **1863.**

$d = \frac{DN}{n}$. **(97.)** Art. **1864.**

$n = \frac{DN}{d}$. **(98.)** Art. **1865.**

$N = \frac{dn}{D}$. **(99.)** Art. **1866.**

WHEEL WORK.

Let $D_1, D_2, D_3, \ldots$ = diameters of driving pulleys;
$d_1, d_2, d_3, \ldots$ = diameters of driven pulleys;
P = power exerted;
W = weight lifted.

Then,

$P = \frac{W \times d_1 \times d_2 \times d_3 \times \ldots}{D_1 \times D_2 \times D_3 \times \ldots}$. **(100.)**

$W = \frac{P \times D_1 \times D_2 \times D_3 \times \ldots}{d_1 \times d_2 \times d_3 \times \ldots}$. **(101.)**

Art. **1867.**

DIAMETER, PITCH, AND SPEED OF GEARS.

Let P = pitch;
T = number of teeth;
D = pitch diameter of the wheel;

Then, $D = \frac{PT}{3.1416}$. **(102.)** Art. **1873.**

$T = \frac{3.1416\,D}{P}$. **(103.)** Art. **1874.**

$P = \frac{3.1416\,D}{T}$. **(104.)** Art. **1875.**

Let R = number of revolutions per minute of the driver;
r = number of revolutions per minute of the driven;
T = number of teeth in the driver;
t = number of teeth in the driven.

Then, $T = \frac{t\,r}{R}$. **(105.)** Art. **1878.**

$t = \frac{T\,R}{r}$. **(106.)** Art. **1879.**

$r = \frac{T\,R}{t}$. **(107.)** Art. **1880.**

$R = \frac{t\,r}{T}$. **(108.)** Art. **1881.**

LAW OF COMBINATION OF PULLEYS.

In any combination of pulleys where one continuous rope is used, a load on the free end will balance a weight on the movable block as many times as great as itself as there are parts of the rope supporting the load—not counting the free end.

HORSEPOWER OF GEARS.

Let p = pitch of teeth of gear (breadth of face is 2½ to 3 p);
s = speed of point on pitch circle in feet per minute;
H = horsepower transmitted by gear.

Then, $H = .01\,s\,p^2$. **(109.)** Art. **1882.**

THE INCLINED PLANE.

When the power acts parallel to the plane:

The power multiplied by the length of the inclined plane equals the weight multiplied by the height of the inclined plane. Art. **1885.**

When the power acts parallel to the base:

The power multiplied by the base is equal to the weight multiplied by the height of the inclined plane. Art. **1885.**

THE SCREW.

Let W = weight lifted by screw;
P = force applied to handle;
p = pitch of screw;
R = radius of circle of force P.

Then, $W = \frac{6.2832\,PR}{p}$. **(110.)**

$P = \frac{p\,W}{6.2832\,R}$. **(111.)**

Art. **1888.**

LAWS OF FRICTION.

Art. **1892.**

(**1**) *Friction is directly proportional to the perpendicular pressure between the two surfaces in contact.*

(**2**) *Friction is independent of the extent of the surfaces in contact when the total perpendicular pressure remains the same.*

(**3**) *Friction increases with the roughness of the surfaces.*

(**4**) *Friction is greater between surfaces of the same material than between those of different materials.*

(**5**) *Friction is greatest at the beginning of motion.*

(**6**) *Friction is greater between soft bodies than between hard ones.*

(**7**) *Rolling friction is less than sliding friction.*

(**8**) *Friction is diminished by polishing or lubricating the surfaces.*

CENTRIFUGAL FORCE.

Let F = centrifugal force in pounds;
W = weight of revolving body in pounds;
R = radius in feet of circle described by center of gravity of revolving body;
N = revolutions per minute of revolving body.

Then, $F = .00034\,W\,R\,N^2$. **(112.)** Art. **1898.**

WORK AND ENERGY.

The force (or resistance) multiplied by the distance through which it acts equals the work. If a weight be raised, the weight multiplied by the vertical height of the lift equals the work. Art. **1902.**

One horsepower *is 33,000 foot-pounds per minute; in other words, it is 33,000 pounds raised vertically one foot in one minute, or 1 pound raised vertically 33,000 feet in one minute, or any combination that will, when multiplied together, give 33,000 foot-pounds in one minute.* Art. **1903.**

Let W = weight of a body in pounds;
v = velocity of body in feet per second;
K = kinetic energy.

Then, $$K = \frac{Wv^2}{64.32}. \quad \textbf{(113.)}$$ Art. **1904.**

BELTS.

Let D = diameter of one pulley;
D_1 = diameter of other pulley;
L = distance between shafts;
B = length of *open* belt.

$$B = 3\tfrac{1}{4}\left(\frac{D + D_1}{2}\right) + 2L. \quad \textbf{(114.)}$$ Art. **1908.**

Let W = width of single belt in inches;
W_1 = width of double belt;
H = horsepower to be transmitted;
S = speed of belt in feet per minute.

Then, $$W = \frac{800\,H}{S}. \quad \textbf{(115.)}$$ Art. **1909.**

$$H = \frac{WS}{800}. \quad \textbf{(116.)}$$ Art. **1910.**

$$W_1 = \tfrac{5}{8}\,W. \quad \textbf{(117.)}$$ Art. **1911.**

TENSILE STRENGTH OF MATERIALS.

Let W = safe load in pounds;
A = area of minimum cross-section;
S = working stress in pounds per square inch (see table of Tensile Strengths of Materials).

$W = A\,S.$ **(118.)** Art. **1934.**

$A = \frac{W}{S}.$ **(119.)** Art. **1935.**

$S = \frac{W}{A}.$ **(120.)** Art. **1936.**

Formulas for the Strength of Chains.

Let W = safe load in pounds;
D = diameter in inches of the iron from which the links are made.

For a stud-link chain,

$W = 18{,}000\,D^2.$ **(121.)** Art. **1938.**

For a close-link chain,

$W = 12{,}000\,D^2.$ **(122.)** Art. **1939.**

Formulas for the Strength of Hemp Ropes.

Let W = maximum working load in pounds;
C = circumference of rope in inches.

Then, $W = 100\,C^2.$ **(123.)** Art. **1941.**

$C = .1\sqrt{W}.$ **(124.)** Art. **1942.**

Formulas for the Strength of Wire Ropes.

Let W = maximum working load in pounds;
C = circumference of rope in inches.

$W = 600\,C^2.$ **(125.)** Art. **1944.**

$C = .0408\sqrt{W}.$ **(126.)** Art. **1945.**

The above formulas are also applicable when computing the safe strength of steel-wire rope by substituting the constant 1,000 for the constant 600, and .0316 for .0408.

CRUSHING STRENGTH OF MATERIALS.

Formula for the Strength of Pillars.

The following formula is applicable to pillars commonly used in practice, the lengths of which are about from 10 to 40 times their least diameter, or, if rectangular, their least thickness as indicated by d:

Let C = crushing strength in tons per sq. in. (see table of Crushing Strengths of Materials);
S = sectional area in inches;
L = length in inches;
d = least thickness of rectangular pillar, or diameter of round pillar in inches;
W = breaking load in tons;
a = constant (see table of Constants for Pillars).

Then, $$W = \frac{C S}{1 + \frac{L^2}{a d^2}}. \qquad \textbf{(127.)} \text{ Art. } \textbf{1951.}$$

TRANSVERSE STRENGTH OF MATERIALS.

Strength of Beams.

d = depth of beam in inches;
w = width of beam in inches;
d_1 = diameter of cylindrical beam in inches;
L = length between supports in feet
= distance between load and fixed end, in the case of cantilevers;
S = safe transverse strength (see table of Constants for Transverse Strength of Beams);
W = safe load in pounds.

Cantilevers. (Load at End.)

$$W = \frac{d^2 w S}{L}. \qquad \textbf{(128.)} \text{ Art. } \textbf{1954.}$$

$$W = \frac{.6 d_1^3 S}{L}. \qquad \textbf{(129.)} \text{ Art. } \textbf{1955.}$$

If the load is uniformly distributed, multiply the results obtained from formulas **128** and **129** by 2.

Beams Supported at the Ends.

$$W = \frac{4\, d^2\, w\, S}{L}. \qquad \textbf{(130.)} \quad \text{Art. } \textbf{1957.}$$

$$W = \frac{4\, d_1^2 \times .6\, S}{L}. \qquad \textbf{(131.)} \quad \text{Art. } \textbf{1958.}$$

If the load is uniformly distributed, multiply the results obtained by 2.

SHEARING STRENGTH OF MATERIALS.

a = area of cross-section in square inches;
S = safe shearing stress (see table of Shearing Strengths of Materials);
W = safe load in pounds.

$$W = a\, S. \qquad \textbf{(132.)} \quad \text{Art. } \textbf{1963.}$$

LINE SHAFTING.

D = diameter of shaft;
R = revolutions per minute;
H = horsepower transmitted;
C = constant (see table of Constants for Line Shafting).

$$H = \frac{D^3 R}{C}. \qquad \textbf{(133.)} \quad \text{Art. } \textbf{1966.}$$

$$R = \frac{C H}{D^3}. \qquad \textbf{(134.)} \quad \text{Art. } \textbf{1967.}$$

$$D = \sqrt[3]{\frac{C H}{R}}. \qquad \textbf{(135.)} \quad \text{Art. } \textbf{1968.}$$

FORMULAS USED IN STEAM AND STEAM-BOILERS.

SPECIFIC HEAT.

W = weight of body in pounds;
t = temperature before heat is applied;
t_1 = temperature after heat is applied;
c = specific heat of body;
U = number of B. T. U. required to raise temperature of body from t to t_1.

$$U = c\, W(t_1 - t). \qquad \textbf{(136.)} \quad \text{Art. } \textbf{1983.}$$

TEMPERATURE OF MIXTURES.

$w, w_1, w_2, \ldots$ = weights of the several substances, respectively;

$c, c_1, c_2, \ldots$ = specific heats of the substances, respectively;

$t, t_1, t_2, \ldots$ = temperatures of the substances, respectively;

T = final temperature of mixture.

$$T = \frac{w\,c\,t + w_1\,c_1\,t_1 + w_2\,c_2\,t_2 + \ldots}{w\,c + w_1\,c_1 + w_2\,c_2 + \ldots}. \qquad \textbf{(137.)} \text{ Art. } \textbf{1987.}$$

PRESSURE AND TEMPERATURE OF STEAM.

Let t = temperature of steam;
p = gauge-pressure of steam.

Then, $t = 199 + 14\sqrt{p}$. **(138.)** Art. **1996.**

$$p = \left(\frac{t - 199}{14}\right)^2. \qquad \textbf{(139.)} \text{ Art. } \textbf{1997.}$$

TOTAL HEAT OF VAPORIZATION.

Let H = total heat of vaporization in B. T. U.;
t = temperature of steam.

Then, $H = 1{,}081.4 + .305\,t$. **(140.)** Art. **1999.**

SAFE WORKING PRESSURE OF BOILERS.

Let t = thickness of plate in inches;
d = diameter of shell in inches;
c = a constant;
p = safe working pressure.

Then, $p = \frac{c\,t}{d}$. **(141.)** Art. **2032.**

The constants to be used in formula **141** are as follows:

Wrought-iron plate, single-riveted joint	10,224
Wrought-iron plate, double-riveted joint	13,152
Steel plate, single-riveted joint	16,608
Steel plate, double-riveted joint	20,688

CHIMNEYS.

Let A = area of chimney;
H = horsepower of boiler;
h = height of chimney.

Then, $h = \left(\frac{H}{3.33A - 2\sqrt{A}}\right)^2$. **(142.)** Art. **2036.**

FORMULAS USED IN STEAM-ENGINES.

INDICATED HORSEPOWER.

I. H. P. = indicated horsepower of engine;
P = mean effective pressure (M. E. P.) in lb. per square inch;
A = area of piston in square inches;
L = length of stroke in feet;
N = number of strokes per minute.

$$\text{I. H. P.} = \frac{PLAN}{33,000}.$$ **(143.)** Art. **2067.**

MEAN EFFECTIVE PRESSURE.

p = gauge-pressure;
k = a constant corresponding to the apparent cut-off (see table of Constants Used in Determining M. E. P.);
M. E. P. = mean effective pressure.

$$\text{M. E. P.} = .9\,[k\,(p + 14.7) - 17].$$ **(144.)** Art. **2069.**

For a condensing engine, subtract the condenser pressure instead of 17.

MECHANICAL EFFICIENCY OF ENGINE.

I. H. P. = indicated horsepower, or total horsepower developed;
N. H. P. = the net horsepower; that is, the horsepower remaining for the performance of useful work = I. H. P. − Friction H. P.;
E_m = mechanical efficiency of engine.

$$E_m = \frac{\text{N. H. P.}}{\text{I. H. P}}.$$ **(146.)** Art. **2076.**

PISTON SPEED.

l = length of stroke in inches;
R = number of revolutions per minute;
S = piston speed in feet per minute.

$$S = \frac{lR}{6}. \qquad \textbf{(145.)} \text{ Art. } \textbf{2071.}$$

FORMULAS USED IN AIR AND AIR COMPRESSION.

AREA UNDER ADIABATIC CURVE.

p = higher pressure;
p_1 = lower pressure;
v = volume corresponding to pressure p;
v_1 = volume corresponding to pressure p_1;
A = area under curve.

$$\text{Then, } A = \frac{p\,v - p_1 v_1}{.41}. \qquad \textbf{(147.)} \text{ Art. } \textbf{2114.}$$

CALCULATION OF THE SIZE OF AN AIR-COMPRESSOR.

Let H = the number of horsepower the engine is to develop;
D = diameter of cylinder in inches;
r = ratio of length of stroke to diameter of cylinder;
P = mean effective pressure per sq. in. on the piston;
N = number of strokes per minute.

$$\text{Then, } D = 79.6 \sqrt[3]{\frac{H}{rPN}}. \qquad \textbf{(148.)} \text{ Art. } \textbf{2152.}$$

FORMULAS USED IN HYDROMECHANICS AND PUMPING.

LIQUID PRESSURE.

a = area of a submerged surface in square inches;
d = distance in inches of center of gravity of surface from surface of liquid;
w = weight of a cubic inch of the fluid in pounds;
p = pressure on surface of liquid, pounds per sq. in.;
P = total pressure on submerged surface in pounds.

$$P = a\,(d\,w + p). \qquad \textbf{(164.)} \text{ Art. } \textbf{2179.}$$

MEAN VELOCITY OF FLOW.

Let Q = the quantity in cubic feet which passes any section in 1 second;
A = the area of the section in square feet;
v = the mean velocity in feet per second.

Then, $Q = A v$. **(165.)** Art. **2185.**

$v = \frac{Q}{A}$. **(166.)** Art. **2185.**

VELOCITY OF EFFLUX OF AN ORIFICE.

Let v = the velocity of efflux in feet per second;
h = the head in feet on the orifice considered;
h_1 = the head equivalent to a pressure p.

$v = \sqrt{2gh}$. **(167.)** Art. **2186.**

$h = \frac{v^2}{2g}$. **(168.)** Art. **2186.**

$h_1 = \frac{p}{.434}$, where h_1 is in feet, and p in pounds per square inch.

$h_1 = \frac{p}{62.5}$, where h_1 is in feet, and p in pounds per square foot.

$h + h_1$ = the *total head.*

$v = \sqrt{2g(h_1 + h)}$. **(169.)** Art. **2187.**

Range $= \sqrt{4hy}$, **(170.)** Art. **2188.**

where y is the vertical height of the orifice above the point where the water strikes.

If a is the area of a large orifice in the bottom of a small vessel whose area is A, the velocity is

$v = \sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}}$. **(171.)** Art. **2189.**

Actual velocity of efflux from a small square-edged orifice:

$v = .98\sqrt{2gh}$. **(172.)** Art. **2190.**

Actual quantity discharged from a small square-edged orifice:

$$Q = .615\,A\,v. \qquad \textbf{(173.)} \text{ Art. } \textbf{2191.}$$

THEORETICAL AND ACTUAL DISCHARGE.

Let Q = theoretical number of cubic feet discharged per second;

v_m = theoretical mean velocity through orifice in feet per second $= Q \div A$;

A = area of orifice in square feet;

h = theoretical head necessary to give a mean velocity v_m;

Q_a = actual quantity discharged in cubic feet per second.

Then, for an orifice in a thin plate, or a square-edged orifice (the hole itself may be of any shape—triangular, square, circular, etc.—but the edges must not be rounded), the actual quantity discharged is

$$Q_a = .615\,Q = .615\,A\,v_m = .615\,A\sqrt{2gh}. \qquad \textbf{(174.)} \text{ Art. } \textbf{2195.}$$

For a discharge through a short tube:

$$Q_a = .815\,Q = .815\,A\,v_m = .815\,A\sqrt{2gh}. \qquad \textbf{(175.)} \text{ Art. } \textbf{2195.}$$

For a discharge through a mouthpiece:

$$Q_a = .97\,Q = .97\,A\,v_m = .97\,A\sqrt{2gh}. \qquad \textbf{(176.)} \text{ Art. } \textbf{2195.}$$

For a discharge through the compound mouthpiece, the area of the orifice being taken as the area of the smallest section:

$$Q_a = 1.5526\,Q = 1.5526\,A\,v_m = 1.5526\,A\sqrt{2gh}. \qquad \textbf{(177.)} \text{ Art. } \textbf{2195.}$$

In these four formulas it is taken for granted that there is a constant head.

FLOW OF WATER THROUGH WEIRS.

If d = the depth of the opening in feet, and b its breadth in feet, the area of the opening is $A = d \times b$, and the theoretical discharge is $Q = d \times b \times v_m = d\,b \times \frac{2}{3}\sqrt{2gh} = \frac{2}{3}\,b\,d\sqrt{2gd}$, the head for this case being taken as d.

The actual discharge is

$$Q_a = .615\ Q = .615 \times \tfrac{2}{3} b d \sqrt{2 g d} = .41\ b \sqrt{2 g d^3}. \qquad (178.)$$

Art. **2198.**

That is, *the actual discharge in cubic feet per second through a weir whose top is on a level with the upper surface of the water, is equal to .41 multiplied by the breadth of the weir, multiplied by the square root of 2 g times the cube of the depth of the weir. All dimensions are to be taken in feet.*

To obtain the mean velocity v_m, divide the actual discharge by the area of the weir, or

$$v_m = \frac{Q_a}{A} = \frac{Q_a}{b d}. \qquad (179.)$$ Art. **2199.**

For a weir whose upper edge is below the level of the upper surface of the water, let h_1 be the depth in feet of the top of the weir below the surface of the water, and h the depth in feet of the bottom of the weir below the surface of the water. The actual discharge Q_a in cubic feet per second is

$$Q_a = .41\ b \sqrt{2 g}\left(\sqrt{h^3} - \sqrt{h_1^3}\right). \qquad (180.)$$ Art. **2201.**

FLOW OF WATER IN PIPES.

The Mean Velocity of Discharge.

For straight cylindrical pipes of uniform diameter:

Let v_m = mean velocity of discharge in feet per second;
h = total head in feet = the vertical distance between the level of the water in the reservoir and the point of discharge;
l = length of pipe in feet;
d = diameter of pipe in inches;
f = coefficient of friction.

Then, $$v_m = 2.315 \sqrt{\frac{h d}{f l + \frac{1}{8} d}}. \qquad (181.)$$ Art. **2203.**

When the pipe is very long, compared with the diameter, the following formula may be used:

$$v_m = 2.315 \sqrt{\frac{h d}{f l}}. \qquad (182.)$$ Art. **2204.**

The Actual Head.

The actual head necessary to produce a certain velocity v_m may be calculated by the formula

$$h = \frac{f l v_m^2}{5.36\, d} + .0233\, v_m^2. \qquad \textbf{(183.)} \text{ Art. } \textbf{2205.}$$

The Quantity Discharged from Pipes.

Let d = the diameter of the pipe in inches; then the discharge Q in gallons per second is

$$Q = .0408\, d^2 v_m. \qquad \textbf{(184.)} \text{ Art. } \textbf{2206.}$$

If the diameter of the pipe and the discharge are known, the mean velocity v_m is

$$v_m = \frac{24.51\, Q}{d^2}. \qquad \textbf{(185.)} \text{ Art. } \textbf{2207.}$$

If the head, the length of the pipe, and the diameter of the pipe are given, to find the discharge use the formula

$$Q = .09445\, d^2 \sqrt{\frac{h d}{f l + .125\, d}}. \qquad \textbf{(186.)} \text{ Art. } \textbf{2208.}$$

To find the value of f, calculate v_m by formula **182**, assuming that $f = .025$, and get the final value of f from the following table (Art. **2209**):

v_m =	0.1	0.2	0.3	0.4	0.5	0.6
f =	.0686	.0527	.0457	.0415	.0387	.0365
v_m =	0.7	0.8	0.9	1	1¼	1½
f =	.0349	.0336	.0325	.0315	.0297	.0284
v_m =	2	3	4	6	8	12
f =	.0265	.0243	.023	.0214	.0205	.0193

POWER NECESSARY TO WORK A PUMP.

Rule.—*In all pumps, whether lifting, force, steam, single or double acting, or centrifugal, the number of foot-pounds of power needed to work the pump is equal to the weight of the water in pounds multiplied by the vertical distance in feet between the level of the water in the well, or source, and the point of discharge, plus the work necessary to overcome the friction and other resistances.* Art. **2219.**

Rule.—*The work done in one stroke of a pump is equal to the weight of a volume of water equal to the volume displaced by the piston during the stroke, multiplied by the total vertical distance in feet through which the water is to be raised, plus the work necessary to overcome the resistances.* Art. **2220.**

DUTY OF A PUMP.

The **duty** *of any pump or pumping-engine is the number of pounds of water raised one foot high for each 100 pounds of coal burned in the boiler.* Art. **2222.**

G = number of gallons discharged per hour,
h = total vertical distance in feet between the level of the water in sump, or other source of supply, and the point of discharge;
W = weight in pounds of coal burned per hour;
D = duty in foot-pounds.

$$D = \frac{835.5\,G\,h}{W}. \qquad (187.)$$ Art. **2223.**

BALANCING THE PUMP-RODS.

Let F = force causing acceleration = weight of pit-work − weight of water column − frictional resistances;
W = total weight to be accelerated = weight of pit-work + weight of water column;
g = 32.16, the acceleration due to gravity;

f = acceleration of pit-work in feet per second;
s = stroke of engine;
t = time occupied in making stroke.

Then, $f = \frac{g F}{W}$. **(188.)** Art. **2248.**

$t = \sqrt{\frac{2 s}{f}}$. **(189.)** Art. **2249.**

CALCULATIONS PERTAINING TO PUMPS.

Head and Pressure.

To find the pressure in pounds per square inch corresponding to any given head of water:

Rule.—*Multiply the head in feet by .434; the result is the pressure in pounds per square inch.* Art. **2289.**

To find the head of water corresponding to a given pressure in pounds per square inch:

Rule.—*Multiply the given pressure in pounds per square inch by 2.304; the result is the head in feet.* Art. **2290.**

Size of Plunger-Cylinder for Given Discharge.

Let G = number of gallons discharged per minute;
S = plunger speed in feet per minute;
d = diameter of cylinder in inches.

Then, the theoretic diameter is given by the formula

$d = 4.95 \sqrt{\frac{G}{S}}$. Art. **2291.**

Since there is always more or less slip of the water past the plungers, it is usual to add $\frac{1}{4}$ of the required number of gallons to the value given to G in the above formula to allow for this slip. Doing so, the formula becomes

$d = 5.535 \sqrt{\frac{G}{S}}$. **(190.)** Art. **2291.**

The Discharge of a Pump.

Allowing for slip,

$G = .03264\ d^2\ S$. **(191.)** Art. **2292.**

The theoretic discharge is

$$G = .0408\ d^2\ S.$$

Horsepower Required for a Given Discharge.

H = horsepower, allowing for friction and slip;
h = height through which water is lifted.

$$H = .00038\,G\,h. \qquad \textbf{(192.)} \text{ Art. } \textbf{2293.}$$

To find the height through which a pump will raise water with a given horsepower:

$$h = \frac{H}{.00038\,G}. \qquad \textbf{(193.)} \text{ Art. } \textbf{2294.}$$

To Find the Size of the Steam or Air Cylinder of a Pump.

Let S = piston speed;
D = diameter of cylinder in inches;
r = ratio between the length of stroke and diameter of cylinder;
l = length of stroke in feet;
N = number of strokes per minute;
H = horsepower;
P = steam or air pressure per sq. in.

Then, $$D = 205\sqrt{\frac{H}{PS}}. \qquad \textbf{(194.)} \text{ Art. } \textbf{2295.}$$

The diameter may also be found by formula **148,**

$$D = 79.6\sqrt[3]{\frac{H}{rPN}}.$$

Having obtained the diameter by either formula **194** or formula **148,** the stroke can be found by multiplying the diameter by the value of the ratio r. In case formula **194** is used, the number of strokes can be found by dividing the piston speed by the length of the stroke in feet.

Sizes of Suction and Delivery Pipes.

The usual practice is to allow a velocity of 200 feet per minute in the suction-pipe and 400 feet per minute in the delivery-pipe.

Let d_1 = diameter of suction-pipe;
d_2 = diameter of delivery-pipe.

Then, $$d_1 = .35\sqrt{G}. \qquad \textbf{(195.)} \text{ Art. } \textbf{2296.}$$

$$d_2 = .25\sqrt{G}. \qquad \textbf{(196.)} \text{ Art. } \textbf{2296.}$$

The pipes may be larger than the values calculated by the above formulas, particularly the suction-pipe, but it is not a good plan to make them any smaller. The larger the pipes are, the less the velocity, and, consequently, the less the frictional resistances.

FORMULAS USED IN MINE HAULAGE.

GRAVITY-PLANES.

Let W_1 = weight in pounds of descending loaded car;
W_2 = weight in pounds of ascending empty car;
W_3 = weight in pounds of hauling rope;
a = percentage of grade, expressed decimally;
F = available gravity force due to coal;
F_1 = total gravity force due to coal;
F_2 = total force required to overcome weight and friction of rope.

$$F_2 = a\,W_3 + \frac{W_3}{40}. \qquad \textbf{(197.)} \text{ Art. } \textbf{2326.}$$

$$F_1 = a\,(W_1 - W_2). \qquad \textbf{(198.)} \text{ Art. } \textbf{2327.}$$

$$F = a\,(W_1 - W_2) - \frac{W_1 + W_2}{40}. \qquad \textbf{(199.)} \text{ Art. } \textbf{2327.}$$

ENGINE-PLANES AND TAIL-ROPE SYSTEMS.

W = total weight of train in pounds;
W_1 = weight of train of empty cars;
w = weight of hauling rope in pounds;
w_1 = difference between weights of hauling rope and tail rope;
a = percentage of grade, expressed decimally;
T = tension in main rope in pounds;
T_1 = tension in tail-rope;
H = horsepower required for haulage;
v = velocity of rope in feet per minute.

$$T = \frac{W + w}{40} + a\,(W + w). \qquad \textbf{(200.)} \text{ Art. } \textbf{2348.}$$

When a return rope is used,

$$T = \frac{W + w}{40} + a\,W. \qquad \textbf{(201.)} \quad \text{Art. } \textbf{2353.}$$

$$T_1 = \frac{W_1 + w}{40} + a\,W_1. \qquad \text{Art. } \textbf{2366.}$$

$$H = \frac{T v}{33{,}000}. \qquad \textbf{(202.)} \quad \text{Art. } \textbf{2353.}$$

If the haulage-roads have a fall towards the bottom of the shaft, the tension in the hauling rope of a tail-rope system is

$$T = \frac{W + w}{40} - a\,(W - w_1). \qquad \textbf{(204.)} \quad \text{Art. } \textbf{2370.}$$

$$T_1 = \frac{W_1 + w}{40} + a\,(W_1 - w_1). \qquad \textbf{(205.)} \quad \text{Art. } \textbf{2370.}$$

ENDLESS-ROPE HAULAGE.

Number of Cars and Distance Apart on Haulage-Band.

O = output of coal, tons per day;
o = weight of coal in tons carried by single car;
o_1 = weight of coal in cars attached to hauling band;
D = distance traveled by a point of the rope in one day;
d = distance in feet traveled by rope from return sheave to hoisting-shaft;
n = number of full cars on main rope band;
r = distance in feet between cars on band.

Then,

$$o_1 = \frac{O d}{D}. \qquad \textbf{(206.)} \quad \text{Art. } \textbf{2405.}$$

$$n = \frac{O d}{D o}. \qquad \textbf{(207.)} \quad \text{Art. } \textbf{2405.}$$

$$r = \frac{d}{n}. \qquad \textbf{(208.)} \quad \text{Art. } \textbf{2405.}$$

Tension on Ropes and Horsepower.

W = weight of loaded cars on one side of rope band;
w_1 = weight of empty cars on ingoing side of rope band;
w = weight of rope band;
a = percentage of grade, expressed decimally;
T = tension in rope in pounds.

Then,

$$T = \frac{W + w_1 + w}{40} + a\,(W - w_1). \qquad \textbf{(210.)} \quad \text{Art. } \textbf{2412.}$$

If the road has a fall towards the shaft, the grade is negative, and the formula becomes:

$$T = \frac{W + w_1 + w}{40} - a\,(W - w_1).$$

RULES AND FORMULAS USED IN HOISTING AND HOISTING APPLIANCES.

SIZE OF ENGINE-CYLINDER.

Rule.—*To find the actual load on the engines, add to the net load 10% of the gross load.* Art. **2473.**

Rule.—*To find the work required of the engines per revolution of the drum, multiply the actual load in pounds by the working circumference of the drum in feet.* Art. **2474.**

Let D = diameter of cylinder in inches;
w = work done per revolution in foot-pounds;
P = mean effective pressure in pounds per square inch;
r = stroke divided by diameter;
r_1 = ratio of gear to pinion in second-motion engine.

$$D = 1.97\sqrt[3]{\frac{w}{Pr}}.$$ **(211.)** Art. **2475.**

For second-motion engine,

$$D = 1.97\sqrt[3]{\frac{w}{Prr_1}}.$$ **(212.)** Art. **2479.**

CONICAL DRUMS.

Let M = the weight of material;
C = the weight of cage and car;
R = the weight of rope;
D = large diameter of drum;
d = small diameter of drum.

$$D = \frac{d\,(M + 2\,C + 2\,R)}{M + 2\,C}.$$ **(213.)** Art. **2491.**

RULES USED IN PERCUSSIVE AND ROTARY BORING.

TEMPERING DRILLS.

In tempering a drill, the following points should be observed:

1. When the bit is dipped in water, it should be moved up and down, or the molecular tension above and below the water-line will be so different that the bit will be liable to break in the same way as the bottom of a glass vessel is cracked by pouring hot water into the vessel.

2. The bit of a drill should not be placed in the incandescent cinders of a fire to be heated, for the cutting edge will be decarbonized and rendered worthless.

3. The bit should be heated a few inches from the cutting edge to prevent decarbonization, and it should not be kept in the fire longer than necessary to heat it to a cherry-red heat.

4. Immediately after removing the bit from the fire, it should be dipped in water for a moment to partially cool it and then rubbed on a stone to remove the outside scale, in order that the colors can be easily distinguished.

5. The colors should advance parallel to the cutting edge, and if in any case they are observed to do otherwise, that portion of the bit to which they are advancing most rapidly should be dipped in water. Frequently it is necessary to dip the bit in water several times to obtain the proper parallelism before the final cooling. If the bit were cooled when the colors were not parallel to its cutting edge but crossed it, the cutting edge would likely be too soft in one place and too brittle in another.

6. The tool dresser should thoroughly understand how iron can be converted into steel by carbonization and steel into iron by the oxidation of a portion of its carbon. For example, if a piece of white-hot iron is buried in powdered charcoal and the air kept away from it, the skin of the iron becomes carbonized and converted into steel, and if, on the

other hand, a bar of red-hot steel is buried in oxide of iron, the skin of the steel becomes decarbonized or converted into malleable iron. In the same way, if the cutting edge of a bit is made red hot in a forge fire and kept at that heat for some time, it will be decarbonized or converted into malleable iron. This is why care should be exercised in heating the drill.

7. The bits of drills give better results when tempered in thick oil or coal-tar than when tempered in water, the reason being that the water rapidly chills the thin parts and the skin of the thick parts, which produces uneven hardness in the bit, while the oil or tar cools the bit more gradually and evenly and renders it more tough. If it is found that a certain bit should be dipped in water when it has a blue color, it should be dipped in oil when it has a purple color. In other words, in order to produce the same degree of hardness while tempering with oil that has been obtained by tempering with water, the bit should be dipped in the oil when it has the color which precedes the one which it has when dipped in water to obtain the best temper. This is due to the fact that the oil cools the bit more slowly. In all cases the oil makes the bit tougher and more reliable than it can be made by the use of water.

8. The best temper for bits made of good steel is produced by dipping the bit in water when it is blue, or in oil when it is a very light blue.

9. The colors are deep and distinct for good steel and scarcely perceptible for poor steel; consequently, a practised eye can determine very accurately the quality of the steel by the depth of the running colors. Art. **30.**

DIAMOND-DRILL CORE RECORDS.

The general rule for the above may be stated as follows:

Rule.—*The value of the record furnished by the diamond-drill core varies inversely as the value per ton of the deposit sought.*

Stating this differently, we have the following

Rule.—*The value of the record furnished by the diamond drill is greater when prospecting for low-grade, uniformly distributed ores than when prospecting for high-grade irregularly distributed ores.* Art. **110.**

FORMULAS USED IN COMPRESSED-AIR COAL-CUTTING MACHINERY.

CALCULATIONS RELATING TO PICK MACHINES.

Let U = work done per stroke;
H = horsepower;
V = velocity of pick in feet per second at moment of impact;
W = weight of pick and piston in pounds;
F = cutting force of pick in pounds;
a = area of piston in square inches;
d = depth of cut per stroke in feet;
g = 32.16;
l = length of stroke in feet;
n = number of strokes per minute;
p = steam pressure in pounds per square inch.

The work done by each stroke of machine is

$$U = \frac{H \times 33,000}{n}. \qquad (1.) \quad \text{Art. } \mathbf{24.}$$

The force of a blow struck by the machine is

$$F = \frac{V^2 W}{2 g d}. \qquad (2.) \quad \text{Art. } \mathbf{25.}$$

The velocity of the pick at impact is

$$V = \sqrt{\frac{2 g a p l}{W}} = \sqrt{\frac{2 g U}{W}}. \qquad (3.) \quad \text{Art. } \mathbf{27.}$$

CALCULATIONS RELATING TO CHAIN-CUTTER MACHINES.

Let F = cutting force of cutter chain in pounds;
P = horsepower of engine;
S = speed of cutter chain in feet per minute.

$$F = \frac{P \times 33,000}{S}. \qquad \textbf{(4.)} \text{ Art. } \textbf{42.}$$

RULES AND FORMULAS USED IN DYNAMOS AND MOTORS.

DIRECTION OF LINES OF FORCE AROUND A CONDUCTOR.

Rule.—*If the current is flowing in the conductor away from the observer, then the direction of the lines of force will be around the conductor in the direction of the hands of a watch.* Art. **26.**

TO DETERMINE THE POLARITY OF A SOLENOID.

Rule.—*In looking at the end of the helix, if it is so wound that the current circulates around the helix in the direction of the hands of a watch, that end will be a south pole; if in the other direction, it will be a north pole.* Art. **29.**

RESISTANCE OF CONDUCTORS.

Let r_1 = original resistance of a conductor;
r_2 = changed resistance;
l_1 = original length;
l_2 = changed length;
a_1 = original sectional area;
a_2 = changed sectional area;
D = original diameter;
d = changed diameter;
k = temperature coefficient;
t = rise or fall in temperature, degrees Fahrenheit.

For a change in the length of a conductor:

$$r_2 = \frac{r_1 l_2}{l_1}. \qquad \textbf{(1.)} \text{ Art. } \textbf{40.}$$

For a change in the sectional area of a conductor:

$$r_2 = \frac{r_1 a_1}{a_2}. \qquad (2.) \quad \text{Art. 41.}$$

For a change in the diameter of a conductor:

$$r_2 = \frac{r_1 D^2}{d^2}. \qquad (3.) \quad \text{Art. 42.}$$

For a rise in the temperature of a conductor:

$$r_2 = r_1 (1 + t\,k). \qquad (4.) \quad \text{Art. 46.}$$

For a fall in the temperature of a conductor:

$$r_2 = \frac{r_1}{1 + t\,k}. \qquad (5.) \quad \text{Art. 47.}$$

RESISTANCES AND TEMPERATURE COEFFICIENTS OF DIFFERENT METALS.

Name of Metal.	Resistance, Microhms per Cu. In.	Relative Resistance.	Temperature Coefficient.
Silver, annealed.......	.5921	1.000	.002094
Copper, annealed......	.6292	1.063	.002155
Silver, hard-drawn....	.6433	1.086	.002094
Copper, hard-drawn...	.6433	1.086	.002155
Gold, annealed........	.8102	1.369	.002028
Gold, hard-drawn.....	.8247	1.393	.002028
Aluminum, annealed..	1.1470	1.935	....
Zinc, pressed.........	2.2150	3.741	.002028
Platinum, annealed....	3.5650	6.022	....
Iron, annealed........	3.8250	6.460	....
Nickel, annealed......	4.9070	8.285	....
Tin, pressed..........	5.2020	8.784	.002028
Lead, pressed.........	7.7280	13.050	.002150
German Silver........	8.2400	13.920	.000244
Antimony, pressed....	13.9800	23.600	.002161
Mercury..............	37.1500	62.730	.000400
Bismuth, pressed......	51.6500	87.230	.001967

CURRENT STRENGTH, ELECTROMOTIVE FORCE, AND RESISTANCE.

Let C = strength of current flowing in a closed circuit;
E = electromotive force;
R = resistance.

$$C = \frac{E}{R}. \qquad (6.) \quad \text{Art. } \mathbf{61}.$$

$$R = \frac{E}{C}. \qquad (7.) \quad \text{Art. } \mathbf{62}.$$

$$E = CR. \qquad (8.) \quad \text{Art. } \mathbf{63}.$$

TO FIND THE AVAILABLE ELECTROMOTIVE FORCE IN A CELL.

Let E = the total generated E. M. F.;
E' = *available* E. M. F. when the circuit is closed;
C = the current flowing when the circuit is closed,
r_i = the internal resistance of the cell.

$$E' = E - C r_i. \qquad (9.) \quad \text{Art. } \mathbf{67}.$$

THE CURRENT AND RESISTANCE IN BRANCHES OF DIVIDED CONDUCTORS.

Let r_1 = resistance of first branch;
r_2 = resistance of second branch;
r_3 = resistance of third branch;
c_1 = current in first branch;
c_2 = current in second branch;
C = sum of the currents in the two branches;
R'' = joint resistance of two branches in parallel;
R''' = joint resistance of three branches in parallel.

$$c_1 = \frac{C r_2}{r_1 + r_2}. \qquad (10.) \quad \text{Art. } \mathbf{69}.$$

$$c_2 = \frac{C r_1}{r_1 + r_2}. \qquad (11.) \quad \text{Art. } \mathbf{69}.$$

$$R'' = \frac{r_1 r_2}{r_1 + r_2}. \qquad \textbf{(12.)} \quad \text{Art. } \textbf{71.}$$

$$R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}. \qquad \textbf{(13.)} \quad \text{Art. } \textbf{72.}$$

ELECTRICAL QUANTITY.

Let Q = quantity of electricity in coulombs;
C = current strength in amperes;
t = time in seconds;

$$Q = C\,t. \qquad \textbf{(14.)} \quad \text{Art. } \textbf{76.}$$

ELECTRICAL WORK AND POWER.

J = electrical work in joules;
F. P. = work in foot-pounds;
Q = quantity of electricity in coulombs;
C = current in amperes;
t = time in seconds during which the current flows;
E = potential, or E. M. F., of circuit;
R = resistance of circuit;
W = power in watts;
H. P. = horsepower.

$$J = C\,E\,t. \qquad \textbf{(15.)} \quad \text{Art. } \textbf{78.}$$

$$J = C^2 R\,t \qquad \textbf{(16.)} \quad \text{Art. } \textbf{78.}$$

$$J = \frac{E^2 t}{R}. \qquad \textbf{(17.)} \quad \text{Art. } \textbf{78.}$$

$$\text{F. P.} = .7373\,J. \qquad \textbf{(18.)} \quad \text{Art. } \textbf{79.}$$

$$W = C\,E. \qquad \textbf{(19.)} \quad \text{Art. } \textbf{80.}$$

$$W = C^2 R. \qquad \textbf{(20.)} \quad \text{Art. } \textbf{80.}$$

$$W = \frac{E^2}{R}. \qquad \textbf{(21.)} \quad \text{Art. } \textbf{80.}$$

$$H.\ P. = \frac{W}{746}. \qquad (22.)\ \text{Art. } \mathbf{81}.$$

$$W = H.\ P. \times 746. \qquad (23.)\ \text{Art. } \mathbf{82}.$$

TO DETERMINE THE DIRECTION OF THE CURRENT GENERATED IN A CONDUCTOR.

Rule.—*Place thumb, forefinger, and middle finger of the right hand so that each will be perpendicular to the other two; if the forefinger points in the direction of the lines of force and the thumb points in the direction towards which the conductor is moving, then the middle finger will point in the direction towards which the current generated in the conductor tends to flow.* Art. **8.**

DETERMINATION OF ELECTROMOTIVE FORCE.

Let E = maximum electromotive force obtained at the brushes;

N = total number of lines of force passing from the north pole through the core to the south pole;

S = number of outside wires on the periphery through which the current flows *in series;*

n = number of complete revolutions per second of the core.

$$E = \frac{2NSn}{10^8}. \qquad (1.)\ \text{Art. } \mathbf{23}.$$

TO DETERMINE THE DIRECTION OF MOTION IMPARTED TO A CONDUCTOR.

Rule.—*Place thumb, forefinger, and middle finger of the left hand each at right angles to the other two; if the forefinger points in the direction of the lines of force and the middle finger points in the direction towards which the current flows, then the thumb will point in the direction of movement imparted to the conductor.* Art. **26.**

EFFICIENCY OF A DYNAMO.

Let I = input of a dynamo;
O = output;
E = efficiency, per cent.

$$\mathsf{E} = \frac{100 \times O}{I}. \qquad \textbf{(2.)} \quad \text{Art. } \textbf{60.}$$

PER CENT. LOSS IN A DYNAMO.

Let L = per cent. loss;
I = input;
O = output;
E = efficiency, per cent.

$$L = \frac{100\,(I - O)}{I}. \qquad \textbf{(3.)} \quad \text{Art. } \textbf{61.}$$

When the output and efficiency are given:

$$I = \frac{100 \times O}{\mathsf{E}}. \qquad \textbf{(4.)} \quad \text{Art. } \textbf{62.}$$

When the input and efficiency are given:

$$O = \frac{I\,\mathsf{E}}{100}. \qquad \textbf{(5.)} \quad \text{Art. } \textbf{63.}$$

RELATION BETWEEN ALTERNATING AND DIRECT-CURRENT VOLTAGE IN ROTARY TRANSFORMERS.

Let $\bar{E}$ = alternating voltage;
V = direct-current voltage.

For single-phase transformers:

$$\bar{E} = .707\,V. \qquad \textbf{(1.)} \quad \text{Art. } \textbf{45.}$$

For three-phase transformers:

$$\bar{E} = .612\,V. \qquad \textbf{(2.)} \quad \text{Art. } \textbf{47.}$$

HORSEPOWER, TORQUE, AND NUMBER OF REVOLUTIONS OF MOTORS.

Let H. P. = horsepower;
T = torque;
S = number of revolutions per minute.

H. P. = .0001904 TS. **(3.)** Art. **62.**

$$S = \frac{\text{H. P.}}{.0001904\,T}. \quad \textbf{(4.)} \text{ Art. } \textbf{62.}$$

$$T = \frac{\text{H. P.}}{.0001904\,S}. \quad \textbf{(5.)} \text{ Art. } \textbf{62.}$$

FORMULAS USED IN ELECTRIC PUMPING, SIGNALING, AND LIGHTING.

CURRENT REQUIRED FOR INCANDESCENT LAMPS.

Let c = current;
c. p. = candle-power of lamp;
E = voltage at which lamp is operated.

$$c = \frac{\text{c. p.} \times 3.5}{E}. \quad \textbf{(1.)} \text{ Art. } \textbf{48.}$$

DIAMETER AND CROSS-SECTION OF WIRES.

Let CM = area of cross-section in circular mils;
d = diameter in mils.

$CM = d^2$. Art. **55.**

AREA OF WIRES TO CARRY A GIVEN CURRENT.

Let C = current supplied over the line;
L = total length of the line *in feet* (i. e., distance to lamps and return);
E = voltage at end of circuit where lights are located;
% = percentage drop (i. e., percentage of voltage at the lamps);
A = area of cross-section of wire in circular mils.

$$A = \frac{10.8 \times L \times C \times 100}{E \times \%}. \qquad (2.) \quad \text{Art. } \mathbf{60.}$$

$$A = \frac{10.8 \times L \times C}{\text{volts drop}}. \qquad (3.) \quad \text{Art. } \mathbf{60.}$$

RULES AND FORMULAS USED IN ASSAYING.

CALCULATING WEIGHT OF GOLD AND SILVER FROM ASSAY BUTTONS.

Gold and Silver.

Rule.—*The weight of the button in milligrams divided by the weight of ore taken in assay tons gives the number of ounces per ton.*

Or, letting w = weight of button in mg. ;
W = weight of ore taken in A. T. ;
N = number of ounces per ton,

we have $N = \frac{w}{W}$. Arts. **30, 120.**

Ores Containing Metallic Scales.

Let A = weight of the pulp in grams ;
B = weight of the scales in grams ;
C = assay value of pulp in ounces of gold or silver per ton (or mg. per A. T.) ;
D = weight of the gold or silver in the scales, in milligrams ;
N = number of milligrams of gold or silver per A. T., or the number of ounces per ton.

Then, $N = \frac{A\,C + 29.166\,D}{A + B}$. Art. **124.**

FORMULAS USED IN PLACER AND HYDRAULIC MINING.

DISCHARGE OF WEIRS.

Let l = length of the weir in feet;
H = measured head in feet;
v = velocity with which the water approaches the weir in feet per second;
h = head equivalent to the velocity with which the water approaches the weir, or a head which would produce a velocity equal to v;
c = coefficient of discharge;
Q = actual discharge in cubic feet per second.

Then the actual discharge for weirs with end contractions is

$$Q = 5.347\,c\,l\,(H + \tfrac{4}{3}h)^{\frac{3}{2}}, \qquad \textbf{(1.)}$$

where the water approaches the weir with a velocity equivalent to the height h, and

$$Q = 5.347\,c\,l\,H^{\frac{3}{2}}, \qquad \textbf{(2.)}$$

where the water has no velocity of approach.

The actual discharge for weirs without end contractions is

$$Q = 5.347\,c\,l\,(H + 1.4\,h)^{\frac{3}{2}}, \qquad \textbf{(3.)}$$

where the water has a velocity of approach, and

$$Q = 5.347\,c\,l\,H^{\frac{3}{2}}, \qquad \textbf{(4.)}$$

where the water has no velocity of approach. Art. **101.**

VELOCITY OF APPROACH.

Let A = the area of the cross-section of the canal;
v = the velocity of approach;
Q = the quantity of water.

Then,

$$v = \frac{Q}{A}.$$

Q may be obtained approximately by assuming that v is equal to zero and applying the formula for the class of weir in question, as given above. Having obtained this quantity Q and from it the value of v, the equivalent head h may be found by the following formula:

$$h = 0.01555\, v^2. \qquad (5.)$$

Since v is small with a properly constructed weir, it is usually neglected unless great accuracy is required. Art. **102.**

RULES USED IN METAL MINING.

FIRING A BLAST BY ELECTRICITY.

To insure success in firing a blast by electricity, the following points should be observed:

1. That the battery wire and primers are suitable to each other. (Never use primers of two different kinds in the same blast.)
2. That the battery is of sufficient power to fire all the caps or primers connected at one time.
3. That the electric fuses or primers are kept in a dry place, and that everything is kept as clean as possible.
4. That all the joints at connections and points of contact of the wires are well made, and that the surfaces are clean. Also, that the joints in one wire do not touch those in another.
5. That the wires do not kink or twist so as to cut the insulation during the process of tamping. (If the insulation is cut, the fuse is useless for wet ground or a wet hole and should be laid to one side.)
6. That the operator's hands do not touch the terminals of the battery when firing.
7. That the battery is not connected to the leading wire or cable until every one is in safety. Art. **152.**

RULES USED IN ORE DRESSING AND MILLING.

LAW OF EQUAL FALLING PARTICLES.

Bodies falling freely in a fluid descend at speeds proportional to their weights divided by the resistance of the fluid.