REINFORCED CONCRETE DESIGN

HAWKESWORTH

D·VAN NOSTRAND COMPANY

GRAPHICAL HANDBOOK

FOR

REINFORCED CONCRETE DESIGN

BY

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PREFACE.

In this work, there is presented a series of plates, showing graphically, by means of plotted curves, the required design for slabs, beams, and columns, under various conditions of external loading, together with practical examples explaining the method of using each plate. The design for most of the more commonly occurring forms of reinforced concrete construction may be ascertained directly from these plates, without performing any of the computations ordinarily required. While practically nothing more than an inspection of the plates is needed to select a design for given conditions, nothing is sacrificed in the way of flexibility by the graphical method, but, on the contrary, an exceptionally wide range of choice is afforded as to the relative proportions of steel and concrete to be used.

The unit stresses prescribed by the Building Code of New York City have been adopted throughout, as a standard, and it is believed that the methods here used are those sanctioned by the best practice at the present time.

This book is expected to appeal chiefly to those architects and engineers whose work in reinforced concrete design is intermittent in its nature, and does not warrant the steady employment of a "concrete engineer." In such offices, the use of a graphical handbook should render it unnecessary to call in expert assistance to solve the majority of problems ordinarily encountered.

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APPENDIX.

INTRODUCTION.

In presenting a handbook for the design of the simpler forms of reinforced concrete construction, such as beams, slabs, and columns, by means of plotted curves, it is not the writer's intention to compete in any way with the several excellent works on reinforced concrete already published. No architect or engineer engaged in the design of concrete work should be without at least one of these volumes of reference, and the comparison of various formulas, different methods of reinforcement, and examples of more complicated construction should be carefully studied before the advantages of a graphical handbook can be appreciated, or the method of using it fully mastered.

To those familiar with such classified computations as have been published in the form of tables, a serious disadvantage will probably have been experienced in the lack of flexibility of this method. For given conditions as to span and loading there can be found in such tables, as a rule, only one or two designs which fulfill the required conditions. When, as in actual practice frequently occurs, it becomes necessary to limit the thickness of a slab or the depth of a beam, the percentage of reinforcement must often be increased beyond that for which the table has been computed.

Most of the tables that have been published are based on a certain percentage of reinforcement which is believed to develop the maximum structural efficiency; that is to say, which will stress both the steel in tension and the concrete in compression to their respective safe limits. This point of maximum structural efficiency, depending on the percentage of steel in tension reinforcement, is clearly shown by the curves presented on these plates, according to the formulas employed. However, in selecting a design from plotted curves it is no more necessary to choose this percentage of reinforcement than any other.

In addition to the condition already mentioned as requiring an increased percentage of reinforcement, there is always the consideration as to the relative cost of steel and concrete. The cost of labor and materials being different in various localities, it is obvious that the maximum economy in cost of construction may be obtained by the use of various percentages of steel, according to the locality. The percentage of steel giving this maximum economy of cost is quite distinct from, though to a certain degree dependent upon, the percentage of steel which produces the maximum structural efficiency. The latter is often erroneously referred to as the "economic ratio," which is misleading in that it implies that by the use of such a ratio the greatest economy in cost will be obtained. Before attempting the design of a concrete structure, it is advantageous, though not essential, to determine what percentages of steel will give the cheapest construction for slabs, beams, and columns respectively, for the given locality, and this percentage should be selected from the plates as frequently as the conditions of the design will permit. For a most excellent discussion in regard to the determination of this "economic cost percentage," the reader is referred to an article written by Capt. John S. Sewell, and presented to the American Society of Civil Engineers on Feb. 21, 1906.

Probably the first point that will be remarked on glancing over the plates here presented is the lack of any designs dealing with concrete structures reinforced both in tension and in compression. It is the writer's belief that compression reinforcement for members subjected to transverse loading is required only in comparatively few cases. The use of "double reinforcement" is only justifiable in girders of far greater depth than is commonly employed, or in cases where the member must be made unusually shallow to fulfill the architectural features of the design. Such reinforcement can in no case give "maximum economy of cost," and it is but seldom that "maximum structural efficiency" is obtained. When conditions arise demanding the use of "double reinforcement," the design should be carefully worked out by accepted formulas for the special case involved. In such a design there will always be conditions that cannot be shown graphically to any advantage, and the subject is one that should receive special treatment, just as a steel plate girder would seldom be designed from a manufacturer's handbook.

It will also be noted that the formulas, from which the curves on Plates VI and VII are plotted, are not applicable for T beams in which the neutral axis lies below the under side of the flange, or table, of the beam. It is believed that such cases are comparatively rare, and the computations required do not lend themselves readily to graphical demonstration. After designing a T beam from the plates, on the assumption that their use is permissible on account of the location of the neutral axis within the flange of the T, the assumption should always be verified by reference to the plate giving the true location of the neutral axis for the design obtained. If it is found that the neutral axis does not lie within the flange as assumed, then either the thickness of the flange must be increased by the necessary amount, or the design computed by means of the formulas given in the Appendix, with sufficient shearing reinforcement introduced between the flange and the stem of the beam. However, as previously stated, the cases in which the latter method must be employed are infrequent, and their treatment sufficiently complicated to exclude them from being expressed in handbook form.

For any given depth of member the ratio of cross-sectional area of metal to the cross-sectional area of the member, often referred to as the percentage of metal, may always be expressed as a certain area of metal per foot of breadth. The advantage of stating the amount of reinforcement as so much area per foot of breadth will be evident when it is remembered that in practice it is usually desired to ascertain at once the sizes of bars, and number of the same, that will give the required amount of reinforcement. For this purpose the above method is particularly adapted, since plates may be prepared as shown in this book, for the conversion of the area so expressed in square inches per foot of breadth, into the requisite sizes of square and round bars, with the spacing required, and vice versa.

A few words may be said here in favor of the method of computation that has been adopted, and the formulas used, the deduction of which is taken up in the Appendix. All the methods of reinforced concrete designing may be divided roughly into two classes. The first consists in making the computations on the basis of ultimate resistance, and then using as a safe working load whatever fraction of the ultimate that is necessary to produce the required factor of safety. This method is open to one serious objection; namely, that while perfectly applicable to members of homogeneous material, it is not always applicable to members of heterogeneous materials, such as a combination of steel and concrete. To those who have given a little study to the subject as presented in the works of reference already referred to, it will have become apparent that the neutral axis for a beam composed of concrete and steel is never in the same place for any two given conditions of loading. Hence, for the case of the loading that will produce failure in a beam, the neutral axis will, it is true, be in a definite and ascertainable position, but this position will not be the same for any other and smaller amount of loading. However, all computations for the resisting moment of a beam at the moment of failure usually have for a factor the distance of the neutral axis from the compression surface under the conditions produced by the ultimate load. Without going too deeply into the mechanics of the subject, it is evident that if the ultimate load be divided by five, for example, with the intention of obtaining this value as a factor of safety, and the beam is loaded to an amount equal to one-fifth of the ultimate load, the neutral axis cannot be in the same position as before. Under this so-called "safe load" either steel or concrete may still possibly be stressed beyond their individual safe limits, on account of the new location of the neutral axis, or in some cases one or the other may not be taking the full amount of stress which it reasonably should, thus producing an uneconomical design. In any event, by this method of designing, it is impossible to say what are the actual stresses in the steel, or in the concrete at the top surface, unless the design is recomputed with due regard to the new position of the neutral axis under the allowable superimposed loading. From this it is also evident that such a method cannot comply with any building code or specifications which state the maximum allowable working stresses in the steel and in the concrete, since, as has been stated, the latter are not considered in making the design. It may be mentioned here that formulas of this nature seem to be in especial favor with the manufacturers of various patented bars. By the introduction of certain factors or other means, it is possible to make the design appear to require more steel than is at all necessary, while the fact that the concrete at the top surface of beams or slabs is stressed in compression far beyond its accepted working value is completely suppressed, since the actual working stresses are seldom investigated by the users of these formulas or tables.

The second method referred to consists in making the design on the basis of maximum allowable superimposed loads, and the actual bending moments thereby induced. The true position of the neutral axis under these conditions of actual loading is obtained, and the resisting moment computed in such a way that the working stresses in the steel and in the concrete cannot be exceeded. Thus it is possible to assign definite values to the maximum stresses allowed for each material, and to make sure that each is taking the amount of stress desired according to the conditions of maximum efficiency, without at the same time exceeding in any case the safe limits proscribed. By this method, which is the one here employed, the design is always determinate in every feature under the actual required conditions.

There will doubtless be some criticism of the values designated as maximum allowable unit stresses for the different materials employed, even if the method of design here used is admitted to be the most rational which can be stated at the present time. The constants employed are those designated in an amendment to the Building Code of New York City, adopted Sept. 9, 1903. Whether or not these allowable stresses are in error on the side of excessive safety is not a question to be discussed here. It is intended only to present a method of rapid designing, using unit stresses which fulfill the requirements of the Building Code of New York City, and will furnish for other localities a conservative design that will be undeniably safe. Under conditions which would warrant an increase in the allowable stresses, it should not be difficult to introduce the required factors that will still permit the plates to be used, with the addition of a few arithmetical reductions. In only one case, namely, that of column design, has it seemed advantageous to provide for more than one condition of allowable unit stress. This has been done to meet the rapidly prevailing theory that the unit stress in compression on a concrete column should be to some extent varied with the unsupported length of the column.

The argument may be advanced that within the limits of working stresses the coefficient of elasticity for concrete under compression is practically constant, or, in other words, the stress-strain curve is a straight line instead of a parabola. There is considerable reason for this belief, when the records of tests on concrete under direct compression are examined. However, the fact has not been quite so clearly demonstrated for the case of compression due to flexure. Furthermore, the use of the constants here designated fixes not only the allowable maximum compressive stress on the concrete, but also the ratio between the coefficients of elasticity of steel and concrete respectively. Both of these assigned values are believed by the writer to be somewhat less than conservative practice requires. The parabolic formula was therefore introduced to offset, to a small extent, the effect produced by these low constants, and the use of the parabolic stress-strain curve would seem to be justified, from its adoption by so large a number of authorities on this subject.

In concluding this brief and necessarily incomplete summary of the theory and purposes of the following plates, the writer wishes to acknowledge his appreciation of the valuable suggestions and assistance of Mr. W. T. Derleth, of the Department of Civil Engineering at Columbia University, and also desires to state his indebtedness to the excellent work on Reinforced Concrete, by Mr. C. F. Marsh, from whose clear and logical presentation of the theory of computation most of the formulas stated in the Appendix have been derived. While these formulas may be of interest to those desiring to investigate more fully the methods here adopted, it is hoped that the plates themselves will attain their greatest usefulness in the offices of architects and engineers whose work does not justify their employment of a specialist in concrete design. After a little practice, which may be obtained by tracing out the examples given with each plate, it should be comparatively easy to solve the great majority of problems encountered in ordinary reinforced concrete design. This should render it unnecessary for the architect or engineer, who may only occasionally be required to design work in reinforced concrete, to call to his aid such contracting firms as may offer gratuitously the services of their "engineering force," and whose motives are not always above suspicion.

Although copies of these plates, now published for the first time, have been used in actual practice for over a year, it is believed that many improvements could still be made, and any changes or corrections which may present themselves to the mind of the reader will be received by the writer with most grateful appreciation. J. H.

NEW YORK, November, 1906.

PLATE I.

Description.

By means of the curve shown on this plate certain numerical constants corresponding to various percentages of tensile reinforcement may readily be ascertained. These constants are the source from which is derived all subsequent values of the resisting moment for concrete members subjected to bending, and reinforced only for the purpose of resisting tension. The percentage of such tensile reinforcement is computed from the ratio of the cross-sectional area of the steel to the area obtained by multiplying the breadth (b) of the concrete member by the distance (h) from the axis of reinforcement to the surface of concrete under maximum compression.

The curve was plotted in the following manner. Various numerical values were assigned to (b) and (h), and various values of the ratio of steel area to the concrete area (bh). For each case the numerical value of (u), the distance of the neutral axis from the surface of the member under maximum compression, was computed. These values of (u) were then inserted in the two formulas for the resisting moment, using the proper working stresses. Thus, for each assumed member and for each percentage of reinforcement, there were obtained two values of the resisting moment. Then one of these values represented the resisting moment at which the steel would be stressed in tension to its maximum allowable safe stress; the other value represented the resisting moment at which the concrete furthest from the neutral axis toward the compression side would be stressed in compression to its maximum allowable safe stress. The lesser of these two resisting moments was therefore chosen as the one at which the stress in neither the concrete nor steel would exceed the allowable value. For each individual case this lesser value of the resisting moment was divided by the quantity (bh^2), or the breadth of the assumed member multiplied by the square of the distance from the axis of the reinforcement to the surface of the concrete under maximum compression. This quotient was found to have a constant value for any given percentage of reinforcement, irrespective of the assumed values of (b) and (h). By laying out these quotients as ordinates, corresponding to various values of the percentage of reinforcement as abscissas, the curve was drawn as shown. The straight-line portion of the curve extends to a percentage of reinforcement up to and including which the allowable stress in the steel is the limiting factor, as determined by using the lesser of the two resisting moments previously referred to. At the percentage where the straight line changes to a curve the two resisting moments are equal, and hence both steel and concrete are stressed to their allowable safe limits. This percentage, therefore, produces the maximum structural efficiency. Beyond this point the limiting factor is the maximum allowable compressive stress in the concrete. This plate need rarely be used in practical work, but to show its entire comprehensiveness the following examples are given.

Use of Plate I.

Example.—What will be the resisting moment (M) for a beam whose breadth (b) is 8 in. and distance (h) of reinforcement from compression surface is 12 in., the area of steel section being 0.96 sq. in.?

Solution.—Percentage of reinforcement $=0.96 \div 8 \times 12 = 1\%$.
Selecting the value 1% at the bottom of the plate, follow the vertical line upward until it intersects the curve. From this point follow the horizontal line to the left to the value of the constant (K) which is found to be 100.

Then, since $M = Kbh^2$,
there results $M = 100 \times 8 \times 144 = 115200$ inch-lbs.,

which is the value of the resisting moment desired.

Example.—Given a floor-slab subjected to a bending moment of 17280 inch-lbs., the total depth of which must not exceed 5 inches. What area of steel section will be required for each 12-inch width?

Solution.—Allowing 1 inch of the total depth for protection below the steel, we have $h=4$ inches.

Then, since $$K=\frac{M}{bh^2},$$

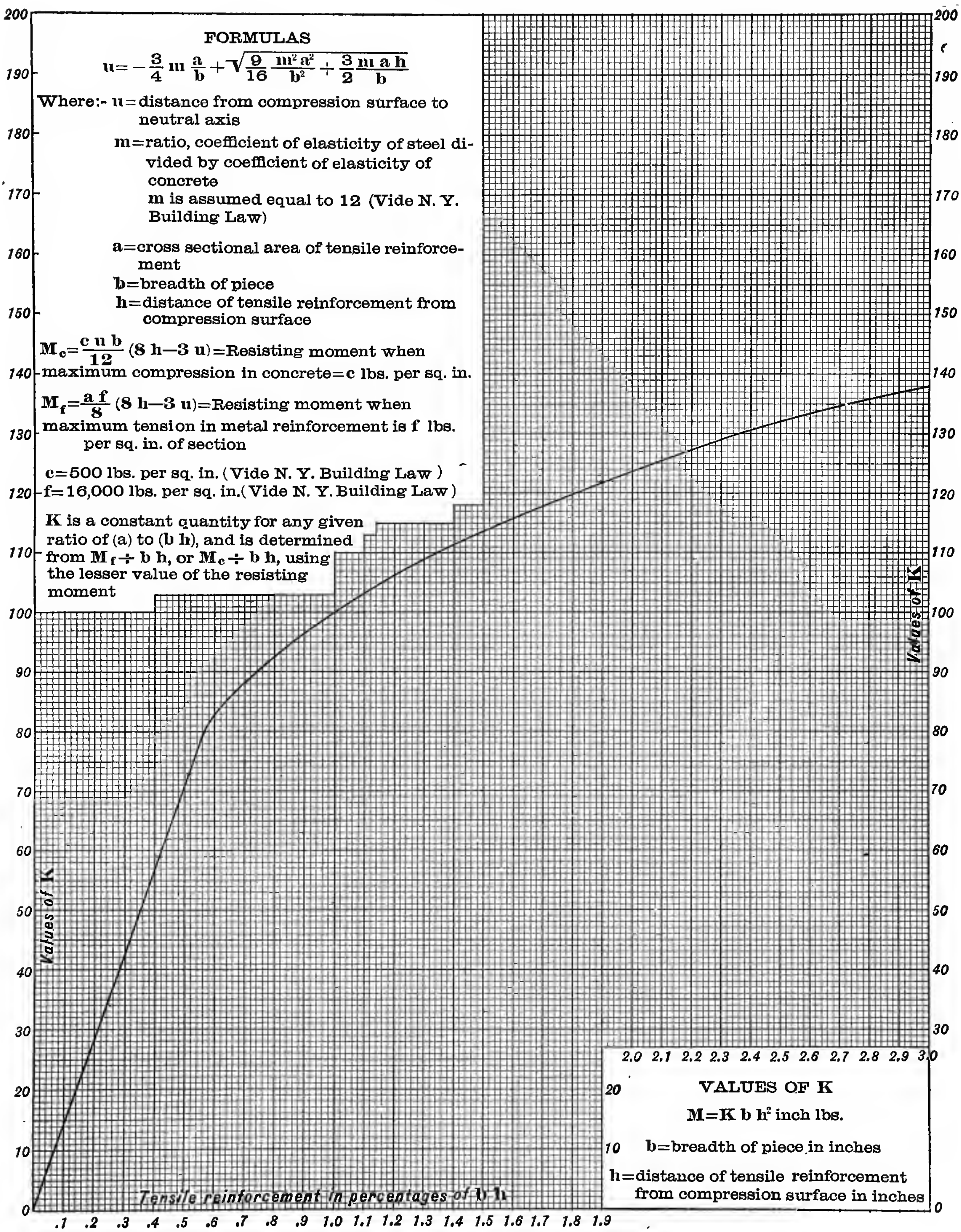
we obtain $$K=\frac{17280}{12 \times 16}=90.$$

Selecting the value of 90 at the left-hand side of the plate, follow the horizontal line to the right to its intersection with the curve. From this point follow the vertical line down to the value 0.74, which is the percentage of steel section needed.

$$0.74\% \text{ of } 12 \times 4 = .3552 \text{ sq. in.,}$$

which is the area of steel section required for each 12-inch width.

PLATE I.


FORMULAS
$u=-\frac{3}{4}m\frac{a}{b}+\sqrt{\frac{9}{16}\frac{m^2a^2}{b^2}+\frac{3}{2}\frac{m\,a\,h}{b}}$
Where:- u=distance from compression surface to neutral axis
m=ratio, coefficient of elasticity of steel divided by coefficient of elasticity of concrete
m is assumed equal to 12 (Vide N. Y. Building Law)
a=cross sectional area of tensile reinforcement
b=breadth of piece
h=distance of tensile reinforcement from compression surface
$M_c=\frac{c\,u\,b}{12}(8\,h-3\,u)$=Resisting moment when maximum compression in concrete=c lbs. per sq. in.
$M_f=\frac{a\,f}{8}(8\,h-3\,u)$=Resisting moment when maximum tension in metal reinforcement is f lbs. per sq. in. of section
c=500 lbs. per sq. in. (Vide N. Y. Building Law)
f=16,000 lbs. per sq. in. (Vide N. Y. Building Law)
K is a constant quantity for any given ratio of (a) to (b h), and is determined from $M_f \div b\,h$, or $M_c \div b\,h$, using the lesser value of the resisting moment
Values of K
Tensile reinforcement in percentages of b h
VALUES OF K
$M=K\,b\,h^2$ inch lbs.
b=breadth of piece in inches
h=distance of tensile reinforcement from compression surface in inches
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3.0

PLATES II AND III.

DESCRIPTION.

THESE plates are intended principally for the design of slabs, reinforced to resist tension only. The left-hand half of the plates gives the curves for values of (h) varying from 3 to 7 inches. To obtain the total depth of any slab, a certain thickness of concrete should be added to protect the reinforcement, usually not more than 1 inch nor less than ½ inch. The ordinates for these curves are the resisting moments (in inch-lbs.) for a strip 12 inches wide, having various percentages of reinforcement, the latter being plotted as abscissas. The percentage of reinforcement is expressed in terms of $b \times h$, or the breadth of the 12-inch strip multiplied by the distance of the reinforcement from the surface under maximum compression. To facilitate the operation of determining the actual area of steel cross-section in the 12-inch strip, certain definite areas of cross-section were selected, and the percentage which they represent has been noted on the curve corresponding to each value of (h), the points noted for any given area being connected simply to enable them to be readily selected.

The right-hand side of the plates shows the external bending moment produced on a 12-inch strip for various spans and loadings. The strip is assumed to be continuous over the supports for the same, and the formula is that proscribed by the Building Code of New York City. The straight lines represent various spans, the length of the span being noted in feet. The abscissas are various loads per linear foot of span, or, since a strip one foot wide is being considered, the loading will be that weight per square foot, including its own weight, which is imposed upon the slab. The external bending moments are plotted as ordinates, and correspond with the ordinates representing the resisting moment at the left-hand side of each plate. It must be noted that the scale is twice as great for Plate II as for Plate III, but this need cause no confusion if the figures are used.

Use of Plate II.

Example.—Given a span of 10 feet for a floor slab extending continuously over a number of girders, with a live load of 60 lbs. per sq. ft.; what depth of slab and what area of steel section per foot of breadth will be required?

Solution.—A little practice in designing will soon enable one to determine by inspection that a slab for this span and loading will not much exceed 4 inches in depth. Or the depth may possibly be limited to 4 inches. In such a case, we may assume 50 lbs. per sq. ft. as the dead load. The live load plus the dead load will then be 110 lbs. Selecting this value at the bottom of the right-hand half of Plate II, follow the vertical line up to its intersection with the line representing a 10-ft. span. From this point follow the horizontal line to the left to its intersection with the curve ($h = 3\frac{1}{2}$ in.) [It should be noted here that this curve was selected, and not, for instance, the curve ($h = 3$ in.), because the percentage corresponding, and shown directly below, is closer to the point of maximum efficiency, as shown by the break in the curve.] The area of metal required is found to be very nearly 0.3 sq. in. per foot of breadth. Adding $\frac{1}{2}$ inch of concrete for protection below the axis of the metal, the total depth is seen to be 4 inches, which gives a dead load of approximately 50 lbs. per sq. ft. as was assumed. The required quantities are therefore:

Depth of slab = 4 inches
Area of steel = 0.3 sq. in. per ft. of breadth.

PLATE II.

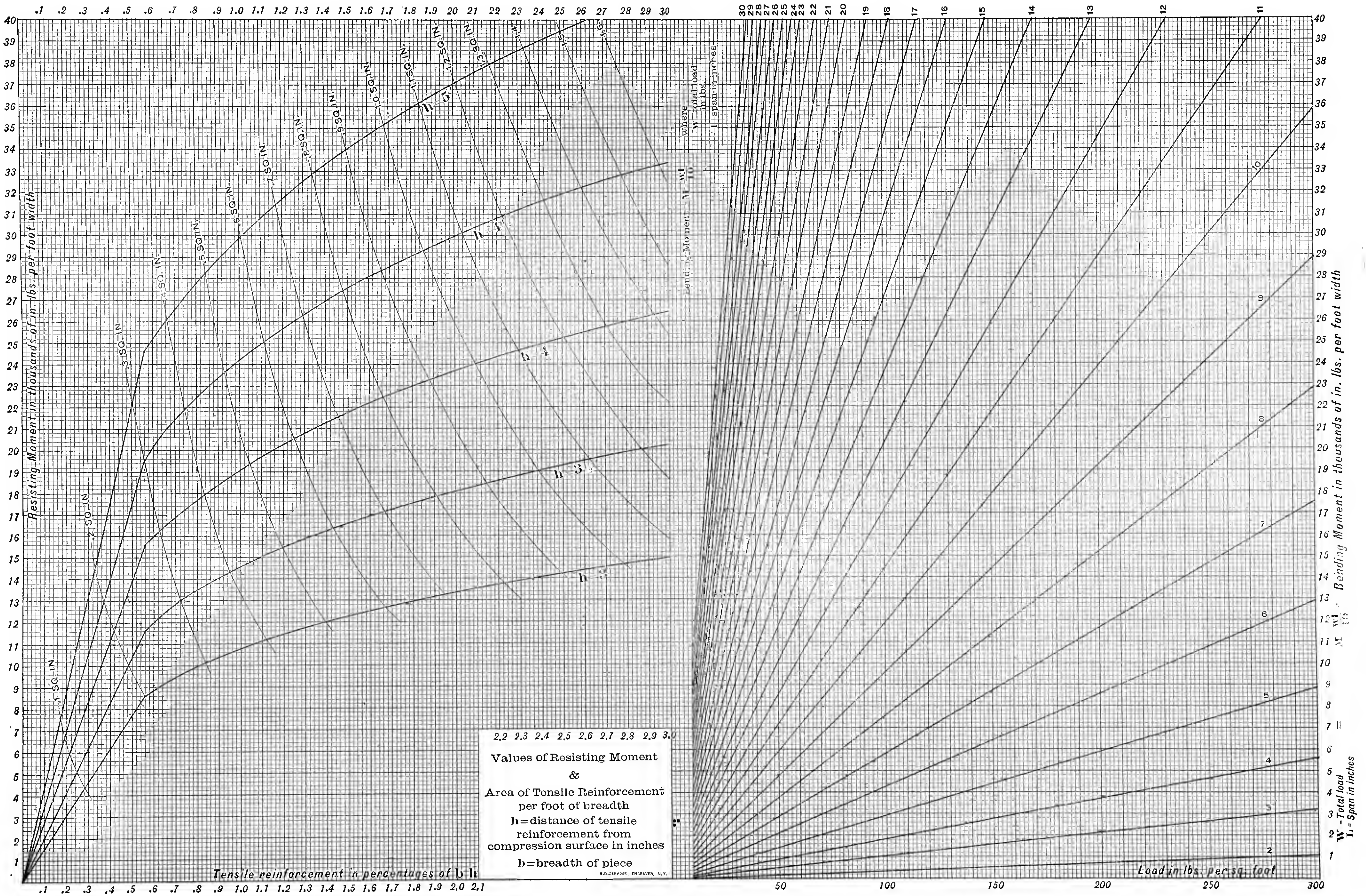
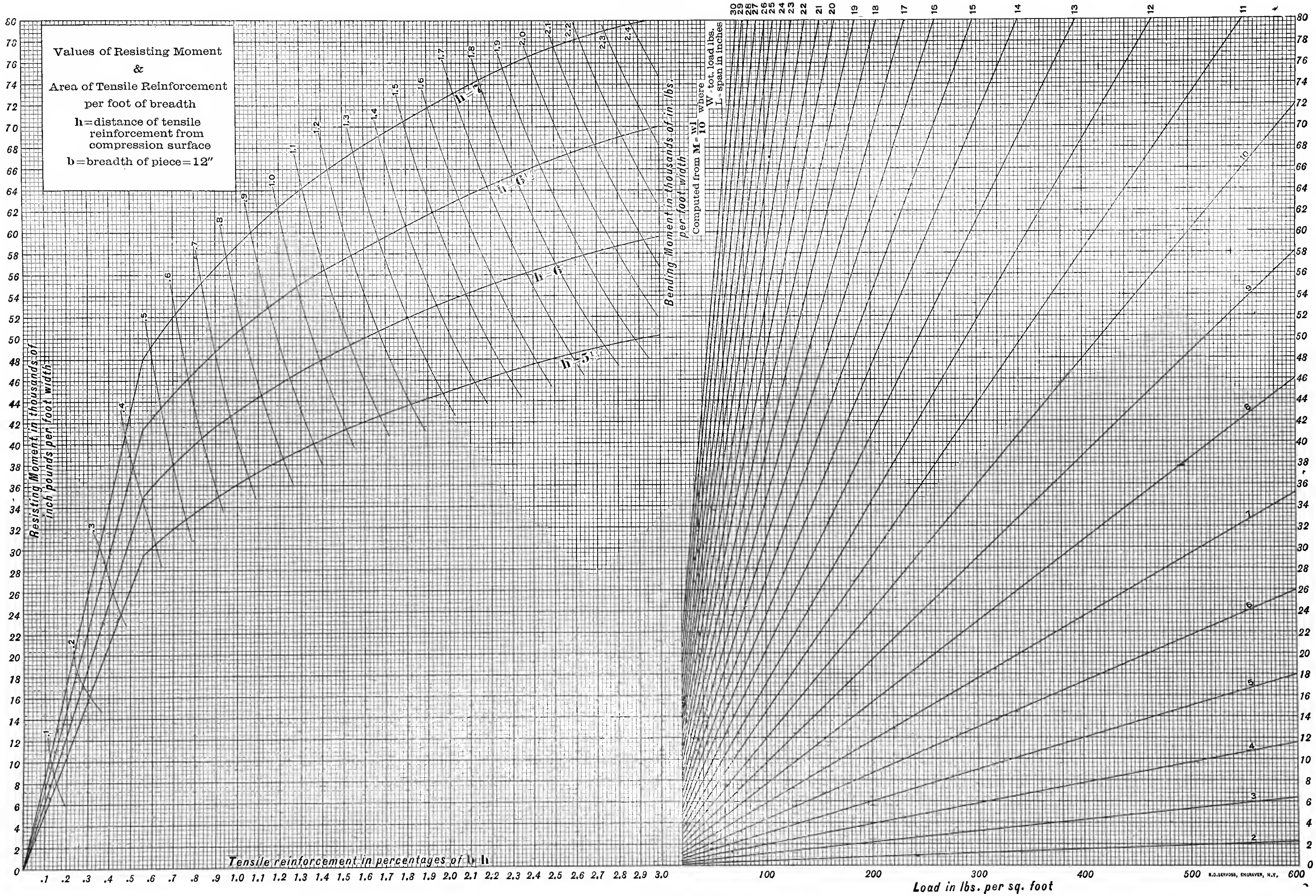

Values of Resisting Moment
&
Area of Tensile Reinforcement
per foot of breadth
h=distance of tensile
reinforcement from
compression surface in inches
b=breadth of piece
Resisting Moment in thousands of in. lbs. per foot width
Tensile reinforcement in percentages of b h
Bending Moment in thousands of in. lbs. per foot width
Load in lbs. per sq. foot
W = Total load
L = Span in inches
.1 SQ. IN.
.2 SQ. IN.
.3 SQ. IN.
.4 SQ. IN.
.5 SQ. IN.
.6 SQ. IN.
.7 SQ. IN.
.8 SQ. IN.
.9 SQ. IN.
1.0 SQ. IN.
1.1 SQ. IN.
1.2 SQ. IN.
1.3 SQ. IN.
50
100
150
200
250
300

PLATE III.



Values of Resisting Moment
&
Area of Tensile Reinforcement
per foot of breadth
h=distance of tensile reinforcement from compression surface
b=breadth of piece=12″
Resisting Moment in thousands of inch pounds per foot width
Tensile reinforcement in percentages of b h
Bending Moment in thousands of in. lbs. per foot width
Computed from $M = \frac{Wl}{10}$ where W = tot. load lbs. L = span in inches
Load in lbs. per sq. foot
R.D.SERVOSS, ENGRAVER, N.Y.

PLATE IV.

Description.

The curves on this plate show the required spacing, center to center, of square bars of various sizes to obtain a given area of metal per foot of breadth along a line perpendicular to the direction of the bars. The abscissas are various areas of cross-section of metal, ranging from 0.1 sq. in. to 3.0 sq. in. per foot of breadth. The corresponding spacing of bars forms the ordinate of the curve at any point. The sizes of bars vary from $\frac{1}{4}$ in. $\times$ $\frac{1}{4}$ in. to $1\frac{1}{4}$ in. $\times 1\frac{1}{4}$ in., this being the range within which will be found most of the slab reinforcements used in practice. For convenience of selection, the curves for bars whose dimensions occur in sixteenths are shown dotted, as these sizes are less frequently employed.

Use of Plate IV.

Example.—Given an area of metal = 0.5 sq. in. per foot of breadth, what spacing will be required if ½ in. × ½ in. bars are to be used?

Solution.—At the bottom of the plate, select the value 0.5. Follow the vertical line upward to its intersection with the curve for ½-in. bars. From this point follow the horizontal line toward the left, where the required spacing is found to be 6 inches.

Example.—Given a floor slab with ⅞ in. × ⅞ in. bars spaced 11½ inches on centers. What area of metal section per foot of breadth is represented?

Solution.—At the left-hand side of the plate select the point midway between 11 and 12, which is, of course, 11½, and follow the horizontal line to its intersection with the curve for ⅞-in. bars. From this point follow the vertical line downward to the value 0.8, which indicates that the area of metal section is 0.8 sq. in. per foot of breadth.

PLATE IV.

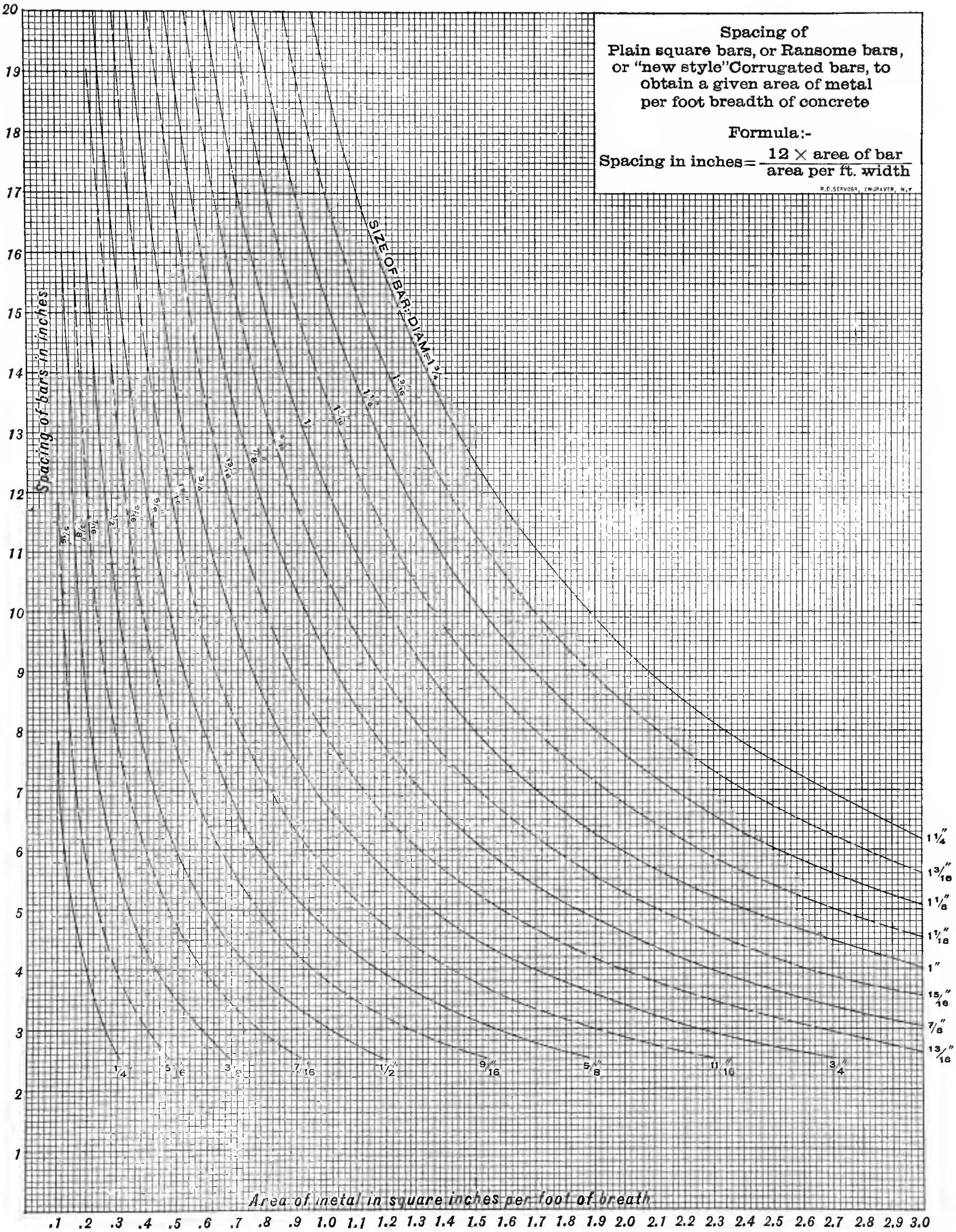

Spacing of
Plain square bars, or Ransome bars, or "new style" Corrugated bars, to obtain a given area of metal per foot breadth of concrete
Formula:-
Spacing in inches = 12 × area of bar / area per ft. width
SIZE OF BAR: DIAM=1 3/4"
Spacing of bars in inches
Area of metal in square inches per foot of breath
1 1/4"
1 3/16"
1 1/8"
1 1/16"
1"
15/16"
7/8"
13/16"
1/4"
5/16"
3/8"
7/16"
1/2"
9/16"
5/8"
11/16"
3/4"
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
.1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3.0

PLATE V.

Description.

This plate provides for the graphical computation of the external bending moment which results when a rectangular footing slab has superimposed upon it a single concentrated column load. It is assumed that the point of application of the concentrated load coincides with the centroid of the rectangular slab. The column load is usually distributed over a portion of the top of the footing slab by means of a base-plate or cap-stone, with whose design we are not concerned. Both the footing and the base-plate may be either square or rectangular in plan. Referring to the shorter dimension of the footing as its width, and to the longer dimension as its length, it is the purpose of the reinforced slab to distribute the load from the base-plate first over the area of a strip whose length is the width of the footing, and whose width is the dimension of the base-plate parallel to the length of the footing. The load on this strip is in turn to be transmitted to a strip running perpendicular to the first strip, whose width is that of the footing and whose length is also that of the footing. Therefore, the maximum bending moment on each strip occurs under the center of the column, and the method of determination is the familiar one employed in the case of a grillage of rolled beams, as contained in the general formula shown on the plate.

Referring to the plate, it will be found that the abscissas are various values of $(l-a)$ in inches, or the difference between the length of the strip under consideration and the length of the base-plate in the same direction. From the condition of symmetry assumed, the value $(l-a)$ is simply twice the projection of the footing beyond the edge of the base-plate. The plotted lines represent various loads from 1 to 80 tons, every fifth line being accentuated for clearness of reference. The ordinates represent the total bending moment in foot-lbs. produced by any given load and given value of $(l-a)$. In this connection it should be noted that the bending moment is directly proportional to the load, so that to obtain the moment due to a load greater than any shown on the plate, it is only necessary to select a load bearing an easy fractional ratio to the required load, and multiply the corresponding bending moment by the same ratio.

Use of Plate V.

Example.—Given a column load of 70 tons, and an allowable soil pressure of 3 tons per sq. ft., what will be the size of base-plate and square footing required, and what will be the maximum bending moment in both directions under the center of the column?

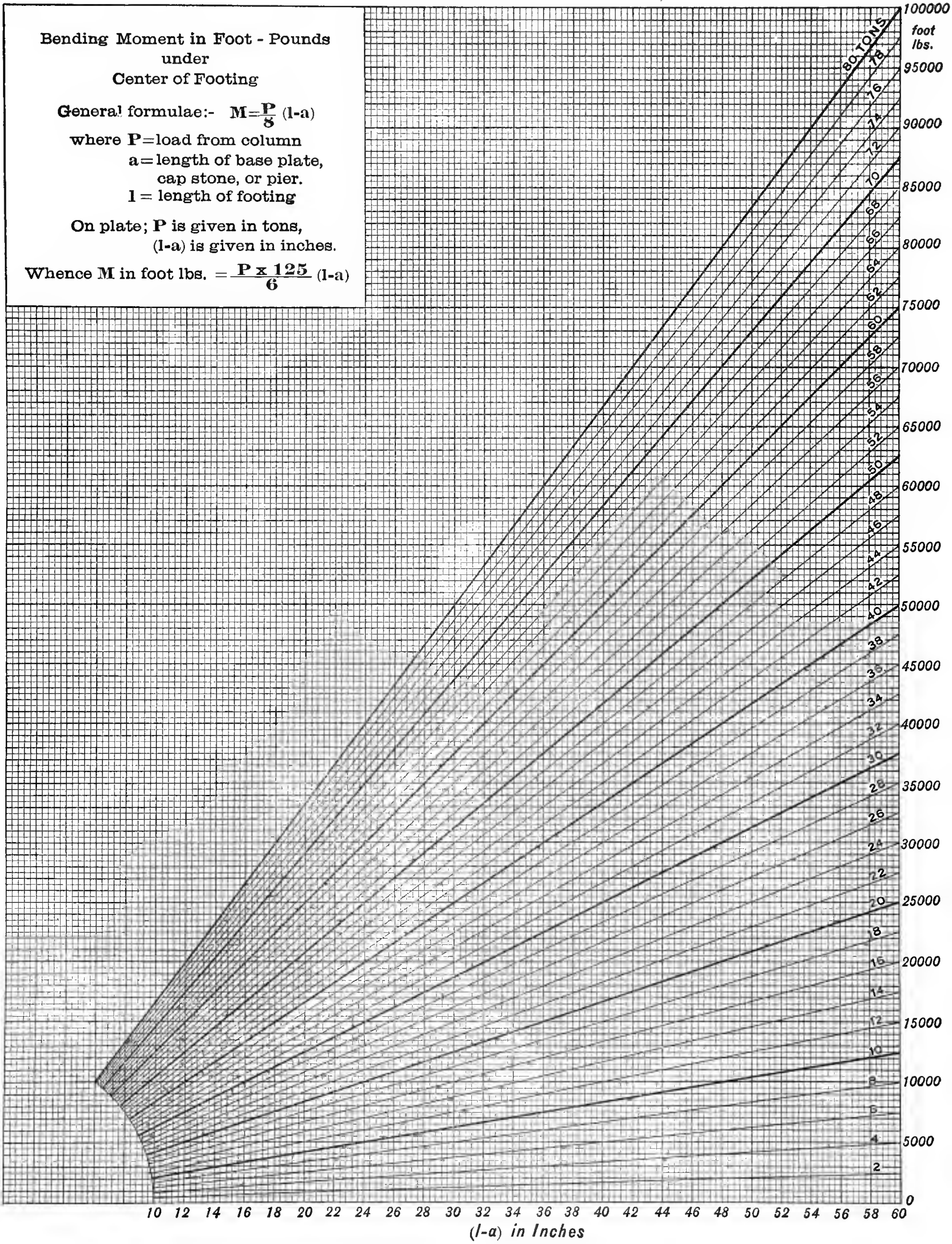
Solution.—Allowing a direct compression of 350 lbs. per sq. in. on the concrete under the base-plate, the area of plate required is $(70 \times 2000) \div 350 = 400$ sq. in. This area will be obtained by a base-plate 20 in. $\times$ 20 in. The area of the footing is approximately determined by dividing 70 tons by 3 tons, giving 23.3 sq. ft. As the slab will be not far from 2 feet thick, its weight may be estimated at 300 lbs. per sq. ft., and for 25 sq. ft. this will give a weight of concrete $= 3.75$ tons, requiring an additional area of 1.25 sq. ft. The total area will, therefore, be $23.3 + 1.25 = 24.55$ sq. ft., and a footing 5 feet square will give this area with sufficient exactness.

We now have the condition of a footing 5 ft. $\times$ 5 ft., with a base-plate 20 in. $\times$ 20 in. The first strip will therefore be 20 inches wide and its length (l) will be 60 inches, with a projection of 20 inches on each end. The value of $(l-a)$ is $60 - 20 = 40$ inches. Select this value at the bottom of the plate, and follow the vertical line upward to its intersection with the line for 70 tons. From this intersection follow the horizontal line to the right, where it will be found that the total bending moment is 58500 ft.-lbs. or $58500 \div 20 = 2925$ ft.-lbs. per inch of breadth.

For the strip running perpendicular to the last one, the value $(l-a)$ is the same, or 40 inches, and the total bending moment is also 58500 ft.-lbs. Although the length (l) happens to be the same, the width is now equal to the length of the strip previously considered, or the full width of the footing, viz., 60 inches. The bending moment is therefore $58500 \div 60 = 975$ ft.-lbs. per inch of breadth.

In conclusion it may be said that the slab must be designed and reinforced in one direction for a bending moment of 2925 ft.-lbs. for every inch of breadth underneath the base-plate, and in the other direction for a bending moment of 975 ft.-lbs. for every inch of breadth across the entire footing.

PLATE V.



Bending Moment in Foot - Pounds
under
Center of Footing
General formulae:- $M=\frac{P}{8}(l-a)$
where P = load from column
a = length of base plate, cap stone, or pier.
l = length of footing
On plate; P is given in tons,
(l-a) is given in inches.
Whence M in foot lbs. $= \frac{P \times 125}{6}(l-a)$
80 TONS
78
76
74
72
70
68
66
64
62
60
58
56
54
52
50
48
46
44
42
40
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
100000 foot lbs.
95000
90000
85000
80000
75000
70000
65000
60000
55000
50000
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60
(l-a) in Inches

PLATES VI AND VII.

DESCRIPTION.

THE general nature of these plates is similar to that of Plates II and III. The curves here shown are applicable to any form of reinforced concrete construction, such as beams or slabs, subjected to transverse stress, and reinforced only to resist tension. The value of (h), the distance from the axis of the tensile reinforcement to the surface under maximum compression, ranges from 8 to 24 inches. The percentages of reinforcement, or ratio of cross-sectional area of metal to the product of (h) multiplied by the breadth, are laid out as abscissas, and range from 0.1 of 1 per cent. to 3.0 per cent. The ordinates are shown as the bending moments in foot-lbs. per inch of breadth, corresponding to any given value of (h), and percentage of reinforcement. In order to show the actual area of metal cross-section, the dotted lines corresponding to various areas per foot of breadth have been plotted to intersect the different curves at the percentage which such areas represent of the area $(h \times 12)$ sq. ins.

It will be noted that while the area of metal is shown per foot of breadth, in conformity with the other plates, the bending moment is stated in foot-lbs. per inch of breadth. The reason for this will be evident when it is remembered that in the design of T beams, a certain width of slab is assumed to act as the compression flange of the T, and this width is determined by some authorities as a certain fraction of the spacing between consecutive beams, and by others as a certain fraction of the span of the beam itself. In either case the width is usually a certain number of inches, which is termed the breadth of the beam, and the total bending moment in foot-lbs. has only to be divided by this number of inches to obtain at once the ordinate at the left-hand side of the plate. The area of metal is of course concentrated in the stem of the T, and when this area is known per foot of breadth of the flange, its total amount for any given breadth in inches may be readily ascertained from Plates VIII and VIII*a*, as will be subsequently explained.

Use of Plate VI.

Example.—Given a footing slab 12 inches thick, with reinforcing bars 10 inches below the top surface, and a bending moment in this direction of 39000 ft.-lbs., the breadth being 50 inches. What area of steel per foot of breadth will be required?

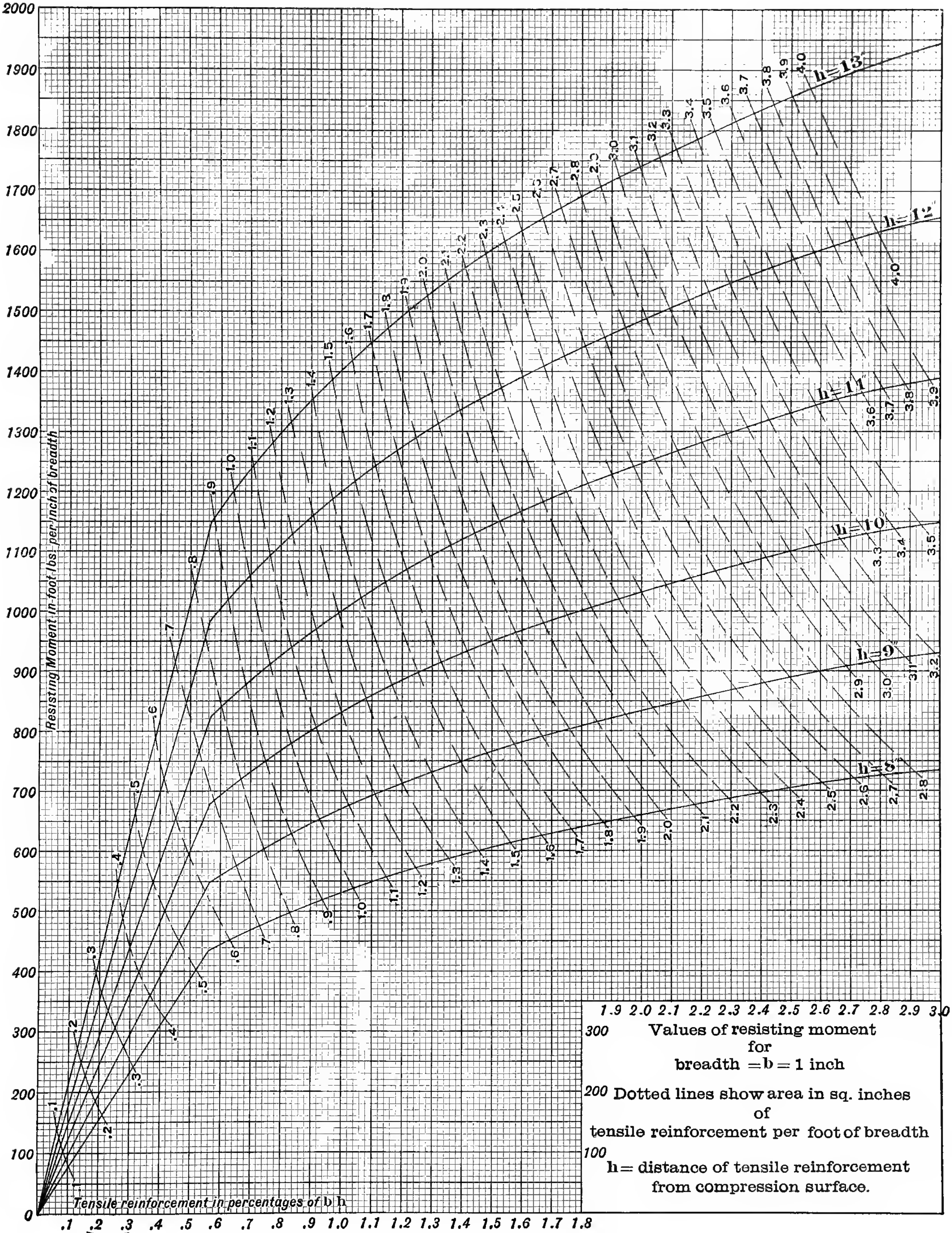
Solution.—The bending moment is here $39000 \div 50 = 780$ ft.-lbs. per inch of breadth. Selecting this value at the left-hand side of the plate, follow the horizontal line to the right to its intersection with the curve for ($h = 10$ inches). The nearest intersecting dotted line shows the required area of steel to be 1.0 inch per foot of breadth.

Example.—Given a T beam whose width of top flange is 48 inches, having an area of tensile reinforcement of 4 sq. in., the bending moment being 57600 ft.-lbs. What will be the required value of (h) and depth of beam?

Solution.—The bending moment is here $57600 \div 48 = 1200$ ft.-lbs. per inch of breadth. The area of reinforcement is seen to be 1 sq. in. per foot of breadth. Selecting the value 1200 at the left-hand side of the plate, follow the horizontal line to the right to its intersection with the dotted curve for 1.0 sq. in. per foot of breadth. This point will be found on or extremely close to the curve for ($h = 13$ in.), which is therefore the value required. Adding 2 inches for protection below the metal, the total depth of beam is found to be 15 inches.

(It must be noted here that the questions of shear, and the location of the neutral axis above or below the bottom line of the flange, are not touched upon, since these points are taken up with plates subsequently described.)

PLATE VI.


Resisting Moment in foot lbs. per inch of breadth
Tensile reinforcement in percentages of b h
h=13
h=12
h=11
h=10
h=9
h=8
Values of resisting moment
for
breadth = b = 1 inch
Dotted lines show area in sq. inches
of
tensile reinforcement per foot of breadth
h = distance of tensile reinforcement
from compression surface.

USE OF PLATE VII.

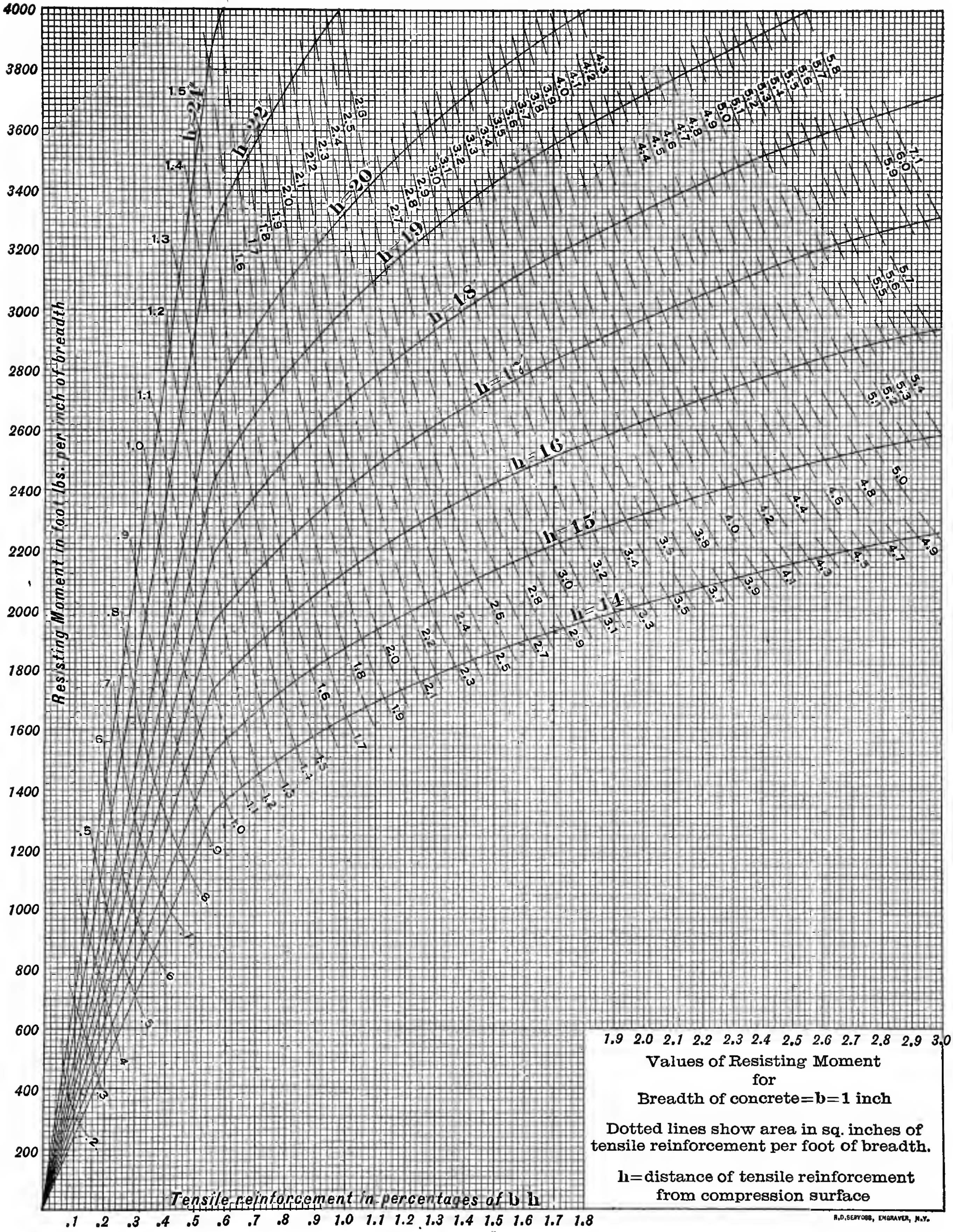
Example.—Given a water-tank with walls 2 feet thick, and reinforced at the point of maximum thrust with vertical steel rods whose area is equivalent to 1.5 sq. in. per linear foot of wall, the rods being placed 4 inches from the outside of the tank. What will be the resisting moment per linear foot in such a case?

Solution.—The value of (h) is here equal to $24-4=20$ inches. Selecting the curve ($h=20$), locate its intersection with the dotted curve for an area of 1.5 sq. inches per foot of breadth. From this point follow the horizontal line to the left, where the resisting moment is found to be 2800 ft.-lbs. per inch of breadth. Therefore the required resisting moment is $12 \times 2800 = 33600$ ft.-lbs. per linear foot of wall.

Example.—Given a T beam whose flange width is limited to 30 inches, with a flange thickness of 4 inches, and an area of steel = 2.75 sq. inches placed at a depth of 14 inches below the top surface. What is the resisting moment?

Solution.—The equivalent area of steel per foot of breadth $= 2.75 \div 2.5 = 1.1$ sq. in. per foot of breadth. Locating the intersection of the curve ($h=14$) with the dotted line for 1.1 sq. in. per foot of breadth, and following the vertical line downwards, the percentage of steel is found to be 0.66 per cent. By computation, or by reference to a subsequent plate, it will be found that the neutral axis in this case will lie approximately at the level of the bottom of the 4-inch flange, so the resisting moment may be taken directly from this plate. Returning to the point of intersection, and following the horizontal line to the left, this resisting moment is found to be 1400 ft.-lbs. per inch of breadth, giving a total resisting moment of $30 \times 1400 = 42000$ ft.-lbs. for the beam, which was the quantity to be determined.

PLATE VII.



R.D.SERVOSS, ENGRAVER, N.Y.

PLATES VIII AND VIII*a*.

DESCRIPTION.

By means of Plate VIII the total area of cross-section of metal for any given breadth, and for any given area of metal section per foot of breadth, may readily be determined. The plotted lines represent areas of cross-section of metal, ranging from 0.1 sq. in. to 4.0 sq. in. per foot of breadth. The total breadth of the member under consideration will be found at the bottom of the plate, in the form of abscissas, the maximum value being 150 inches. The corresponding total areas of metal section are plotted as ordinates.

By means of Plate VIII*a* the number of square bars required to obtain a certain total area of cross-section may be ascertained. The plotted lines represent various sizes of square bars ranging from $\frac{1}{4}$ in. to $1\frac{1}{4}$ in. The number of bars required will be found at the bottom of the plate, laid off as abscissas. The corresponding total areas of metal section are plotted as before in the form of ordinates, agreeing with those for Plate VIII. It will be noted that here, as on Plate IV, for convenience of selection, the lines for bars whose dimensions occur in sixteenths are shown dotted, as these sizes are less frequently employed.

USE OF PLATES VIII AND VIII*a*.

Example.—Given the case of the T beam stated in the example shown for Plate VII, where the width of the beam is 30 inches, and the area of cross-section of the steel is 2.75 sq. inches, what is the equivalent area per foot of breadth?

Solution.—Select the value 30 inches at the bottom of Plate VIII, and follow the vertical line upward until a point is reached horizontally opposite the value 2.75 at the left. This point will be found to lie on or extremely close to the plotted line for 1.1, and the required equivalent area is therefore 1.1 sq. in. per foot of breadth.

Example.—In the foregoing example, what sizes of bars and what number will be required?

Solution.—By laying a straight-edge horizontally across the plate, at the value 2.75, the line so formed will be found to intersect the plotted line for $\frac{3}{4}$-inch bars approximately over the figure 5, and for $\frac{5}{8}$-inch bars exactly over the figure 7. Therefore the required area may be obtained by using five $\frac{3}{4}$-inch bars or seven $\frac{5}{8}$-inch bars.

Example.—Given a footing-slab 50 inches wide and having in this width 20 bars $\frac{1}{2}$ in. $\times \frac{1}{2}$ in. in section, what is the equivalent area per foot of width and what is the total area?

Solution.—Select the value 20 at the bottom of Plate VIII*a*, and follow the vertical line upward to its intersection with the plotted line for $\frac{1}{2}$-in. bars. From this intersection, follow the horizontal line to the left until it intersects the vertical line on Plate VIII, designated as 50 inches at the bottom. This intersection will be found on, or extremely close to, the plotted line indicating that the equivalent area is 1.2 sq. in. per foot of breadth. The total area is read off directly at the left-hand side of the plate, where it is found to be 5 sq. inches.

PLATE VIII.

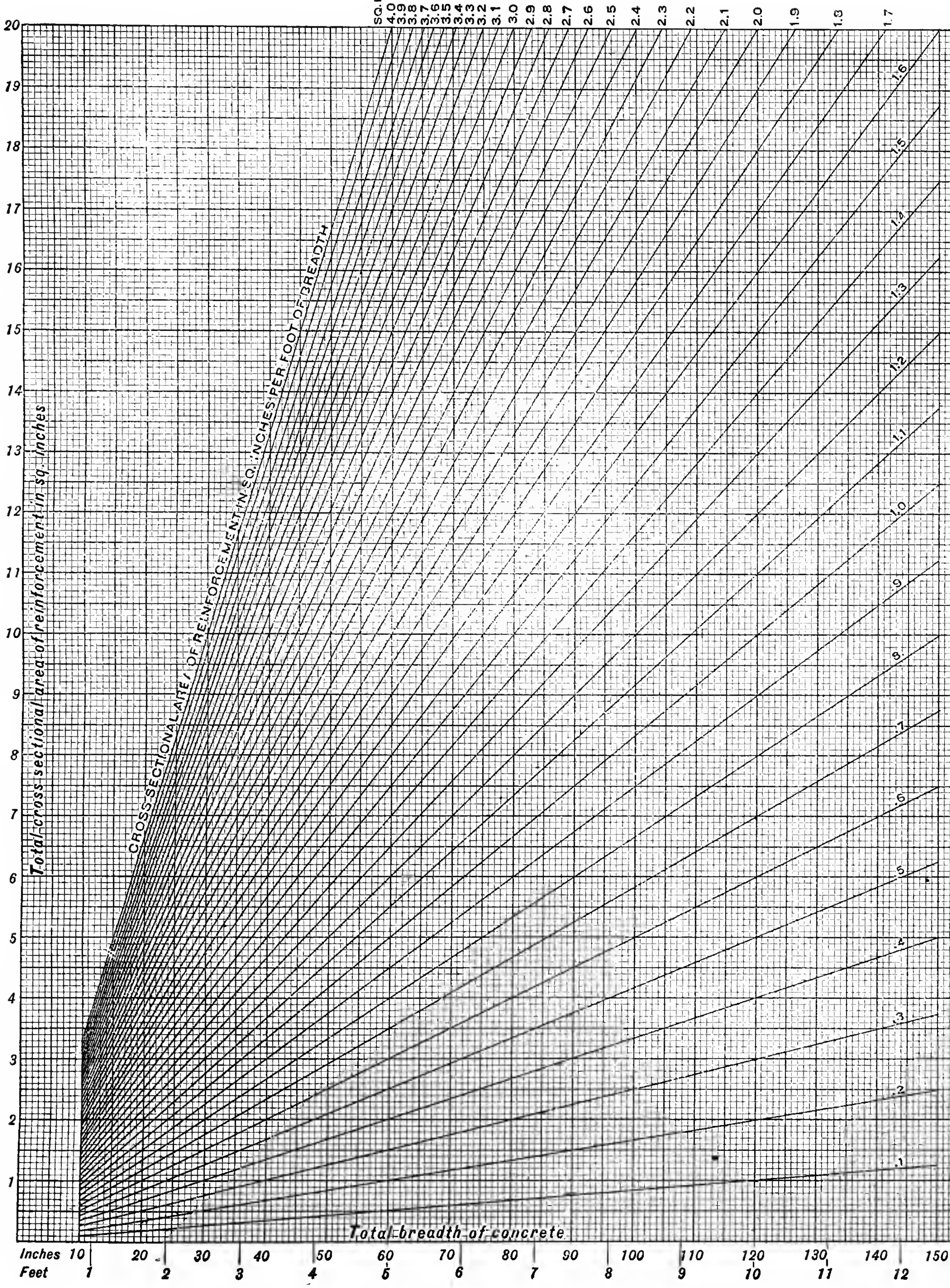


PLATE VIIIa.

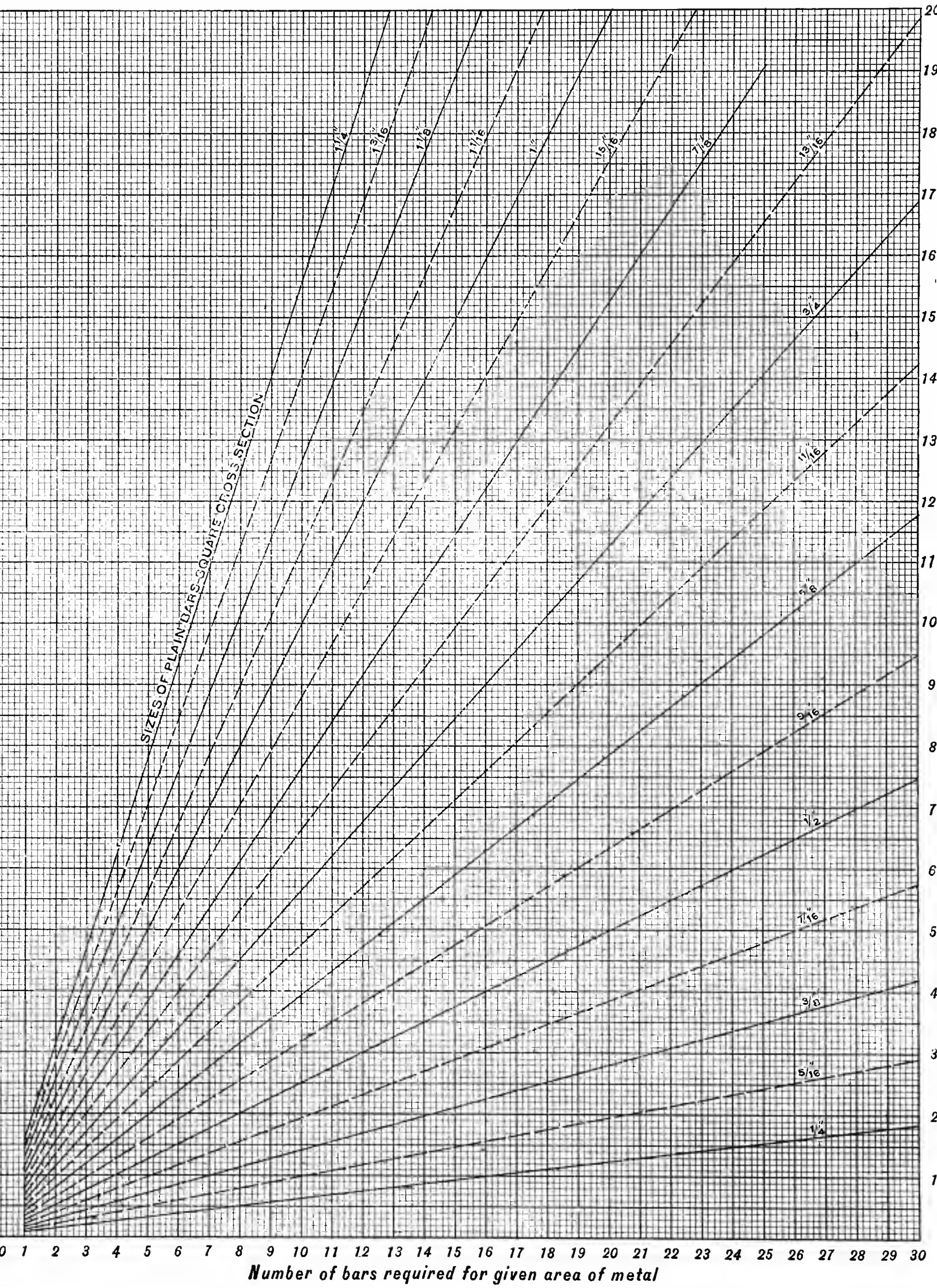


PLATE IX.

Description.

By means of the curves on this plate there may be ascertained the location of the neutral axis for any reinforced-concrete member subjected to maximum safe loading, provided the value of (h), or distance from the axis of the metal to the surface under maximum compression, does not exceed 24 inches. Once the value of (h) and the percentage of reinforcement are assumed, and the member subjected to its maximum allowable bending moment, the distance of the neutral axis from the surface under maximum compression is a fixed and definite quantity. The values of the latter for various values of (h) and percentages of metal are plotted as ordinates, and the percentages of metal are themselves plotted as abscissas. For clearness of selection, the curves for values of (h) which are multiples of 5 have been accentuated.

On a corner of the plate will be found drawn to a small scale a curve whose application is perfectly general irrespective of the value of (h). The abscissas are here the percentages of the value of (h) which constitute the distance of the neutral axis from the compression surface, and the ordinates are various percentages of reinforcement up to 3.0 per cent.

Use of Plate IX.

Example.—Given the case of the T beam described in the example for Plate VII, where the flange width is 30 inches, the flange thickness is 4 inches, the area of steel is 2.75 sq. inches, and the value of (h) is 14 inches. The percentage of steel has been found to be 0.66 per cent of the product 30×14. What will be the position of the neutral axis under the allowable bending moment of 42000 ft.-lbs.?

Solution.—Select the value 0.66 at the bottom of the plate and follow the vertical line upward to its intersection with the curve $(h = 14)$. From this intersection follow across the plate to the left, where the value required is found to lie between 4.05 and 4.1, which is the distance of the neutral axis below the top of the beam. An extremely small increase in the thickness of the flange will therefore permit the design to be accepted as stated.

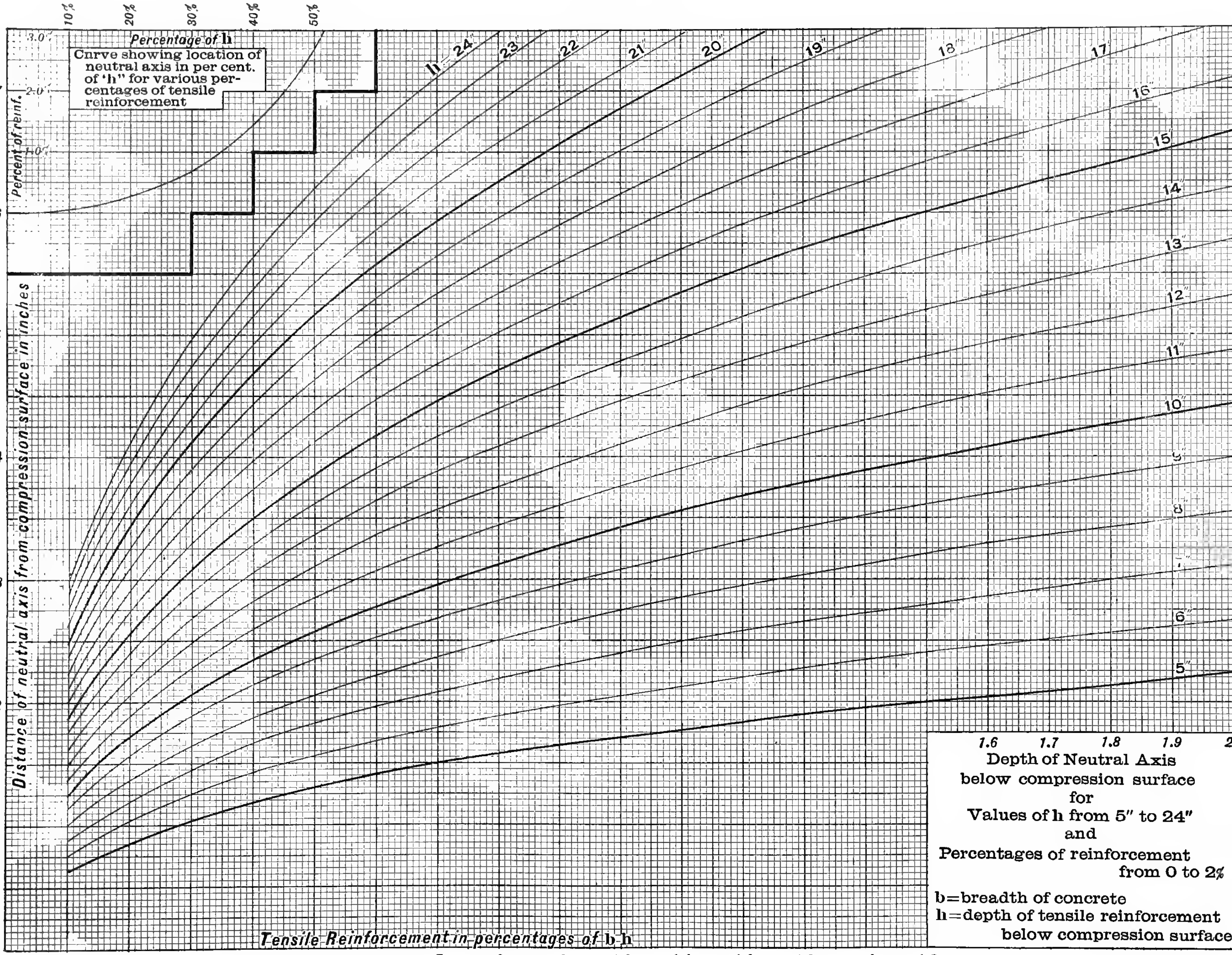
Example.—Given the case of the T beam described in the example for Plate VI, where the flange width is 48 inches, the area of steel is 4 sq. inches, and the value of (h) has been found to be 13 inches; what will be the flange thickness required to include the neutral axis between its top and bottom limits, and permit the design to be used as taken from the plates?

Solution.—The percentage of reinforcement as found from Plates VIII and VI is 0.64 per cent. Select this value at the bottom of Plate IX, and follow the vertical line upward to its intersection with the curve $(h = 13)$. From this intersection follow the line to the left to the value 3.75. The flange thickness, therefore, should be not less than $3\frac{3}{4}$ inches, if the design is to remain as stated.

Example.—Given a rectangular girder 20 inches wide and 40 inches deep, with the axis of the tensile reinforcement 4 inches from the bottom. Where will the neutral axis lie if the reinforcement amounts to 2.0 per cent. of the product 20×36?

Solution.—On the small curve select at the left-hand side the value 2.0, and follow the line to the right to its intersection with the curve. From this point follow the vertical line upward to the value 45.0 per cent. Take 45.0 per cent of the value of (h), or $0.45 \times 36 = 16.2$. The neutral axis, therefore, lies 16.2 inches below the top surface of the beam, when the maximum allowable bending moment is applied.

PLATE IX.



P.D. SERVOSS, ENGRAVER, N.Y.

PLATE X.

Description.

By means of this plate there may be determined the allowable shearing resistance for various areas of cross-section of concrete and steel. The plotted lines on the main portion of the plate represent various loads, acting as external shearing forces, such as the vertical end shear for a beam or slab. The abscissas represent various areas of concrete cross-section, and the ordinates represent various areas of steel cross-section.

In order to obviate the necessity of multiplying the breadth of any given cross-section by its depth, to obtain the area of the section, there has been plotted a series of lines representing various breadths from 3 to 20 inches. Various values for the depth of the cross-section have been laid off as ordinates at the right-hand side, and the product of any given breadth and depth will be found as an abscissa at the bottom of the plate. Every fifth line representing the breadth and also the shear is accentuated for clearness of selection, and the line for a breadth of 12 inches is especially indicated, as this will be frequently required in determining the shearing resistance for slabs, where a strip one foot wide is usually considered.

The plotted lines at the top of the plate are simply the same values of shearing resistance, up to 20000 lbs., laid out to an enlarged scale.

It will be noted that a slight error is introduced by not deducting from a cross-section of given width and depth the area of steel section embedded in it. The operation of selecting the shearing resistance is rendered easier by the assumption that the entire cross-section is of concrete, and that the cross-sectional area of the steel is to be added to this. In order to obtain the exact shearing resistance, the net area of cross-section of concrete would have to be obtained by subtracting from the total area of cross-section of the member the relatively small area occupied by the metal. Fortunately, however, on account of the low value of the allowable unit shear for concrete, this refinement is unnecessary, and, in practically every case, the values may be taken directly as shown on the plate.

USE OF PLATE X.

Example.—Given a T beam whose flange width is 26 inches, with a flange thickness of 5 inches, and with a thickness of stem, or leg of the T, of 7 inches, the total depth being 15 inches. The area of metal cross-section is 4 sq. inches. What will be the maximum allowable vertical shear to which such a beam may be subjected?

Solution.—The total area of cross-section is 26×5 plus $7 \times 10 = 200$ sq. inches. Select this value at the bottom of the plate and follow the vertical line upward. At the same time select the value 4 at the left-hand side of the plate and follow the horizontal line to the right. The intersection of these two lines will be found to lie on the plotted line for 50000 lbs. Therefore, the maximum allowable vertical shear is approximately 50000 lbs.

Example.—Given a footing-slab running continuously under two equally loaded columns. The pressure on the soil is 5 tons per sq. ft., and the distance between the nearest edges of the two column bases is 18 feet. The thickness of the slab is 18 inches. What area of metal will be required per linear foot across the footing, under the edge of the column bases?

Solution.—The total load on a strip one foot wide between the edges of the bases is $5 \times 18 = 90$ tons. The end shear is therefore 90000 lbs. per linear foot across the footing. Select the value of the depth (18 inches) at the right-hand side of the plate, and follow the horizontal line to the left to its intersection with the line (breadth = 12 inches). From this intersection follow the vertical line downward to its intersection with the plotted line for a shear of 90000 lbs. From this point follow the horizontal line to the left to the value 7.9. The area of cross-section of the steel must therefore be equivalent to approximately 8.0 sq. inches per foot of breadth across the footing at the edge of the column base.

PLATE X.

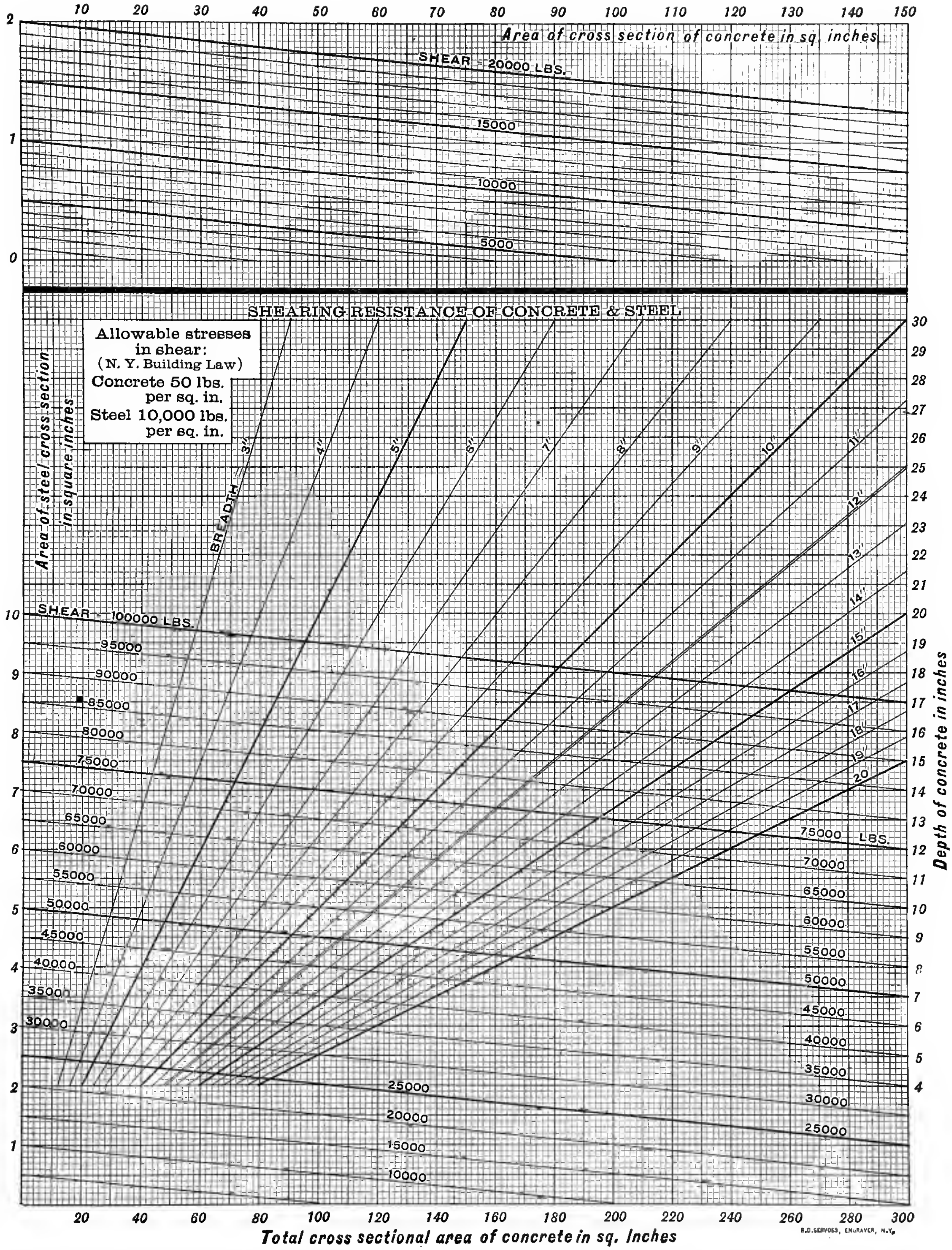

Area of cross section of concrete in sq. inches
SHEAR = 20000 LBS.
15000
10000
5000
SHEARING RESISTANCE OF CONCRETE & STEEL
Allowable stresses
in shear:
(N. Y. Building Law)
Concrete 50 lbs.
per sq. in.
Steel 10,000 lbs.
per sq. in.
Area of steel cross section in square inches
BREADTH = 3"
4"
5"
6"
7"
8"
9"
10"
11"
12"
13"
14"
15"
16"
17"
18"
19"
20"
SHEAR = 100000 LBS.
95000
90000
85000
80000
75000
70000
65000
60000
55000
50000
45000
40000
35000
30000
25000
20000
15000
10000
75000 LBS.
Depth of concrete in inches
Total cross sectional area of concrete in sq. Inches
R.D. SERVOSS, ENGRAVER, N.Y.

PLATE XI.

Description.

By means of the plotted lines on this plate, there may be ascertained the load which can be sustained by square columns of various dimensions from 6 in. × 6 in. to 30 in. × 30 in., and reinforced longitudinally with various sectional areas of steel up to 30 sq. inches. The allowable unit stresses are those proscribed by the Building Code of New York City, and a further requirement states that the ratio of unsupported length to the least side or diameter shall not exceed twelve.

On the plate, the plotted lines will be found to correspond to the various column sizes noted, the ordinates being the total load in tons, while the abscissas are various areas of steel, comprising the total cross-sectional area of longitudinal reinforcement expressed in square inches. It is assumed that the longitudinals are hooped or tied together at intervals not exceeding the least dimension of the column. For clearness of selection the alternate plotted lines have been accentuated.

For convenience in determining the ratio of any given cross-sectional area of longitudinal reinforcement to the cross-sectional area of any given column, the percentages so obtained have been plotted as shown by the dotted lines. For a given percentage of steel section with respect to a given size column the requisite area of steel section will be found vertically below the intersection of the dotted line for the given percentage with the line for the given size of column.

USE OF PLATE XI.

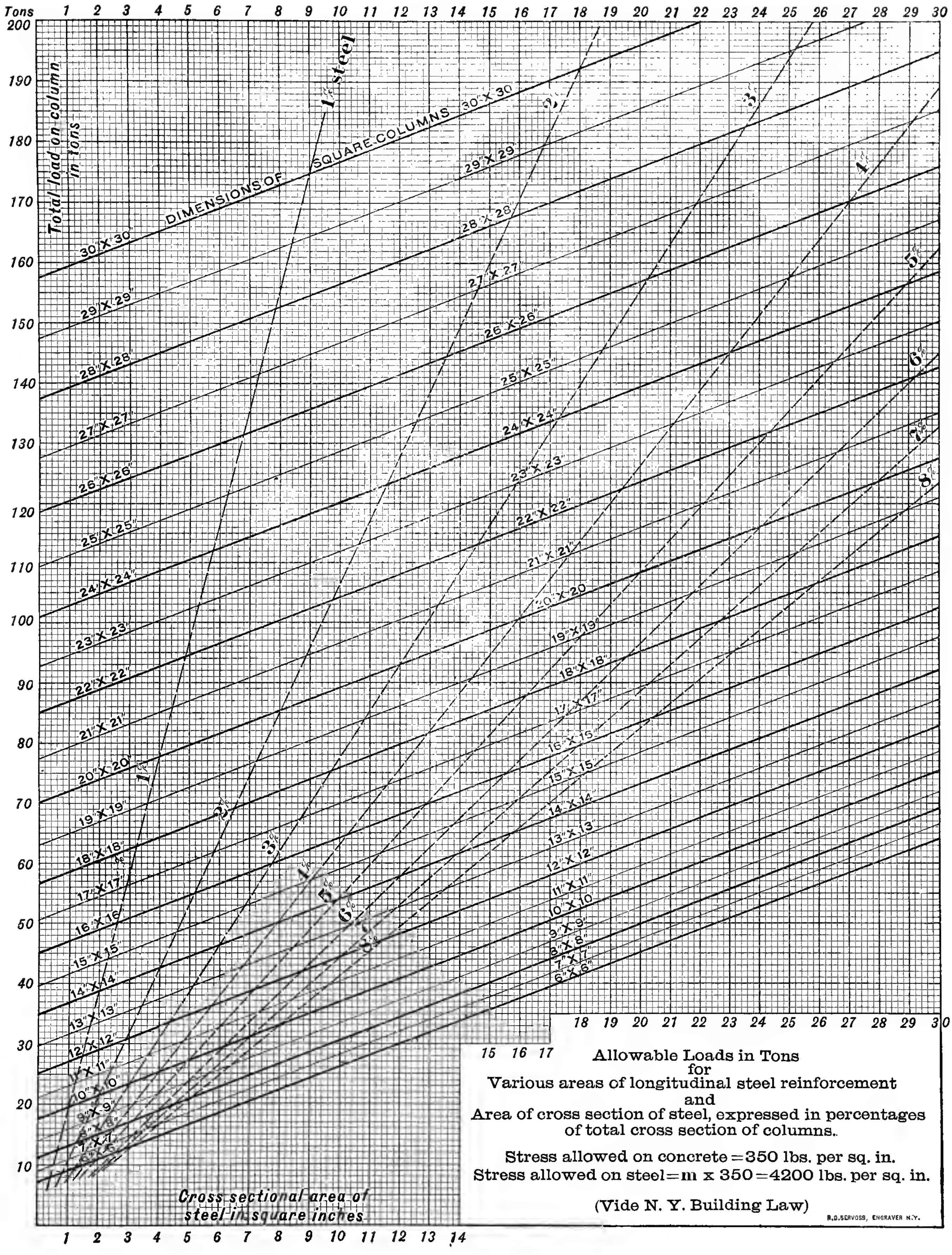
Example.—Given a column load of 75 tons, what size of square column and what total area of steel cross-section is required if the maximum economy of cost is obtained by the use of 3 per cent. reinforcement?

Solution.—Select the value 75 at the left-hand side of the plate, and follow the horizontal line to the right to its intersection with the dotted line for 3 per cent. This will be found on or extremely close to the column line for 18 in. × 18 in. From the point of intersection follow the vertical line downward to the value 9.7. Therefore the required column size is 18 in. × 18 in., and the total cross-sectional area of the steel should amount to 9.7 sq. inches.

Example.—Given a column whose dimensions are 20 in. × 20 in. and whose cross-sectional area of steel amounts to 16 sq. inches. What load will the column sustain and what percentage of steel area is used?

Solution.—Select the value 16 at the bottom of the plate, and follow the vertical line upward to its intersection with the line for a column 20 in. × 20 in. This will be found on or extremely close to the dotted line for 4 per cent., and therefore the steel section amounts to approximately 4 per cent. of the column cross-section. From this intersection follow the horizontal line to the left, where the value of 100.5 is found. The allowable load on this column is therefore 100.5 tons.

PLATE XI.



Tons
Total load on column in tons
Cross sectional area of steel in square inches
DIMENSIONS OF SQUARE COLUMNS
1% steel
2%
3%
4%
5%
6%
7%
8%
30"X30"
29"X29"
28"X28"
27"X27"
26"X26"
25"X25"
24"X24"
23"X23"
22"X22"
21"X21"
20"X20"
19"X19"
18"X18"
17"X17"
16"X16"
15"X15"
14"X14"
13"X13"
12"X12"
11"X11"
10"X10"
9"X9"
8"X8"
7"X7"
6"X6"
Allowable Loads in Tons
for
Various areas of longitudinal steel reinforcement
and
Area of cross section of steel, expressed in percentages
of total cross section of columns.
Stress allowed on concrete = 350 lbs. per sq. in.
Stress allowed on steel = m x 350 = 4200 lbs. per sq. in.
(Vide N. Y. Building Law)
R.D.SERVOSS, ENGRAVER N.Y.

PLATES XII AND XII*a*.

DESCRIPTION.

BY means of Plate XII it is possible to ascertain the equivalent stress which can be sustained by various combinations of square columns and longitudinal steel reinforcement, when various values are assigned to the unit compressive stress which the concrete alone is permitted to sustain. The requirement of the New York Building Code is observed; namely, that the unit stress on the steel longitudinals is to be considered as twelve times the unit stress on the concrete. On the plate values have been assigned to the latter quantity, varying from 350 lbs. per sq. inch to 800 lbs. per sq. inch, and the lines for equivalent stress plotted as shown. By equivalent stress is meant that unit stress, which, if distributed over the cross-section of any given column, will amount to the same total load as the sum of the two quantities obtained by multiplying the concrete area by the unit stress thereon, and by multiplying the steel area by the unit stress thereon. The values of this equivalent stress are laid off as ordinates at the left-hand side of the plate, and the percentages of steel cross-section are plotted as abscissas.

In order to transform the percentages of steel area into actual areas of steel cross-section, it is necessary to assume certain sizes of column cross-section. Lines have accordingly been plotted for various sizes of square columns from 6 in. × 6 in. to 30 in. × 30 in., the abscissas being the percentages of steel at the bottom of the plate. The corresponding ordinates are laid off at the right-hand side of the plate, and show the total area of steel cross-section for any given size column and percentage of steel. Alternate lines have been accentuated for clearness of selection.

By means of Plate XII*a* the necessary number of round bars of given diameter required to obtain a given total cross-sectional area of steel may be readily ascertained. The plotted lines represent various diameters of round bars, the lines for sizes expressed in sixteenths being dotted, as these sizes are less frequently employed in practice. The total areas of metal are laid off as ordinates, reading from the top of the plate downward, to correspond with Plate XII. The number of bars required for any given area will be found as the abscissa at that point.

It should be noted here that this plate is similar to Plate VIII*a*, which has already been described. If it is desired to obtain the required area of steel cross-section by the use of square bars, then Plate VIII*a* should be used instead of Plate XII*a*.

USE OF PLATES XII AND XIIa.

Example.—Given a column load of 117600 lbs., and allowing a unit compression on the concrete of 450 lbs. per sq. inch, what will be the required design of the column if the use of 3 per cent. of steel produces the maximum economy of cost?

Solution.—Select the value 3 per cent. at the bottom of Plate XII, and follow the vertical line upward to its intersection with the plotted line for compression = 450 lbs. per sq. in. From this point follow the horizontal line to the left, where the equivalent stress is found to be 600 lbs. per sq. in. Then the required area of the column section is 117600 ÷ 600 = 196 sq. inches. A column whose dimensions are 14 in. × 14 in. will fulfill this condition. Again, selecting the value 3 per cent. at the bottom of the plate, follow the vertical line upward to its intersection with the plotted line for a column 14 in. × 14 in. in cross-section. From this point follow the horizontal line to the right-hand side of the plate, where the total area of steel section required is seen to be very nearly 6 sq. inches. Follow this horizontal line still further on Plate XIIa, to its intersection with the vertical line from the value 4, at the bottom of the plate. This last intersection will be found to lie on or extremely close to the plotted line for 1⅜-inch bars. Therefore the required area may be obtained by using 4 longitudinal bars, each being 1⅜ inches in diameter. The designated load, therefore, will be sustained by a 14 in. × 14 in. column, reinforced with 4 round steel bars of 1⅜-inch diameter.

Example.—Given a column 15 in. × 15 in. square, reinforced with 6 round steel bars of 1⅜-in. diameter, and a total column load of 211500 lbs., what is the unit stress on the concrete?

Solution.—Select the value 6 at the bottom of Plate XIIa, and follow the vertical line upward to its intersection with the line for 1⅜-in. bars. From this intersection follow the horizontal line to the left, where the total area of steel will be found to be very nearly 9 sq. in. Continue to follow the horizontal line on Plate XII to its intersection with the line for a 15 in. × 15 in. column. This intersection will be found to lie nearly on the vertical line denoted as 4 per cent. Now the equivalent stress is 211500 ÷ (15 × 15) = 940 lbs. per sq. inch. Select this value at the left-hand side of the plate and follow the horizontal line to the right to its intersection with the vertical line previously located; namely, that for 4 per cent. This last intersection will be found to lie on or extremely close to the plotted line for compression = 650 lbs. per sq. inch. Therefore the compressive stress exerted on the concrete portion of the column is very nearly 650 lbs. per sq. inch.

PLATE XII.

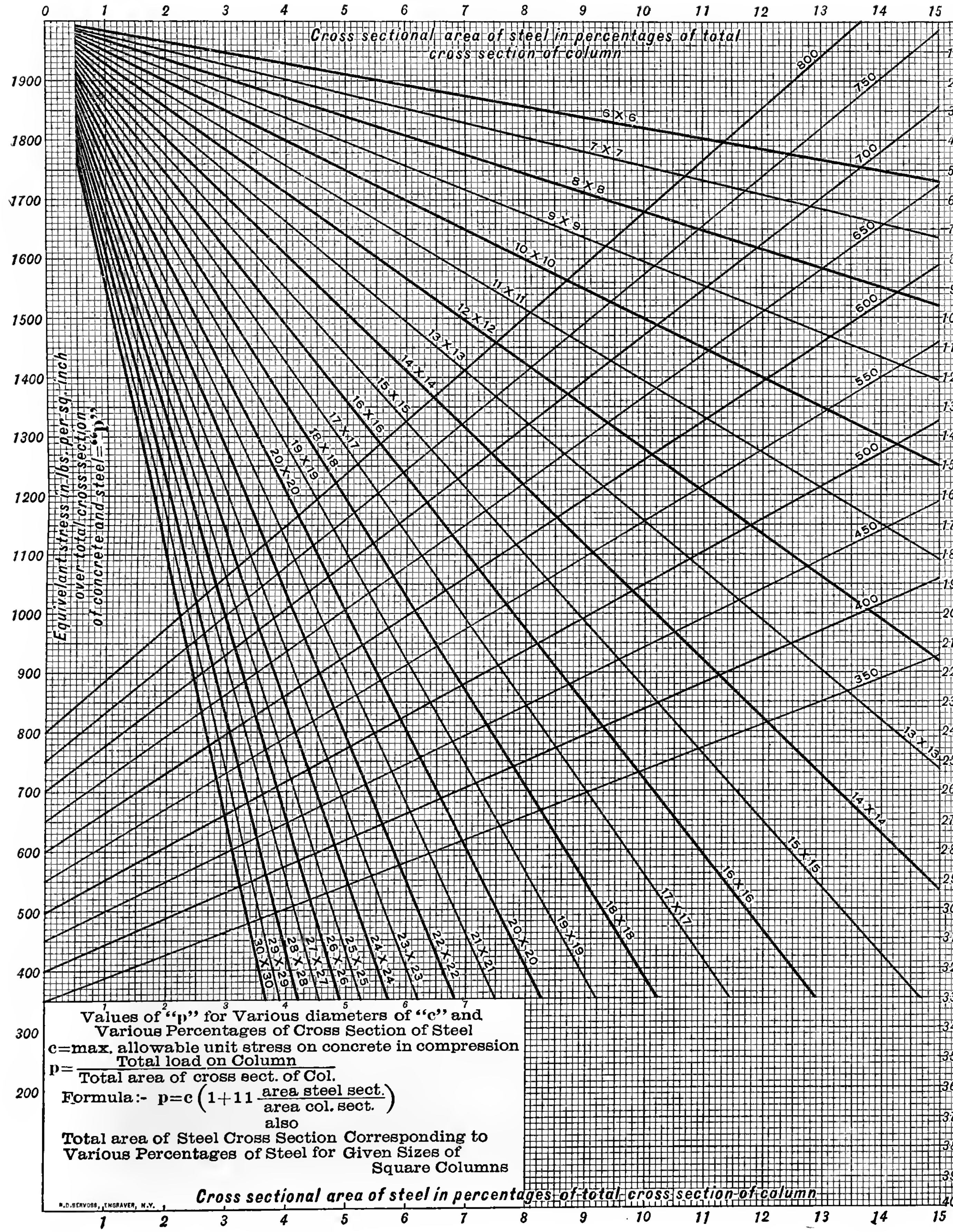


PLATE XIIa.

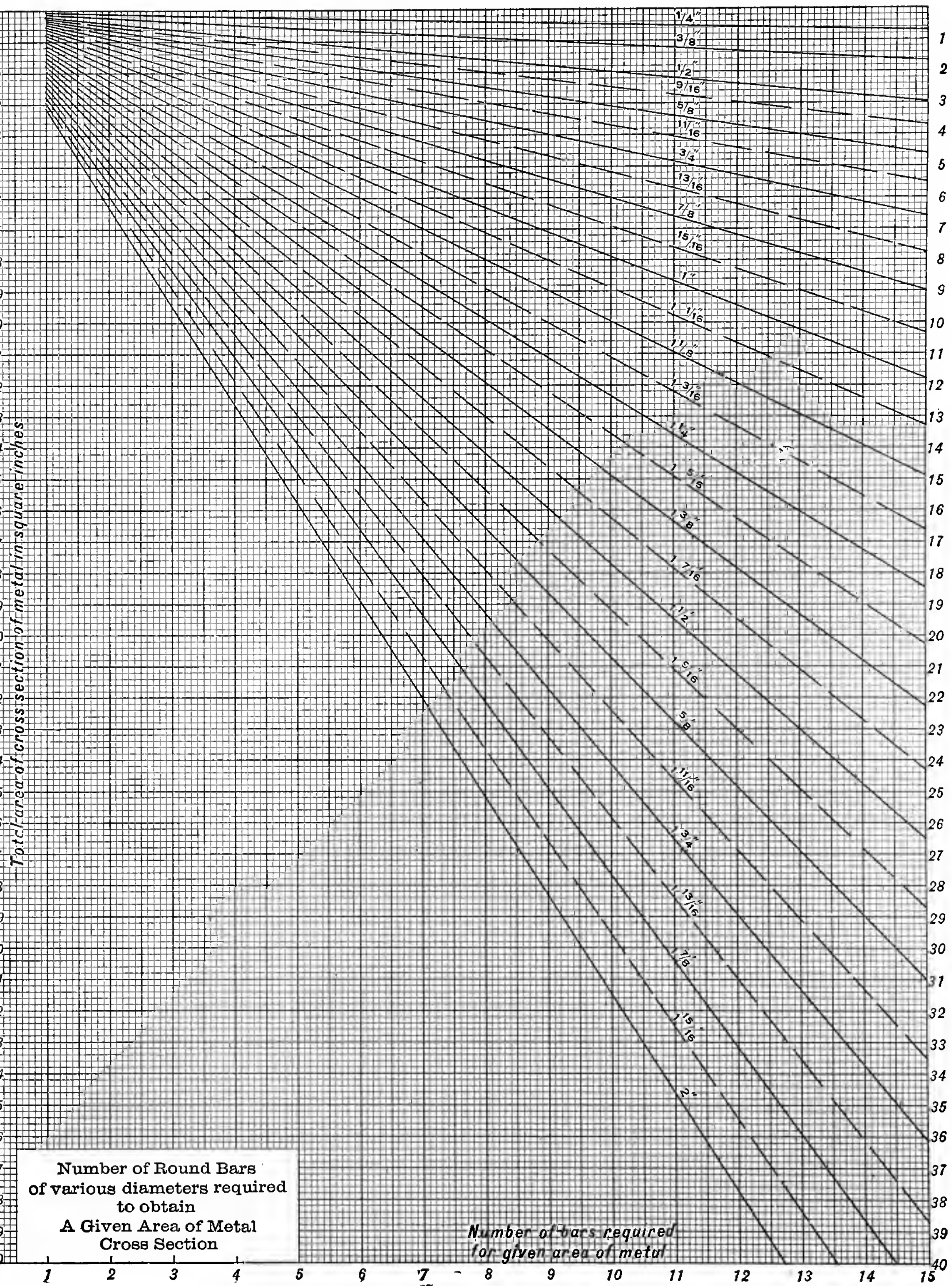


PLATE XIII.

Description.

By means of this plate there may be obtained the complete design of any hooped column, using the allowable stresses designated, and reinforcing longitudinally with six round rods of equal diameter. The column loads, which are stated at the left-hand side of the plate as ordinates, are assumed to be applied directly on the concrete core inside of the hooping, with an intensity of stress equal to 1000 lbs. per sq. inch. The radial thrust is entirely restrained by the hooping and the longitudinal rods. The allowable tension in the hooping, which is assumed to be drawn-steel wire, is 25000 lbs. per sq. inch, and the pitch, or distance between consecutive spirals, is one-sixth of the diameter of the hooped core. The outward or radial thrust being also transmitted to the longitudinal rods, which are tied in by the hooping, there is a tendency on the part of these rods to bulge outward between the spirals, and consequently a bending moment is developed. Assuming a maximum fiber stress of 16000 lbs. per sq. inch, the necessary diameter of rod that will enable it to withstand this bending moment has been computed. Under these circumstances the longitudinal rods are assumed to take no direct compression from the superimposed column load, and on the plate such a condition is referred to as "no excess area." It may be desired, however, to increase the bearing capacity of the column by adding to the cross-sectional area of the rods an amount equal to one, two, or three per cent. of the area of the concrete core. Under these conditions the additional area of steel is considered to be added in the form of an annular shell, around each of the rods required for withstanding the bending moment previously referred to. This additional area is allowed to carry a compressive stress of 12000 lbs. per sq. inch, and, in practice, is obtained not by the addition of a separate shell, but by simply increasing the original diameter of the rods.

The column loads being laid off as ordinates, and the necessary diameters of hooped core as abscissas, the curves have been plotted as shown. The plotted lines, giving the required diameter of longitudinal rods under the four conditions stated, have as abscissas the various diameters of hooped core. The corresponding diameters of the rods themselves are laid off as ordinates at the extreme right of the plate, reading from the top downward.

The diameter of the hooping wire has been computed as a factor of the diameter of the hooped core. Various sizes of wire, expressed in Birmingham Wire Gauge, have been laid off near the right-hand side of the plate. The diameter of hooped core, which exactly corresponds to each size of wire, is indicated by a small circle horizontally opposite each gauge, the diameter of core being found vertically below the circle.

Use of Plate XIII.

Example.—Given a column load of 100 tons, what will be the design of hooped column required, if the longitudinal reinforcement is to take no direct load?

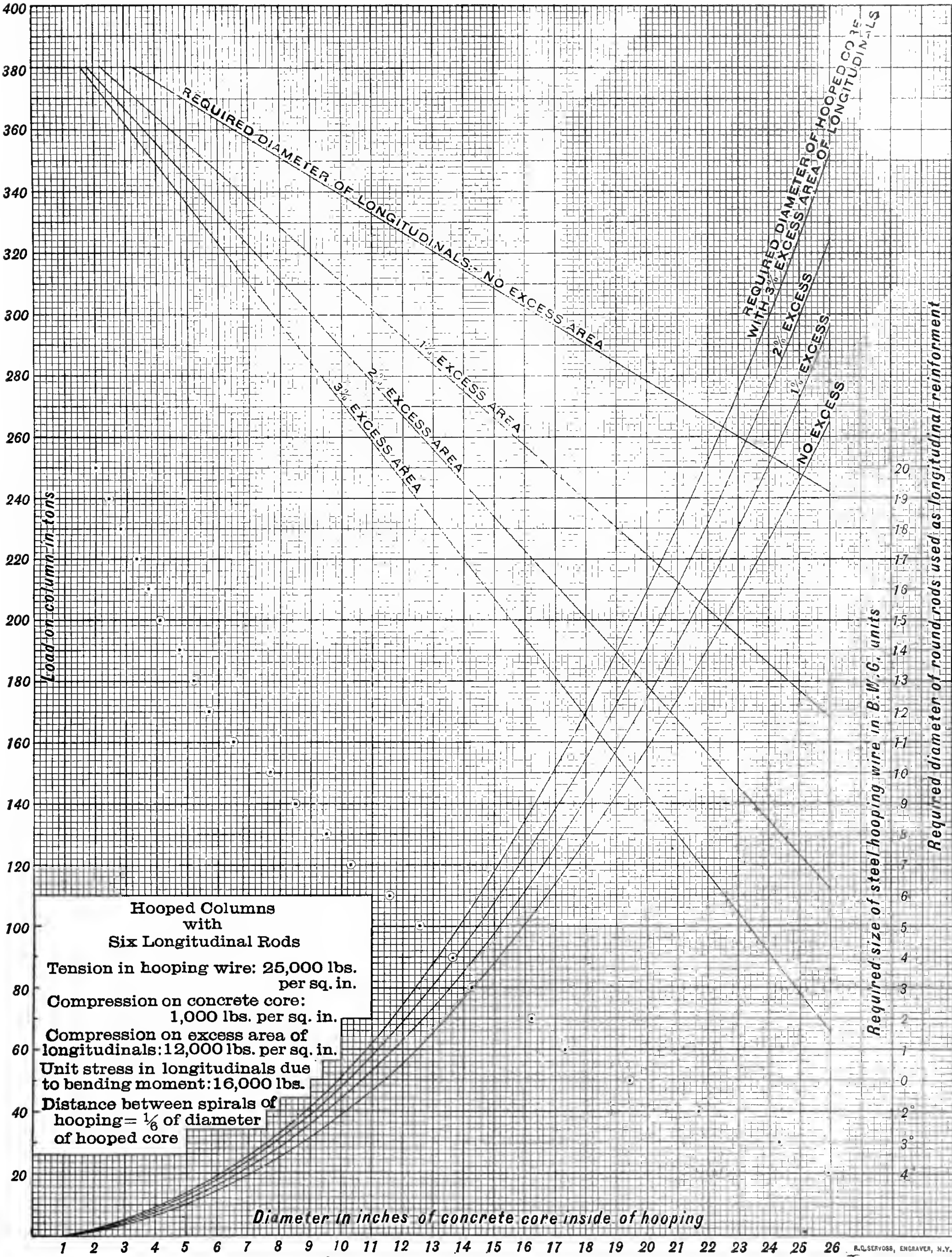
Solution.—Select the value 100 tons at the left-hand side of the plate, and follow the horizontal line to the right to its intersection with the curve designated "no excess." From this intersection follow the vertical line downward to the value 16, at the bottom of the plate, which indicates that the diameter of core should be 16 inches. Following the same vertical line from 16 up to its intersection with the straight plotted line for diameter of rods, with "no excess area," and from this point horizontally across to the extreme right-hand side of the plate, the required diameter is found to be $\frac{5}{8}$ inch. Returning to the vertical line above the value 16, the small circle lying nearest to this line is seen to be the one opposite No. 2, B. W. G.

The complete design of the column should therefore consist of a 16-inch concrete core, with six $\frac{5}{8}$-in. round rods spaced equally around the circumference, and hooped, or wound spirally, with No. 2 steel wire, the spirals being spaced one-sixth of 16 inches, or $2\frac{2}{3}$ inches apart. The hooping should have about 1 in. thickness of concrete as outside protection, making the extreme diameter of the column 18 inches.

Example.—Given a column load of 100 tons as before, what will be the design if the outside diameter is not to exceed 16 inches, allowing as fire protection a minimum thickness of 1 inch of concrete outside of hooping?

Solution.—The diameter of hooped core must not exceed, from the required conditions, $16-2=14$ inches. Select the value of 100 tons at the left-hand side of the plate, and follow the horizontal line to the right. This horizontal line will be found to intersect the curve denoted "3 per cent. excess" at a point vertically over the value 13.8 inches. There will therefore be required an additional area of longitudinals equal to 3 per cent. of the core area, the diameter of which, for convenience, will be taken as 14 inches. Follow the vertical line from 14 upward to its intersection with the plotted line denoted "3 per cent. excess area," and thence to the right-hand side of the plate, where the value $1\frac{1}{8}$ in. is obtained. Returning to the vertical line above the value 14, the small circle lying nearest to this line is seen to be the one opposite No. 3, B. W. G. The conditions of the design, therefore, require that the 14-in. core should have six $1\frac{1}{8}$-in. round rods, hooped with No. 3 steel wire, the spirals being spaced one-sixth of 14 inches, or $2\frac{1}{3}$ inches apart.

PLATE XIII.



Hooped Columns
with
Six Longitudinal Rods
Tension in hooping wire: 25,000 lbs. per sq. in.
Compression on concrete core: 1,000 lbs. per sq. in.
Compression on excess area of longitudinals: 12,000 lbs. per sq. in.
Unit stress in longitudinals due to bending moment: 16,000 lbs.
Distance between spirals of hooping = 1/6 of diameter of hooped core
REQUIRED DIAMETER OF LONGITUDINALS.- NO EXCESS AREA
1% EXCESS AREA
2% EXCESS AREA
3% EXCESS AREA
REQUIRED DIAMETER OF HOOPED CORE WITH 3% EXCESS AREA OF LONGITUDINALS
2% EXCESS
1% EXCESS
NO EXCESS
Load on column in tons
Diameter in inches of concrete core inside of hooping
Required size of steel hooping wire in B.W.G. units
Required diameter of round rods used as longitudinal reinforment
400
380
360
340
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26
1/8" 1/4" 3/8" 1/2" 5/8" 3/4" 7/8" 1" 1 1/8" 1 1/4" 1 3/8" 1 1/2" 1 5/8" 1 3/4" 1 7/8" 2" 2 1/8"
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PLATE XIV.

DESCRIPTION.

THE method of constructing and using this plate is practically the same as that described for Plate XIII. The essential difference consists in the use of eight longitudinal reinforcing rods, spaced equally on the circumference of the hooped core, instead of six rods, as required by the previous plate. The required spacing between successive spirals of the hooping wire is one-eighth of the diameter of the hooped core. It will be noted that an additional curve, denoted "4 per cent. excess," and an additional line, denoted "4 per cent. excess area" have been plotted.

The principal advantage to be derived by the use of eight longitudinals instead of six lies in the possibility of obtaining greater area of cross-section of steel, without increasing the diameter of the rods beyond reasonable limits. This circumstance enables the column to carry greater loads with the same size rods, and the strength is further increased by the closer spacing of the hooping spirals. The use of this plate is suggested whenever the diameter of the hooped core would exceed 16 inches.

It should be noted with regard to Plates XIII and XIV that, as an actual fact, the increase of diameter also increases the resisting moment of the rod. However, as the direct compression allowed is only 75 per cent. of the allowable fiber stress due to bending, it will be apparent that these two assumptions act in opposition, thus tending to eliminate the error which would be introduced by the consideration of either one independently, and the results obtained are believed to be correct for all practical purposes.

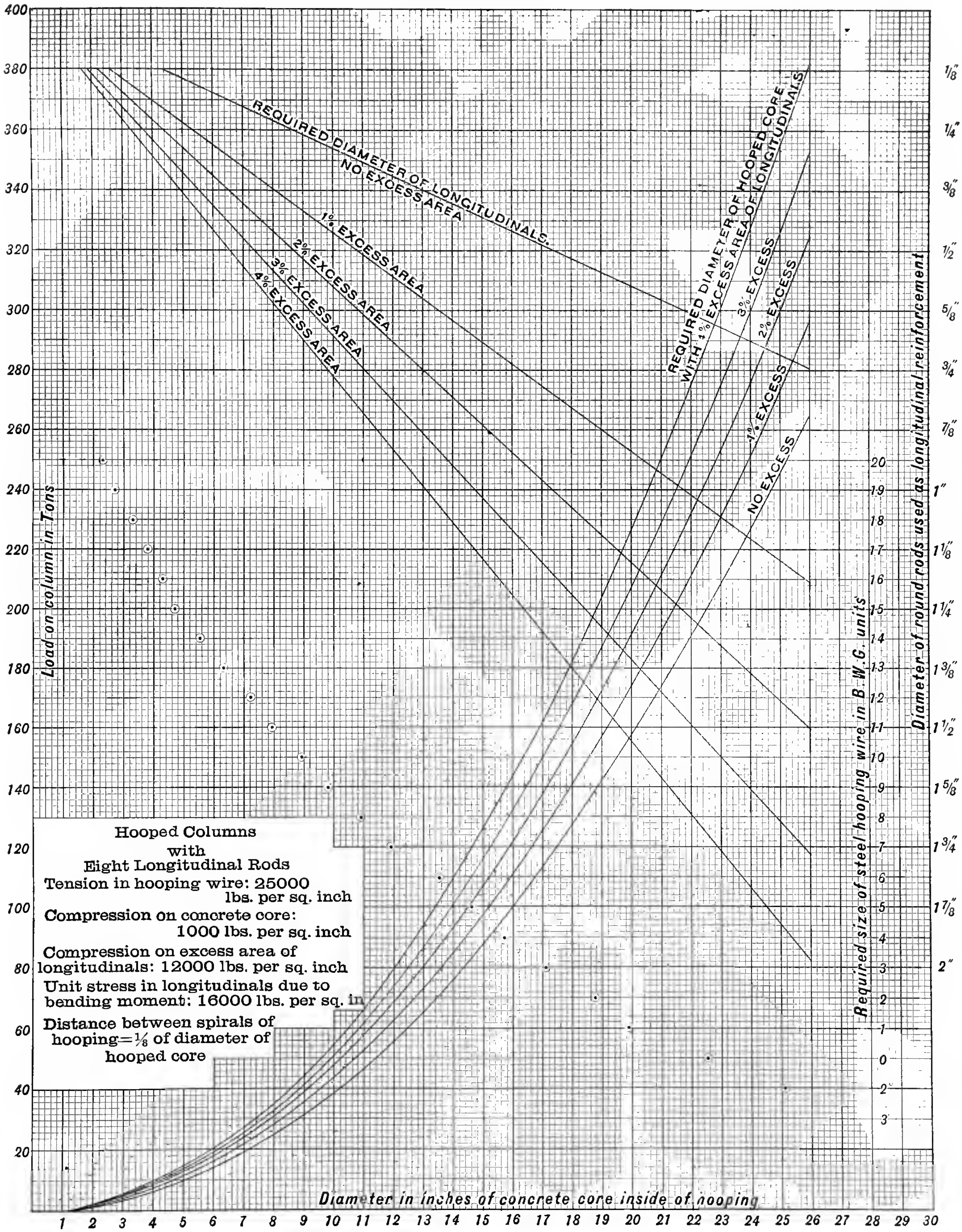
Use of Plate XIV.

Example.—Given a column load of 275 tons, what will be the design of hooped column required, if, from considerations of "maximum economy of cost," the steel area withstanding direct compression is allowed to be 4 per cent. of the core area?

Solution.—Select the value 275 tons at the left-hand side of the plate, and follow the horizontal line to the right to an intersection with the curve designated "4 per cent. excess." From this intersection follow the vertical line downward to the value 22 inches at the bottom of the plate. The core diameter is therefore required to be 22 inches. Following the same vertical line upward from 22 to its intersection with the straight plotted line for diameter of rods, with "4 per cent. excess area," and thence to the right-hand side of the plate, the required diameter is found to be very nearly 1⅝ inches. Returning to the vertical line above the value 22, the small circle lying nearest to this line is seen to be the one opposite No. 0, B. W. G.

The complete design of the column should therefore consist of a 22-inch concrete core, with eight 1⅝-in. round rods spaced equally around the circumference, and hooped with No. 0 steel wire, the spirals being spaced one-eighth of 22 inches, or 2¾ inches apart. Allowing a thickness of 2 inches outside of hooping for fire protection, the outside diameter of the column will be 26 inches.

PLATE XIV.


Hooped Columns
with
Eight Longitudinal Rods
Tension in hooping wire: 25000 lbs. per sq. inch
Compression on concrete core: 1000 lbs. per sq. inch
Compression on excess area of longitudinals: 12000 lbs. per sq. inch
Unit stress in longitudinals due to bending moment: 16000 lbs. per sq. in
Distance between spirals of hooping=1/8 of diameter of hooped core
REQUIRED DIAMETER OF LONGITUDINALS. NO EXCESS AREA
1% EXCESS AREA
2% EXCESS AREA
3% EXCESS AREA
4% EXCESS AREA
REQUIRED DIAMETER OF HOOPED CORE WITH 1% EXCESS AREA OF LONGITUDINALS
3% EXCESS
2% EXCESS
1% EXCESS
NO EXCESS
Load on column in Tons
Diameter in inches of concrete core inside of hooping
Required size of steel hooping wire in B.W.G. units
Diameter of round rods used as longitudinal reinforcement

COMPLETE DESIGN OF A REINFORCED CONCRETE STRUCTURE.

Example.—The required design is that of a five-story building whose extreme outside dimensions are 91 ft. 0 in. × 31 ft. 0 in. The walls are to be of un-reinforced concrete, constructed as bearing walls 12 in. thick, with arched concrete or iron lintels over the doors and windows. The computations, therefore, will be only in connection with the floor and roof systems, interior columns and footings for the same, all to be designed in reinforced concrete. The roof is to be flat, and covered with tile and damp-proofing. A single line of columns will be permitted along the center line of the building. The allowable soil pressure for these interior columns is 4 tons per sq. foot for full live and dead load. The building is to be 60 feet high, the distance from finished floor to finished floor being 12 ft. 0 in. The basement floor will rest directly on the ground, cinder concrete and damp-proofing being laid underneath in the usual manner, the total thickness being 18 inches. Therefore only 4 floors and the roof will be carried on the bearing walls and interior columns. The finished flooring will consist of $1\frac{1}{8}$-in. boards laid on sleepers set in 2 inches of cinder concrete. The columns will show exposed concrete faces and the beams and girders will be exposed on the ceilings. The unit stresses allowed are those prescribed by the Building Code of New York City, and the required live load is 60 lbs. per sq. ft. on floors and 50 lbs. per sq. ft. on the roof.

Solution.—The weight of reinforced concrete is assumed at 150 lbs. per cu. ft. The weight of floor-covering and cinder concrete filling is taken at 20 lbs. per sq. ft. The weight of tile roof-covering is taken at 30 lbs. per sq. ft.

Live load on floors = 60 lbs. per sq. ft.
Live load on roof = 50 lbs. per sq. ft.

As far as possible the percentage of steel used will be that giving the nearest approximation to the "maximum structural efficiency," which, in the present instance, will be assumed equal to the greatest economy of cost.

The spacing of the interior columns and beams is somewhat dependent on the judgment of the designer.

In this case we will use four columns, spacing them 18 ft. apart, center to center, on the long axis of the building. The end columns will therefore be 18 ft. from the center of the bearing wall. The beams will be spaced 9 ft. 0 in. on centers, and run perpendicular to the long axis of the building. The extreme length of the beams will therefore be 15 ft., and every alternate beam will bear directly on a column. The intermediate beams will bear on a girder running between the columns and parallel to the long axis of the building. These girders will have an overall span of 18 ft.

Slab Design.—The total depth of the slab will be approximately 4 or 5 inches. We may therefore assume the weight of the slab itself as 65 lbs. per sq. ft. The total load is therefore $65+20+60=145$ lbs. per sq. ft. The span is 9 ft. and the reinforcements are continuous over the beams. Reference to Plate II gives the value of (h) as 4 inches, and steel as approximately 0.25 sq. in. per foot of breadth. Reference to Plate IV gives this as $\frac{1}{4}$ in. square bars spaced 3 inches on centers. Allowing 1 inch of concrete as protection below the axis of the steel, the total depth of slab is 5 inches, and the weight (62.5 lbs. per sq. ft.) is sufficiently close to the original weight assumed to be considered as a slight error on the side of safety.

Beam Design.—We will assume the flange width as one-sixth of the span of the beam, or $\frac{1}{6}\times 180=30$ in. The weight of the beam itself need not be considered if we use the weight of floor-slab assumed. The beam being built into the bearing wall and continuous through the column, the bending moment may be taken at $\frac{Wl}{10}$.

$$W=(145\times 9\times 15)=19575 \text{ lbs.};$$
$$l=15 \text{ ft.};$$
$$M=29362.5 \text{ ft.-lbs.},$$

or

$$978.75 \text{ ft.-lbs. per inch width.}$$

Reference to Plate VI gives the value of (h) as 12 and steel as 0.82 sq. in. per foot of breadth, or 0.56 per cent. Reference to Plate VIII gives this for a 30-in. width as approximately 2 sq. in. of steel, and from Plate VIII*a* this area may be obtained by using two 1-in. square bars. Allowing $1\frac{1}{2}$ in. of concrete as protection under the steel the total depth of beam is 14 inches.

Reference to Plate IX shows that for a percentage of steel $=0.56$ per cent and (h) $=12$, the distance of the neutral axis below the top surface of the flange is 3.55 inches. Thus the neutral axis falls within the thickness of the slab, and the design will remain as computed.

The stem or leg of the T will be made one-fifth of the assumed flange width, or $\frac{1}{5}\times 30=6$ inches. As the longitudinal reinforcement will be bent up at each end of the beam and anchored to the column, wall, or girder, as the case may be, this width of stem will be ample to provide for horizontal shear between stem and flange.

Girder Design.—The span for the girders is equal to the spacing between columns, or 18 ft. The girders will be built into the bearing wall at each end of the building and continuous through the interior columns, but, as these are principal members of the floor system, the bending moment produced by the concentrated beam load at the center will be computed as though the girder were simply supported at the ends. However, the weight of the girder itself will not be considered, and the span used will be the clear span, or approximately 17 ft.

$$W = 19575 \text{ lbs.};$$
$$l = 17 \text{ ft.};$$
$$M = \frac{Wl}{4} = \frac{19575 \times 17}{4} = 83193.75 \text{ ft.-lbs.}$$

Now the width of the stem of this T girder is limited by the width of the columns. The width or diameter of a square column whose unsupported length is about 11 ft., as in the present case, should be not less than 10 in., to conform with the building code. We will therefore first assume the width of the stem as 10 in., and then consider the flange width as $4 \times 10 = 40$ in. This value is a conservative one, as it constitutes a ratio of between $\frac{1}{5}$ and $\frac{1}{6}$ of the span.

Assuming a flange width of 40 in., we have a bending moment as follows:

$$83193.75 \div 40 = 2079.8 \text{ ft.-lbs. per in. width.}$$

It is desired to maintain a clear head-room of approximately 10 ft. 2 in. from the finished floor to the bottom of the girder. Therefore, allowing for the thickness of sleepers and wood-flooring, the total depth of girder should not exceed 12 ft. 0 in. $-$ (10 ft. 2 in. $+$ 2 in. $+ 1\frac{1}{8}$ in.) $=$ 1 ft. $6\frac{7}{8}$ in. We will therefore assume the value of (h) as 17 inches. Reference to Plate VII gives for $(h) = 17$ in. a reinforcement equivalent to 1.36 sq. in. per foot of breadth, or 0.66 per cent. Reference to Plate VIII gives this for a 40-in. width as 4.5 sq. in. of steel, and from Plate VIII*a* this area may be obtained with sufficient exactness by the use of four $1\frac{1}{8}$-in. square bars. Allowing $1\frac{7}{16}$ inches of concrete as protection under the steel, the total depth of girder is 19 in., or 1 ft. 7 in., which is sufficiently close to the required value of 1 ft. $6\frac{7}{8}$ in. Two of the four bars in the girder should be bent upward at a distance of about 1 ft. 6 in. from the center of the supporting column; one of the remaining two bars should also be bent upward, but at about 3 ft. 6 in. from the center of the column.

Reference to Plate IX shows that for a percentage of steel = 0.66 per cent. and (h) = 17 in., the distance of the neutral axis below the top surface of the flange is 4.85 inches. The neutral axis therefore lies 0.15 in. above the bottom of the flange, and, since it is within the thickness of the slab, the design may remain as computed. However, in the present case a safeguard will be introduced by requiring the placing of 4 stirrups in each girder, consisting of round steel bars bent into a U shape. The bars should be ¼ in. in diameter, the bottom of the U being just below the axis of the tension bars, and the prongs of the U extending to within 1 in. of the top of the flange of the girder, where they should be bent out horizontally to a distance of 3 in. on either side. The distance between either end of the girder and the first stirrup should be about 4 ft. 0 in., and the distance between the first and second stirrups should be about 1 ft. 6 in.

Roof Design.—Since the superimposed load on the roof is 30 + 50 = 80 lbs. per sq. ft., which is the same as in the case of the floor system, the roof design will be precisely the same as that of the floors.

Column Design.—The total load resulting from the weight of a single floor, or from the weight of the roof and bearing on an interior column, is 39150 lbs. Allowing for the weight of the column itself, the load at any floor-level may be assumed at 20 tons. Then the load for the various column lengths will be as follows:

Roof to 4th floor.	20 tons,
4th floor to 3d floor.	40 tons,
3d floor to 2d floor.	60 tons,
2d floor to 1st floor.	80 tons,
1st floor to basement.	100 tons,
Load on footing.	100 tons.

We will assume that for a maximum economy of cost the percentage of steel cross-section of longitudinal reinforcement should not exceed 6 per cent. for ordinary reinforced columns, and that the "excess area" for hooped columns should not exceed 2 per cent. The least dimension of any column should be not less than 10 in., as already stated. The wire used for tying in the longitudinals of ordinary reinforced columns should be not less than No. 8, B. W. G.

Roof to 4th floor.—Reference to Plate XI shows that for a 10 in. × 10 in. column, under a load of 20 tons, 1.2 sq. inches of steel cross-section is required. Reference to Plate XII*a* shows that this may be obtained by the use of four

$\frac{5}{8}$-in. round rods. The rods should be tied together at intervals not exceeding 10 in.

4th floor to 3d floor.—Reference to Plate XI shows that for a load of 40 tons we may use a column 12 in. × 12 in., with 7.6 sq. inches of steel cross-section. Reference to Plate XII shows that this may be obtained by the use of four $1\frac{9}{16}$-in., round rods. The rods should be tied together at intervals not exceeding 12 in.

3d floor to 2d floor.—Reference to Plate XIII shows that for a load of 60 tons and using 2 per cent. "excess area," we may adopt a hooped column whose core diameter is 11 in. For the six longitudinals there will be required rods whose diameter is $\frac{3}{4}$ in. The hooping wire will be No. 6, B. W. G., and the pitch of the spirals will be $\frac{1}{6}$ of 11 in., or $1\frac{5}{6}$ in. Allowing 1 in. of concrete outside of the hooping as a protection, the total outside diameter will be 13 in.

2d floor to 1st floor.—Reference to Plate XIV shows that for a load of 80 tons and using 2 per cent. "excess area" we may adopt a hooped column whose core diameter is 13 in. For the eight longitudinals there will be required rods whose diameter is $\frac{3}{4}$ in. The hooping wire will be No. 6, B. W. G., and the pitch of the spirals will be $\frac{1}{8}$ of 13 in., or $1\frac{5}{8}$ in. Allowing the usual 1-in. thickness of concrete outside the hooping the total outside diameter will be 15 in.

1st floor to Basement.—Reference to Plate XIV shows that for a load of 100 tons we may use a hooped column whose core diameter is 14 in., provided an "excess area" of 3 per cent. is allowed. For this one tier it will be considered justifiable to go beyond the "economic cost percentage," stated as 2 per cent. For the eight longitudinals there will be required rods whose diameter is 1 in. The hooping wire will be No. 5, B. W. G., and the pitch of the spirals will be $\frac{1}{8}$ of 14, or $1\frac{3}{4}$ in. Allowing the usual 1-in. thickness of concrete outside the hooping, the total outside diameter will be 16 in.

It will be noticed in the foregoing design that we have passed from a 12 in. × 12 in. ordinary reinforced column above the 3d floor level to a hooped column of diameter equal to 13 in. below the 3d floor level. This condition need cause no anxiety, if the longitudinals overlap sufficiently and an even and secure bond is obtained between the successive tiers of columns.

Footing Design for Interior Columns.—The bases of the columns of the lowest tier will be made 16 in. square at the point where the column passes below the level of the finished basement floor. Below this point the column base will be formed in pyramidal shape, the sides having a slope of 1 horizontal to 3 vertical. As the total distance from the finished basement floor level to the under side of the cinder concrete on which this floor is laid is 18 in., the dimensions of the column base will be 28 in. × 28 in. at the level of the top of the footing. The direct compression on the surface of the footing-slab will therefore be 255 lbs. per sq. in., which is well within the allowable limit.

By the requirements of the problem we must proportion the area of the footing for a soil pressure of 4 tons per sq. ft. under full live and dead loads. The load being 100 tons, the area of the slab will be approximately 25 sq. ft., and its dimensions 5 ft. × 5 ft. As the slab will probably be in the neighborhood of 2 ft. thick, we may assume its total weight as 25 × 2 × 150 ÷ 2000 = 3.7, or, say 4 tons. We will therefore design for a total load of 104 tons, thus requiring an area of 26 sq. ft. This area may be nearly obtained by using a slab whose dimensions are 5 ft. 1 in. × 5 ft. 1 in. The given load will be divided by 2 for convenience in obtaining the bending moments.

Reference to Plate V shows that for a load of 52 tons and a value of $(l-a)$ equal to $(61-28)$, or 33 inches, the bending moment in each direction is 35750 ft.-lbs. Multiplying by 2 for the actual load of 104 tons, the bending moment is found to be 71500 ft.-lbs.

For a strip in one direction having a width of 28 in. the bending moment is therefore 71500 ÷ 28 = 2554 ft.-lbs. per in. width. Reference to Plate VII shows that for a resisting moment equal to this bending moment the value of (h) required is 19 in., and the steel area should be equivalent to 1.45 sq. in. per foot of breadth. Reference to Plate VIII shows that for a breadth of 28 inches the total area of steel will be 3.4 sq. in., and from Plate VIII*a* this area may be obtained by using six $\frac{3}{4}$-in. sq. bars. The total depth of the footing-slab will be made 2 ft. 0 in., so as to have a sufficient amount of concrete between the steel and the ground below. It should be noted that this was the depth assumed in computing the approximate total load of 104 tons, so that a recomputation will be unnecessary. The six $\frac{3}{4}$-in. sq. bars will then be spaced equally apart under the 28-in. base, and their length should be approximately 5 ft. Although the amount of steel and depth of slab are in excess of the requirements as the edge of the footing is approached, it will be found in actual practice that the saving in cost by the reduction of one or the other or both, on account of the decrease in bending moment, would be more than offset by the additional labor and care involved in making the change. It is therefore customary, though not universally so, to make the depth of slab uniform, and the amount of steel constant, over the entire area of the footing.

We will now consider the reinforcements running perpendicular to the direction of the reinforcements in the narrow strip just designed. The width in this case is 5 ft. 1 in., or 61 ins. The bending moment is now $71500 \div 61 = 1172$ ft.-lbs. per in. width. We will assume that the axis of the steel bars running in this direction is 2 inches above the axis of the bars previously determined. The value of (h) is therefore 17 in., and reference to Plate VII shows that for a resisting moment equal to 1172 ft.-lbs. per in. width, there is required an area of steel equivalent to 0.7 sq. in. per foot of breadth. Reference to Plate VIII shows that for a breadth of 61 inches the total area of steel will be 3.55 sq. in., and from Plate VIII*a* this area may be obtained by using nine $\frac{5}{8}$-in. sq. bars. The nine bars should be spaced equally apart across the entire breadth of the footing, and their length should be approximately 5 ft. If the two tiers or layers of bars which run perpendicular to each other were accurately placed, the minimum distance between the surfaces of an upper and a lower bar at the point of crossing would be $2-(\frac{6}{16}+\frac{3}{8}) = 1\frac{5}{16}$ inches. In actual construction this distance may vary from 1 in. to $1\frac{1}{2}$ in. Each such point should have a wire loop of at least No. 8, B. W. G., passing under and tight against the bottom of the lower bar, and over and tight against the top of the upper bar.

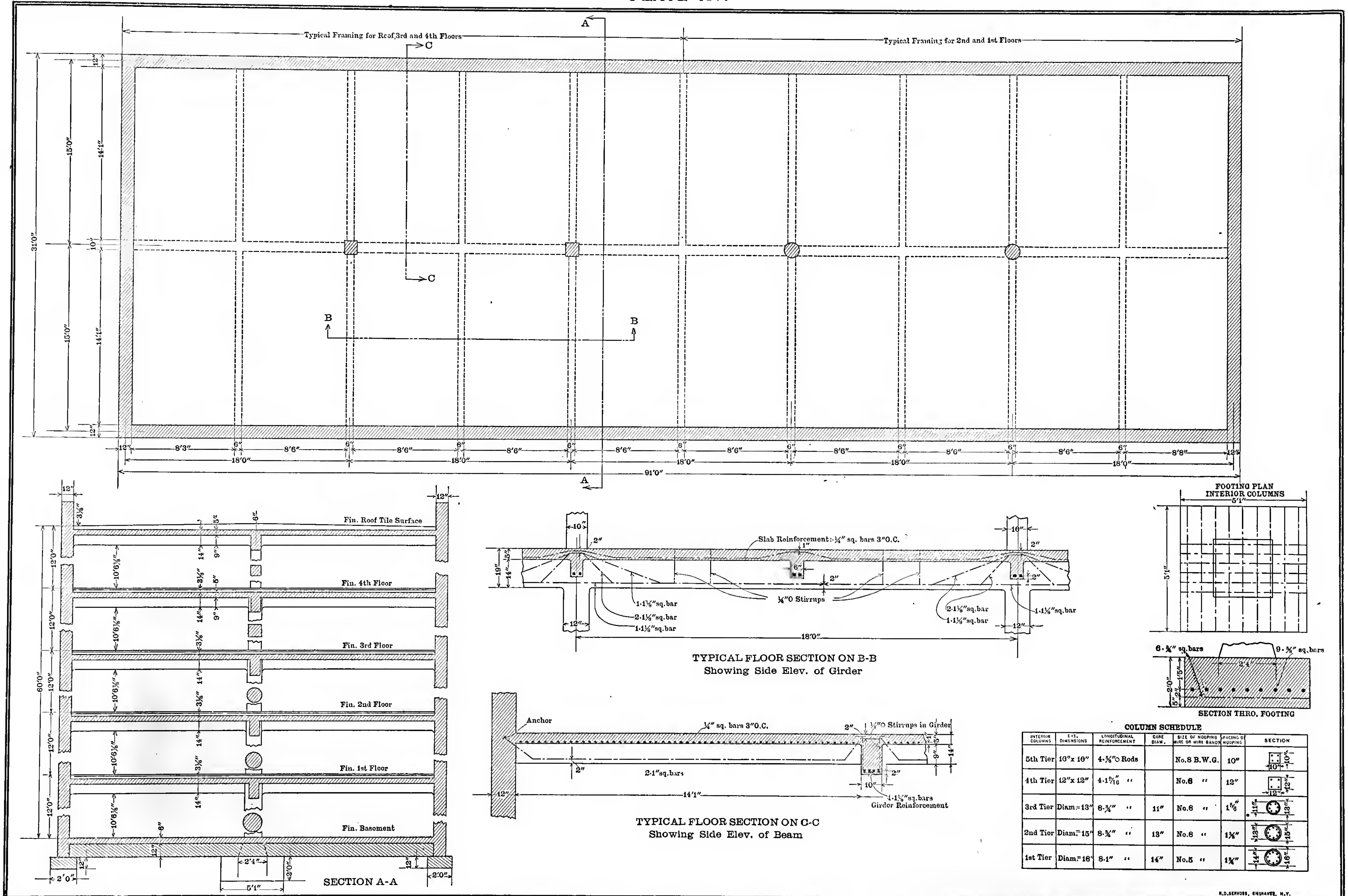
It is advisable now to compute the vertical shear acting on the footing-slab along the edge of the base of the column. The projection of the footing being $\frac{1}{2}(l-a)$, or 16.5 inches, the vertical shear per linear foot along the edge of the column base is $\frac{16.5}{12} \times 4 = 5.5$ tons. On two sides of the base the shearing resistance is supplied by a depth of concrete equal to 24 inches and an area of steel equal to 0.7 sq. in. On the remaining two sides the shearing resistance is supplied by a depth of concrete also equal to 24 inches, and an area of steel equal to 1.45 sq. in. By using these values and referring to Plate X we find the shearing resistance in the first case to be 10.7 tons per linear foot, and in the second case 14.5 tons per linear foot. The design is therefore perfectly safe against shear.

Wall Footings.—The total live and dead load for the wall footings is 7.3 tons per linear foot. The wall being 12 in. thick, we may assume a footing 2 ft. 0 in. wide. The pressure on the soil will then be $7.3 \div 2 = 3.65$ tons per sq. ft. This is somewhat less than was allowed for the interior footings, and is justifiable for the reason that the load on the wall footing contains a larger percentage of dead load than do the loads on the interior column footings. Also if there is any settlement at all, it is desirable to have the settlement of the interior columns slightly greater than that of the walls, as would be the case with the present design.

The projection of the footing on each side of the wall will be 6 in., and the thickness will be made 12 in. Therefore no reinforcement will be required for the wall footings.

Our design is now complete, and a general idea of the relation of the various parts may be obtained by an examination of the drawing shown on Plate XV.

PLATE XV.



COLUMN SCHEDULE

INTERIOR COLUMNS	EXT. DIMENSIONS	LONGITUDINAL REINFORCEMENT	CORE DIAM.	SIZE OF HOOPING WIRE OR WIRE BANDS	SPACING OF HOOPING	SECTION
5th Tier	10"x 10"	4-⅝" O Rods		No.8 B.W.G.	10"	
4th Tier	12"x 12"	4-1 5/16" "		No.6 "	12"	
3rd Tier	Diam.=13"	6-¾" "	11"	No.6 "	1⅝"	
2nd Tier	Diam.=15"	8-¾" "	13"	No.6 "	1⅜"	
1st Tier	Diam.=16"	8-1" "	14"	No.5 "	1⅜"	

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APPENDIX.

As stated in the introduction, it is not intended to present in this book a treatise on the theory of reinforced concrete design. Certain formulas have been used, however, in making up the plates, and for the benefit of those who may care to investigate their accuracy, a brief summary of the derivation of these formulas is given below.

The following hypotheses are first assumed.

1. That the applied forces are perpendicular to the neutral surface in pieces subjected to simple bending.

2. That each fiber acts by itself, not being affected by contiguous fibers.

3. That the reinforcements are disposed with a view of obtaining sufficient homogeneity, or a sufficient mechanical bond is provided, so that we may consider the reinforcements as always being in solid contact with the surrounding concrete, and therefore that both the metal and the surrounding concrete are equally deformed.

4. That the sections remain true planes during bending.

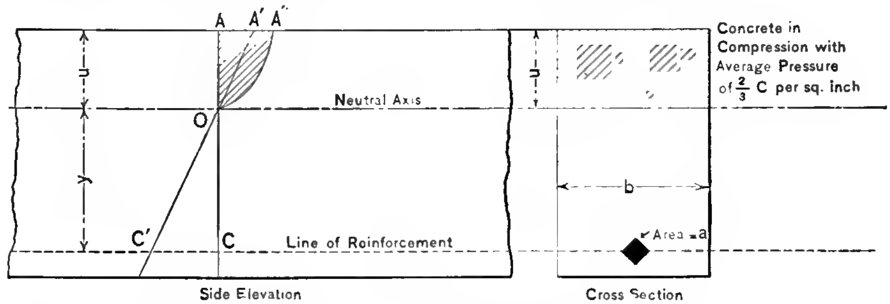
5. That there are no initial stresses, or that with an increasing load commencing from zero the stresses in the different fibers will also commence from zero.

6. That the coefficient of elasticity of the concrete does not remain constant in compression, but that the stress-strain curve is parabolic.

7. That the elongation of the concrete in tension must not exceed .001 of its unit length, and that no tensile resistance is offered by the concrete.

RECTANGULAR MEMBERS WITH A SYSTEM OF TENSION REINFORCEMENT ONLY, AND SUBJECTED TO BENDING.

In the case here shown the compressive resistance is offered by the concrete above the neutral axis, and the tensile resistance is assumed to be supplied only by reinforcement near the bottom of the beam.



Let c = maximum intensity of compressive stress in the concrete under a given load. It is represented by the distance AA'';

f = maximum intensity of tensile stress in the metal under the same load (the area of reinforcement is assumed to be so small with reference to the total area of cross-section of the beam that the stress in the metal is practically uniform);

AA' represent the unit strain, or strain per unit length, in the concrete which is stressed to the amount (c);

$C'C$ represent the unit strain, or strain per unit length, in the metal which is stressed to the amount (f);

E_c represent the coefficient of elasticity of concrete in compression;

E_f represent the coefficient of elasticity of steel in tension;

m = the ratio $\frac{E_f}{E_c}$;

h = distance from compression surface to axis of reinforcement $= u + y$.

Now the total compressive resistance is represented by the area of the parabolic figure AOA'', multiplied by b, the breadth of the beam.

But the area $AOA'' = \frac{2}{3}(AA'')\,u = \frac{2}{3}cu$, and the breadth of beam $= b$.

Therefore the total compressive resistance is equal to $\frac{2}{3}cub$.

The total tensile resistance is evidently the cross-sectional area of the metal multiplied by the uniform intensity of stress thereon. Therefore the total tensile resistance $=af$.

Since the total compressive resistance above the neutral axis must be equal to the total tensile resistance below the same we have

$$\tfrac{2}{3}cub=af. \qquad [1]$$

From the hypothesis of the conservation of plane sections we have

$$\frac{AA'}{OA}=\frac{CC'}{OC} \quad \text{or} \quad \frac{AA'}{u}=\frac{CC'}{y}.$$

But by definition of the values represented by AA' and CC' we have

$$AA'=\frac{c}{E_c} \quad \text{and} \quad CC'=\frac{f}{E_f}.$$

Therefore $$AA':CC'=\frac{c}{E_c}:\frac{f}{E_f}.$$

Combining these equations we have

$$\frac{c}{uE_c}=\frac{f}{yE_f}, \qquad [2]$$

which reduces to

$$f=cm\frac{y}{u}. \qquad [3]$$

The stress-strain curve of the concrete being parabolic, the center of action of the compressive stresses is at a point $\frac{5}{8}$ of the height of OA from O. The lever arm for the resisting moment of the summation of the compressive stresses, with respect to the neutral axis, is therefore represented by $\frac{5}{8}u$.

The center of action of the tensile stresses is at a point distant OC from O The lever arm for the resisting moment of the summation of the tensile stresses, with respect to the neutral axis, is therefore represented by y.

The total resisting moment of the beam is the sum of the moments of the total compressive stresses about the neutral axis, and of the total tensile stresses about the neutral axis. Therefore we have

$$\begin{aligned} M &= \tfrac{5}{8}u\times(\tfrac{2}{3}cub)+y(af) \\ &= \tfrac{5}{12}cu^2b+yaf. \qquad [4] \end{aligned}$$

Introducing the value of f from equation [3] into equation [1] we have

$$\tfrac{2}{3}cub = acm\frac{y}{u},$$

$$\text{or} \quad \tfrac{2}{3}u^2b - may = 0. \qquad [5]$$

Substituting the same value of f in equation [4],

$$M = \frac{c}{u}\{\tfrac{5}{12}u^3b + may^2\}; \qquad [6]$$

also by definition

$$y = h - u. \qquad [7]$$

Substituting this value of y in equation [5],

$$\tfrac{2}{3}u^2b - ma(h - u) = 0,$$

from which

$$u = -\frac{3}{4}m\frac{a}{b} + \sqrt{\frac{9}{16}\frac{m^2a^2}{b^2} + \frac{3}{2}\frac{mah}{b}}. \qquad [8]$$

By substituting equations [5] and [7], in equation [6], we have

$$M = \frac{cub}{12}(8h - 3u). \qquad [9]$$

From equation [1] we have

$$cub = \tfrac{3}{2}af.$$

Substituting this value in equation [9] we have

$$M = \frac{af}{8}(8h - 3u). \qquad [10]$$

It should be noted that the foregoing analysis involves some incongruity. Equation [1] is based on the assumption that the intensity of compressive stress varies according to the parabolic curve, or in other words that the coefficient of elasticity, E_c, is a varying quantity between zero stress and the intensity of stress represented by c. In passing to equation [2], and thence to equation [8], the value of E_c is assumed to be a fixed and constant quantity (as would be the case if the intensity of compressive stress in the concrete varied directly as the distance from the neutral axis), under the assumption that the normal sections remain plane after flexure. The equations are thus analytically faulty, and the discrepancy referred to is mentioned in most of the principal works on the subject. The actual error involved, however, by the use of the equation so derived is so slight that it is regarded by most authorities as negligible, especially when we consider the arbitrary assumptions that have been made, such as the neglect of the tensional resistance of the concrete, and the like.

Therefore it may be stated that equations [8], [9], and [10] are entirely general in their nature, and are applicable to every kind of reinforced concrete member subjected to flexure, provided the assumed width of the member is constant between the compression surface and the neutral axis. Hence the formulas also apply in the case of T beams, or beams supporting slabs, a certain portion of which is supposed to act as part of the beam, *always provided the neutral axis does not lie below the bottom of the slab, or flange, of the* T.

It will be noted that equation [9] gives the resisting moment when the maximum allowable value of c is introduced as the limiting factor, and that equation [10] gives the resisting moment when the maximum allowable value of f is the limiting factor. The lesser of these two resisting moments, when proper working values are assigned to c and f, is the allowable resisting moment of the member in question. On equations [8], [9], and [10] are based Plates I, II, III, VI, VII, and IX.

T-SHAPED BEAMS IN WHICH THE NEUTRAL AXIS LIES BELOW THE UNDER SIDE OF THE SLAB.

BEFORE considering the formulas required for the design of any T beam, irrespective of the position of the neutral axis, the first point to be decided is the width of overlying slab that is to be considered as the flange width of the beam. As the real distribution of the stresses in a monolithic construction is indeterminate, some arbitrary rule is usually adopted in the assumption of this flange width. The rules for making this assumption are divided into two great classes. The first class makes the flange width a factor of the width of the stem, or else of the distance between adjacent beams. The second class makes the flange width a factor of the length of span of the beam itself. As showing the great lack of disagreement between various authorities the following examples are cited:

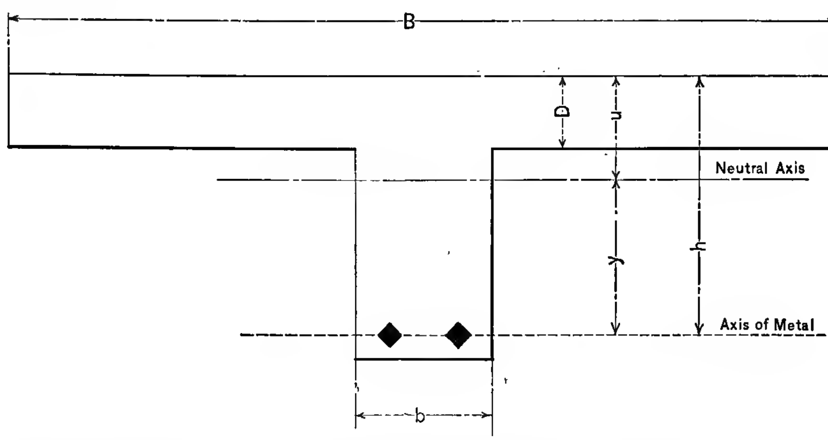
First class.—In an article previously referred to, by Capt. John S. Sewell, published in the Proceedings of the American Society of Civil Engineers for Dec. 1905, the author states that conservative practice would confine the total allowable width of flange to three times the width of the stem or leg of the T.

The building laws of some cities, as Cleveland and Buffalo, allow a width of flange ten times the width of the stem.

In Mr. C. F. Marsh's book on reinforced concrete, he states the allowable width of flange as one-half the distance center to center from the beam in question to the parallel beam nearer to it on either side, the width of the stem being one-sixth of the flange width.

Second class.—In a discussion on the above-mentioned paper by Capt. Sewell, Mr. J. Kreuger, Assoc. M. Am. Soc. C. E., gives as an allowable flange width one-sixth of the length of the beam itself.

These examples might be continued indefinitely, and it is necessary that the designer should select the rule that appears to have either the most logical basis or the most authoritative backing. Whatever may be the actual width of flange assumed, it may be represented by B, and the required formulas deduced.



The quantities a, c, f, E_c, E_f, and m remain as previously defined.

The significance of the values u, y, h, and D will be apparent from the sketch.

If the breadth B were constant between the compressive surface and the neutral axis, we should have from equation [6]

$$M_1 = \frac{c}{u}\{\tfrac{5}{12}u^3B + may^2\}.$$

As the breadth B is not constant, the area subjected to compression is reduced by an amount $(B-b)(u-D)$.

Now let c_1 = intensity of compressive stress at a distance $(u-D)$ from the neutral axis.

At the distance $(u-D)$, the intensity of compressive stress may be considered as very nearly proportional to the distance from the neutral axis. Therefore we have

$$\frac{c_1}{c} = \frac{u-D}{u} \quad \text{or} \quad c_1 = \left(\frac{u-D}{u}\right)c.$$

Let M_2 = the resisting moment which would be produced by the area

$$(B-b)(u-D).$$

Then we have

$$\begin{aligned} M_2 &= \tfrac{2}{3}(u-D)\{\tfrac{1}{2}c_1(u-D)(B-b)\} \\ &= \tfrac{1}{3}(u-D)^2(B-b)\left[\left(\frac{u-D}{u}\right)c\right] \\ &= \tfrac{1}{3}\frac{c}{u}(u-D)^3(B-b). \end{aligned}$$

The actual resisting moment of the concrete in compression equals $M = M_1 - M_2$.

Therefore we have

$$M = M_1 - M_2 = \frac{c}{u}[\tfrac{5}{12}(u^3B) - \tfrac{1}{3}(u - D)^3(B - b) + may^2], \quad . \quad . \quad [11]$$

or

$$M = \frac{c}{u}[\tfrac{5}{12}u(u^2B) - \tfrac{1}{3}(u - D)(u - D)^2(B - b) + may(y)]. \quad . \quad . \quad [12]$$

We also have the relation, which is similar to equation [5],

$$may = \tfrac{2}{3}u^2B - \tfrac{1}{2}(u - D)^2(B - b), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [13]$$

and by definition $y = h - u.$

Substituting in equation [12] we have

$$M = \frac{c}{u}\left[u^2B\left\{\frac{5u}{12} + \frac{2h - 2u}{3}\right\} - \{(u - D)^2(B - b)\}\left\{\frac{u - D}{3} + \frac{h - u}{2}\right\}\right],$$

or in simplified form,

$$M = \frac{c}{6u}[\tfrac{1}{2}u^2B(8h - 3u) - (u - D)^2(B - b)(3h - u - 2D)]. \quad . \quad . \quad . \quad [14]$$

Equation [14] gives an expression for the resisting moment when the maximum allowable value of c is introduced as the limiting factor. The expression contains the known quantities c, B, b, D, and h, and the ascertainable quantity u. Equation [14] therefore corresponds to equation [9], and may be used in the same manner.

The condition of equality between the total compressive stress and the total tensile stress still holds.

The total compressive stress has been found to be

$$\tfrac{2}{3}cuB - \tfrac{1}{2}c_1(u - D)(B - b)$$

or

$$\tfrac{2}{3}cuB - \tfrac{1}{2}c\left(\frac{u - D}{u}\right)(u - D)(B - b).$$

Equating this expression to the tensile resistance, we have

$$\frac{c}{6u}[4u^2B - 3(u - D)^2(B - b)] = af.$$

Whence we have

$$\frac{c}{6u}=\frac{af}{4u^2B-3(u-D)^2(B-b)}. \quad \ldots\ldots \quad [15]$$

Substituting this value in equation [14] we have

$$M=\frac{af}{4u^2B-3(u-D)^2(B-b)}[\tfrac{1}{2}u^2B(8h-3u)-(u-D)^2(B-b)(3h-u-2D)]. \quad [16]$$

Equation [16] gives an expression for the resisting moment when the maximum allowable value of f is introduced as the limiting factor. The expression contains the known quantities a, f, B, b, D, and h, and the ascertainable quantity u. Equation [16] therefore corresponds to equation [10], and may be used in the same manner.

In order to make use of equations [14] and [16], to determine the limiting resisting moment, the value of u must be first ascertained. Equation [13] gives

$$may=\tfrac{2}{3}u^2B-\tfrac{1}{2}(u-D)^2(B-b).$$

Also we have $$y=h-u,$$

whence

$$(\tfrac{2}{3}B-\tfrac{1}{2}B+\tfrac{1}{2}b)u^2+[D(B-b)+ma]u=\tfrac{1}{2}D^2(B-b)+mah$$

or

$$u^2+\frac{D(B-b)+ma}{\tfrac{1}{6}B+\tfrac{1}{2}b}(u)=\frac{\tfrac{1}{2}D^2(B-b)+mah}{\tfrac{1}{6}B+\tfrac{1}{2}b}.$$

Solving for u we obtain

$$u=-\frac{3[D(B-b)+ma]}{B+3b}$$
$$+\frac{3}{B+3b}\sqrt{2\{D(B-b)[\tfrac{2}{3}BD+ma]+ma[\tfrac{1}{3}Bh+bh+\tfrac{1}{2}ma]\}}. \quad [17]$$

Equation [17] corresponds to, and may be used in the same manner as, equation [8]. The same inconsistency will be noted in the deduction of equations [14], [16], and [17] that was observed in the deduction of equations [8], [9], and [10], and the same remarks are applicable. It will be further noted, however, that in the case of these T beams the intensity of compressive stress below the bottom of the flange of the T is assumed to vary, not by the parabolic theory, but directly as the distance from the neutral axis, on account of the low intensity of such stress. This assumption, for the part of the beam in question, has been consistently adhered to in the determination of the subtractive moment, and thus constitutes the only essential difference between these formulas and those deduced by other writers.

MEMBERS WITH LONGITUDINAL REINFORCEMENTS, AND SUBJECTED TO DIRECT COMPRESSION.

A TRANSVERSE section of the member is considered. This is displaced in a direction parallel to itself by a load P.

Let A = total area of transverse section;

a = cross-sectional area of reinforcement;

c = maximum allowable intensity of stress on the concrete;

f = maximum allowable intensity of stress on the metal;

E_c = coefficient of elasticity of concrete under a compression equal to the maximum allowable intensity of compressive stress;

E_f = coefficient of elasticity of metal in compression.

The area of the concrete portion of the transverse section will be $A-a$. We have, therefore,

$$P = c(A-a) + fa. \quad \ldots \ldots \quad [1]$$

As the displacement of the concrete and the reinforcement is identical, we have the relation

$$\frac{c}{E_c} = \frac{f}{E_f} \quad \text{or} \quad f = c\frac{E_f}{E_c}.$$

Let m represent the ratio $\frac{E_f}{E_c}$.

Whence $f = mc$.

Therefore we obtain

$$P = c(A-a) + mca$$

or

$$P = c\{A + (m-1)a\}. \quad \ldots \ldots \quad [2]$$

Let p represent the equivalent intensity of compressive stress over the entire transverse section whose area is A.

Then $$p = \frac{P}{A}.$$

Substituting in equation [2], we obtain

$$p = \frac{c}{A}\{A + (m-1)a\}$$

or

$$p = c\left\{1 + (m-1)\frac{a}{A}\right\}. \quad . \quad . \quad . \quad . \quad . \quad [3]$$

On equations [2] and [3] are based Plates XI and XII.

MEMBERS, WITH LONGITUDINAL REINFORCEMENTS, SUBJECTED TO DIRECT COMPRESSION AND HOOPED TO RESIST INTERNAL PRESSURE.

LET ϕ = the angle of stability of concrete under compression. From the experiments that have been made, this angle may be assumed as 60°;

q = the internal pressure per square inch that is exerted in a radial direction;

P = the intensity of direct compressive stress on the hooped core of concrete;

n = the ratio $\frac{P}{q}$.

Since P is the direct downward pressure on the column, and q is the pressure exerted normally to the surface of the column, we have from the general formula for stability under two sets of forces acting on planes at right angles to each other

$$n=\frac{P}{q}=\frac{1+\sin\phi}{1-\sin\phi}=\frac{1+0.866}{1-0.866}=13.92. \quad . \quad . \quad . \quad . \quad . \quad [1]$$

If we assume P = 1000 lbs. per sq. in., we obtain

$$q=\frac{P}{n}=\frac{1000}{13.92}=71.8 \text{ lbs. per sq. in.}$$

Now, this value of q represents the numerical value of the intensity of radial stress exerted by the concrete, and which, if unrestrained, would cause bulging or swelling.

Let d = diameter of the concrete core in inches;

T = the hoop tension in pounds on a strip 1 inch wide extending around the circumference of the core.

Then from the usual formula for hoop tension we have

$$T=\frac{qd}{2}=\frac{71.8d}{2}=35.9d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [2]$$

Now let the pitch of the spiral hooping, or the distance between successive spirals, equal one-sixth of the diameter of the hooped core. Also let there be six longitudinals, tied in by, and just inside of, the hooping.

Let δ = diameter of the hooping wire in inches;

t = tension in the hooping wire in pounds.

Now from equation [2] we have

$$t=35.9d\times\frac{d}{6}=6d^2 \text{ (approximately).}$$

Allowing a stress of 25,000 lbs. per sq. inch in the hooping, since it will be drawn wire, we have

$$\text{Cross-sectional area of wire}=\frac{6d^2}{25000}.$$

From which we obtain the diameter of wire:

$$\delta=\sqrt{\frac{6d^2}{25000}\times\frac{4}{\pi}}=0.0175d. \quad . \quad . \quad . \quad . \quad . \quad . \quad [3]$$

Let δ_1 = diameter of the round rods used as longitudinals.

The longitudinals are subjected to bending due to the radial stress q. They may be considered as beams having fixed ends, the span being the spacing between consecutive spirals, or $\frac{d}{6}$ inches. The distance on the circumference of the column between the rods is $\frac{\pi d}{6}$. The load in pounds on one of these spans is $\frac{\pi d}{6}\times\frac{d}{6}\times q=\frac{\pi d^2\times 71.8}{36}$. The bending moment is a maximum at the point where the rod passes over the hooping, and is computed from the usual formula $M=\frac{Wl}{12}$.

Therefore we have:

$$W=\frac{\pi d^2\times 71.8}{36} \quad \text{and} \quad l=\frac{d}{6},$$

$$M=\left(\frac{\pi d^2\times 71.8}{36}\right)\times\frac{d}{6}\times\frac{1}{12}=0.80702d^3.$$

The resisting moment is obtained from the usual formula $M=\frac{KI}{y}$. Since the rods are circular in cross-section we have

$$I=0.0491\,\delta_1^4 \text{ and } y=\tfrac{1}{2}\delta_1.$$

K is assumed as 16000 lbs. per sq. inch.

Therefore we have

$$M=16000\times 0.0491\,\delta_1^4\times\frac{2}{\delta_1}=1571.2\,\delta_1^3.$$

Equating the resisting moment and the bending moment we have

$$1571.2\,\delta_1^3=0.08702d^3$$

or

$$\delta_1=0.0381d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [4]$$

In designing a hooped column, the total load must first be divided by the assumed value of P, in this case 1000 lbs. per sq. inch, to obtain the cross-sectional area of the cylindrical concrete core. From this the diameter of the core, d, may be readily determined, and by the use of equations [3] and [4], the diameters of hooped wire and longitudinals respectively may be ascertained for the conditions stated.

In case it is desired to add an excess area to the longitudinals, such additional area may be assumed to take direct longitudinal load, with an intensity of stress equal to mP;

where $m=\text{ratio }\frac{E_f}{E_c}$;

E_f = coefficient of elasticity of steel under compression;

E_c = coefficient of elasticity of concrete under compression at an intensity of stress $=P$.

If the excess area of longitudinals constitutes a known percentage of the area of the concrete core, and is obtained by adding a shell of uniform thickness around the longitudinals which were required simply to resist bending, the increased diameter of rods may be computed as follows:

For 1 per cent. excess area let δ_α represent the new diameter of the longitudinals.

Then the area of a single longitudinal required to resist bending is $\frac{\pi}{4}(0.0381d)^2$.

The excess area to be added is one-sixth of 1 per cent of the core area, or $0.0016667\frac{\pi}{4}d^2$.

The total new area is the sum of these two, or $\frac{\pi}{4}d^2(0.00311828)$, and since the new diameter $=\delta_\alpha$, we have

$$\frac{\pi}{4}\delta_\alpha^2=\frac{\pi}{4}d^2(0.00311828),$$

whence we obtain

$$\delta_\alpha=0.0558d. \qquad [5]$$

Similarly for 2 per cent. excess area, if $\delta_\beta=$the new diameter of longitudinals, we obtain

$$\delta_\beta=0.0691d. \qquad [6]$$

And for 3 per cent. excess area, if $\delta_\Delta=$the new diameter of longitudinals, we obtain

$$\delta_\Delta=0.0803d. \qquad [7]$$

Plate XIII is based on equations [3], [4], [5], [6], and [7].

In certain cases, particularly for heavily loaded columns, it may be desired to use eight longitudinals instead of six, and decrease the pitch of the spiral hooping to one-eighth of the diameter of the hooped core.

Under these conditions, the tension in the hooping wire becomes

$$t=35.9d\times\frac{d}{8}=4.49d^2$$

and equation [3] becomes

$$\delta=\sqrt{\frac{4.49d^2}{25000}\times\frac{4}{\pi}}=0.0151d. \qquad [8]$$

The span of the longitudinals is now $\frac{d}{8}$.

The distance on the circumference of the column between rods is $\frac{\pi d}{8}$.

The load on this span is $\frac{\pi d^2\times71.8}{64}$.

The bending moment is obtained as before from the equation $M=\frac{Wl}{12}$, and we have

$$M=\left(\frac{\pi d^2\times71.8}{64}\right)\times\frac{d}{8}\times\frac{1}{12}=0.03671d^3.$$

The resisting moment is the same as previously obtained. Equating the bending and resisting moments, we have in place of equation [4]:

$$1571.2\delta_1^3 = 0.03671d^3$$

or

$$\delta_1 = 0.02858d. \quad . \quad . \quad . \quad . \quad . \quad . \quad [9]$$

The method of obtaining the diameter of longitudinals with excess area of cross-section is the same as before. Adding one-eighth of 1 per cent. of the core area, the total area of cross-section for one longitudinal is found as follows:

$$\frac{\pi}{4}(0.02858d)^2 + 0.00125\frac{\pi}{4}d^2 = \frac{\pi}{4}d^2(0.00207).$$

If δ_α represents the new diameter of longitudinal we obtain

$$\frac{\pi}{4}\delta_\alpha^2 = \frac{\pi}{4}d^2(0.00207).$$

From which

$$\delta_\alpha = 0.0454d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [10]$$

For 2 per cent. excess area, let δ_β represent the new diameter of longitudinals. Then we have

$$\delta_\beta = 0.0576d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [11]$$

For 3 per cent. excess area:

$$\delta_J = 0.0676d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [12]$$

For 4 per cent. excess area:

$$\delta_\iota = 0.0763d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [13]$$

Plate XIV is based on equations [8], [9], [10], [11], [12], and [13].

It may be remarked that the diameter of the longitudinal reinforcing rods should be computed on the theory of combined bending and direct stress when an excess area of cross-section is added, and assumed to carry direct stress. An inspection of the working stresses used in plotting the plates will, however, probably demonstrate that such a refinement of computation is unnecessary, and on account of the addition of the excess steel on the outside of the amount required for bending resistance the error involved is altogether on the side of safety.

Since the concrete core is regarded simply as a medium for the conversion of vertical stress into radial stress, and the transmission of the same to the hooping, it might be surmised that the value of P could be increased to

1500 lbs. per sq. in., or even 2000 lbs. per sq. in. It is true that the direct stress P on the concrete is immaterial (up to a point at which the substance itself would disintegrate under the load), provided the system of hooping and longitudinals is made sufficiently strong to withstand the radial pressure or thrust. However, on account of the low value of the coefficient of elasticity of concrete under high intensities of stress the deformation might become excessive. In a column extending through several stories, the deformation, or shortening, even under the compressive stress of 1000 lbs. per sq. in. here assumed, would be considerable in the entire height of the building. The concrete core will always tend to bulge out, or swell, between the spirals of the hooping. Probably in the great majority of cases the deformation under a stress of 1000 lbs. per sq. in. represents the limit beyond which it is unadvisable to venture. For very high buildings, the total diminution of length should be computed and the allowable value of P reduced if necessary.

REQUIREMENTS OF THE BUILDING CODE OF NEW YORK CITY IN REGARD TO REINFORCED CONCRETE.

For convenience of reference the following requirements in regard to reinforced concrete construction are quoted from an amendment to the Building Code of New York City, adopted Sept. 9, 1903.

REGULATIONS OF THE BUREAU OF BUILDINGS OF THE BOROUGH OF MANHATTAN, IN REGARD TO THE USE OF CONCRETE-STEEL CONSTRUCTION.

1. The term "concrete-steel" in these Regulations shall be understood to mean an approved concrete mixture reinforced by steel of any shape, so combined that the steel will take up the tensional stresses and assist in the resistance to shear.

2. Concrete-steel construction will be approved only for buildings which are not required to be fireproof by the Building Code, unless satisfactory fire and water tests shall have been made under the supervision of this Bureau. Such tests shall be made in accordance with the Regulations fixed by this Bureau and conducted as nearly as practicable in the same manner as prescribed for fireproof floor-fillings in Section 106 of the Building Code. Each company offering a system of concrete-steel construction for fireproof buildings must submit such construction to a fire and water test.

3. Before permission to erect any concrete-steel structure is issued, complete drawings and specifications must be filed with the Superintendent of Buildings, showing all details of the construction, the size and position of all reinforcing rods, stirrups, etc., and giving the composition of the concrete.

4. The execution of work shall be confided to workmen who shall be under the control of a competent foreman or superintendent.

5. The concrete must be mixed in the proportions of one of cement, two of sand, and four of stone or gravel; or the proportions may be such that the resistance of the concrete to crushing shall not be less than 2000 lbs. per square inch after hardening for twenty-eight days. The tests to deter-

mine this value must be made under the direction of the Superintendent of Buildings. The concrete used in concrete-steel construction must be what is usually known as a "wet" mixture.

6. Only high-grade Portland cements shall be permitted in concrete-steel construction. Such cements, when tested neat, shall, after one day in air, develop a tensile strength of at least 300 lbs. per square inch; and after one day in air and six days in water shall develop a tensile strength of at least 500 lbs. per square inch; and after one day in air and twenty-seven days in water shall develop a tensile strength of at least 600 lbs. per square inch. Other tests, as to fineness, constancy of volume, etc., made in accordance with the standard method prescribed by the American Society of Civil Engineers' Committee may, from time to time, be prescribed by the Superintendent of Buildings.

7. The sand to be used must be clean, sharp grit sand, free from loam or dirt, and shall not be finer than the standard sample of the Bureau of Buildings.

8. The stone used in the concrete shall be a clean, broken trap-rock or gravel, of a size that will pass through a three-quarter-inch ring. In case it is desired to use any other material or other kind of stone than that specified, samples of same must first be submitted to and approved by the Superintendent of Buildings.

9. The steel shall meet the requirements of Section 21 of the Buidling Code.

10. Concrete-steel shall be so designed that the stresses in the concrete and the steel shall not exceed the following limits:

Extreme fiber stress on concrete in compression...	500	lbs. per sq. in.
Shearing stress in concrete..........	50	" " " "
Concrete in direct compression......	350	" " " "
Tensile stress in steel......	16000	" " " "
Shearing stress in steel.....................	10000	" " " "

11. The adhesion of concrete to steel shall be assumed to be not greater than the shearing strength of the concrete.

12. The ratio of the moduli of elasticity of concrete and steel shall be taken as 1 to 12.

13. The following assumption shall guide in the determination of the bending moments due to the external forces. Beams and girders shall be considered as simply supported at the ends, no allowance being made for continuous construction over supports. Floor-plates, when constructed con-

tinuous and when provided with reinforcement at top of plate over the supports, may be treated as continuous beams, the bending moment for uniformly distributed loads being taken at not less than $\frac{WL}{10}$; the bending moment may be taken at $\frac{WL}{20}$ in the case of square floor-plates which are reinforced in both directions and supported on all sides. The floor-plate to the extent of not more than ten times the width of any beam or girder may be taken as part of that beam or girder in computing its moment of resistance.

14. The moment of resistance of any concrete-steel construction under transverse loads shall be determined by formulæ based on the following assumptions:

(*a*) The bond between the concrete and steel is sufficient to make the two materials act together as a homogeneous solid.

(*b*) The strain in any fiber is directly proportionate to the distance of that fiber from the neutral axis.

(*c*) The modulus of elasticity of the concrete remains constant within the limits of the working stresses fixed in these Regulations.

From these assumptions it follows that the stress in any fiber is directly proportionate to the distance of that fiber from the neutral axis.

The tensile strength of the concrete shall not be considered.

15. When the shearing stresses developed in any part of a concrete-steel construction exceed the safe working strength of concrete, as fixed in these Regulations, a sufficient amount of steel shall be introduced in such a position that the deficiency in the resistance to shear is overcome.

16. When the safe limit of adhesion between the concrete and steel is exceeded, some provision must be made for transmitting the strength of the steel to the concrete.

17. Concrete-steel may be used for columns in which the ratio of length to least side or diameter does not exceed twelve. The reinforcing rods must be tied together at intervals of not more than the least side or diameter of the column.

18. The contractor must be prepared to make load tests on any portion of a concrete-steel construction, within a reasonable time after erection, as often as may be required by the Superintendent of Buildings. The tests must show that the construction will sustain a load of three times that for which it is designed without any sign of failure.

Approved September 9th, 1903.